Frederick E. Petry
Vincent B. Robinson
Maria A. Cobb

Fuzzy Modeling with Spatial Information for Geographic Problems

Frederick E. Petry
Vincent B. Robinson
Maria A. Cobb
(Editors)

Fuzzy Modeling with Spatial Information for Geographic Problems

With 135 Figures

Springer

Professor Dr. Frederick E. Petry
Tulane University, Dept.of Electrical Engineering and Computer Science
310 Stanley Thomas Hall, New Orleans LA 70118
USA
e-mail: fep@eecs.tulane.edu

Professor Dr. Vincent B. Robinson
University of Toronto at Mississauga, Dept. of Geography
3359 Mississauga Road North, Mississauga, Ontario
Canada L5L 1C6
e-mail: vbr@geomant.erin.utoronto.ca

Professor Dr. Maria A. Cobb
University of Southern Mississippi, Dept. of Computer Science and Statistics
Hattiesburg MS 39406-5106
USA
e-mail: maria.cobb@usm.edu

Library of Congress Control Number: 2004115949

ISBN 3-540-23713-5 **Springer Berlin Heidelberg New York**

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitations, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

Springer is a part of Springer Science+Business Media
springeronline.com
© Springer-Verlag Berlin Heidelberg 2005
Printed in Germany

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Sponsoring editor: Janusz Kacprzyk
Cover design: Erich Kirchner
Production: Luisa Tonarelli
Typesetting: Final layout by the editors
Printing: Krips, Meppel
Binding: Litges + Dopf, Heppenheim

Printed on acid-free paper 30/2132/LT – 5 4 3 2 1 0

Foreword

The capabilities of modern technology are rapidly increasing, spurred on to a large extent by the tremendous advances in communications and computing. Automated vehicles and global wireless connections are some examples of these advances. In order to take advantage of such enhanced capabilities, our need to model and manipulate our knowledge of the geophysical world, using compatible representations, is also rapidly increasing. In response to this one fundamental issue of great concern in modern geographical research is how to most effectively capture the physical world around us in systems like geographical information systems (GIS). Making this task even more challenging is the fact that uncertainty plays a pervasive role in the representation, analysis and use of geospatial information. The types of uncertainty that appear in geospatial information systems are not the just simple randomness of observation, as in weather data, but are manifested in many other forms including imprecision, incompleteness and granularization. Describing the uncertainty of the boundaries of deserts and mountains clearly require different tools than those provided by probability theory. The multiplicity of modalities of uncertainty appearing in GIS requires a variety of formalisms to model these uncertainties. In light of this it is natural that fuzzy set theory has become a topic of intensive interest in many areas of geographical research and applications

This volume, *Fuzzy Modeling with Spatial Information for Geographic Problems*, provides many stimulating examples of advances in geographical research based on approaches using fuzzy sets and related technologies. It includes chapters on diverse research topics such as spatial directions, geographical interpolation, landscape features and spatial decision systems among others. The editors, Maria Cobb, Vince Robinson and Fred Petry provide a snapshot of current topics of research and should stimulate work in this area and hopefully encourage more cross-disciplinary efforts such as demonstrated by these chapters. The papers published in this volume should be of considerable interest to a broad spectrum of researchers in the fuzzy set and GIS areas as well as those engineers who make use of geospatial information in their applications and systems.

Ronald R. Yager
New York, NY USA
August 25, 2004

Preface

This volume, the companion to *Flexible Querying and Reasoning in Spatio-Temporal Databases* edited by Rita De Caluwe, Guy De Tre, and Gloria Bordogna, focuses on advances in research on approaches to incorporating explicit handling of uncertainty, especially by fuzzy sets, to address geographic problems. Over the past several years interest in the use of fuzzy approaches has grown across a broad spectrum of fields that use spatial information to address geographic problems.

The reasoning about geographic information representing regions, relations, and/or fields is fundamental to any progress in the application of fuzzy sets to modeling geographical problems. There are several papers in this volume that advance our understanding of these fundamental issues. Hans Guesgen builds on his previous work that introduces fuzzy sets into the artificial intelligence community's RCC theory. His results suggest that the formalism developed by converting RCC8 relations into fuzzy sets and applying a fuzzy RCC8 algorithm is robust under uncertainty. Pascal Matsakis and Dennis Nikitenko focus on issues of modeling fuzzy spatial relations. They introduce the Force Histogram (F-histogram) and proceed to illustrate that the F-histogram is a valuable tool for extracting directional and topological relationship information from two spatial objects exploiting a fuzzification of Allen relations.

Much of the research on fuzzy modeling applied to geographical problems is based on a geographic information system that represents information as layers and uses a field based approach to processing the spatially explicit data. Jörg Verstraete and colleagues present their exploration of two types of field based methods for the modeling of fuzzy spatial data. They discuss the extended triangulated irregular networks and extended bitmap models with respect to fuzzy membership values, fuzzy numbers, operations, type-2 fuzzy sets, and possibilistic truth values. Thus, it is an in depth exploration of fuzzy extensions to two very important, fundamental models of geographic information.

Sungsoon Hwang and Jean-Claude Thill model localities as fuzzy regions represented as eggs in the egg-yolk model of spatial representation. Their study illustrates a real world problem domain where fuzzy regions and linguistic variables are shown to be useful in addressing the problem of pinpointing the location of a traffic accident given limited and imprecise (*e.g.* linguistic) information. In the case of emergency dispatch operations, the outcome of this process could have profound consequences.

One of the most common approaches to fuzzy modeling of spatial data for geographical problem solving consists of constructing a fuzzy classification. The fuzzy k-means algorithm has a long history of being applied to geographical problems.

Zhijan Liu and Roy George propose an extension to the fuzzy k-means algorithm to account for both spatial and temporal data. They demonstrate its utility in another important problem area of geographical data analysis, namely data mining, by showing that it is able to identify interesting phenomena with a large weather data set. Cidália Fonte and Weldon Lodwick identify four different sources of fuzziness in their two phase classification procedure. For each source of fuzziness, a method to compute the membership grades for fuzzy geographical entities is presented, based on semantic interpretation of the grades of membership. These semantic interpretations are the likelihood view, the random set view and the similarity view. They show that these semantic interpretations are suitable for construction of fuzzy geographical entities.

Although spatial interpolation is a commonly used technique in geographical analysis, the use of fuzzy spatial interpolation is not yet widespread, especially when incorporating temporal dynamics. Suzana Dragićević presents the potential of using fuzzy set theory to deal with imperfect geographic data and entities when applying GIS based spatial and spatio-temporal interpolation.

Susan Kratochwill and Josef Benedikt present the argument that the uncertainty inherent in geographic information systems is due to the semantics of categorization using linguistic symbols in a process of communication. They go on to present the Talking Space platform for mapping spatial knowledge with uncertainty. Ferdinando Di Martino and colleagues show how the FUZZY-SRA software tool is used to evaluate the reliability of environmental data for the island of Procida.

Landscape features have long been recognized as being inherently fuzzy concepts whose inherent fuzziness has historically been difficult to represent in a manner that is flexible enough to be useful in any but a single problem domain operating at a single scale. Xun Shi and colleagues present a similarity-based method for deriving fuzzy representation of terrain features such as ridges (broad vs narrow), headwaters, and “knobs” that is computationally efficient, effective and flexible. Peter Fisher and colleagues explain the effect that scale has on how landscape features can be modeled using fuzzy sets. This paper represents one of the first to explicitly model landscape morphometry at multiple scales using fuzzy sets.

With the rapid escalation in computational technology and digital geographic data fuzzy modeling of spatial data has become increasingly important in those applications where decision making is of utmost importance. Frank Witlox and Ben Derudder elaborate on fuzzy decision tables as an important addition to qualitative modeling. They show it is possible to explicate the imprecision involved in the decision making process through use of fuzzy decision tables and discuss possible limitations, especially in relation to the use of fuzzy knowledge based systems. Ashley Morris and Piotr Jankowski present the FOOSBALL system that allows for multiple criteria fuzzy queries over an object oriented spatial database. Vince Robinson and Phil Graniero present a computational framework and methodology for modeling small mammals as mobile fuzzy agents making decisions during their dispersal process.

This book has the two aims. One is to stimulate further research in both the theory and application of fuzzy sets to spatial information management and geographic problem solving. The other is to show the advances in research that have matured to the point that we find fuzzy modeling being used by geoscientists, computer scientists, geographers, ecologists, engineers, and others.

Acknowledgements

We would like to thank the authors of the papers. Reviewing and editing their interesting contributions was a pleasure. We would especially like to thank the anonymous referees who so generously gave of their time and expertise under tight deadlines to improve each of the contributions.

Maria Cobb, University of Southern Mississippi
Hattiesburg, Mississippi USA
maria.cobb@usm.edu

Frederick Petry, Tulane University
New Orleans, Louisiana USA
fep@eecs.tulane.edu

Vince Robinson, University of Toronto
Mississauga, Ontario Canada
vbr@geomant.erin.utoronto.ca

September 2, 2004

Contents

Foreword v
Preface vii

Reasoning About Regions, Relations, and Fields

1. Fuzzy Reasoning about Geographic Regions
Hans W. Guesgen 1

2. Combined Extraction of Directional and Topological Relationship Information from 2D Concave Objects
Pascal Matsakis and Dennis Nikitenko 15

3. Field Based Methods for the Modeling of Fuzzy Spatial Data
Jörg Verstraete, Guy De Tré, Rita De Caluwe and Axel Hallez 41

4. Modeling Localities with Fuzzy Sets and GIS
Sungsoon Hwang and Jean-Claude Thill 71

Fuzzy Classification

5. Mining Weather Data Using Fuzzy Cluster Analysis
Zhijian Liu and Roy George 105

6. Modelling the Fuzzy Spatial Extent of Geographical Entities
Cidália Costa Fonte and Weldon A. Lodwick 121

7. Multi-Dimensional Interpolations with Fuzzy Sets
Suzana Dragićević 143

8. Talking Space – A Social & Fuzzy Logical GIS Perspective on Modelling Spatial Dynamics
Susanne Kratochwil and Josef Benedikt 159

9. A Valuation of the Reliability of a GIS Based on the Fuzzy Logic in a Concrete Case Study
Ferdinando Di Martino, Vincenzo Loia, Salvatore Sessa, and Michele Giordano 185

Fuzzy Representations of Landscape Features

10. Fuzziness and Ambiguity in Multi-Scale Analysis of Landscape Morphometry
Peter Fisher, Jo Wood, and Tao Cheng 209

11. Fuzzy Representation of Special Terrain Features Using a Similarity-based Approach
Xun Shi, A-Xing Zhu, and Rongxun Wang 233

Decision Making with GIS and Fuzzy Sets

12. Spatial Decision-Making Using Fuzzy Decision Tables: Theory, Application and Limitations
Frank Witlox and Ben Derudder 253

13. Spatial Decision Making Using Fuzzy GIS
Ashley Morris and Piotr Jankowski 275

14. Spatially Explicit Individual-Based Ecological Modeling with Mobile Fuzzy Agents
Vincent B. Robinson and Phil A. Graniero 299

Index 335

1. Fuzzy Reasoning about Geographic Regions

Hans Werner Guesgen

Computer Science Department, University of Auckland
Private Bag 92019, Auckland, New Zealand
hans@cs.auckland.ac.nz

Abstract. Reasoning about geographic regions, like forests, lakes, cities, etc., often involves uncertainty and imprecision. For example, when we talk about a region like the city of Auckland, we usually do not know exactly the boundaries of that region. Nevertheless, we are able to reason about such a region. Or if we hear on the radio that a cold front is moving in from Antarctica, we can estimate when it will reach New Zealand, although we might not be able to determine with certainty the exact relation between the area covered by the cold front and the one that is referred to as New Zealand.

Recently, the RCC theory has gained a particular interest in the AI research community as formalism to reason about regions. This first-order theory is based on a primitive relation, called connectedness, and uses eight topological relations, defined on the basis of connectedness, to provide a framework to reason about regions. Lehmann and Cohn have introduced an extension to the RCC theory, which deals with imprecision in spatial representations. Our work carries on from there by applying fuzzy sets to the RCC theory and introducing a uniform framework to reason about geographic regions under uncertainty and imprecision.

1.1. Introduction

In the last two decades, the amount of work on formalisms based on spatial relations has increased steadily. Early approaches mainly used extensions of Allen's interval algebra (Allen, 1983) for reasoning about space. In (Guesgen and Hertzberg, 1993), for example, we introduce a form of spatial reasoning that extends Allen's relations to the three dimensions of space by applying very simple methods for constructing higher-dimensional models and for reasoning about them. Freksa (1990) uses the same set of relations and shows that for an important class of problems, only a small subset of all possible combinations of spatial relations can occur. By restricting himself to sets of conceptually neighboring relations, he can restrict the complexity of the constraint satisfaction algorithms significantly.

Hernández (1991) introduces an extension of Allen's approach to represent the spatial features occurring in 2D projections of 3D scenes. He suggests to establish spatial relations between objects by splitting them up into two aspects: projection and orientation. Mukerjee and Joe's work (1990) is similar to Hernández's approach. Objects of a two-dimensional world are characterized by the directions in which the objects are moving and by associating with the objects trajectories along which they are moving.

Kettani and Moulin (1999) use the notion of spatial conceptual maps to generate and describe routes in a qualitative way. Their spatial models are based on the notion of object influence areas. These areas determine how people reason about objects, evaluate metric measures, qualify distances between objects, etc. Musto *et al.* (2000) also use a qualitative approach to describe routes (or courses of motion, as they call them). They use qualitative motion vectors to abstract from irrelevant details of a course of motion.

Recently, the RCC theory (Randell et al., 1992) has gained a particular interest in the AI research community as formalism to reason about regions. This first-order theory is based on a primitive relation, called connectedness, and uses eight topological relations, defined on the basis of connectedness, to provide a framework to reason about regions. Lehmann and Cohn (Lehmann and Cohn, 1994) have introduced an extension to the RCC theory, which deals with imprecision in spatial representations. Our own work (Guesgen, 2002, 2003) carries on from this work by introducing fuzzy sets into the RCC theory. This chapter builds on our previous results and introduces a uniform framework based on fuzzy sets to reason about geographic regions under uncertainty and imprecision.

1.2. The RCC Theory

The idea of using relations to reason about spatio-temporal information dates back at least to the beginning of the eighties, when Allen (1983) introduced an interval logic for reasoning about relations between time intervals. Although Allen's logic can be used to reason about two-dimensional space (Guesgen, 1989), it often leads to counterintuitive results. For example, consider two rectangular-shaped regions, one of which is not aligned to the reference axes (see Figure 1.1). Using Allen's

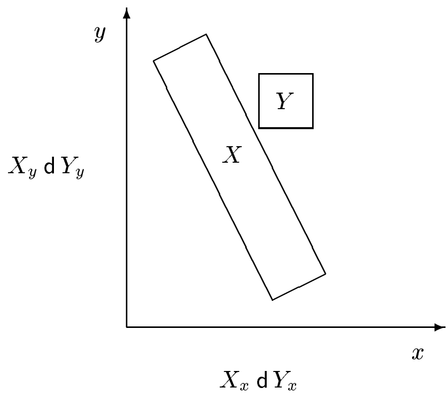


Fig. 1.1. The relations between two rectangles with respect to the x-axis and y-axis, where d denotes the Allen relation during.

relations to describe the relationships between the projections of the rectangles on to the x-axis and y-axis, respectively, would lead to a counterintuitive description: both projections suggest the during relations, although the two rectangles do not have any area in common.

The RCC theory (Randell et al., 1992) avoids this problem by defining the relation between two regions based on their topological properties and therefore independently of any coordinate system. The basis of the RCC theory is the connection relation, C, which is a reflexive and symmetric relation, satisfying the following axioms:

1. For each region X: $\mathsf{C}(X, X)$
2. For each pair of regions X, Y: $\mathsf{C}(X, Y) \rightarrow \mathsf{C}(Y, X)$

From this relation, additional relations can be derived, which can be arranged in a lattice structure as shown in Figure 1.2. From these relations, the RCC8 relations can be selected, which are jointly exhaustive and pairwise disjoint: [1]

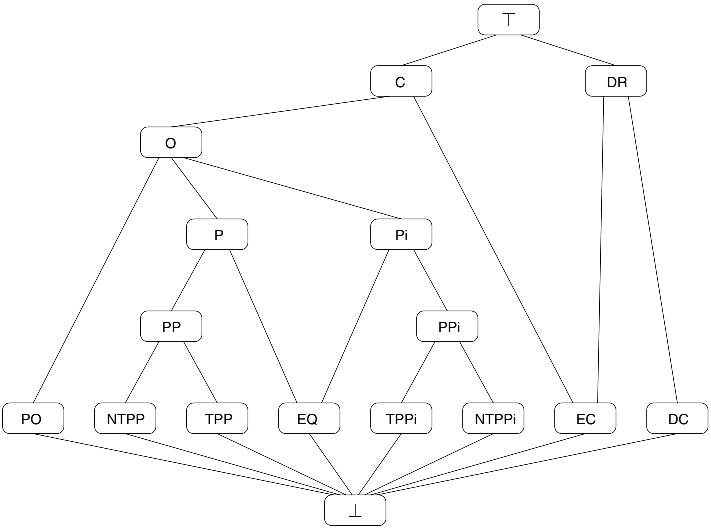
$$\mathsf{RCC8} = \{\mathsf{DC}, \mathsf{EC}, \mathsf{PO}, \mathsf{EQ}, \mathsf{TPP}, \mathsf{TPPi}, \mathsf{NTPP}, \mathsf{NTPPi}\}$$

There are different ways to reason about RCC8 relations. Since the RCC theory is expressed in first-order predicate logic, theorem provers can be used to infer new relations from a set of given ones. More popular, however, is reasoning based on a composition table such as the one shown in Figure 1.3, which describes how relations depend on each other. In particular, given the relation R_1 between the regions X and Y, and the relation R_2 between the regions Y and Z, the composition table determines the relation R_3 between the regions X and Z, i.e., $\mathsf{R}_3 = \mathsf{R}_1 \circ \mathsf{R}_2$. In the case of a set of regions with more than three regions, the composition table can be applied repeatedly to three-element subsets of the set of regions until no more relations can be updated, resulting in a set of relations that is locally consistent.

1.3. Uncertainty and Imprecision in Spatial Relations

Reasoning about space often has to deal with some form of uncertainty or imprecision. Referring to the example used before, when we talk about a region like the city of Auckland, we usually do not know exactly where the boundaries are for that region. Nevertheless, we are perfectly capable of reasoning about such a region, like determining the landmarks that are a part of (TPP or NTPP) Auckland. Or if we hear on the radio that a cold front is moving in from Antarctica, we can estimate when this front "connects" to New Zealand, although we might not be able to determine with certainty whether, at that time, the cold front will be disconnected from (DC), externally connected to (EC), or already partially overlapping (PO) New Zealand. This chapter uses fuzzy logic to deal with this type of issues. Our approach is based on the concept of conceptual neighbors, which was first introduced by Freksa (1992) for Allen relations and later applied to the RCC theory (Cohn et al., 1997; Cohn and Gotts, 1996).

[1] See Figure 1.4 for a graphical illustration of the RCC8 relations.



Relation and its interpretation	**Definition of the relation**
$\mathsf{DC}(X, Y)$ (X disconnected from Y)	$\neg\mathsf{C}(X, Y)$
$\mathsf{P}(X, Y)$ (X part of Y)	$\forall Z[\mathsf{C}(Z, X) \to \mathsf{C}(Z, Y)]$
$\mathsf{PP}(X, Y)$ (X proper part of Y)	$\mathsf{P}(X, Y) \wedge \neg\mathsf{P}(Y, X)$
$\mathsf{EQ}(X, Y)$ (X identical with Y)	$\mathsf{P}(X, Y) \wedge \mathsf{P}(Y, X)$
$\mathsf{O}(X, Y)$ (X overlaps Y)	$\exists Z[\mathsf{P}(Z, X) \wedge \mathsf{P}(Z, Y)]$
$\mathsf{DR}(X, Y)$ (X discrete from Y)	$\neg\mathsf{O}(X, Y)$
$\mathsf{PO}(X, Y)$ (X partially overlaps Y)	$\mathsf{O}(X, Y) \wedge \neg\mathsf{P}(X, Y) \wedge \neg\mathsf{P}(Y, X)$
$\mathsf{EC}(X, Y)$ (X externally connected to Y)	$\mathsf{C}(X, Y) \wedge \neg\mathsf{O}(X, Y)$
$\mathsf{TPP}(X, Y)$ (X tangential proper part of Y)	$\mathsf{PP}(X, Y) \wedge \exists Z[\mathsf{EC}(Z, X) \wedge \mathsf{EC}(Z, Y)]$
$\mathsf{NTPP}(X, Y)$ (X nontangential proper part of Y)	$\mathsf{PP}(X, Y) \wedge \neg\exists Z[\mathsf{EC}(Z, X) \wedge \mathsf{EC}(Z, Y)]$

Fig. 1.2. Spatial relations derived from the connection relation, arranged in a lattice structure.

	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	no info	DR, PO PP	DR, PO PP	DR, PO PP	DR, PO PP	DC	DC	DC
EC	DR, PO PPi	DR, PO TPP, TPi	DR, PO PP	EC, PO PP	PO, PP	DR	DC	EC
PO	DR, PO PPi	DR, PO PPi	no info	PO, PP	PO, PP	DR, PO PPi	DR, PO PPi	PO
TPP	DC	DR	DR, PO PP	PP	NTPP	DR, PO TPP, TPi	DR, PO PPi	TPP
NTPP	DC	DC	DR, PO PP	NTPP	NTPP	DR, PO PP	no info	NTPP
TPPi	DR, PO PPi	EC, PO PPi	PO, PPi	PO, TPP TPi	PO, PP	PPi	NTPPi	TPPi
NTPPi	DR, PO PPi	PO, PPi	PO, PPi	PO, PPi	O	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

Fig. 1.3. Composition table for the RCC8 relations. The entry at row R_1 and column R_2 in the table denotes the possible relations between the regions X and Z, assuming that R_1 is the relation between the regions X and Y, and R_2 the relation between the regions Y and Z.

Two relations on regions X and Y are conceptual neighbors if the shape of X or Y can be continuously deformed such that one relation is transformed into the other relation without passing through a third relation. Figure 1.4 shows the conceptual

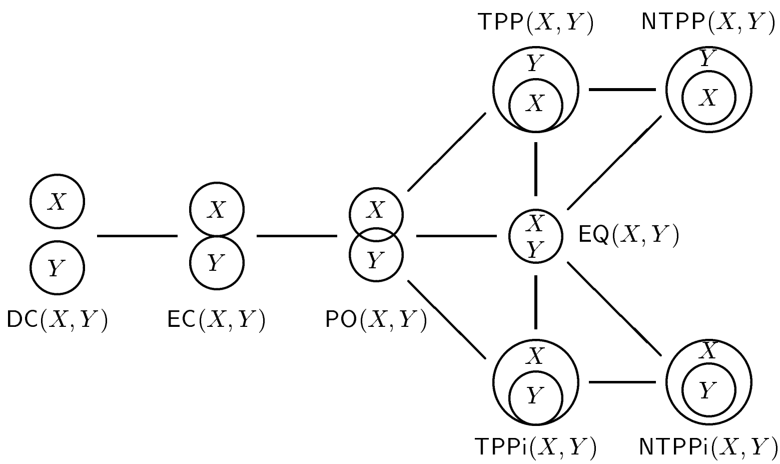


Fig. 1.4. The RCC8 relations arranged in a graphs showing the conceptual neighbors.

neighbors for the RCC8 relations.

The conceptual neighborhood graph can be used to handle uncertainty in reasoning about spatial relations (Guesgen and Hertzberg, 1996). Before we discuss in detail how this can be achieved, we will make an intermediate step and reformulate the relation between two given regions X and Y as a characteristic function:

$$\mu_{\mathsf{R}} : \mathsf{RCC8} \longrightarrow \{0, 1\}$$

The function yields a value of 1 if and only if the argument is equal to the RCC8 relation denoted by the characteristic function:

$$\mu_{\mathsf{R}}(\mathsf{R}') = \begin{cases} 1, \text{ if } \mathsf{R}' = \mathsf{R} \\ 0, \text{ else} \end{cases}$$

For example, if X and Y are externally connected (i.e., $\mathsf{EC}(X,Y)$), the relation between X and Y can be defined by the following characteristic function:

$$\begin{aligned} \mu_{\mathsf{EC}} &\equiv \{(\mathsf{R}, \mu_{\mathsf{EC}}(\mathsf{R})) \mid \mathsf{R} \in \mathsf{RCC8}\} \\ &= \{(\mathsf{DC}, 0), (\mathsf{EC}, 1), (\mathsf{PO}, 0), (\mathsf{EQ}, 0), \ldots\} \end{aligned}$$

The next step is to transform the relation between two regions X and Y into a fuzzy set. A fuzzy set $\tilde{R}$ of a domain D is a set of ordered pairs, $(d, \mu_{\tilde{R}}(d))$, where d is an element of the underlying domain D and $\mu_{\tilde{R}} : D \to [0, 1]$ is the membership function of $\tilde{R}$. In other words, instead of specifying whether an element d belongs to a subset R of D or not, we assign a grade of membership to d. The membership function replaces the characteristic function of a classical subset of D.

In the context of the RCC8 relations, instead of having two classes, one with the accepted relation where μ_R results in 1 and another with the discarded relations where μ_R results in 0, we now assign acceptance grades (or membership grades, to use the term from fuzzy set theory) with the relations.

The question that arises at this point is how to choose the membership grades. Assume, for instance, that the only information we have about the regions X and Y is that they are externally connected. Then it makes sense to assign a larger membership grade to RCC8 relations closer to EC in the conceptual neighborhood graph and a smaller membership grade to relations further away. In other words, if the relation is EC, we assign the membership grade 1; if the relation is a neighbor of EC, we choose a membership grade m_1 with $1 \geq m_1 \geq 0$; if the relation is a neighbor of a neighbor of EC, we assign a grade m_2 with $m_1 \geq m_2 \geq 0$; etc. In the end, the relation between X and Y can be defined by the following membership function:

$$\begin{aligned} \mu_{\widetilde{\mathsf{EC}}} &\equiv \{(\mathsf{R}, \mu_{\widetilde{\mathsf{EC}}}(\mathsf{R})) \mid \mathsf{R} \in \mathsf{RCC8}\} \\ &= \{(\mathsf{DC}, m_1), (\mathsf{EC}, 1), (\mathsf{PO}, m_1), (\mathsf{EQ}, m_2), \ldots\} \end{aligned}$$

Figure 1.5 shows the complete assignment of membership grades.

In some cases, however, we have additional knowledge about the relations between the regions and can take this into consideration when assigning the membership grades. For example, if X and Y are externally connected (i.e., $\mathsf{EC}(X,Y)$) and

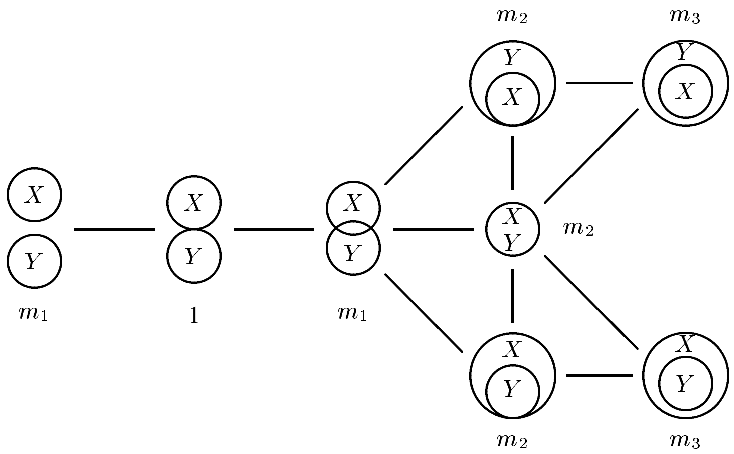


Fig. 1.5. The assignment of membership grades to the $\mathsf{RCC8}$ relations with $\mathsf{EC}(X,Y)$ as reference relation, assuming that we do not have additional knowledge about this relation.

moving towards each other, we would assume that neither $\mathsf{DC}(X,Y)$ nor $\mathsf{EC}(X,Y)$ can be observed in the next time instance, but all the other relations are plausible with decreasing membership grades $m_1 \geq m_2 \geq m_3 \cdots \geq 0$. In this case:

$$\begin{aligned}\mu_{\widetilde{\mathsf{EC}}} &\equiv \{(\mathsf{R}, \mu_{\widetilde{\mathsf{EC}}}(\mathsf{R})) \mid \mathsf{R} \in \mathsf{RCC8}\} \\ &= \{(\mathsf{DC}, 0), (\mathsf{EC}, 0), (\mathsf{PO}, m_1), (\mathsf{EQ}, m_2), \ldots\}\end{aligned}$$

Figure 1.6 illustrates this observation.

In general, movement and deformation is closely related to the notion of direction. The idea of incorporating directions into a static spatial theory is not new. Renz (2001), for example, introduces the directed interval algebra, which uses 26 base relations to describe the relationship between two directed intervals. However, this approach cannot directly be applied to the RCC theory, because movement or deformation is not aligned to a particular axis in this theory (see Figure 1.7). A purely qualitative approach to modeling movements or deformations of regions in the RCC theory, similar to the one used in the directed interval algebra, would lead to descriptions that are too coarse to make meaningful inferences. On the other hand, precise mathematical descriptions of movements or deformations are often too complex. Using membership grades to model movements and deformations is a compromise, which is both powerful and computationally feasible.

Regardless of whether we have additional information or not, determining the actual membership grades $m_1, m_2, \ldots$ can be a problem, since there is no general algorithm for computing the grades. On the other hand, there are experiments showing that membership grades are quite robust, which means that it is not necessary to have precise estimations of these grades (Bloch, 2000). The explanation given for this observation is twofold: first, membership grades are used to describe imprecise

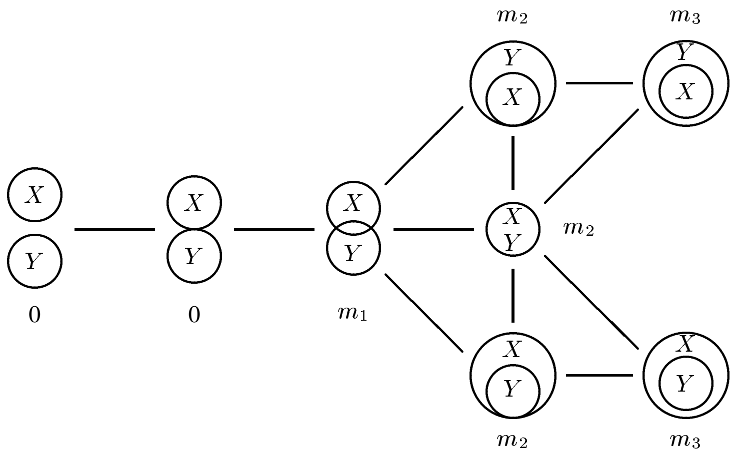


Fig. 1.6. The assignment of membership grades to the $\mathsf{RCC8}$ relations with $\mathsf{EC}(X, Y)$ as reference relation, assuming that X and Y are moving towards each other.

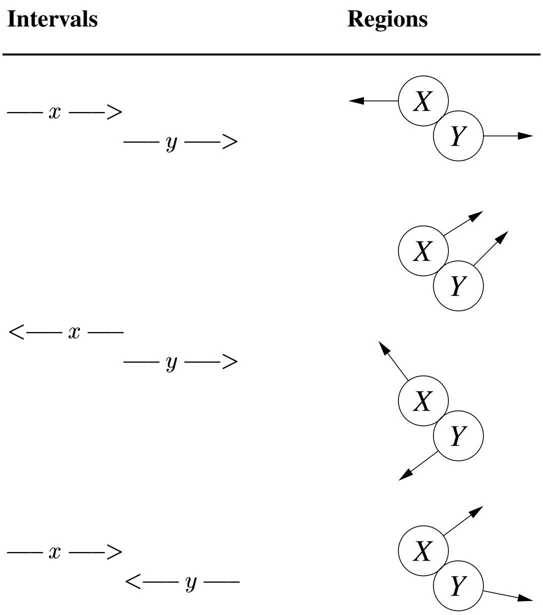


Fig. 1.7. All possible movements/deformations in the directed interval algebra for the meets relation as opposed to some examples of movements/deformations in the RCC theory for the EC relation.

information and therefore do not have to be precise, and second, each individual membership grade plays only a minor role in the whole reasoning process, as it is usually combined with several other membership grades.

If the membership grades are combined by using the min/max combination scheme, as it is the case in the rest of this chapter, we do not need numeric membership grades but can perform reasoning on symbolic values $m_1, m_2, \ldots$, which solves the problem of determining the initial membership grades. The fact that there is an ordering $m_1 \geq m_2 \geq m_3 \cdots \geq 0$ on the grades suffices to guarantee that we can select the largest/smallest grade from a given set of membership grades, which is essentially what fuzzy reasoning is based upon.

So far, we have only considered atomic relations (i.e., single relations between regions). Non-atomic RCC8 relations (i.e., disjunctions of RCC8 relations) can be transformed into fuzzy RCC8 relations by using the same technique as described in the previous section. A non-atomic RCC8 relation is given by a set of atomic RCC8 relations, which is interpreted in a disjunctive way. We therefore transform each atomic relation in the set into a fuzzy RCC8 relation and compute the union of the resulting fuzzy relations.

There are different ways of computing the union, intersection, and complement of fuzzy sets. Here, we have chosen the min/max combination scheme (Zadeh, 1965) to define the membership function of the union, intersection, and complement of fuzzy sets, respectively:

$$\begin{aligned}
\mu_{\widetilde{\mathsf{R}}_1 \cup \widetilde{\mathsf{R}}_2}(\mathsf{R}) &= \max\{\mu_{\widetilde{\mathsf{R}}_1}(\mathsf{R}), \mu_{\widetilde{\mathsf{R}}_2}(\mathsf{R})\} \\
\mu_{\widetilde{\mathsf{R}}_1 \cap \widetilde{\mathsf{R}}_2}(\mathsf{R}) &= \min\{\mu_{\widetilde{\mathsf{R}}_1}(\mathsf{R}), \mu_{\widetilde{\mathsf{R}}_2}(\mathsf{R})\} \\
\mu_{\widetilde{\mathsf{R}}_1^C}(\mathsf{R}) &= 1 - \mu_{\widetilde{\mathsf{R}}_1}(\mathsf{R})
\end{aligned}$$

1.4. Reasoning about Fuzzy Regions

In order to be able to reason about fuzzy RCC8 relations, we have to define the composition of fuzzy RCC8 relations. In the crisp case, the composition of two relations can be represented as a characteristic function of the following form:

$$\mu_{\mathsf{R}_1 \circ \mathsf{R}_2} : \mathsf{RCC8} \longrightarrow \{0, 1\}$$

The function yields a value of 1 for arguments that are elements of the corresponding entry in the composition table of Figure 1.3; otherwise, a value of 0:[2]

$$\mu_{\mathsf{R}_1 \circ \mathsf{R}_2}(\mathsf{R}) = \begin{cases} 1, \text{ if } \mathsf{R} \in \mathsf{R}_1 \circ \mathsf{R}_2 \\ 0, \text{ else} \end{cases}$$

For example, if $\mathsf{R}_1 = \mathsf{EC}$ and $\mathsf{R}_2 = \mathsf{TPPi}$, then the entry for $\mathsf{R}_1 \circ \mathsf{R}_2 = \mathsf{EC} \circ \mathsf{TPPi}$ is DR, which is equivalent to $\{\mathsf{EC}, \mathsf{DC}\}$. The characteristic function of $\mathsf{EC} \circ \mathsf{TPPi}$

[2] Entries in the composition table that are not members of the RCC8 relations are interpreted as disjunctions of the RCC8 relations that they subsume.

is therefore defined as follows:

$$\mu_{\mathsf{EC} \circ \mathsf{TPPi}}(\mathsf{R}) = \begin{cases} 1, \text{ if } \mathsf{R} \in \{\mathsf{EC}, \mathsf{DC}\} \\ 0, \text{ else} \end{cases}$$

Adopting the min/max combination scheme from fuzzy set theory, we can now define the composition $\widetilde{\mathsf{R}}_1 \circ \widetilde{\mathsf{R}}_2$ of two fuzzy RCC8 relations $\widetilde{\mathsf{R}}_1$ and $\widetilde{\mathsf{R}}_2$ as the fuzzy RCC8 relation given by the following membership function:

$$\mu_{\widetilde{\mathsf{R}}_1 \circ \widetilde{\mathsf{R}}_2}(\mathsf{R}) = \max_{\substack{\mathsf{R}'_1, \mathsf{R}'_2 \in \mathsf{RCC8} \\ \mu_{\mathsf{R}'_1 \circ \mathsf{R}'_2}(\mathsf{R})=1}} \{\min\{\mu_{\widetilde{\mathsf{R}}_1}(\mathsf{R}'_1), \mu_{\widetilde{\mathsf{R}}_2}(\mathsf{R}'_2)\}\}$$

The composition of fuzzy relations plays a central role in a number of algorithms for reasoning about fuzzy RCC8 relations. One of these algorithms is an Allen-type algorithm for computing local consistency in networks of fuzzy RCC8 relations. Input to this algorithm is a set of regions and a set of (not necessarily atomic) fuzzy RCC8 relations. The aim of the algorithm is to transform the given relations into a set of relations that are consistent with each other.[3] This is achieved through an iterative process that repeatedly looks at three regions X, Y, and Z, and their fuzzy relations $\widetilde{\mathsf{R}}_1(X, Y)$, $\widetilde{\mathsf{R}}_2(Y, Z)$, and $\widetilde{\mathsf{R}}_3(X, Z)$, computes the composition of two of the relations, and compares the result with the third relation:

$$\widetilde{\mathsf{R}}_3(X, Z) \;\leftarrow\; \widetilde{\mathsf{R}}_3(X, Z) \cap [\widetilde{\mathsf{R}}_1(X, Y) \circ \widetilde{\mathsf{R}}_2(Y, Z)]$$

Figure 1.8 shows pseudocode for the extended algorithm; a more elaborate discussion of the algorithm can be found elsewhere (Guesgen et al., 1994).

The worst-case complexity of the fuzzy RCC8 algorithm is $O(n^3)$, which is the same as the complexity of Allen's original algorithm. However, unlike in the crisp version of Allen's algorithm, no elements are deleted from the fuzzy relations during the run of the algorithm (instead, their membership grades are updated). The reason for this is that the fuzzy RCC8 algorithm does not make a yes/no decision about whether a crisp atomic RCC8 relation is admissible or not, but computes a new membership grade for that relation. The new membership grade is compared with the initial membership grade of the relation. If the new grade is smaller than the initial grade, the membership grade of the relation is updated with the new grade. As a result, the algorithm performs a complete lookup of all table entries and annotates each entry with the minimum of the membership grades of the relations that led to this entry. From the annotated entries, the algorithm selects those whose membership grades are maximal with respect to DC, EC, PO, and so on.

In order to avoid extensive, often redundant, search for the best relation, two different strategies can be exploited. The first strategy avoids extensive search by determining the new membership grades on a best-first basis. This method results from the following considerations:

[3] In this context, consistency means that the membership grades are consistent with each other.

Fuzzy RCC8 Algorithm

- Let $\tilde{\mathcal{R}}$ be a set of fuzzy RCC8 relations between regions $\{X_1, X_2, \ldots, X_n\}$.
- While $\widetilde{\mathcal{R}}$ is not empty:
  1. Select a relation $\widetilde{\mathsf{R}}(X_i, X_j) \in \widetilde{\mathcal{R}}$
  2. $\widetilde{\mathcal{R}} \leftarrow \widetilde{\mathcal{R}} - \{\widetilde{\mathsf{R}}(X_i, X_j)\}$
  3. For $k \in \{1, \ldots, n\}$ with $k \neq i, j$:
     $\widetilde{\mathsf{R}}(X_k, X_j) \leftarrow \widetilde{\mathsf{R}}(X_k, X_j) \cap [\widetilde{\mathsf{R}}(X_k, X_i) \circ \widetilde{\mathsf{R}}(X_i, X_j)]$
     If $\widetilde{\mathsf{R}}(X_k, X_j)$ changed, then $\widetilde{\mathcal{R}} \leftarrow \widetilde{\mathcal{R}} \cup \{\widetilde{\mathsf{R}}(X_k, X_j)\}$
     $\widetilde{\mathsf{R}}(X_i, X_k) \leftarrow \widetilde{\mathsf{R}}(X_i, X_k) \cap [\widetilde{\mathsf{R}}(X_i, X_j) \circ \widetilde{\mathsf{R}}(X_j, X_k)]$
     If $\widetilde{\mathsf{R}}(X_i, X_k)$ changed, then $\widetilde{\mathcal{R}} \leftarrow \widetilde{\mathcal{R}} \cup \{\widetilde{\mathsf{R}}(X_i, X_k)\}$

Fig. 1.8. Extended version of Allen's algorithm for computing local consistency in networks of RCC8 relations. Without loss of generality, we assume that the relation $\widetilde{\mathsf{R}}(X_i, X_j)$ is defined for every $i, j \in \{1, 2, \ldots, n\}$ with $i \neq j$, possibly as universal relation $\{(\mathsf{DC}, 1), (\mathsf{EC}, 1), (\mathsf{PO}, 1), \ldots\}$.

1. A membership grade of 1 in the composed relation can only result from combining relations with a membership grade of 1.
2. The second highest membership grade, m_1, can only result from combining relations which have a membership grade equal to or greater than m_1, and so on.

Search is therefore able to stop as soon as a membership grade has been obtained for each of the RCC8 relations, because, by virtue of the heuristic, the first value obtained must be the maximum.

The second strategy addresses the problem of repeated lookups. During the composition of two fuzzy relations, the same lookup pair of atomic relations is often produced several times. To avoid that a combination of relations is looked up more than once, a hash table is maintained in which pairs are recorded that have already been looked up. Before any two relations are composed, this hash table can be consulted to ensure an equivalent combination has not already been processed.

Research in the area of spatio-temporal reasoning has shown that Allen's algorithm in general only computes local consistency. The same holds for the extended algorithm in Figure 1.8. To obtain a globally consistent network of relations, additional methods have to be used, which usually involve some form of backtracking in the non-fuzzy case. In networks with fuzzy relations, we are seeking some level of optimality, which means that a plain backtracking algorithm is insufficient. Instead, the algorithm must continue after a consistent instantiation is found, if this instantiation is not 'good enough' (in terms of the membership grades of the instantiation). One way to achieve this goal is by applying an optimization technique like branch and bound (Freuder and Wallace, 1992), which operates in the same way as backtracking search with some variations:

1. The best instantiation so far is recorded.
2. A search path is abandoned when it is clear that it cannot lead to a better solution.
3. Search stops when all search paths have been either explored or abandoned, or when a perfect instantiation has been found.

1.5. Conclusion

In many real-world situations, spatial relations between regions are subject to uncertainty and imprecision. For example, it might be the case that we cannot define the boundaries of a region precisely. Or it might be that a region changes over time, due to the fact that the region alters its position or shape. The purpose of this chapter is to introduce a formalism for reasoning about spatial relations that is robust under uncertainty. This is achieved by converting the RCC8 relations into fuzzy sets and applying a fuzzy RCC8 algorithm to the resulting sets.

The chapter focuses on two reasoning techniques: one based on Allen's algorithm, the other on branch and bound techniques. In general, however, reasoning over fuzzy RCC8 relations does not have to be restricted to these techniques. A network of fuzzy RCC8 relations can be viewed as a constraint network, and the problem of finding a consistent instantiation for such a network as a constraint satisfaction problem. This means that in principle any fuzzy constraint satisfaction algorithm (Guesgen and Philpott, 1995) can be used to reason about fuzzy RCC8 relations.

Our formalism has a variety of applications. However, unlike (Cui et al., 1992) for instance, the intention is not to provide a formalism for qualitative simulation, but to provide the basis for reasoning in environments that may (or may not) change from one time instance to the other, or in environments that are not precisely defined at any time. As a result, our formalism does not keep track about the changes in the environment, nor does it allow to reason about sequences of changes. Future work might address these problems.

References

Allen, J (1983) Maintaining knowledge about temporal intervals. *Communications of the ACM*, 26:832–843.

Bloch, I (2000) Spatial representation of spatial relationship knowledge. In *Proc. KR-00*, pp. 247–258, Breckenridge, Colorado.

Cohn, A, Bennett, B, Gooday, J, and Gotts, N (1997) Representing and reasoning with qualitative spatial relations about regions. In Stock, O, editor, *Spatial and Temporal Reasoning*, pp. 97–134. Kluwer, Dordrecht, The Netherlands.

Cohn, A and Gotts, N (1996) The 'Egg-Yolk' representation of regions with indeterminate boundaries. In Burrough, P and Frank, A, editors, *Geographical Objects with Undetermined Boundaries*, GISDATA Series No. 2, pp. 171–187. Taylor and Francis, London, England.

Cui, Z, Cohn, A, and Randell, D (1992) Qualitative simulation based on a logical formalism of space and time. In *Proc. AAAI-92*, pp. 679–684, San Jose, California.

Freksa, C (1990) Qualitative spatial reasoning. In *Proc. Workshop RAUM*, pp. 21–36, Koblenz, Germany.

Freksa, C (1992) Temporal reasoning based on semi-intervals. *Artificial Intelligence*, 54:199–227.

Freuder, E and Wallace, R (1992) Partial constraint satisfaction. *Artificial Intelligence*, 58:21–70.

Guesgen, H (1989) Spatial reasoning based on Allen's temporal logic. Technical Report TR-89-049, ICSI, Berkeley, California.

Guesgen, H (2002) From the egg-yolk to the scrambled-egg theory. In *Proc. FLAIRS-02*, pp. 476–480, Pensacola, Florida.

Guesgen, H (2003) When regions start to move. In *Proc. FLAIRS-03*, pp. 465–469, St. Augustine, Florida.

Guesgen, H and Hertzberg, J (1993) A constraint-based approach to spatiotemporal reasoning. *Applied Intelligence (Special Issue on Applications of Temporal Models)*, 3:71–90.

Guesgen, H and Hertzberg, J (1996) Spatial persistence. *Applied Intelligence (Special Issue on Spatial and Temporal Reasoning)*, 6:11–28.

Guesgen, H, Hertzberg, J, and Philpott, A (1994) Towards implementing fuzzy Allen relations. In *Proc. ECAI-94 Workshop on Spatial and Temporal Reasoning*, pp. 49–55, Amsterdam, The Netherlands.

Guesgen, H and Philpott, A (1995) Heuristics for solving fuzzy constraint satisfaction problems. In *Proc. ANNES-95*, pp. 132–135, Dunedin, New Zealand.

Hernández, D (1991) Relative representation of spatial knowledge: The 2-D case. In Mark, D and Frank, A, editors, *Cognitive and Linguistic Aspects of Geographic Space*, pp. 373–385. Kluwer, Dordrecht, The Netherlands.

Kettani, D and Moulin, B (1999) A spatial model based on the notions of spatial conceptual map and of object's influence areas. In Freksa, C and Mark, D, editors, *Spatial Information Theory: Cognitive and Computational Foundations of Geographic Information Science*, Lecture Notes in Computer Science 1661, pp. 401–416. Springer, Berlin, Germany.

Lehmann, F and Cohn, A (1994) The EGG/YOLK reliability hierarchy: Semantic data integration using sorts with prototypes. In *Proc. 3rd International Conference on Information and Knowledge Management (CIKM-94)*, pp. 272–279, Gaithersburg, Maryland.

Mukerjee, A and Joe, G (1990) A qualitative model for space. In *Proc. AAAI-90*, pp. 721–727, Boston, Massachusetts.

Musto, A, Stein, K, Eisenkolb, A, Röfer, T, Brauer, W, and Schill, K (2000) From motion observation to qualitative motion representation. In Freksa, C, Brauer, W, Habel, C, and Wender, K, editors, *Spatial Cognition II: Integrating Abstract Theories, Empirical Studies, Formal Methods, and Practical Applications*, Lecture Notes in Computer Science 1849, pp. 115–126. Springer, Berlin, Germany.

Randell, D, Cui, Z, and Cohn, A (1992) A spatial logic based on regions and connection. In *Proc. KR-92*, pp. 165–176, Cambridge, Massachusetts.

Renz, J (2001) A spatial odyssey of the interval algebra: 1. directed intervals. In *Proc. IJCAI-01*, pp. 51–56, Seattle, Washington.

Zadeh, L (1965) Fuzzy sets. *Information and Control*, 8:338–353.

2. Combined Extraction of Directional and Topological Relationship Information from 2D Concave Objects

Pascal Matsakis and Dennis Nikitenko

Department of Computing and Information Science
University of Guelph, Guelph, ON N1G 2W1, Canada
matsakis@cis.uoguelph.ca

Abstract. The importance of topological and directional relationships between spatial objects has been stressed in different fields, notably in Geographic Information Systems (GIS). In an earlier work, we introduced the notion of the F-histogram, a generic quantitative representation of the relative position between two 2D objects, and showed that it can be of great use in understanding the spatial organization of regions in images. Here, we illustrate that the F-histogram constitutes a valuable tool for extracting directional and topological relationship information. The considered objects are not necessarily convex and their geometry is not approximated through, e.g., Minimum Bounding Rectangles (MBRs). The F-histograms introduced in this chapter are coupled with Allen's temporal relationships based on fuzzy set theory. Allen's relationships are commonly extended into the spatial domain for GIS purposes, and fuzzy set theoretic approaches are widely used to handle imprecision and achieve robustness in spatial analysis. For any direction in the plane, the F-histograms define a fuzzy 13-partition of the set of all object pairs, and each class of the partition corresponds to an Allen relation. Lots of directional and topological relationship information as well as different levels of refinements can be easily obtained from this approach, in a computationally tractable way.

2.1. Introduction

Space plays a fundamental role in human cognition. In everyday situations, it is often viewed as a construct induced by spatial relations, rather than as a container that exists independently of the objects located in it. A variety of formalisms developed in Artificial Intelligence naturally deal with space on the basis of relations between objects. Geographic Information Systems constitute a wide area of applications for such formalisms. Many authors, from different fields, have stressed the importance of topological (Allen 1983; Clementini and Di Felice 1997; Cohn et al. 1997; Kuipers 1978) and directional relationships (Bloch 1999; Dutta 1991; Krishnapuram et al. 1993; Kuipers and Levitt 1988). Work in the modeling of these relationships for GIS is often based on an extension into the spatial domain of Allen's temporal relationships (Allen 1983). A common procedure is to approximate the geometry of spatial objects by Minimum Bounding Rectangles (Clementini et al. 1994; Nabil et al. 1995; Sharma and Flewelling 1995). A 2D object is then represented as a set of two perpendicular 1D segments and relationships between objects are inferred from relationships between segments. To enhance querying and improve accuracy in relationship determination, however, several alternatives and refinements have been proposed. In (Petry et al. 2002), for instance, MBRs are partitioned into sets of rectangles. Such partitioning results in a finer approximation of the object's true geometry, called Multiple Rectangle Representation.

The need to handle imprecise and uncertain information concerning spatial data has been widely recognized in recent years, e.g., (Goodchild and Gopal 1990), and there has been a strong demand in the field of GIS for providing approaches that deal with such information. Humans often deal with space on a qualitative basis, allowing for imprecision in spatial descriptions when interacting with each other. Qualitative spatial reasoning, a subfield of AI, aims at modeling commonsense knowledge of space (Cohn 1995). Computational approaches for spatial modeling and reasoning, however, can benefit from more quantitative measures. For instance, qualitative composition of positional relations, if iterated over a path of several intermediate positions, introduces too much indeterminacy in the result. The problem can be addressed by coupling qualitative with fuzzy, semi-quantitative knowledge (Clementini 2002). As many authors early emphasized, fuzzy approaches are of great interest for spatial modeling and reasoning (Dutta 1991; Freeman 1975; Robinson 1988; Wang et al. 1990). Research on fuzzy sets and GIS is very active. A recent special issue of *Fuzzy Sets and Systems* (Cobb et al. 2000), for instance, touches on topics as varied as fuzzy objects for GIS, fuzzy spatial queries and landform classification with fuzzy k-means.

In earlier publications, we introduced the notion of the F-histogram (Matsakis 1998; Matsakis and Wendling 1999). It is a generic quantitative representation of the relative position between two 2D objects. It encapsulates structural information about the objects as well as information about their spatial relationships. It is sensitive to the shape of the objects, their orientation and their size. It is also sensitive to the distance between them. Moreover, the F-histogram enables the handling of intersecting, concave, non-connected, unbounded, fuzzy objects as well as of disjoint, convex, bounded, crisp objects. Most work focused on particular F-histograms called force histograms. These histograms offer solid theoretical guarantees and nice geometric properties (Matsakis et al., to appear). They ensure fast and efficient processing of vector data (Skubic et al. 2003) as well as of raster data (Matsakis et al. 2001). Numerous applications have been studied, and new applications continue to be explored. For instance, the histogram of forces lends itself, with great flexibility, to the definition of fuzzy spatial relations. The fuzzy directional relations described in (Matsakis et al. 2001) preserve important relative position properties and can provide inputs to systems for linguistic scene description. One such system has been developed and dedicated to human-robot communication (Skubic et al. 2003). Reference (Matsakis 2002) reviews and classifies work on the histogram of forces. It shows that the notion of the F-histogram can be of great use in understanding the spatial organization of regions in images.

The aim of this chapter is to illustrate that the F-histogram, because of its general properties, constitutes a valuable tool for extracting directional and topological relationship information from two objects. The objects considered here are 2D, crisp, bounded objects, but they are not necessarily convex, nor connected, and they may have holes in them. Their geometry is not approximated through, e.g., centroids, MBRs or convex hulls. The F-histograms described in the present work are coupled with Allen relations using fuzzy set theory. Obviously, the set of Allen relations does not allow all possible topological relationships between 2D concave objects to be described. However, it is a well-known set, of reasonable size, which has been extensively used. For any oriented line, Δ, the Allen relations define a crisp 13-partition of the set of pairs of segments on Δ. For any direction, θ, the F-histograms introduced here define a fuzzy 13-partition of the set of all object pairs, and each class of the partition corresponds to an Allen relation. Lots of directional and topological relationship information as well as different levels of refinements can be easily obtained from this approach, in a computationally tractable way. The notion of the F-histogram is described in Section 2.2 and the way F-histograms are coupled with Allen relations is examined in Section 2.3. Preliminary experiments validate the approach in Section 2.4 and conclusion is given in Section 2.5.

2.2. When Pairs of 2D Objects Are Handled as Pairs of 1D Sections

We describe here the notion of the F-histogram (Section 2.2.2), which was introduced in an earlier work (Matsakis 1998; Matsakis and Wendling 1999). F-histograms include f-histograms (Section 2.2.3) and f-histograms include φ-

histograms (Section 2.2.4). Most of the previous research has focused on force histograms, which are particular φ-histograms and have shown to be of great use in understanding the spatial organization of image objects (Section 2.2.5). First of all, we go over some terms and introduce a few notations (Section 2.2.1).

2.2.1. Terminology and Notations

As shown in Figure 2.1, the plane reference frame is a positively oriented orthonormal frame $(O, \vec{i}, \vec{j})$. For any real numbers α and v, the vectors $\vec{i}_\alpha$ and $\vec{j}_\alpha$ are the respective images of $\vec{i}$ and $\vec{j}$ through the α-angle rotation, and $\Delta_\alpha(v)$ is the oriented line whose reference frame is defined by $\vec{i}_\alpha$ and the point of coordinates (0,v)—relative to $(O,\vec{i}_\alpha,\vec{j}_\alpha)$. The term *object* denotes a nonempty bounded set of points, E, equal to its interior closure[1], and such that for any α and v the intersection $E \cap \Delta_\alpha(v)$ is the union of a finite number of mutually disjoint segments. Note that an object may have holes in it and may consist of many connected components. The intersection $E \cap \Delta_\alpha(v)$, denoted by $E_\alpha(v)$, is a *longitudinal section* of E. Finally, the symbol T denotes the set of all triples $(\alpha, E_\alpha(v), G_\alpha(v))$, where α and v are any real numbers and E and G are any objects.

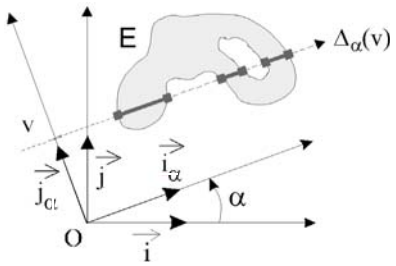


Fig. 2.1. Oriented straight lines and longitudinal sections. Here, $E_\alpha(v)=E \cap \Delta_\alpha(v)$ is the union of three disjoint segments.

[1] In other words, it is a 2D object that does not include any "grafting," such as an arc or isolated point.

2.2.2. F–Histograms

Consider two objects A and B (the *argument* and the *referent*), a direction θ and some proposition $P^{AB}(\theta)$ like "A is *after* B in direction θ," "A *overlaps* B in direction θ," or "A *surrounds* B in direction θ." We want to attach a weight to $P^{AB}(\theta)$. To do so, the objects A and B are handled as longitudinal sections (Figure 2.2).

- For each v, the pair $(A_\theta(v),B_\theta(v))$ of longitudinal sections is viewed as an argument put forward to support $P^{AB}(\theta)$.
- A function F from T into $\mathbb{R}_+$ (the set of non-negative real numbers) attaches the weight $F(\theta,A_\theta(v),B_\theta(v))$ to this argument $(A_\theta(v),B_\theta(v))$.
- The total weight $F^{AB}(\theta)$ of the arguments stated in favor of $P^{AB}(\theta)$ is naturally set to:

$$F^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta,A_\theta(v),B_\theta(v))\,dv.$$

- If the domain of the function F^{AB} so defined is all of $\mathbb{R}$ (the set of real numbers), then F^{AB} is called the F*–histogram associated with* (A,B). This histogram, which is a periodic function with period 2π, is one possible representation of the position of A with regard to B.

$$F^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta,A\cap\Delta_\theta(v),B\cap\Delta_\theta(v))\,dv.$$

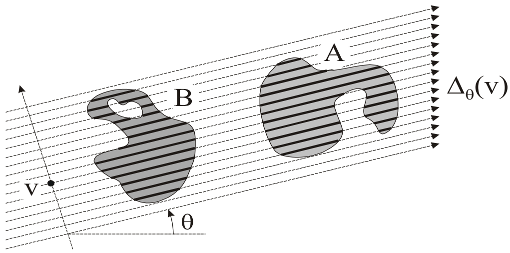


Fig. 2.2. The objects are handled as longitudinal sections:

2.2.3. f–Histograms

There exists one set $\{I_i\}_{i\in 1..n}$ of mutually disjoint segments (and only one) such that $A_\theta(v) = \cup_{i\in 1..n} I_i$. Likewise, there exists one set $\{J_j\}_{j\in 1..m}$ of segments such that $B_\theta(v) = \cup_{j\in 1..m} J_j$. The function F, in charge of the longitudinal sections, might delegate the handling of these segments to some function f, from $\mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+$ into $\mathbb{R}_+$ (Figure 2.3). The case is described below.

- Each (I_i,J_j) is considered an argument put forward to support the proposition $\mathrm{P}^{AB}(\theta)$.
- The function f attaches the weight $f(x_{I_i}, y^{\theta}_{I_i J_j}, x_{J_j})$ to this argument (I_i,J_j)— where x_{I_i} and x_{J_j} denote the lengths of I_i and J_j, and where $y^{\theta}_{I_i J_j}$ characterizes the relative position of I_i and J_j on $\Delta_\theta(v)$ (Figure 2.4).
- $F(\theta,A_\theta(v),B_\theta(v))$ is naturally set to the sum of the weights $f(x_{I_i}, y^{\theta}_{I_i J_j}, x_{J_j})$ of all the (I_i,J_j) arguments: $F(\theta,A_\theta(v),B_\theta(v)) = \Sigma_{i\in 1..n,\, j\in 1..m}\ f(x_{I_i}, y^{\theta}_{I_i J_j}, x_{J_j})$.
- F^{AB} can then be renamed f^{AB} and called the f–*histogram associated with* (A,B).

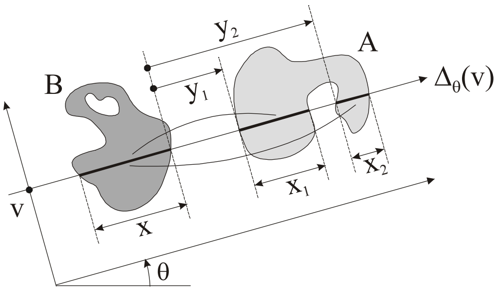


Fig. 2.3. The function F, in charge of the longitudinal sections, might delegate the handling of segments to some function f: $F(\theta,A\cap\Delta_\theta(v),B\cap\Delta_\theta(v)) = f(x_1,y_1,x) + f(x_2,y_2,x)$.

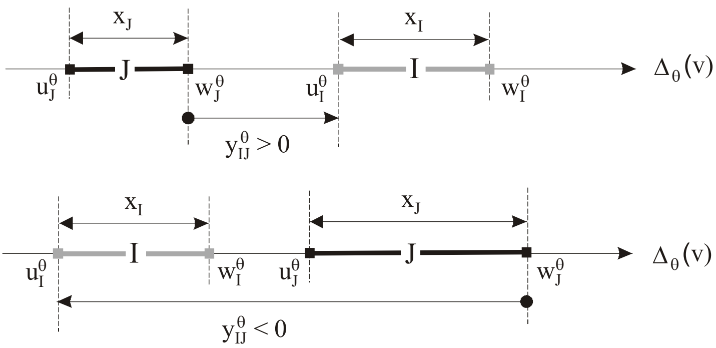


Fig. 2.4. A pair (I,J) of segments on an oriented line $\Delta_\theta(v)$ and the values attached to it.

2.2.4. φ–Histograms

In turn, f, which is in charge of the pairs (I,J) of segments, might delegate the handling of points to another function φ, from $\mathbb{R}$ into $\mathbb{R}_+$ (Figure 2.5). The case is described below.

- Each (M,N), with M in I and N in J, is considered an argument put forward to support the proposition $\mathrm{P}^{AB}(\theta)$.
- The function φ attaches the weight φ(u−w) to this argument (M,N)—where u and w specify the location of M and N on $\Delta_\theta(v)$ and u−w characterizes the relative position of these points on $\Delta_\theta(v)$ (Figure 2.5).
- $f(x_I, y^\theta_{IJ}, x_J)$ is naturally set to the sum of the weights φ(u−w) of all the (M,N) arguments:

$$f(x_I, y^\theta_{IJ}, x_J) = \int_{y^\theta_{IJ}+x_J}^{x_I+y^\theta_{IJ}+x_J} \left(\int_0^{x_J} \varphi(u-w)\,dw \right) du.$$

  Note that:

$$\int_{y^\theta_{IJ}+x_J}^{x_I+y^\theta_{IJ}+x_J} \left(\int_0^{x_J} \varphi(u-w)dw \right)du = \int_{u^\theta_I}^{w^\theta_I} \left(\int_{u^\theta_J}^{w^\theta_J} \varphi(u-w)dw \right)du$$
$$= \int_{u^\theta_J}^{w^\theta_J} \left(\int_{u^\theta_I}^{w^\theta_I} \varphi(u-w)du \right)dw,$$

  where u^θ_I, w^θ_I, u^θ_J and w^θ_J represent the coordinates of the ends of the two segments I and J (Figure 2.4).
- f^{AB} (or F^{AB}) can then be renamed φ^{AB} and called the φ–*histogram associated with* (A,B).

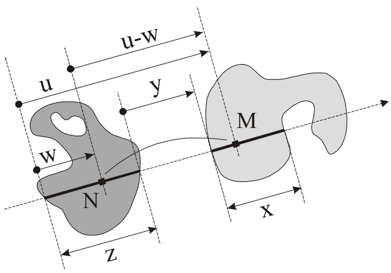


Fig. 2.5. The function f, in charge of the segments, might delegate the handling of points like M and N to some function φ: $f(x,y,z) = \int_{y+z}^{x+y+z} (\int_0^z \varphi(u-w)\,dw)\,du$.

2.2.5. Force Histograms vs. Other F-Histograms

In most previous work, the considered proposition $P^{AB}(\theta)$ is "A is in direction θ of B" (i.e., "A is *after* B in direction θ") and the F-histograms are φ_r-histograms, where r is a real number and φ_r is the function from $\mathbb{R}$ into $\mathbb{R}_+$ defined by:

$$\forall d \in \mathbb{R},\ d \leq 0 \Rightarrow \varphi_r(d)=0 \quad \text{and} \quad d>0 \Rightarrow \varphi_r(d)=1/d^r.$$

The value $\varphi_r^{AB}(\theta)$ can be seen as the scalar resultant of elementary forces. These forces are exerted by the points of A on those of B, and each tends to move B in direction θ (Figure 2.6). The mapping φ_r defines the force fields. As an example, gravitational force fields can be represented by φ_2. This is according to Newton's law of gravity, which states that every particle attracts every other particle with a force inversely proportional to the square of the distance (i.e., d) between them. The argument A and the referent B can then be seen as flat metal plates of uniform density and constant and negligible thickness. A φ_r-histogram is called a *histogram of forces*. It offers solid theoretical guarantees and nice geometric properties. Numerous applications have been studied, and new applications continue to be explored. Reference (Matsakis 2002) reviews and classifies work on the histogram

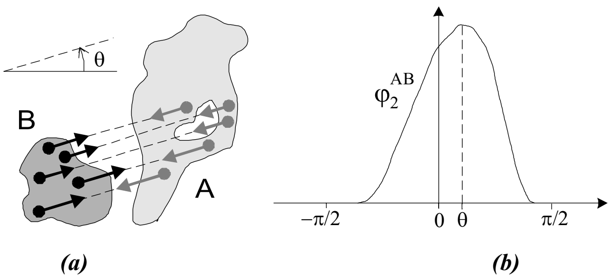


Fig. 2.6. Force histograms. *(a)* $\varphi_r^{AB}(\theta)$ is the scalar resultant of elementary forces (black arrows). Each one tends to move B in direction θ. *(b)* The histogram of gravitational forces associated with (A,B) is one possible representation of the position of A relative to B.

of forces. It touches on varied topics, such as the modeling of spatial relations, spatial indexing mechanisms for medical image databases, pattern recognition, scene matching, linguistic scene description and human-robot communication.

As said above, most work on F-histograms has focused on force histograms. The use of f-histograms that are not φ-histograms, however, was suggested in (Matsakis 1998) for the handling of convex objects. The use of F-histograms that are not f-histograms was suggested in (Matsakis and Andréfouët 2002) with the aim of attaching a weight to the proposition $P^{AB}(\theta) \equiv$ "A *surrounds* B in direction θ." Malki et al. (2002) consider the propositions $P_r^{AB}(\theta) \equiv$ "A *r* B in direction θ," where *r* belongs to the set {>, *mi*, *oi*, *f*, *d*, *si*, =, *s*, *di*, *fi*, *o*, *m*, <} of Allen relations (Figure 2.7). For instance, $P_>^{AB}(\theta)$ is "A is *after* B in direction θ" and $P_o^{AB}(\theta)$ is "A *overlaps* B in direction θ." To attach a weight to these propositions, the authors rely on the research presented in (Matsakis 1998) and propose the use of f-histograms. The thirteen f-histograms are defined by the following functions:

- if y>0 then $f_>(x,y,z)=y/(x+y+z)$
  else $f_>(x,y,z)=0$
- if y=0 then $f_{mi}(x,y,z)=1$
  else $f_{mi}(x,y,z)=0$
- if (y<0 and x+y>0 and y+z>0) then $f_{oi}(x,y,z)=-y(1/x+1/z)$
  else $f_{oi}(x,y,z)=0$
- if (y<0 and x+y>0 and y+z=0) then $f_{si}(x,y,z)=z/x$
  else $f_{si}(x,y,z)=0$
- if (y<0 and x+y>0 and y+z<0) then $f_{di}(x,y,z)=z/x$
  else $f_{di}(x,y,z)=0$
- if (y<0 and x+y=0 and y+z>0) then $f_f(x,y,z)=x/z$
  else $f_f(x,y,z)=0$
- if (y<0 and x+y=0 and y+z=0) then $f_=(x,y,z)=x$
  else $f_=(x,y,z)=0$
- if (y<0 and x+y=0 and y+z<0) then $f_{fi}(x,y,z)=z/x$
  else $f_{fi}(x,y,z)=0$
- if (y<0 and x+y<0 and y+z>0) then $f_d(x,y,z)=x/z$
  else $f_d(x,y,z)=0$
- if (y<0 and x+y<0 and y+z=0) then $f_s(x,y,z)=x/z$
  else $f_s(x,y,z)=0$
- if (y<0 and x+y<0 and y+z<0 and x+y+z>0) then $f_o(x,y,z)=(x+y+z)(1/x+1/z)$
  else $f_o(x,y,z)=0$
- if (y<0 and x+y<0 and y+z<0 and x+y+z=0) then $f_m(x,y,z)=1$
  else $f_m(x,y,z)=0$
- if (y<0 and x+y<0 and y+z<0 and x+y+z<0) then $f_<(x,y,z)=y/(x+y+z)$
  else $f_<(x,y,z)=0$

Only convex objects are actually considered. Moreover, there is no real consistency between the f_r functions and, hence, between the f_r–histograms. For instance, the function $f_>$, which is continuous on its domain and whose range is [0,1], defines a fuzzy relation between aligned segments. The function f_{si} also defines a fuzzy relation between aligned segments; it is not, however, continuous on its domain; its range is [0,1). The function f_{mi} defines a crisp relation; its range is {0,1}. The function f_{oi} defines neither a crisp nor a fuzzy relation; its range is [0,2). In this chapter, we revisit the work of Malki et al. Note that, in their publications, the authors refer to the set of thirteen f_r–histograms as the *histogram of spatial relations*. They also use the term of *orientation histogram* instead of φ-histogram. We do not subscribe to these changes in terminology.

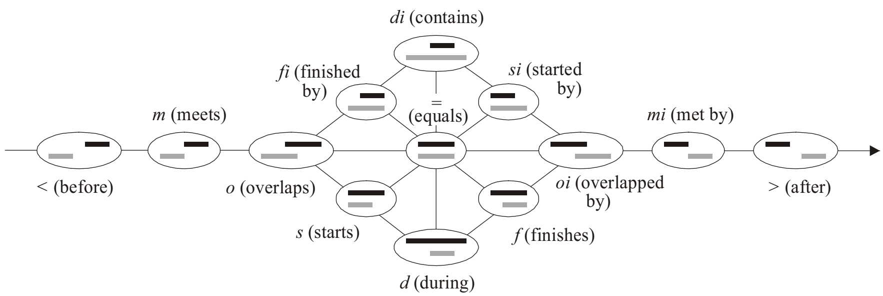


Fig. 2.7. Allen relations (Allen 1983) between two segments on an oriented line. The black segment is the referent, the gray segment is the argument. Two relations r_1 and r_2 are linked if and only if they are conceptual neighbors (Freksa 1992), i.e., r_1 can be obtained directly from r_2 by moving or deforming the segments in a continuous way.

2.3. When F-Histograms Are Coupled With Allen Relations Using Fuzzy Set Theory

Consider an Allen relation r, two objects A and B (convex or not) and a direction θ. The goal of this chapter is to attach an appropriate weight to the proposition $P_r^{AB}(\theta) \equiv$ "A r B in direction θ" (see Section 2.2.5). As discussed in Section 2.2.2, each pair $(A_\theta(v),B_\theta(v))$ of longitudinal sections will be viewed as an argument put forward to support $P_r^{AB}(\theta)$. A function F_r will attach the weight $F_r(\theta,A_\theta(v),B_\theta(v))$ to this argument and the total weight $F_r^{AB}(\theta)$ of the arguments stated in favor of $P_r^{AB}(\theta)$ will be set to:

$$F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta,A_\theta(v),B_\theta(v))\, dv.$$

The question, therefore, is how to define F_r. Let us describe a very simple idea. Consider two segments I and J on an oriented line. We have either IrJ or $\neg$(IrJ). The first case can be rewritten r(I,J)=1 and the second case r(I,J)=0. Now, assume the oriented line is $\Delta_\theta(v)$ and I and J are the longitudinal sections $A_\theta(v)$ and $B_\theta(v)$. There exists one set $\{I_i\}_{i\in 1..m}$ of mutually disjoint segments such that: $I = \cup_{i\in 1..m} I_i$. Likewise, there exists one set $\{J_j\}_{j\in 1..n}$ of segments such that: $J = \cup_{j\in 1..n} J_j$. We could extend the thirteen Allen relations between segments to relations between longitudinal sections and say that r(I,J)=1 (i.e., $F_r(\theta,A_\theta(v),B_\theta(v)=1)$ if and only if there exist two segments I_i and J_j such that $r(I_i,J_j)=1$ (and r(I,J)=0 otherwise). The idea, obviously, is not very satisfactory. For instance, as shown by Figs. 8 to 10, small changes in the longitudinal sections could affect their relationships significantly. As mentioned in Section 2.1, fuzzy set theoretic approaches have been widely used to handle imprecision and achieve robustness in spatial analysis. The issue raised by Figure 2.8 is addressed in Section 2.3.1 by fuzzifying the thirteen Allen relations. The issue raised by Figure 2.9 is addressed in Section 2.3.2 by fuzzifying the longitudinal sections. Section 2.3.3 addresses the last issue (Figure 2.10) and defines the function F_r.

Fig. 2.8. A single pixel at the end of one segment might change the relationships significantly. We may have (>(I,J)=1 and *mi*(I,J)=0 and *oi*(I,J)=0) or (>(I,J)=0 and *mi*(I,J)=1 and *oi* (I,J)=0) or (>(I,J)=0 and *mi*(I,J)=0 and *oi*(I,J)=1).

Fig. 2.9. A missing pixel in the middle of one segment might change the relationships significantly. *(a)* *mi*(I,J)=0 and *oi*(I,J)=1 and *d*(I,J)=0. *(b)* *mi*(I,J)=1 and *oi*(I,J)=0 and *d*(I,J)=1.

Fig. 2.10. A single pixel lost in the middle of nowhere might change the relationships significantly. *(a)* >(I,J)=1 and <(I,J)=0. *(b)* >(I,J)=1 and <(I,J)=1.

2.3.1. Fuzzification of Allen Relations

An Allen relation r can be fuzzified in many ways, depending on the intent of the work. Guesgen (2002), for instance, proceeds in a qualitative manner. Let (I,J) be a pair of segments and let r' be the only (crisp) Allen relation such that Ir'J. Denote by r(I,J) the degree to which the statement IrJ is to be considered true. r(I,J) is chosen as a decreasing function of the conceptual distance between r and r' (i.e., of the distance between r and r' on the graph shown in Figure 2.7). Only a few membership values—which are to be provided by the user—can therefore be taken. Here, we proceed in a quantitative manner. Let α, β, γ and δ be four real numbers such that $\alpha<\beta\leq\gamma<\delta$ and let $\mu_{(\alpha,\beta,\gamma,\delta)}$ be the trapezoid membership function defined on the set of real numbers by:

$$\mu_{(\alpha,\beta,\gamma,\delta)}(u) = \max(\min(\frac{u-\alpha}{\beta-\alpha},1,\frac{\delta-u}{\delta-\gamma}),0)$$

The support of the corresponding fuzzy set is the open interval (α,δ) and the core is $[\beta,\gamma]$: $\mu_{(\alpha,\beta,\gamma,\delta)}(u) \neq 0 \Leftrightarrow u\in(\alpha,\delta)$ and $\mu_{(\alpha,\beta,\gamma,\delta)}(u) = 1 \Leftrightarrow u\in[\beta,\gamma]$. The thirteen Allen relations are fuzzified as shown in Figure 2.11. Each relation, except =, is defined by the min of a few trapezoid membership functions. For instance, the fuzzy relation mi associates with each pair (I,J) of segments the value

$$mi(\mathrm{I,J}) = \mu_{(-a/2,0,0,a/2)}(y)$$

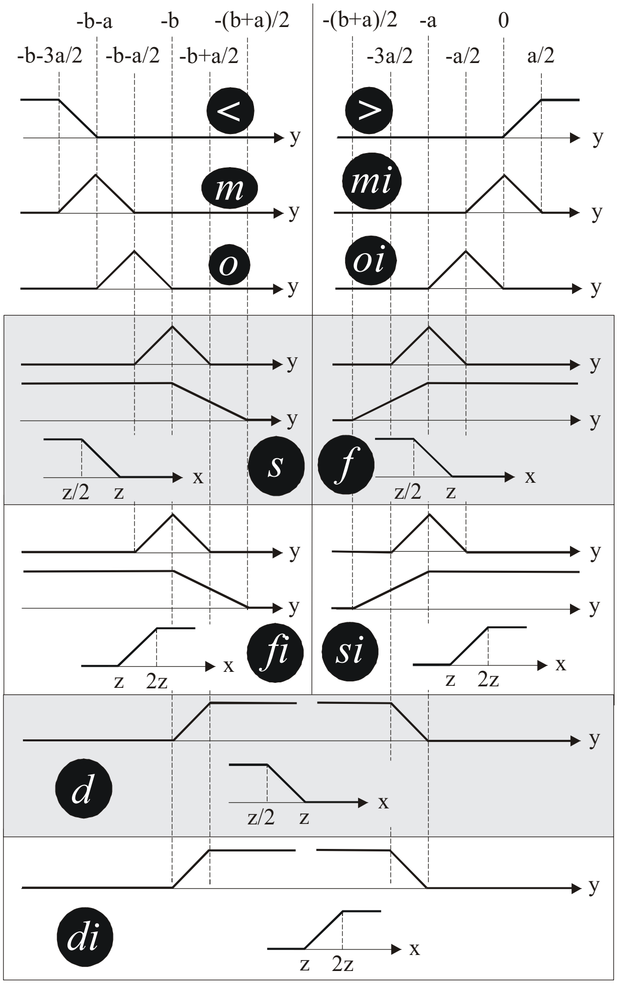
and the relation f associates with each (I,J) the value

$$f(\mathrm{I,J}) = \min\,(\mu_{(-3a/2,-a,-a,-a/2)}(y),\ \mu_{(-(b+a)/2,-a,-a,+\infty)}(y),\ \mu_{(-\infty,z/2,z/2,z)}(x)).$$

Notations are as described in the caption of Figure 2.11. Let $\mathbb{A}$ be the set of all thirteen fuzzy relations. Three properties are worth noticing. First, for any pair (I,J), we have: $\Sigma_{r\in\mathbb{A}}\, r(\mathrm{I,J}) = 1$. This, of course, comes from the definition of = (and it can be shown that = takes its values in [0,1]). Second, for any r in $\mathbb{A}$, there exist pairs (I,J) such that r(I,J)=1. Lastly, for any pair (I,J) and any r_1 and r_2 in $\mathbb{A}$, if $r_1(\mathrm{I,J})\neq0$ and $r_2(\mathrm{I,J})\neq0$ then r_1 and r_2 are direct neighbors in the graph of Figure 2.7.

2.3.2. Fuzzification of Longitudinal Sections

In this section, we address the issue raised by Figure 2.9. The idea is to consider that if two segments are close enough relative to their lengths, then they should be seen, to a certain extent, as a single segment. Let I be the longitudinal section $E \cap \Delta_\theta(v)$ of some object E. Assume I is not empty. There exists one set $\{I_i\}_{i \in 1..n}$ of mutually disjoint segments (and only one) such that: $I = \cup_{i \in 1..n} I_i$. The indexing can be chosen such that, for any i in 1..n−1, the segment I_{i+1} is after I_i in direction θ. Let J_i be the open interval $H(I_i \cup I_{i+1}) - I_i \cup I_{i+1}$, where $H(I_i \cup I_{i+1})$ denotes the convex hull of $I_i \cup I_{i+1}$, i.e., the smallest segment that contains both I_i and I_{i+1}. The longitudinal section I is considered a fuzzy set on $\Delta_\theta(v)$. Its membership function is μ_I and its α-cut is αI. For any point M on any I_i, the value $\mu_I(M)$ is 1. For any point M on any J_i, the value $\mu_I(M)$ is α_i —and, initially, $\alpha_i = 0$. The algorithm presented in Figure 2.12 fuzzifies I by increasing these membership degrees α_i. Note that the maximum number of iterations of the `while` loop is n. An illustration of the fuzzification process is presented in Figure 2.13.



$$= \;=\; 1 - \begin{pmatrix} < + m + o + s + f + d + \\ > + mi + oi + si + fi + di \end{pmatrix}$$

Fig. 2.11. The thirteen fuzzified Allen relations between two segments I and J on an oriented line. Each relation, except =, is defined by the min of a few membership functions (one for <, >, *m*, *mi*, *o*, *oi*; three for *s*, *si*, *f*, *fi*, *d* and *di*). x is the length of I (the argument), z is the length of J (the referent), a is min(x,z), b is max(x,z) and y characterizes the position of I relative to J (see Figure 2.4).

```
c ← 0;
α ← 1;
while α > 0 do

   %--------------- There exists one set {I_i^c}_{i∈1..n_c} of
   mutually disjoint segments (and only one) such that:
   αI = ∪_{i∈1..n_c} I^c . For any i and any j in 1..n_c, with
   i≠j, the length of I_i^c is denoted by x_i^c and the distance
   between I_i^c and I_j^c is denoted by d_ij^c. ---------------%

   for any i in 1..n_c−1 do
         for any j in i+1..n_c do
               β ← α(1− d_ij^c/min(x_i^c,x_j^c));
               for any k in 1..n−1 do
                     if J_k ⊂ H(I_i^c ∪ I_j^c) then
                     α_k ← max{α_k,β};
                     endif;
               endfor;
         endfor;
   endfor;
   α ← max {α_k}_{k∈1..n−1} ∩ [0,α);
   c ← c+1;

endwhile;
```

Fig. 2.12. Algorithm for the fuzzification of a longitudinal section I. The symbol $H(I_i^c \cup I_j^c)$ denotes the convex hull of $I_i^c \cup I_j^c$. The indexing is chosen such that the segments I_i^c and I_{i+1}^c are consecutive in I. The algorithm increases the membership degrees α_k associated with the open intervals $J_k = H(I_k^0 \cup I_{k+1}^0) - I_k^0 \cup I_{k+1}^0$ (initially, all α_k values are zero).

2.3.3. Coupling F-Histograms with Allen Relations

Consider an Allen relation r and the longitudinal sections $A_\theta(v)$ and $B_\theta(v)$ of some objects A and B. We are now able to define the value $F_r(\theta,A_\theta(v),B_\theta(v))$ (see the introductory paragraph of Section 2.3). If $A_\theta(v)=\varnothing$ or $B_\theta(v)=\varnothing$ then $F_r(\theta,A_\theta(v),B_\theta(v))$ is naturally set to 0. Assume $A_\theta(v)\neq\varnothing$ and $B_\theta(v)\neq\varnothing$. Assume $A_\theta(v)$, $B_\theta(v)$ and r have been fuzzified as described in Sections 3.1 and 3.2. There exists a tuple $(\alpha_0,\alpha_1,\ldots,\alpha_c)$ of real numbers such that

$\alpha_0=0<\alpha_1<\alpha_2<\ldots<\alpha_c=1$ and $\{\alpha_k\}_{k\in 0..c} = \{\mu_{A\theta(v)}(M)\}_{M\in\Delta\theta(v)} \cup \{\mu_{B\theta(v)}(M)\}_{M\in\Delta\theta(v)}$ (the set of all membership values in the fuzzy sections $A_\theta(v)$ and $B_\theta(v)$). For any k in 1..c, there exists one set $\{I_i^k\}_{i\in 1..m_k}$ of mutually disjoint segments such that: $\alpha_k\, A_\theta(v) = \cup_{i\in 1..m_k} I_i^k$. Likewise, there exists one set $\{J_i^k\}_{i\in 1..n_k}$ of segments such that: $\alpha_k\, B_\theta(v) = \cup_{i\in 1..n_k} J_i^k$. For any i in $1..m_k$, the length of I_i^k is denoted by x_i^k. For any i in $1..n_k$, the length of J_i^k is denoted by z_i^k. The value $F_r(\theta,A_\theta(v),B_\theta(v))$ is defined as follows:

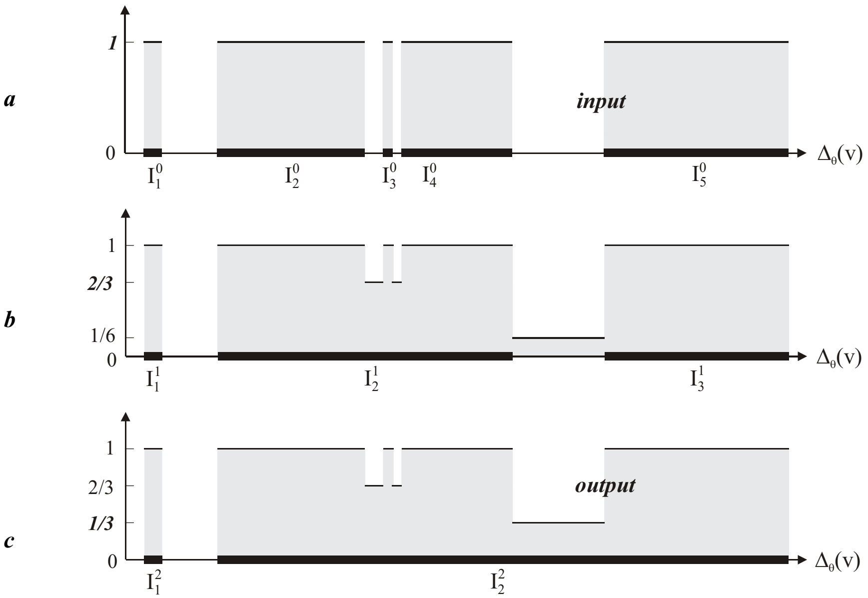


Fig. 2.13. Fuzzification of a longitudinal section I using the algorithm given in Section 2.3.2. Here, I is the union of five segments (n=5). Its membership function μ_I is plotted in *(a)*. We have: $x_1^0=1$ (length of I_1^0), $x_2^0=8$, $x_3^0=1/2$, $x_4^0=6$, $x_5^0=10$, $d_{12}^0=3$ (distance between I_1^0 and I_2^0), $d_{23}^0=1$, $d_{34}^0=1/2$, $d_{45}^0=5$. At the end of the first iteration of the `while` loop, μ_I is as shown in *(b)*. It has been modified because of two pairs of segments: (I_2^0, I_4^0) and (I_4^0, I_5^0). At the end of the second iteration, μ_I is as shown in *(c)*. It has been modified again, because of (I_2^1, I_3^1). The third and last iteration does not bring any changes. The fuzzified longitudinal section is therefore defined by the membership function plotted in *(c)*.

$$F_r(\theta,A_\theta(v),B_\theta(v)) = \Sigma_{k\in 1..c}\ \Sigma_{i\in 1..m_k}\ \Sigma_{j\in 1..n_k}\ [x_i^k\ z_j^k\ (\alpha_k-\alpha_{k-1})]\ r(I_i^k,J_j^k). \qquad (2.1)$$

The issues raised by Figs 2.8 to 2.10 are solved. For instance, since $r(I_i^k,J_j^k)$ is weighted by x_i^k and z_j^k, the emergence of a segment as in Figure 2.10 has no significant impact on $F_r(\theta,A_\theta(v),B_\theta(v))$. Figure 2.14 shows that the emergence of a hole in a segment has no real impact either. Small changes in the longitudinal sections do not affect $F_r(\theta,A_\theta(v),B_\theta(v))$ significantly. Continuity is satisfied and, hence, robustness is achieved. The F_r–histogram associated with (A,B) is as defined in Section 2.2.2:

$$F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta,A_\theta(v),B_\theta(v))\ dv. \qquad (2.2)$$

Remember that the issue raised by Figure 2.9 has led us not to consider a longitudinal section a set of independent segments or points (Section 2.3.2). As a result, the F_r–histograms are neither f-histograms nor φ-histograms. Also note that the sum $\Sigma_{r\in A}\ F_r(\theta,A_\theta(v),B_\theta(v)) = \Sigma_{k\in 1..c}\ \Sigma_{i\in 1..m_k}\ \Sigma_{j\in 1..n_k}\ [x_i^k\ z_j^k\ (\alpha_k-\alpha_{k-1})]$ does not depend on any Allen relation. Therefore:

$$\Sigma_{r\in A}\ F_r^{AB}(\theta) = \Sigma_{r\in A} \int_{-\infty}^{+\infty} F_r(\theta,A_\theta(v),B_\theta(v))\ dv = \int_{-\infty}^{+\infty} \Sigma_{r\in A}\ F_r(\theta,A_\theta(v),B_\theta(v))\ dv$$

does not depend on any Allen relation either. Its value, however, is difficult to interpret. Let us redefine $F_r(\theta,A_\theta(v),B_\theta(v))$ this way [2]:

$$F_r(\theta,A_\theta(v),B_\theta(v)) = \frac{x+z}{w}\ \Sigma_{k\in 1..c}\ \Sigma_{i\in 1..m_k}\ \Sigma_{j\in 1..n_k}\ [x_i^k\ z_j^k\ (\alpha_k-\alpha_{k-1})]\ r(I_i^k,J_j^k)\ , \qquad (2.3)$$

where $x = \Sigma_{i\in 1..m_c}\ x_i^c$, $z = \Sigma_{j\in 1..n_c}\ z_j^c$, and $w = \Sigma_{k\in 1..c}\ \Sigma_{i\in 1..m_k}\ \Sigma_{j\in 1..n_k}\ [x_i^k\ z_j^k\ (\alpha_k-\alpha_{k-1})]$. We now have $\Sigma_{r\in A}\ F_r(\theta,A_\theta(v),B_\theta(v)) = x+z$, and the value $\Sigma_{r\in A}\ F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} \Sigma_{r\in A}\ F_r(\theta,A_\theta(v),B_\theta(v))\ dv$ is the total area of the subregions of A and B that are "facing" each other in direction θ (Figure 2.15). In other words, $\Sigma_{r\in A}\ F_r^{AB}(\theta)$ tells us to what extent the objects are involved in some spatial relationships along direction θ. If this information is judged to be unimportant, the F_r–histograms, of course, can be normalized. Let us denote by $\lceil F_r^{AB} \rfloor$ the histogram F_r^{AB} after normalization. $\lceil F_r^{AB} \rfloor$ is defined by [2]

---

[2] In Eqs. 2.3 and 2.4 we agree that a fraction is 0 if its denominator is 0.

$$\forall\theta\in\mathbb{R},\ \lceil F_r^{AB}\rfloor(\theta) = F_r^{AB}(\theta)\ /\ \Sigma_{\rho\in A}\ F_\rho^{AB}(\theta). \tag{2.4}$$

For a given oriented line $\Delta_\theta(v)$, the Allen relations define a crisp 13-partition of the set of pairs of segments on $\Delta_\theta(v)$. For a given direction θ, the normalized F_r–histograms define a fuzzy 13-partition of the set of all object pairs, and each class of the partition corresponds to an Allen relation.

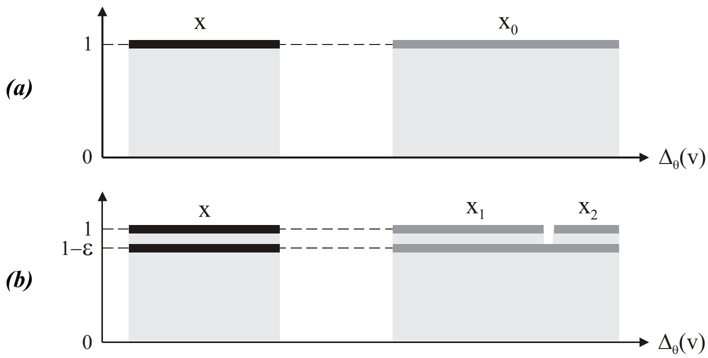


Fig. 2.14. In *(a)*, a missing pixel in the middle of one segment would not have much impact on $F_>(\theta,A_\theta(v),B_\theta(v))$. The way fuzzy relations are weighted (Eq. 2.1), combined with the way longitudinal sections are fuzzified, allow continuity to be satisfied. *(a)* $F_>(\theta,A_\theta(v),B_\theta(v)) = xx_0$. *(b)* $F_>(\theta,A_\theta(v),B_\theta(v)) = xx_1\varepsilon+xx_2\varepsilon+xx_0(1-\varepsilon) = x(x_1+x_2)\varepsilon+xx_0(1-\varepsilon) \approx xx_0\varepsilon+xx_0(1-\varepsilon) = xx_0$.

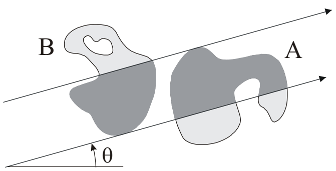


Fig. 2.15. The value $\Sigma_{r\in A}\ F_r^{AB}(\theta)$ is easy to interpret and gives useful information. In this example, $\Sigma_{r\in A}\ F_r^{AB}(\theta)$ is the total area of the two dark gray regions.

2.4. Experiments

In practice, of course, only a finite set of directions θ is considered. For the experiments described in this section, 360 directions were processed (i.e., the angle increment was 1 degree). All objects were stored in raster form. The computation of an F-histogram value, $F^{AB}(\theta)$, is achieved by partitioning the objects into longitudinal sections, i.e., into sets of adjacent pixels (Matsakis 1998; Matsakis and Wendling 1999). The generation of these sections is based on the rasterization of a pencil of parallel lines (Figure 2.2) by means of Bresenham's algorithm in integer arithmetic, which is commonly circuit-coded in visualization systems. The handling of a pair of objects then comes down to the handling of pairs of longitudinal sections, as described by Eq. 2.3. Note that, in a given image, all pairs of objects can be processed simultaneously. Moreover, F-histogram computation is highly parallelizable.

A grayscale value is associated with each Allen relation (Figure 2.16(a)). The referent, B, is always shown in dark gray and the argument, A, in light gray (Figure 2.16(b)). The thirteen F_r–histograms that represent the extracted directional and topological relationship information are plotted in the same diagram (Figure 2.16(c)). The topological relationships along direction θ (on the X-axis) are described by the vector composed of the thirteen $F_r^{AB}(\theta)$ values (on the Y-axis). Usually, most of these values are zero. The histograms are arranged in "layers." For a given θ, the total height of the layers (i.e., $\Sigma_{r \in A}\ F_r^{AB}(\theta)$) represents an area, as described in Section 2.3.3 and Figure 2.15. It tells us to what extent the objects are involved in some spatial relationships along θ. The thirteen normalized F_r–histograms can be plotted in the same way (Figure 2.16(d)). Figs. 16 and 17 show two object pairs and the corresponding diagrams. Figure 2.16(d) and Figure 2.17(c) illustrate well the symmetric nature of the histograms. For any θ, we have:

$$F_{>}^{AB}(\theta) = F_{<}^{AB}(\theta+\pi) \text{ and } F_{mi}^{AB}(\theta) = F_{m}^{AB}(\theta+\pi)$$
$$\text{and } F_{oi}^{AB}(\theta) = F_{o}^{AB}(\theta+\pi) \text{ and } F_{si}^{AB}(\theta) = F_{fi}^{AB}(\theta+\pi) \text{ and } F_{f}^{AB}(\theta) = F_{s}^{AB}(\theta+\pi).$$

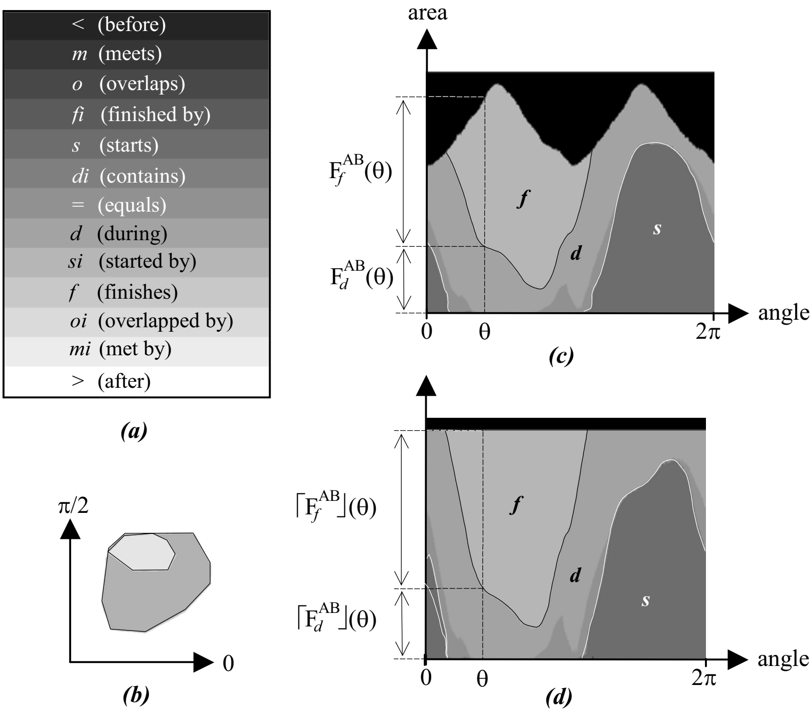


Fig. 2.16. *(a)* Allen relations and attached grayscale values. *(b)* A pair of objects. *(c)* Corresponding F_r–histograms. *(d)* Normalized F_r–histograms.

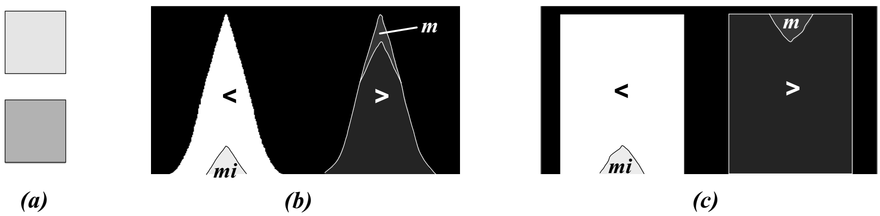


Fig. 2.17. *(a)* A pair of objects. *(b)* Corresponding F_r–histograms. *(c)* Normalized F_r–histograms.

The first series of experiments illustrates how the fuzzy relations defined in Section 2.3.1 are interconnected. It also demonstrates how the prominence of different relations waxes and wanes with the change of distance between the objects. In Figure 2.18(a), the objects are quite far apart, and only the relations *before* and *after* are present. As the distance shortens, Figure 2.18(b) and Figure 2.18(c), *meets* and *met by* appear and become more and more prominent, while *before* and *after* decrease in their importance. Finally, when the objects touch, Figure 2.18(d), *meets* and *met by* perfectly describe the scene.

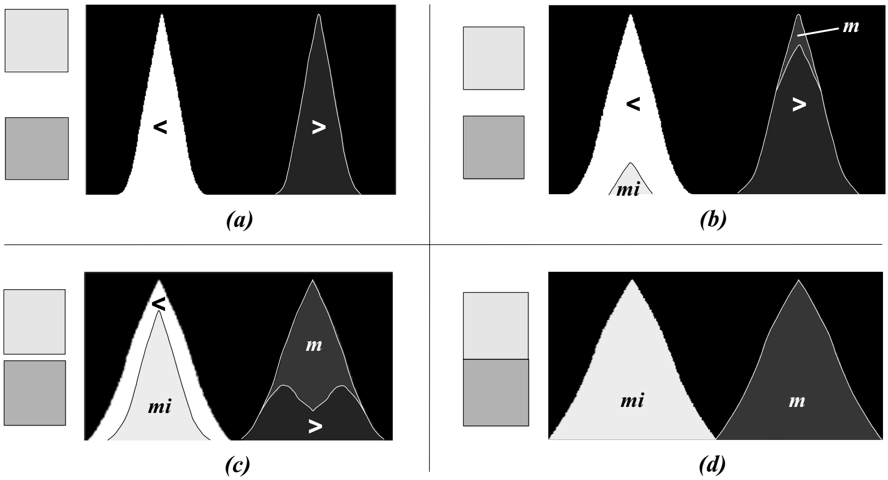


Fig. 2.18. First series of experiments. Interconnection of fuzzy relations and sensitivity to distance. *(a)* Objects far apart. Relations *before* and *after*. *(b)* Objects closer together. Relations *before*, *after*, *meets* and *met by*. *(c)* Objects very close together. *before* and *after* are less prominent, *meets* and *met by* are more. *(d)* Objects touching. Relations *before* and *after* disappear.

In the second series of experiments (Figure 2.19), we examine the relations between two convex objects A and B as A moves towards B, intersects it and, finally, goes through it. In Figure 2.19(a) and Figure 2.19(b), the only relations between A and B are *before* and *after*. As A moves towards B, the support of the two relations becomes wider. Once A intersects B, the relations become more complex and are mainly represented by the symmetric pairs *before – after, meets – met by* and *overlaps – overlapped by* (Figure 2.19(c)). In directions close to horizontal, there are also small contributions from *equals* and its conceptual neighbors *contains* and *during*. Once A is completely in B (Figure 2.19(d)), but still very close to its top edge, the relations *finishes* (when viewed from the bottom) and *starts* (when viewed from the top) become prominent and the relation *during* is consistently present. Once the object A moves further towards the center of B, *during* becomes by far the most important relation (Figure 2.19(e)); *finishes* and *starts* occur only briefly, in the directions where the distances between the edges of A and B are the smallest.

