Seventh Edition

CHAPTER 11

VECTOR MECHANICS FOR ENGINEERS:
DYNAMICS

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Kinematics of Particles

Vector Mechanics for Engineers: Dynamics

Introduction

- Components of Dynamics:
  - *Kinematics*: study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time without reference to the cause of motion.
  - *Kinetics*: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

Vector Mechanics for Engineers: Dynamics

Introduction

In this chapter, we'll study:

- *Rectilinear* motion: position, velocity, and acceleration of a particle as it moves along a straight line.

- *Curvilinear* motion: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

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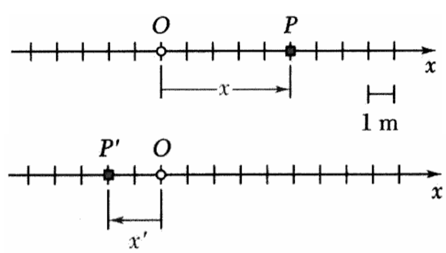
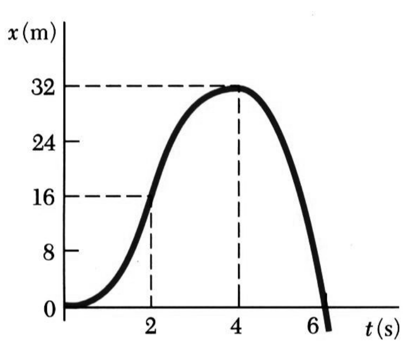
Rectilinear Motion: Position, Velocity & Acceleration





- Particle moving along a straight line is said to be in *rectilinear motion*.

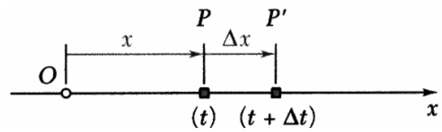
- *Position coordinate* of a particle is defined by positive or negative distance of particle from a fixed origin on the line.

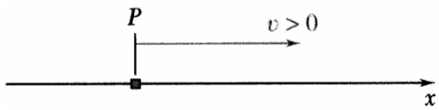
- The *motion* of a particle is known if the position coordinate for particle is known for every value of time t. Motion of the particle may be expressed in the form of a function, e.g.,

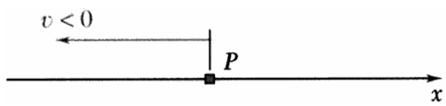
$$x = 6t^2 - t^3$$

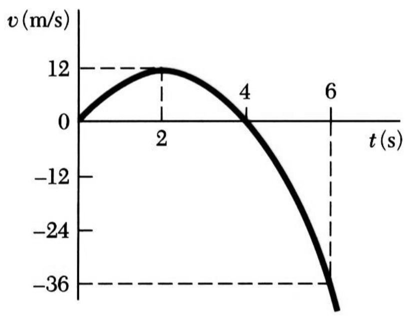
or in the form of a graph x vs. t.

Rectilinear Motion: Position, Velocity & Acceleration









- Consider particle which occupies position *P* at time *t* and *P’* at *t*+*Δt*,

  Average velocity =

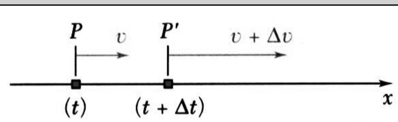
  Instantaneous velocity = *v* =

- Magnitude of velocity is referred to as *particle speed.*

e.g., $x = 6t^2 - t^3$

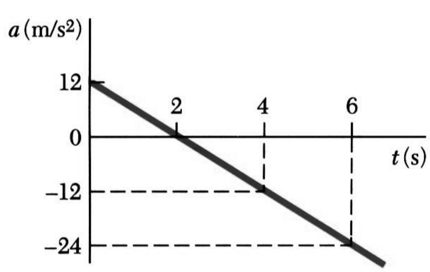
$v =$

Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle with velocity *v* at time *t* and *v’* at *t*+*Δt*,

  Instantaneous acceleration =*a*=



- Instantaneous acceleration may be:
  - positive: increasing positive velocity or decreasing negative velocity
  - negative: decreasing positive velocity or increasing negative velocity.

e.g. $v = 12t - 3t^2$

$a =$

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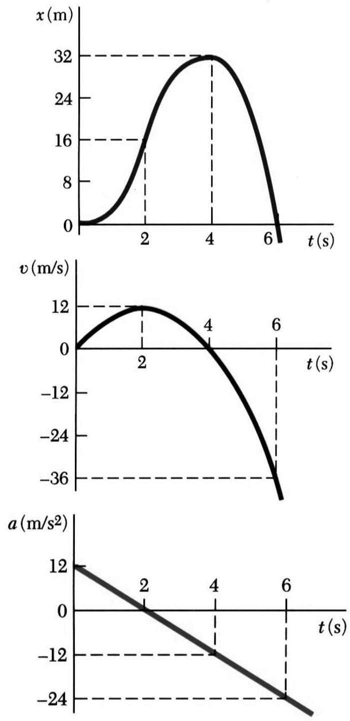
Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle with motion given by

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$$

- at $t = 0$, $x = 0$, $v = 0$, $a = 12$ m/s^2
- at $t = 2$ s, $x = 16$ m, $v = v_{max} = 12$ m/s, $a = 0$
- at $t = 4$ s, $x = x_{max} = 32$ m, $v = 0$, $a = -12$ m/s^2
- at $t = 6$ s, $x = 0$, $v = -36$ m/s, $a = 24$ m/s^2

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Determination of the Motion of a Particle

- Recall, *motion* of a particle is known if position is known for all time t.
- Typically, conditions of motion are specified by the type of acceleration experienced by the particle. Determination of velocity and position requires two successive integrations.
- Three classes of motion may be defined for:
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = f(x)$
  - acceleration given as a function of *velocity*, $a = f(v)$

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Determination of the Motion of a Particle

- Acceleration given as a function of *time*, $a = f(t)$:

$$\frac{dv}{dt} = a = f(t) \qquad dv = f(t)\,dt \qquad \int_{v_0}^{v(t)} dv = \int_0^t f(t)\,dt \qquad v(t) - v_0 = \int_0^t f(t)\,dt$$

$$\frac{dx}{dt} = v(t) \qquad dx = v(t)\,dt \qquad \int_{x_0}^{x(t)} dx = \int_0^t v(t)\,dt \qquad x(t) - x_0 = \int_0^t v(t)\,dt$$

- Acceleration given as a function of *position*, $a = f(x)$:

$$a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} \quad \text{or} \quad a = v\frac{dv}{dx} = f(x)$$

$$v\,dv = f(x)dx \qquad \int_{v_0}^{v(x)} v\,dv = \int_{x_0}^{x} f(x)dx \qquad \tfrac{1}{2}v(x)^2 - \tfrac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)dx$$

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Determination of the Motion of a Particle

