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Technical Report
SCI Publication P164

Design of Steel Portal Frames for Europe

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TECHNICAL REPORT
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Design of Steel Portal Frames for Europe

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FOREWORD

The European Prestandard *ENV 1993-1-1: Eurocode 3: Design of steel structures: Part 1.1 General rules and rules for buildings* was published in 1992. The Steel Construction Institute is in the process of preparing design guides to encourage the use of the Eurocodes during their Prestandard or ENV periods. This publication will form part of a set of design guides that will serve that purpose. It deals with steel portal framed buildings and shows how to choose an efficient frame and prepare calculations to Eurocode 3.

The design principles presented in this publication are applicable for construction throughout the European Economic Area (EU and EFTA), provided that the relevant National Application Document of the country of use is complied with. The design guidance includes additions to the Eurocode text derived from SCI P147 *Plastic design of single storey pitched roof portal frames to Eurocode 3*, which was written as part of the Eureka CIMsteel project.

Eurocode 3 may be more convenient than national standards in certain countries for the design of steel portal frames. However, it is important that designers check that the key requirements of the Eurocode on plastic design, Sections 5.2.1.4, 5.2.5 and 5.2.6.3, are accepted in the National Application Document of the country where the structure is to be erected. In some cases, the relevant national design standard may contain rules that are compatible with this document instead. It is unlikely that Eurocode 3 will be used much for the design of pitched roof portal frames to be constructed in the UK, because BS 5950:1990 *Structural use of steelwork in building: Part 1: Code of practice for design in simple and continuous construction: hot rolled sections* gives greater economy in most cases.

This design guide was prepared by Mr C M King of The Steel Construction Institute with assistance from Mr M S Gough, Mr F J Naji and Mr M Adam Asamoah of The Steel Construction Institute.

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SUMMARY

Single - storey pitched roof steel portal frames are a very economical form of structure for most single-storey buildings for industrial, distribution, retail and leisure purposes.

This publication covers the structural arrangement and calculations for single-storey pitched-roof steel portal frames fabricated from hot rolled I sections. The calculations conform to Eurocode 3, ENV 1993-1-1, with supplementary information where necessary.

Design methods are included for both elastic and plastic frame design. The design approach of Eurocode 3 is described and simplified design equations are presented together with design procedures and worked examples for both plastic and elastic design.

Dimensionnement de portiques en acier en Europe

Résumé

Les portiques simples en acier constituent une forme structurale très économique pour les bâtiments industriels à un seul niveau dans la distribution ou la vente au détail.

Cette publication couvre les conceptions structurales ainsi que les calculs de vérification des portiques simples en acier réalisés à l'aide de profils I laminés à chaud. Les calculs sont établis en conformité avec l'Eurocode 3, ENV 1993-1-1. Des informations complémentaires sont données lorsque cela s'avère nécessaire.

Les méthodes de dimensionnement comportent une analyse élastique ainsi qu'une analyse plastique. L'approche adoptée par l'Eurocode 3 est décrite et des équations de dimensionnement simplifiées sont proposées ainsi que des exemples tant pour le dimensionnement élastique que pour le dimensionnement plastique.

Berechnung von Stahlrahmen für Europa

Zusammenfassung

Eingeschossige Stahlrahmen mit geneigten Dächern sind eine sehr wirtschaftliche Tragwerksform für die meisten eingeschossigen Gebäude im Bereich Industrie, Handel und Freizeit.

Diese Publikation behandelt die Anordnung und Berechnung von eingeschossigen Stahlrahmen mit geneigten Dächern, die aus warmgewalzten I-Profilen hergestellt werden. Die Berechnungen stimmen mit Eurocode 3, ENV 1993-1-1, überein, mit zusätzlicher Information, wo nötig.

Berechnungsverfahren sind sowohl für elastische als auch für plastische Berechnung eingeschlossen. Die Berechnungsweise des Eurocode 3 wird beschrieben und vereinfachte Formeln werden vorgestellt, zusammen mit Berechnungsverfahren und Beispielen für die plastische und elastische Berechnung.

Proyecto de pórticos de acero para Europa

Résumén

Los pórticos simples de acero con dintel a dos aguas son una forma muy económica de estructura para edificios de una planta con fines industriales, de almacenamiento o diversión.

Esta publicación cubre los esquemas estructurales y los cálculos para ese tipo de estructuras fabricadas a partir de perfiles doble te laminadas en caliente.

Los cálculos se ajustan al Eurocódigo 3, ENV 1993-1-1 con información suplementaria cuando ello se estime necesario.

Se incluyen métodos de cálculo tanto para régimen plástico como elástico. Se describe el enfoque del Eurocódigo 3 y se presentan ecuaciones de cálculo simplificadas así como métodos de proyecto y ejemplos desarrollados tanto para regímenes plástico como elástico.

Progettazione per l'Europa di Portali Intelaiati in Acciaio

Sommario

I sistemi intelaiati monopiano multipli in acciaio rappresentano una soluzione tipologica estremamente economica per una gran parte di edifici multicampata ad uso industriale, commerciale, per il tempo libero.

Questa pubblicazione tratta l'organizzazione strutturale e gli aspetti del calcolo per portali multipli in acciaio realizzati con profili ad I laminati a caldo. I calcoli sono conformi all'Eurocodice 3, ENV 1993-1-1 e sono riportate, nella pubblicazione, informazioni complementari sugli aspetti maggiormente significativi.

Viene fatto riferimento ai metodi progettuali sia elastico sia plastico. E' descritto l'approccio progettuale dell'Eurocodice 3 e le equazioni semplificate di progettazione sono presentate unitamente alle procedure progettuali e ad esempi di calcolo riferiti al metodo sia elastico sia plastico.

1 INTRODUCTION

1.1 Background

Single-storey portal frame design uses structural layouts and structural forms that are frequently different from those used in other types of single-storey or multi-storey structures. As a result, many of the calculations used for portal frame design differ from those commonly used for other types of building.

To date, there has been little guidance available on efficient structural layouts for single-storey portal frames. Equally, most current design standards, including Eurocode 3[1], provide very little guidance on checking the structural forms most appropriate to economical single-storey portal frame design.

This publication addresses the requirements of Eurocode 3, gives guidance on efficient structural layouts for single storey portal frames and shows how to use Eurocode 3 for the design calculations. Where Eurocode 3 does not consider criteria that are essential for economical design, this document provides the necessary design methods. Additional guidance on plastic design of portal frames to Eurocode 3 is given in *Plastic design of single-storey pitched-roof portal frames to Eurocode 3*[2].

1.2 Why choose steel portal frames?

Steel portal frames are very efficient and economical when used for single-storey buildings, provided that the design details are cost-effective and the design and analysis assumptions are well chosen. In countries where this technology is highly developed, steel portal frames are the dominant form of structure for single-storey industrial and commercial buildings. In the UK, for example, more than 90% of such buildings have a steel structure, and about half of these are portal frames. A typical single bay portal frame is shown in Figure 1.1.

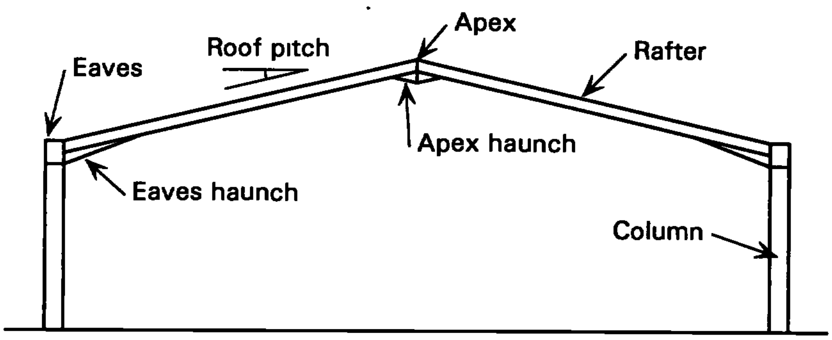


Figure 1.1 *A typical single-bay portal frame*

The advantages of portal frames are:

- Low cost
- Rapid fabrication
- Simple to clad
- Wide range of possible exteriors

- Simple erection
- Ease of maintenance
- Easily adaptable to future needs (additional bays, additional plant or services)
- Large clear spans for a small increase in cost.

The low marginal cost of large clear spans is attractive for three reasons:

- Flexibility of internal layout
- Adaptability to changing use
- Greater range of possible purchasers in the case of sale of the building.

1.3 Different construction traditions

The traditions of different countries may affect the readiness to accept modern steel intensive structures. For example, in Germany the traditional material for single-storey industrial and commercial structures is concrete, whereas steel is often regarded as more appropriate for more architecturally exotic and expensive structures. This results partly from the limited structural calculation facilities in fabricators, the dominance of elastic rather than plastic design, and the common use of connections whose resistances are related to member resistance rather than the actual member forces at the connection. The dominance of concrete is definitely influenced by the acceptance of small clear spans and relatively plain and unimaginative exteriors. It is also influenced by the financial and business position of the concrete industry. It is possible that the high labour costs in Germany also have an adverse effect on the competitive position of steel.

In countries where steel is not currently very competitive, such as Germany and Denmark, the potential for sales of well designed and detailed steel is very high, but considerable effort may be needed to persuade clients and specifiers that steel portal framed buildings are a good choice.

1.4 Scope of this publication

This publication provides guidance to both the specifier and the designer on the design of single-storey steel portal framed buildings.

It covers the structural layouts, detailing and design calculations for steel portal frames fabricated from hot rolled I and H sections with light-gauge steel cladding. This is generally the most economical form of construction for steel portal framed buildings. Many of the general principles are applicable to other forms of construction, e.g. hollow sections, but these are not considered in detail. Both elastic and plastic design are considered, and worked examples of both are presented.

Sections 2 to 8 address the conceptual decisions that must be made to specify a portal framed building that satisfies the client's requirements.

Section 9 gives the basis of design according to the Prestandard ENV 1993-1-1.

Sections 10 to 16 guide the designer through the principles of design to ENV 1993-1-1, to the extent that these are relevant to single-storey portal frames.

Section 17 gives design procedures.

The Appendices present the detailed requirements of Eurocode 3, ENV 1993-1-1[1], in a form suited to the design of single-storey portal frames fabricated from hot rolled sections. They also present information not given in Eurocode 3 but necessary for economical portal frame design.

Important differences between UK practice, including the use of BS 5950-1[3], and ENV 1993-1-1 are given in Appendix K.

Worked examples are given at the end of the publication, showing all the design steps for plastic or elastic design of portal frames.

2 STRUCTURAL FRAME

2.1 General

This Section gives general guidance on the most economical geometry of portal frames constructed from hot rolled I sections, for single- and multi-bay configurations. It gives information about the cost implications of various design possibilities so that specifiers can understand the effects of their decisions.

The optimum geometry of steel portals may be different for each country because of different labour and material costs and different regulations for manufacture and erection. Some typical aspects where there are differences are:

- Health and safety regulations for manufacture and erection
- Fire protection regulations
- Structural design
- Foundation design
- Forms of cladding
- Cladding fixing
- Restraint to the structure from the cladding.

For example, where cladding cannot be assumed to act as the rigid diaphragm that it actually is, additional cross bracing will be required. As a second example, the fire regulations and requirements to use fire protection may force the use of intumescent paint or rigid casings. These may affect the competitive position of the design. In addition, ENV 1993-1-1 is modified by the different National Application Documents (NADs) of each country and these modifications may also affect the cost of the structure.

2.2 Minimum cost

2.2.1 General

The most economical form of portal frame, using hot rolled sections, is usually achieved by the plastic design of a frame with haunches. In a typical portal framed building, purlins and rails provide intermediate restraint to the members and support the roofing and wall sheeting. The use of the haunches allows an economical bolted connection between the rafter and the column, and allows the plastic design bending moment diagram to remain close to the elastic bending moment diagram. As a result, the onset of plasticity normally occurs at loads well above those at the serviceability limit state (SLS) and the plastic rotations are small, even at the ultimate limit state (ULS). Where haunches are not used, the onset of plasticity is likely to occur close to SLS, the plastic rotations at ULS are likely to be severe and the rafter-column connection will be much more expensive. Indeed, the connection may need to be welded, with the rafter splice some distance from the column, creating a column piece that is more awkward to handle and transport than a straight piece.

The economy of the structure will be affected by the requirements for clear spans and the spacing of the frames. There may be little freedom of choice for the positions of the columns, if special plant and equipment have to be accommodated. However, it is worth making a careful study of the possibility of using valley beams to eliminate columns from alternate planes of portal frames, in order to generate a sufficient clear area without using large spans throughout (see Section 2.4). It is also wise to consider whether larger spans or greater height might be a good investment, because they could give greater freedom for future development, reuse or resale.

Extensive studies of the structural cost of single-storey buildings have been made by Horridge and Morris[4] and the results have been reported by Taggart[5], comparing the relative cost of different types of structural forms, e.g. portals, truss and post. The cost of manufacture and erection varies among countries and among fabricators and erectors. However, in countries where labour costs are relatively high, the costs of portals relative to other forms of structure are expected to be generally as in the UK, though they should prove more competitive to fabricate than trusses.

It is important to recognise that:

- The cheapest solution in one country may not be the cheapest in another country.
- The cheapest frame may not produce the cheapest complete building.
- The lowest price solution may not be the least expensive over the life of the building because of:
  - maintenance costs including recladding
  - adaptability for change of use
  - resale price to other users.

2.2.2 The effect of relative costs of labour and materials

The most economical structure will depend on the relative costs of labour and materials in the country of manufacture and the country where the structure is erected.

The lightest steel frames are often designed using plastic design methods and use purlins to stabilise the rafters and columns. The spacing of the purlins will generally be significantly less than that required by the sheeting to carry the imposed loads. Several purlins will carry diagonal bracing members from the purlin to the inner flange of the portal so that the inner flange is stabilised. This is shown in Figure 2.1. Such light frames use a minimum of materials but more labour in manufacture and erection.

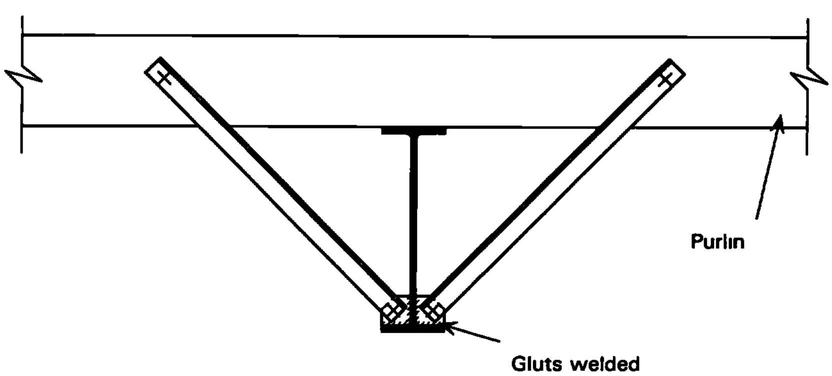


Figure 2.1 *Bracing to inner flanges*

The heaviest steel frames will be required if there is little or no bracing to stabilise the inner flanges. This will normally lead to a requirement for elastic analysis of the structure and to large column and rafter sections to ensure stability of the elements. However, such a design would allow the use of deep deck roofing spanning directly between portals instead of purlins and sheeting. This would reduce the fabrication work and may reduce the site-work. It is possible that this heavy frame design would produce the most economical finished building, if labour costs are high relative to material costs, because such heavy frames use less labour for manufacture than the light frames described above.

Between the lightest and heaviest frames, there lies a range of frames with increasingly heavy sections and decreasing amounts of bracing and secondary structure.

2.3 Frame spacing

Generally, the most economical spacing of frames increases as the span increases. However, the spacing may also be dictated by the length of the structure and the availability of purlins or deep decking adequate for greater spacings.

Typical optimum spacings are

Span of frame (m)	Optimum spacing of frames (m)
25	60
35	75
45	90

These optima are for 1:10 roof slopes with 0,6 kN/m^2 (unfactored) variable snow load and 0,1 kN/m (unfactored) services load using purlins to support the roofing and sheeting rails to support the wall cladding. Typical UK costs are assumed. Further details are available in *Comparative costs of single-storey steel framed structures*[4] and *Single storey buildings*[5].

Where labour costs are high, it may prove more economical to have fewer frames at greater centres. Cold rolled purlins spanning up to 15 m are now available (see Section 3.5.2). However, long span purlins will commonly require secondary stability components, which complicates the erection process. This approach might also lead to heavy wind bracing and longitudinal members.

2.4 Valley beams and "hit" and "miss" frames

In multi-span portal framed buildings, it is common practice to use valley beams to eliminate some internal columns. Most commonly, alternate columns are omitted and the valley of the frame is supported on a valley-beam spanning between the columns of adjacent frames, as shown in Figure 2.2. This arrangement is often referred to as "hit" and "miss" frames, the frames with columns being the "hit" frames. Sometimes more than one column is omitted, though such schemes require very large valley beams and reduce the stiffness and the stability requirements of the structure, even where the remaining complete frames are used to stabilise the frames without columns.

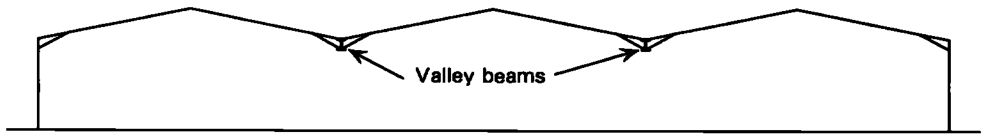


Figure 2.2 *Valley beams*

Valley beams may be simply supported or continuous through the supporting columns. The choice will normally depend on the relative cost of a heavier beam for simply supported construction and the more expensive connection for continuous construction. Continuous construction may cause reductions of clear height near columns, as haunches will probably be required to allow economical bolted beam/column connections. This is not usually a problem.

Careful design is required to ensure that it is possible to fit the valley beam and the rafters onto the column, especially if the column needs stiffeners in the same area. The first choice of column is often too small for the valley beam to fit between the column flanges.

Valley beams often form one or more portals with the columns to provide overall structural stability at right angles to the frames. This avoids the use of cross bracing on the internal column lines, which is often unacceptable for the intended use of the building.

2.5 Roof pitch

The roof pitch adopted depends on the type of roofing, as well as the requirements of the main structural frame. The pitch will affect the economy of the frame and the deflections. For many buildings, the lowest possible roof slope is required, to minimise the area of cladding and the enclosed volume of the building. The type of cladding dictates the minimum practical slope to ensure watertightness.

Where the external face of the roof cladding is steel sheeting, the minimum slope is normally 1:10 to ensure that the laps between lengths of sheeting are easy to waterproof and there is no ponding of water on the roof. This is the most common form of roof cladding for use with purlins. Lower slopes are possible, but only with special verifications or measures to avoid ponding and with sheets with specially engineered joints, or single length sheets to avoid joints. Careful attention must also be paid to the fixings to avoid creating leaks.

Where the external face is a waterproof membrane, the roof slope can be less than 1:10. This is the most common form of roofing where deep profile decking is used to span directly between the purlins.

Systems using a steel external skin with purlins spanning between the portal frames are both visually attractive and economical, which is why they dominate the market in the UK, where steel portal frames are most common.

2.6 Clear internal height

The most economical portal frames made with hot rolled I sections will have haunches at the columns and valley beams. These reduce the rafter size and allow a simple and economical bolted joint at the column/rafter interface, as shown in Figure 2.3.

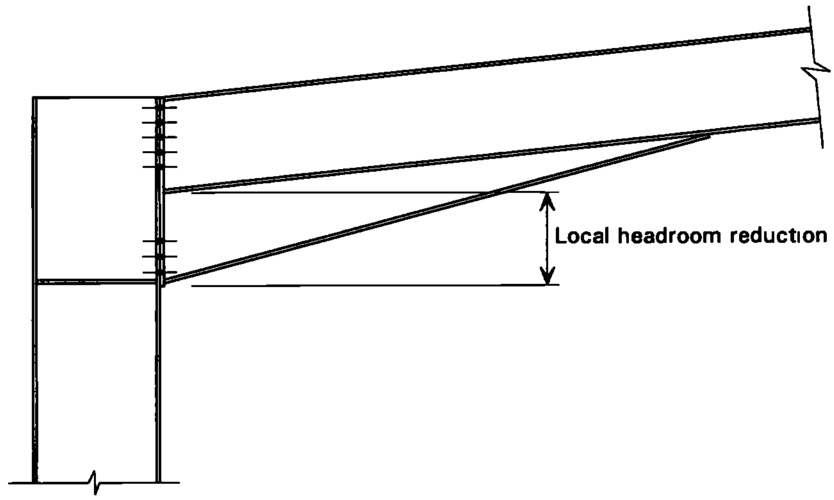


Figure 2.3 *Column/rafter connection showing local headroom reduction*

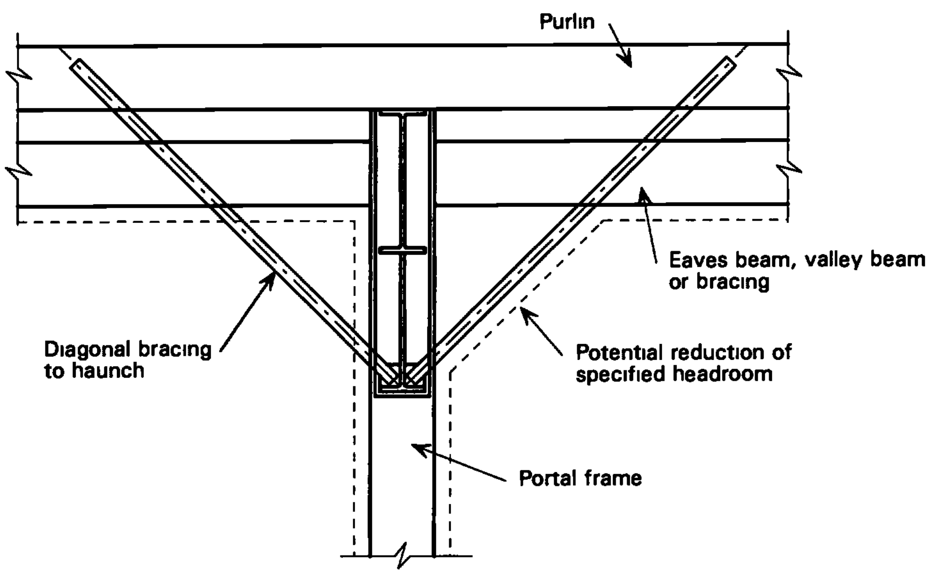


Figure 2.4 *Local headroom reduction*

2.7 Base fixity

Portal bases are generally classified as "pinned" or "fixed", but the actual stiffness or flexibility of nominally pinned or nominally fixed bases should be considered for design to ENV 1993-1-1. The advantage of pinned bases is that the foundation is simplified and so should be less expensive. Fixed bases have the advantage that the frame is stiffened, thereby improving stability.

Partial base fixity can reduce deflections significantly without necessarily affecting foundation costs. A reasonable value of stiffness for nominally pinned bases is given in Appendix A. However, it is very important to agree the approach to partial base fixity with the checking engineer, before proceeding with the design. Where there is any doubt about acceptability, it is safest commercially to assume a truly pinned base.

Where the ground is weak, horizontal reactions generated at the bases might be resisted by the floor slab, if suitable details are provided.

Moment resisting bases might be required for fire regulations, especially at boundaries.

2.8 Horizontal impact on columns

In some buildings, there is a requirement that columns must be able to resist a specified impact loading near the base, in case of vehicle impact. It is common practice to use a reinforced concrete cantilever, rising out of the foundation and cast around the steel column, to resist this loading, as shown in Figure 2.5. The concrete cantilever is designed to resist the impact, ignoring the presence of the steel column. The steel column is designed ignoring the impact.

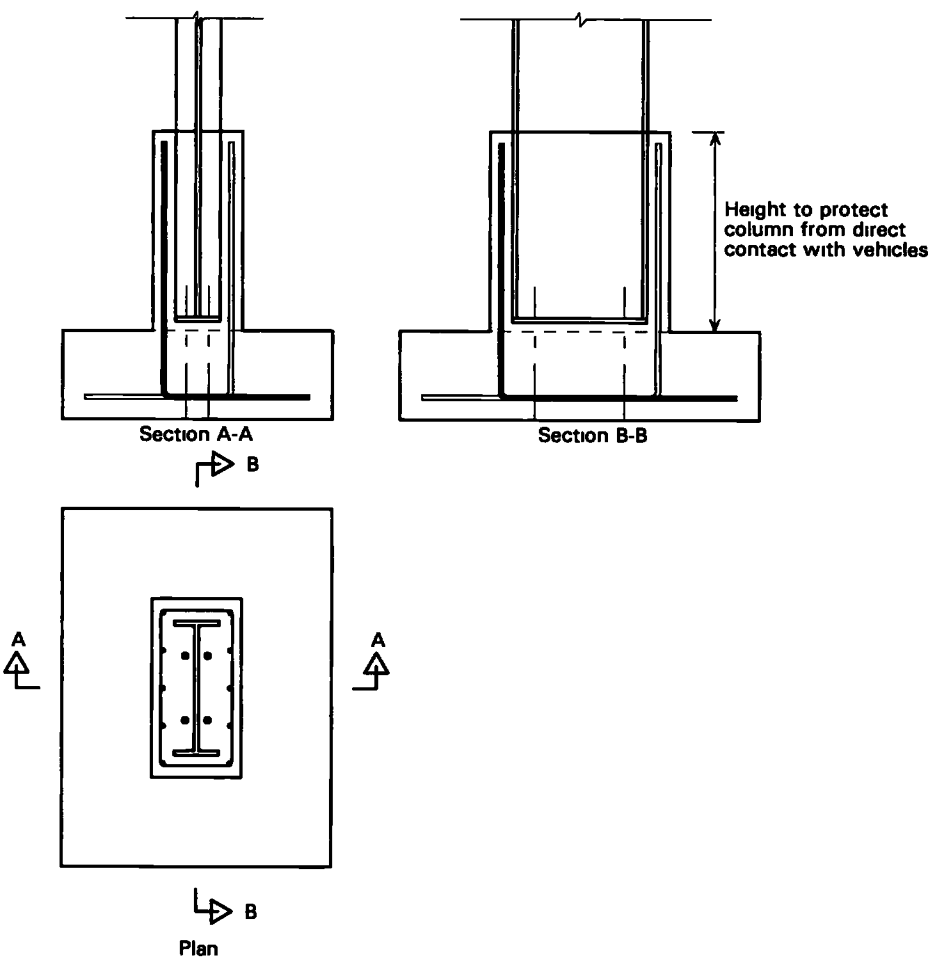


Figure 2.5 *Reinforced concrete cantilever for impact protection*

Note that the horizontal shear reinforcement is not shown in Figure 2.5 but must be provided. The simplest design is probably to use pairs of horizontal "U" bars to form the "stirrups" or "links".

2.9 National differences in loadings

Different intensities of loadings may affect the economy of frame spacings, etc. The intensity of loadings will vary regionally, due to different snow loads and wind loads, so there will be some regional variations in economy due to loading. In addition, different nations have different loading regulations at present, though these should be unified with the finalised set of Eurocodes.

3 CLADDING

3.1 General

It is possible to divide the common systems of cladding into two categories:

(1) Cladding supported on purlins and rails spanning between frames.

Purlins and rails span between the portal frames, forming line supports for the roof and wall cladding, as shown in Figure 3.1. For maximum efficiency and maximum economy, these purlins and rails should be made from thin cold-formed steel (see Section 3.4).

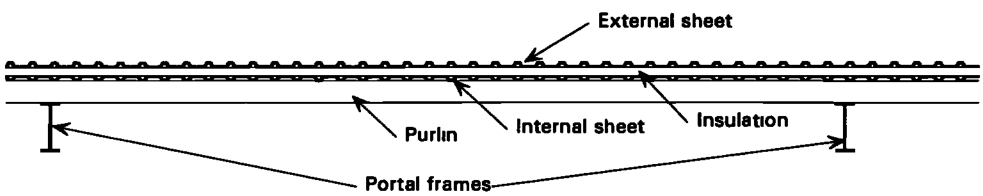


Figure 3.1 *Purlins and rails spanning between the portal frames*

(2) Deep deck and cassettes (liner trays) spanning between frames

Profiled sheets, known as "deep-deck" for the roof and "cassettes" ("liner trays" in UK terminology) for the walls, form an uninterrupted internal steel skin, spanning between the portal frames, supporting the roof and wall cladding. A typical example is shown in Figure 3.2.

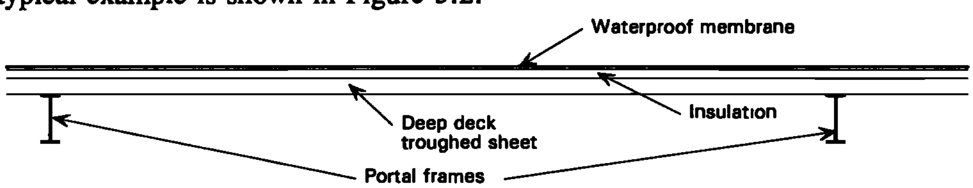


Figure 3.2 *Profiled sheets*

A wide variety of steel cladding products, supported by detailed technical documentation, is available, for example *Colourcoat in building*[6] and *Planung Konstruktion Bemessung*[7].

The fire resistance properties of cladding should be considered prior to selection, particularly where it is required to form part of a fire resistant wall. This is discussed further in Section 8.

3.2 Effects on the cost of the portal frame

The choice of cladding system can have a considerable effect on the economy of the frame, even if the self-weight is unaffected. Therefore, to find the most economical building, the implications of the cladding system on the frame design and detailing should be considered carefully. The combined cost of the frame and cladding should be evaluated together.

Purlins and rails can be used to stabilise the inner flange of the portal frame by the addition of diagonal bracing members, as shown in Figure 3.3. This creates inverted "U frames", which use the bending stiffness of the purlin to resist lateral buckling of the inner flange. This is a much cheaper method of bracing than using plan bracing in the plane of the inner flange.

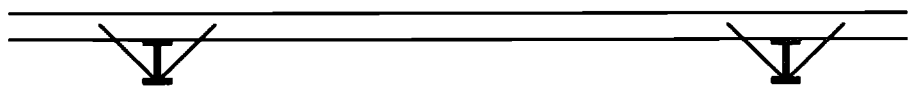


Figure 3.3 *Diagonal bracing members*

The great advantage of frequent bracing to the internal flange is that the members can carry much higher stresses without buckling, providing a more efficient and less expensive structure. For the lowest cost structure, the spacing of the purlins will usually be limited by the need to stabilise the frame member rather than by the strength of the cladding. It is common to use purlins at around 1,6 m centres to stabilise the rafters, depending on the lateral slenderness of the rafter.

By using purlins and rails to stabilise the frame members, the structure can be designed using plastic design criteria without problems of local member instability. This is the most logical design method and often gives the most economical structures. Even where elastic design is specified, the bracing from purlins will allow a significant reduction in section sizes.

3.3 Coated steel cladding systems

3.3.1 General

The most economical cladding systems are usually those with coated steel as the external material. High quality products are available, which are both durable and visually attractive. Coated steel cladding offers architectural possibilities that would be economically impossible with other forms of cladding, such as precast concrete walls or conventional flat roof construction. In addition to offering a wide range of colours, coated steel allows curved surfaces, openings, roof slopes, outstands and other architectural features to be incorporated simply and, therefore, at minimal additional cost.

Coated steel is used in a variety of different cladding systems. These can be categorised as follows:

- Composite (sandwich) panels
- Insulated, site-assembled systems
- Single skin systems.

These categories and their more common subdivisions are described below and the types of fixing systems are described in Section 3.3.5. Information on all aspects can be obtained from Corus (see page 72) and other manufacturers.

3.3.2 Composite (sandwich) panels

Composite panels have external and internal skins of coated steel with a foam core, forming one single bonded unit. The foam core is usually injected and cured between the skins in a continuous production process. The core not only provides insulation but also acts as a structural core, creating a strong and very stiff panel.

Composite panels are available as long panels for roofs (erected with the panel length parallel to the roof slope) and long panels for walls (erected with the panel length vertical). They are also available as wall panels mounted in a rectangular grid system.

3.3.3 Insulated, site-assembled systems

Site assembled, or "built-up", systems comprise an internal liner sheet, a layer of insulation of mineral wool or foam and the external coated steel profile. Where the insulation is compressible or an air gap is required, spacers are needed between the internal liner and the external profile.

This type of system is also used for wall cladding supported by cassettes (liner trays).

3.3.4 Single skin systems

Single skin cladding is used where only a weatherproof envelope is required, with no insulation requirements. This may be appropriate for storage of certain materials or the enclosures for some industrial processes.

3.3.5 Fixings

Fixings can be categorised as:

Concealed fixings

There is a variety of cladding systems in which the fixings are concealed from the external face. This has the two advantages of good appearance and avoiding any possibility of leaks from fixings through the roof.

Perforating fixings

The least expensive cladding systems use fixings that pass through the cladding. When properly installed, there should be no leaks, because the fixings incorporate waterproof seals.

Grid systems

Some composite panel systems for wall cladding use panels mounted in a grid system.

3.4 Purlins and rails

3.4.1 General

The most efficient purlins and rails are cold formed from thin galvanised steel. They are often attached to the frames by use of cleats and other fittings. There are many products available, including several complete systems that include all the necessary fittings. These cold formed members are far more economical than the alternative hot rolled angles, channels or I sections. They have been used in all types of steel portal buildings for many years and have performed so well that they are used for almost all such buildings in the UK.

Information is available from manufacturers and national associations, such as the Cold Rolled Section Association (see page 72).

Some specifications still require the minimum thicknesses of any component to be greater than the normal thicknesses of cold formed members. These requirements should be discussed with the prospective client, because they are usually irrelevant for galvanised members fitted internally.

3.4.2 Cross-section shape

A wide variety of shapes is available. Different manufacturers choose different levels of complexity to increase the efficiency of the weight of steel. For a given size and resistance of section, the number of bends must be increased as the thickness of the steel is decreased. It is important to choose the shape of purlins and rails before doing the detailed drawing of the steelwork, as some of the more complicated shapes do not attach to the simplest cleats. Cold formed purlins spanning up to 15 m are available.

3.4.3 Continuity of purlins and rails

The safe span of a given purlin will be affected directly by the continuity developed at the end of each span. The possible conditions are:

- Continuity of elements
  - single span
  - double span
  - multiple span
- Continuity at connections
  - simple ends
  - sleeved ends
  - overlapping ends.

Different purlin and rail systems are designed for different types of span and connection details. For continuous or semi-continuous purlin systems, the purlins at the ends of the buildings might need to be a heavier section, if the frame centres are the same throughout the building. However, this can be avoided by limiting the load capacity for this condition or by using heavier/longer sleeves or overlaps at the end bay.

3.4.4 Attachment details

The cleats may be bolted or welded to the frames. The choice will depend on the economic circumstances in the particular country and contract under consideration. The decision will also be influenced by the machinery and labour available in the fabricating shop at the time of manufacture. However, it must be appreciated that if the hole size for bolts exceeds a certain diameter (depending on flange width), the strength of the portal frame member will be limited by the reduction of the cross section. Typical attachment details are shown in Figure 3.4.

The cleats may be:

- Short lengths of hot rolled sections (normally angles)
- Bent plate
- Built-up from flat plate
- Individual flat plates, which are especially suitable where purlins are used in pairs back-to-back for increased strength or stiffness. This might be necessary at plastic hinges or at column/rafter connections.

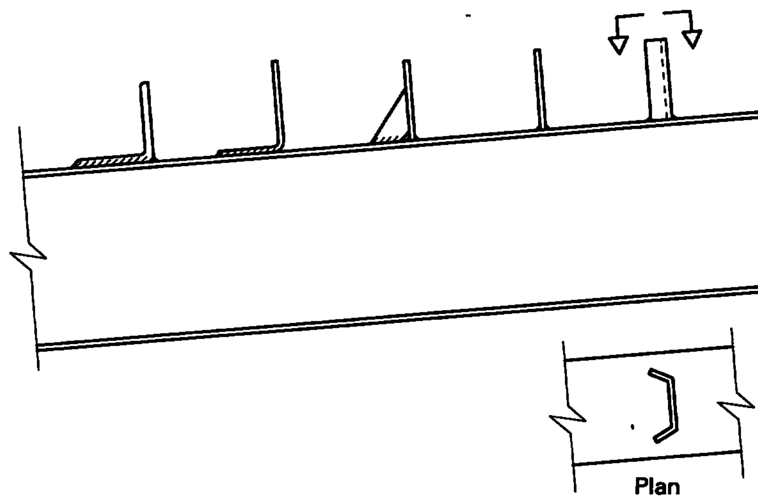


Figure 3.4 *Typical attachment details*

3.4.5 Transverse loads

Cold formed steel purlins and rails are used extensively for sloping roofs and for walls, despite the consequent loading about the weak axis of the section. Loading in this direction is carried by selected systems of intermediate lateral supports and by suspension systems. Typical arrangements for walls are shown in Figure 3.5.

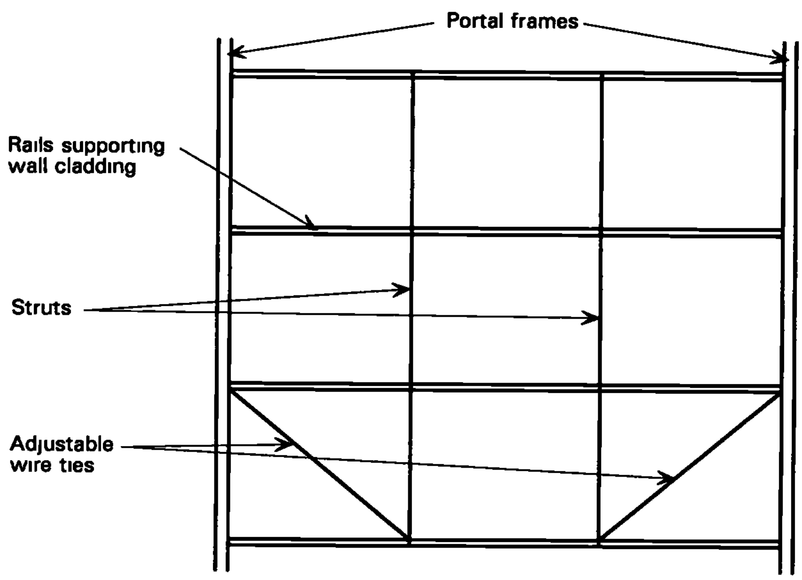


Figure 3.5 *Typical arrangement of wall sheeting rails and supports*

3.4.6 Computer aided design and detailing

A range of computer design and detailing aids is available. These range from "stand-alone" design and detailing packages produced by certain cold formed purlin system manufacturers to integrated detailing packages suitable for design of both the main building frame and the cold rolled purlins, rails, attachments, suspension systems, etc. Information is available from manufacturers and national trade associations, such as the Cold Rolled Section Association (see page 72).

4 ALLOWING FOR FUTURE EXTENSIONS

4.1 General

Steel portal frames with steel cladding are very adaptable for future extensions because the steel frame is easy to modify and the cladding can either be modified or removed. Ideally, future extensions are foreseen at the design stage of the original building, but even when this is not the case, it is relatively easy to add more bays to the structure, making the connections by site drilling or welding to the existing frame. This is one of the great advantages of steel portal frames with steel cladding over other structural forms.

This Section focuses on the design of the original building, when a future extension is foreseen. There are two fundamental cases:

- adding frames to extend the length of the building, as shown in Figure 4.1
- increasing the width of the building by adding new bays, as shown in Figure 4.2.

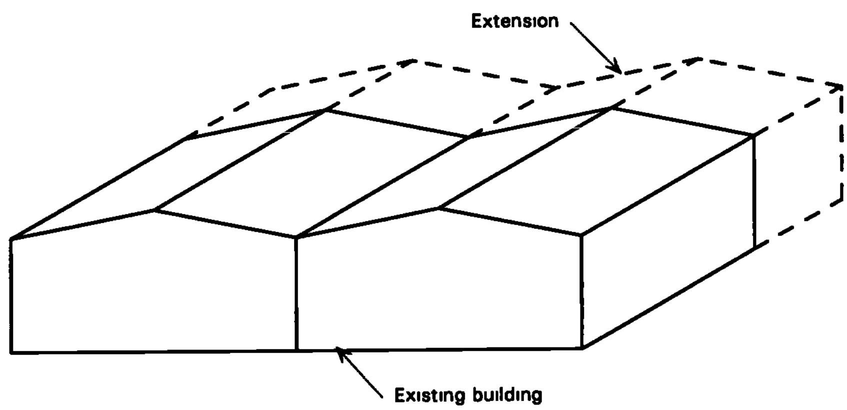


Figure 4.1 *To extend the length of the building*

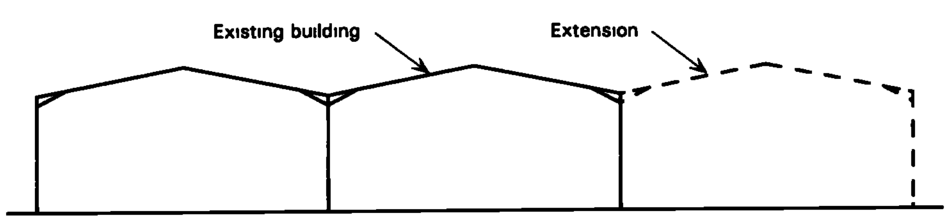


Figure 4.2 *To increase the width of the building*

4.2 Adding frames to extend the length

Normally, the end of the building will not be a portal frame, but a gable wall comprising several vertical columns regularly spaced across the end of the building, each carrying only a small vertical load. If a future extension is to be built, it is prudent to consider the balance of economy of first cost versus possible future disruption, if the gable end has to be replaced to allow erection of a portal. It will normally be better to construct a portal frame to suit the future extended structure. Replacing a light gable end frame will involve stripping the roof back, for one bay if simply supported purlins have been used, but for two bays or more if continuous purlins have been used. It is also likely that a light gable frame would be held in position by bracing in the plane of the rafters, which would also need to be removed, to allow erection of the new portal frame.

4.3 Adding bays to increase the width

It is important to consider the difference between a normal external column and the future requirements of an internal column.

Particular attention should be paid to the following:

- The vertical load on the column will be doubled by the addition of a new bay. It will increase fourfold if alternate frames are to be supported on valley beams, or even more if more frames are supported on valley beams. This will affect the column, the foundation and the holding-down bolts.
- The bending moment in the column will in many cases be reduced by the addition of more bays of a similar span, but this will depend on the national regulations on application of uniformly distributed or "pattern" snow loads.
- ENV 1993-1-1 does not have slenderness ratio limitations for compression members, when the external cladding rails have been removed, which allows greater freedom of section selection. However, the relevant country's National Application Document (NAD) might have a slenderness limitation, as is the case in the UK NAD.
- Detailing the original external column heads for the future extension will add a small initial cost but result in a significant saving later.
- The column head panel must be detailed to serve as a moment resisting knee for the original building configuration (i.e. designed for high shears) and also for the moment connection to the future frames. This may mean that extra flange/web stiffeners have to be added. This can be done when fabricating the original building.

Generally, the external column should be designed for the original building configuration, then checked to ensure that it will function as an internal column in the extended building.

5 LOADING SPECIFICATIONS

5.1 General

The building specifier should define the loads to be applied to the structure or should check and approve the loads proposed by the designer. This Section lists common cases that should be considered.

At present, loadings are subject to the national regulations of each individual country, although, when Eurocode 1 is finalised (i.e. as an EN standard), it is intended that the load intensities and application of loads will be harmonised throughout Europe. Until there is a harmonised loading code, specifiers must be very careful to ensure that they have complied with both the national loading intensities and the national load application regulations.

5.2 Pattern loading effects on multi-bay frames

The size of the internal columns of portal frames is particularly sensitive to the distribution of the load on the roof. Where the loading may be taken as uniformly distributed, the moments might be small for the central column of a symmetrical frame. However, the effect of pattern loading can be severe and even a modest percentage reduction in the load intensity on one span will create a moment that is far more severe in its effect than the relief of axial load. Therefore, the local regulations must be researched carefully and the effect on the design should be considered carefully.

5.3 Service loads and finishes

The choice of design load to allow for service loads and finishes is a delicate matter. If the design load is high, it will allow for a greater range of uses for the building but will make the initial building more expensive. This in turn may make it less competitive to occupy, to rent or for resale. Alternatively, if the design load is low, it may restrict the use of the building and make it harder to find occupants for rent or for resale.

Significant local loads, such as air conditioning plant, should be considered explicitly in the frame design.

5.4 Reuse

It is becoming increasingly common for clients to need to change the use of buildings or to sell them. Therefore, the specifier should consider the original services load specification very carefully. Specifying a low services load may produce a saving in the initial building cost, but may result in a penalty in terms of the difficulty of reuse or resale. However, the specification of an unrealistically high service load may provide few additional opportunities for reuse or resale, while incurring additional building costs.

6 MEMBER AND CONNECTION PROPORTIONS

6.1 General

This Section contains a general description and discussion of good practice in the proportioning of a steel portal frame. It is intended to help the designer and detailer.

The structure may have any proportions provided that it fulfills its purpose and satisfies the appropriate national regulations. However, it is possible to give some indication of proportions that will normally provide an acceptable structure. Additional information on common UK practice will be given in *Design of single-span steel portal frames*[8].

6.2 Columns and rafters

The columns and rafters will usually be I sections rather than H sections, though H sections are sometimes used. The external column sections are normally significantly heavier (in terms of mass/unit length) than the rafter sections, if the rafters are haunched at the rafter/column connections.

The effective length of members for lateral buckling and lateral torsional buckling can be reduced by diagonal stays to purlins and side rails as shown in Figure 2.2. However, the use of purlins for lateral restraint should be agreed with the checking engineer before proceeding. In some countries, this may not be allowed. In many countries it will only be allowed if the purlins are aligned with the nodes of the roof bracing truss and the forces from the bracing loads must be included, when calculating the resistance of the purlins.

The rafter will normally be reinforced by a haunch at the connection to the columns or valley beams, as described in Section 6.3. The column head may need stiffening, as it carries high local loads at the connection to the rafter (tension in some bolts and bearing from the haunch compression flange) and, if there is a high column moment at the connection to the rafter, it will also carry high shear. The need to stiffen depends on the proportions of the column and the connections to the rafter.

6.3 Haunches and other connections

The haunch has a great effect on the economy of the structure by allowing smaller rafter sections. The proportions of the haunch depend on the characteristics of each individual building, especially the size of the rafter. Typical details are shown in Figure 6.1, but it is important to note that these are not definitive and that the only requirement is the ability to carry the loading throughout the design life of the structure. If the environment is clean, dry and protected from the weather, details such as intermittent fillet welds or bolts at very large spacings are usually acceptable.

The length of the eaves haunch of a duo-pitch roof or the upper haunch of a mono-pitch is commonly approximately 1/10 of the span of the portal (Figure 6.2). However, this will vary according to the rafter size and may be considerably longer, especially for elastic design. Duo-pitch apex haunches and lower mono-pitch haunches are usually much shorter. Conventional apex haunches only need a sufficient length to allow a reasonable distribution of load into the bolts.