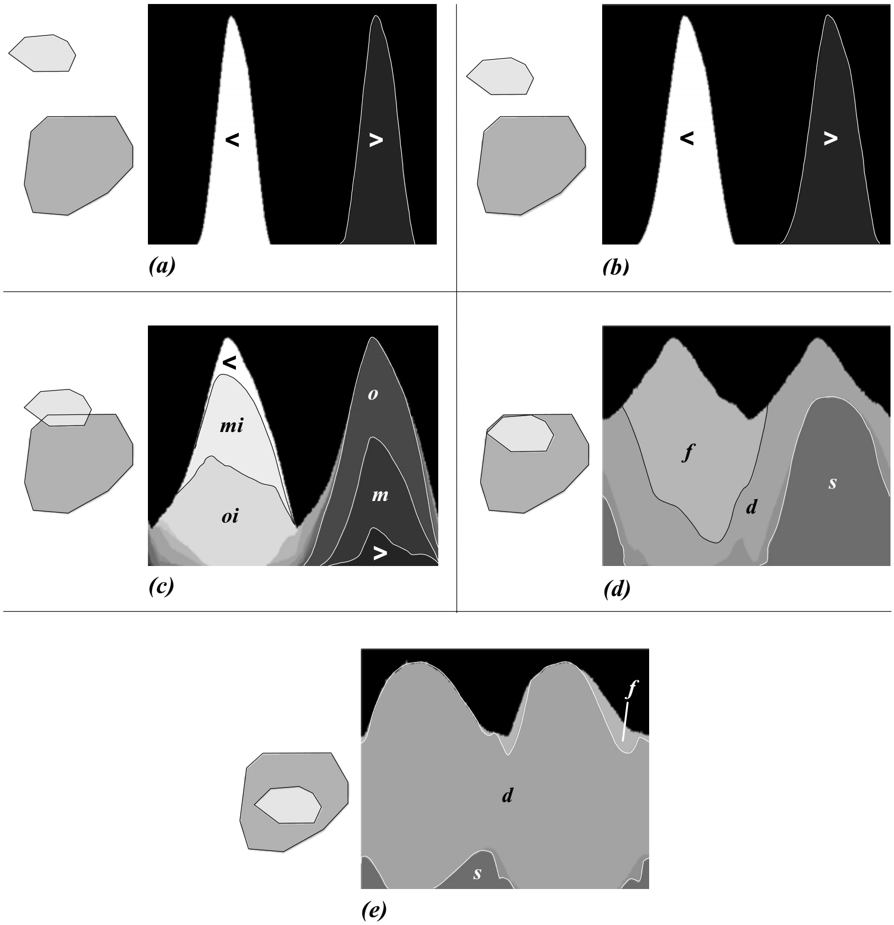


Fig. 2.19. Second series of experiments. Complex relation changes in a dynamic scene between convex objects. The same scene is considered in (Malki et al. 2002).

The final set of experiments involves concave objects. It illustrates how the F_r–histograms allow more complex topological relationships to be described and differentiated. In Figure 2.20(a), a convex object, B, is partially surrounded by a concave object, A. The diagram shows that the relations *before* and *after* coexist equally in the horizontal directions (B is between equidistant, equally thick "arms"). As the direction θ changes, *before* increases and then decreases in prominence, followed in its behavior by *after*. The twin "peaks" in the diagram (for both *before* and *after*) occur when θ passes through B and the arms of A (diagonal directions), whereas the "valleys" occur when θ passes through B and the "body" of A (vertical directions). In Figure 2.20(b), the concave object is asymmetrical, and the histograms are less regular. In the horizontal directions, *before* and *after* do not coexist any longer. Note that small contributions from *meets* and *met by* appear in both experiments due to the proximity of the objects. Also note the complete absence of the pair *overlaps – overlapped by*.

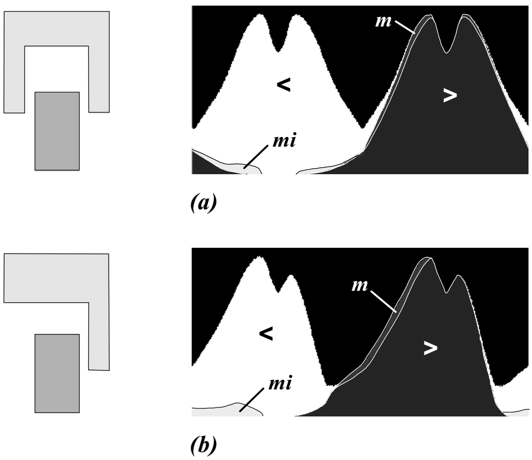


Fig. 2.20. Third series of experiments. Handling of concave objects. The same pairs are considered in (Petry et al. 2002). *(a)* The referent is partially surrounded by the argument. *(b)* The referent is surrounded to a smaller degree.

2.5. Conclusion

The F-histogram is a powerful generic quantitative representation of the relative position between two 2D objects. In this chapter, we have designed a set of thirteen histograms that constitutes a valuable tool for extracting directional and topological relationship information. Imprecision is handled and robustness achieved through fuzzy set theoretic approaches. For any direction in the plane, the F-histograms introduced here define a fuzzy 13-partition of the set of all pairs of objects, where each class of the partition corresponds to an Allen relation. The considered objects are not necessarily convex, nor connected, and they may have holes in them. We have shown that the F-histograms associated with a given pair of objects carry lots of relationship information. For instance, an ambiguity index can be calculated to assess the complexity of the topological relationships along any direction. If so desired, only the Allen relation that represents these relationships the best can be kept (defuzzification). Alternatively, two Allen relations can be kept—the most prominent—and weighed by their corresponding membership degrees. The number of directions to be processed can be chosen according to needs, interests and constraints (e.g., accuracy, computational efficiency). It can be as low as two (horizontal and vertical directions, like for MBRs) and as large as a few hundred (e.g., the increment step of 1° chosen for our experiments). Since directions are handled independently from each other, additional ones can be considered in a second stage, depending on the case in hand (dynamic refinement). The direction for which the ambiguity index is minimum can be searched for. Spatial relationships can be compared from one pair of objects to another, using similarity or distance measures between the vectors of membership degrees in all considered directions. These are avenues that we intend to explore in future work.

References

Allen JF (1983) Maintaining Knowledge about Temporal Intervals. Communications of the ACM 26(11):832-843

Bloch I (1999) Fuzzy Relative Position between Objects in Image Processing: New Definition and Properties Based on a Morphological Approach. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 7(2):99-133

Clementini E (2002) Path Composition of Positional Relations Integrating Qualitative and Fuzzy Knowledge. In: Matsakis P, Sztandera L (eds) Applying Soft Computing in Defining Spatial Relations. Studies in Fuzziness and Soft Computing, Physica-Verlag, 106:17-39

Clementini E, Di Felice O (1997) Approximate Topological Relations. International Journal of Approximate Reasoning 16:173-204

Clementini E, Sharma J, Egenhofer MJ (1994) Modelling Topological and Spatial Relations: Strategies for Query Processing. Computers and Graphics 18(6):815-822

Cobb M, Petry F, Robinson V (2000) Special Issue: Uncertainty in Geographic Information Systems and Spatial Data. Fuzzy Sets and Systems 113(1)

Cohn AG (1995) The Challenge of Qualitative Spatial Reasoning. Computing Surveys 27(3):323-327

Cohn AG, Bennett B, Gooday J, Gotts NM (1997) Representing and Reasoning with Qualitative Spatial Relations about Regions. In: Stock O (ed) Spatial and Temporal Reasoning. Kluwer, pp 97-134

Dutta S (1991) Approximate Spatial Reasoning: Integrating Qualitative and Quantitative Constraints. International Journal of Approximate Reasoning 5:307-331

Freeman J (1975) The Modeling of Spatial Relations. Computer Graphics and Image Processing 4:156-171

Freksa C (1992) Temporal Reasoning Based on Semi-Intervals. Artificial Intelligence 54:199-227

Goodchild M, Gopal S (1990) (eds) The Accuracy of Spatial Databases. Taylor and Francis, Basingstoke, UK

Guesgen HW (2002) Fuzzifying Spatial Relations. In: Matsakis P, Sztandera L (eds) Applying Soft Computing in Defining Spatial Relations. Studies in Fuzziness and Soft Computing, Physica-Verlag, 106:1-16

Krishnapuram R, Keller J, Ma Y (1993) Quantitative Analysis of Properties and Spatial Relations of Fuzzy Image Regions. IEEE Transactions on Fuzzy Systems 1(3):222-233

Kuipers B (1978) Modeling Spatial Knowledge. Cognitive Science 2:129-153

Kuipers BJ, Levitt TS (1988) Navigation and Mapping in Large-Scale Space. AI Magazine 9(2):25-43

Malki J, Zahzah EH, Mascarilla L (2002) Indexation et recherche d'image fondées sur les relations spatiales entre objets. Traitement du Signal 18(4)

Matsakis P (1998) Relations spatiales structurelles et interprétation d'images. Ph. D. Thesis, Institut de Recherche en Informatique de Toulouse, France

Matsakis P (2002) Understanding the Spatial Organization of Image Regions by Means of Force Histograms: A Guided Tour. In: Matsakis P, Sztandera L (eds) Applying Soft Computing in Defining Spatial Relations. Studies in Fuzziness and Soft Computing, Physica-Verlag, 106:99-122

Matsakis P, Andréfouët S (2002) The Fuzzy Line Between Among and Surround. FUZZ-IEEE 2002 (IEEE International Conference on Fuzzy Systems), Honolulu, Hawaii

Matsakis P, Wendling L (1999) A New Way to Represent the Relative Position between Areal Objects. IEEE Transactions on Pattern Analysis and Machine Intelligence 21(7):634-643

Matsakis P, Keller J, Wendling L, Marjamaa J, Sjahputera O (2001) Linguistic Description of Relative Positions in Images. IEEE Transactions on Systems, Man and Cybernetics, Part B 31(4):573-588

Matsakis P, Keller J, Sjahputera O, Marjamaa J (2004) The Use of Force Histograms for Affine-Invariant Relative Position Description. IEEE Pattern Analysis and Machine Intelligence 26(1):1-18

Nabil M, Shepherd J, Ngu AHH (1995) 2D Projection Interval Relationships: A Symbolic Representation of Spatial Relationships. SSD '95 (Advances in Spatial Databases: 42nd Symposium) pp 292-309

Petry F, Cobb M, Ali D, Angryk R, Paprzycki M, Rahimi S, Wen L, Yang H (2002) Fuzzy Spatial Relationships and Mobile Agent Technology in Geospatial Information Systems. In: Matsakis P, Sztandera L (eds) Applying Soft Computing in Defining Spatial Relations. Studies in Fuzziness and Soft Computing, Physica-Verlag, 106:123-155

Robinson V (1988) Implications of Fuzzy Set Theory for Geographic Databases. Computers, Environment, and Urban Svstems 12:89-98

Sharma J, Flewelling FM (1995) Inferences from Combined Knowledge about Topology and Direction. SSD'95 (Advances in Spatial Databases: 42nd Symposium) pp 279-291

Skubic M, Matsakis P, Chronis G, Keller J (2003) Generating Multi-level Linguistic Spatial Descriptions from Range Sensor Readings Using the Histogram of Forces. Autonomous Robots 14(1):51-69

Stoms D (1987) Reasoning with Uncertainty in Intelligent Geographic Information Systems. GIS'87 (2nd Annual International Conference on Geographic Information Systems) pp 693-699

Wang F, Hall GB, Subaryono (1990) Fuzzy Information Representation and Processing in Conventional GIS Software: Database Design and Application. International Journal of Geographical Information Systems 4:261-283

3. Field Based Methods for the Modeling of Fuzzy Spatial Data

Jörg Verstraete, Guy De Tré, Rita De Caluwe and Axel Hallez

Department of Telecommunications and Information Processing, Ghent University, B9000 Ghent, Belgium

Abstract. In this chapter, two different field based techniques for the modeling of fuzzy information spread over a geographic region, are presented and are compared regarding their applicability. The first one is a vector-mode approach, using triangulated irregular networks (or TINs), the second one is a raster (bitmap-mode) approach. Appropriate aggregation operators are defined in both approaches and illustrated by means of examples. The feasibility of the implementation of the operators (by approximation whenever required) is studied. Attention has been paid to the applicability, advantages and disadvantages of both methods in flexible querying.

3.1. Introduction

One of the latest developments in Geographic Information Systems (GIS for short), is handling *uncertain* and *imprecise* information (Morris 2001; Schneider 1999; Zimmerman 1999). At the heart of a GIS is a database in which geospatial data (information on certain locations, e.g. the exact position of a house in a navigation system) and related attribute data (information that is related to a location, e.g. the temperature at the location or the house number) are stored. Special data structures and operations are used to model and manipulate the required data and to query the database in an adequate way.

The geographic information and the related attribute data are often obtained through measurements in the field, by extracting data from images (satellite or aerial) or by applying various sensing techniques (Rigaux et al. 2002). As it is physically impossible to measure values or to record data on every square millimeter, the information is very prone to imprecision or uncertainty. Imprecision occurs when the value of the data is not precisely known, whereas uncertainty occurs when there is doubt concerning the data. This difference is mainly semantical. Basically imprecision and uncertainty can occur (separately or combined) in two different contexts: either imprecision and/or uncertainty in the spatial domain, or imprecision and/or uncertainty concerning the attribute data themselves.

Consider for example that data about a house in a given street are to be registered. Imprecision in the spatial domain can occur when the location of the house with e.g. number 65 is not precisely known, for instance due to limitations in positional measurements as traditional systems use an interpolation between two cor-

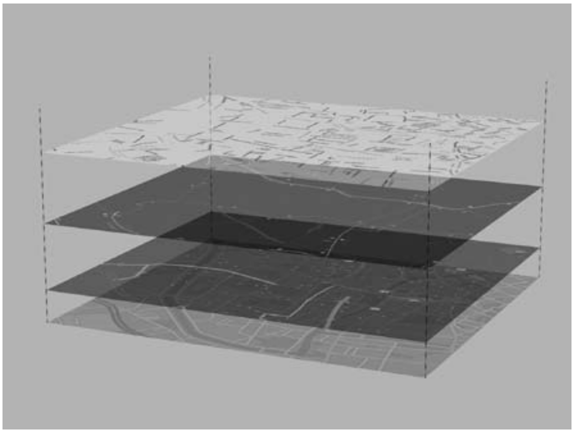


Fig. 3.1. An example of the layer architecture in GIS

ners of a block to determine the location of a given number. Uncertainty in the spatial domain can occur when there is doubt about the position of the particular house. Imprecision in the attribute domain can occur when e.g. it is only known that the number of the house is between 60 and 70. Finally, uncertainty in the attribute domain can occur when one is not sure of the number of the house. Hence, handling these cases of fuzziness and uncertainty more adequately logically leads to more general database models and has to provide for more flexible querying methods (De Tré et al. 2002).

Information in a GIS is generally stored in *layers*, as shown on Figure 3.1. Each layer contains information of one kind: for example the GIS could have a layer containing information about altitudes, another layer containing measured temperatures, etc. In order to answer a query, different kinds of information (thus stemming from different layers) may have to be combined; this operation is called *overlay*. Within a layer, there basically are two main approaches for modeling information (Rigaux et al. 2002, Shekhar et al 2003): an *entity-based* approach and a *field based* approach.

In an entity-based approach, objects and their locations are modeled (e.g. locations of houses, roads, ...) using basic geometric structures such as points, lines and areas (delimited by polygons). A lot of work has already been done in extending entity based approaches, often by using some form of contour lines to model vague regions (Clementini and Di Felice, 1994; Cohn and Gotts, 1994; Gotts and Cohn, 1995; Hallez et al. 2002; Verstraete et al. 2000). These basically are regions with *undetermined boundaries*, or regions located at uncertain or imprecise positions.

In a field based approach, *global* data – i.e. data that are present over the entire area under consideration – are modeled (e.g. temperatures, population densities, ...). For the modeling of this type of data, bitmaps, tessellations (both regular and irregular) and even halfplanes are commonly used.

Both the approaches are used simultaneously in a GIS, each with its specific applications, benefits and drawbacks.

In this chapter, the focus will be on field-based models; both an extension of a tessellation-structure consisting of triangulated irregular networks and an extension of a traditional bitmap-structure are presented. Both are adapted for the modeling of imprecision and/or uncertainty about the attribute data associated with points in the entire area of interest. Section 2 brings a reminder on the classical notions of triangulated irregular networks (with Delaunay triangulations and constrained Delaunay triangulations), thereby introducing notations that will be used throughout the chapter. Section 3 highlights in which way the use of fuzzy set theory can constitute an advantage in handling information associated with triangulated irregular networks (TINs). Section 4 reminds the bitmap models and in section 5 extensions to extended bitmaps (EBs) are discussed. Applicability of ETINs and of EBs is shortly discussed in the corresponding sections 3 and 5, but in section 6 a more in depth discussion is presented. Section 7 concludes the chapter.

3.2. Triangulated Irregular Networks: TINs

3.2.1. Definitions, notations and spatial representation

A Triangulated Irregular Network (TIN) is based on a partition of the two-dimensional space in non-overlapping triangles. This structure is often used in digital elevation models (DEMs). TINs are an example of a field based model, meaning that the TIN is considered to cover the entire map – which might be limited to a region of interest, if necessary. Furthermore, TINs use a vector-mode (Rigaux et al. 2002) approach, more specifically their basic structures are points, edges and triangles. No assumption is made about the distribution and location of the vertices of the triangles (Rigaux et al. 2002).

Definition 2.1. A Tin (as an occurrence of a TIN) is defined by a non-empty finite set of points, connected by non-intersecting straight line segments thereby covering the plane completely with non-overlapping triangles. This can be denoted by means of a triplet containing three finite sets: a set P of points (the vertices of the triangles), a set E of edges (the straight line segments that are the sides of the triangles) and a set T of non-overlapping triangles including their interior (the *tiles* of the TIN).

$$Tin = (P, E, T)$$

Various algorithms can be used to base a TIN on a given set of points. The *Delaunay triangulation* (Shewchuk 1996, 2002) is commonly used. In this case, given a set of points P, the Delaunay triangulation construction algorithms will "build" a network (i.e. define the sets E and T) such that for every triangle in the network, its circumscribing circle does not contain additional points of the set *P*. Apart from a few trivial cases (e.g. four points, each located on the corners of a square), the resulting TIN network is completely and uniquely defined on a given set of points; appropriate definitions also eliminate the trivial cases. Due to this definition, the triangles in the TIN will resemble the equilateral triangle (having all sides equal) as closely as possible (Shewchuk 2002). This property will have a beneficial effect on the interpolation, as degenerate cases caused by narrow, sharp triangles will be avoided.

An interesting extension of the TIN is obtained through the *constrained Delaunay triangulation* (Shewchuk 1996). Instead of defining a network on merely a set *P* of points, this method offers the possibility to specify a set $E' \subseteq E$ of edges to be part of the final triangulation. However, the resulting network does not necessarily satisfy the definition of a Delaunay triangulation: now the circumscribing circle of a triangle in *T* might contain additional points of *P*.

It shall be clear further on in this chapter that, if the networks are to be adopted for representing fuzzy information, even the simplest of operators will resort to the use of constrained Delaunay triangulations. For the remainder of this chapter networks obtained through both the Delaunay triangulation as well as networks obtained through constrained Delaunay triangulation will both be referred to as TINs.

Implementations can require the manipulation of a TIN network. This can be done by adding points to or removing points from the point-set *P*; the changes required to maintain a Delaunay or constrained Delaunay network are localized around the added or removed points. The algorithms to perform the addition or the removal of points extend beyond the scope of this chapter; they are discussed in (Rigaux et al. 2002).

In itself, a TIN is a two dimensional, planar structure. Choosing the XY-plane as a reference, the position of each point in the set *P* is determined by a couple (x,y) consisting of two coordinates. With every point, a value expressing a property can be associated. This associated value can be seen in a third dimension as the third coordinate, *z*, e.g. representing the temperature measured at the related position. Hence, the following notations will be used: $p(x,y)$ for a point in the XY-plane, $p(x,y,z)$ for a point in the XYZ-space. Some algorithms – which will be discussed further on – do not require the associated value to be taken into account; this will be reflected in the used notations. The points of the set *P* are called the *data-points* of the TIN. For points $p(x,y)$ located on an edge or in the interior of a triangle, the associated z-value is calculated by means of linear interpolation in the plane defined by the z-coordinates (i.e. the associated values) of the vertices of this triangle.

3.2.2. The description of the fuzzy spatial data associated with TINs

In traditional geographic databases, TINs are used to model geographically related attribute data over a region of interest. Data have to be available for each point in

P, suitable data for other points of the considered region are obtained through linear interpolation.

In the presented model, the same TIN structures are used, but the modeled attribute values are not necessarily crisp, nor known values. Typical for the models that are presented in the chapter, is that this is accomplished using fuzzy set theory (Dubois and Prade 2000). For instance, this theory allows for expressing a degree (called a membership-grade) in the range [0,1] to indicate the extent to which a property is satisfied or a value is un(certain). For example, the extent to which the temperature in a location can be qualified as *warm* when the temperature at that location is known, can be expressed by a membership grade. Likewise, the extent to which there is certainty about a recorded temperature value can be modeled using membership grades. In general, membership grades can be interpreted in serveral ways: they can be either interpreted as degrees of similarity, degrees of preference, or degrees of uncertainty (Dubois and Prade 1997). Some of them may not be of interest in the context of GIS.

In the following subsections, it will be shown how TINs can be extended to ETINs in four different ways, each one allowing a different interpretation of the data values associated with the TINs, thereby relying on fuzzy set theory and the related possibility theory. The basics of fuzzy set theory can be found in (Zimmerman 1999).

Working with fuzzy set theory implies the definition and use of extended set operators based on the definition of t-norms[1] and t-conorms[2]. The association of membership grades with the points of a TIN has no impact on the interpolation method. However, as the TIN requires data associated with every point, the membership grade 0 should be explicitly denoted in the model; hence associated values will be in the range [0,1]. This implies special care in defining the operators. Associating fuzzy values with points of a TIN does require a slight modification of the interpolation method, as shall be discussed further on.

3.3. Extended Triangulated Irregular Networks: ETINs

3.3.1. ETINs based on membership grades

Definitions

In the presented model, the TIN structure (P, E, T) is extended with a mapping function *f* which characterizes a property *F* regarding a geographic location, for example the property "near Ghent". When modeling this property, a value *1* indicates the location is *near* (or in) Ghent, a value *0* means the location is not at all

---

[1] A t-norm is a commutative, associative, non-decreasing function with 1 as neutral element.

[2] A t-conorm is a commutative, associative, non-decreasing function with 0 as neutral element.

near to Ghent and e.g. a value *0.6* means the location is considered to be more or less near Ghent (all values in [0,1] can occur).

Definition 3.1 An **ETin with membership grades** (as an occurrence of an ETIN*)* is defined by a TIN structure and a mapping function *f*.

$$E\tilde{T}in = [(P,E,T),f]$$

where *f* is defined by

$$\begin{aligned} f: \quad & P \rightarrow [0,1] \\ & p(x,y) \mapsto f(p(x,y)), \forall p \in P \end{aligned}$$

The function *f* associates a membership grade with regard to the property *F* with every point in *P*. In order to determine the membership grades for the other points of the considered region *U*, a definition for the membership grades of these points of the network needs to be given. As said before, the ETIN can be considered as a three dimensional structure; where the X- and Y-axes are interpreted as the domain-axes and the Z-axis represents the membership grade (Verstraete et al. 2002). Calculating the grade for a point $p(x,y) \in U$ is done by firstly determining the triangle in which $p(x,y)$ is located. Suppose that this triangle is defined by the points $p_1(x_1,y_1)$, $p_2(x_2,y_2)$ and $p_3(x_3,y_3)$ of *P*. Considering all points of the region *U*, the membership grades μ_F are defined by linear interpolation of the mapping function *f* as follows

$$\begin{aligned} & \mu_F : U \rightarrow [0,1] \\ & p(x,y) \mapsto \begin{cases} f(p(x,y)) & \text{if } p(x,y) \in P \\ -\dfrac{A}{C}x - \dfrac{B}{C}y - \dfrac{D}{C} & \text{otherwise} \end{cases} \end{aligned}$$

where *U* represents the considered region defined by the X and Y axes, and *A*, *B*, *C* and *D* are the parameters of the equation $Ax+By+Cz+D=0$ of the plane containing the three points $p_1(x_1,y_1,z_1)$, $p_2(x_2,y_2,z_2)$ and $p_3(x_3,y_3,z_3)$ (with the understanding that $z_i=f(x_i,y_i)$, $i=1,2,3$), i.e.

$$\begin{aligned} A &= y_1(z_2 - z_3) + y_2(z_3 - z_1) + y_3(z_1 - z_2) \\ B &= z_1(x_2 - x_3) + z_2(x_3 - x_1) + z_3(x_1 - x_2) \\ C &= x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \\ D &= -Ax_1 - By_1 - Cz_3 \end{aligned}$$

The points $p_1(x_1,y_1)$, $p_2(x_2,y_2)$ and $p_3(x_3,y_3)$ in the XY-plane should not be co-linear, which is guaranteed by the fact that no Delaunay triangulation (or even constrained Delaunay triangulation) would result in a triangulation containing such a degenerate case.

Operations

As an ETIN is an extension of a TIN, again no assumption on the location of its data-points is made, nor do two (or more) ETINs that are intended to be combined need to have the same number of data points or have their points at the same locations.

The ETIN-structure as defined above allows the modeling of membership grades associated with a geographically spread property. As it can be useful to combine data that are present in the GIS, there is a need for combining ETINs, which requires specific operators. To explain the way operators can be defined, defining the intersection and union operators using t-norms and t-conorms (Dubois and Prade 2000) are illustrated.

By way of example, the intersection of two triangulated irregular networks is considered. In every point of $ETin_1$, the associated data represents the degree to which the property F_1, e.g. "near Ghent", is satisfied. Similarly, in every point of $ETin_2$, the associated data represents the degree to which the property F_2, e.g. "along the river Schelde", is satisfied.

If the combination "near Ghent and alongside the river Schelde" is expressed, the two ETINs need to be combined, forming a new ETIN which represents the intersection of the fuzzy sets modeled by the two ETINs.

The intersection of fuzzy sets is performed using a t-norm; a commonly used t-norm is the minimum, which is also used in this example. Informally, when considering the ETINs as three dimensional structures, the desired result of the minimum-operation would be that all the “lowest” points of the two ETINs are retained. These include the data-points, but also the points on edges and inside triangles.

Suppose that we have two ETIN structures $ETin_1$ and $ETin_2$, defined as shown in Figure 3.2.a-b respectively. The data points of $ETin_i$ will be denoted in the three-dimensional space as $p_{j_i}\left(x_{j_i}, y_{j_i}, f_i\left(x_{j_i}, y_{j_i}\right)\right); i = 1,2; j = 1,2,..|P_i|$ (where $|P_i|$ represents the cardinality of P_i); an edge e_{k_i} connecting the points p_{l_i} and p_{m_i} is denoted as $e_{k_i}\left(p_{l_i}, p_{m_i}\right)$. As a shorthand notation, the indices i will be omitted if no confusion is possible. The intersection of the ETINs $ETin_1=[(P_1,E_1,T_1),f_1]$ and $ETin_2=[(P_2,E_2,T_2),f_2]$ (with associated membership functions resp. μ_{F_1} and μ_{F_2}) is by definition obtained by considering the $\min\left(\mu_{F_1}\left(p(x,y)\right),\mu_{F_2}\left(p(x,y)\right)\right)$ of the membership grades of each point $p(x,y)$ in the considered region U. A computable definition of the minimum denoted $ETin_3$ of two ETINs $ETin_1$ and $ETin_2$ can be derived by using the actual definitions of both these arguments. To "build" the resultant network, the set P_3 containing the points that will define $ETin_3$ has to be defined.

P_3 is the union of two sets that will be defined below:

$$P_3 = P_{t_1} \cup P_{t_2}$$

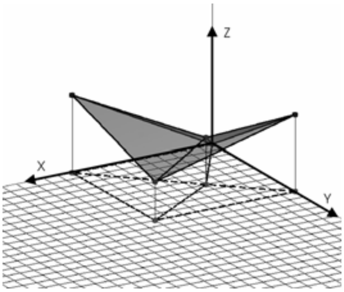


Fig. 3.2.a. ETin_1

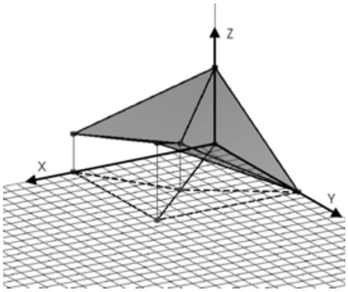
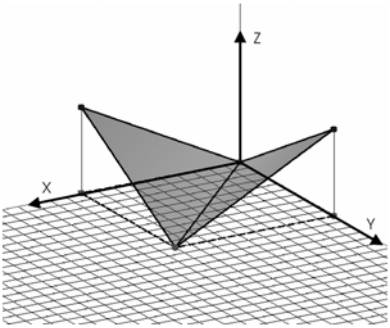


Fig. 3.2.b ETin_2

At first, the points of P_1 that are located "below" $ETin_2$ are added to P_{t_1}. As there is no requirement regarding the relative locations of the data points of both networks, it is possible that values associated with points $p_{j_1}(\mathrm{x}_{j_1}, \mathrm{y}_{j_1}) \in \mathrm{P}_1$ will be compared either with values associated with points $p_{j_2}(\mathrm{x}_{j_1}, \mathrm{y}_{j_1}) \in \mathrm{P}_2$ or with values computed for points $p(\mathrm{x}_{\mathrm{j}_1}, \mathrm{y}_{\mathrm{j}_1}) \notin \mathrm{P}_2$. In either case, if the value $f_1(p_{j_1}(\mathrm{x}_{j_1}, \mathrm{y}_{j_1}))$ is "lower" than or equal to the associated or calculated value in $ETin_2$ at this position $\mu_{F_2}(p(\mathrm{x}_{\mathrm{j}_1}, \mathrm{y}_{\mathrm{j}_1}))$, the point $p_{j_1}(\mathrm{x}_{j_1}, \mathrm{y}_{j_1})$ is contained in P_{t_1}. Completely analogue, the points of P_2 that are "below" $ETin_1$ are added to P_{t_1}.

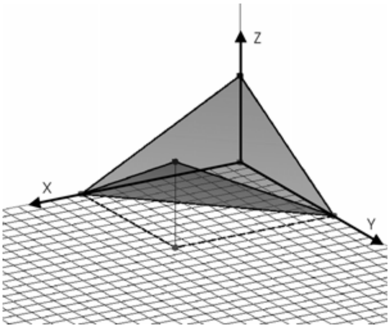
$$
\begin{aligned}
P_{t_1} \quad = \quad & \left\{p_{j_1}(x_{j_1}, y_{j_1}) \in P_1 \middle| f_1(p_{j_1}(x_{j_1}, y_{j_1})) \le \mu_{F_2}(p(x_{j_1}, y_{j_1}))\right\} \\
& \cup \left\{p_{j_2}(x_{j_2}, y_{j_2}) \in P_2 \middle| f_2(p_{j_2}(x_{j_2}, y_{j_2})) \le \mu_{F_1}(p(x_{j_2}, y_{j_2}))\right\}
\end{aligned}
$$

In a next step, the set P_{t_2} of points that result from the intersection of triangles in $ETin_1$ and the edges of triangles in $ETin_2$ and vice versa is determined.



$P_{1'}$	$E_{1'}$
$p_1(0,0,0)$	$e_1(p_1,p_3)$
$p_2(0,100,1)$	$e_2(p_3,p_2)$
$p_3(100,100,0)$	$e_3(p_1,p_2)$
$p_4(100,0,1)$	$e_4(p_3,p_4)$
	$e_5(p_1,p_4)$

Fig. 3.3.a. $\mathrm{ETin}_{1'}$



$P_{2'}$	$E_{2'}$
$p_1(0,0,1)$	$e_1(p_2,p_1)$
$p_2(0,100,0)$	$e_2(p_1,p_4)$
$p_3(100,100,1)$	$e_3(p_4,p_2)$
$p_4(100,0,0)$	$e_4(p_4,p_3)$
	$e_5(p_3,p_2)$

Fig. 3.3.b. $\mathrm{ETin}_{2'}$

$$
\begin{aligned}
P_{t_2} \quad = \quad & \{p(x,y) | p(x,y) \notin P_1 \cup P_2 \wedge \\
& ((\exists e \in E_1 : p(x,y) \in e \wedge \exists t \in T_2 : p(x,y) \in t \wedge \\
& \mu_{F_1}(p(x,y)) = \mu_{F_2}(p(x,y))) \vee \\
& (\exists e \in E_2 : p(x,y) \in e \wedge \exists t \in T_1 : p(x,y) \in t \wedge \\
& \mu_{F_1}(p(x,y)) = \mu_{F_2}(p(x,y))))\}
\end{aligned}
$$

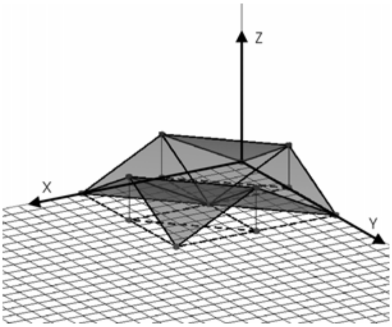
These points were not necessarily present in any of the original ETINs. These points are needed as they determine where an edge of one network "stops" being located "below" the other network.

Together with the locations of the points, their associated values in the new ETIN must be determined. For points in P_{t_1} their associated values are the same as the respective associated values in the ETIN they originate from. For points in P_{t_2} that were obtained as intersection points of *ETin₁* and *ETin₂*, the interpolated values (in either network: in these points the associated values of both networks are equal) are used.

The minimum will then be a new ETIN $ETin_3=[(P_3,E_3,T_3),f_3]$, defined by the points in P_3. Using the set P_3, considering the points not in three dimensions but in the XY-plane (i.e. ignoring the $f_3(p(x,y))$), as the input for a Delaunay triangulation will yield a unique triangulated irregular network.

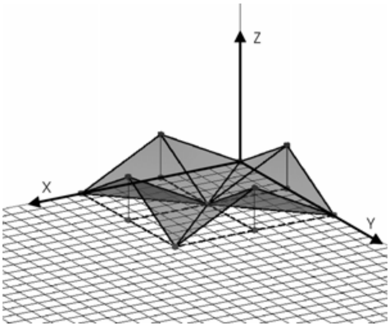
However, defining the set P_3 does not suffice to define the new network: some points in the region may have interpolated values μ_{F_3} that are not the minimum of the respective values in both $ETin_1$ and $ETin_2$. To illustrate where this problem originates from, consider two ETINs $ETin_{1'}$ and $ETin_{2'}$ as defined in Figure 3.3. Although a Delaunay triangulation algorithm would – in this simple case – have yielded the same networks for these same sets of four points, this artificial example best illustrates the problem that can occur in more elaborate (genuine) Delaunay triangulations.

The ETIN that is obtained by applying a Delaunay triangulation algorithm on the set P_3 is shown in Figure 3.4. It can easily be verified that the result is incorrect, considering the expected geometrical minimum as shown in Figure 3.5. Consider for instance the edges $e_2(p_2,p_3)$ and $e_{12}(p_5,p_4)$ of $ETin_{3'}$ in Figure 3.4 that are generated by the triangulation algorithm. The point $p(25,25)$ is located on $e_2(p_2,p_3)$. This is a point with interpolated associated values in each of the three networks ($ETin_{1'}$, $ETin_{2'}$ and $ETin_{3'}$) . As can be clearly seen in Figure 3.4, this point has an associated, calculated membership grade $\mu_{F_{3'}}(p(25,25))=0.5$ in $ETin_{3'}$. However, membership grades for this point in both $ETin_{1'}$ and $ETin_{2'}$ are $\mu_{F_{1'}}(p(25,25))=\mu_{F_{2'}}(p(25,25))=0$; the minimum value should therefore also be equal to 0. This difference is due to the fact that the Delaunay triangulation generates new edges, which are not part of any of the original ETINs, and which



$P_{3'}$	$E_{3'}$
$p_1(50,50,0)$	$e_1(p_6,p_2)$
$p_2(50,0,0.5)$	$e_2(p_2,p_3)$
$p_3(0,50,0.5)$	$e_3(p_3,p_6)$
$p_4(100,50,0.5)$	$e_4(p_1,p_8)$
$p_5(50,100,0.5)$	$e_5(p_8,p_3)$
$p_6(0,0,0)$	$e_6(p_3,p_1)$
$p_7(100,100,0)$	$e_7(p_2,p_1)$
$p_8(0,100,0)$	$e_8(p_1,p_9)$
$p_9(100,0,0)$	$e_9(p_9,p_4)$
	$e_{10}(p_4,p_1)$
	$e_{11}(p_2,p_9)$
	$e_{12}(p_5,p_4)$
	$e_{13}(p_4,p_7)$
	$e_{14}(p_7,p_5)$
	$e_{15}(p_5,p_1)$
	$e_{16}(p_5,p_8)$

Fig. 3.4. $ETin_{3'}$, obtained through an incorrect calculation of min($ETin_{1'}$,$Etin_{2'}$)



$P_{4'}$	$E_{4'}$
$p_1(50,50,0)$	$e_1(p_2,p_1)$
$p_2(50,0,0.5)$	$e_2(p_1,p_6)$
$p_3(0,50,0.5)$	$e_3(p_6,p_2)$
$p_4(100,50,0.5)$	$e_4(p_1,p_8)$
$p_5(50,100,0.5)$	$e_5(p_8,p_3)$
$p_6(0,0,0)$	$e_6(p_3,p_1)$
$p_7(100,100,0)$	$e_7(p_3,p_6)$
$p_8(0,100,0)$	$e_8(p_1,p_9)$
$p_9(100,0,0)$	$e_9(p_9,p_4)$
	$e_{10}(p_4,p_1)$
	$e_{11}(p_2,p_9)$
	$e_{12}(p_1,p_7)$
	$e_{13}(p_7,p_5)$
	$e_{14}(p_5,p_1)$
	$e_{15}(p_4,p_7)$
	$e_{16}(p_3,p_8)$

Fig. 3.5. *ETin*$_{4'}$, obtained through a proper calculation of min(*ETin*$_{1'}$, *ETin*$_{2'}$)

do not satisfy the minimum criterion. Especially when higher accuracy is desired, this result is most likely to be insufficient, which calls for a better approach.

In order to overcome this problem, a set of predefined edges E'_3 will be determined. These edges will be used to force the triangulation algorithm to maintain them, as they are needed in the resulting ETIN; the constrained Delaunay triangulation can then be used to calculate the correct minimum.

Similar to the definition of P_3, E'_3 is determined in a number of steps. At first, the edges obtained through the intersection of the triangles of the two ETINs $ETin_1$ and $ETin_2$ are added to E_{t_1} (Figure 3.6.a)

$$E_{t_1} = \left\{ e'(p_{l'}, p_{m'}) \middle| p_{l'} \in P_{t_2}, p_{m'} \in P_{t_2} \wedge \exists t \in T_1 \cup T_2 : e'(p_{l'}, p_{m'}) \subset t \right\}$$

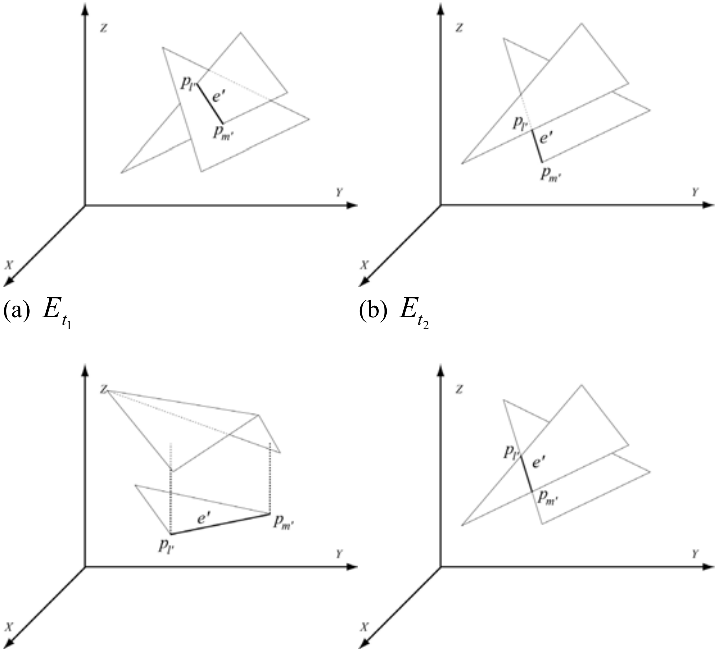
As a next step, all the segments $e'(p_{l'},p_{m'})$ of existing edges $e(p_l,p_m)$ in E_1 or E_2 that connect a point of $p_{l'} \in P_{t_2}$ (i.e. a point obtained through the intersection of a triangle and the edge e of which e' is a segment) with a point $p_{m'} \in P_{t_1}$ (i.e. a point that is definitely part of the minimum) are added (Figure 3.6.b). These segments are in

$$\begin{aligned} E_{t_2} \quad = \quad & \left\{ e'(p_{l'},p_{m'}) \middle| p_{l'} \in P_{t_2} \wedge p_{m'} \in P_{t_1} \wedge \right. \\ & \left. \left(\exists e(p_l,p_m) \in E_1 \cup E_2 : e'(p_{l'},p_{m'}) \subseteq e(p_l,p_m) \right) \right\} \end{aligned}$$

The edges that interconnect two points, that are definitely part of the minimum but are not intersection points (i.e. $p_{l'} \in P_{t_1}$ and $p_{m'} \in P_{t_1}$), and that are an existing edge in either ETIN ($e(p_l,p_m) \in E_1 \cup E_2$) are also added (Figure 3.6.c). These edges are contained in

$$
\begin{aligned}
E_{t_3} \quad = \quad & \{e'(p_{l'},p_{m'}) | p_{l'} \in P_{t_1} \wedge p_{m'} \in P_{t_1} \wedge \\
& \exists e(p_{l'},p_{m'}) \in E_1 \cup E_2 \wedge \forall p_{n'} \in e' : p_{n'} \notin P_{t_2} \wedge \\
& \mu_{F_t}(p_{n'}) = \min(\mu_{f_1}(p_{n'}), \mu_{f_2}(p_{n'}))\}
\end{aligned}
$$

Finally, the edges that interconnect two intersection points $p_{l'} \in P_{t_2}$ and



(a) E_{t_1} (b) E_{t_2}

(c) E_{t_3} (d) E_{t_4}

Fig. 3.6. Construction of *E'*$_3$

$p_{m'} \in P_{t_2}$, are also added (in this case, obviously, no other points of P_{t_2} must be contained in this segment) (Figure 3.6.d). This is expressed by

$$
\begin{aligned}
E_{t_4} \quad = \quad & \{e'(p_{l'},p_{m'}) | p_{l'} \in P_{t_2} \wedge p_{m'} \in P_{t_2} \wedge \\
& (\forall p_{n'} \in e' : p_{n'} \notin P_{t_2} \wedge \mu_{F_t}(p_{n'}) = \min(\mu_{F_1}(p_{n'}), \mu_{F_2}(p_{n'})))\}
\end{aligned}
$$

Hence, E'_3 is defined as

$$
E'_3 = E_{t_1} \cup E_{t_2} \cup E_{t_3} \cup E_{t_4}
$$

In the simplified example (Figure 3.3), the set E'_3 will contain all the edges defining the triangulated irregular network. With more complex networks, edges in E'_3 yield a set of non-overlapping planar polygons. In general, when a Delaunay (or constrained Delaunay) triangulation is applied to a planar polygon, it results in a planar triangulation. In this case triangulating the planar polygons (which is in fact a constrained Delaunay triangulation with the edges of the polygons specified to be part of the result), results in a triangulation that does not exhibit the problems caused by the regular Delaunay triangulation on the non-planar set of points P_3. As can be seen in Figure 3.5 this definition of the minimum is exactly the same as the minimum that should be obtained.

The definition for the union (using the maximum) is obtained in a completely analogue way. Other t-norm and t-conorm operations might not maintain the linearity (i.e. the result of an operation, can yield a result that cannot be presented by a (piecewise) linear model. In this case, the results should be approximated using a piecewise linear model. This kind approximation is often needed (even in non geographic applications) due large amount of calculations required.

3.3.2. ETINs with fuzzy numbers

The ETIN structure as defined above can be used to model crisp membership grades regarding a property spread over a region. The modeling of vague, imprecise or uncertain associated values is however also very useful. Examples of this are modeling inaccurate measurements, and performing analysis involving predictions. As a matter of fact, a huge amount of real life information to be analyzed is dependant on geographical locations (climate analysis, election polls, marketing statistics, ...), and can be represented by fuzzy numbers (Klir and Yuan 1995, Zimmerman 1999).

In this subsection, ETINs based on fuzzy numbers will be considered. Fuzzy numbers with triangular membership functions are the easiest to be considered, as they provide the simplest model for a fuzzy number. Note that the triangular membership function is not to be confused with triangulated networks.

Definition 3.2 An **ETin with fuzzy numbers** is defined by a TIN structure and a mapping function g.

$$\overline{ETin} = [(P,E,T),g]$$

where g is defined by

$$\begin{aligned} g: \quad & P \rightarrow \tilde{\wp}(co(f)) \\ & p(x,y) \mapsto g(p(x,y)), \forall p \in P \end{aligned}$$

with $\tilde{\wp}(co(f))$ the powerset over *co(f)*, this is the set containing all the fuzzy sets over *co(f)*. For fuzzy numbers, the set *co(f)* will be $\mathbb{R}$.

Fuzzy numbers with triangular membership functions are the easiest to be considered, as they provide the simplest model for a fuzzy number. Note that the triangular membership function is not to be confused with triangulated networks.

In order to use fuzzy numbers, the type the associated data value in a point needs to be extended from a simple (crisp) value to a fuzzy set over the domain $\mathbb{R}$. Conceptually, in the presented ETIN-model, this can be accomplished in every point of the considered region by associating a triangular membership function. Three characterizing points are of importance: the two points in which the membership-grade equals 0 that delimit the membership function, and the intermediate point in which the membership grade equals 1. In Figure 3.7 are four examples of such fuzzy numbers, representing values of spatial data. With each data point of the ETIN, a membership function will be associated (Figure 3.8). As only three values are needed to characterize these membership functions, the structure can be represented by means of three ETINs. The "lower" network will connect the "lowest" value points, the "upper" network will connect the "highest" value points; the "middle" network will represent the points where the membership grade is 1, this can be seen on Figure 3.9. It is clear from their definitions, that these three networks can not intersect one another. One understands that the data values represented by the "middle" network are in fact not known with absolute precision, but that the limitation of their precision lies within the values represented by the surrounding "upper" and "lower" networks.

Fuzzy numbers (in this case, triangular fuzzy numbers); are fuzzy sets over the numerical domain (usually $\mathbb{R}$). Mathematically speaking, these sets are the co-domains of the mapping function. Arithmetic operations (addition, product, ...) on fuzzy numbers have been defined; by using the Zadeh extension principle, simple interval-arithmetic can be adopted. There is one drawback though: most operations on triangular fuzzy numbers will result in a more generally shaped fuzzy number; e.g. the product can yield a non triangular piecewise linear membership function. Representing this kind of membership functions requires either a way of approximating them, or an adaptation to make the model better suited for non triangular membership functions, as presented below.

Fuzzy numbers using piecewise linear membership functions are modeled by means of a straightforward extension of the previous approach. The "breakpoints" are no longer limited to three and all have to be involved in the mathematical operations when calculating with the thus defined fuzzy numbers. The problem has been studied in (Kerre and Van Schooten 1988). Discussing this kind of fuzzy numbers (which might occur as results of operations on triangular fuzzy numbers and which are perfectly contained mathematically) is beyond the scope of this chapter.

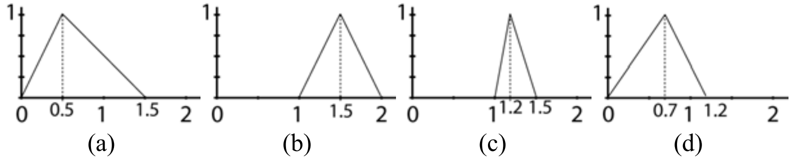


Fig. 3.7. Membership functions in respectively (0,0), (0,100), (100,0) and (100,100)

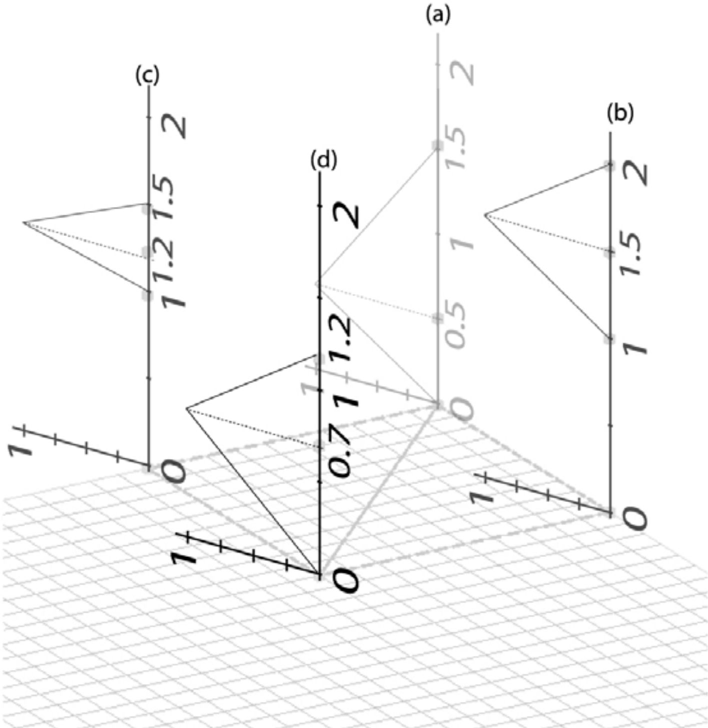


Fig. 3.8. Modeling fuzzy sets using extended triangulated irregular networks

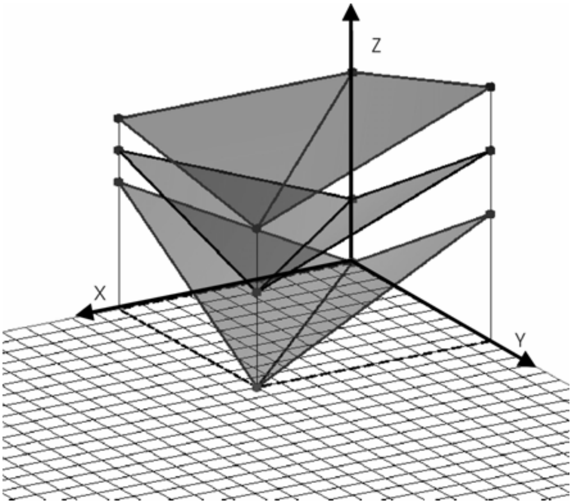


Fig. 3.9. Interpolating between fuzzy sets in extended triangulated irregular networks

3.3.3. ETINs with type-2 fuzzy sets

The ETIN structure with fuzzy numbers can be altered to the uncertainty of membership grades (as presented in section 3.1.) using type-2 fuzzy sets (Mendel 2001). While in this model, type-2 fuzzy sets look similar to fuzzy numbers, there is a huge difference in meaning and interpretation.

Type-2 fuzzy sets are a generalization of regular fuzzy sets, in that they permit imprecision as well as uncertainty regarding the membership grades to be modeled. As such, type-2 fuzzy sets can be used to generalize the model in 3.1. The example used in that section concerned the proposition "near Ghent". This assumes that their is certainty about the extent to which a location is "near Ghent". However, when describing for instance the whereabouts of a person, there might be doubt on where he/she is located. The person could be located near Ghent, but also near Brussels. A type-2 fuzzy set allows this doubt to be modeled: this approach, the membership grade on every location is extended to a "fuzzy" membership grade. As a result, every point will now have an associated fuzzy set over the domain [0,1].

Definition 3.3 An **Extended TIN with type-2 fuzzy sets** is defined by a TIN structure and a mapping function g.

$$\overline{E\tilde{T}in} = [(P, E, T), g]$$

where g is defined by

$$g: \quad P \rightarrow \tilde{\wp}([0,1])$$
$$p(x,y) \mapsto g(p(x,y)), \forall p \in P$$

A similar notation as before is used, with $\tilde{\wp}([0,1])$ being the set of all fuzzy sets over the interval [0,1].

3.3.4. Applicability

ETINs with possibilistic truth values

In the three previous sections, the ETIN structure was presented for the representation of data (membership grades (3.1), fuzzy numbers (3.2) or fuzzy membership grades (3.3)). In this section, attention will go to adapting the structure for use in querying, in order to cope with these types of fuzzyness. In traditional crisp systems, a query condition (e.g. "temperature is more than 30°C") evaluates to true or false for any given crisp data. However, when the model contains fuzzy numbers, this standard boolean logic is insufficient. If the temperature in the above example were to be represented by means of a fuzzy number (e.g. between 26°C and 32°C), the truth cannot be expressed as true or false. Furthermore, there is an interest in using natural language queries (e.g. using propositions such as "warm", "high temperature", ... where this predicate is defined by means of a possibility distribution). As a result, there must be a mechanism to determine the extent to which a value (e.g. a temperature of "between 27°C and 32°C") is considered to match a given fuzzy set (e.g. a set describing the linguistic term "warm").

There are a number of approaches to indicate degrees of truth: membership grades, possibility measures and possibilistic truth values (de Cooman 1995, de Cooman 1999, Dubois and Prade 2001) can be used (Prade 1982, De Tré 2002). The latter case will be explained in further detail. Possiblisitic truth values indicate the truth by means of two values: a degree to indicate the extent to which a proposition is true, and a degree to indicate the extent to which a proposition is false. In order to define possibilistic truth values, the set $\tilde{\wp}(I)$ of all fuzzy sets over the universe *I={True,False}* is considered.

Defintion 3.4 (Possibilistic truth value PTV). While the traditional truth value *t* of a proposition *p*, element of a set of propositions *P* is defined as

$$\begin{aligned} t: \quad P &\rightarrow \quad I \\ p &\mapsto \quad t(p) \end{aligned}$$

the possibilistic truth value $\tilde{t}(p)$ of a proposition $p \in P$ is defined by means of the mapping function $\tilde{t}$

$$\begin{aligned} \tilde{t}: \quad P &\rightarrow \quad \tilde{\wp}(I) \\ p &\mapsto \quad \tilde{t}(p) \end{aligned}$$

With each $p \in P$ a fuzzy set $\tilde{t}(p)$ is associated; the semantics of this associated fuzzy set are defined in terms of a possibility distribution *Π*:

$$\forall x \in I : \Pi_{t(p)}(x) = \mu_{\tilde{t}(p)}(x)$$

i.e.

$$\forall p \in P : \Pi_{t(p)} = \tilde{t}(p)$$

PTVs provide a means for modeling the truth value associated with a given property.

The operator to compare a possibility distribution with a fuzzy set is the IS operator; it is of the form *A* IS *L* and returns a possibilistic truth value, indicating the extent to which the value of a property *A* (e.g. population density), represented by a possibility distribution, matches a linguistic term *L* (e.g. densely populated), represented by a fuzzy set. A possibilistic truth value is associated with every location.

Definition 3.5 An **ETin with PTVs** is defined by a TIN structure and a mapping function *g*.

$$\overline{ETin} = [(P, E, T), g]$$

where *g* is defined by

$$\begin{aligned} g : \quad & P \rightarrow \tilde{\wp}(\{True, False\}) \\ & p(x,y) \mapsto g(p(x,y)) \qquad , \forall p \in P \end{aligned}$$

with $\tilde{\wp}(\{True, False\})$ the powerset over *{True, False}*, this is the set containing all the fuzzy sets over the set *{True, False}*.

Definition 3.6 (*A* IS *L*). *A* IS *L* will yield a possibilistic truth value for which

$$\begin{aligned} \mu_{\tilde{t}(A\,\mathrm{IS}\,L)}(True) &= \sup_{x \in dom(A)} \min(\pi_A(x), \mu_L(x)) \\ \mu_{\tilde{t}(A\,\mathrm{IS}\,L)}(False) &= \sup_{x \in dom(A)} \min(\pi_A(x), 1 - \mu_L(x)) \end{aligned}$$

Apart from the IS operator, common logical operators are extended. For PTVs, the rule for conjuction is $\forall p, q \in P : \tilde{t}(p \text{ AND } q) = \tilde{t}(p) \tilde{\wedge} \tilde{t}(q)$ where

$$\begin{aligned} \tilde{\wedge} : \tilde{\wp}(I) \times \tilde{\wp}(I) &\rightarrow \tilde{\wp}(I) \\ (\tilde{U}, \tilde{V}) &\mapsto \tilde{U} \tilde{\wedge} \tilde{V} \end{aligned}$$

is defined by applying Zadeh's extension principle to the operator $\wedge$:

$$\begin{aligned} \mu_{\tilde{U} \tilde{\wedge} \tilde{V}}(True) &= \sup_{(x,y) \in \{(x,y) | (x,y) \in I \times I \wedge (x \wedge y = True)\}} \min(\mu_{\tilde{U}}(x), \mu_{\tilde{V}}(y)) \\ &= \min(\mu_{\tilde{U}}(True), \mu_{\tilde{V}}(True)) \end{aligned}$$

and

$$\begin{aligned} \mu_{\tilde{U} \tilde{\wedge} \tilde{V}}(False) &= \sup_{(x,y) \in \{(x,y) | (x,y) \in I \times I \wedge (x \wedge y = False)\}} \min(\mu_{\tilde{U}}(True), \mu_{\tilde{V}}(False)) \\ &= \max \begin{pmatrix} \min(\mu_{\tilde{U}}(True), \mu_{\tilde{V}}(False)), \\ \min(\mu_{\tilde{U}}(False), \mu_{\tilde{V}}(True)), \\ \min(\mu_{\tilde{U}}(False), \mu_{\tilde{V}}(False)) \end{pmatrix} \end{aligned}$$

Similarly, the disjunction, negation and other operators can be defined (De Tré 2002). Sometimes it is more useful to use alterative definitions for these operators, as presented in (De Tré et al. 2002b).

The presented extended triangulated irregular network structure can be adopted for the modeling of such PTVs: the ETIN-structure can be extended similarly to what has been done in order to allow the modeling of fuzzy numbers. There, the data in each point had been extended to a fuzzy set over the numerical domain; for the modeling of PTVs over a geographic area, the data in each point will be extended to a possibility distribution over *{True,False}*. Visually, this means that PTVs will yield two triangulated irregular networks (one for *True*, one for *False*).

An important difference between this and the previous models is that the ETIN with PTVs is not stored in the database, it is an intermediate structure used to both evaluate the query (or parts of the query) and to represent the result.

Extended possibistic truth values (De Tré 2002) permit coping with propositions for which a truth value is undefined, which might be the case if the proposition is not applicable. The approach is completely analogue, apart from the fact that the considered domain *{True,False}* is replaced with the domain *{True,False,⊥}*, in which the element $\perp$ represents undefined.

Applications

The extension of TINs to ETINs is useful to model grades of membership over a geographic region, still using a fairly simple structure. From a computational point of view, the structure is interesting as it results in a vector oriented approach, requiring far less storage than similar bitmap techniques. An implementation of the presented operations is quite straightforward; extending the approach towards other uses (fuzzy numbers, PTVs, ...) is more complex, especially when appropriate operators for these uses are considered, as has been discussed in the previous paragraphs.

3.4. Bitmap models

3.4.1. Definition, notation and spatial representation

An alternative technique of representing field based data is the bitmap approach. While the TIN and ETIN structures try to describe a continuous universe by recording information associated with a limited number of "strategic points", the vertices of the triangulation, the bitmap approach retains information that is associated with number of points, grouped in a *cell* of a *mesh* that covers the region of interest. Hence, the bitmap model is a kind of a discreet model, sometimes also called a *spatial resolution model*, where the considered tiles stem from the used partitioning.

Two variants, depending on the shape of the cells, can be distinguished: *fixed* (or regular), if all cells have the same shape and size and *variable* or irregular, if their sizes and/or shapes differ (Rigaux et al. 2002). For clarity reasons, only regular bitmaps will be considered, but the presented techniques can – if needed – be extended to suit irregular bitmaps as well. While only regular tesselations using rectangular cells will be covered here, the presented techniques are not limited to rectangular cells however; sometimes, hexagonal tiling is used as a regular grid. The two dimensional space (limited by the map) is thus partitioned in a finite number of cells. The size of the cells determines the *resolution* at which the data is to be modeled. Basically, a cell is a *convex polygon*, which can be defined by stating that all of the points of a line-segment connecting two points within the polygon are located inside the polygon. Considering the vectors $\vec{p}_1$ and $\vec{p}_2$ as being defined by the origin of the reference frame and the points p_1 and p_2, the requirement of being convex can be defined as follows.

Definition 4.1 (Cell *c*). With the understanding that X is the universe of all the locations (points) considered in the GIS, a subset $c \subseteq X$ is called a cell if it is convex, i.e.

$$\forall p_1, p_2 \in c, \exists p_3 \in c : \frac{\vec{p}_1 + \vec{p}_2}{2} = \vec{p}_3$$

A *grid* – in this context – is a collection of non-overlapping cells (i.e. their interiors are disjoint) that together cover the considered region U in the universe X. A cell thus has a dimension, but the attribute data values of all the points in the cell are represented by one single same value associated with the cell. The cell itself is characterized by one single pair of coordinates in the planar representation. With a refined resolution, more attribute data values will be taken into account. Referring to a more coarse resolution, less attribute values will be taken into account.

Definition 4.2 (Grid *G*).

$$G = \left\{ (c \subseteq X) \middle| \forall c_1, c_2 \in G : c_1 \cap c_2 = \varnothing \wedge \bigcup_i c_i \in G = X \right\},\ i = 1..T$$

where T is the total number of cells in the grid. The following explanation will be made by means of a regular two-dimensional $N \times M$ grid of rectangular cells or *pixels*. A cell has a location in the plane, consisting of two coordinates n and m where $0 \leq n \leq N$ and $0 \leq m \leq M$, and will be denoted $c(n,m)$. A *bitmap* is defined as a set of cells, each representing an associated data value.

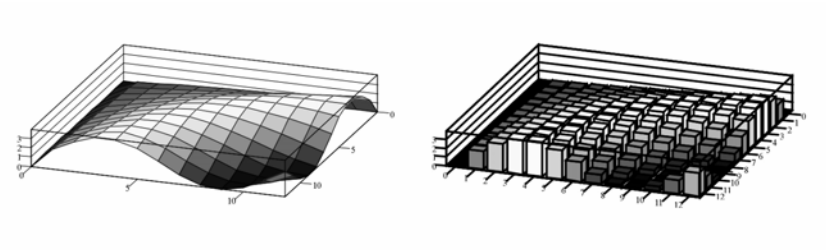
Definition 4.3 (Bitmap *B*)

$$B = (G, f)$$

A bitmap structure B can be used to model a property dependent on a geographic location by approximating the property values in the associated data value of each cell, as shown on Figure 3.10b. The associated value in each cell can for instance be calculated as the average of a finite set of values associated with sample-points within that cell. Again, examples of application fields are the modeling of population densities, temperatures, recorded noise-levels, ...

Using the Z-axis to represent the associated data value, both a continuous three dimensional function and a bitmap-structure approximating the same function can be visualized as shown in Figure 3.10a. Naturally, the more cells the bitmap will contain, the more refined the approximation constituted by the tiles will be.

The bitmap structure as defined above is traditionally used to model crisp data. It does however allow for extensions similar to the ones described for extending the TIN-structure.



(a) (b)

Fig. 3.10. geographically dependent value in continuous (a) and bitmap (b) representation

3.5. Extended bitmap models: EBs

3.5.1. EBs with membership grades

Definition

In a similar approach as in 3.1., the bitmap structure is adapted using membership grades.

Definition 5.1 (Extended bitmap with membership grades $E\widetilde{B}$)

$$E\widetilde{B} = \left(G, \mu_B\right)$$

where μ_F is defined by

$$\begin{aligned} \mu_F : G &\rightarrow [0,1] \\ c &\mapsto \mu_F(c), \forall c \in G \end{aligned}$$

As in 3.1., this structure can be used to model properties regarding a geographic location, e.g. the property "near Ghent".

Suppose we have two different properties F_1 (e.g. "near Ghent") and F_2 (e.g. "alongside the river Schelde"), obtained by sample functions f_1 and f_2 as represented in Figure 3.11; two extended bitmaps $\widetilde{B}_{F_1}$ resp. $\widetilde{B}_{F_2}$ are used to represent the grades μ_{F_1} and μ_{F_2} to which extent the properties are satisfied with respect to the considered area. Sometimes, data from the two extended bitmaps may need

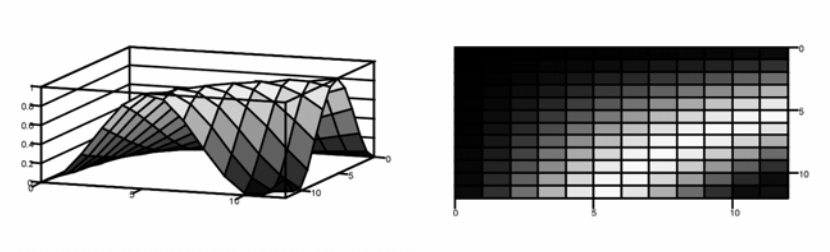


Fig. 3.11.a. Sample function f_1

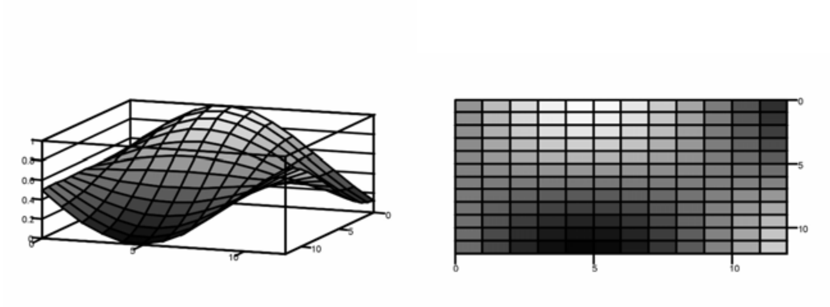


Fig. 3.11.b. Sample function f_2

to be combined, for which any of the common aggregation operations (t-norms, t-conorms, ...) as explained in subsection 3.1.2. are applicable. An example of such a combination is "near Ghent and alongside the river Schelde".

Ideally, the fuzzy bitmaps $\widetilde{B}_{F_1}$ and $\widetilde{B}_{F_2}$ should be based on the same grid and thus both have the same resolution and size. It is possible to use bitmaps that don't use the same grids, but this will require a resampling of one of them in order to match the other grid (target-grid). The resampling can be done by overlaying the target-grid with the bitmap, and then averaging the values for a number of sample points. This resampling entails a loss of precission. The use EBs with fuzzy numbers (as presented further) could prove to be more beneficial in this context. For the sake of argumentation, the fuzzy bitmaps $\widetilde{B}_{F_1}$ and $\widetilde{B}_{F_2}$ are considered to be based on the same grid and therefore both have the same resolution and size; they are defined as shown respectively in Figure 3.11.a-b.

Operations

Operations on cells need special attention. If the new value of a cell is based on the input of a single cell in the same position, the operation is said to be *local*; if the value of a cell is based on the input of a cell and its neighborhood, the operation is said to be *focal*. The *neighborhood* of a cell can be defined in various ways:

either by considering its surrounding cells (eight in total (Rigaux et al. 2002), when using a grid with rectangular cells), or by considering a subset of these (only horizontal and vertical neighboring cells, only diagonally neighboring cells, ...). If a second bitmap is used to define the cells on which calculations are performed, the operation is considered *zonal*; finally, if values in the new raster are based on the values of all the cells in the input, the operation is considered to be *global* (Shekar and Chawla 2003). The aggregation methods described here can be categorized as local operations: the value of a cell in the result is only dependent on the value of the cells on the same position in the input. The operations and functions work cell by cell thus for an operator *o* and the cells $c_i \in \widetilde{B}_i$ this yields:

$$\mu_{F_3}(c_3(n,m)) = o(\mu_{F_1}(c_1(n,m)), \mu_{F_2}(c_2(n,m)))$$

In Figure 3.12.a, the minimum operations on the two bitmap structures represented in Figure 3.11 is illustrated (corresponding to the intersection operator example in subsection 3.1.2. Likewise, in Figure 3.12.b the product of two bitmap structures is shown. Both the three dimensional ideal (continuous) solution and the related bitmap approximation are printed.

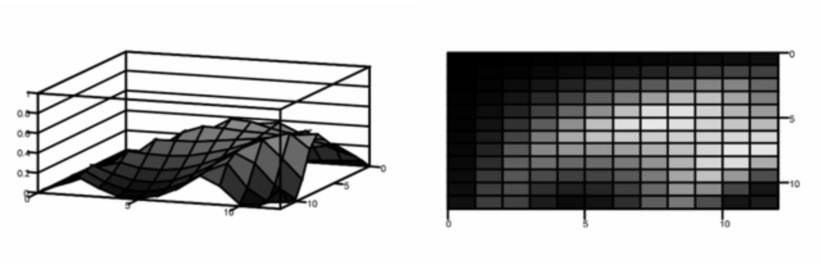


Fig. 3.12.a. minimum: $\min(f_1,f_2)$

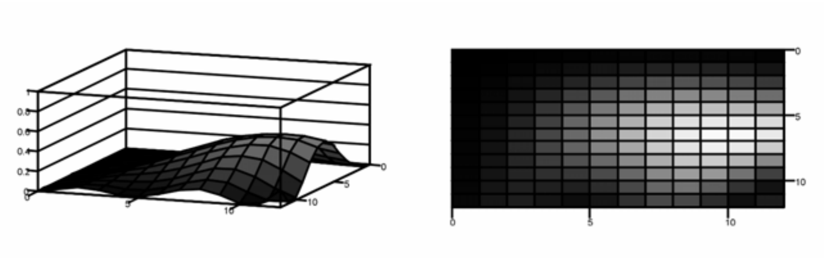


Fig. 3.12.b. product: $f_1 \times f_2$

3.5.2. EBs with fuzzy numbers

Due to the simplicity of the bitmap-structure, the bitmap can easily be extended to represent more complex data. A very useful extension is the use of fuzzy numbers to represent the attribute data value per cell: e.g. this would also allow the modeling of approximate numerical data, which is beneficial for instance in the modeling of predictions or in the analysis of evolutions. Similar to the extension in the ETIN-structure in subsection 4.2., the modelled data – in this case contained within the data field for every cell – are now extended from single values to fuzzy sets. It is important to notice that the co-domain *co(f)* is the domain of the modelled values: in definition 4.3 *f* maps a cell *c* onto an associated value *f(c)*. Extending the associated value from a single value to a fuzzy set requires the fuzzy set to be defined on *co(f)*. The extended bitmap then is

Definition 5.2 (Extended bitmap with fuzzy numbers $\overline{EB}$)

$$\overline{EB} = (G, g)$$

where

$$\begin{aligned} g : G &\to \tilde{\wp}(co(f)) \\ c &\mapsto g(c) \qquad , \forall c \in G \end{aligned}$$

with $\tilde{\wp}(co(f))$ the powerset over *co(f),* i.e. the set containing all fuzzy sets over the set *co(f)*. This association indicates that a fuzzy set is associated with the data field of each cell. For fuzzy numbers, the set *co(f)* will be $\mathbb{R}$.

There is a major conceptual difference between this extended bitmap and the extended bitmap with membership grades as defined previously. Initially, geographic data was modeled using an extended bitmap with membership grades, which indicated e.g. a degree of similarity (or satisfaction) with regard to a given property *F*. In the extended bitmap with fuzzy numbers, there is a new fuzzy set for every cell of the bitmap, which corresponds to the fuzzy number representing the attribute data value associated with the cell. This allows to express that the data value is cursed with imprecision, contained within the defined fuzzy set. This means that if a traditional operator *o* is used to combine that data from two bitmaps, the resulting bitmap then is defined as:

$$\overline{EB} = \left(G, \tilde{o}\left(g_1(c), g_2(c)\right)\right)$$

where the operator *o* is extended using Zadeh's extension principle:

$$\mu_{\tilde{o}(g_1(c), g_2(c))}(x) = \begin{cases} \sup\limits_{(x_1, x_2) \in o^{-1}(x)} \min\left\{\mu_{g_1(c)}(x), \mu_{g_2(c)}(x)\right\} & \\ 0 & \text{if } o^{-1}(x) = \varnothing \end{cases}$$

where $x \in co(f)$.

Zadeh's extension principle is a point wise definition, which implies that it is directly applicable on the extended bitmap model (in essence a discrete model).

The fact that no interpolation is performed between different cells of the bitmap structure, results in a more straightforward extension (in comparison to the similar extension that has been made on TIN-structures). Every traditional mathematical operation that is extended, will work on a per cell basis. The model puts no constraints or limits on the membership functions that can be associated in a data field. One might opt for a triangular fuzzy set representation of the fuzzy numbers, or more generally a piece wise linear function representation, or perhaps even for other types of functions, but it doesn't impact the interpretation, nor the functionality of the bitmap.

This structure now permits the modeling of data such as e.g. temperature, but with imprecise or uncertain values. This also allows for a modeling of the uncertainty and/or imprecission associated with making predictions (e.g. about population densities).

3.5.3. EBs with type-2 fuzzy sets

In the previous section, the associated value in each cell has been represented by a fuzzy number. This approach can be used to generalize the model presented in 4.1. As in section 3.3. – which generalized the model in 3.1. – this will yield a model containing type-2 fuzzy sets (Mendel 2001; Klir and Yuan 1995). The associated value in each cell then is a membership grade which is subject to uncertainty.

Definition 5.3 (Extended bitmap with type-2 fuzzy sets $\overline{E\tilde{B}}$)

$$\overline{E\tilde{B}} = (G, g)$$

where

$$\begin{aligned} g : G &\to \tilde{\wp}([0,1]) \\ c &\mapsto g(c) \quad , \forall c \in G \end{aligned}$$

A similar notation as before is used, with $\tilde{\wp}([0,1])$ being the set of all fuzzy sets over the interval [0,1].

The interpretation is similar to the interpretation discussed in section 3.3.

3.5.4. Applicability

EBs with possibilistic truth values

Similarly to the ETIN structure, the extended bitmap structure was presented for the representation of data (membership grades (4.1), fuzzy numbers (4.2) or fuzzy membership grades (4.3)).

As with the ETIN structure, the introduction of modeling fuzzy information has an impact on the evaluation of queries. Possibilistic truth values will also be used in the context of extended bitmaps to express the degree of truth regarding a condition. A possibilistic truth value can be seen as a fuzzy set over the domain *{True, False}*. Using possibilistic truth values, the bitmap can hence be extended to

$$\overline{EB} = (G, g)$$

where

$$g : G \to \tilde{\wp}(\{True, False\})$$
$$c \mapsto g(c)$$

with $\tilde{\wp}(\{True, False\})$ the powerset of *{True,False}*, i.e. the set containing all fuzzy sets over *{True,False}*.

Hence cell-wise, which comes to point-wise, a possibilistic truth value is associated with the attribute data value to express the degree of truth regarding the value associated with a cell in an query-expression. The modeling of the property *A* – both with crisp or fuzzy values – and the linguistic term *L* have been presented previously. The IS operator for bitmaps works on a per cell basis: the possibility-distribution *A* associated with each cell is matched (using the traditional IS operator) with the fuzzy set *L*. For every cell this will result in a possibilistic truth value, which can be associated with a cell in the resulting extended bitmap as defined above. The other operations (conjunction, disjunction, ...) on possibilistic truth values can also be defined on a per cell basis to suit the bitmap model. Again, the PTVs can be extended to extended possibilistic truth values (De Tré 2002), by using the domain *{True, False, ⊥}* instead of *{True, False}*; where ⊥ means undefined and is used if the proposition is inapplicable.

The EBs with PTVs are not intended for storage within the database, similar to the ETINs with PTVs, this structure is intended for representing (intermediate or final) query results.

A note on data acquisition

An important caveat when using bitmap structures, is the required amount of data: numerical data is required for every cell in the grid (in our examples, N×M values are required). In practice, this amount of data can be generated from a smaller number of points, by interpolating them using a continuous model (e.g. a triangulated irregular network structure as mentioned before). This continuous model can then be used to determine the value(s) that are associated with each cell (e.g. by averaging the data of a number of sample points in that cell). While this poses no theoretical problem, one should be aware that some cells in the bitmap now contain interpolated values, instead of measured values.

Observations

The bitmap structure adopted as described above is suitable for many purposes, both in modeling fuzzy data as in representing results from a fuzzy query (be it on crisp or fuzzy data). Various operations can easily be defined; the translation from the theoretical definitions to implementations proves also to be straightforward. Visualizing an extended bitmap with membership grades is relatively easily accomplished; visualizing an extended bitmap with fuzzy numbers, type-2 fuzzy sets or PTVs is far more challenging, as there basically are four dimensions to be dealt with: two coordinates, a modeled value and its membership grades (i.e. a membership function) per cell.

3.6. Using ETINs and EBs

ETINs and EBs are really different approximation techniques and their use greatly will depend on the way input over the region under consideration is available.

ETINs are called continuous models, as they can draw (approximate) attribute data values for whatever point in the region. To achieve this, interpolation techniques are used on the data values associated with the points in the region for which information is available by input. The mentioned points in the region can be distributed as if in an arbitrary way (corresponding to their availability, e.g. stemming from measurements in the field). EBs are said to be discrete models, as they are developed for regions in which attribute data values are (or become) available meshwise. In practise, the mesh will be or will be forced to be a regular one. No interpolation is used here. Per cell, there is only one (representative) data value under consideration. No interpolation is used here; per cell, there is only one (representative) data value under consideration.

While it is easier to draw a bitmap model from a triangulated irregular network (by interpolation) than vice versa, the latter could be considered as well. Anyway, in any approximation, one has to aim at the best possible approximation at the lowest possible cost. In this context, the two approaches can sometime be used to complement one another. For instance, operations can be applied on a bitmap model and used to have an idea of the more accurate but more computationally intensive solution in a TIN model. The choice of strategy will depend on the availability of the input data, their amount and the time needed to achieve the desired approximation within the limits of precision and certainty aimed at.

In general, to achieve a relatively accurate approximation, bitmap models will rely on a huge amount of data, but benefit from relatively easy calculations; triangulated irregular networks by contrast, will involve a cumbersome number of calculations but provide inherently for a better approximation.

The extended TINs and extended bitmaps have the advantage of offering richer and more realistic semantics at the cost of more calculations and augmented storage capacity in comparison to their non-extended counterparts.

3.7. Conclusion

Two different approaches for representing field-based fuzzy geographic information have been presented, an extended vector-based method using triangulated irregular networks and an extended bitmap model. Each of the presented models has its benefits and its drawbacks. The former is more complex, especially when defining operators, but has the advantage of being a continuous model. This vector-based method allows for an approach that results in a more realistic model for measured data. Defining the operators requires that the interpolation at hand has to be taken into account, which makes this approach more challenging and may require the result to be approximated. Defining the operators requires that the interpolation at hand has to be taken into account, which makes this approach more challenging. In an implementation, this might impact performance negatively. The latter is much simpler for defining operators but is a discrete model. This approach resembles most of the theoretical definitions more closely, which results in a much more straightforward implementation. The two approaches can be said to complement each other; which representation method is the most adequate depends on both the data to be modeled and which operations most frequently need to be applied.

References

Clementini E, Di Felice P (1994) An algebraic model for spatial objects with undetermined boundaries. GISDATA Specialist Meeting - revised version.

Cobb M, Foley H., Petry F and Shaw K (2000). Uncertainty in the Distributed and Interoperable Spatial Information Systems. In: Recent Issues on Fuzzy Databases (eds. G. Bordogna, G. Pasi); Physica-Verlag, Heidelberg, GR, pp 85-108.

Cohn AG, Gotts NM (1994); Spatial regions with undetermined boundaries. In: Proceedings of the Second ACM Workshop on Advances in Geographic Information Systems, pp 52-59.

de Cooman, G (1995) Towards a possiblistic logic. In: Fuzzy set theory and advanced mathematical applications (ed. Ruan D.) Kluwer Academic publishers, Boston, pp 89-133.

de Cooman, G (1999) From possibilistic information to Kleene's strong multi-valued logics. In: Fuzzy sets, logics and reasoning about knowledge (ed. Dubois D.) Kluwer Academic publishers, Boston, pp 315-323.

De Tré G, De Caluwe R, Hallez A, Verstraete J (2002) Fuzzy and Uncertain Spatio-Temporal Database Models: A Constraint-Based Approach. In: Proceedings of the 9th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU 2002, July 1-5, Annecy, France, pp. 1713-1720.

De Tré G (2002) Extended Possibilistic Truth Values. In: International Journal of Intelligent Systems, volume 17, no. 4, April 2002, Wiley Publishers, pp. 427--446.

De Tré G, Hallez A, Verstraete J, Verkeyn A (2002b), Beyond Conjucative Aggregation of Possibilistic Truth Values in Database Systems. In: The Seventh Meeting of the EURO Working Group on Fuzzy Sets – Eurofuse, pp 137-142.

Dubois D, Prade H (1997) The three semantics of fuzzy sets. In: Fuzzy Sets and Systems, volume 90, Elsevier Science, pp 141-150.

Dubois D, Prade H (2000) Fundamentals of Fuzzy Sets. Kluwer Academic Publishers.

Dubois D, Prade H (2001); Possibility theory, probability theory and multiple-valued logics: A clarification. In: Annals of Mathematics and Artificial Intelligence, volume 32, pp 35-66.

Gotts NM, Cohn AG (1995) A mereological approach to representing spatial vagueness, Working Papers, Ninth International Workshop on Qualitative Reasoning; pp 246-255.

Hallez A, Verstraete J, De Tré G, De Caluwe R (2002) Contourline Based Modelling of Vague Regions. In: Proceedings of the 9th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems IPMU 2002, July 1-5, Annecy, France.

Klir GJ, Yuan B (1995) Fuzzy sets and fuzzy logic: Theory and applications. New Jersey, Prentice Hall.

Mendel JM (2001) Uncertain Rule-Based Fuzzy Logic Systems, Introduction and New Directions. Prenctice Hall PTR.

Morris A (2001) Why Spatial Databases Need Fuzziness, In: Proceedings of Nafips 2001, pp 2446-2451.

Prade H (1982) Possibility sets, fuzzy sets and their relation to Lukasiewicz logic. In: Proceedings of the 12th International Symposium on Multiple-Valued Logic, pp 223-227.

Rigaux P, Scholl M, Voisard A (2002) Spatial Databases with Applications to GIS. Morgan-Kaufman Publishers.

Schneider M (1999) Uncertainty management for spatial data in databases : fuzzy spatial data types. In: 6th International Symposium on Advances in Spatial Databases (SSD), LNCS 1651, Springer Verlag; pp 330-351.

Shekhar S, Chawla S (2003) Spatial Databases: A tour. Pearson Education Inc.

Shewchuk JR (1996) Triangle: Engineering a 2D Quality Mesh Generator and Delaunay Triangulator. In: First Workshop on Applied Computational Geometry (Philadelphia, Pennsylvania), Association for Computing Machinery, pp 124-133.

Shewchuk JR (2002) Constrained Delaunay Tetrahedralizations and Provably Good Boundary Recovery; Submitted to the Eleventh International Meshing Roundtable.

Kerre E, Van Schooten A (1988) A Deeper Look on Fuzzy Numbers from a Theoretical as well as from a Practical Point of View. In: Fuzzy Logic in Knowledge-Based Systems, Decision and Control (eds. Gupta M. & Yamakawa T.), Elsevier Science Publishers B.V., North-Holland, pp 173-196.

Vertraete J, Van Der Cruyssen B, De Caluwe R (2000) Assigning Membership Degrees to Points of Fuzzy Boundaries. In: Proceedings of the 19th International Conference of the North American Fuzzy Information Processing Society - Nafips, pp 444-447.

Verstraete J, De Tré G, Hallez A (2002), Adapting TIN-layers to Represent Fuzzy Geographic Information. In: The Seventh Meeting of the EURO Working Group on Fuzzy Sets – Eurofuse, pp 57-62.

Zimmerman H-J (1999), Practical Applications of Fuzzy Technologies, Kluwer Academic Publishers.

4 Modeling Localities with Fuzzy Sets and GIS

Sungsoon Hwang and Jean-Claude Thill

University at Buffalo – The State University of New York at Buffalo
Department of Geography and National Center for Geographic Information and Analysis (NCGIA)
105 Wilkeson Quad
Buffalo, NY 14261
{shwang5,jcthill}@buffalo.edu

4.1 Introduction

It is only recently that the fuzzy-set theoretic approach to spatial objects and their concepts has joined the mainstream of geographic information system (GIS) and science (GIScience). Several reasons account for this. Recent research on spatial objects has revealed that spatial vagueness is inherent in some geographic features (Burrough and Frank 1996). For instance, the boundary between a mountain and a valley is not sharply defined. Furthermore, even if a geographic phenomenon is best described as crisp, humans tend not to reason in a precise manner, but rather in an approximate manner (e.g. they live *near* Chicago). Moreover, perception and cognition vary widely between individuals. Furthermore, information can be incomplete or imprecise due to rough measurements or to our incomplete ability to grasp the scope and detail of spatial objects. In other words, there always exists a gap between the reality and its representation.

We use fuzzy set theory (Zadeh 1965) as a mean to reconcile discrepancies existing between reality and its representation. We discern three representational different levels at which fuzzy set concepts can be applied, namely the ontology, perception, and implementation levels. The (spatial) ontology level pertains to generic concepts inherent in spatial objects. The perception level concerns the mental models used to perceive the environment. The implementation level encompasses the errors that have propagated during system implementation. The combination of spatial vagueness, diverse human perceptions, and implementation errors account for the gap existing between reality and its representation. In general, fuzzy set concepts preserve details (Robinson 2002) whereas traditional (crisp) GIS data models overlook the loss of information by forcing reality into a coarse (in the sense of low resolution) representation. Fuzzy set theory can overcome the gap by providing mechanisms for ontologically and cognitively plausible (Worboys 2001) and error-sensitive (Duckham et al. 2001) representation of the reality. In sum, fuzzy set theory provides a means to address various kinds of uncertainty such as spatial vagueness, human perception, and imperfect information.

This study is part of a larger project aimed at geographically referencing the fatal accident data. Our task is to pinpoint the location where a traffic crash is most likely to have occurred given the limited and imprecise information available on this crash. In our study, georeferencing can be roughly defined as the conversion of the linguistic description of a location to a quantitative specification. As Goodchild (2000) pointed out, effective georeferencing can be a matter of life and death in the case of communication between a caller and an emergency dispatcher. The linguistic description of location is sometimes not clear-cut, not only because many alternate names are used to refer to the same location, but also because the location itself is not well defined. We focus on the problem of determining the location of a certain locality.

We hypothesize that location indeterminacy of localities is caused by spatial vagueness, interpersonal differences in perception, and imperfect information. We compute the value that quantifies location indeterminacy by modeling the indeterminate part of localities by a fuzzy set membership function. We examine the relationship between the value of location indeterminacy and attributes of localities in order to test the stated hypotheses.

The purpose of this research is to show how fuzzy set theory can be properly applied in modeling localities. Also the result will assure whether there exists fuzziness in determining the location of locality. This study develops a fuzzy set membership function for indeterminate boundaries of localities. By testing our hypotheses on the relationship between location determinacy and characteristics of locality, we examine whether fuzzy set theories can capture various kinds of uncertainty at the ontology, perception, and implementation levels.

Modeling localities by fuzzy sets has a definite advantage over a crisp set in that it makes best possible use of sparse information to reconstitute detail. More specifically, fuzzy-set-based localities constitute a closer depiction of reality, such as overlapping memberships of localities. Next, fuzzy set provides a conservative representation tool for individual differences in the perception. Finally, allowing the soft processing (fuzzy set modeling) over the hard data (reference data) can minimize the problems caused by the imperfection of source data.

The remainder of this chapter is organized as follows. In Section 4.2, the specific georeferencing motivating this study is described. We formulate research hypotheses and specify the assumptions on which this study is based. In Section 4.3, we give a brief overview of related research, such as the ontology of spatial objects, the representation of fuzzy regions, and the notion of nearness. In Section 4.4, we define the fuzzy set membership function of localities. The implementation steps in GIS are described in Section 4.5. The analyses of results are given in Section 4.6. We examine if fuzziness is substantial in identifying localities. By looking at the cases that are georeferenced by a fuzzy set modeling, we may or may not find evidence of fuzziness in locality. The hypothesis is examined also. Finally, Section 4.7 concludes this study.

4.2 Locality and the Georeferencing Problem

Figure 4.1 presents the steps involved in the process of georeferencing traffic accidents: from the reporting of accidents, to their storage in the databases, and finally the georeferencing using a geographic information system. Police officers use the coding forms shown in Figure 4.2 to make a permanent record of accident information. The number in the upper right-hand corner of each section of the coding form indicates the column identifier for this information item in the flat files to which the paper-based record is transferred. In Figure 4.2, the locality information of the accident is recorded in the upper section - State (1-2), City (14-17), and County (18-20) - using GSA Geographic Locator Codes (GLC).

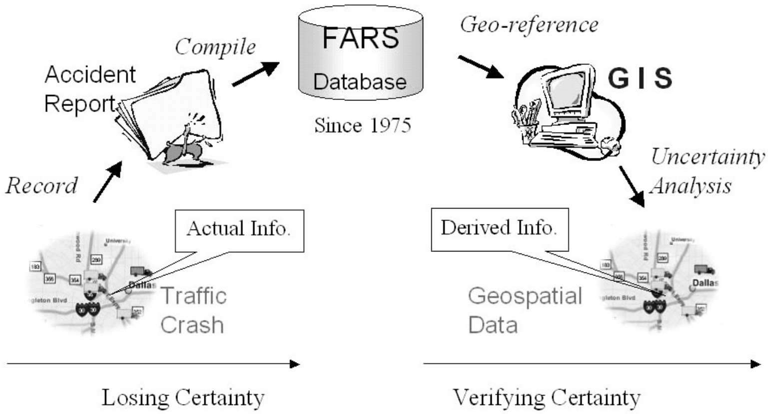
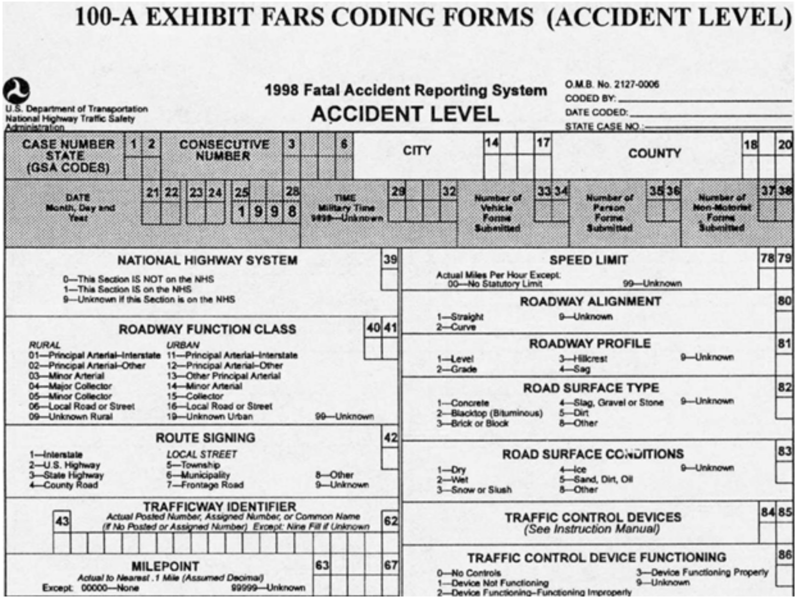


Fig. 4.1. Georeferencing Traffic Accident Data under Uncertainty

As part of the georeferencing process, attention needs to be paid to the manner in which police officers identify the localities. Do they record locality information under the assumption that locality boundaries are determinate? That is, is the location of localities taken as determinate? The answer may be compound. It may depend on the environment (what), on the agent (who), and on the medium (through what).

Environment: It may be much easier to delineate the boundaries if a distinguishable geographic feature, such as a body of water, surrounds a locality (e.g., peninsula, island). Moreover, locality is the result of human conceptualization and demarcation rather than a physical demarcation on the surface of the Earth. Making this kind of spatial concept crisp is part of the repetitive process of conceptualization. It is likely that a person finds it easier to identify the location of a conspicuous locality than that of a less conspicuous locality. Therefore, it can be argued that certain characteristics of localities affect the location indeterminacy, or fuzziness in delineating their boundaries.


100-A EXHIBIT FARS CODING FORMS (ACCIDENT LEVEL)

U.S. Department of Transportation
National Highway Traffic Safety Administration

1998 Fatal Accident Reporting System
ACCIDENT LEVEL

O.M.B. No. 2127-0006
CODED BY:
DATE CODED:
STATE CASE NO.:

CASE NUMBER STATE (GSA CODES) 1 2
CONSECUTIVE NUMBER 3 6
CITY 14 17
COUNTY 18 20

DATE Month, Day and Year 21 22 23 24 25 28 1 9 9 8
TIME Military Time 9999—Unknown 29 32
Number of Vehicle Forms Submitted 33 34
Number of Person Forms Submitted 35 36
Number of Non-Motorist Forms Submitted 37 38

NATIONAL HIGHWAY SYSTEM 39
0—This Section IS NOT on the NHS
1—This Section IS on the NHS
9—Unknown if this Section is on the NHS

ROADWAY FUNCTION CLASS 40 41
RURAL
01—Principal Arterial–Interstate
02—Principal Arterial–Other
03—Minor Arterial
04—Major Collector
05—Minor Collector
06—Local Road or Street
09—Unknown Rural
URBAN
11—Principal Arterial–Interstate
12—Principal Arterial–Other
13—Other Principal Arterial
14—Minor Arterial
15—Collector
16—Local Road or Street
19—Unknown Urban
99—Unknown

ROUTE SIGNING 42
1—Interstate
2—U.S. Highway
3—State Highway
4—County Road
LOCAL STREET
5—Township
6—Municipality
7—Frontage Road
8—Other
9—Unknown

TRAFFICWAY IDENTIFIER 43 62
Actual Posted Number, Assigned Number, or Common Name (If No Posted or Assigned Number) Except: Nine Fill if Unknown

MILEPOINT 63 67
Actual to Nearest .1 Mile (Assumed Decimal)
Except: 00000—None 99999—Unknown

SPEED LIMIT 78 79
Actual Miles Per Hour Except:
00—No Statutory Limit 99—Unknown

ROADWAY ALIGNMENT 80
1—Straight
2—Curve
9—Unknown

ROADWAY PROFILE 81
1—Level
2—Grade
3—Hillcrest
4—Sag
9—Unknown

ROAD SURFACE TYPE 82
1—Concrete
2—Blacktop (Bituminous)
3—Brick or Block
4—Slag, Gravel or Stone
5—Dirt
8—Other
9—Unknown

ROAD SURFACE CONDITIONS 83
1—Dry
2—Wet
3—Snow or Slush
4—Ice
5—Sand, Dirt, Oil
8—Other
9—Unknown

TRAFFIC CONTROL DEVICES 84 85
(See Instruction Manual)

TRAFFIC CONTROL DEVICE FUNCTIONING 86
0—No Controls
1—Device Not Functioning
2—Device Functioning–Functioning Improperly
3—Device Functioning Properly
9—Unknown

Fig. 4.2. FARS (Fatality Analysis Reporting System) Coding Forms (Accident Level) (Source: NHTSA 1995)

Agent: Perceiving the environment requires some form of model through which observation can be made. Human beings form their own mental model through their knowledge, experiences, and preferences. The mental model does not necessarily conform to a formal logic (Johnson-Laird et al. 1998). For instance, a person who has spent all their life in the same city is likely to have a less erroneous (i.e., closer correspondence between the mental model and reality) mental model of this locality than new comers. Even if two persons have the same level of familiarity with the city, they might have different mental models depending on their unique experiences and preferences. In short, there exist inter-personal differences in the perception of locality.

Medium: In the traffic accident example reported above, the coding forms control the granularity and structure of the information transferred from human knowledge. Coding forms in coarse granularity would not receive human knowledge in finer granularity. That is, information is lost through the medium whose resolution does not accommodate the resolution at which knowledge is expressed. For example, police officers can record the specific location of highway accidents using a milepoint (shown in the lower left of coding forms). But the location of the accident on local streets can only be recorded at a rough scale because local streets are not measured by a milepoint. A poorly designed medium can cause ambiguity as well. For example, the 'city' in the coding form can be interpreted as either 'urbanized area' or 'legally recognized and bounded jurisdiction', even if the latter may be intended. Consequently, an imperfect medium can also be responsible for the location indeterminacy of locality.

In Section 4.3 we review related research to examine solutions to the problem of location indeterminacy. We start with a discussion on the ontology of locality and then review appropriate formalisms of locality. Under the circumstances that the boundaries of localities are indeterminate, the membership of a place to a locality is not necessarily binary (i.e., True or False). We propose an approach that uses the notion of nearness to handle borderline cases (i.e. not sure if it is *in*). Environmental factors that are expected to affect the perception of nearness are reviewed so that they can contextualize the fuzzy set membership function of locality.

4.3 Theoretical Underpinnings of 'Locality'

4.3.1 Ontology of Locality

Ontology research in the spatial domain has concentrated on the nature of spatial objects and their boundaries, their mereological (i.e., parts and whole), topological (Egenhofer and Franzosa 1991, Allen 1983), and mereotopological structure and their location in space and time. Smith and Mark (1998) argue that the ontological characterization of geographic reality requires considering three basic aspects: (1) the aspects of *what* spatial objects are, (2) the aspect of *where* spatial objects are, (3) aspects of *scale*. We can distinguish between spatial objects and abstract objects. A spatial object is an object that is located in space and time, while abstract objects are not located in space and time (e.g., numbers and prepositions). Thus, location is an inherent property of spatial objects. Spatial objects of geographic scale are larger than the human body and cannot be perceived within a single perceptual act. For example, forest and ocean are spatial objects of geographic scale whereas a human organ and Mars are not. More often than not, the locations of spatial objects are indeterminate.

Smith (1995) pointed out the fundamental distinction between *bona-fide* and *fiat* (spatial) objects. Roughly, *bona-fide* objects are objects which boundaries coincide with discontinuities of the underlying reality such as 'The planet Earth', human beings, tennis balls, and so on. *Fiat* objects, on the other hand are the result of human conceptualization and demarcation. Examples of *fiat* objects are countries, Federal states, and land property. They also include objects that have *bona-fide* and *fiat* boundary parts such as Mount Everest, dunes, and the Atlantic Ocean.

When climbing Mount Everest it is perfectly clear where the *bona-fide* boundary between rock and air is, but it is indeterminate where the *fiat* boundaries of Mount Everest among its foothills are. *Fiat* boundaries are not directly observable in reality unless they are explicitly marked as, for example, in built environments (Bittner 2000). Obviously, this causes problems when sharing information about such objects with others or when representing such information on a computer.

In general we need to distinguish at least two different kinds of vagueness of concepts that carve out *fiat* objects: (1) the vagueness of the *identity* condition, and (2) the vagueness of the *unity* conditions. Identity is related to the problem of distinguishing one instance of a class from other instances by means of a characteristic property that is unique for it. Unity is related to the problem of distinguishing the parts of an instance from the rest of the world by means of a unifying relation that binds them together (and not involving anything else) (Guarino and Welty 2000). Mount Everest satisfies the identity condition (is this Mount Everest?), but fails to satisfy the unity condition (does this foothill belong to Mount Everest?).

Now let us look at the ontology of locality based on the formal theory described above. Locality is defined as a surrounding or nearby region (WordNet 1.7.1), or a particular place (Webster-Merriam dictionary). Locality is a spatial object of geographic scale and falls into fiat objects. In general, the identity condition of locality holds. However, the unity condition is not necessarily met because the boundaries between localities are not clearly demarcated in mind. Therefore, localities are subject to location indeterminacy. The vague unity condition of locality can suggest that there exist multiple candidates for being the exact spatial region of locality. It allows us to divide the parts of locality in dealing with the multiple candidate situations, which will be discussed in the next section.

4.3.2 Formalism of Locality

A GIS often advocates an entity-oriented view of spatial phenomena. As a result, the vector format has been widely used as a spatial data type. So far, it has been implicitly assumed that the extent, and hence the boundary, of spatial objects is precisely determined. The properties of space are given by attributes whose values are assumed to be constant over the total extent of the objects, whether they are points, lines, or regions (Erwig and Schneider 1997). Increasingly, researchers are beginning to realize that there are many spatial objects in reality that do not have sharp boundaries or whose boundaries cannot be precisely determined. Examples are natural, social, or cultural phenomena with variant properties such as dialectal regions in North America, deserts, vegetation zones, and vernacular localities. We roughly define this kind region as a *fuzzy region*.

According to Schneider (1999), there are at least three possible, related interpretations for a point in a fuzzy region. First, this situation may be interpreted as the degree to which this point *belongs to* some areal feature (being inside or part of). For instance, there is no strict boundary between mountain and valley, and it seems to be more appropriate to model the transition by partial and multiple memberships. Second, the situation may indicate the degree of *compatibility* of the individual point with the attribute or concept represented by the region with indeterminate boundaries. An example is “warm areas” where we must decide for each point whether and to which grade it corresponds to the concept “warm”. Third, this situation may be viewed as the degree of *concentration* of some attribute associated with a fuzzy region at the particular point. An example is air pollution where we can assume the highest concentration in the direct vicinity of power plant, for instance, and lower concentrations with increasing distance from them.

Let us consider the example of traffic accidents. The upper portion of the accident coding form pertains to where the accident occurs. The issue here is the spatial relation between a point (accident) and a region (locality). GIS can find the binary relation (i.e., in or not in) using a point-in-polygon operation. However, what if the boundary of a region is not crisp in reality, unlike the data stored in the GIS database? In this case, it is hard to state whether the accident occurred *in* the locality. Alternatively, it is also plausible to say that the accident occurred *near* some area. Due to the indeterminate boundary, we need to deal with the borderline cases. A crisp data model cannot provide a realistic solution to this situation because the membership has to be either true or false. Fuzzy set theory can provide a better insight on partial or multiple memberships. Therefore, locality can be interpreted as the *belonging to* case in georeferencing applications.

