Design Handbook for

REINFORCED CONCRETE ELEMENTS

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Symbols

a = Concrete cover or half the distance between parallel bars.
or
Distance between points of zero bending moment.
or
The cover over a deformed bar or half the distance between parallel bars whichever is the lesser.
or
Dimension of the critical shear perimeter measured parallel to the direction of M^*_v.
or
Footing outstand.

A_{ct} = Cross-sectional area of uncracked concrete in the tensile zone

A_{st} = Area of tensile reinforcement.

A_b = Cross-sectional area of reinforcing bar.

A_{FN} = Reaction area for punching shear.

A_g = The gross cross-sectional area of a member.

A_m = Area of thin walled section for torsion defined by the median lines of the walls of a single cell.

a_s = Length of support in direction on span.

A_s = The cross-sectional area of the reinforcement $A_{sc} + A_{st}$.

A_{s1} = Tensile area of primary beam. This is usually the area of a singly reinforced beam with the maximum steel ratio p_{max} for which $k_u = 0.4$.

A_{s2} = Tensile area of secondary beam.

A_{sc} = Area of compressive reinforcement.

= Area of reinforcement on the compression side of a column.

A_{st} = Area of tensile reinforcement.

= $A_{s1} + A_{s2}$ for doubly reinforced beams.

$A_{st.min}$ = Minimum area of reinforcement.

A_{sv} = Cross-sectional area of shear reinforcement.

$A_{sv.min}$ = Minimum area of shear reinforcement.

A_{sw} = Area of a single leg of a closed tie used as torsional reinforcement.

A_t = Torsion area defined as the area from the centre of the corner bars of the cross section.

a_v = Distance from section at which shear is being considered to the nearest support.

b = Width of beam.

b = Effective flange width $b_{ef.}$

b = Column width perpendicular to applied moment.

b_{eff} = Effective beam width or effective flange width.

b_{min} = Minimum beam width for a given exposure classification.

b_o = Critical dimension of an opening adjacent to a slab support.

b_v = Effective width of a web for shear.

= b for a rectangular beam.

= b_w for a T-beam or L-beam.

b_w = Width of a web as in a T-beam.

C = Internal compressive force carried by the concrete.

c_{min} = Minimum distance from centroid of reinforcement to exposed concrete face required to satisfy exposure conditions.

D = Overall depth of beam.

D = Column depth in direction of applied moment.

d = Effective depth measured to the resultant tensile force.

d_b = Bar diameter.

D_b = Overall depth of a spandrel beam.

D_c = Smaller column dimension.

d_o = Distance from extreme compression fibre to the centroid of the outermost layer of tensile reinforcement but not less than 0.8D.

d_s = Depth of rectangular stress block $\gamma k_u d$.

D_s = Overall depth of slab or drop panel as appropriate.

d_{sc} = Depth measured to centroid of compressive reinforcement.

e = Load eccentricity measured from plastic centroid.

e' = Load eccentricity measured from tensile reinforcement.

E_c = Modulus of elasticity of concrete.

E_s = Modulus of elasticity of steel reinforcement.

E^* = Design load (or W^*).

E_c = Modulus of elasticity for concrete at 28 days.

E_{cj} = The mean value of modulus of elasticity of concrete at nominated age.
$\sigma^{1.5} \times 0.043\sqrt{f_{cm}}$

E_u = Ultimate earthquake action.

f_c = An intermediate concrete stress.

f_{cs} = Max shrinkage-induced stress on uncracked sections at the extreme fibre where cracking first occurs.

f_{scr} = Tensile stress in the reinforcement (at the cracked section) due to 'short term' serviceability loads under direct loading.

$f_{scr.1}$ = As above but using ψ_s =1.0 (rather than 0.7).

f'_c = 28 day characteristic compressive strength of concrete.

f'_{cf} = Characteristic flexural strength of concrete.

f'_{cf} = Flexural tensile strength of plain concrete.
= $0.6\sqrt{f'c}$

f_{cm} = Mean compressive strength of concrete at relevant age.

f_{cv} = Concrete shear strength.

F_d = Slab design load.

$F_{d.eff}$ = Effective design load for serviceability in kN/m or kN/m^2.

F_{ep} = Load due to earth pressure in kN.

F_{lp} = Load due to liquid pressure.

f_{sc} = Stress in compressive reinforcement.

f_{st} = Stress in tensile reinforcement.

f_{sy} = Yield strength of steel reinforcement.

$f_{sy.f}$ = Yield strength of fitments.

G = Concentrated or total dead load.

g	=	Distributed dead load. or Ratio of distance between outer reinforcement to the overall depth of a column section.
G^R	=	Dead loads resisting instability.
J_t	=	Torsional modulus for the cross section.
k	=	Effective length multiplier.
K	=	Ratio of areas A_{sc}/A_{s2} in design of doubly reinforced beams.
k_s	=	Coefficient to take account of the stress distribution shape in a section prior to cracking (0.6 for flexure & 0.8 for tension).
k_1	=	Second moment of area multiplier.
k_2	=	Deflection constant for rectangular beams.
k_3	=	Slab multiplier.
k_4	=	Deflection constant for slabs.
k_5	=	Special Slab deflection coefficient read from chart D2.
k_b	=	The value of k_u for balanced conditions.
k_{cs}	=	Long-term deflection multiplier (to account for shrinkage & creep).
k_d	=	Depth of N.A. at working/serviceability load conditions.
k_m	=	End moment condition parameter.
k_p	=	Deflection correction for steel ration in beams used with chart D1.
k_u	=	Ratio of depth of NA to beam effective depth d.
k_{uo}	=	Ratio at ultimate strength of the depth of the NA from the extreme compressive fibre to d_o. Symbols k_u is applied for k_{uo} in this text.
L	=	Span of beam between support centrelines.
l	=	Clear distance between webs of parallel beams.
L_n	=	Clear span between inner faces of supports or the clear projection of cantilevers.
l_x	=	Short clear slab panel dimension between supports.
l_y	=	Long clear slab panel dimension between supports.
L_e	=	Effective length of a column.
L_{eff}	=	Effective span of beam, lesser of L and (Ln + D) or (Ln + D/2) for cantilevers.
L_n	=	Clear span between inside of supporting beams, columns or walls.
L_o	=	Span length used in the simplified method, L minus 0.7 times the sum of a_S for each support.
L'_o	=	The smaller value of L_o for adjoining spans.
L_{st}	=	Tensile development length for $f_{st} < f_{sy}$.

$L_{sy.c}$	=	Development length for compressive reinforcement at yield condition.
$L_{sy.t}$	=	Tensile development length i.e. minimum length of embedment required to develop yield strength of a reinforcing bar in tension.
L_t	=	Width of the design strip.
L_u	=	The unsupported length of a column, taken as the clear distance between faces of members capable of providing lateral support to the column.
L_x	=	Shorter effective span of slab supported on four sides.
L_x	=	Short effective span of a slab panel.
L_y	=	Long effective span of a slab panel.
M^*	=	Design moment due to factored loads.
M^*_s	=	Design bending moment (at the Serviceability limit state).
$M^*_{s.1}$	=	As above but using ψ_s =1.0 (rather than 0.7).
M^*_v	=	The unbalanced slab bending moment transferred into the support.
M^*_x and M^*_y	=	Slab design moments in x and y directions.
M_1	=	Effective moment capacity of primary beam.
M_2	=	$M^*–M_1$ the effective moment capacity to be carried by secondary beam.
M_m	=	Positive bending moment at midspan.
M_{NE}	=	Negative moment at exterior support.
M_{NI}	=	Negative moment at interior support.
M_o	=	Total static moment for the span of the design strip.
M_u	=	The ultimate strength in bending at a cross-section of an eccentrically loaded compression member.
M_{ub}	=	The ultimate strength in bending when k_u = 0.545.
M_{ud}	=	Reduced ultimate strength in bending for k_u = 0.4 condition.
M_{uo}	=	Ultimate strength in pure bending.
$M_{uo\ min}$	=	Minimum strength in bending at a critical cross section.
M_y	=	Moment causing initial yield of reinforcement.
N^*	=	Design axial load.
NA	=	Neutral axis.
N_c	=	The buckling load in a column.

N_u	=	The ultimate compressive strength combined with moment M_u.
N_{ub}	=	The ultimate compressive strength when k_u = 0.6.
N_{uo}	=	The ultimate strength of an axially loaded squat columns.
p	=	Reinforcing steel ratio.
P*	=	Concentrated design load.
p_1	=	Tensile steel ratio in primary beam.
p_c	=	Compressive steel ratio.
	=	A_{sc}/bd.
p_{max}	=	Maximum tensile steel ratio for k_u = 0.4 condition.
p_t	=	Total tensile steel ratio.
		A_{st}/bd.
p_v	=	Shear steel ratio $A_{st}/(b_v d_o)$.
Q	=	Concentrated or total live load.
q	=	Distributed live load.
q_1	=	Maximum soil bearing pressure under footing.
q_2	=	Minimum soil bearing pressure under footing.
q_a	=	Permissible soil bearing pressure.
q_u	=	Factored soil bearing capacity.
	=	$1.4q_a$
R	=	Radius of curvature.
r	=	Radius of gyration.
S_u	=	Ultimate action due to combination of various action.
T	=	Internal resultant tensile steel force carried by the reinforcement.
t	=	Flange thickness.
	=	Thickness of slab D_s making up T-beam or L-beam.
t_h	=	Hypothetical thickness used to calculate creep and shrinkage.
	=	$2Ag/u_{e.}$
T*	=	Design torsional moment.
$T_{u.max}$	=	Ultimate torsional strength of a beam limited by crushing failure.
T_{uc}	=	Ultimate torsional strength of a beam without torsional reinforcement.
T_{us}	=	Ultimate torsional strength of a beam with torsional reinforcement.
u	=	Length of critical shear perimeter for two-way action. or Shear perimeter d/2 from face of column.

u_e = Exposed perimeter plus half perimeter of enclosed voids.

u_t = Perimeter of A_t

V_c = Simplified ultimate shear capacity of unreinforced beam.

v'_c = Nominal concrete shear stress capacity.

V_u = Ultimate shear strength.

$V_{u.max}$ = Ultimate shear strength limited by shear crushing.

$V_{u.min}$ = Ultimate shear strength of a beam with minimum shear reinforcement.

V_{uc} = Ultimate shear strength excluding shear reinforcement.

V_{uo} = The ultimate shear strength of a slab where $M^*_v = 0$

V_{us} = Contribution provided by shear reinforcement to the ultimate shear strength of a beam.

w^* = Distributed design load.

W_S = Serviceability wind action.

W_u = Ultimate wind action.

w_x' = Equivalent design load for shorter slab support.

w_y' = Equivalent design load for longer slab support.

x = Smaller dimension of a cross section (or smaller dimension of a rectangular component of a cross section).

x , y = The shorter and longer dimensions respectively of the cross section of the torsion strip or spandrel beam.

y_1 = Larger dimension of a closed rectangular torsion tie.

= AS3600 Concrete Structures Code reference.

β_1 = Shear strength coefficient for comparable increase in shear capacity of shallow beams.

β_2 = Shear strength coefficient for axial load effects.

β_3 = Shear strength coefficient to account for increased strength when concentrated loads are applied near supports (short shear span $a_V < 2d_o$).

β_d = Creep factor for sustained loading.

β_h = The ratio of the longest overall dimension of the effective loaded area, Y, to the overall dimension X, measured perpendicular to Y.

β_x , β_y = Bending moment coefficients for two-way slabs supported by rigid beams and walls.

δ = Deflection obtained from calculations.

δ_b, δ_s	=	Moment magnifiers for braced and sway columns.
Δ	=	Maximum deflection - normally expressed as a fraction eg (D / L).
ε_c	=	Concrete compressive strain.
	=	0.003 at failure.
ε_{cs}	=	Design shrinkage strain (from Section 6.1.7.2 - AS3600).
ε_s	=	Strain in steel reinforcement.
ε_{sc}	=	Strain in compressive reinforcement.
ε_{st}	=	Strain in tensile reinforcement.
ε_y	=	Steel strain at the point of yielding.
Φ	=	Strength reduction factor.
γ	=	Ratio of depth of simplified rectangular stress block to depth of NA.
κ	=	Curvature.
l	=	Design parameter used in conjunction with chart B1.
θ	=	Angle of rotation.
θ_t and θ_v	=	Angle between the concrete compression "strut" and the member axis in the truss model for torsion or shear respectively.
σ	=	Density of concrete in kg/m^3 taken as 2400 kg/m^3 in this book.
ψ_C	=	Live load combination factor for strength.
ψ_L	=	Long-term live load combination factor for serviceability.
ψ_S	=	Short-term live load combination factor for serviceability.
ρ	=	density of concrete (taken as 2400 kg/m^3 in this book)

Preface

First edition

This book is designed to provide an introduction to the design of reinforced concrete elements. The work began as an aid for students to understand the design of concrete elements, not just as a theoretical study, but as a practical operation in the design of structures. In its development it has expanded from the original brief to provide a more complete picture.

The resulting book has become a blend. A blend of theory, Code requirements and Design Aids. It is this blend that helps to provide the balanced process for the design and analysis of concrete components that every engineer and student needs to work with.

It is not possible to comprehend the formulae and the expressions without having an appreciation of the models used to depict the behaviour of concrete elements. The theory in this book presents the basic models in a simplified form. For a more comprehensive understanding of the theory there are a number of excellent Australian books, many of which develop from first principles the models behind the Code requirements.

The Code itself, AS3600, provides the practicing engineer and student with the current "... minimum requirements for the design and construction of concrete structures and members ...". This book has taken some of the fundamental requirements and related them to practical examples to highlight the use of the Code in the design and analysis of elements.

The design office would be incomplete without computer programs and design aids to assist in those tasks which are often repeated. In today's world, engineers and students do not have the time to develop tools to assist them in their tasks. This book contains a large number of design aids which have been developed to provide practical tools for the solution of problems. Many of these aids have evolved during the development of the book. They have already been tried and tested by students. Some, like the column charts in chapter 13, have undergone numerous minor revisions in layout to produce more readable charts. It is always a fine line between developing a design aid and simply providing an expression in simple terms suitable for calculation.

There have been numerous people involved in the development of this book. There are too many people to name individually. The authors, however, are conscious of a debt to all those who have contributed with material, comments, reading and checking. There is also a hidden contribution by those colleagues and fellow engineers whose knowledge and experience has been absorbed by the authors in their careers. The authors wish to specifically thank Standards Australia in referencing relevant Code requirements.

The responsibility of the material in this book is the authors. It has been checked and rechecked within the confines of the printing deadlines.

Both authors wish to acknowledge the part played by their families in the development of the book. Both families have suffered while the authors spent long nights in front of the computer, writing material, developing charts, writing and running programs and printing and plotting. The authors look forward to returning to their families.

Sydney 1992

Second edition

In the light of the 2001 Concrete Structures Code AS3600 and the new and revised AS1170 Loading Codes it was necessary to revised the book to incorporate the Code revisions.

The 2001 AS3600 Code has incorporated 65 MPa concrete and it is proposed that in the forthcoming major Code revision, high strength concretes will also be included.

As a response to the Concrete Code released in 2001 and in anticipation of the introduction of high strength concrete grades, the authors have included 65, 80, 100 and 120MPa high strength concrete grades for the design of columns. Column design charts are included for all strength grades from 25MPa to 120MPa.

The overall philosophy of the book has been maintained. To simplify the use of design tables and design charts, tables and charts have been added in a separate section at the end of the book. Thus where use of design charts is required in any chapter, only the referenced design chart has been included in the solution of the particular example.

The authors regret that the previous co-author, David Hall, was not available to participate in this revision. They wish to acknowledge his contributions. There are other people who also should be acknowledged and thanked with their direct and indirect contributions and while such a list is not within the scope of this preface, the authors' families were perhaps the main contributors and we thank them sincerely.

Argeo Beletich

Paul Uno

January 2003

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chapter

Limit state conditions, loads and load combinations

1.1 Symbols used in This Chapter

E_u	=	Ultimate strength action.
F_{ep}	=	Load due to earth pressure in kN.
F_{lp}	=	Load due to liquid pressure.
G	=	Concentrated or total dead load.
G^R	=	Dead loads resisting instability.
g	=	Distributed dead load.
M^*	=	Design bending moment due to design loads.
N^*	=	Design axial load.
Q	=	Concentrated or total live load.
q	=	Distributed live load.
E^*	=	Design load (or W*)
w^*	=	Distributed design load.
P^*	=	Concentrated design load.
ϕR	=	Design capacity of structural component.
S_u	=	Ulitimate action due to combination of various actions.
W_s	=	Serviceability wind action.
W_u	=	Ultimate wind action.
#	=	AS3600 Concrete Structures Code reference.
ψ_C	=	Live load combination factor for strength.
ψ_S	=	Short-term live load combination factor for serviceability.
ψ_L	=	Long-term live load combination factor for serviceability.

1.2 General Considerations

The AS 3600 Concrete Structures Code is said to be a Limit State design Code. The terminology is comparatively new with design engineers so that there is still some confusion about limit state design approach. The ultimate strength design procedure has not changed, it is only one of the limit state conditions to be satisfied. A limit state is said to have been reached when a structure or a structural element can no longer satisfy any one of a number of limit state conditions. The limit state conditions to be considered in any design will be:

(a) Stability - a structure must be stable to prevent tilting, sliding or overturning.
(b) Strength - a structure and all its structural components must be strong enough to prevent structural failure.
(c) Serviceability - the structure must be serviceable, i.e. it must be able to perform the functions for which it was designed. Deflection is the main serviceability condition of any design. While excessive deflection may not impair the strength of a structure, it may lead to cracking of masonry walls supported by reinforced concrete members, door and window frames may become sufficiently distorted to cause them to jam and exposed slabs may pond water imposing an additional load on the structure.
(d) Durability - the structure must also have a reasonable service and maintenance free life. There is nothing more disconcerting than to find concrete spalling and the reinforcement corroding shortly after construction. Typical causes of durability failure are due to factors such as insufficient concrete cover, lower concrete strength grade and excessive flexural and shrinkage cracking.
(e) Fire resistance - life and property must be safeguarded against fire. While it is not feasible to design a completely fireproof structure, the structure must be capable of safety withstanding the heat generated by fires for a period which will permit evacuation.

When the resistance or performance of a structure is equal to one of the specified performance conditions, the structure is said to have reached the limit state for that condition. The critical condition is the primary limit state. It is apparent that some of the conditions to be satisfied are not load dependent and hence difficult to evaluate. For those conditions which are load dependent, a margin of safety has to be included in the design calculations.

1.3 Load Factors and Load Combinations

A factor of safety is a very simple concept in principle. It is a safeguard against overloading, underestimating of design loads, negative tolerances in material performance and construction processes which may lead to lower strengths. Loads to be considered in the design include dead loads, live loads, wind loads, snow loads, earthquake loads and forces due to structural performance such as differential settlement of foundations, differential temperature effects, as well as material performance such as creep, shrinkage and elastic shortening. While some loads and forces may be determined with a high degree of confidence, others are much more difficult to estimate. For example, the dead loads to be carried by a structure can be calculated quite accurately while live loads cannot be determined with the same degree of accuracy. A group of students were asked to estimate the floor dead loads and live loads for an office construction; predictable the results for the dead loads varied by 10% from the lowest to the highest estimate while the live loads had a variation of 40%. If a universal factor of safety were to be applied for the combined action of dead and live loads, either the structure would be underdesigned and unsafe or overdesigned and uneconomic.

In limit state design the factor of safety is applied indirectly by way of load factors. The magnitude of the load factor depends on how accurately the various types of loads can be estimated. For strength design conditions, the dead load factor is 1.2 and the live load factor is 1.5 if these are the only loads applied to the structure. The summation of the most adverse factored load combinations is called the “design action effect” or simply the “design load” and it is now given a symbol E* (previously W^*) generally or more specifically a symbol which readily identifies the type of design load with a superscript *. For example w^* is used to designate the uniformly distributed design load, N^* is the design axial load, M^* is the design bending moment, V^* is the design shear force and so on. For strength conditions, if the members are proportioned so that their ultimate strength is equal to or greater than the required design strength, the factor of safety against failure is implied by the load factors used to calculate the design load. If wind loads as well as dead and live loads are to be considered then it is necessary to modify the load factors. This becomes apparent in the case of a non-trafficable roof; it is extremely unlikely that the maximum dead load, the maximum live load and the maximum wind load (which has the probability of occurring once in every 50 years) will all be applied at the same time.

The new Structural Design Actions, AS/NZS 1170.0:2002 General Principles Code (previously included in AS1170.1:1989) gives all the load factors and load combinations to be considered for strength, stability and serviceability. The following sections consider these limit state conditions for dead loads G, live loads Q, wind loads W_u, and other superimposed loads. Loads or examples due to earthquake and prestressing are not included because this book does not deal with earthquake or prestressing. Users of this book are referred to appropriate codes for loads not included.

1.3.1 Load Combinations for Strength Design

The design load E^* is taken as the most severe combination of factored loads determined from the following:

Dead Load Only
(a - Permanent Action Only)

$$E^* = 1.35G$$

Dead Loads and Live Loads
(b - Permanent and Imposed Action)

$$E^* = 1.2G + 1.5Q$$

Dead Loads and Long Term Live Loads
(c - Permanent, Arbitrary-Point-in-Time Imposed Action)

$$E^* = 1.2G + 1.5\,\psi_l\,Q$$

Dead Loads, Live Loads and Wind Loads
(d - Permanent, Arbitrary-Point-in-Time Imposed and Wind Action)

$$E^* = 1.2G + \psi_c\,Q + W_u$$

Dead Loads and Wind Load (Reversal)
(e – Permanent and Wind Action Reversal)

$E^* = 0.9G + W_u$

Dead Loads, Live Loads and Earthquake Loads
(f - Permanent, Arbitrary-Point-in-Time Imposed and Earthquake Action)

$E^* = G + \psi_c\, Q + E_u$

Dead Load, Live Load and combinations of Liquid, Snow, Water & Earth Pressures
(g - Permanent, Arbitrary-Point-in-Time Imposed and Earthquake Action)

$E^* = 1.2G + \psi_c\, Q + S_u$

where ψ_c is the live load combination factor tabulated below.

Table 1.1
Live Load Combination Factor ψc

Type of Live load	Combination Factor ψ_c
Distributed Loads only	
Floors	
Residential & Domestic	0.4
Office	0.4
Parking Area	0.4
Retail Area	0.4
Storage Area	0.6
Other	0.6 Unless Otherwise Assessed
Roofs	
Trafficable	0.4
Non-trafficable	0.0
Concentrated Loads	
Floors & Roofs	
Residential & Domestic	As per UDL above
Roofs	
Non-Trafficable	0.0
Machinery (Long Term)	1.2

1.3.2 Load Combinations for Serviceability

Serviceability conditions in this book refer primarily to deflection of flexural members. Deflection is time dependent because of shrinkage and creep phenomena exhibited by concrete. It is thus necessary to consider both the short & long-term effects.

Short-term or Long-term Serviceability Limit States

(a) G

(b) $\psi_S Q$

(c) $\psi_L Q$

(d) W_s

(e) E_s

(f) Other actions

ψ_S and ψ_L are the short-term and long-term live load combination factors - see table 1.2.

Table 1.2
Short Term and Long Term Combination Factors ψ_s and ψ_L

Imposed Load	Short Term factor ψ_s	Long Term factor ψ_L
	Uniformly Distributed Load (Imposed Action)	
Floors		
Residential & Domestic	0.7	0.4
Offices	0.7	0.4
Parking	0.7	0.4
Retail	0.7	0.4
Storage	1.0	0.6
Other	1.0	0.6
<u>Roofs</u>		
Used for floor activities	0.7	0.4
All other roofs	0.7	0.0
	Concentrated Loads (Imposed Actions including Balustrades)	
Floors	1.0	0.6
Domestic housing floors	1.0	0.4
Roofs used as floors	1.0	0.6
All other roofs	1.0	0.0
Balustrades	1.0	0.0
Long Term Machinery	1.0	1.0

1.3.3 Load Combinations for Stability

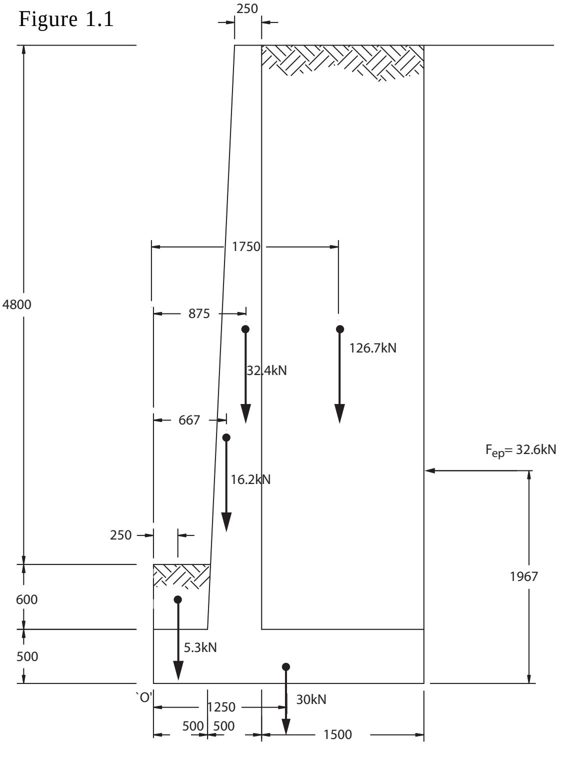
A structure such as a retaining wall must be stable against sliding or overturning. The stability conditions are deemed to be satisfied if;

Combinations that produce a net 'stabilising' effect		Combinations that produce a net 'de-stabilising' effect
0.9 G (i.e. 0.9 x dead load leading to stability)	$\geq$	(i) 1.35 G (i.e. 1.35 x dead load causing instability) or (ii) 1.2 G + 1.5 Q or (iii) 1.2 G + ψ_c Q + W_u or (iv) G + ψ_c Q + E_u or (v) 1.2 G + ψ_c Q + W_u

EXAMPLE 1

Figure 1.1 shows all the component weights and the resultant active earth pressure for a cantilever retaining wall. Check the stability of the retaining wall for overturning and sliding, neglecting the resistive effect of passive earth pressure. A coefficient of friction of 0.55 may be assumed between the base and the sand foundation.

Figure 1.1



(a) For 'Overturning' take moments about pivot point "O" (at the toe of the footing)

0.9 (Restraining Moments) = 0.9 (0.25*5.3 + 0.667*16.2 + 0.875*32.4 + 1.25*30 + 1.75*126.7)

= 269.8 kNm

Overturning moment due to active earth pressure = 1.5 (1.967*1.5*32.6)

= 144.3 kNm

< 269.8 kNm restraining moment

(b) Sliding: Sum of vertical forces = 210.6 kN

0.9 (Sliding resistance) = 0.9*0.55*210.6

= 104.2 kN

Sliding effect due to factored active earth pressure = 1.5*32.6

= 48.9 kN

< 104.2 kN resistive force.

The retaining wall has thus satisfied two of the stability limit state conditions. The designer should check the remaining load combinations. Similarly the designer should check the retaining wall in accordance with AS4678-2002, 'Earth Retaining Structures'.

EXAMPLE 2

A simply supported beam in an office building has a 5.4 m span. The beam is required to carry superimposed dead loads g = 8 kN/m and superimposed live loads q = 12 kN/m. Assuming the weight of the beam is 4.8 kN/m, determine the design loads for (a) strength conditions and (b) the design loads for short-term and long term serviceability conditions.

(a) The distributed design load for strength conditions will be given by:

$$w^* = 1.2g + 1.5q$$
$$= 1.2(8 + 4.8) + 1.5*12$$
$$= 33.4 \text{ kN/m}$$

(b) Short-term Serviceability conditions.

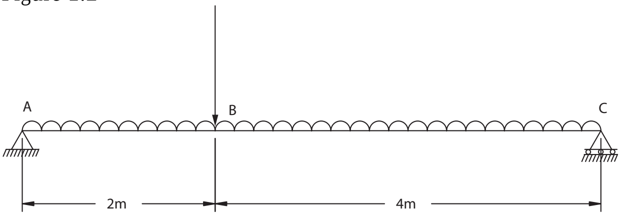
$$w_S = g + \psi_S q$$
$$= (8 + 4.8) + 0.7*12$$
$$= 21.2 \text{ kN/m}$$

Long-term Serviceability conditions.

$$w_L = g + \psi_L q$$
$$= (8 + 4.8) + 0.4*12$$
$$= 17.6 \text{ kN/m}$$

EXAMPLE 3

Figure 1.2



Beam ABC shown in Figure 1.2 supports superimposed uniformly distributed dead load g = 16 kN/m and live load q = 24 kN/m. The beam also supports a concentrated load made up of a 20 kN dead load and 40 kN live load applied at B. Calculate the beam design loads and draw the loading diagram. Using the calculated design loads draw the shear force and bending moment diagrams. The calculations should include estimates for the weight of beam.

Discussion

The weight of concrete beams cannot be neglected as they may constitute between 10% and 15% of the applied service loads. It is common practice to make an initial estimate for the weight of beam. Experienced designers can predict the weight of beam quite accurately while lesser mortals apply various “rules of thumb”. A simple rule used in these notes is;

WEIGHT OF BEAM in (kN/m) = NUMERICAL VALUE OF THE SPAN in (m)

For example a beam spanning 5.6 m, its weight is estimated to be 5.6 kN/m. A check of the beam weight is made when a trial section is chosen in the design but, it will be found that the above rule is conservative in general.

SOLUTION

Estimated weight of beam = 6 kN/m

Uniformly distributed design load $w^* = 1.2(16 + 6) + 1.5*24 = 62.4$ kN/m

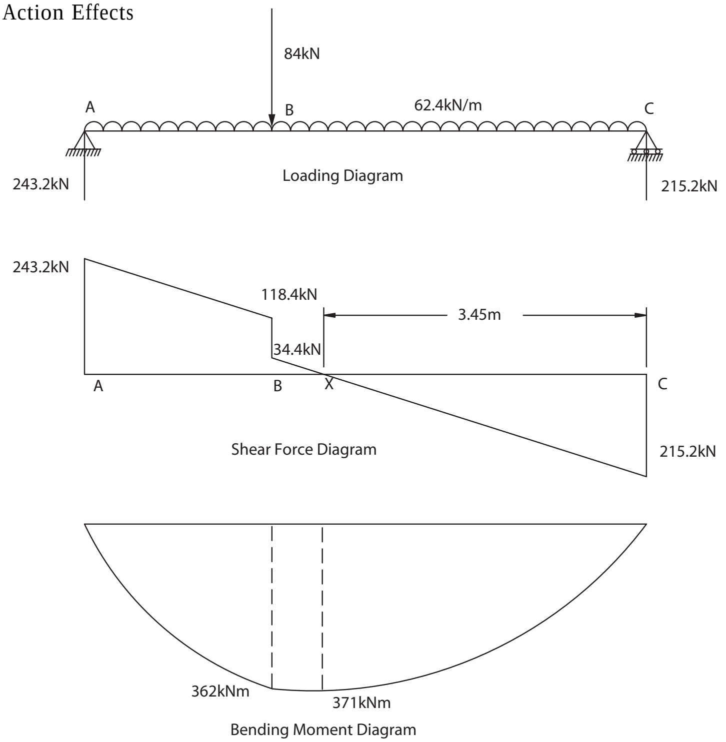
Design concentrated load $W^* = 1.2*20 + 1.5*40 = 84$ kN

The design loading diagram, the shear force diagram and the bending moment diagram are shown in Figure 1.3. The maximum design bending moment M^* at point X is calculated from the shear force diagram.

$$M^* = \frac{3.45 * 215.2}{2} = 371 \quad \text{kNm}$$

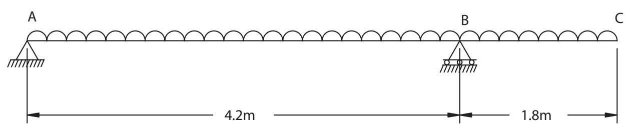
When calculating the load due to self weight, the density of unreinforced concrete is normally taken as 2400 kg/m^3 (23.5 kN/m^3). Since most structures have at least 1% reinforcement (add another 0.63 kN, not allowing for displaced concrete), our minimum self weight of reinforced concrete becomes 24 kN/m^3. If a higher % of reinforcement is present, the self weight should be revised accordingly (refer AS1170.1 Table A1).

Figure 1.3 - Strength Action Effects



Example 4

Figure 1.4



The cantilevered beam shown in Figure 1.4 supports a uniformly distributed dead load g = 20 kN/m which includes the weight of beam and a uniformly distributed live load q = 12 kN/m. Determine the maximum positive and negative design bending moments for the beam.

Discussion

The dead load is a permanent load over the two spans while the live load by its very nature may act over either or both spans. There are four possible loading conditions. Using subscripts to designate the spans over which the loads act, the possible loading conditions are:

(a) g_{ABC}

(b) $g_{ABC} + q_{ABC}$

(c) $g_{ABC} + q_{AB}$

(d) $g_{ABC} + q_{BC}$

SOLUTION

For this simple case the worst conditions can be chosen by inspection. The maximum negative bending moment at B will occur when the cantilever carries a maximum load, i.e. condition (b) and (d).

The design load for the combined dead and live loads is given by:

$$w^* = 1.2*20 + 1.5*12$$
$$= 42 \text{ kN/m}$$

Hence the maximum negative design bending moment at B will be:

$$M^* = -42*\frac{1.8^2}{2} = -68 \text{ kNm}$$

The maximum positive bending moment in span AB will occur when the load over AB is a maximum while the load over the cantilever BC is a minimum i.e. condition (c).

The design load over AB is w^*_{AB} = 42 kN/m due to dead and live loads while the design load over BC due to dead loads only is w^*_{BC} = 1.2*20 = 24 kN/m. The loading diagram is shown in Figure 1.5.

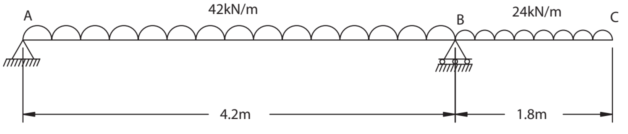


Figure 1.5

The reaction at A is calculated by taking moments about B:

$$R^*_A = \frac{\frac{42*(4.2)^2}{2} - \frac{24*(1.8)^2}{2}}{4.2}$$
$$= 80 \text{ kN}$$

The point of zero shear force occurs 80 / 42 = 1.9 m from A and the maximum bending moment calculated from the area under the shear force diagram will be:

$$M^* = 80\,\frac{1.9}{2}$$
$$= +76 \text{ kNm}$$

It becomes apparent from this example that with multiple spans there can be numerous combinations of dead and live loads applied to various spans. Loading patterns producing maximum bending moments and shear forces in continuous beams will be considered in chapter 9.

PROBLEMS

QUESTION 1

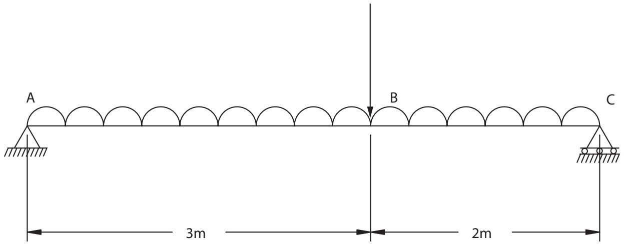
A reinforced concrete column in an office building is required to carry the following axial loads;

Dead Load	G	= 400 kN
Live Load	Q	= 300 kN
Wind Load	W_u	= 360 kN

Determine the column design loads:

QUESTION 2

Beam ABC carries a superimposed uniformly distributed dead load g = 8 kN/m, uniformly distributed live load q = 6 kN/m and a concentrated load applied at B.



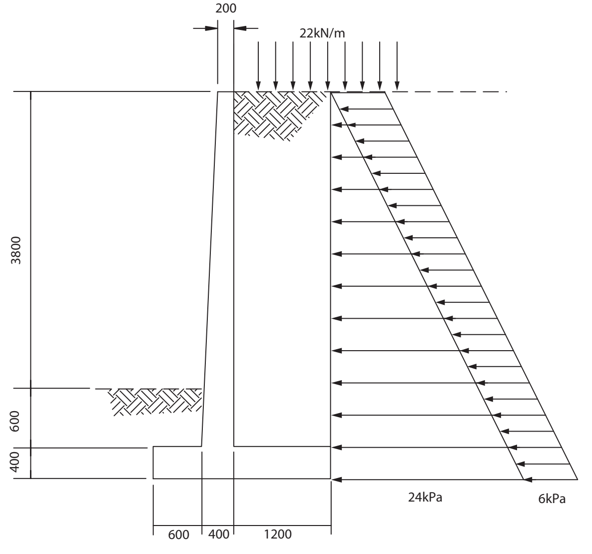
The concentrated load is made up of dead load G = 14 kN and live load Q = 10 kN.

(a) Determine the load action effects for strength design, draw the loading diagram, the shear force diagram and the bending moment diagram.

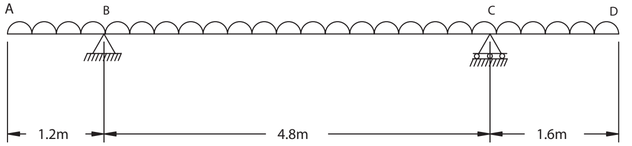
(b) Determine the short-term and the long-term design load actions for serviceability.

QUESTION 3

Check the cantilever retaining wall for overturning stability. Note that a live load surcharge of 22 kN/m is applied on the surface. The soil has a unit weight of 18 kN/m^3. The triangular distribution of active earth pressure varies from zero to 24 kPa and the uniform lateral pressure of 6 kPa is due to the live load surcharge.



QUESTION 4



The double cantilevered beam ABCD carries a total dead load (including its own weight) g = 12 kN/m and a superimposed live load q = 9 kN/m.

Calculate the maximum positive design moment and the maximum negative design moments at the supports.

2

chapter

Singly reinforced concrete beams

2.1 Additional Symbols used in this Chapter

A_{st}	=	Area of tensile reinforcement.
b	=	Width of beam.
C	=	Internal compressive force carried by the concrete.
D	=	Overall depth of beam.
d	=	Effective depth of beam.
ε_c	=	Concrete strain.
ε_s	=	Strain in steel reinforcement.
ε_{sy}	=	Steel strain at the point of yielding.
E_c	=	Modulus of elasticity of concrete.
E_s	=	Modulus of elasticity of steel reinforcement.
f_c	=	An intermediate concrete stress.
f'_c	=	28 day characteristic compressive strength of concrete.
f_{st}	=	An intermediate tensile steel stress within the elastic range.
f_{sy}	=	Yield strength of steel reinforcement.
k_b	=	The value of ku for balanced conditions.
k_u	=	Ratio of depth of NA to beam effective depth d.
L	=	Span of beam between support centrelines.
L_{eff}	=	Effective span of beam L and (Ln + D) or (Ln + D/2) for cantilevers.
L_n	=	Clear span between inner faces of supports or the clear projection of cantilevers.
M_{ud}	=	Reduced ultimate strength in bending for ku = 0.4 condition.
M_{uo}	=	Ultimate strength in pure bending.
NA	=	Neutral axis.
p	=	Reinforcing steel ratio.

p_{max}	=	Maximum tensile steel ratio for k_u = 0.4 condition.
R	=	Radius of curvature.
T	=	Internal resultant tensile steel force carried by the reinforcement.
γ	=	Ratio of depth of simplified rectangular stress block to depth of NA.
Φ	=	Strength reduction factor.
k	=	Curvature.
θ	=	Angle of rotation.

2.2 Material Properties

2.2.1 Concrete

The 1994 of the AS3600 Concrete Structures Code provided for concrete strengths up to 50 MPa but there have been dramatic increases in the strengths of concretes developed in recent years. Concrete strength of 100 and 120 MPa have been readily available for some time. Codes have however not kept up with these rapid developments so that designers using such high concretes strengths have to rely on their own expertise and reliable published research. In the 2001 edition of the AS3600 an additional conrete strength grade with a characteristic strength of 65 MPa was included in the Code and in the proposed review of the Code, three further grades with characteristic strength of 80 MPa, 100 MPa and 120 MPa may be included.

It is now common practice to speak of normal strength concretes (NSC) for strengths up to 50 MPa, high strength concretes (HSC) up to 120 MPa and ultra high strength concretes (UHSC) with strengths exceeding 130 MPa. The standard HSC grades to be provided for in this book are 65, 80, 100 and 120 MPa. On the practical side, use of HSC in flexural members such as beams and slabs are most unlikely as there are no significant advantages; the increase in strength gained with the high strength is minimal. For example, a beam of dimensions b=350, d = 450 reinforced with 4N28mm bars has the following strengths;

Using f'_c = 32 MPa, the design flexural strength ΦM_{uo} = 382 kNm
Using f'_c = 80 MPa, the design flexural strength ΦM_{uo} = 420 kNm

The 80 MPa concrete is 250% stronger than the 32 MPa concrete but, the moment capacity of the beam has only increased by about 20%. It should further be noted that the cost of the 80 MPa concrete is about 50% more expensive then the 32 MPa concrete. In simple terms it may be said that for this particular beam section the unit cost per kNm is 40% more expensive for the 80 MPa concrete compared with the 32 MPa concrete. This is only meant to be a simple illustration as there are many other factors to be considered in carrying out a comparative cost analysis.

On the other hand, use of HSC can appreciably reduce the size of columns and it is in this area where HSC shows significant advantages. The authors have endeavoured to produce a book which is useful to designers and it does not include design material not applicable to designers. It is forseen that NSC will be used in design of all types of flexural members while HSC may be primarily applied to columns in commercial buildings and sporting structures where space is at a premium (or where durability issues dominate the design). HSC concrete design and design aids are only applied to columns in Chapter 13. In the current Code #6.1.1.1 standardised concrete strength grades have been defined whose characteristic strengths f'_c correspond to 20, 25, 32, 40, 50 and 65 MPa. The standard strengths use either normal-class concrete defined in AS1379 "Specification and Supply of Concrete" designated by the prefix N or special class concrete designated by prefix S.

The stress-strain relationship for concrete is not linear as shown in Figure 2.1 (*right*). In the working load range the concrete may reach stresses of approximately $0.5f'_c$. In this range the stress-strain relationship is approximated by a straight line. The slope of the straight line is called the secant modulus of elasticity used to calculate short-term deflections. As shown by Figure 2.1, the stress-strain diagram becomes much more curvilinear as stresses approach the ultimate strength. For gradual long-term strain increment, the stress-strain curve dips below the ultimate strength prior to actual failure.

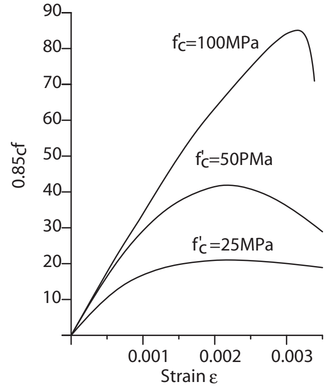


Figure 2.1

2.2.2 Reinforcement

The stress-strain curve for steel is idealised by a continuous yield plateau as shown in Figure 2.2. (*right*) The designer is not interested in the strain hardening part of the curve since concrete and hence member failure will have occurred while the steel reinforcement is in the plastic yield state. The standard grades of reinforcement and the required minimum yield strengths are shown in Table 2.1.

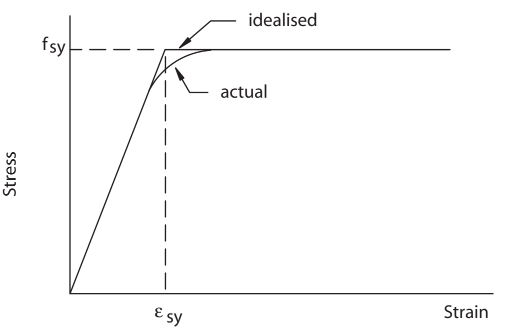


Figure 2.2

Table 2.1 - Standard Grades of Reinforcement

Grade of Reinforcement Type	Grade	Yield Strength f_{sy} in MPa
Plain bars	R250N	250
Deformed bars	D500N	500
Plain & Deformed Hard	R500L &	500
Drawn Wire	D500L	500
Welded Wire Mesh	D500L	500

Table 2.2 - Areas of Reinforcing Bars in mm^2

No. of Bars	Plain R10 Bars	Bar Diameter 12mm	16mm	20mm	24mm	28mm	32mm	36mm
1	80	110	200	310	450	620	800	1020
2	160	220	400	620	900	1240	1600	2040
3	240	330	600	930	1350	1860	2400	3060
4	320	440	800	1240	1800	2480	3200	4080
5	400	550	1000	1550	2250	3100	4000	5100
6	480	660	1200	1860	2700	3720	4800	6120
7	560	770	1400	2170	3150	4340	5600	7140
8	640	880	1600	2480	3600	4960	6400	8160
9	720	990	1800	2790	4050	5580	7200	9180
10	800	1100	2000	3100	4500	6200	8000	10200
11	880	1210	2200	3410	4950	6820	8800	11220
12	960	1320	2400	3720	5400	7440	9600	12240
13	1040	1430	2600	4030	5850	8060	10400	13260
14	1120	1540	2800	4340	6300	8680	11200	14280
15	1200	1650	3000	4650	6750	9300	12000	15300
16	1280	1750	3200	4960	7200	9920	12800	16320
17	1360	1870	3400	5270	7650	10540	13600	17340
18	1440	1980	3600	5580	8100	11160	14400	18360
19	1520	2090	3800	5890	8550	11780	15200	19380
20	1620	2200	4000	6200	9000	12400	16000	20400

Properties of standard welded wire mesh are shown in Table 2.3

Table 2.3 - Standard welded wire mesh

Ref. No.	Area mm^2/m		Longitudinal Wire		Cross Wire	
	Longitudinal Wires	Cross Wires	Size(mm)	Pitch	Size(mm)	Pitch
RL1218	1112	227	11.9	100	7.6	200
RL1118	891	227	10.65	100	7.6	200
RL1018	709	227	9.5	100	7.6	200
RL918	574	227	8.6	100	7.6	200
RL818	454	227	7.6	100	7.6	200
RL718	358	227	6.75	100	7.6	200
SL81	454	454	7.6	100	7.6	100
SL102	354	354	7.6	200	7.6	200
SL92	290	290	8.6	200	8.6	200
SL82	227	227	7.6	200	7.6	200
SL72	179	179	6.75	200	6.75	200
SL62	141	141	6.0	200	6.0	200
SL52	89	89	4.75	200	4.75	200
SL42	63	63	4	200	4	200

2.3 Ultimate Strength Theory

2.3.1 Assumptions

The following assumptions are made in the derivation of strength formulas:

(a) All the concrete on the tension side of the neutral axis (NA) is cracked and it does not contribute to the moment capacity of the beam.

(b) Strain is linear i.e. it is directly proportional to the distance from the NA for all moments up to and including the ultimate bending moment M_{uo}.

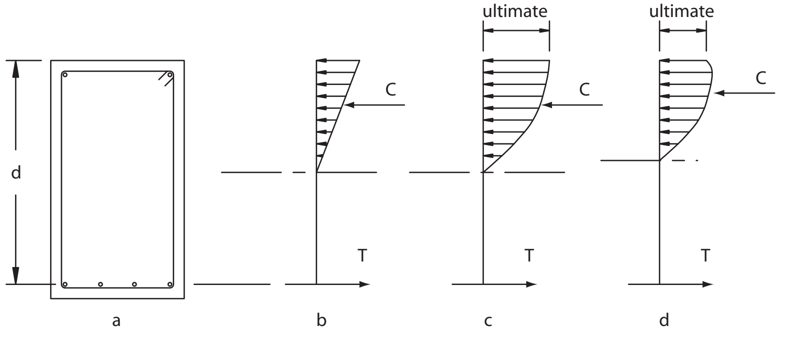
(c) Concrete has crushed when the maximum strain in the concrete section has reached a value of 0.003, i.e. $\varepsilon_c = 0.003$.

(d) The modulus of elasticity for the steel reinforcement is $E_s = 2 \times 10^5$ MPa.

2.3.2 Ultimate Strength Conditions

When a small moment is applied to a reinforced concrete beam, the stress distribution above the NA is almost linear as shown in Figure 2.3b. Gradual increase in the applied moment will cause the stress distribution to become distinctly curvilinear until the maximum stress at the outer fibre has reached the ultimate strength of concrete as shown in Figure 2.3c. At this point the concrete section has not failed since most of the concrete section in compression is stressed below the ultimate strength of concrete. The reinforcing area or the tensile steel ratio is chosen so that the steel has reached its yield capacity at about the time that the maximum outer concrete stress is equal to its ultimate capacity. The tensile force carried by the reinforcement, given by $T = A_{st}f_{sy}$ has reached its limit and it is balanced by the resultant compressive force C carried by the concrete. Internal forces C and T form a couple to balance the externally applied moment. But the concrete has not yet failed; the reinforced concrete section can sustain a larger moment. Increasing the external moment will require an increase in the internal couple but, since the magnitude of the internal forces T and C have reached their limit because the reinforcement is yielding and the tensile force T cannot exceed $A_{st}f_{sy}$. The internal lever arm is increased by shifting C towards the outer fibre and hence reducing the depth of the NA. If the depth of the NA is reduced, a smaller area of concrete is in compression and to maintain internal equilibrium C = T, the concrete stresses must be increased. A greater area of concrete will be subjected to the ultimate stress conditions leading to eventual failure. The stress distribution at failure is depicted by Figure 2.3d which is similar to the stress-strain curve shown in Figure 2.1.

Figure 2.3



The following beam conditions are identified by their ultimate strength actions:

(a) Under-reinforced beams in which the area of tensile reinforcement and hence the steel ratio is such that all the tensile reinforcement will have yielded prior to crushing of the concrete.

(b) Balanced beams in which simultaneous yielding of the tensile reinforcement and crushing of the concrete will occur. Balanced conditions are idealised conditions.

(c) Over-reinforced beams. These are heavily reinforced beams whose brittle failure is due to crushing of the concrete while the steel reinforcement is stressed below its yield strength.

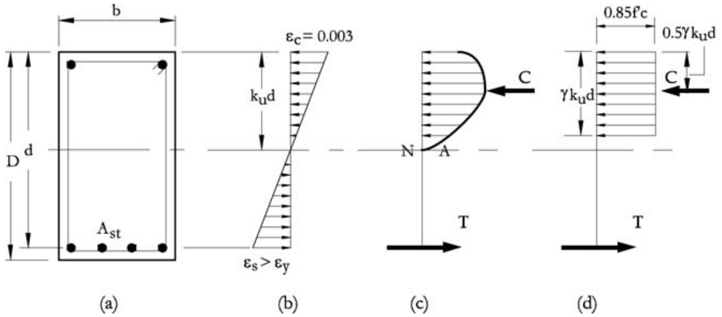
Under-reinforced beams are said to behave in a ductile manner. It is possible to imagine that once the reinforcement has yielded, it continues to elongate rapidly. Since this occurs prior to crushing of the concrete and hence beam failure, very pronounced beam deflection can be observed while the beam is still able to carry the applied loads. The exaggerated beam deflection gives ample warning of impending failure and the ductile beam behaviour allows a redistribution of moments in indeterminate structures. Similar "plastic" behaviour is observed in steel structures.

Over-reinforced beams exhibit brittle failures since concrete, a brittle material, crushes while the reinforcement is still in its elastic stress range. Over-reinforced beams give no warning of failure which is sudden and catastrophic. While over-reinforced beams are stronger, there are no advantages in designing over-reinforced beams because the Code discourages the design of over-reinforced beams by imposing certain design penalties to be considered later.

Balanced beam condition is only a yard stick used to differentiate between under and over reinforced beams.

2.3.3 Derivation of Basic Equations

Figure 2.4



Considering an under-reinforced concrete beam on the verge of failure, Figure 2.4b shows the linear strain diagram with a maximum concrete strain $\varepsilon_c = 0.003$ as defined earlier. Figure 2.4c represents the actual stress diagram which acting on the beam produce a resultant tensile force $T = A_{st}f_{sy}$ and a resultant compressive force of magnitude C equal to the tensile force T. To determine the internal moment of resistance, it will be necessary to evaluate the internal lever arm between T and C. The difficulty becomes apparent in locating the position of C. The compressive stress distribution called the “stress block” has been described by a number of researchers. In Chapter 13 the equation used is the CEB (Comite Europeen du Beton) which is quite a complex equation. For the purposes of determining expressions for the depth of the NA, k_ud and the ultimate moment capacity M_{uo} it is not necessary to evaluate or know the actual stress distribution if a simple empirical method can be applied to determine the magnitude and location of C. One such method accepted by the Code is to replace the actual stress block by a rectangular stress block of uniform stress $0.85f'_c$ and a depth γk_ud. This simplified rectangular stress block is sometimes called the Whitney stress block after the engineer who initially proposed it. The size and shape of the empirical stress block is not important (a triangular stress block could just as easily have been chosen) provided that the magnitude and location of C coincides with that produced by the actual stress block. The rectangular stress block which is universally recognised will be used throughout these notes except for the derivation of the column design charts in Chapter 13.

The value of γ is given in the Code #8.1.2.2 as follows;

$$\gamma = 0.85 \qquad \text{for } f'c \le 28 \text{ MPa}$$

$$\gamma = 0.85 - 0.007(f'c - 28) \qquad \text{for } f'c \ge 28 \text{ MPa}$$

$$\ge 0.65$$

Using the simplified rectangular stress block, the resultant force C is equal to the uniform stress of $0.85f'_c$ acting over a rectangular area of the beam cross-section width b and depth γk_ud, i.e.,

$$C = 0.85f'_c b\gamma k_u d$$

Equating the internal forces (equilibrium condition) C = T and solving for k_u:

$$0.85f'_c b\gamma k_u d = A_{st}f_{sy}$$

$$\boxed{k_u = \frac{1}{0.85\gamma}\left(\frac{A_{st}}{bd}\right)\left(\frac{f_{sy}}{f'_c}\right) \qquad \textbf{(2.1)}}$$

This is the general formula applied to any under-reinforced beam.

For the specific case of balanced conditions the reinforcement has just yielded when the concrete has failed. The depth of the NA for the balanced beam is k_bd. From the geometry of the strain diagram Figure 2.4b we can derive an expression for the depth of the NA in which k_bd replaces k_ud. Thus from similar triangles,

$$\frac{\varepsilon_c}{k_bd} = \frac{\varepsilon_s}{d - k_bd}$$

Multiplying throughout by d,

$$\frac{\varepsilon_c}{k_b} = \frac{\varepsilon_s}{1-k_b}$$

$$\varepsilon_c - \varepsilon_c k_b = \varepsilon_s k_b$$

$$\varepsilon_c k_b + \varepsilon_s k_b = \varepsilon_c$$

$$k_b = \frac{\varepsilon_c}{\varepsilon_c + \varepsilon_s}$$

The steel strain at the point of yielding $\varepsilon_s = f_{sy}/E_s = f_{sy}/2*10^5$ and the concrete strain at failure was specified to be $\varepsilon_c = 0.003$ in the initial assumptions. Thus substituting for ε_c and ε_s in the above expression for k_b gives;

$$k_b = \frac{0.003}{0.003 + \dfrac{f_{sy}}{2*10^5}}$$

$$k_b = \frac{600}{600 + f_{sy}}$$

Most reinforced concrete beams use grade 500N deformed reinforcing bars whose yield strength f_{sy} = 500 MPa. Substituting 500 for f_{sy} in the above equation gives $k_b = 0.545$.

2.3.4 Maximum Value of k_u

The value $k_b = 0.545$ is the limiting or maximum value of k_u for under-reinforced or ductile beams. When $k_u = 0.545$ there is no guarantee that all the reinforcement has yielded; with multiple rows of reinforcement, the reinforcement in the top row may still be in the elastic range. Even with one row of reinforcement, a designer aiming at a value of $k_u = 0.545$ may quite easily end up with an over-reinforced beam by choosing reinforcement whose area is greater than the theoretical area required to give a value of $k_u = 0.545$. The Code #8.1.3 specifies a maximum value of $k_u = 0.4$ which is just under 3/4 of that for a balanced beam. This is to ensure that beams will behave in a ductile manner at ultimate moment conditions.

Maximum $k_u = 0.4$	**(2.2)**

Over-reinforced beams can be made to behave in a ductile manner by the introduction of compressive reinforcement. The addition of compressive reinforcement will increase the internal compressive force and hence the internal tensile force due to increased tensile stresses. Provided that sufficient compressive reinforcement has been included, the tensile reinforcement will reach its yield stress. Beams reinforced with compressive reinforcement as well as tensile reinforcement are referred to as doubly reinforced beams dealt with in chapter 5. #8.1.3 of the Code permits the design of beams whose neutral axis is located so that $k_u > 0.4$ provided that the ultimate moment is reduced to M_{ud} which is the ultimate strength for $k_u = 0.4$ and, a minimum amount of compressive reinforcement is added to the beam given by:

$$\text{Minimum } A_{sc} = 0.01bk_ud \qquad \textbf{(2.3)}$$

There is no strength advantage in designing beams whose $k_u > 0.4$.

2.3.5 Maximum Steel Ratio p_{max}

The steel ratio to satisfy the Code condition for k_u given by equation 2.2 may be determined by equating 2.1 and 2.2.

$$0.4 = \frac{1}{0.85\gamma}\left(\frac{A_{st}}{bd}\right)\left(\frac{f_{sy}}{f'_c}\right)$$

$\dfrac{A_{st}}{bd}$ is the steel ratio p which becomes the maximum steel ratio p_{max} when $k_u = 0.4$.

$$0.4 = \left(\frac{1}{0.85\gamma}\right) p_{max} \left(\frac{f_{sy}}{f'_c}\right)$$

Solving for p_{max} gives:

$$p_{max} = 0.34\gamma\left(\frac{f'_c}{f_{sy}}\right) \qquad \textbf{(2.4)}$$

2.3.6 Beam Ductility

The ductile behaviour of under-reinforced beams was referred to earlier. Having derived some beam relationships, it is now possible to evaluate beam ductility. A ductile beam with a small steel ratio deforms substantially at failure whereas an over-reinforced beam with a large steel ratio exhibits very small deformations right up to failure. A measure of deformation is not the amount of deflection but the curvature of the member usually given the symbol κ. A small deformation has a very large radius of curvature while a large deformation has a small radius curvature. Deformability, curvature or ductility is the inverse of radius of curvature, i.e. 1/R, which we will simply call curvature.

Figure 2.5 shows two similar size beams; beam 'A' is lightly reinforced (small steel ratio) and beam 'B' is heavily reinforced (high steel ratio). Since the depth of the neutral axis k_ud given by equation 2.1 is directly proportional to the steel ratio p (= A_{st}/bd), beam 'A' has a smaller steel ratio p and a smaller depth of neutral axis k_ud and it is the more ductile beam. It may be supposed that for a given size beam, the inverse of the steel ratio 1/p or the inverse of the depth of neutral axis $1/k_ud$ is also a measure of ductility.

Figure 2.5

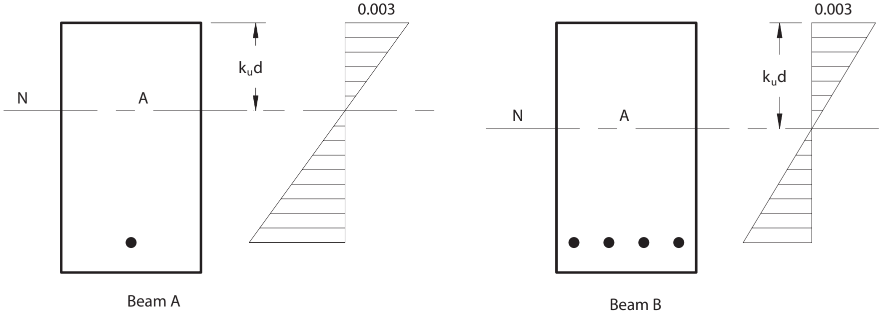


Figure 2.6 will be used to derive beam curvature or ductility relationships. The figure shows a small length x of a deformed reinforced concrete beam on the point of failure (note that the size of the element and its deformed shape have been greatly exaggerated).

The amount of shortening in the top fibre is $x\varepsilon_c$ making the length of the top fibre equal to $x - x\varepsilon_c$ or $x(1 - \varepsilon_c)$ where ε_c is the concrete strain. The compressed length of the top fibre in Figure 2.6 may be calculated from,

$$(R - k_u d)\theta = x(1 - \varepsilon_c)$$

Solving for θ gives:

$$\theta = \frac{x(1 - \varepsilon_c)}{R - k_u d} \qquad \text{(a)}$$

The angle of rotation θ may also be obtained from the geometry of Figure 2.6 as:

$$\theta = \frac{x}{R} \qquad \text{(b)}$$

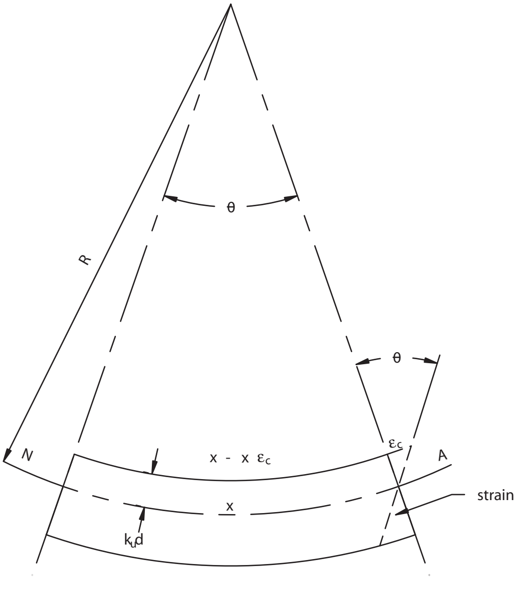


Figure 2.6

Equating the two expressions for θ given by (a) and (b) and solving for curvature 1/R.

$$\frac{x}{R} = \frac{x(1 - \varepsilon_c)}{R - k_u d}$$

$$xR - xk_u d = Rx - Rx\varepsilon_c$$

$$\text{Curvature} = \frac{1}{R} = \frac{\varepsilon_c}{k_u d} \qquad \text{(c)}$$

Since the strain ε_c at failure is taken as a constant of 0.003, curvature is inversely proportional to the depth of the neutral axis as anticipated earlier. From equation 2.1,

$$\frac{1}{k_u} = 0.85\gamma \frac{1}{p} \frac{f'_c}{f_{sy}}$$

Substituting for ε_c and $1/k_u$ in (c), the equation for curvature may be written,

$$\text{Curvature} = \frac{1}{R} = \frac{0.003}{d} \; 0.85\gamma \frac{f'_c}{f_{sy}} \frac{1}{p} \qquad \text{(d)}$$

For a given beam, Curvature = Constant * $\frac{1}{p}$

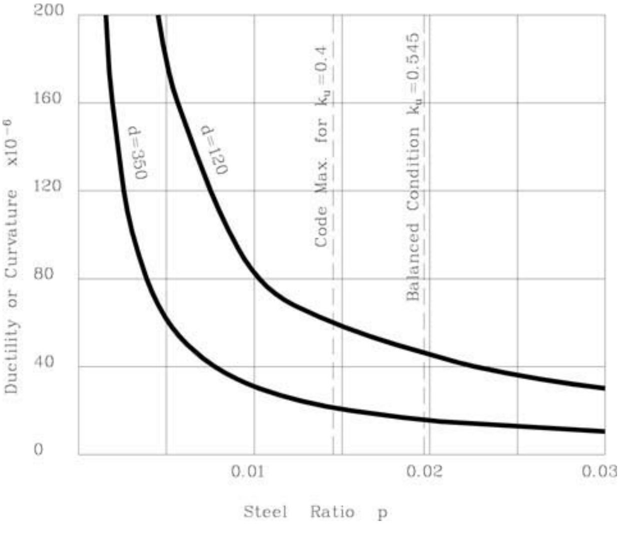
A plot of curvature at collapse or ductility versus the steel ratio for a slab d = 120mm and a beam d = 350mm using f'_c = 25 MPa is shown in Figure 2.7.

The following points may be noted from the plots illustrated by Figure 2.7 (*below*):

(a) The decrease in ductility with increasing depth.

(b) The rapid decrease in ductility with increasing steel ratio.

(c) When the steel ratio reaches the maximum p_{max} corresponding to the Code limit k_u = 0.4, beam ductility approaches the flat portion of the curve, a justification for the limiting k_u.



2.3.7 Ultimate Moment Capacity M_{uo}

Referring to Figure 2.4, the ultimate moment capacity is the moment provided by the couple of the internal forces C and T. Taking moments about C gives,

$$M_{uo} = T(d - 0.5\gamma k_u d)$$

$$= A_{st}f_{sy}(d - 0.5\gamma k_u d)$$

$$= A_{st}f_{sy}d(1 - 0.5\gamma k_u)$$

Substituting the expression for k_u derived by equation 2.1,

$$M_{uo} = A_{st}f_{sy}d\left(1 - 0.5\gamma\frac{1}{0.85\gamma}\frac{A_{st}}{bd}\frac{f_{sy}}{f'_c}\right)$$

$$M_{uo} = A_{st}f_{sy}d\left(1 - \frac{1}{1.7}\frac{A_{st}}{bd}\frac{f_{sy}}{f'_c}\right) \qquad \textbf{(2.5)}$$

The fraction 1/1.7 is sometimes rounded off to decimal 0.6. Equation 2.5 may be simplified by introducing a new symbol z defined by:

$$z = \frac{A_{st}}{bd}\frac{f_{sy}}{f'_c} \qquad \textbf{(2.6)}$$

Making $A_{st}f_{sy} = zbdf'_c$. Sustituting for $A_{st}f_{sy}$ and $\frac{A_{st}}{bd}\cdot\frac{f_{sy}}{f'_c}$ in equation 2.5 gives,

$$M_{uo} = zbdf'_c d\,(1 - \tfrac{z}{1.7})$$

Which is written in the form,

$$M_{uo} = f'_c\, z\left(1 - \frac{z}{1.7}\right)bd^2 \qquad \textbf{(2.7)}$$

2.4 Moment Capacity ΦM_{uo}

Equations 2.5 and 2.7 give the ultimate moment capacity of singly reinforced, under-reinforced beams. These are theoretical values which assume that all physical conditions have been met. Practically it is necessary to make allowances for the possible cummulative adverse effects brought about by:

(a) Variations in concrete strength due to inconsistances in batching, mixing, transporting, compacting and curing of concrete.

(b) Dimensional tolerances in setting up formwork.

(c) Variations in positioning of reinforcement.

The combined negative combination of these conditions may result in a member strength well below that predicted by equation 2.5 or 2.7. Code #2.3 requires that the ultimate moment capacity M_{uo} be reduced to ΦM_{uo} where Φ is called the strength reduction factor. Values of Φ for various strength conditions given by the Code are duplicated in table 2.4. For bending, the reduction factor $\Phi = 0.8$.

Table 2.4 - Strength Reduction Factors Φ

Type of Action Effect	Strength Reduction Factor Φ
(a) Axial force without bending	
(i) tension	0.8
(ii) compression	0.6
(b) Bending without axial tension or compression where:	
(i) ku $\leq$ 0.4	0.8
(ii) ku > 0.4	$\frac{0.8M_{ud}}{M_{uo}} \geq 0.6$
(c) Bending with axial tension	$\Phi + (0.8 - \Phi)\left(\frac{N_u}{N_{uot}}\right)$
	the value of Φ is obtained from (b)
(d) Bending with axial compression where:	
(i) Nu $\geq$ Nub	0.6
(ii) Nu < Nub	$0.6 + (\Phi - 0.6)\left(1-\frac{N_u}{N_{ub}}\right)$
	the value of Φ is obtained from (b)
(e) Shear	0.7
(f) Torsion	0.7
(g) Bearing	0.6
(h) Compression and axial tension in strut and tie action	0.7
(i) Bending shear and compression in plain concrete	0.7
(j) Bending shear and tension in fixings	0.6

ΦM_{uo} is simply called the moment capacity or the effective moment capacity which must at all times be equal to or greater than the design moment M^*.

$$\Phi M_{uo} \geq M^* \qquad \textbf{(2.8)}$$

EXAMPLE 1

The beam cross-section shown in Figure 2.8 (*right*) uses N25 grade concrete and it is reinforced with 3N24 tension bars.

(a) Show that the beam is under-reinforced and calculate the depth of the neutral axis.
(b) Calculate the ultimate and the effective moment capacity.

Data:
b = 300 mm d = 350 mm
f'_c = 25 MPa f_{sy} = 500 MPa
A_{st} = 1350 mm^2

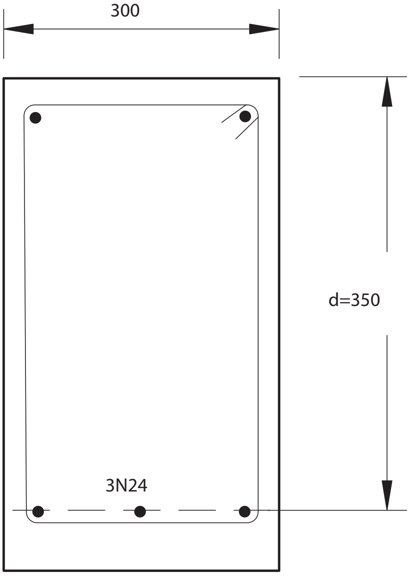


Figure 2.8

SOLUTION

(a) $f'_c < 28$ hence $\gamma = 0.85$.

Steel ratio, $p = \dfrac{A_{st}}{bd} = \dfrac{1350}{300 \times 350} = 0.0129$

From equation 2.1, $k_u = \dfrac{1}{0.85\gamma}\dfrac{A_{st}}{bd}\dfrac{f_{sy}}{f'_c} = \dfrac{1}{0.85\gamma}\,p\,\dfrac{f_{sy}}{f'_c}$

$$k_u = \frac{1}{0.85 * 0.85} 0.0129 \frac{500}{25}$$
$$= 0.357$$
$$< \text{Code maximum of } 0.4$$

The same condition could have been checked by comparing the actual steel ratio with the maximum steel ratio corresponding to $k_u = 0.4$.

$$p_{max} = 0.34\gamma\frac{f'_c}{f_{sy}} = 0.34 * 0.85 \times \frac{25}{500} = 0.0145$$
$$> 0.0129 \text{ the actual steel ratio.}$$

Depth of neutral axis = $k_u d = 0.357 * 350 = 125$ mm

(b) From equation 2.6, $z = p\dfrac{f_{sy}}{f'_c} = 0.0129 * \dfrac{500}{25} = 0.258$

Substituting in equation 2.7 for the ultimate moment capacity,

$$M_{uo} = f'_c z\left(1-\frac{z}{1.7}\right)bd^2$$

$$= 25*0.258\left(1-\frac{0.258}{1.7}\right)*300*350^2*10^{-6}$$

$$= 201 \text{ kNm}$$

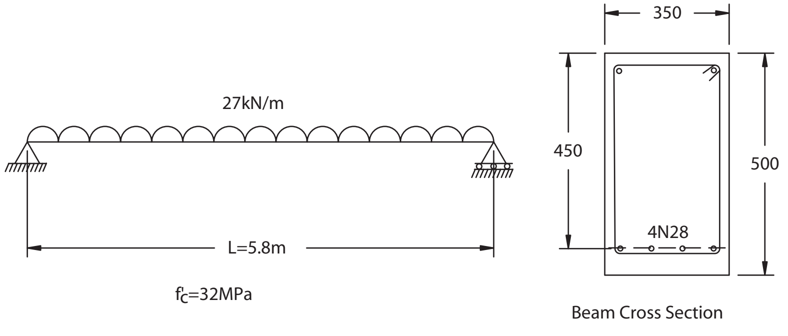
Note that in the equation for M_{uo} all the units are in mm and N, so that the moment will be in Nmm. The multiplier 10^{-6} is included to convert the moment to conventional kNm units.

Effective moment capacity, ΦM_{uo} = 0.8*201 = 160 kNm. The value of 0.8 for Φ was obtained from table 2.4.

EXAMPLE 2

The beam shown in Figure 2.9 carries a superimposed uniformly distributed dead load g = 27 kN/m. Determine the maximum distributed live load which may be applied to the beam.

Figure 2.9



Data:

b = 350 mm	d = 450 mm	D = 500 mm
A_{st} = 2480 mm^2	f'_c = 32 MPa	f_{sy} = 500 MPa

SOLUTION

$f'_c > 28$, therefore $\gamma = 0.85 - 0.007(32 - 28) = 0.822$

Maximum steel ratio, $p_{max} = 0.34\gamma\left(\frac{f'_c}{f_{sy}}\right) = 0.34*0.822\left(\frac{32}{500}\right) = 0.0179$

Actual steel ratio, $p = \left(\frac{2480}{350*450}\right) = 0.0157 < p_{max}$

From equation 2.6, $z = p\left(\frac{f_{sy}}{f'_c}\right) = 0.0157\left(\frac{500}{32}\right) = 0.245$

Moment capacity, ΦM_{uo} $= \Phi f'_c z\left(1-\frac{z}{1.7}\right)bd^2$

$$= 0.8*32*0.245\left(1-\frac{0.245}{1.7}\right)350*450^2*10^{-6}$$

$$= 380 \text{ kNm}$$

The Code condition $\Phi M_{uo} \geq M^*$ is satisfied for the maximum design distributed load w^* when,

$$\frac{w^*L^2}{8} = 380$$

$$w^* = 8*\frac{380}{5.8^2}$$

$$= 90.4 \text{ kN}$$

This is a design distributed load of 90.4 kN/m which would cause the design moment to be equal to the effective moment capacity ΦM_{uo}. Using 24 kN/m^3 for the weight of concrete (assuming 1% steel), the weight of beam = 0.35*0.5*24 = 4.2 kN/m.

Total dead load g = 27 + 4.2 = 31.2 kN/m.

The design load for the factored dead and live loads,

w^* = 1.2*31.2 + 1.5*q

= 39.3 + 1.5q

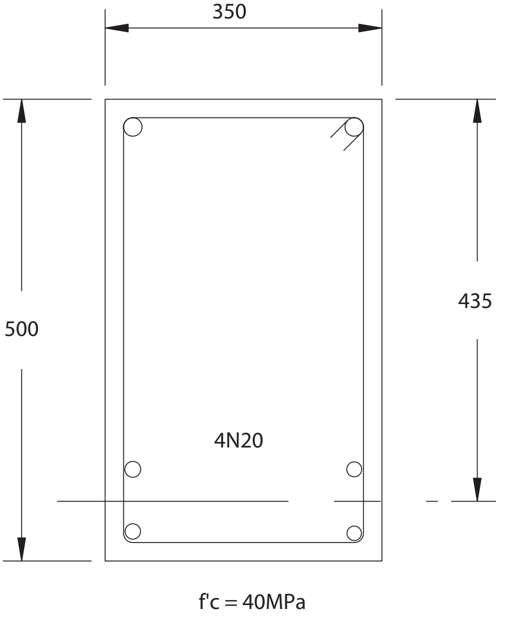
Equating the two values for w^* and solving for the live load q,

$$q = \frac{90.4-39.3}{1.5} = 35.3 \text{ kN/m}$$

That is, the maximum distributed live load which may be applied to the beam is 35.3 kN/m.

PROBLEMS

QUESTION 1



(a) Calculate the effective moment capacity for the beam section shown in figure.

(b) Determine the maximum number of N20 bars which may be added to the beam so that Code condition $k_u \leq 0.4$ is still satisfied and calculate the new effective moment capacity. Note that the reinforcing bars should be kept in two rows placed symmetrically about the vertical axis and they should not be staggered.

(c) Calculate the value of ku for the reinforcement chosen in (b) and use the strain diagram to show that all the reinforcement has yielded for the calculated moment capacity.

(d) The beam in (b) has an effective span L_{eff} = 6.5 m. What is the maximum superimposed dead load which can be applied to the beam if it is required to carry a 24 kN/m distributed live load.

QUESTION 2

Values of γ and the maximum steel ratio p_{max} corresponding to k_u are frequently required in calculations. Complete the following table by calculating p_{max} and γ (rounded off to four decimal places) for the standard grades of concrete.

f_c	f_{sy}	γ	k_u	p_{max}
20	500		0.4	
25	500		0.4	
32	500		0.4	
40	500		0.4	
50	500		0.4	
65	500		0.4	

3

chapter

Beam design

3.1 Additional Symbols used in this Chapter

A_{st}	=	Area of tensile reinforcement.
a	=	Concrete cover or half the distance between parallel bars.
b_{min}	=	Minimum beam width for a given exposure classification.
c_{min}	=	Minimum distance from centroid of reinforcement to exposed concrete face required to satisfy exposure conditions.
f'_{cf}	=	Characteristic flexural strength of concrete.
$M_{uo\ min}$	=	Minimum strength in bending at a critical cross section.

3.2 Durability and Concrete Cover

Durability is one of the primary conditions to be satisfied in any design. It is also one of the limit states. Structures are normally designed for an average life span of 50 years although, some structures such as public monuments are designed for longer life spans while temporary structures may be designed for very short life spans. The designer is aware that what starts out as a temporary structure, often turns out to be a permanent structure; the Eiffel Tower is a classic example. Deterioration of concrete structures during their life span is of primary importance and it may be due to:

(a) Corrosion of the reinforcement and spalling of concrete due to insufficient cover for the degree of imperviousness and aggressiveness of the environmental conditions to which the concrete is exposed.

(b) Chemical or physical breakdown and loss of concrete section caused by direct chemical attack, salt water spray, cycles of freezing and thawing etc. The loss of concrete will reduce cover over the reinforcement and lead to accelerated corrosion of reinforcement.

It does not require the services of an investigative reporter to show that far too many buildings exhibit some signs of deterioration soon after completion. The description “concrete cancer” has been applied to describe the deterioration of concrete structures in major industrial centres. Large sums of money amounting to many millions of dollars are being spent annually on repairs of concrete structures which have deteriorated at a rate not anticipated in their design and construction.

There are many explanations but, one of the principal reasons for reduced durability is lack of adequate concrete cover due to poor design, detailing, construction and supervision. The Code has devoted the whole of section four to the minimum conditions required to satisfy durability. It is an endeavour by the Code Committee to identify the causes and recommend minimum design and construction procedures. The results are; increased concrete strength and increased cover over the reinforcement. The consequences of the recommendations will be more durable although more expensive structures initially. The following section is only concerned with concrete strength and concrete cover.

3.2.1 Concrete Cover for Exposure Classifications

Table 3.1 - Exposure Classifications

(i.e. Table 4.3 from AS3600)

Surface and Exposure Environment	Exposure Classification
1. SURFACES OF MEMBERS IN CONTACT WITH THE GROUND	
(a) Members protected by damp-proof membrane.	A1
(b) Residential footings in non-aggressive soils.	A1
(c) Other members in non-aggressive soils.	A2
(d) Members in aggressive soils.	U
2. SURFACES OF MEMBERS IN INTERIOR ENVIRONMENT	
(a) Fully enclosed within a building except for a brief period of weather exposure during construction.	A1
(b) In industrial buildings, the member being subjected to repeated wetting and drying.	B1
3. SURFACES OF MEMBERS IN ABOVE-GROUND EXTERIOR ENVIRONMENT	
In areas that are;	
(a) Inland (> 50 km from coastline) environment;	
(i) Non-industrial & arid climate.	A1
(ii) Non industrial and temperate climate.	A2
(iii) Non-industrial and tropical climate.	B1
(iv) Industrial and any climate.	B1
(b) Near-coastal (1 km to 50 km from coastline)and any climatic zone.	B1
(c) Coastal (up to 1 km from coastline but excluding tidal and splash zones) and any climatic zone.	B2
4. SURFACES OF MEMBERS IN WATER	
(a) In fresh water.	B1
(b) In sea water -	
(i) permanently submerged.	B2
(ii) in tidal or splash zones.	C
(c) In soft running water.	U
5. SURFACES OF MEMBERS IN OTHER ENVIRONMENTS	
Any exposure environment not otherwise described in items 1 to 4	U

To give the designer a better opportunity to identify the risk of corrosion of the reinforcement, the Code has classified exposures in ascending order of severity. Exposure classifications are designated as A1, A2, B1, B2, and C. Exposure conditions leading to these classifications are described in Table 3.1.

Note:

1 Climatic zones referred to in table 3.1 are shown in Code Figure 4.3

2 Industrial refers to areas within 3 km of industries which discharge atmospheric pollutants.

3 Coastal zones include locations <1 km from shorelines of large expanses of salt water

4 Designations U are undefined and to be determined by the designer.

Well compacted, properly cured concretes are stronger and less porous. The Code requires a minimum curing period and a minimum strength of normal grade concretes for each exposure classification. These minimum requirements are shown in Table 3.2. This is an expedient way of ensuring that stronger and hence less porous concretes are progressively used with increasing severity of exposure.

Table 3.2 - Minimum Strength and Curing Periods

Exposure classification	Minimum Characteristic Strength	Minimum Curing Period	Strength After Minimum Curing Period
A1	20 MPa	3 days	15 MPa
A2	25 MPa	3 days	15 MPa
B1	32 MPa	7 days	20 MPa
B2	40 MPa	7 days	25 MPa
C	50 MPa	7 days	32 MPa

The minimum concrete covers for standard formwork and compaction are shown in Table 3.3 below. For rigid formwork and intense compaction (e.g. precast concrete members using steel forms and form vibrators or vibrating tables) the reader is referred to Code Table 4.10.3.4. The Code does make some concessions when only one surface is externally exposed. In such circumstances, the next lower grade of concrete may be used provided that the cover for that surface is increased by 20 mm if standard formwork and compaction are applied. The increased covers for standard formwork and compaction are shown bracketed in Table 3.3 for each exposure classification.

If concrete is cast against the ground as in footings, concrete cover must be increased by 10 mm if the concrete surface is protected by a damp-proof membrane or by 20 mm otherwise.

Note: Bracketed figures are the appropriate covers for single exterior surfaces when concession relating to the lower strength grade is permitted e.g. exterior surface classification B1, interior surface classification A1, reduced concrete strength grade N25 may be used but cover must be increased from 40 mm to 60 mm.

Table 3.3 - Minimum Cover for Standard Formwork and Compaction

Exposure Classification	Required Cover in mm Characteristic Strength f_c'				
	20 MPa	25 MPa	32 MPa	40 MPa	≥50 MPa
A1	20	20	20	20	20
A2	(50)	30	25	20	20
B1		(60)	40	30	25
B2			(65)	45	35
C				(70)	50

3.2.2 Minimum Member Dimensions

In any beam design it is necessary to check that the chosen reinforcement will fit the beam width. In addition to the cover, the clear spacing between bars must also be established. The Code does not specify what the minimum clear spacing between bars should be, it makes the following statements;

#8.1.7
The minimum clear distance between parallel bars (including bundled bars), ducts and tendons shall be such that concrete can be properly placed and compacted in accordance with Clause 19.1.3.

#9.1.3(d)
Concrete shall be transported , placed and compacted so as to completely fill the formwork to the intended level, expel entrapped air, and closely surround all reinforcement, tendons, ducts, anchorages and embedments.

The Code thus gives the designer the freedom to determine what the clear spacing between parallel bars should be. It is rather unfortunate that the Code has not specified minimum spacing between parallel bars because there will always be designers who will end up with congestion of reinforcement. Congested areas will invariably lead to incomplete compaction around the reinforcement causing loss of strength and loss of bond and leading to premature corrosion of reinforcement.

Experience has shown that a horizontal spacing between parallel bars equal to one and one half times the aggregate size but not less than the bar diameter will ensure that the reinforcing bars are surrounded by concrete and full compaction is achieved. Vertical spacing between horizontal rows of reinforcement is normally obtained by the use of spacer bars; 32 mm spacer bars will provide adequte clearance for compaction. Table 3.4 was produced as a quick and ready means of choosing reinforcement. The table is based on the following conditions;

(a) 20 mm maximum size aggregates are used. The clear horizontal spacing spacing between bars is thus taken as 30 mm for bars up to 28 mm diameter and the bar size for larger bars.

(b) Exposure classification A1. The minimum dimensions b_{min} and c_{min} for exposure classifications other than A1 will need to be adjusted using the additional cover given in Table 3.3

(c) 12 mm stirrups or fitments are used. The minimum cover is measured to the outside of the stirrups.

(d) 32 mm spacer bars are used to separate the horizontal rows of reinforcement.

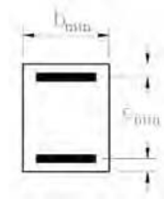


Table 3.4 - Areas of Reinforcemnt and Minimum Beam Dimensions for Exposure Classification A1

Bar Dia.	Number per Row	b_{min}	1 Row Area	1 Row c_{min}	2 Rows Area	2 Rows c_{min}	3 Rows Area	3 Rows c_{min}
	2	118	220		440		660	
	3	160	330		660		990	
12	4	202	440	38	880	60	1320	82
	5	244	550		1100		1650	
	6	286	660		1320		1980	
	2	126	400		800		1200	
	3	172	600		1200		1800	
16	4	218	800	40	1600	64	2400	88
	5	264	1000		2000		3000	
	6	310	1200		2400		3600	
	2	134	620		1240		1860	
	3	184	930		1860		2790	
20	4	234	1240	42	2480	68	3720	94
	5	284	1550		3100		4650	
	6	334	1860		3720		5580	
	2	142	900		1800		2700	
	3	196	1350		2700		4050	
24	4	250	1800	44	3600	72	5400	100
	5	304	2250		4500		6750	
	6	358	2700		5400		8100	
	2	150	1240		2480		3720	
	3	208	1860		3720		5580	
28	4	266	2480	46	4960	76	7440	106
	5	324	3100		6200		9300	
	6	382	3720		7440		11160	
	2	160	1600		3200		4800	
	3	224	2400		4800		7200	
32	4	288	3200	48	6400	80	9600	112
	5	352	4000		8000		12000	
	6	416	4800		9600		14400	
	2	172	2040		4080		6120	
	3	244	3060		6120		9180	
36	4	316	4080	50	8160	84	12240	118
	5	388	5100		10200		15300	
	6	460	6120		12240		18360	

3.2.3 Minimum Steel Ratio

Code #8.1.4.1 requires that reinforced concrete beams have an ultimate strength in bending (M_{uo}) at critcal sections not less than ($M_{uo\ min}$) where

$$(\phi\ M_{uo})_{min} \geq \phi\ 1.2\ Z\ (f'_{cf})$$

$(M_{uo})_{min}$ = Minimum strength in bending at a critical cross section

Z = Modulus of gross (uncracked) section.

$= \dfrac{bD^2}{6}$ for rectangular sections

f'_{cf} = Characteristic flexural strength of concrete.

$= 0.6\sqrt{f'_c}$

The above conditions may however be deemed to have been satisfied if the area of reinforcement is such that the steel ratio is not less than the minimum steel ratio given by:

$$\frac{A_{st}}{bd} \geq 0.22\left(\frac{D}{d}\right)^2 \frac{f'_{cf}}{f_{sy}}$$

3.3 Design

The essential conditions have now been established to proceed with beam design. In any design problem, the material properties, f'_c and f_{sy} are known. There are still three variables to be determined in the design:

(a) the beam width **b**,
(b) the effective depth **d** and
(c) the steel ratio **p** or area of reinforcement $\mathbf{A_{st}}$..

There is however only one strength equation for $\phi\ M_{uo}$ and it cannot be used to solve three unknowns. Consequently it is necessary to assume two of the unknowns and solve for the third unknown. The final product will depend on the initial assumptions; varying the assumptions will alter the final product. The design procedure may take the following steps:

(a) Assume a steel ratio which is more than the minimum but less than the maximum steel ratio. Any steel ratio between these limits is legal although a large steel ratio may lead to steel congestion and shallow beams with large deflections. Small steel ratios will result in very large beams. Initially, a steel ratio $p \approx 0.5\ p_{max}$ means approximately equal to) is a good starting point. Having chosen the steel ratio, the value of z is calculated from equation 2.6.

$$z = p\frac{f_{sy}}{f'_c}$$

(b) The beam must be such as to satisfy equation 2.8,

$$\phi\, M_{uo} \geq M^*$$

Subtituting equation 2.7 for M_{uo},

$$\phi f'_c\, z\left(1-\frac{z}{1.7}\right)bd^2 \geq M^*$$

Using the limiting condition when the two sides are equal and solving for the paramater bd^2 gives,

$$bd^2 = \frac{M^*}{\phi . f'_c\, z\left(1-\dfrac{z}{1.7}\right)} \qquad \textbf{(3.1)}$$

Having assumed a steel ratio **p**, there are still two unknowns to be determined in equation 3.1. There are infinite combinations of dimensions **b** and **d** to satisfy equation 3.1. Beam proportions having a relationship b = 0.6d have been found to produce economic sections which are structurally stable. Thus substituting 0.6d for b in equation 3.1 gives the required effective depth.

$$d = \sqrt[3]{\frac{M^*}{0.6\phi f'_c\, z\left(1-\dfrac{z}{1.7}\right)}} \qquad \textbf{(3.2)}$$

Note that if the beam width b is given, the relationship of b = 0.6d cannot be used. Equation 3.1 would then be applied to determine the required effective depth.

(c) The required area of steel reinforcement is now calculated.

$$A_{st} = pbd$$

(d) The results for b, d and A_{st} thus far are theoretical values based on the assumptions made in (a) and (b). Different assumptions will produce different results since there are an infinite number of solutions. The theoretical values are now converted to practical values by:

(i) Choosing the reinforcement.

(ii) Rounding-off the overall beam dimensions b and D using preferred dimensions of 25 mm increments for dimensions up to 350 mm and 50 mm increments for dimensions greater than 400 mm.

(iii) Steps (i) and (ii) give a trial section whose moment capacity must be checked so that the condition $\phi\, M_{uo} \geq M^*$ is satisfied.

EXAMPLE 1

A beam uses N25 concrete and it is required to carry a design moment $M^* = 225$ kNm. Design the beam for exposure classification A2.

SOLUTION

Maximum steel ratio is obtained from equation 2.4.

$$p_{max} = 0.34\gamma \frac{f'_c}{f_{sy}}$$

$$= 0.34 * 0.85 * \frac{25}{500}$$

$$= 0.014$$

Assume a steel ratio p = 0.01 which is approximately $0.5p_{max}$.

$$z = p\frac{f_{sy}}{f'_c} = 0.01 * \frac{500}{25} \quad = 0.20$$

Assume a beam width to depth ratio such that b = 0.6d.

From equation 3.2, the required effective depth is calculated,

$$d \quad = \sqrt[3]{\left(\frac{225 * 10^6}{0.6 * 0.8 * 25 * 0.16\left(1 - \frac{0.16}{1.7}\right)}\right)}$$

$$= \quad 474 \text{ mm}$$

and b = 0.6*474 = 284 mm

The required area of reinforcement, A_{st} = 0.01*284*474 = 1346 mm^2.

The results so far are purely theoretical based on the assumed steel ratio p = 0.01 and beam proportions such that b = 0.6d. It is now necessary to choose a trial section to match or balance the calculated values. Choose 2N32 bars whose area is 1600 mm^2. The beam dimensions should satisfy preferred dimensions. Since the chosen reinforcement has an area greater than the calculated area, a smaller beam width and/or effective depth may be used. Try beam size b = 300 and D = 500. From Table 3.3, the minimum cover for exposure A2 is 10mm greater than the cover required for exposure A1 on which table 3.4 is based. The adjusted minimum dimensions become;

b_{min} = 160 + 20 = 180 < 300 mm chosen.

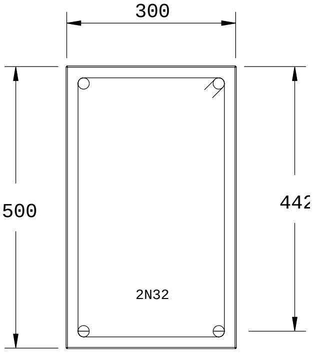
c_{min} = 48 + 10 = 58 mm

Thus, d = 500 - 58 = 442 mm

Calculate the minimum area of steel deemed necessary to ensure $M_{uo} \geq M_{uo\,min}$

$$\frac{Ast}{bd} \geq 0.22\left(\frac{500}{442}\right)^2\left(\frac{0.6\sqrt{25}}{500}\right)$$
$$\therefore p_{min} = 0.0017$$

Figure 3.1 Trial Section



The effective moment capacity for the trial section can now be calculated and compared with the design moment.

Actual steel ratio $p = \dfrac{1600}{300 * 442} = 0.0121$ $(> p_{min} = 0.0017)$
$(< p_{max} = 0.0180)$

$$z = 0.0121\frac{500}{25} = 0.242$$

$$\phi\, M_{uo} = 0.8 * 25 * 0.242\left(1 - \frac{0.242}{1.7}\right)300 * 442^2 * 10^{-6}$$
$$= 243\text{kNm}$$
$$> M^* \;(= 225\text{ kNm})$$

The moment capacity for the trial section matches the design moment reasonably closely in this example. Frequently the difference between the moment capacity and the design moment is greater. The choice of available reinforcement will in most instances not match the calculated area because of discrete sizes of reinforcement. Also the rounded-off beam dimensions will differ from the calculated dimensions. In such circumstances it becomes necessary to compensate for the mismatch in areas and dimensions by judicial adjustments. The reader is invited to repeat the above example by choosing a new trial section if the beam width b = 300 mm is maintained and 3N24 reinforcing bars are used instead. The depth of the trial section should be chosen by the reader and the moment capacity calculated. It may be necessary to repeat the calculations with a new trial section if the moment capacity of the trial section is either less than or much greater than the design moment.

If the minimum strength equation of Section 8.1.4.1 from the Code were used, it would have given an $M_{uo\ min}$ value of 36 kNm which is well and truly satisfied by the value of 276.6 kNm

$$\phi M_{uo\ min} = 0.8 * 1.2 \left(\frac{300(500)^2}{6} \right) \left(0.6\sqrt{25} \right)$$

$$= 36 \text{ kNm}$$

EXAMPLE 2

A 6 m span simply supported exterior beam is required to carry a 26 kN/m superimposed dead load and a 20 kN/m superimposed live load. The beam is part of a commercial complex located in the coastal region (3 km from the sea) south of Sydney. Design the beam using grade N32 concrete if the beam width b = 350 mm is fixed.

SOLUTION

Data: g = 26 kN/m q = 20 kN/m f'_c = 32 MPa L = 6 m b = 350 mm

From Table 3.1, exposure classification = B1.

Assume weight of beam = 6 kN/m

Design load, w^* = 1.2 (26 + 6) + 1.5*20 = 68.4 kN/m

$$Design.Moment, M^* = \frac{68.4 * 6^2}{8} = 307.8 kNm$$

Assume a steel ratio, p = 0.6*p_{max} = 0.60*0.0179

= 0.0107

$$z = p\frac{f_{sy}}{f'_c} = 0.0107 * \frac{500}{32} = 0.167$$

From equation 3.1,

$$bd^2 = \left(\frac{M^*}{\phi f'_c z \left(1 - \frac{z}{1.7} \right)} \right)$$

$$d = \sqrt{\left(\frac{307.8 * 10^6}{0.8 * 32 * 0.167 \left(1 - \frac{0.167}{1.7} \right) * 350} \right)}$$

= 477 mm

Area of reinforcement required, A_{st} = 0.0107*350*477 = 1786 mm^2
From Table 3.4, choose 3N28 bars whose area A_{st} = 1860 mm^2. Applying the increased cover for exposure B1 obtained from Table 3.3, the minimum dimensions read from Table 3.4 become,

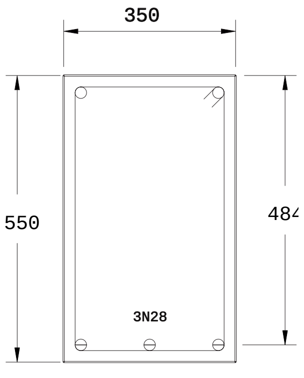
b_{min} = 208 + 2*20 = 248 mm

c_{min} = 46 + 20 = 66 mm

Required total depth of beam, D = 477 + 66 = 543 mm.

Choose D = 550mm, making the effective depth d = 550 - 66 = 484 mm.

Figure 3.2 - Trial Section



For the trial section, $p = \dfrac{1860}{350 * 484} = 0.011$

$$z = 0.011\left(\frac{500}{32}\right) = 0.1716$$

Moment capacity, $\phi\ M_{uo} = 0.8 * 32 * 0.1716\left(1 - \frac{0.1716}{1.7}\right) * 350 * 484^2 * 10^{-6}$

$= 324$ kNm $> M^*$

Actual weight of beam = 0.35*0.55*24 = 4.6 kN/m < Assumed weight of 6 kN/m.
The trial section is satisfactory. If the actual weight of beam is used, the design moment, M^* = 300.2 kNm.

EXAMPLE 3

The overall beam dimensions b = 300 mm and D = 450 mm have been predetermined by the architect. Design the beam for M^* = 200 kNm using N40 grade concrete if the structure is located less than 1 km from the sea.

SOLUTION

There are instances when site conditions or aesthetic considerations dictate the size of a structural member. In such circumstances, for the most efficient beam design using the smallest practicable amount of reinforcement, there is only one possible solution. It is necessary to start the design by estimating the effective depth d which is used to calculate the required area of reinforcement. The reinforcement is then chosen, the value of d is adjusted if necessary and the moment capacity is calculated. If the condition $\phi M_{uo} \geq M^*$ is not satisfied, the process is repeated with a new estimate for d.

Data:

$b = 300$ mm $D = 450$ mm

$f'_c = 40$ MPa $M^* = 200$ kNm

From Table 3.1, exposure classification B2 applies.

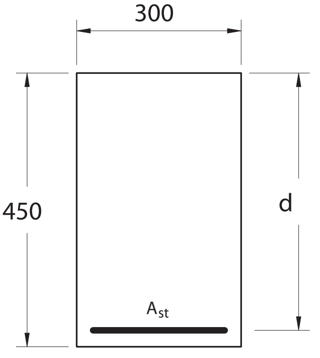


Figure 3.3

It is necessary to estimate the effective depth d for the beam. Assuming that the reinforcement will be made up of say N28 bars placed in one row, from Table 3.4 the minimum value $c_{min} = 46$ mm. From Table 3.3 the minimum covers for exposures A1 and B2 are 20 mm and 45 mm respectively.

Therefore $c_{min} = 46 + 25 = 71$ mm and the estimated effective depth,

$$d = 450 - 71 = 379 \text{ mm}$$

Equation 3.1 may now be solved for z.

$$bd^2 = \frac{M^*}{\phi f'_c z(1 - \frac{z}{1.7})}$$

$$z(1 - \frac{z}{1.7}) = \frac{M^*}{\phi f'_c bd^2}$$

$$1.7z - z^2 = \frac{1.7M^*}{\phi f'_c bd^2}$$

$$z^2 - 1.7z + \frac{1.7M^*}{\phi f'_c bd^2} = 0$$

This is a quadratic in z whose standard solution is,

$$z = \frac{1 \pm \sqrt{1 - \frac{2.4M^*}{\phi f'_c bd^2}}}{1.2}$$

There can only be one solution given by equation 3.3 below.

$$z = \frac{1 - \sqrt{1 - \frac{2.4M^*}{\phi f'_c bd^2}}}{1.2} \qquad \textbf{(3.3)}$$

Evaluating z by substituting in equation 3.3,

$$z = \frac{1 - \sqrt{1 - \frac{2.4 * 200 * 10^6}{0.8 * 40 * 300 * 379^2}}}{1.2}$$

$$= 0.1605$$

But from equation 2.8, $z = \left(\frac{A_{st}}{bd}\right)\left(\frac{f'_c}{f_{sy}}\right)$, the area of reinforement required will be,

$$A_{st} = zbd\frac{f_{sy}}{f'_c} = 0.1605 * 300 * 379\frac{40}{500} \quad = 1460 \text{ mm}^2$$

This is the precise area of reinforcement required for the assumed effective depth of beam d = 379 mm. Choosing the reinforcement from table 3.4, for 3N28 bars give an area A_{st} = 1860 mm^2. The minimum width of beam required to fit the chosen 3N28 bars is read from Table 3.4 for exposure A1 and adjusted for exposure B2 using Table 3.3.

b_{min} = 208 + 2*25 = 258 < b (= 300)

The area of reinforcement provided and the area required are close, however it may be necessary to repeat the calculations using a new estimate for d to achieve an area of reinforcement closer to that provided.

PROBLEMS

QUESTION 1

An external reinforced concrete beam in an industrial area of Sydney is required to carry a 24 kN/m superimposed dead load and a 20 kN/m live load. The beam is simply supported over an effective span L = 6.8 metres.

Design the beam for a given beam width b = 400 mm using the minimum permissible concrete strength grade. Assume initially a steel ratio p = 0.8p_{max}.

QUESTION 2

Repeat question 1 if only N20 bars are available.

QUESTION 3

A reinforced concrete beam has an effective cross-section b = 300 mm and d = 446 mm. If the beam design moment M^* = 320 kNm, calculate the precise area of reinforcement required for a concrete strength f'_c = 40 MPa (do not choose the reinforcement).

QUESTION 4

A 6 m span simply supported reinforced concrete beam ina near-coastal area uses grade N40 concrete. The beam which is exposed to the weather carries a uniformly distributed dead load g = 64 kN/m which includes the weight of beam, and a uniformly distributed live load q = 40 kN/m. Design the beam by determining the required reinforcement if the overall beam dimensions b = 350 mm and D = 600 mm are to be maintained in the design.

4

chapter

Design aids for singly reinforced concrete beams

4.1 Additional Symbols used in this Chapter

λ = Design parameter used in conjunction with chart B1.

4.2 Design Formulae

Designs are most frequently carried out using some form of design aids. It is however important that designers be able to design from formulae derived in chapter three to ensure their understanding of the basic principles. There is always a possibility that a designer may find himself or herself in a situation where design aids are either not available or appropriate. The effective moment capacity of a singly reinforced concrete beam was derived in Chapter 2.

$$\phi M_{uo} = \phi f'_c \, z(1 - \frac{z}{1.7}) bd^2$$

Where $z = \frac{A_{st}}{bd} \frac{f_{sy}}{f'_c} = p \frac{f_{sy}}{f'_c}$

The equation may be simplified by introducing a new symbol λ such that:

$$\phi M_{uo} = \lambda . bd^2 \qquad \textbf{(4.1)}$$

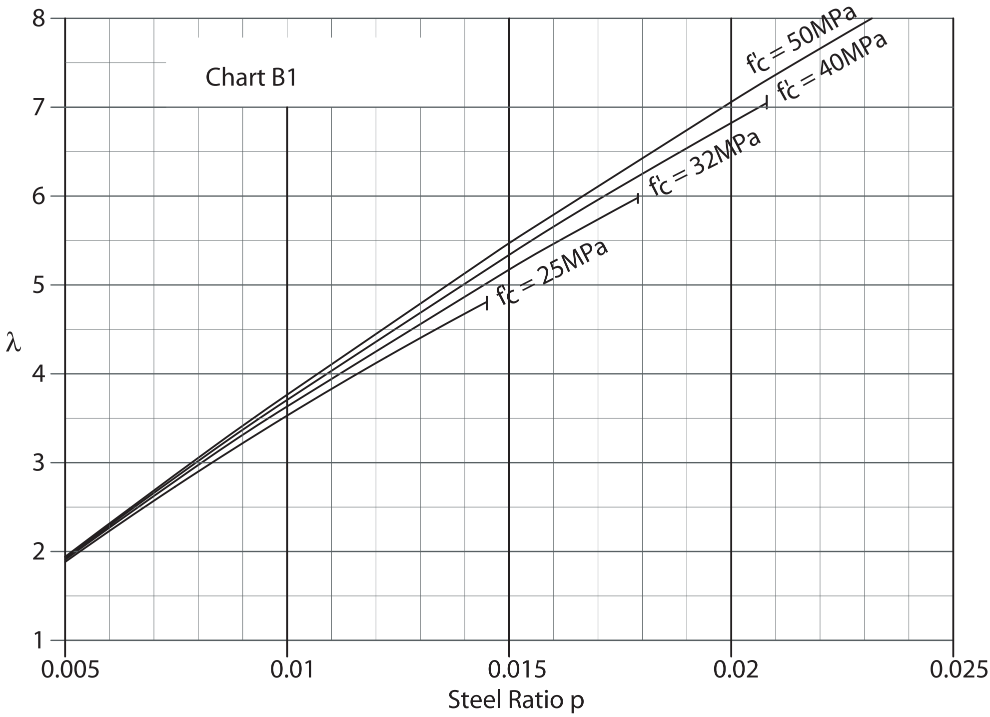
Where:

$$\lambda = \phi f'_c \, z \left(1 - \frac{z}{1.7}\right) \qquad \textbf{(4.2)}$$

Or

$$\lambda = \phi \, p f_{sy} \left(1 - \frac{p}{1.7} \frac{f_{sy}}{f'_c}\right) \qquad \textbf{(4.3)}$$

For a given grade of concrete λ is a function of the steel ratio. A plot of equation 4.2, λ versus p, may be obtained for various grades of concrete. Such plots are produced in design chart B1 for normal grades of concrete for which the steel ratio varies from the minimum to the maximum steel ratio. The advantages of using equation 4.1 in conjunction with design chart B1 become apparent for both design and checking of singly reinforced concrete beams.



4.3 Checking Procedure - Given f'_c, b, d and p or A_{st}

(a) Calculate steel ratio p.

(b) Read value of λ read chart B1

(c) Calculate the effective moment capacity from equation 4.1.

EXAMPLE 1

Determine if the reinforced concrete beam section shown in Figure 4.1 may be used to carry a design moment $M^* = 160$ kNm.

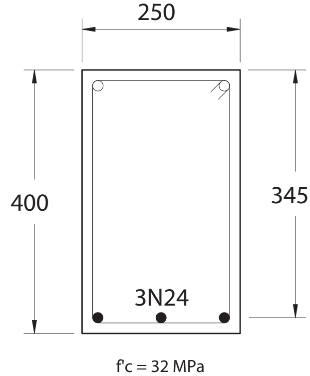


Figure 4.1

SOLUTION

$f'_c = 32$ MPa   b = 250 mm   d = 345 mm

$A_{st} = 1350$ mm^2 $M^* = 160$ kNm

$$p = \frac{1350}{250 * 345} = 0.01565$$

From Chart B1 read λ = 5.4

Effective moment capacity,

$$\phi M_{uo} = \lambda\, bd^2$$
$$= 5.4*250*345^2*10^{-6}$$
$$= 160.6 \text{ kNm}$$
$$> M^* \text{ therefore satisfactory.}$$

4.4 Design Procedure – Given f'c and M*

(a) Choose a steel ratio p not more than p_{max}.

(b) Read λ from Chart B1.

(c) Equating the design moment to the moment capacity given by equation 4.1, $M^* = \lambda bd^2$ the required beam size, $bd^2 = \dfrac{M^*}{\lambda}$

If b is not given then assume b = 0.6d making the required depth,

$$d = \sqrt[3]{\left(\frac{M^*}{0.6\lambda}\right)}$$

and the required width b = 0.6d.

OR

If b is given, the required effective depth,

$$d = \sqrt{\left(\frac{M^*}{b\lambda}\right)}$$

(d) The required steel area, $A_{st} = pbd$.

(e) Choose and round-off overall beam dimensions b and D to obtain a trial section. Note that the minimum cover for the appropriate exposure classification and clear spacing of reinforcement should satisfy the physical size of the trial section.

(f) Check capacity of trial section as for example 1.

EXAMPLE 2

A reinforced concrete beam in an exposure classification A2 uses N40 grade concrete. Design the beam to carry a design moment $M^* = 260$ kNm if the beam width b = 300 mm is to be maintained.

SOLUTION

$f'_c = 40$ MPa $M^* = 260$ kNm   b = 300 mm   Exposure classification A2.

Choose steel ratio, p = 0.016. This is best done from design chart B1 since the permissible working ranges for the steel ratios are readily observed.

From Chart B1 read the value of λ = 5.65

Required effective depth, $d = \sqrt{\left(\frac{M^*}{b\lambda}\right)} = \sqrt{\left(\frac{260 * 10^6}{300 * 5.65}\right)} = 392\text{mm}$

Required area of reinforcement, A_{st} = 0.016*300*392 = 1882 mm^2.

From Table 3.4 choose 4N24 bars for which, b_{min} = 250 mm and c_{min} = 44 mm.

For the trial section shown in Figure 4.2 (*right*),

$$p = \frac{1800}{300 * 406} = 0.0148$$

From chart B1 read λ = 5.3 for p = 0.0148.

Moment capacity of trial section,

$$\phi M_{uo} = \lambda b d^2 = 5.3 * 300 * 406^2 * 10^{-6}$$

$$= 262 \text{ kNm} > M^* (= 260)$$

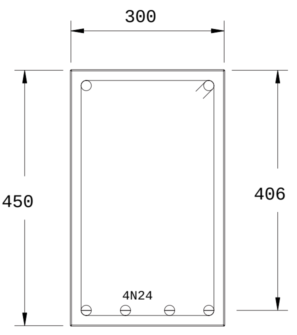
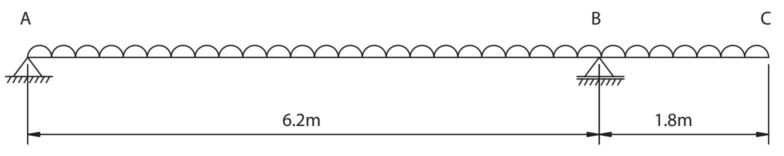


Figure 4.2

EXAMPLE 3

The beam shown in Figure 4.3 is required to support a superimposed dead load equal to 15 kN/m and a superimposed live load equal to 24 kN/m. Design the beam for the maximum positive bending moment if the beam dimensions b = 350 mm and D = 500 mm are to be maintained. The beam will be permanently submerged in sea water. Use the minimum concrete grade required to satisfy exposure conditions.

Figure 4.3



SOLUTION

b = 350 mm D = 500 mm g = (weight of beam + 15) kN/m q = 24 kN/m

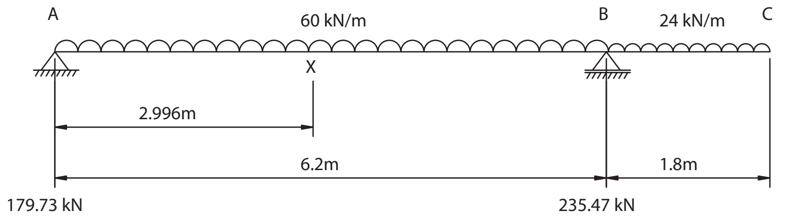
Weight of beam = 0.35*0.5*24 = 4.2 kN/m

The maximum positive bending moment will occur when the live load acts between the supports only. The factored design loads for the maximum positive bending moment shown in Figure 4.4 are calculated from:

From A to B, w^* = 1.2 (4.2 + 15) + 1.5*24 = 59 kN/m

From B to C, w^* = 1.2 (4.2 + 15) = 23 kN/m

Figure 4.4



The maximum positive bending moment occurs at X the point of zero shear force, which is:

$$\frac{179.73}{60} = 2.996 \ \textit{metres from A}$$

The maximum positive design bending moment at X,

$$M^* = \frac{179.73 * 2.996}{2} = 269.2kNm$$

From Table 3.1, exposure classification = B2.

From Table 3.2, minimum grade of concrete = N40.

Assuming one row of N32 bars, from table 3.4, c_{min} = 48 mm for exposure A1. The increased cover of 25 mm required for exposure B2 is obtained from Table 3.3 to give c_{min} = (48 + 25) = 73 mm.

Effective depth d = 500 - 73 = 427 mm.

Required value of $\lambda = \frac{M^*}{bd^2} = \frac{269.2 * 10^6}{350 * 427^2} = 4.22$

From design Chart B1 read the required steel ratio p = 0.0115.

Required area of reinforcement, A_{st} = 0.0115*350*427 = 1719 mm^2.

3-N28 bars have an area A_{st} = 1860 mm^2.

Check b_{min} = 208 + 2*25 = 258 < 350 mm beam width.

There should be no need to check the beam moment capacity since the effective depth is equal to the assumed effective depth and the chosen reinforcement has an area greater than the calculated area. It is however a good practice to check the beam capacity just in case there was an error made in the earlier calculations. Using corrected d = 429

$$p = \frac{1860}{350 * 429} = 0.0124$$

From design Chart B1 read λ = 4.5

Hence moment capacity, $\phi M_{uo} = 4.5*350*429^2*10^{-6}$

$= 290$ kNm > M^* (= 269.2)

4.5 Singly Reinforced Beam (SRB) Design Charts

Charts SRB at the end of the book were drawn in terms of actual beam sizes. These Charts may be used for beam design or analysis of existing beams. The charts are intended for the use of practicing designers or advanced students. To design a beam for a design moment M^*, choose a beam width b and calculate the ratio M^*/b. Choose Chart SRB appropriate to the strength of concrete. For the minimum conditions of $\phi M_{uo} = M^*$, enter the chart with the value of $\frac{\phi M_{uo}}{b}$ equal to the calculated M^*/b. Choose a suitable effective depth d and read the required steel ratio p. This procedure may be repeated with new values of b or d to obtain the best section.

EXAMPLE 4

Design a beam using grade N32 concrete to carry a design moment $M^* = 420$ kNm. Assume exposure A2 will apply.

SOLUTION

Assume a beam width, say b = 400 mm.

$$\frac{M^*}{b} = \frac{420 * 10^6}{400} = 1.05*10^6 \text{ N}$$

From Chart SRB32, for $\frac{\phi M_{uo}}{b} = \frac{M^*}{b} = 1.05*10^6$ N, choose an effective depth d = 500 mm (or any value between the minimum, p_{min}, and the maximum, p_{max}, steel ratio) and read the required steel ratio p = 0.0120.

Required $A_{st} = 0.0120*400*500 = 2400$ mm^2.

Choose reinforcement, 4-N28 bars give $A_{st} = 2480$ mm^2.

Using beam depth D = 550 mm, maximum effective depth d = 550 - 51 = 499 mm (note that $c_{min} = 51$). As the effective depth is reduced, the moment capacity should be checked.

For the beam chosen, p = $\frac{2480}{400 * 499} = 0.0124$.

From Chart SRB32, read $\frac{\phi M_{uo}}{b} = 1.07*10^6$ for p = 0.0124 and d = 499 mm.

Moment capacity, $\phi M_{uo} = 1.07*10^6*400*10^{-6} = 428$ kNm $> M^*$ (= 420).

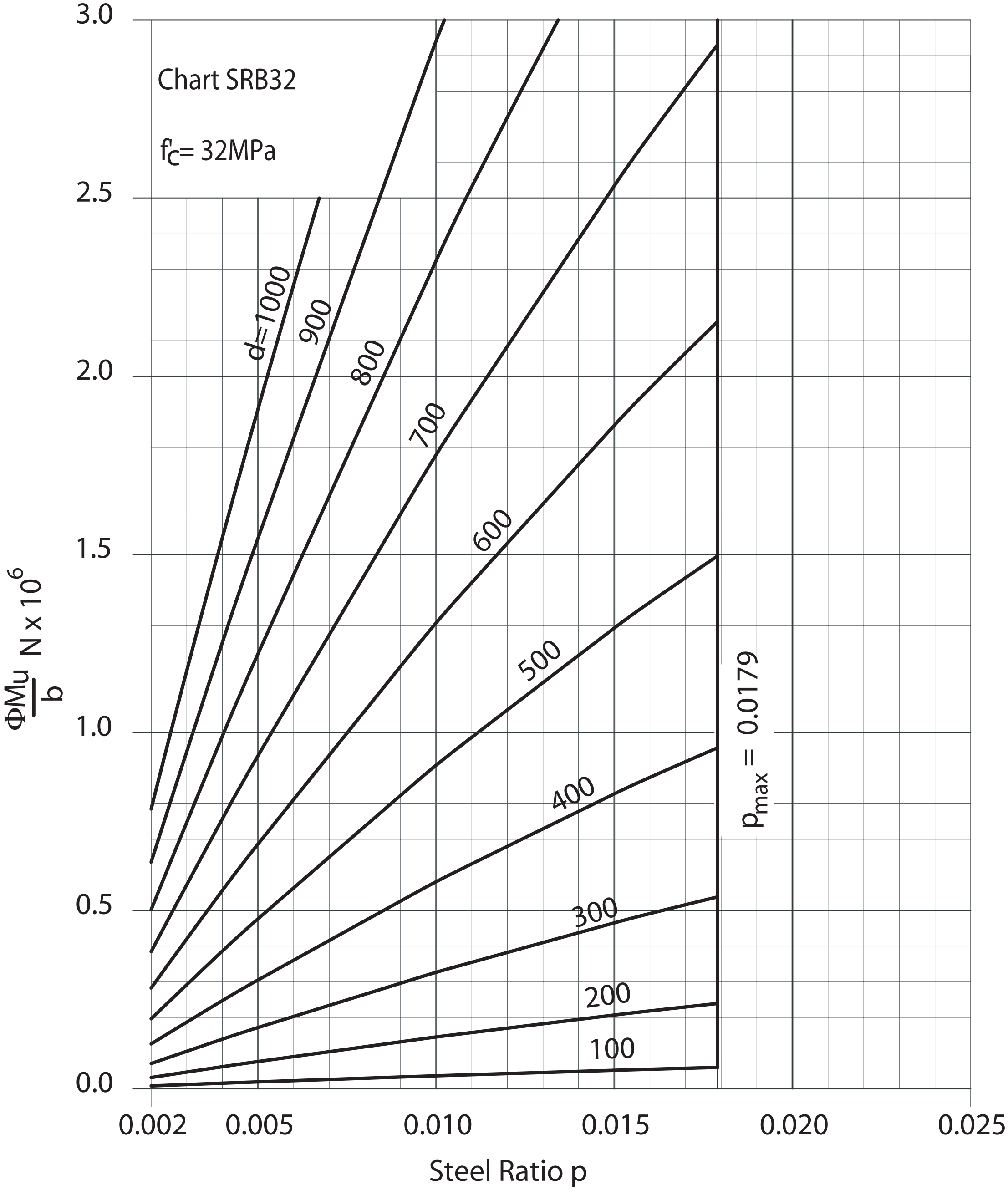


Chart SRB32
f'c= 32MPa
d=1000
900
800
700
600
500
400
300
200
100
pmax = 0.0179
ΦMu/b N x 10^6
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.002
0.005
0.010
0.015
0.020
0.025
Steel Ratio p

PROBLEMS

QUESTION 1

A beam size b = 300, d = 412, D = 500 is reinforced with 8-N20 bars placed in two rows of 4 bars. If the beam uses grade N40 concrete, determine the effective moment capacity:

(a) by calculation using derived formulae
(b) using design Chart B1

to show that the results are comparable.

QUESTION 2

Use design Chart B1 to determine the area of reinforcement which would be required for a beam size b = 350 mm, d = 430 and D = 500. The beam uses grade N40 concrete and it carries a design moment M^* = 270 kNm.

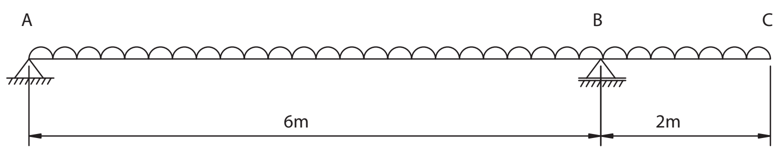
QUESTION 3

A beam in an exposure classification B1 uses grade N32 concrete and it is required to resist a design moment M^* equal to 355 kNm. Design the beam if the external beam dimensions b = 350 and D = 500 must be maintained.

QUESTION 4

The cantivered beam shown below is an external wall beam of a hotel building at Arbel which has a temperate climate. Arbel is an inland town with no industries. The beam uses grade N25 concrete and it is required to carry a superimposed 18 kN/m dead load and a superimposed 22 kN/m live load.

(a) Design the beam for the maximum positive bending moment using an initial estimate of 0.013 for the steel ratio
(b) Use the beam size determined in (a) to design the cantilever for the maximum negative bending moment.



5
chapter

Doubly reinforced beams

5.1 Additional Symbols used in this Chapter

A_{st}	=	Total tensile area.
	=	$A_{s1} + A_{s2}$
A_{s1}	=	Tensile area of primary beam. This is usually the area of a singly reinforced beam with the maximum steel ratio p_{max} for which $k_u = 0.4$.
A_{s2}	=	Tensile area of secondary beam.
A_{sc}	=	Area of compressive reinforcement.
f_{sc}	=	Stress in compressive reinforcement.
d_{sc}	=	Depth measured to centroid of compressive reinforcement.
ε_c	=	0.003 the compressive strain in concrete at failure.
ε_{sc}	=	Compressive strain in A_{sc}.
ε_y	=	Yield strain of reinforcement.
M^*	=	Design moment due to factored loads.
M_1	=	Effective moment capacity of primary beam.
M_2	=	$M^* - M_1$ the effective moment capacity to be carried by secondary beam.
p_c	=	Compressive steel ratio.
	=	$\frac{A_{sc}}{bd}$
p_t	=	Total tensile steel ratio.
	=	$\frac{A_{st}}{bd}$
p_1	=	Tensile steel ratio in primary beam.

5.2 Use of Doubly Reinforced Beams

Doubly reinforced beams are beams with compressive as well as tensile reinforcement. There is little strength advantage in purposely adding compressive reinforcement to a singly reinforced beam when the concrete can carry the internal compressive force required to balance the tensile force. Most beams would in effect include "incidental compressive reinforcement" in the form of hanger bars required to position stirrups. Such incidental compressive reinforcement would be disregarded when it comes to determining the moment capacity. The additional moment capacity obtained by the inclusion of the hanger bars is too small to warrant the additional effort and cost of the calculations which have now become much more involved.

Compressive reinforcement may however be added for the purpose of reducing long-term deflection. If compressive reinforcement is added for the sole purpose of satisfying serviceability, it is disregarded in strength calculations. The effect of compressive reinforcement on serviceability will be considered in Chapter 8 dealing with this topic.

Doubly reinforced beams are required in circumstances where a singly reinforced beam using the maximum steel ratio cannot carry the design moment and beam size cannot be increased either due to physical restrictions or other conditions beyond the control of the designer.

5.3 Strength Equations

Considering a beam of fixed dimensions b and D containing an area of reinforcement A_{s1} so that the steel ratio is equal to the maximum steel ratio.

$$A_{s1} = p_{max}\,bd$$

The effective moment capacity of this beam will be,

$$M_1 = \Phi f'_c\, z\left(1 - \frac{z}{1.7}\right)bd^2 \qquad \text{Where,} \quad z = p_{\max}\left(\frac{f_{sy}}{f'_c}\right)$$

If the design moment M^* is greater than M_1, the beam capacity can be increased while still maintaining beam ductility by additional area A_{s2} of tensile reinforcement which will yield on application of the design moment M^* and an area of compressive reinforcement A_{sc} to balance the tensile force carried by A_{s2}. The forces carried by A_{s2} and A_{sc} form an internal couple whose effective moment is equal to the difference M^* - M_1.

A doubly reinforced beam is shown in Figure 5.1(*below*) (a). Applying the principle of superposition the beam may be regarded as the superposition of a **PRIMARY** beam shown in Figure 5.1(b) and a **SECONDARY** beam shown in Figure 5.1(c). Considering each in turn.

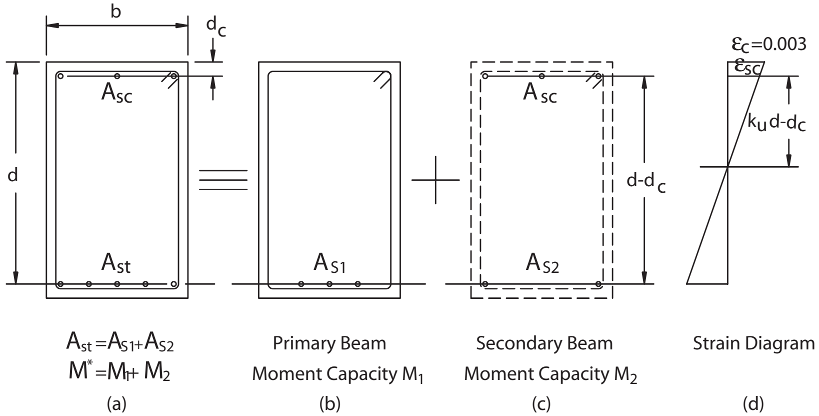


Figure 5.1

PRIMARY BEAM

This is a singly reinforced beam using the maximum steel ratio having the following properties:

Area of reinforcement,

$$A_{s1} = p_{max} bd \qquad \textbf{(5.1)}$$

Where p_{max} is the maximum steel ration given by equation 2.4

$$p_{max} = 0.34\gamma\left(\frac{f'_c}{f_{sy}}\right) \text{for } f'_c \leq 50\text{MPa} \qquad \textbf{(5.2)}$$

Moment capacity,

$$M_1 = \Phi f'_c\, z\left(1 - \frac{z}{1.7}\right)bd^2 \qquad \textbf{(5.3)}$$

Where z is calculated for the maximum steel ratio,

$$z = p_{max}\left(\frac{f_{sy}}{f'_c}\right) \qquad \textbf{(5.4)}$$
$$= 0.34\gamma$$

OR the moment capacity is determined using chart B1,

$$M_1 = \lambda\, b\, d^2 \qquad \textbf{(5.5)}$$

Where λ is read from table 5.1 on page 61, for the maximum steel ratio.

SECONDARY BEAM

The secondary beam is regarded as a 'steel' beam made up of a tensile steel area A_{s2} and a compressive steel area A_{sc}.

The moment capacity to be provided by the secondary beam,

$$M_2 = M^* - M_1 \qquad \textbf{(5.6)}$$

M_2 is equal to the internal moment of resistance due to the couple provided by the reinforcement A_{s2} and A_{sc}. To maintain beam ductility, the additional tensile area A_{s2} must yield at moment M_2. Taking moments about the compressive reinforcement A_{sc} gives,

$$M_2 = \Phi A_{s2} f_{sy}\left(d - d_{sc}\right) \qquad \textbf{(5.7)}$$

Where: $A_{s2} f_{sy}$ = Tensile force and

$(d - d_{sc})$ = Lever arm of internal couple.

Φ = 0.8 (the reduction factor for bending)

Solving for A_{s2}, the required additional area of tensile reinforcement,

$$A_{s2} = \left(\frac{M_2}{\Phi f_{sy}\left(d - d_{sc}\right)}\right)$$

While the additional area of tensile reinforcement A_{s2} is calculated to yield at the design moment and ensure a ductile beam behaviour, the corresponding area A_{sc} of compressive reinforcement may not have yielded at the design moment. To establish the required area A_{sc} it is necessary to evaluate the strain at the level of the compressive reinforcement and compare it with the yield strain.

The compressive strain is calculated from the geometry of the strain diagram shown in Figure 5.1(d).

$$\frac{\varepsilon_{sc}}{k_u d - d_{sc}} = \frac{0.003}{k_u d}$$

$$\varepsilon_{sc} = \left(\frac{k_u d - d_{sc}}{k_u d}\right)0.003$$

$$\varepsilon_{sc} = \left(\frac{k_u - \frac{d_{sc}}{d}}{k_u}\right)0.003 \qquad \textbf{(5.8)}$$

But k_u = 0.4 for maximum steel ratio used in the primary beam and the position of the neutral axis is maintained provided that the areas of reinforcement are not varied from the calcuated areas. Hence the compressive steel strain becomes,

$$\varepsilon_{sc} = \left(\frac{0.4 - \frac{d_{sc}}{d}}{0.4}\right)0.003$$

$$\varepsilon_{sc} = 0.003 - 0.0075\frac{d_{sc}}{d} \qquad \textbf{(5.9)}$$

The steel strain at point of yielding ε_y = 0.0025. Comparing the compressive strain ε_{sc} with the yield strain of 0.0025 will establish if the compressive reinforcement has yielded and hence determine the area of compressive reinforcement.

If $\epsilon_{sc} \geq$ 0.0025 then the compressive reinforcement **HAS** yielded and the required area of compressive reinforcement is give by;

$$A_{sc} = \frac{A_{s2} f_{sy}}{f_{sy} - 0.85 f'_c}.$$

If ϵ_{sc} < 0.0025 then the compressive reinforcement has **NOT** yielded and the required area of compressive reinforcement is calculated from equilibrium condition provided by the internal forces.

$$A_{sc}\left(f_{sc} - 0.85 f'_c\right) = A_{s2} f_{sy}$$

The term $0.85 f'_c$ is to compensate for the concrete displaced by the compressive reinforcement.

$$A_{sc} = \frac{A_{s2} f_{sy}}{f_{sc} - 0.85 f_c^{'}} \qquad \textbf{(5.10)}$$

The reinforcement can then be chosen for areas A_{st} and A_{sc}. To ensure a ductile beam behaviour, the steel ratio for the primary beam p_1, using chosen reinforcement, should be checked that it does not exceed the maximum steel ratio p_{max} within reason. Using the actual areas of reinforcement the steel ratio for the primary beam becomes,

$$p_1 = \frac{A_{st} - A_{sc}\left[\dfrac{f_{sy} - 0.85 f_c^{'}}{f_{sy}}\right]}{bd} \qquad \text{If } \varepsilon_{sc} \geq 0.0025$$

$$p_1 = \frac{A_{st} - A_{sc}\left[\dfrac{f_{sy} - 0.85 f_c^{'}}{f_{sy}}\right]}{bd} \qquad \text{If } \varepsilon_{sc} < 0.0025$$

Care should be exercised in choosing the reinforcement, especially the tensile reinforcement. The doubly reinforced beam is designed for k_u = 0.4 in the primary beam. With the addition of balanced areas A_{s2} and A_{sc}, the position of the neutral axis will remain unchanged i.e. k_u is still 0.4. If however the tensile reinforcement is chosen to be much greater than the calculated value, the neutral axis will be displaced to give a value of k_u greater than 0.4 and the above equations are no longer applicable. It would be necessary to check the capacity of the doubly reinforced beam by determining the position of the neutral axis by successive iterations. The analysis of doubly reinforced beams will be considered later in this chapter. At this point we are only concerned with the design of doubly reinforced beams whose neutral axis parameter k_u = 0.4.

EXAMPLE 1

A reinforced concrete beam section b = 300 mm, D = 400 mm is required to carry a design moment M^* = 300 kNm. The beam uses *N25* concrete and it is located in exposure classification *A2*. Design the beam.

SOLUTION

Data: b = 300 mm D = 400 mm M^* = 300 kNm f'_c = 25 MPa
Exposure *A2*

Assuming one row of *N32* bars are used for both the compressive and tensile reinforcement, use Tables 3.3 and 3.4 to determine depths d and d_{sc}.

$d_{sc} = c_{min}$ = 48 mm from Table 3.4 + 10 mm additional cover for exposure A2 from Table 3.3

= 58 mm

$d = D - c_{min}$ = 400 – 58 = 342 mm

Primary Beam

Tensile area: $A_{s1} = p_{max}\,bd$ = 0.0145*300*342

= 1,488 mm^2

For p_{max} z = 0.34γ = 0.34 x 0.85 = 0.289

Moment capacity, M_1 $= \Phi f'_c\, z\left(1 - \frac{z}{1.7}\right) bd^2$

$$= 0.8 * 25 * 0.289\left(1 - \frac{0.289}{1.7}\right) 300 \times 342^2 \times 10^{-6}$$

= 168.3 kNm

< M^* Hence doubly reinforced beam required.

Secondary Beam

Moment to be carried by reinforcement in secondary beam,

$M_2 = M^* - M_1$

= 300 – 168.3

= 131.7 kNm

Tensile area required,

$$A_{s2} = \frac{M_2}{\Phi f_{sy}\left(d - d_{sc}\right)}$$

$$= \frac{131.7 \times 10^6}{0.8 \times 500 \times (342 - 58)}$$

= 1159 mm^2

Determine strain in compressive reinforcement.

$$\epsilon_{sc} = 0.003 - 0.0075\frac{d_{sc}}{d}$$

= 0.00173

< ϵ_y (= 0.0025)

Hence the compressive reinforcement has NOT yielded.

Stess in compressive reinforcement, $f_{sc} = \epsilon_{sc} E_s$ = 345.6 MPa

Area of compressive reinforcement required,

$$A_{sc} = \frac{A_{s2} f_{sy}}{f_{sc} - 0.85 f'_c}$$

$= 1786$ mm^2

Total tensile area required,

$A_{st} = A_{s1} + A_{s2}$

$= 1488 + 1159$

$= 2647 \text{ mm}^2$

From Table 3.4, it is not possible to choose tensile reinforcement which will fit in one row. It may have been anticipated that for the size of beam more than one row of tensile reinforcement may be required. Assume two rows of N28 bars for tensile reinforcement and one row of N28 bars for the compressive reinforcement and repeat calculations.

From Tables 3.3 and 3.4,

$d = D - c_{min} = 400 - 86 = 314$ mm

$d_{sc} = 56$ mm

Primary Beam

Tensile area $A_{s1} = 0.0145*300*314 = 1366 \text{ mm}^2$

$z = 0.289$ as before.

Moment capacity $M_1 = 0.8*25*0.289(1 - 0.289/1.7)*300*0.314^2$

$= 141.3$ kNm

Secondary Beam

Moment to be carried,

$M_2 = 300 - 141.3 = 158.7$ kNm

Tensile area required,

$$A_{s2} = \frac{158.7 * 10^6}{0.8 * 500(314 - 56)}$$

$= 1538 \text{ mm}^2$

Strain in compressive reinforcement,

$$\epsilon_{sc} = 0.003 - 0.0075\frac{56}{314}$$

$= 0.00166 < 0.0025$

The compressive reinforcement has NOT yielded giving a compressive stress, $f_{sc} = 332$ MPa.

The area of compressive reinforcement required,

$$A_{sc} = \left(\frac{500}{332 - 0.85 \times 25}\right)1538$$

$$= 2475 mm^2$$

Total tensile area required,

$$A_{st} = 1366 + 1538$$
$$= 2904 \text{ mm}^2$$

Alternatively the compressive area could be calculated from,

$$A_{sc} = \frac{M_2}{\Phi(f_{sc} - 0.85f'_c)(d - d_{sc})} \quad \textbf{(5.11)}$$

Substituting in equation 5.11,

$$A_{sc} = \frac{158.7 \times 10^6}{0.8 * (332 - 0.85 \times 25)(314 - 56)}$$
$$= 2475 \text{ mm}^2$$

Choose 4N28 bars in one row and 2N20 bars in the next row for the tensile reinforcement, A_{st} = 3100 mm^2, and 4N28 bars for the compressive reinforcement, A_{sc} = 2480 mm^2. The final beam is shown in Figure 5.2. The steel ratio of the primary beam using the actual reinforcement should be checked against the maximum steel ratio.

Steel ratio of primary beam using reinforcing areas shown in Figure 5.2 (*below*), noting that the compressive reinforcement has NOT yielded,

$$p_1 = \frac{A_{st} - \left(\frac{f_{sc} - 0.85 f_{c'}}{500}\right) A_{sc}}{bd}$$

$$= \frac{3100 - \left(\frac{332 - 0.85 \times 25}{500}\right)2480}{300 * 314}$$

$$= 0.0165 > p_{\max}(= 0.0145)$$

p_1 could be reduced by increasing A_{sc}.

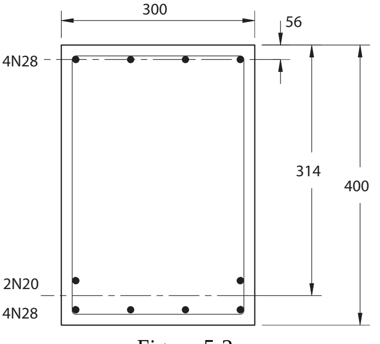


Figure 5.2

5.4 Design Aids for Doubly Reinforced Beams

Primary Beam

Table 5.1 below is useful for deterimining the area of reinforcement A_{s1} and the moment capacity M_1 of the primary beam. The values of z and λ are tabulated for the maximum steel ratio p_{max}.

Table 5.1 - Parameters for Maximum Steel Ratio

Parameter	f_c in MPa					
	20	25	32	40	50	65
γ	0.850	0.850	0.822	0.766	0.696	0.65
p_{max}	0.0116	0.0145	0.0179	0.0208	0.0237	0.0287
z	0.2890	0.2890	0.2795	0.2604	0.2366	0.221
λ	3.838	4.797	5.979	7.057	8.1468	9.998

Secondary Beam

The ratio of compressive area to tensile area is obtained from equations 5.8 and 5.11.

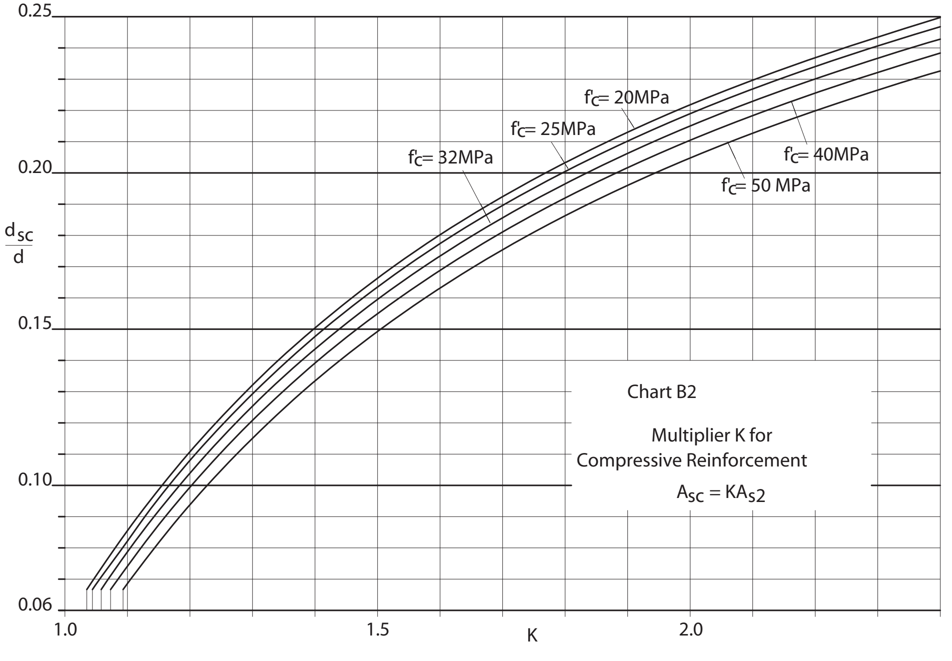
$$\frac{A_{sc}}{A_{s2}} = \frac{f_{sy}}{f_{sc} - 0.85f'_c}$$

So that, $A_{sc} = \left(\frac{f_{sy}}{f_{sc} - 0.85f'_c}\right) A_{s2}$

Or more simply,

$$A_{sc} = K^{*}A_{s2} \qquad \textbf{(5.12)}$$

The value of multiplier K is plotted in Chart B2 for various grades of concrete. For values of $\frac{d_{sc}}{d} \leq 0.0667$, the compressive reinforcement has yielded. This may be shown from equation 5.10 when the compressive strain ϵ_{sc} is equal to the yield strain of 0.0025.



EXAMPLE 2

A reinforced concrete beam using N32 concrete is required to carry a design moment M^* = 465 kNm. The beam is in Exposure classification B1. Design the beam if the beam overall dimensions b = 350 and D = 500 are fixed.

SOLUTION

Data: f'_c = 32 MPa M^* = 465 kNm b = 350 mm D = 500 mm

Exposure classification B1

Assume single row of N24 top (compression) bars and single row of N32 (tension) bars. From Tables 3.3 and 3.4,

d_{sc} = 64 mm

d = 500 - 68 = 432 mm

Primary Beam

From Table 5.1 read,

p_{max} = 0.0179

λ = 5.979

Area of tensile reinforcement in primary beam,

A_{s1} = 0.0179*350*432

= 2706 mm^2

Moment capacity of primary beam,

$M_1 = \lambda b d^2$

= 5.979*350*432^2*10^{-6}

= 390.5 kNm

Secondary Beam

Moment to be carried,

$M_2 = M^* - M_1$

= 465 – 390.5

= 74.5 kNm

Area of tensile reinforcement required,

$$A_{s2} = \frac{M_2}{\Phi f_{sy}(d - d_{sc})} = \frac{74.5 \times 10^6}{0.8 \times 500(432 - 64)} = 506\text{mm}^2$$

$$\frac{d_{sc}}{d} = \frac{64}{432} = 0.148$$

From Chart B2, read $K = 1.42$

Area of compressive reinforcement required,

$$A_{sc} = K^*A_{s2}$$
$$= 1.42^*452$$
$$= 719 \text{ mm}^2$$

Total tensile area required,

$$A_{st} = A_{s1} + A_{s2}$$
$$= 2706 + 506$$
$$= 3212 \text{ mm}^2$$

The available choice of tensile reinforcement from Table 3.4 will require two rows of reinforcement. Assume that the tensile reinforcement is made up of two rows of N24 bars. From Table 3.4, $c_{min} = 92$.

$$d = 500 - 92 = 408 \text{ mm}$$
$$d_{sc} = 64 \text{ mm as before.}$$

Primary Beam

$$A_{s1} = 0.0179^*350^*408$$
$$= 2556 \text{ mm}^2$$

$$M_1 = 5.979^*350^*408^2{}^*10^{-6}$$
$$= 348.3 \text{ kNm}$$

Secondary Beam

$$M_2 = 465 - 348.3 = 116.7 \text{ kNm}$$
$$A_{s2} = \frac{116.7 \times 10^6}{0.8 \times 500(408 - 64)}$$
$$= 848 \text{ mm}^2$$
$$\frac{d_{sc}}{d} = \frac{64}{408} = 0.157$$

From Chart B2, $K = 1.47$

Required compressive area,

$$A_{sc} = 1.47 \times 848 = 1247 \text{ mm}^2$$

Total tensile area required,

$$A_{st} = 2556 + 848 = 3404 \text{ mm}^2$$

For tensile reinforcement, choose 8N24 bars in two rows as shown below giving, A_{st} = 3600 mm^2.

Compressive reinforcement, choose 3N24 bars, A_{sc} = 1350 mm^2.

The final beam is shown in Figure 5.3 below.

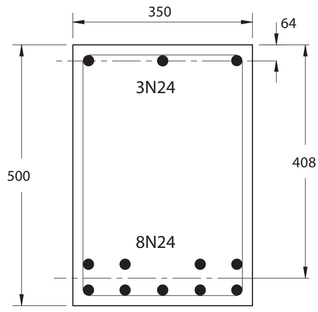


Figure 5.3

5.5 Moment Capacity of Doubly Reinforced Beams

To determine the moment capacity for a given doubly reinforced beam is more complex. The design procedure used so far assumed that the position of the neutral axis is maintained at k_u = 0.4 which is the maximum value permitted by the Code. The addition of compressive reinforcement and tensile reinforcement to the primary beam was balanced to maintain the position of the neutral axis with k_u = 0.4.

In the analysis of doubly reinforced beams, there is no guarantee that the above conditions have been maintained. The value of k_u is unknown and it can only be determined by an iterative procedure. The position of the neutral axis is established when internal equilibrium is achieved as outlined below. The moment capacity is obtained by taking moments about the tensile reinforcement.

1. Assume k_u = 0.4 initially.

2. Calculate compressive steel strain ε_{sc} from equation 5.8 or 5.9.

3. If $\varepsilon_{sc} \geq 0.0025$, the compressive reinforcement has yielded and the compressive stress, $f_{sc} = f_{sy}$ = 500 MPa otherwise the compressive stress $f_{sc} = \varepsilon_{sc} \times 2 \times 10^5$ MPa.

4. Calculate the internal forces,

$$C_c = 0.85f'_c b\gamma k_u d * 10^{-3} \text{ kN}$$
$$C_s = (f_{sc} - 0.85f'_c)A_{sc} \times 10^{-3} \text{ kN}$$
$$T = f_{sy}A_{st} \times 10^{-6} \text{ kN}$$

5. Check for internal equilibrium,

If $C_c + C_s > T$ reduce k_u and repeat from step 2.