The depth of the rafter plus haunch is commonly approximately 1/35 of the span of the portal. This will vary according to the characteristics of each structure and is affected not only by the forces and moments at the end of the rafter but also by the rafter/column connection details and the proportions of the column and column head details.

Haunches are commonly made from a cutting from an I section of a size similar to the rafter (but sometimes heavier) welded to the underside of the rafter. They may also be fabricated from plate. The only requirement for the details is that they should be able to carry the loads that occur in the case under consideration. For example the web welds may be intermittent. Equally, the weld between the haunch flange and rafter flange need not necessarily be a full penetration butt weld on this type of haunch, because the original rafter flange continues in parallel with the haunch bottom flange. As another example, there may be an opening between the haunch and the rafter at the deep end provided that the ability of the design to carry the loads can be demonstrated. This may be done by calculations of Vierendeel action. The total moment of [total haunch shear] × [length of opening] must be resisted by local moments at one or both ends of the opening. These moments will be in the rafter and the haunch cutting at the end of the opening furthest from the column, and in the upper and lower bolt groups at the end by the column. The ordinary design forces must be resisted in addition to these Vierendeel moments.

The rafter/column connections (in common with apex and other connections) are normally bolted for economy, ease of fabrication, ease of transport and ease of erection. The details depend only on the adequacy to carry the connection loads for the design life in the environment, which is usually clean, dry and protected from the weather. In these circumstances, bolts are only needed to carry connection loads rather than to comply with maximum pitch regulations intended to limit distortion from rust in external structures. End plates can be full depth or discontinuous and stiffening is only needed where the end plate or haunched rafter cannot carry the applied loads.

Where the end of the haunch is properly cut to provide a good fit to the end plate, the compressive force in the haunch may be carried by direct bearing on the end plate. The welds do not need to be checked for compression, but only for shear and tension.

6.4 Deflections

The differential deflections, under serviceability loads, between frames of different stiffnesses should be considered, to ensure there is no damage caused to the cladding and no other serviceability failure.

Other effects of deflections should be considered, including the possibility of water lying in gutters or other forms of ponding. These problems can be avoided by the use of suitable pre-sets in more flexible structures.

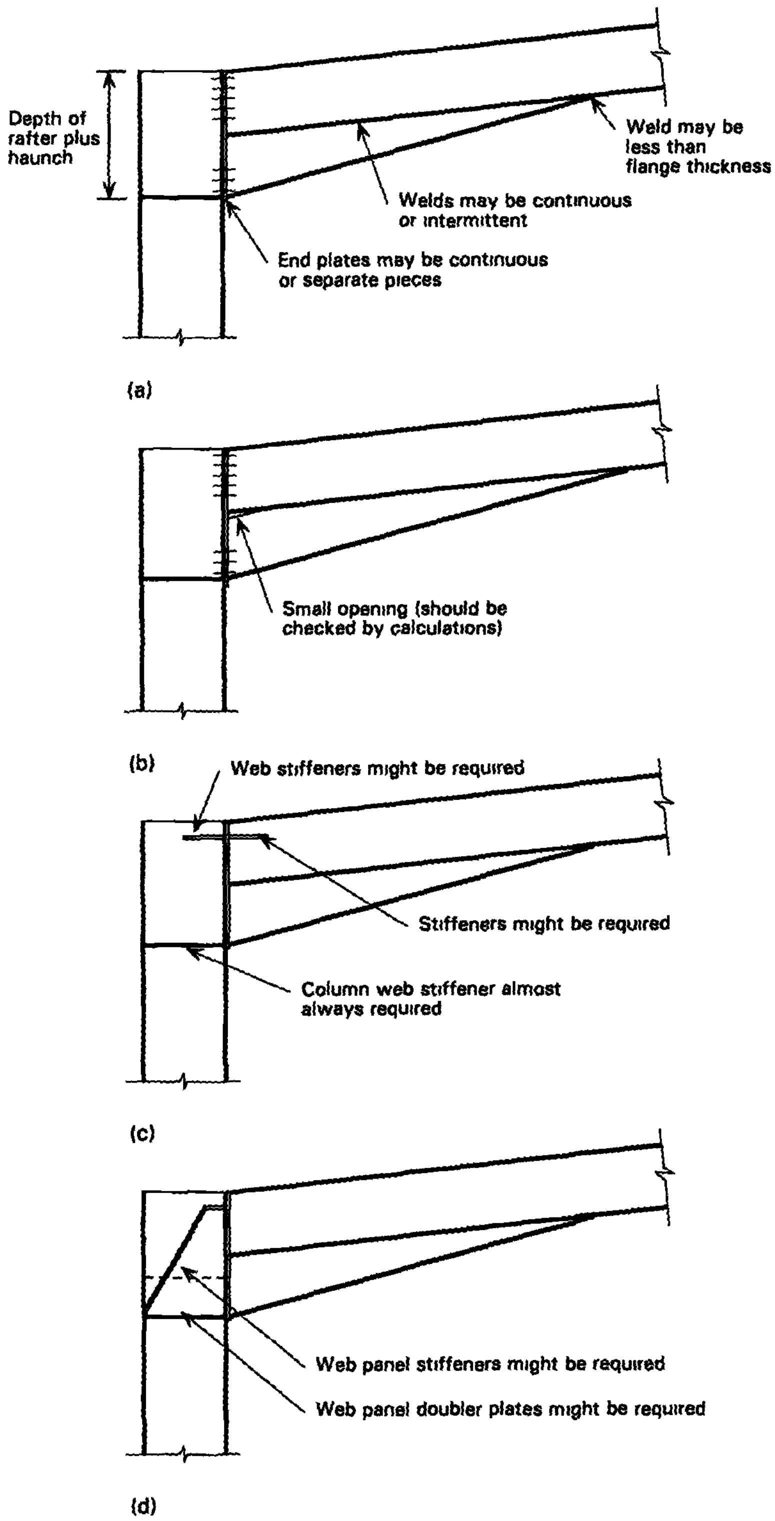


Figure 6.1 *Haunches and other connections*

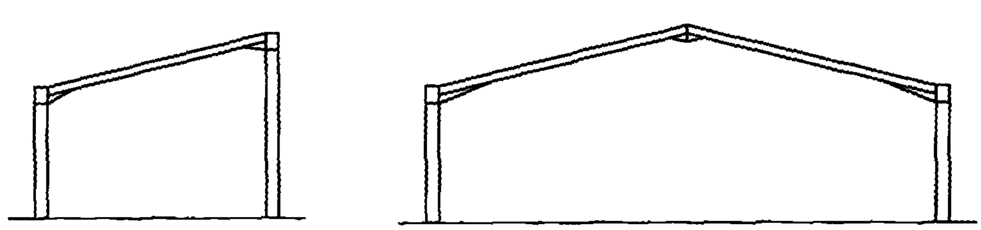


Figure 6.2 *Mono- and duo-pitch portal frames*

7 BRACING ARRANGEMENT

7.1 General

It is essential to provide bracing that is both sufficiently strong and sufficiently stiff at all the points that are assumed to be restrained in the design calculations. This is especially true where the inner flange of the portal frame is in compression. The requirements for bracing vary greatly between countries, and practices that are acceptable in one country may not necessarily be permitted in others. Designers must therefore check with the appropriate authorities in the country under consideration to ensure compliance with the national requirements. Consequently, this publication can only give general guidance not detailed advice. The design criteria given in ENV1993-1-1 are summarised in Section 15.

7.2 Column, rafter and haunch bracing

The columns and rafters (and sometimes the haunches) will normally require intermediate restraint to the compression flanges. The outer flange is easily restrained by the cladding, especially if purlins and sheeting rails are used. The inner flange is also relatively easy to restrain where purlins or sheeting rails are used. The normal method is by diagonal members extending to the inner flange, as shown in Figures 2.1 and 2.4. These diagonal members are designed only to carry the appropriate component on the lateral restraint force. Where purlins and bracing are not used or are insufficient, independent bracing must be provided, for example by tubes in the plane of the bottom flange connected to plan bracing. It may be impossible to provide isolated columns, for example between doorways, or internal columns surrounded by large clear areas, with intermediate bracing.

A point deserving special attention is the inner flange at the column/rafter knee. This point has been assumed to be laterally restrained in the major theoretical research work on portal frames, such as that by Horne *et al.*[9,10], and should therefore always be laterally restrained in real structures, unless the stability of the knee can be otherwise demonstrated by a rigorous stability analysis.

The bottom flanges of haunches sometimes require intermediate bracing. Where this is required, the bracing should connect to the bottom flange by some direct and identifiable stiff load path. Bracing to the middle flange of a haunch should not normally be used without a rigorous stability analysis, except at the shallow end of the haunch, where the middle flange is very close to the bottom flange.

The restraint provided to the flanges is only effective when the structure as a whole is adequately restrained by appropriate overall bracing (e.g. plan bracing or other cross bracing) or diaphragm systems (e.g. stressed skin action of the roof sheets).

7.3 Special considerations for plastic analysis

Special attention is required in plastic design because the extensive plastic zones reduce the stiffness of the elements and make them more sensitive to instability. Eurocode 3 requires that bracing is provided at plastic hinges (see Section 15.5).

It is important to realise that plastic hinges may form, rotate then unload (or cease to exist) as the load is increased. For example, a hinge may form at one end of a haunch, only to disappear and be replaced by a plastic hinge at the other end after a certain rotation. All hinges that rotate before the ultimate limit state (ULS) load is reached, but stop before it is reached, and all hinges that are rotating at ULS must be braced as rotating plastic hinges. If not, an early collapse, triggered by localised instability, is possible. However, hinges that are part of the final collapse mechanism, but do not form before or at ULS loading, do not need to be treated as rotating plastic hinges.

The identification of the load at which hinges form is very difficult and tends to be unreliable, except when elastic-plastic analysis methods are used. Generally, where a rigid-plastic analysis is used instead of elastic-plastic analysis, all hinges in the collapse mechanism should be regarded as rotating hinges at ULS. Other possible hinge locations, for example at the other end of the haunches, should also be braced, unless there is a rigorous analysis to show the hinge formation sequence.

As noted above, EC3 requires bracing at plastic hinges. This is good practice and should be complied with at the design location of the hinges. However, research by Horne *et al.*[11,12,13] and CSC (UK) Ltd[14] shows that plastic hinges may rotate without instability even between restraints, but it is essential that the proportions of the cross section and the spacing of the restraints are compatible with the required rotations.

The restraint forces at plastic hinges should probably be greater than specified in EC3 (see Section 15.5 below).

7.4 Plan bracing

Plan bracing, or an equivalent diaphragm, is required to carry the horizontal forces resulting from the following:

- Wind forces on the gable end
- Stability forces from any columns that are not braced by their own vertical plane bracing system
- Local stability forces from the flanges of the rafters and haunches.

It must be clearly understood that local stability forces from the flanges cannot affect the overall horizontal equilibrium of the roof.

It is common practice to design the plan bracing, e.g. the wind girder at the end of a building, to carry the complete wind load and any required overall instability loads. However, from a consideration of the theoretical deflections of the wind girder, it is clear that in most cases these loads are shared with diaphragm action of the roof cladding, even where the cladding has not been specially detailed for this purpose.

It is common UK practice to assume that the sheeting acts as a diaphragm, which effectively connects the purlins to the nodes of the wind girder. However, in some countries, it is not permitted to assume that the sheeting acts in this way. The regulations for each country must be clearly understood, as these can have a significant effect on layout of the wind girder. Where the sheeting cannot be taken to act as a diaphragm, the wind girder would then be required to provide a node at each purlin line to carry restraint forces from the portal frame flanges. The vertical offset between the plane of the restraint and the plane of the compression flange might need to be considered under very extreme regulations.

Plan bracing will often be required to resist horizontal forces in the planes of the portal frames in "hit" and "miss" structures, i.e. where some planes of portal frames are supported on valley beams rather than columns, as described in Section 6.4 above. The bracing ensures that the horizontal deflections of the "miss" frames are very close to the horizontal deflections of the "hit" frames. Without such bracing, the "miss" frames would deflect more, possibly damaging the cladding at the fixings, unless the "miss" frames were made of stiffer, i.e. deeper, sections. With the plan bracing, the frame instability is averaged out between the "hit" and "miss" frames. The analytical approach to finding the "hit" and "miss" loads in the plan bracing is given in Section 11.10. Plan bracing may be required to stabilise valley beams.

7.5 Vertical-plane bracing

It is essential that the columns of structures are stabilised. In the plane of a portal frame, this stability is provided by the frame action and the moment resisting rafter/column connection. Restraint at right angles to the plane of the portal frame must be provided by additional elements. The classic system is cross bracing between the columns with eaves beams or valley beams to connect all the column heads in the plane to the cross bracing. However, cross bracing is often unacceptable on internal lines of columns or where glazing is required for the full length of the building exterior. In these cases, it is normal to include bracing portals. These are normally formed by additional I-section members in the plane of the bracing portal (fabricated as integral parts of the main portal columns) rigidly framed into the eaves or valley beams.

The design criteria are summarised in Section 15.1.

8 FIRE

8.1 General

The safety of the occupants of the building and the public and property outside must be considered in the case of fire. The regulatory authorities must be satisfied with the design, but thoughtless observance of regulations based entirely on traditional fire protection is likely to prove both unnecessarily expensive and largely irrelevant for most single-storey portal buildings. It is therefore necessary to decide which aspects of fire protection are truly essential, if safe and economical structures are to be built.

8.2 Safety of occupants

Single-storey portal framed buildings have potentially two major fire safety features. Firstly, being single storey, direct escape from such buildings is possible via fire exits, without the need for internal or external stairways. It is usually quite simple to provide many fire exits, distributed around the building, giving short escape routes from all locations. The internal layout of these buildings usually allows easy and free access to such exits. As a result, the complete evacuation of the whole building can be achieved very quickly. Secondly, it is easy to install smoke vents throughout the roof area of a single-storey building so that the occupants are not hindered during their escape, either by poisonous fumes or by a reduction in visibility due to smoke. It is common practice to use special roof vent panels that melt in the presence of hot gases, ensuring that they open automatically, allowing the smoke to escape, without risk of mechanical or electrical failure. Such systems have been used widely for many years.

The safety of the building's occupants will be increased further by a water sprinkler system. Although this will inevitably increase the cost of the building, the additional cost will normally be off-set by reduced insurance premiums for the building user. The use of a sprinkler system might also reduce the cost of fire protection for the structural frame.

While sprinklers reduce the risk from fire, they may not be acceptable in certain buildings due to the risk of water damage. For example, the manufacture and assembly areas for high technology products, such as satellites, do not usually have water sprinkler systems.

The risk to occupants in a very large building may be limited further by partitioning with fire walls. However, adequate means of escape must be provided from each compartment.

8.3 Regulations in different countries and regions

Safety regulations are the responsibility of each national government, and the fire safety regulations vary throughout Europe. Therefore, to avoid expensive additional work, it is essential that the designer understands the requirements of the local regulatory authority in each case. In some countries the regulations vary significantly even between districts, often depending on individual decisions by local regulators. In some cases, the local regulator has the authority to accept waivers of the local fire regulations. This means that the decision depends on the ability of the applicant to convince the regulator that the proposed fire safety measures are adequate for the intended use of the building in its particular location.

Generally, the UK is more flexible in its approach to fire safety than some other countries. In some cases, the authorities might require considerable effort to be persuaded to adopt new methods and might refuse to abandon conservative interpretations of the regulations.

Eurocode rules for fire design of steel structures are given in ENV 1993-1-2[1].

9 BASIS OF DESIGN

9.1 General

9.1.1 Design documents

The design procedures and guidance given in the following Sections of this publication are based on the use of Eurocode 3, Part 1.1, as issued by CEN (reference ENV 1993-1-1: 1992[1]). This prestandard is implemented in the various EU and EFTA countries by means of each country's National Application Document (NAD). The use of ENV 1993-1-1 allows a common design philosophy, although the actual design calculations will vary slightly from country to country, according to the requirements of the particular National Application Document.

Until the complete set of Eurocodes and supporting EN standards is available, reference must be made to certain national standards (for example for loading specifications). These references should be given in the appropriate NAD.

9.1.2 Limit state design

Eurocode 3 is a limit state design code. It requires that the structure be checked for:

- Deflections and other performance criteria at normal (working) loads. These are verified by the serviceability limit state (SLS) checks, which are considered further in Section 10.
- Resistance to extreme loads, which is verified by the ultimate limit state (ULS) checks. The ULS is considered further in Sections 11 to 16.

9.2 National application documents

Until Eurocode 3 Part 1-1 is issued as an EN standard, rather than the current status of the ENV prestandard, it can only be used according to the National Application Document of the country in which the structure is built. The NADs modify the ENV to suit national requirements. However, it is intended that the NADs should be withdrawn when Eurocode 3 appears as an EN standard, so that design can be unified across Europe.

9.3 Design philosophy – elastic or plastic design

This document covers the design of steel portal frames using either elastic or plastic analysis methods.

The most economical structures are often produced using plastic design techniques. These are well established in some countries (e.g. in the UK where these techniques have been used for 40 years) but are rarely used in others. Plastic design has proved to be both economical and safe over four decades on tens of thousands of structures. The possible economies to the client can be significant but these depend on the loadings and on the material, fabrication and erection costs.

9.4 Second-order effects

Efficient portal frames with relatively low roof loads are slender structures and, in some cases, the slenderness is such that second-order effects need to be considered when analysing the structure. In this document, the second-order effects are included by the use of the following approximate methods in ENV 1993-1-1: the amplified sway method for elastic design and the Merchant-Rankine method for plastic design.

Generally, second-order effects must be considered for the ULS, but will have negligible effects at the SLS.

10 SERVICEABILITY LIMIT STATE

10.1 General

The serviceability limit state (SLS) analysis should be performed using the SLS load cases (see Section 10.3), to ensure that the deflections are acceptable at the "working loads". This is normally a first-order analysis. Where the analysis is not elastic-plastic, it should check that there is no plasticity, simply to validate the deflection calculation. Where plasticity occurs under SLS loads, the deflections from plastic deformations should be explicitly included in the analysis.

It is more important to ensure that the deflections etc. are acceptable for the cladding and use of the building than to satisfy arbitrary limits. For example, for brittle cladding such as brickwork, the limit of h /150 on horizontal deflections may be unsuitable, or even unsafe, depending on the details of ties and other restraints.

Section 4.2.1(5) of ENV 1993-1-1 proposes the consideration of second-order effects in deflection calculations. However, as V_{SLS}/V_{cr} will often be well below 0,2 (see Section 11.7.3 and Section 12 below) and the stiffening effect of the cladding has not been considered, previous practice of checking only first-order effects should suffice in all but the most exceptional circumstances.

The deflection limits in Section 4 of ENV 1993-1-1 are in italics, indicating that they are Application Rules. These are not mandatory, but satisfying the limits of these rules may avoid difficult questions from checking authorities and clients.

Guidance on deflection limits can be found in *Steelwork design guide to BS 5950 – Volume 4: Essential data for designers*[15] and *Serviceability design considerations for low rise buildings*[16].

Some differences between ENV 1993-1-1, EC3 and UK practice are given in Appendix K.

10.2 Actions (loads)

European loading specifications are contained in Eurocode 1, however these may not yet be the reference documents for each country that are specified in the National Application Documents (NADs) for EC3. The design loads may be subject to choice in certain cases, for example the services loading. Such loads should be agreed with the specifier (see Section 5).

10.3 General serviceability limit state load combinations

10.3.1 General

The SLS load cases are defined in Sections 2.3.4 and 4 of ENV 1993-1-1.

Structural imperfections should not normally be considered. No partial safety factors are applied to actions for the SLS.

Deflections should be checked for the "rare combination" of actions [see Sections 4.2.1(4) and 2.3.4(2) of ENV 1993-1-1]. This may be done using the Single Expression definition or the Two Expression definition given in Sections 10.3.2 and 10.3.3 of this publication respectively.

10.3.2 Single expression definition of SLS combination

The load combination is as follows:

Characteristic values of the permanent actions $\Sigma G_{k,j}$

+ Full characteristic value of the most unfavourable variable action $Q_{k,1}$

+ Reduced values of all other unfavourable variable actions $\psi_{0,i}\ Q_{k,i}$.

This combination is defined in expression 2.14 of ENV 1993-1-1.

$$\sum_j G_{k,j} + Q_{k,1} + \sum_{i>1} \psi_{0,i}\ Q_{k,i}$$

where $\psi_{0,i}$ is the "combination factor" for the i-th load.

Note that there are different ψ factors for different types of variable action. They should be taken from the appropriate NAD.

The "combination factor" reduces the intensity of loading, when several variable loads are considered to be acting together. There is only a very low probability of all variable actions occurring at full intensity at the same time, so the "combination factor" reduces the loads to an intensity appropriate to the SLS analysis for the combined loads.

10.3.3 Two expression definitions of SLS combinations

This method is intended to be less complicated than the definition in 10.3.2. It was introduced for the benefit of people making calculations by pencil and paper rather than by computer but it is not restricted to use in pencil and paper calculations.

The most unfavourable of either of the two expressions 2.17 or 2.18 of ENV 1993-1-1 must be used.

Expression 2.17 is:

Characteristic values of the permanent actions ($G_{k,j}$)

+ Full characteristic value of the most unfavourable variable action ($Q_{k,1}$)

i.e. $\sum_j G_{k,j} + Q_{k,1}$

Expression 2.18 is:

Characteristic values of the permanent actions ($G_{k,j}$)

+ 0.9 × Full characteristic value of the most unfavourable variable action ($Q_{k,1}$)

i.e. $\sum_j G_{k,j} + 0{,}9 \sum_{i \geq 1} Q_{k,i}$

11 ULTIMATE LIMIT STATE

11.1 General

ENV 1993-1-1 requires that all actions that could occur at the same time are considered together, so frame imperfection equivalent forces (which is the most convenient method of including geometrical imperfections; see 11.6.1 below) and wind loads should be considered as additive to permanent actions and other imposed loads, but with the appropriate combination factor ψ.

However, only the least favourable combination of loads should be considered. For example:

- If wind loads reduce the bending moments, e.g. by the effects of uplift suctions, the combination without wind must be verified.
- Frame imperfection equivalent horizontal forces must be considered in the least favourable direction with respect to each load combination.

Note: The principal differences between current UK terminology and practice (essentially to BS 5950) and that in ENV 1993-1-1 are given in Appendix K.

11.2 Actions (loads)

Shortly, there will be European loading specifications in Eurocode 1. However, these are not yet available, so the reference documents for each country are specified in its National Application Document (NAD) for EC3. The design loads may be subject to choice in certain cases, for example the services loading. Such loads should be agreed with the specifier (see Section 5).

11.3 Structural resistance and static equilibrium

A structure must have adequate resistance to internal forces and bending moments, known as structural resistance. In addition, it must exist in a state of static equilibrium under the action of the applied loads, i.e. it must have adequate resistance to overturning, sliding, uplift, etc. The same load combination expressions are used for both structural resistance and static equilibrium, but the partial safety factors used in these expressions may differ between the structural resistance checks and the static equilibrium checks. These are given in Section 2.3.2.4 of ENV 1993-1-1. Further guidance is available in *Plastic design of single-storey pitched-roof portal frames to Eurocode 3*[2] and *C-EC3 Concise Eurocode 3 for the design of steel buildings in the United Kingdom*[17].

Static equilibrium is rarely critical for portal framed buildings, so is not considered in detail. Examples of cases where static equilibrium might be critical are:

- in high wind speed areas
- in high but narrow buildings, giving high overturning moments
- in lightweight structures with a low pitch roof, generating high uplift forces, which need to be held down by the foundations
- where there are large openings allowing high positive wind pressure inside the building at the same time as high suctions over the roof.

11.4 Partial safety factor format

EC3 uses a partial safety factor format for checking ULS (see Sections 2.3.2 and 2.3.3 of ENV 1993-1-1), in which the partial safety factors allow for the appropriate uncertainties of resistance, analysis and loads. Expressing this in a simplified way, the partial safety factor format checks that:

Structural resistance less than that specified ≥ effect of loads greater than that specified, or

$$\left(\frac{\text{specified resistance}}{\gamma_M\ \gamma_M}\right) \geq \text{effect of} \sum (\text{specified actions} \times \gamma_F)$$

The partial safety factors, γ_M and γ_F, are given as "boxed values", which are currently specified for each country in the National Application Document [see Section 1.3(3) of ENV 1993-1-1]. Note that γ_F is the general symbol for the partial safety factors for actions (e.g. loads), whereas the more specific symbols are γ_G for permanent actions and γ_Q for variable actions.

11.5 Partial safety factors for actions (loads)

Partial factors for actions are:

- permanent actions (dead loads) γ_G
- variable actions (live loads) γ_Q

Note that γ_G and γ_Q have different values and that γ_G depends on whether the action helps the structure to fall down or helps it to stand up. For resistance calculations, γ_G should be taken as the same value throughout the structure for each individual permanent action. Either $\gamma_{G,sup}$ or $\gamma_{G,inf}$ should be taken, depending on which gives the most unfavourable result.

$\gamma_{G,sup}$ is the higher value of γ_G, where 'sup' is an abbreviation of superior meaning higher.

$\gamma_{G,inf}$ is the lower value of γ_G, where 'inf' is an abbreviation of inferior meaning lower.

11.6 Ultimate limit state load combinations

11.6.1 General

The load combinations at ultimate limit state (ULS) are defined in Section 2.3.2.2 of ENV 1993-1-1.

All ULS load combinations must include the effect (if any) of imperfections from any direction (e.g. either from the left or from the right). In particular, in frame analysis, frame imperfections must be considered. These are probably easiest to apply as equivalent horizontal forces, see Section 5.2.4 of ENV 1993-1-1 and Section 3 of *Plastic design of single-storey pitched-roof portal frames to Eurocode 3*[2].

There are two definitions of the ULS load combination for normal loading conditions. These are given in Sections 11.6.2 and 11.6.3 below. Either definition may be used. The magnitudes obtained using the two definitions will often be different, but neither method is unsafe.

There is one definition of the ULS load case for accidental loading. This is given in Section 11.6.4 below.

11.6.2 Single expression definition of load combinations

All actions which can occur at the same time are applied together, as follows:

Characteristic values of the permanent actions ($G_{k,j}$) $\times$ γ_G

+ Full characteristic value of the most unfavourable variable action $Q_{k,1}$ $\times$ $\gamma_{Q,1}$

+ Reduced values of all other unfavourable variable actions ($\psi_{o,i}$ $\times$ $Q_{k,i}$) $\times$ $\gamma_{Q,i}$

where γ_G and γ_Q are as explained in Section11.5.

This is defined in expression 2.9 of ENV 1993-1-1, as:

$$\sum_{j} \gamma_{G,j} G_{k,j} + \gamma_{Q,1} Q_{k,1} + \sum_{i>1} \gamma_{Q,i} \psi_{0,i} Q_{k,i}$$

where $\psi_{0,i}$ is the "combination factor" for the i-th load.

The "combination factor" reduces the intensity of loading when several variable loads are considered to be acting together. There is only a very low probability of all variable actions occurring at full intensity at the same time, so the "combination factor" reduces the loads to an intensity appropriate to the ULS analysis for the combined loads.

Note that there are different ψ factors for different types of variable action. They should be taken from the appropriate NAD.

11.6.3 Two expression definition of normal load combinations

This method is intended to be less complicated than the definition given in Section 11.6.2. It was introduced for the benefit of people doing calculations by pencil and paper rather than by computer, but it is not restricted to use in pencil and paper calculations.

The most unfavourable of either of the two expressions 2.11 or 2.12 in ENV 1993-1-1 must be used.

Expression 2.11 is:

Characteristic values of the permanent actions ($G_{k,J}$) $\times$ $\gamma_{G,J}$

+ Full intensity of most unfavourable variable action

+ Full characteristic value of the most unfavourable variable action ($Q_{k,1}$) $\times$ $\gamma_{Q,1}$

i.e. $\sum_{J} \gamma_{G,J}\, G_{k,J} + \gamma_{Q,1}\, Q_{k,1}$

Expression 2.12 is:

Characteristic values of the permanent actions ($G_{k,J}$) $\times$ $\gamma_{G,J}$

+ 0,9 $\times$ [Full intensity of most unfavourable variable action ($Q_{k,i}$) $\times$ $\gamma_{Q,i}$]

i.e. $\sum_{J} \gamma_{G,J}\, G_{k,j} + 0{,}9 \sum_{i \geq 1} \gamma_{Q,i}\, Q_{k,i}$

γ_G and γ_Q are defined in Section 11.5.

11.6.4 Accidental load combination

The accidental load combinations are defined by expression 2.10 of ENV 1993-1-1.

Characteristic values of the permanent actions ($G_{k,J}$) $\times$ $\gamma_{GA,J}$

+ Design value of the accidental action (A_d)

+ Reduced intensity of most unfavourable variable action ($\psi_{1,1}\, Q_{k,1}$)

+ Reduced intensity of other unfavourable variable actions ($\psi_{2,i}\, Q_{k,i}$)

i.e. $\sum_{J} \gamma_{GA,j}\, G_{k,J} + A_d + \psi_{1,1}\, Q_{k,1} + \sum_{i>1} \psi_{2,i}\, Q_{k,i}$

Note that the "combination factor" to be used for accidental loading is ψ_2. This reduces the intensity further than the ψ_0 factor used for normal loading in Section 11.6.2.

11.7 Analysis for ultimate limit state

11.7.1 General

Normal first-order analysis (with the results modified for slenderness, as described in Sections 12.3 and 12.4, if required) is acceptable except on exceedingly slender frames or certain frames with slender members. The limitations are given in Section 11.7.3. Where structures exceed these limitations, second-order analysis must be used.

Where first-order analysis is used, EC3 requires that the slenderness of frames is explicitly considered and that the analysis results are modified for the more slender frames to allow for the destabilising effects of the frame slenderness. In practice, this will affect the majority of portal frames with relatively low intensity of roof loading. The consideration of slenderness necessitates the calculation of the elastic critical buckling load of the portal, V_{cr}, which is covered in Section 12.2 below. Methods of modifying the results of first-order analysis are given in Section 12.3 for elastic analysis and Section 12.4 for plastic analysis.

11.7.2 Elastic vs. plastic analysis

The methods of analysis fall broadly into two types: elastic analysis and plastic analysis. Elastic analysis often produces less economical structures because it does not allow large-scale plastic redistribution of bending moments, even though these will occur in a suitably braced structure and will enable the structure to carry higher loads than calculated by elastic analysis. Generally, plastic analysis results in more economical structures because plastic redistribution allows slightly smaller members to carry the same loads. However, where haunch lengths of around 15% of the span are acceptable and the lateral loading is small, the elastic bending moment diagram will be almost the same as the plastic collapse bending moment diagram. In such cases, the economy of the design of slender frames could depend on the method of allowing for second-order effects. The simplest of these methods are approximate, so it is possible that elastic analysis will allow the most economical frames in certain cases. The economy of plastic analysis also depends on the bracing system, because plastic redistribution imposes additional requirements on the bracing of members. The overall economy of the frame might, therefore, depend on the ease with which the frame can be braced.

It is recognised that some redistribution of moments is possible even with elastic design assumptions. Section 5.2.1.3(3) of the prestandard ENV 1993-1-1 allows 15%, but this may be reduced to 10% in the EN standard.

11.7.3 First-order vs. second-order analysis

Generally, normal first-order analysis can be used, with modifications to allow for second-order effects in more slender frames.

The deflections of slender frames cause second-order effects, particularly bending moments, in addition to those calculated by normal first-order analysis (see *Plastic design of single-storey pitched-roof portal frames to Eurocode 3*[2]). ENV 1993-1-1 requires that these effects are included in the analysis and gives methods to modify the results of normal first-order analysis. These methods, which are different for elastic or plastic design, are considered in detail in Section 12. Alternatively, second-order analysis may be used where suitable computer software is available.

For very slender frames, or frames including very slender members, carrying both axial loads and bending moments, ENV 1993-1-1 requires that second-order analysis is used.

The slenderness of the frame is measured as the ratio of the ultimate limit state load of the entire frame (V_{Sd}) to the elastic critical buckling load of the entire frame (V_{cr}). The elastic critical buckling load of an entire frame is the equivalent, for a frame structure, of the Euler load for a strut. It is an entirely theoretical value and the true collapse load will be much lower, just as the collapse load of a strut is much lower than the Euler load. The elastic critical buckling load is discussed further in Section 12.2.

The limitations to the use of modified first-order analysis are:

Elastic analysis: $V_{Sd} / V_{cr} \leq 0{,}25$ (ENV 1993-1-1, Section 5.2.6.2)

Plastic analysis: $V_{Sd} / V_{cr} \leq 0{,}20$ (ENV 1993-1-1, Section 5.2.6.3)

where V_{Sd} is the vertical load on the entire frame at ULS
V_{cr} is the elastic critical buckling load for the entire frame, with the same proportion of load as V_{Sd}.

In addition, there are further limitations on the slenderness of the columns (considered as individual members) when plastic analysis is used. The limitations for a member carrying compression and bending are given in Section 5.2.7 of ENV 1993-1-1 as follows:

Braced frames: expression 5.9, which means $N_{Sd} \leq 0{,}16\ N_{cr}$

Unbraced frames: expression 5.10, which means $N_{Sd} \leq 0{,}10\ N_{cr}$

where N_{Sd} is the axial compression force in the column at ULS
N_{cr} is the critical buckling (Euler) load $= \pi^2 EI / L^2$, where L is the storey height.

11.8 Elastic analysis

Elastic analysis is the most common method of analysis for structures in general but will usually give less economical portal structures than plastic analysis. ENV 1993-1-1 allows the plastic cross-sectional resistance, e.g. the plastic moment, to be used with the results of elastic analysis, provided the section class is Class 1 or Class 2. In addition, it allows some redistribution of moments as defined in Section 5.2.1.3(3) of ENV 1993-1-1, although the percentage may be reduced to 10% in the EN. To make full use of this in portal design, it is important to recognise the spirit of the Clause, which was written with continuous horizontal beams of uniform depth in mind. Thus, in a haunched portal rafter, up to 15% (by the ENV) of the bending moment at the shallow end of the haunch could be redistributed, if the moment exceeded the plastic resistance of the rafter and the moments and forces resulting from redistribution could be carried by the rest of the frame. Alternatively, if the moment at the midspan of the portal exceeded the plastic resistance, this moment could be reduced by up to 15% (by the ENV) redistribution, provided that the remainder of the structure could carry the moments and forces resulting from the redistribution.

It is important to understand that the redistribution cannot reduce the moment to below the plastic resistance. To allow reduction below the plastic resistance would be illogical and would result in dangerous assumptions in the calculation of member buckling resistance.

11.9 Plastic analysis

11.9.1 General

Plastic analysis is used for more than 90% of portal structures in the UK and has been in use for 40 years. It is a well established and well proven method of design, even though it is not yet used extensively in continental Europe.

The three common methods of analysis are:

- graphical
- virtual work analysis of rigid-plastic mechanisms
- elastic-perfectly plastic.

The graphical method is explained by Morris and Randall[18] and included in other standard texts. The virtual work analysis of rigid-plastic mechanisms method is explained in many standard texts, for example *The plastic methods of structural analysis*[19].

11.9.2 Graphical method

In the graphical method, the bending moment diagrams are drawn along the members, with the maximum and minimum values of bending moment limited by the plastic resistance of the member at each position (see Figure 11.1 and *Plastic design*[18]). Alternatively, the members may be chosen to suit any statically admissible bending moment diagram. Therefore, the graphical method lends itself to:

- Analysis of very simple structures
- Initial design of any structure.

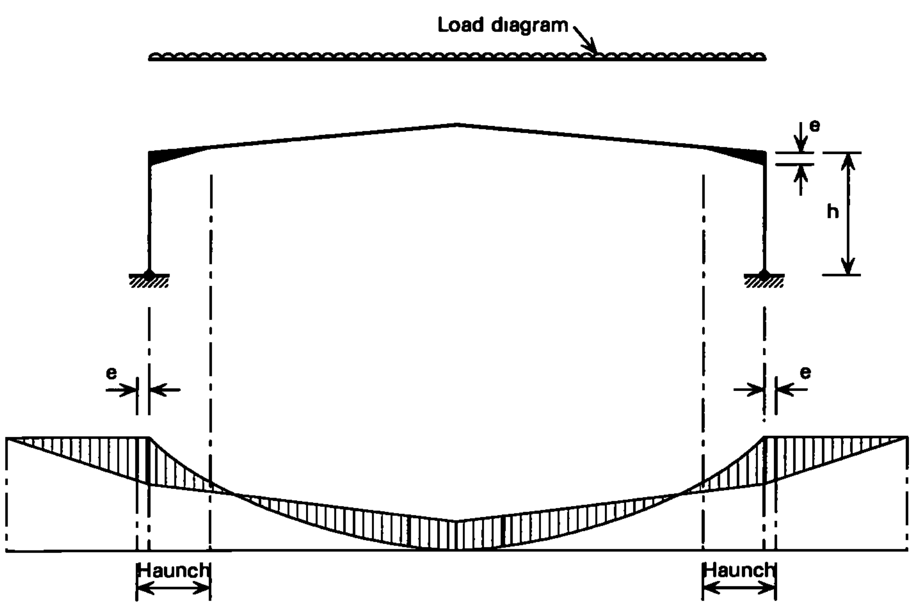


Figure 11.1 *Graphical method*

The graphical method will always find the upper bound of bending moment or lower bound of the load factor, so it is always safe, assuming it is done correctly. However, for all but simple cases, it becomes unwieldy for final analysis.

Where a designer is unfamiliar with a form of structure, the graphical method is recommended for initial design because it helps to develop a 'feel' for the structural behaviour. For all but the simplest of structures, the final analysis should normally be performed using rigid-plastic or elastic-perfectly plastic methods (see Sections 11.9.3 and 11.9.4).

11.9.3 Virtual work of rigid-plastic mechanisms method

The virtual work method calculates the load factor at collapse for a rigid-plastic collapse mechanism (see Figure 11.2 and *The plastic methods of structural analysis*[19]). Extensive structures can be analysed without a computer so the method is often considered to be the classic method.

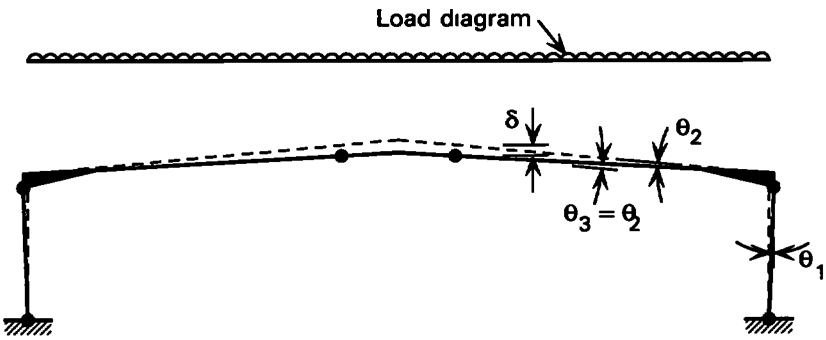


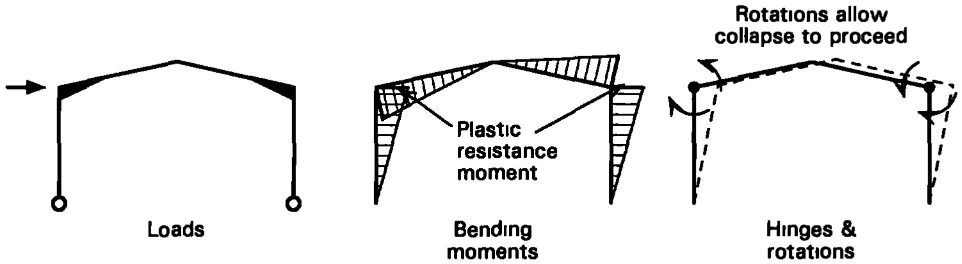
Figure 11.2 *Virtual work method*

Only the collapse mechanism is found, not the order of hinge formation, so the bending moment diagram should not be de-factored for over-strength portals analysed by this method.

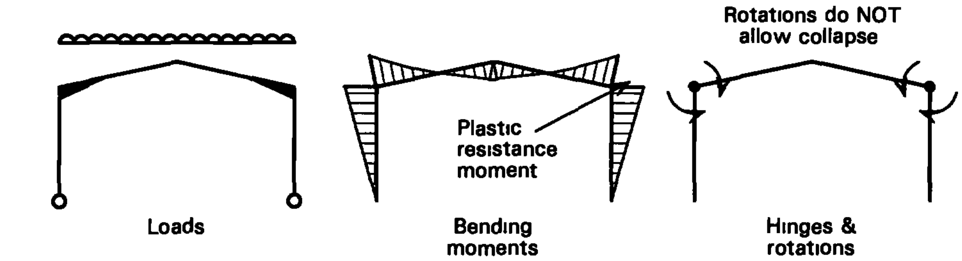
There are three important possible problems with this method:

- The method can only find the load factor for the mechanism analysed. If the true collapse mechanism is not analysed, the load factor will be overestimated. To be safe, the **complete** bending moment diagram of the structure must be constructed for the lowest load factor mechanism, to check that this mechanism is correct.
- The directions of the hinge rotations must be checked carefully to ensure that they are consistent with a mechanism. Examples of a mechanism and a non mechanism are shown in Figure 11.3.
- It will not find any hinges that form, rotate and then cease.

The method is essentially for checking a frame that has already been defined.



(a) Hinge rotations allow collapse to proceed



(b) Hinge rotations do NOT allow collapse to proceed

Figure 11.3 *Hinge rotations consistent with a collapse mechanism*

11.9.4 Elastic-perfectly-plastic method

The elastic-perfectly-plastic method applies loads in small increments and puts plastic hinges into the structure as they form with increasing load (see Figure 11.4). It assumes that the members deform as linear elastic elements past first yield at M_y and right up to the full plastic moment M_p. The subsequent behaviour is assumed to be perfectly plastic without strain hardening. If the appropriate computer software is used, it should be possible to predict hinges that form, rotate then cease, or even unload or reverse, and the software should have this capability. The final mechanism will be the true collapse mechanism (provided that the hinge rotation directions are consistent with the sign of the moment), and will be identical to the lowest load factor mechanism that can be found by the rigid-plastic method.

The method has the following advantages:

- The true collapse mechanism is identified.
- All plastic hinges are identified, including any that might form and cease. Any hinges that cease would not appear in the collapse mechanism but would need restraint.
- Hinges forming at loads greater than ULS can be identified. Where appropriate, the cost of member restraint at these positions could be reduced. This may produce economies in structures where the member resistance is greater than necessary, as occurs when deflections govern the design or when oversize sections are used.
- The true bending moment diagram at collapse, or at any stage up to collapse, can be identified.

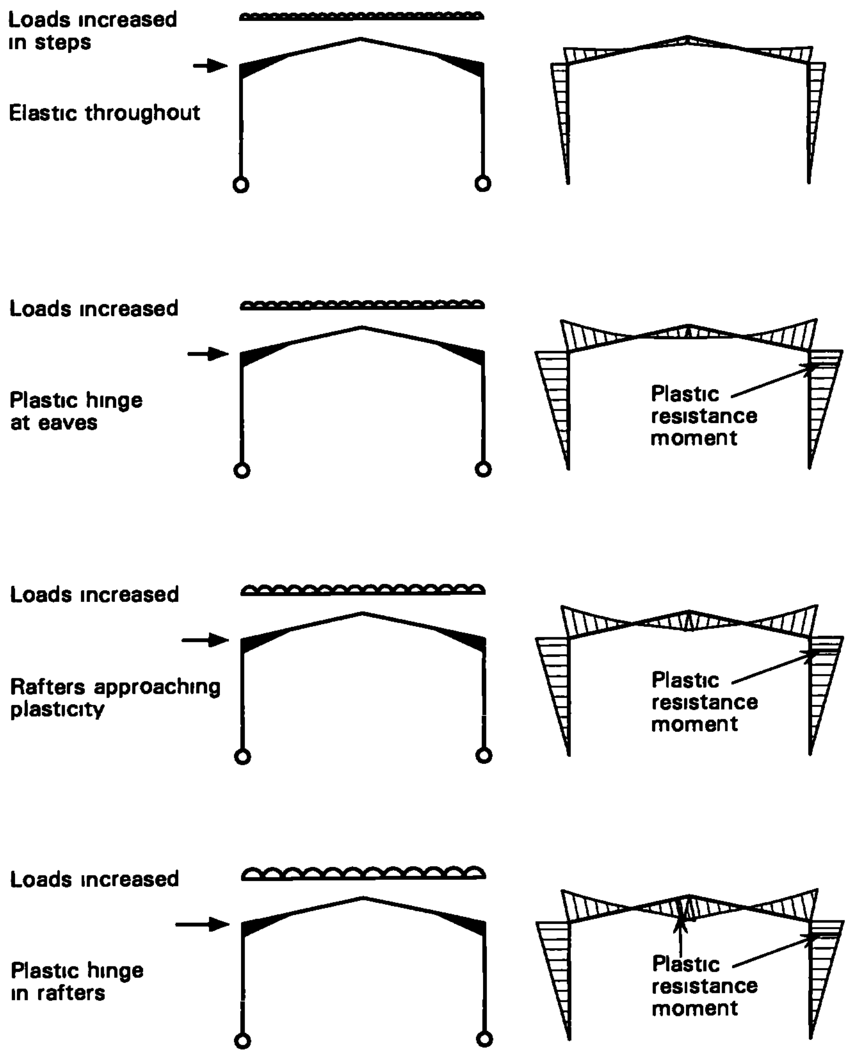


Figure 11.4 *Elastic-perfectly plastic method*

11.10 Analysis of "hit" and "miss" frame buildings

The use of "hit" and "miss" frames in portal structures is described in Section 2.4. The structural analysis of the plan bracing at the eaves and valleys of "hit" and "miss" frames could be performed by 3D software, if such a package were available. However, in general, the available software for plastic analysis is limited to 2D frames. It is therefore necessary to devise a method that allows the use of 2D software for the current 3D application.

It should be remembered that the principle of elastic superposition can be used in first-order elastic analysis, but it is not necessarily valid in plastic analysis and is not strictly valid in second-order elastic or plastic analyses. Therefore, the actual bracing load should be included in the analysis of a fully loaded frame.

Many methods can be used and one method is given below:

(i) Analyse the "hit" frame alone, finding the horizontal deflections, h_{hf}, along the line of the bracing at which the diagonals of the bracing intersect with the portal rafters. This will normally be at the eaves, valleys or apex.

(ii) Analyse the "miss" frame alone, finding the horizontal deflections, h_{mf}, as above.

(iii) Calculate the difference in horizontal deflection, $\delta_f = (h_{mf} - h_{hf})$. This difference will be reduced by the presence of the plan bracing.

(iv) Choose a small trial bracing load, F_t, and apply it in the plane of the bracing as follows:

(a) as a positive force on the "hit" frame, to increase the horizontal deflection from h_{hf} to h_{ht}

(b) as a negative force on the "miss" frame, to decrease the horizontal deflection from h_{mf} to h_{mt}

(c) as a pair of equal and opposite forces F_c / cos θ on the bracing truss to create a deflection b_t in the plane of the truss.

Where the truss is inclined to the horizontal at an angle θ, this will give a horizontal deflection of $b_t / \cos\theta$.

