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DEVELOPMENTS IN GEOTECHNICAL ENGINEERING 34B

PRACTICAL PROBLEMS IN SOIL MECHANICS AND FOUNDATION ENGINEERING, 2

WALL AND FOUNDATION CALCULATIONS,
SLOPE STABILITY

GUY SANGLERAT
GILBERT OLIVARI
BERNARD CAMBOU

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INTRODUCTION

Guy Sanglerat has taught geotechnical engineering at the "Ecole Centrale de Lyon" since 1967. This discipline was introduced there by Jean Costet. Since 1968 and 1970, respectively, Gilbert Olivari and Bernard Cambou actively assisted in this responsibility. They directed laboratory work, outside studies and led special study groups.

In order to master any scientific discipline, it is necessary to apply its theoretical principles to practice and to readily solve its problems. This holds true also for theoretical soil mechanics when applied to geotechnical engineering.

From Costet's and Sanglerat's experiences with their previously published textbooks in geotechnical engineering, which contain example-problems and answers, it became evident that one element was still missing in conveying the understanding of the subject matter to the solution of practical problems: problems apparently needed detailed, step-by-step solutions.

For this reason and at the request of many of their students, Sanglerat, Olivari and Cambou decided to publish problems. Over the years since 1967 the problems in this text have been given to students of the "Ecole Centrale de Lyon" and since 1976 to special geotechnical engineering study groups of the Public Works Department of the National School at Vaulx-en-Velin, where Gilbert Olivari was assigned to teach soil mechanics.

In order to assist the reader of these volumes, it was decided to categorize problems by degrees of solution difficulty. Therefore, easy problems are preceded by one star (★), those considered most difficult by 4 stars (★★★★). Depending on his degree of interest, the reader may choose the types of problems he wishes to solve.

The authors direct the problems not only to students but also to the practicing Civil Engineer and to others who, on occasion, need to solve geotechnical engineering problems. To all, this work offers an easy reference, provided that similarities of actual conditions can be found in one or more of the solutions prescribed herein.

Mainly, the S.I. (Système International) units have been used. But, since practice cannot be ignored, it was deemed necessary to incorporate other widely accepted units. Thus the C.G.S. and English units (inch, foot, pounds per cubic foot, etc.) have been included because a large quantity of literature is based on these units.

The authors are grateful to Mr. Jean Kerisel, past president of the International Society for Soil Mechanics and Foundation Engineering, for having

written the Preface to the French edition and allowing the authors to include one of the problems given his students while Professor of Soil Mechanics at the "Ecole Nationale de Ponts et Chaussées" in Paris. Their gratitude also goes to Victor F.B. de Mello, President of the International Society for Soil Mechanics, who had the kindness to preface the English edition.

The first problems were originally prepared by Jean Costet for the course in soil mechanics which he introduced in Lyon.

Thanks are also due to Jean-Claude Rouault of "Air Liquide" and Henri Vidal of "Reinforced Earth" and also to our Brazilian friend Lucien Decourt for contributing problems, and to Thierry Sanglerat for proofreading manuscripts and printed proofs.

NOTATIONS

The following general notations appear in the problems:

A	:	Skempton's second coefficient (sometimes A refers also to cross-sectional area).
A_f	:	value of A at failure
B	:	footing width (sometimes B refers also to Skempton's first coefficient).
c	:	soil cohesion (undifferentiated)
c'	:	effective cohesion
c''	:	reduced cohesion (slope stability)
c_u	:	undrained cohesion
c_{cu}	:	consolidated-undrained cohesion
C_c	:	compression index
C_u	:	uniformity coefficient, defined as d_{60}/d_{10}
c_v	:	coefficient of consolidation
d	:	soil particle diameter (sometimes: horizontal distance between adjacent, similar structures, as in the case of sub-surface drains)
d_y	:	equivalent diameter of sieve openings in grain-size distribution
D	:	depth to bottom of footings (sometimes D refers to depth to hard layer under the toe of a slope).
e	:	void ratio (sometimes: e refers to eccentricity of a concentrated force acting on a footing)
e_{max}, e_{min}	:	maximum and minimum void ratios
E	:	Young's modulus
E_p	:	pressuremeter modulus
FR	:	friction ratio (static penetrometer test)
g	:	acceleration due to gravity (gravie)
G	:	shear modulus
h	:	hydraulic head
H	:	soil layer thickness (or normal cohesion: $H = c \cot \varphi$)
i	:	hydraulic gradient
i_c	:	critical hydraulic gradient
IP	:	plasticity index
k	:	coefficient of permeability
$k_{a\gamma}$, k_{aq}, k_{ac}	:	active earth pressure coefficients due to overburden, surcharge and cohesion, respectively

$k_{p\gamma}$, k_{pq}, k_{pc}	: passive earth pressure coefficients
$K_{a\gamma}$, K_{aq}, K_{ac}	: active earth pressures perpendicular to a given plane
$K_{p\gamma}$, K_{pq}, K_{pc}	: passive earth pressures perpendicular to a given plane
k_s	: soil reaction modulus
K	: bulk modulus (K_s of soil structure, K_w of water)
K_0	: coefficient of earth pressure at rest
l	: width of an excavation
L	: length of an excavation
m_v	: coefficient of compressibility
M_m	: driving moment
M_R	: resisting moment
M	: bending moment
n	: porosity
n_D	: stability coefficient (slope stability problems)
N_γ, N_q, N_c	: bearing capacity factors for foundation design
P	: concentrated (point) load
p_l	: limit pressure (pressuremeter test)
p_f	: creep pressure (pressuremeter test)
q	: uniformly distributed load (or percolation discharge)
Q	: discharge (or load acting upon a footing)
Q_f	: friction force of pile shaft (total skin friction force)
Q_p	: end-bearing force of pile (total)
q_d	: ultimate bearing capacity of soil under a footing or pile
q_{ad}	: allowable bearing capacity of a footing or pile
R	: radius of a circular footing (or radius of drawdown of a well)
RD	: relative density $(e_{max} - e)/(e_{max} - e_{min})$
r	: well radius (or polar radius in polar coordinate system)
R_p or q_c	: end-bearing on the area of a static penetrometer (cone resistance)
s	: curvilinear abscissa (or cross-sectional area of a thin wall tube, or settlement)
S	: cross-sectional area of a mold or a sample
$S.G.$	: specific gravity
S_t	: degree of saturation
t	: time
T	: shear
T_v	: time factor
u	: porewater pressure
U	: degree of consolidation (or resultant of pore-water pressure forces)
v	: rate of percolation
V	: volume
W	: weight of a given soil volume

w	: water content or settlement
w_l, w_p	: liquid limit, plastic limit
x, y, z	: Cartesian coordinates, with Oz usually considered the vertical, downward axis
α	: angle between orientations, usually reserved for the angle between two soil faces. Also used to classify soils for the purpose of their compressibility from static cone penetrometer test data C.P.T.
β	: slope of the surface of backfill behind a retaining wall (angle of slope)
γ	: unit weight of soil (unspecified)
γ_s	: soil particles unit weight (specific gravity)
γ_{sat}	: saturated unit weight of soil
γ_h	: wet unit weight of soil
γ_w	: unit weight of water = 9.81 kN/m^3.
γ_d	: dry unit weight of soil
γ'	: effective unit weight of soil
γ_{xy}, γ_{yz}, γ_{zx}	: shear strain, twice the angular deformation in a rectangular, 3-dimensional system
δ	: angle of friction between soil and retaining wall surface in passive or active earth pressure problems, or the angle of inclination of a point load acting on a footing
η	: dynamic viscosity of water
ϵ_x, ϵ_y, ϵ_z	: axial strains in a rectangular, 3-dimensional system
ϵ_1, ϵ_2, ϵ_3	: principal stress
ϵ_v	: volumetric strain
θ	: angle of radius in polar coordinates system (sometimes: temperature)
ν	: Poisson's ratio
σ'	: effective normal stress
σ	: total normal stress
σ_x, σ_y, σ_z	: normal stresses in a rectangular, 3-dimensional system
σ_1, σ_2, σ_3	: major principal stresses
σ_m	: average stress
τ	: shear stress
τ_m	: average shear stress
τ_{xy}, τ_{yz}, τ_{zx}	: shear stresses in a rectangular, 3-dimensional system
φ	: angle of internal friction (undefined)
φ'	: effective angle of internal friction
φ''	: reduced, effective angle of internal friction (slope-stability analyses)
φ_{cu}	: angle of internal friction, consolidated, undrained
λ	: slope of a wall from the vertical
ω_β, ω_δ	: auxiliary angles defined by $\sin \omega_\beta = \sin \beta / \sin \varphi$ and $\sin \omega_\delta = \sin \delta / \sin \varphi$

π	:	3.1416
ρ	:	distance from origin to a point in polar coordinate system
ψ	:	angle of major principal stress with radius vector (plasticity problems)

ENGINEERING UNITS

It is presently required that all scientific and technical publications resort to the S.I. units (Système International) and their multipliers (deca, hecta, kilo, Mega, Giga). Geotechnical engineering units follow this requirement and most of the problems treated here are in the S.I. system.

Fundamental S.I. units:

length	: meter (m)
mass	: kilogram (kg)
time	: second (s)

S.I. Units derived from the above

surface	: square meter (m^2)
volume	: cubic meter (m^3)
specific mass	: kilogram per cubic meter (kg/m^3)
velocity (permeability)	: meter per second (m/s)
acceleration	: meter per second per second (m/s^2)
discharge	: cubic meter per second (m^3/s)
force (weight)	: Newton (N)
unit weight	: Newton per cubic meter (N/m^3)
pressure, stress	: Pascal (Pa) $1\ Pa = 1\ N/m^2$
work (energy)	: Joule (J) $1\ J = 1\ N \times m$
viscosity	: Pascal-second* $Pa \times s$

However, in practice, other units are encountered frequently. Table A presents correlations between the S.I. and two other unit systems encountered worldwide. This is to familiarize the readers of any publication with the units used therein. For that purpose also, British units have been adopted for some of the presented problems.

Force (pressure) conversions

Force units	: see Table B
Pressure units	: see Table C
Weight unit	: $1\ kN/m^3 = 0.102\ tf/m^3$

---

*This unit used to be called the "poiseuille", but it has not been officially adopted.

TABLE A
Correlations between most common unit systems

	Système International (S.I.)		Meter-Kilogram system (M.K.)		Centimeter-Gram-Second system (C.G.S.)	
	units	common multiples	units	common multiples	units	common multiples
Length	meter (m)	km	meter (m)	km	cm	m
Mass	kilogram (kg)	tonne (t)	gravie*	—	g	—
Time	second (s)	—	second (s)	—	s	—
Force	Newton (N)	kN	kilogram force (kgf)	tf	dyne	—
Pressure (stress)	Pascal (Pa)	kPa MPa	kilogram force per square meter (kgf/m^2)	t/m^2 kg/cm^2	barye	bar (10^6 baryes)
Work (energy)	Joule (J)	kJ	kilogram meter (kgm)	tf.m	erg	Joule (10^7 ergs)

*Note that 1 gravie = 9.81 kg (in most problems rounded off to 10).

The unit weight of water is: $\gamma_w = 9.81\,kN/m^3$ but it is often rounded off to: $\gamma_w = 10\,kN/m^3$.

Energy units:

1 Joule $= 0.102$ kg.m $= 1.02 \times 10^{-4}$ t.m
1 kgf.m $= 9.81$ Joules
1 tf.m $= 9.81 \times 10^3$ Joules

Dynamic viscosity units:

1 Pascal-second (Pa.s) = 10 poises (Po).

British units:

1 inch	= 0.025 4 m	1 m = 39.370 in.
1 foot	= 0.304 8 m	1 m = 3.280 8 foot
1 square inch	= 6.451 6 cm^2	1 cm^2 = 0.155 sq. in.
1 square foot	= 144 sq. in. = 0.092 9 m^2	
1 m^2	= 10.764 sq. ft.	
1 cubic inch	= 16.387 cm^3	1 cm^3 = 0.061 0 cu. in.
1 cubic foot	= 1728 cu. in. = 0.028 317 m^3	
1 m^3	= 35.314 cu. ft.	
1 pound (lb)	= 4.449 7 Newton = 0.453 59 kgf	
1 Newton	= 0.225 lb = 0.112 4 × 10^{-3} sh. ton. (1 sh. ton. = 2 kip)	
	= 1.003 × 10^{-4} ton.	
1 lb/cu. in.	= 270.27 kN/m^3	

1 lb/cu. ft.	$= 0.156\,99$ kN/m^3
1 kN/m^3	$= 3.7 \times 10^{-3}$ lb/cu. in. = 6.37 lb/cu. ft.
1 lb/sq. in. (p.s.i.)	$= 6.896\,55 \times 10^3$ Pa
1 Pascal	$= 14.50 \times 10^{-5}$ p.s.i.
100 kPa	= 1 bar = 14.50 p.s.i.

TABLE B

Force units conversions

Value of ↓ / expressed in →	Newton	Decanewton	Kilonewton	Kilogram force	Tonne force	Dyne
Newton	1	10^{-1}	10^{-3}	1.02×10^{-1}	1.02×10^{-4}	10^{5}
Decanewton	10	1	10^{-2}	1.02	1.02×10^{-3}	10^{6}
Kilonewton	10^{3}	10^{2}	1	1.02×10^{2}	1.02×10^{-1}	10^{8}
Kilogram force	9.81	9.81×10^{-1}	9.81×10^{-3}	1	10^{-3}	9.81×10^{5}
Tonne force	9.81×10^{3}	9.81×10^{2}	9.81	10^{3}	1	9.81×10^{8}
Dyne	10^{-5}	10^{-6}	10^{-8}	1.02×10^{-6}	1.02×10^{-9}	1

TABLE C

Pressure units conversions

Value of ↓ / expressed in →	Pa	kPa	bar	hbar	barye	kg/cm^2	kg/mm^2	t/m^2	cm of water	atm.
Pascal	1	10^{-3}	10^{-5}	10^{-7}	10	1.02×10^{-5}	1.02×10^{-7}	1.02×10^{-4}	1.02×10^{-2}	9.869×10^{-6}
Kilopascal	10^{3}	1	10^{-2}	10^{-4}	10^{4}	1.02×10^{-2}	1.02×10^{-4}	1.02×10^{-1}	10.2	9.869×10^{-3}
Bar	10^{5}	10^{2}	1	10^{-2}	10^{6}	1.02	1.02×10^{-2}	10.2	1.02×10^{3}	0.9869
Hectobar	10^{7}	10^{4}	10^{2}	1	10^{8}	1.02×10^{2}	1.02	1.02×10^{3}	1.02×10^{5}	9.869×10^{1}
Barye	0.1	10^{-4}	10^{-6}	10^{-8}	1	1.02×10^{-6}	1.02×10^{-8}	1.02×10^{-5}	1.02×10^{-3}	9.869×10^{-7}
kg/cm^2	9.81×10^{4}	9.81×10^{1}	0.981	9.81×10^{-3}	9.81×10^{5}	1	10^{-2}	10	10^{3}	0.9681
kg/mm^2	9.81×10^{6}	9.81×10^{-3}	9.81×10^{1}	0.981	9.81×10^{7}	10^{2}	1	10^{3}	10^{5}	9.681×10^{1}
t/m^2	9.81×10^{3}	9.81	9.81×10^{-2}	9.81×10^{-4}	9.81×10^{4}	0.1	10^{-3}	1	10^{2}	9.681×10^{-2}
cm of water	9.81×10^{1}	9.81×10^{-2}	9.81×10^{-4}	9.81×10^{-6}	9.81×10^{2}	10^{-3}	10^{-5}	10^{-2}	1	9.681×10^{-4}
Atmosphere	1.0133×10^{5}	1.0133×10^{2}	1.0133	1.0133×10^{-2}	1.0133×10^{6}	1.033	1.033×10^{-2}	1.033×10^{1}	1.033×10^{3}	1

Chapter 7

RETAINING WALLS

★*Problem 7.1* Earth pressures on a vertical wall, horizontal backfill, above the water table

A 4 m high wall serves as a retaining-wall for a mass of horizontal flattened dry sand (Fig. 7.1). The dry sand's unit weight is 18.3 kN/m^3 and its internal angle of friction is 36°.

What is the magnitude of the earth force P on a 1 m wide wall slice, assuming that the wall does not deflect? Calculate also the earth force P_1 if the wall deflects sufficiently to generate active (Rankine) pressure conditions in the backfill. Assume that the back face of the wall is frictionless.

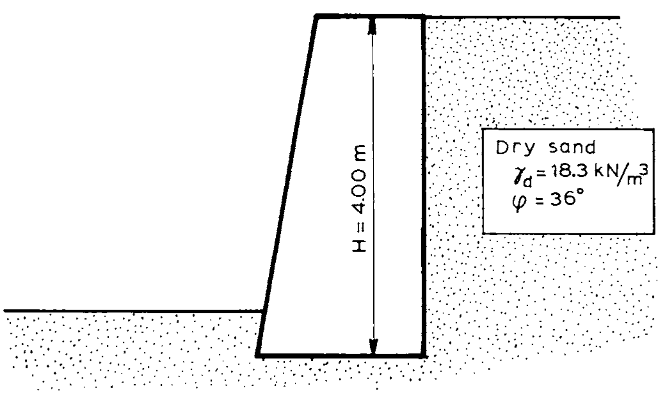


Fig. 7.1.

Solution

If no wall deflection occurs, the earth pressure *at rest condition* prevails, i.e. that pressure P_0, then acting on the wall, may be represented by the Mohr's circle equilibrium condition comprised between the Coulomb's envelopes (Fig. 7.2). In general, for a sand: $0.33 \leqslant K_0 \leqslant 0.7$. (cf. 6.1.4 in Costet-Sanglerat, where the values of K_0 are calculated from empirical formulas.) The pressure distribution on the inner wall face is triangular and because it is assumed that the face is frictionless, the pressures act perpendicular to the wall.

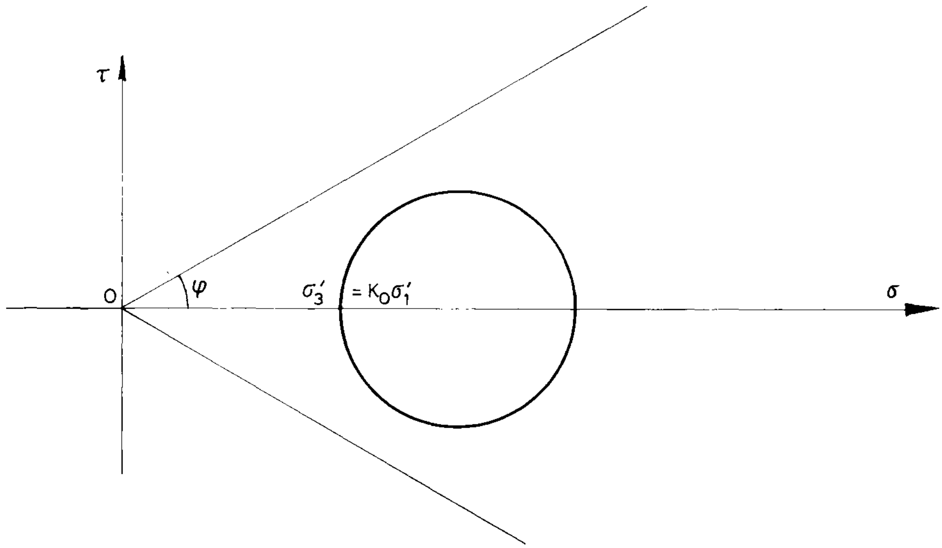


Fig. 7.2.

So, for a 1 m wide wall slice, we have:

$P_0 = \frac{1}{2} K_0 \gamma_d H^2 b$

where $b = 1.00$ m and $\gamma_d = 18.3$ kN/m^3.

K_0 calculated by the formula of Jaky gives: $K_0 = 1 - \sin\varphi'$.

For $\varphi' = 36°$: $\sin\varphi' \simeq 0.588$ and $K_0 \simeq 0.412$.

Then, $P_0 = 0.5 \times 0.412 \times 18.3 \times \overline{4.00}^2 = 60.3$ kN (per meter of wall length).

If we assume that $K_0 = (1 - \sin\varphi')/\cos\varphi'$, as proposed by some authors, $\cos\varphi' = 0.809$ and $K_0 = 0.51$. So, $P_0 = 74.6$ kN (per meter length of wall).

We finally get: 60 kN $< P_0 <$ 74 kN.

Let us now assume that the wall will sufficiently deflect at the top to mobilize a Rankine active pressure (a displacement of the top of the wall of about 1/1000 of the wall height, therefore about 4 mm, as generally occurs for unrestrained walls).

In this case: $P_1 = \tan^2\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)\gamma_d \cdot \frac{H^2}{2} \cdot b$

with $b = 1.00$ m and $\gamma_d = 18.3$ kN/m^3, $\pi/4 - \varphi/2 = 27°$, $\tan 27° = 0.509\,5$, and: $P_1 = 0.5 \times \overline{0.509\,5}^2 \times 18.3 \times \overline{4.00}^2 \simeq 38$ kN (per meter of wall length).

Summary of answers

60 kN/m $< P_0 <$ 74 kN/m; $P_1 = 38$ kN/m.

★★*Problem 7.2* Earth pressure considering the water table on a vertical wall

Assuming the givens of the preceding problem, what is the total resultant earth pressure acting on the wall and its location with respect to the base of the wall, if there is a water table at 1 m below the backfill grade (assume a sand porosity of 0.31) (see Fig. 7.3).

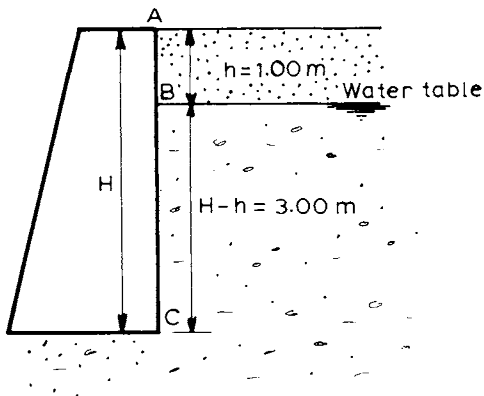


Fig. 7.3.

Solution

From the preceding problem, we have:

$$k_{a\gamma} = \tan^2(27^\circ) = 0.2596, \quad \text{say } 0.26.$$

The buoyant weight of the sand is:

$$\gamma' = \gamma_{sat} - \gamma_w = \gamma_d + n\gamma_w - \gamma_w = \gamma_d - (1-n)\gamma_w, \qquad \text{or:}$$

$$\gamma' = 18.3 - (1-0.31) \times 10.0 = 11.4\,\text{kN/m}^3.$$

The distribution of the stresses behind the wall is (see Figs. 7.3 and 7.5):

On AB: the distribution is triangular and we have:

$$\sigma_A = 0; \qquad \sigma_B = k_{a\gamma} \times \gamma_d \times h = 0.26 \times 18.3 \times 1.00 = 4.76\,\text{kN/m}^2.$$

On BC: the distribution is still triangular, but at B the slope of the hypotenuse changes: here the buoyant weight and the hydrostatic water pressure must be taken into account, as well as the weight of dry sand, to be considered as a uniform surcharge. Therefore:

— pressure due to the buoyant weight of the sand:

$$\sigma'_{1B} = 0; \qquad \sigma'_{1C} = k_{a\gamma} \times \gamma' \times (H-h) = 0.26 \times 11.4 \times 3.00 = 8.89\,\text{kN/m}^2$$

— pressure due to the uniform discharge of the sand (rectangular distribution):

$$\sigma'_{2B} = \sigma'_{2C} = k_q \times q = k_q \times h \times \gamma_d \qquad \text{where: } k_q \simeq \frac{k_{a\gamma}}{\cos(\beta - \lambda)} \quad \text{(Fig. 7.4)}$$

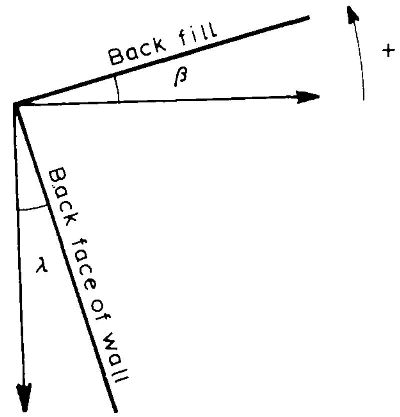


Fig. 7.4.

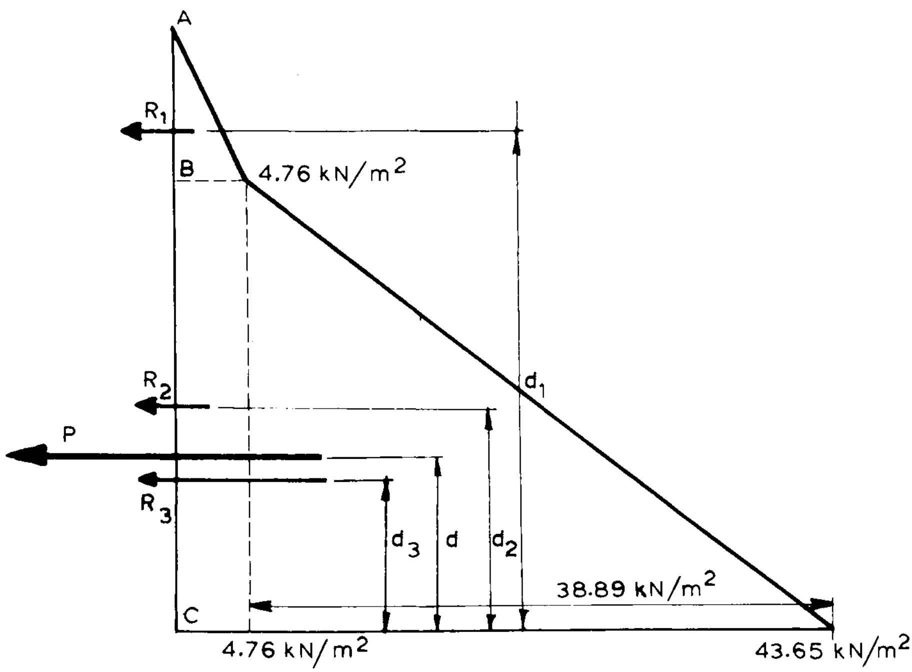


Fig. 7.5.

This equation derived from Coulomb's hypothesis is also valid for Rankine-conditions (6.24 in Costet-Sanglerat); but:

$\beta = \lambda = 0, \quad k_q = k_{a\gamma},$

so: $\sigma_{2B} = \sigma_{2C} = k_{a\gamma} \times h \times \gamma_d = \sigma_B$ computed previously as $= 4.76 \text{ kN/m}^2$
— hydrostatic pressure (triangular distribution):

$\sigma_{3B} = 0; \qquad \sigma_{3C} = (H-h)\gamma_w = 30\,kN/m^2.$

So we end up with the diagram shown in Fig. 7.5: the total force acting on the wall is the resultant of the forces R_1, R_2 and R_3 of that Figure, and we have:

$R_1 = (1/2) \times 4.76 \times 1.00 \times 1.00 = 2.40\,kN$ per meter of wall located at $3.00 + 0.33 = 3.33\,m$ from C (1/3 of AB).

$R_2 = 3.00 \times 4.76 \times 1.00 = 14.3\,kN$ (per meter of wall) acting at 1.5 m distance of C (middle of BC).

$R_3 = (1/2) \times 38.9 \times 3.00 \times 1.00 = 58.3\,kN$ (per meter of wall) acting at 1.00 m distance of C (lower 1/3 of BC).

The resultant force thus is: $P = R_1 + R_2 + R_3 \simeq 75\,kN$ and this force acts at such a distance d from C that:

$$Pd = R_1 d_1 + R_2 d_2 + R_3 d_3,$$

$$d = \frac{2.4 \times 3.33 + 14.3 \times 1.50 + 58.3 \times 1.00}{75.0} \simeq 1.17\,m$$

Summary of answers

$P = 75\,kN$ per meter of wall, $\quad d = 1.17\,m$.

★★*Problem 7.3* Retaining wall with horizontal backfill; overturning stability and sliding stability

Suppose you are asked to determine the stability of the quay wall shown on Fig. 7.6. (It is assumed that the steps of the wall are comparable to a straight line AB because the weight of the soil is not significantly different from that of the concrete in the small triangular areas.)

The base of the foundation's upper part is at the level of the water table and that of the natural soil, in which the footing, completely submerged, is embedded. The retaining-wall supports the soil above the water table.

Assume the following values:

Concrete : unit weight $23\,kN/m^3$

Fill : unit weight $18\,kN/m^3$
internal angle of friction $\varphi_1 = 30°$
cohesion $c = 0$
earth pressure coefficients on AB $(\delta = \varphi$, and $\lambda = 25°)$
$k_{a\gamma} = 0.474 \qquad k_{aq} = 0.522$
surcharge on fill, $q = 10\,kPa$.

Natural soil: buoyant unit weight $11\,kN/m^3$
internal angle of friction $\varphi_2 = 25°$
cohesion $c = 0$
earth pressure coefficients on BC $(\delta = \frac{2}{3}\varphi_2)$

$k_{a\gamma} = k_{aq} = 0.364$

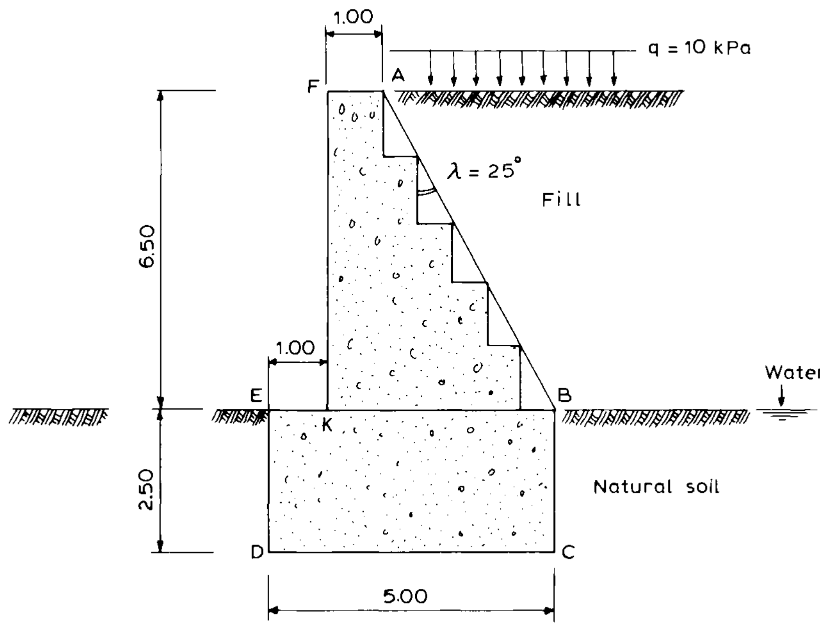


Fig. 7.6

Wall: $h_1 = 6.50\,m \quad h_2 = 2.50\,m$
$FA = 1\ m,\ KB = 4\ m,\ DC = 5\ m.$

As a security precaution, ignore the passive earth pressure on plane ED of the foundation.

Find:

(1) The eccentricity of the resultant force acting on base CD. Is there tension?

(2) The maximum bearing pressure on the foundation soil.

(3) The safety factor against overturning.

(4) The safety factor against lateral sliding (assume the friction coefficient between the bottom of the foundation and the soil is tan φ_2).

Important remark:

As in the Costet-Sanglerat text, $k_{a\gamma}$ and k_{aq} are the coefficients of inclined earth pressure and $K_{a\gamma}$ and K_{aq} the perpendicular acting coefficients ($K_{a\gamma} = k_{a\gamma} \cos \delta$).

Solution

(1) *Calculation of the eccentricity of the resultant acting forces.*

The exterior forces acting on the retaining-wall are:

— the weight of the wall (**W**):

— the hydrostatic force acting on the submerged portion of the wall **Π**;

— the active earth force **P** increased by the value of the lateral force **Q** due to the surcharge imposed by the fill;
— the passive earth force **B** acting on plane *ED* of the foundation;
— the foundation soil reaction **R**.

For the wall to be in equilibrium, the resultant of all these forces must be zero which allows the calculation of the value of the reaction **R**.

For the sake of safety, it is general practice to ignore the passive force **B** acting on the side of the footing. There are two reasons for this. Firstly, the wall displacement is generally not sufficiently large to actually mobilize the passive condition: a displacement of about 0.05 to 0.10 h (h being the height of plane *ED*) would be needed. In our case, this would mean a displacement of 12—25 cm, considerably much more than wall movements associated with the development of active conditions. Secondly, in practice, the possibility of an excavation being made along *ED* after construction, always must be taken into account.

(a) Wall weight and hydrostatic pressure

As indicated above, we assume the back of the wall, *AB*, to be a straight line. Then (Fig. 7.7) we have:

wall: rectangular section *AHKF*

$W_1 = 1.00 \times 6.50 \times 23 = 149.5$ kN (per meter length of wall)

triangular section *AHB*:

$W_2 = \frac{1}{2} \times 3.00 \times 6.50 \times 23 = 224.3$ kN (per meter length of wall)

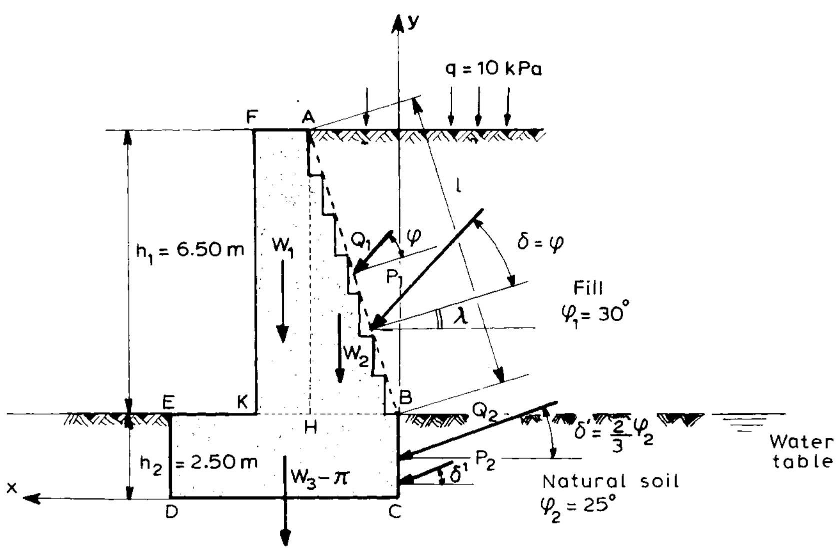


Fig. 7.7.

footing: *BCDE* (taking into account the uplift pressure due to hydrostatic pressure and using the buoyant unit weight of concrete: $\gamma'_{beton} = 13\,kN/m^3$).
$W'_3 = W_3 - \Pi = 2.50 \times 5.00 \times 13 = 162.5\,kN$.

In the axes-system (Cx, Cy) (Fig. 7.7) these forces have following action points:

W_1 : ($x = 3.50$; $y = 5.75$)
W_2 : ($x = 2.00$; $y = 4.67$)
W_3 : ($x = 2.50$; $y = 1.25$).

(b) Forces on the plane *AB*

(b_1) Earth pressure force P_1

$$P_1 = \tfrac{1}{2}\gamma_d l^2 k_{a\gamma} \text{ where } k_{a\gamma} = 0.474, \qquad l = \frac{h_1}{\cos\lambda} = \frac{6.50}{\cos 25^\circ} = 7.17\,m.$$

So, $P_1 = \frac{1}{2} \times 18 \times \overline{7.17}^2 \times 0.474 = 219.3\,kN$ (per m length of wall).
Horizontal component: $P_{1H} = P_1 \cos(\delta + \lambda) = P_1 \cos 55^\circ = 125.8\,kN$ (per m length of wall).
Vertical component: $P_{1V} = P_1 \sin 55^\circ = 179.6\,kN$ (per m length of wall).

Remark

Angle $\delta = \varphi$ has been chosen because when the state of plasticity is developed, *AB* is a line of failure. The portions of soil located to the left of this line and above the steps are not in a plastic equilibrium state. The shear will be that of soil along *AB* and therefore $\delta = \varphi$.

Since the pressure distribution is triangular, the resultant force is located at 1/3 of the height counted from B and along the axes Cx and Cy, at the point of coordinates: $x = 1.00\,m$, $y = 4.67\,m$.

(b_2) Lateral force due to the surcharge of the fill Q_1
The distribution of the stresses working along the 'stem' of the wall is uniform (rectangular diagram).

We then have:
$Q_1 = q \cdot k_{aq} \cdot l = 10 \times 0.522 \times 7.17 = 37.4\,kN$ (per m length of wall): Q_1.
Horizontal component: $Q_{1H} = Q_1 \cos 55^\circ = 21.5\,kN$ (per m length of wall).
Vertical component: $Q_{1V} = Q_1 \sin 55^\circ = 30.6\,kN$ (per m length of wall).

The point through which this force acts is located at half the wall height from *B*, or at $x = 1.50$ and $y = 5.75\,m$.

(c) Forces on the back face of the footing

Plane *BC* is vertical ($\lambda = 0$) and furthermore, from the givens, we know that:
$\delta' = \frac{2}{3}\varphi_2 = \frac{2}{3} \times 25^\circ = 16^\circ 40'$
from which $\cos\delta' = 0.958$, and $\sin\delta' = 0.287$.

Remark

Angle $\delta' = \frac{2}{3}\varphi$ is the usually assumed value in the case of friction between soil and concrete. The footing of the wall is below the water table. Since the hydrostatic pressure acts on both vertical faces of the footing, but in opposite directions, it does not have to be accounted for.

(c_1) Earth pressures: triangular distribution:

$$P_2 = \tfrac{1}{2}\gamma' h_2^2 k'_{a\gamma}$$

$P_2 = 0.5 \times 11 \times \overline{2.50}^2 \times 0.364 = 12.5$ kN (per m length of wall)
Horizontal component: $P_{2H} = 12.5 \times 0.958 \simeq 12$ kN (per m length of wall).
Vertical component: $P_{2V} = 12.5 \times 0.287 \simeq 3.6$ kN (per m length of wall).

The point through which the force acts is at 1/3 up from C on BC, or, in our coordinate system, at $x = 0.0$ and $y = 0.83$ m.

(c_2) Earth pressure due to the surcharge fill and to the mass of earth above the water table: Q_2
The surcharge fill is 10 kPa. The weight of the soil above the water table is: $6.50 \times 18 = 117$ kPa, and the total is: $q' = 127$ kPa.
Therefore, we have:
$Q_2 = q' \cdot h_2 \cdot k'_{aq} = 127 \times 2.50 \times 0.364 = 115.6$ kN (per m length of wall).
Horizontal component: $Q_{2H} = 115.6 \times 0.958 = 110.7$ kN (per m length of wall).
Vertical component: $Q_{2V} = 115.6 \times 0.287 = 33.2$ kN (per m length of wall).

Since the pressure distribution is rectangular, the point of application of the force is half-way up BC, or $x = 0$, $y = 1.25$ m. The resultant of all the forces acting on the wall (with the exception of the soil reaction on the footing) is **F**, and its line of action through plane DC (Fig. 7.8) can be determined. At P, the equivalent force **F**$'$ gives:
M_c = moment of **F** with respect to C = moment of **F**$'$ at $C = F_V \times d$.

Therefore, point P is defined by: $d = M_c/F_V$, where:
$M_c = \Sigma$ moments of exterior forces with respect to C
$F_V = \Sigma$ vertical components of exterior vertical forces.

The eccentricity of P with respect to the axis of symmetry of the footing is $e = |d - DC/2|$ and the resultant **F** goes through the middle third if: $e \leqslant DC/6$.

Table 7A summarizes the calculations:

$$F_V = \Sigma f_v = 783.3 \text{ kN}, \qquad M_c = \Sigma m_c = 2\,463.2 \text{ m} \cdot \text{kN},$$

$$d = 2\,463.2/783.3 = 3.14 \text{ m}$$

from which $e = 3.14 \text{ m} - 2.50 \text{ m} = 0.64$ m.

but $DC/6 = 5.00/6 \simeq 0.83$, then $e < DC/6$.

Assuming a linear distribution of the pressures acting on the bottom of the footing, it follows that the distribution must be trapezoidal. This proves that there are no (uplift) tension forces in the concrete.

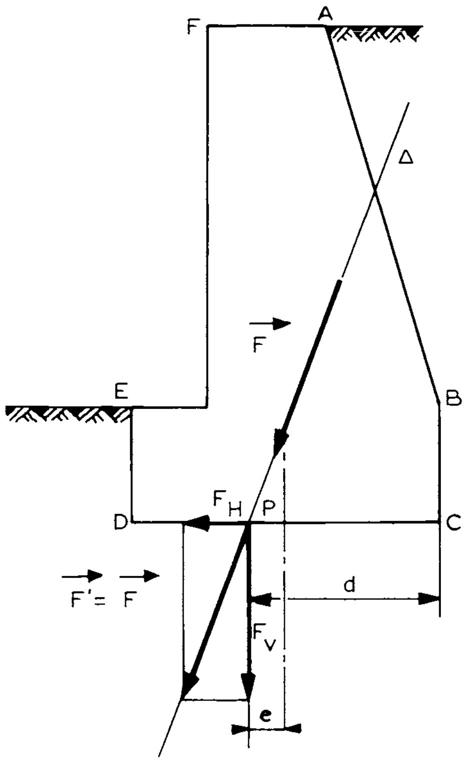


Fig. 7.8.

TABLE 7A

Forces	Forces (kN) vertical	Forces (kN) horizontal	Lever arm by C (m)	Moment by C (kN·m)
P_{1H}		125.8	4.67	587.5
P_{1V}	179.6		1.00	179.6
Q_{1H}		21.5	5.75	123.6
Q_{1V}	30.6		1.50	45.9
P_{2H}		12.0	0.83	10.0
P_{2V}	3.6		0	0
Q_{2H}		110.7	1.25	138.4
Q_{2V}	33.2		0	0
W_1	149.5		3.50	523.3
W_2	224.3		2.00	448.6
$W'_3 = W_3 - \Pi$	162.5		2.50	406.3

(2) *Calculation of the maximal stress in the bottom of the footing*

The stress distribution results in a force **R** which must be in equilibrium with **F**$'$. The usual calculation is to resolve this force in horizontal and vertical components.

For the vertical components, the trapezoidal distribution resultant must equal F_V. Referring to Fig. 7.9, we have:

$$\begin{cases} \dfrac{\sigma_{max} + \sigma_{min}}{2} \times B = F_V & (1) \\[2ex] \tfrac{1}{2}(\sigma_{max} + \sigma_{min}) \times B \times \left(\dfrac{2B}{3} - \dfrac{B}{2}\right) = F_V \times e & (2) \end{cases}$$

from which: $\sigma_{max} = (F_V/B)(1 + 6e/B)$, and therefore:

$$\sigma_{max} = \frac{783.3}{5.00}\left(1 + \frac{6 \times 0.64}{5.00}\right) = 277\,\text{kPa}, \quad \text{or} \quad \sigma_{max} \approx 2.8\,\text{daN/cm}^2.$$

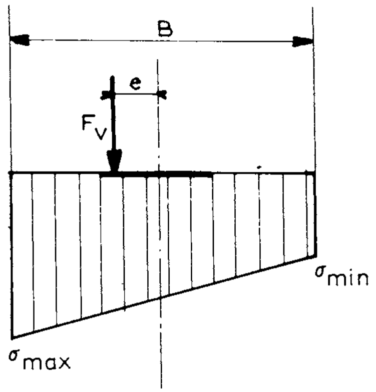


Fig. 7.9.

Remark

For calculating the allowable bearing capacity for an eccentric, inclined load, Meyerhof proposes the following formula for the vertical component of the allowable stress (for sands):

$$q_{vad} = \gamma D + \frac{1}{F}\left\{\frac{1}{2}\gamma B'\left(1 - \frac{\delta}{\varphi}\right)^2 N_\gamma + \gamma D\left[\left(1 - \frac{2\delta}{\pi}\right)^2 N_q - 1\right]\right\}$$

where $B' = B - 2e$ is the decreased width of the footing, e the eccentricity of the load and δ its angle of inclination.

The allowable load then is: $Q_V = B'q_{vad} = 310\,\text{kN/m} < F_V$.

The wall will fail by rupture of the foundation soil.

(3) *Calculation of the safety factor against overturning of the wall*

To estimate the safety factor against overturning of the wall, it is necessary to know the location of the axis of rotation of the wall. If the foundation soil were non deformable, this axis would be through D (Fig. 7.8) at the toe of the footing. Since the soil deforms, the location of the rotation axis is not known and may well vary during the overturning process. Therefore the safety factor varies during the course of the movement.

If it is assumed that the axis of rotation is through point D, we can write:

Moments of stabilizing forces through D:

$$
\begin{array}{lrcl}
W_1: & 149.5 \times 1.50 & = & 224.3 \\
W_2: & 224.3 \times 3.00 & = & 672.9 \\
W_3': & 162.5 \times 2.50 & = & 406.3 \\
P_{1V}: & 179.6 \times 4.00 & = & 718.4 \\
Q_{1V}: & 30.6 \times 3.50 & = & 107.1 \\
P_{2V}: & 3.6 \times 5.00 & = & 18.0 \\
Q_{2V}: & 33.2 \times 5.00 & = & \underline{166.0} \\
 & \Sigma M_1 & = & 2313 \text{ m} \cdot \text{kN}
\end{array}
$$

Moments of overturning forces through D:

$$
\begin{array}{lrcl}
P_{1H}: & 125.8 \times 4.67 & = & 587.5 \\
Q_{1H}: & 21.5 \times 5.75 & = & 123.6 \\
P_{2H}: & 12.0 \times 0.83 & = & 10.0 \\
Q_{2H}: & 110.7 \times 1.25 & = & \underline{138.4} \\
 & \Sigma M_2 & = & 859.5 \text{ m} \cdot \text{kN}
\end{array}
$$

The safety factor against overturning for the condition of an undeformable foundation soil then is:

$$F_r = \frac{\Sigma M_1}{\Sigma M_2} = \frac{2\,313}{859.5} = 2.69 \simeq 2.7 > 1.5.$$

F_r is quite a bit more than 1.5 which is the usually acceptable value of the safety factor. In practice, it is not necessary to control the overturning stability safety factor if the resultant of all forces acting on the wall, passes through the middle third of the foundation. This resultant should, however, be as close as possible to the footing center, when the softness of the foundation soil increases.

(4) *Safety factor against sliding*

Of interest now are the horizontal forces. The horizontal component F_H of $\mathbf{F}'$ must be in equilibrium with the friction force acting against the bottom of the footing.

The general equation for the safety factor against sliding is:

$$F_g = \frac{aB + F_V \tan \delta}{F_H},$$

where a = adherence between soil and footing ($|a| \leqslant c$) and δ = friction angle between them.

For a cohesionless soil, $c = 0$, thus $a = 0$ and:

$$F_g = F_V/F_H \tan \delta$$

if we take $\delta = \varphi_2 = 25°$, then $\tan \delta = 0.466$, and:

$$F_H = 125.8 + 21.5 + 12 + 110.7 = 270\ \text{kN}.$$

Thus, $F_g = (0.466 \times 783.3)/270 = 1.35 < 1.5$.

This means that the safety factor against sliding F_g is too low: the wall geometry should be changed in order to obtain $F_g > 1.5$.

Remark

The safety factors against overturning and against sliding were of course only calculated for learning reasons. In practice, the correct evaluation of the footing consists in considering that it is subjected to an inclined and eccentric load.

The calculation shows that the eccentricity and inclination of the load greatly reduce the allowable bearing pressure. It would be very dangerous to compare the stress $\sigma_{max} = 277$ kPa to the allowable pressure calculated from a vertically applied load (without eccentricity) because this would lead to an unrealistic safety factor.

Summary of answers

(1) $e = 0.64$ m, the resultant passing through the middle 1/3.
(2) $\sigma = 2.8\ \text{daN/cm}^2$ (280 kPa).
(3) $F_r \simeq 2.7$,
(4) $F_g \simeq 1.35$.

The wall will collapse by punching failure.

★★*Problem 7.4* Wall stability without a buttress and with an inclined backfill

Refer to the gravity wall of Fig. 7.6 and assume the back of the wall to be rectilinear through AB. Calculate P_1, Q_1, P_2, Q_2 with the same assumptions as in the preceding problem, but now with a backfill inclined upwards at an angle $\beta = 20°$ with the horizontal.

Compare the results with those obtained for the horizontal backfill ($\beta = 0$) condition of problem 7.3.

Solution

We first must find the earth pressure coefficient of the fill with $\delta/\varphi = 1$, $\beta/\varphi = 20/30 = 0.67$, and $\lambda = 25^\circ$.

It will be noticed that for $\beta/\varphi = 0.6$ and $\beta/\varphi = 0.8$, the Caquot and Kerisel tables do not give the coefficients for the angles of wall λ over 15° and 10° for a $\varphi = 30^\circ$. This is because we are faced with a steeply inclined plane (as mentioned in sect. 5.3.3. of Costet-Sanglerat, vol. 1). The Boussinesq-equilibrium cannot be produced and we therefore must consider the Rankine equilibrium (sect. 5.2.2, vol. 1).

The earth pressure coefficient (normal component) is given by the formula:

$$K_{a\gamma} = \frac{\sin\beta\cos(\theta-\beta)}{\sin\varphi\sin(\omega_\beta+\beta)}[1-\sin\varphi\cos(2\theta+\omega_\beta-\beta)],$$

where: $\theta = \lambda = 25^\circ$, $\beta = 20^\circ$ and $\varphi = 30^\circ$.

$\sin\omega_\beta = \sin\beta/\sin\varphi = \sin 20^\circ/\sin 30^\circ = 0.684$ in which $\omega_\beta = 43.16^\circ$

$\sin\beta = 0.342;\ \cos(\theta-\beta) = \cos 5^\circ = 0.996$

$\sin\varphi = 0.5;\ \sin(\omega_\beta+\beta) = \sin 63.16^\circ = 0.892$

$\cos(2\theta+\omega_\beta+\beta) = \cos(50+43.16-20) = \cos 73.16^\circ = 0.290$

$\sin(2\theta+\omega_\beta-\beta) = \sin 73.16^\circ = 0.957,$

from which: $K_{a\gamma} = \dfrac{0.342\times 0.996}{0.5\times 0.892}[1-0.5\times 0.290] = 0.653.$

The angle α of the earth pressure on the wall face is:

$$\tan\alpha = \frac{\sin\varphi\sin(2\theta+\omega_\beta-\beta)}{1-\sin\varphi\cos(2\theta+\omega_\beta-\beta)} = \frac{0.5\times 0.957}{1-0.5\times 0.290} = 0.560,$$

from which $\alpha = 29.23^\circ$ (α is very close to 30°).

Based on its true inclination, the earth pressure coefficient is:

$k_{a\gamma} = K_{a\gamma}/\cos\alpha = 0.653/\cos 29.23^\circ = 0.748.$

The force P_1 (per m length of wall) is equal to:

$P_1 = \frac{1}{2}\gamma_d\cdot l^2 k_{a\gamma} = \frac{1}{2}\times 18\times \overline{7.17}^2\times 0.748 = 346.1\,\text{kN}.$

The lateral pressure due to the surcharge may be calculated by taking the coefficient: $k_{aq} = k_{a\gamma}/\cos(\beta-\lambda) = 0.748/\cos 5^\circ = 0.750$ and thus: $Q_1 = q\cdot k_{aq}\cdot l = 10\times 0.750\times 7.17 = 53.8\,\text{kN}$ (per m length of wall).

To calculate P_2 and Q_2 a line parallel to the backfill surface is drawn through point B and the earth pressure coefficient is given by the Caquot-Kerisel table:

$\beta/\varphi = 20/25 = 0.8;\ \delta/\varphi = 2/3;\ \varphi = 25^\circ;\ \lambda = 0^\circ,$

from which $k'_{a\gamma} \simeq k'_{aq} = 0.546.$

When, as a first approximation, the soil is assumed to be homogeneous and of unit weight γ'; one finds:
$P_2 = \frac{1}{2}\gamma' h_2^2 k'_{a\gamma} = \frac{1}{2} \times 11 \times \overline{2.50}^2 \times 0.546 = 18.8$ kN (per m length of wall).
$Q_2 = q' \cdot h_2 \cdot k'_{aq} = 127 \times 2.50 \times 0.546 = 173.4$ kN (per m length of wall).

Conclusion
In the case of an inclined backfill at $\beta = 20°$ the lateral forces increase by over 50% in comparison to the $\beta = 0°$ condition.

★★*Problem 7.5* Comparison of lateral forces on a vertical wall with horizontal backfill and different assumptions (Boussinesq equilibrium and graphical method of Culmann)

Referring to the givens of problem 7.1 (wall 4 m high), a vertical-face ($\lambda = 0$) dry sand, an horizontal backfill ($\beta = 0$), $\varphi = 36°$, and assuming $\delta = \frac{2}{3}\varphi$, find:
— the lateral earth forces by the Caquot-Kerisel method;
— the same by the Culmann graphical method;
— the ratio of the two answers above.
N.B. Caquot-Kerisel tables give:

$k_{a\gamma} = 0.247$ *for* $\varphi = 35°$, $\beta = \lambda = 0$ $\delta = \frac{2}{3}\varphi$

$k_{a\gamma} = 0.202$ *for* $\varphi = 40°$, $\beta = \lambda = 0$ $\delta = \frac{2}{3}\varphi$

Assume a linear interpolation for the value of $k_{a\gamma}$ *corresponding to* $\varphi = 36°$.

Solution
By the Caquot-Kerisel method, the tables give:

for $\varphi = 35°$: $k_{a\gamma} = 0.247$; for $\varphi = 40°$: $k_{a\gamma} = 0.202$, and $\Delta k_{a\gamma} = 0.045$.

For $\varphi = 36°$, we get: $k_{a\gamma} = 0.247 - (0.045/5) = 0.238$, and $P_1 = \frac{1}{2} k_{a\gamma} \times \gamma \times h^2$.

$P_1 = 0.5 \times 0.238 \times 18.3 \times \overline{4.00}^2 = 34.8$ kN (per m length of wall).

Calculation by the Culmann method (see Fig. 7.10):
Through point B, draw BD at an angle $\varphi = 36°$ with the horizontal, and BS at an angle ψ with BD, the same as the lateral pressure with the vertical, in this case: $\psi = 90° - \frac{2}{3}\varphi = 90° - (\frac{2}{3} \times 36) = 90 - 24 = 66°$.

Then draw random lines BC, BC_1, BC_2, etc., then Cd, C_1d_1, C_2d_2, etc. . . . parallel to AB and lines de, d_1e_1, d_2e_2, etc. . . . parallel to BS. Measure now the maximum of the ed-lines, here $d_m e_m$ or 2.35 cm. With scale adopted, this translates into $e_m d_m = 1.175$ m from which:

$E_{max} = \frac{1}{2}\gamma h \times (AD/BD) \times e_m d_m$

$AD/BD = \cos 36° = 0.809.$ So:

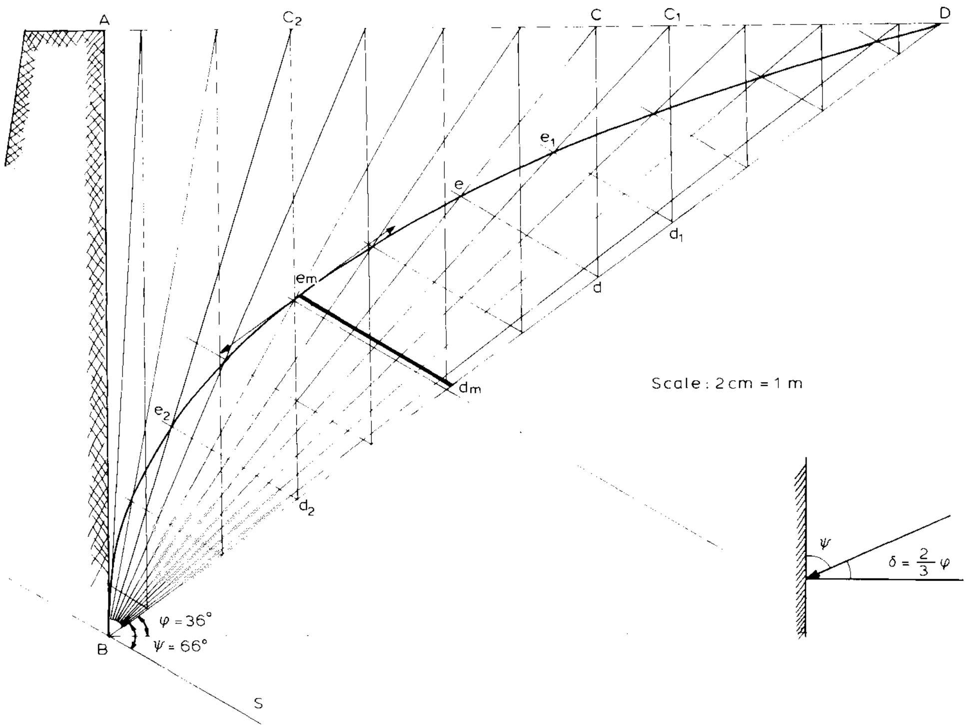


Fig. 7.10.

$E_{max} = 0.5 \times 18.3 \times 4.00 \times 0.809 \times 1.175 = 34.8$ kN (per m length of wall).

Then we have:

$$\frac{k_{a\gamma} \text{ Caquot-Kerisel}}{k_{a\gamma} \text{ Culmann}} \simeq 1 \quad \text{in this particular case.}$$

★★*Problem 7.6* **Detecting errors made in the design of failing retaining structures (ruptures, collapses, etc.) of reinforced concrete or masonry**

The five walls of Fig. 7.11 all failed. Can you identify the causes of these failures?

Solution

— Wall 1. No calculation made. Footing width obviously too narrow. Failure plane at contact face between sand and rock.

— Wall 2. Insufficient drainage of the fill mass and no 'weep holes'. An angle of internal friction of 20° indicates a clayey soil, therefore one which would not easily drain.

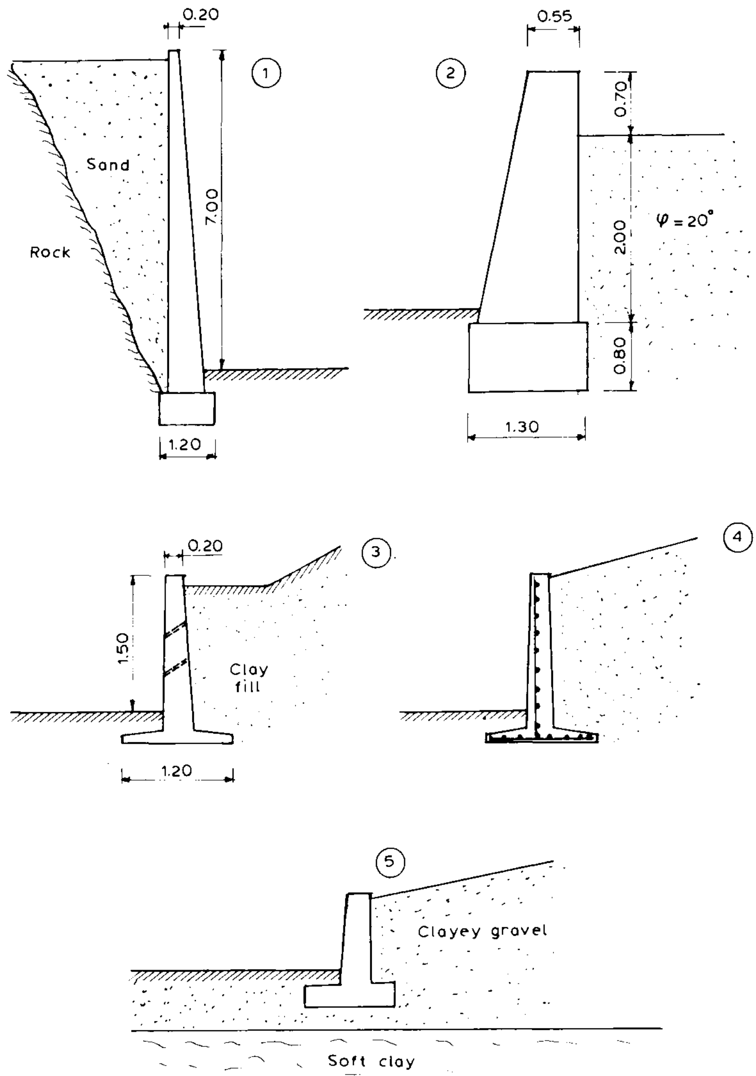


Fig. 7.11.

— Wall 3. Although 'weep holes' are indicated, there is no indication of a drainage blanket in the clay-fill behind the wall.
— Wall 4. Steel reinforcement placed on the compression side of the wall stem, but no steel on the tension side, leading to ruptures in the wall.
— Wall 5. Failure due to deep slip surface. The overall stability was not properly evaluated.

★★*Problem 7.7* **Diagram of stresses behind a gravity wall. Stratified soil and water table. Uniformly loaded backfill**

It is required to draw the distribution of horizontal stress components, acting on the gravity wall of Fig. 7.12(a), knowing that:

— *the inner wall face is straight and inclined 10° with the vertical.*

— *the backfill of the wall is horizontal and uniformly loaded by 20 kPa.*

— *the soil behind the wall consists of 4 distinctly horizontal layers having the properties indicated on Fig. 7.12(a). The lowermost layer is partly submerged to ground water table.*

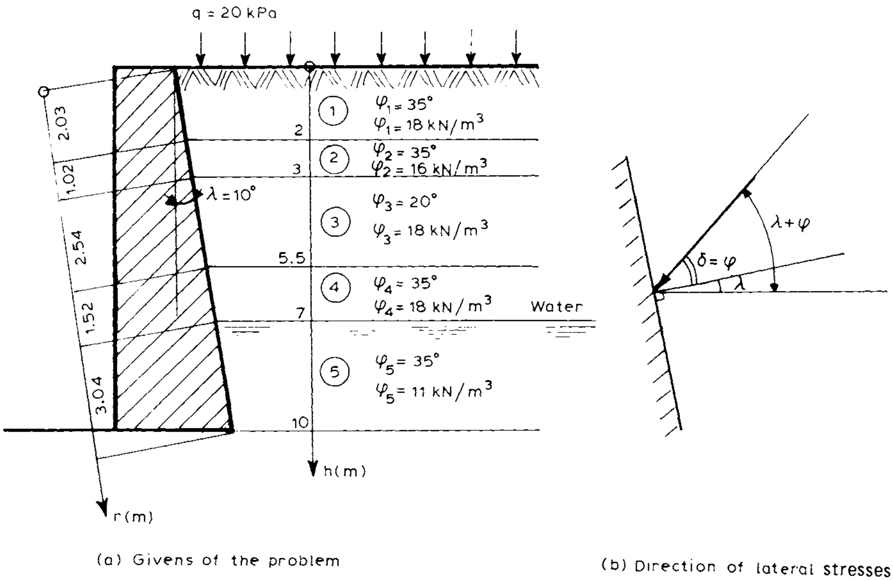


Fig. 7.12.

Solution

The required diagram is shown on Fig. 7.13. The details of the computation are given in Costet-Sanglerat vol. 1, sect. 6.2.5. From Fig. 7.13 it is possible to calculate the safety factors against overturning and against sliding, as described in problem 7.3, provided that the dimensions of the wall are known.

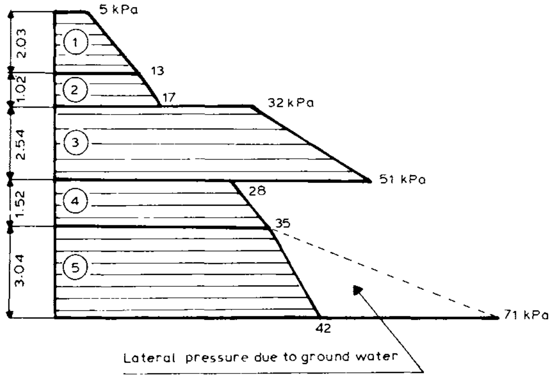


Fig. 7.13. Lateral pressure distribution.

★★★*Problem 7.8* The influence of drainage conditions on the earth pressures acting on a retaining wall

A retaining-wall, 5 m high, supports a horizontal backfill of cohesionless sand (Fig. 7.14). The inner wall face is rough, so assume $\delta = \varphi$ (assume $k_{a\gamma} = 0.308$). The internal angle of friction of the sand (φ) is 30°. The void ratio is 0.53 and the specific gravity of the soil grains is 2.7.

Calculate the lateral earth pressure per length of wall under the following conditions:

(1) The backfill is dry.
(2) Both wall and backfill are completely submerged (quai wall).
(3) The backfill alone is submerged.
(4) The backfill is saturated and drained through a sloping drainage blanket (Fig. 7.15).
(5) The backfill is saturated and drained through a vertical drainage blanket (Fig. 7.16).

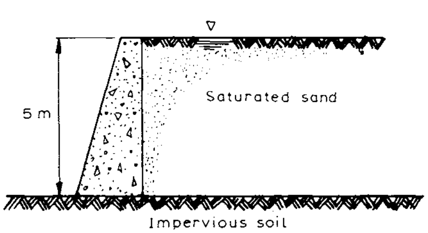


Fig. 7.14. Wall with submerged, undrained backfill.

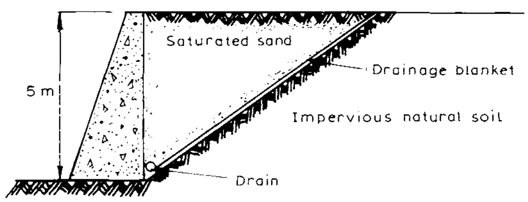


Fig. 7.15. Wall with backfill over the drainage blanket.

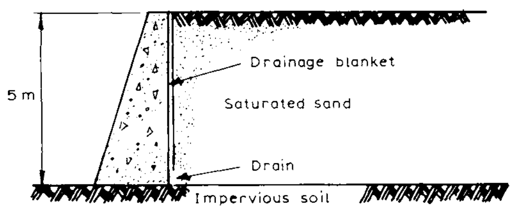


Fig. 7.16. Wall with backfill against vertical drainage blanket.

Solution

(1) Dry-sand backfill

The unit-weight of sand is: $\gamma_d = \dfrac{\gamma_s}{1+e} = \dfrac{27}{1+0.53} = 17.6\ \text{kN/m}^3$.

The lateral earth pressure is:

$$P = \tfrac{1}{2}k_a\gamma_d H^2 = \tfrac{1}{2} \times 0.308 \times 17.6 \times \bar{5}^2 = 67.8\ \text{kN}:$$

$$P_{\text{hor.}} = 67.8 \times \cos 30^\circ = 58.7\ \text{kN}; \quad P_{\text{vert.}} = 67.8 \times \sin 30^\circ = 33.9\ \text{kN}.$$

(2) Both wall and backfill are completely submerged

Here, the submerged or buoyant soil unit-weight must be used:

$$\gamma_h = \gamma_d + \frac{e\gamma_w}{1+e} = 17.6 + \frac{5.3}{1.53} = 21.1\ \text{kN/m}^3$$

$$\gamma' = \gamma_h - \gamma_w = 21.1 - 10 = 11.1\ \text{kN/m}^3.$$

The lateral earth pressure is:

$$P = \tfrac{1}{2}k_a\gamma' H^2 = \tfrac{1}{2} \times 0.308 \times 11.1 \times \bar{5}^2 = 42.7\ \text{kN}:$$

$$P_{\text{hor.}} = 42.7 \times \cos 30^\circ = 37\ \text{kN}; \quad P_{\text{vert.}} = 42.7 \times \sin 30^\circ = 21.4\ \text{kN}.$$

In this case, the hydrostatic pressures act on both sides of the wall, but in opposite directions and therefore cancel themselves.

(3) Backfill alone is submerged

To the calculated buoyant soil pressure must now be added the hydrostatic pressure: $P_{\text{water}} = \gamma_w H^2/2 = 10 \times 5^2/2 = 125\,\text{kN}$. Thus:

$P_{\text{hor.}} = 37 + 125 = 162\,\text{kN}; \quad P_{\text{vert.}} = 21.4\,\text{kN}.$

(4) Backfill saturated and drained through a sloping drainage blanket (Figs. 7.15, 7.17)

In this case, as may occur when a heavy rain falls down, in the backfill there comes to existence a flow net as shown on Fig. 7.17, where flow lines are vertical and equipotential lines are horizontal. Assuming that the drainage blanket is not 'loaded', the pore-water pressures in it are zero as on the free horizontal surface. Therefore the pore pressure is zero throughout the backfill. The calculation is the same as for the case of the dry-sand backfill if we replace the dry unit-weight by the saturated unit-weight.

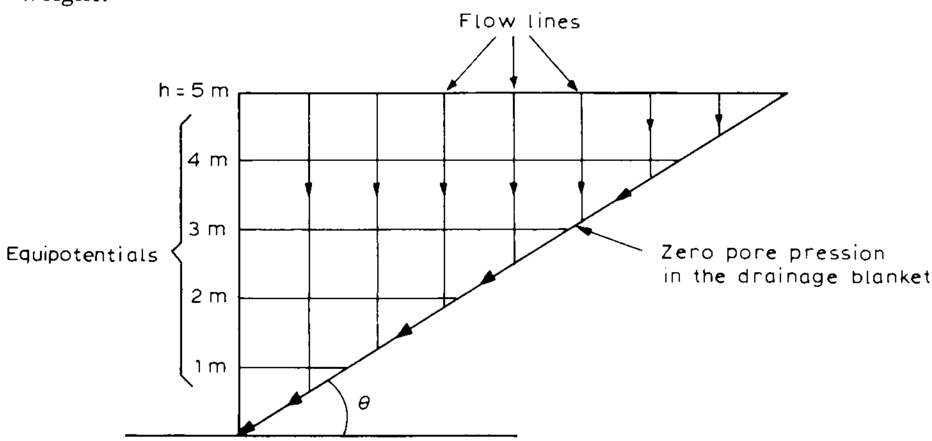


Fig. 7.17. Flow-net due to heavy rainfall over the backfill with a sloping drainage blanket.

Thus:

$P = \frac{1}{2} k_a \gamma_h H^2 = \frac{1}{2} \times 0.308 \times 21.1 \times \bar{5}^2 = 81.2\,\text{kN}:$

$P_{\text{hor.}} = 81.2 \times \cos 30^\circ = 70.3\,\text{kN}; \quad P_{\text{vert.}} = 81.2 \times \sin 30^\circ = 40.6\,\text{kN}.$

(5) The backfill is saturated, but now drained through a vertical drainage blanket

For this situation, the flow net is shown on Fig. 7.18. It is impossible to give a simple mathematical solution. Following the method of Coulomb several soil wedges are tested in order to find one which yields the maximal lateral earth pressure. In each case, pore-water pressure must be evaluated along the failure plane, and the resultant pressure must be calculated by graphical solution, for example. This pore-water pressure must be taken into account in the equilibrium of Coulomb's wedge to calculate the lateral earth pressure.

Because there exist pore pressures all along the boundary, the lateral pressure with a vertical drainage blanket will be larger than that of a sloping drainage blanket.

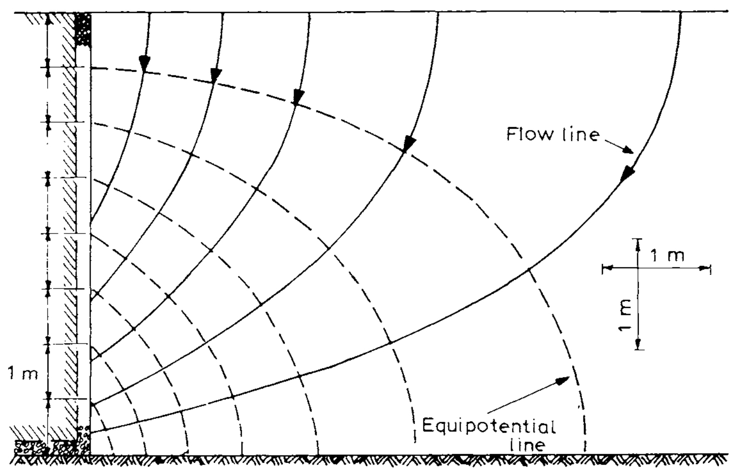


Fig. 7.18. Flow-net for rainwater draining with a vertical blanket.

Fig. 7.19 shows a graphical method to determine pore pressures for a soil wedge whose boundary conditions correspond to an angle of 45° with the horizontal.

Consider an equipotential line, such as *NM*, where the loads at *N* and *M* are equal ($h_N = h_M$). On the other hand, the pore pressure at *N* is zero (no hydrostatic head in the drainage blanket). We then have:

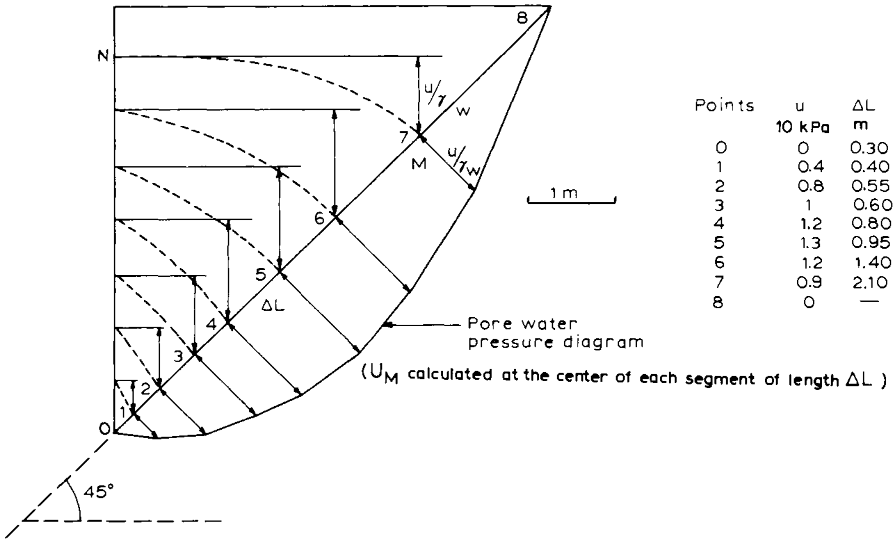
$$h_M = u_M/\gamma_w + z_M, \qquad h_N = z_N$$

where $z_N - z_M = u_M/\gamma_w$

from which each point in the diagram can be analyzed. The resultant of the pore pressure is $U = 60.7$ kN. The equilibrium state of the soil wedge (Fig. 7.20) is then calculated as follows:

$$W = \tfrac{1}{2} \times 5^2 \times 21.1 = 263.8 \text{ kN}.$$

$$\text{Furthermore, } P = \frac{(W - U\cos\theta)\tan(\theta - \varphi) + U\sin\theta}{\sin\delta\tan(\theta - \varphi) + \cos\delta},$$



Points	u 10 kPa	ΔL m	u_m ΔL
0	0	0.30	0.06
1	0.4	0.40	0.24
2	0.8	0.55	0.50
3	1	0.60	0.66
4	1.2	0.80	1.00
5	1.3	0.95	1.19
6	1.2	1.40	1.47
7	0.9	2.10	0.95
8	0	—	—
			U = 6.07·10 KN

Fig. 7.19. Resultant of forces due to pore pressure for $\theta = 45^\circ$.

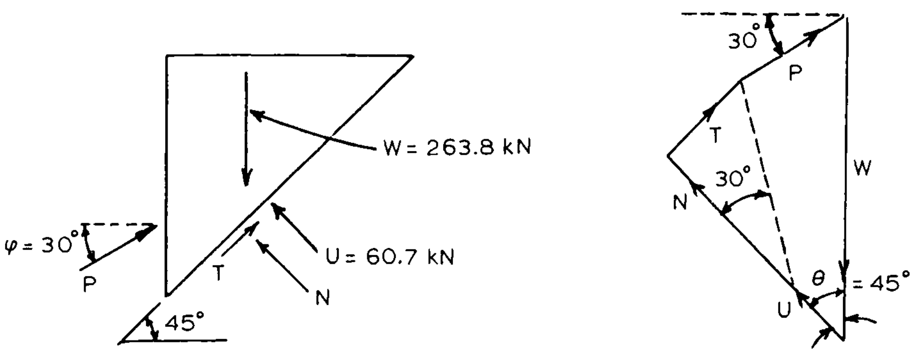


Fig. 7.20. Graphical determination of lateral earth pressure.

where: $\delta = \varphi = 30^\circ$ and $\theta = 45^\circ$:

$\sin\theta = \cos\theta = 0.707$; $\sin\delta = 0.5$, $\cos\delta = 0.86$,

$\tan(\theta - \varphi) = \tan 15^\circ = 0.268$

$$P = \frac{(263.8 - 60.7 \times 0.707) \times 0.268 + 60.7 \times 0.707}{0.5 \times 0.268 + 0.866}$$

$P = 102.1\,\text{kN}$, $P_{\text{hor.}} = 102.1 \cos 30^\circ = 88.4\,\text{kN}$,

$P_{\text{vert.}} = 102.1 \sin 30^\circ = 51.1\,\text{kN}$.

The procedure is repeated for other values of θ and will result in the curve of Fig. 7.21, which gives P as a function of θ, reaching a maximal value for $\theta = 45°$. The value calculated above is the one for which the wall should be designed.

Conclusion

This problem illustrates clearly on the one hand, the importance of providing a drainage for a backfill subject to saturation and, on the other, the influence of the type of the drainage. The value of the pressures increases as follows:

— wall and backfill completely submerged	$P = 42.7$ kN
— dry backfill	$P = 67.8$ kN
— saturated backfill with sloping blanket	$P = 81.2$ kN
— saturated backfill with vertical blanket	$P = 102.1$ kN
— saturated backfill without a blanket	$P = 162.0$ kN

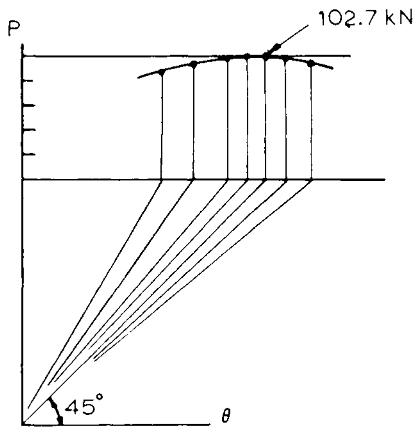


Fig. 7.21. Variation of P as a function of θ.

★★★*Problem 7.9* Analysis of the failure of a reinforced concrete retaining-wall Corrective measure by using rock anchors

A reinforced concrete retaining-wall along a motorway consisted of 21 elements each 6 m in length. Shortly after construction, the wall failed: several elements were pushed over and in others had developed large diagonal cracks. It was observed that most of the drain holes in the wall were plugged up. The wall dimensions are shown on Fig. 7.22.

A review of the construction procedures showed that the excavations for the wall had been done under adverse conditions:

— *already during the excavations numerous seepages had been observed in the cuts;*
— *the graded filter material specified for the drainage blanket had not been used, but was replaced by excavated material;*
— *the wall footings were not bearing on solid rock, in particular not at the toe.*

(1) Analyse the wall stability and explain the observed failures.

(2) Recommend a repair method by tie rods (2 rows) anchored in rock.

The backfill material properties were: $\varphi = 34°$, $\gamma_h = 19\ kN/m^3$, $\gamma' = 11\ kN/m^3$. *The backfill behind the wall was replaced at an angle* $\beta = 34°$ *with the horizontal. Assume that the reinforced concrete unit-weight was* $23\ kN/m^3$ *and the angle of friction between concrete and rock was* $\delta = 30°$.

Solution

(1) Analysis of wall stability

Assumptions used for calculation

Because of the poor quality of the drainage material, the calculation must consider the hydrostatic pressure (assuming that the water level is at the top of the wall).

The earth pressure at the heel of the wall is considered as non-existent since it is encompassed in the rock. However, the hydrostatic pressure acts

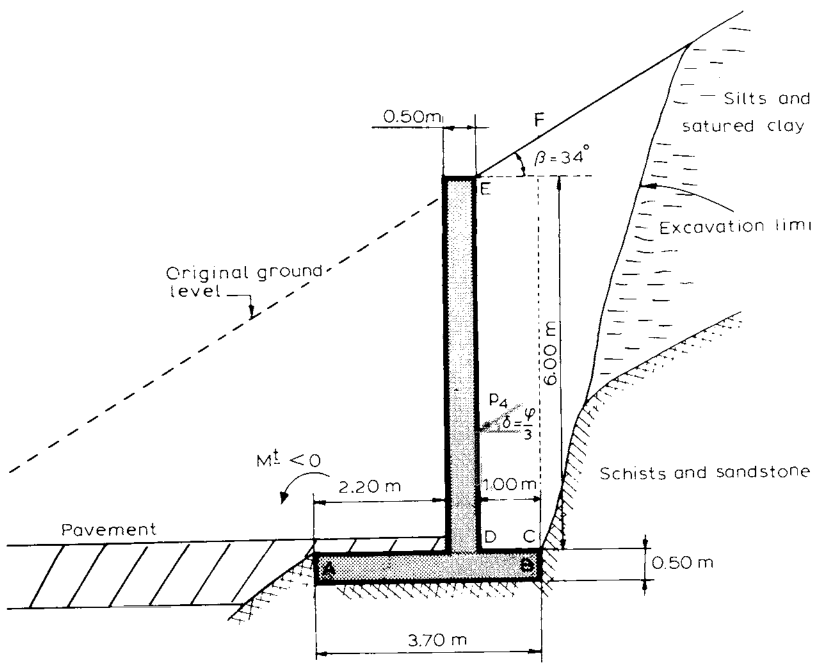


Fig. 7.22. Failed wall.

along *BC* because the rock is fractured. The passive pressure at the toe of the wall also may be overlooked (poor-quality rock).

Assume that the backfill volume *EDCF* is part of the wall weight (Fig. 7.22). Thus the lateral and water pressures on the fictive surface *BCF* which act on the wall and volume of the soil *EDCF* must be calculated.