- Acceleration given as a function of velocity, $a = f(v)$:

$$\frac{dv}{dt} = a = f(v) \qquad \frac{dv}{f(v)} = dt \qquad \int_{v_0}^{v(t)} \frac{dv}{f(v)} = \int_0^t dt$$

$$\int_{v_0}^{v(t)} \frac{dv}{f(v)} = t$$

$$v\frac{dv}{dx} = a = f(v) \qquad dx = \frac{v\,dv}{f(v)} \qquad \int_{x_0}^{x(t)} dx = \int_{v_0}^{v(t)} \frac{v\,dv}{f(v)}$$

$$x(t) - x_0 = \int_{v_0}^{v(t)} \frac{v\,dv}{f(v)}$$

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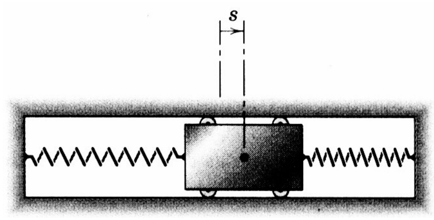
Example: Kinematics of Rectilinear Motion



The spring-mounted slider moves horizontally and has a velocity v_0 in the *s*-direction as it crosses the mid-position where $s=0$ and $t=0$. The acceleration of the slider is given as $a=-k^2s$, where *k* is constant. Determine the expressions for the displacement *s* and velocity *v* as a function of time *t*

- Rectilinear motion
- Given $a(s) = -k^2 s$ (case 2)
- Want $s(t)$ and $v(t)$
- Need to apply chain rule

$$a = \frac{dv}{dt}$$

Integrate both sides

Apply initial conditions $s(0)=0$, $v(0)=v_0$

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Example: Kinematics of Rectilinear Motion

Solve for *v*

- That's $v(s)$, but we want $v(t)$
- Have to find $s(t)$ first then differentiate or solve by substitution

$$v(s) = \frac{ds}{dt}$$

Integrate both sides

Apply initial conditions $s(0)=0$

Solve for *s*

Note: *s* can also be solved by solving the ODE $\ddot{s} + k^2 s = 0$

with the initial conditions. The answers should be the same as above.

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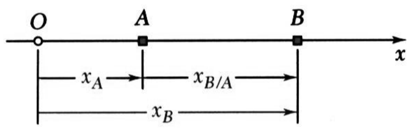
Motion of Several Particles: Relative Motion



- For particles moving along the same line, time should be recorded from the same starting instant and displacements should be measured from the same origin in the same direction.

$x_{B/A} = x_B - x_A =$ relative position of *B* with respect to *A*

$v_{B/A} = v_B - v_A =$ relative velocity of *B* with respect to *A*

$a_{B/A} = a_B - a_A =$ acceleration of *B* relative to *A*

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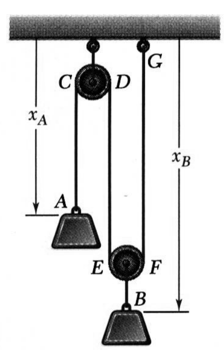
Motion of Several Particles: Dependent Motion



- Position of a particle may *depend* on position of one or more other particles.
- Position of block *B* depends on position of block *A*. Since rope is of constant length, it follows that sum of lengths of segments must be constant.

$x_A + 2x_B =$ constant (one degree of freedom)

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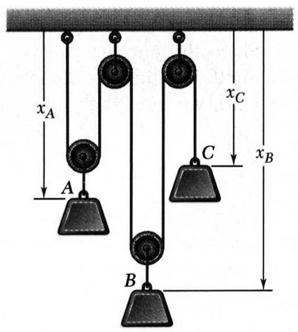
Motion of Several Particles: Dependent Motion



- Positions of three blocks are dependent.

  $2x_A + 2x_B + x_C =$ constant (two degrees of freedom)

- For linearly related positions, similar relations hold between velocities and accelerations.

$$2\dot{x}_A + 2\dot{x}_B + \dot{x}_C = 0 \quad \text{or} \quad 2v_A + 2v_B + v_C = 0$$
$$2\dot{v}_A + 2\dot{v}_B + \dot{v}_C = 0 \quad \text{or} \quad 2a_A + 2a_B + a_C = 0$$

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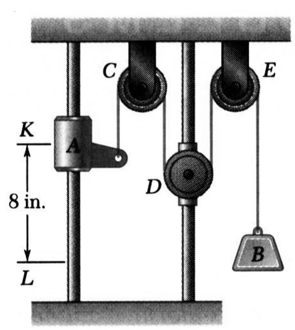
Sample Problem 11.5



Pulley D is attached to a collar which is pulled down at 3 in./s. At $t = 0$, collar A starts moving down from K with constant acceleration and zero initial velocity. Knowing that velocity of collar A is 12 in./s as it passes L, determine the change in elevation, velocity, and acceleration of block B when block A is at L.

SOLUTION:

- Define origin at upper horizontal surface with positive displacement downward.
- Collar A has uniformly accelerated rectilinear motion. Solve for acceleration and time t to reach L.
- Pulley D has uniform rectilinear motion. Calculate change of position at time t.
- Block B motion is dependent on motions of collar A and pulley D. Write motion relationship and solve for change of block B position at time t.
- Differentiate motion relation twice to develop equations for velocity and acceleration of block B.

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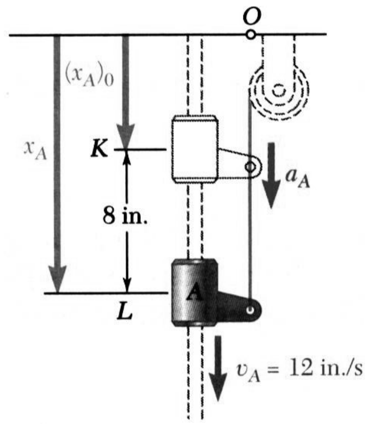
Sample Problem 11.5



SOLUTION:

- Define origin at upper horizontal surface with positive displacement downward.
- Collar *A* has uniformly accelerated rectilinear motion. Solve for acceleration and time *t* to reach *L*.

$$v_A^2 = (v_A)_0^2 + 2a_A\left[x_A - (x_A)_0\right]$$

$$v_A = (v_A)_0 + a_A t$$

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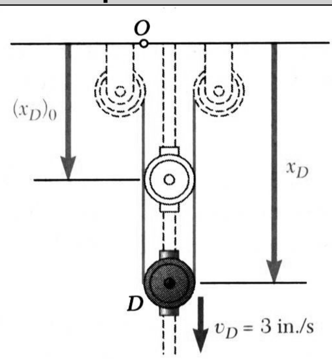
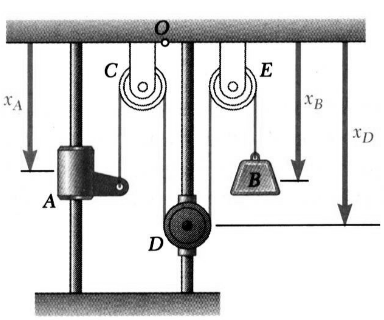
Sample Problem 11.5



- Pulley *D* has uniform rectilinear motion. Calculate change of position at time *t*.

$$x_D = (x_D)_0 + v_D t$$



- Block *B* motion is dependent on motions of collar *A* and pulley *D*. Write motion relationship and solve for change of block *B* position at time *t*.