(v) Calculate the difference in horizontal deflection:

$$\delta_t = (h_{mt} - h_{ht} - b_t / \cos\theta)$$

(vi) Adjust the bracing load from F_t to F_b such that $\delta_t = 0$.

Where elastic analysis is used, the principle of elastic superposition is valid, so that the bracing load F_b is given by:

$$F_b = F_t \times [\delta_f / (\delta_f - \delta_t)]$$

In plastic analysis, the principle of elastic superposition is only valid if the hinges are the same in both the "hit" and the "miss" frames before and after the application of F_b. Therefore, in plastic frames it is important to check the analyses using F_b calculated as above to be sure that the value of F_b is correct. It is also wise to do this with elastic analysis, to ensure that there is no error.

In second-order analyses, either elastic or plastic, the principle of elastic superposition is not valid, though it may give results very close to the correct value of F_b. In these cases, and in plastic analysis where the hinges are not the same, the value of F_b must be applied to the frames and bracing to check the accuracy, and a few iterations may be required.

12 SECOND-ORDER EFFECTS AT ULTIMATE LIMIT STATE

12.1 General

12.1.1 Frame slenderness

The stability of a frame as a whole is affected by the slenderness of the members, because the second-order effects of axial compression increase the deflections beyond the deflections predicted by normal first-order analysis. The increase in deflections is greater for more slender frames. These second-order effects are commonly termed p.δ effects for members and P.Δ effects for the overall frame. Further explanation is given in Appendix A of *Plastic design of single-storey pitched-roof portal frames to Eurocode 3*[2].

Second-order effects increase not only the deflections but also the moments and forces beyond those calculated by first-order analysis. Consequently, the results obtained from first-order analysis have to be modified if the structure is classed as slender. The methods of modification are different for elastic analysis and plastic analysis, as explained below. Although the modifications involve approximations, they are sufficiently accurate within the limits given by ENV 1993-1-1.

The sensitivity of any structure to second-order effects under any individual load case can be assessed by evaluating the ratio of factored applied vertical loads, V_{Sd}, to the elastic critical buckling load of the frame, V_{cr}, under the same distribution of load. The calculation of V_{cr} is treated below. ENV 1993-1-1 gives limitations for the application of modified first-order analysis results. For relatively stiff structures, no modification is required, but where $V_{Sd} / V_{cr} > 0,1$, first-order analysis results must be modified to allow for second- order effects. For very slender structures, only second-order analysis is allowed.

Some differences between EC3 and UK practice are given in Appendix K.

12.1.2 Details that reduce frame stability

Certain details or structural layouts reduce the overall stability of a complete building. Examples of such details are pin-ended props and valley beams.

Portals are sometimes detailed with pin-ended props serving as the internal columns. A pin-ended prop tends to destabilise the whole structure, because any lateral displacement causes the prop to induce an additional lateral load, instead of inducing a restoring shear (as would be generated by a continuous column). It is therefore essential that all pin-ended members are correctly modelled as pin-ended.

Valley beams provide no stabilising effect to the whole structure and should be modelled accordingly (e.g. as sliding supports). If the detailing is poor, valley beams may be unstable and tend to destabilise the whole structure in a similar way to props.

12.2 Elastic critical buckling load

The elastic critical buckling load, V_{cr}, is the load at which the frame would buckle, if the yield stress were infinitely high and the frame material behaviour remained linear elastic however high the loading. Therefore, V_{cr} is an entirely theoretical value, which an ordinary structure could not reach because its failure load would be reduced by the onset of yield (which reduces the stiffness of the structure).

V_{cr} is a very useful value for four reasons, even though it is only theoretical:

- It shows the sensitivity to second-order effects by the ratio V_{Sd} / V_{cr}.
- It can be calculated with fewer steps than a second-order analysis, which must be incremental.
- It allows the calculation of a sufficiently accurate modification factor to be applied to normal first-order analysis in the majority of cases.
- It reflects the sensitivity to second-order effects for each load case, because the value of V_{cr} depends on the distribution of load on the structure for each load case, i.e. each load case has its own V_{cr}.

V_{cr} can be calculated by computer software or by the formulae given in Appendix B below. The use of the formulae is somewhat laborious but not complex. A simple spreadsheet could be created to perform the calculations, following Section 3 of the worked example at the end of publication. Alternatively, V_{cr} can be calculated using stability functions, such as those of Livesely and Chandler[20] and Horne and Merchant[21]. However, the use of stability functions is both laborious and relatively complex.

In ENV 1993-1-1, equations 5.6 and 5.8 are given to calculate V_{Sd} / V_{cr}. However, these equations are appropriate only for normal beam and column frames with no appreciable axial load in the beams. They are **not** appropriate for normal single-storey portal frames because, in these frames, the rafter is generally the critical element in frame stability and the axial load in the rafter is often sufficient to reduce the frame stability significantly.

12.3 Elastic analysis

12.3.1 General

According to Section 5.2.6.2 of ENV 1993-1-1, when a frame is analysed using an elastic method, the second-order effects can be allowed for by using:

(a) first-order analysis, with amplified sway moments

(b) first-order analysis, with sway-mode buckling lengths.

This Section of EC3 was written with normal beam and column multi-storey frames in mind, where second-order effects only occur in sway modes. In the case of portal frames, "sway" includes the spread of the eaves and valley, because these displacements cause P.Δ effects in the same way as sway displacements. (Equally, if the critical buckling mode of a frame is found to be symmetrical, i.e. no overall sway displacement, this buckling mode must be considered when deriving the effective lengths of the members for the appropriate load case.)

The method given in (a) is commonly called the amplified sway method and is generally recommended for single-storey portal design, especially for more asymmetric or complex geometries. The method given in (b) is commonly called the effective length method and should be expected to give a slightly more conservative design.

12.3.2 The amplified sway method

The basis of this method is that the deflections increase due to the second-order effects. Therefore, the moments and forces resulting directly from those deflections increase in proportion to the deflections. Moments and forces that do not result from those deflections do not increase.

The amplified sway method is limited by ENV 1993-1-1 to use where $V_{Sd} / V_{cr} \leq 0{,}25$ [see 5.2.6.2(4) of ENV 1993-1-1].

Two basic methods of analysis are possible. In the first method, amplified horizontal deflections are imposed on the first-order analysis. This is described in 12.3.2a below. In the second method, the unrestrained bending moment diagram and the bending moment diagram of the structure restrained horizontally at the eaves and valleys are combined. This method is explained in 12.3.2b below, with an alternative procedure given in 12.3.2c. Both methods use the ratio V_{Sd} / V_{cr}. In the absence of suitable computer software, it is recommended that the ratio V_{Sd} / V_{cr} is calculated using Appendix B of this document. These calculations can be laborious but are not very complex. It is therefore suggested that simple spreadsheets are created, if such calculations need to be performed frequently.

The procedures described in 12.3.2a, b and c below give identical results. However, 12.3.2a is more easily understood and checked because the increase in deflections is explicit. For the least design effort, the first step alone of 12.3.2b is suggested, but the resulting design will be conservative.

(a) Amplified horizontal deflections

- The first analysis is performed as usual with the ULS loads.
- The eaves and valley deflections are recorded.
- A second analysis of the same frame, but without the ULS loads, is performed, by imposing horizontal deflections at the eaves and valleys of magnitude equal to:

$$(\Delta_{horiz} \text{ first analysis}) \times \{[1 / (1 - V_{Sd} / V_{cr})] - 1\}$$

$$= (\Delta_{horiz} \text{ first analysis}) \times \{1 / [(V_{cr} / V_{Sd}) - 1]\}$$

- The design is performed with the forces from the first analysis and the moments from the first analysis plus the moments from the second analysis.

(b) Combination of unrestrained and restrained bending moment diagrams

- The first analysis is performed as normal, for the portal structure with the ULS loads amplified by $[1 / (1 - V_{Sd} / V_{cr})]$. When using computer software, this is conveniently done by amplifying the partial safety factors on the loads.

Note that this analysis has magnified both the "sway" and "non-sway" components of the bending moment diagram.

- A second analysis is performed with restraints against horizontal translation (but not rotation) at the eaves and valleys, to find the magnification of the non-sway component of the bending moment diagram in the first analysis. For this analysis, the ULS loads are reduced to $\{1 / [(V_{cr} / V_{Sd}) - 1]\} \times$ the original ULS load. When using computer software, this reduction can easily be achieved by reducing the partial safety factors on the loads.
- The design is performed with the output (axial force, moment and shear) of the first analysis minus the output of the second analysis.

(c) Alternative combination of unrestrained and restrained bending moment diagrams

- The first analysis is performed with restraints against horizontal translation (but not rotation) at the eaves and valley positions, using ULS loads $\times$ $\{1/[(V_{cr} / V_{Sd}) - 1]\}$.
- The horizontal restraint reactions at the eaves and valley are recorded.
- The second analysis is performed on the normal portal structure, using 1,0 $\times$ ULS loads plus horizontal loads at the eaves and valleys, equal in magnitude but **opposite** in direction to the eaves and valley reactions calculated from the first analysis. This additional loading increases the spread of the eaves caused by the gravity loads.
- The design is performed using the output from the second analysis.

12.3.3 The effective length method

This method is suitable in cases where the correct effective lengths, corresponding to the overall frame buckling failure mode, can be calculated. In calculating the effective lengths, the influence of the entire frame, including the effects of the axial forces in the rafters, must be considered. Individual members should not, therefore, be analysed in isolation from the rest of the structure.

The effective length data in Annex E of ENV 1993-1-1 were derived for multi-storey frames with negligible axial loads in the floor beams. They have not been validated for single-storey portals, where the axial forces in the rafters can affect the stability of the structure significantly. This is particularly true in the case of pitched roof portals.

The effective lengths can be derived from the elastic critical buckling load of the entire frame, V_{cr}, which is described in Section 12.2 and can be calculated using the formulae in Appendix B.

$$\ell_{\text{eff in-plane}} = \left(\frac{\pi^2 EI_{\text{in-plane}}}{N_{Sd}} \times \frac{V_{Sd}}{V_{cr}} \right)^{0,5}$$

where N_{Sd} is the axial compression in the member at ULS.

However, it is unnecessary to calculate ℓ_{eff} because the effective relative slenderness $\bar{\lambda}_{eff}$ for in-plane buckling can be calculated directly from:

$$\bar{\lambda}_{\text{eff in-plane}} = \left(\frac{Af_y}{N_{Sd}} \times \frac{V_{Sd}}{V_{cr}} \beta_A \right)^{0,5}$$

where $\beta_A = A_{eff} / A$ as given in Section 5.5.1.1 of ENV 1993-1-1.

The derivations of these formulae are given in Appendix J.

When using the effective length method, the cross-section checks and connection checks must also recognise the increase in moments due to second-order effects. Section 5.2.6.2(8) of ENV 1993-1-1 proposes an amplification in sway moments of at least 1,2, unless a smaller value is shown to be adequate by analysis. (The amplified moments are calculated explicitly by the amplified sway method, which is why that method is recommended.) For the effective length method, a simple and safe approximation to the amplification factor would be to use $1/(1 - V_{Sd} / V_{cr})$ in all cases. This would make the design rather more conservative than the amplified sway method.

When Appendix B is used to obtain V_{Sd} / V_{cr}, the worst (i.e. highest) of the values, calculated for the different sub-frames, must be used. The values of ℓ_{eff} and $\bar{\lambda}_{eff}$ will therefore be based on the worst V_{Sd} / V_{cr}, possibly resulting in an over-conservative design. For this reason, the amplified sway method may prove to be more economical, especially for complex frames.

12.4 Plastic analysis

12.4.1 General

The treatment outlined below is taken from Section 5.2.6.3 of ENV 1993-1-1. It is important to ensure that this Section of the ENV has not been rejected in the relevant National Application Document (NAD). Where it has been rejected, it is possible to proceed by full second-order analysis but this is complicated. It may be possible to find compatible analysis methods in the relevant national design standard instead.

When plastic methods are used for frame analysis, EC3 permits the use of first-order analysis modified by the Merchant-Rankine formula (see Appendix D), to allow for second-order effects when $V_{Sd} / V_{cr} \leq 0{,}20$.

For plastic analysis, the Merchant-Rankine criterion can be expressed as:

$$\text{Collapse load factor} = \frac{\text{collapse load}}{\text{ULS load}} \times \left(1 - V_{Sd} / V_{cr}\right) \geq 1$$

Note: When V_{Sd} is a net upward load, e.g. from a wind uplift that is greater than the coexistent gravity loads, $(1 - V_{Sd} / V_{cr})$ is taken as 1,0.

It should be noted that the Merchant-Rankine approach, applied to the bare frame, will yield conservative results because it ignores the beneficial effects of the cladding. Even without cladding, Merchant-Rankine tends to be conservative.

Where forces and moments have been calculated using the Merchant-Rankine criterion, only out-of-plane buckling need be checked, as discussed in Appendix D. The in-plane buckling deformations are accounted for by the Merchant-Rankine formula.

As expressed in ENV 1993-1-1, the method of application is not perfectly clear, but Merchant-Rankine may be applied by three methods:

- modified loading
- modified load factor (where elastic-plastic analysis is used)
- modified section capacity.

These three methods are described below.

The term load factor is used here to describe the ratio of applied load / ULS design load at any particular stage in the analysis. The collapse load factor is the load factor required to produce collapse by a plastic mechanism. The collapse load factor must not be less than 1,0, and will usually be slightly larger.

12.4.2 Modified loading to comply with Merchant-Rankine

The Merchant-Rankine criterion can be applied by performing the plastic analysis of the portal with all the applied loads, including self weight, increased by the factor:

$$\left(\frac{1}{1 - V_{Sd}/V_{cr}}\right)$$

This method is convenient for all methods of analysis. For computer analysis, it is convenient to increase the partial safety factor on the loads by multiplying by $1/(1 - V_{Sd}/V_{cr})$.

12.4.3 Modified elastic-plastic analysis to comply with Merchant-Rankine

The Merchant-Rankine criterion can be applied to first-order elastic-plastic analysis by using the factor

$$(1 - V_{Sd} / V_{cr})$$

as a reduction factor applied to the load factor calculated at each hinge formation. This can be expressed as:

Load factor = {Load factor from first-order analysis} $\times$ $(1 - V_{Sd} / V_{cr})$

This method is particularly appropriate for first-order elastic-plastic analysis, which is an incremental analysis procedure. The portal frame is considered to be loaded with the standard ULS load combination, with load increments applied as usual, but the load factor at each hinge formation is reduced (see Figure 12.1). An additional benefit of elastic-plastic analysis by this method is that safe results are obtained, even if the frame remains entirely elastic at ULS.

Note that this method enables the computer to calculate the bending moments at ULS, which may be less than the collapse mechanism load, resulting in a very economical frame.

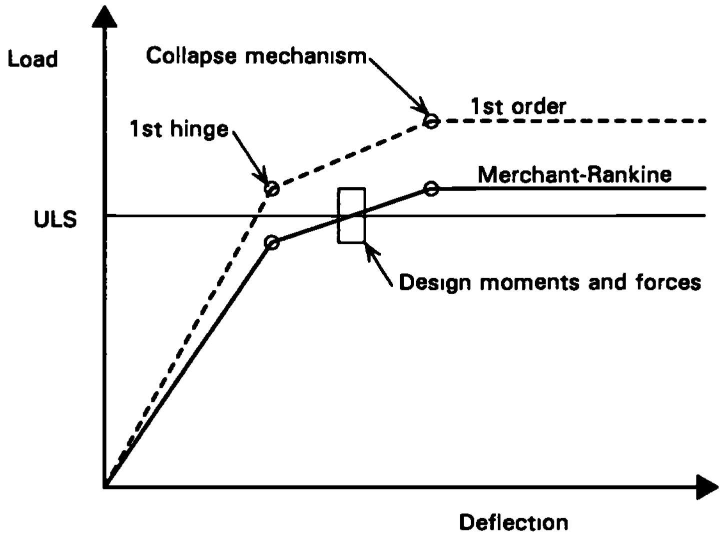


Figure 12.1 *Load/deflection diagram with reduction by Merchant-Rankine*

12.4.4 Modified section capacity to comply with Merchant-Rankine

The method, as currently given in Section 5.2.6.3(4) of ENV 1993-1-1, implies the following procedure:

(i) Choose initial portal sections.

(ii) Calculate the critical buckling load V_{cr} of the portal.

(iii) Calculate the factor V_{Sd}/V_{cr}.

(iv) Comply with Merchant-Rankine by dividing the plastic capacity of the sections by:

$$\left(\frac{1}{1 - V_{Sd}/V_{cr}}\right)$$

(v) Perform the rigid-plastic analysis of the portal frame using the plastic capacities from (iv).

(vi) Check the collapse load factor of (v) ≥ 1,0.

(vii) Either

(a) Increase all internal moments and forces (i.e. bending moment, shear force and axial force) by:

$$\left(\frac{1}{1 - V_{Sd}/V_{cr}}\right)$$

to generate a consistent set of internal forces for member stability checks

or (b) Perform checks on the first-order moments and forces using reduced resistances.

13 CROSS-SECTION RESISTANCE AND BUCKLING RESISTANCE

13.1 General

ENV 1993-1-1 requires that the resistance of cross sections and the member buckling resistance are checked by separate calculations. Additional checks are required for the resistance of webs to shear buckling and buckling due to transverse loads.

The form of calculation is generally dependent on the classification of the cross section.

Some differences between ENV 1993-1-1 and UK practice are given in Appendix K.

13.2 Classification of cross sections

The form of calculation in ENV 1993-1-1, for both cross-section resistance and member buckling resistance, depends on the classification of the cross section. The class of a section is the worst class of either the flanges or the web.

It is important to note that the classification depends on both the geometry of the cross section and the moments and forces at the cross section. For example, a typical I-beam will be Class 1 under pure moment but Class 4 under pure axial loading, and under combined loading may be any of Classes 1, 2, 3 or 4 depending on the proportions of axial load and bending moment at the cross section under consideration.

The classes denote the following structural behaviour:

Class 1 can support a rotating plastic hinge without loss of resistance.

Class 2 can develop full plastic moment but with very limited rotation capacity.

Class 3 can develop yield in extreme fibres but local buckling prevents development of plastic moment.

Class 4 local buckling will occur at stresses below first yield.

The geometrical and material requirements for member ductility in plastic portals are given in Appendix G of this publication. In portals using plastic design, the columns and rafters will generally be chosen to be Class 1 or Class 2, which are both able to develop plastic resistance. The only element that may be different is the haunch web, which may be Class 3, according to Section 5.3.3 of ENV 1993-1-1.

ENV 1993-1-1 does not give a method of classification for sections carrying less than the full bending and/or axial resistance. The following method is recommended:

Take N = N_{Sd} from the global analysis

Take M = the maximum moment that the cross section can resist when applied together with N_{Sd}.

Classify the section on the basis of N and M as calculated above.

13.3 Member ductility for plastic design

For plastic design, particular attention must be paid to the cross section of the members (and material properties), especially at hinges and in the haunches. The provisions of ENV 1993-1-1 are given in Appendix G of this publication.

If haunches are used, as shown in Figures 1.1 and 2.1, the collapse bending moment diagram and the elastic bending moment diagram will be very similar. This limits the plastic rotations in the collapse mechanism, so that stability under rotation is not as onerous as in portal frames without haunches.

13.4 Cross-section resistance

Cross-section resistance is treated in Section 5.4 of ENV 1993-1-1.

Due to strain hardening, the interaction of a modest axial force and/or shear force with the bending moment does not reduce the resistance to the extent predicted by pure plastic theory. This is recognised by EC3.

The calculations for Class 1 and Class 2 cross sections are given in Appendix C of this document in a simplified format.

Sections for which flanges are Class 1 or Class 2, but for which the web is Class 3, may be checked according to Appendix E of this publication.

13.5 Member buckling resistance

13.5.1 General

Member stability is considered in terms of a non-dimensional relative slenderness, $\bar{\lambda} = (N_{p\ell} / N_{cr})^{0,5}$ for compression and $\bar{\lambda}_{LT} = (M_{p\ell} / M_{cr})^{0,5}$ for lateral torsional buckling. This is similar to the common practice of using (effective length)/(radius of gyration).

Member buckling resistance is checked using the same buckling curve for both axial compression and lateral torsional buckling. The only difference is the length of plateau of slenderness for which there is no reduction for buckling. For compression the limit is $\bar{\lambda}$ = 0,2 and for lateral torsional buckling it is $\bar{\lambda}_{LT}$ = 0,4, which gives a step from the plateau at yield stress down to the buckling curve, as shown in Figure 13.1. For plastic design, particular attention must be paid to the stability and ductility of the cross section of the members, especially at hinges and in the haunches. The provisions of ENV 1993-1-1 are given in Appendix G of this publication.

13.5.2 Members without plastic hinges

Member buckling resistance is treated in Section 5.5 of ENV 1993-1-1.

The requirements for plastic portal frames are given in Appendix G of this publication. In portal frames using plastic design, the columns and rafters will generally be chosen to be Class 1 or Class 2, which have the same rules for checking resistance. The only element that might be different is the haunch web, which can be Class 3, according to Section 5.3.3 of ENV 1993-1-1.

The calculations for Class 1 and Class 2 cross sections are given in Appendix D of this publication in a simplified format.

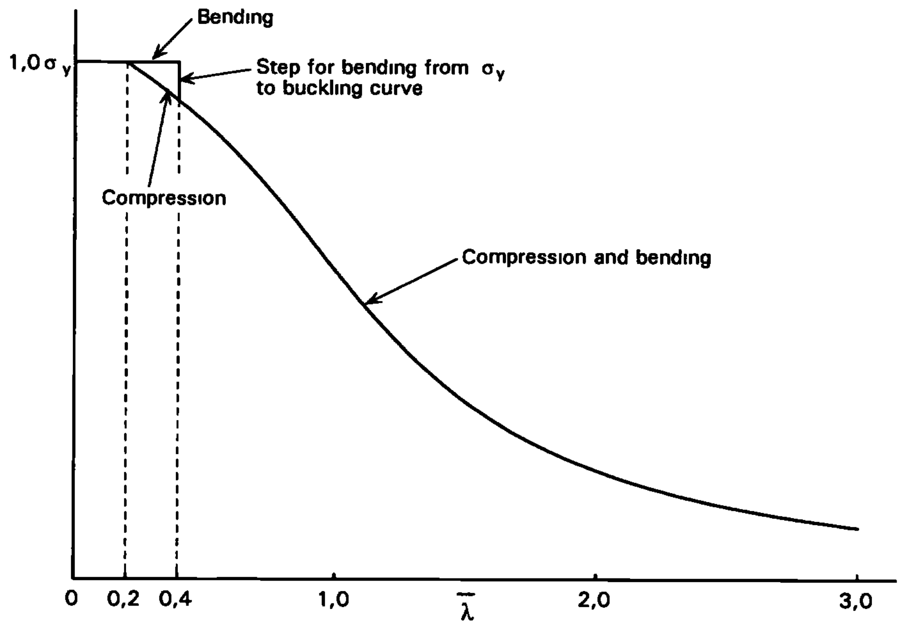


Figure 13.1 *Typical buckling curve*

13.5.3 Members with plastic hinges

ENV 1993-1-1 gives insufficient guidance on the stability of members with plastic hinges, so additional guidance is given in Appendix D of this publication. Section 15 gives guidance for the design of restraints required at plastic hinge locations.

Haunches should not contain plastic hinges within their length, as explained in Section 13.5.5.

13.5.4 Restrained members with an unrestrained compression flange

ENV 1993-1-1 does not explain how to allow for the stability afforded by this arrangement of restraint, but Appendix F.3 of this publication shows how this may be done using the methodology of ENV 1993-1-1 or that standard.

13.5.5 Haunches

The original research by Horne *et al.*[9,10], on which the advice in this publication is based, specifically excluded plastic hinges within the length of the haunch. Instead, it was assumed that the hinge formed in the rafter, adjacent to the shallow end of the haunch. Therefore, the haunches should be sized so that there are no hinges within the length of the haunch at or below ULS.

13.6 Web buckling resistance to shear and transverse forces

The resistance of webs to buckling caused by shear and transverse forces is given in Sections 5.6 and 5.7 of ENV 1993-1-1. In single-storey portal frames fabricated from hot rolled sections it is very rare for web buckling to be a design criterion.

14 CONNECTIONS

14.1 General

Connections are treated in Section 6 of ENV 1993-1-1.

The applied forces must include:

- second-order effects, if there are any
- the effects of imperfections (see Section 5.2.4 of ENV 1993-1-1)
- the effects of connection flexibility in the case of semi-rigid connections (see Section 6.9 of ENV 1993-1-1). Normally, portal connections will be detailed so that they are rigid, not semi-rigid.

Connections in portal frames will normally be classed as rigid partial-strength connections or rigid full-strength connections, according to Section 6.4 of ENV 1993-1-1. The rigidity is assisted by the depth of the connection produced by the long haunches at the eaves and valley, and the short haunches used at the apex connections.

The design of the connections between the principal frame members (eaves, apex, valley) should recognise the possibility that the plastic moment of resistance of the member is greater than the calculated value. To avoid rupture of the connections, if the members are over-strength, the non ductile parts (the bolts and welds) should be designed for the effects of the possible higher moment, unless this can be proved to be irrelevant, e.g. by ensuring that there is connection rotation capacity. In normal end-plate connections, there is no need to design the end plate for the effects of the higher moment, as the end plates are ductile. Appendix K9 explains why this has not been explicitly calculated in UK practice.

ENV 1993-1-1 gives a possible method of allowing for over-strength members by using 1,2 $\times$ the analysis results, as in Section 6.4.3.2 of ENV 1993-1-1. This is the simplest design procedure.

A more economical design might be achieved by calculating the increase in the forces at ULS that the connections would experience if the frame were to remain elastic. This increase is the reciprocal of the fraction of ULS loading at which the first hinge forms. For example, if the first hinge is calculated to form in the column at 0,90 $\times$ ULS, based on the minimum guaranteed yield strength of the member, the maximum haunch connection design moment at ULS is 1/0,90 = 1,11 $\times$ the calculated moment, even if the yield strength is so high that the member remains elastic at ULS.

Some differences between EC3 and UK practice are given in Appendix K.

14.2 Bolted connections

Bolted connections are covered by Section 6.5 of ENV 1993-1-1. The rules for positioning the holes, e.g. spacing, end distance and edge distance are given in Section 6.5.1. It should be noted that the Principles are all contained within

Section 6.5.1.1 (as explained in Section 1.2 of ENV 1993-1-1), while the remaining Sections of 6.5.1 contain the Application Rules.

Haunched connections will normally be Category D according to Section 6.5.3 of ENV 1993-1-1. These are connections with non-preload bolts, up to and including grade 10.9.

The design resistance of bolts is treated in Section 6.5.5 of ENV 1993-1-1, with additional rules for countersunk bolts in Section 6.5.7.

Prying forces must be allowed for in the calculation of the force in the bolt (see Section 6.5.9 of ENV 1993-1-1). Prying forces are not allowed for in either the partial safety factor γ_{Mb} or in the strengths in Table 3.3 of ENV 1993-1-1. This is discussed further in Appendix K of this publication.

14.3 Welded connections

Welded connections are covered by Section 6.6 of ENV 1993-1-1.

The resistance of fillet welds is given in Section 6.6.5 of ENV 1993-1-1. The resistance per unit length may be calculated either by the simple formula given in Section 6.6.5.3 of ENV 1993-1-1 or by the more complicated method presented in Annex M of ENV 1993-1-1.

14.4 Splices

Splices are treated in Section 6.8 of ENV 1993-1-1.

14.5 Beam-to-column connections

Beam-to-column connections are treated in Section 6.9 of ENV 1993-1-1. This Section might appear to be so full of detail as to be very expensive in design time, but the following comments will help to avoid unnecessary work.

14.5.1 Rotational stiffness

A beam-to-column connection may be classified as rigid (or nominally pinned) on the basis of particular or general experimental evidence, or on **significant experience of previous satisfactory performance in similar cases** or by calculation based on test evidence. This is stated in Section 6.9.6.2 Paragraph 2 of ENV 1993-1-1 and is a Principle. Paragraphs 3 to 7 are only Application Rules and do not override paragraph 2. Therefore, portal joints of traditional proportions may be considered to be rigid, as has always been the case in UK practice, since there has certainly been significant experience of previous satisfactory performance in this case.

14.5.2 Moment resistance

In plastic design, the formation of plastic hinges limits the bending moments and so limits the moments at the connections. Where members have higher resistances than assumed in the design (most commonly due to the yield stress being greater than the guaranteed minimum), the moments may be higher than the analysis would suggest. This is discussed in Section 14.1 above.

14.5.3 Calculations

Annex J of ENV 1993-1-1:1992/A2:1998 includes the yield line method applied to connection design, which should enable designers to reduce connection costs below those required in countries that have traditionally required very conservative connection design.

14.6 Bases

Column base design is considered in Section 6.11 of ENV 1993-1-1. The requirements can be satisfied by following Annex L.

15 BRACING

15.1 General

Bracing is required to resist lateral loads, principally wind loads, and the destabilising effects of the imperfections defined in Section 5.2.4 of ENV 1993-1-1. This bracing must be correctly positioned and have adequate strength and stiffness to justify the assumptions made in the analysis and member checks.

Section 5.2.4 of ENV 1993-1-1 allows the imperfections to be described either as geometrical imperfections or as equivalent horizontal forces.

The equivalent horizontal forces, which cause the forces in the bracing, do not increase the **total** load on the **whole** structure, because they form a self equilibrating load case.

15.2 Overall column bracing

15.2.1 General

Bracing must be provided to hold the columns upright and resist loads, such as wind loading, occurring at right angles to the frame. Columns are assumed to be built slightly out of vertical, and the simplest method to account for this effect is by means of the equivalent horizontal forces shown in Figure 5.2.3 of ENV 1993-1-1 and considered further in Section 3 of *Plastic design of single-storey pitched-roof portal frames to Eurocode 3*[2]. These forces can occur in any direction, but are considered to act in one direction at a time.

The bracing is designed to resist wind loads plus equivalent horizontal forces. The equivalent forces are given by Section 5.2.4.3 of ENV 1993-1-1, and amount to approximately 0,5% vertical forces causing axial compression. Care must be taken when detailing the bracing, as details that reduce the stiffness (e.g. by deflection from local bending of plates, rings or angles) might increase the imperfection loads due to second-order effects.

If the columns carry a nett tensile force, as in an uplift load case due to wind, this loading does not destabilise the structure, so may be neglected in calculating the equivalent forces.

15.2.2 Portal bracing

The term "portal bracing" is commonly used to describe a system of bracing comprising portals instead of cross bracing to provide the restraint normal to the main frames. An example of portal bracing is shown in Figure 15.1. It is frequently used to provide lateral stability to the top of the internal columns, because the use of cross bracing would result in an unacceptable restriction to freedom of use. It is also used at exterior walls, where cross bracing might obstruct windows, doors, etc. These bracing portals are classified as "bracing systems" in Section 5.2.5.3 of ENV 1993-1-1.

The bracing portals are designed to resist the total equivalent horizontal forces from all the columns that rely on those portals for stability, plus the relevant wind load.

The bracing portals are subject to frame stability considerations similar to those explained in Section 12. However, some simplifications are possible.

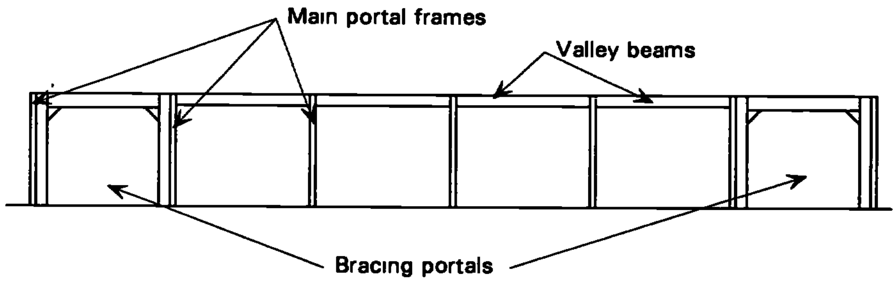


Figure 15.1 *Braced portals*

First-order analysis results need not be modified to allow for second-order effects where $V_{Sd} / V_{cr} \leq 0{,}1$ ("non-sway" to Section 5.2.5.2 of ENV 1993-1-1). This is most likely in practice. In this case, V_{Sd} is the sum of the vertical loads on all the columns that rely on the bracing portal for stability and V_{cr} is the critical load for the whole group of columns restrained by the bracing portal.

V_{Sd} / V_{cr} can be calculated using the formulae in Section B.4 of Appendix B, taking $N_c = 0{,}5\ V_{Sd}$ and $s = 0{,}5 \times$ portal span. Generally, due to the short span of bracing portals and the relative stockiness of the beam, the effect of axial load on the stiffness of the beam can be ignored and the following approximation can be used:

$$V_{Sd} / V_{cr} = (V_{Sd} / h)(\delta_h / H)$$

where

h is the column height of the bracing portal

δ_h / H is the horizontal sway stiffness of the portal, δ_h being the horizontal deflection of the portal beam due to the horizontal force H applied at the neutral axis of the beam. It may be convenient to use H = (wind + equivalent horizontal forces).

If $V_{Sd} / V_{cr} > 0{,}1$ (i.e. it is a sway frame according to Section 5.2.5.2 of ENV 1993-1-1), second-order effects must be considered, as in either Section 12.3 or 12.4 of this publication, as appropriate. Generally, elastic design will be used and the first step of 12.3.2(b) is the only analysis step that is required, because the loading is entirely sway loading.

Section B.3 does not apply, as arching action is not involved.

15.3 Plan bracing

Bracing must be provided in the planes of the rafters to give strength and stiffness to the restraint points assumed in the member buckling checks. In addition, the bracing must resist any forces applied at right angles to the frames, such as wind loads.

The loads in the plan bracing due to imperfections are given by Section 5.2.4.4 and Figure 5.2.5 of ENV 1993-1-1. The deflection of the bracing is considered in the ENV 1993-1-1 method, if the deflection is relatively large. Adequately fixed sheeting will help to resist plan bracing loads, but the use of the sheeting for this purpose might be prohibited in certain countries and its effectiveness might be difficult to demonstrate.

15.4 Bracing to inner flanges

Bracing to the inner flanges is often most conveniently formed by diagonal struts from the purlins or sheeting rails to gluts welded to the inner flange and web, as shown in Figure 2.1. If flats are used as diagonals, only one of the pair will be effective, so the strength may be impaired. Alternatively, a member fitted to the web of the column with an end plate connection may be more convenient. The proportions of the member and connection must then be chosen to avoid reducing the overall stiffness of the restraint by the local flexibility in the web of the column.

The effectiveness of such bracing depends on the stiffness of the system, especially the stiffness of the purlins. The effect of purlin flexibility on the bracing is shown in Figure 15.2. Where the proportions of the members, purlins and spacings differ from proven previous practice, the effectiveness should be checked. This can be done using the formula given in Section 15.5, or other methods, such as may be found in bridge codes for U-frame action.

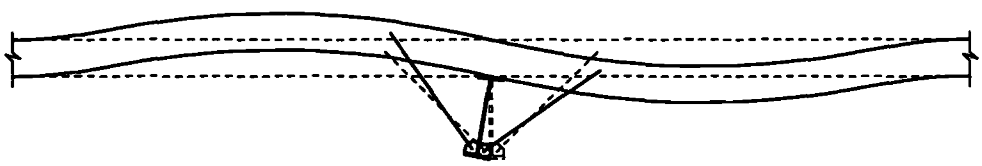


Figure 15.2 *Effect of purlin flexibility on bracing*

The compression flange of eaves and valley haunches must be braced at the column-haunch connection, unless stability analysis or test data prove it to be unnecessary. This restraint is needed, because the haunch stability checks are based on full torsional restraint at this point.

15.5 Bracing at plastic hinges

Section 5.2.1.4(4) of ENV 1993-1-1 recommends that bracing should be provided to both tension and compression flanges at or within 0,5h of the calculated plastic hinges, where h is the depth of the member (see Figure 15.3). This is good practice, even though Horne and Ajmani[11,12,13] and CSC (UK) Ltd[14] have shown that, for appropriate proportions of members, hinges have a large rotation capacity even when they occur mid-way between points of restraint.

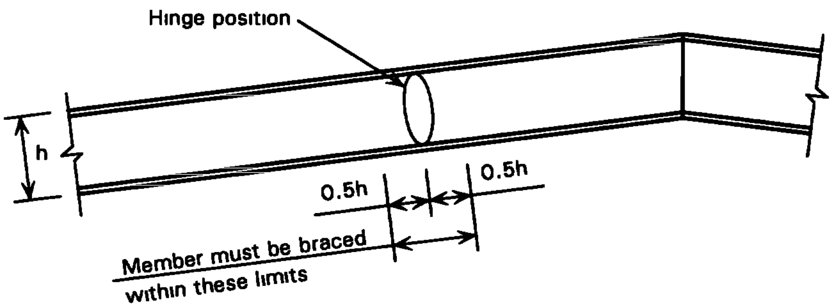


Figure 15.3 *Bracing at plastic hinges*

ENV 1993-1-1 does not specifically consider the strength and stiffness requirements of bracing at plastic hinges, which are more onerous than for elastically designed elements. It is recommended that the bracing to a plastic hinge should be designed assuming that the compression flange exerts a lateral load of 2,5% (plastic moment resistance/depth of section) at right angles to the web of the member.

Where the plastic hinge is braced by diagonals from the purlins (see Figure 2.1), the stiffness of the "U-frame" formed by the purlin and diagonals is especially important. Where the proportions of the members, purlins or spacings differ from proven previous practice, the effectiveness should be checked. In the absence of other methods, the stiffness check may be based on the work of Horne and Ajmani[13]. Thus, the support member (the purlin or sheeting rail) should have I_s such that:

$$\frac{I_s}{I_f} \geq \frac{f_y}{190 \times 10^3} \; \frac{B\,(L_1 + L_2)}{L_1\, L_2}$$

where f_y is the yield stress of the portal frame member

I_s is the second moment of area of the supporting member (purlin or sheeting rail) about the axis parallel to the longitudinal axis of the frame member (i.e. the purlin major axis in normal practice)

I_f is the major axis second moment of area of the frame member

B is the span of the purlin or sheeting rail

L_1 and L_2 are the distances either side of the plastic hinge to the eaves (or valley) or points of contraflexure, whichever are the nearest to the hinge (see Figure 15.4).

Hinges that form, rotate then cease, or even unload and rotate in reverse, must be fully braced. However, hinges that occur in the collapse mechanism but rotate only above ULS need not be considered as plastic hinges for ULS checks. These hinges are easily identified by elastic-plastic or graphical analysis, but are not shown by virtual-work (rigid-plastic) analysis. However, it is important to note that the mathematics of the analysis can calculate the presence of hinges that form then disappear at the same load factor. This indicates that no rotation takes place and, therefore, no hinge occurs. In these cases, it is not necessary to provide the usual restraint associated with plastic hinges; only the restraints for normal elastic stability are required.

Analysis cannot account for all of the section tolerances, residual stresses and material tolerances. Care should be taken to brace points where these effects could affect the hinge positions, e.g. the shallow end of the haunch instead of the top of the column. Wherever the bending moments come close to the plastic moment capacity, the possibility of a hinge should be considered.

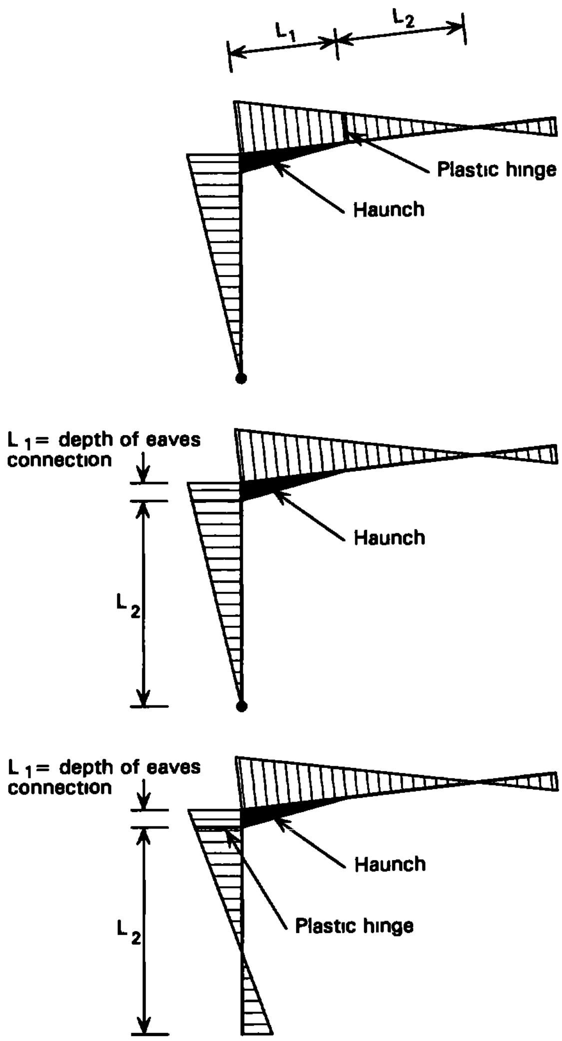


Figure 15.4 *Lengths for checking "U-frame" stability*

16 PORTALS WITH MEZZANINE FLOORS

16.1 General

Mezzanine floors are frequently required within single-storey portal buildings to provide office space. They are not usually designed as fully continuous moment resisting structures and usually rely on the main portal columns for their stability and for support of one end of the floor and ceiling beams.

16.2 Overall stability

The overall stability in the plane of the portal is usually provided by the portal columns. For this reason, the frame imperfection equivalent horizontal force (Section 5.2.4.3 of ENV 1993-1-1) from the whole mezzanine width must be applied in the plane of the portal to the portal column, in addition to the appropriate vertical reaction from the mezzanine. This force should be applied at the level of the mezzanine.

The overall stability perpendicular to the plane of the portal may be provided entirely by the portal columns, or partly by a bracing system in the plane of the mezzanine columns.

These initial imperfections apply in all horizontal directions, but need only be considered in one direction at a time.

16.3 Application of mezzanine loads to columns

The appropriate loads from the mezzanine must be applied to the columns. Guidance on the point of application of the load is given in ENV 1993-1-1, Annex H *Modelling of building structures for analysis*. The essential philosophy of the rules is that the assumptions made in the design of the members and connections should be consistent. There is a wide choice of assumptions as follows:

(1) Beams should be modelled as simply supported by the columns. Generally, beams in simple frames are single span, but where they are physically continuous at their supports they may be modelled as continuous beams.

(2) The spans of the beams should be determined on the basis of support locations that are consistent with the assumptions made in modelling the columns and the joints.

(3) The supports of the beams may generally be modelled as pinned connections, located either:

   (a) on the centreline of the column, provided that the connection can resist the moments derived from the distance between the connection to the beam and the centreline of the column

(b) at the face of the column, provided that the connection can resist the moments derived from the distance between the connection to the beam and the face of the column

(c) at the centroid of a group of bolts or welds connecting the beam web to supporting cleats or brackets, provided that the column is designed for any moments resulting from the eccentricity

d) at the centreline of a supporting bracket under the beam, provided that the column is designed for any resulting eccentricity.

(4) The supports of beams on cap-plates of columns should be modelled as located at the face of the column, unless detailed otherwise.

(5) Generally, columns in simple framing are physically continuous at the level of the connections of the beams. In such cases, they may be modelled as continuous members.

(6) Alternatively, continuous columns should be modelled as axially loaded simple struts. In this case, the moments due to eccentricities of the beam supports should be added.

(7) The design of the components of the beam-to-column joints should be consistent with the assumptions made in modelling the beams and the columns.

16.4 Member design

The procedure for the design of members is similar to that for the portal frame members. When selecting effective lengths, particularly of the mezzanine columns, care should be taken to observe the requirements of the appropriate National Application Document.

17 DESIGN PROCEDURES

17.1 Choice between elastic and plastic design

The following considerations will help choose between elastic and plastic design.

Plastic design will normally allow the most economical structures, especially for multi-bay frames fabricated from hot rolled sections. The acceptability of plastic design and the proposed method of analysis should be checked, as design traditions differ from country to country.

Elastic design will normally be sufficient for frames for which deflection is the governing criterion. These will include frames that are relatively tall.

Elastic design is recommended for the design of tied portal frames.

Elastic design is a common method in all countries, whereas plastic design may be accepted with reluctance and with onerous requirements for stability or for analysis methods, e.g. second-order analysis only.

17.2 Plastic design – ultimate limit state

The following procedure is suggested:

(1) Define frame geometry, determine loads, load combinations, γ factors and ψ factors (see Section 11).

(2) Choose trial sections and trial haunch lengths by selecting beam sections that have resistances at least equal to the following:

$$\text{Rafter } M_{pl} = \frac{wL^2}{24}$$

$$\text{Haunch } M_{pl} = \frac{wL^2}{10}$$

$$\text{Column } M_{pl} = \frac{wL^2}{12} \times \frac{\text{height to bottom of haunch}}{\text{(height to centreline of rafter)}}$$

where w is the maximum ULS gravity load/unit length along the span and L is the span of the portal.

The haunch length should be chosen to optimise the overall portal structure. A length of $L / 10$ from the column face is a reasonable initial choice, but the proportions of the haunch generally depend of the characteristics of each individual building, especially the size of the rafter (see Section 6.3). A haunch length of $L / 10$ will normally place the first hinge in the top of the column. A rather longer haunch will place the first hinge at the sharp end of the haunch.

(3) Calculate frame imperfection equivalent forces, referring to Section 5.2.4.3 of ENV 1993-1-1 and Section 15 of this publication. This can be by a preliminary frame analysis (which is necessary for all but the simplest buildings) or by a suitable approximation.

(4) Perform plastic analysis of the frame (see Section 12.4) assuming:

(a) no reduction in plastic moment of resistance from coexistent axial and shear forces. This approach may need modifying, where axial loads are high, e.g. in tied portals, in portals with steep slopes and in portals with heavy roofing loads.

(b) A trial value of $V_{Sd} / V_{cr} = 0,12$ unless a better estimate is possible.

Note that in uplift cases, the members might be subject to axial tension. In this case, there will be no destabilisation of the frame and V_{Sd} / V_{cr} can be taken as zero.

(5) Calculate an accurate value of V_{Sd} / V_{cr} (see Section 12 and Appendix B).

(6) If (accurate V_{Sd} / V_{cr}) >(trial V_{Sd} / V_{cr}) or if a more refined design is required, return to Step 4.

Note: For relatively slender frames, it is often wise to check the deflections at the serviceability limit state (SLS), as explained in Section 17.4, before checking the buckling resistance.

(7) For the columns check that:

(a) The classification is Class 1 or Class 2 as appropriate, as explained in Section 13 and Appendix C; Sections 13.1, 13.2, 13.3 and Appendix C.2 are particularly relevant.

(b) The cross-sectional resistance is adequate, as explained in Section 13 and Appendix C; Sections 13.4 and Appendix C.4 and C.6 are particularly relevant.

(c) The resistance of the member to minor axis buckling between lateral restraints is adequate, as explained in Section 13 and Appendix D, together with the appropriate parts of Appendix F; Section 13.5 is particularly relevant.