Now the next natural question is how we can represent a fuzzy region given insufficient knowledge about the grade of indeterminate parts of localities. Locality is the result of human conceptualization unlike natural process such as weather pattern or a soil type. Consequently the semantic import model (Robinson 1988, Fisher 2000) using expert knowledge may not be a plausible option. An empirical model can be properly applied to locality, but it requires a sufficient amount of data from which the generally accepted model can be derived. While the issue of deriving fuzzy set membership is deferred to the next section, we take care here of delimiting the vague parts of a fuzzy region using lower (definitely in) and upper (possibly in) bounds of locality. The approximation of these bounds rests on the categorization of the geographic space into three components. That is, locality can be divided into three parts: *core*, *boundary*, and *exterior*, as shown in Figure 4.3. The three parts respectively relate to those parts that definitely belong, perhaps belong, and definitely do not belong to a specified locality. We can assign the membership value 1 to each point of the *core*, value 0 to each point of the *exterior*, and value in [0, 1] (somewhere between completely true and completely false) to each point of the *boundary*. This model can approximate many different situations as discussed in Section 2. Some of these situations are enumerated as representations based on incomplete information (e.g., missing locality information in the reference data), conflicting information (e.g., conflicting zone types), and changing information (e.g., changing perception of locality, or changing environments such as urban growth, and urban sprawl over time), as well as representations of inherently vague concepts (e.g., the indeterminacy of location) (Worboys 2001).

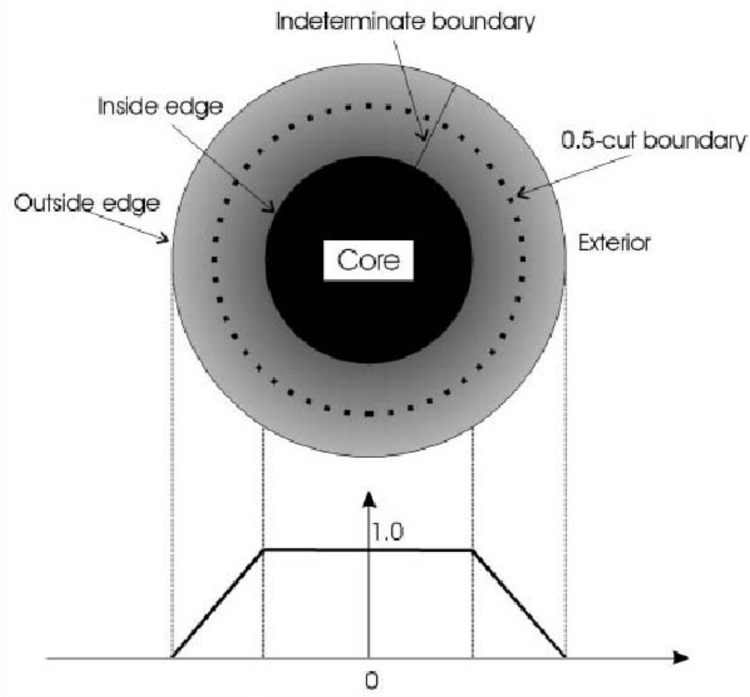


Fig. 4.3. Representation of a Fuzzy Region and its Fuzzy Set Membership Function (Source: Zhan and Lin 2003)

The problem now boils down to delineating those three parts, and defining the grade of fuzzy set membership function. Assigning fuzzy set membership value for the *boundary* part (i.e., perhaps belong) is similar to determining how close the point (accident) is to the area (locality). Therefore, examining the notion of nearness can give important insights into the fuzzy set membership function. More specifically, the next section examines the factors that influence the human perception of nearness. We derive factors (or concepts) from the theory of nearness, and then we apply them to the fuzzy set membership function of locality, which is the main concern of this paper.

4.3.3 Nearness

According to Gahegan (1995), human perception and cognition of nearness (or proximity) is influenced by the following: (1) in the absence of other objects, humans reason nearness in a geometric fashion. Absolute distance is the major factor that affects nearness. Furthermore, the relationship between distance and nearness can be approximated by a linear relationship (some researches suggest an S-shaped function). (2) When other objects of the same type are introduced, nearness is judged in part by relative distance. For example, London and Milan may be considered to be close in the absence of Paris. But they are considered to be far apart when Paris is introduced. In a linguistic term, Milan is close to London. Milan is far from London compared to Paris. (3) Distance is affected by the size of the area being considered. That is, the reference frame plays a significant role in comparing distances between objects. For instance, Milan may be far from London in a European reference frame, but they are close to each other in a world reference frame. Therefore, the scale of the reference frame influences the perceived distance.

To get further insight into the perception of nearness, we can consider the intervening opportunities (Stouffer 1940) model based on the principle of the least effort. This model states that individuals consider opportunities that are closest to them first, and if they find them unacceptable they will go on to the next closest opportunity or opportunities, and so on. By delineating the opportunity set by their proximity relationship to the reference location, we can obtain a set of sequentially embedded neighborhoods. In the context of the spatial object 'locality', it suggests that nearness can be significantly affected by the order of neighborhoods. In other words, nearness can be qualitatively defined as the order (or lag) of neighborhoods. In Figure 4.4, C is considered to be closer to A than H because C is in the 2^{nd} order neighborhood and H is in the 3^{rd} order neighborhood even though they are apart from A with similar distances. This conceptualization implies that the relative location of neighborhoods determines the extent of nearness surface. Other contextual factors important for judgments on nearness include connection paths between places, attractiveness of objects, type of activity to be undertaken, traffic conditions, transportation mode, as well as personal characteristics (Yao and Thill 2005).

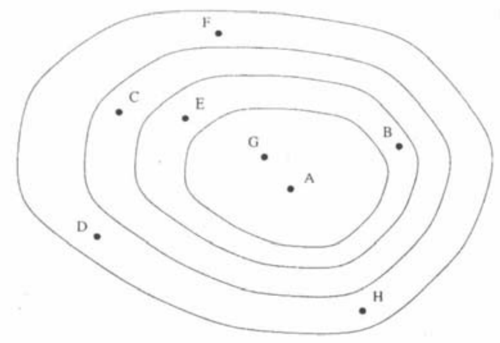


Fig. 4.4. Nearness Illustrated as a Set of Neighborhoods (Source: Guesgen and Albrecht 2000)

4.4 Modeling Localities by Fuzzy Sets

4.4.1 Formal Definition of Locality

Let us consider a locality indexed by l. This locality is a fuzzy region denoted by $\tilde{A}_l$.$\tilde{A}_l$ is composed of the following three parts: *Core*, *Boundary*, and *Exterior*. These parts are defined as crisp regions (regular closed sets) denoted as reg_c. Also let $\Re^2$ be the two dimensional geographic space and let $\mu_{\tilde{A}l}$ be the fuzzy set membership function of $\tilde{A}_l$. $\tilde{A}_l$ and $\mu_{\tilde{A}l}$ are defined as follows:

$$\tilde{A}_l = Core(\tilde{A}_l) \vee Boundary(\tilde{A}_l) \vee Exterior(\tilde{A}_l)$$
$$Core(\tilde{A}_l) = reg_c\ (\{(x,y) \in \Re^2 \mid \mu_{\tilde{A}l}\,(x,y) = 1\})$$
$$Exterior(\tilde{A}_l) = reg_c\ (\{(x,y) \in \Re^2 \mid \mu_{\tilde{A}l}\,(x,y) = 0\})$$
$$Boundary(\tilde{A}_l) = reg_c\ (\{(x,y) \in \Re^2 \mid 0 < \mu_{\tilde{A}l}\,(x,y) < 1\})$$

The *core* identifies the part that definitely belongs to $\tilde{A}_l$. The *exterior* determines the part that definitely does not belong to $\tilde{A}_l$. The indeterminate character of $\tilde{A}_l$ is summarized in the *boundary* of $\tilde{A}_l$ in a unified and simplified manner. The *core* and *boundary* can be adjacent with a common border, and *core* and/or *boundary* can be empty. When the *boundary* is an empty set, $\tilde{A}_l$ becomes a crisp region. Thus, a crisp region is a special case of a fuzzy region.

The generalized membership function of *Boundary*($\tilde{A}_l$) is a weighted average of neighboring points in the geographic space:

$$\mu_{\tilde{A}l}\,(x,y) = \sum_{i=1}^{n} w_i(x,y)\ \mu_{\tilde{A}i}\,(x,y)$$

where $\Sigma\ w_i(x,y) = 1$ and $w_i(x,y) \propto h_i^{-1}$ in which h_i is the distance from (x,y) to $(x,y)_i$. The nonzero weight $w_i(x,y)$ is only given to the points that meet the specific criteria to be described in the next section. The fuzzy set membership value of *Boundary*($\tilde{A}_l$) can be seen as a z-value in the plane that is fitted to *core* and *exterior*. Therefore, delineating *core* and *exterior* determines the extent and grade of fuzzy set membership function in *boundary*. Additionally, *0.5-cut boundary* is introduced as a way to make the interpolation compact. The delineation of *core*, *0.5-cut boundary*, and *exterior* is formalized using predicates that are introduced in the next section.

4.4.2 Formal Properties of Locality

Every locality implicitly has the *Resolution-Level*. Let an instance of locality be x. The *Resolution-Level*(x) (or RL (x)) is defined as a spatially hierarchical structure of administrative boundaries or communities. Suppose we have n members of the *Resolution-Level* say RL_1, RL_2, …, RL_n. Each member of the *Resolution-Level* must be in the hierarchical order; thus the order is consistently assigned to each member of the *Resolution-Level*(x) as denoted by the subscript. We use {*State*, *County*, *TownorCity*, *Place*} as the members of *Resolution-Level* for our applications, where n is 4, and each member has the resolution order (Liu and Satur 1999). Denote the resolution order of *RL* (x) as *Resolution-Order*(*RL*(x)) (or *RO*(*RL*(x))). For example, *RO*(*RL*(x)) is 1 when the *Resolution-Level* of locality x is *State*, and so on. *Difference*(*RO*(*RL*(x)), *RO*(*RL*(y))) returns the difference in resolution order between two resolution levels

$$\forall\ x\ \textit{Resolution-Level}(x) = \{RL_1, RL_2, \ldots RL_n\}$$

$$\forall\ x, y \rightarrow \text{Domain of Locality in the United States}$$

$$\forall\ x\ \textit{Resolution-Level}\ (x) = \{\textit{State, County, TownorCity, Place}\}$$

$$\exists\ x\ \textit{Place} \subseteq \textit{TownorCity} \subseteq \textit{County} \subseteq \textit{State}$$

$$\textit{Resolution-Order}(RL(x)) \quad \begin{cases} 1 \text{ if } RL\ (x) = \textit{State} \\ 2 \text{ if } RL\ (x) = \textit{County} \\ 3 \text{ if } RL\ (x) = \textit{TownorCity} \\ 4 \text{ if } RL\ (x) = \textit{Place} \end{cases}$$

$$\textit{Difference}(RO(RL(x)), RO(RL(y))) = |\ RO(RL(x)) - RO(RL(y))\ |$$

Regarding the relationship between two localities x and y, we can distinguish two types of geographic neighborhoods: *horizontal* and *vertical* neighborhood. A horizontal neighborhood refers to a competing neighborhood in the same *Resolution-Level* while a vertical neighborhood is a compatible neighborhood in the different *Resolution-Level*. For example, San Francisco and Los Angeles is the horizontal neighborhood while San Francisco and California is the vertical neighborhood. Denote horizontal neighborhood as *HN* and vertical neighborhood as *VN* in terms of 2-ary predicates. They satisfy symmetric properties, and can be extended to n-ary predicates. If the resolution level of x and y is the same, they are horizontal neighborhood. If the resolution level of x and y is not the same, they are vertical neighborhood.

$$[RL\ (x) = RL\ (y)] \Rightarrow HN(x, y)$$

$$[RL\ (x) \neq RL\ (y)] \Rightarrow VN(x, y)$$

Locality x has a *Proximity-Order* with respect to other locality y. *Proximity-Order* (x, y) quantifies the ordinal proximity between x and y. Similar to geographic neighborhood, we distinguish two dimensions of proximity order: *Horizontal-Proximity-Order*(x, y) and *Vertical-Proximity-Order*(x, y). *Horizontal-Proximity-Order*(x, y) is 1 when a region x meets other region y where their neighborhood relation is horizontal. The definition of predicate *Meet* (x, y) follows that of Egenhofer and Franzosa (1991).

$$\textit{Horizontal-Proximity-Order}\ (x, y) \rightarrow \mathbb{N}$$
$$HN(x, y)\ \wedge\ Meet(Core(\tilde{A}_x), Core(\tilde{A}_y)) \Rightarrow HPO\ (x, y) = 1$$
$$HN(y, z)\ \wedge\ Meet(Core(\tilde{A}_y), Core(\tilde{A}_z)) \Rightarrow HPO\ (y, z) = 1$$

Horizontal-Proximity-Order(x, z) is 2 by transitivity if $HPO\ (x, y) = 1$ and $HPO\ (y, z) = 1$.

$$[HPO\ (x, y) = 1] \wedge [HPO\ (y, z) = 1] \Rightarrow HPO\ (x, z) = 2$$

The function *FirstOrderHNGroup* of x is defined as the sum of all y whose horizontal proximity order with respect to x is 1. Similarly, the function *SecondOrderHNGroup* of x is defined as the sum of all z whose horizontal proximity order with respect to x is 2.

$$FirstOrderHNGroup(x) = \textstyle\sum^{n}_{i=1} y_i \text{ where} \forall\ y_i\ [\ HPO\ (x, y_i) = 1\ ]$$
$$SecondOrderHNGroup(x) = \textstyle\sum^{n}_{i=1} z_i \text{ where} \forall\ z_i\ [\ HPO\ (x, z_i) = 2\ ]$$

Vertical-Proximity-Order(x, y) is 1 when a region x is inside another region y, and y is the one-level-higher vertical neighborhood of x. The predicate *Inside*(x, y) is not the same as *Inside* (y, x), thus the predicate is non-symmetric unlike *Meet* (x, y).

$$\textit{Vertical-Proximity-Order}\ (x, y) \rightarrow \mathbb{N}$$

$VN(x, y)$
$RO(\ RL(y)\ ) > RO(\ RL(x)\ )$
$Difference(\ RO(\ RL(x)\ ), RO(\ RL(y)\ )\ ) = 1$
$Inside(Core(\tilde{A}_x), Core(\tilde{A}_y))$
$\Rightarrow VPO\ (x, y) = 1$

$VN(y, z)$
$RO(\ RL(z)\ ) > RO(\ RL(y)\ )$
$Difference(RO(RL(y)), RO(RL(z))) = 1$
$Inside(Core(\tilde{A}_y), Core(\tilde{A}_z),)$
$\Rightarrow VPO\ (y, z) = 1$

Vertical-Proximity-Order(x, z) is 2 by transitivity if $VPO\ (x, y) = 1$ and $VPO\ (y, z) = 1$.

$$[VPO\ (x, y) = 1] \wedge [VPO\ (y, z) = 1] \Rightarrow VPO\ (x, z) = 2$$

The function *FirstOrderVNGroup* of x is defined as the sum of all y whose vertical proximity order with respect to x is 1. Similarly, the function *SecondOrderVNGroup* of x is defined as the sum of all z whose vertical proximity order with respect to x is 2.

$$[VPO\ (x, y) = 1] \Leftrightarrow FirstOrderVN(x, y)$$
$$[VPO\ (y, z) = 1] \Leftrightarrow FirstOrderVN(y, z)$$
$$[\mathrm{VPO}\ (x, z) = 2] \Leftrightarrow SecondOrderVN(x, z)$$
$$FirstOrderVNGroup(x) = \textstyle\sum^{n}_{i=1} y_i \text{ where } \forall\ y_i\, [\ VPO\ (x, y_i) = 1\ ]$$
$$SecondOrderVNGroup(x) = \textstyle\sum^{n}_{i=1} z_i \text{ where } \forall\ z_i\ [\ VPO\ (x, z_i) = 2\ ]$$

To illustrate the point, let us consider locality ‘Syracuse’ as a fuzzy region. *Resolution-Level* of Syracuse is *TownorCity*. Vertical neighborhoods of Syracuse are ‘Onondaga’ in the *Resolution-level* of *County*, and ‘New York’ in the *Resolution-level* of *State*. Horizontal neighborhoods of ‘Syracuse’ are other localities in the same *Resolution-level* such as ‘Clay’, ‘De Witt’, and ‘Pompey’ shown in Figure 4.5. Let ‘Syracuse’ be x, then *Horizontal-Proximity-Order*(x, ‘Salina’) is 1 whereas *Horizontal-Proximity-Order*(x, ‘Clay’) is 2. Thus, *FirstOrderHN*(x, ‘Salina’) and *SecondOrderHN*(x, ‘Clay’) hold. In Figure 4.5, it can be seen that *FirstOrderHNGroup*(x) is {‘Salina’, ‘Geddes’, ‘De Witt’, ‘Onondaga’} and *SecondOrderHNGroup*(x) is {‘Clay’, ‘Lysancer’, ‘Van Buren’, Camillus’, ‘Marcellus’, ‘Otisco’, ‘La Fayette’, ‘Pompey’, ‘Manlius’, Cicero’}.

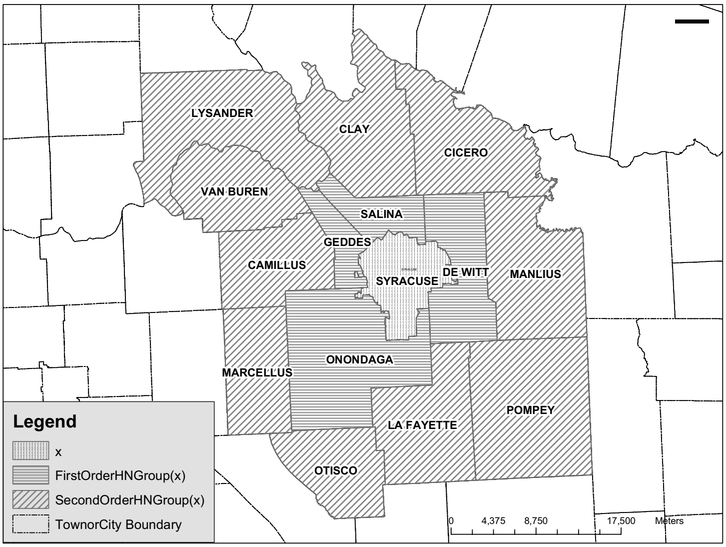


Fig. 4.5. Illustration of *FirstOrderHNGroup*(x) and *SecondOrderHNGroup*(x)

Likewise, an example of vertical neighborhoods is given in Figure 4.6. Let 'Syracuse' be *x*, then *Vertical-Proximity-Order*(*x*, 'Onondaga') is 1 whereas *Vertical-Proximity-Order*(*x*, 'New York') is 2. Thus, *FirstOrderVN*(*x*, 'Onondaga') and *SecondOrderVN*(*x*, 'New York') hold. *FirstOrderVNGroup*(*x*) is 'Onondaga' and *SecondOrderVNGroup*(*x*) is 'New York'.

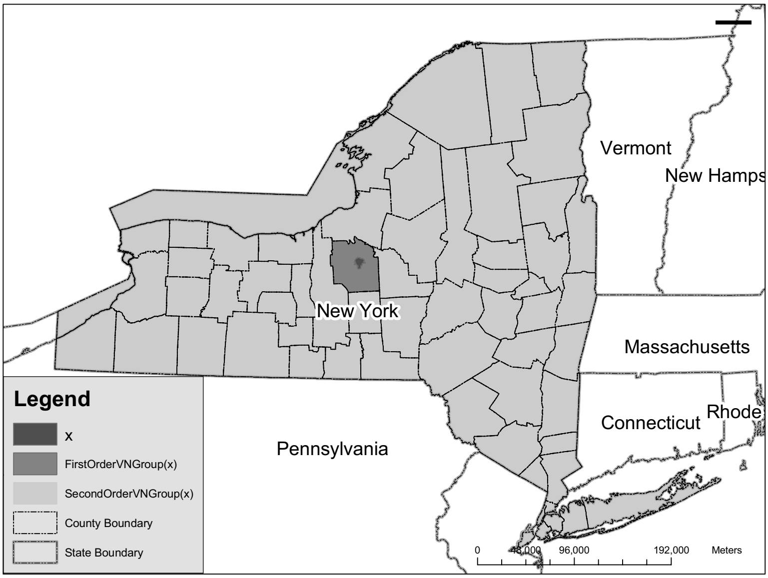


Fig. 4.6. Illustration of *FirstOrderVNGroup*(*x*) and *SecondOrderVNGroup*(*x*)

4.4.3 Fuzzy Set Membership Function of Locality

The indeterminate parts of locality *l* can be replaced with predicates defined above. The combinations of *FirstOrderHNGroup*(*l*) and *SecondOrderHNGroup*(*l*) comprise the (indeterminate) *boundary* of locality as a fuzzy region. Within *boundary*, the outside edge of *FirstOrderHNGroup*(*l*) serves as *0.5-cut boundary* is defined as. Additionally, the extent of locality as a fuzzy region is delimited by *FirstOrderVNGroup*(*l*).

$$Boundary(\tilde{A}_l) = FirstOrderHNGroup(l) \vee SecondOrderHNGroup(l)$$
$$\text{where } 0.5\text{-}cut\ boundary\ (\tilde{A}_l) = \text{outside-edge}\ (FirstOrderHNGroup(l))$$
$$\tilde{A}_l \subseteq FirstOrderVNGroup(l)$$

Accordingly, the fuzzy set membership value can be redefined as follows:

$$(x,y) \in \Re^2$$

$$\mu_{\tilde{A}l}(x,y) \begin{cases} 0 & \text{if (x,y) is at } \textit{Exterior}(\tilde{A}_l) \\ [0, 0.5] & \text{if (x,y) is in } \textit{SecondOrderHNGroup}(l) \\ [0.5, 1] & \text{if (x,y) is in } \textit{FirstOrderHNGroup}(l) \\ 1 & \text{if } (x,y) \text{ is in } \textit{Core}(\tilde{A}_l) \end{cases}$$

To compute the fuzzy set membership value in *Boundary*($\tilde{A}_l$), we create Delaunay Triangulation whose nodes are comprised of any vertices on *core*, *0.5-cut boundary*, and *exterior*. The membership value is obtained by intersecting a vertical line with the plane defined by the three nodes of the triangle (Figure 4.7). In Figure 4.7, any location within this plane is guaranteed to have a fuzzy set membership value between 0.5 and 1.

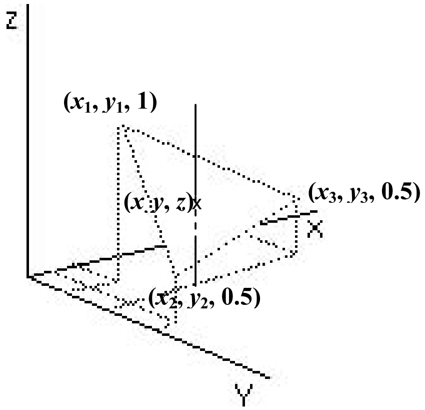


Fig. 4.7. TIN Surface Created to Interpolate the Membership Value

In Figure 4.7, the generalized equation for linear interpolation of a point (x, y, z) in a triangle facet is:

$$Ax + By + Cz + D = 0$$

where A, B, C, and D are constants determined by the coordinates of the triangle's three nodes. Thus, the fuzzy set membership value can be obtained from the following equation given the x- and y- coordinates:

$$\mu_{\tilde{A}l}(x,y) = (-Ax - By - D) / C$$

4.5 Implementing the Fuzzy Locality Model in GIS

In this section, we describe the procedures used to georeference 8631 fatal accident records in New York State during the period 1996-2001. The accident records are matched against the reference data such as road network and locality layers. The records are matched on the basis of how similar two features (trafficway and locality) recorded in the accident database are to those in the reference data. Fuzzy locality layers are created (derived) from (crisp) locality layers according to the formalism presented in Section 4.4. The overall quality of a match is computed as the average of two similarity scores. The road segment with the best score is chosen as the best candidate. If the score of the best candidate is above a preset threshold, the georeferencing result is accepted. Otherwise, we reject the result.

4.5.1 Data Sets

Source Data

The Fatality Analysis Reporting System (FARS) contains data on a census of fatal traffic crashes within the 50 States, the District of Columbia, and Puerto Rico. To be included in FARS, a crash must involve a motor vehicle traveling on a trafficway customarily open to the public and result in the death of a person (occupant of a vehicle or a non-occupant) within 30 days of the crash.

In Table 4.1, we identify the data fields that may be of use for georeferencing purposes. The rightmost column of the table labels them. More particularly, RG stands for “relevant to georeferencing” while RGBI indicates “relevant to georeferencing, but incomplete.” For instance, data field TRAFFICWAY IDENTIFIER is definitely used for geo-referencing purposes, whereas SPECIAL JURISDICTION is not likely to be used because most values are left blank.

Table 4.1. Location-related Fields in the FARS Accident Data

ACCIDENT LEVEL DATA FIELDS	TYPE	START	LENGTH	GEORef.
STATE	N	1	2	RG
CITY	N	13	4	RG
COUNTY	N	17	3	RG
TRAFFICWAY IDENTIFIER	A/N	42	20	RG
MILEPOINT	N	62	5	RG
SPECIAL JURISDICTION	N	67	1	RGBI
RELATION TO JUNCTION	N	71	2	RG
LATITUDE	A/N	116	8	RGBI
LONGITUDE	A/N	124	9	RGBI

Reference Data

Table 4.2 lists the reference data that we used for georeferencing accident records of the FARS database. The selection of reference data is based on two primary considerations. First, we consider how relevant/compatible the data is to the source data. Second, we compromise between data accessibility and data quality. For instance, TIGER/LINE roads are chosen over other road network data that may be more accurate because TIGER is in the public domain.

Table 4.2. Sources of Reference Data

Relevant FARS Field	Organization	Layer Name	Data Source (URL or product) / Metadata Documentation
Tway_id	DOT-FHWA	NHPN	http://www.fhwa.dot.gov/planning/nhpn http://www.fhwa.dot.gov/planning/nhpn/docs/metadata.html
	US Census Bureau	TIGER	http://arcdata.esri.com/data/tiger2000/tiger_download.cfm http://www.census.gov/geo/www/tiger/rd_2ktiger/tlrdmeta.txt
	NY State GIS clearinghouse: DOT	CLASS	https://www.nysgis.state.ny.us/s_dot/regional.html http://www.nysgis.state.ny.us/repository/dotlist.htm
County	NY State GIS clearinghouse: DOT	Nyshore	https://www.nysgis.state.ny.us/s_dot/data/state/nyshore.zip (user login required) http://www.nysgis.state.ny.us/gis3/data/dot.nyshore.html
City	NY State GIS clearinghouse: DOT	Nybndry	https://www.nysgis.state.ny.us/s_dot/data/state/nybndry.zip (user login required) http://www.nysgis.state.ny.us/gis3/data/dot.nybndry.html
	Caliper	Ccplacec	Academic TransCAD® version 3.5d (program CD) N/A
	GSA	GSA GLC table	http://www.gsa.gov/attachments/GSA_PUBLICATIONS/extpub/glcout_1.zip N/A

4.5.2 Data Preprocessing

It is necessary to preprocess the reference data from multiple sources. Major tasks include (1) transforming different coordinate systems (for example, NHPN and Nybndry are stored in UTM whereas TIGER and Ccplacec are in decimal degrees) into a single unified one, (2) creating locality layers against which the locality code in FARS is matched, (3) populating the field used to join FARS table to a spatial data (i.e., writing GSA code to locality layer), and (4) assembling necessary spatial data. We only describe the second and third ones due to the relevance to the content in this chapter.

Creating Locality Layers

We define four different *Resolution-Levels* in Section 4.2, including *State*, *County*, *TownorCity*, and *Place*. It is necessary to create locality layers that correspond to those levels respectively (Liu and Satur 1999). Three locality layers are created: COUNTY for the *Resolution-Level County*, PLACE_PL for *TownorCity*, and PLACE_PT for *Place* (Layer names are capitalized to tell them apart from other names). Nyshore (see Table 4.2) is renamed to the layer COUNTY.

PLACE_PL is extracted from Nybndry. The field CITY in FARS accidents (see Table 4.1) loosely refers to a place name. The field does not distinguish between different types of zones. On the contrary, the reference data Nybndry distinguishes them. The *region* (i.e., Arc/Info feature class) coverage Nybndry consists of multiple layers (including county, town, city, village, airport, and so on) to which the value of the field CITY can be related. For example, the Town of Amherst (156) and the City of Buffalo (750) are coded in the same field CITY in FARS accidents without regard for the fact that these localities belong to different classes of administrative units. On the other hand, the reference data Nybndry stores towns and cities as distinct layers. For matching purposes, the Town and City layers in Nybndry are merged into a new polygon layer called PLACE_PL. They can be merged because those layers do not overlap.

Layer PLACE_PT is extracted from Ccplacec, which contains the centroids of places. Only features appropriate to our applications are selected to populate this new layer. First, features within New York State are selected. In a second stage, features that are included in the GSA geographic code table are selected out of this first set. Since PLACE_PT does not have a polygon topology, Thiessen polygons are created from the point features in order to approximate their polygon boundaries. Locality layers are illustrated in Figure 4.8.

When it comes to the difference between PLACE_PL and PLACE_PT, PLACE_PL refers to administrative boundaries whereas PLACE_PT refers to the place with vague boundaries. For example, ‘New York’ can be found in PLACE_PL while ‘Greenwich Village’ can be found in PLACE_PT.

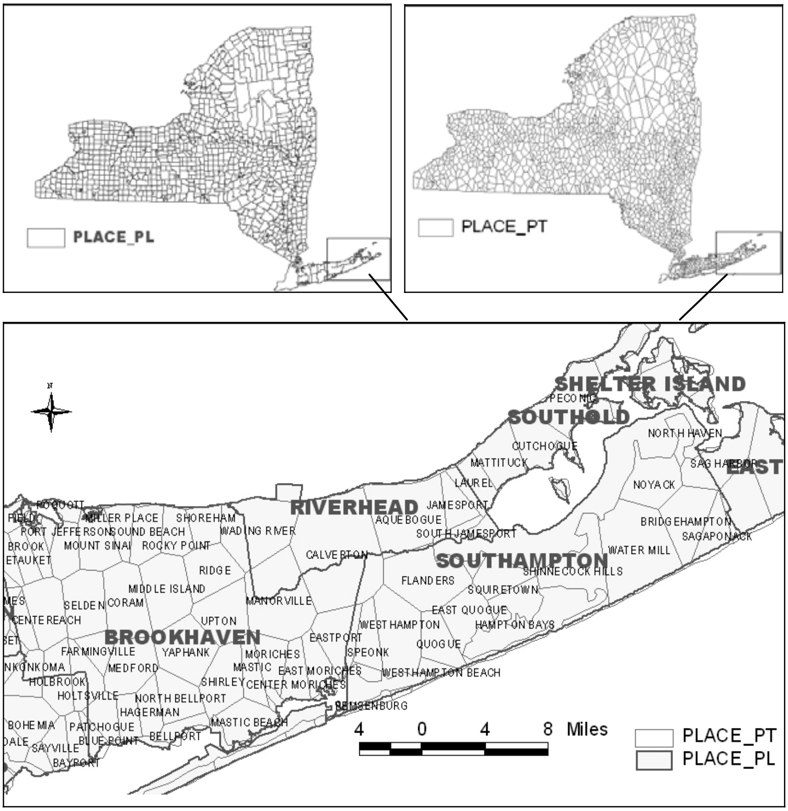


Fig. 4.8. Locality Layers: PLACE_PL and PLACE_PT

Writing GSA Codes to Locality Layers

The locality-related fields of the FARS database are populated with GSA codes, but entities of the locality layers do not contain a GSA code. In order to match each FARS data record to entities of the locality layers, we need to create a key filed for joining. The GSA code table is used to relate locality (in FARS) to locality layers, as depicted in Figure 4.9. In this figure, GSA City code 0010 can be derived and written to locality layers using the match through the field NAME between the GSA code table and the Reference GeoData layer.

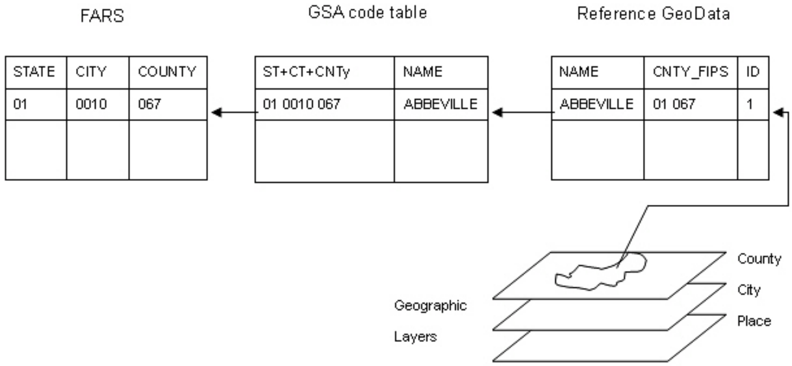


Fig. 4.9. Writing GSA Codes to Locality Layers

4.5.3 Georeferencing Procedures

Given the intrinsic differences between the coding schemes of accidents on highways and on local streets, the georeferencing problem is divided into two procedures implemented in parallel. One is to use dynamic segmentation tied to the highways' linear referencing system (LRS) of mile points, while the other involves a similarity-based matching on local streets. The two types of matching lead to different spatial resolutions under which crashes can be positioned. Dynamic segmentation identifies the exact point of location while the local street matching merely identifies the road segments on which the crash is most likely to have occurred. (Accidents have not been reported in an address-style for more than two decades in FARS databases. NHTSA is in the process of switching to a longitude/latitude geographic coding system that is fully compatible with contemporary GPS-based data collection techniques). Simplified matching rules are specified in Figure 4.10. In this figure, the notation "≅" is roughly defined as equivalence, similarity, consistency, or fuzzy proximity, depending on the case in hand.

Since the fuzzy set modeling of locality is only used in local streets matching, we exclusively focus on the local streets matching hereunder. The whole procedure of local streets matching is fully automated using Arc/Info 8.1 AML. Due to the rather low quality of FARS accident data (i.e. there are not many exact matches), similarity-based procedures have been implemented to account for the reliability of the matches. Each potential match is rated on the basis of a similarity score. The similarity score accounts for two features, namely the trafficway and the locality. In order to compute a trafficway similarity score, we have developed a thesaurus and a complete set of associational rules that account for the incompatibility between source and target (or reference) datasets. See Hwang and Thill

(2003) for more detail. With regard to the locality similarity score, the locality code in FARS is matched to that of fuzzy locality layers with a certain fuzzy set membership value. In contrast to a (crisp) locality layer where a single membership is assigned to each feature, a fuzzy locality layer can have *n*-possible memberships where *n* is the total number of features (Stefanakis et al. 1999). Thus, the match to the fuzzy locality layers yields any similarity score between 0 and 1 whereas a match to the (crisp) locality layer would yield a score that is either 0 or 1.

■ *Dynamic Segmentation*

[(FARS.TWAY_ID ≅ NHPN.SIGN1) OR (FARS.TWAY_ID ≅ NHPN.SIGN2) OR (FARS.TWAY_ID ≅ NHPN.SIGN3) OR (FARS.TWAY_ID ≅ NHPN.LNAME)]
AND (NHPN.BEGMPT ≤ FARS.MILEPT ≤ NHPN.ENDMPT)
AND (FARS.CITY ≅ PLACE_PT.CITY_GSA)
AND (FARS.COUNTY = COUNTY.COUNTY_FIPS)

■ *Local Streets Matching*

[FARS.TWAY_ID ≅ TIGER.(FEDIRP+FENAME+FETYPE+FEDIRS)] OR (FARS.TWAY_ID ≅ NHPN.LNAME) OR (FARS.TWAY_ID ≅ CLASS.NAME)
AND (FARS.CITY ≅ PLACE_PL.CITY_GSA) OR (FARS.CITY ≅ PLACE.PT.CITY_GSA)
AND (FARS.COUNTY = COUNTY.COUNTY_FIPS)

Fig. 4.10. Simplified Matching Rules Used in Georeferencing FARS Accidents

Fuzzy locality layers are created from the original locality layers PLACE_PL and PLACE_PT. For instance, when the original locality layer consists of 10 localities, ten fuzzy locality layers are created, each of which corresponds to an original locality feature. Each fuzzy locality layer has the fuzzy set membership value 1 for the core of the locality, and any value between 0 and 1 for the boundary of the locality. To illustrate this point, suppose that we create a fuzzy locality layer for locality *l*, say Buffalo. First, a fixed membership value is set to the *core*, the *0.5-cut boundary*, and the *exterior* of *l*. According to the conditions stated in Section 4.3, *core* is derived from the outside edge of *l*; the *0.5-cut boundary* is the outside edge of the set of first-lagged localities surrounding *l*; *exterior* is the outside edge of a set of second-lagged localities surrounding *l*. Next, the fuzzy set membership value is set to 1 for the *core*, 0.5 for the *0.5-cut boundary*, and 0 for the *exterior*. Third, a triangulated irregular network (TIN) is built on the vertices of these three line features. Fourth, the fuzzy set membership value is interpolated on the TIN facet. The steps of creating fuzzy locality layers are illustrated in Figure 4.11. Consequently, a continuous membership value between 1 and 0 is computed along the *boundary* region (Wang and Hall 1996) as shown in Figure 4.12.

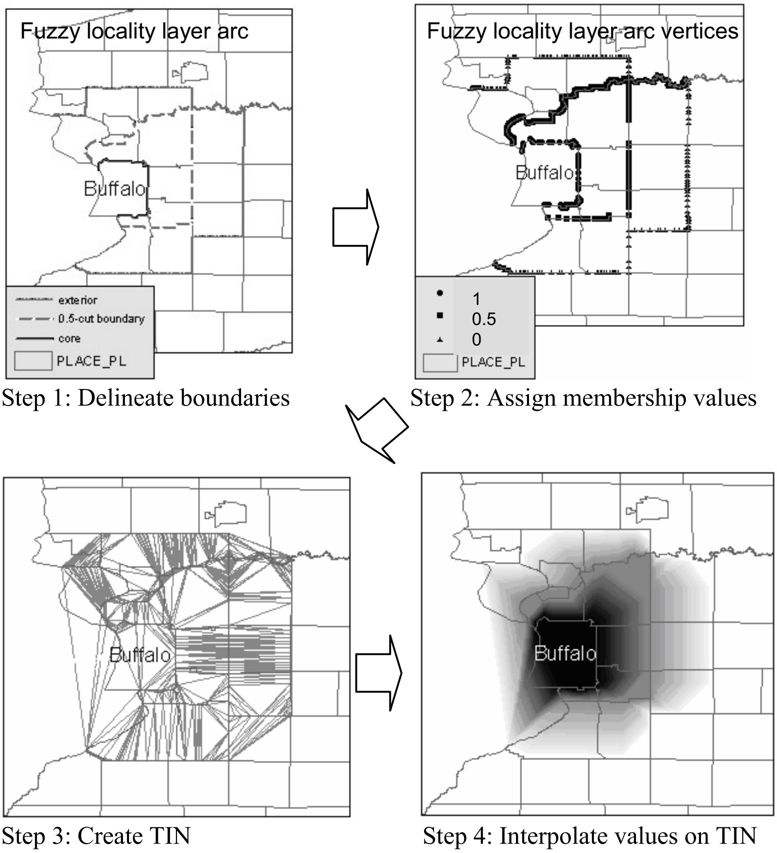


Fig. 4.11. Creating Fuzzy Locality Layers

The membership value is not only determined by the Euclidean distance, but also by the proximity order of neighboring localities. Because horizontal neighborhoods that compose the fuzzy parts of a locality are in the same resolution level, the membership value is also scale-dependent. In sum, Euclidean distance, neighborhood relation, and scale determine the fuzzy set membership value of locality. In Figure 4.12, the locality of Buffalo has a full membership in its *core*, a partial membership in its *boundary* (e.g. Amherst), and has no membership beyond its *exterior* (e.g. Alden).

We can illustrate the similarity-based georeferencing procedure using the example in Figure 4.12. Let us consider a FARS record where TWAY_ID is "Millersport Hwy", and CITY corresponds to "Buffalo". The reference data layers contain a record named Millersport Hwy, but this entity is in the nearby Town of

Amherst instead of the City of Buffalo. For this best match, the matching score is 0.5, which is the average of the trafficway score of 1 and of the locality score of 0. The latter score would be assigned by virtue of Millersport Hwy being outside of the Buffalo city limits. However, the matching score will be at least 0.5 (average of 1 and [0.5, 1]) with the similarity-based matching described above since the locality score is greater than zero considering the nearness of Amherst to Buffalo.

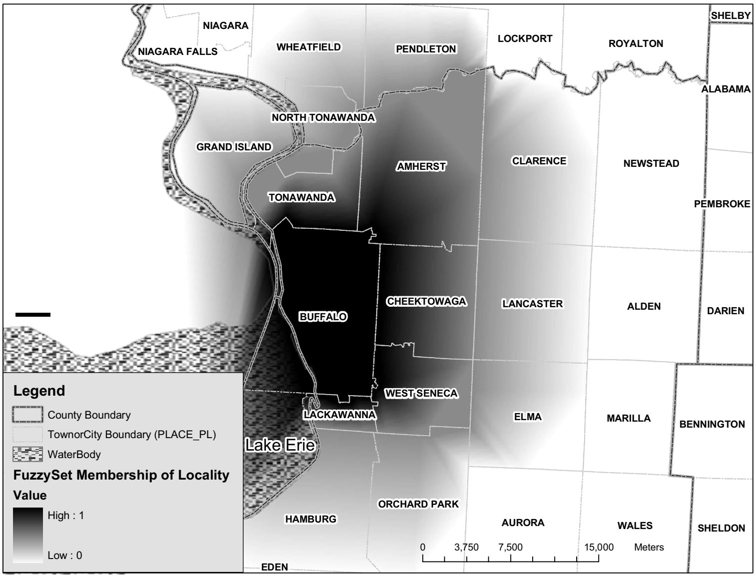


Fig. 4.12. Fuzzy Set Membership of the Locality of Buffalo

It may be useful to compare the grade of fuzzy proximity membership function used in the computation of the similarity score with other possible measures. Figure 4.13 illustrates the setting where the arrow that stretches outwards from the geographic center of the locality of Buffalo is depicted as an x-axis in Figure 4.14. The first option is to use distance as a measure of dissimilarity. Another option is to consider buffers delineated around the *Core*.

To measure the distance between a polygon (locality) and a point (accident), a representative point of polygon, such as the centroid, is required. This leads to a loss of geographic detail. Moreover, distance option ignores the anomalous proximity surface that is created by neighborhood relations. For instance, if the arrow (in Figure 4.13) is drawn along a different direction, the membership values will be adjusted to new neighborhood relations in the case of fuzzy proximity whereas the distance option maintains the membership value regardless of the direction. Creating multiple buffers around the *Core* does not require a centroid, but proximity decreases in an abrupt manner along the buffer boundary. As for the distance option, it does not take into account the anomalous geometry of proximity. Moreover, the buffer interval has to be set based on the absolute distance while the fuzzy proximity can be determined by relative distances. Figure 4.14 shows the differences in membership grades under each of the three options.

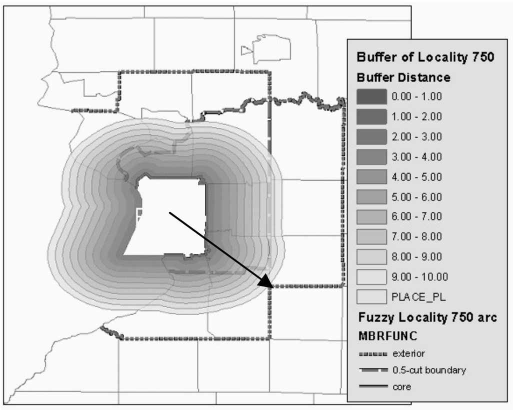


Fig. 4.13. The Setting for the Proximity Measures in a Geographic Space Where the Arrow is the Transect Depicted in Figure 4.14.

In Figure 4.14, the x-axis represents the distance from the centroid of the *Core* of the locality while the y-axis represents the fuzzy set membership value. If the arrow is drawn along another direction in Figure 4.13, the membership value will be adjusted to the new neighborhood relations only in the case of fuzzy proximity, while distance- and buffer-based measures keep the same uniform membership value.

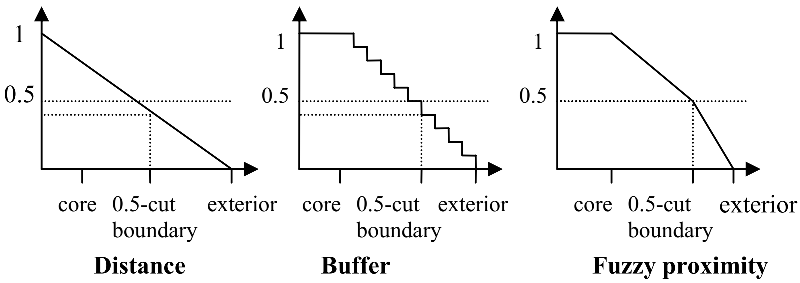


Fig. 4.14. Differences in Membership Grade between Three Proximity Measures

The grades of the first two measures are given by the Euclidean distance in a non-scalable way without regard for qualitative relations to neighboring regions. On the contrary, the grade of fuzzy proximity is determined by the Euclidean distance that has been accommodated to the size and qualitative relations to surrounding localities in the same resolution level.

4.6 Analysis of Results

4.6.1 Georeferencing Results

Table 4.3 is the cross-tabulation of georeferencing results by year and matching score range. 'DS' refers to instances geocoded by dynamic segmentation on the highway system. Score 1 means an exact match, whereas scores less than 1 indicate that a less than perfect match was achieved. The proportion of DS cases has noticeably increased since 1998, and so have overall similarity scores. This suggests that a matching procedure based on similarity outperforms an exact-match procedure when data to be georeferencing is of rather low quality.

Table 4.3. Cross-tabulation of Georeferenced Cases by Year and by Matching Score

Score_Class	1996	1997	1998	1999	2000	2001
DS	85	55	435	484	460	534
1	227	429	519	516	542	520
0.90 - 0.99	111	188	205	204	169	154
0.80 - 0.89	130	342	59	93	75	60
0.70 - 0.79	33	71	35	35	34	26
0.60 - 0.69	14	34	33	27	18	18
0.50 - 0.59	168	103	39	34	19	33
0.40 - 0.49	66	63	17	22	11	17
0.30 - 0.39	176	64	9	15	11	6
0.20 - 0.29	64	11	2	1	0	1
0.10 - 0.19	173	11	0	0	0	1
0.00 - 0.09	16	0	0	0	0	0
Unmatched	187	139	51	42	30	55
Total	1450	1510	1404	1473	1369	1425

4.6.2 The Evidence of Fuzzy Locality

A locality similarity score (from now we will call this the locality score for simplicity) is the fuzzy set membership value of locality, as discussed in Section 4.5. Therefore, we can examine the evidence of fuzzy locality by looking at locality scores less than 1. During the six-year period 1996-2001, 8631 fatal accidents occurred in the State of New York. Of this total, we now focus on 5460 cases to examine the contribution of the notion of fuzzy locality to the geocoding process. (The remaining 3171 cases are accounted for by DS georeferencing or are unmatched cases which do not use fuzzy locality modeling at all). In Figure 4.15, we distinguish georeferenced cases by reference data (either Place_PL or Place_PT) and by crispness of locality (either In or Near), for illustration purposes. We depict crisp cases (score = 1) on the left side of the figure, and fuzzy cases (score less than 1) on the right side. Fuzzy locality accounts for 12.4% (677/5460) of the total number of georeferenced cases. It shows there is fuzziness involved in identifying locality, but not too prominently. Anyhow, the match rate has increased from 86% (7450/8631) to 94% (8127/8631) thanks to fuzzy locality modeling.

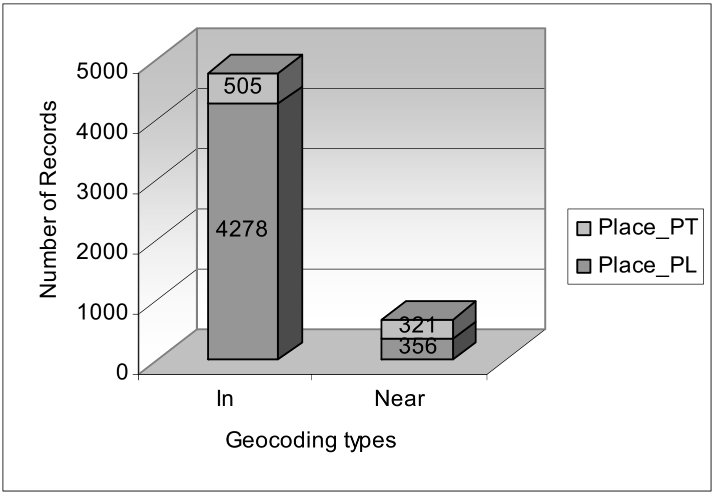


Fig. 4.15. Proportion of Fuzzy Locality Relative to Crisp Locality

We also examine how the locality scores are distributed in case of fuzzy locality. To control for the effect of poor data quality, we only consider cases whose matching quality is reasonably high (matching score is above 0.7). As expected, in the majority of instances, locality is perceived with little fuzziness and only a short distance from the target locality (Figure 4.16). Of the 329 fatal crashes for which fuzzy locality modeling enhances georeferencing, the location of the target locality x is identified within the *FirstOrderHNGroup*(x) 75% (247/329) of the time, and within the *SecondOrderHNGroup*(x) 25% (82/329) of the time.

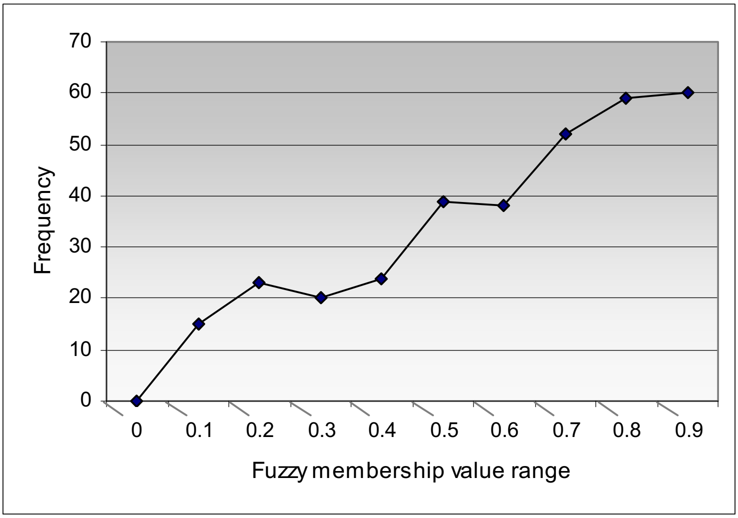


Fig. 4.16. Histogram of Locality Fuzzy Set Membership Values below 1 (Fatal Crashes with a Matching Score over 0.7)

4.6.3 The Effect of Locality Characteristics on Location Indeterminacy

We examine if the location determinacy is affected by certain characteristics of localities. For this purpose, we test two hypotheses. (1) How does the degree of urbanization affect the location determinacy of a locality? Is a big city more location determinate or not? (2) Is the location determinacy of a locality the outcome of human conceptualization? For instance, do people find it easier to identify the location of a well-established city?

The location indeterminacy value of each locality is computed as follows: First we compute the total number of accident cases (t) for each locality. Next we compute the number of fuzzy locality cases (f) for each locality as the number of accidents whose locality fuzzy membership value is less than one. Then divide f by t. For example, suppose that ten fatal traffic crashes occurred in Buffalo. Among them, suppose three cases turn out to be georeferenced outside of the legal boundary of the stated locality. In which case, the location indeterminacy value is 0.3. Thus, the location indeterminacy value, which is a characteristic specific to the locality, is the proportion of fuzzy locality cases to total cases. The higher the value is, the more indeterminate its location is. Sixty-four percent of all localities (657/1031) have zero location indeterminacy. To test the stated hypotheses, we only consider the remaining 374 cases for which some indeterminacy is revealed by the fuzzy set model of locality.

For the localities corresponding to entities of the PLACE_PL data layer (211 cases), the location indeterminacy value is graphed against population in Figure 4.17. The scatter plot suggests that larger localities are more location determinate. People may find it easier to identify the location of urban areas than rural areas. It is likely that urbanization strengthens location determinacy whereas the location indeterminacy value in smaller and more recently settled areas follow more of a "hit or miss" pattern. Curve fitting to the data reveals a clear inverse and non-linear relationship (R-square = 42.71%). Table 4.4 lists the 15 largest localities, and their population as well as their location indeterminacy value.

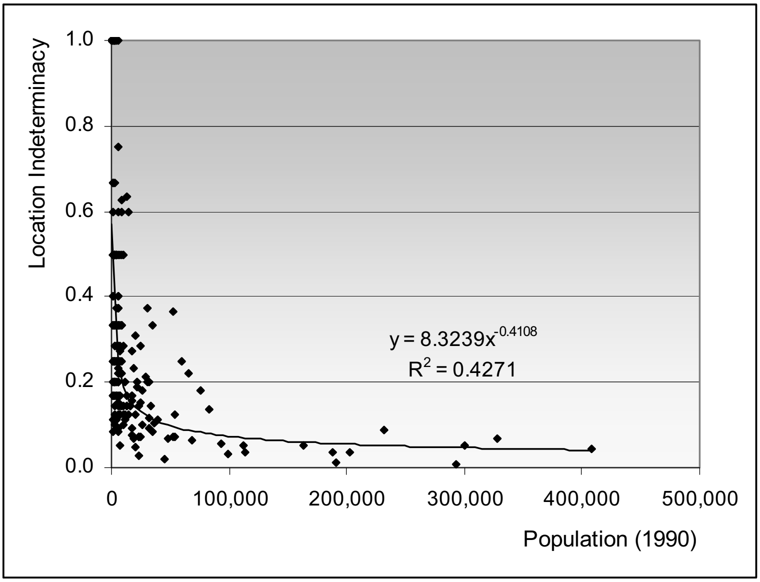


Fig. 4.17. Scatter Plot between Location Indeterminacy and Population

We need to be careful when interpreting the location indeterminacy index because location indeterminacy is measured on the basis of a very small number of accidents, possibly because a large proportion of recorded accidents were georeferenced by means of dynamic segmentation. Alternatively, localities with large population may yield a low indeterminacy index simply due to the large denominator, just as localities with small population may yield a high value due to small denominator. However, when we control for the number of accidents per locality, the pattern remains pretty much the same. We can not produce an equivalent indeterminacy-population graph for localities that are matched against PLACE_PT because there is no consistent population statistic for these types of spatial entities.

Table 4.4. Location Indeterminacy Index for Selected Localities

NAME (PLACE_PL)	1990 Population	#TotalCases	#FuzzyCases	Location Indeterminacy
HEMPSTEAD	725639	283	8	0.028
BROOKHAVEN	407779	201	9	0.045
BUFFALO	328123	102	7	0.069
ISLIP	299587	137	7	0.051
OYSTER BAY	292657	106	1	0.009
ROCHESTER	231636	78	7	0.090
BABYLON	202889	81	3	0.037
HUNTINGTON	191474	93	1	0.011
YONKERS	188082	55	2	0.036
SYRACUSE	163860	37	2	0.054
SMITHTOWN	113406	55	2	0.036
AMHERST	111711	38	2	0.053
CHEEKTOWAGA	99314	29	1	0.034
RAMAPO	93861	18	1	0.056
TONAWANDA	82464	22	3	0.136

It may be insightful to provide maps showing the pattern of location indeterminacy index grouped by region. The 211 localities considered above are grouped into 11 regions that correspond to New York State Department of Transportation (NYSDOT) administrative boundaries. Location indeterminacy is averaged for each region, which is tabulated in the column labeled AVELOCINDET in Table 4.5. According to the map in Figure 4.18, regions 1, 6, and 9 show a higher degree of location indeterminacy compared to regions encompassing the New York metropolitan area. The map confirms the existence of a direct relationship between location determinacy and urbanization. The more urbanized the area, the more location-determinate the locality because urban areas provide more landmarks on which the judgment on location can be made. In addition, it is less likely that a location would be indeterminate when clearly defined bodies of water abound as on Long Island and the rest of the New York metropolitan area. Finally, metropolitan areas have been settled and developed for a longer period of time so that it is likely that a stronger sense of community identity has established than in other areas. The sense of community that has been formed for centuries may contribute to the higher degree of location determinacy. In that sense, locality as a *fiat* spatial object seems to find some empirical validity here.

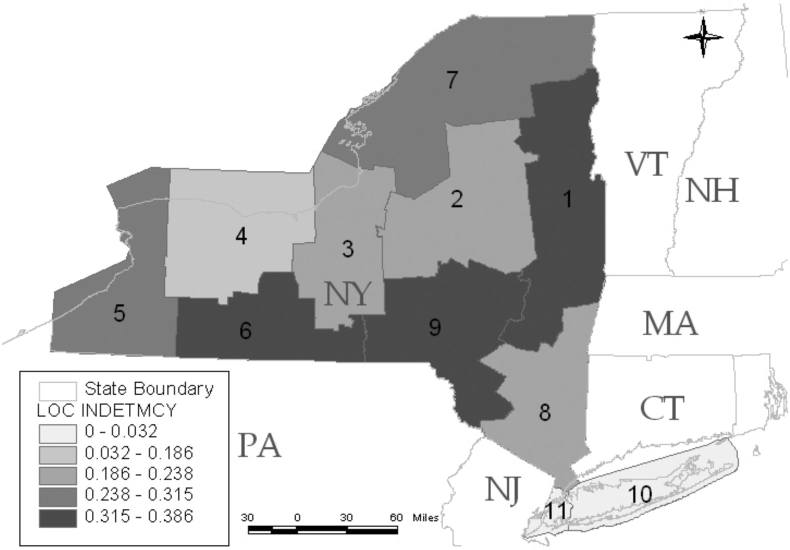


Fig. 4.18. Location Indeterminacy Index Grouped by NYSDOT Regions

Table 4.5. Average Location Indeterminacy Index Grouped by NYSDOT Regions and Number of Localities Included in Fuzzy Cases in Each Region

REGION	OFFICE SITE	AVELOCINDET	#LOCALITY
1	Albany	0.3860	84
2	Utica	0.2376	52
3	Syracuse	0.2192	67
4	Rochester	0.1856	82
5	Buffalo	0.2829	67
6	Hornell	0.3734	54
7	Watertown	0.3147	64
8	Poughkeepsie	0.2195	76
9	Binghamton	0.3576	67
10	Hauppauge	0.0316	14
11	New York	0.0000	0

4.7 Conclusions

Fuzzy set theory has been used to improve the georeferencing of traffic accident data. Georeferencing has been performed using a similarity measure due to imperfect information available in source datasets. To obtain the similarity measure of locality features, locality (e.g. city, town, and place) is modeled by fuzzy sets such that membership values can be set to the similarity measures. Thanks to this approach, the rate of success of the georeferencing procedure is enhanced from 86 % of all fatal accidents to 94 %.

Locality is seen as a fuzzy region and is represented as eggs in the *egg-yolk* representations (also known as *core-boundary* representation). The *Yolk* (or *core*) has a full membership of locality whereas the remaining parts (egg minus yolk or wide boundary) have partial membership. The vague parts are composed of a set of the first- and second-order surrounding localities at the same resolution level. Consequently neighborhood relation and scale determine the vague parts of locality.

To implement the concept of fuzzy locality in GIS, fuzzy locality layers are created from the GIS databases such as city/town boundary and place centroid. With a crisp conceptualization of locality, a single membership value would be assigned to each feature of the locality layers. In a fuzzy set approach, however, multiple membership values can be assigned to each feature of fuzzy locality layers. The fuzzy set membership value is set to 1 for the *core* part, and 0 for the *exterior* part of locality. The membership value is set to 0.5 on the edge of a set of the first-order surrounding localities (spatial lag of one). Then, the membership value is linearly interpolated between the core and the 0.5-cut boundary (so [0.5, 1]), and between the 0.5-cut boundary and the exterior (so [0, 0.5]). Therefore, the fuzzy set membership function of locality is defined as the combination of 4 different piecewise linear functions (0. [0, 0.5], [0.5,1], 1) that depend on proximity with respect to neighboring localities. Uncertainty involved in determining the location of locality is thus captured in the vague parts through the notion of partial membership.

Fuzzy perception and cognition of localities accounts for 12.4% of all accident cases examined in this study. That is, people actually designated a locality outside of their legal boundary 12.4% of the time. Fuzziness in determining the location of locality turns out to be concentrated in areas close to the legal boundary. The magnitude of fuzziness is modeled to decrease linearly with proximity.

The contribution to the determination of locality can be explained at three different levels: (1) Ontology level: Some spatial objects may be inherently vague. The spatial extent of a locality can be indeterminate because this locality is the result of human conceptualization. (2) Perception level: There exist individual differences in perceiving the environment. Individuals have different mental maps (Gould and White 1986, Thill and Sui 1993) depending on their experiences, preferences, and knowledge. Maybe their decision to determine the location of a certain locality has relied on their error-prone mental maps rather than survey knowledge (Golledge et al. 1995). (3) Implementation level: Imperfect information such as imprecise measurements, ambiguity, and incomplete value induces more fuzziness.

We attempted to address several questions with regard to the location determinacy of locality. Is the location determinacy inherent in locality? If so, can a certain characteristic of locality actually affect the location determinacy? Or is it just the result of rough measurements (e.g., poorly designed coding form)? The indicator of location indeterminacy is obtained from the proportion of fuzzy locality cases to the total cases in each locality. The relationship between population and location indeterminacy follows a negative exponential curve in general. The more urbanized the locality is, the more determinate its location is. This may be due to the fact that urban settings provide more landmarks on which perception and cognition of spaces and places can be made. This result actually fits the perception level hypothesis: mental maps are less erroneous in urban area than in rural area. Acquiring spatial knowledge in urban settings is quite different from rural settings. It can also be noted that natural environments such as bodies of water help determine the boundaries of localities, as evidenced by the low location indeterminacy index on Long Island and the rest of the New York metropolitan area. A sense of community can also play an important role in location determinacy even though it is hard to measure. Generally speaking, a historical city is more likely to satisfy identity condition with regard to location determinacy than other cities, not only because it has long been recognized as a distinct spatial entity, but also because it helps build a sense of community.

This study is significant by the wide range of domains of Geographic Information Science to which the methods and concepts discussed can be applied, including georeferencing, modeling fuzzy regions in GIS, and data quality control. Most importantly, the study attempts to formalize how to assign the fuzzy set membership value to a fuzzy region such as locality, which is often pointed out as a weakness in applying fuzzy set theory (Duckham 2001). Furthermore, the study supports the proper application of fuzzy set theory to spatial concepts, such as the indeterminacy of location. In sum, fuzzy set theory provides a mechanism to address various kinds of uncertainty by preserving the detail that would have been truncated in a crisp set.

This study has several limitations. We introduce a resolution level to make a fuzzy proximity measure scale-dependent, but the use of administrative boundaries, as a member of resolution level, does not necessarily represent the concrete existence of localities. Moreover, the assumption that a higher resolution level would exhibit a lower magnitude of fuzziness does not necessarily hold. The assumption turns out to exclude many appropriate parts of the locality entity. Furthermore, algorithms have not been tested for the cost that may trade off the effectiveness obtained from fuzzy locality modeling. When it comes to testing hypotheses, it was hard to distinguish between different forces that cause fuzziness in ontology, perception, and implementation level. They are treated as rather mixed. Finally, in the analysis of results, some cases presented may be prone to misinterpretation because of the small number problem.

Acknowledgements

This research was supported in part by the Center for Transportation Injury Research (CenTIR) in Buffalo, NY with funding from the Federal Highway Administration (FHWA) and technical oversight by the National Highway Traffic Safety Administration (NHTSA).

References

Allen JF (1983) Maintaining knowledge about temporal intervals. Communications of the ACM 26(11): 832-843

Bittner T (1999) On ontology and epistemology of rough location. In: Freksa C, Mark D (eds) Spatial information theory: A theoretical basis for GIS (COSIT '99) Lecture Notes in Computer Science (LNCS) Vol 1661, Springer, Hamburg, pp 433-448

Bittner T (2000) A qualitative formalization of built environments. In: Proceedings of DEXA2000, LNCS Vol 1873, Springer-Verlag, Berlin-Heidelberg

Burrough P, Frank AU (1996) (eds) Geographic Objects with Indeterminate Boundaries. GISDATA Series II. Taylor and Francis, London

Duckham M, Mason K, Stell JG, Worboys MF (2001) A formal approach to imperfection in geographic information. Computers, Environment and Urban Systems 25: 89-103

Egenhofer M, Franzosa R (1991) Point-set topological spatial relations. International Journal of Geographical Information Systems 5(2): 161-174

Fisher P (2000) Sorites paradox and vague geographies. Fuzzy Sets and Systems 113: 7–18

Erwig M, Schneider M (1997) Vague regions. In: 5th International Symposium on Advances in Spatial Databases (SSD'97), LNCS Vol 1262, Springer, Berlin, pp 298-320

Gahegan M (1995) Proximity operators for qualitative spatial reasoning, In: Frank, AU, and Kuhn W (eds) Spatial Information Theory: A Theoretical Basis for GIS. LNCS Vol 988, Springer, Berlin, Germany, pp 31-44

Golledge RG, Dougherty V, Bell S (1995) Acquiring spatial knowledge: survey versus route-based knowledge in unfamiliar environments. Annals of the Association of American Geographers 85(1): 34-158

Goodchild M (2000) GIS and transportation. Geoinformatica 4(2): 127-140

Gould P, White R (1986) Mental Maps, Harmondsworth, Penguin

Guarino N, Welty C (2000) Ontological analysis of taxonomic relationships. In: Laender A, Storey V (eds) Proceedings of ER-2000: The International Conference on Conceptual Modeling, Lecture Notes in Computer Science (LNCS) Vol 1920, Springer-Verlag, Berlin, pp. 210-224

Guesgen HW, Albrecht J (2000) Imprecise reasoning in geographic information systems. Fuzzy Sets and Systems 113: 121-131

Hwang S, Thill J-C (2003) Georeferencing historical FARS accident data. A preliminary report. National Center for Geographic Information and Analysis, University at Buffalo, Amherst, NY

Johnson-Laird PN, Girotto V, Legrenzi P (1998) Mental Models: A Gentle Guide for Outsiders http://www.si.umich.edu/ICOS/gentleintro.html

Liu Z-Q, Satur R (1999) Contextual fuzzy cognitive map for decision support in geographic information systems. IEEE Transactions on Fuzzy Systems 7(5): 495-507

NHTSA (1995) FARS 1996 Coding and Validation Manual. National Center for Statistics and Analysis, National Highway Traffic Safety Administration, Department of Transportation, Washington, D.C.

Robinson VB (1988) Some implications of fuzzy set theory applied to geographic databases. Computers, Environment, and Urban Systems 12: 89-97

Robinson VB (2002) A perspective on geographic information systems and fuzzy sets. In: Proceedings of North American Fuzzy Information Processing Society, IEEE, New Orleans, LA, p 1-6

Schneider M (1999) Uncertainty management for spatial data in databases: fuzzy spatial data types. In: Guting RH, Papadias D, Lochovsky F (eds) SSD'99, LNCS Vol 1651, Springer-Verlag, Heidelberg, pp 330-351

Smith B (1995) On drawing lines on a map. In: Frank AU, Kuhn W (eds) Spatial Information Theory: A Theoretical Basis for GIS. LNCS Vol 988, Springer, Berlin, Germany, pp 475-484

Smith B, Mark DM (1998) Ontology and geographic kinds. In: Proceedings of international symposium on spatial data handling (SDH'98), Vancouver, Canada

Stouffer SA (1940) Intervening opportunities: a theory relating mobility and distance. American Sociological Review 5: 845-867

Stefanakis E, Vazirgiannis M, Sellis T (1999) Incorporating fuzzy set methodologies in a DBMS repository for the application domain of GIS. International Journal of Geographical Information Science 13(7): 657-675

Thill J-C, Sui DZ (1993) Mental maps and fuzzy preferences. Professional Geographer 45: 264-276

Wang F, Hall GB (1996) Fuzzy representation of geographic boundaries in GIS. International Journal of Geographical Information Systems 10(5): 573-590

Worboys MF (2001) Nearness relations in environmental space. International Journal of Geographical Information Science 15(7): 633-651

Yao X, Thill J-C (2005) How far is too far? – A statistical approach to context-contingent proximity modeling. Transactions in GIS 9(2), forthcoming

Zadeh LA (1965) Fuzzy sets. Information and Control 8: 338-353

Zhan FB, Lin H (2003) Overlay of two simple polygons with indeterminate boundaries. Transactions in GIS 7(1): 67-81

5. Mining Weather Data Using Fuzzy Cluster Analysis

Zhijian Liu and Roy George

Army Center for Research in Information Science
Department of Computer Science
Clark Atlanta University
Atlanta, GA 30314
email: rkavil@bellsouth.net

Abstract. The need to analyze the vast quantities of weather data collected has led to the development of new data mining tools and techniques. Mining this data can produce new insights into weather, climatological and environmental trends that have both scientific and practical significance. This chapter discusses the challenges posed by weather databases and examines the use of fuzzy clustering for analyzing such data. It proposes the extension of the fuzzy K-Means clustering algorithm to account for the spatio-temporal nature of weather data. It introduces an unsupervised fuzzy clustering algorithm, based on the fuzzy K-Means and defines a cluster validity index which is used to determine an optimal number of clusters. These techniques are validated on weather data in the South Central US, and global climate data (sea level pressure). It is seen that the algorithm is able to identify and preserve interesting phenomena in the weather data.

5.1. Introduction

Satellites, weather stations, and sensors are collecting large volumes of geospatial data on a wide range of parameters. Examples of such geospatial data include earth science data describing spatiotemporal phenomena, multi-dimensional sequences of images in geographic regions, the evolution of natural phenomena, etc. However, despite the importance of such geospatial data sets it is only recently that there have been efforts to develop appropriate data mining and knowledge discovery in databases tools and techniques suitable for data analysis.

The spatio-temporal domain is complex (Gahegan 2001) and characterized by high volumes of data. For example several global landcover/landuse maps have terabytes of information, requiring computationally intense analysis techniques. Interesting signals and data are often masked by stronger signals caused by local effects, such as seasonal variations. The coupling between different regions of the globe also introduces complexities in behavior. These effects make analysis of this data difficult. Non-uniformity in data gathering and sampling sometimes require indirect measurements and interpolation, which lead to the introduction of model artifacts. Formulating geographic knowledge, and applying it to knowledge discovery and data mining is also difficult. This level of complexity is evident in data mining and knowledge discovery in databases techniques for this domain which have encompassed a wide range of models and techniques (Roddick and Spiliopoulou 1999). These techniques vary from the visual oriented approach (Openshaw 1984), the generation of geographic association rules and sequence rules (Koperski and Han 1995), clustering (Steinbach et al. 2003; Smyth et al. 2000), to combinational approaches (Gahegan et al. 2001).

The application of fuzzy logic techniques to data mining and knowledge discovery in the weather domain has several advantages. Imprecision and uncertainty in this domain is present at several levels. Attribute ambiguity occurs when class membership is partial or unclear. Attribute ambiguity is a severe problem in remotely sensed data(Mohan 2000), such as aerial photography, which is often interpreted inconsistently. Spatial vagueness emerges when the sampling resolution is not fine enough to identify boundary locations exactly, where gradual transitions occur between classes, or when there is location uncertainty. Clustering is a technique which helps in the analysis of such large data sets through the definition of regions that have similar properties. However, conventional hard clustering (Steinbach et al. 2003) is inappropriate when faced with the ambiguities in weather data measurement. The data often is incomplete or has errors in measurement, and the spatial and attribute ambiguities that are characteristic to this data introduce further difficulties into the analysis. Fuzzy clustering is more appropriate for this data with its ability to naturally incorporate these real world issues. The ability to produce soft boundaries permits improved interpretation capabilities.

There are several interesting applications for this research effort. At a regional scale, these clustering techniques may identify micro-climatic regions. The knowledge of these regions can be used to improve operations planning and decision support. The presence of such microclimatic regions is subjective even for domain experts. Fuzzy clustering techniques are better suited to providing a basis for these interpretations in comparison to hard partitioning. The use of clustering techniques is also useful for tuning weather prediction models. These models while good predictors of long term weather fail spectacularly in predicting short term weather patterns (the Santa Ana winds, for instance). Terrain and diurnal heating effects cannot be effectively modeled in these models and the ability to recognize such weather anomalies can be used as corrections to the forecasts produced by the weather models. The development of data mining and knowledge discovery in databases techniques can thus lead to the understanding and prediction of such effects.

Clustering techniques have been widely used in data mining and knowledge discovery in databases and are ideal for understanding weather data. Fuzzy clustering is an extension of the classical clustering technique and has been used to solve numerous problems in the areas of pattern recognition and fuzzy model identification (MacQueen 1967). A variety of fuzzy clustering methods have been proposed and several of them are based upon distance criteria. Fuzzy K-Means clustering has been widely used for understanding patterns especially where the clusters might overlap. It has been applied to landuse/landcover classification (Mohan 2000), gene cluster identification (Gash and Eisen 2002) and the classification of water chemistry data (Guler et al. 2002). An overview of clustering in the spatio-temporal domain is given in Section 5.2. The application of fuzzy K-Means clustering to spatio-temporal data as a data mining technique and a new algorithm for Unsupervised fuzzy K-Means (UKFM) clustering are described in Section 5.3. It is shown that the UKFM clustering technique is able to capture interesting features in climatic data both in the regional weather domain, and also in the global climatic domain (indicating good scalability for the algorithm). Section 5.4 examines future directions and concludes the chapter.

5.2. Clustering in the Spatio-Temporal Domain

Clustering is the process where feature vectors are grouped into clusters. Given a set of data points each with a set of feature vectors, clustering groups the data points into clusters such that data points in one cluster are similar to one another while those in separate clusters are dissimilar to one another. The process of clustering is to assign the feature vectors into the clusters based on a similarity measure. The choice of cluster centers (or prototypes) is crucial to the clustering process. The similarity measure should discriminate against feature vectors that are farther away from the cluster center in favor of vectors that are closer. Several different clustering techniques have been introduced by various authors. However, the challenge is in applying them to domain specific problems particularly the identification of similarity measures that are appropriate to the problem. In this effort we report on the application of the K-Means clustering technique and its fuzzy variations to the spatio-temporal domain.

5.3. K-Means Clustering

The K-Means clustering algorithm is used widely to partition data points into coherent clusters (Forgy 1965; MacQueen 1967). The K-Means algorithm assumes the existence of K coherent clusters. The algorithm may be summarized as:

Step 1: Randomly select k points as the initial centroids for clusters.
Step 2: Assign all data points to the cluster with the most similar centroid.
Step 3: Recompute the centroid of each cluster.
Step 4: If the centroids do change, go to Step 2; otherwise stop.

The implementation of this algorithm can vary with different choice of the measure of similarity (or distance).

In the spatio-temporal domain each data point (in space) may be viewed as a vector consisting of time series parameters. The similarity in this domain may be measured by Pearson's correlation coefficient p through using Pearson Distance

|1-p| (Luke n.d.). For variables (vectors) X and Y, Pearson Coefficient of Correlation is defined as

$$p = \frac{\sum (x \cdot y) - n \cdot \overline{X} \cdot \overline{Y}}{\sqrt{(\sum x^2 - n \cdot \overline{X}^2) \cdot (\sum y^2 - n \cdot \overline{Y}^2)}} \tag{5.1}$$

$\overline{X}$, $\overline{Y}$ are the mean of variable X and Y respectively; $\sum (x \cdot y)$ is the sum of the product of all ordered pairs; and n is the number of ordered pairs (data points). $\sum x^2$ is the sum of the squares of all values of the variable X, and $\sum y^2$ is the sum of the squares of all values of the variable Y.

5.3.1. Fuzzy K-Means Clustering

The classical K-Means clustering does a hard partition (0 or 1) on the data. The Fuzzy K-Means (deGruijter and McBratney 1988) is a more expressive clustering technique. It computes the degree of membership of a data point in a cluster ($\mu \in [0,1]$). For the spatio-temporal domain this permits flexibility in interpretation of regions that are at the outer limits of a cluster. The hard partition can be viewed as a fuzzy partition with a truth value of either 0 (false) or 1 (true).

Fuzzy K-Means clustering tries to minimize the within cluster sum of square errors function under the following conditions:

$$\sum_{k=1}^{c} \mu_{ik} = 1, \; i=1,2,\ldots\ldots,n;$$

$$\sum_{i=1}^{n} \mu_{ik} > 0, \; k = 1,2,\ldots\ldots,c \tag{5.2}$$

μ is the membership function and $\mu_{ik}\, \varepsilon\, [0,1]$ $i = 1,2,\ldots\ldots,n;$ c is the cluster number. It is defined by the following objective function:

$$J = \sum_{i=1}^{n} \sum_{k=1}^{c} \mu_{ik}^{\varphi} d^2(x_i c_k) \tag{5.3}$$

where n is the number of data points, c is the number of clusters, c_k is the vector representing the centroid of cluster k, x_i is the vector representing individual data point i and $d^2(x_i,c_k)$ is the squared distance between x_i and c_k according to a chosen definition of distance, which for simplicity is denoted by d^2_{ik}. φ is the fuzzy exponent and ranges from 1 to ∞. It determines the degree of fuzziness of the final solution, i.e. the degree of overlap between groups. With φ =1, the solution is a hard partition. As φ approaches infinity the solution approaches its highest degree of fuzziness.

The minimization of the objective function, J, provides the solution for the membership function.

$$\mu_{ik} = \frac{d_{ik}^{-2/(\varphi-1)}}{\sum_{j=1}^{c} d_{ij}^{-2/(\varphi-1)}}, \; i=1,2,.....,n; \; k=1,2,.....,c \tag{5.4}$$

$$c_k = \frac{\sum_{i=1}^{n} \mu_{ik}^{\varphi} x_i}{\sum_{i=1}^{n} \mu_{ik}^{\varphi}}, \; k = 1,2,....., c \tag{5.5}$$

The Fuzzy K-Means algorithm is initialized by the following:

- a: Choose the number of clusters: c, with $1<c<n$.
- b: Select a value for the fuzziness exponent φ, with $\varphi>1$.
- c: Choose a definition of distance in the variable space.
- d: Select a value for the stopping criterion e (e= 0.001 gives good convergence.)

The steps of the K-Means algorithm are as below:

Step1: Initialize M = $\{ \mu_{ik} \}$ = $M^{(0)}$, with random memberships or with memberships from a hard K-Means partition.

Step2: Start iteration with it =1.

Step3: Calculate C = $C^{(it)}$ using equation [5] and $M^{(it-1)}$

Step4: Calculate M=$M^{(it)}$ using equation [4] and $C^{(it)}$.

If numerical overflow occurs with d_{ik} close or equal to 0, m_{ik} is set to 1.

Step5: Compare $M^{(it)}$ to $M^{(it-1)}$. If $||M^{(it)} - M^{(it-1)}|| < e$, then stop; otherwise it=it+1, go to Step 3.

Since the data being clustered is spatio-temporal, the Pearson Distance $|1-p|$ is used in the initialization step c. The initialization of membership function matrix *M* has impact on the resulting clustering, and so multiple runs of clustering should be carried out when random initialization is used.

The spatio-temporal domain for this research effort is weather data. The Navy Operational Global Atmospheric Prediction System (NOGAPS) (Baker et al. 1988) is a global weather forecasting model. NOGAPS uses conventional observations (surface, rawinsonde, pibal, and aircraft), and various forms of satellite observations. Besides the wind observations derived from the various operational processing centers for the geostationary satellites, the NOGAPS also utilizes high density multispectral wind observations. The data granularity is on 1 degree X 1 degree scale, 4 times daily. This effort used a four year subset of the data of Southern United States.

Weather information may be considered as spatial time series data. The frequency and density of measurement make these data sources large. These data are dominated by seasonal effects, (for example, summers are warm and winters cold) that have to be removed before the data are mined for interesting non-seasonal patterns. Several techniques may be used for the removal of seasonal patterns

(Steinbach et al. 2003) - this effort uses the monthly Z-score transformation. All weather data are first standardized for each month, i.e., monthly means and standard deviations are obtained, and all data parameters are subtracted from their corresponding means and divided by the standard deviation. The resulting data is a time series with the effects of seasonal patterns removed. Figures 5.1 and 5.2 shows the results of the Fuzzy K-Means clustering on the Temperature and Precipitation data respectively (for brevity, later discussions have only the Precipitation results). The results of data mining are displayed using the ARCINFO geographical information system.

From a domain standpoint (Climatic Atlas of the US, 1931 - 1960), these clusters correspond to known weather features. The weather off the coast of Florida is dominated by a permanent air mass called the Bermuda high. This modifies the weather in Florida, distinguishing it from the Gulf Coast. The Appalachian range (high elevation) produces modifications in the weather, except in the south - west part of the range, which is more typically affected by the Gulf effects. The weather in the interior is composed of several clusters, however their significance is not completely understood. These multiple clusters are an artifact of the clustering technique that apriori decided that there were 5 clusters in the data. A better solution would be to have an unsupervised clustering approach where the number of clusters is determined either using a domain independent approach or through the use of cluster determining domain knowledge. Both techniques would require the clusters to be validated, in the former case through validity measures or in the latter case through subject matter expertise. The Unsupervised Fuzzy K-Means Clustering technique was developed using the domain independent approach.

5.3.2. Unsupervised Fuzzy K-Means Clustering (UFKM)

A problem with the K-Means clustering (including fuzzy version) is that the number of clusters, k, has to be specified apriori. This creates a problem- a small value of k, would aggregate many natural clusters, hiding desirable features. On the other hand a larger value of k would result in the creation of several trivial clusters. In either case the clustering does not result in optimal detection of all interesting features, leaving the user to guess the minimal number of clusters that reveal all significant features. Unsupervised clustering can eliminate the need to guess the number of clusters. A large initial set of k clusters are initially assumed by the algorithm. The algorithm eliminates trivial clusters, and merges clusters that are similar at each step. These steps are repeated until the number of clusters is “minimal”. A validity index is computed at each step and is used to decide, post-priori the stage of clustering that is capable of conserving pattern information.

Xie and Beni (Baker et al. 1988) proposed a compactness - separation validity index that is independent of the number of clusters. The validity index is defined as:

$$S = \frac{\sum_{k=1}^{c} \sum_{i=1}^{n} \mu_{ik}^2 d_{ik}^2}{N \min_{j,k}(d^2(c_j, c_k))}, j,k = 1,2,....., c \tag{5.6}$$

d_{ik} is the distance of each point in the cluster to the centroid, $d(c_j, c_k)$ is the difference between the centroids of the j and k clusters. The validity index is a compactness separateness measure and uses the minimum distance between cluster centers to evaluate the separation of the clusters.

The Xie - Beni validity index used is extended to accommodate the spatio-temporal nature of the data through the incorporation of the Pearson Distance. The index is reformulated as:

$$S = \frac{\sum_{k=1}^{c} \sum_{i=1}^{n} \mu_{ik}^2 (1 - p_{ik})}{n \min_{j,k}(1 - p_{jk})}, j,k = 1,2,....., c \tag{5.7}$$

where p_{ik} is pearson coefficient of x_i and c_k, and p_{ik} is pearson coefficient of c_j and c_k.

A lower value of S indicates a better clustering.

The Unsupervised Fuzzy K-Means algorithm is as follows:

a. Choose the initial number of clusters
   - ($k = \sqrt{n}$ is a good guess, where n is number of data points)

b. Develop a clustering using the Fuzzy K-Means

c. Merge those clusters that satisfy the following rules:
   - Pearson Coefficient of Correlation of their virtual centroids, $p \geq 0.5$

d. Calculate validity index.

e. Repeat step b and c until stopping condition is reached.

A virtual centroid is the computed centroid of a cluster, i.e., it does not necessarily correspond to an actual measurement point. For nearly empty clusters, the corresponding member function will be assigned 0.0 and the original value will be added to the member function for one of the other clusters. A set of clustering schemes with clustering number ranging from k_0 to 2, are obtained and an optimal clustering can be chosen based on the validity index. Additional rules featuring domain related information may be incorporated into the algorithm to preserver interesting clusters even though they would be candidates for merging based on the domain independent approach.

The initial number of clusters is chosen to be 11 (Figure 5.4), and the UFKM algorithm reduces this number iteratively. The result with 8 clusters (Figure 5.5) gives the optimal validity index - however, the results when shown to meteorologists were not easily explained. It is possible that the 8 clusters reveal regions that are differentiable only through a sophisticated analysis, a case where the capabilities of the cluster analysis outstrips the ability to interpret them. (At a philosophical level, these “confusing” discoveries are fundamental to data mining. One component is the rediscovery of known phenomena, the discovery of other phenomena that are not known or recognized, but could eventually be significant) Figure 5.6 shows 4 clusters case as determined by the UFKM algorithm. The validity index values for different numbers of clusters are shown in Figure 5.3. Note that the optimal clustering is obtained when the measure is at a minimum. These clusters correspond to the significant climatological regions in the selected region of the US discussed in the previous section.

5.3.3. Application of UFKM to Global Weather Data

There are strong connections between the ocean, atmosphere, and land. This leads to connections, "teleconnections", between climatic phenomena between geographically widely separated points. For example, the El Nino phenomenon, off the coast of South America has been linked to droughts in Australia, and anomalous weather patterns in the US. Ocean Climate Indices (OCIs) have been developed to capture these teleconnections, and as a technique to understand climate. Initially, these OCI's were discovered using techniques such as Principal Components Analysis and Singular Value Decomposition. However these techniques are capable of discovering only the strongest of these OCIs, and are hard to interpret. Recently, there have been some studies on "discovering" OCIs through the use of clustering techniques (Steinbach et al. 2003; Smyth et al. 2000). Ertoz et al (2002) have applied the S-Nearest Neighbor clustering technique to detect these indexes. The SNN technique relies on finding nearest neighbors, and computing the similarity between pairs of points. This similarity is then used as a basis of finding core points, which become the basis of the clusters. Use of the core points, while it leads to rapid clustering could fail to reveal weaker OCIs. The UFKM considers all points for clustering ensuring that even weaker clusters are captured. Further, the computation of the virtual centroid is an essential component of the UFKM clustering, which avoids the arbitrary fixing of centroids to conform to actual points. This section reports the preliminary use of the UFKM clustering to detect OCIs. Computing the correlation between the clusters and the OCIs is a measure of the strength of the clustering algorithm.

The UKFM "discovered" the Southern Ocean Index (SOI), an OCI, in global sea level pressure taken between 1982 and 1993. It is seen that the algorithm can detect clusters of interest, with results similar to that reported by Steinbach (Steinbach et al. 2003). The correlation of the cluster centroids with the Southern Ocean Index (SOI) is high (Figures 5.7, 5.8, 5.9). It should be noted that while the UKFM iteratively reduces the number of cluster, it still can capture important features such as the SOI, even at very coarse cluster level (Figure 5.9). Other further tests have shown that the UFKM algorithm can capture even weaker OCIs such as the North Atlantic Oscillation, with highly coarse clusters. This further establishes that UKFM algorithm is capable of preserving important features.

5.4. Conclusion

The proliferation of weather data sources has necessitated the development of new data mining tools and techniques applicable to this domain. Mining this data can produce new insights into weather and climatological trends that can aid in applications such as operations planning. Data mining techniques may also be used to reveal anomalous climatic regions that may be used to modify large scale weather models to include locally relevant phenomena. This chapter proposed the use of fuzzy clustering as a data mining technique that can overcome the problems found in geographical data sets. Several new contributions are proposed in this chapter. The fuzzy K-Means clustering methodology was extended to the spatio-temporal domain. A new unsupervised fuzzy clustering algorithm (UKFM) was introduced, and a cluster validity index that determines the optimal number of clusters was defined. UKFM was applied to two large weather sets- regional weather in the South-East USA, and global sea level pressure. The UKFM was able to rediscover interesting climatic phenomena in both cases. In the case of the regional weather, known climatic phenomena were found by the technique, while in the case of the global sea level pressure, the UKFM algorithm was able to track the SOI, an Ocean Climatic Index.

There are several extensions to this work. The reduction of clusters in UKFM is done through the use of distance measures between cluster centroids. An alternate method of deriving significant clusters would be to use other geographic data sets for cluster validation. We are currently investigating the extension of UKFM through the use of such multi-data sets and initial results are promising. From an application standpoint, this combined technique may be used in other geographical/environmental problems such as land/cover use, pollution studies, climatic anomalies, etc. The identification of microclimatic regions is an area that needs further study. The centroid represents the characteristic properties within a cluster. These characteristic properties may be used to detect microclimatic regions and anomalies within the area of analysis. The UKFM is computation intensive, as it considers all points for clustering. Improving the efficiency of the clustering algorithm would be another fruitful area of research.

Acknowledgements

The authors would like to thank Prof. Vipin Kumar of the University of Minnesota for sharing his insights into clustering of earth science data and for the global climate data set.

References

Forgy, E., Cluster Analysis of Multivariate Data: Efficiency Versus Interpretability of Classifications. Biometry, 1965. 21(785).

Gahegan, M., Data Mining and Knowledge Discovery in the Geographical Domain: Intersection of Geospatial Information and Information Technology, National Academies, Computer Science and Telecommunications Board White Paper.September 2001.

Gahegan, M., et al., The Integration of Geographic Visualization with Knowledge Discovery in Databases and Geocomputation. Cartography and Geographic Information Systems, Special Issue on Research Challenges in Geovisualization, 2001. 28(1): p. 29-44.

Gasch, A.P. and M.B. Eisen, Exploring the Conditional Coregulation of Yeast Gene Expression through Fuzzy K-Means Clustering. Genome Biology Research, 2002: p. 0059.1-0059.22.

Guler, C., G.D. Thyne, and e. al., Evaluation of Graphical and Multivariate Statistical Methods for Classification of Water Chemistry Data. Hydrogeology Journal, 2002. 10(4): p. 455-474.

Koperski, K. and J. Han. Discovery of Spatial Association Rules in Geographic Information Databases. in 4th International Symposium on Large Spatial Databases (SSD95). 1995. Maine.

MacQueen, J.B. Some Methods for Classification and Analysis of Multivariate Observations. in 5th Berkeley Symp. on Probability and Statistics. 1967. Berkeley.

Mohan, B.K., Integration of Irs-1a L2 Data by Fuzzy Logic Approaches for Landuse Classification. International Journal of Remote Sensing, 2000. 21(8): p. 1709-1713.

Openshaw, S., The Modifiable Areal Unit Problem. CATMOG 38, 1984.

Roddick, J.F. and M. Spiliopoulou, A Bibliography of Temporal, Spatial, and Spatio-Temporal Data Mining Research. SIGKDD Explorations, 1999. 1(1).

Smyth, P., K. Ide, and M. Ghil, Multiple Regimes in Nothern Hemisphere Height Fields Via Mixture Model Clustering. Atmospheric Science, 2000. 56: p. 3704-3723.

Steinbach, M., et al. Discovery of Climate Indices Using Clustering. in Knowledge Discovery in Databases. 2003. Washington D. C.

Luke, B.T. Pearson's Correlation Coefficient in http://members.aol.com/btluke/pearson.html.

deGruijter, J.J. and A.B. McBratney, A Modified Fuzzy K Means for Predictive Classification, in Classification and Related Methods of Data Analysis, H.H. Bock, Editor. 1988, Elsevier Science: Amsterdam. p. 97-104.

Baker, N., et al., The Navy Operational Global Atmospheric Prediction System: A Brief History of Past, Present, and Future Developments . 1998.

Ertoz, L., M. Steinbach, and V. Kumar. A New Shared Nearest Neighbor Clustering Algorithm and Its Applications. in Workshop on Clustering High Dimensional Data and its Applications, SIAM Data Mining. 2002. Arlington, VA.

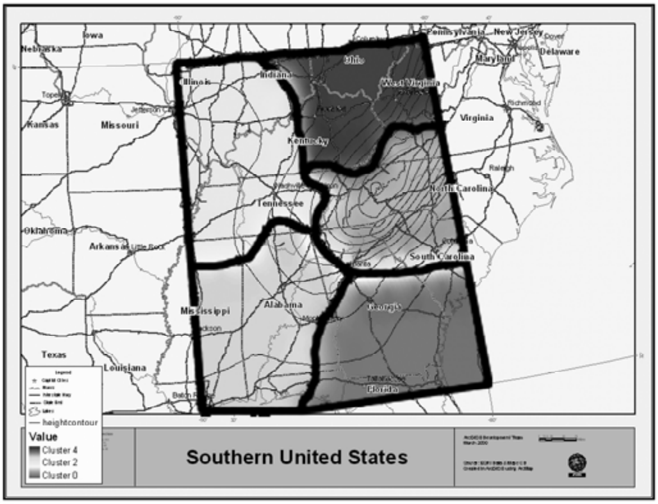


Fig. 5.1. Fuzzy K-Means clustering for temperature, 5 clusters

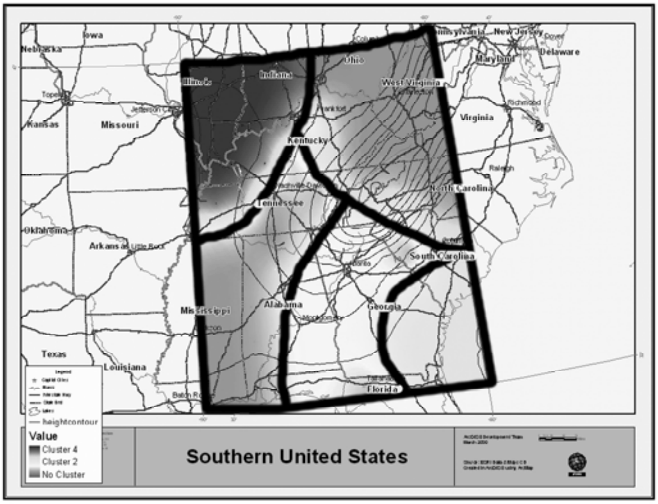


Fig. 5.2. Fuzzy K-Means clustering for precipitation, 5 clusters

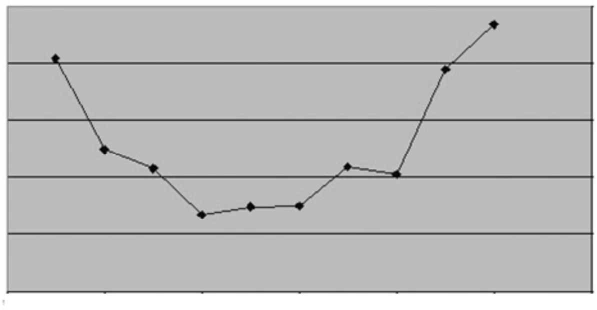


Fig. 5.3. Validity Measure for Different Numbers of Clusters

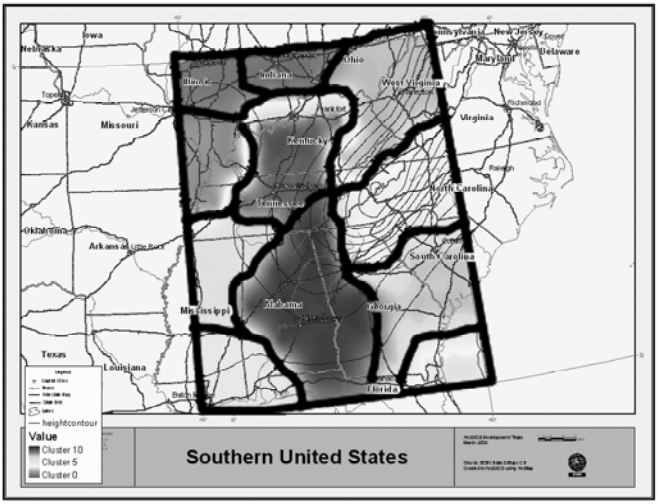


Fig. 5.4. UFKM clustering for precipitation, initial, 11 clusters

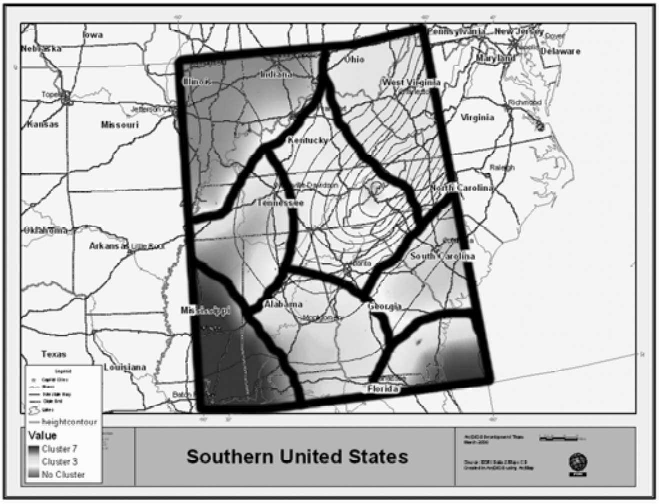


Fig. 5.5. UFKM clustering for precipitation 8 clusters.

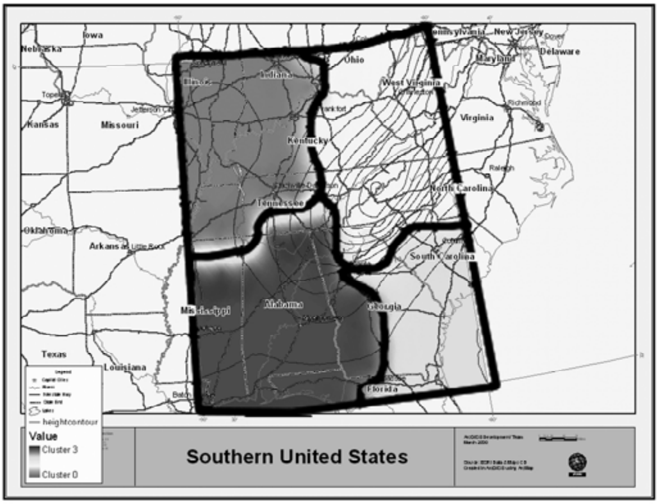


Fig. 5.6. UFKM clustering for precipitation 4 clusters

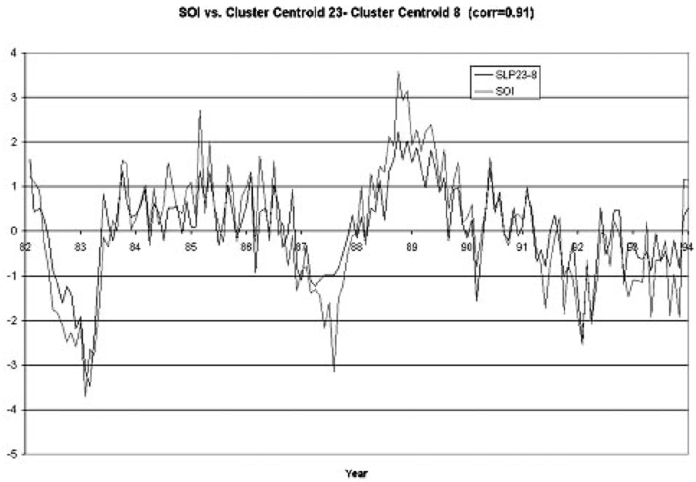


Fig. 5.7. Correlation of Initial Clustering of Sea Level Pressure (1982 - 1993) with SOI (90 clusters)

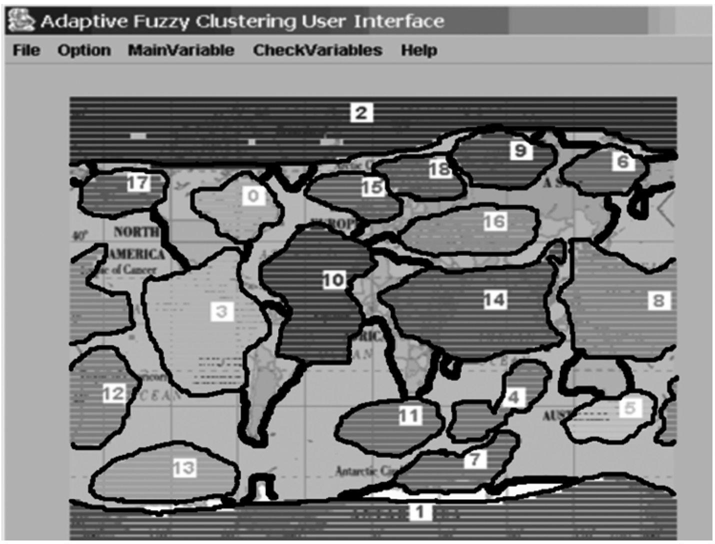


Fig. 5.8. Sea Level Pressure (1982 - 1993) UKFM Final Clustering (19 Clusters)

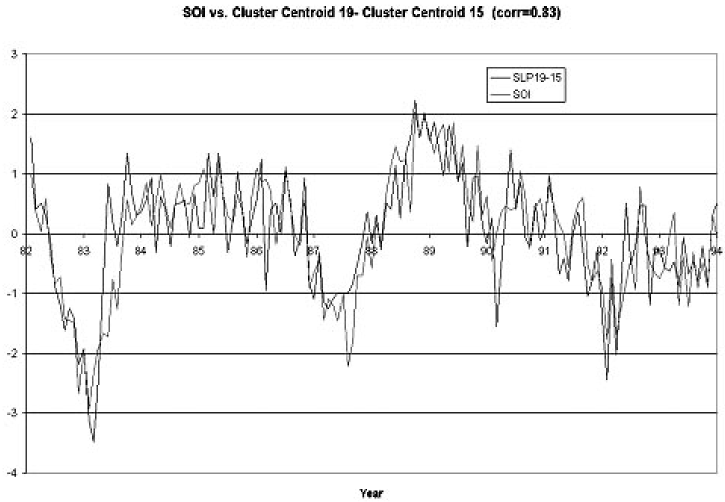


Fig. 5.9. Correlation of Final Clustering (19 clusters) with SOI

6. Modelling the Fuzzy Spatial Extent of Geographical Entities

Cidália Costa Fonte[1] and Weldon A. Lodwick[2]

[1] Secção de Engenharia Geográfica, Departamento de Matemática da Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Apartado 3008, 3001-454 Coimbra, Portugal

[2] Mathematics Department, University of Colorado at Denver, Campus Box 170 – P.O.Box 173364, Denver, Colorado USA 80217-3364

Abstract. In several situations the spatial extent of geographical entities is uncertain or fuzzy. In such cases the entities may be represented using fuzzy sets through the construction of the herein called "fuzzy geographical entities". Four sources of fuzziness are identified in the process of constructing geographical entities characterized by predefined attributes through the classification of a tessellation. For each, a method to compute the membership grades to the fuzzy geographical entity is proposed, based on the appropriate semantic interpretation of the grades of membership. The interpretations used are the likelihood view of membership grades, the random set view and the similarity view. A practical example is presented for each case.