If $C_c + C_s < T$ increase k_u and repeat from step 2.

If $C_c + C_s = T$ calculate moment capacity.

6. Moment capacity, take moments about tensile reinforcement

$$\Phi M_{uo} = \Phi[C_c(d - 0.5\gamma k_u d) + C_s(d - d_{sc})]$$

$$= \Phi\left(C_c d(1 - 0.5\gamma k_u) + C_s d(1 - \frac{d_{sc}}{d})\right)$$

$$= \Phi d\left(C_c(1 - 0.5\gamma k_u) + C_s(1 - \frac{d_{sc}}{d})\right)$$

Note: The above procedure is not complete since it assumes that all tensile reinforcement has yielded and it is concentrated at depth *d*. Also there is no provision made to reduce the moment capacity when k_u exceeds the maximum of 0.4 required by the Code. A classic example of these conditions exists in columns subjected to pure bending.

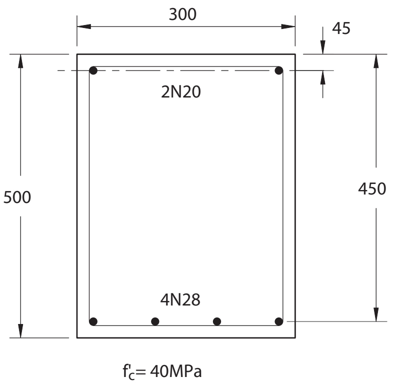
EXAMPLE 3

The reinforced concrete beam shown in Figure 5.4 (*below*) was designed as a singly reinforced beam using 4N28 bars. The 2N20 top bars were added as hanger bars for the shear reinforcement and as compressive bars to reduce long-term deflection. The top bars were not included in the strength design.

(a) Neglecting the hanger bars, determine the effective moment capacity fo the beam as a singly reinforced beam.

(b Since the 2N20 top bars are in effect compressive reinforcement, determine the effective beam moment capacity as a doubly reinforced beam and comment on the results.

Figure 5.4



SOLUTION

(a) Singly reinforced beam.

$$\gamma = 0.766$$

$$p = \frac{2480}{300 * 450} = 0.0184 < p_{\max}(= 0.0208)$$

From Chart B1, $\lambda = 6.3$

Moment capacity, $\Phi M_{uo} = 6.3\times300\times450^2\times10^{-6} = 382.7$ kNm

$$k_u = \frac{1}{0.85\times0.766}0.0184\frac{500}{40}$$

$$= 0.3532$$

Depth of NA, $k_u d$ = 0.3532*450 = 158.9 mm

(b) As a doubly reinforced beam. The effect of the compressive reinforcement will be to reduce the depth of the neutral axis i.e. k_u will be reduced. A starting point in this example could be a value for k_u less than 0.3292 obtained for the singly reinforced beam. For the sake of uniformity, the given procedure will be used with an initially assumed value for k_u = 0.4.

$$\frac{d_{sc}}{d} = \frac{45}{450} = 0.1$$

Assume, k_u = 0.4

Stress in compressive reinforcement for this condition is 500 MPa

Internal compressive and tensile forces,

C_c = 0.85*40*300*0.766*0.4*450*10^{-3}

= 1406 kN

C_s = (500 - 0.85*40)*620*10^{-3}

= 289 kN

T = 500*2480*10^{-3}

= 1240 kN

The total internal compression $C_c + C_s$ = 1695 kN is greater than the internal tension of 1240kN. The value of k_u will have to be reduced to balance the internal forces.

Try k_u = 0.25.

Strain in compression reinforcement,

$$\varepsilon_{sc} = \left(\frac{k_u - \frac{d_{sc}}{d}}{k_u}\right)\times0.003 = \left(\frac{0.25-0.1}{0.25}\right)\times0.003 = 0.0018$$

$$< 0.0025$$

The compression reinforcement has NOT yielded.

Stress in compressive reinforcement,

$$f_{sc} = 0.0018 \times 2 \times 10^5$$
$$= 360 \text{ MPa}$$

Internal forces,

$$C_c = 0.85 \times 40 \times 300 \times 0.766 \times 0.25 \times 450 \times 10^{-3}$$
$$= 879 \text{ kN}$$

$$C_s = (360 - 0.85 \times 40) \times 620 \times 10^{-3}$$
$$= 202 \text{ kN}$$

$$T = 1240 \text{ kN as before.}$$

$$C_c + C_s = 879 + 202$$
$$= 1081 \text{ kN}$$
$$< T \quad (= 1240)$$

The position of the neutral axis has now been under-estimated. The value of k_u needs to be increased. Using a value of 0.290 for k_u gives the following results:

$$\epsilon_{sc} = 0.001966$$
$$f_{sc} = 393.1 \text{ MPa}$$
$$C_c = 1019.6 \text{ kN}$$
$$C_s = 222.6 \text{ kN}$$
$$C_c + C_s = 1242.2 \text{ kN}$$
$$T = 1240 \text{ kN}$$

The depth of the neutral axis,

$$k_u d = 0.290 \times 450 = 130.5 \text{ mm}$$

The moment capacity is now determined by taking moments about the tensile reinforcement.

$$\Phi M_{uo} = \Phi\left(C_c(d - 0.5\gamma k_u d) + C_s(d - d_{sc})\right)$$
$$= 0.8 \times \left((1019.6 \times \left(450 - 0.5 \times 0.766 \times 130.5\right) + 222.6 \times \left(450 - 45\right)\right) \times 10^{-3}$$
$$= 398.4 \ kNm$$

Comments: The 2N20 hanger bars caused a sizeable decrease in the depth of the neutral axis from 158.9 mm to 130.5 mm but the moment capacity was only increased by 4.1% from 382.7 kNM to 398.4 kNm. This explains why incidental compressive reinforcement is frequently not included in strength calculations.

5.6 Design Charts DRB

Design of doubly reinforced concrete beams can be simplified by the development of design charts. These are Charts DRB-25, DRB32, DRB-40 and DRB-50 corresponding to 25, 32, 40 and 50 MPa concrete grades are included at the end of the book. Charts DRB (doubly reinforced beams) are total design charts which allow a designer to directly choose the steel ratios p_t and p_c required for a beam of given size to carry a design moment M^* for any ratio of d_{sc}/d.

The design procedure is made up of the following steps:

(a) For the given design conditions, f'_c, M^*, b and D, assume d_{sc} and d and calculate d_{sc}/d and $M^*/(bd^2)$.

(b) From chart corresponding to f'_c, enter chart with values of d_{sc}/d and $\Phi M_{uo}/(bd^2)$ $(=M^*/(bd^2))$ and read the required steel ratios p_t and p_c.

(c) Required areas of reinforcement $A_{st} = p_t bd$, $A_{sc} = p_c bd$.

(d) Choose the reinforcement.

(e) If d_c and d for chosen reinforcement are different to the assumed values, repeat steps (b), (c) and (d) using new ratio d_c/d.

EXAMPLE 4

Choose the reinforcement for a beam size $b = 350$ mm, $D = 500$ mm to carry a design moment $M^* = 520$ kNm using grade N32 concrete assuming exposure classification *A2*.

SOLUTION

Assume $d_{sc} = 50$ mm and d = 400 mm to give $d_{sc}/d = 50/400 = 0.125$.

$$\frac{M^*}{bd^2} = \left(\frac{520 * 10^6}{350 * 400^2}\right) = 9.29\,MPa$$

From Chart DRB-32 for $d_{sc}/d = 0.0125$ and $\Phi M_{uo}/(bd^2) = 9.29$ MPa read the required steel ratios $p_t = 0.0276$ and $p_c = 0.0126$. Figure 5.5 shows the construction lines required to read the chart.

Required areas of reinforcement $A_{st} = 0.0276 \times 350 \times 400$

$= 3864 \text{ mm}^2$

$A_{sc} = 0.0126 \times 350 \times 400 = 1764 \text{ mm}^2$

6N32 bars in two rows of three bars gives $A_{st} = 3720 \text{ mm}^2$ and 4N24 bars gives $A_{sc} = 1800 \text{ mm}^2$. From Table 3.4 adjusted dimensions for exposure A2 are $d = 419$ mm and $d_{sc} = 49$ mm. Repeating the procedure using new values of d and d_{sc},

$$d_{sc} / d = 49/419 = 0.117$$

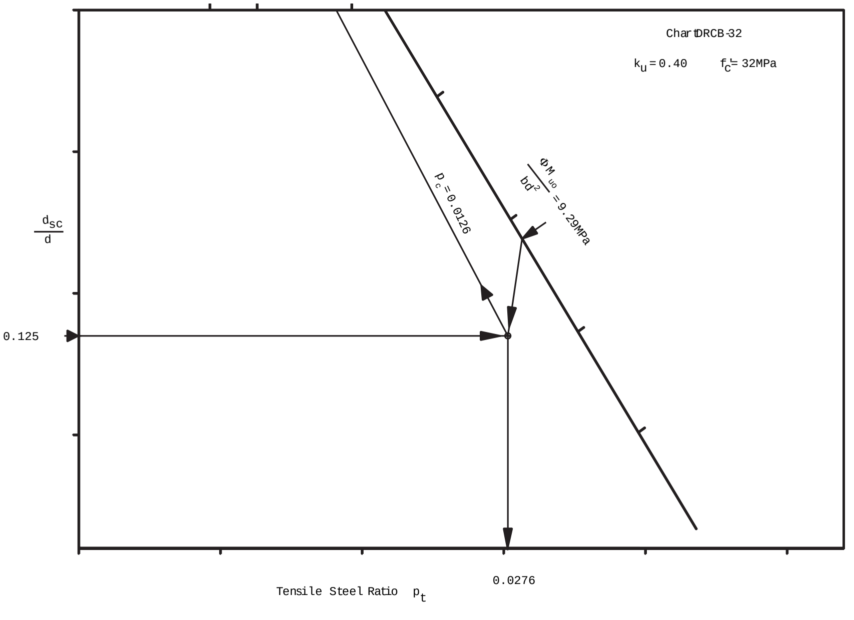
$$\Phi M_{uo} / bd^2 = 8.46 MPa$$

$Chart$ DRCB-32 read P_t = 0.0250. P_c = 0.009

$$Required\, A_{st} = 0.0250 \times 350 \times 419 = 3666$$

$$Required\, A_{sc} = 0.009 \times 350 \times 419 = 1320$$

Figure 5.5



Choose 6N28 bars giving A_{st} = 3720 mm^2, and 3N24 bars giving A_{SC} =1350 mm^2

Check primary beam steel ratio.

$$p_t = \left(\frac{3720 - 1350}{350 * 419} \right)$$

$$= 0.0162 < p_{max} (= 0.0179)$$

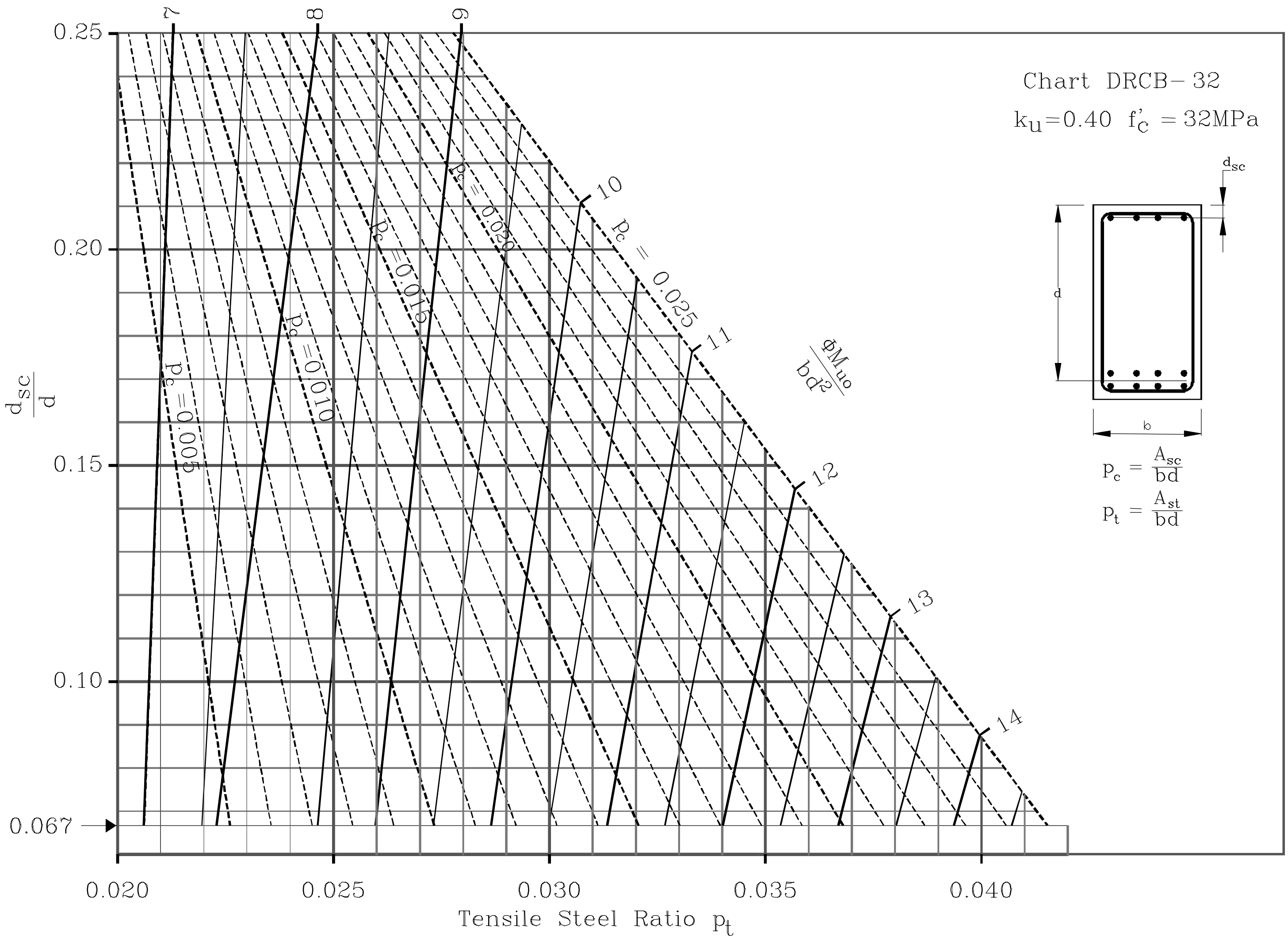
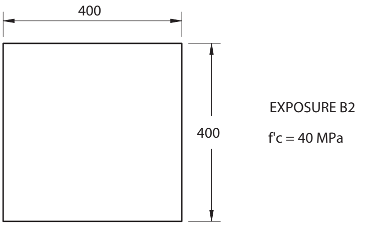


Chart DRCB-32
k_u=0.40 f'_c = 32MPa
d_{sc}
d
b
$p_c = \frac{A_{sc}}{bd}$
$p_t = \frac{A_{st}}{bd}$
$\frac{\phi M_{uo}}{bd^2}$
7
8
9
10
11
12
13
14
p_c = 0.005
p_c = 0.010
p_c = 0.015
p_c = 0.020
p_c = 0.025
$\frac{d_{sc}}{d}$
0.25
0.20
0.15
0.10
0.067
0.020
0.025
0.030
0.035
0.040
Tensile Steel Ratio p_t

PROBLEMS

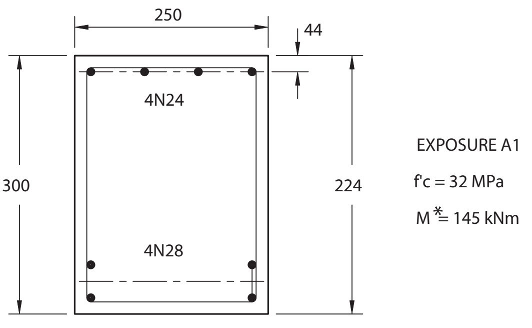
QUESTION 1



(a) Show that for the beam section shown a singly reinforced beam only is needed to carry a design moment M* = 282 kNm and choose reinforcement.

(b) If the design moment for the beam in part (a) is increased to 299 kNm, show that while theoretically a singly reinforced beam may be used, the choice of reinforcement is such that compressive reinforcement will have to be added to satisfy Code requirements. What is the required minimum compressive reinforcement ?

QUESTION 2

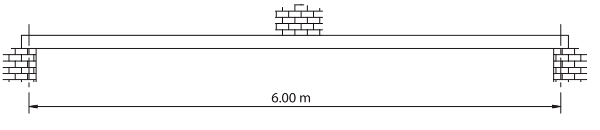


The beam shown appears in a drawing but the design calculations cannot be found. Using the beam dimensions for the conditions shown on the drawing, go through the normal design step procedure to show that the beam satisfies strength conditions.

QUESTION 3

Determine the depth of the NA and the effective moment capacity for the beam given in question 2. Note that evaluating k_u within $\pm$ 0.005 will achieve adequate accuracy.

QUESTION 4



A simply supported beam with a 6 m effective span is used in the exterior wall over an opening for a basement carpark of a commercial building 10 km from the coast. The beam supports a total dead load g = 32 kN/m which includes its own weight and a live load q = 40 kN/m. Because of physical restrictions the beam has a square section 400 mm by 400 mm. N40 grade concrete is used.

(a) Design the beam for the maximum bending moment using N24 reinforcing bars only.

(b) How far from the support centrelines could the compressive reinforcement be theoretically terminated and what will be the tensile reinforcement carried through to the supports.

(c) Draw the beam showing all the relevant details.

6

chapter

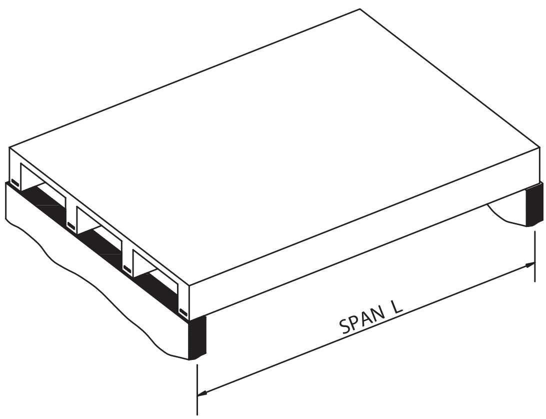
T-beams and L-beams

6.1 Additional Symbols used in this Chapter

a	=	Distance between points of zero bending moment.
b	=	Effective flange width b_{ef}.
b_w	=	Width of web.
d_s	=	Depth of rectangular stress block.
	=	$\gamma k_u d$
t	=	Flange thickness.
	=	Thickness of slab D_s making up T-beam or L-beam.
L	=	Span of T-beam or L-beam.
l	=	Clear distance between webs of parallel beams.

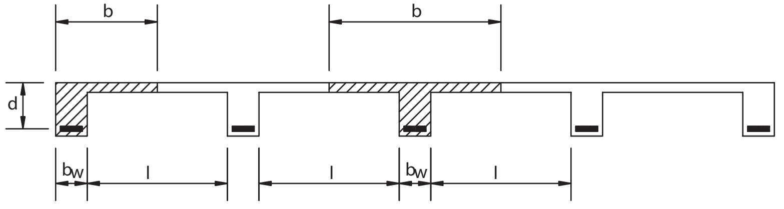
6.2 Effective Flange Width

Floor slabs are generally supported by integrally cast beams. While the floor slab is designed to span the parallel supporting beams, in the direction of the span L shown in Figure 6.1 (*right*), portion of the slab is considered to make up the beam and increase the load carrying capacity.



The beams in the direction of span L are made up of a web and a flange which is part of the slab on each side of the web to give the beams a T or L shape as shown in Figure 6.2 (*below*). It may be anticipated that the flange width b should extend to centre of each slab i.e. $l/2$ either side of the web. This is a reasonable supposition and it is the case in most instances. However an excessively thin flange may not be very effective because it may buckle under a relatively small moment. The Code #8.8.2 limits the flange width to a maximum effective flange width b_{ef} herein given the symbol b.

Figure 6.2



T-beams b $= b_w + 0.2a$

L-beams b $= b_w + 0.1a$

Where; a = distance between points of zero bending moment

$= 0.7L$ for continuous beams.

Note that the flange outstand on either side of the web cannot exceed $l/2$.

6.3 When are T-beams, T-beams?

Consider a rectangular beam shown in Figure 6.3 (*right*) in which some of the concrete below the neutral axis has been removed. It is quite obvious that the moment capacity of such a beam has not been affected since the concrete below the neutral axis is assumed to be fully cracked and it does not contribute towards the moment capacity of the beam. The removal of some of the concrete below the neutral axis has altered the beam shape but the beam flexural strength is still that of a rectangular beam. This is really a rectangular beam in 'disguise' of a T-beam and it is designed as a rectangular beam.

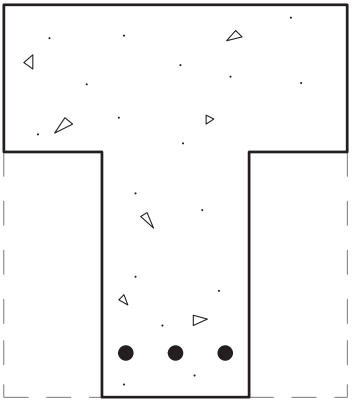
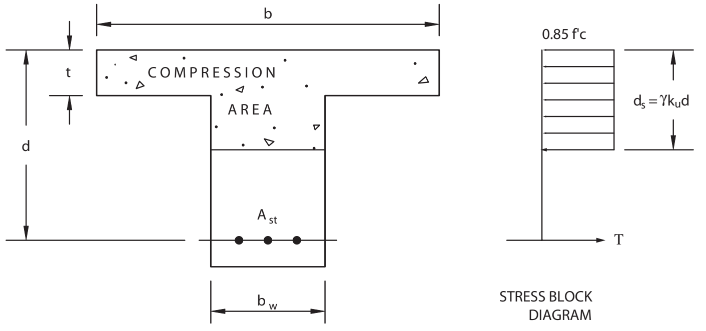


Figure 6.3

However, if the concrete removed in Figure 6.3 extended into the compression region above the neutral axis as shown in Figure 6.4, then the moment capacity of the beam is affected. The beam is a true T-beam and it can no longer be designed as a rectangular beam.

Figure 6.4



All references made to T-beams apply equally to L-beams. To design a T-shaped beam it is first necessary to determine the depth of the stress block by treating the beam as a rectangular beam *b* by *d*. If the depth of the stress block is within the flange, then the beam is designed as a rectangular beam *b* by *d*. If the depth of the stress block is below the flange and in the web then a separate design procedure must be addopted. For a rectangular beam, the effective moment capacity which must be at least equal to the design moment, may be obtained by taking moments about the tensile reinforcement. Referring to Figure 6.4, for a rectangular beam *b* by *d* when the effective moment capacity $\Phi M_{uo} = M^*$ the design moment,

$$M^* = \Phi C\,(d - 0.5\,\gamma k_u d)$$

$$= \Phi^* 0.85 f'c\, b\gamma k_u d(d - 0.5\gamma k_u d)$$

Let $d_s = \gamma k_u d$ the depth of the stress block.

$$M^* = \Phi^* 0.85 f'_c b d_s (d - 0.5 d_s)$$

Solving for the depth of stress block d_s,

$$\frac{M^*}{\Phi \times 0.85 f'_c\, b} = d_s(d - 0.5\, d_s)$$

$$= d_s d - \frac{d_s^2}{2}$$

Transposing all the terms to one side and multiplying by 2 gives,

$$d_s^{\,2} - 2dd_s + \frac{2M^*}{0.85\Phi f'_c\, b} = 0$$

This is a quadratic in d_s whose standard solution is,

$$d_s = \frac{2d \pm \sqrt{(2d)^2 - 4 \times \dfrac{2M^*}{0.85\Phi\, f'_c\, b}}}{2}$$

$$= d \pm \sqrt{d^2 - \frac{2M^*}{0.85\Phi f'_c\, b}}$$

There is only one solution for the depth of the stress block given by,

$$d_s = d - \sqrt{d^2 - \frac{2M^*}{0.85\Phi f'_c b}} \qquad \textbf{(6.1)}$$

The depth of the stress block calculated from equation 6.1 is compared with the flange thickness.

If $d_s \leq t$ the stress block is in the flange and the beam is designed as a **rectangular** beam b by d.

If $d_s > t$ the stress block is in the web and beam has to be designed as a true **T-beam**. The design procedure will be developed on the following pages.

6.4 Determining Depth of Stress Block Using Design Aids

Substitute $\lambda\, b\, d^2$ for M^* in equation 6.1,

$$d_s = d - \sqrt{d^2 - \frac{2\lambda b d^2}{0.85\Phi f'_c b}}$$

$$= d - d\sqrt{1 - \frac{2\lambda}{0.85\Phi f'_c}}$$

Dividing both sides by the effective depth d gives,

$$\frac{d_s}{d} = 1 - \sqrt{1 - \frac{2\lambda}{0.85\Phi f'_c}} \qquad \textbf{(6.2)}$$

It may be noted that d_s/d is a dimensionless ratio. On the right hand side λ and f'_c have the same units of N/mm^2, so the right hand side is also dimensionless as expected.

The d_s/d ratio may be read from the design chart B3 on page 77 which gives a plot of d_s/d versus λ for standard concrete strength grades.

EXAMPLE 1

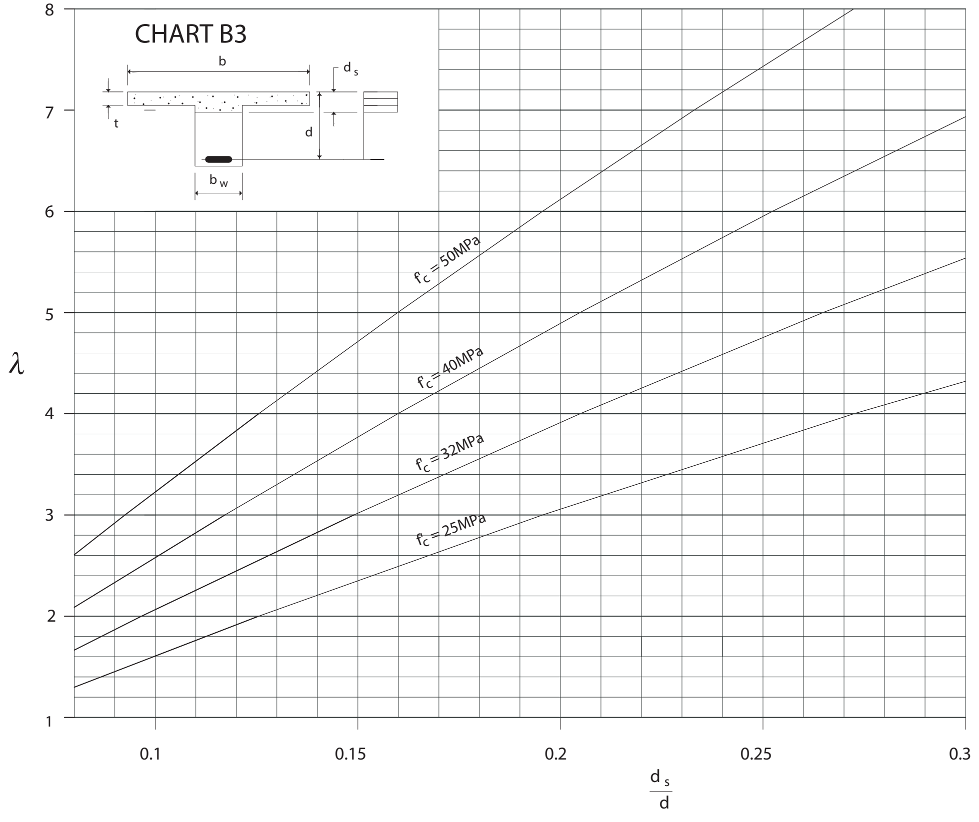
Figure 6.2 represents the cross-section of a beam and slab construction with the following properties;

f'_c = 25 MPa $\quad f_{sy}$ = 500 MPa $\quad L$ = 7000 mm $\quad t$ = 75 mm $\quad b_w$ = 350 mm

D = 500 mm $\quad d$ = 420 mm $\quad l$ = 1600 mm $\quad$ Exposure classification A1

The slab supports a superimposed dead load (not including weight of construction) g = 6 kPa, and a superimposed live load q = 15 kPa.

(a) Calculate the design load and hence the design bending moment to be carried by the intermediate beams.



(b) Determine the effective flange width for the T-beams.

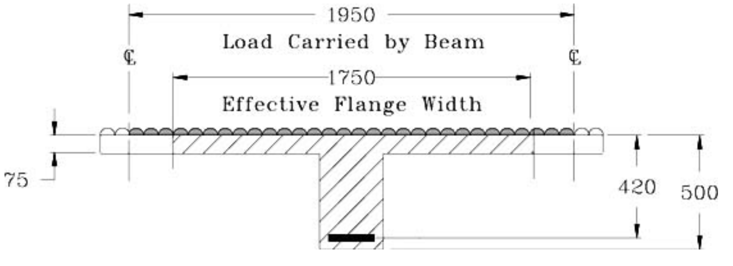
(c) Will the intermediate beams be designed as T-beams or rectangular beams?

(d) If the intermediate beams are to be designed as rectangular beams, then choose the reinforcement.

SOLUTION

(a) Each beam carries the superimposed loads extending to the centreline of each panel as shown in Figure 6.5 (*below*).

Figure 6.5



Superimposed dead load per metre length of beam,

$g = 1.95\times6 = 11.7$ kN/m

Superimposed live load per metre length of beam,

$q = 1.95\times15 = 29.25$ kN/m

Weight of 1 metre length of beam,

$= 0.075\times1.95\times24 + 0.425\times0.35\times24$

$= 7.08$ kN/m

Design load,

$w^* = 1.2\times(11.7 + 7.08) + 1.5\times29.25$

$= 66.4$ kN/m

Design bending moment,

$$M^* = \frac{66.4 * 7^2}{8} = 406.7 \text{ kNm}$$

(b) Effective flange width,

$b = b_w + 0.2L$

$= 350 + 0.2*7000$

$= 1750$mm

The flange outstand is $0.1\times7000 = 700$ mm which is less than half the distance between parallel beams.

(c) For a rectangular beam size b = 1750 mm and d = 420 mm, the depth of the stress block may be calculated from equation 6.1,

$$d_s = d - \sqrt{d^2 - \frac{2M^*}{0.85\Phi f'_c b}}$$

$$420 - \sqrt{\left(420^2 - \frac{2 * 406.7 * 10^6}{0.85 * 0.8 * 25 * 1750}\right)}$$

$= 34$mm

$\leq$ t

Alternatively using Chart B3,

$$\lambda = \frac{M^*}{bd^2} = \frac{406.7 * 10^6}{1750 * 420^2} = 1.32$$

From Chart B3 read $d_s/d = 0.08 < t/d(=75/420=0.179)$ i.e. stress block is in the flange and the beam is designed as a rectangular beam whose width is equal to the effective flange width b = 1750 mm.

(d) From Chart B1 read the required steel ratio corresponding to $\lambda = 1.32$ falls below the range, use p = 0.005.

Area of tensile reinforcement required,

$$A_{st} = pbd = 0.005 \times 1750 \times 420$$
$$= 3675 \text{ mm}^2$$

From Table 3.4 select 6N28 bars placed in two rows of three bars.

6.5 Design of T-Beams

Design of true T-beams (or L-beams) is similar to doubly reinforced beams in which the compressive reinforcement is replaced by the concrete flanges. The superposition principle will be used to derive design formulae.

Referring to Figure 6.6 (*below*), the T-beam may be regarded as the superposition of the "primary" beam (b) and "secondary" beam (c).

Each beam will be considered separately.

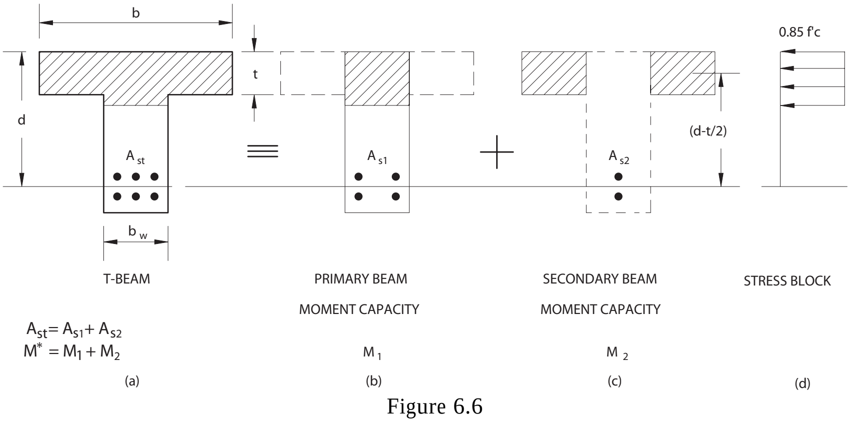


Figure 6.6

SECONDARY BEAM

Compressive force provided by the flanges,

$$C_f = 0.85 f'_c (b - b_w)\, t$$

Taking moments about the tensile reinforcement, the effective moment capacity of the secondary beam,

$$M_2 = \Phi C_f (d - \frac{t}{2})$$

$$\boxed{M_2 = \Phi 0.85 f'_c (b - b_w)\, t\, (d - \frac{t}{2}) \qquad \textbf{(6.3)}}$$

The effective moment capacity may also be determined by taking moments about the centroid of the flanges.

$$M_2 = \Phi A_{s2} f_{sy} (d - \frac{t}{2})$$

But the value of M_2 is already known from equation (6.3), therefore the area A_{s2} may be determined by solving for A_{s2}.

$$A_{s2} = \frac{M_2}{\Phi f_{sy}\left(d - \frac{t}{2}\right)} \qquad \textbf{(6.4)}$$

PRIMARY BEAM

The effective moment capacity to be carried by the primary beam is,

$$M_1 = M^* - M_2$$

Since this is a rectangular beam, the effective moment capacity is give by the equation,

$$M_1 = \lambda \, b_w \, d^2$$

Transposing to determine the value of λ for the primary beam,

$$\lambda = \frac{M_1}{b_w d^2}$$

The steel ratio p may now be read from chart B1 and the tensile area required by the primary beam will be,

$$A_{s1} = p \, b_w \, d$$

The total tensile area required by the T-beam,

$$A_{st} = A_{s1} + A_{s2}$$

Finally it is just a matter of choosing the reinforcement for A_{st}. As with doubly reinforced beams a check should be made that the steel ratio of the primary beam using the **actual** area A_{st} of the selected reinforcement does not exceed the maximum steel ratio.

$$\frac{A_{st} - A_{s2}}{b_w d} \leq p_{\max}$$

EXAMPLE 2

Redesign the intermediate T-beam in example 1 if the design moment $M^* = 900$ kNm.

Data: $f'_c = 25$ MPa $M^* = 900$ kNm $b = 1750$ mm $d = 420$ mm

$t = 75$ mm $b_w = 350$ mm Exposure Classification A1.

SOLUTION

For a rectangular beam b = 1750 mm and d = 420 mm carrying an ultimate design moment M^* = 900 kNm,

$$\lambda = \frac{900 * 10^6}{1750 * 420^2} = 2.915$$

$$\frac{t}{d} = \frac{75}{420} = 0.18$$

From Chart B3 for λ= 2.915 read, d_s/d = 0.19 > t/d

That is, $d_s > t$. The depth of the stress block is greater than the flange thickness so the beam must be designed as a true T- beam.

SECONDARY BEAM

Effective moment capacity,

$$M_2 = 0.85\Phi f'_c\,(b - b_w)\, t\left(d - \frac{t}{2}\right)$$

$$= 0.85 \times 0.8 \times 25(1750 - 350) \times 75 \times \left(420 - \frac{75}{2}\right) \times 10^{-6}$$

$$= 682.8 \text{ kNm}$$

Area of tensile reinforcement required,

$$A_{s2} = \frac{M_2}{\Phi f_{sy}\left(d - \frac{t}{2}\right)}$$

$$= \frac{682.8 \times 10^6}{0.8 \times 500\left(420 - \frac{75}{2}\right)}$$

$$= 4463 \text{ mm}^2$$

PRIMARY BEAM

Moment to be carried by primary beam,

$$M_1 = M^* - M_2$$

$$= 900 - 682.8$$

$$= 217.2 \text{ kNm}$$

$$\lambda = \frac{M_1}{b_w d^2}$$

$$= \frac{217.2 * 10^6}{350 * 420^2} = 3.52$$

From Chart B1 read required steel ratio corresponding to λ

$$p = 0.01$$

Area of reinforcement required for primary beam,

$$A_{s1} = pb_w d$$
$$= 0.01 \times 350 \times 420$$
$$= 1470 \text{ mm}^2$$

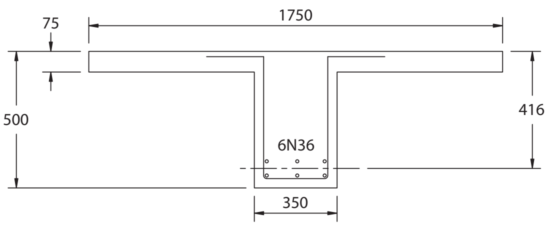
Total tensile area required for the T-beam,

$$A_{st} = A_{s1} + A_{s2}$$
$$= 1470 + 4463 = 5933 \text{ mm}^2$$

From Table 3.4 choose 6N36 bars place in two rows of 3 bars whose area $A_{st} = 6120$ mm^2. Checking the steel ratio for the primary beam,

$$p_1 = \frac{6120 - 4463}{350 \times 416}$$
$$= 0.0114 < p_{max}$$

Figure 6.7



6.6 T-Beam Design Charts

At the end of the book design Charts TB.10 to TB.20 have been drawn for t/d ratios of 0.10, 0.12, 0.14, 0.16, 0.18 and 0.20. Each chart plots the b/b_w ratio versus the steel ratio p for a series of stresses $\Phi M_{uo}/(bd^2)$ and the standard concrete stress grades. The required steel ratio for a given condition is not greatly affected by the t/d value. For example, a T-beam whose ratio $b/b_w = 4.0$ uses concrete strength $f'_c = 25$ MPa. For a moment capacity ΦM_{uo} such that $\Phi M_{uo}/(bd^2) = 4.0$ MPa, the required steel ratio p = 0.0106 when $t/d = 0.16$ and $p = 0.0105$ when $t/d = 0.18$. The smaller t/d ratio gives slightly higher values of p. The main effects of the t/d ratio are the moment value or the $\Phi M_{uo}/(bd^2)$ stress which causes the depth of the stress block d_s to be equal to the flange thickness t and the limit of b/b_w for a maximum value of $k_u = 0.4$. These limits are shown on the charts. For intermediate t/d ratios the charts may be interpolated.

It is important to realise that b is the flange width and that the steel ratio p is in terms of the flange width and not the web width. Ratios of b/b_w less than 1.0 designate inverted T-beams or T-beams with negative flanges which are designed using the same procedures.

EXAMPLE 3

Determine the area of reinforcement for the beam shown in Figure 6.8.

SOLUTION

Data: $b = 150$ $b_w = 350$ $d = 540$ $t = 100$ $f'_c = 25$ MPa $M^* = 175$ kNm

$$\frac{M^*}{bd^2} = \frac{175 * 10^6}{150 * 540^2} = 4.0 \text{ MPa}$$

$$\frac{t}{d} = \frac{100}{540} = 0.185 \qquad \frac{b}{b_w} = \frac{150}{350} = 0.43$$

Using Chart T-18A for t/d = 0.18, read the required steel ratio p = 0.0114 for $\frac{\Phi M_{uo}}{bd^2}$ = 4.00 MPa and b/b_W = 0.43.
Using the smaller value of t/d, the steel ratio read from T-18A will be slightly conservative, a more accurate value can be obtained by interpolating between the results obtained from Charts T-18 and T-20 although the difference may not be discernable.

Hence the required tensile area,

$$A_{st} = 0.0114*150*540$$
$$= 923 \text{ mm}^2$$

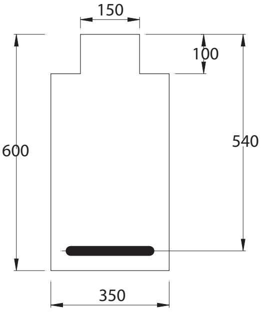
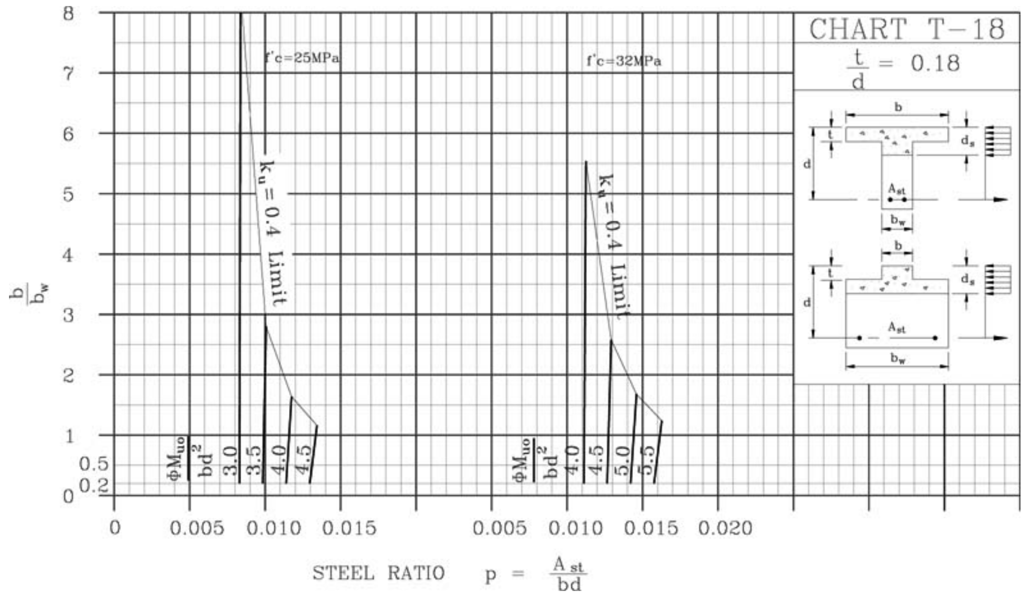


Figure 6.8



PROBLEMS

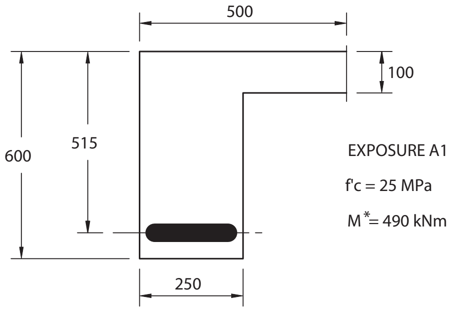
QUESTION 1

The L-beam shown below is part of a beam and slab construction. Determine the depth of the stress block to show that the beam is to be designed as an L-beam and design the beam. The flange width shown may be taken as the effective flange width.

f'_c = 25 MPa

M^* = 490 kNm

Exposure Classification A1



QUESTION 2

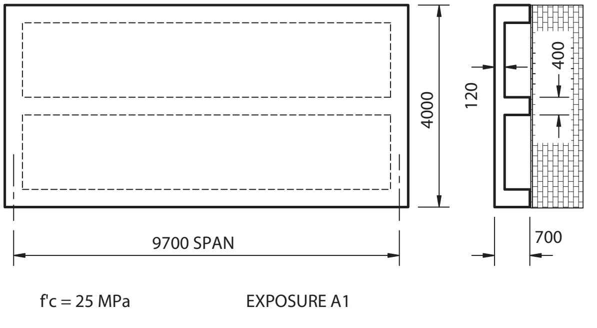
Figures shown below represent an integral beam and slab construction which is required to support a superimposed dead load (not including the weight of construction) g = 25 kPa and a superimposed live load q = 37 kPa.

(a) Determine the design load and hence the design moment to be carried by the intermediate beam.

(b) Determine the effective flange width for the T-beam and the L-beams.

(c) Design the intermediate T-beam. Note that because of the large moment there will likely be more than one row of reinforcement.

f'_c = 25 MPa, Exposure Classification A1



7

chapter

Developmental length and termination of reinforcement

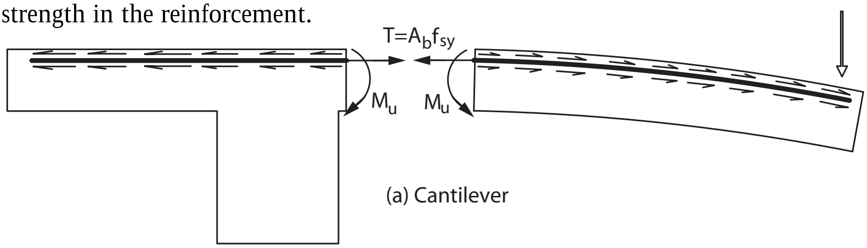
7.1 Additional Symbols used in this Chapter

A_b	=	Cross-sectional area of reinforcing bar.
a	=	The cover over a deformed bar or half the distance between parallel bars whichever is the lesser.
d_b	=	Bar diameter.
ϵ_c	=	Concrete strain.
$L_{sy.c}$	=	Development length for compressive reinforcement at yield condition.
$L_{sy.t}$	=	Tensile development length i.e. minimum length of embedment required to develop yield strength of a reinforcing bar in tension.
L_{st}	=	Tensile development length for $f_{st} < f_{sy}$.
M_y	=	Moment causing initial yield of reinforcement.

7.2 Development Length

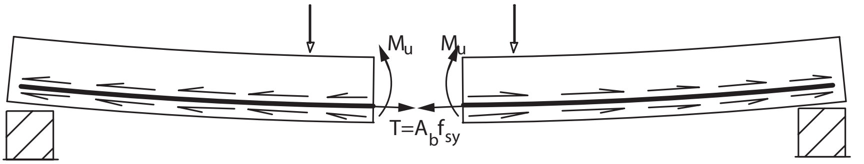
Determining the size of a beam and the reinforcement required to carry a given design moment is but one part of the overall design process. A reinforced concrete beam can only function if the reinforcement is effectively bonded to the concrete and the length of embedment is sufficient to carry the tensile force in the reinforcement. Considering Figure 7.1a and 7.1b, in each case the reinforcement is chosen to yield at the point of maximum bending moment. On either side of the maximum bending moment, the reinforcing bars must be embedded for a sufficient length to develop the tensile yield strength in the reinforcement.

Figure 7.1



(a) Cantilever

Figure 7.1



(b) Simply Supported Beam

The Code #13.1.2.1 gives the tensile development length as,

$$L_{sy.t} = \frac{k_1 k_2 f_{sy} A_b}{(2a + d_b)\sqrt{f'_c}} \geq 25 k_1 d_b \quad \textbf{(7.1)}$$

Where:

k_1 = 1.25 for "top" bars i.e. bars with more than 300mm of concrete cast below the bar.
= 1.0 otherwise.

k_2 = 1.7 for bars in slabs and walls if the clear distance is 150mm or more.
= 2.2 for longitudinal bars in beams and and columns with fitments.
= 2.4 for any other longitudinal bars.

A_b = Cross-sectional area of reinforcing bar.

2a = The lesser of twice the cover to the deformed bar and the clear distance between parallel bars.

Some explanations of the multipliers k_1 and k_2 may be in order. Bleeding of freshly placed concrete is a form of segregation. While excessive bleeding can be reduced with proper mix design, it cannot be eliminated altogether. As concrete settles, bleed water will accumulate under the reinforcing bars to leave voids after hardening and drying out of the concrete. The deeper the concrete pour, the greater will be settlement. Reinforcing bars near the top of a beam may not be fully surrounded by concrete causing a loss of bond. Such losses can only be compensated by increasing the the contact area i.e. by increasing the length of embedment. The Code considers that if the depth of concrete below the reinforcing bars is greater than 300 mm, settlement of concrete may cause 25% loss in bond which must be compensated by increasing the length of embedment.

Bond between the reinforcing bars and the concrete is the result of chemical adhesion, friction and positive bearing between the concrete and the deformations on the surface of the reinforcing bars. The effect the reinforcing bars placed in tension is a shear force over the surface area of the reinforcing bars known as bond stresses. A secondary effect is a radial bursting pressure in the concrete around the reinforcing bars. This is analogous to the pressure in a water pipe which may cause the pipe to split longitudinally. Bond failure may take the form of pulling out of the reinforcing bars or more commonly longitudinal splitting of the concrete either between bars or from the reinforcing bars to the surface. If the bars are placed close together, the bursting pressures between the bars will combine and increase the possibility of longitudinal cracks between the bars. The Code states that if the bars in walls or slabs are placed closer than 150 mm, the development length is increased with the larger multiplier k_2 = 2.4 rather than the value of 1.7 when the bars are placed more than 150 mm apart.

Table 7.1 is drawn to allow direct reading of the tensile developmental length $L_{sy.t}$ given by equation 7.1. The following notes should be read in conjunction with Table 7.1:

TABLE 7.1 TENSILE DEVELOPMENT LENGTH Lsy.t

Tensile Development Length $L_{sy.t}$ in mm for Standard Compaction

		Bar Size						
f′c	a	N12	N16	N20	N24	N28	N32	N36
	15	592	961	1382	1841	2337	2853	3400
	20	478	790	1151	1554	1993	2457	2953
	25	401	670	987	1344	1737	2157	2609
	30	345	582	864	1184	1540	1923	2338
	35	303	514	768	1058	1383	1734	2117
25	40	300	461	691	956	1255	1579	1934
	45	300	417	628	872	1148	1450	1781
	50	300	400	576	802	1059	1340	1650
	55	300	400	531	742	982	1246	1537
	60	300	400	500	646	858	1092	1352
	65	300	400	500	606	807	1028	1275
	15	523	850	1221	1628	2065	2522	3005
	20	423	698	1018	1373	1762	2171	2610
	25	354	592	872	1188	1536	1907	2306
	30	305	514	763	1046	1361	1699	2066
	35	300	454	678	935	1222	1533	1871
32	40	300	407	611	845	1109	1396	1710
	45	300	400	555	771	1015	1281	1574
	50	300	400	509	709	936	1184	1458
	55	300	400	500	656	868	1101	1359
	60	300	400	500	610	809	1029	1271
	70	300	400	500	600	758	965	1195
	15	468	760	1092	1456	1847	2255	2688
	20	378	624	910	1228	1576	1942	2334
	25	317	530	780	1062	1374	1705	2063
	30	300	460	683	936	1217	1520	1848
	35	300	406	607	836	1093	1371	1674
40	40	300	400	546	756	992	1249	1529
	45	300	400	500	690	908	1146	1408
	50	300	400	500	634	837	1059	1304
	55	300	400	500	600	776	985	1215
	60	300	400	508	600	724	920	1137
	65	300	400	500	600	700	863	1069
	15	419	680	977	1302	1652	2017	2404
	20	338	558	814	1099	1409	1737	2088
	25	300	474	698	950	1229	1525	1845
	30	300	411	611	837	1089	1359	1653
50	35	300	400	543	748	978	1226	1497
	40	300	400	500	676	887	1117	1368
	45	300	400	500	617	812	1025	1259
	50	300	400	500	600	749	948	1167
	55	300	400	500	600	700	881	1087

Notes on Table 7.1

(1) Intermediate values may be interpolated.

(2) All tabulated and plotted values are for k_1 = 1.0 and k_2 = 2.2.

(3) For top bars the development length is calculated by multiplying tabulated and the plotted values by 1.25 (the value of k_1 for top bars).

(4) For slabs in which the clear spacing between reinforcing bars is more than 150 mm, k_2 = 1.7. The tabulated and the plotted values may be adjusted by multiplying by 1.7/2.2 = 0.773.

(5) For slabs in which the clear spacing between reinforcing bars is 150mm or less, and for beams and columns without fitments, k_2 = 2.4. The tabulated and the plotted values may be adjusted by multiplying by 2.4/2.2 = 1.09.

The development lengths are tabulated for discrete concrete covers. In beams and columns the concrete cover for longitudinal reinforcement will exceed the minimum cover because of fitments.

For plain bars used as fitments with diameters $d_b \le 13$ mm, the development length is taken as,

$$L_{sy.t} = 40d_b \ge 300\text{mm}$$

For hard drawn wire the development length is taken as,

$$L_{sy.t} = 50d_b$$

7.3 Reinforcement Stressed Below Yield Stress

The Code requires that the tensile force carried by the reinforcement must be developed by bond at any section. At a point in the beam where the bending moment is say one half of the maximum bending moment, the reinforcement will only be stressed to one half of the yield stress. The tensile force carried by the reinforcing bars will also be halved and the length of embedment L_{st} required to develop the tensile force will only be one half of $L_{sy.t}$ the development length required at the point of maximum bending moment where the reinforcement is at its yield point. Thus the development length required at any section, where the reinforcement is not fully stressed, may be calculated as the stress ratio of $L_{sy.t}$. The Code requires that this length be not less than 12 bar diameters.

$$L_{st} = L_{sy.t}\frac{f_{st}}{f_{sy}} \ge 12d_b$$

7.4 Hooks and Cogs

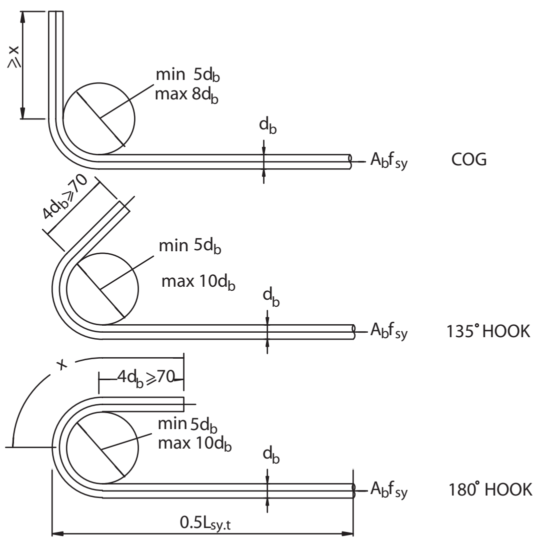
Occasions frequently arise when it is not possible to provide a length of embedment equal to the development length requirement. Two possible courses of action may be taken.

(a) Reduce the size of the reinforcement. For example a beam in exposure classification A1 uses grade N25 concrete. The concrete cover which satisfies Code requirement is 20 mm. The required tensile reinforcing area is 1260 mm^2. Either 4N20 bars or

2N28 bars may be used having similar areas of 1256 and 1232 mm^2. However from Table 7.1 the development length required for the N20 bars is 1151 mm while for the N28 bars 1993 mm is the minimum development length. If the beam is 3000 mm long, then it is obvious that N28 reinforcing bars could not be used since the available length of embedment of reinforcement from the point of maximum bending moment (at mid-span) is only 1500 less end cover.

(b) The second choice is to hook the reinforcing bars at the ends. The pull-out resistance will be increased substantially if a hook is present at the end of a reinforcing bar. The contribution of a standard hook is given by the Code #13.1.2.4 as the equivalent resistance provided by a straight length of bar equal to $0.5L_{sy.t}$. In other words with a hook a length of only $0.5L_{sy.t}$ is required to be embedded. This length is measured to the outside of the hook. For the example used in (a) with N28 bars, the length of embedment required from mid-span will be 0.5×1993 = 946 mm. This could easily be accommodated.

Figure 7.2 Standard Hooks and Cogs



The Code considers a standard hook as being a hook with a 135° or 180° bend plus a straight extension of $4d_b$ but not less than 70 mm, or a cog made up of a 90° bend having the same total length as that provided by a 180° hook. This is shown in Figure 7.2. The minimum diameter of the pin around which reinforcing bars are bent is give by #19.2.3.2 as $5d_b$. The minimum extensions of the straight portion for cogs required to satisfy Code requirements are shown in Table 7.2. These are based on an internal diameter of bends equal to $5d_b$. The anchorage provided by hooks and cogs will be lost if the radius of curvature of the bends is excessively large. If the internal diameter of a bent bar is equal to $10d_b$ or greater, the actual length of bar measured around the curve is used to determine development length. With cogs, the Code specifies a maximum internal bend diameter of $8d_b$ while with hooks the maximum implied bend diameter is a diameter less than $10d_b$.

Table 7.2 Minimum Straight Extension for

Cogs Based on a $5d_b$ Internal Diameter

Bar Size	Min. Straight Length x (mm)
N12	120
N16	135
N20	160
N24	190
N28	225
N32	255
N36	285

Even with cogs or hooks there are instances when the available length of embedment is insufficient to develop the tensile force carried by the reinforcement. Common practices use some form of end plates or anchor plates welded to the reinforcement as shown in Figure 7.3 (*below*). The end plate is fillet or even butt welded to the reinforcing bars.

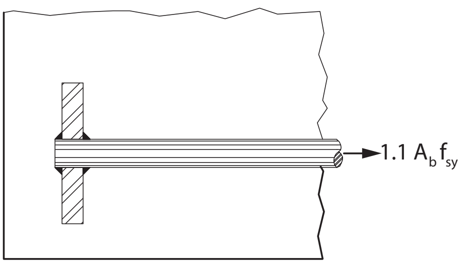


Figure 7.3

7.5 Curtailment of Tensile Reinforcement

Some reinforcement may be terminated at sections along the beam where it is no longer required. For example, in a simply supported beam carrying uniformly distributed loads, one half of the tensile reinforcement provided at mid-span may be theoretically terminated 0.146L from each support. This is the position in the beam where the bending moment is one half the mid-span moment. The length of the terminating reinforcement would need to be at least equal to the development length $L_{sy.t}$. either side of mid-span to satisfy development length.

It is assumed here that the reinforcement has only yielded at the point of maximum bending moment. In fact, the tensile reinforcement reaches its yield point when the bending moment is about 5% smaller than the ultimate bending moment. This implies that at ultimate moment conditions the tensile reinforcement has yielded over a length of beam where the bending moment is greater than $0.95M_{uo}$.

The CEB/FIB concrete stress equation was used to calculate the ratio of the bending moment at first yield of the reinforcement to the ultimate moment M_y / M_{uo}. The results are shown in Table 7.3.

Table 7.3 Moment Ratios for f'_c = 32 MPa

$\frac{p}{p_{max}}$	ε_c	f_c	$\frac{M_y}{M_{uo}}$
0.1	0.0004	11.8	0.964
0.2	0.0006	16.0	0.964
0.3	0.0008	19.0	0.964
0.4	0.0010	21.2	0.965
0.5	0.0012	22.9	0.969
0.6	0.0013	24.3	0.973
0.7	0.0015	25.4	0.977
0.8	0.0017	26.2	0.982
0.9	0.0019	26.7	0.988
1.0	0.0021	26.9	0.993

Table 7.3 gives the moment ratios over the full range of steel ratios. As shown in the table the yielding moment M_y is only slightly less than the ultimate bending moment M_{uo}. However, even a 4% difference can have important implications in the termination of reinforcement and the required development length. This is illustrated by example 1 below.

EXAMPLE 1

The simply supported beam shown in Figure 7.4 carries a design load w*. If the yield moment $M_y = 0.964M_{uo}$, determine the region over which the reinforcement has yielded at ultimate strength conditions.

$$\text{Yield moment, } M_y = 0.964M_{uo} = 0.964\,\frac{w^*L^2}{8}$$

Equating the bending moment distance y from the support to the yield moment,

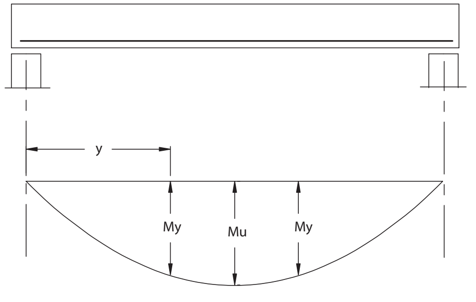
$$\frac{w^*L}{2}y - \frac{w^*y^2}{2} = 0.964\,\frac{w^*L^2}{8}$$

$$L\,y - y^2 = 0.241L^2$$

$$y^2 - L\,y + 0.241L^2 = 0$$

$$y = 0.5L \pm 0.095L$$

Figure 7.4

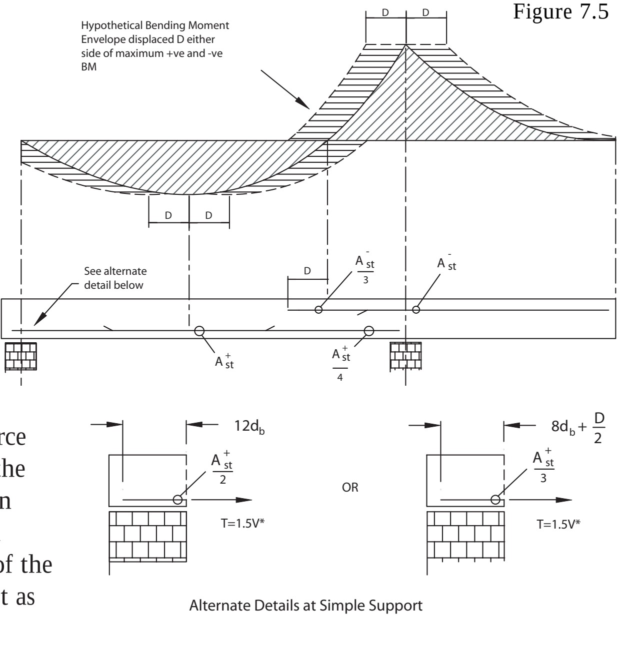


That is, the yielding moment occurs 0.095L either side of mid-span so that the reinforcement has yielded within a 0.19L central region of the beam. The reinforcement will need to extend a distance $L_{sy.t}$ on either side to satisfy anchorage. If the beam is reinforced with more than one row of reinforcement, then the bottom row will yield even earlier. Practically it may be taken that at ultimate conditions the tensile reinforcement has yielded in the central 0.25L of a simply supported beam. For all positive reinforcing bars the development length $L_{sy.t}$ should be provided for sections displaced 0.125L either side of the maximum bending moment.

The Code #8.1.8.1 requires that termination and anchorage of flexural reinforcement be based on a hypothetical bending moment diagram by displacing the positive and negative bending moments distance D either side of the maximum bending moment. The Code condition shown in Figure 7.4 is based on a beam truss analogy. The author feels that this may not always be conservative.

Figure 7.5 (*right*) summarises Code #8.1.8 dealing with termination and anchorage of positive and negative reinforcement at simple and restrained or continuous supports.

Figure 7.5



Positive reinforcement at a simple support:

Sufficient positive reinforcement must be carried to enable it to develop a tensile force of 1.5V* at the face of the support. V* is the design shear force at the critical section d from the face of the support or at the support as per section 8.4.2(b).

Either a minimum of one half of the tensile positive reinfo-rcement required at mid-span must extend 12db or the equivalent anchorage past the face of the support,

OR

one third of the tensile positive reinforcement must be carried a distance $8d_b$+D/2 past the face of the support.

Positive reinforcement at a restrained or continuous support:

At least one quarter of the positive reiforcement must be continued past the near face of the support.

Negative reinforcement:

At least one third of the total negative reinforcement must extend a distance D past the point of contraflexure.

Curtailment or termination of tensile reinforcement can setup severe stress conditions, especially if a large number of reinforcing bars are terminated at the one section. Bursting pressure in the concrete may lead to longitudinal splitting and shear failure due to reduced dowell action as the tensile force and the shearing resistance shared by a large number of reinforcing bars is suddenly transferred to a smaller number of reinforcing bars. The Code #8.1.8.4 requires that where tensile reinforcement is terminated one of the following conditions must be satisfied;

Not more than one quarter of the maximum tensile reinforcement is terminated within any distance 2D.
OR
At the cut-off point the shear capacity of the beam ΦV_u is not less than 1.5 times the design shear force V^* .
OR
Shear reinforcement in the form of stirrups with an area $(A_{sy} + A_{sv.min})$ is provided for a distance D along the terminating bars from the cut-off point.

7.6 Continuous Beams

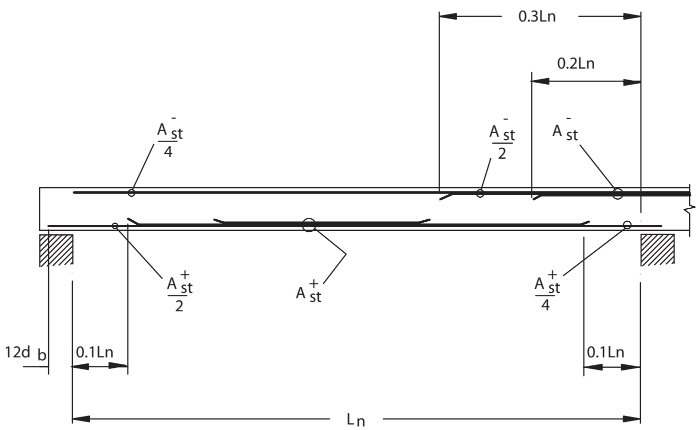


Figure 7.6 - Termination and anchorage of tensile reinforcement in continuous beams where the longer span is ≤ 1.2 times the shorter span in any two adjacent spans and the live load q ≤ 2× the dead load g

For continuous beams which would normally be analysed using bending moment and shear force coefficients, Code compliance is simplified by #8.1.8.6 summarised in Figure 7.6. To satisfy shear conditions, not more than one quarter of the maximum tensile reinforcement may be terminated within a distance 2D.

7.7 Development Length of Bundled Bars (#13.1.4)

Bundles of 2, 3 and 4 reinforcing bars may be used in heavily reinforced members where congestion of reinforcement may otherwise result. Naturally with bundles of 3 and 4 bars, not all the surface area of the individual bars will be surrounded by concrete. Because of the predictable loss in bond, the Code requires that the development length of the largest bar in the bundle be increased by:

(a) 20% for a 3-bar bundle and

(b) 33% for a 4-bar bundle.

7.8 Development Length for Mesh in Tension (#13.1.5)

The development length of the longitudinal wires in welded wire mesh is satisfied by the embedment of at least two transverse wires with the closer wire 25 mm from the critical section.

7.9 Development Length of Compressive Reinforcement (#13.1.3)

The development length required to develop the yield strength f_{sy} of compressive reinforcement, $L_{sy.c}$, is taken as $20d_b$. Hooks may not be considered to contribute to the development length.

7.10 Lapped Splices for Tension Bars (#13.2.2)

The minimum length of tension lap splices is $L_{sy.t}$, the development length for tensile reinforcement given by equation 7.1 and Table 7.1.

7.11 Lapped Splices for Reinforcing Mesh in Tension (#13.2.3)

The minimum length of lap splices is taken to be at least equal to the development length i.e. two transverse wires.

7.12 Lapped Splices for Compression Reinforcement (#13.2.4)

The minimum length of lap splice is taken as the compressive developmental length $L_{sy.c}$ but not less than 300 or $40d_b$ where d_b is the diameter of the smaller bar.

EXAMPLE 2

A cantilevered reinforced concrete beam shown in Figure 7.7 supports dead and live loads which produce design load conditions 1 and 2 shown in the figure. Grade N32 concrete is used for the beam. The bending moment envelope for the two loading conditions (live load over cantilever and live load between supports) is also shown in the figure. The beam has been designed for the maximum positive and maximum negative bending moments. Cross-sections for the maximum moment conditions are shown in Figure 7.7 as well. In accordance with the Code, curtail as much reinforcement as possible and draw a final layout of the longitudinal reinforcement. The reaction force at A is 228 kN for loading condition 1 and 108 kN for loading condition 2.

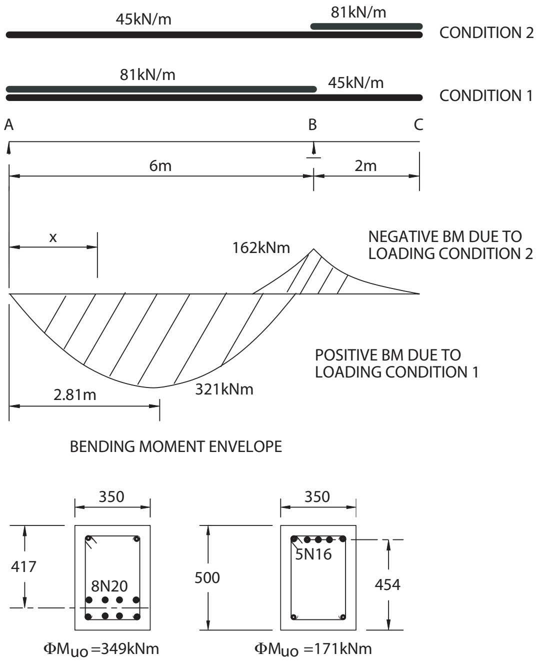
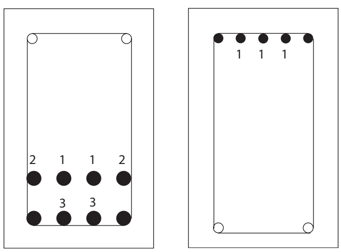


Figure 7.7

SOLUTION

The order in which the positive and negative reinforcement will be terminated is shown in Figure 7.8 (*below*).



CURTAILMENT OF POSITIVE REINFORCEMENT

(1) Termination of No.1 bars (6 bars remaining).

Problems involving termination of reinforcement may be simplified by assuming that the moment capacity is proportional to the area of the reinforcement. This simplification will yield conservative results by slightly underestimating the beam moment capacity. The theoretical position of curtailment is obtained by equating the bending moment expression (in terms of the unknown distance x from support A) to the moment capacity.

Moment capacity with 6N20 bars,

$$M_6 = \frac{6}{8} \times 349 = 262 \text{ kNm}$$

The bending moment at any distance x from support A,

$$R_A x - w \frac{x^2}{2} = Mx$$

Solving the quadratic in x,

$$x = \frac{R_A \pm \sqrt{R_A^2 - 2 \times M_x w^*}}{w^*} \qquad \textbf{(7.2)}$$

This is a general equation to be used for determining distance x from the support where reinforcing bars may be terminated. Substituting for R_A and Mx and solving for x when two of the reinforcing bars may be terminated.

$$x = \frac{228 \pm \sqrt{228^2 - 2 \times 262 \times 81}}{81}$$

$$= 1.6 \text{ m and } 4.0 \text{ m}$$

(2) Termination of bars No.2 (4 bars remaining).
Moment capacity of section with remaining 4 bars,

$$M_2 = \frac{4}{8} \times 349 = 174.5 \text{ kNm}$$

Substituting in equation 7.2 to deterime distances x where the additional two bars may be terminated.

$$x = \frac{228 \pm \sqrt{228^2 - 2 \times 174.5 \times 81}}{81}$$

$$= 0.91 \text{ m and } 4.72 \text{ m}$$

(3) Termination of bars No.3 (2 bars remaining).
Moment capacity of section with remaining 2 bars,

$$M_3 = \frac{2}{8} \times 349 = 87.3 \text{ kNm}$$

The theoretical position where these bars may be terminated,

$$x = \frac{228 \pm \sqrt{228^2 - 2 \times 87.3 \times 81}}{81}$$

$$= 0.41 \text{ m and } 5.22 \text{ m}$$

It should be noted that at the simple support A, one half of the reinforcement must be carried a minimum distance $12d_b$ (=240 mm) past the face of the support and a minimum of one quarter of the positive reinforcement must be carried to the interior or moment resisting support B. Thus it will only be possible to terminate bars No.3 near support B.

Curtailment of Negative Reinforcement

While the Code does not make any reference to this aspect, negative reinforcement should not be terminated in a cantilever. Since one third of the negative reinforcement must be carried a minimum distance D past the point of contraflexure, not more than three reinforcing bars can be curtailed. Bars No.1 will be curtailed at the one section. This exceeds the condition that not more than one quarter of the reinforcement may be terminated within a distance 2D. It then becomes necessary to check the beam for shear. This will be left until later when shear in beams has been covered.

Negative moment capacity of beam with remaining 2 bars,

$$M_1 = \frac{2}{5}171 = 68.4 \text{ kNm}$$

Equation 7.2 may again be used to calculate the theoretical position of curtailment. Note however that the maximum negative moment is due to loading condition 2 and the corresponding loads (w* = 45 kN/m) and reactions (R_A = 108 kN) must be used.

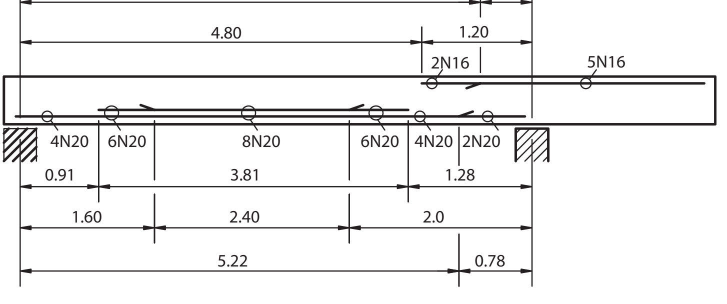
$$x = \frac{108 + \sqrt{108^2 - 2 \times 68.4 \times 45}}{45}$$
$$= 4.05 \text{ m}$$

It is also necessary to determine the point of contraflexure. This may be determined from equation 7.5 when the moment is zero.

$$x = \frac{2 \times 108}{45} = 4.8 \text{ m}$$

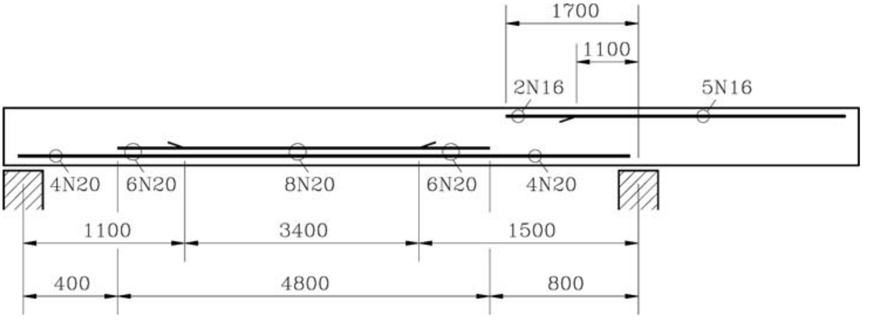
The calculated theoretical points of curtailment are shown in Figure 7.9 (*below*).

Figure 7.9



The actual termination of reinforcement must satisfy the Code requirement of a hypothetical bending moment envelope obtained by displacing the actual bending moment envelope a uniform distance D each side of the maximum positive and negative bending moments. This simply means that the reinforcement will be carried a distance D past the theoretical points of cut-off calculated above. The new layout and distribution of longitudinal reinforcement is shown in Figure 7.10 (*below*).

Figure 7.10



It now only remains to check that development lengths have been provided. The concrete cover using 12mm stirrups is 38mm for the nagative reinforcement and 47mm for the positive reinforcement. Assuming 20mm minimum cover, the actual concrete side cover for the longitudinal reinforcement is 32mm.

$$\text{Clear spacing} = \frac{350 - 2\times 32 - 4\times 20}{3} = 68 \text{ mm } < 94 \text{ (Twice the cover)}$$

Assuming N32 spacer bars between each row of positive reinforcement, the clear spacing between bars is 32 mm which is less than the horizontal spacing of 68 mm and less than twice the cover of 94 mm. The development length will therefore be calculated for a = 32/2 = 16 mm. From Table 7.1 $L_{sy.t}$ = 1180 mm by interpolation between a = 15 mm and a = 20 mm.

Similarly the clear spacing between the negative reinforcing bars is calculated to be (350 - 2*32 - 5*16)/4 = 51.5 mm. This is less than twice the 38 mm cover. The required development length for the N16 negative reinforcement is obtained from Table 7.1 for a = 51.5/2 = 26 mm.

$L_{sy.t}$ = 576 mm

The negative reinforcing bars are top bars with more than 300 mm of concrete cast below the bars. The development length must therefore be increased by factor k_1.

$L_{sy.t}$ = 1.25*576 = 720 mm

The reinforcement layout in Figure 7.10 should now be checked to ensure that the above development lengths are available for all positive and negative reinforcement.

PROBLEMS

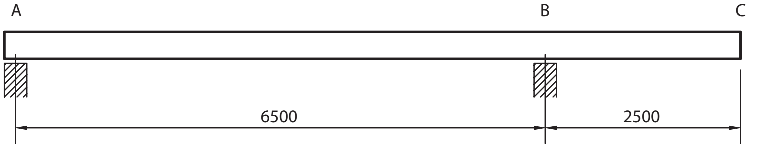
QUESTION 1

The cantilever beam shown below supports a uniformly distributed dead load g = 38 kN/m (including weight of beam) and a uniformly distributed live load q = 25 kN/m. Grade N40 concrete is to be used for the beam which is in Exposure Conditions B1. The beam supports are 200 mm wide.

(a) Draw the bending moment envelope to include live load applied between the supports (condition 1) and live load applied on the cantilever only (condition 2).

(b) The beam size b = 300 mm, D = 550 mm is to be used for the full length of the beam. Choose the reinforcement for the maximum positive and the maximum negative bending moments given that only N24 bars are available to you.

(c) Terminate as many bars as possible so that Code conditions are satisfied and show the final beam details on an appropriate drawing.



8

chapter

Serviceability

8.1 Additional Symbols used in this Chapter

$A_{st.min}$	=	Minimum area of reinforcement.
A_{ct}	=	Cross-sectional area of uncracked concrete in the tensile zone.
b_{eff}	=	Effective beam width or effective flange width.
ϵ_{cs}	=	Design shrinkage strain (from Section 6.1.7.2 - AS3600).
$\epsilon_{cs.b}$	=	Basic shrinkage strain.
Ec	=	Modulus of elasticity for concrete at 28 days.
Ecj	=	The mean value of modulus of elasticity of concrete at nominated age.
	=	$\rho^{1.5}*0.043\sqrt{f_{cm}}$
$F_{d.eff}$	=	Effective design load for serviceability in kN/m or kN/m^2.
f_{cm}	=	Mean compressive strength of concrete at relevant age.
f_{cs}	=	Max shrinkage-induced stress on uncracked sections at the extreme fibre where cracking first occurs.
f_{scr}	=	Tensile stress in the reinforcement (at the cracked section) due to 'short term' serviceability loads under direct loading.
$f_{scr.1}$	=	As above but using ψ s =1.0 (rather than 0.7).
k_1	=	Second moment of area multiplier.
k_2	=	Deflection constant for rectangular beams.
k_3	=	Slab multiplier.
k_4	=	Deflection constant for slabs.
k_5	=	Special Slab deflection coefficient read from chart D2.
k_p	=	Deflection correction factor for steel ratio in beams.
k_{cs}	=	Long-term deflection multiplier (to account for shrinkage & creep).
k_s	=	Coefficient to take account of the stress distribution shape in a section prior to cracking (0.6 for flexure & 0.8 for tension).

k_d	=	Depth of N.A. at working/serviceability load conditions.
L_{eff}	=	Effective span, also noted as L_{ef}.
	=	Lesser of (L_n + D) and L. = (L_n + D/2) for a cantilever.
L_n	=	Clear span between beams, columns or walls (cantilever=clear projection).
L_x	=	Shorter effective span of slab supported on four sides.
L_y	=	Longer effective span of a slab supported on four sides.
M^*_s	=	Design bending moment (at the Serviceability limit state).
$M^*_{s.1}$	=	As above but using ψ s =1.0 (rather than 0.7).
w*	=	Design load used for strength conditions.
t_h	=	Hypothetical thickness used to calculate creep and shrinkage.
	=	$2A_g/u_e$
u_e	=	Exposed perimeter plus half perimeter of enclosed voids.
δ	=	Deflection obtained from calculations.
Δ	=	Maximum deflection.
ρ	=	Density of concrete in kg/m^3, taken as 2400 kg/m^3 in these notes.

8.2 Serviceability Considerations

A structure and all its structural components must perform the functions for which they are designed. Deflection is one of the main criteria to be satisfied. The results of excessive deflection may:

(a) Produce a feeling of concern in the safety of a structure.

(b) Cause excessive distortion in window and door openings rendering them unserviceable (they may become permanently stuck).

(c) Cause cracking of masonry walls and spalling of finishes such as tiles.

(d) Cause ponding of water in exposed suspended slabs when drainage is inadequate.

The other serviceability criterion is flexural cracking, which is also associated with durability. Cracks in the concrete are unsightly and excessive cracks will allow water to reach the reinforcement and promote corrosion. Limiting of flexural or other cracks may be achieved by good detailing practice. .

8.3 Deflection

It is a comparatively simple matter to calculate the deflection of say a simply supported steel beam carrying a uniformly distributed load w. The material and section properties are known and it is only a matter of applying a standard deflection formula.

$$\text{Deflection} = \frac{5wL^4}{384\,EI}$$

In reinforced concrete, however, the following problems must be considered:

(a) Since a reinforced concrete flexural member is made up of steel reinforcement and concrete, what value should be use for the modulus of elasticity E? This is further complicated by the fact that concrete is not a perfectly elastic material.

(b) The flexural member is cracked below the neutral axis. Taking this into account and the difference in the properties of the materials, how is the second moment of area I to be calculated?

(c) With the flexural cracks occurring at variable spacings along the beam, in between the cracks, the flexural member is much stiffer due to the stiffness of the uncracked portions. The stiffening effect of the uncracked portions is to reduce deflection. The second moment of area should be increased to account for increased stiffness.

(d) Assuming the problems noted in (a), (b) and (c) are solved, the deflection formula may be used to calculate the immediate deflection upon loading. Shrinkage and creep are inherent properties of concrete which lead to substantial additional long term deflection (about twice the initial deflection). Shrinkage and creep rates depend on many factors such as the type of cement, size and properties of the aggregates, additives, water cement ratio, curing conditions, distribution of reinforcement, duration and intensity of loading, just to name a few.

It is quite apparent from the forgoing discussion that deflection can never be accurately calculated. There are three ways to satisfy serviceability:

(a) Deemed to comply condition.

(b) Deflection calculated by simplified calculations.

(c) Deflection determined by refined calculations.

Deflection limitations are given by Code Table #2.4.2, duplicated in Table 8.1 below.

Table 8.1 - Limits for Calculated Deflection of Beams and Slabs

Type of Member	Deflection to be Considered	Deflection Limitation Δ/L_{eff}	
		for Spans Notes 1 and 2	for Cantilevers Notes 3
All members	The total deflection	1/250	1/125
Members supporting masonry partitions	The deflection which occurs after the addition or attachment of the partitions.	1/500 where provision is made to minimise the effect of movement, otherwise 1/1000	1/250 where provision is made to minimise effect of movement, otherwise 1/500
Bridge members	The live load and impact deflection	1/800	1/400

Notes:

1 In flat slabs, the deflection to which the above limits apply is the theoretical deflection of the line diagram representing the idealised frame.

2. Deflection limits given may not safeguard against ponding.

3. For cantilevers, the value of Δ/L_{ef} given in this table applies only if the rotation at the support is included in the deflection calculations.

8.4 Deemed to Comply Conditions #8.5.4 and #9.3.4

This is by far the simplest method since it does not involve deflection calculations. Deflection is deemed to have satisfied limitations give by Table 8.1 if the ratio effective span to effective depth L_{eff}/d does not exceed the maximum ratio given by the following equations for beams and slabs:

$$\frac{L_{eff}}{d} = \left(\frac{k_1 \frac{\Delta}{L_{eff}} b_{eff} E_c}{k_2 F_{d.eff}} \right)^{1/3} \quad \text{for beams} \tag{8.1}$$

$$\frac{L_{eff}}{d} = k_3 k_4 \left(\frac{1000 \frac{\Delta}{L_{eff}} E_c}{F_{d.eff}} \right)^{1/3} \quad \text{for slabs} \tag{8.2}$$

Where; L_{eff} = Effective span.

$\frac{\Delta}{L_{eff}}$ = Deflection limit given by Table 8.1.

$F_{d.eff}$ = The effective design load in kN/m for beams and kN/m^2 for slabs given by:
= $(1 + k_{cs})g + (\psi_s + \psi_l k_{cs})q$ for total deflection and,
= $k_{cs} g + (\psi_s + \psi_l k_{cs})q$

for deflection which occurs after the addition or attachment of partitions (i.e. the incremental deflection).

k_{cs} = Long term deflection multiplier i.e. the additional long-term deflection is calculated by multiplying the immediate deflection by k_{cs}.
= $2 - 1.2 \frac{A_{sc}}{A_{st}} \geq 0.8$

k_1 = Second moment area multiplier.
= $0.02 + 2.5p$ for rectangular sections where $p \geq 0.005$.
= $0.1 - 13.5p$ (but ≤ 0.06) for rect. sections where $p < 0.005$.

k_2 = Deflection constant.
= $\frac{5}{384}$ for simply supported beams,
= $\frac{2.4}{384}$ for end spans in continuous beams,
= $\frac{1.5}{384}$ for interior spans in continuous beams.

k_3 = 1.0 for one-way slabs and for two way rectangular slabs carrying uniformly distributed loads where $q \leq g$.
= 0.95 for two-way flat slabs without drop panels.
= 1.05 for two-way flat slabs with drop panels which extend

at least L/6 in each direction on each side of a support centreline and have an overall depth not less than 1.3D where D is the slab thickness without the drop panel.

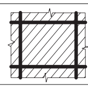
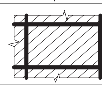
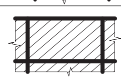
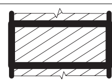
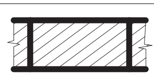
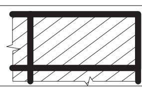
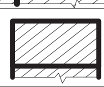
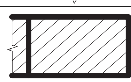
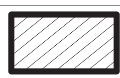
k_4 = The deflection constant taken as:

(a) 1.6 for simply supported one-way slabs.
(b) For continuous one-way slabs (where the span ratio of adjoining spans does not exceed 1.2 and the end spans are no longer than an interior span), the value of k_4 may be taken as 2.0 for an end span or 2.4 for interior spans.

(c) For simply supported two-way slab panels, the value of k_4 = 2.5 may be used provided that $q \leq g$ (see Table 8.2, No. 9.)

(d) For rectangular slabs supported on four sides by walls or beams subjected to uniformly distributed loads where $q \leq g$, the value of k_4 is taken from Table 8.2 below.

Table 8.2 - Slab System Multiplier k4 for Rectangular Slabs Supported on Four Sides

Edge Conditions		Deflection Constant k_4 $\frac{L_y}{L_x}$			
		1.0	1.25	1.5	2.0
1. Four edges continuous		4.00	3.40	3.10	2.75
2. One short edge discontinuous		3.75	3.25	3.00	2.70
3. One long edge discontinuous		3.75	2.95	2.65	2.30
4. Two short edges discontinuous		3.55	3.15	2.90	2.65
5. Two long edges discontinuous		3.55	2.75	2.25	1.80
6. Two adjacent edges discontinuous		3.25	2.75	2.50	2.20
7. Three edges discontinuous, one long edge continuous		3.00	2.55	2.40	2.15
8. Three edges discontinuous, one short edge continuous		3.00	2.35	2.10	1.75
9. Four edges discontinuous		2.50	2.10	1.90	1.70

8.5 Beam Deflections: Deemed-to-Comply

Chart D1 is a plot of equation 8.1 in terms of $(b_{eff}, F_{d.eff})$ for:

(a) Deflection limitation $\dfrac{\Delta}{L_{eff}} = \dfrac{1}{250}$

(b) Second moment area multiplier $k_1 = 0.02 + 2.5p$ if $p \geq 0.005$, or
$= 0.1 - 13.5p$ (but ≤ 0.06) if $p < 0.005$

(For beams with a steel ratio p = 0.01 the value of $k_1 = 0.045$. For beams whose steel ratio is other than 0.01, a correction factor k_p read from the bottom of the chart can be applied to the L_{ef} /d ratio).

(c) Simply supported beams where $k_2 = \dfrac{5}{384}$

(d) End spans of continuous beams where $k_2 = \dfrac{2.4}{384}$

(e) Interior spans of continuous beams where $k_2 = \dfrac{1.5}{384}$

(f) Assumed concrete density of 2400 kg/m^3 is used for calculating the modulus of elasticity of concrete.

Example 1

In a retail construction a simply supported reinforced concrete beam whose cross-section is shown in Figure 8.1 (*right*) has an effective span L_{eff} = 4800 mm. The beam is designed to support a total dead load (including its own weight) g = 16 kN/m and a live load q = 12 kN/m.

(a) Check the beam for total deflection. It may also be assumed that the beam is loaded after a 28 day curing period.

(b) Check the beam for incremental deflection. It may be assumed that the beam supports internal masonry partitions for which there is no provision made to minimise the effects of deflection; i.e. small deflection may lead to cracking of the partitions.

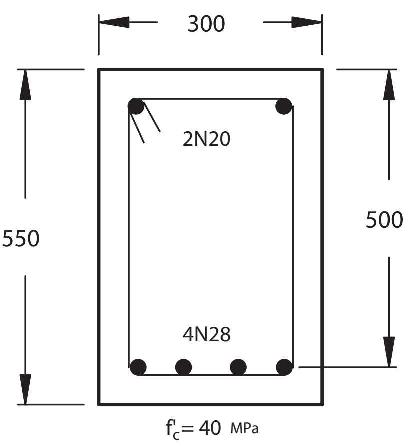


Figure 8.1

SOLUTION

(a) From Table 8.1, for maximum total deflection, $\frac{\Delta}{L_{eff}} = \frac{1}{250}$

$k_2 = \frac{5}{384}$ for simply supported beams.

$L_{eff} = 4800$ $\quad$ $p = 2480/(300 \times 500) = 0.0165$

$b_{eff} = 300$ $\quad$ $k_1 = 0.02 + 2.5 \times 0.0165 = 0.0613$

$\psi_s = 0.7$ from table 1.2

$\psi_l = 0.4$ from table 1.2

$k_{cs} = 2 - 1.2\frac{620}{2480} = 1.7$ $\quad$ $E_c = 34500$ MPa

$F_{d.eff} = (1 + k_{cs})g + (\psi_s + \psi_l\ k_{cs})q$

$= (1 + 1.7)16 + (0.7 + 0.4*1.7)12$

$= 59.8$ kN/m or N/mm

$$\frac{b_{eff}}{F_{d.eff}} = \frac{300}{59.8} = 5.0$$

From chart D1 read maximum $\frac{L_{eff}}{d}$ ratio for total deflection of $\frac{\Delta}{L_{eff}} = \frac{1}{250}$

$\frac{L_{eff}}{d} = 13.3$ $\quad$ $k_p = 1.11$

Maximum $\frac{L_{eff}}{d} = 1.11 \times 13.3 = 14.8$

Alternatively substitute into the formula: $\frac{L}{d} = \left[\frac{0.061\left(\frac{1}{250}\right)300 * 34500}{\left(\frac{5}{384}\right) * 59.8}\right]^{1/3} = 14.8$

Actual $\frac{L_{eff}}{d} = \frac{4800}{500} = 9.6 < 14.8$ SATISFACTORY

That is, the total deflection criteria has been satisfied.

(b) From Table 8.1 the incremental deflection criteria (this is the deflection which occurs after the attachment of the partitions) is $\frac{\Delta}{L_{eff}} = \frac{1}{1000}$

$F_{d.eff} = k_{cs}\ g + (\psi_s + \psi_l\ k_{cs})q$

$= 1.7*16 + (0.7 + 0.4*1.7)12 = 43.8$

$$\frac{b_{eff}}{F_{d.eff}} = \frac{300}{43.8} = 6.85$$

From chart D1 for $\frac{\Delta}{L_{eff}} = \frac{1}{250}$

$\frac{L_{eff}}{d} = 14.8$

Adjust $\dfrac{L_{eff}}{d}$ for $\dfrac{\Delta}{L_{eff}} = \dfrac{1}{1000}$ by multiplying by 0.63.

Maximum $\dfrac{L_{eff}}{d}$ = 0.63*1.11 x 14.8 = 10.3 > 9.6 actual.

The beam satisfies the incremental deflection requirement. If the deflection limitation is exceeded by a small amount a designer would most likely elect to apply a check by calculating beam deflection (using the Code 'simplified' method - section 8.5.3) rather than redesigning the beam.

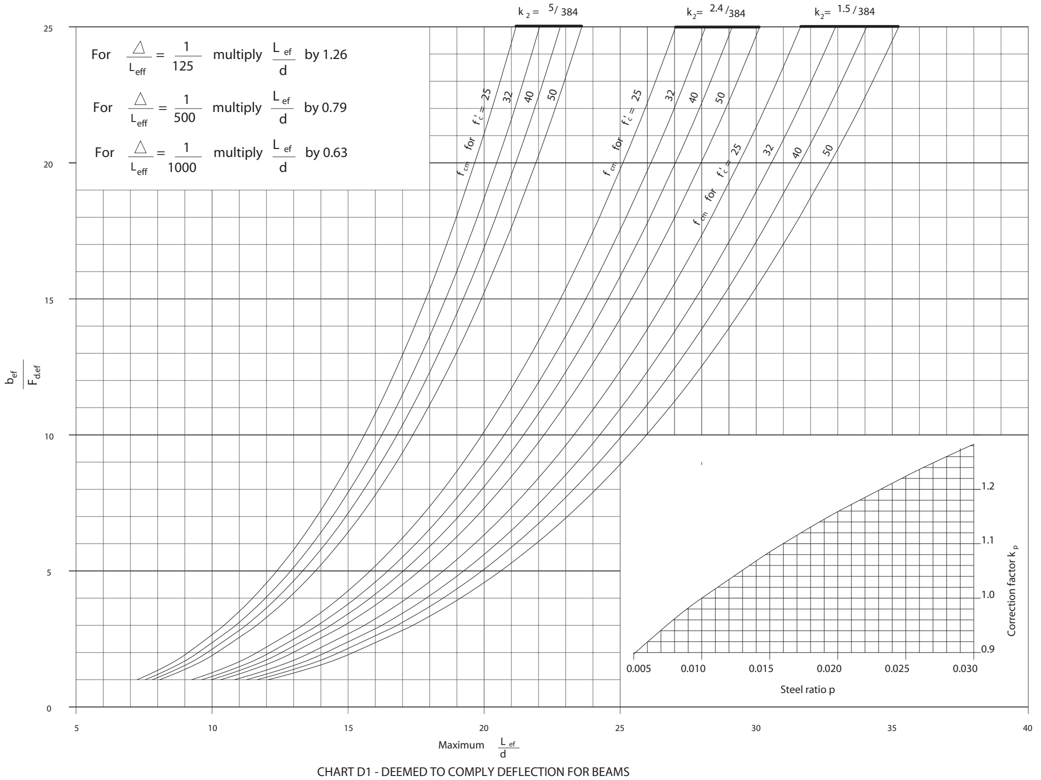


CHART D1 - DEEMED TO COMPLY DEFLECTION FOR BEAMS

With slender beams, the serviceability criteria frequently dictates the beam design. That is to say that the serviceability limit state is the primary limit state. The designer should therefore choose the beam depth to satisfy serviceability and then design the beam for strength (the secondary limit state). This is illustrated by Example 2. It may be worth while noting that compression reinforcement in the form of hanger bars will be assumed, from now on (say 2N20 bars).

EXAMPLE 2

Design a simply supported beam spanning 6.5 m to carry superimposed dead and live loads g = 20 kN/m and q = 24 kN/m respectively. The beam uses grade N50 concrete and it is located in Exposure Classification A2. The beam is an external wall beam supporting a masonry wall. It may be assumed that a number of expansion joints have been included in the wall to minimise wall damage due to excessive deflection. Assume p=0.01 to make correction k_p = 1.0.

SOLUTION

Estimate of beam depth required for serviceability.

Assume, b = 350

k_{cs} = 1.8

Weight of beam = 6 kN/m

From Table 1.2, the short-term and long-term live load multipliers for an office construction,

ψ_s = 0.7

ψ_l = 0.4

Estimated effective load for serviceability.
For total deflection, the effective load,

$$F_{d.eff} = (1 + 1.8)26 + (0.7 + 0.4*1.8)24$$
$$= 106.9 \text{ kN/m or N/mm}$$

$$\frac{b_{eff}}{F_{d.eff}} = \frac{350}{106.9} = 3.27$$

From Chart D1 read $\frac{L_{eff}}{d}$ = 12

Hence required $d = \frac{6500}{12} = 542$

For incremental deflection, the effective load,

$$F_{d.eff} = 1.8*26 + (0.7 + 0.4*1.8)*24$$
$$= 80.9 \text{ kN/m}$$

$$\frac{b_{eff}}{F_{d.eff}} = \frac{350}{80.9} = 4.33$$

From Chart D1 read maximum $\frac{L_{eff}}{d}$ = 13.2

Adjusted maximum $\frac{L_{eff}}{d}\left(for\frac{\Delta}{L_{eff}} = \frac{1}{500}\right)$ = 0.79*13.2 = 10.4

Required effective depth = $\frac{6500}{10.4}$ = 625

The incremental deflection governs.

Assume beam depth D = 700 and design beam for strength. If serviceability is not satisfied, increase the area of compressive reinforcement.

STRENGTH DESIGN

Weight of beam = 0.35*0.7*24 = 5.9 say 6 kN/m which is equal to the assumed weight at start of question.

Design load, $w^* = 1.2(6 + 20) + 1.5*24$

$= 67.2$ kN/m

Design moment, $M^* = \dfrac{67.2*(6.5)^2}{8}$

$= 355$ kNm

Assume d = 650 for one row of reinforcement.

Required, $\lambda = \dfrac{355*10^6}{350*650^2} = 2.4$

From Chart B1, read required steel ratio p = 0.0062

Required tensile reinforcing area,

$A_{st} = 0.0062*350*650$

$= 1410 \text{ mm}^2$

3N28 bars give $A_{st} = 1860 \text{ mm}^2$

Checking trial beam, $d = 700 - c_{min} = 700 - 46 = 654$ mm

$p = \dfrac{1860}{350*654} = 0.0081$

$\lambda = 3.1$

Moment capacity $\phi M_{uo} = 3.1*350*6542*10^{-6}$

$= 464 \text{ kNm} > M^* (= 362 \text{ kNm})$

Now recheck trial beam for serviceability. From initial calculations it was established that incremental deflection will govern.

Assuming 2N20 compression bars, $k_{cs} = 2 - 1.2 \dfrac{620}{1860} = 1.60$

Design load, $F_{d.eff} = 1.6*26 + (0.7 + 0.4*1.6)*24 = 73.76$ kN/m

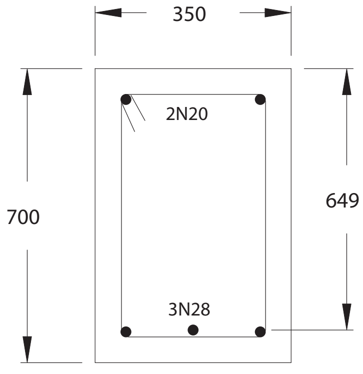
$$\frac{b_{eff}}{F_{d.eff}} = \frac{350}{73.76} = 4.75$$

From chart D1 for $\dfrac{\Delta}{L_{eff}} = \dfrac{1}{250}$, $\dfrac{L_{eff}}{d}$ = 13.0 and k_p = 0.96

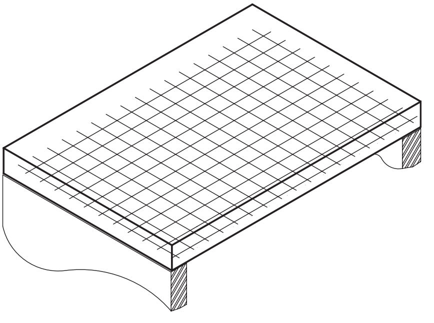
Maximum $\dfrac{L_{eff}}{d}$ = 0.79 x 0.96 x 13.6
= 10.3

Actual $\dfrac{L_{eff}}{d} = \dfrac{6500}{654}$ = 9.9 < 10.3 maximum.

Figure 8.2



8.6 Simply Supported One-Way Slabs



The slab shown in Figure 8.3 (*right*) is a one-way slab because the bending action is in one direction between the supports. The slab is designed as a rectangular beam of unit (one metre) width. In addition to the main longitudinal reinforcement, secondary transverse reinforcement is provided to prevent the formation of cracks caused by temperature and in particular shrinkage movements.

If a slab is unrestrained in the transverse direction the change in length due to temperature variations and shrinkage strains would be free to take place and theoretically there would not be any need to reinforce the slab in the transverse direction. In reality most slabs will be restrained either by attachment to other structural members or by frictional restraint provided by the supporting members.

Shrinkage cracks in unreinforced or inadequately reinforced slabs tend to be localised and quite large. The purpose of the reinforcement is to reduce temperature and shrinkage movements and redistribute cracks over a wide area. Temperature and shrinkage cracking will not be visible if sufficient reinforcement is provided. Shrinkage cracks extend the full depth of a slab so that they are much more likely to be the cause of leaking roofs.

8.7 Crack Control for Flexure #8.6.1 & #9.4.1

In the previous Code flexural cracks were considered to be controlled if the centre-to-centre spacing of reinforcement did not exceed the lesser of 2.5D or 500 mm. In the new Code these absolute values have been removed, whereby a beam or slab in flexure is only 'deemed to comply' to the crack control requirements of the Standard if the minimum area of reinforcement $A_{st.min}$ is greater than or equal to $3k_sA_{ct}/f_s$, and if the calculated steel stress f_{scr1} is less than $0.8f_{sy}$. If the beam or slab is exposed to weather conditions longer than the usual brief period during construction then other Code provisions need to be satisfied. More severe exposure conditions require the designer to check the stresses in the reinforcement based upon bar size and bar spacing. The steps in this process are summarized here:

	Steps	Formula / Clause
1	Determine $A_{st.min}$	$Ast_{min} = 3\ k_sA_{ct}\ /\ f_s$
2	Calculate A_{ct}	# 8.6.1 (I)
3	If $A_{st.min} > A_{st}$ then $A_{st} = A_{st.min}$	-
4	Determine M^*_s and M^*_{s1}	# 8.6.1 (b)
5	Calculate steel stress f_{scr} after concrete cracks	
6	If $f_{scr} > f_s$ using Table 8.6.1 (A) then check bar spacing	Table 8.6.1.A
7	If $f_{scr} > f_s$ using Table 8.6.1 (B) then redesign	Table 8.6.1.B
8	Redesign by either increasing bar size or reducing bar spacing ensuring Clause 8.6.1.(ii) is satisfied	# 8.6.1 (ii)
9	If beam is under Direct loading then calculate $f_{scr.1}$	# 8.6.1.(iv)
10	If $f_{scr.1} > 0.8\ f_{sy}$ then revise design	-

TABLE 8.6.1 (A) Maximum Steel Stress for Flexure in Beams

Bar Diameter (mm)	Max Steel Stress (MPa)
10	360
12	330
16	280
20	240
24	210
28	185
32	160
36	140
40	120

TABLE 8.6.1 (B) Maximum Steel Stress for Flexure (or Tension) in Beams

Centre to Centre Spacing (mm)	Max Steel Stress (MPa)
50	360
100	320
150	280
200	240
250	200
300	160

EXAMPLE 2A

Check the beam in Figure 8.2 against the crack control requirements for flexure of Clause 8.6.3 in AS3600-2001. Assume the beam is subjected to direct loading and exposed to external weather conditions for its design life. Concrete strength $f'_c = 50$ MPa and $E_c = 38{,}000$ MPa ($f_{cm} = 56.5$ MPa).

STEP 1

To determine $A_{st.min}$ we first have to calculate A_{ct}. If we ignore the presence of the reinforcement then A_{ct} for rectangular sections is bD/2 (= 122,500 mm^2). The beam is in flexure thus k_s=0.6. The maximum permitted steel stress f_s for N28 bars is 185 MPa (Table 8.6.1.A).

STEPS 2 & 3

$$A_{st.min} = \frac{3.k_s.A_{ct}}{f_s}$$

$$= \frac{3 * 0.6 * 122500}{185}$$

$$= 1192 \text{ mm}^2$$

STEP 4 < 1860 mm^2 (3 N28) ∴ no need to increase A_{st}.

STEP 5

To calculate the tensile stress in the reinforcement at a cracked section under short term serviceability loads f_{scr} we must calculate the design bending moment at the serviceability limit state M_s^*, the neutral axis depth kd and the I_{cr}.

$$f_{scr} = \frac{n.M_s^*.(d - kd)}{I_{cr}}$$

where

n	=	Es / Ec
	=	200,000 / 38,000
	=	5.26
p	=	A_{st} / bd
	=	1860 / (350*649)
	=	0.0082
np	=	0.0431
k	=	d ($\sqrt{(np)^2 + 2np}$ - np) -see equation 8.9
	=	165 mm
I_{cr}	=	$b(kd)^3/3 + n A_{st} (d - kd)^2$ –see equation 8.10
	=	$350(165)^3/3 + 5.26*1860*(649-165)^2$
	=	2816×10^6 mm^4
M^*_s	=	M* (G +ΨsQ) / (1.2G + 1.5Q)
	=	362*(26+0.7*24) / (1.2*26 + 1.5*24)
	=	230.5 kNm
f_{scr}	=	$n M^*_s (d – kd) / I_{cr}$
	=	$5.26*226.2 \times 10^6 (649 – 165) / 2816 \times 10^6$
	=	208.4 MPa

STEP 6

Determine if the calculated steel stress f_{scr} exceeds the maximum steel stress f_s for bar size based upon Table 8.6.1(A).

f_s = 185 MPa (for N28 bars)

$\therefore f_{scr} > f_s$

STEP 7

The maximum stress value from Table 8.6.1(A) for bar size was exceeded; however the Code gives the designer a second chance by calculating the bar spacing and using the maximum permitted steel stress from Table 8.6.1(B) as the new limit on steel stress. (refer to Note 2 AS3600-2001 page 90 & 104).

Spacing = [350 – 2(10 ties) – 1(28 bars) – 2(20 cover)]/2

= 130 mm (centre to centre)

f_s = 300 MPa (for 125 spacing from Table 8.6.1B)

$\therefore f_{scr} < f_s$ OK for flexure (thus redesign not req'd)

STEP 8

Even though the resultant steel stress is less than the permitted steel stress based upon bar spacing, the Code still requires that minimum bar spacing values be checked – see AS3600 #8.6.1.(ii). Fortunately this requirement is satisfied since distance from side or soffit of the beams to centre of nearest longitudinal bar is < 100 mm and the centre to centre spacing of bars near the tension face is < 300 mm

STEP 9

For beams under 'direct loading' (i.e. due to direct superimposed loads as opposed to loads from restraint or thermal effects), the designer needs to calculate the tensile stress in the reinforcement at a cracked section under short term serviceability loads $f_{scr.1}$ where full dead load and live load act (i.e. ψ_s =1.0)

$M^*_{s.1}$ = $M^* (G + \Psi_s Q) / (1.2G + 1.5Q)$

= 362*(26+1.0*24) / (1.2*26 + 1.5*24)

= 269.4 kNm

$f_{scr.1}$ = $n M^*_s (d - kd) / I_{cr}$

= $5.26 * 269.4 \times 10^6 (649 - 165) / 2816 \times 10^6$

= 243.5 MPa

$0.8f_{sy}$ = 400 MPa

$f_{scr.1} < 0.8f_{sy}$

$\therefore$ Design OK for Crack Control

8.8 Crack Control for Shrinkage and Temperature Effects #9.4.3

Control of cracks due to shrinkage effects and temperature variations is achieved by providing a minimum amount of reinforcement to reduce the size of cracks and prevent localised movement and propagation of cracks. The minimum areas of reinforcement are given below.

8.9 Minimum Reinforcement Required in Secondary Direction

(a) For Unrestrained Slabs - i.e. slabs free to expand and contract in the secondary direction. If the slab width is less than 2.5 metres, no secondary reinforcement is required. Otherwise the minimum area of secondary reinforcement is given by:

$$A_{st.min} = 1.75\ bD \times 10^{-3} \tag{8.3}$$

(b) For Restrained Slabs - the minimum area of reinforcement depends on the degree of crack control (how important is it) and the severity of exposure. The minimum areas are given by the following equations:

FOR EXPOSURE CLASSIFICATIONS A1 AND A2

$$A_{st.min} = 1.75\ bD \times 10^{-3} \text{ for minor control} \tag{8.4}$$

$$A_{st.min} = 3.5\ bD \times 10^{-3} \text{ for moderate control} \tag{8.5}$$

$$A_{st.min} = 6.0\ bD \times 10^{-3} \text{ for strong control} \tag{8.6}$$

FOR EXPOSURE CLASSIFICATIONS B1, B2 AND C

$$A_{st.min} = 6.0\ bD \times 10^{-3} \text{ for strong control} \tag{8.7}$$

(c) Partially Restrained Slabs - the minimum area of reinforcement will be somewhere between that given by (a) and (b) above. The designer must exercise his/her professional judgement in determining the degree of restraint.

8.10 Minimum Flexural Steel Ratio

The minimum strength requirement in bending is similar to that required for beams i.e. 1.2 times the mininum strength in bending $M_{uo\ min}$ (this is the minimum moment capacity just prior to cracking). This can be satisfied with a minimum steel ratio obtained by using the formula $\frac{A_{st}}{bd} \geq 0.22 \left(\frac{D}{d}\right)^2 \left(\frac{f'_{cf}}{f_{sy}}\right)$. This minimum steel ratio will apply to one-way slabs.

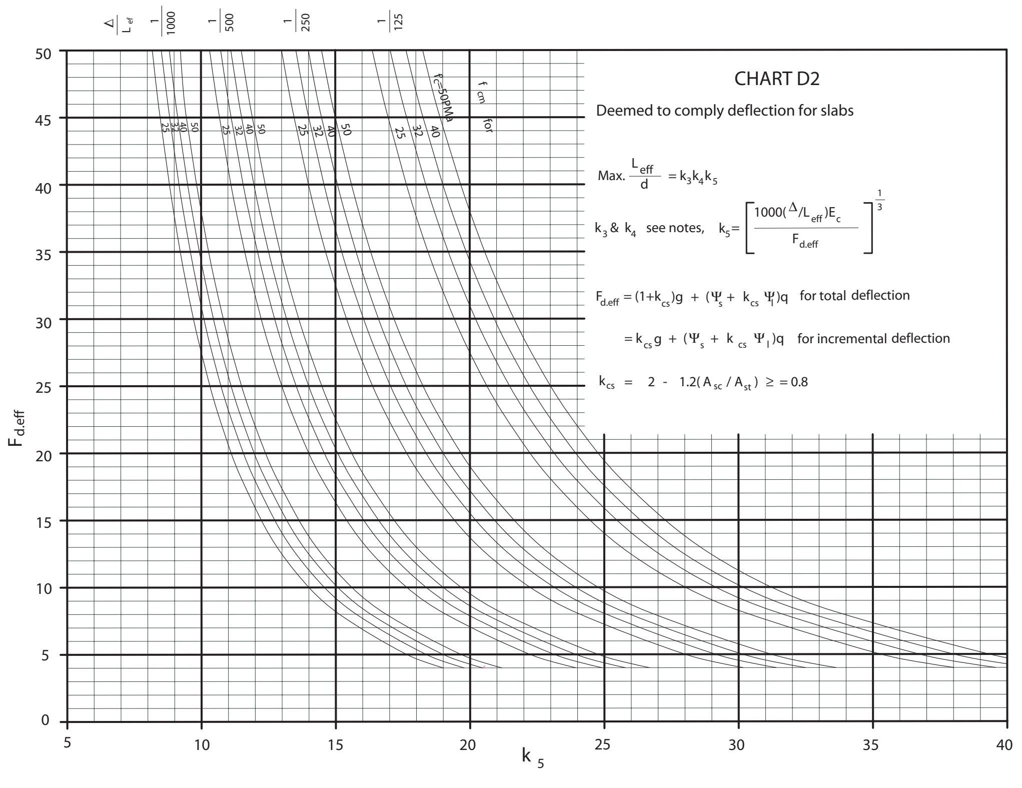
For continuous two-way slabs the Code requirement is deemed to have been satisfied by providing a minimum steel ratio of 0.0025 for slabs supported by columns and 0.002 for slabs supported by beams or walls (Code section 9.1.1).

8.11 Slab Deflection

Chart D2 below is a graphic representation of the bracketed term in equation 8.2 given the symbol k_5. Thus the maximum effective length to effective depth ratio required to satisfy serviceability is given by:

$$\frac{L_{eff}}{d} = k_3\, k_4\, k_5 \qquad \textbf{(8.8)}$$

Serviceability condition for slabs is in most instances the primary limit state. A designer should therefore always begin by estimating the minimum effective depth required to satisfy serviceability. The design of simply supported slabs making use of Chart D2 is illustrated by Example 3 (*below*).



EXAMPLE 3

Figure 8.3 is a representation of a simply supported slab L = 4.6 m span, 3 m wide for a retail store. Design the slab to carry a 2.5 kPa dead load and a 4 kPa live load using N32 grade concrete for exposure conditions A2. Assume that the slab will be required to support partitions for which some control of damage due to deflection is provided.

SOLUTION

Data: $f'c = 32$ MPa $g = 2.5$ kPa $q = 4$ kPa

$L = 4.6$ m $B = 3$ m Exposure A2

$\psi_s = 0.7$ (Table 1.2)

$\psi_l = 0.4$ (Table 1.2)

For total deflection, maximum $\dfrac{\Delta}{L_{eff}} = \dfrac{1}{250}$ (table 8.1)

For deflection after addition of partitions $\dfrac{\Delta}{L_{eff}} = \dfrac{1}{500}$

(a) Determine Depth of Slab for Serviceability

Estimate 4.8 kN/m^2 weight of slab (assumed 200 mm deep) to make the total dead load $g = 2.5 + 4.8 = 7.3$ kN/m^2.

Determine effective loads $F_{d.eff}$ to be used for deflection criteria, the slab is assumed to have tensile reinforcement only so that $k_{cs} = 2$.

$$
\begin{aligned}
F_{d.eff} &= (1 + k_{cs})g + (\psi_s + \psi_l k_{cs})q \quad \text{(for total deflection)} \\
&= (1 + 2)7.3 + (0.7 + 0.4*2)4 \\
&= 27.9 \text{ kN/m}^2
\end{aligned}
$$

$$
\begin{aligned}
F_{d.eff} &= k_{cs}\, g + (\psi_s + \psi_l k_{cs})q \quad \text{(for deflection after the attachment of partitions)} \\
&= 2*7.3 + (0.7 + 0.4*2)4 \\
&= 20.6 \text{ kN/m}^2
\end{aligned}
$$

From Chart D2 read, $k_5 = 16.4$ for $\Delta/L_{eff} = 1/250$ and $F_{d.eff} = 27.9$ and, $k_5 = 14.4$ for $\Delta/L_{eff} = 1/500$ and $F_{d.eff} = 20.6$. The incremental deflection will govern because of the smaller value of k_5.

Multiplier $k_3 = 1.0$ for one-way slabs and $k_4 = 1.6$ for simply supported slabs .

$$
\begin{aligned}
\text{Maximum,} \quad \frac{L_{eff}}{d} &= k_3\, k_4\, k_5 \\
&= 1.0*1.6*14.4 \\
&= 23.0
\end{aligned}
$$

Hence the minimum required effective depth,

$$d = \frac{4600}{23.0} = 200 \text{ mm}$$

From Table 3.3, minimum cover required is 25 mm. Assuming N20 bars, depth of slab required, D = 200+10+25 = 235 mm, say 250 mm.

(b) Design Slab for Strength

Weight of slab w_{swt} = 0.25*24 = 6 kN/m

Design load, w^* = 1.2(2.5 + 6) + 1.5*4

= 16.2 kN/m

Design moment, $M^* = \dfrac{16.2 * 4.6^2}{8}$

= 42.9 kNm per m width of slab.

Estimated effective depth d = 250 - 25 -10

= 215mm (for N20 bars)

$$\lambda = \frac{42.9 * 10^6}{10^3 * 215^2} = 0.93$$

Using Chart B1 and λ = 1 (close enough to 0.93), read off a p value of 0.0025.

$$\text{Minimum steel ratio} = \frac{Ast}{bd} > 0.22\left(\frac{D}{d}\right)^2 \frac{f'_{cf}}{f_{sy}} = 0.22\left(\frac{250}{215}\right)^2 \frac{0.6\sqrt{32}}{500}$$

$$= 0.0020 < 0.0025$$

Note also Clause 9.1.1 of AS3600 requires that the minimum A_{st}/bd be not less than 0.002 for a slab supported by walls or beams. This is satisfied in this example.

Required area of reinforcement, A_{st} = 0.0025*1000*215

= 538 mm^2 per m width of slab

The reinforcement may be chosen from Table 8.3 which tabulates the equivalent area in mm^2/m width for various spacings and sizes of reinforcement. From the table, initially select N20 bars at 500 mm centres to give an area A_{st} = 620 mm^2. Now check the crack control requirements of the AS3600 assuming the slab is "*fully enclosed within a building except for a brief period of weather exposure during construction where wider cracks can be tolerated*". Clause 9.4.1 (i) of AS3600 has been satisfied on the previous page. Clause 9.4.1 (iii) requires that the centre to centre spacing of bars in each direction shall not exceed the lesser of 2.0 D or 300 mm. In this example we must therefore adopt N20 bars at 300 mm spacing or use N16 at 300 centres whose area is 667 mm^2 per metre width.

Table 8.3 - Areas of Reinforcement mm^2 / m width

Bar Spacing	Bar Sizes 12 mm	16 mm	20 mm	24 mm	28 mm	32 mm
100	1 100	2 000	3 100	4 500	6 200	8 000
125	880	1 600	2 480	3 600	4 960	6 400
150	733	1 333	2 067	3 000	4 133	5 333
175	629	1 143	1 771	2 571	3 543	4 571
200	550	1 000	1 550	2 250	3 100	4 000
225	489	889	1 378	2 000	2 756	3 556
250	440	800	1 240	1 800	2 480	3 200
275	400	727	1 127	1 636	2 255	2 909
300	367	667	1 033	1 500	2 067	2 667
325	338	615	954	1 385	1 908	2 462
350	314	571	886	1 286	1 771	2 286

The slab should now be checked for serviceability since the weight of slab is greater than the initially assumed weight.

Total dead load g = 2.5 + 6 = 8.5 kN/m^2

For total deflection $F_{d.eff}$ = 3*8.5 + 1.5*4 = 31.5 kN/m ^{2}

From Chart D2 for $\frac{\Delta}{L_{eff}} = \frac{1}{250}$, k_5 = 15.8

Maximum, $\frac{L_{eff}}{d}$ = 1.0*1.6*15.8= 25.3 > $\frac{4600}{215}$ = 21.4 actual.

For deflection after attachment of partitions,

$F_{d.eff}$ = 2*8.5 + 1.5*4 = 23 kN/m 2

From Chart D2 for $\frac{\Delta}{L_{eff}} = \frac{1}{500}$, k_5 = 13.9

Maximum, $\frac{L_{eff}}{d}$ = 1.0*1.6*13.9 = 22.3 > 21.4 actual

SECONDARY REINFORCEMENT

Assuming partial restraint and that moderate crack control is required, choose steel ratio for temperature and shrinkage mid-way between equations 8.3 and 8.5 for exposure A2. The minimum required area of reinforcement,

$$A_{st.min} = \left(\frac{1.75+3.5}{2}\right) bD \times 10^{-3}$$

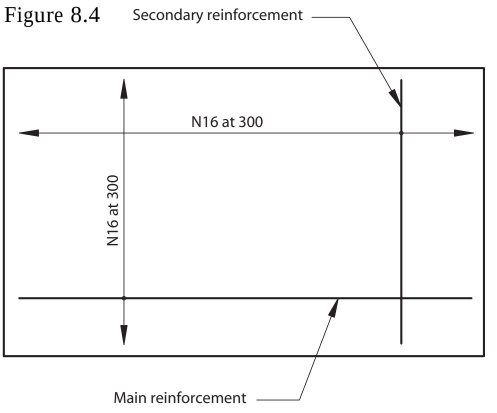
$$= 2.63*1000*250 \times 10^{-3}$$

$$= 658 \text{ mm}^2 \text{ per m width}$$

From Table 8.3, select N16 bars at 300 mm centres to give an area A_{st} = 667 mm^2 per metre width of slab.

If the slab had been fully restraint and strong control against cracking had been required, the secondary reinforcement may have been greater than the main longitudinal reinforcement.The final slab is shown in Figure 8.4 (*right*).

Figure 8.4



It was discovered in the last example that Chart B1 is not suitable for slabs because of the small moments carried by slabs. A table of moment capacities for slabs could be produced. Such a table is not really practical because of the variations in concrete cover for different exposures and concrete grades. Chart S1 is used instead of a table. Chart S1 is drawn for f'_c = 25 MPa but it can be safely used for higher grade concretes. For example at the lower range, for a slab with d=70 mm and M*=10 kNm, the required steel ratios are:

p = 0.00544 for f'_c = 25 MPa,
p = 0.00536 for f'_c = 32 MPa (1.5% below that for N25 grade) and,
p = 0.00530 for f'_c = 40 MPa (2.5% below that for N25 grade).

Similarly at the higher range for a slab d=260 mm and M*=150 kNm, the required required steel ratios are;

p = 0.00596 for f'_c = 25 MPa,
p = 0.00586 for f'_c = 32 MPa (1.7% below that for grade N25) and,
p = 0.00579 for f'_c = 40 MPa (2.9% below that for grade N25).

Chart S1 is conservative and the differences are indeed very small. The designer may choose to apply a correction by deducting 1.5% from the steel ratio read from S1 for both N32 and N40 grade concrete. Nevertheless, Chart S2 for f'_c = 32 MPa is also included as this may be a more common grade in slabs.

Slabs are frequently reinforced with reinforcing mesh whose yield strength f_{sy} is also 500 MPa. Standard reinforcing meshes and their areas are shown in Table 8.4 page 124. Note that these bar diameters and areas are different to the previous grades of undeformed (or smooth wire) fabric that had a yield of 450 MPa (eg F72, F918 etc).

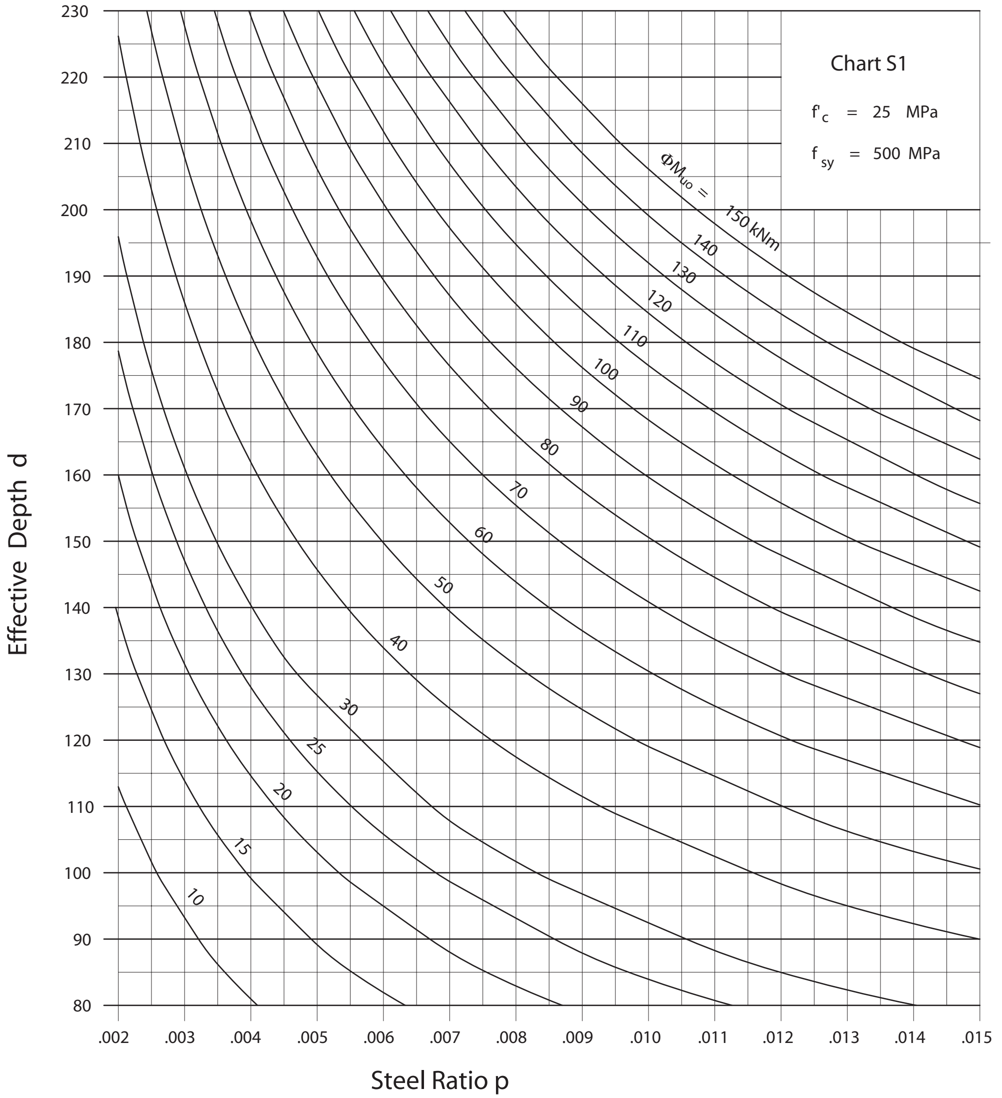


Chart S1
f'c = 25 MPa
f sy = 500 MPa
ΦMuo = 150 kNm
140
130
120
110
100
90
80
70
60
50
40
30
25
20
15
10
Effective Depth d
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
.002
.003
.004
.005
.006
.007
.008
.009
.010
.011
.012
.013
.014
.015
Steel Ratio p

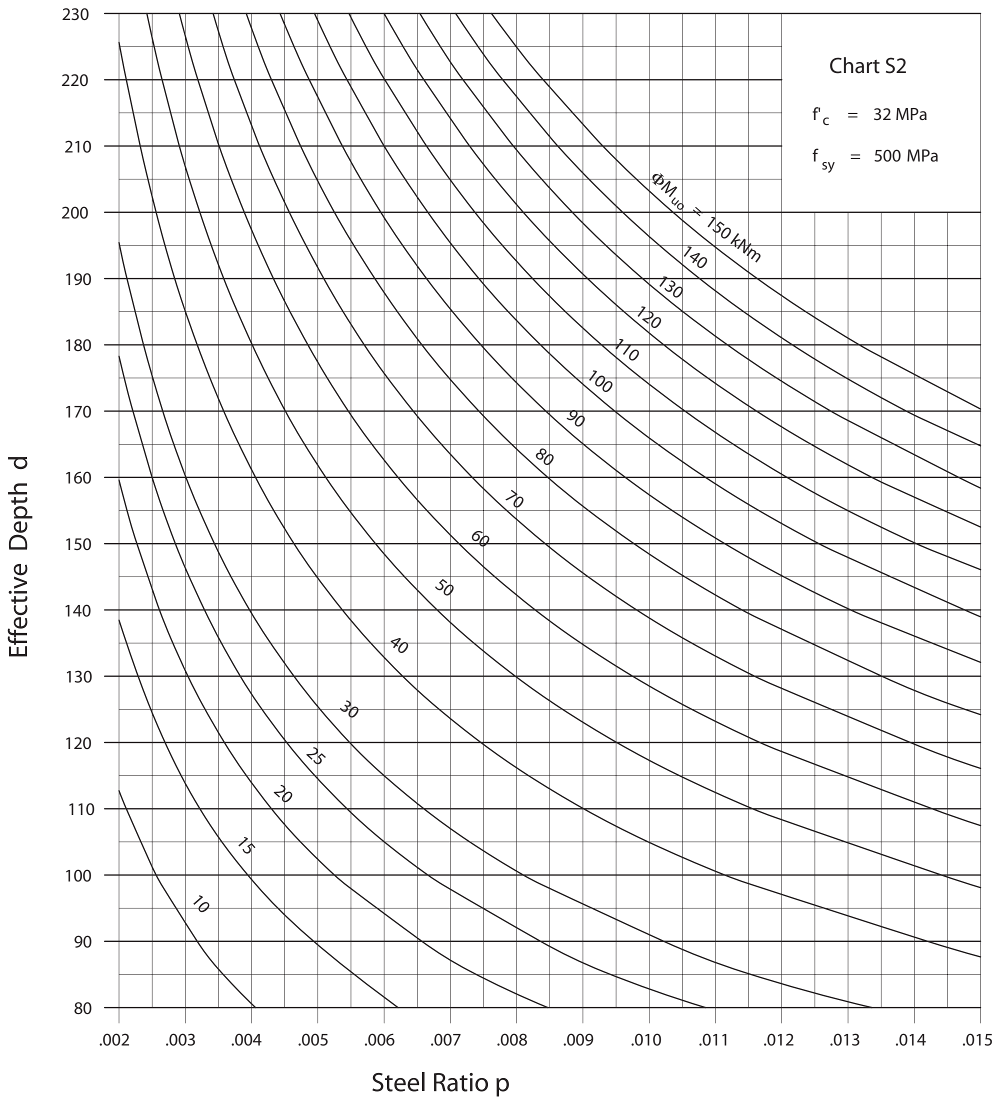


Chart S2
f'c = 32 MPa
f sy = 500 MPa
ΦMuo = 150 kNm
140
130
120
110
100
90
80
70
60
50
40
30
25
20
15
10
Effective Depth d
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
.002
.003
.004
.005
.006
.007
.008
.009
.010
.011
.012
.013
.014
.015
Steel Ratio p

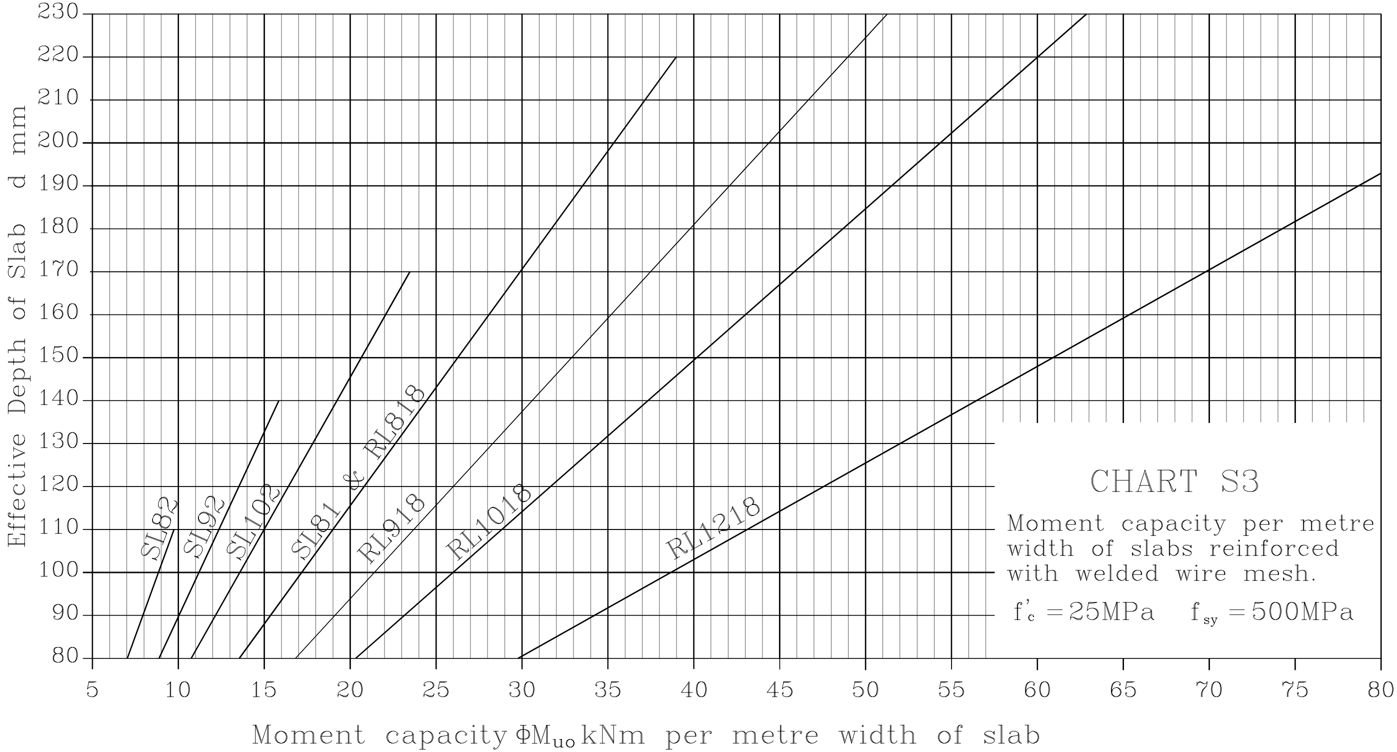


CHART S3
Moment capacity per metre width of slabs reinforced with welded wire mesh.
f'c = 25MPa   fsy = 500MPa
Effective Depth of Slab d mm
Moment capacity ΦMuo kNm per metre width of slab
SL82
SL92
SL102
SL81 & RL818
RL918
RL1018
RL1218
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80

Table 8.4 - Standard Reinforcing Meshes

	Area mm^2 /m		Longitudinal Wires		Cross Wires	
Ref.No.	Longitudinal Wires	Cross Wires	Size	Pitch	Size	Pitch
			Rectangular Meshes			
RL1218	1112	227	11.9	100	7.6	200
RL1118	891	227	10.65	100	7.6	200
RL1018	709	227	9.5	100	9.5	200
RL918	574	227	8.6	100	7.6	200
RL818	454	227	7.6	100	7.6	200
RL718	358	227	6.75	100	7.6	200
			Square Meshes			
SL81	454	454	7.6	100	7.6	100
SL41	126	126	4	100	4	100
SL102	354	354	9.5	200	9.5	200
SL92	287	287	8.6	200	8.6	200
SL82	227	227	7.6	200	7.6	200
SL72	179	179	6.75	200	6.75	200
SL62	141	141	6.0	200	6.0	200
SL52	89	89	4.75	200	4.75	200
SL42	63	63	4	200	4	200
SL63	94	94	6	300	6	300
SL53	59	59	4.75	300	4.75	300
			Trench Meshes			
L12TM	1112	65	11.9	100	5.0	300
L11TM	899	65	10.7	100	5.0	300
L8TM	454	65	7.6	100	5.0	300

The moment capacities for slabs using reinforcing mesh are plotted on Chart S3. Chart S3 on page 123 is a much more useful alternative since it enables the designer to enter the chart with the slab effective depth and design moment and read directly the required reinforcing mesh. Example 4 illustrates the use of Table 8.4, serviceability chart D2 and moment Charts S2 and S3.

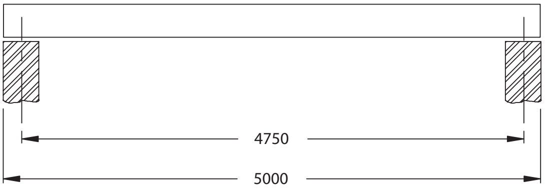
EXAMPLE 4

Figure 8.5 shows a simply supported slab for a domestic building. The slab supports a superimposed dead load g = 1.5 kPa and a live load q = 2 kPa.

(a) Design the slab for exposure classifications A1 using N32 grade concrete and reinforcing mesh if precautions are taken to ensure that the partitions will not be unduly affected by deflection.

(b) Redesign the slab if a layer of compressive reinforcing mesh equal to the tensile reinforcing mesh is provided.

Figure 8.5



SOLUTION

Data: $f'_c = 32$ MPa $f_{sy} = 500$ MPa $L = 4.75$ m
$g = 1.5$ kPa $q = 2$ kPa Exposure A1
$\psi_s = 0.7$ for serviceability from Table 1.2
$\psi_l = 0.4$ for serviceability from Table 1.2

(a) Assume D = 250 mm for the purpose of estimating weight of slab.

Weight of slab = 0.25*24 = 6 kN/m^2

Total dead load, $g = 1.5 + 6 = 7.5\ kN/m^2$

Live load, $q = 2\ kN/m^2$

Since there is no compressive reinforcement, the long-term deflection multiplier $k_{cs} = 2$.

For total deflection $\left(\dfrac{\Delta}{L_{eff}} = \dfrac{1}{250}\right)$, the effective design load,

$$
\begin{aligned}
F_{d.eff} &= (1 + k_{cs})g + (\psi_s + \psi_l k_{cs})q \\
&= (1 + 2)7.5 + (0.7 + 0.4*2)2 \\
&= 25.5\ kN/m^2
\end{aligned}
$$

From Chart D2 read $k_5 = 16.9$

For incremental deflection $\left(\dfrac{\Delta}{L_{eff}} = \dfrac{1}{500}\right)$, the effective design load,

$$
\begin{aligned}
F_{d.eff} &= k_{cs}\, g + (\psi_s + \psi_l k_{cs})q \\
&= 2*7.5 + (0.7 + 0.4*2)2 \\
&= 18\ kN/m^2
\end{aligned}
$$

From Chart D2 read $k_5 = 15.1$

The incremental deflection will govern.

$k_3 = 1$ for one-way slabs and $k_4 = 1.6$ for simply supported slabs.

$$
\begin{aligned}
\text{Maximum } \frac{L_{eff}}{d} &= k_3\, k_4\, k_5 \\
&= 1.0*1.6*15.1 \\
&= 24.1
\end{aligned}
$$

Required effective depth, $d = \dfrac{4750}{24.1} = 197.$

Allowing approximately 25 mm from the centre of reinforcement to the underside of the slab, the required depth D = 222. Since the slab depth and weight has been over-estimated,

Try D = 220 mm.

Weight of slab = 0.22*24 = 5.3 kN/m^2

Total g = 1.5 + 5.3 = 6.8 kN/m^2

For total deflection $\left(\dfrac{\Delta}{L_{eff}} = \dfrac{1}{250}\right)$, the effective design load,

$F_{d.eff} = 3*6.8 + 1.5*2 = 23.4$ kN/m^2

From Chart D2, $k_5 = 17.4$

For incremental deflection $\left(\dfrac{\Delta}{L_{eff}} = \dfrac{1}{500}\right)$, the effective design load,

$F_{d.eff} = 2*6.8 + 1.5*2 = 16.6$ kN/m^2

From Chart D2, $k_5 = 15.5$. Using the smaller value of k_5,

Maximum $\dfrac{L_{eff}}{d} = 1.6*15.5 = 24.8$

Required, $d = \dfrac{4750}{24.8} = 192$ mm

Required, D = 192 + 5 (half bar diameter) + 20 (cover)

= 217 < 220 mm (the assumed value).

The design load for strength conditions for a 1 m wide strip,

$w^* = 1.2*6.8 + 1.5*2$

$= 1.2$ kN/m

Design moment for 1 m width of slab,

$$M^* = \frac{11.2\,(4.75)^2}{8}$$

= 31.6 kNm per m width of slab.

From Chart S2, the required p = 0.0022, which is greater than the calculated minimum steel ratio of 0.0019 (ie using D=220 & d=195)

Required $A_{st} = 0.0022*1000*195$

= 430 mm^2 per m width of slab.

From Table 8.4, choose SL81 mesh whose area is 454 mm^2 per metre width of slab.

Check slab depth, (the longitudinal wires are ≈ 8 mm diameter),

$$D = 195 + (8/2) + 20 \approx 220 \text{ mm}$$

(b) Using equal tensile and compressive reinforcement, the long-term deflection multiplier will be,

$$k_{cs} = 2 - 1.2\,\frac{A_{sc}}{A_{st}} = 2 - 1.2 = 0.8$$

Assume D = 170 mm, the weight of slab = 0.17*24 = 4.1 kN/m^2

Total dead load g = 4.1 + 1.5 = 5.6 kN/m^2

For total deflection $\left(\dfrac{\Delta}{L_{eff}} = \dfrac{1}{250}\right)$, the effective design load,

$$F_{d.eff} = (1 + 0.8)5.6 + (0.7 + 0.4*0.8)2$$
$$= 12.12 \text{ kN/m}^2$$

From Chart D2, k_5 = 21.7

For incremental deflection $\left(\dfrac{\Delta}{L_{eff}} = \dfrac{1}{500}\right)$, the effective design load,

$$F_{d.eff} = 0.8*5.6 + (0.7 + 0.4*0.8)2$$
$$= 6.52 \text{ kN/m}^2$$

From Chart D2, k_5 = 21.2. Using the smaller value of k_5,

Maximum $\dfrac{L_{eff}}{d}$ = 1.6*21.2 = 33.9

Minimum required d = $\dfrac{4750}{33.9}$ = 140 mm

For strength design considering a 1 m wide strip, the design load,

$$w^* = 1.2*5.6 + 1.5*2$$
$$= 9.7 \text{ kN/m}$$

Design moment,

$$M^* = \frac{9.7\,(4.75)^2}{8}$$
$$= 27.4 \text{ kNm}$$

From Chart S2, the required steel ratio p = 0.0037, which is greater than the minimum steel ratio of 0.0019.

Required area of longitudinal reinforcement,

A_{st} = 0.0037*1000*145

= 537 mm^2 per m width of slab.

Choose N12 bars @ 200 mm with an area A_{st} = 550 mm^2 per metre.

Alternatively the reinforcing mesh may be chosen directly from Chart S3. Enter chart with effective depth d = 140 mm and design moment M* = 28.2 kNm to read RL918 (580 mm^2) as the required reinforcing mesh.

Check depth of slab.

D = 145 + (9/2) + 20

≈ 170 mm (which is approximately equal to the assumed depth).

Comments: The following points are noted as a result of example 3 and example 4;

(a) Serviceability requirements are invariably the governing criteria for slabs.

(b) Singly reinforced, simply supported, one-way slabs are rather thick when the "deemed to comply" serviceability condition is used to determine the depth of slab. This is especially so when incremental deflection limitation of L/1000 is applied.

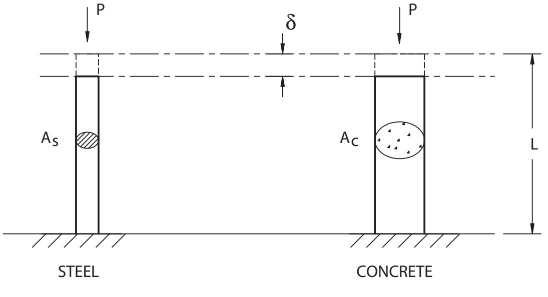
(c) Incremental deflection is in most cases the criterion for serviceability conditions.

(d) Slab depths can sometimes be reduced by providing a layer of compressive reinforcement. The cost of the additional reinforcement may be offset by the thinner slab, reduced weight and increased headroom.

8.12 Transformed Sections

Transformed sections are sometimes a convenient means of converting cross sectional areas of composite materials to an equivalent or transformed area of one material type. In reinforced concrete, the steel reinforcement is transformed to an equivalent area of concrete. Transformed section areas are then used to calculate the second moment of area of reinforced concrete sections required for deflection calculations.

Figure 8.6



A short steel section of area A_s, length L and modulus of elasticity E_s subjected to a load P will deflect an amount δ_s given by;

$$\delta_s = \frac{PL}{E_s A_s}$$

A similar concrete section will deflect an amount δ_c given by;

$$\delta_c = \frac{PL}{E_c A_c}$$

The transformed concrete section is a concrete area which will give a deflection equal to the steel section. Equating the two deflection expressions and solving for the transformed concrete area A_c,

$$\frac{PL}{E_c A_c} = \frac{PL}{E_s A_s}$$

$$E_c A_c = E_s A_s$$

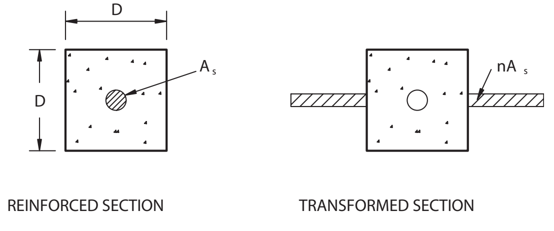
$$A_c = \frac{E_s}{E_c} A_s$$

$$= nA_s$$

Where $n = E_s / E_c$ is called the "modular ratio".