For elements with a plastic hinge within the length or at either end, Appendix D.4 should be used (which contains the same check as Appendix F.4).

For elements resisting moments and forces close to the plastic resistance, Appendix D.4 might need to be used, because Appendix D.3.4 cannot be satisfied.

For elements without plastic hinges and not very highly stressed, Appendix D.3.4 should be used, together with Appendix F.2, with L taken as the distance between lateral restraints, e.g. between purlins.

(d) Resistance of member to minor axis buckling between torsional restraints is adequate, as explained in Section 13 and Appendix D, together with the appropriate parts of Appendix F.

The method of checking is similar to 7c above, but Appendix F.3 should be used instead of Appendix F.2.

(8) For the rafters check that:

(a) The classification is Class 1 or Class 2 as appropriate, as step 7a above.

(b) The cross-sectional resistance is adequate, as step 7b above, but without Appendix C.6.

(c) The resistance of the member to minor axis buckling between lateral restraints is adequate, as step 7c above.

(d) The resistance of the member to minor axis buckling between torsional restraints is adequate, as step 7d above.

(9) For the haunches check that:

(a) The classification is Class 1 or Class 2 as appropriate, as step 7a above, but using Appendix E where the web slenderness is Class 3.

(b) The cross-sectional resistance is adequate, as step 7b above, but checking at several cross sections within the length of the haunch (both ends, quarter, mid-span and three-quarter points) is recommended. Appendix E.2 or Appendix E.3 should be used where the web slenderness is Class 3.

(c) The resistance of the member to minor axis buckling between lateral restraints is adequate, as Step 7c above, but giving special attention to the effect of the taper. These effects are described in Section 13.5.4, Appendix D.3.4.2, Appendix D.5 and Appendix E.4.

(d) The resistance of the member to minor axis buckling between torsional restraints is adequate, as Step 7d above, but giving special attention to the effect of the taper as noted in Step 9c above and to Appendix F.3.3.2b and F.3.3.2c.

(10) Check web buckling resistance to shear and transverse forces, as explained in Section 13.6.

(11) Check the connections, as described in Section 14.

(12) Check the restraints, as described in Section 15.

17.3 Elastic design – ultimate limit state

The following procedure is suggested:

(1) Define frame geometry, determine loads, load combinations, γ factors and ψ factors (see Section 11).

(2) Choose trial sections and trial haunch lengths by selecting beam sections that have resistances at least equal to the following:

$$\text{Rafter } M_{pl} = \frac{wL^2}{24}$$

$$\text{Haunch } M_{pl} = \frac{wL^2}{10}$$

$$\text{Column } M_{pl} = \frac{wL^2}{10} \times \frac{\text{height to bottom of haunch}}{\text{(height to centreline of rafter)}}$$

where w is the maximum ultimate limit state gravity load/unit length along the span and L is the span of the portal.

The haunch length should be chosen to optimise the overall portal structure. A length of $L / 10$ from the column face is a reasonable initial choice, but the proportions of the haunch generally depend on the characteristics of each individual building (see Section 6.3). A rather longer haunch will normally allow the rafter section to be reduced.

(3) Calculate frame imperfection equivalent forces, referring to Section 5.2.4.3 of ENV 1993-1-1 and Section 15 of this publication. This can be by a preliminary frame analysis (which is necessary for all but the simplest buildings) or by a suitable approximation.

(4) Perform elastic analysis of the frame (see Section 12.3) assuming:

(a) no reduction in plastic moment of resistance from coexistent axial and shear forces. This approach may need modifying where axial loads are high, e.g. in tied portals, in portals with steep slopes and in portals with heavy roofing loads.

(b) trial $V_{Sd} / V_{cr} = 0,12$ unless a better estimate is possible.

(5) Calculate an accurate value of V_{Sd} / V_{cr}, as explained in Section 12 and Appendix B.

(6) If (accurate V_{Sd} / V_{cr}) > (trial V_{Sd} / V_{cr}) or if a more refined design is required, return to Step 4.

Note: For relatively slender frames, it is often wise to check the deflections at the SLS, as explained in Section 17.4, before checking the buckling resistance.

(7) For the columns check that:

(a) The classification is Class 1 or Class 2 as appropriate, as explained in Section 13 and Appendix C; Sections 13.1, 13.2, 13.3 and Appendix C.2 are particularly relevant.

(b) The cross-sectional resistance is adequate, as explained in Section 13 and Appendix C; Sections 13.4 and Appendix C.4 and Appendix C.6 are particularly relevant.

(c) The resistance of the member to minor axis buckling between lateral restraints is adequate, as explained in Section 13 and Appendix D, together with the appropriate parts of Appendix F; Section 13.5 is particularly relevant.

For elements resisting moments and forces close to the plastic resistance, Appendix D.4 might need to be used, because Appendix D.3.4 cannot be satisfied.

Generally, Appendix D.3.4 should be used, together with Appendix F.2 with L taken as the distance between lateral restraints, e.g. between purlins.

(d) The resistance of the member to minor axis buckling between torsional restraints is adequate, as explained in Section 13 and Appendix D, together with the appropriate parts of Appendix F.

The method of checking is similar to 7c above, but Appendix F.3 should be used instead of Appendix F.2.

(8) For the rafters check that:

(a) The classification is Class 1 or Class 2 as appropriate, as Step 7a above.

(b) The cross-sectional resistance is adequate, as step 7b above, but without Appendix C.6.

(c) The resistance of the member to minor axis buckling between lateral restraints is adequate, as Step 7c above.

(d) The resistance of the member to minor axis buckling between torsional restraints is adequate, as Step 7d above.

(9) For the haunches check that:

(a) The classification is Class 1 or Class 2 as appropriate, as Step 7a above, but using Appendix E where the web slenderness is Class 3.

(b) The cross-sectional resistance is adequate, as Step 7b above, but checking at several cross sections within the length of the haunch (both ends, quarter, mid-span and three-quarter points are recommended). Appendix E.2 or Appendix E.3 should be used where the web slenderness is Class 3.

(c) The resistance of the member to minor axis buckling between lateral restraints is adequate, as Step 7c above, but giving special attention to the effect of the taper. These effects are described in Section 13.5.4, Appendix D.3.4.2, Appendix D.5, and Appendix E.4.

(d) The resistance of the member to minor axis buckling between torsional restraints, as Step 7d, but giving special attention both to the effect of the taper as noted in Step 9c above and to Appendix F.3.3.2b and Appendix F.3.3.2c.

(10) Check web buckling resistance to shear and transverse forces, as explained in Section 13.6.

(11) Check the connections, as described in Section 14.

(12) Check the restraints, as described in Section 15.

17.4 Serviceability limit state

For relatively slender frames, it is normally wise to check the deflections at the SLS before checking the member buckling resistance at the ULS, i.e. after step 6 in Sections 17.2 and 17.3.

The following procedure is suggested:

(1) Determine SLS load combinations and ψ factors, as described in Section 10.

(2) Perform first-order analysis.

(3) Check that $V_{Sd} / V_{cr} < 0{,}1$ to allow second-order effects to be ignored (see Section 12 and Appendix B).

(4) If $V_{Sd} / V_{cr} > 0{,}1$, re-analyse including magnification factor, as described in Section 12.

(5) If an elastic analysis method is used, check that the structure remains elastic.

(6) Check that the deflections do not impair the function of building, including the cladding and doors.

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APPENDIX A COLUMN BASE STIFFNESS

According to Section 5.2.3.3 of ENV 1993-1-1, analysis should take account of the rotational stiffness of the bases. To avoid soil mechanics calculations, the following rules from modern UK practice are suitable. However, these might not be known or accepted in other countries and the relevant National Application Document and the local regulatory authorises should be consulted.

It is important to distinguish between column base resistance and column base stiffness. Column base resistance is only relevant to elastic-plastic or rigid-plastic calculations of frame resistance, not to deflections. Column base stiffness is relevant to elastic-plastic or elastic frame analysis for resistance and deflection. However, the base stiffness can only be included if the column foot and base details have sufficient resistance to sustain the calculated moments and forces.

A.1 Pinned and rocker bases

Where a true pin or rocker is used, the rotational stiffness is zero.

A.2 Fixed bases

Bases considered as "fixed" are not infinitely rigid and the rotational stiffness may conveniently be taken as equal to the column stiffness, as in Clause 5.1.3 of BS5950-1[3] and *Advisory Note AD097 Nominal base stiffness*[22]. This means that the base may be modelled as a spring with rotational stiffness of $4EI_{column}/L_{column}$.

If the program cannot accept a rotational spring, the simplest solution is to add a dummy member, continuous with the column and pinned at the far end, of stiffness EI_{column} and of length 0,75 L_{column} (to allow for the pinned end which reduces the stiffness) (Figure A.1).

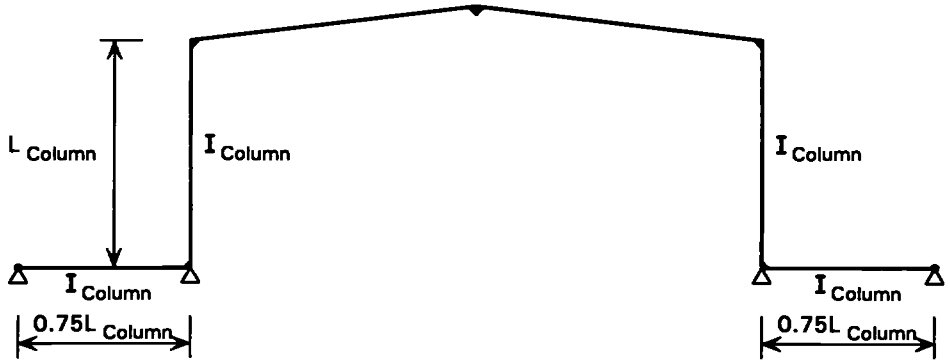


Figure A.1 *Alternative modelling of fixed base flexibility*

Note that the reaction at the pinned end of the dummy member will affect the reaction at the column base. This must be "corrected" by taking the base reaction equal to the axial force in the column, which equals the sum of the reactions at the base and the pinned end of the dummy member.

A.3 Nominally pinned bases

A.3.1 Ultimate limit state

Most simple connections from column to base, e.g. an end plate with four holding down bolts, are not truly pinned but have some rotational stiffness. This may be taken as 10% of the column stiffness, as in Clause 5.1.3 of BS5950-1[3], which means that the base may be modelled as a spring with rotational stiffness of $0,4_{EIcolumn} / L_{column}$. However, the designer should establish the implications of using such a base moment because it may have to be included in the foundation design.

If the program cannot accept a rotational spring, the model can be made as in Section A.2 above **but** with I of the dummy member equal to $0,1\,I_{column}$. Note that the same "correction" of column base reaction must be made as in Section A.2 above.

A.3.2 Serviceability limit state

For the serviceability limit state, the stiffness may be taken as 20% of the column stiffness, as in Clause 5.1.3 of BS 5950-1[3] and *Advisory Note AD090 Deflection limits - portal frames*[23].

APPENDIX B ELASTIC CRITICAL BUCKLING LOAD

B.1 General

General explanations of the elastic critical buckling load, V_{cr}, and its use are given in Section 12.1 above. V_{cr} can be calculated by special computer software modules. However, these are not commonly available in structural design offices. Alternatively, V_{cr} may be found by the use of stability functions, e.g. those of Livesley and Chandler[20] and Horne and Merchant[21], but this method can be unwieldy and is only recommended for highly irregular frames or to resolve special problems.

A method of calculation that does not require a computer is presented in this Appendix. The method is derived from work by Davies[24,25] with extensions to allow for the slight flexibility (which must be included to comply with Section 5.2.3.3 of ENV 1993-1-1) of nominally rigid bases and the modest stiffness of nominally pinned bases, as recommended in Appendix A of this publication.

B.2 Simplified method

This method establishes an upper bound of V_{Sd} / V_{cr} for each load case, which can be used to calculate the second-order effects.

B.2.1 Approximations

The elastic critical buckling load is not affected significantly by the distribution of transverse load along the members, so only the axial loads need be considered. This has been demonstrated by Horne and Merchant[21] and is used extensively in practice.

The maximum axial load in each member is assumed to act along its full length and the stiffening effect of haunches is ignored. These are both conservative assumptions.

Axial loads should be calculated from an elastic analysis. They may be calculated by computer analysis or by using standard results, see *Steel designers manual*[26], Rahmenformeln[27] and Mehrstielige Rahmen[28] assuming fully pinned/fixed bases for the buckling analysis of nominally pinned/fixed bases. Any slight error will be offset by the two conservative assumptions above.

B.2.2 General procedure

The frame is considered as a series of subdivisions (see Figure B.1) including:

(i) Rafter pairs, see Section B.3.

(ii) External column + rafter, see Section B.4.

(iii) Internal column + rafter each side, see Section B.5.

(iv) Equivalent frame for frames with props or valley beams, see Section B.6.

For each ultimate limit state load combination analysed, V_{Sd} / V_{cr} should be found for each of the above subdivisions and the worst (i.e. highest) V_{Sd} / V_{cr} should be used throughout the structure for that particular load combination. (The very highest V_{Sd} / V_{cr} could be used for all load combinations, but it could prove wasteful.)

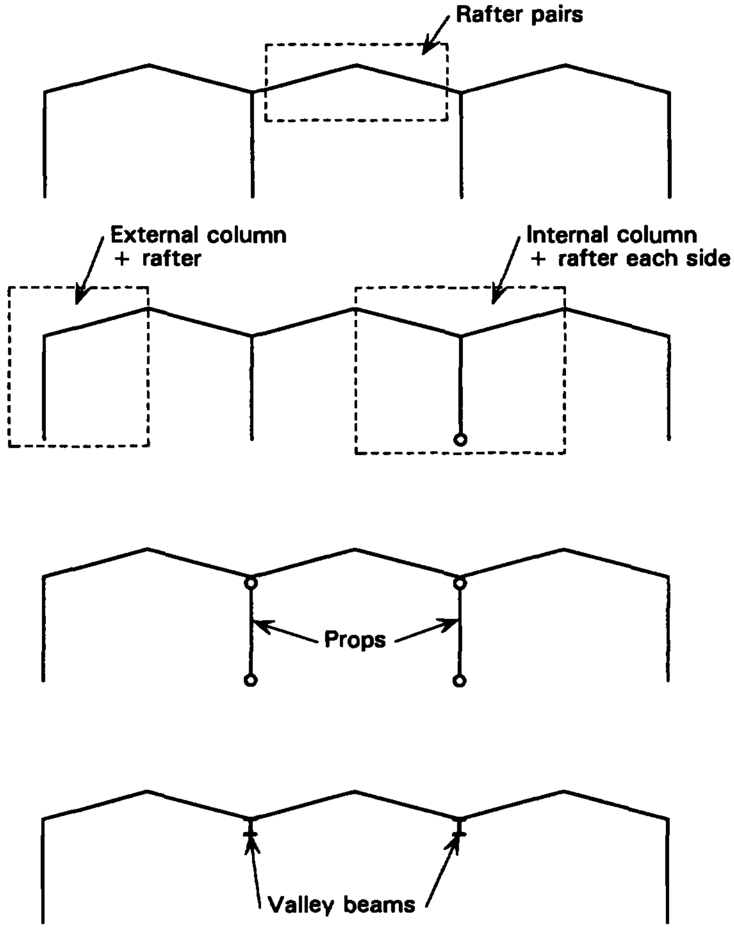


Figure B.1 *Sub-divisions of frames for analysis without computer*

B.3 Rafter pairs

This method checks that the "arch" formed by the rafters does not collapse, as shown in Figure B.2.

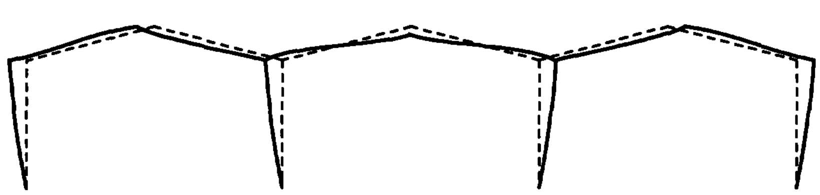


Figure B.2 *Arching failure of rafters*

This method was developed by Horne[29] and forms the basis of the rules of Clause 5.5.3.3 of BS5950-1[3]. It has been re-expressed and re-examined by Davies[25].

For roof slopes in the range $0 \leq \theta_r \leq 20°$

$$V_{sd} / V_{cr} \approx \left(\frac{L}{D}\right)\left[\frac{\Omega - 1}{55,7(4 + L / h)}\right]\left(\frac{I_r}{I_c + I_r}\right)\left(\frac{f_{yr}}{275}\right)\left(\frac{1}{\tan 2\theta_r}\right)$$

where L is the span of the bay

D is the minimum depth of the rafters

h is the column height

I_c is the minimum second moment of area of the column (taken as zero if the column is not rigidly connected to the rafter)

I_r is the minimum second moment of area of the rafters

f_{yr} is the yield strength of the rafters

Ω is the arching ratio, which equals the ratio of factored vertical load on the rafters to the vertical load that could cause failure (by a plastic mechanism) of the rafter treated as a fixed-ended beam of span L. For haunched rafters, without plastic moments within the length of the haunch, use the reduced span between haunches.

θ_r is the roof slope if symmetrical, or else $\theta_r = \tan^{-1}\left(\frac{2h_1}{L}\right)$

h_1 is the height of the apex above the top of the columns.

When $\Omega \leq 1,0$, there is no possibility of this "arch" type of failure.

B.4 External column and rafter

The method is based on work by Davies[24,25] with modifications to include an explicit column base stiffness in Sections B.4.2 and B.4.3 below.

B.4.1 Truly pinned bases

For truly pinned bases, or bases with rockers (see Section A.1)

$$V_{Sd} / V_{cr} = \frac{s}{3EI_r}\left[0,3\ N_r\ s + \left(1 + \frac{1,2}{R}\right) N_c\ h\right]$$

This may be expressed in terms of the rafter and column Euler buckling loads as:

$$V_{Sd} / V_{cr} = \left[\left(\frac{N_r}{N_{r.cr}}\right) + (4 + 3,3R)\left(\frac{N_c}{N_{c\,cr}}\right)\right]$$

where E is the Young's modulus of steel = 210 kN/mm²

I_r is the rafter inertia in the plane of the portal (normally this is I_y in EC3)

I_c is the column inertia in the plane of the portal (normally this is I_y in EC3)

s is the rafter length along the slope (eaves to apex)

h is the column height

N_c is the axial compression in column from elastic analysis

N_r is the axial compression in rafter from elastic analysis

$$N_{c\,cr} = \frac{\pi^2 E I_c}{h^2} = \text{the Euler buckling load of the column}$$

$$N_{r.cr} = \frac{\pi^2 E I_r}{s^2} = \text{the Euler buckling load of the rafter}$$

$$R = \frac{\textit{column stiffness}}{\textit{rafter stiffness}} = \frac{\left(\frac{I_c}{h}\right)}{\left(\frac{I_r}{s}\right)} = \frac{I_c\, s}{I_r\, h}$$

B.4.2 Nominally pinned bases

For nominally pinned bases, allowing for some slight rigidity as in Section A.3:

$$V_{Sd} / V_{cr} = \frac{s}{(4{,}2 + 0{,}4R)\, EI_r} \left[0{,}42\, N_r\, s + \left(1{,}16 + \frac{1{,}2}{R} \right) N_c\, h \right]$$

This may be expressed in terms of the rafter and column Euler buckling loads as:

$$V_{Sd} / V_{cr} = \frac{1}{(1 + 0{,}1R)} \left[\left(\frac{N_r}{N_{r.cr}} \right) + (2{,}9 + 2{,}7R) \left(\frac{N_c}{N_{c.cr}} \right) \right]$$

where E, I_r, I_c, s, h, N_c, N_r, $N_{c\,cr}$, $N_{r\,cr}$ and R have the same meanings as in Section B.4.1.

B.4.3 Nominally fixed bases

For nominally fixed bases, allowing for some slight flexibility as in Section A.2:

$$V_{Sd} / V_{cr} = \frac{1}{5\, E(10 + 0{,}8R)} \left[\frac{5N_r\, s^2}{I_r} + (2{,}6R + 4) \frac{N_c h^2}{I_c} \right]$$

This may be expressed in terms of the rafter and column Euler buckling loads as:

$$V_{Sd} / V_{cr} = \frac{1}{(1 + 0{,}08R)} \left[\left(\frac{N_r}{N_{r.cr}} \right) + (0{,}8 + 0{,}52R) \left(\frac{N_c}{N_{c\,cr}} \right) \right]$$

where E, I_r, I_c, s, h, N_c, N_r, $N_{c\,cr}$, $N_{r\,cr}$ and R have the same meanings as in Section B.4.1.

B.5 Internal column and rafter each side

The method is similar to that described in Section B.4, but modified to allow for internal columns.

B.5.1 Truly pinned bases

For truly pinned bases, or bases with rockers as in Section A.1

$$V_{Sd} / V_{cr} = \left[\left(\frac{N_{r\ell}}{N_{r\ell.cr}} \right) R_\ell + \left(\frac{N_{rr}}{N_{rr\,cr}} \right) R_r + (4 + 3{,}3\ R_2) \left(\frac{N_c}{N_{c.cr}} \right) \right]$$

which in the case of identical rafter forces, sections and lengths gives

$$V_{Sd} / V_{cr} = \left[\left(\frac{N_r}{N_{r.cr}} \right) + (4 + 3{,}3\ R_2) \left(\frac{N_c}{N_{c\,cr}} \right) \right]$$

where E, I_r, I_c, s, h, N_c, N_r, $N_{c.cr}$ and $N_{r\,cr}$ have the same meanings as in Section B.4.1

and
- $I_{r\ell}$ is the left hand rafter inertia in the plane of the portal
- I_{rr} is the right hand rafter inertia in the plane of the portal
- s_ℓ is the left hand rafter length along the slope (valley to apex)
- s_r is the right hand rafter length along the slope (valley to apex)
- $N_{r\ell}$ is the axial compression in left-hand rafter from elastic analysis
- N_{rr} is the axial compression in right-hand rafter from elastic analysis
- $N_{r\ell\,cr}$ is the Euler buckling load of left-hand rafter $= \pi^2 EI_{r\ell} / s_\ell^2$
- $N_{rr\,cr}$ is the Euler buckling load of right-hand rafter $= \pi^2 EI_{rr} / s_r^2$

$$R_\ell = \frac{\text{left hand rafter stiffness}}{\text{total rafter stiffness}} = \frac{EI_{r\ell} / s_\ell}{(EI_{r\ell} / s_\ell + EI_{rr} / s_r)}$$

$$R_r = \frac{\text{right hand rafter stiffness}}{\text{total rafter stiffness}} = \frac{EI_{rr} / s_r}{(EI_{r\ell} / s_\ell + EI_{rr} / s_r)}$$

$$R_2 = \frac{\text{column stiffness}}{\text{total rafter stiffness}} = \frac{EI_c / h}{(EI_{r\ell} / s_\ell + EI_{rr} / s_r)}$$

B.5.2 Nominally pinned bases

For nominally pinned bases, allowing for some slight rigidity as in Section A.3:

$$V_{Sd} / V_{cr} = \frac{1}{(1 + 0{,}1R_2)} \left[\left(\frac{N_{r\ell}}{N_{r\ell\,cr}} \right) R_\ell + \left(\frac{N_{rr}}{N_{rr\,cr}} \right) R_r + (2{,}9 + 2{,}7R_2) \left(\frac{N_c}{N_{c.cr}} \right) \right]$$

which in the case of identical rafter forces, sections and lengths gives

$$V_{Sd} / V_{cr} = \frac{1}{(1 + 0{,}1R_2)} \left[\left(\frac{N_r}{N_{r\,cr}} \right) + (2{,}9 + 2{,}7\ R_2) \left(\frac{N_c}{N_{c\,cr}} \right) \right]$$

where $I_{r\ell}$, I_{rr}, s_ℓ, s_r, $N_{r\ell}$, N_{rr}, $N_{r\ell\,cr}$, $N_{rr\,cr}$, R_ℓ, R_r and R_2 have the same meanings as in Section B.5.1.

B.5.3 Nominally fixed bases

For nominally fixed bases, allowing for some slight flexibility as Section A.2:

$$V_{sd} / V_{cr} = \frac{1}{(1 + 0{,}08R_2)} \left[\left(\frac{N_{r\ell}}{N_{r\ell\,cr}} \right) R_\ell + \left(\frac{N_{rr}}{N_{rr\,cr}} \right) R_r + (0{,}8 + 0{,}52R_2) \left(\frac{N_c}{N_{c\,cr}} \right) \right]$$

which in the case of identical rafter forces, sections and lengths gives

$$V_{sd} / V_{cr} = \frac{1}{(1 + 0{,}08R_2)} \left[\left(\frac{N_r}{N_{r\,cr}} \right) + (0{,}8 + 0{,}52\ R_2) \left(\frac{N_c}{N_{c.cr}} \right) \right]$$

where $I_{r\ell}$, I_{rr}, s_ℓ, s_r, $N_{r\ell}$, N_{rr}, $N_{r\ell\,cr}$, $N_{rr\,cr}$, R_ℓ, R_r and R_2 have the same meanings as in Section B.5.1.

B.6 Portal frame with props or valley beams

This method was developed by Davies[24,25] but is modified in B.6.2 and B.6.3 below to include an explicit column base stiffness. The method assumes that all the valleys are supported either by props or by valley beams.

A simple equivalent frame with one column pinned top and bottom is used, representing an end bay loaded by a share of the prop loads (normally 50%) on the pin-ended column. Assuming that the internal column load is twice the external column load, the equivalent frame prop load is nN_c

where N_c is the axial compression in the external column from elastic analysis
n is the total number of props in the frame.

The rafter beyond the first bay contributes little to the sway stability, so is ignored.

Valley beams do not add appreciably to the stability of the portal frame and do not destabilise it when detailed properly with a rigid connection to the eaves. Therefore, rigidly connected valley beams make no contribution to n. A portal with valley beams but no props has $n = 0$.

B.6.1 Truly pinned bases

For truly pinned bases, or bases with rockers as in Section A.1

$$V_{Sd} / V_{cr} = \frac{s_2}{3EI_r} \left[0{,}3N_r\ s_2 + \left(\frac{1{,}2}{R_p} + 1 \right) (n + 1) N_c\ h \right]$$

which may be expressed in terms of the Euler loads as

$$V_{Sd} / V_{cr} = \left[\left(\frac{N_r}{N_{2r\,cr}} \right) + (4 + 3{,}3\ R_p)\ (n + 1) \left(\frac{N_c}{N_{c.cr}} \right) \right]$$

where the symbols have the same meaning as in B.4 except:

$$R_p = \frac{\text{stiffness of column}}{\text{stiffness of rafter pair}} = \frac{\left(\frac{I_c}{h}\right)}{\left(\frac{I_2}{s_2}\right)} = \frac{I_c\, s_2}{I_2\, h}$$

For rafters of equal cross section and equal length

I_2 is the rafter inertia in the plane of the frame
s_2 is the length of rafter pair (i.e. eaves to apex to valley)

but for asymmetrical arrangements of rafters, I_2 / s_2 is the value that gives the true ratio of column stiffness to stiffness of the pair of rafters (length = sum of rafter lengths, i.e. eaves to apex to valley) for rotation about the eaves.

$N_{2r\,cr}$ is the Euler critical buckling load of the pair of rafters adjacent to the external column

$$N_{2r\,cr} = \frac{\pi^2 EI}{(s_2)^2}$$

for a symmetrical pair of rafters.

B.6.2 Nominally pinned bases

For nominally pinned bases, allowing for some slight rigidity as in Section A.3:

$$V_{Sd} / V_{cr} = \frac{1}{(1 + 0{,}1\, R_p)} \left[\left(\frac{N_r}{N_{2r\,cr}} \right) + (2{,}9 + 2{,}7\, R_p)\,(n + 1) \left(\frac{N_c}{N_{c.cr}} \right) \right]$$

where the symbols have the same meaning as in B.6.

B.6.3 Nominally fixed bases

For nominally fixed bases, allowing for some slight flexibility as in Section A.2:

$$V_{sd} / V_{cr} = \frac{1}{(1 + 0{,}08\, R_p)} \left[\left(\frac{N_r}{N_{2r.cr}} \right) + (0{,}8 + 0{,}52\, R_p)\,(n + 1) \left(\frac{N_c}{N_{c\,cr}} \right) \right]$$

where the symbols have the same meaning as in B.6.

APPENDIX C CROSS-SECTION RESISTANCE

C.1 Partial safety factor for resistance γ_M

In EC3, the cross-section resistance is taken as $\left(\dfrac{\text{Nominal resistance}}{\gamma_{M1}}\right)$

Values of partial safety factor for resistance, γ_M, are given in ENV 1993-1-1 Section 5.1.1. They are given as "boxed" values. The values are confirmed or modified by the relevant NAD.

The values given in ENV 1993-1-1 are listed in Table C.1 below.

Table C.1 *Partial safety factors*

Failure mode	EC3 "boxed" values
γ_{M0} squash	1,10
γ_{M1} local buckling	1,10
γ_{M2} rupture at bolt holes	1,25

C.2 Classification of cross sections

In EC3, cross sections are classified according to the relative thickness of the flanges and web, together with the magnitude of the bending moment and axial compression on the section. The Eurocode classification according to the slenderness of flange or web elements is given in ENV 1993-1-1 Table 5.3.1.

In portal frames designed by plastic design methods, the columns and rafters must be Class 1 or Class 2. These two classes have the same rules for checking resistance. The only element that may be different is the haunch web, which may be Class 3, according to ENV 1993-1-1 Section 5.3.3. The geometrical and material requirements for member ductility in plastic portal frames are given in Appendix G of this publication.

ENV 1993-1-1 does not give a method of classification for sections carrying less than the full bending and/or axial resistance. The following method is recommended in this publication.

Take N = N_{Sd} from the global analysis

Take M = the maximum moment that the cross section can resist when applied together with N_{Sd}.

Classify the section on the basis of N and M as calculated above.

C.3 Section properties

The rules for calculating the net section properties are given in Section 5.4.2 of ENV 1993-1-1.

C.4 Resistance checks

The cross-sectional resistance of the critical sections must be checked for tension, compression, shear and bending, according to the criteria given below. Combinations of bending with shear and/or axial are treated in Section C.4.4.

The checks in this Section consider only cross sections with flanges and webs in Class 1 or Class 2, with symmetry in the plane of the frame. Cross sections with a Class 3 web are considered in Appendix E. For haunches in general, see Section C.5.

All forces and moments are the ultimate limit state values (i.e. factored).

The symbols used in this Section are as follows:

A	is the gross cross-sectional area
$A_{f\,net}$	is the net flange area
A_{net}	is the net cross-sectional area = gross area reduced by holes. Detailed Clauses, including rules for staggered holes, are given in Section 5.4.2 of ENV 1993-1-1
A_v	is the shear area, see Section 5.4.6 of ENV 1993-1-1
b	is the overall breadth
d	is the depth of the web
f_y	is the design yield strength, see Table 3.1 of ENV 1993-1-1
f_u	is the design ultimate strength, see Table 3.1 of ENV 1993-1-1
h	is the overall depth
M_{sd}	is the applied bending moment
$M_{pl\,Rd}$	is the plastic moment of resistance
N_{Sd}	is the applied axial force (tension or compression)
$N_{pl\,Rd}$	is the plastic cross-sectional design resistance to axial force
$N_{u\,Rd}$	is the ultimate cross-sectional design resistance to axial force
r	is the root radius
t_f	is the flange thickness
t_w	is the web thickness
V_{Sd}	is the applied shear force, for cross-sectional resistance checks

[There is a possible confusion with (total ULS vertical load), which has the same symbol in overall stability checks.]

$V_{pl\,Rd}$	is the plastic cross-sectional design resistance to shear force.

C.4.1 Tension

Check $N_{Sd} \ngtr N_{pl\,Rd} = Af_y \,/\, \gamma_{M0}$ (see Section 5.4.3 of ENV 1993-1-1).

If there are holes for fasteners, check $N_{Sd} \ngtr N_{u\,Rd} = 0{,}9\, A_{net} f_u \,/\, \gamma_{M2}$

Wherever ductility is necessary, check $N_{u\,Rd} \geq N_{pl\,Rd}$

C.4.2 Compression

Check $N_{Sd} \ngtr N_{pl\,Rd} = Af_y \,/\, \gamma_{M0}$ (see Section 5.4.4 of ENV 1993-1-1).

Fastener holes need not be allowed for, except for oversize or slotted holes.

C.4.3 Shear

Check $V_{Sd} \not> V_{pl\,Rd} = A_v\,(f_y\,/\,\sqrt{3})\,/\,\gamma_{M0}$ (see Section 5.4.6 of ENV 1993-1-1)

where A_v is the shear area and may be taken as:

(i) $A - 2bt_f + (t_w + 2r)\,t_f$ or, for simplicity, 1,04 ht_w for rolled I sections, load parallel to web

(ii) $\Sigma(dt_w)$ for welded I sections, load parallel to web.

Shear buckling may be limiting if $d\,/\,t_w > 69(235\,/\,f_y)^{0.5}$ (see Appendix H).

Fastener holes can be ignored if $A_{v\,net} \geq (f_y\,/f_u)\,A_v$.

C.4.4 Bending moment

(a) Holes for fasteners (see Section 5.4.5.3 of ENV 1993-1-1)

(i) Fastener holes in the tension flange need not be allowed for, provided that for the tension flange:

$$0{,}9\,(A_{f\,net}\,/\,A_f) \geq (f_y\,/f_u)\,(\gamma_{M2}\,/\,\gamma_{M0})$$

(ii) When $A_{f\,net}/A_f$ is less than this limit, the simplest solution is to assume a reduced flange area in the calculation of the bending modulus that satisfies the limit.

(iii) Fastener holes in the tension zone of the web need not be allowed for, provided that the limit given in C.4.4.a.i is satisfied for the complete tension zone, comprising the tension flange plus the tension zone of the web.

(iv) Fastener holes in the compression zone of the cross section need not be allowed for, except for oversize and slotted holes.

(b) For low shear and low axial load (see Section 5.4.5 of ENV 1993-1-1),

i.e. $V_{Sd} \not> 0{,}5\,V_{pl\,Rd}$

and $N_{Sd} \not> 0{,}5\,A_{web}\,f_y$ and $\not> 0{,}25\,N_{pl\,Rd}$,

check $M_{Sd} \not> M_{pl\,Rd} = W_{pl}f_y\,/\,\gamma_{M0}$.

Note: This means that there is no reduction in the plastic moment of resistance, provided that the coexistent shear force and axial force are less than the limits defined above.

(c) For high shear and low axial load (see Section 5.4.7 of ENV 1993-1-1),

i.e. $V_{Sd} > 0{,}5\,V_{pl\,Rd}$

and $N_{Sd} > 0{,}5\,A_{web}\,f_y$ and $> 0{,}25\,N_{pl\,Rd}$, check $M_{Sd} \not> M_{V\,Rd} = \left(W_{pl} - \dfrac{\rho A_v^2}{4t_w} \right) f_y \,/\, \gamma_{M0}$

where $\rho = (2V_{Sd} / V_{pl\,Rd} - 1)^2$.

(d) For low shear and high axial load (see Section 5.4.8.1 of ENV 1993-1-1),

i.e. $V_{Sd} \not> 0{,}5\ V_{pl\,Rd}$

and $N_{Sd} > 0{,}5\ A_{web} f_y$ or $> 0{,}25\ N_{pl\,Rd}$,

and for standard rolled I and H sections,

check $M_{Sd} \not> M_{Ny\,Rd} = M_{pl\,Rd}\,(1 - n) / (1 - 0{,}5a)$ but $M_{Ny.Rd} \not> M_{pl.y.Rd}$,

where $n = N_{Sd} / N_{pl\,y\,Rd}$ and $a = (A - 2bt_f) / A$ but $a \not> 0{,}5$.

(e) For high shear and high axial load (see Section 5.4.9 of ENV 1993-1-1),

i.e. $V_{Sd} > 0{,}5V$

and $N_{Sd} > 0{,}5A_{web} f_y$ or $N_{sd} > 0{,}25\ N_{pl.Rd}$

reduce the effective web thickness to $(1 - \rho)t_{web}$, where ρ is defined in C.4.4.c above, then check the new effective section as in C.4.4.d above.

C.5 Special considerations for haunches

In the analysis model, the haunch neutral axis is often assumed to be on the same line as the rafter neutral axis and the moment generated by the eccentricity of the rafter axial force is ignored. In most structures the critical design cases are (dead load + snow) or (dead load + wind), so the eccentricity moment reduces the haunch moment and it is safe to ignore it. In rare cases, e.g. forced displacements of the bases, the eccentricity moment may increase the haunch moment.

Haunches made by welding rolled beam section cuttings or fabricated Tees to the underside of the rafter have an internal flange. This flange helps to carry axial loads and moments, but the effect is complicated to calculate because the relative position of the flange varies along the length of the haunch, from the bottom flange to mid-height. For this reason, it will generally be convenient to ignore the internal flange, except for its stabilising effect on web buckling and on lateral torsional buckling.

C.6 Special considerations for knee panels

The knee panel is the section of the column to which the rafter haunch connects. It is subject to high shear forces due to the rafter moment in addition to a high bending moment. Generally the moment resistance of the column member will be reduced by the coexistent shear force. In practice, this reduction in resistance is only critical for a small proportion of the height of the knee, for example 20–25%, as the moment in the knee increases to a maximum at the bottom of the haunch in simple portals, as shown in Figure C.1. In multi-bay portals with bays of differing heights, a similar problem could occur at the top of the knee panel.

In the UK, where plastic design has been used for this form of portal frame for 40 years, the reduction in moment of resistance is ignored. The shear resistance of the web of the knee is checked. The reduced moment of resistance in the knee is not checked.

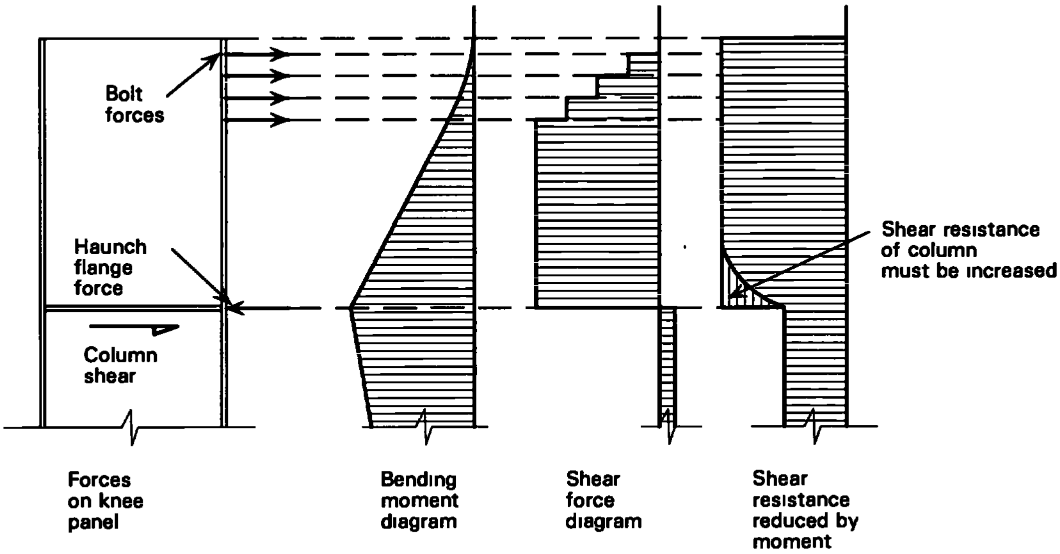


Figure C.1 *Forces, bending moment and shear in knee panel*

The reason why there is no record of poor performance of knees using UK practice is that, provided that there is adequate restraint, the moment resistance of I sections is typically 30% higher than the plastic moment of resistance. In the case of the knee, the compression flange is well restrained by the haunch end-plate and the bolts at the bottom of the haunch, so both the flange and the web can develop stresses well in excess of the yield stress, as a result of strain hardening.

However, to avoid problems with checking engineers who are unaccustomed to plastic design, it might be wise to increase the shear resistance of the web for the small proportion of the knee in question (just above the horizontal stiffener at the haunch bottom flange). This can be achieved using any logical arrangement of diagonal stiffeners, including Morris stiffeners or web doubler plates, as shown in Figure C.2. The stiffeners or doubler plates must be proportioned to provide sufficient shear resistance to leave adequate moment resistance in the column section.

Note that the haunch bottom bolt positions may be affected by web doubler plates.

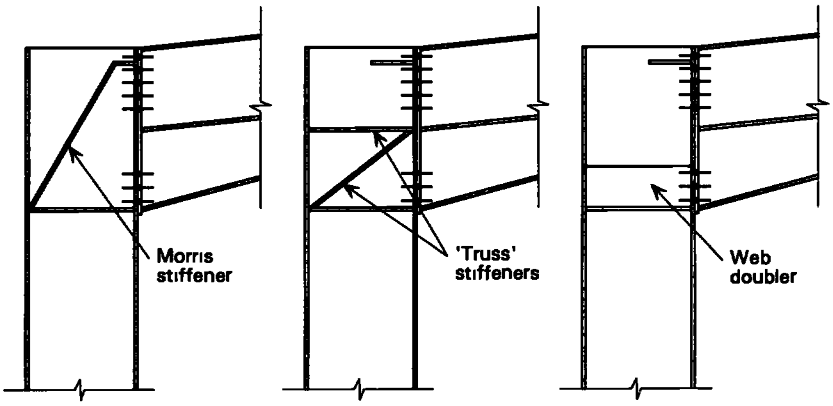


Figure C.2 *Knee strengthening*

APPENDIX D MEMBER STABILITY AND BUCKLING RESISTANCE

The stability of members depends on adequate bracing. The position, strength and stiffness of bracing are discussed in Section 15. ENV 1993-1-1 does not consider the more onerous requirements of plastic design, but guidance is given below. Elements that do not include a plastic hinge may be checked for stability using the normal rules of EC3.

Purlins and cladding increase the stability of the members, even when the compression flange is not restrained but the tension flange is restrained. Guidance is given in Appendix F of this publication to enable the designer to benefit from this increased stability, which is not treated explicitly in ENV 1993-1-1.

Member stability must be checked between points of restraint appropriate to each possible mode of buckling. Lateral buckling and lateral torsional buckling should be checked between points of lateral restraint to the compression flange, or points of lateral-torsional restraint. Where the stabilising effects of the purlins and cladding are included, the stability must also be checked between the purlins.

Where the stiff axis buckling has been accounted for either by the use of Merchant-Rankine in the frame analysis or by use of a second-order analysis including equivalent initial imperfections, it need not be checked between restraints for major axis buckling.

Slenderness and lateral torsional buckling are treated in Appendix F, both with and without intermediate lateral restraint. Lateral buckling with restraint to one flange is also treated in Appendix F.

Shear buckling is treated in Appendix H.

Some differences between EC3 and UK practice are given in Appendix K.

D.1 Partial safety factor for resistance γ_M

In EC3, the buckling resistance is taken as $\left(\dfrac{\text{Nominal resistance}}{\gamma_{M1}}\right)$

The value of partial safety factor for buckling resistance, γ_M, is given in Section 5.1.1 of ENV 1993-1-1 as $\gamma_{M1} = 1,10$. This is a "boxed" value to show that it needs to be checked against the NAD of the country in which the structure is to be built, and modified where appropriate.

D.2 Classification of cross sections

In ENV 1993-1-1 cross sections are classified according to the relative thickness of the flanges and web, together with the magnitude of the bending moment and axial compression on the section, see Table 5.3.1 in ENV 1993-1-1.

In portal frames designed by plastic design methods, the columns and rafters must be Class 1 or Class 2. These two classes have the same rules for checking resistance. The only element that might be different is the haunch web, which can be Class 3 according to Section 5.3.3 of ENV 1993-1-1. The requirements for plastic portals are given in Appendix G of this document.

ENV 1993-1-1 does not give a method of classification for sections carrying less than the full bending and/or axial resistance. The following method is recommended in this publication.

Take N = N_{Sd} from the global analysis

Take M = the maximum moment that the cross section can resist when applied together with N_{Sd}.

Classify the section on the basis of N and M as calculated above.

D.3 Members without plastic hinges

D.3.1 Compression

Where the forces and moments have been calculated using the Merchant-Rankine criterion, as in Section 12.4, only out-of-plane buckling need be checked.

For members with flanges and webs in Class 1, Class 2 or Class 3:

Check $N_{Sd} \ngtr N_{b\,Rd} = \chi\, A\, f_y \,/\, \gamma_{M1}$ (see Section 5.5.1 of ENV 1993-1-1),

where χ can be obtained from Section 5.5.1.2 or Table 5.5.2 of ENV 1993-1-1. Alternatively, the values of $f_c = \chi f_y$ can be found in Table 5.14 of C-EC3[17] taking $\beta_A = 1{,}0$.

$\bar{\lambda} = (A.f_y \,/\, N_{cr})^{0{,}5} = \lambda \,/\, [93{,}9(235 \,/\, f_y)^{0{,}5}] = \lambda \,/\, [86{,}8(275 \,/\, f_y)^{0{,}5}]$

where λ is the slenderness of the buckling mode $\ell \,/\, i$ (traditionally $\ell \,/\, r$ in UK practice), where the effective lengths ℓ are given in Section D.3.5.

The appropriate buckling curve a, b, c or d depends on the type of section and the plane of buckling, as shown in Table 5.5.3 of ENV 1993-1-1.

For tapered members, the greatest value of λ can be used, or Section 5.5.1.3 of ENV 1993-1-1 should be followed.

For members with intermediate restraint along only the tension flange, N_{cr} may be found from Appendix F.3.2, or λ may be found from Appendix F.3.4. Where these values are used, $N_{b\,Rd}$ must also be checked for buckling between the intermediate restraints.

D.3.2 Lateral-torsional buckling

For members with flanges and webs in Class 1 or Class 2:

Check $M_{Sd} \ngtr M_{b\,Rd} = \chi_{LT}\, W_{pl\,y} f_y \,/\, \gamma_{M1}$ (see Section 5.5.2 of ENV 1993-1-1) where χ_{LT} is found from Section 5.5.2 or Table 5.5.2 of ENV 1993-1-1, but with $\chi_{LT} = 1{,}0$ for $\bar{\lambda}_{LT} \leq 0{,}4$.

Alternatively, values of $f_b = \chi_{LT} f_y$ can be found in Tables 5.18a and 5.18b of C-EC3[17] taking $\beta_w = 1{,}0$.

$$\bar{\lambda}_{LT} = (W_{pl\,y} f_y \,/\, M_{cr})^{0,5} = \lambda_{LT} \,/\, [93{,}9(235 \,/\, f_y)^{0,5}] = \lambda_{LT} \,/\, [86{,}8(275 \,/\, f_y)^{0,5}]$$

where M_{cr} or λ_{LT} may be found from Annex F of ENV 1993-1-1 or Appendix F of this publication, in which the use of bending moment shape correction factors is shown.

The effective lengths are given in Section D.3.5. For rolled sections, use Curve a of Table 5.5.2 of ENV 1993-1-1.