To determine the overturning stability, bending moments can be calculated with respect to *A* (at the toe), because the foundation soil can be assumed to be rigid (rock) and the center of rotation will be at point *A*. It also can be assumed that a limit Rankine-equilibrium condition exists on plane *CF*.

The stress tensor at depth *h* may be represented by a Mohr's circle as shown on Fig. 7.23. The pole of this circle can be easily constructed and then the failure lines of Rankine equilibrium can be drawn (Fig. 7.24).

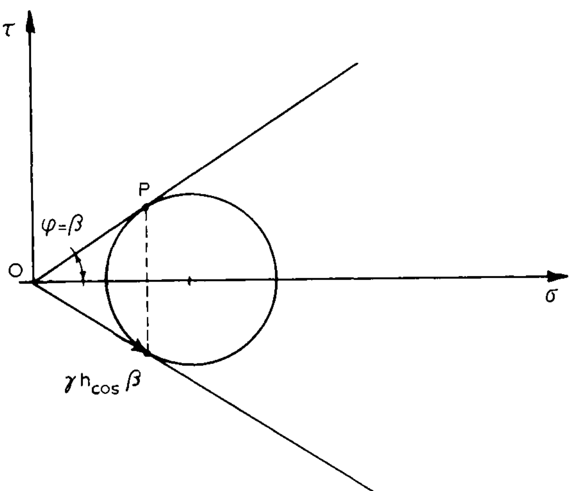


Fig. 7.23.

Stability calculation

The first failure line which intersects the wall is the line *CF*. Thus the Rankine equilibrium condition will be modified only between the back face of the wall and *CF* plane. It is therefore justified to calculate the Rankine earth pressure acting on plane *CF*. In practice, the lateral earth pressure calculated as above, is slightly overestimated because the critical failure wedge intersects the rock zone which cannot slip.

The stress acting on a vertical face along *CF* is $\gamma h \cos\beta$ (see Fig. 7.23). It is inclined upwards at an angle $\beta = 34°$ with the normal to *CF*. Thus we get the earth pressure coefficients:

horizontal: $k_{ah} = \cos^2\beta = \cos^2(34°) = 0.687$

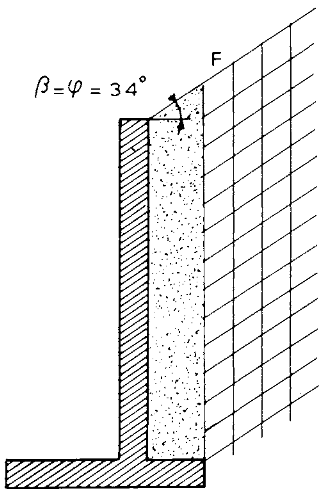


Fig. 7.24.

vertical: $k_{av} = \cos\beta \sin\beta = \cos(34°)\sin(34°) = 0.463$

Overturning stability and sliding stability are studied using results of Table 7B (calculations are made per one meter of wall length). The stabilizing moments are assumed to be positive and the overturning moment negative. Uplift pore pressures are disregarded.

TABLE 7B

Forces (kN) (per m length)	Lever arm about A (m)	Moment A (kN · m) (per m length)
Weight of concrete and soil		
$p_1 = 6 \times 1 \times 21 = 126$ kN	3.20	+403
$p_2 = 0.5 \times 1 \times 23 = 11.5$ kN	3.20	+36.8
$p_3 = 0.5 \times 6.5 \times 23 = 74.7$ kN	2.45	+183
$p_4 = 0.5 \times 2.2 \times 23 = 25.3$ kN	1.1	+27.8
Earth pressure on CF		
$p_V = \frac{1}{2} \times 11 \times (6.67)^2 \times 0.463 = 113$ kN	3.7	+419
$p_H = \frac{1}{2} \times 11 \times (6.67)^2 \times 0.687 = 168$ kN	2.77	−465
Earth pressure on BF		
$p_{water} = \frac{1}{2} \times 10 \times (7.17)^2 = 257$ kN	2.39	−614.2

Overturning stability:

$$F_R = \frac{\Sigma(M/A > 0)}{\Sigma(M/A < 0)} = \frac{1070}{1079} = 0.99.$$

The safety factor against overturning is less than 1 and thus overturning is a certainty. In addition, because the footing does not bear entirely on solid rock, the centre of rotation will shove back to a point under the footing instead of to point A, and this in turn will still decrease the safety factor.

Sliding stability

The sum of the vertical forces is equal to 350.5 kN (per m of wall length), the sum of the horizontal forces is 425 kN. The angle of friction between the concrete and the rock is 30°, therefore: $F_G = 350.5 \times \tan(30°)/425 = 0.47$: the wall would also fail in sliding.

To conclude, the lack of drainage behind the wall causes it to be unstable and creates two modes of failure, namely by overturning and sliding.

The various wall panels underwent important displacements of varying magnitudes as a consequence of the bedrock quality. This caused the panels to interact with each other while they, theoretically, were supposed to act independently of each other. Since no reinforcing was designed to resist the bending moments, cracks developed in the outer face of the panels.

Remark

Assuming that a proper drainage had been installed and that the rock was sound, the forces acting on the wall would have been (per meter of wall length):

$$p_V = \tfrac{1}{2} \times 19 \times (6.67)^2 \times 0.463 = 196\,\text{kN}$$

$$p_H = \tfrac{1}{2} \times 19 \times (6.67)^2 \times 0.687 = 290\,\text{kN}$$

$$p_1 = 19 \times 6 \times 1 = 114\,\text{kN}$$

Moment with respect to A due to $p_V = 722\,\text{kN} \cdot \text{m}$
Moment with respect to A due to $p_H = 803\,\text{kN} \cdot \text{m}$
Moment with respect to A due to $p_1 = 365\,\text{kN} \cdot \text{m}$.

The safety factor against overturning is: $F_R = 1335/803 = 1.7$ (acceptable) and the coefficient against sliding would have been: $F_G = 420 \tan 30°/290 = 0.84$, which is too low.

The designer probably assumed the presence of a passive pressure at the toe. If the rock there had been sound, sliding could not occur. The errors in the design consisted of: (1) unrealistic appraisal of the rock quality; (2) a poor construction practice (faulty drainage blanket).

(2) Corrective measures

The first step to repair would be, as far as possible, to improve the drainage of the backfill by clearing out the plugged up drain holes and by adding a drainage blanket. If this would not be possible, it would be required to set up for the fortification a calculation, taking into account the water pressure acting on the wall.

For instance, two rows of rock anchors may be placed, one located just above the footing, to prevent sliding, and the other at height z above the base. Each row of anchors is assumed to equal a tension T (per m length of wall). To realise a safety factor of 1.5 against sliding and overturning, we would have:

$F_G = (201 + 2\,T)/425 = 1.5,$

or $T = \frac{1}{2}[(1.5 \times 425) - 201] = 218\,\text{kN}$

and $F_R = (1070 + T \times 0.5 + T \times z)/1079 = 1.5$

from which:

$z = [(1.5 \times 1079) - 1070 - (218 \times 0.5)]/218$: assume $z = 2$.

In this calculation, it is assumed that the placement (thus: the tension) of the anchors did not alter the magnitude of the earth pressures. This corrective method only seeks to avoid further failures and not to replace the wall to its original design position. (This would engender passive pressures.)

The calculation neither did account for the poor rock quality at the toe of the wall. It is therefore not possible to determine the point of rotation. This unknown is partly taken care of by seeking a design yielding a safety factor of 1.5 which can be dangerous. It could also be taken care of by increasing

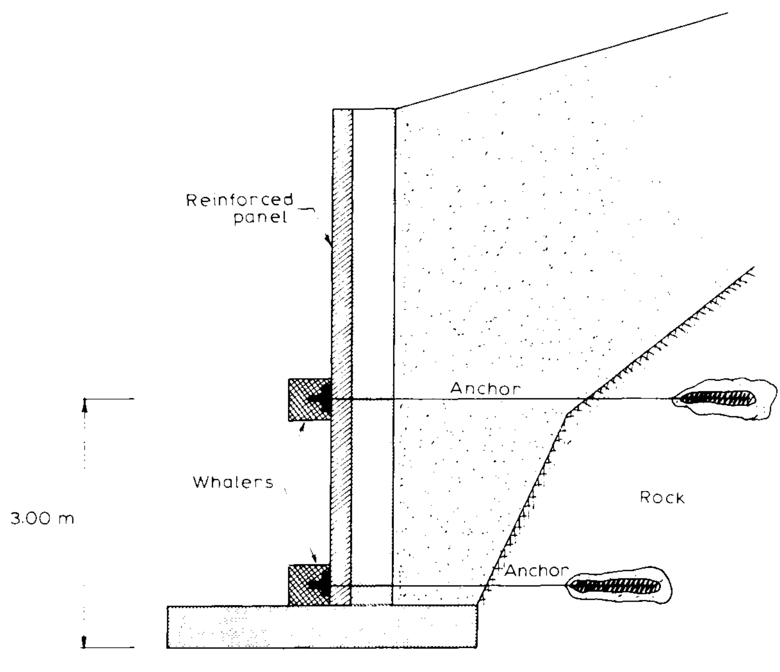


Fig. 7.25. Remedial methods of support.

the load which the upper anchor is designed to take or by increasing the height of the row.

To conclude, the remedial measure for the wall (Fig. 7.25) would be:
— placing a reinforced concrete panel against the center wall face;
— installing two whalers located just above the footing and 3 m above the base, respectively;
— installing two anchor lines deriving their tension in the bedrock and on the whalers, designed to withstand a tension of 218 kN per m of length of wall.

★★★*Problem 7.10* Design of a reinforced-earth retaining-wall with horizontal backfill

A motorway is planned to cross an unstable slope as shown on Fig. 7.26. It is proposed to construct the pavements on engineered fill placed over the unstable areas and to support the fill by a retaining-wall.

Two solutions are being considered, one with a conventional reinforced-concrete wall, the other with a reinforced-earth structure.

(1) List the conditions favorable for the choice of a reinforced-earth design.

(2) In a general manner, what are the problems that could affect the performance of such a structure?

(3) The height of the reinforced-earth wall must be $H = 20$ m. Assume the wall thickness to be $L = 0.8\,H$ (generally accepted value).

Backfill and fill of the wall consist of the same material whose unit-weight is $18\,kN/m^3$ and angle of internal friction $\varphi = 35°$. It is assumed that the backfill is not surcharged.

The earth-reinforcements consist of aluminum strips of 6 cm width and 2 mm in thickness. The elastic limit of the aluminum is 2.5×10 kPa. In order to account for the effects of corrosion and for the safety-factor criteria, ($\sigma'_{adm} = 2/3\,\sigma'_e$), only half of the cross-section of each strip is assumed to be effective which counterbalances the area reduction in the strip connections.

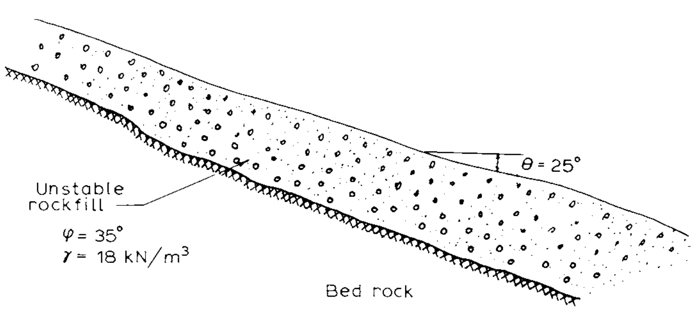


Fig. 7.26.

The technology of the wall surface elements imposes the following added restrictions: The strip layers are laid 0.25 m apart. The reinforcement can only be attached every 50 cm to the wall panels.

Design the wall to meet the safety-factor criteria. Assume all backfill and fill to be sand.

The following assumptions are necessary to determine the internal stability of the wall:

— the principal stresses near the wall skin are horizontal and vertical;

— the vertical stresses in the wall mass along any line of elevation h is uniform over a width $L - 2e$, where e is the eccentricity of the resultant of the forces acting at that elevation. The coefficient of friction between the soil and the reinforcement is 0.2.

Because of the spacing of the tie points between reinforcing and wall skin (every 0.50 m), it is necessary to attach a larger number of strips than strictly required. Calculate the corresponding safety factor which, in any event, cannot be less than 1.5.

Solution

(1) The stability of reinforced-concrete walls would have been very difficult to guarantee because they would have imposed heavy, concentrated loads on the foundation soils. It would have been necessary to anchor the foundation into the underlying bedrock. Small movements in the unstable soils above would have sheared the anchors. Retaining-structures of reinforced earth, however, can be supported directly by unstable masses because they can withstand small deflections.

(2) There are basically 3 types of problems related to the stability of a reinforced-earth wall:

(a) The overall wall-mass stability of the slope. This problem is the same as that encountered with reinforced-concrete walls. It can be analyzed by the 'circular slide' method. For the present problem, it is assumed that this overall stability has already been assessed.

(b) The wall stability under the lateral pressure of the fill. This is similar to the classical retaining-wall problem (external stability).

(c) The problem of internal stability of the wall that determines the dimensions and the spacing of the reinforcements.

(3) *External stability.* Since we have assumed that the wall has an overall stability, let us look at its *external* stability.

Assuming a Rankine equilibrium state behind the wall, the earth pressure on the vertical face is horizontal. We then have:

Active pressure: $P = k_a \cdot \gamma(H^2/2)$

$$\varphi = 35°, \quad \delta = 0, \quad k_a = 0.27$$

$$P = 0.27 \times 18(20^2/2) \simeq 970\,\text{kN}.$$

The resultant of the forces applied to the foundation of the wall will have the following components:

horizontal = 970 kN, vertical = $\gamma \cdot H \cdot L$ = 5760 kN.

The resultant will act at a distance e from the center of the footing so that $e = (970 \times 6.6)/5750 = 1.1$ m.

Since the resultant falls within the middle third of the wall footing, the wall is safe against overturning.

If we assume a coefficient of friction of 0.3 between the wall and the foundation soil, the safety factor against sliding will be:
$(5750 \times 0.3)/970 = 1.8$, which is satisfactory.

A failure through punch is not likely because of the relatively high angle of internal friction of the foundation soil, $\phi = 35°$. The external stability of the wall is satisfactory.

(4) *Internal stability*

(a) Tension in the reinforcement. For determining the *internal* stability, we have to consider the tension stresses in the reinforcements and the length of the reinforcing elements. As for the tension stresses, we must first evaluate the vertical stresses acting at a depth of h from the top of the wall (Fig. 7.27). The vertical stress is due to the overburden above h and to the earth pressure of the fill being retained. The resultant of the forces applied at this level has the following components (per m length of wall):

$R_v = W = \gamma h L$ along the vertical,

$R_h = P = k_a \gamma (h^2/2)$ along the horizontal,

the eccentricity of the resultant is:

$$e = \frac{k_a h^2 \gamma h/3}{2\gamma h L} = \frac{k_a h^2}{3 \times 2 \times L}$$

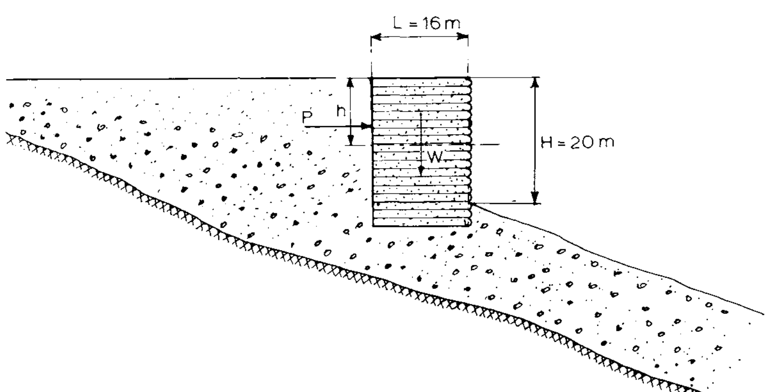


Fig. 7.27.

In accordance with Meyerhof's hypothesis, we assume that the stress distribution is uniform over a width $L - 2e$. The magnitude of the vertical stress σ_v is:

$$\sigma_v = \frac{\gamma h L}{L - 2e} = \frac{\gamma h L}{L - k_a(h^2/3L)}$$

If we assume that the soil at the contact with the wall skin is in a state of active pressure, then the horizontal stress is: $\sigma_h = k_a \sigma_v$. If we now assume that the line of reinforcement at depth h is designed to withstand all the horizontal stresses at that level and above height Δh, the tension in the reinforcing elements must be equal to:

$$T = \sigma_h \times \Delta h = k_a \sigma_v \Delta h = k_a \gamma h \frac{\Delta h}{1 - (1/3)k_a(h/L)^2}$$

for a wall length of 1 m.

The maximal reinforcing tensions will occur at the bottom of the wall. The tensions there are:

$h = H = 20\,\text{m}$, $L = 16\,\text{m}$, $\gamma = 18\,\text{kN/m}^3$, $k_a = 0.27$, $\Delta h = 0.25\,\text{m}$.

Thus $T = 0.27 \times 18 \times 20 \times 0.25/[1 - (0.27/3)(20/16)^2] = 28.3\,\text{kN}$

For a length of 1 m of wall, it will be necessary to design the reinforcement to withstand a tensile stress of 28.3 kN.

The cross-sectional area of each reinforcing element, assuming each to be 6 cm wide and 2 mm thick, and taking into account the safety factor, will be: $\frac{1}{2}(6 \times 10^{-2} \times 2 \times 10^{-3}) = 6 \times 10^{-5}\,\text{m}^2$. Each element can resist a tension of: $6 \times 10^{-5} \times 250 \times 10^6 = 15\,\text{kN}$.

At the bottom of the wall, one element will have to be placed every 50 cm. Let us now compute the height h, from which only one element per m will be required.

We have: $15 = 0.27 \times 18 \times h_1 \times 0.25/[1 - (0.27/3)(h_1/16)^2]$.

For h less than 16 m, the term $(0.27/3)(h_1/16)^2$ may be neglected. A simple calculation leads to: $h_1 = 12.0\,\text{m}$.

One reinforcing element per m will suffice for a height of 8 m upward and one element every 2 m from 14 m and up.

The tension diagram for the entire wall, 1 m in length is shown on Fig. 7.28.

The safety factor obtained with this design is:

F = area $ABCDEFGO$/area $ACEHO$:

Area $ABCDEFGO = 30 \times 8.5 + 15 \times 5.5 + 7.5 \times 6 = 255 + 83 + 45 = 383$, the area $ACEHO$ varies little from the area of triangle AHO or: $30 \times (20/2) = 300$.

Therefore the safety factor is: $F = 383/300 = 1.28$.

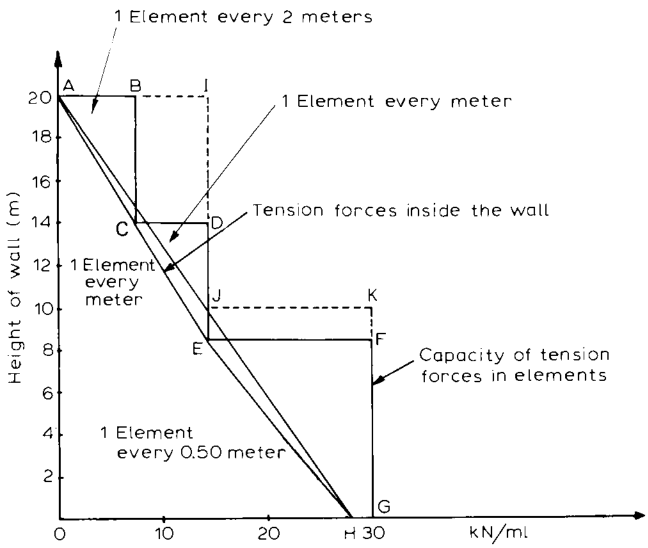


Fig. 7.28.

This coefficient is too low. To increase it, it is for instance possible to place strips at 0.50 m intervals half-way up the wall and at meter intervals in the upper part of the wall.

The safety factor becomes:

$$F = \frac{AIJKG}{AOH} = \frac{30 \times 10 + 15 \times 10}{300} = \frac{450}{300} = 1.5,$$

which is acceptable.

(b) *Length* of reinforcing elements. The vertical stress σ is very close to the value of γh. If f is the coefficient of friction between the soil and the reinforcing element and b is the element width, the adhesion requirement is: $T < 2bf\gamma hL_a$, where T is the tensile force on the strip and L_a its length over which adhesion acts. Therefore we need:

$$L_a > \frac{T}{2bf\gamma \cdot h} \simeq \frac{k_a \Delta h}{2bfn}$$

where n is the number of elements per meter.

Usually the value of 0.2 is assumed for the friction coefficient between soil and strip, and thus:

$$L_a > \frac{0.27 \times 0.25}{2 \times 6 \times 10^{-2} \times 0.2 \times 1} = 2.8\,\text{m}$$

This length should be added to the width of Coulomb's edge, at the elevation h, i.e.:

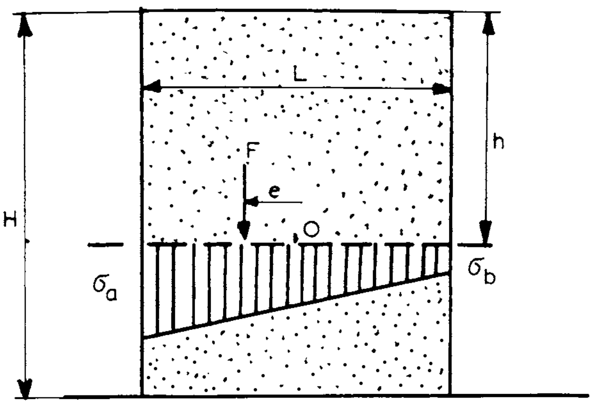


Fig. 7.29.

$$L_c = (H-h)\tan\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)$$

for $h = 0$: $(L_c)_{\max} = 10.4\,\text{m}.$

Therefore, the maximum length of the reinforcing elements is: $2.80 + 10.40 = 13.20\,\text{m}$. This condition is fulfilled since the wall's thickness is 16 m.

Remark

The assumptions made above to calculate the tension in the reinforcing strips are somewhat arbitrary and other assumptions could be made, in particular for the calculation of the tensions in the strips (see Fig. 7.29), where a trapezoidal vertical stress distribution is assumed over the width of the wall. Force F is the vertical component of the resultant force applied at the level h, and is in equilibrium with the stresses of trapezoidal distribution:

$$F = \frac{L}{2(\sigma_1 + \sigma_2)}, \qquad F \cdot e = \tfrac{1}{2}(\sigma_a - \sigma_b)L \times (\tfrac{1}{2}L - \tfrac{1}{3}L)$$

or: $F \cdot e = (\sigma_a - \sigma_b)L^2/12$

which gives: $\sigma_a + \sigma_b = 2F/L, \quad \sigma_a - \sigma_b = 12F \cdot e/L^2$

from which: $\sigma_a = (F/L)(1 + 6e/L), \quad \sigma_b = (F/L)(1 - 6e/L).$

The vertical component of the resultant, F, is γhL, and its eccentricity:

$e = k_a h^2/6L$, from which:

$\sigma_a = \gamma \cdot h[1 + k_a(h/L^2]$ and $\sigma_b = \gamma \cdot h[1 - k_a(h/L)^2]$.

If we assume, as we did above, that the soil pressure against the skin is active, then the horizontal pressure:

$\sigma_H = k_a \sigma_v = k_a \sigma_a = k_a \gamma \cdot h[1 + k_a(h/L)^2]$,

while the maximum stress at the base of the wall is:

$k_a \cdot \gamma \cdot H[1 + k_a(H/L)^2]$

The tension in a row of strips at the wall base, per meter length of wall is:

$$T_{max} = k_a \gamma \cdot H \cdot \Delta H[1 + k_a(H/L)^2]$$
$$= 0.27 \times 18 \times 20 \times 0.25[1 + 0.27(20/16)^2] = 34.55 \text{ kN}.$$

The magnitude found on the basis of this assumption, therefore is about 20% higher than in the previous assumption of uniform stress over width $L - 2e$: $T_{max} = 28.3$ kN.

Calculation of the tension force in the reinforced strips by the method of 'Coulomb's wedge'

This method consists in considering the triangle of reinforced earth bounded by the potential rupture planes AC passing through the basis of the wall (Fig. 7.30).

It is assumed in the method that the soil between the strips is in a plastic equilibrium along the potential failure plane.

The forces acting on the prism are:

— the weight of the soil wedge: $W = \frac{1}{2}\gamma H^2 \cot\theta$;

— reaction R of the soil on plane AC (this reactant is inclined by an angle φ with respect to the normal AC);

— the total tension force T_t in the strips at the different intersection points with AC (this force is horizontal).

The equilibrium of the three forces requires that:

$$T_t = \tfrac{1}{2}\gamma H^2 \cot\theta \cdot \tan(\theta - \varphi)$$

T_t is a function of angle θ. This function has a maximum for $\partial T_t/\partial\theta = 0$, which gives $\theta = \frac{\pi}{4} + \frac{\varphi}{2}$,

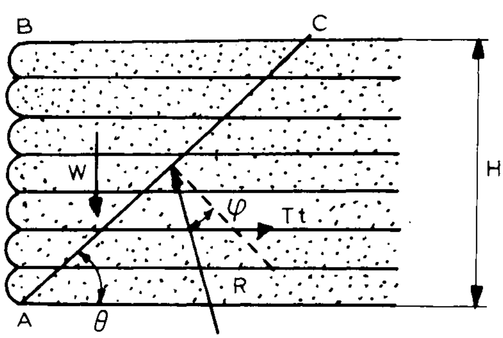


Fig. 7.30.

from which:

$$T_{t\max} = \frac{1}{2}\tan^2\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)\gamma H^2 = \frac{1}{2}k_a\gamma H^2$$

One more assumption can be made regarding the distribution of tension forces in the reinforcing strip, namely that it is triangular. The forces that undergo the greatest tension are the ones located near the base of the wall. For a wall length of 1 m, the tension in the bottom row of strip is:

$$T = k_a\gamma H\Delta H = 0.27 \times 18 \times 20 \times 0.25 = 24.3\,\text{kN}.$$

This tension is about 15% less than that calculated with the assumption of a uniform stress distribution over the width $L - 2e$: $T = 28.3\,\text{kN}$.

★★★*Problem 7.11* Design of a reinforced earth retaining-wall with a reinforced concrete skin and a sloped, surcharged backfill

The dimensions of the reinforced earth structure are shown on Fig. 7.31, and the skin consists of reinforced concrete slabs of the type shown in Fig. 7.32. The internal angle of friction of the fill is 35°, its unit-weight is $\gamma_1 = 20\,kN/m^3$. *The horizontal portion of the backfill supports a uniformly distributed load of 10 kPa.*

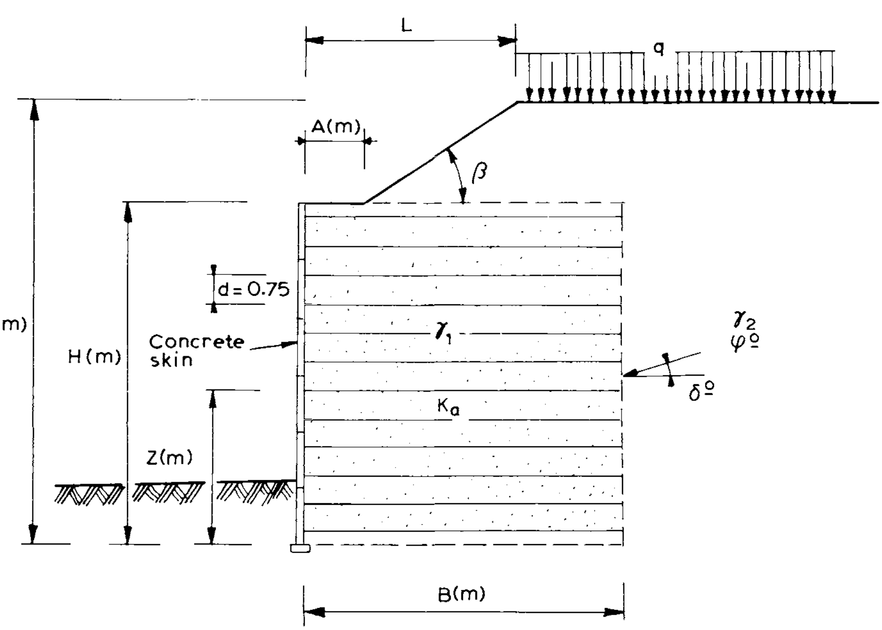


Fig. 7.31.

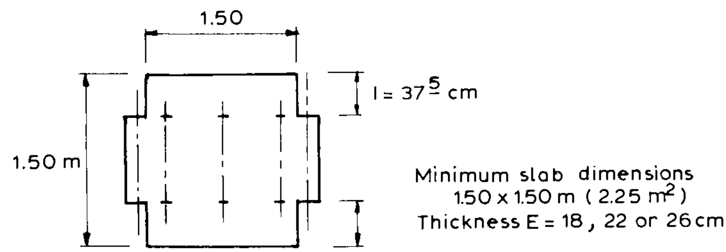


Fig. 7.32.

The reinforcing strips are smooth and made of galvanized steel, 60 × 3 and 80 × 3 mm. For considering corrosion loss, assume thickness of the strips to be 2 mm. The allowable tensile stress σ_a' is less or equal to 2/3 of the elastic limit or 1.6×10^5 kPa.

To calculate the strip spacings, assume the gross cross-section, taking into account that experimental results on models and full-size test walls indicate that the maximum tension in the strips occurs some distance away from the skin. The coefficient of friction between soil and strips is $f = 0.4$.

Assume the coefficient of active pressure of the soil backfill is $k_a = 0.30$ and its unit weight $\gamma_2 = 20\,kN/m^3$. Determine:

(1) The stresses imposed on the foundation soils by the reinforced-earth mass.

(2) The stresses taken by the reinforcing strips at the various rows.

(3) The safety factor F against friction between soil and strips.

(4) The thickness of the reinforced-concrete slabs (skin) assuming, that the allowable tensile stress in the concrete is $\sigma_b = 500$ kPa.

Assume that the inclination of the active force behind the wall is $\delta = 0$ and use Meyerhof's formula to calculate the vertical stress in the foundation soil. Verify that the safety factor against adherence is at least 2.

Solution

(1) Stresses transmitted to the foundation soil

The coefficient of earth pressure in the soil behind the retaining-structure is calculated from the Rankine formula:

$$K_{a\gamma} = k_{a\gamma} = \tan^2\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) = \tan^2(45^\circ - 17.5^\circ) = 0.27.$$

Since $\beta = \lambda = 0$, $\quad K_{aq} = K_{a\gamma} = k_{a\gamma} = k_{aq} = K$.

All computations are made for a wall length of 1 m without taking into account the soil in front of the wall. For the buried section at the toe, the concrete slabs and strips will be placed as determined by the lowest wall portion. Using the notations of Fig. 7.33, the horizontal component of the earth pressure is:

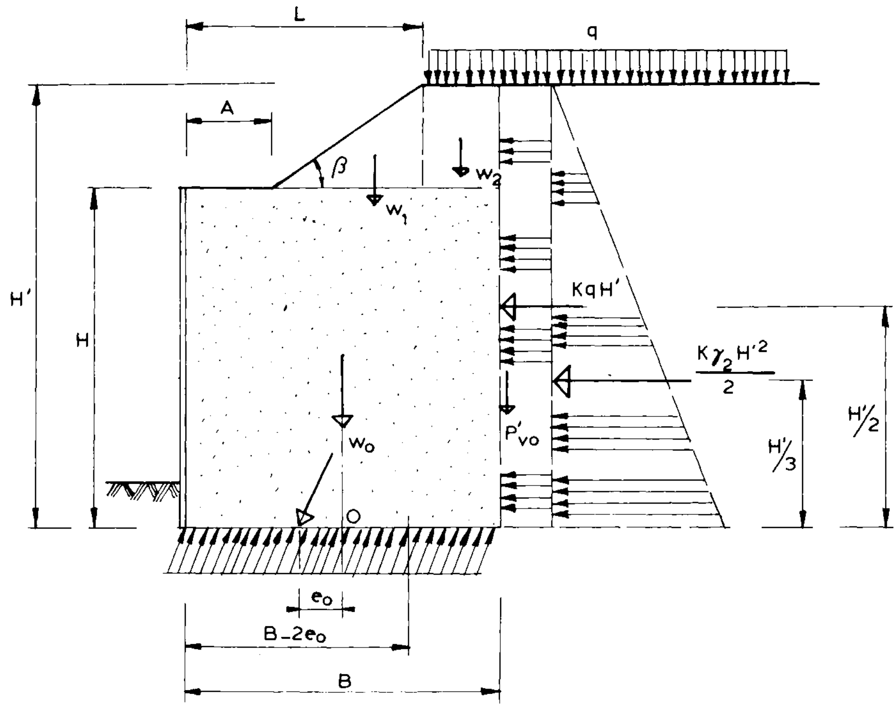


Fig. 7.33.

$$P_{ho} = \tfrac{1}{2}K\gamma_2 H'^2 + KqH'.$$

Total vertical load is:

$$W_0 = w_0 + w_1 + w_2 + P_{vo}$$
$$= BH\gamma_1 + \tfrac{1}{2}(L-A)(H'-H)\gamma_2 + (B-L)[q + (H'-H)\gamma_2] + P_{vo}$$

where: $L = A + (H' - H)\cot\beta$ and P_{vo} = the vertical component of the earth force $= P_{ho}\tan\delta$.

The overturning moment with respect to O is:

$$M_{ro} = \tfrac{1}{6}K\gamma_2 H'^3 + \tfrac{1}{2}KqH'^2.$$

The righting moment with respect to O:

$$M_{so} = w_1\,\frac{2A + 4L - 3B}{6} + w_2\,\frac{L}{2} + P_{vo}\,\frac{B}{2},$$

$$M_{so} = \frac{1}{2}\left[(2A + 4L - 3B)\,\frac{w_1}{3} + Lw_2 + BP_{vo}\right]$$

from which the resulting moment is: $M_0 = M_{so} - M_{ro}$.

The eccentricity of the resultant with respect to the middle of the width of the wall mass is: $e_0 = M_0/W_0$.

With Meyerhof's formula, the vertical stress in the foundation soil is: $\sigma_m = W_0/(B - 2e_0)$.

The safety factor against sliding of the base depends on the ratio P_{ho}/W_0.

(2) Stresses taken by the reinforcing strips

At level z of a row of strips, the weight W of the wall section located above the row is: $W = w + w_1 + w_2 + P_v$:
or:

$$W = B(H - z)\gamma_1 + \frac{L - A}{2}(H' - H)\gamma_2 + (B - L) \times [q + (H' - H)\gamma_2] + P_v$$

with $P_v = P_h \tan\delta$ and where P_h is the horizontal component of the pressure exerted on this section, or:

$$P_h = \tfrac{1}{2}K\gamma_2(H' - z)^2 + Kq(H' - z)$$

The positive moment with respect to the middle of the wall base is:

$$M_1 = \tfrac{1}{6}K\gamma_2(H' - z)^3 + \tfrac{1}{2}Kq(H' - Z)^2$$

and the negative moment is:

$$|M_2| = \tfrac{1}{2}[(2A + 4L - 3B)\tfrac{1}{3}w_1 + Lw_2 + BP_v],$$

from which the resulting moment is:

$$M = M_1 - |M_2|$$

with $e = M/W$, and in accordance with Meyerhof's formula, the vertical principal stress in the wall at this level is: $\sigma_1 = W/(B - 2e)$.

The horizontal stress, $\sigma_3 = k_a\sigma_1$, is balanced by the reinforcing strips located at that level. These strips are in a row containing N strips of width b for 2.25 m^2 (slab area) or n per m^2, in such a manner that tension T in one strip is: $T = \sigma_3/n = 2.25\,\sigma_3/N$ per m^2 and the stress in the strip is: $\sigma_a = T/\omega < \bar{\sigma}_a'$ where ω = cross-sectional area and $\bar{\sigma}_a'$ = allowable stress.

(3) The safety factor for adherence

This factor, $F = B/L_m \geqslant 2$
where $L_m = T/2b \cdot \sigma \cdot f$ (minimal length) and σ is the vertical stress in the strip or:

$$\sigma = W/B \text{ and } F = 2bBf(N/2.25)(\sigma/\sigma_3) \geqslant 2.$$

(4) Thickness of the reinforced-concrete slabs (skin)

The tensile stress in the concrete over a width l above the strip row which constitutes the tie of the slab must be calculated (Fig. 7.32).

Two conservative assumptions are made:

(a) the horizontal stress exerted on the slab is equal to σ_3 as invalidated by experimental observations;

(b) the compressive stress due to the weight of the skin is considered equal to σ_1, which is less than the weight of the concrete skin, the flexural moment is: $m = \sigma_3 l^2/2$,
the section modulus is: $I/v = E^2/6$ (per unit-length of wall)
with E is skin thickness.

$$\sigma_b = mv/I - \sigma_1 = 3\sigma_3(l^2/E^2) - \sigma_1 \quad \text{or:}$$

$$\sigma_b = [3(l^2/E^2) - (1/0.3)]\sigma_3 \quad \text{because } \sigma_1 = \sigma_3/0.3.$$

Take $E = 18$ cm, when $\sigma_3 < 52$ kPa, then:

$$\sigma_b = \left[3\left(\frac{37.5}{18}\right)^2 - \frac{1}{0.3}\right]52 = 503.7\,\text{kPa} \simeq 500\,\text{kPa}$$

or: $E = 22$ cm, when $52\,\text{kPa} < \sigma_3 < 92.5\,\text{kPa}$

$$\sigma_b = \left[3\left(\frac{37.5}{22}\right)^2 - \frac{1}{0.3}\right]92.5 = 498\,\text{kPa} < 500\,\text{kPa}$$

or: $E = 26$ cm, when $92.5 < \sigma_3 < 165$ kPa

$$\sigma_b = \left[3\left(\frac{37.5}{26}\right)^2 - \frac{1}{0.3}\right]165 = 479.7\,\text{kPa} < 500\,\text{kPa}.$$

Numerical applications are shown on Table 7C.

Remark:

At present, the design of reinforced-earth structures is being modified with the tendency to reach limit-state conditions. These results are very close to those obtained by the above described "classical" method.

TABLE 7C

I. Givens:

Geometry:

H (m)	H' (m)	A (m)	$\cot\beta$	B (m)	q (kPa)
9.75	14.00	0.80	1.50	9.00	10

Fill properties:

γ_1 (kN/m^3)	k_a	γ_2 (kN/m^3)	φ (°)	δ (°)	K
20	0.30	20	35.00	0.00	0.27

TABLE 7C (continued)

II. Loads transmitted to the foundation soil:

P_{ho} (kN) (per m length)	W_0 (kN) (per m length)	$P_{ho} W_0$	e_0 (m)	$B - 2e_0$ (m)	σ_m (kPa)
567	2199.3	0.26	0.90	7.21	305.2

III. Slab and strip designs:

Level of strip lines z (m)	Horizontal stresses σ_3 (kPa)	Skin thickness E (cm)	Strips		Vertical stresses σ_v (kPa)	Safety factor with respect to adherence F
			No.	Section		
0.38	87	22	8	80 × 3	236.8	5.57
1.13	78.6	22	7	80 × 3	221.8	5.06
1.88	70.8	22	7	80 × 3	206.8	5.23
2.63	63.5	22	6	80 × 3	191.8	4.64
3.38	56.7	22	5	80 × 3	176.8	3.99
4.13	50.3	18	5	80 × 3	161.8	4.12
4.88	44.2	18	4	80 × 3	146.8	3.40
5.63	38.5	18	4	80 × 3	131.8	3.51
6.38	33.0	18	4	60 × 3	116.8	2.72
7.13	27.7	18	4	60 × 3	101.8	2.82
7.88	22.7	18	4	60 × 3	86.8	2.93
8.63	17.9	18	4	60 × 3	71.8	3.08
9.38	13.3	18	4	60 × 3	56.8	3.29

Chapter 8

SHEETPILE WALLS

★★★*Problem 8.1* Design of a sheetpile wall. Comparison of two design methods: the classical method of plasticity, and the design by elasto-plasticity

The sheetpile wall shown on Fig. 8.1 must be designed. A row of anchors, not shown on the figure but located between points A and B, should be incorporated for stability of the upper section. The design of these anchors and their effect are not to be considered in this problem.

The soil has the following characteristics:

— *dry unit-weight: 18 kN/m*^{3}
— *buoyant unit-weight: 11 kN/m*^{3}
— *angle of internal friction:* $\varphi = 33°$
— *cohesion* $c = 0$.

Active and passive earth pressure coefficients are: $K_a = k_a = 0.296$; $K_p = k_p = 6.81$; $(\delta = 0)$.

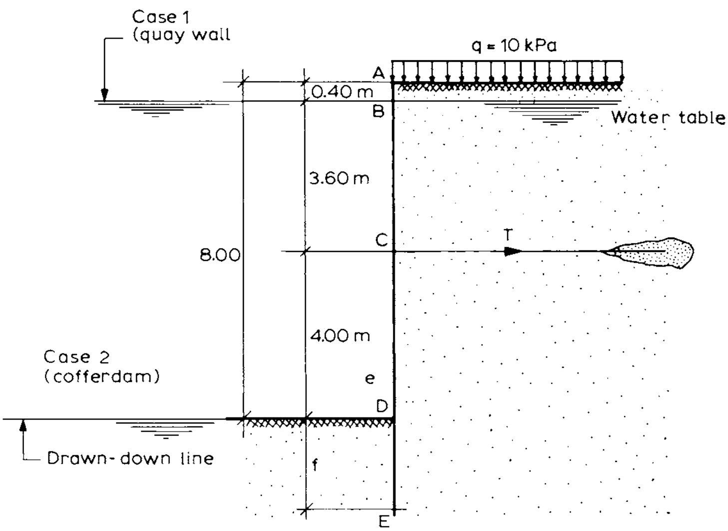


Fig. 8.1.

In addition, the soil behind the wall supports a uniform load of 10 kPa. Passive forces act at the toe of the wall.

Find: (1) the depth of penetration of the sheets for a safety factor of 2 applied to passive forces; (2) the tension force in the anchor rod, assumed to be horizontal; (3) the maximum moment in the sheetpile.

Two conditions must be considered: case 1: the water level is the same on both sides of the sheets (quay-wall situation); case 2: the water table is drawn down to the dredge level (cofferdam situation).

Remark

At the end of the solution, the results of the Rido computer program are presented for the elasto-plastic method.

Solution

Method 1, Plasticity method (classical method): case 1. Water level is the same on both sides of the sheets.

(1) Firstly, the earth pressure diagram must be determined. Behind the wall, we have active earth pressure conditions:

— between A and B:

$\sigma_p = k_a \gamma_d h + k_q \cdot q$ at depth h, for $h \leqslant 0.40$ m,

or, since $k_q = k_a$ (because $\beta = \lambda = 0$):

$\sigma_p = k_a(\gamma_d h + q)$

— between B and E:

$\sigma_p = \sigma_B + k_a \gamma'(h - 0.40)$ at depth h, with $h > 0.40$ m.

Passive pressures act at the toe of the wall and, with a safety factor of 2, we get:

— between D and E:

$\sigma_b = \frac{1}{2} k_p \gamma' x$ at distance x from the dredged level.

The hydrostatic pressures on either side of the sheetpile cancel. The following stresses are calculated, which are shown on Fig. 8.2.

Active pressures:

$\sigma_{pA} = K_q \cdot q = 0.296 \times 10 = 2.96 \simeq 3.0$ kPa

$\sigma_{pB} = \sigma_{pA} + K_a \gamma_d h_1 = 2.96 + 0.296 \times 18 \times 0.40 = 5.09 \simeq 5.1$ kPa

$\sigma_{pD} = \sigma_{pB} + K_a \gamma' h_2 = 5.09 + 0.296 \times 11 \times 7.60 = 29.83 \simeq 29.8$ kPa

$\sigma_{pE} = \sigma_{pD} + K_a \gamma' f = 29.8 + 0.296 \times 11 \times f = 29.8 + 3.26f.$

Passive pressures:

$\sigma_{bE} = \frac{1}{2} K_p \gamma' f = 0.5 \times 6.81 \times 11 \times f = 37.46f.$

A 3rd-degree equation from the penetration f is obtained by considering that the resulting moment at point C is zero. Table 8A (counterclockwise moments are positive) summarizes the moments. The stresses apply to one unit length of wall.

Therefore, the sought equation, $\Sigma M_i = 0$, is:

$11.4f^3 + 53.5f^2 - 119.2f - 139.3 = 0.$

The only positive root of this equation is: $f = 2.275$ or say $f = 2.30$ m.

(2) The tension in the anchor is: $T = \Sigma F_i$, which is computed from the data of Table 8A, by taking $f = 2.275$ m.

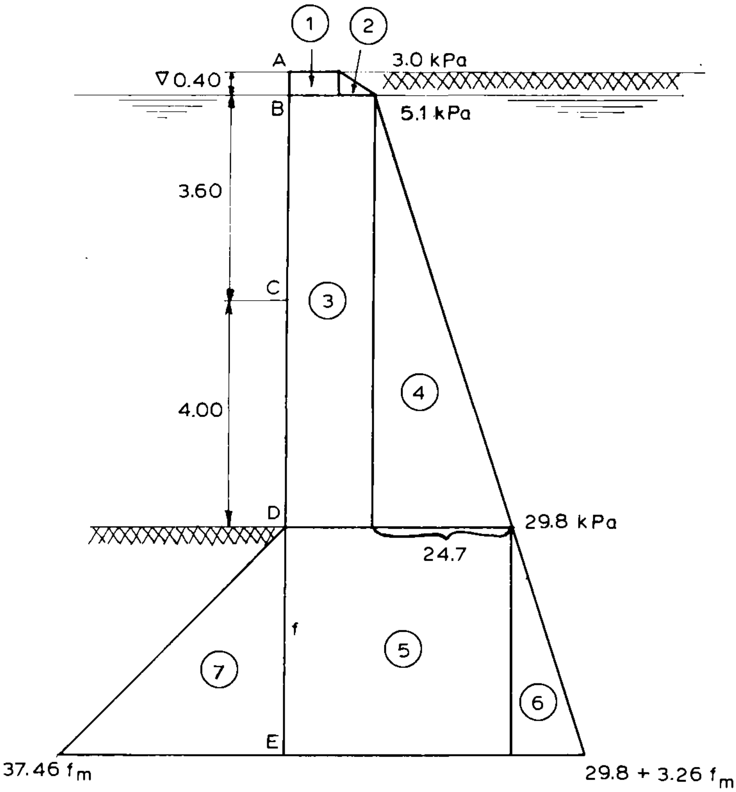


Fig. 8.2.

TABLE 8A

Zone	Resultant force F_i (kN)	Lever arm with respect to C d_i (m)	Moment with respect to C M_i (kN · m)
1	$3.0 \times 0.40 = +1.2$	$+3.80$	$+4.56$
2	$\frac{1}{2} \times (5.1 - 3.0) \times 0.40 = +0.42$	$+3.73$	$+1.57$
3	$5.1 \times 7.60 = +38.76$	-0.20	-7.75
4	$\frac{1}{2} \times (29.8 - 5.1) \times 7.60 = +93.86$	-1.47	-137.69
5	$+29.8f$	$-4.00 - \frac{1}{2}f$	$-119.20f - 14.90f^2$
6	$\frac{1}{2} \times 3.26f \times f = 1.63f^2$	$-4.00 - \frac{2}{3}f$	$-6.52f^2 - 1.09f^3$
7	$-\frac{1}{2} \times 37.46f \times f = -18.73f^2$	$-4.00 - \frac{2}{3}f$	$+74.92f^2 + 12.49f^3$

$$T = 1.20 + 0.42 + 38.76 + 93.86 + 29.8 \times 2.275 + $$
$$+ (1.63 - 18.73) \times \overline{2.275}^2 \simeq 113.5\,\text{kN}$$

or $T = 114\,\text{kN}$ per m of length.

Remark. We would have found $T = 112$ kN with $f = 2.30$ m.

(3) We know that the *maximal bending moment* occurs near the level of the dredged line, therefore, between C and D (Fig. 8.2). The maximal moment occurs where shear is zero. The shear diagram is calculated from the bottom of the sheets and with the notations shown on Fig. 8.3, we have: $T_D = F_5 + F_6 + F_7$, and $F_5 = 29.8 \times f = +67.80$ kN, $F_6 = 1.63 \times f^2 = +8.44$ kN, $F_7 = -18.73 \times f^2 = -96.94$ kN.

So $T_D = +67.80 + 8.44 - 96.94 = -20.70$ kN.

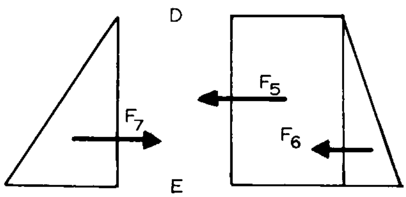


Fig. 8.3.

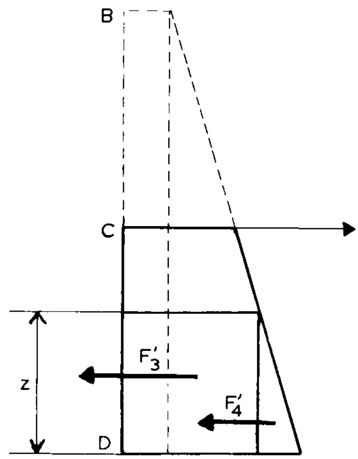


Fig. 8.4.

It is easy to verify that T_D is negative, which corresponds to $|F_7| > F_5 + F_6$: the maximal bending moment occurs between C and D as anticipated.

Let Z (Fig. 8.4) be the elevation of the pile section under study above the dredge level. Then we have, following the notation of Fig. 8.4:

$$F'_3 = [29.8 - (24.7/7.60)Z] \cdot Z = 29.8Z - 3.25Z^2$$

and: $F'_4 = \frac{1}{2}Z \cdot (24.7/7.60)Z = 1.625Z^2$

Therefore: $F'_4 + F'_3 = 29.8Z - 1.625Z^2$. Condition $T(Z) = 0$ can be written as: $T_D + F'_3 + F'_4 = 0$, or: $-1.625Z^2 + 29.8Z - 20.7 = 0$.

The root of this equation lies between the values 0 and 4:

$Z = 0.723$, say 0.72 m.

From this we derive: $F'_3 = 19.90$ kN, $F'_4 = 0.86$ kN and:

$$M(Z) = |F_7| \times \left(\frac{2f}{3} + Z\right) - F_5\left(\frac{f}{2} + Z\right) - F_6\left(\frac{2f}{3} + Z\right) - F'_3\,\frac{Z}{2} - 2F'_4\,\frac{Z}{3}.$$

With $f = 2.275$ and $Z = 0.72$, $M(Z) = 64.5$ kN $\cdot$ m (per m of sheetpile wall) say:

$M_{\max} = 65$ kN $\cdot$ m.

Case 2. Cofferdam-condition.

A cofferdam condition prevails when the water table is drawn down to the dredge level. The calculation method for this condition is analogous to that of case 1, but now the hydrostatic pressure must be taken into account since the water is present on one side of the sheetpiles.

(1) Pressure diagram acting on the wall *behind* the sheetpiles (Fig. 8.5):

— Between A and B: $\sigma_p = K_a(\gamma_d h + q)$ for $0 \leqslant h \leqslant 0.40\,\text{m}$.

— Between B and D: $\sigma_p = \sigma_B + K_a\gamma'(h - 0.40) + \gamma_w(h - 0.40)$.

The term $\gamma_w(h - 0.40)$, represents the hydrostatic pressure.

— Between D and E: $\sigma_p = \sigma_D + K_a\gamma' x$,

where x is the elevation from the dredge level: $x = h - 8.00$.

In front of the sheet piles, it is not necessary to account for the hydrostatic pressure since it was not accounted for beneath point D behind the wall.

For a safety factor of 2 against the passive resistance, we have:

— Between D and E: $\sigma_b = \frac{1}{2}K_p\gamma' x$

where x is the distance from the dredge level.

The following values were obtained for the construction of the pressure diagram shown on Fig. 8.5 (it was assumed that $\gamma_w = 10\,\text{kN/m}^3$).

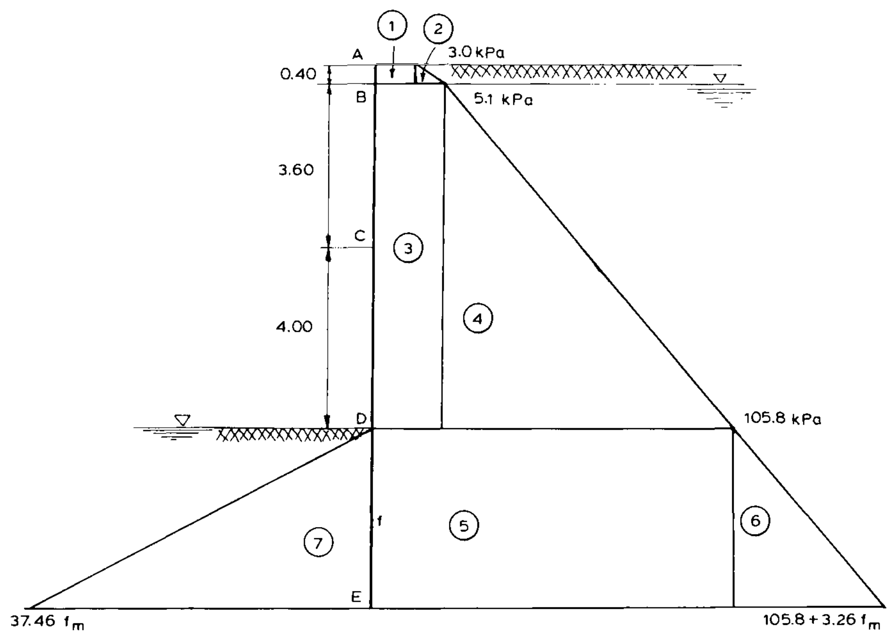


Fig. 8.5.

Active pressures:

$$\sigma_{pA} = K_a q = 0.296 \times 10 = 2.96 \simeq 3.0\,\text{kPa}$$

$$\sigma_{pB} = \sigma_{pA} + K_a \gamma_d h_1 = 2.96 + (0.296 \times 18 \times 0.40) = 5.09 \simeq 5.1\,\text{kPa}$$

$$\sigma_{pD} = \sigma_{pB} + K_a \gamma' h_2 + \gamma_w h_2 = 5.09 + (0.296 \times 11 \times 7.60) + 10 \times 7.60$$

$$\simeq 105.8\,\text{kPa}$$

$$\sigma_{pE} = \sigma_{pD} + K_a \gamma' f = 105.8 + (0.296 \times 11 \times f) = 105.8 + 3.26f.$$

TABLE 8B

Zone	Resultant force F_i (kN)	Lever arm with respect to C d_i (m)	Moment with respect to C M_i (kN · m)
1	$3.0 \times 0.40 = +1.2$	$+3.80$	$+4.56$
2	$\frac{1}{2} \times (5.1 - 3.0) \times 0.40 = +0.42$	$+3.73$	$+1.57$
3	$5.1 \times 7.60 = +38.76$	-0.20	-7.75
4	$\frac{1}{2}(105.8 - 5.1) \times 7.60 = 382.66$	-1.47	-562.51
5	$+105.8f$	$-4.00 - \frac{1}{2}f$	$-423.20f - 52.9f^2$
6	$\frac{1}{2} \times 3.26f \times f = 1.63f^2$	$-4.00 - \frac{2}{3}f$	$-6.52f^2 - 1.09f^3$
7	$-\frac{1}{2} \times 37.46f \times f = -18.73f^2$	$-4.00 - \frac{2}{3}f$	$+74.92f^2 + 12.49f^3$

Passive pressure:

$$\sigma_{bE} = \tfrac{1}{2}K_p \gamma' f = 0.5 \times 6.81 \times 11 \times f = 37.46f.$$

Once again a third-degree equation is obtained for the embedment f. The sum of the moments about point $C = 0$, gives the value of f.

From Table 8B, the equation for the condition $\Sigma M_i = 0$ is:

$$11.4f^3 + 15.5f^2 - 423.20f - 564.13 = 0,$$

which gives: $f = 6.08$ m or $f = 6.10$ m.

The embedment in this case is deeper than when the water table is not drawn down.

(2) The tension force in the anchor is: $T = \Sigma F_i$, which is calculated from the data of Table 8B, with $f = 6.08$ m. We get then $T = 434.2$ kN, say $T =$ 434 kN.

So, the tension in the anchor is about 4 times greater than in case 1, where the water table was the same at both sides.

(3) At the bottom of the excavation, the shear force is: $T_D = F_5 + F_6 + F_7$ where:

$$F_5 = 105.8f = 105.8 \times 6.08 = 643.26\,\text{kN}$$

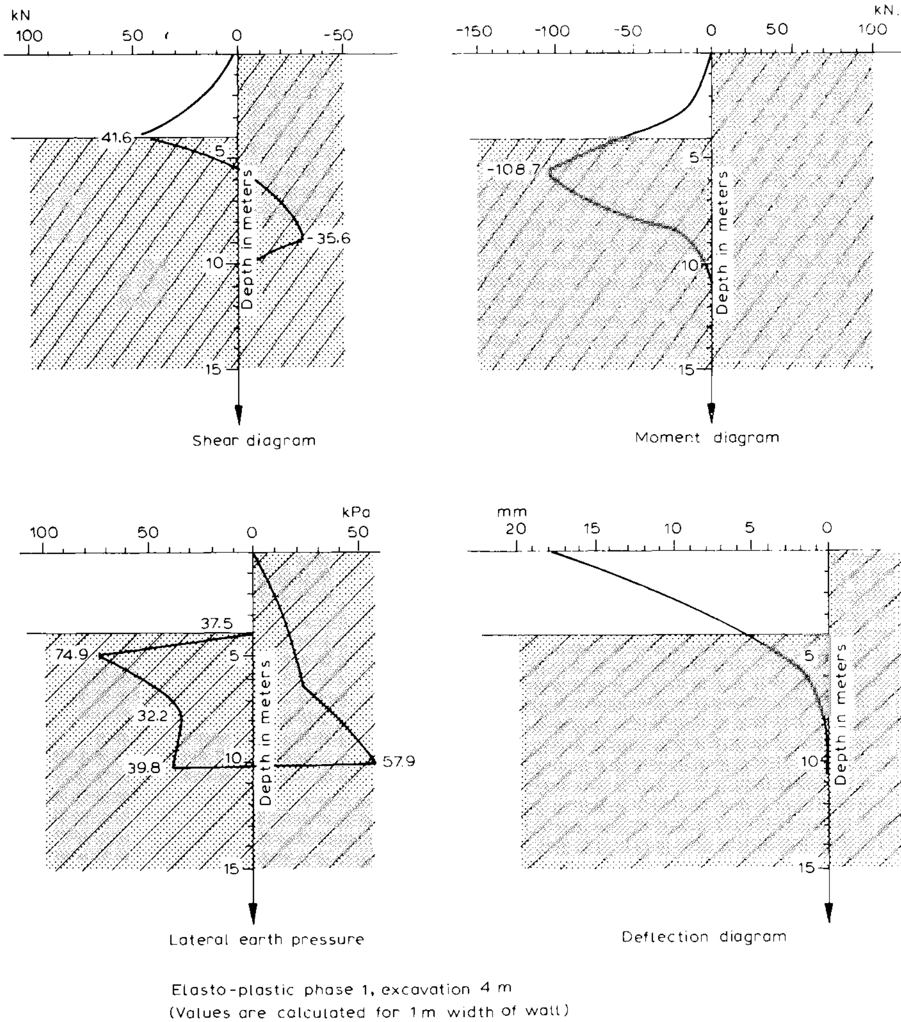


Fig. 8.6. Sheet-pile wall (for 1 m width of wall) without water drawdown (embedment 2.30 m).

$$F_6 = 1.63f^2 = 1.63 \times \overline{6.08}^2 = 60.26\,\text{kN}$$

$$F_7 = -18.73f^2 = -18.73 \times \overline{6.08}^2 = -692.38\,\text{kN, or:} \quad T_D \simeq 11\,\text{kN}.$$

It is quite nil, and the maximal bending moment may be assumed to be at that elevation.

Its value there is:

$$M_{\max} = -643.26 \times (6.08/2) - 60.26 \times (2/3) \times 6.08 + 692.38 \times \times (2/3) \times 6.08 = 606.47$$

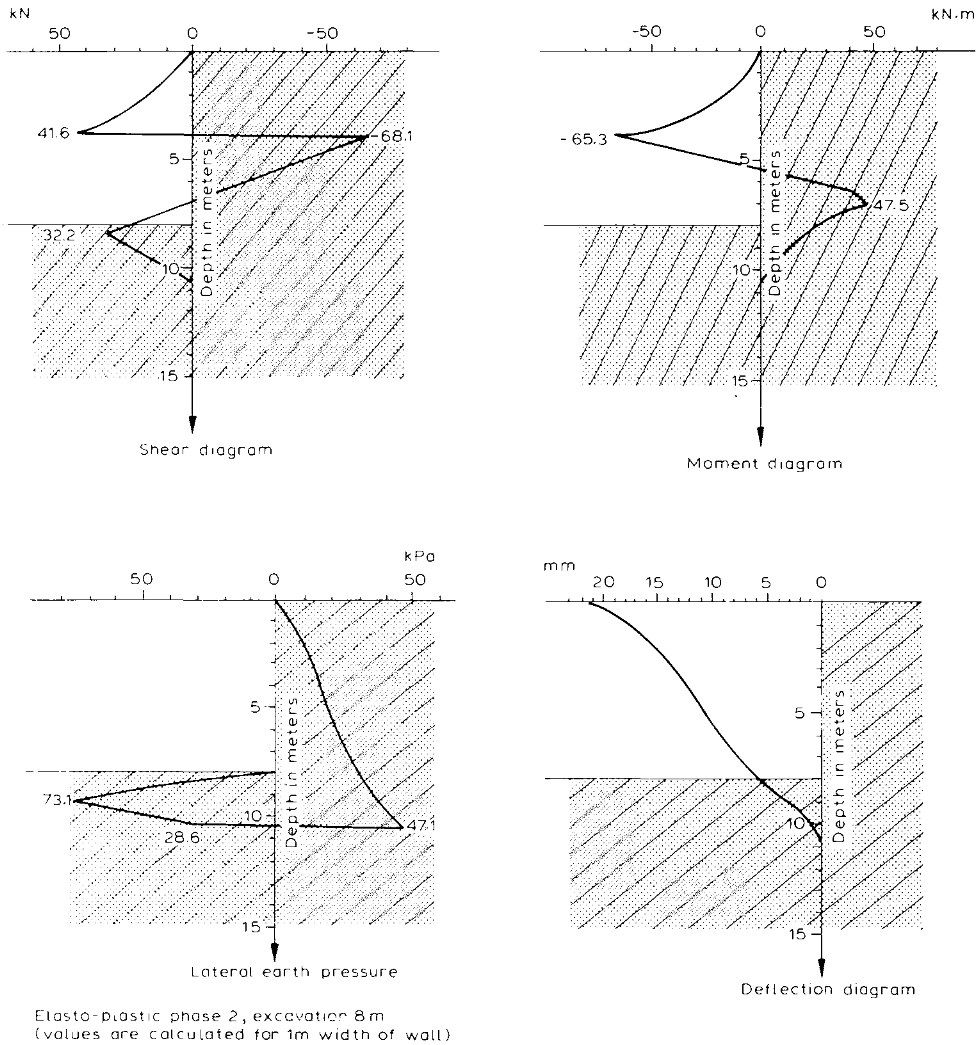


Fig. 8.7. Sheet-pile wall without water drawdown (embedment 2.30 m).

or: $M_{\max} = 606\ \text{kN} \cdot \text{m}$ (per m of length)

So the maximum bending moment is about 9 times greater than in the preceding situation, where the water table was not the same on both sides.

Elasto-plastic calculations

These computations are too complex to be carried out by hand. One must resort to a computer. We may refer here especially to the Rido program which was developed by Fages in Lyon for the subway construction.

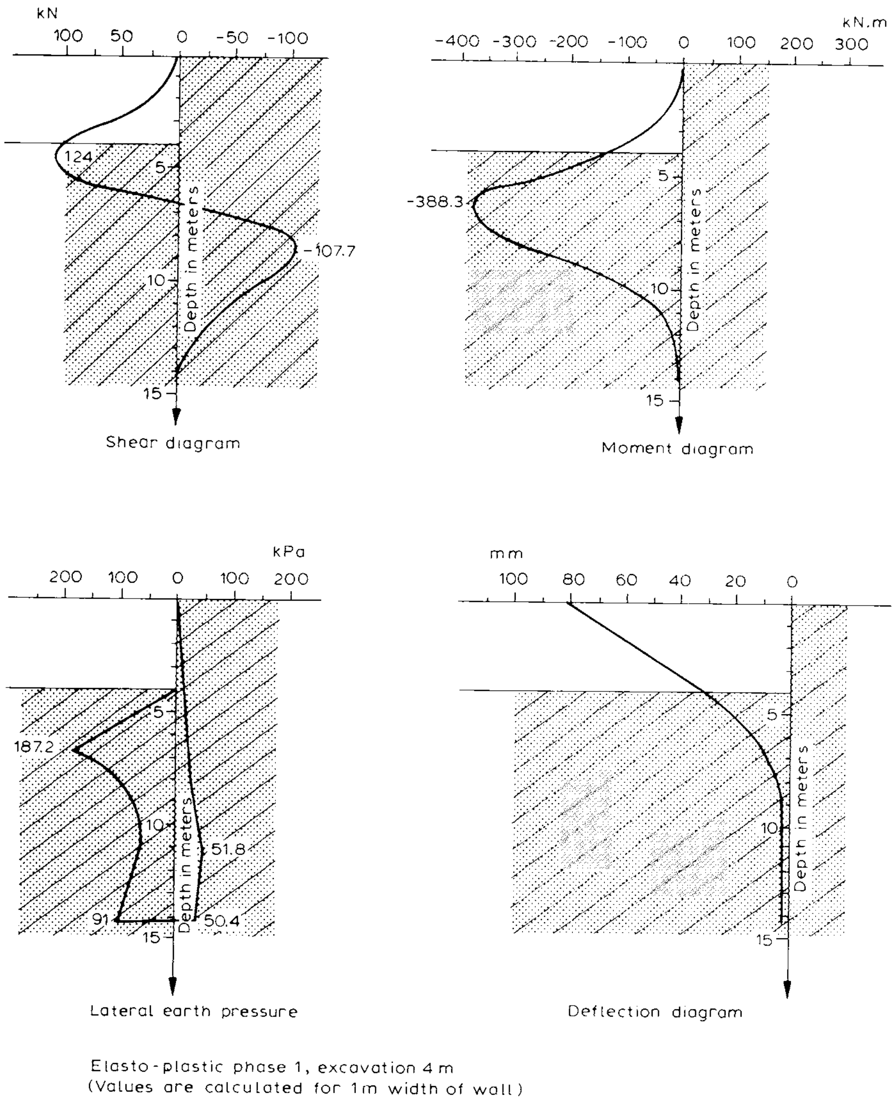


Fig. 8.8. Sheet-pile wall with water drawdown (embedment 6.10 m).

This program, that was conceived for slurry trench walls with several levels of anchors, takes into consideration deformations caused by partial excavation by slices, where anchors are placed as the excavation progresses. The program is also applicable to sheetpile walls with anchors, giving for each slice of excavation the stresses of the soil on the wall, the bending moments, shear and deflection diagrams as well as the tension in the anchors. Parts of the results of such an analysis are presented in Figs. 8.6 to 8.9. It will be

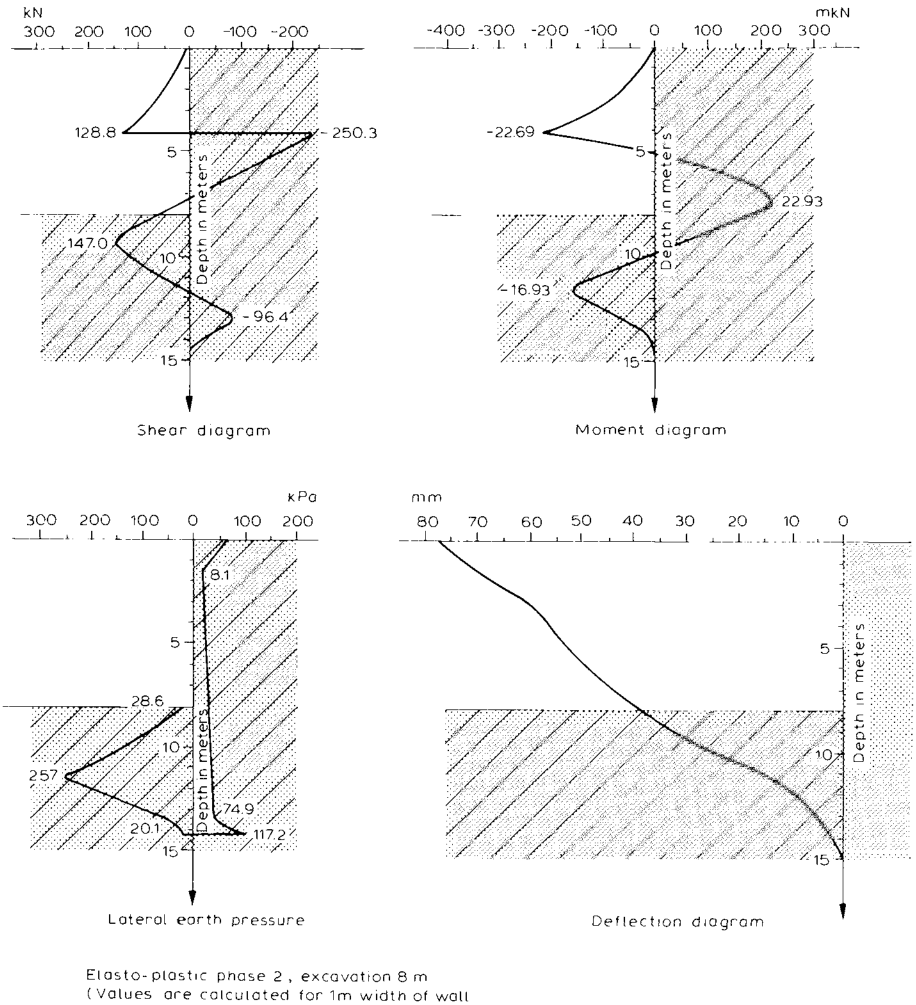


Fig. 8.9. Sheet-pile wall with water drawdown (embedment 6.10 m).

seen that they more closely approximate the real behaviour of the sheet-piles than those obtained by the above described classical method, which only considers the final stress conditions, without accounting for the deflections occurring during the phased excavation.

Remarks

Case 1. Sheetpile wall without water drawdown (Figs. 8.6 and 8.7). In elasto-plastic condition, the maximal bending moment is developed during the initial excavation phase made to install the anchors (108.7 kN · m width).

The tension in the anchor is very close to that computed by the plasticity method (109.7 kN instead of 114 kN per m of length).

Case 2. Sheetpile wall with water drawdown (Figs. 8.8 and 8.9). Once again the maximum bending moment is developed during the excavation of the first upper 4 m (388.3 kN · m per m length of wall). The tension in the anchors is very close (Fig. 8.9) to that computed by the plasticity method (379.1 kN instead of 434 kN per m length). It must be pointed out that the classical method yields somewhat more conservative results, comparing them to phase 2.

Summary of answers

Case 1: $f = 2.30$ m, $T = 114$ kN (per m length of wall)

$M_{max} = 65$ kN (per m length of wall)

Case 2: $f = 6.10$ m, $T = 434$ kN (per m length of wall)

$M_{max} = 606$ kN (per m length of wall).

★★*Problem 8.2* Design of an anchor system

The anchors of problem 8.1 are installed as shown on Fig. 8.10. Their spacing is 1.60 m.

Find:

(1) Height H of the deadman (assume a safety factor of 1 and that the groundwater table is at the backfill level).

(2) The location of the tie point of the anchor at the deadman and the maximal bending moment in the deadman.

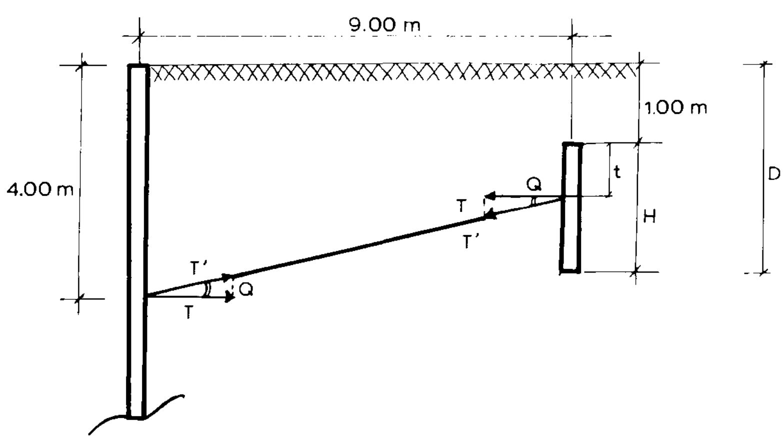


Fig. 8.10.

(3) The modulus of resistance of the anchor rod on the deadman.

(4) The cross-sectional area of the anchor rod (for the last two parts, assume the allowable stress of steel to be 1.6 × 10^5 kPa).

Solution

(1) The resistance provided by the deadman is equal to the difference between the passive earth pressure acting on the fore face and the active pressure acting on the opposite face (Fig. 8.11).

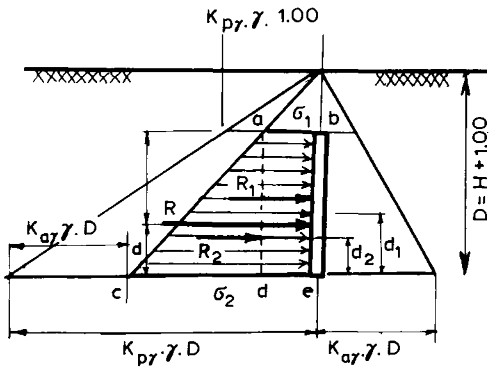


Fig. 8.11.

From the givens of problem 8.1, and assuming the water table to be at the level of the backfill, we have:

$\gamma K_{a\gamma} = 11 \times 0.296 = 3.26\,\mathrm{kN/m^3}$ and

$\gamma K_{p\gamma} = 11 \times 6.81 = 74.91\,\mathrm{kN/m^3}$.

The resultant **R** may be divided into $\mathbf{R}_1$ and $\mathbf{R}_2$ components, with $\mathbf{R}_1$ corresponding to rectangle *abed* and $\mathbf{R}_2$ to triangle *acd* (Fig. 8.11). These horizontal forces have a magnitude of:

$$R_1 = \sigma_1 \times H = [(K_{p\gamma} - K_{a\gamma})\gamma \times 1.00] \times H = 71.65 \times H$$

$$R_2 = (1/2)(\sigma_2 - \sigma_1) \times H = (1/2)[(K_{p\gamma} - K_{a\gamma})\gamma H]H = 35.83H^2$$

$$R = R_1 + R_2 = (71.65 + 35.83H)H$$

Since the deadmen form a continuous wall, the anchors transmit a shear T′ per unit length whose horizontal component T must be equal to **R**.

From the preceding problem, $T = 114$ kN per meter, therefore we have: $35.83H^2 + 71.65H - 114 = 0$. Solving for the positive root of the equation: $H = 1.045$ or:

$H = 1.05$ m.

Remark

With $H = 1.05\,\text{m}$, we have $D = 2.05\,\text{m}$ or $H > D/2$. From sect. 7.2.8 of Costet-Sanglerat, it can be seen that the method yields conservative results, since for $H > D/2$, experience shows that the structure behaves as if the deadman extended to the surface of the fill.

(2) The tie point of the anchor to the deadman must be the same as that where resultant **R** is applied. The location of that point is obtained by taking the moments of $\mathbf{R}_1$ and $\mathbf{R}_2$ with respect to the toe of the deadman wall:

$R_1 d_1 + R_2 d_2 = Rd$ (Fig. 8.11)

$d_1 = 1.05/2 = 0.525$; $d_2 = 1.05/3 = 0.35$

$R_1 = 71.65 \times 1.05 = 75.23$

$R_2 = 35.83 \times \overline{1.05^2} = 39.50$

$R = \overline{114.73}$ and

$d = (75.23 \times 0.525 + 39.50 \times 0.350)/114.73 \simeq 0.46$

from which: $t = H - d = 1.05 - 0.46 = 0.59\,\text{m}$

$t = 0.59\,\text{m}$.

The maximal bending moment in the deadman is in the section normal to the anchor point (S). Following the notation of Fig. 8.12, we get:
$M = \phi_1 \delta_1 + \phi_2 \delta_2$.

The magnitude of the stress developed at the level of section(s) by the deadman (passive—active pressures) is: $\sigma/1.59 = 71.65/1$ (cf. Figs. 8.11, 8.12), and:

$\sigma = 71.65 \times 1.59 = 113.92\,\text{kPa}$

$\phi_1 = 71.65 \times 0.59 = 42.27\,\text{kN}$

$\delta_1 = 0.5 \times 0.59 = 0.295$

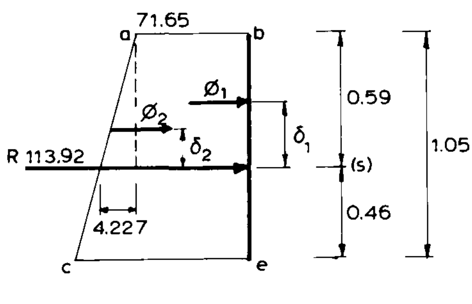


Fig. 8.12.

$\phi_2 = 0.5(113.92 - 71.65) \times 0.59 = 12.47\,\text{kN}$

$\delta_2 = (1/3)(0.59) = 0.196$

from which $M_{\max} = 42.27 \times 0.295 + 12.47 \times 0.196 = 14.9\,\text{kN}\cdot\text{m}$

$M_{\max} = 14.9\,\text{kN}\cdot\text{m}$ (per m length)

(3) From the classical hypothesis (see sect. 7.2.5, Costet-Sangletat), the maximum bending moment of the deadman wall is $M'_{\max} = AL^2/10$, where A is the tension force in the anchor for 1 m length of deadman and L is the spacing of the anchors, or: $M_{\max} = 114 \times \overline{1.60}^2/10 = 29.18\,\text{kN}\cdot\text{m}$.

Checking, $M_R = I/v \geqslant M_{\max}/\sigma_{\text{adm}}$.

Since: $\sigma_{\text{adm}} = 1.6 \times 10^5\,\text{kPa}$, $M_R \geqslant 29.18/(1.6 \times 10^5) \simeq 1.82 \times 10^{-4}\,\text{m}^3$

or $M_R \geqslant 182\,\text{cm}^3$ (per m of length).

From Table 1 of sect. 7.1.1 of Costet-Sanglerat it can be seen that a Larssen I member would suffice ($M_R = 500\,\text{cm}^3$ per m of length).

(4) Referring to Fig. 8.13, we have: $\tan\theta = (4.00 - 1.59)/9.00 = 0.268$, or: $\cos\theta = 0.966$.

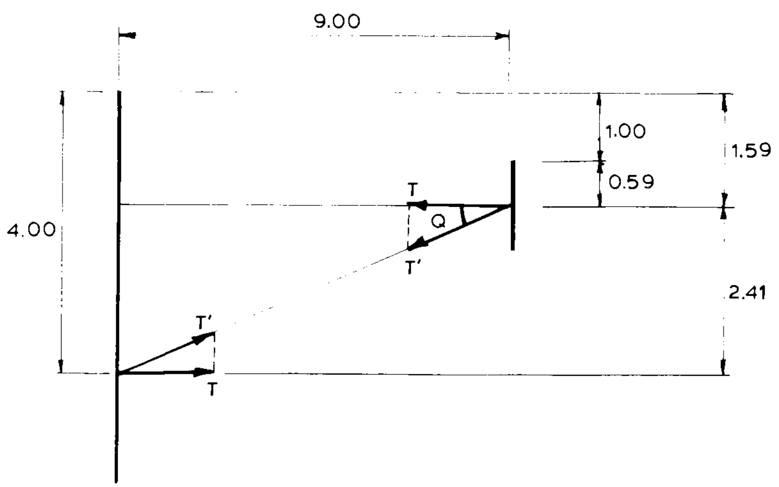


Fig. 8.13.

The force transmitted by the anchors is:

$T = T/\cos\theta = 1.14/0.966 = 118\,\text{kN}$ per m of length.

Taking into account the spacing of the anchors of 1.60 m, each anchor transmits a load of: $T'' = T' \times 1.60 = 189\,\text{kN}$. The allowable steel stress is

1.6×10^5 kPa. Hence, the cross-sectional area of steel for each anchor is:

$s = 189/(1.6 \times 10^5) = 1.18 \times 10^{-3}\,\text{m}^2 \qquad s = 11.8\,\text{cm}^2.$

Remark

For the above computation to be valid, one must verify that the active and passive pressure zones for wall and deadmen do not overlap (see sect. 7.2.5 of Costet-Sanglerat). The assumption is made that the zone behind the sheetpile wall has an apex where the residual stress on the wall is nil. In addition, a check should be made that the upper part of the deadman wall is located below an imaginary line passing through the zero residual stress point and making an angle φ with the horizontal.

Assuming that the zero residual stress point is close to the zero moment point, we get with graph VII-7 of sect. 7.2.2 of Costet-Sanglerat:

$a/d = 0.05 \qquad \text{for } \varphi = 33^\circ$ (Fig. 8.14) $\qquad$ or:

$a \simeq 0.40\,\text{m}; \quad h = d + a = 8.40\,\text{m}$

$L_1 = h \tan(\pi/4 - \varphi/2) = 8.40 \times 0.543 = 4.56\,\text{m}$

$L_2 = D \tan(\pi/4 + \varphi/2) = 2.05 \times 1.842 = 3.78\,\text{m}$

$L_1 + L_2 = 4.56 + 3.78 = 8.34\,\text{m} < 9.00\,\text{m}$

So $L_1 + L_2 < L$; the first necessary condition is met.