  Total length of cable remains constant,

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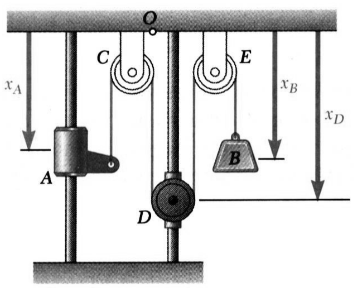
Sample Problem 11.5



- Differentiate motion relation twice to develop equations for velocity and acceleration of block *B*.

$$x_A + 2x_D + x_B = \text{constant}$$
$$v_A + 2v_D + v_B = 0$$

$$a_A + 2a_D + a_B = 0$$

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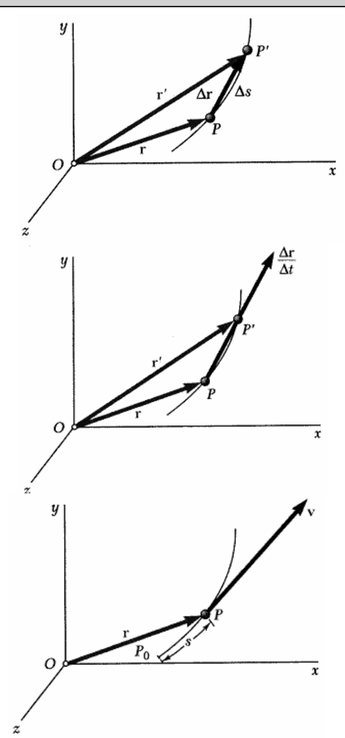
Curvilinear Motion: Position, Velocity & Acceleration

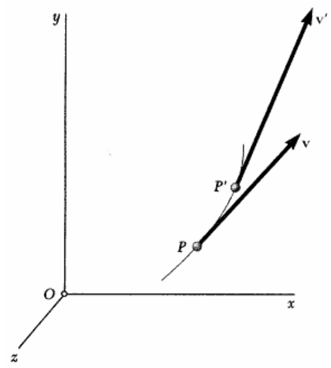


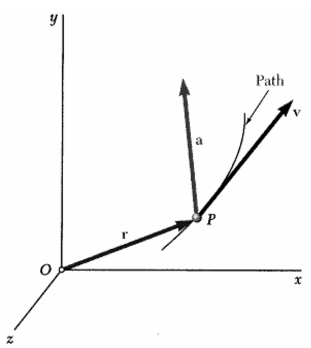
- Particle moving along a curve other than a straight line is in *curvilinear motion*.
- *Position vector* of a particle at time *t* is defined by a vector between origin *O* of a fixed reference frame and the position occupied by particle.
- Consider particle which occupies position *P* defined by $\vec{r}$ at time *t* and *P'* defined by $\vec{r}\,'$ at $t + \Delta t$,

  instantaneous velocity (vector)

  instantaneous speed (scalar)

Curvilinear Motion: Position, Velocity & Acceleration



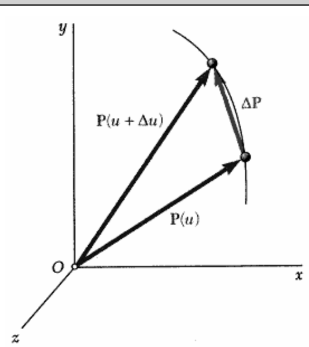


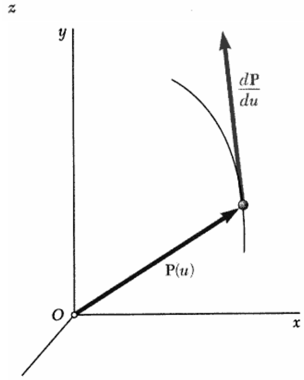
- Consider velocity $\vec{v}$ of particle at time t and velocity $\vec{v}'$ at $t + \Delta t$,

  instantaneous acceleration (vector)

- In general, acceleration vector is not tangent to particle path and velocity vector.

Review: Derivatives of Vector Functions





- Let $\vec{P}(u)$ be a vector function of scalar variable u,

$$\frac{d\vec{P}}{du} = \lim_{\Delta u \to 0} \frac{\Delta \vec{P}}{\Delta u} = \lim_{\Delta u \to 0} \frac{\vec{P}(u+\Delta u) - \vec{P}(u)}{\Delta u}$$

- Derivative of vector sum,

$$\frac{d\left(\vec{P}+\vec{Q}\right)}{du} = \frac{d\vec{P}}{du} + \frac{d\vec{Q}}{du}$$

- Derivative of product of scalar and vector functions,

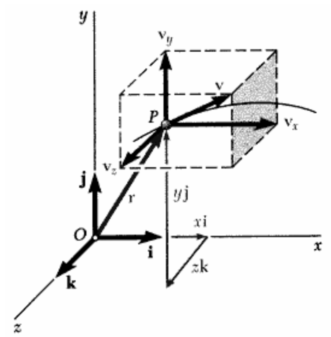
$$\frac{d\left(f\vec{P}\right)}{du} = \frac{df}{du}\vec{P} + f\frac{d\vec{P}}{du}$$

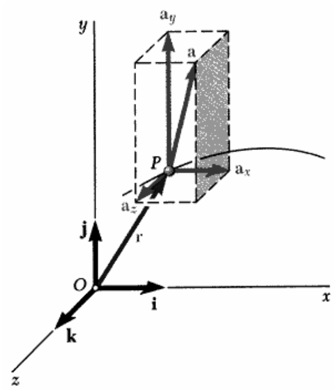
- Derivative of *scalar product* and *vector product,*

$$\frac{d\left(\vec{P}\bullet\vec{Q}\right)}{du} = \frac{d\vec{P}}{du}\bullet\vec{Q} + \vec{P}\bullet\frac{d\vec{Q}}{du}$$

$$\frac{d\left(\vec{P}\times\vec{Q}\right)}{du} = \frac{d\vec{P}}{du}\times\vec{Q} + \vec{P}\times\frac{d\vec{Q}}{du}$$

Rectangular Components of Velocity & Acceleration





- When position vector of particle P is given by its rectangular components,

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

- Velocity vector,

$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$
$$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

- Acceleration vector,

$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$
$$= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