For welded sections, use Curve c of Table 5.5.2 of ENV 1993-1-1, except where the member is a haunch with a middle flange to which a beam cutting is welded (as in Figure 2.1), and the middle flange is neglected for calculation of stresses, so Curve a is appropriate.

For tapered members, the maximum value of λ_{LT} can be used, which will normally be found at the deepest end in sections with uniform flanges and uniform web thickness. Alternatively, Appendix F.3 may be used, where intermediate tension flange restraint exists. The buckling resistance should be checked at the location where the calculated elastic compressive stress is greatest.

For members with intermediate restraint along only the tension flange, M_{cr} may be found from Appendix F.3.3. Alternatively, λ_{LT} may be found from Appendix F.3.4 of this publication. Where these values are used, $M_{b\,Rd}$ must also be checked for buckling between the intermediate restraints, using F.3.1 or F.3.2.

D.3.3 Bending and axial tension

The tension forces are generally ignored in buckling checks (see Section 5.5.3 of ENV 1993-1-1). If they are included, they must be applied with combination factor ψ_{vec}. Section 2.3.3.1 of ENV 1993-1-1 gives suggested values, but reference must be made to the relevant National Application Document for the country in which the structure is to be built.

D.3.4 Bending and axial compression

Eurocode 3 requires that two buckling checks are performed for members subject to both axial load and moment. These are given in Section 5.5.4 of ENV 1993-1-1. The first considers overall buckling together with bending; the second considers minor axis buckling together with lateral torsional buckling.

These checks are given below, simplified by the assumption that there is no minor axis bending. This is the most common case.

Overall buckling with bending

Where the forces and moments have been calculated using the Merchant-Rankine criterion, as in Section 12.4 of this publication, only out-of-plane buckling need be checked, as D.3.4.2 below. The in-plane buckling deformations are accounted for by the mode shape of the buckling mode used in the Merchant-Rankine criterion. In other cases, e.g. normal elastic design, overall in-plane buckling must be checked.

Where there is no applied minor axis bending for Class 1 and Class 2 sections:

Check $N_{Sd} / N_{b.Rd\,min} + k_y M_{y\,Sd} / M_{pl\,y} \not> 1$ (see Section 5.5.4 of ENV 1993-1-1),

where k_y may be safely taken as 1,5 or may be found from Section 5.5.4(1) of ENV 1993-1-1
N_{sd} may be taken conservatively as the maximum axial load
$N_{b.Rd\,min} = \chi_{min} A f_y / \gamma_{M1}$, see D.3.1
$M_{y\,Sd}$ in uniform sections is taken as maximum $M_{y\,Sd}$.

In tapered sections, the check should be made at the point where the calculated elastic compressive stress is greatest, using λ calculated from the section giving the lowest value.

Minor axis and lateral torsional buckling

Where the forces and moments have been calculated using the Merchant-Rankine criterion, as in Section 12.4 of this publication, only out-of-plane buckling need be checked.

Where there is no applied minor axis bending for Class 1 and Class 2 sections:

Check $N_{Sd} / N_{b\,Rd\,z} + k_{LT} M_{y\,Sd} / M_{b\,Rd} \not> 1$ (see Section 5.5.4 of ENV 1993-1-1),

where k_{LT} may be safely taken as 1,0 or may be found from Section 5.5.4(2) of ENV 1993-1-1
N_{sd} may be taken conservatively as the maximum axial load
$N_{b\,Rd\,z} = \chi_z A f_y / \gamma_{M1}$ (see D.3.1)
$M_{y\,Sd}$ in uniform sections is taken as maximum $M_{y\,Sd}$
$M_{b\,Rd}$ is defined in D.3.2.

In tapered sections, the check should be made at the point where the calculated elastic compressive stress is greatest, using λ calculated from the section giving the lowest λ value and λ_{LT} calculated from the section giving the lowest λ_{LT} value. Alternatively, Appendix F.3 may be used where intermediate tension flange restraint exists. In sections with uniform flanges and uniform web thickness, the lowest values of λ and λ_{LT} are normally found at the deepest section.

As an alternative to the above: Check by the stable length check of D.4, which is derived from F.4. This check is especially suitable for sections loaded to near their full plastic resistance.

Where members are restrained along the tension flange but not along the compression flange, the member must be checked both between the torsional restraints and between the intermediate lateral restraints.

D.3.5 Effective lengths

Effective lengths may normally be taken as follows:

Out-of-plane buckling

Out-of-plane buckling includes minor axis buckling from pure compression (lateral flexural buckling) and lateral torsional buckling from bending moments about the major axis.

The following must be checked:

(a) distance between intermediate lateral restraints

(b) distance between torsional restraints.

In-plane buckling

(a) Where the amplified sway method is used, as described in Section 12.3.2, the effective length is the distance from base to eaves for columns and from eaves (or valley) to apex for rafters.

(b) Where the effective length method is used, the in-plane effective length and in-plane slenderness are calculated using the method described in Section 12.3.3.

D.4 Members with plastic hinges or near plastic hinges

The stability of members containing plastic hinges that are required to rotate is not treated in ENV 1993-1-1. The following guidance is equivalent to Clause 5.3.5 of BS5950-1[3] **but** with an allowance for the shape of the bending moment diagram and coexistent axial load.

To ensure adequate rotation capacity of a plastic hinge, the slenderness of the adjacent members must be limited to avoid premature lateral or lateral-torsional instability. This includes hinges that form, rotate and then cease or even unload or reverse. Hinges that form above ultimate limit state need not be considered as rotating hinges, but some allowance, say 5% M_p, should be made for differences between the analytical model and the structure's true yield stresses, including the effect of residual stresses, etc.

Lateral restraint for all rotating hinges should be provided to both flanges.

The slenderness $\bar{\lambda}_{plastic}$ between restraint points should be limited according to the formulae presented in Appendix F.4 of this publication. Note that for members restrained along the tension flange, research in the UK has shown that $\bar{\lambda}_{plastic}$ may be higher, as explained in Appendix I of *Plastic design of single-storey pitched-roof portal frames to Eurocode 3*[2].

Expressing stability in terms of a stable length of member:

$$L_{cmax} = 0,4\ (C_p)^{0,5}\ \lambda_1\ i_{LT}$$

where C_p is the equivalent plastic moment factor given by

$$C_p = \left[\frac{1}{\dfrac{M_h}{C_1 W_{pl\,y} f_y} + \dfrac{4N_h}{Af_y} \left(\dfrac{i_{LT}}{i_z} \right)^2} \right]$$

and M_h is the moment at the hinge (or at the critical section if there is no hinge)

N_h is the axial load at the hinge (or at the critical section if there is no hinge)

L_c is the length along compression flange

i_{LTC} is the effective radius of gyration for lateral torsional buckling.

For sections with $i_{zc} \geq i_{zt}$, i_{LTC} may be taken as i_z / 0,9 using i_z for the complete cross section.
For sections with $i_{zc} < i_{zt}$, i_{LTC} may be taken as i_{zc} / 0,9

i_{zc} is the radius of gyration about the z axis of the compression flange and half the web

i_{zt} is the radius of gyration about the z axis of the tension flange and half the web

λ_1 $= (\pi^2 E / f_y)^{0,5}$.

C_1 = 1,0, except for cases where the plastic hinges are accurately known, such as in columns complying with Section 5.2.7 of ENV 1993-1-1 or in members with pronounced moment gradients (such as at the eaves and valleys of common portals under usual loads), where C_1 is defined in Table F.1.1 or Table F.1.2 of ENV 1993-1-1 for k = 1,0.

For tapered sections, properties are evaluated at the point of maximum $\bar{\lambda}_{plastic}$. This will generally be at the shallow end, because structures are usually designed so that the plastic hinge is at this location. However, the hinge might occur away from the shallow end, if the ratio of (applied effects / resistance) is constant along the member or increases towards the deep end. C_1 should be taken as 1,0, unless more reliable data are available.

D.5 Special considerations for haunches

Where the haunch is made by welding an I section cutting to the underside of the rafter, the haunch has a middle flange, as shown in Figure 2.1.

The middle flange, even when ignored for section property calculations, should be remembered when classifying the web, because it reduces the web slenderness. Also, the middle flange will carry any residual stresses from welding the cutting to the rafter, so the same compression and bending curves may be used as for the basic rafter or cutting section.

The basic research and test work on stability of tapered haunches used in this document was on haunches with only top and bottom flanges. Therefore, where calculations use the "c" and "q" factors from Appendix F.3, the middle flange should be ignored for the calculation of stresses for buckling resistance. However, the middle flange may be included in the calculation of I_t.

Where Appendix F.3 is not used, it must be remembered that the position of the middle flange, varying from edge to centre, affects the warping behaviour. Therefore, it is easier to ignore the middle flange for bending stability calculations (except for increasing I_t), even though this is conservative. For axial stability calculations, the middle flange may be included, where Appendix F.3 is not used. However, the effect of axial compression is small, so the improved accuracy may not be worth the effort of calculating another set of section properties.

APPENDIX E CROSS SECTIONS WITH CLASS 3 WEBS AND CLASS 1 OR CLASS 2 FLANGES

E.1 General

Cross sections with Class 3 webs may be checked as Class 3 sections, following Section 5.4.5.1(2) of ENV 1993-1-1, using an elastic stress distribution. However, this may prove uneconomical, as the flanges will normally be Class 1 or Class 2.

This Appendix explains the Application Rule of Section 5.3.4(5) of ENV 1993-1-1, which refers to a cross section with a Class 1 or Class 2 compression flange but a Class 3 web. This allows the section to be treated as Class 2 by reducing the effective web section using a method given in ENV 1994-1-1[30].

Class 3 webs should only be used away from plastic hinges, and will normally only be used in elastic design or in the haunches in plastic design.

Some differences between EC3 and UK practice are given in Appendix K.

E.2 Effective web section

The relevant part of ENV 1994-1-1 for determining the effective web section is Section 4.3.3.1(3). This rule applies only to sections with Class 1 or Class 2 flanges.

According to EC4, a Class 3 web may be represented by a Class 2 effective web, by assuming that the depth of web that resists compression is limited to 20 $t\epsilon$ adjacent to the compression flange and adjacent to the new plastic neutral axis. This is the neutral axis of the section based on the flanges plus the effective web and is shown in Figure E.1. Note that the omission of a section of web means that the plastic neutral axis is displaced from the position in the gross section. The cross-section resistance checks are then performed as for true Class 2 sections using this effective section.

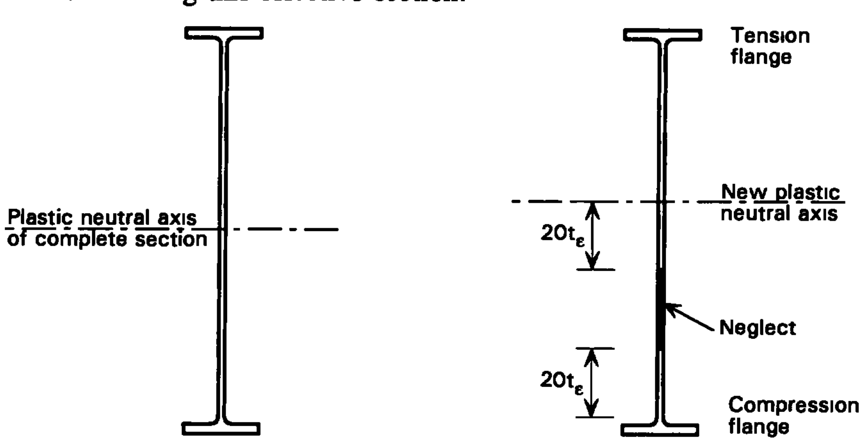


Figure E.1 *General case*

The above rule minimises the discontinuity of resistance between Class 2 and Class 3 webs. There will be a small discontinuity, because the Class 2 limit on web slenderness is 83 $t_w\epsilon$, whereas the 20 $t\epsilon$ limit reduces the web area for symmetric sections when the depth exceeds 80 $t_w\epsilon$.

E.3 Haunches with an internal flange

Haunches are commonly fabricated by welding a beam cutting to the underside of the rafter. Thus, the haunch has an internal flange that divides the web into two smaller panels.

In the typical case of hogging moment in the haunch, the upper web panel will be in tension and the lower web panel will be in compression. The same approach can be used as in E.2 above, where the lower web exceeds 40 $t_w\epsilon$ in depth, as shown in Figure E.2. The limit for webs wholly in compression is 38 $t\epsilon$ for normal I section beams, whereas the 20 $t\epsilon$ rule will give 40 $t\epsilon$ for the haunch lower web panel. However, this is acceptable, because the haunch lower web panel is restrained by the upper flange, the upper web panel and the internal flange.

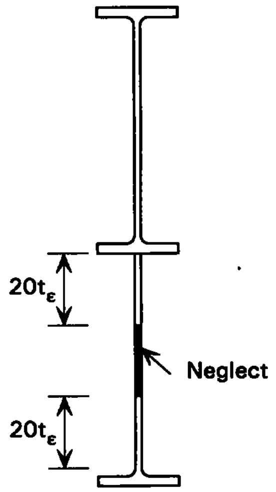


Figure E.2 *Haunch with internal flange*

It might be desirable, in calculating section properties, to ignore the internal flange, if possible, as its position relative to the external flanges varies along the length of the haunch. This variation in position might add complexity to the calculation of the position of the neutral axis, the section properties and the eccentricity of the rafter thrust. However, the presence of the flange does simplify the calculation of the neutral axis because this will generally remain within the flange at the deeper sections.

E.4 Buckling resistance with a Class 3 web

E.4.1 General

The slenderness of the web is included in the buckling resistance by inclusion of values of the EC3 parameters β_A and β_w, which are less than unity, as shown below.

E.4.2 Compression

Where the forces and moments have been calculated using the Merchant-Rankine criterion, as in Section 12.4, only out-of-plane buckling need be checked.

Check $N_{Sd} \not> N_{b\,Rd\,z} = \chi_z A_{eff} f_y / \gamma_{M1}$ (see Section 5.5.1 of ENV 1993-1-1)

where A_{eff} = area of the cross section using the effective web section from Sections E.2 or E.3.

χ is found from Section 5.5.1.2 or Table 5.5.2 of ENV 1993-1-1. Alternatively, the values of $f_c = \chi f_y$ can be found in Table 5.14 of *C-EC3 - Concise Eurocode 3 for the design of steel buildings in the United Kingdom*[17].

Take $\bar{\lambda} = (\beta_A A.f_y / N_{cr})^{0,5} = (\beta_A)^{0,5} \lambda / [93,9 (235 / f_y)^{0,5}]$

$= (\beta_A)^{0,5} \lambda / [86,8 (275 / f_y)^{0,5}]$

where λ is the slenderness of the buckling mode l/i
A is the gross cross-sectional area
$\beta_A = A_{eff} / A$.

In all other respects, the check is as D.3.1 above.

E.4.3 Lateral-torsional buckling

Check $M_{Sd} \not> M_{b\,Rd} = \chi_{LT} W_{eff\,pl\,y} f_y / \gamma_{M1}$ (see Section 5.5.2 of ENV 1993-1-1),

where $W_{eff\,pl\,y}$ is the plastic modulus of the cross section using the effective web section of E.2 or E.3.

Take $\bar{\lambda}_{LT} = (\beta_w W_{pl\,y} f_y / M_{cr})^{0,5} = (\beta_w)^{0,5} \lambda_{LT} / [93,9 (235 / f_y)^{0,5}]$

$= (\beta_w)^{0,5} \lambda / [86,8 (275 / f_y)^{0,5}]$

where $\beta_w = W_{eff\,pl\,y} / W_{pl\,y}$
and $W_{pl\,y}$ is the plastic modulus of the gross cross section.

M_{cr} or λ_{LT} may be found from ENV 1993-1-1 Appendix F.3.1 or Appendix F.3.2 of this publication.

In all other respects, the check is as D.3.1 above.

E.4.4 Bending and axial tension

See Section 5.5.3 of ENV 1993-1-1.

The tension forces are generally ignored in buckling checks. If they are included, they must be applied with a reduction factor ψ_{vec}, see Section 2.3.3.1 of ENV 1993-1-1 and the relevant National Application Document.

E.4.5 Bending and axial compression

Where the forces and moments have been calculated using the Merchant-Rankine criterion, as in Section 12.4, only out-of-plane buckling need be checked.

Where the Merchant-Rankine formula is not used, as in Section 12.3, the in-plane buckling must be checked. However, as the haunch normally only forms 20 to 25% of the apex to eaves/valley length, the in-plane buckling check is normally based on the rafter section of that length. This is conservative, as it ignores the slight stiffening effect of the haunch on the overall rafter buckling.

The checks for bending and axial compression are as D.3.4, but using $N_{b\,Rd}$ and $M_{b\,Rd}$ calculated using the section properties and slenderness obtained by the methods in D.3.

APPENDIX F LATERAL TORSIONAL BUCKLING

F.1 Elastic critical moment

The elastic critical moment for lateral-torsional buckling of a beam of uniform symmetrical cross section with equal flanges, under standard conditions of restraint at each end, loaded through its shear centre is given by:

$$M_{cr} = \frac{C_1 \pi^2 E I_z}{L^2} \left(\frac{I_w}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z} \right)^{0,5}$$

where $G = \dfrac{E}{2(1 + \nu)}$

- C_1 is found from Appendix F, Tables F.1.1 and F.1.2 of ENV 1993-1-1, with $k = 1{,}0$
- I_t is the torsion constant
- I_w is the warping constant
- I_z is the second moment of area about the minor axis
- L is the length of the beam between points which have lateral restraint.

The standard conditions of restraint at each end are:

- restrained against lateral movement
- restrained against rotation about the longitudinal axis
- free to rotate in plan.

For more information, see Annex F of ENV 1993-1-1.

F.2 Slenderness

F.2.1 General

The slenderness ratio $\bar{\lambda}_{LT}$ for lateral-torsional buckling is given by:

$$\bar{\lambda}_{LT} = (\lambda_{LT} / \lambda_1) (\beta_w)^{0,5}$$

where
- $\lambda_1 = \pi (E / f_y)^{0,5} = 93{,}9\epsilon$
- $\epsilon = (235 / f_y)^{0,5}$ (f_y in N/mm²)
- $\beta_w = 1$ for Class 1 or Class 2 cross sections
- $\beta_w = W_{eff\,pl\,y} / W_{p\ell\,y}$ for effective Class 2 cross sections (see Appendix E).

The geometrical slenderness ratio λ_{LT} for lateral-torsional buckling is given for all classes of cross section by:

$$\lambda_{LT} = (\pi^2 E W_{p\ell\,y} / M_{cr})^{0,5}.$$

F.2.2 Uniform doubly symmetric sections

For beams with uniform doubly symmetric cross sections, for cases with end-moment loading or transverse loads applied at the shear centre and no end fixity, the value of λ_{LT} can be obtained from:

$$\lambda_{LT} = \frac{L\left(\dfrac{W_{p\ell\,y}{}^2}{I_z\,I_w}\right)^{0,25}}{(C_1)^{0,5}\left(1 + \dfrac{L^2\,GI_t}{\pi^2\,EI_w}\right)^{0,25}}$$

which can also be written:

$$\lambda_{LT} = \frac{L\,/\,i_{LT}}{(C_1)^{0,5}\left(1 + \dfrac{(L\,/\,a_{LT})^2}{25,66}\right)^{0,25}}.$$

where $a_{LT} = (I_w\,/\,I_t)^{0,5}$.

For a plain I or H section (without lips):

$I_w = I_z\,h_s^2\,/\,4$

where $h_s = h - t_f$.

For a doubly symmetric cross section, the value of i_{LT} is given by:

$i_{LT} = (I_z\,I_w\,/\,W_{p\ell\,y}{}^2)^{0.25}$

or with a slight approximation by:

$i_{LT} = [I_z\,/\,(A - 0,5\,t_w\,h_s)]^{0,5}$

F.2.3 Approximations for rolled I or H sections

For rolled I or H sections with uniform doubly symmetric cross sections, for cases with end-moment loading or transverse loads applied at the shear centre and no end fixity, the following conservative approximations can be used:

$$\lambda_{LT} = \frac{L\,/\,i_{LT}}{(C_1)^{0,5}\left[1 + \dfrac{1}{20}\left(\dfrac{L\,/\,i_{LT}}{h\,/\,t_f}\right)^2\right]^{0,25}}$$

$$\text{or } \lambda_{LT} = \frac{0,9\,L\,/\,i_z}{(C_1)^{0,5}\left[1 + \dfrac{1}{20}\left(\dfrac{L\,/\,i_z}{h\,/\,t_f}\right)^2\right]^{0,25}}$$

Values of λ_{LT} and i_{LT} for rolled UB and UC sections are tabulated in *Introduction to Concise Eurocode 3 (C-EC3) – with worked examples*[31].

F.2.4 Approximation for other I or H sections

For any plain I or H section with equal flanges, with uniform doubly symmetric cross sections, for cases with end-moment loading or transverse loads applied at the shear centre and no end fixity, the following approximation is conservative:

$$\lambda_{LT} = \frac{L / i_z}{(C_1)^{0,5} \left[1 + \frac{1}{20} \left(\frac{L / i_z}{h / t_f} \right)^2 \right]^{0,25}}$$

F.3 Buckling of restrained members with an unrestrained compression flange

F.3.1 General

This Section deals with the buckling of members, or portions of members, between effective torsional restraints to both flanges, which are restrained by intermediate restraints to the tension flange in such a way as to leave the compression flange unrestrained (see Figure F.1).

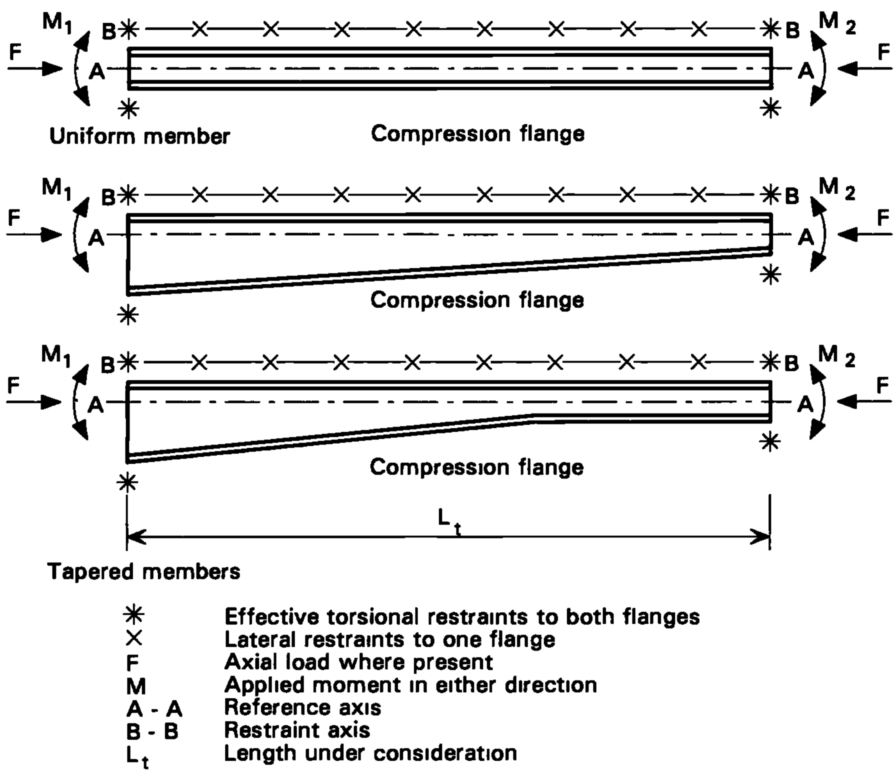


Figure F.1 *Restrained members with an unrestrained compression flange*

ENV 1993-1-1 requires that full lateral restraint should be provided to both the compression flange and the tension flange at plastic hinge locations, or, where this is impracticable, within $D/2$ of the hinge location, where D is the depth of the member. This restraint is required at all hinges that rotate at loads up to the ultimate limit state, including hinges that form, rotate and then cease to rotate as other hinges develop.

Adjacent to plastic hinges, the members should be checked for hinge rotation capacity using the recommendations given in Appendix D.4 of this publication between intermediate restraints and the recommendations given in Appendix F.3.5 of this publication between torsional restraints.

Elements which do not adjoin a plastic hinge should be verified using the recommendations of Section 5.5 of ENV 1993-1-1 between the intermediate tension flange restraints and the recommendations of Section 5.5 of ENV 1993-1-1, modified by this Appendix, between torsional restraints. However, where Section 5.5.4 of ENV 1993-1-1 is not satisfied, because of the moments and axial force approaching the plastic resistance, the check of Appendix F.4 may prove that the member is stable.

Tapered or haunched members should not contain plastic hinges within the tapered length and should be braced at the inner corner of the knee. If a plastic hinge is to be developed, the bracing stiffness should comply with Section 15.4 of this publication.

F.3.2 Basis

Uniform sections

For uniform sections, symmetrical about the minor axis, the elastic critical load for pure axial compression, N_{cr}, under the standard conditions for this form of restraint (see below), has been derived by Horne and Ajmani[32] and is given by:

$$N_{cr} = \frac{1}{i_s^2}\left(\frac{\pi^2 EI_z a^2}{L_t^2} + \frac{\pi^2 EI_w}{L_t^2} + GI_t\right)$$

$$= \alpha\frac{\pi^2 EI_z}{L_t^2} + \frac{GI_t}{i_s^2}$$

$$\text{where } \alpha = \left(\frac{a^2 + \frac{I_w}{I_z}}{i_s^2}\right)$$

$$G = \frac{E}{2(1+\nu)}$$

$$i_s^2 = i_y^2 + i_z^2 + a^2$$

I_t is the torsion constant

I_z is the second moment of area about the minor axis

I_w is the warping stiffness $= I_z\ (h_s\ /\ 2)^2$ for doubly symmetric I sections

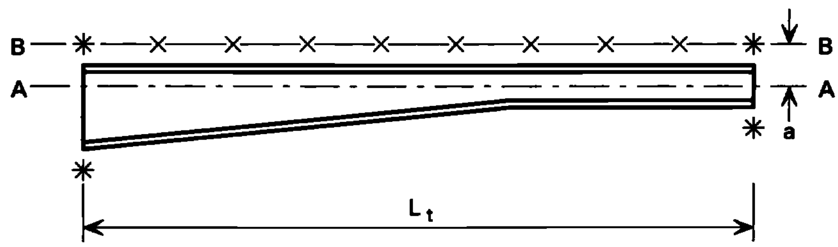
L_t is the length of the beam between points that have lateral restraint to both flanges (see Figure F.1)

a is the distance between the restrained longitudinal axis (e.g. the centroid of the purlins) and the shear centre of the beam (see Figure F.2)

h is the overall depth of the section

h_s is the distance between the shear centres of the flanges.

For doubly symmetric and nearly doubly symmetric I section beams, $\alpha \approx 1{,}0$ when a = 0,75h.



*	Effective torsional restraints to both flanges
×	Lateral restraints to one flange
A - A	Reference axis
B - B	Restraint axis
a	Distance between reference axis and restraint axis
L_t	Length under consideration

Figure F.2 *Restraint axis and reference axis*

For uniform sections, symmetrical about the minor axis, the elastic critical moment for lateral torsional buckling, M_{cr0}, for a pure uniform moment under the standard conditions for this form of restraint (see below), is given by:

$$M_{cr0} = \left(\frac{i_s^2}{2a}\right) N_{cr}$$

The standard conditions for this form of restraint, shown in Figure F.1, are:

(i) At each end of the element length L_t

- restrained against lateral movement
- restrained against rotation about the longitudinal axis
- free to rotate in plan.

(ii) Along the tension flange:

- restrained against lateral movement
- free to rotate about all axes.

The restraint to the tension flange need not be continuous, provided that the spacing of these restraints is such that the member is stable against buckling, assuming it had restraint to both flanges at such points.

Tapered or haunched sections

For sections with uniform flanges that are symmetrical about the minor axis, N_{cr} and M_{cr0} are calculated as in Section F.3.2.1, but with the following modifications.

To define a, the distance between the restrained longitudinal axis (e.g. the centroid of the purlins) and the shear centre of the beam, the depth of the section must be defined. For $\bar{\lambda}_{LT} \leq 1,0$ and using the "c" factors of Section F.3.3, the shear centre of the shallowest section may be used, as shown in Figure F.2. In all other cases, the shear centre of the deepest section should be used.

F.3.3 Formula for the elastic critical moment

For sections with uniform flanges that are symmetrical about the minor axis

$$M_{cr} = \left(\frac{1}{m_t \; c^2} \right) M_{cr0}$$

where M_{cr0} is defined in Section F.3.2.

For haunched or tapered members, M_{cr0} is calculated using the section properties of the shallow end.

For tapered members with $\bar{\lambda}_{LT} > 1,0$, c should be taken as 1,0 and the maximum $\bar{\lambda}_{LT}$ should be used, which will normally come from the deepest end, unless more accurate solutions are available.

For members with a (third) internal flange, Section F.3.2 should be used with I_w and I_z calculated ignoring the internal flange, but with I_t including the internal flange. m_t and c are defined below.

Equivalent uniform moment factor m_t for members of uniform depth without intermediate loads between lateral-torsional restraints

This case will not often be used, because ENV 1993-1-1 load cases apply to all types of loading simultaneously (see Section 10.6 above). Therefore, in almost all load cases, the purlins (in the roof cladding) or sheeting rails (in the wall cladding) will be applying gravity or wind loads. These loads are intermediate loads, so the method of this section is not directly applicable, because it assumes a "straight-line" bending moment diagram between the bending moments at the two ends of the element. However, where an approximate bending moment diagram, which is conservative in terms of buckling, can be constructed using a "straight line", this approximate bending moment may be used with the method of this section to calculate m_t.

m_t should be obtained from Table F.1. These values are from BS5950-1[3].

ψ_t is the ratio of the smaller end moment to the larger. Moments that produce compression on the unrestrained flange should be taken as positive. When $\psi_t < -1$, the value of ψ_t should be taken as -1, as shown in Figure F.3.

$$y = \left[\left(\frac{\pi^2 \, EI_z}{L_t^2} \right) / \, N_{cr} \right]^{0,5} = \frac{\lambda_a}{(L_t \; / \; i_z)}$$

where N_{cr} is the critical load defined in Section F.3.2
λ_a is the axial slenderness λ defined in Section F.3.4.

Equivalent uniform moment factor m_t for all other cases

This formula, derived by Singh[33], is applicable in all cases, especially when the bending moment diagram is not a straight line between the torsional restraints defining the ends of the element.

$$m_t = \frac{1}{12}\left(\frac{M_{c.Rd}}{M_{Sd}}\right)_{min}\left(\frac{M_{Sd1}}{M_{c\,Rd1}} + \frac{3M_{Sd2}}{M_{c\,Rd2}} + \frac{4M_{Sd3}}{M_{c\,Rd3}} + \frac{3M_{Sd4}}{M_{c.Rd4}} + \frac{M_{Sd5}}{M_{c\,Rd5}} + 2\,\mu_{SE}\right)$$

Table F.1 *Equivalent uniform moment factor, m_t*

ψ_t \ y	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2
-1,0	1,00	0,76	0,61	0,51	0,44	0,39	0,35	0,31	0,28	0,26	0,24	0,22	0,21
-0,9	1,00	0,78	0,63	0,52	0,45	0,40	0,36	0,32	0,30	0,28	0,26	0,24	0,23
-0,8	1,00	0,80	0,64	0,53	0,46	0,41	0,37	0,34	0,32	0,30	0,28	0,27	0,26
-0,7	1,00	0,81	0,66	0,55	0,47	0,42	0,39	0,36	0,34	0,32	0,30	0,29	0,28
-0,6	1,00	0,83	0,67	0,56	0,49	0,44	0,40	0,38	0,36	0,34	0,33	0,32	0,31
-0,5	1,00	0,85	0,69	0,58	0,50	0,46	0,42	0,40	0,38	0,37	0,36	0,35	0,34
-0,4	1,00	0,86	0,70	0,59	0,52	0,48	0,45	0,43	0,41	0,40	0,39	0,38	0,37
-0,3	1,00	0,88	0,72	0,61	0,54	0,50	0,47	0,45	0,44	0,43	0,42	0,41	0,41
-0,2	1,00	0,89	0,74	0,63	0,57	0,53	0,50	0,48	0,47	0,46	0,45	0,45	0,44
-0,1	1,00	0,90	0,76	0,65	0,59	0,55	0,53	0,51	0,50	0,49	0,49	0,48	0,48
0,0	1,00	0,92	0,78	0,68	0,62	0,58	0,56	0,55	0,54	0,53	0,52	0,52	0,52
0,1	1,00	0,93	0,80	0,70	0,65	0,62	0,59	0,58	0,57	0,57	0,56	0,56	0,56
0,2	1,00	0,94	0,82	0,73	0,68	0,65	0,63	0,62	0,61	0,61	0,60	0,60	0,60
0,3	1,00	0,95	0,84	0,76	0,71	0,69	0,67	0,66	0,65	0,65	0,65	0,64	0,64
0,4	1,00	0,96	0,86	0,79	0,75	0,72	0,71	0,70	0,70	0,69	0,69	0,69	0,69
0,5	1,00	0,97	0,88	0,82	0,78	0,76	0,75	0,75	0,74	0,74	0,74	0,74	0,74
0,6	1,00	0,98	0,91	0,85	0,82	0,81	0,80	0,79	0,79	0,79	0,79	0,79	0,79
0,7	1,00	0,98	0,93	0,89	0,87	0,85	0,85	0,84	0,84	0,84	0,84	0,84	0,84
0,8	1,00	0,99	0,95	0,92	0,91	0,90	0,90	0,89	0,89	0,89	0,89	0,89	0,89
0,9	1,00	1,00	0,98	0,96	0,95	0,95	0,95	0,95	0,95	0,94	0,94	0,94	0,94
1,0	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00

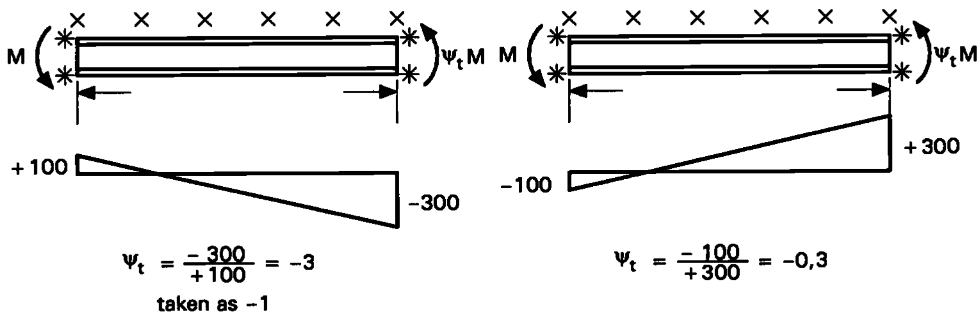


Figure F.3 *Value of ψ_t*

M_{Sd1} to M_{Sd5} are the values of the applied moments at the ends, the quarter points and mid-length of the length between effective torsional restraints, as shown in Figure F.4. Only positive values of M_{Sd} should be included. M_{Sd} is positive when it produces compression in the unrestrained flange.

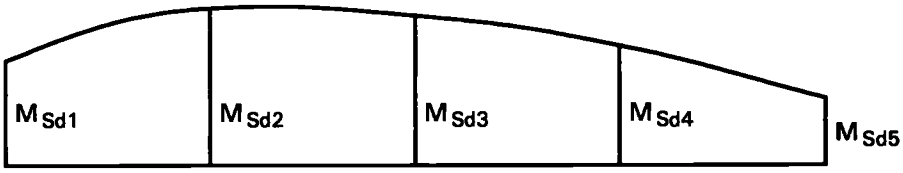


Figure F.4 *Intermediate moments*

$M_{c\,Rd1}$ to $M_{c\,Rd5}$ are the cross-section moment resistances of the sections corresponding to M_{Sd1} to M_{Sd5} as given below.

$\dfrac{M_{SdS}}{M_{c\,RdS}}$ is the greatest of $\dfrac{M_{Sd2}}{M_{c\,Rd2}}, \dfrac{M_{Sd3}}{M_{c\,Rd3}}, \dfrac{M_{Sd4}}{M_{c.Rd4}}$

$\dfrac{M_{SdE}}{M_{c\,RdE}}$ is the greater of $\dfrac{M_{Sd1}}{M_{c\,Rd1}}, \dfrac{M_{Sd5}}{M_{c\,Rd5}}$

$$\mu_{SE} = \left(\frac{M_{SdS}}{M_{c\,RdS}} - \frac{M_{SdE}}{M_{c.RdE}} \right)$$

Only positive values of μ_{SE} should be included.

$\left(\dfrac{M_{c\,Rd}}{M_{Sd}}\right)_{min}$ is the minimum value of $\left(\dfrac{M_{c\,Rd}}{M_{Sd}}\right)$ occurring at points 1 to 5

where $M_{c\,Rd} = W_{p\ell}\, f_y \,/\, \gamma_{M0}$ for Class 1 or Class 2 cross sections (see Section 5.4.5.2 of ENV 1993-1-1).

and W_{pl} is the plastic modulus about the y-axis for Class 1 and Class 2 cross sections or $W_{eff\,pl\,y}$ for haunches complying with Appendix E.

Equivalent section factor c

For uniform depth members, $c = 1,0$.

For tapered I section beams with $\bar{\lambda}_{LT} \leq 1,0$, without plastic hinges in the member length being checked, $c = c_0$,

where c_0 is given in Table F.2 below and comes from *The stability of tapered and haunched beams*[9].

Table F.2 *Section factor, c_o*

D/t_f	r			
	1,5	2,0	2,5	3,0
20	1,162	1,271	1,355	1,425
22	1,128	1,219	1,290	1,350
24	1,108	1,186	1,249	1,304
26	1,094	1,164	1,222	1,272
28	1,084	1,149	1,202	1,249
30	1,077	1,137	1,187	1,232
32	1,072	1,128	1,176	1,219
34	1,067	1,121	1,167	1,208
36	1,064	1,115	1,160	1,200
38	1,061	1,110	1,154	1,193
40	1,059	1,106	1,149	1,187
42	1,057	1,103	1,144	1,182
44	1,055	1,100	1,141	1,178

c_o applies to tapered beams (q = 1,0)

- r is the ratio of end depths d_2 / d_1 of tapered and haunched beams where
- d_1 is the minimum distance between centroids of the external flanges (see Figure F.2)
- d_2 is the maximum distance between centroids of the external flanges (see Figure F.2)
- D is the overall depth of the I section at the shallow end of the haunch
- t_f is the average thickness of the two external flanges

Note that c_0 is valid only for $\bar{\lambda}_{LT} \ngtr 1,0$. Where $\bar{\lambda}_{LT} > 1,0$, the buckling can be checked using the greatest $\bar{\lambda}_{LT}$, which will normally occur at the deepest end of uniform width members without steps in the flange or web thicknesses.

For haunched I section beams with $\bar{\lambda}_{LT} \leq 1,0$, without plastic hinges in the member length being checked

$$c = 1 + (c_o - 1)\sqrt{q}$$

$$\text{where} \quad q = \frac{\text{tapered length of haunch in element}}{\text{total length of element}}$$

F.3.4 Slenderness

$\bar{\lambda}$ is defined by Section 5.5.1.2(1) of ENV 1993-1-1 using N_{cr} as defined in F.3.2 of this publication.

$\bar{\lambda}_{LT}$ is defined by Section 5.5.2(5) of ENV 1993-1-1 or F.2.1 using M_{cr} as defined in F.3.3 of this publication.

Note that for $\bar{\lambda}_{LT} \leq 1,0$ and using the "c" factors in F.3.3, the shear centre of the section is taken as at the shallow end, but for $\bar{\lambda}_{LT} > 1,0$ calculations are based on the geometry at the deepest end and c is taken as 1,0.

The axial slenderness λ is given by:

$$\lambda = \frac{L/i_z}{\left(\alpha + \frac{I_t\, L_t^2}{2{,}6\ \pi^2\ I_z\ i_s^2}\right)^{0{,}5}}$$

The bending slenderness λ_{LT} is given by:

$$\lambda_{LT} = \left(m_t^{0{,}5} c\right) \left(\frac{W_{pl\,y}}{A}\ \frac{2a}{i_s^2}\right)^{0{,}5} \lambda$$

where λ is the axial slenderness defined above with a conservative approximation

$$\lambda_{LT} = \left(m_t^{0{,}5} c\right) \left(\frac{h_s a}{i_s^2}\right)^{0{,}5} \lambda$$

F.3.5 Stability adjacent to plastic hinge

To ensure adequate rotation capacity of a plastic hinge, the slenderness of the adjacent members must be limited to avoid premature lateral or lateral-torsional instability. Tapered members checked to Appendix F.3 should not contain plastic hinges, because that condition was specifically excluded from the basic research by Horne *et al.*[9]

$$\text{Check} \quad L \leq 0{,}4 \left[\frac{1}{\frac{M_{Sd}}{C_1\ W_{p\ell y} f_y} + \frac{4N_{Sd}}{Af_y}\left(\frac{i_{LT}}{i_z}\right)^2}\right]^{0{,}5} \lambda_1\ i_{LT}$$

where L is the distance between torsional restraints (derived in Section F.4).

F.4 Derivation of stable length

The following derivation of the stable length of a member containing a plastic hinge is conservative for low axial loads. However, the formula below provides an interaction of axial compression and moment over the complete range of possibilities and gives $\bar{\lambda}_{plastic\ z} \leq 0{,}2$ and $\bar{\lambda}_{plastic\ LT} \leq 0{,}4$, which agrees with EC3. It adopts a linear form of combination, because the combination of axial compression and moment is linear in Section 5.5.4 of ENV 1993-1-1 at low slenderness. In this way, the principles of EC3 are demonstrably adhered to.

$$\bar{\lambda}_{LT} = \left(\frac{M_{p\ell}}{C_1\ M_{cro}}\right)^{0{,}5} \qquad \bar{\lambda}_z = \left(\frac{N_{pl}}{N_{cr}}\right)^{0{,}5}$$

$$\therefore \text{ take } \bar{\lambda}_{plastic} = \left[\frac{M_{Sd}}{M_{p\ell}}\ \frac{M_{p\ell}}{C_1\ M_{cro}} + \left(\frac{\bar{\lambda}_{plastic\ LT}}{\bar{\lambda}_{plastic\ z}}\right)^2 \frac{N_{Sd}}{N_{p\ell}} \times \frac{N_{p\ell}}{N_{cr}}\right]^{0{,}5}$$

$$= \left[\frac{M_{Sd}}{M_{p\ell}} (\bar{\lambda}_{LT})^2 + \left(\frac{\bar{\lambda}_{plastic\ LT}}{\bar{\lambda}_{plastic\ z}}\right)^2 \frac{N_{Sd}}{N_{p\ell}} (\bar{\lambda}_z)^2\right]^{0.5}$$

When $\bar{\lambda}_{LT} = 0,4$ and $\bar{\lambda}_z = 0,2$ (the limiting slenderness values for buckling in ENV 1993-1-1), this reduces to:

$$\therefore \bar{\lambda}_{plastic} = \left[\frac{M_{Sd}}{M_{p\ell}} (\lambda_{LT})^2 + 4 \frac{N_{Sd}}{N_{p\ell}} (\lambda_z)^2\right]^{0.5} \times \left(\frac{1}{\lambda_1}\right)$$

To derive an expression that requires less calculation, write:

$$N_{cr} = \left(\frac{N_{pl}}{N_{cr}}\right)^2 = \pi^2 E A \left(\frac{i_z}{L_z}\right)^2$$

$$M_{cr0} \approx \pi^2 E W_{p\ell\,y} \left(\frac{i_{LT}}{L_{LT}}\right)^2$$

$$\therefore \bar{\lambda}_{plastic} = \left[\frac{M_{Sd}}{C_1 \pi^2 E W_{p\ell\,y} \left(\frac{i_{LT}}{L_{LT}}\right)^2} + \left(\frac{\bar{\lambda}_{pLT}}{\bar{\lambda}_{pz}}\right)^2 \frac{N_{Sd}}{\pi^2 EA \left(\frac{i_z}{L_z}\right)^2}\right]^{0.5}$$

Taking $L_{LT} = L_Z = L$ = distance between restraints

$$\bar{\lambda}_{plastic} = \left[\frac{M_{Sd}}{C_1 W_{p\ell\,y} f_y} + \left(\frac{\lambda_{pLT}}{\lambda_{pz}}\right)^2 \frac{N_{Sd}}{Af_y} \left(\frac{i_{LT}}{i_z}\right)^2\right]^{0,5} \left(\frac{f_y}{\pi^2 E}\right)^{0,5} \left(\frac{L}{i_{LT}}\right)$$

$$\therefore L \leq \bar{\lambda}_{plastic} \left[\frac{1}{\frac{M_{Sd}}{C_1 W_{p\ell\,y} f_y} + \left(\frac{\bar{\lambda}_{pLT}}{\bar{\lambda}_{pz}}\right)^2 \frac{N_{Sd}}{Af_y} \left(\frac{i_{LT}}{i_z}\right)^2}\right]^{0,5} \left(\frac{\pi^2 E}{f_y}\right)^{0,5} i_{LT}$$

Section 5.5.2(7) of ENV 1993-1-1 defines the limit for stability with respect to lateral torsional buckling as $\bar{\lambda}_{LT} \leq 0,4$ and the equivalent limit with respect to lateral flexural buckling is $\bar{\lambda} \leq 0,2$.

$$\therefore \quad \bar{\lambda}_{plastic} = \bar{\lambda}_{plastic\ LT} = \bar{\lambda}_{pLT} = 0,4$$

$$\bar{\lambda}_{plastic\ z} = 0,2$$

$$\therefore \quad \left(\frac{\bar{\lambda}_{pLT}}{\bar{\lambda}_{pz}}\right)^2 = \left(\frac{0,4}{0,2}\right)^2 = 4$$

Writing $\lambda_1 = \left(\frac{\pi^2 E}{f_y}\right)^{0,5}$

$L \leq 0,4$

C_1 may conservatively be taken as 1,0

i_{LT} for hot rolled I sections may be taken as $\left(\frac{i_z}{0,9}\right)$.

APPENDIX G MEMBER DUCTILITY FOR PLASTIC DESIGN

For plastic design, the members must be sufficiently stable at the plastic hinges to undergo considerable rotation without either local buckling or rupture. This is ensured by limiting the cross-section geometry and the material properties.

G.1 Cross-section geometry

The geometric limitations given in Section 5.3.3 of ENV 1993-1-1 relate to:

(i) Thickness (or more correctly slenderness ratio) of flat elements, e.g. flange or web in I sections

(ii) Symmetry.

G.1.1 Thickness of elements

The thickness of each element must be such that it does not shed load as a result of local buckling under the stresses and/or plastic deformations it is required to sustain.

Depending on the thickness of the elements, EC3 classifies cross sections in terms Class 1, 2, 3 or 4, which are defined in Section 5.3.2 and Table 5.3.1 of ENV 1993-1-1.

At locations of plastic hinges that are required to rotate appreciably at or below ultimate limit state, the cross section should be Class 1. However, for small rotations, Class 2 members could be accepted, if the redistribution from the elastic bending moment is not more than 15%, as Section 5.2.1.3 of ENV 1993-1-1. This is likely to be reduced to 10% for the EN.