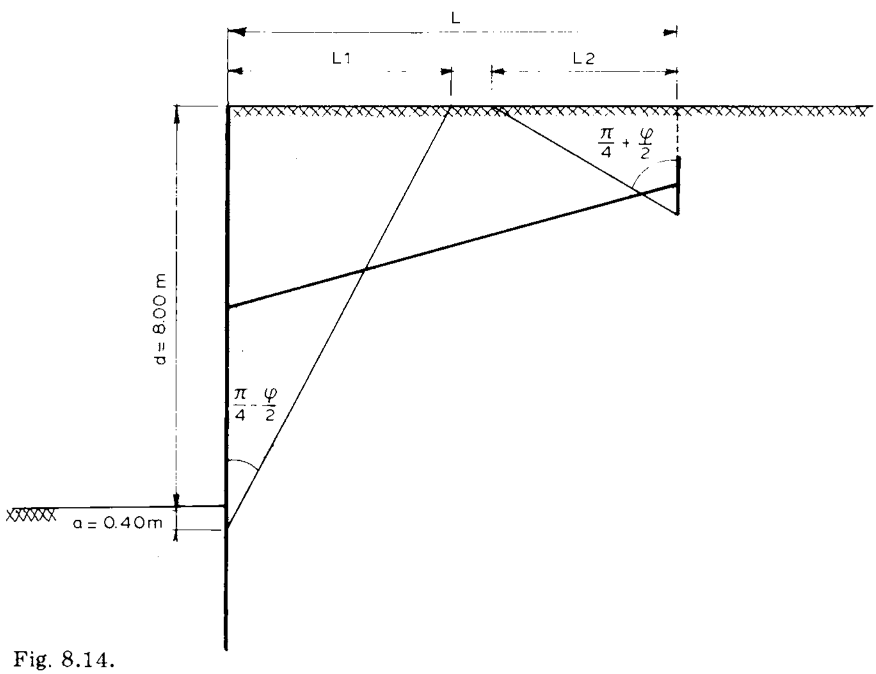


Fig. 8.14.

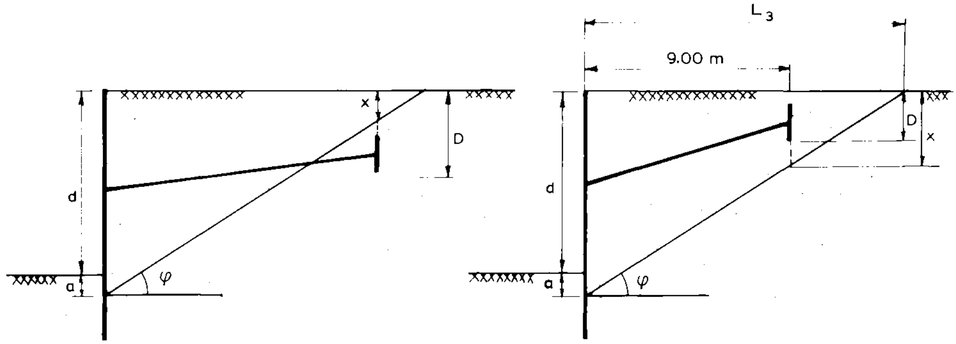


Fig. 8.15.

To check the second (see Fig. 8.15), we have $a + d = 8.40$ m,

$L_3 = 8.40/\tan\varphi = 12.93\,\text{m}, \quad x/(L_3 - 9.00) = \tan 33^\circ = 0.649$

from which: $x = (12.93 - 9) \times 0.649 = 2.55$ m and $D = H + 1 = 2.05$ m, i.e., the second condition is not met because $x > D$ and we should have $x = 1.00$ m. As indicated in sect. 7.2.5 of Costet-Sanglerat, the two above conditions are too restrictive and the anchors can be shorter. We assume, therefore, that meeting condition 1 is sufficient to satisfy the needs of the design.

★★*Problem 8.3* **Design of a sheetpile wall with anchors by the Blum method**

Refer to the givens for the quay wall in the first case of problem 8.1, but assume the wall to be embedded with passive pressures acting at the toe. By utilizing Blum's method, (point of zero bending moment corresponds to point of zero residual stress).

Find:

(1) the location of the zero moment point, b. the anchor tension, and c. the total embedment of the sheets (with a safety factor of 1 in passive pressure);

(2) the bending moment M_1 at the anchor tie-in location, b. the maximum bending moment M_2 between the anchor and the point of zero moment, and c. the maximal bending moment M_3 below the point of zero moment.

Solution

(1) a. The point of zero residual stress in the sheetpile wall must be determined. Refer to Fig. 8.2 of problem 8.1 and call elevation a that of the point sought with respect to the excavation lines. We then have: $29.8 + 3.26a = 2 \times 37.46a$, where the coefficient 2 of the right side of the equation is due to

the fact that the safety factor on the passive pressure is 1 in this case and that of the diagram is 2. We then find: $a = 29.8/71.66 \simeq 0.416$

say $a = 0.42\,\text{m}$.

Remark

Graph VII-7 of Ch. 7.2.2 of Costet-Sanglerat gives for $\varphi = 33°$:

$a/d \simeq 0.05$ or: $a = 0.05 \times 8.00 = 0.40\,\text{m}$,

but the rule: $a = 0.1d$ (for $25° < \varphi < 35°$), gives: $a = 0.80\,\text{m}$.

For a safety factor of 2 on the passive pressure, the preceding calculation gives: $a = 29.8/34.2 \simeq 0.87\,\text{m}$.

b. *Anchor tension*

Tension T is obtained by summing the moments of force acting on the sheetpile wall above the point of zero residual stress and equating the sum to zero. Refer to Fig. 8.16 to make up Table 8C (per meter length).

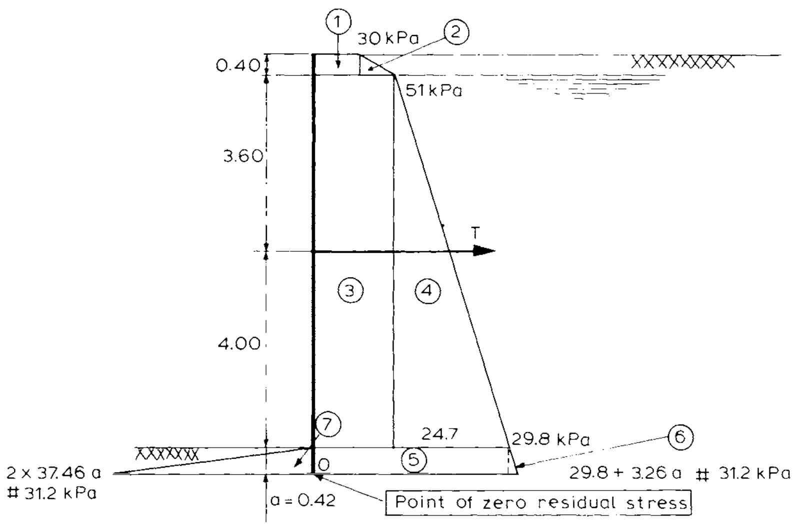


Fig. 8.16. For the sake of clarity, not to scale.

It is easy to see that tension T in the anchor rod, whose lever arm is 4.42 m, is given by: $4.42 \times T = \Sigma M_i$,

i.e., $T = 463.23/4.42 = 104.8\,\text{kN}$,

$T = 105\,\text{kN}$ per m of length.

c. *Total embedment of the sheetpiles*

Assume (Fig. 8.17) $f_0 = a + b$.

Length b is determined by part OO' of the wall equated to a simply supported beam of span, with reaction forces T_o at point O and C at point O'. The

TABLE 8C

Zone	Corresponding force F_1 (kN)	Lever arm d_i (m)	Moment with respect to O (kN · m)
1	$3 \times 0.40 = 1.2$	8.22	+9.86
2	$\frac{1}{2} \times (5.1 - 3) \times 0.40 = 0.42$	8.15	+3.42
3	$5.1 \times 7.60 = 38.76$	4.42	+171.32
4	$\frac{1}{2} \times 24.7 \times 7.60 = 93.86$	2.95	+276.89
5	$29.8 \times 0.42 = 12.52$	0.21	+2.63
6	$\frac{1}{2} \times 3.26 \times 0.42^2 = 0.29$	0.14	+0.04
7	$-2 \times \frac{1}{2} \times 37.46 \times 0.42^2 = -6.61$	0.14	−0.93
			$M_i = 463.23$

point of zero moment is assumed to coincide with the point of zero residual stress. The loads on the beam are distributed over a triangle and therefore we get: $C = 2T_0$. Then follows the calculation of T_0:

$$T_0 = T_0 + T = \Sigma_i Fi, \qquad T_0 = \Sigma_i Fi - T.$$

With the values of Table 8C:

$$\Sigma_i Fi = 140.44 \simeq 140\,\text{kN}$$

$$T = \underline{105\,\text{kN}}$$

$$T_0 = 35\,\text{kN}.$$

Following further the notation of Fig. 8.17, we get:

$$p_2 = \cancel{29.8} + 3.26a + 3.26b - 2 \times \cancel{37.46a} - 2 \times 37.46b.$$

(The crossed-out terms cancel out in consequence of the definition of a.) Thus we have: $p_2 = 3.26b - 74.92b = -71.66b$: $\quad |p_2| = 71.66b$.

The moment of the forces applied to beam OO' is zero at O', so we get: $T_0 b = \frac{1}{2}p_2 b \times b/3 \qquad$ or $\qquad T_0 = p_2 b/6 = 71.66b^2/6$

but: $T = 35\,\text{kN}$,

then: $b = \sqrt{6 \times 35/71.66} = 1.71\,\text{m}$

and $f_0 = a + b = 0.42 + 1.71 = 2.13\,\text{m}$.

One accepts in general: $f = 1.2f_0$, from which $f = 1.2 \times 2.13 = 2.55\,\text{m}$,

say: $f = 2.60\,\text{m}$.

(2) Bending moments M_1, M_2 and M_3

The upper portion of the sheetpile wall (above the point of zero moment)

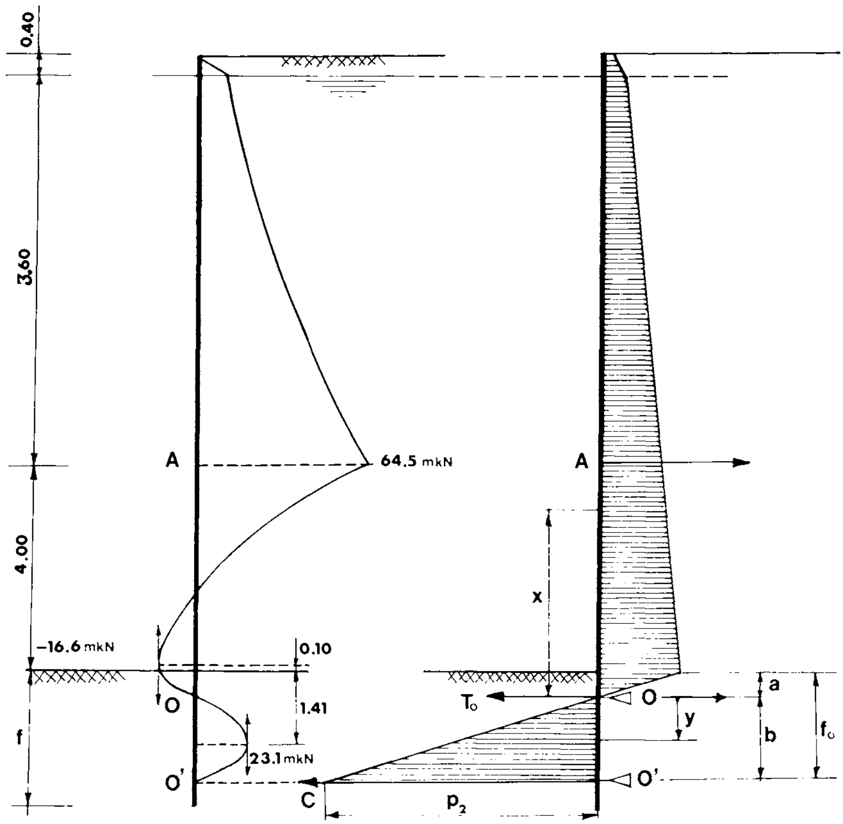


Fig. 8.17.

is analyzed as a simple beam supported at O and A* where the anchors tie in. The load diagram is shown on Fig. 8.18.

As for the lower part $O'O$ of the pile wall, it is analyzed as a simply at O and O' supported beam (Fig. 8.17).

a. Moment M_1 at the tie point of anchors is:

$$M_1 = 1.2 \times 3.80 + 0.42 \times 3.73 + 5.1 \times (\overline{3.60}^2/2) + 11.7 \times (\overline{3.60}^2/6)$$

$$= 64.45 \text{ kN} \cdot \text{m};$$

$$M_1 = 64.5 \text{ kN} \cdot \text{m per m of length.}$$

*This point A corresponds to the old point C of Fig. 8.1 which was changed to avoid confusion with the passive force C.

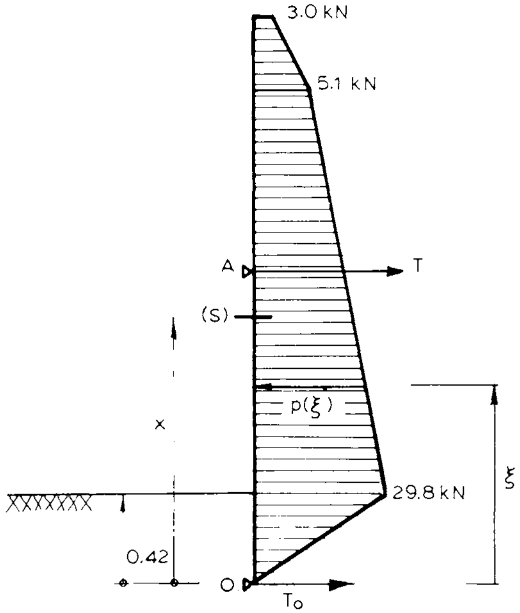


Fig. 8.18.

b. The load per unit length between O and A is expressed by the following formulas:

$p(\xi) = (29.8/0.42)\xi = 70.95\xi \quad \text{for } 0 < \xi < 0.42\,\text{m}$

$p(\xi) = 31.2 - 3.26\xi \quad \text{for } 0.42 < \xi < 4.42\,\text{m}.$

The shear in beam OA is:

$$T(x) = T_0 - \int_0^x 70.95\xi \cdot d\xi \quad \text{for } 0 < x < 0.42\,\text{m}$$

$$T(x) = T_0 - \int_0^{0.42} 70.95\xi \cdot d\xi - \int_{0.42}^{x} (31.2 - 3.26\xi)d\xi \quad \text{for } x > 0.42\,\text{m}.$$

the value of $T(x)$ is 0 for $x = 0.52$ m.

The corresponding magnitude of the bending moment is:

$$M_{0.52} = -T_0 \times 0.52 + \tfrac{1}{2} \times 0.42 \times 29.8 \times ((0.42/3) + 0.10) + $$
$$+ 29.5 \times (\overline{0.10}^2/2) + 0.3 \times (\overline{0.20}^2/3)$$

which gives, since $T_0 = 35$ kN,

$M_{0.52} = -16.55$ kN for 1 m of length.

This value corresponds to the extreme of the moment diagram between O and A.

It is the value of M_2 asked for:

$M_2 \simeq -16.6\,\text{kN}\cdot\text{m}$ per m of length.

c. The maximal bending moment below the point of zero moment is given by the simply supported beam OO' (Fig. 8.17). From the first part of Problem 8.3, we had: $p_2 = -71.66b$ and the shear in the wall at level y, counted positive in the downward direction O, is given by:

$$T(y) = T_0 - \int_0^y p_2 \times \frac{y}{b}\,dy = T_0 - \frac{71.66}{2}\,y^2 \quad \text{or} \quad T(y) = 35 - 35.83y^2.$$

The shear is zero at a point whose elevation is $y = \sqrt{35/35.83} = 0.99\,\text{m}$. The bending moment M_3 is obtained by integration:

$$M_3 = T_0(y) - 35.83(y^3/3) \quad 35 \times 0.99 - 35.83 \times (\overline{0.99}^3/3)$$

$$= 23.06\,\text{kN}\cdot\text{m}$$

$M_3 = 23.1\,\text{kN}\cdot\text{m}$ per m of length.

Summary of answers:

(1) $a = 0.42\,\text{m}$, $T = 105\,\text{kN}$, $f = 2.60\,\text{m}$

(2) $M_1 = 64.5\,\text{kN}\cdot\text{m}$, $M_2 = -16.6\,\text{kN}\cdot\text{m}$, $M_3 = 23.1\,\text{kN}\cdot\text{m}$
all per unit length.

★★*Problem 8.4* Design of an anchored sheetpile wall by the method of Tschebotarioff

The same sheetpile wall, as in Problem 8.1, is to be designed by the method of Tschebotarioff. Limit the analysis to case 1 (quay wall). Find:
(1) the embedment of the sheets;
(2) the tension in the anchors;
(3) the maximal bending moment.

Solution

(1) Embedment of sheets

Tschebotarioff's method applies to driven piles on one side of which a non-cohesive fill is placed. It does not apply to driven sheets and a soil dredged on one side. The method suggests to assume an embedment of $f = 0.43d$. Using the notations of Fig. 8.19, we get: $f = 0.43 \times 8 = 3.44\,\text{m}$.

(2) Anchor tension

The method also recommends to take the following earth pressure values:

$$K_a' = K_a(1 - 0.3(t/d)) = K_a(1 - 0.3 \times \tfrac{4}{8})$$

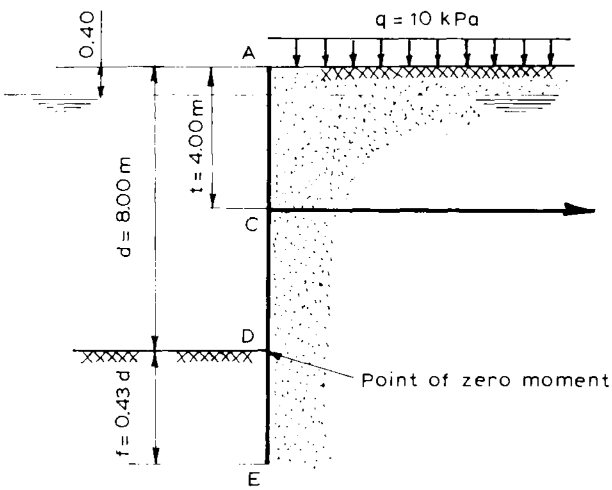


Fig. 8.19.

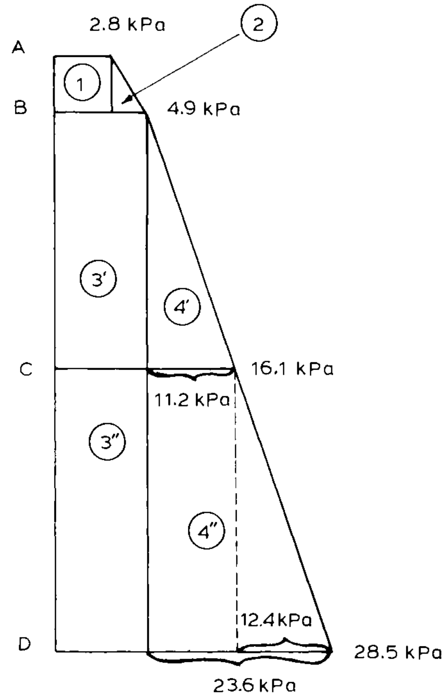


Fig. 8.20.

$K'_a = 0.85 K_a$.

To find K_a, it is recommended to use $\varphi = 30°$ and $\delta = 0$. The tables in Caquot-Kerisel give $k'_a = 0.333$ from which $K'_a = k'_a = 0.283$ (since $\delta = 0$).

The following stresses are then computed which enables us to draw a load diagram as that of Fig. 8.20.

$$\sigma_A = 0.283 \times 10 = 2.83 \text{ kPa}$$

$$\sigma_B = 2.83 + 0.283 \times 18 \times 0.40 = 4.87 \text{ kPa}$$

$$\sigma_C = 4.87 + 0.283 \times 11 \times 3.60 = 16.08 \text{ kPa}$$

$$\sigma_D = 16.08 + 0.283 \times 11 \times 4.00 = 28.53 \text{ kPa}.$$

To determine the tension in the anchor, we write that the sum of the moments is zero with respect to point D, from which Table 8D can be made, for unit lengths of wall. The tension is: $T \times 4 = 380.89$, $T = 95.22$

say: $T = 95$ kN per m of length.

(3) Maximal bending moment

The method of Tschebotarioff assumes that the maximal moment occurs

TABLE 8D

Zone	Corresponding force F_i (kN)	Lever arm (m)	Moment with respect to D M_i (kN · m)
1	$2.8 \times 0.40 = 1.12$	7.80	8.74
2	$\frac{1}{2} \times 2.1 \times 0.4 = 0.42$	7.73	3.25
3′	$4.9 \times 3.60 = 17.64$	5.80	102.31
3″	$4.9 \times 4 = 19.60$	2	39.20
4′	$\frac{1}{2} \times 11.2 \times 3.60 = 20.16$	5.20	104.83
4″	$11.2 \times 4 = 44.8$	2	89.60
	$\frac{1}{2} \times 12.45 \times 4 = 24.9$	1.33	33.12
			381.05

in the section of the pile above the dredge line, most likely between C and D. Let x be the vertical distance of the maximal moment point to C. The shear then is:

$$T(x) = -4.9 \times 3.60 - \tfrac{1}{2} \times 11.2 \times 3.60 - 2.8 \times 0.40 - \tfrac{1}{2} \times 2.1 \times 0.40 + $$

$$+ T_{\text{anchorage}} - 16.1 \times - \int_0^x \frac{12.4}{4} \zeta \, d\zeta$$

which yields: $T(x) = 1.55x^2 + 16.1x - 55.9$. $T(x)$ is zero for $x = 2.75$ m, the corresponding bending moment is:

$$M_{\max} = 1.12(3.80 + 2.75) + 0.40(3.73 + 2.75) + 17.64(1.80 + 2.75) +$$

$$+ 19.60(1.20 + 2.75) + 16.1 \times 2.75 \times \frac{2.75}{2} + \frac{2.75}{3} \times \int_0^{2.75} \frac{12.4}{4} \zeta \, d\zeta$$

$$- 95.2 \times 2.75 = -22.57 \text{ kN} \cdot \text{m}$$

$$M_{\max} = -22.6 \text{ kN} \cdot \text{m} \qquad \text{per meter length.}$$

Conclusion

The magnitude of the moment by the method of Tschebotarioff varies little from that of Blum (see Problem 8.3), but in the latter's method, the moment occurred in the embedded portion and had the opposite direction. Both methods yield the same size pile requirements and the maximum bending moments in both cases are at the tie point of the anchor (M_1 = 6.5 kN · m per m of length) (Compare Problem 8.3).

Summary of answers
(1) $f = 3.44$ m; (2) $T = 95$ kN; (3) $M_{max} = -22.6$ kN · m
(T and M_{max} are given by m of length).

★★★★*Problem 8.5* Design of fender piles

Two identical fender-pile systems must be designed, each composed of a metal pile with the butt at El. + 8.0. The characteristics of the design are given in Table 8E.

TABLE 8E

Elevation	D (cm) External diameter	e (mm) Thickness	σ_e (MPa) Elastic limit
+ 8.00			
	150	15	240
+ 1.00			
	150	15	470
− 2.95			
	150	18	600
− 6.90			
	150	25	600
− 10.85			
	150	26	600
− 22.70			
	150	18	600
− 27			

The piles have varying moments of inertia with depth and are to be placed in a soil as that illustrated on Fig. 8.21. It is anticipated that the sand layer from −8.5 to −13 m will be dredged.

The point is the determination of the lengths of the piles so that they will be stable under the following loading conditions:

— *ship docking loads:* *the energy applied to the pile, when the ship moves against it, is 500 kN · m, with the centroid of the applied load acting at El. + 4.75.*
Under the impact, the pile deflection at El. + 4.75 must be limited to 1.40 m and the stresses in the pile cannot exceed the elastic limit of steel.

— *ship mooring:* *a horizontal load of 600 kN is applied to a pile at EI + 8.0. Under the action of this force, the stresses in the pile cannot exceed 80% of the elastic limit.*

Part 1

To simplify the problem, assume the following:

— *pile is circular with a constant inertia and an elastic modulus constant. Take El.* $= 6.29 \times 10^6$ *kPa* $\times$ m^4.

— *modulus of soil reaction* k_s *is constant for the depth of embedment.*

For the simplified solution solve for the ship docking load only.

(1) Write down the differential equation for the deformation of a fender

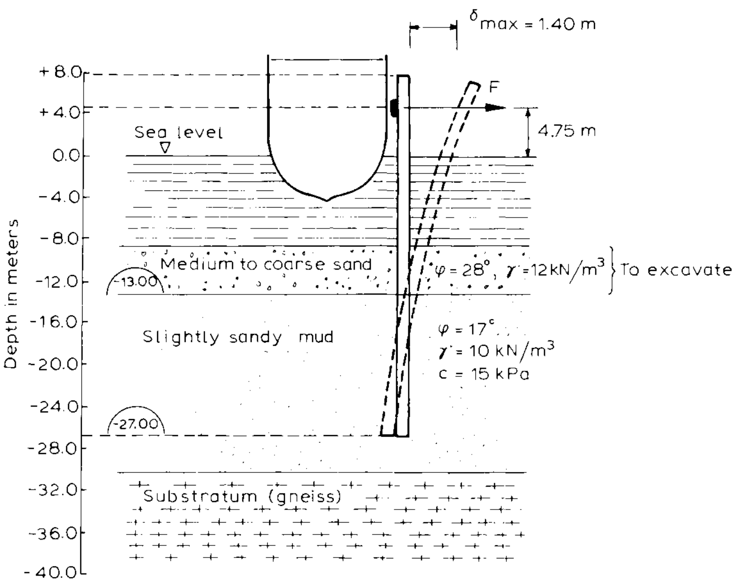


Fig. 8.21.

pile for its section below grade. By the method of finite differences, replace this differential equation by a system of linear equations, taking into account the limiting conditions imposed by the problem givens (the buried portion of the pile will be segmented into 4 parts of equal length). Consider the following 2 cases: (a) the modulus of soil reaction is $k_s = 20$ kPa/cm and the tip of the pile is embedded in the gneiss; (b) the modulus of soil reaction is 80 kPa/cm and the tip of the pile is at a depth of 27 m below grade.

The solution of the linear equations will allow the plotting of the fender pile deformation over its embedded depth. Determine the deflection at El. + 4.75 m under the action of 500 kN·m. Use the method of interpolation.

(2) With the above results, draw the moment diagram along the pile and the normal stress diagram in the soil during ship docking loads, as a function of depth. What can be concluded?

Part 2

With the givens above (variable pile inertias and elastic limits) is the differential equation of Part 1 still valid? What can be said about the modulus of soil reaction k_s in the clay?

Assuming the clay to have a plasticity index of 45%, give the equation for the undrained cohesion as a function of depth.

The solution necessarily requires the use of a computer. For illustration *purposes, the computation method is presented here for the ship docking load and mooring conditions, assuming two possibilities for the mechanical properties of the soil. Results of the more sophisticated approach can be compared with the simplified method of Part 1.*

Solution

Part 1

(1) Let us call δ the deflection of the pile at El. + 4.75 during ship docking loads. Let F be the force exerted by the ship and W be the corresponding energy. These values are related by:

$$W = \tfrac{1}{2}F\delta \tag{1}$$

On the other hand, the horizontal force F applies at El. + 4.75 is equivalent to the system of (F, Γ) at El. -13.00 (upper limit of the mud) to: $\tau(F, \Gamma)$, with $\Gamma = -Fl_0$ and $l_0 = 13.00 + 4.75 = 17.75$ m.

First, we will determine the deflection of the fender pile below the dredge line by analogy to an elastic beam (see sect. 9.4.2. Costet-Sanglerat). To better understand the action of the soil on the pile shaft, let's first look at the action on an infinitely rigid screen being translated (Fig. 8.22).

The analysis presented below is two-dimensional. It is the only possible one for calculation by hand. It is a first approximation to the problem which in fact is a 3-dimensional one.

Under these conditions, on one side of the screen we have passive pressures

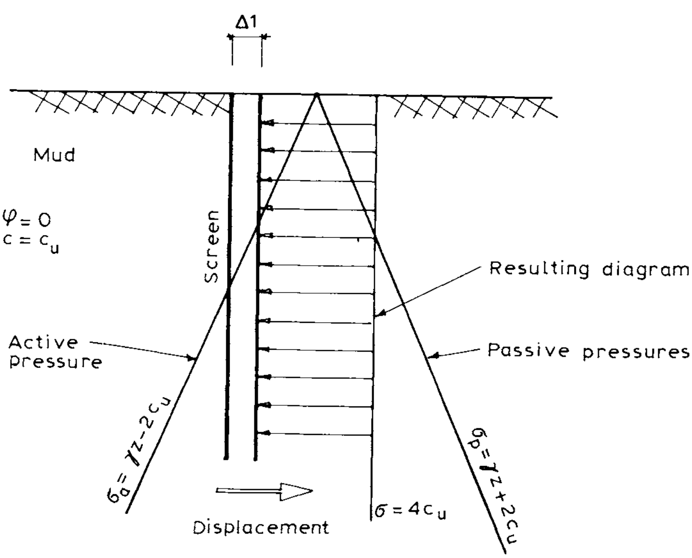


Fig. 8.22.

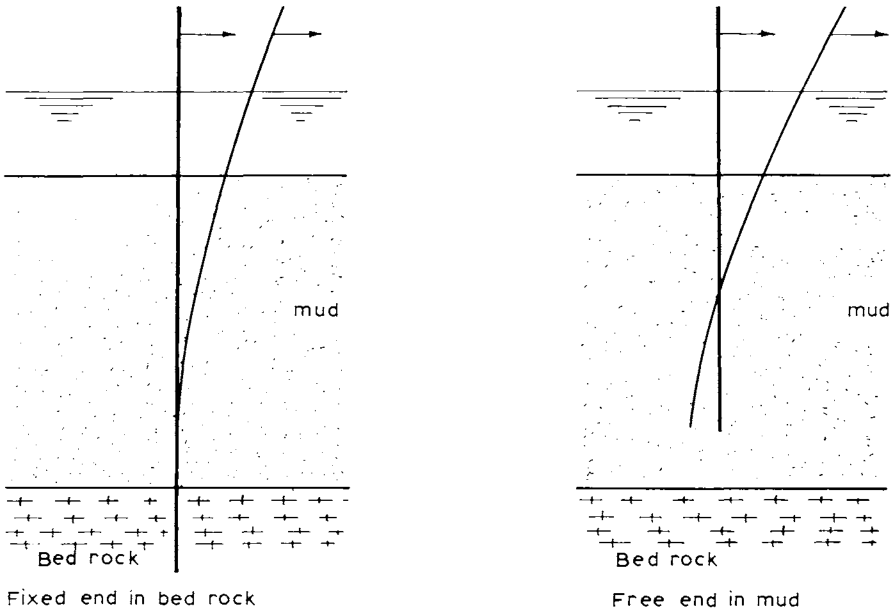


Fig. 8.23.

which resist the motion and on the opposite side we have active pressure conditions.

Under short-term loading (dynamic loads) $\varphi = 0$, $c = c_u = s_u$ and we have:

active pressure $\sigma_a = \gamma z - 2c_u$,

passive pressure $\sigma_p = \gamma z + 2c_u$.

The net resulting pressure thus is: $\sigma = \sigma_p - \sigma_a = 4c_u$.

Assume c_u to be constant with depth. The stress distribution is uniform with depth. In reality, the shaft of the fender pile is flexible, cohesion c_u increases with depth z and the shaft deflections depend on the boundary conditions at the base of the pile (fixed or free end, Fig. 8.23). The deformations of the shaft are not everywhere large enough to generate full passive and full active pressures.

The problem is thus very complex. As a simplified analysis, let us assume that the reaction along the shaft is uniform and that it corresponds to the modulus of soil reaction k_s (constant). Under these conditions, the differential equation for the deflections of the pile below grade, is, if B is the width of the pile:

$$\mathrm{d}^4 v/\mathrm{d}z^4 + (k_s B/\mathrm{El.})v = 0. \tag{2}$$

For the following boundary conditions:
(1) at the surface (point O), bending moment $M_0 = \Gamma$, shear $T_0 = F$;
(2) at the lowest point (point B), free-end condition $M_B = T_B = 0$,
fixed-end condition $v_B = 0$, $(\mathrm{d}v/\mathrm{d}z)_B = 0$,

the bending moment is given by

$$-M/EI = \mathrm{d}^2v/\mathrm{d}z^2 \tag{3}$$

and the shear by:

$$T/EI = \mathrm{d}^3v/\mathrm{d}z^3 \tag{4}$$

Dividing section OB into 4 segments of equal length h (Fig. 8.24) we assume that function $v(z)$ is given by the numerical values v_i at the points equidistant of elevation $z_i = z_0 + ih$ (axis Oz being positive downward).

The successive derivatives of v may then be approximately calculated by the following finite-differences equations:

$$(\mathrm{d}v/\mathrm{d}z)_{zi} = (v_{i+1} - v_i)/h$$

$$(\mathrm{d}^2v/\mathrm{d}z^2)_{zi} = (v_{i+1} - 2v_i + v_{i-1})/h^2$$

$$(\mathrm{d}^3v/\mathrm{d}z^3)_{zi} = (v_{i+2} - 3v_{i+1} + 3v_i - v_{i-1})/h^3$$

$$(\mathrm{d}^4v/\mathrm{d}z^4)_{zi} = (v_{i+2} - 4v_{i+1} + 6v_i - 4v_{i-1} + v_{i-2})/h^4$$

All that remains to do now is to write the finite-differences equation (obtained by replacing $\mathrm{d}^4v/\mathrm{d}z^4$ by its approximate value) for each point 1, 2 and 3 (Fig. 8.24) and to write the boundary conditions at points O and B numbered 0 and 4.

We then obtain $3 + 2 \times 2 = 7$ equations with 7 unknowns which can be reduced to a system of 5 equations with 5 unknowns easily solved by desk-top computer.

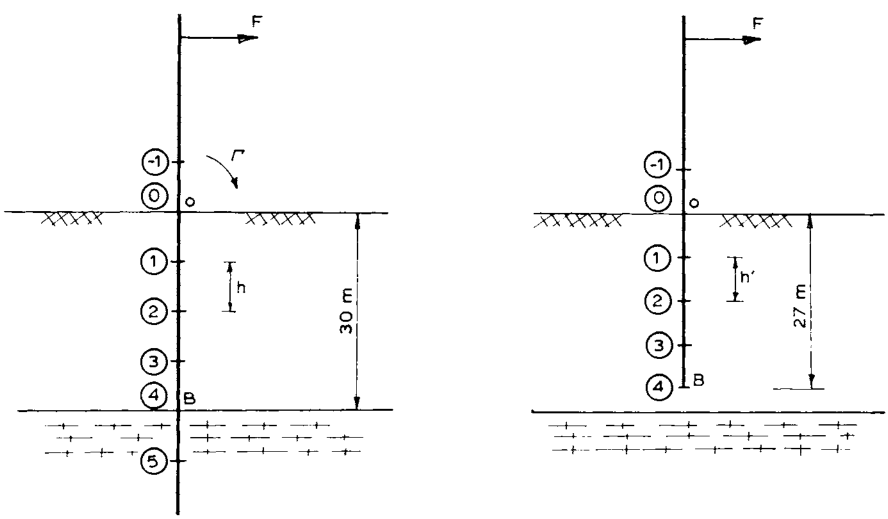


Fig. 8.24.

(a) Fixed-end condition (tip at − 35 m)
Finite-difference equations give:

$$v_3 - 4v_2 + (6+K)v_1 - 4v_0 + v_{-1} = 0$$
$$v_4 - 4v_3 + (6+K)v_2 - 4v_1 + v_0 = 0$$
$$v_5 - 4v_4 + (6+K)v_3 - 4v_2 + v_1 = 0$$

with: $K = h^4 k_s B/EI$

The boundary conditions allow us to write:

$$v_1 - 2v_0 + v_{-1} = -\Gamma h^2/EI$$
$$v_2 - 3v_1 + 3v_0 - v_{-1} = Fh^3/EI$$
$$v_4 = 0$$
$$v_5 = v_3 \quad \text{(equivalent to } (\mathrm{d}v/\mathrm{d}z)_4 = 0\text{).}$$

This leads us to the system of 5 equations with 5 unknowns.

$$\text{(I)}\begin{cases} v_3 - 4v_2 + (6+K)v_1 - 4v_0 + v_{-1} = 0 \\ -4v_3 + (6+K)v_2 - 4v_1 + v_0 = 0 \\ (7+K)v_3 - 4v_2 + v_1 = 0 \\ v_1 - 2v_0 + v_{-1} = -\Gamma h^2/EI \\ v_2 - 3v_1 + 3v_0 - v_{-1} = Fh^3/EI \end{cases}$$

With the numerical values of the problem, we get:

$$h = l/4 = 17/4 = 4.25\,\mathrm{m}, \quad B = 1.50\,\mathrm{m}$$
$$k_s = 20\,\mathrm{kPa/cm} = 2 \times 10^3\,\mathrm{kPa/m}$$
$$EI = 6.29 \times 10^6\,\mathrm{kPa} \times \mathrm{m}^4$$

from which: $K = (\overline{4.25}^4 \times 2 \times 10^3 \times 1.50)/(6.29 \times 10^6) = 0.1556.$

If we fix a value for F, we get $\Gamma = -Fl_0$ and we can then solve system (I) above, which gives for each point the shaft deformations. Deflection δ at point A of application of force F, whose elevation is + 4.75 is (with the notations of Fig. 8.25) obtained by the first Bresse formula (neglecting the deflection due to shear):

$$\delta = v_0 - \omega_0(z - z_0) + \int_{s_0}^{s} \frac{M(z-\eta)}{EI}\,\mathrm{d}\eta$$

We thus have:

$$\delta = v_0 - \omega_0 l_0 + \int_{O}^{l_0} \frac{F(l_0-\eta)^2}{EI}\,\mathrm{d}\eta = v_0 - \omega_0 l_0 + \frac{l_0^3 F}{3EI}$$

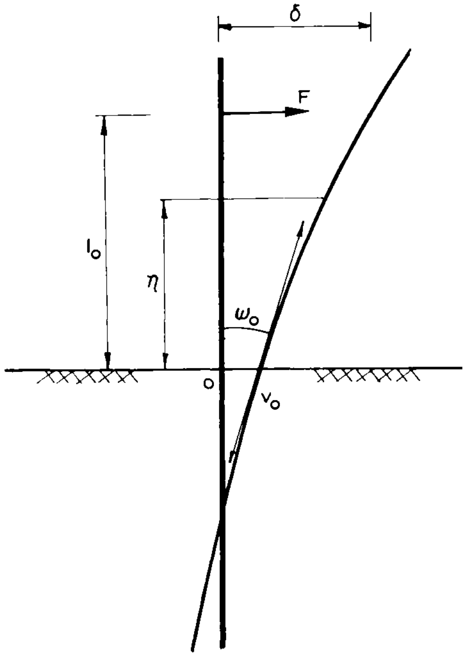


Fig. 8.25.

but: $\omega_0 = (dv/dx)_0 \simeq (v_1 - v_{-1})/2h$

from which: $\delta = v_0 - [(v_1 - v_{-1})/2h] \cdot l_0 + l_0^3 F/3EI$

This is the value of δ for the selected value of F. The energy applied to the fender pile during docking action of the ship is $W = (1/2)F\delta$.

The solution is obtained by an iterative process starting with the value of $F = (2 \times 500)/1.40 \simeq 715$ kN.

The obtained results are:

For $F = 715$ kN:

$v_{-1} = 0.366$ m, $v_0 = 0.216$ m, $v_1 = 0.103$ m, $v_2 = 0.036$ m,

$v_3 = 0.005$ m, $v_4 = 0$; $\delta = 0.975$ m and $W = 349$ kN · m.

For $F = 1000$ kN:

$v_{-1} = 0.512$ m, $v_0 = 0.303$ m, $v_1 = 0.145$ m, $v_2 = 0.050$ m,

$v_3 = 0.008$ m, $v_4 = 0$, $\delta = 1.366$ m and $W \simeq 683$ kN · m.

By interpolation we get, for $W = 500$ kN · m:

$F = 844$ kN, say $F = 850$ kN, $\delta = 1.18$ m,

$v_{-1} = 0.441$ m, $v_0 = 0.261$ m, $v_1 = 0.125$ m, $v_2 = 0.043$ m,

$v_3 = 0.007$ m, $v_4 = 0$.

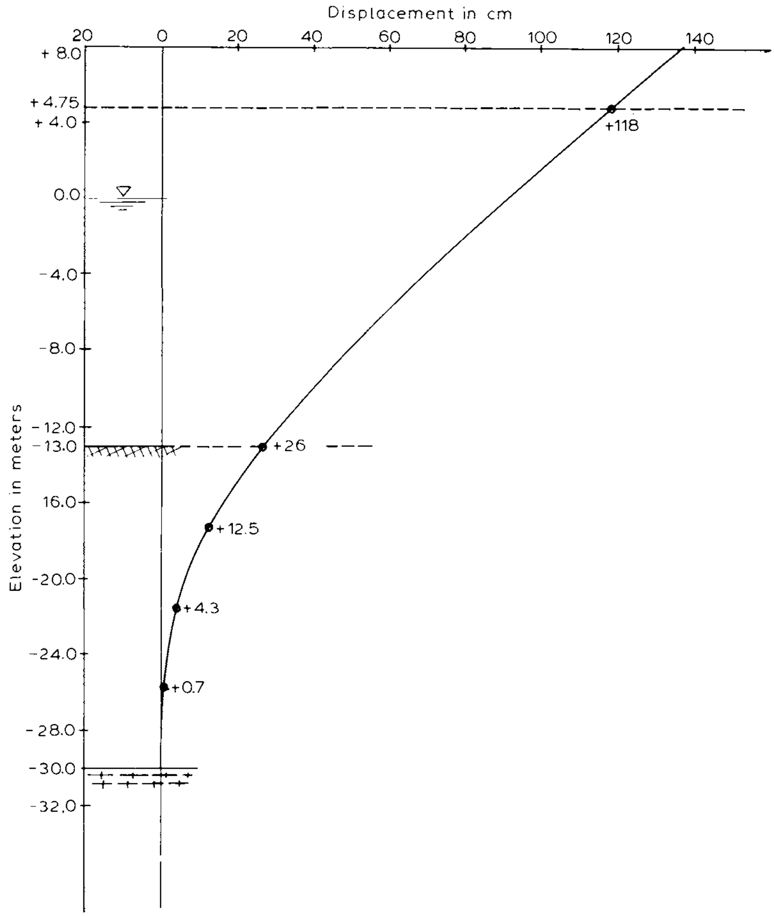


Fig. 8.26. Deflection of pile (parameters 1: tip at −35 m, fixed-end condition, ship docking case).

The corresponding diagram is shown on Fig. 8.26.

(b) For the free-end condition (pile-tip in clay at −27 m)
The equations of finite differences are:

$$v_3 - 4v_2 + (6 + K')v_1 - 4v_0 + v_{-1} = 0$$

$$v_4 - 4v_3 + (6 + K')v_2 - 4v_1 + v_0 = 0$$

$$v_5 - 4v_4 + (6 + K')v_3 - 4v_2 + v_1 = 0$$

For $K' = h'^4 k_s' b/\mathrm{El}$. the boundary conditions give:

$$v_1 - 2v_0 + v_{-1} = -\Gamma h'^2/EI$$

$$v_2 - 3v_1 + 3v_0 - v_{-1} = Fh'^3/EI$$

$$v_5 - 2v_4 + v_3 = 0$$

$$v_5 - 3v_4 + 3v_3 - v_2 = 0$$

from which the system of 5 equations with 5 unknowns becomes:

$$\text{(II)}\begin{cases} 4v_4 - 7v_3 + (6 + K')v_1 - 4v_0 + v_{-1} = 0 \\ -(5 + K')v_4 + (8 + 2K')v_3 - 4v_1 + v_0 = 0 \\ 2v_4 + (K' - 3)v_3 + v_1 = 0 \\ v_1 - 2v_0 + v_{-1} = -\Gamma h'^2/EI \\ -v_4 + 2v_3 - 3v_1 + 3v_0 + v_{-1} = Fh'^3/EI \end{cases}$$

For the numerical values of the problem:

$h' = l/4 = 14/4 = 3.50\,\text{m}$, $B = 1.50\,\text{m}$,

$k_s' = 80\,\text{kPa/cm} = 8 \times 10^3\,\text{kPa/m}$, $EI = 6.29 \times 10^6\,\text{kPa} \times \text{m}^4$

and: $K' = (\overline{3.50}^4 \times 8 \times 10^3 \times 1.50)/(6.29 \times 10^6) = 0.2863$.

From here on, calculations are identical to the preceding case. The final solution is:

For $F = 715\,\text{kN}$:

$v_{-1} = 0.291\,\text{m}$, $v_0 = 0.167\,\text{m}$, $v_1 = 0.069\,\text{m}$,

$v_2 \simeq 0.001\,\text{m}$, $v_3 = -0.053\,\text{m}$, $v_4 = -0.106\,\text{m}$,

$\delta = 0.941\,\text{m}$ and $W \simeq 336\,\text{kN} \cdot \text{m}$.

For $F = 1000\,\text{kN}$:

$v_{-1} = 0.408\,\text{m}$, $v_0 = 0.235\,\text{m}$, $v_1 = 0.097\,\text{m}$,

$v_2 \simeq 0.001\,\text{m}$, $v_3 = -0.074\,\text{m}$, $v_4 = -0.149\,\text{m}$,

$\delta = 1.319\,\text{m}$ and $W \simeq 660\,\text{kN} \cdot \text{m}$.

By interpolation from $W = 500\,\text{kN} \cdot \text{m}$ we get the following values: $F = 859\,\text{kN}$, say $F = 860\,\text{kN}$, $\delta = 1.16\,\text{m}$,

$v_{-1} = 0.350\,\text{m}$, $v_0 = 0.202\,\text{m}$, $v_1 = 0.083\,\text{m}$,

$v_2 = 0.001\,\text{m}$, $v_3 = -0.063\,\text{m}$, $v_4 = -0.128\,\text{m}$.

The corresponding diagram is shown on Fig. 8.27.

(2) The diagram of bending moments along the shaft is obtained from the preceding calculations from eqn. (3):

$$\frac{d^2 v}{dz^2} = -\frac{M}{EI}$$

which is for point i:

$$M_i = (EI/h^2)(v_{i+1} - 2v_i + v_{i-1})$$

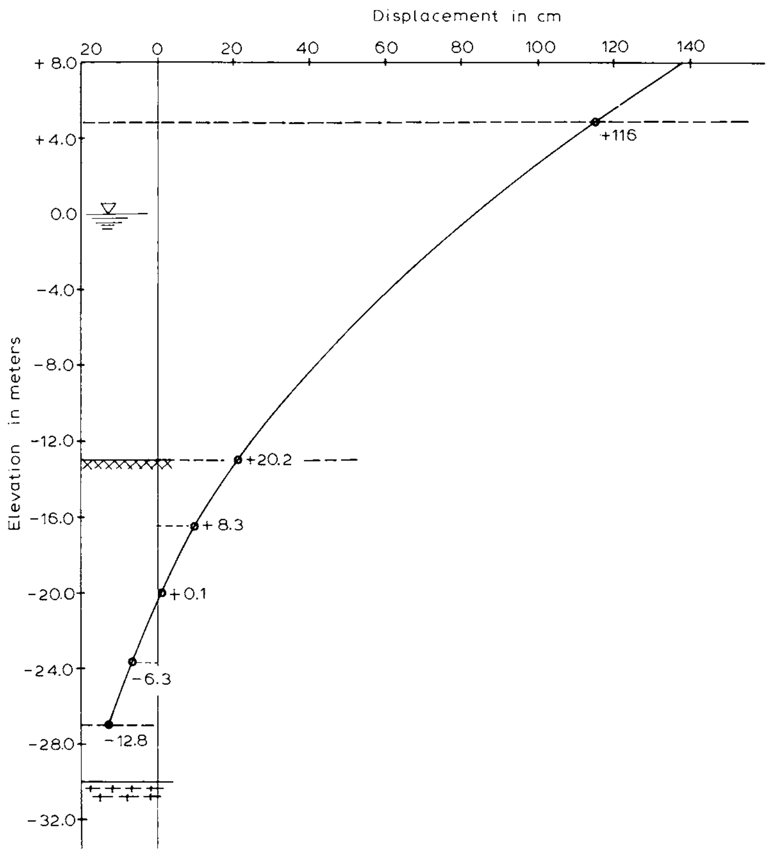


Fig. 8.27. Deflection of pile (parameters 2: tip at −27 m in the mud), ship docking case.

The resulting diagrams are presented on Fig. 8.28. The normal stress developed in the soil at point i has a value of: $\sigma = k_s v_i$, from which the solid-line diagrams of Fig. 8.29 are drawn. The passive pressure being mobilized in the clay has a value of: $\sigma_p = \gamma z + 2c_u$ (short-term loading since it is a dynamic condition).

With $c_u = 30$ kPa (assumed constant) and $\gamma = 20\ \text{kN/m}^3$, we have:

$\sigma_p = 20z + 20$ kPa (z in m).

For the free-end condition at −27 m, assume a cohesion 4 times larger*, then
$\sigma_p = 20z + 120$ kPa, from which the passive pressure diagram is drawn and shown in a dotted line on Fig. 8.29, which gives the upper limits of soil susceptible for mobilization.

By comparing the two diagrams (Figs. 8.28 and 8.29), it is obvious that the soil reaction relied upon in the simplified method cannot be mobilized in the upper portion of the mud. The computation must be altered to take

*See p. 78

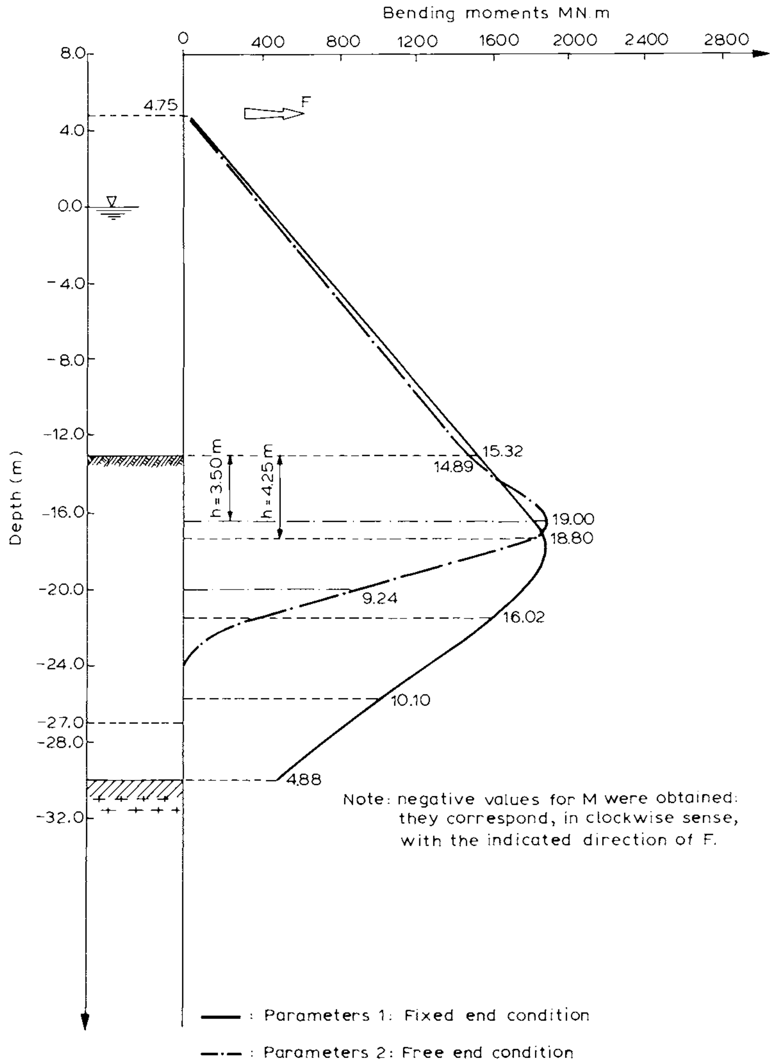


Fig. 8.28. Diagram of bending moment (simplified method: ship docking conditions).

into account a modulus of the soil reaction k_s, which varies with depth and is limited by the passive pressure available at each level.

Part 2

For the real conditions studied here, the pile inertia varies with depth z. Although the givens of the problem do not specify it, it is possible that the modulus of elasticity varies also. We have thus $I = I(z)$ and $E = E(z)$.

As a consequence, the differential equation of Part One is no longer valid

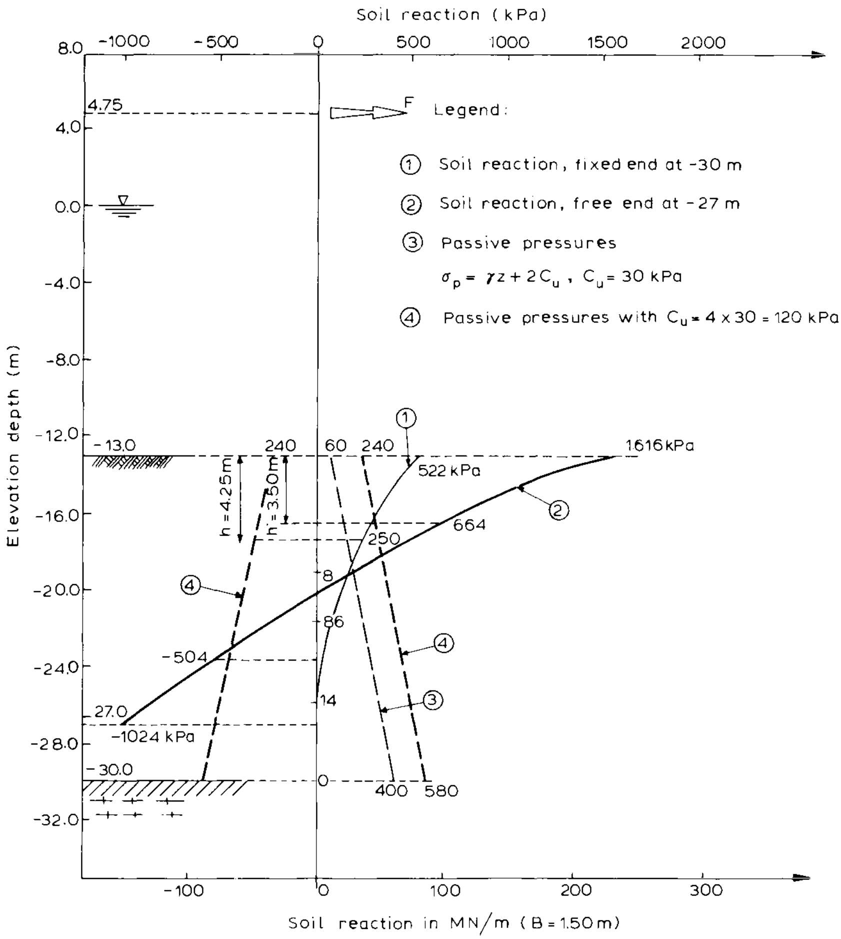


Fig. 8.29. Soil reaction and passive pressure diagram (ship docking conditions).

since it assumes both E and I to be constant. Furthermore, it was demonstrated that an increasing modulus of soil reaction k_s should be considered. Finally, the undrained cohesion of the clay, assumed to be normally consolidated, is a function of depth and may be related to Skempton's equation (see sect. 4.2.2 of Costet-Sanglerat): $c_u/\sigma_c = 0.11 + 0.37 IP$.

With $IP = 45\%$, we get: $c_u/\sigma_c = 0.11 + 0.37 \times 0.45 \simeq 0.27$, say: $c_u = 0.27\sigma_c$.

On the other hand, $\sigma_c = \sigma_o'$ since the soil is normally consolidated, therefore: $\sigma_c = (13 - 8.50) \times 12 + 10z = 54 + 10z$ kPa,

and: $c_u = 0.27(54 + 10z) \simeq 14.6 + 2.7z$ kPa (z in m).

Computer calculations in the elasto-plastic method

(1) Method of analysis. This method is that of R. Marche (Ref. 16). The deformation, bending moments and soil reactions in the pile have been

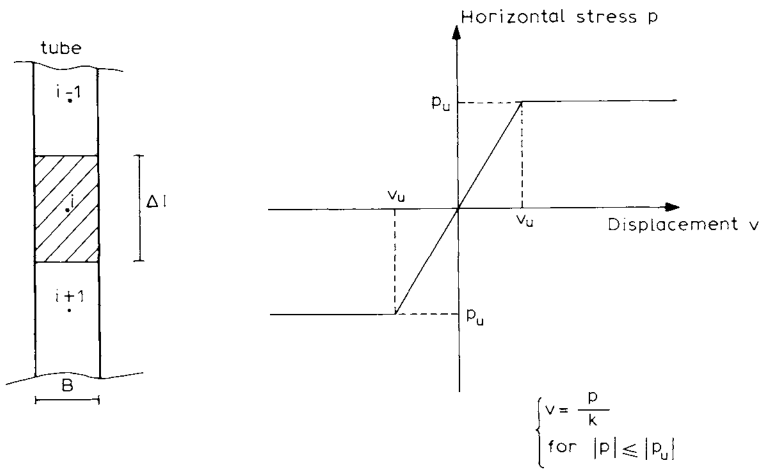


Fig. 8.30.

calculated by the analogy of a beam supported on an elastic foundation. The program takes into account the variation of the moment of inertia of the pile with depth and the elasto-plastic behaviour of the soil. This behaviour, as well as its defining parameters, are shown on Fig. 8.30.

The soil reaction normal to the section i of the pile is $P_u = \Delta l \times B \times p_u$, $v = P_u/k$, where Δl = length of pile section (m), B = width of pile (m), p_u = ultimate soil stress, k = modulus of soil reaction (p_u and k varying with depth), v = horizontal displacement of pile (m). Vertical loads and torsion are neglected.

The behaviour of the pile during docking was evaluated by calculating the energy of deformation associated to different displacements at El. + 5.0. The deformations, bending moments and soil reactions corresponding to the energy of 500 kN · m were obtained by linear interpolation from points obtained with energy levels above and below 500 kN · m.

(2) Soil parameters. The available givens do not allow a rational evaluation of strength parameters k and p_u because values of c and φ are missing for the mud. Two sets of parameters were considered for this problem.

The first set was obtained from the assumption that the mud is normally consolidated with $c_u/\sigma'_o = 0.25$ where σ'_o is the effective overburden stress. Values of k and p_u were then obtained from the values of c_u.

The second set was determined by our experience, that tells us that calculations based on a two-dimensional analysis seriously underestimate the results. In order to obtain realistic answers, the values of k and p_u must be increased by a factor of 4.

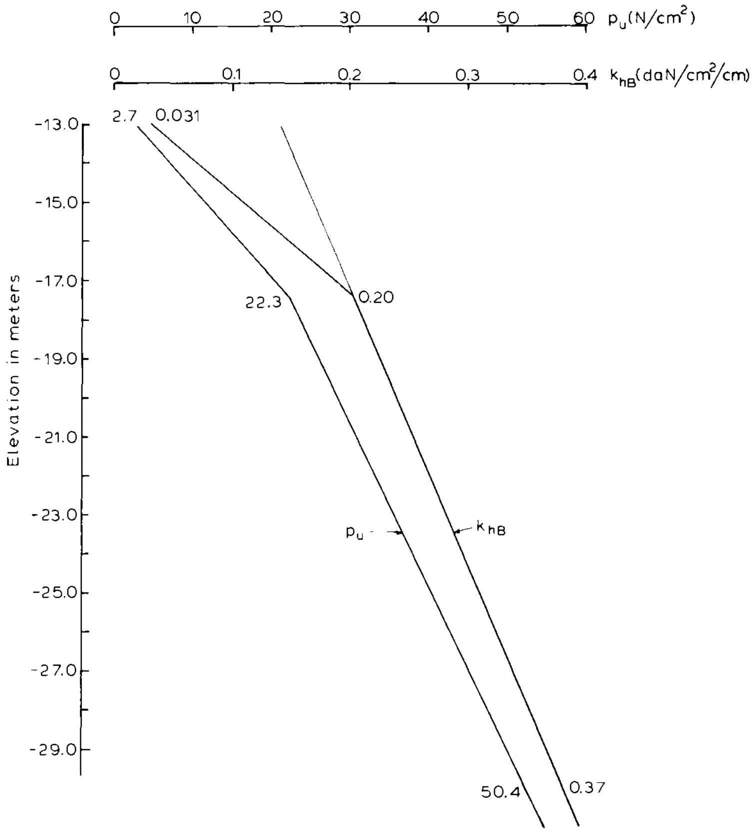


Fig. 8.31.

2.1. The parameters of set 1:

(a) The undrained shear resistance of the clay.

We have $c_u/\sigma'_o = 0.25$. This relation is very close to that obtained by taking the $PI = 45\%$ which yields $c_u = 0.27\,\sigma_o$.

Therefore:

at -13.0 m: $\sigma'_o = 4.5 \times 12 = 54$ kPa, and $c_u = 13.5$ kPa

at -30.0 m: $\sigma'_o = 54 + 17.0 \times 10 = 224$ kPa, and $c_u = 56$ kPa.

(b) Modulus of soil reaction of the clay.

In accordance with the recommendation of Marche, we should use:

$k_{hB} = k_{hl}(B_1/B)$ where $B_1 = 30$ cm, $B = 150$ cm, so $k_{hB} = k_{hl}/5$.

From the relation of c_u and k_{hl} proposed by Marche, we get:

$k_{hl} = 69$ kPa/cm for $c_u = 13.5$ kPa,

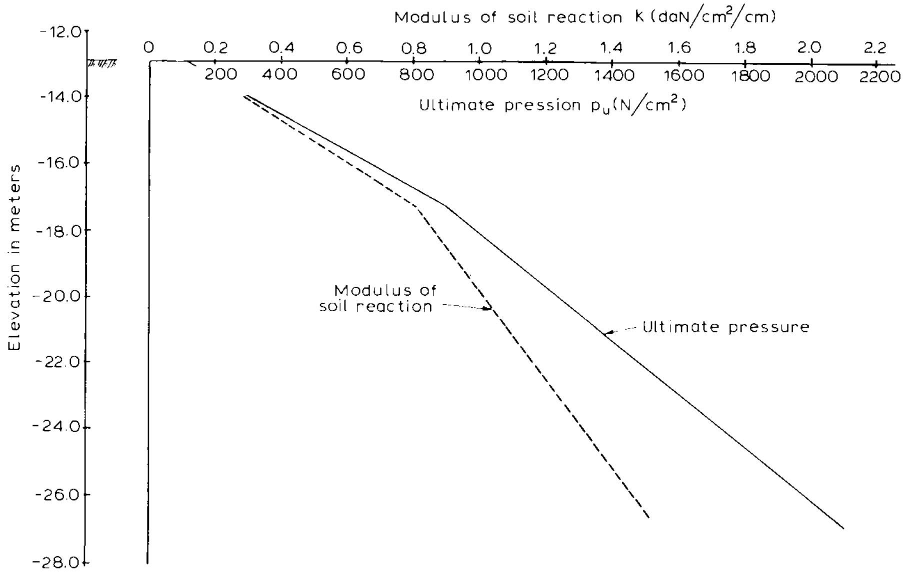


Fig. 8.32.

$k_{hl} = 183\,\text{kPa/cm}$ for $c_u = 56\,\text{kPa}$

and at -13.0 m: $c_u = 13.5\,\text{kPa}$, $k_{hB} = 69/5 = 14\,\text{kPa/cm}$

at -30.0 m: $c_u = 56\,\text{kPa}$, $k_{hB} = 183/5 = 37\,\text{kPa/cm}$.

Furthermore, we should consider a reduction in the modulus for depths less than $3B$. We write:

$k'_{hB} = C_m k_{hB}$

with: $C_m = (2 + 7z/z_c)/9$ and:

$z_c = 3B = 3 \times 1.5\,\text{m} = 4.5\,\text{m}$, from which:

at -13.0 m: $C_m = 2/9$, $k_{hB} = (2/9) \times 14 = 3.1\,\text{kPa/cm}$

at -17.5 m: $C_m = 1.0$, $k_{hB} = 20\,\text{kPa/cm}$.

(c) The ultimate pressure. The Marche method gives:

at -13.0 m: $c_u = 13.5\,\text{kPa}$, $p_u = 2 \times 13.5 = 27\,\text{kPa}$

at -17.5 m: $c_u = 24.8\,\text{kPa}$, $p_u = 9 \times 24.8 = 223\,\text{kPa}$

at -30.0 m: $c_u = 56\,\text{kPa}$, $p_u = 9 \times 56 = 504\,\text{kPa}$,

from which is drawn Fig. 8.31.

2.2. The parameters of set 2. The magnitudes of k and p_u are those above multiplied by 4, from which is drawn Fig. 8.32.

TABLE 8F

(a) Moments of inertia

Elevation	e (mm)	I (m^4)	EI (N/cm^2)
+8.0 to −2.95	15	0.01929	405 130
−2.95 to −6.90	18	0.02301	483 237
−6.90 to −10.85	25	0.03151	661 790
−10.85 to −22.70	26	0.03271	686 879
−22.70 to −26.65	18	0.02301	483 237

Note: $I = (\pi/64)[D^4 - (D - 2e)^4]$; $E = 2.1 \times 10^5$ MPa; e = wall thickness $D = 150$ cm.

(b) Resistance

Elevation	σ_e (MPa)
+8.00 to +1.00	240
+1.00 to −2.95	470
−2.95 to −26.65	600

2.3. Pile characteristics. The pile characteristics are the moment of inertia and the elastic limit of steel. Even an important variation of EI has little effect on the deformation, the bending moments and the soil reactions (see Table 8F).

(3) Results of the analysis, parameters set 1, pile tip at El. − 27 m

3.1. Docking. The results of the computer output for the deformations are presented on Fig. 8.33. The deformation at the pile butt is 5.5 m.

3.2. Mooring. The computer results may be summarized as follows:

Calculation no. 1:
— deformation at El. + 5.0: $\delta = 1.40$ m
— reaction at El. + 5.0: $F = 566$ kN
— energy absorbed by the pile: $E = \frac{1}{2}F\delta = 396$ kN · m

Calculation no. 2:
— deformation at El. + 5.0: $\delta = 2.00$ m
— reaction at El. + 5.0: $F = 626$ kN
— energy absorbed by the pile: $E = \frac{1}{2}F\delta = 626$ kN · m

Interpolation for E = 500 kN · m:
— energy absorbed by the pile: $E = 500$ kN · m.
— deformation at + 5.0:

$$\delta = 1.40 + (500 - 396)/(626 - 396) \times (2.0 - 1.4) = 1.67\text{ m}$$

— reaction at + 5.0: $F = (2 \times 500)/1.67 = 598$ kN.

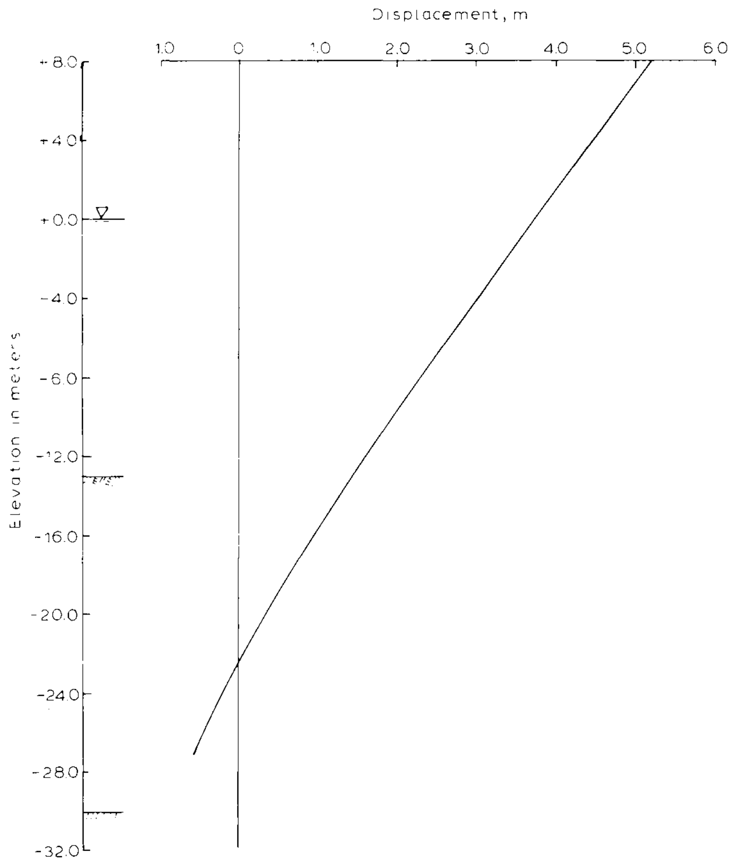


Fig. 8.33. Pile displacement during mooring (tip at −27 m, parameters 1).

3.3. Conclusion. The pile deformations during docking and mooring are too large, so the pile should be embedded in the underlying bedrock.

(4) Results of the analysis, parameters set 1, fixed-end pile at depth −35.0 m

4.1. Assumptions. Assume an embedment of 5.0 m into the rock (= 3 times the pile diameter). The results of the analysis indicate that a shallower embedment of about 3 m into the rock could be recommended. A pile wall thickness of 26 mm is justified from El. − 11 to El. − 35.0.

4.2. Docking. The variations of the deformations, the bending moment and the soil reactions are presented on Figs. 8.34, 8.35 and 8.36.

4.3. Mooring. The variation of the deformation energy of the pile as a function of the deformation at El. + 5.0 is presented on Fig. 8.37. An applied energy of 500 kN · m corresponds to a displacement of 1.14 m at

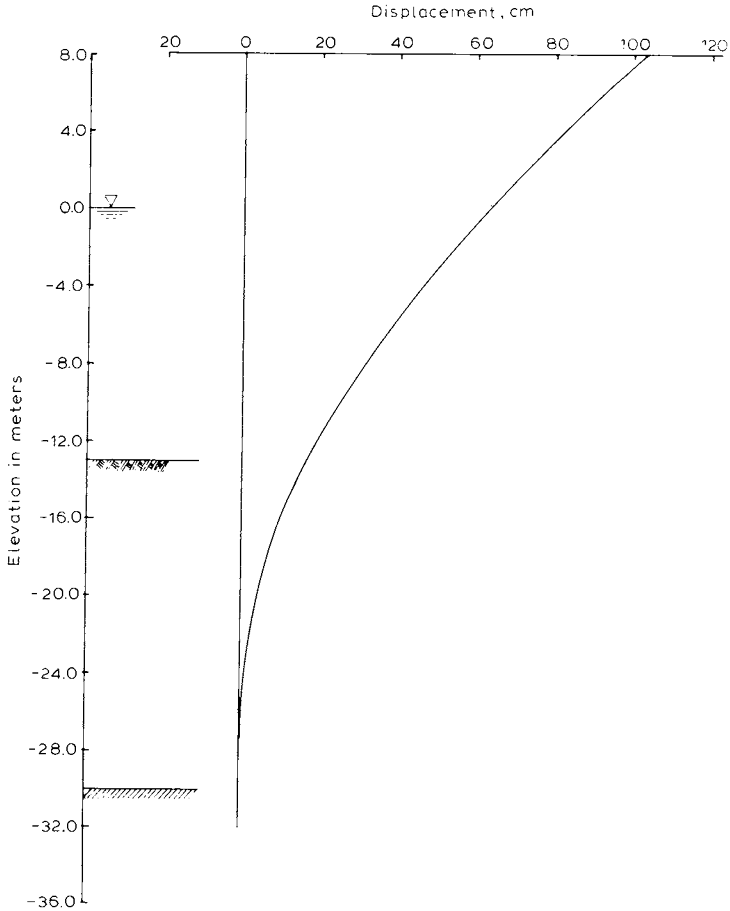


Fig. 8.34. Pile displacement during mooring (parameters 1: tip at -35 m).

El. + 5.0. The variations of deformation, bending moments and soil reactions are presented on Figs. 8.38, 8.39 and 8.40.

4.4. Conclusion. The pile deformation and soil reactions during both docking and mooring conditions are allowable and the stability of the fender pile is adequate.

(5) Results of the analysis, parameter set 2, free-end pile at depth -27 m
The variations of the displacement, the bending moment and soil reaction are presented on Figs. 8.41, 8.42 and 8.43.

5.1. Docking
Calculation no. 1:
— displacement imposed at El. + 5.0: $\delta = 0.8$ m

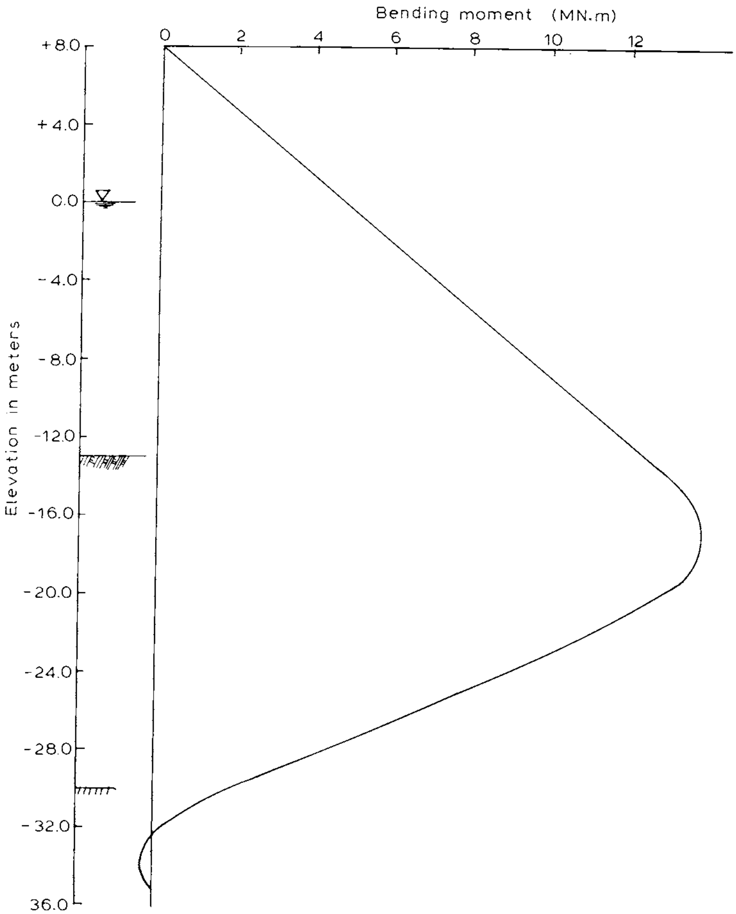


Fig. 8.35. Bending moment during mooring (parameters 1: tip at − 35 m).

— reaction at El. + 5.0: $F = 814$ kN
— energy absorbed by pile: $E = \frac{1}{2}F\delta = 330$ kN · m.

Calculation no. 2:
— displacement imposed at El. + 5.0: $\delta = 1.40$ m
— reaction at El. + 5.0: $F = 1360$ kN
— energy absorbed by pile: $E = \frac{1}{2}F\delta = 950$ kN · m.

Interpolation for $E = 500$ kN · m:
— energy absorbed by pile: $E = 500$ kN · m
— deformation at El. + 5.0:

$$\delta = 0.80 + (500 - 330)/(950 - 330) \times (1.40 - 0.80) = 0.96 \text{ m}$$

— reaction at El. + 5.0: $F = (2 \times 500)/0.96 = 1040$ kN.

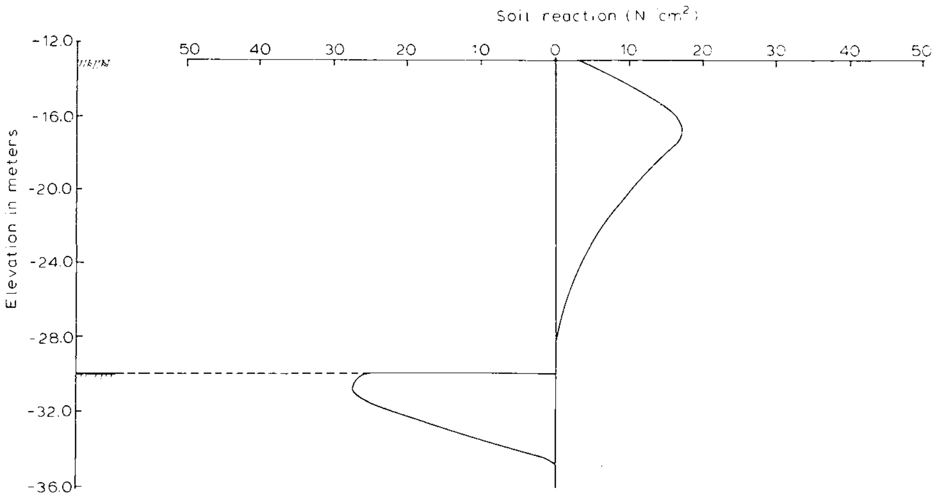


Fig. 8.36. Soil reaction during mooring (parameters 1: tip at − 35 m).

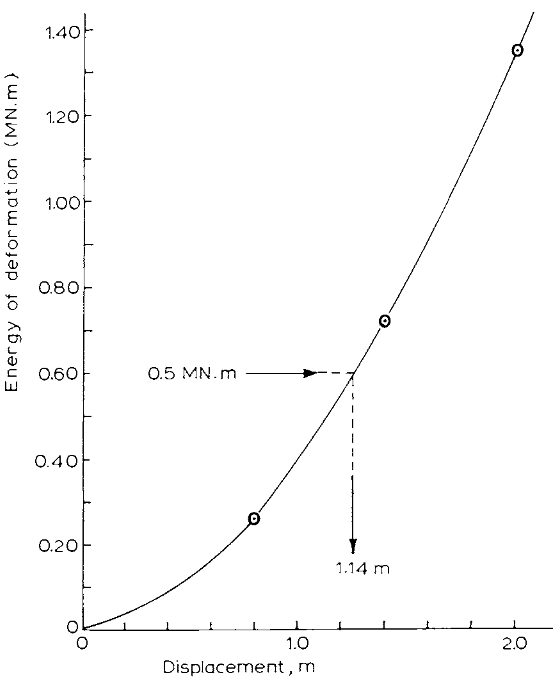


Fig. 8.37. Variation of the energy of pile deformation as a function of imposed deformation (parameters 1: tip at − 35 m).

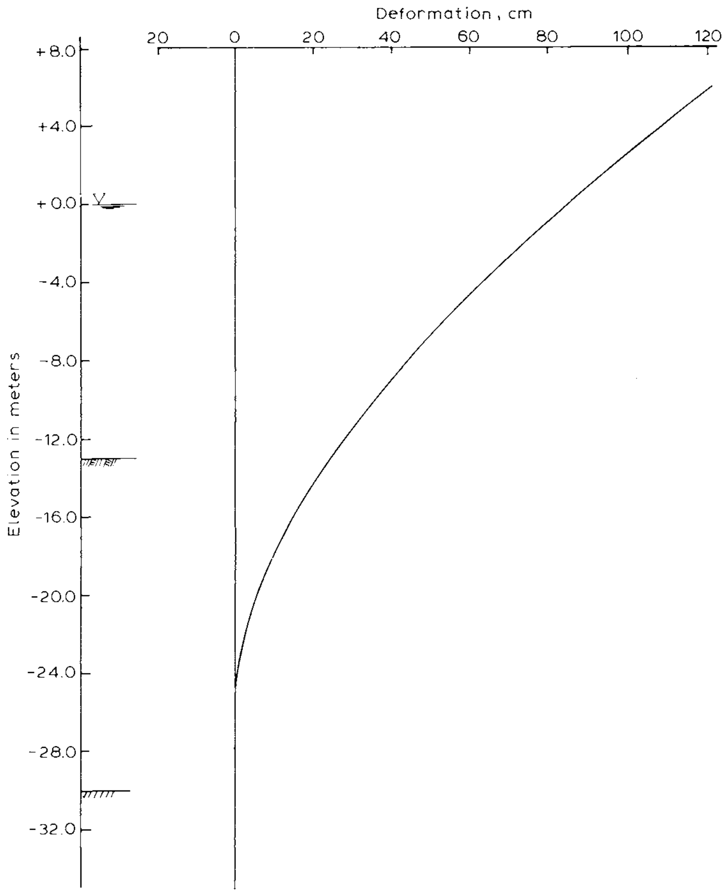


Fig. 8.38. Pile displacement during docking (parameters 1: tip at −35 m).

Variations of deformation, bending moments and soil reaction are presented on Figs. 8.44, 8.45 and 8.46.

5.2. Conclusion. Pile deformations and soil reactions during mooring conditions are allowable and the stability of the fender pile for this loading condition is adequate.