6.1. Introduction

Geographical entities are characterized by an attribute and by a spatial extent, which is the region of the geographical space where that attribute exists. In this chapter we address the construction of geographical entities characterized by predefined attributes. These attributes are defined in terms of other attributes or characteristics observed in the geographical space, called here *base attributes*, and are therefore referred to as *derived attributes*. Each derived attribute corresponds to a set of values of the base attributes used to define it. For example, the attribute "forest" may be defined as a set of values of the levels of radiance of a multispectral satellite image, or the attribute "hilly regions" as a set of terrain altitudes.

We assume that the geographical space is divided into a tessellation formed by regions r_i, that may be cells in a raster representation or polygons in a vector representation, and that to each region is assigned a value of the base attributes. The construction of geographical entities characterized by derived attributes requires a prior classification of the regions according to the values of the base attributes assigned to them and the sets of values corresponding to the derived attributes. The geographical entities are formed aggregating contiguous regions to which the classification assigned the same derived attribute.

A derived attribute is defined in terms of one or several base attributes. For example, "hilly regions" may be defined only as a function of the terrain altitude, but the attribute "regions with high risk of erosion" may be defined as a function of slope and vegetation coverage. When more than one base attribute is used to define a derived attribute connectives have to be used. The most used connector is the logical operator "and". For clarity reasons, we will, in general, consider that only one base attribute A_b (measurable and taking values on a scale Z) is used to define each derived attribute *"A"*. When this is not the case, the space defined by the several base attributes has a dimension equal to the number of base attributes used.

The classification required to construct the geographical entities is performed in two phases.

Phase 1: Identify which z values of the base attribute A_b correspond to the derived attribute "*A*". In many situations these values correspond to an interval $Z_A = \left[\underline{z}, \overline{z}\right]$, and therefore $\underline{z}$ and $\overline{z}$ have to be identified for each derived attribute.

Phase 2: Identify if the value of base attribute A_b in each region r_i, denoted by $z\left(r_i\right)$, belongs or not to the set of values defining "*A*". That is, it has to be verified whether $z\left(r_i\right) \in \left[\underline{z}, \overline{z}\right]$ or not. In the affirmative case region r_i will be classified as "*A*" and in the negative case as "*not A*" (see Figure 6.1).

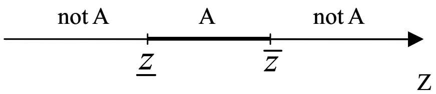


Fig. 6.1. *Z* values defining attribute *"A"*

Traditionally this classification is boolean. That is, each region r_i of the geographical space is classified as belonging or not to each of the derived attributes. It is assumed that the object representation is appropriate to represent the geographical entities corresponding to the derived attribute, and the influence of any uncertainties or errors in the classification on the spatial extent of the geographical entities is ignored. Note that an incorrect classification of the regions r_i produces changes in the position of the obtained geographical entities. That is, the errors and uncertainty of the classification will be reflected in the spatial location of the geographical entities, and therefore transposed to the geographical space.

In this chapter, we model the influence of these uncertainties, errors, or the unsuitability of the object representation on the spatial location of the geographical entities. Fuzzy sets are used to represent the spatial extent of geographical entities, generating the herein called fuzzy geographical entities.

Definition: A *fuzzy geographical entity (FGE) E* is a geographical entity whose position in the geographical space is defined by the fuzzy set $E = \left\{\text{Regions belonging to geographical entity E}\right\}$, with membership function

$\mu_E(r_i) \in [0,1]$ defined for every region r_i in the space of interest. The membership value one represents full membership. The membership value zero represents no membership, and the values in between correspond to membership grades to E, decreasing from one to zero.

The representation using FGEs is based on the construction of the membership function $\mu_E(r_i)$, so its construction is of prime importance.

Several authors, such as Lowell (1994), Burrough (1996), Zhang (1996) and Cheng & Molennar (1999), have already suggested this type of representation. Even though the approach with fuzzy sets seams to be convenient for many applications, often no semantic justification is given to the construction of the membership functions, and their construction is, to a large extent, left to the imagination of the user.

We will identify four different sources of fuzziness in the described classification procedure. For each, a method to compute the membership grades to the fuzzy geographical entities is presented, based on the appropriate semantic interpretation of the grades of membership. The interpretations used are the likelihood view of membership grades, the random set view and the similarity view (Fonte 2003, Bilgiç and Turksen 2000, Dubois *et al.* 2000).

6.2. Construction of membership functions

6.2.1. Sources of fuzziness in the classification procedure

Even though the classification procedure described in section 1 is simple, some difficulties may arise. In phase 1 two sources of fuzziness may be identified:

1A. There may be several acceptable versions of the values $\underline{z}$ and $\overline{z}$ corresponding to attribute *"A"*.

1B. The derived attribute may not be easily characterized by values $\underline{z}$ and $\overline{z}$, because the transition between "*A*" and "*not A*" is not abrupt but gradual.

In phase 2 two possible sources of fuzziness may also be identified:

2A. There are errors associated to the value $z(r_i)$, which may contribute to an erroneous classification;

2B. There are several measurements of $z(r_i)$, some belonging to the interval $[\underline{z}, \overline{z}]$ and others not, which leads to uncertainty in the classification.

In this chapter these sources of fuzziness are addressed separately, but they can combine themselves into more complex cases (Cheng *et al.* 1997).

6.2.2. Several acceptable values for $\underline{z}$ and $\overline{z}$

Source of fuzziness 1A corresponds to the case where there are several versions of the values $\underline{z}$ and $\overline{z}$ that correspond to attribute *"A"*. That is, there are several versions of the set $Z_A = \left[\underline{z},\overline{z}\right]$. Therefore, as illustrated in Figure 6.2, for some *z* values there will be ambiguity regarding the classification as "*A*" or "*not A*".

Examples of this source of fuzziness are, for example, when experts determine interval Z_A, and several experts have different opinions about its amplitude, or when the amplitude of Z_A is estimated using several procedures and different procedures determine different intervals, such as in the supervised classification of multispectral remote sensing images.

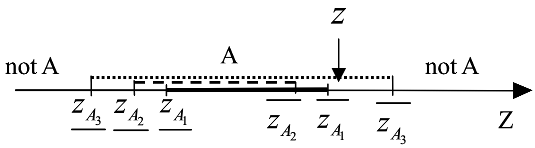


Fig. 6.2. Three versions $Z_{A_j} = \left[\underline{z}_{A_j},\overline{z}_{A_j}\right], j = 1,,3,$ of the set Z_A corresponding to attribute *"A"*. Consequently, the classification of *z* as "*A*" or "*not A*" is ambiguous

In this case, a fuzzy set Z_A may be considered, and the random set view of fuzzy sets (see Bilgiç and Turksen 2000 or Dubois and Prade 1989) may be used to compute the grades of membership of the *z* values to Z_A.

A finitely discrete *random set* defined in *X* is a set $\Re = \left\{\left(S_j, m_j\right) \mid j=1,\dots, n\right\}$, where $S_1,\dots,S_n \subseteq X$ is a family $\mathcal{F}$ of distinct non-empty subsets of *X*, and m_j is an application from $\mathcal{F}$ to [0,1] such that $m_j = m\left(S_j\right)$ and $\sum_{j=1}^{n} m_j = 1$.

Any random set $\Re = \left\{\left(S_j, m_j\right) \mid j=1,\dots, n\right\}$ allows the construction of a fuzzy set *S* on elements $x \in X$ (*e.g.* Dubois and Prade 1991) such that for all $x \in X$

$$\mu_S\left(x\right) = \sum_{x \in S_j} m_j \ . \tag{6.1}$$

To apply this interpretation of membership functions within this context, consider the random set $\mathcal{A} = \left\{\left(Z_{A_j}, m_j\right) \mid j=1,\dots, n\right\}$, where the sets Z_{A_j} are the several versions of the set Z_A, and the values m_j are weights assigned to the sets Z_{A_j} such that $\sum_{j=1}^{n} m_j = 1$. Then, the membership function of every value z to the fuzzy set Z_A is given by:

$$\mu_{Z_A}(z) = \sum_{z \in \left[\underline{z_{A_j}}, \overline{z_{A_j}}\right]} m_j \ . \tag{6.2}$$

In this case, the first phase of the classification generates a fuzzy set Z_A of the z values corresponding to the derived attribute *"A"*. The second phase requires that each region r_i be classified according to the values $z_i = z(r_i)$. Since fuzzy set Z_A assigns degrees of belonging of the z_i values to *"A"*, these degrees of belonging can be transposed to the geographical space considering

$$\mu_{E_A}(r_i) = \mu_{Z_A}(z_i) \, . \tag{6.3}$$

Then, for each region r_i of the geographical space, a membership grade to the fuzzy geographical entities E_A, characterized by attribute *"A"*, is obtained.

Example

This example addresses the supervised classification of multispectral remote sensing images. Frequently the classification of the same image with different methods using data from the same training stage produces different results. This happens because in some cases different classification methods assign different classes to the same levels of radiance. That is illustrated in Figure 6.3 for two dimensions (two bands), where the results of the training stage for three classes are shown. Note that with the parallelepiped classification method the levels of radiance corresponding to point r are classified as belonging to class 2 and with the minimum distance to mean method to class 1.

Instead of choosing just one classification method, the different results produced by the several methods may be used to generate FGEs. The membership grades, in this case, represent the classification uncertainty. Figure 6.4 shows the results of the classification of a small part of a multispectral remote sensing image into three classes with three different methods, namely the "minimum distance to mean", the "maximum likelihood method" and the "parallelepiped classifier".

The levels of radiance obtained in the training stage for each class correspond to a cluster of points in the base attributes space, which has dimension equal to the number of bands used. Each classification method defines in the base attribute space a region corresponding to each class based on those clusters. As each method defines the regions differently, there are as many versions of the set Z_{A_k} corresponding to classes A_k $(k = 1,...,3)$ as classification methods. Since three classification methods were used, we have three versions of each set Z_{A_k} $(k = 1,...,3)$. Then, for each class A_k we can define a random set $\Re_{A_k} = \left\{\left(Z_{A_{kj}}, m_{kj}\right) \mid j = 1,...,3\right\}$ whose elements are the several versions of Z_{A_k}. If all methods are considered equally appropriate to classify each class, equal weights are assigned to each set $Z_{A_{kj}}$, that is $m_{kj} = 1/3$.

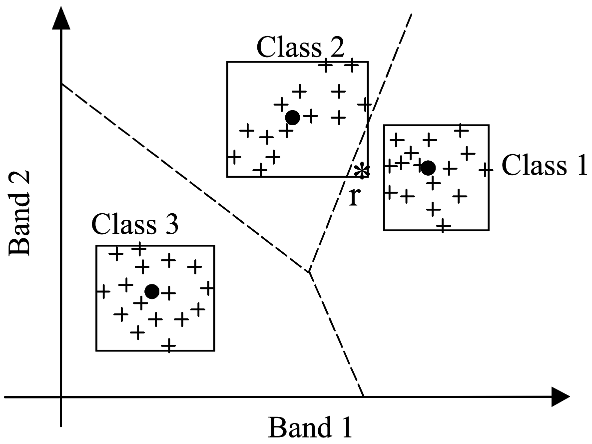


Fig. 6.3. Scheme of the supervised classification of a multispectral remote sensing image with two methods. The symbol (+) represents the values obtained in the training stage for each class, (●) represents the mean of the values obtained in the training stage for each class, (□) represents the regions assigned to each class by the "parallelepiped classification method" and (---) the limits of the regions assigned to each class by the "minimum distance to mean method".

Since to each pixel r_i corresponds a position z_i in the space of the base attributes, corresponding to the levels of radiance registered for that pixel in the several bands, according to Eqs. (6.2) and (6.3), the grade of membership of each pixel to the fuzzy geographical entities characterized by the attributes A_k is given by:

$$\mu_{E_{A_k}}(r_i) = \mu_{\widetilde{Z_{A_k}}}[z_i] = \sum_{z_i \in Z_{A_{kj}}} m_{kj} . \tag{6.4}$$

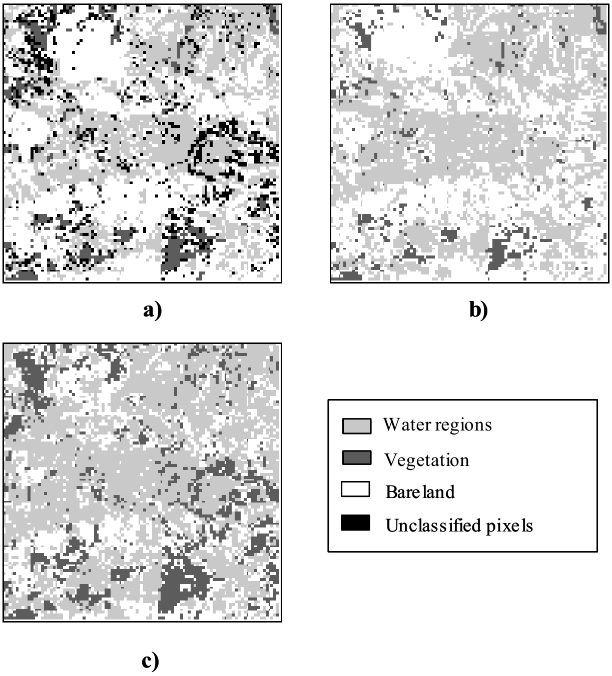


Fig. 6.4. Results of the classification of a multispectral image with three different methods: **a)** parallelepiped classifier, **b)** minimum distance to mean classifier and **c)** maximum likelihood classifier

The resulting FGEs are represented in Figure 6.5. They can now be processed to get more information about the classification uncertainty. For example, regions with greater uncertainty, corresponding to grades of membership closer to 0.5, or the regions classified as having partial membership to all classes may be identified (Fonte 2003).

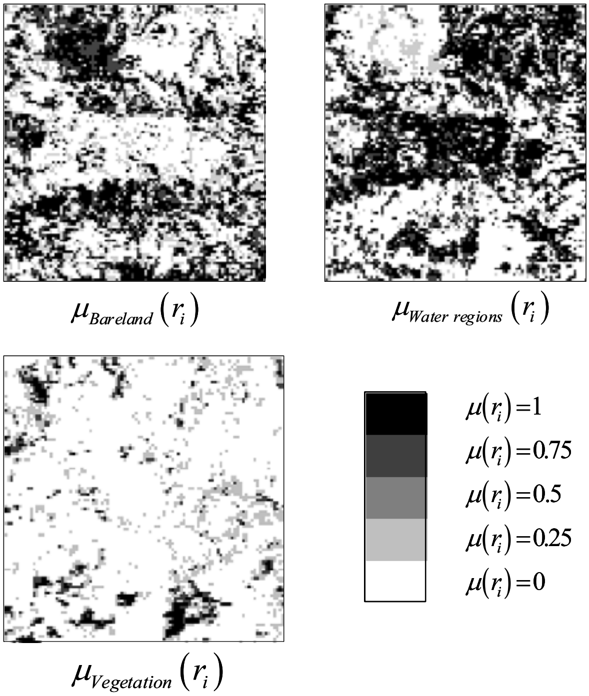


Fig. 6.5. Degrees of membership to the FGEs characterized by attributes “bareland”, “water regions” and “vegetation”

6.2.3. Gradual transition between ”A” and “not A”

For the source of fuzziness 1B), since there isn’t an abrupt transition between “*A*” and “*not A*” but a gradual one, it is difficult to identify the values $\underline{z}$ and $\overline{z}$. The gradual transition between belonging and not belonging to the set of values corresponding to “*A*” may also be modelled using a fuzzy set Z_A .

Even though it is difficult to identify values corresponding to the transition between “*A*” and “*not A*”, in most of these situations it is possible to identify a set of z values that correspond perfectly to attribute *“A”*. These values correspond to the core of the fuzzy set Z_A . The degrees of membership of the z values to Z_A may then be interpreted as a degree of similarity between the z values observed and the ones that ideally define attribute *“A”*, that is, the z values belonging to the core of

Z_A. The interpretation of grades of membership to a fuzzy set based on the similarity view (Bilgiç and Turksen 2000) is then appropriate for this situation.

The similarity view interprets grades of membership as a quantification of the similarity of the observed characteristics to the ideal ones. So, the first step is to determine what are the ideal values $Z_{A_{id}} = \left[\underline{z_{id}}, \overline{z_{id}}\right]$ corresponding to attribute *"A"*, which have to be known, computed or obtained from experts. Then, the degrees of similarity between the other values and the ideal ones are determined as a function of the distance between them. The distance between the ideal values and the other *z* values is the Euclidian distance measured over the Z-axis between the z values and the extreme points of interval $Z_{A_{id}}$ (see Figure 6.6).

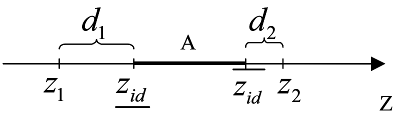


Fig. 6.6. Quantities d_1 and d_2 are respectively the distances of values z_1 and z_2 to the set of ideal values $Z_{A_{id}} = \left[\underline{z_{id}}, \overline{z_{id}}\right]$ corresponding to attribute "*A*"

The distance of the values *z* to the set of the ideal values $Z_{A_{id}} = \left[\underline{z_{id}}, \overline{z_{id}}\right]$ is defined by

$$d\left(z, Z_{A_{id}}\right) = \begin{cases} \min\left[d\left(z, \underline{z_{id}}\right), d\left(z, \overline{z_{id}}\right)\right] & \Leftarrow z \notin \left[\underline{z_{id}}, \overline{z_{id}}\right] \\ 0 & \Leftarrow z \in \left[\underline{z_{id}}, \overline{z_{id}}\right]. \end{cases} \tag{6.5}$$

The degrees of membership of the *z* values to Z_A depend on distance *d*. That is, $\mu_{Z_A}(z) = f\left[d\left(z, Z_{A_{id}}\right)\right]$, where function *f* translates the degree of variation of the membership grades with *d*, and is application dependent.

Since $0 \le d\left(z, Z_{A_{id}}\right) < +\infty$, function *f* must be a decreasing function such that:

$$\begin{cases} f(0) = 1 \\ \lim\limits_{d \to +\infty} f(d) = 0. \end{cases} \tag{6.6}$$

An example of a function satisfying these requirements is $f(d) = e^{-ad}$, where *a* is a parameter that influences the slope of the membership function (see Figure 6.7).

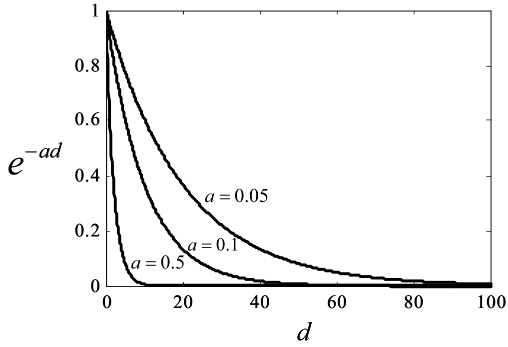


Fig. 6.7. Function $f(d) = e^{-ad}$, with $a = 0.5$, $a = 0.1$ and $a = 0.05$

However, for most GIS applications it is neither necessary nor appropriate to consider that grades of membership to Z_A take positive values for infinitely large distances to the ideal interval, since for distances larger than a certain value, the similarity is so small that it is not appropriate to consider positive degrees of membership to attribute *"A"*. Then, an application dependent value $d_{\max}$ has to be obtained such that $\forall z : d\left(z, Z_{A_{id}}\right) \geq d_{\max} \Rightarrow \mu_{Z_A}(z) = f\left[d\left(z, Z_{A_{id}}\right)\right] = 0$. The existence of a value $d_{\max}$ simplifies the construction of function *f*, as it must be decreasing and such that:

$$\begin{cases} f(0) = 1 \\ d \geq d_{\max} \Rightarrow f(d) = 0. \end{cases} \tag{6.7}$$

So far the degrees of similarity have been considered only as a function of the distance between the *z* values and the ideal set $Z_{A_{id}}$. Although, in some cases, the degrees of similarity between the *z* values and the ideal set $Z_{A_{id}}$ may also depend of whether $z < \underline{z_{id}}$ or $z > \overline{z_{id}}$, since different degrees of variation may be considered for both sides of the ideal interval. In this case, different functions are defined for each side of interval $Z_{A_{id}}$, creating asymmetric membership functions $\mu_{Z_A}(z)$ (see Figure 6.8).

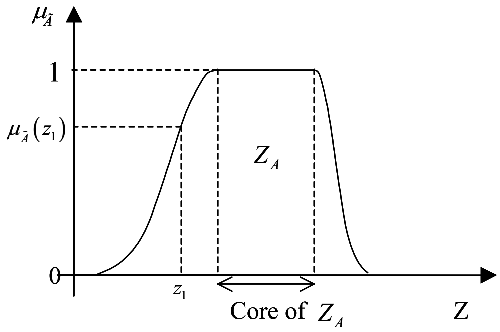


Fig. 6.8. Asymmetric fuzzy set Z_A . The core of Z_A is the set of *z* values that ideally define attribute *"A"*

In this case, as in the previous one, the first phase of the classification generates a fuzzy set Z_A of the z values corresponding to the derived attribute *"A"*. So, the second phase of the classification is done, as in the previous section, considering for each region r_i

$$\mu_{E_A}(r_i) = \mu_{Z_A}(z_i) . \tag{6.8}$$

This method of computing grades of membership to a derived attribute is basically equivalent to the "semantic import approach" proposed by Burrough (1996), but in their approach the notion of distance between the ideal and the observed values is only implicit, and no semantic interpretation of the membership grades is given.

Burrough and McDonnell (1998) show examples of membership functions built with the "semantic import approach".

6.2.4. Errors in the measurement of $z(r_i)$

The source of fuzziness 2A) no longer refers to the identification of the values corresponding to the derived attribute, but to errors in the values $z(r_i)$, which may contribute to an erroneous classification of regions r_i. The aim is to identify how the errors in $z(r_i)$ will influence the classification, and consequently the positional extent of the obtained geographical entity.

Hisdal (1988) presented an interpretation of the grades of membership to a fuzzy set based on likelihoods, where an estimation of the measurement error is used to compute grades of membership. Hisdal considers this method to simulate human language. Three types of experiments were considered:

1. Labeling (from the set "very tall", "tall" and "short" people, John is tall)

2. Yes/No experiment (is John tall?)
3. Grade of membership experiment (what is the degree of belonging of John to the set of tall people?)

Hisdal (1988)considers that the degree of belonging of an element to an attribute λ (ex: tallness) is evaluated analysing one or several of the elements characteristics. For example, to determine if a man is tall or not it's height has to be evaluated. To each attribute corresponds a crisp set of values of those characteristics. That is, the set of values u corresponding to attribute λ is a crisp interval $\Delta u_\lambda = \left[\underline{u_\lambda}, \overline{u_\lambda}\right]$. The fuzziness in this assignment results from errors in the evaluation of the element characteristics. That is, the set of height values determining if a man is tall or not is well defined, and the fuzziness results from errors in the estimation of a man's height. Hisdal considers that, for an element with a characteristic corresponding to the exact value u_e, the estimation of the error associated to the considered value u is given by a probability distribution $P(u|u_e)$, and the grade of membership of the element to the set Δu_λ is given by:

$$\mu_\lambda(u_e) = \int_{\underline{u_\lambda}}^{\overline{u_\lambda}} P(u \mid u_e) du \ . \tag{6.9}$$

That is, the membership grades correspond to the area of the shaded region represented in Figure 6.9.

Even though this method was developed in a very different context, it may be used to construct FGEs.

The set of values $Z_A = \left[\underline{z}, \overline{z}\right]$ corresponding to attribute "A" is a crisp set. Let

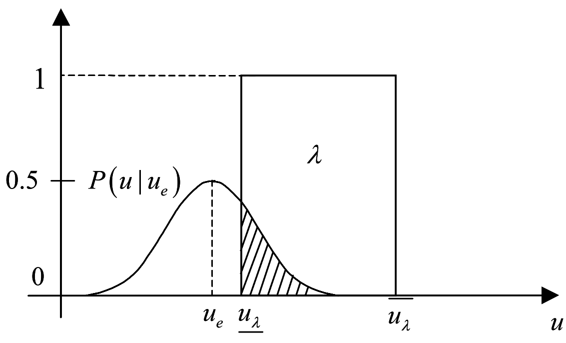


Fig. 6.9. The area of the shaded region is the grade of membership of the element with characteristic u_e to the set λ, considering that the error of the estimation u is $P(u|u_e)$

us now consider that there are errors associated to the values $z(r_i)$, and let us denote by $EF\left[z \mid z_e(r_i)\right]$ a function representing the error in $z(r_i)$. When statistical information is available about the estimated values $z(r_i)$, then $EF\left[z \mid z_e(r_i)\right]$ may be a probability distribution and

$$\mu_{E_A}(r_i) = \int_{\underline{z}}^{\overline{z}} EF\left[z \mid z_e(r_i)\right] dz \,. \tag{6.10}$$

When there is no information available to build a probability distributions, depending on the information available, the error functions may also be represented by intervals, fuzzy intervals[1] or fuzzy numbers[2]. In these cases, for the membership grades to belong to the interval $[0,1]$ a normalization is necessary, and the grades of membership are given by

$$\mu_{E_A}(r_i) = \frac{\int_{\underline{z}}^{\overline{z}} EF\left[z \mid z_e(r_i)\right] dz}{\int_{z_{\min}}^{z_{\max}} EF\left[z \mid z_e(r_i)\right] dz} \tag{6.11}$$

where $z_{\min}$ and $z_{\max}$ are respectively the smaller and larger value that $z(r_i)$ can take according to the error estimation.

Example

One of the variables used to define the "National Ecological Reserve of Portugal" (NER) is the terrain slope. It is determined that regions with a slope larger than 25% should be included in the NER. The identification of a region as belonging to the NER has important implications, since for example no construction is allowed, and the terrains have a much lower commercial value. Therefore, it is useful to estimate the influence of errors in the slope on the regions classified as having a slope larger than 25%. The region of Lousã village was used for this example. A Digital Elevation Model (DEM) with a 10 m resolution was build from the 5 m interval contours at the scale 1:10 000 (see Figure 6.10). A map of the slope was then built, and the regions with a slope large than 25% were identified (see Fig 6.11).

---

[1] A fuzzy interval is a normal fuzzy set defined on the real line, with bounded support and such that all alpha cuts are closed intervals (*e.g.* Dubois *et al.* 2000b)

[2] A fuzzy number is a fuzzy interval whose core has only one element (*e.g.* Klir and Yuan 1995)

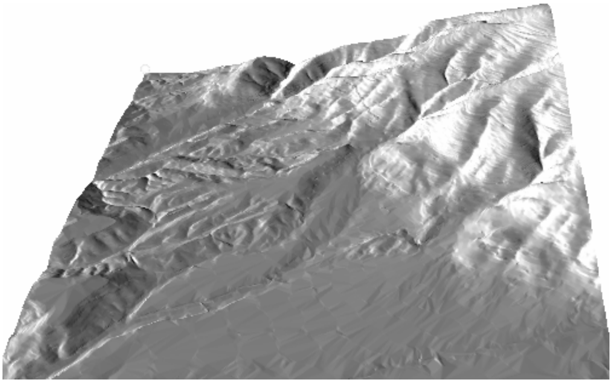


Fig. 6.10. Digital elevation model of the region of Lousã village

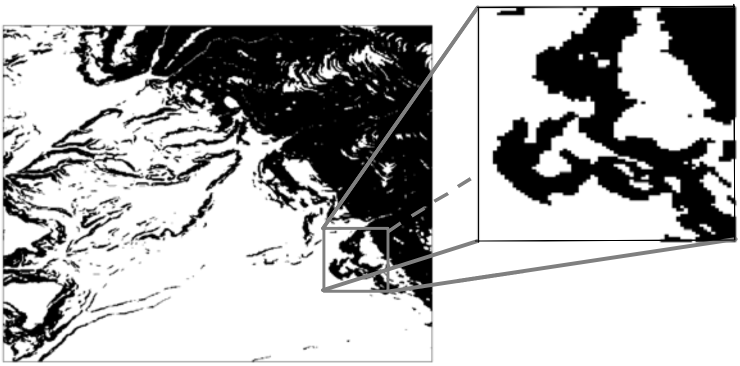


Fig. 6.11. Boolean classification of the regions with a slope larger than 25 %

An estimation of the errors in altitude h is used to evaluate the errors in the slope. The information about the contour accuracy states that, when considering a representative sample of points on the contours, in 90% of the cases the altitude error should be smaller than half of the contour interval. Then, considering that the global error in the altitude follows a gaussian distribution (Longley *et al.* 2001), the error in the altitude at each cell $r_{i,j}$ of the DEM (where index i denotes the line number and j the column number) could be expressed by a gaussian probability distribution with mean equal to $h\left(r_{i,j}\right)$ and standard deviation corresponding to a 90% confidence interval with amplitude 2.5 m. However, in this case, the computation of the error in the slope requires performing calculations with the probability distributions. Then, since these computations are complex and the computations with triangular fuzzy numbers are relatively simple, the probability distribution might be substituted by a possibility distribution expressed in terms of a symmetric triangular fuzzy number, with core equal to $h\left(r_{i,j}\right)$ and support equal to $h\left(r_{i,j}\right) \pm 2.5m$ (see Figure 6.12) (*e.g.* Dubois *et al.* 2000, Klir and Yuan 1995).

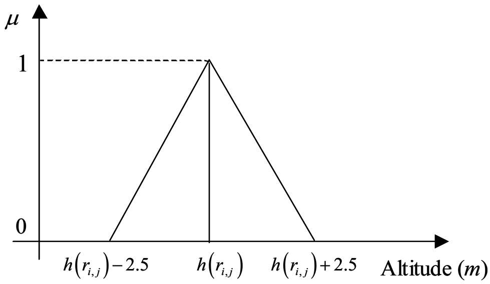


Fig. 6.12. Triangular fuzzy number $\widetilde{h\left(r_{ij}\right)}$ representing the estimation of altitude error in the DEM

MatlabTM and the Intlab (Rump 1998) were used to propagate the error in the altitude to the slope using fuzzy operations (Kaufman and Gupta 1985, Klir and Yuan 1995) . The algorithm used to compute the slope at cell $r_{i,j}$ was:

$$\widetilde{z\left(r_{i,j}\right)} = \left[\left(\frac{\delta h_{i,j}}{\delta x}\right)^2 + \left(\frac{\delta h_{i,j}}{\delta y}\right)^2\right]^{1/2} \tag{6.12}$$

and the derivatives were estimated considering only four neighbours, that is

$$\frac{\delta h_{i,j}}{\delta x} = \frac{\widetilde{h_{i+1,j}} - \widetilde{h_{i-1,j}}}{2d} \tag{6.13}$$

$$\frac{\delta h_{i,j}}{\delta y} = \frac{\widetilde{h_{i,j+1}} - \widetilde{h_{i,j-1}}}{2d} \tag{6.14}$$

where d represents the distance between the cells centre.

This generates, for every grid cell, a fuzzy number that represents the fuzzy slope. The grades of membership of every grid cell to “slope larger than 25%” were then computed using Eq. (6.11) and correspond to the percentage of the area of the function translating the error inside the interval $[25, +\infty)$ (see Figure 6.13). Figure 6.14 shows the obtained result. Notice that a comparison with Figure 6.11 shows clearly that regions classified as having slope larger than 25% with the boolean classification, may actually have smaller slopes and *vice versa*.

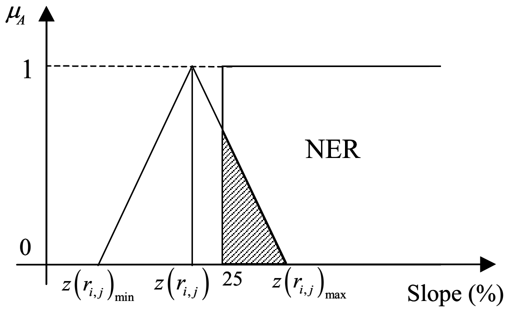


Fig. 6.13. The grades of membership of the slope $\widetilde{z(r_{i,j})}$ to the attribute "slope larger than 25%" corresponds to the shaded area

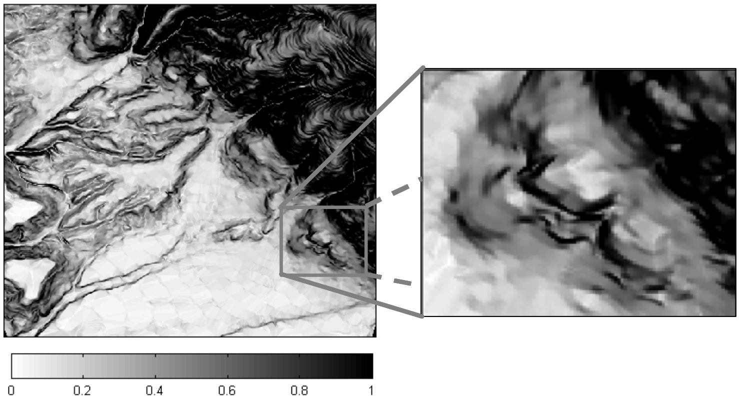


Fig. 6.14. Fuzzy classification of the regions with slope larger than 25 %

6.2.5. Several measurements of $z(r_i)$

The source of fuzziness presented in 2B) concerns the case where there are several measurements of $z(r_i)$, some belonging to interval $[\underline{z}, \overline{z}]$ and others not.

In this case, two distinct situations can be identified.

1. The various values of $z(r_i)$ translate errors. That is, they correspond to the same geographical reality, and are due for example to measurement errors, to the use of different analysis techniques or to different interpolation methods. In this case, either there is only one set of observations and the different values are an outcome of different processing techniques, or there are several measurements, all done in a period of time were no changes in the reality are expected.

In this case, the several values of $z(r_i)$ can be used to estimate the errors in $z(r_i)$ and we are therefore in the case of the previous section.

2. The several values of $z(r_i)$ don't translate errors but a change over time of the base attribute values at region r_i. In this case, there are several sets of observations collected at different times, and the discrepancies between the observed values translate a variation of the real conditions on the geographical space. Therefore, the several values of $z(r_i)$ should not be treated as errors. However, if temporal data are to be used together, FGEs that represent this variation over time may be constructed using the random set view of fuzzy sets. The boolean classification of each set of observations generates a version of the geographical entities corresponding to each attribute of interest. If *n* sets of observations were made, *n* versions of the geographical entities are obtained. Then, the random set $\mathcal{E}_A = \left\{ \left(E_{A_j}, m_j \right) \mid j = 1, \ldots, n \right\}$ may be considered, where each E_{A_j} represents a version of the geographical entity, and the m_j are weights assigned to each set of observations, such that $\sum_{j=1}^{n} m_j = 1$. The membership grades of each region to the FGE corresponding to attribute *"A"* is given by:

$$\mu_{E_A}(r_i) = \sum_{r_i \in A_j} m_j \ . \tag{6.15}$$

Example

The geographical position of some water bodies changes over time. For example, the location of the coast line varies continuously, and some rivers may also be subject to considerable positional variation. The water bodies near the village of Constância were chosen as an example. In this region rivers Tejo and Zêzere meet (see Figure 6.15) and, frequently, there are floods that submerge part of the village. The water level is continuously measured at the hydrometric station of Almourol, located in the vicinity. The means of the daily results from 1982 to 1990 (INAG 1999) were used to represent the change in the river position during that period of time. A total of 2557 observations were used. Figure 6.16 shows the variation of the water level during the considered period of time. A DEM of the region was created from the contours of the 1:25 000 map of the Army Geographical Institute of Portugal (IGeoE), to identity the regions submerged during the several water levels registered. To each water level corresponds a version of the waterbody. Some of those versions are shown in Figure 6.17.

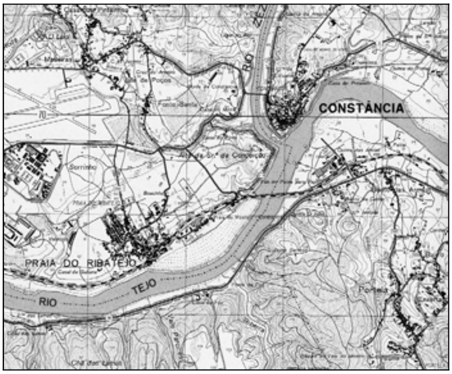


Fig. 6.15. Reduced representation of the 1:25 000 IGeoE map of the region of Constância

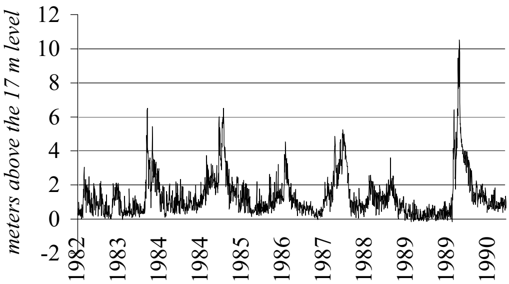


Fig. 6.16. Water level registered in Almourol hydrometric station from 1982 to 1990

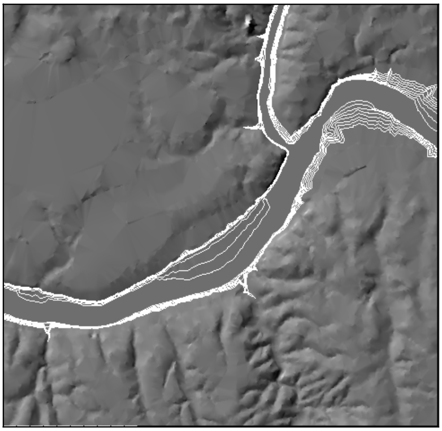


Fig. 6.17. Several versions of the outer limits of the water body

A random set $\mathcal{WB} = \{(WB_j, m_j), j = 1,...,n\}$ formed by all versions of the water body can now be considered. Equal weights were assigned to all versions, that is, it was considered that $m_j = 1/2557$, for $j = 1,\ldots,2557$. According to Eq. (6.15), the grade of membership of each cell r_i to the water body is then given by:

$$\mu_{WB}(r_i) = \sum_{r_i \in WB_j} m_j = \sum_{j=1}^{s} m_j \tag{6.16}$$

where s is the number of times that cell r_i was submerged. This value is computed as a function of the altitude at each cell r_i, considering that, for each water level, all cells with a lower altitude are submerged. The value s assigned to each altitude value is shown in Figure 6.18.

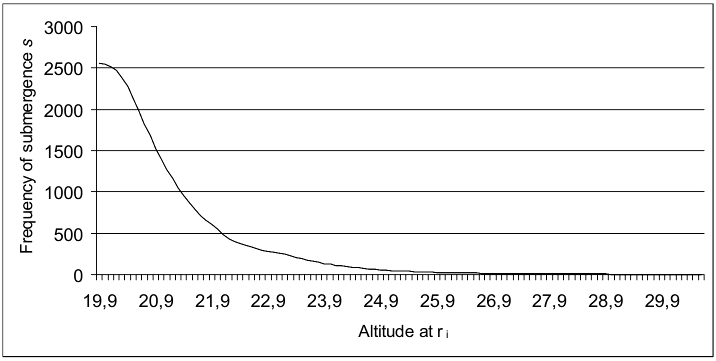


Fig. 6.18. Number of times that points with altitude $z(r_i)$ were submerged during the considered period of time

Figure 6.19 shows the FGE representing the water body. The representation of these water bodies as a FGE enables the spatial variation of the rivers position to be stored, allowing the users to know that the rivers have frequent floods, the amplitude of the most severe flood, and also other information, such as their relative frequency.

This type of representation is useful to represent the position of entities that move regularly in the geographical space, such as dunes, watercourses or other natural phenomena.

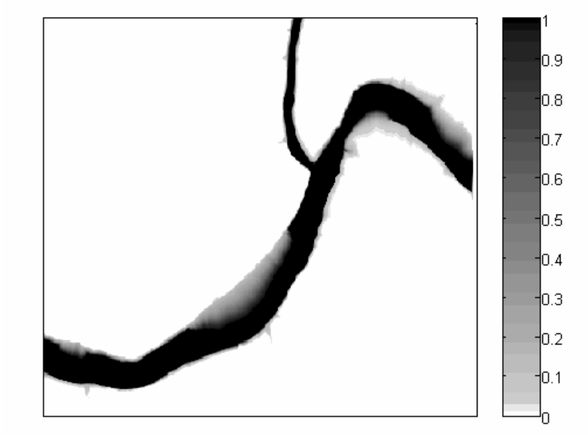


Fig. 6.19. FGE representing the position of rivers Tejo and Zêzere

6.3. Conclusions

This chapter focuses on the construction of geographical entities characterized by predefined attributes, which are derived from the values of other attributes, called base attributes, in such a way that each derived attribute corresponds to a set of values of the base attributes. We have considered the geographical space divided into a tessellation of regions, and the values of the base attributes in each region to be known. In these circumstances, the construction of geographical entities presupposes the previous classification of the regions into the derived attributes characterizing the geographical entities. The geographical entities are then formed aggregating contiguous regions to which the same derived attribute was assigned.

Associated with this simple procedure several types of difficulties have been identified that may influence the spatial extent of the obtained geographical entities. This influence has been modelled using fuzzy sets, and building the herein called fuzzy geographical entities (FGEs). The spatial extent of a FGE is expressed by the grades of membership of the regions forming the tessellation to the geographical entity, and therefore the computation of the grades of membership is a key issue.

One of the characteristics of fuzzy sets is that the grades of membership may have several meanings. While this may be a drawback, this characteristic can also be considered as an advantage, as they are versatile and have a wide range of applications. Although, this versatility also allows the use of many methods to compute grades of membership, depending on the application, the available information, and often on the imagination of the user. So, it is useful to point out methods to evaluate the grades of membership in each context using wide accepted methods and identifying the semantic interpretation associated to them.

Four sources of fuzziness in the classification procedure have been identified. For each, a method to compute the grades of membership to the fuzzy geographical entity has been presented, based on an appropriate semantic interpretation. The interpretations used are the likelihood view of membership grades, the random set view and the similarity view. These interpretations are usually used in other contexts, but it has been shown that they are suitable to be applied in the construction of geographical entities with fuzzy spatial extent. Three practical examples of their application have been presented to show their applicability.

References

Bilgiç T, Türkşen I (2000) Measurement of Membership functions: Theoretical and empirical work. In: Dubois D, Prade H (eds) Fundamentals of Fuzzy Sets. The Handbook of Fuzzy Sets Series, Kluwer Acad. Publ., 195-230

Burrough P, (1996) Natural Objects with Indeterminate Boundaries. In: Burrough P, Frank A (eds) Geographic Objects with Indeterminate Boundaries. Taylor & Francis, London

Burrough P, MacDonnell R (1998) Principles of Geographical Information Systems. Oxford University Press, Oxford

Cheng T, Molenaar M, Bouloucos T (1997) Identification of fuzzy objects from field observation data. In: Hirtle S C, Frank A U (eds) Proceedings of Conference of Spatial Information Theory (COSIT'97), Spatial Information Theory: A Theoretical Basis for GIS, Lecture Notes in Computer Sciences. Berlin: Spring-Verlag, 1329: 241–259

Cheng T, Molenaar M (1999) Objects with Fuzzy Spatial Extent. Photogrammetric Engineering & Remote Sensing, 65: 797-801

Dubois D, Prade H (1989) Fuzzy sets, probability and measurement. European Journal of Operational Research, 40: 135-154

Dubois D, Prade H (1991) Random sets and fuzzy interval analysis. Fuzzy Sets and Systems, 42: 87-101

Dubois D, Nguyen H, Prade H (2000) Possibility theory, probability and fuzzy sets. In: Dubois D, Prade H (eds) Fundamentals of Fuzzy Sets. The Handbook of Fuzzy Sets Series, Kluwer Acad. Publ., 343-438

Dubois, D., Prade, H., (2000b), Fuzzy interval analysis. In Dubois D, Prade H (eds) Fundamentals of Fuzzy Sets. The Handbook of Fuzzy Sets Series, Kluwer Acad. Publ.,483-581.

Fonte C (2003) Entidades Geográficas Difusas – Métodos de Construção e Processamento. Ph.D. Thesis, Coimbra University

Hisdal E (1988) Are grades of membership probabilities?. Fuzzy Sets and Systems 25: 325-348

INAG (1999) Homepage of the Portuguese Water Institute, Accessed 12 of September 1999 http://www.inag.pt.

Kaufmann A, Gupta M (1985) Introduction to Fuzzy Arithmetic: Theory and Applications. Van Nostrand, NY.

Klir G, Yuan B (1995) Fuzzy Sets and Fuzzy Logic-Theory and Applications. Prentice Hall PTR, New Jersey.

Longley P, Goodchild M, Maguire D, Rhind D (2001) Geographical Information Systems and Science. John Wiley & Sons. New York.
Lowell K (1994) An uncertainty-based spatial representation for natural resources phenomena. Advances in GIS research 2: 933-944
Rump S (1998) INTLAB – INTerval LABoratory. Technical University Hamburg, Germany. (URL http://www.ti3.tu.harburg.de/rump/intlab/)
Zhang J (1996) A Surface Approach to the Handling of Uncertainties in an Integrated Spatial Database Environment. Cartographica 33: 23-31

7. Multi-Dimensional Interpolations with Fuzzy Sets

Suzana Dragićević

Spatial Analysis and Modelling Laboratory, Department of Geography, Simon Fraser University, 8888 University Drive, Burnaby, BC, V5A 1S6 Canada
email: suzana_dragicevic@sfu.ca

Abstract. Geographic phenomena are continuous and dynamic but are often represented with data that are static, discrete and crisp. Interpolation is a technique that uses discrete sample data to generate a continuous spatial representation of geographic phenomena. Further, fuzzy set theory represents one of the avenues to overcome the problems of static and crisp data representations. This chapter explores the benefits of integrating fuzzy sets theory and spatio-temporal interpolation techniques within geographical information systems (GIS) to address the multidimensionality of geographic phenomena. The fundamental theory of spatial interpolation using geographic sample data, together with an assessment of the inexactness of such data is presented. Moreover, fuzzy interpolation methods that use the concepts of fuzzy data, fuzzy numbers and fuzzy arithmetic to generate fuzzy surfaces are elaborated. Four case studies that use GIS-based fuzzy set reasoning to build multidimensional spatial or spatio-temporal interpolation methods are discussed.

7.1. Introduction

The general trend of research efforts has focused on the development of robust spatial interpolation techniques and their full integration within geographic information system (GIS) frameworks. Mostly these techniques use measured sample geographic data that deal with two or three spatial dimensions. Geographic data can be measured at a given location (x, y and z coordinate), at different instants of time (t) and with one or several attributes. The measured data is stored in a GIS database as entities of the form points, lines, and areas. These entities are used to model 2 and 2.5-dimensional geographic space as planes, and 3-dimensional geographic space as volumes. The temporal dimension provides 1-dimensional information and is usually considered as an attribute associated to each spatial layer stored in the GIS database.

Geographic phenomena are dynamic and represent the process of change with space and time as inseparable dimensions. In order to study, analyze and predict such phenomena, the spatial and temporal dimensions must be taken into account when they are representation. The GIS stores data as discrete and unique digital layers represented as either vector or raster spatial data models. As such the GIS generally give a static representation of dynamic geographic phenomena. One practical way of making the dynamics of geographic phenomena more explicit is to make use of spatio-temporal interpolation to transform spatial data from discrete to continuous representations (Peuquet, 2002). Only a limited number of papers in the published literature have focused on developing spatio-temporal interpolation methods. Geographic data are also characterized by measurements irregularities arising from data collection or manipulation errors, which raise the issue of uncertainty in the results obtained from existing interpolation techniques. Moreover, the nature of geographic entities have inherent vagueness and hence their crisp representation in the current GIS databases and modeling techniques is not always appropriate.

The objective of this chapter is to present the potential of using fuzzy set theory to deal with imperfect geographic data and entities when applying GIS based spatial and spatio-temporal interpolation. The concept of fuzzy numbers, fuzzy arithmetic, and fuzzy possibility theory provides the flexibility to represent the uncertainty in location of sample points, their attribute values, and temporal component of the change process. The focus is on presenting different aspects of the application of fuzzy set theory when dealing with multidimensional interpolation methods. The discussion begins with the fundamental concepts of spatial interpolation and crisp based data representation, followed by an examination of the nature of data used in the interpolation techniques. The basic fuzzy interpolation methods where fuzzy data is used to generate fuzzy surfaces are then presented. These concepts are developed in four illustrative examples that use fuzzy set reasoning to build spatial or spatio-temporal interpolation methods. The advantages of the approach are also highlighted and discussed.

7.2. Interpolation and GIS

Interpolation can be described broadly as a mathematical process for estimating the unknown value of a function at a given point based on a set of given discrete point values or sub-areas (Lam, 1983). The challenge in interpolation is to find the function that represents the entire curve or surface and that is able to reasonably predict the values for other points or sub-areas. An infinite number of functions may satisfy these conditions and hence additional constraining factors need to be imposed to define the character of different interpolation methods (Mitas and Mitasova, 1999). Further, depending on the dimensions of the dynamic geographic phenomena under study, multidimensional interpolation is further classified as univariate, bivariate, trivariate, and quadvariate.

Consider a given discrete set of data points $x_0, x_1, x_2, \ldots x_{n-1}, x_n$ on a line, then *univariate* interpolation represents a curve $y = f(x)$ (Figure 7.1a). This type of interpolation is useful for dealing with time-series data such as temperature variations at a single location. Extending this to a plane gives the data points

$$(x_0, y_0), (x_1, y_1), \ldots, (x_{n-1}, y_{n-1}), (x_n, y_n)$$

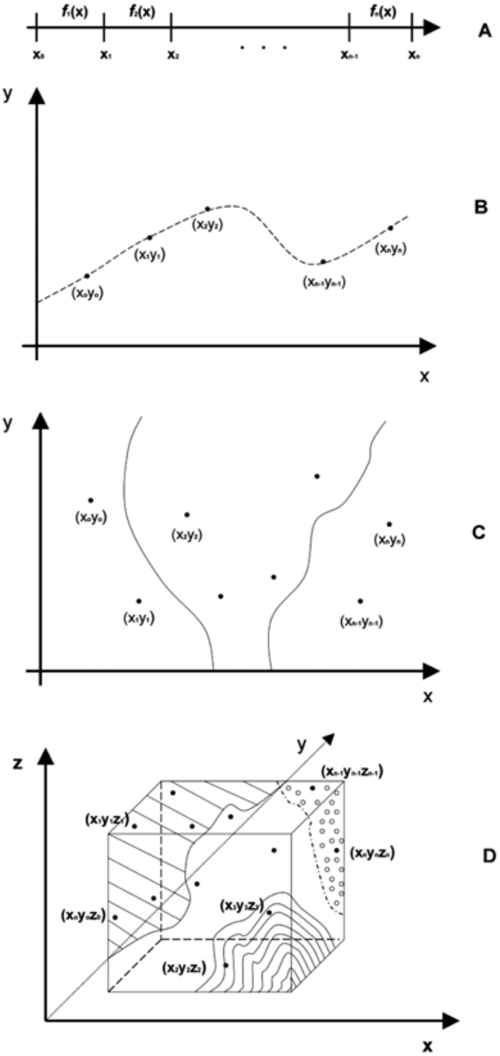


Fig. 7.1. Multi-dimensional interpolations problems:(a)1-D, (b) 2- D, (c) 2.5-D, (d) 3-D

and the interpolation is then *bivariate* creating a 2 or 2.5 dimensional continuous surface (Figure 7.1b and c) where the estimation function is $z = f(x,y)$. This approach is useful when a surface of terrain elevation is constructed from the measured values of elevation sample points at different locations. For example, in the case where a geological model of the earth subsurface has to be created, the geographic data forms 3 dimensions – location (x,y) and depth (z). Here, *trivariate* or

3-D interpolation is required to create the volume $v = f(x,y,z)$ (Figure 7.1d). When the time dimension is combined with the 3-D geographic data, a *quadvariate* or 4-D interpolation model of volume $w = f(x,y,z,t)$ is created. For example, the time changes in the variation of chemical contamination of the ocean in the coastal zone can be represented with a 4-D interpolation model. The temporal dimension can also be combined with the other spatial dimensions (1D, 2D, 2.5D).

The 2-D, 2.5-D and 3-D interpolations are usually referred to in the literature as *spatial interpolation*. The most frequently used spatial interpolation methods are associated with the conditions used to generate the interpolation function (Mitas and Mitasova, 1999). As an example, the local neighbourhood approach is implemented as inverse distance weighting (IDW), tessellation (Thiessan/Voronoi polygons) and Delaunay triangulation (TIN) interpolation techniques. Geostatistical approaches use kriging while the variational approach is used in splines interpolation techniques. For GIS based implementations, a comprehensive review and comparative analysis of a variety of spatial interpolation methods are given by Burrough (1986) and Lam (1983). Moreover, recent approaches have been developed for large databases using spherical methods (Robenson and Willmott, 1996), optimization principles (Koike et al., 1998) and artificial neural networks (Merwin et al., 2002; Rigol et al., 2001).

When the time and spatial dimensions interact, which is typical of dynamic geospatial phenomena, *spatio-temporal interpolation* becomes essential. *Temporal interpolation* can be seen as an extension of existing interpolation techniques to include the time domain where estimation of new data values or set of values between two adjacent points in time are considered (Zhang and Hunter, 2000). However, the practical challenges of spatio-temporal interpolations are not often well elaborated in the literature. This can be explained by the fact that the paradigm of time implementation in current GIS is not yet fully resolved (Peuquet, 2002). In this regard, interpolation methods using splines (Mitasova et al., 1995) or kriging (Miller, 1997) are used in 4-dimensional GIS. A fuzzy set approach was used for GIS based temporal and spatio-temporal interpolation of land-use change (Dragicevic and Marceau, 1999; Dragicevic and Marceau, 2000). Genetic algorithms have been used to simulate the spatio-temporal behavior of geographic patterns and entities (Shibasaki and Huang, 1996). Li and Revesz (2002) use finite element method for interpolation to examine spatio-temporal real estate changes.

Integrating spatial interpolation procedures within GIS software frameworks is important as it provides easy access to the procedures and establishes baselines for comparisons of interpolation model outputs. Varekamp et al. (1996) argues that GIS users need to know more about the theoretical background of various interpolation techniques to be able to apply and choose the most appropriate for their research question. The open source software movement, including web-based spatial interpolation applications, has also provided increased opportunities for access to interpolation methods. However the more computationally intensive interpolation methods are usually performed within statistical software environments that in many cases do not have the capability to represent the spatial arrangement of the output results. The integration of spatio-temporal interpolation methods in GIS software frameworks is an ongoing area of research that holds solutions for spatial statistics and analytical data analysis in general.

7.3. Characteristics of Geographical Data and Entities

The geographic data used in interpolation procedures contain different types of errors that introduces uncertainty in the obtained results thereby having significant consequences on the validity of the outputs (Phillips and Marks, 1996). Two major problems of standard interpolation techniques and related to the geographic data and entities are identified as: originating from the nature of the data collected, and related to data representation in GIS.

Diverse geographical phenomena have different spatial distribution characteristics that are modeled using interpolation. These phenomena are composed of entities that are continuous and boundless. Their representation in a GIS database is a crisp discretization in space and time with attribute values only provided for sample points or areas that are delimited with boundaries (Laurini and Pariente, 1996). Further geographic entities are characterized by their inherent vagueness or fuzziness (Burrough and Frank, 1996). For example, the boundaries of shorelines, valleys or mountains, soil or forest type classes, humidity or nutrition soil quantities are all spatially and temporarily delimited in a gradual and often inhomogeneous manner. Data used to represent geographic entities in GIS are measured with different quality, at an insufficient amount, at inappropriate sampling points, at irregularly scattered measurement sites; and sometimes they are derived from other data sources at various measurement scales. This creates errors associated with these data, and these are combined with other sources of errors related to GIS such as data representation, conversions, management, and analysis functions. Management of uncertainty is essential for interpolation techniques and often relies on probabilistic methods to errors approximation and value maps of the error (Goodchild and Jeansoulin, 1998; Heuvelink, 1998). However if there is luck of appropriate sampling data, and if there is an inherent inexactness or imprecision of the location or measured value of the attribute, the formal or empirical computation of probabilities cannot be used. Hence fuzzy set theory provides a useful solution to this situation (Bardossy et al., 1989; Fisher, 1999; Schneider, 1999; Zadeh and Kacprzyk, 1993).

The representation of geographic data and entities in the GIS databases use Boolean sets. The Boolean model of space assumes that boundaries are crisp and at a particular location a geographic entities belongs fully to one and only one set (Figure 7.2a). In the case of crisp data representation, the probability epsilon band is used to characterize the uncertainty of representing the crisp spatial boundaries (Mark and Csillag, 1989). Many researchers have argued that the traditional crisp approach is not the most appropriate for representing geographic data and entities in GIS databases (Burrough and Frank, 1996; Leung, 1987). Fuzzy set theory provides useful tools to deal with the representation of geographic data, entities and the boundaries in a GIS framework (Wang and Hall, 1996).

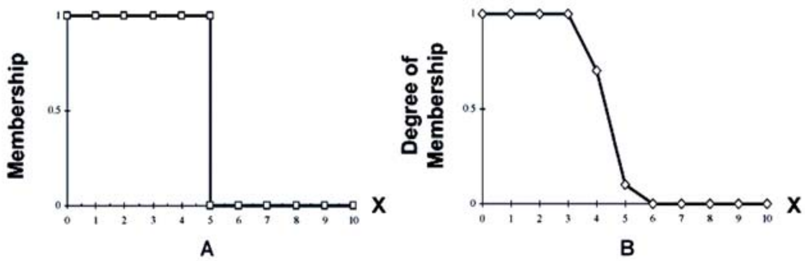


Fig. 7.2. Boolean versus Fuzzy sets

Fuzzy set theory has been developed by (Zadeh, 1965) and deals with classification of elements or phenomena that have continuous values. The classes of ele ments or phenomena are represented as gradual transitions. The fuzzy set provides a way to express the degree of membership to a particular class or set (Figure 7.2b). Fuzzy sets are seen as a generalization of classical set theory and, because of its ability to represent degrees of membership, is a more appropriate approach for representing geographical data in which geographic entities can belong to multiple classes. The degree of belonging to a particular set or class of data or entities is determined through the membership function. Robinson (2003) provides a comprehensive review on the use of fuzzy set theory in GIS.

The five different categories of spatial interpolation methods identified by Ozdamar et al. (1999) are as follows:

- point versus areal
- gradual versus abrupt interpolators
- global versus local
- approximate versus exact
- stochastic versus deterministic

However these classical spatial interpolation methods use the crisp data and do not address circumstances such as luck or inappropriateness of sampling data or their inherent uncertainty and fuzziness. The next section discusses the fundamentals of fuzzy interpolation in dealing with crisp and uncertain data.

7.4. Fuzzy Interpolation

The description of fuzziness and uncertainty of geographic data can be done using formal concepts such as *fuzzy points*, *fuzzy lines* and *fuzzy polygons* (Figure 7.3a-c). Kaufmann and Gupta (1985) define *fuzzy numbers* and extended the concept of fuzzy sets to *fuzzy arithmetic* (Anile et al., 2000). The *fuzzy point* in a coordinate system can therefore be expressed through *fuzzy numbers* in the *x*- and *y-axis* using two membership functions $\mu(x)$ and $\mu(y)$ (Figure 7.3a). The fuzzy points described by fuzzy numbers can represent location and/or attribute uncertainty. The concepts of a *fuzzy line* and a *fuzzy polygon* (Figure 7.3b and c) are used to represent the geographic entities with fuzzy boundaries where fuzzy function can describe the gradual transition between different entities - for example land-use or soil qualities classes (Figure 7.3d). Moreover, Brimicimbe (1998) proposed a *fuzzy coordinate system* to represent the locational uncertainty of geographic data and features within a GIS. The fuzzy coordinate system behaves in a similar way to the conventional coordinate system but allows the flexibility to define the fuzzy boundaries of geographic features. The easting and northing for example are defined by fuzzy numbers and expected degree of membership, so the coordinates are represented by a three dimensional concept (x, y, μ). In order to handle the temporal dimension the coordinate system can be extended to 4-dimensions or an even higher dimensional space.

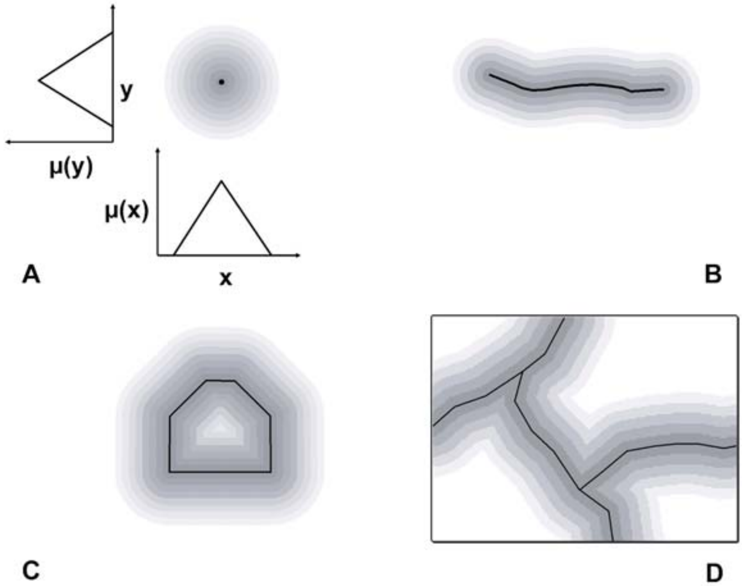


Fig. 7.3. Fuzzy point, line, polygon and map classes

These concepts depend heavily on how the fuzzy membership functions define the degree of belonging to the specific set, class, or transition. Definition of fuzzy membership functions can depend on subjective factors such as expert knowledge or can be obtained from numerical categorizations derived from measurements (Robinson, 1988).

In classical interpolation theory, it is expected that measured data are precisely known together with the estimated error. This provides the basis for generating the function that represents the entire curve or surface. In reality the geographical data are a combination of fuzzy and crisp data types. Table 7.1 shows the different types of geographic data and categories of fuzzy representation that can be encountered in spatial or spatio-temporal interpolations. Type I is typically used for classical interpolation techniques that deals only with crisp data. Other types can have different combination of crisp or fuzzy categories and therefore need to rely on the application of fuzzy based interpolation techniques. When interpolation data are not sets of real numbers but ranges of values whose distribution within the range are qualitative, sample data have to be determined with a theory of possibility (Zadeh, 1978). For example, geological data collected from wells where it is not obvious from the sample description the exact component percentages of clay, sand, or silt. Another example is the mapping of ocean floor depths using sonar measurements where terrain models contain unpredictable variability of the ocean floor landscape (Lodwick and Santos, 2003).

Table 7.1. Different type of geographic data that can be used in the spatio-temporal interpolation

Type of geographic data	**Location**		**Attribute**	**Time**
	x	**y**	**z**	**t**
I	*crisp*	*crisp*	*crisp*	*crisp*
II	*crisp*	*crisp*	*fuzzy*	*crisp*
III	*crisp*	*crisp*	*crisp*	*fuzzy*
IV	*crisp*	*crisp*	*fuzzy*	*fuzzy*
V	*fuzzy*	*fuzzy*	*crisp*	*crisp*
VI	*fuzzy*	*fuzzy*	*crisp*	*fuzzy*
VII	*fuzzy*	*fuzzy*	*fuzzy*	*crisp*
VIII	*fuzzy*	*fuzzy*	*fuzzy*	*fuzzy*

The concept of fuzzy interpolation is derived from gradual rules that in fact fully capture the interpolation process (Dubois and Prade, 1992). The mathematical formulations are given on the basis of linear interpolation that uses fuzzy and precisely known – crisp data (Dubois and Prade, 1994). The concept has roots in the fuzzy Lagrange interpolation theorem (Lowen, 1990).

The interpolation of fuzzy data can also be achieved by using fuzzy splines (Anile et al., 2000; Kaleva, 1994). The generation of consistent two- and three-dimensional surfaces from fuzzy data and fuzzy digital elevation models by using cubic splines is presented in Lodwick and Santos (2003). Anile et al. (2003) construct an inter-visibility maps by using a fuzzy terrain model than can be coupled to a GIS. A linear fuzzy interpolation is used as a forecasting method for product price and sales (Chen and Wang, 1999).

Fuzzy kriging interpolation has been proposed by Diamond (1989) and further extended by use of fuzzy variograms by Bardossy et al. (1990). Piotrowski et al., (1996) applied a fuzzy kriging approach to model glacial thickness at the regional scale. In order to illustrate the concepts and provide some advantages of the use of fuzzy set theory in spatial and/or spatio-temporal multidimensional interpolations four examples are presented in next section.

7.5. Examples of Fuzzy Spatial Interpolations

7.5.1.2-Dimensional Fuzzy Voronoi Spatial Interpolation

The area stealing technique, which is based on the integration of Voronoi diagram and fuzzy set theory, has been applied to surface map representations (Lowell, 1994). Advantages of the approach as applied to forestry includes better estimation of two forest parameters, representation of local variations, and gradual transition zones of different forest types.

The traditional forestry thematic maps are typically represented by a set of polygons representing thematic categories (e.g. forest types) with Boolean boundaries where each polygon is assigned specific attributes (e.g. wood volume). The polygons are derived from inputs such as aerial photographs. The attributes are quantified from measurements at specific locations. A drawback of such a representation lies in the fact that the polygon boundaries cannot be deemed certain due to different interpretations of the aerial photograph data. In addition, the distributions of attribute values over surfaces are not reliable because of an insufficient amount of ground measurements. Since most geographic attributes are not of a continuous nature, spatial interpolation is required to create the continuous surface with respect to the selected attribute and to represent the transition zones between polygons. These problems are resolved using fuzzy Voronoi diagrams.

Given a set of data points at known locations and with defined attributes of interest related to the forest type, the first step consists of constructing Voronoi diagrams around these points (Boots, 1999). The so-called Theissen polygons are created for each point clearly defining the boundaries between areas that correspond to different categories. In the next step, a "query point" in placed in the Voronoi diagram and a new diagram reconstructed as if the query point was one of the original data points. Thus, new polygons are delineated containing the area-stolen from the original polygons. The percentage of the stolen area from each polygon constitutes the fuzzy membership value for a thematic category represented by the corresponding original polygon. If a grid of query points is processed over the entire surface at regular intervals, a series of grid points with fuzzy membership values are produced for each geographic category. Linear interpolation can then be used to produce a continuous surface that can be stored in a raster GIS format. The attributes of interest are evaluated at any location on the defined fuzzy map by multiplying the mean estimated volume of the particular attribute for each geographic category by the corresponding fuzzy membership value over all geographic categories.

7.5.2.2-Dimensional Spatial Interpolation Based on Fuzzy Function Estimator

The dynamic fuzzy-reasoning based estimator (DFFE) model developed by Sun and Davidson (1996) has been used for interpolation of irregularly scattered uncertain data distributed in space (Gedeon et al., 2003). DFFE relies on a rule-based system of inference where concepts from fuzzy set theory are integrated. For a multi-dimensional input vector, that consists of spatial and other variables, a set of neighboring points is extracted from the database which contains complete past observations. The closeness of the given input to the neighboring points is then examined and evaluated with respect to "parallel" and "close" geometric conditions. Fuzzy membership functions are used to determine the membership value of the "parallel" and "close" fuzzy sets with respect to each neighboring point. The closest neighboring point is used to infer the spatial output value (e.g. rainfall at the particular location). It is important to note that fuzzy membership functions, which are used in the DFFE model, depend on functional parameters that are usually determined by trial-and-error or cross-validation. However, an approach based on genetic algorithm theory was derived to optimize both "parallel" and "close" fuzzy membership functions in order to determine more realistic values for the functional parameters (Huang et al., 1998).

7.5.3.3-Dimensional Spatial Interpolation Based on Fuzzy Neural Networks

Soil geology interpolation is an important activity in determining the geologic formation in regions identified as construction sites. The conventional methods used to infer subsurface soil geology are limited to digging pits, boreholes and trenches. But geological information observed in this way is considered as only moderately reliable due to uncertainties in the data collection process. This has led to the development of various methods for subsurface soil geology interpolation with the goal to improve geological assessments thereby contributing to cost reduction and better operational planning (Kumar et al., 2000). The interpolation method is based on the application of artificial neural networks combined with fuzzy set theory. In this application, the method was used to determine the core soil geology structure for a rock-fill dam construction project. The subsurface soil geology was interpolated between two cross-boreholes with known geological data. The main advantage of this combined interpolation method is the capacity to capture and represent the uncertainties of geological data.

The multi-layer preceptor (MLP) neural network model used by the authors was chosen for its ability to create generalized non-linear mapping for a given set of input-output data (Haykin, 1998). A three-layer model was adopted (input, hidden and output layer) for the soil geology interpolation. The inputs consisted of three-dimensional spatial coordinates (x, y, z) and geological variables including skyline elevation, rock-bed elevation and soil thickness. The output was represented by a finite set of geological classes such as riverbed, terrace, volcanic layers etc.

Fuzzy set theory was introduced by defining the neural network activation function as a fuzzy membership function of input variables. The usage of the fuzzy set theory was fully justified as geological data are fuzzy in nature thereby rendering the results from applying classical crisp computational methods unreliable. Thus the outputs of the fuzzy MLP neural network are the occurrence possibilities of each geology class for the given input. The final output is expressed as the most possible geology class determined from the composite maximum technique as a class with the highest degree of membership.

7.5.4.3-Dimensional Fuzzy Spatio-Temporal Interpolation

In typical raster GIS databases spatial data are represented by a series of snapshot layers that correspond to particular instants in time. As a result, the information about the change that occurred in the interval between two consecutive snapshots is not available. When spatial change occurred in the past, it is often difficult to obtain the missing information from data sources such as maps or remote sensing images. A possible solution consists of performing a fuzzy based temporal interpolation between two consecutive and known snapshot layers registered in the raster GIS database (Dragicevic and Marceau, 2000). Using fuzzy probable trajectories of gradual progressing from one class to another, the degrees of membership in a specific class at a particular intermediate space-time location are calculated using fuzzy set membership functions. As an illustration, Figure 7.4 depicts the gradual transition from class type 1 to class type 2 in the period of 10 years. Different membership functions based on expert knowledge are developed for different possible class changes (e.g. forest to agricultural, forest to urban, agricultural to road etc) to depict realistic estimation of time needed for a change to take place.

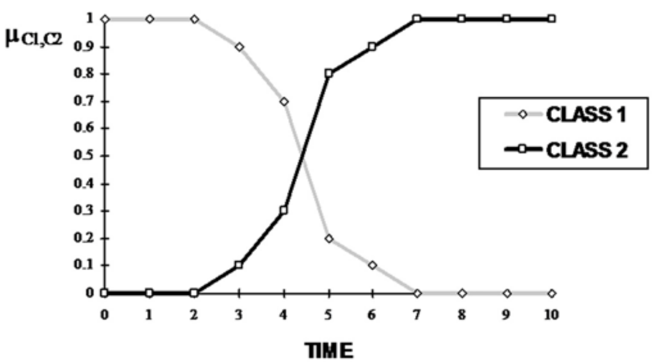


Fig. 7.4. Temporal interpolation based on fuzzy set transitions

In order to simplify the simulation of land-use change, spatial inverse distance weighing interpolation has been used to depict the spatial change process in the fuzzy spatio-temporal interpolation (Dragicevic and Marceau, 1999). The simulation of a gradual change between two consecutive snapshot layers is accomplished in three steps. The first step is to determine the number of intermediate layers between two consecutive snapshot layers captured at distinct time instances *T1* and *T2*. These layers correspond to the shortest possible transition time for a cell to change from one geographic class to another. The second step is to establish the generic layer based on the inverse distance weighting method that contains the information about the change of spatial boundaries between the two explicitly recorded snapshot states. The third step consists of implementing fuzzy logic to generate the missing information about the change of geographic entities in the intermediate layers through the analysis of the generic and two basic snapshot layers. Fuzzy membership functions are developed for simple transitions such as rural to urban land-use classes.

As an illustration of spatio-temporal fuzzy interpolation, Figure 7.5 depicts the simulation of rural-to-urban transition from the year 1990 to year 2000 at a hypothetical study site. The known stages of the land-use boundaries are stored in the GIS raster database (Figure 7.5 a and b). A generic layer contains 10 urban boundaries corresponding to each year for newly created intermediate layers (Figure 7.5c). The generated map for year 1997 is presented in Figure 5d. The gradual variation is reflected by the value of the membership degree such that the greater the value the higher the degree of belonging to class "rural" or "urban". The values of the change are given by the statement R->U: 0.7U+0.3R which mean that at the seventh year of change the cell value in question has 0.7 degree of belonging to urban land use and 0.3 degree of belonging to rural. In addition it is possible to calculate the amount of changes for transitions of other land-use classes such as agricultural to urban (A->U) or forest to urban (F->U). Three scenarios simulating the different possible outcomes of geographic entity change are proposed: scenario with the shortest duration, longest duration, and variable duration of transitions. Further, measures of dynamics of change were proposed that describe different spatio-temporal speed and mechanisms of change such as diffusive and road-influenced (Dragicevic et al., 2001). The main advantage of this proposed technique is the use of fuzzy set theory to handle temporal uncertainties of discrete GIS databases. However the use of more complex model

7.6. Summary

This study represents an overview of the status of research in the field of multidimensional interpolation methods in GIS that are related to geographic phenomena and that exploit the virtues of fuzzy set theory. Four examples that use fuzzy sets in the process of interpolation are provided: two dimensional spatial interpolation based on fuzzy Voronoi diagrams, fuzzy function estimator, three dimensional spatial interpolation based on fuzzy neural networks, and finally GIS based fuzzy spatio-temporal interpolation.

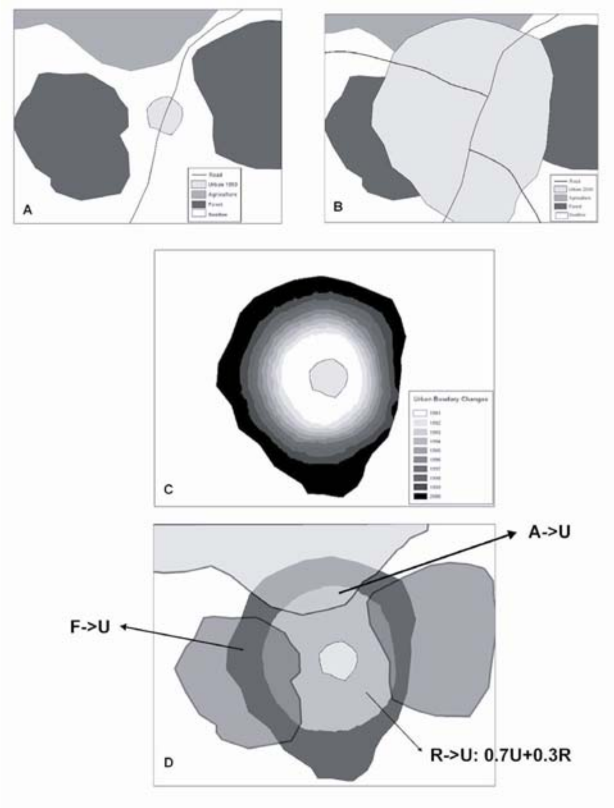


Fig. 7.5. Fuzzy spatio-temporal interpolation for land use changes: (a) land use at year 1990, (b) land use at year 2000, (c) generic layer, and (d) generated map of urban transitions for year 1997

Anile et al. (2000) suggest that fuzzy sets theory function already represents a form of interpolation. Thus, in geographical applications the fuzzy membership function can also be considered as a spatial, temporal or spatio-temproral interpolator. The multidimensional interpolation based on fuzzy set theory is a process of determining the best spatial and temporal behavior of the geographic feature in their transition from one geographic entity or a class to another and by using the uncertain and fuzzy geographic data to determine the transition.

The spatial and spatio-temporal interpolation methods based on fuzzy sets provides the flexibility to develop different models of spatial evolution and temporal dynamics based on diverse fuzzy membership functions. Nevertheless, current GIS do not as yet fully handle the concept of fuzzy numbers, fuzzy operators or fuzzy interpolation. Furthermore, current GIS still have difficulties in dealing with the temporal dimension of geographic data. This opens the potential to exploit these concepts and challenges through the development of robust fuzzy spatio-temporal interpolation procedures fully integrated within a GIS framework.

Acknowledgements

This study is supported by an individual research grant (250518-02) from the Natural Sciences and Engineering Research Council (NSERC) of Canada. The author thanks Vincent Robinson for inviting this contribution. Two anonymous reviewers gave valuable comments on an earlier version of the manuscript. The assistance of Shivanand Balram and Christopher Bone, Spatial Analysis and Modeling Laboratory, Simon Fraser University is appreciated.

References

Anile AM, Falcidieno B, Gallom G, Spagnuolo M Spinello S (2000) Modeling uncertain data with fuzzy B-splines. Fuzzy Sets and Systems, 113: 397-410.

Anile MAP, Furno P, Gallo G, Massolo A (2003) A fuzzy approach to visibility maps creation over digital terrains. Fuzzy Sets and Systems, 135: 63-80.

Bardossy A, Bogardi I, Kelly WE (1989) Geostatistics utilizing imprecise (fuzzy) information. Fuzzy Sets and Systems, 31: 311-328.

Bardossy A, Bogardi I, Kelly WE (1990) Kriging with imprecise (fuzzy) variograms. 1: Theory. Mathematical Geology, 22(1): 63-79.

Boots B (1999) Spatial tessellation. In: P. Longley, M. Goodchild, D. Maguire and D. Rhind (Editors), Geographical Information Systems: Principles and Applications. Wiley and Sons, New York, 503-526.

Brimicimbe A (1998) A fuzzy coordinate system for location uncertainty in space and time. In: S. Carver (Editor), Innovations in GIS 5. Taylor and Francis, UK.

Burrough PA (1986) Principles of GIS for Land Resources Assessment. Clarendon Press, Oxford.

Burrough PA, Frank A (1996) Geographic Objects with Intermediate Boundaries. Innovations in GIS 2. Taylor and Francis, London.

Chen T, Wang MJ (1999) Forecasting methods using fuzzy concepts. Fuzzy Sets and Systems, 105: 339-352.

Diamond P (1989) Fuzzy Kriging. Fuzzy Sets and Systems, 33: 315-332.

Dragicevic, S. and Marceau, D.J., 1999. Spatio-temporal interpolation and fuzzy logic for GIS simulation of rural-to-urban transition. Cartography and Geographical Information Science, 26(2): 125-137.

Dragicevic, S. and Marceau, D.J., 2000. A fuzzy set approach for modelling time in GIS. International Journal of Geographical Information Science, 14(3): 225-245.

Dragicevic S, Marceau DJ, Marois C (2001) Space, time, and dynamics modeling in historical GIS databases: a fuzzy logic approach. Environment and Planning B: Planning and Design, 28: 545-562.

Dubois D, Prade H (1992) Gradual inference rules in approximate reasoning. Information Sciences, 61: 103-122.

Dubois D, Prade H (1994) On fuzzy interpolation, Proceedings of the Third International Conference on Fuzzy Logic, Neural Networks and Soft Computing, Japan.

Fisher P (1999) Models of uncertainty in spatial data, Geographic information systems: Principles and applications. Wiley, New York, 191-205.

Gedeon TD, Wong PM, Wong KW, Huang Y (2003) Spatial interpolation using fuzzy reasoning. Transitions in GIS, 7(1): 55-66.

Goodchild M, Jeansoulin R (1998) Data Quality in Geographical Information: From Errors to Uncertainty. Hermes, Paris.

Haykin S (1998) Neural Networks - A Comprehensive Foundation. Prentice-Hall, Upper Sandle River.

Heuvelink GBM (1998) Error Propagation in Environmental Modelling. Taylor and Francis, New York.

Huang Y, Gedeon T, Wong P (1998) Spatial interpolation using fuzzy reasoning and genetic algorithms. Journal of Geographic Information and Decision Analysis, 2: 223-33.

Kaleva O (1994) Interpolation of fuzzy data. Fuzzy Sets and Systems, 61: 63-70.

Kaufmann, A. and Gupta, M.M., 1985. Introduction to Fuzzy Arithmetic: Theory and Applications. VonNostrand Reinhold, New York.

Koike K, Shiraishi Y, Verdeja E, Fujimura K (1998) Three-dimensional interpolation and lithofacies analysis of granular composition data for earthquake-engineering of shallow soil. Mathematical Geology, 30(6): 733-759.

Kumar JK, Konno M, Yasuda N (2000) Subsurface soil-geology interpolation using fuzzy neural network, Journal of Geotechnical Engineering, pp. 632-639.

Lam, N., 1983. Spatial interpolation methods: a review. The American Cartographer, 10(2): 129-149.

Laurini R, Pariente D (1996) Towards a field-oriented language: first specification. In: P.A. Burrough and A. Frank (Editors), Geographic objects with indeterminate boundaries. GisData 2. Taylor and Francis, London.

Leung Y (1987) On the imprecision of boundaries. Geographical Analysis, 19: 125-151.

Li, L. and Revesz, P., 2002. A comparison of spatio-temporal interpolation methods. In: M. Egenhofer and D.M. Marks (Editors), GIS Science 2002. Springer-Verlag, Berlin, pp. 145-160.

Lodwick WA, Santos J (2003) Constructing consistent fuzzy surfaces from fuzzy data. Fuzzy Sets and Systems, 135: 259-277.

Lowell K (1994) A fuzzy surface cartographic representation for forestry based Voronoi diagram area stealing. Canadian Journal of Forest Research, 24(9): 1970-1980.

Lowen R (1990) A fuzzy Lagrange interpolation theory. Fuzzy Sets and Systems, 34: 33-38.

Mark DM, Csillag F (1989) The nature of boundaries on 'area-class' maps. Cartographica, 26(1): 65-78.

Merwin DA, Cromley RG, Civco DL (2002) Artificial neural networks as a method of spatial interpolation for digital elevation models. Cartography and Geographic Information Science, 29(2): 99-110.

Miller EJ (1997) Towards a 4D GIS: Four-dimensional interpolation utilizing kriging. In: Z. Kemp (Editor), Innovations in GIS 4. Taylor and Francis, London, pp. 181-197.

Mitas L, Mitasova H (1999) Spatial interpolation. In: P. Longley, M.F. Goodchild, D.J. Maguire and D.W. Rind (Editors), Geographical Information Systems: Principles and Applications. Wiley and Sons, New York, 481-792.

Mitasova H, Mitas L, Brown BM, Gerdes DP, Kosinivsky I (1995) Modeling spatially and temporally distributed phenomena: New methods and tools for GRASS GIS. International Journal of Geographical Information Systems, 9(4): 433-436.

Ozdamar L, Demirham M, Ozpinar A (1999) A comparison of spatial interpolation methods and a fuzzy areal evaluation scheme in environmental site characterization. Computer, Environment and Urban Systems, 23(5): 399-422.

Peuquet DJ (2002) Representations of Space and Time. The Guilford Press, New York.

Phillips, D. and Marks, D., 1996. Spatial uncertainty analysis: propagation of interpolation errors in spatially distributed models. Ecological Modelling, 91(1): 21229.

Piotrowski JA, Bartels F, Skalski G, Schmidt G (1996) Geographical regionalization of glacial aquitard thickness in northwestern Germany, based on fuzzy kriging. Mathematical Geology, 28: 437-452.

Rigol J, Jarvis C, Stuart N (2001) Artificial neural networks as a tool for spatial interpolation. International Journal of Geographical Information Science, 15:323-343.

Robenson S, Willmott C (1996) Spherical spatial interpolation and terrestrial air temperature variability. In: M.F. Goodchild, L. Steyaert and B. Parks (Editors), GIS and Environmental Modeling: Progress and Research Issues. GIS World, Fort Collins, 111-115.

Robinson VB (1988) Some implications of fuzzy set theory applied to geographical databases. Computers, Environment and Urban Systems, 12: 89-97.

Robinson VB (2003) A perspective on the fundamentals of fuzzy sets and their use in geographical information systems. Transitions in GIS, 7(1): 3-30.

Schneider M (1999) Uncertainty management for spatial data in databases: fuzzy spatial data types. In: R.H. Guting, D. Papadias, and F. Lochovsky (Editor), SSD'99. Springer-Verlag, Germany, 330-351.

Shibasaki R, Huang, S (1996) Spatio-temporal interpolation by integrating observation data and a behavioural model. In: M. Kraak and M. Molenaar (Editors), SDH'96, Advances in GIS Research II. Taylor and Francis, Delft, The Netherlands, 655-666.

Sun W, Davidson VJ (1996) Dynamic fuzzy-reasoning-based function estimator. Fuzzy Sets and Systems, 79: 357-366.

Varekamp C, Skidmore AK, Burrough PAB (1996) Using public domain geostatistical and GIS software for spatial interpolation. Photogrammetric Engineering and Remote Sensing, 62(7): 845-854.

Wang F, Hall GB (1996) Fuzzy representation of geographical boundaries in GIS. International Journal of Geographical Information Systems, 10(5): 573-590.

Zadeh LA (1965) Fuzzy Sets. Information and Control, 8: 338-353.

Zadeh LA (1978) Fuzzy sets as a basis for a theory of possibility. Fuzzy Sets and Systems, 1: 3-28.

Zadeh LA, Kacprzyk J (1993) Fuzzy Logic for the Management of Uncertainty. Wiley, New York.

Zhang W, Hunter G (2000) Temporal interpolation of spatially dynamic objects. GeoInformatica, 4(4): 403-418.

8. Talking Space – A Social & Fuzzy Logical GIS Perspective On Modelling Spatial Dynamics

Susanne Kratochwil[1] and Josef Benedikt[2]

[1] Vienna University of Technology
Institute for Sociology in Spatial Planning and Architecture (ISRA)
Karlsplatz 13, A-1040 Vienna, Austria
Susanne.Kratochwil@tuwien.ac.at
http://isra.tuwien.ac.at

[2] GEOLOGIC Dr. Benedikt
Lerchengasse 34/3, A-1080 Vienna, Austria
benedikt@geologic.at
http://www.geologic.at

Abstract. *Talking Space* is drafted as a GIS-based communication platform to map spatial knowledge, which contains inherent uncertainty. This uncertainty is argued to be due to the semantics of categorization using linguistic symbols as applied in a communication process, which is argued to create and shape space and spatial phenomena. Inherent uncertainty is nothing to be eliminated but is an indispensable part in communicating (spatial or non-spatial) knowledge and therefore needs to be *talked* about. Space is shaped in a deterministic and objective way – yet, in all probability, this overlooks the perceptions, assessments and interests of many space protagonists. The formation and information of actors in space implies relations among different points of view. Perception and assessment of space is understood inadequately. Conventional planning and GIS do not meet requirements on communicating space. GIS is sometimes even referred to as *socially empty space*[1]. This emptiness may be filled with our ability to talk about space, to perceive space and talk about perceptions and ***to visualize what we are talking about***. This paper is proposing perspectives on different notions of spatial phenomena and their impact on creating spatial knowledge while limiting ourselves to the logical and techno-logical requirements of GIS. Alternative views on spatial categories and their contribution to communication in space are introduced. Three settings are used to develop our perspectives with respect to a *Talking Space*. An *Introduction* (A) focuses on challenges of visualizing relations among individual perspectives in space, which is shaped and constructed by social actors. Social as well as cognitive differences among social actors in geographical space are at the core of a *Talking Space Development*. In

[1] See *Casey, Pederson (1995)*

Communicating Spatial Knowledge (B) theoretical foundations are introduced, which are necessary in a *Talking Space* to draft social perspectives on constructing space. It deems necessary to open up the notion of space to Social Science in enabling all actors in space to comprehend spatial phenomena. Theoretical issues on constructing space are discussed. The third setting describes implications of the perspectives introduced in (A) and (B) to a *Talking Space Environment* (C). This kind of environment will be discussed as a framework of symbols, models and codes addressing social construction, logical proceedings and visual engagements, respectively. Examples using the notion of a *Meeting Point* and the modeling of *noise* are used to support the arguments.

8.1. Introduction (A)

Recent discussions in Spatial Planning overlook perception, assessment and interest of many actors, which leads to a loss of acceptance and efficiency of GI technology. The success of modeling spatial dynamics in a Decision Support System relies heavily on the ability of GI-technology to map and to communicate space. This is particularly true for human activities. *Spatial Knowledge* of social values and the empirical notion of object in space result in new perspectives in space-based disciplines such as Social Science, Geography and Spatial Planning.

Spatial Knowledge needs to be developed within communication, which relates to tasks in Spatial Planning. Information processes which are able to represent vague categories have to be formalized. Social and cognitive differences in the perception and assessment of space between individuals, social groups or institutions[2] have to be described and communicated on issues like *noise annoyance* in actual applications of Spatial Planning. A consensus building process will be improved by visualizing disagreement and extending technology's ability for modeling information processes. Linguistic terms are of particular importance in that kind of processes.

Due to the growing amount of (measured) data and increasingly powerful storage concepts and facilities, the quest for accessing information and meaning became an important issue in mapping knowledge. To retrieve appropriate information different kinds of visualization techniques have been used. Extracting spatial relations and information using maps have always been a domain for Geographers[3] interested in advancing Geographic Information Technology. To retrieve information and meaning, perspectives among different scientific communities

---

[2] Example: Planning Authorities or Regional Manager, Political Parties or local Pressure Groups.

[3] *Skupin, Buttenfield* (1997); quoted in *Fabrikant, Buttenfield,* (2001:266): *"(...) the projection of elements of a high dimensional information into a low-dimensional, potentially experiential, representational space*"

need to be shared who have not been using space as a top research area in the first place[4].

A concept like *Talking Space* does not see the meaning (or the semantics) of the database limited to retrieval of content by sophisticated means of (carto)graphical tools but as a process of producing information which actors can view, perceive and communicate. If we think of perception as an interactive and creative process we are able to share perceptual worlds. A visualized meaning offers the possibilities to talk about those perspectives in the end.

The task is to develop a suitable process for modeling social reality with respect to the perception of space and to apply it to planning context. The perception and assessment of space is becoming heterogeneous and subject to new structural forces. An improved recording of (subjective) space construction and communication can put Spatial Planning in a situation at which knowledge of an area is established and meaningful intervention is applied.

This paper covers work-in-progress on communicating *Spatial Knowledge*. High priority is given to questions which increase knowledge of cognitive spatial representations and a formalization of its representations. GIS rely heavily on technical issues which are to be linked with human cognition and dynamic social space of interactions. A socio-spatial approach demands a dual relationship between people and space. According to social factors[5] people act within and react to space. Within space people organize their actions according to the meaning of constructed space. On this account we want to query our ability to work with symbols, models and codes as it could be developed in Geography, Urban Planning and Social Science summed up in the quest for *visualization of what we are talking about*. *MacEachren* (1995) suggested that *visualization tools can take advantage of human vision, the propensity to categorize and human facilitates which are involved in cognitive processes*. Concepts of spatial representation which allow the use of a mechanism of recognition are of interest in shaping a *Talking Space*. Inherent mental elements corresponding to places and locations should be communicated. Therefore an actor-centered design approach is preferred over conventional GIS analytical design in dealing with spatial phenomena. Actor's preferences, views and attitudes should be developed, documented and exchanged among various protagonists in space.

Knowledge of the *object world* actually sees and reflects the appearance of situations, subjects and picture. Emotional response and relations to individual experiences are encompassed. *Implicit* knowledge contributes a lot to this situation. Transforming data into information and into knowledge is based on people's strength in recognition, perception and description which puts key research problems to discover the way information is processed.

[4] Example: *Imaging* could be an interesting interdisciplinary challenge

[5] Example: gender, class, race, etc.

8.1.1. Spatial Processes & Spatial Knowledge

Medyckyi-Scott & Hearnshaw (1993), Frank & Kuhn (1995), Nyerges et al. (1995) share a common view in that "research on the cognition of geographic information has been identified as being important in decision making, planning, and other areas involving human related activities in space." Fabrikant & Buttenfield (2001) calling for more interdisciplinary approaches involving Cognitive Psychology, Geographic Information Science, Cartography, Urban and Environmental Planning, and Computer Science. GIS scientists have growing concerns on the impact of spatial data handling by various societal groups. In accordance to UCGIS (1996) studies place high priorities on ways that organizations, groups and individuals manipulate geographic information to model the world. The need for a general theory of spatial relationships and spatial concepts has gained importance.[6]

Cartography and Geography share a long history of activities on information processes. Sociology frequently questions information space, knowledge construction and facilitation of decision-making. In all disciplines *space is a representational strategy* (*Crang* & *Thrift*, 2000). In a *Talking Space* no social process exists without geographical extensions vice versa.

Linking human understanding with a formal system like GIS involves communication as well as interaction and presentation. From a technical communication is defined by the transfer of information to the user. Therefore communication is both spatial[7] and non-spatial[8]. Recent literature focuses on map-like interfaces to communicate information in geodatabases to users.

Based on a social science perspective this supposition is not appropriate. Instead of the term *user* "*actor*" will be used. The *involvement of actors* requires a different approach from those associated with an *engagement with users*[9]. Without actor-participation any information procession tends to be meaningless. A question arises on the existence of boundaries between *spatial* and *non-spatial* for non-experts being both users and actors.

In addition, using the term *spatialization* with respect to information retrieval and knowledge processing may also cause controversial applications. One possibility is to use spatialization as a mathematical, semantical or geographical transformation. *Skupin* & *Buttenfield* (1997) defines it with *"(...) the projection of elements of a high-dimensional information space into a low-dimensional, potentially*

[6] See *Fabrikan & Buttenfield* (2001)

[7] Involving static and dynamic mapping techniques

[8] Including charts, graphs etc.

[9] There are fundamental differences between philosophies which underpin information and communication technology and, for example, perspectives behind qualitative research. Computing technology assumes a positivistic approach to the *world* that sees it as being composed of objects. Qualitative research is rooted in an understanding of the *social world* that sees human action as being the force that creates what we perceive. It becomes the goal therefore to try and see things from the perspective of human actors. One consequence is a greater sensitivity on the ambiguities of interpretative meaning that social interaction has for its participants.

experiential, representational space." [10] In the context of a Social Science based approach *spatialization* causes another interpretation. *Shields* (1997) uses the term *social spatialization "(...) to designate the ongoing social construction of the spatial level of the social imaginary (collective mythologies, presuppositions) as well as interventions in the landscape (for example, the built environment). This term allows us to name an object of study which encompasses both the cultural logic of the spatial and its expression and elaboration in language and more concrete actions, constructions and institutional arrangements.*"[11]

"It is interaction in space, not perception of space, which is considered a fundamental building block for the acquisition of spatial knowledge".[12] These arguments will not easy the problem of interpreting spatialization. Interaction is important but visualizations of communication processes are means easier to understand. This is true for sources of understanding, too, which actually is due to interaction in space based on perception. A concept that is associated with interdependency of space-perception-interaction producing Spatial Knowledge is seen more rewarding in communicating spatial phenomena[13].

8.2. Communicating Spatial Knowledge (B)

The importance of language can not be overstated. It is language that gives actors the experience of their *being-in-the-world*[14]. Even if we are all experts in space and talk about spatial dimensions, we do not have the slightest idea of what others are talking about, to say it colloquially. Space in the context of a *Talking Space* is a vague product of communication. *"Imagine a geographer and a spatial planner trying establishing the dimensions of a planned date in a conversation. This is an encounter shaped by different cognitive abilities, and by the concepts of two disciplines that both work with the hard-to-pin-down object of space, walking the tightrope between rationality and irrationality - on the search for a go-between solution. (...) It is an inherent characteristic of a date that it touches upon different senses: A date implies something to be recognized (a symbol), something to be perceived (a model), something to be described (a code)."*[15]

More importance is given to the transaction of meanings in people's communication at an early stage of an information process rather than to predefined symbols represented already in fixed speech, visual arts or architecture. Instead of proposing a map made of geographical *objective* features, a map of vague *Meeting*

[10] Quoted in *Fabrikant, Buttenfield* (2001:266)
[11] *Shields* (1997:188)
[12] *Golledge, Stimson* (1997:159); quoted in *Fabrikant, Buttenfield* (2001)
[13] Example: People perceive their own home-landscape by taking the physical landscape and warping it to a perceived political landscape. How do they communicate this?
[14] See *Gadamer* (1976)
[15] *Kratochwil & Benedikt* (2000:239)

Points (Dates) can be created. It becomes necessary to perceive objects around us but also to perceive our own self as vague *computations of points*.[16]

According to *Flusser* (1999) perceived reality is a tiny detail from the field of possibilities surging around us, which our nervous system has realized through *computation*. The author proposes a city as a *meeting of energies* that can set forth new knowledge and focuses on the potential of a city being its connectivity rather than on an urban space as several buildings fastened together. Points like this would no longer be related to space and time. Unstable territories are driven forward subjectively and materialize into a corpus, knowledge, visionary images, feelings and perceptions, which can be interpreted as an anthropological concept of a city: A net formed by a field of inter-subjective relationships. *Flusser* (1999) argues that modern societies are in flux, with traditional linear and literary epistemologies being challenged by global circulatory networks and a growth in visual stimulation. *Flusser* posits that these changes will radically alter the ways cultures define themselves and deal with each other. *Flusser's* arguments about communication and identity are rooted in the concept of self-determination and self-realization through the recognition of the other.

8.2.1. Theoretical Key Concepts around Space-production

A certain *Meeting Point* is as vague as *Space* itself. We need ideas around Space-production first. Different disciplines do space differently.[17] Space is not just a surface for activity. In a conceptual framework of *Talking Space* human actors are seen as the producers and carriers of space. Space produces transformations on each occasion that is put into effect. The transformation process leads to *meaning, acting* and *observing*, which *construct*[18] *Spatial Knowledge*.[19]

---

[16] "*What we need to take into account, though, is that our understanding is not based upon a point in space. Much rather, points are a symbol for the information that is being transported. In other words, there is no single reason why we would depict the concept of "Meeting Point" with a single point. "Meeting Point" is an invented category that has to meet a number of expectations - what we seek is an image of this point.*" *Kratochwil & Benedikt* (2000:241)

[17] In *Geography* and *Sociology* a questioning around materiality starts. In *Media Theory* it tends to be a shift to primarily visual media. Space is increasingly seen as a socially produced set of manifolds instead of a container of actions. Epistemological practices also illustrate the role of knowledge which is always placed and localized. (see *Crang, Thrift,* 2000). In *Planning* and *GI-Sciences* among others, empirical space is defined by dimensional measurements and by trigonometric descriptions of geometrical relationships between objects. Effects of contingent positions and geometric relationships between persons or objects which are expressed via distance – it is space in its most limited perspective.

[18] The constructed nature of knowledge has been described from a number of different epistemological positions, including *Radical Constructivism* (*Foerster, Glasersfeld*), *Semiotics* (*Eco*), the *Sociology of Knowledge* (*Fleck, Khun*), and *Science Studies* (*Knorr-Cetina*, 1981). In our case the theoretical framework follows particularly *meaning* and *acting* associated with *Symbolic Interactionism* (see *Blumer*, 1969) and with *Structur-Habitus-Praxis-*

Meaning & Acting

Human Beings act on *meanings*. This simple view turns out to be quite complex because meanings cannot be taken for granted nor does a neutral link exist among factors[20] responsible for human behavior. The *source of meanings* is fundamental.[21] Meaning of an object, a situation or space itself arises in a process of interaction between people.

Acting within a social context comprises an interpretative process. This is reflected in at least two steps. First, one person indicates to oneself the situation toward which the person is acting. One person produces *self-interaction* for this reason. Second, interpretation becomes part of handling a meaning. The person selects and transforms meaning facing a particular situation, i.e., no *objective* or *natural* application of meaning exists.

A tremendous variety of pictures of environments have to be managed and communicated. Even if persons are referring to the same (precise) location, this type of space will offer different environments. To talk about such a location comprises not only dimensions of communication but dimensions of perception. Meaning has to be formed and space will be transmitted through a process of indication. Persons indicate lines of action to each other and interpret the indications made by others. To observe the process under which construction is happening is not *objective* at all.

Acting could be explained also in consulting the *Structur-Habitus-Praxis-Concept* by French sociologist *Pierre Bourdieu* (1982). By *subjectivism* he refers to approaches to human life and action that locate causes of social behaviour in individual will, conscious decision-making and lived experience. By *objectivism* he names approaches that set out regularities as structure, law, and systems of relationships. As one result (out of many), *objectivism* could be seen as a form of idealism with regularities dependent on the subjectivity of the observer. According to *Bourdieu* social practises arise from the operation *Habitus*.

Instead of a dichotomy between *objectivism* and *subjectivism* he proposes a *Theory of Practise* which refers to the ongoing mix of human activity. In general it

Concept, (see *Bourdieu, 1982*); *observing* with *Second-Order-Systems* (*Second-Order-Cybernetic*, *Foerster, Maturana*).

[19] Knowledge depends on the position of the epistemic subject not only in a metaphoric but also spatial sense. The birds-eye-perspective is ideal (and used in many mapping procedures) but embodied subjects can not ever take. Although the integration of, for instance, two perspectives lead to new form of perception. Interactivity pertains to the transactions between subject and its object. Without interacting with the environment we are, for example, not able to see.