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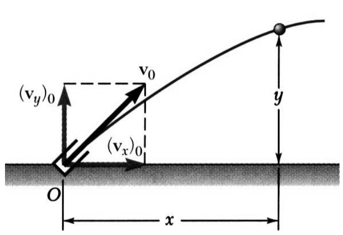
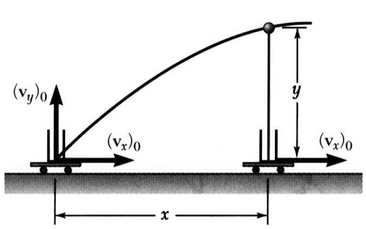
Rectangular Components of Velocity & Acceleration





- Rectangular components particularly effective when component accelerations can be integrated independently, e.g., motion of a projectile,

$$a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = -g \qquad a_z = \ddot{z} = 0$$

with initial conditions,

$$x_0 = y_0 = z_0 = 0 \qquad (v_x)_0, (v_y)_0, (v_z)_0 = 0$$

Integrating twice yields

$$v_x = (v_x)_0 \quad v_y = (v_y)_0 - gt \qquad v_z = 0$$
$$x = (v_x)_0 t \quad y = (v_y)_0 y - \tfrac{1}{2}gt^2 \quad z = 0$$

- Motion in horizontal direction is uniform.
- Motion in vertical direction is uniformly accelerated.
- Motion of projectile could be replaced by two independent rectilinear motions.

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Example: Kinematics in Rectangular Coordinates

The position vector of a radio-operated airplane is given as

$$\vec{r} = (1.5t^2 + 3t)\hat{i} + (1.5t - t^2)\hat{j} + 1.2t^2\hat{k} \quad \text{ft.}$$

where t is in seconds. The operator stands at the origin of the coordinate system with z-axis directly upwards. At t = 2 s., determine:

(a) the (x,y,z) projections of the velocity and acceleration.

(b) the speed of the airplane.

(c) the magnitude of the displacement of the airplane from t = 0 s.

(d) the distance travel from t = 0 s.

(a) Want $\vec{v}(2), \vec{a}(2)$

(b) speed = magnitude of velocity

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Example: Kinematics in Rectangular Coordinates

(c) First find the displacement

then find the magnitude

(d) distance traveled

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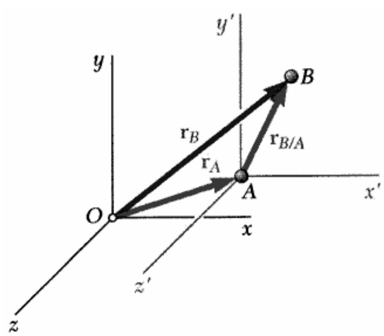
Motion Relative to a Frame in Translation



- Designate one frame as the *fixed frame of reference*. All other frames not rigidly attached to the fixed reference frame are *moving frames of reference*.
- Position vectors for particles *A* and *B* with respect to the fixed frame of reference *Oxyz* are $\vec{r}_A$ and $\vec{r}_B$.
- Vector $\vec{r}_{B/A}$ joining *A* and *B* defines the position of *B* with respect to the moving frame *Ax'y'z'* and

  $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$
- Differentiating twice,

  $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$ $\quad \vec{v}_{B/A} =$ velocity of *B* relative to *A*.

  $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ $\quad \vec{a}_{B/A} =$ acceleration of *B* relative to *A*.
- Absolute motion of *B* can be obtained by combining motion of *A* with relative motion of *B* with respect to moving reference frame attached to *A*.

Vector Mechanics for Engineers: Dynamics

Note on Vector Notations and Vector Usage

•The book uses bold faced letter to denote vectors, i.e., ***a***.

•Bold is impractical with handwriting, so use symbols such as: $\vec{a}, \hat{a}, \tilde{a}, \underline{a}$

•Variables with no special symbols denoting them as vectors will be interpreted as a scalar. A scalar with the same variable name as a vector is the magnitude of that vector. For example:

v_A will be interpreted as $\left|\vec{v}_A\right|$

ω as $\left|\vec{\omega}\right|$

•Be especially careful with vector equations. If you write the relative velocity equation as:

$$v_B = v_A + v_{B/A}$$

The equation above is normally incorrect. You will get points off!

•Worse, if you treat the above equation as a scalar equation (adding up the magnitudes even though the vectors are not parallel), you will get no credit at all.

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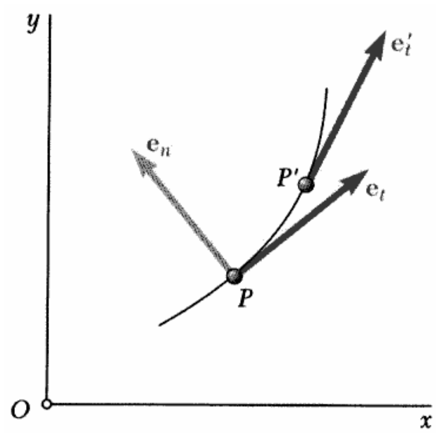
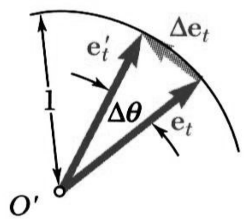
Tangential and Normal Components (Path Coordinates)





- Velocity vector of particle is tangent to path of particle. In general, acceleration vector is not. Wish to express acceleration vector in terms of tangential and normal components.

- $\hat{e}_t$ and $\hat{e}_t'$ are tangential unit vectors for the particle path at *P* and *P'*. When drawn with respect to the same origin, $\Delta\vec{e}_t = \vec{e}_t' - \vec{e}_t$ and $\Delta\theta$ is the angle between them.

$$\Delta e_t = 2\sin(\Delta\theta/2)$$

$$\lim_{\Delta\theta\to 0}\frac{\Delta\vec{e}_t}{\Delta\theta} = \lim_{\Delta\theta\to 0}\frac{\sin(\Delta\theta/2)}{\Delta\theta/2}\vec{e}_n = \vec{e}_n$$

$$\vec{e}_n = \frac{d\vec{e}_t}{d\theta}$$

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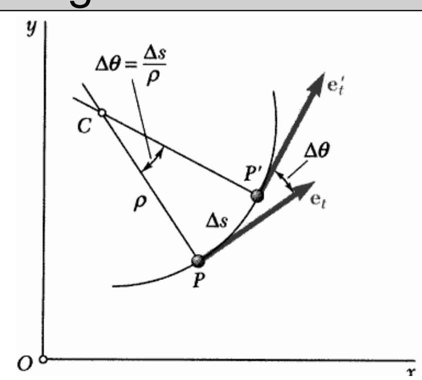
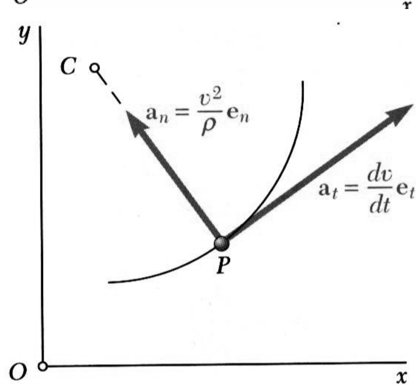
Tangential and Normal Components