If the section is a square concrete section size D reinforced with steel of area A_s, the whole section may be converted to a concrete section in which the steel area is replaced by a thin rectangular concrete area of magnitude nA_s at the level of the reinforcement.

Figure 8.7



The total concrete area in the transformed section will be:

$$A_c = D^2 + nA_s$$

It is common to allow for the concrete area displaced by the reinforcement. Adjusting the above formula,

$$A_c = D^2 - A_s + nA_s$$

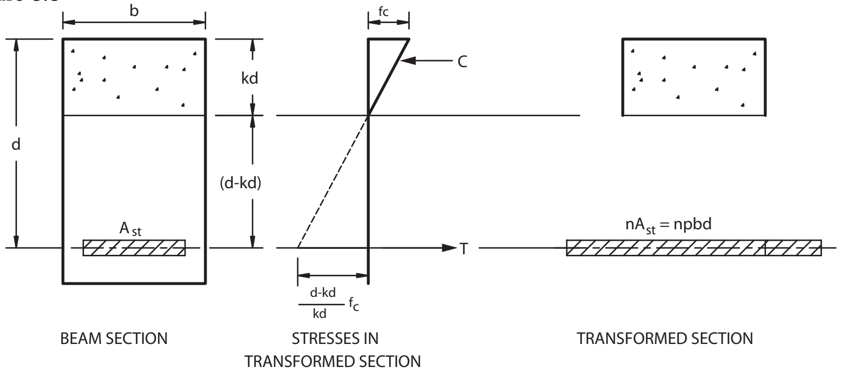
$$= D^2 + (n - 1)A_s$$

i.e. transformed area $(n - 1)A_s$ allows for the concrete displaced by the reinforcement.

8.12.1 Transformed Beam Section - Cracked Beams

Transformed sections will be used to determine the second moment of area, I_{cr}, of a singly reinforced cracked beam. The stresses at which flexural cracking occurs are very small so that the stress distribution may be assumed to be linear as shown in Figure 8.8 (*below*).

Figure 8.8



The depth of the neutral axis is found by equating the internal forces C = T.

$$0.5\ f_c\ b\ kd = n\ A_{st}\ \frac{d - kd}{kd}\ f_c$$

In terms of the tensile steel ratio, A_{st} = pbd

$$0.5\ b\ kd = npbd\ \frac{d-kd}{kd}$$

$$0.5^*(kd)^2 = npd^2 - npdkd$$

$$(kd)^2 = 2npd^2 - 2npdkd$$

$$(kd)^2 + 2npdkd - 2npd^2 = 0$$

$$kd = \frac{-2npd + \sqrt{(2npd)^2 + 8npd^2}}{2}$$

(using the quadratic equation formula)

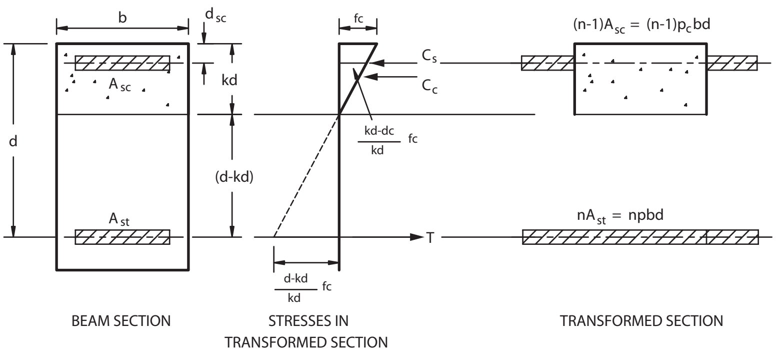
$$\boxed{kd = d(\sqrt{(np)^2 + 2np} - np) \qquad \textbf{(8.9)}}$$

Using the transformed section, the second moment area of the cracked section is found by taking moment areas about the neutral axis.

$$I_{cr} = \frac{b(kd)^3}{12} + b\ kd \left(\frac{kd}{2}\right)^2 + nA_{st}\ (d - kd)^2$$

$$\boxed{I_{cr} = \frac{b(kd)^3}{3} + n\ A_{st}\ (d - kd)^2 \qquad \textbf{(8.10)}}$$

Figure 8.9



For a doubly reinforced beam, the transformed area is shown above as Figure 8.9. The depth of the neutral axis and the second moment area of the cracked section are found as for singly reinforced beams. The derived formulae are given below.

$$kd = d\left(\sqrt{[np+(n-1)p_c]^2 + 2\left(np+(n-1)p_c\,\frac{d_{sc}}{d}\right)} - [np+(n-1)p_c]\right) \quad \textbf{(8.11)}$$

$$I_{cr} = b\,\frac{(kd)^3}{3} + nA_{st}(d - kd^2) + (n - 1)A_{sc}(kd - d_{SC}) \quad \textbf{(8.12)}$$

Where: p = Tensile steel ratio = $\dfrac{A_{st}}{bd}$

p_c = Compressive steel ratio = $\dfrac{A_{sc}}{bd}$

Note that for the tensile reinforcement the transformed area is nA_{st} since the concrete below the neutral axis is cracked i.e. the tensile reinforcement does not displace any concrete because the concrete does not carry tensile forces while the transformed area of the compressive reinforcement is $(n - 1)A_{sc}$ to allow for the stress carrying concrete displaced by the reinforcement.

8.12.2 Effective Second Moment of Area I_{eff}

The second moment of area to be used for calculating the immediate or elastic deflection cannot be the second moment of area of the cracked section, I_{cr}, since it ignores the stiffening effect of concrete between flexural cracks. The Code #8.5.3 gives a formula for an effective moment of area, I_{ef} which is weighted to account for the stiffening effect of the uncracked concrete. This is the well known “Bransons Formula”.

$$I_{ef} = I_{cr} + (I - I_{cr})\left(\frac{M_{cr}}{M_s}\right) \leq I_{e.max} \quad \textbf{(8.13)}$$

Where: $I_{e.max}$ = Second moment of area of the gross section which may conservatively be taken as the second moment of area of the gross concrete section by neglecting the reinforcement.

= $0.6\,I$ if $p < 0.005$

= I if $p \geq 0.005$

I = Second moment of Area of gross section.

M_{cr} = Cracking moment.

= $Z\,(f'_{cf} - f'_{cs})$ or $\dfrac{f'_{cf}\, I}{y_t}\,(f''_{cf} - f'_{cs})$

$f_{'cs}$ = $\left(\dfrac{1.5p}{1+50p}\right) E_s\, \epsilon_{cs}$

ϵ_{cs} = Concrete design shrinkage strain (from AS3600 Clause 6.1.7)

M_s = Maximum bending moment calculated for the short-term serviceability loads.

y_t = Distance from centroid of gross section to the extreme tension fibre.

Z = Section Modulus

= $\left(\dfrac{I}{y_t}\right)$ or $\left(\dfrac{I}{D/2}\right)$ for uncracked section.

f'_{cf} = Characteristic flexural strength of concrete.

$$f'_{cf} = 0.6\sqrt{f'_c} \qquad \textbf{(8.14)}$$

Sustituting for f'_{cf} the equation for the cracking moment becomes,

$$M_{cr} = Z\,(0.6\sqrt{f'_c} - f_{cs}) \qquad \textbf{(8.15)}$$

If one wants an approximate value of the effective I value (I_{ef}) without carrying out detailed calculations as described above then the Code provides two simple but conservative formulae as an alternative, namely

$I_{ef} = (0.02 + 2.5p)bd^3$ when $p \geq 0.005$, or

$I_{ef} = (0.01 - 13.5p)bd^3 \leq 0.06$ when $p < 0.005$

For simply supported members, the effective second moment of area to be used in the deflection formula is that given by equation 8.13. For continuous members or members with end restraints the effective second moment of area to be used in the deflection formula is a weighted average second moment of area given by equations 8.16 and 8.17 in (a) and (b) below.

(a) One end simply supported and one end restrained,

$$I_{eff} = \frac{I_M + I_L}{2} \text{ or } \frac{I_M + I_R}{2} \qquad \textbf{(8.16)}$$

(b) Both ends restrained,

$$I_{eff} = \frac{I_M}{2} + \frac{I_L + I_R}{4} \qquad \textbf{(8.17)}$$

Where: $I_M = I_{eff}$ at mid-span.

$I_L = I_{eff}$ at left support.

$I_R = I_{eff}$ at right support.

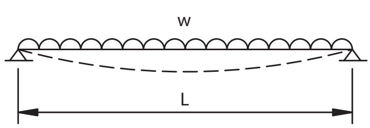
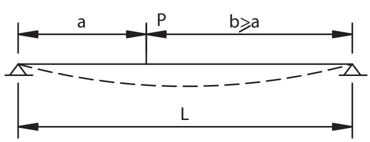
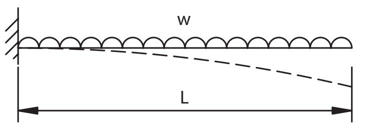
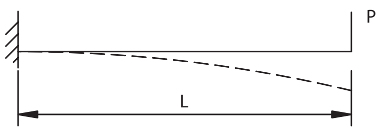
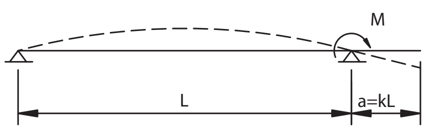
8.12.3 Elastic Deflection Formulae

The immediate deflection is calculated using standard deflection formulae which may be expressed in the form:

$$\delta = K \frac{ML^2}{EI_{eff}} \tag{8.18}$$

The maximum moment M and the multiplier K for common loading conditions are shown in the Table 8.5 below.

Table 8.5 - Deflection Multipliers

LOAD CONDITION	MOMENT	K
1. 	$\frac{wL^2}{8}$	$\frac{5}{48}$
2. 	$\frac{Pab}{L}$	$\frac{L[3 - (2a/L)^2]}{48b}$
3. 	$\frac{wL^2}{2}$	$\frac{1}{4}$
4. 	PL	$\frac{1}{3}$
5. 	M	-1/16 at mid-span 3a/L at cantilever

It may be necessary to superimpose two or more deflection conditions such as in a cantilevered beam to obtain the additional deflection due to end rotation.

The immediate deflection, δ_s, for the short-term serviceability loads w_s is calculated from,

$$w_s = g + \psi_s\, q \tag{8.19}$$

The additional or incremental deflection due to sustained loading, δ_{inc}, is obtained using the long-term serviceability loading w_l calculated from,

$$w_l = g + \psi_l\, q \tag{8.20}$$

To calculate the additional or incremental deflection, δ_{inc}, the Code permits the use of the long-term multiplier, k_{cs}. The incremental deflection is obtained by multiplying the elastic deflection due to the long-term serviceability loading by the long term multiplier k_{cs}.

Note that the total deflection, $\delta_{tot} = \delta_s + \delta_{inc}$, must not exceed the maximum deflection of L_{ef} /250 given in Table 8.1. The incremental deflection due to shrinkage and creep effects is also limited if masonry partitions are supported.

8.12.4 Beam Deflection – Simplified Calculation Method

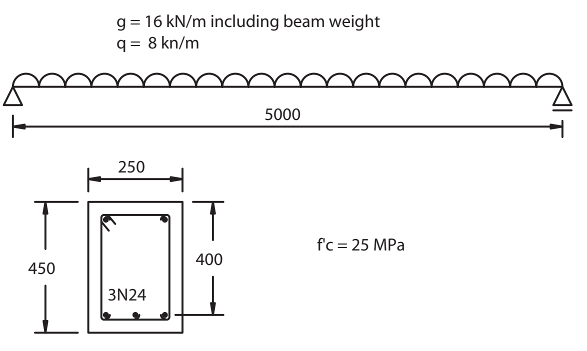
The previous few sections have provided the necessary information to calculate the deflection of a beam more precisely than the 'deemed to comply' span to depth ratio method. This is because we now have a better estimate of the true or effective I value taking into account factors such as (i) degree of cracking (ii) concrete shrinkage (iii) amount of reinforcement and (iv) support conditions.

The following example will illustrate how to calculate the deflection of a singly reinforced beam using the 'simplified' method as nominated in AS3600 section 8.5.3.

EXAMPLE 5

Calculate the immediate (short term) deflection and the total (short + long term) deflection for the simply supported, singly reinforced concrete beam shown in Figure 8.10 (*right*). The beam is part of an office construction and it supports a masonry partition.

Figure 8.10



SOLUTION

Modulus of elasticity of concrete (for f'_c = 25 MPa, the f_{cm} is 27.5 MPa – see Table 8.6)

$$E_c = (\rho)^{1.5}\ 0.043\ \sqrt{f_{cm}}$$

$$= (2400)^{1.5}\ 0.043\ \sqrt{27.5}$$

$$= 27{,}500 \text{ MPa}$$

Modular ratio,

$$n = \frac{E_s}{E_c} = \frac{200*10^3}{27{,}500} = 7.27$$

Steel ratio,

$$p = \frac{1350}{250*400} = 0.0135$$

$$np = 7.27*0.0135 = 0.098$$

Second moment area of gross section,

$$I = \frac{bD^3}{12} = \frac{250*(450)^3}{12} = 1900*10^6 \text{ mm}^4$$

Modulus of section of gross section,

$$Z = \frac{1900*10^6}{\left(450/2\right)} = 8.44*10^6 \text{ mm}^3$$

Depth of neutral axis of cracked section,

$$kd = d\left(\sqrt{(np)^2 + 2np} - np\right)$$

$$= 0.3554 \text{ x } 400$$

$$= 142 \text{ mm}$$

Second moment of area of cracked section,

$$I_{cr} = \frac{b(kd)^3}{3} + nA_{st}\,(d - kd)^2$$

$$= \frac{250*(142)^3}{3} + 7.27*1350*(400 - 142)^2$$

$$= 892*10^6 \text{ mm}^4$$

Concrete flexural-tensile stress induced by shrinkage strain ϵ_{cs}

$$f_{cs} = \left(\frac{1.5p}{1+50p}\right)E_s\epsilon_{cs}$$

where ϵ_{cs} is a function of the hypothetical thickness t_h (see AS3600 #1.7 - Notation)

$$t_h = \left(\frac{2Ag}{u_e}\right)$$

$$= \left(\frac{2(250x450)}{250+2(450)}\right)$$ assuming the top of the beam is not exposed to drying

= 195 mm (say 200)

From AS3600 Figure 6.1.7.2 (Interior Environment), choose $k_1 = 0.3$ (use 56 days as a guide since shinkage tests on fresh concrete are based on 56 day results. Designers may choose an alternative period for 'short term' based upon their own job conditions).

$\epsilon_{cs} = 0.3*\epsilon_{cs.b}$ (short term shrinkage – say 56 days)

$= 0.3*850$

$= 255 \times 10^{-6}$ (i.e. 255 microns)

$$f_{cs} = \left(\frac{1.5x0.0135}{1+50x0.0135}\right)200{,}000 \text{ x } 255\text{x}10^{-6}$$

$= 0.62$ MPa

Cracking moment,

$$M_{cr} = Z\ (0.6\sqrt{fc} - f_{cs})$$

$$= 8.44*106\ (0.6\sqrt{25} - 0.62)$$

= 20.1 kNm

Short-term, loading

$$w_s = g + \psi_s q$$

$= 16 + 0.7*8 = 21.6$ kN/m

Maximum serviceability bending moment for short-term loading,

$$M_s = \frac{21.6*(5)^2}{8} = 67.5 \text{ kNm}$$

Effective second moment area (sometimes incorrectly called 'moment of inertia'),

$$I_{ef} = I_{cr} + (I - I_{cr})\left(\frac{M_{cr}}{M_s}\right)^3$$

(but $\le I_{e.max}$)

$$= \left(892+(1900-892)\left(\frac{20.1}{67.5}\right)^3\right)*10^6$$

$= 918*10^6$ mm^4

Since $p = 0.0135 > 0.005$ then $I_{e.max}$ = I gross

Immediate deflection for short-term service loading,

$$\delta_s = K \frac{ML^2}{EI_{eff}} \text{ where K = 5/48, (or just } \frac{5wL^4}{384 EI_{ef}} \text{ for simply supported beams)}$$

$$= \frac{5}{48} \frac{67.5 * 10^6 * (5000)^2}{27500 * 918 * 10^6} = 7.0 \text{ mm}$$

Sustained loading,

$$w_l = g + \psi_l q$$

$$= 16 + 0.4*8 = 19.2 \text{ kN/m.}$$

We must now recalculate ϵ_{cs} and f_{cs} in longer term since concrete shrinkage will be greater as will be the tensile stresses induced by these shrinkages. Using Table 6.1.7.2 from AS3600 for a 30 year life and t_h = 195 mm, the new ε_{cs} will be 670 microstrain (i.e. k_1= 0.78). The new f_{cs} will be 1.63 MPa, the new M_{cr} will be 11.6 kNm and thus the long term I_{ef} will be 897 x 10^6 mm^4. Substituting this I_{ef} into the standard deflection equation now gives 7.1 mm (hardly any difference to the initial value – this is because the Ief is almost equal to the I_{cr} in both short and long term cases).

By direct proportions, the immediate deflection due to the sustained loading will be;

$$\delta_{sus} = \frac{19.2}{21.6} 7.1 = 6.3 \text{ mm (alternatively recalculate } \delta_s \text{ using } w_l = 19.2 \text{ in the formula)}$$

The long-term deflection multiplier k_{cs} = 2 since there is no compressive reinforcement. Hence the incremental deflection due to sustained loading (i.e. the deflection that occurs after the attachment of masonry walls or partitions) will be:

$$\delta_{inc} = k_{cs}\,\delta_{sus} = 2*6.3 = 12.6 \text{ mm} \left(= \frac{L}{397} \right) \text{which does not satisfy L / 500.}$$

Total deflection,

$$\delta_{tot} = 7.1 + 12.6 = 19.7\text{mm} \left(= \frac{L}{253} \right) \text{which barely satisfies L / 250.}$$

The engineering designer has a few options: (a) totally redesign the beam to satisfy the requirements of AS3600 Table 2.4.2; (b) make minor changes to the design eg increase the f'_c (which increases E_c) or provide compression reinforcement (which reduces k_{cs}); (c) accept the design as the masonry wall supported may not be a feature wall and as such minor cracking may be tolerable; or (d) carry out a refined calculation as per AS3600 #8.5.2.

EXAMPLE 6

The continuous beam ABCD shown in Figure 8.11 is part of an office building. The bending moment diagram shown is for the short-term serviceability loading. Check the deflection in the end span AB if the beam supports masonry partitions for which there is some provision made to reduce the effects of deflection. Refer to Table 8.6 for f_{cm} and E_c values.

f'_c = 32 MPa g = 42 kN/m q = 30 kN/m

SOLUTION

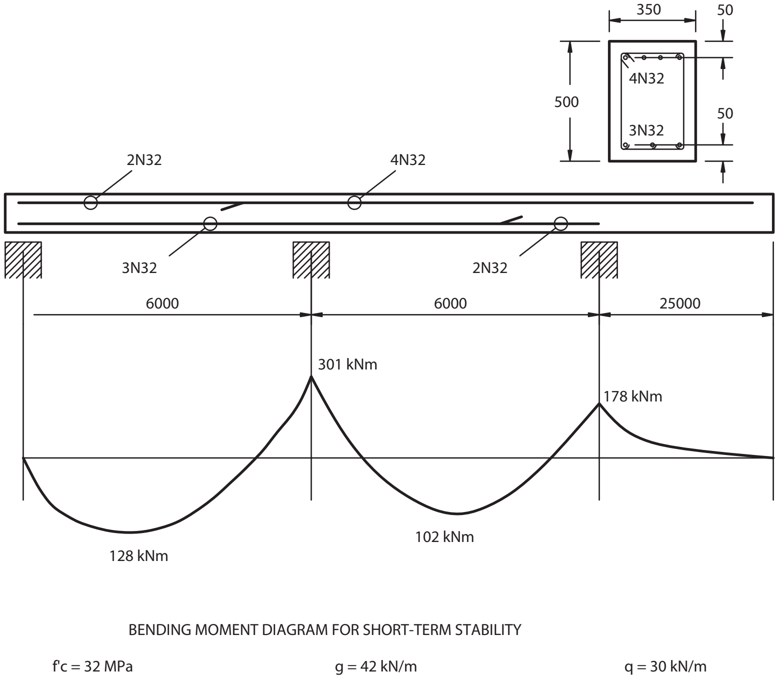
Modulus of elasticity of concrete,

$$E_c = (2400)^{1.5}\, 0.043\, \sqrt{37.5}$$
$$= 31{,}000 \text{ MPa}$$

Modular ratio,

$$n = \frac{2 * 10^5}{31000} = 6.45$$

Figure 8.11



Only the section properties in the middle of the beam and the right hand support will be determined. The beam is simply supported at A so that its stiffness is effectively zero at A.

PROPERTY	MID-SPAN	RIGHT SUPPORT
b	= 350mm	= 350mm
d	= 450mm	= 450mm
d_{sc}	= 50mm	= 50mm
A_{st}	= 2400mm^2	= 3200 mm^2
$p = \frac{A_{st}}{bd}$	= 0.0152	= 0.0203
np	= 0.098	= 0.131
A_{sc}	= 1600mm^2	= 2400mm^2
$p_c = \frac{A_{sc}}{bd}$	= 0.0102	= 0.0152
$(n-1)p_c$	= 0.056	= 0.083

$$kd = d\left(\sqrt{[np+(n-1)p_c]^2 + 2\left(np+(n-1)p_c\frac{d_{sc}}{d}\right)} - [np+(n-1)p_c]\right)$$

kd	= 148 mm	= 161 mm

$$I_{cr} = \frac{b(kd)^3}{3} + nA_{st}(d - kd)^2 + (n - 1)A_{sc}(kd - d_{sc})^2$$

I_{cr}	= 1874*10^6 mm^4	= 2372*10^6 mm^4
$I = \frac{bD^3}{12}$	= 3650*10^6 mm^4	= 3650*10^6 mm^4
$Z = \frac{I}{\left(D/2\right)}$	= 14.6*10^6 mm^3	= 14.6*10^6 mm^3
$t_h = \frac{2Ag}{u_e}$	= 200 mm (u_e for 4 sides)	= 200 mm [u_e =2(500+350)]
ε_{cs} (say 30yrs)	= 670*10^{-6} (microstrain)	= 670*10^{-6} (microstrain)
$f_{cs(dr)} = f_{cs(sr)}$ -	= 0.58 MPa (doubly reinf)	= 0.51 MPa (doubly reinf)
$(A_{sc}/A_{st})f_{cs(sr)}$...but $\geq 1/4\ f_{cs(sr)}$	(suggested minimum)	
where $f_{cs(sr)}$ = $[1.5p/(1+50p)]E_s\epsilon_{cs}$	= 1.74 MPa	= 1.52 MPa
$M_{cr} = Z\,(0.6\sqrt{f'_c} - f_{cs})$	= 41.1 kNm	= 42.1 kNm
M_s	= 178 kNm	= 235 kNm
$I_{ef} = I_{cr} + (I - I_{cr})\left(\frac{M_{cr}}{M_s}\right)^3$	= 1896*10^6 mm^4	= 2379*10^6 mm^4

The effective second moment of area to be used in the deflection formulae:

$$I_{ef} = \frac{I_M + I_R}{2}$$

$$I_{ef} = \frac{1896 + 2379}{2} \text{ x } 10^6$$

$$= 2138 * 10^6 \text{ mm}^4$$

Short-term and long-term serviceability loading,

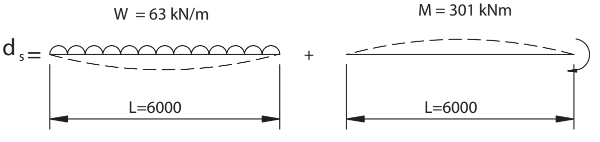
$$w_s = 42 + 0.7*30$$

$$= 63 \text{ kN/m}$$

$$w_l = 42 + 0.4*30$$

$$= 54 \text{ kN/m}$$

The immediate deflection due to short term loading is calculated from equation 8.17 by superposition of loading conditions 1 and 2 shown in Table 8.5.



$$M = \frac{w_s L^2}{8} \qquad M = 235 \text{ kNm}$$

$$= \frac{63 * 6^2}{8}$$

$$= 283.5 \text{ kNm}$$

$$K = \frac{5}{48} \qquad K = -\frac{1}{16}$$

$$\delta s = \frac{5}{48}\left(\frac{283.5 * 10^6 * (6000)^2}{31000 * 2138 * 10^6}\right) - \frac{1}{16}\left(\frac{235 * 10^6 * (6000)^2}{31000 * 2138 * 10^6}\right)$$

$$= 16.0 - 8.0 \qquad \textbf{(8.19)}$$

$$= 8.0 \text{ mm}$$

INCREMENTAL DEFLECTION

Long-term deflection multiplier,

$$k_{cs} = 2 - 1.2 \; \frac{1600}{2400}$$

$$= 1.20$$

Immediate deflection due to long-term sustained loading may be calculated as direct loading proportion of the immediate deflection calculated for the short-term loading.

$$\delta_{sus} = \frac{54}{63} \times 8.0$$
$$= 6.9 \text{ mm}$$

Hence incremental deflection due to sustained loading,

$$\delta_{inc} = k_{cs}\, \delta_{sus}$$
$$= 1.2 * 6.9$$
$$= 8.3 \text{ mm} \quad (= L / 723)$$
$$< \frac{L}{500} \quad (= 12 \text{ mm}) \quad \therefore \text{SATISFACTORY}$$

Total deflection,

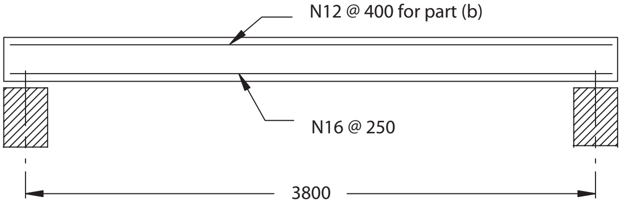
$$\delta_{tot} = 8.0 + 8.3$$
$$= 16.3 \text{ mm } (< L / 250) \text{ i.e. } 24 \text{ mm} \quad \therefore \text{SATISFACTORY}$$

Table 8.6 Elastic Modulus values for various grades of concrete

Strength Grade - f_c (MPa)	Mean strength - f_{cm} (MPa)	Elastic modulus E_c (MPa)
20	24	25000
25	29.5	27500
32	37.5	31000
40	46	34500
50	56.5	38000

PROBLEMS

QUESTION 1



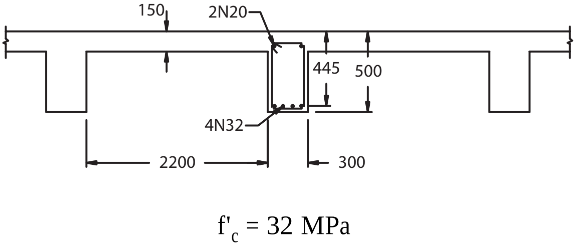
f'_c = 25 MPa   D = 200 mm   d = 170 mm

The simply supported slab for a domestic building shown above carries a total dead load g = 6 kPa (including weight of slab) and a live load q = 3 kPa. The slab supports masonry partitions for which provision is made to minimise the effects of movement.

(a) If the slab is only reinforced with N16 bars at 250 centres, check the slab for serviceability.

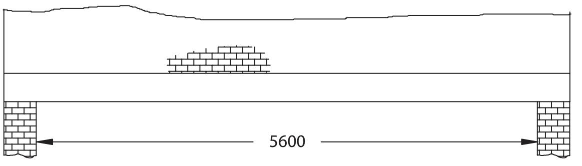
(b) If N12 compression bars at 400 centres are added to the slab, what will be the new live load which may be applied to the slab.

QUESTION 2



For the office floor construction shown right, the T-beams are simply supported over a span L = 6500. The effective flange width for the T-beams is b_{eff} = 1600. Check the T-beams for deflection if the superimposed dead load on the slab g = 2 kPa and the live load q = 3 kPa. It may be assumed that provision is made to limit deflection effects on the masonry partitions supported by the construction.

QUESTION 3

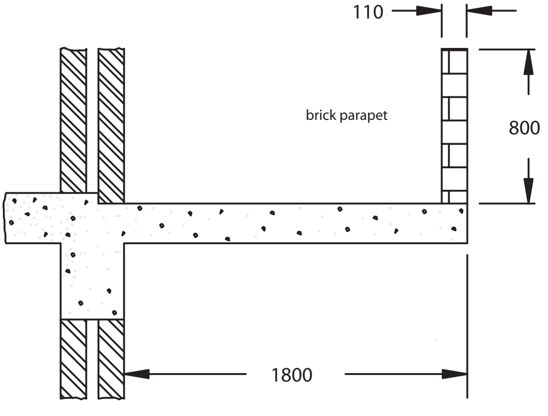


The figure to the right shows the arrangement of an external masonry wall supported by a reinforced concrete beam. The construction is part of a retail store on a beach promenade. Loads carried by the beam are made up of a 20 kN/m dead load and 5 kN/m live load. There is no provision made to reduce deflection effects on the masonry wall construction. Using grade N40 concrete and a beam width b = 350:

(a) Design the beam as a singly reinforced beam using 2N20 hanger bars in the top of the beam.

(b) Redesign the beam if 4N28 compression bars are included in the beam to reduce deflection only.

QUESTION 4



The figure shows an external balcony in a domestic building. The balcony is to be designed to carry a 1.0 kPa superimposed dead load and a 3kPa live load. Design the reinforced concrete balcony using N32 concrete in exposure classification A2. Check that serviceability has been satisfied for total deflection only using simplified deflection calculations. It may be assumed that the supporting beam provides torsional rigidity to prevent rotation.

9

chapter

Continuous beams and continuous one-way slabs

9.1 Additional Symbols used in this Chapter

F_d = Uniformly distributed design load.

L_n = Clear span between inside faces of supports.

9.2 Loading Combinations

Beams and one-way slabs which are continuous over two or more spans may be analysed by any of the well known methods such as three moment equation, slope deflection, moment distribution, and so on. The difficulty is not in the analysis but in the shear volume of work due to the loading possibilities. While the dead load is permanent, the live load can be applied to any one span or combination of spans. The maximum design shear force and bending moment in any span can only be determined after the analysis has been carried out for each dead load and live load combination. Considering a 3-span continuous beam ABCD shown in Figure 9.1, there are 8 possible loading combinations. Whatever method of analysis is applied, the beam is indeterminate to the second degree requiring two simultaneous equations to be solved for each loading condition.

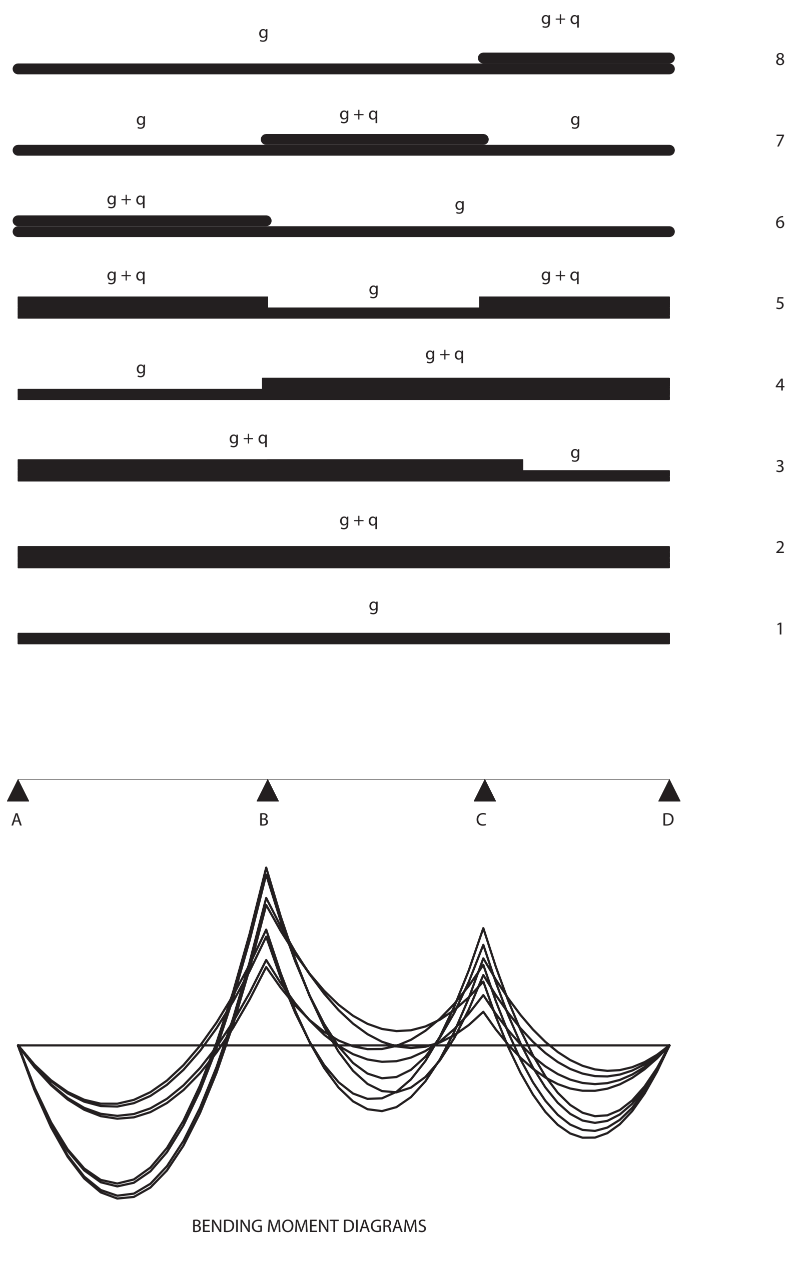
As shown in the figure, there are 8 possible loading conditions. To determine the support moments for all loading conditions it will be necessary to solve 2 simultaneous equations 8 times. In each case it is still necessary to calculate the maximum bending moment and the shear forces for each span. Figure 9.1 also shows the bending moment diagram for each loading condition drawn on the same baseline. A line drawn to enclose all the bending moment diagrams represents the “bending moment envelope”. A shear force envelope is obtained in the same manner. Figure 9.2 shows the shear force and the bending moment envelopes for a 3-span continuous beam drawn by a computer program.

If the number of spans is increased to say 8 continuous spans, to cover all possible dead and live load combinations, it would be necessary to solve 7 simultaneous equations 256 times. A manual solution is obviously not the answer. There are two possibilities for carrying out such analysis:

(a) a computer program,
(b) an empirical solution using bending moment and shear force coefficients.

Ideally the computer program should also plot the bending moment and shear force envelopes as shown in Figure 9.2 for example 1.

Figure 9.1 - Loading Combinations for a 3-span Continuous Beam



EXAMPLE 1

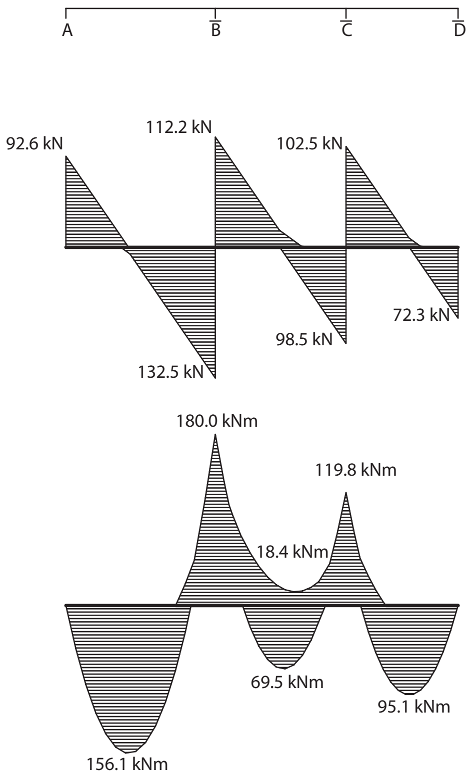
The following computer results are obtained for a three-span continuous beam ABCD carrying a 10 kN/m dead load and a 10 kN/m live load. The spans are 8 m, 7 m and 6 m respectively.

Number of spans ______________	3	
Left end fixed _________________	N	
Left end cantilever _____________	N	
Right end fixed ________________	N	
Right end cantilever ____________	N	
Span in m, AB _________________	8	
BC ________________	7	
CD ________________	6	
Uniformly distributed live loads Y or N _	Y	
Dead loads in kN/m	AB __________	10
	BC __________	10
	CD __________	10
Live loads in kN/m	AB __________	10
	BC __________	10
	CD __________	10
Dead load and live load factors	1.25, 1.5	

SUPPORT	MAXIMUM REACTION	MAX SHEAR FORCE LEFT	RIGHT	MAXIMUM MOMENT	DISTANCE FROM SUPPORT TO LEFT
A	+92.6 kN		+92.6kN		
In span AB		+ 156.1kNm	+3.37 m		
B	+244.7 kN	-132.5kN	+112.2kN	-180.0kNm	
In span BC		+69.5kNm	+3.63 m		
				-18.4kNm	+4.39 m
C	+201.0 kN	-98.5kN	+102.5kN	-119.8kNm	
	In span CD		+95.1kNm	+3.37 m	
D	+72.3 kN	-72.3kN			

Tabulated Results

Figure 9.2 - Shear Force and Bending Moment Envelopes



9.3 Bending Moment and Shear Force Coefficients for Continuous Beams and Continuous One-Way Slabs

Design Bending Moment,

$$M^* = B.M.Coefficient * F_d * (L_n)^2 \quad \textbf{(9.1)}$$

Design Shear Force,

$$V^* = S.F.Coefficient * F_d * L_n \quad \textbf{(9.2)}$$

Code clause #7.2 permits shear force and bending moment coefficients to be used for calculating design shear forces and design bending moments provided that the following conditions are satisfied:

(a) The ratio of the longer to the shorter span in any two adjacent spans does no exceed 1.2.

(b) The loads are uniformly distributed.

(c) The live load q does not exceed twice the dead load g.

(d) Members are of uniform cross-section.

(e) The reinforcement is arranged in accordance with Figure 7.9, redrawn in Figure 9.5, (note that for beams at least one quarter of the negative reinforcement provided at the support must be extended over the full span) and Figure 9.6 for slabs.

(f) Bending moments at supports are caused only by the action of loads applied to the beam or slab.

The shear force and bending moment coefficients are shown in Figure 9.3 for 2 spans and Figure 9.4 for 3 or more spans.

Figure 9.3 -
S.F. and B.M. Coefficients for 2 Spans

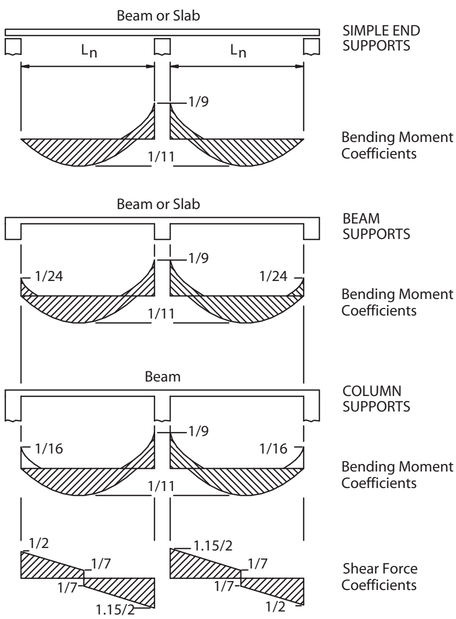


Figure 9.4 -
S.F. and B.M. Coefficients for 3 or More Spans

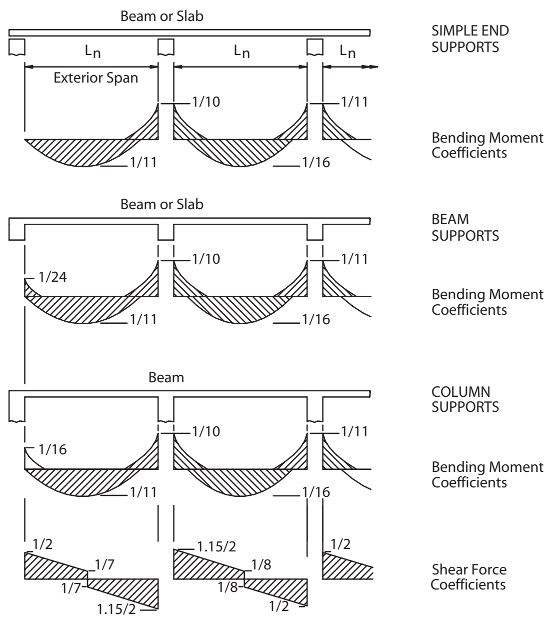


Figure 9.5 -
Arrangement of Reinforcement for Continuous Beams

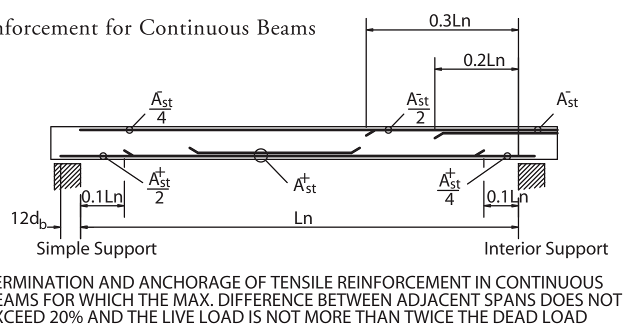
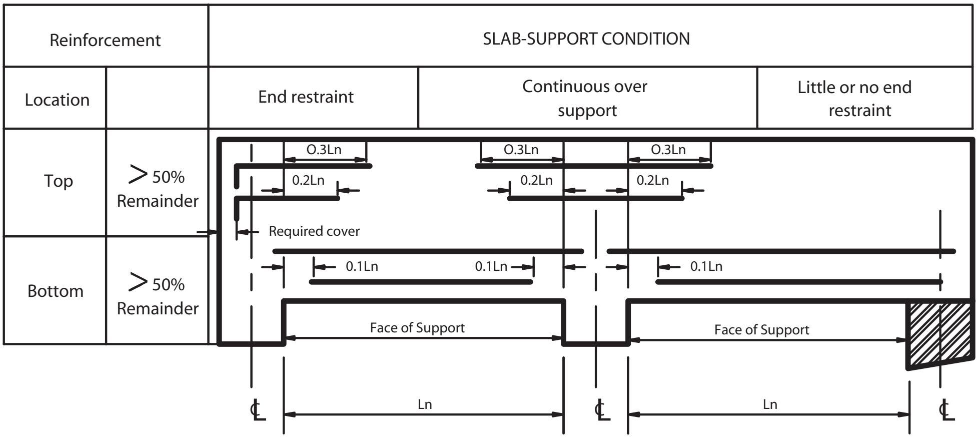


Figure 9.6 - Arrangement of Reinforcement for Continuous Slabs



EXAMPLE 2

The continuous 3-span beam used in example 1 is redrawn in Figure 9.7 (*right*). The beam carries a total dead load (including the weight of beam) g = 10 kN/m and a live load q = 10 kN/m.

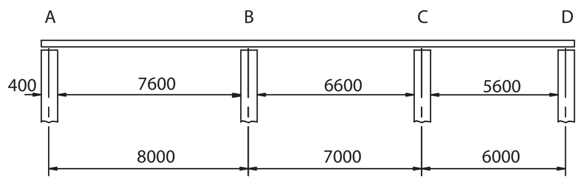


Figure 9.7

Using bending moment coefficients,calculate all negative and positive bending moments and compare the results with example 1.

SOLUTION

Design load, $F_d = 1.2\times10 + 1.5\times10 = 2.7$ kN/m

Moment Condition	Coeff.	L_n	$M^* = \text{Coeff.} \times F_d \times (L_n)^2$	Results from Example 1
Negative BM s				
Support B				
Exterior Face	1/10	7.6	156.0 kNm	180.0 kNm
Interior Face	1/10	6.6	117.6 kNm	180.0 kNm
Support C				
Interior Face	1/10	6.6	117.6 kNm	119.8 kNm
Exterior Face	1/10	5.6	84.6 kNm	119.8 kNm
Positive BM s				
Span AB	1/11	7.6	141.8 kNm	151.1 kNm
Span BC	1/16	6.6	73.5 kNm	69.5 kNm
Span CD	1/11	5.6	77.0 kNm	95.1 kNm

Comments:

(1) The bending moments obtained using bending moment coefficients appear to under-estimate the theoretical results in example 1.

(2) The theoretical results are based on knife-edge supports where in reality the support width, or more precisely the clear span between support faces as compared to the centreline span, must affect the bending moment; the wider the support, the lesser will be the bending moment.

(3) In practice a redistribution of moments will occur, depending on the extent of flexural cracking over the supports. A relaxation of negative bending moments over the supports will produce a corresponding increase in the positive bending moment between supports.

Code #7.6.8 permits redistribution of moments. For steel ratios equal to or less than $0.5p_{max}$ i.e. $k_u \leq 0.2$, the negative moments over interior supports may be reduced by up to 30% with a corresponding increase in the positive bending moment. The permissible redistribution is reduced for higher steel ratios and no redistribution is allowed if the steel ratio is equal to or greater than the maximum steel ratio i.e. $k_u.\geq 0.4$. With doubly reinforced beams, redistribution is permitted provided that the steel ratio in the primary beam does not exceed the maximum steel ratio.

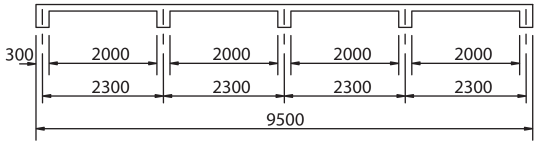
(4) The central span BC in example 1 also showed a negative moment between supports. This is indirectly catered for by Code #8.1.8.6 which requires that at least one quarter of the negative moment tensile steel reinforcement is extended over the whole span.

(5) Continuous members which do not fall within the Code guidelines ($q \leq 2g$ and 20% maximum span difference between adjacent spans) or contain cantilevered ends may not be designed by using bending moment and shear force coefficients.

EXAMPLE 3

In an office building a continuous one-way slab over four equal spans is supported by integral beams as shown in Figure 9.8. Design the slab using N32 grade concrete and welded wire mesh reinforcement for the maximum negative and maximum positive bending moments. The reinforcement so determined will be extended over the full slab. The slab supports masonry partitions for which there is no provision made for deflection. The superimposed dead and live loads are: g = 2 kPa and q = 3 kPa. Exposure classification A1 may be assumed.

Figure 9.8



SOLUTION

Data: $g = 2$ kPa $q = 3$ kPa $f'_c = 32$ MPa $f_{sy} = 500$ MPa

Exposure Classification A1.

From Table 1.2, $\psi_s = 0.7$ and $\psi_l = 0.4$

From Table 8.1, the deflection limitations are;

Maximum total deflection $\dfrac{\Delta}{L_{ef}} = \dfrac{1}{250}$

Maximum incremental deflection $\dfrac{\Delta}{L_{ef}} = \dfrac{1}{1000}$

SERVICEABILITY

Try 150 mm thick slab. Weight of slab = $0.15 \times 24 = 3.6$ kN/m^2.

Total dead load $g = 2 + 3.6 = 5.6$ kN/m^2

Assuming initially the same negative and positive reinforcement. The slab will in effect carry compressive reinforcement since the required reinforcement for the maximum negative and positive bending moments will be carried over the full slab. The value of the long-term deflection multiplier k_{cs} will thus become 0.8 since the reinforcing areas $A_{sc} = A_{st}$ based on initial assumption.

For total deflection, the design service load,

$$\begin{aligned} F_{d.ef} &= (1 + k_{cs})g + (\psi_s + \psi_l\, k_{cs})q \\ &= (1 + 0.8)5.6 + (0.7 + 0.4\times0.8)3 \\ &= 13.14 \text{ kN/m}^2 \end{aligned}$$

From Chart D2 for $\dfrac{\Delta}{L} = \dfrac{1}{250}$, read $k_5 = 21.1$

For incremental deflection, the design service load,

$$\begin{aligned} F_{d.ef} &= k_{cs}g + (\psi_s + \psi_l\, k_{cs})q \\ &= 0.8\times5.6 + (0.7 + 0.4\times0.8)3 \\ &= 7.54 \text{ kN/m}^2 \end{aligned}$$

From Chart D2 for $\dfrac{\Delta}{L_{ef}} = \dfrac{1}{1000}$, read $k_5 = 16.2$.

From Chapter 8, $k_3 = 1.0$ and $k_4 = 2.1$ for end spans.

$$\begin{aligned} \text{Maximum } \frac{L_{ef}}{d} &= k_3\, k_4\, k_5 \\ &= 1.0\times2.1\times16.2 \\ &= 34.0 \end{aligned}$$

Using the centreline spacings of supporting beams as conservative estimate for the effective span, the minimum required effective depth of slab will be,

$$d = \frac{2300}{34} = 68 \text{ mm}$$

Use d = 75 mm to give depth of slab D = 100 mm.

DESIGN STRENGTH FOR A 1m WIDE STRIP

Weight of slab = 0.1×24 = 2.4 kN/m

Total dead load; g = 2.4 + 2 = 4.4 kN/m

Design load; F_d = 1.2×4.4 + 1.5×3

= 9.8 kN/m

Using bending moment coefficients shown in Figure 9.4,

Maximum negative design bending moment,

$$M = -\frac{1}{10} 9.8 \times 2^2 = -3.9 \text{ kNm per m width of slab.}$$

Maximum positive design bending moment,

$$M = +\frac{1}{11} 9.8 \times 2^2 = +3.6 \text{ kNm per m width of slab.}$$

The design moments are too small to be read from Chart S2. SL82 mesh provides an area of 227 mm^2 per metre width of slab and a steel ration p = 0.003. The moment capacity using SL82 mesh is:

$$\Phi M_{uo} = 0.8 \times 25 \times \frac{0.003 \times 500}{25} \times \left(1 - \frac{0.003 \times 500}{25 \times 1.7}\right) \times 1000 \times 0.075^2$$

$$= 6.5 \text{ kNm}$$

$$> M^*$$

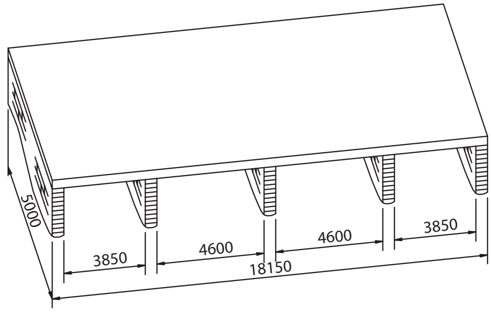
The slab should now be checked for the minimum steel ratio and for crack control.

PROBLEMS

QUESTION 1

An exposed reinforced concrete deck supported by 250 mm wide cross-walls is shown at the right. The structure is located at Hornsby in NSW, which is approximately 20 km from the coastline.

In addition to its own weight, the deck is required to carry a live load q = 5 kPa.



Future plans exist to enclose the deck, in which case it would also be required to carry a 1 kPa superimposed dead load primarily due to the masonry partitions to be installed. Provision will be made in the construction of the partitions to minimise the effects of movement.

Design the slab using grade N40 concrete and reinforcing fabric. It may be assumed that the negative reinforcement required at the supports will be extended over the full length of the slab. It may also be assumed that the slab is partially restrained against movement due to temperature and shrinkage effects.

10 chapter

Shear and Torsion

10.1 Additional Symbols used in this Chapter

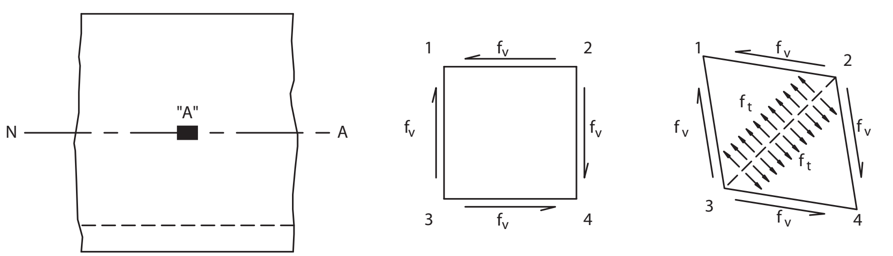
A_m	=	Area of thin walled section for torsion defined by the median lines of the walls of a single cell.
A_{sv}	=	Cross-sectional area of shear reinforcement.
$A_{sv.min}$	=	Minimum area of shear reinforcement.
A_{sw}	=	Area of a single leg of a closed tie used as torsional reinforcement.
A_t	=	Torsion area defined as the area from the centre of the corner bars of the cross section.
a_v	=	Distance from section at which shear is being considered to the nearest support.
b_v	=	Effective width of a web for shear.
	=	b for a rectangular beam.
	=	b_w for a T-beam or L-beam.
b_w	=	Width of a web as in a T-beam.
d_o	=	Distance from extreme compression fibre to the centroid of the outermost layer of tensile reinforcement but not less than 0.8D.
f_{cv}	=	Concrete shear strength.
$f_{sy.f}$	=	Yield strength of fitments.
J_t	=	Torsional modulus for the cross section.
p_v	=	Shear steel ratio $A_{st}/(b_v d_o)$
T_{uc}	=	Ultimate torsional strength of a beam without torsional reinforcement.
$T_{u.max}$	=	Ultimate torsional strength of a beam limited by crushing failure.
T_{us}	=	Ultimate torsional strength of a beam with torsional reinforcement.
T^*	=	Design torsional moment.
u	=	Length of critical shear perimeter for two-way action.

u_t	=	Perimeter of A_t.
V_c	=	Simplified ultimate shear capacity of unreinforced beam.
v'_c	=	Nominal concrete shear stress capacity.
V_u	=	Ultimate shear strength.
$V_{u.max}$	=	Ultimate shear strength limited by shear crushing.
$V_{u.min}$	=	Ultimate shear strength of a beam with minimum shear reinforcement.
V_{uc}	=	Ultimate shear strength excluding shear reinforcement.
V_{us}	=	Contribution provided by shear reinforcement to the ultimate shear strength of a beam.
x	=	Smaller dimension of a cross section (or smaller dimension of a rectangular component of a cross section).
y	=	Larger dimension of a cross section (or larger dimension of a rectangular component of a cross section).
y_1	=	Larger dimension of a closed rectangular torsion tie.
β_1	=	Shear strength coefficient for comparable increase in shear capacity of shallow beams.
β_2	=	Shear strength coefficient for axial load effects.
β_3	=	Shear strength coefficient to account for increased strength when concentrated loads are applied near supports (short shear span $a_v < 2d_o$).
θ_t , θ_v	=	Angle between the concrete compression "strut" and the member axis in the truss model for torsion or shear respectively.

10.2 Shear Failure Models

It is useful to consider some aspects of the behaviour of concrete in shear to give the reader a better appreciation of the design process. Considering an uncracked reinforced concrete member shown in Figure 10.1, a small element "A" taken at the level of the neutral axis is only subjected to boundary shear stresses f_v since there are no bending stresses.

Figure 10.1



(a) Element "A" at the Neutral Axis

(b) Enlarged Element "A"

(c) Diagonal Tensile Stresses

The enlarged element "A" and the boundary shear stresses are shown in Figure 10.1(b). The effect of the shear stresses will be to distort the element and produce tensile stresses f_t (in this case principal tensile stresses) across the diagonal 2-3 of the element as shown in Figure 10.1(c). The principal diagonal tensile stresses act on the principal plane 2-3 inclined at 45^o to the neutral axis. Above or below the neutral axis, compressive or tensile stresses exist in combination with shear stresses which will alter the slope of the principal plane on which the tensile stresses act.

If the element is located at a section near a simple support, the moments are small and flexural cracks are not likely to occur. Cracking of the member, if it takes place, will occur along diagonal lines parallel to the principal planes. The propagation of the diagonal cracks above the neutral axis is affected by the horizontal compressive stresses due to bending and direct vertical compressive stresses due to applied loads and in particuar concentrated loads. Axial tension or compression applied to the member may also have a marked effect. Shear failure initiated by diagonal tension cracks will be resisted by a dowel action in the longitudinal tensile reinforcement (the degree of this resistance will depend on the amount and the size of the tensile reinforcement) and direct shear resisted by the uncracked concrete above the diagonal tension crack. A failure condition is shown in Figure 10.2.

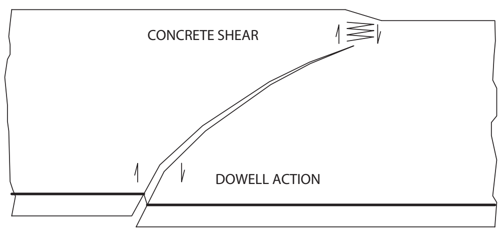


Figure 10.2

Shear reinforcement in the form of vertical stirrups or inclined shear reinforcement and bent-up bars will not prevent the formation of diagonal tension cracks. Only vertical stirrups will be considered in this chapter because they are the most frequently used form of shear reinforcement. Shear reinforcement is in reality tensile reinforcment which ties together the beam on either side of the diagonal tension crack as shown in Figure 10.3 . Shear reinforcement will thus increase the beam shear capacity by providing an additional shear resisting component. It is apparent from Figure 10.3 that the capacity of the shear reinforcement will depend on the area of shear reinforcement available within a potential diagonal crack length and effective anchorage of the shear reinforcement on either side of the diagonal crack.

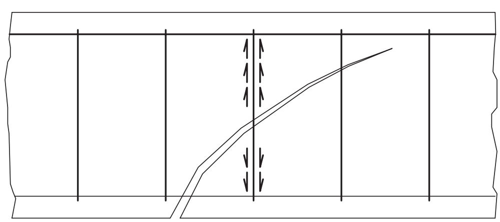
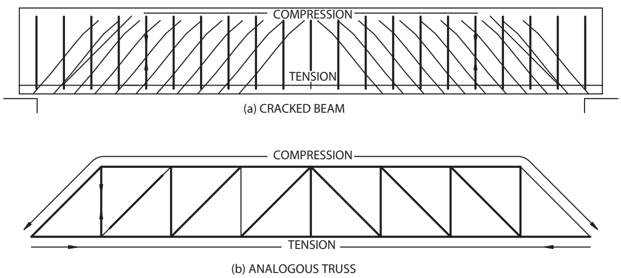


Figure 10.3

In addition to the direct concrete shear resistance and dowel action, shear resistance is also provided by the ragged shape of the diagonal crack. Diagonal shear cracking generally occurs in conjunction with flexural cracking and the terminology "flexural-shear cracking" is frequently employed. The mechanics of shear failure is very complex and to a large degree it is still not sufficiently well understood to enable practical design rules to be formulated to specifically include all the shear resistance components. In 1962 the ACI-ASCE Committee on "Shear and Diagonal Tension" reported that it could not clearly define the shear failure mechanism. Considerable research on shear has been carried out since and a lot of experimental data is available but it is still not possible to formulate simple design rules to account for the contributions to shear resistance provided by all the beam components.

The shear strength of a beam also depends on such factors as the shear span and the depth of beam. In addition to the classic analysis, the Code permits the use of analysis based on the truss analogy for determining ultimate shear capacity. The truss analogy will only be considered here as a simple means of understanding beam behaviour. As a simplification, it is assumed that flexural-shear cracks occur at regular intervals at 45^{o} as shown in Figure 10.4a .

Figure 10.4 - Beam Truss Analogy



The beam functions as a conventional truss with:

(a) the top compression chord formed by the uncracked concrete,

(b) by the main longitudinal reinforcement acting as the bottom tension chord,

(c) the stirrups acting as vertical ties and,

(d) the uncracked concrete bands separated by diagonal tension cracks acting as web compression members.

The analogous truss is shown in Figure 10.4b. The main difference between the beam and the truss is that in the beam the uncracked concrete and the tensile reinforcement can transmit shear while in the pin-jointed truss only the web members can transmit shear. In the analogous truss the shear capacity will depend on:

(a) The yield strength and the area of the stirrups within a potential shear crack which depends on the spacing of stirrups.

(b) The crushing strength of the inclined concrete web members. Web crushing may be a possibility in beams with very thin webs and it is the limiting consideration and the upper limit for beams.

The design shear strength of a beam is ϕV_u where the ultimate shear strength V_u is made up of a beam component V_{uc} and a shear reinforcing component V_{us}.

$$V_u = V_{uc} + V_{us} \tag{10.1}$$

The beam component V_{uc}, which is the ultimate shear strength of the unreinforced beam, has to incorporate all the shear strength components already mentioned. This can only be done by an empirical formula based on experimental data and it is given by equation 10.2.

$$V_{uc} = \beta_1 \beta_2 \beta_3 b_v d_o \left(\frac{A_{st} f'_c}{b_v d_o} \right)^{1/3} \quad \textbf{(10.2)}$$

Where: $\beta_1 = 1.1 \left(1.6 - \frac{d_o}{1000}\right) \geq 1.1$

β_2 = Factor for axial forces which will not be considered.

1.0, or

1 – (N* / 3.5 Ag) for significant axial tension

1 + (N* / 14 Ag) for significant axial compression

$\beta_3 = \frac{2d_o}{a_v} \leq 2$ for large concentrated load applied < $2d_o$ from support.

d_o = Distance from extreme compression to outer layer of tensile reinforcement.

Equation 10.2 may be simply written as,

$$V_{uc} = \beta_1 \beta_2 \beta_3 V_c \quad \textbf{(10.3)}$$

Where V_c is a conservative value of V_{uc} (no axial tension) assuming values of 1.0 for β_1, β_2 and β_3 and it is given by equation 10.4.

$$V_c = b_v d_o \left(\frac{A_{st} f'_c}{b_v d_o} \right)^{1/3} \quad \textbf{(10.4)}$$

The beam shear capacity is obtained by multiplying equation 10.4 by the reduction factor $\phi = 0.7$ for shear.

$$\phi V_c = \phi\, b_v d_o \left(\frac{A_{st} f'_c}{b_v d_o} \right)^{1/3}$$

Rearranging the equation,

$$\frac{\Phi V_c}{b_v\ d_o} = \Phi \left(\frac{A_{st}\ f'_c}{b_v\ d_o} \right)^{1/3} \quad \textbf{(10.5)}$$

Let, p_v = Shear steel ratio defined by equation 10.6.

$$p_v = \frac{A_{st}}{b_v d_o} \quad \textbf{(10.6)}$$

and let v'_c = Nominal concrete shear stress capacity defined by equation 10.7.

$$v'_c = \frac{\phi V_c}{b_v d_o} \qquad \textbf{(10.7)}$$

Equation 10.5 may be re-written to give the nominal concrete shear stress capacity in terms of the shear steel ratio.

$$v'_c = \phi \left(p_v f'_c\right)^{1/3} \qquad \textbf{(10.8)}$$

Chart V1 on page 160 is a plot of equation 10.8.

10.3 Contribution to Shear Capacity by Vertical Stirrups

The contribution provided by shear reinforcement in the form of vertical stirrups is simply the tensile capacity of the number of vertical stirrups contained within a potential shear crack. Assuming shear cracks inclined at 45^o to the longitudinal direction so that the horizontal projection of the inclined cracks can be taken to be approximately equal to d_o, the number of vertical stirrups included in the horizontal projection will be d_o/s where s is the stirrup spacing. Multiplying the effective number of stirrups d_o/s by the area A_{sv} of the stirrups (note 2 legs of stirrups make A_{sv}) gives the shear area within the potential diagonal crack. The shear area $A_{sv}d_o/s$ multiplied by the yield strength of the stirrups $f_{sy.f}$ represents the ultimate tensile resistance or the ultimate shear contribution V_{us} provided by the shear reinforcement. The ultimate shear capacity of the stirrups is given by equation 10.9.

$$V_{us} = \frac{A_{sv} f_{sy.f} d_o}{s} \qquad \textbf{(10.9)}$$

Multiplying both sides by the reduction factor ϕ (=0.7) for shear,

$$\phi V_{us} = \phi \frac{A_{sv} f_{sy.f} d_o}{s}$$

Rearranging the equation in a form to make it suitable for the development of a design aid by dividing both sides by d_o.

$$\frac{\Phi V_{us}}{d_o} = \frac{\Phi A_{sv} f_{sy.f}}{s} \qquad \textbf{(10.10)}$$

Design Chart V2 is a plot of equation 10.10 for N12 stirrups. This chart will be used to determine the required stirrup spacing to carry the excess shear which is the difference between the design shear force and the unreinforced concrete beam capacity, $V^* - \phi V_{uc}$.

There are still a number of Code conditions which must be satisfied before a systematic design procedure is outlined followed by worked examples.

10.4 Maximum Ultimate Shear Strength #8.2.6

The maximum shear force is limited by crushing of the inclined concrete compression members considered in the analogous truss. The ultimate shear strength V_u cannot exceed $V_{u.max}$ given by,

$$V_{u.max} = 0.2 f_c b_v d_o \tag{10.11}$$

10.5 Maximum Design Shear Force Near a Support #8.2.4

The maximum design shear force V^* is taken "at the face of the support". This is a major change from the previously accepted position which was at a distance d from the face of the support. The concrete Code AS3600-2001 does state that the maximum transverse shear near a support can be taken at a distance d_o from the face of the support provided that the following conditions are satisfied:

(i) diagonal cracking cannot take place at the support or extend into it

(ii) no concentrated loads exist closer than $2d_o$ from the face of the support

(iii) the value of $\beta_3 = 1.0$

(iv) transverse shear reinforcement required at d_o from the support is continued unchanged to the face of the support.

The Code also states that the longitudinal tensile reinforcement required at d_o from the face of the support be continued onto the support and that it be fully anchored past the face of the support.

10.6 Requirement for Shear Reinforcement #8.2.5

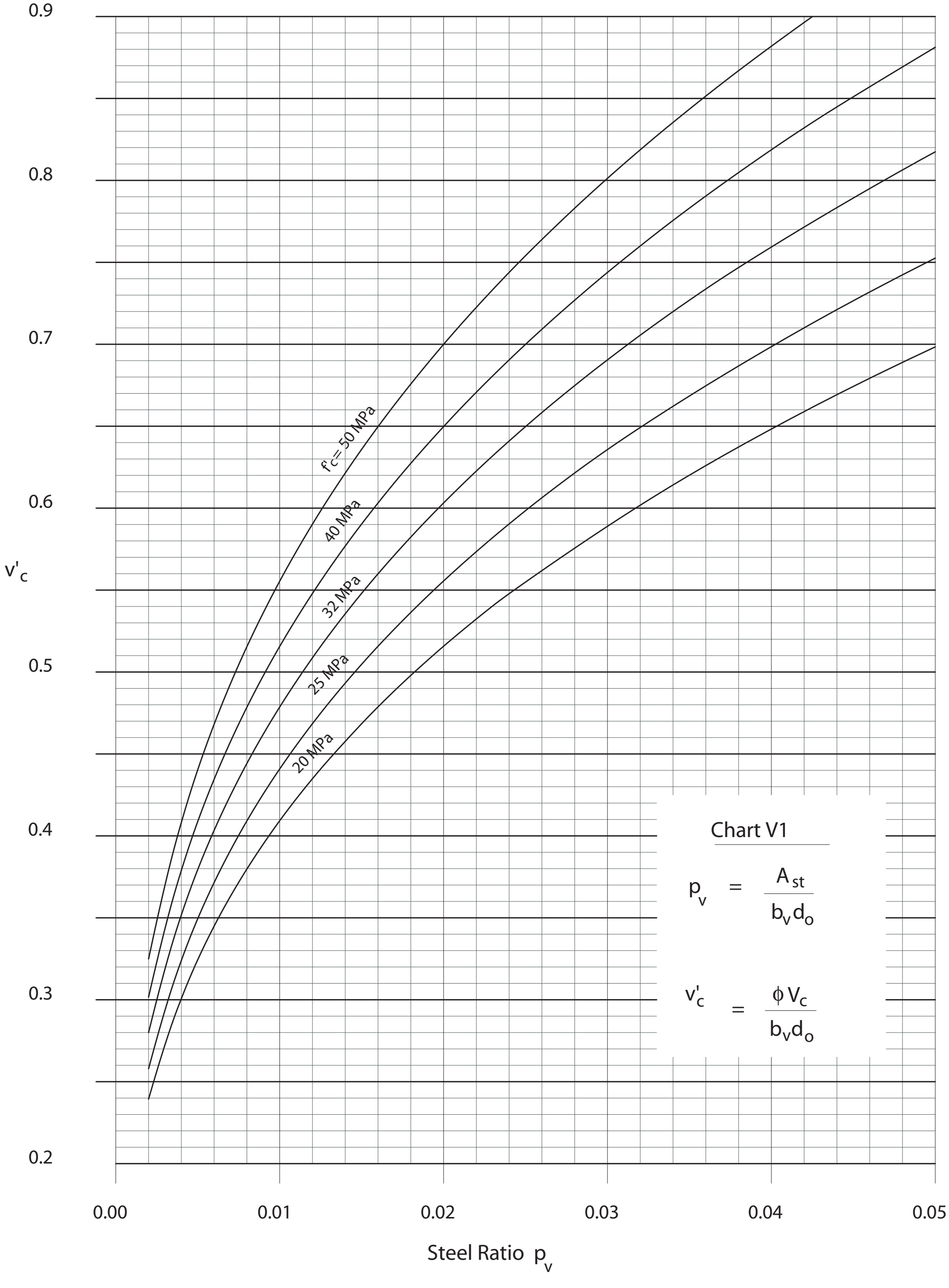
The conditions governing the inclusion of shear reinforcement are as follows:

(a) $V^* \leq 0.5\phi V_{uc}$	No shear reinforcement is required except for deep beams (i.e D>750) for which the minimum shear area $A_{sv.min}$ must be provided.
(b) $0.5\phi V_{uc} < V^* \leq \phi V_{u.min}$	Minimum shear area $A_{sv.min}$ must be provided.
(c) $V^* \leq \phi V_{uc}$	No shear reinforcement is required for shallow members (where $D \leq x$ where x is the greater of 250 or $(b_w/2)$.
(d) $V^* > \phi V_{u.min}$	Shear reinforcement must be provided.

10.7 Minimum Shear Reinforcement #8.2.8

The minimum area of shear reinforcement to be provided in a beam is given by:

$$A_{sv.min} = \frac{0.35 b_v s}{f_{sy.f}} \tag{10.12}$$


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
v'c
f'c = 50 MPa
40 MPa
32 MPa
25 MPa
20 MPa
Chart V1
pv = Ast / bv do
v'c = φ Vc / bv do
0.00
0.01
0.02
0.03
0.04
0.05
Steel Ratio pv

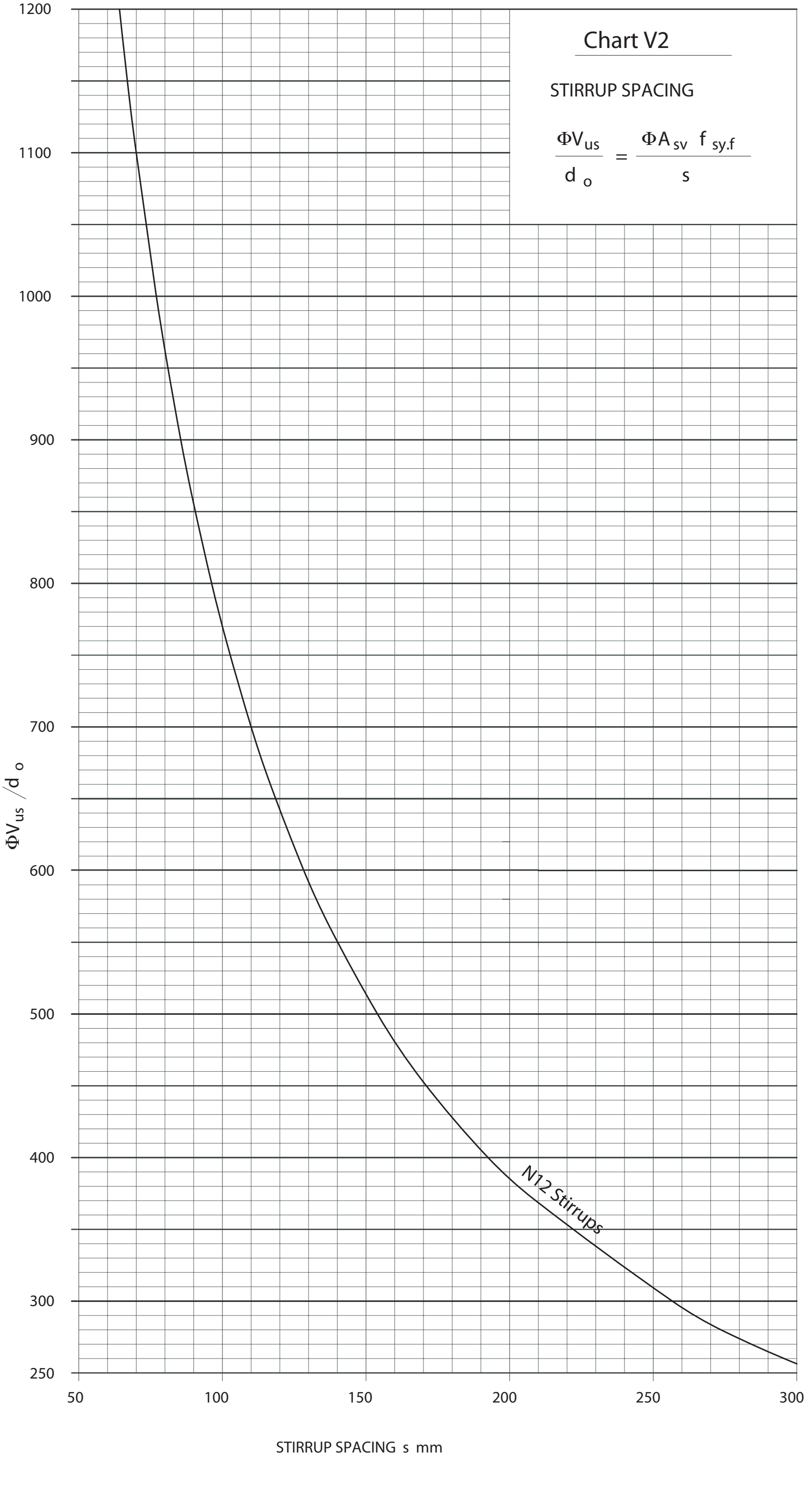


Chart V2
STIRRUP SPACING
$\frac{\Phi V_{us}}{d_o} = \frac{\Phi A_{sv} f_{sy.f}}{s}$
N12 Stirrups
$\Phi V_{us} / d_o$
1200
1100
1000
900
800
700
600
500
400
300
250
50
100
150
200
250
300
STIRRUP SPACING s mm

10.8 Shear Strength with Shear Reinforcement #8.2.9

The ultimate shear strength of a beam provided with minimum shear reinforcement is given by:

$$V_{u.min} = V_{uc} + 0.6b_v\, d_o \qquad \textbf{(10.13)}$$

(Note the $0.6b_v d_o$ section of the above formula is the steel shear strength contribution as the 0.6 value is in units of kN/mm^2). The derivation of the $V_{u.min}$ formula comes from the basic design shear strength formula $V_u = V_{uc} + V_{us}$ as defined in # 8.2.2. The shear strength contribution by the shear reinforcement V_{us} is defined by the formula given in # 8.2.10:

$$V_{us} = \left(\frac{A_{sv}\, f_{sy.f}\, d_o}{s} \right) \cot\left(\theta_v\right)$$

substituting $A_{sv.min} = \left(\frac{0.35\, b_v\, s}{f_{sy.f}} \right)$ and using $\theta_v = 30°$, V_{us} becomes $0.6\, b_v\, d_o$.

10.9 Spacing and Distribution of Shear Reinforcement #8.2.12.2-3

The maximum spacing of shear reinforcement is taken as 0.5D but not more than 300mm except when the design shear force is less than or equal to the minimum shear force, $V^* \leq \phi V_{u.min}$, the spacing may be increased to 0.75D or 500mm, whichever is the lesser.

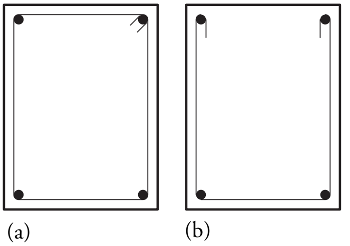
The shear reinforcement required at any section must be carried a minimum distance D in the direction of decreasing shear.

10.10 Anchorage of Shear Reinforcement #8.2.12.4

Since shear reinforcement is in tension it needs to be anchored on each side of potential diagonal cracks. The Code now requires that shear reinforcement develop its yield strength at any point in the stirrup legs and as such nominates various 'deemed to comply' conditions. These include using hooks with 135° and 180° hooks plus having the hook extend a distance of 10 d_b or 100 mm (whichever is greater) into the centre of the concrete element. If the hook is located in the tension zone, the original calculated spacing s of the stirrups (or ties) must now be reduced to 0.8s (i.e brought 20% closer together). Finally any fitment cogs are not allowed to be anchored in the cover zone (i.e. usually 20 to 70 mm from any surface) of the concrete. Welding of the fitments is considered suitable anchorage.

Closed shear reinforcement shown in Figure 10.5a should be used in preference to the open U-shaped shear reinforcement shown in Figure 10.5b because it provides a much more rigid reinforcing cage securing the longitudinal reinforcement and it is effective in resisting torsion.

Figure 10.5

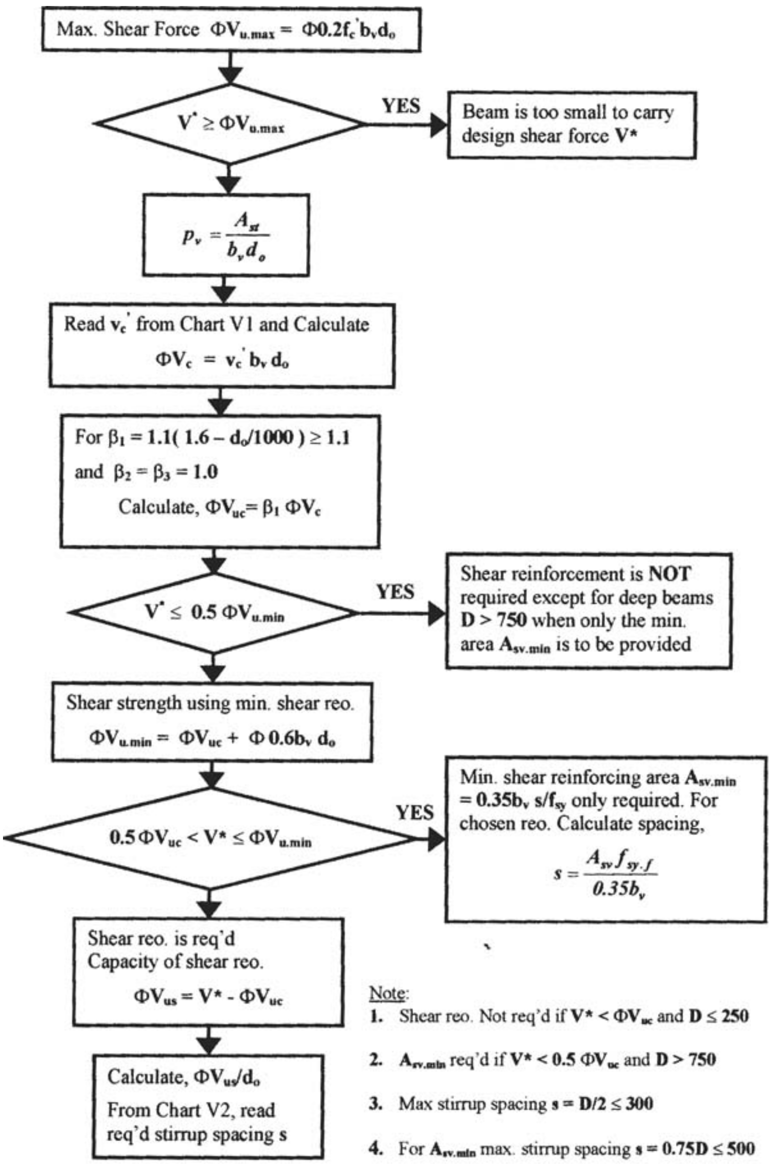


(a) Closed Fitment

(b) Open Fitment

10.11 Design Procedure

The design procedure for shear reinforcement using vertical stirrups is outined by the flow chart below.



SHEAR DESIGN FLOW CHART

EXAMPLE 1

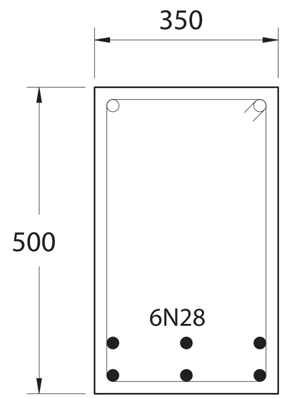
For the reinforced concrete beam shown in Figure 10.6 calculate:

(a) the maximum permissible shear force $V_{u.max}$

(b) the unreinforced beam shear capacity ϕV_{uc}

(c) the beam shear capacity $V_{u.min}$ using minimum shear reinforcement and,

(d) determine the spacing of N12 stirrups at a section where the design shear force V*= 280 kN.



f'_c = 32 MPa
Exposure A2

Figure 10.6

SOLUTION

Data: b_v = 350 D = 500 A_{st} = 3720 mm^2
Exposure A2

A_{sv} = 220 mm^2 $f_{sy.f}$ = 500 MPa f'_c = 32 MPa

Minimum cover to shear reo. = 25 mm

Design shear force V^* = 280 kN

(a) Maximum permissible shear force.

Shear depth d_o = 500 - 25(cover) - 12(stirrup) - 14(half bar)
= 449 mm

$V_{u.max} = 0.2\, f'_c\, b_v\, d_o$
$= 0.2*32*350*449*10^{-3}$
= 1005 kN

(b) Unreinforced beam shear capacity

Shear steel ratio $p_v = \dfrac{A_{st}}{b_v d_o}$

$= \dfrac{3720}{350 * 449} = 0.0237$

From Chart V1 read v'_c = 0.64 MPa

$\phi V_c = v'_c\, b_v\, d_o = 0.64*350*449*10^{-3}$
= 100.6 kN

A value of 100.6 kN may be taken as a conservative value of the unreinforced beam shear capacity ϕV_{uc} provided there are no axial tension forces. A better estimate is obtained if multiplying factor β_1 is included.

$$\begin{aligned}\beta_1 &= 1.1(1.6 - \frac{d_o}{1000})\\ &= 1.1(1.6 - \frac{449}{1000})\\ &= 1.266\end{aligned}$$

Adjusted beam shear capacity,

$$\begin{aligned}\phi V_{uc} &= \beta_1 \phi V_c\\ &= 1.266*100.6\\ &= 127.4 \text{ kN}\end{aligned}$$

(c) Having chosen N12 shear reinforcement as the minimum shear reinforcement as well as the shear reinforcement to carry shear forces where required, the only difference will be in the spacing. The spacing required to satisfy the minimum shear reinforcement, from equation 10.12:

$$\begin{aligned}s &= \frac{A_{sv} f_{sy.f}}{0.35 b_v}\\ &= \frac{220 * 500}{0.35 * 350}\\ &= 898 \text{ mm}\end{aligned}$$

This exceeds the maximum spacing which is the lesser of 300 mm and 0.5D, i.e. 250 mm, thus N12 stirrups at 250 mm centres will be used for the minimum shear reinforcement. This is a hefty minimum shear reinforcement. Much smaller bars could have been used.

Beam shear capacity using minimum shear reinforcement,

$$\begin{aligned}\phi V_{u.min} &= \phi V_{uc} + \phi\ 0.6 b_v d_o\\ &= 127.4 + 0.7*0.6*350*449*10^{-3}\\ &= 193.4 \text{ kN}\end{aligned}$$

(d) Required capacity of shear reinforcement,

Minimum shear capacity to be provided by the shear reinforcement,

$$\begin{aligned}\phi V_{us} &= V^* - \phi V_{uc}\\ &= 280 - 127.4\\ &= 152.6 \text{ kN}\end{aligned}$$

$$\begin{aligned}\frac{\phi.V_{us}}{d_o} &= \frac{152.6 * 10^3}{449}\\ &= 340\end{aligned}$$

From Chart V2 read required spacing of N12 stirrups, s = 226 say 220 mm.

10.12 Torsion in Beams

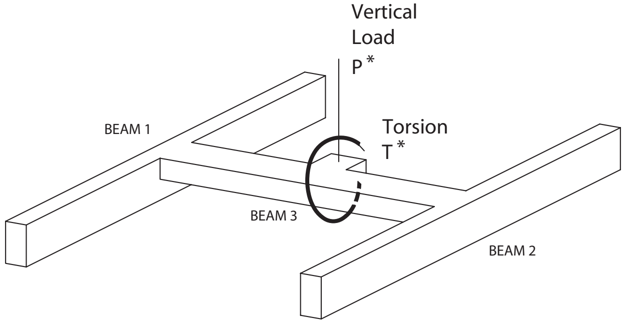
Torsion is the action of a load eccentric to the longitudinal axis of a beam. Torsion within a structure can be classified in two forms:

(1) Statically Determinate Torsion

This torsion is related to a normal statically determinate structure, such as a simple beam. Using the equations of statics there is a resultant torsion (twisting) action on the beam due to the applied forces.

A typical instance of this type of action is shown in Figure 10.7.

Figure 10.7

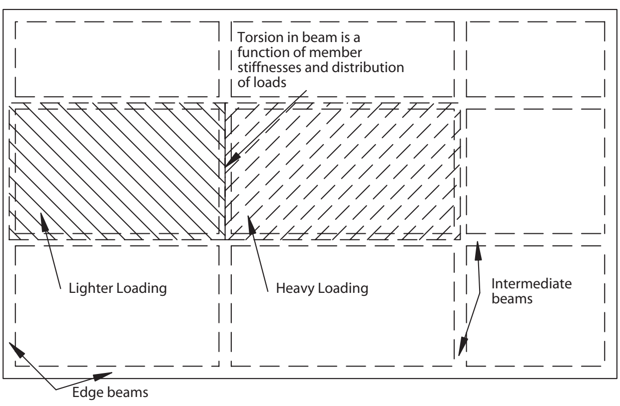


(2) Torsion Induced in Statically Indeterminate Structures.

With statically indeterminate structures consisting of columns, beams and slabs there are often residual torsional forces due to the redistribution of forces within members in relation to their stiffnesses. For example, a slab floor subject to differing load patterns may redistribute the slab moments into beam torsion due to moment rotation at the beam support. A typical instance of this type of action is shown in Figure 10.8.

This book deals only with statically determinate torsion. The Code indicates that if the "torsional reinforcement requirements of Clauses 8.3.7 and the detailing requirements of Clause 8.3.8 are satisfied." it is permissible to disregard the effect of indeterminate torsion.

Figure 10.8



10.13 Torsion Action

The action of torsion in a beam causes a twisting of the cross section along the longitudinal axis. This twisting causes a spiral cracking pattern to develop as illustrated in Figure 10.9 (*below*).
To resist the torsion stresses there are three components of the cross section:

(1) Diagonal Compressive Stresses parallel to the spiral cracks.
(2) Transverse Tension Stresses in the closed reinforcing ties.
(3) Tension Stresses in the Longitudinal Reinforcement in the corners of the ties.

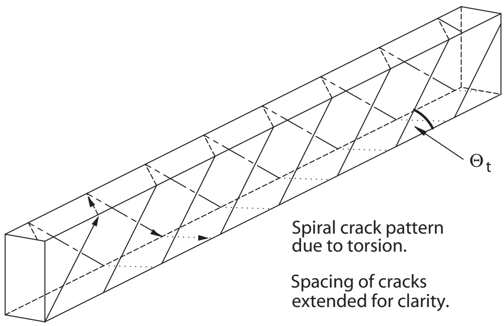


Figure 10.9

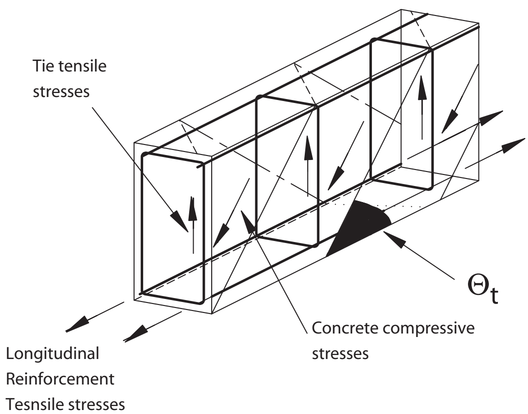


Figure 10.10

It is possible to model this behaviour as a space truss. This is similar in principle to the truss analogy used to represent the behaviour of shear. Figure 10.10 shows these actions diagrammatically.

Torsion and shear both cause diagonal compressive stresses and also transverse tension stresses in the closed ties. This is reflected in the equations given in the Code for Torsion, which include shear as a component. The longitudinal tension forces in the corner bars do not have a direct shear component.

In nearly all cases torsion will occur in conjunction with bending and shear action. The designer must then consider the combined action of shear and torsion as well as considering the additional tensile component of torsion in the longitudinal steel.

10.14 Equations for Torsion Effects in Beams

The Code equations are given below in the sequence the designer would approach the design. A flow chart is shown in Figure 10.11 which combines the design process for bending, shear and torsion.

TORSIONAL STRENGTH LIMITED BY WEB CRUSHING #8.3.3
Equation 10.14 combines the shear and torsion actions and limits them to prevent the crushing of the concrete in a diagonal direction.

$$\frac{T^*}{\phi T_{u.max}} + \frac{V^*}{\phi V_{u.max}} \leq 1 \qquad \textbf{(10.14)}$$

Where: $\phi = 0.7$

$T_{u.max} = 0.2f'_c J_t \; J_t$

$= 0.4x^2 y$ for rectangular sections

OR

$J_t = 0.4\ \Sigma(x^2\ y)$ for T, L or I shaped sections

OR

$J_t = 2A_m\ b_w$ for thin walled sections.

REQUIREMENTS FOR TORSIONAL REINFORCEMENT #8.3.4 (A)

The next stage is to determine if torsional and shear reinforcement is required. The code gives three conditions. Torsional and shear reinforcement is not required if any **ONE** of the three equations are satisfied.

$$T^* < 0.25\ \phi T_{uc}$$

OR

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 0.5 \qquad \textbf{(10.15)}$$

OR

$$\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 1.0$$

The latter applies only where D < (the greater of 250 mm and $b_w/2$)

T_{uc} is defined for beams without prestress in equation 10.16.

$$T_{uc} = J_t(0.3\sqrt{f'_c}\ ) \qquad \textbf{(10.16)}$$

TORSIONAL REINFORCEMENT #8.3.4 (B)

If torsional reinforcement is required by equation 10.15 then the amount of reinforcement for both shear and torsion shall be sufficient to satisfy equation 10.17.

$$\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} \leq 1 \qquad \textbf{(10.17)}$$

T_{us} is defined for beams without prestress in equation 10.18.

$$T_{us} = f_{sy.f}\ (A_{sw}/s)\ 2A_t \cot \theta_t \qquad \textbf{(10.18)}$$

Where: A_t = the area of a polygon whose vertices are at the centre of the longitudinal bars at the corners of the cross section. This is normally a rectangle formed by lines joining the centres of the four longitudinal corner bars.

θ_t = the angle between the longitudinal axis of the member and the diagonal compressive struts. This is taken as 45 degrees as a conservative value. For a more accurate value the angle varies linearly from 30 degrees (for $T^* = \phi T_{uc}$) to 45° (for $T^* = \phi T_{u.max}$). Equation 10.19 shows this relationship.

$$\theta_t = 30 + 15\left(\frac{(T^* - \phi T_{uc})}{(\phi T_{u.max} - \phi T_{uc})}\right) \qquad \textbf{(10.19)}$$

The value of θ_t can conservatively be taken as 1.0 (this corresponds to an angle of 45 degrees).

It should be noted that in the design process equation 10.18 can be expanded and a unique solution determined for the spacing of the ties. An alternative is to use the design charts on a trial and error basis. Both alternatives are shown in the design example.

Minimal Torsional Reinforcement #8.3.7

IF torsion reinforcement is required all of the shear reinforcement required by Clause 8.2.8 from the Concrete Code shall be provided in the form of closed ties.

The closed ties shall be continuous around all sides of the cross-section and anchored so as to develop full strength at any point. The spacing of the closed ties shall *not* be greater than the lesser of 0.12 u_t and 300 mm.

Longitudinal Torsional Reinforcement #8.3.6

Additional longitudinal reinforcement is required to resist the tensile forces generated by the torsion action which extends the length of the member along the longitudinal axis. In theory this action causes a tensile action in all longitudinal bars (both in the tension and compression zone).

(a) The additional area of tensile longitudinal reinforcement shall be calculated using equation 10.20.

$$A_s(\text{tors}) = 0.5\,\frac{f_{sy.f}}{f_{sy}}\,\frac{A_{sw}}{s}\,u_t \cot^2\theta_t \qquad \textbf{(10.20)}$$

(b) The additional area of longitudinal reinforcement in the compressive zone shall be calculated using equation 10.21

$$A_s(\text{tors}) = \left(0.5\,\frac{f_{sy.f}}{f_{sy}}\,\frac{A_{sw}}{s}\,u_t \cot^2\theta_t\right) - \left(\frac{M^*}{\phi f_{sy}(d - 0.5\gamma k_u d)}\right) \qquad \textbf{(10.21)}$$

10.15 Flow Chart for Combined Bending, Shear and Torsion

Figure 10.11 illustrates the design process for concrete beams with a combined action of bending, shear and torsion.

Note that the design of the shear reinforcement is integral with the torsion details. Hence the full shear design process should not be undertaken in the shear design phase. Rather the basic factors calculated and incorporated in the torsion design phase.

10.16 Notes on Torsional Reinforcement

Section 8.3.8 of the Code specifies the details for torsional reinforcement.

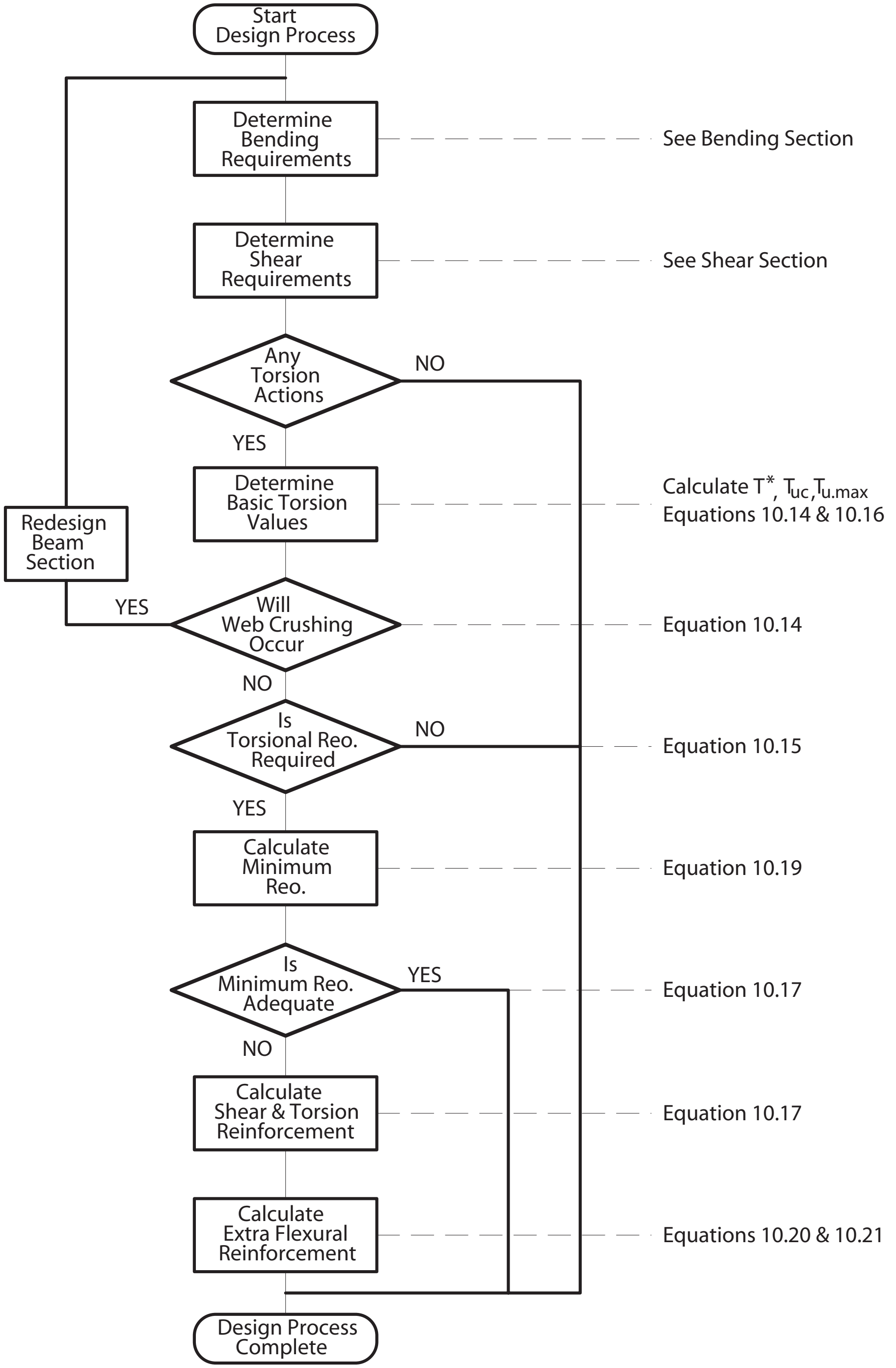
(a) Torsional reinforcement consists of both closed ties and longitudinal top and bottom reinforcement.

(b) The closed ties shall be continuous around all sides of the cross section and the ends anchored so that at any point on the ties the full strength can be developed.

The spacing of the closed tie shall not exceed the lesser of $0.12u_t$ and 300 mm.

(c) Longitudinal reinforcement shall be placed such that at least one bar is at each corner and is as close as possible to the corner of the closed ties.

Figure 10.11 Beam Torsion Design Flow Chart



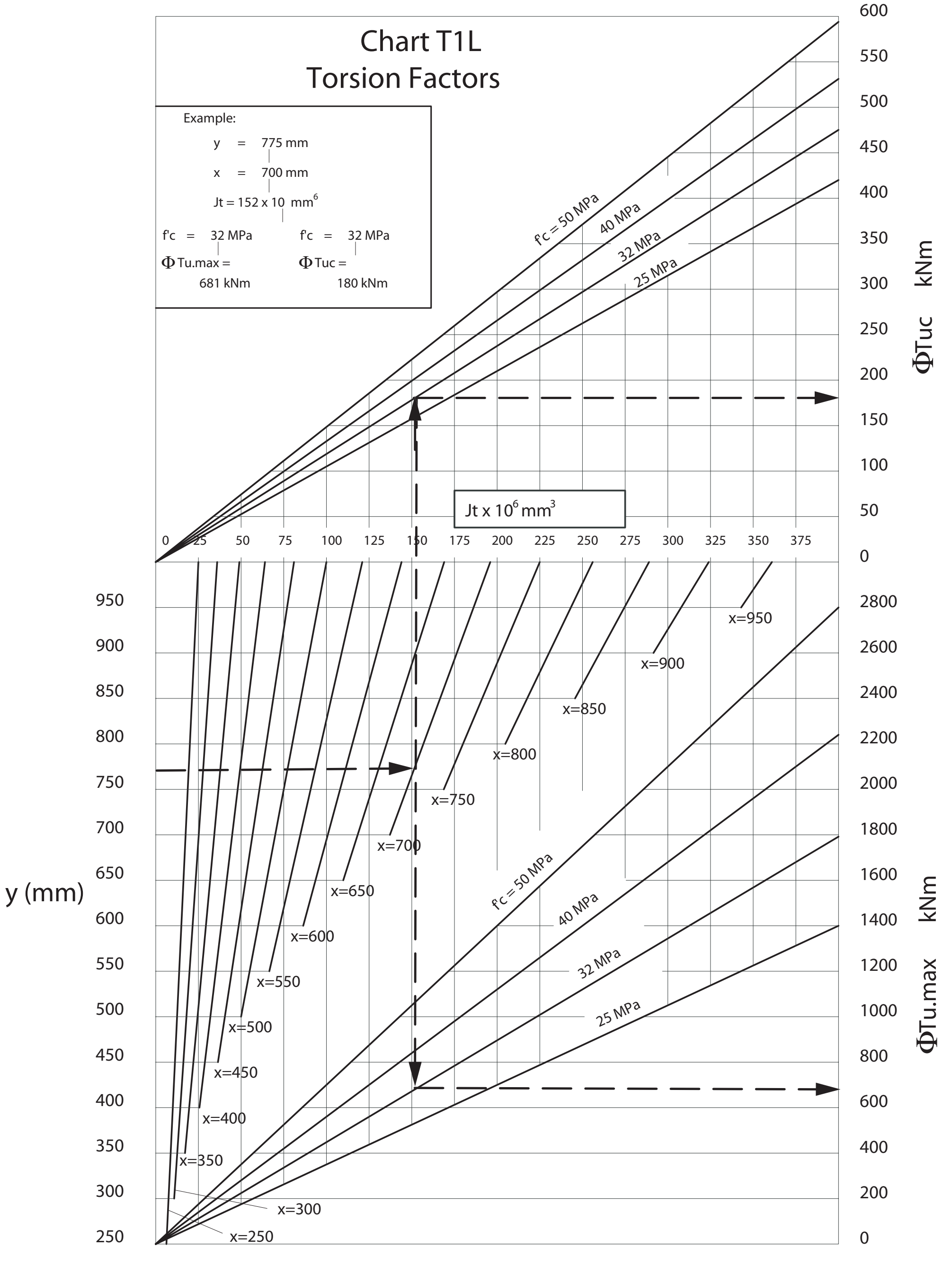


Chart T1L
Torsion Factors
Example:
y = 775 mm
x = 700 mm
Jt = 152 x 10 mm6
f'c = 32 MPa
Φ Tu.max =
681 kNm
f'c = 32 MPa
Φ Tuc =
180 kNm
f'c = 50 MPa
40 MPa
32 MPa
25 MPa
Jt x 10^6 mm^3
0 25 50 75 100 125 150 175 200 225 250 275 300 325 350 375
ΦTuc kNm
0 50 100 150 200 250 300 350 400 450 500 550 600
y (mm)
250 300 350 400 450 500 550 600 650 700 750 800 850 900 950
x=250
x=300
x=350
x=400
x=450
x=500
x=550
x=600
x=650
x=700
x=750
x=800
x=850
x=900
x=950
ΦTu.max kNm
0 200 400 600 800 1000 1200 1400 1600 1800 2000 2200 2400 2600 2800

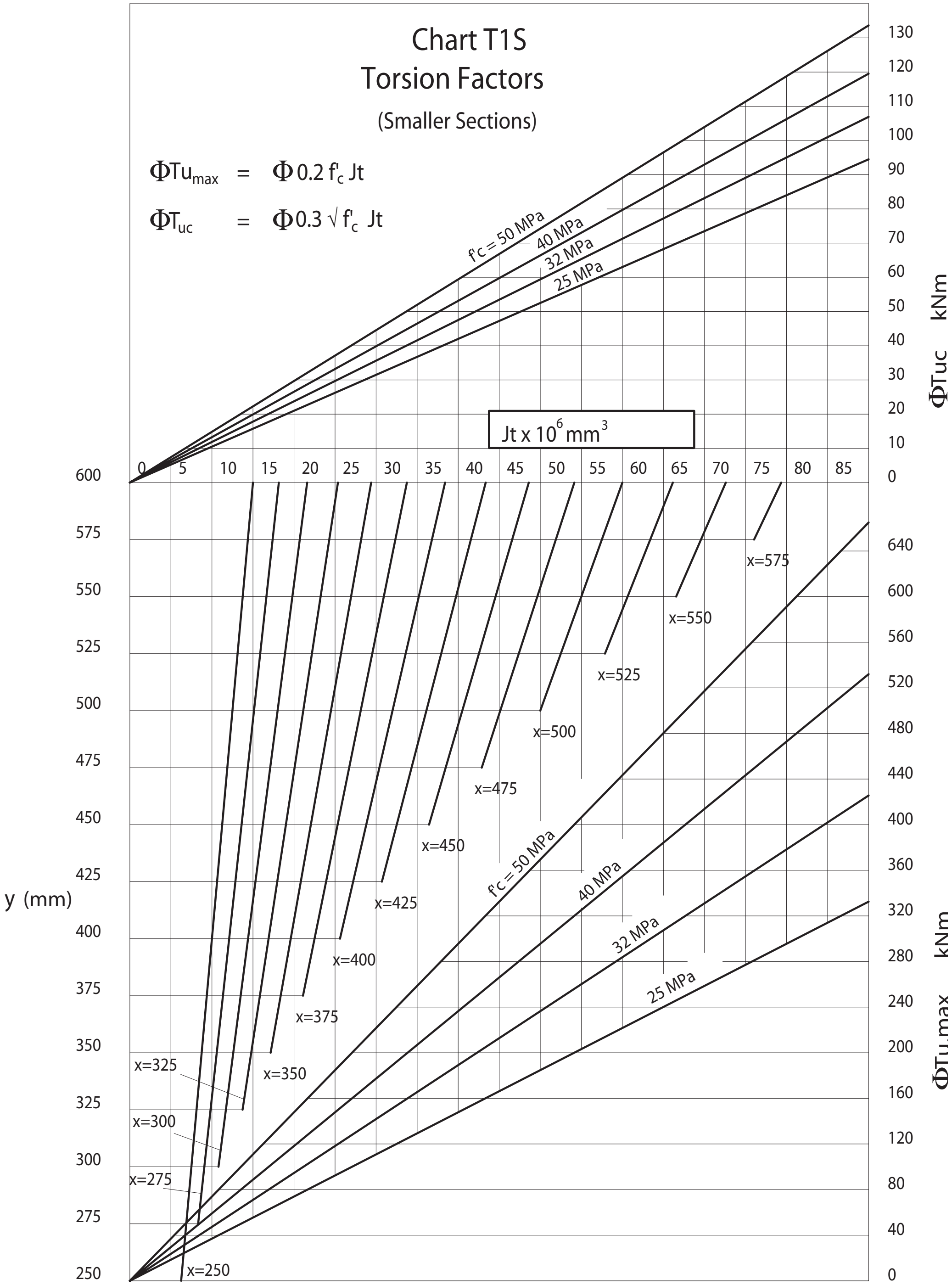
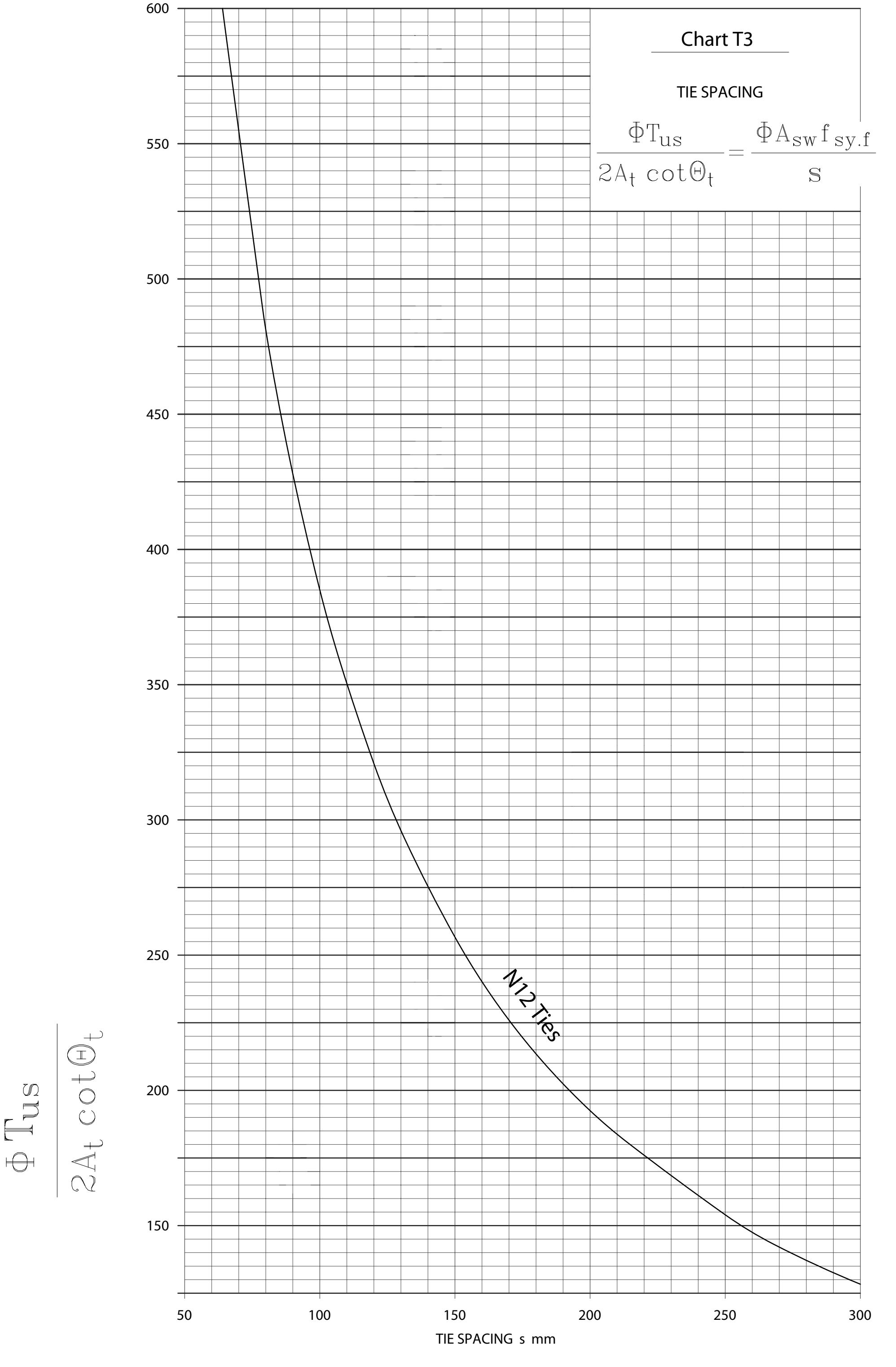


Chart T1S
Torsion Factors
(Smaller Sections)
ΦTu_max = Φ 0.2 f'_c Jt
ΦT_uc = Φ 0.3 √f'_c Jt
f'c = 50 MPa
40 MPa
32 MPa
25 MPa
Jt x 10^6 mm^3
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85
ΦTuc kNm
0 10 20 30 40 50 60 70 80 90 100 110 120 130
y (mm)
250 275 300 325 350 375 400 425 450 475 500 525 550 575 600
x=250
x=275
x=300
x=325
x=350
x=375
x=400
x=425
x=450
x=475
x=500
x=525
x=550
x=575
ΦTu.max kNm
0 40 80 120 160 200 240 280 320 360 400 440 480 520 560 600 640

Chart T3 Spacing of Closed Ties for Torsion Reinforcement

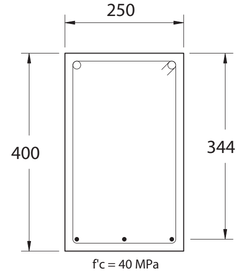


EXAMPLE 2

The reinforced concrete beam shown in Figure 10.12 is subject to the following loads:

(a) Bending Moment M* = 45 kNm
(b) Shear force V* = 110 kN
(c) Torsion force T* = 14 kNm

Check whether the beam can carry the loads satisfactorily and determine the reinforcement details to satisfy bending, shear and torsion requirements.



Exposure B1

Figure 10.12

Data

f'_c = 40 MPa b = 250 mm D = 400 mm Use N28 bars M^* = 45 kNm
Minimum cover to reinforcement = 30 mm

1. BENDING

d = 400 - 30 - 12 - 28/2 = 344 mm

$M^* \leq \phi M_{uo}$

$$\lambda = \frac{\phi M_{uo}}{bd^2}$$

$$= \frac{M^*}{bd^2} = \frac{45 * 10^6}{250 * 344^2} = 1.521$$

From Chart B1 p = 0.0038

A_{st} (bending) = p b d
= 0.0038 * 250 * 344
= 327 mm^2

2. SHEAR

d_o = 344 mm b_v = 250 mm

(a) Maximum permissible shear force

$\phi V_{u.max}$ = 0.7x 0.2f'_c b_v d_o
= 0.7*0.2*40*250*344*10^{-3} = 482 kN

(b) Adjusted beam shear capacity

$$p_v = \frac{A_{st}}{b_v d_o}$$

$$= \frac{1240}{250 * 344} = 0.0144$$

From Chart V1 ($f'_c = 40$) read $v'_c = 0.583$

$\phi V_c \quad = v'_c \, b_v \, d_o$

$= 0.583 * 250 * 344 * 10^{-3} = 50.12$ kN

$\beta_1 \quad = 1.1(1.6 - \dfrac{d_o}{1000})$

$= 1.1(1.6 - \dfrac{344}{1000})$

$= 1.382$

$\phi V_{uc} = \beta_1 \, \phi V_c = 1.382 * 50.12$

$= 69.2$ kN

(c) Beam shear capacity using minimum shear reinforcement

$\phi V_{u.min} = \phi V_{uc} + \phi 0.6 \, b_v \, d_o$

$= 69.2 + 0.7 * 0.6 * 250 * 344 * 10^{-3}$

$= 105.3$ kN

3. TORSION

$x = 250$ mm $y = 400$ mm $J_t = 0.4 * 250^2 * 400 = 10 * 10^6$

(a) Maximum permissible torsion force

$\phi T_{u.max} \quad = \phi 0.2 \, f'_c \, J_t$

$= 0.7 * 0.2 * 40 * 10$

$= 56$ kNm

(b) Beam torsion capacity

$\phi T_{uc} \quad = \phi \, 0.3 \sqrt{f'_c} \, J_t$

$= 0.7 * 0.3 * \sqrt{40} * 10$

$= 13.3$ kNm

These values can be read directly from Chart T1S

4. COMBINED SHEAR AND TORSION

(a) Check section strength

$$\frac{T^*}{\phi . T_{u.\max}} + \frac{V^*}{\phi . V_{u.\max}} \leq 1.0$$

$$\frac{14}{56}+\frac{110}{482} = 0.48 \quad \leq 1.0 \qquad \text{..... section size OK}$$

(b) Check unreinforced section capacity

$0.25\ \phi T_{uc} = 3.35$ $\qquad T^* = 14 > 3.35$

$$\frac{T^*}{\phi T_{uc}}+\frac{V^*}{\phi V_{uc}} \leq 0.5$$

$$\frac{14}{13.3}+\frac{110}{69.2} = 2.64 \qquad (> 0.5)$$

Both criteria are exceeded thus shear and torsion reinforcement is required.

(c) Determine shear/torsion reinforcement

$$\frac{T^*}{\phi T_{uc}}+\frac{V^*}{\phi V_{uc}} \leq 1.0$$

(i) Using the charts:

Trial 1 - adopt 50% contribution from both shear and torsion

Shear $V^* = 110$ kN $\cot \theta_v = 1$ (conservative method)

$d_o = 344$ mm $\quad f_{sy.f} = 500$ MPa $\quad A_{sv} = 220\ mm^2$ (for 2 legs of N12 ties)

Required value $\dfrac{V^*}{\phi V_{us}} = 0.5$

thus $\phi V_{us} = \dfrac{V^*}{0.5} = \dfrac{110}{0.5} = 220$ kN

$$\frac{\phi V_{us}}{d_o \cot\theta_v} = \frac{220 * 10^{-3}}{344} = 640$$

From Chart V2, spacing = 120 mm

Torsion

$T^* = 14$ kNm $\quad \cot \theta_t = 1$ (conservative method) $\quad d_o = 344$ mm

$f_{sy.f} = 500$ MPa $\quad A_{sw} = 110\ mm^2$

$A_t = (400 - 2 * 30 - 2 * 12 - 28) * (250 - 2 * 30 - 2 * 12 - 28)$

$= 39744\ mm^2$

Required value $\dfrac{T^*}{\phi T_{us}} = 0.5$ (assumed)

thus $\qquad \phi T_{us} = \dfrac{T^*}{0.5} = \dfrac{14}{0.5} = 28$ kNm

$$\frac{\phi T_{us}}{2A_t \cot \theta_t} = \frac{28 * 10^6}{2 * 39744 * 1} = 352$$

From Chart T3, spacing = 107 mm

Simplify the spacing to 100 mm.

SHEAR

From Chart V2 for N12 at 100 mm spacing

$$\frac{\phi V_{us}}{d_o \cot \theta_v} = 770$$

$$\begin{aligned} \phi V_{us} &= d_o \cot \theta_v * 770 \\ &= 344 * 1 * 770 * 10^{-3} \\ &= 265 \text{ kN} \end{aligned}$$

TORSION

From Chart T3 for N12 at 100 mm spacing

$$\frac{\phi T_{us}}{2A_t \cot \theta_t} = 370$$

$$\begin{aligned} \phi T_{us} &= 2 A_t \cot\theta_t * 370 \\ &= 2 * 39744 * 1 * 370 * 10^{-6} \\ &= 29.4 \text{ kN} \end{aligned}$$

$$\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} = \frac{14}{29.4} + \frac{110}{265}$$

$$= 0.89 \text{ ....OK}$$

Use N12 closed ties at 100 mm spacings

Check minimum spacing as the lesser of 0.12 u_t and 300 mm

u_t = 2*(400 - 2*30 - 2*12 - 28) + 2*(250 - 2*30 - 2*12 - 28)

= 852 mm

$\therefore$ 0.12 u_t = 0.12 * 852 = 102 mm (≈100) ....OK *(ii) By equations (using the non-conservative approach):*

SHEAR

$$\theta_v = 30 + 15\left(\frac{(V^* - \phi V_{u.\min})}{(\phi V_{u.\max} - \phi V_{u.\min})}\right)$$

$$= 30 + 15\left(\frac{110-105.3}{482-105.3}\right) = 30.01°$$

$\cot \theta_v = 1.73$

$$\phi V_{us} = \frac{\phi A_{sv} f_{sy.f} d_o \cot\theta}{s} = \frac{0.7 * 220 * 500 * 344 * 1.73 * 10^{-3}}{s}$$

$$= \frac{45824}{s}$$

TORSION

$$\theta_t = 30 + 15\left(\frac{(T^* - \phi T_{uc})}{(\phi T_{u.\max} - \phi T_{uc})}\right)$$

$$= 30 + 15\ \frac{14-13.3}{56-13.3} = 30.24°$$

$\cot \theta_t = 1.715$

$$\phi T_{us} = \frac{\phi A_{sw} f_{sy.f} 2A_t \cot\theta_t}{s}$$

$$= \frac{0.7 * 110 * 500 * 2 * 39744 * 1.715 * 10^{-6}}{s}$$

$$= \frac{5248}{s}$$

$$\frac{T^*}{\phi T_{us}} + \frac{V^*}{\phi V_{us}} = \frac{14}{\left(\frac{5248}{s}\right)} + \frac{110}{\left(\frac{45824}{s}\right)}$$

$$= \left(\frac{s}{374}\right) + \left(\frac{s}{416}\right) \le 1.0$$

$$= 0.0027s + 0.0024\,s \le 1.0$$

$$\therefore s = \frac{1.0}{0.0051} = 197 \text{ mm}$$

Note the larger spacing because of the inclusion of the cot θ effect. The value without this (i.e. using θ = 45°) is 115 mm. These values are greater than the minimum thus use 100 mm.

USE N12 closed ties at 100 mm spacings

5. TORSION AND BENDING

Additional reinforcement shall be designed to resist the force of:

$$0.5\ f_{sy.f}\left(\frac{A_{sw}}{s}\right)u_t \cot^2\theta_t = 0.5*500*\left(\frac{110}{100}\right)*852*1.73^2 *10^{-3}$$

$$= 701\ \text{kN}$$

$$A_{st}(\text{tors}) = \frac{701*10^3}{500} = 1403\ \text{mm}^2$$

Bottom longitudinal reinforcement:

$$A_{st} = A_{st}(\text{bend}) + A_{st}(\text{tors})$$
$$= 327 + 1403$$
$$= 1730\ \text{mm}^2\ \ (3\ \text{N28 - }1860\ \text{mm}^2)$$

Use 3 N28 bars in the bottom

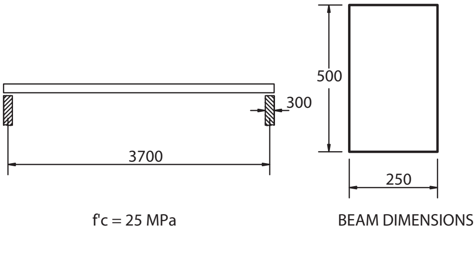
Top longitudinal reinforcement:

$$A_{st} = -\ A_{st}(\text{bend}) + A_{st}(\text{tors})$$
$$= -\ 327 + 1403$$
$$= 1076\ \text{mm}^2\ \ (2\ \text{N28 - }1240\ \text{mm}^2)$$

Use 2 N28 bars in the top

PROBLEMS

QUESTION 1



f'_c = 25 MPa

The beam shown above carries superimposed dead loads g = 76 kN/m and superimposed live loads q = 100 kN/m. The beam which is exposed to the weather is located inland, 80 km from the coast, in a non-industrial environment and arid climate.

(a) Choose the longitudinal reinforcement for moment conditions using N28 reinforcing bars.
(b) Determine the spacing of N12 stirrups at the critical section. Vary the spacing along the beam.
(c) Draw the beam showing all details.

11

chapter

Two-way slabs

11.1 Additional Symbols used in this Chapter

a	=	Dimension of the critical shear perimeter measured parallel to the direction of M^*_v.
a_s	=	Length of support in direction on span.
b_o	=	Critical dimension of an opening adjacent to a slab support.
D_b	=	Overall depth of a spandrel beam.
D_s	=	Overall depth of slab or drop panel as appropriate.
F_d	=	Slab design load.
L_n	=	Clear span between faces of supports.
L_o	=	Span length used in the simplified method, L minus 0.7 times the sum of a_s for each support.
L'_o	=	The smaller calue of L_o for adjoining spans.
L_t	=	Width of the design strip.
l_x	=	Short clear slab panel dimension between supports.
l_y	=	Long clear slab panel dimension between supports.
L_x	=	Short effective span of a slab panel.
L_y	=	Long effective span of a slab panel.
M_m	=	Positive bending moment at midspan.
M_{NE}	=	Negative moment at exterior support.
M_{NI}	=	Negative moment at interior support.
M_o	=	Total static moment for the span of the design strip.
M^*_v	=	The unbalanced slab bending moment transferred into the support.
M^*_x	=	Slab design moments in x directions.
M^*_y	=	Slab design moments in y directions.

V_{uo}	=	The ultimate shear strength of a slab where $M^*_v = 0$.
w^*	=	Unit slab design load used as alternative symbol for F_d.
w_x'	=	Equivalent design load for shorter slab support.
w_y'	=	Equivalent design load for longer slab support.
x and y	=	The shorter and longer dimensions respectively of the cross section of the torsion strip or spandrel beam.
β_h	=	The ratio of the longest overall dimension of the effective loaded area, Y, to the overall dimension X, measured perpendicular to Y.
β_x β_y	=	Bending moment coefficients for two-way slabs supported by rigid beams and walls.

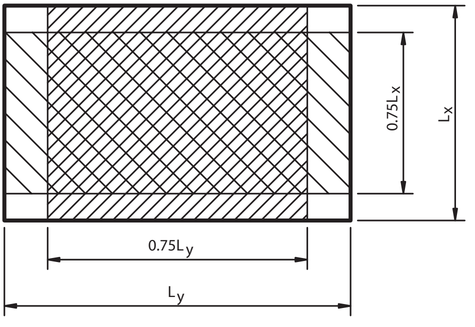
11.2 Rectangular Two-Way Slabs

As the name implies, two-way slabs transmit bending action in two directions between the supports. There are a number of two way slabs to be considered.

(a) Slabs supported by rigid beams or walls. The beams are designed separately as T and L-beams.
(b) Slabs supported by columns with drop panels around the columns. The drop panels are thickened portions of the slab designed to carry the high shear forces around the columns. Such slabs are referred to as flat slabs.
(c) Slabs supported by columns without drop panels. Such slabs are called flat plates.

Two-way slabs conventionally refer to slabs supported by rigid beams or walls. Type (b) and (c) slabs are usually specifically referred to as flat slabs and flat plates. The Code permits two-way slabs to be designed using a rigorous design procedure. This frequently requires the use of sophisticated computer programs utilising finite element analysis techniques and it is outside of the scope of this book. The alternate method provided for in the Code is the simplified method using bending moment coefficients with deemed to comply arrangement of reinforcement. This latter method which has been widely used and well proven over many years, will be used here.

11.3 Two-Way Rectangular Slabs Supported by Rigid Beams and Walls



For the purpose of determining strength of two-way rectangular slabs, the slab is divided into a middle strip, equal to three quarters of the effective span, and two edge strips in each direction as shown in Figure 11.1 (*right*). Most of the bending action in each direction is carried by the middle strips. The edge strips are not designed in the normal sense; the small bending action taken by the edge strips is considered to be adequately catered for by the minimum steel ratio 0.002 as per #9.1.1(b). The positive design bending moments per unit width (1 m) in each direction are given by:

$$M^*_x = \beta_x F_d (L_x)^2 \quad \textbf{(11.1)}$$

$$M^*_y = \beta_y F_d (L_x)^2 \quad \textbf{(11.2)}$$

Where; β_x and β_y are bending moment coefficients.

L_x and L_y is the shorter and longer effective span.

F_d is the design load in kPa or kN/m for a unit width.

Note that both equation 11.1 and 11.2 is in terms of span L_x.

It does not require a great deal of imagination to realise that the bending moment carried per unit width in each direction will depend on the spans L_x and L_y. A simple analogy is to consider two elastic bands stretched between supports at right angles with a point load applied at the intersection of the elastic bands. When the lengths are equal, $L_x = L_y$, the proportion of the load carried by each band will also be equal. If L_x is made shorter than L_y, the shorter band will carry a higher proportion of the load. As L_x is progressively made shorter, the proportion of the load carried by the shorter band will continue to increase while the load carried by the longer band will continue to decrease. In the ultimate, when the longer band length L_y is many times longer than L_x, the longer band will hardly carry any load while the shorter band carries most of the load. The condition being approached is that of a one-way action. Bending action in two-way slabs is very similar to this analogy.

Table 11.1 on the next page is a table of moment coefficients β_x and β_y given by the Code for two-way slabs to be used in equations 11.1 and 11.2. It is noted that when the span ratio $L_y/L_x = 2.0$, a slab is considered to approach a one-way slab and it is designed as such. The minimal bending in the long direction is adequately catered for by the minimum steel ratio 0.002. Note that while the bending moment coefficient β_x. varies with the span ratio L_y/L_x the coefficient β_y is constant.

As an alternative to Table 11.1, Chart S4, following the table, is a plot of the bending moment coefficients. The chart illustrates how the bending moment coefficients tend assymptotically towards a horizontal line near $L_y/L_x = 2.0$.

The negative bending moment at a continuous support is given as;

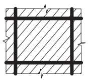
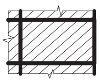
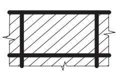
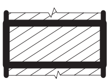
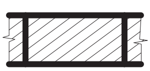
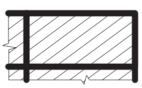
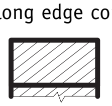
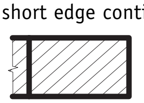
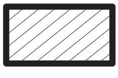
1.33*(The maximum positive bending moment)

The negative bending moment at a discontinuous support is given as;

0.50*(The maximum positive bending moment)

When the negative bending moments on either side of a support differ due to difference in the spans or edge conditions in the adjacent spans, the Code permits the re-destribution of the out of balance moments in proportion to the slab stiffnesses.

Table 11.1 - Bending Moment Coefficients for Slabs Supported on Four Sides

Edge Conditions	Short Span Coefficient β_x Values L_y / L_x								Long-Span Coefficient β_y for All Values of L_y / L_x
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	‡2.0	
1. Four edges continuous 	0.024	0.280	0.320	0.350	0.370	0.400	0.400	0.480	0.024
2. One short edge discontinuous 	0.028	0.032	0.036	0.038	0.041	0.043	0.047	0.050	0.028
3. One long edge discontinuous 	0.028	0.035	0.041	0.046	0.050	0.054	0.061	0.066	0.028
4. Two short edges discontinuous 	0.034	0.038	0.040	0.043	0.045	0.047	0.050	0.053	0.034
5. Two long edges discontinuous 	0.034	0.046	0.056	0.065	0.072	0.078	0.091	0.100	0.034
6. Two adjacent edges discontinuous 	0.035	0.041	0.046	0.051	0.055	0.058	0.065	0.070	0.035
7. Three edges discontinuous one long edge continuous 	0.043	0.049	0.053	0.057	0.061	0.064	0.069	0.074	0.043
8. Three edges discontinuous one short edge continuous 	0.043	0.054	0.064	0.072	0.078	0.084	0.096	0.105	0.043
9. Four edges discontinuous 	0.056	0.066	0.074	0.081	0.087	0.093	0.103	0.111	0.056

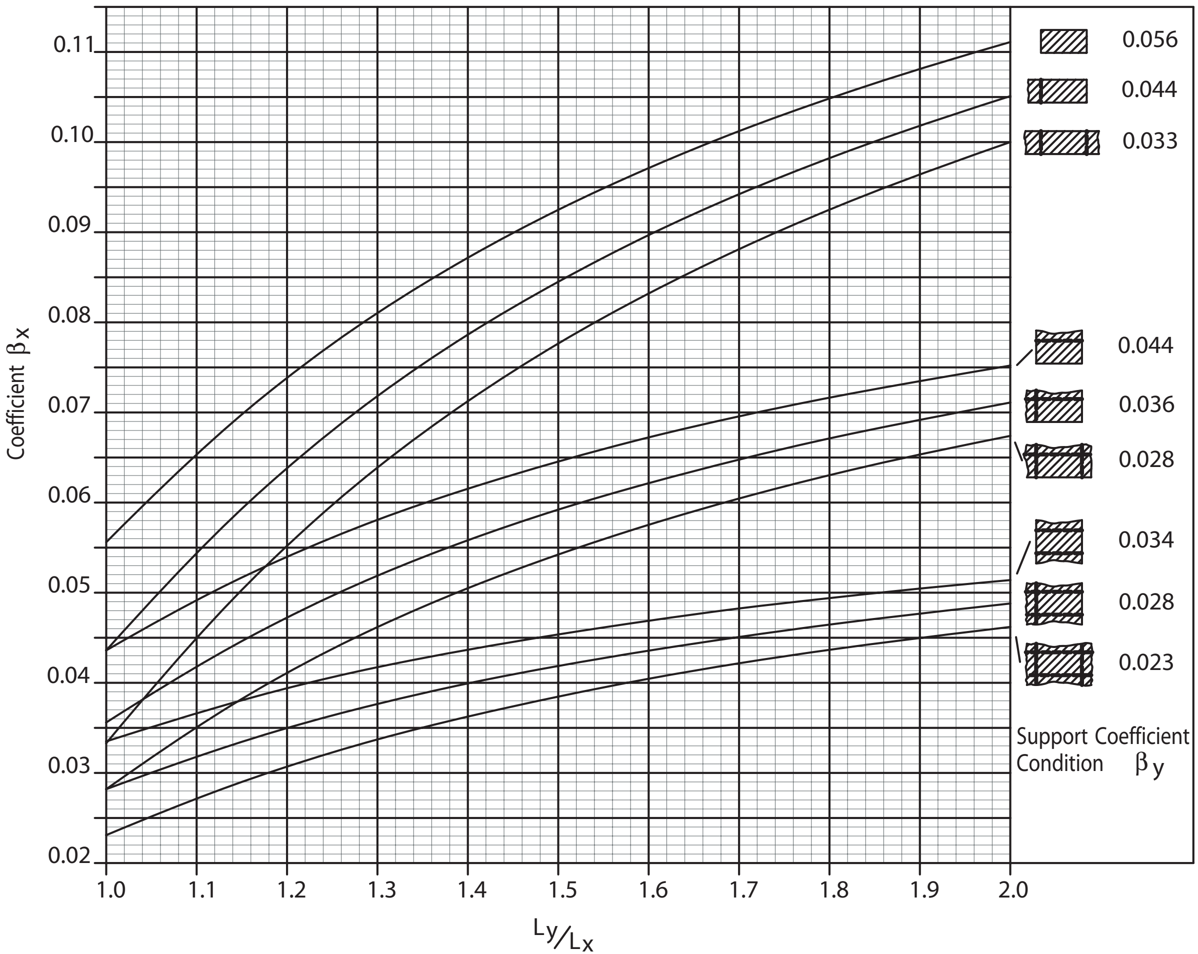
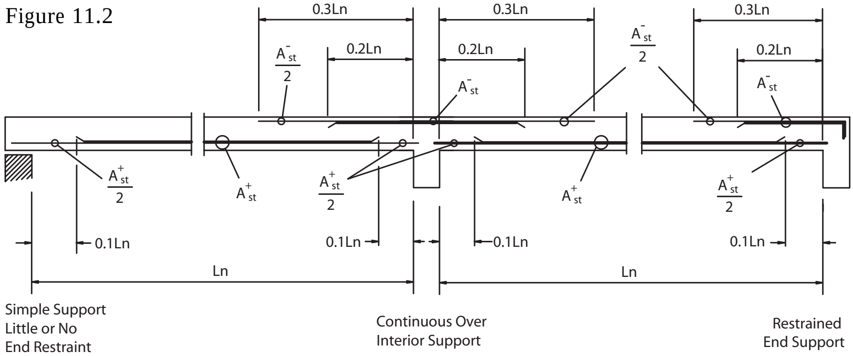


CHART S4 - Bending Moment Coefficients for Rectangular Slabs Supported on Four Sides

11.4 Deemed to Comply Arrangement of Reinforcement #9.1.3.3

A deemed to comply arrangement of reinforcement required by the Code is shown in Figure 11.2.

Figure 11.2



11.5 Torsional Reinforcement at Corners of Restrained Edges

In addition to the deemed to comply arrangement of flexural reinforcement shown above, torsional reinforcement must be provided in both the top and bottom of the slab at corners not free to lift where one or both edges are discontinuous. The Code # 9.1.3.3 requires the reinforcement in each face to be made up of two layers perpendicular to the edges of the slab and extending a minimum distance of 0.2 times the shorter span. The area of each layer should be not less than:

(a) $0.75A_{st}$ when both edges are discontinuous and,

(b) $0.50A_{st}$ when only one edge is discontinuous.

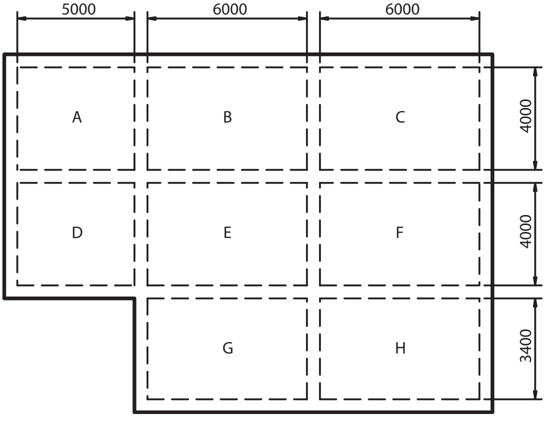
Where A_{st} is the area of the maximum positive moment reinforcement required at mid-span. Any other reinforcement provided may be considered as part of the torsional reinforcement.

A slab must still satisfy serviceability requirements as well as shear strength conditions.

EXAMPLE 1

The rectangular slab system for an office building shown in Figure 11.3 is supported by 400 mm wide and 600 mm deep beams. Design slab panel "A" using grade N32 concrete and steel reinforcing mesh for a superimposed dead load g = 1 kPa and a live load q = 4 kPa. It may be assumed that the slab does not support masonry walls. Masonry walls in the building are applied directly to the beams. Exposure classification A2 may be assumed.

Figure 11.3



SOLUTION

Data: f'_c = 32 MPa f_{sy} = 500 MPa g = 1 kPa q = 4 kPa

Exposure Classification A2

SERVICEABILITY

Assume 150 mm thick slab.

Weight of slab = 0.15*24 = 3.6 kN/m^2

Total dead load g = 1 + 3.6 = 4.6 kN/m^2

The effective length is taken as the lesser of the distance between support centrelines and the clear span L_n + D. It is sufficient to take the effective spans as the span between support centrelines. Thus the effective spans are:

L_x = 4000 + 400 = 4400

L_y = 5000 + 400 = 5400

$$\frac{L_y}{L_x} = \frac{5400}{4400} = 1.23$$

From Table 1.2 the short-term live load factor ψ_s = 0.7.

Also from Table 1.2 the long term live load factor ψ_l = 0.4.

For total deflection $\dfrac{\Delta}{L_{eff}} = \dfrac{1}{250}$

Assuming tension reinforcement only, the long-term deflection multiplier k_{cs} = 2.

The effective load for for total deflection,

$F_{d.eff}$ = (1+2)*4.6 + (0.7+0.4*2)*4 = 19.8 kN/m^2

From Chart D2, k_5 = 18.4

(where E_c =31,000 MPa for 32 MPa)

From page 104, k_3 = 1.0

From Table 8.2 for edge condition 6 and L_y/L_x = 1.23 (by interpolation), k_4 = 2.79

The maximum span to depth ratio for total deflection,

$$\frac{L_{eff}}{d} = k_3\, k_4\, k_5 = 1.0*2.79*18.4 = 51.4$$

Required effective depth, $d = \dfrac{4400}{51.4} = 85.6\,\text{mm}$

The assumed 150mm thick slab is adequate, although a thinner slab could be used.

STRENGTH

For a 1 m wide slab strip.

Design load, F_d = 1.2*4.6 + 1.5*4 = 11.5 kN/m

From chart S4 for L_y/L_x = 1.23, β_x = 0.049 and β_y = 0.036

Maximum positive design bending moments.

$$M^*_x = \beta_x\, F_d\, (L_x)^2$$
$$= 0.049*11.5*(4.4)^2 = +\,10.9 \text{ kNm}$$
$$M^*_y = \beta_y\, F_d\, (L_x)^2$$

OR

$$M^*_y = \frac{\beta_y}{\beta_x}\, M^*_x \;=\; \frac{0.036}{0.049}\, 10.9 = +\,8.0 \text{ kNm}$$

Maximum negative design bending moments.

At continuous edges,

M^*_x = - 1.33*10.9 = - 14.5 kNm

M^*_y = - 1.33*8.0 = - 10.6 kNm

At discontinuous edges,

M^*_x = - 0.5*10.9 = - 5.5 kNm

M^*_y = - 0.5*8.0 = - 4.0 kNm

Reinforcement

Effective depth of slab for reinforcement in short direction

d = 150 - 25(cover) - 6(half bar assuming 12 mm bars)
= 119 mm

Effective depth of slab for reinforcement in long direction,

d = 119 - 12(bar dia.)

= 107 mm

The reinforcing fabric may now be chosen from Chart S3 as shown in the following table. Note that by using Chart S3, selection of meshes will satisfy the minimum steel ratio.

M_x	M_y	d	Mesh Chosen From Chart S3
- 14.5	-	119	SL102
+ 10.9	-	119	SL92
- 5.5	-	119	SL92*
-	- 10.6	107	SL92
-	+ 8.0	107	SL82*
-	- 4.0	107	SL82*

(* minimum reinforcement 0.0020 bD as per Code)

Alternatively the reinforcing areas could have been determined from Chart S2. The required areas will be calculated for the maximum positive reinforcement since the area of the torsional reinforcement is a proportion of the maximum positive area required for bending.

M^*_x = + 10.9 kNm and d = 119 mm
From Chart S3 read required steel ratio p = 0.0020.
Required A_{st} = 0.0020*1000*119 = 238 mm^2/m.
M^*_y = + 8.0 kNm and d = 107 mm

From Chart S2 required steel ratio p = 0.0020 which is greater than the minimum steel ratio as per # 9.1.1 from AS3600.

Required A_{st} = 0.0020*1000*107 = 214 mm^2/m

The reinforcing mesh determined above are shown in Figure 11.4. It would not be a good engineering practice to end up with a proliferation of reinforcing mesh. The number of mesh types should be kept to a minimum. The final arrangement of reinforcement which satisfies the deemed to comply arrangement given in Figure 11.2 is shown in Figure 11.5

For torsional reinforcement at corners, with two edges discontinuous, the required area:

$$= 0.75*A_{st} = 0.75*238 = 179 \text{ mm}^2$$

The SL72 mesh provided top and bottom has an area of 179 mm^2. However the minimum area of steel 0.0020bD required by the Code is still 238 mm^2 i.e. SL82

With one edge discontinuous, the required area;

$$= 0.5*238 = 120 \text{ mm}^2$$

Again the SL62 square mesh is sufficient (but again the Code minimum requires that SL82 be used).

Figure 11.4 - Reinforcing Mesh as Calculated

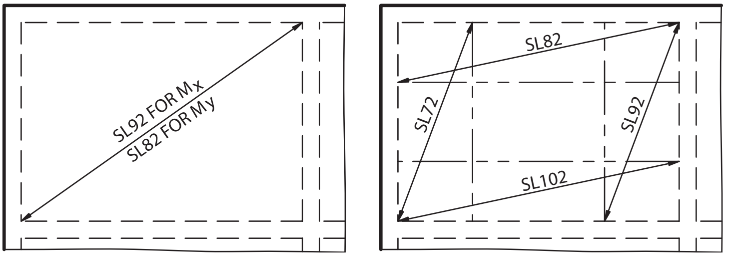
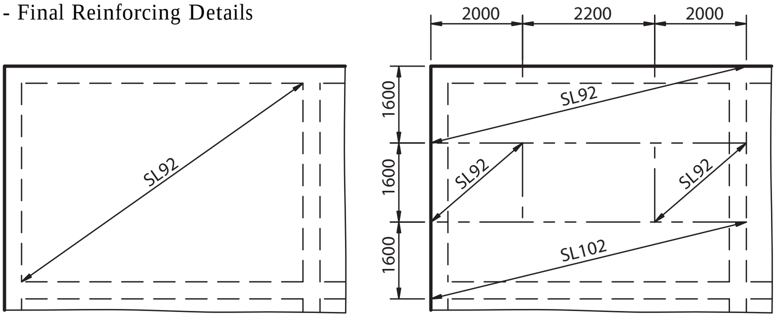


Figure 11.5 - Final Reinforcing Details



11.6 Equivalent Uniformly Distributed Loads

The tributary areas shown in Figure 11.6 may be used to calculate shear forces in slabs. The loads carried by supporting walls or beams, an equivalent uniformly distributed load w' is applied. The value of w' is calculated by dividing each slab panel into triangular and trapezoidal loaded areas obtained by lines drawn at 45^o at each corner as shown in Figure 11.6.

The value of w' is calculated by applying the triangular or trapezoidal load to the supporting beams. An expression for w' for the short span beam supporting a triangular load shown in Figure 11.7 is derived below.