[20] Do factors as attitudes, stimuli, perception or social position and roles, norms, rules explain behavior? Or: Are psychological elements as transfer of feelings, association of ideas more important?

[21] The meaning of a *Meeting Point* (for example a date or a Rendezvous) in space could be an intrinsic part of the *Meeting Point* itself, for example, the *Meeting Point* being (at) a church. As part of *objective* reality a church is a church without any type of information-process involved.

proves that people act deliberate and with ostentation in context of their *Habitus*[22]. With this type of acting people start to position themselves in society, i.e. signals of belonging and distinction. *Habitus* is dependent on complexity and structure of resources/constraints which *Bourdieu* calls *capital*.[23]

Observing

Conversation with the *world* is based on *concepts of world* and rely on the linkage to others. Connections to others are necessary because of useful references, for example, a reference to our *seeing*.[24] Concepts will be *recognized, perceived* and *described* through a continuous circular process that actors refine until accordance or replacement of concepts could be reached. The developed balance as a type of consensus will be achieved through a *recursive operation*.[25] In conversation we make a selection among possibilities.[26] It is a *circular system*. *"There is a word for language, namely "language"; there is a word for word, namely "word". If you don't know what "word" means, you can look it up in a dictionary. I did that. I found it to be an "utterance". I asked myself, what is an "utterance"? I looked it up in the dictionary. The dictionary said it means: "to express through words." So we are back were we started. Circularity: A implies A."*[27]

---

[22] Reservoir of targets and values; refers to the total ideational environment of a person including the person's beliefs and dispositions.
Example: Conceptual use of space is class specific and discipline based. Spatial boundaries or zones, for instance, *critical suburban areas* are constructions ideologically coded into cartographic conventions. Planners (Model-Makers) never locate simply natural features. Model-Makers play a role defined by a conventional structure and they have their own spatiality. It could be understood as the interaction of groups following their own goals in accordance with a representation of the consequences of their interaction.

[23] *Bourdieu* differs between *economic* (in the sense of confidence and validity), *social* and *cultural capital*. Casually he mentions also the *symbolic capital* which belongs to all forms of capital.

[24] Examples: To see the own face needs a reflecting surface; or: any speaker needs a listener. That offers multiple worlds and particularly multiple visual worlds that stimulate decisions-in-action daily life. In this manner we are dependent on our own designs of thoughts and meanings because of assimilation within the collection of already existing concepts and concepts of others. Perceptions contain a beholder's share and it derives from an observer's expertise. It includes a single perspective which needs to be explained to others. But – both perception and language are ambiguous.

[25] *Heinz von Foerster* examined recursion and found a formulation for an example of recursion that tended to stabilize at one value (*Eigenwert*; an *Eigen*-Function generating an *Eigen*-Value). In our case *Meeting Point* could be seen as a stabilized value (consensus) reached through recursive operations during communication processes; *Meeting Point* as *Eigenwert*; proceeded through a recursive operation. (see *Kratochwil*, 2001a, 2001b)

[26] The process through selection comprises contingent meanings. Relations are embedded in the accumulation of shared meanings which produce information. To define a situation actors require data as well as rules of selection, categorization and organization.

[27] *Foerster,* (1991a)

An utterance is always situated in a relation and dependent on the relationship of other utterances. Meanings always occur as a part of dialogue between two utterances at least. We accept *dialogism* as a method; therefore we have to look for explanations around constructions. Constructions as *symbolic media* make it possible to share meaning and allow coming to similar conclusions. Constructions can be examined not only by *who* is defining something but also by *how* it is defined. We emphasize operations which are able to identify actors to understand what distinctions actors (*Observing Systems*) make.

The operative mandate, following the work of *Heinz von Foerster* (1999), is *observe the observer.* It gives an epistemological foundation to replace a *Subject/Object View*[28] with *Self-Referential Observations*[29]. *Self-Reference* is a basic condition for constructing and observing. Including the observer marks a difference between *First-Order-Observations* of constructions that describe something and *Second-Order-Observations* of social constructions, recognizing that a describer is implied in his own construction. How a self-referential system constructs is implied in how space is observed. All knowledge is defined in some way by culture and since the observer is always part of a specific culture, he can never be objective - there is *no observing without an observer*.[30] Only the observing system determines how it relates to its observations.

Associations with a Talking Space

Due to circularity within the space-production-process it is of no particular relevance which step comes first - *meaning, acting* or *observing*. Space-production is a circular system which is determined through its own production[31]. This approach results in *Self-Referential Circular Observations* (*Second-Order-Observations*) which replace *Subject/Object Views* (*First-Order-Observations*). To support our arguments the next figures will guide the reader to the concept of a *Talking Space* and related environments.

---

[28] Observer is the Subject and the Observed is the Object; *First-Order-Observation*

[29] Observer is implied in Observations; *Second-Order-Observations*

[30] *Maturana, Varela* (1980) develop the *autopoietic system* which receives input from environment but has the ability to operate internally in such a way as to continuously recreate the whole. The internally operations constitute an observing system.
Example: If the scientist as an observer is no longer *objective* (neutral) and external to the object of investigation, it influences the object. It will become part of a feedback-circle, i.e. becoming part of an active involvement. It implies that information does not come from outside as an objective reality, but it is constructed inside the respective system: in the human being itself.

[31] Example: Physical space could be produced via classification schemes with various ideological divisions as *good* and *bad* areas. Significance, meanings and associations around topographical features are embedded in different types of representation. Representation means reproduction of relations, implicated in symbolic and conceptual formations. Conceptual formations could be involved in ideologies

Fig. 8.1 shows a system representing the human brain which is seen as a *black box* and is operating on a stimulus-reaction-scheme.[32] Space is a container full of data and associated information to be retrieved, while relationships among data and information are determined through *objective* descriptions[33].

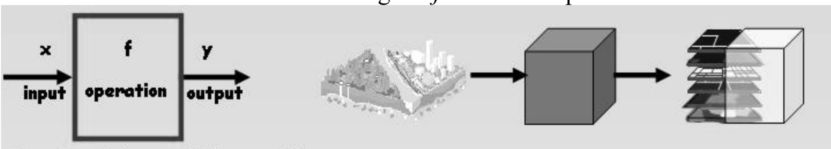


Fig. 8.1. *Subject/Object View* on Space; *First-Order-Observation* (Observer is the Subject and the Observed is the Object)

Unlike the *Subject/Object* View a *Talking Space* does not focus on the expert's view on space but on space created by the observer being an actor in a process of creating space itself. A strict logical model of classifying information according to some criteria leaves no room for knowledge deviations[34].

Compared to Fig. 8.1 the actor[35] in Fig. 8.2 is a circular interactive device incorporating individual transformation processes. A problem will not be solved because a problem exists. A problem will be solved because it is constructed first by the observer. Problem-solving is problem-generating the first place. A common validity for *objective* descriptions or representations does not exist.

Talking Space is a conceptual design in which the observer and the observed are structurally coupled to each other. Although relations and exchanges can be depicted, information processions are based on individual cognitive operations in the first place. Interconnections (based on communication) to other systems makes space-production apparent and *observable* to others.[36] This results in an *Observe the Observer* View on space as depicted in Fig. 8.3.

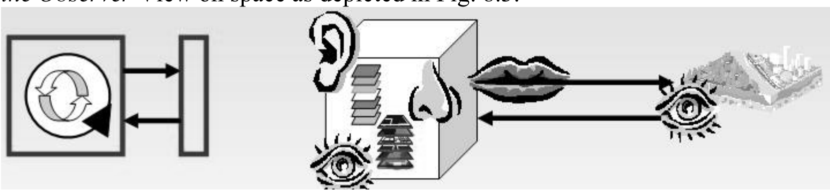


Fig. 8.2. One Actor: *A Self-Referenced Circular Observation View*; *Second-Order-Observation* (Observer is implied in own Observation)

[32] We use that kind of logic on modeling space albeit other explanations have become feasible in the scientific community.
[33] Like *noise annoyance* is described by *decibel*
[34] Logical implications will be discussed at a later point
[35] Actors in our figures are symbolized through cubes, filled with eyes, nose, ear (representing perception) and two different types of *layers*; one for social characteristics and individual concepts, experiences, memory, bibliography, and the other layers represent spatial based data-sets.
[36] See *Foerster* (1999) - "*The listener and not the speaker will decide what is talked about*."

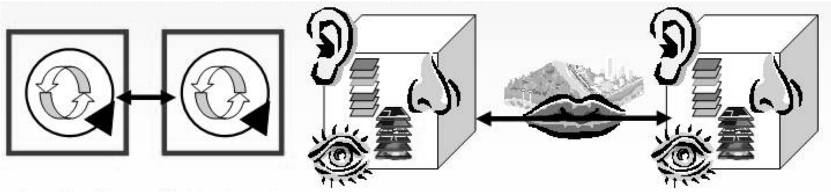


Fig. 8.3. Two Actors – "*Observe the Observer*"

8.3. Talking Space Environments (C)

The concept of traditional maps leads to modern interface techniques. Traditional maps are constructed from geometric primitives, associated with map symbols and displayed on a flat surface. They are results of a certain projection of a higher-dimensional configuration into a two-dimensional space.[37] *Talking Space Environments* require interactive means to enable communication of n-dimensional information on the basis of a diversity of media-systems in which *meaning-parties* can be mapped. Mapping should allow any type of actors to recognize that judgment was being suspended throughout a modeling process. Actors are engaged in activities displaying their concern about meanings and get response from others.[38] Different meanings have to be shown differently on the computer screen, the person has to be enabled to easily change his or her perspective – difference is made visible. *Implicit* knowledge of the actor becomes essential. Knowledge becomes *explicit* in the sense that it is embodied in the technical design. People's *tacit knowledge*[39] can be extracted and made *explicit* through visibility. Knowledge is

---

[37] Long history of mapping shows quite different types of uses and definitions. For example, maps could be seen as *description* or *representation*, as *picture* or *plain figure*, *delineation* or *tablet*, a *drawing* or *portray*, an *abstract* or *chart*, a *graphic document* or *pilot's eye view*, a *topographic transfer* or *set of signs*, a *symbol* or a *formal system*, a *spatial analogue* or *device*, a *scale model* or an *information structure*, a *mirror* or an *abstraction*, a *communication tool* or *medium*, an *image* or *dissociated transcript*, a *matrix* or …
… "*The outcome of man's desire to give geographical expression to his knowledge or his ideas concerning the characteristics and distributions of the earth's feature*" (*North*, 1933)

[38] That means information in flexible sense contexts, in order to create information from data which can be used as knowledge. This path requires cognitive and social reciprocation, i.e. a reciprocal relationship in a circular process (see *Foerster*, 1997, 1999). These systems are dependent on an actor's exchange so that modes of perception and knowledge achievements can be adapted to each other. Thus also shapes the person's own language and perspectives, build into language. If one transforms perspectives, explanation of what stays the same, and what will change in relation to the *Point of Reference*, becomes necessary.

[39] This idea contains the term *Tacit Knowledge* which was formalized by *Michael Polanyi* more than 50 years ago. *Tacit Knowledge* is a product of individual experience and not a source of common knowledge. Each of us has a tacit understanding which allows us to respond to different situations differently but, in general appropriately. *Polanyi* (1958) was

embedded in meaningful patterns of experience, i.e., a secondary context of relevant criteria.

Talking Space Environment as a communication platform gains more importance with the transaction of meanings during people's activities. Data, information and knowledge deem distinct approaches to a space related communication process. Data are coded systems of symbols and dependent on observation and need transformations to be information and to become knowledge[40]. As long as no exchange among actors takes place, space is not ascertainable.

The geographic notion *"(...) no two things can occupy the same point in space and time*"[41] as the basis for locating a geographic entity in space, has tremendous effects! If two actors decide to meet somewhere in space, they might have a church or any other location, which is visible, in mind. Still, the *Meeting Point* is not an obvious place. Information is embedded in a primary context of relevant criteria. Modes of perception have to be adapted to each other, because actors will talk about their *Personal Meeting Point.* To make a communication in space effective we **need to visualize what we are talking about!**

In *Talking Space* a technical environment is proposed which enables Researchers, Planners, Geographers and other actors on and in space to reflect their work in a comprehensive way and employ tacit as well as explicit knowledge to identify ideas and meanings. It is designed to enhance unreliable quantitative data sets and to stimulate observations by actors.

8.3.1. Logical Perspectives

Looking for a *Meeting Point* is a spatial decision-making-process. A classical logical model does not necessarily lead towards a successful decision or as *Lotfi A. Zadeh* put it - *"When the complexity of a system increases our ability to make precise and thus significant statements about its behavior decrease accordingly. Precision and significance cancel each other out beyond a certain degree of complexity"*[42].

---

concerned on the process of recognizing and making a commitment to ideas which may result from understanding and knowledge.

[40] According *Willke* (1998) are following considerations necessary: a) Data depend on observation (Data are created and constructed by observation); b) Data are coded systems of symbols. 1) Data become information by: "*embedding a primary context of relevant criteria, which held for a certain system";* 2) Information transfer requires the *systems to have identical criteria of relevance*; 3) Information becomes knowledge by "*embedding in a secondary context of relevant criteria. The secondary context does not consist of criteria of relevance like the primary one, but from meaningful patterns of experience. These patterns are stored in a special part of our memory and they are available. Knowledge without memory is impossible, but not everything that comes out of memory is knowledge."*

[41] *Golledge* (1995) quoted in *Fabrikant* (2001:268)

[42] *Zadeh* quoted in *Yager* (1987)

Classical models[43] do not allow for extensions on representations of individual truth or knowledge beyond to what seems to be a contradiction or a paradoxon at best. Varying degrees of *truthness* which refer to the extent of an object representing the uncertainty due to semantic and pragmatic use of linguistic variables[44] are of interest in a *Talking Space*. A fuzzy truth in the sense that truth of an object cannot be determined is not an appropriate mean in developing a *Talking Space*. Talking about a *Meeting Point* we refer to a location in space with *varying degrees of correctness or truth.* All of the actors are forced to talk about a truth that is unique to a geographic location. Data in GIS would support the exemplification and the information *in* a point in space so to extract meaning of a data point with respect to its relevance as a *Meeting Point*. The logical model in a GIS does require data to represent the full point or not. There is no room for uncertainty being a source of information. Uncertainty in conventional GIS analysis is due to the lack of precision of a point in space in a layer environment covering spatial features. Uncertainty is not considered a feature along the way of gaining information and knowledge on a point in space becoming a *Meeting Point*, for example.

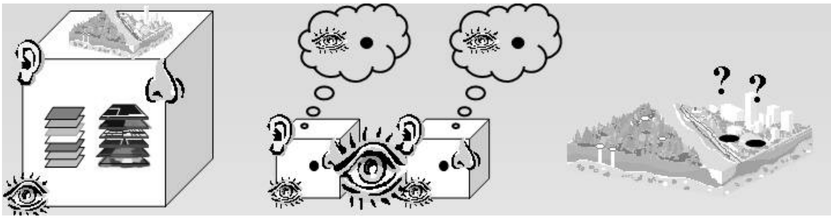


Fig. 8.4. Point Features (*Meeting Point*) based on Data

Fig. 8.4 (from left to right) gives the reader an idea, on how a *Meeting Point* is discussed within the logical model of a GIS. A relationship between data and information is straightforward. Data represent *objective* knowledge and carry all the information associated with a geographical category.

In the left part of Fig. 8.4 perceptual, linguistic and related references to the notion of a *Meeting Point* are describing a unique point in space. Therefore concepts

43 All representations and analytical GIS features rely upon a logical model of the *Aristotelian type.* At the core of this logic and, in fact, of any kind of logic is the concept of truth. Truth in various aspects of GIS scheme development is restricted to {0,1}. Classical logic is designed for perfect symbols, objects and features in space. It is designed as a unique model that can be applied to any situation at any time and applies to the representation level as well as the analytical and reasoning level. Our knowledge is made up of a long row of questions (or hypotheses) answered by *yes* or *no* and thus being very precise. At several occasions this is very awesome, time and cost consuming and does not necessarily reflect our actual knowledge on an issue. Some early artificial intelligence applications assume the human mind is limited because it cannot calculate as many true and false operations as a machine thus making the human mind less intelligent. We do not agree to that.

44 Example: Modeling of *noise annoyance*

of spatial metaphors like distance in modeling so called *semantics* of large geodatabases are possible.

The mid part of Fig. 8.4 shows two actors, both of them having a certain perception and cognitive idea of a meeting. The right part of Fig. 8.4 visualizes how *implicit* knowledge is applied *explicitly* to data layers provided by a GIS.

From a *Subject/Object View*, space is clearly defined by its logical representations[45]. This also can be seen in Fig. 8.1 in viewing *Meeting Point* as a result of a (crisp) functional relationship between input and output. The only uncertainty left is based on the distance, for example, of a unique meaning represented in a GIS based space modeling process.

The logical design of a *Talking Space*, however, has to take into consideration that uncertainty is a major source of information that makes it easy for us to make a decision in space[46]. Uncertainty is referred to as fuzziness or vagueness and is taking place at all levels of spatial analysis, that is, in geometrical as well as representational issues. This discussion is of course not limited to a *Meeting Point* but to any spatial dynamic process involving social activity.

Uncertainty stems from the fact that in some aspects we just are not certain whether something is true or not[47]. If you cannot estimate the truth for a single point you may broaden the spatial band of truth creating intervals[48]. It has to be noted that this paper does not get into the different notions of probability (logical, frequentistic, subjective)[49] used for modeling uncertainty. As many authors in the field of fuzzy logic have pointed out, probability is not necessarily the appropriate concept to deal with uncertainty in a fuzzy logical context[50]. Probability[51] does not extend the ability of GIS to visualize the truthness of an object or a situation that is questioned by the meaning of different actors. Nevertheless, probability bands[52] are used to fuzzify spatial data and thus making decisions based upon that data more intelligent. A fuzzy logical perspective in a *Talking Space* is being devel-

[45] Being points, lines or areas and associated attributes
[46] Without precision as a tool's basis in determining truth of a location
[47] In common GIS applications these uncertainties are treated as errors that propagate throughout GIS analysis and have to be eliminated or to be dealt with, at least. Advanced geostatistical approaches exist to handle uncertainty whether expressed as probabilistic, crisp or fuzzy sets in GIS. These techniques are deeply rooted in probability theory estimating the *probability of truth*.
[48] Interval statistics
[49] A detailed account on how different kinds of probabilities are handled in Soft Computing can be found in *Spies (1993)* as well as publications of the *Studies in Fuzziness and Soft Computing* Series by Springer Publishing Services among a great variety of related publications.
[50] Note: Ambiguity occurs when values are associated with multiple attributes and no certain decision criterion is available; vagueness or fuzziness is a concept that is associated with the problem of making sharp distinctions in the world. Whereas fuzzy measures extend probability measures in modeling ambiguity, measures of fuzziness have been developed to focus on fuzzy/vague problems. (*Klir & Folger*, 1988)
[51] Whether its used for data analysis or reasoning purposes
[52] As has been done by geographers a while ago with e.g. ε-bands

oped towards a flexible form of representing symbols and models of a constantly changing space, which is focusing on the vagueness of linguistic categories rather than on aspects of ambiguity and eliminating uncertainty.[53] The changes are not caused by different levels of precision but due to the different space various actors are referring to. As long as we employ precise symbols, probability and error we are unable to adequately address vague/fuzzy concepts that are associated with the linguistic/spatial notion of a *Meeting Point* in space. What makes it fuzzy (or vague) are the semantics of using linguistic variables[54]. The crucial issue is not the possibility of specification but the handling of uncertainty due to semantics of using language.

Uncertain knowledge encompasses communication phenomena where people have some elements of choice in an information process, a choice within the constraints imposed by cognitive, social and physical space conditions. *Black* (1937) argued that the problem is a gap between human (linguistic) understanding and the scientific mode of expression, i.e. scientific results are easily misinterpret for human beings, a statement also expressed by *Bertrand Russell* in his frequently quoted phrase: *"All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life, but only to an imagined celestial existence.*[55] In order to overcome these gaps without having to forsake formal science a formalism was developed by *Black* which forms the basis for understanding *Fuzzy Set Theory* as it was formulated by *Zadeh* almost 30 years later and used as logical perspective in a *Talking Space Environment.* As defined by *Black*, *vagueness* is not understood by the uncertainty of lacking data or incomplete knowledge[56], but the uncertainty that comes with the complexity of the

[53] A *fuzzy* logic does not exist, since all methods employed are crisp in a mathematical sense and do only address vague and ambiguous representations, symbols, models and codes. It may sound simple but it is a common misunderstanding in using fuzzy sets. We do not talk about fuzzy points but about points representing symbols and points that represent invented, cognitive and linguistic categories. As *Varzi* (2001) has pointed out, "*to say a geographic"* object like a *Meeting Point "is vague mistakenly infers that the product of representation is vague because the representation process is vague*".

[54] Vagueness has been an issue in linguistics, philosophy and mathematics for a very long time and has become a focus in geography quite recently. *Vagueness* is an aspect of fuzziness which is above all expressed in the activity of the influence of cognitive modeling on the results of forming categories. *Vagueness* is defined via the semantics of categories in the context of linguistic categorization. The *Problem of Vagueness* is formulated with regard to interpretations of reality and how they are learnt by scientists in their language and by daily usage of language. *Black* (1937) assumes that is in general not possible to clearly define a linguistic category determinably in terms of its use, i.e. unambiguously via the allocation of its characteristics. There are always areas in which clear allocation to categories based on occurring characteristics is impossible. In the case of the traditional *Fuzzy Set Theory* (*Zadeh*, 1965) the problem dealt with the allocation of known elements to the categories which represent them.

[55] *Russell* quoted in *Black* (1937)

[56] Explicit

systems to be investigated[57]. *Uncertain knowledge is on no account the same as uncertainty about knowledge.*[58]

The concept of a fuzzy set being at the core of Fuzzy Set Theory, is a very simple concept in the first place: The *information* contained in a geographical or spatial category is not in the datum but in the meaning of the datum to a particular linguistic category. It thus expresses the degree to which a point in space[59] is contributing to the explanation of a problem or a category without providing a solution to a problem beforehand as fathomed in precise symbols of geographic coordinates. At the same time it contributes to its negation, which actually is of interest in a fuzzy logic perspective for modelling a dynamic component through acting in space. Fuzzy logic, however, does not confuse gradual transitions from truth with arbitrariness. It rather makes a gradual transition from *truth* to *non-truth* visible.[60] This leaves room for negotiating paradox situations. We can logically refer to the degree of contribution of an element rather than using a measured value in explaining the object.

The ability to focus on aspects that deal with uncertainty as a product of different opinions views and actors greatly enhances the notion of space in a logical GIS model. *Noise*, for example, is restricted no longer to the view of a single man or woman (like the GIS expert) but to a more general view of actors involved in working on spatial situations. In addition, the importance (information) of a linguistic category like *noise* is no longer restricted to the measurement of the level of decibel. *Noise* is a phenomenon that is described by symbols of *annoyance* and communicates actor's views on *annoyance*.[61]

In the case of a *Meeting Point* there is no single attribute that fully describes the relationship between a coordinate in space (a point, a line, an area) and a *Meeting Point*. A perception-based, linguistic and formal representation of a geographic category shapes our understanding of space (see also Fig. 8.4). This understanding cannot be reflected by ordinary sets since they represent only *one* possibility in a whole range of solutions. *Information* is what we understand thus a future geographic *information* system will have to take into account this situation.

A practical implication is modeling the meaning of *noise* or retrieving the information on *noise* from data. Streets, buildings or neighbors are only one instance of *noise* rather than being a unique representation of *noise annoyance*. Many GIS applications define *noise annoyance* by the amount of decibel measured and associated with a street. The street becomes *annoying* which neglects the aspect of the possibility of belonging to and having an explanatory and defining effect to other *noise annoyances*. This simple thought makes fuzzy sets or, in fact, any extension of classical sets, attractive in modeling geographical categories

---

[57] Implicit

[58] See *Spies* (1993, 1994)

[59] Like a (x,/y) coordinate

[60] This concept of a measure of fuzziness addressing vagueness has been formally described in *Klir & Folger* (1988)

[61] It is more important to discuss the relationship between *decibel and noise* rather than to model *noise by the amount of decibel.*

Different forms of a *multivalent logic*[62] have been used by scientists who have concerned themselves with the possibilities of extending *logical calculations* whose subject matter was the *phenomenon of vagueness* of language as a communication medium.[63] Everything we work on, space, attributes, and demography has a vague component in it. This is not a unique claim of geography or planning since *vagueness is a pervasive phenomenon of human thought and language*[64].

Set theoretical and logical approaches can contribute to a better understanding of geographical phenomena and thus holding more information than simple data and their (forced) cognitive, linguistic and formal classifications.

In proposing a fuzzy logical perspective we extend the question of "*how loud is loud?*", "*how noise is noise?*" to a spatial problem: "*How Austria is Austria*"?

The actual GIS space is only one of many instances in modeling and talking about Austria. That has to be considered when talking about spatial models. It is necessary to develop a spatial relationship not only between points, lines and polygons but also between linguistic categories and points, lines and areas. The kind of information that comes with this is, however, most crucial in determining the uncertainty of space in terms of planning issues in, for example, suburban development with suburb being not a fixed space.

In classical GIS Analysis[65] map layers refer to "*Austria*" being a space uniquely representing knowledge of different actors on e.g. *noise annoyance*. It is not taken into account that, for example, "*Austria*“ is being different for each actor, but the concept of *noise annoyance* being a very precise concept to anybody. In current (fuzzy) GIS applications, "*Austria*" is seen as being crisp[66] and *noise annoyance* being the *fuzzy* part. Space itself, however, to whom the relations are applied to, holds fuzziness, too due to its knowledge and space creation.

Talking Space takes a map[67] not as result but an attribute to use spatial knowledge like *high living quality* as a basis for spatial decision-making-processes. The Location "*Austria*" is not uniquely associated with an administrative unit but an ever-changing model of spatial relationships.

---

[62] At the start of the 20th century the Polish mathematician *Lukasziewicz* was among the first to develop a multi-valued logic to extended the scientific possibilities for describing facts as true or false {0,1} by providing a third possibility, which could be interpreted as "*a little bit true*". He expressed this knowledge through a number from the interval [0,1], namely through 0.5.

[63] Vagueness is not in a point because a coordinate specification is impossible but it is in the semantics of this point in representing the various degrees of a coordinate contributing to the understanding of, for example, a *Meeting Point*. A point associated with a (x/y) coordinate is a function of what is being expressed in the meaning of attributes for a certain category. A (x/y) coordinate is becoming *one part* but it is not *the* solution being a *Meeting Point* as represented in a GIS layer and if so we have to develop new ways of associating spatial knowledge towards geographical features like points, lines and areas.

[64] See *Varzi* (2001)

[65] Using either fuzzy or crisp sets

[66] As determined by its boundaries

[67] Or a single GIS layer

8.3.2. Visual Perspectives

Any kind of perspective in GI science has always to be evaluated against its potential on visualization. Visualizing of what actors are talking about is an essential part in communicating space thus becoming a practical necessity in making a *Talking Space* useful as proposed in this paper. GIS are considered to be one of the mapping tools of the digital age. GIS are well suited to store, analyze and display data that refer to a location in space. As has been pointed out frequently, GIS are, however, referred to as socially empty space.

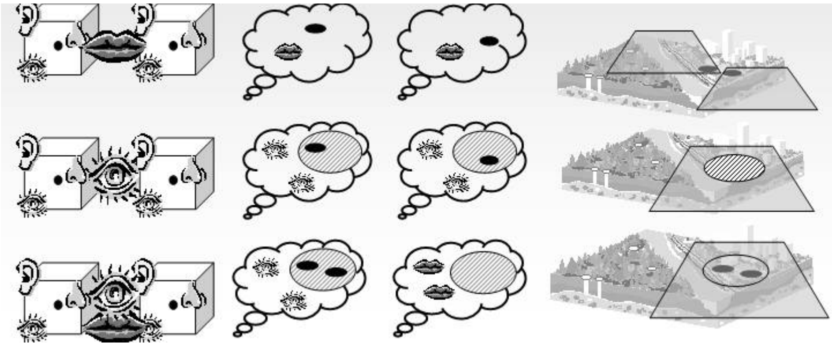


Fig. 8.5. Point Features (*Meeting Point*) based on Information

Fig. 8.5 sketches (from top to bottom and left to right) three steps on visualizing information processes and space creation by two actors creating a *Meeting Point*. It addresses the issues that need to be considered with visualising features of a social space like a *Meeting Point*. The *Meeting Point* is talked about in a different environment using different criteria in shaping space and thus representing different spaces, which may be defined socially (first step).

A *Meeting Point* is socially constructed and does not necessarily reflect particular geographic coordinates but may address very different views of a layer (or even two layers). Thus the geographical metaphor of distance is not applicable in retrieving information on the semantics of a *Meeting Point*. It deems necessary to develop different map layers (first and second step).

This is mainly due because mapping issues in GIS are focusing on the relationship between data in a technical sense. The visualization is limited to representing the knowledge implied by the experts on space revealing or retrieving information that is *in the data*, using semantics that are limited to the expert's concepts on space. The visualization process needs to be generalized to open up possibilities of displaying the creation of new relationships between data, geometrical features and the actors in space. This is implying a shift from mapping geographical coordinates with associated attributes (or vice versa[68]) towards knowledge coordinates (third step).

GIS must be able to visualize space in its n-ary dimensions to allow actors to immediately see *the space they talk about* as a GIS layer or whatever means are appropriate to get a view of actor's opinion and thus actor's creation of space. This involves an integrated approach of different media like photos, paintings, maps, satellite images among others. The link between the human actors in space and geographical information technologies has to focus on visualizing the relationships while *talking* on space.

Talking has been suspicious as a source of knowledge representation due to implicit and explicit uncertainties involved in the semantic representation of what is talked about. Some semantic aspects in modeling uncertainty have been visualized with non-spatial extensions using sophisticated tools like *VisCovery*[69] which are based on fuzzy *Neural Networks* of the *Kohonen Type*, also known as *Self-Organizing Maps* (SOM). *fuzzyTECH*[70] is fuzzy rule based developer environment on control problems to show the impact of vague linguistic variables on non-spatial decision-making-processes. Some of these approaches have been used to improve information retrieval in large geographic databases[71].

A visual engagement in *Talking Space Environment* is not only aiming at retrieving information from large amount of databases but at building connections between the semantics of communication in space and the inherent uncertainty of geographic objects in databases. A visual space will not let you take a different view on relationships existing in large databases but allows you to create your own relations and immediately see your space evolving to finally see what actors in space are talking about. A visual space is not limited to the visualization of what experts know of and talk about.

An example on how a *Talking Space* may be used by two actors as a visual platform on communicating knowledge within a classical GIS layer environment is drafted in Fig. 8.6.

Meeting Point is not something *out there* that can be measured, classified and associated with geometrical features resulting in a single GIS layer, e.g. points (explicit data). *Meeting Point* is a complex phenomenon shaping space through its protagonists, who are all experts of *Meeting Points* being able to precisely categorize, for example, *point as symbol, model and codes* in their individual environments. The implicit assessment on *Meeting Points* by the human actors is considered the actual spatial knowledge a *Talking Space* is referring to.

GIS becoming a part of an *actor-oriented-space-perception* and has to be extended from a mere *drawing* tool towards a representation technology of spatial knowledge as discussed in previous sections. Data levels represent structures and relations between database elements being spatial or not. Actors visualize their personal opinion and include this opinion into the GIS analysis as additional in-

---

[68] See *Dodge & Kitchin (2001)*
[69] Details on Eudaptic's homepage www.eudaptics.com
[70] Details on INFORM's homepage www.fuzzytech.com
[71] See *Engeli (2001), Fabrikant & Buttenfield (2001)*

formation layers. The example in Fig. 8.6 is related to the 2D visualization, because many GIS actors, at least in the field of current Spatial Planning, are still using this kind of representation.

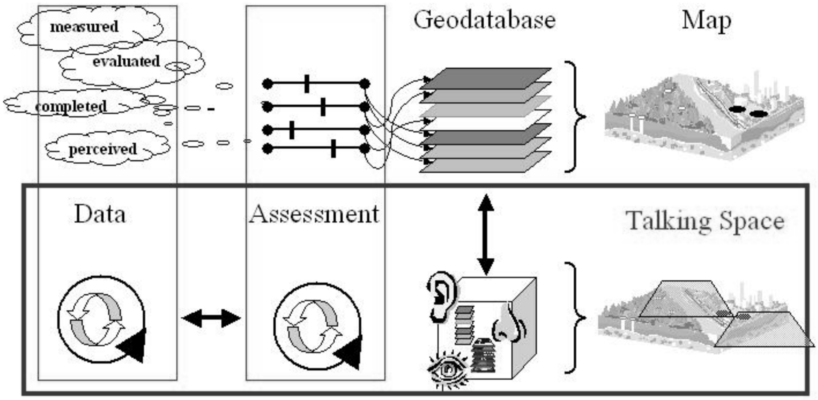


Fig. 8.6. A diagram on how *Talking Space* is using Spatial Knowledge by making use of logical and visual generalizations on mapping spatial data

The extension/representation of geographical coordinates becoming knowledge coordinates is a major goal for a fuzzy logical GIS perspective as described in this paper.

Visualizing *noise* that may have an impact on categories like *annoyances* has to focus on a tool that allows a flexible association of weights and relations to (geographical) coordinates and thus being able to present the vagueness induced by semantic aspects of linguistic categorization. Due to the usability of slider bars in representing gradual transitions we have chosen three examples on how the relationship between objects and actors may look like. The following examples have been chosen because of their usability in modeling weights and relations via an easy to operate interface. Other visual approaches in modeling information landscapes or using spatial metaphors in retrieving knowledge[72] are examples on spatial interfaces as well. Highly sophisticated mapping approaches[73] do not represent communication in space but are supporting our views on actor's space. The visual examples are two of many approaches in the field of mapping sciences. A review of all developments on mapping in various fields would go beyond the scope of this paper. We have chosen these two due to their interdisciplinary viewpoint[74]. The examples are very down-to-earth approaches. It is not necessarily spatial metaphors that are of interest but the usability as a practical platform of expressing opinions. The point of choosing these examples is not the complex interface and

---

[72] *Fabrikant & Buttenfield (2001), Dodge & Kitchen (2001)*

[73] Including *Descartes, Knowledge Territory & MapModels*

[74] Computer Science, Design and Architecture

the use of metaphors[75] but the necessity of handling weights and relations in a transparent way.

Example Descartes[76]

Descartes is an Online Spatial Decision Support system using -
"(...) parallel coordinate plots to consider simultaneously multiple attributes[77]*. It includes several horizontal axes, one axis per each attribute under consideration. The length of an axis represents the value range of the corresponding attribute. Hence, an attribute value can be represented by a position on the axis. An object is represented connecting the positions on the neighboring axes corresponding to the attribute values for this objects* "[78](see **Fig. 8.7.**a).

Differences in relative weights of criteria are reflected by the variation of lengths of axes. The concept has also been applied to non spatial interface programming (**Fig. 8.7.**b) in constructing knowledge bases by evaluating relationships (slider bars on the left) and immediately displaying the ever-changing query results (on the right hand). This example (**Fig. 8.7.**b) is taken from *Engeli* (2001)[79]. The visualization using slider bars has been applied to a large database on books and papers in the field of architecture. The sliders explore the quality of relations among the references in books and articles defined using another interface not shown in this figure. It is useful to assist the actor in a weighted keyword search and more important "(…) *it allows the user to observe intermediate and changing results, like items that are about to be included or excluded from a set rather than focusing only on the resulting sets."* [80]

[75] From left=false to right=true

[76] *Descartes* is a Java based GIS development by the *Knowledge Discovery Team* (KD) at the *Frauenhofer Institute of Autonomous Intelligent Systems*, Germany.

[77] Usually more than two

[78] See *Descartes Online Help*; detailed description please visit http://ais.gmd.de/index.html

[79] Figure 7b is taken from an application called *Knowledge Territory* which focuses on the visualisation of relationships to improve the comprehensiveness of large amounts of data. Goals are, among others, to collect subjective information on papers and to visualize relations between (paper) references, for details see *Engeli* (2001)

[80] *Engeli* (2001:142)

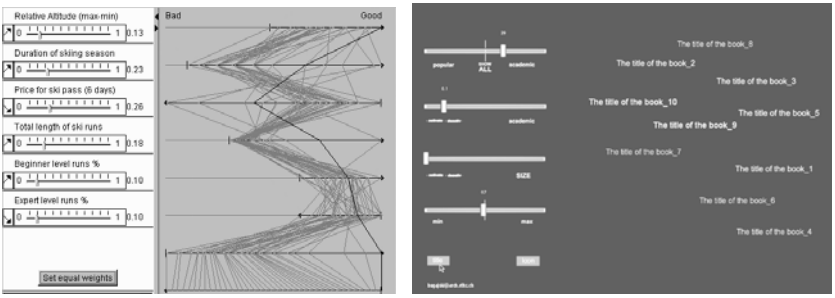


Fig. 8.7. a) SDSS Descartes b) Quality of Relations between References

In terms of a *Talking Space* this model would build relations between different actors, who would turn the sliders. If *noise* is an issue, different perceptions of *noise* would be able to be displayed. Each actor and his (changing) association with *noise annoyance* can be displayed immediately. *Space-production based on noise-perception* does get some more attention, since the building of space is turning on a more flexible tool in displaying uncertainty.

Example MapModels[81]

MapModels is an ArcView 3 GIS extension and extends the possibilities of representing linguistic variables to geometrical features following a map algebraic approach using a graphical user interface. The flow-chart based interface allows the actor to handle spatial information as a basis for visualizing relationships among geometrical features which can be extended to fuzzy logical principles.[82]

Figure 8.8 shows screenshots of the *MapModels Slide Bar Tool* (Figure 8.8b) and the model interface (Figure 8.8a) which employs all kinds of spatial functionality including a monitor on intermediate maps showing the impact of each modeling step, which is especially useful in adding to the transparency of a complex model. Flexible association of weights and relations to (geographical) coordinates can be displayed thus making *MapModels* a promising tool to bridge the gap between social, logical and visual perspectives in a *Talking Space Environment*. *MapModels* is by no means a fully developed tool to visualize the perspectives but a good starting point in making uncertain knowledge transparent.

---

[81] *MapModels* is an ArcView 3.x extension that incorporates map algebra and fuzzy logical algebraic extensions. It is a flow chart based model and consists of defined set of elements and rules adding to a transparency necessary in advancing GIS becoming a spatial decision support tool. *MapModels* use spatial operators, multi criteria tools on decision making, overlays and neighborhood analysis. *MapModels* allow the actor to focus on geographic phenomena and their relationships without the need of extensive programming knowledge unless desired. For a detailed study on the functionalities of *MapModels* please refer to *Riedl & Kalasek* (1998)

[82] *Riedl & Kalasek* (1998)

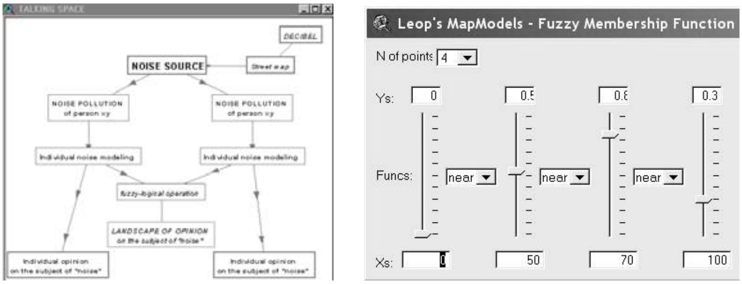


Fig. 8.8. *MapModels* Modeling Interface and Slider Based Weighting Approach

Outlook

Talking Space is a perspective on the use of GIS being a platform for different opinions and their relations to linguistic categories symbols, models, and codes. The success of modeling spatial dynamics in a Decision Support Systems relies heavily on the ability of GI-technology to communicate space. Space is no longer considered as something *out there* but something to be recognized, to be perceived and to be described by space protagonists. This is particularly true for human activities, which are, however, often overlooked in modeling spatial dynamics. Flexible forms of perception, assessment and interests of space protagonists tend to be ignored in their impact on shaping space. Space production of social values and the empirical notion of object in space result in new perspectives in space-based disciplines such as Social Science, Geography and Spatial Planning.

Talking Space is drafted to assist in adding some transparency to the recognition, perception and visualization of uncertain knowledge being a useful component in human decision making on complex situations. It aims at visualizing vagueness caused by the semantics of linguistic categorization via relationships among symbols, models and code development in space related disciplines. Possible assessments do not prejudice but create new GIS levels which are evaluated against conceptions of participants and their interplay in the context of a discourse than to explicit factual data. Therefore GIS is being developed to become a digital platform in representing spatial knowledge.

This paper presented perspectives on the basis of sociological and fuzzy logical issues in using GIS technology to enhance spatial decision-making-processes resulting in the creation and visualization of space shaped by the social actors. None of the envisioned environments has been technically realized yet. Future steps need to consider communication platforms including a great variety of scientific and non-scientific members of the ever growing community of spatial experts to jointly work on different communications about the same space GIScience is aiming at, too.

Acknowledgment

Talking Space is an ongoing communication and research process between a Spatial Planner, a Geographer, a Computer Scientist and a Sociologist.

The authors thank Computer-Scientist *Leopold Riedl, who* developed *MapModels* and inspired much of the necessary visualization process in a *Talking Space*.

We also thank Sociologist *Jens Dangschat* whose continuous support, critical remarks and feedback greatly enhances the ongoing dialogue on perspectives in communicating space.

References

Benedikt J, Kratochwil S (2002) "*Die Wirklichkeit der Raumplanung. Werkzeuge für ein neues Raumverständnis* ", CORP 2002, 7th International Symposium on Information Technology in Urban and Spatial Planning and Impacts of ICT on Physical Space, Vienna University of Technology, in: CORP 2002, Vienna, pp. 207-213

Benedikt J, Reinberg S, and Riedl L (2002) "*A GIS application to enhance cell-based information modeling*", in: Information Sciences 142 (2002) pp.151-160

Benedikt J, Riedl L, and Reinberg S (1999) "Sprechen Sie Geographie? – Operationalisierung natürlich-sprachlicher geographischer Konzepte mit visuell-sprachlichen EDV-Werkzeugen", in: Strobl, Plaschke (Eds.): "Angewandte Geographische Informationsverarbeitung XI", Wichmann Verlag, Heidelberg, pp. 23-33

Benko G , Strohmayer (Eds., 1997) *"Space and Social Theory. Interpreting Modernity and Postmodernity"*; Blackwell Publisher Ltd., Oxford, UK, 1997

Black M (1937) "*Vagueness. An exercise in Logical Analysis*"; Philosophy of Science, pp.427-455; 1937

Blumer H (1969) *"Symbolic Interactionism. Perspective and Method"*; University of California Press, Berkeley, Los Angeles, London, 1986

Bourdieu P (1982) "*Die feinen Unterschiede. Kritik der gesellschaftlichen Urteilskraft*"; Frankfurt am Main: Suhrkamp

Casey L, Pederson T (1995) "*Urbanizing GIS: Philadelphia's Strategy to Bring GIS to Neighborhood Planning*"; in: Proceedings of the 1995 ESRI International User Conference, ESRI Press, CA, USA.
online: http://gis.esri.com/library/userconf/proc95/to150/p107.html

Crang M, Thrift N (Eds., 2000) "*thinking space*", London: Routledge

Descartes Online Help; http://ais.gmd.de/index.html

Dodge M, Kitchin R (2001) *Atlas of Cyberspace*. Addison Wesley 2001, 286 pp

Engeli M (Eds., 2001) "*bits and spaces. Architecture and Computing for Physical, Virtual, Hybrid Realms. 33 Projects by Architecture and CAAD, ETH Zürich*", Birkhäuser Verlag, Basel, Boston, Berlin, 2001

Erwig M, Schneider M (1997) "*Vague Regions*", in: Proceedings of the 5th Int. Symposium On Advances in Spatial Databases (SSD'97), LNCS 1262, pp.298-320

Fabrikant SI, Buttenfield BP (2001) "*Formalizing Semantic Spaces For Information Access*"; Annals of the Association of American Geographers, 91 (2), Oxford: Blackwell Publishers, 2001

Flusser V (1999) "*Die Stadt als Wellental in der Bilderflut*" in "*Auf dem Weg zur telematischen Informationsgesellschaft*", in Medienkultur, Fischer Verlag, Frankfurt am Main, 1999

Foerster Hv (1991a) "*Ethics and Second-Order-Cybernetics*"; Opening Address for the International Conference, Systems and Family Therapy: Ethics, Epistemology, New Methods, Paris, October 1991; http://shr.stanford.edu:80/shreview/4-2/text/foerster.html (last accessed 26.08.1999)

Foerster Hv (1997) "*Wissen und Gewissen: Versuch einer Brücke.*", Suhrkamp Taschenbuch Verlag Wissenschaft (876), Frankfurt am Main, 1993

Foerster Hv (1999) "*Sicht und Einsicht. Versuche zu einer operativen Erkenntnistheorie*", Reihe: Konstruktivismus und systemisches Denken, Carl-Auer-Systeme Verlag, Heidelberg, 1999

Frank AU, Kuhn W (Eds., 1995) "*Spatial Information Theory: A Theoretical Basis for GIS (COSIT 1995)*"; Vol. 988, Lecture Notes in Computer Science, Berlin: Springer Verlag

Gadamer HG (1976) "*Philosophical Hermeneutics*"; California: University of California Press

Golledge RG (1995) "*Primitives of Spatial Knowledge*"; in Nyerges et al. (1995); cit. in Fabrikant, Buttenfield (2001:268)

Golledge RG, Stimson RJ (1997) "*Spatial Behavior: A Geographic Perspective*", New York: Harper & Row, 1997; cit in Fabrikant, S.I. & Buttenfield, B.P. (2001)

Harvey D (1990) "*The Condition of Postmodernity*", Oxford: Blackwell.

Hocevar A, Riedl L (2003) "*Vergleich verschiedener multikriterieller Berwertungsverfahren mit MapModels*"; available online only:
www.corp.at/corp2004/archiv/papers/CORP2003_papers.htm

Jiang, H., Eastman, J.R. (2000): "*Application of fuzzy measures in multi-criteria evaluation in GIS*" in: Int. Journal of Geographic Information Science 2000, Vol.14, No.2., pp. 173-184

Klare J,Van Swaaij L (2000) "*Atlas der Erlebniswelten*", Eichborn Verlag. 2000

Klir GJ, Folger T (1988) "*Fuzzy Sets, Uncertainty and Information*"; Prentice Hall, Englewood Cliffs, NJ, 1988

Knorr-Cetina KD (1981) "*The Manufacture of Knowledge: A Essay on the Constructivist and Contextual Nature of Science*"; Oxford: Pergamon Press

Kratochwil S, Benedikt J (2000) "*Blind Date – Dimensions of a Rendezvous*", 23rd International Wittgenstein Symposium: „*Rationality and Irrationality*"; in: Contributions of the Austrian Ludwig Wittgenstein Society, Volume VIII (1), Berit Brogaard (Eds.), Kirchberg am Wechsel, 2000

Kratochwil S (2001a) "*Raumkybernetik. (0/0/0).(0/0/1).(0/1/0).(1/0/0).(0/0/0)*", (*Space & Cybernetic*) Doctoral Thesis, Faculty for Spatial Planning and Architecture, Vienna University of Technology, 2001

Kratochwil S (2001b) "*Epistemische Raum(ver)suche – zwischen Wissensmanagement und Raumplanung*", Master Thesis (Library and Information Science), Zentrum für Informationsmanagement und Technische Dokumentation, Donau-Universität Krems, 2001

MacEachren AM (1995) "*How maps work: representation, visualization, and design*"; New York: The Guilford Press, 1995

Maturana HR, Varela FJ (1980) "*Autopoiesis and Cognition*"; Reidel, Dordrecht

Medyckyi-Scott D, Hearnshaw HM (eds., 1993) "*Human Factors in Geographical Information Systems*"; London: Belhaven Press

Narayanan N, Hübscher R (1998) "*Visual Language Theory: Towards a Human-Computer Interaction Perspective*", in: Marriott, Meyer (Eds.): "*Visual Language Theory",* Springer Verlag, Berlin, 1998

North FJ (1933) "*Maps, their history and uses, with special reference to Wales*", Cardiff, 1933; http://www.usm.maine.edu/~maps/essays/andrews.htm (last accessed 17.08.2003); "*Definitions of the word map, 1649-1996*"

Nyerges et al. (1995) "*Cognitive Aspects of Human-Computer Interaction for Geographic Information Systems*"; Dordrecht, The Netherlands: Kluwer Academic Publishers

Polanyi M (1958) "*Personal Knowledge: Towards a Post-Critical Philosophy*"; London: Routledge, 1958

Riedl L, Kalasek R (1998) "*MapModels- Programmieren mit Datenflußgraphen*" in: Strobl/Dollinger (Eds., 1998) Angewandte Geographische Informationsverarbeitung: Beiträge zum achten AGIT-Symposium Salzburg 1998, Wichmann, Heidelberg, S.279-288

Riedl L; Vacik H, Kalasek R (2000) "*MapModels: A new approach for spatial suport in silvicultural decision making*", Poster presented at the International Conference "*The application of scientific knowledge to decision making in managing forest ecosystems*", 3.-7. May 1999, Asheville, NC, USA, extended abstract in: *Computers and Electronics in Agriculture,* Vol. 27, pp 407-412

Shields R (1997) "*Spatial Stress and Resistance: Social Meanings of Spatialization*"; in Benko, G. & Strohmayer (Eds., 1997)

Skupin A, Buttenfield BP (1997) "*Spatial Metaphors for Display of Information Science*"; Proceedings, AUTO-CARTO 13, April 1997, Seattle, WA; cit. in Fabrikant, Buttenfield (2001)

Spies M (1993) "*Unsicheres Wissen*"; Spektrum Akademischer Verlag GesmbH, Heidelberg 1993

Spies M (1994) "*Repräsentation unsicheren Wissens*", in - Best, H. et al (Eds., 1994): "*Informations- und Wissensverarbeitung in den Sozialwissenschaften. Beiträge zur Umsetzung neuer Informationstechnologien*", Westdeutscher Verlag, Opladen, 1994

Stanitzek W, Vosskamp W (Eds./2001) "*Schnittstelle. Medien und kulturelle Kommunikation*", DuMont Buchverlag, Köln, 2001

UCGIS (University Consortium for Geographic Information Science) (1996) "*Research Priorities for Geographic Information Science*"; Cartography and Geographic Information System 23 (2); cit. in Fabrikant, Buttenfield (2001)

Varzi AC (2001) "*Vagueness in Geography*", in: Philosophy & Geography 4(1), 2001, pp.49-65

Verkeyn A, Botteldooren D (2002) "*Annoyance Prediction with Fuzzy Rule Bases*", in - Da Ruan et al. (Eds., 2002): "Computational Intelligent Systems for Applied Research", Proceedings of the 5th International FLINS Conference, World Scientific, Singapore

Willke H (1998) "*Systemisches Wissensmanagement*", Lucius & Lucius Verlagsgesellschaft, Stuttgart

Yager RR (eds., 1987) "*Fuzzy Sets and Applications*", selected papers by L. A. Zadeh; John Wiley & Sons, New York, 1987

Yager RR (1988) "*On Ordered Weighted Averaging Aggregation Operations in Multi-crietria Decision Making*"; in - IEEE Transactions on Systems, Man and Cybernetics, 8(1) 1988, pp.183-190

Zadeh LA (1965) "*Fuzzy Sets*", Information and Control 8, pp.338-353, 1965

Zukin S (1995) "*The Cultures of Cities*", Oxford: Blackwell

9. A Valuation of the Reliability of a GIS Based on the Fuzzy Logic in a Concrete Case Study

Ferdinando Di Martino[1], Vincenzo Loia[2], Salvatore Sessa[3], and Michele Giordano[3]

[1]Datitalia Processing S.p.A., Via G. Porzio 4, Isola G5, 80139 Napoli, Italy
email:dimartino@datitalia.it

[2]Dipartimento di Matematica e Informatica, Università degli Studi di Salerno, Via S. Allende, 84081 Baronissi (Salerno), Italy
email:loia@unisa.it

[3]Dipartimento di Costruzioni e Metodi Matematici in Architettura, Università degli Studi di Napoli "Federico II", Via Monteoliveto 3, 80134 Napoli, Italy
email:sessa@unina.it, mikgiordano@tiscali.it

Abstract. The great difficulty of evaluating the quality of the applications concerning environmental problems, mainly for the heterogeneity of input data and their indeterminateness in the error estimations, is a well known problem. The usage of the Fuzzy Logic can be adequate in the treatment of this kind of information, especially when using approximate linguistic labels to define the input data. We have applied this idea in a previous work and here proposed for the study of a GIS of the PROCIDA island (located near Napoli), realized with technology of the Environmental Systems Research Institute and implemented by means of a software tool called FUZZY-SRA.

9.1. Introduction

The concept of "reliability", associated to a territorial information, is understood as a measure of the quality of this information evaluated with tools which are not of deterministic nature, but based on analysis of uncertain and partial data. In a previous work (Di Martino et al. to appear) , we implemented a GIS using a software tool called FUZZY-SRA (Fuzzy Spatial Reliability Analysis) for studying the geographic map of the vulnerability of aquifers realized by utilizing the DRASTIC model (Di Martino et al. to appear) in which was defined the so-called **DPPI** (Drastic Potential Pollution Index), with respect to a determined zone, given from the pondered sum of seven hydro-geological parameters $\mathbf{R_j}$ (see Nielsen (1991) and Ward and Elliot (1995) for more details), each with rating R_j and weight w_j**:**

$$\text{DPPI} = \sum_{j=1,7} w_j \tag{9.1}$$

Roughly speaking, any parameter $\mathbf{R}_j$ has two weights (Table 9.1) and the choice depends obviously from the problem: industrial pollution or agricultural pollution.

Table 9.1. Weights for parameters in DRASTIC model

Parameters	Industrial Weight	Agricultural Weight
DEPTH	5	5
RECHARGE	4	4
AQUIFER MEDIA	3	3
SOIL MEDIA	2	5
TOPOGRAPHY	1	3
IMPACT VADOSE ZONE	5	4
CONDUCTIVITY	3	2

A classification of the polluted zones based on the **DPPI** is only qualitative, hence it is impossible to evaluate the reliability of this classification using traditional statistical methods. The approach in Di Martino et al. (to appear) is indeed based on the Fuzzy Logic, by dividing the area under study into iso-reliable zones, that is in zones where the parameters and the weights of Table 9.1 have (quasi) constant value in accordance to the evaluation of the experts. For each iso-reliable zone we defined the (index of) reliability to be the middle point M_μ of the support [a,b] of a triangular fuzzy number (for short, TFN) $\mu:\mathbf{R}\to[0,1]$, where $\mathbf{R}$ denotes the set of real numbers, (for literature involving fuzzy numbers and related operations we refer to the books of Kaufmann and Gupta (1985) and Mansur (1995)) such that

$$\mu(x) = \begin{cases} 0.0 & \text{if } x \le a \\ \dfrac{x-a}{M_\mu - a} & \text{if } a < x \le M_\mu \\ \dfrac{b-x}{b-M_\mu} & \text{if } M_\mu < x \le b \\ 0.0 & \text{if } b < x \end{cases} \tag{9.2}$$

In the sequel the above TFN is represented by $\mu=(a,M_\mu,b)$, and, when no misunderstanding can arise, we omit the subscript μ in M_μ. The TFN μ (Di Martino et al. to appear) was the final output obtained from algebraic operations executed on the TFNs λ_j (inputs), j=1,…,7, representative, in the same iso-reliable zone, of each parameter R_j. The membership functions of each λ_j were established by considering also the related weights w_j of Table 9.1. The operations used, for combining the above TFNs λ_j, are defined inside an algebraic structure already known in literature (Gisolfi and Loia 1995) and integrated in the mentioned software FUZZY-SRA.

This algebraic structure, recalled in section 9.2, is also used here for the analysis of a GIS of the Procida island (requested by the Local Administration for the resolution of some urban problems, such as, for instance, the location of a public school) in which the following four parameters were taken in consideration:

- spot elevations $\mathbf{S_1}$,
- contour lines $\mathbf{S_2}$,
- buildings $\mathbf{S_3}$,
- network streets $\mathbf{S_4}$.

The whole geographic area of the Procida island was divided in six iso-reliable zones, that is in zones having (quasi) homogeneous values of the parameters $\mathbf{S_j}$ and related weights w_j, j=1,…,4, in accordance to their goodness measured and evaluated from expert surveyors. For each parameter $\mathbf{S_j}$, in each iso-reliable zone, we defined the (index of) reliability to be the middle point of a TFN, obtained combining two other TFNs representing two sub-parameters $\mathbf{P_{1j}}$ and $\mathbf{P_{2j}}$ to which was assigned a weight w_{hj} (h=1,2) representing the relevance of each one with respect to the other one in the same zone for obtaining $\mathbf{S_j}$. The membership functions of the fuzzy sets representing these sub-parameters and the parameters (fuzzy attributes which become layers inside the internal structure of the GIS) and related weights were supplied from the experts and shown in the sections 9.3, 9.4, 9.5 and 9.6, respectively. Successively we passed to calculate the (index of) global reliability of each iso-reliable zone starting from the (indexes of) reliability of each parameter $\mathbf{S_j}$, always using the software tool FUZZY-SRA. The final results are contained in Section9.7 and conclusions in Section 9.8.

9.2. Definition of the algebraic structure

In this section we recall the main properties of the used algebraic structure (Gisolfi and Loia 1995).

9.2.1. The algebraic structure

Let U be the universe of discourse and $\{\alpha_1, \alpha_2,\dots, \alpha_n\}$ be an ordered n-tuple of linguistic labels, each composed from one or more linguistic modifiers and a variable, as, e.g., "α_1 = False","α_2 = More or Less Good",…,"α_i = Good","α_{i+1} = Very Good",…, "α_n = Completely Good", and each represented by suitable TFNs denoted also with α_i , i = 1,2,…,n. Let A be a fuzzy attribute, that is a map $A : U\rightarrow \{\alpha_1, \alpha_2,\dots, \alpha_n\}$, represented by a string of the following type:

$$A = [a_n]^{\alpha}{}_{n}\ [a_{n-1}]^{\alpha}{}_{n-1} \ldots [a_1]^{\alpha}{}_{1} \tag{9.3}$$

where $a_i = A^{-1}(\alpha_i)$ is a subset of U, also called “class” in the sequel. If $A^{-1}(\alpha_i)=$ Ø, then we write $a_i = [-]$. Let B be another fuzzy attribute represented by the following string:

$$B = [b_m]^{\beta_m}\ [b_{m-1}]^{\beta_{m-1}} \ldots [b_1]^{\beta_1} \tag{9.4}$$

where the symbols have a similar meaning to those in Eq. 9.2. In accordance with Gisolfi and Loia (1995), we define the operation Δ between A and B by setting

$$C = (A\ \Delta\ B) = [c_{m+n-1}]^{\gamma}{}_{m+n-1}\ [c_{m+n-2}]^{\gamma}{}_{m+n-2} \ldots [c_1]^{\gamma}{}_{1} \tag{9.5}$$

where, by assuming $n \geq m$ without loss of generality, the subsets $\{c_i\}$ are given by Eq. 9.6 where $i=1,\ldots,m+n-1$:

$$c_i = \begin{cases} \bigcup_{j=1,\ldots,i} (a_{i-j+1} \cap b_j) & \text{if } 1 \leq i \leq m-1 \\ \bigcup_{j=1,\ldots,m} (a_{i-j+1} \cap b_j) & \text{if } m \leq i \leq n-1 \\ \bigcup_{j=i-n+1,\ldots,m} (a_{i-j+1} \cap b_j) & \text{if } n \leq i \leq m+n-1 \end{cases} \tag{9.6}$$

As suggested by Gisolfi and Loia (1995), the subsets c_i can be calculated by using a simple rule based on the usual arithmetical multiplication. The TFNs γ_i, for $i=1,\ldots,m+n-1$, are indeed given by

$$\gamma_i = \begin{cases} \dfrac{1}{k1+k2} * \sum_{j=1}^{i} d2_j * d1_{i-j+1} * (k1 * \alpha_{i-j+1} + k2 * \beta_j) \\ \text{if } 1 \leq i \leq m-1 \\ \dfrac{1}{k1+k2} * \sum_{j=1}^{m} d2_j * d1_{i-j+1} * (k1 * \alpha_{i-j+1} + k2 * \beta_j) \\ \text{if } m \leq i \leq n-1 \\ \dfrac{1}{k1+k2} * \sum_{j=i-n+1}^{m} d2_j * d1_{i-j+1} * (k1 * \alpha_{i-j+1} + k2 * \beta_j) \\ \text{if } n \leq i \leq m+n-1 \end{cases} \tag{9.7}$$

being the above coefficients d_i, for $i=1,\ldots,m+n-1$, defined by Eq. 9.8.

The index $d1_i$ (resp., $d2_i$) represents the number of subsets $\{a_i\}$ (resp., $\{b_i\}$) of the string A (resp., B) involved in the operation of union performed to obtain the subsets $\{c_i\}$ of the resulting fuzzy attribute C, whereas the index k1 (resp., k2) stands for the total number of subsets $\{a_i\}$ of A (resp., $\{b_i\}$ of B) involved in the operation of intersection which gives the subsets $\{c_i\}$ of C.

The following example shall clarify the above concepts and definitions. Let {A,B,A'} be the set of three parameters (fuzzy attributes), U={O1,O2} be two iso-reliable zones and $\alpha_4 = \beta_4 = Cv$, $\alpha_3 = \beta_3 = V$, $\alpha_2 = \beta_2 = Mv$, $\alpha_1 = \beta_1 = F$ be four TFNs with the linguistic labels, in decreasing order in accordance to their meaning, shown in Table 9.2:

$$d_i = \begin{cases} \sum_{j=1}^{i} d2_j * d1_{i-j+1} & \text{if } 1 \le i \le m-1 \\ \sum_{j=1}^{m} d2_j * d1_{i-j+1} & \text{if } m \le i \le n-1 \\ \sum_{j=i-n+1}^{m} d2_j * d1_{i-j+1} & \text{if } n \le i \le m+n-1 \end{cases} \tag{9.8}$$

Table 9.2. The TFNs of the linguistic labels

Label	Description	a	M	b
Cv	Optimum Reliability	0.80	0.90	1.00
V	Good Reliability	0.65	0.75	0.80
Mv	Sufficient Reliability	0.55	0.60	0.65
F	Mediocre Reliability	0.45	0.50	0.55
Sc	Scanty Reliability	0.35	0.40	0.45
Bd	Bad Reliability	0.20	0.30	0.35
Nl	Null Reliability	0.00	0.10	0.20

Suppose the following strings:

$$\begin{aligned} A &= [O1]^{Cv}\,[-]^{V}\,[-]^{Mv}\,[O2]^{F} \\ B &= [-]^{Cv}\,[O1,O2]^{V}\,[-]^{Mv}\,[-]^{F} \\ A' &= [O2]^{Cv}\,[-]^{V}\,[-]^{Mv}\,[O1]^{F} \end{aligned}$$

This means that in the zone O1 (resp., O2) the parameter A (resp. A') has a optimum reliable measure but not A' (resp. A) which has a mediocre reliable measure, whereas the parameter B has good reliable measure in both zones. In order to obtain the string (A Δ B), now we calculate the subsets c_i by taking in account that m=n=4:

$$\begin{array}{r} [O1]\;[-]\;[-]\;[O2] \\ [-]\;[O1,O2]\;[-]\;[-] \\ \hline [-]\;[-]\;[-]\;[-] \end{array}$$

$$\begin{array}{ccccccc}
 & & [-] & [-] & [-] & [-] & \\
 & [O1] & [-] & [-] & [O2] & & \\
[-] & [-] & [-] & [-] & & & \\
\hline
[-] & [O1] & [-] & [-] & [O2] & [-] & [-]
\end{array}$$

In the calculation of the TFNs γ_i, we put for brevity:

$$a_{i,j} = d2_j * d1_{i-j+1} * (k1 * \alpha_{i-j+1} + k2 * \beta_j) \tag{9.9}$$

We observe that $k1= k2=1$, $d2_i = d1_i = 1$ for every $i = 1,\ldots,4$ and thus we have:

$$\begin{array}{ccccccc}
 & & & [0.80,1.00,1.20] & [0.65,0.70,0.80] & [0.55,0.60,0.65] & [0.45,0.50,0.55] \\
 & & & [0.80,1.00,1.20] & [0.65,0.75,0.80] & [0.55,0.60,0.65] & [0.45,0.50,0.55] \\
\hline
 & & & a_{4,1} & a_{3,1} & a_{2,1} & a_{1,1} \\
 & & a_{4,2} & a_{3,2} & a_{2,2} & a_{1,2} & \\
 & a_{4,3} & a_{3,3} & a_{2,3} & a_{1,3} & & \\
a_{4,4} & a_{3,4} & a_{2,4} & a_{1,4} & & & \\
\hline
\gamma_7 & \gamma_6 & \gamma_5 & \gamma_4 & \gamma_3 & \gamma_2 & \gamma_1
\end{array}$$

where

$a_{1,1} = 1*1* (1*[0.45,0.50,0.55]+1*[0.45,0.50,0.55]) = [0.90,1.00,1.10]$
$a_{2,1} = 1*1* (1*[0.55,0.60,0.65]+1*[0.45,0.50,0.55]) = [1.00,1.10,1.20]$
$a_{3,1} = 1*1* (1*[0.65,0.70,0.80]+1*[0.45,0.50,0.55]) = [1.10,1.20,1.35]$
$a_{4,1} = 1*1* (1*[0.80,1.00,1.20]+1*[0.45,0.50,0.55]) = [1.25,1.50,1.75]$
$a_{1,2} = 1*1* (1*[0.45,0.50,0.55]+1*[0.55,0.60,0.65]) = [1.00,1.10,1.20]$
$a_{2,2} = 1*1* (1*[0.55,0.60,0.65]+1*[0.55,0.60,0.65]) = [1.10,1.20,1.30]$
$a_{3,2} = 1*1* (1*[0.65,0.70,0.80]+1*[0.55,0.60,0.65]) = [1.20,1.30,1.45]$
$a_{4,2} = 1*1* (1*[0.80,1.00,1.20]+1*[0.55,0.60,0.65]) = [1.35,1.60,1.85]$
$a_{1,3} = 1*1* (1*[0.45,0.50,0.55]+1*[0.65,0.70,0.80]) = [1.10,1.20,1.35]$
$a_{2,3} = 1*1* (1*[0.55,0.60,0.65]+1*[0.65,0.70,0.80]) = [1.20,1.30,1.45]$
$a_{3,3} = 1*1* (1*[0.65,0.70,0.80]+1*[0.65,0.70,0.80]) = [1.30,1.40,1.60]$
$a_{4,3} = 1*1* (1*[0.80,1.00,1.20]+1*[0.65,0.70,0.80]) = [1.45,1.70,2.00]$
$a_{1,4} = 1*1* (1*[0.45,0.50,0.55]+1*[0.80,1.00,1.20]) = [1.25,1.50,1.75]$
$a_{2,4} = 1*1* (1*[0.55,0.60,0.65]+1*[0.80,1.00,1.20]) = [1.35,1.60,1.85]$
$a_{3,4} = 1*1* (1*[0.65,0.70,0.80]+1*[0.80,1.00,1.20]) = [1.45,1.70,2.00]$
$a_{4,4} = 1*1* (1*[0.80,1.00,1.20]+1*[0.80,1.00,1.20]) = [1.60,2.00,2.40]$

Furthermore we have that

$d_1=d1_1*d2_1=1*1=1$
$d_2= d1_1*d2_2+d1_2*d2_1=1*1+1*1=2$
$d_3=d1_3*d2_1+d1_2*d2_2+d1_1*d2_3=1*1+1*1+1*1=3$
$d_4=d1_4*d2_1+d1_3*d2_2+d1_2*d2_3+d1_1*d2_4=1*1+1*1+1*1+1*1=4$
$d_5=d1_4*d2_2+d1_3*d2_3+d1_2*d2_4=1*1+1*1+1*1=3$

$d_6=d1_4*d2_3+d1_3*d2_4=1*1+1*1=2$
$d_7=d1_4*d2_4=1*1=1$

Therefore we have the following TFNs:

$\gamma_1=(1/2)*a_{1,1}=[0.45,0.50,0.55]$
$\gamma_2=(1/4)*(a_{2,1}+a_{1,2})=[0.50,0.55,0.60]$
$\gamma_3=(1/6)*(a_{3,1}+a_{2,2}+a_{1,3})=[0.55,0.90,1.00]$
$\gamma_4=(1/8)*(a_{4,1}+a_{3,2}+a_{2,3}+a_{1,4})=[0.61,0.70,0.80]$
$\gamma_5=(1/6)*(a_{4,2}+a_{3,3}+a_{2,4})=[0.66,0.76,0.88]$
$\gamma_6=(1/4)*(a_{4,3}+a_{3,4})=[0.72,0.85,1.00]$
$\gamma_7=(1/2)*a_{4,4}=[0.80,1.00,1.20]$

Then we deduce the following resulting fuzzy attribute:

$$A \Delta B = [-]^{\gamma_7}[O1]^{\gamma_6}[-]^{\gamma_5}[-]^{\gamma_4}[O2]^{\gamma_3}[-]^{\gamma_2}[-]^{\gamma_1} \tag{9.10}$$

Considering the composition (A Δ B) Δ A', we have that k1 = 2 and the new $d1_i$, for i = 1,…,7, are then $d1_1 = 1$, $d1_2 = 2$, $d1_3 = 3$, $d1_4 = 4$, $d1_5 = 3$, $d1_6 = 2$, $d1_7 = 1$ while it is k2 = 1 and $d2_1 = 1$, $d2_2 = 1$, $d2_3 = 1$, $d2_4 = 1$. Since now n=7 and m = 4, the fuzzy attribute (AΔB)ΔA' has 10 classes to which 10 TFNs are associated and built with the above formulas. It is possible to show (Gisolfi and Loia 1995) that (AΔB)ΔA' = A'Δ(AΔB) but we omit this fact for brevity.

9.2.2. The weights of the attributes

The first step, which precedes the above mentioned operations over the strings, consists in the determination of the weights of each attribute connected to a fixed zone because they can vary by changing zone. Strictly speaking, the above model implies the necessity to build a mean of the weights of the zones which have the same linguistic label in an attribute. This mean shall be the weight of that linguistic label, which in turn is multiplied for the middle point of the TFN, representing the same label, giving a number q, of which we consider the smallest integer contained in it, i.e. INT(q). At the right of the same linguistic label, thus we create INT(q) new linguistic labels "approximated" with the procedure of the Subsection 9.2.3. For example, we consider six zones O1, …, O6 in which the fuzzy attribute S1 has received six values with the related weights W1 in accordance to Table 9.3. Then if U = {O1,O2,O3,O4,O5,O6}, then the fuzzy attribute S1 is represented by the string:

$$\mathbf{S1} = [01,03,04]^{Cv}\,[02]^{V}\,[05]^{Mv}\,[06]F \tag{9.11}$$

and consider the linguistic label Cv. For simplicity, let us denote with W_{1i} the weight of the attribute S1 for the zones Oi with i =1,3,4. Then the mean value $W_{1,Cv}$ for Cv is equal to 2, to be multiplied for 1.0 (cfr. Table 9.2) giving $N_{1,Cv} =$

INT($W_{1,Cv}$ * 1.0) = 2 which represents the number of new linguistic labels, inserted at the right of Cv. Other new linguistic labels shall be not inserted at the right of the three remaining labels since we have, with evident meaning of the symbology, $W_{1,V} = W_{12} = 1$, $W_{1,Mv} = W_{15} =1$ and $W_{1,F}=W_{16}=1$ obtaining $N_{1,V}$ = INT($W_{1,V}$ * 0.7) = 0, $N_{1,Mv}$ = INT($W_{1,Mv}$* 0.6) = 0 and $N_{1,F}$ = INT($W_{1,F}$ * 0.5) = 0. Then we obtain the following finer string for the attribute S1:

$$\mathbf{S1} = [01,03,04]^{Cv}\,[-]^{Cv,2}\,[-]^{Cv,1}\,[02]^{V}\,[05]^{Mv}\,[06]^{F} \qquad (9.12)$$

Table 9.3. Values for W1

ID	S1	W1
O1	Cv	3
O2	V	1
O3	Cv	2
O4	Cv	1
O5	Mv	1
O6	F	1

This methodology gives the advantage to improve the position of the objects (in our case study, the iso-reliable zones) in the set of the attributes, just bearing in mind the new linguistic labels to which the objects can be associated. The calculation of the membership functions for the TFNs, representing the new linguistic labels, is made in the following way:

> Let β be the considered linguistic label present in the the attribute **Si** and let $N_{i,\beta}$ be the number of the new linguistic labels obtained with the above procedure. Let α be the linguistic label immediately following β in the linguistic labels of **Si.** For every t = 1,…, $N_{i,\beta}$, we put $a_{\beta,t} = a_\alpha + t*(a_\beta - a_\alpha)/(N_{i,\beta}+1)$ and similarly for $M_{\beta,t}$ and $b_{\beta,t}$. Then $[a_{\beta,t},M_{\beta,t},b_{\beta,t}]$ is the TFN representative of the linguistic label β,t.

Returning to the example discussed above, we have the differences $a_{Cv}- a_V$ = =0.25, $M_{Cv}- M_V$ =0.30, $b_{Cv} - b_V$ =0.20 to be divided for the number ($N_{1,Cv}$ + 1), where $N_{1,Cv}$ =2.We obtain thus $(a_{Cv} - a_V)/(N_{1,Cv} + 1) = 0.083$, $(M_{Cv}- M_V)/(N_{1,Cv} + 1) = 0.10$ and $(b_{Cv}-b_V)/(N_{1,Cv} + 1) = 0.66$. Then we deduce that $a_{Cv,2} = a_V + 2 * (a_{Cv} - a_V)/(N_{1,Cv} + 1) = 0.65 + 2 * 0.083 = 0.816$, $a_{Cv,1} = a_V + 1 * (a_{Cv} - a_V)/(N_{1,Cv} + 1)= 0.65 + 1* 0.083 = 0.733$. Similarly we obtain $M_{Cv,2}$ =1.30, $M_{Cv,1}$ =1.00 and $b_{Cv,2}$ =1.40, $b_{Cv,1}$=1.10, that is [0.816,1.30,1.40] and [0.733,1.00,1.10] are the respective TFNs representative of Cv,2 and Cv,1.

9.2.3. Approximation of the linguistic labels

Some TFNs obtained in the final fuzzy attribute, after the successive composition of several strings, must be reconverted in linguistic labels, which can be approximated to known TFNs using the following procedure:

> Let β be the TFN to be approximated and α, γ be TFNs known (that is the meaning of their linguistic labels is known) such that $M_\alpha \le M_\beta \le M_\gamma$. By setting $d = M_\gamma - M_\alpha$ and if $M_\alpha \le M_\beta \le M_\alpha + d/10$, then we put $\beta = \alpha$; if $M_\alpha + d/10 < M_\beta \le M_\alpha + 3d/10$, then we say β is "Next To α" and we write $\beta = NT[\alpha]$; if $M_\alpha + 3d/10 < M_\beta \le M_\alpha + 7d/10$, then we say β is "Included Between α and γ" and we write $\beta = IB[\alpha,\gamma]$; if $M_\alpha + 7d/10 < M_\beta \le M_\alpha + 9d/10$, then we say β is "Before To γ" and we write $\beta = BT[\gamma]$; if $M_\alpha + 9d/10 < M_\beta \le M_\gamma$, then we put $\beta = \gamma$. For instance, taking in account the TFNs of Table 9.3, let $\beta = \gamma_6$ of Subsection 2.1. Since $M_V \le M_\beta \le M_{Cv}$ and $d=0.30$, it is easily seen that $\beta = IB[V,Cv]$.

We note that no matter of comparison between a_α, a_β, a_γ and similarly for b_α, b_β, b_γ is requested in this procedure.

9.3. The layer "Spot Elevations" S1

This parameter (layer) is related to the spot elevations (shortly, SE) of each iso-reliable zone and it is performed from two sub-parameters $\mathbf{P}_{11}$ and $\mathbf{P}_{21}$, represented from the values (see Tables 9.4 and 9.5) "d_{11}=density of uncoded SE" and "d_{21}=density of coded SE per ha", respectively (ha stands for hectare=10000 m^2). In other words, we have

$$d_{11} = \text{uncoded SE}/(\text{uncoded SE} + \text{coded SE})$$

and

$$d_{21} = (\text{coded SE}/\text{area iso-reliable zone}) * 100.$$

Table 9.4. Layer "SPOT ELEVATIONS": values of the sub-parameter $\mathbf{P}_{11}$

Zone	Uncoded SE	Coded SE	Total SE	d_{11}
O1	20	332	352	0.056818
O2	19	505	524	0.036260
O3	12	157	169	0.071006
O4	5	73	78	0.064103
O5	33	412	445	0.074157
O6	16	170	186	0.086022

By taking in account the TFNs of Table 9.2 and the evident fact that the sub-parameter $\mathbf{P}_{21}$ has greater relevance than $\mathbf{P}_{11}$ in the performance of the parameter **S1** (the weight $\mathbf{w_{21}}$ of $\mathbf{P_{21}}$ increases if the area of the iso-reliable zone increases), the experts, considering the values d_{11} and d_{21} as middle points of the support of TFNs, have suggested the following TFNs and related weights, reported in Table 9.6, for both sub-parameters:

Table 9.5. Layer "SPOT ELEVATIONS": values of the sub-parameter $\mathbf{P_{21}}$

Zone	Coded SE	Area zone (m^2)	d_{21}
O1	332	747649.73	0.044406
O2	505	1330929.13	0.037943
O3	157	369991.46	0.042433
O4	73	345378.48	0.021136
O5	412	968092.37	0.042558
O6	170	409442.32	0.041520

Table 9.6. Layer "SPOT ELEVATIONS": labels of the TFNs and weights of $\mathbf{P}_{11}$ and $\mathbf{P}_{21}$

Zone	d_{11}	Labels of $\mathbf{P}_{11}$	w_{11}	d_{21}	Labels of $\mathbf{P}_{21}$	w_{21}
O1	0.056818	Mv	2	0.044406	Cv	4
O2	0.036260	Cv	3	0.037943	Mv	5
O3	0.071006	Sc	2	0.042433	V	3
O4	0.064103	F	1	0.021136	Nl	3
O5	0.074157	Bd	2	0.042558	V	4
O6	0.086022	Nl	2	0.041520	V	3

Hence we can define the following strings:

$$\mathbf{P}_{11} = [O2]^{Cv}\ [O1]^{Mv}\ [O4]^{F}\ [O3]^{Sc}\ [O5]^{Bd}\ [O6]^{Nl} \tag{9.13a}$$
$$\mathbf{P}_{21} = [O1]^{Cv}\ [O3,O5,O6]^{V}\ [O2]^{Mv}\ [O4]^{Nl} \tag{9.13b}$$

to which it is possible to apply the analysis of the weights of Subsection 9.2.2 and afterwards the algebraic operations described in Subsection 9.2.1. Successively we apply the algorithm of approximation of Subsection 9.2.3 to the TFNs obtained and, by omitting all these calculations for brevity, we limit ourselves to say that the final string **S1** has many empty classes with labels like $[\text{-}]^{V}$, $[\text{-}]^{Mv}$, $[\text{-}]^{Sc}$. Then, by avoiding to write these uninteresting classes in the final string **S1**, we have that

$$\mathbf{S1} = [O1,O2]^{IB[V,Cv]}\,[O3]^{IB[F,Mv]}\,[O5]^{F}\,[O6]^{BT[F]}\,[O4]^{Bd}, \tag{9.14}$$

where the linguistic labels are represented by the TFNs included in the Table 9.7, which reports the description of all TFNs used in this case study. Figure 9.1 gives a graphical representation of the layer **S1**.

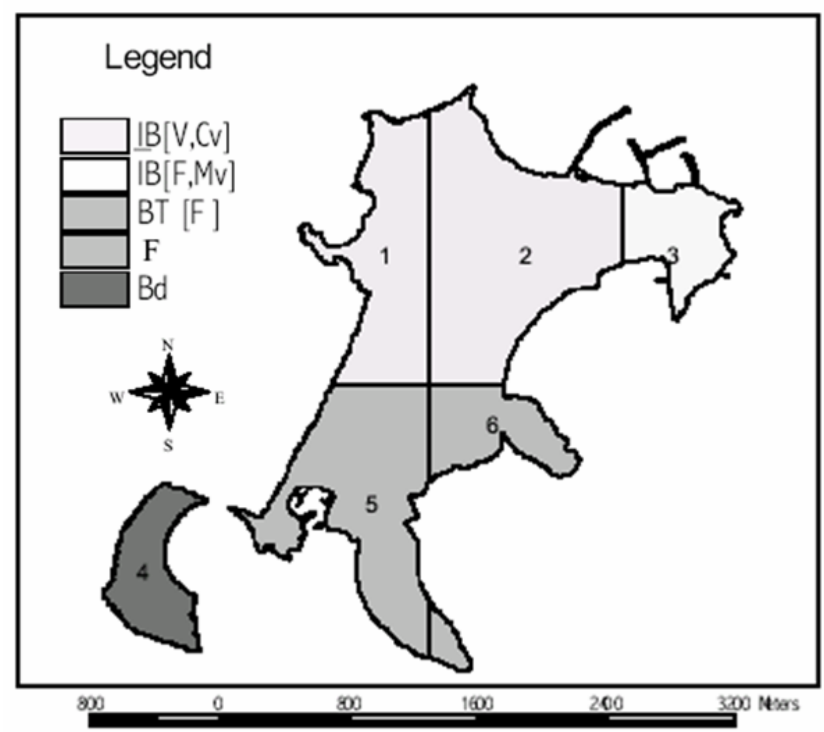


Fig 9.1. Layer "SPOT ELEVATIONS"

Table 9.7. Labels of the TFNs used in this case study

LABEL	DESCRIPTION	a	M	b
Cv	Optimum	0.80	1.00	1.20
NT[Cv]	Quasi Optimum	0.77	0.93	1.10
IB[V,Cv]	Very Very Good	0.73	0.85	1.00
BT[V]	Very good	0.70	0.78	0.90
V	Good	0.65	0.70	0.80
NT[V]	Quasi Good	0.62	0.68	0.75
IB[Mv,V]	Very Very Sufficient	0.60	0.65	0.70
BT[Mv]	Very Sufficient	0.58	0.62	0.67
Mv	Sufficient	0.55	0.60	0.65
NT[Mv]	Quasi Sufficient	0.52	0.57	0.63
IB[F,Mv]	Very Very Mediocre	0.50	0.55	0.60
BT[F]	Very Mediocre	0.48	0.52	0.57
F	Mediocre	0.45	0.50	0.55
NT[F]	Quasi Mediocre	0.42	0.48	0.53
IB[Sc,F]	Very Scanty	0.40	0.45	0.50
BT[Sc]	Very Very Scanty	0.38	0.42	0.47
Sc	Scanty	0.35	0.40	0.45
NT[Sc]	Quasi Scanty	0.32	0.38	0.42
IB[Bd,Sc]	Very Bad	0.28	0.35	0.40
BT[Bd]	Very Very Bad	0.25	0.32	0.37
Bd	Bad	0.20	0.30	0.35
NT[Bd]	Quasi Bad	0.15	0.25	0.32
IB[Nl,Bd]	Quasi Null	0.10	0.20	0.28
BT[Nl]	Very Quasi Null	0.05	0.15	0.25
Nl	Null	0.00	0.10	0.20

9.4. The layer “Contour Lines” S2

Dividing the area (expressed in m^2) of any iso-reliable zone in hectares, it was calculated the number of contour lines per hectare and afterwards the related mean, thus obtaining the mean density **C** of the contour lines for ha in each iso-reliable zone O_i, i=1,…,6. Afterwards, in each zone O_i, it was calculated the meanslope **S**, that is the mean of all the slopes (measured in radiants) per hectare. Since **C** increases if each zone O_i has strong variations of altimetry, that is **S** achieves an high value, then we can deduce that the parameter **S2** is performed from two sub-parameters $\mathbf{P}_{12}$ and $\mathbf{P}_{22}$, respectively, represented from

d_{12} = mean density of the number of contour lines for ha/meanslope

and

d_{22} = standard deviation (number of contour lines for ha/meanslope).

The sub-parameter $\mathbf{P}_{12}$ has greater relevance than $\mathbf{P}_{22}$ in the performance of the parameter **S2,** in other words, we have $w_{12} \geq w_{22}$. Then, always considering the linguistic labels of the TFNs of Table 9.2, the experts have suggested the following TFNs and related weights, reported in the successive Table 9.9. Then we can consider the following strings:

$$\mathbf{P}_{12} = [O4]^{Cv} [O6]^{F} [O1]^{Bd} [O2,O3,O5]^{Nl} \quad (9.15a)$$
$$\mathbf{P}_{22} = [O4]^{Cv} [O6]^{V} [O2,O5]^{F} [O1]^{Bd} [O3]^{Nl} \quad (9.15b)$$

Table 9.8. Layer “CONTOUR LINES”: values of the sub-parameters $\mathbf{P}_{12}$ and $\mathbf{P}_{22}$ which are normalized.

Zone	Mean density of contour lines	Meanslope	d_{12}	d_{22}
O1	0.0671	0.2113	0.317558	0.446207
O2	0.0286	0.1066	0.268293	0.501266
O3	0.1092	0.3976	0.274648	0.425268
O4	0.3950	0.6655	0.593539	0.585138
O5	0.0557	0.2146	0.259553	0.497379
O6	0.1502	0.3525	0.426099	0.552639

to which it is possible to apply the algorithms of Section 9.2, giving the final string:

$$\mathbf{S2} = [O4]^{Cv}\,[O6]^{IB[Mv,V]}\,[O1]^{BT[Bd]}[O2,O5]^{NT[Bd]}[O3]^{Nl} \tag{9.16}$$

where the labels have the meaning given in Table 9.7. Figure 9.2 gives a graphical representation of the layer **S2.**

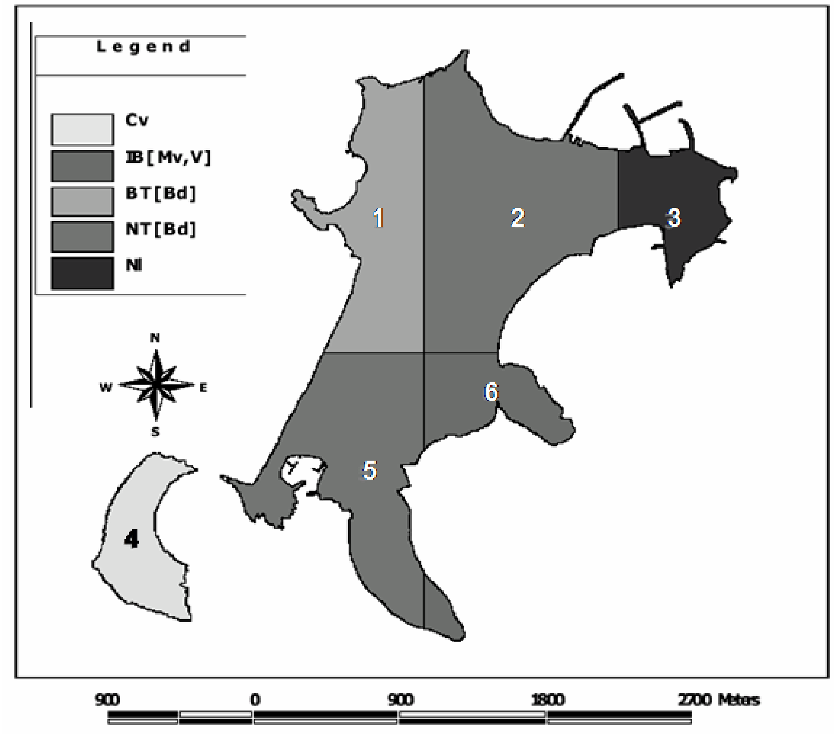


Fig. 9.2. Layer "CONTOUR LINES"

Table 9.9. Layer "CONTOUR LINES": labels of the TFNs and weights of $\mathbf{P}_{12}$ and $\mathbf{P}_{22}$

Zone	d_{12}	Labels of P_{12}	w_{12}	d_{22}	Labels of P_{22}	w_{22}
O1	0.317558	Bd	2	0.446207	Bd	2
O2	0.268293	Nl	3	0.501266	F	2
O3	0.274648	Nl	2	0.425268	Nl	1
O4	0.593539	Cv	2	0.585138	Cv	1
O5	0.259553	Nl	3	0.497379	F	2
O6	0.426099	F	2	0.552639	V	1

9.5. The layer "Buildings" S3

This layer derives from a comparison between the perimeters of all the buildings of the whole island supplied by the Local Administration (here denoted with the symbol "Perimeter_j"for the j-th building of the iso-reliable zone under study) and the perimeters graphically measured (here denoted with the symbol "Length(Shape)_j") on the polygons representing the buildings itself. Further, since the Local Administration did not supply data about the area of all the buildings in the planimetry of the island, we have considered the area Area_j of any building, graphically measured, as equivalent to that one of a circle having the length of the circonference equal to the known perimeter of the building itself. Then, in order to perform the parameter **S3,** we have considered in each iso-reliable zone O_i**,** i=1,…,n, two sub-parameters $\mathbf{P}_{13}$ and $\mathbf{P}_{23}$ **,** respectively, represented from "d_{13} = mean of the absolute value of the difference [Perimeter_j - Length(Shape)_j]" and "d_{23} = mean of the absolute value of the difference [Area_j – (Perimeter_j^2 /4π)]". Strictly speaking, we have used Eq. 9.17 and 9.18

$$d_{13} = \frac{\sum_{j=1}^{n} \left| Perimeter_j - Length(Shape)_j \right|}{n} \tag{9.17}$$

$$d_{23} = \frac{\sum_{j=1}^{n} \left| Area_j - \frac{perimeter_j^{\,2}}{4\pi} \right|}{n} \tag{9.18}$$

where n is the number of the buildings in the iso-reliable zone Oi under consideration. If we put for brevity:

$$SUM1 = \sum_{j=1}^{n} \left| Perimeter_j - Length(Shape)_j \right| \quad (9.19)$$

and

$$SUM2 = \sum_{j=1}^{n} \left| Area_j - (Perimeter_j^{\;2} / 4\pi) \right| \quad (9.20)$$

we have the results for $\mathbf{P}_{13}$ and $\mathbf{P}_{23}$ given in Table 9.10. Here the sub-parameter $\mathbf{P}_{13}$ has more greater relevance than $\mathbf{P}_{23}$ in the performance of **S3,** in other words, we have $w_{13} > w_{23}$. Then, always considering the linguistic labels of the TFNs of Table 9.2, the experts, normalizing the values of d_{13} and d_{23}, have suggested the TFNs and related weights reported in the successive Table 9.11. Then, by using the representation under form of strings, we have:

$$\mathbf{P}_{13} = [O1,O5,O6]^{Cv}\,[O4]^{V}\,[O2]^{Mv}\,[O3]^{Nl} \quad (9.21a)$$
$$\mathbf{P}_{23} = [O1,O4,O5,O6]^{Cv}\,[O2]^{Mv}\,[O3]^{Nl} \quad (9.21b)$$

from which, after applying the algorithms of Section 9.2, we get the final string:

$$\mathbf{S3} = [O1,O5,O6]^{Cv}\,[O4]^{IB[V,Cv]}\,[O2]^{Mv}\,[O3]^{Nl} \quad (9.22)$$

with the labels having the meaning given in Table 9.7. Figure 9.3 gives a graphical representation of the layer **S3**, in which we note that the (index of) reliability of the zone O3 is null because $\mathbf{d}_{23}$ assumes a very high value, hence it does not give contribution to perform **S3**.

Table 9.10. Layer “BUILDINGS”: values of the sub-parameters $\mathbf{P}_{13}$ and $\mathbf{P}_{23}$

Zone	**n**	**SUM1**	$\mathbf{d}_{13}$	**SUM2**	$\mathbf{d}_{23}$
O1	667	2717.34	4.0740	59873.01	89.7646
O2	1806	12689.95	7.0266	302649.9	167.5802
O3	549	6526.71	11.8884	179219.2	326.4466
O4	14	66.05	4.7179	1229.65	87.8321
O5	1319	5866.23	4.4475	122516.83	92.8861
O6	261	1137.15	4.3569	23349.32	89,4610

Table 9.11. Layer "BUILDINGS": labels of the TFNs and weights of $\mathbf{P}_{13}$ and $\mathbf{P}_{23}$

Zone	$\mathbf{d}_{13}$	Labels of $\mathbf{P}_{13}$	$\mathbf{w}_{13}$	$\mathbf{d}_{23}$	Labels of $\mathbf{P}_{23}$	$\mathbf{w}_{23}$
O1	0.3426	Cv	2	0.2749	Cv	1
O2	0.5910	Mv	4	0.5133	Mv	2
O3	1.0000	Nl	2	1.0000	Nl	1
O4	0.3968	V	2	0.2690	Cv	1
O5	0.3741	Cv	3	0.2845	Cv	2
O6	0.3664	Cv	2	0.2740	Cv	1

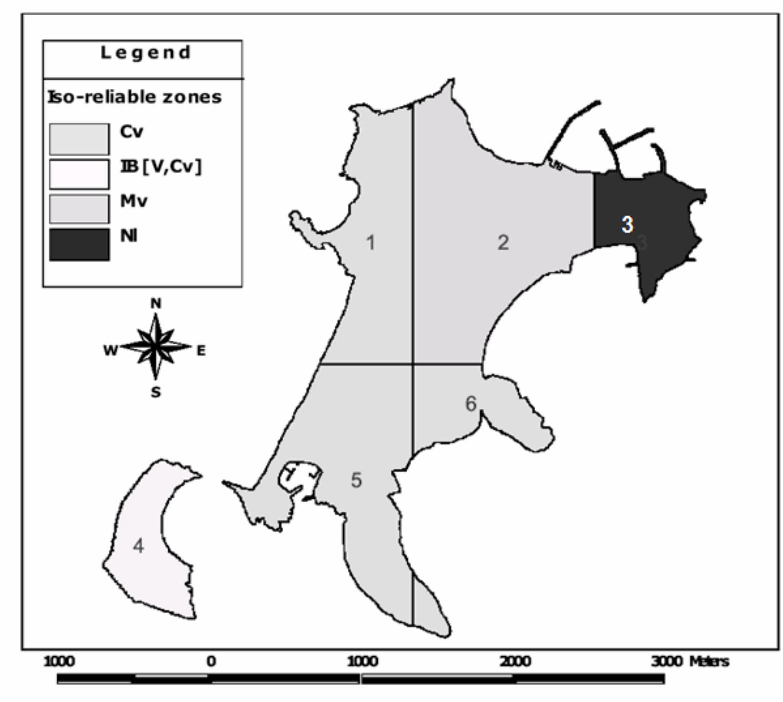


Fig. 9.3. Layer "BUILDINGS"

9.6. The layer " Network Streets" S4

This network is realized by taking in account roads, retaining walls, pathways, enclosures, etc. Since there are also streets which lead to buildings, then it is necessary to establish a buffering area **B** (see Figure 9.4) in such a way that if it does not intersect the polygons representing these buildings. Thus, in order to perform the layer **S4,** in each iso-reliable zone O_i, i=1,…,6, we need to know two sub-parameters $\mathbf{P}_{14}$ and $\mathbf{P}_{24}$, respectively, represented from

d_{14} = density of area of the polygons (buildings included in the zone Oi) outside **B**

and

d_{24} = standard deviation of the areas of the same polygons outside **B**

If n is the number of buildings of the zone O_i outside **B** and if A_j is the area (in m^2) of the j-th building of the zone O_ii outside **B**, by setting A=area of the zone O_i, we have the following formulas:

$$d_{14} = \frac{\sum_{j=1}^{n} A_j}{A} \tag{9.23}$$

$$d_{24} = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left[\frac{A_j}{A} - \frac{d_{14}}{n}\right]^2} \tag{9.24}$$

Note that d_{14}/n is the mean of the area of a polygon representing a building outside **B**. It is clear that if d_{24} achieves a high value, then there are many buildings not intersected from the buffering area and hence noteworthy information is missing for this network (in other words, the whole information on the streets leading to these buildings is completely absent). An optimal situation should consist in a low value d_{14} of the sub-parameter $\mathbf{P}_{14}$ and by taking in consideration the areas of the polygons outside the buffering area, we have the values for d_{14} and d_{24} given in Table 9.12. Note that the values of n are obviously lower of those ones appearing in Table 9.10 except the zone **O4** corresponding to the small island of Vivara in which the streets are completely absent. Low values of d_{14} correspond to TFNs with optimal or good reliability label and, by considering that the sub-parameter $\mathbf{P}_{14}$ is certainly more important than $\mathbf{P}_{24}$ (that is $w_{14} \geq w_{24}$), the experts have suggested the TFNs and related weights given in Table 9.13.

Table 9.12. Layer "NETWORK STREETS": values of the sub-parameters $\mathbf{P}_{14}$ and $\mathbf{P}_{24}$

Zone	n	d_{14}	d_{24}
O1	639	0.0088	0.0933
O2	1776	0.0083	0.0906
O3	540	0.0480	0.2137
O4	14	0.0006	0.0235
O5	1293	0.0114	0.1063
O6	242	0.0053	0.0723

Table 9.13. Layer "NETWORK STREETS": TFNs and weights of $\mathbf{P}_{14}$ and $\mathbf{P}_{24}$

Zone	d_{14}	Labels of $\mathbf{P}_{14}$	w_{14}	d_{24}	Labels of $\mathbf{P}_{24}$	w_{24}
O1	0.0088	V	2	0.2749	Sc	2
O2	0.0083	V	3	0.5133	Sc	2
O3	0.0480	Nl	2	1.0000	Cv	1
O4	0.0006	Cv	2	0.2690	Nl	1
O5	0.0114	V	3	0.2845	F	2
O6	0.0053	Cv	2	0.2740	Sc	1

Then, by using the representation under form of strings, we have:

$$\mathbf{P}_{14} = [O4,O6]^{Cv}\,[O1,O2,O5]^{V}\,[O3]^{Nl} \tag{9.25a}$$
$$\mathbf{P}_{24} = [O3]^{Cv}\,[O5]^{F}\,[O1,O2,O6]^{Sc}\,[O4]^{Nl} \tag{9.25b}$$

from which, after applying the algorithms of Section 9.2, we deduce the final string:

$$\mathbf{S4} = [O4]^{BT[V]}\,[O1]^{IB[Mv,V]}\,[O2]^{Mv}\,[O3,O5,O6]^{IB[F,Mv]} \tag{9.26}$$

with the labels having the meaning given in Table 9.7. Figure 9.5 gives a graphical representation of the layer **S4**.

Fig. 9.4. The buffering area of the “NETWORK STREETS”

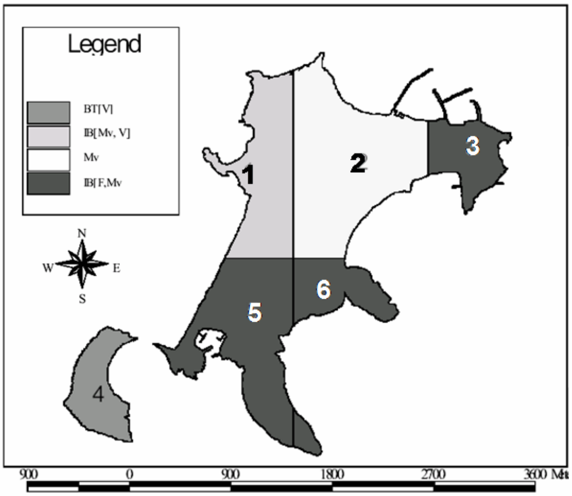


Fig. 9.5. Layer “NETWORK STREETS”

9.7. Global Reliability

By using again the software tool FUZZY-SRA, we passed to calculate the (index of) global reliability of each iso-reliable zone starting from the (indexes of) reliability of each parameter $\mathbf{S_j}$,. In other words, we have reconsidered the above strings:

$$\mathbf{S1} = [O1,O2]^{IB[V,Cv]}\,[O3]^{IB[F,Mv]}\,[O5]^{F}\,[O6]^{BT[F]}\,[O4]^{Bd} \tag{9.27a}$$
$$\mathbf{S2} = [O4]^{Cv}\,[O6]^{IB[Mv,V]}\,[O1]^{BT[Bd]}[O2,O5]^{NT[Bd]}[O3]^{Nl} \tag{9.27b}$$
$$\mathbf{S3} = [O1,O5,O6]^{Cv}\,[O4]^{IB[V,Cv]}\,[O2]^{Mv}\,[O3]^{Nl} \tag{9.27c}$$
$$\mathbf{S4} = [O4]^{BT[V]}\,[O1]^{IB[Mv,V]}\,[O2]^{Mv}\,[O3,O5,O6]^{IB[F,Mv]} \tag{9.27d}$$

Following the opinion of the experts, if we call w_j (j=1,2,3,4) the weight assigned to the layer **Sj** in the iso-reliable zone **Oi (**i=1**,…,**6**),** it was decided generally to give a relevant value to w_1 and w_3 with respect to w_2 and w_4 because the layers **S1** and **S3** were considered from the Local Administration more important than the layers **S2** and **S4**. The weights are given in the following Table 9.14:

Table 9.14. Weights of the single layers in each iso-reliable zone

Zone	w_1	w_2	w_3	w_4
O1	2	1	2	2
O2	3	1	4	3
O3	2	1	2	1
O4	2	2	1	1
O5	3	1	3	2
O6	2	2	1	1

Hence, applying the algorithms of Section 2, we got the following final string:

$$\mathbf{S} = [O1,O6]^{NT[IB[Mv,V]]}\,[O4]^{BT[Mv]}\,[O2,O5]^{NT[Mv]}\,[O3]^{NT[NT[Sc]]} \tag{9.28}$$

where the linguistic labels represent the TFNs listed in Table 9.15

:

Table 9.15. Weights of the single layers in each iso-reliable zone

Zone	w_1	w_2	w_3	w_4
O1	2	1	2	2
O2	3	1	4	3
O3	2	1	2	1
O4	2	2	1	1
O5	3	1	3	2
O6	2	2	1	1

Bearing in mind the TFNs of Table 9.7, it appear evident the meaning of the (indexes of) global reliability of each iso-reliable zone. These indexes (which we have approximated) are listed in Table 9.16:

Table 9.16. Description of the final TFNs outputs

Label	a	M	b
NT[IB[Mv,V]]	0.619867	0.640555	0.657656
BT[Mv,V]	0.600945	0.620907	0.638046
NT[Mv]	0.551823	0.570301	0.587773
NT[NT[Sc]]	0.354477	0.369847	0.392198

Table 9.17. Values of the indexes of reliability for each iso-reliable zone

Zone	Index of S1	Index of S2	Index of S3	Index of S4	Global Index
O1	0.85	0.32	1.00	0.65	0.640
O2	0.85	0.25	0.60	0.60	0.570
O3	0.55	0.10	0.10	0.55	0.370
O4	0.30	1.00	0.85	0.78	0.621
O5	0.50	0.25	1.00	0.55	0.570
O6	0.52	0.65	1.00	0.55	0.640

As appears in Figure 9.6, it is evident the minor global reliability of the zone **O3** with respect to the remaining ones.