Haunches should be checked to Section 5.3.3(5) of ENV 1993-1-1 (see Figure G.1).

(i) The web thickness must not change within a distance $2d$ to each side of the hinge, where d is the clear depth of the web at the plastic hinge.

(ii) The compression flange must be Class 1 within a distance $2d$ to each side of the hinge, as defined above.

(iii) The compression flange must be Class 1 to each side of the hinge up to a distance x, such that:

$$\frac{\text{Moment at x}}{\text{Reduced plastic moment of resistance at x}} \ngtr 0{,}8$$

where the reduced plastic moment of resistance is the plastic moment of resistance as reduced by coincident axial force and shear force.

(iv) Elsewhere the compression flange must be Class 1 or Class 2, and the web should be Class 1, 2 or 3.

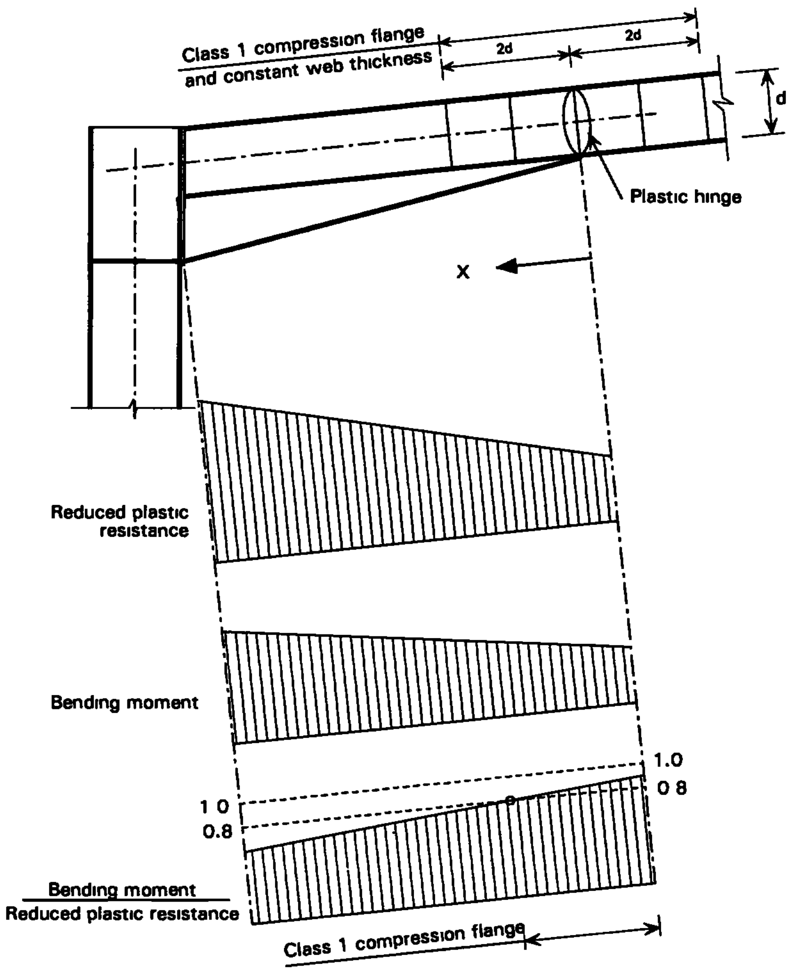


Figure G.1 *Haunch thickness requirements*

G.1.2 Symmetry

At plastic hinges, the cross section should be symmetrical about the plane of the frame, as required in Section 5.3.3(1) of ENV 1993-1-1. This is always true in portals made from hot rolled I sections.

G.2 Material properties

The material property requirements of EC3 are given in Section 3 of ENV 1993-1-1. Special requirements for plastic analysis are given in 3.2.2.2, but the grades of steel listed in Table 1 of ENV 1993-1-1 will fulfil these requirements. (Note that the designation of structural steels has now been revised and the relevant strength grades are now designated S235, S275 and S355.) Other grades to EN 10025 are not suitable and other grades to EN 10113 may not be suitable. The suitability of S420 and S460 steel has not been considered in this publication.

APPENDIX H SHEAR BUCKLING RESISTANCE

This Appendix provides guidance on the use of EC3 for sections for which the webs are sufficiently slender to be prone to shear buckling. This is very rare in portal frames fabricated from hot rolled I sections.

Shear buckling resistance is covered in Section 5.6 of ENV 1993-1-1.

For normal portals, the tension field method should be ignored, as it relies on web stiffeners, which are not economic for portal frames. Therefore, the designer need only consider the simple post-critical method given in Section 5.6.3 of ENV 1993-1-1.

The interaction of shear force, bending moment and axial force for the simple post-critical method is covered in Section 5.6.7.2 of ENV 1993-1-1.

If simple calculations of the shear force in the haunch show this force to be so large as to cause problems, it may be possible to demonstrate that the angle between the flange forces gives a considerable reduction of shear in the web. The greater inclination of the haunch bottom flange (compared with the top flange) provides a vertical component of force, which relieves the shear in the haunch. As this shear relief depends on the magnitude of the moment, the worst case of high shear and low moment must be checked, in addition to the worst case of high shear and high moment. The true cross-section properties must be used for these calculations (modified by the connection capacity of the haunch/rafter welds if these are light), as the bottom flange force is reduced by any inner flange near the shallow end of the haunch.

APPENDIX I THE MERCHANT-RANKINE FAILURE CRITERION

I.1 Classic Merchant-Rankine

The Merchant-Rankine failure criterion predicts the reduction in the capacity of a structure below its plastic collapse capacity due to its tendency to buckle. It was developed during the 1950s and 1960s by Horne[34] and others and is now a mature and well-accepted criterion. It is included in ENV 1993-1-1 in an unconventional form, which is not always helpful.

The later modification to the Merchant-Rankine formula by Wood[35] is not included in EC3 at present.

The Merchant-Rankine failure criterion is:

$$\frac{1}{\lambda_{fail}} = \frac{1}{\lambda_{plastic}} + \frac{1}{\lambda_{crit}} \tag{I.1}$$

where $\lambda_{fail} = \dfrac{\text{failure load}}{\text{applied load}} = \dfrac{V_{fail}}{V_{Sd}}$

$$\lambda_{plastic} = \frac{\text{plastic analysis collapse load}}{\text{applied load}} = \frac{V_{plastic}}{V_{Sd}}$$

$$\lambda_{crit} = \frac{\text{elastic critical buckling load}}{\text{applied load}} = \frac{V_{cr}}{V_{Sd}}$$

for the first critical mode under the applied load.

Putting the applied load equal to the ultimate limit state load, λ_{fail} must not be less than 1,0. In the above form, the criterion is very simple to apply to check the capacity of the structure, but a consistent set of axial forces, shear forces and bending moments must also be calculated for member stability checks.

I.2 Merchant-Rankine in Eurocode 3

In Section 5.2.6.3(4) of ENV 1993-1-1, the criterion is re-expressed as a factor that should be applied to the forces and moments of the plastic collapse mechanism, assuming $\lambda_{fail} = 1,0$. This method of application is very confusing and is easily applied wrongly in a way that penalises stronger structures. Three methods of applying Merchant-Rankine are given in Section 12.4.

The derivation of the EC3 factor is as follows:

$$\frac{1}{\lambda_{fail}} = \frac{1}{\lambda_{plastic}} + \frac{1}{\lambda_{crit}}$$

$$\therefore \quad \frac{1}{V_{fail}} = \frac{1}{V_{plastic}} + \frac{1}{V_{cr}}$$

$$\therefore \quad \frac{1}{V_{plastic}} = \frac{1}{V_{fail}} - \frac{1}{V_{cr}} = \frac{V_{cr} - V_{fail}}{V_{fail}\ V_{cr}}$$

$$\therefore \quad V_{plastic} = \frac{V_{fail}\ V_{cr}}{V_{cr} - V_{fail}} = V_{fail}\left(\frac{V_{cr}}{V_{cr} - V_{fail}}\right)$$

$$= V_{fail}\left(\frac{1}{1 - \frac{V_{fail}}{V_{cr}}}\right)$$

When $V_{fail} = V_{Sd}$

$$V_{plastic} = V_{Sd}\left(\frac{1}{1 - \frac{V_{Sd}}{V_{cr}}}\right)$$

i.e. $V_{Sd} = V_{plastic}\left(1 - \frac{V_{Sd}}{V_{cr}}\right)$

APPENDIX J EFFECTIVE LENGTH AND EFFECTIVE SLENDERNESS

J.1 General

The effective length and effective slenderness are directly related, because they are both functions of the elastic critical buckling load.

J.2 Effective length

The effective length is the length which gives

$$\frac{\pi^2 EI}{\ell_{\text{eff}}^2} = N_{\text{cr cs}}$$

where $N_{\text{cr cs}}$ is the theoretical elastic critical buckling load of the member under relevant axial force and forming part of the complete structure. It is the axial force in the member when the structure reaches its elastic critical buckling load, V_{cr}, and is given by:

$$N_{\text{cr cs}} = N_{\text{Sd}} \times \frac{V_{\text{cr}}}{V_{\text{Sd}}} = \frac{N_{\text{Sd}}}{(V_{\text{Sd}} / V_{\text{cr}})}$$

Combining gives $\dfrac{\pi^2 EI}{\ell_{\text{eff}}^2} = \dfrac{N_{\text{Sd}}}{(V_{\text{Sd}} / V_{\text{cr}})}$

and thus $\ell_{\text{eff}} = \left(\dfrac{\pi^2 EI}{N_{\text{Sd}}} \times \dfrac{V_{\text{Sd}}}{V_{\text{cr}}} \right)^{0,5}$

J.3 Effective slenderness

For an individual member, EC3 gives rules for evaluating its buckling resistance in terms of the relative slenderness $\bar{\lambda} = (N_{\text{p}\ell} / N_{\text{cr}})^{0,5}$.

The effective slenderness for a member in a complete structure is $\bar{\lambda}_{\text{eff}} = (N_{\text{p}\ell} / N_{\text{cr cs}})^{0,5}$.

From J.2 above, $N_{\text{cr cs}} = N_{\text{Sd}} / (V_{\text{Sd}} / V_{\text{cr}})$

$$\therefore \quad \bar{\lambda}_{\text{eff}} = \left[\frac{N_{\text{p}\ell}}{N_{\text{Sd}}/(V_{\text{Sd}}/V_{\text{cr}})} \right]^{0,5} = \left[\left(\frac{N_{\text{pl}}}{N_{\text{Sd}}} \right) \left(\frac{V_{\text{Sd}}}{V_{\text{cr}}} \right) \right]^{0,5}$$

Putting $N_{\text{p}\ell} = \beta_{\text{A}} A f_y$ gives $\bar{\lambda}_{\text{eff}} = \left[\left(\dfrac{Af_y}{N_{\text{Sd}}} \right) \left(\dfrac{V_{\text{Sd}}}{V_{\text{cr}}} \right) (\beta_{\text{A}}) \right]^{0,5}$

APPENDIX K DIFFERENCES BETWEEN EUROCODE 3 AND UK PRACTICE

K.1 General

There are a number of differences between EC3 and previous UK practice, which need to be clearly understood by designers who are familiar with the latter but intend to use the former. Many of the principal differences affecting portal frame construction are noted below. Reference to BS 5950 in this Section implies BS 5950-1[3].

K.2 Axes

The axis convention is different. It is explained briefly below, but for greater detail Section 1.6.7 and Figure 1.1 of ENV 1993-1-1 should be examined carefully.

- The xx axis is along the length of the member.
- The yy axis is at right angles to the stiff plane (traditionally it was the xx axis).
- The zz axis is at right angles to the weak plane (traditionally it was the yy axis).

K.3 Definitions

Many definitions differ from previous UK practice:

- Actions - In Eurocodes, loads are called "actions" (derived from the action on the structure of external influences such as gravity, wind, temperature etc.). Direct actions are applied forces and indirect actions are settlement, temperature loads etc.
- Resistance - The load bearing capacity of a member, a cross section, a fastener or a weld is called the "resistance". The term "capacity" is reserved for deformation and rotation.
- Effects of actions - The term "effect of actions" includes the stresses and deflections, as well as the internal forces and moments.
- Internal forces and moments - The internal forces and moments (sometimes called the action effects) are the axial force, shear force, bending moments and torque in a member, as distinct from the external forces and moments applied to it.

There is a list of definitions in Section 1.4 of ENV 1993-1-1.

K.4 Partial safety factors

Eurocode 3 may appear to differ from previous practice in BS 5950 in the clear use of a partial safety factor γ_M to decrease resistance, as well as partial safety factors γ_G and γ_Q to increase loads. However, in BS 5950 the partial safety factor for resistance does exist in principle, but is set at 1,0 for steel. It is incorporated in the tabulated values in the same way as the respective partial safety factors for bolts, welds etc.(see Clause 2.1.1 and Appendix A of BS 5950-1[3]).

K.5 Load combinations

BS 5950 uses one set of load factors for a combination of (dead + live) loads, but a lower load factor for (dead + live + wind) loads.

In principle, EC3 requires that all variable actions (live loads, wind loads etc.) are considered in the same load combination, but includes a reduction factor ψ on all variable actions, except the most unfavourable (see equation 2.9 of ENV 1993-1-1). For this purpose, the floor loads on all floors, including any reduction for area, are one variable action, but the roof load, due to snow, is a separate, independent variable action like the wind load. Where it is not possible to identify the most unfavourable load by inspection, several different load combinations must be applied, each assuming a different load is the most unfavourable to find which combination is most unfavourable.

Eurocode 3 allows simplified load combinations (see equations 2.11 and 2.12 of ENV 1993-1-1). These are simplifications that may sometimes be slightly less conservative compared with equation 2.9, but are safe in all cases. The EC3 load combinations are given in Sections 10.3 and 11.6 of this publication for serviceability and ultimate limit state respectively.

K.6 Explicit consideration of second-order effects

EC3 requires that second-order effects are explicitly considered in analysis, either by second-order analysis or by modifications to classic first-order analysis, as explained in Section 12.3 for elastic analysis and Section 12.4 for plastic analysis.

For single-storey portal frame design, the sway-check method in BS 5950 uses deflection checks to assess the stiffness. If the stiffness is too low, it has to be increased. If the stiffness is above a certain limit, second-order effects are ignored. These checks are not entirely reliable, as shown by Davies[24,25]. The sway-check method in BS 5950 ignores second-order effects in portal frames for $V_{Sd} / V_{cr} \leq 0{,}2$ ($\lambda_{cr} \geq 5$ in BS 5950 terminology), which appears to be less demanding than both EC3 and multi-storey building design to BS 5950. However, there is justification for this. Firstly, in most portals, deflection is in a symmetrical mode for which λ_{cr} is approximately twice the magnitude of the sway λ_{cr} estimated by BS 5950, so the actual $V_{Sd} / V_{cr} \leq 0{,}1$ when the estimated $V_{Sd} / V_{cr} \leq 0{,}2$. Secondly, in portals in which deflection is in an overall sway mode, the cladding on the gable ends and roof provide very significant stiffness (except for clip-fix systems) so the actual V_{cr} of the clad structure will be approximately twice the V_{cr} for the bare frame.

K.7 Prying in bolts

BS 5950 allows for prying in the bolts by reducing the tabulated values below the actual bolt strength as measured in pure tension. ENV 1993-1-1 does not include such a reduction in the values in Table 3.3, so prying effects must be explicitly included in the calculations where appropriate.

K.8 Base fixity

It is common practice in the UK to assume that column bases are truly pinned for the ULS bending moment diagram, where they are actually nominally connected. Section 5.2.3.3 of ENV 1993-1-1 requires consideration of the actual flexibility. In the case of nominally pinned bases, the assumption of a truly pinned base is conservative, so is acceptable. However, for "fixed" bases the actual flexibility must be considered. The same approach as in Clause 5.1.3 of BS 5950-1 is recommended in Appendix A of this publication.

K.9 Overstrength effects

United Kingdom design has always used higher partial safety factors for bolts and welds than for other parts of the structure, producing heavier connections than in some other countries. Therefore, the effects of overstrength members are catered for in BS 5950 without increasing the design forces in the bolts and welds. With ENV 1993-1-1, a specific increase must be used, as explained in Section 14.1 above.

Worked Example Number 1

Contents

Note:

References to Sections and Appendices in the Worked Example refer to this publication.

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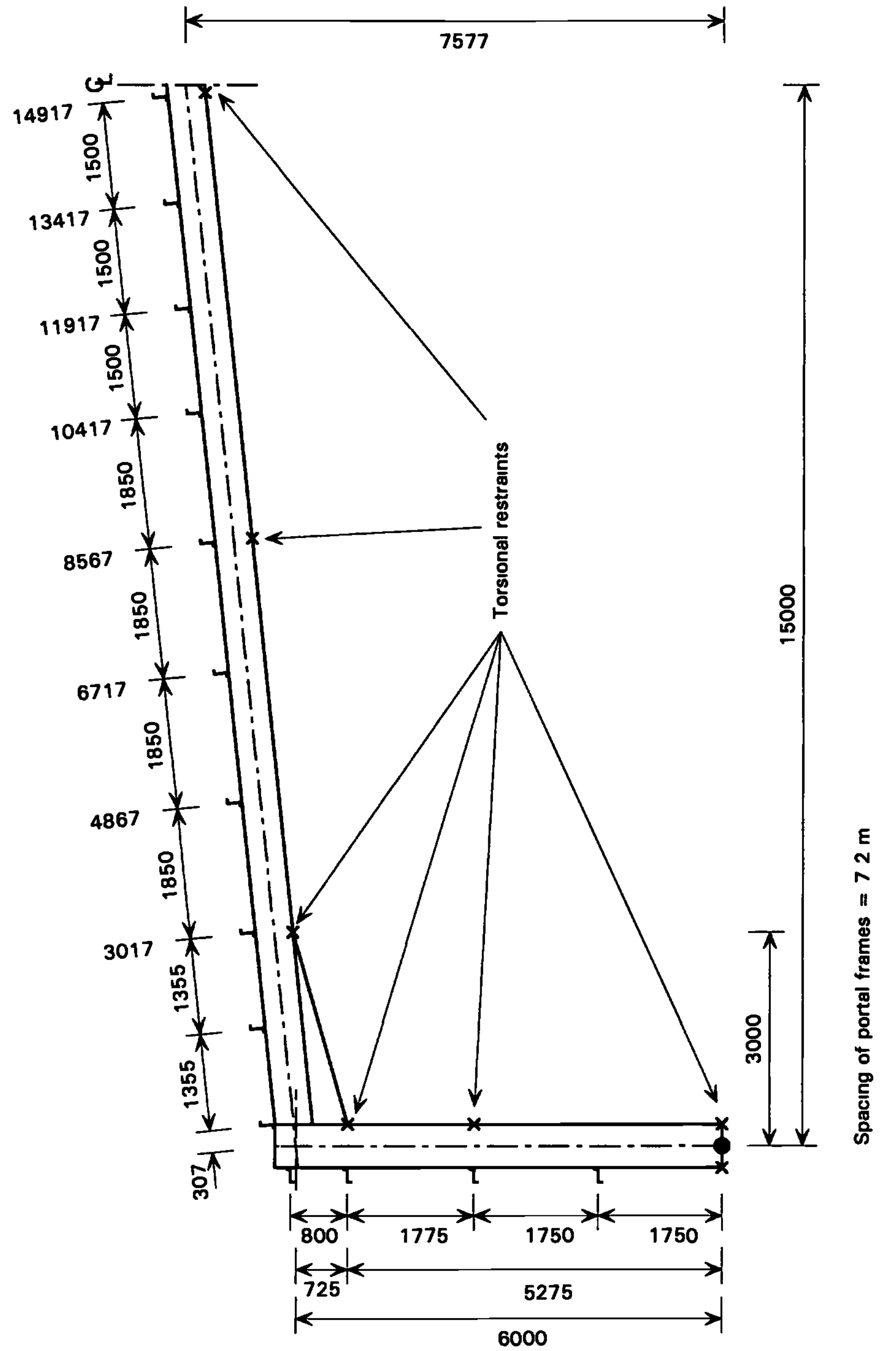
The Steel Construction Institute	Job No: *BCB 588*	Page *1* of *54*	Rev *A*
Silwood Park, Ascot, Berks SL5 7QN Telephone: (01344) 623345 Fax: (01344) 622944	Job Title *Portals for Europe*		
	Subject *EC3 plastic portal: General*		
CALCULATION SHEET	Client *BS/DETR*	Made by *CMK*	Date
		Checked by *MSG*	Date

1. GENERAL

1.1 Frame geometry



1.2 Gravity loads

Assume the following load conditions:

The snow load in many countries will be greater than in the UK. The value of 0.75 kN/m² is for an example but the relevant national code must be used

roofing = 0,20 kN/m² × 7,2 m = 1,44 kN/m on plan
services = 0,20 kN/m² × 7,2 m = 1,44 kN/m on plan
snow = 0,75 kN/m² × 7,2 m = 5,40 kN/m on plan

From preliminary sizing calculations, member self weights are given by:

rafter = 30 × 82 kg/m = 2460 kg
haunch = 3 × 82 kg/m = 246 kg
column = 6 × 113 kg/m = 678 kg
Total = 3384 kg

1.3 Partial safety factors

The ENV gives the following:

Partial safety factors for loads — *Section 11.4 and NAD*

$\gamma_G = 1{,}35$
or $\gamma_G = 1{,}00$ when dead load reduces stresses in the section being checked
$\gamma_Q = 1{,}50$

Partial safety factors for resistance — *ENV 5.1.1*

$\gamma_{M0} = 1{,}10$
$\gamma_{M1} = 1{,}10$

Check the above values in the relevant National Application Document (NAD)

1.4 Combination factor ψ — *Section 11.6*

The combination ψ must be found from Eurocode 1.1 Part 1.1 or the relevant NAD. Note that where the NAD specifies a value for ψ_o, this value must be used instead of the value from Eurocode 1

The value in Eurocode 1 Part 1.1, ENV 1991-1-1: 1996 Table 9.3 is 0.7 generally, but 1.0 for structures supporting storage loads

Note that in this example, the wind load always reduces the effects of roof load. Therefore, the critical design combinations are:

(i) Maximum gravity loads without wind, causing maximum sagging moment in the rafter and maximum hogging moment in the haunches

(ii) Maximum wind with minimum gravity loads, causing maximum reversal of moment compared with case 1.4i. The worst wind case might be from either transverse wind or longitudinal wind, so both must be checked

Because maximum gravity load relieves the wind load, the worst combination does not include maximum gravity together with maximum wind. Therefore, the combination factor ψ is never applied in this example

2. ULTIMATE LIMIT STATE ANALYSIS

2.1 Load combination No. 1: dead + snow

2.1.1 Loads

The loads are the gravity loads calculated in 1.2 above

2.1.2 Frame imperfection equivalent horizontal forces:

Section 15.1, 15.2, 17.2(3) ENV 5.2.4.3

$$\phi = k_c\, k_s\, \phi_0$$

where $\phi_0 = 1 / 200$

$$k_c = (0{,}5 + ½)^{0{,}5} = 1{,}0$$
$$k_s = 1{,}0$$

giving $\phi = 1{,}0 \times 1{,}0 \times 1 / 200 = 1 / 200$

The column loads could be calculated by a frame analysis, but a simple calculation based on plan areas is suitable for single storey portals

(i) Permanent loads (unfactored):

rafter	=	15 × 0,82	=	12,3 kN
roofing	=	15 × 1,44	=	21,6 kN
services	=	15 × 1,44	=	21,6 kN
½ side sheeting	=	0,5 × 6,0 × 1,44	=	4,3 kN
½ side columns	=	0,5 × 6,0 × 1,25	=	3,8 kN
		Total	=	63,6 kN

(ii) Variable loads (unfactored):

snow = 15 × 5,40 = 81,0 kN

Thus the unfactored equivalent horizontal forces are given by:

permanent/column	=	63,6 / 200	=	0,318 kN
variable/column	=	81,0 / 200	=	0,405 kN

Note: EC3 requires that all loads that could occur at the same time are considered together, so frame imperfection forces and wind loads should be considered as additive to permanent loads and variable loads with the appropriate load combination factor, ψ — *Section 11.6*

2.1.3 Partial safety factors and second-order effects

These calculations use Merchant-Rankine to account for second-order effects. This is allowed in ENV 1993-1-1 for $V_{Sd} / V_{cr} \leq 0,20$ — *ENV 5.2.6.3, Section 12.4*

If the software has no input request for a magnification factor for second-order effects, it is most convenient to allow for second-order effects by modifying the partial safety factors for the loads. This is done below

Assume for preliminary calculations that $V_{Sd} / V_{cr} = 0,12$ — *Section 17.2(4)(b)*

Then the Merchant-Rankine factor

$$\frac{1}{1 - V_{Sd}/V_{cr}} = \frac{1}{1 - 0,12} = 1,136$$

Therefore the modified partial safety factors are:

$$\gamma_G = 1,35 \times 1,136 = 1,53$$
$$\gamma_Q = 1,50 \times 1,136 = 1,70$$

2.1.4 Analysis — *Section 17.2(4)*

In this example, the bases have been assumed to be truly pinned as Appendix A.1 for simplicity

If software is used that has been written to perform calculations to BS 5950 and not EC3, care should be taken to avoid using BS 5950 member sizing routines, which differ from EC3

A major difference between BS 5950 and EC3 is that EC3 does not require a reduction of plastic moment of resistance for low axial loads. This an aspect of EC3 that is more economical than BS 5950 — *Appendix C.4.4*

If the software allows the user to define the plastic moment of resistance directly, the EC3 plastic moment of resistance can be entered *as the "user defined" to avoid use of the BS 5950 reduction of plastic moment of resistance.*

Steel grade used is S275 — ENV 3.2.2.1

Assume sections are Class 1, then check later

Column: 610 × 229 × UB113 has $t_f \leq 40$ mm, $\therefore f_y = 275$ N/mm²

$$M_p = W_{pl.y}\, f_y / \gamma_{M0} = \frac{3287 \times 275}{1,1 \times 10^3}$$

$$= 821,8 \text{ kNm}$$

Rafters: 533 × 210 × UB82 has $t_f \leq 40$mm, $\therefore f_y = 275$ N/mm²

$$M_p = W_{pl.y}\, f_y / \gamma_{M0} = \frac{2058 \times 275}{1,1 \times 10^3} = 514,5 \text{ kNm}$$

Load factor	Hinge number	Span no.	Member	Position (m)	Hinge status
0,897	1	1	RH column	5,275	Formed
1,005	2	1	LH rafter	13,955	Formed

Although hinge 1 occurs at a load factor ≤ 1,0, a mechanism is not formed until the second hinge has formed. Therefore this combination of section sizes is suitable as preliminary sections

A diagram of bending moments, shear and axial forces is given opposite for load factor 1,0, which is the condition at ultimate limit state

The bending moments in the columns are shown to reduce from the level of the bottom of the haunch to the top of the column. This is the true bending moment in the column when the haunch to column connection is a bolted connection on the inner vertical face of the column

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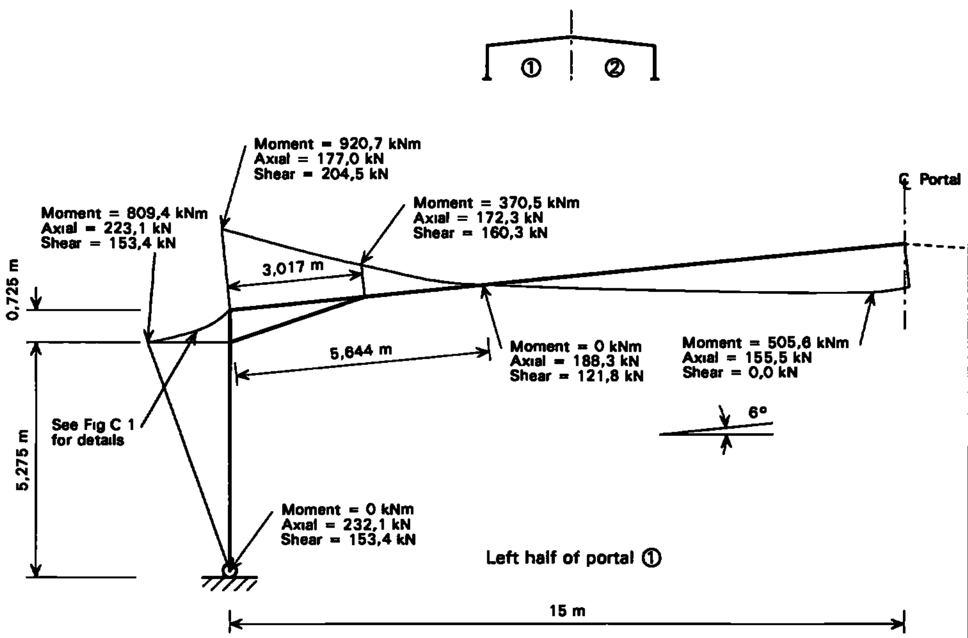
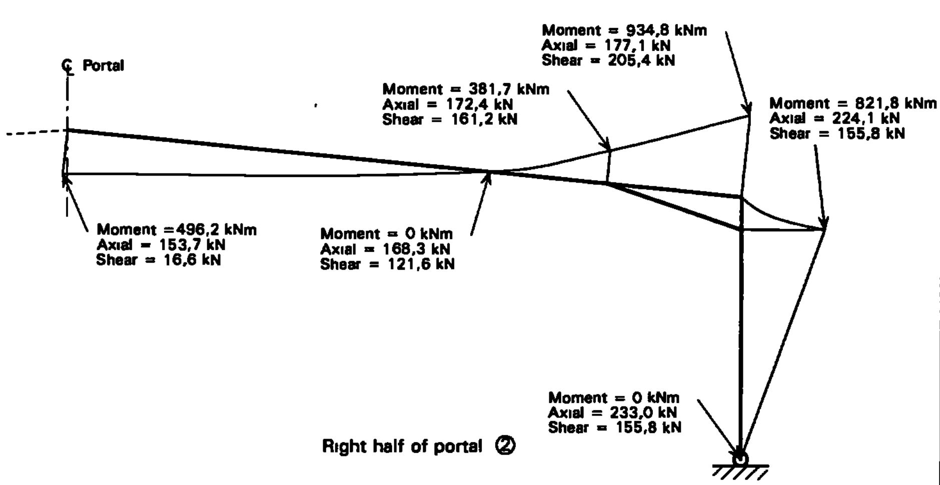
CALCULATION SHEET

Load combination no. 1:
bending moment, shear and axial force diagram





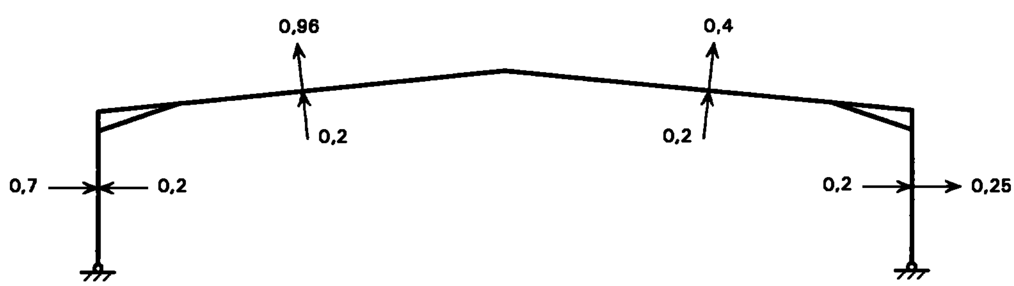
2.2 Load combination no. 2: dead +transverse wind

2.2.1 Loads

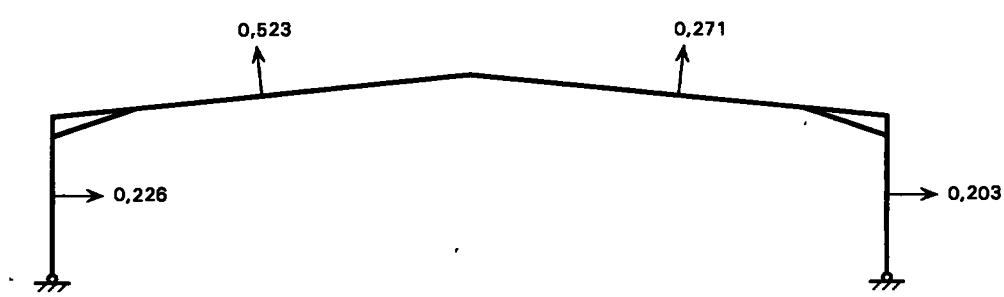
Assume the following transverse wind loading

The wind coefficients have been converted into wind pressures by multiplying by the dynamic pressure given below:

q_e = 0,451 kN/m²



Wind coefficients for transverse wind



Area wind loads for transverse wind
units = kN/m²

For this load combination, the wind loads applied to the structure result in a net upward force on the roof. In this case, therefore, imposed loads such as services and snow are neglected to ensure that the least favourable load combination is considered

The unfactored loads acting on the structure are given by:

wind load	*- LH column*	=	*0,226 × 7,2*	=	*1,63 kN/m*
	- LH rafter	=	*–0,523 × 7,2*	=	*–3,77 kN/m*
	- RH rafter	=	*–0,271 ×7,2*	=	*–1,95 kN/m*
	- RH column	=	*–0,203 ×7,2*	=	*–1,46 kN/m*
dead loads	*- columns*	=	*1,13*	=	*1,13 kN/m*
	- rafters	=	*0,82*	=	*0,82 kN/m*
	- sheeting	=	*1,44*	=	*1,44 kN/m*

Note:

1) *Various variable loads should be neglected under certain wind load cases. In this case, the snow and service loads would have a favourable but temporary influence on the applied loading and as a result have been omitted from this load combination*
2) *The load factors applied to other permanent loads are taken as 1,0 as they are favourable actions, as in accordance with Section 2.3.3.1 of EC3*

2.2.2 Frame imperfection equivalent horizontal forces — *Section 17.2(3)*

The frame imperfection factor (EC3, Section 5.2.4.3) may be omitted from this load combination because the net force is tension (from uplift), from which does not destabilise the structure — *Section 15.2.1*

2.2.3 Partial safety factors and second-order effects

The Merchant-Rankine factor is not applied to the loads as the load combination results in an uplift load case causing tension in the members. It is therefore incorrect to apply the Merchant-Rankine factor, which allows for the destabilising effects of axial compression in the members — *Section 12.4.1*

Therefore the partial safety factors for loads are:

$\gamma_G = 1{,}0$

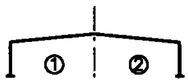
$\gamma_Q = 1{,}5$

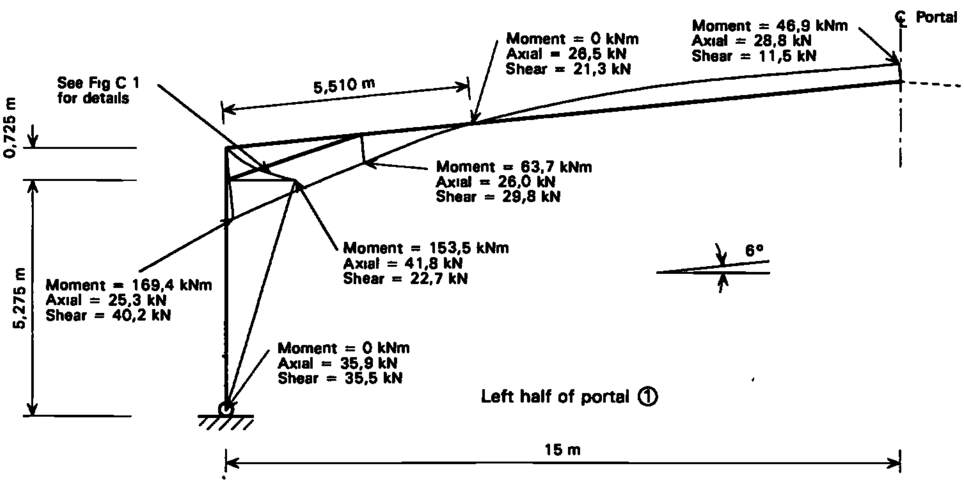
2.2.4 Analysis — *Section 17.2(4)*

The collapse load factor = 5,931, which is greater than load case no. 1. Therefore this load case is not the critical case for cross-sectional resistance, but member stability for this case must be checked because the moments are in the opposite sense to load case no. 1

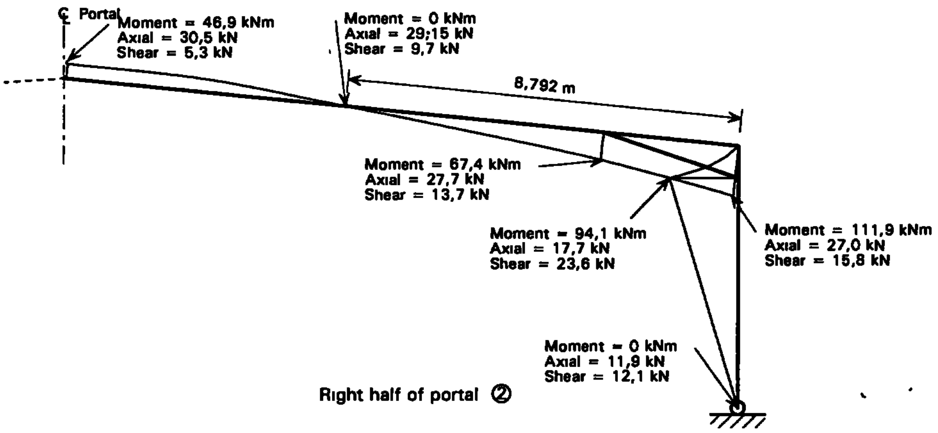
The bending moments from this load case are shown below

Load combination no. 2:
bending moment, shear and axial force diagram





Left half of portal ①



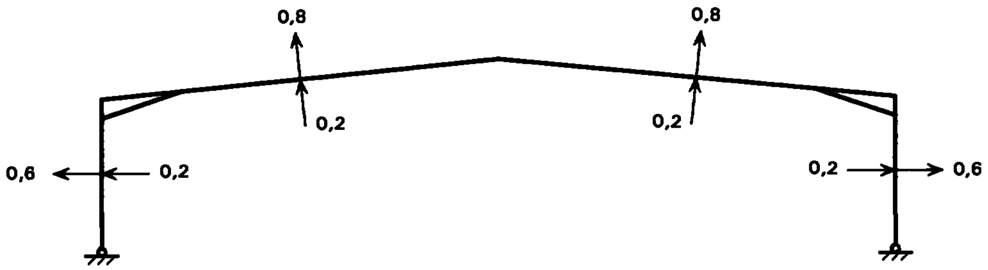
Right half of portal ②

2.3 Load combination no. 3: dead + longitudinal wind

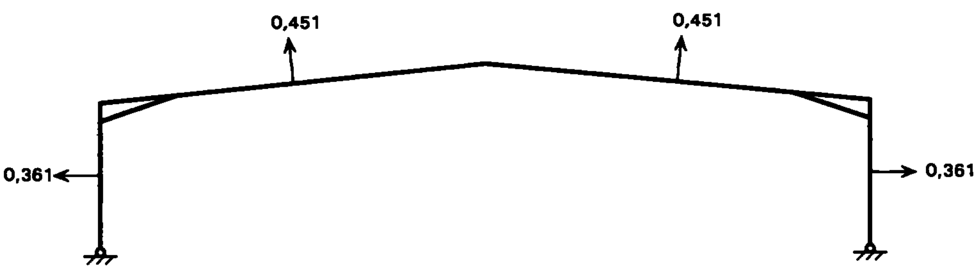
2.3.1 Loads

The wind coefficients have been converted into wind pressures by multiplying by the dynamic pressure given below:

$q_e = 0{,}451 \text{ kN/m}^2$



Wind coefficients for longitudinal wind



Area wind loads for longitudinal wind
units = kN/m²

In this case the wind loads applied to the structure result in a net upward force on the roof as in load case no. 2

The unfactored loads acting on the structure are given by:

wind load	*– LH column*	=	*–0,361 × 7,2*	=	*–2,60 kN/m*
	– LH rafter	=	*–0,451 × 7,2*	=	*–3,25 kN/m*
	– RH rafter	=	*–0,451 × 7,2*	=	*–3,25 kN/m*
	– RH column	=	*–0,361 × 7,2*	=	*–2,60 kN/m*
dead loads	*– columns*	=	*1,13*	=	*1,13 kN/m*
	– rafters	=	*0,82*	=	*0,82 kN/m*
	– sheeting	=	*1,44*	=	*1,44 kN/m*

See "Notes" for load combination no. 2 above

2.3.2 Frame imperfection equivalent horizontal forces

As load combination no. 2 above

2.3.3 Partial safety factors and second-order effects

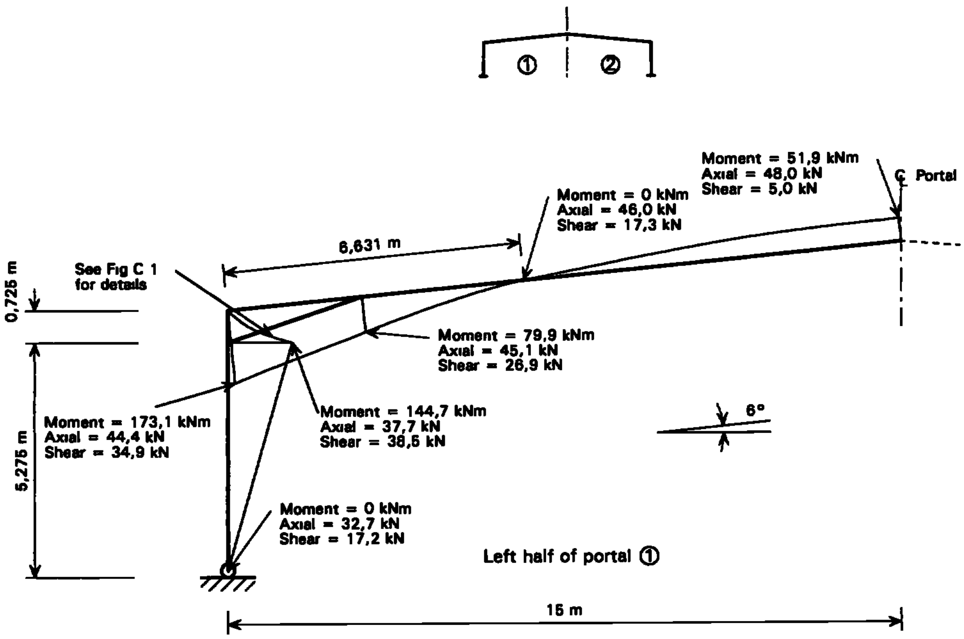
As load combination no. 2 above

2.3.4 Analysis

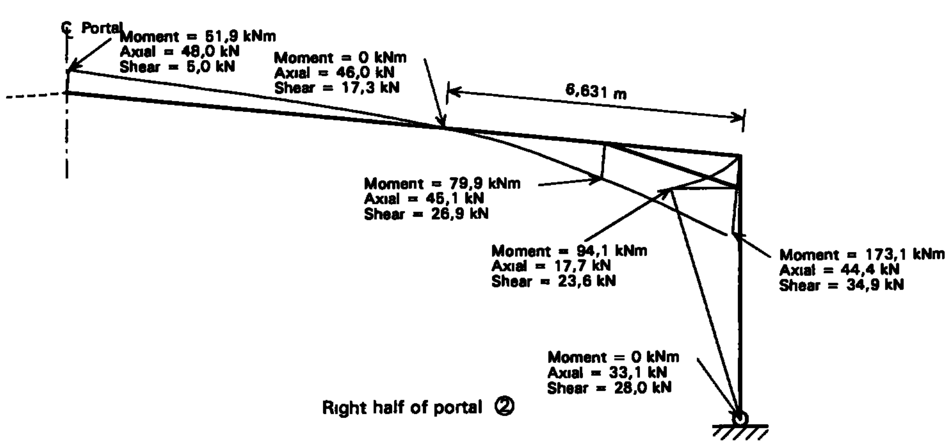
The collapse load factor = 6,421 which is greater than load case no. 1. Therefore this load case is not the critical case for cross-sectional resistance, but member stability for this case must be checked because the moments are in the opposite sense to load case no. 1

The bending moments from this load case are shown below

Load combination no. 3:
bending moment, shear and axial force diagram



Left half of portal ①



Right half of portal ②

3. CALCULATE V_{sd} / V_{cr}

Section 17.2(5)

These checks use the ENV partial safety factors 1,35 and 1,5 not the increased values used to allow for second-order effects in the ultimate limit state analysis

3.1 Load combination no. 1

3.1.1 Rafter 'snap through'/arching stability check

ENV 5.2.6.1
Section 12.1
Appendix B.3

$$\frac{V_{Sd}}{V_{cr}} = \left[\frac{L}{D}\right]\left[\frac{\Omega - 1}{55{,}7\,(4 + L/h)}\right]\left[\frac{I_r}{I_c + I_r}\right]\left[\frac{f_{yr}}{275}\right]\left[\frac{1}{\tan 2\theta_r}\right]$$

Where:

$L = 30$ m $\quad D = 0{,}528$ m

$h = 6$ m $\quad I_r = 47520$ cm^4

$I_c = 87380$ cm^4 $\quad \theta_r = 6^{\circ}$

$f_{yr} = 275$ N/mm^2

Ω = *factored vertical load/* F_{feb}

$\Rightarrow F_{feb}$ = *max. vertical load to cause failure of rafter treated as fixed ended beam (plastic moment,* $M_p = wl^2/16$*)*

$F_{feb} = 16 \times M_p / L = 16 \times 514{,}5 / (30 - 6) = 343{,}0$ kN

From page 2, factored vertical loads:

Roofing	=	1,35 × 1,44	=	1,94 kN/m
Services	=	1,35 × 1,44	=	1,94 kN/m
Rafter	=	1,35 × 0,82	=	1,11 kN/m
Snow	=	1,50 × 5,40	=	8,10 kN/m
		Total	=	13,09 kN/m

$\Rightarrow \Omega = 13{,}09 \times (30 - 6) / 343{,}0 = 0{,}92 \leq 1{,}0$

therefore snap through will not occur and need not be considered further for this example

3.1.2 Sway stability check:

Assuming truly pinned bases — *Appendix B.4.1*

$$\frac{V_{Sd}}{V_{cr}} = \left[\frac{N_r}{N_{r.cr}} + (4 + 3{,}3R)\left(\frac{N_c}{N_{c.cr}}\right)\right]$$

Note:

(1) The axial compression in the columns and rafters is found by elastic analysis for the critical load combination. This can be undertaken on the same elastic-plastic package as was used for the main analysis, by applying only a nominal percentage of the ultimate load, for example 10%. From this, the axial force characteristics can be determined then scaled up to the appropriate level, for example by multiplying by 1 if only 10% of ultimate load was applied

(2) The load factors applied to determine the axial forces in the columns for stability checks should not take account of the Merchant-Rankine modification

N_c = *Axial compression in column at column mid-height*
(Axial from half side cladding $= 1{,}35 \times 4{,}3 = 5{,}8$ *kN)*
$= 201{,}2 + 5{,}8$
$= 207{,}0$ kN