Figure 11.6

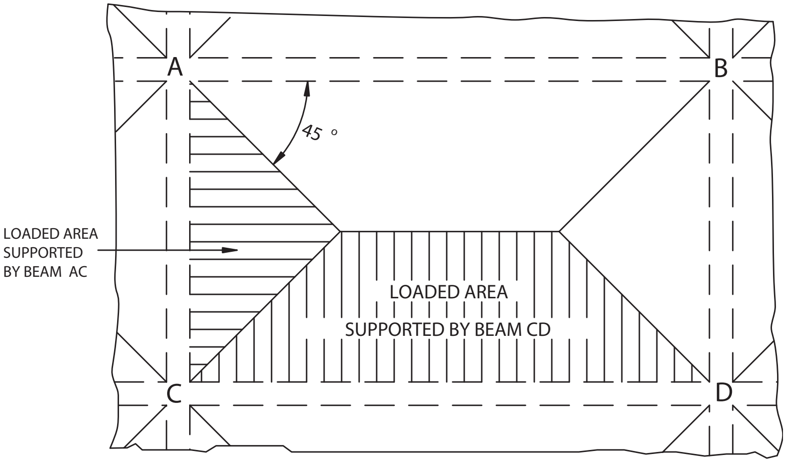
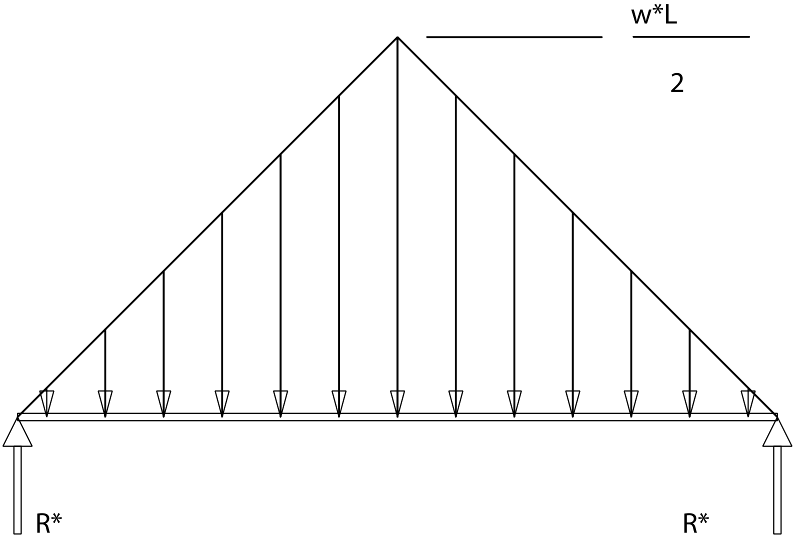


Figure 11.7



Distributed load at mid-span = $\dfrac{w^* l_x}{2}$ kN/m

Support reactions, R^* = One half the triangular area.

$$= \frac{1}{2}\frac{l_x}{2} w^* \frac{l_x}{2}$$

$$= w^* \frac{(l_x)^2}{8}$$

Taking moments about mid-span,

$$M^* = w^* \frac{(l_x)^2}{8}\frac{l_x}{2} - w^* \frac{(l_x)^2}{8}\frac{l_x}{6}$$

$$= w^* \frac{(l_x)^3}{24} \qquad \text{(a)}$$

For a uniformly distributed load w_x', the mid-span moment is,

$$M^* = w_x' \frac{(l_x)^2}{8} \qquad \text{(b)}$$

Equating the two expressions and solving w_x' the equivalent load carried by the short span beam,

$$w_x' = w^* \frac{l_x}{3} \qquad \textbf{(11.3)}$$

Applying the trapezoidal load to the long span, an expression for the equivalent uniformly distributed load w_y' may be derived to be given by,

$$w'_y = w^* l_x \left[\frac{3 - \left(\frac{l_x}{l_y}\right)^2}{6} \right] \qquad \textbf{(11.4)}$$

The equivalent loads given by equations 11.3 and 11.4 may now be used for checking the slab for shear near the supports or for designing the supporting beams. Note that the total equivalent uniformly distributed load to be applied to a beam will be the sum of the equivalent uniformly distributed loads contributed by the slabs on either side of the beam.

EXAMPLE 2

Calculate the equivalent uniformly distributed load which may be used for designing the supporting beam between panel "A" and panel "B" in Figure 11.3 if the slab design load w^* (including the weight of slab) is 11.75 kN/m^2.

SOLUTION

Considering panel "A"

This is the short span beam in the panel with l_x = 4.0 m.

From equation 11.3, the equivalent distributed load,

$$w_x' = w^* \frac{l_x}{3} = \frac{11.75 * 4}{3} = 15.7 \text{ kN/m}$$

Considering panel "B"

This is also the short span beam in panel "B" having the same span as panel "A", and hence the same equivalent load w_x' = 15.7 kN/m.

The total equivalent load applied to this beam will hence be 2*15.7 = 31.4 kN/m. Note however that the load over the beam width (0.4*11.75 = 4.7 kN/m) and the weight of beam must be added to the equivalent load of 31.4 kN/m to obtain the design load for the beam.

11.7 Simplified Slab Design Method

The Simplified Slab Design Method is one of three methods of rectangular slab systems which are outlined in AS3600 Code. They are:

(a) Simplified Slab Design

The Simplified slab design method provides a simple approach for the design of regular flat slabs and plates and other rectangular two way slab systems. It is limited in application but simple to apply.

(b) Idealised Frame Method of Slab Design

The Idealised Frame Method of Slab Design provides the designer with a more flexible structural model to determine the slab moments. This method also provides for the inclusion of vertical forces into the design. Many of the restrictions of the Simplified Method do not apply to the Idealised Frame. The analysis of the “Frame” requires a basic knowledge of two dimensional frame analysis. This does provide the designer more control in modelling the slab structure. The Simplified Method is used to provide the distribution of moments across the strip and for reinforcing details.

(c) Collapse Load Method of Analysis and Design of Slabs.

The Collapse Load Method involves the use of plastic design and collapse mechanism in the determination of the failure capacities of slabs. This method has the advantage of being readily applicable to slabs of any shape. Separate checks must be made on shear and deflections.

This chapter limits itself to details of the Simplified Method of Slab Design. It should be appreciated that in the design process the designer may need to use any of the three methods or a combination of them.

11.8 Criteria for Application of Simplified Slab Design Method

This method is intended to apply to multiple span reinforced two way slab structures with a rectangular grid support system.

These include:

(a) Flat Plates
(b) Flat Slabs
(c) Beam and Slabs
(d) Slabs with Thickened Slab Band (Band Beam)
(e) Waffle Slabs

The restrictions on the application of this method are listed in Code section 7.4.1.

(a) There shall be at least two continuous spans in each of the two principal directions.

(b) The supporting grid is rectangular. Individual supports can be offset a maximum of 10% of the span length in the direction of offset, from the grid line.

(c) The ratio of the longer span to the shorter span of any portion of the slab enclosed by the centrelines of the supporting members, shall not be greater than 2.0.

(d) Within the design strips, shown in Figure 11.8, in either direction, the lengths of successive spans shall not differ by more than one third of the longer span. The end spans shall not exceed the adjacent internal spans.

(e) Lateral forces on the structure are not resisted by the slab system but by shear walls or braced frames.

(f) Vertical loads are essentially uniformly distributed.

(g) The live load applied to the slab shall not exceed the twice the dead load.

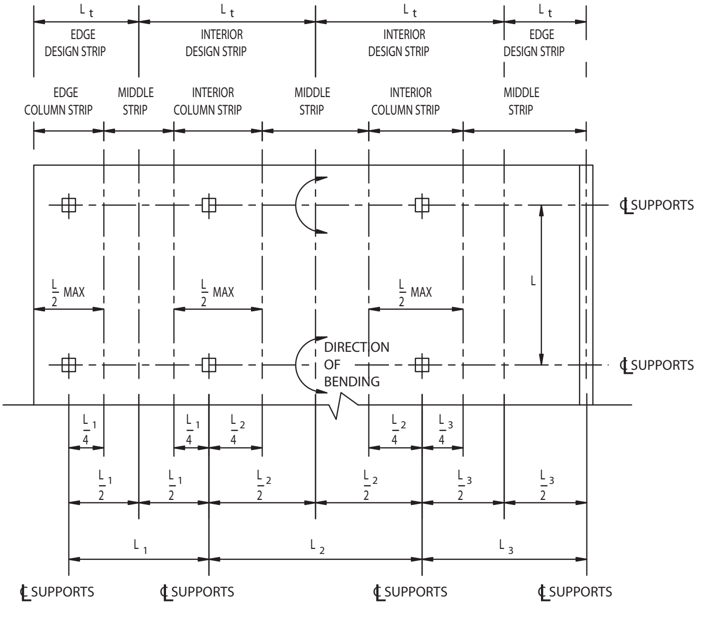
(h) The reinforcment shall be arranged in accordance with Code requirements. See Sections 11.11 and 11.15

11.9 Geometry of the Simplified Slab System

The geometry of the Simplified Slab System is shown in Figure 11.8 (*below*). Each of the two principal directions of the two-way action is considered separately. For each principal direction the slab is divided into design strips (interior and exterior). Within each of these design strips a further division is made into a column strip and two middle strips.

The moments are calculated for each principal direction and each design strip. Each design strip consists of a number of spans which are analysed in turn. Then the moments are distributed between the column strip and the middle strips for each span.

Figure 11.8



11.10 Calculation of Moments

(a) Division of Slab into Design Strips

For each of the principal directions, the slab is first divided up into design strips as shown in Figure 11.8.

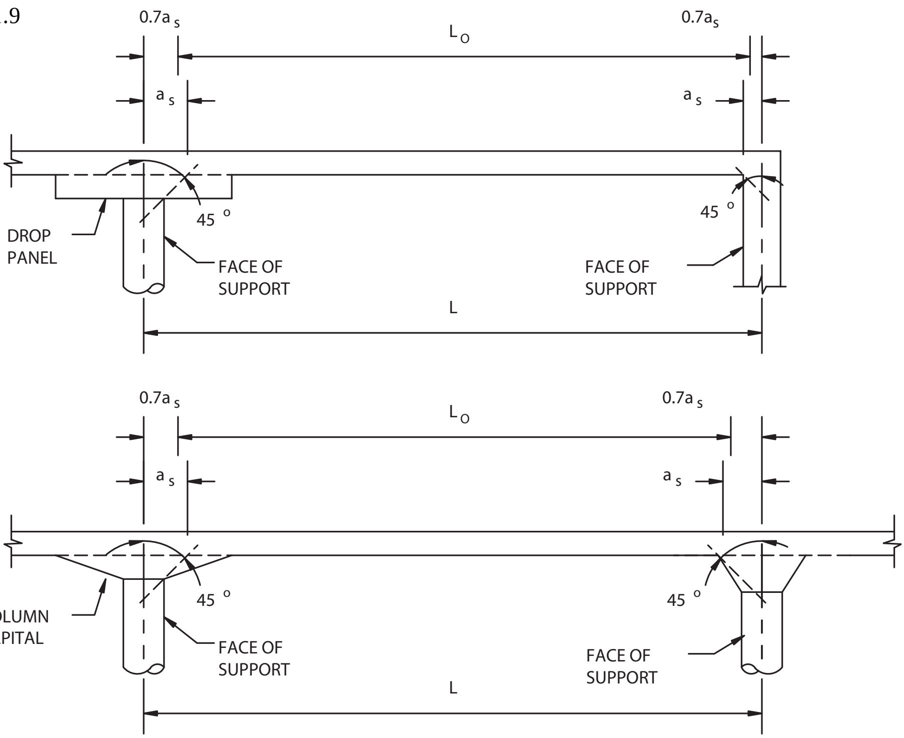
(b) Calculation of Static Moment for Each Span

For each of the spans within the design strips the static moment is calculated using equation 11.5.

$$M_0 = \frac{F_d L_t L_0^2}{8} \tag{11.5}$$

Figure 11.9 illustrates the method of calculating L_0 for various support conditions.

Figure 11.9



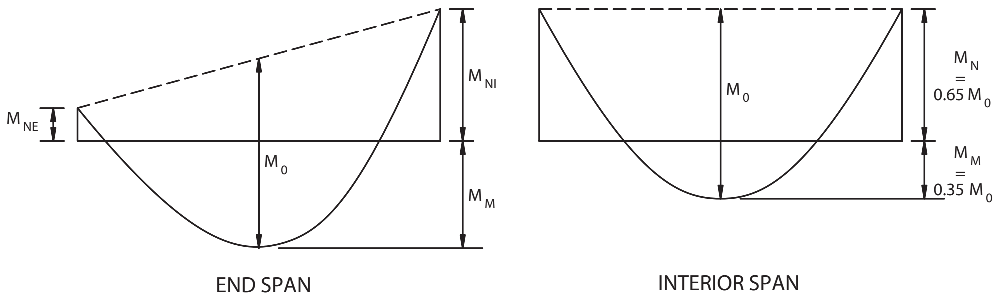
(c) Design Moments within the Span

The total static moment shall be distributed between the midspan positive moment and the end negative moments in accordance with Table 11.2 for end spans. For interior spans for all types of slab systems the negative moment factor shall be 0.65 and the positive moment factor shall be 0.35. These details are shown in Figure 11.10.

Table 11.2 - End Span Moment Distribution for Flat Slabs

Type of slab system and edge rotation restraint	Exterior negative M_{NE}	Positive moment M_M	Interior negative moment M_{NI}
Flat slabs with exterior edge unrestrained	$0.0M_0$	$0.60M_0$	$0.80M_0$
Flat slabs with exterior edge restrained by columns only	$0.25M_0$	$0.50M_0$	$0.75M_0$
Flat slabs with exterior edge restrained by spandrel beams and columns	$0.30M_0$	$0.50M_0$	$0.70M_0$
Flat slabs with exterior edge fully restrained	$0.65M_0$	$0.35M_0$	$0.65M_0$
Beam and slabconstruction	$0.15M_0$	$0.55M_0$	$0.75M_0$

Figure 11.10 Moment Distribution within Spans



(d) Design Moments within Column and Middle Strips

The three design moments for each span shall then be distributed transversely across the design strip in accordance with Table 11.3.

Table 11.3 - Distribution of Moment Between Column and Middle Strip

Bending Moment	Moment Factor for Column Strip	Moment Factor for Middle Strip
Negative Moment - Interior Support	0.60 to 1.00	0.40 to 0.00
Negative Moment - Exterior Support	0.75 to 1.00	0.25 to 0.00
Positive Moment - All Spans	0.50 to 0.70	0.50 to 0.30

The design moments for the middle strip are calculated by adding together the moments for the two adjoining halves from adjacent design strips. For middle strips which are adjacent to and parallel with an edge supported by a wall the value of the design moment shall be twice the value of design moment for the adjacent half middle strip for the adjoining interior design strip (see Figure 11.8).

11.11 Moment Reinforcement Design and Detail

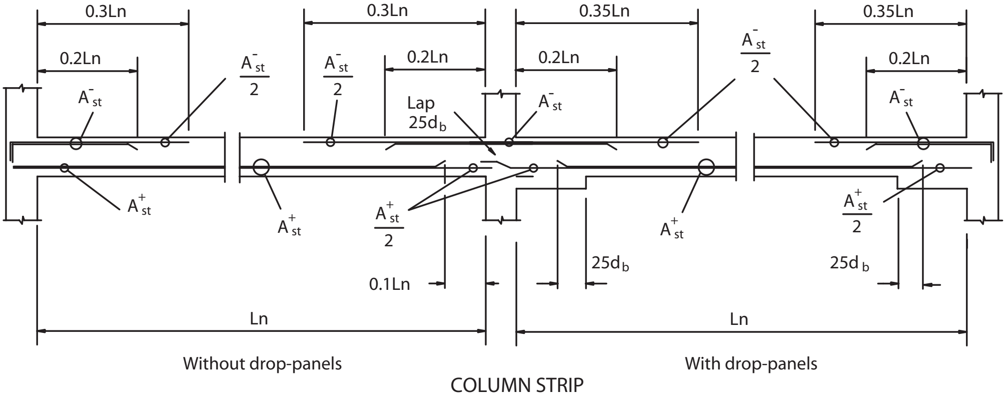
Figure 11.11 shows the reinforcement arrangement which is deemed to comply with section 9.1.3 of the Code.

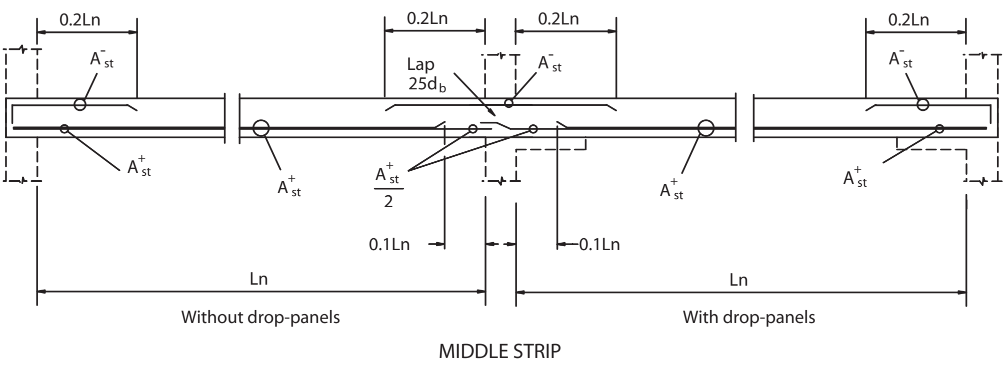
In addition all slab reinforcement perpendicular to a discontinuous edge shall extend beyond the supporting member as follows:

(a) Positive Reinforcement - not less than 150 mm beyond the supporting edge or to the edge of the slab if there is no supporting member.

(b) Negative Reinforcement - such that the calculated force is developed at the support face in accordance with section 13.1 of the Code.

Figure 11.11 - Reinforcing Details





11.12 Shear Consideration

There are two principal forms of shear which act in a slab system. The first form is the localised effect of shear at columns, the second is the normal beam shear action across the width of the slab. This section discusses the localised effect of shear at columns. The beam shear can be checked using the methods discussed in Chapter 10 and in Section 11.6 for the distribution of load for shear calculations.

The action in the vicinity of the column is complicated by the combined action of shear and moment. The principal shear effect is punching shear. The conventional model for punching shear is shown in Figure 11.12.

The shear perimeter is defined by an outline in plan which is at a distance d away from the boundary of the area of effective support (previously d/2) normally the cross section of the column under. The outline is similar to the area of support with two differences:

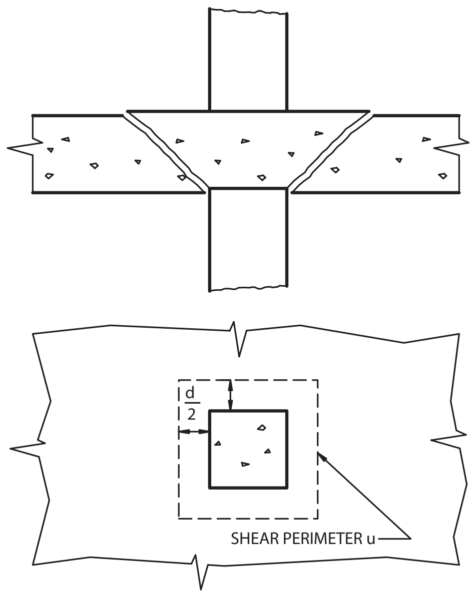


Figure 11.12

(1) The outline does not follow reentrant corners. The outline ignores the point on the cross section which is the reentrant point. Figure 11.15 shows some typical shear perimeters, one of which is an example of an "L" shaped support. The inside corner of the "L" shape is ignored.

(2) Portions of the critical shear perimeter is ignored where a critical opening occurs. A critical opening is defined as an opening through the thickness of a slab which has an edge or portion of an edge located with a clear distance to the critical shear perimeter less than 2.5 b_0. Where b_0 is effective opening dimension. This is illustrated in Figure 11.13 where a circular opening and a rectangular opening are shown.

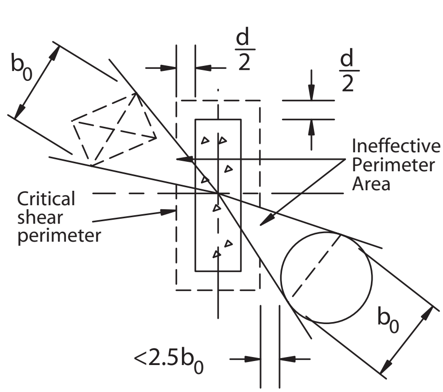


Figure 11.13

In considering the capacity in shear of the slab, at the support, due account must be taken of any transfer of moment from the slab into the support. The term M^*_v designates the amount of moment which is transferred from the slab system into the support in the direction being considered. For the simplified method the code designates:

(a) For Interior supports M^*_v shall be determined from the unbalanced moment transferred from the slab to the support. The Code specifies the minimum value of this moment in equation 11.6.

$$M^*_v = 0.06[(1.2g+0.75q)\,L_t(L_o)^2 - 1.2g\,L_t(L'_o)^2] \quad \textbf{(11.6)}$$

Where L'_o is the smallest value of the adjoining spans.

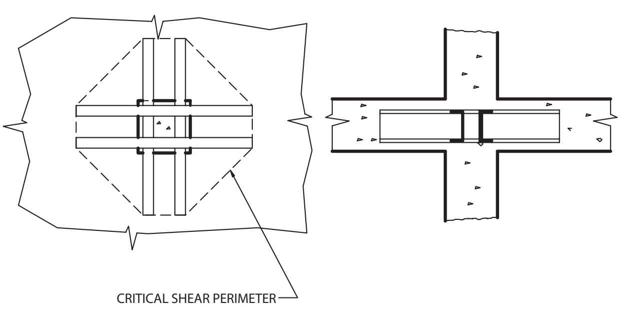
(b) For Exterior supports M^*_v is the actual moment.

11.13 Shear Capacity where Bending Moment $M^*_v = 0$

Although in the simplified method M^*_v should always be non-zero the value V_{uo} is a basic factor which is used in some cases.

Where there is no transfer of moment from the slab to the support there are two design cases. These relate to the inclusion or non inclusion of a shear head. A shear head usually consists of steel sections, universal beams, columns or channels embedded in the slab. This is illustrated in Figure 11.14.

Figure 11.14 - Shear Head Arrangement



1. NO SHEAR HEAD.

The shear capacity of a slab for two way punching at a support without a shear head and ignoring prestress is given by equation 11.7

$$\phi V_{uo} = \phi u d f_{cv} \quad \textbf{(11.7)}$$

Where: $f_{cv} = 0.17\left(1 + \dfrac{2}{\beta_h}\right)\sqrt{f'_c} \leq 0.34\sqrt{f'_c}$

β_h = the ratio of the larger effective support dimension (Y) to the overall dimension (X) measured perpendicular to Y

2. SHEAR HEAD PRESENT

The shear capacity of a slab for two way punching at a support with a shear head and ignoring prestress, is given by equation 11.8.

$$\phi V_{uo} = \phi u d(0.5\sqrt{f'_c}) \qquad \textbf{(11.8)}$$

11.14 Capacity where Bending Moment $M^*_v > 0$

Where M^*_v is not zero the transfer of moment into the support must be included in the determination of punching shear and torsional effects at the support. The expressions which relate to the shear capacity are derived from considering the shear and torsion on the side of the support at the slab level. The condition which must be satisfied is $\frac{T^*}{\phi T_{uc}} + \frac{V^*}{\phi V_{uc}} \leq 1.0$ The following expressions derived for this condition relate to the different cases of shear in slabs at supports. If the transfer of moment from the slab to the column or support is significant it is necessary to resist the torsional effect with either a torsion strip or a spandrel beam. These are illustrated in Figures 11.15 and 11.16.

Figure 11.15

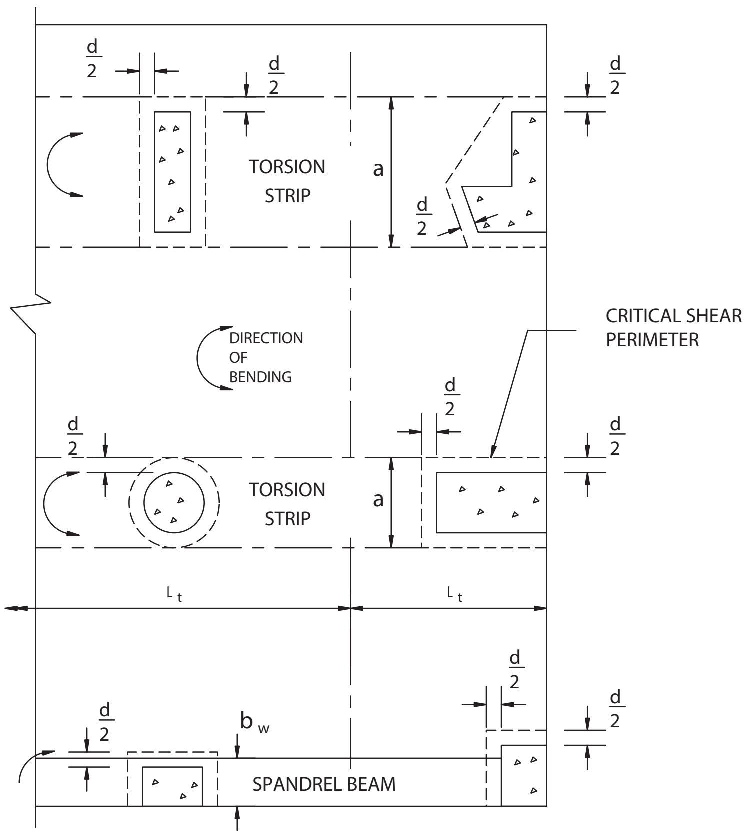
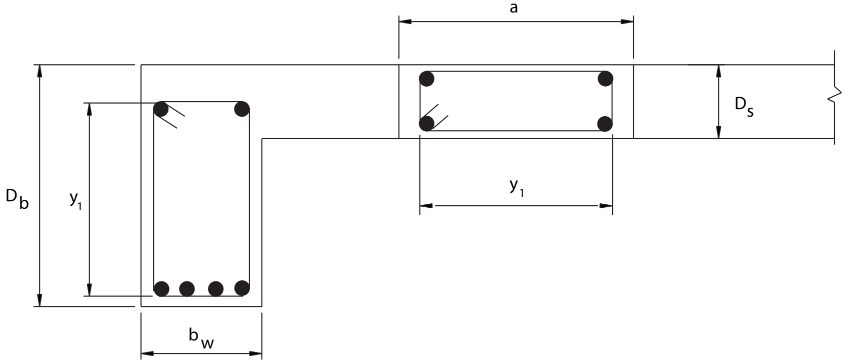


Figure 11.16



There are four cases considered in the Code.

(1) The shear capacity of a slab at a support where there are no closed ties in the torsion strip or spandrel beam is given in equation 11.9.

$$\phi V_u = \phi \left(\frac{V_{uo}}{1.0 + \left(\frac{u M_V^*}{8 V^* a d} \right)} \right) \tag{11.9}$$

(2) The shear capacity of a slab at a support where the torsion strip contains torsional reinforcement in the form of closed ties as detailed in Section 11.15 and as defined in equation 11.14, is given in equation 11.10.

$$\phi V_{u.min} = \phi \left(\frac{1.2 V_{uo}}{1.0 + \left(\frac{u M_V^*}{2 V^* a^2} \right)} \right) \tag{11.10}$$

(3) The shear capacity of a slab at a support where the spandrel beam contains torsional reinforcement in the form of closed ties as detailed in Section 11.15 and as defined in equation 11.14, is given in equation 11.11

$$\phi V_{u.min} = \phi \left(\frac{D_b}{D_s} \right) \left(\frac{1.2 V_{uo}}{1.0 + \left(\frac{u M_V^*}{2 V^* a b_w} \right)} \right) \tag{11.11}$$

(4) The shear capacity of a slab at a support where the torsion strip or spandrel beam contains torsional reinforcement in the form of closed ties as detailed in Section 11.15 and in excess of the minimum required by equation 11.14, is given in equation 11.12

$$\phi V_u = \phi V_{u.min} \sqrt{\frac{A_{sw} f_{sy.f}}{0.2 y_1 s}} \qquad \textbf{(11.12)}$$

The value of ϕV_u shall not exceed the maximum as specified in equation 11.13.

$$\phi V_{u.max} \leq 3\phi V_{u.min} \sqrt{x/y} \qquad \textbf{(11.13)}$$

11.15 Shear Reinforcement Details

If closed ties are required within the torsion strip or spandrel beam then the minimum area is given by equation 11.14.

$$A_{sw} \geq 0.2 \frac{y_1 s}{f_{sy.f}} \qquad \textbf{(11.14)}$$

The closed ties used as shear reinforcement shall be as detailed in Figure 11.16. They shall also comply with the following Code requirements (Section 9.2.6).

(1) the ties shall extend along the torsion strip or spandrel beam a distance of not less than L_t / 4 from the face of a support or concentrated load. The first tie shall be located with in a distance of 0.5 x spacing 's' from the face of support.

(2) The spacing, s, of the closed ties shall not exceed the greater of 300 mm and D_b or D_s as applicable.

(3) There shall be at least one longitudinal bar at each corner of the closed tie.

11.16 Deflection

The deflection calculations are as detailed in Chapter 8. Chart D2 can be used to check on deemed to comply conditions. The following points are reiterated to emphasize the particular aspects relevant to the simplified method.

(1) k_3 = 0.95 for two way flat slabs without drop panels.

= 1.05 for two way flat slabs with drop panels.

The drop panels shall extend a distance L/6 in each direction on each side of a support centreline.

The overall depth of the drop panel shall be at least 1.3D where D is the slab thickness without the drop panel.

(2) k_4 = 2.0 for end spans

= 2.4 for interior spans

This applies where in the adjoining spans the ratio of the longer span to the shorter span does not exceed 1.2. The end span shall not be longer than an interior span.

11.17 Design Process

The design process for the simplified method can be outlined as follows:

(1) The Code places restrictions on the application of the method. These restrictions should be verified before proceeding with any analysis or design.

(2) The slab size should be estimated (for design only).

(3) The analysis should be carried out in both the x and the y direction.

(4) The moments in the direction of the strip and the transverse moments calculated.

(5) The reinforcing steel calculated.

(6) The slab deflection is calculated. If the deflection exceeds the code requirements then the slab is redesigned.

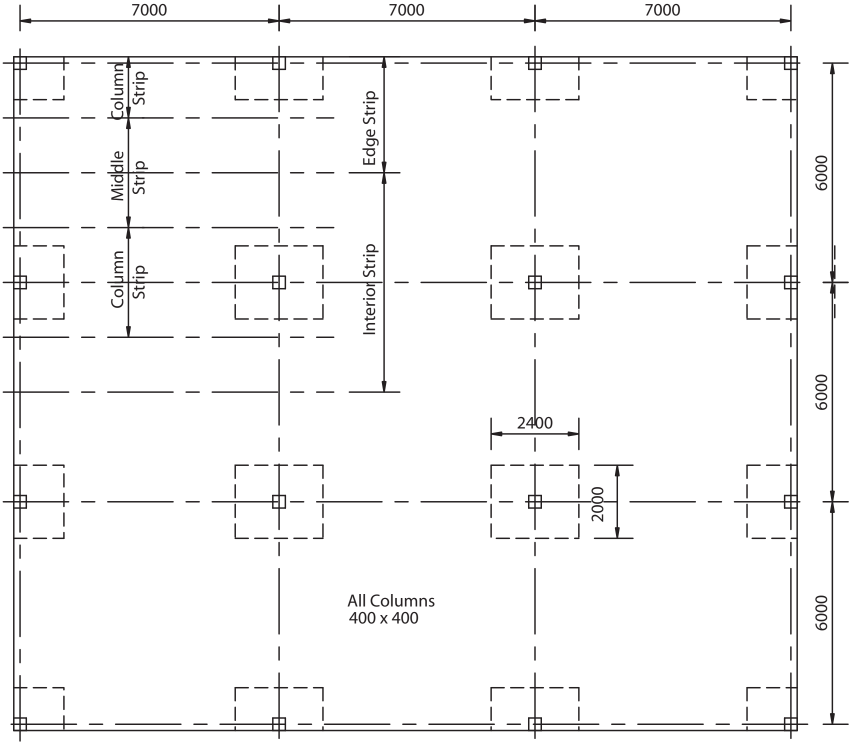
(7) The shear capacity of the slab is checked.

(8) Final reinforcement details are then determined.

Example 3

A simple slab floor system is shown in Figure 11.17. Design the floor system using the simplified design method. In this example the complete problem will not be worked out. Typical aspects will be detailed only.

Figure 11.17



DATA

The floor system is used for offices with a design live load of 3 kPa.

The slab will not support any masonry walls or features and there are no fittings which will be damaged by deflection.

f'_c = 40 MPa.   Use N12 bars where possible.

Required cover A1 - 20 mm

CALCULATIONS

(a) Determine design strip dimensions - see Figure 11.7

(b) Determine drop panel extent - based on L/6

x direction   L/6 = 7000/6 = 1167 mm * 2 = 2333 mm

Round off to   2400 mm overall.

y direction   L/6 = 6000/6 = 1000 mm * 2 = 2000 mm

2000 mm overall

See Figure 11.17

(c) Determine approximate slab thickness. There are a number of techniques and aids available to estimate the depth. Chart D2 in chapter 8 can be used as follows.

L_{eff} /d = $k_3k_4k_5$ Data L_{eff} = 7000

k_3 = 1.05,   Drop panels k_4 = 2.1,   End span k_{cs} = 2.0 (No A_{sc} at midspan)

Live Load = 3 kPa,   Dead Load = 6 kPa (estimate 250 average D)

Long Term Loading $F_{d.eff}$ = (1 + 2)*6 + (0.7 + 2*0.4)*3 = 22.5 kPa

Δ/L_{eff} = 1/250

From Chart D2 (Chapter 8) k_5 = 18.3

Incremental Loading $F_{d.eff}$ = 2*6 + (0.7 + 2*0.4)*3 = 16.5 kPa

Δ/L_{eff} = 1/500

From Chart D2 (Chapter 8) k_5 = 16.1

Therefore using the lesser value of k_5,   d = 7000/(1.05*2.1*16.1) = 197 mm

D = 197 + 12/2 + 20 = 223   say 230 mm

Drop panel depth 1.3D = 230 * 1.3 = 300 mm

Check average concrete thickness = 230 + 2.4*2.0*(300-230)/(7*6)

= 238 say 240 mm

Try new dead load, g = 24 * 0.24 = 5.76 kPa

Check incremental deflection only which was earlier established to govern.

Incremental Loading, $F_{d.eff}$ = 2 * 5.76 + (0.7 + 2*0.4)*3 = 16.0 kPa

Δ/L_{eff} = 1/500

From Chart D2 (Chapter 8) k_5 = 16.3

Therefore using the new value of k_5,
d = 7000/(1.05*2.1*16.3) = 195 (say 200 mm)

D = 200 + 12/2 + 20 = 226 say 230 mm

Drop panel depth 1.3D = 230 * 1.3 = 300 mm ...... OK

(d) Slab Analysis

X-DIRECTION - LONG SPAN

L_t = 6.0 L = 7.0 F_d = 1.2*5.76+1.5*3 = 11.4 kPa

To calculate L_0, first calculate a_s = 270 mm see Figure 11.9

L_o = 7000 - 2*(0.7*270) = 6622 mm

M_o = 11.4*6.0*6.622^2/8 = 375 kNm

INTERNAL STRIP

Note C- indicates column strip M - indicates middle strip

Internal Span		End Span		
Distribution of Moment along the Span				
Negative End Mmt	Positive Moment	Neg. Ext	Pos. Mmt	Neg. Int
0.65*375	0.35*375	0.25*375	0.5*375	0.75*375
-244	130	-94	188	-281

Distribution of Moment Across the Strips									
C 0.75	M 0.25	C 0.5	M 0.5	C 1.0	M 0.0	C 0.5	M 0.5	C .75	M .25
-183	-61	65	65	-94	0	94	94	-211	-70

L_t = 3.2 L = 7.0 F_d = 1.2*5.76+1.5*3 = 11.4 kPa

L_o = as before = 6622 mm

$$M_o = \frac{11.4 * 3.2 * 6.622^2}{8} = 200 \text{ kNm}$$

EXTERNAL STRIP

Internal Span		End Span		
Distribution of Moment along the Span				
Negative End Mmt	Positive Moment	Neg. Ext	Pos. Mmt	Neg. Int
0.65*200	0.35*200	0.25*200	0.5*200	0.75*200
-130	70	-50	100	-150

Distribution of Moment Across the Strips									
C 0.75	M 0.25	C 0.5	M 0.5	C 1.0	M 0.0	C 0.5	M 0.5	C .75	M .25
-98	-32	35	35	-50	0	50	50	-112	-38

Y-DIRECTION - SHORT SPAN

$L_t = 7.0$ $L = 6.0$ $F_d = 1.2*5.76+1.5*3 = 11.4$ kPa

To calculate L_o - first calculate a_s = 270 mm, see Figure 11.9

$$L_o = 6000 - 2*(0.7*270) = 5622 \text{ mm}$$

$$M_o = \frac{11.4 * 7.0 * 5.622^2}{8} = 316 \text{ kNm}$$

INTERNAL STRIP

Internal Span		End Span		
Distribution of Moment along the Span				
Negative End Mmt	Positive Moment	Neg. Ext	Pos. Mmt	Neg. Int
0.65*316	0.35*316	0.25*316	0.5*316	0.75*316
-205	110	-79	158	-237

Distribution of Moment Across the Strips

C 0.75	M 0.25	C 0.5	M 0.5	C 1.0	M 0.0	C 0.5	M 0.5	C .75	M .25
-154	-51	55	55	-79	0	79	79	-178	-59

$L_t = 3.7$ $L = 6.0$ $F_d = 1.2*5.76+1.5*3 = 11.4$ kPa

L_o = as before = 5622 mm

$$M_o = \frac{11.4 * 3.7 * 5.622^2}{8} = 167 \text{ kNm}$$

EXTERNAL STRIP

Internal Span		End Span		
Distribution of Moment along the Span				
Negative End Mmt	Positive Moment	Neg. Ext	Pos. Mmt	Neg. Int
0.65*167	0.35*167	0.25*167	0.5*167	0.75*167
-109	58	-42	84	-125

Distribution of Moment Across the Strips

C 0.75	M 0.25	C 0.5	M 0.5	C 1.0	M 0.0	C 0.5	M 0.5	C .75	M .25
-82	-27	29	29	-42	0	42	42	-94	-31

Each of these moments acts across the width of the particular strip and adjoining middle strips are added together.

(e) Reinforcement Selection

For each of these moments the appropriate area of steel can be calculated. The following checks must be made.

Minimum reinforcement is 0.0025 bD (as per # 9.1.1(a) needs to be calculated for four cases - two in each direction. The long span bars will be laid closest to the external face as this is the main bending direction.

The value of d in the long span for the drop panel is 300 - 20-12/2 = 274 mm.

In the slab d = 230 - 20 -12/2 = 204 mm.

The value of d in the short span for the drop panel is 300 – 20 -12/2 -12 = 262 mm.

In the slab d = 230 - 20 -12/2 -12= 192 mm.

Check long span - column moment at internal column M* = -211 kNm.

Strip is 3m wide - M* per m width = -211/3 = -70 kNm.

Check minimum steel p = 0.0025 bD

Therefore required A_{st} is 0.0025*1000*274 = 685 mm^2/m

Capacity of this section is $0.8*685*500*274 (1 - 685*500/(1.7*1000*274*40))*10^{-6}$

= 74 kNm/m

N12 at 125 centres give a capacity of 76 kNm/m.

The remaining sections can be completed.

(f) Shear Capacity at Columns

Check in x-direction.

The shear load on an internal column is 7*6*11.7 = 492 kN

Out of balance slab moments are 217 - 188 = 29 kNm

Check minimum M^*_v using equation 11.6

Minimum $M^*_v = 0.06\ [\ (1.2*5.76 + 0.75*3)*6*6.622^2 - 1.2*5.76*6*6.622^2\ ]$

= 35.5 kNm ..... use minimum

Use average d for both directions d = (274+262)/2 = 268 mm

a = b = 400 + d = 668 mm   u = 4 * 668 = 2672 mm

$f_{cv} = 0.17*(1+2)\sqrt{40} = 0.51\sqrt{40}$ <0.34)$\sqrt{40}$ therefore use $0.34\sqrt{40}$ = 2.15 MPa

$\phi V_{uo} = 0.7*2672*268*2.15*10^{-3}$ = 1078 kNm from equation 11.7

Using equation 11.9

$$\phi V_{u.min} = \frac{1078}{1 + \frac{35.5 * 2672}{8 * 492 * 10^{-3} * 668 * 268}} = 950 \text{ kN} > 492 \quad \text{....OK}$$

Check for minimum area of ties using equation 11.13.

PROBLEMS

QUESTION 1

Derive the expression for the equivalent load w_y' given by equation 11.4.

QUESTION 2

(a) Use the information given in example 1 to design slab panel "F" if the panel supports masonry walls which will be constructed to reduce the effects of deflection.

(b) Calculate the equivalent uniformly distributed load to be carried by the supporting beam between panels "H" and "F".

(c) Check the slab for shear near supports . The slab may be treated as a wide beam (say consider one metre width) and the shear requirements for beams applied.

(d) Show all the final slab details on a suitable drawing.

QUESTION 3

Complete the flat slab design in example 3.

(a) Determine steel reinforcement throughout the slab.

(b) Check edge columns for shear.

(c) Draw a section showing location and extent of reinforcement.

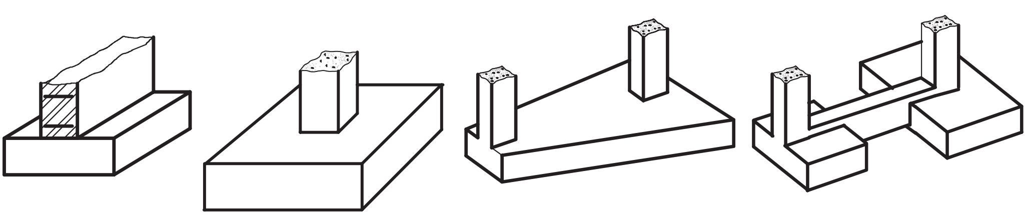
12 chapter Footings

12.1 Additional Symbols used in this Chapter

a	=	Footing outstand.
A_{FN}	=	Reaction area for punching shear.
f'_{cf}	=	Flexural tensile strength of plain concrete.
	=	$0.6\sqrt{f'_c}$
q_1	=	Maximum soil bearing pressure under footing.
q_2	=	Minimum soil bearing pressure under footing.
q_a	=	Permissible soil bearing pressure.
q_u	=	Factored soil bearing capacity.
	=	$1.4q_a$.
u	=	Shear perimeter d/2 from face of column.
V_{uc}	=	Ultimate punching shear strength.

12.2 Types of Footings

Figure 12.1



(a) Strip Footing (b) Spread or Pad Footing (c) Combined Footing (d) Strap or Cantilever Footing

Footings are used to transfer concentrated or linear loads from columns or walls to the foundations. The size of the footing is governed by the soil bearing capacity. There are many types of footings, Figure 12.1 illustrates the more common types.

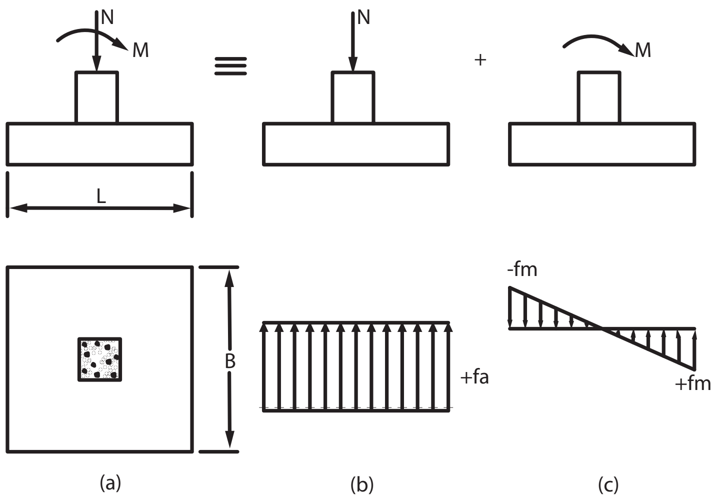
12.3 Spread Footings

Considering the general case where the column transmits an axial load N and a moment M to the footing, it is necessary to proportion the footing so that the maximum soil bearing pressure under the footing does no exceed the permissible soil bearing capacity.

The bearing pressure distribution under the footing can be determined as the summation of the uniform bearing pressure due to the axial load N and the linearly varying pressure due to the moment M as shown in Figure 12.2.

The bearing pressure distribution under the footing can be determined as the summation of the uniform bearing pressure due to the axial load N and the linearly varying pressure due to the moment M as shown in Figure 12.2.

Figure 12.2



Due to the axial load, the bearing pressure will be uniform given by:

$$f_a = \frac{N}{BL} \qquad \text{(a)}$$

Due to the moment the bearing pressure will vary linearly from a maximum tensile or negative value $-f_m$ to zero at the centreline and to a maximum compressive or positive value $+f_m$. The maximum values may be calculated from the usual moment–stress relationship.

$$f_m = \pm \frac{M}{Z}$$

Where; Z = Modulus of section of footing $= \dfrac{BL^2}{6}$

The moment M is produced by the load N acting at an eccentricity $e = M/N$ from the column centreline, i.e. $M = N*e$.

Substituting for M and Z in the equation for f_m gives;

$$f_m = \frac{6Ne}{BL^2} \qquad \text{(b)}$$

The combined effect is the summation of equations (a) and (b) to give the maximum and minimum bearing values q_1 and q_2 at the extremities.

$$q_1 = \frac{N}{BL} + \frac{6Ne}{BL^2}$$

$$q_2 = \frac{N}{BL} - \frac{6Ne}{BL^2}$$

The two equations are frequently represented by,

$$q_{1,2} = \frac{N}{BL} \pm \frac{6Ne}{BL^2}$$

Which may be further simplified in the form,

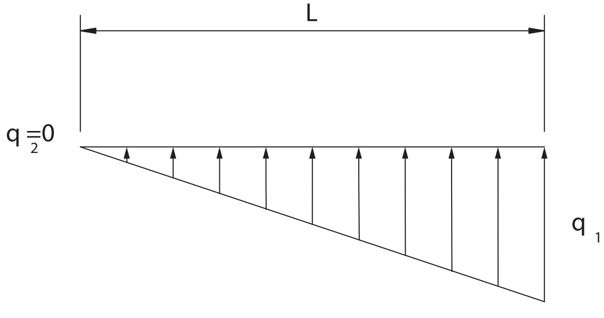
$$q_{1,2} = \frac{\mathrm{N}}{\mathrm{BL}^2}(\mathrm{L} \pm 6\mathrm{e}) \qquad \textbf{(12.1)}$$

It may be seen that q_2 can be positive or negative or even zero. q_2 will be zero when the terms in the brackets are zero, that is when:

$$(L - 6e) = 0$$

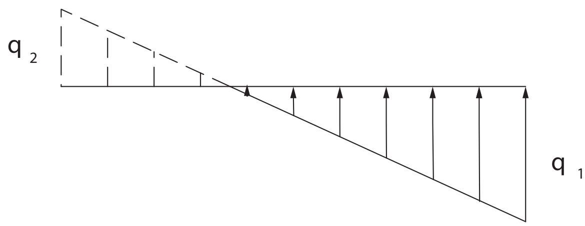
Which is when: $e = \frac{L}{6}$

When $e = L/6$ the pressure distribution will vary from zero to a maximum as shown in Figure 12.3 (*below*).



When $e > L/6$, q_2 will be negative. But the contact bearing pressure between the footing and the soil cannot be negative except for soil adhesion and possible short-term suction. The bearing pressure distribution is shown in Figure 12.4.

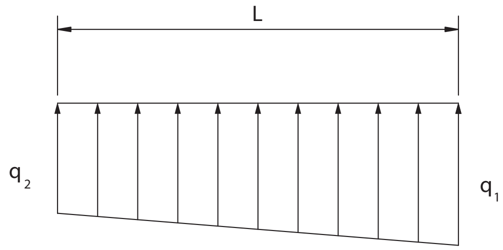
Figure 12.4



Such conditions lead to an inefficient footing since a portion of the footing is not in contact with the soil. Also q_1 will tend to be excessively high.

When $e < L/6$, both q_1 and q_2 are positive. In other words the footing is in direct contact with the soil over its full length as shown in Figure 12.5.

Figure 12.5



A designer will always proportion the footing to ensure that the footing is fully in contact with the soil, for which the soil pressure distribution is shown in Figure 12.5. This is achieved when the eccentricity e < L/6 from the column centreline, i.e. the eccentricity is within the middle third of the footing. The central third is frequently referred to as the kern.

EXAMPLE 1

A column transmits an axial load $N = 600$ kN and a moment $M = 78$ kNm to the footing. Determine the length of the footing L if the width $B = 2$ m and the maximum permissible soil bearing capacity $q_a = 150$ kPa.

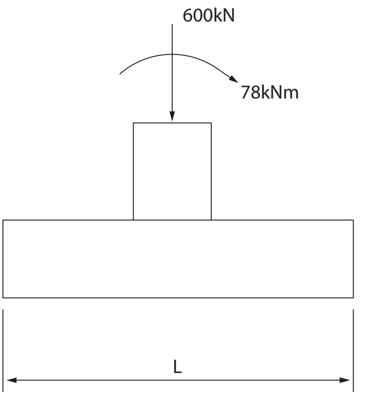


Figure 12.6

SOLUTION

Eccentricity of loading, $e = \dfrac{M}{N} = \dfrac{78}{600} = 0.13\text{m}$

Substituting 150 for q_1 in equation 12.1,

$$150 = \frac{600}{2L^2}\ (L + 6 \times 0.13)$$

Solving for the required length L;

$$300L^2 = 600 \times L + 600 \times 6 \times 0.13$$
$$L^2 - 2 \times L - 1.56 = 0$$
$$\text{L} = 2.60\ \text{m}$$

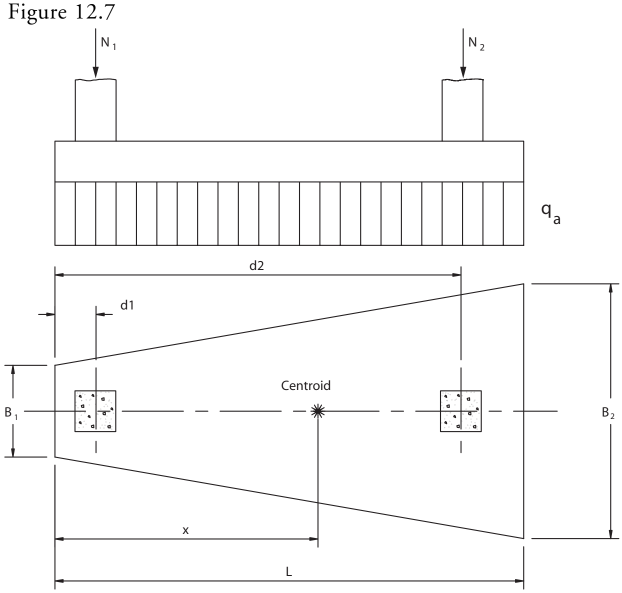
Use the derived value of 2.6 m for L in equation 12.1 as a check on the soil bearing pressure and plot the final distribution.

$$q_{1,2} = \frac{600}{2 \times (2.6)^2}\left(2.6 \pm 6 \times 0.13\right)$$
$$q_1 = 150\ \text{kPa}$$
$$q_2 = 81\ \text{kPa}$$

12.4 Combined Footings and Strap Footings

In commercial buildings the exterior columns are frequently placed close to or hard against the building boundaries to maximise use of land since commercial land is very expensive. The column sits right on the edge of the footing causing eccentric loading and excessively high soil bearing pressure. It is possible however to combine the exterior and the first interior column footings to act as a single footing. The footing is called a combined footing and it is proportioned to give uniform bearing pressure. This is achieved if the resultant column load coincides with the centre of area of the footing. The shape of the footing is frequently trapezoidal as shown in Figure 12.7 (*right*).

Figure 12.7



Referring to Figure 12.7, the position of the centroid of the trapezoidal area may be derived to be given by;

$$x = \frac{B_1 + 2B_2}{3(B_1 + B_2)} L$$

The position of the resultant of the column loads is obtained by taking moments about the left edge and it is given by:

$$x = \frac{N_1 d_1 + N_2 d_2}{N_1 + N_2}$$

Equating the two expressions for x:

$$\frac{B_1 + 2B_2}{3(B_1 + B_2)} L = \frac{N_1 d_1 + N_2 d_2}{N_1 + N_2} \qquad \text{(a)}$$

The required area of footing is calculated using the permissible soil bearing pressure q_a.

$$A = \frac{N_1 + N_2}{q_a} \qquad \mathbf{(12.2)}$$

The area of the trapezoidal shape is also given by:

$$A = \frac{B_1 + B_2}{2} L \qquad \text{(b)}$$

From which,

$$B_1 + B_2 = \frac{2A}{L} \qquad \text{(c)}$$

and,

$$B_2 = \frac{2A}{L} - B_1 \qquad \mathbf{(12.3)}$$

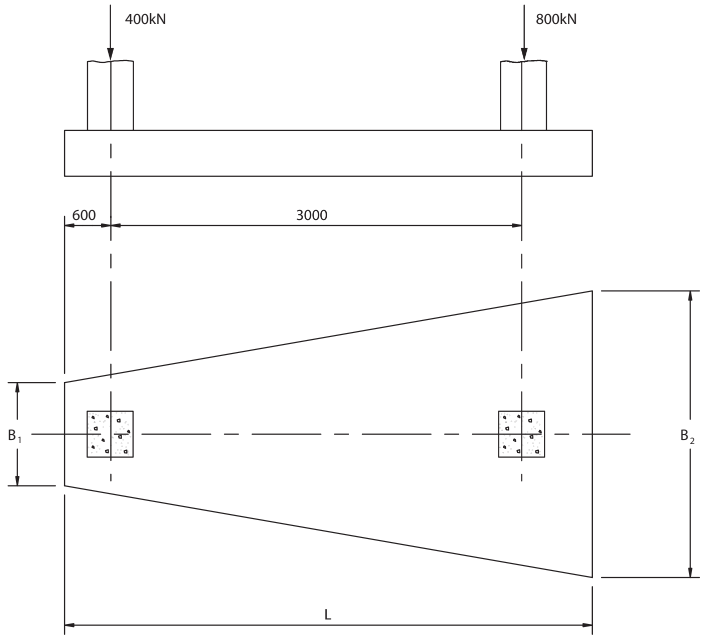
The two equations (a) and (b) relate three unknowns B_1, B_2 and L. It is necessary to assume one of the unknowns, usually L, and solve simultaneously for the remaining unknowns B_1 and B_2. Substituting for $(B_1 + B_2)$ and B_2 in equation (a) and solving for B_1 gives,

$$B_1 = \frac{4A}{L} - \left(\frac{6A}{L^2}\right)\left(\frac{N_1 d_1 + N_2 d_2}{N_1 + N_2}\right) \qquad \mathbf{(12.4)}$$

EXAMPLE 2

Referring to the combined footing shown in Figure 12.8, determine suitable size of footing for a permissible soil bearing pressure q_a = 200 kPa and footing length L = 4200.

Figure 12.8



SOLUTION

d_1 = 0.6 m N_1 = 400 kN L = 4.2 m

d_2 = 3.6 m N_2 = 800 kN q_a = 200 kPa

Area of footing required from equation 12.2,

$$A = \frac{400+800}{200} = 6 \text{ m}^2$$

Footing dimension B_1 from equation 12.4,

$$B_1 = \frac{4\times 6}{4.2} - \left(\frac{6\times 6}{4.2^2}\right)\left(\frac{400\times 0.6+800\times 3.6}{400+800}\right) = 0.41m$$

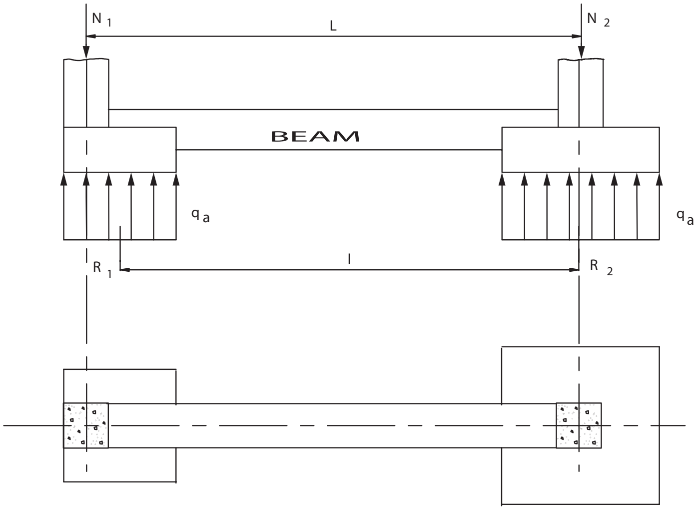
Substituting for B_1 in equation 12.3,

$$B_2 = \frac{2\times 6}{4.2} - 0.41 = 2.45 \text{ m}$$

When $B_1 = B_2$, the combined footing is a rectangular footing. What would be the required size of a rectangular footing in this example. Try it. (Answer: B = 1.15 m and L = 5.2 m).

If the columns are too far apart, a combined footing becomes excessively large. Instead two footings connected by a rigid beam are used. Such footings are called strap or cantilever footings, as shown in Figure 12.1(d) and Figure 12.9. The philosophy is similar to combined footings. To obtain uniform bearing pressure the centroid of the two footings (excluding the connecting beam) is made to coincide with the resultant of the column loads.

Figure 12.9



Reaction forces R_1 and R_2 act through the centroid of each footing. From equilibrium conditions:

$$R_1 + R_2 = N_1 + N_2$$

$$R_2 = N_1 + N_2 - R_1 \quad \text{(d)}$$

Taking moments about N_2 (and R_2 since they coincide),

$$R_1 = N_1\, L$$

$$R_1 = \frac{N_1 L}{l}$$

Substituting for R_1 in (d),

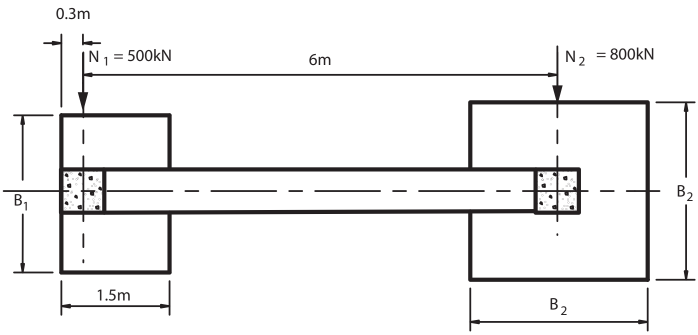
$$R_2 = N_1 + N_2 - \frac{N_1 L}{l}$$

$$= N_1\left(1 - \frac{L}{l}\right) + N_2$$

EXAMPLE 3

Determine the required footing dimensions for the strap footing shown in Figure 12.10 if the maximum permissible soil bearing pressure q_a = 150 kPa.

Figure 12.10



SOLUTION

N_1 = 500 kN L = 6.0 m

N_2 = 800 kN l = 6.3 - = 5.55 m

$$R_1 = \frac{N_1 L}{l}$$

$$= \frac{500 \times 6.0}{5.55}$$

$$= 540.5 \text{ kN}$$

Therefore the required footing dimension B_1 under the 500 kN load

$$B_1 = \frac{540.5}{1.5 \times 150} = 2.40 \text{ m}$$

Under the 800 kN load, the reaction force;

$$R_2 = N_1\left(1 - \frac{L}{l}\right) + N_2$$

$$= 500\left(1 - \frac{6.0}{5.55}\right) + 800$$

$$= 759.5 \text{ kN}$$

The required footing dimension,

$$B_2 = \sqrt{\frac{759.5}{150}}$$

$$= 2.25 \text{ m}$$

12.5 Design of Footings Supporting Axially Loaded Columns

There are three failure conditions to be considered:

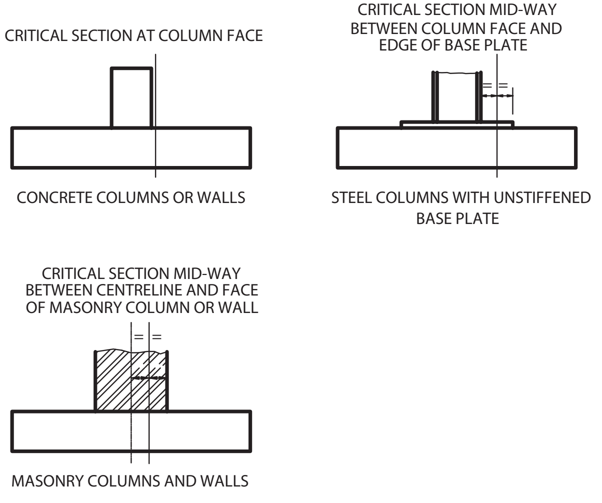
(1) Bending,
(2) Bending shear (or one-way shear),
(3) Punching shear (or two-way shear).

(1) CRITICAL SECTION FOR BENDING.
The critical section for the maximum bending moment is illustrated in Figure 12.11 below.

(2) CRITICAL SECTION FOR BENDING SHEAR.
The critical section is taken a distance d from the critical section for bending. This is a one-way shear action taken across the full width of the footing.

(3) CRITICAL SECTION FOR PUNCHING SHEAR.
This is referred as two-way action. The critical section is taken along a perimeter u distance $d/2$ from the critical section for bending.

Figure 12.11



12.5.1 Design Considerations

All the problems in this chapter are concerned with footings supporting axial column loads only. In practice this is frequently the case. To enable a column to transmit a moment to the footing, fixed ended conditions are implied. It is only necessary for the footing to rotate, by uneven settlement, a very small amount (on average one half of one degree) to make the column end behave as a pin-ended column.

Most of the problems will assume a reinforced concrete column with the critical section for bending at the face of the column. Design procedure for footings supporting masonry or steel columns will be identical except that the position of the critical section will be different.

12.5.2 Factored Bearing Soil Capacity q_u

Footings are designed in bending as wide cantilever beams supporting uniformly distributed loads. Since all designs will be carried out by the ultimate strength design method, the permissible soil bearing pressure q_a is replaced by a factored soil bearing value $q_u = 1.4 * q_a$ where 1.4 is the average dead and live load factor applied to the column loads. It must be stressed that q_u is NOT the ultimate soil bearing capacity; it may be thought of as the "permissible" soil bearing value for ultimate load conditions.

For a 1 m width of footing shown in Figure 12.12, the design bending moment at the critical section will be,

$$M^* = \frac{q_u a^2}{2}$$

The footing can be designed for bending as a rectangular beam 1 m wide.

The bending shear force V^*, i.e. the shear force across the 1 m width of footing, is the shear force distance d from the critical section for bending.

$$V^* = (a - d)\, q_u \text{ kN per m width of footing}$$

Figure 12.12

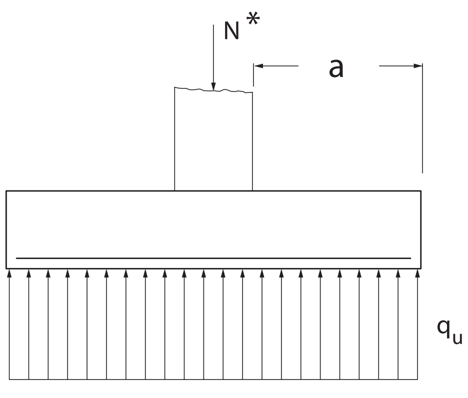
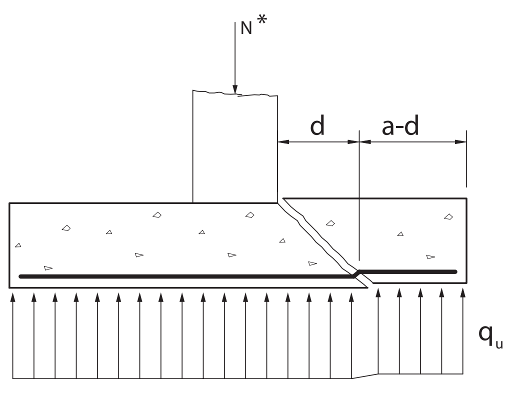


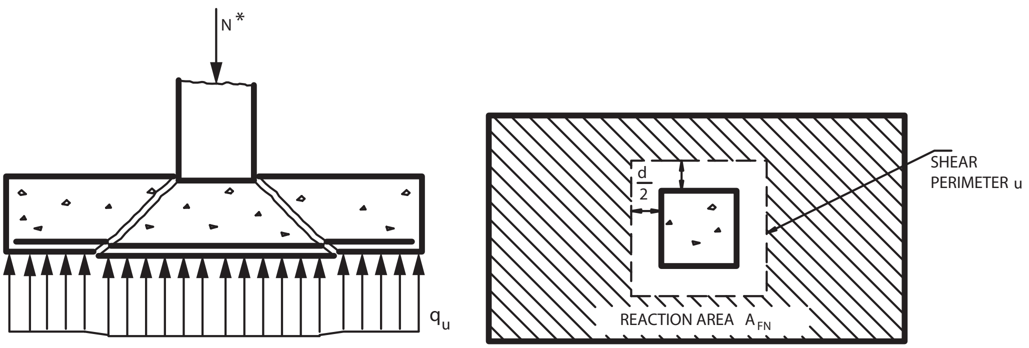
Figure 12.13



The effective shear capacity ΦV_{uc} is determined as for beams. Bending shear is frequently a critical condition in footings. That is, if a footing is designed for bending first, bending shear may not be satisfied so that it may be necessary to either increase the depth of the footing or increase the steel ratio to give a shear capacity ΦV_{uc} at least equal to the design shear force V^*.

Finally there is always a possibility that the column may literally punch through the footing as illustrated in Figure 12.14.

Figure 12.14



The punching shear force will be equal to the bearing pressure q_u acting on the reaction area A_{FN}.

$V^* = q_u A_{FN}$

where $A_{FN} = B \times L - (c + d)^2$

The nominal shear stress due to punching shear acts on the shear area around the perimeter *u*. The Code #9.2.3 gives the shear capacity for two way action, i.e. punching shear,

ΦV_{uc} = Shear area × Shear stress

$= \Phi \text{ u d } f_{cv}$

Where u*d is the punching shear area and f_{cv} is the maximum shear stress given by:

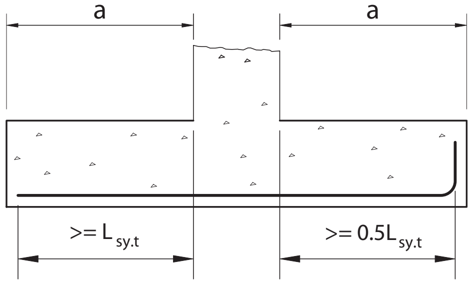
$$f_{cv} = 0.17\left(1 + \frac{2}{\beta_h}\right)\sqrt{f'_c} \leq 0.34\sqrt{f'_c}$$

Where: β_h = Ratio of the larger column dimension Y to the smaller column dimension X.

The value of f_{cv} will be equal to the maximum of $0.34\sqrt{f'_c}$ for all rectangular columns whose larger dimension $Y \leq 2X$.

Table 12.1 - Minimum Footing Outstands for Developmental Length

Bar Size	Minimum Footing outstand a: Straight Bars	Minimum Footing outstand a: Cogged Bars
N16	440	240
N20	580	310
N24	780	410
N28	1010	520
N32	1260	650
N36	1530	780



When choosing the reinforcement it is necessary to check that development length $L_{sy.t}$ for straight bars or $0.5L_{sy.t}$ for cogged or hooked bars is satisfied. Table 12.1 gives the minimum footing outstand a required to satisfy development length. The table is based on factor $k_2 = 1.7$, 40 mm end cover, $f'c = 25$ MPa and $f_{sy} = 500$ MPa.

EXAMPLE 4

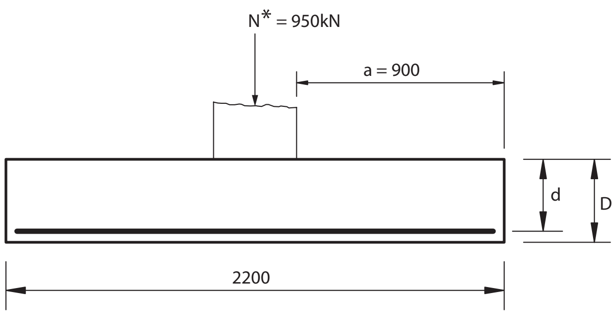
A 400 mm square reinforced concrete column transmits a design load $N^* = 950$ kN to a square footing. Design the footing using grade N25 concrete if the factored soil bearing capacity $q_u = 200$ kPa.

SOLUTION

Size of square footing required,

$$L = \sqrt{\frac{950}{200}} = 2.18 \text{ m say } 2.2 \text{ m}$$

Figure 12.15



Bending moment at face of column (critical section for bending),

$$M^* = 200\,\frac{(0.9)^2}{2} = 81 \text{ kNm/m width}$$

Choose a steel ratio of say $p = 0.005$.

From Chart B1, read $\lambda = 2.0$.

Minimum required effective depth,

$$d = \sqrt{\frac{M^*}{\lambda . b}} = \sqrt{\frac{81 \times 10^6}{2.0 \times 1000}} = 200 \text{ mm}$$

Required area of reinforcement,

$$A_{st} = 0.005 \times 1000 \times 200 = 1000 \text{ mm}^2\text{/m width.}$$

Check footing for bending shear.

Shear force at critical section,

$$V^* = q_u\,(a - d)$$
$$= 200\ (0.9 - 0.200)$$
$$= 140 \text{ kN/m width}$$

From chart V1 for $p_v = 0.005$, $v'_c = 0.35$ MPa.

$$\Phi V_c = 0.35 \times 1000 \times 200 \times 10^{-3}$$
$$= 70 \text{ kN}$$

$$\beta_1 = 1.1\left(1.6 - \frac{d_0}{1000}\right) \geq 1.1$$

$$= 1.1\left(1.6 - \frac{200}{1000}\right) = 1.54$$

Shear capacity; $\Phi V_{uc} = b_1\ \Phi V_c = 1.54*70$

$= 108$ kN

$< V^*$ (140 kN) UNSATISFACTORY

There are two ways to rectify this problem.

(a) Increase the effective depth of the footing to increase the shear capacity. This will be a trial and error procedure.

Choosing steel ratio $p = 0.003$ and effective depth $d = 300$ mm gives a moment capacity $\Phi M_u = 108$ kNm.

For bending shear, from Chart V1, $v'_c = 0.30$ MPa.

$$\beta_1 = 1.1\left(1.6 - \frac{300}{1000}\right) = 1.43$$

Shear capacity,

$$\Phi V_{uc} = 1.43 \times 1000 \times 300 \times 0.30 \times 10^{-3} = 129 \text{ kN}$$

Design shear force,

$$V^* = 200 \times (0.9 - 0.30)$$
$$= 120 \text{ kN}$$
$$< \Phi V_{uc} \text{ (129 kN) SATISFACTORY}$$

Check footing for punching shear.

Punching shear force,

$$V^* = 200 \, [(2.2)^2 - (0.70)^2] = 870 \text{ kN}$$

Perimeter for punching shear,

$$u = 4 \times 700$$
$$= 2800 \text{ mm}$$

For a square column sections ($X = Y$), the shear strength of concrete for two-way shear,

$$f_{cv} = 0.34\sqrt{f'_c}$$
$$= 0.34\sqrt{25}$$
$$= 1.7 \text{ MPa}$$

Thus the punching shear capacity,

$$V_{uc} = 0.7 \times 2800 \times 300 \times 1.7 \times 10^{-3}$$
$$= 1000 \text{ kN}$$
$$> V^* \text{ (870 kN) SATISFACTORY}$$

Total area of reinforcement required,

$$A_{st} = 0.003 \times 2.2 \times 10^3 \times 300$$
$$= 1980 \text{ mm}^2$$

Choose 7N20 bars to give $A_{st} = 2170 \text{ mm}^2$.

Check reinforcement for anchorage. From Table 12.1, minimum outstand required for development length of straight bars is a = 580 mm while the available outstand is 900 mm.

(b) Increase the steel ratio to make $\Phi V_{uc} \geq V^*$.

From Chart V1, for a steel ratio p = 0.012, v'_c = 0.47 MPa.

Shear capacity of footing,

$$\Phi V_{uc} = 1.54 \times 200 \times 10^3 \times 0.47 \times 10^{-3}$$
$$= 145 \text{ kN}$$
$$> V^* \text{ (140 kN) SATISFACTORY}$$

The required area of reinforcement,

A_{st} = $0.012 \times 1000 \times 200$

= 2400 mm^2/m width of footing.

Total tensile area required,

A_{st} = 2.2×2400

= 5280 mm^2

8N32 bars give A_{st} = 6400 mm^2.

Check the reinforcement for anchorage. From Table 12.1, the minimum outstand a = 1260 mm for straight bars and 650 mm for cogged bars. Use N32 cogged bars.

Check punching shear.

Reaction area,

A_{FN} = $(2.2)^2 - (0.600)^2$

= 4.48 m^2

Punching shear force,

V^* = $q_u A_{FN}$

= 200×4.48

= 896 kN

Shear perimeter (noting that 600 is one side of the shear perimeter)

u = 4×600

= 2400 mm

Shear strength, f_{cv} = 1.7 MPa

Hence punching shear capacity,

$\Phi V_{uc} = \Phi\, u\, d\, f_{cv}$

= $0.7 \times 2400 \times 200 \times 1.7 \times 10^{-3}$

= 571 kN

< V^* (892 kN) UNSATISFACTORY

It will still be necessary to increase the depth of the footing to satisfy punching shear. However, a smaller steel ratio will be required for the increased depth.

12.6 Design Aids

The required steel ratio for a given depth of footing or the required depth for a specified steel ratio may be governed by bending or bending shear. Most frequently, the bending shear condition is the governing criteria. For a given effective depth d of footing and steel ratio p, it is a simple process to calculate the maximum outstand a for both the bending and bending shear conditions. The lesser value of a is the governing condition. This is the process used to plot charts F1 to F20 in the design section at the end of the book. From charts F, the required steel ratio for any effective depth d and footing outstand a may be read directly. The designer need not be concerned with the governing criteria.

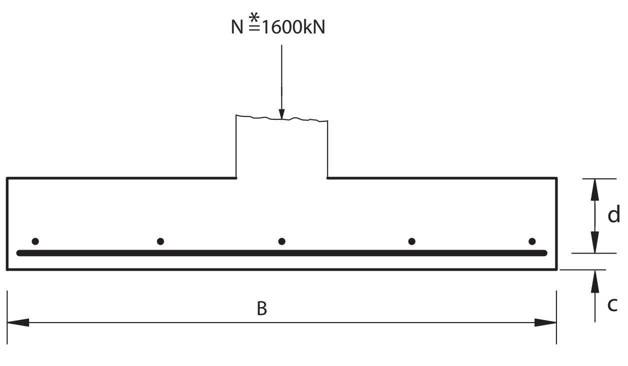
The punching shear condition must however be checked separately. Chart FP is used for punching shear. The punching shear capacity ΦV_{uc} for any shear perimeter u and depth d may be read directly from chart FP and compared with the design punching shear force V^*. This chart may be used to initially establish the approximate minimum effective depth required to satisfy punching shear.

While in the past footings were commonly designed using grade N20 concrete, the exposure conditions are such that grade N20 concrete may only be used in domestic construction. Grade N25 concrete will be the most common grade to be used in footings and it is the grade chosen in the design charts.

EXAMPLE 5

A 350 mm square column carries a design load $N^* = 1600$ kN. Design a square footing using N25 concrete for a factored soil bearing capacity $q_u = 300$ kPa. The footing is in non-aggressive soil and it is cast against a damp-proof membrane.

Figure 12.16



SOLUTION

Size of square footing required,

$$B = \sqrt{\frac{1600}{300}}$$

$= 2.3$ m

Minimum cover required for exposure classification A1 is 30 mm. Assuming N32 bars, the distance from the centre of top layer of reinforcement to the underside of the footing,

$$c = 30 + 32 + 32/2$$
$$= 78 \text{ say } 80 \text{ mm}$$

The footing depth will be chosen initially for punching shear. Try D = 550 mm to give d = 470 mm.

Reaction area: $A_{FN} = (2.3)^2 - (0.82)^2$

$$= 4.62 \text{ m}^2$$

Punching shear force,

$$V^* = 300 \times 4.62 = 1386 \text{ kN}$$

Punching shear perimeter,

$$u = 4 \times 820$$
$$= 3280 \text{ mm}$$

From Chart FP (page 234), read punching shear capacity of footing,

$$\Phi V_{uc} = 1850 \text{ kN}$$
$$> V^* (= 1400 \text{ kN}) \text{ SATISFACTORY}$$

Footing outstand, $a = \dfrac{2300-350}{2}$ = 975 mm

From chart F3 (page 234), required steel ratio p = 0.007.

Total area of reinforcement required,

$$A_{st} = 0.007 \times 2300 \times 470$$
$$= 7667 \text{ mm}^2$$

Choose 13N28 bars whose area A_{st} = 8060 mm^2.

From Table 12.1, bars will need to be cogged to satisfy development length condition (available outstand is 975 mm and the required minimum outstand for cogged bars is 520 mm).

Comments on Example 5.

(a) The footing was designed for the lesser effective depth, i.e. using the effective depth for the top row of reinforcement. Applying the resulting reinforcement will yield conservative results for the bottom row of reinforcement. Designers frequently use the average effective depth for the design of footings. This approach seams reasonable for punching shear but the author feels that when the outstands in the two directions are not the same, as in rectangular footings, the reinforcement in each direction should be determined by using the corresponding effective depth.

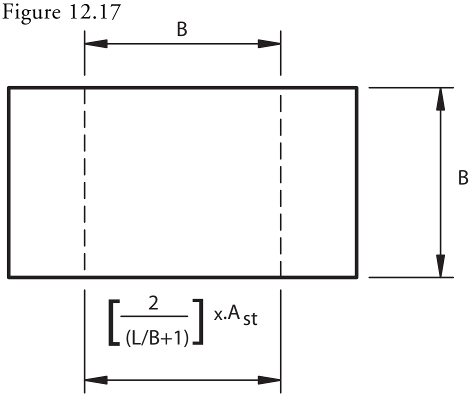
(b) The depth of footing must satisfy the compressive development length $L_{sy.c}$ for column starter bars. This is frequently the criteria governing the depth of footing.

(c) In the selection of footing size, there was no provision made for the additional bearing pressure due to the weight of footing. This is normally very small and it is frequently disregarded unless the footing is very deep or the soil bearing value is very small. In example 5, the additional factored bearing pressure due to the weight of footing will be, 1.25*0.5*24 = 15 kPa. The size of footing should be determined using the net bearing pressure (q_u - unit weight of footing). Taking the factored weight of footing into consideration in example 5, the required size of footing will be,

$$B = \sqrt{\frac{1600}{300-15}} = 2.37$$

12.7 Further Design Considerations

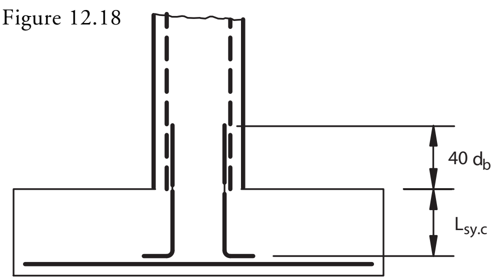
Figure 12.17



Rectangular footings may be required if site conditions limit one of the footing dimensions. With rectangular footings a greater proportion of the bending moment in the short direction is carried by portion of the footing adjacent to the column because of the stiffening effect provided by the column. A larger portion of the total short reinforcement should be placed in the central band of width *B* equal to the short footing dimension (shown in Figure 12.17 (*right*). Commonly $2/(L/B+1)$ of the total short reinforcement is placed in the central band and the remainder equally distributed between the outer strips.

12.8 Column to Footing Load Transfer

Figure 12.18



The axial load carried by the column is transferred by end bearing and dowel action. The area of the dowels does not have to be equal to the area of the column reinforcement. While there are no Code requirements, it is suggeted that a minimum dowel area of $0.005 \times A_g$ be adopted which is specified by the American Concrete Institute Code. The dowels must extend into the footing a minimum distance equal to the compressive development length $L_{sy.c} = 20d_b$, and they must extend into the column a minimum distance equal to $40d_b$ which is the compressive lap splice length as per # 13.2.4 of AS 3600. These conditions are shown in Figure 12.18.

The maximum concrete bearing stress given by #12.3 is, $\Phi 0.85f'_c \sqrt{\frac{A_2}{A_1}}$ but not greater than $\Phi 2f'_c$.

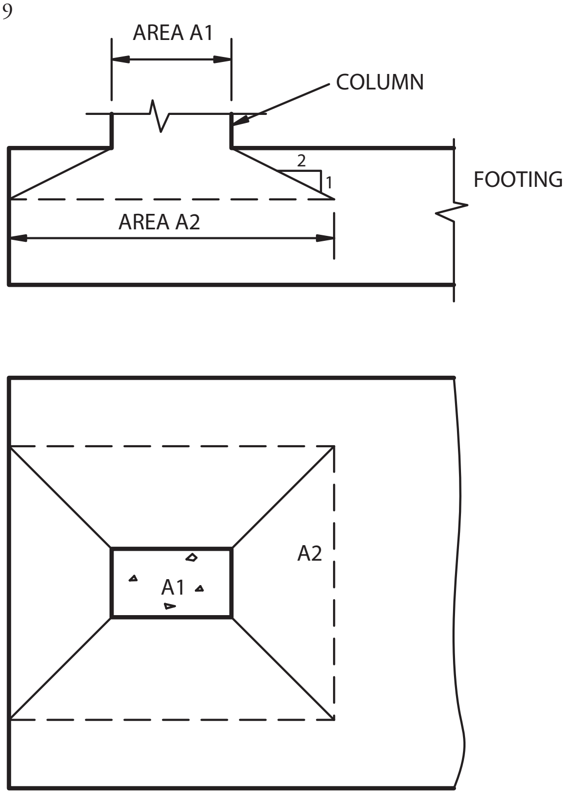
Where: $\Phi = 0.6$

A_1 = Contact bearing area and

A_2 = The largest concentric surface area at the base of a frustum of a pyramid whose sides slope 2 units transverserly for each longitudinal unit as shown in Figure 12.19.

In concentrically loaded pad footings it will be found that the minimum area of dowels only will be required for most cases unless the columns are extremely large columns carrying large loads. If the columns transmit moments to the footings, the contact bearing area at the column-footing interface is reduced to the depth of the neutral axis. Some of the dowels will thus be in tension requiring the tensile forces carried by the column reinforcement to be transfered to the footing entirely by dowel action.

Figure 12.19

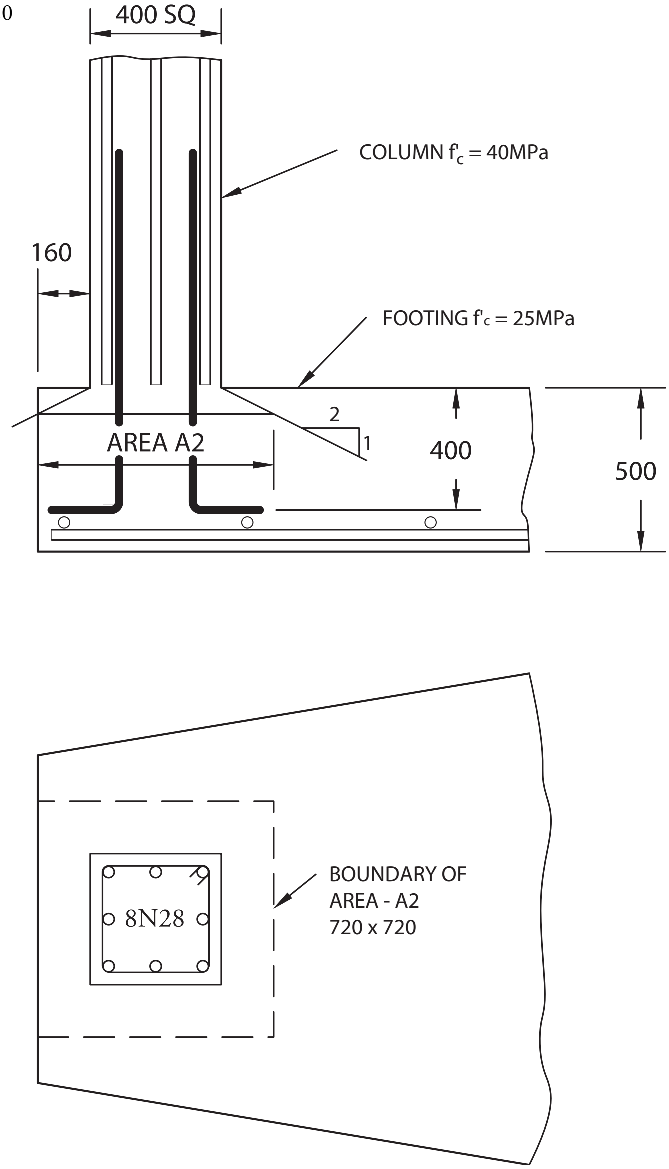


EXAMPLE 6

Figure 12.20 shows the outer column supported by a combined footing. The column uses N40 concrete and it is reinforced with 8N28 longitudinal bars. The footing uses N25 concrete.

Determine the size and number of dowels required to be placed in the footing to transmit the full potential column axial load capacity.

Figure 12.20



SOLUTION

Column

Steel area = $8 \times 620 = 4960$ mm^2
Load capacity of reinforcement = $4960 \times 400 \times 10^{-3} = 1984$ kN
Net area of concrete = $400 \times 400 - 4960 = 155.04 \times 10^3$ mm^2
Ultimate load capacity of concrete = $0.85 \times 40 \times 155.04 \times 10^3 * 10^{-3}$
= 5271 kN

Ultimate column load capacity = 1984 + 5271 = 7255 kN

The reduction factor for columns $\Phi = 0.6$

Effective column capacity = $0.6 \times 7255 = 4353$ kN

It will be learned in chapter 13 that the above value is the ΦN_{uo} value.

Footing

Maximum concrete bearing stress = $\Phi 0.85 \times f'_c \sqrt{\frac{A_2}{A_1}}$

$= 0.6 \times 0.85 \times 25 \sqrt{\frac{720}{400}}$

= 22.95 MPa

$< \Phi * 2f'_c$ (= 30 MPa)

Therefore concrete bearing capacity of footing,

$= 22.95 \times 400^2 \times 10^{-3}$

= 3672 kN

Compressive load to be carried by dowels,

= 4353 - 3672 = 681 kN

Area of dowels required = $\frac{681 \times 10^3}{500}$ = 1362 mm^2

Minimum area of dowels = $0.005 \times 400^2 = 800$ mm^2

< 1362 mm^2 calculated area

7N16 dowels give an area A_s = 1400 mm^2

Required compressive development length = $20 \times 16 = 320$ mm which is greater than the available depth in footing of 400 mm.

Splice length required above footing = $40 \times 16 = 640$ mm

PROBLEMS

QUESTION 1

A rectangular reinforced concrete column 300 mm by 600 mm supports a 360 kN dead load and a 580 kN live load. The supporting footing uses grade N25 concrete. Underground services make it necessary to restrict the footing dimension parallel to the 300 mm column dimension to 2400 mm.

(a) For a soil bearing value q_u = 200 kPa, determine the size of the footing required.

(b) Design the footing for a depth D = 450 mm.

QUESTION 2

An unreinforced concrete footing using grade N25 concrete is required for a 300 mm wide masonry wall transmitting a design load N^* = 280 kN/m. Determine the depth of the footing required if the factored soil bearing capacity q_u = 100 kPa.

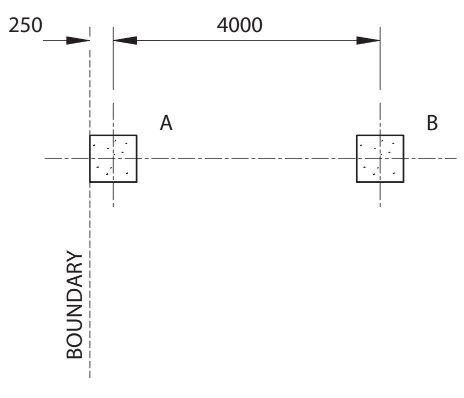
QUESTION 3

Figure shows the position of two 500 mm square columns. Column A supports a design load N^* = 800 kN and column B supports a design load N^* = 1450 kN. Determine the required footing dimensions for,

(a) a trapezoidal combined footing of length L = 5000 mm and,

(b) a rectangular combined footing.

The factored soil bearing capacity q_u = 400 kPa.



QUESTION 4

Determine the size of a strap footing required in question 3 if the length of the footing perpendicular to the boundary under column A is 1500 mm.

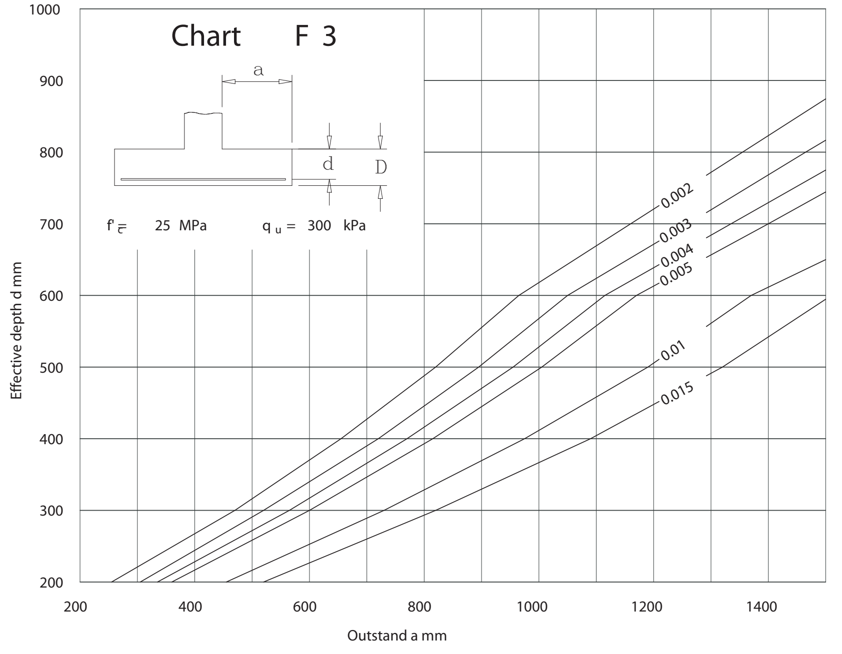


Chart F 3
a
d
D
f'c = 25 MPa
q u = 300 kPa
0.002
0.003
0.004
0.005
0.01
0.015
Effective depth d mm
Outstand a mm
1000
900
800
700
600
500
400
300
200
200
400
600
800
1000
1200
1400

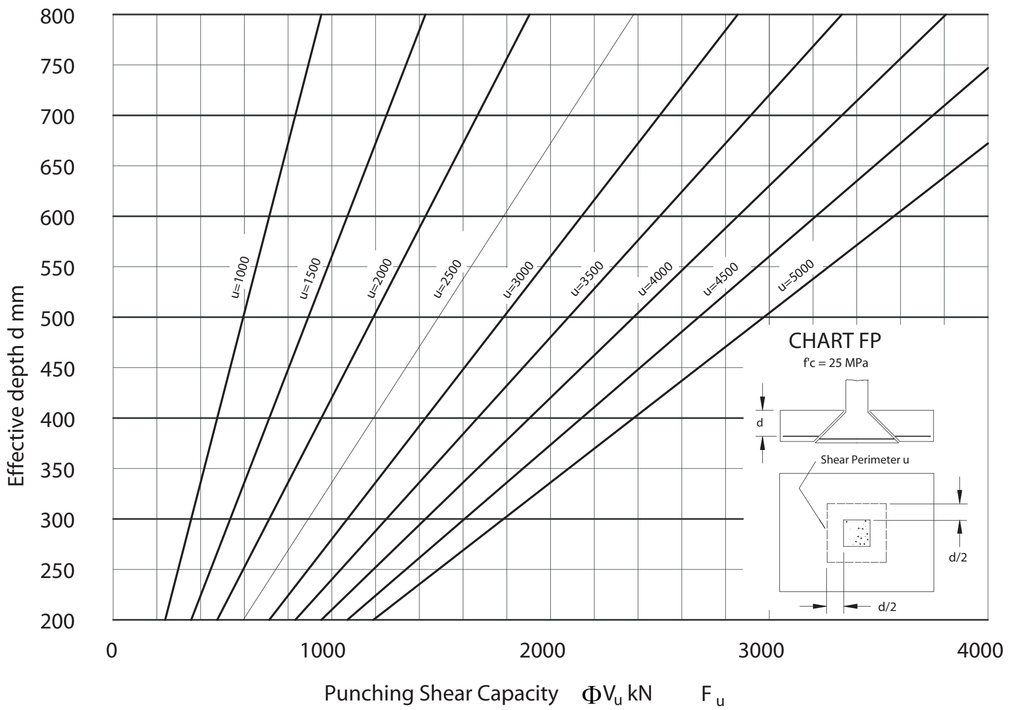


CHART FP
f'c = 25 MPa
d
Shear Perimeter u
d/2
d/2
u=1000
u=1500
u=2000
u=2500
u=3000
u=3500
u=4000
u=4500
u=5000
Effective depth d mm
Punching Shear Capacity ΦVu kN Fu
800
750
700
650
600
550
500
450
400
350
300
250
200
0
1000
2000
3000
4000

13

chapter

Columns

13.1 Additional Symbols used in this Chapter

A_g	=	The gross cross-sectional area of a member.
A_s	=	The cross-sectional area of the reinforcement = $A_{sc} + A_{st}$.
A_{sc}	=	Area of reinforcement on the compression side.
A_{st}	=	Area of reinforcement on the tension side.
b	=	Column width perpendicular to applied moment.
D	=	Column depth in direction of applied moment.
D_c	=	Smaller column dimension.
d	=	Effective depth measured to the resultant tensile force.
d_{sc}	=	Distance from extreme compression fibre to the centroid of the outer compression reinforcement.
e	=	Load eccentricity measured from plastic centroid.
e'	=	Load eccentricity measured from tensile reinforcement.
f_{sc}	=	Stress in compressive reinforcement.
p	=	Total steel ratio A_s/bD.
f_{sy}	=	Yield strength of steel reinforcement.
f_{st}	=	Stress in tensile reinforcement.
g	=	Ratio of distance between outer reinforcement to the overall depth of a column section.
k_{uo}	=	Ratio at ultimate strength of the depth of the NA from the extreme compressive fibre to d_o. Symbols k_u is applied for k_{uo} in this text.
k	=	Effective length multiplier.
k_m	=	End moment condition parameter.
L_e	=	Effective length of a column.
L_u	=	The unsupported length of a column, taken as the clear distance between faces of members capable of providing lateral support to the column.

M_u	=	The ultimate strength in bending at a cross-section of an eccentrically loaded compression member.
M_{ub}	=	The ultimate strength in bending when k_u = 0.545.
M_{ud}	=	The reduced ultimate strength in bending when k_u is reduced to 0.4.
N_c	=	The buckling load in a column.
N_u	=	The ultimate compressive strength combined with moment M_u.
N_{ub}	=	The ultimate compressive strength when k_u = 0.6.
N_{uo}	=	The ultimate strength of an axially loaded squat columns.
r	=	Radius of gyration.
β_d	=	Creep factor for sustained loading.
δ_b, δ_s	=	Moment magnifiers for braced and sway columns.
ε_{sc}	=	Strain in compressive reinforcement.
ε_{st}	=	Strain in tensile reinforcement.
ε_{sy}	=	Strain in reinforcement at point of yielding.

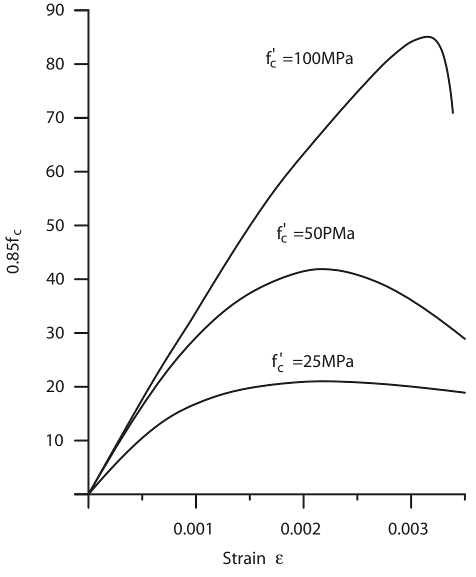
13.2 High Strength Concrete

For NSC (normal strength concrete) the stress-strain behaviour of concrete is well known and while there are a number of empirical formulae defining the stress-strain relationship, the one used by the authors is the CEB stress equation. The interesting part of stress-strain relationship of NSC is the fact that the maximum ultimate stress, f_{cu}, for all these grades occurs fairly consistently at a strain of ε_{cu} = 0.0022.

For HSC (high strength concrete) grades the same stress-strain relationship does not apply and the maximum ultimate stress, f_{cu}, occurs at higher strains ε_{cu}. Figure 13.1 below is a plot of stress-strain for 25, 50 and 100 MPa concretes. It may be observed that maximum ultimate stress for the 100 MPa concrete occurs at a strain which is in the region of 0.003 which is significantly higher then either the 25 or 50 MPa concretes. A further point of particular interest is the fact that the strain ε_{cu} of 0.0022 for NSC is less then the yield strain for steel ε_y = 0.0025. The figure may also be used to illustrate that for the HSC the stress strain curve is almost linear up to the maximum stress while for NSC the stress-strain relationship is distinctly curvilinear.

The shape of the stress-strain curve is also an indication of the "ductility" of concrete. This is shown by flatness of the curve following the peak stress. The plateau reached by the stress-strain curve is analogous to elesto-plastic material such as steel. The unloading part of the curve for the 25 MPa concrete is very flat, implying that there is an extended post peak loading region providing for extensive movement and rotation prior to failure. The flat plateau region of the curve will allow for redistribution of forces in a structural framework and provide for visual warning of potential failures. For the 100 MPa HSC, the unloading part of the curve is almost vertical which implies sudden failure as would be expected in a brittle material. The unloading part of the HSC can however be flattened with the introduction of effective lateral confinement of the column core using closely spaced steel ties or helical reinforcement. The HSC plot also illustrates why there is a strong argument for adopting equivalent triangular rather than rectangular stress block for the design of columns and beams.

Figure 13.1



The CEB-FIB equation used for normal strength concretes ($f_c^{'} \leq 50$ MPa) is given by equation 13.1 below.

$$f_c = \left(\frac{\left(a_1 - a_2 \varepsilon_c\right)\varepsilon_c}{1 + a_3 \varepsilon_c} \right) f_o \qquad \textbf{(13.1)}$$

Where: f_c = Concrete stress at strain ε_c

$a_1 = 39000(f_o + 7)^{-0.953}$

$a_2 = 206600$

$a_3 = 65600\,(f_o + 10)^{-1.085} - 850$

$f_0 = 0.85\, f'_c$

For high strength concretes ($f_c^{'} > 50$ MPa) the stress-strain equation is that developed by Collins, Mitchell and MacGregor[9] given by equation (13.2) below.

$$\frac{f_c}{f_c^{'}} = \frac{\varepsilon_c}{\varepsilon_c^{'}} \times \frac{n}{n - 1 + \left(\dfrac{\varepsilon_c}{\varepsilon_c^{'}} \right)^{nk}} \qquad \textbf{(13.2)}$$

Where:

$$k = 1.0 \qquad for \quad \frac{\varepsilon_c}{\varepsilon_c'} \le 1.0$$

$$k = 0.67 + \frac{f_c'}{62} \qquad for \quad \frac{\varepsilon_c}{\varepsilon_c'} > 1.0$$

$$n = 0.8 + \frac{f_c'}{17}$$

$$\varepsilon_c' = \frac{f_c'}{E_c}\frac{n}{n-1}$$

$E_c =$ Initial tangent modulus of elasticity of concrete. Note that this is not the E_c given in the AS3600 Code which is the secant modulus of elasticity of concrete.

$$E_c = 3320\sqrt{f_c'} + 6900$$

13.3 Column Strength Equations

The main function of a column is to transfer loads and moments from beams and slabs to the footings and foundations of a structure. While columns primarily carry compressive loads, they must also be able to carry moments. Even with the best intentions, axially loaded columns do not exist in practice. The Code #10.1.2 requires that columns be designed for a minimum eccentricity of 0.05*D*. The term beam-columns is a term frequently used to describe the axial load and moment actions transmitted by columns.

The primary longitudinal reinforcement serves to increase the compressive strength of a column and to provide the internal tensile force required to transmit moments. Considering initially a theoretical axially loaded short stocky reinforced concrete column shown in Figure 13.2 (*right*).

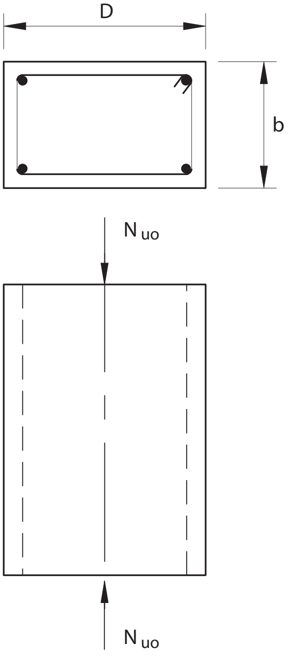


Figure 13.2

The ultimate axial load capacity N_{uo} (at zero moment) is achieved by the concrete reaching its ultimate strength of $0.85f'_c$ and the reinforcement reaching its yield strength f_{sy}. The ultimate load capacity is given by equation (a).

$$N_{uo} = 0.85f'_c bD + f_{sy} A_s \quad \text{(a)}$$

A maximum concrete stress of $0.85f'_c$ is used instead of f'_c to account for the size difference between a laboratory tested cylinder and the full size of a column. In the laboratory there is greater control to achieve uniformity of the 100 mm by 200 mm or 150 mm by 300 mm test cylinders while in an actual column, uniformity of loading, uniformity of construction and consistency of material properties is much more difficult to achieve even in a laboratory. There are other arguments which suggest that the multiplier 0.85 is made up two factors, one to account for the size difference as noted above and the other to account that spalling of concrete cover which occurs prior to failure.

bD is the gross area A_g of the column and A_s is the total area of reinforcement. A more precise estimate is obtained by using the net concrete area $(A_g - A_s)$ in the first term.

$$N_{uo} = 0.85f'_c(A_g - A_s) + f_{sy} A_s \qquad \text{(b)}$$

Equation (b) is conventionally written with the concrete stress $0.85f'_c$ subtracted from the yield stress in the second term. This is shown in equation 13.3 which may be obtained by collecting the like terms in equation (b).

$$\boxed{N_{uo} = 0.85f'_c A_g + (f_{sy} - 0.85f'_c)A_s \qquad \textbf{(13.3)}}$$

Equation 13.3 assumes that the reinforcement has yielded at the time of the concrete reaching its maximum stress of $0.85f'_c$. In discussing material properties in Section 13.2 it was shown that for normal strength concretes ($f'_c \leq 50\ MPa$) the maximum strength occurs at a fairly consistent concrete strain ε_c = 0.0022 which is less than the yield strain of the steel reinforcement ε_{sy} = 0.0025. Strictly speaking there is no guarantee that either material has reached its maximum capacity at the point of failure. The correct solution could be obtained by considering the combined axial load-strain (N - ε) diagram. Rewriting the general axial load equation in terms of the combined strain ε,

$$\mathrm{N} = \textit{(Concrete Stress from CEB - FIB Equation)}\,\mathrm{A_c} + \varepsilon \mathrm{E_s A_s} \qquad \text{(c)}$$

The maximum value of $N = N_{uo}$ may be obtained by either differentiating equation (c) with respect to the composite strain ε, solving for ε which will be the ultimate strain ε_{uo} corresponding to N_{uo} or by numerical iteration of equation (c) to obtain the maximum value of $N = N_{uo}$. One would expect that the solution obtained for ε_{uo} will lie between the value for maximum concrete strain for plain concrete, ε_c = 0.0022 and the yield strain of steel ε_{sy} = 0.0025. So what is all the fuss about you may ask.

There are a couple of simplified solutions for obtaining N_{uo}. The first is to just use equation 13.3, which assumes that both materials have reached their respective maximum capacities. This can be slightly unconservative but the error is only small. The second solution is to assume that both materials have reached a strain of 0.0025 which is the yield strain of steel. That is, it will be necessary to calculate the concrete stress f_c from the CEB-FIB equation for a strain ε_c = 0.0025. This is on the unloading part of the stress-strain curve so that the result for N_{uo} will be conservative. The following example will illustrate these considerations.

EXAMPLE 1

Determine the maximum axial load capacity of a 350 mm square, short column made using 40 MPa concrete and reinforced with four N32 bars.

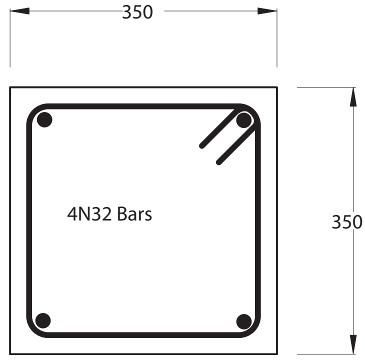


Figure 13.3

SOLUTION

(a) Simplified solution using equation 13.3

$$N_{uo} = \left[0.85\times 40\times 350\times 350 \; + \; 3200\times\left(500-0.85\times 40\right)\right]\times 10^{-3}$$

$$= 5{,}656 \text{ kNm}$$

(b) Assuming that the concrete strain $\varepsilon_c = \varepsilon_{sy} = 0.0025$ at the ultimate load,

Concrete stress, $$f_c = \left[\frac{\left(a_1 - a_2\times\varepsilon_c\right)\varepsilon_c}{1+a_3\times\varepsilon_c}\right]\times f_o$$

$$a_1 = 39000\left(0.85\times 40+7\right)^{-0.953}$$

$$= 1{,}133$$

$$a_2 = 206{,}600$$

$$a_3 = 65600\left(0.85\times 40+10\right)^{-1.085} - 850$$

$$= 231$$

$$f_c = \left[\frac{\left(1{,}133 - 206{,}600\times 0.0025\right)\times 0.0025}{1+231\times 0.0025}\right]\times 0.85\times 40$$

$$= 33.2 \text{ MPa}$$

Maximum ultimate axial load,

$$N_{uo} = \left[\left(350\times 350-3200\right)\times 33.2+3200\times 500\right]\times 10^{-3}$$

$$= 5{,}560 \text{ kN}$$

(c) The axial load at any strain ε of the composite stress-strain diagram is given by;

$$N = \left[\frac{\left(a_1 - a_2\varepsilon\right)\varepsilon}{1+a_3\varepsilon}\right] f_o A_c + \varepsilon E_s A_s = \left[\frac{a_1\varepsilon - a_2\varepsilon^2}{1+a_3\varepsilon}\right] f_o A_c + \varepsilon E_s A_s$$

Differentiating with respect to strain and equating to zero,

$$\frac{dN}{d\varepsilon} = \left[\frac{-a_2 a_3 \varepsilon^2 - 2a_2\varepsilon + a_1}{\left(1 + a_3\varepsilon\right)^2}\right] f_o A_c + E_s A_s = 0$$

Let $k = \dfrac{E_s A_s}{f_o A_c}$

Solving for ε,

$$\left(-a_2 a_3 \varepsilon^2 - 2a_2\varepsilon + a_1\right) + k\left(1 + a_3\varepsilon\right)^2 = 0$$

$$\left(-a_2 a_3 \varepsilon^2 - 2a_2\varepsilon + a_1\right) + k\left(1 + 2a_3\varepsilon + a_3^2\varepsilon^2\right) = 0$$

$$\left(ka_3^2 - a_2 a_3\right)\varepsilon^2 + \left(2ka_3 - 2a_2\right)\varepsilon + a_1 + k = 0$$

From earlier solution,

$$a_1 = 1{,}133 \quad a_2 = 206{,}600 \quad a_3 = 231 \text{ and } k = \frac{200{,}000 \times 3{,}200}{0.85 \times 40 \times \left(350^2 - 3200\right)} = 158$$

Substitute constants and solve quadratic for ε,

$$39.29\varepsilon^2 + 0.34\varepsilon - 0.0013 = 0$$

$$\varepsilon = 0.00287 \quad > \varepsilon_{sy}$$

That is, the reinforcement has yielded and the maximum ultimate axial compressive load, N_{u0} is achieved at the steel yield strain of 0.0025. The result for N_{u0} = 5,560 kN is the same as that achieved for (b). A plot of the individual material axial loads and the combined axial load is shown in Figure 13.4 (*right*).

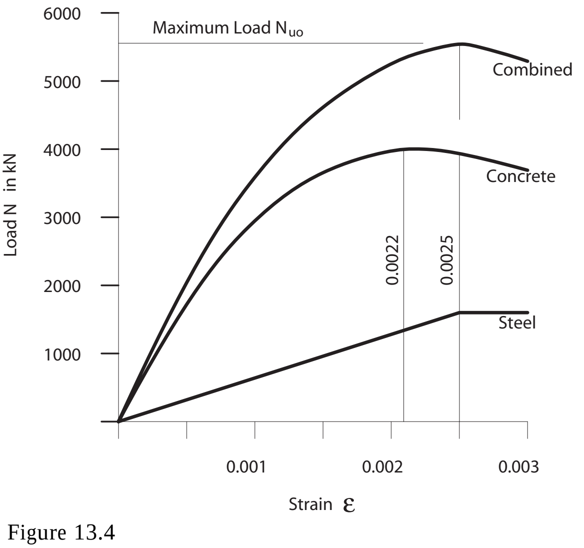


Figure 13.4

The results for the three solutions were:

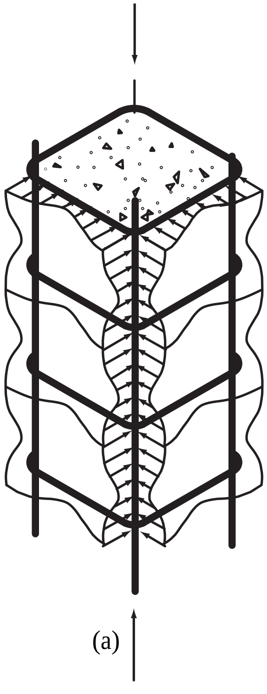
N_{uo} = 5,656 kN Simplest solution overestimates by 2.8%
N_{uo} = 5,560 kN Equals exact solution
N_{uo} = 5,560 kN Exact solution

As can be seen by the above results, the differences between the exact solution and the simplest solution given by equation 13.1 is small and within acceptable orders of accuracy. The simplest solution (a) is very attractive and is one which could be applied to a manual design. The column design charts developed at the end of this book have however been drawn using exact solutions.

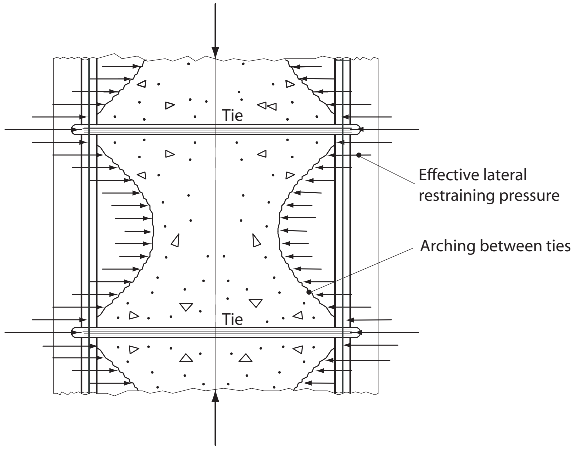
13.4 Constraining Effects of Transverse Reinforcement

Transverse reinforcement in the form of rectangular ties, circular ties and helical reinforcement required by the Code makes up a steel cage which can be fabricated off site. The transverse reinforcement restrains the longitudinal reinforcement and most importantly it acts to prevent the longitudinal bars from buckling prematurely. The transverse reinforcement will also act as lateral constraint to the concrete which has the potential of increasing the maximum concrete strength of the core contained within the transverse reinforcement by as much as 50% depending on the yield strength, amount and spacing of the transverse reinforcement. Figure 13.5(a) below illustrates the concept of constraining effects of ties in a square column.

Figure 13.5



(a)

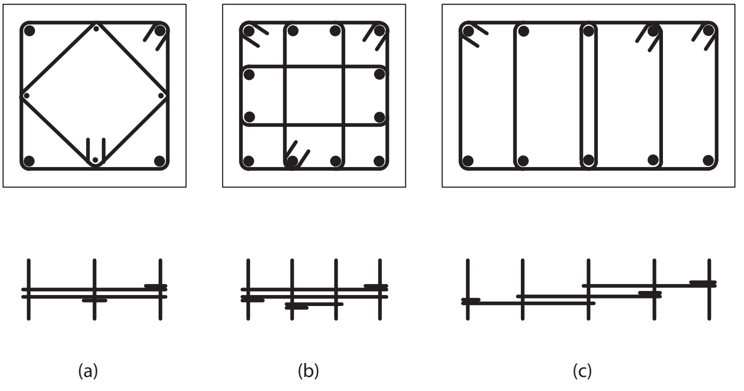


(b)

An added complication is the arching effect in between the ties shown in Figure 13.5(b). The overall implication of the transverse reinforcement as far as constraining the core is that the constraining stresses are greatest adjacent to the ties and in line with the return legs. The magnitude of the constraining stresses reduce non linearly with the reduction of the effective core as shown above. Some researches have developed empirical formulae for effective constraining stresses applied uniformly to the full core. Designing of columns for constraining effects of transverse reinforcement is still a very complex procedure and it may be some time before practical design formulae are developed. Never the less, ensuring that columns behave in a ductile manner is essential in earthquake prone areas. Ductility is achieved with the inclusion of transverse reinforcement specified in terms of minimum spacing or more appropriately in terms of volume of transverse reinforcement per unit length.

It may be readily appreciated that single closed square or rectangular ties may be effective restraints to the longitudinal reinforcement but they do not provide very effective constraint to the concrete core. On the other hand, closely spaced helical reinforcement will create a very effective constraint to the concrete core approaching that of a tubular casing. It has been shown that square or rectangular ties can also be effective provided that there are multiple ties placed to maximise constraining effects. Some examples of effective tie arrangements are shown below.

Figure 13.6



Recently there have been numerous publications of research projects which develop the quantity or volume of transverse reinforcement and the stress-strain relationships of constrained concrete cores to be used in design. In such cases, the concrete cover is ignored as it spalls off prior to the maximum load being reached by the core as the lateral reinforcement becomes effective in constraining the concrete core. The Code makes no provision for the design of transverse reinforcement which is effective in increasing the compressive strength of the concrete core.

A designer may choose to take advantage of the higher strengths available with high strength concretes combined with further increases in concrete strengths achieved by effective lateral constraint provided by transverse reinforcement. Use of say 100 MPa concrete could produce a maximum compressive strength of up to 150 MPa in the core leading to very ductile and efficient space saving columns. There are however some disadvantages:

(a) designs are much more complex,

(b) there are no current Code design guides so the designer will need to be very competent and up to date with latest research,

(c) spacing of the transverse reinforcement can be very close leading to construction difficulties such as placing and compacting concrete and providing continuity of longitudinal reinforcement at column, beam and slab connections,

(d) increased costs in construction and supervision,

(e) high strength concretes are much more impervious with finer discontinuous water channels which can lead to explosive failures at high temperatures.

The choice to the designer is:

(a) Use the full concrete section but ignore potential strength benefits of the constraining effects of transverse reinforcement.

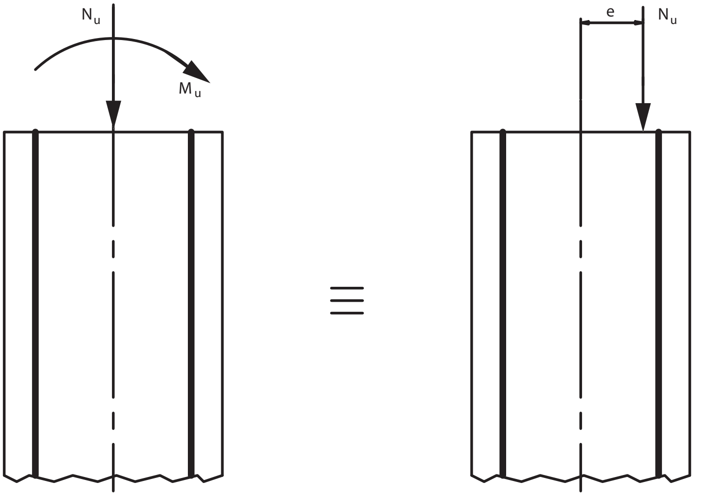
(b) Design the column as a section made up of the core contained within the transverse reinforcement and ignore all concrete cover. The transverse reinforcement will need to be designed as an effective "sleeve" which is mobilised with the development of hoop tension during loading to restrain the core and increase the stress-strain behaviour of the core. There have been a number of research papers published recommending mathematical models for stress-strain relationship of constrained concrete columns.

13.5 Combined Moment and Axial Load Condition

Figure 13.7 shows a column section subjected to an ultimate axial load N_u and an ultimate moment M_u which is the same as an eccentric load N_u applied at an eccentricity e to produce the same moment, i.e. $N_u e = M_u$. The eccentricity e is measured from the geometric centroid of a symmetric column which coincides with the plastic centroid. In an unsymmetric column section it is necessary to calculate the position of the plastic centroid defined as the point in the cross-section through which the ultimate load N_{uo} (given by equation 13.3) would need to be applied to cause simultaneous uniform crushing of the concrete and yielding of all the reinforcement. Since this chapter is primarily concerned with symmetric sections, eccentricity of loading will be measured from the geometric centroid.

As a matter of convenience the reinforcement furthest from the eccentric load is called the tensile reinforcement A_{st} (even though it may be in compression at small eccentricities) while the reinforcement on the load side is referred to as the compressive reinforcement A_{sc}. When a column is reinforced on four sides the distinction is not quite so evident.

Figure 13.7



13.6 Strength Interaction Diagrams

It may be readily appreciated from Figure 13.7 that there is an infinite combination of loads N_u and moments M_u (or loads N_u and eccentricities e) defining the strength of a given column section. A plot of ultimate loads and moments which would just cause a section to fail is typically shown in Figure 13.8 and it is called a load-moment interaction diagram.

There are a number of salient points on the interaction diagram which should be noted and are explained below with the aid of a strain diagram.

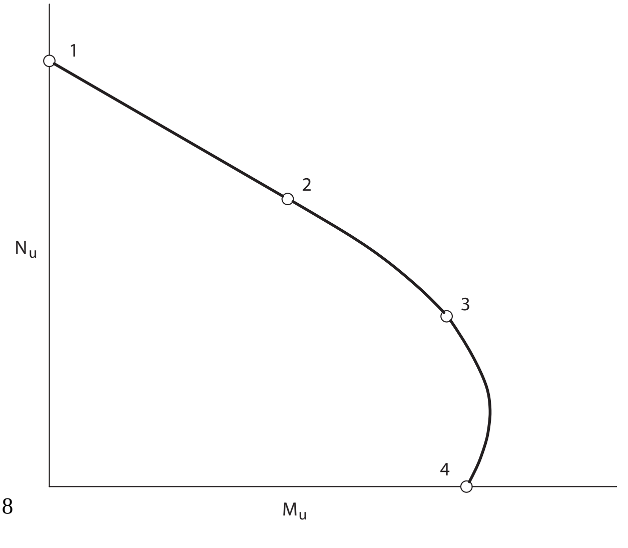
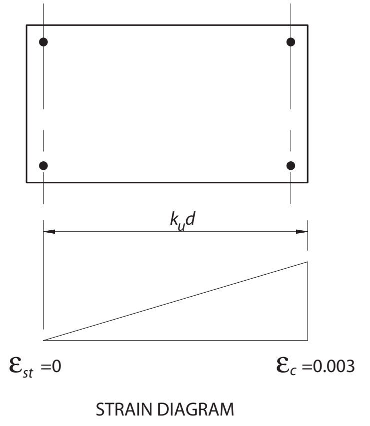


Figure 13.8

Point 1 - This is the ultimate strength N_{uo} at zero eccentricity given by equation 13.1.

Point 2 At this point the eccentricity is such that the N.A. coincides with the tensile reinforcement i.e. $k_u = 1.0$.

Figure 13.9

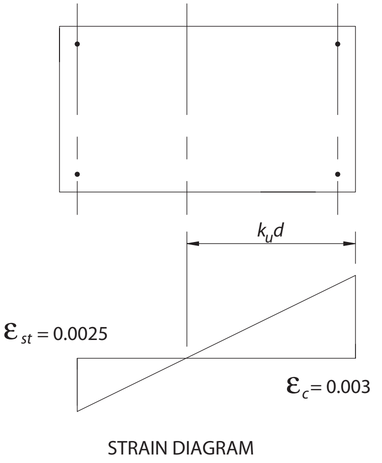


Between point 1 and 2, $k_u > 1.0$ so that the tensile reinforcement will be in compression.

Point 3 The position of the neutral axis is such that $k_u = 0.545$. For a column reinforced on two faces only, this would be referred to as the balance point at which the load capacity is N_{ub} and the moment capacity is M_{ub}. At this point the tensile reinforcement has just reached its yield stress f_{sy} and the eccentricity is denoted as e_b. For columns reinforced on four faces, the strength values are still designated as N_{ub} and M_{ub} obtained for $k_u = 0.545$.

Between 2 and 3 the stress in the tensile reinforcement f_{st} is less than the yield stress f_{sy}.

Figure 13.10



Point 4 This is the strength of the section in pure bending. It is calculated as the strength of a doubly reinforced beam for which the value of k_u is not known and it must be calculated iteratively.

13.7 Strength Equations - Columns Reinforced on Two Faces

The Code #10.6.2 permits the use of the simplified rectangular stress block previously used for beams. The strain is assumed to vary linearly from a maximum value of 0.003 when the neutral axis falls within the column section. Figure 13.11 illustrates the external and internal forces and stresses on a column section reinforced on two faces with applied eccentricity of loading so that $k_u \leq 1.0$.

Referring to Figure 13.10, the resultant internal forces in the concrete C_C, the compressive reinforcement C_S and the tensile reinforcement T, may be determined from:

$$C_C = 0.85f'_C b\gamma k_u d$$

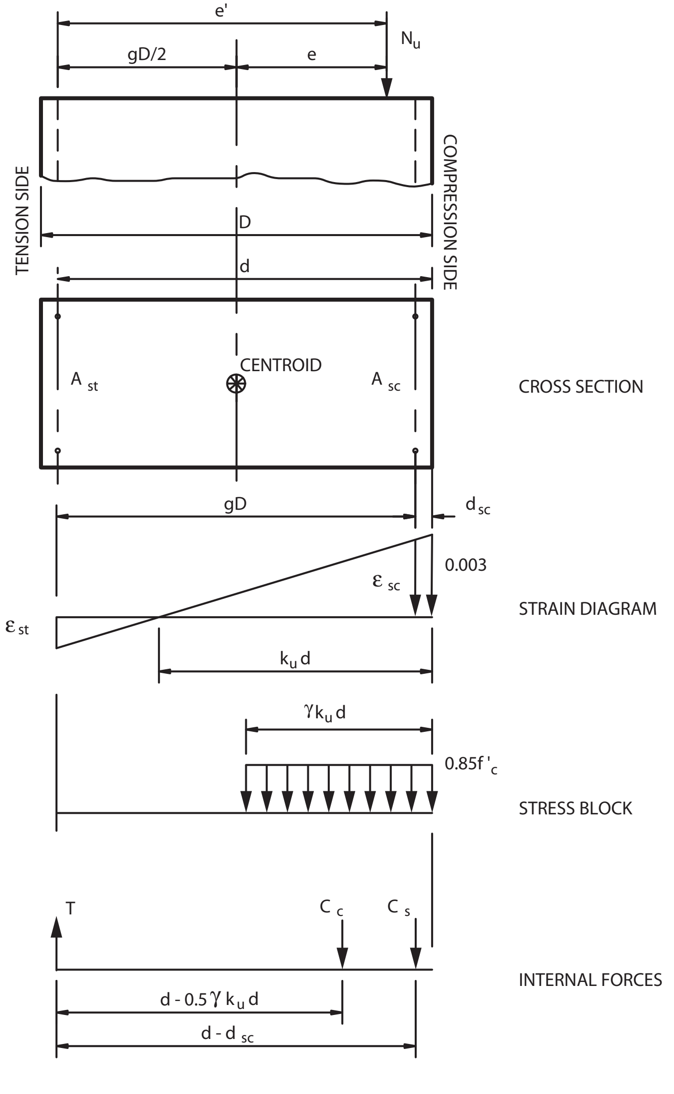
$$C_S = A_{SC} f'_{SC}$$

Allowing for the concrete displaced by the reinforcement,

$$C_S = A_{SC}(f_{SC} - 0.85f'_C)$$

$$T = A_{st}\, f_{st}$$

Figure 13.11



The compressive and the tensile reinforcement may or may not have yielded. It is necessary to calculate strains from the strain diagram.

$$\varepsilon_{sc} = 0.003 \frac{k_u d - d_{sc}}{k_u d}$$

$$f_{sc} = E_s \varepsilon_{sc} = 2 \times 10^5 \times \varepsilon_{sc} \leq f_{sy}$$

$$\varepsilon_{st} = 0.003 \frac{d - k_u d}{k_u d} = 0.003 \frac{1 - k_u}{k_u}$$

$$f_{st} = 2 \times 10^5 \varepsilon_{st} \leq f_{sy}$$

From equilibrium conditions, $N_u = C_c + C_s - T$

$$N_u = 0.85 f'_c b \gamma k_u d + A_{sc}(f_{sc} - 0.85 f'_c) - A_{st} f_{st} \qquad \textbf{(13.4)}$$

Taking moments about the tensile reinforcement,

$$N_u e' = C_c(d - 0.5\gamma k_u d) + C_s(d - d_{sc})$$

Substituting for C_c and C_s gives;

$$N_u e' = 0.85 f'_c b \gamma k_u d^2 (1 - 0.5\gamma k_u) + A_{sc}(f_{sc} - 0.85 f'_c)(d - d_{sc}) \qquad \textbf{(13.5)}$$

Dividing equation 13.3 by 13.2 gives the eccentricity e',

$$e' = \frac{N_u e'}{N_u} \qquad \textbf{(13.6)}$$

The eccentricity of loading from the column centroid,

$$e = e' - \frac{gD}{2} \qquad \textbf{(13.7)}$$

The column moment capacity,

$$M_u = N_u e \qquad \textbf{(13.8)}$$

Equations 13.4, 13.5, 13.6, 13.7 and 13.8 can be applied to determine the load N_u and moment M_u capacity for any value of $k_u \leq 1.0$ of a given column. The results may be used to plot an interaction diagram. By varying the amount of reinforcement, a series of interaction diagrams can be obtained for the one column size.

The above equations are valid for columns reinforced on two faces with $k_u \leq 1.0$ and a maximum concrete strain $\varepsilon_c = 0.003$. When $k_u > 1.0$, the maximum concrete strain is varied between 0.003 for $k_u = 1.0$ (point 2 on the interaction diagram) and 0.002 when k_u is equal to infinity, that is, the zero moment condition (point 1 on the interaction diagram). The interaction diagram between points 1 and 2 is drawn by a connecting straight line.

EXAMPLE 2

Determine the ultimate load and moment capacity of the column section shown in Figure 13.12 for the points shown in Figure 13.8 and sketch the load-moment interaction diagram.

SOLUTION

Point 1 zero moment condition. Applying equation 13.3,

$$N_{uo} = [0.85*25 \times 400 \times 500 + 4960 \times (500 - 0.85 \times 25)] \times 10^{-3}$$
$$= 6625.6 \text{ kN}$$

Point 2 $k_u = 1.0$, $k_u d = 425$ and the tensile stress $f_{st} = 0$.

Data: $f'_c = 25$ MPa

$b = 400$
$D = 500$
$d_{sc} = 75$
$gD = 350$
$d = 425$
$\gamma = 0.85$
$A_{sc} = A_{st} = 2480 \text{ mm}^2$

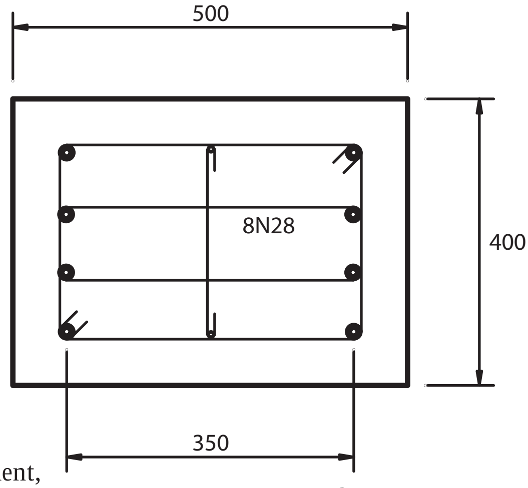


Figure 13.12

The strain in the compressive reinforcement,

$$\varepsilon_{sc} = 0.003 \frac{425 - 75}{425} = 0.00247 < \varepsilon_{sy}$$

The compressive stress,

$$f_{sc} = 2 \times 10^5 \times 0.00247 = 494 \text{ MPa} < f_{sy} \ (= 500 \text{ MPa})$$

From equation 13.2, the ultimate load capacity,

$$N_u = 0.85 \times 25 \times 400 \times 0.85 \times 1 \times 425 + 2480 \times (494\text{-}0.85 \times 25) - 2480 \times 0$$
$$= 4.243 \times 10^6 \text{ N or } 4243 \text{ kN}$$

Evaluating equation 13.3,

$$N_u e' = 0.85 \times 25 \times 400 \times 0.85 \times 1 \times 425^2(1\text{- } 0.5 \times 0.85 \times 1) + 2480 \times (494\text{-}0.85 \times 25)(425\text{-}75)$$
$$= 1.161 \times 10^9 \text{ Nmm or } 1.161 \times 10^6 \text{ kNmm}$$

$$e' = \frac{1.161 \times 10^6}{4243} = 274 mm$$

Eccentricity measured from the column centroid,

$$e = e' - \frac{gD}{2} = 274 - 175 = 99mm$$

Ultimate Moment capacity,

$$M_u = 4243 \times 99 \times 10^{-3} = 420 \text{ kNm}$$

Point 3 k_u = 0.545, this is the balance point when $f_{st} = f_{sy}$.

Strain in compressive reinforcement,

$$\varepsilon_{sc} = 0.003\left(\frac{0.545 \times 425 - 75}{0.545 \times 425}\right) = 0.00203 < \varepsilon_{sy}$$

$$f_{sc} = 0.00203 \times 2^5 = 406MPa$$

Following the procedure used for point 2 gives the following results:

$$N_u = 0.85 \times 25 \times 400 \times 0.85 \times 0.545 \times 425 + 2480 \times (406\text{-}0.85 \times 25) - 2480 \times 500$$

$$= 1388 \text{ kN}$$

$N_u e'$ = 0.85 × 25 × 400 × 0.85 × 0.545 × 425^2 (1-0.5 × 0.85 × 0.545) + 2480 × (406-0.85 × 25) × (425-75)

$$= 880457 \text{ kNmm}$$

$$e' = \frac{880457}{1388} = 634mm$$

$$e = 634 - 175 = 459 \text{ mm}$$

$$M_u = 1388 \times 0.459 = 637 \text{ kNm}$$

Point 4 This is the pure moment condition for a doubly reinforced beam. Since the position of the neutral axis i.e. the value of k_u is unknown, it will be necessary to estimate the value of k_u and continue to refine the estimate. The correct value of k_u is obtained when the internal forces are in equilibrium. When k_u was 0.545 the resultant internal force was a sizeable 1388 kN.

Try k_u = 0.2 and calculate N_u from equation 13.2.

$$\varepsilon_{sc} = 0.003\left(\frac{0.2 * 425 - 75}{0.2 * 425}\right) = 0.000353$$

$$< \varepsilon_y (= 0.0025)$$

$$f_{sc} = 2 \times *10^5 \times 0.000353 = 70.6 \text{ MPa}$$

$$\varepsilon_{st} = 0.003\frac{1 - 0.2}{0.2}$$

$$= 0.012 > 0.0025 \text{ hence } f_{st} = 500 \text{ MPa.}$$

Considering the strain equation, the tensile reinforcement will yield for values of

$k_u \leq 0.545.$

N_u = [0.85*25*400*0.85*0.2*425 + 2480*(70.6-0.85*25) - 2480 × 500] × 10^{-3}

= - 503 kN

Since the value of N_u has changed sign in going from k_u = 0.545 to k_u = 0.2, try k_u slightly greater say 0.24.

$$\varepsilon_{sc} = 0.003 \frac{0.24 \times 425 - 75}{0.24 \times 425} = 0.000794$$

f_{sc} = 158.8 MPa

N_u = [0.85 × 25 × 400 × 0.85 × 0.24 × 425 + 2480 × (158.8-0.85 × 25) - 2480 × 500] × 10^{-3}

= -162 kN

Further iteration gives the following:

For k_u = 0.262, f_{sc} = 195.9 MPa,

N_u = [0.85 × 25 × 400 × 0.85 × 0.262 × 425 + 2480 × (195.9-0.85 × 25) - 2480 × 500] × 10^{-3}

= 804.5 + 433.1 – 1240

= -2.4 kN which is near enough to zero

Taking moments about the tensile reinforcement, the moment capacity may be calculated,

M_u = 804.5 × (425-0.5 × 0.85 × 0.262 × 425) × 10^{-3} + 433.1 × (425-75) × 10^{-3}
= 303.8 + 151.6 = 455.4 kNm

The load-moment interaction diagram for the column section is plotted in Figure 13.13.

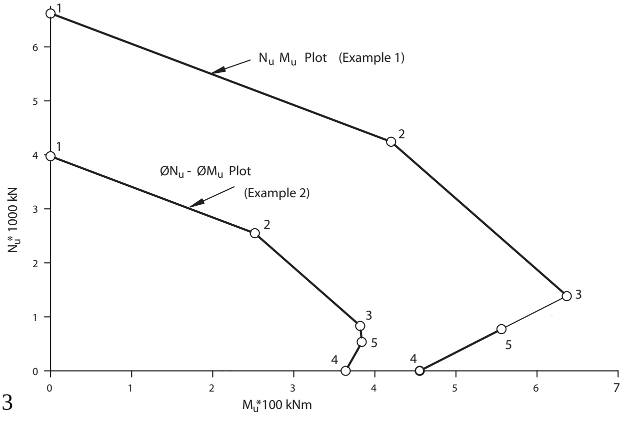


Figure 13.13

13.8 Reduction Factor Φ

The reduction factor for beam-columns given in the Code lies between 0.6 and 0.8 depending on the moment influence. Point 3 in the interaction diagram was referred to earlier as the balance point. At the balance point the value of k_u = 0.545 and the outer layer of tensile reinforcement has reached its yield strength. Between point 1 and point 3, the tensile reinforcement has not yielded and the column is said to exhibit a primary compression mode of failure. Columns are the most important elements in a structure. A beam failure in a structure is very undesirable but the failure tends to be localised. A column failure can be catastrophic by comparison. The reduction factor for primary compression failure is given in the Code as Φ = 0.6.

At point 4 in the interaction diagram, the member is in pure bending for which the usual reduction factor Φ = 0.8 is applied. Column sections in pure flexure are frequently over-reinforced. Code #8.1.3 specifies that for beams with the neutral axis parameter k_u > 0.4, the design strength for bending shall be taken as ΦM_{ud} where M_{ud} is the ultimate strength in bending for a cross-section with k_u = 0.4 and the tensile force has been reduced to balance the reduced compressive force. This simply means that the Code will not permit beam strength greater than that for which k_u = 0.4 to be used while the reduction factor Φ =0.8 is applied. If M_u is the actual ultimate strength of an overreinforced beam, the real or effective reduction factor is,

$$\Phi_o = \frac{\Phi M_{ud}}{M_{uo}} = 0.8\frac{M_{ud}}{M_{uo}} \geq 0.6$$

Between points 3 and 4, the reduction factor will vary from 0.6 to 0.8 or Φ_0 (whichever is the lesser) and its value is interpolated between these points using the following equations.

When $k_u \leq 0.4$ and $N_u < N_{ub}$ $\qquad \Phi = 0.6 + 0.2\left(1 - \frac{N_u}{N_{ub}}\right)$

When $k_u > 0.4$ and $N_u < N_{ub}$ $\qquad \Phi = 0.6 + (\Phi_0 - 0.6)\left(1 - \frac{N_u}{N_{ub}}\right)$

The effect of the continuous change in the reduction factor is to cause a discontinuity in the interaction diagram at the balance point with increased values in moment capacities below the balance point.

EXAMPLE 3

Calculate the reduced ΦN_u and ΦM_u for the column section in example 2 and replot the reduced interaction diagram on Figure 13.13.

SOLUTION

Point 1 $\quad \Phi N_{uo}$ = 0.6*6625.6 = 3975 kN

Point 2 $\quad \Phi N_u$ = 0.6*4243 = 2546 kN

ΦM_u = 0.6*420 = 252 kNm

Point 3 $\Phi N_{ub} = 0.6*1388 = 833$ kN

$\Phi M_{ub} = 0.6*637 = 382$ kNm

Point 4 Since the section in pure bending is not over- reinforced $k_u = 0.262 < 0.4$ the reduction factor $\Phi = 0.8$ as for beams.

$\Phi M_u = 0.8 \times 455.4 = 364$ kNm

Point 5 It is worth while obtaining an additional point between $N_u = 0$ and N_{ub} for the exercise of calculating the reduction factor Φ and to obtain a better plot of the interaction curve. Determine the co-ordinates for say a point with $k_u = 0.4$ which is about mid-way between k_{ub} and k_u for M_{uo}.

$$k_u d = 0.4 \times 425 = 170$$

$$f_{sc} = 0.001676 \times 2 \times 10^5 = 335.3 \text{ MPa}$$

$$N_u = [0.85 \times 25 \times 400 \times 0.85 \times 170 + 2480(335.3 - 0.85 \times 25) - 2480 \times 500]*10^{-3}$$

$$= 1228.2 + 778.8 - 1240$$

$$= 767 \text{ kN}$$

$$N_u e' = 1228.2 \times (425 - 0.5 \times 0.85 \times 170) + 778.8 \times (425 - 75)$$

$$= 705850 \text{ kNmm}$$

$$e' = \frac{705850}{767} = 920 \text{ mm}$$

$$e = 920 - 175 = 745 \text{ mm}$$

$$M_u = 767 \times 745 \times 10^{-3}$$

$$= 571 \text{ kNm}$$

Since $k_u < 0.4$,

$$\Phi = 0.6 + 0.2 * (1 - \frac{N_u}{N_{ub}})$$

$$= 0.6 + 0.2 * (1 - \frac{767}{1388}) = 0.69$$

The value of Φ could have been anticipated since the equation for Φ (when $k_u \le 0.4$) is a linear variation between 0.6 and 0.8.

The load and moment capacity is now calculated.

$$\Phi N_u = 0.69 \times 767 = 529 \text{ kN}$$

$$\Phi M_u = 0.69 \times 571 = 394 \text{ kNm}$$

EXAMPLE 4

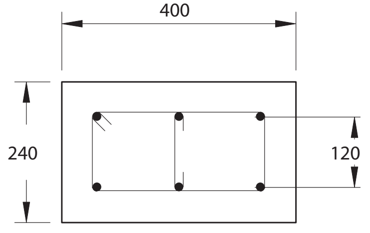
Figure 13.14 shows the cross-section of a column reinforced with 6N32 bars. The column uses grade N32 concrete and it is reinforced to carry moments about the weak axis.

(a) Calculate the ultimate load N_u and ultimate moment M_u for the balance condition when $k_u = 0.545$ and for pure moment condition.

(b) Calculate the value of Φ_o.

(c) Determine the load and moment capacities ΦN_u and ΦM_u between $N_u = 0$ and N_{ub} when $k_u = 0.52$.

Figure 13.14



SOLUTION

Data: $b = 400$ $D = 240$ $d = 180$ $d_{sc} = 60$ $f'_c = 32$ MPa

$\Upsilon = 0.822$ $A_{st} = A_{sc} = 2400$ mm^2

(a) Balance point for $k_u = 0.545$

$k_u d = 0.545 \times 180 = 98.1$ mm

$f_{st} = 500$ MPa

$$e_{sc} = 0.003 \frac{98.1 - 60}{98.1} = 0.001165$$

$f_{sc} = 0.001165 \times 2 \times 10^5 = 233$ MPa

Ultimate load for balance conditions,

$$N_{ub} = [0.85 \times 32 \times 400 \times 0.822 \times 98.1 + 2400 \times (233 - 0.85 \times 32) - 2400 \times 500] \times 10^{-3}$$
$$= 877.3 + 493.9 - 1200$$
$$= 171.2 \text{ kN}$$

$$N_{ub}e' = 877.3 \times (180 - 0.5 \times 0.822 \times 98.1) + 493.9 \times (180 - 60)$$
$$= 181810 \text{ kNmm}$$

$$e' = \frac{181810}{171.2} = 1062 \text{ mm}$$

$$e = 1062 - 60 = 1002 \text{ mm}$$

Hence ultimate moment for balance conditions,

$$M_{ub} = 171.2 \times 1002 \times 10^{-3}$$
$$= 171.5 \text{ kNm}$$

N_u = 0 condition

The value of k_u must be determined by iteration until equation 13.4 gives N_u = 0.

For k_u = 0.495:

$k_u d$ = 0.495 × 180 = 89.1 mm

$$f_{sc} = 0.003 \times \frac{89.1 - 60}{89.1} \times 2 \times 10^5$$

= 196 MPa

$$N_u = \left[0.85 \times 32 \times 400 \times 0.822 \times 89.1 + 2400(196 - 0.85 \times 32) - 2400 \times 500\right] \times 10^{-3}$$

= 796.9 + 405.1 – 1200

= 2.0 kN which is close to zero

Ultimate moment,

$$M_u = \left[796.9 \times (180 - 0.5 \times 0.822 \times 89.1) + 405.1 \times (180 - 60)\right] \times 10^{-3}$$

$$= 162.9 kNm$$

(b) M_{ud} condition for k_u = 0.4

Since the value of k_u for $N_u = 0$ is greater than 0.4, it will be necessary to calculate M_{ud} for k_u = 0.4 so that the reduction factor Φ_0 may be determined.

$k_u d$ = 0.4 × 180 = 72 mm

$$f_{sc} = 0.003 \times \frac{72 - 60}{72} \times 2 \times 10^5 = 100 \text{ MPa}$$

Taking moments about the tensile reinforcement,

$$M_{ud} = [\,0.85 \times 32 \times 400 \times 0.822 \times 72(180 - 0.5 \times 0.822 \times 72)$$
$$+ 2400 \times (100 - 0.85 \times 32) \times (180 - 60)\,] \times 10^{-6}$$
$$= 117.8 \text{ kNm}$$

$$\Phi_0 = 0.8 \frac{M_{ud}}{M_u} = 0.8 \times \frac{117.8}{162.9} = 0.579$$

< 0.6 hence use 0.6

Moment capacity at N_u = 0

ΦM_u = 0.6 × 162.9 = 97.7 kNm

(c) For the intermediate point between the balance point (k_u = 0.545) and pure moment condition (k_u = 0.495) of k_u = 0.52

$k_u d$ = 0.52 × 180 = 93.6 mm

$$f_{sc} = 0.003 \times \frac{93.6-60}{93.6} \times 2 \times 10^5 = 215.4\,MPa$$

$$N_u = \left[\left(0.85 \times 32 \times 400 \times 0.822 \times 93.6\right) + 2400\left(215.4 - 0.85 \times 32\right) - 2400 \times 500\right]$$

$$= 837.1 + 451.7 - 1200$$

$$= 88.8\,kN$$

$$N_u e' = 837.1(425 - 0.5 \times 0.822 \times 93.6) + 451.7(180 - 60)$$

$$= 172679 \text{ kNmm}$$

$$e' = \frac{172679}{88.8} = 1945 \text{ mm}$$

$$e = 1945 - 60 = 1885 \text{ mm}$$

$$M_u = 88.8 \times 1885 \times 10^{-3}$$

$$= 167.4 \text{ kNm}$$

Since $k_u > 0.4$ and N_u is less than N_{ub}, the reduction factor Φ is calculated from:

$$= 0.6 + (\Phi_o - 0.6)\left(1 - \frac{N_u}{N_{ub}}\right) = 0.6 \quad \text{since } \Phi_0 = 0.6$$

Hence the design moment capacity,

$$\Phi M_u = 0.6 \times 167.4 = 100.4 \text{ kNm}$$

And the design load capacity,

$$\Phi N_u = 0.6 \times 88.8 = 54.2 \text{ kN.}$$

13.9 Column Design

If column interaction charts are not available, the design procedure would be to select a column size and the reinforcement. For the chosen column, the moment capacity ΦM_u is calculated when the load capacity $\Phi N_u = N^*$.