General conclusion

The available information on soil parameters is not sufficient to allow for a final design of the structure. Because of this lack of information, it was necessary to assume two sets of conditions:

(1) The first assumption was that of a two-dimensional calculation. The mud is assumed to be normally consolidated and to have a shear strength

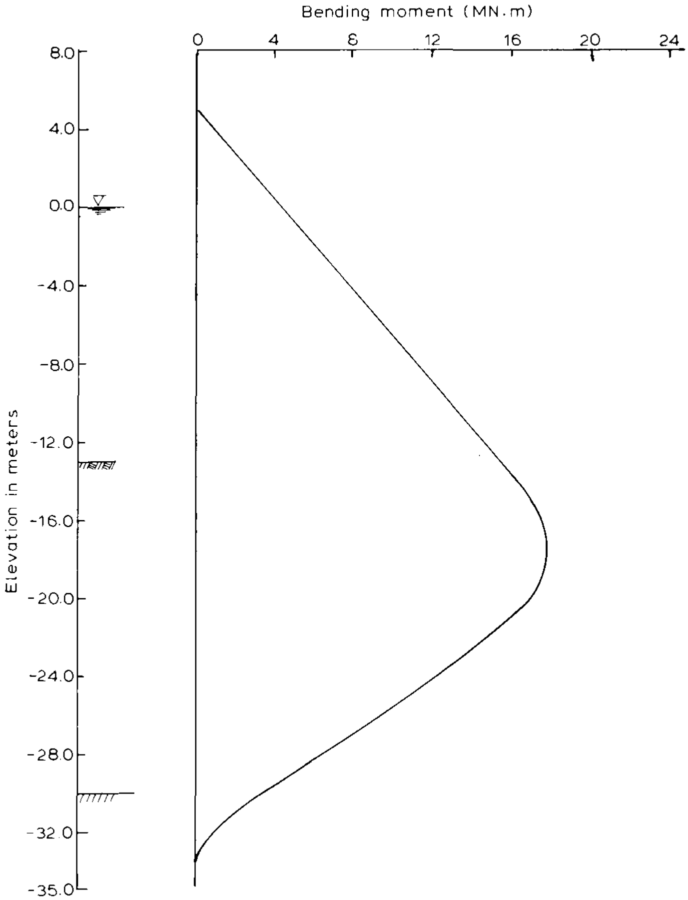


Fig. 8.39. Bending moment during docking (parameters 1: tip at −35 m).

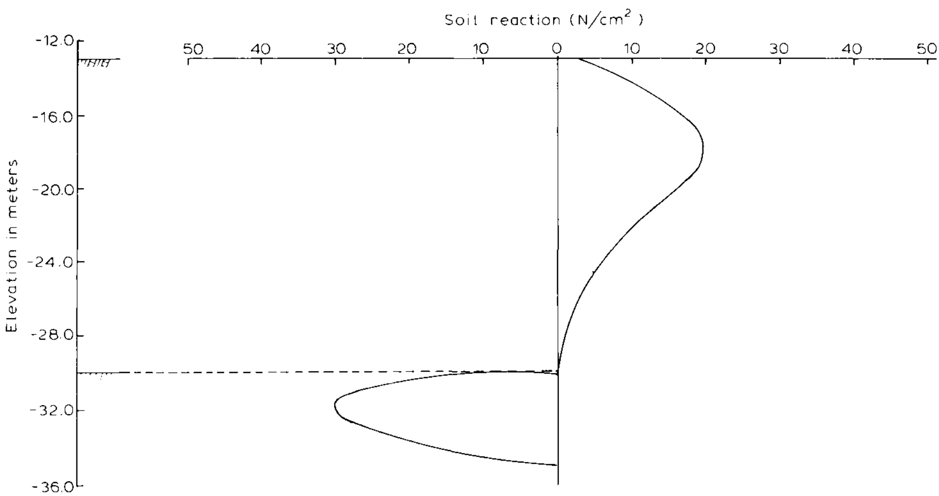


Fig. 8.40. Soil reaction during docking (parameters 1: tip at −35 m).

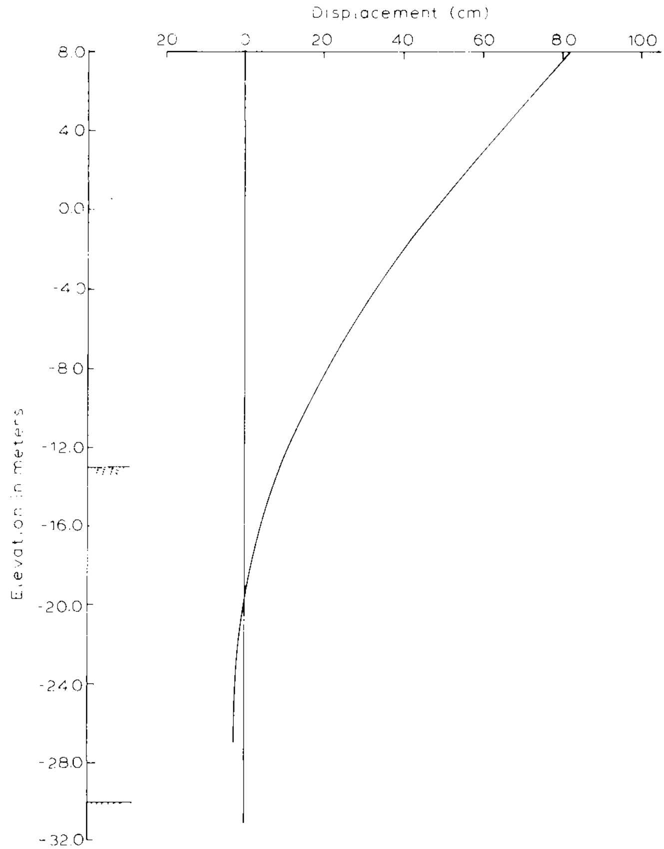


Fig. 8.41. Pile displacement during mooring (parameters 2: tip at −27 m).

that increases as a function of depth $c_u/\sigma'_o = 0.25$ where σ'_o is the effective overburden pressure. The basis for estimating k and p_u gives values of k from 0.03 daN/cm^3 at El. − 13.0 to 0.37 daN/cm^3 at El. − 27 and gives values of p_u from 0.27 daN/cm^3 at El. − 13.0 to 5.04 daN/cm^3 at El. − 27.0. Under those conditions, it is necessary to provide for an embedment of the pile of 3 m into the underlying bedrock in order to ensure good fender-pile performance.

(2) The second assumption is based on practical experience which shows that the pile behaves as if the values of k and p_u were 4 times larger than those assumed above. Under those conditions, the fender pile performs satisfactorily with an embedment of 14 m in the clay.

Given the reduced available information of the problem to work with, only the first assumption guarantees an acceptable behaviour of the fender pile. The second assumption and its conclusions could only be realistically

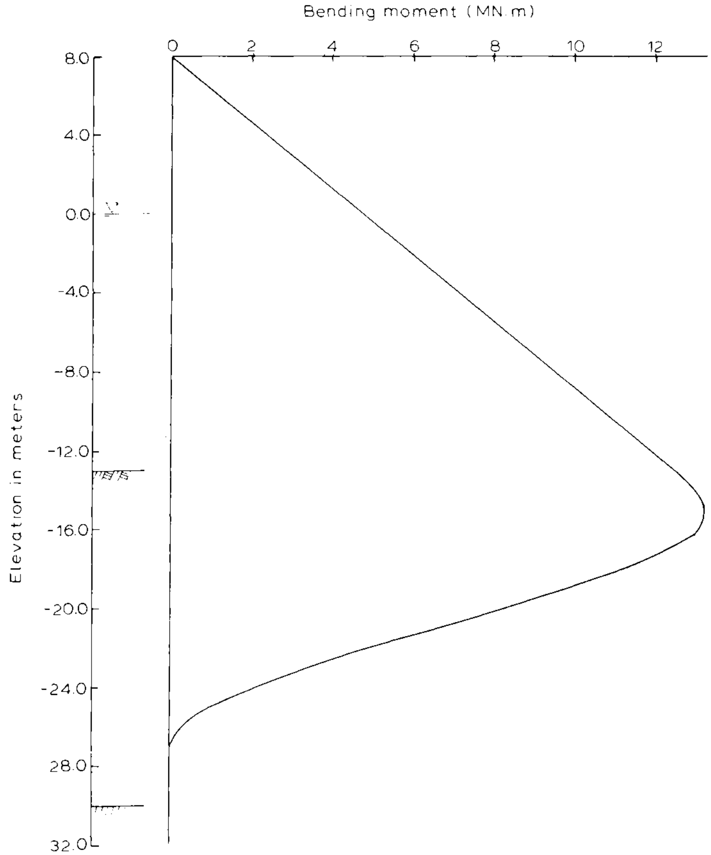


Fig. 8.42. Bending moment of pile during mooring (parameters 2: tip at −27 m).

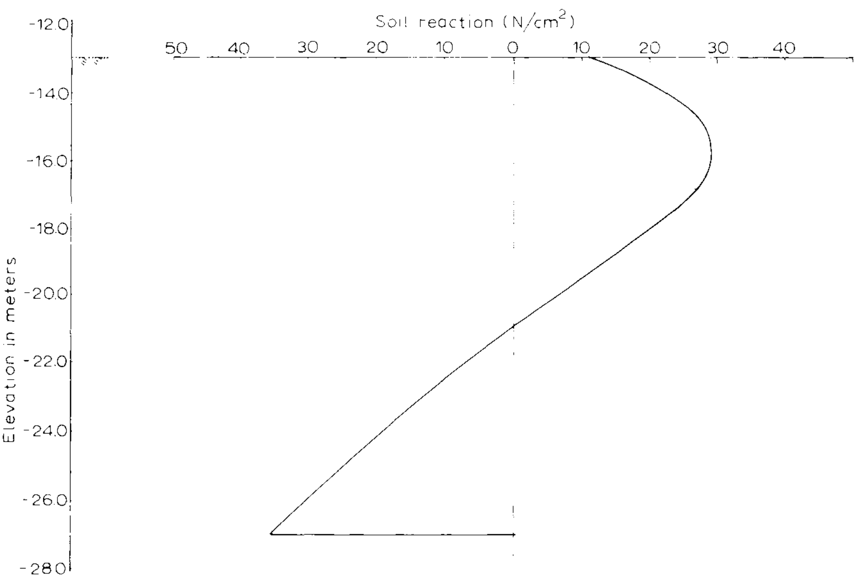


Fig. 8.43. Soil reaction during mooring (parameters 2: tip at −27 m).

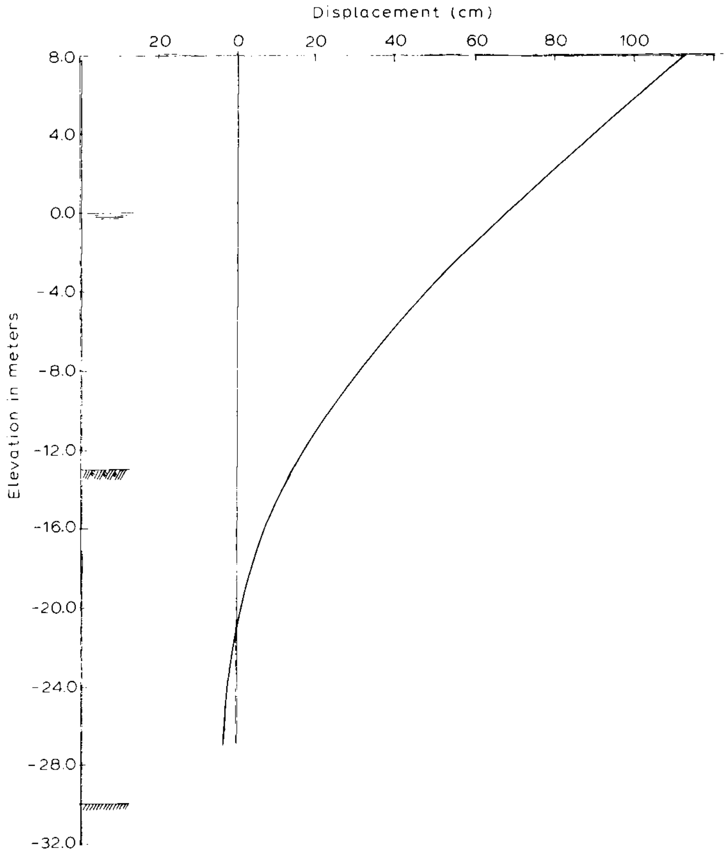


Fig. 8.44. Displacement of pile during docking (parameters 2: tip at −27 m).

considered if a site study confirmed the data. Such a study should consider two borings with a pressuremeter testing every meter at the location of the pile, or with recovery of undisturbed samples with laboratory testing of the soils of various layers. It should be noted that in the case of embedment in rock, the results obtained by the simplified method are in good accordance with those obtained by computer calculation.

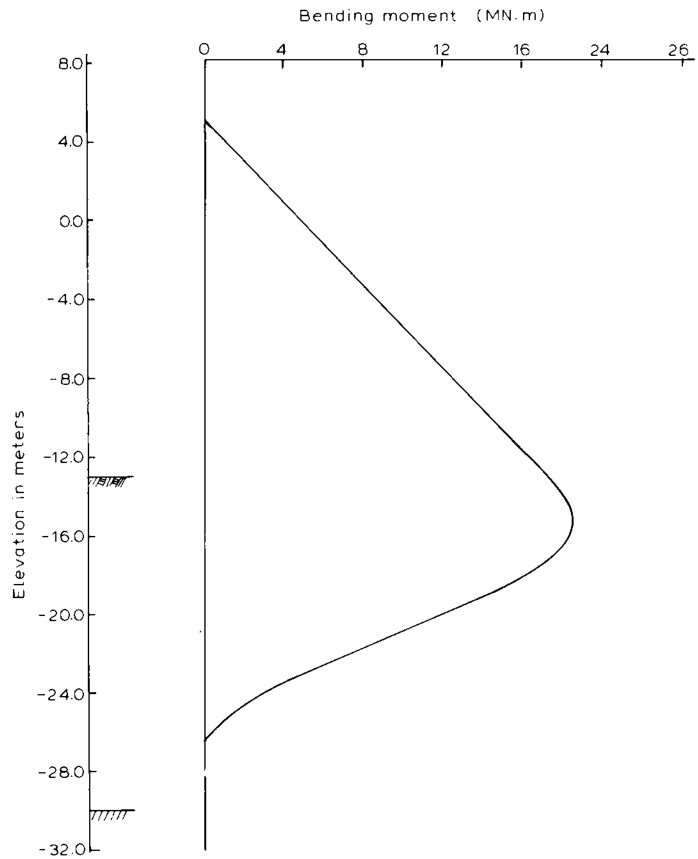


Fig. 8.45. Bending moment of pile during docking (parameters 2: tip at −27 m).

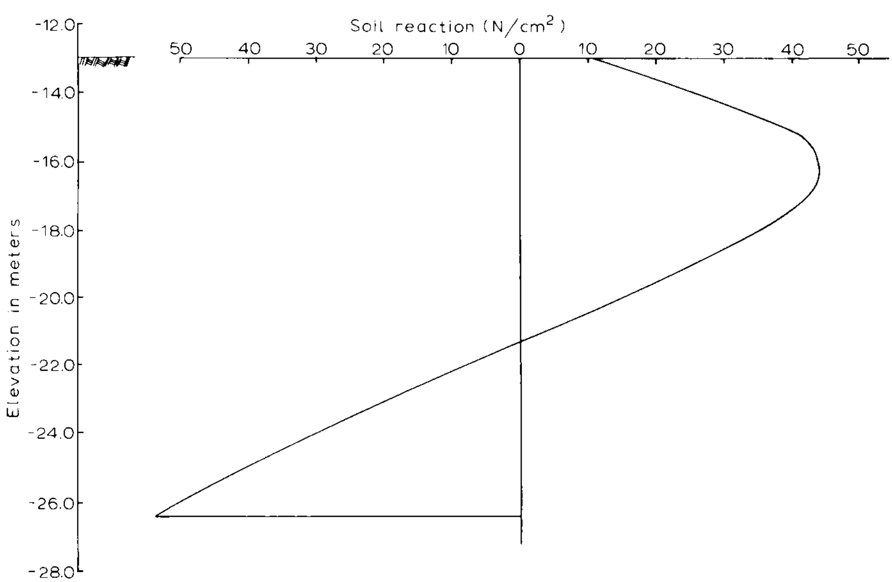


Fig. 8.46. Soil reaction on the pile during docking (parameters 2: tip at −27 m).

★★★*Problem 8.6* **Uplift of an excavation bottom**

It is necessary to determine the short-term stability of the bottom of an excavation of large length, whose width is B and its depth is H. The excavation is to be made in a homogeneous, cohesive soil of cohesion c_u and unit weight γ.

The excavation is shored and buttressed so that the sides may be assumed not to deform. The incipient failure plane therefore can be assumed to be a plane parallel to the shoring located at a horizontal distance x away from it, as well as the plane of symmetry with respect to the longitudinal axis of the excavation (Fig. 8.47).

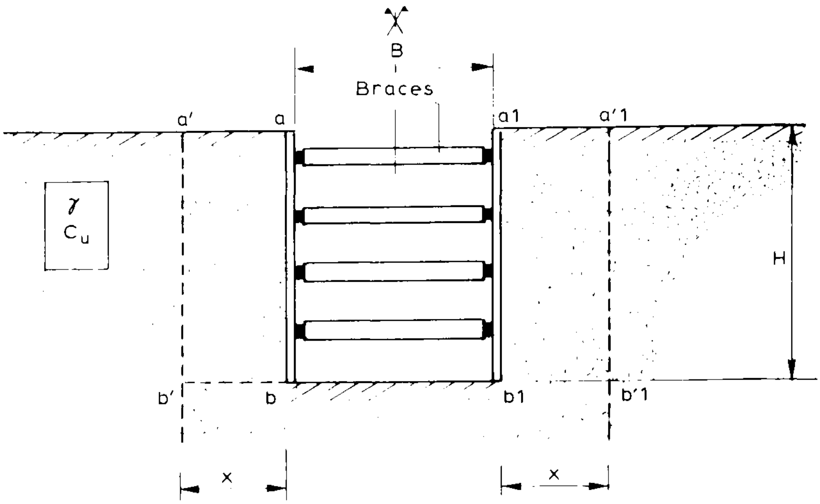


Fig. 8.47.

Assume further that the shoring does not extend down beyond the bottom of the excavation and, therefore, cannot impart any friction resistance to the soil mass it retains and which could become unstable.

(1) Determine the equilibrium of the soil of aa'bb' as defined on Fig. 8.47 which could fail along plane a'b' and assume R to be the reaction of the soil pressure on bb'.

(2) Evaluate R from the results for the shallow footing design. Find the limit value of x and determine the value of the safety factor F. Compare the results (according to the plastic diagrams used by Caquot or Terzaghi, for example).

(3) Suggest another approach which would not consider the soil mass aa'bb'. What are your conclusions?

(4) What happens to the safety factor if the soil mass supports a uniform surface load of magnitude q?

(5) Use the above results for a two-soil-layered system as shown on Fig. 8.48, the shoring being only for the upper layer $H_1 (\gamma_1 c_{u1})$ and the bottom

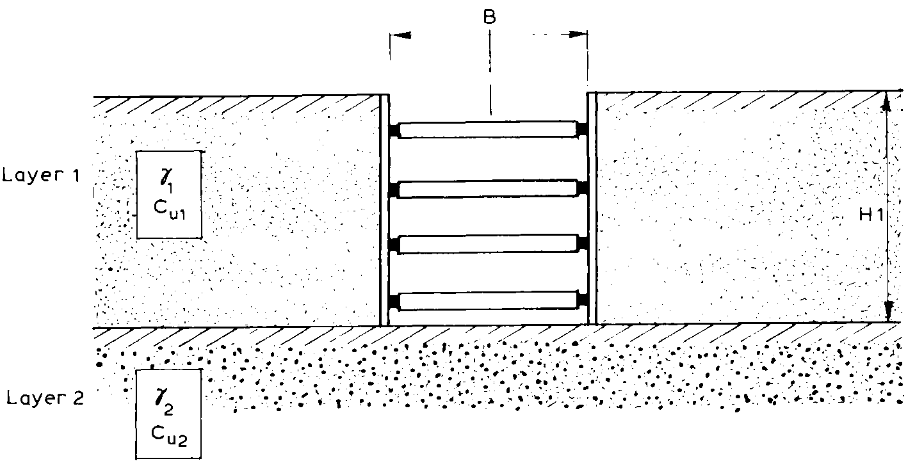


Fig. 8.48.

of the excavation corresponding to the upper limit of the lower soil layer ($\gamma_2 c_{u2}$) whose thickness may be assumed infinite.

(6) Numerical application. Compare the values of the safety factor corresponding to the above conditions for the following case: a two-layer system with an upper layer of thickness 6.5 m and c_{u1} = 20 kPa and a lower layer that is infinitely deep with c_{u2} = 25 kPa. The unit weights are both 20 kN/m^3. The bottom of the proposed excavation is at 6.50 m and has a width of 15 m. Assume the length to be infinite.

(7) Determine the safety factor by the method described above against uplift of the excavation bottom for the case shown on Fig. 8.49. The excavation is assumed to be rectangular in plan with a length equal to twice the width.

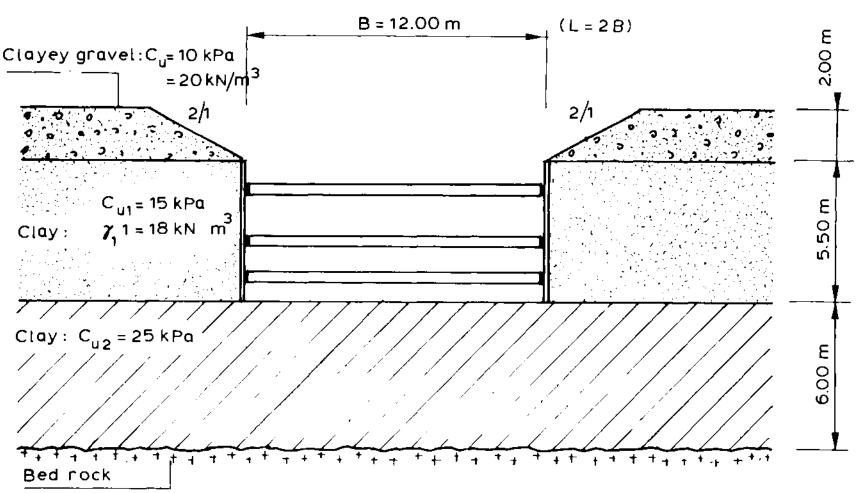


Fig. 8.49.

Solution

(1) The excavation is assumed to have an infinite length and therefore the problem reduces to a two-dimensional solution. We can then consider a typical 1 m wide slice of trench. Since we are dealing with a purely cohesive soil, no friction is assumed between the shoring and the soil mass. Furthermore, if we assume that there is no adhesion between them, there is no vertical force component to consider along plane ab. The earth pressure is horizontal. The forces which determine the equilibrium of the soil mass are shown on Fig. 8.50 and are:

— the mass of the soil: $W = \gamma H x$
— the shear stress along vertical plane $a'b'$: $T = c_u H$
— the reaction of the underlying soil on plane bb': R (this will be calculated below in answering question 2.)

For a width x of soil mass, there exists a reaction R which meets the criteria $R + T = > W$, because otherwise failure would occur.

The safety factor is given then by

$$F = (R + T)/W \tag{1}$$

Terzaghi expresses the safety factor from a slightly different concept, as:

$$F' = R/(W - T) \tag{2}$$

which leads to different results. It appears more logical, to adopt formula (1)

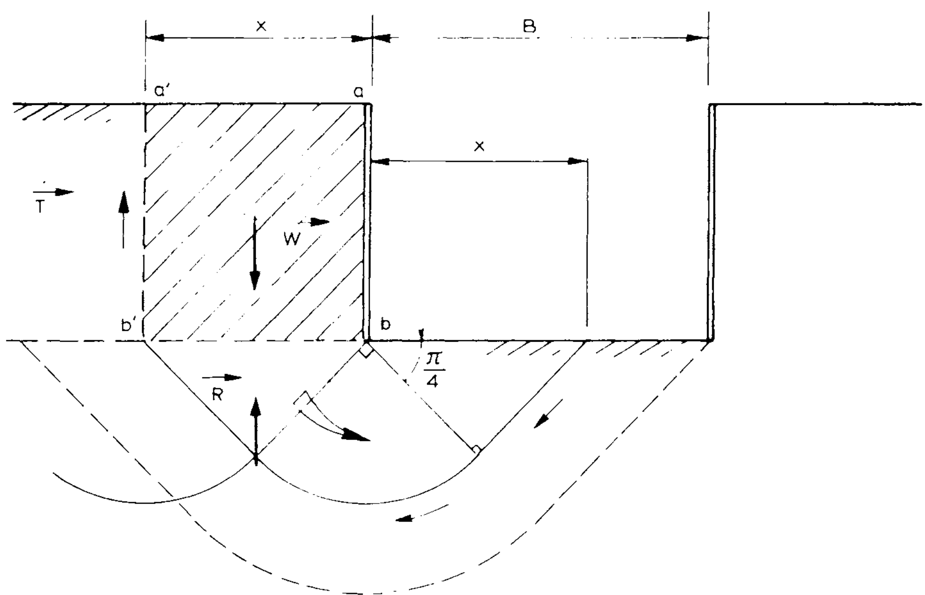


Fig. 8.50

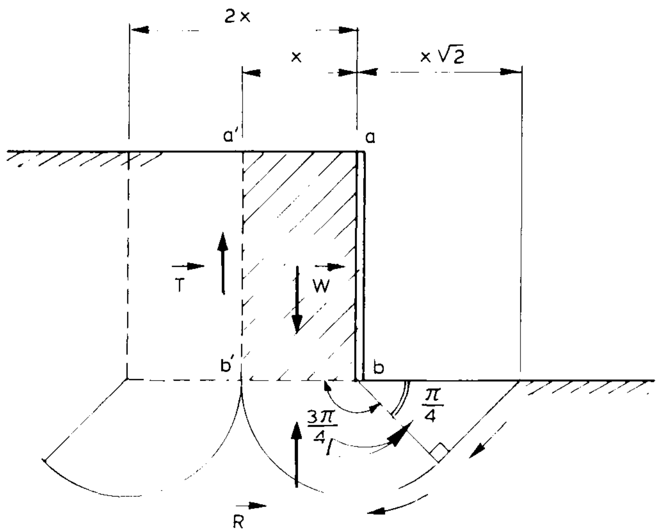


Fig. 8.51.

in which R and T are the resisting forces which keep the soil mass from falling into the excavation. However, the classical formula encountered in many books is based on F'.

(2) To calculate R, we may try to transfer the results of the shallow footing calculations to this particular case, and consider the mass $aa'bb'$ as a load acting on the underlying soil as a footing would. Let's take for instance, the Caquot plasticity graph (see Fig. 8.50). We can say that R is equal to the bearing capacity of a strip footing of width x, resting at the surface of the ground ($D = 0$) over a purely cohesive soil, since the failure would occur along the shortest line on the excavation side of the plane.

The bearing capacity for such a condition is:

$$q_d = c_u N_c \qquad \text{with} \qquad N_c = \pi + 2 = 5.14$$

Thus: $R = 5.14 x c_u$

and:

$$F = \frac{5.14 x c_u + H c_u}{\gamma H x} = \frac{5.14 c_u}{\gamma H} + \frac{c_u}{\gamma x}$$

This shows that the safety factor decreases as x increases, but we must remember that up to now we have assumed that the imaginary strip-footing was at the surface and that the failure plane daylighted in the trench bottom. From the plasticity graph, it is seen that the failure planes daylight in the trench for $0 < x < B$ conditions only (see Fig. 8.50). The minimum value of the safety factor therefore appears when $x = B$.

Finally, we have:

$$F = \frac{5.14c_u}{\gamma H} + \frac{c_u}{\gamma B} \tag{3a}$$

$$F = \frac{5.14B + H}{BH} \cdot \frac{c_u}{\gamma} \tag{3b}$$

Notes

(1) If we had used the Terzaghi plasticity graph instead of Caquot's (see Costet-Sanglerat, vol. 2, p. 152) results would have been different. In that case, we assume R to be half the bearing capacity of a strip footing of width $2x$; the value of N_c is that given by Terzaghi:

$$N_c = (3\pi/2) + 1 = 5.71$$

and: $R = \frac{1}{2} \times 5.71 \times 2xc_u = 5.71xc_u$

The net of failure lines gives:

$$x = B\cos\frac{\pi}{4} = B\frac{\sqrt{2}}{2} \quad \text{(Fig. 8.51)}$$

or:

$$F = \frac{5.71c_u}{\gamma H} + \frac{\sqrt{2}c_u}{\gamma B} \tag{4a}$$

$$F = \frac{5.71B + \sqrt{2}H}{BH} \cdot \frac{c_u}{\gamma} \tag{4b}$$

(2) If, in addition, we consider Terzaghi's definition of the safety factor (formula 2) as: $F = R/(W - T)$
we get:

$$F' = \frac{5.71Bc_u}{\gamma HB - H\sqrt{2}c_u} \tag{4c}$$

or, with the maximum depth corresponding to $F = 1$:

$$H_{\text{lim}} = \frac{5.71Bc_u}{\gamma B - c_u\sqrt{2}} \tag{5}$$

which is the formula frequently encountered in the technical literature.

(3) A different method of analysis may be considered referring to the theory of shallow footings.

In the case of a strip footing of width B, embedded to depth D in a purely cohesive soil, we get:

$$q_d = \gamma D + c_u N_c \text{ since } N_q = 1 \qquad (\varphi = 0)$$

For our case, where $D = H$:

$$q_d = \gamma H + c_u N_c$$

and $q_{ad} = \gamma H + (c_u N_c)/F$, which may be written as:

$$p_0 = p_1 + \frac{c_u N_c}{F} \qquad \text{where } p_0 > p_1$$

In the case of a cut of great length, width B and depth H, the same analysis could be made with $p_0 < p_1$ (p_0 and p_1 should be changed in the result). In the case of $p_0 = 0$ we get:

$$p_0 = p_1 - \frac{c_u N_c}{F} = 0$$

from which:

$$F = c_u N_c / \gamma H \tag{6a}$$

since we assumed $p_1 = \gamma H$.
Assuming a value of N_c from Caquot's theory, we finally get:

$$F = 5.14 c_u / \gamma H \tag{6b}$$

Conclusions

If we compare the various theories of the preceding questions, we may conclude that the safety factor from formula (6) is the greatest. Indeed, in the first instance, the safety factor is increased by $c_u/\gamma B$ or $\sqrt{2}c_u/\gamma B$, depending on the plasticity net chosen (formulas 3 and 4).

By the same token, if we use Terzaghi's formula (4) we may write:

$$F' = \frac{5.71 B c_u}{\gamma H B - H\sqrt{2}c_u} = \frac{\dfrac{5.71 B c_u}{\gamma H B}}{1 - \dfrac{H\sqrt{2}c_u}{\gamma H B}} = \frac{F}{1 - \dfrac{\sqrt{2}c_u}{\gamma B}}$$

hence $F' > F$ (assuming $(\sqrt{2}c_u > 1)/\gamma B$).

We note that in this latter method, there is a limit value for B which is $B = c_u\sqrt{2}/\gamma$ below which the analysis has no more significance (we find that $F' < 0$ and an upper height-limit which is negative).

According to this method, there could never be a stability for trench depths less than $\sqrt{2}c_u/\gamma$. This contradiction comes from the bad definition of the safety factor. With the proposed formula (1), this difficulty disappears. It also does not exist in the method used for solving question 3.

Remark

It is normal that the value of the safety factor F of question 3 is the largest. This is because the shear resistance along the failure planes was not taken into account between the level of the bottom of the trench and the upper soil level. Thus, it was assumed that no increase in N_c occurred with depth. We could, however, count on such a linear increase as a function of depth. For instance, Skempton proposed such an increase in N_c with depth with 5.14 for $D/B = 0$ and to 7.5 for $D/B > 5$ or:

$$N_c = 5.14 + 0.472D/B \quad \text{for } D/B \leqslant 5$$

$$N_c = 7.5 \quad \text{for } D/B \geqslant 5$$

We then have for $H \leqslant 5B$:

$$F = \frac{c_u[5.14 + 0.472(H/B)]}{\gamma H} = \frac{5.14c_u}{\gamma H} + 0.472\frac{c_u}{\gamma B} \tag{7}$$

which is comparable to formulas (3) and (4). In spite of this increase of N_c, we still get minimum values for the safety factor.

(4) The formulas used above must be modified when the soil mass is acted upon by a surface uniform surcharge of $Q = q \times x$, which is added to the weight of the mass $aba'b'$, if we use the first approach, or to add stress q to γH for the second one.

For the first instance then we have:

$$F = \frac{5.14c_u}{\gamma H + q} + \frac{H}{B}\cdot\frac{c_u}{\gamma H + q} \quad \text{(Caquot)} \tag{8a}$$

$$F = \frac{5.71c_u}{\gamma H + q} + \sqrt{2}\cdot\frac{H}{B}\cdot\frac{c_u}{\gamma H + q} \quad \text{(Terzaghi)} \tag{8b}$$

and for the second approach:

$$F = \frac{c_u \cdot N_c}{\gamma H + q} \tag{9}$$

(5) For a two-layer system, R is calculated from the cohesion values of the underlying layer, whereas W and T are those pertaining to the upper layer. We then have:

$$F = \frac{5.14c_{u2}}{\gamma_1 H_1} + \frac{c_{u1}}{\gamma_1 B} \quad \text{from (3)}$$

$$F = \frac{5.71Bc_{u2}}{\gamma_1 H_1 B - H_1 \cdot \sqrt{2}\,c_{u1}} \quad \text{from (4c)}$$

$$H_{\text{lim}} = \frac{5.71 c_{u2}}{\gamma_1 - (c_{u1} \cdot \sqrt{2})/B} \qquad \text{from (5)}$$

$$F = \frac{c_{u2} N_c}{\gamma_1 H_1} \qquad \text{from (6)}$$

In this last instance, the upper layer cohesion has no influence.

On the other hand, the increase of N_c proposed in question 3 can only be made unless if c_{u1} of the upper layer is higher than c_{u2} of the lower layer. For the opposite case, it is advisable not to increase N_c if c_{u1} is considerably lower than c_{u2}.

(6) Numerical application

$c_{u1} = 20\,\text{kPa}, \quad c_{u2} = 25\,\text{kPa}, \quad \gamma_1 = \gamma_2 = 20\,\text{kN/m}^3$

$B = 15\,\text{m}$ (infinite length), $H = 6.50\,\text{m}$

The results are:

— Method 1 (Caquot graph)

$$F = \frac{5.14 \times 25}{20 \times 6.50} + \frac{20}{20 \times 15} \simeq 1.06$$

— Method 2 (Terzaghi graph):

$$F = \frac{5.17 \times 25}{20 \times 6.50} + \frac{20 \times \sqrt{2}}{20 \times 5} \simeq 1.19$$

— Method 3 (Terzaghi):

$$F' = \frac{8.71 \times 15 \times 25}{20 \times 6.5 \times 15 - 6.5 \times \sqrt{2} \times 20} \simeq 1.21$$

$$H_{\text{lim}} = \frac{5.71 \times 25}{20 - \dfrac{20 \times \sqrt{2}}{15}} = 7.88\,\text{m}.$$

— Method 4 (without increasing N_c):

$$F = \frac{5.14 \times 25}{20 \times 6.50} \simeq 0.99$$

In actual fact, even though c_{u1} is lower than c_{u2} we may increase N_c. We propose the following approach using Skempton's coefficient:

$$N_c = 5.14 + \frac{c_{u1}}{c_{u2}} \times 0.472 \times \frac{H}{B}$$

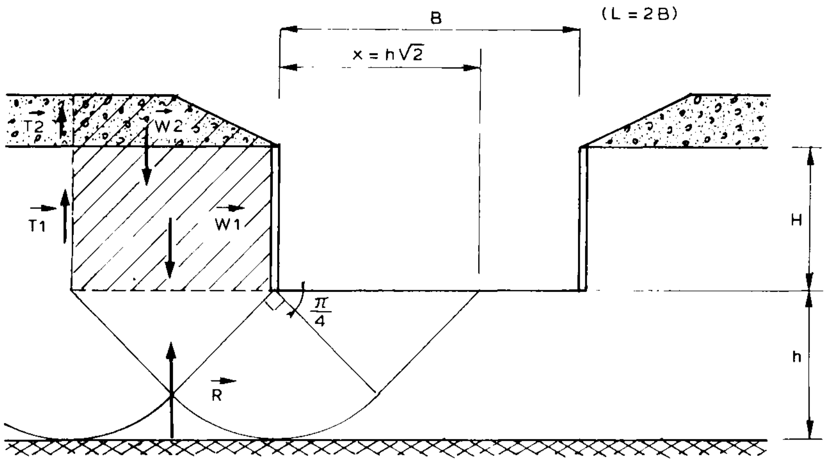


Fig. 8.52.

$$\text{or here: } N_c = 5.14 + \frac{20}{25} \times 0.472 \times \frac{6.5}{15} = 5.30$$

$$\text{so: } F = \frac{5.30 \times 25}{20 \times 6.50} \simeq 1.02$$

The safety factor thus is very close to 1: the trench bottom is about to lift up.

(7) Let us adopt method 1 to solve this problem, taking into account the increase in bearing capacity $(1 + 0.2B/L)$ (see Costet-Sanglerat, Vol. II) due to the fact that the trench is no longer infinite. On the other hand, there is a bedrock substratum through which the failure planes will not go. This limits the value of x to $h\sqrt{2}$, as seen on Fig. 8.52.

We have, finally, for a slice of 1 m length:

$$x = h\sqrt{2} = 6.00 \times \sqrt{2} \simeq 8.50\,\text{m}$$

$$R = xc_{u2}\left(1 + 0.2\frac{B}{L}\right)N_c = 8.5 \times 25\left(1 + 0.2 \times \frac{12}{24}\right) \times 5.14 \simeq 1201.5\,\text{kN}$$

$$T = 2 \times 10 + 15 \times 5.50 = 102.50\,\text{kN}$$

Weight of soil mass $aba'b'$:

fill (clayey gravel) $= W_1 = \frac{1}{2}(4.50 + 8.50) \times 2.00 \times 20 = 260\,\text{kN}$

soft clay layer $W_2 = 8.50 \times 5.50 \times 18 = 841.5\,\text{kN}$,

$W = W_1 + W_2 = 1101.5\,\text{kN}$

and finally:

$$F = \frac{1201.5 + 102.5}{1101.5} = 1.18$$

The safety factor is low. For temporary support design, a safety factor between 1.5 and 2 is preferred. The finally adopted safety factor depends on the level of confidence of the engineer in evaluating the soil characteristics and the knowledge of the possible surcharges.

Final remark

There are other approaches but all are based on the same principle. The essential difference between the methods resides in the plasticity graph choice. The methods of Bjerrum and Eide (1956), Tschebotarioff (1973), the NAVFAC DH7 (1971) method mentioned in "Foundation Engineering Handbook" of H.F. Winterkorn and H.Y. Fang are other possible choices.

As for the remarks made above regarding the safety factor, the most common method used is that of method 3 (formula 6) with correction for N_c depending on the depth and shape of the trench. *Formulas 4c and 4d, which correspond to the historical formulas of Terzaghi (1943) should, in our opinion, be abandoned.*

Finally, question 7 shows that in certain instances, a simple plasticity graph must be chosen if conditions do not match those found in the available tables or charts.

Chapter 9

SLURRY WALLS

★★*Problem 9.1* Slurry wall stability during construction

A trench for a slurry wall is excavated in a cohesive soil where $c_u = 35$ kPa/cm^2, $\gamma_{sat} = 20$ kN/m^3. The trench sides are stabilized by filling the excavation with a bentonite slurry whose unit-weight is $\gamma_B = 11$ kN/m^3. Find the theoretical depth to which the trench may be excavated without caving. Assume its length to be infinite.

Solution

Short-term loading conditions apply. The cohesive soil is very impervious and the trench stability is critical for only a short time.

Let us find the height H_p of the trench corresponding to the development of the plastic conditions at the excavation bottom.

At point A (Fig. 9.2) we have: $\sigma_1 = H\gamma_{sat}$, $\sigma_3 = H\gamma_B$ for conditions at A to become plastic, we must have:

$\sigma_1 - \sigma_3 = 2\,c_u = H_p(\gamma_{sat} - \gamma_B)$ (Fig. 9.2).

The height corresponding to this will then be: $H_p = 2c_u/(\gamma_{sat} - \gamma_B)$.

The state of plasticity is reached when large deformations occur. Two conditions may develop: either there occurs a redistribution of the stresses in

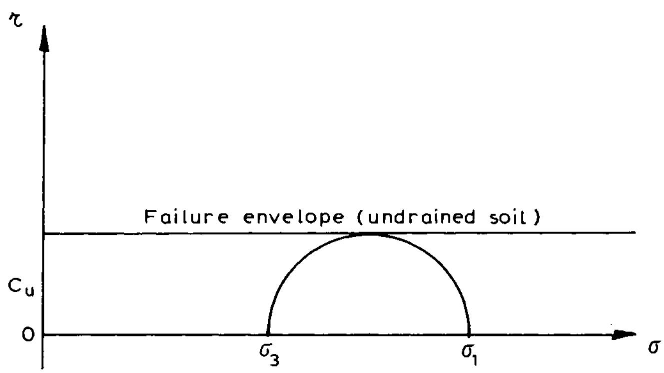


Fig. 9.1.

the upper soil zones, still in elastic state, or cracks appear in the soil and no stress redistribution can occur.

Case 1—stress redistribution occurs,

When height H_p is exceeded, σ_1 increases but $(\sigma_1 - \sigma_3)$ can no longer increase (plastic limit). Therefore, the stresses shown in the shaded area of Fig. 9.3 must be taken into account by an increase in stress in the upper zone, still in an elastic state of stress.

If the stress redistribution occurs completely, the entire trench height

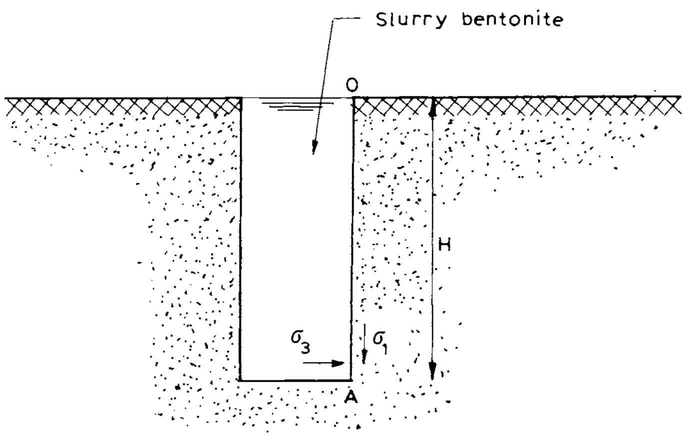


Fig. 9.2.

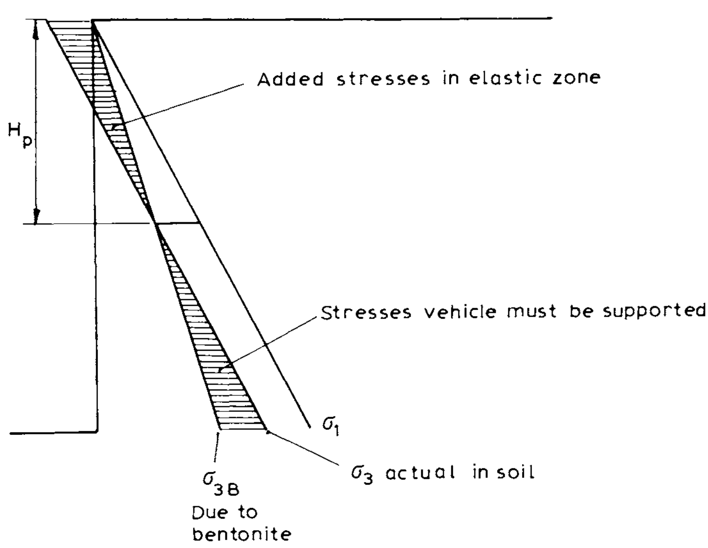


Fig. 9.3.

can become in a state of plasticity:

$$H_M = 2H_p = 4\,c_u/(\gamma_{sat} - \gamma_B) \quad \text{or:} \quad H_M = 140/9 = 15.5\,\text{m}$$

Case 2—fissures appear

Cracks appear in the soil as soon as the state of plasticity is reached at trench bottom; then failure occurs, for: $H_p = 2c_u/(\gamma_{sat} - \gamma_B) = 7.8$ m.

In reality, slurry wall excavation trenches can reach considerably deeper than the calculated value above. This is because the trench length is only a few meters and the boundary conditions do improve the stability and allow for greater depths (Compare sect. 8.3 in Costet-Sanglerat).

★★★*Problem 9.2* Design of a slurry wall with pre-stressed anchors

A prefabricated retaining-wall, as shown on Fig. 9.4, must be designed. The soil characteristics as well as the active and passive pressure coefficients are shown on the figure. A row of prestressed anchors is proposed at 2.5 m depth.

The soil above the wall supports a load of 10 kPa. The groundwater table is at − 4m.

The soil on the excavation side will be grouted from a level of 9 to 11.5 m to avoid uplift of the excavation bottom during dewatering.

(1) A first excavation phase will bring the level to 3 m depth, where anchors will be placed. Draw the passive and active pressure diagrams and calculate the maximal bending moment in the slab for this first construction stage.

(2) After tensioning of the anchors, a second excavation phase will follow, reaching a depth of 7.30 m. Draw the active-passive pressures diagram and calculate the maximal bending moment in the slab and the tension in the anchor rods to allow the excavation of phase 2 to take place.

Solution

A. CALCULATION FOR PLASTIC CONDITIONS

(1) Phase 1. Excavation to 3 m depth

1.1. Determination of the active-passive pressure diagram

From 0—3.70 m:

$$p_0 = 0.41 \times 10 = 4.1\,\text{kPa}$$

$$p_{(-3)} = 0.41(10 + 18 \times 3) = 26.2\,\text{kPa}$$

$$p_{(-3.70+\epsilon)} = 0.41(10 + 18 \times 3.70) = 31.4\,\text{kPa}$$

$$b_{(-3.70+\epsilon)} = 3.54 \times 18 \times 0.70 = 44.6\,\text{kPa}$$

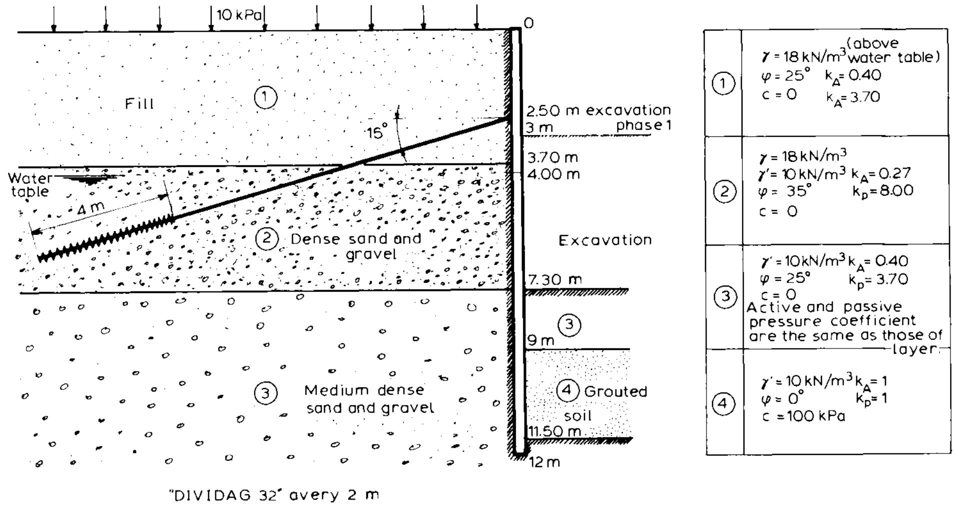


Fig. 9.4. Cross-section of wall and soils, as well as their mechanical characteristics.

$(b - p)_{(-3.70+\epsilon)} = 13.2\,\text{kPa}.$

From -3.70 to -7.30 m:

$p_{(-3.70-\epsilon)} = 0.27 \times (10 + 18 \times 3.70) = 20.7\,\text{kPa}$

$b_{(-3.70-\epsilon)} = 6.51 \times 18 \times 0.70 = 82\,\text{kPa}$

$(b - p)_{(-3.70-\epsilon)} = 61.3\,\text{kPa}$

$p_{(-4)} = 20.7 + 0.27 \times 18 \times 0.3 = 22.2\,\text{kPa}$

$b_{(-4)} = 82 + 6.5 \times 18 \times 0.3 = 117.2\,\text{kPa}$

$(b - p)_{(-4)} = 95\,\text{kPa}$

$p_{(-7.30)} = 22.2 + 0.27 \times 10 \times 3.3 = 31.1\,\text{kPa}$

$b_{(-7.30)} = 117.2 + 6.51 \times 10 \times 3.30 = 332\,\text{kPa}$

$(b - p)_{(-7.30)} = 300.9\,\text{kPa}.$

The diagram is shown on Fig. 9.5.

Notes. When a discontinuity occurs due to a change in soil layer, $+\epsilon$ indicates that one is immediately in the upper layer and $-\epsilon$ indicates that one is immediately in the lower layer.

From the similar triangles 2 and 3 (see Fig. 9.5) the point of zero stress is at depth -3.3 m approximately.

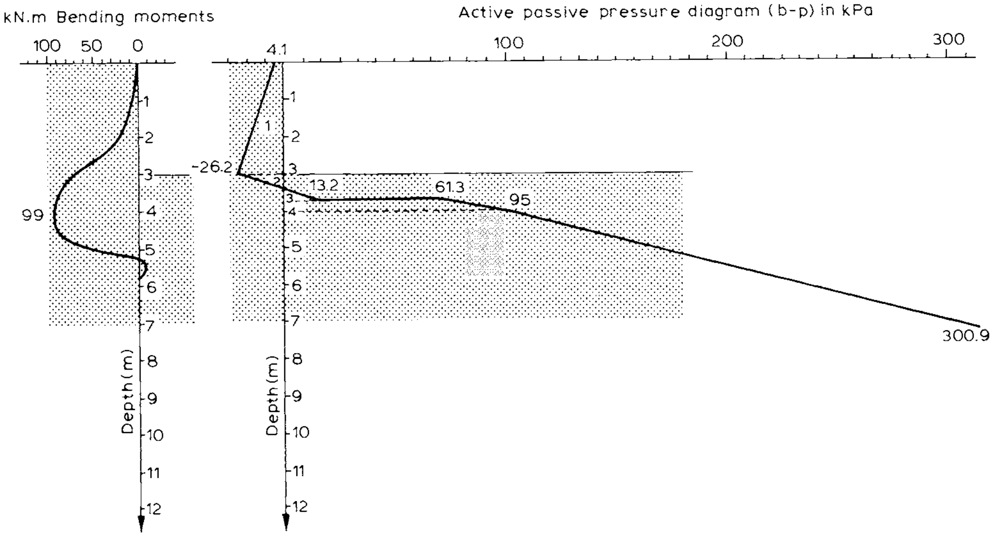


Fig. 9.5. Plastic condition, phase 1: excavation to 3 m of depth.

1.2. Maximal bending moment

$R_1 = \dfrac{4.1 + 26.2}{2} \times 3 = 45.5$ kN (inferior R-numbers correspond to areas shown on Fig. 9.5)

$$R_2 = \frac{26.2 \times 0.33}{2} = 4.3 \text{ kN}$$

Σ of the active pressures = 49.8 kN

$$R_3 = (13.2 \times 0.37)/2 = 2.4 \text{ kN}$$

$$R_4 = \frac{61.3 + 95}{2} \times 0.3 = 23.4 \text{ kN}$$

Σ of the passive pressures = 25.8 kN

Point of zero shear. At level -4 m, it is necessary to find a net-shear (passive less active) equal to: $49.8 - 25.8 = 24$ kN.

From -4 m:

$$(b-p) = q = 95 + [(300.9 - 95)/3.30]\,x$$

where x is the ordinate measured from -4 m.

$$(b-p)x = 24 \text{ and } 95\,x + 62.4\,x^2 = 24$$

from which $x = 0.22$ m.
The point of zero shear is located at -4.22 m. Thus, the maximal bending moment is:

$$M = 4.1 \times 3 \times 2.72 + \frac{22.1 \times 3}{2} \times 2.22 + \frac{26.2 \times 0.33}{2} \times$$

$$\times 1.11 - 13.2\,\frac{(3.7 - 3.33)}{2} \times 0.64 - \frac{61.3 + 95}{2} \times$$

$$\times 0.3(0.15 + 0.22) - \frac{108.7 + 95}{2} \times 0.22 \times 0.10 = 99\,\text{kN.m.}$$

The diagram of bending moments is shown on Fig. 9.5.

(2) Phase 2—excavation to 7.30 m of depth

2.1. Active-passive earth pressure diagram:

From 0 to −3.70 m:

$p_0 = 0.41 \times 10 = 4.1\,\text{kPa}$

$p_{(-2.50)} = 0.41 \times (10 + 18 \times 2.50) = 22.6\,\text{kPa}$

$p_{(-3.70+\epsilon)} = 0.41 \times (10 + 18 \times 3.70) = 31.4\,\text{kPa}$

From −3.70 to −7.30 m:

$p_{(-3.70-\epsilon)} = 0.27 \times (10 + 18 \times 3.70) = 20.7\,\text{kPa}$

$p_{(-4)} = 20.7 + 0.27 \times 18 \times 0.3 = 22.2\,\text{kPa}$

$p_{(-7.30+\epsilon)} = 22.2 + 0.27 \times 10 \times 3.30 + 10 \times 3.30 = 64.1\,\text{kPa}$

From −7.30 to −9 m:

$p_{(-7.30-\epsilon)} = 0.41 \times (10 + 18 \times 4 + 10 \times 3.30) + 10 \times 3.30 = 80.2\,\text{kPa}$

$p_{(-9+\epsilon)} = 80.2 + 0.41 \times 10 \times 1.70 = 87.2\,\text{kPa}$

$b_{(-9+\epsilon)} = 3.54 \times 10 \times 1.70 = 60.2\,\text{kPa}$

$(p - b)_{(-9+\epsilon)} = 27\,\text{kPa}.$

From −9 to −11.50 m.
Since $\varphi = 0$, the passive pressure is calculated directly from Mohr's diagram (see Problem 9.1) and so we get: $b = \gamma' z + 2c_u = 10z + 200$
where z = depth measured from the bottom of the excavation

$b_{(-9-\epsilon)} = 10 \times 1.70 + 200 = 217\,\text{kPa}$

$p_{(-9-\epsilon)} = 60.2\,\text{kPa}$

$(b - p)_{(-9-\epsilon)} = 156.8\,\text{kPa}$

$b_{(-11.50+\epsilon)} = 10(11.50 - 7.30) + 200 = 242\,\text{kPa}$

$p_{(-11.50+\epsilon)} = 60.2 + 0.41 \times 10 \times (11.50 - 9) = 70.5\,\text{kPa}$

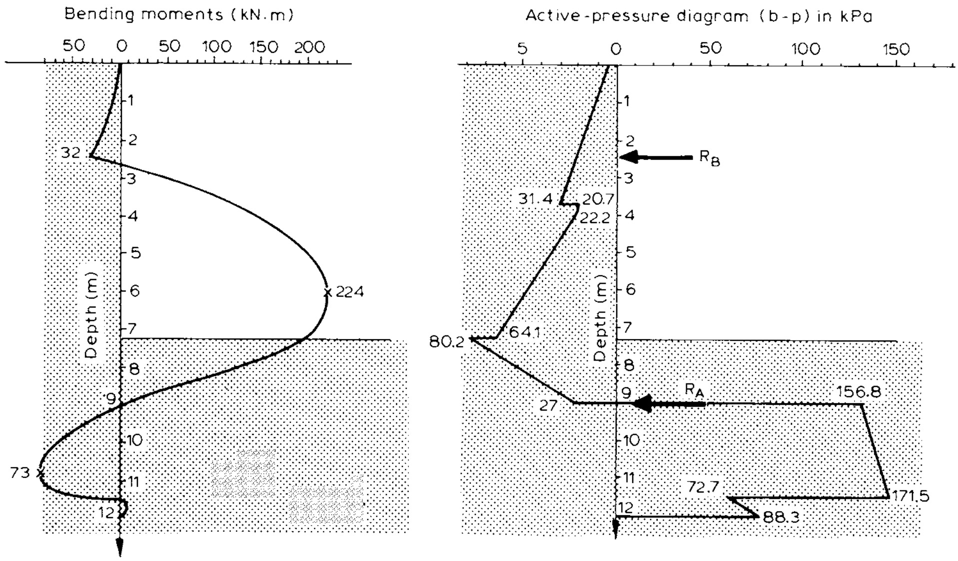


Fig. 9.6. Plastic condition, phase 2: excavation to 7.3 m of depth.

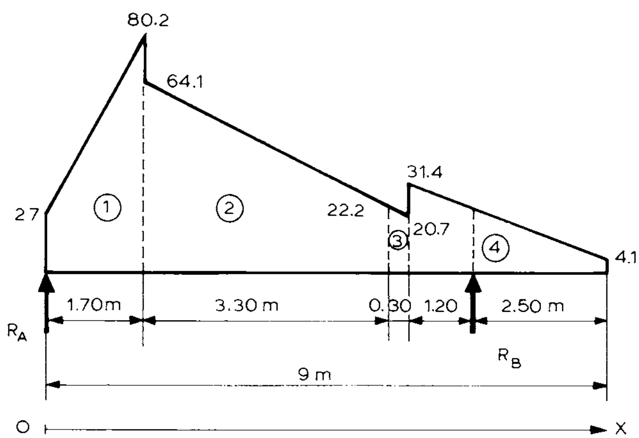


Fig. 9.7. Loads per unit of length in kN/m.

$$(b-p)_{(-11.50+\epsilon)} = 171.5 \text{ kPa}.$$

From -11.50 to -12 m:

$$b_{(-11.50-\epsilon)} = 3.41 \times 10 \times 4.20 = 143.2 \text{ kPa}$$

$$p_{(-11.50-\epsilon)} = 70.5 \text{ kPa}$$

$$(b-p)_{(-11.50-\epsilon)} = 72.7 \text{ kPa}$$

$$b_{(-12)} = 143.2 + 3.54 \times 10 \times 0.50 = 160.9\,\text{kPa}$$

$$p_{(-12)} = 70.5 + 0.41 \times 10 \times 0.50 = 72.60\,\text{kPa}$$

$$(b-p)_{(-12)} = 88.3\,\text{kPa}.$$

The active-passive pressure diagram is presented on Fig. 9.6.

2.2. Stresses in the slab

A modified method of Blum will be used, (see sect. 7.2, Costet-Sanglerat): the point of zero bending moment corresponds to the point of zero stress.

2.2.1. The equivalent beam must be computed (shown on Fig. 9.7)

$$R_1 = \frac{27 + 80.2}{2} \times 1.70 = 91.1\,\text{kN}$$

$$R_2 = \frac{64.1 + 22.2}{2} \times 3.30 = 142.4\,\text{kN}$$

$$R_3 = \frac{20.7 + 22.2}{2} \times 0.30 = 6.4\,\text{kN}$$

$$R_4 = \frac{31.1 + 4.1}{2} \times 3.70 = 65.7\,\text{kN}$$

$$\Sigma R = 305.6\,\text{kN}$$

Finally, we get:

$$R_A = 151.8\,\text{kN}, \qquad R_B = 153.8\,\text{kN}.$$

The tension in the anchors is then: $A = (153.8 \times 2)/\cos 15^\circ = 318.5\,\text{kN}$ for a 2 m spacing of the anchors inclined 15° with the horizontal.

Maximal bending moment. This occurs where shear is zero, somewhere between $x = 1.70$ m and $x = 5$ m (the direction of the x axis is indicated on Fig. 9.7). Between $x = 1.70$ m and $x = 5$ m (s designates the load per m), we get:

$$s = 64.1 - \frac{(64.1 - 22.2)}{3.30}(x - 1.70)$$

$$s = 85.7 - 12.7x$$

$$T = 151.8 - 91.1 - \frac{(85.7 - 12.7x) + 64.1}{2} \times (x - 1.70) = 0$$

from which:

$$12.7x^2 - 171.39x + 376.06 = 0$$

the roots of which equation are: $x_1 = 2.76$ m, $x_2 = 10.73$ m.
The value here to consider is $x = 2.76$ m, from which $s = 50.6$ kN.

$$M = -151.8 \times 2.76 + 27 \times 1.70 \times 1.91 + \tfrac{1}{2} \times 53.2 \times 1.70 \times 1.63$$
$$+ 50.6 \times 1.06 \times 0.53 + \tfrac{1}{2} \times 13.5 \times 1.06 \times 0.71$$

$M = -224$ kN.m at level $-9 + 2.76 = -6.24$ m.

Moment at anchor point.
$M = 4.1 \times 2.5 \times 2.5/2 + (18 \times 2.5)/2 \times 2.5/3 = +32$ kN.m.

The diagram of bending moments for phase 2 is presented on Fig. 9.6.

2.2.2. Equivalent lower beam (see Fig. 9.8):

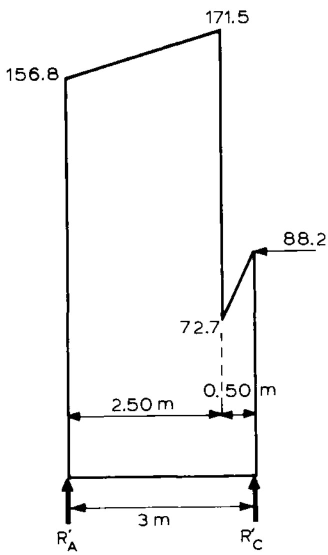


Fig. 9.8. Load per unit of length in kN/m.

$$R'_A + R'_c = \frac{156.8 + 171.5}{2} \times 2.50 + \frac{72.7 + 88.2}{2} \times 0.50 = 450.6 \text{ kN}$$

$$3R'_c \simeq 156.8 \times 2.5 \times 1.25 + \tfrac{1}{2} \times 14.7 \times 2.50 \times 1.67 + \tfrac{1}{2}(72.7 + 88.2) \times$$
$$\times 0.50 \times 2.75 = 631.2 \text{ kN}$$

$R'_c \simeq 210.4$ kN, $\quad R'_A \simeq 240.2$ kN:

$R'_A > R_A$: there is to much embedment.

Let us evaluate the order of magnitude of the bending moment (overestimated because of the excess embedment). Assume the load on the beam to be a uniformly distributed load:

$s = 450.6/3 = 150.2$ kN (per m of length):

$M_{max} = (150.2 \times 3^2)/8 = 169$ kN.m

Remark

The minimum required embedment can be calculated: assuming $R'_A = R'_c$, we get: $R'_A + R'_c = 2R'_A = 303.6$ kN.

Assuming further the constant load is $q = 157$ kPa per m, the passive pressure is needed over a depth of $303.6/157 = 1.93$ m. The embedment is: $f = 1.93 + 1.70 = 3.63$ m; the moment is $M = (157 \times 1.93^2)/8 = 73$ kN.m.

2.3. Conclusion

Calculation in plasticity condition is a long and drawn-out process, even when based on simplifying assumptions. Furthermore, the results greatly differ from reality, because plasticity is only developed when substantial deformations are allowed to occur, which is not the case for a rigid wall or slab and even less than for steel sheetpiles.

B. CALCULATIONS IN ELASTO-PLASTIC CONDITIONS

The calculations in elasto-plastic conditions are too complex to be carried out by hand. A computer must be used. We used the Rido-program developed in Lyon by Fages for the local subway construction.

This program takes into account deformations caused by progressive excavation whereas under plasticity conditions each phase had to be considered independently. For each phase the stresses exerted on the wall

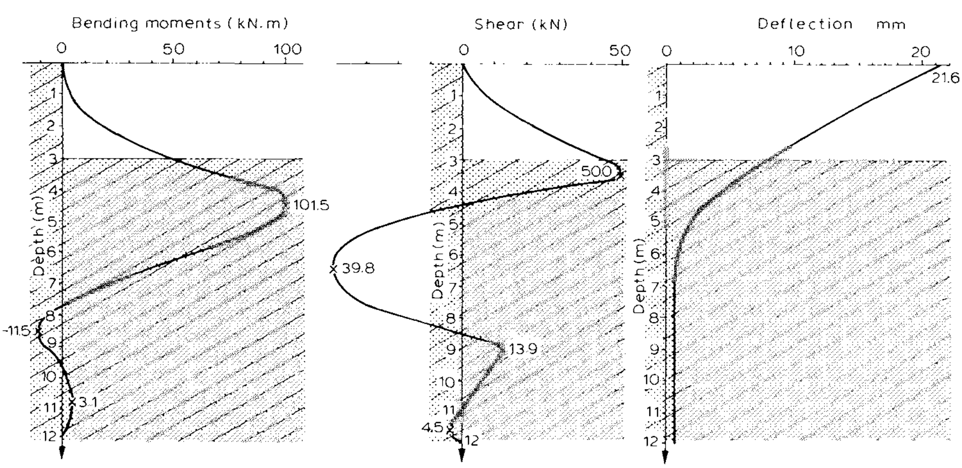


Fig. 9.9. Elasto-plastic condition, phase 1: excavation to 3 m of depth.

by the soil are given, as well as bending moments, shear, deflections and tension in the anchors.

Results are presented on Figs. 9.9, 9.10 and 9.11. They can be compared with those obtained for plasticity conditions (part A).

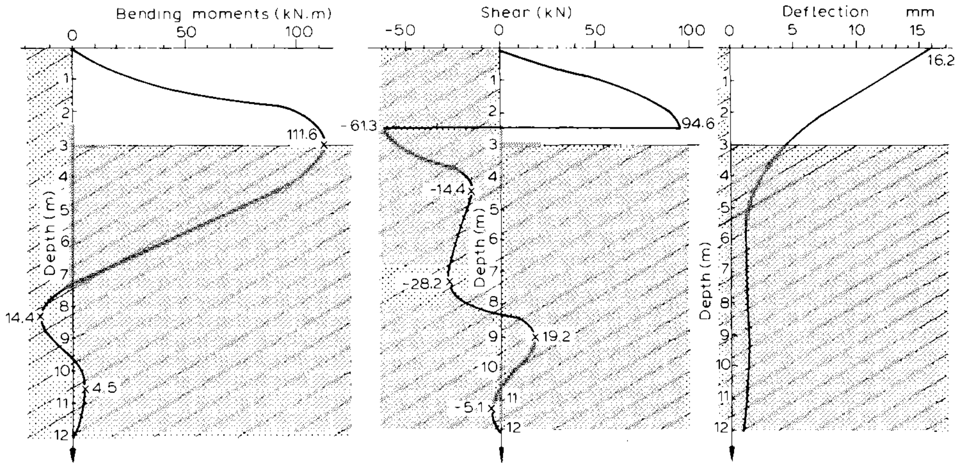


Fig. 9.10. Elasto-plastic condition, phase 2: excavation to 3 m of depth (active anchors).

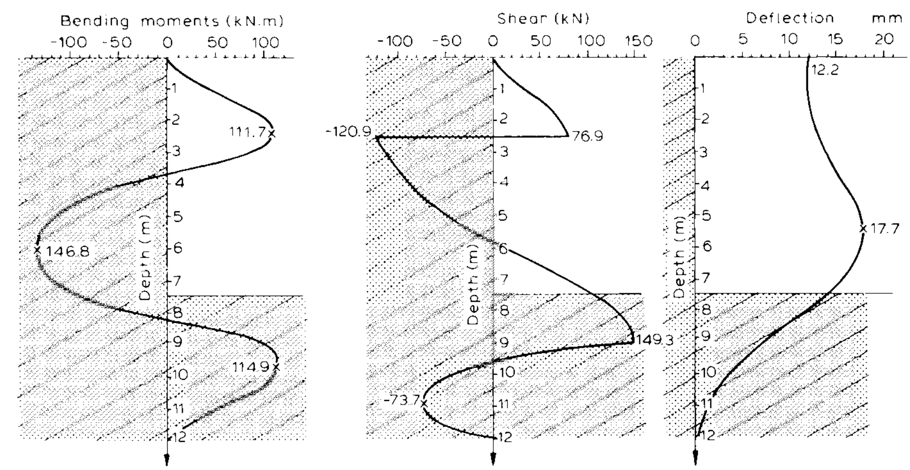


Fig. 9.11. Elasto-plastic condition, phase 3: bottom at 7.30 m of depth.

Remarks

Phase 1. The maximal bending moment in elasto-plastic conditions (101.5 kN.m) is close to the value found by assuming plastic conditions (99 kN.m).

Phase 2 (bottom of excavation at 7.30 m). Here, considerable differences occur. The maximal bending moment in plastic conditions is 224 kN.m whereas it is only 147 kN.m for elasto-plastic situations. The difference may be accounted for by the fact that the elasto-plastic method takes into consideration the deformations which have occurred in the wall after the completion of Phase I, which causes moments to decrease. Calculations in elasto-plastic conditions more closely approximate real conditions.

★★★*Problem 9.3* Self-sustaining slurry wall

Calculate the stability of the wall shown on Fig. 9.12. Is the embedment sufficient?

In order to limit the horizontal deflection at the top of the wall that would cause cracking of the adjacent building, assume a low value of passive pressure.

Although the wall is rough, assume $\delta = 0$. This is equivalent to apply a reduction factor to the passive pressure which would be usually: $\delta = -2/3\varphi$.

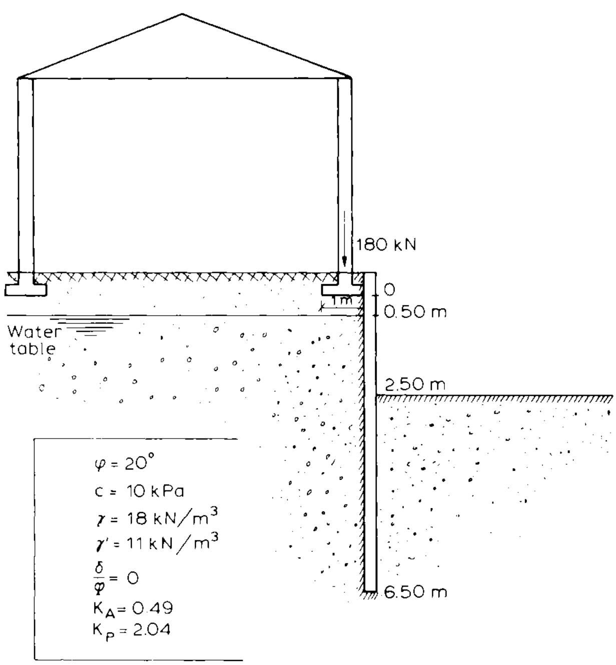


Fig. 9.12. Cross-section of walls and encountered soils with their mechanical characteristics (loads per m of wall length).

Solution

First we must determine the earth pressure diagram acting on the wall as well as the stresses due to the footing load. The simplest method to accomplish this is to assume a rectangular stress distribution as proposed by Graux (in his "Deep Foundation and Excavations", Vol. I, p. 196) and shown on Fig. 9.13.

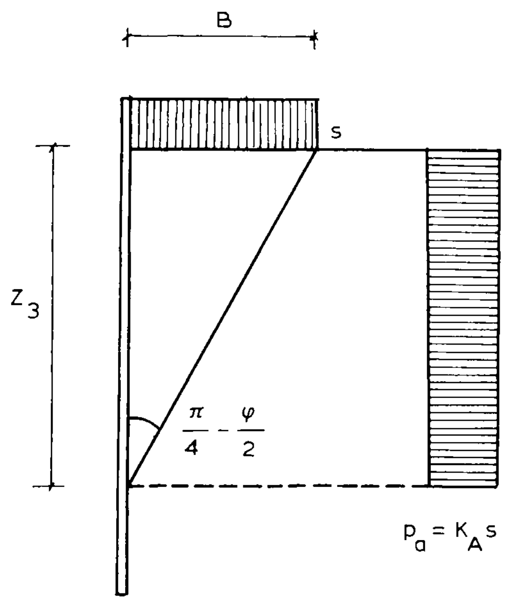


Fig. 9.13.

The active pressure on the wall is taken equal to that of a uniformly distributed load of infinite length but whose influence is limited to depth Z_3, corresponding to the failure wedge below the footing:

$Z_3 = B \tan(\pi/4 + \varphi/2) = 1 \times \tan(45^\circ + 10^\circ) = 1.43\,\text{m}$,

say $Z_3 = 1.5\,\text{m}$.

Let s represent the surcharge due to the footing, then the lateral earth pressure on the wall is: $p_a = K_A s = 0.49 \times 180/1 = 88.2\,\text{kPa}$, as is shown on diagram 1 of Fig. 9.14.

Lateral earth pressures

Take as the origin, the bottom level of the footing. Between that level and the upper soil surface, the soil acts as a surcharge $s' = 18 \times 0.5 = 9\,\text{kPa}$.

At depth h from the bottom of the footing, the lateral pressure then is:

$\sigma = k_a(\gamma \cdot h + s') + [(k_a - 1)/\tan \varphi]c$ (valid if $\delta/\varphi = 0$)

$\sigma = 0.49\ (\gamma \cdot h + 9) + [(0.49 - 1)/0.36] \times 10$

$\sigma = 0.49\ \gamma h + 4.4 - 14.2 = 0.49\ \gamma h - 9.8$

For $0 \leqslant h \leqslant 0.5$ m, $\gamma = 18$ kN/m^3, and $\sigma = 0.88\,h - 0.98 < 0$, so the lateral earth pressure is zero.

From $h = 0.5$ m (groundwater level) we get:

$\sigma = \sigma_{0.50} + k_a\ \gamma'(h - 0.50)$

$\sigma = 8.8 \times 0.50 - 9.8 + 0.49 \times 11\ (h - 0.50)$

$\sigma = 4.4 - 9.8 + 5.4\,h - 2.7 = 5.4\,h - 8.1$, so

there are no lateral pressures until depth $h_o = 8.1/5.4 = 1.5$ m: the situation is shown by diagram 2 on Fig. 9.14.

For $h = 6.5$ m, we get $\sigma = 27$ kPa.

Water pressure

Between El. -0.5 m and the bottom of the excavation at -2.50 m the pressure due to the water is $\sigma = (h - 0.5)\gamma_w$.

From the bottom of the excavation, the residual water pressure due to the difference in the head on the two sides of the wall remains constant and is equal to $p = 20$ kPa, as shown on diagram 3 of Fig. 9.14.

Passive earth pressure from the bottom of the excavation

With $\delta/\varphi = 0$, the passive earth pressure is equal to:

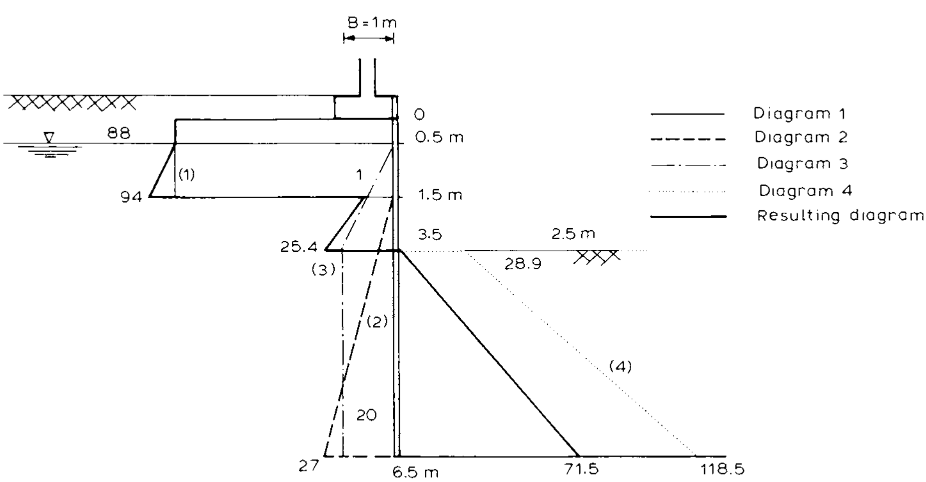


Fig. 9.14. Stresses in kPa.

$$\sigma = k_p \gamma'(h - 2.50) + [(k_p - 1)/\tan\varphi]\, c$$
$$= 2.04 \times 11(h - 2.50) + (1.04/0.36)10$$
$$= 22.4(h - 2.50) + 28.9$$

$h = 2.50, \qquad \sigma = 28.9$ kPa

$h = 6.50, \qquad \sigma = 118.5$ kPa,

from which is drawn diagram 4 of Fig. 9.14. The resulting net diagram (diagram 5) is obtained by superposition of all 4 diagrams, with the following values:

$h = 0 \qquad \sigma = 88$ kPa

$h = 0.50 \qquad \sigma = 88$ kPa

$h = 1.50 - \epsilon \qquad \sigma = 88 + 10 = 98$ kPa

$h = 1.50 + \epsilon \qquad \sigma = 10$ kPa

$h = 2.50 - \epsilon \qquad \sigma = 5.4 + 20 = 25.4$ kPa

$h = 2.50 + \epsilon \qquad \sigma = 5.4 + 20 - 28.9 = -3.5$ kPa

$h = 6.50 \qquad \sigma = 27 + 20 - 118.5 = -71.5$ kPa,

the latter two being negative stresses due to passive pressure.

The resultant of the active forces, per meter of wall is:

$$P = 88 \times 1.5 + \frac{1 \times 10}{2} + 1 \times 10 + \frac{15.4 \times 1}{2} = 132 + 5 + 10 + 7.7$$
$$= 154.7 \text{ kN (per m)}$$

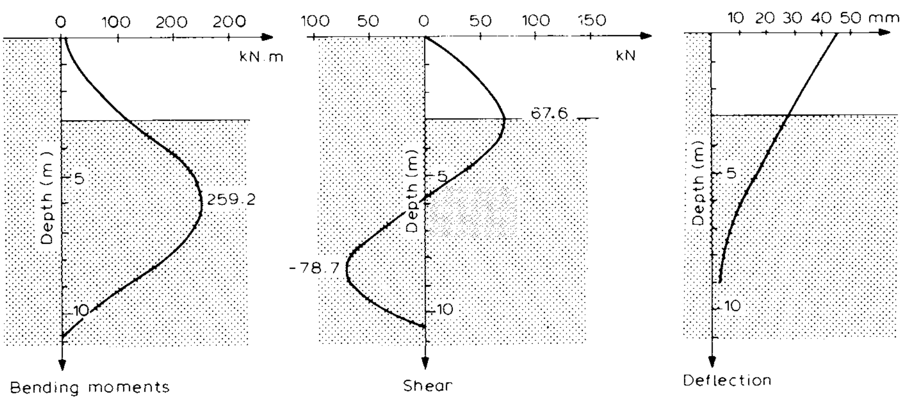


Fig. 9.15.

The resultant of the passive forces is: $B = 3.5 \times 4 + (68 \times 4)/2 = 14 + 136 = 150$ kN (per m).

The passive force just barely counteracts the active force. It is apparent that there cannot be equilibrium of the moments and therefore the stability of the wall must be doubted because of insufficient penetration. This conclusion was confirmed by the elasto-plastic computer method using the Rido-program. It also indicated that the embedment should go another 2.9 m (see results obtained on Fig. 9.15 for a wall height of 10.4 m instead of 7 m).

★★★*Problem 9.4* Wall buttressed by floors

The stability of an excavation for a 4-story deep parking structure was assured by a wall constructed after the slurry method.

Bracing of the wall was realised by the floors of the structure butting into the wall and built during the several phases of excavation. The applied technique therefore, consisted of building the floors on grade and excavating below them once finished.

The wall, the soil properties and the bracings are shown on Fig. 9.16, as well as the phases of excavation; assume $\delta = 0$ (see problem 9.3).

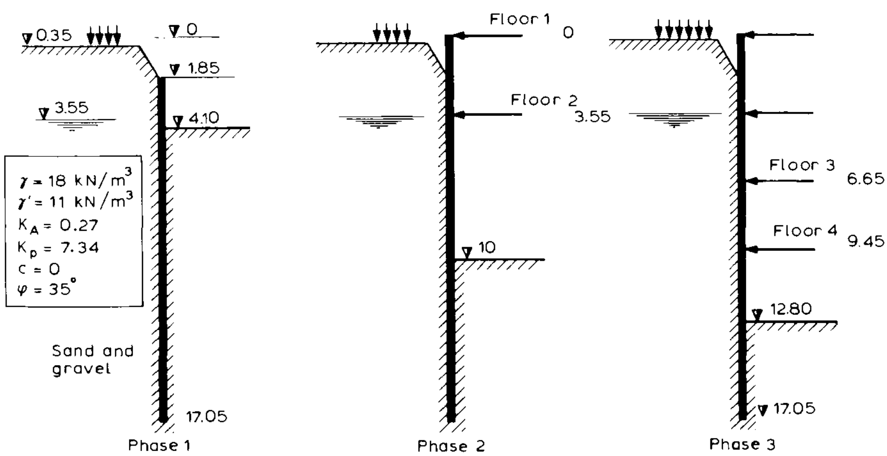


Fig. 9.16.

The elasto-plastic method was used by means of a computer with the Rido-program. Results are presented on Figs. 9.17, 9.18 and 9.19, while the method of plasticity was used in accordance with examples of previous problems. The results of that method are presented on Figs. 9.20, 9.21 and 9.22.

Verify quickly the order of magnitude of the reactions at the floor levels

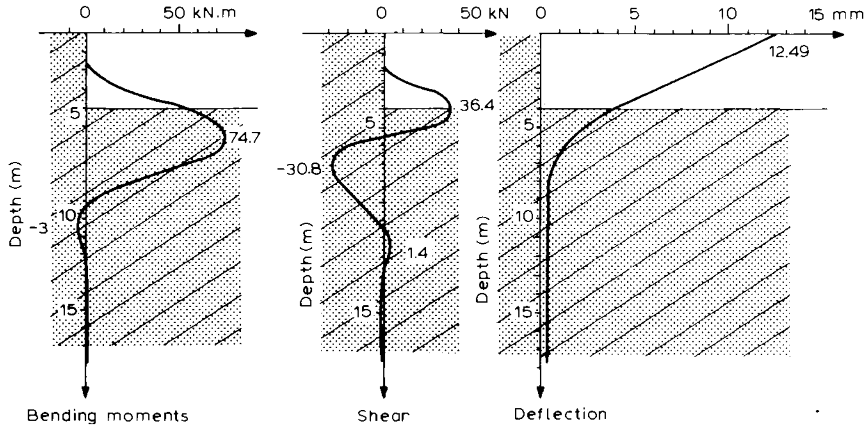


Fig. 9.17. Elasto-plastic method, phase 1: excavation at 4.10 m of depth.