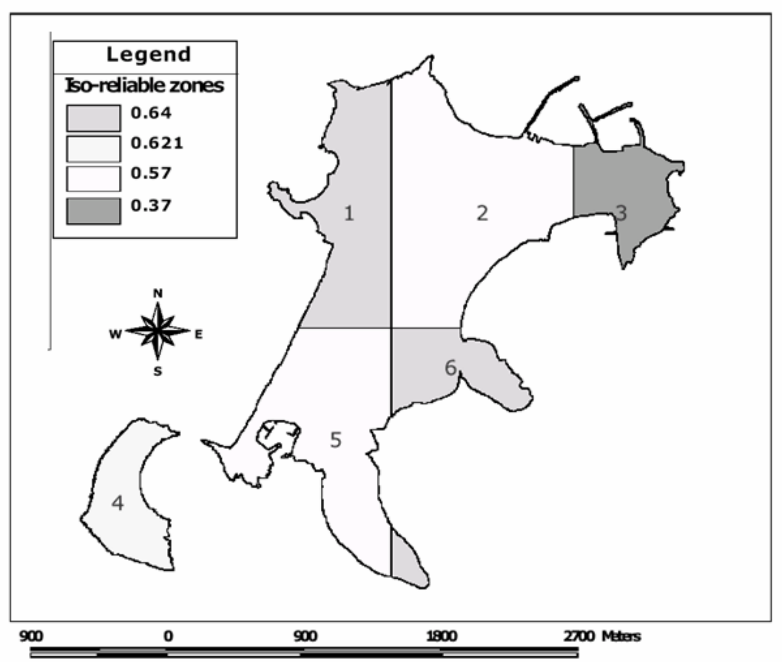


Fig. 9.6. Global reliability of each zone

9.8. Conclusions

It is well known the great difficulty for evaluating the quality of the applications concerning environmental problems, mainly due to the heterogeneity of input data and their indeterminateness in the error estimations. Then the usage of the Fuzzy Logic can be adequate in the treatment of this kind of information, especially when using approximate linguistic values to define the input data (Bardossy and Duckstein 1995, Di Martino et al. to appear) via suitable TFNs.

Here we also have used TFNs for defining the (indexes of) global reliability of the parameters involved. These parameters become layers inside the internal structure of our GIS, where the calculations are executed by using suitable algebraic operations and implemented inside our GIS via the software tool FUZZY-SRA (Gisolfi and Loia 1995).

References

Bardossy A, Duckstein L (1995) Fuzzy Rule-Based Modelling with Applications to Geophysical, Biological and Engineering Systems, CRC Press, Boca Raton.

Di Martino F, Loia V, Sessa S (to appear) A Fuzzy-Based Tool for Modelization and Analysis of the Vulnerability of Aquifers: a case study, International Journal of Approximate Reasoning..

E.S.R.I. (1994) ARC/INFO Data Management: Concepts, Data Models, Database Design and Storage., Environmental Systems Research Institute, Inc., Redlands.

E.S.R.I. (1996) Arcview GIS: The Geographic Information System for Everyone, Environmental Systems Research Institute, Inc., Redlands.

E.S.R.I. (1997) Understanding GIS: The ARC/INFO Method, Environmental Systems Research Institute, Inc. Redlands.

E.S.R.I. (2001) MapObjects GIS and Mapping Components, Environmental Systems Research Institute, Inc., Redlands (California).

Gisolfi A, Loia V (1995) A Complete, Flexible Fuzzy - Based Approach to the Classification Problem, Internat. J. Approx. Reasoning 13:151-183.

Kaufmann A, Gupta MM (1985) Fuzzy Mathematical Model in Engineering and Management Science, North Holland, Amsterdam (1985).

Mansur, JM (1995) Fuzzy Sets and Economics: Applications of Fuzzy Mathematics to Non-Cooperative Holigopoly, Edward Elgar, London.

Nielsen DM (ed.) (1991) Practical Handbook of Ground-Water Monitoring, Lewis Publishers, Boca Raton.

Ward AD, Elliot WJ (eds.) (1995) Environmental Hydrology, Lewis Publishers, Boca Raton.

10. Fuzziness and Ambiguity in Multi-Scale Analysis of Landscape Morphometry

Peter Fisher[1], Jo Wood[2], and Tao Cheng[3]

[1] Department of Geography, University of Leicester, Leicester, LE1 7RH, United Kingdom
email: pete.fisher@le.ac.uk

[2] School of Informatics, City University, London, United Kingdom
email: jwo@soi.city.ac.uk

[3] Department of Land Surveying and Geo-Informatics,The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong
email: lstc@polyu.edu.hk

Abstract. Recent research on the identification of landscape morphometric units has recognised that those units have a vague spatial extent which may be modelled by fuzzy sets. To date most such have looked at the landscape at a single resolution although scale dependence is one of the reasons the concepts are vague. The fact is that the allocation of landscape elements to morphometric classes is ambiguous, and in this chapter we exploit the ambiguity of multi-resolution classification as the basis of the morphometric classes as fuzzy sets. We explore this idea with respect to both the mountains around *Ben Nevis* in Scotland and the dynamic environment of a coastal dunefield. The results in the first example show that the landscape elements identified correspond to landmarks named in a placename database of the area, although many more peaks are found than are named in the available database. In the second case multi-temporal data on a dynamic coastal dunefield is used to show results for fuzzy set and fuzzy logic analysis to identify patterns of change which contrast with more traditional change analysis. Both examples provide new insights over the types of analysis which are currently available in Geographical Information Systems, and the manipulation of scale to parameterise membership of the fuzzy set is a uniquely geographical method in fuzzy set theory.

10.1. Introduction

Published geographical applications of fuzzy set theory have used approaches to membership parameterisation which are grounded in classic models, referred to by Robinson (1988) as Semantic Import Models where some *a priori* knowledge is used to assign certain values of memberships to particular indicator variables such as elevations (Usery, 1996; Cheng and Molenaar, 1999a, 1999b), or the Similarity

Relation Model where a multivariate dataset is assembled and some method of multivariate fuzzy classification applied (Burrough et al., 2000, 2001; MacMillan et al., 2000; Irvin et al., 1997). These methods are comparable to methods used widely in fuzzy set research (Klir and Yuan, 1995; Kruse et al., 1994). Both approaches rely on the idea that applying a Boolean classification in either the multiple or single variable situation involves a sharp boundary where a sharp boundary may not be appropriate. Ultimately these approaches are both grounded in sorites paradox (the paradox of the heap and semantic vagueness (Williamson 1994; Sainsbury 1995; Fisher 2000a, 2000b). The threshold value associated with a Boolean set assignment is trivial because it cannot be clearly, precisely and unarguably identified, so that preserving some information on the boundary condition into the classification is beneficial. In this paper, we introduce a alternative geographically based approach to parameterising memberships of fuzzy sets which is grounded in ambiguity and related therefore to the epistemic approach to vagueness (Williamson, 1994)

Specifically, in this paper we explore information on landscapes derived from Digital Elevation Models (DEMs). Among the simpler derivatives from such data is the assignment of a location to a geomorphic, or morphometric unit. For a target grid cell in the DEM, valus in the surrounding 3 x 3 cell area are used to assign that target cell to one of 6 classes: pit, peak, pass, channel, ridge, and plane (Figure 10.1; Peucker and Douglas, 1975; Evans, 1980; Wood, 1996b). Many such units in the real (rather than digital) landscape are hard to define as Boolean entities in terms of either their elevation or their spatial extent (Fisher and Wood, 1998; Usery, 1996; Wood, 1996a, 1996b). Thus a pixel in a raster grid can be assigned to the morphometric class *peak*, but, because it is a peak at one scale of the landscape, it is not always a peak (Figure 10.2). If a landscape feature is defined simply from the elevation at a location together with those elevations in the area immediately surrounding the location as controlled by the arbitrary resolution of a DEM, it will have nothing more than a possible meaning for either landscape processes or how people perceive the landscape. People do not see a mountain of a particular name as a point feature. The summit of the mountain, usually the highest point, tends to have the name associated with it, but that does not really describe the landscape as we experience it and is not sufficient to define the spatial extent of the mountain (Fisher and Wood, 1998). The mountain is part of the continuum of the surface of the earth, but it is a part of that continuum which is recognized and recalled through the act of naming it. It is a semantic construct that has been used to exemplify the philosophical discussion of vague objects (Burgess, 1990; Williamson, 1994; Sainsbury, 1989, 1995; Varzi, 2001).

In the case of morphometry (the measurement of form) one reason for the vagueness of the morphometric class is the geographic distance over which a class can be considered to persist that is to say the scale of measurement (Tate and Wood, 2002). We use the variation in feature definition with the scale of measurement as the basis of the membership. What may be a peak at one scale may be another morphmetric class at another, say a ridge (Figure 10.2). So is it a peak or a ridge? The assignment at each scale is specific and clearcut, but because the class may differ for different scales, the answer is ambiguous. The location is to some degree both a peak and a ridge. Indeed, many locations may be classed as all six morphometric units depending solely on the scale of measurement.

In this paper, we propose an approach to fuzzy set membership based on the ambiguity in multi-scale classification of a landscape to morphometric classes, and we briefly explore the consequences of this in two instances – one looking at regional scale static morphometry in the *Ben Nevis* area of Scotland, and the second exploring temporal changes in a coastal dune field in North-west England (Figure 10.3). In Section 2 we outline the general method of analysis, in Sections 3 and 4 the two particular applications are the principal focii. Some necessary theoretical developments over and above that given in Section 2 is specified in Section 4. In Section 5 we present conclusions and suggestions for further work.

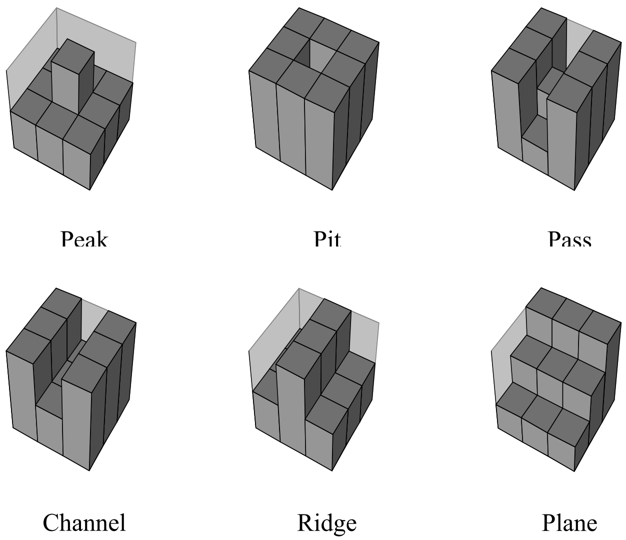


Fig. 10.1. Morphometric units

10.2. Theory and method of multi-scale analysis

Let L denote a Boolean or crisp set of the morphometric class to which a location in the landscape measured at a particular scale can be assigned. In the model of morphometry used here (Evans, 1980; Wood, 1996a, 1996b), six possible valuations of ***L*** at location ***x*** exist **[ridge, peak, pass, channel, pit, planar]**. These six classes represent all the permutations of the first and second derivatives of a surface in two orthogonal directions. Let us indicate this set of six classes by the symbol **[A]**. We can represent this in Equation 10.1.

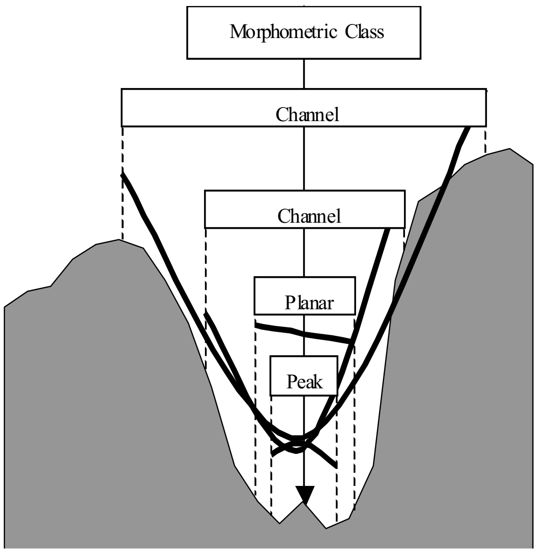


Fig. 10.2. Changing morphometric classes with scale

Fig. 10.3. The location of the study areas in northwest England.

$$L_x = [\mathbf{A}] \tag{10.1}$$

If we give a numberical valuation to the membership of the set equal to 1 or 0 such that 1 indicates that the set is true in a particular case, and 0 indicates that it is untrue, then because there are six possible valuations of **A,** it follows that for five values of **A**, the membership of the Boolean set, $\boldsymbol{m_{Ax}}$, of that class is given in equation 10.2, and for only one possible valuation of **A** the membership of the Boolean set is unity (Equation 10.3).

$$m_{Ax} = 0 \tag{10.2}$$

$$m_{Ax} = 1 \tag{10.3}$$

The allocation of a location to a class is not, however, persistent under different scales of measurement (Wood 1996a, 1996b). Just because the value of $\mathrm{m}_{\mathrm{Ax|s1}}$ = 1 for a certain landform class, it does not follow that $m_{Ax|s2}$ =1 or that $m_{Ax|s3}$ =1 for the same class, where s1, s2, etc indicate different scales of measurement (Figure 10.2). There is therefore ambiguity as to which class a location belongs. The membership of a fuzzy morphometric class, **A**, can therefore be given in equation 10.4, for each of **A**, where ***n*** is the number of scales of analysis.

$$\mu_{Ax} = \frac{\sum_{i=1}^{n} m_{Ax|s_i}}{n} \tag{10.4}$$

Equation 10.4 weights each scale of analysis equally, and is the approach used in discussion here. Differential weightings can be envisaged and easily implemented, but the values of the weightings associated with particular scales requires careful consideration.

To execute the multi-scale analysis we have used the method of Wood (1996a; 1996b), where the surface is modelled as a gridded DEM and then locally interpolated as a quadratic surface centred on an expanding window of gridded cell values. Morphometric analysis is then performed on the generalised surface over a range of window sizes. Other generalisation operators are possible (Fisher, 1996), and the exact outcome will be dependent on the generalisation operator. We used the Landserf software to execute these operations (Wood, 2002a). This was supported with the Idrisi 32 (Eastman 1999) and ArcGIS packages for map algebra and cartography, respectively.

10.3. The *Ben Nevis* Area

10.3.1. Introduction

As mentioned in Section 1, the definition of the vague concept "a mountain" is used to illustrate philosophical discussions of vagueness (Burgess, 1990; Sanisbury, 1989; Varzi, 2001), and is therefore an appropriate first issue to motivate analysis here. We have chosen to explore the definition of the extent of the highest mountain of the British Isles, *Ben Nevis*, and other mountains in its vicinity, the Grey Corries and the Mamores ranges (Figure 10.4).

The Ordnance Survey's 50m resolution Panorama^{TM} gridded DEM of the area was used. Two 20 x 20 km tiles were mosaiced to derive the DEM used, but a subset of the total area was used in analysis and is shown in Figure 10.4. The DEM was originally derived from contours digitised from 1:50,000 maps. To support some interpretation we used the Panorama^{TM} contour data as well. This area of Britain is well known to mountain climbers, and in particular it contains 19 of the Munros (Figure 10.4; Table 10.1). The Munros are a number of Scottish mountains over 3000ft, and were first listed by Sir Hugh Munro in his Tables published in 1891 (Bearhop,1997; Bennett, 1999). Munros are used by hillwalkers as trophies, and "Munro bagging" (walking/climbing to the summits of Munro's) is the leisure activity of a whole community. The Ordnance Survey DEM data used here was supplemented by a database of waypoints from peaks of the mountains (Seymour, 2003).

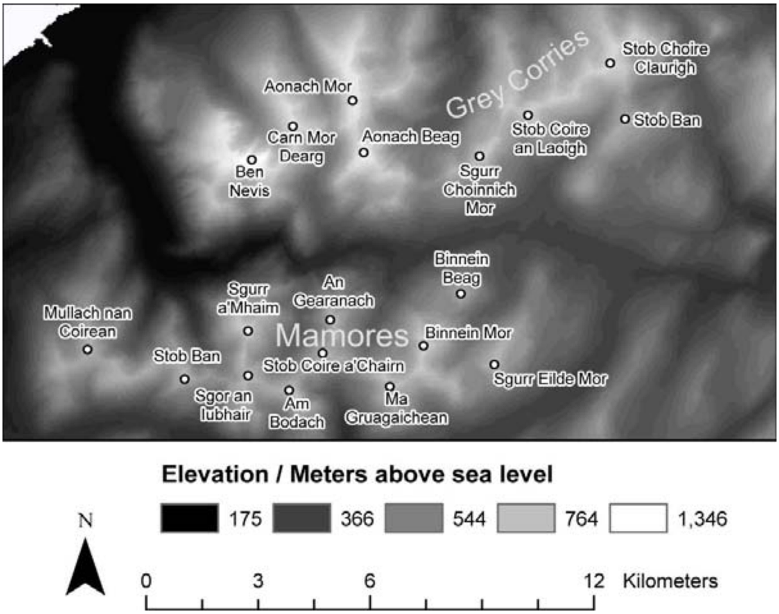


Fig. 10.4. A DEM *Ben Nevis* and the Mamores derived from parts of 2 Panorama tiles of the Ordnance Survey 50m resolution DEM (© Crown Copyright Ordnance Survey. An EDINA Digimap / JISC supplied service). Superimposed are the names of the Munros from waypoint files (Seymour, 2003).

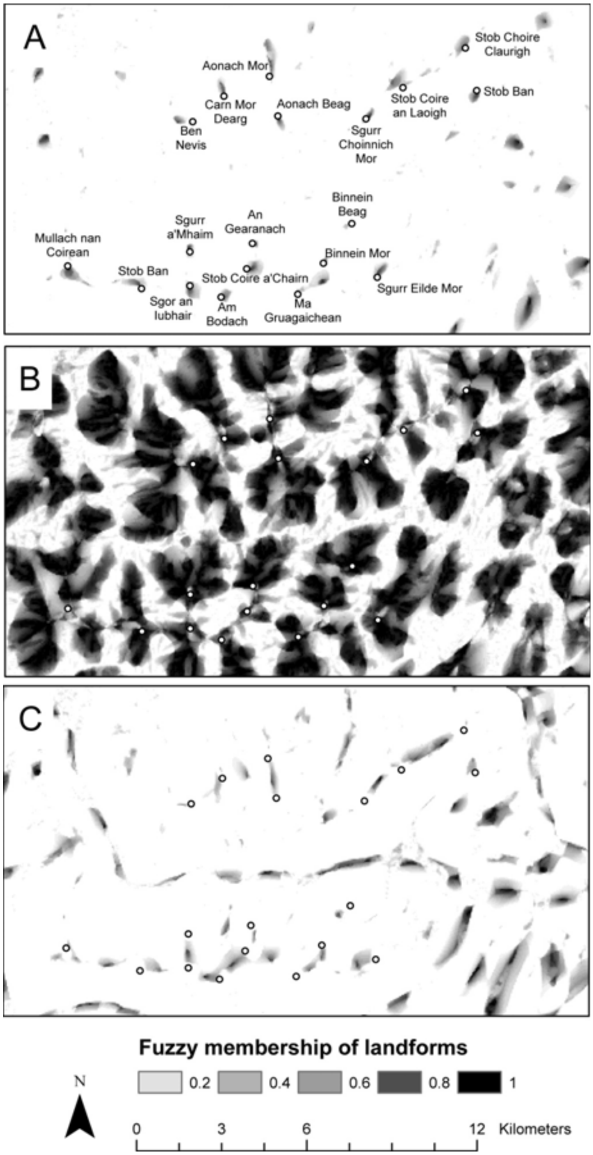


Fig. 10.5. Memberships of three fuzzy morphometric landforms: A) Peakness; B) Ridgeness; and C) Passness. The locations of Munro summits from waypoint lists are shown in A. The feature extractions shown in this figure used a threshold of 4^{o} for the peaks over a range of 21 scales.

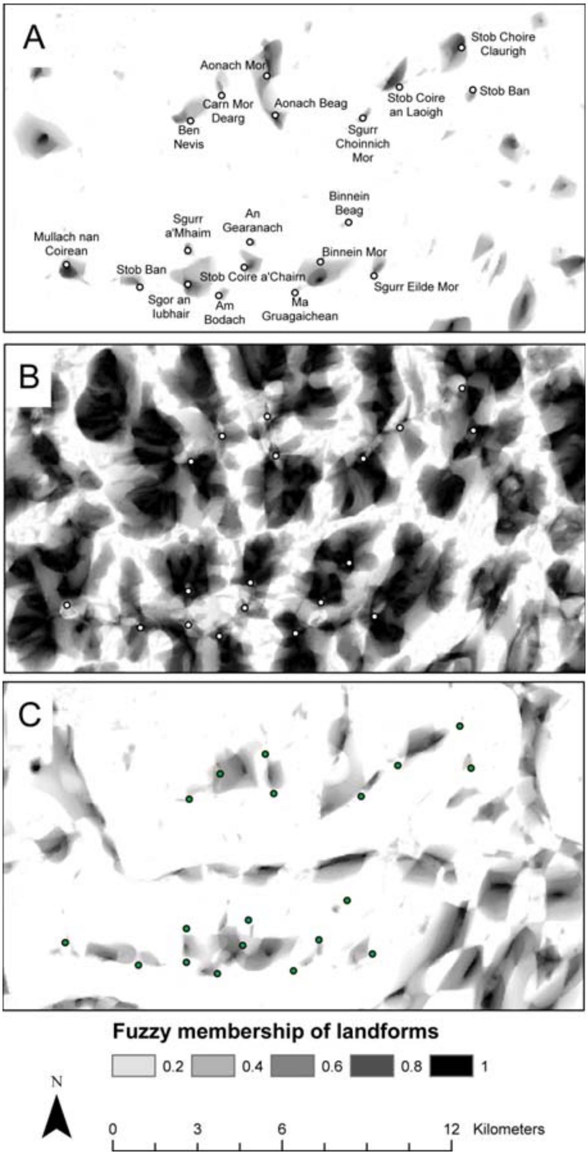


Fig. 10.6. Memberships of three fuzzy morphometric landforms: A) Peakness; B) Ridgeness; and C) Passness. The locations of Munro summits from waypoint lists are shown in A. The feature extractions shown in this figure used a threshold of 4° for the peaks over a range of 37 scales.

The DEM was processed as outlined above using Landserf (Wood, 2002a). Generalization filter sizes from 3 x 3 to 43 x 43 pixels were used, and then 3 x 3 to 75 x 75 pixels, and morphometric classes extracted which can be considered representative of size from 100 m to 2100 m, and 100 m to 3700 m, the first identifying local-to-meso scale features, and the second local-to-regional features. The Boolean assignment of locations to morphometric classes, particularly Peaks and Pits, is problematic because the method encoded in Landserf uses a best-fit surface through an area, and is intended to assign features without a slope to one or the other. However, in the polynomial approximation used, most locations have a slope, and so very few would be assigned to peak or pit classes. A threshold is therefore included such that locations which would otherwise be considered a peak or pit, but which have a slope less than the threshold are assigned to the category peak or pit. In the analysis reported here, the thresholds of 4^{o} was used, following exploration of the alternatives. This is large enough that peaks are present, and threshold values a little larger include only a few more pixels in the peak class. The analysis of each scale classified every pixel into one of 6 morphometric classes and was saved. The extent of each of the six landforms at each scale was then separated into binary coded geographic databases. Across the range of scales the values were then added and divided by the number of scales to yield the memberships (Equation 10.4).

10.3.2. The Degree of Peakness

Using the local-to-meso scale range we derived the membership images of peakness, ridgeness, and passness, shown in Figure 10.5. The extents of all six morphometric classes were determined, but these three relate most to the degree to which a location is a mountain. For the local-to-regional scales, the equivalent images are shown in Figure 10.6. Figure 10.7A shows the degree of peakness in those areas over 3000ft (914m). It is possible to see that all the named Munro peaks are identifiable by a region with some degree of peakness in both analyses. The maximum degree of membership in the fuzzy set peak within the zone associated with the named Munros is listed in Table 10.1. The majority of Munros have a relatively high membership in the concept peak, with the majority having memberships over 0.5. Only *Na Gruagaichean* and *Binnein Beag* have memberships less than 0.5 in the local-to-meso scale analyses. Most peaks have a lower membership of peakness in the analysis of local-to-regional scales. It can be seen that in 2/3 of the Munros the number of times a location is a peak increased in the larger number of analyses (peaks identified in the local-to-meso scale analysis continue to be identified as peaks in the local-to-regional scale analysis), although the membership of peakness may be smaller.

Table 10.1. The maximum values of memberships of the fuzzy set of Peakness in the vicinity of Munro summits. In each of the two ranges of scales analysed, the maximum number of times a peak was identified as well as the maximum membership is listed for each peak.

Munro Name	Local-to-meso scales		Local-to-regional scales	
	Count	Membership	Count	Membership
	Ben Nevis and the Grey Corries			
Ben Nevis (Figure 10.8C)	15	0.71	21	0.57
Carn Mor Dearg	14	0.67	14	0.38
Aonach Mor	16	0.76	30	0.81
Aonach Beag	18	0.86	32	0.86
Sgurr Choinnich Mor (Figure 10.8A)	19	0.90	21	0.57
Stob Coire an Laoigh	13	0.62	29	0.78
Stob Choire Claurigh	13	0.62	29	0.78
Stob Ban	17	0.81	17	0.46
	Mamores			
Mullach nan Coirean (Figure 10.8B)	17	0.81	33	0.89
Stob Ban (Mamores)	16	0.76	21	0.57
Sgurr a'Mhaim	17	0.81	19	0.51
Am Bodach	19	0.90	19	0.51
Stob Coire a'Chairn	19	0.90	24	0.65
An Gearahach	11	0.52	11	0.30
Na Gruagaichean (Figure 10.8D)	7	0.33	7	0.19
Binnein Mor (Figure 10.8D)	12	0.57	16	0.43
Binnein Beag	6	0.29	6	0.16
Sgurr Eilde Mor	17	0.81	21	0.57

In some cases, the area of highest memberships is displaced from the location of the peak (e.g. *Ben Nevis*, itself), while in others the peak is located in the area of high peakness (e.g. *Carn Mor Dearg*). Some peaks are very extensive (e.g. *Mullach nan Coirean*), while others are rather small (e.g. *Sgurr Choinnich Mor*). Some peaks are singular (their zone of high peakness is surrounded by a zone with no degree of peakness (e.g. both the *Stob Ban* in the Grey Corries and the *Stob Ban* in the Mamores), while others merge with other peaks (e.g. Aonach Mor and Aonach Beag).

The allocation of peaks to the Munros is somewhat arbitrary. The hill or mountain has to be popularly acknowledged and recognised as a mountain, and over 3000ft but there are more candidate peaks over 3000ft than there are Munros (Bennet, 1999). Possible Munros are listed in separate tables and are known as Tops. In the Mamores and Grey Corries, the majority of Tops have only very small memberships of the set peak in either of the ranges of scales analysed. In all but three cases, the memberships are well below 0.5. Furthermore, most Tops do not show the continuing identification as peaks in the regional scale analyses; the number of times a location is a peak in the local-to-meso scale analysis is equal to the number of times in a local-to-regional scale analysis (Table 10.2). Indeed only, *Sgor an Lubhair* and *Binnean Mor* – South Top are identifiable as pronounced regional Peaks, competing for the degree of peakness with the Munros. It is also clear that many locations which are not over 3000ft possess a large membership of peakness. These are not explored further here, but some possibly interesting patterns are apparent.

Table 10.2. The highest values of memberships of the fuzzy set of Peakness in the vicinity of Top summits. In each of the two ranges of scales analysed, the maximum number of times a peak was identified as well as the maximum membership is listed for each peak.

Top Name	Local-to-meso scales		Local-to-regional scales	
	Count	Membership	Count	Membership
Ben Nevis and the Grey Corries				
Carn Dearg – Northwest Top (2)	5	0.24	5	0.14
Carn Dearg Meadhoach (7)	14	0.67	16	0.43
Carn Dearg – Southwest Top(66)	3	0.14	3	0.08
Stob Choire Bhealach (26)	1	0.05	1	0.03
Sgurr a'Bhuic (130)	2	0.10	2	0.05
Stob an Cul Choire (39)	3	0.14	3	0.08
Torn na Sroine (218)	2	0.10	2	0.05
Sgurr Choinnich Beag (131)	4	0.19	4	0.11
Stob Coire Easain (33)	Part of the Munro, *Stob Coire an Labigh*			
Beinn na Socaich (76)	2	0.10	2	0.05
Caisteil (23)	7	0.33	7	0.19
Stob Coire Cath na Sine (34)	4	0.19	4	0.11
Stob a'Choire Leith (24) *Stob Coire na Gaibhre* (143)	Parts of the Munro, *Stob Coire Claurigh*			
Stob Coire Dhonhnuill (15)	3	0.14	3	0.08
Mamores				
Sgor an Lubhair (82)	17	0.81	33	0.89
Mullach nan Coirean (220)	10	0.48	10	0.27
Stob Choire a'Mhail (97)	4	0.19	Part of Top *Sgor an Lubhair*	
An Garbhanach (113)	1	0.05	Part of Munro *Stob Coire a'Chairn*	
Na Guagaichean – Northwest Top (54)	Not identifiable			
Binnean Mor – South Top (41; Figure 10.8D)	18	0.86	34	0.92
Sgor Eilde Beag (146)	4	0.19	4	0.11

Above all, and assuming that a mountain should to some extent correspond to a morphometric peak, this analysis presents a plan view of the spatial extent of the peaks. Figures 10.5A, 10.6A, and 10.7A show those areas which to some degree correspond to a morphometric peak and so give one view of the fuzzy extent of the mountains. Four particular peaks are shown in detail in Figure 10.8, where the memberships of peakness are compared with the contour maps and the topographic maps of the areas. The four peaks illustrate various different relationships between the degree of peakness and Peakness, the Munros, and the Tops. Fig 8A and 8B show *Sgurr Choinnich Mor* and *Mullach nan Coirean*, respectively. The two Munros are clearly associated with areas of high degrees of peakness. In the case of the first, the zone of large peakness is directly over the top of the mountain, as indicated in the contours, but in the second, The zone of large values is associated with the southern flank of the mountain. This is due to a zone of elevated ridgeness on the part of the mountain forming a ridge to the north. *Ben Nevis* itself (Figure 10.8C) suffers from the same issue. The ridge on which *Ben Nevis* lies, is a pronounced feature, and subsumes the peak where it is named, but the peak is definitely revealed in the morphometric analysis to the north. In another case (Figure 10.8D), the Munros *Binnein Mor* and *Na Gruagaichean* are just visible as areas of slightly to moderately elevated peakness, but peakness in the area is dominated by the peak associated with the Top, *Binnean Mor* – South Top (1062m). Like other Tops, this peak is a candidate Munro, but not agreed as a Munro. It is actually higher than *Na Gruagaichean*, and with its symmetrically radiating ridges it forms a morphometric zone which matches well to the concept peak, and this is reflected in the degree of peakness. Detailed examination shows that most areas which are recognizable as peaks in the morphometric analysis can be broadly associated with named peaks. Not all summits correspond with peaks, however, and frequently they are not in exactly the same position as the summit. Rather the summit may be subsumed morphometrically within a larger ridge. However, in all cases, for the observer of the mountain, the area of the "peak" associated with a summit can be mapped out by the analysis presented here. Where *Ben Nevis* occurs as a recognizable peak for example is shown by the map of peakness for *Ben Nevis* (Figure 10.8C) although the location does not correspond to the location of the summit of *Ben Nevis*. Furthermore, the mountain is mapped out in this endeavour

10.3.3. The Ridges

The peaks of the Grey Corries and the Mamores are interconnected in a system of ridges and passes, as in other mountain ranges. The distribution of ridgeness is visible in Figures 10.5B, and 6B and of Passness in Figures 5C and 6C. Ridgeness, in particular is a spatially dispersed phenomenon revealing large memberships over large areas. Using a revised algorithm designed to identify the network features of the landscape (Wood, 1998), the analysis was redone using only the local-to-meso scale range. Figure 10.7B shows the distribution of the membership of the fuzzy set of network ridges over these scales, and Figure 10.7C shows the distribution of ridges from the Collins Map of the Munros (Collins, 1998). It is apparent that these ridges are much more restricted in their spatial extent, than those from the previous analysis, and the figure also shows that the ridges identied correspond remarkably with the walker's ridges linking the Munros. However, there are prominent ridges (large membership) which do not form part of the walker's network, (for example the extent of the ridge shown running out from *Sgurr Eilde Mor*, into the southeast corner of the study area) and parts of the walker's network where the network ridge is very spatially diffuse (for example, between *Ben Nevis* and *Carn Mor Dearg* to the north west).

10.3.4. Summary

This section has demonstrated that it is possible to define the extent of an indistinct spatial object, based on the ambiguity of multiscale analysis of morphometric classes (hills and mountains) and relate them to known named summits and other landscape features. The correspondence between peakness and the Munros indicates that the analysis yields results which have some correspondence to human perception of the landscape. Further work needs to explore the extent of this correspondence as well as examining the sensitivity of the method to thresholds and differential weightings of scales. We present a more thorough analysis of the pattern and arrangement of the fuzzy morphometry for the English Lake District elsewhere (Fisher et al. 2004).

10.4. The Ainsdale coastal sand dunes

10.4.1. Introduction

Cheng and Molenar (1999a, 1999b) have presented fuzzy analyses of changing coastal landforms. They show that it is possible to model the coastal landforms with fuzzy sets using the Semantic Import Model to define the fuzzy extent of landforms, including beach, foreshore and dune. To parameterise the fuzzy sets they use elevation above datum, together with expert opinion of the height at which different landscape units occur. They demonstrate the change in these landforms over time. This approach works well in the beach environment, but once landforms are on dryland the analysis is less useful. Indeed, in the work of Cheng and Molenaar (1999a, 1999b) a small area is classified as foreshore at one point when it is surrounded by what is classified as dunes. This is a most unusual situation and does not correspond to any prior ideas about how dunes and beach interact. The area is almost certainly an area of sand removal by the wind: a blowout or interdune area. In the present research we use the multi-scale approach already outlined to examine process in a coastal dunefield.

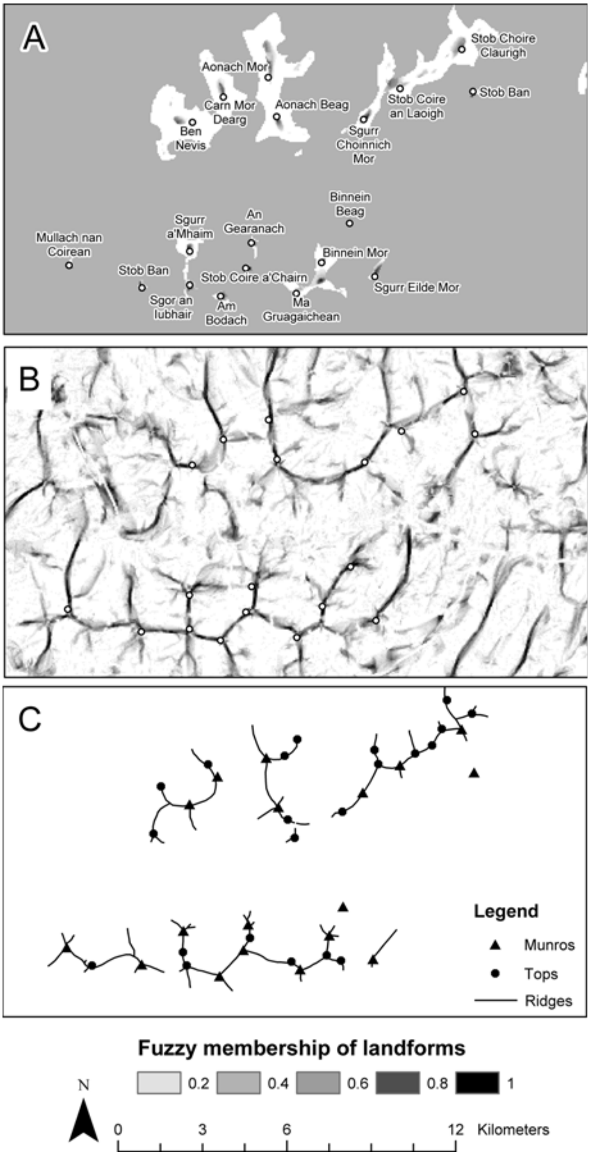


Fig. 10.7. A) The Peakness of the the land which is over 3000ft (914 m), the Munros (the area shown in solid grey is the land below 3000ft); B) The fuzziness of network ridges in the study area, and C) ridges digitized from the Collins Munro Map (Collins, 1998).

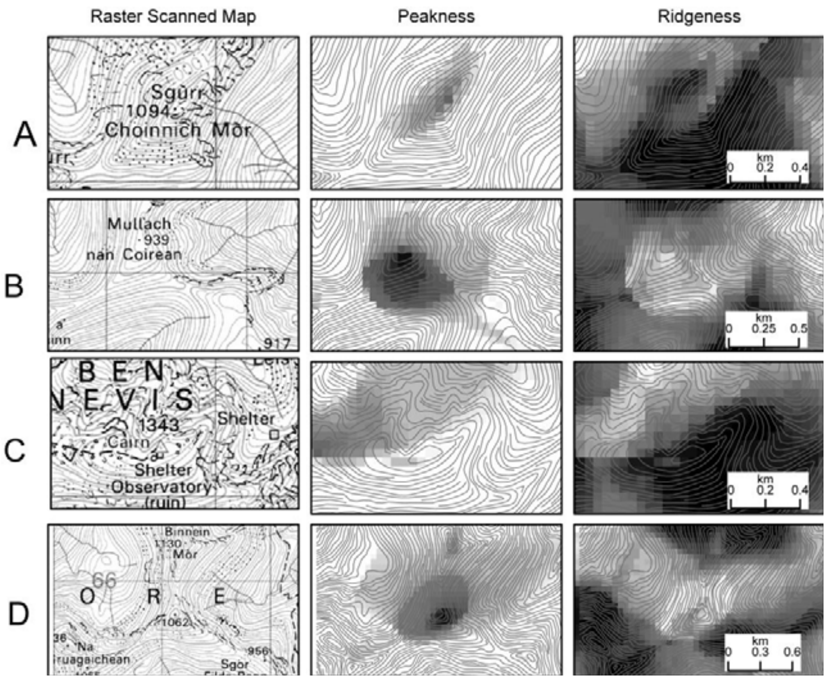


Fig. 10.8. Details of four Peaks showing three areas identified as having large values of peakness compared with the Ordnance Survey Raster Scanned 1:50,000 mapping and the 1:50,000 DEM contours of the same areas (© Crown Copyright Ordnance Survey. An EDINA Digimap / JISC supplied service). As in Figure 10.6, the threshold slope for peak differentiation is 4°.

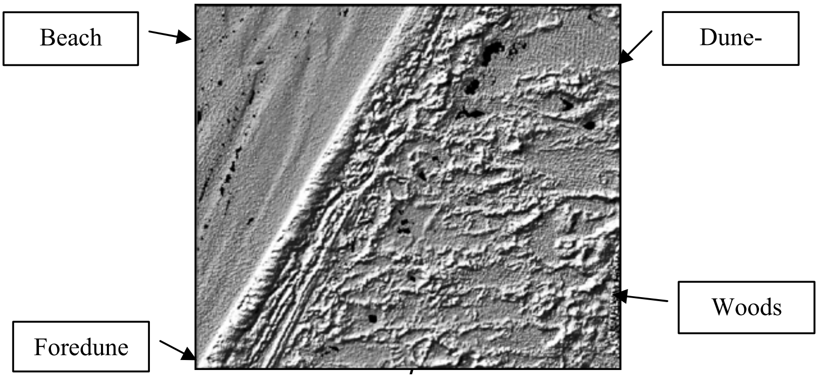


Fig. 10.9. LiDAR DEM of the Ainsdale Sands in 1998. The Foredune is clearly visible running diagonally to the left of the images, as are degrading parabolic dunes. The area shown is 1.5 km by 1 km. Areas in black in these and other figures of the area have no LiDAR return on at least one date due to standing water and wet ground.

For the work presented here, the study site is located in northwest England (Figure 10.3) at Ainsdale Sands. High resolution DEMs have been collected from this area by the UK Environment Agency (EA) by LiDAR, in 1998 and 2000. The EA's interest in the area is due to its being a Site of Special Scientific Interest, as well as the general importance of the environment as a major and threatened habitat type and as a defence against coastal flooding. The data were made available as 2m resolution DEMs registered to the Ordnance Survey National Grid for each of the three dates. The full DEM covers an area 2 x 2 km, but a subset 1.5 x 1 km is examined here (Figure 10.9). Small areas in each image, frequently the same areas, gave no reading for the LiDAR sensor due to the standing water or wet soil absorbing the infra-red light of the sensor. These areas are shown in black in Figure 10.9 and most subsequent figures. These areas are either on the beach or in interdune areas where wetness is to be expected. In running the Landserf analysis of the landforms, we used generalisation windows from 3 x 3 cells to 49 x 49 cells representing landforms with magnitudes from 4 to 96 m; that is to say that there are 24 different realisations (versions) of the morphometry for each cell in the DEM. In this instance features such as channels and ridges are of most interest, and so the default 1^0 slope tolerance for identification as pit, peak or pass is used in this analysis.

In the DEM it is possible to see two principal areas, the beach to the northwest (top left, Figure 10.9) and the dunefield over the remainder of the area. The beach is characterised by a variety of broad low relief linear features trending from southwest to northeast. The major foredune is then encountered being the dominant feature with this trend, and a series of small parallel ridges follows. These are a series of more or less in-undated previous foredunes. Finally a series of parabolic dunes and blow-outs can be distinguished as curved ridges. To the extreme eastern edge of the area, a small patch of woodland is just apparent.

10.4.2. Additional theory

Sand dunes have definite properties, which include a crest, a windward and a leeward side. The sides of a dune will typically be planar slopes, but these are widespread in the nearshore environment typifying the beach as well as the dunes. At this stage in our study we can therefore model the dune by the dune-crest, being the union of the memberships of the ridge and the peaks in the dunefield. Although the Union is frequently taken as the maximum function in fuzzy set theory, it seems more appropriate here to use the sum of the two sets. Thus the membership of the dune crest is given by the bounded sum :

$$\mu_{dune\_crest} = \min \left(1, \quad \mu_{ridge} + \mu_{peak}\right) \tag{10.5}$$

It follows that the degree to which a location is not a dune-crest at any time is given by the negation operation:

$$\mu_{dune\_crest}' = \left(1 - \mu_{dune\_crest}\right) \tag{10.6}$$

It is therefore possible to determine the area over which dune crest has spread between time t1 and t2, by taking the intersection of the fuzzy set *not dune-crest* at time t1, and a *crest* at time t2. The normal fuzzy set intesection, the minimize operation, will not suffice in this instance. One location with membership 0.5 at both t1 and t2 will have membership in this intersection of 0.5, when no change has occurred (min (0.5, 1-0.5) = 0.5). However, there are number of alternative operations of most fuzzy set operations, and, in the context of change detection, the bounded difference (Equation 10.7; Klir and Yuan 1995:63) serves better.

$$\mu_{dune\_gain} = \max \left(0, \quad \mu_{dune\_crest}'_{t1} + \mu_{dune\_crest}{}_{t2} - 1\right) \tag{10.7}$$

It is also possible to model loss of dune-crest area, as the intersection (bounded difference) of the area which is *dune-crest* at time t1, but is *not dune-crest* at time t2 (Table 10.3).

We can also model the interdune area since the only reason for channels and pits in the study area is the existence of slack areas between dunes. Therefore similarly to Equation 10.7, we find that the membership of dune slack is given by:

$$\mu_{dune\_slack} = \mu_{channel} + \mu_{pit} \tag{10.8}$$

These can then be subjected to similar logical operations as the membership of the dune ridge (Equations 10.6 and 10.7). Thus it is possible to model the movement of the dune system as four separate fuzzy sets derived from the different intersection operations (Table 10.3).

Table 10.3 The various intersection operations for change in the dune system

Crest	Gain	Not_Dune_Crest $_{t1}$	∩	Dune_Crest $_{t2}$
	Loss	Dune_Crest $_{t1}$	∩	Not_Dune_Crest $_{t2}$
Slack	Gain	Not_Dune_Slack $_{t1}$	∩	Dune_Slack $_{t2}$
	Loss	Dune_Slack $_{t1}$	∩	Not_Dune_Slack $_{t2}$

10.4.3. Results

Elevation changes are relatively common across the area as should be expected. These will partly be due to error in the LiDAR readings, but also due to real changes. Most are small (within 10 cm), but exceptionally they go up to 2.5 m. The beach features have undergone systematic change, but due to the frequency of change in this inter-tidal area little significance can be attributed to this. Changes in the dunefield are generally not great, but are localised, and follow a southwest to northeast trend similar to the foredune, although there are few large changes. The wooded area is very noticeable because of the amount of change. Here the difference is more likely due to variations in the tops of the trees, the positions of the trees due to the wind and the light-feature interaction when recording the last Figure 10.10 A-C shows the memberships of the fuzzy sets channel, planar slope and ridge classes of morphometry, respectively, for 1998 after implementation of equation 10.4. Memberships of the other three classes are too small even at their highest to be visible in a printed map, just as they are not visible in the modal morphometric class (Figure 10.10A). Large values are very localised showing the continuity of dune-crests and interdune-slacks. Furthermore, the patterns of the ridges and crests conform with what would be expected in a dune field, with the ridges showing parabolic forms with the open end to the seaward. It is noticeable that a number of the larger areas giving no data due to water have halo effects of relatively large memberships of ridge, but these are a artifacts consequent on the zone of “No Data” return of the LiDAR signal. The analyses for 1999 and 2000 show remarkably similar characteristics at the large scale.

Figures 10.11A and B show the gain and loss, respectively, of the fuzzy dune crest, based on the intersections listed in Table 10.3 (after Equation 10.7). Relatively large areas are seen to have increased their dune crestness to some degree (gain), the main area being on the foredune. Losses of the extent of the dune crest are slight, and can generally be interpreted as edge effects around no-data areas. Change in the dune slack is also generally slight, but both loss and gain of slack has occurred, to some degree, in the vicinity of the wooded area to the east of the study site.

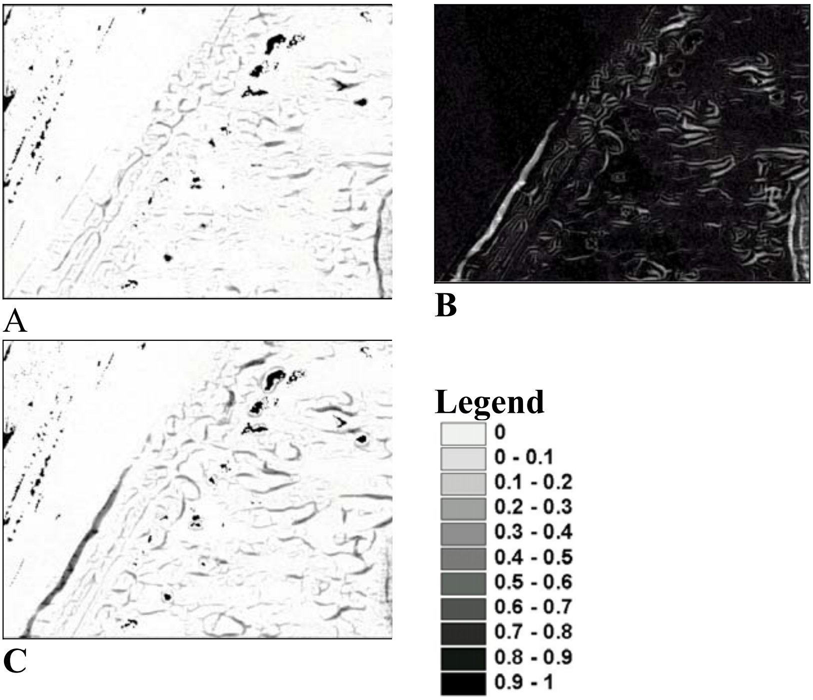


Fig. 10.10. Membership of the fuzzy sets A) channelness, B) planarity, and C) ridgeness in 1998 – memberships of peakness, pitness and passness were too low to be visible in these images. Black indicates membership 1 and white membership 0. Other areas shown in black have no LiDAR return on at least one date due to standing water and wet ground.

10.4.4. Summary

A traditional change analysis of the coastal dune field might focus on the change in elevation. Through the morphometric analysis presented here, it is possible to extract far more information from the DEM, than when considering elevation alone. Specifically, from the analysis of form over time gains can be seen in the foredune ridge, and in the slack around the woodland.

10.5. Conclusion

Research presented in this paper has shown the possibility and utility of defining landscape phenomena using a novel method of multi-scale analysis which can be used to model ambiguous objects which are vague for scale reasons. Initially the

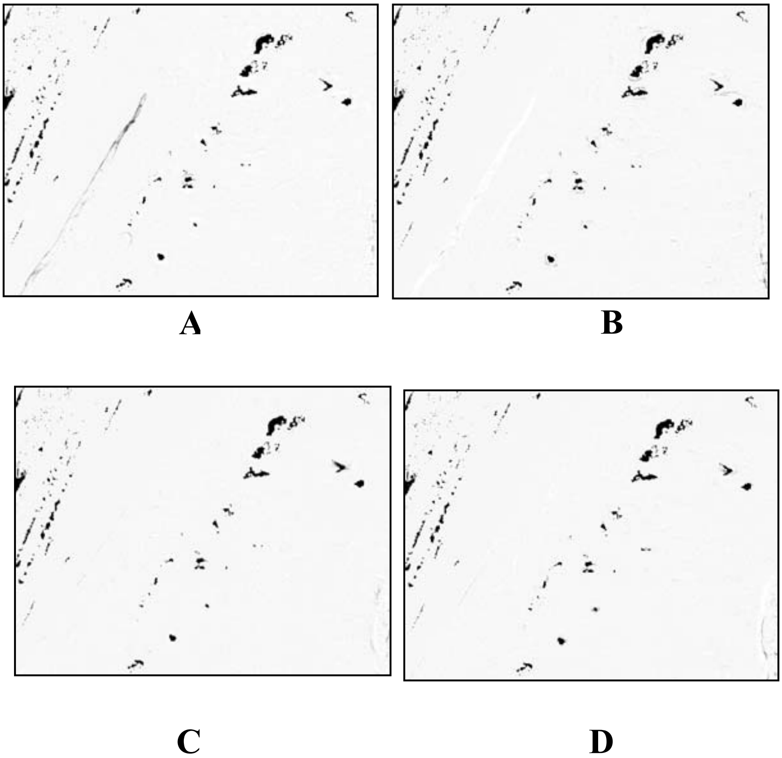


Fig. 10.11. Dune change between 1998 and 2000. Membership of A) gain in Dune Crest, B) loss of Dune Crest, C) gain of Dune Slack, and D) loss of Dune Slack.

paper examined the case of landscape phenomena which are conceived to be static such as hills and mountains. It was found that in the morphometric analysis used the class peak is identifiable as corresponding to many known mountains and hills. Similarly passes are identifiably associated with landscape objects identified and named by people, and included in a national toponym databases.

In the second part of the paper, the method was used for the high resolution analysis of dynamic landforms, specifically a coastal dunefield. Although the analysis fails to identify the positions of dunes, it is possible to identify dune ridges and slacks, and to monitor their changing positions. The analysis was shown to provide significant insights over a more conventional analysis of the accumulation and erosion of material as is reflected in changing elevations. The analysis draws attention to completely different parts of the dunefield. A coastal system is relatively stable, although subject to change. Not only will future work look at automated feature extraction in the environment and integration with alternative data sources giving alternative fuzzy locational indications of the dunes, but also at more mobile environments such as desert dunefields.

The analysis is not without problems owing to parameterisation of the morphometric feature identification procedure, the use of a very partial toponym database and the lack of association between either morphometric peak and mountain or morphometric ridge and dune. However, the method shows great promise and will be investigated further with respect to other areas, and other applications. The method articulated here has been shown to be successful in the limited contexts of analysis presented providing new insights in both examples.

Acknowledgements

We wish to acknowledge the use of Ordnance Survey data under JISC licence agreements through EDINA, and to Emma Sutton for promptly answering questions related to the data. We would like to thank Al Duncan and Kyle Brown of the Environment Agency for their cooperation and supplying the Ainsdale data used in this paper. Thanks too to Kate Moore for assistance with diagram preparation.

References

Bearhop DA (1997) Munro's Tables. Scottish Mountaineering Club

Bennett, D (ed) (1999) The Munros: Hillwalker's Guide Volume 1. Scottish Mountaineering Club

Burgess JA (1990) The sorites paradox and higher-order vagueness. *Synthese* 85:417-474

Burrough PA van Gaans PFM, and MacMillan RA (2000) High-resolution landform classification using fuzzy k-means. *Fuzzy Sets and Systems* 113: 37-52

Burrough PA, Wilson JP, van Gaans PFM, and Hansen AJ (2001) Fuzzy *k*-means classification of topo-climatic data as an aid to forest mapping in the Greater Yellowstone Area, USA. *Landscape Ecology* 16:523–546

Cheng T and Molenaar M (1999a) Objects with fuzzy spatial extent. *Photogrammetric Engineering and Remote Sensing* 63:403-414

Cheng T and Molenaar M (1999b) Diachronic analysis of fuzzy objects. *GeoInformatica* 3: 337-356.

Collins (1998) Munro Map: All Scotland's mountains over 3000 feet. Harper Collins Publishers, London.

Eastman JR (1999) *Idrisi32, Reference Guide* Clark University, Worcester.

Evans IS (1980) An integrated system of terrain analysis and slope mapping. *Zeitschrift fur Geomorphologie* Suppl-Bd: 36 274-295

Fisher, PF (1996) Propagating effects of database generalization on the viewshed. *Transactions in GIS* 1(2): 69-81

Fisher PF (2000a) Fuzzy modeling. In Openshaw S, Abrahart R and Harris T (eds) *Geocomputting*, Taylor & Francis, London, pp 161-186
Fisher PF (2000b) Sorites paradox and vague geographies. *Fuzzy Sets and Systems* 113:7-18
Fisher PF and Wood J (1998) What is a mountain? or the Englishman who went up a Boolean geographical concept and realised it was fuzzy. *Geography* 83:247-256
Fisher PF, Wood J and Cheng T (2004) Where is Helvellyn? Fuzziness of multiscale landscape morphometry. *Transactions of the Institute of British Geographers* 29: 106-128
Gale S (1972) Inexactness fuzzy sets and the foundation of behavioral geography. *Geographical Analysis* 4: 337-349
Irvin BJ, Ventura SJ and Slater BK (1997) Fuzzy and isodata classification of landform elements from digital terrain data in Pleasant Valley, Wisconsin. *Geoderma* 77: 137-154
Klir GJ and Yuan B (1995) *Fuzzy Sets and Fuzzy Logic: Theory and Applications,* Prentice Hall, Englewood Cliff
Kruse R, Gebhardt J and Klawonn F (1994) *Foundations of Fuzzy Systems,* John Wiley and Son, Chichester
MacMillan RA, Pettapiece WW, Nolan SC and Goddard T W (2000) A generic procedure for automatically segmenting landforms into oandform elements using DEMs, heuristic rules and fuzzy logic. *Fuzzy Sets and Systems* 113: 81-109
Ordnance Survey (1998) The English Lakes: South Western area, Coniston, Ulverston and Barrow-in-Furness. 1:25,000 Outdoor Leisure 6. Ordnance Survey, Southampton
Peucker TK and Douglas DH (1974) Detection of surface specific points by local parallel processing of discrete terrain elevation data. *Computer Graphics and Image Processing* 4: 375-387
Robinson VB (1988) Some implications of fuzzy set theory applied to geographic databases. *Computers, Environment and Urban Systems* 12: 89-97
Sainsbury RM (1989) What is a vague object? *Analysis* 49: 99-103
Sainsbury RM (1995) *Paradoxes* 2nd Edition, University Press, Cambridge
Seymour, J (2003) *GB Waypoints: Munros.* http://www.itatwork.freeserve.co.uk/waypoints/munros/index.htm Accessed 20 April 2003
Tate N and Wood J (2001) Fractals and scale dependencies in topography. In Tate N and Atkinson P (eds) *Modelling Scale in Geographical Information Science,* Wiley, Chichester, pp 35-51
Usery EL (1996) A conceptual framework and fuzzy set implementation for geographic features. In Burrough PA and Frank A (eds) *Geographic Objects with Indeterminate Boundaries*, Taylor & Francis, London, pp 87-94
Varzi AC 2001 Vagueness in Geography *Philosophy and Geography* 4 49-65
Williamson T (1994) *Vagueness*. Routledge, London
Wood J (1996a) Scale-based characterisation of digital elevation models. In Parker D (ed) *Innovations in GIS 3*, Taylor & Francis, London, pp 163-175
Wood J (1996b) *The Geomorphological Characterisation of Digital Elevation Models.* Unpublished PhD Thesis, Department of Geography, University of Leicester. (http://www.soi.city.ac.uk/~jwo/phd) Accessed 13 September 2002
Wood J (1998) Modelling the continuity of surface form using digital elevation models. In: *Proceedings of the 8th International Symposium on Spatial Data Handling,* Simon Fraser University, Burnaby, British Columbia, pp 725-736
Wood J (2002a) *Landserf: Visualisation and Analysis of Terrain Models* , URL: http://www.soi.city.ac.uk/~jwo/landserf/) Accessed 13 September 2002

Wood J (2002b) Visualizing the structure and scale dependency of landscapes. In: Fisher P and Unwin D (eds), *Virtual Reality in Geography*, Taylor & Francis, London, pp 163-174

11. Fuzzy Representation of Special Terrain Features Using a Similarity-based Approach

Xun Shi[1,4], A-Xing Zhu[2,3], and Rongxun Wang[3]

[1] Geography Department, Dartmouth College, 6017 Fairchild, Hanover, NH 03755, USA

[2] State Key Laboratory of Resources and Environmental Information System, Institute of Geographical Sciences and Natural Resources Research, Chinese Academy of Sciences

[3] Department of Geography, University of Wisconsin-Madison, 550 North Park Street, Madison, Wisconsin 53706, USA

[4] Corresponding author: email: xun.shi@dartmouth.edu; Tel: (603) 646-0884; Fax: (603) 646-1601

Abstract. Fuzzy representation of terrain positions can be useful in environmental modeling process, especially in soil-landscape studies. Existing methods for deriving this representation from a digital elevation model (DEM) are often neither effective nor efficient, especially when dealing with some special terrain positions that have only regional or local meanings. This paper presents a similarity-based method for deriving fuzzy representation of special terrain features. This method has two general steps. The first is to find the typical locations (cases) of a specified terrain position and assign full fuzzy membership to these typical locations. The typical locations can be identified in two ways: they can be located by using a set of simple rules based on the geomorphologic definition of the terrain position; or they can be pinpointed or delineated directly by experts using a GIS visualization tool. With the typical locations identified, the next step is to compute the similarities between these typical locations and other landscape locations, and the derived similarity values are then used to approximate the fuzzy memberships of those locations for being the terrain position. This process is applied to some special terrain features in two study areas: one in Wisconsin and the other in Tennessee. In the Wisconsin study area, this method is used to derive the fuzzy representations of broad ridge, narrow ridge, and headwater. In the Tennessee study area, this method is used to derive the fuzzy membership of being a "knob". The resultant fuzzy representations are realistic and meaningful and the whole process is computationally efficient, which indicates that this similarity-based (cased-based) method can be an effective and flexible approach to deriving fuzzy representations of terrain features.

11.1. Introduction

Information about terrain features (e.g., broad ridges and narrow ridges) can be very useful in environmental modeling, especially in soil-landscape studies (Miline 1935, Troeh 1964, Ruhe and Walker 1968, Huggett 1975, Conacher and Dalrymple 1977, Hole and Campbell 1985, Pennock et al. 1987, Zhu et al. 2001). The spatial manifestation of terrain features is that the transition from one terrain feature to its adjacent feature is often not abrupt, but rather gradual. Quantifying this graduation can be a prerequisite to understanding and representing in detail the spatial processes and their interactions over land surface. Fuzzy representation is an effective approach to achieving this quantification (Irvin et al. 1997, MacMillan et al. 2000, Ventura and Irvin 2000).

Three approaches to deriving fuzzy representation of terrain features from digital elevation models (DEM) have been proposed. Skidmore (1990) used Euclidean distances of a given location to the nearest streamline and ridgeline to represent the location's relative position. The main problem of this approach is that the Euclidean distance is often not sufficient to represent local morphological characteristics. Irvin et al. (1997) performed a continuous classification of terrain features using the fuzzy *k*-mean method. As a basically unsupervised classification, the fuzzy *k*-mean method sometimes has difficulty to produce results that satisfactorily match domain experts' (e.g., soil scientists) views on landscapes. MacMillan et al. (2000) developed a sophisticated and comprehensive rule-based method for fuzzy classification of terrain features. However, this method requires intensive terrain analysis operations and has a high demand for user's knowledge of local landform, which might limit its use in some real-world applications.

This chapter describes an alternative method for deriving fuzzy representation of terrain features. The general idea of this method is to derive fuzzy membership of a test location for being a specific terrain feature based on the location's similarity to the typical locations of that terrain feature. The approach is illustrated using four special terrain features in two study areas: broad ridge, narrow ridge, and headwater in a study area in Wisconsin and "knob" in a study area in Tennessee. These terrain features are considered to be "special" in the sense that they do not belong to any established terrain models (e.g., five-component model (Ruhe and Walker, 1968)) but only have local meanings in certain areas and in certain applications. Since they are not components of a classification system (terrain model) that exhausts the geographic and attribute spaces of a study area, they receive little attention from the digital terrain analysis community. But these special terrain features have very unique meanings to soil-landscape analysts since unique soil conditions often exist at these locations.

11.2. The Similarity-based Approach

This similarity-based approach has two general steps: first, find the typical locations of a specified terrain feature and assign these typical locations full fuzzy memberships for being that terrain feature; and second, calculate the similarities between these typical locations and other landscape locations. These similarity values are then used as the fuzzy membership values of other landscape locations for being the specified terrain feature. The idea of utilization of typical examples to perform classification can be traced back to the research in case-based reasoning in artificial intelligence area (e.g., Aamodt and Plaza 1994, Kolodner 1993, Leake 1996, Watson 1997) and exemplar-based classification in cognitive science (e.g., Medin, Dewey, and Murphy 1983, Smith and Minda 1998). Readers familiar with soil-landscape study may also associate this method to some previous work in this area. For example, Pike (1988) used samples to classify landslide-terrain types with DEM and Lagacherie et al. (1995) proposed a method to conduct soil mapping based on the rules developed from a reference area and the samples observed in the new area. We consider that Pike performed a conventional supervised classification: use samples to develop a set of classifying rules (expressed by the thresholds of variables involved). The present method, while is capable of utilizing samples, does not attempt to develop a set of rules beforehand, but performing classification by *locally* and *individually* comparing a test location and those typical locations (Some authors in artificial intelligence area refer to this strategy as *lazy learning*. See Aha 1997 for details). The difference between the present method and Lagacherie et al's is that the present method does not require reference areas.

11.2.1. Finding Typical Locations

Generally, there are two ways to find the typical locations: a definition-based way and a knowledge-based way. For some terrain features that have clear geomorphologic definitions, it may be possible to develop a set of simple rules based on the definitions and use these rules to locate the typical locations. In some cases, there exist algorithms to locate the typical locations, e.g., ridgelines and streamlines. However, for a terrain feature that only has a local meaning, finding the typical location may require knowledge from local experts. The experts can express their knowledge through manually delineation using a GIS visualization tool. Note that in this step, crisp logic is applied – the objective is to identify a location that is a typical location of the specified feature.

11.2.2. Calculating Similarity

The similarities of any other location to those located typical locations are evaluated based on a set of selected terrain attributes (e.g., elevation, slope gradient, curvatures, etc.). Therefore, the process of assigning fuzzy membership value to a location consists of three steps: first, evaluate the similarity between a test location and a typical location at the individual terrain attribute level; second, integrate the similarities on individual terrain attributes to obtain the overall similarity between the test location and a typical location; and third, integrate the test location's similarities to all the typical locations to obtain the final fuzzy membership of the test location for being the terrain feature under concern. This process can be represented by

$$s_{ij} = \underset{t=1}{\overset{n}{T}} \ \underset{v=1}{\overset{m}{\{P_t}}}[E_t^v(z_{ij}^v, z_t^v)]\} \tag{11.1}$$

where s_{ij} is the fuzzy membership value for location (i, j) being a specific terrain feature; n is the number of identified typical locations of the terrain feature and m is the number of terrain attributes taken into account; z_{ij}^v is the value of the vth terrain attribute at location (i, j) and z_t^v is the corresponding value associated with the tth typical location; E is the function for evaluating the similarity on the vth terrain variable and this function can be specific for terrain attribute v and typical location t; P is the function for evaluating the overall similarity at the location level and can be specific for typical location t; T is the function for deriving the final fuzzy membership value based on the similarities between site (i, j) and all the typical locations for the specified terrain feature.

Evaluating similarity on an individual terrain attribute

There can be various choices for function E in Eq. 11.1. Since most terrain attributes have continuous values, here we focus on the functions that handle such values. Burrough (1989) and Burrough et al. (1992) discussed five fuzzy membership functions (models) that can be applied to continuous variables. MacMillan et al. (2000) reduced these five models to three models, which are illustrated in Figure 11.1. Model *a* (Figure 11.1a) is a generic model. It describes the scenario in which a *central* value exists. Precisely at this central value, the similarity value achieves its peak. The farther away from the central value, the lower the similarity value, and this variation is symmetric on the two sides of the central value. This model is often referred to as the *bell-shaped* model. Models *b* and *c* are for the scenarios in which the variation of similarity is asymmetric. In model *b*, there is a threshold. When the value of the variable under concern is below this threshold, the similarity stays at the maximum value; when the value of the variable is above this threshold, the similarity value decreases gradually. This model is called the *z-shaped* model. Model *c* is a reverse image of model *b*, and is referred to as *s-shaped* model. The *bell-shaped*, *s-shaped*, and *z-shaped* models are often referred to as π, S+, and S- functions in the fuzzy set literature, respectively. In this research, the central value in model *a* and the thresholds in models *b* and *c* are the terrain attribute value at the typical location (i.e., z_t^v in Eq. 11.1).

There can be various mathematical ways to construct a *bell-shaped* model. Burrough et al. (1992) used a mathematical function that can be found in Kandel (1986), which is as follows (adapted from Burrough et al. 1992):

$$s_{ij,t}^v = \frac{1}{1 + \left[\left(z_{ij}^v - z_t^v\right)/d\right]^2} \tag{11.2}$$

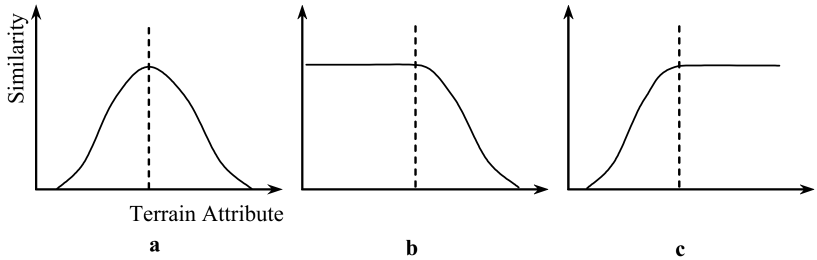


Fig. 11.1. An illustration of various functions (models) that can be applied to continuous variables (Adapted from Burrough et al. 1992 and MacMillan 2000)

where $s_{ij,t}^{v}$ is the similarity between location (*i, j*) and typical location *t* on variable *v*; z_{ij}^{v} and z_{t}^{v} have the same meanings as those in Eq. 11.1; and *d* is a *dispersion index* (Burrough et al. 1992) for adjusting the shape of the function curve and the position of the *crossover points* (Kandel 1986). A *crossover point* is the value of a terrain variable at which fuzzy membership is 0.5, if the fuzzy membership is defined on a 0-1 range. Models *b* and *c* are constructed by simply replacing either half (left or right) of model *a* with a constant linear function. The use of functions in this form by Burrough et al. (1992) was followed by MacMillan et al. (2000). Alternatively, McBratney and Odeh (1997) used a Gaussian-style function that can be found in Jang and Gulley (1995) to describe a *bell-shaped* curve (adapted from McBratney and Odeh 1997):

$$S_{ij,t}^{v} = e^{-\left(\frac{\left|z_{ij}^{v}-z_{t}^{v}\right|}{1.44\sigma^{2}}\right)} \qquad (11.3)$$

where $s_{ij,t}^{v}$, z_{ij}^{v}, and z_{t}^{v} have the same meanings as those in Eq. 11.2; *e* is the base of the natural logarithm (2.71828…); σ is the lower *crossover point* (i.e., *cross point* on the left half of the curve). None of these authors explained their choices.

In this research, we follow McBratney and Odeh (1997) to use a Gaussian-style function, mainly because the normal-distribution function is generally appealing and also because the function is relatively convenient to expand to a form that allows the user to easily control the shape of the curve. The function we used is as follows (Burt 2000):

$$s = e^{-p} \qquad (11.4)$$

where *p* is calculated with the equation below:

$$p = \left(\frac{\left|z_{ij}^{v}-z_{t}^{v}\right|}{w/(-\ln(k))^{\frac{1}{r}}}\right)^{r} = -\left(\frac{\left|z_{ij}^{v}-z_{t}^{v}\right|}{w}\right)^{r}\ln(k) \qquad (11.5)$$

Therefore, Eq. 11.4 may take the following form:

$$S_{ij,t}^{v} = e^{\left(\left|z_{ij}^{v}-z_{t}^{v}\right|/w\right)^{r}\ln(k)} \qquad (11.6)$$

where $s_{ij,t}^{v}$, z_{ij}^{v}, and z_t^{v} have the same meanings as those in Eqs. 11.2 and 11.3, and w, k, and r are parameters for controlling the shape of the model curve. The w and k work together to perform the function similar to that performed by d in Eq. 11.2, but give the user more controlling power: The user can adjust the curve shape by specifying that if the difference between the variable values at two locations is w, the similarity should be k. The value of r controls the width of the flat top part of the curve and the steepness of the side part of the curve.

Eq. 11.6 can be expanded for more general situations: The two halves of the function curve do not have to be symmetric:

$$\begin{cases} s_{ij,t}^{v} = e^{\left(\left|z_{ij}^{v}-z_t^{v}\right|/w_1\right)^{r_1}\ln(k_1)} & if\, z_{ij}^{v} < z_t^{v} \\ s_{ij,t}^{v} = 1 & if\, z_{ij}^{v} = z_t^{v} \\ s_{ij,t}^{v} = e^{\left(\left|z_{ij}^{v}-z_t^{v}\right|/w_2\right)^{r_2}\ln(k_2)} & if\, z_{ij}^{v} > z_t^{v} \end{cases} \tag{11.7}$$

The idea of Eq. 11.7 is to use two sets of w, k, and r to define the two halves of the curve. In Eq. 11.7, w_1, k_1, and r_1 define the half of the curve when the terrain attribute value at a given location is smaller than the value at the typical location (the left half of the curve), and w_2, k_2, and r_2 define the half of the curve when the terrain attribute variable value is greater than the value at the typical location (the right half of the curve). When $w_1 = w_2$, $r_1 = r_2$, and $k_1 = k_2$, Eq. 11.7 is equivalent to Eq. 11.6; when w_1 and/or k_1 increase, Eq. 11.7 approaches a *z-shaped* model; Similarly, when w_2 and/or k_2 increase, Eq. 11.7 approaches an *s-shaped* model.

Eq. 11.7 cannot be directly applied to a cyclic variable (e.g., the slope aspect). However, the difference between angle degrees can be measured with the following equation group (Burt, personal communication):

$$\begin{aligned} \rho &= \boldsymbol{max}((z_{ij}^{v}-z_t^{v}),\ -360-(z_{ij}^{v}-z_t^{v}))\quad if\, z_{ij}^{v} \le z_t^{v} \\ \rho &= \boldsymbol{min}((z_{ij}^{v}-z_t^{v}),\ 360-(z_{ij}^{v}-z_t^{v}))\quad if\, z_{ij}^{v} > z_t^{v} \end{aligned} \tag{11.8}$$

Besides this linear function, a nonlinear function, described below, can also be used:

$$\begin{aligned} \rho' &= 1 - \cos\,(z_{ij}^{v} - z_t^{v})\quad if\, z_{ij}^{v} \le z_t^{v} \\ \rho' &= 1 + \cos\,(z_{ij}^{v} - z_t^{v})\quad if\, z_{ij}^{v} > z_t^{v} \end{aligned} \tag{11.9}$$

Replace z_{ij}^{v} - z_t^{v} in Eq. 11.6 and Eq. 11.7 with ρ in Eq. 11.8 or ρ' in Eq. 11.9, then Eq. 11.6 and Eq. 11.7 can be applied to cyclic variables.

Evaluating overall similarity between two locations

Function P in Eq. 11.1 is for integrating the information at the individual terrain attribute level. The result from this function is the overall similarity between a test location and a typical location. Choices for function P may include the weighted-average method, distance method, ID3 method, and limiting-factor method.

The weighted-average method can be represented as follows:

$$s_{ij,t} = \frac{\sum_{v=1}^{m} w_t^v \times s_{ij,t}^v}{\sum_{v=1}^{m} w_t^v} \tag{11.10}$$

where $s_{ij,t}$ is the overall similarity between locations (i, j) and typical location t; $s_{ij,\ t}^{v}$ is the similarity on vth terrain attribute; w_t^v is the weight of the vth terrain attribute and it can be specific for t; and m is the number of terrain attributes involved in the computation.

The (dis)similarity between two locations can also be represented by their distance in attribute space. Possible measurements of distance may include Minkowski Distance and Mahalanobis Distance. Minkowski Distance is calculated as follows:

$$D_{ij,t} = \left(\sum_{v=1}^{m} \left(s_{ij,t}^{v} \right)^{q} \right)^{\frac{1}{q}} \tag{11.11}$$

where $D_{ij,t}$ is the distance; $s_{ij,\ t}^{v}$ has the same meaning as that in Eq. 11.10; and q is a user-specified parameter. Mahalanobis Distance is calculated as follows:

$$D_{ij\ ,t}^{2} = (X_{ij}-X_t)'C_{ij,t}^{-1}(X_{ij}-X_t) \tag{11.12}$$

where D is the distance; X_{ij} and X_t are terrain attribute vectors consisting of the values on individual terrain attributes; and $C_{ij,t}^{-1}$ is covariance matrix of X_{ij} and X_t. If Mahalanobis distance is used, function E in Eq. 11.1 is not needed.

The ID3 (Quinlan 1993) method requires terrain attributes to be ordered according to their importance in the evaluation. The evaluation starts from the most important terrain attribute. If the two locations are considered similar in terms of the current terrain attribute, the evaluation goes on to the next terrain attribute; otherwise, the evaluation stops. For each terrain attribute, a threshold can be set to determine if the two locations are similar enough on this attribute. The overall similarity can be defined based on the number of terrain attributes "passed" by the two locations.

The limiting-factor method is based on the limiting-factor principle in ecology. According to this principle, the formation or development of an ecological feature, such as vegetation or soil, is controlled or determined by the least favoring factor in its environment. Technically, this method is applied by using the fuzzy minimum operator for the function P in Eq. 11.1, which selects the smallest value from all the similarity values on individual attributes as the overall similarity value at the location level. Following Zhu and Band (1994), the limiting-factor method is employed in the case studies presented in this chapter. While the limiting factor method is probably the easiest and simplest choice for function P, more research, nevertheless, is needed to find out the most appropriate way to integrate the influences of different terrain attributes.

Integrating similarity from multiple typical locations

Similarity can be calculated between a given location and each of the typical locations for the terrain feature under concern. Function T in Eq. 11.1 is for integrating similarity values from multiple typical locations and obtaining the final fuzzy membership for being the terrain feature. One way to achieve this integration is to use the inverse-distance method:

$$s_{ij} = \frac{\sum_{t=1}^{n} (d_{ij,t})^{-r} s_{ij,t}}{\sum_{t=1}^{n} (d_{ij,t})^{-r}} \tag{11.13}$$

where s_{ij} is the integrated fuzzy membership value, $d_{ij,t}$ is the geographic distance between the test location and the tth typical location; $s_{ij,t}$ is the similarity between locations (i, j) and typical location t; n is the number of the typical locations involved; and r is the decay factor.

Apparently, when calculating the fuzzy membership of a given location for being a specific terrain feature, it is neither efficient nor reasonable to involve all the typical locations for that terrain feature in the study area. For example, a typical location in one sub-watershed may have little influence to a test location in another sub-watershed. Therefore, it is necessary to define *relevant* typical locations for a test location and only use those relevant typical locations to calculate the fuzzy membership for that test location. One method to define relevant typical locations is to define the *maximum influence distance* – a typical location is relevant only when the geographic distance between this typical location and the test location is shorter than the *maximum influence distance*. In other words, the *maximum influence distance* allows each typical location to influence the classification of locations only within its defined control distance. When measuring the distance between a typical location and a test location, the distance along the terrain surface can be more meaningful than the Euclidean distance in characterizing local topographic features (Figure 11.2). We call the distance along terrain surface *surface distance*.

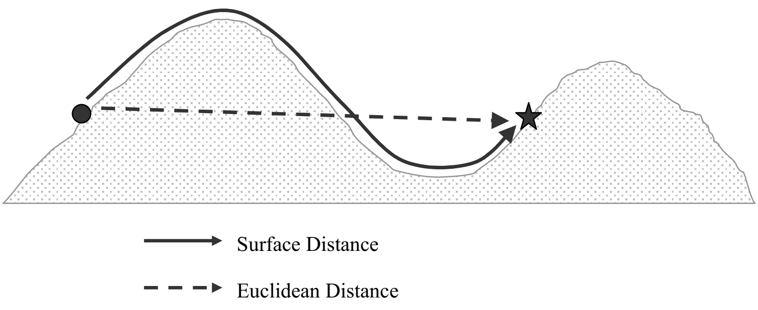


Fig. 11.2. Surface Distance vs. Euclidean Distance

Before sending the similarity value on each individual typical location ($s_{ij,t}$) to function T, if necessary, the similarity value, which is calculated in attribute space, can be adjusted using the geographic distance:

$$s_{ij,t}' = s_{ij,t}\, e^{\left(d_{ij,t}/w_t'\right)^{r'} \ln(k')} \tag{11.14}$$

where $s_{ij,t}'$ is the adjusted similarity between test location (i,j) and typical location t; $s_{ij,t}$ is the similarity value from function P; $d_{ij,t}$ is the geographic distance between the two locations; w_t' is the *maximum influence distance* of typical location t; r' performs the same function as that of r in Eq. 11.5; and k' is a very small number (e.g., 0.0001). Eq. 11.14 defines a function curve by specifying that a given location at the *maximum influence distance* to typical location t will have a very small similarity value to t.

11.3. Deriving Fuzzy Representations of Some Special Terrain Features

The four special terrain features dealt with in this research are specified by the soil scientists in two soil mapping research projects. The soil scientists believe that these special terrain features have strong association with certain soils, thus the information about these features can be highly useful in mapping those soils.

11.3.1. Study Areas and Data

There are two study areas in this research. The study area for broad and narrow ridges and headwaters is the Pleasant Valley area, a small watershed in southwestern Wisconsin. This study area is located in the eastern portion of the Driftless Area, which was not directly overridden by continental ice sheets during the Quaternary. The topography in this area is primarily narrow and alluvial valleys, steep slopes, and broad ridges (Irvin et al., 1997). The size of the Pleasant Valley study area is about 10 km^2. The data for this study area is a 9.1-m (30-ft) DEM interpolated from a TIN provided by the Natural Resources Conservation Service (NRCS) Dane County (Wisconsin) Office.

The study area for “knobs” is the northwestern corner of Mt. Guyot Quadrangle (Tennessee) in terms of USGS 7.5-minute topographic maps. The area is located in the Appalachian Mountains and is part of the Great Smoky Mountains National Park. The geological type is Pigeon siltstone and the variation of topography is relatively gentle with the range of around 440m to 1100m. The size of this study area is about 15 km^2. The data for this study area is the USGS 7.5-minute DEM with 10-m resolution.

11.3.2. Broad ridge and narrow ridge

The soil scientists working on a soil survey project in the Pleasant Valley area expect to see two different soil series on two different types of ridges in the Oneota geological region: New Glarus on broad ridges and Dunbarton on narrow ridges. New Glarus has a deeper soil profile than Dunbarton does. Therefore, automated classification of different types of ridges should be able to facilitate the soil mapping process.

There are two steps in finding typical locations for broad and narrow ridges: first, find typical locations for ridges, and then, label these ridge locations as "broad ridge" or "narrow ridge" according to the definition given by the soil scientists. In this research, typical ridge locations are found using Peucker and Douglas' (1975) algorithm. The result from this algorithm usually contains noise. To reduce the noise, certain "cutting" rules can be applied. For the Pleasant Valley area, those "ridge locations" whose elevations are below 278 m or slope gradients are greater than 6% are considered to be noise.

For the Pleasant Valley area, soil scientists defined a broad ridge as any ridge

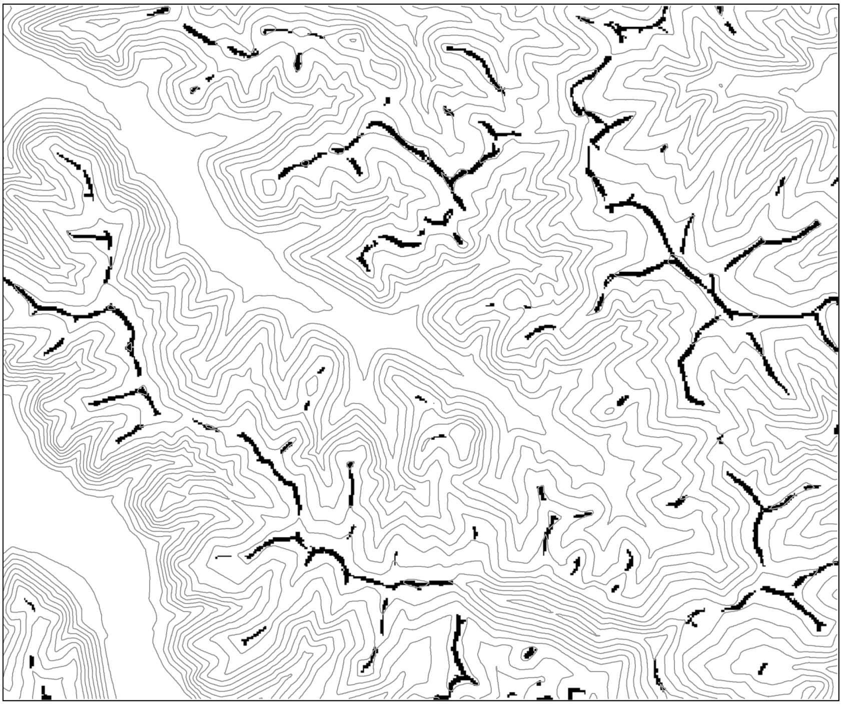


Fig. 11.3. Ridgelines of broad ridges in the Pleasant Valley study area

that has a width greater than 25 m and a continuous flat area greater than 930 m^2 in size. Ridges that do not meet these criteria were defined as narrow ridges. To implement this definition, for each ridge pixel, first the size of its surrounding contiguous flat area (slope gradient < %6) is measured, and if the size is equal to or smaller than 930 m^2, this ridge pixel will be labeled as narrow-ridge pixel; if the

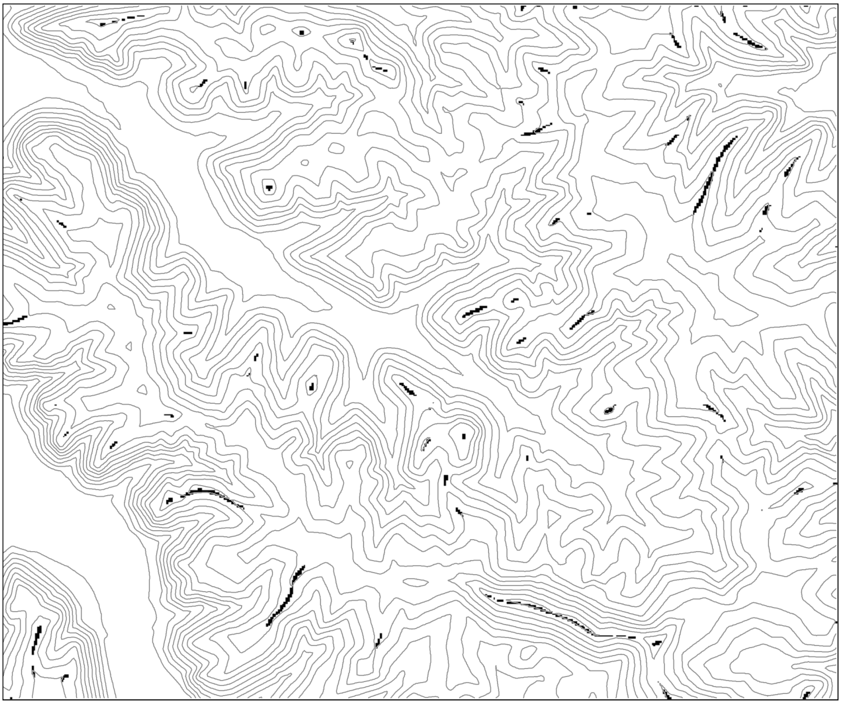


Fig. 11.4. Ridgelines of narrow ridges in the Pleasant Valley study area

size is greater than 930 m^2, the widths of the flat area along north-south, west-east, northwest-southeast, and northeast-southwest directions will be measured, and if the widths along all four directions are greater than 25 m, the ridge pixel will be labeled as broad-ridge pixel; otherwise, labeled as a narrow-ridge pixel. Since normally it is not reasonable that a single broad-ridge pixel is surrounded by narrow-ridge pixels or a narrow-ridge pixel is surrounded by broad-ridge pixels, a post-process filter was applied to convert single isolated cells into the type of the surrounding pixels. The results are shown in Figures 11.3 and 11.4.

Figures 11.5 and 11.6 show the fuzzy representations of broad and narrow ridges derived based on the two types of ridgelines. The selection of terrain attributes for classifying terrain features have been discussed by many authors (e.g., Irvin et al. 1997, MacMillan et al. 2000). The selection of terrain attributes for similarity evaluation in this study is based on those discussions. However, it turns out that to generate fairly appealing results the present method is able to use much fewer terrain attributes than what were used in previous research and the use of more terrain attributes does not significantly improve the results. The parameter

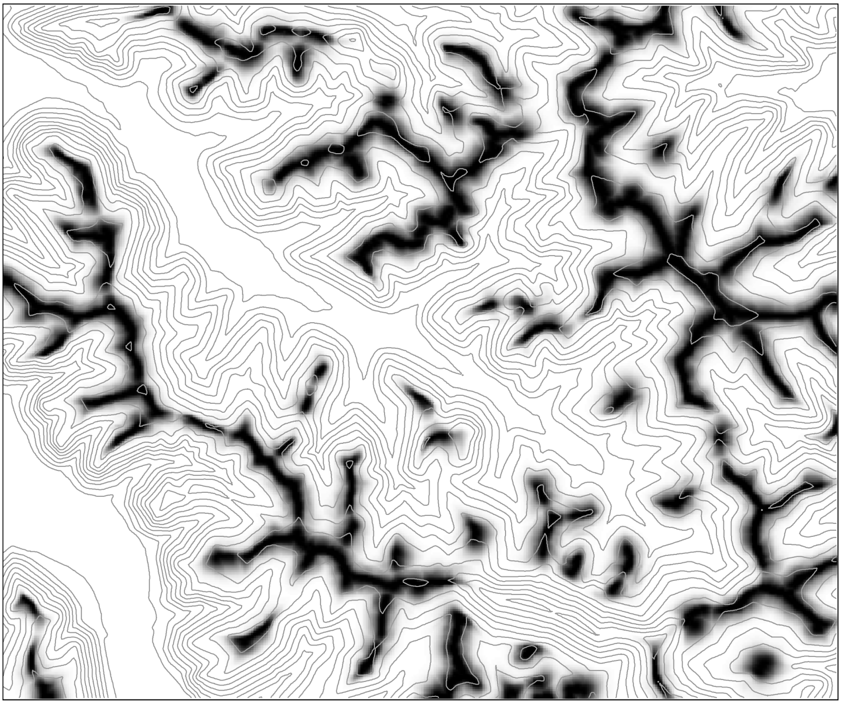


Fig. 11.5. Fuzzy representation of broad ridges in the Pleasant Valley study area. Three terrain attributes, elevation, slope gradient, and profile curvature, are used to evaluate the similarity. The parameters in Eq. 11.7 for these three attributes are as follows: elevation: *s-shaped* model, $w_1 = 6$ m, $k_1 = 0.5$, $r_1 = 2$; slope gradient: *z-shaped* model, $w_2 = 8\%$, $k_2 = 0.5$, $r_2 = 2$; profile curvature: *z-shaped* model, $w_2 = 0.005$, $k_2 = 0.5$, $r_2 = 2$. The limiting-factor method is used for function P in Eq. 11.1. The *maximum influence distance* is 245 m, r in Eq. 11.13 is 2, and r' in Eq. 11.14 is 2.

values were set through experiments. After each experiment, the inference result was visually evaluated and the parameter values were adjusted accordingly.

Using the contour lines as a reference, we find that the fuzzy representations shown in Figures 11.5 and 11.6 match common expectations well. The contour lines demonstrate that most of the major ridges in this watershed have relatively wide and flat tops and can be considered to be broad ridges. The contour lines also show that most small ridges, especially those branches of the major ridges, can be interpreted as narrow ridges. Besides those small ridges, two major narrow ridges occur in the upper-left and lower-right corners of the watershed. The present method correctly captures these features and the resulting fuzzy membership values vary naturally in accordance with the elevation and local landform. Note that in this research "broad ridge" and "narrow ridge" are considered as two types of ridges, and the fuzzy memberships for both types of ridges are representing the variation from ridge to non-ridge, but not the variation from broad ridge to narrow ridge. When defining broad and narrow ridges, we still use crisp logic (with the thresholds 25 m and 930 m^2).

11.3.3. Headwater

The typical locations of headwater are found using a method inspired by Tribe's methods (Tribe 1992a, 1992b): First, a simulated stream network is derived using the algorithm of O'Callaghan and Mark (1984); Second, a weighted average of normalized profile and planform curvatures is calculated for each streamline pixel; Third, this weighted average value is smoothed by averaging the values of a certain number of consecutive streamline pixels; finally, starting from the upper end of each streamline, check this smoothed value along the streamline and the first peak value indicates the location of headwater. This method may mistakenly label some locations in flood plains as headwaters. A simple "cutting" rule that all the headwaters must be above a specified elevation (in our case, 260 m) is used to remove this kind of noise, but at the cost that some real headwater locations are also being removed. Figure 11.7 shows the fuzzy representation of headwaters derived based on the typical locations found using this method. In most places the representation is reasonable. However, it is easy to find that in some places real headwaters are not identified (e.g., location A), in some places identified headwaters are not typical (e.g., location B), and in some places the positions of headwaters are either too upperstream (e.g., location C) or too downstream (e.g., location D). The main reason for these problems is that using O'Callaghan and Mark's (1984) algorithm, our method locates the heads of streams solely based on upper drainage area. Without considering local morphological factors, sometimes the heads of streams located by our method are not appropriate for locating headwaters.

11.3.4. Knob

Knob is a special type of ridge in the Mt. Guyot study area. Local soil scientists define knobs as flat, isolated, and low elevation small benches in Pigeon siltstone area. These benches have a strong association with Brasstown soil series.

The typical locations of knobs were identified by local soil scientists as lines on a 3D view of the terrain surface. The results from this process are shown in Figure 11.8. The fuzzy representation derived using the similarity-based approach is shown in Figure 11.9. Although the validity of the fuzzy representation needs to be further studied and examined, we find the gradation of fuzzy membership over the area matches our expectation. The Brasstown soil series is a fine textured soil that occurs on the top of the knobs where soil erosion is less severe. The severity of the erosive process is highly related to how these knobs transition into other landform positions. Often, a smoother transition results in a gradual increase in

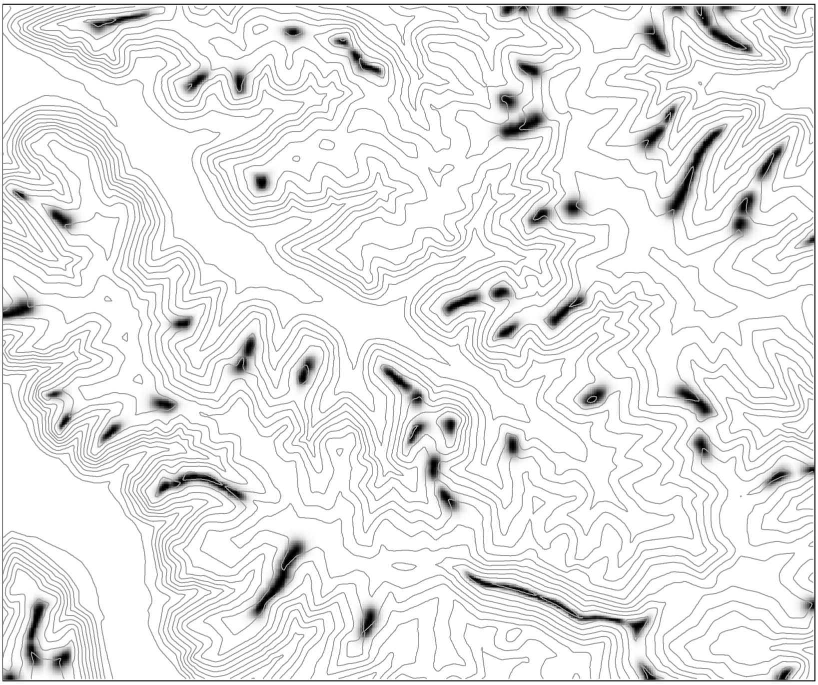


Fig. 11.6. Fuzzy representation of narrow ridges in the Pleasant Valley study area. Three terrain attributes, elevation, slope gradient, and profile curvature, are used to evaluate the similarity. The parameters in Eq. 11.7 for these three attributes are as follows: elevation: *s-shaped* model, $w_1 = 4.6$ m, $k_1 = 0.5$, $r_1 = 2$; slope gradient: *z-shaped* model, $w_2 = 10\%$, $k_2 = 0.5$, $r_2 = 2$; profile curvature: *z-shaped* model, $w_2 = 0.008$, $k_2 = 0.5$, $r_2 = 2$. The limiting-factor method is used for function P in Eq. 11.1. The *maximum influence distance* is 150 m, r in Eq. 11.13 is 2, and r' in Eq. 11.14 is 2.

the severity of erosive process. The current fuzzy representation depicts areas of smooth transition from areas of abrupt transition around the areas of typical knobs.

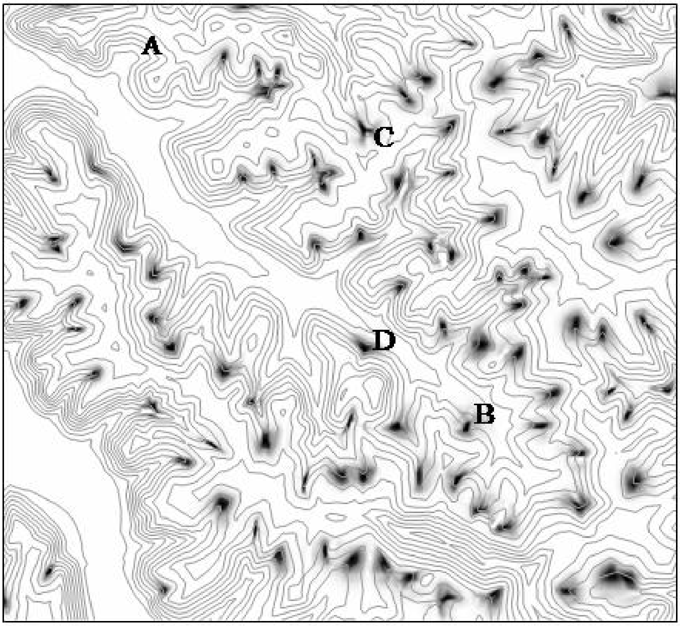


Fig. 11.7. Fuzzy representation of the headwaters in the Pleasant Valley study area. To find typical locations, the threshold for extracting streamlines using O'Callaghan and Mark (1984) algorithm is 14,000 m^2; profile and planform curvatures are normalized to the same range and are assigned equal weights in the inference; smooth range is 5 pixels; and the elevation of a typical location must be greater than 260 m. Four terrain attributes, elevation, slope gradient, profile curvature, and planform curvature, are used to evaluate the similarity. The parameters in Eq. 11.7 are as follows: elevation: *bell-shaped* model, $w_1 = w_2 = 6$ m, $k_1 = k_2 = 0.5$, $r_1 = r_2 = 2$; slope gradient: *bell-shaped* model, $w_1 = w_2 = 6\%$, $k_1 = k_2 = 0.5$, $r_1 = r_2 = 2$; profile curvature: *s-shaped* model, $w_1 = 0.0015$, $k_1 = 0.5$, $r_1 = 2$; planform curvature: *s-shaped* model, $w_1 = 0.015$, $k_1 = 0.5$, $r_1 = 2$. The limiting-factor method is used for function P in Eq. 11.1. The *maximum influence distance* is 300 m, r in Eq. 11.13 is 2, and r' in Eq. 11.14 is 2.

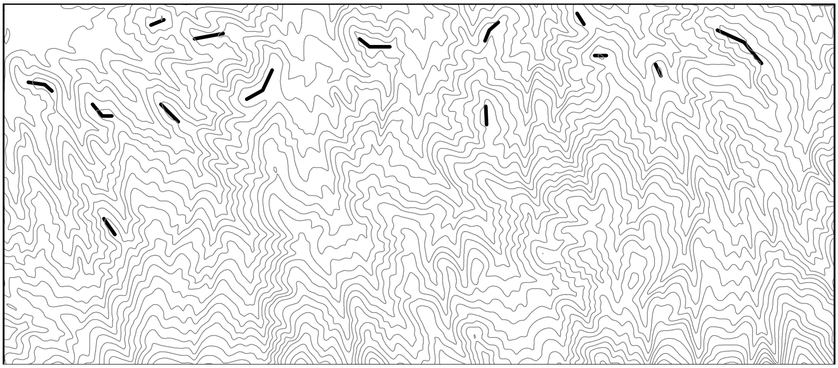


Fig. 11.8. Typical locations for knobs in the Mt. Guyot study area

11.4. Summary

This paper discusses a similarity-based method for deriving fuzzy representations of some special terrain features, including broad ridge, narrow ridge, headwater, and "knob". There are two general steps in this method: first, locating those typical locations for a terrain feature and second, comparing other locations with these typical locations. Typical locations for a terrain feature can be found in an automatic way based on the established definition or rules for that terrain feature (e.g., Puecker and Douglas' algorithm for ridge). In practice, the possibility of using certain algorithms to automatically identify typical locations will always be first explored. When there is no explicit definition or the definition is very hard to technically implement, local experts can manually delineate the typical locations using a GIS tool. We consider this flexibility as an advantage of this method, as it allows the method to be capable of dealing with highly special and/or subjective landform features. The second step is to derive the fuzzy memberships by computing the similarities between a test location and those typical locations. There are many options for constructing the similarity-evaluating functions at different levels (individual terrain attribute, individual typical location, and multiple typical locations) and in different aspects (attribute and spatial).