Example 5

Choose a reinforced concrete column section to carry a design load $N^* = 1400$ kN and design moment $M^* = 250$ kNm. Use N25 concrete and assume exposure A2.

Solution

Choosing the column section is just a matter of experience, there are no hard and fast rules although aesthetics frequently dictate member sizes. For this example, a 450 deep and 350 wide column section reinforced with 6N32 bars has been chosen as shown in Figure 13.15.

Data: b = 350 mm

A_{st} = A_{sc} = 2400 mm^2

d_{sc} = 60 mm

M^* = 250 kNm

D = 450 mm

f'_c = 25 MPa

Exposure A2

d = 390 mm

N^* = 1400 kN

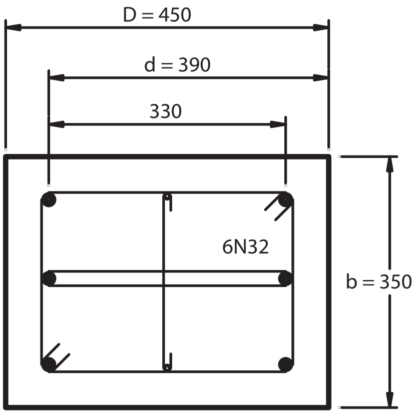


Figure 13.15

The procedure adopted here is to determine the neutral axis parameter k_u which will produce a load capacity ΦN_u equal to the design load N^*. Once the neutral axis parameter k_u is determined by trial and error from equation 13.4, the eccentricity and the moment capacity can be calculated from equations 13.3 to 13.6.

Try k_u = 0.6, $k_u d = 0.6 \times 390 = 234$

Stress in compresive reinforcement,

$$f_{sc} = 600 \times \frac{k_u d - d_{sc}}{k_u d} = 600 \times \frac{234 - 60}{234} = 446 < f_{sy}$$

$$f_{st} = 600 \times \frac{d - k_u d}{k_u d} = 600 \times \frac{390 - 234}{234} = 400 < f_{sy}$$

Substituting in equation 13.4

ΦN_u = 0.6 × [0.85 × 25 × 350 × 0.85 × 234 + 2400 × (446 - 0.85*25) - 2400 × 400] × 10^{-3} = 923 kN < N^* (= 1400)

Increase k_u to increase ΦN_u, say try k_u = 0.7

$k_u d$ = 273

f_{sc} = 468.1 MPa

f_{st} = 257.1 MPa

Substituting in equation 13.4,

ΦN_u = 0.6 × [0.85 × 25 × 350 × 0.85 × 273 + 2400(468.1 - 0.85 × 25) - 2400 × 257.1]*10^{-3}
= 1309 kN

Increase k_u to 0.727, $k_u d$ = 283.5

$$
\begin{aligned}
\Phi N_u &= 0.6 \times (1792.2 + 1084.2 - 540.7) \\
&= 1075.3 + 650.5 - 324.4 \\
&= 1401.4 \text{ kN}
\end{aligned}
$$

$$
\begin{aligned}
N_u \times e' &= 1075.3 \times (390 - 0.5 \times 0.85 \times 283.5) + 650.5 \times (390 - 60) \\
&= 504472 \text{ kNmm}
\end{aligned}
$$

$$e = \frac{504472}{1401.4} - 165 = 195 \text{ mm}$$

Moment capacity,

$$\Phi M_u = \Phi N_u \times e = 1401.4 \times 195 \times 10^{-3} = 273.3 \text{ kNm}$$

The chosen section has a combined load-moment capacity ΦN_u = 1401kN ^{a} N^* and ΦM_u = 273.3 kNm > M^* = 250 kNm hence the chosen section is satisfactory.

13.10 Design Charts

The load-moment interaction charts considered so far are limited in their application to specific sized columns. Dividing equation 13.4 by the column dimensions bD will reduce the equation to a stress N_u/bD versus the total steel ratio p (= A_s/bD).

$$\frac{N_u}{bD} = \frac{0.85f'_c b\gamma k_u d}{bD} + \frac{A_{sc}\left(f_{sc} - 0.85 * f'_c\right)}{bD} - \frac{A_{st} f_{st}}{bD}$$

Defining, $g = \dfrac{\text{dis tance } \textit{between outer reinforcement}}{D}$

The effective depth $d = \dfrac{D(1+g)}{2}$

$$\frac{A_{sc}}{bD} = \frac{A_{st}}{bD} = \textit{Half the total steel ratio} = \frac{p}{2}$$

The equation now becomes,

$$\frac{N_u}{bD} = \frac{0.85f'_c b\gamma k_u d(1+g)}{2} + \frac{p\left(f_{sc} - 0.85f'_c\right)}{2} - \frac{pf_{si}}{2}$$

$$\boxed{\frac{N_u}{bD} = \frac{0.85f'_c b\gamma k_u d(1+g)}{2} + \frac{p\left(f_{sc} - f_{st} - 0.85f'_c\right)}{2}} \qquad \textbf{(13.9)}$$

Similarly dividing equation 13.3 by bD^2 will also reduce that equation to a stress.

$$\frac{N_u e'}{bD^2} = \frac{0.85f'_c \gamma k_u d^2(1-0.5\gamma k_u)}{bD^2} + \frac{A_{sc}(f_{sc} - 0.85f'_c)(d - d_{sc})}{bD^2}$$

$$d^2 = \frac{D^2\left(1+g\right)^2}{4}$$

$$d - d_{sc} = D\frac{1+g}{2} - D\frac{1-g}{2} = gD$$

Substituting in the equation,

$$\frac{N_u}{bD}\frac{e'}{D} = 0.85f'_c b\gamma k_u(1+g)^2(1-0.5\gamma k_u) + \frac{p\left(f_{sc} - 0.85f'_c\right)g}{2} \qquad \textbf{(13.10)}$$

Dividing equation 13.7 by D,

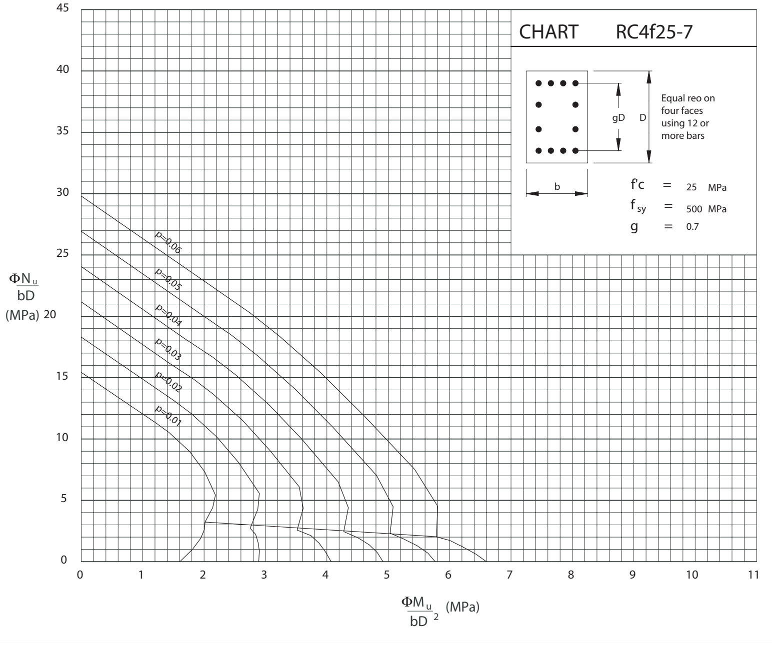
$$\frac{e}{D} = \frac{e'}{D} - \frac{g}{2} \qquad \textbf{(13.11)}$$

Dividing equation 13.10 by 13.9 gives the ratio e'/D to be used in equation 13.11. And finally dividing equation 13.8 by bD^2,

$$\frac{M_u}{bD^2} = \frac{N_u}{bD} \times \frac{e}{D} \qquad \textbf{(13.12)}$$

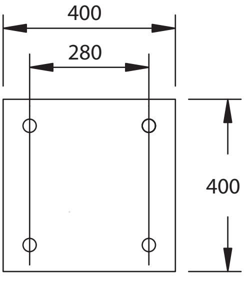
The interaction diagrams may now be drawn using equations 13.9 to 13.12. Such interaction diagrams will be in terms of stresses N_u/bD and M_u/bD^2 and the total steel ratio $p = A_s/bD$. A typical interaction diagram is shown in Figure 13.16. A full set of interaction diagrams is contained at the end of the book.

Figure 13.16



EXAMPLE 6

The column section whose ratio g=0.7 is shown in Figure 13.17 (*right*). The column is required to carry a design axial load N^*=1850 kN and a design moment M^*=160 kNm. Choose the column reinforcement for a grade N25 concrete.



SOLUTION

Chart RC2f25-7 in the design charts at the end of the book will be used for g = 0.7 and f'_c = 25 MPa.
Stresses due to design loading,

$$\frac{N^*}{bD} = \frac{1850 \times 10^3}{400 \times 400} = 11.56 MPa$$

$$\frac{M^*}{bD^2} = \frac{160 \times 10^6}{400 \times 400^2} = 2.5 MPa$$

Enter Chart RC2f25-7 and read the required steel ratio p = 0.022.

Required area of reinforcement,

$$A_s = p\, b\, D = 0.022 \times 400 \times 400$$
$$= 3520 \text{ mm}^2$$

Choose 6N28 bars (3 bars on each face) whose area is 3720 mm^2.

13.11 Design of Short Columns

Short columns are defined as columns whose slenderness is such that the additional moment due to slenderness effects is minimal and it may be disregarded in the design procedure. The Code #10.3.1 defines short columns by the slenderness ratio.

(a) For braced columns,

$$Maximum\ \frac{L_e}{r} = 60\left(1 + \frac{M^*_1}{M^*_2}\right)\left(1 - \frac{N^*}{0.6 * N_{uo}}\right) \geq 25 \quad \textbf{(13.13)}$$

That is, a short column is defined by a slenderness which is less than the greater of the above values.

(b) For unbraced columns,

$$Maximum\ \frac{L_e}{r} = 22 \quad \textbf{(13.14)}$$

Where:

L_e = effective length.

r = radius of gyration.

= $0.3D$ for rectangular columns (D is the column dimension in the direction in which stability is being considered).

= $0.25D$ for circular columns.

M^*_1 = the lesser end moment.

M^*_2 = the larger end moment.

The ratio M^*_1/M^*_2 is taken to be negative when the column is bent in single curvature and positive when the column is bent in double curvature. When the larger moment M^*_2 is equal to or less then the minimum design moment of $0.05DN^*$ the ratio of M^*_1/M^*_2 is taken to be -1.0.

Column design will generally fall in the category of short columns. Slender columns in braced and unbraced frames are subjected to additional moments due to the P-Δ effect and they will be considered at the end of this chapter.

13.12 Short Columns with Small Axial Loads

When the axial load N^* in short columns is less than $0.1f_c'A_g$, the axial force may be disregarded and the column section is designed for bending only.

13.13 Short Braced Columns with Small Bending Moments

The bending moments in short interior columns of a braced rectangular structure may be disregarded if the following conditions are satisfied:

(a) The ratio of the longer to the shorter length of any two adjacent spans does not exceed 1.2.

(b) The column loads are primarily due to distributed loads whose intensities are such that the live load q is not more than twice the dead load g.

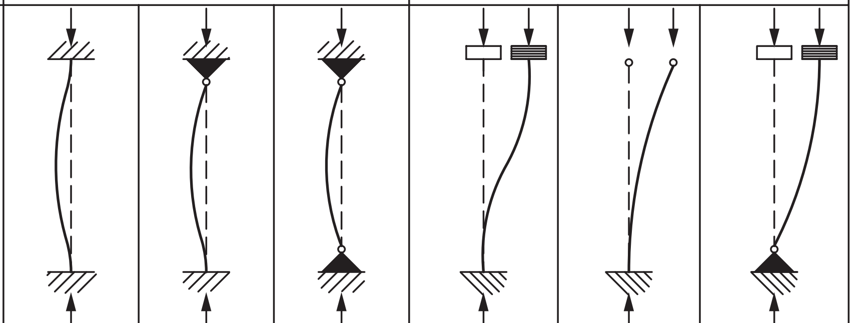
(c) The column sections are prismatic and they are symmetrically reinforced.

If the above conditions are satisfied, the design axial strength of the column may be taken as $0.75\Phi N_{uo}$.

13.14 Effective Length of Columns $L_e = kL_u$

The effective length of columns with simple end restraints may be determined using the effective length multiplier k given in Figure 13.18.

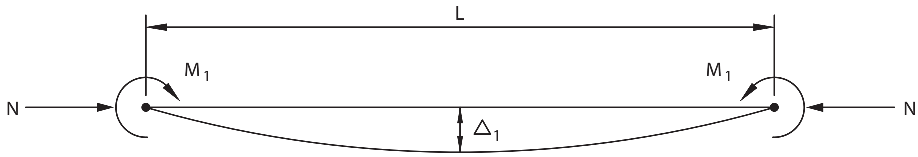
Figure 13.18

	No Sway or Braced Column			Sway or Unbraced Column		
Buckled Shape						
Effective length factor (k)	0.70	0.85	1.00	1.20	2.2	2.2
Symbols for end restraint conditions	= Rotation fixed, translation fixed = Rotation free, translation fixed			= Rotation fixed, translation free = Rotation free, translation free		

13.15 Moment Magnifier for Braced Columns

Consider a compression member subjected to an axial load N and end moments M_1 shown in Figure 13.19. The end moments will cause the member to deflect an amount Δ_1 at mid-height. Due to the deflection Δ_1 the moment is increased by an amount $M_2 = \Delta_1 N$ since the load N is now eccentric to the member at mid-height.

Figure 13.19



The additional moment M_2 will cause a further deflection Δ_2 and a corresponding increase in moment $M_3 = \Delta_2 N$. This process is continued until stable conditions are achieved and a maximum deflection Δ_{max} is reached given by,

$$\Delta_{max} = \frac{M_{max}}{N_c} \qquad \text{(a)}$$

Where M_{max} is the final maximum moment given by,

$$M_{max} = M_1 + \Delta_1 N + \Delta_2 N + \Delta_3 N + \ldots\ldots\ldots$$
$$= M_1 + \Delta_{max} N \qquad (b)$$

and N_c is Euler's buckling load given by,

$$N_c = \frac{\pi^2 EI}{L_e^2} \qquad (c)$$

Substituting for Δ_{max} in (b),

$$M_{max} = M_1 + M_{max}\left(\frac{N}{N_c}\right)$$

Solving for M_{max} gives,

$$M_{max} = \left(\frac{1}{1-\dfrac{N}{N_c}}\right)M_1$$

The term $1/(1-N/N_c)$ is the moment magnifier for a column bent in single curvature due to equal end moments. In the Code, the magnification factor for a braced column is δ_b which also includes the effects of end moments M^*_1 and M^*_2. Computer programs are available which will calculate the increased moment. This is called a second order analysis. If such programs are not readily available to the designer, the Code #10.4.2 permits the use of the moment magnifier δ_b given by:

$$\delta_b = \frac{k_m}{1-\dfrac{N^*}{N_c}} \qquad \textbf{(13.15)}$$

Where; k_m = End moment condition parameter given by:

$$k_m = 0.6 - 0.4\frac{M_1^*}{M_2^*} \geq 0.4 \qquad \textbf{(13.16)}$$

The ratio of the smaller end moment to the larger end moment M^*_1/M^*_2 is negative if the column is bent in single curvature and positive when the column is bent in reverse curvature. If the minimum moment of $0.05DN^*$ exceeds the larger applied design moment, the value of M^*_1/M^*_2 is taken to be -1.0 and k_m becomes 1.0.

N_c = Buckling load given by:

$$N_c = \left(\frac{\pi}{L_e}\right)^2\left(\frac{182 d_o \Phi M_{ub}}{1+\beta_d}\right) \qquad \textbf{(13.17)}$$

$182 d_o \, \Phi M_{ub}$ = Measure of the stiffness EI of the member.

d_o = Depth from compression fibre to outermost tension reinforcement.

β_d = Factor for creep effects due to sustained axial loads. It may be disregarded for short columns whose slenderness ratio $L_e/r \leq 40$ and for columns with small axial loads where $N^* \leq M^*/2D$.

$$\beta_d = \frac{G}{G+Q} \tag{13.18}$$

ΦM_{ub} = Balanced moment capacity for k_u = 0.545. This value would normally be read from the column interaction diagram.

Since designs will be carried out using design charts which are in terms of stresses, the buckling load equation is converted to buckling stress. Dividing by the column area gives,

$$\frac{N_c}{bD} = \frac{\left(\dfrac{\pi}{L_e}\right)^2 \left(\dfrac{200 d_o \Phi M_{ub}}{1+\beta_d}\right)}{bD}$$

which may be written in the format of equation 13.19,

$$\frac{N_c}{bD} = \left(\frac{\pi}{L_e}\right)^2 D \left(\frac{200 d_o \dfrac{\Phi M_{ub}}{bD^2}}{1+\beta_d}\right) \tag{13.19}$$

EXAMPLE 7

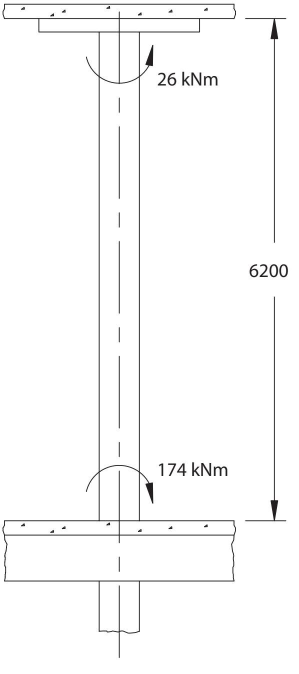
A rectangular column shown in Figure 13.20 has cross-sectional dimensions b = 350 D = 400, and it uses grade N32 concrete. The column is reinforced on two faces to give the ratio g = 0.7. End conditions may be taken as free to rotate at the top end and not free to rotate at the lower end. Lateral sway is prevented.

The column is required to carry an axial design load N^* = 1855 kN made up of 507 kN dead load G and 831 kN live load Q. The two design end moments of 26 kNm and 174 kNm will cause the column to deform in single curvature.

(a) Calculate the column moment magnification factor.

(b) Determine the required steel ratio and choose the reinforcement.

Figure 13.20



SOLUTION

Data: $b = 350$ $D = 400$ $d_0 = 340$ $g = 0.7$
$f'_c = 32$ MPa $L_u = 6200$ $N^* = 1855$ kN $M_1^* = 26$ kNm
$M_2^* = 174$ kNm

Column effective length using effective length multiplier $k = 0.85$ from Figure 13.18,

$$L_e = kL_u = 0.85*6200 = 5270 \text{ mm}$$

Radius of gyration, $r = 0.3*400 = 120$ mm

Slenderness ratio, $\frac{L_e}{r} = \frac{5270}{120} = 43.9$

Minimum design moment $= 0.05DN^*$
$= 0.05*0.4*1855$
$= 37.1$ kNm
> 26 kNm applied design moment

Use $M_1^* = 37.1$ kNm

Axial stress due to design load,

$$\frac{N^*}{bD} = \frac{1855 \times 10^3}{350 \times 400} = 13.25 \text{ MPa}$$

For the larger design moment,

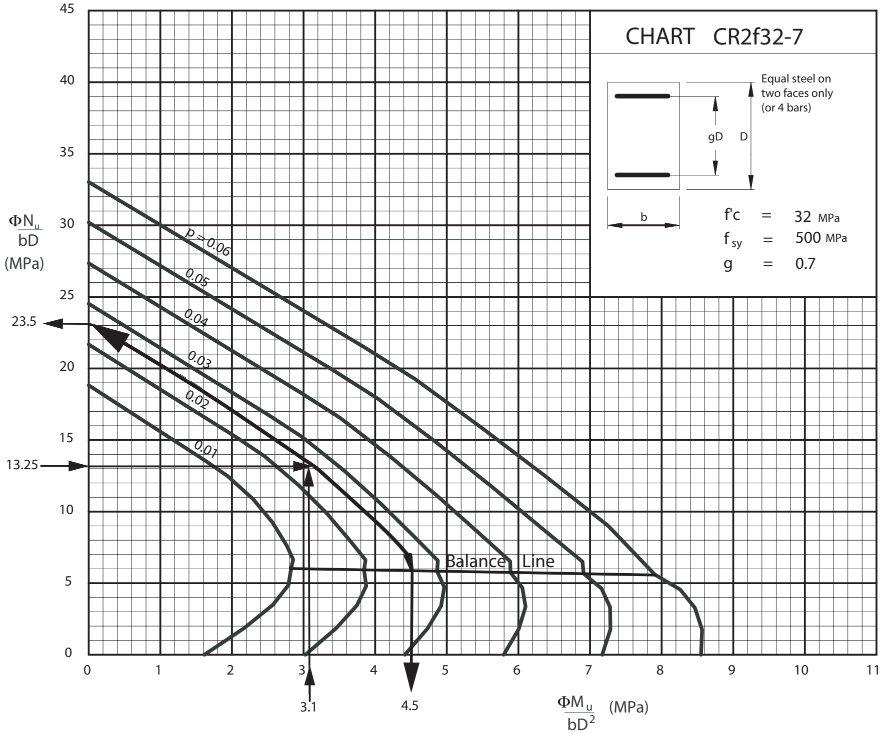
$$\frac{M^*}{bD^2} = \frac{174 \times 10^6}{350 \times 400^2} = 3.1 \text{ MPa}$$

On Chart CR2f32-7 reproduced in Figure 13.21, locate point using above stresses to give p = 0.026 which is an initial approximation for the steel ratio since it does not account for moment magnification. For this steel ratio read,

$$\frac{\Phi N_{uo}}{bD} \left(= \frac{0.6 N_{uo}}{bD} \right) = 23.5 \text{ MPa}$$

$$\frac{\Phi M_{ub}}{bD^2} = 4.5 \text{ MPa}$$

Figure 13.21



To determine the maximum slenderness ratio for a short column,

$$60\left(1+\frac{M^*_1}{M^*_2}\right)\left(1-\frac{N^*}{0.6N_{uo}}\right)=60\left(1-\frac{37.1}{174}\right)\left(1-\frac{13.25}{23.5}\right)$$

$$=20.6<25$$

$$\therefore \mathrm{L/r} = \text{for a short column}$$

The column will have to be designed as a slender column since the slenderness ratio of 43.9 exceeds the maximum value of 25 for a short column.

To calculate the buckling stress N_C/bD, the creep factor β_d will have to be included since the slenderness ratio is greater than 40.

$$\beta_d=\frac{G}{G+Q}=\frac{500}{500+820} \qquad = 0.379$$

$$\frac{N_c}{bD}=\left(\frac{\pi}{L_e}\right)^2 D\left(\frac{182d_o\dfrac{\Phi M_{ub}}{bD^2}}{1+\beta_d}\right)$$

$$=\left(\frac{\pi}{5270}\right)^2 400\left(\frac{182\times 340\times 4.5}{1+0.379}\right)$$

$$=28.7MPa$$

To calculate the moment magnification factor δ_b, the end moment ratio M^*_1/M^*_2 is negative since the column is bent in single curvature.

$$k_m = 0.6 - 0.4\frac{M_1^*}{M_2^*}$$

$$= 0.6 + 0.4\frac{37.1}{174}$$

$$= 0.685$$

$$\delta_b = \frac{k_m}{1 - \dfrac{N^*}{N_c}} = \frac{0.685}{1 - \dfrac{13.25}{28.7}} = 1.28$$

Magnified bending stress $\delta_b \dfrac{M^*}{bD^2}$ = 1.28*3.1 = 3.97 MPa.

Using the magnified bending stress, from chart RC2f32-7, read new steel ratio p = 0.035. This initial value may be accepted and it will be a conservative estimate of the required steel ratio. The more accurate result will be obtained by repeating the design steps using the new steel ratio until the same magnification factor and steel ratio obtained in two consecutive iterations. Repeating the procedure with the new steel ratio of 0.035,

For p = 0.035, $\dfrac{\Phi N_{uo}}{bD}$ = 25.8 MPa and $\dfrac{\Phi M_{ub}}{bD^2}$ = 5.4 MPa

$$\frac{N_c}{bD} = \left(\frac{\pi}{5270}\right)^2 400\left(\frac{182 \times 340 \times 5.4}{1 + 0.379}\right)$$

$$= 34.4 \ \ MPa$$

$$\delta_b = \frac{0.685}{1 - \dfrac{13.25}{34.4}} = 1.11$$

Magnified stress $\delta_b \dfrac{M^*}{bD^2}$ = 1.11 × 3.1 = 3.4 MPa

Further iterations will converge on δ_b = 1.16 with a magnified stress, $\delta_b \dfrac{M^*}{bD^2}$ = 1.16 × 3.1 = 3.6 MPa and a requred steel ratio p = 0.032. The required area of reinforcement,

A_s = 0.032 × 350 × 400 = 4480 mm^2

Use 6N32 (A_s = 4800 mm^2). It is not always possible to closely match the reinforcement to the required area and it is not a good practice to use mixed size reinforcement.

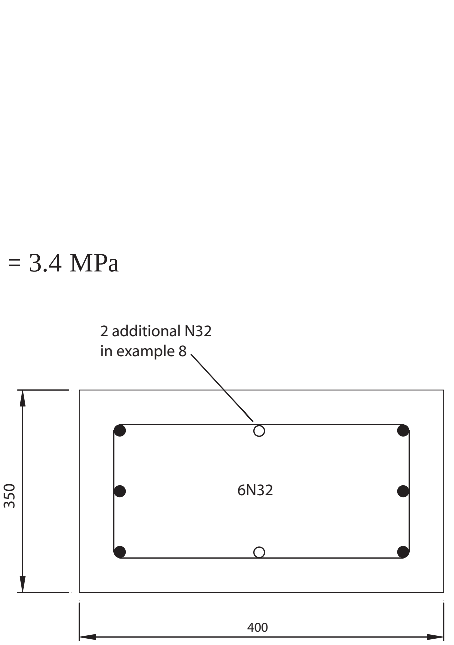


Figure 13.22

13.16 Biaxial Bending

Columns are frequently subjected to design moments applied about both axes. Even when the moment is applied about one axis (uniaxial bending) the minimum moment condition of $0.05DN^*$ will ensure that columns are designed for biaxial bending. Design of columns in biaxial bending is much more complex. The required design aids are three dimensional interaction diagrams. Biaxial bending problems for rectangular columns can be simplified by the use of the combined orthogonal interaction formula given in #10.6.5 of the Code.

$$\left(\frac{M_x^*}{\Phi M_{ux}}\right)^{\alpha_n} + \left(\frac{M_y^*}{\Phi M_{uy}}\right)^{\alpha_n} \leq 1.0 \qquad \textbf{(13.20)}$$

Where: M^*_x, M^*_y = Design moments magnified where applicable.

$$\alpha_n = 0.7 + 1.7\frac{N^*}{0.6N_{uo}}$$ Between the limits, $1.0 \leq \alpha_n \leq 2.0$

The interaction formula will generally be worked out in terms of stresses because the design charts are in terms of stresses.

Columns subjected to biaxial bending are commonly reinforced on four faces using 8, 12 or 16 bars distributed to give equal reinforcement on each face. Design charts RC4f##-4 to RC4f##-9 for columns equally reinforced on four faces using 12 reinforcing bars are included at the end of this chapter. Charts RC2f may still be used for columns carrying small moments M^*_y applied about the weak axis. Only the corner bars are considered for moments M^*_y as shown in the following example 7.

EXAMPLE 8

Check the column in example 7 for biaxial bending when the minimum design moment is applied about the weak axis.

SOLUTION

Data: D = 350 mm b = 400 mm f'_c = 32 MPa L_e = 5270 mm

N^* = 1855 kN N^*/bD = 13.25 MPa k_m = 0.685 $M_x^*/bD^2 = 3.1$ MPa

For bending about the x-axis

$$p = \frac{4800}{350 \times 400} = 0.034$$

From chart RC2f32-7, for p = 0.034 and $\dfrac{\Phi N_u}{bD}$ = 13.25 MPa:

$$\frac{\Phi M_{ux}}{bD^2} = 3.9 \text{ MPa}$$

$$\frac{\Phi M_{ubx}}{bD^2} = 5.3 \text{ MPa}$$

$$\frac{N_c}{bD} = \left(\frac{\pi}{5270}\right)^2 400\left(\frac{182\times 340\times 5.3}{1+0.379}\right) = 33.8 \quad MPa$$

$$\delta_b = \frac{0.685}{1-\frac{13.25}{33.8}} = 1.13$$

Magnified bending stress $\delta_b \frac{M_x^*}{bD^2}$ = 1.13 × 3.1 = 3.5 MPa

<u>For bending about the y-axis</u>

Using the 4 corner bars only since charts RC2f will be used for equal reinforcement on 2 faces.

Minimum design moment about y-axis,

M^*_y = 0.050 × 0.35 × 1855 = 32.5 kNm

$$\frac{M_y^*}{bD^2} = \frac{32.5\times 10^6}{400\times 350^2} = 0.663 \text{ MPa}$$

Using the same cover, depth to reinforcement, d_o = 350 - 60 = 290 mm

Steel ratio using the 4N32 corner bars, $p = \frac{4\times 800}{350\times 400} = 0.023$

$$g = \frac{350-2\times 60}{350} = 0.66$$

The g ratio falls between two charts, Chart RC2f32-6 will be used since the lesser value of g gives conservative results. Alternatively both Charts RC2f32-6 and RC2f32-7 may be used and the results interpolated for g = 0.66.

From Chart RC2f32-6 for p = 0.023, $0.6N_{uo}/bD$ = 22.5 MPa (= $\Phi N_{uo}/bD$ at zero moment) and $\Phi M_{ub}/bD^2$ = 3.6 MPa.

Buckling stress,

$$\frac{N_c}{bD} = \left(\frac{\pi}{5270}\right)^2 350\left(\frac{182\times 290\times 3.6}{1.379}\right) = 17.1 \ MPa$$

For equal end moments k_m = 1.0 since the ratio M^*_1/M^*_2 is taken to be -1.0.

Magnification factor,

$$\delta_b = \frac{1}{1-\frac{13.25}{17.1}} = 4.44$$

Magnified bending stress,

$$\delta_b \frac{M_y^*}{bD^2} = 4.44 \times 0.663 = 2.9 \text{ MPa}$$

From chart RC2f32-6 for p = 0.023 and $\frac{\Phi N_u}{bD}$ = 13.25 MPa,

$$\frac{\Phi M_{uy}}{bD^2} = 3.6 \text{ MPa}$$

Exponential to be used in interaction formula,

$$\alpha_n = 0.7 + \frac{1.7 * 13.25}{22.5} = 1.70$$

$$\left(\frac{M_x^*}{\Phi M_{ux}}\right)^{\alpha_n} + \left(\frac{M_y^*}{\Phi M_{uy}}\right)^{\alpha_n} = \left(\frac{3.5}{3.8}\right)^{1.7} + \left(\frac{2.9}{3.6}\right)^{1.7}$$

$$= 1.19$$

$$> 1.0 \text{ UNSATISFACTORY}$$

The column section will need to be increased or additional reinforcement added. Add two more N32 reinforcing bars for bending about the weak axis. In this case the column is equally reinforced on 4 faces using 8 bars. Charts CR4f are drawn for columns equally reinforced on 4 faces using 12 or more reinforcing bars. Use of Charts CR4f is illustrated in example 9. A conservative result may be obtained using Charts CR2f.

It will be necessary to recalculate the stress conditions for bending about the y-axis using the increased steel ratio for 8N32 bars.

Steel ratio $p = \frac{6400}{350 \times 400} = 0.046$

From Chart RC2f32-6 for p = 0.046, $\frac{\Phi N_{uo}}{bD}$ = 29 MPa and $\frac{\Phi M_{ub}}{bD^2}$ = 5.5 MPa and buckling stress $\frac{N_c}{bD}$ =33.8 as for bending about x-axis.

Magnification factor,

$$\delta_b = \frac{1}{1 - \frac{13.25}{33.8}} = \mathit{1.64}$$

Magnified design bending stress,

$$\delta_b \frac{M_y^*}{bD^2} = 1.64 \times 0.663 = 1.09 \text{ MPa}$$

From Chart RC2f32-6 for the steel ratio p = 0.046 and axial stress $N^*/(bD)$ = 13.25 MPa, $\Phi M_{uy}/(bD^2)$ = 3.6 MPa.

Exponential index for the interaction formula,

$$\alpha_n = 0.7 + \quad 1.7\frac{13.25}{29} \quad = 1.48$$

$$\left(\frac{M_x^*}{\Phi M_{ux}}\right)^{\alpha_n} + \left(\frac{M_y^*}{\Phi M_{uy}}\right)^{\alpha_n} = \left(\frac{3.5}{3.8}\right)^{1.48} + \left(\frac{1.09}{3.6}\right)^{1.48}$$

$$= 1.06$$

The result of 1.06 is close to the limit of 1.0 and it may be accepted considering the conservative approach adopted by using the design chart C2f3-6for cloumn with equal reinforcement on two facts.

< 1.0 satisfactory

The procedure will need to be refined or the steel ratio will have to be increased to satisfy biaxial bending conditions.

EXAMPLE 9

The square column section shown in Figure 13.23 is equally reinforced on all four faces using a total of 12 reinforcing bars. For the data given in the figure choose the reinforcement assuming that the applied moments cause the column to deform in single curvature and that the column is laterally restrained at the ends.

Data:

$g = 0.6$	$L_e = 3060$ mm
$G = 320$ kN	$M^*{1x} = 35$ kNm
$M^*{1y} = 10$ kNm	$d_o = 280$
$f'_c = 25$ MPa	$N^* = 950$ kN
Q = 250 kN	$M^*{2x} = 90$ kNm
$M^*{2y} = 25$ kNm	

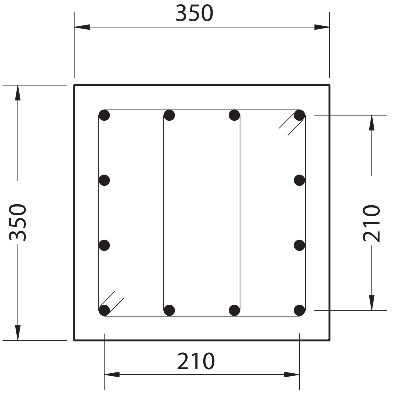


Figure 13.23

SOLUTION

$$\frac{N^*}{bD} = \frac{950 \times 10^3}{350 \times 350} = 7.8 \text{ MPa}$$

Bending about the X-Axis

$$\frac{M_x^*}{bD^2} = \frac{90 \times 10^6}{350 \times 350^2} = 2.1 \text{ MPa}$$

From Chart CR4f25-6, required steel ratio $p = 0.012$.

To determine if the column is short or slender, for $p = 0.012$ read $\Phi N_{uo}/(bD) =$ 16.5 MPa and $\Phi M_{ub}/(bD^2) = 2.4$ MPa.

For a short column,

$$\text{Maximum} \quad \frac{L_e}{r} = 60\left(1 + \frac{M_1^*}{M_2^*}\right)\left(1 - \frac{N^*}{0.6N_{uo}}\right)$$

$$= 60\left(1 - \frac{35}{90}\right)\left(1 - \frac{7.8}{16.5}\right)$$

$$= 19.3 < 25$$

Therefore maximum slenderness ratio for a short column is 25.

Radius of gyration, $r = 0.3 \times 350 = 105$ mm

Actual slenderness $L_e/r = 3060/105 = 29.1 > 25$ for a short column. This is a slender column and it will have to be designed as a slender column which includes moment magnification due to slenderness effects.

Buckling stress,

$$\frac{N_c}{bD} = \left(\frac{\pi}{3060}\right)^2 350(200 \times 280 \times 2.4) = 49.6\text{MPa}$$

Note that the creep factor has been ignored since the slenderness ratio was less than 40.

To determine the magnification factor, the end moment condition parameter,

$$k_m = 0.6 - 0.4\left(\frac{-35}{90}\right) = 0.756$$

Magnification factor,

$$\delta_b = \frac{0.756}{\left(1 - \frac{7.8}{49.5}\right)} = 0.90 < 1 \quad \text{Hence no moment magnification}$$

From Chart CR4f25-6, required steel ratio, p = 0.012.

Required area of reinforcement, $A_s = 0.012 \times 350 \times 350 = 1{,}470$ mm^2

Bending about the Y-Axis

Minimum design bending moment = $0.05 \times 0.35 \times 900$

= 15.75 kNm

The maximum design moment for bending about y-axis, $M_y^* = 25$ kNm

Maximum design bending stress, $\frac{M_y^*}{bD^2} = \frac{25 \times 10^6}{350 \times 350^2} = 0.58$ MPa

From Chart CR4f25-6, the required steel ratio for bending about the y-axis $p < 0.01$

As for bending about the x-axis, there is no moment magnification.

As for bending about the x-axis, there is no moment magnification.

Required area of reinforcement = 0.01 × 350 × 350 = 1,225 mm^2

Choosing the Reinforcement and Checking Interaction Formula

For the biaxial bending condition the area of reinforcement calculated for bending about the x-axis and about the y-axis may be added together and the reinforcement chosen for the total area. This approach is usually conservative and it does not require checking of the interaction formula. In this example the reinforcement will be chosen to satisfy the interaction formula.

The initially estimated area of reinforcement will be the sum of the areas calculated for bending about the x and y axis.

A_s = 1470 + 1225 = 2695 mm^2

Steel ratio p = $\frac{2695}{350 \times 350}$ = 0.022.

From Chart RC4f25-6, for p = 0.022 and $\Phi N_u/(bD) = N^*/(bD)$ = 7.8 MPa the bending stress capacity $\Phi M_u/(bD^2)$ = 2.9 MPa. This will be the same for bending about both the x and y axes because of the square column section. Also from the same chart the axial stress capacity at zero moment condition $\Phi N_{uo}/bD$ = 19 MPa.

Exponential index for interaction formula,

$$\alpha_n = 0.7 + 1.7\,\frac{7.8}{19} = 1.40$$

Applying the interaction formula,

$$\left(\frac{M_x^*}{\Phi M_{ux}}\right)^{\alpha_n} + \left(\frac{M_y^*}{\Phi M_{uy}}\right)^{\alpha_n} = \left(\frac{2.1}{2.9}\right)^{1.4} + \left(\frac{0.58}{2.9}\right)^{1.4}$$

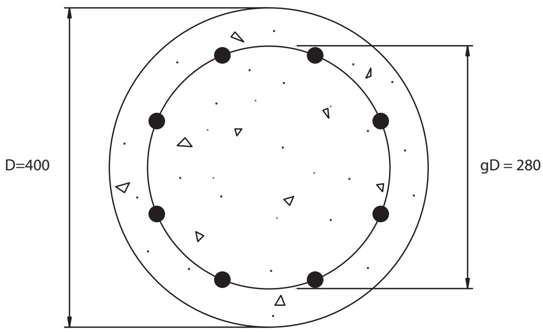
$= 0.74 < 1.0$ **SATISFACTORY**

EXAMPLE 10 CIRCULAR COLUMNS

Circular columns are ideal for biaxial bending. The applied moments are combined vectorially and the column is designed using the resultant moments as a uniaxially bent column.

A 400 mm diameter circular column shown in Figure 13.24 is reinforced with 8 bars placed on a 280 mm pitch circle diameter. The column uses grade N25 concrete. Choose the reinforcement for the following conditions:

Figure 13.24



L_e = 3250 mm N^* = 1540 kN

M^*_{1x} = 40 kNm M^*_{2x} = 75 kNm M^*_{1y} = 25 kNm M^*_{2y} = 64 kNm

Note: M^*_{1x} and M^*_{1y} are applied to the same column end.

Solution

Combine the end moments to determine the resultant end moments.

$$M^*_1 = \sqrt{40^2 + 25^2} = 41.2 \text{ kNm}$$

$$M^*_2 = \sqrt{75^2 + 64^2} = 98.6 \text{ kNm}$$

Gross column area, $A_g = \dfrac{\pi \times 400^2}{4} = 125.7 \times 10^3 \text{ mm}^2$

$$g = \frac{280}{400} = 0.7$$

$$\frac{N^*}{A_g} = \frac{1540 \times 10^3}{125.7 \times 10^3} = 12.25 \text{ MPa}$$

$$\frac{M^*}{A_g D} = \frac{98.6 \times 10^6}{125.7 \times 10^3 \times 400} = 1.96 \text{ MPa}$$

From Chart CC8B25-7 at the end of the book read p = 0.029 and for p = 0.029 read $\dfrac{\Phi N_{uo}}{A_g}$ = 21 MPa.

Radius of gyration, r = 0.25*400 = 100 mm

Slenderness ratio, $\dfrac{L_e}{r} = \dfrac{3250}{100} = 32.5$

The maximum slenderness ratio for a short column,

$$60\left(1+\frac{M*_1}{M*_2}\right)\left(1-\frac{N^*}{0.6N_{uo}}\right)=60\left(1+\frac{47.2}{98.6}\right)\left(1-\frac{12.25}{21}\right)$$

$$= 37.0 > 32.5$$

The column is thus designed as a short column whch does not require moment magnification i.e. the required steel ratio is read directly from the design chart.

Required area of reinforcement, $A_S = 0.029 \times 125.7 \times 10^3 = 3645\ \text{mm}^2$

Use 8N24 reinforcing bars whose area = 3616 mm^2

13.17 Column Reinforcing Details

(a) Minimum and Maximum Steel Ratio

The longitudinal steel ratio should be not less than 0.01. While the Code does not specify a maximum steel ratio, it gives a warning that if the steel ratio exceeds 0.04, the designer must satisfy himself that steel congestion will not occur (especially at junctions and around splices) and that proper placing and compaction of concrete can be achieved.

(b) Lateral Restraint of Longitudinal Reinforcement

The longitudinal compression reinforcing bars are compression elements which may buckle unless they are restrained at regular intervals by ties or continuous helical reinforcement. Helical reinforcement also functions much like a tension membrane; it prevents the concrete from bursting under compression so that the load carrying capacity of the column is increased. The AS3600 Code does not provide additional load carrying capacity for columns with helical reinforcement, it in effect considers that ties have the same restraining capacity provided by helical reinforcement. The Code requires that restraint be provided for all corner bars and every longitudinal bar if the spacing between bars is more than 150 mm or every alternate bar if the bar spacing is 150 mm or less. The minimum size of ties and helical reinforcement is given in Table 13.1. Effective restraint is provided by bends in ties with an included angle of 135° or less or between two 135° fitment hooks. The minimum spacing of ties and general Code requirements discussed above are shown in Table 13.1 and Figure 13.25.

Table 13.1 Minimum Size of Ties and Helices

Longitudinal Bar Size	Minimum Size of Fitment
Up to N20 single bars	6 mm
N24 to N36 single bars	10 mm
Bundled bars	12 mm

Figure 13.25

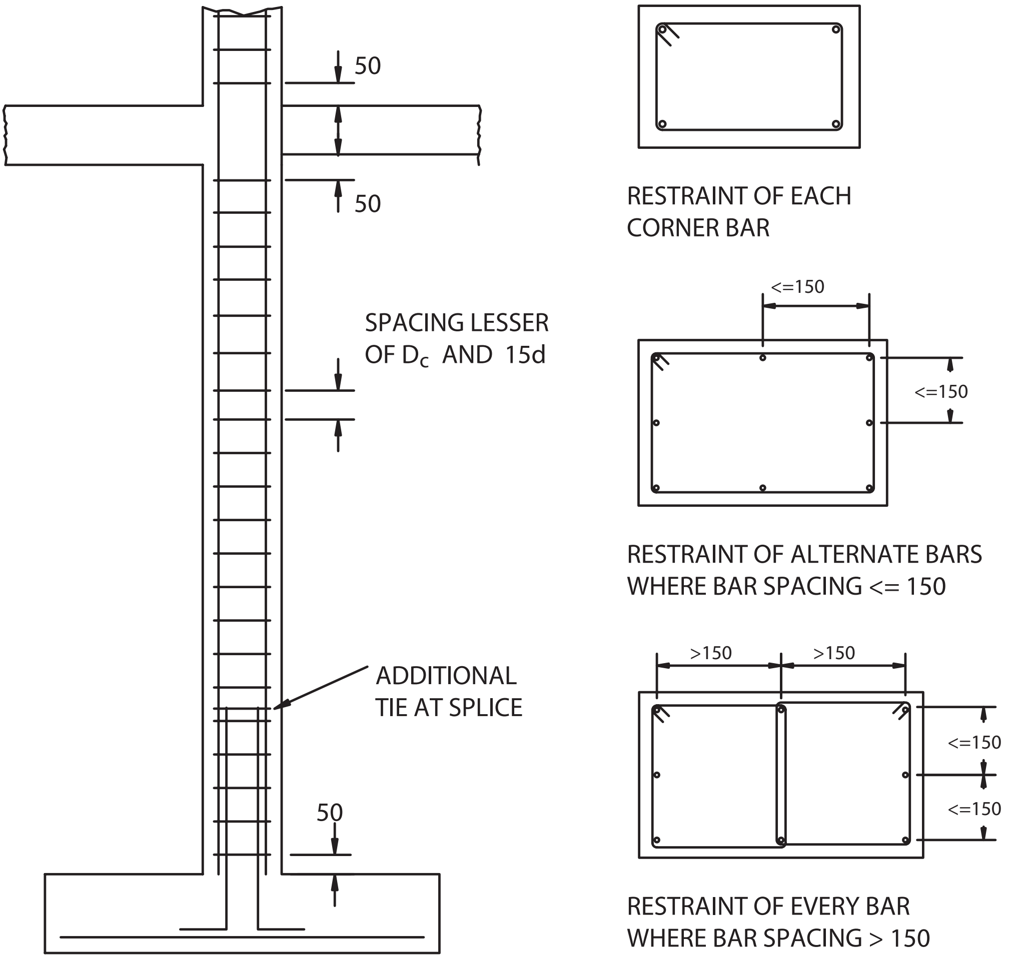

50
50
SPACING LESSER OF D_c AND 15d
ADDITIONAL TIE AT SPLICE
50
RESTRAINT OF EACH CORNER BAR
<=150
<=150
RESTRAINT OF ALTERNATE BARS WHERE BAR SPACING <= 150
>150
>150
<=150
<=150
RESTRAINT OF EVERY BAR WHERE BAR SPACING > 150

Figure 13.26

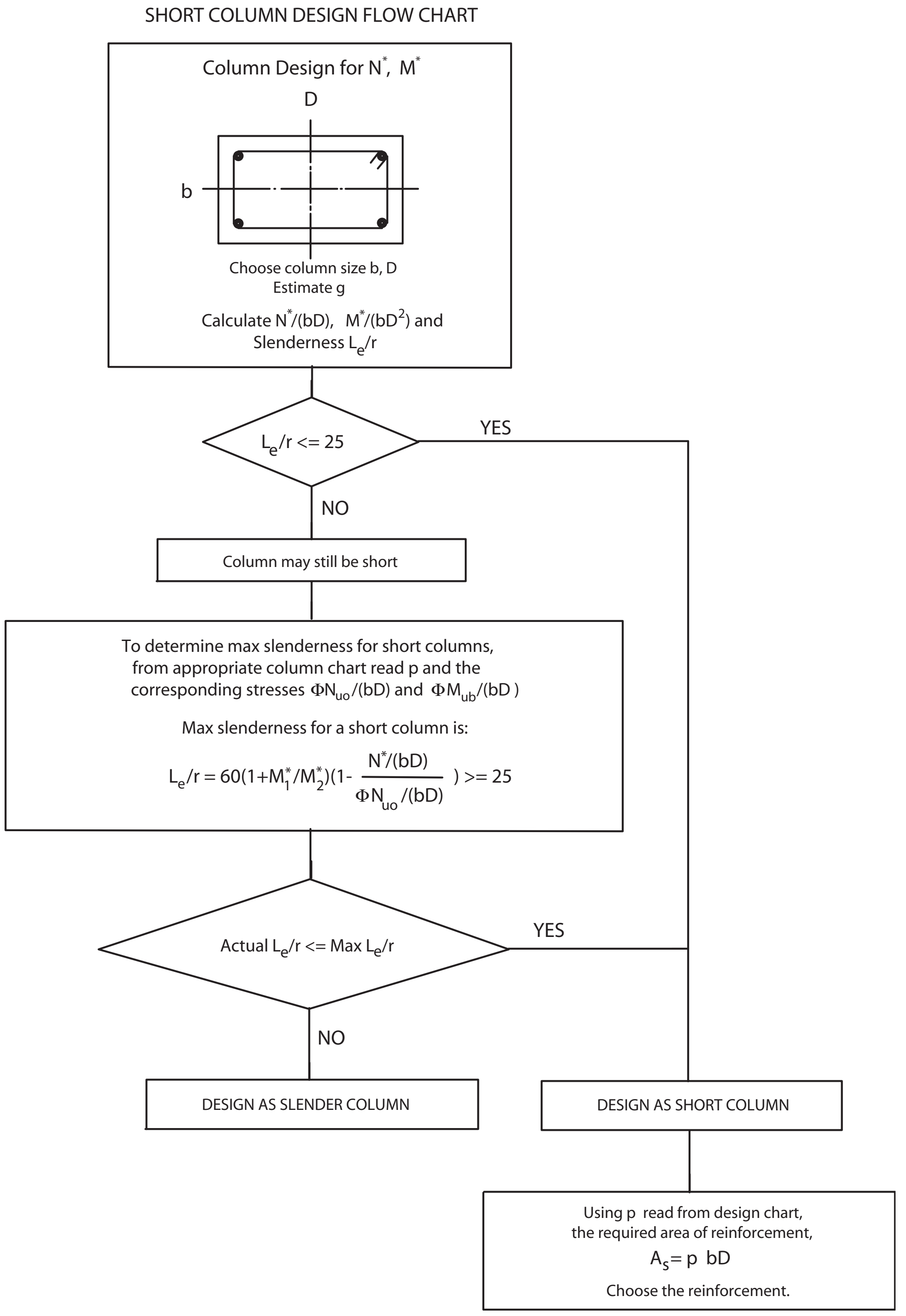
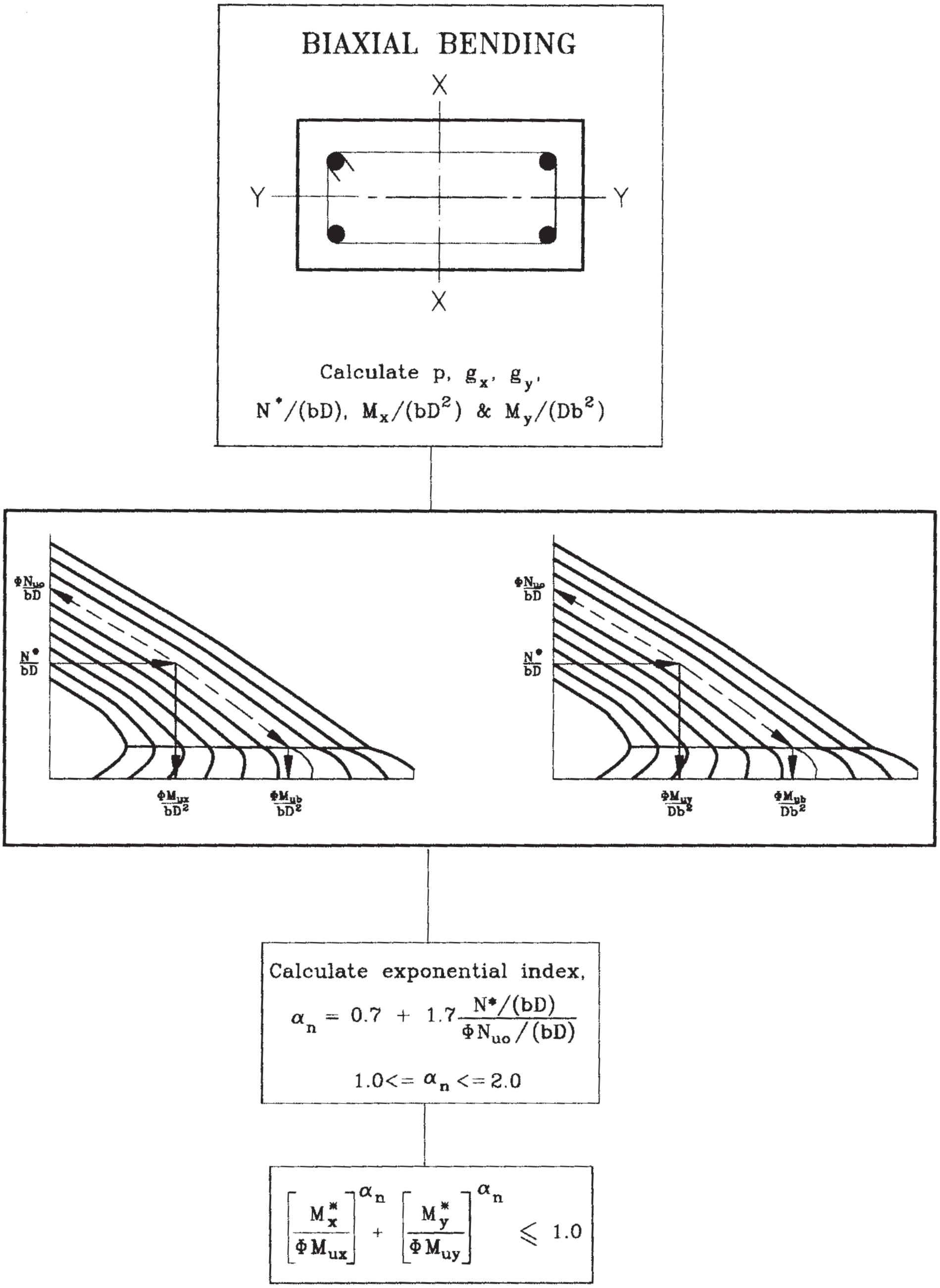

SHORT COLUMN DESIGN FLOW CHART
Column Design for N^*, M^*
D
b
Choose column size b, D
Estimate g
Calculate $N^*/(bD)$, $M^*/(bD^2)$ and
Slenderness L_e/r
$L_e/r <= 25$
YES
NO
Column may still be short
To determine max slenderness for short columns,
from appropriate column chart read p and the
corresponding stresses $\Phi N_{uo}/(bD)$ and $\Phi M_{ub}/(bD)$
Max slenderness for a short column is:
$L_e/r = 60(1+M_1^*/M_2^*)(1-\frac{N^*/(bD)}{\Phi N_{uo}/(bD)}) >= 25$
Actual $L_e/r <=$ Max L_e/r
YES
NO
DESIGN AS SLENDER COLUMN
DESIGN AS SHORT COLUMN
Using p read from design chart,
the required area of reinforcement,
$A_s = p\ bD$
Choose the reinforcement.

Figure 13.27



Biaxial Bending Flow Chart

13.18 Fire Design

The Building Code of Australia provides mandatory requirements for the construction of buildings and often the elements that form the building. Fire requirements have become a major focus of this Code (and its predecessor Ordinance 70). Originally building elements such as walls, columns etc merely had to provide structural adequacy for a minimum period (eg a 3 hr fire rating meant the wall should not collapse after 3 hrs of fire exposure) so that people could egress from a building.

This rating was then modified to account for structural integrity and insulation i.e. to ensure the wall did not crack or transfer heat through it (such that it posed a fire risk on the protected side of the wall).

The original requirements for columns design under fire merely required the designer to work out the cover to longitudinal reinforcement and the minimum column dimension then use a graph from the Standard (# Figure 5.6.3) to read off a fire resistance period (primarily for structural adequacy). The design parameters for columns exposed to fire have now been extended from the previous version of AS3600-1994.

The requirements of structural integrity and insulation were not necessary as columns are usually totally surrounded by fire. Beams and slab design however requires the designer to calculate structural integrity and insulation values (using tables from AS3600 based upon cover or slab thickness) as these elements are usually only exposed to fire from one side.

As mentioned in the previous section, even though fire resistance period (FRP) is comprised of three levels (fire resistance levels – FRL) and always in the order - Structural Adequacy, Structural Integrity and Insulation (eg 180/120/120), the only important FRL for columns is Structural Adequacy. The old Code only used Figure 5.6.3 to provide the FRP for columns yet did not take into account important parameters such as (a) length of column (b) reinforcement % (c) applied load (d) concrete grade (e) aspect ratio.

The AS3600-2001 Code provides a new formula to account for these variables. Since these parameters are now being accounted for, designers should find that the FRP derived from the formula gives FRP values less than those derived from the graph. Since Figure 5.6.3 is more conservative (so as to account for all the other variables) the new Code has retained this graph and now called it the Deemed to Comply condition for column fire design.

$$\mathrm{FRP(min)} = \frac{k\, f'_c{}^{1.3} D_c{}^{3.3} D_g{}^{1.8}}{10^5 (N^*)^{1.5} L_e{}^{0.9}}$$

The value of k relates to the reinforcement ratio. If $p\% < 2.5\%$ then $k = 1.5$, however is $p\% \geq 2.5\%$ then $k = 1.7$. An example has been provided below to give designers the opportunity to compare FRP values derived using the formula vs the graph.

EXAMPLE 11

A column 500 x 500 with an effective length of 5000 mm is totally engulfed by a fire. The column has an applied design axial load of 4000 kN, contains 32 MPa concrete and is reinforced with 8-N20 bars (cover being 30 mm). Determine the fire resistance period

(FRP) using the formula and compare this with the deemed to comply value using the graph from AS3600.

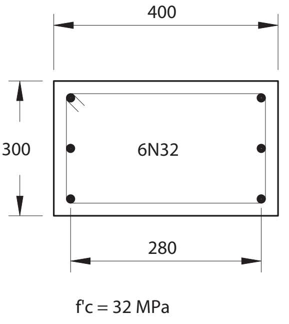
N^*	=	4000 kN
D_c	=	500 mm
D_g	=	500 mm
f'_c	=	32 MPa
A_{st}	=	8-N20
	=	2500 mm^2
p %	=	[2500 / (500 x 500)} x 100
	=	1.0 %
$\therefore k$	=	1.5
$f'_c{}^{1.3}$	=	90.5
$D_c{}^{3.3}$	=	806.5×10^6
$D_g{}^{1.8}$	=	72.1×10^3
$N^{*\,1.5}$	=	253×10^3
$L_e{}^{0.9}$	=	2133
$\therefore$ FRP	=	146 min (use 120 min standard level)

Using Code Figure 5.6.3 (with cover to main reinforcement of 30 mm and minimum column dimension of 500 mm), the actual point corresponds to 105 min. FRP values however are only ever quoted in the standard levels 30, 60, 90, 120, 180 or 240 minutes so our 105 value would fall back to the 90-minute level. It can thus be seen that using the formula provided an extra 30 minutes of fire rating when compared to the deemed to comply figure.

PROBLEMS

QUESTION 1

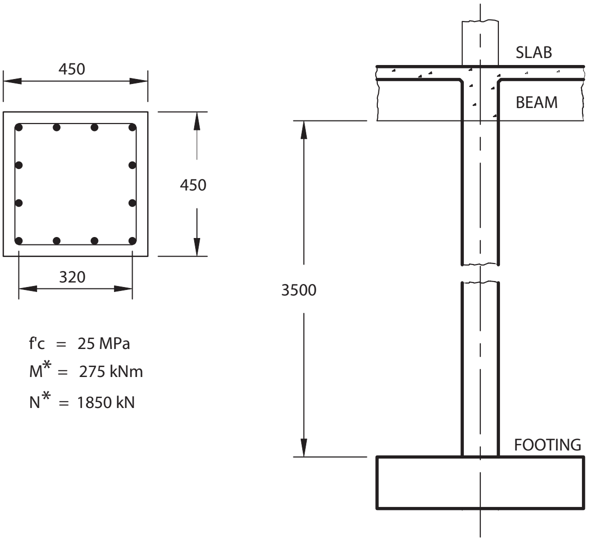
The rectangular column section (b = 300, D = 400, g = 0.7) shown uses grade N32 concrete and it is reinforced with 6Y32 bars placed along the 300mm ends. Calculate effective load and moment capacities at $\Phi M_u = 0$, $k_u = 0$, $k_u = 0.6$, $N_u = 0$, and when $N_u = N_{ub}/2$ and plot the results on a suitable graph. Clearly show the resulting values of ΦN_u and ΦM_u for each point on the graph.



QUESTION 2

The square column shown above is framed into concrete floor beams so that it is rotationally restrained at the top end. At the lower end, the column is supported by a pad footing which is not rotationally restrained. The whole framework is restrained from sidesway by shear walls.

The column has a square cross-section and it is to be reinforced with 12 reinforcing bars evenly distributed between the four faces. Choose the reinforcement for the column.

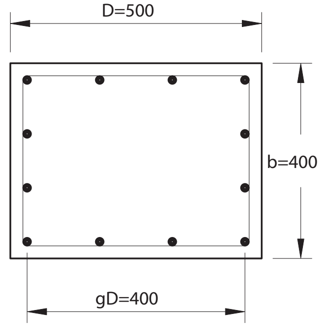


QUESTION 3

The column in question 2 is also subjected to a design moment M^*_y = 100 kNm applied about the y-axis. Check the column for biaxial bending if it is reinforced with 12N32 bars.

QUESTION 4

The following data applies to a braced column bent in single curvature whose cross-section is shown below:



f'_c = 32 MPa

G = 680 kN

Q = 900 kN

N^* = 2200 kN

M^*_1 = 90 kNm

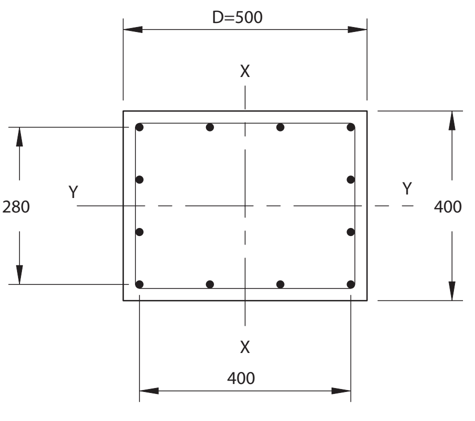
M^*_2 = 250 kNm

g = 0.8 Choose the reinforcement for the column for the following conditions;

(a) L_e = 3600 mm
(b) L_e = 7200 mm

QUESTION 5

The column in question 4 has an effective length L_e = 3600 mm for buckling about both axes. The column is reinforced with 12N32 bars. Check the column for biaxial bending.



f'_c = 32 MPa

G = 680 kN Q = 900 kN

N^* = 2200 kN

M^*_{1x} = 90 kNm

M^*_{2x} = 250 kNm

g_x = 0.8 g_y = 0.7

M^*_{1y} = 40 kNm

M^*_{2y} = 120 kNm

DESIGN TABLES AND CHARTS

Table 2.2 - Areas of Reinforcing Bars in mm^2

No. of Bars	Plain R10 Bars	Bar Diameter 12mm	16mm	20mm	24mm	28mm	32mm	36mm
1	80	110	200	310	450	620	800	1020
2	160	220	400	620	900	1240	1600	2040
3	240	330	600	930	1350	1860	2400	3060
4	320	440	800	1240	1800	2480	3200	4080
5	400	550	1000	1550	2250	3100	4000	5100
6	480	660	1200	1860	2700	3720	4800	6120
7	560	770	1400	2170	3150	4340	5600	7140
8	640	880	1600	2480	3600	4960	6400	8160
9	720	990	1800	2790	4050	5580	7200	9180
10	800	1100	2000	3100	4500	6200	8000	10200
11	880	1210	2200	3410	4950	6820	8800	11220
12	960	1320	2400	3720	5400	7440	9600	12240
13	1040	1430	2600	4030	5850	8060	10400	13260
14	1120	1540	2800	4340	6300	8680	11200	14280
15	1200	1650	3000	4650	6750	9300	12000	15300
16	1280	1750	3200	4960	7200	9920	12800	16320
17	1360	1870	3400	5270	7650	10540	13600	17340
18	1440	1980	3600	5580	8100	11160	14400	18360
19	1520	2090	3800	5890	8550	11780	15200	19380
20	1620	2200	4000	6200	9000	12400	16000	20400

Table 2.3 - Standard welded wire mesh

Ref. No.	Area mm^2/m		Longitudinal Wire		Cross Wire	
	Longitudinal Wires	Cross Wires	Size(mm)	Pitch	Size(mm)	Pitch
RL1218	1112	227	11.9	100	7.6	200
RL1118	891	227	10.65	100	7.6	200
RL1018	709	227	9.5	100	7.6	200
RL918	574	227	8.6	100	7.6	200
RL818	454	227	7.6	100	7.6	200
RL718	358	227	6.75	100	7.6	200
SL81	454	454	7.6	100	7.6	100
SL102	354	354	7.6	200	7.6	200
SL92	290	290	8.6	200	8.6	200
SL82	227	227	7.6	200	7.6	200
SL72	179	179	6.75	200	6.75	200
SL62	141	141	6.0	200	6.0	200
SL52	89	89	4.75	200	4.75	200
SL42	63	63	4	200	4	200

Table 2.4 - Strength Reduction Factors Φ

Type of Action Effect	Strength Reduction Factor Φ
(a) Axial force without bending	
(i) tension	0.8
(ii) compression	0.6
(b) Bending without axial tension or compression where:	
(i) ku $\leq$ 0.4	0.8
(ii) ku > 0.4	$\frac{0.8 M_{ud}}{M_{uo}} \geq 0.6$
(c) Bending with axial tension	$\Phi + (0.8 - \Phi)\left(\frac{N_u}{N_{uot}}\right)$
	the value of Φ is obtained from (b)
(d) Bending with axial compression where:	
(i) Nu $\geq$ Nub	0.6
(ii) Nu < Nub	$0.6 + (\Phi - 0.6)\left(1 - \frac{N_u}{N_{ub}}\right)$
	the value of Φ is obtained from (b)
(e) Shear	0.7
(f) Torsion	0.7
(g) Bearing	0.6
(h) Compression and axial tension in strut and tie action	0.7
(i) Bending shear and compression in plain concrete	0.7
(j) Bending shear and tension in fixings	0.6

Table 3.1 - Exposure Classifications *(i.e. Table 4.3 from AS3600)*

Surface and Exposure Environment	Exposure Classification
1. SURFACES OF MEMBERS IN CONTACT WITH THE GROUND	
(a) Members protected by damp-proof membrane.	A1
(b) Residential footings in non-aggressive soils.	A1
(c) Other members in non-aggressive soils.	A2
(d) Members in aggressive soils.	U
2. SURFACES OF MEMBERS IN INTERIOR ENVIRONMENT	
(a) Fully enclosed within a building except for a brief period of weather exposure during construction.	A1
(b) In industrial buildings, the member being subjected to repeated wetting and drying.	B1
3. SURFACES OF MEMBERS IN ABOVE-GROUND EXTERIOR ENVIRONMENT	
In areas that are:	
(a) Inland (> 50 km from coastline) environment;	
(i) Non-industrial & arid climate.	A1
(ii) Non industrial and temperate climate.	A2
(iii) Non-industrial and tropical climate.	B1
(iv) Industrial and any climate.	B1
(b) Near-coastal (1 km to 50 km from coastline)and any climatic zone.	B1
(c) Coastal (up to 1 km from coastline but excluding tidal and splash zones) and any climatic zone.	A1
4. SURFACES OF MEMBERS IN WATER	
(a) In fresh water.	B1
(b) In sea water -	
(i) permanently submerged.	B2
(ii) in tidal or splash zones.	C
(c) In soft running water.	U
5. SURFACES OF MEMBERS IN OTHER ENVIRONMENTS	
Any exposure environment not otherwise described in items 1 to 4	U

To give the designer a better opportunity to identify the risk of corrosion of the reinforcement, the Code has classified exposures in ascending order of severity. Exposure classifications are designated as A1, A2, B1, B2, and C. Exposure conditions leading to these classifications are described in Table 3.1.

Notes:

1 Climatic zones referred to in Table 3.1 are shown in Code Figure 4.3
2 Industrial refers to areas within 3 km of industries which discharge atmospheric pollutants.
3 Coastal zones include locations <1 km from shorelines of large expanses of salt water.
4 Designations U are undefined and to be determined by the designer.

Table 3.3 - Minimum Cover for Standard Formwork and Compaction

Exposure Classification	Required Cover in mm Characteristic Strength f c				
	20 MPa	25 MPa	32 MPa	40 MPa	≥50 MPa
A1	20	20	20	20	20
A2	(50)	30	25	20	20
B1		(60)	40	30	25
B2			(65)	45	35
C				(70)	50

Table 3.4 - Areas of Reinforcemnt and Minimum Beam Dimensions for Exposure Classification A1

Bar Dia.	Number per Row	b_{min}	1 Row Area	1 Row c_{min}	2 Rows Area	2 Rows c_{min}	3 Rows Area	3 Rows c_{min}
	2	118	220		440		660	
	3	160	330		660		990	
12	4	202	440	38	880	60	1320	82
	5	244	550		1100		1650	
	6	286	660		1320		1980	
	2	126	400		800		1200	
	3	172	600		1200		1800	
16	4	218	800	40	1600	64	2400	88
	5	264	1000		2000		3000	
	6	310	1200		2400		3600	
	2	134	620		1240		1860	
	3	184	930		1860		2790	
20	4	234	1240	42	2480	68	3720	94
	5	284	1550		3100		4650	
	6	334	1860		3720		5580	
	2	142	900		1800		2700	
	3	196	1350		2700		4050	
24	4	250	1800	44	3600	72	5400	100
	5	304	2250		4500		6750	
	6	358	2700		5400		8100	
	2	150	1240		2480		3720	
	3	208	1860		3720		5580	
28	4	266	2480	46	4960	76	7440	106
	5	324	3100		6200		9300	
	6	382	3720		7440		11160	
	2	160	1600		3200		4800	
	3	224	2400		4800		7200	
32	4	288	3200	48	6400	80	9600	112
	5	352	4000		8000		12000	
	6	416	4800		9600		14400	
	2	172	2040		4080		6120	
	3	244	3060		6120		9180	
36	4	316	4080	50	8160	84	12240	118
	5	388	5100		10200		15300	
	6	460	6120		12240		18360	

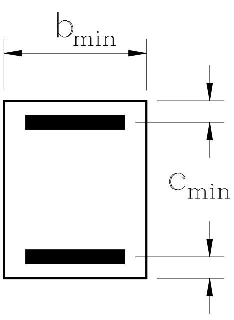


TABLE 7.1 TENSILE DEVELOPMENT LENGTH Lsy.t

		Tensile Development Length Lsy.t in mm for Standard Compaction						
		Bar Size						
f′c	a	N12	N16	N20	N24	N28	N32	N36
	15	592	961	1382	1841	2337	2853	3400
	20	478	790	1151	1554	1993	2457	2953
	25	401	670	987	1344	1737	2157	2609
	30	345	582	864	1184	1540	1923	2338
	35	303	514	768	1058	1383	1734	2117
25	40	300	461	691	956	1255	1579	1934
	45	300	417	628	872	1148	1450	1781
	50	300	400	576	802	1059	1340	1650
	55	300	400	531	742	982	1246	1537
	60	300	400	500	646	858	1092	1352
	65	300	400	500	606	807	1028	1275
	15	523	850	1221	1628	2065	2522	3005
	20	423	698	1018	1373	1762	2171	2610
	25	354	592	872	1188	1536	1907	2306
	30	305	514	763	1046	1361	1699	2066
	35	300	454	678	935	1222	1533	1871
32	40	300	407	611	845	1109	1396	1710
	45	300	400	555	771	1015	1281	1574
	50	300	400	509	709	936	1184	1458
	55	300	400	500	656	868	1101	1359
	60	300	400	500	610	809	1029	1271
	70	300	400	500	600	758	965	1195
	15	468	760	1092	1456	1847	2255	2688
	20	378	624	910	1228	1576	1942	2334
	25	317	530	780	1062	1374	1705	2063
	30	300	460	683	936	1217	1520	1848
	35	300	406	607	836	1093	1371	1674
40	40	300	400	546	756	992	1249	1529
	45	300	400	500	690	908	1146	1408
	50	300	400	500	634	837	1059	1304
	55	300	400	500	600	776	985	1215
	60	300	400	508	600	724	920	1137
	65	300	400	500	600	700	863	1069
	15	419	680	977	1302	1652	2017	2404
	20	338	558	814	1099	1409	1737	2088
	25	300	474	698	950	1229	1525	1845
	30	300	411	611	837	1089	1359	1653
50	35	300	400	543	748	978	1226	1497
	40	300	400	500	676	887	1117	1368
	45	300	400	500	617	812	1025	1259
	50	300	400	500	600	749	948	1167
	55	300	400	500	600	700	881	1087

Table 8.1 - Limits for Calculated Deflection of Beams and Slabs

Type of Member	Deflection to be Considered	Deflection Limitation Δ/L_{eff} for Spans Notes 1 and 2	for Cantilevers Notes 3
All members	The total deflection	1/250	1/125
Members supporting masonry partitions	The deflection which occurs after the addition or attachment of the partitions.	1/500 where provision is made to minimise the effect of movement, otherwise 1/1000	1/250 where provision is made to minimise effect of movement, otherwise 1/500
Bridge members	The live load and impact deflection	1/800	1/400

Notes:

1 In flat slabs, the deflection to which the above limits apply is the theoretical deflection of the line diagram representing the idealised frame.

2 Deflection limits given may not safeguard against ponding.

3 For cantilevers, the value of D/L_{eff} given in this table applies only if the rotation at the support is included in the deflection calculations.

Table 8.3 - Areas of Reinforcement mm2 / m Width

Bar Spacing	Bar Sizes 12 mm	16 mm	20 mm	24 mm	28 mm	32 mm
100	1 100	2 000	3 100	4 500	6 200	8 000
125	880	1 600	2 480	3 600	4 960	6 400
150	733	1 333	2 067	3 000	4 133	5 333
175	629	1 143	1 771	2 571	3 543	4 571
200	550	1 000	1 550	2 250	3 100	4 000
225	489	889	1 378	2 000	2 756	3 556
250	440	800	1 240	1 800	2 480	3 200
275	400	727	1 127	1 636	2 255	2 909
300	367	667	1 033	1 500	2 067	2 667
325	338	615	954	1 385	1 908	2 462
350	314	571	886	1 286	1 771	2 286

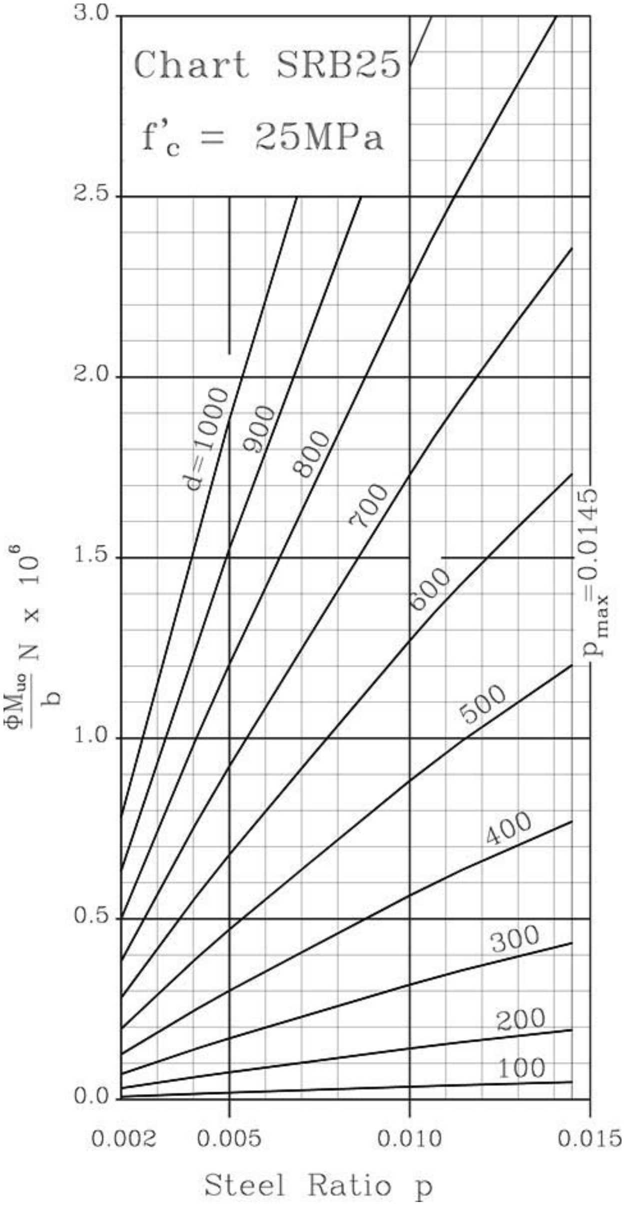


Chart SRB25
f'c = 25MPa
d=1000
900
800
700
600
500
400
300
200
100
pmax=0.0145
ΦMuo/b N x 10^6
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.002
0.005
0.010
0.015
Steel Ratio p

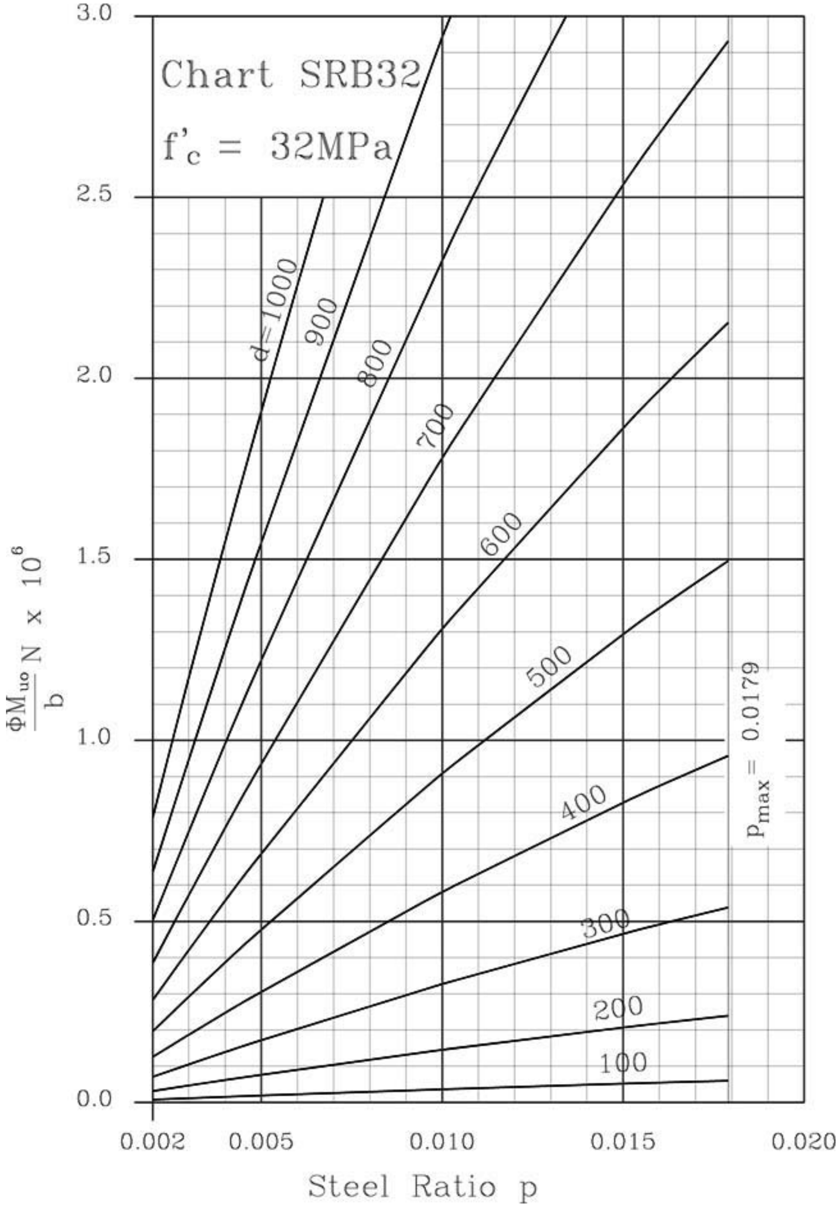


Chart SRB32
f'c = 32MPa
d=1000
900
800
700
600
500
400
300
200
100
pmax = 0.0179
ΦMuo/b N x 10^6
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.002
0.005
0.010
0.015
0.020
Steel Ratio p

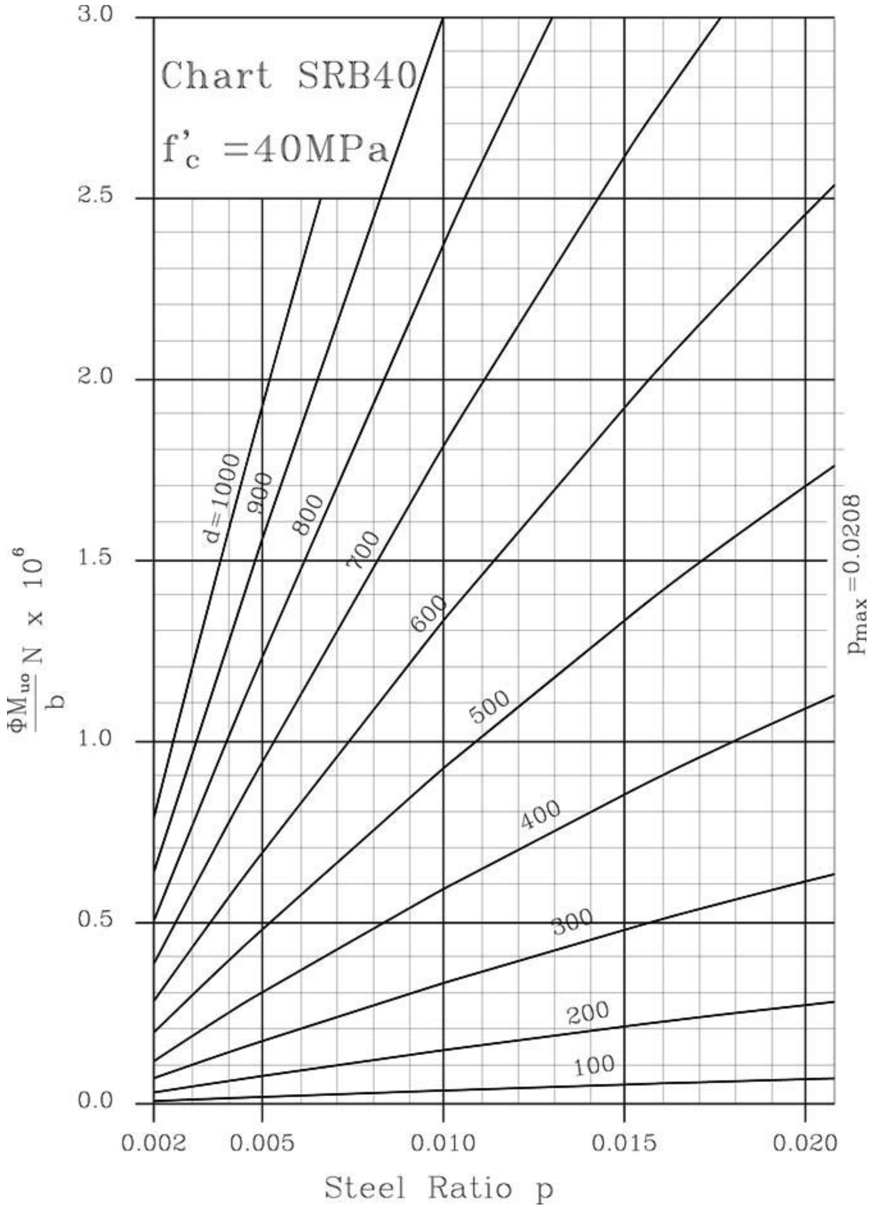


Chart SRB40
f'c =40MPa
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ΦMuo/b N x 10^6
d=1000
900
800
700
600
500
400
300
200
100
pmax =0.0208
0.002
0.005
0.010
0.015
0.020
Steel Ratio p

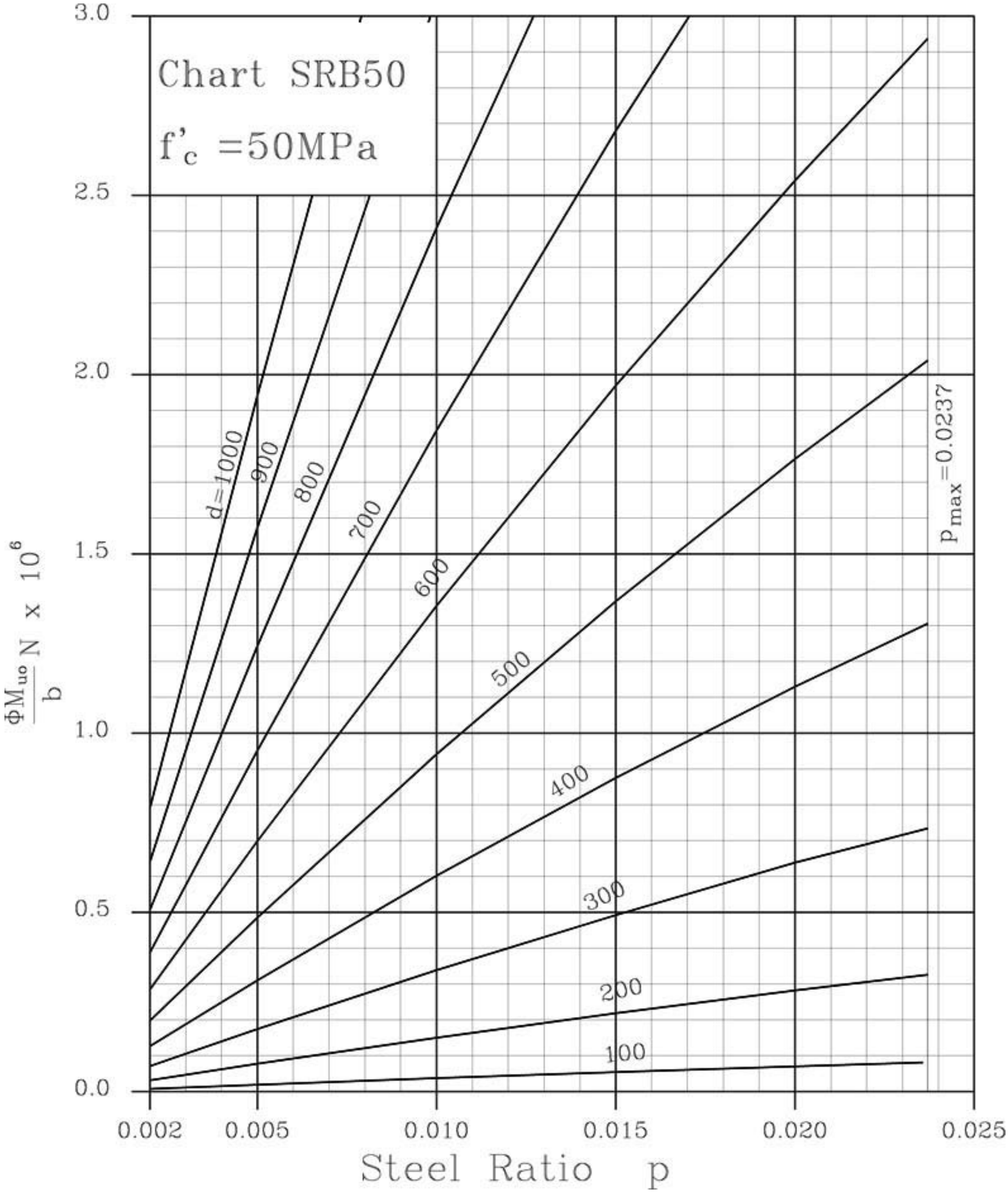


Chart SRB50
f'c =50MPa
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\frac{\Phi M_{uo}}{b}$ N x 10^6
d=1000
900
800
700
600
500
400
300
200
100
p max =0.0237
0.002
0.005
0.010
0.015
0.020
0.025
Steel Ratio p

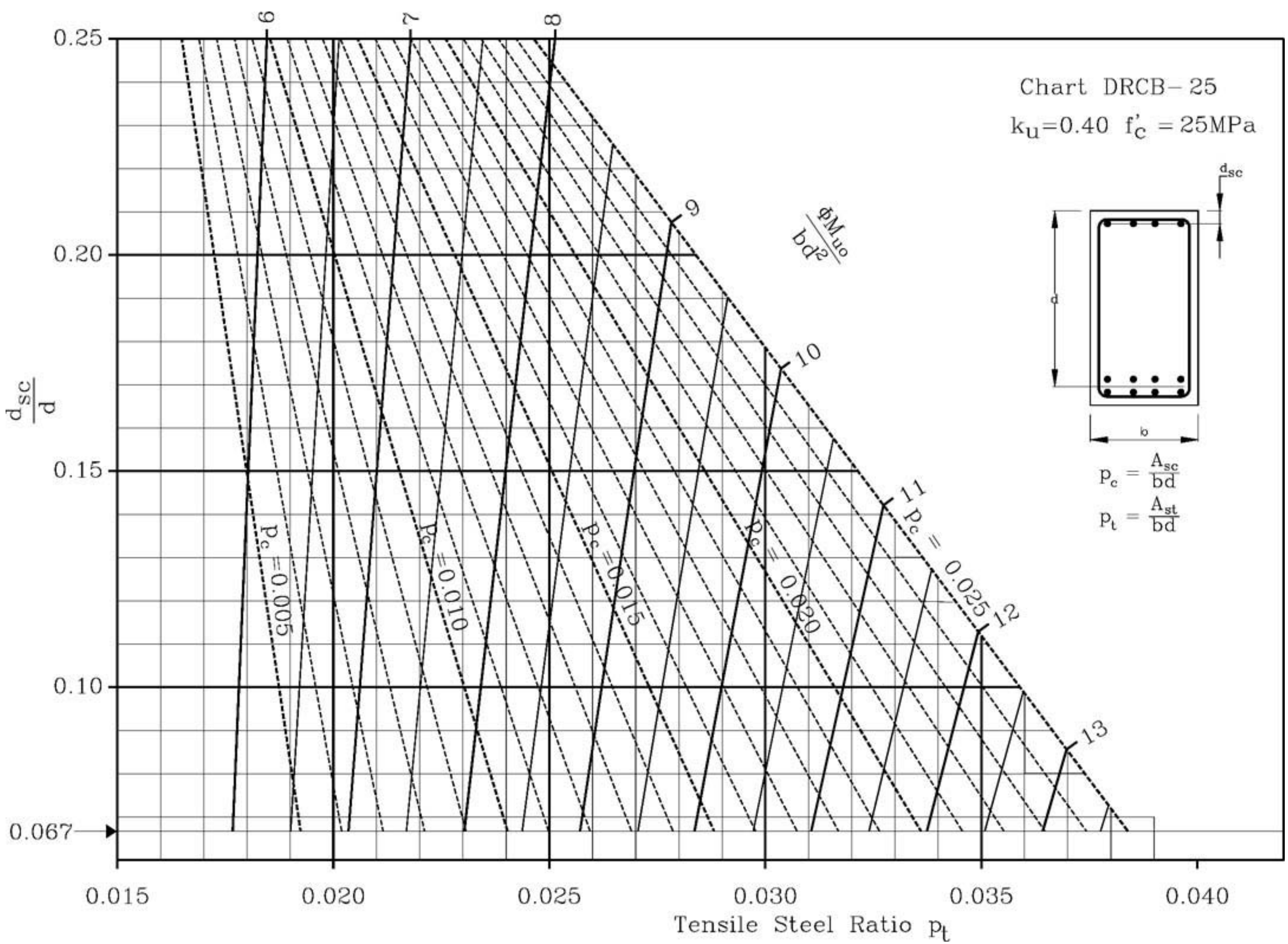


Chart DRCB-25
k_u=0.40 f'_c = 25MPa
$\frac{\phi M_{uo}}{bd^2}$
6
7
8
9
10
11
12
13
p_c = 0.005
p_c = 0.010
p_c = 0.015
p_c = 0.020
p_c = 0.025
d_{sc}
d
b
$p_c = \frac{A_{sc}}{bd}$
$p_t = \frac{A_{st}}{bd}$
$\frac{d_{sc}}{d}$
0.25
0.20
0.15
0.10
0.067
0.015
0.020
0.025
0.030
0.035
0.040
Tensile Steel Ratio p_t

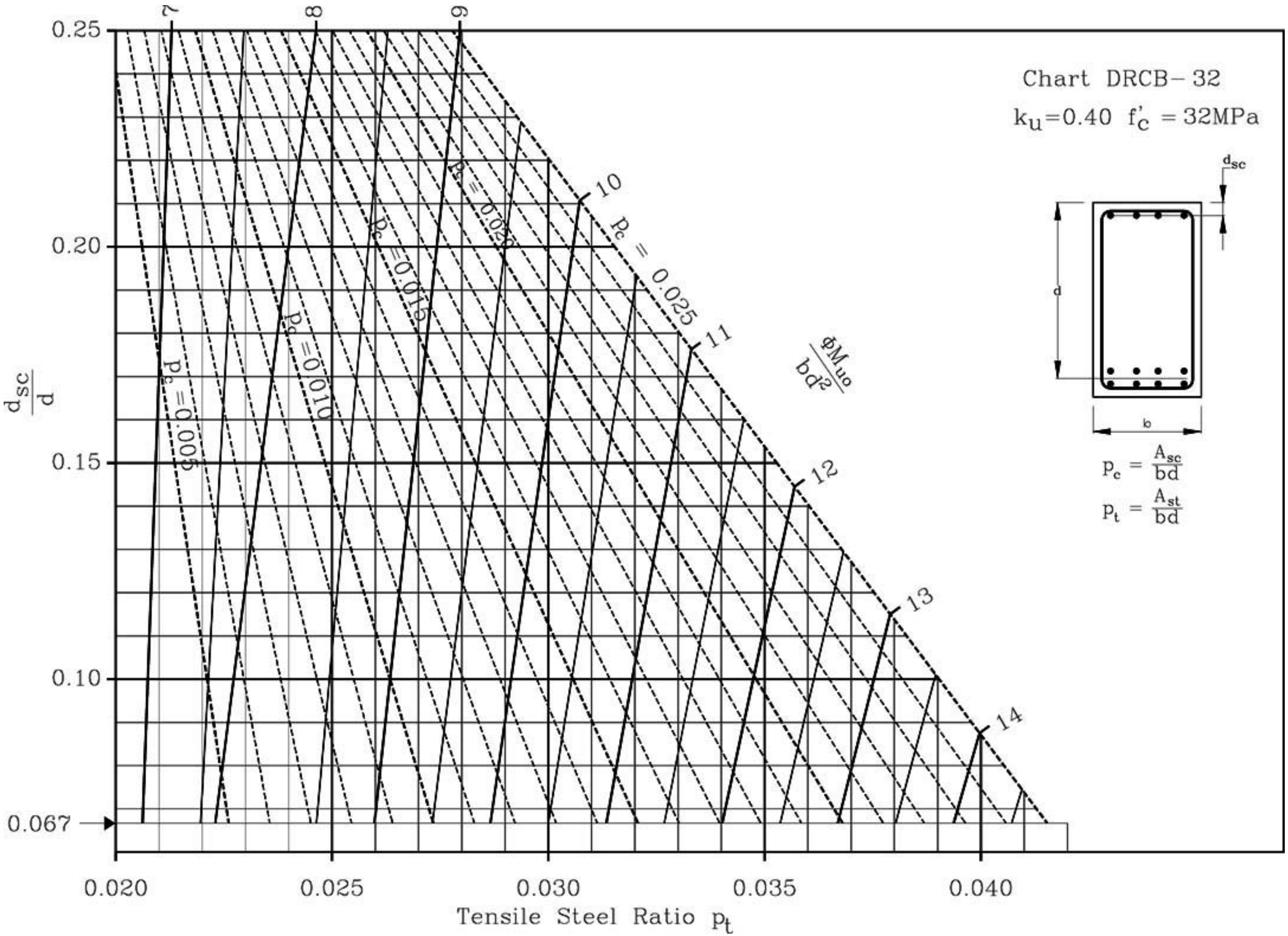


Chart DRCB– 32
k_u=0.40 f'_c = 32MPa
d_{sc}
d
b
$p_c = \frac{A_{sc}}{bd}$
$p_t = \frac{A_{st}}{bd}$
$\frac{\phi M_{uo}}{bd^2}$
7
8
9
10
11
12
13
14
p_c = 0.005
p_c = 0.010
p_c = 0.015
p_c = 0.020
p_c = 0.025
$\frac{d_{sc}}{d}$
0.25
0.20
0.15
0.10
0.067
0.020
0.025
0.030
0.035
0.040
Tensile Steel Ratio p_t

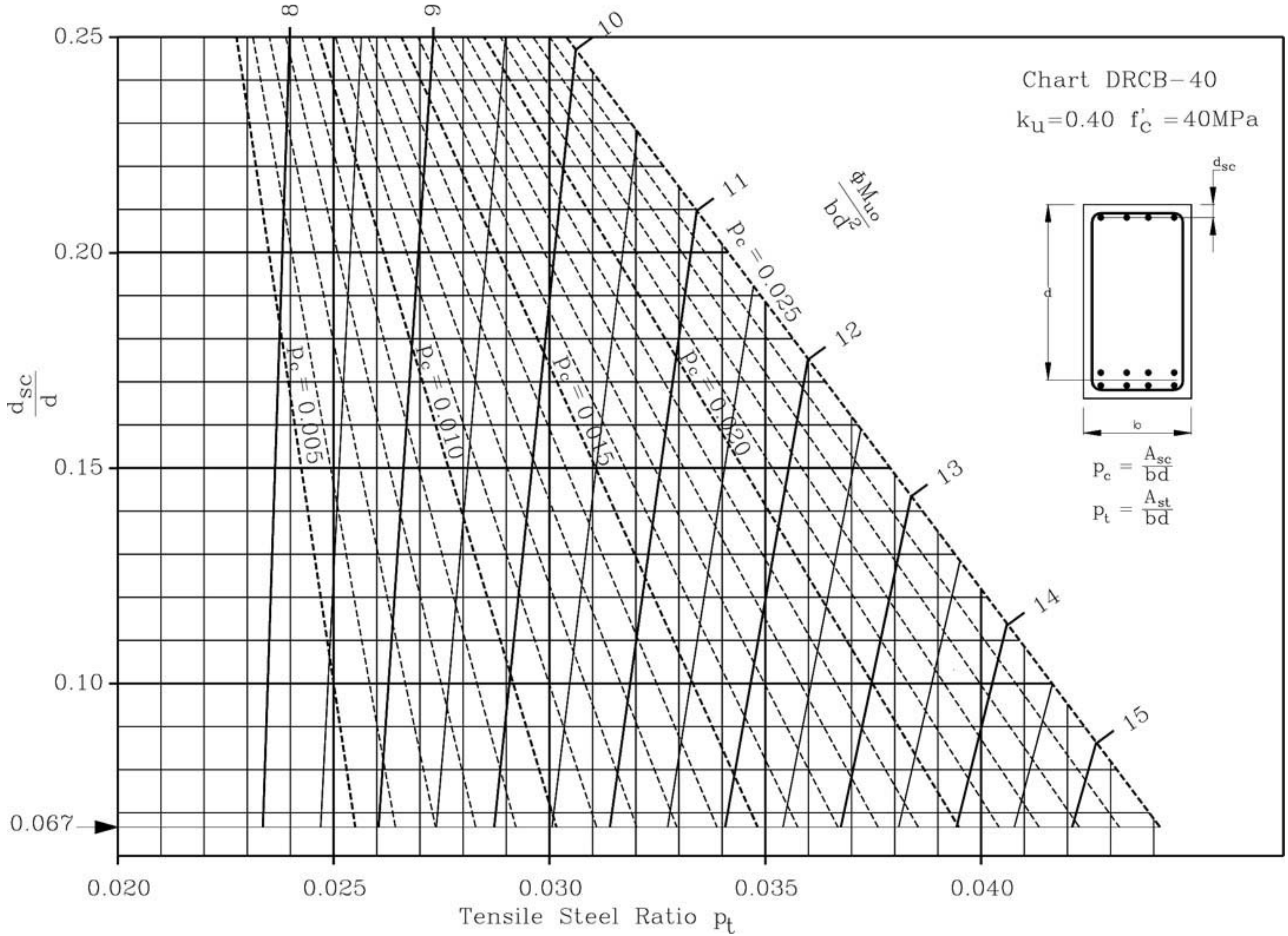


Chart DRCB-40
$k_u = 0.40$ $f'_c = 40$MPa
$\frac{\phi M_{uo}}{bd^2}$
8
9
10
11
12
13
14
15
$p_c = 0.005$
$p_c = 0.010$
$p_c = 0.015$
$p_c = 0.020$
$p_c = 0.025$
$\frac{d_{sc}}{d}$
0.25
0.20
0.15
0.10
0.067
0.020
0.025
0.030
0.035
0.040
Tensile Steel Ratio p_t
d_{sc}
d
b
$p_c = \frac{A_{sc}}{bd}$
$p_t = \frac{A_{st}}{bd}$

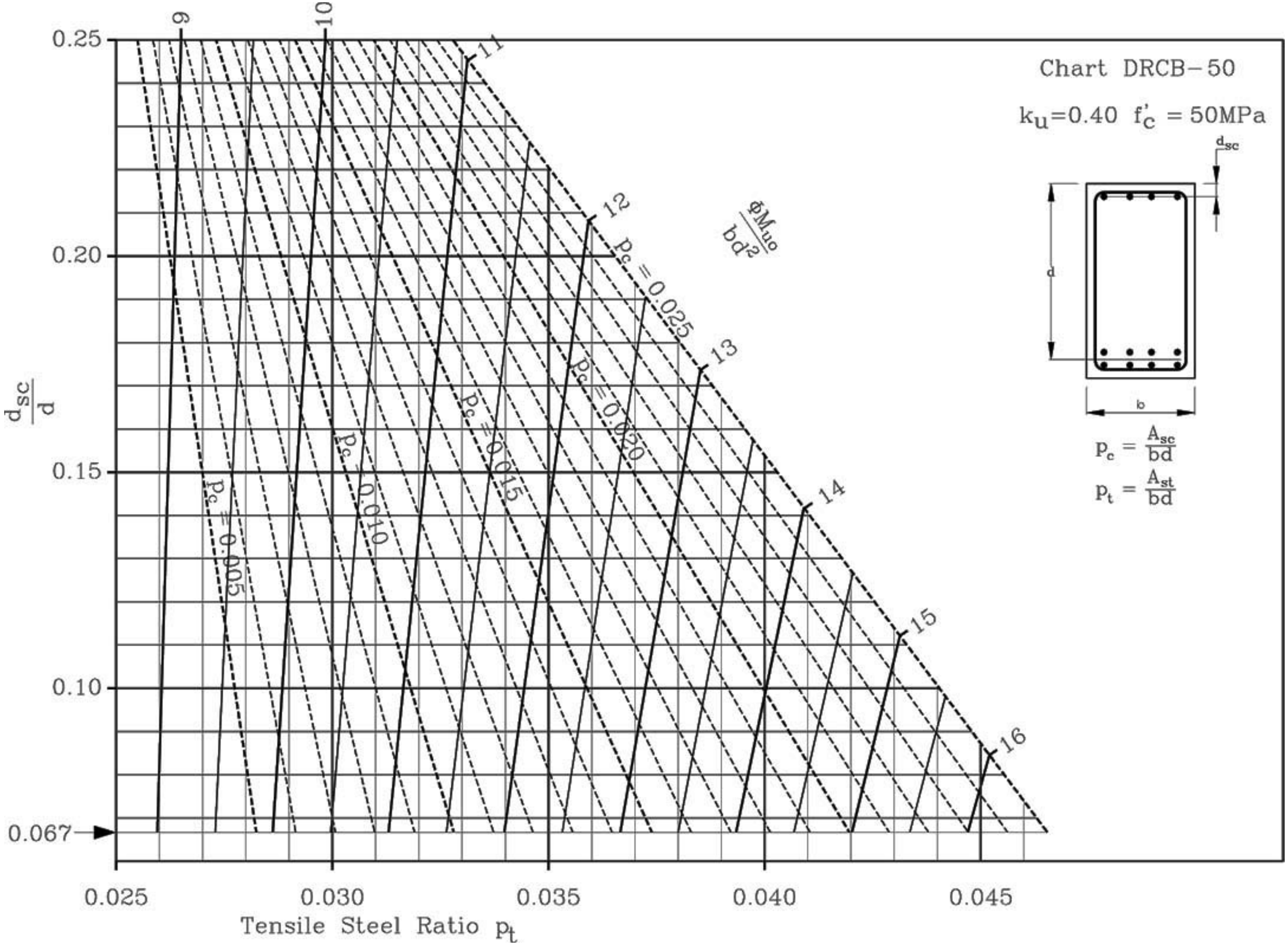


Chart DRCB−50
k_u=0.40 f'_c = 50MPa
d_{sc}
d
b
$p_c = \frac{A_{sc}}{bd}$
$p_t = \frac{A_{st}}{bd}$
$\frac{\phi M_{uo}}{bd^2}$
9
10
11
12
13
14
15
16
p_c = 0.005
p_c = 0.010
p_c = 0.015
p_c = 0.020
p_c = 0.025
$\frac{d_{sc}}{d}$
0.25
0.20
0.15
0.10
0.067
0.025
0.030
0.035
0.040
0.045
Tensile Steel Ratio p_t

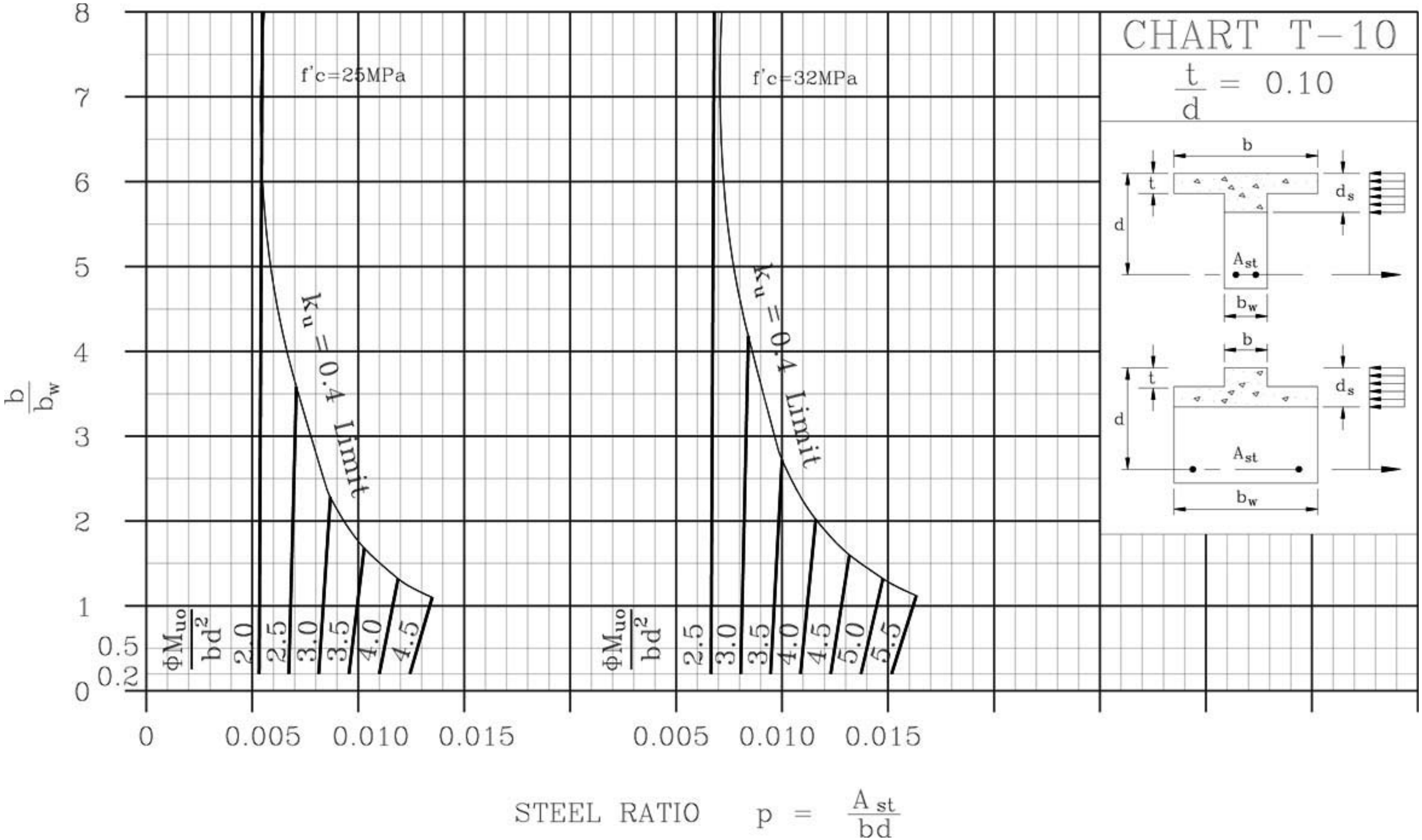


CHART T−10
t/d = 0.10
f'c=25MPa
f'c=32MPa
ku = 0.4 Limit
ku = 0.4 Limit
ΦMuo/bd²
2.0
2.5
3.0
3.5
4.0
4.5
ΦMuo/bd²
2.5
3.0
3.5
4.0
4.5
5.0
5.5
b/bw
8
7
6
5
4
3
2
1
0.5
0.2
0
0
0.005
0.010
0.015
0.005
0.010
0.015
b
t
d
ds
Ast
bw
b
t
d
ds
Ast
bw
STEEL RATIO p = Ast/bd

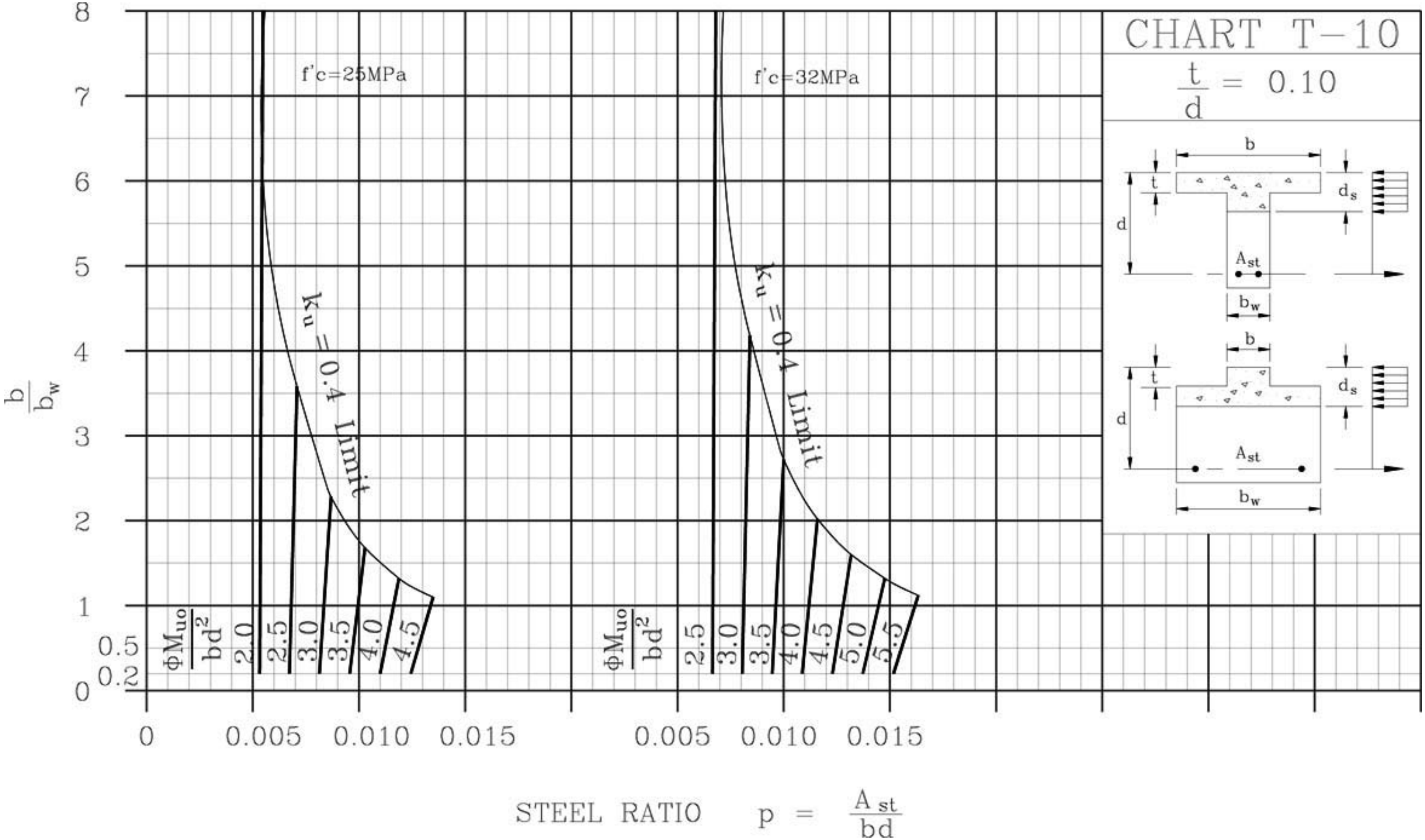


CHART T−10
$\frac{t}{d} = 0.10$
f'c=25MPa
f'c=32MPa
$k_u = 0.4$ Limit
$k_u = 0.4$ Limit
$\frac{\Phi M_{uo}}{bd^2}$
2.0 2.5 3.0 3.5 4.0 4.5
$\frac{\Phi M_{uo}}{bd^2}$
2.5 3.0 3.5 4.0 4.5 5.0 5.5
$\frac{b}{b_w}$
0 0.2 0.5 1 2 3 4 5 6 7 8
0 0.005 0.010 0.015 0.005 0.010 0.015
STEEL RATIO $p = \frac{A_{st}}{bd}$
b
t
d
d_s
A_{st}
b_w

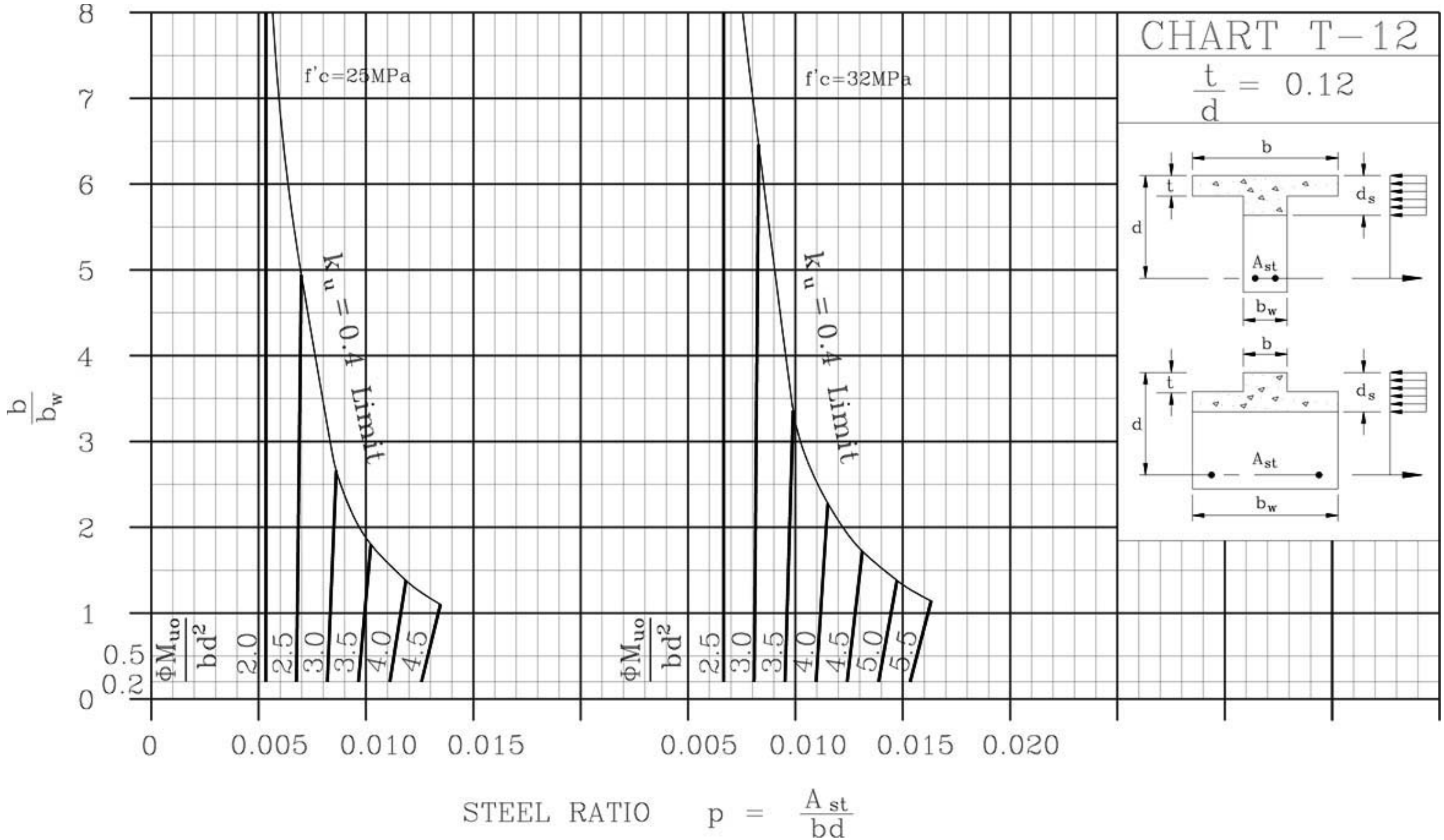


CHART T–12
t/d = 0.12
f'c=25MPa
f'c=32MPa
ku = 0.4 Limit
ku = 0.4 Limit
ΦMuo/bd²
2.0
2.5
3.0
3.5
4.0
4.5
ΦMuo/bd²
2.5
3.0
3.5
4.0
4.5
5.0
5.5
b/bw
8
7
6
5
4
3
2
1
0.5
0.2
0
0
0.005
0.010
0.015
0.005
0.010
0.015
0.020
STEEL RATIO p = Ast/bd
b
t
ds
d
Ast
bw
b
t
ds
d
Ast
bw

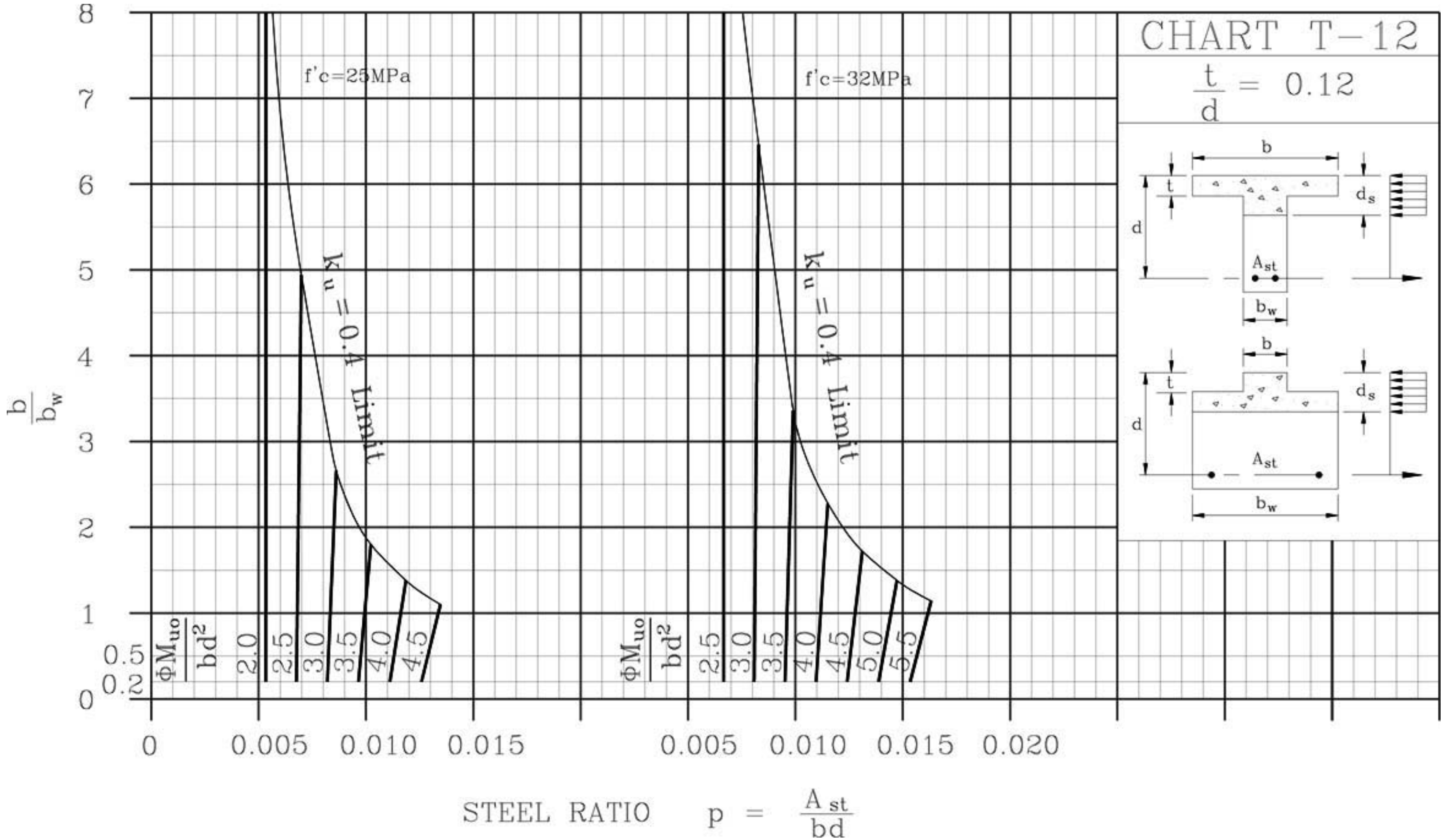


CHART T−12
$\frac{t}{d} = 0.12$
f'c=25MPa
f'c=32MPa
$k_u = 0.4$ Limit
$\frac{b}{b_w}$
$\frac{\Phi M_{uo}}{bd^2}$
2.0 2.5 3.0 3.5 4.0 4.5
2.5 3.0 3.5 4.0 4.5 5.0 5.5
8 7 6 5 4 3 2 1 0.5 0.2 0
0 0.005 0.010 0.015
0.005 0.010 0.015 0.020
b
t
d
d_s
A_{st}
b_w
STEEL RATIO $p = \frac{A_{st}}{bd}$

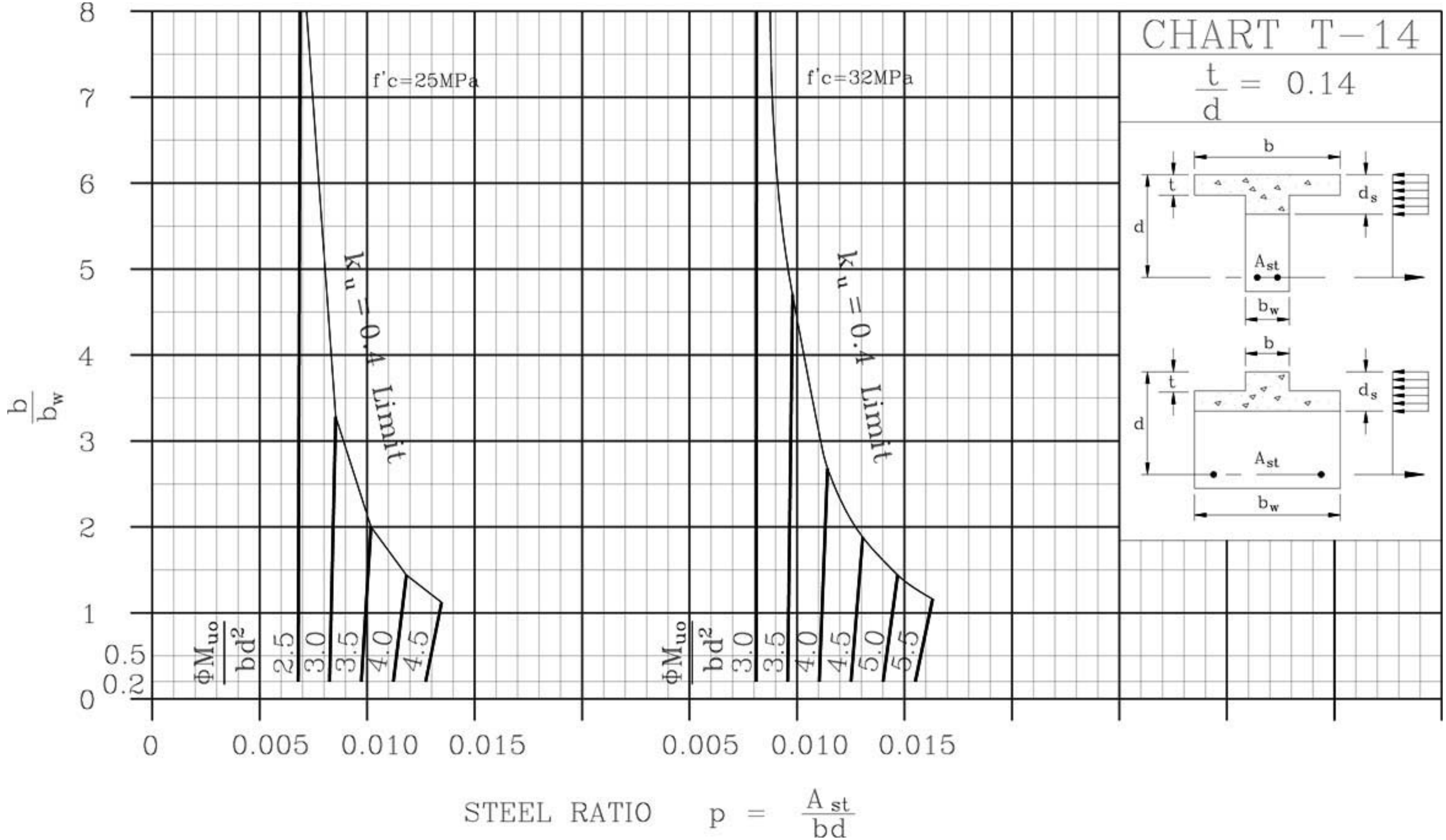


CHART T–14
$\frac{t}{d} = 0.14$
b
t
d_s
d
A_{st}
b_w
f'c=25MPa
f'c=32MPa
$k_u = 0.4$ Limit
$k_u = 0.4$ Limit
$\frac{\phi M_{uo}}{bd^2}$
2.5
3.0
3.5
4.0
4.5
3.0
3.5
4.0
4.5
5.0
5.5
$\frac{b}{b_w}$
8
7
6
5
4
3
2
1
0.5
0.2
0
0.005
0.010
0.015
STEEL RATIO $p = \frac{A_{st}}{bd}$

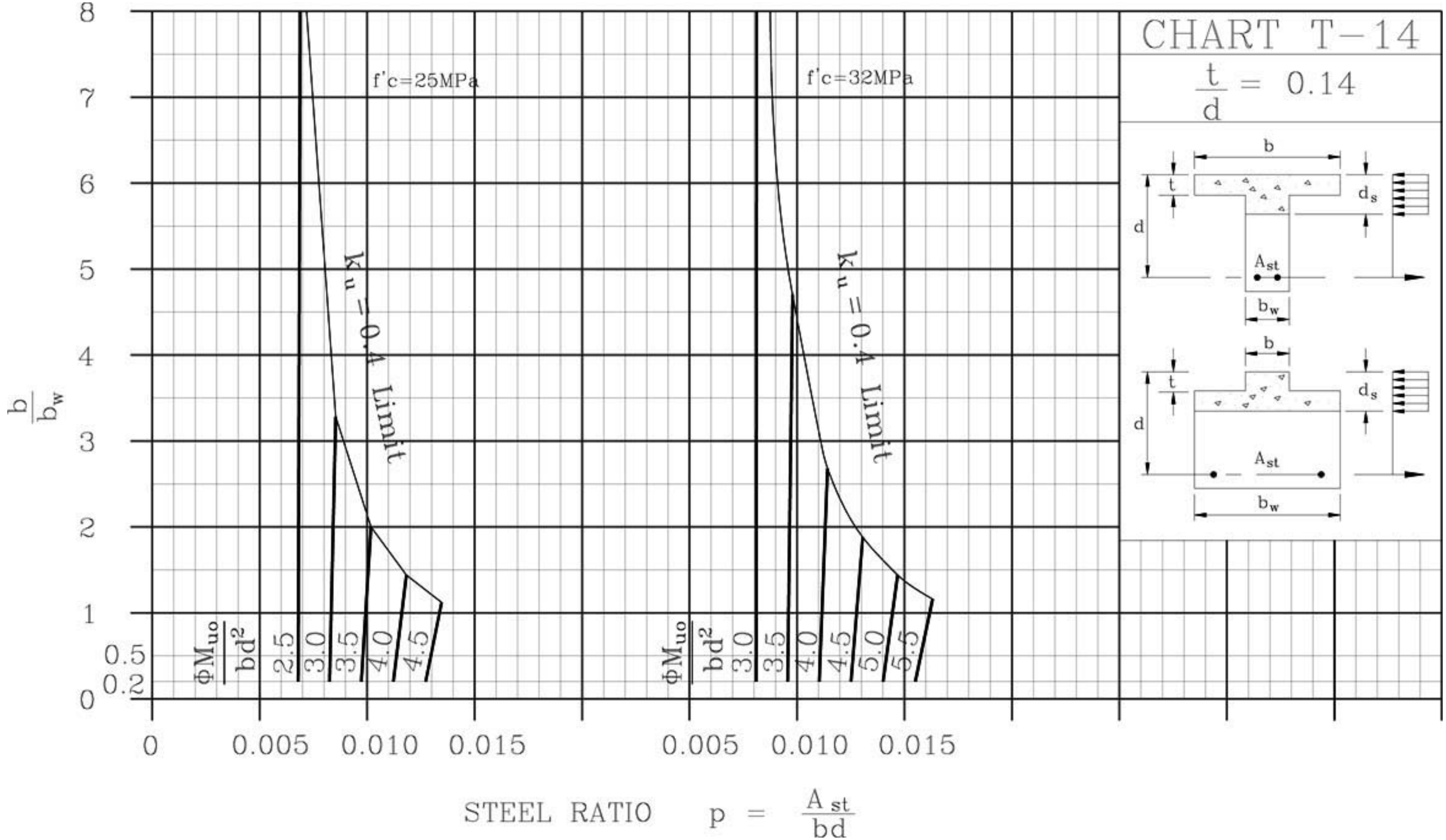


CHART T-14
t/d = 0.14
f'c=25MPa
f'c=32MPa
$k_u = 0.4$ Limit
$k_u = 0.4$ Limit
$\frac{\phi M_{uo}}{bd^2}$
2.5
3.0
3.5
4.0
4.5
3.0
3.5
4.0
4.5
5.0
5.5
b/b_w
b
t
d
d_s
A_st
b_w
0
0.005
0.010
0.015
0.005
0.010
0.015
0
0.2
0.5
1
2
3
4
5
6
7
8
STEEL RATIO p = A_st/bd

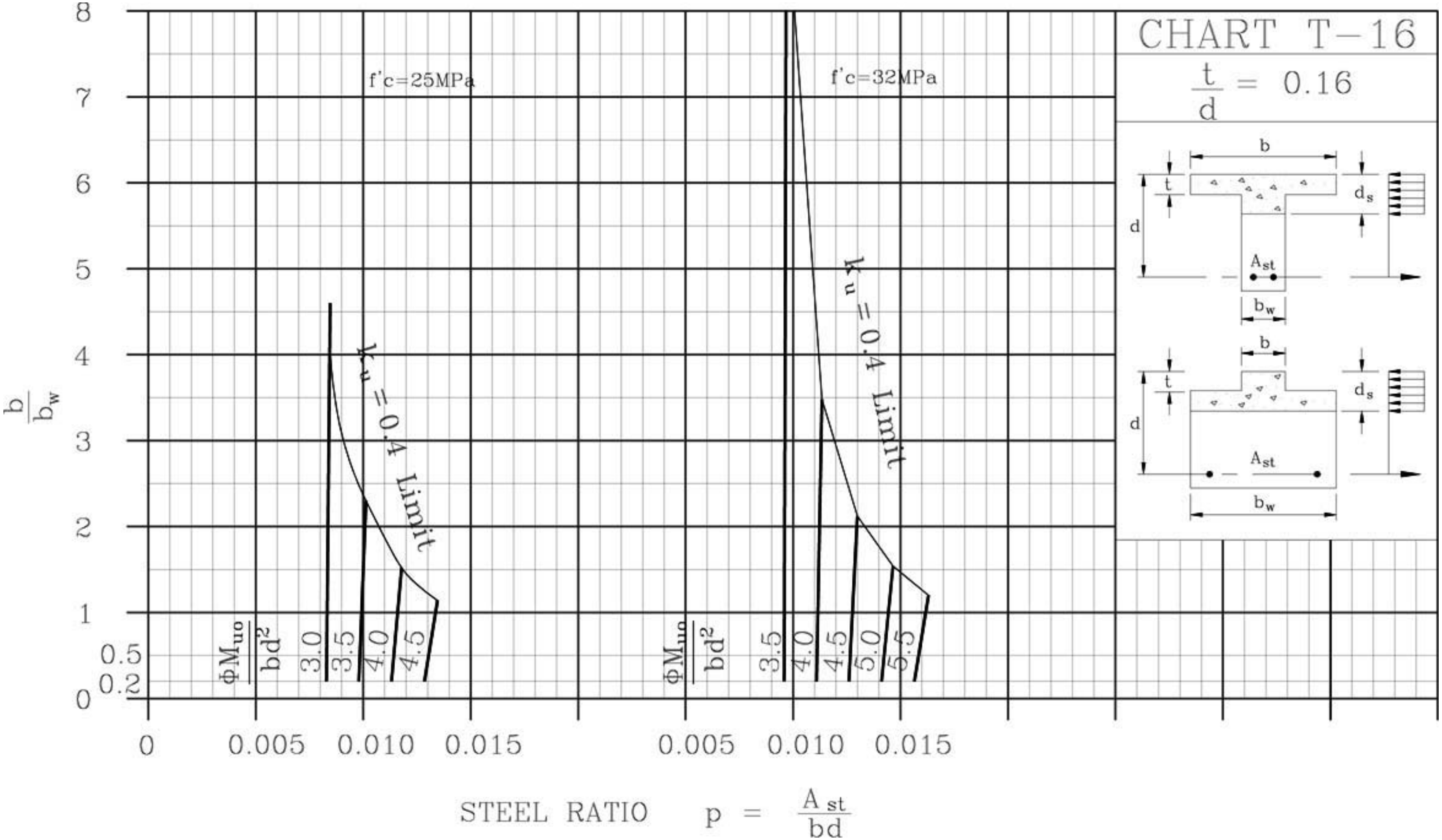


CHART T−16
t/d = 0.16
f'c=25MPa
f'c=32MPa
ku = 0.4 Limit
ku = 0.4 Limit
b/bw
ΦMuo/bd²
3.0
3.5
4.0
4.5
ΦMuo/bd²
3.5
4.0
4.5
5.0
5.5
b
t
d
ds
Ast
bw
0
0.005
0.010
0.015
0.005
0.010
0.015
0
0.2
0.5
1
2
3
4
5
6
7
8
STEEL RATIO p = Ast/bd

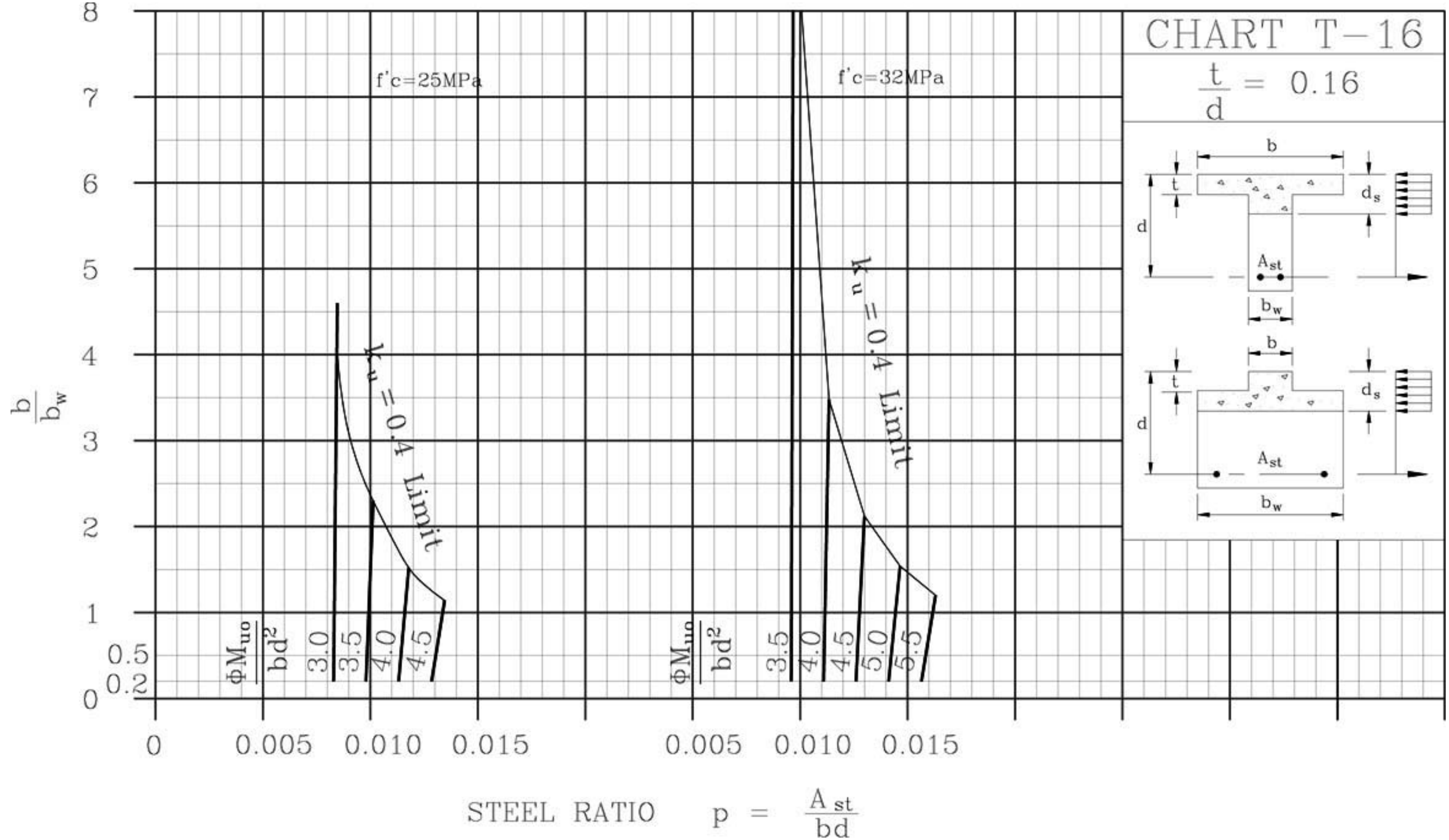


CHART T−16
t/d = 0.16
f'c=25MPa
f'c=32MPa
$k_u = 0.4$ Limit
$k_u = 0.4$ Limit
$\frac{\phi M_{uo}}{bd^2}$
3.0
3.5
4.0
4.5
$\frac{\phi M_{uo}}{bd^2}$
3.5
4.0
4.5
5.0
5.5
$\frac{b}{b_w}$
0
0.2
0.5
1
2
3
4
5
6
7
8
0
0.005
0.010
0.015
0.005
0.010
0.015
STEEL RATIO p = $\frac{A_{st}}{bd}$
b
t
d_s
d
A_{st}
b_w
b
t
d_s
d
A_{st}
b_w

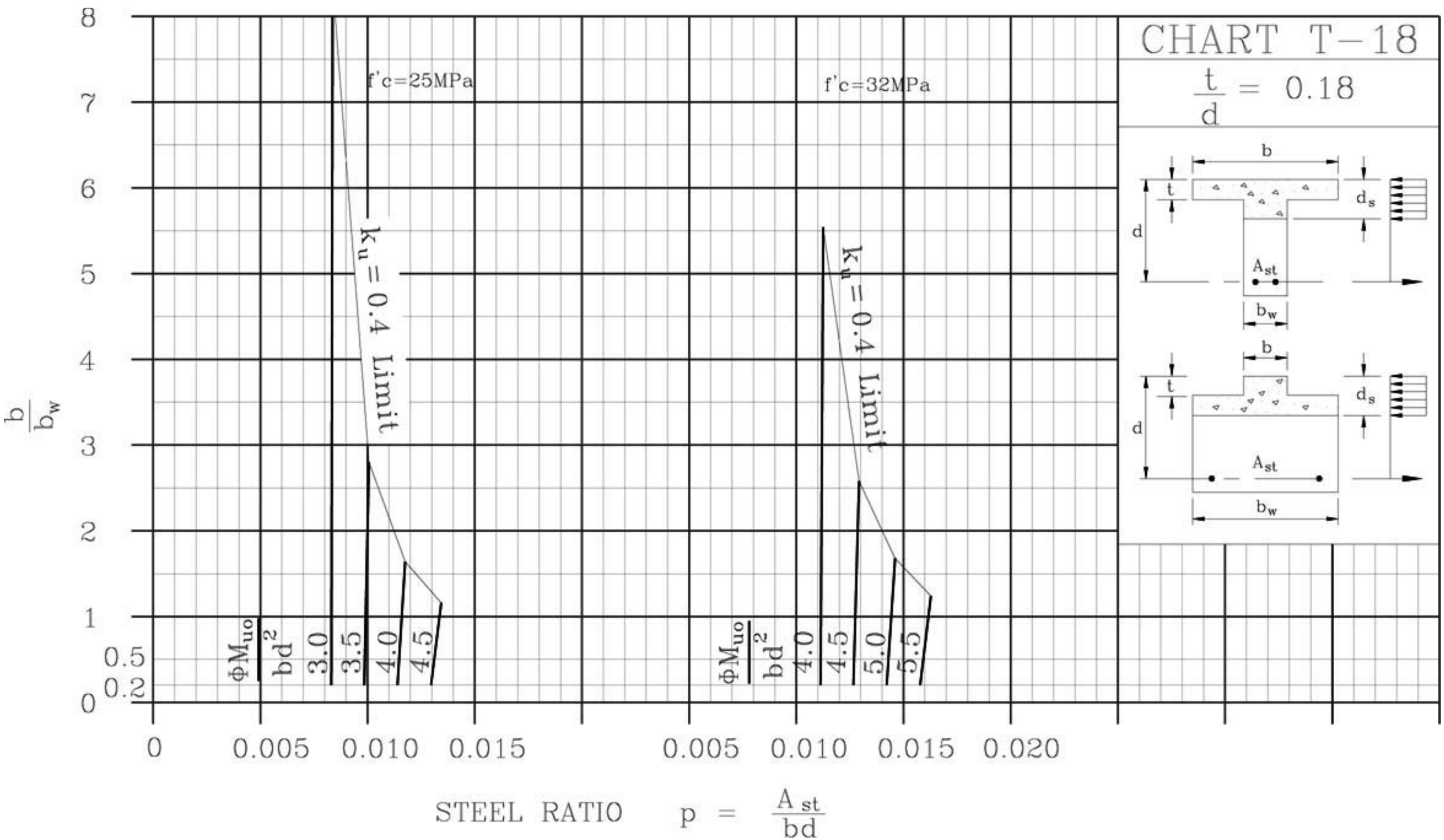


CHART T-18
t/d = 0.18
f'c=25MPa
f'c=32MPa
ku = 0.4 Limit
ku = 0.4 Limit
ΦMuo/bd²
3.0
3.5
4.0
4.5
ΦMuo/bd²
4.0
4.5
5.0
5.5
b/bw
8
7
6
5
4
3
2
1
0.5
0.2
0
0
0.005
0.010
0.015
0.005
0.010
0.015
0.020
STEEL RATIO p = Ast/bd
b
t
ds
d
Ast
bw
b
t
ds
d
Ast
bw