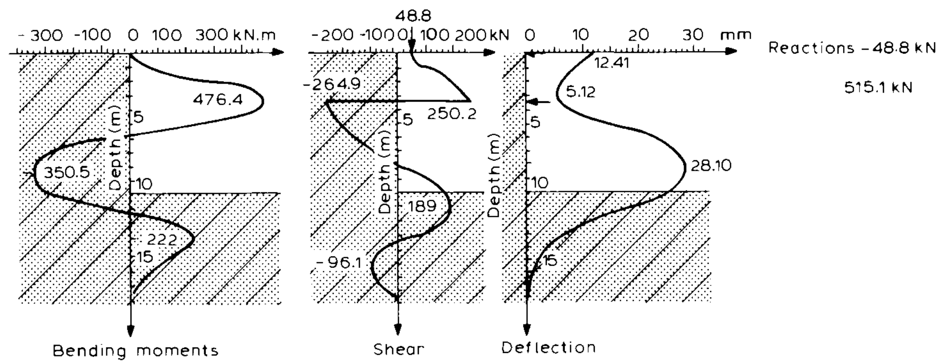


Fig. 9.18. Elasto-plastic method, phase 2: excavation at 10 m of depth.

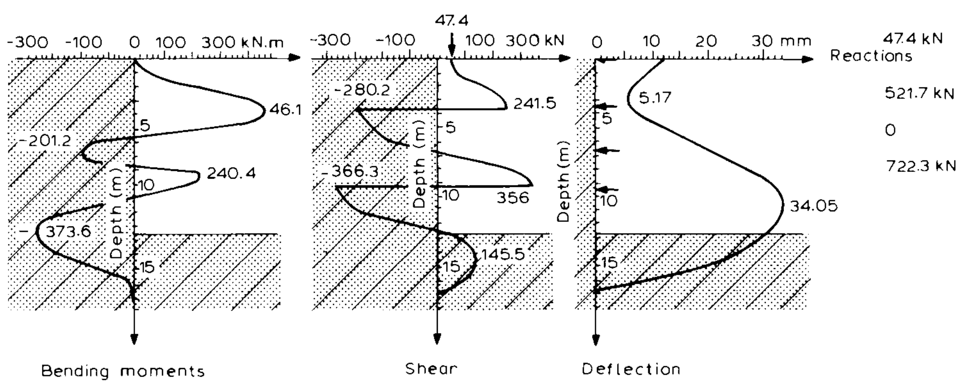


Fig. 9.19. Elasto-plastic method, phase 3: excavation at 12.80 m of depth.

at the end of the construction by using the active pressure diagram of Terzaghi for a cohesionless soil (see sect. 7.5.2., Costet-Sanglerat).

Compare the results of the various methods. What are your conclusions?

Solution

The simplified diagram is shown on Fig. 9.23. The free wall height is: $H = 12.80 - 0.35 = 12.45\,\text{m}$.

The maximum stress at the bottom of the footing is (plastic calculation):

$$\sigma = 0.27 \times [10 + (3.20 \times 18 + 9.25 \times 11)] = 45.7\,\text{kPa}$$

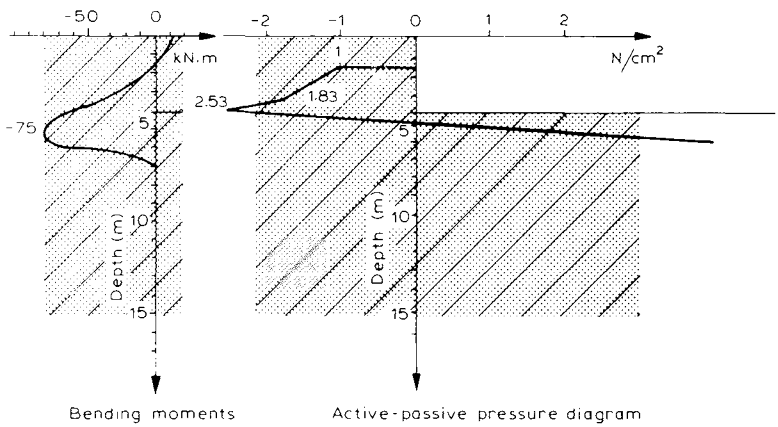


Fig. 9.20. Plastic condition, phase 1: excavation at 4.10 m of depth.

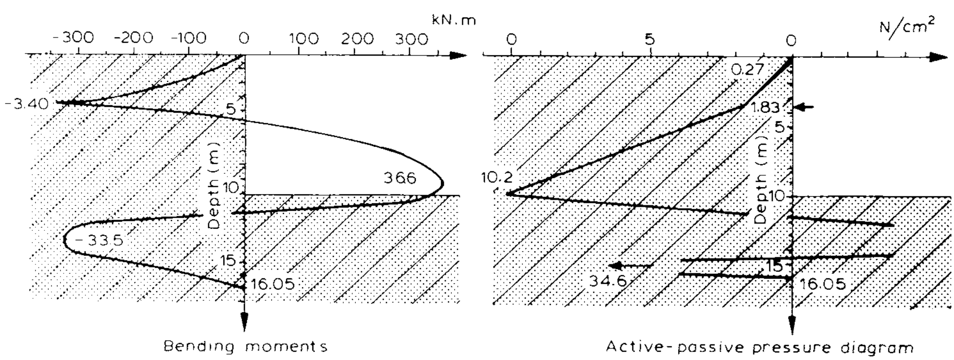


Fig. 9.21. Plastic condition, phase 2: excavation at 10 m of depth.

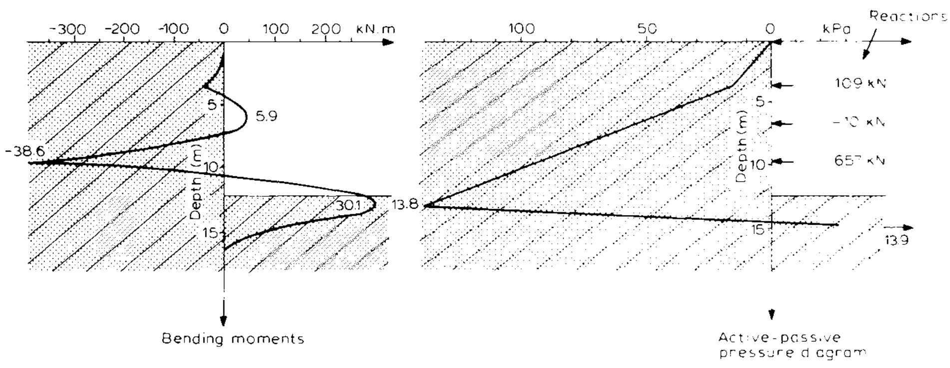


Fig. 9.22. Plastic condition, phase 3: excavation at 12.80 m of depth.

Use a trapezoidal diagram (see fig. VII-30 of Costet-Sanglerat) with a maximum stress equal to: $\sigma = 0.8 \times 45.7 = 36.6$ kPa, to which the water pressures must be added, leading to results as shown on Fig. 9.23.

For a first approximation, we assume that each floor supports a load corresponding to each adjacent half-floor height. Then we get:
Floor at level 0:

$$R_0 = 21.2 \times (1.8 - 0.35)/2 = 15.4\,\text{kN}$$

Floor at level 3.55:

$$R_{3.55} = [(36.6 + 21.2)/2]\,(2.85 - 1.8) + [36.6(3.55 - 2.85)] + [(36.6 + 52.1)/2]\,(5.1 - 3.55) = 30.3 + 25.6 + 68.7 = 124.7\,\text{kN}$$

Floor at level 9.45:

$$R_{9.45} = [(104.1 + 81.6)/2] \times (10.3 - 8.05) + [(104.1 + 92.5/2)] \times \times (12.8 - 10.3) = 208.9 + 245.8 = 454.7\,\text{kN.}$$

The resultant of these reactions is $R = 792$ kN.

Table 9A reviews the values, in kN, for each floor reaction in accordance with the 3 methods of evaluation used. It is obvious that the plastic method does not give a true appreciation of the complexity of the rigid wall with multi-level bracing. On the other hand, the trapezoidal distribution of Terzaghi corresponds to the envelope of the maximum stresses observed in flexible walls. For the case of a braced, rigid wall, we could consider a trapezoidal load distribution with an earth pressure coefficient of K_0 at rest to account for the small deflection that occurs (see deformations on Fig. 9.17). The last entry of Table 9A gives the values of this condition.

The total of the reactions shows that the earth pressure is very close to that of an at-rest condition.

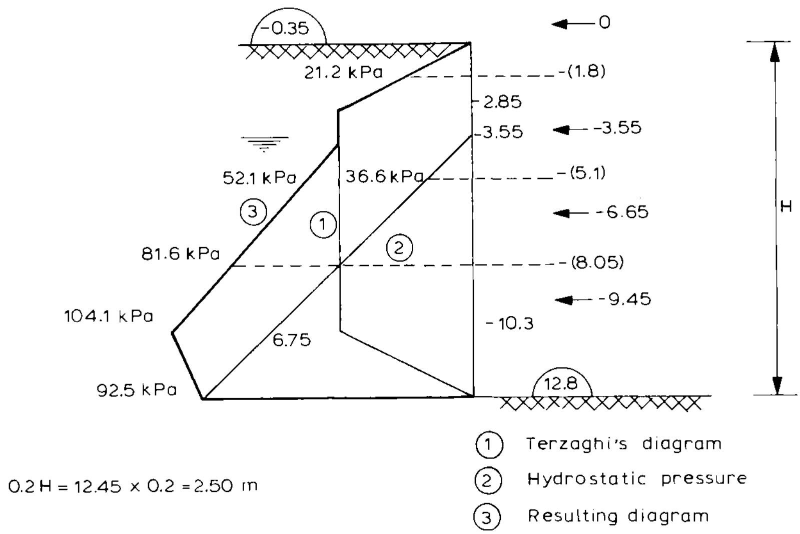


Fig. 9.23.

TABLE 9A

Comparison of floor reactions (in kN)

Floor No.	Floor Level (m)	Elasto-plastic condition (Rido)	Plastic condition (continuous beam)	Terzaghi method	Trapezoidal diagram with K_0
1	0	− 47.4	0	15.4	22.8
2	− 3.55	+ 521.7	109	124.7	220.8
3	− 6.65	0	− 1	197.2	289.2
4	− 9.45	722.3	657	454.7	563.9
Total reactions		1196.6	765	792	1102.4

Chapter 10

SHALLOW FOOTINGS

★*Problem 10.1* Allowable bearing capacity of a strip footing on sand

Find the bearing capacity of a strip footing on sand of 1 m in width resting on a sand of unit-weight 16.5 kN/cm^3 and having an angle of internal friction of $\varphi = 35°$.
What is the allowable stress?

Solution
The bearing capacity of a strip footing is given by:

$$q_d = \tfrac{1}{2}\gamma B N_\gamma + \gamma D N_q + c N_c.$$

For a sand, $c = 0$ and for $\varphi = 35°$, $N_\gamma = 47.9$ (see Table II in Costet-Sanglerat).
On the other hand, $D = 0$, so:

$$q_d = 16.5 \times 0.5 \times 47.9 = 395\,\text{kPa} = 3.9\,\text{daN/cm}^2.$$

The last value is the bearing capacity, or the failure load. The allowable is one third of this value, for a safety factor of 3. Therefore:

$$q_d = 1.3\,\text{daN/cm}^2.$$

★★*Problem 10.2* Evaluation of the bearing capacity factor N_γ

(1) A circular plate of 1.05 m in diameter is placed on a sand of density 1.65 and it is loaded. Failure occurred when the plate was loaded to withstand a pressure of 15 daN/cm^2. Determine the value of the bearing capacity factor N_γ.
(2) The angle of internal friction of the sand was measured in a triaxial test and found to be $\varphi = 39°$. Compare this value with the theoretical value of φ corresponding to N_γ calculated as asked above.

Solution
(1) The bearing capacity of a strip footing is: $q_d = \frac{1}{2}\gamma B N_\gamma + \gamma D N_q + c N_c$. In this case, it reduces to the first term since both D and c are zero. For a circular footing, the term N_γ must be multiplied by a shape factor (equal to 0.8 according to most authors). We then have:

$q_d = \frac{1}{2}\gamma B N_\gamma \times 0.8$

with $q_d = 15\,\text{daN/cm}^2$, $B = 105\,\text{cm}$ and $\gamma = 16.5\,\text{kN/m}^3 = 1.65 \cdot 10^{-3}\,\text{daN/cm}^3$ and

$N_\gamma = (2 \times 15 \times 10^3)/(105 \times 0.8 \times 1.65) = 216.$

(2) The theoretical value of φ corresponding to $N_\gamma = 216$ is about 44°. The difference between the two values of φ is important. For $\varphi = 39°$, N_γ is equal to 83. It may be surprising to find different values for N_γ based on experience ($N_\gamma = 216$) and on theory ($N_\gamma = 83$). Four remarks are in order regarding this question.

(a) The shape factor of 0.8 to N_γ was determined in a semi-empirical manner and it only is known as an approximate value.

(b) The plastic criteria of a soil may be represented in the axial system σ_1, σ_2 and σ_3 by a surface $f(\sigma_1, \sigma_2, \sigma_3) = 0$. During triaxial testing, we impose a condition of $\sigma_2 = \sigma_3$, i.e., the plastic criterion then becomes the intercept of $f(\sigma_1, \sigma_2, \sigma_3) = 0$ and of the plane $\sigma_2 = \sigma_3$. Let this curve be C_1.

For the strip footing of infinite length (for which values of N_γ are calculated) it can be assumed that the deformation along the footing axis is zero (plane strain). This condition allows translation into a condition between σ_1, σ_2 and σ_3, say $g(\sigma_1, \sigma_2, \sigma_3) = 0$. The plastic criterion then becomes the intercept curve of the two surfaces, $f(\sigma_1, \sigma_2, \sigma_3) = 0$ and $g(\sigma_1, \sigma_2, \sigma_3) = 0$. Let that curve be C_2.

There are no evident reasons why C_1 and C_2 should be the same. Therefore, calculating the factor N_γ from the angle of internal friction ϕ from a triaxial test, means introducing a systematic error, which is very important, especially for compact sands, because it can be shown (Costet-Sanglerat, Ch. 9.3.1.) that N_γ is highly dependent on values of φ. The angle of friction should actually be measured under conditions similar to the field strain conditions (plain strain). Such tests were performed by Tcheng and Iseux and showed that the angle of internal friction in plain strain is higher than found by triaxial tests.

(c) The value of N_γ given in the tables of Costet-Sanglerat, was calculated by the method of Caquot and Kerisel which assumes a rigid wedge of soil acting as a whole on the footing. This assumption does not really represent the actual field conditions, and so introduces some error in N_γ (see Table 9.3.1. of Costet-Sanglerat).

(d) Finally, it should be mentioned that it is very difficult to obtain undisturbed samples of sand for triaxial testing with a density identical to that in the field. This method can introduce substantial errors in the evaluation of the angle of internal friction.

Summary of answers

$N_\gamma = 216$; $\varphi = 44°$.

★*Problem 10.3* Bearing capacity of a strip footing embedded in sand

A sand's characteristics are as follows: density = 1.70, $\varphi = 30°$.
Find the allowable bearing pressure for a strip footing, 1.10 m wide, and founded at a depth of 1.40 m in the soil. Assume a safety factor of 3.

Solution

The allowable bearing capacity under a strip footing with a vertical and axial load is:

$$q_{ad} = \frac{\gamma(B/2)N_\gamma + \gamma D(N_q - 1) + cN_c}{F} + \gamma D.$$

For $\varphi = 30°$, we get $N_\gamma = 21.8$ and $N_q = 18.4$ (table II, sect. 9.2.1. in Costet-Sanglerat). Therefore, with $c = 0$:

$$q_{ad} = (17 \times 0.55 \times 21.8 + 17 \times 1.4 \times 17.4)/3 + (17 \times 1.4)$$

$$q_{ad} = 230\,\text{kPa} = 2.3\,\text{daN/cm}^2.$$

Fig. 10.21 and Table 10C, presented at the end of this chapter, give the detailed values of the coefficients N_γ, N_q and N_c. The values are highly comparable to those in Costet-Sanglerat. Only the values of N_γ are slightly lower which, in practice, has little impact.

★*Problem 10.4* Bearing capacity of a strip footing embedded in a cohesive soil

Same problem as 10.3, but now it is assumed that the sand has a small cohesion of 0.1 daN/cm².

Solution

We use the same formula of problem 10.3, but now cohesion c is no longer zero:

$$q_{ad} = \frac{\gamma(B/2)N_\gamma + \gamma D(N_q - 1) + cN_c}{F} + \gamma D$$

For $\varphi = 30°$, we get (table II, sect. 9.2.1 of Costet-Sanglerat):
$N_\gamma = 21.8, N_q = 18.4, N_c = 30.1$

$$q_{ad} = \frac{17 \times 0.55 \times 21.8 + 17 \times 1.4 \times 17.4 + 10 \times 30.1}{3} + 17 \times 1.4$$

$$q_{ad} = 330\,\text{kPa} = 3.3\,\text{daN/cm}^2.$$

★*Problem 10.5* **Bearing capacity of a square footing on a cohesionless or cohesive soil**

Same questions as in 10.3 and 10.4, but now for a square footing, 1.10 m on the side.

Solution

The allowable pressure under a square footing under a vertical and axial load is given by:

$$q_{ad} = \left[(1 - 0.2B/L)\gamma \frac{B}{2} N_\gamma + \gamma D(N_q - 1) + (1 + 0.2B/L)cN_c\right] \frac{1}{F} + \gamma D$$

For a square footing, $B/L = 1$. In general, a safety factor of 3 is used.
For $\varphi = 30°$, we get (table II, sect. 9.2.1. of Costet-Sanglerat):

$$N_\gamma = 21.8, \quad N_q = 18.4, \quad N_c = 30.1$$

For a cohesionless sand, we have:

$$q_{ad} = [(0.8 \times 17 \times 0.55 \times 21.8) + (17 \times 1.4 \times 17.4)]1/3 + (17 \times 1.4)$$

$$q_{ad} = 216\,\text{kPa} = 2.2\,\text{daN/cm}^2.$$

For a sand with slight cohesion, we get:

$$q_{ad} = [(0.8 \times 17 \times 0.55 \times 21.8) + (17 \times 1.4 \times 17.4) + (10 \times 1.2 \times 30.1)]1/3 + (17 \times 1.4)$$

$$q_{ad} = 336\,\text{kPa} = 3.4\,\text{daN/cm}^2.$$

Notice the important increase due to cohesion.

★★*Problem 10.6* **Comparison of footings and mat foundation**

Consider a six-storey building over a basement whose faces A_1 and A_2 impart, at the foundation level, loads of 0.29 and 0.36 MN/m, respectively. Each column in row A_3, spaced 3.75 m apart, carries 1.1 MN. The building's length is 38 m (see Fig. 10.1).

The building rests on a dense gravel bed ($\gamma_d = 1.65\gamma_w$, $\phi = 35°$, $\gamma' = 1.02\gamma_w$), 9 m thick, that is underlain by a soft clay, normally consolidated, of more than 20 m in thickness and whose properties are: $\varphi = 0$, $c_u = 0.3$ daN/cm² (30 kPa).

The finish grade of the basement is 2 m below the natural grade. The groundwater table is at −8 m below the natural grade.

Two types of foundations are being considered for the building. Either

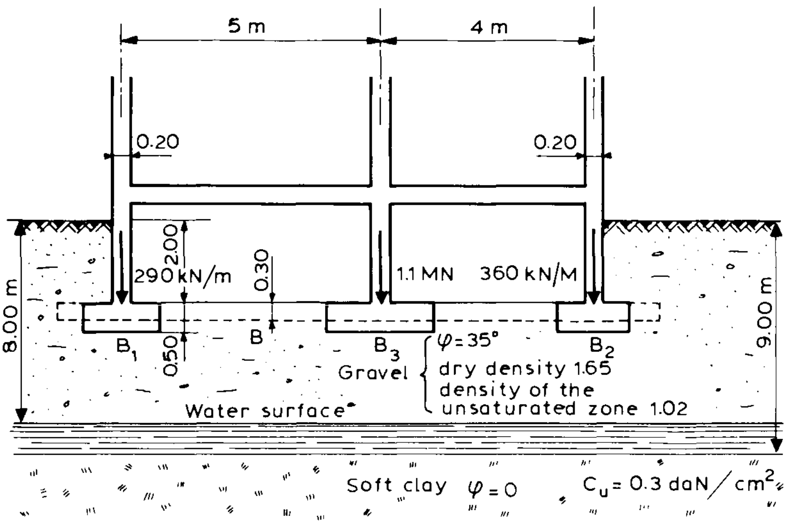


Fig. 10.1.

strip footings at B_1 and B_2 and square footings at B_3 of 50 cm in thickness, or a mat foundation of 0.30 m thickness.

Compare the two schemes of foundation for which we have to determine the width of the footings and of the mat. Specify the width of the mat lip beyond the building line. Assume the unit weight of concrete to be 2.4.

Solution

(a) Shallow strip and square footings

(1) In order to determine if consideration must be given to the underlying clay layer, consider the ratio h/B: gravel thickness/foundation width.

If we assume that the influence of the soft clay is negligible, we could expect an allowable pressure of the order of $4\,\text{daN/cm}^2$ for a footing on solid, dense gravel. For a load of 400 kN we get $B = 1$ m for the strip footing. For an isolated footing with a load of 1 MN, B is $= 1.60$ m. In either case, we would have $h/B > 3.5$ (load is given by unit of length of building).

Referring to the results obtained by Tcheng (sect. 9.3.7. of Costet-Sanglerat) the assumption is valid and the structure would behave as if the clay did not exist.

(2) The *bearing capacity* of a strip footing is:

$$q_d = \tfrac{1}{2}\gamma B N_\gamma + \gamma D N_q + c N_c$$

which is valid for an embedment which is equal on both footing sides. In this case, the smallest embedment must be considered which is 0.5 m in the

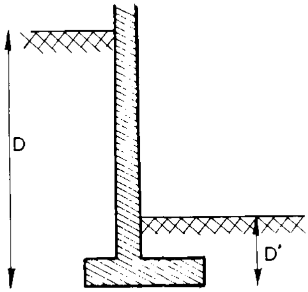


Fig. 10.2.

basement. For the sake of safety, assume D to be 0.5 m and call it D' (Fig. 10.2). To calculate the allowable stress, use a safety factor of 3 for the bearing pressure less the overburden pressure at the footing level counted from the original grade. We will then have:

$$q_{ad} = \gamma D + \tfrac{1}{3}(0.5\gamma B N_\gamma + \gamma D' N_q - \gamma D + cN_c).$$

Remarks

The soil is the same over depth D' and under the footing. This means that the coefficient N_q may be increased (table III, sect. 9.2.2, Costet-Sanglerat).

The groundwater table being at -8.00 m, we must use the dry unit-weight of gravel in the calculations.

(3) For the *isolated footings*, use:

$$q_{ad} = \gamma D + \tfrac{1}{3}(0.4\gamma B N_\gamma + \gamma D' N_q - \gamma D + 1.2cN_c)$$

taking into account the shape factor (see sect. 9.5.1. of Costet-Sanglerat) with $B = L$.

For *practical computations*, see tables II and III, Ch. 9, of Costet-Sanglerat.

For $\varphi = 35°$, we get $N_\gamma = 47.9$ and $N_q = 33.3 \times 1.245 = 41.5$ since we have a gravel $c = 0$.

Footing B_1:

$$q_{ad} = 16.5 \times 2.5 + \tfrac{1}{3}(0.5 \times 16.5 \times 47.9 \times B_1 + 16.5 \times 0.5 \times 41.5 - 16.5 \times 2.50):$$

$$q_{ad} = (132B_1 + 142)\,\text{kPa}$$

Total load on the footing in kN per m of length:

— superstructure	=		290
— weight of footing: $24 \times 0.5 \times B_1$	=	$12B_1$	
— weight of soil outside excavation:			
$200 \times 1.65 \times (B_1/2 - 0.10)$	=	$16.5B_1$	$-$ 3.3
total	=	$28.5B_1$	+ 287.7

The allowable bearing pressure under the footing is equal to the stress at the level of the footing: $B_1(132B_1 + 142) = 29B_1 + 287$, from which we get $B_1 = 1.10$ m.

The actual bearing pressure then is $q_1 = 2.9$ daN/cm^2.

Footing B_2

The same method is used and we find $B_2(132B_2 + 142) = 29B_2 + 357$ and $B_2 = 1.27$ m say $B_2 = 1.25$ m.

The actual stress is $q_2 = 3.1$ daN/cm^2.

Footing B_3

$$q_{ad} = 16.5 \times 2.5 + \tfrac{1}{3}(0.4 \times 16.5 \times 47.9 \times B_3 + 16.5 \times 0.5 \times 41.5 - 16.5 \times 2.50)$$

$$q_{ad} = (105B_3 + 142)\,\text{kPa}.$$

Total load on the footing:

— superstructure		1100
— weight of footing: $24 \times 0.50 \times B_3^2$	$12B_3^2$	
	total	$12B_3^2 + 1100$

So we have:

$$B_3^2(105B_3 + 153) = 12B_3^2 + 1100.$$

The root of this third-degree equation has a value between

$B_3 = 1.80$ m and $B_3 = 1.85$ m. Thus $B_3 = 1.85$ m.

The actual bearing pressure is $q_3 = 3.3$ daN/cm^2.

Remarks

(1) For the sake of simplicity, it is often assumed that for footings with unequal embedment on each side, the influence of the large embedment allows one to neglect the weight of the soil outside of the excavation.

The scattering of the results of bearing capacity calculations depending on which theory one uses, justifies this simplification.

(2) The spread between q_1, q_2 and q_3 being small, there is no reason to be afraid that differential settlements would be a problem.

(b) Mat foundation

The width, B, of a mat is at least that of the building. It will therefore be over 9.00 m. The gravel thickness below the mat is only 6.7 m. Therefore, $h/B < 1$ and the underlying clay layer will feel the building load. The bearing capacity in this lower soil layer will govern the design.

The mat plan dimensions will be close to those of the building (about 40 m by 10 m). The dead loads of the structure and the mat will be transmitted to the clay through the 6.7 m of gravel. The vertical stress increase

$\Delta\sigma$ at the clay level will not be uniform. It may be estimated to be $0.75\,q$ along the axis of the mat from the graph of fig. III-8 in Costet-Sanglerat.

For a simple calculation and still being on the conservative side assume that $\Delta\sigma$ is uniform and equal to its maximum value, that is: $\Delta\sigma = 0.75\,q$.

Assume the load to be applied uniformly over a rectangular area of width B' and length L', so that: $L' \times B' \times 0.75q = L \times B \times q$,
or: $L'B' = BL/0.75 = 1.33BL$. We may assume that $B/L = B'/L'$,
which gives $B' = 1.15B$ and $L' = 1.15L$.

The allowable bearing pressure of the clay will then be:

$$q_{ad} = \gamma D + \frac{5.14\,[1 + 0.2(B'/L')]\,c_u}{F}$$

The calculation assumes no drainage conditions which is conservative, with $\varphi = 0$. So $N = 0$, $N_q = 1$ and $N_c = \pi + 2 = 5.14$, $B'/L' = B/L$. If we, furthermore, estimate L equal to 40 m, then:

$$q_{ad} = 16.5 \times 8 + 20.2 \times 1 + \frac{5.14\,[1 + 0.2\,(B/40)] \times 30}{3}$$

$$q_{ad} = 205 + 0.26B.$$

We see that the previously estimated value of B' does not appear in the equation.

The actual bearing pressure imparted at -9.00 m, keeping in mind the above assumptions, is:

$$\text{structure loads:}\quad \left[290 + 360 + \frac{1100}{3.75}\right]\frac{0.75}{B \times 1.00} = \frac{707.5}{B \times 1.00}\ \text{kPa}$$

$$\text{mat weight: } 24 \times 0.3 \times 0.75 \times 1.00 = 5.4\ \text{kPa}$$

$$\text{weight of gravel under the mat: } (5.70 \times 16.5 + 1.00 \times 20.2) \times 1.00 =$$

$$= 114.2\ \text{kPa}$$

Assuming that the allowable pressure is equal to the applied pressure, we can write: $707.5/B + 5.4 + 114.2 = 205 + 0.26B$ or: $707.5/B = 0.26B + 85.4$.

This second-degree equation has a root of: $B = 8.06$ m which is less than the building width. Therefore, B will be at least equal to 9.2 m (building width).

Let us now calculate the distance to the resultant of the loads on face A_1. Let that distance be x:

$$x(290 + 360 + 1100/3.75) = 5(1100/3.75) + 360 \times 9, \text{ from which}$$

$$x = 5\ \text{m}.$$

The bary center is in line of the column row A_3. Therefore, the computation of the bearing capacity must consider the load eccentricity, which is: $E = 5.10 - 9.2/2 = 0.50$ m.

Meyerhoff's theory leads to a uniformly loaded area subjected to a load $\bar{q}_d$ applied to an equivalent width B_e of:

$$B_e = B - 2E = 9.20 - 2 \times 0.50 = 8.20 \text{ m and}$$

$$\bar{q}_d = (1 - 2e)\gamma \frac{BN\gamma}{2} + \gamma DN_q + cN_c$$

For this example: $N_\gamma = 0$ and $\bar{q}_d = q_d$.

The rest of the computation remains valid. Since $B_e > 8.06$ m it is not necessary to increase the width of the mat beyond 9.20 m, at least as far as the bearing capacity is concerned. If differential settlements could be a problem, centering the load would present an advantage. An additional length of 1.10 m on side A_2, taking into account the 0.10 m on side A_1 (needed to accommodate forms thickness) would then give a width of the mat of $B = 10.40$ m.

★★*Problem 10.7* Comparison of settlements of a footing and of a mat supporting a building over a two-layer system

Take the same givens of Problem 10.6 and calculate settlements of the two foundation schemes by assuming, on the one hand, that the gravel causes no settlements and, on the other hand, the properties of the soft clay deposit are: $\gamma_s = 2.78\gamma_w$, $w = 44\%$, $w_L = 48\%$, $\gamma_h = 1.8\gamma_w$.

Solution

(a) Footings

No consolidation test was performed to determine the parameters of the clay. Empirical correlations, therefore, must be used which relate the liquid limit w_L with the consolidation characteristics of the soft clay.

We will calculate the settlements for a 20 m thick clay layer. Indeed, for larger depths, the vertical stresses in the clay are very low ($\Delta\sigma < 10$ kPa) and the corresponding settlements would be very small.

The first step is to determine the increase in vertical beneath the footings at the level of the upper boundary of the clay layer. For the strip footings B_1 and B_2 we will not take into account the stress increase due to the adjacent footings because its magnitude is small. For the square footings B_3 on the other hand, we must take into consideration the proximity of the strip footings. For the strip footings, at the upper clay boundary, we get (Giroud's tables, Vol. 1, II.4 and Vol. 2, IV.1):

$\Delta\sigma_1 \;=\; \Delta\sigma_2 \;=\; 0.4\,\mathrm{daN/cm^2}$

For the square footings, if we consider the influence of the adjacent strip footings, we get:

$\Delta\sigma_3 \;=\; 0.17 + 2 \times 0.18 \;=\; 0.53\,\mathrm{daN/cm^2}$

At the lower boundary of the clay layer, the vertical stress increase is about: $\Delta\sigma = 0.07\,\mathrm{daN/cm^2}$ (for either strip or square footings).

The variation of the increase of stresses is substantial enough to justify the consideration of two 10-m thick layers.

Upper 10-m layer. Consider the midpoint of this layer. The stress increases are:
— strip footings: $\Delta\sigma_1 = 0.2\,\mathrm{daN/cm^2}$
— square footings: $\Delta\sigma_3 = 0.048 + 2 \times 0.075 = 0.2\,\mathrm{daN/cm^2}$

The settlement due to the consolidation of this 10-m layer may be estimated by evaluating the coefficient of compressibility from the correlation with the liquid limit of Skempton: $C_c = 0.009\,(w_L - 10)$ which gives: $C_c = 0.009\,(48 - 10) \simeq 0.34$.

Settlements due to consolidation are estimated from:

$$\frac{\Delta h}{h} = \frac{C_c}{1 + e_0} \cdot \log \frac{\sigma_0' + \Delta\sigma}{\sigma_0'}$$

where σ_0 is the effective overburden pressure at mid-height of the clay layer under consideration. It is:

$$\sigma_0' \;=\; 16.5 \times 8.00 + 10.2 \times 1.00 + \gamma_a' \times 5$$

the buoyant unit-weight of the clay being $\gamma_a' = \gamma_h - 10 = 8\,\mathrm{kN/m^3}$ (if we assume, as is logical, that the clay layer is saturated).

Therefore:

$$\sigma_0' \;=\; 142.2 + 40 \simeq 182\,\mathrm{kPa} \simeq 1.8\,\mathrm{daN/cm^2}$$

The initial void ratio is:

$$e_0 \;=\; w\gamma_s/\gamma_w \;=\; 0.44 \times 2.78 \;=\; 1.22$$

Hence:

$$\frac{\Delta h}{h} = \frac{0.34}{1 + 1.22} \log \frac{1.8 + 0.2}{1.8} \;=\; 0.007$$

For the upper 10.m layer then, $\Delta h = 1000 \times 0.007 = 7\,\mathrm{cm}$ (for both strip and square footings).

Lower 10-m layer. At mid-height of this layer, the stress increases are:
— strip footings: $\Delta\sigma_1 = 0.12\,\mathrm{daN/cm^2}$
— square footings: $\Delta\sigma_1 = 0.12\,\mathrm{daN/cm^2}$
and we have:

$$\frac{\Delta h}{h} = \frac{C_c}{1 + e_0} \cdot \log \frac{\sigma_0' + \Delta\sigma}{\sigma_0'}$$

where $\sigma_0' = 16.5 \times 8 + 10.2 \times 1 + 8 \times 15$

$= 142.3 + 120 = 262\,\text{kPa} = 2.6\,\text{daN/cm}^2$

and $\Delta h/h = 0.34/2.22 \cdot \log\,[(2.62 + 0.12)/2.62] = 0.003$

For the lower 10-m layer, $\Delta h = 3$ cm.

The total settlement under the footing loads will be of the order of 10 cm. It will be clear that, because the settlement was calculated from an empirical relation, it gives only an order of magnitude of the real settlement.

(b) Mat foundation

The load at the level of the mat is:

Superstructure load: $(290 + 360 + 1100/3.75) \times 38.00$	=	35 847 kN
Mat slab weight: $24 \times 0.3 \times 10.60 \times 38.40$	=	2 931 kN
Total weight	=	38 778 kN

The weight of the excavated soil can now be deducted as:

$16.5 \times 2.30 \times 9.20 \times 38 = 13\,267$ kN.

The stress increase at the mat level is:

$$\Delta\sigma = \frac{38\,778 - 13\,267}{38.4 \times 10.6} = 62.7\,\text{kPa} = 0.63\,\text{daN/cm}^2$$

As was the case for the footings, we will consider 2 layers each 10 m thick.

Upper layer. Consider a point at mid-height. The stress increase is:

$\Delta\sigma = 0.488 \times 0.63 = 0.31\,\text{daN/cm}^2$, from which

$\Delta h/h = (0.34/2.22)\log\,[(1.82 + 0.31)/1.82] = 0.010$

So, $\Delta h = 10$ cm for the upper layer.

Lower layer. Again consider the point at mid-height of this layer. The stress increment is:

$\Delta\sigma = 0.248 \times 0.63 = 0.16\,\text{daN/cm}^2$, from which

$\Delta h/h = (0.34/2.22)\log\,[(2.62 + 0.16)/2.62] = 0.004.$

So, $\Delta h = 4$ cm for the second layer.

The total settlement under the mat foundation thus will be of the order of 14 cm. Note that in this case, the settlement due to the stress increase of the mat foundation exerting a stress of $0.70\,\text{daN/cm}^2$ is greater than the settlement due to pressure of the footings of 2.9 and $3.4\,\text{daN/cm}^2$. This may be called the "bi-layer paradox".

★★*Problem 10.8* **Settlement of a mat on a two-layered system**

Take the givens of problems 10.6 and 10.7 and assume that the gravel layer is 25 m thick, that the water table is at 1 m below the level of the natural ground and that the clay properties are:

$c = 10\,\text{kPa}, \quad \gamma_a = 13\,\text{kN/m}^3, \quad w = 60\%, \quad w_L = 72\%$

Would the shallow footings present any kind of advantage?

Solution

Preliminary remark. The presence of the groundwater 1 m below the existing ground surface dictates the choice of a mat foundation. To consider individual footings would require special design features to decrease the hydrostatic pressures on the basement floor slab and the water tightness of the joints between the floor slab and the footings would always be a problem.

The mat foundation in this instance is less expensive and technically more reliable.

Mat foundation. As for the preceding problem, we will take the mat dimensions as 10.6 m by 38.40 m, which were found to be adequate.

We calculate the increase in vertical stresses at the level of the mat:

building load	32 047 kN
mat weight	2 930 kN
uplift force 38.4 × 9.2 × 13	−4 590 kN
Net force	30 387 kN

Deduction of the weight of the excavated soil:

$(16.5 \times 1 + 10.2 \times 1.30) \times 9.20 \times 38 = 10\,450\,\text{kN}.$

The stress increase at the mat level then is:

$\Delta\sigma = (30\,387 - 10\,450)/(38.4 \times 10.6) = 49\,\text{kPa}:$

$\Delta\sigma = 0.49\,\text{daN/cm}^2$

As for the preceding problem, two 10-m layers of clay will be considered.

Upper layer. The stress increase at the center of this layer is:

$\Delta\sigma = 0.11\,\text{daN/cm}^2$ (see fig. III.8, sect. 3.2.2, Costet-Sanglerat)

In order to estimate C_c, we use Skempton's empiric relation:

$C_c = 0.009\,(w_L - 10) = 0.009\,(72 - 10) = 0.56$

$e_0 = w(\gamma_s/\gamma_w) = 0.60 \times 2.78 = 1.67$

The settlement is calculated from:

$$\frac{\Delta h}{h} = \frac{C_c}{1 + e_0} \log \frac{\sigma_0' + \Delta\sigma}{\sigma_0'}$$

σ_0' = effective overburden stress at mid-height in the clay layer before excavation

$\sigma_0' = (16.5 \times 1) + (10.2 \times 24) + \gamma_a' \times 5,$

where the buoyant weight of the clay is $\gamma_0' = \gamma_h - 10 = 6.7\,\text{kN/m}^3$ (clay is saturated).

Therefore: $\sigma_0' = 300\,\text{kN/m}^2 = 3\,\text{daN/cm}^2$ and

$\Delta h/h = (0.56/2.67) \log[(3 + 0.11)/3] = 0.0032$

the settlement in the upper layer is $\Delta h_1 = 3.2$ cm.

Lower layer. The stress increase at the center of this layer is:

$\Delta\sigma = 0.06\,\text{daN/cm}^2$

$\sigma_0' = 367\,\text{kN/m}^2 = 3.67\,\text{daN/cm}^2$

and $\Delta h/h = (0.56/2.67) \log[(3.67 + 0.06)/3.67] = 0.0014$

the settlement in the lower layer is $\Delta h_2 = 1.4$ cm.

The total settlement in both layers of the mat foundation is of the order of 5 cm.

★★★★*Problem 10.9* Elastic and plastic equilibrium in a soil under a strip footing

Take a strip footing of width B, resting at a depth D in a soil whose angle of internal friction is φ, cohesion c and of unit weight γ.

(1) Give the formula for the principal stresses at a point M (θ, Z) in the soil assuming elastic equilibrium, and the state of stresses in the soil is isotropic.

As it will be remembered, the principal stresses developed at point M(θ, Z) of a semi-infinite elastic body, due to a uniform load q at the surface, spread over an infinitely long strip are given by the Boussinesq formulas with the notations of Fig. 10.3: $\sigma_1 = (q/\pi)(\theta + \sin\theta)$, $\sigma_3 = (q/\pi)(\theta - \sin\theta)$.

The values calculated are approximate values applicable to shallow footings. Why is it necessary to assume isotropic conditions for initial state?

Find the locus (L) of the points in the body where the shear stresses are highest and determine graphically the faces upon which the maximum shear acts. By studying Mohr's circle at locus L, show that the plastic state is initiated at the edges of the footing.

Determine the orientation of the planes of maximum shear at the corners of the footing.

(2) From Coulomb's equation, write the equation $Z = f(\theta)$ of locus (C) of

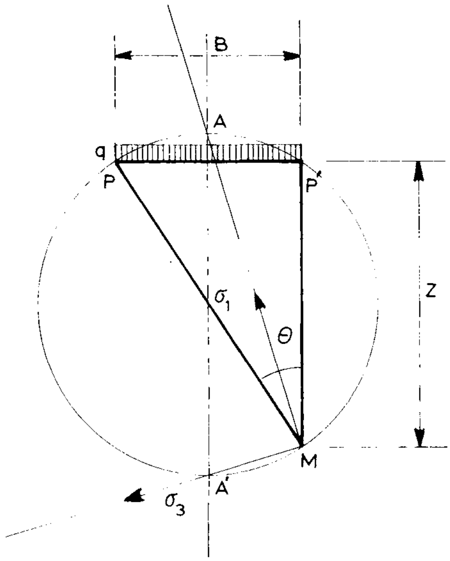


Fig. 10.3.

the points in the body that are the boundary between the elastic and plastic zones. Construct the locus for the following values:

$\varphi = 30°$, $c = 0$ *(cohesionless soil)*, $\gamma = 16.5\,\mathrm{kN/m^3}$,

$D = 0.20\,\mathrm{m}$, $q = 0.8\,\mathrm{daN/cm^2}$, and $q = 0.6\,\mathrm{dN/cm^2}$.

Assume $B = 1$ m for the graph.

(3) Write an equation for the condition where no point of the body is in limit equilibrium. Show that this condition prevails under:
$q_0 = \gamma \cdot D \cdot \alpha + H(\alpha - 1)$, *(Fröhlich's formula),*
where $H = c \cot \varphi$

$$\text{and} \quad \alpha = \frac{\pi}{\cot \varphi - (\pi/2 - \varphi)} + 1$$

Calculate the values of α for $\varphi = 10°$, $20°$, $30°$, and $45°$. Comment on this formula.

Study the particular case for a footing of zero embedment resting on sand and resting on clay (purely cohesive soil).

Compare the value of q_0 (calculated with the numerical values of question (2) with the bearing capacity q_d of the same footing computed by the classical formula. Do the same for a footing at the surface on a purely cohesive soil. What are your conclusions?

(4) Write the equation for the condition that locus (C) intersects the axis Δ of the foundation.

Indicate the graphical method which allows to calculate θ corresponding to the common points of (C) and Δ. Let q_{lim} be the value of q corresponding to the case where (C) presents a double point on Δ.

Calculate q_{lim} with the numerical data of question 2 and construct curve C_{lim}.

Compare q_{lim} with q_d of the preceding question. Indicate on the drawing the zones which are in elastic and plastic equilibrium and compare this to the drawing made to establish the classical formula of the bearing capacity for the plastic condition.

Remark: It is assumed that when plastic zones appear, the stress-field for elastic conditions does not change.

Solution

(1) For the initial stress state assumed to be isotropic, the stress at a point M of depth Z under the footing is: $\sigma_i = \gamma(Z + D)$. The footing creates a stress variation along PP' which is: $\Delta\sigma_v = q - \gamma D$.

At point $M(\theta, Z)$, the new stresses are obtained by assuming two equilibrium states exist. From which we have:

$$\left.\begin{aligned} \sigma_1 &= \gamma(Z + D) + (q - \gamma D)/\pi\ (\theta + \sin\theta) \\ \sigma_3 &= \gamma(Z + D) + (q - \gamma D)/\pi\ (\theta - \sin\theta) \end{aligned}\right| \quad (1)$$

It is necessary to assume isotropic conditions, otherwise the mathematics would become too complex. When isotropic conditions are assumed, all the directions of the stress tensors are principal directions, which justifies the addition of stresses of formula (1). If this assumption is not made, the initial stress condition would be:

$$\sigma_v = \gamma(Z + D), \text{ and } \sigma_h = K_0\gamma(Z + D)$$

and the tensors representing the initial state of stresses and the state of stresses due to the footing load would not have the same principal directions at point M. Equation (1) then could not be written.

On the other hand, Boussinesq equations correspond to a condition of a footing load at the surface acting on a semi-infinite mass. No account is taken of the embedment of the footing. This leads to acceptable results in the case of light loads and shallow depths.

Formulas (1) allow us to calculate the radius of Mohr's circle:

$$R = (\sigma_1 - \sigma_3)/2 = [(q - \gamma D)/\pi]\ \sin\theta \qquad (2)$$

We also know that for a given Mohr's circle, the maximal value of shear stress is equal to the radius of the circle.

Locus (L) of the points in the soil mass where shear stresses are the largest, corresponds to $\theta = \pi/2$ from (2). Locus (L) is a half circle whose diameter is the base of the footing (Fig. 10.4). Formulas (1) allow us also to calculate the abscissa p of the center of Mohr's circle:

$p = (\sigma_1 + \sigma_3)/2 = \gamma(Z + D) + [(q - \gamma D)/\pi]\theta$.

For $\theta = \pi/2$, we get: $p = \gamma Z + \frac{1}{2}(q + \gamma D)$.

As point M conscribes L, Mohr's circle radius remains constant but its center translates: Z varies from 0 to $B/2$ (Fig. 10.5).

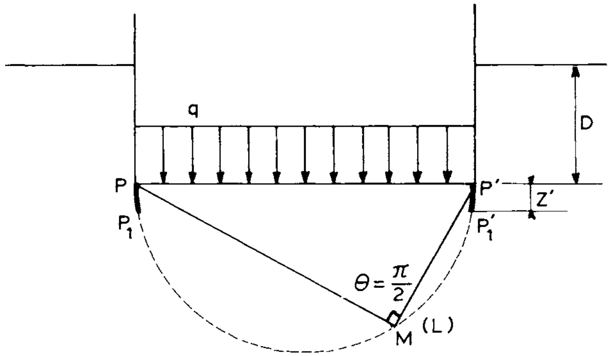


Fig. 10.4.

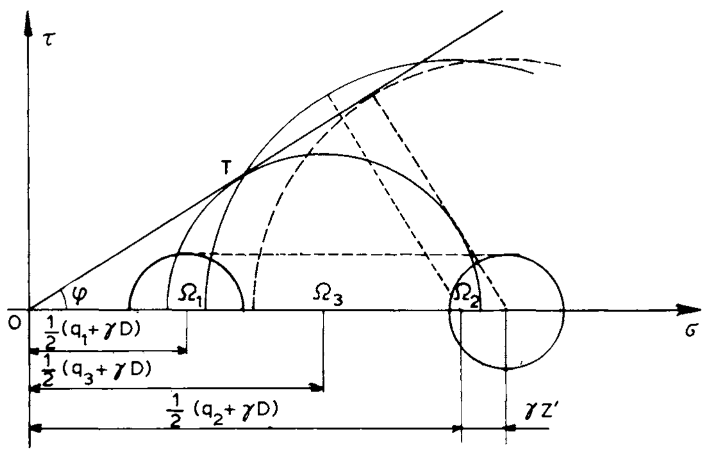


Fig. 10.5.

For a given condition of footing size and soil type (γ and D) we see that, according to the magnitude of q, the following 3 cases may be considered:

case 1: q is low: None of the Mohr's circles intersect the failure envelope. Elastic equilibrium prevails everywhere in the mass.

case 2: q is high: an infinite number of Mohr's circles intersect the failure envelope. These are the circles corresponding to the points of L whose elevation lies between D and $D + Z'$.

case 3: $q = q_3$: only one Mohr's circle is tangent to the failure envelope; it corresponds to $Z' = 0$, therefore to points P and P' of L located at the edge of the footing.

This clearly shows that the plastic state is initiated at the edges of the footing and progresses from there.

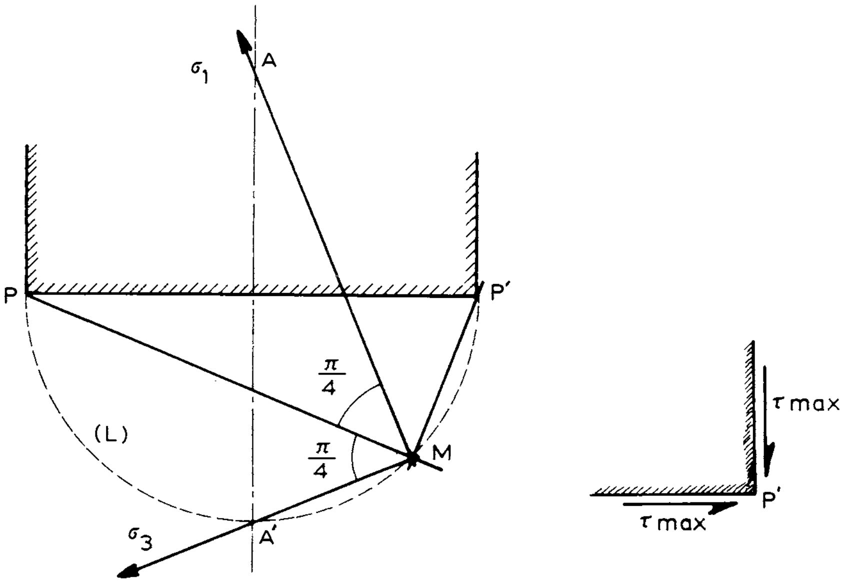


Fig. 10.6.

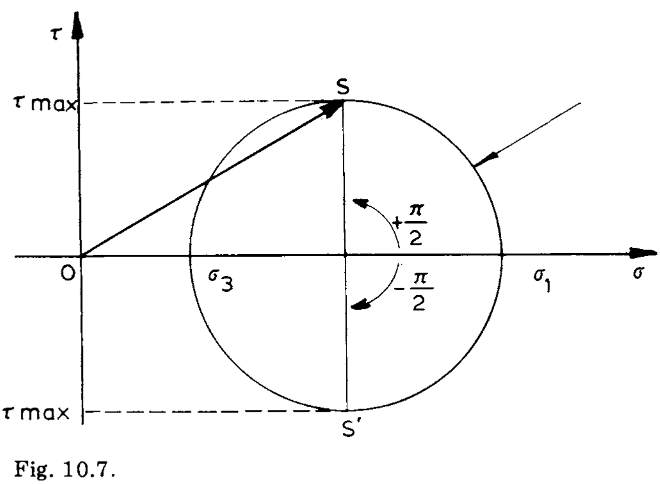


Fig. 10.7.

The maximum shear stresses correspond to the stress vectors whose extremity lies on S or S' on Mohr's circle (Fig. 10.7). The planes' orientation corresponds to an angle of $\pi/4$ with the planes of the principal stresses.

According to Boussinesq's theory (see Fig. 10.3), the latter go through the edges A and A' of the vertical diameter of locus L; therefore, the planes in which the maximum shear stresses act go through the corners P and P'

of the footing. In particular, when point M coincides with P or P', the planes correspond to the bottom and vertical faces of the footing (Fig. 10.6).

(2) Coulomb's criterion (Fig. 10.8) considers the Mohr's circle tangent to the failure envelope:

$R = (H + p) \sin \varphi$ where $H = c \cot \varphi$

$$\text{or:} \quad \frac{\sigma_1 - \sigma_3}{2} = \left(\frac{\sigma_1 + \sigma_3}{2} + H\right) \sin \varphi \tag{3}$$

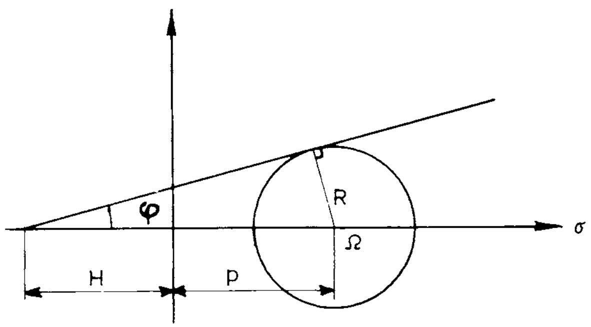


Fig. 10.8.

The equation for the locus of the points in the mass which correspond to the boundary between the elastic and plastic zones is written in equation (3). Replacing the values of p and R in that equation, we get:

$$\frac{q - \gamma D}{\pi} \sin \theta = \left(\gamma [Z + D] + \frac{q - \gamma D}{\pi} \theta + H\right) \sin \varphi$$

which gives:

$$Z = -D + \frac{q - \gamma D}{\gamma \pi} \left(\frac{\sin \theta}{\sin \varphi} - \theta\right) - \frac{H}{\gamma} \tag{4}$$

when γ, H and φ are known for a given soil. For a particular footing, D is given.

Hence, the locus of the points of limit equilibrium is a curve C depending on the value of q whose equation is of the form $Z = F(\theta, q)$.

Points $M(\theta, Z)$ of C are constructed by taking the intersection of the straight horizontal line through Z and of the arc, the locus of the points from which the footing base PP' is seen through an angle θ (only considering the points inside the mass of course and even to $Z > 0$ for which the Boussinesq conditions apply) (Fig. 10.9). C has the same axis of symmetry Δ as the footing. For the graphical construction, note that: $d = (B/2) \cot \theta$.

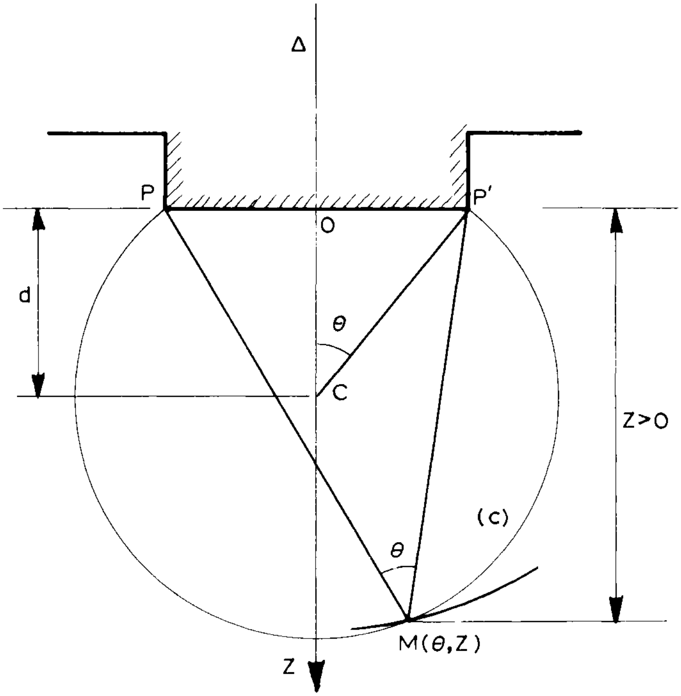


Fig. 10.9.

Numerical application

$\varphi = 30°$, $\sin\varphi = 0.5$;

sandy soil: $c = 0$, $H = c\cot\varphi = 0$

$\gamma = 16.5\,\text{kN/m}^3$

$D = 0.20\,\text{m}$

Equation (4) becomes:

$$Z = F(\theta, q) = -0.20 + [(q - 3.3)/51.84]\,(2\sin\theta - \theta)$$
$$= -0.2 + k(2\sin\theta - \theta).$$

Detailed calculations are following for $q = 0.6\,\text{daN/cm}^2$ (60 kPa) and for $q = 0.8\,\text{daN/cm}^2$ (80 kPa):

$q = 0.6\,\text{daN/cm}^2 = 60\,\text{kPa}$ gives $k_1 = (60-3.3)/51.84 = 1.09$

$q = 0.8\,\text{daN/cm}^2 = 80\,\text{kPa}$ gives $k_2 = (80 - 3.3)/51.84 = 1.48.$

Calculations are summarized on Table 10A.

Fig. 10.10 shows the family of curves (C) for various values of q for the

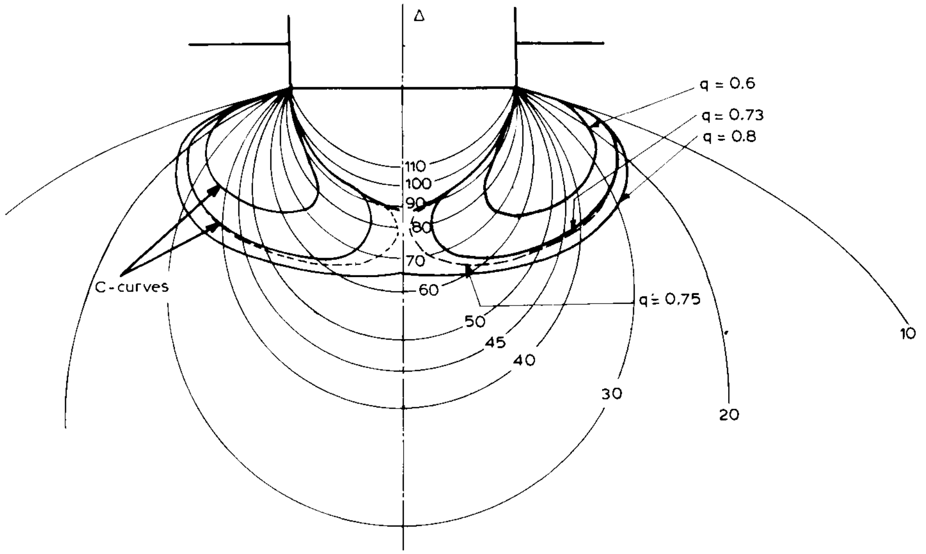


Fig. 10.10

TABLE 10A

($\varphi = 30^\circ$; $c = 0$; $\gamma = 16.5\ \mathrm{kN/m^3}$; $D = 0.20\ \mathrm{m}$)

θ°	d (in m) = $(B/2)\cot\theta$ with $B = 1$ m	$2\sin\theta - \theta$ (rad)	$q = 0.6\ \mathrm{daN/cm^2}$ = 60 kPa		$q = 0.8\ \mathrm{daN/cm^2}$ = 80 kPa	
			$k_1(2\sin\theta - \theta)$ ($k_1 = 1.09$)	Z (m)	$k_2(2\sin\theta - \theta)$ ($k_2 = 1.48$)	Z (m)
10	2.84	0.1727	0.188	<0	0.256	0.06
20	1.37	0.3349	0.365	0.17	0.496	0.30
30	0.87	0.4764	0.519	0.32	0.705	0.51
40	0.60	0.5875	0.640	0.44	0.870	0.67
45	0.50	0.6288	0.685	0.49	0.931	0.73
50	0.42	0.6593	0.719	0.52	0.976	0.78
60	0.29	0.6848	0.746	0.55	1.014	0.81
70	0.18	0.6577	0.717	0.52	0.973	0.77
80	0.09	0.5734	0.625	0.43	0.849	0.65
90	0	0.4292	0.468	0.27	0.635	0.44
100	−0.09	0.2243	0.244	0.04	0.332	0.13

case of a cohesionless soil with $\varphi = 30^\circ$ and $\gamma = 16.5\ \mathrm{kN/m^3}$ for $D = 0.20$ m. The graph can be achieved by symmetry.

(3) In order for all the points in the mass to be below the limit equilibrium, the lowest point of the curve (C) must have zero elevation. In fact, we should say "in order that no point at a depth greater than D in the soil

mass reach the limit equilibrium" etc. . . . Indeed, no statement can be made regarding what occurs between the surface and the level of the footing, because in this zone Boussinesq's formulas are not applicable.

Let us identify points on (C) of maximum elevation. The condition

$$\frac{dZ}{d\theta} = \frac{q-\gamma D}{\gamma\pi}\left(\frac{\cos\theta}{\sin\varphi}-1\right) = 0$$

gives: $\theta = \pi/2-\varphi$

The locus of the points on curve C with a horizontal tangent is an arc of a circle.

Replacing the value of θ thus obtained in eqn. (4), we get:

$$Z_{\max} = -D+\frac{q-\gamma D}{\gamma\pi}\left(\cot\varphi-\frac{\pi}{2}+\varphi\right)-\frac{H}{\gamma} = 0$$

which condition can also be written:

$$q_0 = \gamma D(\alpha)+H(\alpha-1) \tag{5}$$

where $\alpha = \dfrac{\pi}{\cot\varphi-\left(\dfrac{\pi}{2}-\varphi\right)}+1,$ i.e. Fröhlich's formula.

Values of α are given in Table 10B:

TABLE 10B

φ	10°	20°	30°	35°	40°	45°
α	1.74	3.06	5.59	7.71	10.85	15.64

The width of footing B does not enter into the equation. For a footing at the surface ($D=0$) on sand ($H=0$), formula (5) gives zero stress. For a footing at the surface on a purely cohesive soil (no angle of internal friction) we get, on the other hand:

$$H(\alpha-1) = \lim\left[\frac{\pi c\cot\varphi}{\cot\varphi-[(\pi/2)-\varphi]}\right]_{\varphi\to 0} \approx \pi c$$

For the numerical values of the second question, we have

$$q_0 = 16.5\times 0.20\times 5.59 = 18.4\,\text{kPa} = 0.18\,\text{daN/cm}^2.$$

The bearing capacity of the footing after the classical formula:

$$q_d = \tfrac{1}{2}B\gamma N_\gamma+\gamma DN_q+cN_c$$

obtained by plastic theory is ($\varphi=30^\circ, c=0, N_\gamma=21.8, N_q=18.4$):

$$q_d = [(16.5\times 1.00/2)\times 21.8]+(16.5\times 0.20\times 18.4) = 241\,\text{kPa} = 2.4\,\text{daN/cm}^2.$$

The correction factor of N_q was not taken into account to disregard the layer above the bottom of the footing.

For a footing at the surface on a purely cohesive soil, we know that

$$q_d = cN_c = c(\pi + 2) = 5.14c.$$

We thus get: $q_d/q_0 = 2.4/0.18 = 13.3$ for the cohesionless condition $\varphi = 30°$, $\gamma = 16.5\ \mathrm{kN/m^3}$ and $q_d/q_0 = 5.14/3.14 = 1.64$ for a footing at the surface of a purely cohesive soil.

(4) At a point Δ of the axis of the footing, we have (Fig. 10.9):

$$B/2Z = \tan(\theta/2) \quad \text{or} \quad Z = \tfrac{1}{2}B\cot(\theta/2)$$

Replacing this value in equation (4), we get an equation for θ, the roots of which give θ values corresponding to points on C located on the axis Δ.

Equation (4) becomes then:

$$\frac{B}{2}\cot\frac{\theta}{2} = -D + \frac{q-\gamma\pi}{\gamma\pi}\left(\frac{\sin\theta}{\sin\varphi} - \theta\right) - \frac{H}{\gamma} \tag{6}$$

which can be written:

$$a(q)\theta + b = a'(q)\sin\theta + b'\cot(\theta/2) \tag{6 bis}$$

with:

$$a(q) = 2(q-\gamma D), \quad b = 2\pi c\cot\varphi + 2\pi\gamma D,$$

$$a'(q) = 2(q-\gamma D)/\sin\phi, \quad b' = -\gamma\pi B.$$

Roots of this equation may be obtained graphically in plane (θ, y) at the intersection of line D, whose equation is: $y = a(q)\theta + b$, and of curve (Γ),

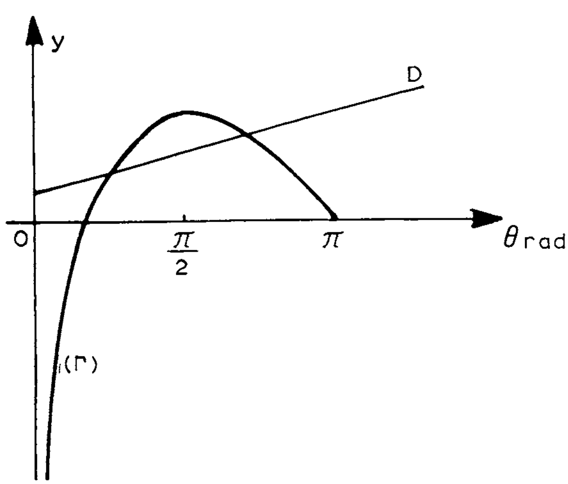


Fig. 10.11.

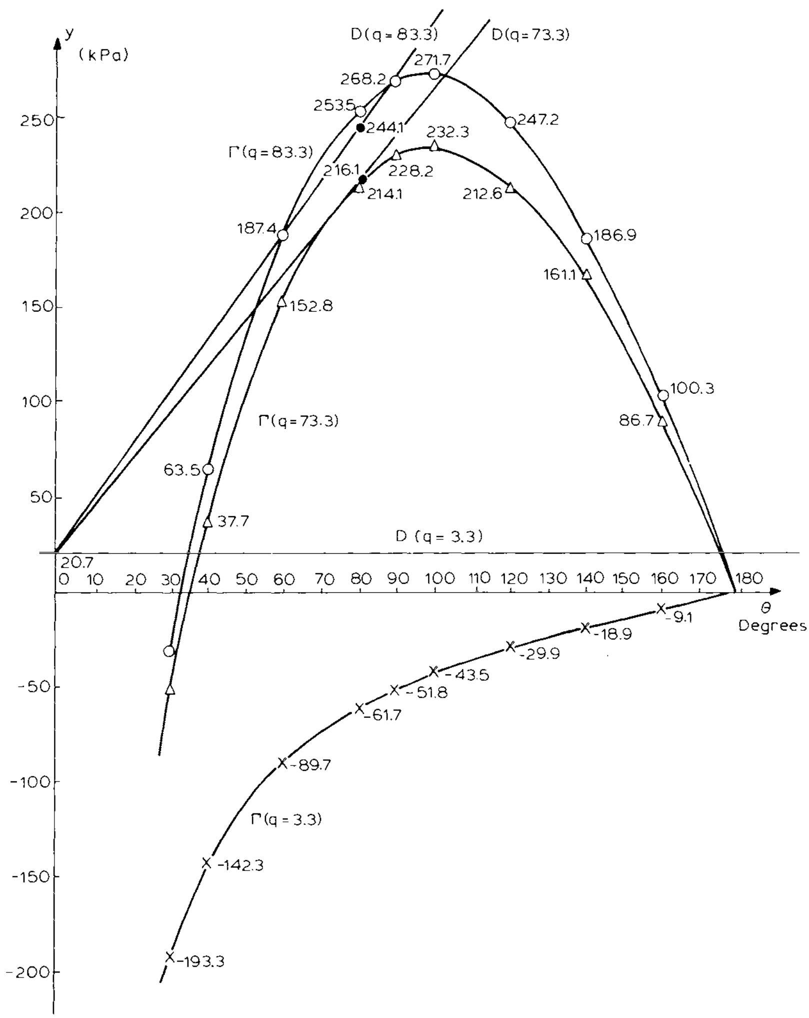


Fig. 10.12.

whose equation is: $y = a'(q) \sin\theta + b' \cot(\theta/2)$, both equations depending on parameter q.

Depending on the value of q, equation (6) may have zero, one or two solutions. (Fig. 10.11).

By a succession of approximations, we can so find the value of q for which D is tangent to Γ, which corresponds to a double root of equation (6).

With the numerical values above ($\varphi = 30°$, $c = 0$, $\gamma = 16.5\,\mathrm{kN/m^2}$, $D = 0.20\,\mathrm{m}$ and $B = 1.00\,\mathrm{m}$) we get:

$$a(q) = 2(q - 3.3)$$

$$b = 2\pi \times 3.3 = 20.73$$

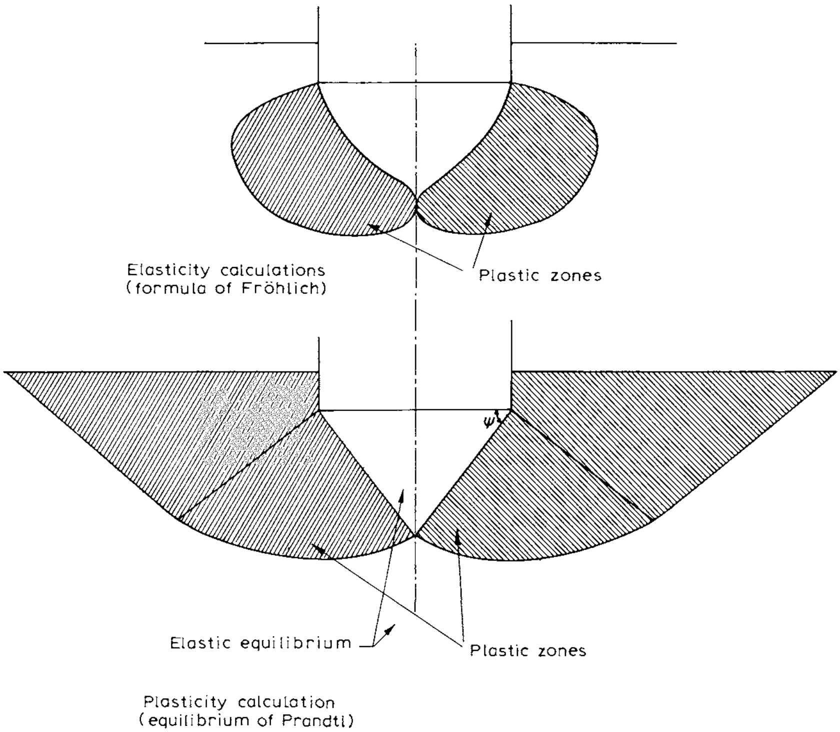


Fig. 10.13.

$$a'(q) = 2(q - 3.3)/\sin 30^\circ = 4(q - 3.3)$$

$$b' = -\pi \times 16.5 = -51.84$$

If the angle θ is in degrees, the equation of D becomes (Fig. 10.12)

$$y = 2(q - 3.3)(\pi/180)\theta^\circ + 20.73$$

On Fig. 10.12, both lines D and Γ correspond to the values of q: $q = 3.3$ kPa, $q = 83.3$ kPa and $q = 73.3$ kPa. A good graphical approximation gives $q_{\text{lim}} = 75$ kPa or 0.75 daN/cm^2.

If we compare q_{lim} and q_d obtained from the classical formula for bearing capacity, we find: $q_d/q_{\text{lim}} = 240/75 = 3.2$, which is very close to the value of the safety factor normally considered in foundation designs.

The shapes of plastic zones and corners under the footing may be compared, on Fig. 10.13, with the classical representation usually considered in plasticity. In the latter case, the failure lines are developed in a plastic zone of larger dimension, which could account for the higher values of q_d.

★★★*Problem 10.10* Design of a shallow footing based on laboratory test results

A preliminary design is needed for a 10-storey building whose planned dimensions are 70 m by 12 m and whose total weight is 60 MN (6000 tf). A soil investigation performed at the site shows a plastic clay layer from 0—10 m depth overlaying a silt layer. The groundwater table is located at 2.00 m below natural grade. Four undisturbed samples were recovered and tested in the laboratory. The results were:

— sample 1, depth —1 m, $\gamma_d = 15\ kN/m^3$, $w = 23\%$, *consolidated undrained triaxial test:* $c = 4 \times 10^4$ *pascals,* $\varphi = 10°$.

— sample 2, depth —3 m, $\gamma_d = 14.7\ kN/m^3$, $w = 31\%$, *unconfined compression test (strain rate 1 mm/min),* $R_c = 7.8 \times 10^4$ *pascals.*

— sample 3, depth —5 m, $\gamma_d = 15\ kN/m^3$, $w = 30\%$, *consolidated drained triaxial test:* $c = 3.8 \times 10^4$ *pascals,* $\varphi = 18°$.

— sample 4, depth —8 m, $\gamma_d = 15\ kN/m^3$, $w = 30\%$, *unconfined compression test (strain rate 1 mm/min),* $R_c = 8.2 \times 10^4$ *pascals.*

Only shallow footings are to be considered.

(1) Does each type of test made on the samples appear appropriate? How should the test results be used?

(2) Design a foundation for the building on strip footings. Two footing widths are to be considered (1 m and 2 m) and for each, determine the bearing capacity as a function of depth (to 4 m). Assume that bearing capacities are affected by the groundwater table when it is located one and a half times as deep below the footing as the footing is wide.

What are your conclusions?

(3) Design a mat foundation for the building. What are your conclusions?

Solution

(1) Both short- and long-term stability are to be considered.

Short term stability. We must use the results of the tests under undrained conditions. The unit-weight of soil to consider in the computations is the wet unit-weight. Therefore, use the consolidated undrained triaxial test results (c.u. tests) and the unconfined compression test results (because of the large strain rate, the test is considered undrained).

The degree of saturation of the samples is: $S_r = w/(1/\gamma_d - 1/\gamma_s)$.

for Sample 1: $S_r = 0.78$ for Sample 3: $S_r = 1.00$

for Sample 2: $S_r = 1.00$ for Sample 4: $S_r = 1.00$.

Sample 1 is not saturated; it is therefore necessary to perform a triaxial test in order to determine the undrained angle of friction.

For the saturated samples 2 and 4, the unconfined compression test results are acceptable to determine the undrained cohesion, since the angle of friction is zero.

Long term stability. The results of drained tests must now be used. The unit-weight to consider is the buoyant weight and we, therefore, will utilize the results of the consolidated-drained triaxial test.

To conclude, it appears that the test program was well conceived for a complete analysis of the proposed foundation schemes.

(2) The short-term bearing capacity is usually the most critical and will be considered first. The increase of stress corresponding to the limit foundation stress is given by: $q_d - \gamma D = \frac{1}{2}\gamma B N_\gamma + cN_c + \gamma D(N_q - 1)$.

For the soil parameters at a depth between 0 and 2 m, we have:

$$\gamma_h = \gamma_d(1 + w) = 15 \times 1.23 = 18.5\,\mathrm{kN/m^3}$$

$c = 4 \times 10^4$ Pa, $\varphi = 10^\circ$, $N_c = 8.3$, $N_q = 2.5$, $N_\gamma = 1$ (see table II, sect. 9.2.1. of Costet-Sanglerat or Fig. 10.21 and Table 10C at the end of this chapter).

We have:

$$\begin{aligned} q_d - \gamma D &= \tfrac{1}{2}\gamma B N_\gamma + cN_c + \gamma D(N_q - 1) \\ &= 9250B + 3.3 \times 10^5 + 27\,700D \end{aligned}$$

$$\left.\begin{aligned} \text{for } B &= 1\,\mathrm{m}, \quad q - \gamma D = 3.31 \times 10^5 + 27\,700D \\ B &= 2\,\mathrm{m}, \quad q - \gamma D = 3.48 \times 10^5 + 27\,700D \end{aligned}\right. \qquad (1)$$

where q is in pascals and D in meters.

For the soil conditions below 2 m depth, we have:

$$\gamma_h = \gamma_d(1 + w) = 14.8 \times 1.31 = 19.4\,\mathrm{kN/m^3}$$

$$c = R/2 = 3.9 \times 10^4 \text{ pascals}, \quad \varphi = 0^\circ \text{ (saturated soil)}$$

Note, however, that for the embedment factor in the bearing capacity formula, we must take into account the weight of the soil above the level of the footing bottom.

With $\varphi = 0^\circ$, we get $N_\gamma = 0$, $N_c = 5.1$, $N_q = 1$. The increase of the limit stress will, therefore, be:

$$q_d - \gamma D = 5.1 \times 3.9 \times 10^4 = 19.9 \times 10^4\,\mathrm{kN/m^2} \qquad (2)$$

where q is in pascals and D is in m.

We will assume, as do L'Herminier, Tcheng and Obin, that the bearing capacity of the soil is affected by the proximity of the water table whenever $H = 1.5B$ or less. (Fig. 10.14).

For a 1 m wide foundation, formula (1) above must be considered for 0—0.5 m, formula (2) from 2 m depth. For the depth increment between 0.5 and 2 m, we will assume a smooth progression as indicated by the curve of Fig. 10.15.

The presence of the water table only intervenes in the term $(q - \gamma D)$

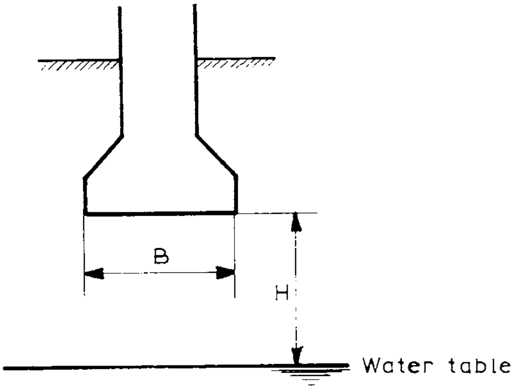


Fig. 10.14.

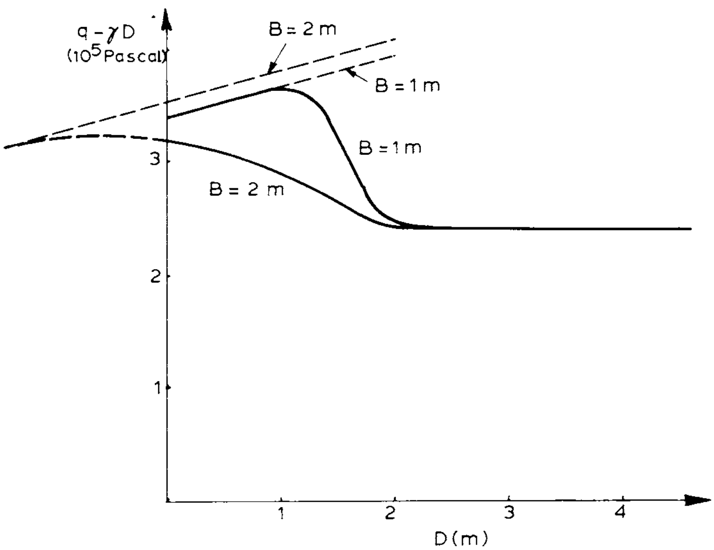


Fig. 10.15.

corresponding to the increase of stress in the soil. We thus will draw the curves $q - \gamma D = f(D)$ corresponding to the increase of the limit stress in the soil (Fig. 10.14).

The 1 m wide footing allows a higher bearing capacity than the 2 m wide footing. Under the best conditions and for a 1 m wide footing, we can count on an increase of ultimate stress of 3.5×10^5 pascals. For a safety factor of 3, the allowable bearing capacity is $q_{adm} = \gamma D + (3.5/3) \times 10^5$ Pa, and for an embedment of 0.5 m, therefore: $q_{adm} = 1.25 \times 10^5$ Pa.

The area of the footing corresponding to an average stress of 1.25×10^5 Pa is: $(60 \times 10^6)/(1.25 \times 10^5) = 480\,\mathrm{m}^2$.

The total plan area of the building is $70 \times 12 = 840\,m^2$. This solution would require that the total spread footings area be more than half the plan area of the proposed building. The strip footing scheme, therefore, is not an economic solution and involves a risk because no account was made of the interaction of adjacent footings on each other. The above computations then are no longer valid. *The solution of strip footings should be abandoned.*

(3) Because of the large dimensions of the mat, we must consider the soil conditions below the level of water table. There, we have:

$\gamma_h = 19.40\,kN/m^3, \quad c = 3.9 \times 10^4\,Pa, \quad \varphi = 0°$ (saturated soils).

Note, however, that in the embedment term of the bearing capacity formula, and for mat depths less than 2 m, we must use the value $\gamma_h = 18.5\,kN/m^3$. Therefore, we get, for $D < 2\,m$: $N_c = 5.13, N_q = 1, N_\gamma = 0$ ($\varphi = 0°$) and the stress increase is $q_d - \gamma D = 3.9 \times 5.14 \times 10^4 = 20 \times 10^4$ Pa.

The uniform stress corresponding to the weight of the building is:

$(60 \times 10^6)/(70 \times 12) = 7.2 \times 10^4$ Pa.

This stress must be less than or equal to the allowable stress, which is:

$q_{adm} = \gamma D + (20/3) \times 10^4 = 7.2 \times 10^4$ Pa,

Hence $\gamma D = 0.5 \times 10^4$ Pa and $D = (0.5 \times 10^4)/18\,500 = 0.27\,m$.

The foundation may be at 27 cm depth. However, in order to account for frost action, the mat should be designed for 80 cm embedment. *For the short-term stability, a mat at a depth of 80 cm is acceptable.*

For the long-term stability, the consolidated-drain triaxial test results must be used:

$c = 3.8 \times 10^4$ Pa, $\quad \varphi = 18°$,

$\gamma' = [(\gamma_s - \gamma_w)/\gamma_s] \times \gamma_d = 0.62, \quad \gamma_d = 9.3\,kN/m^3$

and $N_\gamma = 3.5, N_q = 5.3, \quad N_c = 13.1$, from which:

$q_d - \gamma D = (9300 \times 6 \times 3.5) + (18\,500 \times 0.8 \times 4.3) + (3.8 \times 10^4 \times 13.1)$:

$q_d - \gamma D = 7.56 \times 10^5$ Pa.

If we consider a safety factor of 3, the allowable stress is:

$q_{adm} = \gamma D + (7.56/3) \times 10^5 = 2.7 \times 10^5$ Pa.

This stress is larger than that exerted by the building. The mat at 80 cm depth is an acceptable solution. (This depth will prevent frost heave in winter, which varies depending on geographical locations).

A complete evaluation of the foundation should incluse an assessment of settlements calculated from the results of triaxial or consolidation tests or from in situ tests with the static penetrometer or the pressuremeter.

Summary of answers

The testing program was good. The strip footings should not be adopted. A mat foundation is recommended at a depth of 0.80 m.

★★*Problem 10.11* Shallow footings on a two-layer system

Design a mat foundation 18 m wide and 70 m long for the support of a building. The depth of embedment is to be 1 m. The upper soil layer consists of a 6 m thick clay, overlying a silty sand of great depth. (Fig. 10.16).

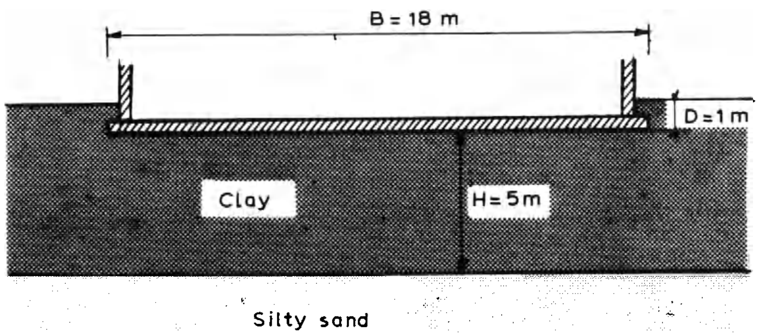


Fig. 10.16.