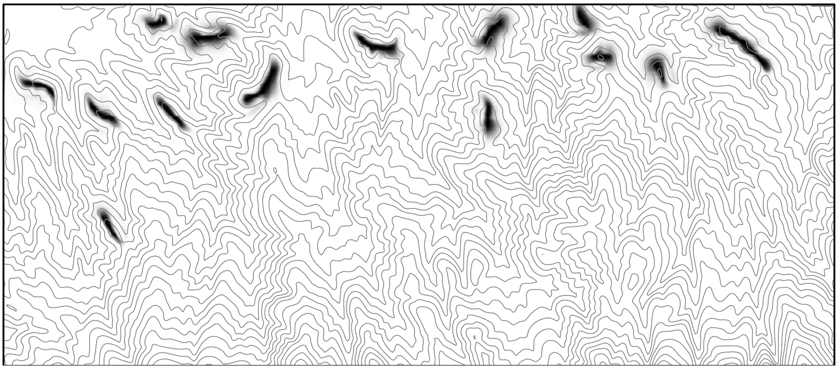


Fig. 11.9. Fuzzy representations of the knobs in the Mt. Guyot study area. Two terrain attributes, elevation and slope gradient, are used to evaluate the similarity. The parameters in Eq. 11.7 for these two attributes are as follows: elevation: $w_1 = 200$ m, $k_1 = 0.5$, $r_1 = 2$, $w_2 = 50$ m, $k_2 = 0.5$, $r_2 = 2$; slope gradient: *z-shaped* model, $w_2 = 10\%$, $k_2 = 0.5$, $r_2 = 2$. The limiting-factor method is used for function P in Eq. 11.1. The *maximum influence distance* is 225 m, r in Eq. 11.13 is 2, and r' in Eq. 11.14 is 2.

Two basic ideas underlie this method. Firstly, we use specific real locations selected as being typical of the feature, rather than abstract rules, as benchmarks in deriving fuzzy membership values. Secondly, we localize the operations, comparing a given location only to nearby typical locations. On one hand, the landscape in the real world can be highly complex even within a small area and any general rules might not be able to accurately cover all specific situations; On the other hand, what is really meaningful in practice (e.g., soil mapping) is the *relative* landform and position in a local context, so it makes sense to do the classification locally. We believe that the ideas and methods presented in this chapter can also be applied to geomorphic features other than the ones discussed here.

Acknowledgements

The research described here are funded by the following institutions and agencies: Dartmouth College, Institute of Geographical Sciences and Natural Resources Research, Chinese Academy of Sciences (through its One Hundred Person Award to A-Xing Zhu), Natural Resources Conservation Service, USDA (Agreement No. 69-5F48-9-00186) and National Park Service, Department of Interior.

References

Aamodt A, Plaza E (1994) Case-based reasoning: Foundational issues, methodological variations, and system approaches. AI Communications 7:39-52

Aha DW (1997) Editorial. Artificial Intelligence Review 11:7-10

Burrough PA (1989) Fuzzy mathematical methods for soil survey and land evaluation. J Soil Science 40:477–492

Burrough PA, MacMillan RA, van Deursen W (1992) Fuzzy classification methods for determining land suitability from soil profile observations and topography. J Soil Science 43:193–210

Burt JE (2000) personal communication. Department of Geography, University of Wisconsin-Madison, Madison, Wisconsin

Conacher AJ, Dalrymple JB (1977) The nine unit landsurface model: an approach to pedogeomorphic research. Geoderma 18:1–54

Hole FD, Campbell JB (1985) Soil Landscape Analysis. Rowman & Allanheld, Totowa, NJ.

Huggett RJ (1975) Soil landscape systems: a model of soil genesis. Geoderma 13:1–22

Irvin BJ, Ventura SJ, Slater BK (1996) Fuzzy and isodata classification of landform elements from digital terrain data in Pleasant Valley, Wisconsin. Geoderma 77:137–154

Kandel A (1986) Fuzzy mathematical techniques with applications. Addison-Wesley, Boston, MA

Kolodner J (1993) Case-Based Reasoning. Morgan Kaufmann Publishers, San Mateo, CA

Lagacherie P, Legros JP, Burrough PA. (1995) A soil survey procedure using the knowledge of soil pattern established on a previously mapped reference area. Geoderma 65:283–301

Leake DB (1996) CBR in context: the present and future. In D.B. Leake (eds) Case-based reasoning: Experiences, lessons, and future directions. MIT Press, Cambridge, MA, pp 3-30

MacMillan RA, Pettapiece WW, Nolan SC, Goddard TW (2000) A generic procedure for automatically segmenting landforms into landform elements using DEMs, heuristic rules and fuzzy logic. Fuzzy Sets and Systems 113:81–109

Medin DL, Dewey GI, Murphy TD (1983) Relationships between item and category learning: Evidence that abstraction is not automatic. J. of Experimental Psychology: Learning, Memory, and Cognition 9:607-625

Miline G (1935) Some suggested units of classification and mapping particularly for East African soils. Soil Research 4:183–198

O'Callaghan JF, Mark DM (1984) The extraction of drainage networks from digital elevation data: Computer Vision. Graphics and Image Processing 28:323–344

Peucker TK, Douglas DH (1975) Detection of surface specific points by local parallel processing of discrete terrain elevation data. Computer graphics and Image Processing 4:375–387

Pike RJ (1988) The geometric signature: Quantifying landslide-terrain types from digital elevation models. Mathematical Geology 20:491-510

Ruhe RV, Walker PH (1968) Hillslope models and soil formation, II: open systems. In: Transactions of the Ninth Congress of the International Soil Science Society, Adelaide, Australia, pp 551–560

Skidmore AK (1990) Terrain position as mapped from a gridded digital elevation model. Int J Geographical Information Systems 4:33–49

Smith JD, Minda JP (1998) Prototypes in the mist: The early epochs of category learning. J. of Experimental Psychology: Learning, Memory, and Cognition 24:1411-1436

Troeh FR (1964) Landform parameters correlated to soil drainage. Soil Science Society of America Proceedings 28:808–812

Tribe A (1992a) Automated recognition of valley lines and drainage networks from grid digital elevation models: a review and a new method. J Hydrology 139:263–293

Tribe A (1992b) Problems in automated recognition of valley features from digital elevation models and a new method toward their resolution. Earth Surface Processes & Landforms 17:437–454

Ventura SJ, Irvin BJ (2000) Automated landform classification methods for soil-landscape studies. In: Wilson JP, Gallant JC (eds) Terrain analysis: Principles and applications. John Wiley & Son, New York, pp 267–294

Watson I (1997) Applying case-based reasoning: Techniques for enterprise systems. Morgan Kaufman Publishers, San Mateo, CA

Zhu AX, Band LE (1994) A knowledge-based approach to data integration for soil mapping. Can. J. Remote Sens. 20: 408-418

Zhu AX, Hudson B, Burt JE and Lubich K (2001). Soil mapping using GIS, expert knowledge and fuzzy logic, Soil Science Society of America Journal, Vol. 65, pp. 1463-1472.

12. Spatial Decision-Making Using Fuzzy Decision Tables: Theory, Application and Limitations

Frank Witlox and Ben Derudder

Ghent University, Department of Geography, Krijgslaan 281 (S8), B-9000 Gent, Belgium

Abstract. In this paper the basic principles of decision-making using fuzzy decision tables (FDTs) are explained and illustrated. The main emphasis is on introducing standard notations and definitions. The point of departure is the crisp decision table formalism and its inability to deal with imprecision and vagueness. As a potential solution, elements of the theory of fuzzy sets are used to develop a new modelling technique, known as FDTs. The properties of FDTs are formally described and illustrated.

12.1. Introduction

A spatial decision-making problem, for instance, the issue of selecting a suitable location site, can be modelled from different perspectives. Especially in economic research, several authors have suggested to use stated or revealed preference and choice models to predict the probability that a particular location will be chosen as a function of its locational and non-locational attributes (Timmermans 1986; Moore 1988; Henley et al. 1989; McQuaid et al. 1996). Such algebraic models do have the appeal of theoretical rigour, mathematical sophistication and an associated error theory. However, the application of such models is characterised by many problems, including high multi-collinearity among explanatory variables, complexity in the sense of a large number of influential attributes, and the fact that algebraic equations by definition cannot capture all theoretical notions.

A modelling approach that avoids these problems is qualitative modelling. The quintessence of this approach is to represent the spatial decision-making process in terms of a set of IF, THEN … ELSE expressions (Witlox, 2000a). These logical expressions, also called productions or decision rules, have sufficient flexibility to represent a wider variety of complex decision rules. Often techniques such as decision plan nets (DPNs) or decision tables (DTs) are used for data representation (Witlox 1995; Witlox and Timmermans 2000). An important problem associated with DPN's and DT's is the "crisp" (or exact) nature of these decision rules. These rules are based on the so-called principle of dichotomy and follow the law of the excluded middle. Although this feature seems highly desirable, it can be a potential drawback if the objective is to model complex human behaviour. A too rigid decision-making process could be the result. To illustrate, consider the following simple statement “the distance to the highway is long”. Such a statement is abound with vague and imprecise concepts that are difficult to translate into more precise rules without losing some of its meaning. For example, the statement “the distance is equal to 1000 m.” does not explicitly state whether the distance is considered as long or not. And even if we would state that a 1000 m. distance is long, would 999 m. be not considered as long? Clearly, handling this kind of non-crisp or “fuzzy” information in qualitative modelling approaches implies the development of new tools. One such potential tool, on which we would like to elaborate in this paper, is the fuzzy decision tables (FDTs).

The remainder of the paper is organised as follows. In section 2, a brief outline is given of the crisp DT formalism. This section is necessary in order to better understand the FDT formalism that is extensively explained in section 3. In that section, some basic definitions, an overview of the different types of FDTs, and the outline of a decision-making process using FDTs are made clear. In section 4, we discuss some of the pros and cons of an FDT approach. Finally, in section 5, the findings are summarized, and avenues for future research are discussed.

12.2. The crisp DT formalism

A crisp decision table (DT) is informally defined by Verhelst (1980) as “(…) a table representing the exhaustive set of mutual exclusive conditional expressions within a pre-defined problem area”. To illustrate, assume that a decision needs to be taken with respect to allowing a certain production level (level 1, level 2, or level 3). The level allowed depends upon the outcome of two criteria (CS_1: Safety of the production process ($S<50$, $S\geq50$) and CS_2: Distance to a residential area ($D<25$, $25\leq D\leq35$, $D>35$)). Represented in a DT, we obtain the following result:

Table 12.1. An example of a crisp DT

CS_1 Safety production process	$S<50$	$S\geq50$	$S<50$	$S\geq50$	$S<50$	$S\geq50$
CS_2 Distance to residential area	$D<25$	$D<25$	$25\leq D\leq35$	$25\leq D\leq35$	$D>35$	$D>35$
AS_1 Allowed production level	level 1	level 2	level 2	level 2	level 2	level 3

As can be noted from Table 12.1, each DT contains four parts. The upper left part of the table lists the condition subjects CS_i for $i = 1, ..., c$ which are the criteria for the decision making process. The universe of discourse CD_i for each condition i is the set of all possible values that the condition can attain. The condition-state set for each condition i consists of the possible states of the condition:

$$CT_i = \left\{S_{i1}, S_{i2}, \ldots, S_{it_i}\right\} \tag{12.1}$$

where t_i is the number of categories for the i-th condition and S_{ij} determines a subset of CD_i. The condition space of a DT is the Cartesian product of the condition state sets CT_i, as follows:

$$\begin{aligned} \mathrm{SPACE}(\mathrm{C}) &= \mathrm{CT}_1 \times \mathrm{CT}_2 \times \ldots \times \mathrm{CT}_\mathrm{c} \quad && \text{for c} > 1 \\ &= \mathrm{CT}_1 && \text{for c} = 1 \end{aligned} \tag{12.2}$$

An element of SPACE(C) is an ordered c-tuple and is called a condition entry (CE). The set of CE's which are present in the DT defines the domain of the DT and is denoted as DOM(DT).

The bottom left part of a DT contains the action subjects AS_i for $i = 1, ..., a$, which represent the terms in which decision outcomes are expressed. For each action i, the action state set AT_i contains the possible values action i can attain:

$$AT_i = \left\{m_{i1}, m_{i2}, \ldots, m_{it_i}\right\} \tag{12.3}$$

The action space of a DT is defined as the Cartesian product of the action state sets:

$$\begin{aligned} \mathrm{SPACE}(\mathrm{A}) &= \mathrm{AT}_1 \times \mathrm{AT}_2 \times \ldots \times \mathrm{AT}_\mathrm{a} \quad && \text{for a} > 1 \\ &= \mathrm{AT}_1 && \text{for a} = 1 \end{aligned} \tag{12.4}$$

An element of SPACE(A) is an ordered a-tuple called an action entry (AE).

As Wets (1998) has shown, a DT can be defined in different ways as a relation, a function or a matrix. The matrix definition suits the purpose of the discussion and can be written as follows. Let n be the number of columns and c the number of conditions. Then, the condition part of a DT can be defined in matrix notation as:

$D = (d_{ij})$, $\quad i = 1, ..., c$ and $j = 1, ..., n$

where $d_{ij} = \cup_{x \in S_{ij}} x$

The action part can be written as:

$E = (e_{ij})$, $\quad i = 1, ..., a$ and $j = 1, ..., n$

where $e_{ij} = m_{ij}$

A DT defines the relation between condition space and action space. Formally:

$$DT = (dt_{ij}) = \begin{pmatrix} D \\ E \end{pmatrix} \tag{12.5}$$

where,

$dt_{ij} = d_{ij}$, $\quad i = 1, ..., \mathrm{c}$ and $j = 1, ..., n$

$= e_{(i-c)j}$, $\quad i = c + 1, ..., c + a$ and $j = 1, ..., n$

The three key-properties of crisp DT's are consistency, exclusivity and completeness. The properties can be formally defined as follows. Let D^j denote the j-th column of D and E^j the j-th column of E. Then, *consistency* can be defined as:

a DT is consistent $\Leftrightarrow \forall\ (D^j, D^k)$: if $\forall(d_{ij}, d_{ik})$: $d_{ij} \cap d_{ik} \neq \varnothing$ then $E^j = E^k$
where $i = 1, ..., c$ and $j, k = 1, ..., n$. The DT is consistent because there is no intersecting pair of columns in the DT of which the action parts differ.

The property of *exclusivity* can be defined as:

a DT is exclusive $\Leftrightarrow \forall\ (D^j, D^k)$: if $j \neq k$ then $\exists(d_{ij}, d_{ik})$: $d_{ij} \cap d_{ik} = \varnothing$
where $i = 1, ..., c$ and $j, k = 1, ..., n$. A DT meets the exclusivity constraint because for every pair of columns there is at least one condition of which the condition states exclude each other.

Finally, a DT is *complete* if it meets the following two constraints:

$\mathrm{DOM(DT)} = \mathrm{SPACE}(C)$ and $\forall E^j$: $\exists\ e_{ij} \in AT_i$

The DT is complete since every CE is included in the condition part of the DT and in every column at least one action is specified.

The crisp decision table formalism has some advantageous properties. Firstly, because they are exclusive, consistent and complete, DTs return for every possible case within the domain a response. This behaviour is not guaranteed by traditional decision trees or by production systems and represents a clear advantage of DTs for any modelling purpose. Secondly, the DT provides a suitable formalism for representing various types of interactions between variables, such as conditional relevance and conceptual interaction. Within each column, a partition of a condition can be defined independently of other columns. Conditional relevance captures this notion that particular requirements are relevant only for particular condition states. Conceptual interaction implies that different locational profiles may be equally suitable for site selection. Compared to other formalisms (decision tree, decision plan nets), the crisp DTs also offer several advantages. For instance, a DT provides a schematic view of the inference process of a decision-making procedure. It also offers a more compact visual presentation, and is more efficient and effective than the decision tree with respect to checking the information input on completeness, correctness, and consistency. Moreover, a DT is easier to manipulate and satisfies a number of logical constraints.

An important point that deserves more attention is the fact that conventional, crisp DTs are unable to adequately deal with decision imprecision or vagueness. Although sharply defined discrete categorizations imply an accurate and precise decision-making, in many real time problems this property proves to be a too stringent and severe assumption to impose on the decision maker. Hence, with the use of crisp DTs, a decision making process is forced upon the decision-maker that only appears to produce exact matching results because the imprecision is not made explicit. The result is an inflexible, and too hard of a decision-making process.

In order to solve this problem, the crisp decision table formalism will be enhanced to incorporate elements of the theory of fuzzy sets (Zadeh 1965).

12.3. The fuzzy DT formalism[1]

The purpose of fuzzifying the crisp DT formalism is to be able to take into account the effects of imprecision and vagueness that are present in a human decision-making process. Therefore, it is investigated here whether it is possible to define a fuzzy decision table (FDT) using a classical DT format, and what the advantages and limitations of such an approach would be.

The main difference between a crisp DT and a FDT is that in a FDT the condition states and action states can be expressed by fuzzy linguistic terms (Vanthienen et al. 1996; Wets et al. 1996a, 1996b, 1996c; Witlox, 1998; Witlox and Timmermans 2000; Wets and Witlox, 2002). Thus, to each condition or action state a fuzzy set is assigned. The resulting FDT is shown in Table 12.2. Note that Table 12.2 is the fuzzified version of Table 12.1.

Table 12.2. An example of an FDT

CS_1 Safety production process	low	high	low	high	low	high
CS_2 Distance to residential area	short	short	average	average	long	long
AS_1 Allowed production level	low	medium	medium	medium	medium	high

In what follows, we first introduce some basic definitions. Next, we discuss different kinds of FDTs and conclude with a discussion on the way in which a decision-making process evolves in a FDT environment.

12.3.1. Some basic definitions

Based on the definitions of crisp DTs stated above, FDTs and some related concepts are defined. To avoid repetition, we will only explicitly define those concepts, which differ in a fuzzy context.

Definition 1: Fuzzy condition states

$CT = \{CT_1, CT_2, \ldots, CT_c\}$ is the set of condition states

with $CT_i = \{CT_{i1}, CT_{i2}, \ldots, CT_{it_i}\}$ and CT_{ij} is a fuzzy set defined on CD_i.

In the FDT depicted in Table 12.2, CT = {Safety of production process, Distance to residential area} = {{low, high}, {short, average, long}}. Next, for each linguistic term, a fuzzy membership function needs to be specified. Such a mem-

[1] This part of the paper is mainly based on Wets and Witlox (2002).

bership function assigns to each object of the set a degree of membership ranging from zero (non-membership of the set) to one (full-membership of set)

Selecting a suitable membership function is by no means an easy task. This is partly explained by the fact that the choice of a membership function is (i) context-dependent (i.e. devised for a specific, individual problem), and (ii) for the same context it depends on the observer (different observers have different opinions). The fact that there exists a diversity of opinions on membership function specification indicates the controversy of the subject. As a result, leading fuzzy researchers and scientists (e.g. Dubois and Prade 1980; Kandel 1986; Dombi 1990; Hellendoorn 1990; Turksen 1991; Zimmermann 1991; Kerre 1993; Cox 1994; Tzafestas 1994) have made great efforts to define a number of representative standard membership functions (e.g. sigmoid or *S*-curve, logistic or *L*-curve, π-curve, Beta-curve, Gaussian-curve, and triangular, shouldered and trapezoidal fuzzy sets) that can be used to represent different fuzzy concepts (i.e. increasing concepts, decreasing concepts, "about" or "close to" representation, and fuzzy numbers).

Turning to the FDT in Table 12.2, five membership functions need to be defined: two reflecting the decreasing concepts “low” and “short”, two reflecting the increasing concepts “high” and “long”, and one reflecting the concept “average”. First, concepts like “low” and “short” (i.e. intrinsic decreasing notions) are best represented by a so-called decline curve as it is evident that lowness and shortness are inversely proportional to length and distance. Here, different options are possible. Either an *L*-curve is used, which is a quadratic monotonic continuous curve which can be defined in terms of three parameters: the complete membership value (α), the zero membership value (γ), and the inflexion point (β), or an open ended on the left trapezium can be used. Second, the concepts “high” and “long” are intrinsically increasing notions because highness and longness are proportional to length and distance. Consequently, these concepts are best represented by a so-called growth curve (e.g. a sigmoid or *S*-curve, which is simply complementary of an *L*-curve, or an open ended trapezium on the right). Third, a concept like “about average” can best be represented by means of a so-called bell-shaped curve or a triangular curve. Typical of this class of functions is that they represent the approximations of a central value (or plateau if truncated). For the present contribution, we opt to use trapezium and triangular membership functions. The motivation for this choice is that these types of membership functions are easy to calculate and they still represent fairly accurately the underlying meaning of the concept. Figures 12.1 and 12.2 represent the membership functions for the condition “Safety of production process” and “Distance to residential area”.

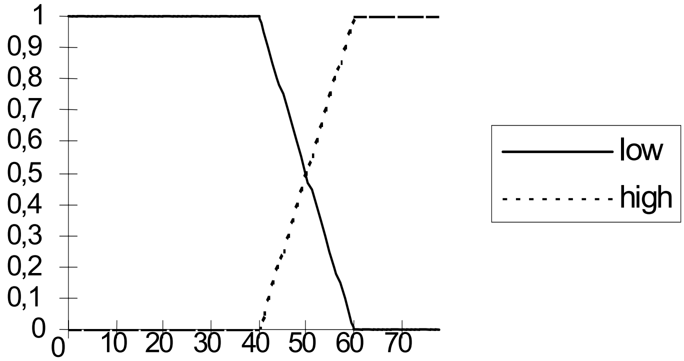


Fig. 12.1. The membership functions for the condition CS_1 "Safety of production process"

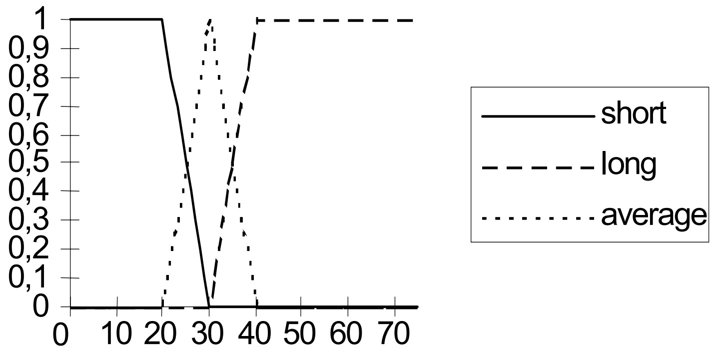


Fig. 12.2. The membership functions for the condition CS_2 "Distance to residential area"

An important difference with crisp DTs is that FDTs allow that the states of a condition overlap. In practical fuzzy systems, it is common that two (sometimes three) neighbouring states overlap. The overlap between two states is mostly limited to 25% (Cox 1994). Another constraint often adopted in practical applications is that the states of a fuzzy variable must sum to one (fuzzy partitioning)[2]. We will also accept this constraint. Although this constraint limits the possibilities of the knowledge engineer to express the knowledge of expert, we argue that fuzzy partitioning is a natural extension of a crisp condition. For example, consider a limited entry condition with states <50 and ≥50. It seems fairly reasonable to state that at the boundary value the membership value in each fuzzy state is 0.5. Furthermore,

[2] For an extensive overview on the discussion of the sum-to-one criterion, see Witlox (1998).

the more a value belongs to one set, the less it belongs to the other. A natural way to express this behaviour is to use the sum to one rule.

The overlap between fuzzy states has some impact on the desirable properties of the DT such as completeness and exclusivity. To guarantee completeness and exclusivity of a condition in an FDT, the following two constraints (listed in Definition 2 and Definition 3) have to be fulfilled for every CT_i (Wets and Witlox 2002).

Definition 2: Completeness of a condition

A fuzzy condition is complete $\Leftrightarrow$ supp $\bigcup_{j=1}^{t_i} CT_{ij} = CD_i$.

This definition indicates that a condition in the FDT is only complete if, for each value, in its universe of discourse, there is at least one state which has a membership function value > 0 (i.e. the support (supp) or height of the membership function should be greater than 0). This definition holds independently of the *s*-norm used to express the union. This can easily be verified. Because the definition holds if the max operator is used to represent the union of two fuzzy sets, and using the property that $s(a, b) \geq \max(a, b)$, it can be concluded that the definition holds for all *s*-norms.

Definition 3: Exclusivity of a condition (strict)

$\forall\ CT_{ij}, CT_{ik} \in CT_i$: if $j \neq k$ then $CT_{ij} \cap CT_{ik} = \varnothing$ where $j, k = 1, \ldots, t_i$.

This definition is very strict. It is more in correspondence with the nature of fuzzy set theory to allow for some overlap between the states. In general, it holds that in order to convert a series of individual fuzzy regions into a continuous surface, each fuzzy set may overlap its neighbouring set. Thus, the exclusivity criterion has to be redefined.

A possible solution, as advocated in Wets (1998) and Wets and Witlox (2002), tries to ensure that pairwise states of a condition are more different than similar. In this way, it seems that the interesting properties of fuzzy set theory and DTs could be integrated. Formally, exclusivity of a condition in an FDT can then be defined as follows.

Definition 4: Exclusivity of a condition

$\forall\ CT_{ij}, CT_{ik} \in CT_i$: if $j \neq k$ then $SM(CT_{ij}, CT_{ik}) \leq \alpha$

where SM is a similarity measure, $j, k = 1, \ldots, t_i$ and α a predefined threshold.

This definition states that we have to pairwise compare the states of a condition and if the similarity is smaller than a predefined threshold α we would accept that those two condition states can occur in the FDT. Next, we will investigate two problems associated with this definition: selecting an appropriate SM and choosing an appropriate value for α.

First of all, we have to define an adequate similarity measure for fuzzy sets. In the literature, several definitions of *SM*s can be found (Wets, 1998). The choice of an appropriate *SM* seems to be application dependent. With respect to our problem, i.e. determining when two fuzzy sets are allowed as states for a fuzzy condition, different results may be obtained when using different *SM*s as illustrated by the following example. Consider the fuzzy sets $A = 0.9 / 1 + 1 / 2 + 0.9 / 3$ and $B = 0.1 / 1 + 1 / 2 + 0.1 / 3$. We will use the following

$$SMs\text{: } S = \sup_{x \in U} \mu_{A \cap B}(x) \tag{12.6}$$

$$T = 1 - \sum_{i=1}^{n} \left| a_i - b_i \right| \Big/ n \tag{12.7}$$

to express the intersection (Chen, Yeh and Hsiao, 1995). Hence, $S_{A,\,B}$ = max (0.1, 1, 0.1) = 1 and $T_{A,\,B} = 1 - (0.8 + 0 + 0.8) / 3 = 0.47$.

According to the first *SM* the fuzzy sets *A* and *B* have maximum similarity, while according to the second *SM* the similarity is 0.47. Hence, the outcome of the problem whether these two states are sufficiently different to be used in the FDT depends on the value of α chosen by the expert.

A second problem with the proposed approach is determining a good value for α. An intuitively appealing value for α is 0.5. However, no sound justification for this value can be given. Consider the previous example. According to the first *SM* the two fuzzy sets would be treated as equal while according to the second *SM* they are more different than they are equal. When we put this in FDT terminology this would mean that in the one case we would allow the expert to specify the fuzzy states *A* and *B* for a condition, while in the other case we would not. Clearly this would create a great deal of confusion for the expert, which is not desirable. It would be reasonable to state that it is impossible to find a single value for α, which would be appropriate in all cases. This would mean that we would leave this problem to the expert. He would then specify which value for α should be used depending on the situation.

In conclusion, we may state that the property of exclusivity cannot be enforced anymore. This means that we must accept the drawbacks of the proposed approach and regard it as a rule of thumb. Thus, the decision whether two states of a condition are acceptable in an FDT is delegated to the expert. We have to rely on the common sense of the expert. If, however, some crisp conditions also occur in an FDT, the property of exclusivity can be enforced for these conditions.

Identical to defining fuzzy conditions, fuzzy actions can be defined. This means that each action state of the decision table is represented by a fuzzy set.

Definition 5: Fuzzy action states

$AT = \{AT_1, AT_2, \ldots, AT_a\}$ is the set of action states with $AT_i = \{AT_{i1}, AT_{i2}, \ldots, AT_{it_i}\}$ and AT_{ij} is a fuzzy set.

In Table 12.2 the action states are equal to AT = {Allowed production level} = {{low, medium, high}}. Also, in this case, fuzzy membership functions have to be assigned to each of three fuzzy action states (Figure 12.3).

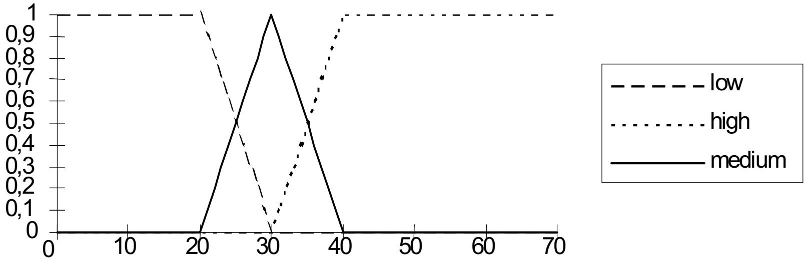


Fig. 12.3. The membership functions for the condition AS_1 “Allowed production level”

Based on the above-mentioned definitions, an FDT can also be defined as a matrix (like in the crisp case). The only difference is that *CE* and *AE* consist of fuzzy sets.

Definition 6: FDT as a matrix

Given a matrix *D* denoting the condition part of a FDT, a matrix *E* denoting the action part of a FDT, and a parameter *n* denoting the number of columns in a FDT, then the FDT can be defined as:

$$FDT = \left(fdt_{ij}\right) = \begin{pmatrix} D \\ E \end{pmatrix} \tag{12.8}$$

where $fdt_{ij} = d_{ij}$, $i = 1, \ldots, c$ and $j = 1, \ldots, n$

$= e_{(i\text{-}c)j}$, $i = c+1, \ldots, c+a$ and $j = 1, \ldots, n$

d_{ij} and $e_{(i\text{-}c)j}$ are fuzzy sets defined on their respective domains.

For the example given in Table 12.2, this yields the following result:

$$\text{FDT} = \begin{pmatrix} \text{low} & \text{high} & \text{low} & \text{high} & \text{low} & \text{high} \\ \text{short} & \text{short} & \text{average} & \text{average} & \text{long} & \text{long} \\ \text{low} & \text{medium} & \text{medium} & \text{medium} & \text{medium} & \text{high} \end{pmatrix}.$$

In the previous section three important properties of crisp DTs were identified: completeness, exclusivity and consistency. We will now check whether these properties hold in a fuzzy context and how they should be defined. With respect to the property of *completeness*, we can quite easily adapt the definition in the crisp case to the fuzzy case.

Definition 7: Completeness of an FDT

Let E^j denote the j^{th} column of *E*. Then, a complete DT has to satisfy the following two constraints:

DOM(DT) = SPACE(*C*)

$\forall E^j: \exists e_{ij}: \text{supp}(e_{ij}) \neq \varnothing$

where $i = 1, \ldots, a$ and $j = 1, \ldots, n$.

For the FDT depicted in Table 12.2, it can be readily noted that this FDT is complete. The second property is *exclusivity*. We know that a fuzzy condition cannot be exclusive unless we accept the strict definition of exclusivity.

Definition 8: Exclusivity of an FDT (strict)

Let D^j denote the j^{th} column of D and n the number of columns of the FDT. Then,

an FDT is exclusive $\Leftrightarrow \forall\ (D^j, D^k)$: if $j \neq k$ then $\exists\ (d_{ij}, d_{ik})$: $d_{ij} \cap d_{ik} = \varnothing$

where $i = 1, \ldots, c$ and $j, k = 1, \ldots, n$.

In the case that fuzzy sets are defined strict, i.e. no overlap is allowed, exclusivity of an FDT can be defined in the same way as for DTs. However, if fuzzy condition states are defined more in the line with the basic principle of fuzzy set theory, i.e. they may overlap, we can only rely on an approach based on *SM*s which may give an indication whether two fuzzy states are sufficiently different or not (Vanthienen and Wets 1996). Given that in an FDT the fuzzy conditions are conjunctively combined, it follows that exclusivity cannot be guaranteed. In a way, this is the price we have to pay for the enhanced flexibility that decision-making using fuzzy set theory offers.

Definition 9: Exclusivity of an FDT

Let D^i denote the i^{th} column of D and n the number of columns of the FDT. Then, an FDT is exclusive $\Leftrightarrow$ if $i \neq j$ then $SM(d_{1i}, d_{1j})\ T\ SM(d_{2i}, d_{2j})\ T \ldots T\ SM(d_{ci}, d_{cj}) \leq \alpha$

where $i, j = 1, \ldots, n$, SM is a similarity measure and T is a t-norm.

The same objections that can be made with respect to the exclusivity of a fuzzy condition, can be made here with respect to the selection of SM and α. Moreover, the choice of the t-norm may influence the result. If the minimum operator is taken as t-norm, only the smallest value of the *SM*s will influence the result. However, if the product operator is chosen, all the *SM*s will contribute to the result. For the FDT, which is taken here as an example (Table 12.2), it can be noted that the check of exclusivity using the inconsistency measure as SM and the min operator as t-norm, the maximum similarity between two columns is equal to 0.5. Thus, for values of α equal or larger than 0.5 we would accept that the FDT is exclusive. The same result holds if another t-norm is used because min is the largest t-norm.

The third property is *consistency*. A DT is consistent if, for each combination of condition values, only one action pattern has been specified. Consistency can be ensured if single hit DTs are used, since this means that each combination of condition values occurs only once in this type of DTs. Furthermore, even in multiple hit DTs, if they do not contain too many conditions, inconsistency can be detected rather easily by visual inspection. Unfortunately, in a fuzzy environment, a more complicated picture is found. In an FDT, columns are not different or equal but they are equal to some extent. Thus, the crisp definition of consistency is not sufficient anymore. To determine whether an FDT is consistent, we have to view it as a set of rules, which will be checked with respect to consistency.

Definition 10: Consistency of an FDT

A FDT is consistent $\Leftrightarrow$ the set of gradual (certainty) rules it represents is consistent.

In the literature the verification of fuzzy rules has received little attention. Some authors (Cox 1994) even argue that it is an advantage of fuzzy set theory that it can deal with conflicting knowledge provided by the experts. Although inference under inconsistency can still be performed in fuzzy logic, we think that it remains important to detect whether some inconsistency is present in the model or not. It is then up to the expert to decide whether the inconsistency should be removed or not. According to Gottwald and Petri (1995) a number of papers tackling the problem of inconsistency in fuzzy rule bases rely on *SM*s. The more the condition parts of the rules are alike, the more the conclusion parts of the rules should be alike in order to be consistent.

12.3.2. Types of FDTs

As already stated above, a DT consists of conditions and actions. Since both can be fuzzified or not, four types of FDTs are possible: conditions crisp - actions crisp, conditions fuzzy - actions crisp, conditions crisp - actions fuzzy and conditions fuzzy - actions fuzzy. Note also that some mixed forms (i.e. certain conditions or actions in the condition or action space of the DT are crisp while others are fuzzy) are possible, but these will not be investigated here. With respect to these different types of FDTs two remarks have to be made.

First, when the conditions and the actions in a FDT are all crisp, implying that all fuzzy membership grades of all elements are restricted to the traditional set $\{0,1\}$, the result is again the classical, Boolean crisp set, and the resulting table is of course a classic, crisp DT. This means that all crisp DTs can be interpreted as FDTs. This characteristic is known as the "extension principle" (Zadeh 1975). It effectively establishes that fuzzy sets are a true generalization of classical set theory. In fact, by this reasoning all crisp sets are fuzzy sets of that very special type; and there is no conflict between both methods.

Second, working with sharply defined categorizations of the condition and action states implies a crisp and precise decision-making. By substituting these crisp condition and action states for their fuzzy counterparts, the overall result is a fuzzy decision output. Thus, while in a crisp environment only one action configuration is possible (i.e. an "all-or-nothing" or a binary $\{0,1\}$ decision), in a fuzzy environment, more than one action configuration, each with a degree in $[0,1]$, may be chosen. The associated membership function value gives an indication of the degree of precision (or imprecision) with which a decision can be taken.

12.3.3. Decision-making using FDTs

In the previous section, different types of FDTs were considered. To illustrate the decision-making using FDTs, we assume the case that both conditions and actions are fuzzy. The remaining types of FDTs can be considered as special cases of this type, and consultation can be performed in exactly the same way. In addition, for these alternative types of FDTs, it is in most cases possible to make the decision-making considerably simpler than in the case discussed here. For example, if the condition part of the FDT is crisp, then only one column matches a given combination of condition values. Thus, the decision-making can be carried out as in the crisp case.

Fuzzy decision-making in a FDT can be performed by considering the FDT as a set of fuzzy rules. Fuzzy decision-making is also to allow fuzzy consultation of DTs (Vanthienen et al. 1996). However, a decision or action configuration cannot be taken by merely checking whether a column of the table perfectly matches a given combination of condition values. Instead, the degree of matching between the given combination of condition values and each column should be evaluated. As result, more than one action configuration may be chosen. To illustrate how the appropriate decision can be taken in a FDT, we will use the FDT depicted in Table 12.2.

Determining the degree of matching

To determine the degree of matching between a given combination of condition values and a column in the FDT, similarity measures (*SM*s) can be used. Wets (1998) found that the following *SM* performed rather well:

$$SM(A', A) = \sup_i \min\left[\mu_{A'}(x_i), \mu_A(x_i)\right] \tag{12.9}$$

where $\mu_{A'}(x_i)$ and $\mu_A(x_i)$ are the membership function grades of fuzzy sets A' and A at support point x_i. This *SM* is a special case of the so-called class of *T*-similarity measures as proposed in Turksen and Tian (1995). In their paper, "min" in the above expression is replaced with a general *t*-norm. More formally, this class of *SM*s is defined as:

$$SM(A', A)\big|_T = \sup_i T\left[\mu_{A'}(x_i), \mu_A(x_i)\right] \tag{12.10}$$

To denote the similarity between a given combination of condition values d' and the condition part of a column in the FDT, denoted as d, we use the term *overall similarity measure* (*OSM*). Thus, an *OSM* can be defined as follows.

Definition 11: Overall similarity measure

$OSM(d', d) = SM(d', d)$.

To compute which columns match, based on the given combination of condition values, an exhaustive search can be performed. In this case, an *OSM* for each column in the FDT has to be computed and compared with a threshold value α to determine whether a column should influence the decision-making or not. In general, calculating this *OSM* is very complicated since for an n-antecedent system it involves n-dimensional matrix operations. This is due to the fact that the n-variable AND relations in the condition part of a column need to be computed. Because of the complex calculations, Turksen and Tian (1995) have proposed a simplification. They prove that if the same *t*-norm is used to calculate the *t*-similarity measure and the connective AND, the *n*-dimensional matrix operations can be reduced to *n* one-dimensional column operations. In terms of FDT terminology, this means that we have to compute *c* one-dimensional operations. More formally,

Definition 12: Simplified computation of *OSM*

$OSM(d', d) = SM(d'_1, d_1)\,|_T T\; SM(d'_2, d_2)\,|_T T \ldots T\; SM(d'_c, d_c)\,|_T.$

Besides simplifying the computation of the *OSM*, Turksen and Tian (1995) also prove that the number of *OSM*s which have to be computed can be substantially reduced. For instance, given the property that "min" is the largest *t*-operator, it can be proven that the value of an *OSM* is always smaller than the minimum of the *SM*s for a given column in the FDT and a given combination of condition values. As a result, it is no longer necessary that all the *OSM*s are calculated, but instead what is called by Turksen and Tian a *two-level tree search* can be performed. First, all the *SM*s are calculated. Then, all the columns are checked whether they contain a linguistic term associated with the *SM*, which has a smaller value than α. In this case, the *OSM* of this column needs not be computed since we can be sure that its value will be lower than α.

Table 12.3. The calculation of the *SM*s

d_i	d'_i	Min
low	10	1
high	10	0
short	27	0.3
average	27	0.7
long	27	0

For example, consider the situation where the safety of the production process is 10 and the distance to residential area is 27. First, we will calculate the necessary *SM*s. Next, the *OSM*s are calculated with $\alpha > 0$. In the following table, these results are depicted. The symbol "-" denotes that it was not necessary to calculate this *OSM*.

Table 12.4. The calculation of the OSMs

Column	Min
1	0.3
2	-
3	0.7
4	-
5	-
6	-

Influence of parameters in the decision-making process

In the previous section, we explained how decision-making using FDTs can be performed. During the decision-making process several decisions have to be made. For example, which *t*-norm will be chosen, what is the influence of the value of the threshold, how does the type of input influence the decision outcome, etc. In this section, we will give an indication how each of these parameters may influence the decision chosen by using some examples.

Influence of the t-norm. To illustrate the influence of the *t*-norm in the decision-making process, we will use the FDT of the previous section. In addition to using the min operator, we will also use the product operator. To illustrate the influence of a *t*-norm, we will use a situation where the safety of the production process is 55 and the distance is 36. First, the necessary *SM*s have to be calculated.

Table 12.5. The influence of the t-norm: calculation of the *SM*s

d_i	d'_i	Min	Product
low	55	0.25	0.25
high	55	0.75	0.75
short	36	0	0
average	36	0.6	0.6
long	36	0.4	0.4

Table 12.5 shows that in this case the *t*-norm does not make any difference. The reason is that in the given combination of condition values, each value is a crisp set and not a fuzzy set. As a result, only one value in this set has a membership value > 0. Next, the *OSM*s are calculated, and their results depicted in Table 12.6.

Table 12.6. The influence of the t-norm: calculation of the *OSM*s

Column	Min	Product
1	-	-
2	-	-
3	0.25	0.15
4	0.6	0.45
5	0.25	0.1
6	0.4	0.3

Suppose that we use Gödel implication instead of min, and Goguen in case of the product and combine all columns with a positive degree of matching with a given combination of condition values (Wets 1998). The resulting aggregated output fuzzy sets can then be computed. The result is depicted in Figure 12.4.

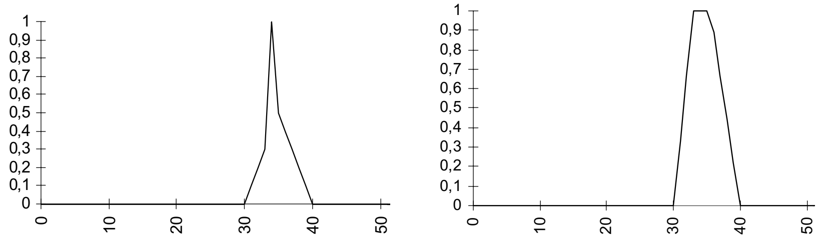


Fig. 12.4. The aggregated output using min (left) and product (right)

In Figure 12.4, it can be noted that the aggregated output fuzzy set has more characteristics of the medium membership function than of the high membership function. Intuitively, this is a result we expect because of the degree of column matching. After all, the specified action *medium* rates higher than the degree of matching with the column where the specified action is *high*. Independently of the *t*-norm used, it can be seen that the shape of the membership function of aggregated output corresponds reasonably well with our intuition. Both membership functions are situated at the right of the medium membership function and left from the high membership function. In case of the product, the core is wider and the curve is smoother. This is mainly because the product operator takes into account all matching factors, whereas the min operator only uses a single one.

Influence of the threshold value. To investigate the influence of the threshold value on the decision-making, we will start from the situation where the safety of the production process is 55 and the distance is 36. In the previous section, we have already computed the necessary *OSM*s. The influence of different threshold values on the output can be seen in Figure 12.5.

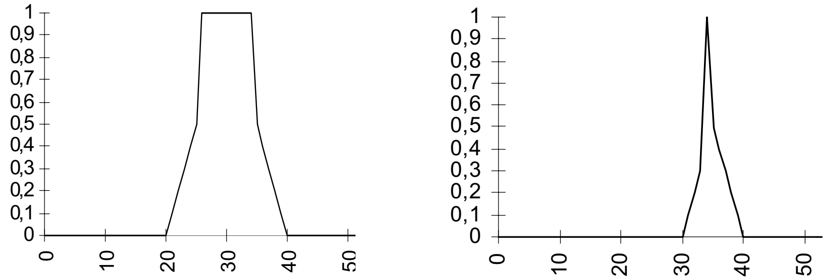


Fig. 12.5. The aggregated output with threshold value 0.6 (left) and 0.4 (right)

If the threshold value of 0.6 is used, the membership function of the aggregated output fuzzy set reflects the expected action *medium*. Only the core of the newly created action is wider than that of the original action *medium*. This is logical because the given combination of condition values did not match completely with a column in the FDT. Thus, it seems fair to generate a less restrictive action.

Next, we consider the situation where the threshold value is lowered to 0.4. A lower threshold means that, in general, more columns in the FDT will influence the decision-making. Besides column 4, column 6 is involved in the decision-making process. In Figure 12.5, it can be seen that, if the threshold value 0.6 is used, the shape of the membership function of the output fuzzy set is quite different from the generated output. As explained in the previous section, this is because the action *high* specified in column 6 contributes to the output fuzzy set.

If we further lower the threshold value to 0.25, the number of columns used in the decision process increases from 2 to 4. In this case, columns 3 and 5 are involved also in the decision-making process. However, if we compute the aggregated output membership function, we can conclude that this membership function is equal to the one we computed in the previous case. As a conclusion, we can state that although more columns are involved in the decision-making the generated output does not differ because the influence of the additional columns is not substantial.

Influence of the type of input. Until now, we have only investigated situations where the input is numeric. This situation occurs most frequently. However, it is also possible that the given combination of input values contains fuzzy values. For example, if we cannot measure exactly the safety of the production process involved, we may accept as input that the safety is *low*. Low is a linguistic term, which can be characterised by a fuzzy membership function. Note that this type of input can also occur if the conditions in the FDT are crisp. The same procedure, as will be illustrated in the more general case of fuzzy conditions, can be applied. To illustrate this type of reasoning, we will start from the following situation. Suppose that the given combination of input values is such that the distance to the residential area is 60 and the safety of the production process is more-or-less high. The linguistic expression *more-or-less high* can be interpreted as the hedge, which is attached to the original linguistic notions *high* (Zadeh 1975; Yager 1982). The membership function representing more-or-less high is depicted in Figure 12.6. We have also depicted the original membership function to show the difference between the original membership functions and the input given by the user (Figure 12.6).

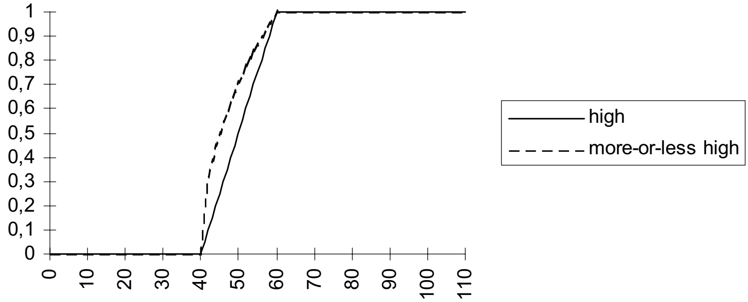


Fig. 12.6. User membership function vs. predefined membership function

After the input values are specified, we have to determine which action has to be executed for this input. Until now, to obtain the correct result we determined the degree of matching between the given combination of input values and each column in the FDT. Subsequently, the columns with a degree of matching higher than the threshold value were selected. Then, the output fuzzy sets for each column were computed, and finally these output fuzzy sets were aggregated to become the final action. It may seem that the same procedure can be used in this case, i.e. with fuzzy inputs but, unfortunately the situation is not that easy.

In a fuzzy rule base, we have to make a distinction between local inference and global inference. Recall that local inference first makes the inference with individual rules and subsequently aggregates the results, while global inference first aggregates the rules and subsequently makes the inference. In general, the global approach yields a more accurate result when implications are used which comply with the classical implication (Wets 1998).

Next, we have to compute the output fuzzy set, which is depicted in Figure 12.7. It can be seen that the output fuzzy set represents a linguistic notion falling between medium and high.

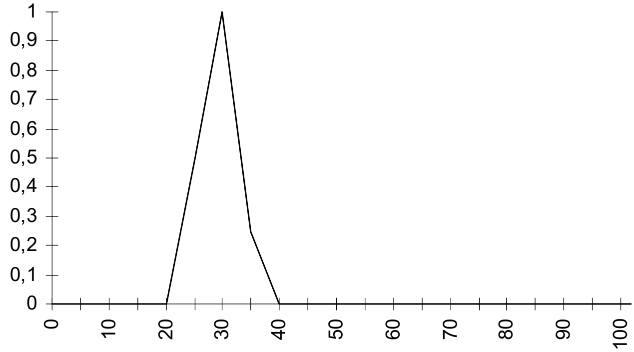


Fig. 12.7. The aggregated output action

12.4. Discussion

An interesting point of discussion involves the evaluation of some of the typical pros and cons of a model based on FDTs. In particular, it will be worthwhile to discuss what we have actually gained and, equally important, potentially lost if a spatial modelling approach is followed based on the concept of crisp and fuzzy DTs

First, several arguments have been put forward, which may justify the introduction of fuzzy concepts in a crisp DT. For one, the problem of the strict boundaries between discrete condition states is solved. Also, the problem of vagueness common in human decision-making is dealt with. After all, conventional DTs only appear to produce an accurate or non-fuzzy decision output because the vagueness inherent in decision-making is not noticed or assumed to be non-existent. By contrast, in an FDT, condition state transition occurs gradually and the vagueness is made mathematically explicit. As a result, a more flexible modelling technique is acquired. Note, however, that the gain of having a more flexible and less strict or softer decision-making process is at the expense of being able to reach exact decisions. This is because the consultation of a FDT produces a fuzzy decision output. Therefore, one of the main differences between crisp and fuzzy DTs is their decision output. In the crisp case, consultation results in a single, exact matching outcome; in the fuzzy case, a degree of membership indicates the matching between the given condition combination and each column. Thus, in an FDT, each of the columns has an influence on the decision to be taken, whereby the associated membership function value gives some indication of the degree of precision (or imprecision) with which that decision is taken. Following a probabilistic interpretation of fuzzy set theory (Stallings 1977; Haack 1979; Cheeseman 1986, 1988; Laviolette et al. 1995), this fuzzy action state membership value can be interpreted as the probability with which a particular decision rule will be selected.

Besides the important difference in consultation and decision output, crisp and fuzzy DTs also substantially vary in terms of complying with a number of desirable decision table properties. In particular, in the fuzzy case, there exists the problem of table contraction and expansion, the problem of checking for consistency, the problem of fulfilling the Law of the Excluded Middle, and the problem of complying with the property of distributivity. Although most of these problems can be solved if additional assumptions are made - assumptions with respect to the selection of operators, the sum-to-one criterion, and the interpretation of membership values - and if specific use is made of certain complementary techniques, it should be clear that the concept of FDTs needs to be interpreted with great care due to its overall complexity.

In sum, it is argued here that the introduction of fuzzy concepts in DTs offers some interesting possibilities with respect to modelling a more flexible decision-making process, and making explicit the vagueness in that decision-making process. However, it also has to be accepted that FDTs do not offer identical gains crisp DTs have vis-à-vis other relational modelling techniques.

Although at the heart of fuzzy set theory applications lies the selection of the membership function, in a number of fuzzy studies this particular issue is not really considered as important. Usually, the researcher simply selects and assigns a standard membership function to those concepts that need to be fuzzified. These membership functions model the states in an FDT. While it is still fairly easy to understand what happens when one membership function is changed, it becomes difficult to understand what the impact will be of changing the membership functions of the states of several conditions. Of course, this is not only a problem of FDTs, but of fuzzy reasoning in general.

12.5. Conclusions

This paper introduces the concept of working with fuzzy decision tables. The point of departure is the crisp decision table formalism. One of the major advantages of crisp DTs is that they can be easily checked with respect to completeness and consistency. It can be noted that also in a FDT the completeness of represented knowledge is quite easy to check. However, this is no longer true for consistency. It is still possible to check the FDT for consistency, but it is a tedious process involving the use of special techniques to check a fuzzy rule base. The FDT formalism itself does not facilitate the checking.

In summary, introducing fuzziness solves the problem of the strict boundaries in the DTs, but also ads more complexity in the reasoning process, the verification process and the comprehensibility of the FDT.

It was also illustrated how the decision-making using FDTs can be performed. The examples given in this paper show that it is possible to explicate the imprecision involved in the decision-making process by means of FDTs. In most cases, the computed results correspond well with our intuition. While this conclusion indicates that FDTs are an interesting technique in the decision-making involving imprecision , the overall picture may be less positive. One has to ask the following question: Does the decision-making using FDTs add something to the decision-making using fuzzy KBS? One may argue that properly constructed FDTs are consistent and complete, hence it is possible to avoid decision-making errors. But in order to ensure these properties, the same techniques are needed as those used to verify fuzzy KBS. Furthermore, the visualisation qualities of FDT are less important than those of its crisp counterpart because the most important aspect in fuzzy reasoning is not the table itself, but the membership function attached to the respective conditions. Of course, these membership functions can be depicted but the same holds for a fuzzy KBS. Does this mean that FDTs should not be used at all and that we should use only fuzzy KBS? In our view, FDTs can be useful when only a few fuzzy conditions are added to DTs. However, with the increasing number of fuzzy conditions the advantages of the FDT formalism will fade.

References

Cheeseman P (1986) Probabilistic versus fuzzy reasoning. In: Kanal LN, Lemmer JF (eds) Uncertainty in Artificial Intelligence. Amsterdam, Elsevier Science Publishers, Machine Intelligence and Pattern Recognition Series (vol. 4), pp 85-102

Cheeseman P (1988) An inquiry into computer understanding. Computational Intelligence 4(1):58-142 (with 23 commentaries and a reply)

Chen S, Yeh M, Hsiao P (1995) A comparison of similarity measures of fuzzy values. Fuzzy Sets and Systems 72:79-89

Cox E (1994) The Fuzzy Systems Handbook. A Practitioner's Guide to Building, Using, and Maintaining Fuzzy Systems. Academic Press, London

Dombi J (1990) Membership function as an evaluation. Fuzzy Sets and Systems 35:1-21

Dubois D, Prade H (1980) Fuzzy Sets and Systems. Theory and Application. Academic Press, Boston

Gottwald S, Petri U (1995) An algorithmic approach toward consistency checking for systems of fuzzy control rules. Proceedings of the 3rd European Congress on Intelligent Techniques and Soft Computing, pp 682-687

Haack S (1979) Do we need 'fuzzy logic'?. International Journal of Man-Machine Studies 11: 437-445

Hellendoorn H (1990) Reasoning with Fuzzy Logic. Technische Universiteit Delft, Delft

Henley A, Carruth A, Thomas A, Vickerman R (1989) Location choice and labour market perceptions: a discrete choice study. Regional Studies 23(5): 431-445

Kandel A (1986) Fuzzy Mathematical Techniques with Applications. Addison-Wesley Publishing Company, Reading (Mass.)

Kerre EE (1993) Basic principles of fuzzy set theory for the representation and manipulation of imprecision and uncertainty. In: Kerre EE (ed) Introduction to the Basic Principles of Fuzzy Set Theory and some of its Applications. Communication & Cognition, Gent, pp 1-158

Laviolette M, Seaman W, Barrett JD, Woodall WH (1995) A probabilistic and statistical view of fuzzy methods. Technometrics 37(3): 249-292 (with 6 commentaries and a reply)

McQuaid RW, Leitham S, Nelson JD (1996) Accessibility and location decisions in a peripheral region of Europe: a logit analysis. Regional Studies 30(6):579-588

Moore L (1988) Stated preference analysis and new store location. In: Wrigley N (ed) Store Choice, Store Location and Market Analysis. Routledge, London, pp 203-224

Stallings W (1977) Fuzzy set theory versus Bayesian statistics. IEEE Transactions on Systems, Man, and Cybernetics 7(2): 216-219

Timmermans H (1986) Locational choice behaviour of entrepreneurs: an experimental analysis. Urban Studies 23:231-240

Turksen IB (1991) Measurement of membership functions and their acquisition. Fuzzy Sets and Systems 40:5-38

Turksen IB, Tian Y (1995) Two-level tree search in fuzzy expert systems. IEEE Transactions on Systems, Man, and Cybernetics 25:555-568

Tzafestas SG (1994) Fuzzy systems and fuzzy expert control: an overview. The Knowledge Engineering Review 9(3):229-268

Vanthienen J, Wets G (1996) Fuzzy decision tables: modeling of V&V issues. ECAI-96 Workshop Validation Verification and Refinement of Knowledge-Bases, pp 36-40

Vanthienen J, Wets G, Chen G (1996) Incorporating fuzziness in the classical decision table formalism. Intelligent Systems 11:879-891

Verhelst M (1980) De Praktijk van Beslissingstabellen. Kluwer, Deventer

Wets G (1998) Decision Tables in Knowledge-Based Systems: Adding Knowledge Discovery and Fuzzy Concepts to the Decision Table Formalism. Acco, Leuven

Wets G, Witlox F (2001) Decision-making using fuzzy decision tables: theory and illustration. Belgium Journal of Operations Research, Statistics and Computer Science 41(3/4): 139-166

Wets G, Witlox F, Timmermans H, Vanthienen J (1996a) A fuzzy decision table approach for business site selection. 5th IEEE International Conference on Fuzzy Systems, pp 1605-1610

Wets G, Witlox F, Timmermans H, Vanthienen J (1996b) Choice modelling by means of fuzzy decision tables. 4th European Congress on Intelligent Techniques and Soft Computing, pp 1335-1339

Wets G, Witlox F, Timmermans H, Vanthienen J (1996c) Locational choice modelling using fuzzy decision tables. Biennial Conference of the North American Fuzzy Information Processing Society, pp 80-84
Witlox F (1995) Qualitative housing choice modelling: decision plan nets versus decision tables. Netherlands Journal of Housing and the Built Environment 10(3):209-237
Witlox F (1998) Modelling Site Selection: A Relational Matching Approach based on Fuzzy Decision Tables. Technische Universiteit Eindhoven, Faculteit Bouwkunde, Eindhoven
Witlox F (2000a) Towards a relational view on industrial location theory. Journal of Economic and Social Geography 91(2): 135-146
Witlox F (2000b) Modelling site suitability of harbour-related companies using multidimensional fuzzy decision tables: the case of Antwerp. Belgium Journal of Operations Research, Statistics and Computer Science 40(3/4):147-168
Witlox F (2001) Computer-based systems in urban planning: 1. Theory, concepts and practical relevance. The Land – Journal of the International Land Use Society 5(3):173-190
Witlox F (2002) Computer-based systems in urban planning: 2. Matisse, A relational knowledge-based decision support system. The Land – Journal of the International Land Use Society 6(1):23-44
Witlox F, Timmermans H (2000) Matisse: A knowledge-based system for industrial site selection and evaluation. Computers, Environment and Urban Systems 24(1):23-43
Witlox F, Timmermans H (2002) Representing locational requirements using conventional decision tables: theory and illustration. Geographical & Environmental Modelling 6(1):59-79
Yager RR (1982) Linguistic hedges: their relation to context and their Experimental realization. Cybernetics and Systems 13:357-374
Zadeh LA (1965) Fuzzy sets. Information and Control 8(3):338-353
Zadeh LA (1975) The concept of a linguistic variable and its application to approximate reasoning - Part I. Information Sciences 8(3):199-249
Zimmermann H-J (1991) Fuzzy Set Theory and its Applications (2nd Edition). Kluwer Academic Publishers, Boston

13. Spatial Decision Making Using Fuzzy GIS

Ashley Morris[1] and Piotr Jankowski[2]

[1]DePaul University
School of CTI
Chicago, IL, USA
ashley@ashley.morris.com

[2]Department of Geography
San Diego State University
San Diego, CA, USA
piotr@typhoon.sdsu.edu

Abstract. Geographic Information Systems (GIS) and spatial databases are inherently suited for fuzziness, because of the uncertainty inherent in the assimilation, storage, and representation of spatial data. One of the most fertile GIS development areas is integrating multiple criteria decision models into GIS querying mechanisms. The classic approach for this integration has been to use Boolean techniques of decision making with crisp representations of spatial objects to produce static maps as query answers. This paper examines a prototype system, FOOSBALL, which integrates both multiple attribute querying and a fuzzy object-oriented GIS. FOOSBALL addresses many of the inherent weaknesses of current systems by implementing: 1) fuzzy set membership as a method for representing the performance of decision alternatives on evaluation criteria, 2) fuzzy methods for both criteria weighting and capturing geographic preferences, and 3) a fuzzy object oriented spatial database for feature storage. This makes it possible to both store and represent query results more precisely. The end result of all of these enhancements is to provide spatial decision makers with more information so that their decisions will be more informed, and thus, more correct.

13.1. Introduction

Geographic Information Systems (GIS) and spatial databases are inherently suited for fuzziness (George et al 1996). This is due to the fact that there are many geographic objects with uncertain boundaries, and fuzziness is a natural way to represent this uncertainty, vagueness, or inaccuracy (Goodchild 1990, Couclelis 1996, Burrough 1996). There has been a considerable amount of research regarding spatial databases/GIS and fuzziness (George et al 1996, George et al 1997, Usery

1996, Wang 1998), and some work on object-oriented (OO) spatial databases and fuzziness (Morris 1998, George et al 1992). As the database paradigm is shifting to object oriented databases (OODB), it is imperative that we see how OO can play a part in fuzziness, and spatial data.

A GIS combines the techniques of digital mapping with database technology to support and provide a wide range of applications (Maguire et al 1991). GIS is a general term for a number of approaches to the management of cartographic and spatial information. GIS can be also be thought of as a toolbox with a rich set of techniques and functions for spatial data editing, management, analysis, modeling, and visualization supported by standardized APIs allowing system customization (Longley et al 2001). This is clear in systems such as ARC/INFO, which consists of two truly independent products: ARC is the digital mapping package, and INFO is the spatial database interfaced to ARC.

In a spatial database, a spatial object may have four additional types of attributes. These are spatial (location) data (*where* an object is), temporal data (*when* an object is), thematic (attribute) data (*what* an object is), and scale data (*how* an object is) (Feuchtwanger 1993).

One way that spatial databases can provide efficient query operations is by storing the features in *layers*. A layer is a group of spatial features, or a usable subdivision of a dataset, generally containing objects of certain classes (rivers, roads, mountains). Layers are often combined to solve a query. This operation is called an *overlay* (Walker 1993), since multiple layers may be superimposed so that the resultant set would contain the features of all combined layers. It is similar to a typical set union operation. Of course, layers may also be combined with difference and intersection operations.

A typical spatial query would be a user requesting display of the mountain layer, then mousing over to and clicking upon a specific mountain on the GIS presented map. Upon the click, the database would be queried, and the textual results, such as the name, latitude and longitude, and height of the mountain would be displayed.

A more complex spatial query would be "Display all skiable mountains within ten kilometers of an airport" (Morris and Petry 1998). This query would require that several manipulations be done to the spatial data. First, we could begin the overlay process by displaying our mountain layer, second, eliminate all non-skiable mountains, third, overlay a layer consisting of all airports, next, draw circles from each airport with a ten kilometer radius, and finally, highlight all of the mountains left which were within the ten kilometer radius.

13.2. Spatial decision making and MCDM

Spatial decision making is an everyday process for almost everyone. Choosing to walk to the library on a certain sidewalk rather than another one is a typical example. This decision is typically made ad hoc, without any formal analysis. Most spatial decisions are made this way, and they are often based on heuristics and internalized preferences of decision options. One may choose a certain path because of the slope of the hills, the view, the condition of the path, and so on. This cognitively simple technique can be explained by there being a relatively small "decision equity" at stake. The cost of a poor decision at this level may be trivial (the hill is steeper than expected, the view is not as scenic as hoped).

However, as GIS are becoming more widely used for making critical decisions in many disciplines (Armstrong 1992, Carver 1991, Jankowski 1995), the decision equity becomes critical. As this decision equity heightens in cost, more sophisticated approaches need to be used.

Many spatial decision making problems such as site selection or land use allocation require the decision maker to consider the impacts of choice-alternatives along multiple dimensions in order to choose the best option (Jankowski et al 1999). The decision making process, involving policy priorities, tradeoffs, and uncertainties, can be aided by Multiple Criteria Decision-Making (MCDM) methods (Jankowski 1995).

This research in spatial decision analysis (Malczewski 1999), also known as Multi-Criteria Evaluation (Jiang 2000), assists decision makers by evaluating multiple choice alternatives using multiple decision criteria. (Malczewski, 1999a) proposes that visualization is critical in spatial decision making. Since visualization is something that GIS do well, it has been a natural marriage to attempt to integrate MCDM tools with GIS. By integrating these tools with an underlying fuzzy GIS, we hope to attain the enhancements that have been realized in traditional MCDM techniques (Czogala 1990, Fodor and Roubens 1994). Fuzzy MCDM uses fuzzy ranking methods and/or fuzzy multiple attribute decision making methods to enhance the technique (Carlsson and Fuller 1996).

Spatial MCDM differs from traditional MCDM in that the proposed solutions are typically presented to the user in the form of a map with multiple overlays (Jankowski et al 1997, Jankowski et al 2001, Jankowski 1995). This differs from traditional MCDM in that the visualization problems presented via the map raise many new issues.

Another problem encountered with MCDM-GIS integration is that the decision weighting techniques have assigned weights to the criteria somewhat arbitrarily, or have assumed that the criteria were strictly Boolean. What is required is an analysis technique that will allow continuous or fuzzy functions to be assigned fuzzy values.

When decisions that must be made are not based upon crisp or binary criteria, it is important to convey this to the decision maker. A GIS that supports MCDM should also be able to *represent* fuzzy queries (either queries that involve geographic objects with indeterminate boundaries, or queries with fuzzy operators or fuzzy terms) in a visual manner. This way the decision maker will be cognizant that the query results are not Boolean, but do in fact have shades of gray.

Currently, MCDM-GIS integration has resulted in various query solutions, ranging from maps based on non-compensatory decision rules and crisp feature visualization to geographical spaces integrated with sophisticated compensatory decision techniques (Malczewski 1999, Jiang 2000). Studies have shown that using maps during the analytical process (where the specialized maps depict the solutions of multiple criteria decision models) plays only a limited support role in the decision making process (Jankowski et al 1997, Lotov et al 1999). So it may seem that maps are not adequate for decision problem exploration. However, (Casner 1991) has shown that different graphical representations are needed to support different information requirements. We pose that the limitations may lie in the integration of and representation of the map; and by integrating fuzziness into every critical area of this integration, we will provide better and more accurate decision making tools.

13.3. How fuzziness can support spatial databases

In recent years, several models have been proposed which provide for enriching database models to allow the user to deal with fuzzy and uncertain data. Many of these models have been targeted toward the object-oriented database model, so as to reap the benefits offered by this paradigm (Buckles and Petry 1995, Petry 1996, Van Gyseghem and DeCaluwe 1997).

None of these models have specifically provided for the inclusion of spatial data in their model. Spatial databases and Geographic Information Systems (GIS) are among the most exciting new technologies today. However, one of the hidden drawbacks of spatial databases is that the spatial data stored in the database is inherently uncertain (Goodchild 1990, Couclelis 1996). Therefore, since uncertainty is an inescapable attribute of a spatial database, it behooves us to design a framework for these spatial databases that will provide for fuzziness.

Our idea is to introduce a mechanism for facilitating the use of fuzzy data into an OO spatial database. We believe that by implementing fuzziness into a spatial database, we will provide:

- the GIS community with the flexibility needed and required by their field (Goodchild 1990, Morris et al 1999), and will provide a framework by which GIS researchers can effectively represent and handle uncertainty
- the OO community with a prototype of how fuzziness can naturally be implemented within the OO framework, and will show how spatial data is a natural fit for an object oriented database.

Much spatial data is inherently uncertain (Morris and Petry 1998, Burrough 1996, Frank 1996). GIS modelers and others in the GIS community realize this, and they seek a way to model it. Fuzzy logic is a proven way to model uncertainty (Katinsky 1994).

13.3.1. Fuzzy OO features

One advantage of the OO paradigm for GIS is that it is immaterial what type an object is. Thus it may be stored internally as raster-based, vector-based, or feature-based, and it will be transparent to the user (Woodsford 1995). This also provides for a more natural way to deal with combined raster and vector data.

This also means that objects may have more than one geometry. While this may seem impractical, an example where this may be useful is when displaying a ski lift gondola at different resolutions. At very fine resolutions, it would be possible to display a geometry that showed the number of seats available on the chair lift. At lower resolutions (higher scales), we might not want to show that much detail. Rather than having the representation mechanism introduce more uncertainty when displaying the object, the use of multiple geometries allows us to limit the amount of detail presented, and possibly limit the uncertainty introduced in the display mechanism.

Another possibility of the advantage of multiple geometries with an object comes when we are dealing with objects that have their geometries derived from different sources. If we use conflation (or expert opinion) to determine that an object derived from multiple sources is truly the same object, then we can assign different geometries to that object. We will then be able to use the best one when representing that object, depending upon the scale, the derived layer that into which this object fits, or any other criteria. This is possible, although it is up to the modeler to perform the conflation; it is not a function of our model.

So duplicate objects are not desirable, but the framework outlined in (Morris 1998) allows for multiple representations of the same spatial object in two ways. First, a feature may be stored in the database as a single object, with several representations of its spatial characteristics. Obviously, the GIS modeler has to determine that the multiple representations are in fact, the same feature. Therefore we may assume that the issue of conflation was handled manually by the modeler (George et al 1992). Second, the database may store several features that are actually multiple representations of the same feature. If the GIS modeler did not determine that these multiple objects *were* the same feature, then it would be out of scope for the framework mechanism to perform conflation.

13.3.2. Where uncertainty occurs in spatial databases

Uncertainty can occur in numerous places in a spatial database, and can be represented quite well using the fuzzy concept of partial membership.

Queries

An example of a fuzzy query is shown by one of our previous examples: "Display all skiable mountains within ten kilometers of an airport." In section 1, we showed how a GIS may resolve this query using crisp data. Now we will discuss how uncertainty may arise in this query.

There are three terms or phrases in this query that may lead to uncertainty: *skiable mountains*, *airport*, and *within ten kilometers*.

First, the thematic layer which contains *skiable mountains* may consist of crisp data. However, if one is Franz Klammer (1976 Olympic downhill gold medalist), or a heli-skier, who takes a helicopter to the summit of a mountain where ski lifts do not go, then their definition of *skiable* may differ from the norm.

Secondly, the concept of "airport" may be fuzzy as well. One may be looking for simply a dirt air strip, where a tiny two passenger plane could land, as opposed to a multi-runway tarmac.

A third way uncertainty may exist within this query would be the fuzziness in the semantics of the person posing the query. Even though they may ask for skiable mountains within ten kilometers of an airport, they may want to know all skiable mountains within walking distance, driving distance, taxi distance, or some other distance depending upon the circumstances. Also, the person posing the query may not know how the GIS works when trying to satisfy the query. It would be simple for the GIS to draw circles around the airports with ten kilometer radii, but the person posing the query may want to know the mountains within ten kilometers by road, which is a very different query indeed.

A more classic example of a fuzzy query would be to alter the query to actually include fuzzy terms. An example of this would be: "Display all skiable mountains near an airport." This query contains the fuzzy term "near", which could return a solution set with a degree of membership of 1 for every mountain less than 9 kilometers from an airport, and a degree of membership of 0 for every mountain more than 20 kilometers from an airport (see Figure 13.1). Every mountain between 9 and 20 kilometers from an airport would have a variable degree of membership. These fuzzy terms can be used in a GIS whether or not the objects are stored in a fuzzy manner (Morris 1998, Morris and Petry 1998).

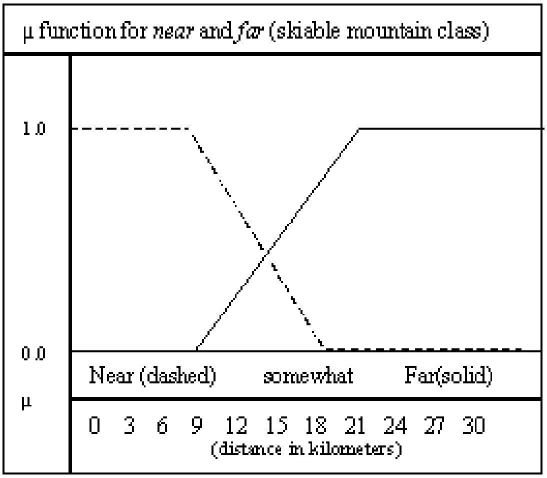


Fig. 13.1. Membership Graph

Objects

There are many examples of how fuzziness can exist in the *objects* in a GIS. (Morris and Petry 1998, Couclelis 1996) These are:

- Resolution
- Missing Data
- Uncertain Data
- Object Identification
- Boundaries
- Temporality
- Object Grouping
- Geostatistics
- Multi-dimensional Fuzziness
- Objects with Fluid Boundaries
- Groups or collections of Objects

Fuzzy Sets

Fuzzy selection procedure should be included if for no other reason than because it is much less sensitive to errors or missing values in the data than crisp, discrete selection.

When we want to represent a point in an object in our GIS, the point will surely represent a 0 dimensional X,Y,Z and possibly temporal coordinate. The point will have no size, but will be a representation of a place. However, when we have to represent the point to the user, we must do so in a manner that the user can visualize. When we show our results, we must tell the user about the uncertainty introduced to the *representation* by the GIS.

We are primarily interested in capturing the fuzziness of the data in the context of every object, as opposed to implementing fuzzy querying and retaining the crispness of the objects. As stated earlier, we believe that it is in every aspect of the object where fuzziness may be introduced. Also, even if we were to implement a fuzzy query mechanism, when it came time to actually represent the objects, we ourselves would introduce uncertainty in the representation. So by incorporating the fuzziness into the objects themselves, we are providing the most plausible representation possible to the user.

13.4. How to manage uncertainty in spatial data

GIS researchers are making a strong demand to provide for approaches that deal with inaccuracy and uncertainty in GIS (Goodchild 1990). This ability has been recognized as vital to the long term viability of GIS technology. There are many places in a GIS where the potential for inaccurate or uncertain data may occur. Our investigation will focus on the aspects that lend themselves to modeling via fuzzy set techniques, within the object-oriented framework. As we will see later, the object-oriented framework actually aids in our handling of fuzziness and uncertainty within a spatial database.

One advantage of building our GIS on top of an OODB is that by providing for fuzziness in the OODB, we have, by definition, provided for fuzziness in the OOGIS. Also, any imprecision or uncertainty that is built into our database architecture, would extend into the particular GIS application that we are trying to implement. We would just have to include the degree of set membership and other data into every object that we want to have fuzzy capabilities as we store them in our OOGIS.

A GIS can include the use of fuzzy terms for queries, regardless of how the data is stored. Whether data, entities, or objects are stored with uncertainty or in fuzzy sets, they can still be queried using fuzzy terms. If this approach is taken, then the "back end"; the database storage mechanism, does not need to represent fuzziness in any way shape or form. This means that any traditional or any commercial spatial database may be used.

13.4.1. Fuzzy Object-oriented Data Model

The traditional method for handling uncertainty in a database has been to remove the uncertainty manually, and forcing the data to conform to precise values. The problems with this approach are many and self-evident, not the least of which in a GIS sense is the problem of missing data being represented as crisp, and resulting in erroneous representations of results.

The similarity based approach (George et al 1997) provides for a richer data model that can support fuzziness and uncertainty. This is done by allowing specialization and instantiation, as well as object-class and subclass-superclass relationships that allow partial inheritance. One advantage of this technique is permitting a more accurate knowledge representation of the universe of discourse. Another advantage is that it now becomes possible to perform fuzzy retrievals via the query language. This way, the crisp retrieval merely becomes a special case (subset) of imprecise retrieval.

Note that this model preserves the underlying features of the object-oriented paradigm, and also provides for uncertainties in hierarchies. Impreciseness and uncertainty are both represented in this model, and they are distinguished from each other. The fact that uncertainty and imprecision are both supported by the model lends itself to implementation of a spatial database that is inherently uncertain.

13.4.2. Objects and Classes

Our data model supports spatial data, which is implemented as a superset of regular object data. Therefore our model is capable of storing non-spatial data, as well as spatial data. Spatial data (features) are represented as a special type of object, with additional attributes.

Similar objects are grouped together to form a class. For example, we may have a 'mountain' class. This is different from a 'layer'. In typical GIS terminology, a *layer* is a group of topologically related features. As we can use the OODB notion of collections to group any number of objects together, in our system a layer is simply a *collection* of features. This has a profound effect upon our model, in that layers can either be generated "on the fly", or may be stored in a persistent collection. In practice, the layers used by the GIS often but not always correspond to the classes in the underlying object database.

The idea that layers do not correspond to classes may sound counter-intuitive, but this is necessary to distinguish between classes, which are used by the database, and layers, which are used by the GIS. For example, a layer may consist of all features on a mountain (trails, lifts, restaurants, mines…). This would not be a class, as the thing that binds these objects together has nothing to do with their non-spatial attributes, but their *location*. This layer will be constructed by performing a spatial operation on all spatial objects in the map space. The result query set will then be a collection, which may then be treated by the GIS as a layer.

13.5. Decision Making using a fuzzy OOGIS

As outlined in (Jiang 2000), we can overcome some of the problems of Boolean MCDM by using fuzzy sets. Even if we are looking at land allocation, where our resultant set will tell us simply if the land is suitable or not (a Boolean, or *crisp* choice), by using fuzzy set theory we will be able to incorporate more relevant criteria into our decision making process than we would be able to do without it. For example, land allocation suitability is considered a fuzzy concept expressed as a fuzzy set membership (Burrough et al 1992, Hall et al 1992).

There are three strong reasons why we would want to implement these fuzzy concepts into our model (Jiang 2000). First, the use of fuzzy set membership provides a strong logic for the process of standardization, and is much better fit for the process of set membership functions than that of linear rescaling. Second, we can represent continuous scaling accurately in fuzzy sets. This is an impossibility in Boolean sets. Third, when we are representing objects with uncertain boundaries in our database, fuzzy spatial models can accurately represent these features, and traditional GIS can not (Morris 1998, George et al 1996, Usery 1996, Morris et al 1999).

13.5.1. Fuzzy spatial databases

An obvious advantage of having fuzziness permeate the data storage, algorithms, and visualization is that fuzzy sets give us an unlimited set of values to satisfy variables. These may be dynamically (Jankowski et al 2001) altered and displayed. When used with colors, hue, intensity, chroma, and shading, we can use these values to demonstrate relative membership values much better, and thus aid in decision making.

If we simply allow for fuzziness in the algorithms (Jiang 2000), but not in the underlying spatial database (Morris 1998, Morris and Petry 1998), the representation may provide for alterations in visualization quanta, but it will still be represented as discrete data sets. The use of an underlying fuzzy spatial database will provide a more accurate and more dynamic approach to the visualization analysis.

13.5.2. Cognitive Complexity

Even though (Jankowski et al 2001) showed that maps frequently play a limited role in decision making, it also showed that there were much better results when a facilitator led the decision making sessions. The role of the facilitator was simply to guide the groups through the problem exploration and resolution, and assist in the use of maps and MCDM models. This is consistent with findings from other studies with group decision support systems (Chun and Park 1998). We speculate that since the facilitator acted to simplify and synchronize the maps and decision criteria, a system that better integrated maps and MCDM tools could lead to a better decision making process. Also, since a fuzzy GIS can often yield more accurate results than a non-fuzzy GIS (Usery 1996, Wang 1998), we propose that a total integration of MCDM within a fuzzy GIS can provide the best support for decision makers (Mackay and Robinson 2000, Morris and Jankowski 2000).

(Malczewski 1999a) states that the main purpose for using maps in multiple criteria spatial decision analysis is to consider the geographic location when exploring the best compromise for a decision problem. Candidate solutions can be depicted as a scatterplot so that every point represents the performance of a decision option on the respective two criteria (Jankowski et al 1999). Thus, we can depict decision options along with the underlying spatial relationships as a geographic decision space on a map. These visualization techniques have limitations (Malczewski 1999a), but we propose that these are due mainly to the static nature of the maps being displayed. By having dynamic interactive depiction of criteria and decision spaces within a technique tolerant of uncertainty, we can more effectively represent the decision situation.

Also, the cognitive complexity of a multiple criteria spatial decision problem indicates that it is difficult for decision makers to consistently assign weights that reflect the decision maker's perception of the relative importance of the criteria. Assigning weights for any fuzzy or continuous field will become even more arbitrary, as decision makers may attach varying degrees of importance to the same criteria at different times (Kirkwood 1997). Some ways to compensate for this are through the implicit representation of preferences through criteria tradeoffs and aspiration levels (Jankowski et al 1999, Lotfi et al 1992), and also by conducting an interactive dialogue with the user or decision maker (Robinson 1990, Robinson 2000).

One assumption frequently made is that evaluation criteria are independent. A system which could determine dependencies based upon implicit criterion trade-offs could better manage these dependencies. Also, if a decision maker knew about existing dependencies and could account for them when assigning weights, a more accurate representation could be presented. Fuzziness can help in this area, especially if iterative fuzzy techniques are used (Berthold 1999). As the user continuously makes decisions, and the decisions are explicitly or implicitly ranked, the system can autonomously determine weights and ranks for explicit criteria.

The technique described by Robinson (2000) guides the user through a series of yes/no questions. An example would be where a user is shown a map, and the user is asked "Is town *A* ***near*** town *B*?" The user must answer either yes or no. From this series of questions, a tree is constructed. If two users have identical answers, the resultant trees will be identical.

Once these trees are constructed, we have the option of applying either the user's own personal preferences to the dataset, or through multiperson concept construction, we can generate one of several consensus methods. This multiperson concept construction can be explored using the agreement, global evidence, combined agreement and global evidence, and Zimmerman methods outlined in (Robinson 2000). Regardless of what technique is used, we are able to more precisely determine the value of fuzzy terms such as near, close_to, remote_from using these techniques.

13.5.3. Capturing Geographical Preferences

Ideally, a GIS with MCDM capabilities should offer decision makers the most information to aid them in choosing criteria, and not impose any preferences of the system architect (Morris 1998). The ability of a GIS with MCDM capabilities to simultaneously represent decision spaces and criteria values, as well as allowing the user to manipulate the displays, will provide for the best choices not only on the basis of attribute data, but also geography.

13.5.4. Rough and Fuzzy Techniques

Ahlqvist et al (1998) describes how rough sets can be used to determine the core and boundary of geographic objects with uncertain boundaries. The core of an object is that area of objects that have full (1.0) membership in that class of objects. The boundary is the perimeter beyond which an object has no (0.0) membership in the class of objects.

By using rough sets, Ahlqvist et al (1998) are able to represent that area of an object about which we have absolute certainty, the area about which we have absolute negative certainty, and the area that is uncertain.

A classic example of the core and boundary problem is determining where a forest begins. Is it determined based on a hard threshold of trees per hectare? This may be the political boundary, but it is not likely the natural definition. If our spatial database can represent the outlying trees as being partial members of the forest, then the decision maker will see these features as being partial members on the display. Thus, all algorithms and criteria may be applied to these partial members as well as to the core forest.

The model we are proposing (Morris and Jankowski 2000) also represents core and boundary, but allows the user to select any number of alpha-cuts for partial membership values. This technique works for both raster and vector based GIS. This has the advantage of allowing the modeler to use either classic fuzzy sets (n alpha-cuts) or rough sets (core and boundary). Our system incorporates these fuzzy features, fuzzy algorithms, and facilities for interactive display and manipulation of the criterion outcome space and graphics. This allows the GIS modeler to represent features as crisp or fuzzy objects with any number of alpha cuts.

13.6. Current Work

At this point, two projects by the authors are incorporating and integrating this technology to provide more accurate tools for MCDM GIS.

The DECADE system, as described more thoroughly by Jankowski et al (2001), has been developed on the basis of the dynamic mapping software Descartes (Andrienko and Andrienko 1999). These tools implement the concept of integrating dynamic mapping with multiple criteria spatial decision making.

The FOOSBALL (Fuzzy Object Oriented Spatial Boundary and Layer) system (Morris et al 1999) is a prototype system that integrates fuzzy object oriented databases and spatial data with fuzzy operations and display methods. The system may be found at http://ashleymorris.com/gis/index.html.

13.7. FOOSBALL

Originally, FOOSBALL was created as a proof-of-concept exercise, to show that the work described in Morris (1998) was feasible. As the work has progressed, the concept has evolved from merely being a proof of concept to being a viable system for storage and representation of fuzzy objects (Morris 2003).

13.7.1. Fuzziness as a function of the object, not the user

The objects stored in this OODBMS can be stored as either crisp or fuzzy objects, and the associated quantifiers (such as *near* an object) attached to the objects can be crisp or fuzzy.

While fuzzy set theory has traditionally applied weighting techniques depending upon the user‘s perception of the fuzzy terms (Berthold 1999), it is our position that when dealing with spatial objects, the values assigned to fuzzy terms are more dependent upon the *object*, rather than the *user*.

For example, if a query stated “Display all houses *near* a fire hydrant“, we would assume that the fuzzy term *near* would be a function of the fire hydrant, and that the definition of *near* in this case would be fairly standard for every user. However, if we were to pose the query “Display all houses near a toxic waste dump“, we would have a totally different definition of *near*. Where *near* a fire hydrant could mean 50 meters, *near* a toxic waste dump could mean 50 kilometers. FOOSBALL allows the users to define operations such as *near* for classes of objects, and then that definition would be inherited by all instances of that object.

13.7.2. FOOSBALL’s technique for storing objects with indeterminate boundaries

Traditionally, one of the disadvantages of any fuzzy DBMS has been poor performance. We believe that we have addressed the performance issue in several ways. First, we are using a standard commercial OODBMS, which has sufficient performance capabilities built into the system. Second, we are using vector based objects rather than raster based. Vector based GIS typically have better performance than raster based. Third, the mechanism for implementing the framework described by Morris (1998) has not been to assign individual membership values to individual pixels (as would have been the case in a raster-based model), but to provide a varying number of alpha cuts for every spatial object represented with a vector data model.

By providing for a varying number of alpha cuts to represent varying degrees of membership for every feature, we are allowing the GIS modeler to use as much or as little fuzziness as is required. So, for example, if the modeler wanted to represent shorelines, then the modeler could draw the sea boundary at high tide as having membership 1, and the boundary at low tide as having membership 0.5. Thus, by providing for a single alpha cut, the system allows the use of a fuzzy boundary. If dealing with soil types, the modeler could have many boundaries, representing many alpha cuts, with many membership values.

The FOOSBALL system will then represent both fuzzy features and fuzzy operations in two ways. Either the colors can be constant values across the same membership value (alpha cut), or the colors can gradually fade from alpha cut to alpha cut. This allows the modeler to better evaluate decision options, as different techniques are better suited to different problems.

The richness of fuzzy algorithms allows decision makers to choose the membership functions that best represent their data. (Jiang 2000) discusses how these algorithms are used in the IDRISI commercial GIS system. This allows the decision maker to interactively choose an algorithm that best helps them to make decisions. However, IDRISI stores only objects with crisp boundaries, and does not store objects with indeterminate boundaries. In FOOSBALL, which provides for the storage of fuzzy features (Morris 1998, George et al 1996, Usery 1996), we can apply these algorithms not only to the part of the feature that has full membership (core), but also to any outlying portions of the feature (boundary).

13.7.3. Example queries using FOOSBALL 1

If we were to use the query "Display all houses near a fire hydrant" with a strictly Boolean definition for near, such as within 100 meters, then the result from our query might look like this:

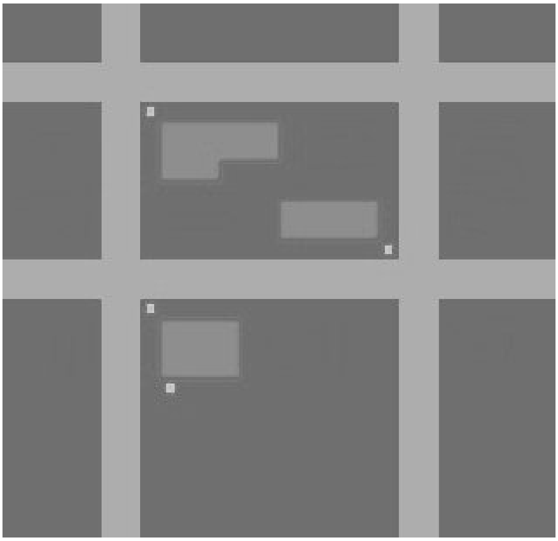


Fig. 13.2. Boolean query in FOOSBALL 1

In Figure 13.2, the dots at the locations represent fire hydrants, the irregular shaped rectangles are houses, and they all lie upon city blocks with streets. The only houses and hydrants represented in this query result are those that meet the Boolean criteria.

Now, let us change the definition of near to be fuzzy instead of Boolean. Assume that fire trucks carry hoses of 50, 75, and 100 meters. So *near* in this case would have a different definition. A house within 50 meters would have a membership value of 1.0 in the set of houses *near* a fire hydrant, and would be in the *core*. Houses within 75 meters would have a membership value of 0.66, and houses within 100 meters would have a membership value of 0.33. Any houses greater than 100 meters away would have a membership of 0.0, and would be outside the *boundary*.

Figure 13.3 depicts the query response as a map showing *all* buildings, but displaying several levels of concentric circles over each fire hydrant. The circle in the center is considered to be the "core" boundary, inside of which everything has a membership of 1.0 (or full membership) in the class "near a fire hydrant." The areas that are more transparent than the core are places where the membership in the class "near a fire hydrant" are less then 1.0 and represent the alpha cuts. In this case the second circle represents the alpha cut of membership 0.66, and the outer circle represents membership 0.33. As powerful as this ability is, it may be more useful for decision makers to be able to visualize the query results in a less coarse representation.

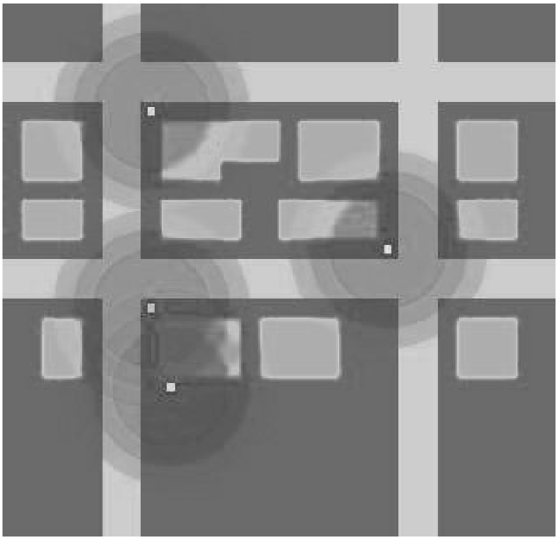


Fig. 13. 3. Fuzzy Alpha Cut Query.

In Figure 13.4, we used the same definition of near as used for our three alpha cut example. The only modification was in the representation. Open GL, the open graphics library initiated by Silicon Graphics Inc., allows us to smoothly fade between objects. So we faded between the boundaries of every alpha cut so that the concept of partial membership was even more visible. The FOOSBALL system allows the user to toggle between fades and crisp alpha cuts. In our preliminary testing, decision makers appreciated the ability to toggle between smooth fades and crisp alpha cuts, as it provided additional information from a visual perspective.

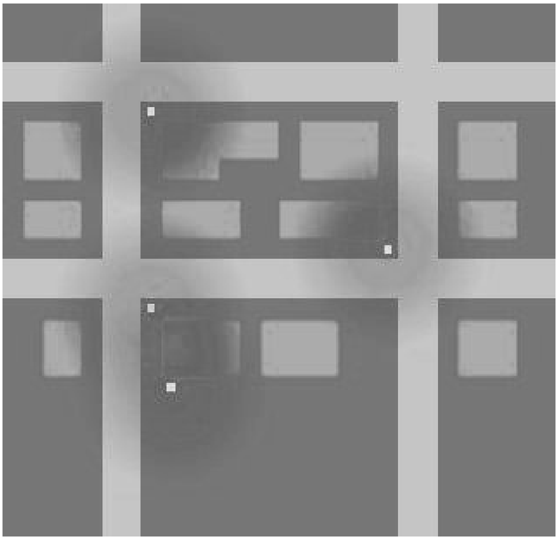


Fig. 13.4. Fuzzy Continuous Query

Any number of alpha cuts may be represented. For these examples, we used concentric circles for representation of varying membership values. FOOSBALL also supports objects with no core (soil classes), irregular shaped objects, both convex and concave polygons, and objects with no determinable boundary.

Figures 13.2-4 show how the FOOSBALL system will represent fuzziness as a function of the object. These examples have used a single criterion. When we begin using multiple criteria, FOOSBALL will allow the user many options.

Typically, MCDM in FOOSBALL is a three-step process. In the first stage, a single criterion is applied to our scene. This is exemplified by the Figures 13.2-4. Typically, each of the criteria is then displayed on the map, and the user will assign a different color to each. The user has the option of toggling between whether the operations/objects should be Boolean (Figure 13.2), fuzzy with alpha cuts (Figure 13.3), or fuzzy continuous (Figure 13.4). FOOSBALL will then produce a map showing the representations of the multiple criteria.

Secondly, each criterion will be weighted. These weights will then be multiplied by the membership values determined in the first step. The user has the option of displaying these weighted maps.

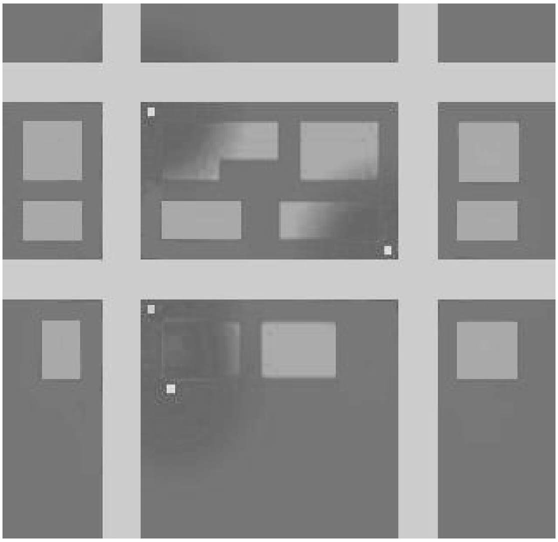


Fig. 13.5. Fuzzy Continuous Query with Boolean Constraint in FOOSBALL1

Thirdly, a final map is produced which will calculate the membership values of all objects in the final fuzzy set. Typically, the final membership value will be the sum of the object's membership values of all previous criteria divided by the number of criteria. This is a fairly naïve and simplistic approach, and we are in the process of using other techniques, such as OWA (Ordered Weighted Averaging) (Yager 1988). Also, as we are using these multiple criteria to make a single decision, we represent the final coverage map with a single color denoting membership in the final fuzzy set.

Another feature of FOOSBALL is the ability to perform queries with Boolean constraints as well as fuzzy constraints. In Figure 13.5, we used the same query as depicted in Figure 13.4, but we added the Boolean constraint that fire hoses cannot cross streets.

An advantage of using Open GL is that color transparencies are represented in the range 0.0 – 1.0, so we are able to directly map the membership values to display values. For this publication, please note that the representations are in grayscale, while the FOOSBALL system uses color.

13.7.4. Second generation of FOOSBALL

Our initial implementation was successful in the sense that it showed us that it was possible to implement our ideas in a GIS environment. The ability to represent some degree of fuzziness was well received in our informal reviews (Morris 2000). Especially well received by our reviewers was the ability to toggle the different modes of representation (crisp, fuzzy with crisp alpha cuts, fuzzy with continuous representation of crisp alpha cuts) as detailed in Figures 2-5.

Enhancements

We realized that we needed to enhance FOOSBALL to provide added functionality. The enhancements for the second phase of the project were to:

- Change the data storage facility from flat files to ObjectStore, a commercial off-the-shelf (COTS) object-oriented database
- Create a multiple document interface allowing for multiple views of the same data
- Provide better support for crisp queries
- Provide the ability to define fuzzy terms on a class/object basis
- Provide management of features with irregularly shaped alpha-cuts
- Provide management of fuzzy spatial terms with irregularly shaped regions
- Provide better support for MCDM.

Development Environment

Our development environment remained basically the same as the original FOOSBALL application, with the addition of Microsoft Foundation Classes (MFC) and Object Store as programming tools.

ObjectStore allowed us to truly store the geographic features as objects, following the fuzzy spatial object class hierarchy proposed in (Morris 1998, 2003). Basically, this hierarchy states that every type of spatial object (feature) is a subtype of the class *Feature*. The only difference between a spatial object and a non-spatial object is that a spatial object has additional attributes to indicate the geographical position of the object, and the scale of the object. Please note that by *scale* of the object, we may also include additional information such as sampling rate, data quality, and other aspects of how well the data describes the object.

New Interface

The screen capture of FOOSBALL2 shows the new interface. The upper left window displays the hierarchy of objects in the domain. A “T” before the name indicates that the object is a type of object, or a class of objects. Types can be created dynamically by the user and can be subtypes of any other type in the tree. Types can be thought of in this context as classes in the traditional object oriented sense, as subtypes inherit all attributes of their parent types. We use the term *type* over *class* in this context, as a membership grouping in a query is denoted by the term *class*.

An “O” before a tree entry means that the item is an object. Objects are always of a particular subtype, and their supertype is always that of feature. An object can be thought of as an instantiation of a type. *Types* contain data structure definitions, whereas *objects* contain the actual value data in those structures.

As this article will be printed in grayscale, we do not demonstrate the ability to provide multiple views of the same data. It *can* be done, and scaling can also be performed. Also, multiple views can provide different coloring for the features being displayed.

We are researching, both formally and through informal reviews, the differences that color changes can have on reviewers making spatial decisions.

Queries

The query is displayed in the lower left window of the screen shot. This window will show the user the criteria entered by means of the query entry window. In this particular screen shot the criteria have been defined as "Near Fire Hydrants" and "Not Streets." This is a form of the natural language query "Show all features near fire hydrants that are on the same side of the street as the hydrant." This would be important if one did not want to have a hose cross the street in case of fire.

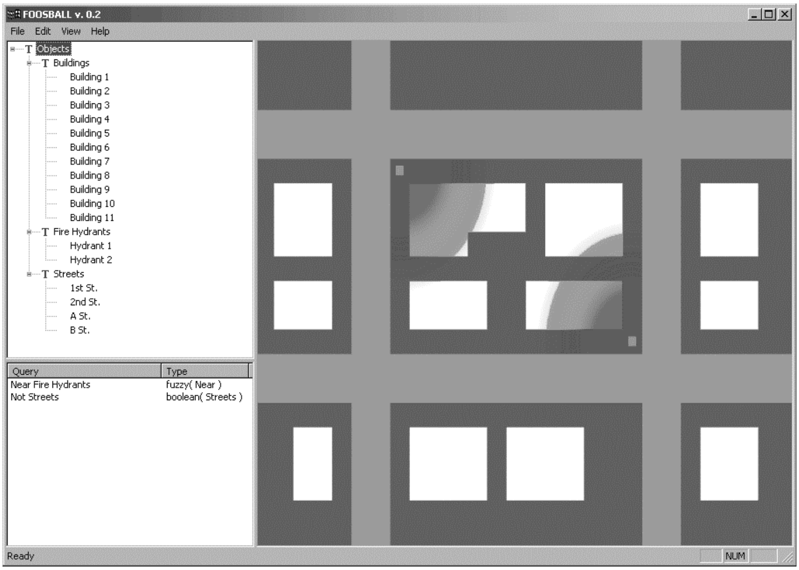


Fig. 13.6. Screenshot of the FOOSBALL v. 0.2 system

The "Not Streets" operator works similar to a "minus" operator, except that it creates a new boundary that the query must consider, namely, that the streets become a boundary. In this case the system checks the value of "near" specified for the type "fire hydrants", and draws the blue fuzzy membership values accordingly. Note that in this case, the display defaults to "fuzzy with continuous display of crisp alpha cuts."

Fuzzy spatial terms

As described in Morris and Jankowski (2000) and Morris (2003), the fuzzy spatial term "near" is associated with a spatial object, not with a user. This may seem counter-intuitive at first, but consider that "near" a city and "near" a star take on such wildly different values that they cannot be grouped together as like quantifiers. Thus, the GIS modeler, when entering new classes, must define the values for fuzzy spatial terms he wishes to use for each particular type. We anticipate allowing individual users to "tune" the definitions of these fuzzy spatial terms as they become more accustomed to the system.

In this example, we have displayed crisp data with fuzzy query results. Since FOOSBALL is built upon an underlying fuzzy OOGIS, it is possible to store and display objects with fuzzy boundaries.

Alpha cuts and fuzzy value mapping

Alpha cuts are used in FOOSBALL to construct areas of an object that have less than 1.0 membership in the object's class.

The second generation of the FOOSBALL software allows the user to define any number of alpha cuts for each individual object. If an object has no alpha cuts, it is simply treated as a crisp object. Also, spatial objects with no core (no membership values of 1.0) can be stored. This is a requirement when dealing with objects such as soil types, which may have partial membership in any number of classes, and no full membership in any one class.

Alpha cut boundaries may be drawn by the user with a mouse, but FOOSBALL also supports automatic generation of alpha cuts. This is typically done from raster sampled data, and is handled internally by a selectable "fuzziness" function applied to each sampled data point. The function determines the values of membership for each sampled data point of a feature based on an initial +/- setting supplied for each feature, and a set number of alpha cuts (supplied by the user). This +/- value can be thought of as the error value for each data point. Functions that can be selected for mapping this value are: logarithmic, linear, square, and cubic. The application of the +/- error value and the fuzziness function is called Fuzzy Value Mapping. The user may select any number of alpha cuts. This would allow every pixel to be treated as an individual spatial object, with its own individual membership values in the class.

These two mechanisms (alpha cuts and fuzzy value mapping) provide a method for the user to better visualize fuzzy data sets.

Criteria Weighting

As stated in Morris and Jankowski (2000) it is difficult if not impossible for human decision makers to weight criteria consistently among queries and multiple decision makers. Our hope is that the underlying fuzzy mechanisms and the ability to propose fuzzy queries will minimize the need for the human decision maker to manage criteria weights.

This problem gets particularly difficult when dealing with multiple criteria and fuzzy objects. At this point FOOSBALL can represent such queries, but the visual representation is somewhat obfuscated. A query such as "Display all soil types with dense pine trees" will have multiple overlapping result regions, and it is exceedingly difficult for a naïve user to interpret the results. The FOOSBALL system can currently perform these queries, but as mentioned earlier, we intend to work further with informal reviewers to ascertain the optimal visual and color combinations to present to the user.