- With the velocity vector expressed as $\vec{v} = v\vec{e}_t$ the particle acceleration may be written as

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\vec{e}_t + v\frac{d\vec{e}_t}{dt} = \frac{dv}{dt}\vec{e}_t + v\frac{d\vec{e}_t}{d\theta}\frac{d\theta}{ds}\frac{ds}{dt}$$

but

$$\frac{d\vec{e}_t}{d\theta} = \vec{e}_n \qquad \rho\, d\theta = ds \qquad \frac{ds}{dt} = v$$

After substituting,

$$\vec{a} = \frac{dv}{dt}\vec{e}_t + \frac{v^2}{\rho}\vec{e}_n \qquad a_t = \frac{dv}{dt} \quad a_n = \frac{v^2}{\rho}$$

- Tangential component of acceleration reflects change of speed and normal component reflects change of direction.
- Tangential component may be positive or negative. Normal component always points toward center of path curvature.

No eqn for r!

Vector Mechanics for Engineers: Dynamics

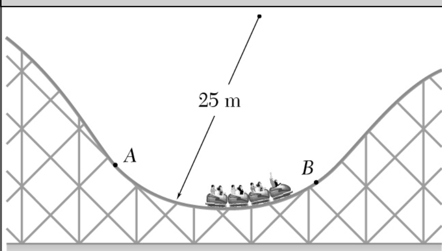
Tangential and Normal Components

- What happens to these equations when the object is moving along a circular path?

- The path (n-t) coordinates do expand to cover 3-D motion, but we won't be studying them in this class.

Vector Mechanics for Engineers: Dynamics

Problem 11.135



Determine the maximum speed that the cars of the roller-coaster can reach along the circular portion AB of the track if the normal component of their acceleration cannot exceed 3g.

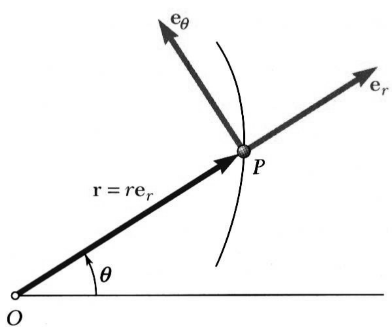
SOLUTION:

•The tangential acceleration is not given, but it is irrelevant.

•Normal acceleration and speed are related.

•Solve for v_{max}

Radial and Transverse Components (Polar Coordinates)



$$\vec{r} = r\vec{e}_r$$

$$\frac{d\vec{e}_r}{d\theta} = \vec{e}_\theta \qquad \frac{d\vec{e}_\theta}{d\theta} = -\vec{e}_r$$

$$\frac{d\vec{e}_r}{dt} = \frac{d\vec{e}_r}{d\theta}\frac{d\theta}{dt} = \vec{e}_\theta \frac{d\theta}{dt}$$

$$\frac{d\vec{e}_\theta}{dt} = \frac{d\vec{e}_\theta}{d\theta}\frac{d\theta}{dt} = -\vec{e}_r \frac{d\theta}{dt}$$

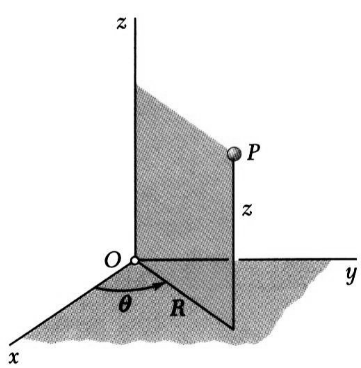
- When particle position is given in polar coordinates, it is convenient to express velocity and acceleration with components parallel and perpendicular to *OP*.
- The particle velocity vector is

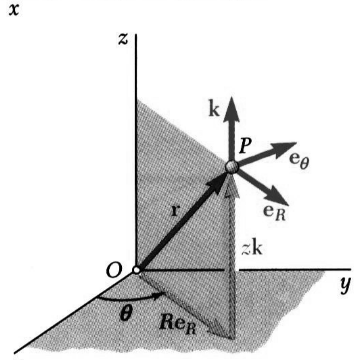
$$\vec{v} = \frac{d}{dt}(r\vec{e}_r) = \frac{dr}{dt}\vec{e}_r + r\frac{d\vec{e}_r}{dt} = \frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta$$
$$= \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta$$

- Similarly, the particle acceleration vector is

$$\vec{a} = \frac{d}{dt}\left(\frac{dr}{dt}\vec{e}_r + r\frac{d\theta}{dt}\vec{e}_\theta\right)$$
$$= \frac{d^2r}{dt^2}\vec{e}_r + \frac{dr}{dt}\frac{d\vec{e}_r}{dt} + \frac{dr}{dt}\frac{d\theta}{dt}\vec{e}_\theta + r\frac{d^2\theta}{dt^2}\vec{e}_\theta + r\frac{d\theta}{dt}\frac{d\vec{e}_\theta}{dt}$$
$$= \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

Radial and Transverse Components (Cylindrical Coordinates)





- When particle position is given in cylindrical coordinates, it is convenient to express the velocity and acceleration vectors using the unit vectors $\vec{e}_R, \vec{e}_\theta$, and $\vec{k}$.
- Position vector,

$$\vec{r} = R\,\vec{e}_R + z\,\vec{k}$$

- Velocity vector,

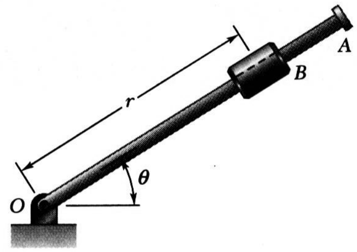
$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{R}\,\vec{e}_R + R\dot{\theta}\,\vec{e}_\theta + \dot{z}\,\vec{k}$$

- Acceleration vector,

$$\vec{a} = \frac{d\vec{v}}{dt} = \left(\ddot{R} - R\dot{\theta}^2\right)\vec{e}_R + \left(R\ddot{\theta} + 2\dot{R}\dot{\theta}\right)\vec{e}_\theta + \ddot{z}\,\vec{k}$$

Vector Mechanics for Engineers: Dynamics

Sample Problem 11.12



Rotation of the arm about O is defined by $\theta = 0.15t^2$ where θ is in radians and t in seconds. Collar B slides along the arm such that $r = 0.9 - 0.12t^2$ where r is in meters.

After the arm has rotated through 30°, determine *(a)* the total velocity of the collar, *(b)* the total acceleration of the collar, and *(c)* the relative acceleration of the collar with respect to the arm.