Soil characteristics are as follows:

clay: $c = 0.25\,\mathrm{daN/cm^2}$, $\varphi = 10°$, $\gamma_h = 19\,\mathrm{kN/m^3}$

silty sand: $c = 0$, $\varphi = 30°$, $\gamma_h = 18\,\mathrm{kN/m^3}$.

What is the allowable bearing capacity?

Use the results of Mandel and Salençon which give the correction factors for N_γ, N_q and N_c for a two-layer system consisting of a compressible layer over a rock substratum.

For this particular problem, and for $\varphi = 10°$, $B/H = 18/5 = 3.6$, the correction factors are for N_γ: $\epsilon_\gamma \simeq 1$, for N_q: $\epsilon_q \simeq 1.3$, for N_c: $\epsilon_c \simeq 1.5$. These results are valid if it is considered that friction is totally mobilized at the two-layer interface.

Solution

It is not possible to theoretically calculate the bearing capacity of a heterogeneous soil. Computations are made possible through simplifying assumptions (Giroud, Obin), but the results should be considered carefully. Here we will give the upper and lower limits of the results.

First calculation assumption. Let's assume that the soil is homogeneous and consists only of clay. We then have: $c = 0.25\,\mathrm{daN/cm^2}$, $\varphi = 10°$, $\gamma_h = 1.9 \times 10^{-3}\,\mathrm{daN/cm^3}$.

Frequently used tables give: $N_\gamma = 1$, $N_q = 2.5$, $N_c = 8.3$.

The stress increase corresponding to the ultimate stress will then be:

$$\begin{aligned} q_d - \gamma D &= \tfrac{1}{2}\gamma B N_\gamma + \gamma D(N_q - 1) + cN_c \\ &= (1.9 \times 10^{-3} \times 9 \times 10^2 \times 1) + (1.9 \times 10^{-3} \times 10^2 \times 1.5) \\ &\quad + (0.25 \times 8.3) = 1.7 + 0.29 + 2.08 \simeq 4.1\,\mathrm{daN/cm^2}. \end{aligned}$$

This value is the lower limit to take into account for ultimate stress.

Second calculation assumption. Assuming that the soil is homogeneous and consists of silty sand only, we have: $c = 0$, $\varphi = 30°$, $\gamma_h = 1.8 \times 10^{-3}\,\mathrm{daN/cm^3}$ and $N_\gamma = 21.8$, $N_q = 18.4$, $q_d - \gamma D = 35.5 + 3.3 = 38.6\,\mathrm{daN/cm^2}$.

This value is the upper limit to consider for ultimate stress increase.

Third calculation assumption. We now assume that the clay layer is underlain by bedrock, assumed rigid. Then, we have: $c = 0.25\,\mathrm{daN/cm^2}$, $\varphi = 10°$, $\gamma_h = 1.9 \times 10^{-3}\,\mathrm{daN/m^3}$, $N_\gamma = 1$, $N_q = 2.5$, $N_c = 8.3$.

Introducing the correction factors of Mandel and Salençon we can write:

$$q_d - \gamma D = \tfrac{1}{2}\gamma B N_\gamma \epsilon_\gamma + D\gamma(N_q\epsilon_q - 1) + cN_c\epsilon_c$$

where: $\epsilon_\gamma = 1$, $\epsilon_q = 1.3$, $\epsilon_c = 1.5$ for this soil type and the geometry of the bi-layer system.

$q_d - \gamma D = 1.7 + 0.43 + 3.12 = 5.25\,\mathrm{daN/cm^2}$

This represents another upper limit, closer to the reality, for the increase of the stress.

Finally, we have: $4.1\,\mathrm{daN/cm^2} < q_d - \gamma D < 5.25\,\mathrm{daN/cm^2}$.

The upper and lower limits are close to each other and the average value may be retained, $q_d - \gamma D = 4.8\,\mathrm{daN/cm^2}$, without detriment to the design.

The allowable stress will then be:

$q_{ad} = \gamma D + (4.8/3) = 1.79\,\mathrm{daN/cm^2}$

Using the graphs of Giroud, Tran Vo Nhiem and Obin for a two-layer system, the ultimate stress increase is: $q_d - \gamma D \simeq 5.5\,\mathrm{daN/cm^2}$. This slightly higher than the upper limit calculated by the method of Mandel and Salençon, because a different plastic zone is considered by either case.

To conclude, it should be remembered that the presence of a sand layer under the mat only slightly increases the ultimate bearing capacity.

Summary of answers

The allowable stress below the mat foundation is $1.6\,\mathrm{daN/cm^2}$.

★★★*Problem 10.12* Circular mat design for the support of a stack (shallow footing with an eccentric, inclined load by the method of Tran Vo Nhiem)

A smoke stack for a heating plant is to be supported on a circular mat foundation deriving its bearing from a thick clayey silt layer. The ground-water table is located at −20 m. The stack is 35 m high and weighs about 2 MN (200 tf). Wind load imparts a horizontal force of 0.18 MN (18 tf) at mid-height of the stack.

Design a circular mat foundation 0.50 m thick to support the structure.

The clayey silt properties are, between 8 and 20 m depths:
$\gamma_h = 19\,kN/m^3$, $c = 1.5 \times 10^4$ *Pa (0.15 bar),* $\varphi = 20°$.

Solution

The diameter of the circular foundation must be determined. Assume a diameter of the order of 8 m. The weight of the footing will then be about 0.60 MN, if the concrete unit-weight is $23\,\mathrm{kN/m^3}$.

Assume further that the bottom of the mat will be located at a depth of 1 m for reasons of frost action on the soil. The resultant of forces acting on the base of the stack then is: $\sqrt{(2.6)^2 + (0.18)^2} \simeq 2.6$ MN. This force is inclined by angle α to the vertical, such that $\tan\alpha = 0.18/2.6 \simeq 0.07$ and $\alpha \simeq 4°$.

Let e be the eccentricity of the resultant on the base. The equilibrium of forces about the center of the base requires then that $0.18 \times d = 2.6 \times e \times \cos\alpha$, where d is the distance between the point of application of the wind load and the base of the foundation.

We have: $d = 18.5$ m, from which $e = (0.18/2.6) \times (18.5/0.997) = 1.30$ m. We must calculate the allowable bearing capacity for a circular footing supporting an eccentric load where $e = 1.3$ m and inclined at 4° with the vertical.

No simple theoretical solution exists and only approximations can be made. We preferred to set up our computations as for the case of a strip footing, inserting empirical correction factors.

In the case of an inclined, eccentric load acting on a strip footing, only a portion B' of the width B of the footing is considered to bear actually on the soil. B' is defined as:

$$\frac{B'}{B} = \frac{1 \pm (2e/B)}{1 \pm (2e_M/B)}$$

where e_M is the optimal eccentricity for the given load inclination.

In this instance, e_M/B is of the order of 0.002 and therefore negligible (see table, in sect. 9.3.3 of Costet-Sanglerat). We may therefore write: $B' = B \pm 2e$.

Respecting sign conventions, e in this case is negative and $e < e_M$; we must therefore consider the + sign in the above formula.

Finally, $B' = B + 2(-1.30) = B - 2.60$.

The bearing capacity factor N_γ must furthermore be corrected by a coefficient i_γ which depends on the inclination α of the applied force. The tables of sect. 9.3.3 of Costet-Sanglerat indicate for i_γ the value $i_\gamma = 0.9$. The bearing capacity of a strip footing would then be given by:
$q_d = \gamma(B'/2)i_\gamma N_\gamma + \gamma D N_q + cN_c$.

Since the footing is circular, shape factors must be introduced for N_γ, N_q, N_c. In agreement with Terzaghi and Peck, Costet and Sanglerat propose as multipliers 0.8 for N_γ, 1 for N_q and 1.2 for N_c.

We thus have:

$$q_d = \gamma(B'/2) \times 0.8 i_\gamma N_\gamma + \gamma D N_q + 1.2 c N_c.$$

For a safety factor of 3 applied to the stress increase we get:

$$q_{ad} = \gamma D + \frac{\gamma B' \times 0.4 \times i_\gamma N_\gamma + \gamma D(N_q - 1) + 1.2 c N_c}{3}.$$

For $\varphi = 20°$, the tables give $N_\gamma = 5$, $N_q = 6.4$, $N_c = 14.8$. Therefore: $q_{ad} = 11\,400 B' + 142\,000$.

Assuming that this stress is applied only over a circle of radius B', then the bearing force is: $Q = (q_{ad} \times \pi \times B'^2)/2$. The actual load is equal to 2.6×10^6 N. The value of B' is given by the equation:

$$2.6 \times 10^6 = [(11\,400 B' + 142\,000) \times \pi \times B'^2]/2$$

$$\text{or: } (1.655 \times 10^6)/B'^2 = 114\,000 B' + 142\,000.$$

This equation can be solved graphically, by drawing in Fig. 10.17 the curves: $y_1 = 114\,000 B' + 142\,000$, and $y_2 = (1.655 \times 10^6)/B'^2$. The intersection of the two curves corresponding to $B' > 0$ indicates the solution desired.

Let us calculate a few points of each curve. When $B' = 0$, $y_2 \to \infty$; when $B \to \infty$, $y_2 \to 0$.

For $B' = 2$ we get $y_2 = 4.14 \cdot 10^5$

$B' = 4$ $\quad y_2 = 1.03 \cdot 10^5$

$B' = 6$ $\quad y_2 = 4.6 \cdot 10^4$

For $B' = 0$ $\quad y_1 = 1.42 \cdot 10^5$

$B' = 4$ $\quad y_1 = 1.88 \cdot 10^5$.

Therefore, $B' = 3.1$ m (Fig. 10.17) and $B = B' + 2.6 = 5.7$.
The smoke stack may be supported on a circular mat 6 m in diameter.

The weight of the mat was originally overestimated (8 m instead of 6 m). The dimension of 6 m is therefore on the safe side.

Furthermore, the ratio of the mat dimension to the distance to the water table, which is 3, allows us to justify the values of the soil properties used in the equations.

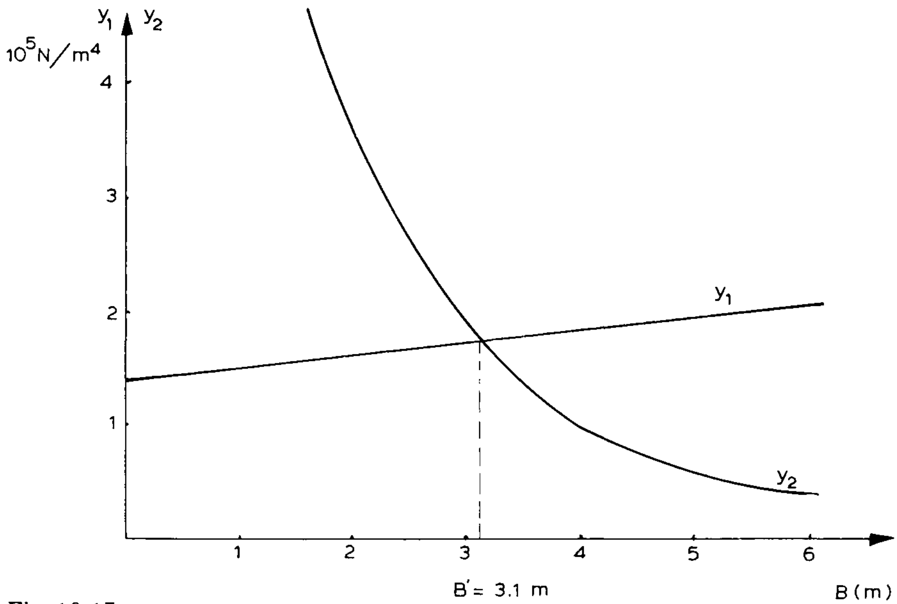


Fig. 10.17.

It should be noted that a complete foundation analysis would require making an estimate of settlement. Such an estimation would be based on triaxial or consolidation test results or on results of in situ tests such as of the static penetrometer or pressuremeter.

Summary of answers

The smoke stack can be supported on a circular, 6 m in diameter mat foundation located at 1 m below grade.

★★★*Problem 10.13* **Design of footings on swelling clay. Evaluation of swelling pressures and computations of possible differential uplifts**

A single storey house is to be constructed on a swelling clay, 3 m thick, overlying dense, non-swelling substratum. The exterior and interior bearing walls are 40 cm thick and transmit a load of 40 kN and 60 kN (per running meter) to the foundation. Test results on undisturbed samples of the swelling clay indicated: $\gamma_d = 17\,kN/m^3$, $w = 11.90\%$.

The unconfined compression $R_c = 300\,kPa$.

A swelling test was made in which the water content of the clay increased from 11.9% to 25%, when the volume changes, in %, were a function of the applied loads as follows:

$\Delta V/V = 3.9\%$ *for* $\sigma_v = 20\,\text{kPa}$

$\Delta V/V = 2.8\%$ *for* $\sigma_v = 30\,\text{kPa}$

$\Delta V/V = 1.6\%$ *for* $\sigma_v = 50\,\text{kPa}$

$\Delta V/V = 1\%$ *for* $\sigma_v = 70\,\text{kPa}$.

(1) By drawing a graph, estimate the clay swelling pressure.*

(2) Assuming that swelling cannot occur, design the footings for the external and internal walls. Is such a solution acceptable if swelling should occur?

(3) Determine what practical solutions may be considered in the footing design in order to account for swelling.

(4) Evaluate differential swelling which would occur, should the footing be designed to have plan dimensions of 40 cm wide by 1 m long, embedded at 70 cm with a center-to-center spacing of 1.80 m for both exterior and interior walls.

Solution

(1) Estimation of swelling pressure

As is done for an oedometric diagram, we can plot the test results on semi-log paper. Volume variations are plotted against the log of pressure. Fig. 10.18 shows, by extrapolation, that the pressure required to prevent swelling, is about 130 kPa. So the swelling pressure is 130 kPa.

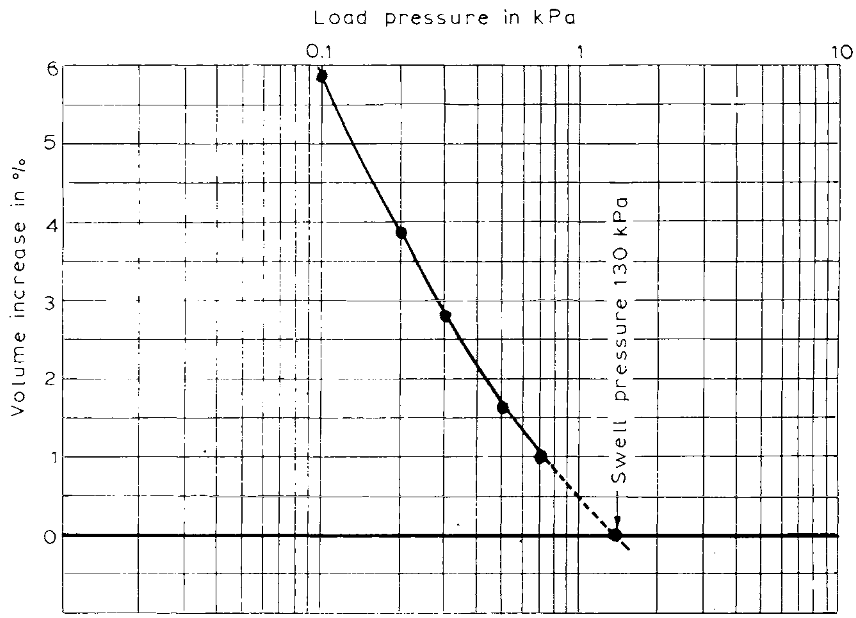


Fig. 10.18. Determination of swelling pressure.

*The swelling pressure is equal to the vertical stress σ that must be applied on the sample in a consolidometer to maintain constant volume.

(2) Footing design

The footings must satisfy 5 conditions: (1) adequate bearing, regardless of swell; (2) frost prevention; (3) allowable total settlement; (4) allowable differential settlement; (5) no uplift in case of swell.

(a) As indicated in the givens, we will first assume that no swelling of the clay can occur. Since frost action varies with geographical regions, we further assume that an embedment of 0.70 m is sufficient. We will also make a short-term bearing capacity analysis, which in most instances is unfavourable. Since the clay is saturated, its undrained cohesion is: $c_u = R_c/2$ or $c_u = 150$ kPa.

The bearing capacity of a strip footing, 0.40 m wide embedded to 0.70 m is: $q_d = \gamma D + 5.14c_u$, with: $D = 0.70$, and $\gamma = \gamma_d(1 + w) = 17 \times (1 + 0.119) \simeq 19\,\text{kN/m}^3$. So we get:
$q_d = (19 \times 0.70 + 5.14 \times 150) \simeq 784$ kPa
and the allowable bearing pressure will be:

$$q_{ad} = 19 \times 0.70 + (5.14 \times 150)/3 = 270.3 \text{ or } 270\,\text{kPa}.$$

Taking $\gamma_b = 25\,\text{kN/m}^3$ for the specific weight of concrete, the stress applied to the soil under a 0.40 m wide footing at 0.70 m depth is:

$$q_1 = 25 \times 0.70 + (40/0.40) = 117.5 \text{ or } 118\,\text{kPa for exterior walls.}$$

$$q_2 = 25 \times 0.70 + (60/0.40) = 167.5 \text{ or } 168\,\text{kPa for interior walls.}$$

Both values are considerably less than the allowable stress.

Let us now evaluate the settlement. A resistance to unconfined compression of 300 kPa (3 bars) means that the clay is very stiff (see table VII, sect. 1.5.5, Costet-Sanglerat). The oedometric modulus E' may be estimated at a minimum of 6000 kPa (60 bar) (see table I, sect. 3.4.2 of Costet-Sanglerat), which is certainly inferior to the actual value.

The settlement is calculated from: $\Delta h/h = -\Delta\sigma/E'$, the stress increase, $\Delta\sigma$, is obtained from graph III-3 in sect. 3.2.2 of Costet-Sanglerat. Here, we have $z/B = 1.15/0.40 = 2.875$, which gives $\Delta\sigma \simeq 0.22q'$.
The stress increases q' at the level of the footings are:

$$(25 - 19) \times 0.70 + 40/0.40 \simeq 104\,\text{kPa (exterior walls)}$$

$$(25 - 19) \times 0.70 + 60/0.40 \simeq 154\,\text{kPa (interior walls)}$$

from which: $\Delta\sigma_1 = 0.22 \times 104 \simeq 23$ kPa and: $\Delta\sigma_2 = 0.22 \times 154 \simeq 34$ kPa.
So we finally obtain:

$$|(\Delta h_1)| = 230(23/6000) = 0.88\,\text{cm}$$

$$|(\Delta h_2)| = 230(34/6000) = 1.30\,\text{cm}.$$

The differential settlement will thus be: $1.3 - 0.88 = 0.42$ cm.

Referring to table I, in Ch. 9 of Costet-Sanglerat, it is seen that the total settlement corresponds to the values presently adopted for masonry walls. The limit value for differential settlements is $L/1000$, which gives $L = 0.42 \times 1000 = 420$ cm or 4.2 m which is convenient since L represents the distance between exterior and interior walls.

To conclude, and in the absence of swell, the footings may be placed directly on the clay at a depth of 0.70 m for both exterior and interior walls, without having to increase their width, which is 0.40 m.

(b) As for the uplift due to swell, the final footing design also requires that uplift consequences be evaluated. If the water content of the clay goes from 11.90% to 25%, uplift pressure could be as high as 130 kPa. As a consequence, the buried portion of the footing would undergo an upward lateral friction due to the surrounding soil uplift (Fig. 10.19). On the other hand, floor uplift could occur where they are on grade.

According to Fu Hua Chen, the lateral upward friction may be assumed to be 15% of the swell pressure. The unit friction is: 0.15×130 kPa $= 19.5$ or about 20 kPa. We may also assume that dead loads correspond to 80% of the total load of 40 kN and 60 kN or, 32 and 48 kN, respectively (all loads are per unit-length).

The strip footings of 0.40 m width and at 0.70 m embedment would undergo, per m of length, an uplift force of (Fig. 10.19):

— uplift at base: $0.40 \times 1.00 \times 130 = 52$ kN

— uplift on sides: $2 \times 0.70 \times 1.00 \times 130 \times 0.15 = 27.3$ kN.

or a total uplift of $52 + 27.3 = 80$ kN per linear m. This uplift force is far superior to the sum of footing weights and dead weights.

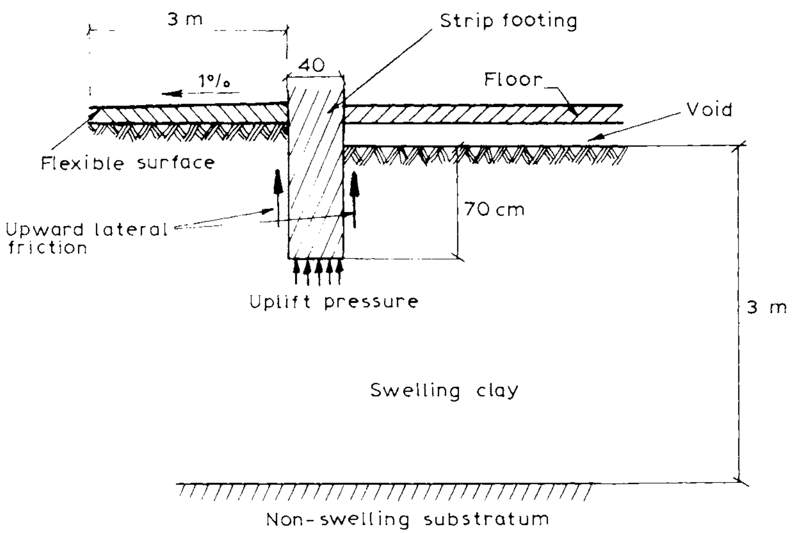


Fig. 10.19.

PROBLEM 10.13

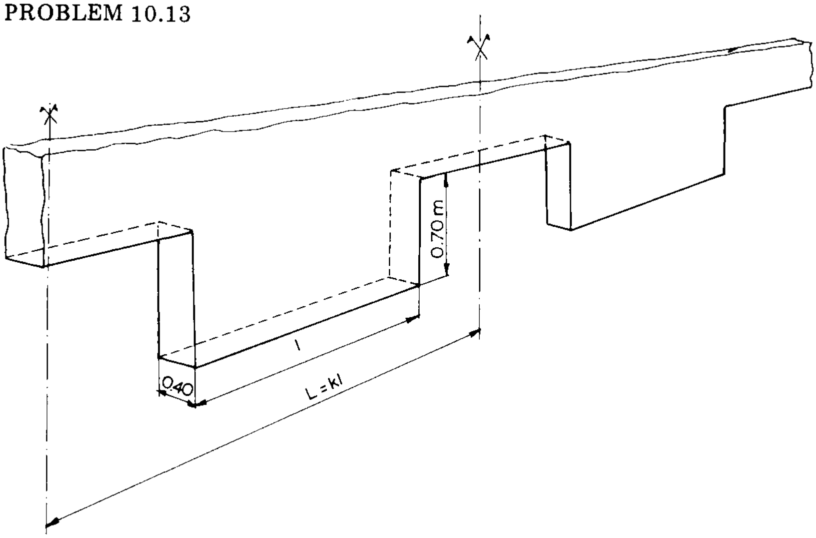


Fig. 10.20.

The footing weight (per m) is only: $0.40 \times 0.70 \times 1.00 \times 25 = 7$ kN, which gives $32 + 7 = 39$ kN for exterior walls, $48 + 7 = 55$ kN for interior walls. Thus it is certain that in the event of an increase in water content (raising water table, leak in water drains, leak in sprinkling systems or excessive watering of lawns) the 0.4 m wide footings would be uplifted. *This is not acceptable.*

(3) To avoid uplift, the strip footings may be replaced by an isolated footing of equal width (Fig. 10.20). Let l be the length of such a footing and $L = kl$ the wall width supported by the footing.

We will only work on the problem for interior walls. The loads of the list below are expressed in kilonewtons:

load transmitted by the wall to the footing ($48 \times kl$)	$= 48kl$
weight of footing ($0.40 \times 0.70 \times l \times 25$)	$= 7l$
uplift force under footing ($0.40 \times l \times 130$)	$= 52l$
uplift on vertical sides of footing ($2 \times 0.70 \times l \times 130 \times 0.15$)	$= 27.3l$
($2 \times 0.40 \times 0.70 \times 130 \times 0.15$)	$= 10.9$

The condition for no uplift is: $48kl + 7l \geqslant 52l + 27.3l + 10.9$

$$l(48k - 72.3) \geqslant 10.9. \qquad (1)$$

In addition, under the maximum load applied, and if the clay should not swell, the stress cannot exceed the maximum allowable soil bearing pressure: $q \leqslant q_{ad}$. But we have:

$q = (60kl + 7l)/0.40l$

and:

$$q_{ad} = \gamma D + \frac{5.14(1 + 0.2B/l)c_u}{3} = 19 \times 0.70 + \frac{5.14(1 + 0.2 \times 0.40/l)150}{3}$$

or: $q_{ad} = 270.3 + 20.6/l$.

The second condition to meet, thus is:

$(60kl + 7l)/0.40l \leqslant 270.3 + (20.6)/l$ or: $l(60k - 101.1) \leqslant 8.2$. (2)

Condition (1) dictates that $k \geqslant 72.3/48$, say: $k \geqslant 1.51$. Let us try $k = 2$. Condition (1) then gives: $l \geqslant 0.46$ m, and condition (2): $l \leqslant 0.43$ m.

These two conditions are not compatible. Let us then try $k = 1.8$: condition (1) yields: $l \geqslant 0.77$ m and condition (2): $l \leqslant 1.19$ m.

Both conditions are satisfied if we take for instance: $l = 1.00$. Hence: $L = kl = 1.80$ m. A similar computation for the exterior wall leads to the following inequalities:

$l(32k - 72.3) \geqslant 10.9$ (1′)

$l(40k - 101.1) \leqslant 8.2$. (2′)

These conditions are both satisfied by $k = 2.7$, $l = 1.00$ m and $L = kl = 2.7$ m.

Once again, both total and differential settlements must be checked, which would require making a consolidation test. We will assume here that settlements are small.

To conclude, the interior wall footings may consist of embedding the walls in the clay layer in order to obtain masses 40 cm wide and 70 cm high and of 1.00 m length, 1.80 m center-to-center spacing for interior walls and of 2.70 m spacing for the exterior walls. These footings must be reinforced with vertical steel to prevent rupture under the uplift loads.

Naturally, the base of the footing will also have to be reinforced as well as the grade beams connecting them. In addition, a 10-cm void should be provided between the bottom of the grade beam and the underlying soil in order to prevent uplift pressures acting on the lower face of the beams. By the same token, all floors will have to be structural types (Fig. 10.19).

An apron will have to be constructed around the house with a slope inclined outward of about 3 m in width in order to drain the surface water away from the footings.

Remark: In the event where the inequalities of this problem cannot be resolved, another type of foundation would have to be considered, such as short piles or drilled cast-in-place piers.

(4) Since the foundations have been designed so that the dead weights are larger than the uplift pressure, no danger exists from differential uplift.

Let us assume that the builder decides to construct the exterior footings with the same spacing as that of the inside footings (1.80 m instead of 2.70 m). He could decide to do so (erroneously for sure) because he reasons that the footing sizes provide adequate bearing capacity. Let us see what the consequence would be:

with $l = 1.00$ and $k = 1.80$ instead of 2.70, the dead weight on each footing is:

$$W = 32kl + 7l = 32 \times 1.8 \times 1.00 + 7 \times 1.00 = 64.6\,\text{kN}$$

and the uplift pressure is:

$$(52 + 27.3)l + 10.9 = 79.3 \times 1.00 + 10.9 = 90.2\,\text{kN}$$

Uplift could then occur if the water content of the soil increased accidentally from 11 to 25%. Let us try to roughly estimate the uplift. At the end of the swelling, the generated stresses in the clay are in equilibrium with the dead weight of the structure or 64.6 kN. Let v' be the load on the clay.* We then can write:

$$\text{uplift of the lateral faces} = 2 \times 0.70 \times 1.00 \times v' \times 0.15 = 0.21v'$$

$$2 \times 0.40 \times 0.70 \times v' \times 0.15 = 0.084v'$$

$$\text{uplift on the base} = 0.40 \times 1.00 \times v' = 0.40v'$$

and: $(0.21 + 0.084 + 0.40)v' = 64.6\,\text{kN}$

or: $v' = 64.6/0.694 = 93\,\text{kPa}$.

From the diagram of Fig. 10.18, the swell would be of the order of 0.7% or, for safety's sake, say 1%. The swelling occurs over the 2.30 m thickness of the clay between the footing bottoms and the substratum.

The uplift could therefore be of $230 \times 1\% = 2.30$ cm which represents the differential settlement between two adjacent footings. One could be wetted whereas the other would not. This shows that even though the builder meant well, he undertook a considerable risk.

Remark about the Table 10C

Generally, the values of the coefficients N_q and N_c, proposed by various authors, are very close to each other, because both have an analytical base (Costet-Sanglerat, sect. 9.2.2). On the other hand, values of the coefficient N_γ vary considerably with the authors. This is due to the fact that the possibility exists to consider several failure modes under the footings. These correspond,

* v' is less than the swelling pressure v since a swelling occurred.

TABLE 10C

Values of bearing capacity factors N_γ, N_q and N_c as functions of φ

φ	N_γ Terzaghi	N_γ Caquot Kérisel	N_γ Biarez Nhiem	N_q	N_c
0°	0	0	0	1.00	5.14
1°			0.00	1.09	5.38
2°			0.01	1.20	5.63
3°			0.03	1.31	5.90
4°			0.05	1.43	6.19
5°		0.2	0.09	1.57	6.49
6°			0.14	1.72	6.81
7°			0.19	1.88	7.16
8°			0.27	2.06	7.53
9°			0.36	2.25	7.92
10°	0.546	1.00	0.47	2.47	8.34
11°			0.60	2.71	8.80
12°		1.40	0.76	2.97	9.28
13°			0.94	3.26	9.81
14°		1.97	1.16	3.59	10.37
15°		2.3	1.42	3.94	10.98
16°		2.73	1.72	4.34	11.63
17°			2.08	4.77	12.34
18°		3.68	2.49	5.26	13.10
19°			2.97	5.80	13.93
20°	3.44	4.97	3.54	6.40	14.83
21°			4.19	7.07	15.81
22°		6.73	4.96	7.82	16.88
23°			5.85	8.66	18.05
24°		9.03	6.89	9.60	19.32
25°		10.4	8.11	10.66	20.72
26°		12.1	9.53	11.85	22.25
27°			11.2	13.20	23.94
28°		16.4	13.1	14.72	25.80
29°			15.4	16.44	27.86
30°	18.1	21.9	18.1	18.40	30.14
31°			21.2	20.63	32.67
32°		29.8	25.0	23.18	35.49
33°			29.4	26.09	38.64
34°		40.8	34.7	29.44	42.16
35°		47.9	41.1	33.30	46.12
36°		56.8	48.8	37.75	50.59
37°			58.2	42.92	55.63
38°		79.8	69.6	48.93	61.35
39°			83.4	55.96	67.87
40°	102	113	100	64.20	75.31
41°			120	73.90	83.86
42°		165	144	85.37	93.71
43°			173	99.01	105.1
44°		244	209	115.3	118.4

TABLE 10C Continued

φ	N_γ Terzaghi	N_γ Caquot Kérisel	N_γ Biarez Nhiem	N_q	N_c
45°	284	299	254	134.9	133.9
46°		369	309	158.5	152.1
47°			379	187.2	173.6
48°		576	467	222.3	199.3
49°			578	265.5	229.9
50°		915	720	319.1	266.9
51°			900	386.0	311.8
52°		1540	1140	470.3	366.7
53°			1450	577.5	434.4

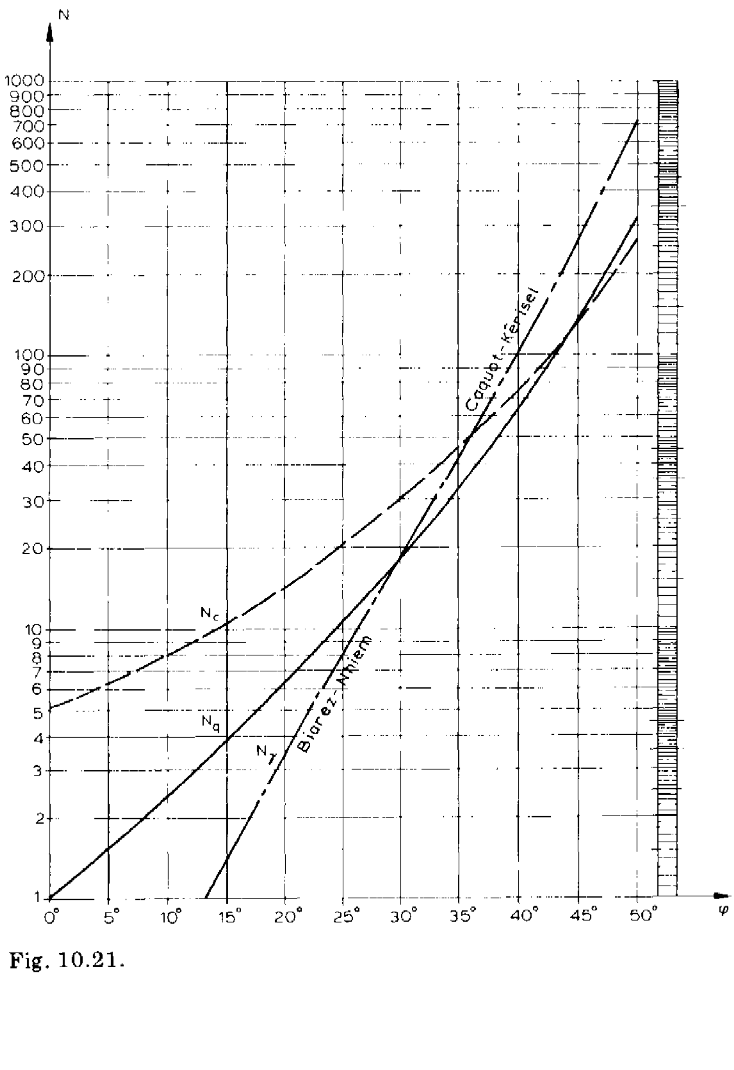


Fig. 10.21.

in particular, to varying values of the angle ψ which defines the wedge of rigid soil under the footing (see Fig. IX.5 and IX.7, Costet-Sanglerat). Table 10C gives values proposed by Terzaghi, which suppose that $\psi = \varphi$, by Caquot and Kerisel who have adopted $\psi = \pi/4 + \varphi/2$, and finally by Biarez and Nhiem who have sought to determine the minimum wedge of passive pressure. Most geotechnical engineers adopt the values of N_γ corresponding to those proposed by Caquot-Kerisel (compare Fig. 10.21).

★★*Problem 10.14* Evaluation of the bearing capacity and settlement of a shallow footing on a cohesive soil from results of a pressuremeter test

A rectangular shallow footing is 2 m by 4 m in plan dimensions and embedded at 1.50 m below grade. It bears on a layer of homogeneous clay of infinite thickness. The unit weight of the clay is $\gamma_h = 18\ kN/m^3$. The water level is sufficiently far below the bottom of the footing that it can be ignored (Fig. 10.22).

A standard-pressuremeter test was performed in the clay and yielded the following results: pressiometric modulus $= E_p = 8.7 \times 10^3$ kPa, limit pressure $= p_l = 7.9 \times 10^2$ kPa.

Assuming that the horizontal total pressure p_0, at rest at the level of the footing, is 10 kPa, calculate the bearing capacity of this footing and estimate its settlement. What should be the allowable bearing pressure that limits the settlement to 2 cm?

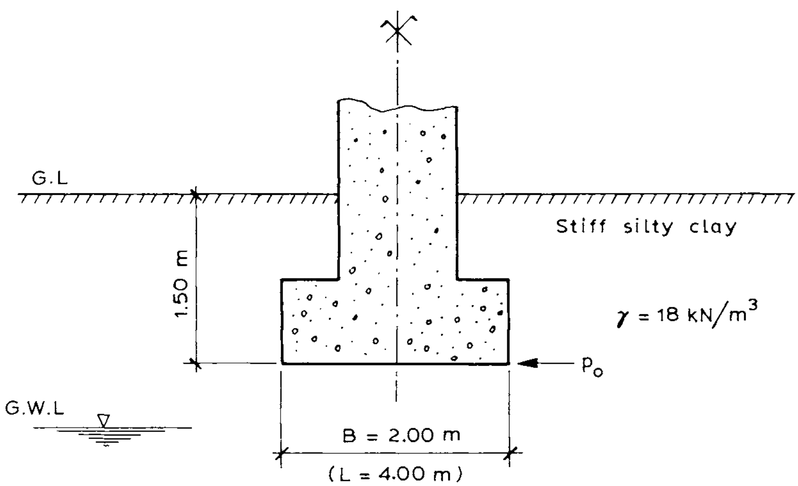


Fig. 10.22.

Solution

In the case of a homogeneous soil, the ultimate bearing capacity of a shallow footing is given by Ménard's formula, as a function of the limit pressure p_l:

$$q_d = q_0 + k(p_l - p_0)$$

where q_0 and p_0 are, respectively, the total vertical stress at the footing periphery after construction, and the total horizontal stress in the soil at rest at the time of the pressuremeter test.

The coefficient k is the bearing capacity factor, depending on the shape of the footing and the nature of the soil type.

The net limit pressure is defined as: $p_l^* = p_l - p_0$. Finally, for a safety factor $F = 3$, the allowable bearing stress would be: $q_{ad} = q_0 + (k/3)p_l^*$.

Referring to table II of sect. 12.2.3 in Costet-Sanglerat (Vol. 2) and Fig. 6.39 of the present book (Vol. 1), we find:

silty clay ($0 < p_l^* < 12$ bar), soil type I: $\dfrac{D}{B} = \dfrac{1.50}{2.00} = 0.75$ or:

$\dfrac{h_e}{R} = 1.50, \quad \dfrac{L}{2R} = \dfrac{4}{2} = 2,$ from which: $k = 1.28$.

In this instance, $q_0 = 18 \times 1.50 = 27$ kPa. The limit stress is then: $q_d = 27 + 1.28(7.9 - 0.1) \times 10^2 = 1025$ kPa and the allowable stress is:

$$q_{ad} = 27 + \frac{1.28}{3} \times 7.8 \times 10^2 = 360 \text{ kPa (3.6 bar)}$$

To evaluate the settlement of a shallow footing on a homogeneous soil, Ménard proposes the following formula:

$$s = \frac{1.33}{3E} pR_0 \left(\lambda_2 \frac{R}{R_0}\right)^{\alpha} + \frac{\alpha}{4.5E} p\lambda_3 R,$$

where:

p = average uniform stress due to the footing on the soil: $p = q_{ad} - q_0$;

R = half the width of the footing ($R > 30$ cm);

R_0 = reference width equal to 30 cm;

E = pressiometric modulus of the homogeneous soil;

α = a coefficient depending on soil type and its state of consolidation (see Table 6K in Vol. 1);

λ_2 and λ_3 = shape factors of the footing (see Table 6L in Vol. 1).

This formula is applicable for an embedded footing where the depth of embedment is at least 1 diameter ($h > 2R$). If this is not the case, the settlement s should be increased by 10% for $h = R$ and by 20% for $h = 0$.

Thus, we have: $E_p/p_l = (8.7 \times 10^3)/(7.9 \times 10^2) = 11$.

The clay is normally consolidated, and $\alpha = 2/3$, we have also $L/2R = 4/2 = 2$.

Then $\lambda_2 = 1.53$ and $\lambda_3 = 1.2$.

On the other hand: $p = q_{ad} - q_0 = 360 - 27 = 333$ kPa, then:

$$s = \frac{1.33 \times 3.33 \times 10^2}{3 \times 8.7 \times 10^3} \times 30 \times \left(1.53 \times \frac{100}{30}\right)^{2/3} +$$

$$+ \frac{0.667 \times 3.33 \times 10^2}{4.5 \times 8.7 \times 10^3} \times 1.2 \times 10^2$$

or: $1.51 + 0.68 = 2.19$ cm.*

But here, since $R < h < 2R$, we must increase this value by 5%, so we get:

$s = 2.19 \times 1.05 = 2.3$ cm.*

As for the question about the bearing capacity that limits the settlement to 2 cm, we have:

$q_{\text{allowed}} \leqslant q'_{ad} = q_{ad}(2/2.3) = 313$ kPa (= 3.1 bar).

Summary of answers:

$q_d = 1025$ kPa (10.25 bar), $q_{ad} = 360$ kPa (3.6 bar), $s = 2.3$ cm.

Settlement limits the magnitude of the bearing capacity to 313 kPa (3.1 bar).

★★*Problem 10.15* Evaluation of bearing capacity and settlement of a shallow footing on a cohesionless soil from results of a pressuremeter test

A square footing, 4 m by 4 m in plan dimensions, is located at a depth of 6 m in a layer of homogeneous sandy gravel of large thickness. The water table is at 0.50 m below the level of natural grade. The unit weight of the saturated gravel is 20.2 kN/m^3*. Above the water table, the moist soil unit weight is* $\gamma_h = 17.6$ *kN/m*^{3} *(Fig. 10.23).*

The coefficient of earth pressure at rest K_0 *is assumed to be 0.5. A pressuremeter test performed in the gravel yielded the following results: pressiometric modulus =* $E_p = 1.18 \times 10^4$ *kPa; limit pressure* $p_l = 1.25 \times 10^3$ *kPa.*

Calculate the bearing capacity of the footing and estimate its settlement. The settlement should not exceed 2.5 cm.

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*The 1st and 2nd term in the equation for s represent, respectively, the influence of the deviator, and the spherical components of the stress tensor.

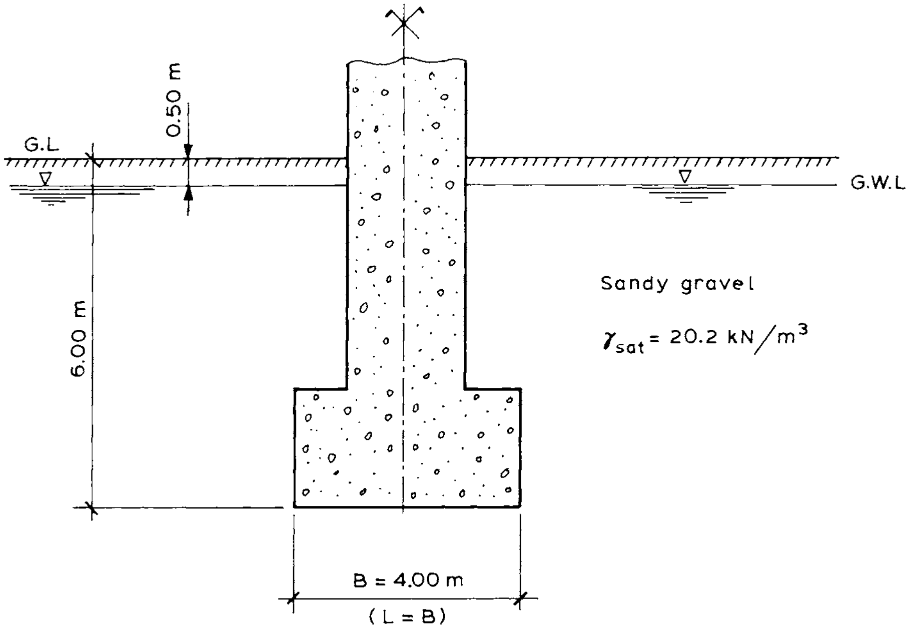

Fig. 10.23.

Solution

As indicated in problem 10.14, the ultimate bearing capacity of a footing on homogeneous soil is given by Ménard's formula:

$$q_{ad} = q_0 + (k/3)p_l^*$$

where p_l^* is the net limit pressure defined by $p_l^* = p_l - p_0$. In this case, we have:

$$q_0 = 0.50 \times 17.6 + 5.50 \times 20.2 = 119.9\,\text{kPa}, \text{ say } 120\,\text{kPa, or } 1.2\,\text{bar.}$$

— the effective vertical stress at the bottom level of the footing is:

$$\sigma_v' = q_0 - \gamma_w h_w = 120 - 5.50 \times 10 = 65\,\text{kPa.}$$

— the effective horizontal stress at that same level is:

$$\sigma_h' = K_0 \sigma_v' = 0.5 \times 65 = 32.5\,\text{kPa.}$$

and finally:

$$p_0 = \sigma_h' + \gamma_w h_w = 32.5 + 55 = 87.5\,\text{kPa, say } 88\,\text{kPa or } (0.88\,\text{bars}).$$

Graph 6.39 (see Volume 1) gives the value of k: $L/2R = 1$ (square footing), $h/R = 6/2 = 3$, from which: $k = 1{,}52$,

$$p_l^* = p_l - p_0 = (1.25 - 0.088) \times 10^3 = 1.162 \times 10^3 \text{ kPa},$$

$$q_d = 120 + 1.52 \times 1.162 \times 10^3 = 1.89 \times 10^3 \text{ kPa (or 18.9 bar)}.$$

$$q_{ad} = 120 + (1.52/3) \times 1.162 \times 10^3 = 0.709 \times 10^3,$$

$$\text{say } 0.71 \times 10^3 \text{ kPa (7.1 bar)}.$$

As indicated in Problem 10.14, in a homogeneous soil, the settlement may be estimated from:

$$s = \frac{1.33}{3E} pR_0 \left(\lambda_2 \frac{R}{R_0}\right)^{\alpha} + \frac{\alpha}{4.5E} p\lambda_3 R$$

This formula is applicable to footings embedded at least 1 diameter ($h > 2R$), which is the case here.

We have, on the other hand:

$$\frac{E_p}{p_l} = \frac{1.18 \times 10^4}{1.25 \times 10^3} = 9.44.$$

Table 6K gives $\alpha = 1/4$ for sands and gravels. For a square footing, Table 6L gives: $\lambda_2 = 1.12$ and $\lambda_3 = 1.1$.

Finally, $p = q_{ad} - q_0 = (0.709 - 0.120) \times 10^3$ kPa or $p = 0.589 \times 10^3$ kPa (5.89 bar).

Settlement is evaluated at:

$$s = \left[\frac{1.33 \times 5.89 \times 10^2}{3 \times 1.18 \times 10^4}\right] \times 30 \times \left(1.12 \times \frac{200}{30}\right)^{0.25}$$

$$+ \left[\frac{0.25 \times 5.89 \times 10^2}{4.5 \times 1.18 \times 10^4}\right] \times 1.1 \times 200$$

or $s = 1.10 + 0.61 = 1.71$ cm, which is less than 2.5 cm.

The magnitude of the stress is alright at $q_{ad} = 7.1 \times 10^2$ kPa (7.1 bar), corresponding to a column load of $7.1 \times 10^2 \times 4 \times 4 = 11\,344$ kN, say 1134 t.f.)

Summary of answers:

$q_d = 1.89 \times 10^3$ kPa (18.9 bar)

$q_{ad} = 7.1 \times 10^2$ kPa (7.1 bar)

Total column load: $F = 11\,344$ kN (1134 t.f.),
settlement $s = 1.71$ cm < 2.5 cm.

★★*Problem 10.16* Bearing capacity and settlement calculations of a mat foundation on a two-layer system from pressuremeter test results

A very long mat foundation of width B = 30 m is located at a depth of 0.80 m on a two-layer soil consisting of a layer of silt underlain by a silty sand which overlays a schist bedrock. The groundwater table is at 1.80 m. (see Fig. 10.24).

A soil exploration was made which included Ménard pressuremeter tests, the results of which are given on the diagram of Fig. 10.25.

1. Determine the bearing capacity of the mat.

2. Estimate its settlement assuming that the actual loading corresponds to the allowable load calculated in 1.

Solution

(1) The mat may be considered as a shallow footing of great width. The bearing capacity is then given by the formula:

$$q_d = q_0 + k(p_l - p_0) = q_0 + kp_l^*$$

Taking into account the small embedment of the mat, the vertical overburden pressure at the bottom of the foundation level may be ignored, i.e. $q_0 = 0$.

Therefore, for a safety factor of 3, we have: $q_{ad} = (k/3)p_l^*$.

In this instance, p_l^* in the geometric mean of the net values $(p_l - p_0)$ over the whole thickness of the compressible layers, because the layers are thin with respect to the width of the mat (see rule R_4 of the general notice in Ménard D.60).

We then have:

$$p_l = \sqrt[7]{2.0 \times 2.1 \times 2.0 \times 2.3 \times 3.8 \times 3.9 \times 4.5} = 2.78 \text{ bars} \quad \text{or} \quad 278 \text{ kPa}$$

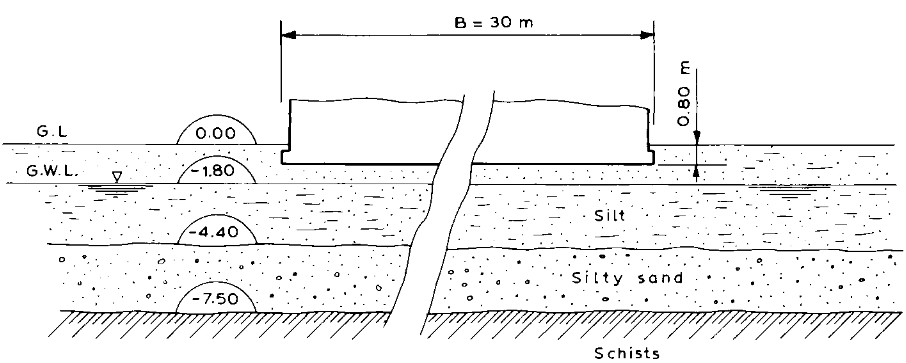

Fig. 10.24.

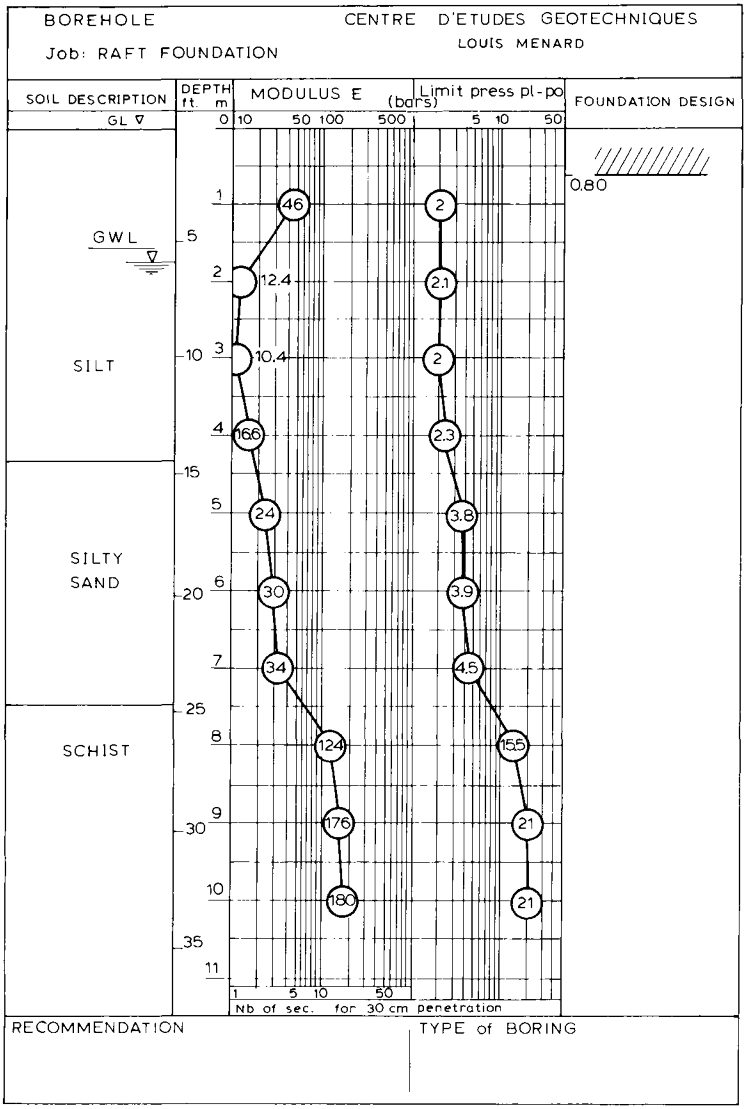

Fig. 10.25.

The equivalent embedment h_e is equal to the real embedment h, therefore: ($h_e = 0.80/15 = 0.05$) from which $k = 0.8$ (graph 6.39: notice that the length of the mat does not enter into the calculation).

Finally: $q_{ad} = (0.8 \times 278)/3 = 74$ kPa (or 0.74 bar).

(2) Settlements from consolidation of a two-layer system are computed

from the following formula (Ménard: rule 15, note D.60):

$$s = \int_0^h \frac{\alpha(z)\beta(F)p(z)}{E(z)}\, dz = \sum_{i=1}^{n} \frac{\alpha_i \beta_i p_i}{E_i} \Delta z_i$$

in which:
$p(z)$ is the vertical stress at depth z due to the structural load imposed on the soil;
$E(z)$ is the pressuremeter modulus at depth z;
$\alpha(z)$ is a coefficient related to the soil type and its state of consolidation for a layer located at an average depth z (see Table 6K);
$\beta(F)$ is a coefficient related to the safety factor F chosen. We take usually:

$$\beta(F) = \frac{2}{3}\frac{F}{F-1} \quad \text{for} \quad F < 3$$

$$\beta(F) = 1 \quad \text{for} \quad F \geqslant 3.$$

Both layers must be studied.

(a) In the *silt* layer z varies from 0.80 m to 4.40 m. Disregarding the test results at depth 1 m, the other tests give values of E/p_l^* lower than 14. Therefore, (see Table 6K) $\alpha = 1/2$ for this layer.

What would be the value of F if only the silt layer was present? We then would have:

$$F_1 = \frac{q_l}{q_{ad}} = \frac{kp_{le}}{q_{ad}} \quad \text{with} \quad k = 0.8.$$

So, for the silt layer, we find:

$$p_{le}^* = \sqrt[4]{2.0 \times 2.1 \times 2.0 \times 2.3} = 2.1 \text{ bar, or } 210 \text{ kPa}$$

and: $F_1 = \dfrac{0.8 \times 210}{74} = 2.27$

from which: $\beta(F) = 2/3 \times 2.27/(2.27-1) = 1.19$.

Finally, because the compressible layers are thin with respect to the width of the mat, we may assume that stress $p(z)$ over the whole depth of the soil layer, remains equal to the stress under the mat, that is to q_{ad}.

The settlement obtained for the silt layer, therefore, is:

$$s_1 = \sum_{i=1}^{n} \frac{\alpha\beta q_{ad}}{E_i} \Delta z_i = \alpha\beta q_{ad} \sum_{i=1}^{n} \frac{\Delta z_i}{E_i}$$

$$s_1 = \frac{1}{2} \times 1.19 \times 74 \left(\frac{1.50 - 0.80}{4600} + \frac{1.00}{1040} + \frac{1}{1.240} + \frac{4.40 - 3.50}{1660}\right)$$

$s_1 \;=\; 0.108\,\text{m}$, say 10.8 cm.

(b) In the *silty-sand* layer, z varies from 4.40 to 7.50 m and E/p_l ranges from 6 to 8.

Therefore, we take $\alpha = 1/3$ (see Table 6K). Then

$$p_{le}^* \;=\; \sqrt[3]{3.8 \times 3.9 \times 4.5} \;=\; 4.06\,\text{bar, or } 406\,\text{kPa}$$

$$F_2 \;=\; \frac{kp_{le}^*}{q_{ad}} \;=\; \frac{0.8 \times 406}{74} \;=\; 4.39 > 3 \quad \text{therefore,} \quad \beta(F) \;=\; 1$$

and we get:

$$s_2 \;=\; \frac{1}{3} \times 1 \times 74 \left[\frac{5.50 - 4.40}{2400} + \frac{1.00}{3000} + \frac{1.00}{3400}\right] \;=\; 0.027\,\text{m.}$$

The total settlement thus is: $s = s_1 + s_2 = 10.8 + 2.7 = 13.5$ cm.

Summary of answers

(1) $q_{ad} = 7.4$ kPa with safety factor of 3.
(2) Settlement $s = 13.5$ cm.

Problem 10.17 **Bearing capacity of shallow foundations from static penetrometer tests**

See problems 6.4, 6.8, 6.9, 6.10 and 6.11 in Volume 1.

Chapter 11

DEEP FOUNDATIONS

★★*Problem 11.1* Design of a driven pile in homogeneous sand from static penetrometer test data

Determine the allowable soil bearing under a driven pile of 1 m in diameter. The upper soil consists of soft clay and is underlain by a medium dense sand represented by the penetration diagram of the static test of Fig. 11.1. The test was performed with the Gouda-penetrometer and a Delft-cone.

Compare the allowable stresses under this pile, when it is driven to levels A and B.

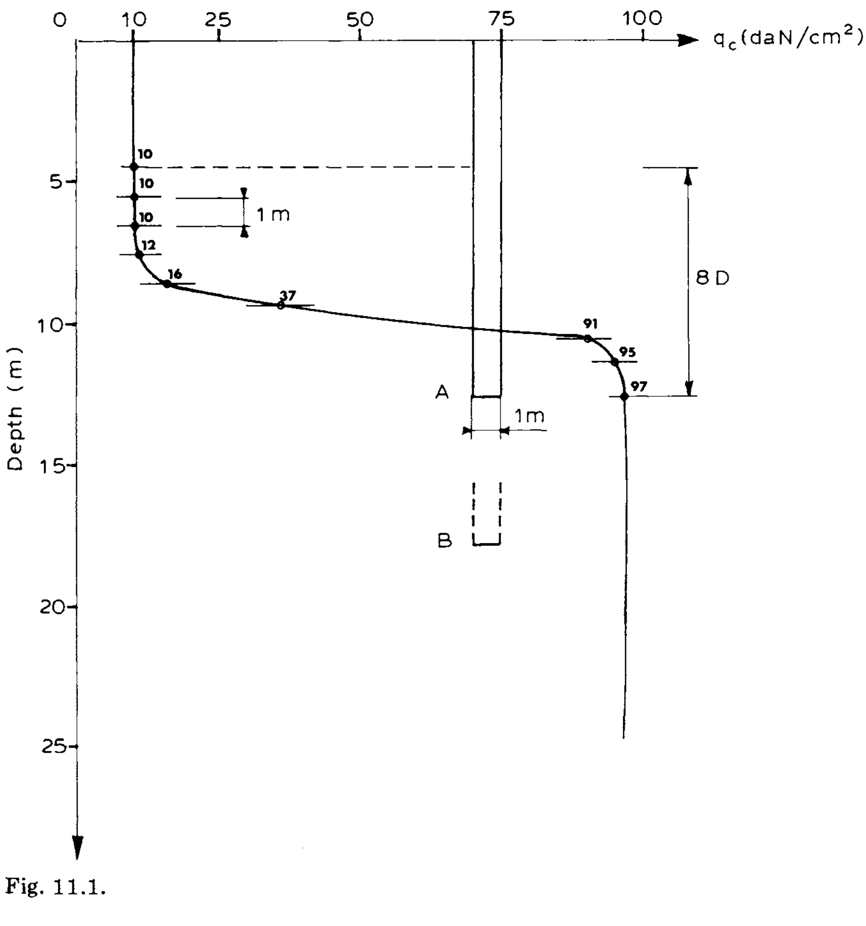

Fig. 11.1.

Solution

The Dutch method of analysis may be used which consists of determining the average point resistance on 8 pile diameters above the pile base, or q_{c_1}, and the average over 4 pile diameters below the pile base, or q_{c_2}.

The ultimate stress is: $q_d = (q_{c_1} + q_{c_2})/2$, and the allowable stress is:

$q_{ad} = q_d/2 = (q_{c_1} + q_{c_2})/4$, for a safety factor equal to 2.

Neglecting the lateral skin friction, we can determine the allowable stress under the pile tip in the following manner:

— *at level A:*

$$q_{c_1} = (10 + 10 + 10 + 12 + 16 + 37 + 91 + 95 + 97)/9 = 378/9 =$$
$$= 42\,\text{daN/cm}^2$$
$$q_{c_2} = 97\,\text{daN/cm}^2$$

from which: $q_{ad} = 139/4 \simeq 35\,\text{daN/cm}^2 = 3500\,\text{kPa}$.

— *at level B:*

$$q_{c_1} = 97\,\text{daN/cm}^2 = 9700\,\text{kPa}; \quad q_{c_2} = 100\,\text{daN/cm}^2 = 10\,000\,\text{kPa}$$
$$q_{ad} = q_{c_2}/2 = 50\,\text{daN/cm}^2 = 5000\,\text{kPa}.$$

Note the large difference of allowable stress depending on the depth to which the pile is driven into the homogeneous sand layer.

★★*Problem 11.2* **Design of a pile driven in a heterogeneous soil from static penetrometer test data**

Assume that the soil described in problem 11.1 contains, at a depth of 22 m, a loose layer as indicated on the graph of Fig. 11.2. Under this condition, determine what the allowable stress would be under a 1-m diameter pile driven to levels A and B, identical to those of problem 11.1.

Solution

If the pile is driven to level A, its bearing capacity is not influenced by the less compact soil layer at a depth of 22 m. Consequently, as found in problem 11.1, the allowable soil stress is 3500 kPa.

At level B, however, near the layer of lower resistance, the consequences of lowering the pile tip must be evaluated to a depth of 3.5—4 meters below the pile tip.

Following the recommendations of the Dutch, whenever a poor-quality soil layer is encountered at a depth of 4 diameters below the pile tip, and

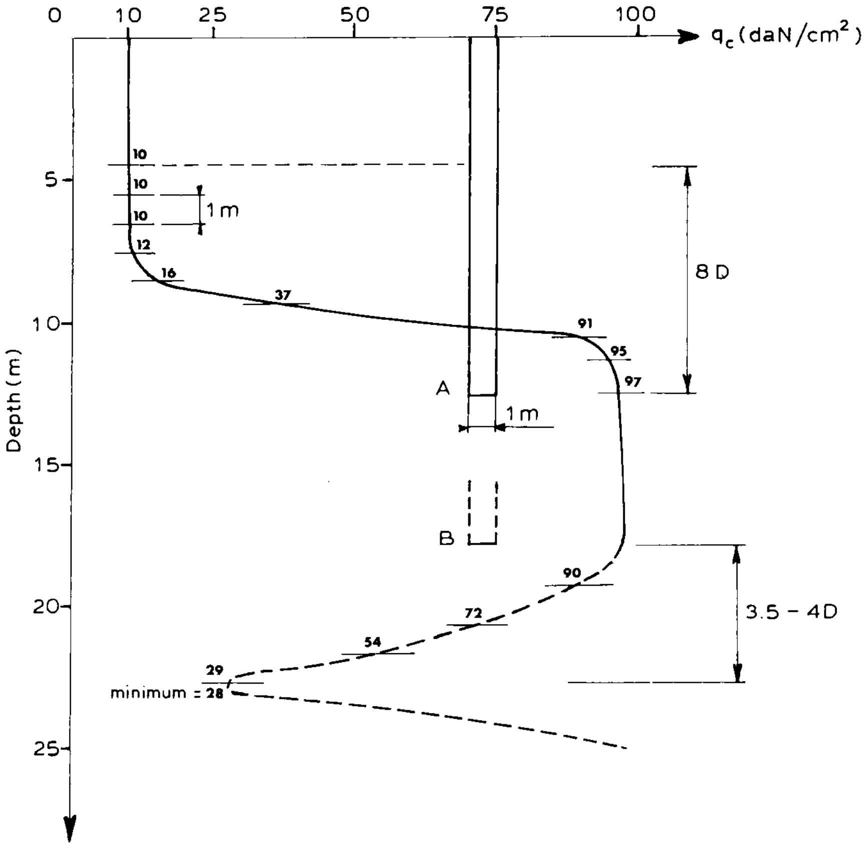

Fig. 11.2.

when over this depth n cone-penetrometer test readings were made, then q_{c_2} is:

$$q_{c_2} = (q_1 + q_2 + \ldots + q_n + nq_{\text{mini}})/2n$$

$$q_{c_1} = 97\,\text{daN/cm}^2 = 9700\,\text{kPa}$$

$$q_{c_2} = [90 + 72 + 54 + 29 + (4 \times 28)]/8 = 357/8 = 44.6\,\text{daN/cm}^2 =$$

$$= 4460\,\text{kPa}$$

$$q_d \simeq 141.6/2 = 70.8\,\text{daN/cm}^2 = 7080\,\text{kPa}$$

from which: $q_{ad} \simeq 35\,\text{daN/cm}^2 = 3500\,\text{kPa}$

It is apparent that for a condition as shown on Fig. 11.2, lowering the pile tip to level B does not increase the bearing capacity of the pile. From an economic view, there is no advantage in driving the pile below level A.

★★*Problem 11.3* **Design of drilled piles from static penetrometer test data (Andina penetrometer)**

At a site along the Mediterranean coast, two static-dynamic penetrometer tests were performed with the Andina device. Both tests yielded very similar results as typified by the graph of Fig. 11.3.

Determine the bearing capacity of drilled piles, 50 cm or 1 meter in diameter, driven 13 or 14 m, respectively, below grade.

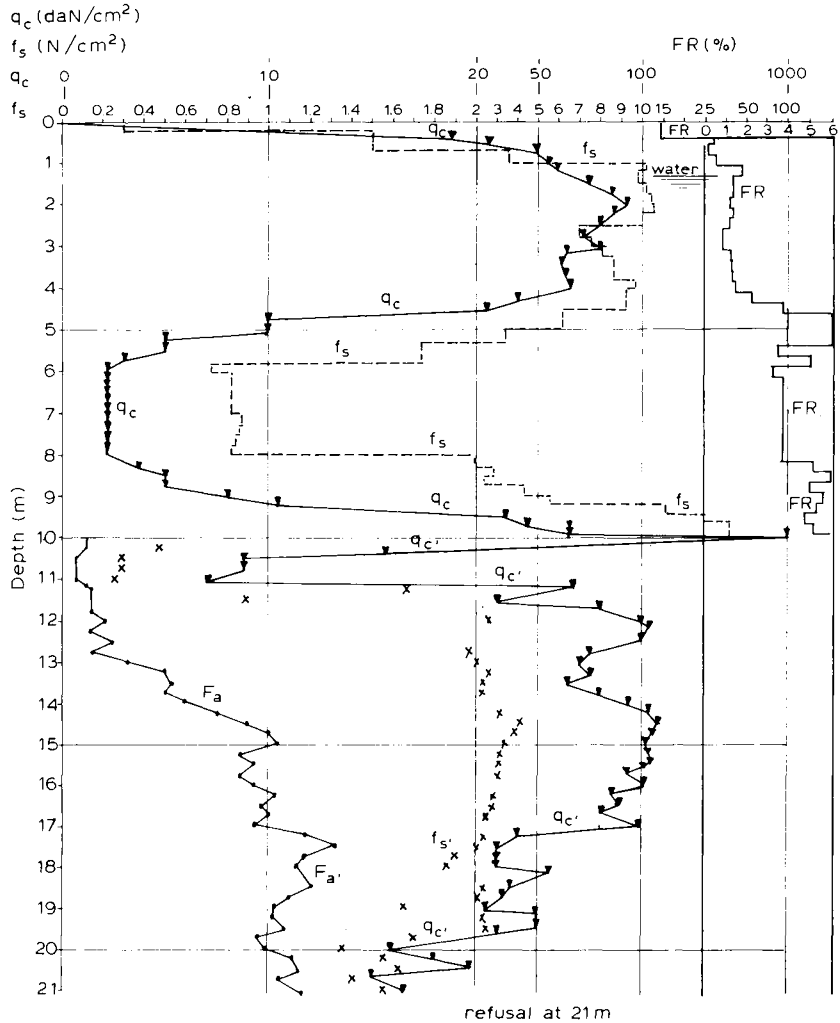

Fig. 11.3.

Solution

To determine the bearing capacity at the tip, the following formula is used (Ref. 22): $q_d = [q_{c_0} + (q_{c_1} + q_{c_2})/2]/2$ where:
q_{c_1} = minimum point resistance over a depth of 4 diameters above the base of the pile;
q_{c_2} = average over the same depth;
q_{c_0} = average along the critical embedment (in practice, over 8 pile diameters above the tip elevation).

(1) *50-cm diameter pile to 13 m*
We have:

$$q_{c_1} = 68 \text{ bars} = 6800 \text{ kPa}$$

$$q_{c_2} = (72 + 75 + 68 + 80 + 92 + 120 + 145 + 130 + 110)/9 = 892/9$$
$$\simeq 99 \text{ bars} = 9900 \text{ kPa}$$

$$q_{c_0} = (72 + 76 + 100 + 126 + 100 + 84 + 30 + 62 + 7 + 8.5 + 8.5 +$$
$$+ 14 + 100 + 60 + 45 + 35 + 10.5 + 8)/18 = 946.5/18 = 52.6 \text{ bars}$$
$$= 5260 \text{ kPa}$$

from which the allowable stress is:

$$q_{ad} = [52.6 + (68 + 99.1)/2]/4 = 34 \text{ bars} = 3400 \text{ kPa}.$$

However, in the above calculations, skin friction was not taken into consideration. This friction can be accounted for over a length L-$8D$-D, where L is the pile length, D is the pile diameter. ($8D$: above the tip, D: at the pile top).

Considering the value of R_f obtained from the Andina penetrometer test on the cone friction sleeve, we can assume for a first approximation that the useful friction may be 10 kPa (a slice-by-slice evaluation would yield about the same value).

This friction acts over an area $\pi D \times 8.50 = 13.35 \text{ m}^2$, i.e., a total force of 133.5 kN was applied to the net cross-section of the pile of 0.196 m^2, then: $\sigma_f = 133.5/0.196 = 680$ kPa. This means that the allowable stress under a 50-cm diameter pile, 13 m long, is: $q_{ad} = 3400 + 680 = 4100$ kPa.

Taking into account the fact that $q_{c'}$ of the diagram (Fig. 11.3) in the bearing layer is divided by 2, we will not apply for the drilled pile the usual 30 to 50% reduction of the allowable bearing ($q_{c'}$ is the point resistance measured with the small point of the Andina penetrometer).

(2) *50-cm diameter pile to 14 m depth*
For this pile we have:

q_{c_1} = 80 bars = 8000 kPa

q_{c_2} = (92 + 120 + 145 + 130 + 110 + 112 + 110 + 96 +102)/9 =

= 1017/9 = 113 bars = 11 300 kPa

q_{c_0} = (92 + 80 + 68 + 75 + 72 + 76 + 100 + 126 + 100 + 84 + 30 +

+ 62 + 7 + 8.5 + 8.5 + 14 + 100)/17 = 1103/17 = 65 bars =

= 6500 kPa

from which the allowable stress is:

q_{ad} = [65 + (80 + 113)/2]/4 = 40.4 bars = 4040 kPa.

The surface of the pile on which the lateral friction acts is: $\pi \times 0.5 \times 9 =$ 14.14 m^2. The corresponding load, applied to a pile section of 0.196 m^2 is: 141 kN, from which: $\sigma_f = 141/0.196 = 720$ kPa and $q_{ad} \simeq 4040 + 720 =$ 4760 kPa.

(3) *1-m diameter pile, 14 m long*

A computation similar to the one above, gives:

q_{c_1} = 30 bars = 3000 kPa

q_{c_2} = (92 + 120 + 145 + 130 + 110 + 112 + 110 + 96 + 102 + 86 +

+ 88 + 80 + 102 + 42 + 30 + 30 + 30)/17 = 1505/17 = 88.5 bars

= 8850 kPa

q_{c_0} = (92 + 80 + 66 + 75 + 72 + 76 + 100 + 126 + 100 + 84 +

+ 30 + 62 + 7 + 8.5 + 8.5 + 14 + 100 + 60 + 45 + 35 + 10.2 +

+ 8 + 5 + 5 + 4 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2)/33 =

= 1289.2/33 = 39.1 bars = 3910 kPa, from which

q_{ad} = [39.1 + (30 + 88.5)/2]/4 = 24.6 bars = 2460 kPa.

The lateral friction acts over an area of: $\pi \times 1 \times 5 = 15.70$ m^2. This corresponds to a load of $15.7 \times 10 = 157$ kN applied over a pile cross-section of 0.785 m^2. So the net stress due to skin friction is $\sigma_f = 157/0.785 = 200$ kPa and $q_{ad} = 2460 + 200 = 2660$ kPa.

(4) *1-m diameter pile, 14 m long*

As before:

q_{c_1} = 67 bars = 6700 kPa

$$q_{c_0} = (72 + 75 + 67 + 80 + 92 + 120 + 145 + 130 + 110 + 112 + 110 + 96 + 102 + 86 + 80 + 102 + 42)/17 = 1621/17 = 95.4 \text{ bars} = 9540\,\text{kPa}$$

$$q_{c_2} = (72 + 76 + 100 + 126 + 100 + 84 + 30 + 63 + 7 + 8.5 + 8.5 + 14 + 100 + 60 + 45 + 35 + 10.2 + 8 + 5 + 5 + 4 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 3.5 + 5 + 5)/33 = 992.7/33 = 30.1 \text{ bars} = 3010\,\text{kPa}$$

$$q_{ad} = [30.1 + (67 + 95.4)/2]/4 = 27.8 \text{ bars} = 2780\,\text{kPa}.$$

surface of lateral friction $= 4 \times 3.14 = 12.5\,\text{m}^2$, from which

$\sigma_f = 125/0.785 = 160\,\text{kPa}$. The total allowable stress is:

$q_{ad} = 2780 + 160 = 2940\,\text{kPa}$.

Conclusion

This problem shows, once again, that in a heterogeneous soil, one should never recommend an allowable stress beneath a pile without specifying also the pile diameter, since this stress is dependent upon the diameter. In this instance, notice that to account for the penetration diagram, it is logical that:

(1) There is no gain to increase the diameter from 0.5 to 1 m regardless of the length, 13 or 14 m.

(2) To lengthen the pile from 13 to 14 m would be of interest for the 0.5-m diameter pile but would be disadvantageous for the 1-m diameter pile.

This once more indicates the danger of having pre-conceived ideas on the length of embedment of piles in heterogeneous soils.

★*Problem 11.4* **Design of a pile driven into a three soil layer system from static penetrometer test data**

For this problem, refer to problem 6.12

★★*Problem 11.5* **Design of a driven pile on the basis of static formulae**

A preliminary study requires the determination of the bearing capacity of a driven pile, 32 cm in diameter and 9 m long, in a soil whose geotechnical profile is shown on Fig. 11.4.

The soils have the following mechanical and physical properties:

— soft silt: wet unit-weight, $\gamma = 17\,kN/m^3$*, buoyant unit-weight,* $\gamma' = 10\,kN/m^3$.

— loose clean sand: buoyant unit-weight, $\gamma' = 11\ kN/m^3$, angle of internal friction $\varphi = 30°$.
— medium to dense clean sand: buoyant unit-weight, $\gamma' = 11\ kN/m^3$, angle of internal friction $\varphi = 35°$.

The groundwater level is 2 m below the soft silt grade. Assume that the upper layer of soft silt will never be loaded and therefore will never create negative skin friction.

Is the reinforced concrete pile diameter adequate?

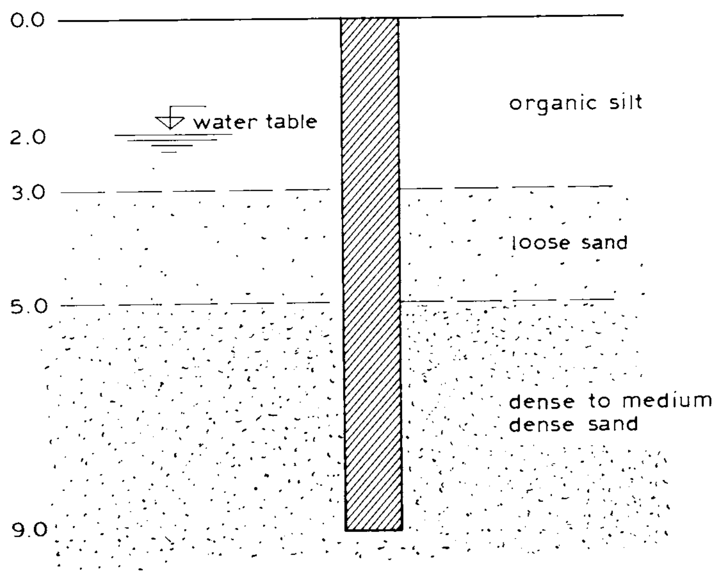

Fig. 11.4.

Solution

The soils being cohesionless, all computations can be based on effective stresses using drained parameters.

(1) Caquot-Kerisel method

The first order of work is to determine the critical embedment given by the following formula: $D_c = (B/4)N_q^{2/3}$
where B = diameter of the pile, $N_q = 10^{N \tan \varphi}$, and $2.7 \leqslant N \leqslant 3.7$, depending on pile diameter.

For a 32-cm pile, Caquot and Kerisel propose $N = 2.7$. Thus, for $\varphi = 35°$, $N_q = 77.7, D_c = 1.5$ m.

The value of N_q corresponding to the shallow footing condition is 33.3 for $\varphi = 35°$. This indicates that the recommendation of Yves Lacroix (of Woodward and Clyde, New York, personal communication) that:
N_q (pile) = $2N_q$ (shallow footings) is correct and on the safe side.

Ultimate point resistance of a pile

The ultimate stress at the tip is: $q_d = p_0 N_q$,
where p_0 = effective overburden stress at the level of the pile tip, or:
$p_0 = 17 \times 2 + 10 \times 1 + 11 \times 6 = 110\,\text{kPa}$.

The pile embedment into the medium dense to dense sand ($h = 4\,\text{m}$) is greater than the critical embedment $D_c = 1.5\,\text{m}$. So, we can consider the values of $N_q = 77.7$ and $q_{\text{ult}} = 110 \times 77.7 = 8550\,\text{kPa}$.

The ultimate load on the pile then is: $Q_p = 8550 \times 0.08 = 684\,\text{kN}$.*

Ultimate lateral friction along the pile shaft

After the method of Caquot-Kerisel, lateral friction is disregarded along the critical embedment into the bearing layer.

At depth z, the unit skin friction at failure (Costet-Sanglerat) is:
$f = \alpha \cdot \gamma \cdot z$ where $q = k_{p\gamma} \cdot \sin \delta$.**

Since the pile is driven, passive pressures in the sand may be assumed to have developed (this assumption is often contested and at times may be on the unsafe side).

Ultimate skin friction along the pile length in the loose sand

$$Q_{f1} = p \int_{3.00}^{5.00} \alpha \cdot \gamma \cdot z \cdot dz$$

where p = perimeter of the pile, here: 1 m.

To calculate Q_{f1}, we may consider the effective overburden pressure at mid-height of the layer of loose sand (at 4 m depth):

$$\sigma'_4 = 17 \times 2 + 10 \times 1 + 11 \times 1 = 55\,\text{kPa}$$

Therefore, for a loose sand layer of 2 m: $Q_{f1} = P \times \alpha \times \sigma'_4 \times 2$, for $\varphi = 30^\circ$ and $\delta = -2\varphi/3$, coefficient α is 1.9, then: $Q_{f1} = 1 \times 1.9 \times 55 \times 2 = 209\,\text{kN}$.

Ultimate skin friction along the pile in the dense sand layer

This friction will only be calculated for a length of 2.5 m because skin friction is not assumed to act over the depth equal to the critical embedment depth.

At a depth of 6.25 m, the effective overburden stress is:

$$\sigma'_{6.25} = 17 \times 2 + 10 \times 1 + 3.25 \times 11 = 79.75\,\text{kPa}$$

For $\varphi = 35^\circ$ and $\delta = -\,2\varphi/3$, coefficient α is 3.3.

Then: $Q_{f2} = 1 \times 3.3 \times 79.75 \times 2.5 = 658\,\text{kN}$.

---

*The method proposed by P. Foray and A. Puech (J. ITBTP, 339, May 1976) gives, assuming a relative density of 0.7 in the bearing layer and 0.4 in the loose sand layer, an ultimate bearing value of 675 kPa at the tip and 263 kN skin friction.

**The formula was derived for a homogeneous soil. It remains valid for stratified soils if $k_{p\gamma} = k_{pq}$, which is here the case.

Therefore, the total skin friction along the whole pile length is:

$$Q_f = Q_{f1} + Q_{f2} = 209 + 658 = 867\,\text{kN}.$$

Allowable load on pile

Using a safety factor of 3 to Q_f and Q_p:

$$Q_{ad} = Q_p/3 + Q_f/3 = 684/3 + 867/3 = 517\,\text{kN}, \quad Q_{ad} = 517\,\text{kN}.$$

Remark

If we assume $\varphi = 38^\circ$ instead of 35° for the medium dense to dense sand layer, we get:

$$N_q = 129, D_c = 2\,\text{m}, Q_p = 110 \times 129 \times 0.08 = 1135\,\text{kN},$$

$$Q_{f1} = 209\,\text{kN}, \quad Q_{f2} = 739\,\text{kN}, \quad Q_f = 209 + 739 = 948\,\text{kN}.$$

Allowable load on pile: $Q_{ad} = 1135/3 + 948/3 = 694$ kN.

If we change the value of φ by 3° of the bearing layer, the allowable load rises from 517 kN to 694 kN, so with an increase of some 34%.

Any error on the evaluation of φ may have a considerable influence on the allowable pile load.

*(2) Other method (see Fond document 1972)**

Ultimate stress at the pile tip: $q_d = \gamma_d N_q$

N_q is computed from graphs made for shallow footings. It is the minimum N_q referred to in the preceding question (Fig. 10.2 and Table 10C). For $\varphi = 35^\circ$, $N_q = 33.30$: $q_d = 110 \times 33.30 \simeq 3660$ kPa.

Ultimate load at the pile tip: $Q_p = 3660 \times 0.08 = 293$ kN.