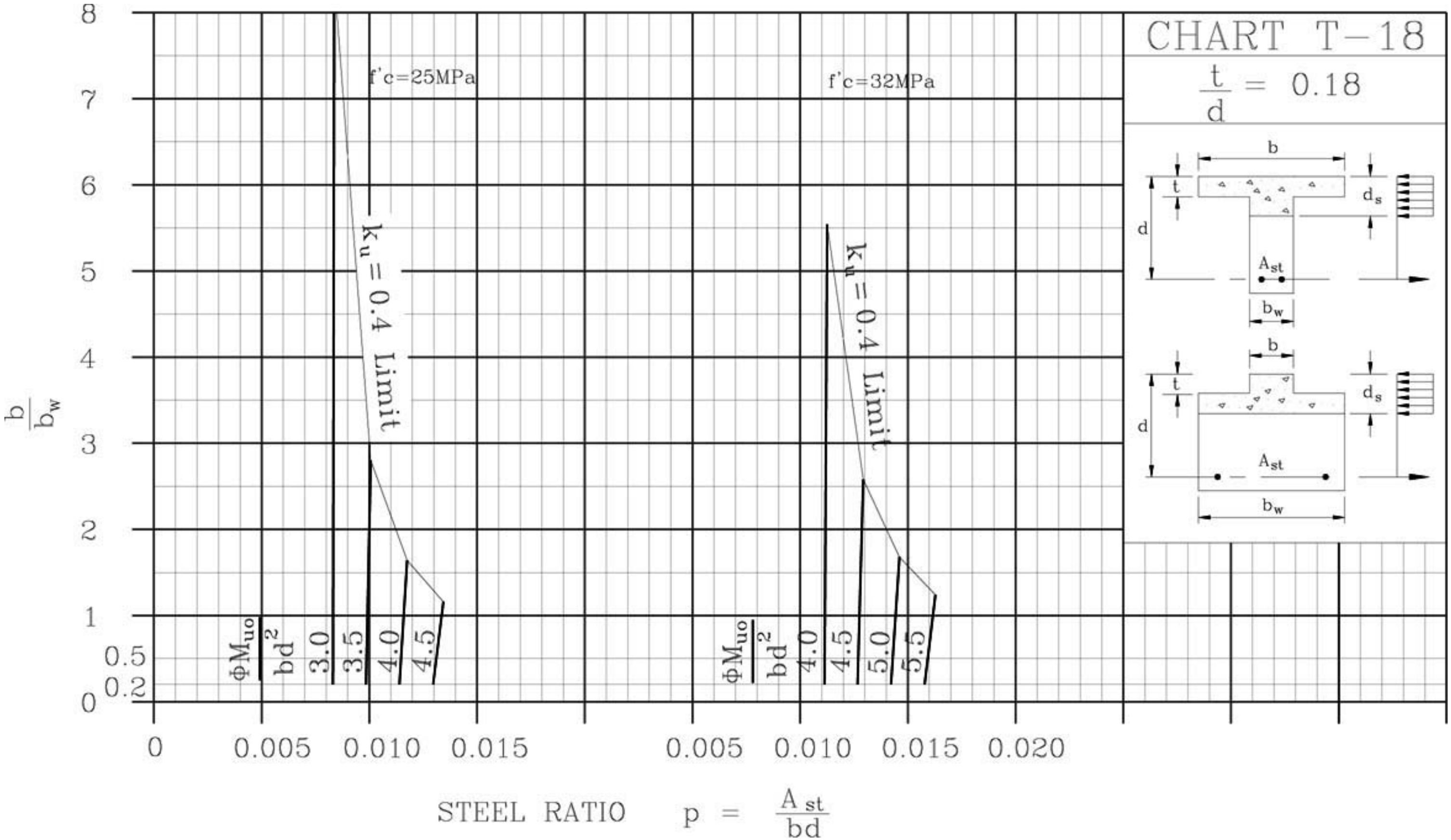


CHART T–18
$\frac{t}{d} = 0.18$
f'c=25MPa
f'c=32MPa
$k_u = 0.4$ Limit
$k_u = 0.4$ Limit
$\frac{\phi M_{uo}}{bd^2}$
3.0
3.5
4.0
4.5
$\frac{\phi M_{uo}}{bd^2}$
4.0
4.5
5.0
5.5
$\frac{b}{b_w}$
8
7
6
5
4
3
2
1
0.5
0.2
0
0
0.005
0.010
0.015
0.005
0.010
0.015
0.020
STEEL RATIO $p = \frac{A_{st}}{bd}$
b
t
d
d_s
A_{st}
b_w
b
t
d
d_s
A_{st}
b_w

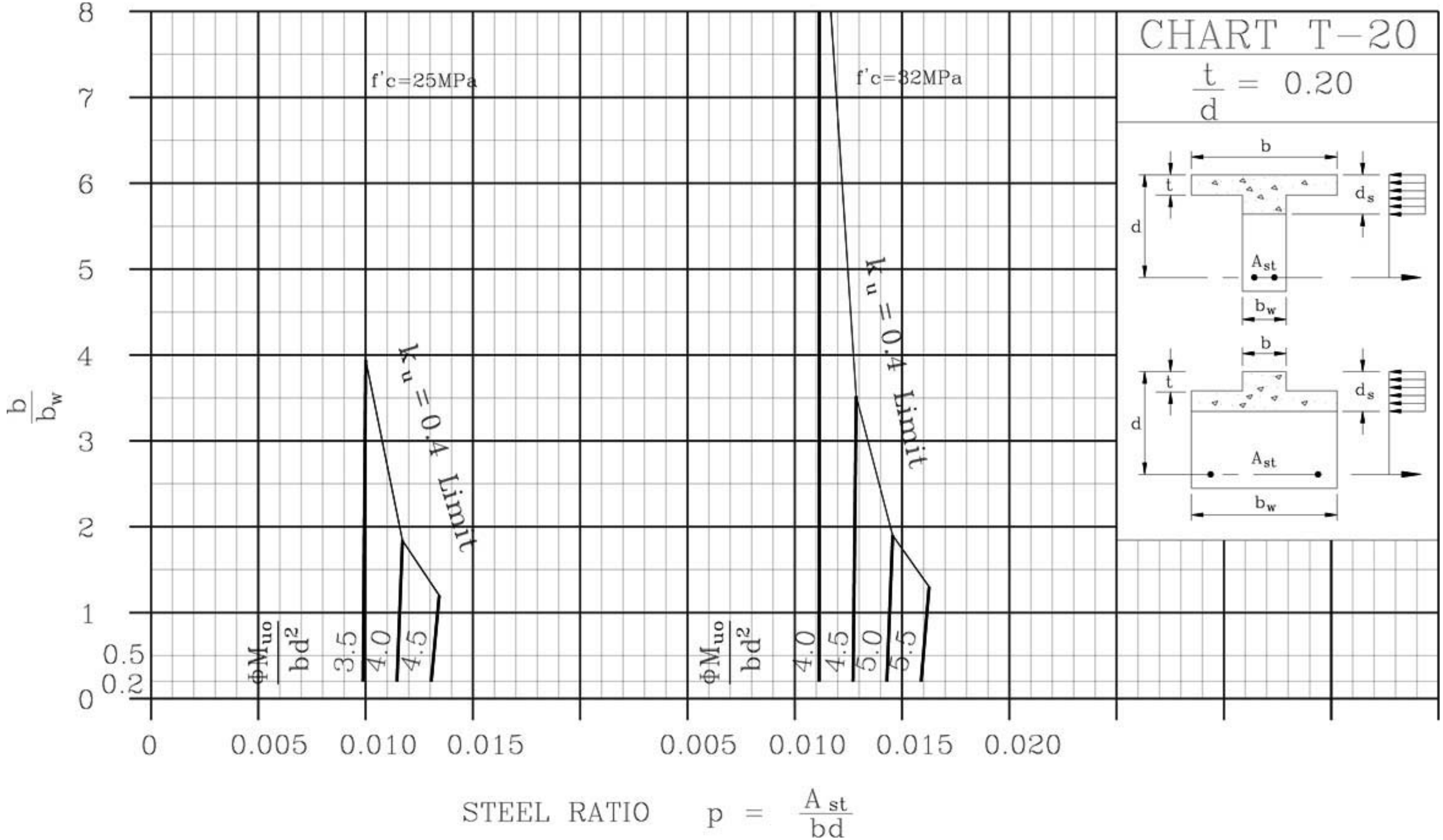


CHART T-20
t/d = 0.20
f'c=25MPa
f'c=32MPa
k u = 0.4 Limit
k u = 0.4 Limit
ΦMuo/bd²
3.5
4.0
4.5
ΦMuo/bd²
4.0
4.5
5.0
5.5
b/bw
0
0.2
0.5
1
2
3
4
5
6
7
8
0
0.005
0.010
0.015
0.005
0.010
0.015
0.020
STEEL RATIO p = Ast/bd
b
t
d
ds
Ast
bw
b
t
d
ds
Ast
bw

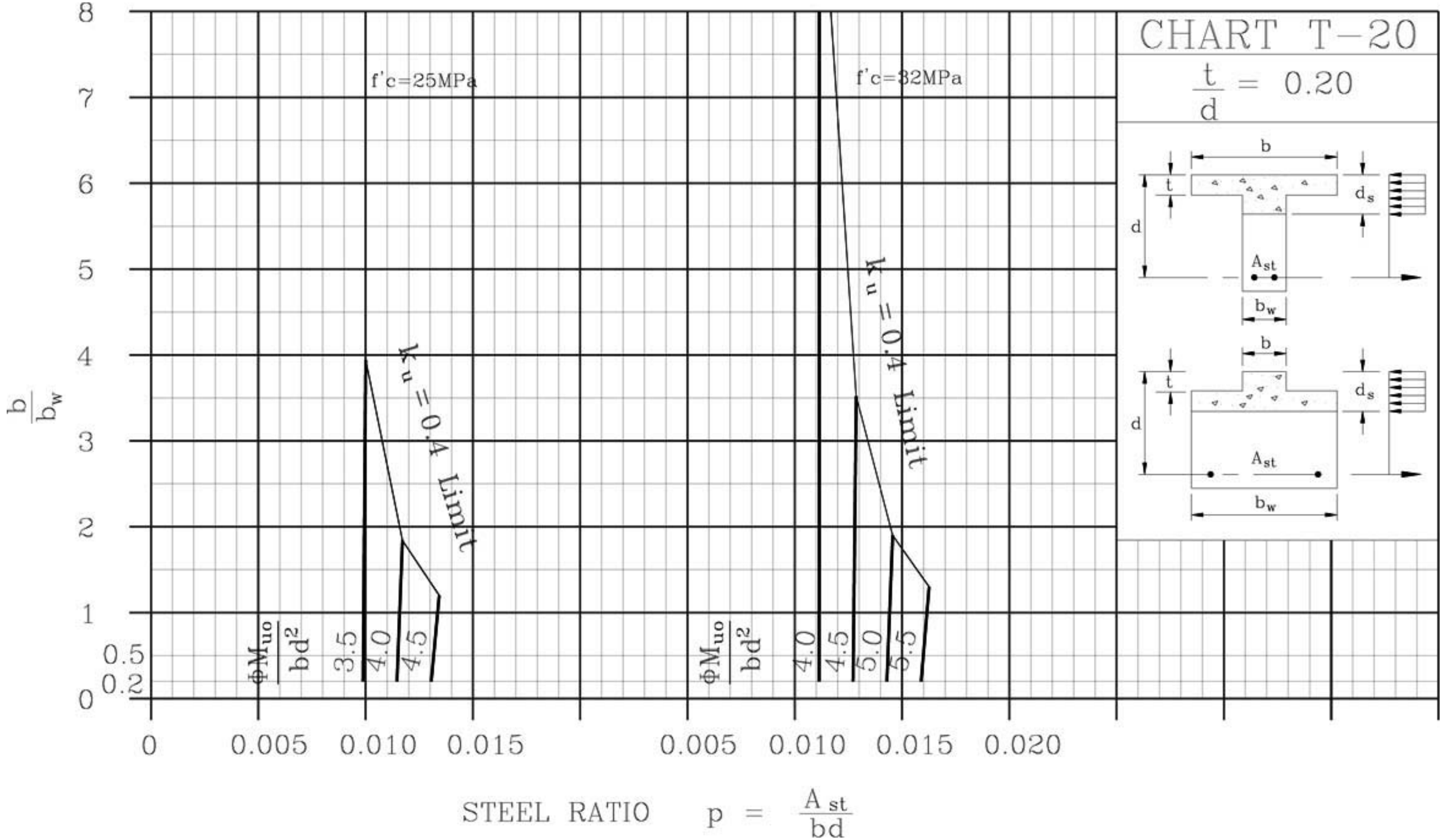


CHART T-20
t/d = 0.20
f'c=25MPa
f'c=32MPa
ku = 0.4 Limit
ku = 0.4 Limit
ΦMuo/bd²
3.5
4.0
4.5
ΦMuo/bd²
4.0
4.5
5.0
5.5
b/bw
8
7
6
5
4
3
2
1
0.5
0.2
0
0
0.005
0.010
0.015
0.005
0.010
0.015
0.020
STEEL RATIO p = Ast/bd
b
t
d
ds
Ast
bw
b
t
d
ds
Ast
bw

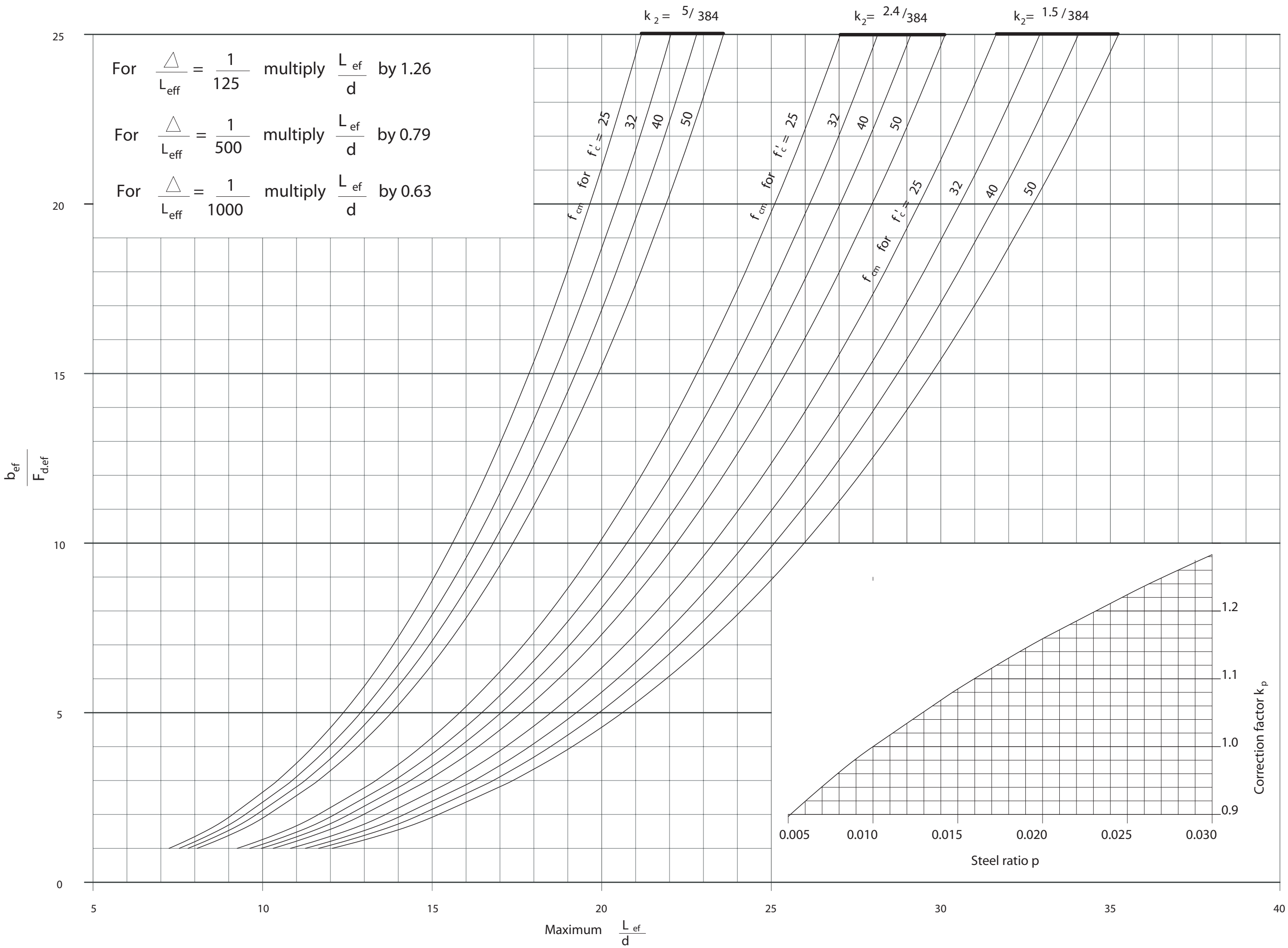


CHART D1 - DEEMED TO COMPLY DEFLECTION FOR BEAMS

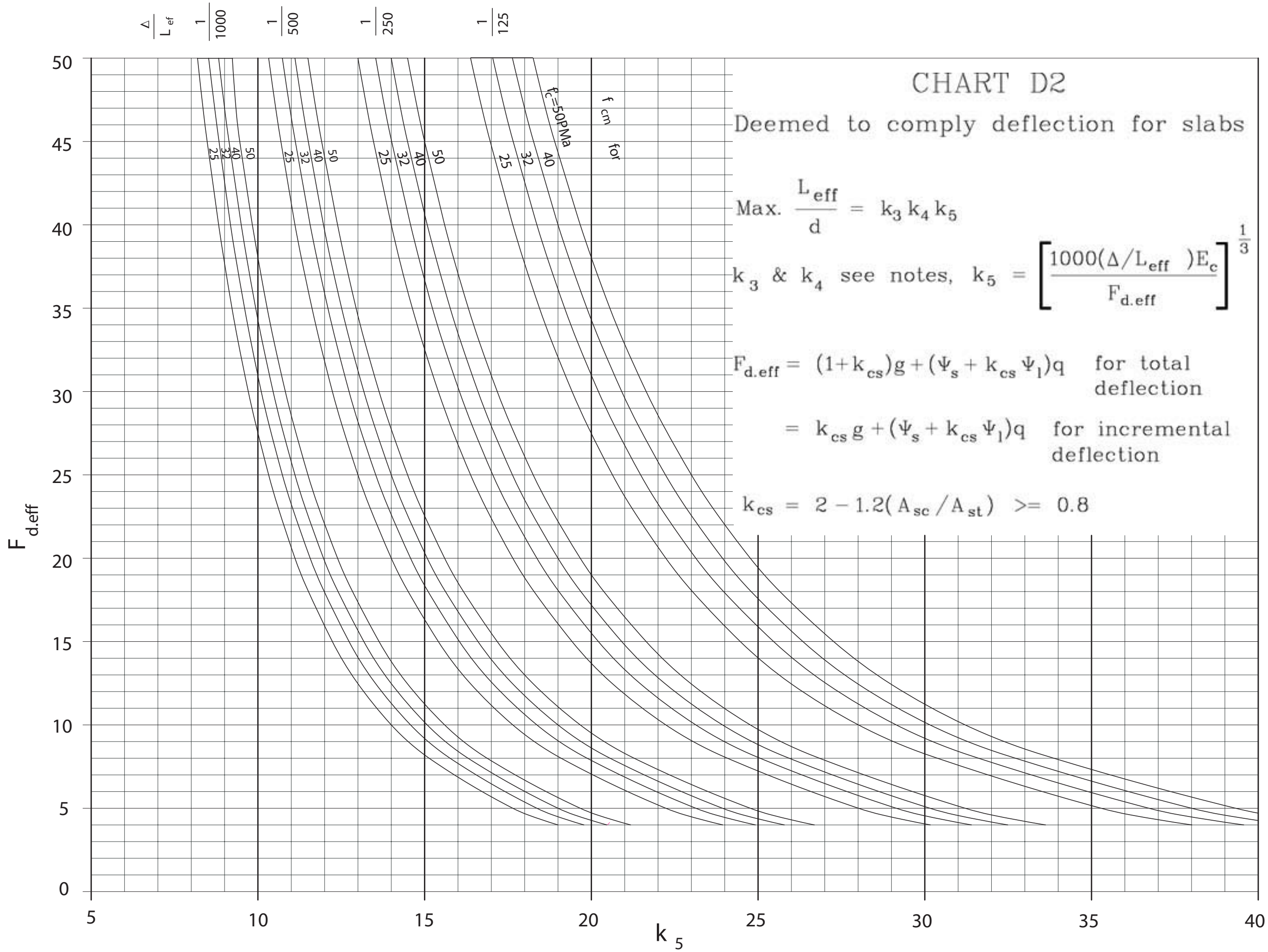


CHART D2
Deemed to comply deflection for slabs
Max. $\frac{L_{eff}}{d} = k_3 k_4 k_5$
k_3 & k_4 see notes, $k_5 = \left[\frac{1000(\Delta/L_{eff})E_c}{F_{d.eff}}\right]^{\frac{1}{3}}$
$F_{d.eff} = (1+k_{cs})g + (\Psi_s + k_{cs}\Psi_l)q$ for total deflection
$= k_{cs}g + (\Psi_s + k_{cs}\Psi_l)q$ for incremental deflection
$k_{cs} = 2 - 1.2(A_{sc}/A_{st}) >= 0.8$
Δ / L_{ef}
1/1000
1/500
1/250
1/125
f_{cm} for f_c=50PMa
25
32
40
50
$F_{d.eff}$
k_5
0
5
10
15
20
25
30
35
40
45
50

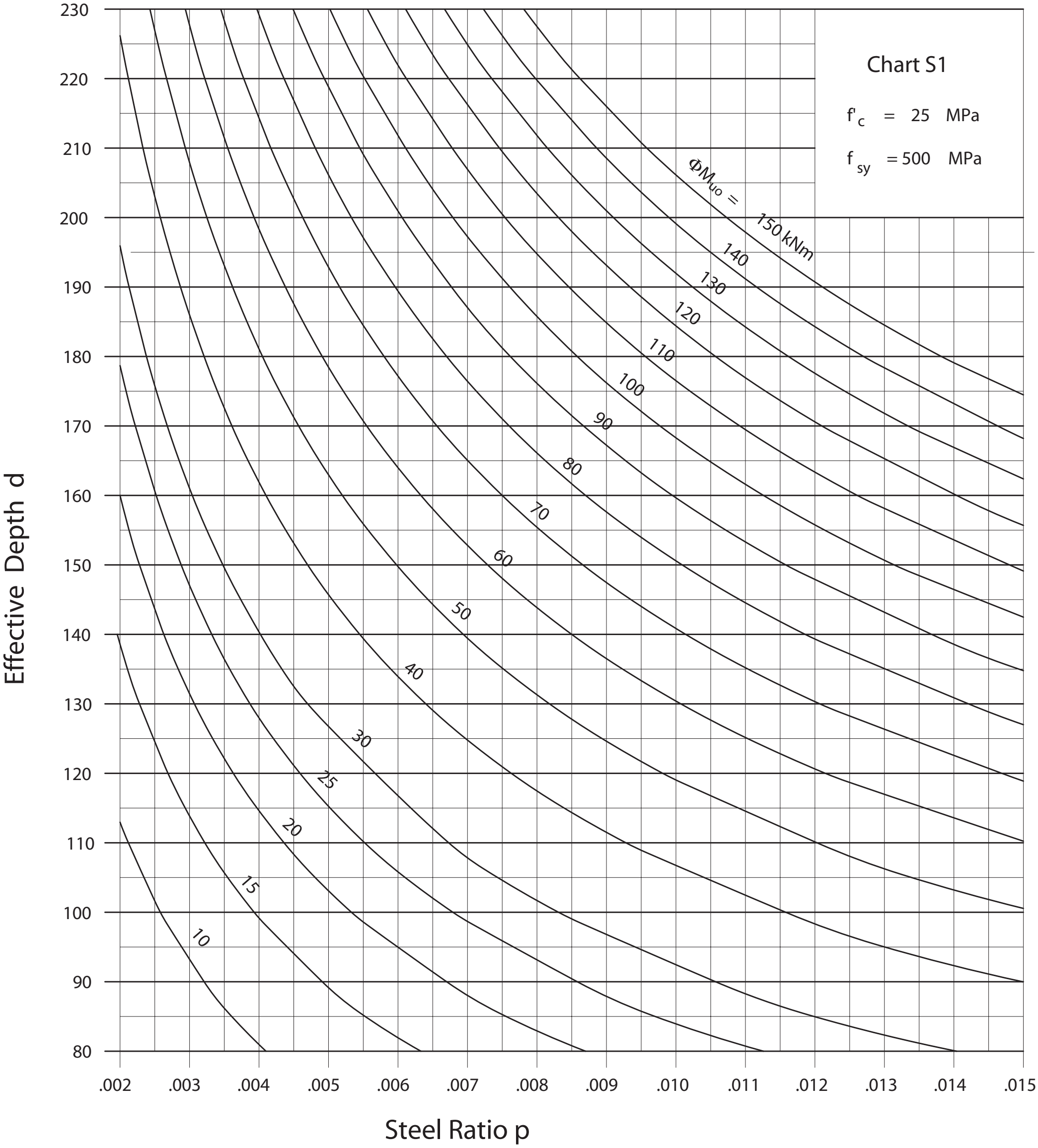


Chart S1
f'c = 25 MPa
fsy = 500 MPa
ΦMuo = 150 kNm
140
130
120
110
100
90
80
70
60
50
40
30
25
20
15
10
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
Effective Depth d
.002
.003
.004
.005
.006
.007
.008
.009
.010
.011
.012
.013
.014
.015
Steel Ratio p

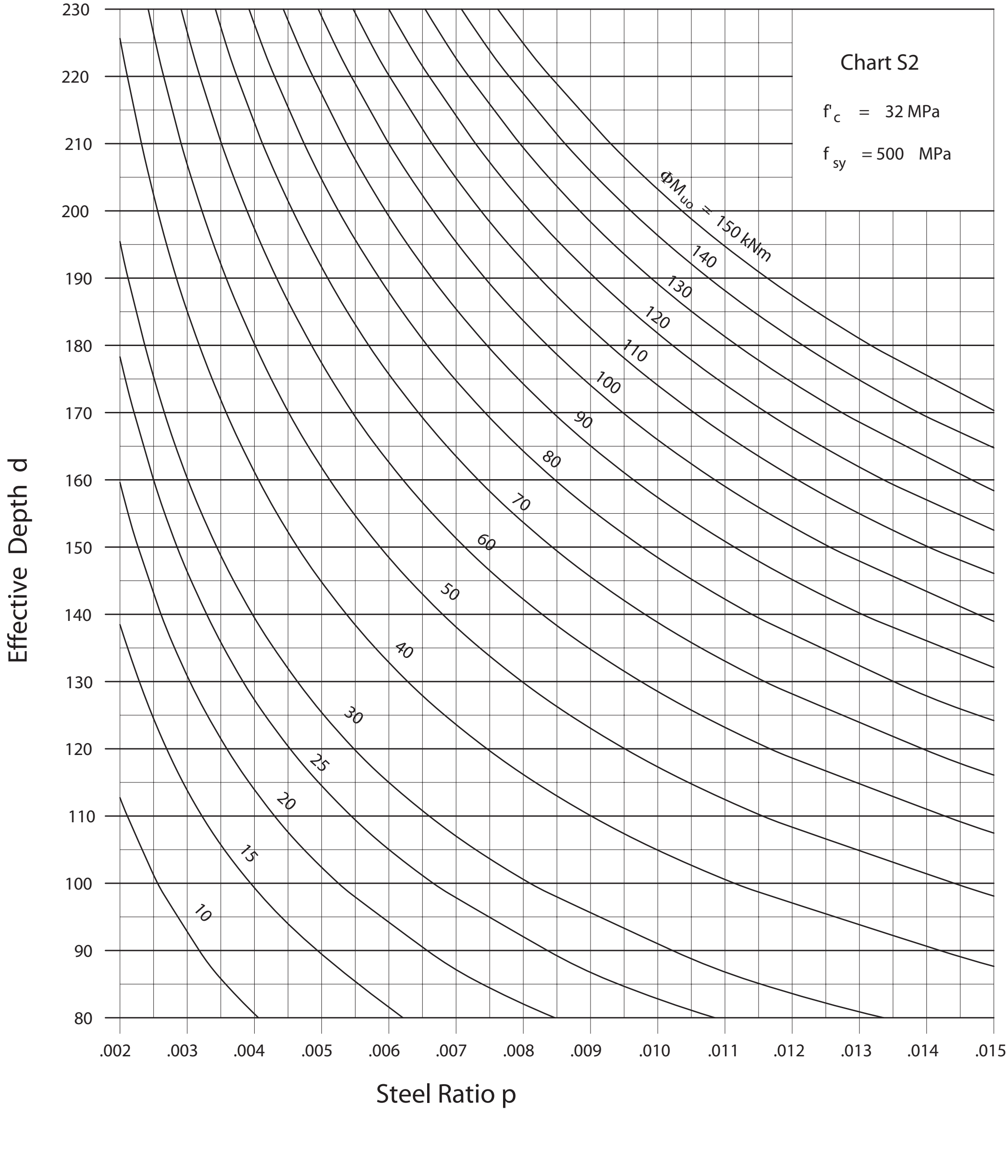


Chart S2
f'c = 32 MPa
f sy = 500 MPa
ΦMuo = 150 kNm
140
130
120
110
100
90
80
70
60
50
40
30
25
20
15
10
Effective Depth d
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
Steel Ratio p
.002
.003
.004
.005
.006
.007
.008
.009
.010
.011
.012
.013
.014
.015

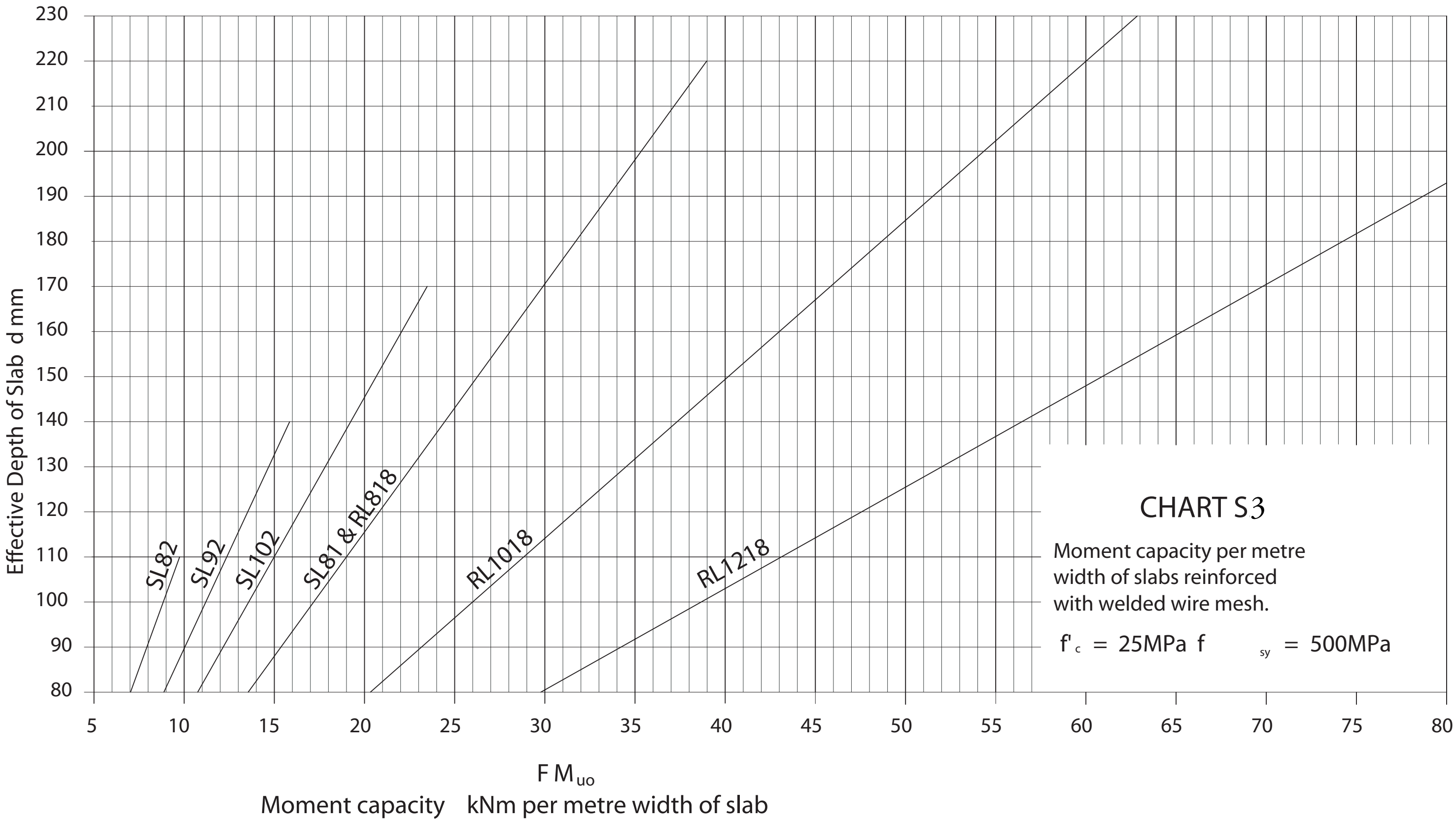
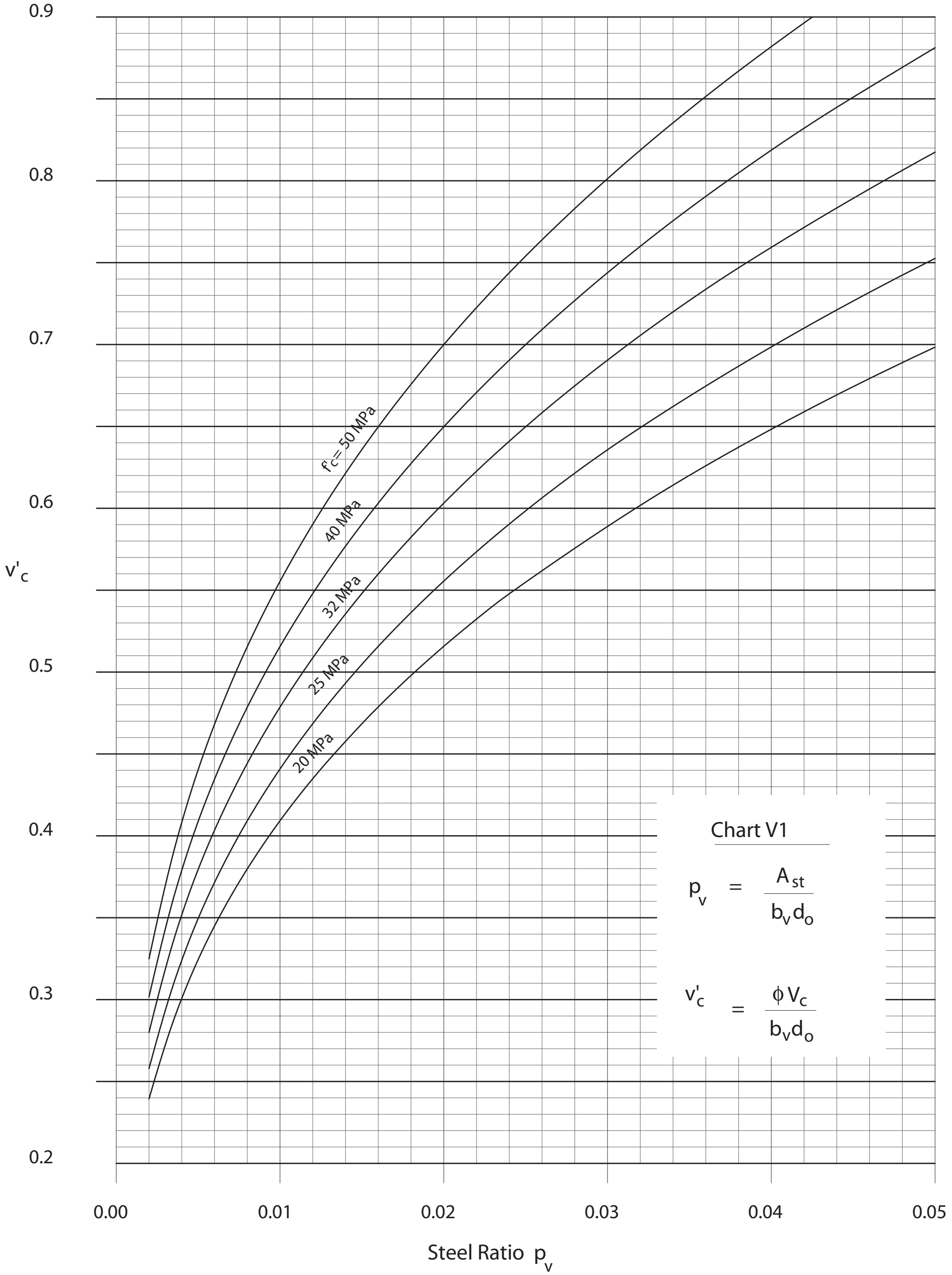


CHART S3
Moment capacity per metre width of slabs reinforced with welded wire mesh.
f'c = 25MPa fsy = 500MPa
Effective Depth of Slab d mm
230
220
210
200
190
180
170
160
150
140
130
120
110
100
90
80
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
SL82
SL92
SL102
SL81 & RL818
RL1018
RL1218
F Muo
Moment capacity kNm per metre width of slab



0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
v'c
f'c = 50 MPa
40 MPa
32 MPa
25 MPa
20 MPa
Chart V1
$p_v = \frac{A_{st}}{b_v d_o}$
$v'_c = \frac{\phi V_c}{b_v d_o}$
0.00
0.01
0.02
0.03
0.04
0.05
Steel Ratio p_v

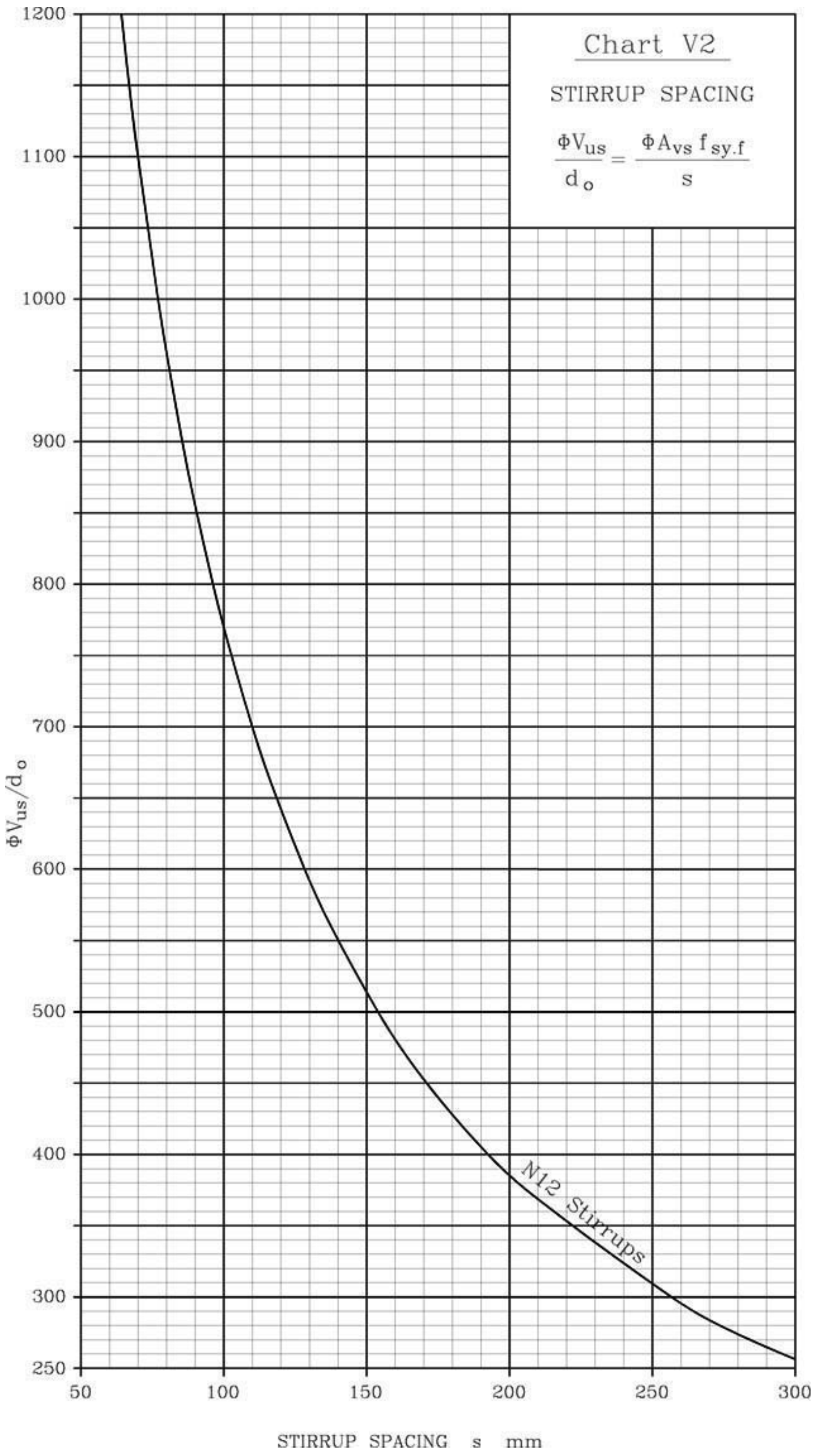


Chart V2
STIRRUP SPACING
$\frac{\phi V_{us}}{d_o} = \frac{\phi A_{vs} f_{sy.f}}{s}$
N12 Stirrups
$\phi V_{us}/d_o$
1200
1100
1000
900
800
700
600
500
400
300
250
50
100
150
200
250
300
STIRRUP SPACING s mm

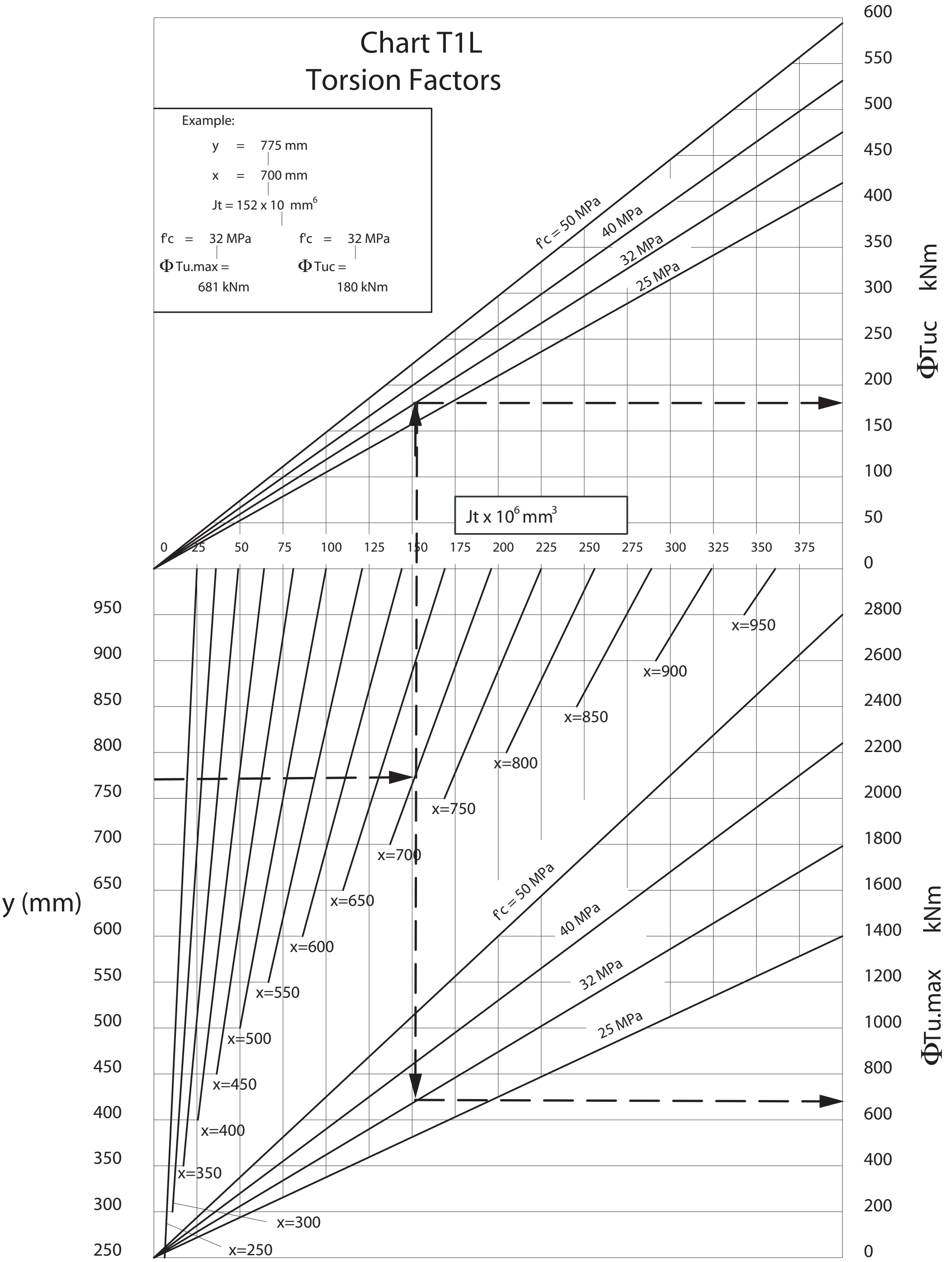


Chart T1L
Torsion Factors
Example:
y = 775 mm
x = 700 mm
Jt = 152 x 10 mm6
f'c = 32 MPa
ΦTu.max =
681 kNm
f'c = 32 MPa
ΦTuc =
180 kNm
f'c = 50 MPa
40 MPa
32 MPa
25 MPa
Jt x 10⁶ mm³
0 25 50 75 100 125 150 175 200 225 250 275 300 325 350 375
600 550 500 450 400 350 300 250 200 150 100 50 0
ΦTuc kNm
y (mm)
950 900 850 800 750 700 650 600 550 500 450 400 350 300 250
x=950
x=900
x=850
x=800
x=750
x=700
x=650
x=600
x=550
x=500
x=450
x=400
x=350
x=300
x=250
2800 2600 2400 2200 2000 1800 1600 1400 1200 1000 800 600 400 200 0
ΦTu.max kNm

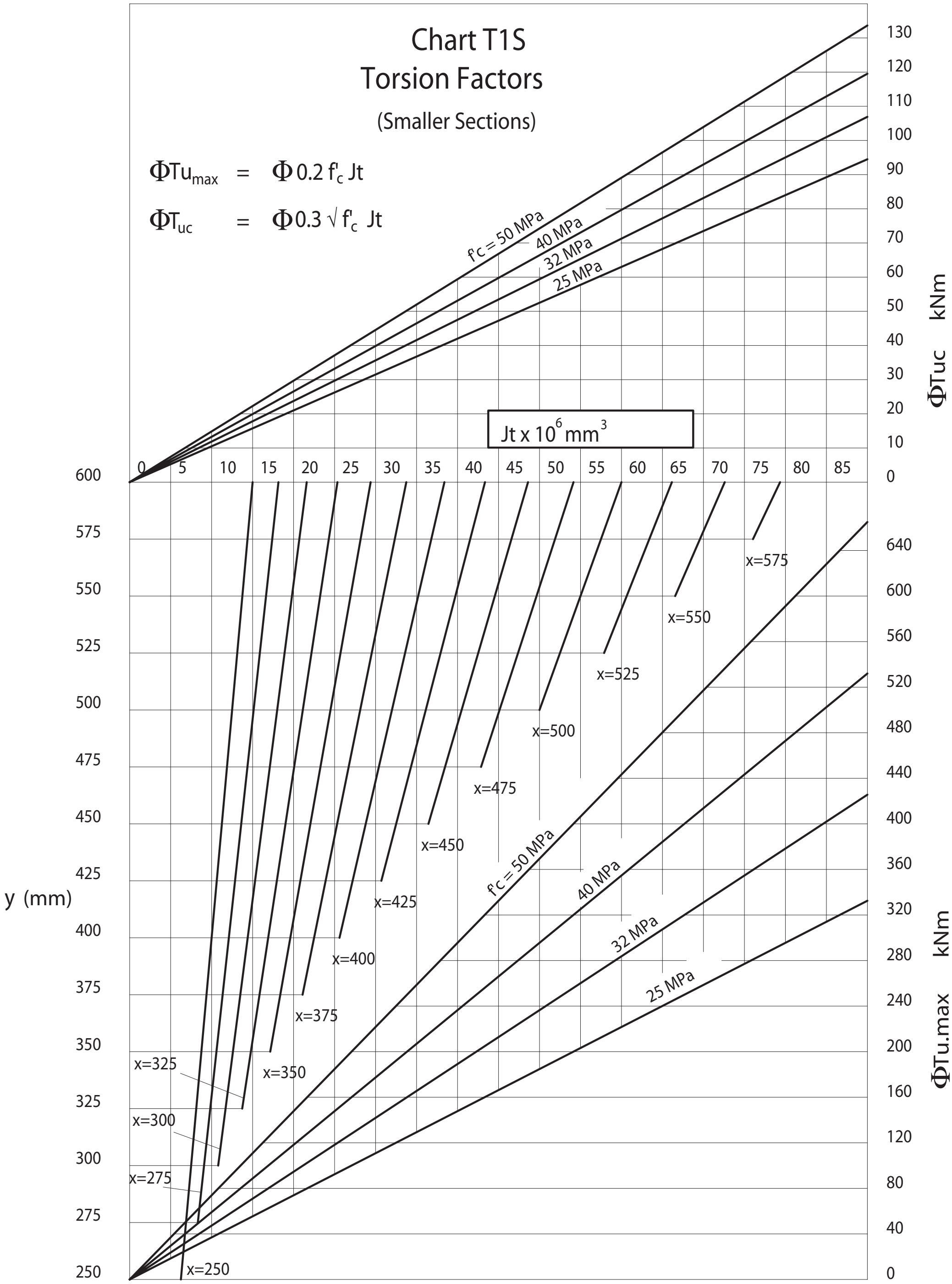


Chart T1S
Torsion Factors
(Smaller Sections)
$\Phi Tu_{max} = \Phi\, 0.2\, f'_c\, Jt$
$\Phi T_{uc} = \Phi\, 0.3 \sqrt{f'_c}\, Jt$
f'c = 50 MPa
40 MPa
32 MPa
25 MPa
Jt x 10^6 mm^3
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
ΦTuc kNm
ΦTu.max kNm
y (mm)
x=250
x=275
x=300
x=325
x=350
x=375
x=400
x=425
x=450
x=475
x=500
x=525
x=550
x=575

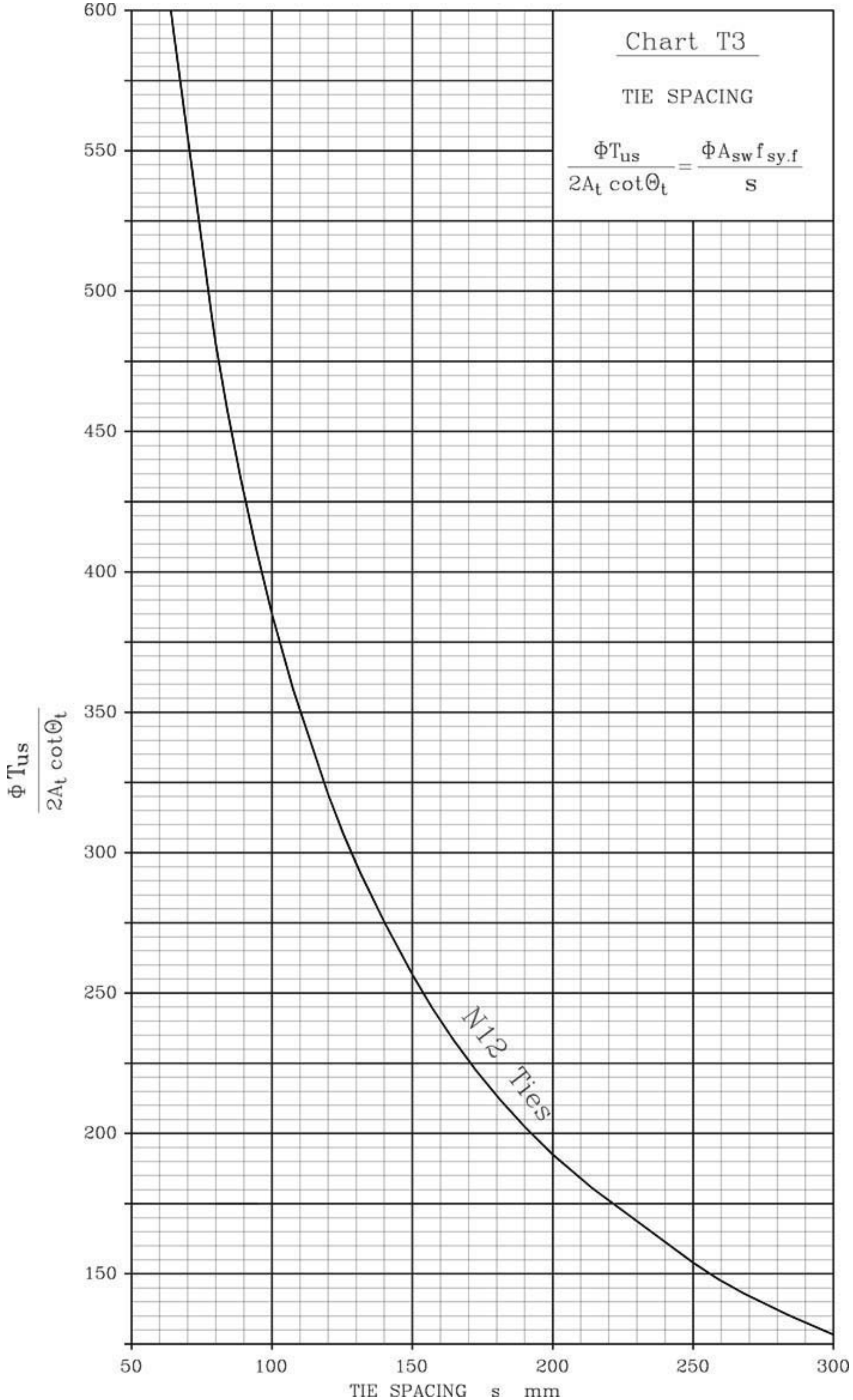


Chart T3
TIE SPACING
$\frac{\Phi T_{us}}{2A_t \cot\Theta_t} = \frac{\Phi A_{sw} f_{sy.f}}{s}$
N12 Ties
$\frac{\Phi T_{us}}{2A_t \cot\Theta_t}$
600
550
500
450
400
350
300
250
200
150
50
100
150
200
250
300
TIE SPACING s mm

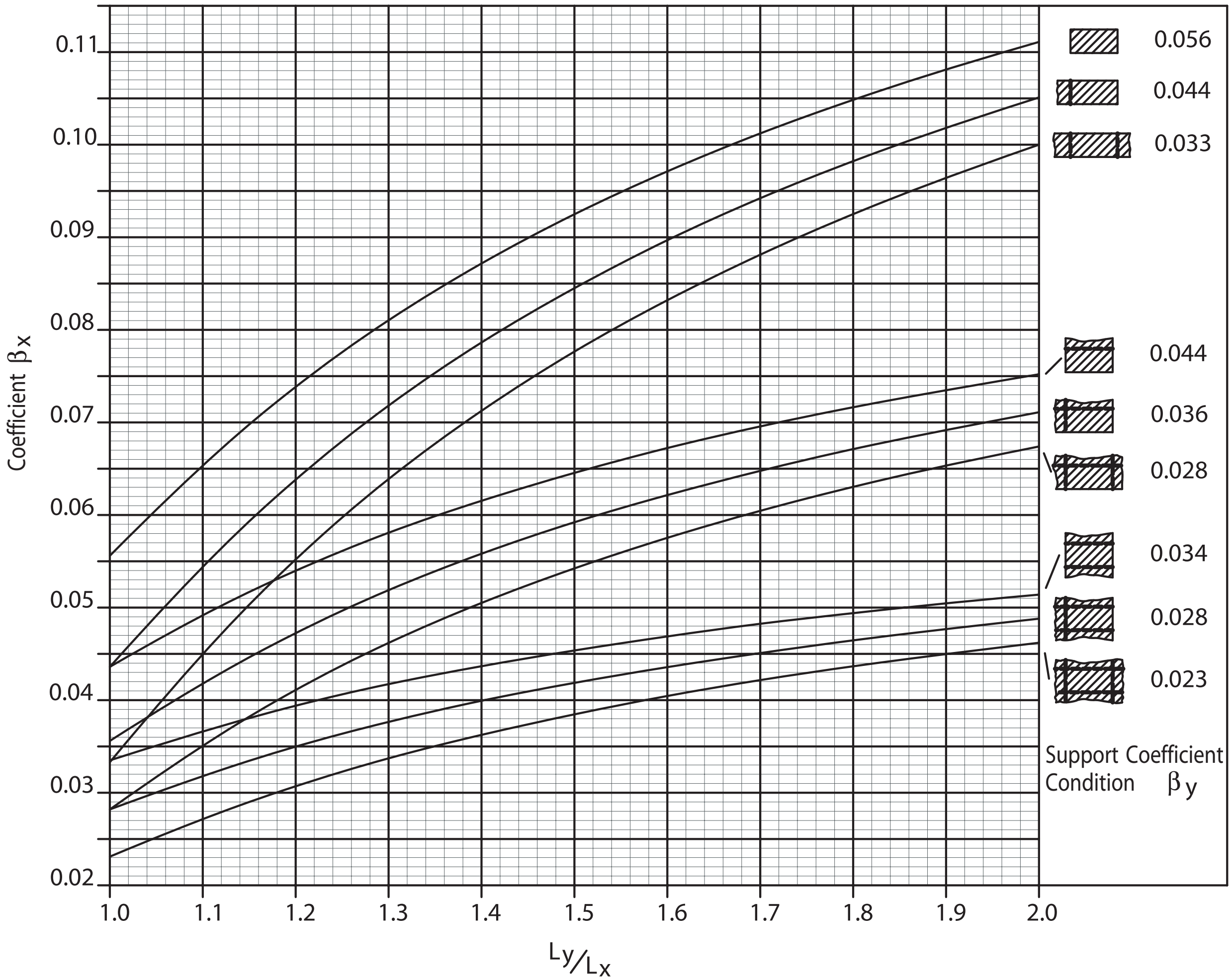


CHART S4 - Bending Moment Coefficients for Rectangular Slabs Supported on Four Sides

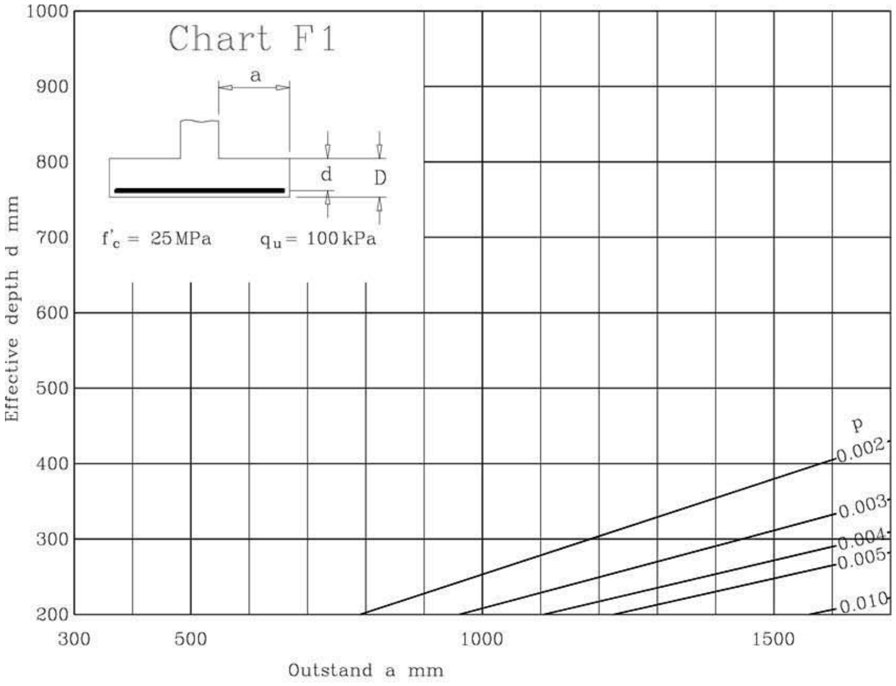


Chart F1
a
d
D
f'c = 25 MPa
qu = 100 kPa
p
0.002
0.003
0.004
0.005
0.010
Effective depth d mm
Outstand a mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500

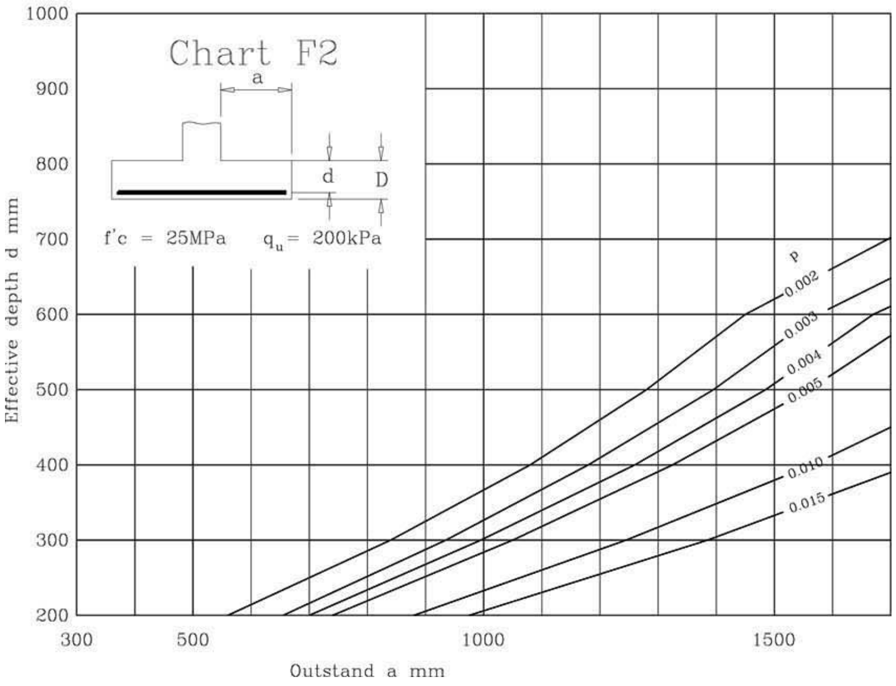


Chart F2
a
d
D
f'c = 25MPa
qu = 200kPa
p
0.002
0.003
0.004
0.005
0.010
0.015
Effective depth d mm
Outstand a mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500

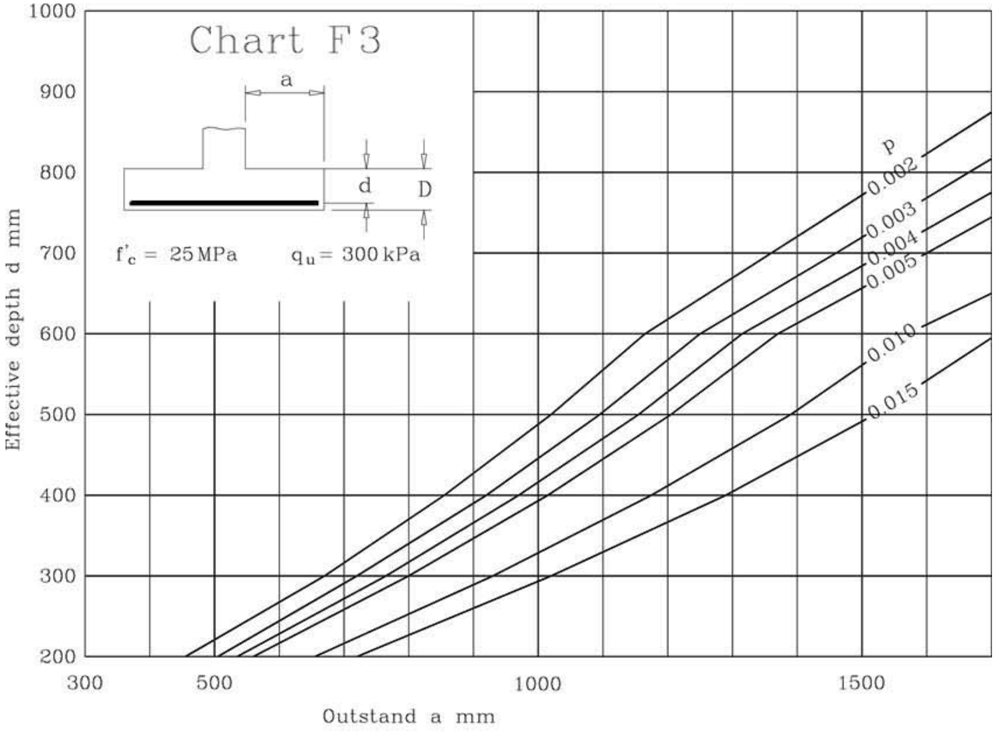


Chart F3
a
d
D
f'c = 25 MPa
qu = 300 kPa
p
0.002
0.003
0.004
0.005
0.010
0.015
Effective depth d mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500
Outstand a mm

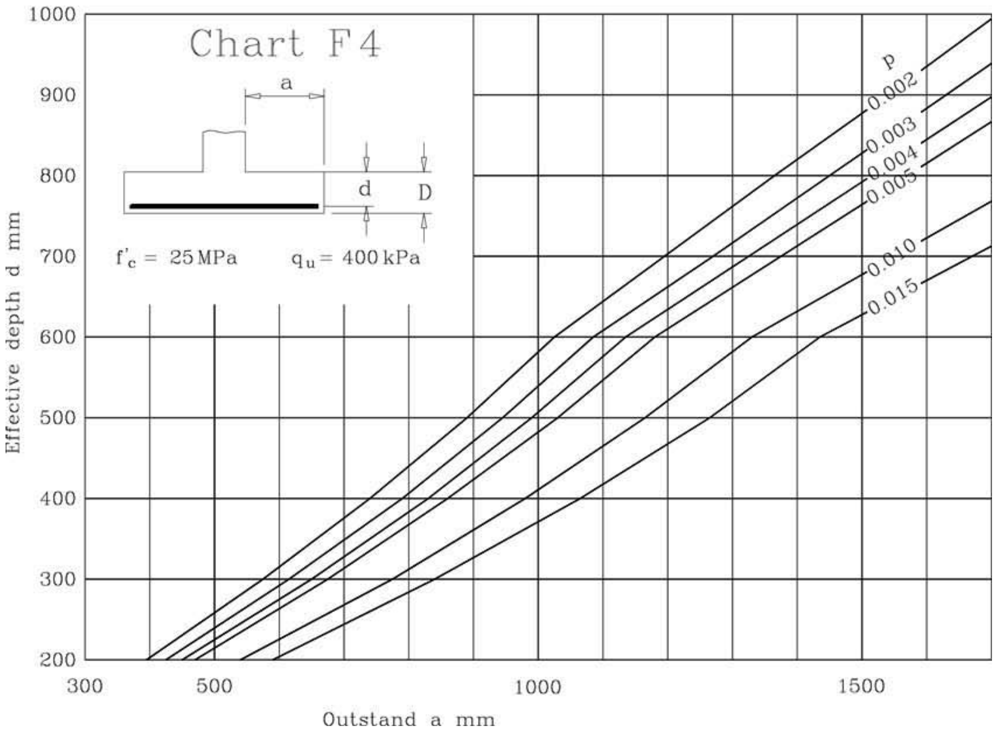


Chart F4
a
d
D
f'c = 25 MPa
qu = 400 kPa
p
0.002
0.003
0.004
0.005
0.010
0.015
Effective depth d mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500
Outstand a mm

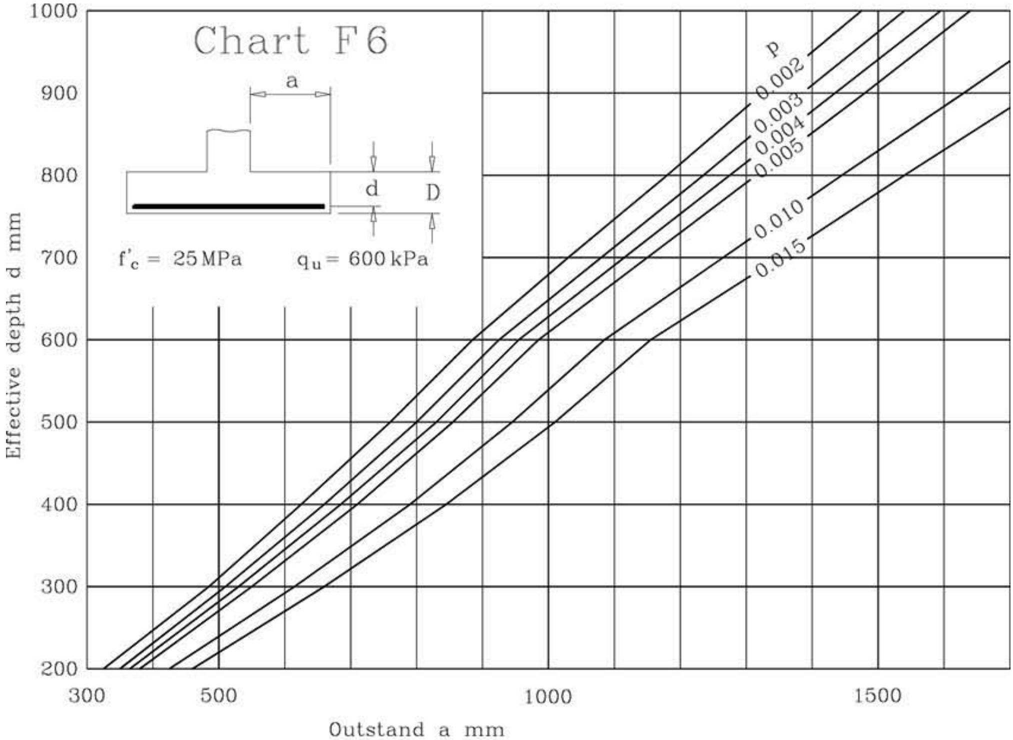


Chart F6
a
d
D
f'c = 25 MPa
qu = 600 kPa
p
0.002
0.003
0.004
0.005
0.010
0.015
Effective depth d mm
Outstand a mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500

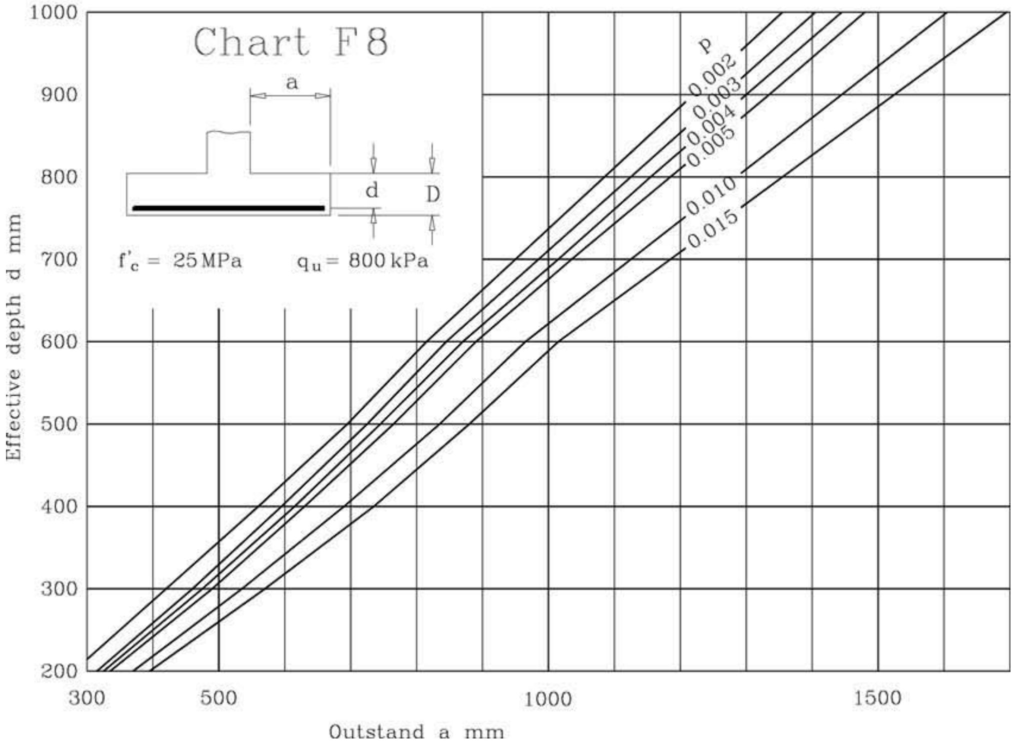


Chart F8
a
d
D
f'c = 25 MPa
qu = 800 kPa
p
0.002
0.003
0.004
0.005
0.010
0.015
Effective depth d mm
Outstand a mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500

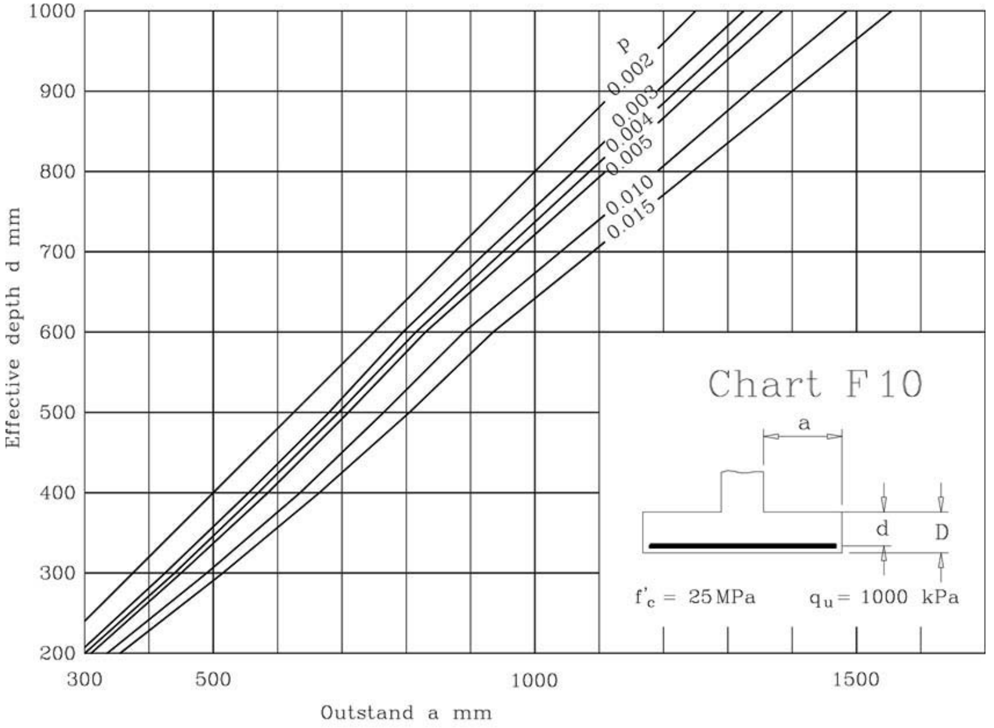


Chart F10
p
0.002
0.003
0.004
0.005
0.010
0.015
a
d
D
f'c = 25 MPa
qu = 1000 kPa
Effective depth d mm
Outstand a mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500

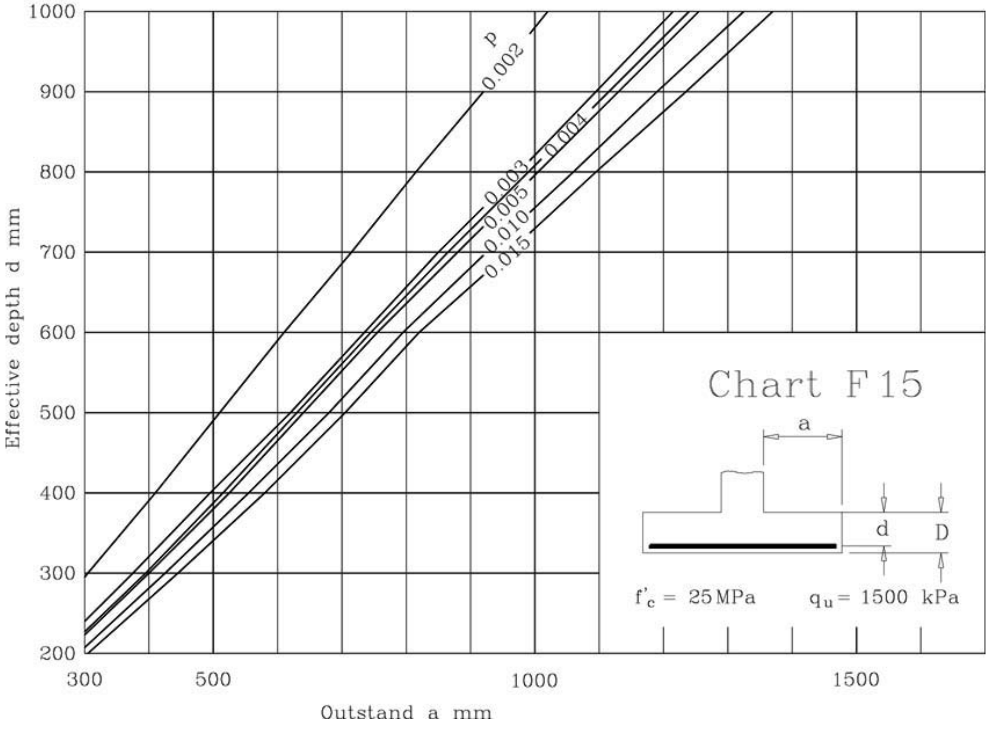


Chart F15
p
0.002
0.003
0.004
0.005
0.010
0.015
a
d
D
f'c = 25 MPa
qu = 1500 kPa
Effective depth d mm
Outstand a mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500

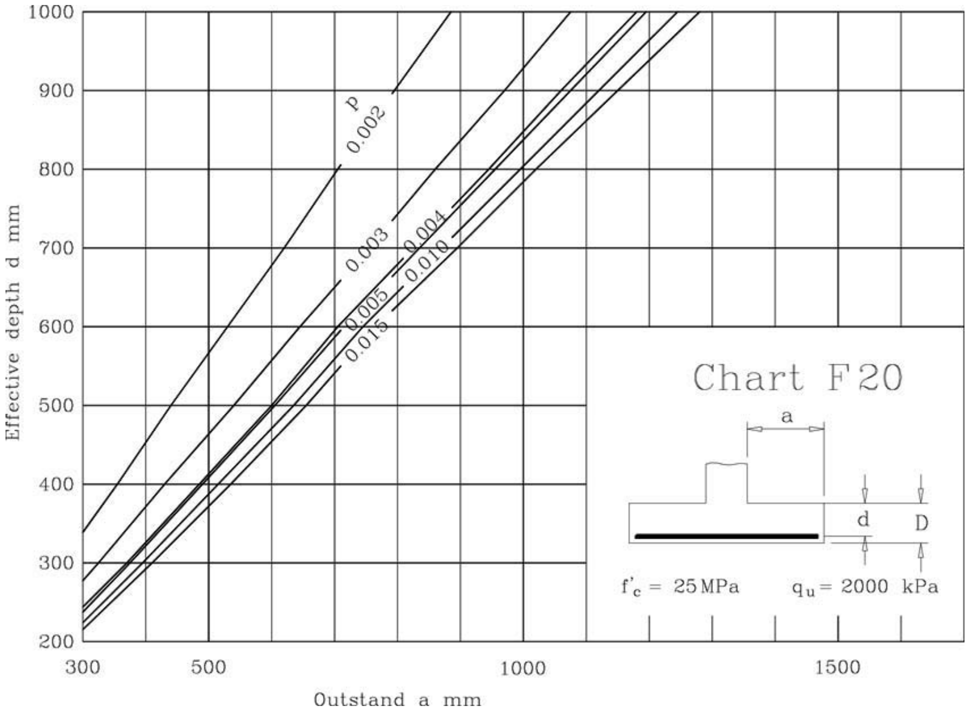


Chart F20
a
d
D
f'c = 25 MPa
qu = 2000 kPa
p
0.002
0.003
0.004
0.005
0.010
0.015
Effective depth d mm
Outstand a mm
1000
900
800
700
600
500
400
300
200
300
500
1000
1500

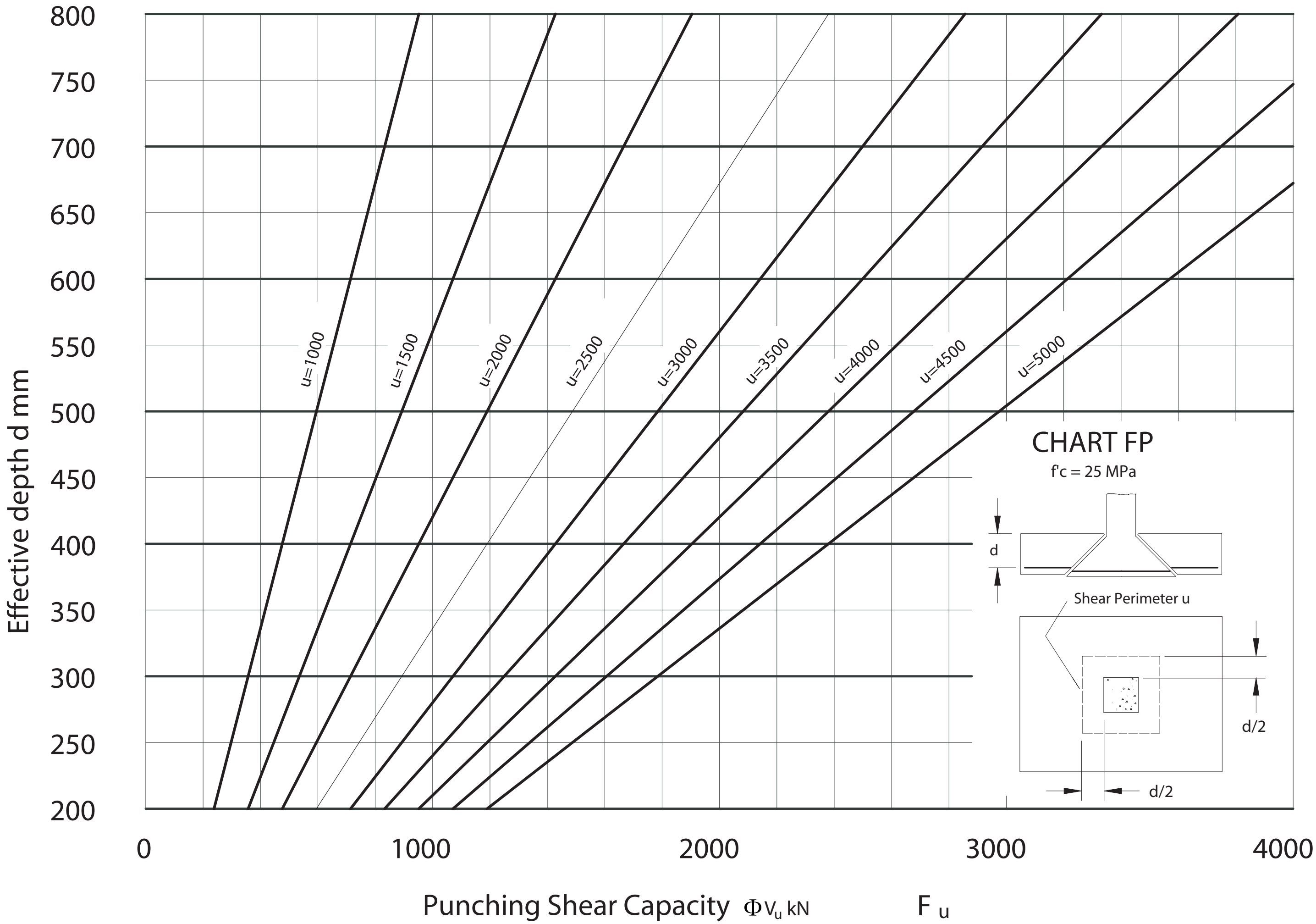


CHART FP
f'c = 25 MPa
Effective depth d mm
800
750
700
650
600
550
500
450
400
350
300
250
200
0
1000
2000
3000
4000
Punching Shear Capacity ΦV_u kN
F_u
u=1000
u=1500
u=2000
u=2500
u=3000
u=3500
u=4000
u=4500
u=5000
d
Shear Perimeter u
d/2
d/2

DESIGNATION OF COLUMN DESIGN CHARTS

Rectangular Columns (RC)

f'_c g

R C f -

2 = Equal reinforcement on 2 faces
4 = Equal reinforcement on 4 faces using 12 or more bars

Circular Columns (CC)

f'_c g

C C 8 b -

Using 8 or more bars

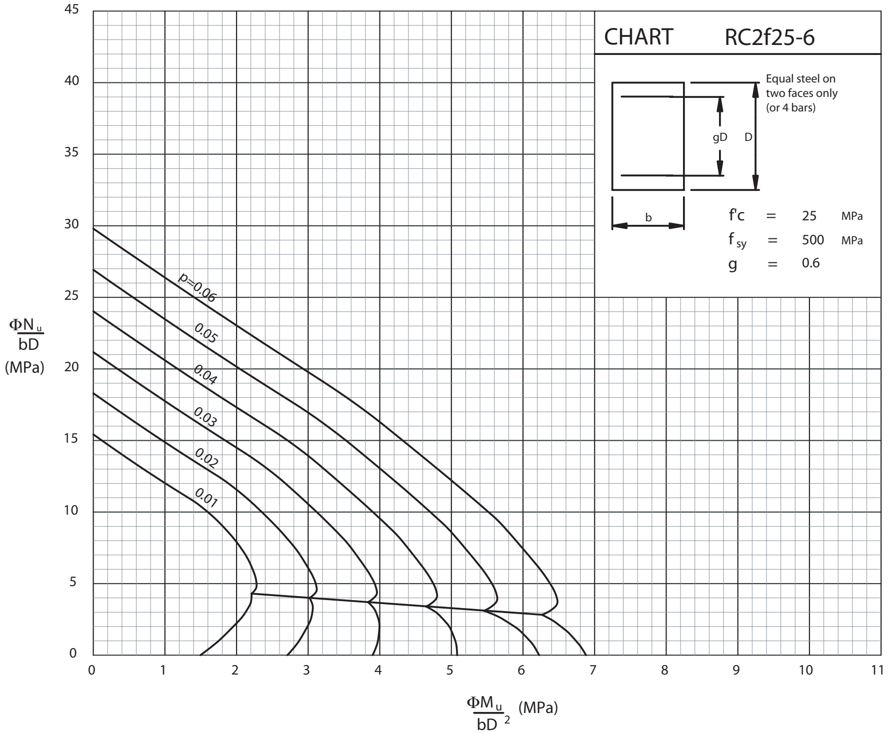


CHART RC2f25-6
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 25 MPa
f_{sy} = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{bD^2}$ (MPa)

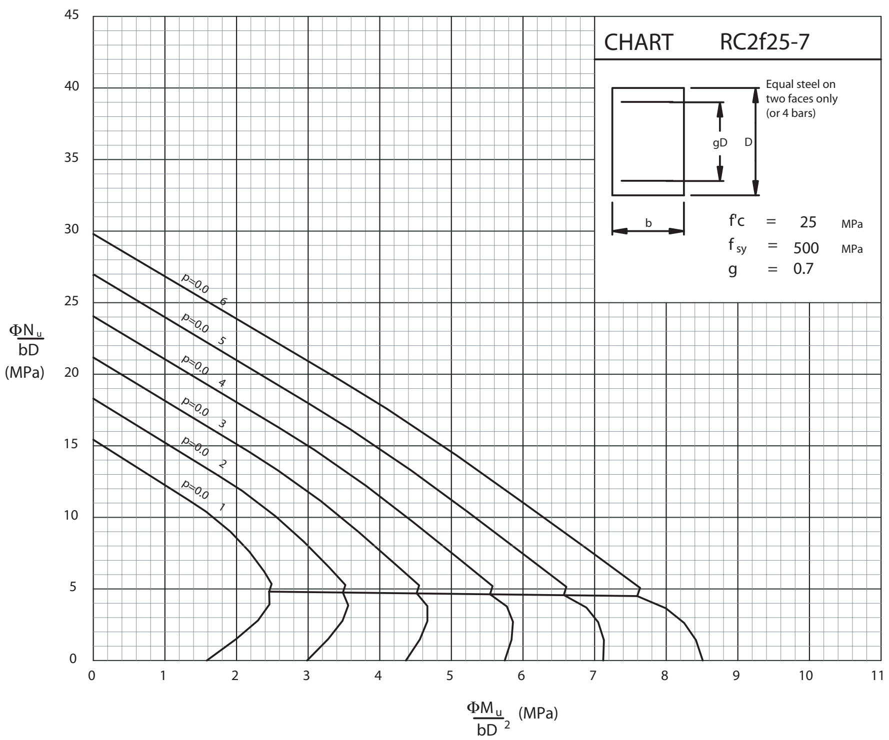


CHART RC2f25-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 25 MPa
f_{sy} = 500 MPa
g = 0.7
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{bD^2}$ (MPa)

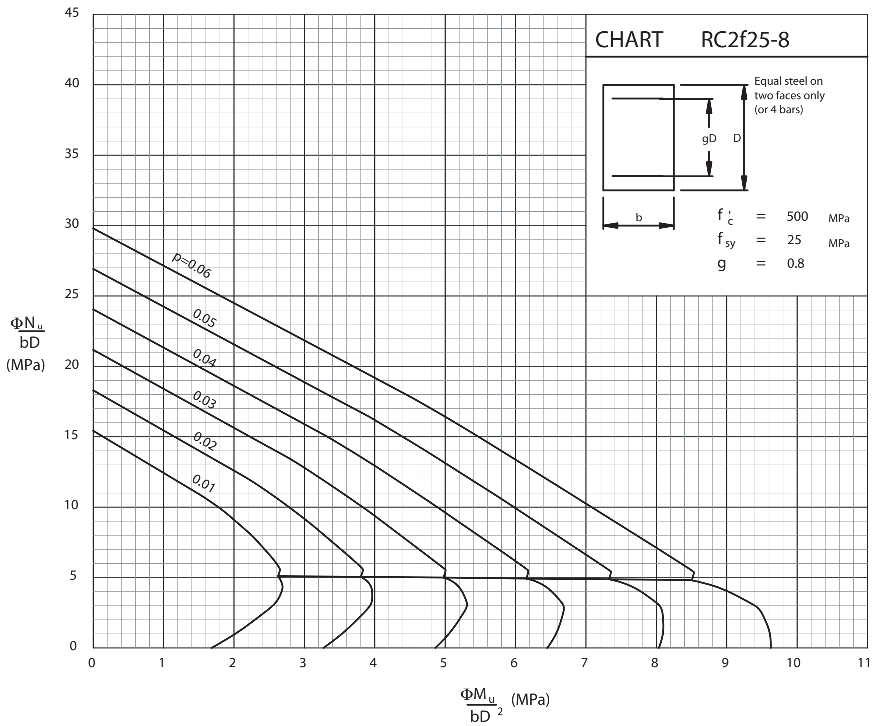


CHART RC2f25-8
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 25 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)
0
5
10
15
20
25
30
35
40
45
0
1
2
3
4
5
6
7
8
9
10
11

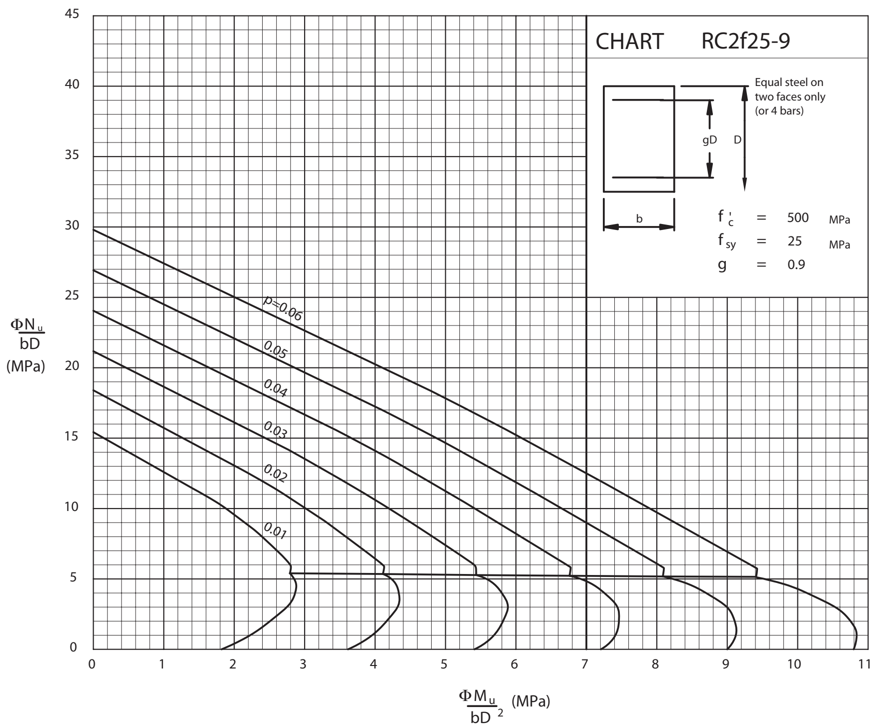


CHART RC2f25-9
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 25 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)
0
5
10
15
20
25
30
35
40
45
0
1
2
3
4
5
6
7
8
9
10
11

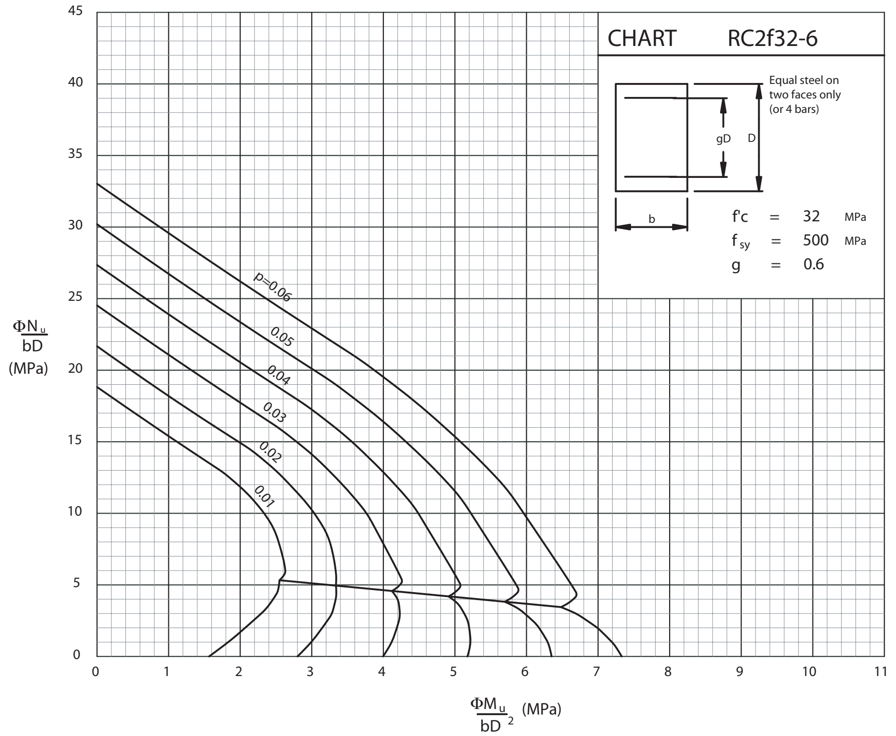


CHART RC2f32-6
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 32 MPa
f sy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD2 (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

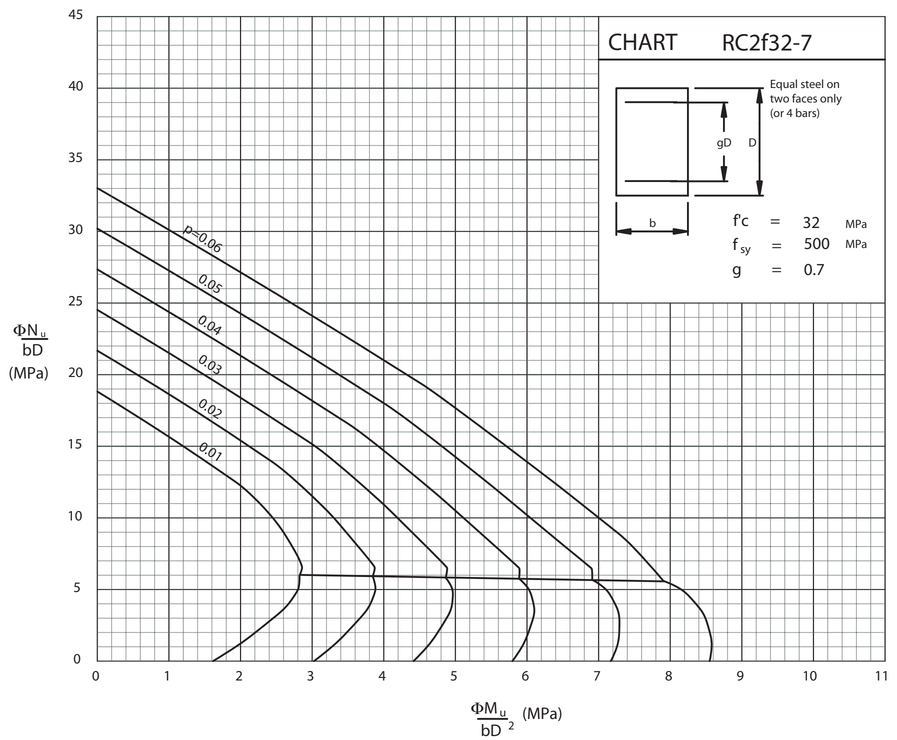


CHART RC2f32-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 32 MPa
f sy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD2 (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

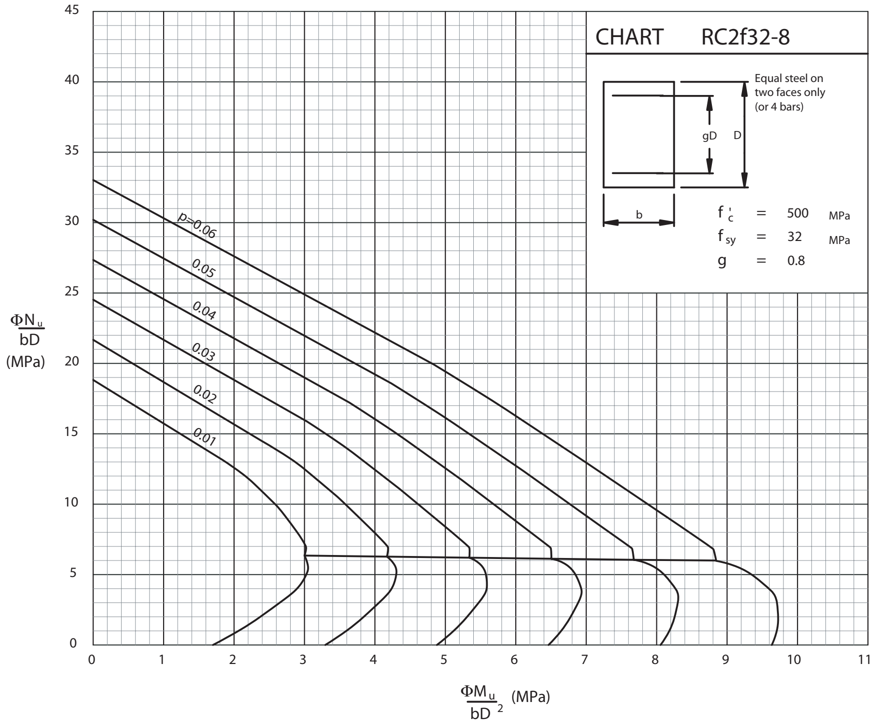


CHART RC2f32-8
Equal steel on two faces only (or 4 bars)
gD
D
b
f'_c = 500 MPa
f_{sy} = 32 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{bD^2}$ (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

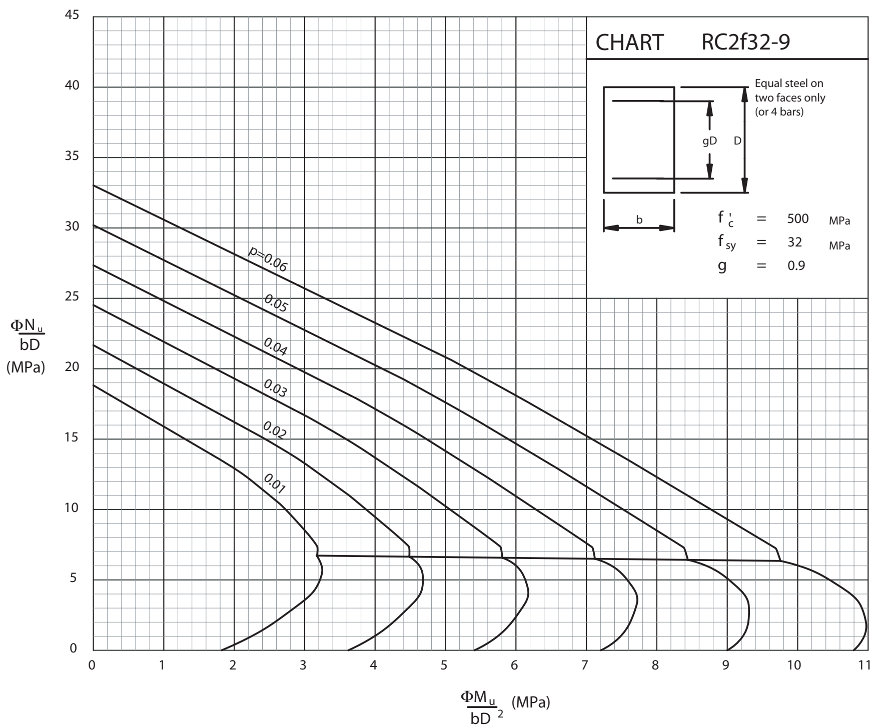


CHART RC2f32-9
Equal steel on two faces only (or 4 bars)
gD
D
b
f'_c = 500 MPa
f_{sy} = 32 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{bD^2}$ (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

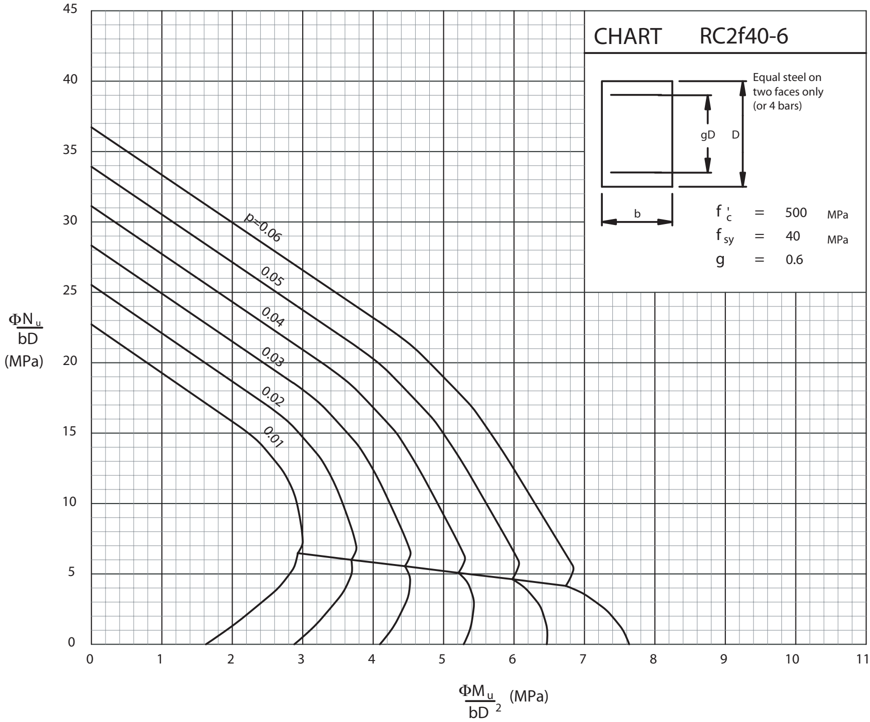


CHART RC2f40-6
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 40 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD2 (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

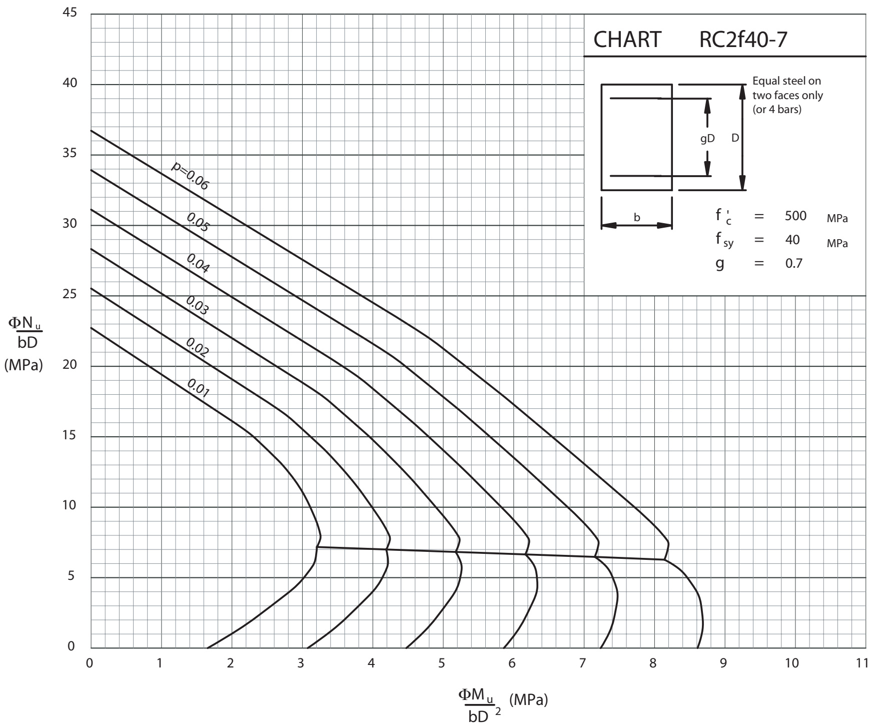


CHART RC2f40-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 40 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD2 (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

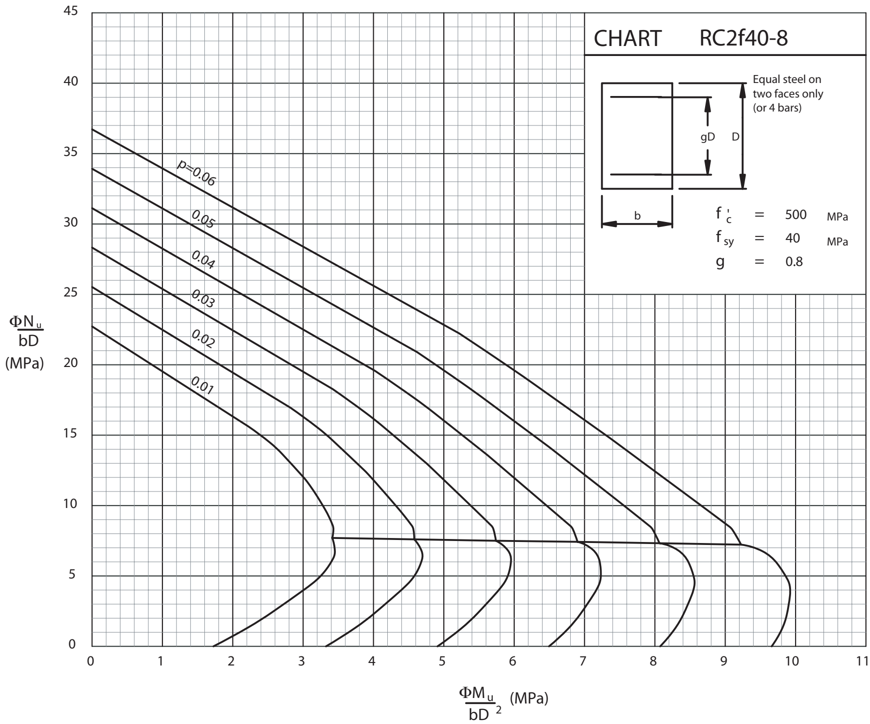


CHART RC2f40-8
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 40 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD2 (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

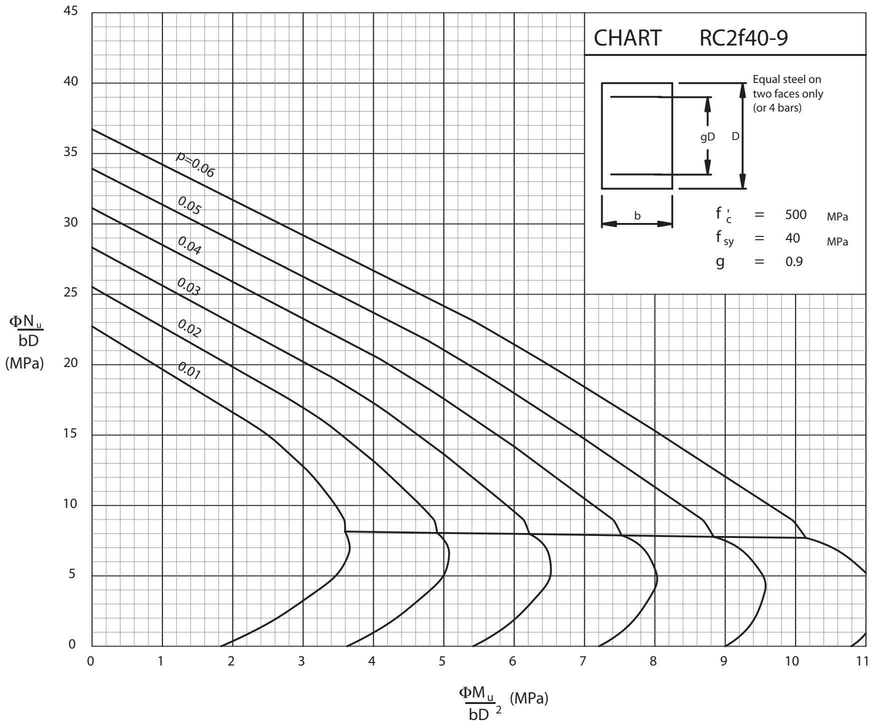


CHART RC2f40-9
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 40 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD2 (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

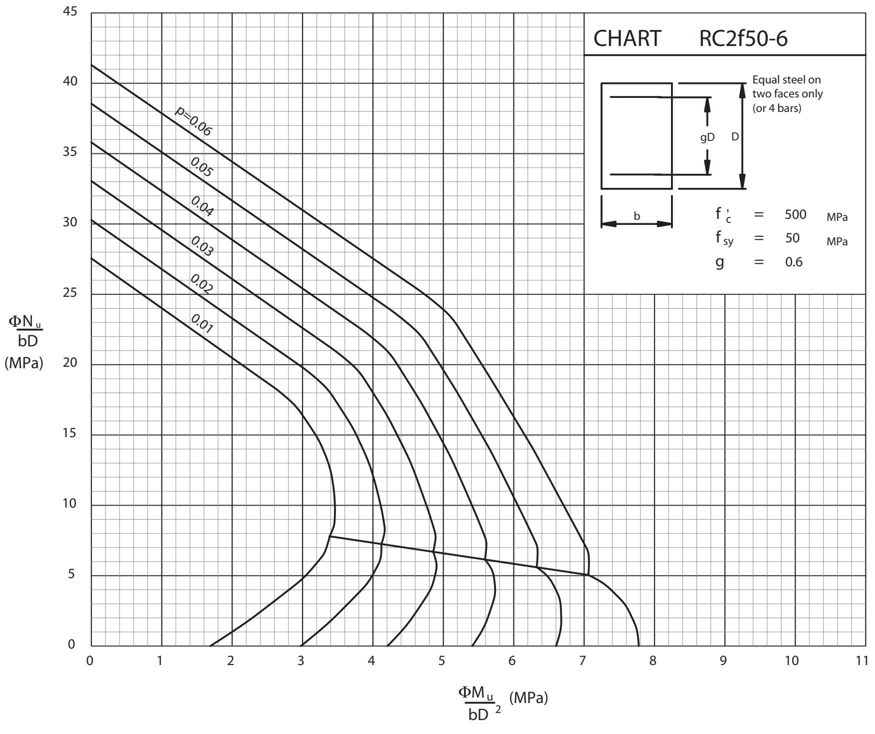


CHART RC2f50-6
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 50 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD² (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

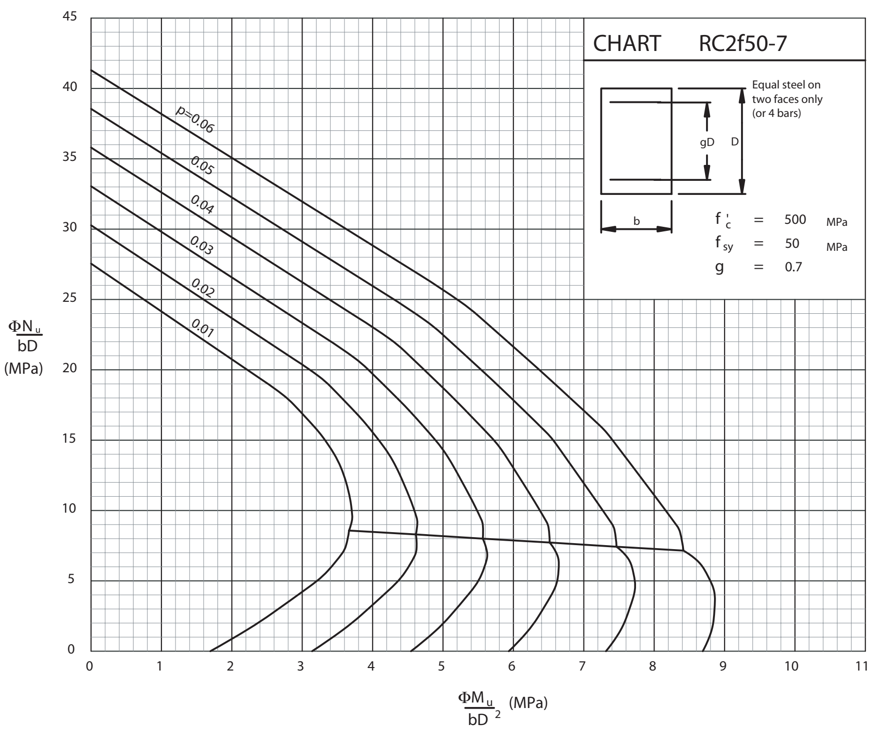


CHART RC2f50-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 50 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD² (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

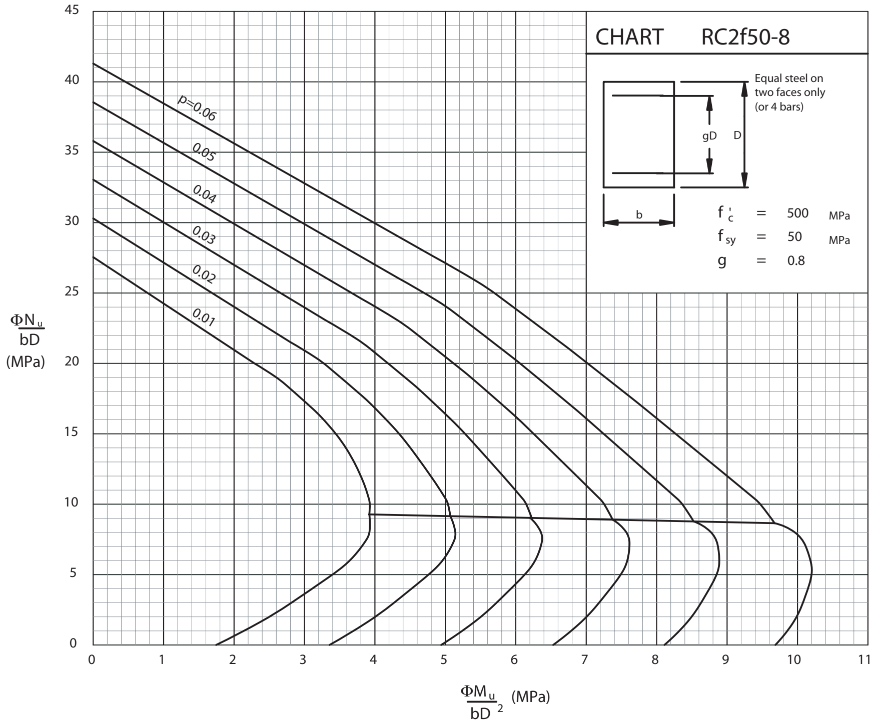


CHART RC2f50-8
Equal steel on two faces only (or 4 bars)
gD
D
b
f'_c = 500 MPa
f_{sy} = 50 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{bD^2}$ (MPa)
0
5
10
15
20
25
30
35
40
45
1
2
3
4
6
7
8
9
11

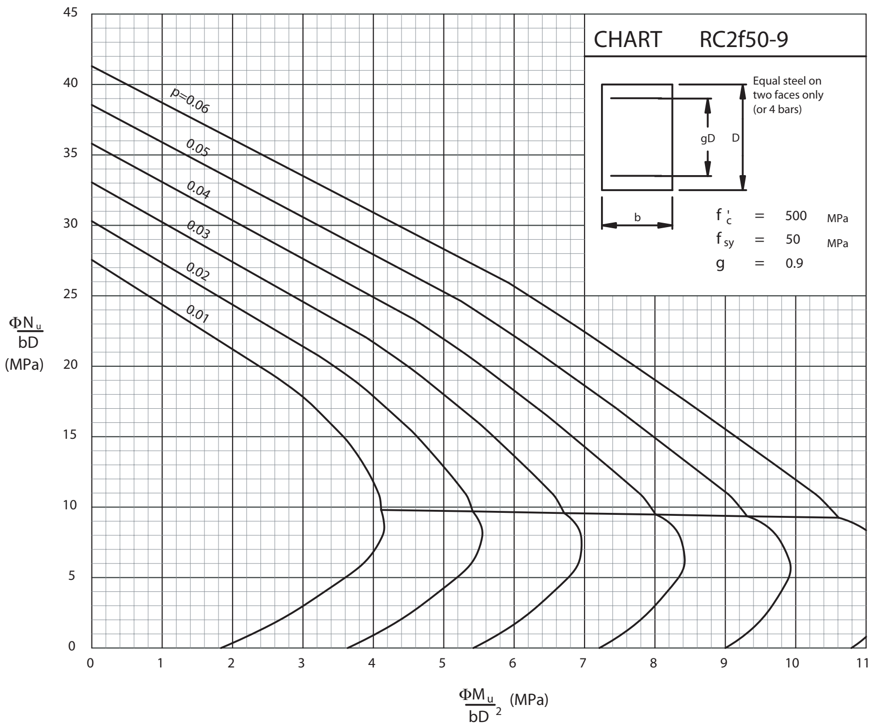


CHART RC2f50-9
Equal steel on two faces only (or 4 bars)
gD
D
b
f'_c = 500 MPa
f_{sy} = 50 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{bD^2}$ (MPa)
0
5
10
15
20
25
30
35
40
45
1
2
3
4
6
7
8
9
11

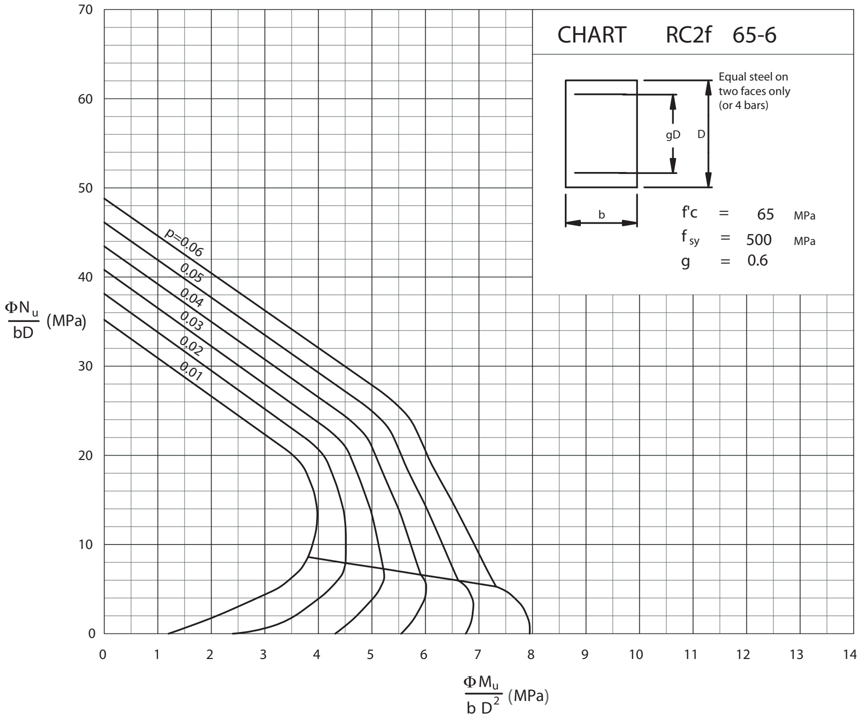


CHART RC2f 65-6
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 65 MPa
f_{sy} = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)

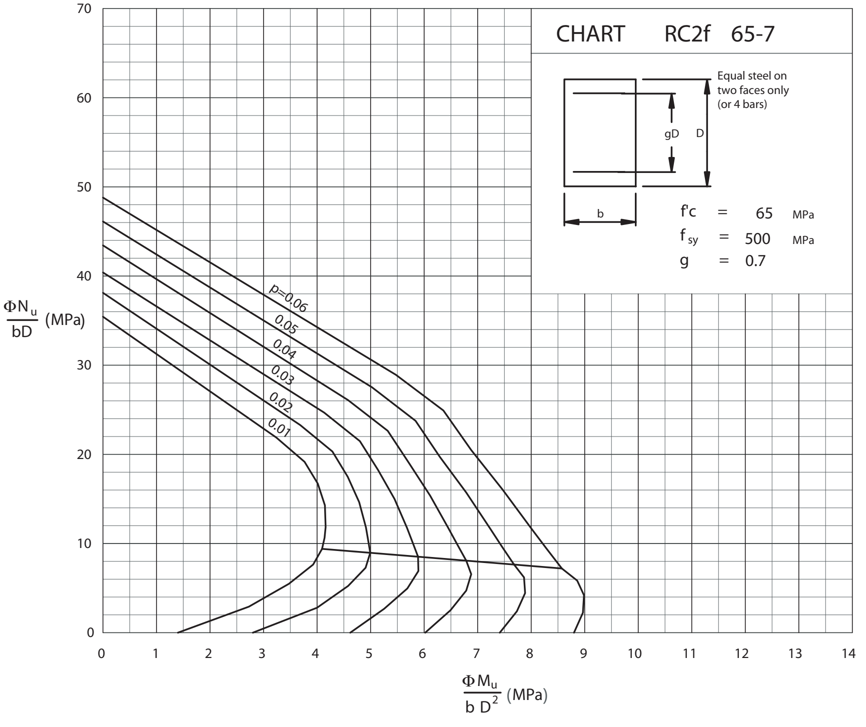


CHART RC2f 65-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 65 MPa
f_{sy} = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)

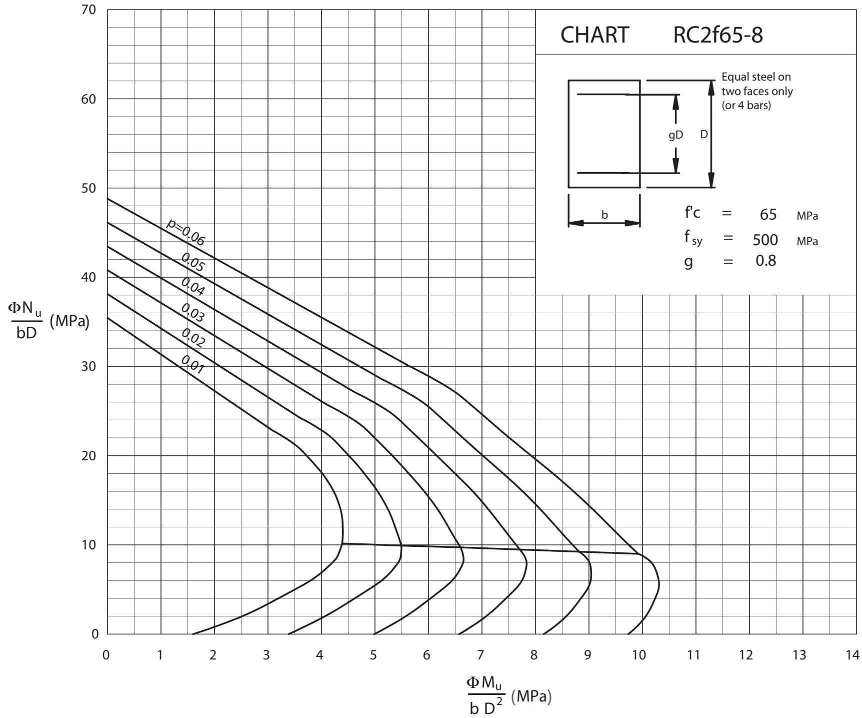


CHART RC2f65-8
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 65 MPa
f_{sy} = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)
0
10
20
30
40
50
60
70
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

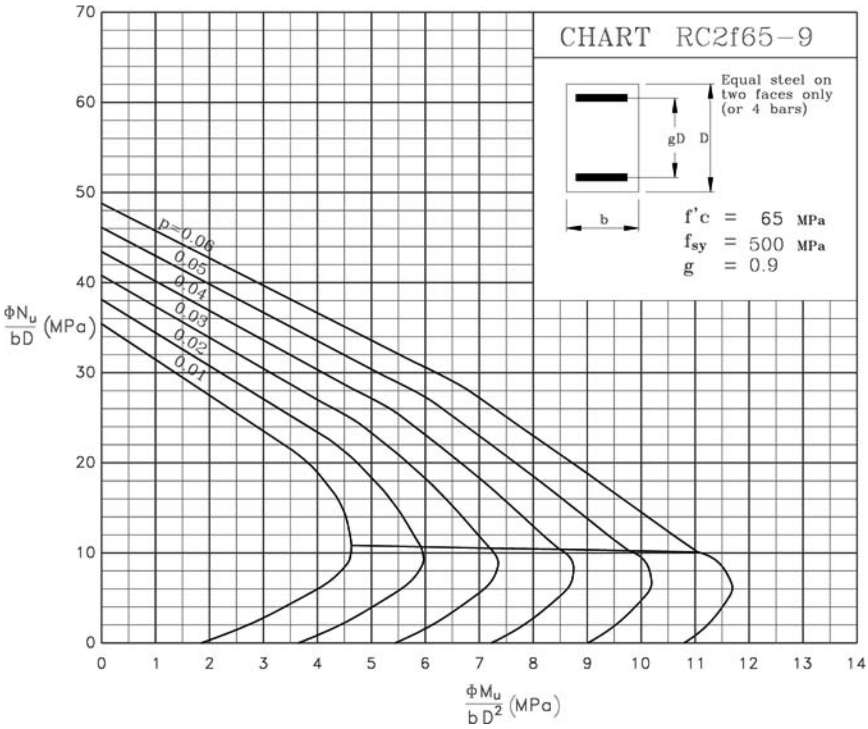


CHART RC2f65-9
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 65 MPa
f_{sy} = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)
0
10
20
30
40
50
60
70
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14

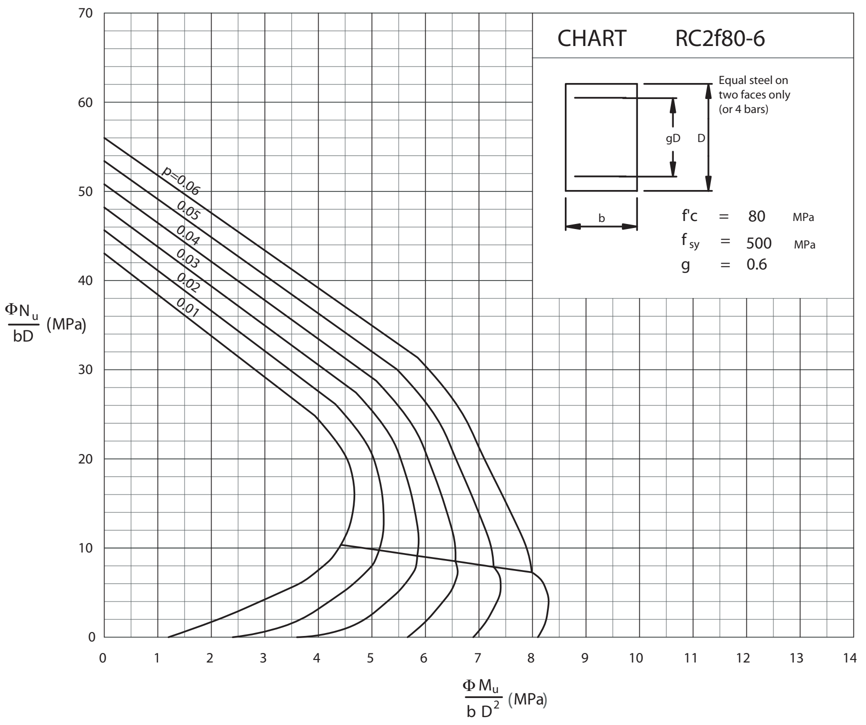


CHART RC2f80-6
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 80 MPa
f_{sy} = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)

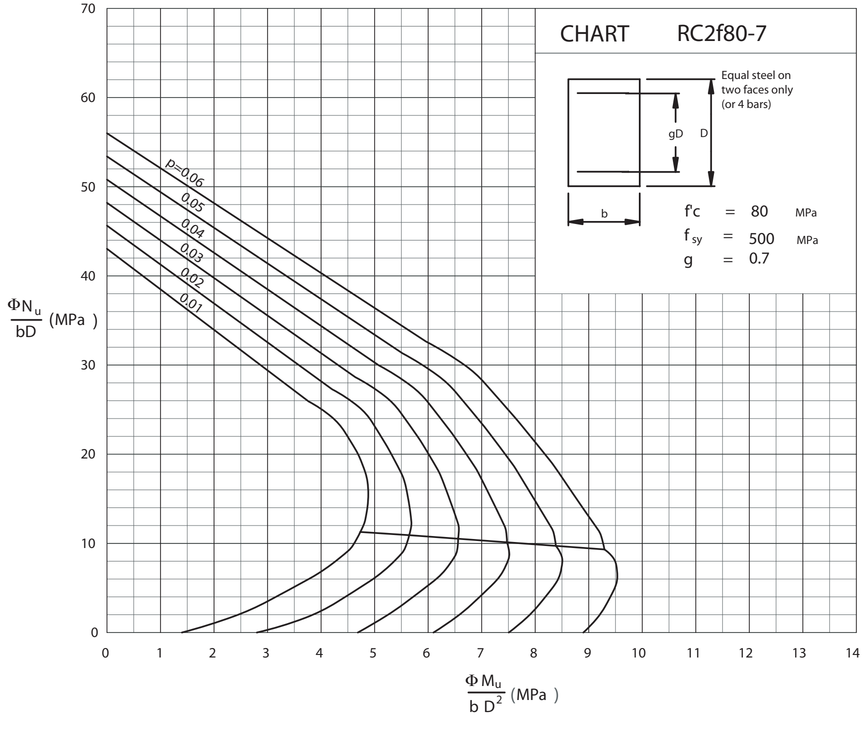


CHART RC2f80-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 80 MPa
f_{sy} = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)

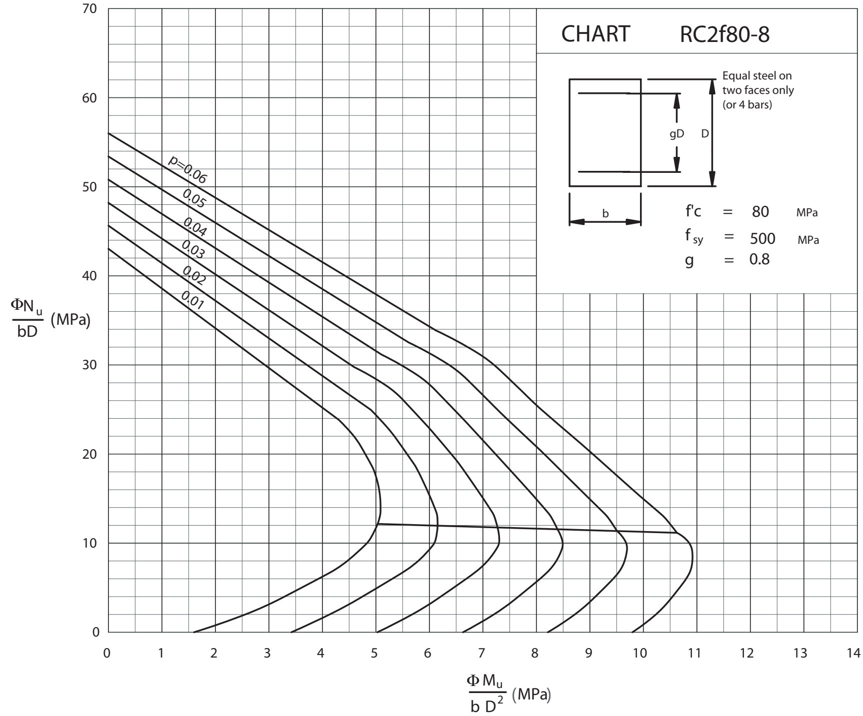


CHART RC2f80-8
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 80 MPa
f_{sy} = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{bD^2}$ (MPa)
0
10
20
30
40
50
60
70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

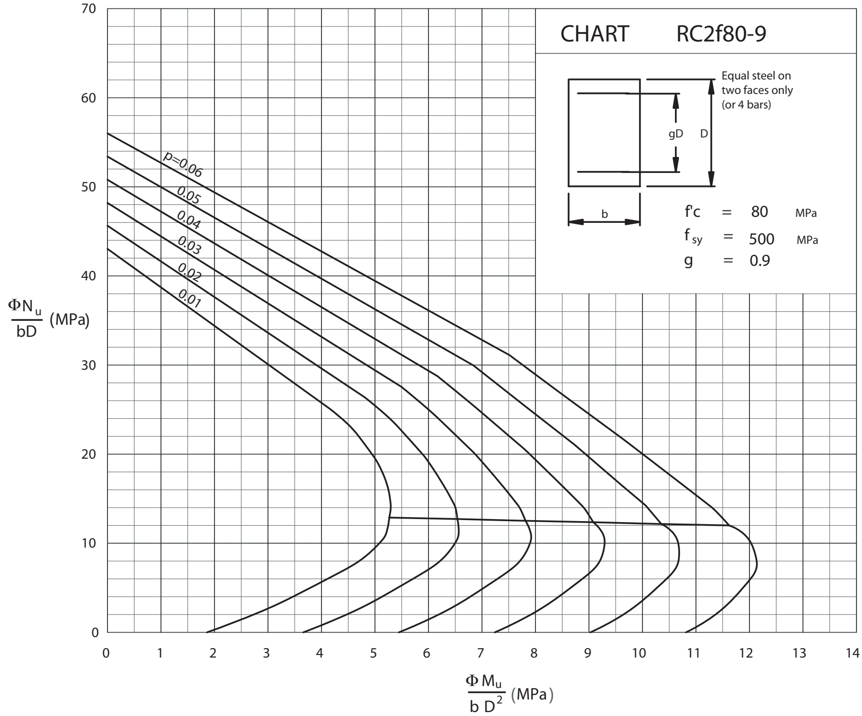


CHART RC2f80-9
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 80 MPa
f_{sy} = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{bD^2}$ (MPa)
0
10
20
30
40
50
60
70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

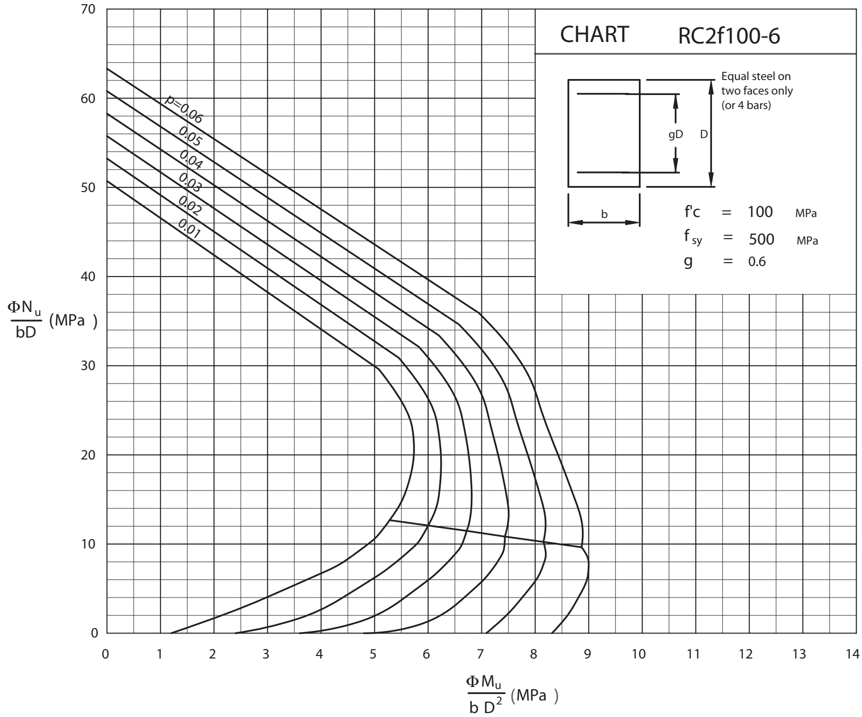


CHART RC2f100-6
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 100 MPa
f_{sy} = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

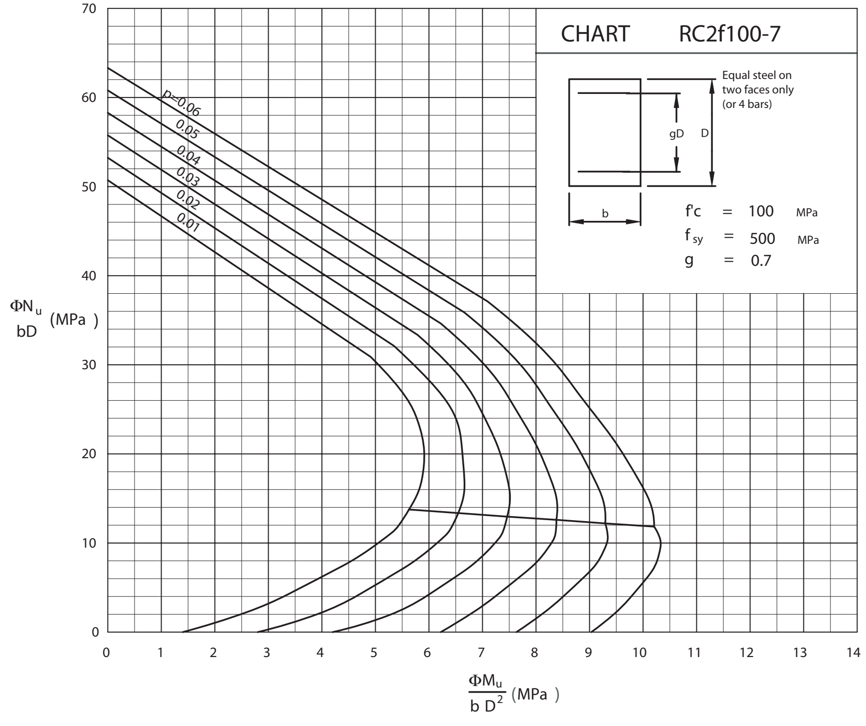


CHART RC2f100-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 100 MPa
f_{sy} = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

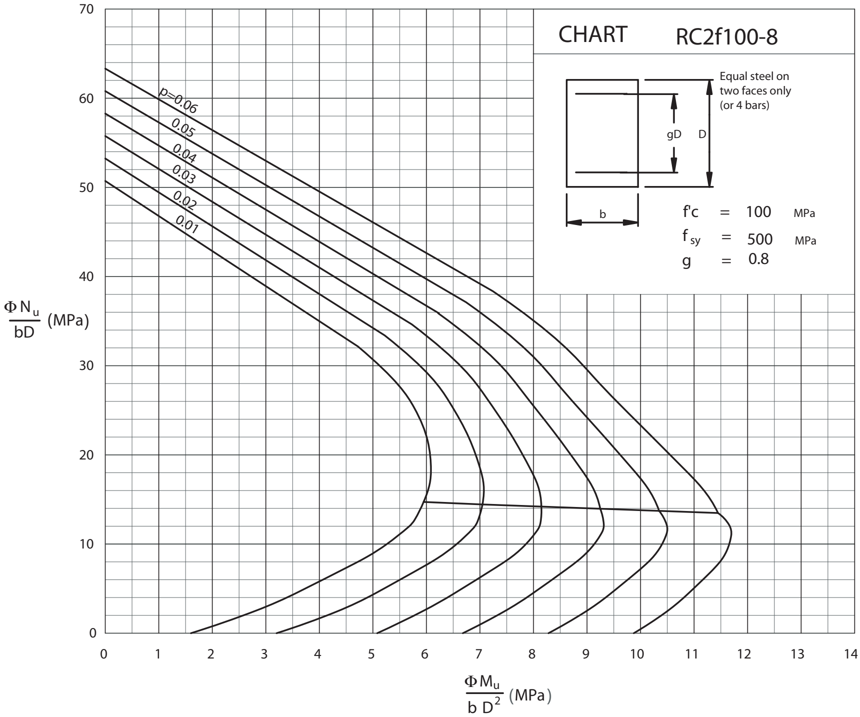


CHART RC2f100-8
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 100 MPa
f_{sy} = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