SOLUTION:

- Evaluate time t for $\theta = 30°$.
- Evaluate radial and angular positions, and first and second derivatives at time t.
- Calculate velocity and acceleration in cylindrical coordinates.
- Evaluate acceleration with respect to arm.

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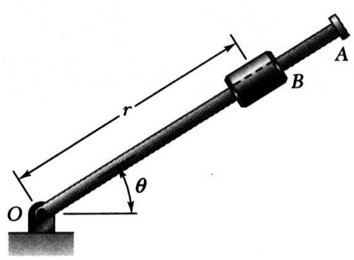
Sample Problem 11.12



SOLUTION:

- Evaluate time t for $\theta = 30°$.

$$\theta = 0.15t^2$$
$$=$$

- Evaluate radial and angular positions, and first and second derivatives at time t.

$$r = 0.9 - 0.12t^2 = 0.481\,\text{m}$$
$$\dot{r} =$$
$$\ddot{r} =$$

$$\theta = 0.15t^2 = 0.524\,\text{rad}$$
$$\dot{\theta} =$$
$$\ddot{\theta} =$$

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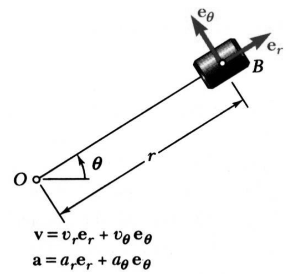
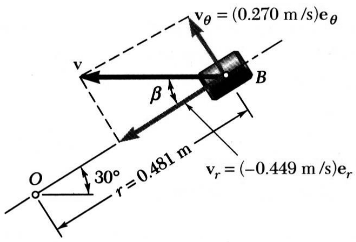
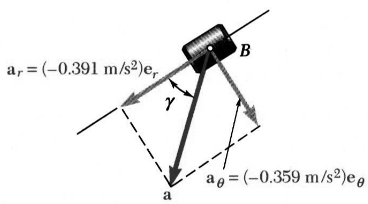
Sample Problem 11.12







- Calculate velocity and acceleration.

$$v_r =$$

$$v_\theta =$$

$$v = \sqrt{v_r^2 + v_\theta^2} \qquad \beta = \tan^{-1}\frac{v_\theta}{v_r}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$=$$
$$=$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$=$$
$$=$$

$$a = \sqrt{a_r^2 + a_\theta^2} \qquad \gamma = \tan^{-1}\frac{a_\theta}{a_r}$$

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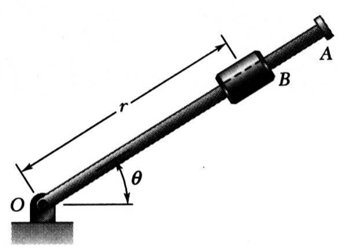
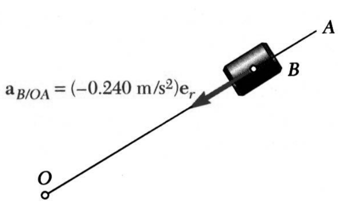
Sample Problem 11.12



- Evaluate acceleration with respect to arm.

  Motion of collar with respect to arm is rectilinear and defined by coordinate *r*.



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CHAPTER 12

VECTOR MECHANICS FOR ENGINEERS: DYNAMICS

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Modified for Engr 51 at Cuesta College:
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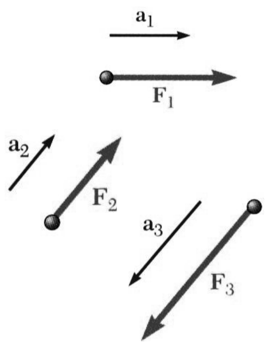
Kinetics of Particles: Newton's Second Law

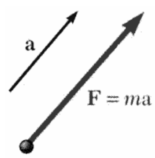
Introduction

- Newton's first and third laws are sufficient for the study of bodies at rest (statics) or bodies in motion with no acceleration.
- When a body accelerates (changes in velocity magnitude or direction), Newton's second law is required to relate the motion of the body to the forces acting on it.
- Newton's second law:
  - A particle will have an acceleration proportional to the magnitude of the resultant force acting on it and in the direction of the resultant force.
  - The resultant of the forces acting on a particle is equal to the rate of change of linear momentum of the particle.
  - The sum of the moments about *O* of the forces acting on a particle is equal to the rate of change of angular momentum of the particle about *O*.

Newton's Second Law of Motion



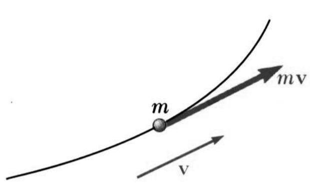
- *Newton's Second Law*: If the *resultant force* acting on a particle is not zero, the particle will have an acceleration *proportional to the magnitude of resultant* and in the *direction of the resultant*.
- Consider a particle subjected to constant forces,
- When a particle of mass *m* is acted upon by a force $\vec{F}$, the acceleration of the particle must satisfy
  $$\vec{F} = m\vec{a}$$



- Acceleration must be evaluated with respect to a *Newtonian frame of reference*, i.e., one that is not accelerating or rotating.
- If force acting on particle is zero, particle will not accelerate, i.e., it will remain stationary or continue on a straight line at constant velocity.

Linear Momentum of a Particle

- Start with the equation $\sum \vec{F} = m\vec{a}$
- Replacing the acceleration by the derivative of the velocity yields



- *Linear Momentum Conservation Principle*:
  If the resultant force on a particle is zero, the linear momentum of the particle remains constant in both magnitude and direction.

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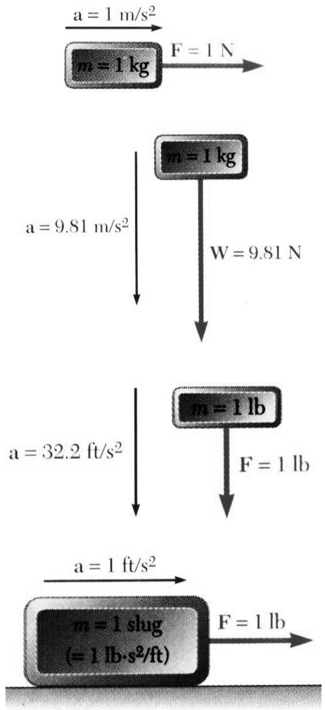
Systems of Units (Review)



- Of the units for the four primary dimensions (force, mass, length, and time), three may be chosen arbitrarily. The fourth must be compatible with Newton's 2nd Law.