Ultimate skin friction along the pile

At depth z, the unit skin friction at failure is:

$$\tau = K \tan \varphi_a \sigma'_z$$

where: K = coefficient depending on the method of installation of the pile and the compactness of the sand, φ_a = friction angle pile—soil; σ'_z = effective overburden stress at z, equal to $\Sigma\gamma' z$.

If we take Broms coefficient as proposed in the Fond 1972 document for driven piles, we get: $\varphi_a = \frac{3}{4}\varphi$, $K = 1$ for loose sand, $K = 2$ for compact sand.

Ultimate skin friction over the loose sand layer

$$Q_f = p \cdot K \cdot \tan 22^\circ 5 \int_{3.00}^{5.00} \sigma'_z \cdot dz$$

---

*Published by L.C.P.C.-SETRA (French Ministry of Equipment) in Paris.

To calculate Q_f, we consider the vertical overburden stress at mid-height of the loose sand layer, or 4 m depth:

$$Q_f = 1 \times 1 \times 0.41 \times 55 \times 2 = 45\,\text{kN}.$$

Ultimate skin friction over the medium to dense sand layer

$$Q_f = p \cdot K \cdot \tan 26^\circ \int_{5.00}^{9.00} \sigma_z' \cdot \mathrm{d}z.$$

To calculate Q_f, consider the effective vertical overburden stress at mid-height of this layer, or 7 m depth:

$$Q_f = 1 \times 2 \times 0.49 \times 88 \times 4 = 345\,\text{kN}$$

from which the ultimate skin friction over the pile length is:

$$Q_f = 45 + 345 = 390\,\text{kN}.$$

Allowable load on pile

With this method, a safety factor of 3 is used for the tip ultimate bearing, and of 2 for the ultimate skin friction:

$$Q_a = Q_p/3 + Q_f/2, \qquad Q_a = 293/3 + 390/2 = 293\,\text{kN}.$$

With the method of Caquot-Kerisel we obtained for this load: $Q_a = 517$ kN.

Note the considerable difference between the two methods.

We see that, when loaded, the compressive stress in the concrete calculated from Q_{ad} is acceptable, but it is too high during driving (calculation with Q_d), so the diameter of the concrete pile should be increased or a steel pipe pile should be used.

The first result is certainly too optimistic in the evaluation of skin friction, because it assumes that passive pressures are mobilized along the full length of the pile. In reality, passive pressure is a seldom reached maximum condition (that depends on the driving method and the sand compactness).

The second result is certainly too pessimistic for the evaluation of the tip resistance. It is underestimated by using the shallow footing formula.

Conclusion

As a general rule, avoid calculating the bearing capacity of driven piles from laboratory test results on soil samples, because the design theories are too uncertain and lead to greatly varying results depending on the method used.

In addition, for sandy soils, the recovery of an undisturbed sample is almost impossible, particularly below the water table. Moreover in the case of highly heterogeneous soils, even good samples may not be representative of the whole of the various layers affected by the pile. It is, therefore, very difficult to assign correct values of the angle of internal friction φ. This shows the interest in calculating the bearing capacity of piles from the results of in-situ tests and, in certain cases (driven piles in sands) from the driving formulas (Costet-Sanglerat, sect. 10.12).

★★*Problem 11.6* Allowable bearing capacity of a pile from pressuremeter test results.

Consider a pile of diameter $2R = 0.60\,m$, drilled to $8\,m$, as indicated on Fig. 11.5.

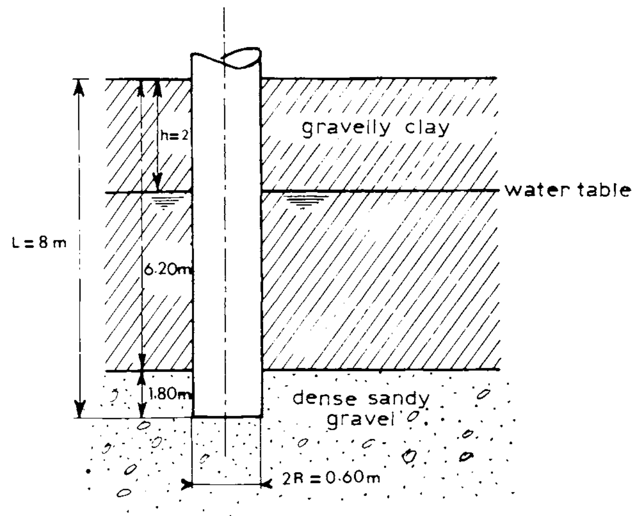

Fig. 11.5.

A pressuremeter testing programme yielded the following data:
— gravelly clay: average limit pressure = $3.5\,daN/cm^2$ (350 kPa)
— sandy gravel: average limit pressure = $12\,daN/cm^2$ (1200 kPa)
— level of water table: 2 m below grade.

Calculate the allowable soil bearing pressure under the pile, assuming that the saturated unit-weight of soils is $22\,kN/m^3$. Use the graphs of Ménard, reproduced on Figs. 11.6 and 11.7 as well as table II, sect. 12.2.3 of Costet-Sanglerat.

It should also be referred to the notes of Ménard published in 1967 on the use of the pressuremeter and to the Fond document 72 (LCPC SETRA) published by the French Ministry of Equipment and Housing.

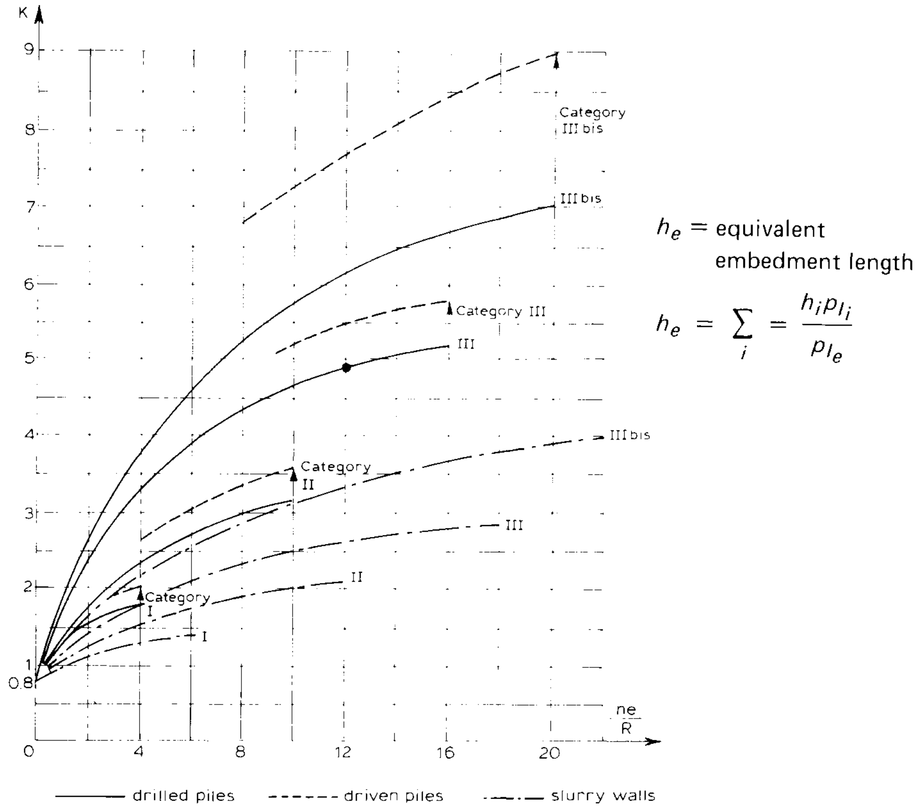

Fig. 11.6. Bearing factors k for deep foundations, (after Ménard).

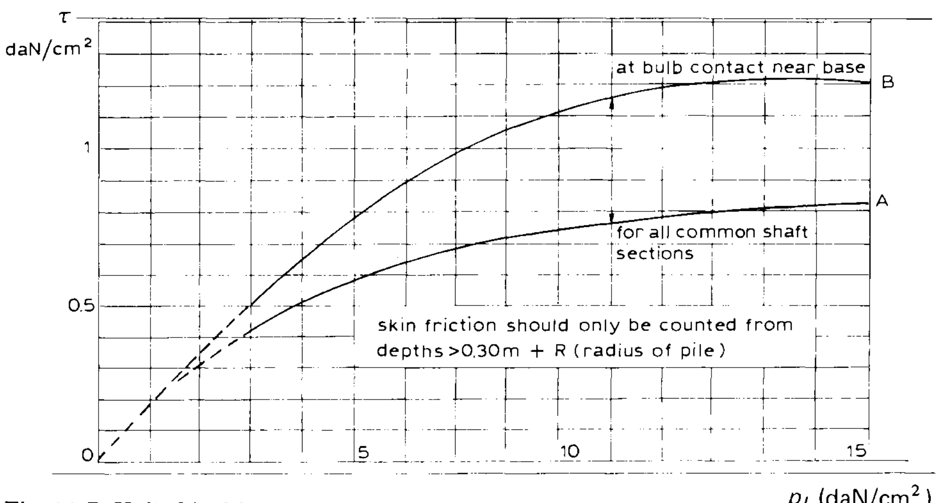

Fig. 11.7. Unit skin friction at failure (after Ménard).

Solution

The interpretation of classical pressuremeter tests recommended by Ménard leads us to consider 3 factors to estimate the allowable load of a deep foundation: tip resistance, current skin fraction and increased skin friction.

Tip resistance

This resistance is estimated from the same formula as used for shallow footings:

$q_d = q_0 + k(p_l - p_0)$ (cf. Costet-Sanglerat, 12.2.3)

A safety factor of 3 is used on the bearing capacity factor k which is calculated from the graph proposed by Ménard, of Fig. 11.6.

Current skin friction

Skin friction is considered over the length of the pile, less the lower portion over a length of 3 pile diameters and the upper portion: $(0.3\,\text{m} + R)$, where R is the pile radius.

It is estimated from the Ménard graph of Fig. 11.7, which gives the unit skin friction τ_c as a function of the limit pressure. It is used with a safety factor of 2.

Increased skin friction

Over the length of 3 radii near the pile tip, Ménard estimates that the loading of the pile causes the soil to tightening against the pile shaft and that, as a consequence, skin friction increases.

The unit skin friction τ_a is determined from the graph of Fig. 11.7 and it is used with a safety factor of 2.

From there, the allowable load on the pile is:

$$Q_{ad} = \pi R^2 \left[q_0 + \frac{k}{3}(p_l - p_0) \right] + [L - (3R + 0.3 + R)]\,\frac{2\pi R\tau_c}{2} + \\ + 3R\,\frac{2\pi R\tau_a}{2}$$

with: q_0 = total vertical stress at the pile tip; p_0 = total horizontal stress of the silt at rest at the test elevation (assumed performed at the pile tip).

$$q_0 = \gamma_{\text{sat}} \times L,\ p_0 = K_0[\gamma_{\text{sat}} \times 2.00 + \gamma'(L - 2.00)] + \gamma_w(L - 2.00)$$

because the clay is saturated by capillarity above the water table.

Now, assuming $K_0 = 0.5$:

$$q_0 = 8 \times 22 = 176\,\text{kPa},$$

$$p_0 = 0.5 \times 22 \times 2.00 + 10 \times 6.00 + 0.5 \times 12 \times 6.00 = 118\,\text{kPa},$$

$$p_l = 12\,\text{daN/cm}^2 = 12 \times 10^2\,\text{kPa} = 1200\,\text{kPa}.$$

The embedment length is given by (see Fond. 72): $h_e = \Sigma h_i p_{li}/p_{le}$ with in the case where $2R < 1$ m:
$pl_e = \sqrt[3]{p_{l1} p_{l2} p_{l3}} = \sqrt[3]{p^3_{l\,grave}} = p_{l\,grave} = 12\,\text{daN/cm}^2$ (1200 kPa) because the 3 levels of +1, 0 and −1, at the base, are in the gravel; therefore:

$$h_e = (1.80 \times 12 + 6.20 \times 3.5)/12 = 3.60 \quad \text{and} \quad h_e/R = 12$$

from which, with the aid of the graphs (soil category III, table II, sect. 12.2.3. of Costet-Sanglerat):

$$k = 4.9, \tau_c \neq 0.8\,\text{daN/cm}^3 = 80\,\text{kPa},$$

$$\tau_a \simeq 1.2\,\text{daN/cm}^2 = 120\,\text{kPa},$$

and finally: $Q_{ad} \simeq 549 + 491 + 102 = 1142\,\text{kN}$

This soil allowable pressure is also allowable for the concrete of the pile since the corresponding concrete stress would be $40\,\text{daN/cm}^2$, inferior to the value of $50\,\text{daN/cm}^2$ usually considered allowable.

★★*Problem 11.7* Design of a pile and pier drilled into a swelling clay

(1) Consider a pile with diameter $2r = 30$ cm and a total length $D = 4.50$ m, embedded over a length $d = 3$ m into a non-swelling soil and over a length D—d in an upper swelling soil layer (Fig. 11.8). Calculate the minimum stress p that the structural dead load must exert at the tip to avoid problems due to swelling. Assume that the swell pressure v of the upper soil is 1 daN/cm^2 and that the friction at uplift is 15% of the swell pressure.*

Assume also that, in the event that the upper soil layer does not swell, the properties of the non-swelling soil at depth are good enough to prevent a bearing failure.

(2) Determine the new value of p in the case where the swelling clay layer is 6 m thick, but the length of pile remains at 4.50 m.

(3) In Colorado (U.S.A), 39 houses were supported on 10-inch diameter shafts, embedded 7 ft. in swelling clay whose swell pressure was measured at 10 000 p.s.f. and loaded to 20 kips.

Severe uplift occurred. Explain why. N.B. 1 kip = 1000 pounds, 1 p.s.f. = 1 pound per square foot.

Solution

(1) The lower end of the shaft is in a non-swelling soil. The total uplift force V is: $V = 2\pi r(D{-}d) f v$ where $f = 0.15$.

Note the similarity between the ratio of f and v and the "friction ratio" of penetrometer tests (see Refs. 29 and 30); f here corresponds to $FR = 15\%$.

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*Swell pressure may be determined in the laboratory in the following manner: submerge a soil sample in a consolidation mold, measure the vertical stress which must be applied to maintain constant volume (see Problem 10.13).

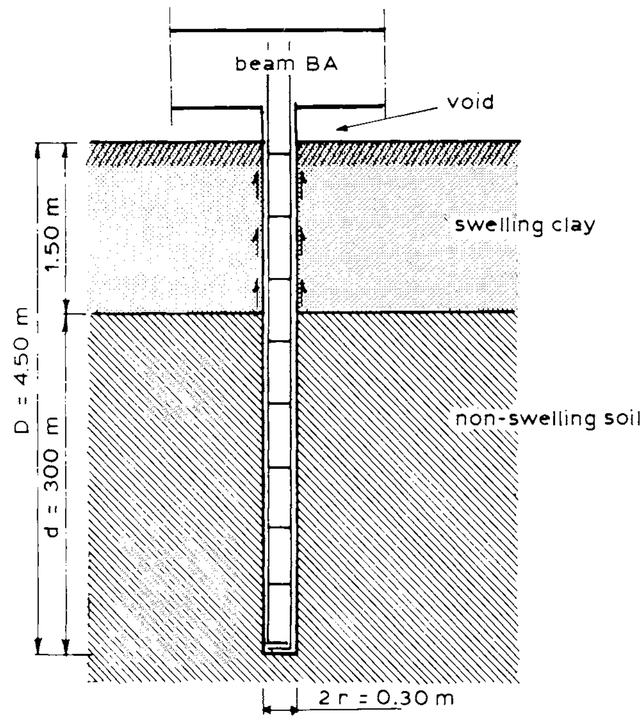


Fig. 11.8.

Let p be the stress at the pile tip due to the weight of the structure, if we neglect lateral friction over length d (non-swelling soil).
No uplift can occur when:

$\pi r^2 p \geqslant 2\pi r f v(D-d)$ or $p \geqslant [2(D-d)fv]/r$,

from which: $p \geqslant [2 \times (4.50 - 3.00) \times 0.15 \times 100]/0.15 = 300\,\text{kPa} =$
$= 3\,\text{daN/cm}^2$.

In practice, we can recommend $3\,\text{daN/cm}^2$ because lateral friction S over height d (which can be assumed to be half of the cohesion value of the non-swelling clay) provides a margin of safety. Indeed, if we account for friction, we get:

$$p \geqslant \frac{2(D-d)fv}{r} - \frac{2\pi r f_m d}{\pi r^2}$$

with f_m = shear stress at the pile—clay interface or:

$p \geqslant (2/r)[fv(D-d) - f_m d]$.

Let us find the value of f_m for which the term in the brackets would become zero:

$$f_m = \frac{fv(D-d)}{d} = \frac{0.15 \times 100 \times 1.50}{3.00} = 7.5\,\text{kPa}.$$

Taking $\beta = 0.5$ (see sect. 10.4.1, Costet-Sanglerat) and $\varphi' = 0$ for the underlying clay layer, we see that all we need is a cohesion of 15 kPa (0.15 daN/cm^2) to annul the uplift by embedment into the lower layer. The pier should, however, have tension steel.

(2) If the pile is embedded over its entire length into a swelling clay, the total uplift force V is obtained by adding to the uplift friction force acting on the lateral surface, the uplift force acting under the base, or:

$$V = 2\pi rDfv + \pi r^2 v.$$

To avoid uplift, we must have:

$$p \geqslant (2Dfv/r) + v$$

or: $p \geqslant (2 \times 4.50 \times 0.15 \times 100)/0.15 + 100$

or: $p \geqslant 1000\,\text{kPa}$* (10 daN/cm^2).

Note that for lightly loaded structures, it is rare that p be so high, which explains the frequent occurrence of problems in these types of structures in swelling clays. It should be noted that very often, the height of clay susceptible to an increase in moisture content, does not exceed 1.50 m. However, in the case of a water pipe leakage in swelling clays, the zone of soil affected may reach 4.50 m and even more, exceptionally 6 m. There is an advantage therefore to resort to belled piers, as discussed in problem 11.8.

(3) From the preceding question, the total uplift pressure is:

$$V = 2\pi rDfv + \pi r^2 v = \pi \phi Dfv + \pi(\phi^2/4)v.$$

Knowing that 1 ft = 12 inches and 1 kip = 1000 pounds ($\simeq$ 4.45 kN):

$$V = \pi \times (10/12) \times 7 \times 0.15 \times 10\,000 + (\pi/4) \times (10/12)^2 \times 10\,000 =$$
$$= 32\,942 \text{ pounds}$$

say: $V \simeq 33$ kips.

Since the load on each pier is 20 kips, uplift occurs. Problems should have been expected in the 39 houses.

---

*We will assume, as for the preceding question, that in the event that swelling does not occur, the soil properties are such that no bearing failure will occur.

★★★*Problem 11.8* **Enlarged base pile design in a swelling clay**

A pile has a diameter of $2r = 0.30\,m$ and goes through an upper soil layer consisting of a swelling clay, 2 m thick. The pile has an enlarged base whose diameter is $2R = 0.80\,m$ and is 0.2 m thick. It is embedded 1.2 m into the underlying soil layer which has no swell characteristics (Fig. 11.9). The permanent load on the pile is 83 kN.

The characteristics of the two soil layers are:

— upper layer (swelling clay): unit weight, $\gamma = 20\,kN/m^3$, swell pressure $v = 500\,kPa$.

— lower layer (non-swelling clay): unit-weight $= 20\,kN/m^3$, undrained cohesion, $c_u = 120\,kPa$, drained cohesion, $c' = 19\,kPa$, effective angle of friction, $\varphi' = 20°$.

Assume the soils are saturated by capillarity and that friction to uplift is 15 percent of the swell pressure:

(1) Check that diameter and depth of the embedment of the tip are sufficient to prevent uplift.

(2) Verify the foundation stability in the event that no uplift occurs. What can be concluded? Neglect the friction to lateral faces of the footing.

Solution

(1) Assume the downward direction positive (Fig. 11.10). Let us define P, the permanent load applied to the foundation (weight of pile and footing included), V, the vertical uplift force and P_0, the weight of the soil cylinder of diameter $2R$ above the footing.

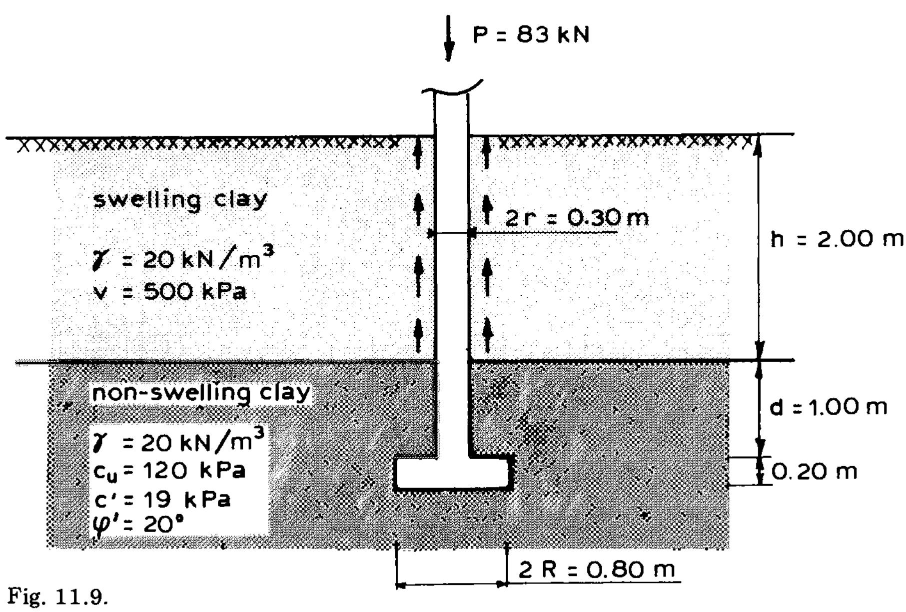


Fig. 11.9.

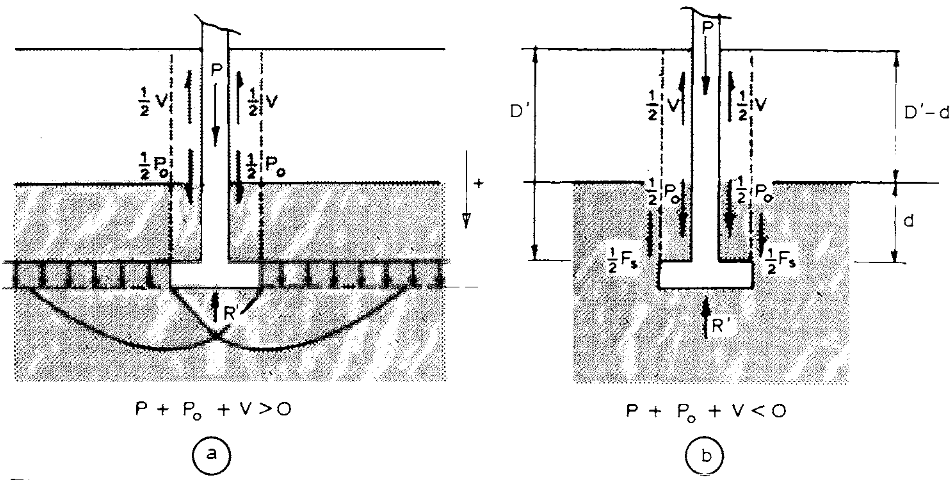


Fig. 11.10.

Two cases must be considered:

(a) First case (Fig. 11.10a): $P + P_0 + V > 0$. The footing behaves in the classical manner of shallow footings because $D/B = 3.20/0.80 = 4.0$. We can then, with adequate safety, apply the theory to this condition.

(b) Second case (Fig. 11.10b): $P + P_0 + V < 0$. In this case, the uplift force will have a tendency to push up the soil cylinder located above the footing.

Shear stresses will be developed along the lateral surface of this cylinder, with a resultant F_s opposing this uplift. Taking into account the swelling of the upper clay, which will cause a displacement upwards of each of its points, shear stresses cannot be developed in the swelling clay. The resultant F_s corresponds only to the shear stresses generated in the lower layer. Finally, we have: $P + P_0 + F_s + V + R' = 0$,
where R' = soil reaction under the footing, with $0 \leqslant R' \leqslant q_{ad}S$,
S is the surface area of the footing.

For the first question then, we must consider the mechanism of the second case.

Let τ_m be the average shear stress (long-term) computed at mid-height of the lower non-swelling layer:

$$\tau_m = c' + \sigma_0 \tan \varphi' = c' + \gamma(h + d/2) \tan \varphi'$$

The unit weight to consider, is γ and not γ' because the soil is saturated by capillarity. We than have:

$$F_s = 2\pi R\, d\tau_m ,$$

$$P_0 = \pi(R^2 - r^2)D'\gamma,$$

$$V = 2\pi r(D' - d)fv.$$

Replacing with the numerical values of the givens:

$$\tau_m = 19 + 20(2.00 + 1.00/2) \tan 20° = 37.2\,\text{kPa}$$

$$F_s = 2\pi \times 0.40 \times 1.00 \times 37.2 = +93.5\,\text{kN}$$

$$P_0 = \pi(\overline{0.40}^2 - \overline{0.15}^2) \times 3.00 \times 20 = +25.9\,\text{kN}$$

$$V = 2\pi \times 0.15 \times 2.00 \times 0.15 \times 500 = -141.4\,\text{kN}.$$

We will have: $P + P_0 + V < 0$ when $P < 141.4 - 25.9 = 115.5\,\text{kN}$. Since $P = 83\,\text{kN}$, we do indeed have this condition.

Taking a safety factor of 3 against shear strength, we get:

$$P + P_0 + 1/3F_s + V + R' = 0 \quad \text{or}$$

$$R' = 83 + 25.9 + 93.5/3 - 141.4 = -1.3\,\text{kN}.$$

The reaction R' of the soil on the footing is upward, the action of the footing on the soil is downward, therefore no uplift occurs and the footing size is acceptable.

The stress exerted on the soil in this case is:

$$q = \frac{R'}{S} = \frac{1.3 \times 4}{\pi \times \overline{0.80}^2} \simeq 2.6\,\text{kPa}.$$

(2) The calculation must consider the short-term (undrained) condition under the permanent and "living" load the longterm (drained) condition under dead load only. The behaviour of the footing is shown on Fig. 11.10a.

(a) Stresses under the footing

— under permanent dead load:

$$q_p = \frac{P + P_0}{S} = \frac{83 + 25.9}{\pi \times \overline{0.80}^2/4} = \frac{108.9}{0.503} = 216.5\,\text{kPa}$$

— under dead and "living" loads:

$$q_s = \frac{P + P_0 + P_s}{S} = 216.5 + \frac{P_s}{0.503}.$$

(b) Allowable stresses under the footing

— undrained condition, clay saturated ($\varphi = 0$):

$$q_{ad} = \gamma D + \frac{1.2\,c_u N_c}{3} = 20 \times 3.20 + \frac{1.2 \times 120 \times 5.14}{3} \simeq 310\,\text{kPa}$$

— drained condition (clay is still saturated by capillarity):

$$q_d = (0.8/2)\,\gamma B N_\gamma + \gamma D N_q + 1.2c' N_c$$

We have: $\varphi' = 20°$, from which $N_\gamma = 5$, $N_q = 6.4$, $N_c = 14.8$.

Therefore:

$$q_d = (0.5 \times 0.8 \times 20 \times 0.80 \times 5) + (20 \times 3.20 \times 6.4) + (1.2 \times 19 \times 14.8) \simeq$$
$$\simeq 779\,\text{kPa}$$

$\gamma D = 20 \times 3.20 = 64$ kPa from which

$q_{ad} = 64 + (779 - 64)/3 \simeq 302$ kPa.

(c) Conclusion

Longterm condition: $q_{ad} = 302$ kPa > 216.5,

short-term condition: $310 \geqslant 216.5 + P_s/0.503$, from which:

$P_s \leqslant (310 - 216.5) \times 0.503$ or $P_s \leqslant 47$ kN.

The "living" load should not exceed 47 kN, which is about 57% of the dead load. We are, therefore, far above the 20% usually admissible.

Remarks

(a) The greatest advantage of piles or piers with an enlarged base is that the resistance against uplift does not run the risk of being affected by the loss of friction, which could occur for any one of many reasons along the shaft of the pile in the zone of height d.

On the other hand, load P_0 is only slightly affected by variations in the water content.

Piers with an enlarged base are particularly well adapted for high swelling clays ($v > 5$ daN/cm^2) when the substratum is rocky and not too deep, and if a water table could exist.

(b) A very efficient solution to prevent uplift due to swelling clays is to provide a layer of vermiculite or glass wool along the shaft, between the pile and soil, some 3 to 5 cm thick (Fig. 11.11). The pile should have tension steel to prevent shaft rupture and a void of about 10 cm must be designed under the beams to allow the soil to swell.

(c) Clays with a plasticity index of over 30 should be suspected of having swell potential.

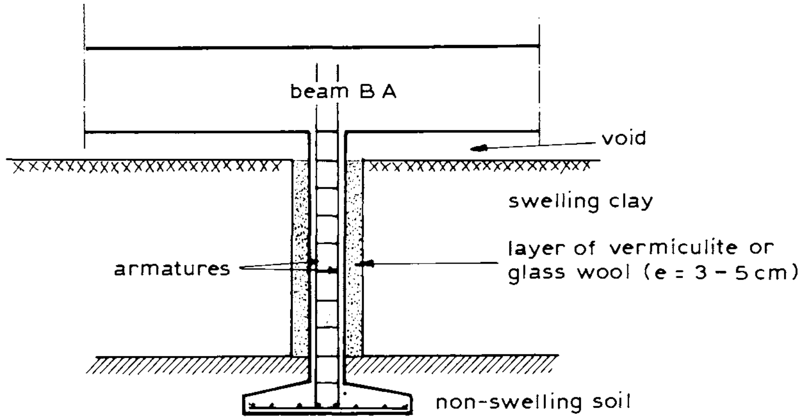


Fig. 11.11.

★*Problem 11.9* Calculation of the bearing capacity of a pile from the S.P.T. (Standard Penetration Test)

This problem is reported in 6.18.

★★*Problem 11.10* Bearing capacity of a driven pile from static-penetration dynamic-penetration tests and S.P.T. Comparison with in situ pile load test

During the second European Symposium on Penetration Tests (E.S.O.P.T. II) held in Amsterdam in May, 1982, the problem of determining the bearing capacity

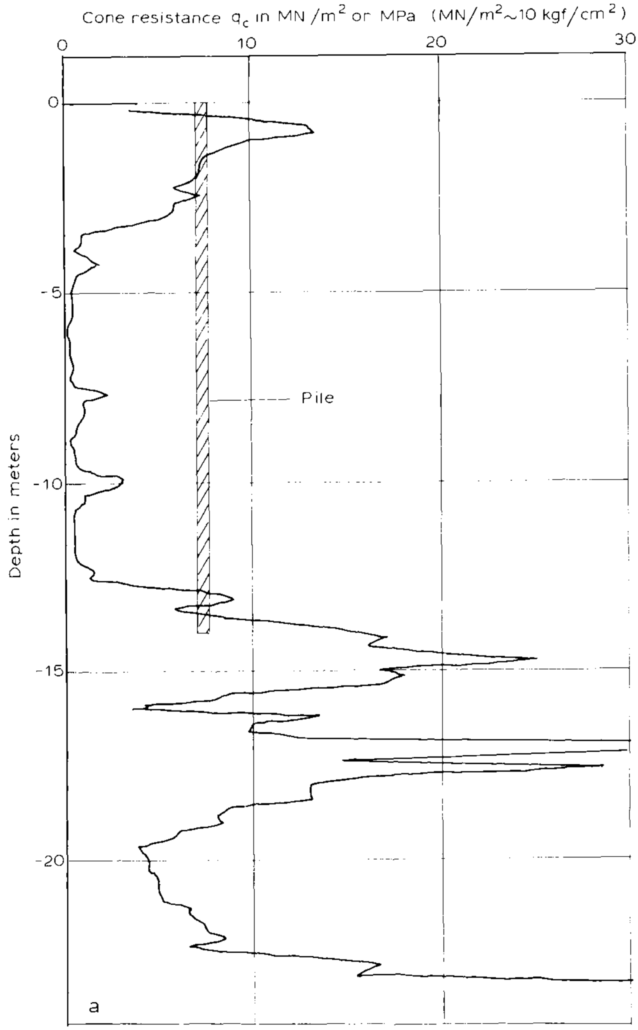


Fig. 11.12a. Static cone resistance.

of a pile from in situ static and dynamic penetration test and S.P.T. tests was presented. The data came from a site in Amsterdam where a pile had been driven and on which, in the presence of some of the symposium attendees, a cyclic loading was applied until failure occurred. The problem is presented here because of its instructional interest.

Determine the ultimate and allowable bearing capacities of a square pile

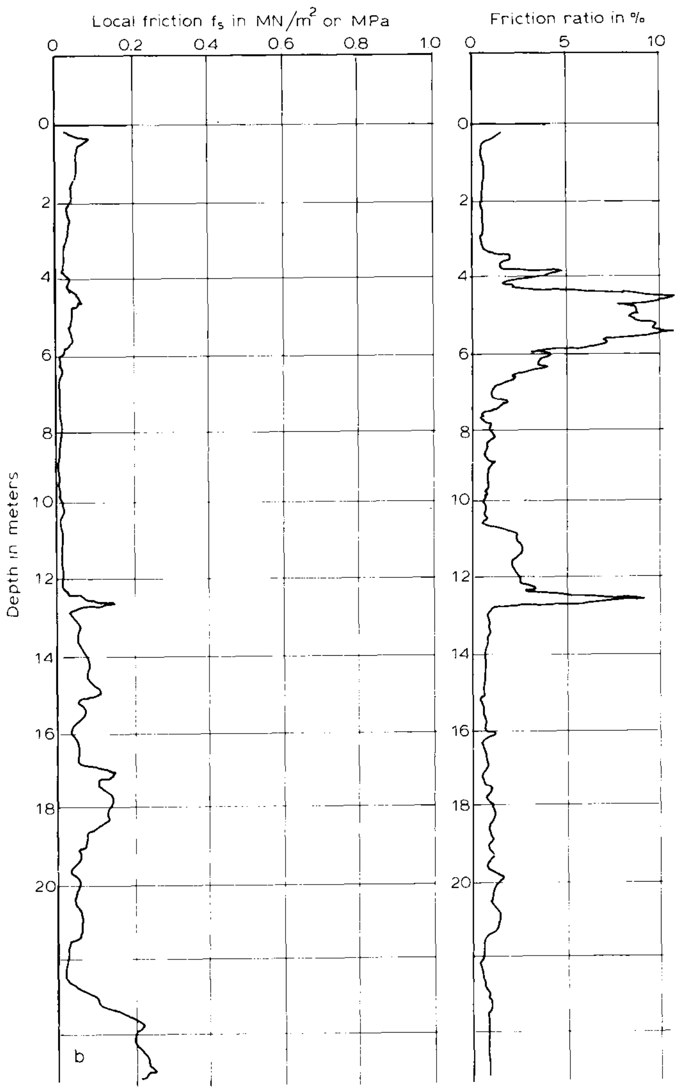


Fig. 11.12b. Local friction cone resistance and friction ratio.

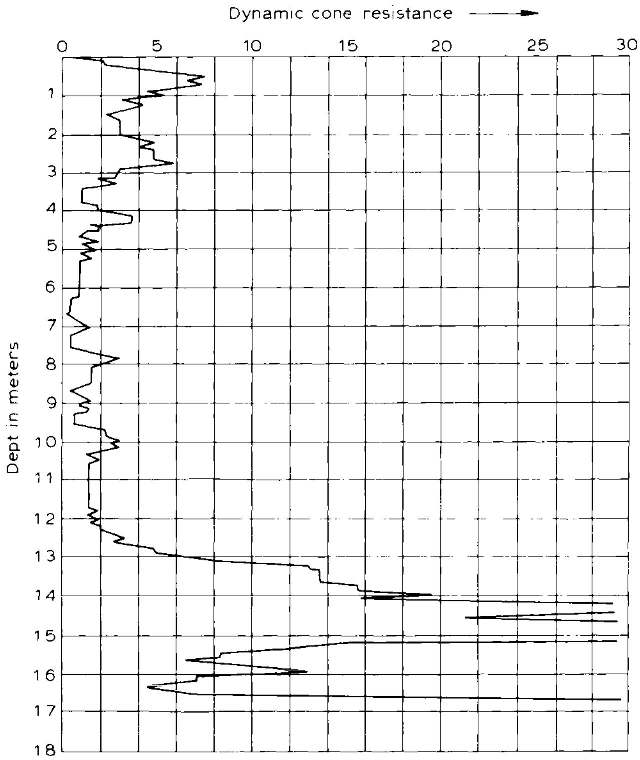


Fig. 11.13. Dynamic penetration.

of 25 cm on the side. The pre-fabricated pile was driven to El. − 14 m in a soil whose cross-section is given in Table 11A.

The evaluation may be made by different methods, based respectively on:

— static penetration diagram obtained with a simple electric cone (10 cm² section) with friction sleeve, in accordance with international standards. Cone resistance, lateral unit friction and friction ratio are indicated on the diagrams of Fig. 11.12a and 11.12b.

— dynamic penetration of Fig. 11.13, where penetration was obtained in accordance with international standards DPA, using drilling mud. (Rod diameters, 40 mm; point diameter, 61.8 mm; 64 kg hammer; drop height of 73 cm, dynamic resistance computed in accordance with the conventional Dutch formulae). Ref. [13], [22] and [23].

— S.P.T. tests, presented on Fig. 11.14.

Compare the results obtained with the pile loading record, shown on Fig. 11.15.

ESOPT II CASE STUDY I
Pile Prediction
Site : AMSTERDAM
Result of boring

TABLE : 11 - A

depth m.	boring result	samplenr (SPT)	Soil description	sand fraction µm	d_{50}
1					
NAP		1	sand	90 - 420	210
		2	sand	90 - 350	210
−1		3	sand	90 - 420	250
−2		4	peat – clayey		
		5	peat – clayey		
−3		6	peat		
−4		7	peat		
−5		8	clay – some peat		
−6		9	sandy clay		
		10	clayey sand		
−7		11	clayey sand		
−8		12	clayey sand		
−9		13	sand – some clay		
		14	clayey sand (some organic pieces)		
−10		15	organic clay		
−11		16	clay peat		
−12		17	sand – small pieces of clay and peat	60 - 210	175
		18	sand – small pieces of clay and peat	60 - 210	175
−13		19	sand – slightly silty – some pieces of clay	105 - 250	210
−14		20	sand – slightly silty – some pieces of clay	75 - 350	210
−15		21	sand – slightly silty – some pieces of clay	90 - 710	500
−16		22	sand – slightly clayey and silty		
		23	sand – slightly clayey and silty		
−17		24	sand and some clay		
		25	clayey sand		
−18					

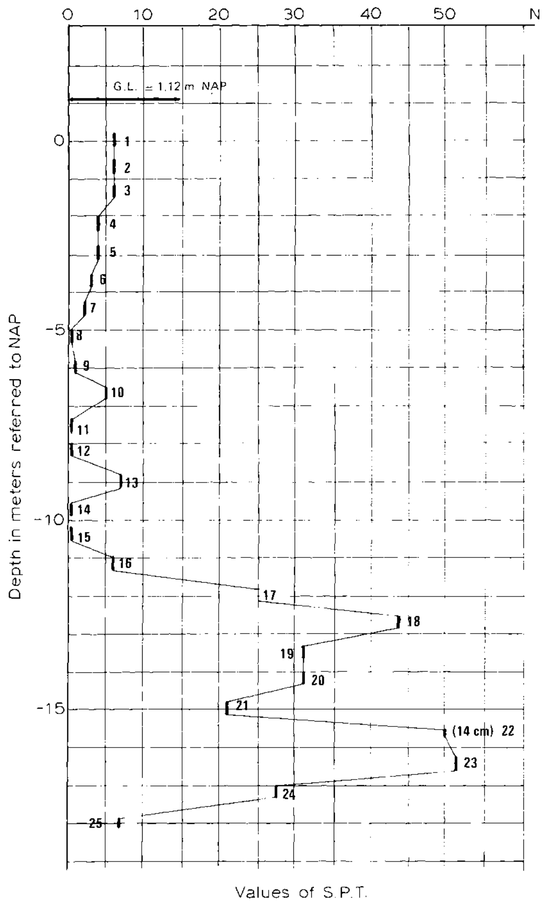


Fig. 11.14.

Solution

(a) Cone penetrometer (C.P.T.)

Since the cone resistance decreases greatly 1 m below the driven pile tip, the bearing capacity must be computed, for safety's sake, in accordance with the method indicated in problem 6.12:

$$q_{ul} = (q_{c_1} + q_{c_2})/2.$$

$$q_{c_2} = (q_1 + q_2 + q_n + n\, q_{mini})/2n,$$

with $4D = 1$ m under the pile tip we get:

$$q_{c_2} = [16.4 + 21 + 25 + 17 + 17 + 10 + (6 \times 10)]/12 = 166.4/12 =$$
$$= 13.8\ \mathrm{MN/m^2}$$

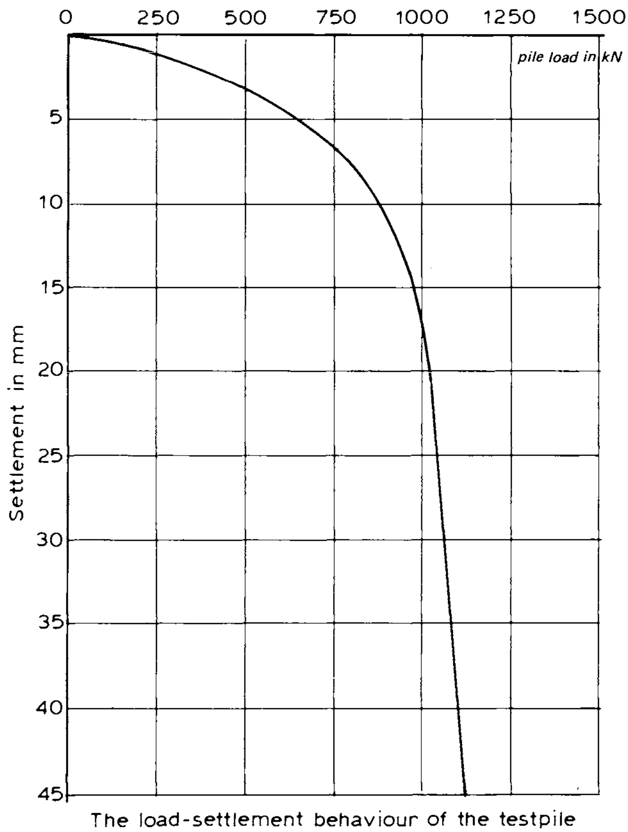


Fig. 11.15.

and for $8D = 2$ m above the pile tip:

$$q_{c_1} = (16.4 + 17 + 12 + 8 + 6 + 9 + 5 + 1.5 + 1.5)/9 = 8.5\,\mathrm{MN/m^2}$$

The ultimate pressure at the pile tip would thus be:

$$q_{ul} = (8.5 + 13.8)/2 = 11.15\,\mathrm{MN/m^2}$$

and the ultimate point bearing capacity $Q_p = 0.25 \times 0.25 \times 11\,150 \simeq 700$ kN.

From Fig. 11.12b, the average lateral unit friction may be evaluated as follows:

from 0 to 3 m, $f_s = 0.04\,\mathrm{MN/m^2}$

3 to 6 m, $f_s = 0.06\,\mathrm{MN/m^2}$

6 to 12 m, $f_s = 0.01\,\mathrm{MN/m^2}$

12 to 14 m, $f_s = 0.07\,\mathrm{MN/m^2}$

If we let p be the perimeter of the pile and h the height of the layer, the ultimate capacity due to the friction alone is:

$$Q_S = \Sigma ph\, f_s = 4 \times 0.25\, \Sigma\, hf_s$$
$$= 1[3 \times 40 + 3 \times 60 + 6 \times 10 + 2 \times 60] = 490\,\text{kN}$$

and the ultimate total load is $Q_{ul} = 700 + 480 = 1180\,\text{kN}$.

This must be compared to the ultimate pile load test of 1100 kN (see Fig. 11.15). The allowable load may then be computed as follows: $Q = 700/2 + 480/3 = 510\,\text{kN}$. The overall safety factor is $1100/510 = 2.15$.

(b) Dynamic penetrometer

From the dynamic penetration test data (Fig. 11.13) the ultimate pile tip resistance may be computed as for the static cone data, by:

$$q_{d_1} = (20 + 15 + 14 + 14 + 10 + 5 + 3 + 2 + 2)/9 = 9.4\,\text{MN/m}^2$$
$$q_{d_2} = (20 + 21 + 20 + 20 + 14 + 8 + 6 \times 8)/12 = 12.5\,\text{MN/m}^2$$
$$q_{ul} = (9.4 + 12.5)/2 = 10.95\,\text{MN/m}^2$$

from which $Q_p = 684\,\text{kN}$.

This value is very close to that from the static test, because the dynamic test was performed with mud in the hole, in accordance with the DPA standards (International Standards). Ref. [8].

On the other hand, the dynamic test yields no useful data for the value of the lateral friction that thus must be roughly estimated. For a first approximation, the formula $f_s = 0.01 q_d$ may be used:

$f_s = 0.01\, q_d$, from which

for 0—3 m $f = 0.04\,\text{MN/m}^2$

for 3—6 m $f = 0.015\,\text{MN/m}^2$

for 6—12 m $f = 0.01\,\text{MN/m}^2$

for 12—13 m $f = 0.04\,\text{MN/m}^2$

for 13—14 m $f = 0.10\,\text{MN/m}^2$

and, finally:

$$Q_s = 4 \times 0.25\,(3 \times 40 + 3 \times 15 + 6 \times 10 + 1 \times 1 + 1 \times 10)$$
$$= 120 + 45 + 60 + 11 = 236\,\text{kN}$$

from which $Q_{ul} = 920\,\text{kN}$.

If we were to take into account the soil types as evidenced by the S.P.T. (see Table 11A) we could possibly, for the cohesive soil layers, accept a friction ratio higher than 1%. This would increase the value of Q_s.

(c) S.P.T.

The bearing capacity of the pile from the S.P.T.-data may be computed as follows:

According to L. Decourt in his paper "Prediction of the bearing capacity of piles based exclusively on N values of the S.P.T." (ESOPT II, Amsterdam, 1982) the ultimate bearing capacity (Q_u) of a pile is given by: $Q_u = Q_p + Q_s$ were Q_p is the ultimate point bearing capacity and Q_s is the ultimate load capacity due to friction along the shaft.

c.1. Point bearing capacity

To estimate the point bearing capacity an average of the three N values around the pile tip is taken. In the present case:

$$\bar{N}_p = (25 + 44 + 31)/3 = 33.33.$$

The ultimate point stress is given by:

$$q_p = \bar{N}_p K,$$

where K is the soil coefficient taken from the table below for K-values:

Soil type	$K(\mathrm{t/m^2})$	
clays	12	
clayey silts*	20	
sandy silts*	25	
sands	40	(After Decourt [9].)

*residual soils

Then: $q_p = 33.33 \times 40 = 1.333\, tf/\mathrm{m}^2$, and the point bearing capacity will be:

$$Q_p = q_p A_p = 1.333 \times 0.0625 = 83.3\, tf = 817.1\,\mathrm{kN}$$

c.2. Shaft friction capacity

There is no need to take into account the soil type (clay, silt, sand, etc.) met along the shaft. It is enough to consider the average $\bar{N}$ value along the shaft. But the N-values taken for the estimation of point bearing capacities must not be considered for the estimation of shaft friction. N values smaller than 3 shall be considered as equal to 3 and N values greater than 50 shall be made equal to 50. In the present case we have assumed that between zero and 1.12 m of depth there was one N value equal to 6. Thus we have:

$$\Sigma N = 74$$

$$\bar{N} = 74/17 = 4.35$$

The friction along the shaft is given by:

$$q_s = (\bar{N}/3) + 1\ (tf/\mathrm{m}^2),$$

$$q_s = (4.35)/3 + 1 = 2.45\, tf/\mathrm{m}^2 = 24.03\,\mathrm{kN/m^2}, \text{ then:}$$

$$Q_s = A_s q_s = 4 \times 0.25 \times 14.12 \times 2.45 = 34.6\,tf = 339.3\,\mathrm{kN}$$

The ultimate pile bearing capacity will then be:

$$Q_u = Q_p + Q_s = 83.3 + 34.6 = 117.9\,tf = 1156.5\,\mathrm{kN}$$

This prediction, made by L. Decourt before the symposium, has been found to be the best for the bearing capacity calculations of the tested pile.

Summary of answers:

Ultimate pile bearing capacity: C.P.T. = 1180 kN.
Dynamic penetration: 920 kN; S.P.T. = 1156 kN.
Loading test = 1150 kN;

★★*Problem 11.11* **Determination of bearing capacity and settlement estimates of semi-deep foundations based on pressuremeter tests**

A drilled pier of 1.2 m in diameter and 3 m deep is excavated in a very thick dense silt, as shown on Fig. 11.16. The groundwater table is at 2 m below the bottom of the pier.

A geotechnical investigation made with the standard pressuremeter gave results as summarized on the diagram of Fig. 11.17. Assume that at the bottom of the pier p_0 = 25 kPa.

1. Determine the allowable soil bearing pressure below the pier and the net bearing capacity at the top of the pier. 2. Evaluate the settlement of the pier.

How are the results affected if we assume that the water table could rise above the bottom of the pier?

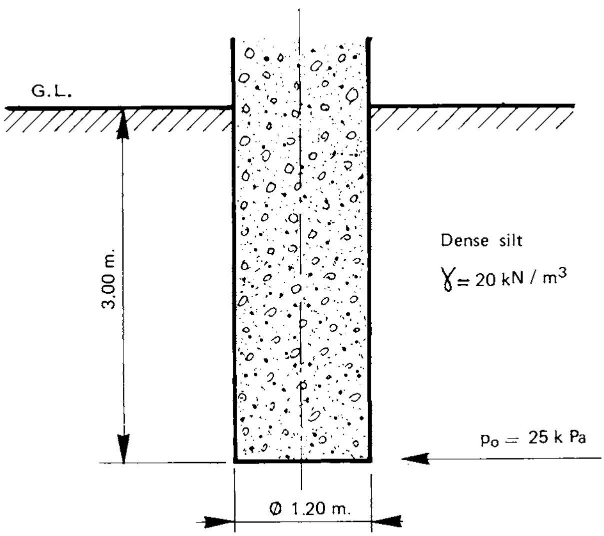


Fig. 11.16.

Solution

1. The equivalent limit pressure is calculated from the values of the limit pressure obtained between depths $-3R$ and $+3R$ about the bottom of the pier, that is from -1.20 to -4.80 m since $R = 0.60$ m.

$p_{le} = \sqrt[3]{1300 \times 1700 \times 2400} = 1690$ kPa, or 16.9 bars.

(We verify in this case that the differences between the values of p_l do not exceed 30%, as required by Ménard.)

For the calculation of the equivalent embedment depth, h_e, we should take, over the initial 50 cm of depth, half of the value of the limit pressure measured at 1 m, that is: 750 kPa (7.5 bars).

We then have: $h_e = \dfrac{\Sigma p_{li} hi}{p_{le}}$

$$h_e = \frac{0.5 \times 750 + 1 \times 1500 + 1 \times 1300 + 0.5 \times 1700}{1690} = 2.38\,\text{m}.$$

and: $h_e/R = 2.38/0.6 = 3.97 > 3$

The bearing capacity factor k is obtained from the graph of Fig. 11.6. The soil corresponds to category II, a dense silt with $p_l > 1200$ kPa (see Table II in sect. 12.2.3 of Costet-Sanglerat, Vol. II).

For a drilled pier then: $k \simeq 2.3$, and finally:

$q_{ad} = k(p_{le} - p_0)/3$ or:

$q_{ad} = 2.3 \times (1690 - 25)/3 = 1277$ kPa (12.8 bars).

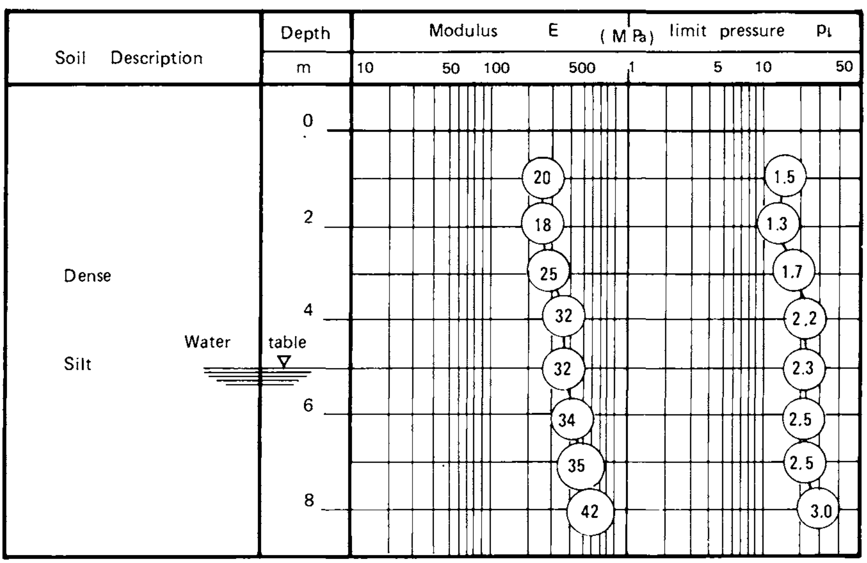


Fig. 11.17.

Remarks

The skin friction was neglected, contrary to what would be done for a pile, because we are dealing with a semi-deep foundation of small embedment. For deeper embedments, skin friction could be considered.

On the other hand, the bearing coefficient is computed from a graph which applies to deep foundations. That for shallow footing is limited to a value of $h_e/R = 3$.

The bearing capacity therefore is:

$$Q_{ad} = \pi\phi^2/4 \times 1277 \simeq 1444\ \text{kN}.$$

However, at the top of the pier, the weight of the concrete should be subtracted with γ concrete $\simeq 24\ \text{kN/m}^3$:

$$Q_{ad} = 1444 - (\pi\phi^2/4) \times D \times \gamma_b = 1444 - (\pi/4) \times 1.2^2 \times 3 \times 24 =$$

$$= \simeq 1363\ \text{kN, or about } 136\ tf.$$

2. For an isolated, semi-deep foundation of radius $R < 1$ m and for which $h_e/\text{R} < 5$, Ménard proposed to use the following formula (pressuremeter rule 4):

$$s = c_q \frac{q'}{2E}\, 30\left(\lambda_d \frac{R}{30}\right)^{\alpha} \quad \text{(for } 30\ \text{cm} < R < 100\ \text{cm)}$$

where q' = stress on top of the pier, E = pressuremeter modulus of the soil, α = soil structure coefficient (see Table 6K), λ_d = shape coefficient of the foundation ($\lambda_d = 1$ for a circular section and $\lambda_d = 1.13$ for a square section), c_q = embedment coefficient given by:

$$c_q = \frac{1}{0.8 + 0.1\,(h_e/R)}$$

For our case then:

$q' = 1277 - 3 \times 25 = 1205$ kPa,

$E = 250$ bars $= 25\,000$ kPa $\qquad \alpha = 2/3$

$E/p_l = 250/17 > 14$ (overconsolidated dense silt) (see Table 6K),

$\lambda_d = 1$ (circular section),

from which

$c_q = 1/(0.8 + 0.1 \times 3.97) = 0.835.$

From which: $s = 0.835\,(1205/2 \times 25\,000) \times 30\,(1 \times 60/30)^{2/3} = 0.96$ cm.

The settlement will be of the order of 1 cm. If the groundwater table rises above the bottom of the footing, Ménard indicates that for a dry silt soil with $E/p_l \simeq 20$, the modulus should be reduced by 20 to 40% (the factor increases with the value of E/p_l).

Let's assume here a reduction factor of 20% ($E/p_l \simeq 14.7$), then consequently $s \simeq 1.2$ cm. This is still within reasonable limits and we can conclude that the pier design is acceptable.

Summary of answers:

$q_{ad} = 1277$ kPa (12.8 bar)

$\overline{Q_{ad}} = 1363$ kN (136 *tf*)

$s \simeq 1$ cm (if the water table rises: $s \simeq 1.2$ cm).

Chapter 12

SLOPES AND DAMS

★★*Problem 12.1* Failure of a vertical cut

Consider a vertical cut of height H, in a clayey soil whose undrained cohesion is c_u and saturated unit-weight is γ_{sat}.

Evaluate the different possible modes of failure.

(1) Study the hypothesis of failure occurring on a circular arc centered at mid-height.

(2) Study the hypothesis of a failure occurring on a circular arc at the top of the cut.

(3) Study the hypothesis of a plane failure occurring through the toe of the cut.

(4) Compare the above possibilities to the results obtained in Problem 5.7.

Solution

All the following computations are based on short-term conditions; they therefore, are taking into account c_u for cohesion and an angle of internal friction $\varphi = 0$. Assume the soil to be saturated.

(1) Circular arc failure centered at mid-height

The center of gravity of the semi-circle AMB (Fig. 12.1) is located at point G in such a way that:

$$CG = \frac{4}{3} r \frac{\sin^3(\alpha/2)}{\alpha - \sin\alpha}$$

where $\alpha = \pi$: the angle at the center of sector AMB. Hence $CG = (4/3\pi)r$.

The driving moment due to the weight of a 1 m long slice whose thickness is limited by the semi-circle AMB will be:

$$M_m = \gamma_{sat}\pi \frac{r^2}{2}\frac{2H}{3\pi} = \gamma_{sat}\frac{H^3}{12}.$$

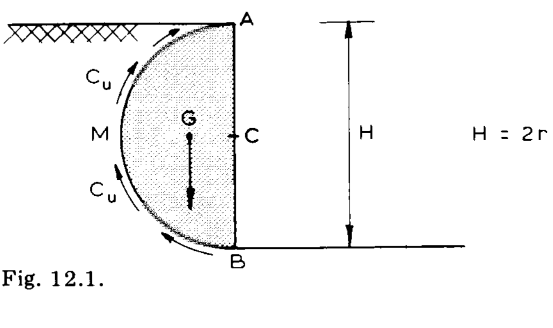


Fig. 12.1.

The resisting moment due to the soil cohesion along the assumed failure surface is: $M_r = \pi c_u r^2 = \pi c_u (H^2/4)$. The safety factor in this case is equal to: $F = M_r/M_m = 3\pi c_u H^2/\gamma_{sat} H^3 = 3\pi c_u/\gamma_{sat} H$.

(2) Circular arc failure centered at the top of the cut (Fig. 12.2)

The center of gravity of the portion of the circle (cross-hatched) *DMB* is located at point G_2 so that:

$$AG_2 = \frac{4}{3} r \frac{\sin^3(\alpha/2)}{\alpha - \sin\alpha}$$

$\alpha = \widehat{DAB} = \pi/2$, from which $AG_2 = 4/3r(0.353/0.570) = 0.826r$.

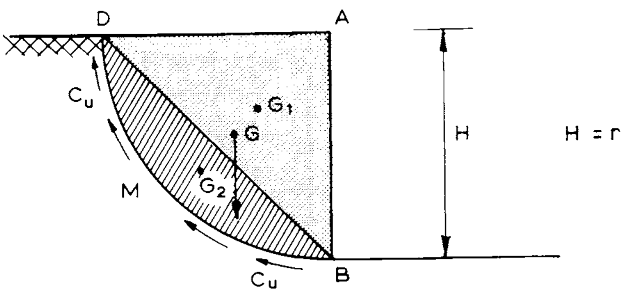


Fig. 12.2.

The center of gravity of triangle *ADB* is located at G_1, so that:

$$AG_1 = \frac{2}{3} r \frac{\sqrt{2}}{2} = r\frac{\sqrt{2}}{3}$$

The center of gravity of the total section *ADMB* will then be located at point *G* so that:

$$AG\frac{\pi r^2}{4} = AG_1 \frac{r^2}{2} + AG_2\left(\pi\frac{r^2}{4} - \frac{r^2}{2}\right)$$

$$AG = \frac{4}{\pi}\left[r\frac{\sqrt{2}}{6} + 0.826\left(\frac{\pi}{4} - 0.5\right)r\right], \qquad \text{then } AG = 0.6r.$$

The driving moment due to the weight of section *ADMB* is equal to:

$$M_m = \gamma_{sat}\frac{\pi r^2}{4} AG \sin\frac{\pi}{4} = 0.33\gamma_{sat} r^3.$$

The resisting moment due to cohesion along the failure surface is equal to: $M_r = c_u \pi r^2/2 = 1.57 c_u r^2$. The safety factor will then be equal to:

$$F = \frac{M_r}{M_m} = \frac{1.57}{0.33}\frac{c_u}{\gamma_{sat} r} = 4.75\frac{c_u}{\gamma_{sat} H}.$$

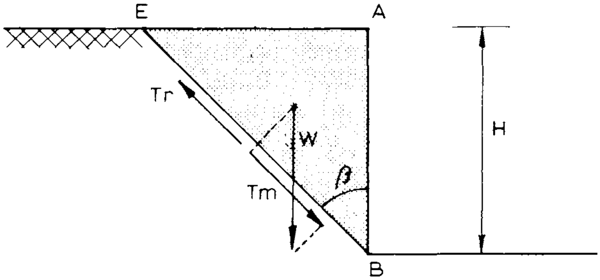


Fig. 12.3.

(3) Plane failure

Consider the plane failure surface through B, at the toe of the cut. The weight of triangle AEB (Fig. 12.3) is equal to: $W = \gamma_{sat}(H^2/2)\tan\beta$.

The driving force projected on the sliding plane EB is:

$$T_m = W\cos\beta = \gamma_{sat}\frac{H^2}{2}\tan\beta\cos\beta = \gamma_{sat}\frac{H^2}{2}\sin\beta.$$

The resisting force in the sliding plane is equal to: $T_r = (H/\cos\beta)c_u$. The safety factor will then be:

$$F = \frac{T_r}{T_m} = \frac{2Hc_u}{\gamma_{sat}H^2\cos\beta\sin\beta} = \frac{4c_u}{\gamma_{sat}H\sin(2\beta)}.$$

The F coefficient will be minimum when $\sin(2\beta)$ is maximum, or when $\beta = \pi/4$. In that case, the safety factor is: $F = 4c_u/\gamma_{sat}H$.

(4) Conclusion

The first observation made on the three calculations of the safety factor is that the term $c_u/\gamma_{sat}H$ appears. This explains why certain graphs for slope stability analysis are based on this term.

The most unfavourable circle of failure of the two cases considered is the circle centered at the top in A of the cut. It can be observed that the expression for the safety factor in that case is slightly higher than that obtained for the plane failure hypothesis. However, in reality the circle centered at the top of the cut is certainly not the most unfavourable one: a more complex method of calculation can prove that the most critical circle is centered above the top of the cut.

In Problem 5.7, we found that the maximum cut height is $H = 4c_u/\gamma_{sat}$ which corresponds to a safety factor of 1 for the case of a plane failure. The two methods of calculation thus are seen to lead to the same answer.

Remark

In the case of an excavation, one should consider also the problem of uplift of the excavation bottom.

★★★*Problem 12.2* Plane failure

A natural slope has an inclination $\theta = 15°$ with the horizontal. Borings have indicated the presence of a clayey silt underlain by a fractured limestone layer (Fig. 12.4).

The groundwater table is parallel to the ground surface and 2 m deep.

(1) Assuming a plane failure, show that the most likely failure plane would occur at a depth of 8 m.

(2) An excavation of slope 1/1 must be made through the silt to the limestone for the construction of a road. Calculate the safety factor against sliding assuming a plane failure.

The clayey silt properties are:

— unit weights: above the water table: $\gamma_h = 18\,kN/m^3$, below the water table: $\gamma_{sat} = 20\,kN/m^3$;

— cohesion and angle of internal friction (effective values from drained triaxial test): $c' = 20\,kPa$, $\varphi' = 15°$.

(3) In the event that the safety factor is not high enough, recommend a solution to increase its value.

Solution

(1) Stability calculation must be made in long-term conditions using the drained soil parameters c' and φ'. Consider point P at depth z (Fig. 12.5). Let σ be the vertical total stress acting on a face oriented parallel to the free

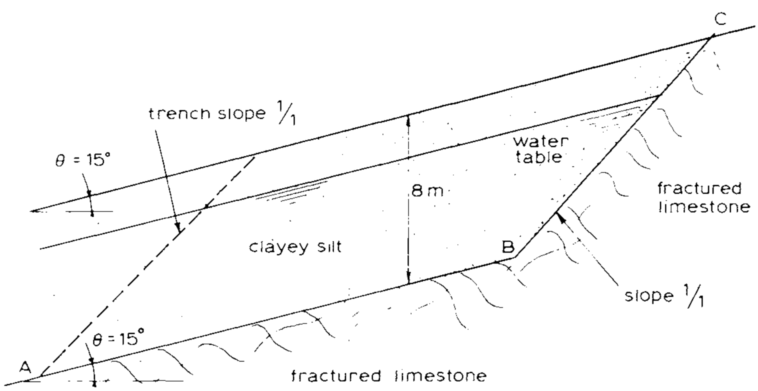


Fig. 12.4.

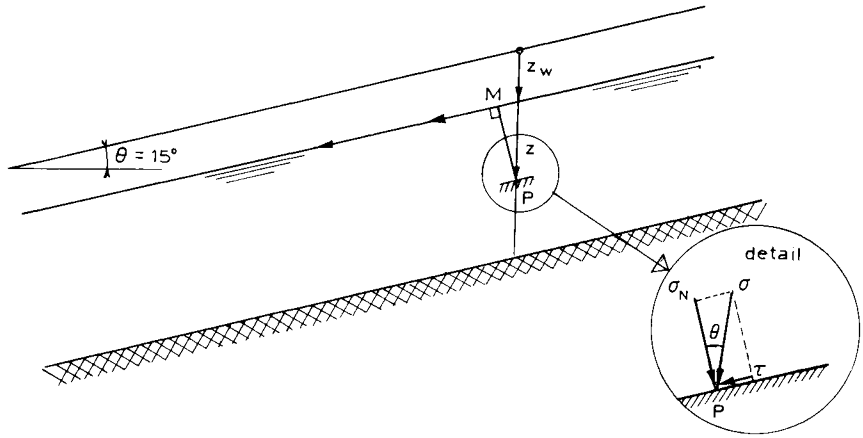


Fig. 12.5.

surface:

$\sigma = [\gamma_h z_w + \gamma_{sat}(z - z_w)] \cos\theta.$

The normal stress to this face, σ_N is equal to:

$$\sigma_N = [\gamma_h z_w + \gamma_{sat}(z - z_w)] \cos^2\theta$$

and the shear stress acting along this face is:

$$\tau = [\gamma_h z_w + \gamma_{sat}(z - z_w)] \cos\theta \sin\theta.$$

Let us determine the pore-water pressure at P, call it u_p. The free groundwater table line corresponds to a flow line. Therefore, the line MP perpendicular to the flow line is an equipotential line (Fig. 12.5). Therefore: $h_M = h_p = (u_M/\gamma_w) - z_M = (u_p/\gamma_w) - z_p$ and since $u_M = 0$,

$$u_p = \gamma_w(z_p - z_M) = \gamma_w PM \cos\theta = \gamma_w(z - z_w)\cos^2\theta$$

This means that the effective normal stress at P on a plane parallel to the free surface is equal to:

$$\sigma'_N = [\gamma_h z_w + \gamma_{sat}(z - z_w) - (z - z_w)\gamma_w] \cos^2\theta$$

$$\sigma'_N = [\gamma_h z_w + \gamma'(z - z_w)] \cos^2\theta$$

γ' being the buoyant weight of the clayey silt.

The maximum shear stress allowable in the silt is equal to:

$$\tau_M = c' + \sigma'_N \tan\varphi' = c' + [\gamma_h z_w + \gamma'(z - z_w)] \cos^2\theta \tan\varphi'.$$

The safety factor against sliding along the plane is then:

$$F = \frac{\tau_M}{\tau} = \frac{c' + [\gamma_h z_w + \gamma'(z - z_w)] \cos^2\theta \tan\varphi'}{[\gamma_h z_w + \gamma_{sat}(z - z_w)] \cos\theta \sin\theta}$$

$$\text{or: } F = \frac{c'}{[\gamma_h z_w + \gamma_{sat}(z - z_w)] \cos\theta \sin\theta} + \frac{[\gamma_h z_w + \gamma'(z - z_w)] \tan\varphi'}{[\gamma_h z_w + \gamma_{sat}(z - z_w)] \tan\theta}$$

The first term is a decreasing function of z (z is the denominator). The second term is equally a decreasing function of z because $\gamma_{sat} > \gamma'$, therefore F decreases when z increases.

So, the most likely failure plane is located at the bottom of the clayey silt layer, that is to say at -8 m.

(2) When the trench will be excavated, and if we neglect adhesion forces between the silt and the limestone along plane BC, the value of the safety factor calculated above remains valid, therefore:

$$F = \frac{c' + [\gamma_h z_w + \gamma'(z - z_w)] \tan\varphi'}{[\gamma_h z_w + \gamma_{sat}(z - z_w)] \tan\theta}$$

where: $\varphi' = 15°$, $c' = 20$ kPa, $\theta = 15°$, so $F = 1.12$..

This safety factor is quite low.

(3) In order to increase the safety against sliding, the silt could be drained in order to lower the surface of the groundwater table. If the groundwater is lower by 4 m: $z_w = 6$ m, and the new value for the safety factor is: $F = 1.4$. Therefore, the safety of the slope against sliding may be increased by lowering the groundwater table in the silt.

In this example, we only considered a plane failure. In order to completely assess the problem, the trench slope stability should also be analyzed by the circular slice method.

In the event that the silt would be completely drained, the safety factor could be estimated from the graphs XI-12, Sect. 11.2.2 of Costet-Sanglerat.

For $\beta = 45°$ and $\varphi' = 15°$ and assuming that the failure circle would pass through the toe of the slope: $c''/\gamma H = 0.08$. In order to avoid confusion with the drained parameters c' and φ', designate c'' and φ'' the reduced characteristics where:

$$c'' = \frac{c'}{F} \quad \text{and} \quad \tan\varphi'' = \frac{\tan\varphi}{F}$$

If we take $\gamma = \gamma_h = 18\,\text{kN/m}^3$, we obtain the stability for a cohesion value c'' equal to: $c'' = 11.5$ kPa.

As a result, and for the condition of the silt being completely drained, the safety factor is $20/11.5 = 1.74$, therefore sufficient.

★★★*Problem 12.3* Dam stability (global method)

It is proposed to construct a dam for a reservoir to retain water in a small touristic area. The dike is to be built of homogeneous soil very well *compacted. Determine the stability of the dam: (1) at the end of construction, (2) after filling the reservoir.*

The construction soil consists of a clayey silt whose mechanical properties are as follows:

— undrained cohesion, $c_u = 0.4$ daN/cm^2 (40 kPa)
— cohesion and angle of internal (effective) friction (c.d. triaxial test):
$c' = 0.25$ daN/cm^2 (25 kPa), $\varphi' = 10°$
— saturated unit-weight = 18 kN/m^3.

The flow net after filling the reservoir is shown on Fig. 12.6.

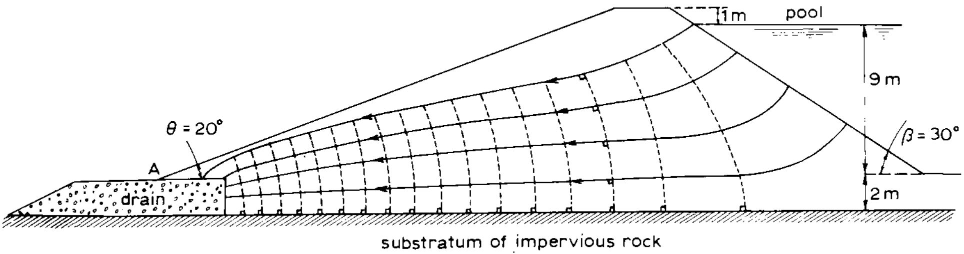


Fig. 12.6. Dam dimensions and flow net after fill of the pool.

Solution

(1) At end of construction

It is obviously the upstream slope which is critical. A short-term computation must be performed because at the end of construction, the pore-water pressure will not have had time to dissipate.

From the graphs XI-14 of Costet-Sanglerat, we can see that if there is a deep circular failure, we have:

$$n_D = (9 + 2 + 1)/(9 + 1) = 1.20, \quad \text{then:} \quad c''/\gamma H = 0.15$$

with c'' cohesion corresponding strictly to the stability $c'' = 0.15 \times 18 \times 10 = 27$ kPa $= 0.27$ daN/cm^2.

The safety factor obtained with the undrained cohesion will then be: $F = 4/2.7 = 1.5$. This is sufficiently high.

(2) After filling the reservoir

Here we must study the downstream slope and the long-term stability must be considered. Indeed we can assume that excess hydrostatic pressures due to construction have had time to dissipate.

Graphs XI-13 of Costet-Sanglerat allow us to determine the most critical failure circle passing at the toe of the dike: $\beta = 20°$, $\varphi' = 10°$, therefore $\alpha = 40°$, $\beta = 18°$.

The graphical construction of the circle results in a radius $R = 25.2$ m (Fig. 12.7).

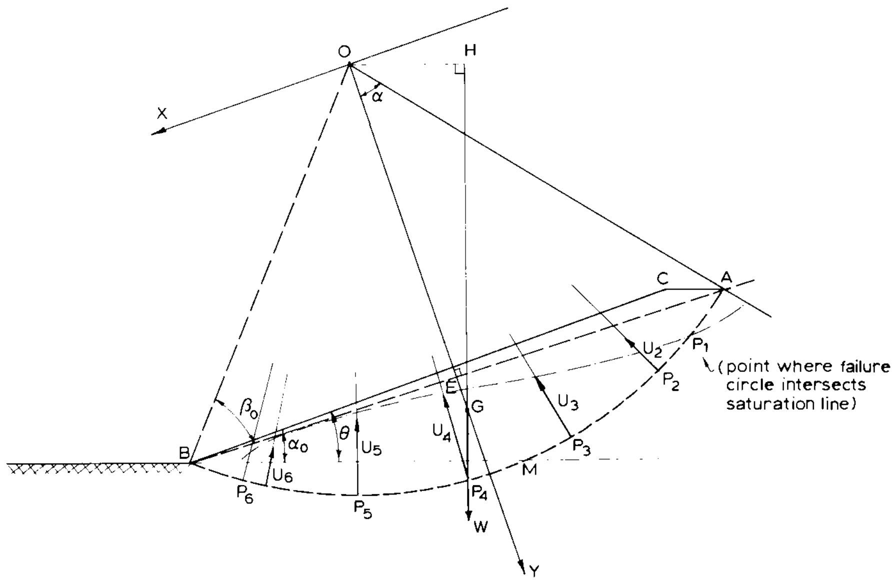


Fig. 12.7.

Let us define the safety factor F by: $\tan \varphi'' = \tan \varphi'/F$ and $c'' = c'/F$ where c'' and φ'' are the critical values of cohesion and friction assuring limit equilibrium condition.

Forces acting on the mass $ACBM$ (Fig. 12.7) are: the weight $\mathbf{W}$, acting through the center of gravity, G, the resultant $\mathbf{U}$ of the forces due to the pore-water pressure applied to the circle BMA, and the resultant of the contact forces on the circle ABM.

Consider the sliding face at a point located on the failure circle (Fig. 12.8).

The components of the stress applied to this face are: σ_N and $\tau = c'' + \sigma_N \tan \varphi''$. The resultant of the contact forces will then be a force $\mathbf{Q}$ such that:

$$\mathbf{Q} = \int_{-\alpha}^{+\alpha} R\,(c'' + \sigma_N \tan \varphi'')\tau \, \mathrm{d}\delta + \int_{-\alpha}^{+\alpha} R\,\sigma_N \, \mathrm{d}\delta$$

where δ is an integration variable.

Let $\mathbf{C}$ be the force defined by:

$$|\mathbf{C}| = \int_{-\alpha}^{+\alpha} Rc''\tau \, \mathrm{d}\delta = \int_{-\alpha}^{+\alpha} \frac{c'R \cos \delta}{F} \, \mathrm{d}\delta = \frac{2c'R \sin \alpha}{F}$$

and whose direction is OX, perpendicular to the bissectrix OY of angle OAB.

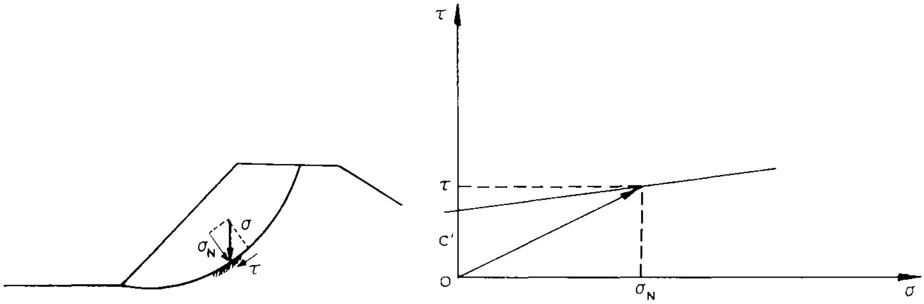


Fig. 12.8.

In addition, let:

$$\mathbf{R}_N = \int_{-\alpha}^{+\alpha} R\,\sigma_N\,\mathrm{d}\delta$$

$$\mathbf{R}_T = \int_{-\alpha}^{+\alpha} R\,(\sigma_N \tan\varphi'')\,\tau\,\mathrm{d}\delta$$

$\mathbf{R}_N$ and $\mathbf{R}_T$ are perpendicular and $|\mathbf{R}_T| = |\mathbf{R}_N| \tan\varphi''$.

Q being the resultant of the contact forces on ABM, we have:

$$\mathbf{Q} = \mathbf{C} + \mathbf{R}_N + \mathbf{R}_T.$$

For equilibrium condition, we obtain:

$$\mathbf{W} + \mathbf{U} + \mathbf{C} + \mathbf{R}_N + \mathbf{R}_T = 0.$$

We write now down the equation for the sum of the moments with respect to O, applied to the mass $ACBM$, is zero.

— moment due to weights **W**: $W \times OH$, where OH is the lever arm of weight **W** with respect to O;

— moment due to the pore-water pressure $= 0$;

— moment due to the tangential stresses, $= M_T$;

In each point of the circle along the failure line, the tangential stress is:

$$c'' + \sigma_N \tan\varphi'' = \frac{c'}{F} + \sigma_N \cdot \frac{\tan\varphi'}{F}.$$

Let M_c be the moment due to the term c'':

$$M_c = \int_{-\alpha}^{+\alpha} \frac{c'R^2}{F}\,\mathrm{d}\delta = \frac{2c'\alpha R^2}{F}$$

and let M_φ be the moment due to the term $\sigma_N \tan\varphi''$:

$$M_\varphi = \int_{-\alpha}^{+\alpha} R^2 |\sigma_N| \tan\varphi'' \, d\delta$$

$$M_\varphi = R \tan\varphi'' \int_{-\alpha}^{+\alpha} |\sigma_N| R \, d\delta = R \frac{\tan\varphi'}{F} \int_{-\alpha}^{+\alpha} |\sigma_N| R \, d\delta$$

where: $\int_{-\alpha}^{+\alpha} |\sigma_N| R \, d\delta$,

corresponding to the distribution of the normal stresses is difficult to determine, but for which we can write:

$$\int_{-\alpha}^{+\alpha} |\sigma_N| R \, d\delta \leqslant \left| \int_{-\alpha}^{+\alpha} \sigma_N R \, d\delta \right| = |\mathbf{R}_N|$$

therefore R_N is the minimal of $\int_{-\alpha}^{+\alpha} |\sigma_N| R \, d\delta$.

M_φ being a stabilizing moment, we may take a minimum value for M_φ (which adds to the safety).

The moment equilibrium with respect to O may be written as:

$$|\mathbf{W}| \cdot OH = \frac{R \tan\varphi' |\mathbf{R}_N|}{F} + \frac{2c'\alpha R^2}{F} \tag{1}$$

In addition, the equilibrium of the forces gives:

$$\mathbf{W} + \mathbf{U} + \frac{2c'R \sin\alpha}{F} + \mathbf{R}_N + \mathbf{R}_T = 0 \tag{2}$$

and furthermore, $|\mathbf{R}_T|/|\mathbf{R}_N| = \tan\varphi'/F$.

The solution of the simultaneous equations must be made by successive approximation.

Assume an arbitrary value for F (1 for instance) and solve graphically equation (2). From this, the value of R_N is obtained which is then transposed into equation (1) which permits the calculation of a new value of F. If the calculated value is considerably different from the assumed F value, the procedure may even be repeated with the new value of F.

Numerical application

$W = W_1 + W_2$.

W_1 = weight of a unit slice corresponding to the portion of the circle ABM,
W_2 = weight of the slice of soil corresponding to triangle ACB.

$$W_1 = \left(\frac{2\pi\alpha}{180} - \sin 2\alpha\right)\frac{R^2}{2}\gamma = \left(2\pi\frac{40}{180} - 0.985\right)\frac{(25.2)^2}{2} \times 18 \cdot 10^3 =$$

$$= 235 \times 10^4 \text{ N.}$$

$$W_2 = 30 \times 10^4 \text{ N.}$$

$$W = 265 \times 10^4 \text{ N.}$$

The center of gravity of the portion of the circle ABM is G, so that:

$$OG_1 = \frac{2}{3}R\frac{\sin^3\alpha}{\alpha - (\sin 2\alpha/2)} = 21.7 \text{ m.}$$

The center of gravity of the total mass is determined from the location of G_1 and G_2 (center of gravity of triangle ACB). OH is the projection of OG on the horizontal, from which $OH = 6.8$ m.

Determination of U

The pore-water pressures will give rise to forces acting on the failure surface between points B and P_1 (Fig. 12.7).