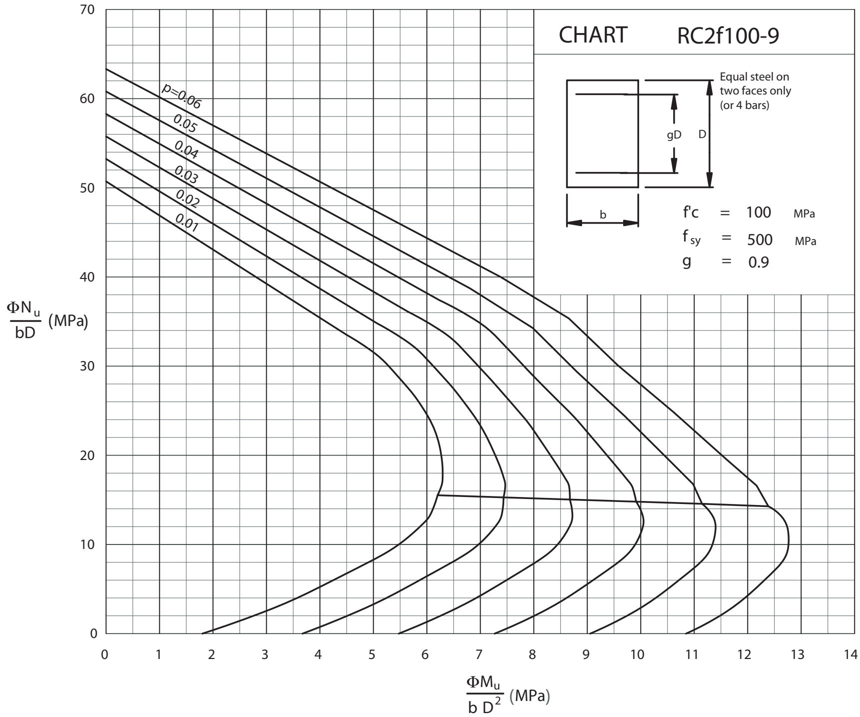


CHART RC2f100-9
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 100 MPa
f_{sy} = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{bD}$ (MPa)
$\frac{\Phi M_u}{b D^2}$ (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

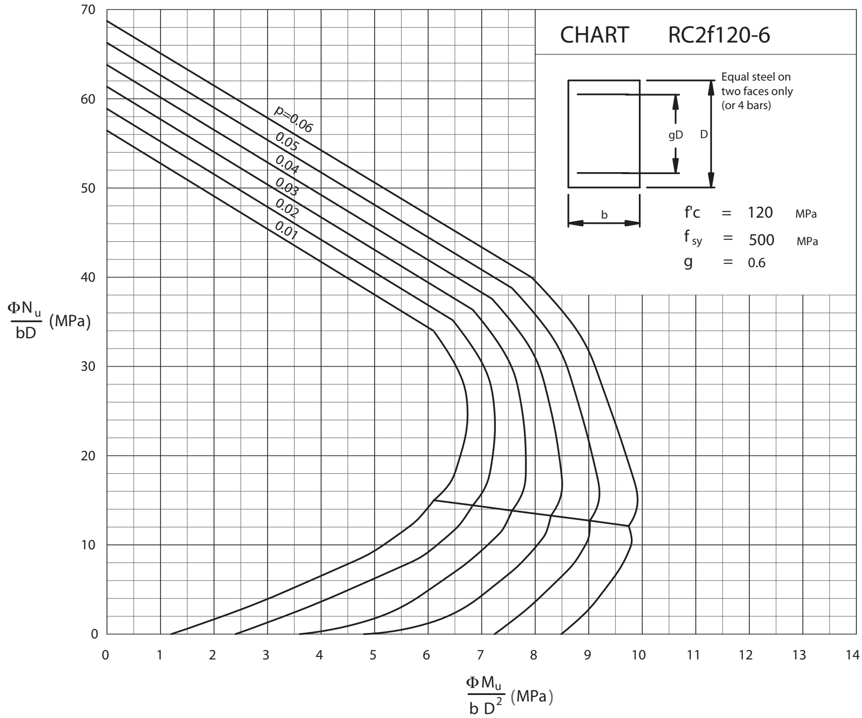


CHART RC2f120-6
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 120 MPa
f sy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 10 20 30 40 50 60 70

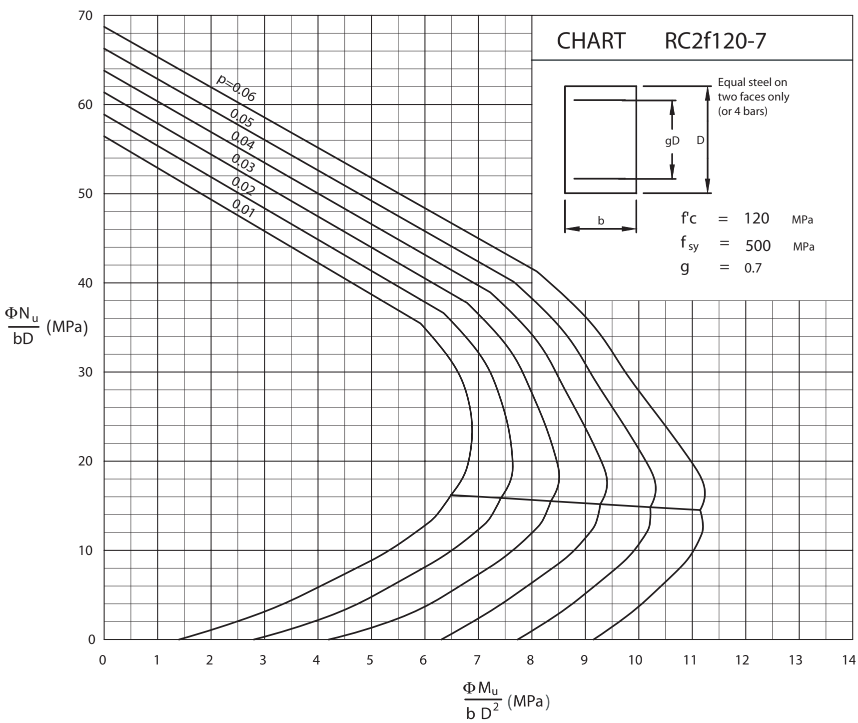


CHART RC2f120-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 120 MPa
f sy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 10 20 30 40 50 60 70

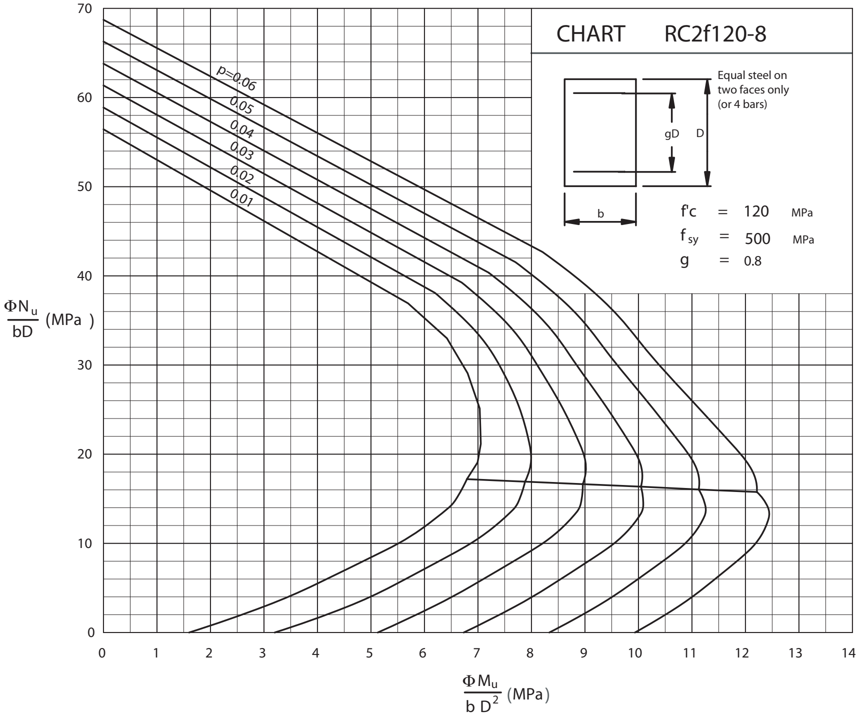


CHART RC2f120-8
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 120 MPa
f sy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 10 20 30 40 50 60 70

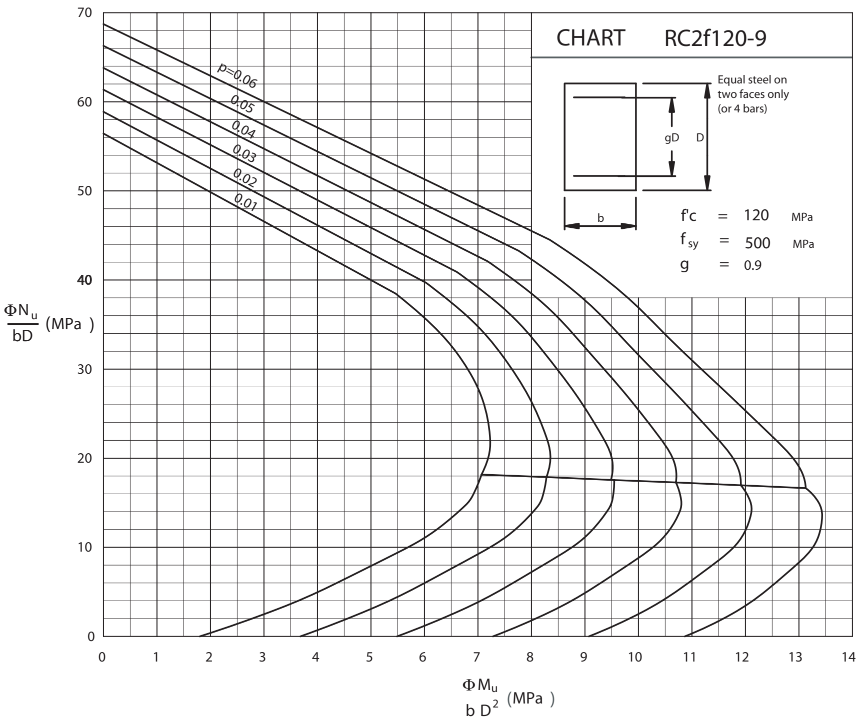


CHART RC2f120-9
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 120 MPa
f sy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 10 20 30 40 50 60 70

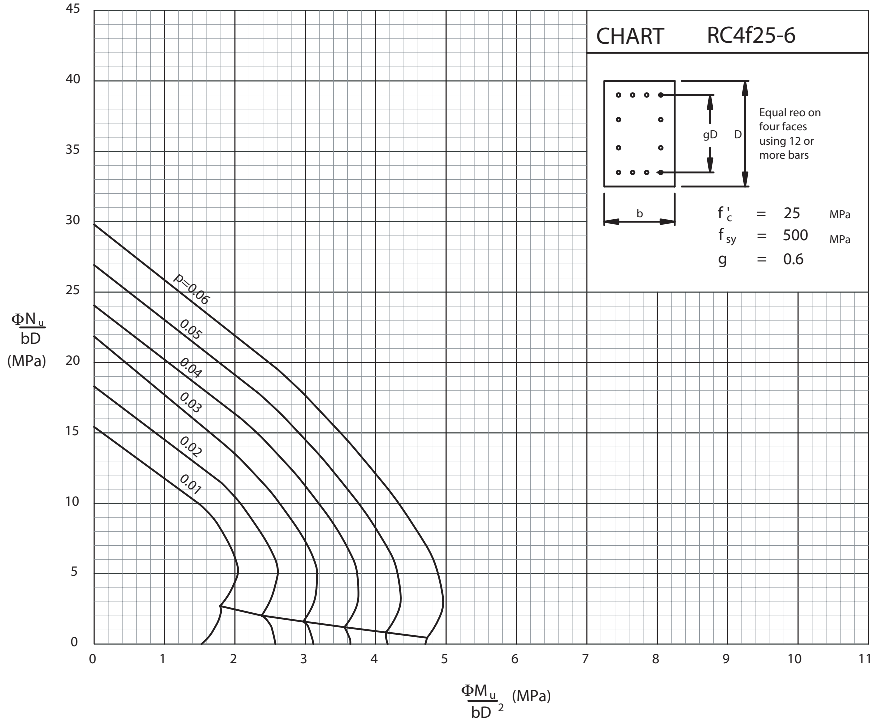


CHART RC4f25-6
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 25 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

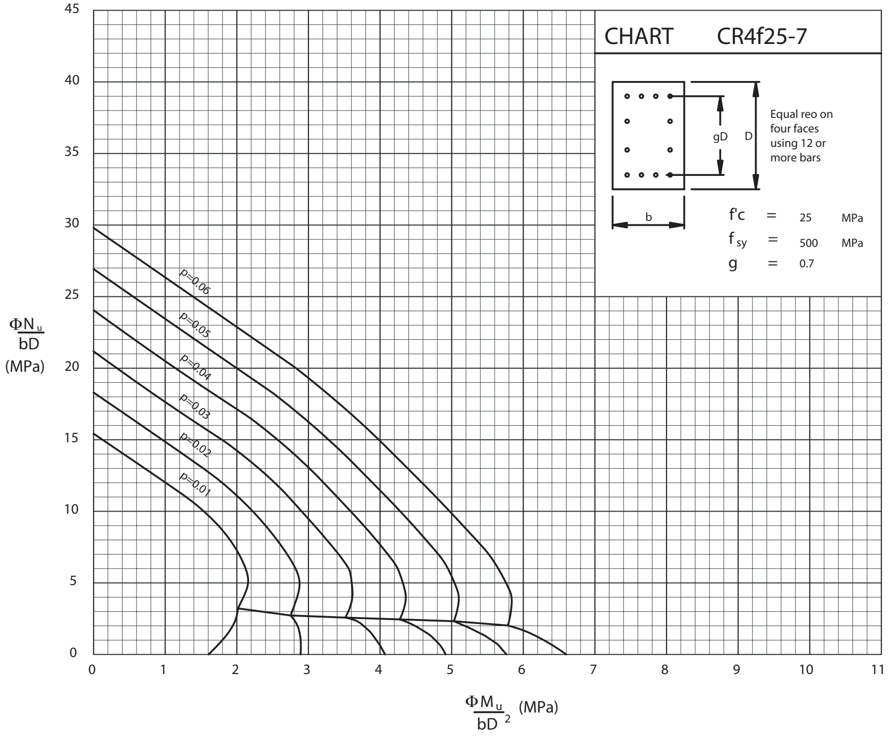


CHART CR4f25-7
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 25 MPa
fsy = 500 MPa
g = 0.7
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

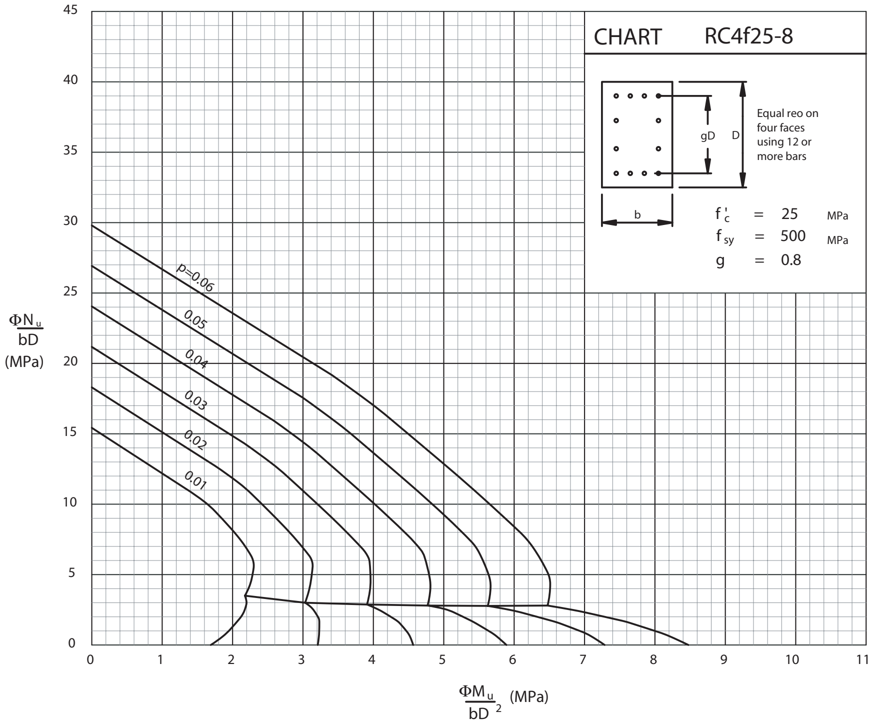


CHART RC4f25-8
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 25 MPa
fsy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

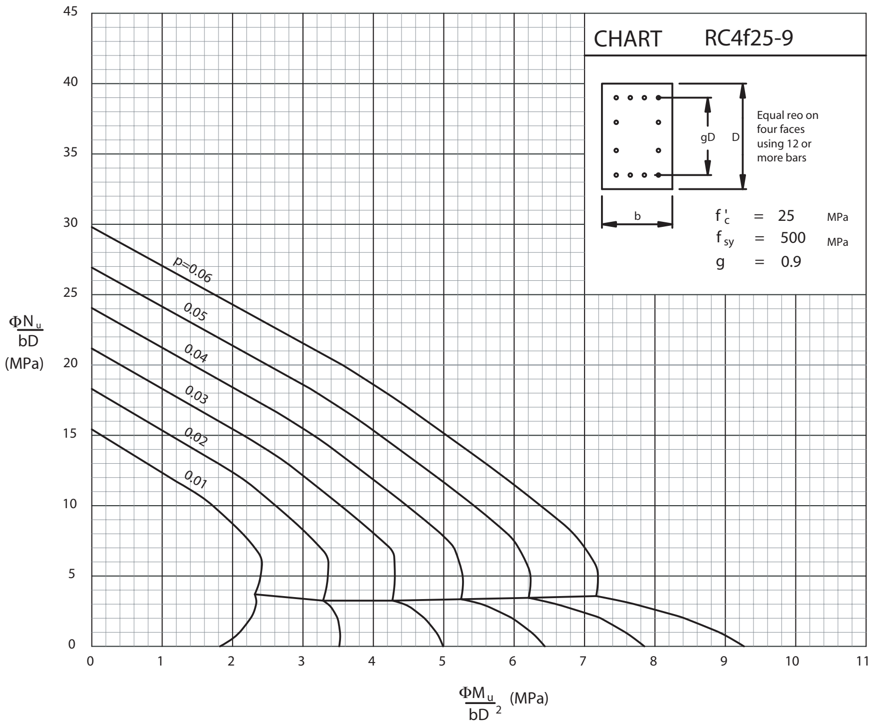


CHART RC4f25-9
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 25 MPa
fsy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

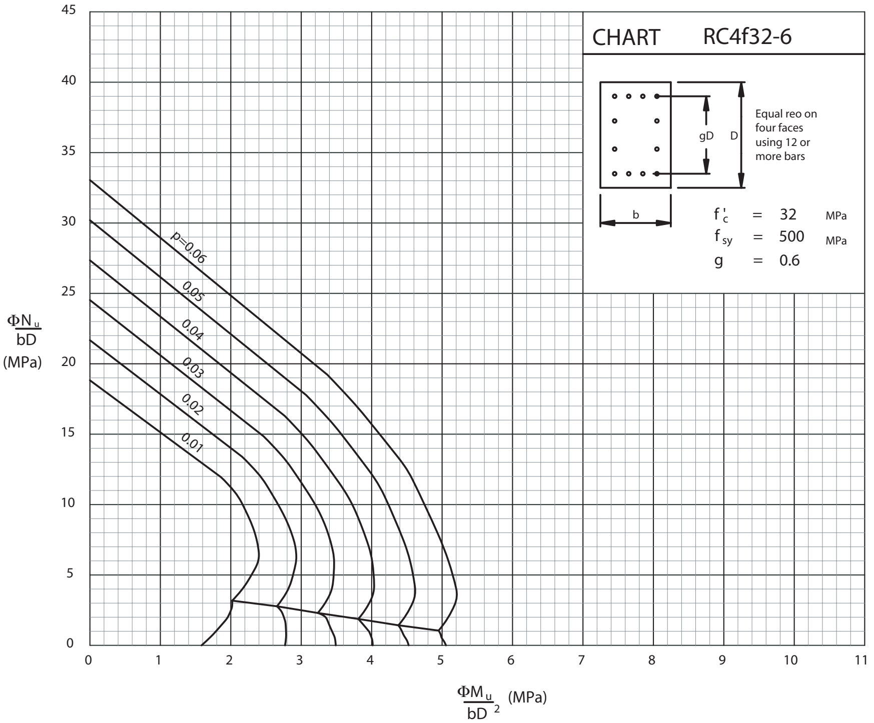


CHART RC4f32-6
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 32 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD2 (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

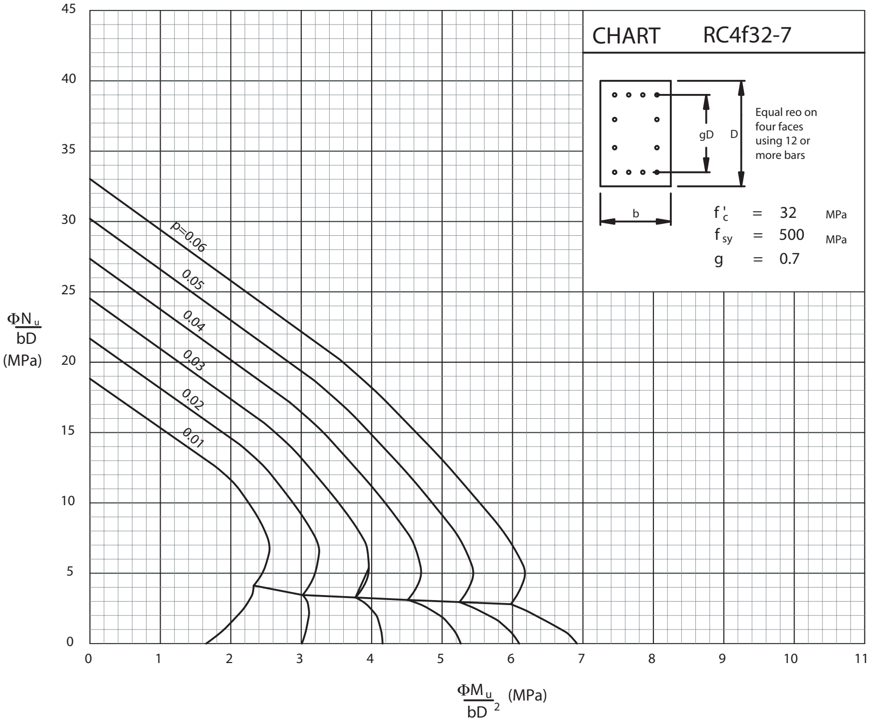


CHART RC4f32-7
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 32 MPa
fsy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD2 (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

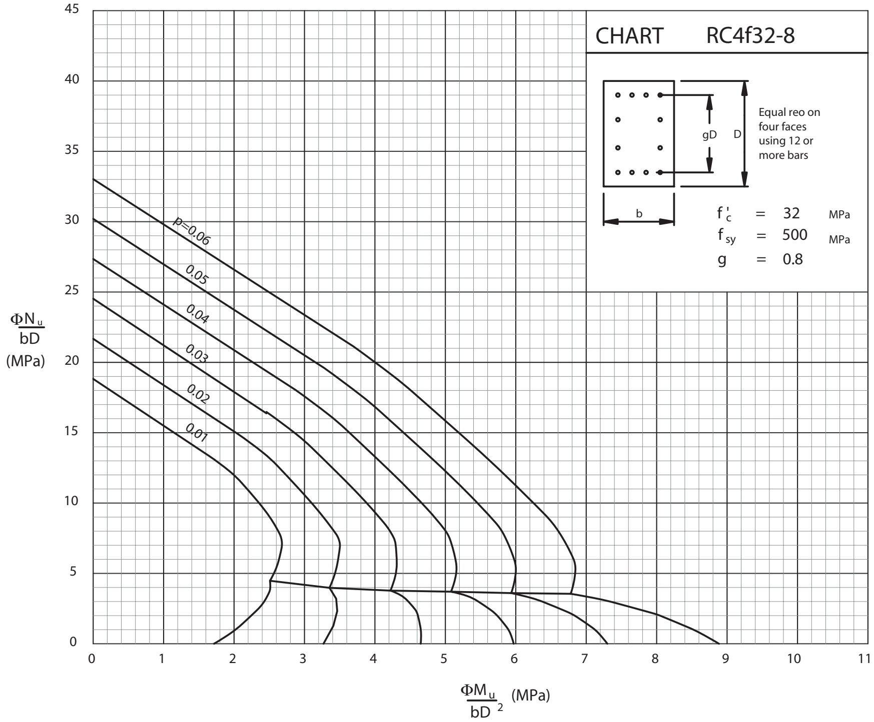


CHART RC4f32-8
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 32 MPa
fsy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD2 (MPa)
0
1
2
3
4
5
6
7
8
9
10
11
5
10
15
20
25
30
35
40
45

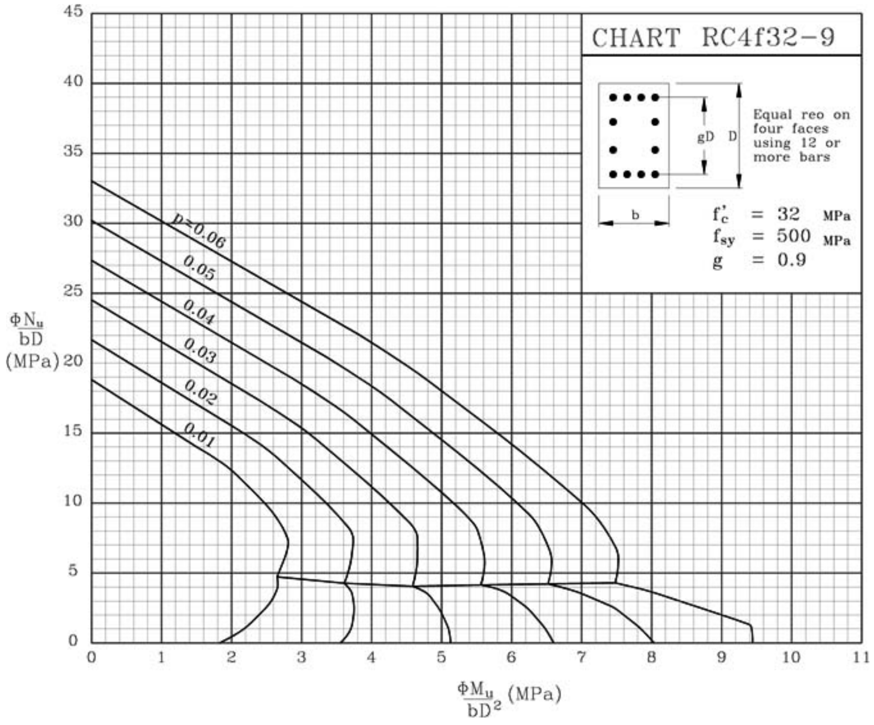


CHART RC4f32-9
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 32 MPa
fsy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/bD (MPa)
ΦMu/bD2 (MPa)
0
1
2
3
4
5
6
7
8
9
10
11
5
10
15
20
25
30
35
40
45

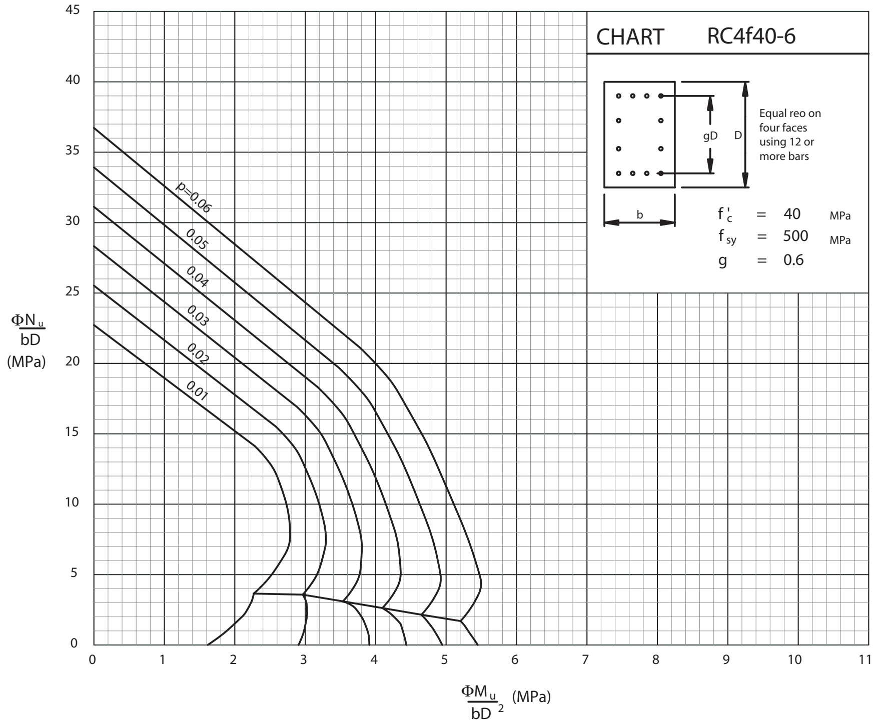


CHART RC4f40-6
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 40 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD2 (MPa)
0
1
2
3
4
5
6
7
8
9
10
11
5
10
15
20
25
30
35
40
45

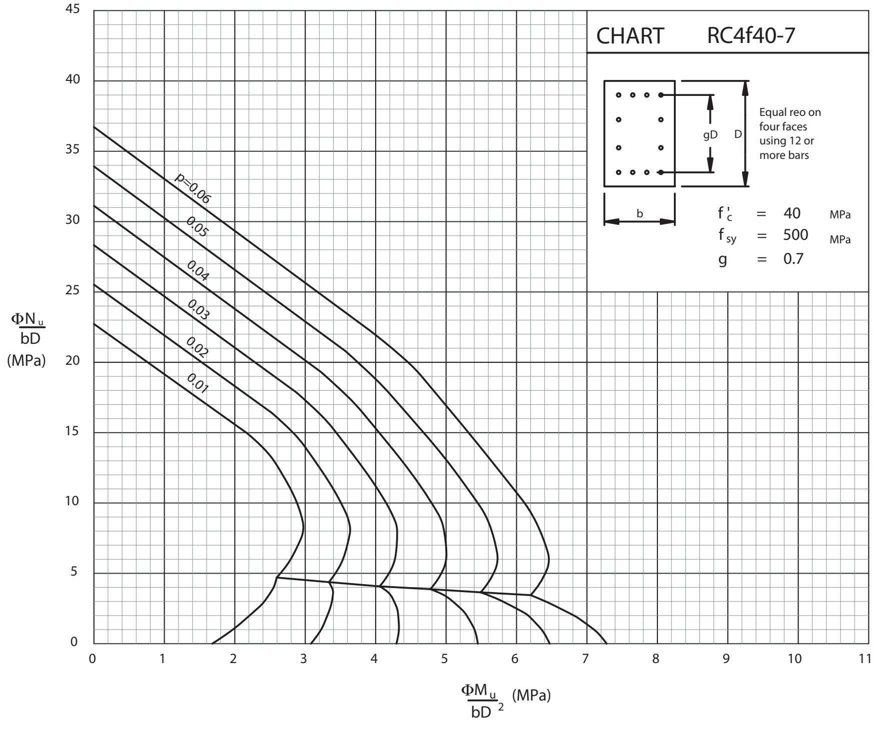


CHART RC4f40-7
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 40 MPa
fsy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD2 (MPa)
0
1
2
3
4
5
6
7
8
9
10
11
5
10
15
20
25
30
35
40
45

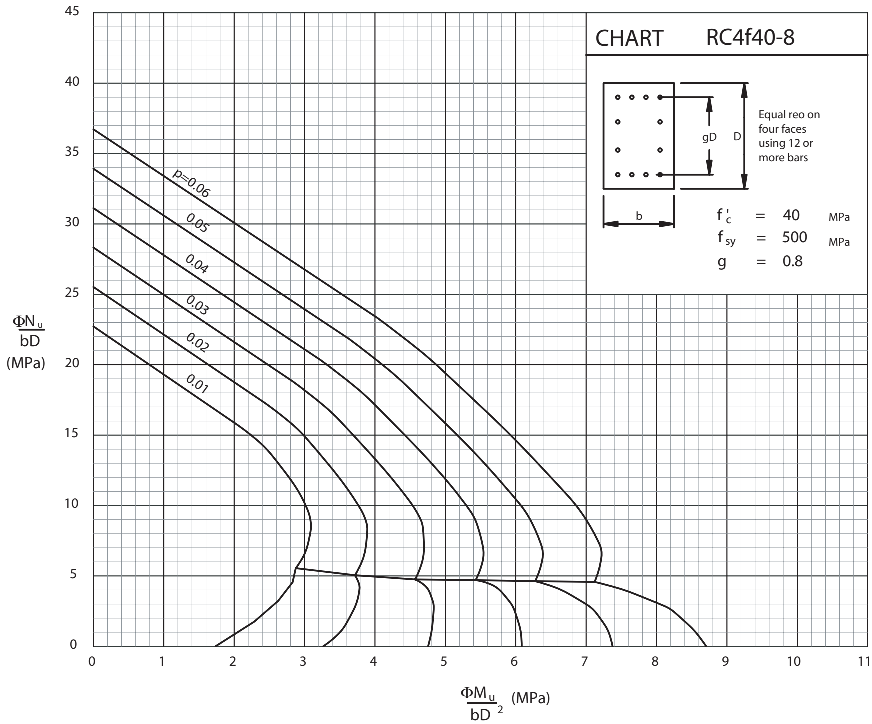


CHART RC4f40-8
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 40 MPa
fsy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)
0 1 2 3 4 5 6 7 8 9 10 11
0 5 10 15 20 25 30 35 40 45

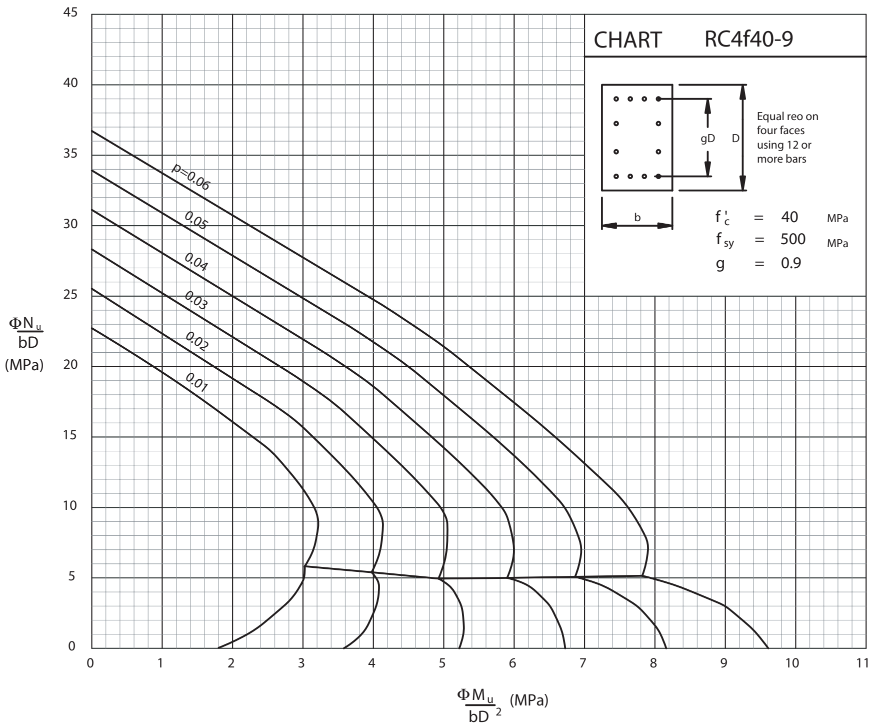


CHART RC4f40-9
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 40 MPa
fsy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)
0 1 2 3 4 5 6 7 8 9 10 11
0 5 10 15 20 25 30 35 40 45

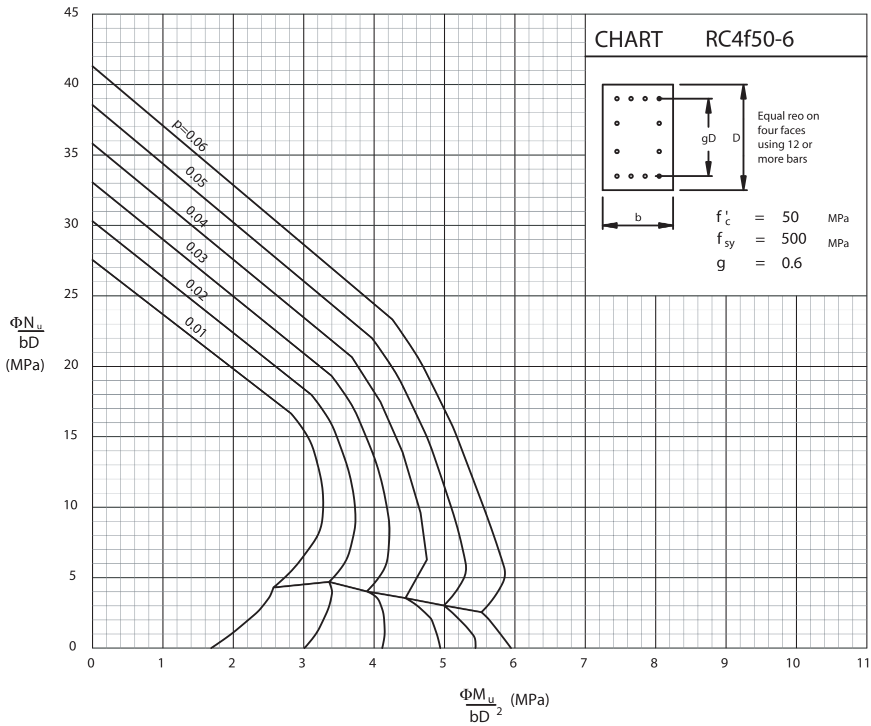


CHART RC4f50-6
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 50 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)

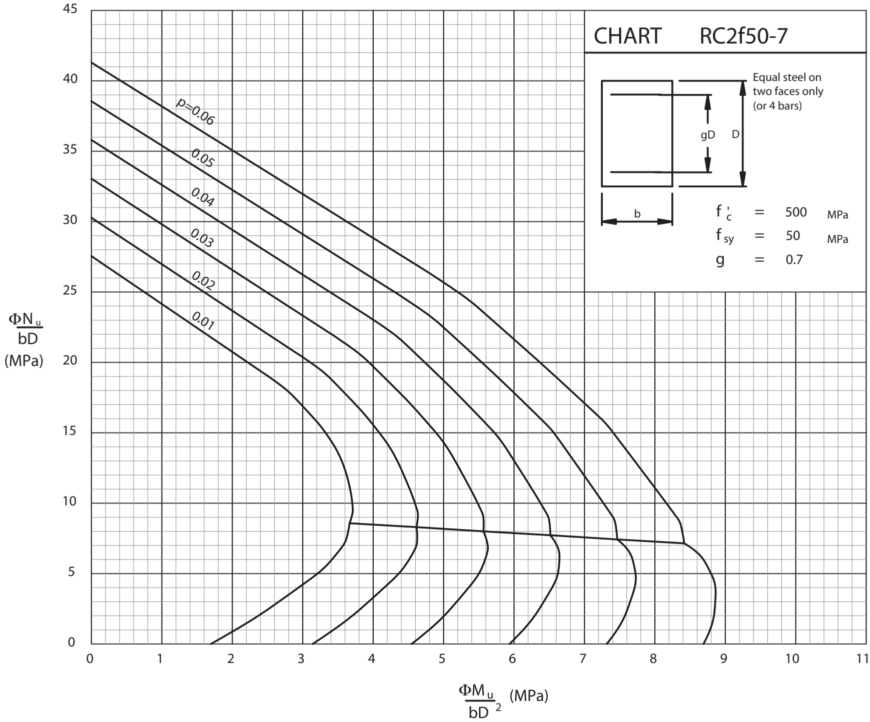


CHART RC2f50-7
Equal steel on two faces only (or 4 bars)
gD
D
b
f'c = 500 MPa
fsy = 50 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD² (MPa)

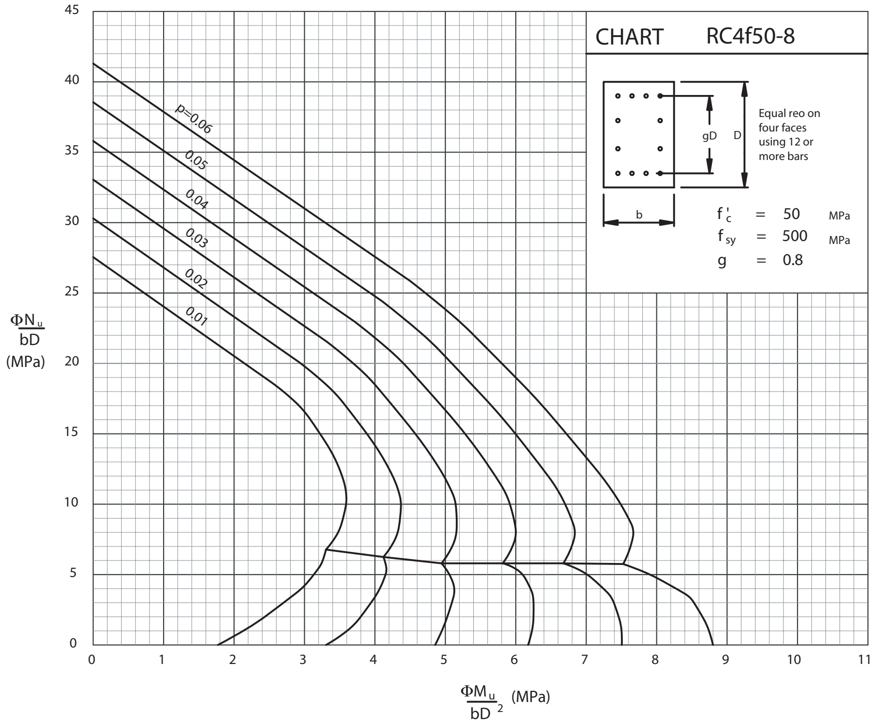


CHART RC4f50-8
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 50 MPa
fsy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD 2 (MPa)
0
5
10
15
20
25
30
35
40
45
0
1
2
3
4
5
6
7
8
9
10
11

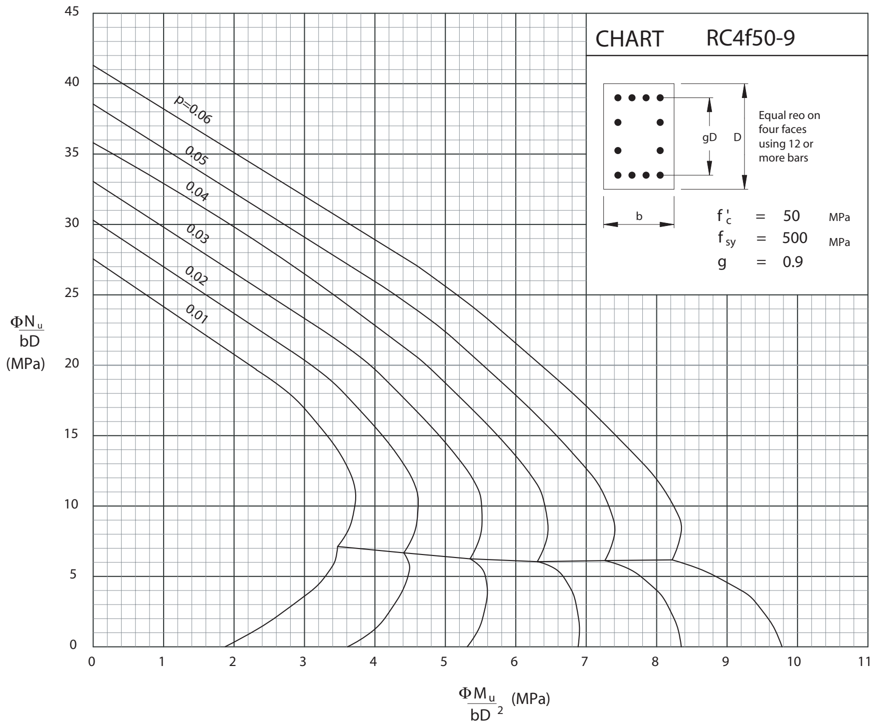


CHART RC4f50-9
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 50 MPa
fsy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / bD 2 (MPa)
0
5
10
15
20
25
30
35
40
45
0
1
2
3
4
5
6
7
8
9
10
11

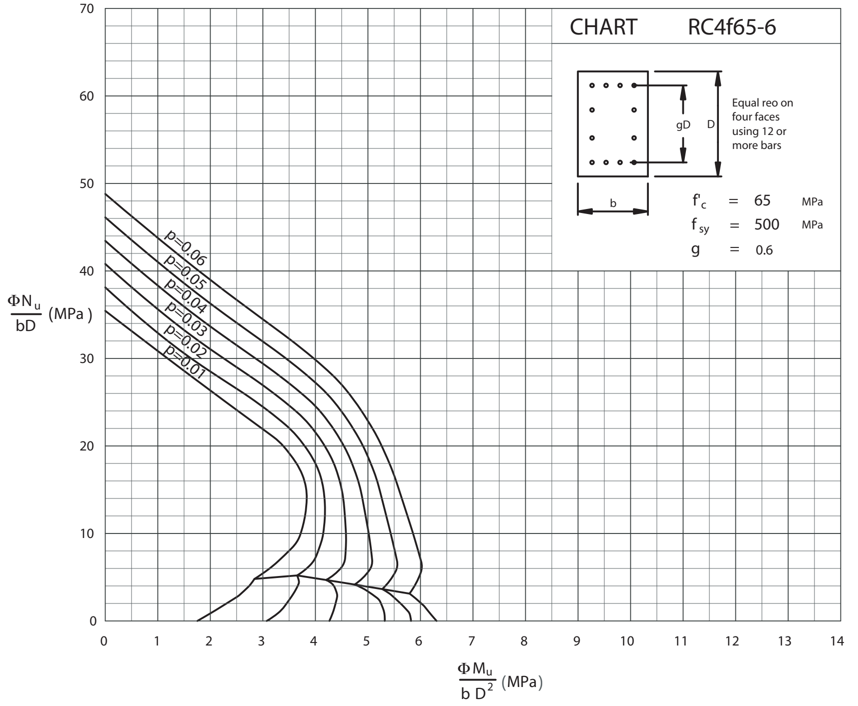


CHART RC4f65-6
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 65 MPa
fsy = 500 MPa
g = 0.6
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu/bD (MPa)
ΦMu/bD² (MPa)
0
10
20
30
40
50
60
70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

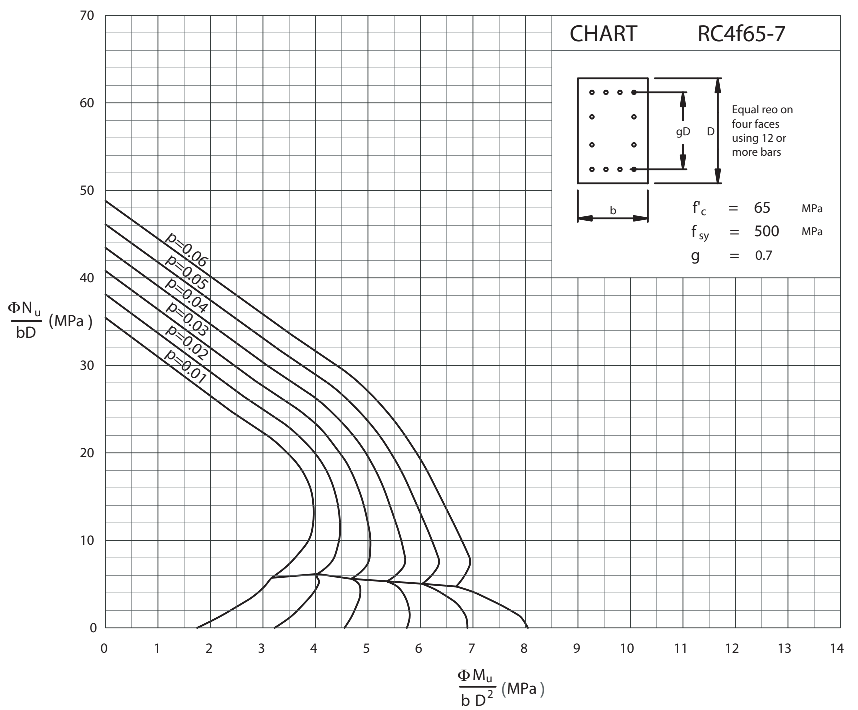


CHART RC4f65-7
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 65 MPa
fsy = 500 MPa
g = 0.7
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu/bD (MPa)
ΦMu/bD² (MPa)
0
10
20
30
40
50
60
70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

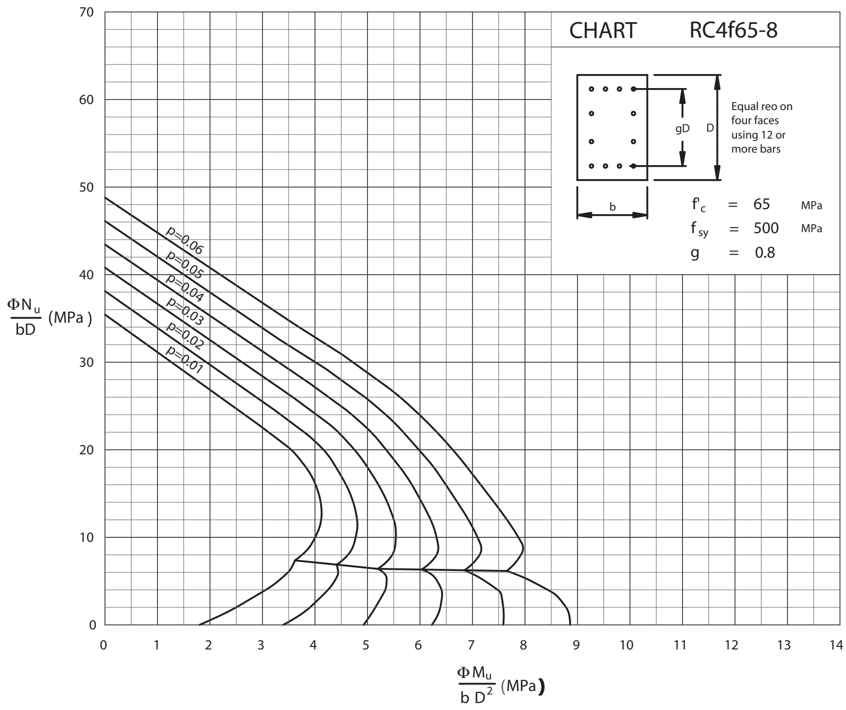


CHART RC4f65-8
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 65 MPa
f sy = 500 MPa
g = 0.8
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

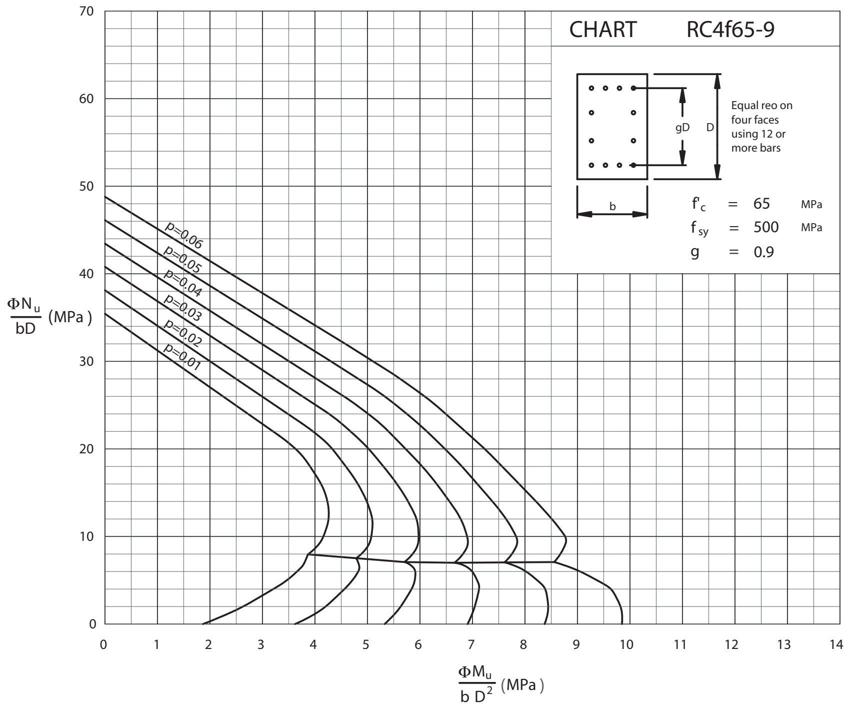


CHART RC4f65-9
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 65 MPa
f sy = 500 MPa
g = 0.9
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

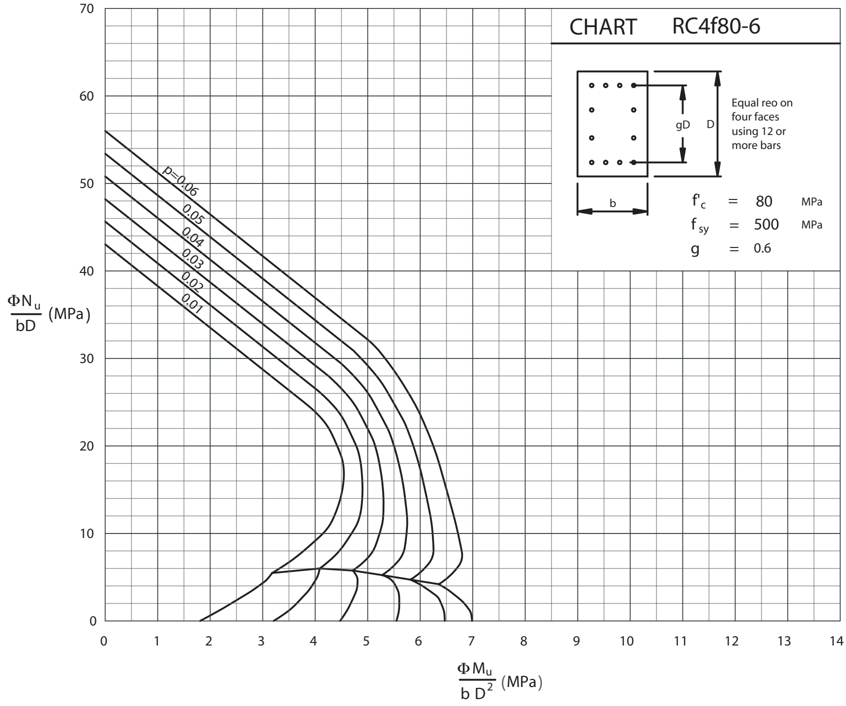


CHART RC4f80-6
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 80 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D² (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

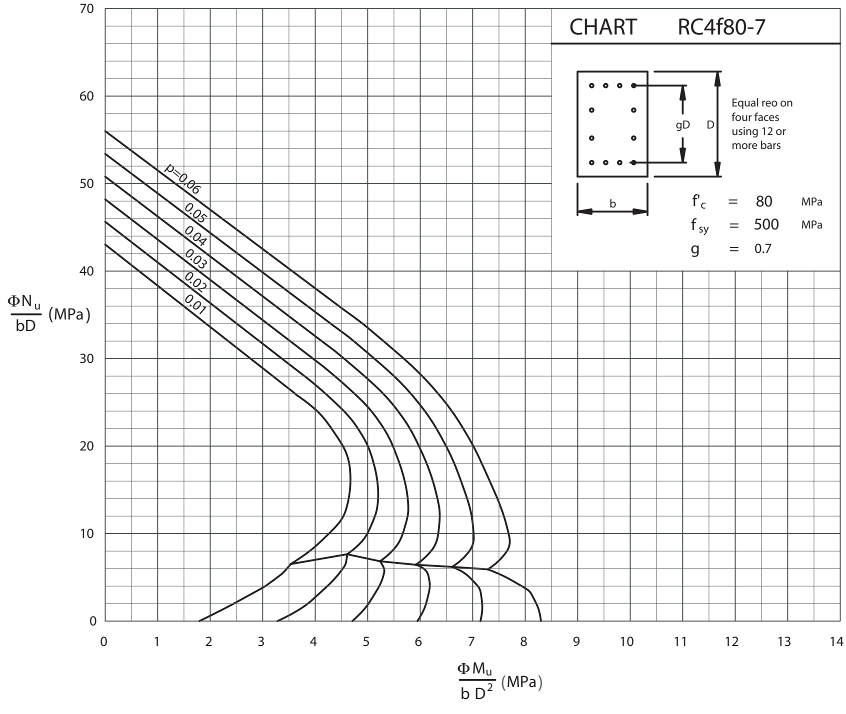


CHART RC4f80-7
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 80 MPa
fsy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D² (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

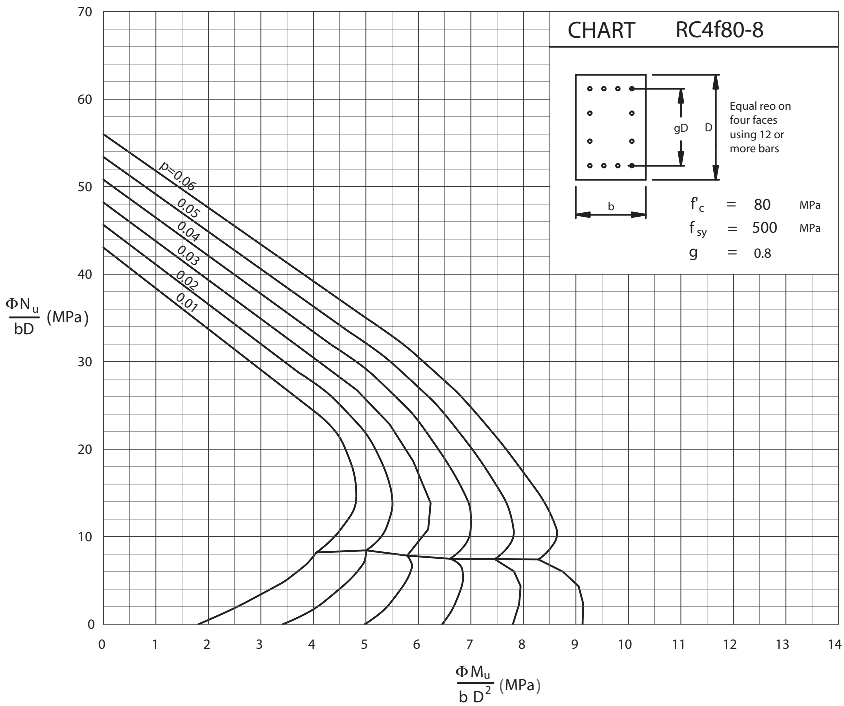


CHART RC4f80-8
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 80 MPa
f sy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D² (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

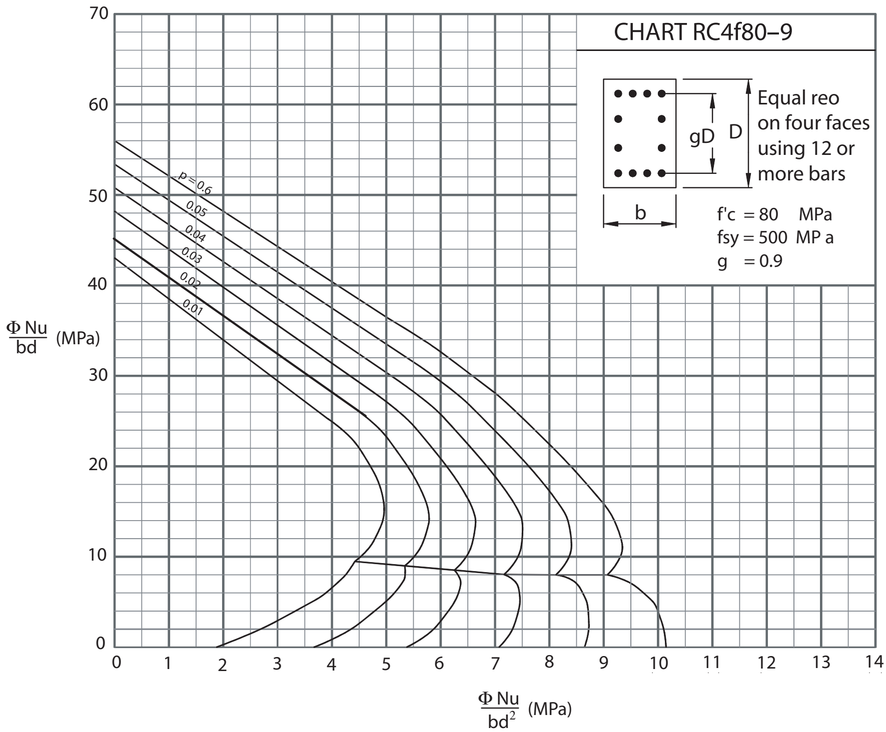


CHART RC4f80–9
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 80 MPa
fsy = 500 MP a
g = 0.9
p=0.6
0.05
0.04
0.03
0.02
0.01
Φ Nu / bd (MPa)
Φ Nu / bd² (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

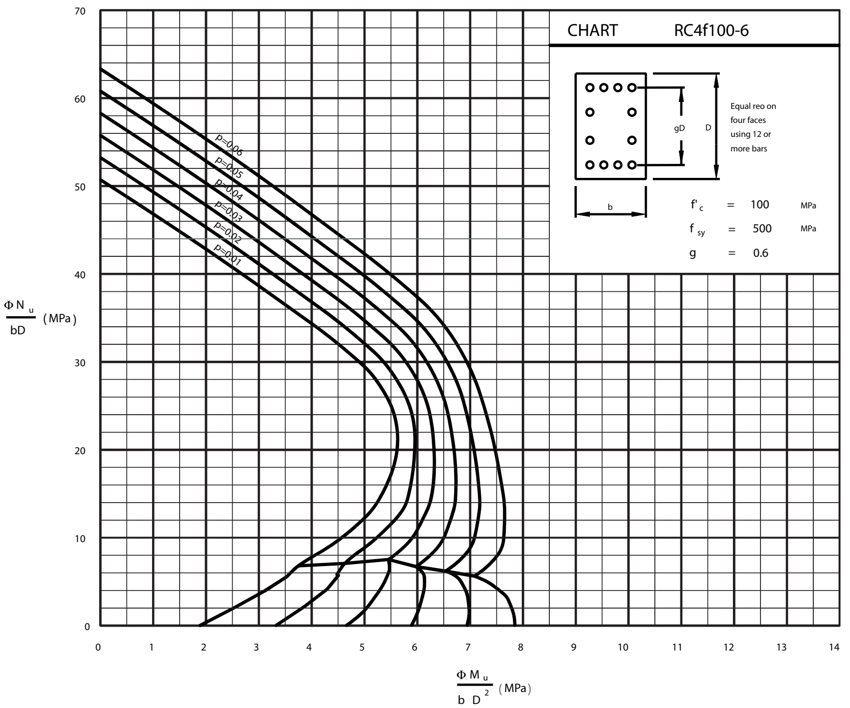


CHART RC4f100-6
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 100 MPa
fsy = 500 MPa
g = 0.6
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu/bD (MPa)
ΦMu/bD² (MPa)

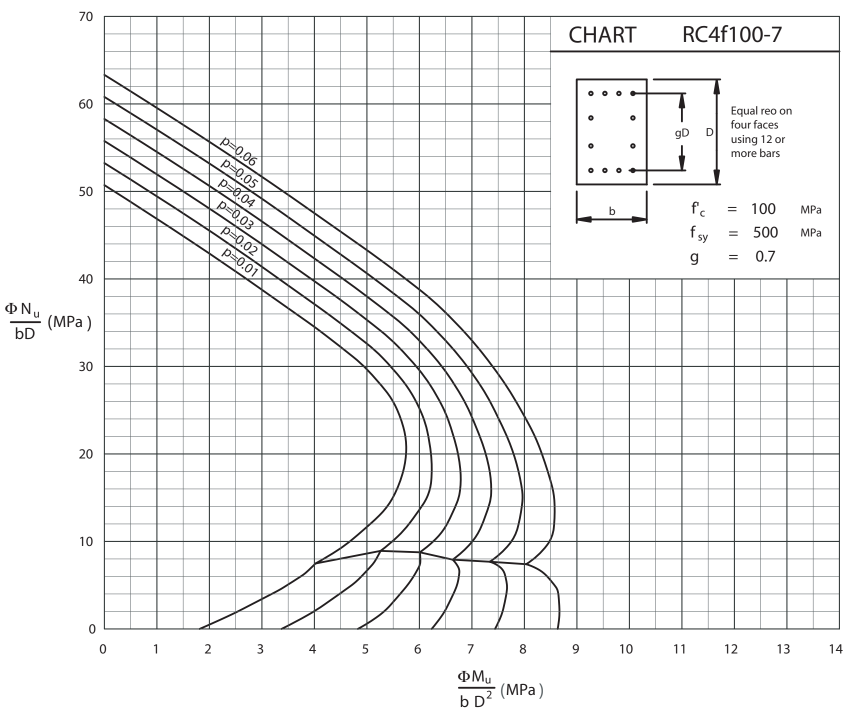


CHART RC4f100-7
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 100 MPa
fsy = 500 MPa
g = 0.7
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu/bD (MPa)
ΦMu/bD² (MPa)

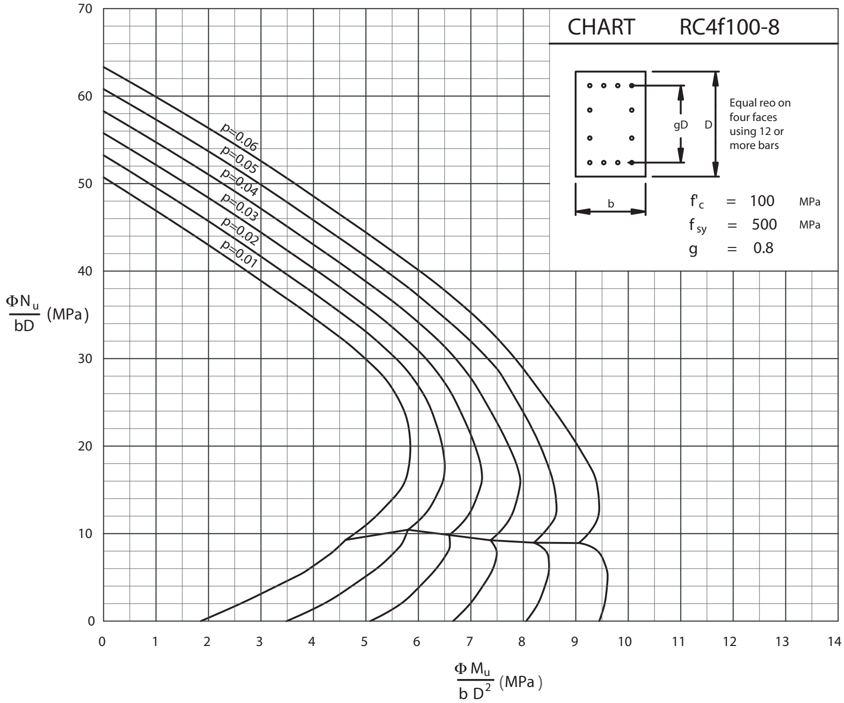


CHART RC4f100-8
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 100 MPa
fsy = 500 MPa
g = 0.8
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu/bD (MPa)
ΦMu/bD² (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

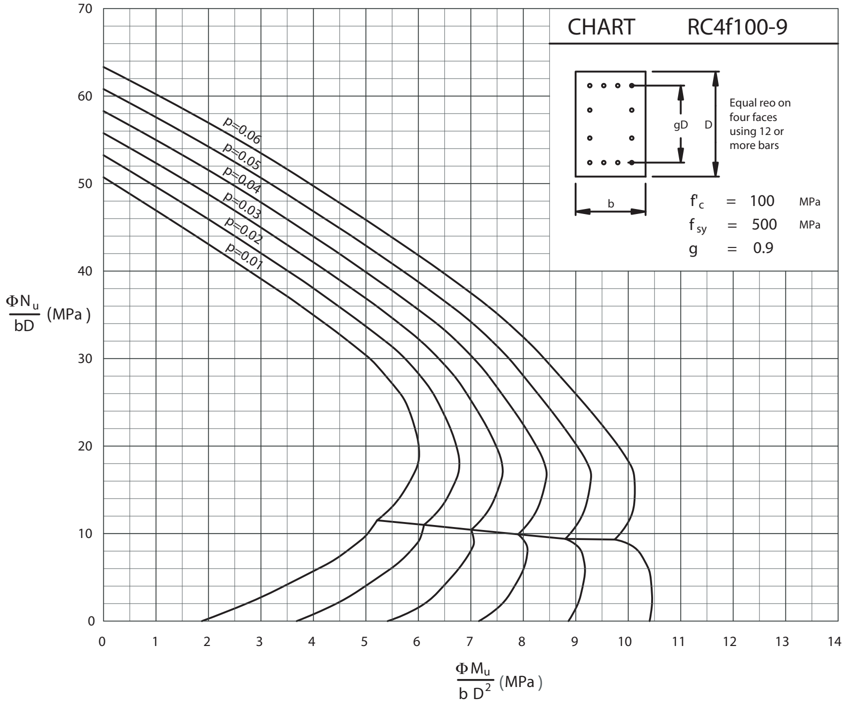


CHART RC4f100-9
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 100 MPa
fsy = 500 MPa
g = 0.9
p=0.06
p=0.05
p=0.04
p=0.03
p=0.02
p=0.01
ΦNu/bD (MPa)
ΦMu/bD² (MPa)
0 10 20 30 40 50 60 70
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

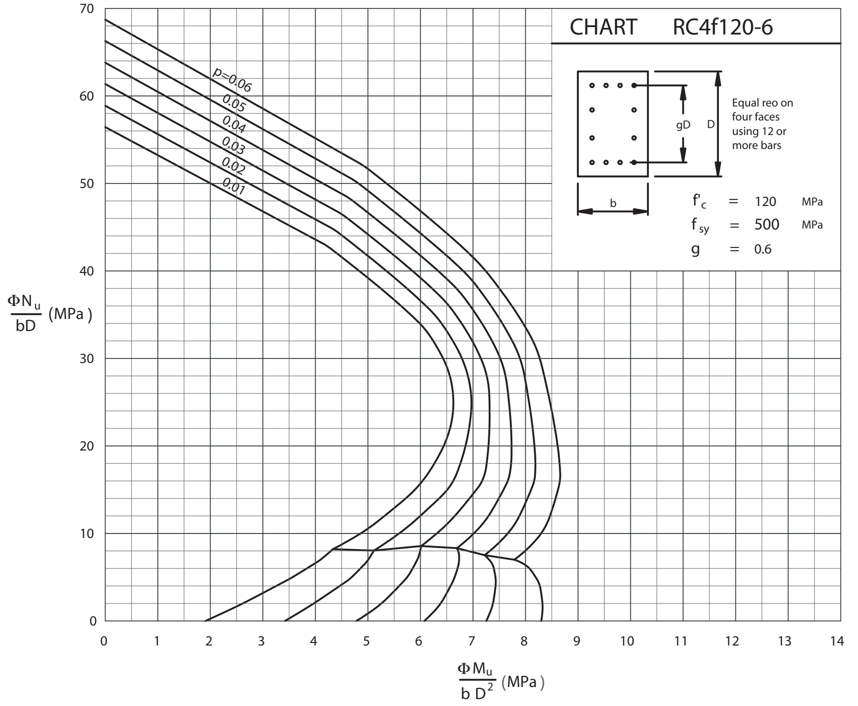


CHART RC4f120-6
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 120 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)

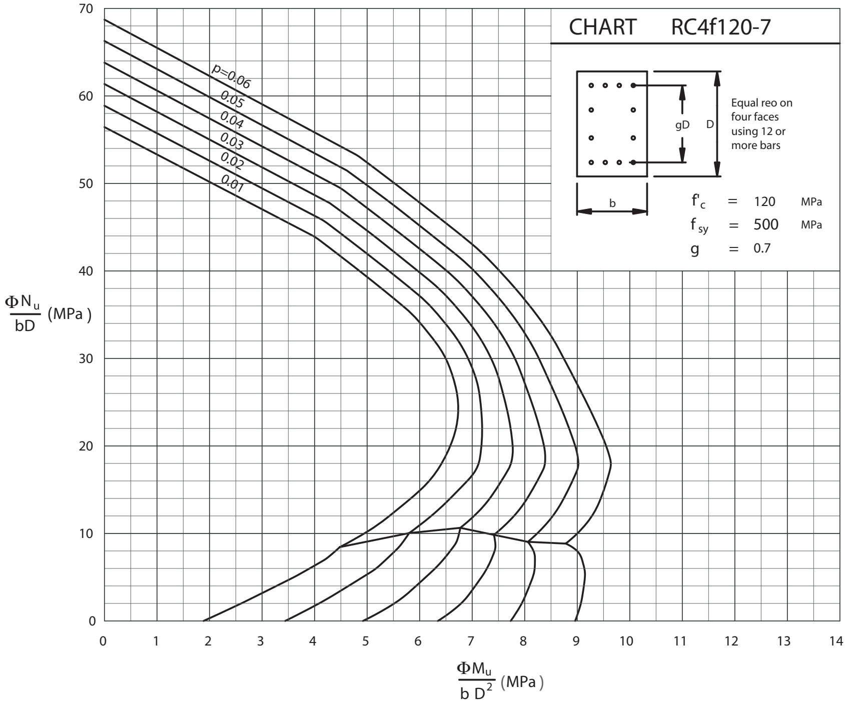


CHART RC4f120-7
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 120 MPa
fsy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)

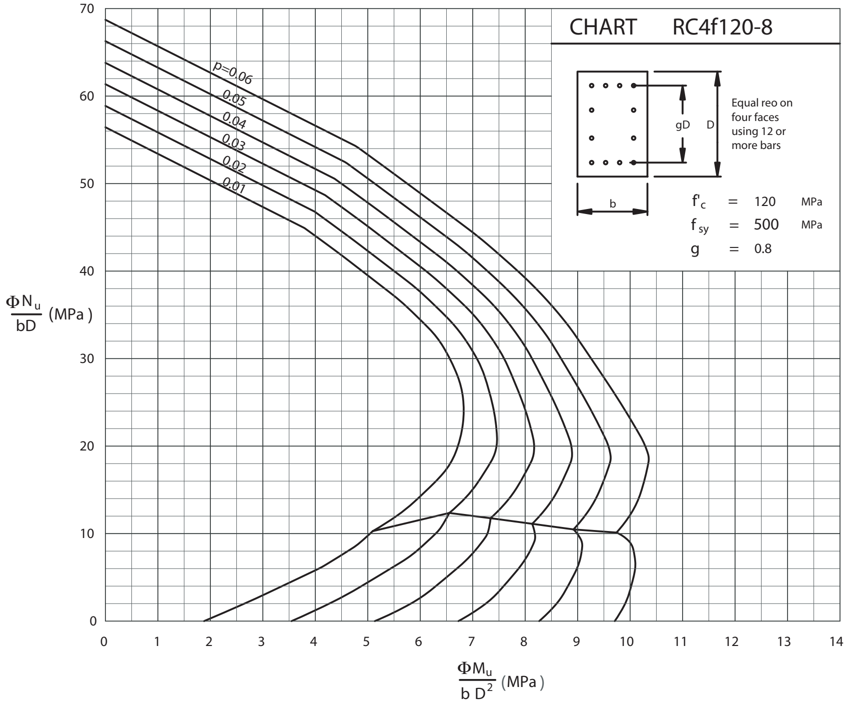


CHART RC4f120-8
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 120 MPa
f sy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 10 20 30 40 50 60 70

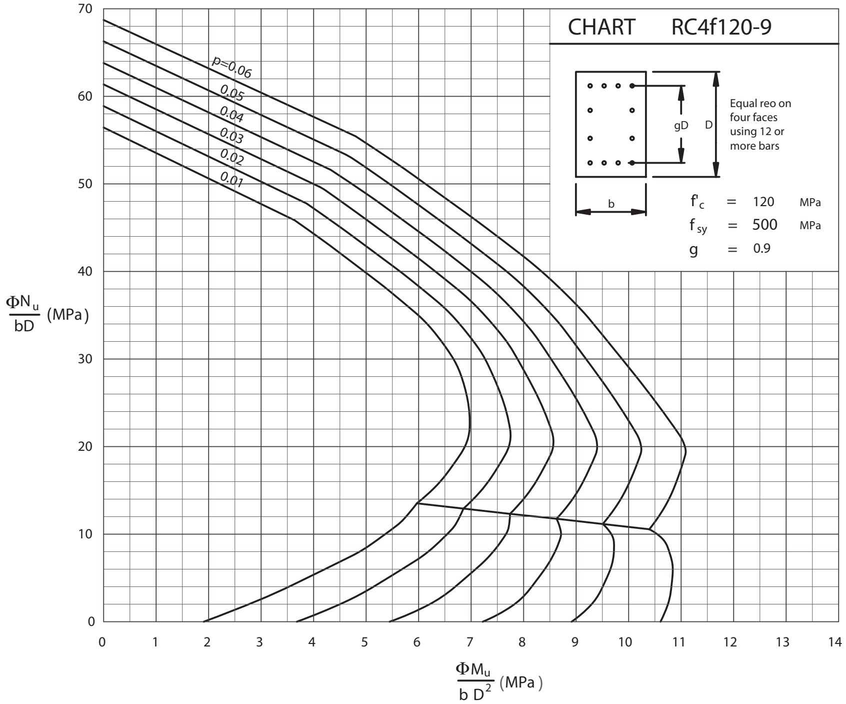


CHART RC4f120-9
Equal reo on four faces using 12 or more bars
gD
D
b
f'c = 120 MPa
f sy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / bD (MPa)
ΦMu / b D2 (MPa)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
0 10 20 30 40 50 60 70

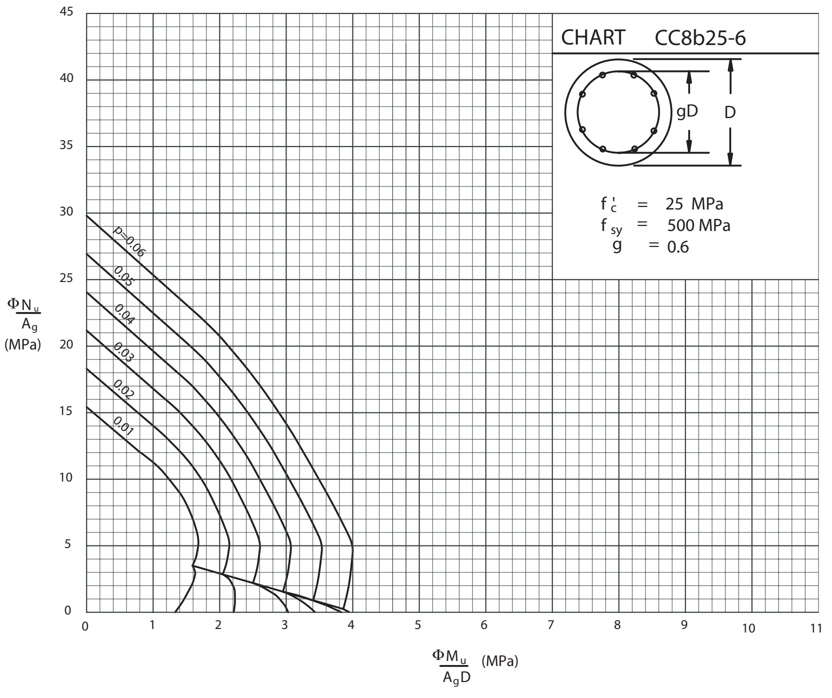


CHART CC8b25-6
gD
D
f'c = 25 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/Ag (MPa)
ΦMu/AgD (MPa)
0
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11

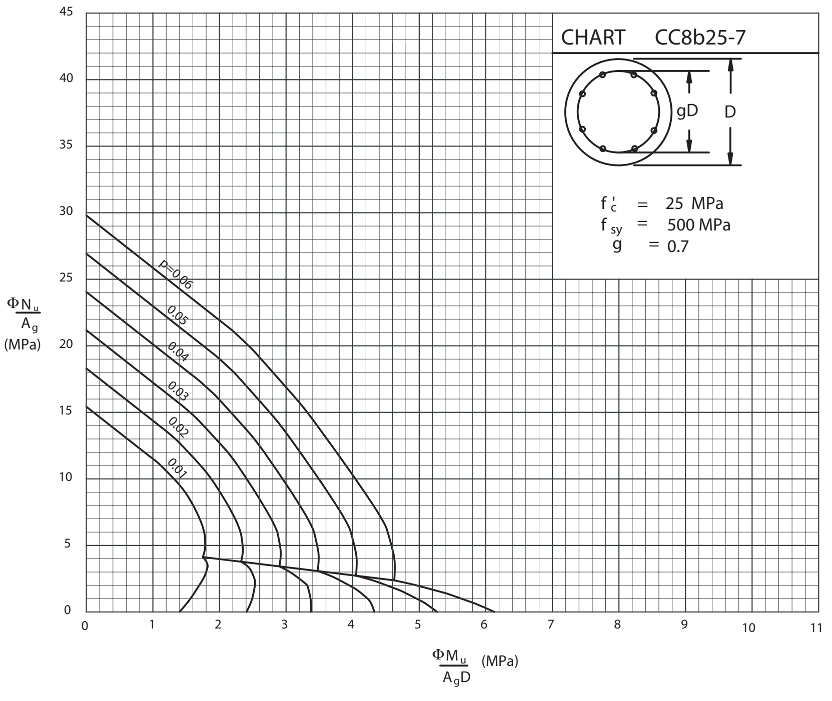


CHART CC8b25-7
gD
D
f'c = 25 MPa
fsy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/Ag (MPa)
ΦMu/AgD (MPa)
0
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11

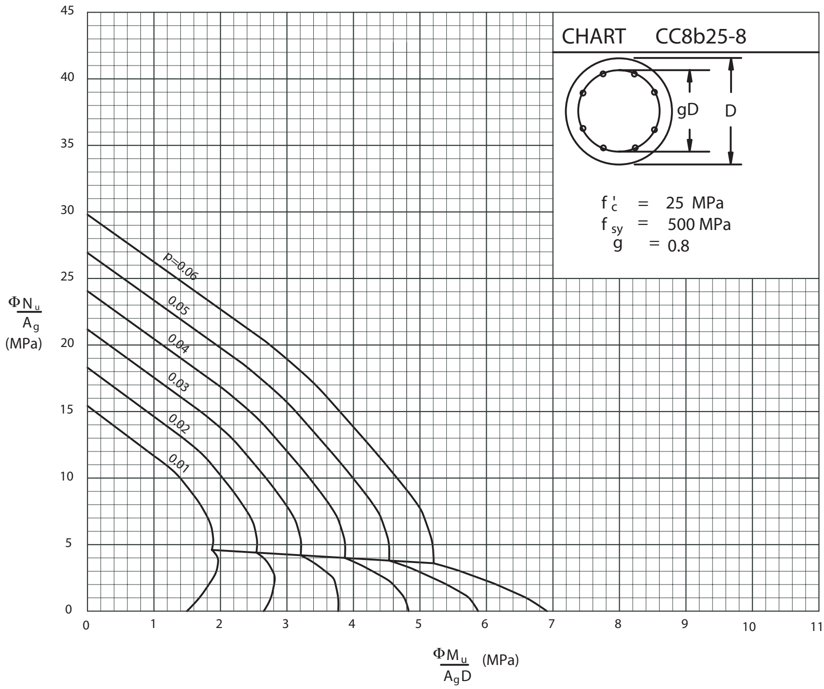


CHART CC8b25-8
gD
D
f'c = 25 MPa
f sy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
0
5
10
15
20
25
30
35
40
45
0
1
2
3
4
5
6
7
8
9
10
11

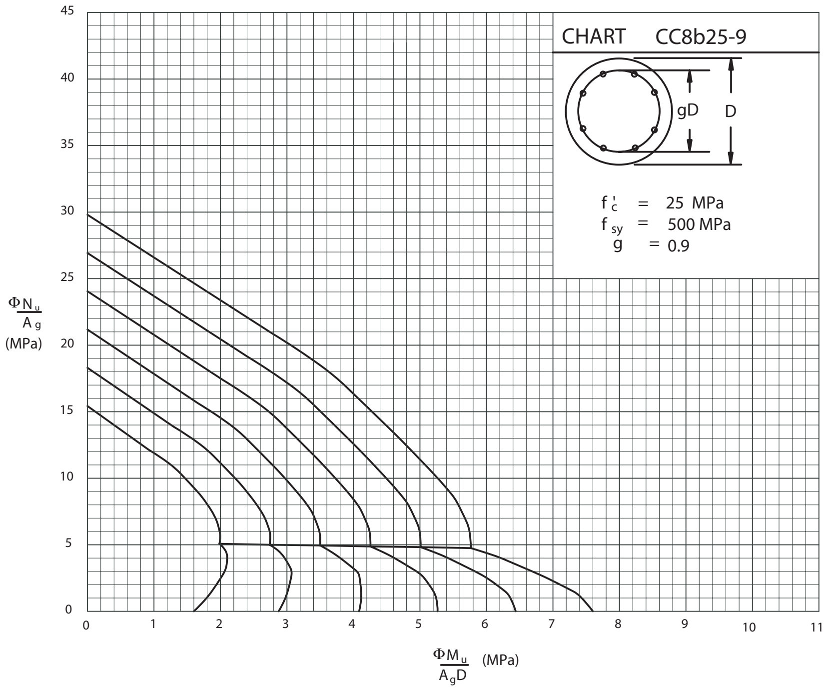


CHART CC8b25-9
gD
D
f'c = 25 MPa
f sy = 500 MPa
g = 0.9
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
0
5
10
15
20
25
30
35
40
45
0
1
2
3
4
5
6
7
8
9
10
11

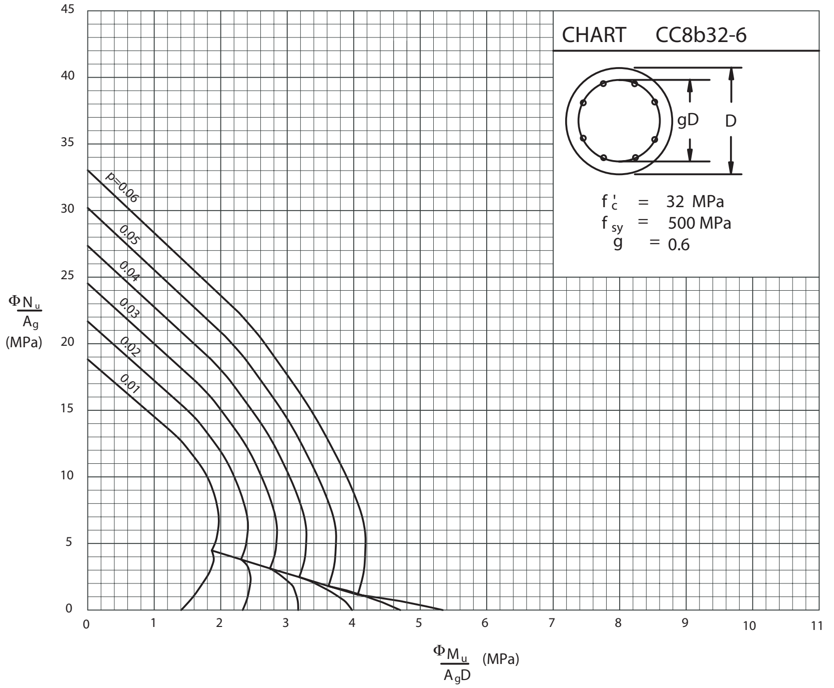


CHART CC8b32-6
gD
D
f'c = 32 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
0
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11

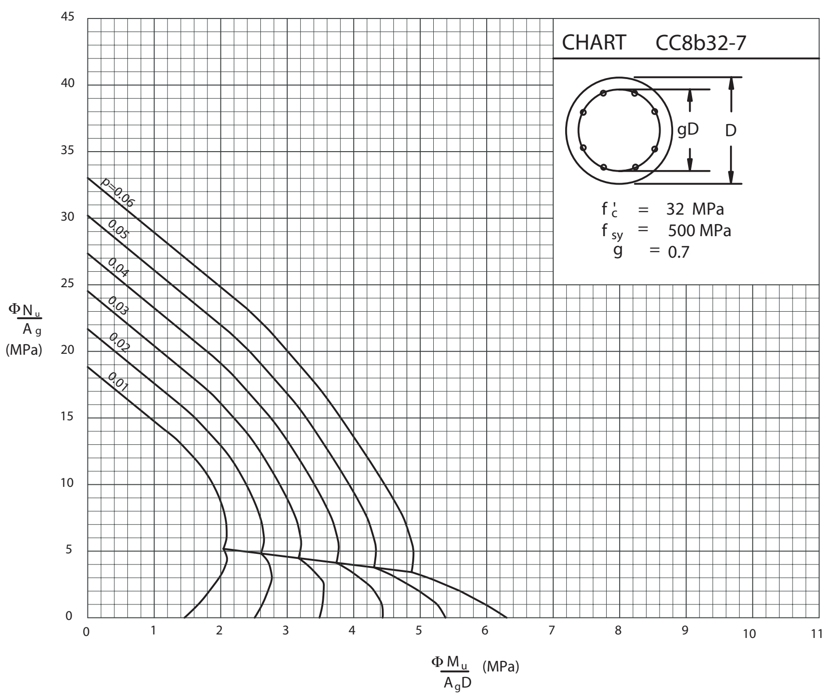


CHART CC8b32-7
gD
D
f'c = 32 MPa
fsy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
0
5
10
15
20
25
30
35
40
45
1
2
3
4
5
6
7
8
9
10
11

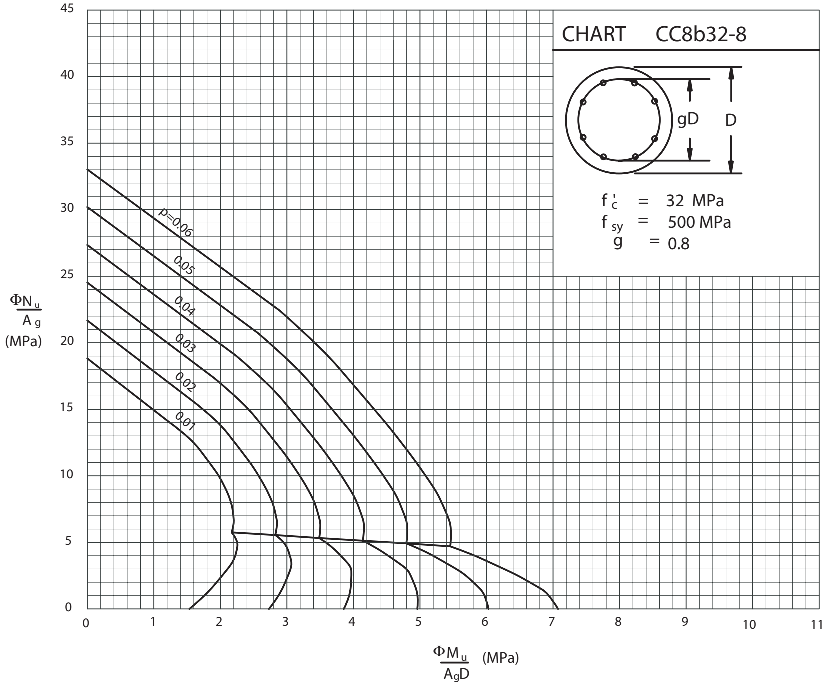


CHART CC8b32-8
gD
D
f'c = 32 MPa
fsy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
0
5
10
15
20
25
30
35
40
45
0
1
2
3
4
5
6
7
8
9
10
11

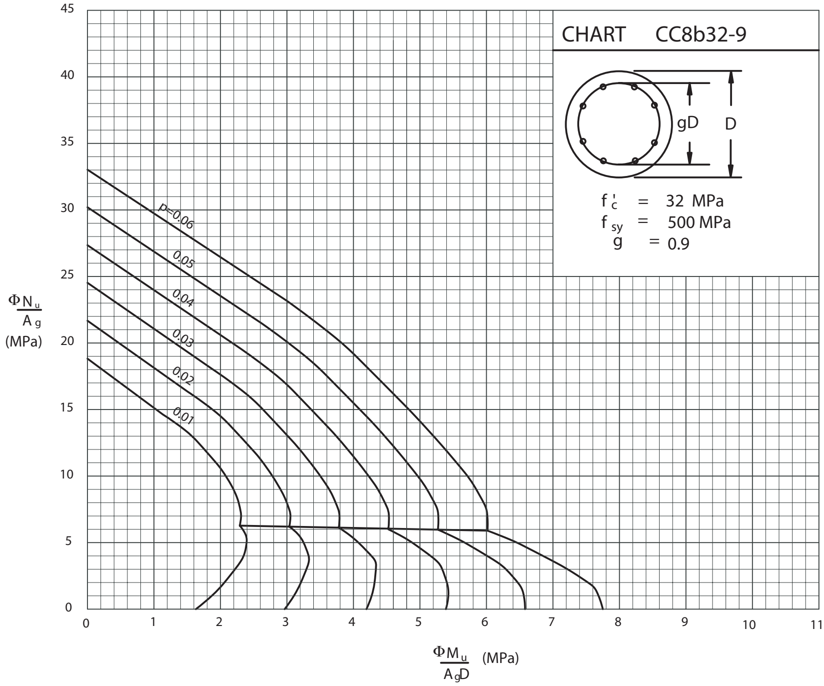


CHART CC8b32-9
gD
D
f'c = 32 MPa
fsy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
0
5
10
15
20
25
30
35
40
45
0
1
2
3
4
5
6
7
8
9
10
11

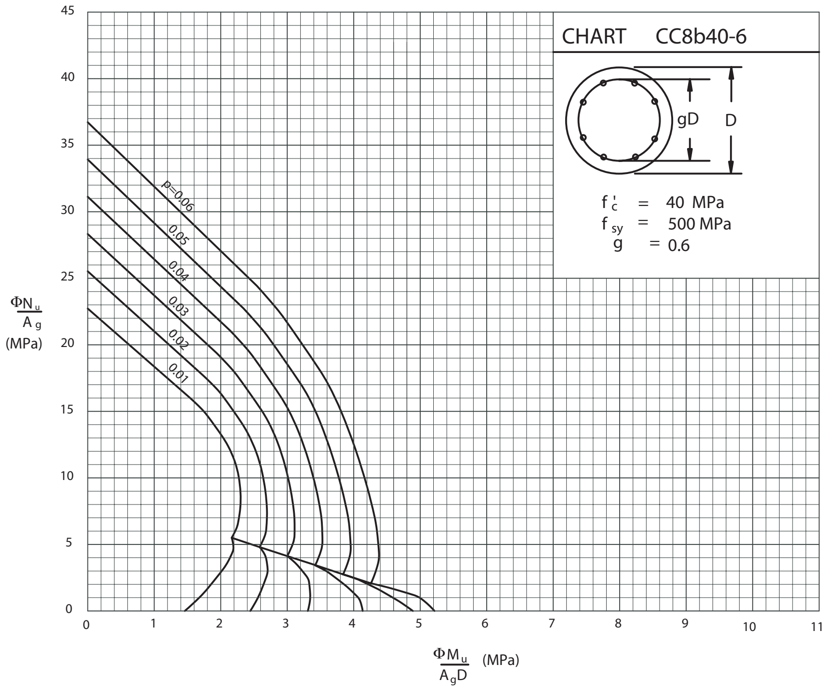


CHART CC8b40-6
gD
D
f'c = 40 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)

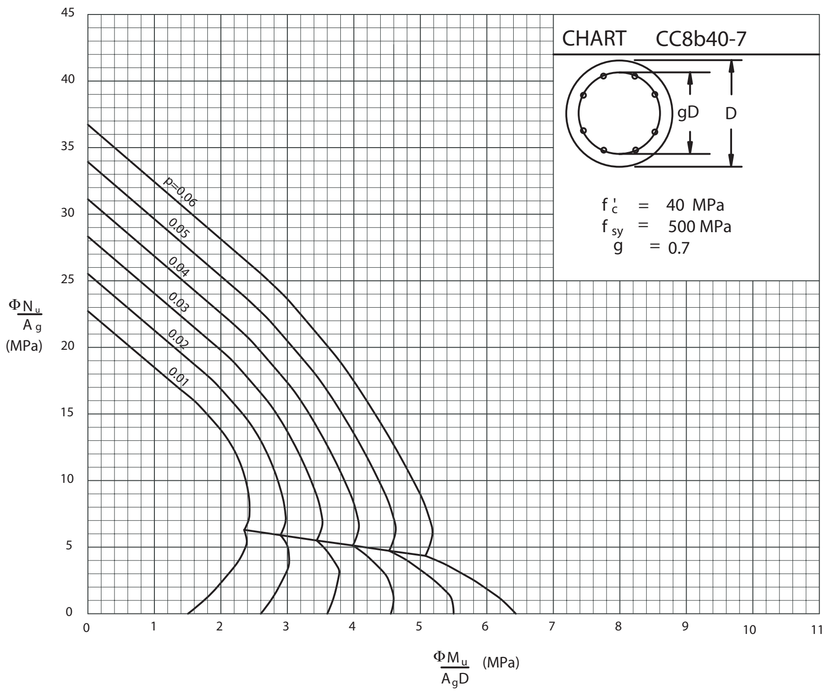


CHART CC8b40-7
gD
D
f'c = 40 MPa
fsy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)

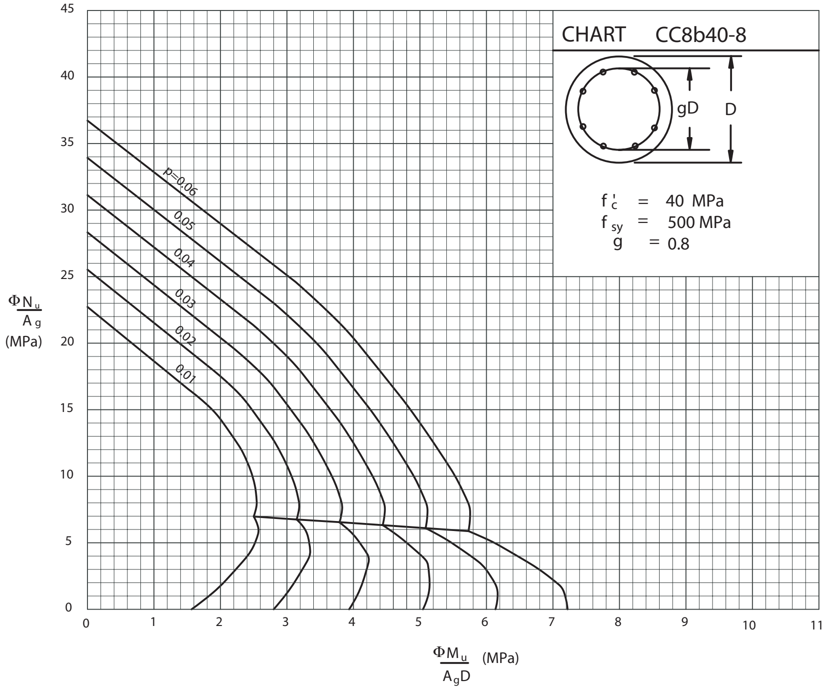


CHART CC8b40-8
gD
D
f'_c = 40 MPa
f_{sy} = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{A_g}$ (MPa)
$\frac{\Phi M_u}{A_g D}$ (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

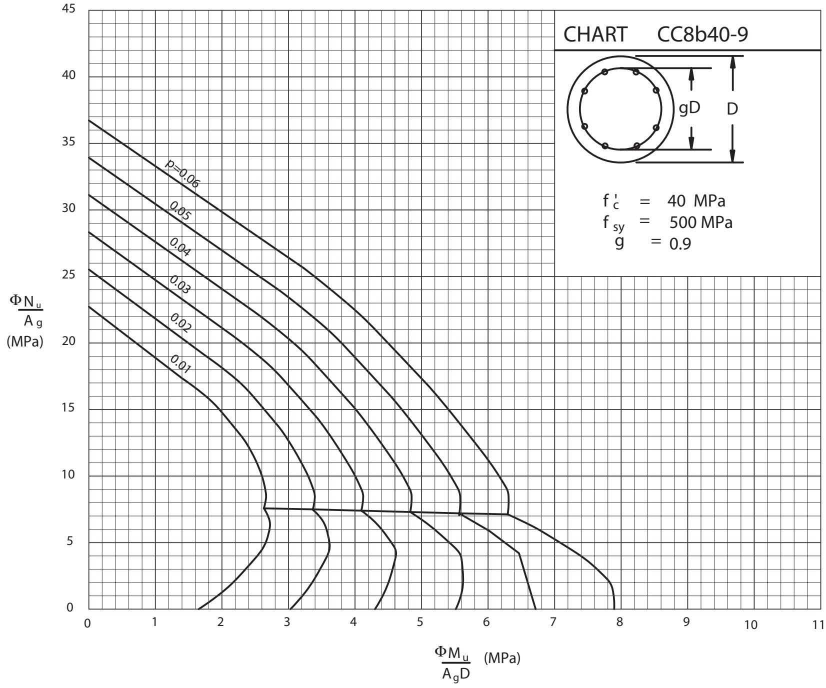


CHART CC8b40-9
gD
D
f'_c = 40 MPa
f_{sy} = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
$\frac{\Phi N_u}{A_g}$ (MPa)
$\frac{\Phi M_u}{A_g D}$ (MPa)
0 5 10 15 20 25 30 35 40 45
0 1 2 3 4 5 6 7 8 9 10 11

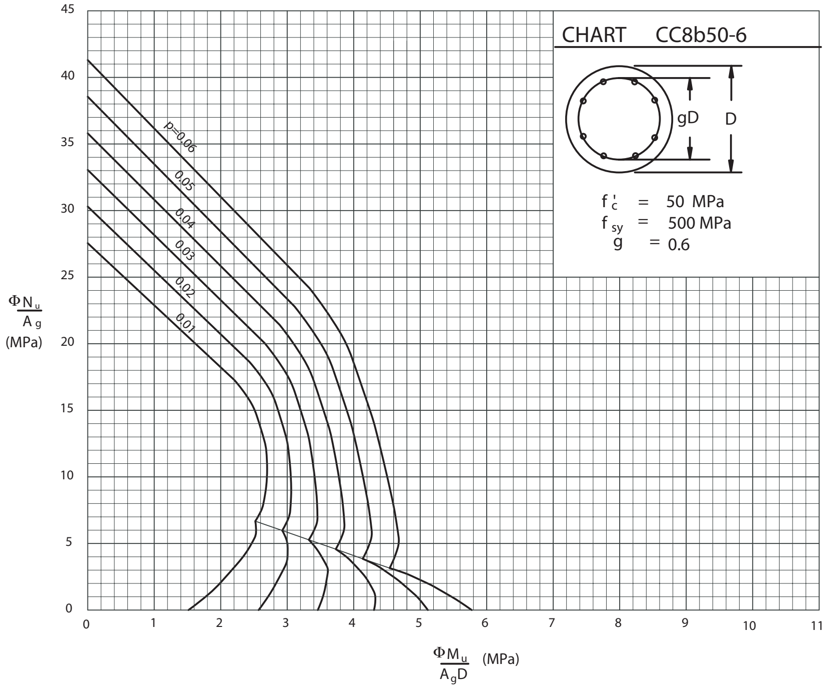


CHART CC8b50-6
gD
D
f'c = 50 MPa
fsy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)

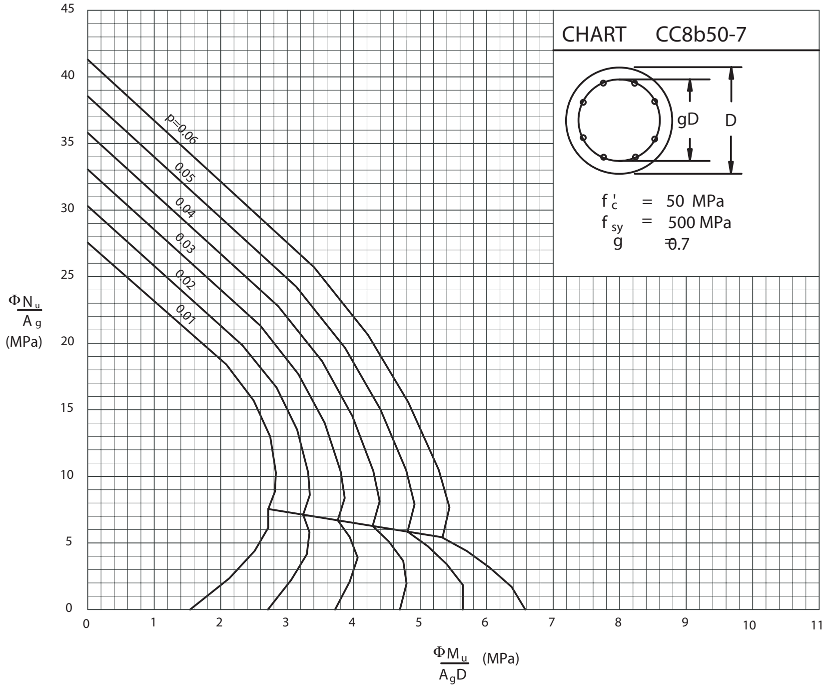


CHART CC8b50-7
gD
D
f'c = 50 MPa
fsy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)

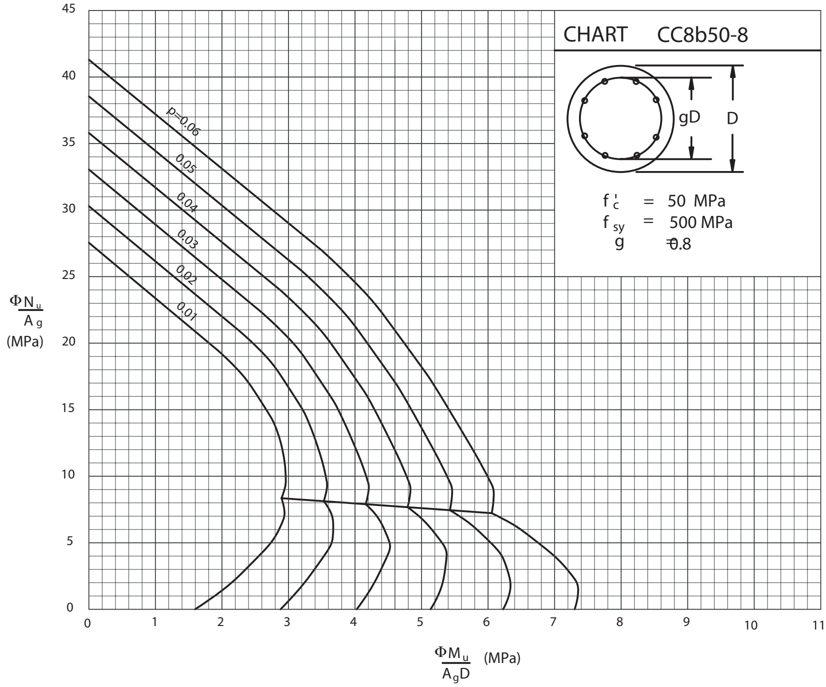


CHART CC8b50-8
gD
D
f'c = 50 MPa
fsy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/Ag (MPa)
ΦMu/AgD (MPa)
0
1
2
3
4
5
6
7
8
9
10
11
5
10
15
20
25
30
35
40
45

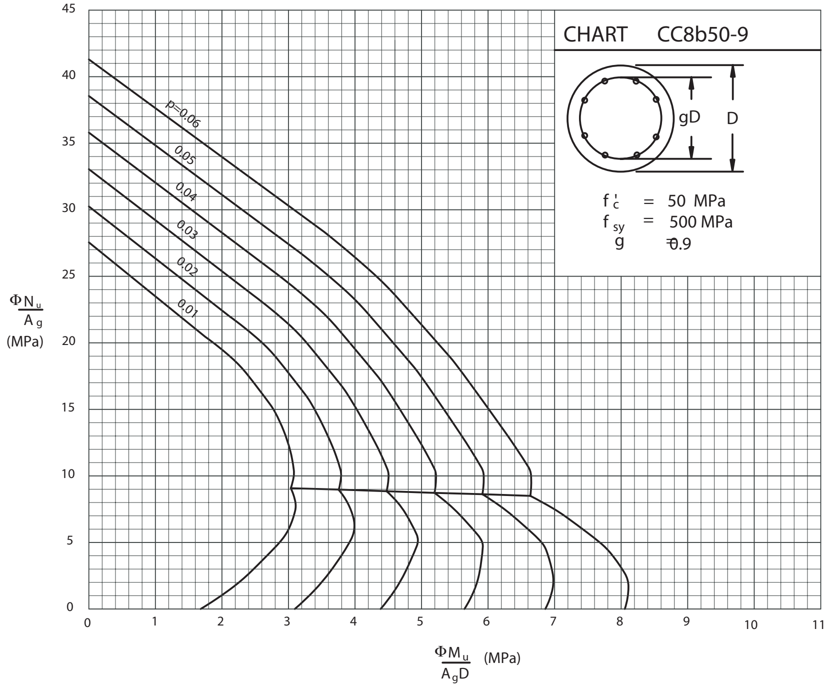


CHART CC8b50-9
gD
D
f'c = 50 MPa
fsy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu/Ag (MPa)
ΦMu/AgD (MPa)
0
1
2
3
4
5
6
7
8
9
10
11
5
10
15
20
25
30
35
40
45

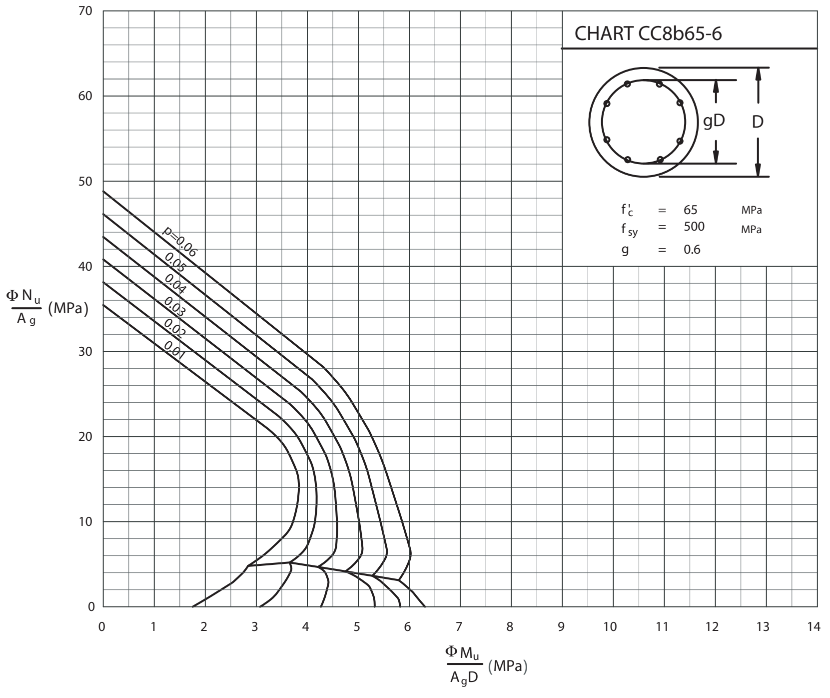


CHART CC8b65-6
gD
D
f'c = 65 MPa
f sy = 500 MPa
g = 0.6
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)

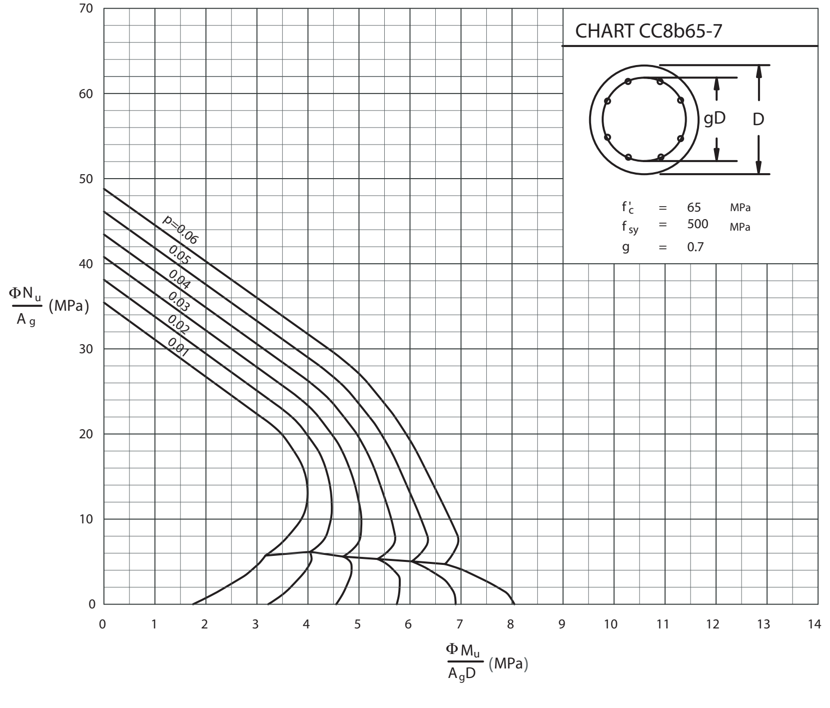


CHART CC8b65-7
gD
D
f'c = 65 MPa
f sy = 500 MPa
g = 0.7
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)

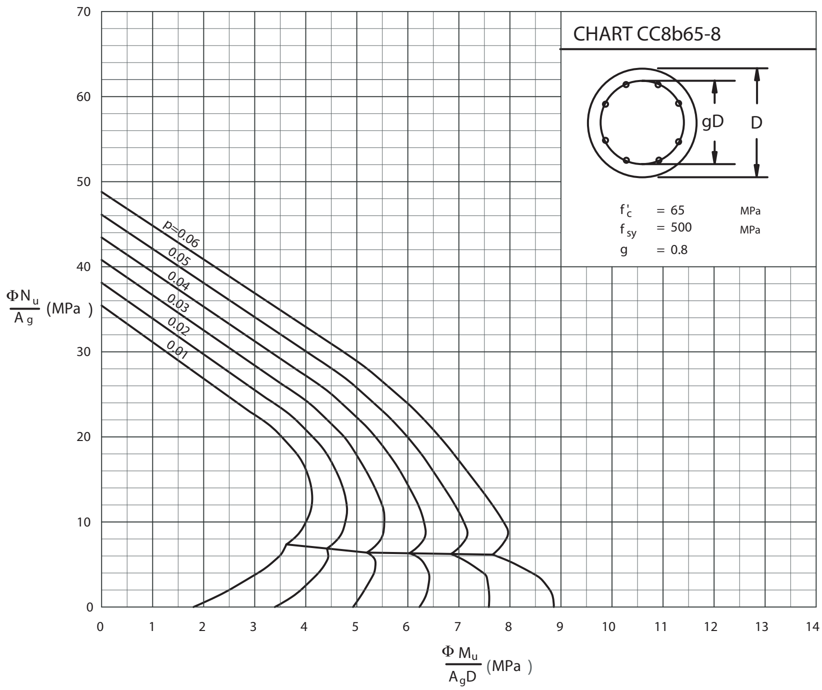


CHART CC8b65-8
gD
D
f'c = 65 MPa
fsy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
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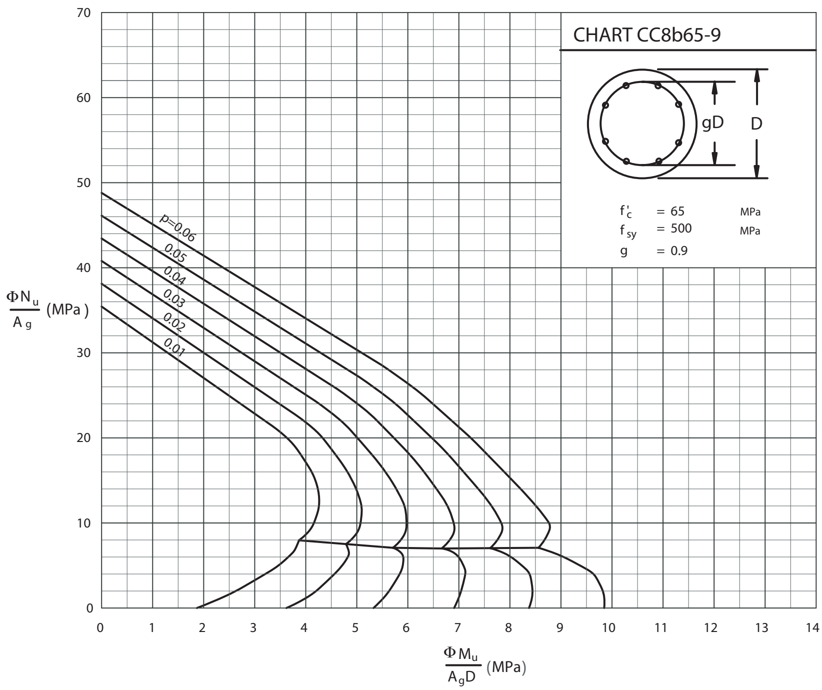
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ΦMu / AgD (MPa)
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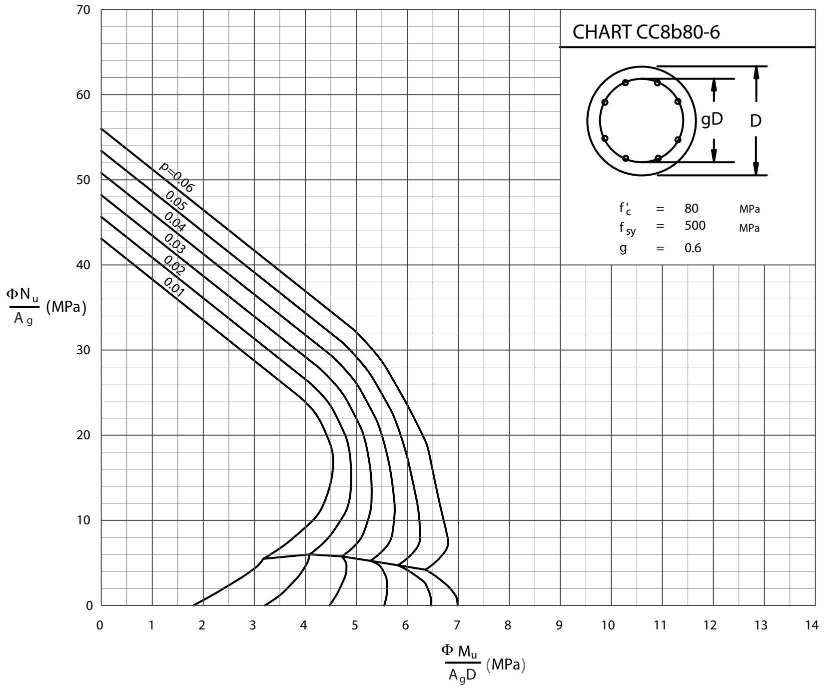
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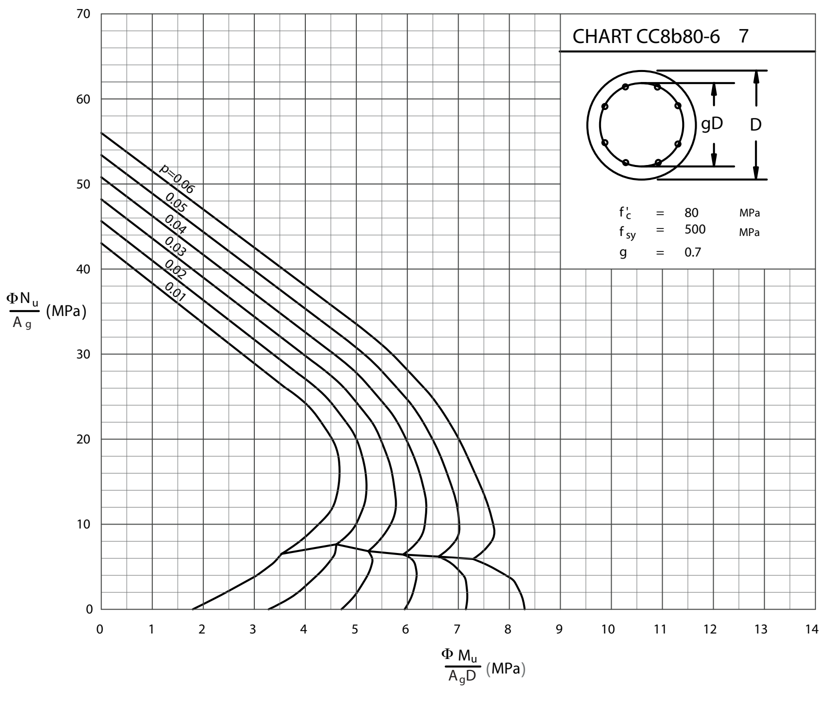
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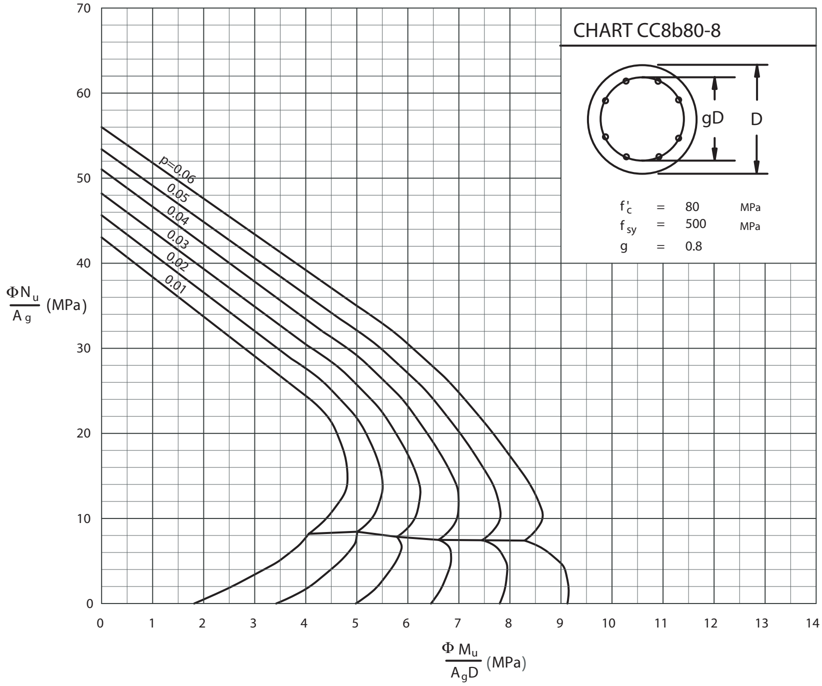
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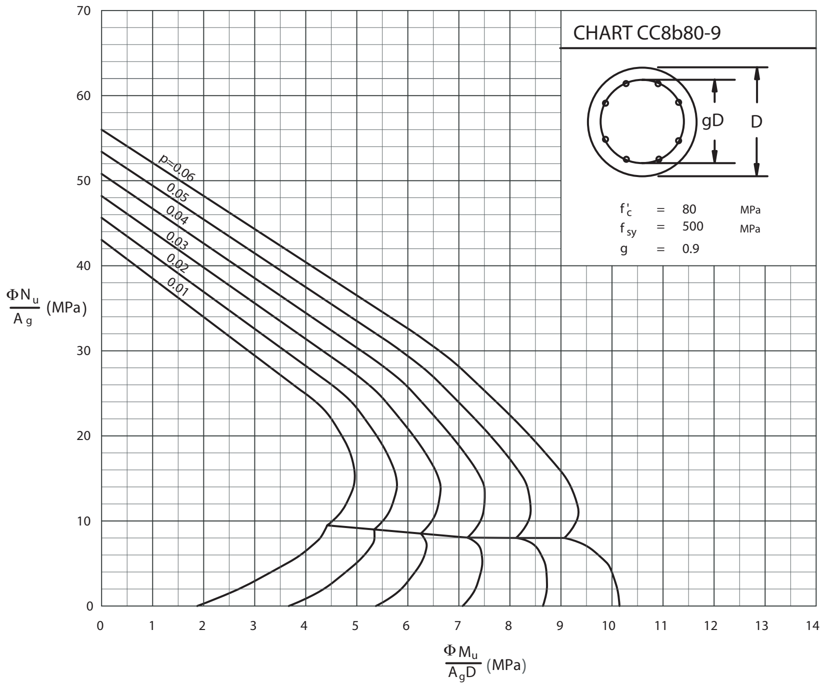
ΦMu / AgD (MPa)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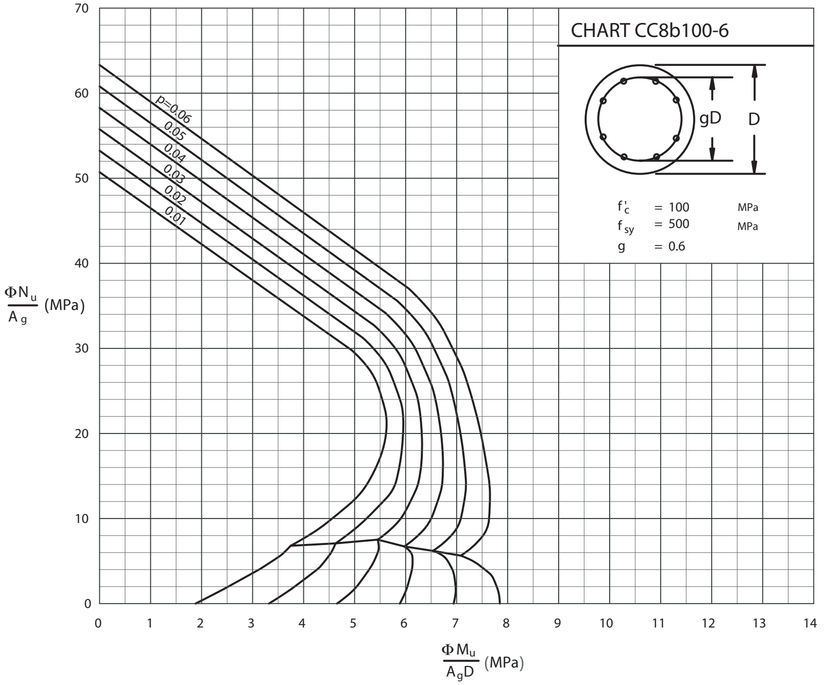
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)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p=0.06
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ΦNu/Ag (MPa)
ΦMu/AgD (MPa)
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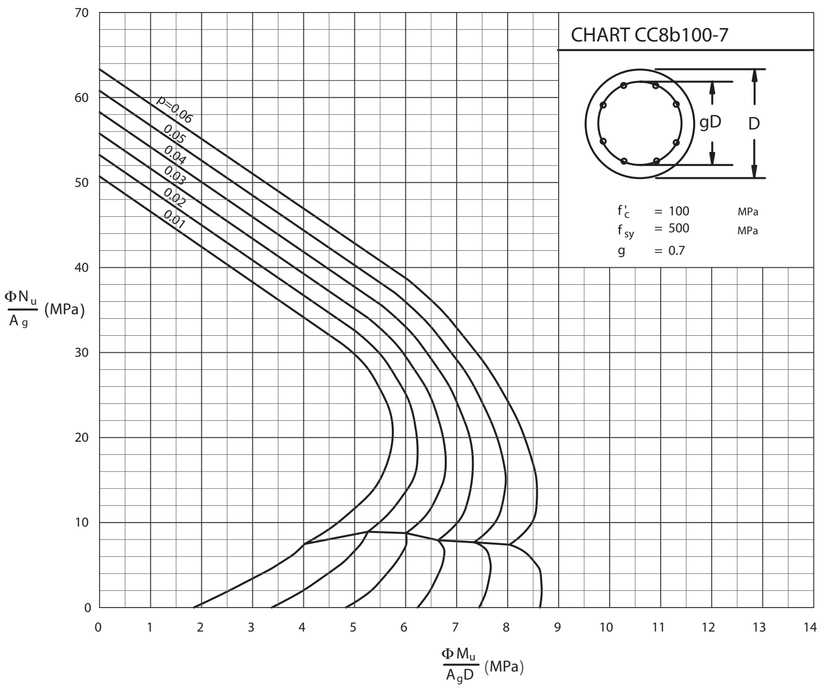
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ΦNu/Ag (MPa)
ΦMu/AgD (MPa)
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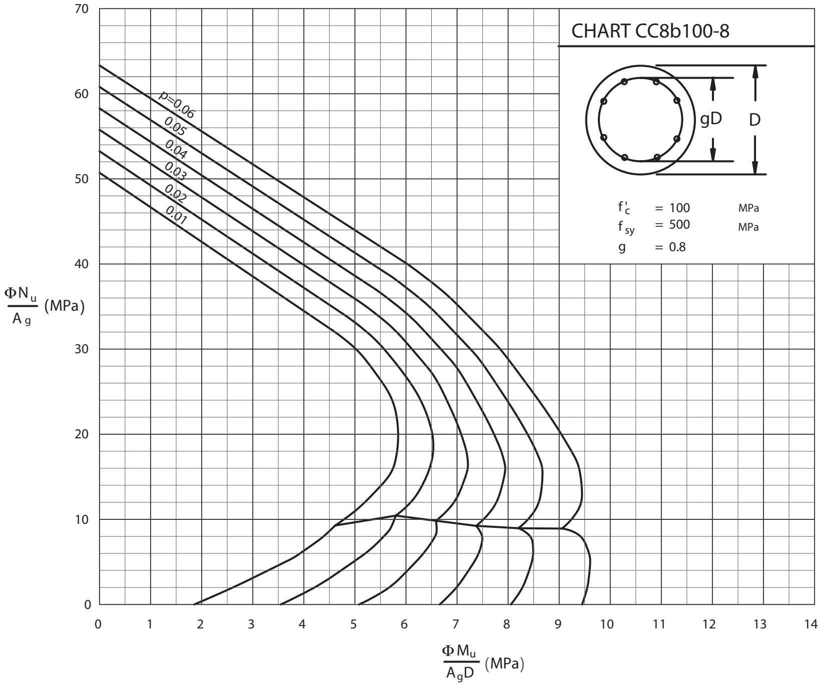
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fsy = 500 MPa
g = 0.8
p=0.06
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ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
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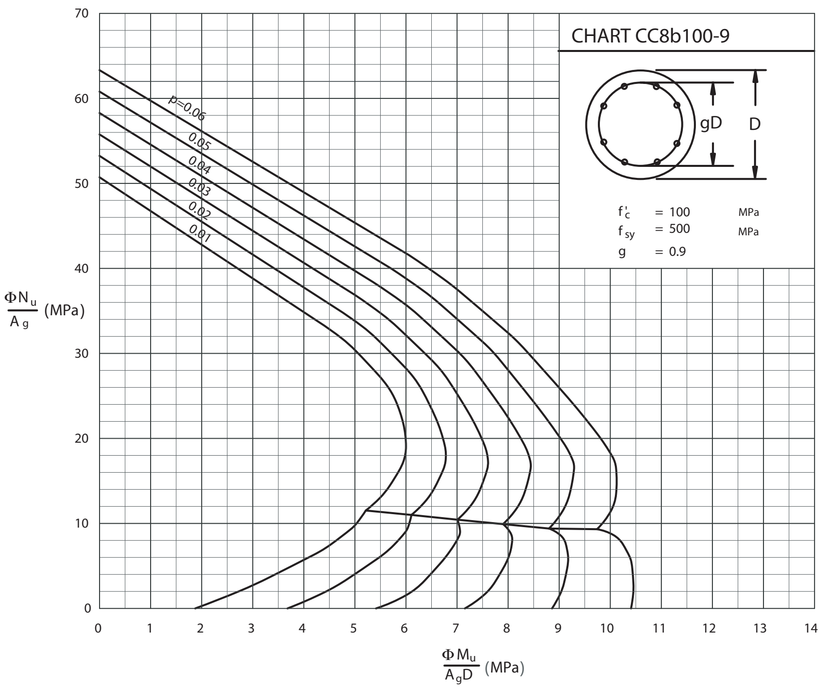
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fsy = 500 MPa
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p=0.06
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ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
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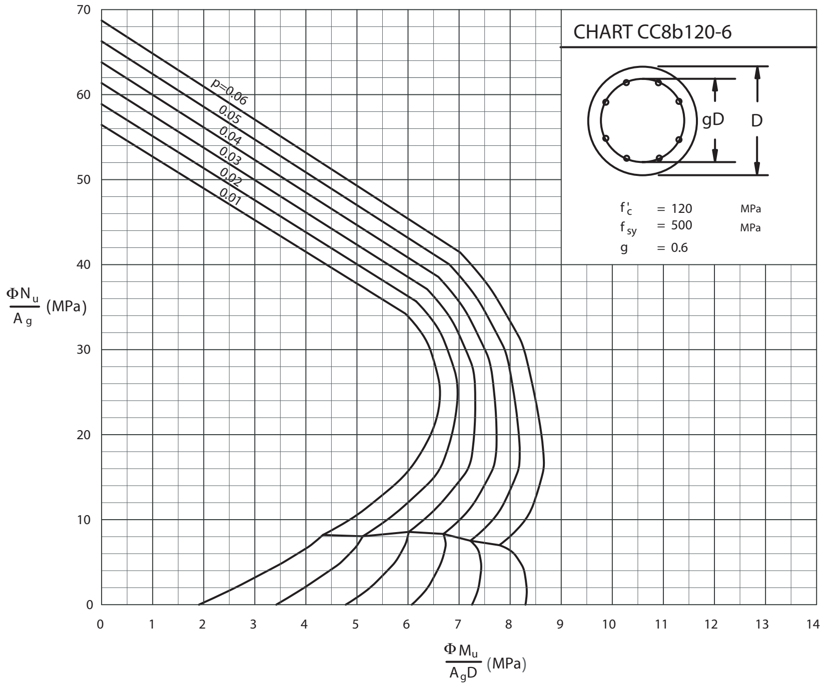
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0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
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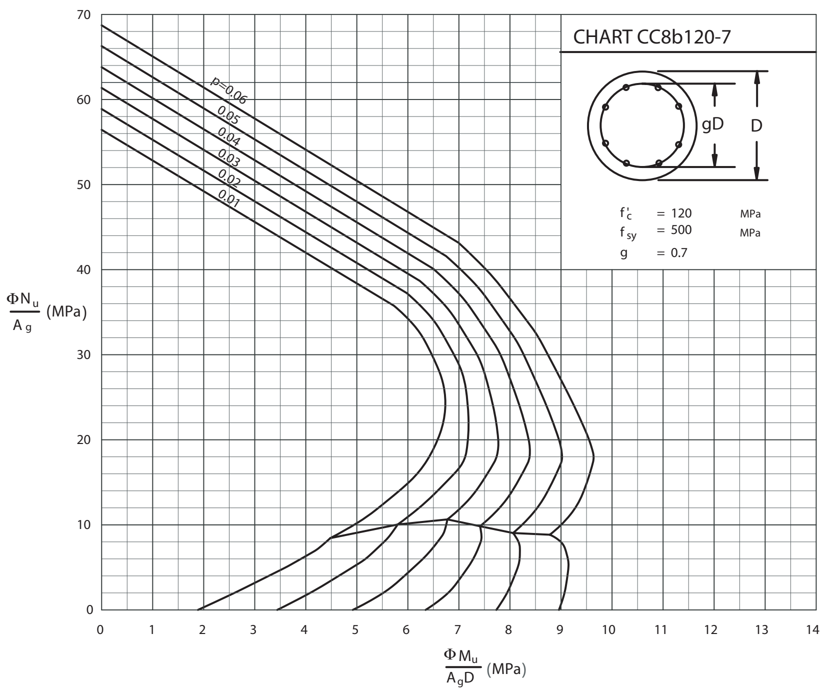
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fsy = 500 MPa
g = 0.7
p=0.06
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ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
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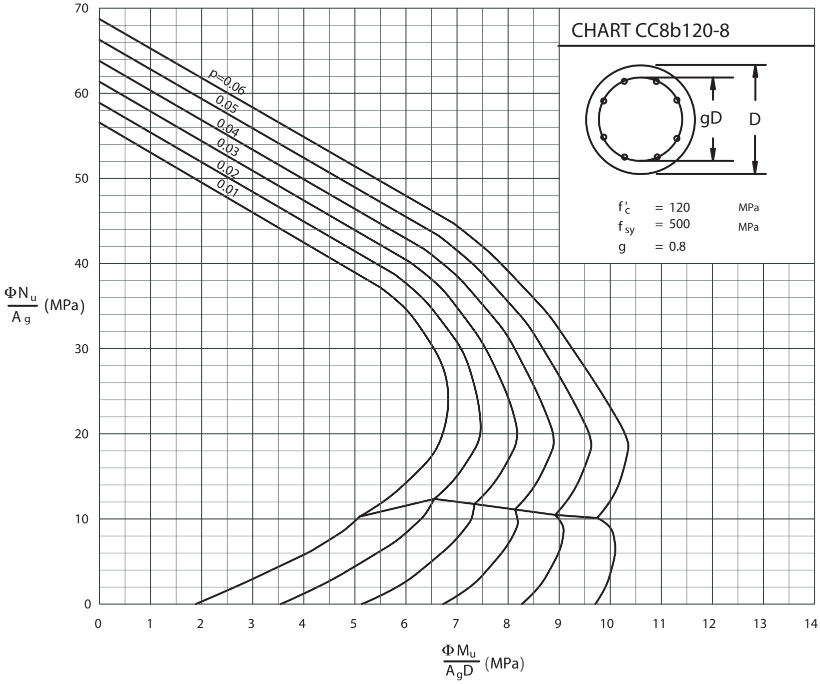
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gD
D
f'c = 120 MPa
fsy = 500 MPa
g = 0.8
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
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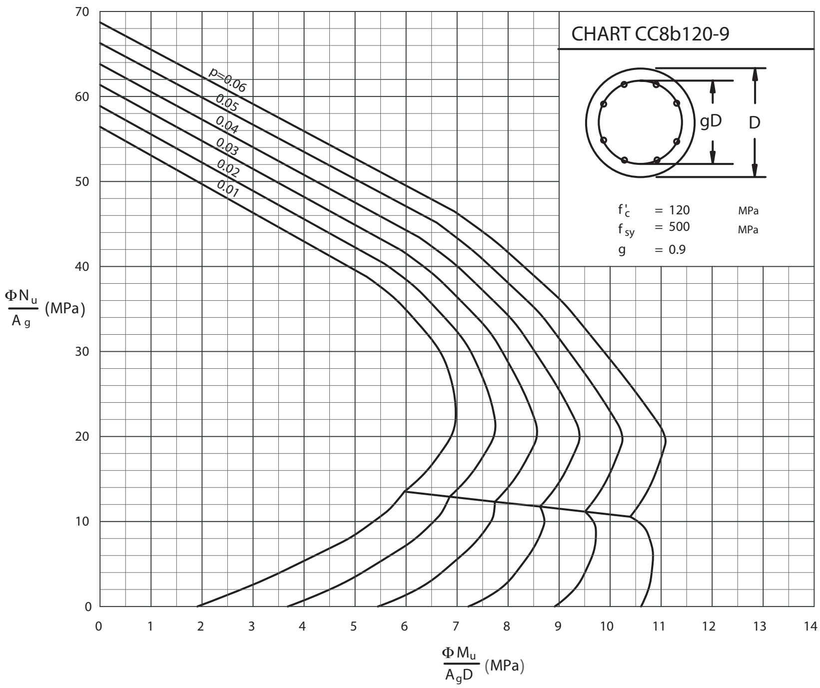
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D
f'c = 120 MPa
fsy = 500 MPa
g = 0.9
p=0.06
0.05
0.04
0.03
0.02
0.01
ΦNu / Ag (MPa)
ΦMu / AgD (MPa)
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0 1 2 3 4 5 6 7 8 9 10 11 12 13 14

Page xi – Amended text highlighted in **bold**

A_{s1} = Tensile area of primary beam. This is usually the area of a singly reinforced beam with maximum steel ratio $\mathbf{p_{max}}$ for which $k_u = 0.4$.

Remove following;

~~A_{se} = Area of reinforcement on compression side~~.

Page xii – Remove following;

~~b_w = Width of web~~.

~~d = Effective depth of beam~~.

Page xiii – Remove crossed out material and add/amend text highlighted in **bold**;

~~d_{se} = Distance from extreme compression fibre to the centroid of the outer compression reinforcement~~.

$E_{c\mathbf{j}}$ = The mean value of modulus of elasticity of concrete at nominated age.

= $\rho^{1.5} \times 0.043\sqrt{\mathbf{f_{cm}}}$

$\mathbf{E_u}$ = Ultimate earthquake action.

$\mathbf{f_c}$ = An intermediate concrete stress.

~~F_{st} = An intermediate tensile steel stress within the elastic range~~.

Page xiv – Amended text highlighted in **bold**

$\mathbf{G^R}$ = Dead loads resisting instability

L_{ef} = Effective span of beam**, lesser of** L and ($L_{\mathbf{n}}$ + D) or ($L_{\mathbf{n}}$ + D/2) for cantilevers.

L_o = Span length used in simplified method, L – 0.7 times the sum of $\mathbf{a_s}$ for each support.

Pages xiv and 235 – Add symbol definition k_{uo}

k_{uo} = Ratio at ultimate strength of the depth of the NA from the extreme compressive fibre to d_o. Symbols k_u is applied for k_{uo} in this text.

Page xv – Remove following;

~~L_y = Longer effective span of slab supported on four sides~~.

~~M_{ud} = The reduced ultimate strength in bending when k_u is reduced to 0.4~~.

Page xvi – Add/amend text highlighted in **bold**

p_v = Shear steel ratio $\mathbf{A_{st}}/(b_v d_o)$.

S_u = Ultimate action due to combination of various actions.

Page xvii - Remove crossed out material
- add/amend text highlighted in **bold**
- replace symbols b_1, b_2, b_3 and d by β_1, β_2, β_3 and δ

~~V_{uc} = Ultimate punching shear strength.~~

~~w^* = Design load used for strength conditions.~~

~~w^* = Unit slab design load used as alternative symbol for F_d.~~

W_s = Serviceability wind action.

W_u = Ultimate wind action.

β_1 = Shear strength coefficient for comparable increase in shear capacity of shallow beams.

β_2 = Shear strength coefficient for axial load effects.

β_3 = Shear strength coefficient to account for increased strength when concentrated loads are applied near supports (short shear span $a_v < 2d_o$).

δ = Deflection obtained from calculations.

Page xviii – Replace symbols d_b, d_s, D, F and g by δ_b, δ_s, Δ, Φ, γ and add ρ

δ_b, δ_s = Moment magnifiers for braced and sway columns.

Δ = Maximum deflection – normally expressed as a fraction, eg Δ/L.

Φ or ϕ = Strength reduction factor.

γ = Ratio of depth of simplified rectangular stress block to depth of NA.

ρ **= density of concrete (taken as 2400 kg/m^3 in this book)**

Page 1 - Add the following symbols;

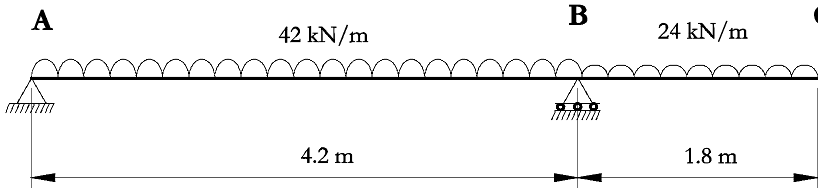
E_u = Ultimate strength action.