- *International System of Units* (SI Units): base units are the units of length (m), mass (kg), and time (second). The unit of force is derived,

$$1\,\mathrm{N} = (1\,\mathrm{kg})\left(1\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 1\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$$

- *U.S. Customary Units*: base units are the units of force (lb), length (ft), and time (second). The unit of mass is derived,

$$1\,\mathrm{lbm} = \frac{1\,\mathrm{lb}}{32.2\,\mathrm{ft}/\mathrm{s}^2} \qquad 1\,\mathrm{slug} = \frac{1\,\mathrm{lb}}{1\,\mathrm{ft}/\mathrm{s}^2} = 1\frac{\mathrm{lb}\cdot\mathrm{s}^2}{\mathrm{ft}}$$

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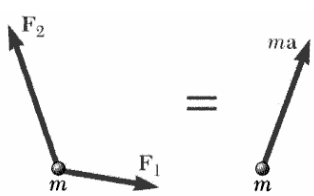
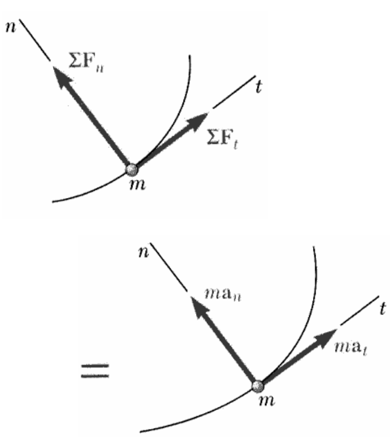
Equations of Motion (The Force-Mass-Acceleration Method)



- Newton's second law provides

$$\sum \vec{F} = m\vec{a}$$

- Solution for particle motion is facilitated by resolving vector equation into scalar component equations, e.g., for rectangular components,



- For tangential and normal components,

- We can do the same with radial and transverse components.

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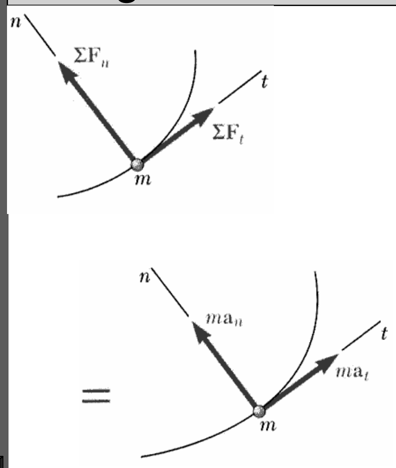
Using the Force-Mass-Acceleration (FMA) Method



- First, define the coordinates used in the problem. They can be drawn separately, or integrated into the diagrams. (The book often skips this step. You don't get to. If you mention *x*-direction, define it.)
- Draw the Free-Body Diagram (FBD) to see all forces acting on the body.
- Draw the Mass-Acceleration Diagram (MAD), sometimes called Kinematics Diagram (KD). The acceleration should be broken down into components according to the coordinates.
- Relate the two diagrams.
- Often we have to analyze kinematics information. Sometimes to find the acceleration, or sometimes to find the motion given the acceleration.

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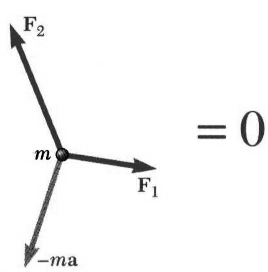
Dynamic Equilibrium (Don't use!)



- Alternate expression of Newton's second law,

$$\sum \vec{F} - m\vec{a} = 0$$

$$-m\vec{a} \equiv \textit{inertial vector}$$

- With the inclusion of the inertial vector, the system of forces acting on the particle is equivalent to zero. The particle is in *dynamic equilibrium*.
- Inertia vectors are often called *inertial forces* as they measure the resistance that particles offer to changes in motion, i.e., changes in speed or direction.
- Inertial forces may be conceptually useful but are not like the contact and gravitational forces found in statics.

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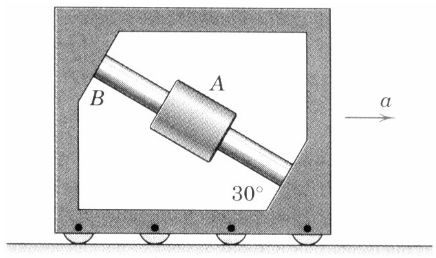
Example: FMA in Rectangular Coordinates



The collar A is free to slide along the smooth shaft B mounted in the frame. The plane of the frame is vertical. Determine the horizontal acceleration *a* of the frame necessary to maintain the collar in a fixed position on the shaft.

SOLUTION:

- Rectangular (x-y) coordinates fit well with this problem, though n-t would work just as well.
- We are interested in the collar, so we'll draw the FBD and MAD diagrams for it.
- The collar position is fixed relative to the shaft/frame, therefore its acceleration is also *a*.
- We can write 2 equations relating the FBD and the MAD. (for the x and y components)
- The unknowns are the normal force, *N*, between the shaft and the collar, and the acceleration, *a*.

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Example: FMA in Rectangular Coordinates

SOLUTION:

- We can write the $\sum \vec{F} = m\vec{a}$ equation as two rectangular component equations.

$\sum F_x = ma_x$:

$\sum F_y = m_A a_y = 0$:

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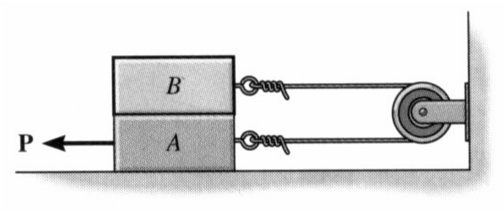
Example: FMA in Constrained System



Each of the two blocks shown has a mass *m*. The coefficient of kinetic friction at all surfaces of contact is μ. If a horizontal force **P** moves the blocks, determine the acceleration of the bottom block.

SOLUTION:

- We have to analyze each block separately. So draw FBD and MAD for each of them.
- Write the kinematic relationships for the dependent motions and accelerations of the blocks.
- We can write 4 equations relating the FBD and the MAD. (the x and y components of each block)
- The unknowns are acceleration (of both; they are related), the tension in the cable, the normal force N_A between block A and the ground, and the normal force N_B between blocks A and B.

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Example: FMA in Constrained System

SOLUTION:

- Write the kinematic relationships for the dependent motions and accelerations of the blocks.

- Write equations of motion for each block.

$B: \sum F_y = m_B a_y$ $\qquad$ $\sum F_x = m_B a_x :$

$A: \sum F_y = m_A a_y :$ $\qquad$ $\sum F_x = m_A a_x :$

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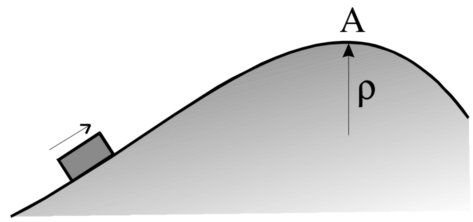
Example: Curvilinear FMA



Determine the maximum speed, *v*, which the sliding block may have as it passes point A without losing contact with the surface

SOLUTION:

- Pick the coordinate system for the problem.
- Translate "losing contact".
- We're only interested in the dynamics at point A, not as it travels there. Only have to draw the FBD and MAD diagram for point A.