Divide the arc of the circle BP_1 in 5 equal segments over the length of each we will assume that pore-water pressure is constant and equal to that at the center of the arc length; be it P_2, P_3, P_4, P_5 and P_6, the centers of the 5 segments.

The value of u is calculated from the equipotential lines:
$u = (h - z)\gamma_w$
h = water head
z = level with respect to a reference plane (horizontal plane passing through B for example).
We will then have:

$$u(P_2) = (7.7 - 5.0)\gamma_w = 0.27 \text{ daN/cm}^2 \text{ (27 kPa)}$$

$$u(P_3) = 0.55 \text{ daN/cm}^2 \text{ (55 kPa)}$$

$$u(P_4) = 0.615 \text{ daN/cm}^2 \text{ (61.5 kPa)}$$

$$u(P_5) = 0.45 \text{ daN/cm}^2 \text{ (45 kPa)}$$

$$u(P_6) = 0.1 \text{ daN/cm}^2 \text{ (10 kPa)}$$

the segment on which these pressures act has a length equal to:

$$P_2P_3 \simeq 7 \text{ m.}$$

Therefore, the corresponding forces will be:

$$U_2 = 18.9 \cdot 10^4 \text{ N}$$

$$U_3 = 38.5 \cdot 10^4 \text{ N}$$

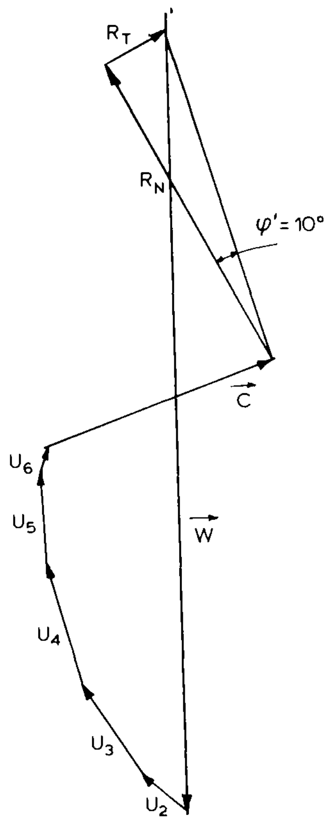


Fig. 12.9. Graphic solution of eq. (2) for R_N.

$U_4 = 43 \cdot 10^4$ N

$U_5 = 31.5 \cdot 10^4$ N

$U_6 = 7 \cdot 10^4$ N.

The directions of these forces $\mathbf{U}_i$ are determined by the radius of the circle ending at P_i.

Assuming a value of $F = 1$, we can solve equation (2) graphically (Fig. 12.9).

$|\mathbf{W}| = 265 \times 10^4$ N (vertical direction)

$$|\mathbf{C}| = \frac{2C'R \sin \alpha}{F} = 83 \times 10^4 \text{ N} \quad (OX \text{ direction})$$

The graphical solution (Fig. 12.9) gives $R_N = 116 \cdot 10^4$ N. Transposing this value in equation (1), we get:

$$F = \frac{R \tan \varphi' |R_N| + 2c'\alpha R^2}{|\mathbf{W}| OH} =$$

$$= \frac{25.2 \times 0.176 \times 116 + 2.5 \times 2 \times 0.698 \times (25.2)^2}{267 \times 6.8}$$

from which $F = 1.50$.

Then taking a value of $F = 1.50$, another diagram is drawn of the forces similar to the preceding one. We then get $R_N = 120 \times 10^4$ N. Transposing this new value in the equation:

$$F = \frac{R \tan \varphi' |R_N| + 2c'\alpha R^2}{|\mathbf{W}| OH}$$

we finally obtain: $F = 1.51$. This value is very close to the preceding one and may be considered as the solution to both equations. This value near 1.50 may be considered as acceptable for the safety against sliding.

★★★*Problem 12.4* Dam stability (method of slices)

Solve Problem 12.3 for the downstream slope stability analysis by the method of slices (Fellenius' method, for instance).

Solution

Using Fellenius' method, the failure circle is defined in the same manner as in Problem 12.3 (Fig. 12.7). The failure zone is now divided into 5 slices, each of width b. We have:

$$BA = 2R \sin \alpha = 2R \sin (40°) = 32.4 \text{ m}$$

The horizontal projection of BA is then:

$$BA \cos \beta_0 = BA \cos (18°) = 30.8 \text{ m}$$

thus we can consider 5 slices each 6.15 m long in the horizontal direction (Fig. 12.10).

In Fellenius' method, it is assumed that each slice is in equilibrium under the action of its weight, of the lateral forces (which cancel) and of the reaction along failure line.

The safety factor is computed solely from the equilibrium of the moments with respect to the center of the circle.

Considering slice i (Fig. 12.11), the driving moment due to this slice is equal to: $M_{mi} = W_i \sin \theta_i R$.

The resisting moment due to the slice is:

$$M_{ri} = \left(\frac{cb}{\cos \theta_i} + N_i \tan \varphi \right) R$$

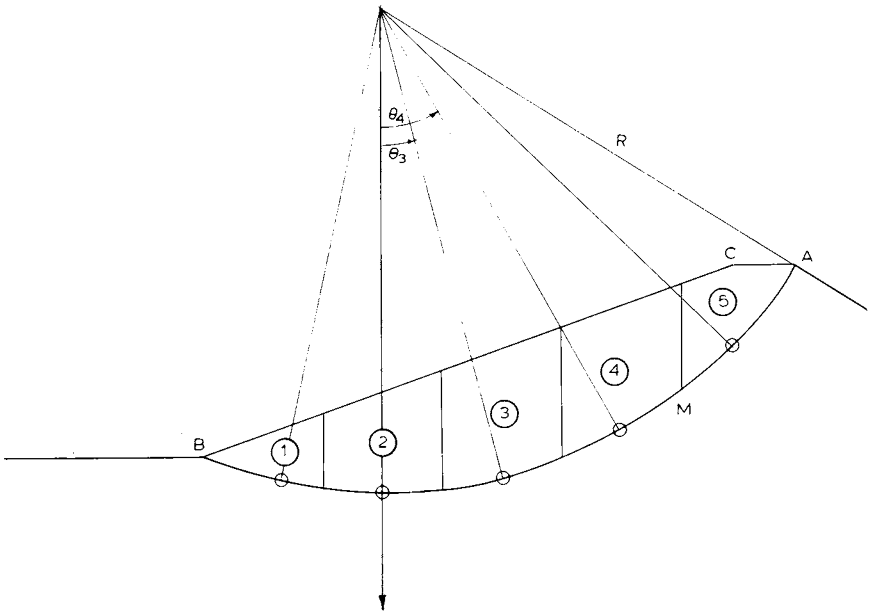


Fig. 12.10.

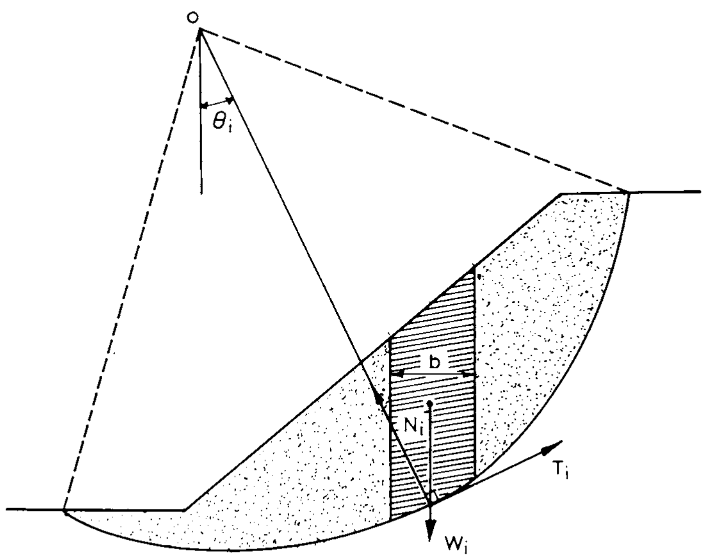


Fig. 12.11.

TABLE 12A

	Slice numbers					Total (10^4 N)
	1	2	3	4	5	
θ_i (degrees)	−13.5	0	13.5	28	44	—
W_i (10^4 N)	20.5	56	76.4	75	41.2	—
u_i (kN/m^2) or kPa	0	22.5	60	52.5	20	—
$(W_i \cos^2\theta_i - u_i b)$ (10^4 N)	19.4	42	35.4	26.2	9	—
$[cb + (W_i \cos^2\theta_i - u_i b)\tan\varphi]\dfrac{1}{\cos\theta_i}$	19.3	22.8	22.2	22.7	23.6	110.6
$W_i \sin\theta_i$ (10^4 N)	−4.8	0	17.8	35	28.6	76.6

where N_i is the normal component of the reaction on the failure plane, $N_i = W_i \cos\theta_i - u_i(b/\cos\theta_i)$, u_i being the pore-water pressure, normal to the failure surface, which is computed at the center of the base of each slice.

We then have: $u_i = (h_i + z_i)\gamma_w$.

Overall, we obtain the formula for the safety factor:

$$F = \frac{\sum_{i=1}^{n}\left[[cb + (W_i \cos^2\theta_i - u_i b)\tan\varphi]\dfrac{1}{\cos\theta_i}\right]}{\sum_{i=1}^{n} W_i \sin\theta_i}$$

This factor may be computed from the data of Table 12A where values of θ_i and W_i have been graphically determined based on Fig. 12.10.

Finally, we obtain: F = 110.6/76.6 = 1.44.

This value is in good agreement with the value found in Problem 11.3 by the global method.

The slight difference between the two results is due to the different hypotheses made for each of the computations and also due to the lack of accuracy inherent in graphic solutions.

The above safety factor may seem a little low since the allowable lower limit is usually $F = 1.5$. Two solutions may be considered to increase the value of F. The downstream slope could be flattened slightly (1 or 2°) or use could be made of burrow material whose mechanical properties are somewhat better than those studied.

★★★*Problem 12.5* **Stability of a dam with impervious core. Comparison of results from computer calculation and slice method**

Check the stability of a dam with an impervious core as shown on Fig. 12.12 (which gives the geometry and material properties).

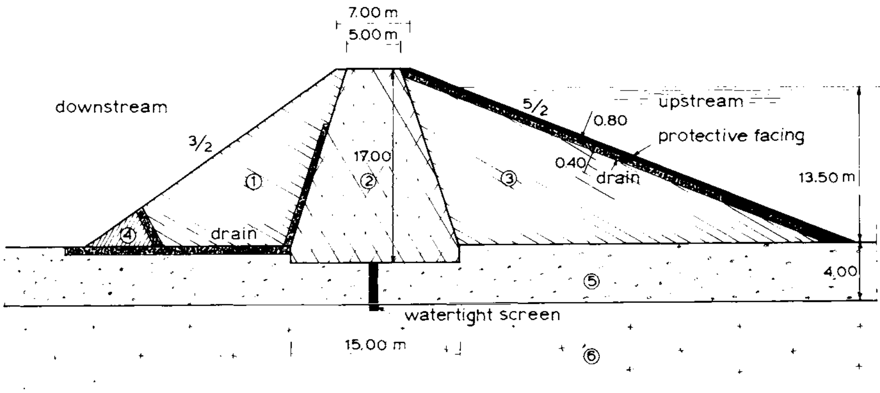


Fig. 12.12. Cross-section of dam (scale 1/670).

Two calculations must be made:

— *downstream slope stability with maximum pool;*

— *upstream slope stability under rapid drawdown condition.*

In order to simplify the verification, for each case (illustrated in Figs. 12.13 and 12.15) the coordinates of the center and radii of the critical circles are given. These were determined by computer. The lines of saturation are shown also.

All that is required is to determine the safety factors corresponding to each of these critical circles by the method of Fellenius, and to compare the results with those of the computer program.

Computer method

The "Lease" IBM-program was used, partly based on the simplified Bishop-method and partly on the "normal" method.

The advantage of the computer is that a large number of critical circles can be studied (100 to 200) and thus it is possible to quickly find a very good approximation of the most critical one.

For the two cases under consideration, the minimal safety factors were found, being: 1.45 for the upstream slope under rapid drawdown, and 1.69 for the downstream slope at maximum pool.

Solution

(a) Stability of the downstream slope at maximum pool

The dam is heterogeneous and therefore the slice method is best suited for the stability analysis. The Fellenius method is used which is practical for calculations by hand.

The portion of the circle in the dam may be divided into 6 slices as indicated on Fig. 12.13. As in the preceding problem, the safety factor is given

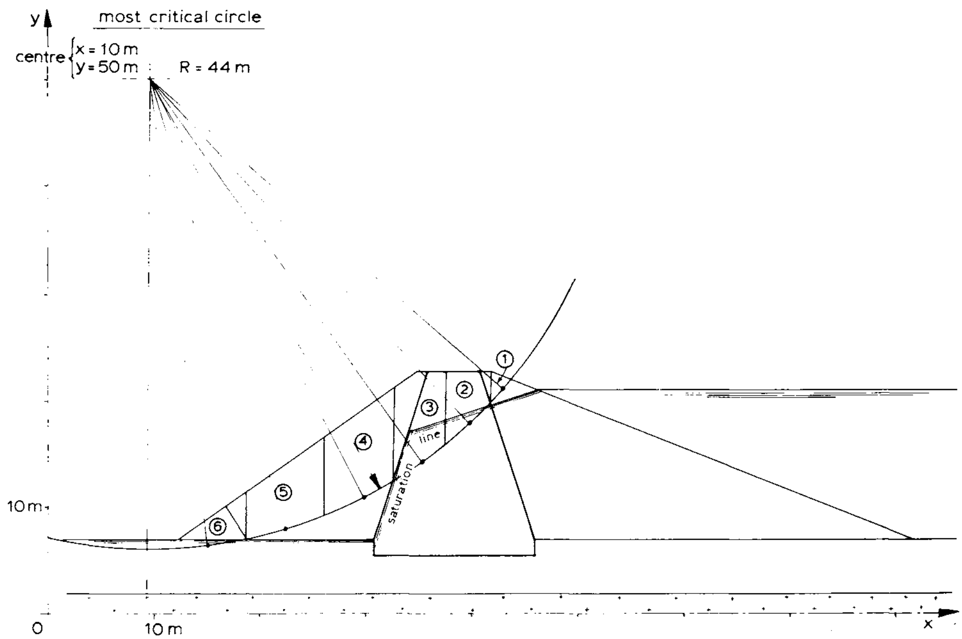


Fig. 12.13. Stability check of downstream slope, maximum pool.

by the formula:

$$F = \frac{\sum_{i=1}^{n} \left[[c_i b_i + (W_i \cos^2 \theta_i - u_i b_i) \tan \varphi] \frac{1}{\cos \theta_i} \right]}{\sum_{i=1}^{n} W_i \sin \theta_i}$$

We do not dispose over the flow net, we know only the free-water surface line, but we can make a simplifying assumption and say that the flow lines are parallel to the free-water surface line. AB is an equipotential line (perpendicular to the free-water surface, Fig. 12.14); the pore-water pressure at a point A may then be calculated by: $h_B = h_A = z_A + u_A/\gamma_w = z_B + u_B/\gamma_w$; $u_B = 0$. Thus, $u_A = (z_B - z_A)\gamma_w$.

For the slice No. 6, the downstream part of the failure circle has been neglected, because in this zone the failure surface is very near the ground surface. In fact, because the failure line is not a perfect circle, it will pass through the toe of the downstream slope.

The factor F may be computed from the data of Table 12B, from which $F = 264/164 = 1.5$.

Taking into account the simplifying assumption, we can say that this value

TABLE 12B

	Slice numbers						Total (10^4 N)
	1	2	3	4	5	6	
b_i (m)	2.08	4.18	4.78	6.50	7.16	7.46	
θ_i (degrees)	48.5	43	35	28.2	17	7	
W_i (10^4 N)	6.08	43.22	82.64	110	90	31	
u_i (10^4 Pa)	0	0	0.6	0	0	0	
$W_i \cos^2\theta_i - u_i b_i$ (10^4 N)	2.67	19.35	52.6	85.45	82.30	30.53	
$[c_i b_i + (W_i \cos^2\theta_i - u_i b_i)\tan\phi] \times \dfrac{1}{\cos\theta_i}$ (10^4 N)	7.3	24.4	45.8	74	67	26	246
$W_i \sin\theta_i$ (10^4 N)	4.55	29.47	47.4	52.48	26.3	3.78	164

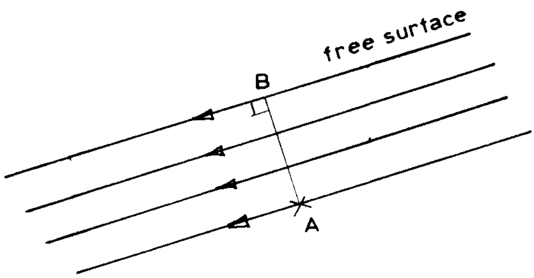


Fig. 12.14.

is in good agreement with the value of $F = 1.64$ found by computer program.

These values of F are sufficient to ensure the stability of the slope ($F \geqslant 1.5$).

(b) Upstream slope stability under rapid drawdown condition

Once again, because the dam is heterogeneous, the slice method of Fellenius is used. The portion of the dam in the critical circle may be divided into 5 slices as indicated on Fig. 12.15.

As for the previous problem, the safety factor is determined from:

$$F = \frac{\sum_{i=1}^{n} \left[[c_i b_i + (W_i \cos^2\theta_i - u_i b_i)\tan\varphi] \dfrac{1}{\cos\theta_i} \right]}{\sum_{i=1}^{n} W_i \sin\theta_i}.$$

Again, the flow net is not available, but only the free-water surface line. The pressures, after rapid drawdown, may be evaluated from Bishop's hypothesis, that is:

u_A (after rapid drawdown) $= u_A$ (before drawdown) $- \gamma_w h_w$ (Fig. 12.16).

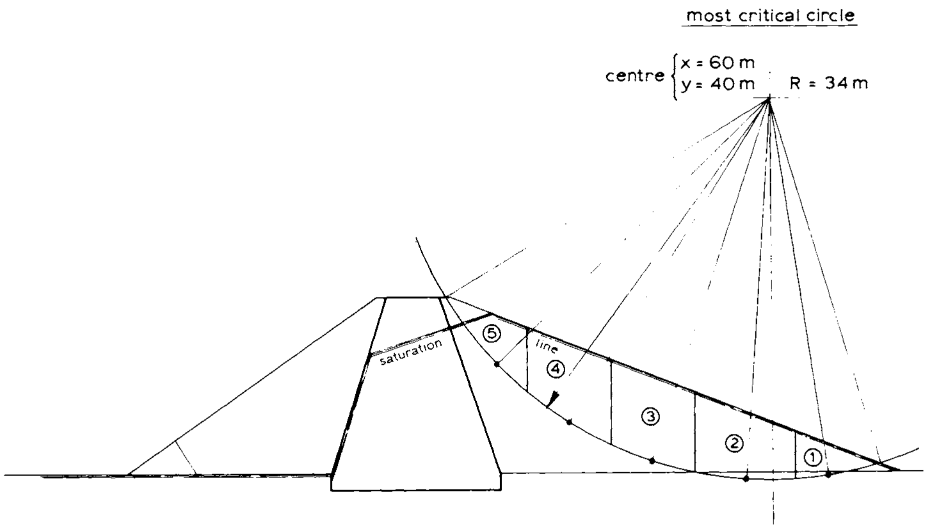


Fig. 12.15. Stability check of upstream slope: rapid drawdown.

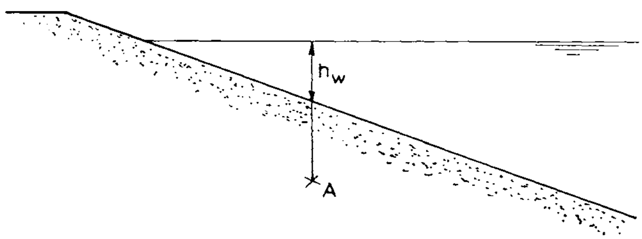


Fig. 12.16.

We assume that at maximum pool, the pressures in the sandy gravel soil (3) correspond to the hydrostatic pressures (the material is relatively pervious: the head losses through the soil are neglected for a first approximation).

Then it is easy to determine the pore-water pressures before and after drawdown. The global calculation of F is done as before, based on Table 12C that finally gives: $F = 159/123 = 1.29$.

The influence of the riprap was overlooked in the above calculation. Its angle of internal friction is high (60°), so the real safety factor is certainly greater than 1.29. We may consider it to be $F = 1.30$. A computer analysis of this condition with assumptions closer to the real conditions showed that $F = 1.45$.

Taking into account the simplifying assumptions of the manual method, we may still consider the two values of the coefficient to be close enough.

Usually, it is desirable to work with a safety factor of over 1.5.

TABLE 12C

	Slice numbers					Total (10^4 N)
	1	2	3	4	5	
b_i (m)	7.46	8.96	7.46	7.31	7.22	
θ_i (degrees)	−8	4	18	32	46	
W_i (10^4N)	37.42	109.5	114.6	103	42.7	
u_i (10^4 Pa)	3	6.1	7.5	6.9	3	
$W_i \cos^2\theta_i - u_i b_i$ (10^4N)	14.3	54.3	47.5	23.9	1.1	
$[c_i b_i + (W_i \cos^2\theta_i - u_i b_i) \tan\varphi] \times \dfrac{1}{\cos\theta_i}$	16	50	44	31.2	17.6	159
$W_i \sin\theta_i$ (10^4 N)	−5.2	7.6	35.4	54.5	30.7	123

In this case, we may be satisfied with a safety factor of the order of 1.3 to 1.45, because the computation is made for a *rapid drawdown* condition, which occurs very rarely during the life of a dam. Under these conditions, a slightly higher risk is acceptable than that for maximal pool conditions. (An empty pool condition where a slope failure would occur, would cause considerably less damage.)

Remark

In each of the calculations, the result of the Fellenius-method is inferior by about 0.15 compared to that of the computer.

★★★★*Problem 12.6* Design of a retaining-wall on unstable slope

Consider a natural slope of $\theta = 25°$, consisting of a clayey silt layer whose thickness $h = 8$ m, overlying a bedrock substratum parallel to the ground surface. The properties of the silt were obtained from laboratory tests and are:

— *wet unit weight: $\gamma = 18$ kN/m^3*
— *effective maximum angle of internal friction = $Q'_{peak} = 20°$*
— *effective maximum cohesion: $c' = 25$ kPa*
— *residual angle of internal friction $\varphi_{res} = 18°$*
— *residual cohesion $c_{res} = 10$ kPa*

A retaining wall must be designed for a proposed highway along the slope. The wall is to be founded on the bedrock and capable of supporting the slope. The length L of the slope, measured on the slope from toe to crest is 100 m (Fig. 12.17).

(1) Determine the stability of the slope prior to construction.

(2) Since excavations into the slope for the construction of the wall may

trigger instability, the residual shear strength parameters of the silt must be used for the stability calculation.

(a) Show that under this condition, the natural slope becomes unstable.

(b) Compute the limit depth z, to which the bedrock should be located to have a stability with a safety factor of 1. The computation done for this must overlook the presence of the wall.

(c) Calculate the minimum value of the force Q_m parallel to the free surface and applied to an imaginary vertical plane which would restore the slope stability.

(3) Suppose that the wall construction will mobilize this force Q_m. Show, by proposing a plastic equilibrium net above the wall, that the limit available force is a passive pressure B which must be defined. Is the stability insured regardless of length L? Analyse the different possible cases.

(4) Compute the force exerted by the soil on the wall. What are your conclusions?

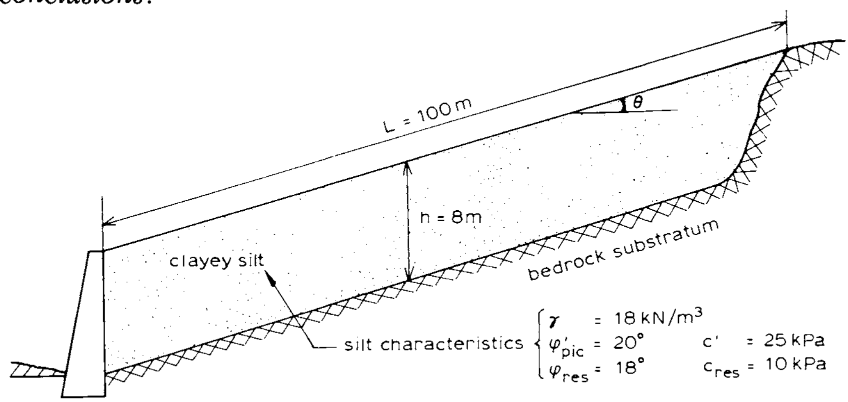


Fig. 12.17.

Solution

(1) Following the steps of Problem 12.2, and without draining conditions ($Z_W = Z$), we get:

$$F = \frac{c'}{\gamma h \cos\theta \sin\theta} + \frac{\tan\varphi'}{\tan\theta} \tag{1}$$

which, for this case, gives:

$$F = \frac{25}{18 \times 8 \times \cos 25^\circ \sin 25^\circ} + \frac{\tan 20^\circ}{\tan 25^\circ} = 1.23$$

$$F > 1.$$

The natural slope is stable.

(2a) Going back over the calculations with the residual shear strength parameters, we get:

$$F = \frac{10}{18 \times 8 \times \cos 25^\circ \sin 25^\circ} + \frac{\tan 18^\circ}{\tan 25^\circ} = 0.88.$$

Since the safety factor is less than 1, the slope is unstable.

(2b) Z_1 is obtained immediately if we say $F = 1$ in formula (1), using the residual shear parameters and $Z_1 = h$. We get: $Z_1 = 4.78$ m.

Note that by a different analytical method, we again find the value of Z_1, calculated in Problem 5.6.

(2c) Referring to Fig. 12.18 and writing the equilibrium equation for the soil volume located upslope from the imaginary plane AC, we have:

— tangential component of weight $\mathbf{W}$ of the soil mass likely to slide:

$W_T = W \sin\theta = \gamma \cdot hL \cos\theta \sin\theta$

— normal component of weight $\mathbf{W}$:

$W_N = W \cos\theta = \gamma \cdot hL \cos^2\theta$

— shear resistance in the soil (assume perfect adherence between soil and rock, which implies that the sliding plane occurs in the soil mass):

$$T = c'L + W_N \tan\varphi' = c'L + \gamma \cdot hL \cos^2\theta \tan\varphi'.$$

The overall stability is: $T + Q_m = W_T$, from which:

$$Q_m = \gamma \cdot hL \cos\theta \sin\theta - \gamma \cdot hL \cos^2\theta \tan\varphi' - c'L$$

and for this case: $Q_m \simeq 673$ kN.

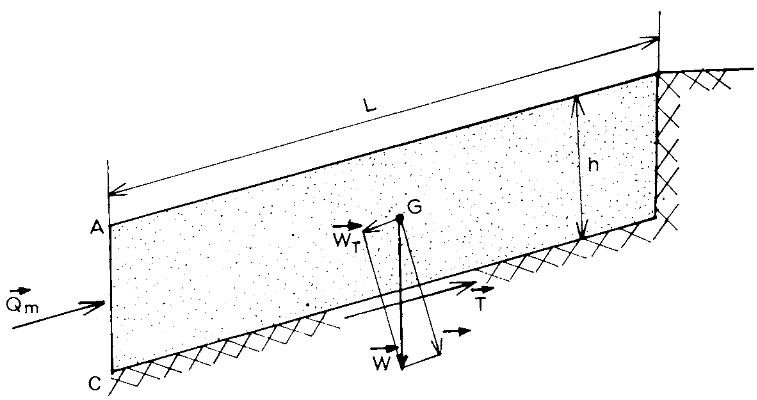


Fig. 12.18.

(3) Preliminary remark

For the following computation, and in order to simplify notations, c and φ will stand for c_{res} and φ_{res}.

Let us now consider the initially unstable slope retained by a wall, and let us study its influence on the mass above it. We assume that the wall stabilizes the slope. We know that in the case of cohesive soil (c, φ) whose free surface has an angle $\theta > \varphi$ with the horizontal, the limit equilibrium can only occur if depth h of the mass is less than the critical depth Z_1 (see Problem 5.6). In

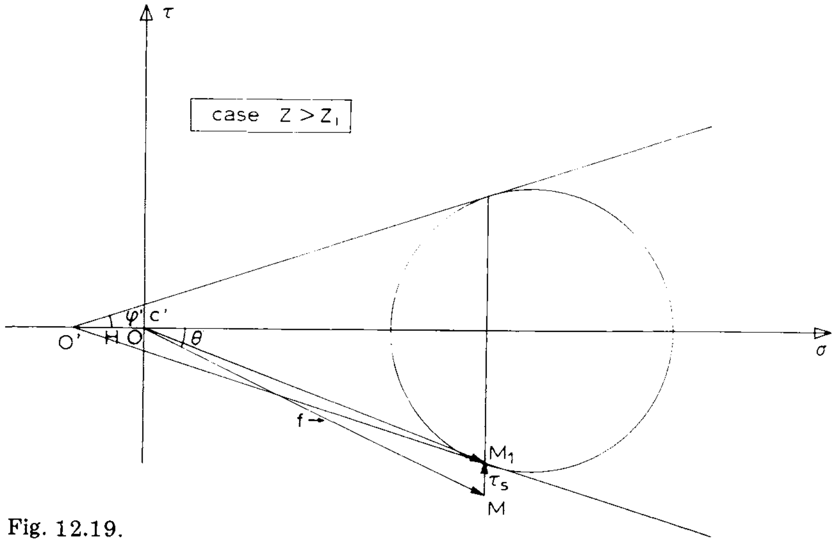


Fig. 12.19.

this instance $h > Z_1$ (see question 2b), therefore, in the absence of a wall, equilibrium cannot exist since limit equilibrium can not occur if $Z_1 < Z \leqslant h$. To admit that the wall stabilizes the slope, is also to admit that behind the wall a limit equilibrium can exist (which we suppose to be the Rankine limit equilibrium) to depth h. Therefore, in all points of a soil mass to depth $Z > Z_1$, the Mohr's circle must be tangent to the failure envelope of the soil. This condition exists if the extremity M of the stress vector f acting on a face parallel to the ground slope is brought back to M_1 on the failure envelope. This amounts then to assume that the influence of the wall consists of an additional shear stress τ_s on the plane considered. (Fig. 12.19). This stress τ_s may be computed by writing the overall equilibrium of the mass.

The pole method (see Problem 5.1) gives the lines of failure in the mass in limit equilibrium state.

In the zone $Z < Z_1$, we have a choice between two possible limit equilibrium conditions since in this instance extremity M of the stress vector f acting on a face parallel to the ground slope is located inside the failure envelopes.

In view of the fact that large displacements would occur before equilibrium condition is developed, we must only consider the upper Rankine equilibrium (see Fig. 12.10) which corresponds to a passive pressure being developed behind the wall.

With a similar reasoning as that in Problem 5.6, the failure lines net in zone $Z < Z_1$, may be drawn. We find that for $Z = 0$ the tangents to these lines make an angle $(\pi/4 - \varphi/2)$ with the free surface, and that for $Z = Z_1$ one family of lines has a tangent parallel to the free surface, whereas the other family of lines has a tangent with an angle $\pi/2 - \varphi$ with the free surface. (Fig. 12.21).

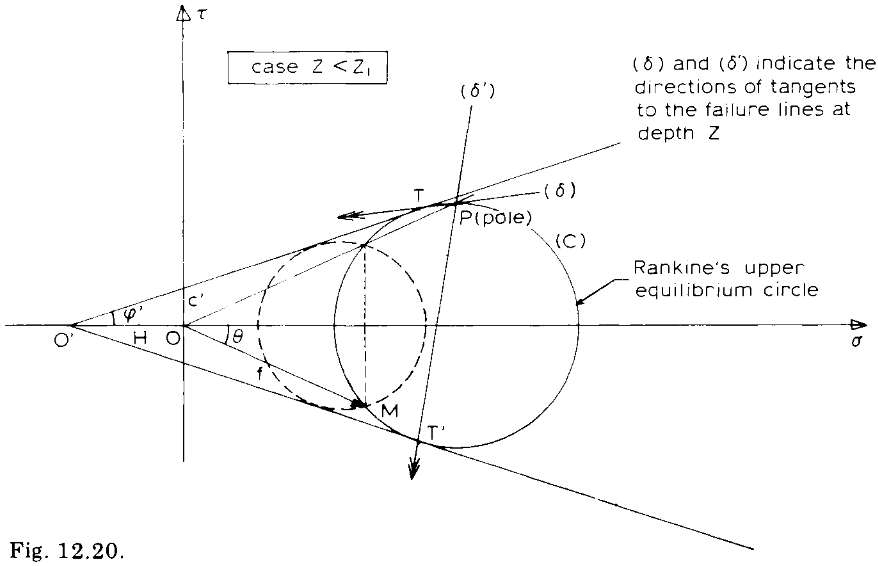


Fig. 12.20.

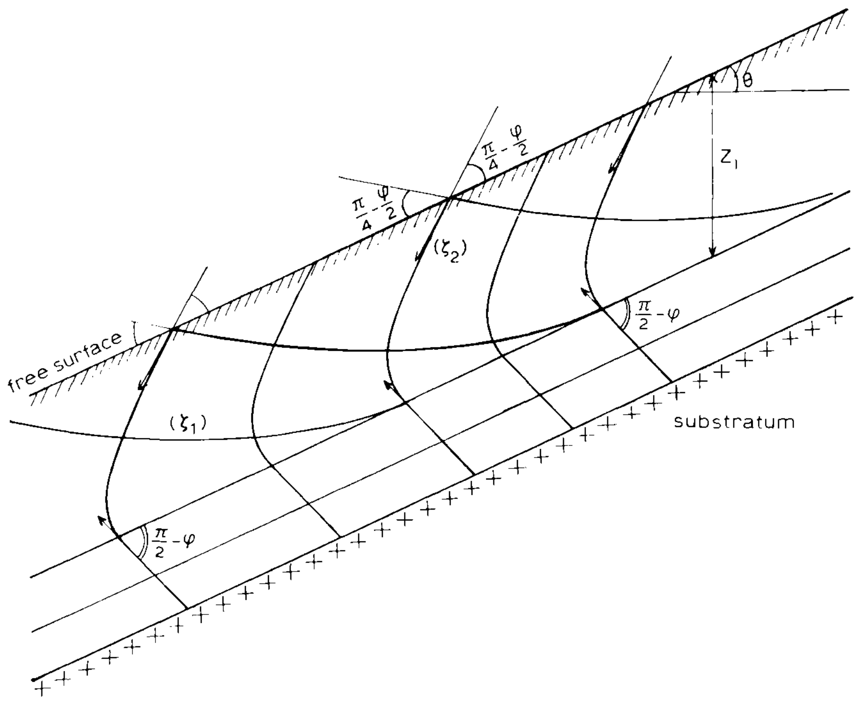


Fig. 12.21.

In the zone $Z > Z_1$, the pole method shows immediately that the failure lines are straight lines, one family of lines being parallel to the free surface, whereas the lines of the second family make an angle $(\pi/2 - \varphi)$ with the free surface (Fig. 12.21). A plastic equilibrium diagram may then be drawn which consists of 3 zones (Fig. 12.22) which shows, above the wall, a stable equilibrium zone (zone 1) and two plastic equilibrium zones, the upper Rankine equilbrium (zone 2) and the limit equilibrium due to the presence of the wall (zone 3).

Force **B** mobilized to insure equilibrium may be evaluated by calculating the resultant of the passive stresses acting on the imaginary vertical plane ABC drawn in the plastic zone and passing through point C where the first failure line crosses the substratum (Fig. 12.22). On part AB of the plane, corresponding to $Z \leqslant Z_1$, a passive stress b is developed which is calculated by considering the upper Rankine equilibrium circle (Fig. 12.23). On part BC of this plane, corresponding to $Z > Z_1$, a stress q is developed with an inclination δ which is calculated from the Mohr diagram of Fig. 12.24.

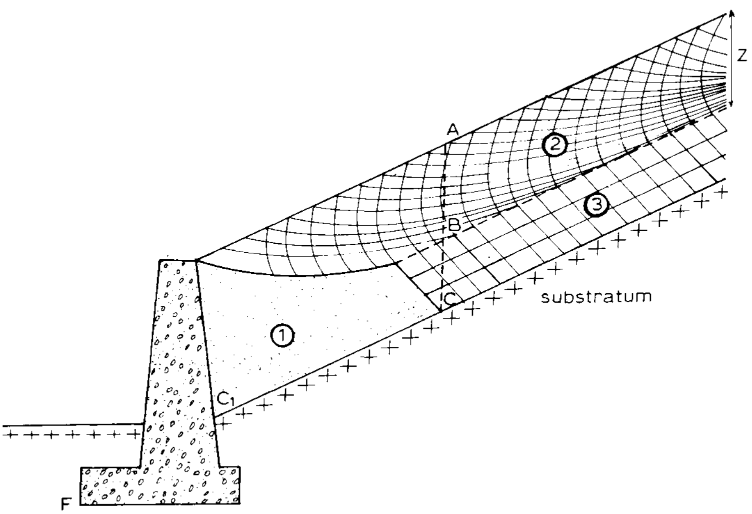


Fig. 12.22.

Evaluation of b

Let p and R be the abscissa of the center and the radius of the Mohr's circle for passive conditions. By writing that this circle passes through the extremity M (σ_0, τ_0) of the stress vector **f** acting on a face parallel to the ground slope, we get an equation of the second degree in p whose largest root corresponds to the passive circle sought.

The general equation for the passive pressure circle is:

$$(\sigma - p)^2 + \tau^2 = R^2.$$

Coordinates of M are: $\sigma_0 = \gamma \cdot h \cos^2\theta$ and $\tau_0 = \gamma \cdot h \cos\theta \sin\theta$.
Condition for tangency to the failure envelope:

$$R/(H + p) = \sin\varphi \text{ (with } H = c \cot\varphi).$$

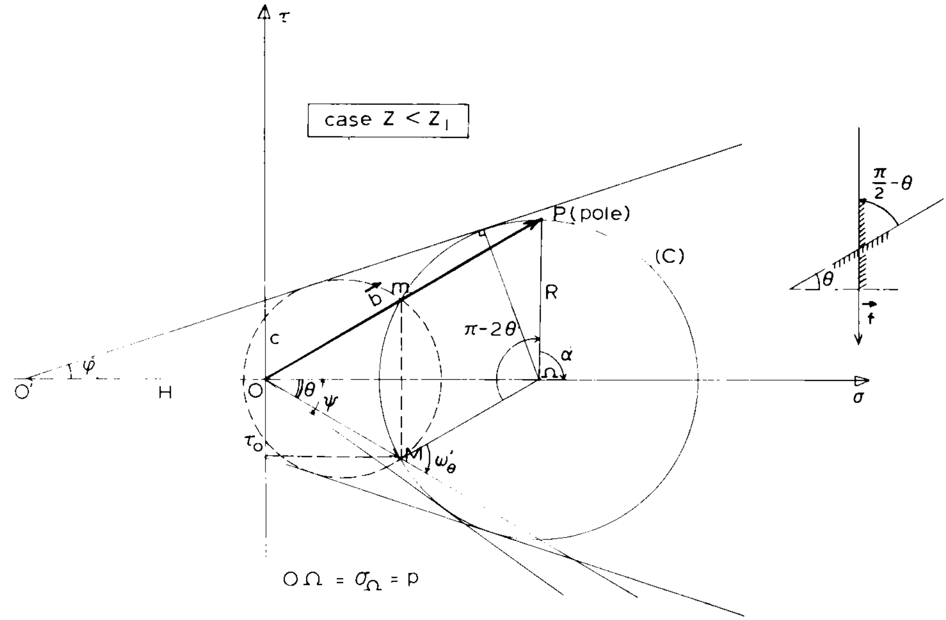


Fig. 12.23. Case $Z < Z_1$.

Writing that (C) passes through M:

$$(\sigma_0 - p)^2 + \tau_0^2 = (H + p)^2 \sin^2\varphi,$$

then the second-degree equation desired is:

$$p^2 \cos^2\varphi - 2(\sigma_0 + H \sin^2\varphi)p + \sigma_0^2 + \tau_0^2 - H^2 \sin^2\varphi = 0$$

$$\Delta' = (\sigma_0 + H \sin^2\varphi)^2 - \cos^2\varphi[\sigma_0^2 + \tau_0^2 - H^2 \sin^2\varphi] = 0.$$

From which:

$$p = \frac{\sigma_0 + H \sin^2\varphi + \sqrt{\Delta'}}{\cos^2\varphi}$$

$$R = (H + p) \sin\varphi.$$

Note: we can verify that $\Delta' = 0$ for $Z = Z_1$ (double root).

Finally, we obtain the normal component of the passive stress:

$$\sigma_B = p + R \cos\alpha,$$

where: $\pi - \alpha = \pi - 2\theta - (\omega'_\theta - \theta) = \pi - (\theta + \omega'_\theta)$

from which: $\alpha = \theta + \omega'_\theta$,

the auxiliary angle ω'_θ being defined by (Fig. 12.23):

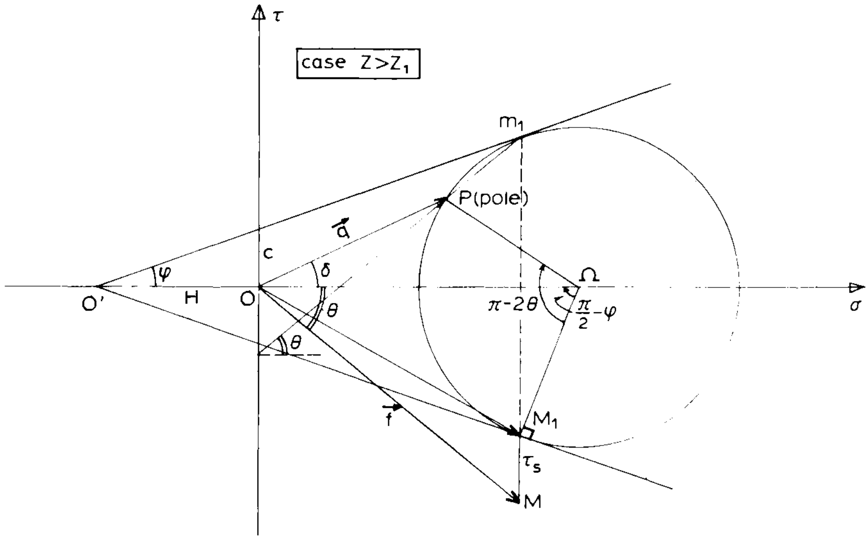


Fig. 12.24. Case $Z > Z_1$.

$$\omega'_\theta = \arcsin \frac{\sin \theta}{\sin \psi} = \arcsin \frac{p \sin \theta}{R} \; *$$

from which: $\sigma_B = p + R \cos (\theta + \omega'_\theta)$
and the passive stress is: $b = \sigma_B / \cos \theta$.

The following table gives the results of the computation for different values of Z.

Z (m)	0	1.00	2.00	3.00	4.00	$4.78 = Z_1$
b (kPa)	24.9	44.2	59.1	70.6	77.5	69.5

Evaluation of q

Referring to Fig. 12.24, we get the following relationship:

$$\beta = \pi - 2\theta - (\pi/2 - \varphi) = (\pi/2) - (2\theta - \varphi)$$

from which, the components of q are:

$$\sigma_q = q \cos \delta = p - R \cos \beta = p - R \sin (2\theta - \varphi),$$

*In determining ω'_θ it must be taken into account that for the values of Z such as $Z_0 < Z < Z_1$, ω'_θ is greater than $\pi/2$. The value Z_0 corresponds to the minimum of R/p. In this case $Z_0 \simeq 4.50$ m.

$\tau_q = q \sin \delta = R \sin \beta = R \sin (2\theta - \varphi)$.

Abscissa p of the center of a Mohr's circle and its radius R are determined by writing that it is tangent at M_1 to the failure envelope, which translates into:

$$R/(p + H) = \sin \varphi$$

$$p - R \cos (\pi/2 - \varphi) = \sigma_v = \gamma \cdot h \cos^2 \theta$$

or:

$$R - p \sin \varphi = c \cos \varphi$$

$$p - R \sin \varphi = \gamma \cdot h \cos^2 \theta$$

from which:

$$p = \frac{\gamma \cdot h \cos^2 \theta + c \cos \varphi \sin \varphi}{\cos^2 \varphi}$$

$$R = \frac{\gamma \cdot h \cos^2 \theta \sin \varphi + c \cos \varphi}{\cos^2 \varphi}$$

from which: $q = \sqrt{\sigma_q^2 + \tau_q^2}$ and $\tan \delta = \tau_q / \sigma_q$.

Finally, the component of q in the direction parallel to the ground slope, or q_θ is given by $q_\theta = q \cos (\theta - \delta)$. The following table ($c = c_{res} = 10$ kPa, $\varphi = \varphi_{res} = 18°$) summarizes the results of the calculations for various values of Z.

Z (m)	4.78 (Z_1)	5.00	6.00	7.00	8.00
q (kPa)	69.5	72.7	86.9	101.1	115.4
δ	25°	24° 7	23° 5	22° 6	22°
q_θ (kPa)	69.5	72.7	86.9	101	115.2

Fig. 12.25 shows the diagram of stresses acting along the imaginary vertical plane ABC. Integrating the passive force increments, gives the passive force B:

$$B = Q_B + Q'_B \quad \text{with} \quad Q_B = \int_0^{Z_1} b \cdot dZ \quad \text{and} \quad Q'_B = \int_{Z_1}^{h} q_\theta \cdot dZ.$$

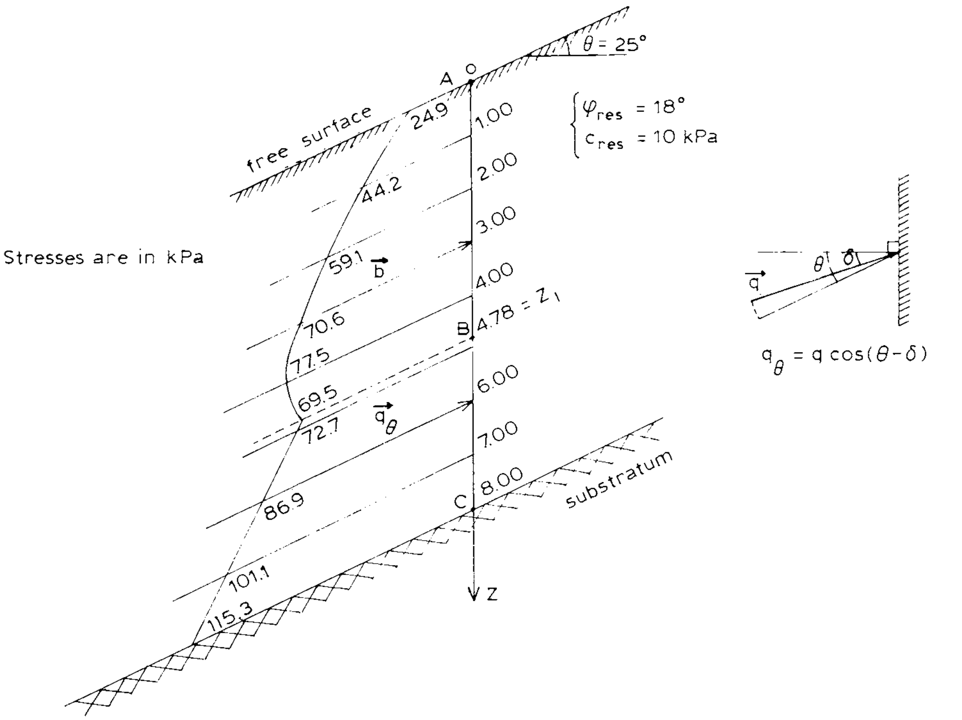


Fig. 12.25.

$$Q_B \simeq \left(\frac{24.9}{2} + 44.2 + 59.1 + 70.6 + \frac{77.5}{2}\right) \times 1.00 +$$

$$+\left(\frac{77.5 + 69.5}{2}\right) 0.78 \simeq 282\,\text{kN}$$

$$Q'_B \simeq \left(\frac{69.5 + 72.7}{2}\right) \times 0.22 + \left(\frac{72.7}{2} + 86.9 + 101 + \frac{115.3}{2}\right) \times 1.00 \simeq$$

$$\simeq 298\,\text{kN}, \quad \text{from which } B \simeq 580\,\text{kN}.$$

We previously saw (question 2) that the required force Q_m to establish equilibrium has a value of 673 kN (this for a slope length of 100 m measured along the slope).

Force B, therefore, will guarantee equilibrium of slope length L', such that: $L' = 100 \times (B/Q_m) = 100 \times (580/673) \simeq 86.2$ m.

But this length L' is to be measured from the intersection C of the first failure line of zone 3, with the substratum. Distance C_1C between the toe

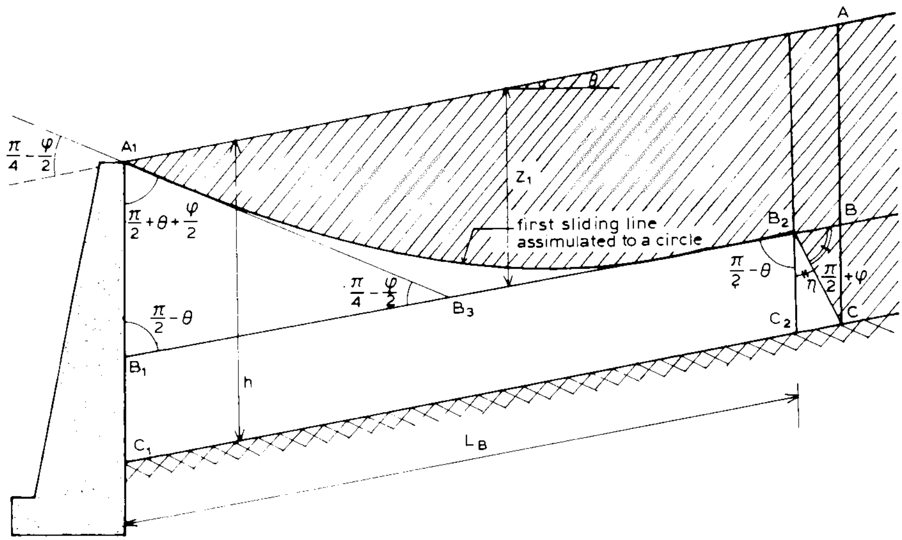


Fig. 12.26.

of the wall C_1 and point C, corresponds to the base of zone 1 in stable equilibrium. The computation of length C_1C is done by assimilating the first failure-line A_1B_2 of the passive equilibrium in zone 2 to an arc of circle passing through A_1 at the top of the wall and drawing a tangent at B_2 parallel to the free surface. Under these conditions, referring to Fig. 12.26, we get:

$$B_1B_2 = B_1B_3 + B_3B_2$$

or $B_3B_2 = A_1B_3$ (tangents to a circle),

and in the triangle $A_1B_1B_3$:

$$\frac{A_1B_3}{\sin(\pi/2 - \theta)} = \frac{B_1B_3}{\sin(\pi/4 + \theta + \varphi/2)} = \frac{A_1B_1}{\sin(\pi/4 - \varphi/2)} \text{ from which}$$

$$A_1B_3 = \frac{\cos\theta}{\sin(\pi/4 - \varphi/2)} \times A_1B_1 \simeq 1.542Z_1$$

$$B_1B_3 = \frac{\sin(\pi/4 + \theta + \varphi/2)}{\sin(\pi/4 - \varphi/2)} \times A_1B_1 \simeq 1.670Z_1$$

$$B_1B_2 \simeq 3.21Z_1 = 3.21 \times 4.78 = 15.34\,\text{m}.$$

and in triangle B_2C_2C: $\dfrac{B_2C_2}{\sin(\pi/2 + \varphi)} = \dfrac{C_2C}{\sin\eta}$

where: $\eta = \pi - (\pi/2 - \theta) - (\pi/2 + \varphi) = \theta - \varphi$

from which: $C_2C = \dfrac{\sin(\theta - \varphi)}{\cos\varphi} \times B_3C_2 = \dfrac{\sin(\theta - \varphi)}{\cos\varphi}(h - Z_1)$

$C_2C = 0.128\,(8.00 - 4.78) = 0.41\,\text{m}$

$C_1C = C_1C_2 + C_2C = B_1B_2 + C_2C = 15.34 + 0.41 \simeq 15.75\,\text{m}.$

Let $C_1C = L_B$.

Thus, $L_B + L' = 15.75 + 86.20 = 101.95\,\text{m} > 100\,\text{m} = L.$

Then the equilibrium of the unstable slope is hardly obtained in this case.

In general, two cases must be considered: In the case where $L_B + L' < L$, equilibrium cannot exist even if the wall is stable and fixed: the soil mass would flow over the wall.

But, in the case where $L_B + L' > L$, the mass is stabilized by the wall, provided that the wall is designed to withstand the forces applied to it.

(4) Let us now calculate the applied force to the wall (for a one meter length), or F_p. We get F_p by studying the equilibrium of the parellelopipede A_1C_1CA. On face A_1C_1, force-$\mathbf{F}_p$ acts, on face AC force-**B**. On face C_1C, force $T_d = (\tau - \tau_{\max}) \times L_B$ acts, which corresponds to the deficit of shear along plane C_1C (part of the weight component W_T not being equalized by the shear stress in the soil along C_1C) (Fig. 12.27).

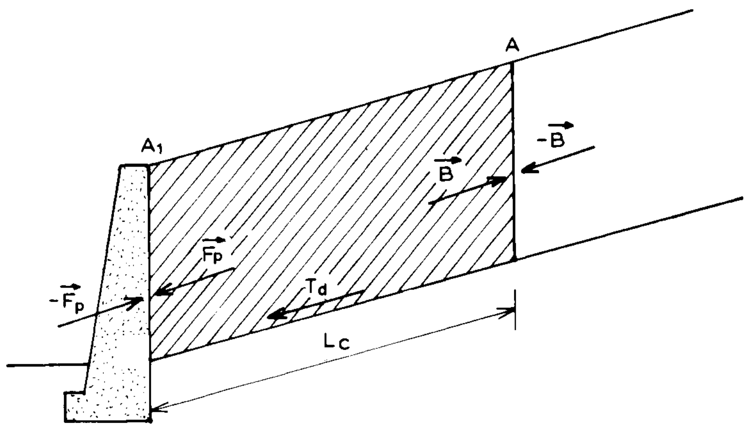


Fig. 12.27

So: $|\mathbf{F}_p| = |\mathbf{T}_d| + |\mathbf{B}|$

$$T_d = [\gamma \cdot h \cos\theta \sin\theta - (c' + \gamma \cdot h \cos^2\theta \,\tan\varphi')]L_B$$

$$T_d = [18 \times 8 \times \cos 25^\circ \times \sin 25^\circ - (10 + 18 \times 8 \times \cos^2 25^\circ \times \tan 18^\circ)] \times$$
$$\times\ 15.75 = 105.9\,\text{kN} \simeq 106\,\text{kN}$$

from which $F_p = 106 + 580 = 686\,\text{kN}$, *value greater than the passive pressure which can be developed by the soil mass.*

If slope length L of the unstable side is such that $L < (L_b + L')$ the equilibrium of the soil mass behind the wall does not require the total mobilization of the passive force, and force F_p is correspondingly less. It is computed by considering the overall equilibrium of the slope.

To conclude, we observe that *it is not always possible to stabilize an unstable slope by a retaining-wall, because soil may flow over the wall if the length of the slope is long enough.*

On the other hand, the wall stability must be ensured by calculating force F_p as shown above, considering an adequate safety factor. *A serious error would be made if F_p was considered to be the Rankine passive pressure.* Note that the calculation based on the table for earth pressure coefficients and on the theorem of the corresponding state is not possible here since in the associated cohesionless soil mass we have $\theta > \varphi$.

Remark

Blondeau and Virollet reported in special issue II (March 1976) of the *Bulletin de Liaison des Laboratoires des Ponts et Chaussées* on a similar problem, giving a solution in which no account is made of the passive force which we called Q'_B acting on BC of the imaginary plane. Their simplification we consider not permissible for the case studied where $(h - Z_1)$ is of the same order of magnitude as Z.

★★*Problem 12.7* Embankment stability on a compressible soil

An embankment design is being considered for a preliminary study of a highway. The embankment is proposed to be 7 m high, to have side slopes of 1.5 horizontal to 1 vertical and to consist of a gravelly soil whose properties are as follows: internal angle of friction: $\varphi = 35°$, cohesion $= 0$, unit weight $19\,kN/m^3$.

The embankment would be constructed on a soft clay layer of 6 m thickness and having the following characteristics:

average undrained shear strength: $c_u = 20\,kPa$,
average consolidated undrained angle of friction φ_{cu}, with $\tan \varphi_{cu} = 0.22$,
consolidation coefficient $c_v = 4 \times 10^{-7}\,m^2/s$.

The clay layer is underlain by a fractured pervious bedrock. The groundwater table is at existing ground surface.

With the charts shown on Fig. 12.28 where φ = angle of friction of the fill material, c = cohesion of the natural clay layer, F = safety factor, N = stability factor $= c/\gamma H$, specify the method of construction which would yield a stability with $F \geqslant 1.5$ (short-term consideration).

Solution

Since the design is preliminary, simplifying assumptions can be made. A final design would require a more rigorous analysis.

(1) Short-term stability of the embankment constructed in one phase

With the notations of Fig. 12.29, we have:

$D/H = 6/7 = 0.86, \quad N = c/\gamma H = 0.15$

From stability charts, we have:

for $N = 0.2$, $F = 1.12$,

for $N = 0.1$, $F = 0.6$,

so for $N = 0.15$, $F < 1$

The conclusion is that the embankment cannot be stable (short-term) if it is constructed in one phase.

(2) Analysis of the embankment stability built in various phases, phase one: (short term)

Assume that the first phase consists of building the embankment to half its proposed height, then:

$D/H = 6/3.5 = 1.7, \quad N = c/\gamma H = 0.30$

From the graphs we get:

for $N = 0.3$, $F = 1.61$.

The conclusion here is that the embankment may be constructed to half its proposed height ($F > 1.5$).

(3) Analysis of consolidation due to the embankment constructed to half its planned height

The time required to achieve 99% consolidation of the clay is: $t = T_v h^2/c_v$. Since the clay layer is drained over its two boundaries (fractured bedrock), $h = 3$ m (half thickness of clay layer), then $t = 2 \times (9/4) \times 10^7 = 4.5 \times 10^7$ s, or $t = 520$ days, say 1.5 years.

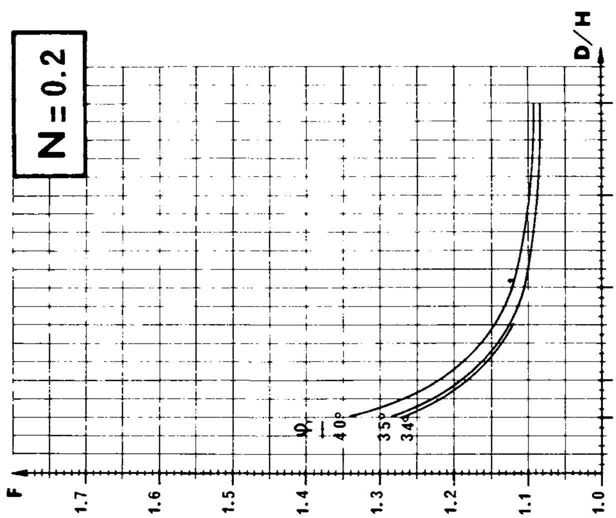
Taking into account the normal times of construction of a highway, it does not appear practical to wait that long time for a consolidation to occur. The time required to achieve 70% consolidation is:

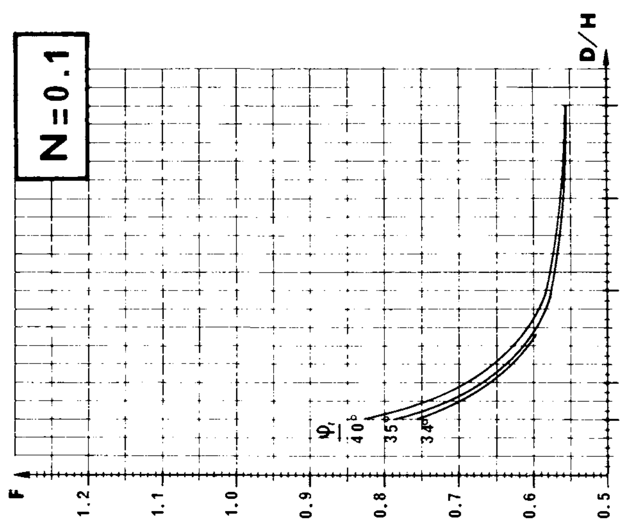
$t = T_v h^2/c_v = (0.4/4) \times 9 \times 10^7 = 0.9 \times 10^7 = 104$ days

or 3.5 months. This is acceptable.

Hence, at the end of 3.5 months, we may assume that the clay layer would be 70% consolidated. The degree of consolidation is not, of course, uniform throughout the height of the clay layer, but, for finding the increase of c_u, the following calculation is based as a first approximation on the assumption that the degree of consolidation is at any point equal to its average value say 70%.

The effective stress increment due to the load of the 3.5 m of embankment is: $\Delta\sigma' = 3.5 \times 19 \times 0.7 = 46.55$ kPa.


N = 0.2
F
D/H
ϕ′
40°
35°
34°
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.3
0.5
1.0
1.5
2.0


N = 0.1
F
D/H
ϕ′
40°
35°
34°
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.3
0.5
1.0
1.5
2.0

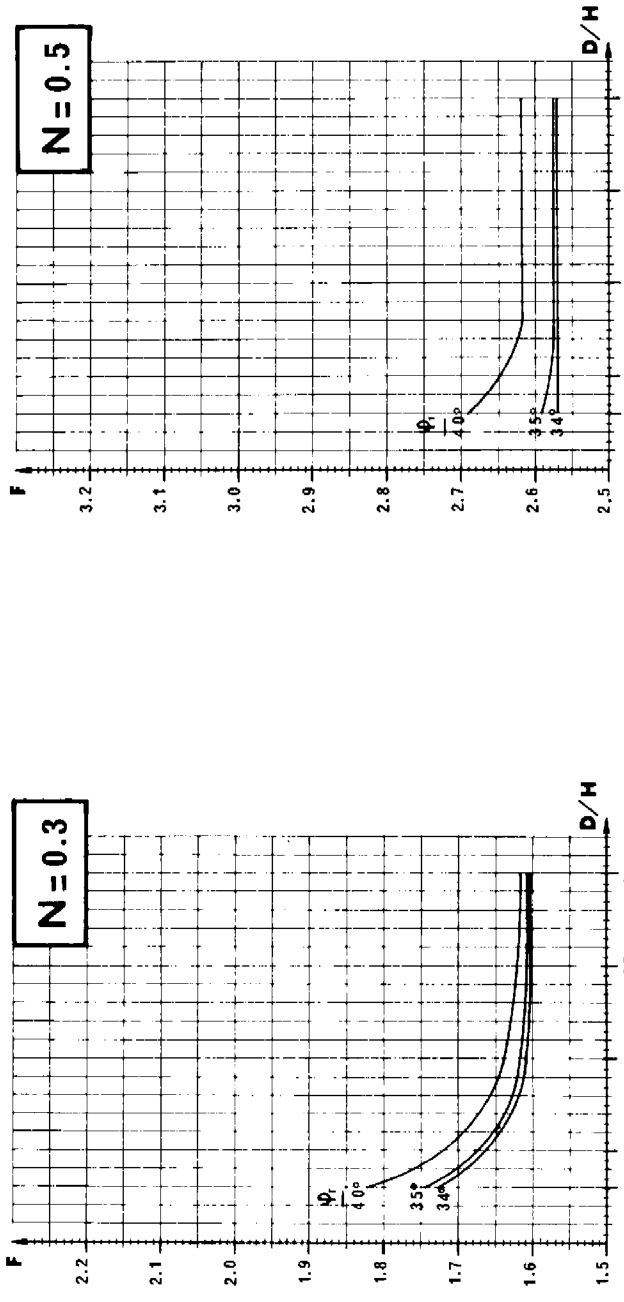


Fig. 12.28. After Pilot and Moreau. Slope 3/2. $N = c_u/\gamma H$. (c_u: undrained shear strength of clay; φ_r: friction angle of the fill).

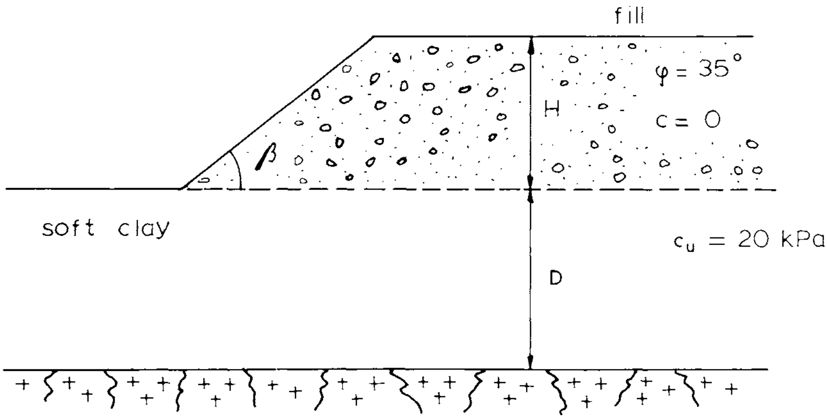


Fig. 12.29.

Assume that this stress increment is uniform over the entire clay layer including where failure circle could occur (a somewhat optimistic assumption). The increase in the undrained cohesion will then be:

$$\Delta c_u = \Delta\sigma_1' \times \tan\varphi_{cu} = 46.55 \times 0.22 = 10.2\,\text{kPa}$$

Therefore, at the end of 3.5 months, we may consider that $c_u = 20 + 10.2 = 30.25$ kPa.

(4) Analysis of the short-term stability of the second phase (second fill layer of 2 m thickness)

If the balance of the embankment were to be constructed in the second phase, F would be = 1.2, say too low. Therefore, the second phase should only consist of placing an additional 2 m of fill. We then would have: $D/H = 6/5.5 = 1.09$, $N = c_u/\gamma H = 30.25/(19 \times 5.5) = 0.29$ which (from the graphs) gives a safety factor of the order of 1.55, which is acceptable.

As done above, we must wait for about 3.5 months so that the degree of consolidation will be 70% (assuming that c_v remains constant).

(5) Analysis of the short-term stability of the third phase (additional 1.5 m of fill)

At the end of the second phase (additional 2 m of fill placed since 3.5 months) the increase of the average effective stress in the clay will be about:

$$\Delta\sigma_3' = 0.7 \times 2 \times 19 = 26.6\,\text{kPa}$$

Then the increase of the cohesion of the clay is:

$$\Delta c_u = 26.6 \times 0.22 = 5.85\,\text{kPa}$$

Hence:

$c_u = 30.25 + 5.85 = 36.1 \text{ kPa}$

and therefore $D/H = 6/7 = 0.86$,

$N = c_u/\gamma H = 36.1/19.7 = 0.271$

which gives a safety factor close to 1.5. Once again, this is acceptable.

The method of construction should therefore consist of 3 phases each allowing a placement of 3.5 m, 2 m and 1.5 m of fill thickness with consolidation times of 3.5 months between phases.

Remarks

(1) Because of the preliminary nature of the evaluation, numerous simplifications were made.

(2) Problem only dealt with the embankment stability, the final design should further evaluate the embankment settlements.

(3) If the calculated time intervals between loadings cannot fit into the construction schedule, fills with enlarged berms can be applied at reduced time intervals. Enlarged berm will require more materials.

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INDEX

References beginning with numbers 1 to 6 = Volume 1.
References beginning with numbers 7 to 12 = Volume 2.
References correspond to the problem numbers.

Absi, Garnier, Giroud, 6.15
Absi's theory, 3.14
Allowable load, 11.5; 11.6
Allowable stress, under a footing, 6.3; 6.4; 6.5; 6.7; 6.8; 6.11; 6.12; 6.17; 10.1; 10.3; 10.4; 10.5; 10.6; 10.7; 10.10; 11.1; 11.2; 11.3; 11.8
Anchors, 7.9; 8.1; 8.2; 9.2
—, for retaining walls, 7.9
Anchor wall, 8.2
Anchored wall, 9.4
Andina, penetrometer, 6.5; 6.8; 6.9; 6.10; 6.11; 6.12; 6.14
Angle of internal friction, 4.1; 4.2; 4.3; 4.4; 4.5
Anisotropy, flow through, 2.12
Apparent cohesion, 4.17
Atterberg limits, 1.6; 1.7; 6.19

Bearing capacity, 10.1; 10.2; 10.3; 10.4; 10.5; 10.6; 10.7; 10.9; 10.10; 10.14; 10.15
— — factor, 5.5; 6.17; 11.6
Bending moment diagram, 8.1; 8.3; 8.5; 9.2; 9.3; 9.4
Bentonite, trench with, 9.1
Bevac, dynamic penetrometer, 6.8
Bishop, coefficients of, 4.16
Blum, method of, 8.3; 9.2
Bottom heave, 8.6
Boussinesq, equilibrium, 7.3; 7.5
—, formula of, 10.9
Broms, coefficients of, 11.5

Capillary rise, 2.10; 2.14; 2.15
Caquot and Kerisel (method of), 3.17; 11.5
Casagrande, chart of, 1.7
—, device, 4.10
—, formula of, 2.3
Circular footing, 10.1; 11.8; 6.23
Circular mat, 10.12
Coefficient of compressibility, 3.16
Coefficient of consolidation, 3.8; 3.9; 3.10; 3.11; 3.13; 3.14
Coefficient of earth pressure at rest, 4.17; 7.1
Coefficient of permeability, 2.1; 2.2; 2.3; 2.4; 2.18; 3.9
Cohesion, 4.2; 4.3; 4.4; 4.5; 6.3; 6.4
Compaction, 1.9
—, maximum, 1.14
Compression curve, 3.1; 3.2; 3.11
— index, 3.2; 3.11; 3.12; 3.16; 4.4
Cone penetrometer, 6.3; 6.4; 6.11
Consolidation, 3.8; 3.10; 3.11; 3.13; 3.14; 3.19
— curve, 3.11
— pressure, 3.4; 3.11; 3.15; 3.17
— test, oedometric, 3.1; 3.2; 3.11; 3.12; 3.15; 4.12; 6.19; 6.20
Cordary, method of, 6.14
Correction of a grain-size distribution, 1.8
Coulomb's criteria, 10.9
— method, 7.10
— wedge, 7.10

Cubic dilatation, 4.13
Cullman's method, 7.5
Cut in clay, 8.6

Dam, homogeneous, 12.3; 12.4
—, seepage under, 2.12
— with core, 12.5; 4.18
Deformation tensor, 4.13
Deformations, of a wall, 8.1; 8.5; 9.2; 9.3; 9.4
Degree of consolidation, 3.8; 3.13; 3.16; 3.19
Delft (penetrometer cone), 6.3; 6.4; 6.7
Density, 1.1; 1.3; 1.4; 6.19
—, relative, 1.12
Deviator, 4.13; 4.14
Differential uplift, 10.13
Dike, 12.3; 12.4; 12.5
Drain, sand, 3.13
—, toe, 2.11
Drainage, behind a wall, 7.8
—, blanket, 2.11
Drains, vertical, 2.16
Drawdown, 2.16
Durmeyer (penetrometer), 6.5
Dynamic penetration test, 6.5; 6.6; 6.7; 6.8
Dynamic penetrometer, 11.10

Earth pressures, 7.1; 7.2; 7.3; 7.4; 7.5; 7.6; 7.7; 7.8; 7.9; 7.10; 7.11; 8.1; 8.2; 8.3; 8.4; 9.2; 9.3; 9.4
Effective diameter, 1.5
Effective stresses, 2.6; 2.7; 2.14; 2.16; 4.5; 4.6
Elasto-plasticity, 8.1; 8.5; 9.2; 9.4
Elements, of reinforced earth, 7.11
Embankment stability, 12.7
Embedment, 9.2
—, critical, piles, 11.5
—, of sheet piles, 8.1; 8.3; 8.4
Excavation, limit height of, 5.7; 9.1; 12.1
— piping into, 2.9

Failure envelope, 4.4
— lines, 5.4; 5.5; 5.6; 5.13
— plane, 12.1; 12.2
Failures, 7.6; 7.9; 11.8
Fender pile, 8.5
Fill, preloading, 3.11; 3.14; 6.13
— settlement, 3.11; 3.13; 6.13; 6.14; 6.16
Filter blanket, 2.13
Filters, 2.13
Finite elements, 2.12; 3.19; 8.5
Flow, 2.5; 2.11; 2.16
— net, 2.12; 7.18; 12.3
—, of a leak, 2.12; 2.13
—, plane, 2.9; 2.11; 2.12; 2.13; 7.8; 12.2
Foundations, deep, 11.1—11.9; 6.11; 6.12; 6.19
—, shallow, 3.5; 3.6; 3.17; 3.18; 3.20; 6.3; 6.4; 6.7; 6.8; 6.9; 6.10; 6.11; 6.13; 6.14; 6.15; 6.17; 10.1—10.13
Freezing probe, 2.19
— of soils, 2.19
Friction, lateral, 11.5; 11.6
— ratio, 6.1; 6.3
Fröhlich's formula, 10.9
Fugro, penetrometer, 6.4

Geuze, rule of, 6.12
Giroud, diagrams of, 6.9; 6.11; 6.13; 6.15; 10.7; 10.11
Global method, 12.3
Gouda (penetrometer), 6.3; 6.7; 6.8; 6.13; 6.16
Grain size, 1.5; 1.13; 1.14; 2.12; 6.19
— distribution, 1.5
— — curve, 1.5

Hazen's coefficient, 1.5
Heave, of the bottom of a cut in clay, 8.6

Height, limit of excavation, 5.7
Henkel's coefficient, 4.14; 4.15
H.R.B. classification, 1.6
Hydraulic gradient, 2.11
— —, critical, 2.8

Ice wall (freezing), 2.19
Inclined and eccentric load, 7.3; 10.12
Influence coefficient, 3.5
In-situ loading test, 11.10
Interaction of neighbouring footings, 3.5; 3.18; 6.10
Isochrones, 3.19; 6.16
Isolated footing, 6.22; 6.23; 10.2; 10.6; 10.7; 10.8; 10.13
Isotropic tensor, 4.13; 4.14

Limit equilibrium, 5.2; 5.3; 5.5; 5.8
Long-term calculations, 10.10; 11.8

Mandel and Salencon, method of, 10.11
Mat foundation, 6.3; 6.11; 10.6; 10.7; 10.8; 10.10; 10.11
Menard's graphs, 11.6
Meyerhof's formula, 6.17
— method, 6.12; 7.3; 7.10; 7.11
Mixture, grain size of, 1.13
Mohr's circle, 4.1; 4.2; 4.3; 4.4; 4.6; 4.12; 5.1; 5.2; 5.3; 5.4; 5.5; 5.6; 5.8; 10.9

Newmark's chart, 3.18; 6.16
Normally consolidated clay, 3.3; 6.20

Oedometric diagram, 3.1; 3.2; 3.11; 3.12; 3.15; 3.17; 6.20
Oedometric modulus, 3.1; 3.2; 3.12; 4.8
Optimum (Proctor), 1.9
Organic matter content, 1.10
Organic, soils, 1.10
Overconsolidated clay, 3.4; 3.11; 3.12; 3.15

Parez, penetrometer, 6.15
Passive earth pressure, 8.1; 8.2; 8.3; 8.4; 9.2; 9.3; 9.4
Penetrometer, with mud, 6.6
—, Andina, 6.4; 6.5; 6.8; 6.9; 6.10; 6.11; 6.12; 6.14; 6.20
—, Bevac, 6.8; 6.9
—, Durmeyer, 6.5
—, Gouda, 6.3; 6.7; 6.8; 6.13; 6.16
—, Sermes, 6.6; 6.7
Penetrometer, dynamic, 6.2; 6.5; 6.6; 6.7; 6.8; 6.9
—, static, 6.1; 6.4; 6.7; 6.8; 6.9; 6.10; 6.11; 6.12; 6.13; 6.14; 6.15; 6.16; 6.20; 11.1; 11.2
—, static-dynamic, 6.19, 11.3
Permeability, coefficient, 2.1; 2.2; 2.3; 2.4; 2.18; 3.9
Permeameter, constant head, 2.1; 2.18
—, variable head, 2.29
Phase construction of fill, 12.7
— excavation, 9.2; 9.4
Pier (drilled), 11.7
Pile bearing capacity, 11.10; 11.11
Piles, 6.12; 6.19
—, drilled, 11.3; 11.6; 11.7
—, driven, 11.1; 11.2; 11.5
—, enlarged base, 11.8
Piping, 2.9; 2.17
— condition, 2.17
Plane strain, 4.16
Plasticity chart, Casagrande graph, 1.17
— index, 1.6
Plate bearing test, 3.7
Poisson's ratio, 4.8; 4.9
Pole, of Mohr's circle, 5.1
Porosity, 1.1; 1.4; 1.9
Prandtl's wedge, 5.5
Prefabricated wall, 9.2

Preloading, 3.11; 3.14; 6.13
Pressure (limit), 6.21; 11.6
Pressuremeter, 6.19; 6.21; 6.22; 6.23; 11.6
— modulus, 6.21; 6.22; 6.23
— test, 6.19; 6.21; 6.22; 10.14; 10.15; 11.11
Prestressed anchors, slurry wall, 9.2
— —, sheet piles, 8.1; 8.2; 8.3; 8.4
Principal directions, 4.10; 5.1
Principal stresses, 4.10; 4.12; 4.18
Proctor diagram, 1.9
— test, 1.9
Pumping test, 5.5

Radius of influence, 2.5
— of freezing, 2.19
Raft, 10.16
Rankine equilibrium, 5.2; 5.3; 5.4; 5.5; 7.1; 7.9; 7.11
Rapid drawdown, 12.5
Reaction modulus, 8.5
Regular grain arrangement, 1.14; 2.15
Reinforced earth, 7.10; 7.11
Retaining wall, 7.1; 7.2; 7.3; 7.4; 7.5; 7.6; 7.7; 7.8
— —, gravity, 7.3; 7.4; 7.6; 7.7
— —, reinforced earth, 7.10; 7.11
Rido program, 8.1
Rigidity of footings, 6.11

Safety factor of reinforcement, reinforced earth, 7.10; 7.11
— —, wall sliding, 7.3; 7.9; 7.10; 12.1; 12.2; 12.3; 12.4; 12.5
Sand liquefaction, 2.9; 2.13
Saturation curve, 1.9
—, degree of, 1.2; 1.9; 1.10
Schmertmann's method, 6.10
Schneebeli's rods, 1.14
Sedimentometry, 1.11
Sermes penetrometer, 6.6; 6.7
Settlements, 3.3; 3.4; 3.6; 3.11; 3.16; 3.17; 3.18; 3.20; 6.9; 6.10; 6.11; 6.13; 6.14; 6.15; 6.16; 6.17; 6.20; 6.23; 10.7; 10.8; 12.7
Shallow footing, 10.14
Shear, resistance, 4.5
— strength test, Casagrande box, 4.10; 4.12; 6.19
— stress, diagrams, 8.1; 9.2; 9.3; 9.4
Sheet pile wall, 8.1—8.4
— piles, 8.1—8.4
Short term, calculations for, 6.3; 10.10; 11.8
Sieving, 1.5
Simple point, penetrometer, 6.4; 6.11
Skempton's coefficient A_f, 4.11
— (Bishop and) coefficients, 4.16
— formula, 3.3; 3.4; 10.7
Slice method, 12.4
Slip circle, 12.1; 12.3; 12.4; 12.5
Slope, drainage in, 2.11
—, failure of, 12.2; 12.3; 12.4; 12.5
— stability, 12.2; 12.3; 12.4; 12.5
Slurry wall, 9.1; 9.2; 9.3
S.P.T., 6.17; 6.18; 6.19; 11.10
Square footing, 3.5; 3.18; 10.5; 10.6; 10.7; 10.8
Stability against overturning (of a wall), 7.3; 7.9; 7.10
—, internal, reinforced earth, 7.10
Static penetration test, 6.1; 6.2; 6.3; 6.4; 6.10; 6.11; 6.12; 6.14; 6.15; 6.16; 6.19; 6.20
— penetrometer, 11.10
Stiffness coefficient, 3.7
Stratified soil, 7.7
Stress path, 4.12; 4.18
— tensors, 4.18; 5.1; 5.2; 5.3; 5.4; 5.5; 5.6
Stresses under footings, 3.5; 3.18
Swelling, of clays, 10.13; 11.7; 11.8
Strip footing, 6.4; 6.7; 6.10; 6.11; 6.13; 6.15; 6.19; 10.1; 10.3;

Strip footing, 10.4; 10.6; 10.7; 10.8; 10.9; 10.10; 10.13
Superposition, limit equilibrium, 5.8
Surface tension, 2.15

Terzaghi's diagram, 9.4
— equation, 3.19
— formula, 2.3
Terzaghi and Peck's charts, 6.17
Time of consolidation, 3.8
— factor, 3.8; 3.9; 3.10; 3.11; 3.16; 3.19
Tip resistance, penetrometer, *see* Penetrometer
— —, piles, 11.5; 11.6
Total stresses, 4.6
Tran Vo Nhiem, 10.12
Triaxial test, 3.17; 4.1; 4.2; 4.3; 4.5; 4.6; 4.7; 4.9; 4.10; 4.11; 4.12; 4.15; 4.17; 6.19
Tri-layer system, 6.12; 6.14
Two-layer system, 3.19; 6.9; 10.7; 10.8; 10.11
Tschebotarioff's method, 8.4

Ultimate load, 11.5
Unconfined compression (resistance to) 4.7; 6.19
Uniformity coefficient (Hazen's) 1.5
Unit weight, 1.1; 1.3; 1.10; 1.14

Void, air, 1.9
— ratio, 1.1; 1.2; 1.4; 1.10; 1.12; 1.14; 3.15

Water content, 1.1; 1.2; 1.4; 6.19
Weightless material, 5.5
Well, pumping, 2.5

Young's modulus, 4.8