S_u = Ultimate action due to combination of various actions.

W_s = Serviceability wind action.

W_u = Ultimate wind action.

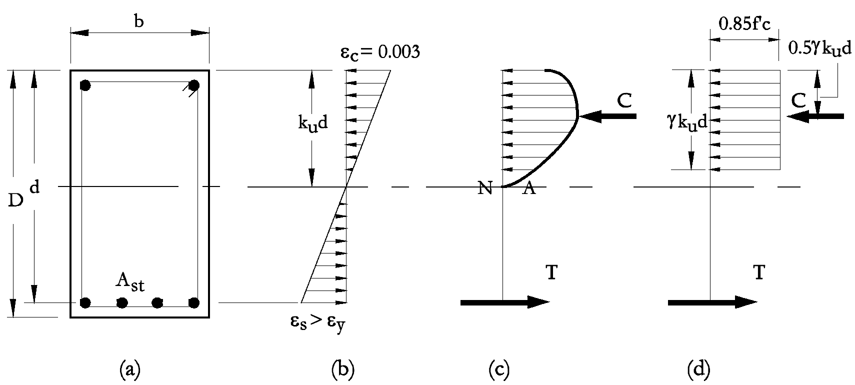
Page 10 Figure 1.5 – Replace 43 kN/m UDL by 42 kN/m and 25kN/m UDL with 24kN/m



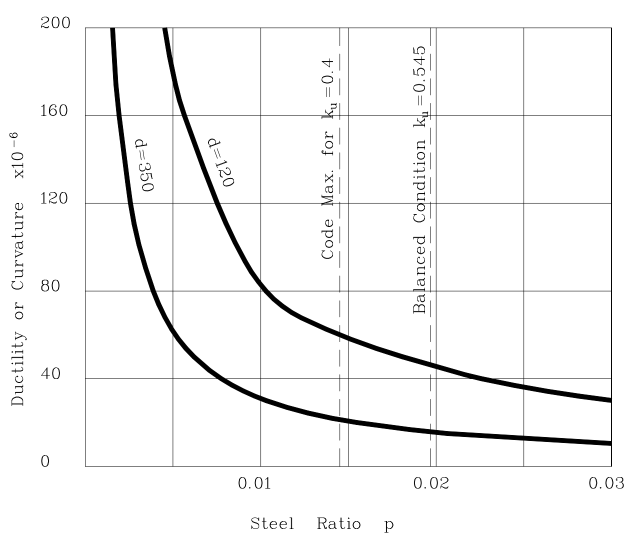
Page 17 – Amended text highlighted in **bold**

(b) Strain is linear i.e. it is directly proportional to the distance from the NA for all moments up to and including the ultimate bending moment $\mathbf{M_{uo}}$.

Page 18 Figure 2.4 alter dimension $\gamma k_u d$ and identify figures (a), (b), (c) and (d)



Page 23 – Replace Figure 2.7 with following



Page 25 and Page 285 Table 2.4 Replace symbol ‡ with ≥

(d) Bending with axial compression where:

(i) $N_u \geq N_{ub}$

Page 26 – Altered value shown highlighted in **bold**

$$p_{max} = 0.34\gamma \frac{f'_c}{f_{sy}} = 0.34 \times \mathbf{0.85} \times \frac{25}{500} = \mathbf{0.0145}$$

Page 28 – Add **bd²** to equation as shown **bold**

$$\Phi M_{uo} = \Phi f'_c\, z \left(1 - \frac{z}{1.7}\right) \mathbf{bd^2}$$

Page 32 – Table 3.1, altered classification shown highlighted in **bold**

(c) Coastal (up to 1 km from coastline but excluding tidal and splash zones) and any climatic zone. **B2**

Page 34 – Table 3.3, heading, alteration shown highlighted in **bold**

Required Cover in mm
Characteristic Strength $\mathbf{f'_c}$

Page 36 – Altered symbols shown highlighted in **bold**

$$\frac{A_{\mathbf{st}}}{bd} \geq 0.22 \left(\frac{D}{d}\right)^2 \frac{\mathbf{f'_{cf}}}{\mathbf{f_{sy}}}$$

Page 41 – Altered data shown highlighted in **bold**

Required total depth of beam, D = **477 + 66 = 543** mm.

.

.

The trial section is satisfactory. If the actual weight of beam is used, the design moment, M^* = **300.2** kNm

Page 45 Equation 4.3 – Corrected equation reads,

$$\lambda = \Phi\, p\, f_{sy} \left(1 - \frac{p}{1.7} \frac{f_{sy}}{f'_c}\right)$$

Page 56 – Remove crossed out text and add new formula

If $\varepsilon_{sc} \geq$ **0.0025** then the compressive reinforcement **HAS** yielded and the required area of compressive reinforcement ~~is equal to the tensile area~~ is given by;

~~$A_{sc} = A_{S2}$~~

$$\mathbf{A_{sc} = \frac{A_{s2} f_{sy}}{f_{sy} - 0.85 f'_c}}$$

Page 57 – Modify equation for p_1 (modification shown in **bold**) when $\varepsilon_{sc} \geq 0.0025$

$$p_1 = \frac{A_{st} - A_{sc}\left[\mathbf{\frac{f_{sy} - 0.85 f'_c}{f_{sy}}}\right]}{bd} \qquad \text{If } \varepsilon_{sc} \geq 0.0025$$

Page 60 – Equation 5.11, add Φ to denominator

$$A_{sc} = \frac{M_2}{\mathbf{\Phi}\left(f_{sc} - 0.85 f'_c\right)\left(d - d_{sc}\right)} \qquad \mathbf{5.11}$$

Page 61 Table 5.1 – Replace data for 65MPa concrete grade shown in **bold**

Parameter	f'_c in MPa					
	20	25	32	40	50	65
γ	0.850	0.850	0.822	0.766	0.696	**0.65**
p_{max}	.0116	0.0145	0.0179	0.0208	0.0237	**0.0287**
z	0.2890	0.2890	0.2795	0.2604	0.2366	**0.221**
λ	3.838	4.797	5.979	7.057	8.147	**9.998**

Page 62 – Altered data shown in **bold**

d_{sc} = 64mm

d = 500 - **68** = **432**mm

Area of tensile reinforcement in primary beam,

A_{s1} = 0.0179×350×**432**

= **2706** mm²

$M_1 = \lambda b d^2$

= 5.979×350×**432**^{2}×10^{-6}

= **390.5** kNm

$M_2 = M^* - M_1$

= 465 – **390.5**

= **74.5** kNm

Area of tensile reinforcement required,

$$A_{s2} = \frac{M_2}{\Phi f_{sy}(d - d_{sc})} = \frac{\mathbf{74.5} \times 10^6}{0.8 \times 500(\mathbf{432} - 64)} = \mathbf{506}\,\text{mm}^2$$

$$\frac{d_{sc}}{d} = \frac{64}{\mathbf{432}} = \mathbf{0.148}$$

Page 63 – Altered data shown in **bold**

From chart B2 read $K = \mathbf{1.42}$

Area of compressive reinforcement required,

$$A_{sc} = K \times A_{s2}$$
$$= \mathbf{1.42} \times \mathbf{506}$$
$$= \mathbf{719}\ \text{mm}^2$$

Total tensile area required,

$$A_{st} = A_{s1} + A_{s2}$$
$$= \mathbf{2706} + \mathbf{506}$$
$$= \mathbf{3212}\ \text{mm}^2$$

.
.
.

Total tensile area required,

$$A_{st} = 2556 + \mathbf{848} = \mathbf{3404}\ \text{mm}^2$$

Page 64 – Correct text shown in **bold** and in Figure 5.3 replace 9N24 by 8N24 bars

For tensile reinforcement, choose **8N24** bars in two rows as shown below giving, $A_{st} = \mathbf{3600}\ \text{mm}^2$.

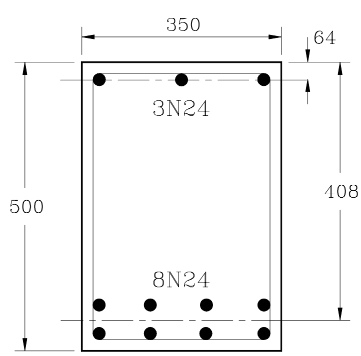


Figure 5.3

Page 68 – Altered data shown in **bold**

From chart DRCB-32 for $d_{sc}/d = 0.125$ and $\Phi M_{uo}/(bd^2) = 9.29$ MPa read the required steel ratios $p_t = \mathbf{0.0276}$ and $p_c = \mathbf{0.0126}$. Figure 5.5 shows the construction lines required to read the chart.

Required areas of reinforcement $A_{st} = \mathbf{0.0276} \times 350 \times 400$

$$= \mathbf{3864}\ \text{mm}^2$$

A_{sc} = **0.0126**×350×400 = **1764** mm^2

6N28 bars in two rows of three bars gives A_{st} = **3720** mm^2 and **4N24** bars gives A_{sc} = **1800**mm^2. From Table 3.4 adjusted dimensions for exposure A2 are d = **419**mm and d_{sc} = **49**mm. Repeating the procedure using new values of d and d_{sc},

Page 69 – Altered data shown in **bold**

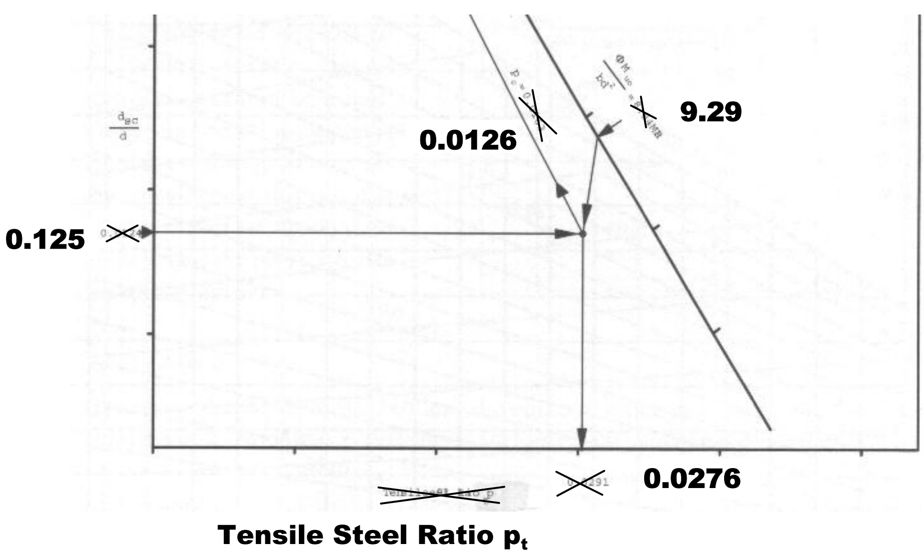
$d_{sc} / d = \mathbf{49/419} = \mathbf{0.117}$

$\Phi M_{uo} / bd^2 = \mathbf{8.46}$ MPa

Chart DR**C**B-32 read $p_t = \mathbf{0.0250}$, $p_c = \mathbf{0.009}$

$\text{Required } A_{st} = \mathbf{0.0250} \times 350 \times \mathbf{419} = \mathbf{3666}$

$\text{Required } A_{sc} = \mathbf{0.009} \times 350 \times \mathbf{419} = \mathbf{1320}$

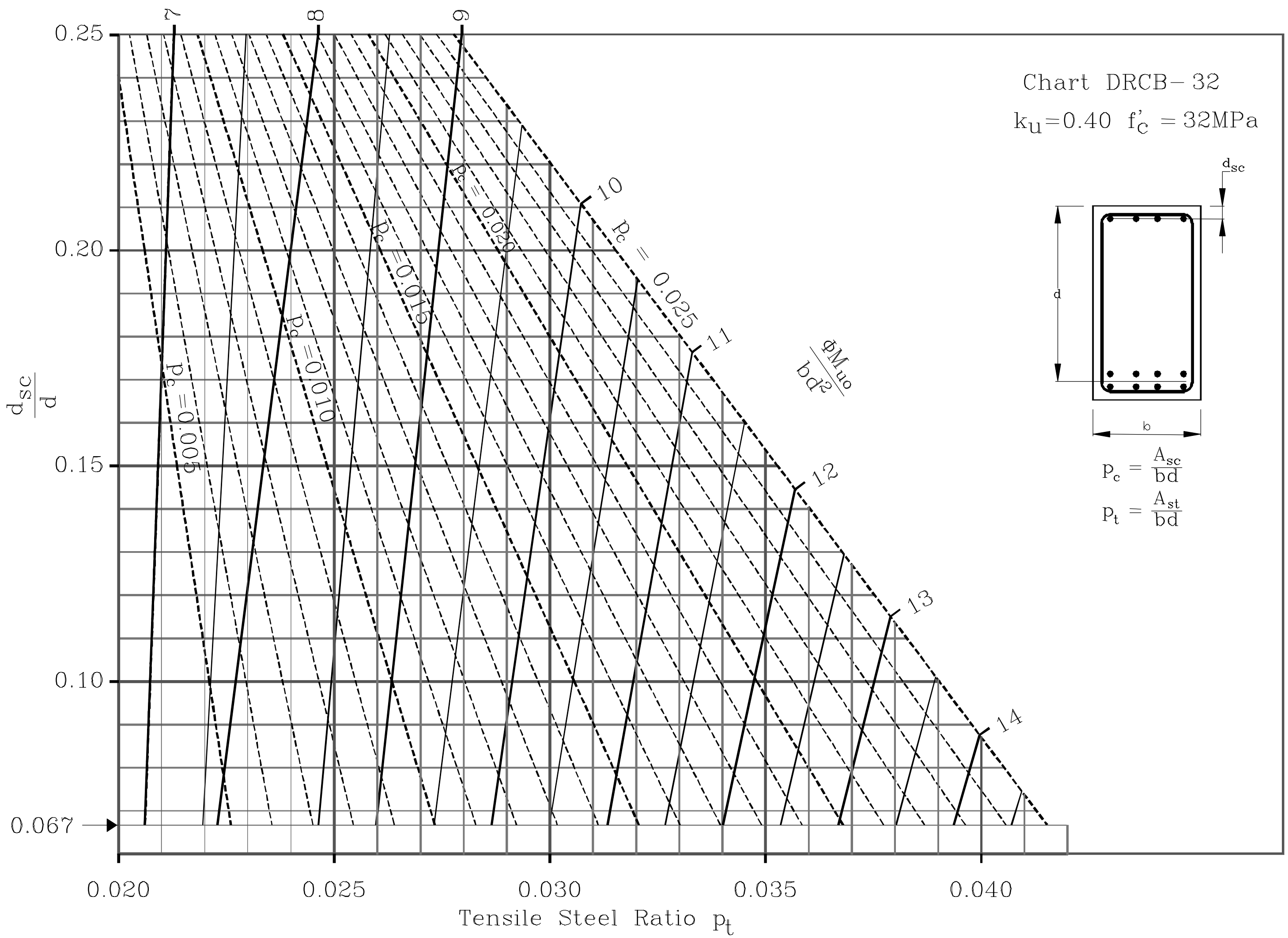


Choose **6N28 bars** giving A_{st} = **3720**mm^2, **and 3N24 bars giving A_{sc} =1350**mm^2

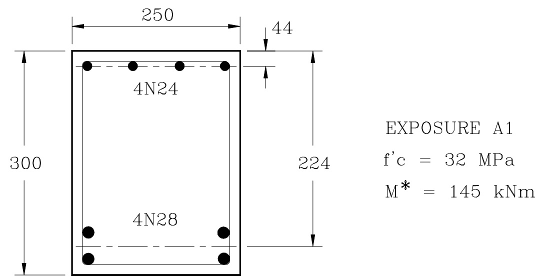
Check primary beam steel ratio.

$$p_t = \left(\frac{\mathbf{3720} - \mathbf{1350}}{350 \times \mathbf{419}}\right)$$
$$= \mathbf{0.0162} < p_{max} \; (= 0.0179)$$

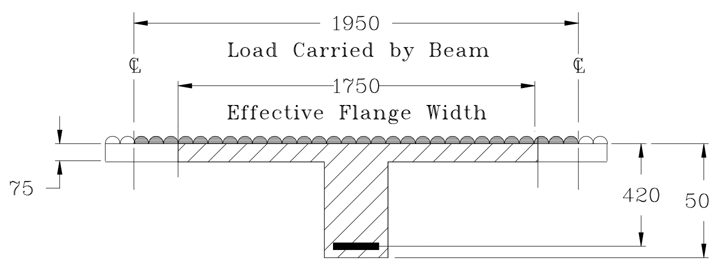
Page 70 – Replace Chart DRCB – 32 with following



Page 71 Question 2 – Adjust effective depth dimension to centroid of reo.



Page 77 Figure 6.5 – Amend figure



Page 82 – Amendment shown in **bold**

From Table 3.4 choose 6N36 placed in two rows of **3** bars whose area $A_{st} = 6120$ mm^2.

Page 83 – Amended text shown **bold** and replace chart T-18A

Example 3.

Determine the area of reinforcement for the beam shown in Figure 6.8.

Solution

Data: $b = 150$ $b_w = 350$ $d = 540$ $t = 100$ $f_c' = 25$ MPa $M^* = \mathbf{175}$ kNm

$$\frac{M^*}{bd^2} = \frac{\mathbf{175} \times 10^6}{\mathbf{150} \times 540^2} = \mathbf{4.0} \text{ MPa}$$

$$\frac{t}{d} = \frac{100}{540} = 0.185 \qquad \frac{b}{b_w} = \frac{150}{350} = 0.43$$

Using chart T-18A for t/d = 0.18, read the required steel ratio p = **0.0114** for $\Phi M_{uo}/(bd^2)$ = **4.00** MPa and b/b_w = 0.43.

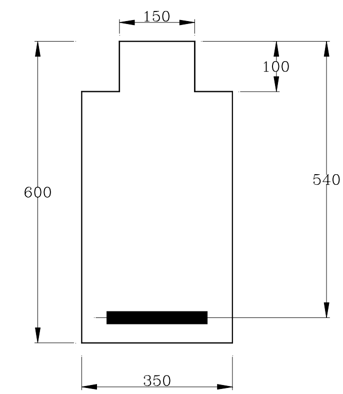


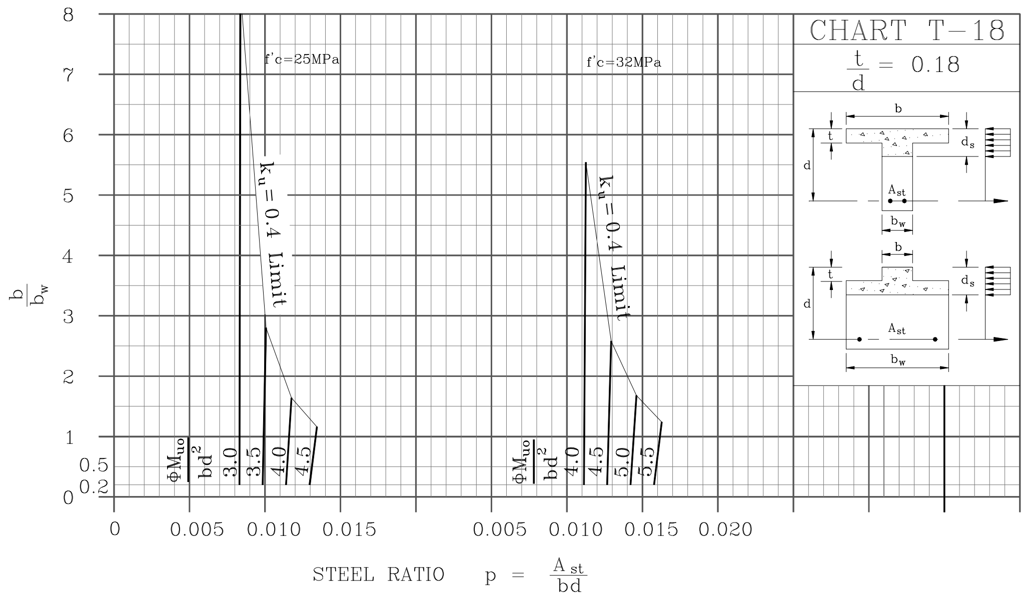
Figure 6.8

Using the smaller value of t/d, the steel ratio read from T-18A will be slightly conservative, a more accurate value can be obtained by interpolating between the results obtained from charts T-18 and T-20 although the difference may not be discernable.

Hence the required tensile area,

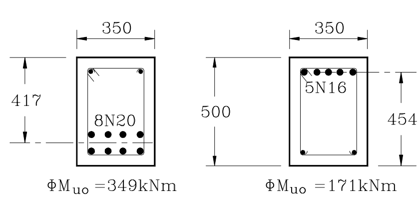
$$A_{st} = \mathbf{0.0114} \times 150 \times 540$$

$$= \mathbf{923}\ \text{mm}^2$$



Page 95 Example 2 – Add text shown in **bold**
- Alter effective depth dimensions in Figure 7.7

shown in the figure. Grade **N32** concrete is used for the beam ~~which is in an Exposure Classification A1~~. The bending

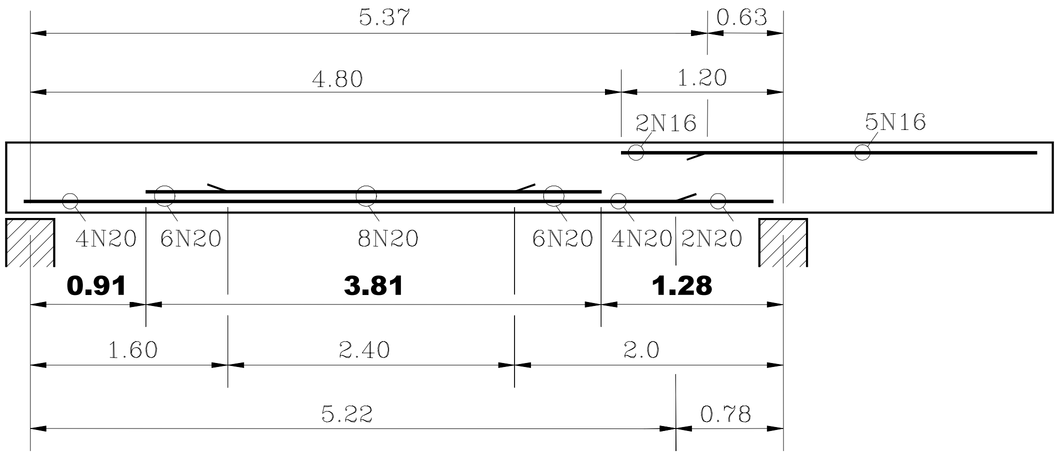


Page 96 – Example 2 - Correction shown in **bold**

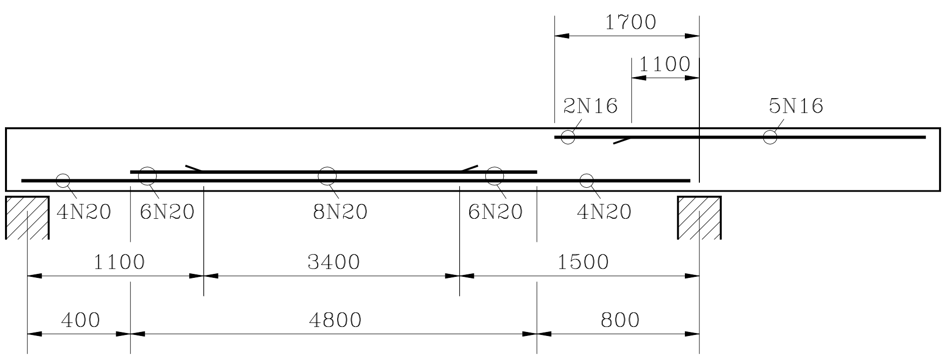
$$x = \frac{228 \pm \sqrt{228^2 - 2 \times 174.5 \times 81}}{81}$$

$$= \mathbf{0.91}\ \text{m and}\ \mathbf{4.72}\ \text{m}$$

Page 97 Figure 7.9 – Corrected dimensions shown in **bold**



Page 98 Figure 7.10 – Amended figure shown below



Page 98 – Amended values shown in **bold**

It now only remains to check that development lengths have been provided. The concrete cover ~~for Exposure Condition A1 is 20mm. Since at this stage~~ using 12mm stirrups is 38mm for the negative reinforcement and 47mm for the positive reinforcement. ~~are assumed,~~ **Assuming 20mm minimum cover,** the actual concrete side cover for the longitudinal reinforcement is 32mm.

$$\text{Clear spacing} = \frac{350-2\times 32-4\times 20}{3} = 68\text{mm} < \mathbf{94} \quad \text{(Twice the cover)}$$

Assume N32 spacer bars between each row of positive reinforcement, the clear spacing between bars is 32 mm which is less than the horizontal spacing of 68 mm and less than twice the cover of **94** mm. The development length will therefore be calculated for a = 32/2 =16 mm. From Table 7.1 $L_{sy.t}$ = **1180** mm by interpolation ...

Similarly the clear spacing between the negative reinforcing bars is calculated to be (350-2×32-5×1.5 mm. This is less than twice the **38** mm cover. The required development length for N16 negative reinforcement is obtained from Table 7.1 for a = 51.5/2 = 26mm.

$L_{sy.t}$ = **576** mm

The negative reinforcing bars are top bars with more than 300mm of concrete cast below the bars. The development length must therefore be increased by factor k_1.

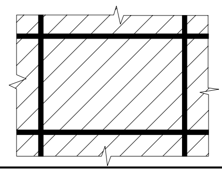
$L_{sy.t} = 1.25 \times \mathbf{576} = \mathbf{720}$ mm, ~~say 710 mm~~

Page 99 Question 1 – Change Grade N32 concrete to Grade **N40**

Page 103 Note 3. – Change D/L_{ef} to read $\mathbf{\Delta/L_{ef}}$

Page 105 Line 7 – Change “then” to “than”

Page 105 Table 8.2 Edge Condition 1 should read as follows;

1. Four edges continuous		4.00	3.40	3.10	2.75

Page 107 – Amendment shown in **bold**

$$k_{cs} = 2 - 1.2\frac{620}{2480} = 1.7 \qquad E_c = \mathbf{34500\ \ MPa}$$

Page 113 – Corrections shown in **bold**

STEP 1

To determine $A_{st.min}$ we first have to calculate A_{ct}. If we ignore the presence of the Reinforcement then $\mathbf{A_{ct}}$ for rectangular section ……………..

STEPS 2 & 3

$$A_{st.min} = \frac{3\mathbf{k_s}A_{ct}}{\mathbf{f_s}}$$

.
.
.

STEP 5

To calculate the tensile stress in the reinforcement at a cracked section under short term Serviceability loads f_{scr} we must calculate the design bending moment at the serviceability Limit state $\mathbf{M_s^*}$ the neutral axis depth kd and the $\mathbf{I_{cr}}$.

$$f_{scr} = \frac{n\,\mathbf{M_s^*}\,(d - kd)}{I_{cr}}$$

Page 116 and 311 – On Chart D2 change D/L_{ef} to Δ/L_{ef} along top of chart

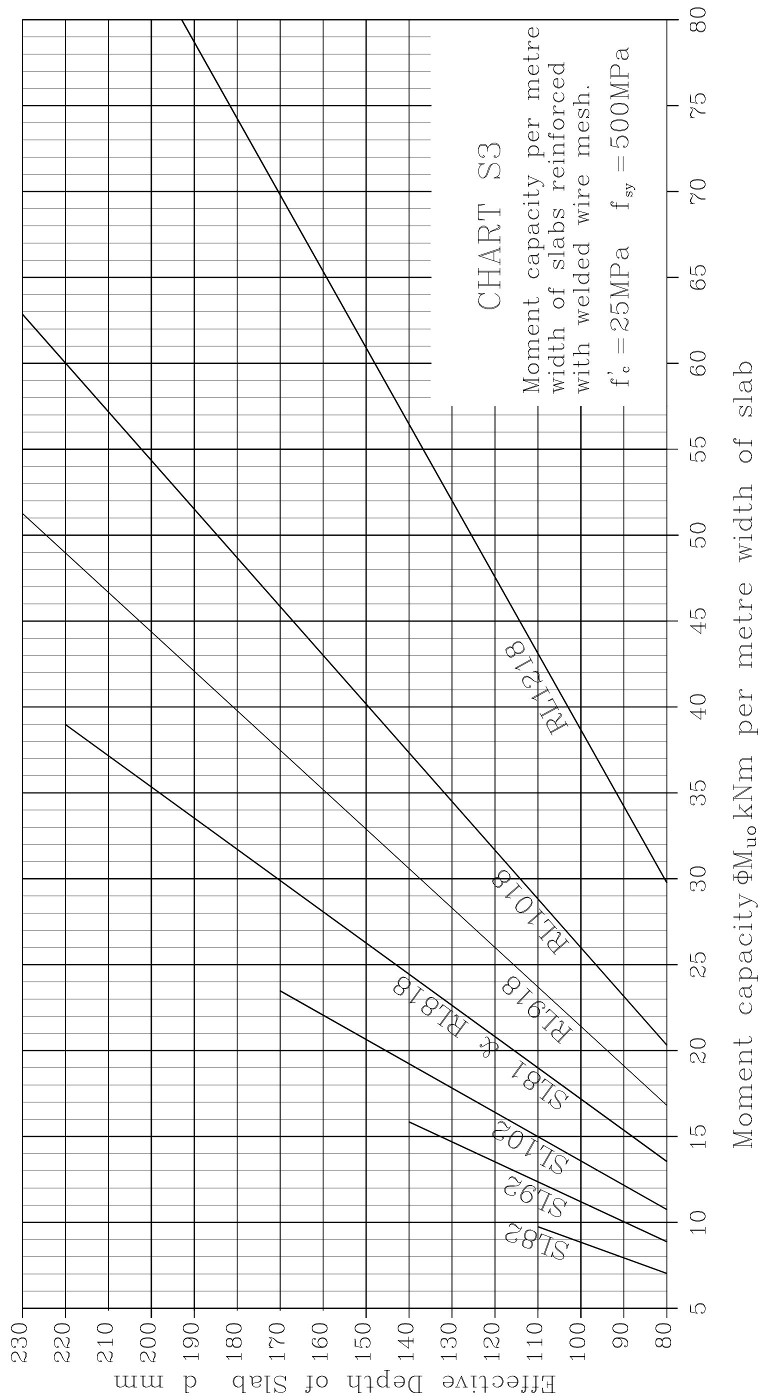
Page 118 Correction shown in **bold**

From Table **3.3**, minimum ………….

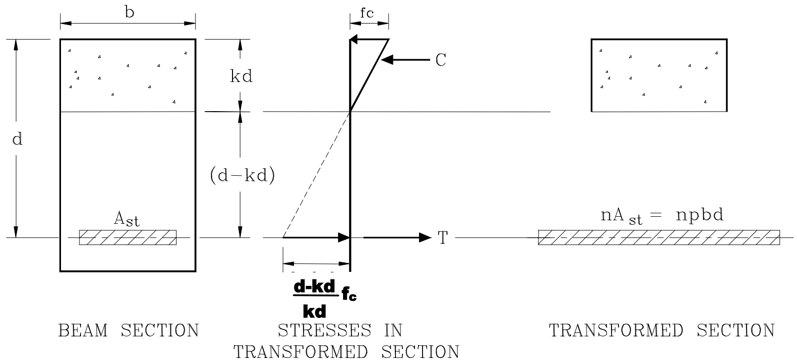
Pages 123 and 314 - Replace chart S3 with chart on following page

OR

simply plot line for RL918 mesh from 16.8 kNm for d=80mm to 51.3 kNm for d-230mm on existing chart.



Page 130 Figure 8.8 – Change stress at level of reinforcement shown in **bold**



Page 131 Equation 8.12 – Corrections shown in **bold**

$$I_{cr} = b\frac{(kd)^{\mathbf{3}}}{3} + nA_{sc}\left(d - kd^2\right) + (n-1)A_{sc}\left(kd - \mathbf{d_{sc}}\right)$$

Page 137 Replace e_s by $\varepsilon_{\mathbf{cs}}$

From AS3600 for a 30 year life and t_h = 195 mm, the new $\varepsilon_{\mathbf{cs}}$ will be 670 microstrain

Page 139 – Corrections shown in **bold**

A_{sc} = $1600mm^{\mathbf{2}}$

.
.
.

$f_{cs(dr)} = f_{cs(sr)} - (A_{sc}/A_{st})f_{cs(sr)}$…but $\geq f_{\mathbf{cs(sr)}}$ (suggested minimum)

	= 0.58 MPa (doubly reinf)	= 0.51 MPa (doubly reinf)
Where $\mathbf{f_{cs(sr)}}$ **=** $[1.5p/(1 + 50p)]E_c\varepsilon_{cs}$	= 1.74 MPa	= 1.52 MPa

Page 151 – Correction shown in **bold**

DESIGN STRENGTH FOR A **1m** WIDE STRIP

Page 154 – Alter symbols b_1, b_2 and b_3 to $\beta_{\mathbf{1}}$, $\beta_{\mathbf{2}}$ and $\beta_{\mathbf{3}}$.

Page 159 Section 10.5 - Altered symbol shown in **bold**

Shear near a support can be taken at a distance $\mathbf{d_o}$ from the face of the support provided

Page 162 Section 10.8 – Altered text shown in **bold**

$$V_{us} = \left(\frac{\mathbf{A_{sv}}\, f_{sy.f}\, d_o}{s}\right)\mathrm{Cot}(\theta_v)$$

substituting $\mathbf{A_{sv.min}} = \left(\dfrac{0.35 b_v s}{f_{sy.f}}\right)$ and …………..

Page 163 Flow Chart – Change $\Phi\mathbf{V_c}$ and $\Phi\mathbf{V_{uc}}$ shown in **bold**

Read $\mathbf{v_c}'$ from Chart V1 and Calculate

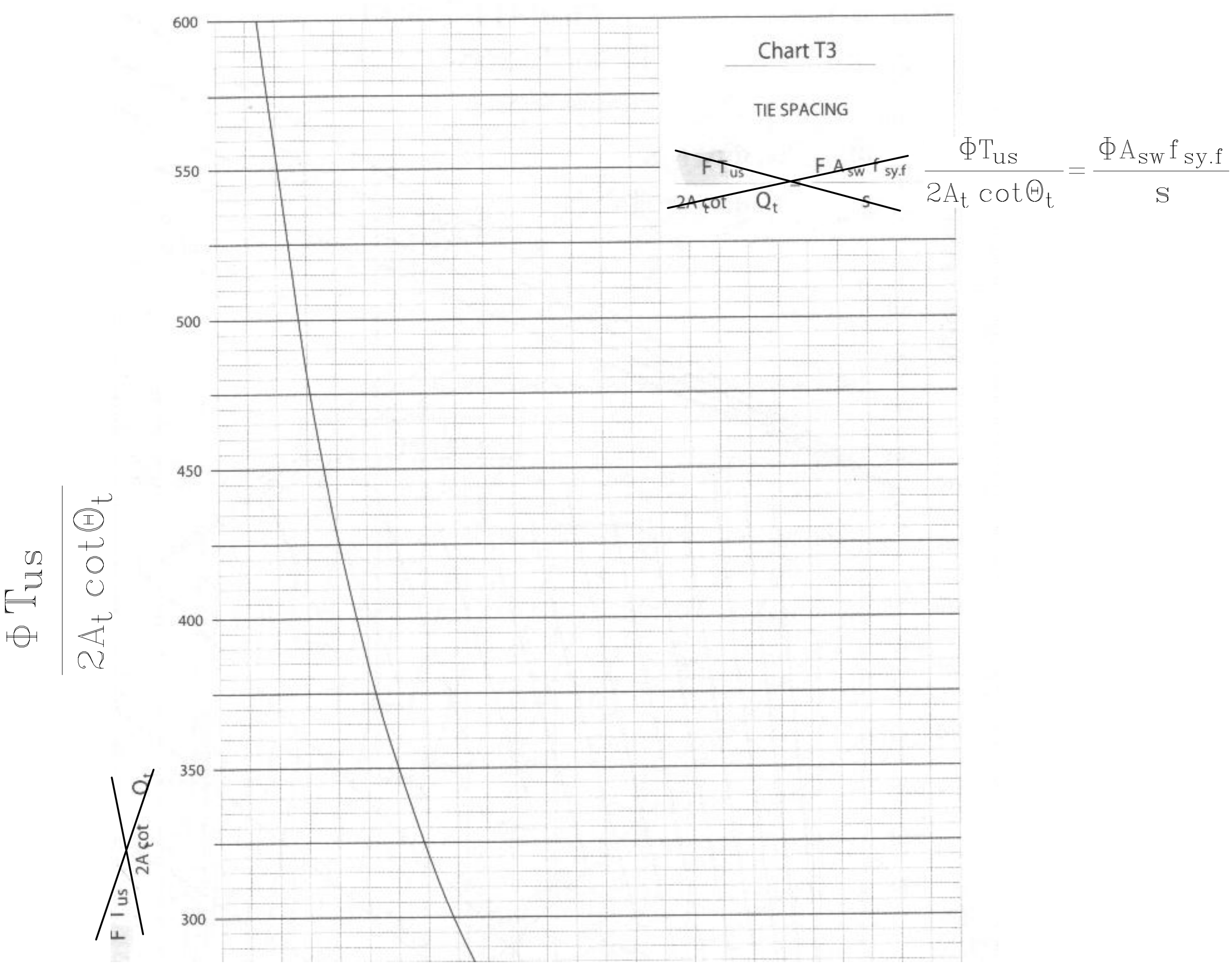
$\Phi\mathbf{V_c} = v_c' \, b_v \, d_o$

For $\beta_1 = 1.1(1.6 - d_o/1000) \geq 1.1$

and $\beta_2 = \beta_3 = 1.0$

Calculate, $\Phi\mathbf{V_{uc}} = \beta_1 \, \Phi V_c$

Page 174 Alter Text in Chart Heading and Vertical Axis Format



Page 179 – Replace £ with ≤

$= 0.0027s + 0.0024s \leq 1.0$

Page 189 - Correction shown in **bold**

Data: $f_c' = 32$ MPa $f_{sy} = \mathbf{500}$ MPa $g = 1$ kPa $q = 4$ kPa

Page 190 - Correction shown in **bold**

Required effective depth, $d = \frac{4400}{\mathbf{51.4}} = \mathbf{85.6}$ mm

Page 216 – Correction shown in **bold**

Unknowns B_1 and B_2. Substituting for ($\mathbf{B_1}$ + B_2) and B_2 in equation (a) and solving for

Page 217 – Correction shown in **bold**

Or cantilever footings as shown in Figure 12.1**(d)** and Figure 12.9. The philosophy is

Page 222 Figure 12.14 – Change to lower case **u** after SHEAR PERIMETER

Page 231 Figure 12.20 – Change 8Y28 to **8N28**

Page 233 Question 3 Correction shown in **bold**

The factored soil bearing capacity $q_{\mathbf{u}} = 400$ kPa.

Pages 234 and 326 Chart FP – Horizontal axis symbol shown in **bold**

Punching Shear Capacity $\Phi \mathbf{V_u}$ kN

Pages 239 – Replace symbol *e* by ε in line,

Rewriting the general axial load equation in terms of the combined strain ε,

Page 244 – Corrections shown in **bold**

(given by equation **13.3**) would need to be applied to cause simultaneous uniform

Page 247 Figure 13.11 – Corrections shown in **bold**

Replace N_{uo} by $\mathbf{N_u}$

Replace $d - 0.5\,\gamma\,k\,d_u$ by $\mathbf{d - 0.5}\,\gamma\,\mathbf{k_u\,d}$

Page 248 Fourth equation – Equation to read as follows;

$$f_{st} = 2 \times 10^5 \varepsilon_{st} \leq f_{sy}$$

Page 249 - Line 6 replace 13.1 by **13.3** Line 27 replace 1161×10^6 by $\mathbf{1.161} \times 10^6$

Page 253 – Amend text shown in **bold**

$N_u e' = 1228.2 \times (425 - 0.5 \times \mathbf{0.85} \times 170) + 778.8 \times (425 - 75)$

$= \mathbf{705850}$ kNmm

$e' = \dfrac{\mathbf{705850}}{767} = \mathbf{920}$ mm

$e = \mathbf{920} - 175 = \mathbf{745}$ mm

$M_u = 767 \times \mathbf{745} \times 10^{-3}$

$= \mathbf{571}$ kNm

.
.
.

$\Phi = \dots\dots\dots\dots$

$= 0.6 \mathbf{+} 0.2 \times \left(1 - \dfrac{767}{1388}\right) = 0.69$

.
.
.

$\Phi M_u = 0.69 \times \mathbf{571} = \mathbf{394}$ kNm

Page 254 – Amend text shown in **bold**

$N_{ub} e' = \dots\dots\dots\dots$

$= 181810$ kN**mm**

Page 255 - In line 2 replace 13.2 by **13.4**

Page 256 – Amend text shown in **bold**

$N_u e' = 837.1 \times (425 - 0.5 \times \mathbf{0.822} \times 93.6) + 451.7 \times (180 - 60)$

$= \mathbf{172679}$ kNmm

$e' = \dfrac{\mathbf{172679}}{88.8} = \mathbf{1945}$ mm

$e = \mathbf{1945} - 60 = \mathbf{1885}$ mm

$M_u = 88.8 \times \mathbf{1885} \times 10^{-3}$

$= \mathbf{167.4}$ kNm

.
.
.

$\Phi M_u = 0.6 \times \mathbf{167.4} = \mathbf{100.4}$ kNm

Page 257 - In line 12, line 18 and line 25, replace 13.2 by **13.4**

Page 258 - Amendments shown **bold**

$\Phi N_u \times e' = 1075.\mathbf{3} \times (390 - .5 \times 0.85 \times 283.5) + 650.5 \times (390-60)$
$= \mathbf{504472}$ kNmm

$$e = \frac{\mathbf{504472}}{1401.4} - 165 = 195 \text{ mm}$$

Page 258 - Section 13.10 line 2, replace 13.2 by **13.4**

Page 263 Equation 13.17 - Replace $200d_o$ by $\mathbf{182d_o}$

Page 264 - Line 1 and subsequent Equation 13.19 - Replace $200d_o$ by $\mathbf{182d_o}$
- Example 7 changed dead and live loads shown **bold**

The column is required to carry an axial design load $N^* = 1855$ kN made up of **507** kN dead laod G and **831** kN live

Page 266 Calculations for N_c/bD – Corrected data is highlighted in **bold**

$$\frac{N_c}{bD} = \left(\frac{\pi}{L_e}\right)^2 D \left(\frac{\mathbf{182}\, d_o \dfrac{\Phi M_{ub}}{bD^2}}{1+\beta_d}\right)$$

$$= \left(\frac{\pi}{5270}\right)^2 400 \left(\frac{\mathbf{182} \times 340 \times 4.5}{1+0.379}\right)$$

$$= \mathbf{28.7} \text{ MPa}$$

Page 267 - Altered data is highlighted in **bold**

$$\delta_b = \frac{k_m}{1 - \dfrac{N^*}{N_c}} = \frac{0.685}{1 - \dfrac{13.25}{\mathbf{28.7}}} = \mathbf{1.28}$$

Magnified bending stress $\delta_b \dfrac{M^*}{bD^2} = \mathbf{1.28} \times 3.1 = \mathbf{3.97}$ MPa

Using the magnified bending stress, from chart CR2f32-7, read new steel ratio $p = \mathbf{0.035}$. This initial value may be accepted and it will be a conservative estimate of the required steel ratio. The more accurate result will be obtained by repeating the design steps using the new steel ratio until the same magnification factor and steel ratio obtained in two consecutive iterations. Repeating the procedure with the new steel ratio of **0.035**,

For $p = \mathbf{0.035}$, $\dfrac{\Phi N_{uo}}{bD} = \mathbf{25.8}$ MPa and $\dfrac{\Phi M_{ub}}{bD^2} = \mathbf{5.4}$ MPa

$$\frac{N_c}{bD} = \left(\frac{\pi}{5270}\right)^2 400\left(\frac{\mathbf{182} \times 340 \times \mathbf{5.4}}{1+0.379}\right)$$

$$= \mathbf{34.4} \text{ MPa}$$

$$\delta_b = \frac{0.685}{1-\frac{13.25}{\mathbf{34.4}}} = \mathbf{1.11}$$

Magnified stress $\delta_b \dfrac{M^*}{bD^2} = \mathbf{1.11} \times \mathbf{3.1} = \mathbf{3.4}$ MPa

Further iterations will converge on $\delta_b = \mathbf{1.16}$ with a magnified stress,

$\delta_b \dfrac{M^*}{bD^2} = \mathbf{1.16} \times 3.1 = \mathbf{3.6}$ MPa and a required steel ratio $p = \mathbf{0.032}$. The required area of reinforcement,

$$A_s = \mathbf{0.032} \times 350 \times 400 = \mathbf{4480} \text{ mm}^2$$

Pages 268 Example 8 - Amendments shown highlighted in **bold**

Check the column in example **7** for biaxial bending when the minimum design moment is applied about the weak axis.

.
.
.

$$\frac{\Phi M_{ux}}{bD^2} = \mathbf{3.9} \text{ MPa}$$

$$\frac{\Phi M_{ubx}}{bD^2} = \mathbf{5.3} \text{ MPa}$$

Pages 269 - Amendments shown highlighted in **bold**

$$\frac{N_c}{bD} = \left(\frac{\pi}{5270}\right)^2 400\left(\frac{\mathbf{182} \times 340 \times \mathbf{5.3}}{1+0.379}\right) = \mathbf{33.8} \text{ MPa}$$

$$\delta_b = \frac{0.685}{1-\frac{13.25}{\mathbf{33.8}}} = \mathbf{1.13}$$

Magnified bending stress $\delta_b \dfrac{M_x^*}{bD^2} = \mathbf{1.13} \times 3.1 = \mathbf{3.5}$ MPa

.
.
.

From Chart RC2f32-6 for p=0.023, $0.6N_{uo}/bD$ = 22.5 MPa (=$\Phi N_{uo}/bD$ at zero moment) and $\Phi M_{ub}/bD^2$ = **3.6** MPa.

$$\frac{N_c}{bD} = \left(\frac{\pi}{5270}\right)^2 350\left(\frac{\mathbf{182} \times 290 \times \mathbf{3.6}}{1.379}\right) = \mathbf{17.1} \text{ MPa}$$

$$\delta_b = \frac{1}{1 - \frac{13.25}{\mathbf{17.1}}} = \mathbf{4.44}$$

Pages 270 - Amendments shown highlighted in **bold**

Magnified bending stress,

$$\delta_b \frac{M_y^*}{bD^2} = \mathbf{4.44} \times 0.663 = \mathbf{2.9} \text{ MPa}$$

$$\vdots$$

$$\frac{\Phi M_{uy}}{bD^2} = \mathbf{3.6} \text{ MPa}$$

Exponential to be used in interaction formula,

$$\alpha_n = 0.7 + \frac{1.7 * 13.25}{22.5} = 1.70$$

$$\left(\frac{M_x^*}{\Phi M_{ux}}\right)^{\alpha_n} + \left(\frac{M_y^*}{\Phi M_{uy}}\right)^{\alpha_n} = \left(\frac{\mathbf{3.5}}{5.3}\right)^{1.7} + \left(\frac{\mathbf{2.9}}{\mathbf{3.6}}\right)^{1.7}$$

$$= \mathbf{1.19}$$

> 1.0 UNSATISFACTORY

The column section will need to be increased or additional reinforcement added. Add two more N32 reinforcing bars for bending about the weak axis. In this case the column is equally reinforced on 4 faces using 8 bars. Charts CR4f are drawn for columns equally reinforced on 4 faces using 12 or more reinforcing bars. Use of charts CR4f is illustrated in example **9**. A conservative result may be obtained using charts CR2f.

It will be necessary to recalculate the stress conditions for bending about the y-axis using the increased steel ratio for **8**N32 bars.

Steel ratio $p = \frac{\mathbf{6400}}{350 \times 400} = \mathbf{0.046}$

From chart CR2f32-6 for $p = \mathbf{0.046}$, $\frac{\Phi N_{uo}}{bD} = \mathbf{29}$ MPa and $\frac{\Phi M_{ub}}{bD^2} = \mathbf{5.5}$ MPa and

buckling stress $\frac{N_c}{bD} = \mathbf{33.8}$ determined earlier.

Magnification factor,

$$\delta_b = \frac{1}{1 - \frac{13.25}{\mathbf{33.8}}} = \mathbf{1.64}$$

Magnified design bending stress,

$$\delta_b \frac{M_y^*}{bD^2} = \mathbf{1.64}\times 0.663 = \mathbf{1.09}\text{ MPa}$$

From chart CR2f32-6 for the steel ratio p = **0.046** and axial stress $N^*/(bD)$ = 13.25 MPa, $\Phi M_{uy}/(bD^2)$ = **3.6** MPa.

Page 271 – Amendments shown highlighted in **bold**

Exponential index for the interaction formula,

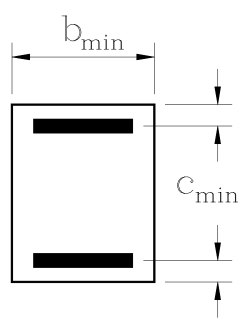
$$\alpha_n = 0.7 + 1.7\frac{13.25}{\mathbf{29}} = \mathbf{1.48}$$

$$\left(\frac{M_x^*}{\Phi M_{ux}}\right)^{\alpha_n} + \left(\frac{M_y^*}{\Phi M_{uy}}\right)^{\alpha_n} = \left(\frac{\mathbf{3.5}}{3.8}\right)^{\mathbf{1.48}} + \left(\frac{\mathbf{1.09}}{\mathbf{3.6}}\right)^{\mathbf{1.48}}$$

$$= \mathbf{1.06}$$

The result of 1.06 is close to the limit of 1.0 and it may be accepted considering the conservative approach adopted by using the design chart C2f3-6 for columns with equal reinforcement on two faces.

Page 287 Table 3.4 Modify Sketch



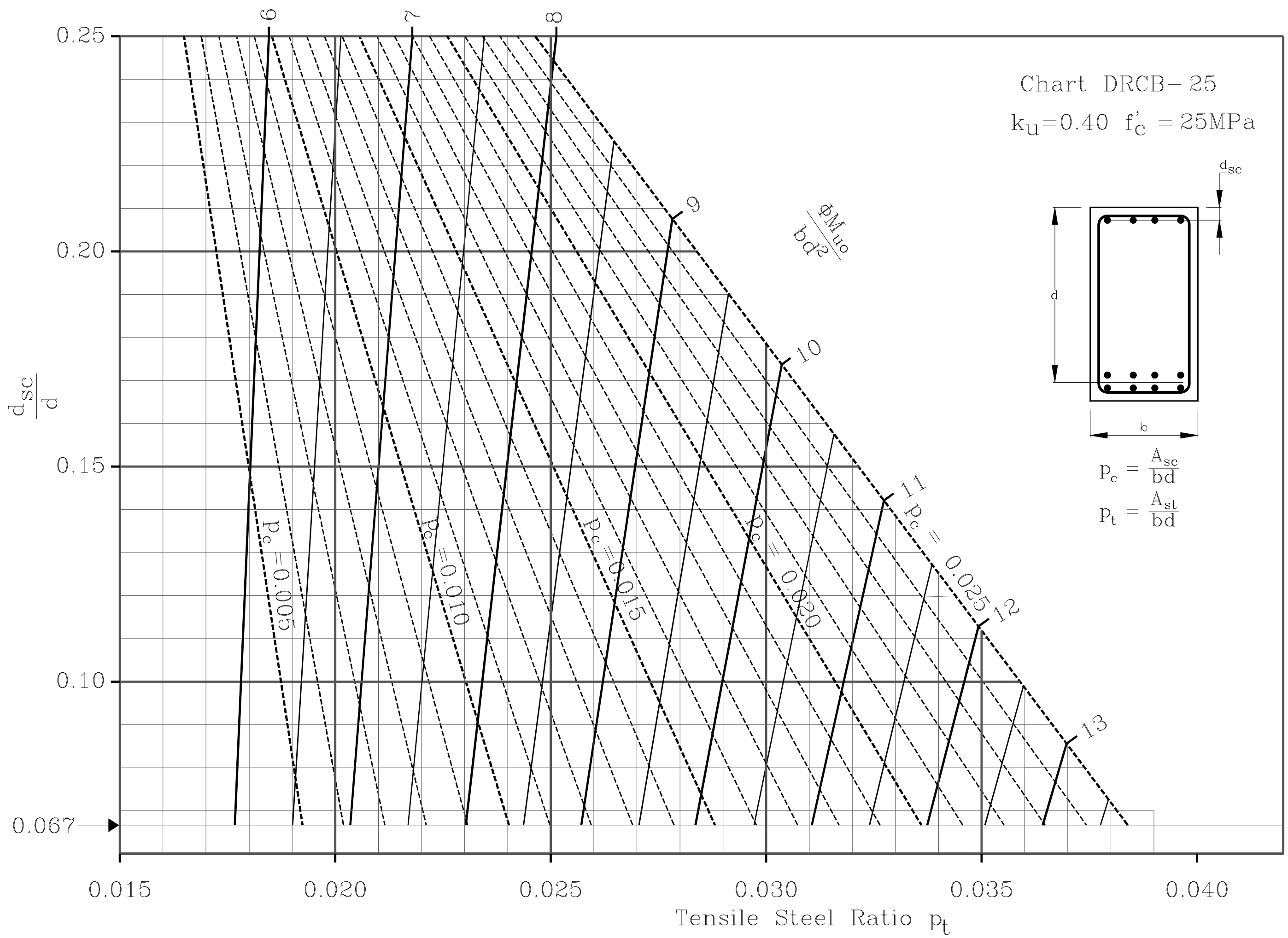
Pages 362 to 377 –Circular Column Charts;

Horizontal Axis should read, $\frac{\mathbf{\Phi M_u}}{\mathbf{A_g D}}$ and Vertical Axis should read, $\frac{\mathbf{\Phi N_u}}{\mathbf{A_g}}$

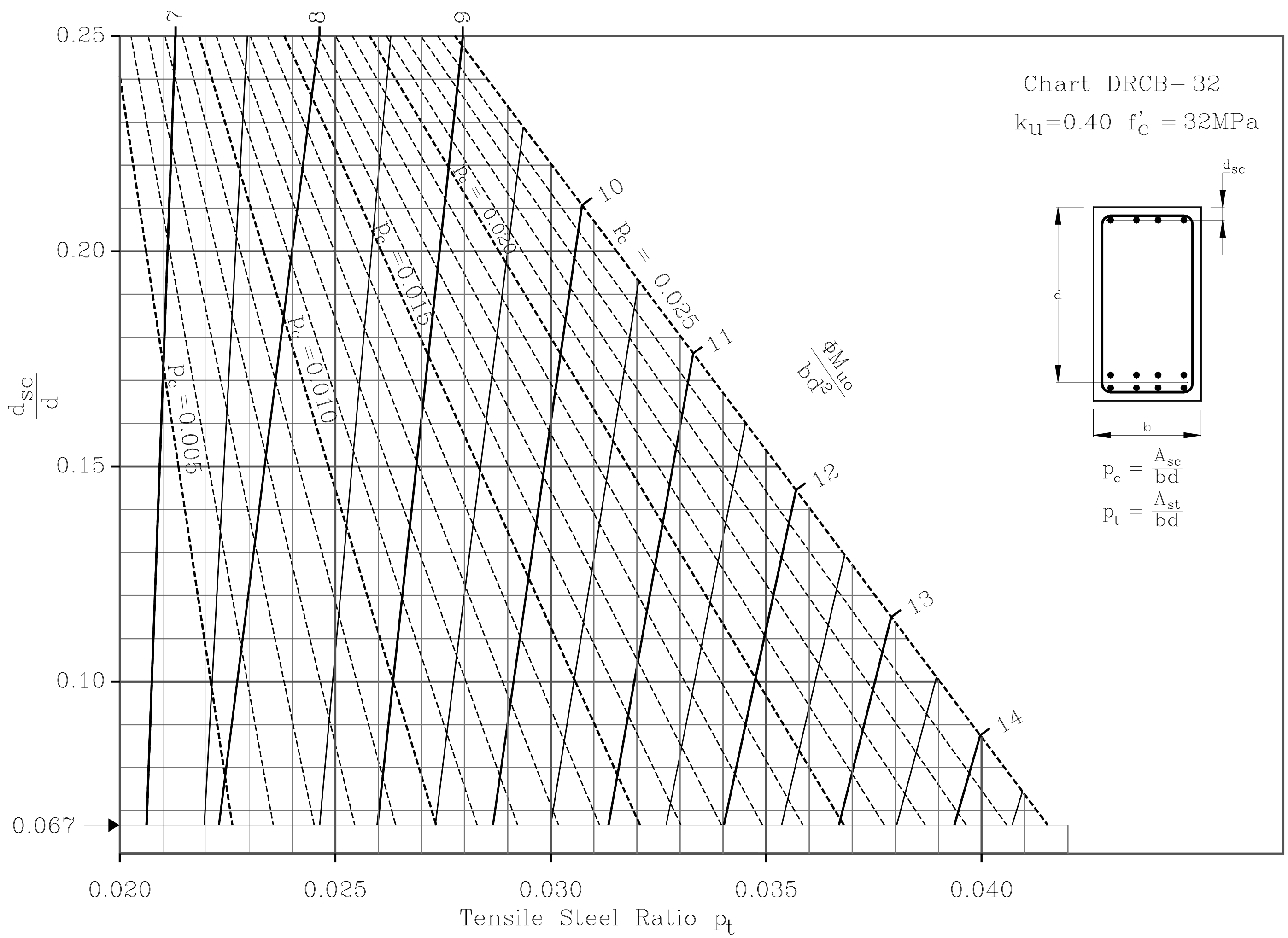
Page 385 – Add page number shown in **bold**

Web Crushing **167**

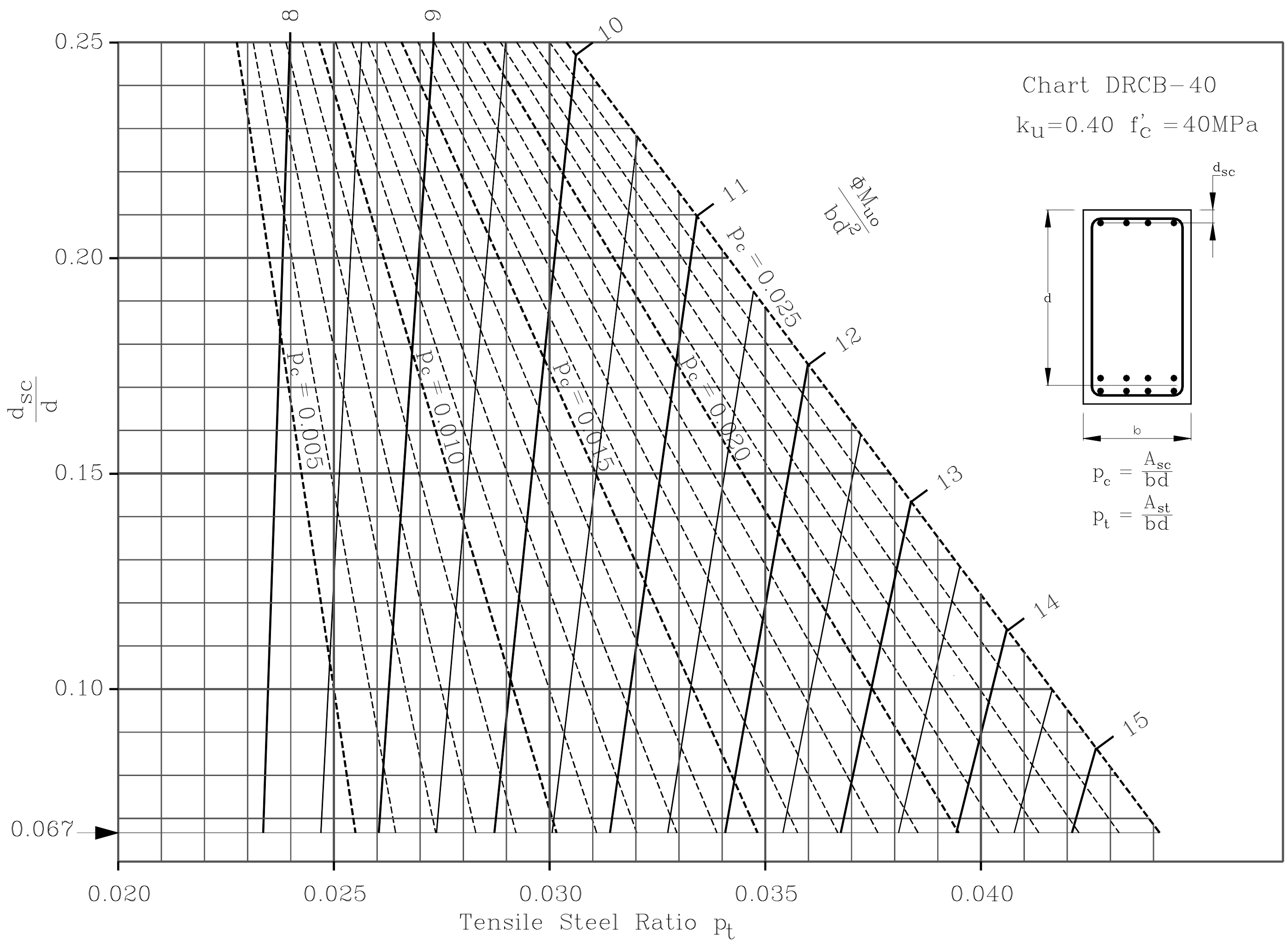
Pages 294 Replace Chart DRCB - 25



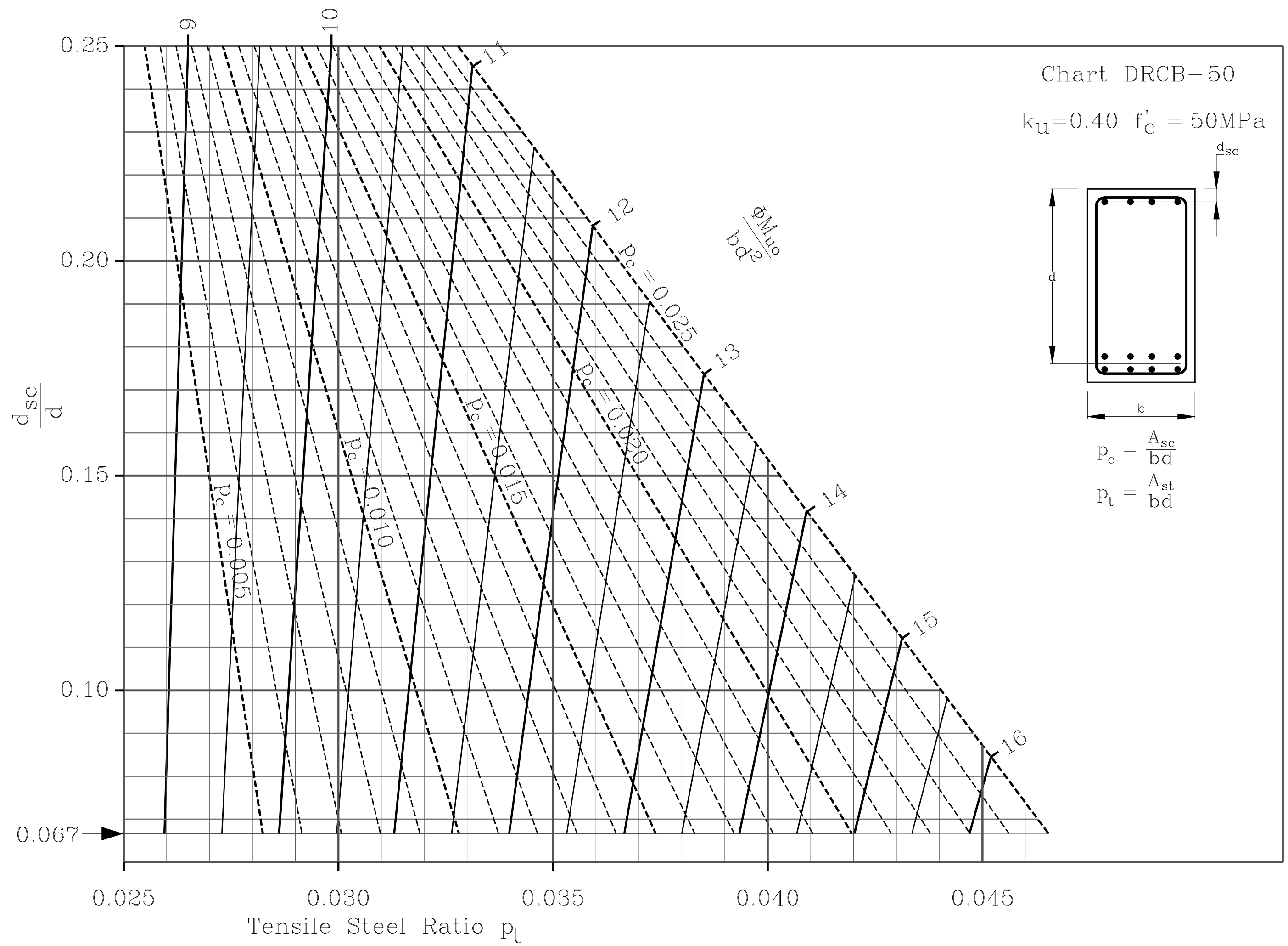
Page 295 Replace Chart DRCB - 32



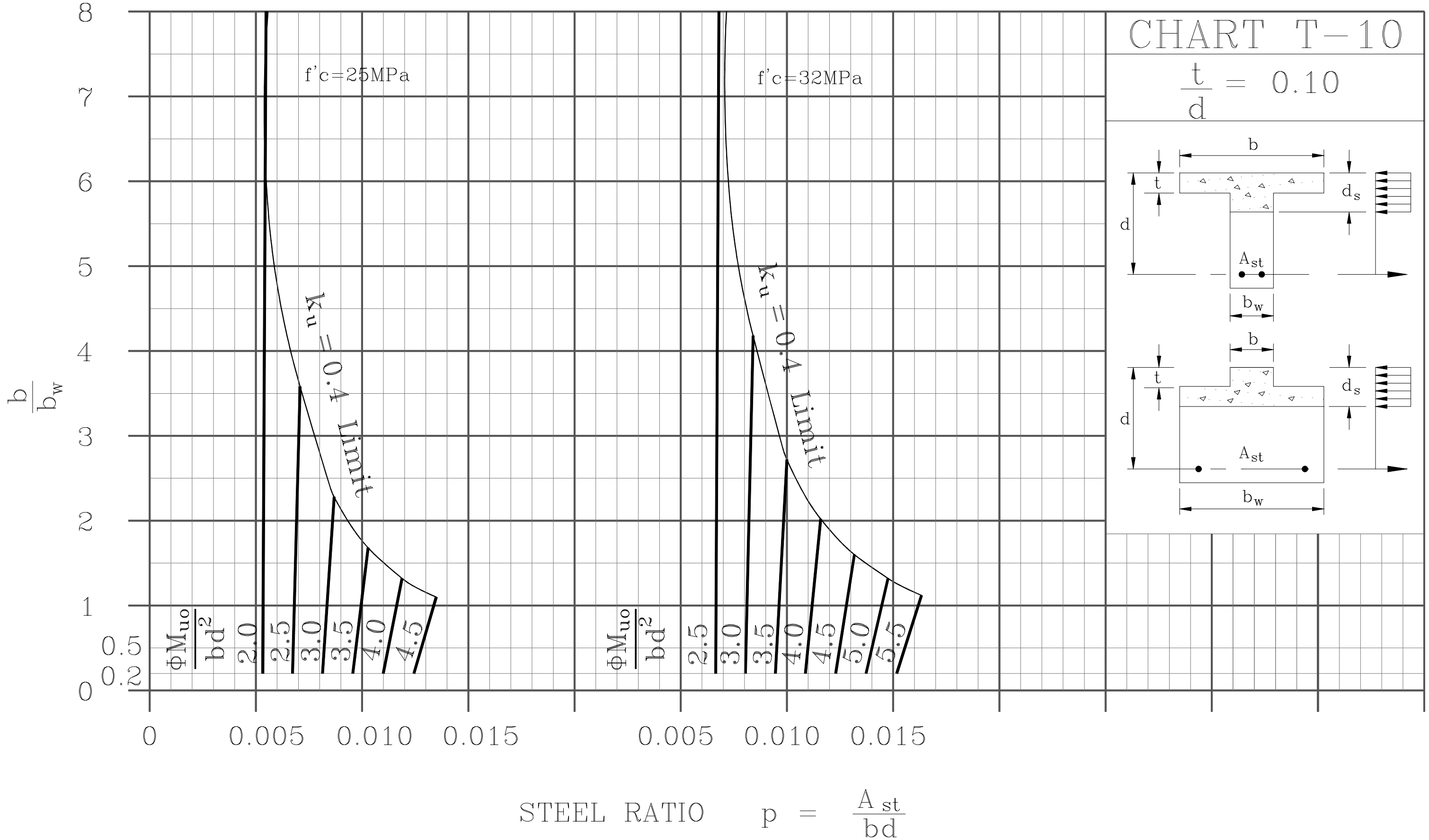
Page 296 Replace Chart DRCB - 40



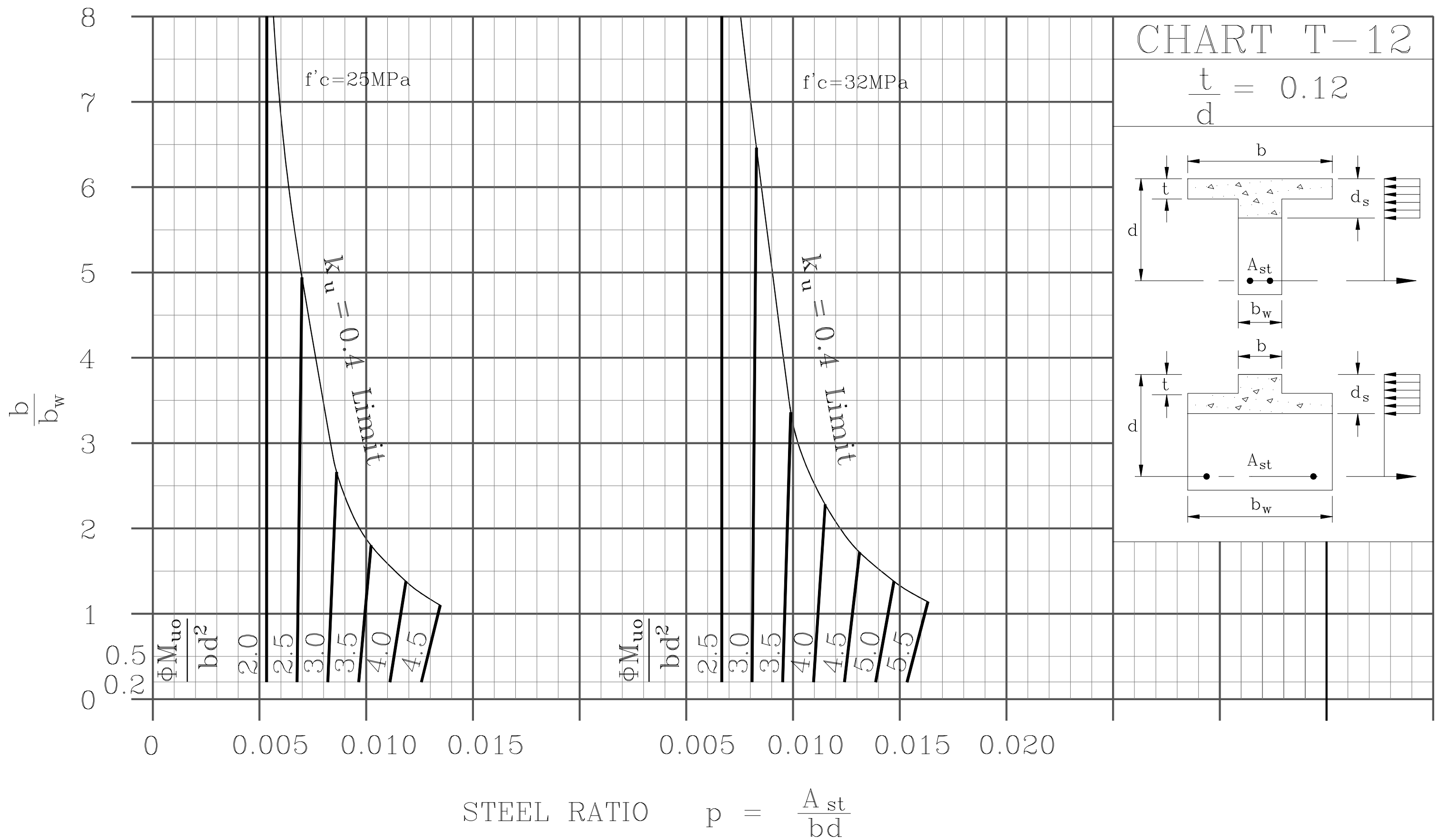
Page 297 Replace Chart DRCB - 50



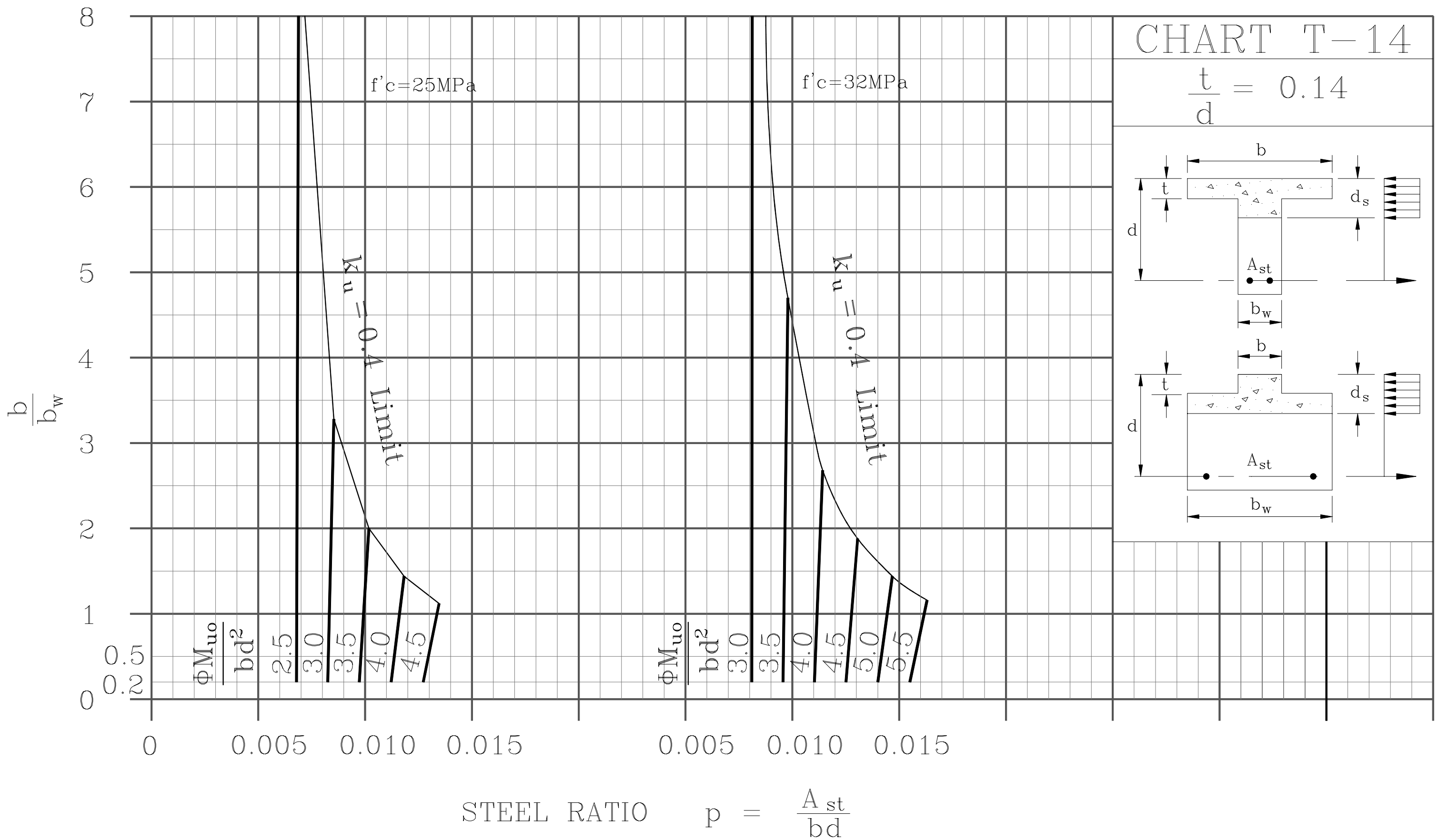
Page 298/299 Replace With Single T-Beam Chart T-10



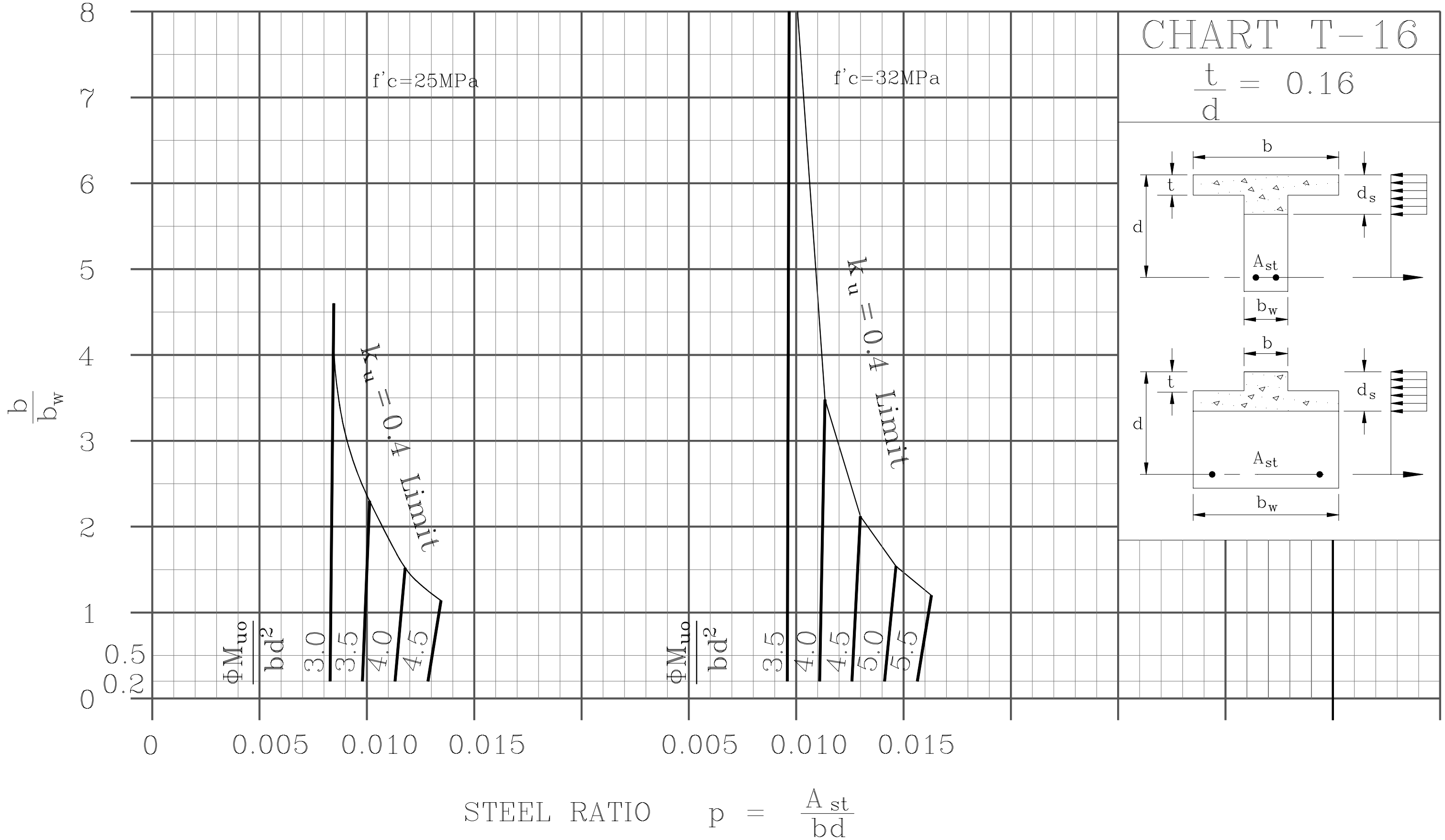
Page 300/301 Replace With Single T-Beam Chart T-12



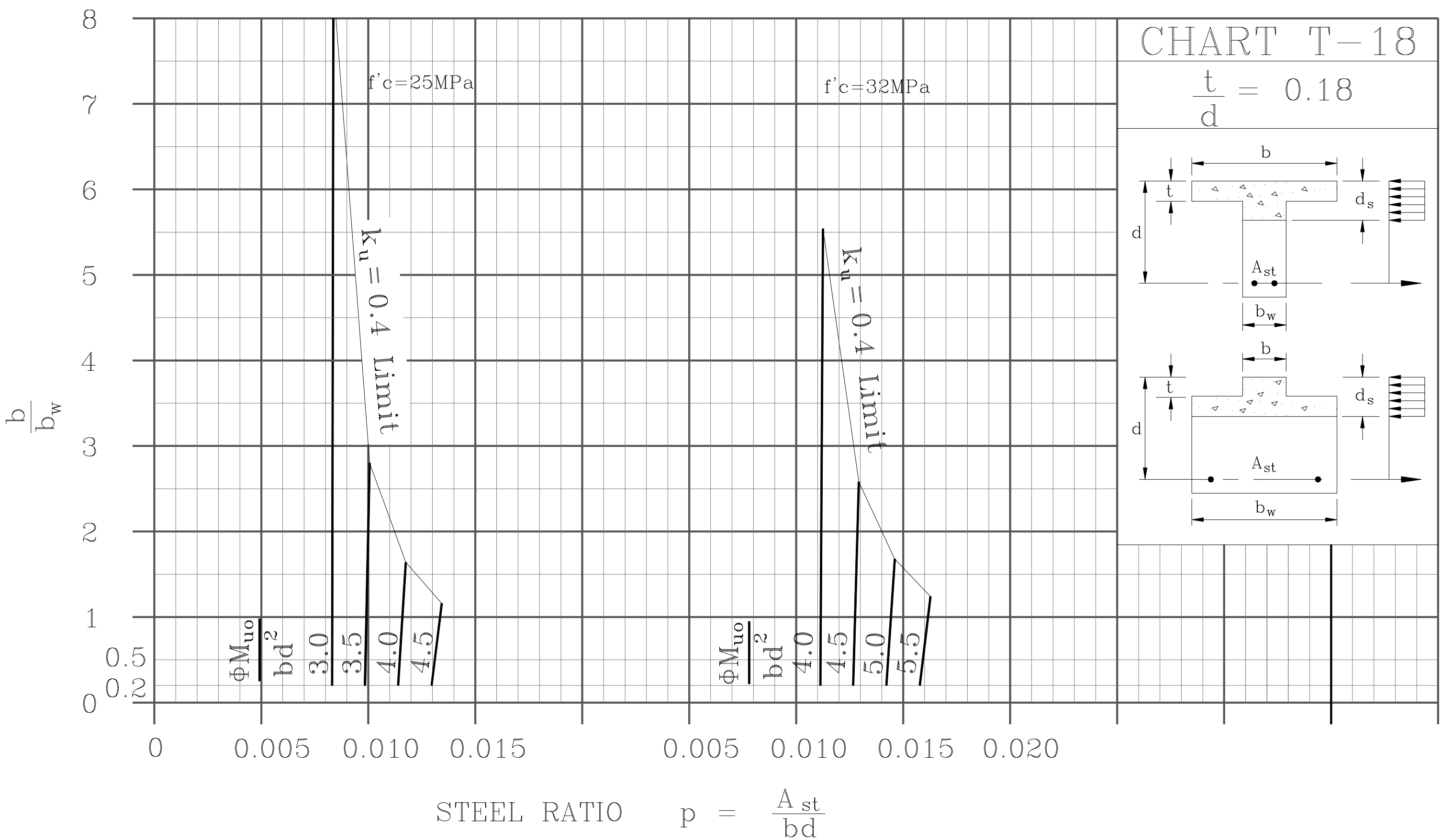
Pages 302/303 - Replace with single T-Beam Chart T–14



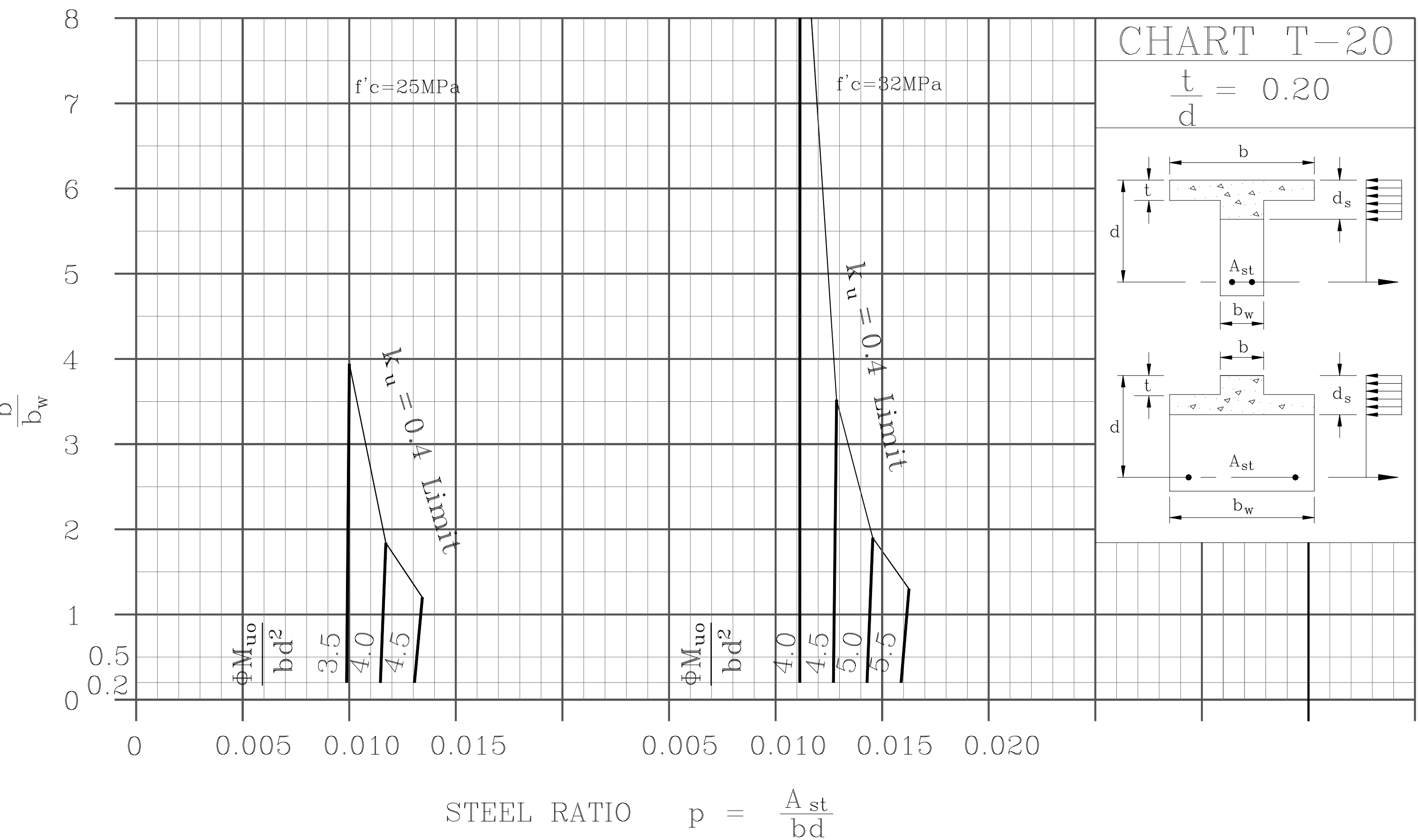
Pages 304/305 - Replace with single T-Beam Chart T–16



Pages 306/307 - Replace with single T-Beam Chart T–18



Pages 308/309 - Replace with single T-Beam Chart T–20



Page 311 Chart D2 – Replace title block

CHART D2

Deemed to comply deflection for slabs

Max. $\dfrac{L_{eff}}{d} = k_3 k_4 k_5$

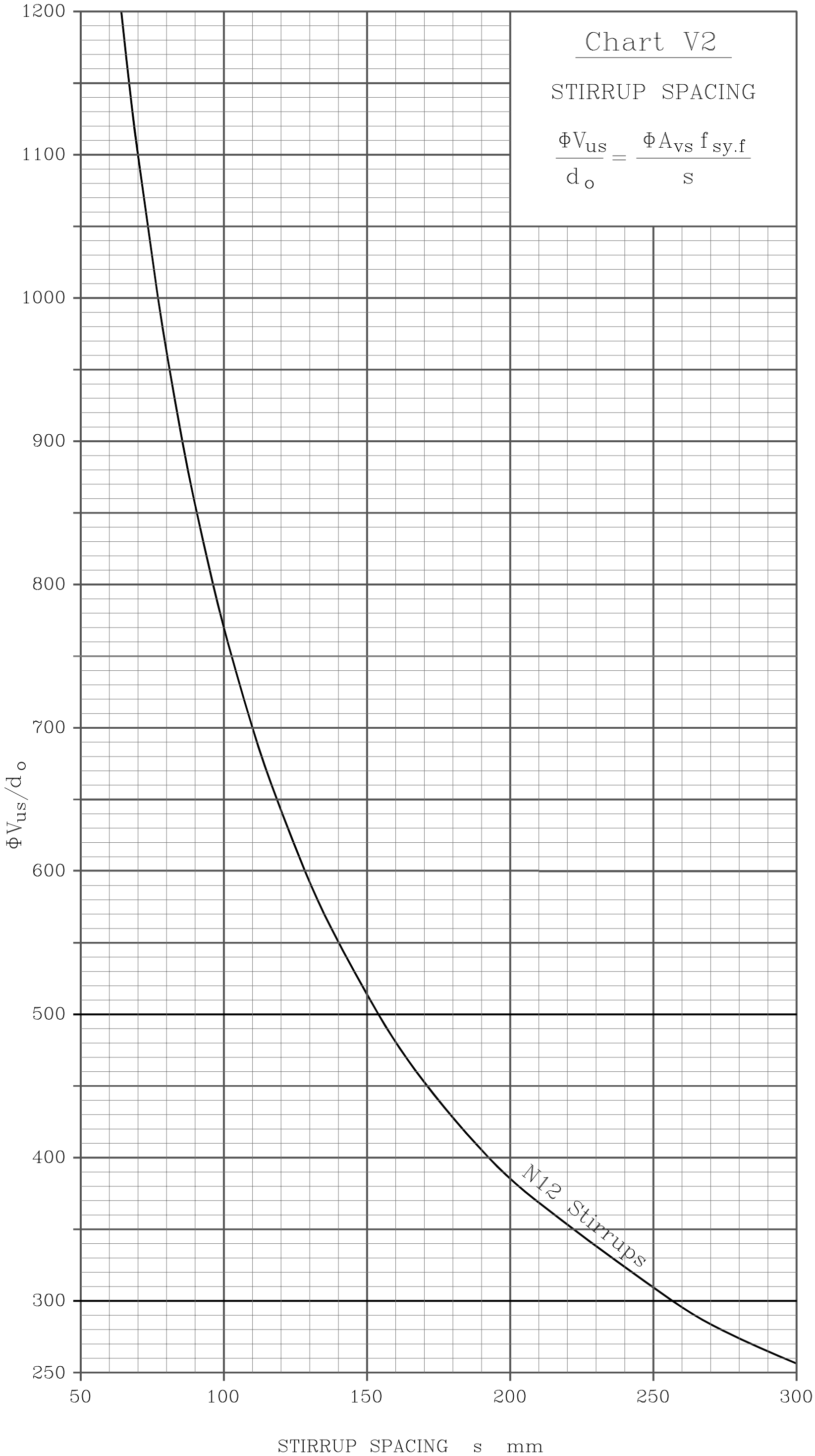
k_3 & k_4 see notes, $k_5 = \left[\dfrac{1000(\Delta/L_{eff})E_c}{F_{d.eff}}\right]^{\frac{1}{3}}$

$F_{d.eff} = (1+k_{cs})g + (\Psi_s + k_{cs}\Psi_l)q$ for total deflection

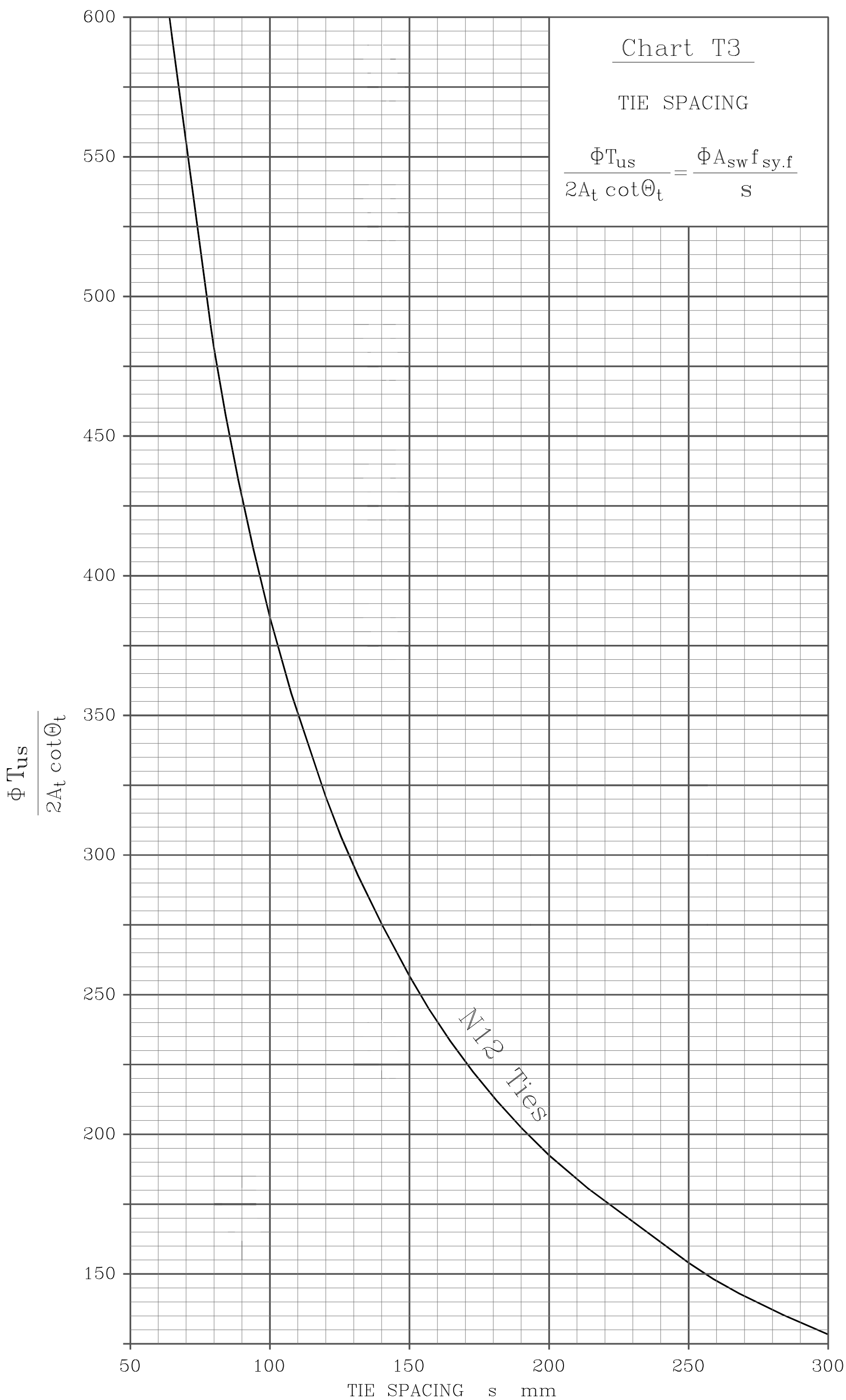
$= k_{cs}\,g + (\Psi_s + k_{cs}\Psi_l)q$ for incremental deflection

$k_{cs} = 2 - 1.2(A_{sc}/A_{st}) >= 0.8$

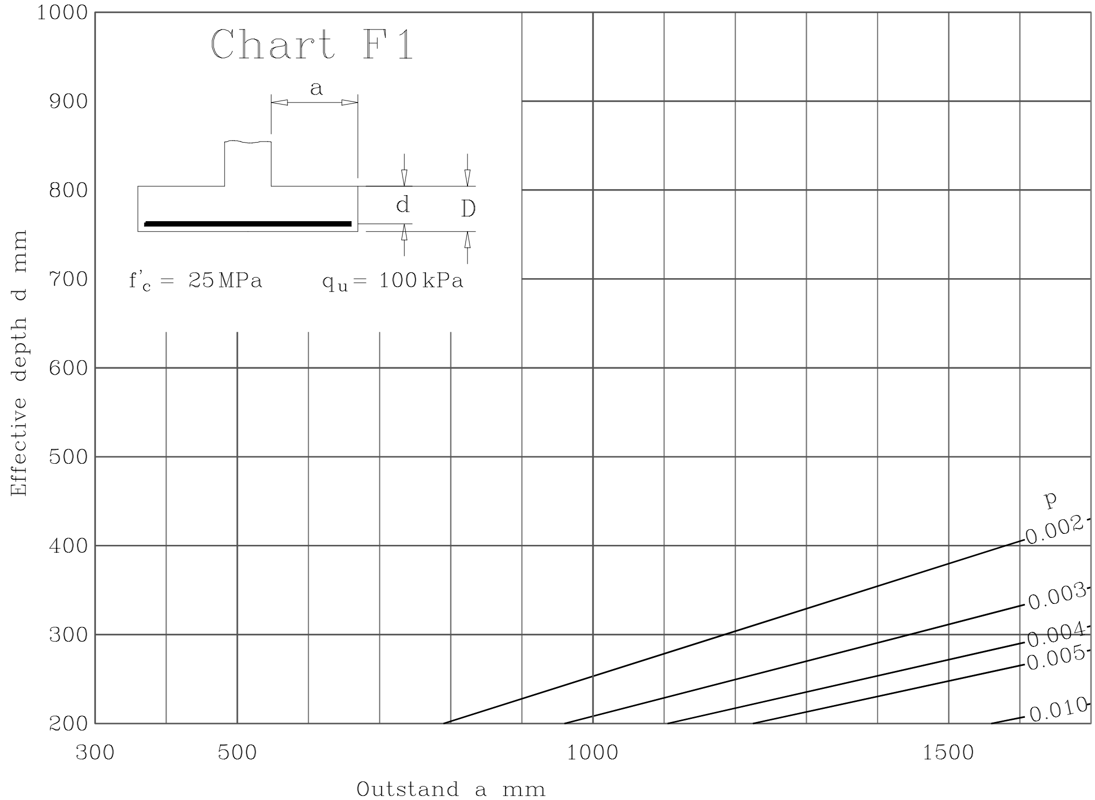
Page 316 replace Chart V2

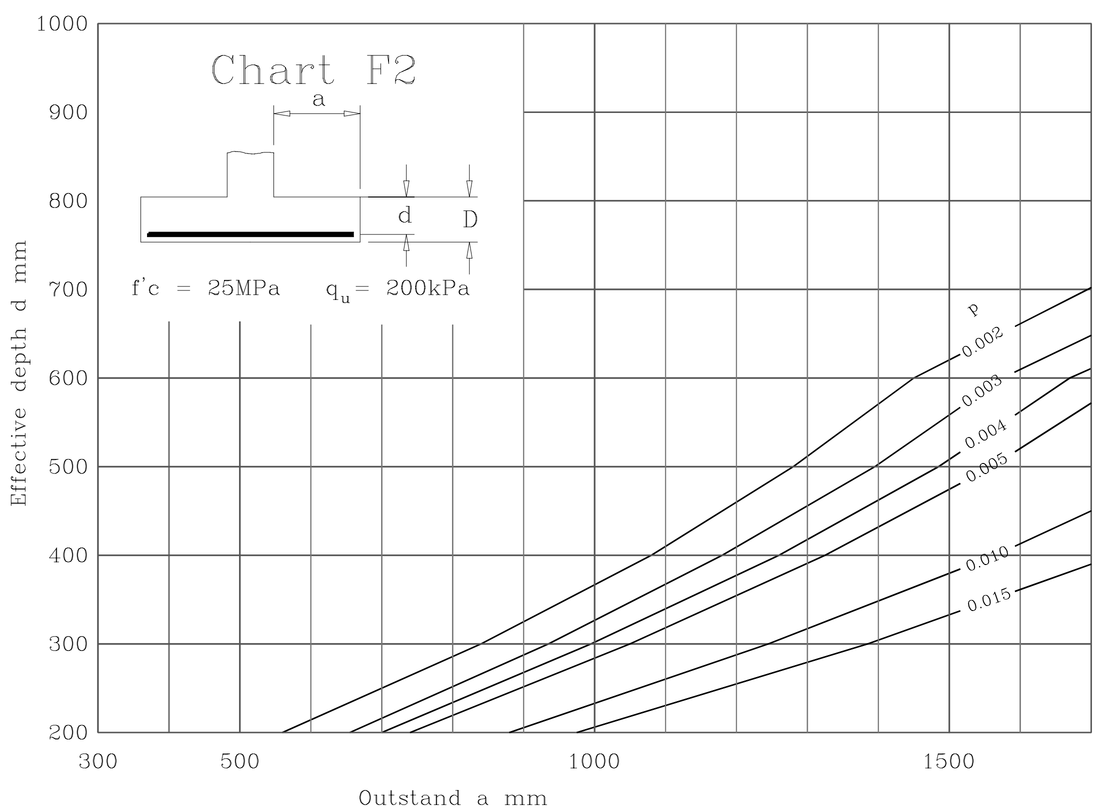


Page 319 replace Chart T3

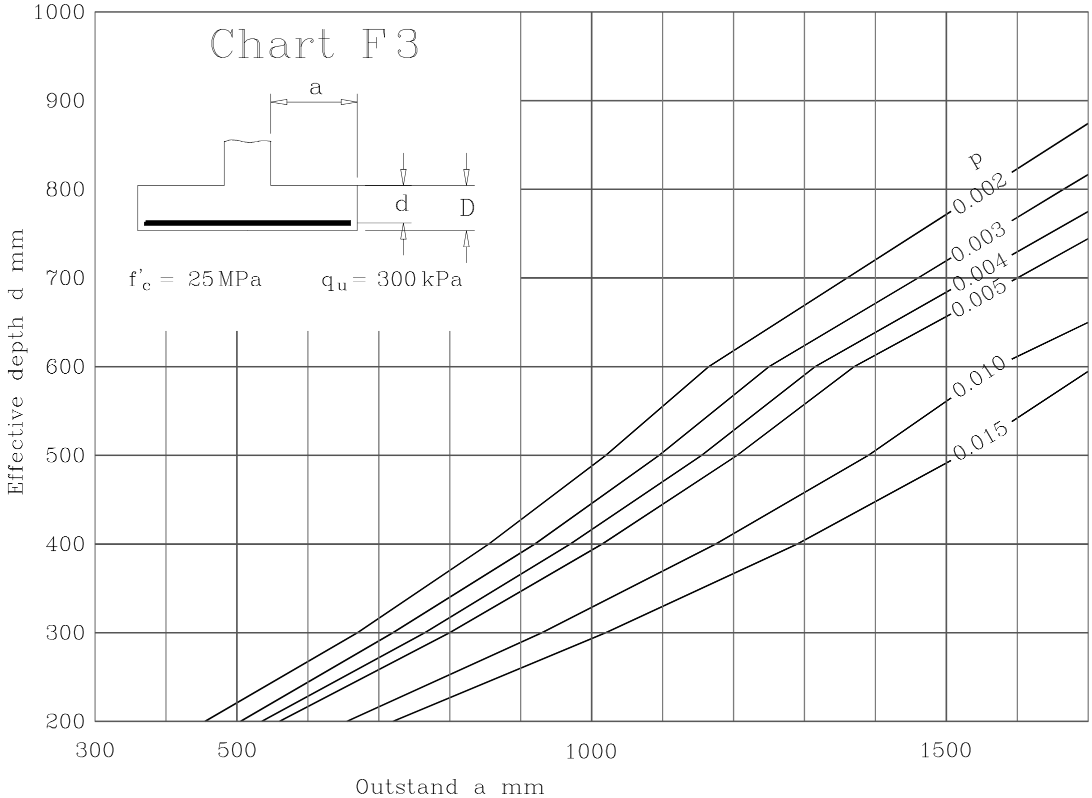


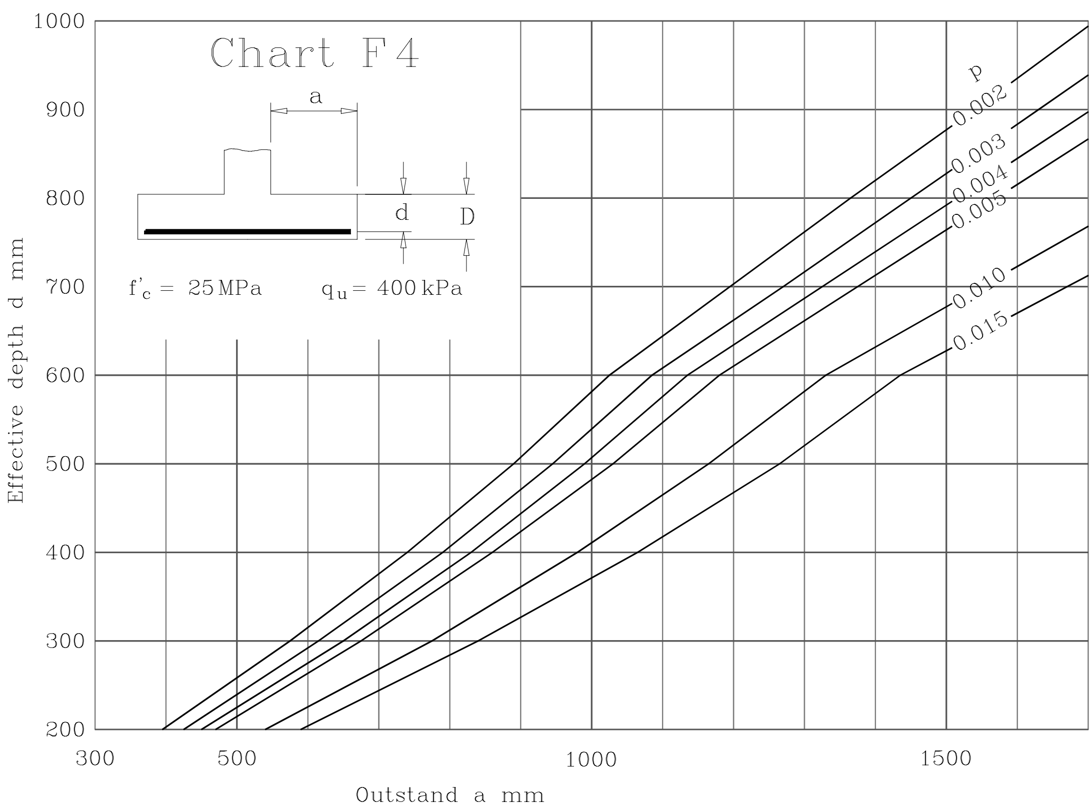
Page 321 Replace charts F1 and F2



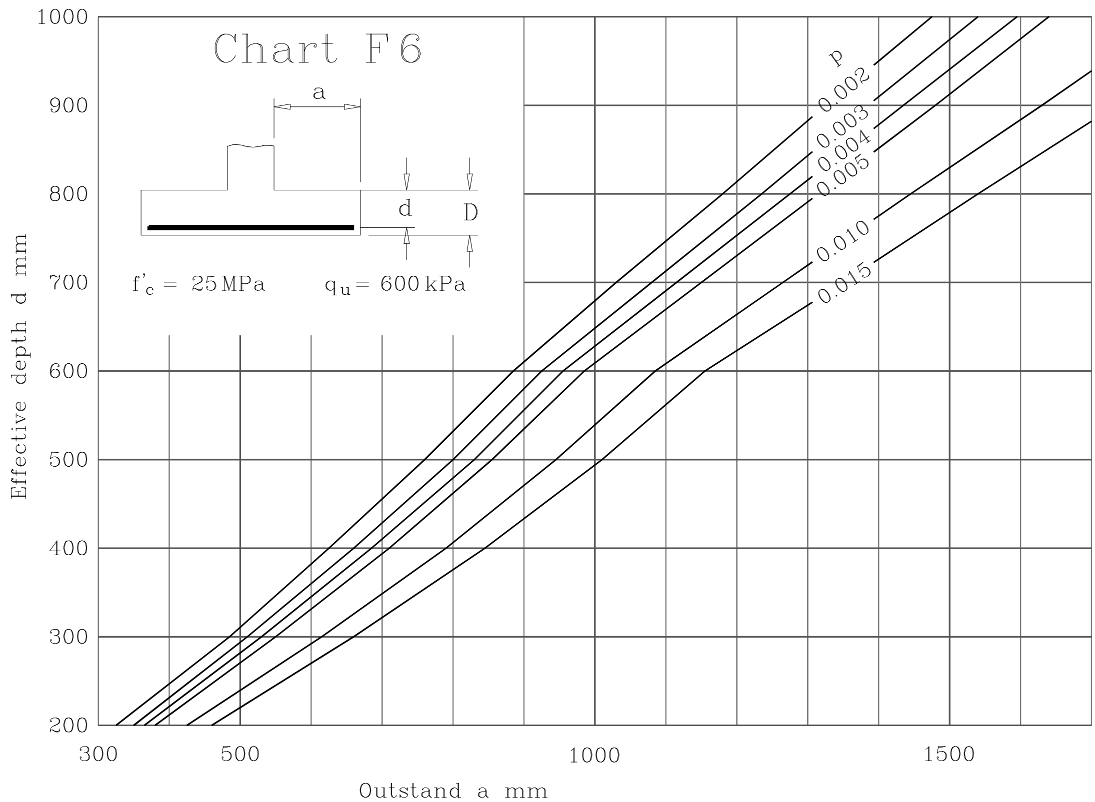


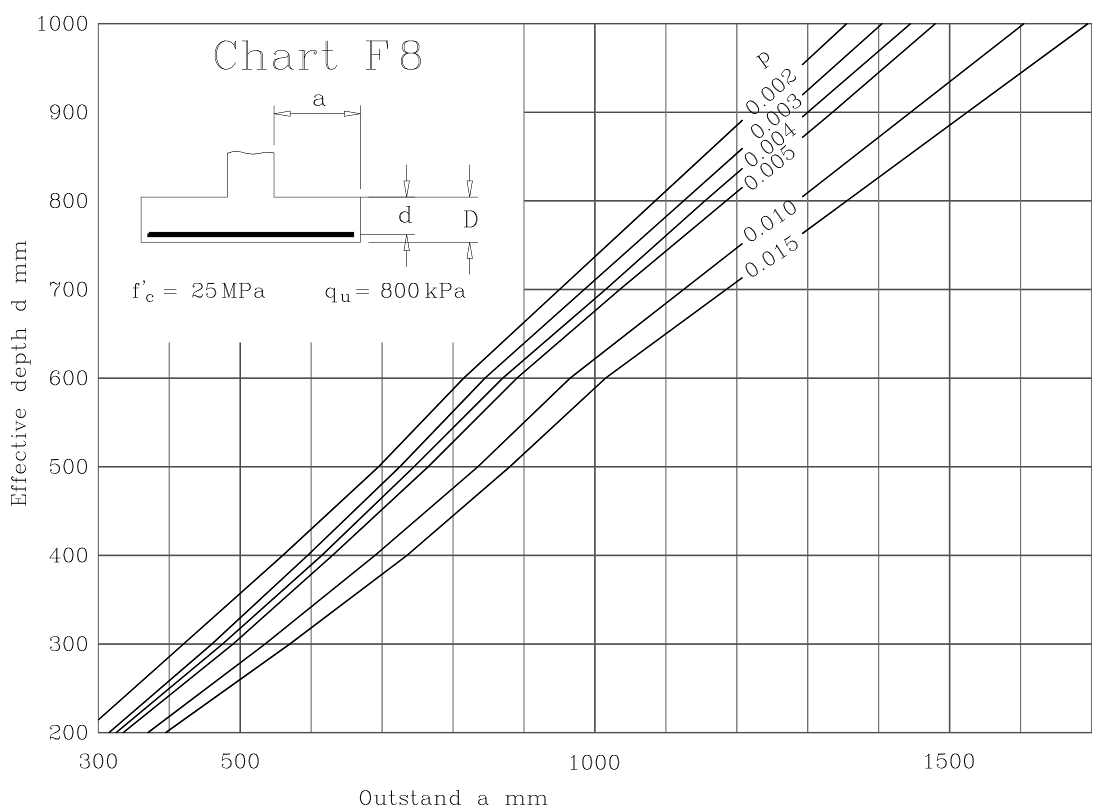
Page 322 Replace charts F3 & F4



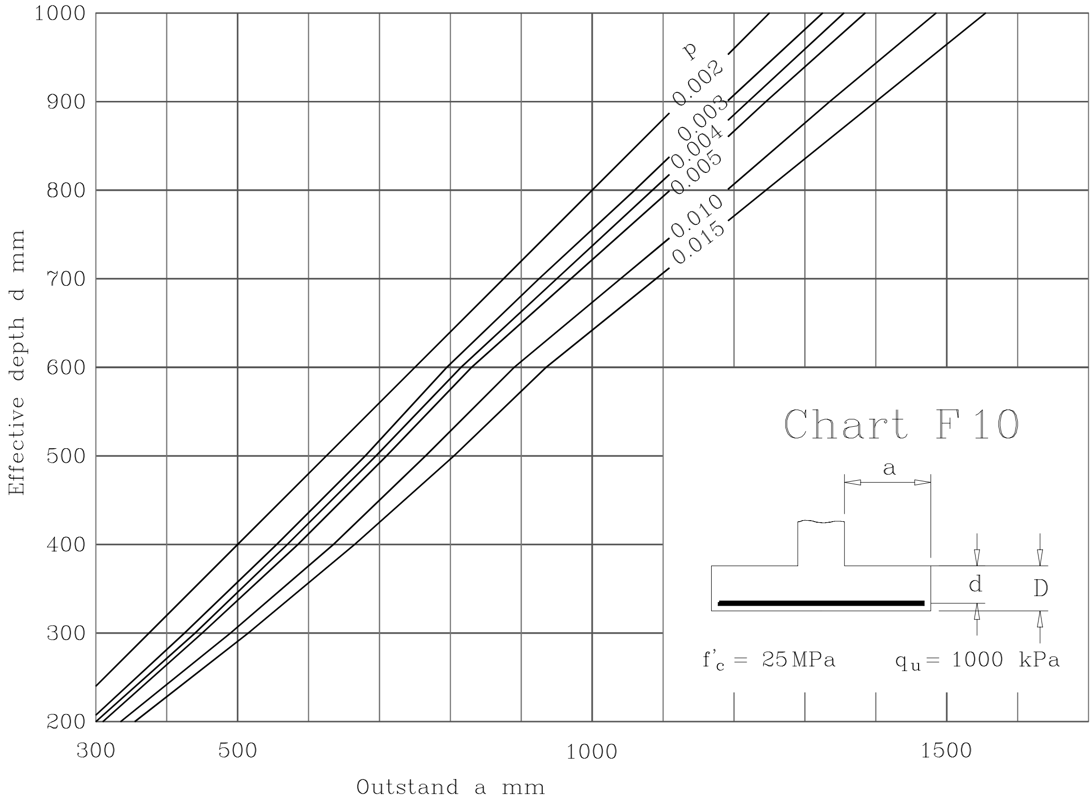


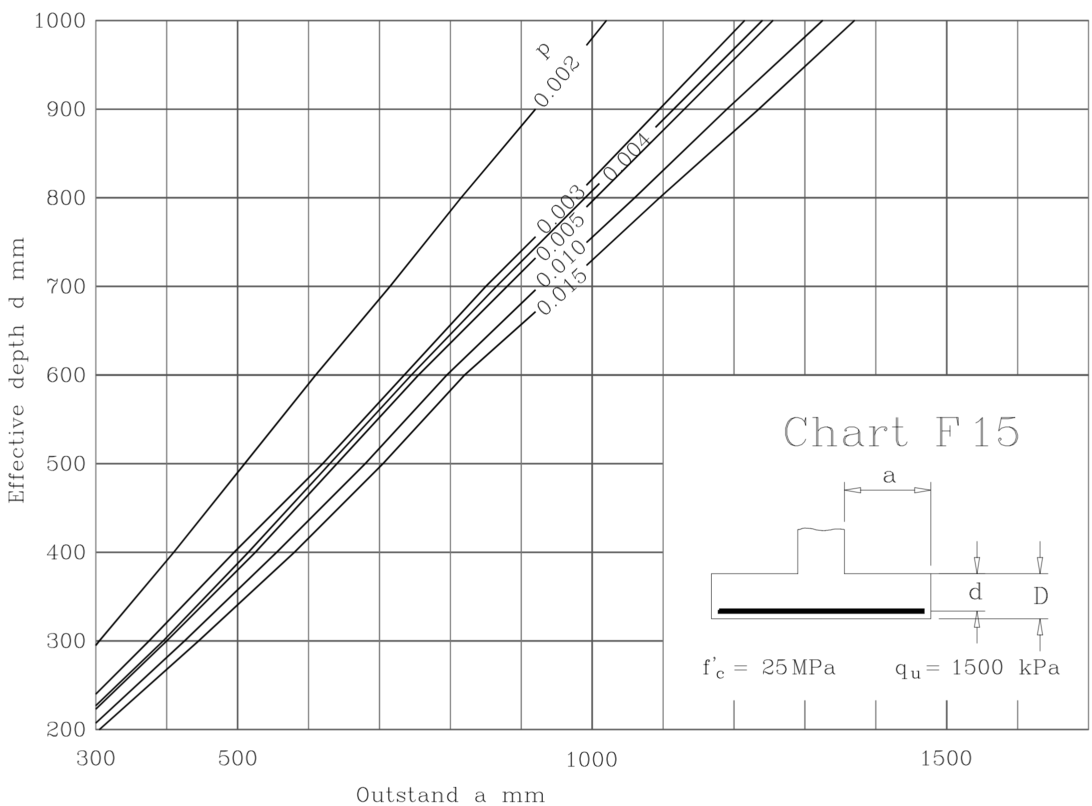
Page 323 Replace charts F6 and F8



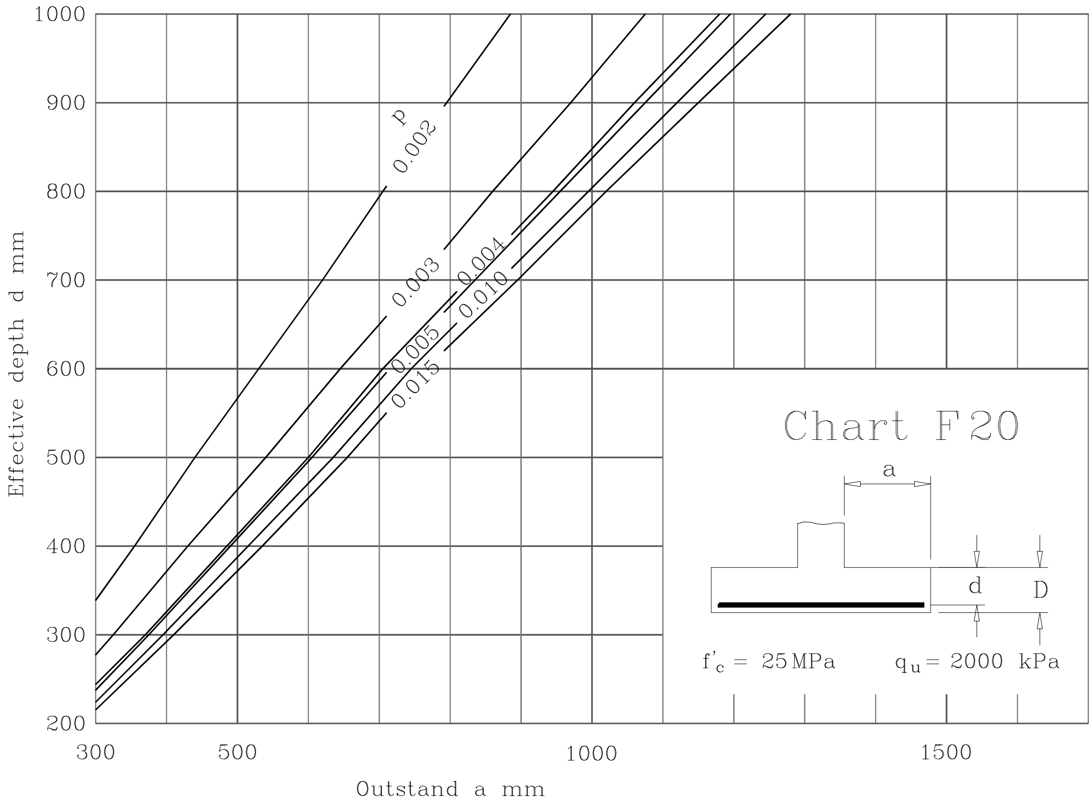


Page 324 Replace charts F10 and F15

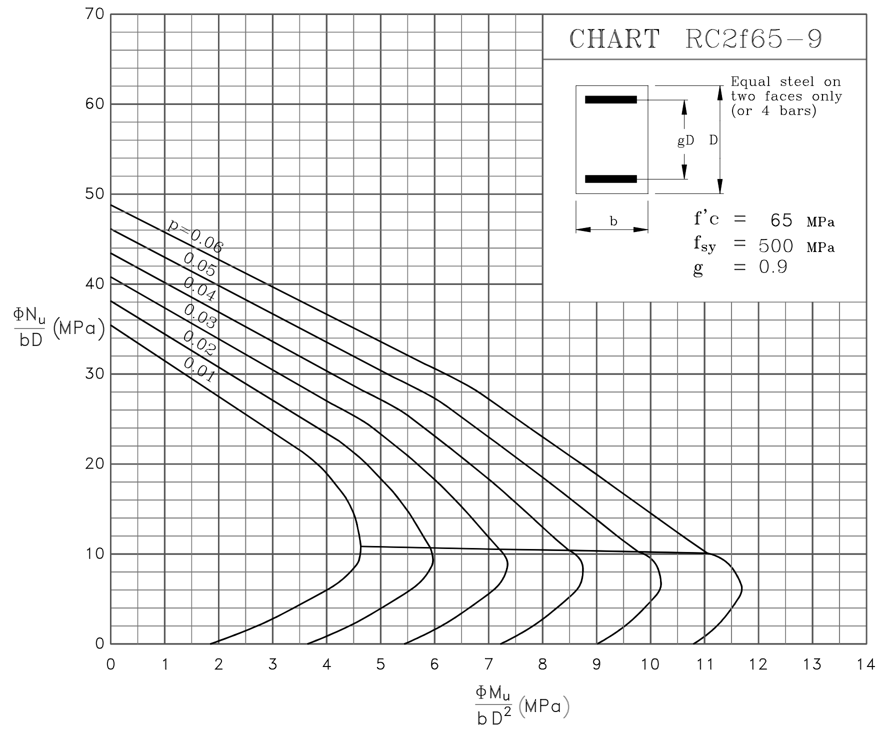




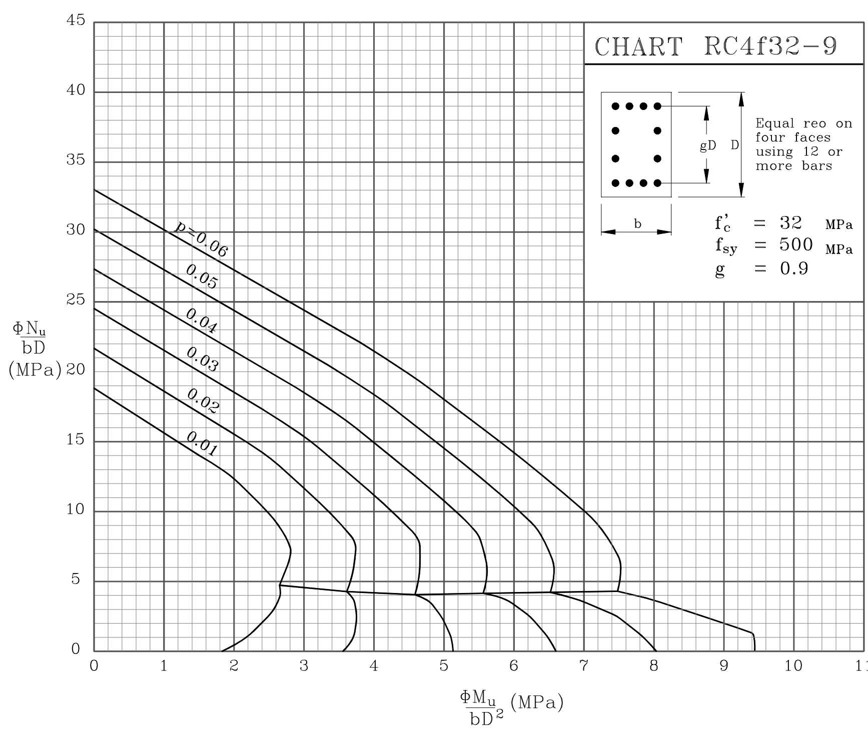
Page 325 Replace chart F20



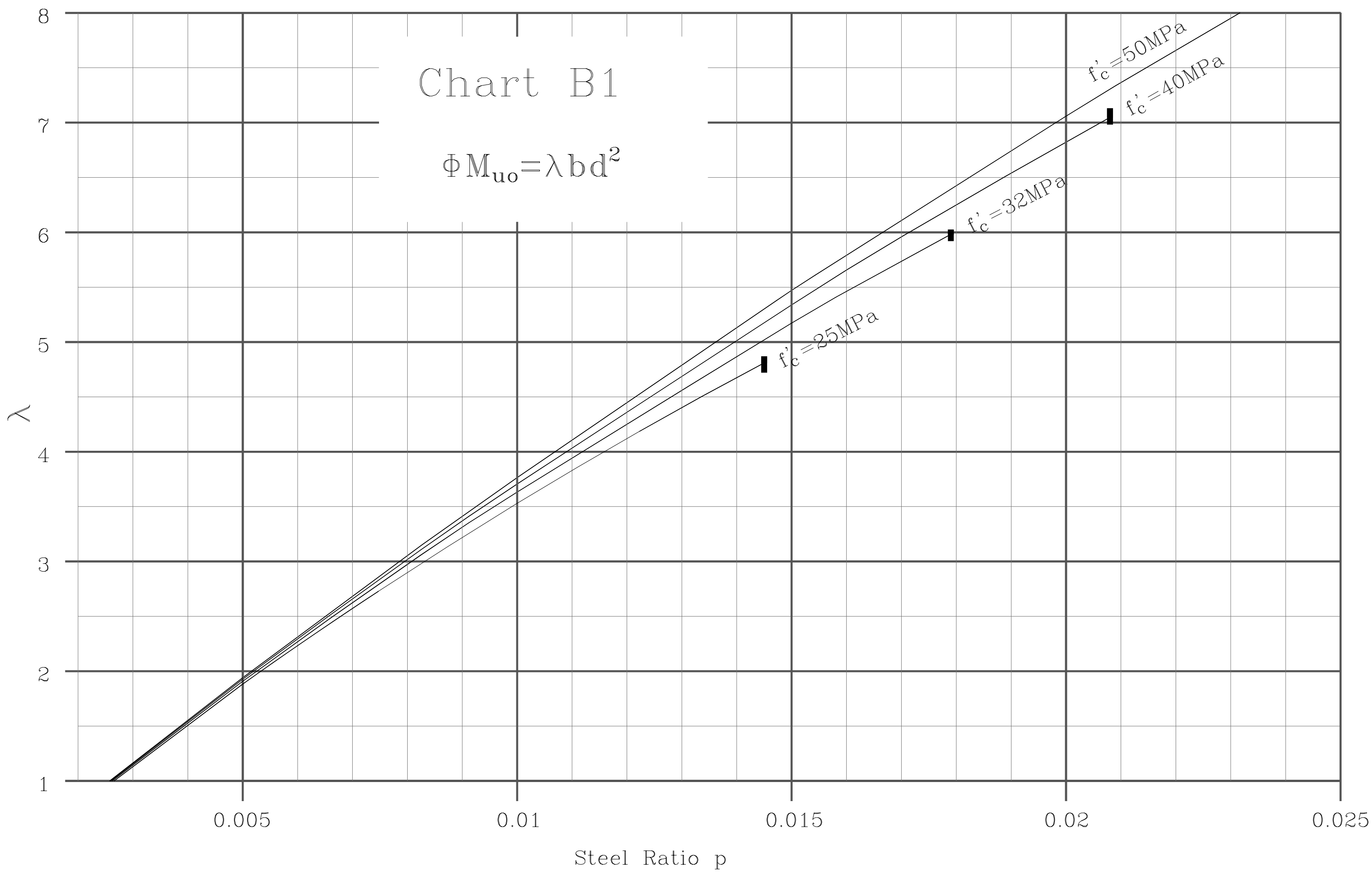
Page 339 replace Chart RC4f65-9 with Chart RC2f65-9 below



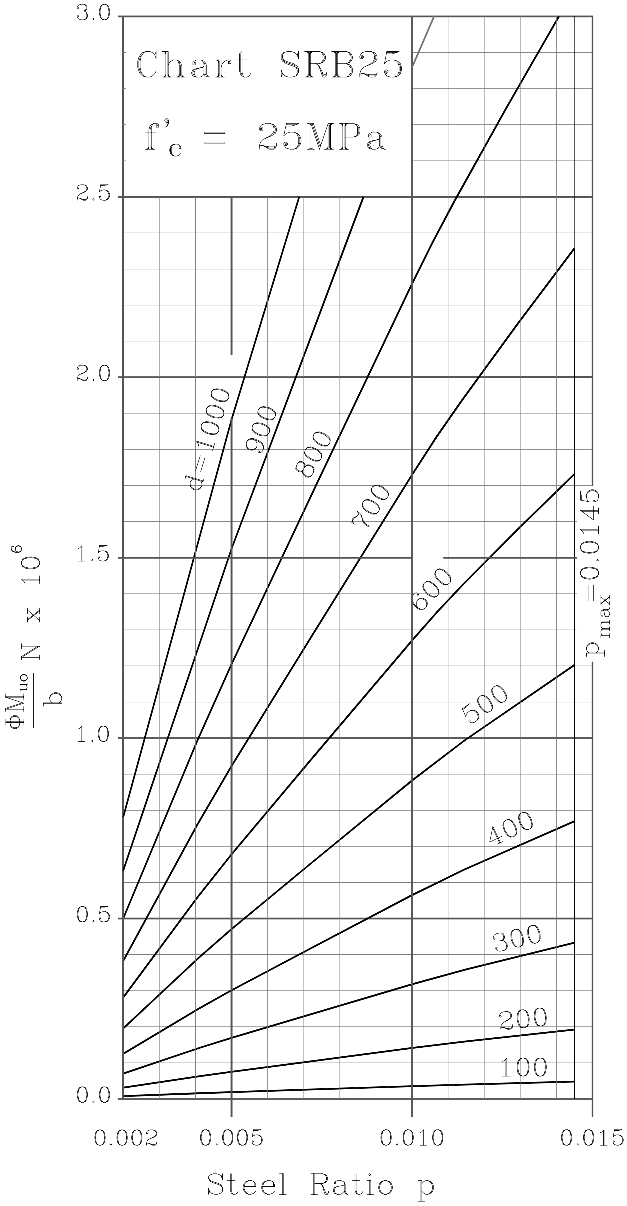
Page 349 replace Chart RC2f32-9 with RC4f32-9 below



Design Section Following Page 283 – Enlarged Chart B1 may be added



Optional – Charts SRB25 – SRB50 on pages 290-293 have been redrawn for relevant steel ratios



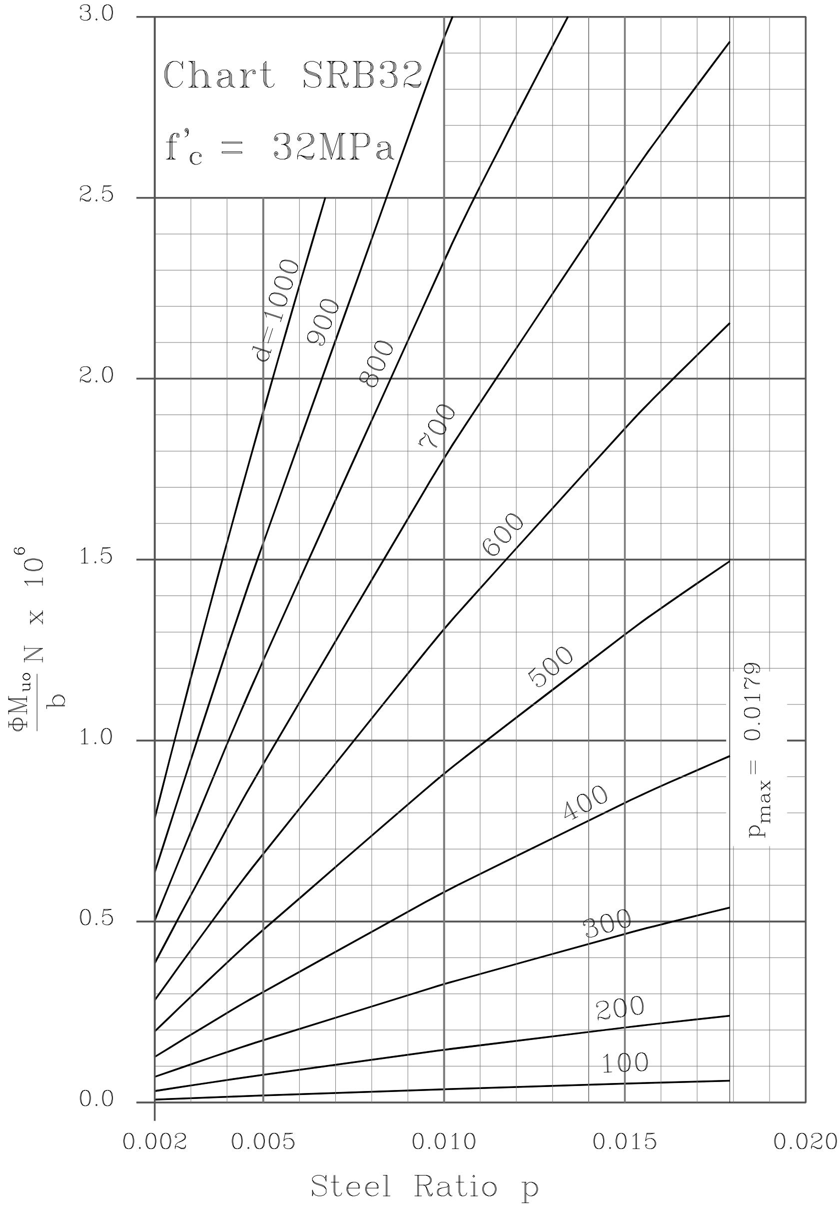


Chart SRB32
f'c = 32MPa
d=1000
900
800
700
600
500
400
300
200
100
pmax = 0.0179
ΦMuo/b N x 10^6
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.002
0.005
0.010
0.015
0.020
Steel Ratio p

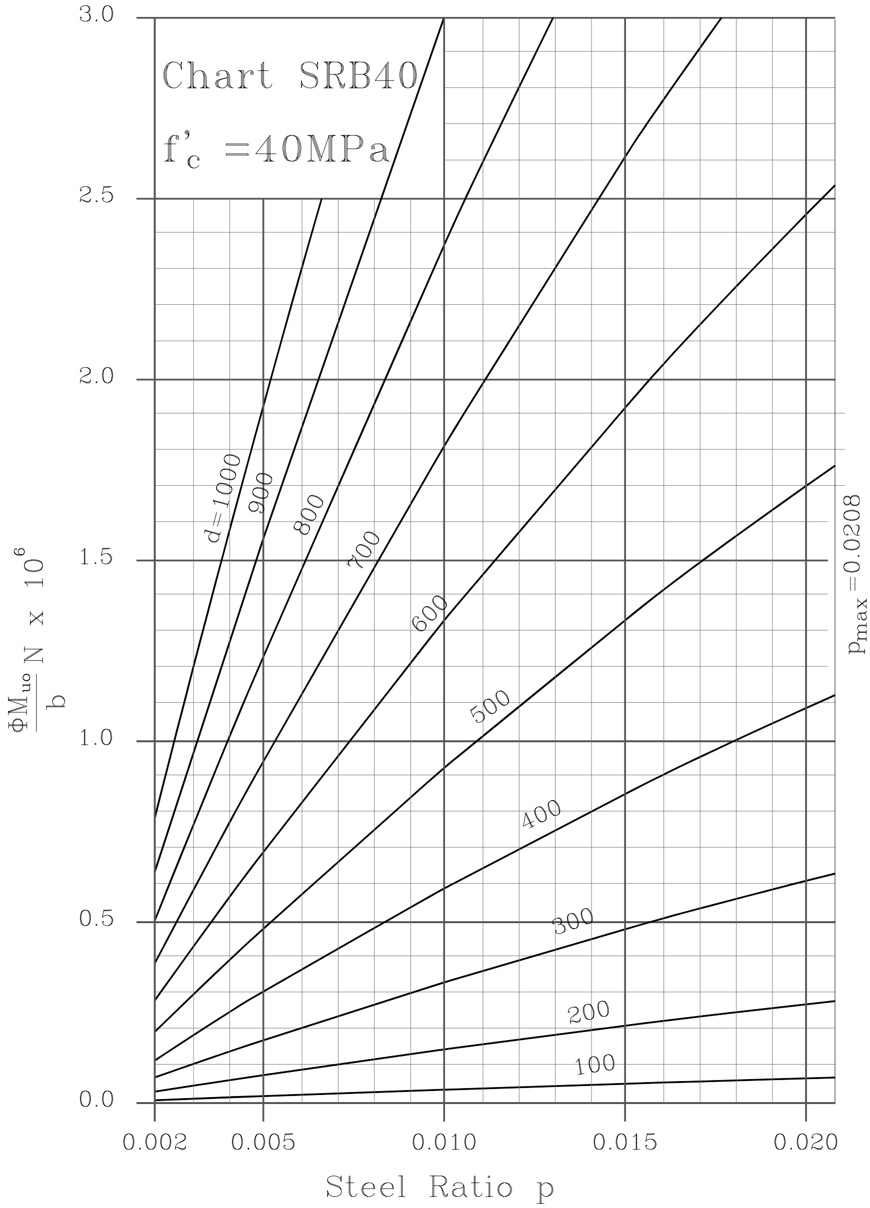


Chart SRB40
f'c =40MPa
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ΦMuo/b N x 10^6
d=1000
900
800
700
600
500
400
300
200
100
pmax =0.0208
0.002
0.005
0.010
0.015
0.020
Steel Ratio p

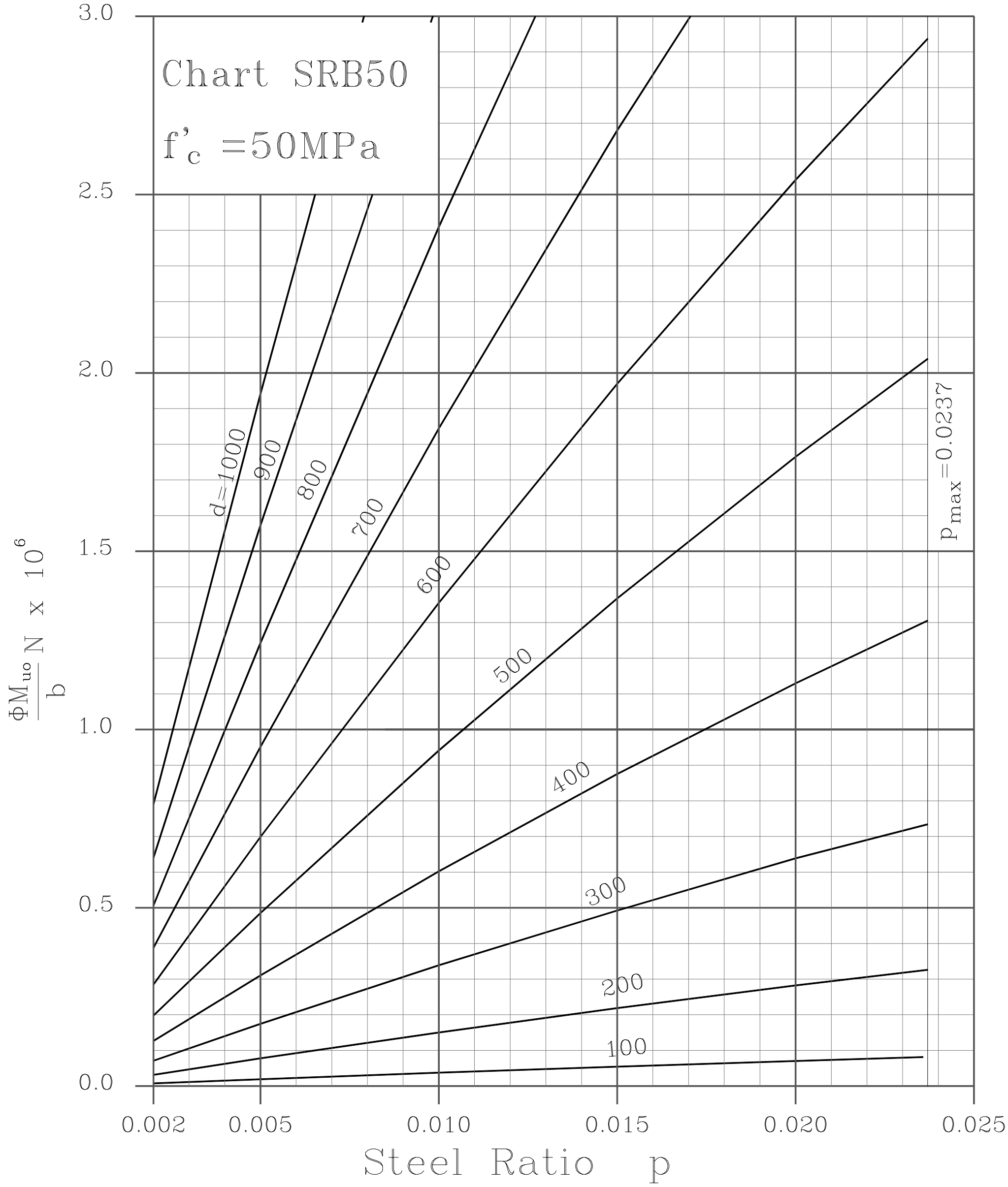


Chart SRB50
f'c =50MPa
3.0
2.5
2.0
1.5
1.0
0.5
0.0
ΦMuo/b N x 10^6
d=1000
900
800
700
600
500
400
300
200
100
pmax=0.0237
0.002
0.005
0.010
0.015
0.020
0.025
Steel Ratio p

Errata Slip

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Corrections exists for the pages listed below. Please refer to page listings of each amendment.

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A PDF of all corrections is available from UNSW Press; please contact enquiries@newsouthpublishing.com

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