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Example: Curvilinear FMA

SOLUTION:

- There could be friction and therefore a_t but they are both irrelevant in this particular problem.

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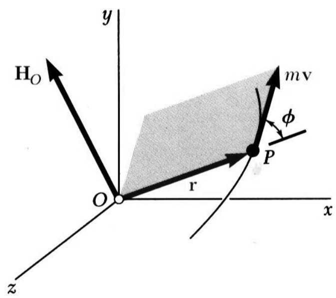
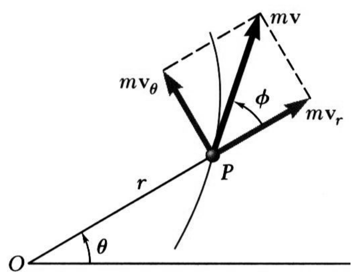
Angular Momentum of a Particle





- $\vec{H}_O = \vec{r} \times m\vec{v} =$ *moment of momentum* or the *angular momentum* of the particle about *O*.
- $\vec{H}_O$ is perpendicular to plane containing $\vec{r}$ and $m\vec{v}$

$$\vec{H}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$

- Derivative of angular momentum with respect to time,
- It follows from Newton's second law that the sum of the moments about *O* of the forces acting on the particle is equal to the rate of change of the angular momentum of the particle about *O*.

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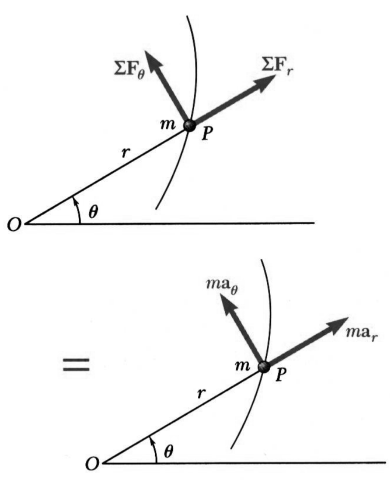
Eqs of Motion in Radial & Transverse Components



- Consider particle at *r* and θ, in polar coordinates,

$$\sum F_r = ma_r = m\left(\ddot{r} - r\dot{\theta}^2\right)$$
$$\sum F_\theta = ma_\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$$

- This result may also be derived from conservation of angular momentum,

$$H_O = mr^2\dot{\theta}$$

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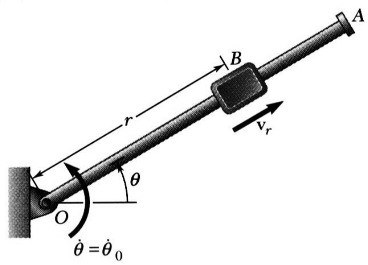
Sample Problem 12.7



A block *B* of mass *m* can slide freely on a frictionless arm *OA* which rotates in a horizontal plane at a constant rate $\dot{\theta}_0$.

Knowing that *B* is released at a distance r_0 from *O*, express as a function of *r*

a) the component v_r of the velocity of *B* along *OA*, and

b) the magnitude of the horizontal force exerted on *B* by the arm *OA*.

SOLUTION:

- First, we want v_r which is $\dot{r}$. We don't have *r* as a function of time, so we have to find v_r some other way.
- With the information we have, we can write the radial and transverse equations of motion for the block.
- The radial equation contains $\ddot{r}$ which can be integrated to find v_r.
- The transverse equation can be used to find an expression for the force on the block.

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Sample Problem 12.7

- Integrate $\ddot{r}$ to find an expression for the radial velocity.

SOLUTION:

- Write the radial and transverse equations of motion for the block.

$\sum F_r = m a_r$:

$\sum F_\theta = m a_\theta$:

- Use the transverse equation to find an expression for the force on the block.

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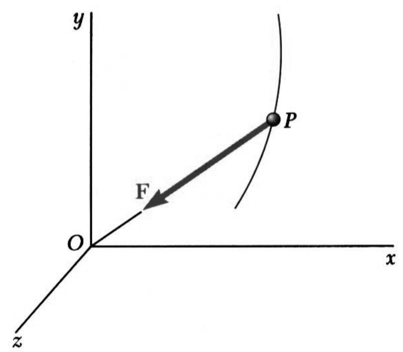
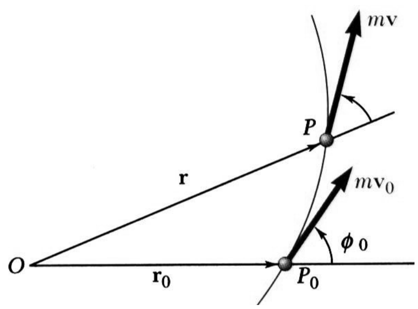
Vector Mechanics for Engineers: Dynamics

Conservation of Angular Momentum



- When only force acting on particle is directed toward or away from a fixed point *O*, the particle is said to be *moving under a central force*.
- Since the line of action of the central force passes through *O*, $\sum \vec{M}_O = \dot{\vec{H}}_O = 0$
- Position vector and motion of particle are in a plane perpendicular to $\vec{H}_O$.
- Magnitude of angular momentum,

$$H_O = rmv\sin\phi = \text{constant}$$
$$= r_0 m v_0 \sin\phi_0$$
or
$$H_O = mr^2\dot{\theta} = \text{constant}$$



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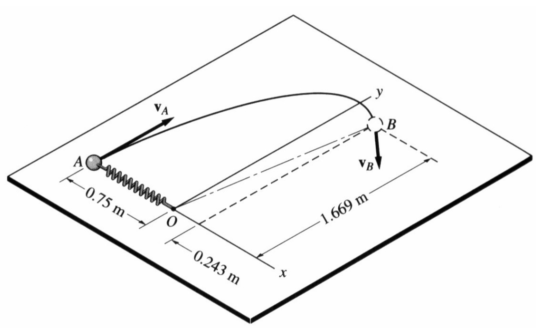
Example: Conservation of Angular Momentum



The particle, connected by a spring to the fixed point *O*, slides on the frictionless, horizontal table. The particle is launched at A with the velocity v_A in the y-direction. If the velocity of the particle at B is

$\vec{v}_B = 3.66\hat{i} - 5.72\hat{j}$ m/s , determine v_A.

SOLUTION:

- Since there is no friction and the motion is horizontal, the only force acting on the particle is the spring force.
- The spring force is always directed to or from point O, that means the particle is moving under a central force.
- The particle's angular momentum is constant. Equate the angular momentum at *A* and *B* and solve for v_A.

Example: Conservation of Angular Momentum

SOLUTION:

- The particle's angular momentum is constant. Equate the angular momentum at A and B and solve for v_A.