Also, the cognitive complexity of a multiple criteria spatial decision problem indicates that it is difficult for decision makers to consistently assign weights that reflect the decision maker's perception of the relative importance of the criteria. Assigning weights for any fuzzy or continuous field will become even more arbitrary, as decision makers may attach varying degrees of importance to the same criteria at different times (Kirkwood 1997). Some ways to compensate for this are by implicit representation of preferences through criteria tradeoffs and aspiration levels (Jankowski et al 1999, Lotfi et al 1992), and also by conducting an interactive dialogue with the user or decision maker (Robinson 1990).

13.8. Conclusions and Future development

Uncertainty occurs. It can arise because of the data values, the semantics of the data, the type of hierarchy being modeled, and the nature of the data itself. GIS and spatial databases have relied on the Boolean well-defined set, and this classical set theory has proven to be insufficient for the demands of spatial data. Unlike earlier research, ours provides for fuzziness not only in the thematic attributes of spatial data, but also in the location (spatial) attributes, as well as supporting fuzzy queries.

We also believe that fuzzy object oriented databases are a natural fit for the storage of spatial data, and the advantages offered by the fuzzy OO paradigm give many additional benefits to spatial databases. This tight integration of MCDM within a GIS backed by a fuzzy OO spatial database provides for the strongest possible marriage of the disciplines, and the most accurate presentation to the decision maker.

Our wholesale acceptance of the Windows platform was an acceptable tradeoff, as we decided that to truly make an impact in the GIS community, we needed to port the application to a platform more used in the GIS world, that of ArcView.

When we made this decision, an object-oriented scripting language proprietary to ArcView, *Avenue*, was the language being used to create *extensions* to the GIS. An ArcView extension is a plug-in to the GIS which allows features to be added to the basic GIS. Since that time, with the introduction of ArcView 8.1, Microsoft's *Visual Basic for Applications* is the language to be used for creating custom applications or extensions. We have not yet attempted to create ArcView extensions using Visual Basic, but we believe that it should be an easier port than Avenue.

13.9. Acknowledgement

Thanks to Karl Riley for his programming work for FOOSBALL.

13.10. References

Ahlqvist, O., J. Keukelaar, K. Oukbir, 1998: "Using rough classification to represent uncertainty in spatial data" SURC 98 -- The 10th Annual Colloquium of the Spatial Information Research Center University of Otago, Dunedin, New Zealand.

Andrienko, G., Andrienko, N. 1999. "Interactive Maps for Visual Data Exploration", International Journal of Geographical Information Science, 13(4), 355-374.

Armstrong, M.P., Densham, P.J., Lolonis, P., and Ruston, G. 1992. Cartographic displays to support locational decision making. Cartography and Geographic Information Systems, 19(2): 154-164.

Berthold, M. 1999. Fuzzy Logic. In Intelligent Data Analysis, M. Berthold, D. Hand eds. Milan: Springer-Verlag, 269-298.

Buckles, B. P., Petry, F. E., "Fuzzy Databases in the New Era". Proceeding of FUZZ-IEEE/IFES '95 Workshop on Fuzzy Database Systems and Information Retrieval, 1995.

Burrough, P.A., MacMillan, R.A., VanDeursen, W., 1992. Fuzzy classification methods for determining land suitability from soil profile observations and topography. Journal of Soil Science, 43, 193-210.

Burrough, P. A. "Natural Objects With Indeterminate Boundaries", *Geographic Objects with Indeterminate Boundaries* (Burrough, P., and Frank, A., eds.), 3-28, Taylor and Francis, London, 1996.

Carlsson, C., Fuller, R., "Fuzzy multiple criteria decision making: recent developments." Fuzzy sets and systems 78, 139-153, 1996.

Carver, S. 1991. Integrating Multicriteria Evaluation with GIS. International Journal of Geographical Information Systems, 5:321:339.

Casner, S.M. 1991. A Task-analytic Approach to the Automated Design of Graphic Presentations. ACM Transactions on Graphics, 10, 2 (Apr. 1991), 111-151.

Chun, K. J., Park, H.K. 1998. Examining the conflicting results of GDSS research. Information and Management, 33:313-325.

Couclelis, H. 1996. Towards an Operational Typology of Geographic Entities with Ill-defined Boundaries, *Geographic Objects with Indeterminate Boundaries* (eds, P. Burrough and Frank), 45-56, Taylor and Francis, London, UK.

Czogala, E., Multi-criteria decision-making by means of fuzzy and probabilistic sets, Fuzzy Sets and Systems 36, 235-244, 1990.

Feuchtwanger, M., *Towards a Geographic Semantic Database Model*. Unpublished Ph.D. Thesis, Simon Fraser University, Vancouver, 1993.

Fodor, J.C., Roubens, M., Fuzzy Preference Modelling and Multicriteria Decision Support. Kluwer, Dordrecht, 1994.

Frank, A. "The Prevalence of Objects with Sharp Boundaries in GIS", *Geographic Objects with Indeterminate Boundaries* (Burrough, P., and Frank, A., eds.), 29-39, Taylor and Francis, London, 1996.

George, R., Buckles, B., Petry, F., Yazici, A., "Uncertainty Modeling in Object-Oriented Geographical Information Systems", *Proceedings of Conference on Database and Expert System Applications* (DEXA 92), 77-86, 1992.

George, R., Petry, F.E., Buckles, B.P., Radharkrishnan, S., 1996. Fuzzy Database Systems – Challenges and Opportunities of a New Era. International Journal of Intelligent Systems, Vol 11. John Wiley and Sons.

George, R., Yazici, A., Petry, F. E., Buckles, B. P., "Modeling Impreciseness and Uncertainty in the Object-Oriented Data Model - A Similarity-Based Approach", *Fuzzy and Uncertain Object-Oriented Databases: Concepts and Models*, pp. 63-95. World Scientific: New Jersey, 1997.

Goodchild, M., Gopal, S. eds. *The Accuracy of Spatial Database*s, Taylor and Francis, Basingstoke, UK, 1990.

Hall, G.B., Wang, F., Suvaryono. 1992. Comparison of Boolean and fuzzy classification methods in land suitability analysis by using geographical information systems. Environment and Planning A, 24, 497-516.

Jankowski, P. 1995. "Integrating GIS and Multicriteria Decision Making Methods." International Journal of Geographical Information Systems.g(3):251-273.

Jankowski, P. and Ewart, G. 1996. Spatial decision support system for health practitioners: selecting a location of rural health practice. Geographical Systems, 3(2):279-299.

Jankowski, P., Nyerges, T. L., Smith, A., Moore, T. J., Horvath, E. 1997. Spatial Group Choice: a SDSS tool for collaborative spatial decision making. International Journal of Geographic Information Systems, 11(6): 577- 602.

Jankowski, P., Lotov, A., Gusev, D. 1999. Application of multiple criteria trade-off approach to spatial decision making, in Jean- Claude Thill (ed.) Spatial Multicriteria Decision Making and Analysis: A geographic information sciences approach. Aldershot: Ashgate.

Jankowski, Piotr; Natalia Andrienko; Gennady Andrienko. Map-centred exploratory approach to multiple criteria spatial decision making. International Journal of Geographical Information Science 15(2):101 -- 127. 2001.

Jiang, H., Eastman, J.Ronald. 2000. Application of fuzzy measures in multi-criteria evaluation in GIS. International Journal of Geographical Information Science, Casner 1991(2):173-184.

Katinsky, M., *Fuzzy Set Modeling in Geographic Information Systems*, Unpublished Master's Thesis, University of Wisconsin-Madison, Madison, WI, 1994.

Kirkwood, C. W. 1997. Strategic Decision Making: Multiobjective Decision Analysis with Spreadsheets. Belmont, CA, Duxbury Press.

Lotfi, V., Stewart, T.J., and Zionts, S. 1992. An aspiration-level interactive model for multiple criteria decision making. Computers Operations Research, 19:671-681.

Longley, P.A., Goodchild, M.F., Maguire, D.J., and Rhind, D.W.. Geographic Information Systems and Science. Chichester: Wiley & Sons. 2001

Lotov, A. V., Bourmistrova, L. V., and Bushenkov, V. A. 1999. Efficient strategies: An application in water quality planning. In: Decision Analysis and support for sustainable development, edited by G. Kersten, Z. Mikolajuk, and A. Yeh, Kluwer, 145-166.

Mackay, D.S. and V.B. Robinson. 2000. A multiple criteria decision support system for testing integrated environmental models. *International Journal of Fuzzy Sets and Systems,* 113(1), 53-67.

Maguire, D. J., Goodchild, M., Rhind, D. W., *Geographical Information Systems: Principles and Applications*, Longman, 1991.

Malczewski, J. 1999. GIS and Multicriteria Decision Analysis. New York: John Wiley & Sons.

Malczewski, J. 1999. Visualization in multicriteria spatial decision support systems. Geomatica, 53(2):139-147.

Morris, A., Petry, F.E., Cobb, M. 1998. Incorporating Spatial Data into the Fuzzy Object Oriented Data Model. Proceedings of Seventh International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU98). Editions EDK, Paris, 604-611.

Morris, A. and Petry, F.E. , "Design of Fuzzy Querying in Object-Oriented Spatial Data and Geographic Information Systems", *Proceedings of 1998 Conference of the North American Fuzzy Information Processing Society* (NAFIPS 98), 165-169, 1998.

Morris, A., Foster, J., Petry, F.E. 1999. Providing Support for Multiple Collection Types in a Fuzzy Object Oriented Spatial Data Model. Proceedings of 18th International Conference of the North American Fuzzy Information Processing Society. IEEE Press: Piscataway, NJ. 824-828.

Morris, A., Jankowski, P. 2000. Combining Fuzzy Sets and Databases in Multiple Criteria Spatial Decision Making. In: Flexible Query Answering Systems: Recent Advances, edited by H.Larsen, J. Kacprzyk, S. Zadrozny, T. Andreasen, H. Christiansen. Kluwer, 103-116.

Morris, A. Trials conducted in 2000-2001 at the University of Idaho and DePaul University.

Morris, A. "A Framework for Modeling Uncertainty in Spatial Databases". *Transactions in GIS*, January 2003, vol. 7, no. 1, Blackwell Publishing Ltd, Oxford., pp. 83-101(19)

Petry, F. E., *Fuzzy Databases: Principles and Applications*. Kluwer Academic Publishers, Boston, 1996.

Robinson, V.B. 1990. Interactive Machine Acquisition of a Fuzzy Spatial Relation, Computers and Geosciences 16(6), 857-872.

Usery, E.L. 1996. A Conceptual Framework and Fuzzy Set Implementation for Geographic Features. *Geographic Objects with Indeterminate Boundaries* (eds, P. Burrough and Frank), 71-86, Taylor and Francis, London, UK.

Van Gyseghem, N., De Caluwe, R., "The UFO Database Model: Dealing with Imperfect Information", *Fuzzy and Uncertain Object-Oriented Databases: Concepts and Models*, pp. 123-186. World Scientific: New Jersey, 1997.

Walker, R., AGI Standards Committee GIS Dictionary, Association for Geographic Information, 1993.

Wang, F., "A Fuzzy Grammar-Based Natural Language User Interface for Spatial Queries". *Proceedings of Seventh International Conference on Information Processing and Management of Uncertainty in Knowledge Based Systems* (IPMU 98), Editions EDK, Paris, 1998.

Woodsford, P. A., "The significance of Object-Orientation for GIS", *Proceedings*: *IUSM Conference*, Hanover, Germany, 1995.

Yager, R.R., "On ordered weighted averaging aggregation operators in multi-criteria decision making," IEEE Transactions on Systems, Man and Cybernetics 18, 183-190, 1988.

14. Spatially Explicit Individual-Based Ecological Modeling with Mobile Fuzzy Agents

Vincent B. Robinson[1] and Phil A. Graniero[2]

[1] Department of Geography
University of Toronto
Mississauga, ON L5L 1C6
Canada
email: vbr@geomant.erin.utoronto.ca

[2] Department of Earth Sciences
University of Windsor
Windsor, ON N9B 3P4
Canada
email :graniero@uwindsor.ca

Abstract. Previous theoretical work illustrated how fuzzy spatial relations can be used to control the movement of mobile agents in spatially explicit individual-based ecological models (Robinson 2002). We present a computational framework and methodology for modeling small mammals as mobile agents making decisions during the dispersal process. It is shown how this object-oriented framework can accommodate the uncertainty of geographic information as well as the inherent fuzziness of the decision process. A fuzzy decision making model is presented along with its corresponding crisp equivalent. Using a realistic landscape, simulations are used to explore model behavior relative to fuzzy compensatory and noncompensatory aggregation operators. Simulations are used to compare fuzzy versus crisp model behaviors. Results are used to evaluate relative strengths and weaknesses of each. It is shown that this approach can be used for developing individual-based models to address spatially explicit ecological problems that are dependent on being based in a geographic information systems environment.

14.1. Introduction

The potential for incorporating of fuzzy logic into spatially explicit, individual-based ecological models of animal movement has been portrayed as an approach to controlling foraging, exploratory, and dispersal movements in spatially explicit models (Robinson 2002). Although fuzzy set approaches have been used to model animals in relation to their habitat (Burgman et al. 2001;Cao 1995;Rickel et al. 1998) and in modeling spatially explicit mobile agents (Graniero and Robinson 2003;Lim et al. 2002;Petry et al. 2002), there have been few, if any, attempts to model the movement of animal objects in a spatially explicit, individual-based model of dispersal. Based on the theoretical discussion of Robinson (2002), this chapter illustrates how the fuzzy control of dispersal agents can be implemented in an object-oriented modeling framework for spatially explicit ecological modeling.

Dispersal is a component of vertebrate behavioural systems that substantially contributes to the colonization of vacant habitats in fragmented landscapes. In most species and most dispersing individuals, dispersal takes place before first reproduction and is termed natal dispersal (Howard 1960). It is usually considered the single largest, often only, long-distance movement made by individual animals and is generally accepted as the major agent of gene flow among populations (Sutherland et al. 2000;Wiklund 1996). Therefore, it plays a critical role in the spatial dynamics of populations, including population spread, recolonization, and gene flow. It is a central focus of conservation issues for many vertebrate species. Estimates of the tendency to disperse and dispersal distances are used to predict the likelihood of a given species colonizing a vacant habitat or crossing a fragmented landscape (Wolff 1999). Estimates of dispersal patterns and distances are also used in spatially explicit population viability models (Lamberson et al. 1994;Schumaker 1996). Although dispersal, particularly natal dispersal, is an important component of mammalian behavioural systems, it has been noted that dispersal distances are rarely studied directly for mammals. Therefore, data on dispersal distances are often obtained from studies on demography or from data obtained inadvertently in radio telemetry or mark-recapture studies. Much data are anecdotal (Wolff 1999;Koenig et al. 1996). The uncertainty of dispersal data is particularly important when formulating models of dispersal. It has been suggested, by simulation studies, that errors in dispersal parameters have significant effects on predicted dispersal success (Ruckelshaus et al. 1997). Given the importance of dispersal modeling and the underlying uncertainty regarding the parameters of dispersal models, it has been theorized how to use the logic of fuzzy spatial relations to control the movement of animal objects in simulations of movement about a landscape (Robinson 2002).

We present a computational framework and methodology to explore the possibility of fuzzy logic for modeling small mammals as mobile agents making decisions during the natal dispersal process. First, we place this effort within the context of information-based approaches to ecological modeling. This is followed by a concept level presentation of the relationship between information-based ecological modeling approaches, geographic information systems(GIS), and modeling of movement. To provide tangible illustrative examples of this approach a simulation model of the natal dispersal behavior of eastern gray squirrels (*Sciurus carolinensis*) is developed. Therefore, we briefly present the rationale for choosing a small mammal as the focus of our modeling effort. It is followed by an overview of our object-oriented approach to modeling animal objects as mobile agents. A fuzzy decision making model for natal dispersal is presented that serves as a plausible base model. Using a realistic landscape derived from a GIS database, we illustrate the similarities and differences in movement behavior as a function of crisp, compensatory, noncompensatory aggregation operators along with a corresponding crisp equivalent. Simulations are used to compare fuzzy versus crisp model behaviors. The results suggest that this approach can be used for developing individual-based models to address spatially explicit ecological problems that are dependent on being based in a geographic information systems environment.

14.1.1. Information-based Approach to Spatially Explicit Ecological Modeling

Spatially explicit ecological models are used to examine plausible connections between landscape patterns and species viability (Ruckelshaus et al. 1997). In an information-based approach to modeling the movement of animals such models may link behavioral ecology with landscape-level ecological processes (Lima and Zollner 1996). Furthermore, it has been argued that the difficulties of incorporating different levels of habitat heterogeneity, individual differences and local interactions into mathematical models suggest that a general theory of individual-based ecology is impossible and that we have no choice but to develop detailed computer-supported models (Lomnicki 1999).

Figure 14.1 illustrates where our approach fits among the continuum of information-based approaches to ecological modeling. Our effort develops models that fall somewhere between the intermediate level and those studies attributing considerable cognitive abilities. In particular, the models considered here generally assume the animals have knowledge about their nearby landscape and no information about the greater landscape They can include a degree of spatial memory and use methods/techniques drawn from the broad field of artificial intelligence. In this region of the continuum (see box in Figure 14.1) information about the landscape is essential to the model-building enterprise. Spatially explicit descriptions of landscapes are now commonly represented in a GIS. Thus, it now seems quite natural that the use of models that connect animal populations to “maps” stored in a GIS have become a prominent feature in the field of conservation biology (Ruckelshaus et al. 1997: 1305). In general, spatially-explicit population models use a GIS database to configure the layout of available habitat and then apply a detailed simulation of individual organisms moving through the landscape (see Figure 14.2). Due to enabling spatial and computational technologies these models allow one to describe a landscape in as much detail as a GIS database can support (Holt et al. 1995). Because they are individual-based models (IBMs), they can represent realistic behavior with parameters that reflect mechanisms thought to be responsible for a species being at risk in fragmented habitat (Ruckelshaus et al. 1997). An important point to be remembered is that although IBMs usually make more realistic assumptions than do state variable models, the aim of the IBM exercise is not realism but modeling (Grimm 1999).

14.1.2. Conceptual Framework for Spatially Explicit Ecological Modeling

Figure 14.2 illustrates the major components of a spatially explicit model and the relationship between each of them. Of critical importance in all the models is some representation of the landscape. It may be derived from extensive field observations and/or inferred from sources such as land cover maps, or even satellite remote sensing data. Here the landscape will be treated as a spatial database from which the animal objects will receive information about their surroundings. The landscape is not modeled as changing in any significant manner during the simulation process. This is usually due to the already complex nature of the model that introducing this level of complexity would obscure behaviors that are the focus of the research effort. Furthermore, landscape dynamism may or may not be a plausible phenomenon to include in the model depending on the temporal and spatial scale of problem. For example, over the time period of natal dispersal of a small mammal it is unlikely landscape processes would alter the distribution of critical variables. However, should a simulation of natal dispersal span many generations

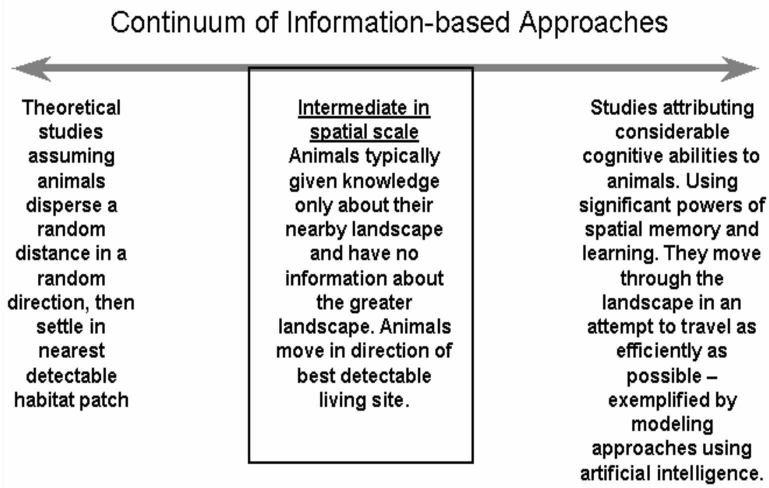


Fig. 14.1. Continuum of information-based approaches to modeling movement (including dispersal) of animals (based on Lima and Zollner 1996; Robinson 2002). Box represents region of continuum that best describes our modeling efforts

then a case can be made for a dynamic landscape. For example, Hale et al. (2001) report how fragmentation of a forested landscape led to the separation of red squirrel populations for long enough to make them genetically distinct. However, over many generations the landscape also changed leading to defragmentation that facilitated dispersal to the point where there was substantial genetic mixing. With future development, our GIS-based approach will allow for the explicit modeling of landscape change as well as spatially explicit behavior of simulated animal objects.

In this framework (Figure 14.2), animal objects pose spatial queries to the landscape to acquire information. However, like their counterparts in the real world, they are able to acquire information about the landscape within a certain distance determined by the animals perceptual range (Mech and Zollner 2002;Zollner 2000)or finite range of vision (Fahse et al. 1998).That information is then processed to determine the specifics of which movement behavior to pursue. Robinson (2002) discussed how foraging and exploratory movements could be modeled using fuzzy logic to control movement decisions. Here we concentrate on a different kind of movement, namely natal dispersal.

Considering the results of their simulations Ruckelshaus et al. (1997) suggested that errors in dispersal parameters have much larger consequences for predicting

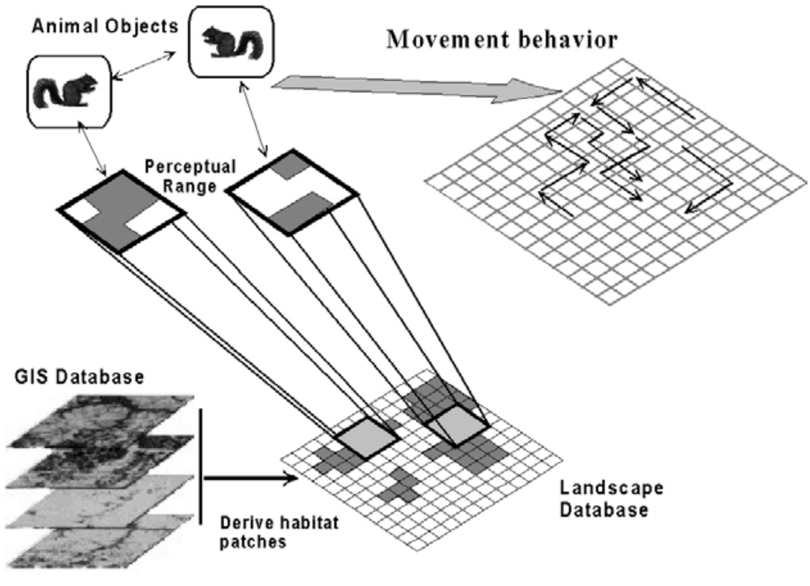


Fig. 14.2. Conceptual framework showing relationship between geographic information system (GIS) database, the landscape database describing the spatial distribution of habitat, animal objects, their perceptual range, and their movements over the landscape (Robinson 2002)

dispersal success than did errors in landscape classification. Their conclusions suggest that uncertainty surrounding dispersal parameters is a significant problem that ecological models and modelers must face. Indeed, there are crucial parameters in models of movement, such as perceptual range, that can not be precisely derived from field and/or experimental work. (Mech and Zollner 2002). In a GIS modeling effort, a fuzzy membership function was used to model the likelihood that a grid cell can be reached by a red squirrel (*Sciurus vulgaris*) starting from one of the source areas. Results using the fuzzy approach were more consistent with field data (DeGenst et al. 2001). Their results are suggestive of the potential use of fuzzy sets in the parameterization of spatially explicit models. Ruckelshaus et al. (1997) noted that in spatially explicit population models, landscape classification could affect demographic processes through an effect on patch carrying capacity *(i.e.*, habitat quality). This could increase the importance of landscape classification in spatially explicit models that have a strong demographic component or submodel(s). Although we intend to build towards demographically relevant models, this work focuses upon simulating natal dispersal movement of a single generation. In a study of landscape connectivity of red squirrel (*Sciurus vulgaris*) dispersal landscape classification errors were simulated using monte carlo techniques. In this case, the fuzzy set approach appeared to make more ecological sense relative to observed squirrel behavior than did the nonfuzzy results (DeGenst et al. 2001).

14.1.3. Small Mammals as Subjects of Landscape-scale Model Building

Although various vertebrate groups have been used to test hypotheses at the landscape scale, Barrett and Peles (1999) feel that small mammals are an ideal taxonomic group to serve as models for addressing landscape scale questions. They present compelling reasons for using small mammals as models for addressing landscape scale questions. First, detailed information is known regarding the biology and natural history of numerous species of small mammals, especially at the organismal, population, and community levels. In addition, the roles and niches of member species functioning in old-field, grassland, and forest ecosystems is known. This level of knowledge is what is needed in order to develop individual-based models for spatially explicit ecological models that address landscape scale problems. Secondly, using live-trapping and radio telemetry we can identify small mammals, follow their lives, and monitor their patterns of movement. Such studies have provided insights into dispersal behavior and supplied us with knowledge of why a particular species predominantly selects a particular patch in a landscape (Barrett and Peles 1999). Thirdly, small mammals live in relatively small spatial areas, have short lives, and typically disperse from their natal areas upon reaching adulthood (natal dispersal). These characteristics allow small mammal ecologists to develop knowledge about processes of colonization, extinction, dispersal, and persistence. The tendency to disperse from their natal areas upon reaching adulthood is an especially important factor in our choice of animal to simulate. In short, many important details of life histories of numerous species of small mammals are well known. Because of the good work and sound research on the part of ecologists who study small mammals, it turns out that the knowledge base on how small mammals live their lives make them favored subjects for the study of landscape-level processes (Barrett and Peles 1999).

In his discussion of how fuzzy logic can be used to control the movement of animal objects in a spatially explicit ecological model Robinson (2002) used several common species of *sciuridae* to illustrate his points. Of those species he considered, we focus on one well known sciurid, namely the eastern gray squirrel (*Sciurus carolinensis*). Much is known about this small mammal's behavioural ecology. In some regions, such as southern Illinois, dispersal of gray squirrels is an important issue because fragmentation of habitat has led to a noticeable decrease in their population level (Nixon et al. 1978). In addition, the dispersal of this species can have effects on other sciurids such as the red squirrel (*Sciurus vulgaris*) (Wauters et al. 2000). Thus, not only does it have desirable qualities as a modeling subject, such as an extensive knowledge base about its ecological behavior and strong natal dispersal behaviour, but may also be of some conservation significance.

14.1.4. The GIS database

In this exploratory study we chose to use a single realistic landscape that is well-known to one of the authors (Robinson and Cetin 2001). The two data layers that will be used describe the habitat-relevant land cover and topography. The land cover layer is based on the Kentucky GAP Project. Because topography can influence how visible a location is to an animal object, a digital elevation model (DEM) is another layer in the GIS database. Elevation data were generated from the USGS 7.5 minute Digital Elevation Models (DEM), with a cell size of 30m x 30m. Both the land cover and elevation data layers were made available to us by the Mid-America Remote Sensing Center (MARC). The 30m resolution of the DEM may seem rather coarse in comparison with the size and home range size of this focal organism. Although the DEM, and land cover layers could have been resampled at any finer resolution, we chose to maintain the original resolution for the illustrative purposes of this paper as well as being representative of commonly utilized data. For example, use of the 30m DEM data has been used in habitat modeling of animals of even smaller size than gray squirrels, namely neotropical songbirds (Dettmers and Bart 1999). Other studies of similar sized species have also used raster-based GIS data at similar levels of resolution varying from 20m-30m in resolution (Hale et al. 2001;DeGenst et al. 2001;Sheperd and Swihart 1995). As the relatively new 10m DEM products of the United States Geological Survey become available it should be possible to develop spatial databases that are of much finer resolution. It would be an interesting exercise to investigate the degree of sensitivity these models would exhibit as a function of the resolution differences.

The study area is an 11 km by 11km subset taken from a larger GIS database for Western Kentucky. Because species like the gray squirrel tend to have dispersal distances well under 5,000 meters (Wolff 1999) the study area is large enough to accommodate the simulation of gray squirrel natal dispersal movements. In fact, it has been reported that the maximum distance a squirrel has returned to a home range after translocation is about 4.5 km (Bowman et al. 2002).

Figure 14.3 shows the distribution of major vegetation types of most importance to gray squirrels. This 11km by 11km landscape provides sufficient size while not being too large to be computationally burdensome for this exploratory study. Importantly, it contains a gradation, moving west to east, from fragmented to nonfragmented oak/deciduous forest.

14.2. ECO-COSM: An Object Oriented Approach to Spatially Explicit Modeling

A computing environment that supports development of spatially explicit individual-based modeling should support, among other requirements, mobility, evaluating and interacting with other individuals, and acquiring and maintaining knowledge about the surrounding landscape (Westervelt 2002). The object-oriented approach has been demonstrated to be a superior approach for developing spatially explicit models (Bian 2000;Rickel et al. 1998;Westervelt 2002;Westervelt and Hopkins 1999). The object oriented approach has been combined with GIS and agent based models in a variety of settings (Gimblett 2002). One object-oriented approach to spatially explicit simulation of animals used an object class scheme where an AnimalInfo class performed the function of accessing/changing facts about a given animal type. Thus, it provided a common area for animals and animal support classes to communicate with each other (Westervelt and Hopkins 1999). It also provided a means for accessing facts associated with all other animal entities. In the same scheme, LandInfo provided the animal's view of its environment by accessing the GIS information. In many simulations, an animal is able to collect information, or view, its surroundings within a certain perceptual range. They note that it is possible to combine both the AnimalInfo and LandInfo object classes so that one object class handles all queries for information from individuals. In a sense, Graniero and Robinson's (2001) concept of a spatial probe follows a similar conceptual approach to Westervelt and Hopkins' AnimalInfo and LandInfo scheme. However, there are fundamental differences in the way probes are incorporated into the modeling structure.

The approach taken here uses the Extensible Component Objects for Constructing Observable Simulation Models (ECO-COSM) system loosely coupled with GRASS, an open source GIS, and ArcGIS. ECO-COSM is a simulation modeling framework used to build spatially explicit ecological models (Graniero 2001). Its component-based structure allows a model design to evolve by replacing or adding individual model components that change the overall behavior. The simulation framework provides a library of modular software objects that manage the structure of space and time within a simulation model. It includes mechanisms to handle concurrent activity among objects within the simulation. Objects that have embedded assumptions about the spatial or temporal structure of the simulated world are packaged into replaceable modules. This feature provides the modeler superior control over simulation behavior.

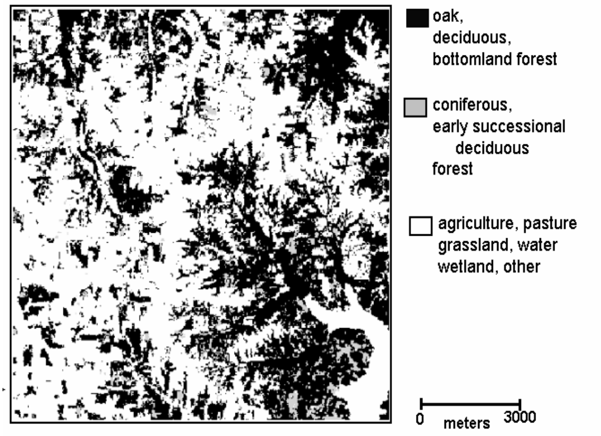


Fig. 14.3. This is the study area landscape showing the distribution of major land cover types grouped according to their ranking as preferred habitat for gray squirrels. Black includes oak forest, mixed oak, deciduous forest, and oak/deciduous bottomland forest. Note that the majority of area colored in black is classified as oak or oak/hickory forest in the Kentucky GAP data set. The gray areas are of lesser preference and are composed primarily of coniferous forest and early succession deciduous forest. The white areas are of little habitat value to gray squirrels.

The framework of each simulation program is comprised of a Simulation object that contains three interacting subsystems: Scheduling, Modeling, and Instrumentation (see Figure 14.4). The Simulation object is used to describe the overall structure of and relationships between the components comprising the model proper. It also looks after the mechanics of executing the simulation and managing the overhead required to acquire and release any computing resources needed to run the program.

The anchor is the Scheduling subsystem that is composed of two primary objects, the Clock and the Schedule. Each program is constrained to include only one instance of each of these objects. Any object in the simulation program may access the Clock's time or place actions on the Schedule. The Schedule object keeps track of all pending actions. It decides which action should occur next and triggers that event. Currently the scheduling is an event-driven structure, but discrete time step models may be constructed by adding "step" actions at every time step.

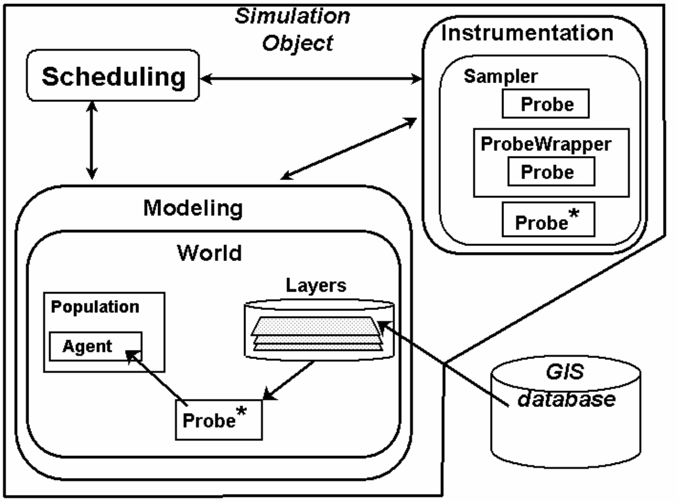


Fig. 14.4. The main subsystems of the simulation framework. Note that in the World object the Agents cannot know about Layers except through a Probe in the Instrument interface.

The Modeling subsystem provides the main components for constructing the simulated world. The spatial and temporal structure of the world is defined by the specific choice of object modules. The primary high-level object is the World, which organizes the model components into collections of "landscape" objects and "individual agent" objects. The "landscape" collection is made up of Layers representing various attributes of the study area's extent. Layers are typically represented using a Grid. Although Grids can be generated and their grid cell values populated entirely within the simulation, Grids can also reference an external, abstracted GridSource to set the grid geometry and populate the grid cell values. Using the specialized GridSource, called GrassAsciiGridSource, GIS data layers from the GRASS GIS can be used to create a simulated World. Other GridSource specializations could be constructed to import GIS data from any GIS software package. For example, an EsriAsciiGridSource might import data layers exported from the ArcGIS GRID module, or other GridSource specializations might directly read and write native GIS formats. Each Layer can have a StepRule which, when triggered by the Schedule, can calculate a new state for each grid cell based on the current state of the cell and its neighbors, as well as the state in other Layers at the corresponding location. This allows the landscape to evolve following ecological processes operating in the simulated ecosystem.

The "individual agent" collection is organized into one or more Populations, each of which contains zero or more Agents. A Population is used to group Agents that share common traits, with a separate Population for each type of Agent. For example, this is useful in ecological modeling for treating prey agents separately from predator agents. Populations can also be used to organize Agents that are of similar type, but in different fundamental states, e.g. squirrels that are active, dead, out-of-bounds, or settled into a home range. In addition, population-level monitoring is useful for controlling the simulation Schedule. For example, it may be used to add a Terminate Action when there are no Agents left in the "active" Population.

An Agent is a model component that operates autonomously, located on the landscape and obtaining information about other agents or the local landscape to make decisions about changes in its own state, movement on the landscape, or changes to the local state of one or more landscape Layers. Access to information about other model components is controlled by Probe objects, as described in the Instrumentation subsystem below. All Agent specializations share a similar information-access and processing structure, but differ in the specific details of their decision-making algorithms, which evoke important differences in behavior across Agent types. Each individual instance of a particular type of Agent shares the same decision-making algorithm, but variation in individual response to its surroundings is easily achieved by using different values for fundamental parameters, or by using different information-gathering 'filters' that modify the individual's perception of their surroundings.

The Instrumentation subsystem provides the information-access structures that allow model components to discover the state of other components in a controlled and safe fashion, ensuring the consistency and integrity of the model's overall operating state. The ability to collect data from the running model is made possible by the Probe / Probeable interface mechanism. Many of the objects in the Modeling subsystem implement the Probeable interface as well as fulfill their own modeling functions. Probes can only be created by Probeable objects; a request is made to the target Probeable object via its getProbe() method, specifying the desired type of Probe using a keyword. Each type of Probe is designed to query a specific aspect of the Probeable object's state. Whenever the Probe's probe() method is invoked (e.g., by a ProbeCommand on the Schedule, or by an Agent requiring current information about another object) the Probeable's appropriate private data access method is invoked. The result is passed to the Probe, which in turn passes the result to the object using the Probe. Using this structure, a Probeable object only exposes attributes that are deemed "public knowledge" to external objects. External objects never have direct access to the Probeable object's state, which means that they cannot accidentally change the object due to programming errors.

A ProbeWrapper is a specialized Probe that has another Probe embedded within it. A ProbeWrapper is used to modify the 'pure' result retrieved from a Probeable object in some way. For example, the land cover type observed at a distance may be subject to random misclassification due to limits of perceptual range. Alternatively, the state's description scheme may be modified to suit the purpose of the observer: the grid cell may be described as 'mature oak' in the land cover Layer, but the observing Agent may perceive it as 'suitable location for inhabiting'. Since ProbeWrappers are also Probes in their own right, an object (such as an Agent) can use either 'pure' Probes or Probes that are modified by ProbeWrappers transparently, with no knowledge of the difference. By wrapping Probes in slightly different ways for different individual Agents of a certain type, it is possible for the modeler to introduce variation in an individual's ability to perceive the world while using the same basic decision-making process. ProbeWrappers may be nested as deeply as desired, so highly sophisticated perceptual 'filters' may be constructed. In addition, some specialized ProbeWrapper objects can take the results of many nested Probes and combine their results together in some fashion, creating views of the modeled world and its components at different scales of observation.

The Instrumentation subsystem also allows the modeler to 'instrument' the operating simulation model in order to monitor the model's evolution and collect data for later analysis. A Sampler is made up of a set of one or more Probes that perform the actual queries about system state. The Sampler will typically take the Probe results and format them in an organized fashion for output to a file on disk, or for periodic output to the computer console to inform the user on progress. Data files produced by a Sampler may be used in other separate analysis programs to generate summary statistics from a large number of model runs.

The Simulation object acts as the core engine of the simulation model. It manages the interaction of the components in the three subsystems. The setup() method structures the simulation appropriately for the desired model, attaches any instrumentation desired, and acquires any necessary memory or file resources required for the model. The run() method is very simple: until the Schedule is finished, it will trigger the next pending item on the Schedule. The teardown() method releases any memory or file resources and gets ready for program termination. The Simulation object may be instantiated and executed as an independent, stand-alone program. It can also act as a 'pure' object that is contained in a larger program, such as a simulation experiment which executes many instances of the Simulation object, each one of which has slight variations in its selection and configuration of model components.

14.3. Using Fuzzy Sets to Control Natal Dispersal Movement

A computing environment that supports development of spatially explicit individual-based modeling should support, among other requirements, mobility, evaluating surrounding individuals, interactions with other individuals, and acquiring/maintaining knowledge (Westervelt 2002). One object-oriented approach to spatially explicit simulation of animals used an object class scheme where an AnimalInfo class was used for accessing/changing facts about a given animal type thus providing a common area for animals and animal support classes to communicate with each other. It also provided a means for accessing facts associated with all other animal entities. In the same scheme, LandInfo provided the animal's view of its environment by accessing the GIS information. In many simulations, an animal is able to collect information, or view, its surroundings within a certain perceptual range. They note that it is possible to combine both the AnimalInfo and

LandInfo object classes so that one object class handles all queries for information from individuals (Westervelt and Hopkins 1999). In a sense, with important differences, that is the approach taken in the computational environment we use for implementing spatially explicit ecological models.

The dispersal movement process of each object consists of two major decisions – movement and residence. If the object is to move from its current location then it must decide on a destination location. Once at the new location it will need to assess its surroundings to gather information that is used to make a residence decision. In other words, has the object found a suitable location or will it need to continue the dispersal movement. In the following sections we present a simple fuzzy decision making process for each decision. The basic decision model used here is one in which relevant goals and constraints are expressed in terms of fuzzy sets, and a decision is determined by an appropriate aggregation of the fuzzy sets (Bellman and Zadeh 1970). Fundamental to either the movement or residence decision, is information about the surrounding landscape and conspecifics. This is usually described as a perceptual range (Mech and Zollner 2002) or finite range of vision (Fahse et al. 1998). Because an animal's perceptual range represents its informational window onto the larger landscape, it determines how much of the area surrounding the individual it can perceive. In the spatially explicit simulation model outlined in Figure 14.2 this is tantamount to the perceptual range being a spatial constraint on a query to the GIS database.

14.3.1. Perceptual Range as Fuzzy Spatial Relation

An important controlling parameter in many spatially explicit simulation models is the perceptual range of individuals. Perceptual range is the distance from which a particular landscape element can be perceived as such. An animal's perceptual range represents its informational window onto the larger landscape. This determines how much of the area surrounding the individual it can perceive in terms of habitat quality and other conspecifics. Models incorporating perceptual range have typically specified them as crisp sets (Baum and Grant 2001;Fahse et al. 1998;Railsback et al. 1999;South 1999;Westervelt and Hopkins 1999). However, like the example of margin widths used in a study of species density of foliage dwelling spiders using a fuzzy rule-based model, applying crisp boundaries to the concept of a perceptual range may not be biologically meaningful (Klosterman 1998). Research on the landscape level perceptual abilities of forest sciurids suggest that the perceptual range of the sciurids varies according to body size (Zollner 2000;Mech and Zollner 2002). Results of field-based research do not offer a single crisp limit for the spatial extent of the perceptual range of gray squirrels. It is generally reported that the perceptual range of a gray squirrel is 300-400 meters (Mech and Zollner 2002;Zollner 2000). There are other variables involved such as height above the horizon of the forest relative to the squirrel. Thus, if we were to set the perceptual range at 400.0 meters and say that 400.5 meters is not within the perceptual range we would be forced to draw an artificially sharp distinction that may have little basis in the behavior ecology of the animal. Thus, we specify the perceptual range as a fuzzy set.

14.3.2. The Movement Decision Model

The basic decision model used here is one in which relevant goals (***G***) and constraints (***C***) are expressed in terms of fuzzy sets, and a decision is determined by an appropriate aggregation of the fuzzy sets (Bellman and Zadeh 1970;Klir and Yuan 1995). In the movement decision model the constraints consist of two major sets of locations. One set is composed of those locations that are within the visible perceptual range (Ψ). The other constraint relates to distance from conspecifics (***F***)

The Constraint Set

At this stage in developing our modeling environment, we model the constraint set***, C,*** as a function of visible perceptual range and conspecific spacing. Visible perceptual range constrains the animal to consider information about the landscape that falls with its limits of perception. Conspecifics may constrain locational choices due to defended territory, or the converse due to social grouping (Wolff 1999).

Visible Perceptual Range. Let $X = \{x\}$ a finite set of locations bounded by the limits of the study area. Let d_x^c be the euclidean distance from the location of the dispersing animal object, *c,* to location x. *P(x)* is the fuzzy set defining the perceptual range for a single individual. Based on Zollner (2000) and Mech and Zollner (2002) Eq. 14.1 defines the fuzzy set representing the membership of a location within the 'ideal' perceptual range (Figure 14.5).

$$P(x;\beta,\theta) = \mu_p(x) \quad = \begin{cases} 1 & if \quad d_x^c \le \beta \\ \theta(\beta - d_x^c) + 1 & if \quad \beta < d_x^c < \beta + 1/\theta \\ 0 & if \quad \beta + 1/\theta \le d_x^c \end{cases} \tag{14.1}$$

In Figure 14.5 note that the parameters have been set such that any location within 90 meters has the highest membership value for being within the perceptual range and that as it declines somewhere between 400 and 600 meters is where it finally reaches zero. At this point it is worth noting that some research (Mech and Zollner 2002; Zollner 2000) suggests that the 300-400m perceptual range for arboreal sciurids may be an underestimate. Thus, it seems that the membership curve in Figure 14.5 would appear to be quite plausible, perhaps even a bit conservative.

$P(x)$ determines the extent over which there is some information about the landscape that can be perceived and information is retrieved from the GIS database. Subsequent operations are confined to this fuzzy geographic region only over locations where $x \in {}^{0+}P(x)$. ${}^{0+}P(x)$ is the support of fuzzy set *P* which means it

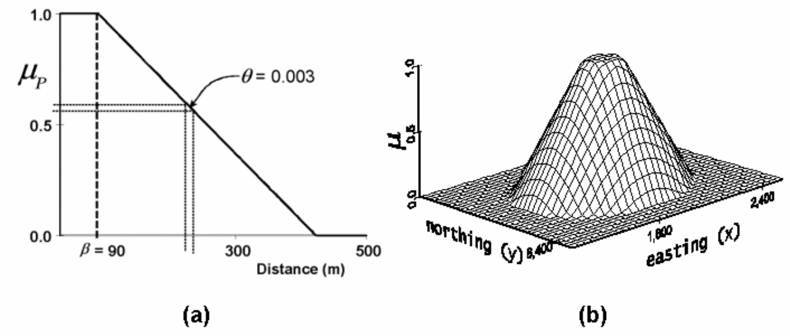


Fig. 14.5. (a) the open-form membership function for μ_p where $\beta = 90$ meters and $\theta = 0.003$, (b) the result of applying the membership function to locations surrounding an individual animal.

is a crisp set containing all the elements of $\boldsymbol{X}$ having a nonzero membership in P (Klir and Yuan 1995).

Because squirrels must perceive potential destinations over a terrain with varying elevation it is reasonable to further qualify their perceptual range with those locations falling in the fuzzy set of *line-of-sight*. Typically, not all the locations that are a member of ${}^{0+}P(x)$ are visible by a squirrel assumed to be viewing the landscape from the treetops. A good approximation for a mature oak forest is 15m. For these experiments, we assume differing heights of vegetation depending on the land cover type at a location. For example, in the canopy of an oak forest height = 15m, whereas pasture would be height = 0.1m. The canopy height is added to the local elevation to obtain an absolute elevation for the squirrel. An alternative might be to select canopy heights from possibilistic distributions, e.g. oak forest is 12-18m in height.

Let $L : X \to [0,1]$ be the fuzzy set describing the degree to which location x is visible to a particular squirrel. The membership function for L is defined by Eq.14.2 as a closed-form triangular function (Figure 14.6a). Where los_x^c is the angle at which location *x* is visible from location *c*. It is based on output style of the GIS package GRASS where $90°$ is looking straight ahead. Below the line of sight is less than $90°$ and above the line of sight is greater than $90°$. If the local terrain creates a physical obstruction to visibility between *c* and *x*, then $L = 0$.

$$L(x;\alpha,\beta,\gamma) = \mu_L(x) = \max(\min\left(\frac{los_x^c - \alpha}{\beta - \alpha}, \frac{\gamma - los_x^c}{\gamma - \beta}\right),0) \qquad (14.2)$$

The degree to which a cell is both visible and falls within the perceptual range is defined by $\Psi = P \cap L$ (Figure 14.6b). This operation takes into account both

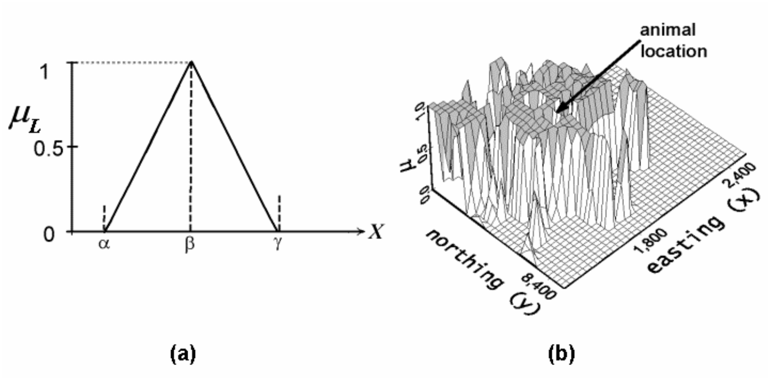


Fig. 14.6. (a) the closed-form triangular membership curve for line-of-sight membership of fuzzy set L where $\alpha = 45°; \beta = 90°; \gamma = 135°$, (b) an example of applying the membership function to the results of a line-of-sight analysis for locations surrounding an individual animal.

the level plain perceptual distance and the potential effect topography may have on the ability of an object to perceive a location.

Conspecifics. Territory defended by conspecifics can impede the movement of animals, especially if all suitable space is occupied and individuals are not able to cross undefended space (Wolff 1999). Gray squirrels do defend home territory. However, it is common for males to have overlapping home ranges (Allen 1987) making their boundaries less than crisp and their home range territories akin to fuzzy regions. In the case of this species, natal dispersers would tend to avoid conspecifics. Thus, a constraint on the movement decision is to avoid stopping at a location too close to one or more conspecifics.

$$F(x) = \mu_F(x) = 1.0 - \left(\bigcup_{k=1}^{c} \mu_{NC}^{k}(x) \right) \tag{14.3}$$

If a conspecific is within a squirrels' visible perceptual range (*i.e.*, $k \in^{0+} \Psi$.) then let d_i^k be the distance from conspecific *k* to location *i*. This is used to estimate the membership of each location in the set of *far_from_conspecific* (***F***). The fuzzy set of *far_from_conspecific* (***F***) is defined in Eq. 14.3 where the fuzzy set NC^k is the fuzzy set near conspecific *k* and $\mu_{NC}^{k}(x)$ is the degree to which *x* is near conspecific *k* as defined in Eq 14.4.

$$NC^k(x;\alpha,\beta) = \mu_{NC}^k(x) = \begin{cases} \dfrac{\beta - d_x^k}{\beta - \alpha} & \alpha \le d_x^k \le \beta \\ 0 & otherwise \end{cases} \tag{14.4}$$

In Eq 14.4 d_x^k is the distance from conspecific k to x. Plausible parameter values are $\alpha = 0$ and $\beta = 300$. The area of home range can vary from 0.72 to as great as 6 ha with the more typical areas being around 1 ha (Allen 1987).

It is worth noting that the parameters of Eq.s 14.3 and 14.4 can easily be adjusted to represent the behavior system of other more social species such as prairie dogs and ground squirrels. They are attracted to conspecifics (Wolff 1999).

Constraint Set. If we let C be the fuzzy set of constraints on the movement decision then it can be defined as $C = \Psi \cap F$. In effect we are constraining the search to those locations that are in the visible perceptual range and far from a competing conspecific.

The Goal Set

Habitat. One of the major goals of a move by an individual is to reach a location that is perceived to be habitat. In the case of this species that habitat would be forest. We use the crisp classification function in Eq. 14.5 because it is unlikely, es-

$$A(x) = \mu_A(x) = \begin{cases} 1 & if \quad forest \\ 0 & if \quad nonforest \end{cases} \tag{14.5}$$

pecially towards the edge of the perceptual range, that squirrels can evaluate vegetation in any detailed manner. Once an individual has moved to a location then, through exploratory movement, an evaluation of the habitat can become more detailed and the subtleties of how well the vegetation relates to habitat requirements can be taken into account.

Dispersal Imperative. Eq. 14.6 describes the membership function for the dispersal imperative set $I : X \to [0,1]$. In our model $\alpha = 0$ and β is the distance of the farthest location in Ψ that has a non-zero membership value. In other words, the farthest distance that the animal can theoretically perceive to some, non-zero, degree.

$$\mathrm{I}(x) = \mu_I(x) = \max\left(\min\left(1, \frac{d_x^c - \alpha}{\beta - \alpha} \right), 0 \right) \tag{14.6}$$

This membership function is constructed to reflect the imperative of finding a home as far from the current location as possible. Given the constraint of perceptual range, this seems plausible for gray squirrels.

Goal Set. The degree to which a location is a member of the goal set is defined by $G = A \cap \mathrm{I}$. In effect, the goal of an individual is to find a location as near the edge of the perceptual range as possible that is forested.

The Decision Set

On the first, initial, move the degree to which each location within the perceptual range falls in the decision set (D) is defined by $D = C \cap G$. Movement is to the location with the highest value for ***D,*** *i.e.,* $\kappa = \{x \in X \mid \mu_D(x) = \max\ D\}$. However, given the nature of the problem it is possible that more than one location will have the same, maximum, value. In our case, should there be ties, the first one in the list is chosen. On moves beyond the first one there is the question of directional bias. This is a topic that has received some attention in the ecological literature.

To simulate more realistic moves and reduce redundancies many simulations use correlated random walks e.g. (Schumaker 1996;Zollner and Lima 1999). An alternative systematic search strategy based on search theory had been proposed where the simplest example is to have a rule to move always in the same direction (Dusenbery 1989). Results of a simulation study comparing search strategies for landscape-level movement suggest that a simple and effective search rule for landscape-explicit models would involve straight or nearly straight movements (Zollner and Lima 1999). Therefore, on subsequent moves, the previous direction of movement will bias the decision.

Let ***B*** be the fuzzy set representing the degree to which a location falls within the set of ***direction_to_move***. Because direction is a circular measure, let q_p be the direction, in radians, of the move to the current location and $q(x)$ be the direction, in radians, from the current location (κ) to location *x*. Then ***B*** can be defined by Eq. 14.7, where the exponent ρ functions much like a *hedge* that constricts or expands the shape of the function. For our purposes, we assume $\rho = 2$.

$$B(x; q_p, q(x), \rho) = \mu_B(x) = \left[\left(\left(\frac{\cos(q_p) + \cos(q(x))}{2}\right)^2 + \left(\frac{\sin(q_p) + \sin(q(x))}{2}\right)^2\right)^{0.5}\right]^{\rho} \qquad (14.7)$$

On subsequent moves we can define the decision set as $D = (C \cap G) \cap B$. Again, movement is to the location with the highest value for ***D,*** *i.e.* $\kappa = \{x \in X \mid \mu_D(x) = \max\ D\}$. In our case, should there be ties, a random location amongst the candidate set (***D***) is chosen.

14.3.3. The Residence Decision Model

Once the animal has moved to a location, it must then decide whether it is a location suitable for stopping its dispersal movement. In other words, is it suitable for staying and maintaining a home range. Like the movement decision model, this is one in which relevant goals (***G***) and constraints (***C***) are expressed in terms of fuzzy sets, and a decision is determined by an appropriate aggregation of the fuzzy sets (Bellman and Zadeh 1970;Klir and Yuan 1995). In the residence decision model the animal is constrained by whether or not its current location is sufficiently spatially separated from conspecifics that a home range can be established while the goal is to have habitat of sufficient area. Finally, a decision rule is applied to the decision set that leads to the animal either taking up residence at the location or attempting a move to another location.

Since this work is focused on modeling natal dispersal, we use the residence decision primarily as a stopping rule. In the future we plan to incorporate a subsystem that would include exploratory movement so that the agent explores the vicinity around its destination and uses that information in a more sophisticated decision process, than presented here, to setup a home range or not. However, at the present we have simplified the decision to address just a few key criteria that have been suggested by the literature (Allen 1987;Wolff 1999).

The Constraint Set

The constraint set (***FC***) is a function of spatial separation from surrounding conspecifics. This construct measures how far an animal is from conspecifics, yet allowing for overlap in home ranges (something common in gray squirrels). For simplicity, we use the same perceptual range (*P*) as explained above as the limit of the landscape information available to the animal. In Eq. 14.8 let $\mu^c_{Far}(\kappa)$ be the membership of location κ in the fuzzy set ***Far*** *from conspecific c* defined by

$$\mu^c_{Far}(\kappa; d_c(\kappa), \beta, \theta) = \begin{cases} 1.0 - \left\{ \dfrac{1.0}{\left[1 + \dfrac{d_c(\kappa) - \beta^c_{Far}}{\theta^c_{Far} - \beta^c_{Far}} \right]} \right\} & if \quad d_c(\kappa) \geq \beta^c_{Far} \\ 0.0 & if \quad d_c(\kappa) < \beta^c_{Far} \end{cases} \tag{14.8}$$

where $d_c(\kappa)$ is the distance from conspecific c (c = 1…k) and the current location (κ) of the individual animal. It is an open membership function with β^c_{Far} representing the limit of a hypothetical core (*i.e.* membership =1) and θ^c_{Far} is the distance at which membership = 0.5. Our working example will be $\beta^c_{Far} = 30m$, $\theta^c_{Far} = 100$. The mean area of gray squirrel home ranges varies from just under 1ha to 6ha (Allen 1987) which is considered largely a function of population density and habitat quality. Based on local experience, we will use a home range area of around 1 ha. as the basis for constructing ***Far***. With $\beta^c_{Far} = 30$ and $\theta^c_{Far} = 100$ the core of a conspecific home range would be approximately 1 ha which corresponds to the area it would be expected to defend most vigorously. Since overlapping home ranges are common in the species, the membership function in Eq. 14.8 allows for it with those locations where $\mu^c_{Far}(\kappa) < 0.5$. Note that $\theta^c_{Far} = 100$ may be valid for certain densities but for areas of high quality habitat with densities approaching the maximum indicated in the literature, then a value of 60 may be more in order. In other words, this parameter may be adjusted as more information about the landscape becomes available to the modeler and/or to the animal agent. Because there are usually several conspecifics in the surrounding area, to define the fuzzy set ***Far*** *from nearest conspecific*, we use $FC = \underset{c}{\sim}\mu^{c}_{Far}(\kappa)$.

The Goal Set

The quality of the habitat at the animal's location and the area of the habitat patch are combined to define the goal set. This is a simplification made for illustrative purposes, in particular the importance of oak/deciduous forest has been emphasized.

Habitat. Membership values associated with particular land cover types describe the degree to which a land cover type, typically found in a GIS database, can be considered quality habitat for a gray squirrel at location κ which is where the squirrel has moved. Eq. 14.9 shows the membership values that were assigned to land cover types found in our GIS database. The values were based on an interpretation of the research literature, our knowledge of the study area, and the land cover data in the GIS database. For example, locations classified as oak in the GIS database because of the importance assigned to hard mast producing trees in the habitat suitability model derived from research literature on this species (Allen 1987). The oak/deciduous bottomland cover type was assigned similar but slightly lower membership value because deciduous species provide must less food during the winter months thus diluting the high suitability score of a pure hard mast producing location. This logic is followed through to the other classes. It may be worth noting, that it is not unusual for the source of such membership functions to be reported as based on professional judgment without detailing a formal knowledge acquisition procedure (Rickel et al. 1998;DeGenst et al. 2001). Although the theoretical specification of how fuzzy logic can be used to describe vegetation communities and habitat in a form suitable for expert system development (Moraczewski 1993) has been described, it has not be implemented. There have been many approaches to specifying membership functions and incorporating fuzzy logic in ecological models that have a habitat-related component (Robinson 2003;Salski et al. 1996;Hobbs et al. 2002), including characterization of uncertainty in habitat suitability indices (Burgman et al. 2001). It is beyond the scope of this paper to develop a comprehensive habitat modeling methodology. This work does present a modeling framework within which mapping from a GIS database to fuzzy representations of habitat can be directly utilized by an ecological model. Hopefully this will spur further development in this area by explicitly illustrating the utility of such a methodology in ecological modeling.

$$LC(\kappa) = \mu_{LC}(\kappa) = \begin{cases} 1.0 & if \quad oak \\ 0.9 & if \quad oak / deciduous\_bottomland \\ 0.75 & if \quad deciduous \\ 0.0 & if \quad conifer \\ 0.0 & if \quad early\_successional\_deciduous \\ 0.0 & if \quad wetland, pasture, grassland, agriculture \\ 0.0 & if \quad water \end{cases} \tag{14.9}$$

Area. Not only is the specific land cover type important in evaluating the quality of a location as habitat the area of a habitat patch may also be a determining factor. We use the size of an oak/deciduous forest patch as an important factor in the residence decision. Minimum habitat area is the minimum amount of contiguous habitat that is required before an area will be occupied. Allen (1987) notes that information pertaining to the minimum habitat area for gray squirrels was not located in the literature. It remains an elusive number. However, he notes that literature suggests the mean minimum home range for the gray squirrel is at least 0.49 ha. For the purposes of his habitat suitability model, it was assumed that an area of less than 0.4 ha is unsuitable (Allen 1987), i.e., classified as not habitat. Note that even in this case the researcher did not set the lower limit at 0.49 ha, the mean, but relaxed it a bit to 0.4 ha to allow for the variation, or uncertainty, regarding the precise number. For example, if one set it at 0.49 but a patch was 0.48 why disqualify it? Therefore, In Eq. 14.10 we define a fuzzy set, ***HA,*** to express the degree to which a location falls within the class of minimum habitat area by setting the parameters $\alpha_{HA} = 0.3$ and $\beta_{HA} = 2.0$. In other words, any patch less than 0.3 ha is clearly too small while any patch greater than 2 ha is clearly large enough. Any patches whose area falls between 0.3 ha and 2.0 ha will be a member of ***HA*** but to a degree somewhere between 0 and 1. The area measurement is based on the size of patches formed from contiguous cells that have been classified as oak, deciduous, or oak/deciduous bottomland. Let $farea(\kappa)$ be the area in hectares of the oak-deciduous forest patch that location κ falls within.

$$HA(\kappa) = \mu_{HA}(\kappa) = \max\left(0, \min\left(1, \left[\frac{farea(\kappa) - \alpha_{HA}}{\beta_{HA} - \alpha_{HA}}\right]\right)\right) \tag{14.10}$$

Goal Set. The degree to which location κ falls in the goal set is defined by $H = LC \cap HA$. In effect, H is a measure of the degree to which the current animal location is habitat and contained within a large enough patch of habitat. We recognize that this may be a simplification, hence additional spatially explicit landscape measures are planned for future elaboration of the residence decision. Nevertheless, it is instructive as to how such information can be extracted from a GIS and used by a mobile object in a simulation.

The Decision Set

The membership of location κ in set *residence location* is defined as $R = H \cap FC$. We set the decision rule for residence versus to not reside as

IF $R \geq 0.5$ THEN **reside**

ELSE **move**

Although we use the 0.5 as the threshold in this instance, in future developments it may be possible to incorporate a dynamic adjustment so that ***R*** decreases as the number of steps increase. This would reflect a relaxation of expectations as the animal's stored energy is depleted during the dispersal. Once the individual decides to reside then it would be up to other mechanisms, home range/foraging, exploration, or predation to determine whether it survives. However, this is beyond the scope of this particular paper.

14.4. Effects of Aggregation Operators on Mobile Agent Behavior

In this section, an example application of the ECO-COSM implementation is used to investigate the differences in agent behavior as a function of both aggregation operator and conspecific density. The ECO-COSM framework was used to run simulations of squirrel agent behavior in the study area.

14.4.1. Modeling with different aggregation operators and conspecific density

In the presentation of the decision models there was little mention of the specific aggregation operators that will be used. For example, the residence decision set is defined as $R = H \cap C$. In this aggregation is $\cap$ the typical *min* operator or a compensatory aggregation operator? Robinson (2003) has noted that few studies in GIS using fuzzy logic have compared the use of different operators. This is especially true with regard to the few studies that have used fuzzy sets in the decision making process of agents in spatially explicit models (e.g,, Itami 2002). Here we use an illustrative example to explore the differences in agent behavior. Each decision model was implemented using noncompensatory, compensatory and Yager aggregation operators. In addition, a crisp logic version of each was also contructed. Tables 14.1 and 14.2 detail how the operators are defined in our model for the movement and residence decisions respectively. Note that there are at least two types of compensatory operators. The Yager connective is used in addition to the more common algebraic product.

One of the important spatially explicit variables in the simulation is the density of conspecifics. Territoriality can impede movement, especially if all suitable space is occupied and individuals are not able to cross undefended space. When the density of conspecifics reaches such a level, it may result in what is called a social fence (Wolff 1999). To vary the distribution of conspecifics a dense base distribution was created by generating a random set of cells that are at least 50 meters apart. These cells were constrained to only be located in land cover that corresponds to gray squirrel habitat. To generate different levels of density each variation was based on eliminating a certain percentage of conspecifics from the dense base distribution. The specific conspecifics to be eliminated were chosen at random. Figure 14.7 shows four spatial distributions of conspecifics representing different densities from the base distribution where 0 percent were eliminated to those where 20, 40, and 60 percent respectively were eliminated. In essence, we are creating *conspecific holes* when we eliminate a certain number of conspecifics from the densest distribution.

14.4.2. Using the ECO-COSM Framework

To explore the effects of aggregation operators and the density of conspecifics, the model described in Section 14.3 was implemented in the ECO-COSM framework (Figure 14.8). The aggregation types associated with movement and residence decision models were implemented as populations of individual squirrel agents. For example, in Figure 14.8 the population of active noncompSquirrel agents are agents that are actively making movement decisions based on the noncompensatory models described in Table 14.1. To decide on whether to stay or move on the noncompSquirrel uses the residence decision model described in Table 14.2. The AgentSampler gathers information about the state of each individual agent at each time step and writes it to an archive on disk that is subsequently used to generate shapefiles that can be used for visualization in ArcGIS, as well as to generate summary statistics from a large number of simulation replications.

The various squirrel agents interact with the landscape as represented by the Layers via a number of Probes (Figure 14.9). A squirrel agent acquires information about the land cover Layer via three different perceptual filters, namely ‘patch area’, ‘mobility’, and ‘habitat’; in each case a ProbeWrapper modifies the land cover class retrieved from the land cover Layer. Similarly, the Squirrel perceives canopy elevation via ‘line-of-sight’ and ‘visibility’ perceptual filters using ProbeWrappers.

In the case of evaluating conspecifics, the Squirrel Agent uses the same ‘conspecifics’ Probe to identify where conspecifics are located within its perceptual range. The Squirrel uses the information provided by this Probe in different ways, depending whether it is making a movement decision or a residence decision.

14.4.3. Simulation Results

For this illustrative example, 21 starting points were used with four agents (one using each type of aggregation operation) dispersing from each starting point. For convenience, a separate simulation was executed for each starting point scenario, and the four squirrel agents starting from the same spot were modeled simultaneously. Because the agent code does not include Probes associated with other agents, and the agents do not modify the landscape in any way, the agent movements may be treated as completely independent behavior. In other words, each of the four simulations can be run at the same time without interfering with each other in any way.

As noted in the ecological literature, successful dispersal should be limited by energy reserves (Zollner and Lima 1999). Therefore, in these simulations, if an agent did not find a location that met residence requirements within 10 steps then it was placed in the 'dead' population. Those squirrel agents that moved to a location within 300m of the boundary of the study area were placed in the 'out of bounds' population. We chose to use absorbing boundaries rather than reflecting

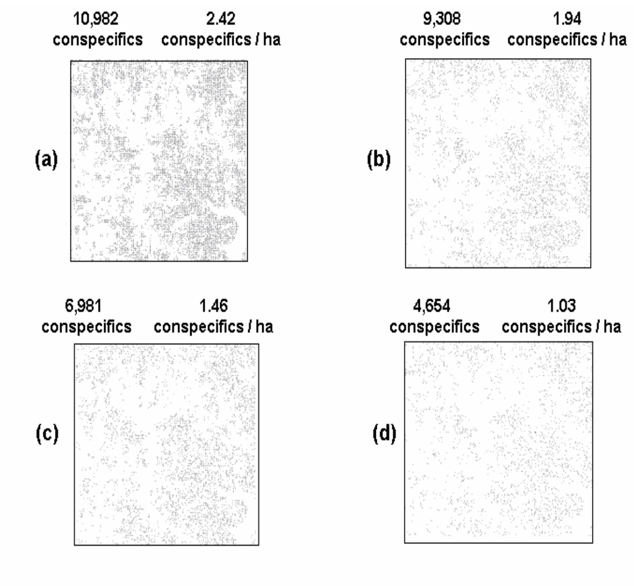


Fig. 14.7. Four spatial distributions and densities of conspecifics; (a) is the densest with a minimum distance of 50m separating conspecifics in oak/deciduous forest. It is the base distribution, therefore 0% of the base distribution has been randomly eliminated, (b) 20% of the base distribution has been randomly eliminated, (c) 40% of the base distribution has been randomly eliminated, and (d) 60 % of the base distribution has been randomly eliminated.

boundaries, or a torus-like landscape (Tischendorf and Fahrig 2000) because real landscapes are roughly 2-dimensional and likely to have patch-containing edges that are not apparent to our squirrel agents (Zollner and Lima 1999). Simulations were run for each of the conspecific landscapes shown in Figure 14.7 and each of the aggregation types in Tables 14.1 and 14.2.

Table 14.1. . Summary of different aggregation operators used in the movement decision models.

Visible Perceptual Range: $\Psi = P \cap L$

compensatory: $\Psi = P \cdot L$

noncompenstory: $\Psi = \min(P, L)$

Yager:

$$\Psi = 1 - \min\left\{ \left[(1-P)^p + (1-L)^p\right]^{1/p}, 1 \right\}, for \quad p = 2$$

crisp model: $\Psi = {}^{0.5}P \wedge {}^{0+}L$

Contraint set: $C = \Psi \cap F$

compensatory: $C = \Psi \cdot F$

Yager:

$$C = 1 - \min\left\{ \left[(1-\Psi)^p + (1-F)^p\right]^{1/p}, 1 \right\}, for \quad p = 2$$

noncompensatory: $C = \min(\Psi, F)$

crisp: $C = \Psi \wedge {}^{0.5}F$

Goal set: $G = A \cap \mathrm{I}$

compensatory: $G = A \cdot \mathrm{I}$

Yager:

$$G = 1 - \min\left\{ \left[(1-A)^p + (1-I)^p\right]^{1/p}, 1 \right\}, for \quad p = 2$$

noncompensatory: $G = \min(A, \mathrm{I})$

crisp: $G = {}^{0.5}A \wedge {}^{0.5}\mathrm{I}$

Decision Set: $D = \left(C \cap G\right) \cap B$

compensatory: $D = \left(C \cdot G\right) \cdot B$

Yager:

$$CG = 1 - \min\left\{ \left[(1-C)^p + (1-G)^p\right]^{1/p}, 1 \right\}, for \quad p = 2$$

$$D = 1 - \min\left\{ \left[(1-CG)^p + (1-B)^p\right]^{1/p}, 1 \right\},$$

noncompensatory: $D = \min\left\{ \min\left(C, G\right), B \right\}$

crisp: $D = \left({}^{0.5}C \wedge {}^{0.5}G\right) \wedge {}^{0.5}B$

Locational Decision: $\kappa = \left\{ x \in X \mid \mu_D(x) = \max D \right\}$

In the case of ties, we randomly choose κ from the list.

Table 14.2. Summary of different aggregation operators used in the residence decision models.

Goal set: $H = LC \cap HA$

compensatory: $H = LC \cdot HA$

Yager:

$$H = 1 - \min\left\{ \left[(1-LC)^{p} + (1-HA)^{p}\right]^{1/p}, 1 \right\}, \textit{for} \quad p = 2$$

noncompensatory: $H = \min(LC, HA)$

crisp:

$$H = \begin{cases} 1 & \textit{if} \quad LC \geq 0.5 \quad \textit{and} \quad HA \geq 0.5 \\ 0 & \textit{otherwise} \end{cases}$$

Constraint set: $C = \underset{c}{\sim} \mu^{c}_{Far}(\kappa)$

compensatory, noncompensatory, Yager:

$$C = \min_{c}\left\{\mu^{c}_{Far}(x)\right\}$$

crisp:

$$C = \underset{c}{\sim} \mu^{c}_{Far}(\kappa) \qquad \textit{where} \quad `\mu^{c}_{Far}(\kappa) = \begin{cases} 1 & \textit{if} \quad \mu^{c}_{Far}(\kappa) \geq 0.5 \\ 0 & \textit{if} \quad \mu^{c}_{Far}(\kappa) < 0.5 \end{cases}$$

Decision set: $R = H \cap FC$

compensatory: $R = H \cdot FC$

Yager:

$$R = 1 - \min\left\{ \left[(1-H)^{p} + (1-FC)^{p}\right]^{1/p}, 1 \right\}, \textit{for} \quad p = 2$$

noncompensatory: $R = \min(H, FC)$

Decision Rule:

IF $R \geq 0.5$ THEN **reside**
ELSE **move**

crisp:

$$R = \begin{cases} 1 & \textit{if} \quad H \geq 0.5 \quad \textit{and} \quad C \geq 0.5 \\ 0 & \textit{otherwise} \end{cases}$$

Figure 14.10 illustrates the movement behavior of a single squirrel agent. Note that the squirrel agent using the crisp decision models is shown locating in agricultural/pasture land. According to the decision models above such locations would have a membership in either the movement or residence decision of 0.0. This behavior results from the situation where the max(D) = 0.0. When this is the case, unless the 'fence' rule is used a location is chosen at random. Using the 'fence' rule when such a situation occurs the agent is placed in the population of 'fenced' agents, i.e. agents who have no non-zero elements in their decision set(D). In this set of simulations, we disabled the 'fence' rule so as to highlight the behavioral differences between the populations of 'active' agents. Figure 14.10 illustrates a common difference between the crisp and fuzzy agents movement patterns. It was common for crisp agents to choose such marginal locations while the fuzzy agents tended to be better at locating and moving to habitat patches.

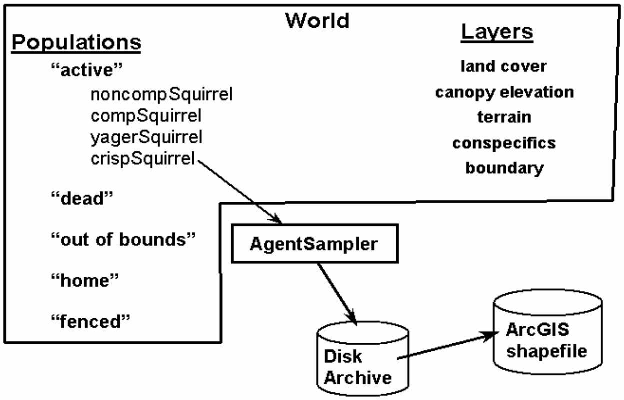


Fig. 14.8. The World object in ECO-COSM as implemented for running simulations of squirrel agent behavior.

In Figure 14.10 the crisp agent was the only agent that did not find a home after 10 time steps. This is representative of aggregate results. In all cases, except the densest conspecific distribution, the crisp agents had the lowest percentage of success (see Table 14.3). When using the base, *i.e.* densest, distribution of conspecifics no squirrel agent, regardless of decision model, successfully found a "home." Thus, it is a good worst-case scenario. As we progress to the case where 60 percent of the conspecifics have been eliminated it is noteworthy that the fuzzy agents tended to have a 90+ percent success while only about two-thirds of the crisp agents were successful. In general, fuzzy agents were consistently more successful in finding homes than were the crisp agents.

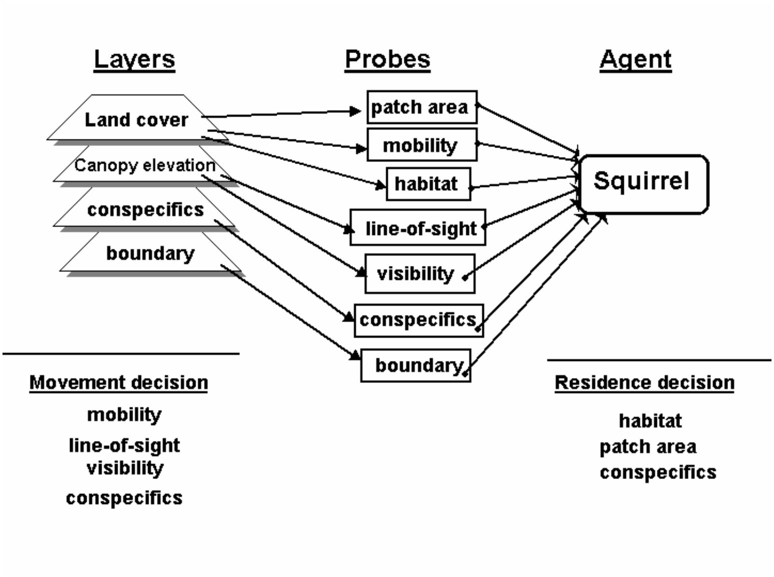


Fig. 14.9. The relationship between Layers, Probes, Agents and decision models, movement and residence, for a squirrel agent

Table 14.4 shows that when using the crisp decision models agents, on average moved fewer steps than the compensatory type operators when the density of conspecifics is lowered. On this measure, the crisp and noncompensatory models performed in a similar fashion. Although it appears as though crisp-thinking agents found suitable homes quicker, remember that in Table 14.3 they were on average much less successful in finding homes.

In the ecological literature it is usually the straight-line distance between the start and end of a dispersal movement that is used (Wolff 1999). The mean and maximum straight-line distances generated by the simulations (Tables 15.5 and 14.6) are within the range suggested by the literature (Wolff 1999;Bowman et al. 2002). In fact, one could argue that they tend to be on the low end of estimates. Nevertheless, as a preliminary test of the ECO-COSM framework the results are not out of bounds with reports from the field. When non-compensatory squirrels found homes, which was quite often, they tended to travel a shorter distance than the agents using either the compensatory or Yager decision models -- both on average, and maximum (Tables 14.5 and 14.6). This was especially true when there were fewer potential locations available for taking up residence. It is also interesting to note that agents using the crisp and noncompensatory decision models were the ones who on average tended to travel the least distance to a home location.

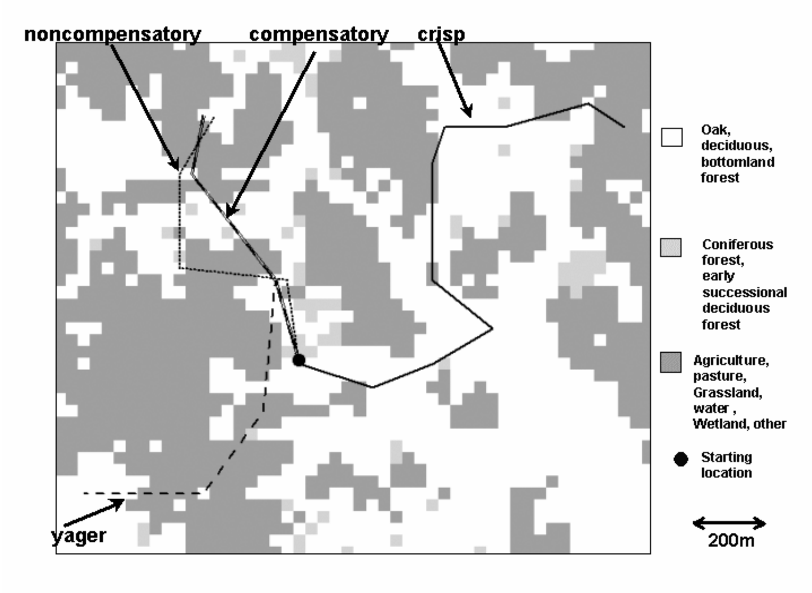


Fig. 14.10. Paths taken by mobile agents as a function of the decision models in Tables 14.1 and 14.2

In contrast to the straight-line dispersal distance, we also measured the meandering-path distance of each squirrel agent by measuring the length of the straight line segment connecting each successive move. It was beyond the scope of this work to model the path-finding behavior. Nevertheless, it does provide a complementary measure to straight-line distance. Among the agents using the fuzzy decision models there is no clear tendency for one to have a greater or lesser mean distance in all scenarios (Table 14.7). The relatively low mean distance for the crisp model agents that found a home in the 40% scenario is an artifact of how few squirrel agents actually found a home (see Table 14.3). This also led to a similar artifact when considering the maximum meandering-path dispersal distance (Table 14.8). In no case did the maximum dispersal distance to find a home exceed 2,800 meters. Considering the species and landscape used in these scenarios, this is not an unreasonable result.

Table 14.3. Percentage of squirrel agents finding a home (i.e., success rate) in relation to density of conspecifics. Note that in one 20% scenario a crisp squirrel went out of bounds; other than that, all squirrels either found a home or reached the 10 time step limit.

	Conspecific Holes 0%	20%	40%	60%
compensatory	0.0	28.6	90.5	95.2
crisp	0.0	0.0	9.5	66.7
non-compensatory	0.0	28.6	71.4	90.5
yager	0.0	28.6	61.9	95.2

Table 14.4. Average number of time steps for a squirrel to find a home.

	Conspecific Holes 0%	20%	40%	60%
compensatory		3.8	4.7	3.2
crisp			3.0	3.9
non-compensatory		2.8	3.3	2.6
yager		3.8	4.1	3.5

Table 14.5. Mean straight-line dispersal distance (m) for squirrel agents who found a home, and for all squirrel agents.

	Conspecific Holes 0%		20%		40%		60%	
	home	all	home	all	home	all	home	all
compensatory		1481.5	616.6	1285.2	881.6	918.0	482.8	532.6
crisp		481.5		570.3	394.7	582.6	427.6	461.1
non-compensatory		920.2	425.7	695.1	480.9	513.2	497.4	549.9
yager		704.3	774.6	852.2	781.4	1048.2	590.1	652.4

The ratio between the meandering distance and straight-line distance is used to provide an index of sinuosity. A greater value will indicate long meandering path relative to straight-line distance separating the starting location and the ending location. When considering all agents, regardless of whether a home is found, those using the crisp decision models, on average, take noticeably more sinuous paths. No obvious pattern emerged to distinguish sinuosity among different types of fuzzy squirrel agents. Considering agents that found a home location the mean sinuosity was closer to the straight-line distance (Table 14.9).

On average each time a squirrel agent using crisp decision models moved from one location to another, it was a shorter distance than taken by the agents using the

Table 14.6. Maximum straight-line dispersal distance (m) for squirrel agents that found a home, and for all squirrel agents.

	Conspecific Holes							
	0%		20%		40%		60%	
	home	all	home	all	home	all	home	all
compensatory		2176.8	1210.8	2227.3	1916.7	1919.3	1510.8	1529.7
crisp		1207.8		908.0	536.7	1075.4	937.7	937.7
non-compensatory		1731.2	649.0	1755.7	1339.3	1339.3	1571.8	1571.8
yager		1355.3	1441.6	1891.0	1916.7	2083.9	1223.8	1898.6

Table 14.7. Mean meandering-path dispersal distance (m) for squirrel agents that found a home, and for all squirrel agents.

	Conspecific Holes							
	0%		20%		40%		60%	
	home	all	home	all	home	all	home	all
compensatory		2404.6	821.5	1886.9	1161.4	1284.5	700.7	769.1
crisp		1832.9		1723.8	488.4	1536.4	738.7	1105.4
non-compensatory		2463.3	656.5	1917.2	815.1	1247.7	617.8	797.1
yager		2594.6	953.0	1934.0	1013.6	1519.1	798.5	866.9

Table 14.8. Maximum meandering-path dispersal distance (m) for squirrels who found a home, and for all squirrel agents.

	Conspecific Holes							
	0%		20%		40%		60%	
	home	all	home	all	home	all	home	all
compensatory		2703.5	1481.3	2640.1	2796.5	2796.5	1680.2	2153.6
crisp		2228.2		2057.1	379.7	2042.9	1738.2	2075
non-compensatory		2796.4	1006.1	2851.7	2419.4	2648.9	2571.2	2626.8
yager		3141.1	1840.1	2851.3	2103.5	2696.5	2205.5	2233.4

fuzzy decision models (Table 14.10). The mean step size for agents using the fuzzy decision models tended to be in the middle to slightly less than the middle of the perceptual range.

14.5. Concluding Discussion

This chapter illustrated how fuzzy mobile agents can be incorporated into an object-oriented individual-based modeling framework, namely ECO-COSM. Fuzzy decision models for movement and residence were developed using noncom-

Table 14.9. Mean sinuosity of dispersal path for squirrel agents who found a home, and for all squirrel agents. A value of 1.0 shows a perfectly straight path; higher numbers show more meandering or doubling back

	Conspecific Holes							
	0%		20%		40%		60%	
	home	all	home	all	home	all	home	all
compensatory		2.1	1.3	1.5	1.3	1.4	2.0	2.0
crisp		6.0		4.1	1.2	3.6	1.7	2.5
non-compensatory		3.6	1.6	3.7	2.0	2.7	1.1	1.3
yager		4.9	1.2	5.4	1.3	1.5	1.6	1.6

Table 14.10. Average step size (m) for squirrel agents that found a home, and for all squirrel agents.

	Conspecific Holes							
	0%		20%		40%		60%	
	home	all	home	all	home	all	home	all
compensatory		240.5	208.4	224.8	247.0	246.8	222.9	222.5
crisp		183.3		176.6	159.2	164.1	196.3	192.2
non-compensatory		246.3	221.5	236.3	252.9	247.2	230.6	232.4
yager		259.5	237.5	234.0	253.3	246.0	226.0	225.9

pensatory, compensatory, and crisp aggregation operators. Simulations are used to explore model behavior relative to fuzzy compensatory and noncompensatory aggregation operators. Although the simulations were illustrative of the approach, it is instructive to note that, in general, the fuzzy agents were consistently more successful in finding home locations than were the crisp agents. In addition, the fuzzy agents behaved in a reasonably plausible fashion, more so than did agents using crisp decision models. Furthermore, the simulations showed that fuzzy agents were consistently more successful in finding homes than were the crisp agents. Their behavior in this regard was within the general bounds of plausibility suggested by the ecological literature.

Further development within the ECO-COSM framework seems warranted by this exercise. Simulations that are more extensive will be designed so that we can assess the:

1. sensitivity of the model(s) to changes in critical parameters such as perceptual range;
2. utility of fuzzy sets in addressing the computational issues associated with IBMs. For example, can fuzzy sets effectively address the known problems of complexity in these types of models as demographic, foraging, and other modules are added;
3. sensitivity of the fuzzy agents to landscape configuration as opposed to conspecifics and other factors

A more highly developed residence decision model can be developed, in conjunction with an exploratory movement model, that incorporates knowledge of how the species may react to the arrangement of landscape elements. Other capabilities to develop would be exploratory behavior, including some form of spatial memory, foraging behavior, demographics so multigenerational simulations can be done in a meaningful manner, and predation.

One of the reasons for using individual-based modeling approaches is to capture the variations in individual behavior (Grimm 1999). Often this is done using random draws (Tischendorf and Fahrig 2000). This work demonstrates an alternative. It is possible to have a population of squirrel agents where each agent uses the same general set of decision models, however the specific fuzzy aggregation mechanisms can vary to produce individualistic variations in behavior.

Results of this exercise demonstrate that this approach can be used for developing spatially explicit individual-based models to address spatially explicit ecological problems that are dependent on spatially explicit information derived from a GIS. The ECO-COSM framework was able to work in a loosely coupled architecture with two GIS packages. Since ECO-COSM is, by design, extensible, the framework can be further developed. For example, Probe objects can be used to filter the information an agent receives. Thus, it could be used to model the inherent fuzziness in the land cover classifications typically derived from remote sensing technology and used in spatially explicit models. Thus, ECO-COSM may serve well as a flexible computational laboratory within which experiments are conducted to investigate a host of issues related to GIS databases, spatially explicit ecological models, and fuzzy information processing.

Acknowledgements

Partial support in the form of operating research grants to each of the authors from the Natural Sciences and Engineering Research Council (NSERC) of Canada is gratefully acknowledged. We are especially grateful to Professor Haluk Cetin and the Mid-America Remote Sensing Center for graciously providing the digital elevation and Kentucky GAP land cover datasets. Comments on an earlier manuscript by Linda Robinson improved the quality of this chapter.

References

Allen AW (1987) *Habitat Suitability Index Models: Gray Squirrel, revised.* U.S. Fish Wildlife Service Biological Report 82 10.135 U.S. Dept. of the Interior Washington, D.C. (First printed as: FWS/OBS-82/10.19, July 1982)

Barrett GW and Peles JD (1999) *Landscape Ecology of Small Mammals*. Springer-Verlag , New York

Baum KA and Grant WA (2001) Hummingbird foraging behavior in different patch types: simulation of alternative strategies. *Ecological Modelling*

137:201-209
Bellman RE and Zadeh LA (1970) Decision-making in a fuzzy environment. *Management Science* 17:141-164
Bian L (2000) Object-oriented representation for modelling mobile objects in an aquatic environment. *International Journal of Geographical Information Science* 14:603-623
Bowman J, Jaeger JAG and Fahrig L (2002) Dispersal distance of mammals is proportional to home range size. *Ecology* 83:2049-2055
Burgman MA, Breininger BW, Duncan BW and Ferson S (2001) Setting reliability bound on habitat suitability indices. *Ecological Applications* 11:70-78
Cao G (1995) The definition of the niche by fuzzy set theory. *Ecological Modelling* 77:65-71
DeGenst A, Canters F and Gulink H (2001) Uncertainty modeling in buffer operations applied to connectivity analysis. *Transactions in GIS* 5:305-326
Dettmers R and Bart J (1999) A GIS modeling method applied to predicting forest songbird habitat. *Ecological Applications* 9:152-163
Dusenbery DB (1989) Ranging strategies. *Journal of Theoretical Biology* 136:309-316
Fahse L, Wissel C and Grimm V (1998) Reconciling Classical and Individual-Based Approaches in Theoretical Population Ecology: A Protocol for Extracting Population Parameters from Individual-Based Models . *The American Naturalist* 162:838-852
Gimblett HR (2002) *Integrating geographic information systems and agent-based modeling techniques for simulating social and ecological processes*. Oxford University Press , New York
Graniero PA (2001) *The Effect of Spatiotemporal Sampling Strategies and Data Acquisition Accuracy on the Characterization of Dynamic Ecological Systems and Their Behaviours*. Ph.D. dissertation, University of Toronto
Graniero PA and Robinson VB (2003) A real-time adaptive sampling method for field mapping in patchy, heterogeneous environments. *Transactions in GIS* 7:31-54
Grimm V (1999) Ten years of individual-based modelling in ecology: what have we learned and what could we learn in the future? *Ecological Modelling* 115:129-148
Hale ML, Lurz PWW, Shirley MDF, Rushton S, Fuller RM and Wolff K (2001) Impact of landscape management on the genetic structure of red squirrel populations. *Science* 293:2246-2248
Hobbs BF, Ludsin SA, Knight RL, Ryan PA, Biberhofer J and Ciborowski JJH (2002) Fuzzy cognitive mapping as a tool to define management objectives for complex ecosystems. *Ecological Applications* 12:1548-1565
Holt RD, Pacala SW, Smith TW and Liu J (1995) Linking contemporary vegetation models with spatially explicit animal population models. *Ecological Applications* 5:20-27
Howard WE (1960) Innate and environmental dispersal of individiual vertebrates. *American Midland Naturalist* 63:152-161
Klir GJ and Yuan B (1995) *Fuzzy sets and fuzzy logic: theory and applications*.

Prentice-Hall , Upper Saddle River NJ

Klosterman RE (1998) Computers Applications in Planning. *Environment and Planning B: Planning and Design Anniversary Issue* 32-36

Koenig WDD, Van Vuren D and Hooge PN (1996) Detectability, philopatry and the distribution of dispersal distances in vertebrates. *Trends in Ecology and Evolution* 11:514-517

Lamberson RH, Noon BR, Voss C and McKelvey K (1994) Reserve design for territorial species: the effects of patch size and spacing on the viability of the Northern Spotted Owl. *Conservation Biology* 8:185-195

Lim K, Deadman P, Moran E, Brondizo E and McCracken S (2002) Agent-based simulations of household decision making and land use change near Altamira, Brazil. In: Gimblett HR(ed) *Integrating geographic information systems and agent-based modeling techniques for simulating social and ecological processes* , Oxford University Press, Oxford, pp 277-310

Lima SL and Zollner PA (1996) Towards a behavioral ecology of ecological landscapes. *Trends in Ecology and Evolution* 11:131-135

Lomnicki A (1999) Individual-based models and individual-based approach to population ecology. *Ecological Modelling* 115:191-198

Mech SG and Zollner PA (2002) Using body size to predict perceptual range. *Oikos* 98:47-52

Moraczewski IR (1993) Fuzzy logic for phytosociology 1. syntaxa as vague concepts. *Vegetatio* 106:1-11

Morris A (2003) A framework for modeling uncertainty in spatial databases. *Transactions in GIS* 7:83-103

Nixon CM, Havera SP and Greenberg RE (1978) *Distribution and abundance of the gray squirrel in Illinois*. 105 Illinois Natural History Survey Champaign, Illinois

Petry FE, Cobb MA, Ali D, Angryk R, Paprzycki M, Rahimi S, Wen L and Yang H (2002) Fuzzy spatial relationships and mobile agent technology in geospatial information systems. In: Matsakis P and Sztandera LM(ed) *Applying Soft Computing in Defining Spatial Relations*, Physica-Verlag, Heidelberg, pp 121-155

Railsback SF, Lamberson RH, Harvey BC and Duffy WE (1999) Movement rules for individual-based models of stream fish. *Ecological Modelling* 123:73-89

Rickel BW, Anderson B and Pope R (1998) Using fuzzy systems, object-oriented programming, and GIS to evaluate wildlife habitat. *AI Applications* 12:31-40

Robinson VB (2002) Using fuzzy spatial relations to control movement behavior of mobile objects in spatially explicit ecological models. In: Matsakis P and Sztandera LM (eds) *Applying Soft Computing in Defining Spatial Relations*, Physica-Verlag, Heidelberg, pp 158-178

Robinson VB (2003) A perspective on the fundamentals of fuzzy sets and their use in geographic information systems. *Transactions in GIS* 7:3-30

Robinson, VB and Cetin, H (2001) An exploratory geographical analysis of forest land-cover transitions in the LBL national recreation area 1973-1995. In: *Proceedings of the American Society of Photogrammetry and Remote Sens-*

ing Conference, St. Lois, MO, pp CD-ROM

Ruckelshaus M, Hartway C and Kareiva P (1997) Assessing the data requirements of spatially explicit dispersal models. *Conservation Biology* 11:1298-1306

Salski A, Franzle O and P. Kandzia(editors) (1996) Fuzzy logic in ecological modeling (Special Issue). *Ecological Modelling* 85:1-98

Schumaker NH (1996) Using landscape indices to predict habitat connectivity. *Ecology* 77:1210-1225

Sheperd BF and Swihart RK (1995) Spatial dynamics of foz squirrels (Sciurus niger) in fragmented landscapes. *Canadian Journal of Zoology* 73:2098-2105

South A (1999) Extrapolating from individual movement behavior to population spacing patterns in a ranging mammal. *Ecological Modelling* 117:343-360

Sutherland GDA, Harestad AS, Price K and Lertzman KP (2000) Scaling of natal dispersal distances in terrestrial birds and mammals. *Conservation Ecology* 4:[online]URL: http://www.consecol.org/vol4/iss1/art16

Tischendorf L and Fahrig L (2000) How should we measure landscape connectivity? *Landscape Ecology* 15:633-641

Wauters LA, Lurz PW and Gurrell J (2000) Interspecific effects of grey squirrels (*Sciurus carolinensis*) on the space use and population demography of red squirrels (*Sciurus vulgaris*). *Ecological Research* 15:271-284

Westervelt JD (2002) Geographic information systems and agent-based modeling. In: Gimblett HR(ed) *Integrating geographic information systems and agent-based modeling techniques for simulating social and ecological processes*, Oxford University Press, Oxford, pp 83-103

Westervelt JD and Hopkins LD (1999) Modeling mobile individuals in dynamic landscapes. *International Journal of Geographical Information Science* 13:191-208

Wiklund CG (1996) Determinants of dispersal in breeding merlins (*Falco columbarius*). *Ecology* 77:1920-1927

Wolff JO (1999) Behavioral Model Systems. In: Barrett GW and Peles JD(ed) *Landscape Ecology of Small Mammals*, Springer, New York, pp 11-26

Zollner PA (2000) Comparing the landscape level perceptual abilities of forest sciurids in fragmented agricultural landscapes. *Ecology* 80:1019-1030

Zollner PA and Lima SL (1999) Search strategies for landscape-level interpatch movements. *Ecology* 80:1019-1030

Index

action space, 255, 264
Allen relation(s) 2,3, 17, 23-29, 32, 34, 38

base attributes, 121, 122, 125, 127, 140
bitmap(s), 41, 42, 43, 59-67
boundary, 73, 75-78, 80, 84, 85, 91, 92, 94, 97, 98, 101, 106, 210, 259, 285- 290, 293, 321
buffering area, 202, 204
buildings, 164, 174, 187, 199- 202, 289, 291

cluster
- centers, 107, 111
- validity index, 105, 110-113

cognitive complexity, 284, 295
conceptual neighborhood, 3, 5, 6
constraint set, 312, 314, 317, 324
contour lines, 42, 187, 197- 199, 244, 245
core, 26, 77, 80, 85, 91, 101, 112, 128, 131, 135, 152, 159, 171, 174, 268, 285- 290, 294, 317

data
- mining, 105, 106, 107, 110, 113
- model, 71, 77, 143, 284, 298

decision
- maker(s), 275, 277, 284- 289, 294, 295
- making, 162, 165, 170, 175, 177, 181, 182, 253, 254, 256, 263 272, 276- 278, 281, 283, 284, 286, 299, 300, 308-310, 313, 320
- plan net (DPN), 253, 256
- table (DT), 253-257, 260-264, 270- 272

Delaunay triangulation, 43, 44, 46, 50, 51, 53, 85
derived attributes, 121, 122, 140
digital elevation model (DEM), 43, 150, 210, 214, 215, 218, 224, 225, 228, 233, 234, 305
directed interval algebra, 7, 8
directional relationships, 15, 16

ecological model, 299-304, 307, 308, 310, 318, 331
ETIN, 43, 45, 46, 47, 49-54, 56, 57, 59, 64, 66, 67
exterior, 77, 80, 85, 91, 92, 101

Fatality Analysis Reporting, 74, 86
F-histogram, 15, 16, 17, 19, 20, 23, 24, 29, 31, 33, 38
field-based model(s), 43, 67
FOOSBALL, 275, 286-295
fuzzy
- clustering, 105, 106, 113
- condition, 257, 260, 263, 269, 272
- coordinate system, 148, 149
- decision table (FDT), 253, 254, 257-272
- function estimator, 152, 154
- geographical entity, 122, 131, 137, 140
- interpolation 143, 144, 148, 150, 155, 156
- K-means, 105-113
- line, 148, 149
- locality, 91, 96, 97, 101, 102
- morphometric class, 213
- morphometric landform, 216, 217
- neural networks, 152, 155
- polygons, 148
- proximity, 90, 93, 94, 95, 102

region, 9, 72, 76, 80, 84, 101, 261, 314
representation, 150, 233, 234, 241, 243-248, 318
spatial reliability, 185
splines, 146, 150
state(s), 259, 260, 261, 263
Voronoi diagram, 151, 154

geographic distance, 210, 240, 241
global reliability, 187, 205, 206, 207, 208
goal set, 315, 317, 319, 323, 324

headwater, 233, 234, 246, 249

individual-based models, 299, 301, 304, 331
iso-reliable zone, 186, 187, 189, 192, 193, 194, 197, 199, 202, 205, 206
interval logic, 2

knob, 233, 234, 241, 245-249

layer(s), 33, 42, 86, 91, 101, 143, 153, 155, 168, 171, 172, 175-178, 187, 205, 206, 208, 276, 279, 283, 305, 307, 308, 321, 326
likelihood view, 121, 123, 141
linguistic
label(s), 185, 187, 189, 191, 192, 193, 196, 197, 200, 205
scene description, 17, 22
variable(s), 171, 173, 177, 180
location indeterminacy, 72, 76, 97, 99, 102
longitudinal section(s) , 18, 19, 20, 21, 24, 25, 27, 29-33

maximum influence distance, 240, 241, 244, 247, 248
meeting point, 160, 164, 165, 166, 170-174, 176, 177
mereological, 75
minimum bounding rectangle (MBR), 15, 16,17, 38,
morphometric class, 209-213, 218, 222, 227
morphometry 209, 210, 211, 212, 222, 225, 227
movement decision, 303, 311, 313, 314, 321, 323
multi-dimensional interpolation, 143, 145
multiple criteria decision making, 277, 278, 285, 286, 295
multi-scale analysis, 211, 214, 228
multispectral remote sensing, 124, 125, 126

network streets, 187, 202, 203, 204

object oriented 275, 276, 278, 282, 286, 292, 295, 299, 300, 306, 310, 329
ocean climate indices (OCI), 112
ontology, 71, 72, 75, 102
overall similarity measure (OSM), 265-268

radiance, 121, 125, 126
random set view, 121, 123, 124, 137, 141
RCC theory, 1,2,3,7,8
RCC8 relation, 3, 5-12
reliability, 185, 186, 187, 189, 190, 200, 202, 205- 208
residence decision, 310, 311, 316, 318-321, 324, 325, 330
resolution-level, 80, 81, 88, 92, 95, 102

ridge(s), 210- 212, 221, 222, 226, 227, 228, 230, 233, 234, 241-246, 248
ridgeline(s), 234, 235, 242, 243
rough sets, 285, 286

similarity
score, 86, 91, 95, 96,
measure, 101, 107, 261, 266, 267
value, 233, 235, 236, 239, 240, 241
view, 121, 123, 129, 141
simulation, 154, 299-303, 305-311, 315, 319-322, 325, 326, 330, 331
social space, 161, 176
spatial
databases, 275, 276, 278, 279, 282, 283, 284, 286, 295, 305
interpolation, 143, 144, 146, 148, 151, 152, 154
planning, 160, 161, 178, 181
relations, 1, 3, 15, 16, 17, 22, 24, 33, 38, 160, 162, 175, 284, 299, 300
explicit, 299-307, 310, 311, 319, 320, 331
spatio-temporal interpolation, 143, 144, 146, 150-155
spot elevations, 187, 193, 194, 195
surface distance, 240

terrain attribute, 235, 236, 238, 239, 240, 244, 249
tessellation, 43, 121, 140, 146
TIN, 43, 47, 53, 58, 60, 65, 68, 91, 92, 146, 242
t-norm, 45, 47, 53, 62, 264, 268, 269
topological relationship(s), 2, 15, 17, 33, 37, 38
triangular fuzzy number, 54, 134, 186
Type-2 fuzzy sets, 56, 62, 65, 66

weather data, 105, 106, 109, 110, 112, 113