N_r = *Axial compression in rafter at shallow end of haunch*
$= 171{,}7$ kN

$N_{c.cr} = \pi^2 EI_c / h^2$
$= \pi^2 \times 210 \times 87380 \times 10^4 / (6{,}0^2 \times 10^6)$
$= 50256$ kN

$N_{r.cr} = \pi^2 EI_r / s^2$
$= \pi^2 \times 210 \times 47520 \times 10^4 / (15{,}083^2 \times 10^6)$
$= 4325$ kN

$R = I_c S / I_r h$
$= (87380 \times 15{,}083) / (47520 \times 6{,}0)$
$= 4{,}62$

$$\frac{V_{Sd}}{V_{cr}} = \left[\frac{171.7}{4325} + (4 + 3{,}3 \times 4{,}62)\left(\frac{207{,}0}{50256}\right)\right]$$

$= 0{,}12 \Rightarrow$ *structure classified as a 'sway' case*

Therefore second-order effects must be accounted for, as is done by Merchant-Rankine — *ENV 5.2.5.2, Section 12.1, Appendix K.6*

Maximum value of V_{Sd} / V_{cr} *calculated above* $= 0{,}12$

Original assumption $= 0{,}12$

Therefore the original assumption was correct and the calculation need not be repeated

If however, nominally pinned bases were used instead: — *Appendix B.4.2*

$$\frac{V_{Sd}}{V_{cr}} = \left[\frac{N_r}{N_{r.cr}} + (2{,}9 + 2{,}7R)\left(\frac{N_c}{N_{c.cr}}\right)\right] \times \left[\frac{1}{(1 + 0{,}1R)}\right]$$

$$\frac{V_{Sd}}{V_{cr}} = \left[\frac{171.7}{4325} + (2{,}9 + 2{,}7 \times 4{,}62)\left(\frac{207{,}0}{50256}\right)\right]\left[\frac{1}{1 + 0{,}462}\right]$$

$= 0{,}07 \leq 0{,}1 \Rightarrow$ *do not need to allow for second-order frame effects*

Hence a more economic solution is possible

3.2 Other load combinations

Each load combination should be checked unless the worst case can be identified by inspection and the lowest value of V_{Sd} / V_{cr} *is acceptable for all load combinations* — *Appendix B.2.2*

In this example, both load combinations no. 2 and no. 3 are uplift cases, so there are no overall instability effects

4. COLUMN: 610 × 229 UB113

Load combination no. 1 is clearly the worst case for the column for axial force, bending moment and shear force as well as restraint to the compression flange

Therefore, the column checks need to be made for only load combination no. 1

M_{Sd} = 821,8 kNm
V_{Sd} = 155,8 kN
N_{Sd} = 233,1 kN (conservative, as coincident axial = 224,1 kN)

Section properties

h	= 607,3 mm	i_{LT}	= 55,5 mm
b	= 228,2 mm	a_{LT}	= 1630 mm
t_w	= 11,2 mm	I_{yy}	= 87380×10^4 mm^4
t_f	= 17,3 mm	I_{zz}	= 3434×10^4 mm^4
d	= 547,3 mm	I_t	= 111×10^4 mm^4
λ_1	= 86,8	$W_{pl.y}$	= 3287×10^3mm^3
i_y	= 246 mm	A	= 14400 mm^2
i_z	= 48,8 mm		

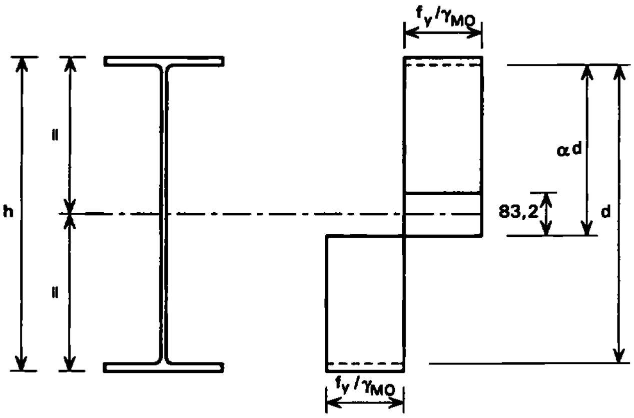
4.1 Classification — Section 17.2(7)(a)

Ensure the section is Class 1 to accommodate plastic hinge formation

Web check from ENV 1993-1-1 Sheet 1 of Table 5.3.1: — Appendix G.1.1

Appendix C.2

ENV 5.3.2



Plastic stress distribution in web

Web is under combined axial and bending forces, so find α :

Depth of stress block at yield stress resisting axial load

$= N_{Sd} / (f_y \times t_w / \gamma_{m0}) = 233{,}2 \times 10^3 / (275 \times 11{,}2 / 1{,}1) = 83{,}2$ *mm*

$\therefore \alpha d \quad = \quad d / 2 + 83{,}2 / 2$

$\therefore \alpha \quad = \quad 0{,}5 + (83{,}2 / 2)d$
$\quad = \quad 0{,}5 + [(83{,}2 / 2) / 547{,}6] \quad = \quad 0{,}577$

$\therefore$ *for Class 1, limiting* $d / t_w \quad = \quad 396\,\epsilon / (13\alpha - 1)$

$\quad = 396 \times 0{,}92 / (13 \times 0{,}576 - 1) \quad = \quad 56{,}2$

actual $d / t_w \quad = \quad 547{,}3 / 11{,}1 \quad = \quad 49{,}3$ → *web is Class 1*

<u>Flange check from ENV 1993-1-1 Sheet 3 of Table 5.3.1:</u>

for Class 1, limiting $c/t_f \; = \; 10\epsilon \quad = \quad 10 \times 0{,}92 \quad = \quad 9{,}2$

actual $c/t_f \; = \; 114{,}1 / 17{,}3 \quad = \quad 6{,}6$ → *flange is Class 1*

4.2 Cross-sectional resistance — *Section 17.2(7)(b)*

The frame analysis assumed that there is no reduction in the plastic moment resistance from interaction with shear force or axial force. This assumption must be checked because it is more onerous than checking that the cross-sectional resistance is sufficient — *Appendix C.4.4(b)*

Load combination no. 1 is clearly the worst load combination

Max. shear force, $_{Vsd} \quad = \quad 155{,}8$ *kN*
Max. axial force, $_{Nsd} \quad = \quad 233{,}0$ *kN*

<u>Check that the plastic moment of resistance, $M_{pl.Rd}$, is not reduced by the coincident shear force</u> — *Appendix C.4.4(b)*

Check $\quad V_{Sd} \ngtr 0{,}5\, V_{pl.Rd}$

$A_v \; = \; 1{,}04\, ht_w \quad = \quad 1{,}04 \times 607{,}3 \times 11{,}1 \quad = \quad 7011\ mm^2$

Appendix C.4.3

$V_{pl.Rd} = A_v\,(f_y / \sqrt{3}) / \gamma_{MO}$
$= 7011\,(275 / \sqrt{3}) / 1{,}1$
$= 1012$ kN

$\therefore 0{,}5\ V_{pl.Rd} = 0{,}5 \times 1012 = 506$ kN

Max $V_{Sd} = 155{,}8$ kN

$\therefore V_{Sd} \not> 0{,}5\ V_{pl.Rd}$ *so the plastic moment of resistance is not reduced by the coexistent shear force*

Appendix C.4.4.(b)

Check that the plastic moment of resistance, $M_{pl.Rd}$, is not reduced by the coincident axial force

Check:

(i) $N_{Sd} \not> 0{,}5 \times$ *plastic tensile resistance of the web*

$A_{web} = A_{gross} - \sum A_{flanges} = 14{,}400 - 2 \times 228{,}2 \times 17{,}3$
$= 6{,}504\ \text{mm}^2$

$\therefore 0{,}5 \times A_{web} \times f_y / \gamma_{MO} = 0{,}5 \times 6{,}504 \times 275 / 1{,}1 = 813$ kN

Max. $N_{Sd} = 233{,}0$ kN

$\therefore N_{Sd} \not> 0{,}5 \times$ *plastic tensile resistance of web*

(ii) $N_{Sd} \not> 0{,}25 \times$ *plastic tensile resistance of the section*

$0{,}25 \times A \times f_y / \gamma_{MO} = 0{,}25 \times 14400 \times 275 / 1{,}1 = 900$ kN

Max. $N_{Sd} = 233{,}0$ kN

$\therefore N_{Sd} \not> 0{,}25 \times$ *plastic tensile resistance of section*

Checks (i) and (ii) show that the plastic moment of resistance is not required by the coexistent axial force

Therefore, the effects of shear and axial force on the plastic resistance moment can be neglected according to EC3 and the frame analysis assumption is validated

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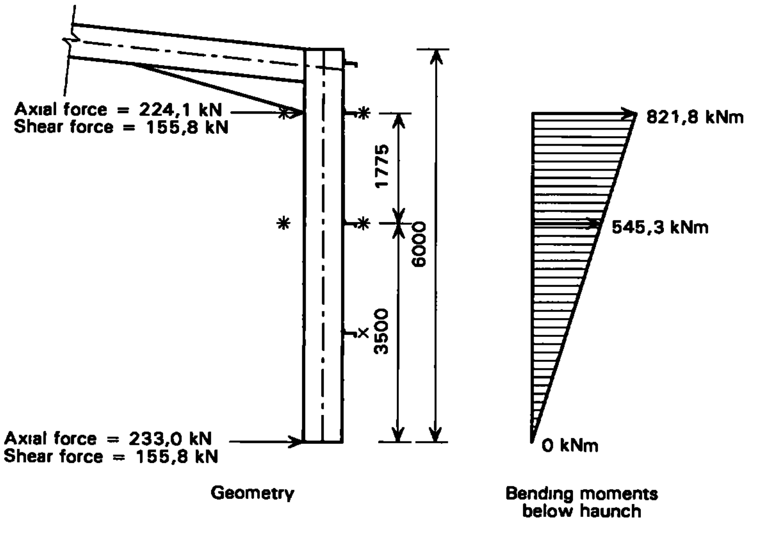
4.3 Buckling between intermediate restraints — *Section 17.2(7)(c)*

Note that this design uses the Merchant-Rankine criterion, so only out-of-plane buckling remains to be checked — *Appendix D.3.4*

For members with plastic hinges, EC3 gives insufficient guidance for member buckling checks. Therefore Appendix D of this document is used in addition

The critical column bending moment diagram is from load combination no. 1 in this structure, causing a plastic hinge to occur at the underside of the haunch

Therefore, find the stable length with a plastic hinge — *Appendix D.4*



Moments, forces and restraints

Max. restraint spacing at a plastic hinge $= 0{,}4\,(C_p)^{0{,}5}\,\lambda_1\,i_{LT}$

Where: $$C_p = \left[\frac{1}{\dfrac{M_{hinge}}{C_1 W_{pl.y} f_y} + \dfrac{4N_{hinge}}{Af_y}\left(\dfrac{i_{LT}}{i_z}\right)^2}\right]$$

For a first approximation, ignore the axial force

$\therefore$ *take* $C_p = 1$ ***for initial trial***

∴ spacing $= 0{,}4 \times 1{,}0 \times 86{,}8 \times 55{,}5 = 1927$ *mm*

Reduce spacing slightly to allow for the coincident axial force

∴ try restraint spacing at 1775 mm centres:

Moment at first side rail beneath haunch $= 545{,}3$ *kNm*

To determine a more accurate value of C_p, a value of C_1 is required and can be obtained from ENV 1993-1-1 Annex F, Table F.1.1 — *ENV Table F.1.1*

$\psi = 545{,}3 / 821{,}8 = 0{,}664$
$k = 1{,}0$
at $\psi = 0{,}75, \quad C_1 = 1{,}141$
at $\psi = 0{,}5, \quad C_1 = 1{,}323$

$$\rightarrow C_1 = 1{,}141\,(0{,}664 - 0{,}5) / (0{,}75 - 0{,}5) + 1{,}323\,(0{,}75 - 0{,}664) / (0{,}75 - 0{,}5)$$
$$= 0{,}748 + 0{,}455$$
$$= 1{,}204$$

using the approximation $i_{LT} \approx i_z / 0{,}9$

$M_{hinge} = 821{,}8$ kNm
$N_{hinge} = 224{,}2$ kN

$$C_p = \left[\frac{1}{\dfrac{821{,}8 \times 10^6}{1{,}204 \times 3287 \times 10^3 \times 275} + \dfrac{4 \times 224{,}2 \times 10^3}{14400 \times 275}\left(\dfrac{1}{0{,}9}\right)^2}\right]$$

$$= 1 / (0{,}755 + 0{,}279) = 0{,}965$$

$$\rightarrow L_{cmax} = 0{,}4\,(C_p)^{0,5}\,\lambda_1\,i_{LT} = 0{,}4 \times (0{,}965)^{0,5} \times 86{,}8 \times 55{,}5$$
$$= 1893 \text{ mm}$$
Appendix D.4

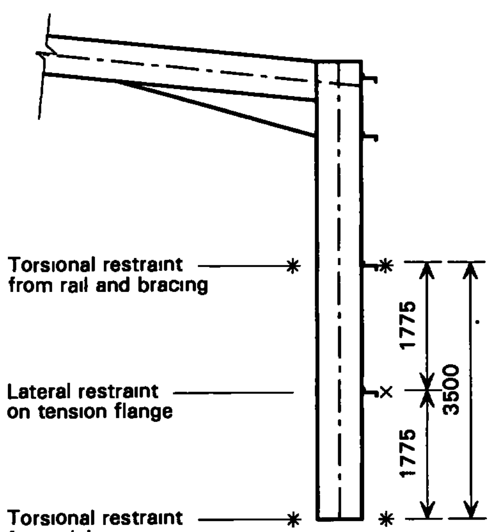
Therefore, 1775 mm is OK

If 1775 mm spacing ensures stability between intermediate restraints at the top of the column where the maximum bending moment occurs, then a spacing of 1750 is OK for rails below 3,5 m height up column, where the moment is lower

4.4 Buckling between torsional restraints — *Section 17.2(7)(d)*

The check assumes that the outer flange, which is the tension flange, has intermediate restraint between the inner flange restraints. The stability of the column between the intermediate restraints has been checked above, because 1775 mm spacing with a plastic hinge at one end is much less stable than 1750 mm with much lower bending moments

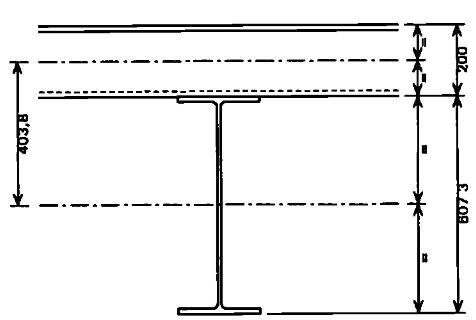
If there were no intermediate restraints, this part of the column would be checked to D.3.4 using slenderness calculated without the benefit of F.3



Column between torsional restaints

(a) Calculate slendernesses λ and λ_{LT} — *Appendix F.3.4*

Assume side rail depth = 200 mm



Distance from column shear centre to centre of side rail, a — Appendix F.3.2

$$a = 607{,}3 / 2 + 200 / 2 = 403{,}8 \text{ mm}$$

$$i_s^2 = i_y^2 + i_z^2 + a^2$$

$$i_s^2 = 246^2 + 48{,}8^2 + 403{,}8^2 = 225952 \text{ mm}^2$$

distance between shear centres of flanges

$$= 607{,}3 - 17{,}3 = 590{,}0 \text{ mm}$$

$$\alpha = \left(\frac{a^2 + \dfrac{I_w}{I_z}}{i_s^2}\right)$$

using the simplification for doubly symmetric I sections

$$I_w = I_z (h_s / 2)^2$$

$$\alpha = \frac{a^2 + (h_s/2)^2}{i_s^2}$$

$$= [403{,}8^2 + (590{,}0 / 2)^2] / 225952$$
$$= 1{,}107$$

The slenderness of the column is given by: — Appendix F.3.4

$$\lambda = \frac{L_t/i_z}{[\alpha + (I_t L_t^2 / 2{,}6\ \pi^2\ I_z\ i_s^2)]^{0,5}}$$

$$= \frac{3500/48{,}8}{[1{,}107 + (111 \times 3500^2 / 2{,}6 \times \pi^2 \times 3434 \times 225952)]^{0,5}}$$

$$= \frac{71{,}7}{(1{,}107 + 0{,}068)^{0,5}}$$

$$= 66{,}15$$

$\lambda_{LT} = (m_t^{0,5}c)\,[(W_{pl.y}\,/\,A)(2a\,/\,i_s^2)]^{0,5}\lambda$ — *Appendix F.3.4*

where: m_t is obtained from F.3.3 because load combination no. 1 does not apply lateral loads to the walls, so there are no intermediate loads — *Appendix F3.3*

$\psi_t = 0\,/\,545,3 = 0$ — *Appendix F3.3*

$y = \lambda\,/\,(L_t\,/\,i_z) = 66,15\,/\,(3500\,/\,48,8) = 0,922$

$\rightarrow m_t = 0,53$

$c = 1,0$

$\lambda_{LT} = (0,53^{0,5})[(3287 \times 10^3\,/\,14400)\,(2 \times 403,8\,/\,225952)]^{0,5}\,66,15$

$= 0,657 \times 66,15$

$= 43,5$

For load combinations including lateral loads, e.g. wind, m_t should be obtained from F.3.3.1.2 because there will normally be intermediate loads from the intermediate sheeting rails between 3,5 m and ground level. These sheeting rails also provide the intermediate restraints to the tension flange of the column, between the torsional restraints

(b) Calculate buckling resistance for axial force — *Appendix D.3.1*

$N_{b.Rd} = \chi A f_y\,/\,\gamma_{M1}$ — *ENV Table 5.5.3*

$\chi_{min} = 1\,/\,[\phi + (\phi^2 - \bar{\lambda}^2)^{0,5}]$

$\phi = 0,5\,[1 + a\,(\bar{\lambda} - 0,2) + \bar{\lambda}^2]$

$h\,/\,b = 607,3\,/\,228,2 = 2,66$

→ curve b for hot rolled I sections

$\rightarrow a = 0,34$ — *ENV Table 5.5.1*

$\bar{\lambda} = \lambda\,/\,\lambda 1$ — *ENV 5.5.1.2*

$= 66,15\,/\,86,8$

$= 0,7621$

$\phi = 0,5[1 + 0,34\,(0,7621 - 0,2) + 0,7621^2]$

$= 0,886$

$\chi_z = 1\,/[0,886 + (0,886^2 - 0,7621)^{0,5}]$

$= 0,7475$

$N_{b.Rd} = \chi A f_y / \gamma_{M1}$
$= 0{,}7475 \times 14400 \times 275 / 1{,}1 \quad = \quad 2691 \text{ kN}$

(c) Calculate buckling resistance for bending — *Appendix D.3.2*

$M_{b.Rd} = \chi_{LT} W_{pl.y} f_y / \gamma_{M1}$

$\bar{\lambda}_{LT} = 43{,}5 / 86{,}8$ — *ENV 5.5.2(5)*
$= 0{,}5012$

$\chi_{LT} = 1 / [\phi_{LT} + (\phi_{LT}^2 - \bar{\lambda}_{LT}^2)^{0,5}]$ — *ENV 5.5.2(2)*

$\phi_{LT} = 0{,}5\,[1 + a_{LT}(\bar{\lambda}_{LT} - 0{,}2) + \bar{\lambda}_{LT}^2]$
$= 0{,}5\,[1 + 0{,}21\,(0{,}5012 - 0{,}2) + 0{,}5012^2]$
$= 0{,}6572$

$\chi_{LT} = 1 / [0{,}6572 + (0{,}6572^2 - 0{,}5011^2)^{0,5}]$
$= 0{,}9239$

$M_{b.Rd.y} = \chi_{LT} W_{pl.y} f_y / \gamma_{M1}$
$= 0{,}9239 \times 3287 \times 10^3 \times 275 / 1{,}1$
$= 759{,}2 \text{ kNm}$

(d) Calculate buckling resistance to combined axial and bending — *Appendix D.3.4*

$N_{Sd} / N_{b.Rd.z} + k_{LT} M_{y.Sd} / M_{b.Rd.y} \leq 1{,}0$

$\psi = 0{,}0$ — *ENV Fig 5.5.3*

$\beta_{M.LT} = 1{,}8 - 0{,}7\psi = 1{,}8 - 0{,}7 \times 0{,}0 = 1{,}8$ — *ENV 5.5.4(7)*

$\mu_{LT} = 0{,}15\,\bar{\lambda}_z \beta_{M.LT} - 0{,}15 \quad \text{but } \mu_{LT} \leq 0{,}9$
$= 0{,}15 \times 0{,}7621 \times 1{,}8 - 0{,}15 \quad = \quad 0{,}056$ — *ENV 5.5.4(2)*

$k_{LT} = 1 - [\mu_{LT} N_{Sd} / (\chi_z A f_y)] \quad \text{but } k_{LT} \leq 1{,}0$ — *ENV 5.5.4(2)*
$= 1 - [0{,}056 \times 233{,}0 \times 10^3 / (0{,}7475 \times 14400 \times 275)]$
$= 0{,}996$

$N_{Sd} / N_{b.Rd.z} + k_{LT} M_{y.Sd} / M_{b.Rd.y} = (233{,}0/2691) + (0{,}996 \times 545{,}3/759{,}2)$

$= 0{,}087 + 0{,}715$
$= 0{,}80 < 1{,}0 \qquad \therefore$ *column OK*

5. RAFTER

For the rafters it is not often clear which is the critical load combination, especially for buckling. Each load combination must therefore be checked

Summary of member properties: 533 × 210 UB82

h	= 528,3 mm	i_z	= 43,8 mm
b	= 208,7 mm	i_{LT}	= 50,1 mm
t_w	= 9,6 mm	a_{LT}	= 1610 mm
t_f	= 13,2 mm	$W_{el.y}$	= 1799×10^3 mm^3
d	= 476,5 mm	$W_{el.z}$	= 192×10^3 mm^3
I_{yy}	= $4,752 \times 10^8$ mm4	$W_{pl.y}$	= 2058×10^3 mm^3
I_{zz}	= $2,004 \times 10^7$ mm^4	$W_{pl.z}$	= 300×10^3 mm^3
i_y	= 213 mm	A	= 10500 mm^2

5.1 Classification

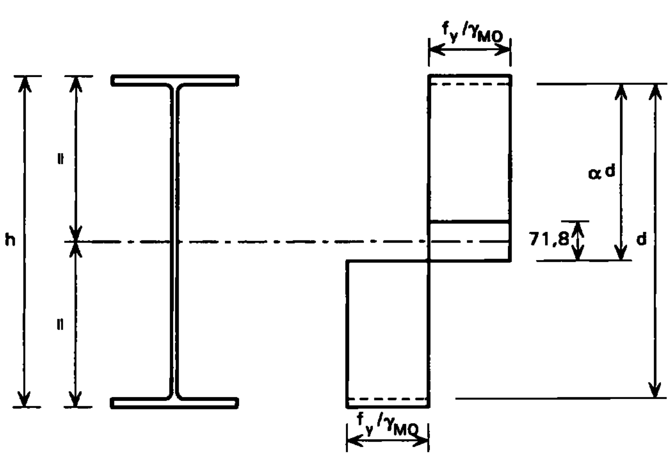
Section 17.2(8a) Appendix G.1.1

Ensure the section is Class 1 to accommodate plastic hinge formation. The rafter is expected to be Class 1 because it is a UB section in S275 steel. Therefore, the only likely problem is the axial compression, which is clearly worst in load combination no. 1, so the classification is checked for only load combination no. 1

Web check from ENV 1993-1-1 Sheet 1 of Table 5.3.1:

ENV 5.3.2 Appendix C.2

Web is under combined axial and bending forces, so find α:



Plastic stress distribution in web

Depth of stress block at yield stress resisting axial load

$= N_{Sd} / (f_y \times t_w / \gamma_{MO}) = 172{,}4 \times 10^3 / (275 \times 9{,}6 / 1{,}1)$

$= 71{,}8$ *mm*

$\therefore \alpha_d = d / 2 + 71{,}8 / 2$

$\therefore \alpha = 0{,}5 + (71{,}8 / 2) / d$

$= 0{,}5 + (71{,}8 / 2) / 476{,}5 = 0{,}575 \quad \alpha > 0{,}5$

$\therefore$ *for Class 1*

limiting $d / t_w = 396\,\epsilon / (13\alpha - 1)$

$= 396 \times 0{,}92 / (13 \times 0{,}575 - 1) = 56{,}3$

actual $d/t_w = 476{,}5 / 9{,}6 = 49{,}6 \rightarrow$ *web is Class 1*

__Flange check from ENV 1993-1-1 Sheet 3 of Table 5.3.1:__

for Class 1, limiting $c/t_f = 10\epsilon = 10 \times 0{,}92 = 9{,}2$

actual $c / t_f = 104{,}4 / 13{,}2 = 7{,}9 \rightarrow$ *flange is Class 1*

5.2 Cross-sectional resistance — *Section 17.2(8b)*

The frame analysis assumed that there is no reduction in the plastic moment resistance from interaction with shear force or axial force. This assumption must be checked because it is more onerous than checking that the cross-sectional resistance is sufficient — *Appendix C.4.4(b)*

Load combination no. 1 is clearly the worst load combination

Max. shear force, $V_{Sd} = 161{,}2$ *kN at haunch tip*
Max. axial force, $N_{Sd} = 172{,}4$ *kN at haunch tip*

__Check that the plastic moment of resistance, $M_{pl.Rd}$, is not reduced by the coincident shear force__ — *Appendix C.4.4(b)*

Check $V_{Sd} \ngtr 0,5\ V_{pl.Rd}$

$A_v = 1,04\ ht_w = 1,04 \times 528,3 \times 9,6 = 5275\ mm^2$

$V_{pl.Rd} = A_v\ (f_y / \sqrt{3}) / \gamma_{MO}$ — *Appendix C.4.3*

$= 5275\ (275 / \sqrt{3}) / 1,1$
$= 761\ kN$

$\therefore 0,5\ V_{pl.Rd} = 0,5 \times 761 = 381\ kN$

Max $V_{Sd} = 161,2\ kN$

$\therefore V_{Sd} \ngtr 0,5\ V_{pl.Rd}$ *so the plastic moment of resistance is not reduced by the coexistent shear force*

Check that the plastic moment of resistance, $M_{pl.Rd}$, is not reduced by the coincident axial force — *Appendix C.4.4.(b)*

Check:

(i) $N_{Sd} \ngtr 0,5 \times$ *plastic tensile resistance of the web*

$A_{web} = A_{gross} - \sum A_{flanges} = 10500 - 2 \times 208,7 \times 13,2$
$= 4990\ mm^2$

$\therefore 0,5 \times A_{web} \times f_y / \gamma_{MO} = 0,5 \times 4990 \times 275 / 1,1 = 624\ kN$
Max $N_{Sd} = 172,4\ kN$

$\therefore N_{Sd} \ngtr 0,5 \times$ *plastic tensile resistance of web*

(ii) $N_{Sd} \ngtr 0,25 \times$ *plastic tensile resistance of the section*
$0,25 \times A \times f_y / \gamma_{MO} = 0,25 \times 10500 \times 275 / 1,1 = 656\ kN$
Max $_{Nsd} = 172,4\ kN$
$\therefore N_{Sd} \ngtr 0,25 \times$ *plastic tensile resistance of section*

Checks (i) and (ii) show that the plastic moment of resistance is not reduced by the coexistent axial force

Therefore, the effects of shear and axial force on the plastic resistance moment can be neglected according to EC3 and the frame analysis assumption is validated

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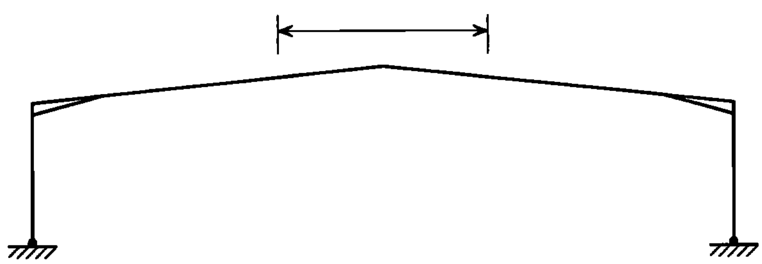
5.3 Buckling between intermediate restraints

Section 17.2(8)(c)

By inspection, worst case is near apex in left hand rafter, because this has the highest bending moment in the rafter

5.3.1 Stable length check for high bending moment



Rafter under highest bending moment

If the approach described by section F.4 of this document is used, which is for sections containing a plastic hinge or is so highly loaded that is close to containing a plastic hinge, a stable length can be derived without many time consuming checks. The stable length approach is based on $\bar{\lambda}_{LT} \leq 0{,}4$ so no further checks for lateral torsional buckling are required

Appendix D.4

$$L = 0{,}4\left[\frac{1}{\left(\dfrac{M_{Sd}}{C_1\, W_{pl.y.fy}}\right) + \left(\dfrac{4N_{Sd}}{Af_y}\right)\left(\dfrac{i_{LT}}{i_z}\right)^2}\right]^{0,5} \lambda_1\, i_{LT}$$

M_{Sd} = 505,6 kNm
N_{Sd} = 155,5 kN
i_{LT} = 50,1
C_1 = 1,0

Taking $i_{LT} / i_z \approx 1 / 0{,}9$

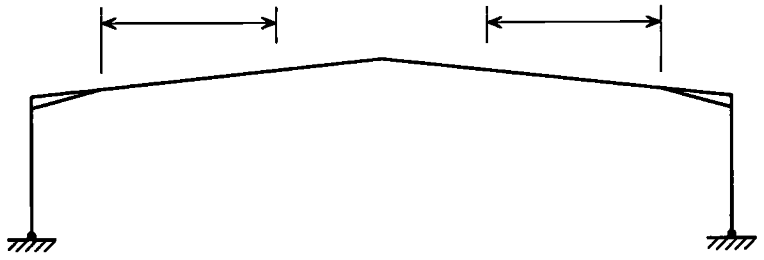
$$L=0{,}4\left[\frac{1}{\left(\dfrac{505{,}6\times 10^3}{1\times 2058\times 275}\right) + \left(\dfrac{4\times 155{,}5\times 10^3}{10500\times 275}\right)\left(\dfrac{1}{0{,}9}\right)^2}\right]^{0,5} 86{,}8\times 50{,}1$$

$= 1616$ mm

5.3.2 Combined axial and moment check for lower bending moments

Appendix D.3.4

Where bending moment is lower, the purlin spacing can be increased:



Rafter under lower bending moments

Next critical case is in right hand rafter. Try purlin spacing at 2000 mm centres

Check for lateral torsional buckling between purlins:

$M_{Sdmax.y}$ = *381,7 kNm*
$N_{Sd.max}$ = *172,4 kN*

(a) Calculate buckling resistance to axial force

Appendix D.3.1

$l = 2000$ mm
$\lambda_z = 2000 / 43{,}8 = 45{,}66$
$\bar{\lambda}_z = 45{,}66 / 86{,}8 = 0{,}526$
$\phi = 0{,}5\,[1 + \alpha\,(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2]$
$= 0{,}5\,[1 + 0{,}34\,(0{,}526 - 0{,}2) + 0{,}526^2]$
$= 0{,}694$
$\chi_z = 1 / [\phi + (\phi^2 - \bar{\lambda}^2)^{0,5}]$
$= 1 / [0{,}694 + (0{,}694^2 - 0{,}526^2)^{0,5}]$
$= 0{,}872$

$N_{b.Rd.z} = 0{,}872 \times 10500 \times 275 / 1{,}1$
$= 2289$ kN

(b) Calculate buckling resistance to bending moment

Appendix D.3.2

Appendix F.2

$$\lambda_{LT} = \frac{L/i_{LT}}{(C_1)^{0,5}\left[1 + \dfrac{(L/a_{LT})^2}{25{,}66}\right]^{0,25}}$$

a_{LT} and i_{LT} are given in EC3 section property tables (see, for example, Steel designers' manual[26])

Where tables are not available, formulae for a_{LT} and i_{LT} are given in F.2

Take $C_1 = 1,0$ (conservative)

$$\therefore \lambda_{LT} = \frac{2000/50,1}{(1,0)^{0,5}\left[1 + \dfrac{(2000/1610)^2}{25,66}\right]^{0,25}} = \frac{39,9}{1,015} = 39,3$$

$$\bar{\lambda}_{LT} = 39,3 / 86,8 = 0,453$$
$$\phi_{LT} = 0,5 [1 + \alpha_{LT} (\bar{\lambda}_{LT} - 0,2) + \bar{\lambda}^2_{LT}]$$
$$= 0,5 [1 + 0,21 (0,453 - 0,2) + 0,45^2]$$
$$= 0,629$$
$$\chi_{LT} = 1 / [\phi_{LT} + (\phi_{LT}^2 - \bar{\lambda}^2_{LT})]^{0,5}$$
$$= 1 / [0,629 + (0,629^2 - 0,453^2)^{0,5}]$$
$$= 0,939$$
$$M_{b.Rd.y} = 0,939 \times 2058 \times 10^3 \times 275 / 1,1$$
$$= 483,1 \text{ kNm}$$

(c) Calculate buckling resistance to combined axial and bending — Appendix D.3.4

Check $N_{sd} / N_{b.Rd.z} + k_{LT} M_{y.Sd} / M_{b.Rd} \leq 1,0$

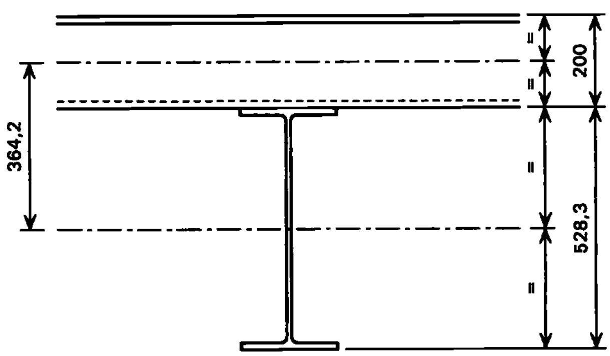
Take $k_{LT} = 1,0$ (conservative)

$$(172,4 / 2289) + (1,0 \times 381,7 / 483,1) = 0,865 \leq 1,0 \therefore OK$$

$\therefore$ rafter is stable between intermediate restraints

5.4 Buckling between torsional restraints — Section 17.2(8d)

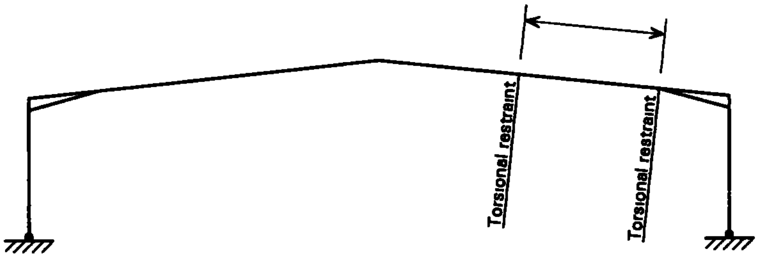
Where the bottom flange of the rafter is in compression, the stability must be checked between the torsional restraints (e.g. restraint to bottom and top flanges). For first trial, assume rotational restraints are positioned at approximately quarter span intervals (see diagram in Section 1.1 of this worked example) — Appendix D.3, F.3



5.4.1 Load combination no. 1

Worst case between RH haunch tip and quarter span rotational restraint – between 3,017 m and 8,567 m (taking hogging moments positive):

$M_{Sd.haunch}$ = 381,7 kNm
$M_{Sd.2}$ = 172,2 kNm
$M_{Sd.3}$ = –9,2 kNm
$M_{Sd.4}$ = –162,3 kNm
$M_{Sd.restraint}$ = –287,2 kNm



Worst buckling from gravity loads

(a) Calculate slendernesses λ and λ_{LT}

The purlins provide intermediate restraints assumed for the method of F.3. so, λ_{LT} may be calculated using Appendix F.3.4 of this document

Appendix F.3.4

$$\lambda = \left[\frac{L_t / i_z}{\left(\alpha + \dfrac{I_t \, L_t^2}{2{,}6 \, \pi^2 \, I_z \, i_s^2} \right)^{0{,}5}} \right]$$

Where:

purlin depth $=$ 200 mm

$\rightarrow \quad a = 528{,}3 / 2 + 200 / 2$ (see Figure 5.1)

$= 364{,}2$ mm

$i_s^2 = i_y^2 + i_z^2 + a^2$

$= 213^2 + 43{,}8^2 + 364{,}2^2$

$= 179929 \text{ mm}^2$

$$\alpha = \left(\frac{a^2 + \dfrac{I_w}{I_z}}{L_s^2} \right)$$

for a doubly symmetric I section:

$$I_w = I_z (h_s/z)^2$$

$$\therefore \quad \alpha = \left[\frac{a^2 + \left(\dfrac{h_s}{2}\right)^2}{i_s^2} \right]$$

Appendix F.3.2

which gives:

$\alpha = \{364{,}2^2 + [0{,}5 (528{,}3 - 13{,}2)]^2\} / 179929$

$= 1{,}106$

$$\lambda = \left[\frac{5550/43{,}8}{\left(1{,}106 + \dfrac{51{,}5 \times 10^4 \times 5550^2}{2{,}6\pi^2\, 2{,}004 \times 10^7 \times 179929} \right)^{0,5}} \right]$$

$= 112{,}1$

Appendix D.3.1

$$\lambda_{LT} = (m_t^{0,5}\, c) \left(\frac{W_{pl.y}}{A} \times \frac{2a}{i_s^2} \right)^{0,5} \lambda$$

Appendix F.3.4

Where:

$$m_t = \frac{1}{12}\left(\frac{1}{381,7}\right)[(381,7 + 3 \times 172,2 + 2\ (381,7)]$$

Appendix F.3.3

Note: $M_{c.Rd}$ *is constant throughout uniform section only positive values included*

$\therefore m_t = 0,363$

$c = 1,0$

$$\lambda_{LT} = (0,363^{0,5})\left(\frac{2058 \times 10^3 \times 2 \times 364,2}{10500 \times 179929}\right)^{0,5} 112,1$$

$= 60,1$

(b) Calculate buckling resistance to axial force

Appendix D.3.1

$\bar{\lambda} = 112,1 / 86,8 = 1,291$

$\therefore \chi_z = 0,4315$

$N_{b.Rd} = 0,4315 \times 10500 \times 275 / 1,1$
$= 1133$ kN

(c) Calculate buckling resistance to bending moment

Appendix D.3.2

$\bar{\lambda}_{LT} = 60,1 / 86,8 = 0,692$

$\chi_{LT} = 0,8517$

$M_{b.Rd.z} = 0,8517 \times 2058 \times 10^3 \times 275 / 1,1 \times 10^{-6}$

$= 438$ kNm

(d) Calculate buckling resistance to combined axial and bending

Appendix D.3.4

In this case, k_{LT} *is calculated accurately for maximum economy:*

$$k_{LT} = 1 - \frac{\mu_{LT}\, N_{Sd}}{\chi_z\, A f_y} = 1 - \frac{\mu_{LT}\, N_{Sd}}{N_{b.Rd.z} \times \gamma_{M1}}$$

$$\therefore \mu_{LT} = 0{,}15\, \bar{\lambda}_z\, \beta_{M.LT} - 0{,}15$$

$$\beta_{M.LT} = \beta_M = \beta_{M.\psi} + (\beta_{M,Q} - \beta_{m.\psi})$$

ENV Fig. 5.5.3

$$M_Q = wL^2 / 8 = 14{,}83 \cos 6° \times (5{,}55)^2 / 8 = 56{,}8 \text{ kNm}$$

$$\Delta_M = 381{,}7 + 287{,}2 = 668{,}9$$

$$\beta_{M,Q} = 1{,}3$$

$$\beta_{M,\psi} = 1{,}8 - 0{,}7\psi = 1{,}8 - 0{,}7\,(-287{,}2 / 381{,}7)$$

$$= 2{,}33$$

$$\therefore \beta_{M.LT} = 2{,}33 + \frac{56{,}8}{668{,}9}(1{,}3 - 2{,}33) = 2{,}24$$

$$\therefore \mu_{LT} = 0{,}15 \times 1{,}291 \times 2{,}24 - 0{,}15 = 0{,}284$$

$$\therefore k_{LT} = 1 - \frac{0{,}284 \times 172{,}4}{1133 \times 1{,}1} = 0{,}961$$

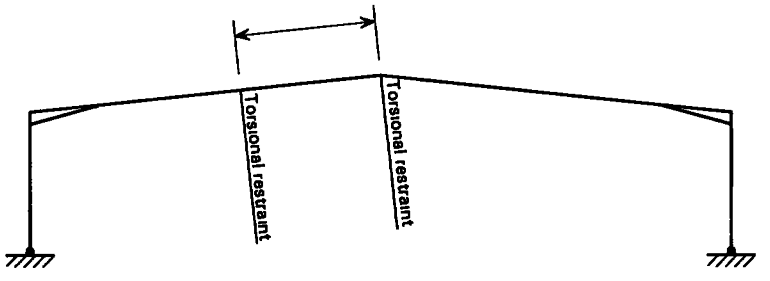
$$\therefore N_{sd} / N_{b.Rd.z} + k_{LT} M_{sd} / M_{b.Rd} \leq 1{,}0$$

$$172{,}4 / 1133 + 0{,}961 \times 381{,}7 / 438 = 0{,}99 \leq 1{,}0 \therefore OK$$

5.4.2 Load combination no. 2

Check for lateral torsional buckling in LH rafter under load combination no. 2. Critical span is between apex and quarter span torsional restraint

Appendix D.3.4



Worst buckling from uplift

(a) Calculate slendernesses λ and λ_{LT}

Effective length of rafter = 14917 – 8567 = 6350 mm

As purlins are positioned within the length under consideration, λ_{LT} may be calculated using Appendix F.3.4 of this document — *Appendix F.3.4*

From load combination no. 1 calculations in 5.4.1 above, with effective length = 6350 mm

$$\lambda = \left[\frac{6350/43{,}8}{\left(1{,}106 + \dfrac{51{,}5 \times 10^4 \times 6350^2}{2{,}6\ \pi^2\ 2{,}004 \times 10^7 \times 179929}\right)^{0{,}5}}\right]$$

$$= 125{,}7$$

$$\lambda_{LT} = (m_t^{0,5} c)\left[\frac{W_{pl.y}}{A} \times \frac{2a}{i_s^2}\right]^{0,5} \lambda$$ — *Appendix F.3.4*

The rafter is not highly stressed under this load case and the moments do not vary greatly along the length considered, so take m_t = 1,0 (conservatively) to simplify the calculations

$m_t = 1{,}0$

$c = 1{,}0$

$$\lambda_{LT} = (1{,}0^{0.5})\left(\frac{2058 \times 10^3 \times 2 \times 364{,}2}{10500 \times 179929}\right)^{0,5} 125{,}7$$

$\lambda_{LT} = 112{,}0$

(b) Calculate buckling resistance to axial force

$\bar{\lambda} = 125{,}7 / 86{,}8 = 1{,}45$

$\chi_z = 0{,}3620$

$N_{b.Rd} = 0{,}3620 \times 10500 \times 275 / 1{,}1$ — *Appendix D.3.1*

$= 950{,}1 \text{ kN}$

(c) Calculate buckling resistance to bending moment

$\bar{\lambda}_{LT} = 112{,}0 / 86{,}8 = 1{,}29$

$\chi_{LT} = 0{,}4760$

$\therefore M_{b.Rd.z} = 0{,}4760 \times 2058 \times 10^3 \times 275 / 1{,}1$ — *Appendix D.3.2*

$= 244{,}9 \text{ kNm}$

(d) Calculate buckling resistance to combined axial and bending

Check $N_{Sd} / N_{b.Rd} + k_{LT} M_{Sd} / M_{b.Rd} \leq 1{,}0$ — *Appendix D.3.4*

Take $k_{LT} = 1{,}0$ *(conservative)*

$25{,}0 / 950{,}1 + 1{,}0 \times 66{,}1 / 244{,}9 = 0{,}30 \leq 1{,}0 \therefore$ *OK*

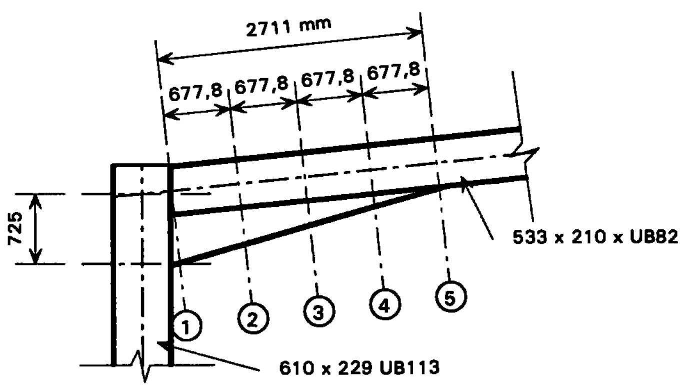
5.4.3 Load combination no. 3

By inspection, load combination no. 3 is not critical

6. HAUNCH

The bending moments and plastic modulus (and effective plastic modulus as Appendix E where appropriate) of the section are required at end and quarter points for the stability checks, so these points are also used to check the stresses along the haunch

Determine section properties of haunched beam – at four positions across haunch:



Haunch geometry

The highest rafter bending moment occurs near mid span and it is clear that the rafter is not critical at the end of the haunch. Therefore, the calculation of elastic section properties for this case is approximate, ignoring the middle flange. In cases where the haunch is more critical, including the middle flange could prove worthwhile

Position	*Lower web depth (mm)*
1	*489*
2	*367*
3	*245*
4	*122*

Section properties neglecting middle flanges							
Position	*Distance from column face (mm)*	*$Area_{gross}$ (mm^2)*	*$Area_{eff}$ (mm^2)*	*$A_{web\,eff}$ (mm^2)*	*$W_{pl\,y}$ (mm^3)*	*$W_{eff\,pl\,y}$ (mm)*	$\beta = \frac{W_{eff\,pl\,y}}{W_{pl\,y}}$
1	*0*	*15025*	*13846*	*8334*	*5124×10^3*	*4784×10^3*	*0,934*
2	*678*	*14071*	*13200*	*7688*	*4401×10^3*	*3831×10^3*	*0,870*
3	*1356*	*12682*	*12682*	*7170*	*3434×10^3*	*3434×10^3*	*1,0*
4	*2033*	*11629*	*11629*	*5989*	*2766×10^3*	*2766×10^3*	*1,0*

Moments and axial forces acting at each of the four positions are as follows:

$M_{Sd.1}$ = *873,1 kNm at 0,304 m from intersection of rafter and column*

$N_{Sd.1}$ = *176,6 kN*

$M_{Sd.2}$ = *740,1 kNm at 0,982 m from intersection of rafter and column*

$N_{Sd.2}$ = *175,6 kN*

$M_{Sd.3}$ = *613,9 kNm at 1,661 m from intersection of rafter and column*

$N_{Sd.3}$ = *174,5 kN*

$M_{Sd.4}$ = *494,5 kNm at 2,339 m from intersection of rafter and column*

$N_{Sd.4}$ = *173,5 kN*

$M_{Sd.5}$ = *381,7 kNm*

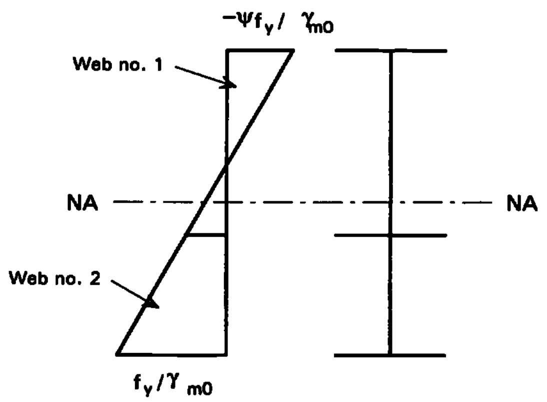
$N_{Sd.5}$ = *172,4 kN*

6.1 Classification

Section 17.2(9a) ENV 5.3

The flanges are Class 1, as shown in the rafter checks

The web can be divided into two, and classified according to the stress and geometry of each:



Division of web into web no. 1 and web no. 2

By inspection web no. 1 will be class 3 or better (because it is mainly in tension)

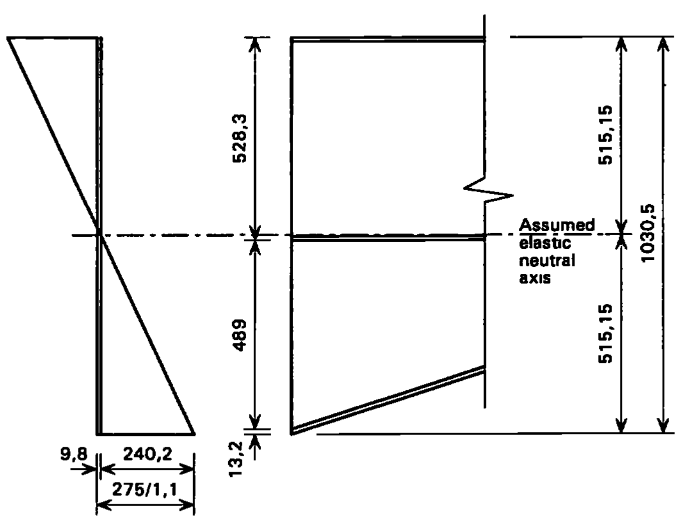
Classify web no. 2:

(a) Find maximum stress from bending that could coexist with actual axial force

Stress in web caused by axial load:

Using gross area including middle flange

$\sigma_N \;=\; 176{,}6 \,/\, 17{,}949 \;=\; 9{,}8\ N/mm^2$



Elastic stress distribution in web

.: stress available to resist bending:

$\sigma_M \;=\; (f_y \,/\, \gamma_M) - \sigma_N$

$\;=\; (275 \,/\, 1{,}1) - 9{,}8 \;=\; 240{,}2\ N/mm^2$

(b) Classify web assuming $\sigma_M = 240{,}2\ N/mm^2$ *is present*

.: coexistent stress at top of haunch would be:

$\sigma \;=\; \sigma_M - \sigma_N \;=\; 240,2 - 9,8 \;=\; 230,4$

Total depth = rafter + web + bottom flange

$= 528,3 + 489 + 13,2 \;=\; 1030,5$

∴ depth from neutral axis to underside of middle flange

$= 528,3 - (1030.5 / 2) = 528,3 - 515,2 \;=\; 13,1$ *mm*

∴ bending + axial at top of haunch cutting

$= 240,2 \times (13,1 / 515,2) + 9,8$

$= 6,1 + 9,8 \;=\; 15,9$ *N/mm² compression*

Distance from assumed elastic neutral axis to top of root radius on bottom flange of haunch cutting

$= 515,3 - 12,7 - 13,2 \;=\; 489,3$

∴ bending + axial at top of root radius on bottom flange of haunch cutting

$240,2 \times (489,3 / 515,2) + 9,8 \;=\; 228 + 9,8 \;=\; 238$ *N/mm²*

For class 3 check, determine ψ: — *ENV Table 5.3.1 Sheet 1*

$\psi \;=\; 15,9 / 238 \;=\; 0,067$

depth of web excluding roof radius $= 489 - 12,7 \;=\; 476$ *mm*

Class 3 limit for $\psi > -1$

$42 \times 0,92 / (0,67 + 0,33 \times 0,067) \;=\; 55,8$

$d/t_w \;=\; 476 / 9,6 \;=\; 49,6 \not> 55,8 \rightarrow$ *web is class 3*

6.2 Cross-sectional resistance — *Section 17.2(9b)*

For the stability checks given in this document for tapered haunches to remain valid, the tapered haunch must not contain a plastic hinge

6.2.1 Shear

The depth of the web between flanges is not greater than in the rafter, so shear buckling is not a problem in the haunch

The shear in the rafter has been checked in 5.2 above, showing $V_{Sd} \not> 0,5\ V_{pl.Rd}$. In the haunch, the shear area A_v increases more than the applied shear V_{Sd}, so the shear force has no effect by inspection

The tables provided below give the axial and moment resistance of the haunch section at various positions from the column face. A series of checks is carried out to determine whether the cross-sectional moment resistance $M_{c.Rd}$ is reduced by coexistent axial force. Positions 1 to 5 are checked to find M_{Sd} / $M_{c.Rd}$

Appendix C.4.4(b),(d)

6.2.2 Axial and bending

Table 6.2.2.1

Position	Distance* (mm)	N_{Sd} (kN)	A_{eff} (mm²)	$N_{pl.Rd}$† (kN)	$A_{web.eff}$	$A_{web.eff} f_y$
1	0	176,6	13846	3462	8334	2292
2	678	175,6	13200	3300	7688	2114
3	1356	174,5	12682	3171	7170	1972
4	2033	173,5	11629	2907	5989	1647
5	2711	172,4	10500	2625	4818	1325

** Distance taken along slope from column face*
† $N_{pl.Rd} = A_{eff} f_y / \gamma_{M0}$ and f_y = 275 N/mm²

Appendix C.4.4(b), E.4.3

Position	Distance (mm)	M_{Sd} (kNm)	Is N_{Sd} > 0,5 $A_{web.eff} f_y$	Is N_{Sd} > 0,25 $N_{pl.Rd}$	$M_{c.Rd}$ (= $M_{pl.y.Rd}$) (kNm)	Is $M_{Sd} > M_{c.Rd}$?
1	0	873	No	No	1196	No
2	678	740	No	No	958	No
3	1356	614	No	No	859	No
4	2033	495	No	No	692	No
5	2711	382	No	No	515	No

** $M_{pl.y.Rd} = W_{pl.y} f_y/\gamma_{M0}$ for Class 1 and 2 sections*
but = $W_{eff.pl.y} f_y/\gamma_{M0}$ for sections to Appendix E

The loading on the haunch is a combination of axial load, shear and bending. By inspection, the applied shear force is small relative to the shear capacity of the section and need not therefore be considered. Check positions 1 to 5 to find M_{Sd} / $M_{c.Rd}$, where $M_{c.Rd}$ is the cross-sectional moment of resistance. With low coexistent axial force and shear force, $M_{c.Rd} = M_{pl.Rd}$ — *Appendix C.4.4(b)*

i) *Position 1:* $\frac{873}{1196} = 0{,}730 < 1 \therefore OK$

ii) *Position 2:* $\frac{740}{958} = 0{,}772 < 1 \therefore OK$

iii) *Position 3:* $\frac{614}{859} = 0{,}715 < 1 \therefore OK$

iv) *Position 4:* $\frac{495}{692} = 0{,}715 < 1 \therefore OK$

v) *Position 5:* $\frac{382}{515} = 0{,}742 < 1 \therefore OK$

→ *No plastic hinges in haunch*

6.3 Buckling between intermediate restraints

Section 17.2(9)(c)

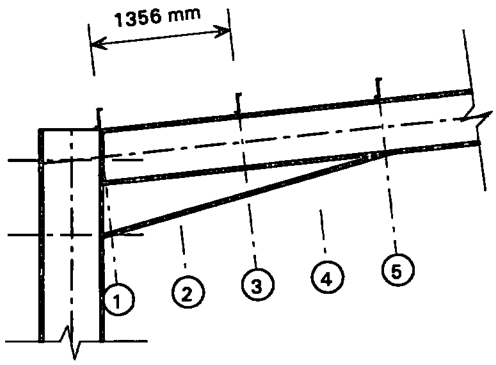
Assuming a purlin is positioned at the mid-length of the haunch, intermediate buckling should be checked between the column and purlin, and between purlin and haunch tip. Overall buckling checks should be carried out for the haunch as a whole

Using the approach suggested in Appendix E of this document, the following effective section properties are required: — *Appendix E.4*

1) *Effective area*
2) *Effective plastic section modulus*

Table 6.2.2.1 gives effective section properties at the start and mid span of the haunch, calculated for the haunch neglecting the "middle" flange, but remembering its stabilising effect on the web — *Appendix E.3*

6.3.1 Check between column flange and mid-haunch purlin — *Appendix E.4*



Buckling on deep end of haunch

From 6.2 above, Position 2 is the most critical cross section, having $M_{sd} / M_{c.Rd} = 0,772$. Therefore, the resistance is calculated using the area and modulus at this cross section, together with the axial force and bending moment at this cross section

Note that χ_z and χ_{LT} are calculated at the deepest end because this gives the most conservative results where the flanges are of constant section and the web is of constant thickness along the haunch

(a) Calculate buckling resistance to axial force — *Appendix E.4.2*

$$N_{b.Rd.z} = \chi_z A_{eff} f_y / \gamma_{M1}$$

$$l = 1356 \text{ mm}$$

$$\beta_A = Area_{eff} / Area$$

$$= 13846 / 15025 = 0,922$$

$$I_z = 2,004 \times 10^7 \text{ neglecting the middle flange}$$

$$i_z = (2,004 \times 10^7 / 15025)^{0,5} = 36,5$$

$$\lambda = 1356 / 36,5 = 37,15$$

$$\bar{\lambda} = \lambda(\beta_A)^{0,5} / \lambda_1$$

$$= 37,15 \times 0,922^{0,5} / 86,8 = 0,411$$

$\chi_z = 0{,}9217$ — ENV Table 5.5.2 Curve b

$A_{eff} = 13200\ mm^2$

$\therefore N_{b.Rd.z} = 0{,}9217 \times 13200 \times 275 / 1{,}1 \quad = \quad 3042\ kN$

(b) Calculate buckling resistance to bending moment — Appendix E.4.3

$M_{b.Rd} = \chi_{LT} W_{eff.pl.y} f_y / \gamma_{M1}$

$\beta_w = W_{eff.pl.y} / W_{pl.y}$

$= 4784 / 5124 \quad = \quad 0{,}934$

$$\lambda_{LT} = \frac{L/i_{LT}}{(C_1)^{0,5}\left(1 + \dfrac{(L/a_{LT})^2}{25{,}66}\right)^{0,25}}$$

$a_{LT} = (I_w / I_t)^{0,5}$

$I_w = I_{ztb} h_s^2 / 4$

$= 2004 \times 10^4 \times (528{,}3 + 489 - 13{,}2)^2 / 4$

$= 5{,}05 \times 10^{12}\ mm^6$

where I_{ztb} is for the top and bottom flanges only $\approx I_{z.rafter}$

$I_t = I_{t.rafter} + I_{t.cutting}$

$I_{t.cutting}$ may be taken as $0{,}5 \times I_t$ of section from which the cutting is taken, because I_t is dominated by the flange

$\therefore I_t = 51{,}5 \times 10^4 + 0{,}5 \times 51{,}5 \times 10^4$

$= 77{,}25 \times 10^4\ mm^4$

$\therefore a_{LT} = (5{,}05 \times 10^{12} / 77{,}25 \times 10^4)^{0,5} \quad = \quad 2557\ mm$

$$\therefore i_{LT} = \left(I_{ztb} \cdot I_w / W^2_{p\ell.y}\right)^{0,25}$$

$$= [2004 \times 10^4 \times 5{,}05 \times 10^{12} / (4957 \times 10^3)^2]^{0,25}$$

$= 45{,}0$ *mm where* $W_{p\ell.y}$ *for the deepest section is used to find the smallest* i_{LT} *to give the greatest* λ_{LT}

taking $C_1 = 1{,}0$

$$\therefore \lambda_{LT} = \frac{1356 / 45{,}0}{(1{,}0)^{0,5}\left(1 + \frac{(1356/2557)^2}{25{,}66}\right)^{0,25}}$$

$$= 30{,}0$$

Note that $L/i_{LT} = 30{,}1$*, which shows that the approximation* $\lambda_{LT} = L/i_{LT}$ *is normally adequate for checking between intermediate restraints*

$$\bar{\lambda}_{LT} = \lambda(\beta_w)^{0,5} / \lambda_1$$

$$= 30{,}0 \times 0{,}934^{0,5} / 86{,}8 = 0{,}334$$

$\bar{\lambda}_{LT} \ngtr 0{,}4$*, therefore*

$\chi_{LT} = 1{,}0$ — *Appendix D.3.2*

$$W_{eff.p\ell.y} = 3831 \times 10^3 \text{ mm}^3$$

$$\therefore M_{b.Rd} = 1{,}0 \times 3831 \times 10^3 \times 275 / 1{,}1 = 958 \text{ kNm}$$

(c) Calculate buckling resistance to combined axial and bending — *Appendix E.4.5*, *Appendix D.3.4*

Check $N_{Sd} / N_{b.Rd.z} + k_{LT} M_{Sd} / M_{b.Rd} \leq 1{,}0$

$$N_{Sd} = 175{,}6 \text{ kN}$$

$$M_{Sd} = 740{,}1 \text{ kNm}$$

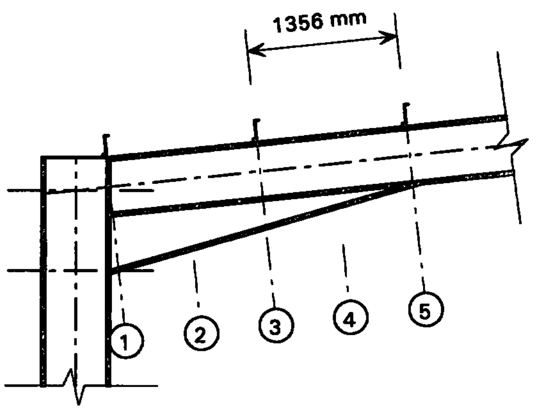
$N_{Sd} / N_{b.Rd.z} + k_{LT} M_{Sd} / M_{b.Rd} \leq 1,0$

Taking $k_{LT} = 1,0$ *(conservative)*

$\therefore (175,6 / 3042) + (1,0 \times 740,1 / 958) = 0,058 + 0,773$

$= 0,831 < 1,0$ *OK*

6.3.2 Check between mid-haunch purlin and haunch tip



Buckling on shallow end of haunch

From 6.2 above, Position 5 is the most critical cross section, having $M_{Sd} / M_{c.Rd} = 0,742$. *Therefore, the resistance is calculated using the area and modulus at this cross section, together with the axial force and bending moment at this cross section*

Note that χ_z *and* χ_{LT} *are calculated at the deepest end because this gives the most conservative results where the flanges are of constant section and the web is of constant thickness along the haunch*

(a) Calculate buckling resistance to axial force — *Appendix E.4.2*

$N_{b.Rd.z} = X_z A_{eff} f_y / \gamma_{M1}$
$l = 1355$ *mm*
$\beta_A = A_{eff} / A = 1,0$
$I_z = 2,004 \times 10^7$ *neglecting the "middle" flange*
$i_z = (2,004 \times 10^7 / 12682)^{0,5} = 39,8$
$\lambda = 1355 / 39,8 = 34,1$
$\bar{\lambda} = 34,1 / 86,8 = 0,393$
$\chi_{min} = 0,929$

$A = 10500\ mm^2$ neglecting the "middle" flange

$\therefore N_{b\,Rd.z} = 0{,}929 \times 1{,}0 \times 10500 \times 275 / 1{,}1 = 2440\ kN$

(b) Calculate buckling resistance to bending moment — Appendix E.4.3

$M_{b.Rd} = \chi_{LT}\, \beta_w W_{pl.y} f_y / \gamma_{M1}$

$\beta_w = W_{eff.pl.y} / W_{pl.y} = 1{,}0$

From check between column and mid-haunch purlin above:

$I_w = I_{ztb}\, h_s^2 / 4$

$= 2004 \times 10^4 \times (528{,}3 + 245 + 13{,}2)^2 / 4$

$= 3{,}10 \times 10^{12}\ mm^6$

$I_t = 77{,}25 \times 10^4\ mm$

$\therefore a_{LT} = (I_w / I_t)^{0,5}$

$= (3{,}10 \times 10^{12} / 77{,}25 \times 10^4)^{0,5} = 2003\ mm$

$i_{LT} = [I_{ztb}.I_w / W_{pl.y}^2]^{0,25}$

$= [2004 \times 10^4 \times 3{,}10 \times 10^{12} / (3434 \times 10^3)^2]^{0,25}$

$= 47{,}9\ mm$

Using the slightly conservative approximation $\lambda_{LT} = L / i_{LT}$

$\lambda_{LT} = 1355 / 47{,}9 = 28{,}3$

$\bar{\lambda}_{LT} = \lambda_1 / \lambda_{LT}$

$= 28{,}3 / 86{,}8 = 0{,}326$

$\bar{\lambda}_{LT} \ngtr 0{,}4$ *therefore* $\chi_{LT} = 1{,}0$

$W_{pl.y} = 2058 \times 10^3\ mm^3$

$M_{b.Rd} = 1{,}0 \times 2058 \times 10^3 \times 275 / 1{,}1 = 514{,}5\ kNm$

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Fax (01344) 622944

(c) Calculate buckling resistance to combined axial and bending

Check: $N_{Sd} / N_{b.Rd.z} + k_{LT} M_{Sd} / M_{b.Rd} \leq 1{,}0$ — *Appendix E.4.5*

$N_{Sd} = 172{,}4$ kN

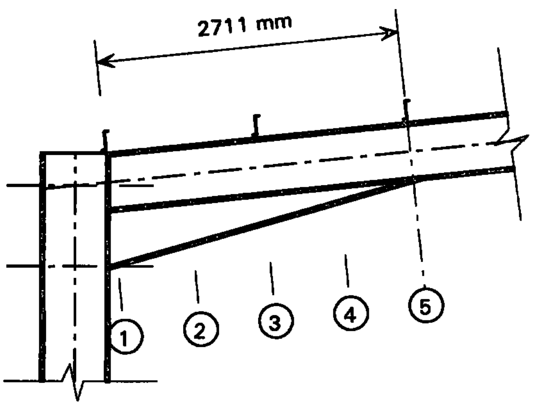
$M_{sd} = 381{,}2$ kNm

taking $k_{LT} = 1{,}0$ *(conservative)*

$(172{,}4 / 2440) + (1{,}0 \times 381{,}2 / 514{,}5) = 0{,}071 + 0{,}741$

$= 0{,}812 < 1 \therefore$ OK

6.4 Buckling between torsional restraints — *Section 17.2(9)(d)*



Haunch buckling length

Section 6.2 of this example shows that the critical section in the haunch is Position 2 because M_{Sd} / M_{Rd} *is maximum at that point. Therefore, check the resistance based on the forces, moments and resistances at that point* — *Appendix D.3.4*

(a) Calculate slendernesses λ *and* λ_{LT}

χ_z *and* χ_{LT} *are calculated from Appendix F.3 for which the constants accounting for the effect of the taper are based on the shallow end of the haunch*

$l = 2711$ mm
$I_z = 2{,}004 \times 10^7$ mm^4 *(ignoring the middle flange)*

$i_z = (2{,}004 \times 10^7 / 10500)^{0,5} = 43{,}7$

$\alpha = 1{,}106$ *as Section 5.4a because the properties for the shallow end of the haunch are used when using the date in Appendix F.3*

The axial slenderness (for restrained tension flange) is given by:

Appendix F.3.4

$$\lambda = \frac{L / i_z}{\left(\alpha + \dfrac{I_t L^2}{2{,}6\pi^2 I_z i_s^2}\right)^{0,5}}$$

$$\lambda = \frac{2711 / 43{,}7}{\left(1{,}106 + \dfrac{77{,}25 \times 10^4 \times 2711^2}{2{,}6\pi^2 \times 2{,}004 \times 10^7 \times 179929}\right)^{0,5}}$$

$$= \frac{62{,}0}{(1{,}106 + 0{,}061)^{0,5}} = \frac{62{,}0}{1{,}080} = 57{,}4$$

Appendix F.3.4

$$\lambda_{LT} = (m_t^{0,5} c)\left(\frac{W_{pl.y}}{A} \times \frac{2a}{i_s^2}\right)\lambda$$

Calculate m_t

Appendix F.3.3

$$\left(\frac{M_{c.Rd}}{M_{Sd}}\right)_{min} = \left(\frac{M_{Sd}}{M_{c.Rd}}\right)_{max}$$

$= 0{,}773$ *at position 2 from 6.2 above*

$\left(\dfrac{M_{Sd}}{M_{c.Rd}}\right)$ *for positions 1 to 5 has been found in 6.2 above*

$$m_t = \frac{1}{12}\left(\frac{1}{0{,}773}\right)(0{,}731 + 3 \times 0{,}773 + 4 \times 0{,}716 + 3 \times 0{,}714 + 0{,}741)$$

Note that $\mu_{SE} = \left(\dfrac{M_{\mathrm{Sd.S}}}{M_{\mathrm{c.Rd.S}}} - \dfrac{M_{\mathrm{Sd.E}}}{M_{\mathrm{c.Rd.E}}}\right)$ *is negative so is not included*

$\therefore m_t = 0{,}948$

Calculate c_o

$D / t_f = 528{,}2 / 13{,}2 = 40{,}0$ — *Table F.2*

$r = d_2 / d_1 = [725 + (528{,}2 / 2) - 13{,}2] / (528{,}3 - 13{,}2)$

$= 1{,}89$

$c_o = 1{,}096$

$$\lambda_{LT} = \left(0{,}948^{0,5} \times 1{,}096\right)\left(\frac{2059 \times 10^3 \times 2 \times 364{,}2}{10500 \times 179929}\right)^{0,5} 57{,}4$$

$$= 1{,}067 \times 0{,}891 \times 57{,}4 = 54{,}6$$

(b) Calculate buckling resistance to axial force

$\bar{\lambda} = \lambda / \lambda_1 \quad 57{,}4 / 86{,}8 = 0{,}661$ — *ENV Table 5.5.2 Curve b*

$\chi_z = 0{,}805$

$A = 13200$ mm^2 *(neglecting the middle flange)*

$\therefore N_{b.Rd.z} = 0{,}805 \times 1{,}0 \times 13200 \times 275 / 1{,}1 = 2657$ kN

(c) Calculate buckling resistance to bending moment

$\bar{\lambda}_{LT} = 54{,}6 / 86{,}8$ — *ENV Table 5.5.2 Curve a*

$= 0{,}629$

$\chi_{LT} = 0{,}878$

$W_{pl.y} = 3831 \times 10^3$ mm^3 *(neglecting the middle flange)*

$\therefore M_{b.Rd} = 0{,}878 \times 1{,}0 \times 3831 \times 10^3 \times 275 / 1{,}1$

$= 841$ kNm

(d) Calculate buckling resistance to combined axial and bending

Check $N_{Sd} / N_{b.Rd.z} + k_{LT} M_{Sd} / M_{b.rd} \leq 1{,}0$ at point of maximum $M_{Sd} / M_{n.Rd}$, which occurs at Position 2 in this case, as shown in Section 6.2 of this worked example

$N_{Sd} \quad = \quad 175{,}6 \text{ kN}$

$M_{Sd} \quad = \quad 736{,}8 \text{ kNm}$

Taking $k_{LT} \quad = \quad 1{,}0$ *(conservative)*

$(175{,}6 / 2657) + (1{,}0 \times 736{,}8 / 841) = 0{,}066 + 0{,}876$
$= 0{,}942 < 1$, *OK*

∴ haunch is stable between torsional restraints

7. COMPARISON WITH BS5950

For comparison purposes, design of a portal frame with identical geometrical and loading specifications was carried out using the UK code of practice BS 5950: 1990

A summary of the section sizes for each design are shown below:

Design no.	*Design code*	*Variation in loading*	*Column size*	*Rafter size*	*Haunch length (m)*
1	*EC3*	*Benchmark*	*610×229 UB113*	*533×210 UB82*	*3*
2	*BS5950*	*As benchmark*	*610×229 UB113*	*457×191 UB74*	*3*
3	*BS5950*	*As benchmark but snow load 0,6 kN/m²*	*610×229 UB101*	*457×191 UB67*	*3*

Worked Example Number 2

Contents

Page No.

Note:

References to Sections and Appendices in the Worked Example refer to this publication.

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1. GENERAL

The loads, geometry and partial safety factors are generally as for the plastic portal worked example

Note that for elastic analysis, the partial safety factors are not modified by Merchant-Rankine

Only the essential differences between elastic and plastic portals are shown in this example, so only one load combination is considered and only the in-plane buckling check is shown. All the other checks are as for a plastic portal

Note that the elastic portal bending moment diagram has higher moments at the haunches. This causes greater problems of instability in the haunch and also in the column and rafter near the haunch, unless the section sizes are increased. For example, in this elastic portal the rafter is the same size as in the plastic portal, so additional torsional restraint will be required near the haunch

Rafter: 533 × 210 UB82

h	=	528,3 mm	i_z	=	43,8 mm
b	=	208,7 mm	i_{LT}	=	50,1 mm
t_w	=	9,6 mm	a_{LT}	=	1610 mm
t_f	=	13,2 mm	$W_{el.y}$	=	1800 cm^3
I_{yy}	=	$4{,}752 \times 10^8$ mm^4	$W_{el.z}$	=	$0{,}192 \times 10^6$ mm^3
I_{zz}	=	$2{,}004 \times 10^7$ mm^4	$W_{pl.y}$	=	2059 cm^3
A	=	10500 mm^2	$W_{pl.z}$	=	$0{,}3 \times 10^6$ mm^3
i_y	=	213,0 mm			

Column: 686 × 254 UB125

h	=	677,9 mm	i_z	=	52,4 mm
b	=	253,0 mm	i_{LT}	=	60,3 mm
t_w	=	11,7 mm	a_{LT}	=	2030 mm
t_f	=	16,2 mm	$W_{el.y}$	=	$3{,}481 \times 10^6$ mm^3
I_{yy}	=	$11{,}800 \times 10^8$ mm^4	$W_{el.z}$	=	$0{,}346 \times 10^6$ mm^3
I_{zz}	=	$4{,}383 \times 10^7$ mm^4	$W_{pl.y}$	=	$3{,}994 \times 10^6$ mm^3
A	=	15900 mm^2	$W_{pl.z}$	=	$0{,}542 \times 10^6$ mm^3
i_y	=	272,0 mm			

2. ULTIMATE LIMIT STATE ANALYSIS

2.1 Load combination no. 1

First order linear elastic analysis gives the following from load combination no. 1 defined in the plastic portal worked example

Haunch max. M_{Sd} = 891,9 kNm at the intersection of the rafter and column

Column max. N_{Sd} = 197,1 kN
max. V_{Sd} = 148,7 kN

Rafter max. N_{Sd} = 169,5 kN
max. Vsd = 180,4 kN

2.2 Stability check

2.2.1 Check for arching/snap through stability

ENV 5.2.6
Section 12.1
Appendix B.3

$$\frac{V_{Sd}}{V_{cr}} = \left(\frac{L}{D}\right)\left(\frac{\Omega - 1}{55,7\ (4 + L/h)}\right)\left(\frac{I_r}{I_c + I_r}\right)\left(\frac{f_{yr}}{275}\right)\left(\frac{1}{\tan 2\phi_r}\right)$$

$L = 30,0$ m
$D = 0,528$ m
$h = 6,0$ m
$F_{vl} = (1,44 + 1,44 + 0,82) \times 1,35 + (5,4 \times 1,5) = 13,1$ kN/m
$F_{fail} = 16M_p / l^2$
$= 16 \times 2,058 \times 10^6 \times 10^{-6} \times 275 / 1,1 / 24^2 = 14,03$
$\Omega = 13,1 / 14,03 = 0,94 \leq 1,0$

∴ Stable – no further check necessary

2.2.2 Check sway stability check, assuming pinned feet:

Appendix B.4

The calculation uses the results of the first-order linear elastic analysis given in 2.1 above

$$\frac{V_{Sd}}{V_{cr}} = \left[\left(\frac{N_r}{N_{r.cr}}\right) + (4 + 3,3R\left(\frac{N_c}{N_{c.cr}}\right)\right]$$

Where:

$$R = \frac{I_c\, s}{I_r\, h} = \frac{11{,}8 \times 10^8 \times 15{,}083}{4{,}752 \times 10^8 \times 6{,}0} = 6{,}240$$

$$N_{cr} = \frac{\pi^2 EI_c}{h^2} = \frac{\pi^2 \times 2{,}1 \times 11{,}8 \times 10^{13} \times 10^{-3}}{6000^2}$$

$$= 67867 \text{ kN}$$

$$N_{r.cr} = \frac{\pi^2 EI_r}{s^2} = \frac{\pi^2 \times 2{,}1 \times 4{,}752 \times 10^{13}}{15083^2}$$

$$= 4325 \text{ kN}$$

N_c = 197,9 + 5,8 (self weight of column and cladding)

$$= 203{,}7 \text{ kN}$$

$$N_r = 169{,}5 \text{ kN}$$

$$\frac{V_{Sd}}{V_{cr}} = \left[\left(\frac{169{,}5}{4325}\right) + (4 + 3{,}3 \times 6{,}24)\left(\frac{203{,}7}{67867}\right)\right] = 0{,}113$$

$$\therefore \frac{V_{Sd}}{V_{cr}} > 0{,}1$$ *Section 12.1*

∴ first-order analysis must be modified to allow for second-order effects

2.3 Amplified sway

Because $V_{Sd} / V_{cr} \ngtr 0{,}25$, the amplified horizontal deflection method may be used: *Section 12.3.2(a)*

Sway amplification factor $= [1 / (1 - V_{Sd} / V_{cr})] = 1{,}127$ *ENV 5.2.6.2 (3), (4)*

$\therefore [1 / (1 - V_{Sd} / V_{cr})] - 1 = 0{,}127$

∴ the second-order effects are included by adding the bending moments from imposing additional eaves/valley horizontal deflections
= 0,127 × (first order analysis deflection)

∴ adding the increments of sway bending moment from amplified sway to the moments and forces from the first-order analysis, the design moments and forces are:

Columns	max. M_{sd}	=	863,6 kNm
	max. N_{sd}	=	197,9 kN
	max. V_{Sd}	=	150,7 kN
Rafter	max. M_{Sd}	=	972,8 kNm
	max. N_{Sd}	=	169,5 kN
	max. V_{Sd}	=	181,2 kN

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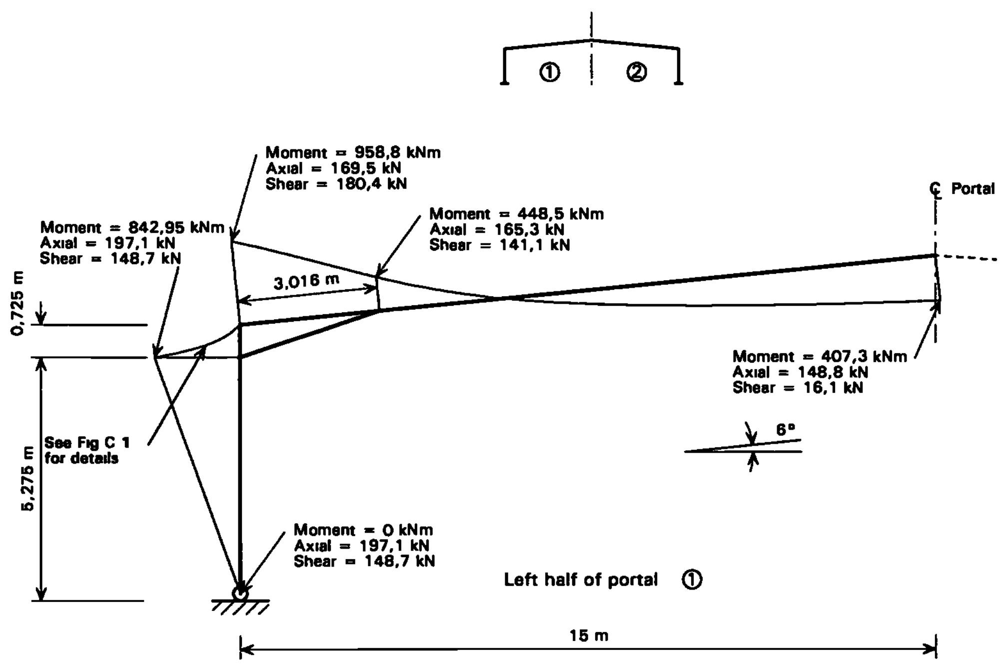
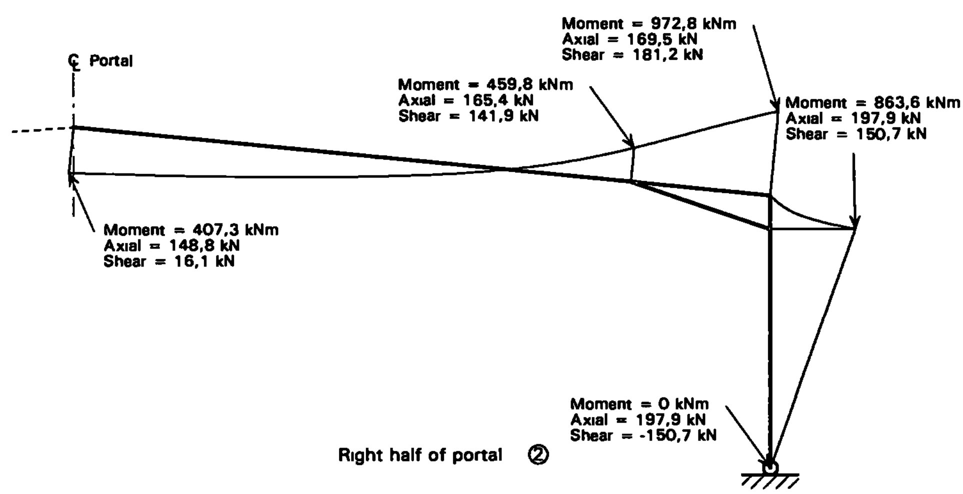
Combination no. 1:
bending moment, shear and axial force diagram
(modified ultimate limit state)





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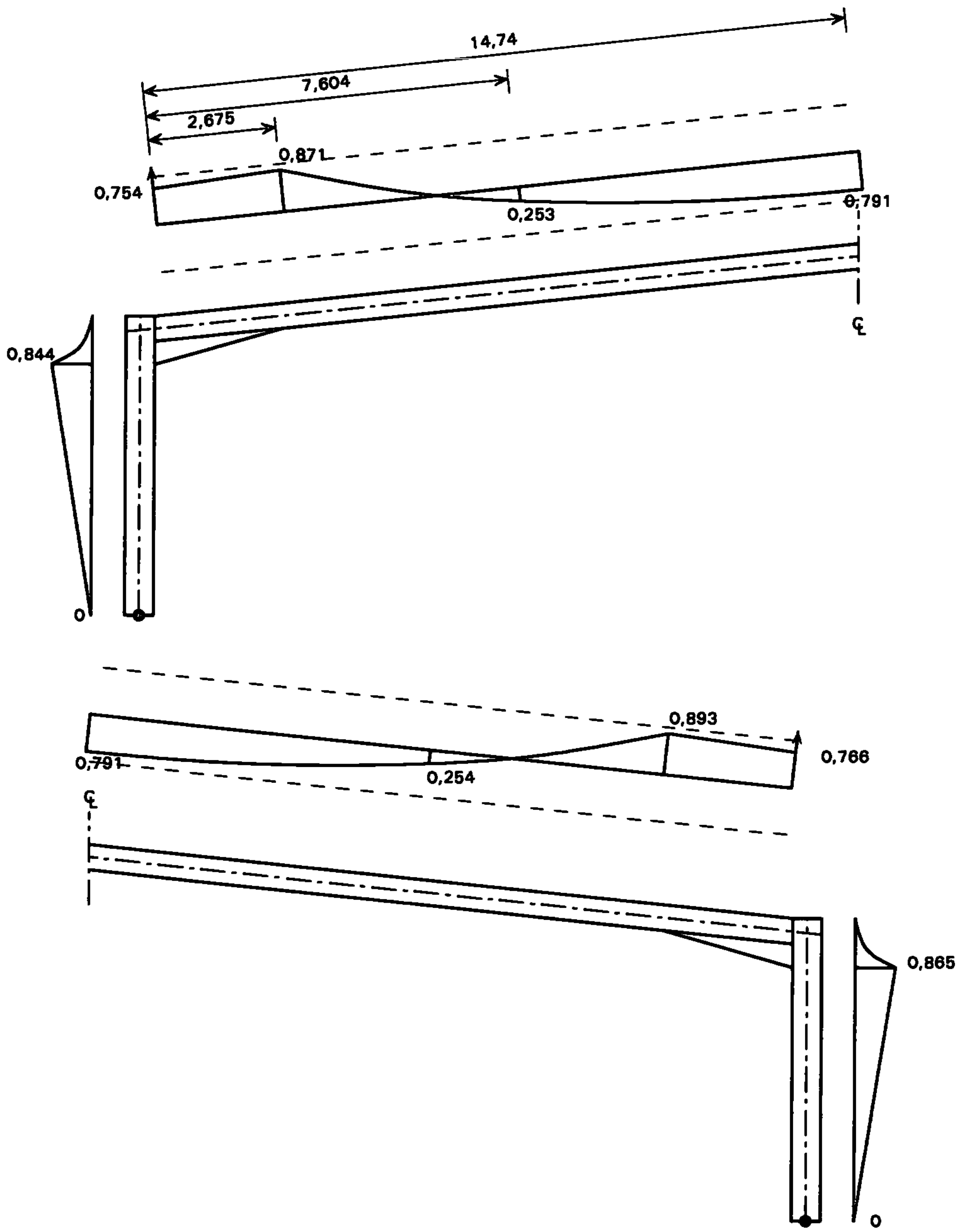
$M_{Sd} / M_{c\,Rd}$ relationship within rafter and column



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3. COLUMN

3.1 Classify section

The checks are as the plastic portal worked example

3.2 Cross-sectional resistance

The checks are as the plastic portal worked example

3.3 In-plane buckling

Check $N_{Sd} / N_{b.Rd.y} + k_y M_{y.Sd} / M_{pl.y} \leq 1{,}0$

From 2.3 above:

$\max. M_{Sd} = 863{,}6$

$\max. N_{Sd} = 197{,}9$

$\max. {}_{VSd} = 150{,}7$

(a) Calculate buckling resistance to axial force — *Appendix D.3.1*

Effective length of column $= L$

Appendix D.3.5

$$\therefore \lambda_y = L / i_y = \frac{6000}{272{,}0} = 22{,}1$$

$$\lambda_1 = \pi (E / f_y)^{0,5} = 6{,}8$$

$$\therefore \bar{\lambda}_y = \frac{\lambda_y}{\lambda_1} = \frac{22{,}1}{86{,}8} = 0{,}254$$

$$\phi = 0{,}5\,[1 + \alpha(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2]$$

Identify buckling curve

$h / b = 528{,}3 / 208{,}7 = 2{,}5 > 1{,}2$ — *ENV Table 5.5.3*

$t_f \quad = \quad 13{,}2 \text{ mm} < 40 \text{ mm}$

Buckling about yy axis

$\therefore$ *use buckling curve "a"*

$\therefore \alpha \quad = \quad 0{,}21$ — *ENV Table 5.5.1*

$\therefore \phi \quad = \quad 0{,}5\,[1 + 0{,}21\,(0{,}254 - 0{,}2) + 0{,}254^2] \quad = \quad 0{,}538$

$\therefore \chi_y \quad = \quad 1 / [\phi + (\phi^2 - \bar{\lambda}^2)^{0,5}] = 1 / [0{,}538 + (0{,}538^2 - 0{,}524^2)^{0,5}]$
$\quad = \quad 0{,}988$

$N_{b.Rd.y} \quad = \quad \chi_y\, A\, f_y / \gamma_{M1} \quad = \quad 0{,}988 \times 15900 \times 275 / 1{,}1$
$\quad = \quad 3927 \text{ kN}$

(b) Determine bending parameters

Determine β_m: — *ENV Fig 5.5.3*

$M_1 \quad = \quad 863{,}6 \text{ kNm}$

$\psi M_1 \quad = \quad 0 \text{ kNm}$

$\therefore \psi \quad = \quad 0{,}0$

$\beta_{M,\phi} \quad = \quad 1{,}8 + (0{,}7 \times 0{,}0) \quad = \quad 1{,}8$

$\Rightarrow \mu_y \quad = \quad \bar{\lambda}_y\,(2\beta_{MY} - 4) + [(W_{pl.y} - W_{el.y}) / W_{el.y}]$

$$= \quad 0{,}254(2 \times 1{,}8 - 4) + \left(\frac{3994 - 3481}{3481}\right) = -0{,}102 + 0{,}147$$

$$= \quad 0{,}045$$

$$\Rightarrow k_y \quad = \quad 1 - \frac{\mu_y N_{Sd}}{\chi_y A f_y} = 1 - \left(\frac{-0{,}045 \times 197900}{0{,}988 \times 15900 \times 275}\right) = 1 - 0{,}002$$

$k_y \quad = \quad 0{,}998 \le 1{,}5$

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$M_{pl.y} = W_{pl.y} \times f_y / \gamma_{M0} = 3994 \times 10^3 \times 275/1{,}1 = 999\ kNm$

(c) Calculate buckling resistance to combined axial and bending

$N_{Sd} / N_{b.Rd.y} + k_y M_{y.Sd} / M_{pl.y} = 197{,}9 / 3927 + (0{,}998 \times 863{,}6 / 999)$ — *Appendix D.3.4*

$= 0{,}050 + 0{,}863$
$= 0{,}913 < 1{,}0 \therefore OK$

3.4 Buckling between intermediate restraints

The checks are as Appendix D.3.4 or D.4, as in the plastic portal worked example

3.5 Buckling between torsional restraints

The checks are as Appendix D.3.4 or D.4, as in the plastic portal worked example

4. RAFTER

4.1 Classify section

The checks are as the plastic portal worked example

4.2 Cross-sectional resistance

The checks are as the plastic portal worked example

4.3 In-plane buckling

Check $N_{sd} / N_{b.Rd.y} + k_y \, M_{y.Sd} / M_{pl.y} \leq 1{,}0$ — *Appendix D.3.4*

This check is for uniform sections and the bending moment diagram must be modified to allow for the change of section along the rafter and haunch. The calculation is based on the maximum values of $M_{Sd} / M_{c.Rd}$ shown in the figure in Section 2.3 above

From 2.3 above:

$$\text{max. } N_{Sd} = 169{,}5 \text{ kN}$$
$$\text{max. } M_{Sd} = 181{,}2 \text{ kN}$$

where M_{Sd} is shown on the bending moment diagram

(a) Calculate buckling resistance to axial force

$$\lambda_y = L / i_y = \frac{15083}{213{,}0} = 70{,}8$$

$$\lambda_1 = \pi (E / f_y)^{0,5} = 86{,}8$$

$$\therefore \bar{\lambda}_y = \frac{\lambda_y}{\lambda_1} = \frac{70{,}8}{86{,}8} = 0{,}816$$

$$\phi = 0{,}5 \, [1 + \alpha (\bar{\lambda} - 0{,}2) + \bar{\lambda}^2]$$

Identify buckling curve

$$h / b = 528{,}3 / 208{,}7 = 2{,}5 > 1{,}2$$

ENV Table 5.5.3

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$t_f \quad = 13{,}2 \text{ mm} < 40 \text{ mm}$

Buckling about yy axis

$\therefore$ *use buckling curve "a"*

$\therefore \alpha \quad = \quad 0{,}21$ — *ENV Table 5.5.1*

$\therefore \phi \quad = \quad 0{,}5\,[1 + 0{,}21\,(0{,}816 - 0{,}2) + 0{,}816^2] \quad = \quad 0{,}8976$

$\therefore \chi_y \quad = \quad 1 / [\phi + (\phi^2 - \bar{\lambda}^2)\; 0{,}5]$

$\quad = \quad 1 / [0{,}8976 + (0{,}8976^2 - 0{,}816^2)^{0{,}5}] \quad = \quad 0{,}786$

$N_{b.Rd.y} \quad = \quad \chi_y\, A\, f_y / \gamma_{M1} \quad = \quad 0{,}786 \times 10500 \times 275 / 1{,}1 \quad = \quad 2063 \text{ kN}$

(b) Determine bending parameters

Determine β_m — *ENV Fig 5.5.3*

Figure 5.5.3 does not include the shape of bending moment diagram shown in $M_{sd} / M_{c.Rd}$. *The application of Figure 5.5.3 is therefore approximate, but it is applied so as to be conservative. By assuming 459,8 kNm at the end of the 15,083 m length of rafter, the sagging portion of the assumed bending moment is longer than in reality, so the added imperfection (due to curvature from the bending moment) is greater than in reality, giving a conservative value for buckling resistance*

$M_1 \quad = \quad 459{,}8 \text{ kNm}$

$\psi M_1 \quad = \quad -407{,}3 \text{ kNm}$

$\therefore \psi \quad = \quad -0{,}866$

$\beta_{M,\psi} \quad = \quad 1{,}8 + (0{,}7 \times 0{,}866) \quad = \quad 2{,}42$

$\Delta M \quad = \quad 459{,}8 + 407{,}3 \quad = \quad 867{,}1$

$$M_Q \quad = \quad \frac{wl^2}{8} \quad \text{where} \quad \frac{w(2l)^2}{8} \quad = \quad \Delta M \quad = \quad 867{,}1$$

$= \frac{1}{4}\,(867{,}1) \quad = \quad 216{,}8\ kNm$

$\beta_M \quad = \quad \beta_{M,\psi} + \frac{M_Q}{\Delta M}(\beta_{M,Q} - \beta_{M,\psi}) = 2{,}42 + \frac{216{,}8}{867{,}1}(1{,}3 - 2{,}42) = 2{,}14$

$\Rightarrow \mu_y \quad = \quad \bar{\lambda}_y\,(2\beta_{MY} - 4) + [(W_{pl.y} - W_{el.y}) / W_{el.y}]$

$= \quad 0{,}816(2 \times 2{,}14 - 4) + \left(\frac{2059 - 1800}{1800}\right) = 0{,}228 + 0{,}144$

$= \quad 0{,}372$

$\Rightarrow k_y \quad = \quad 1 - \frac{\mu_y\,N_{Sd}}{\chi_y\,Af_y} = 1 - \left(\frac{0{,}372 \times 169500}{0{,}786 \times 10500 \times 275}\right) = 1 - 0{,}03$

$k_y \quad = \quad 0{,}97 \le 1{,}5$

$M_{pl.y} \quad = \quad W_{pl.y} \times f_y / \gamma_{M0} \quad = \quad 2059 \times 10^3 \times 275 / 1{,}1 \quad = \quad 515\ kNm$

(c) Calculate buckling resistance to combined axial and bending:

$N_{Sd} / N_{b.Rd.y} + k_y\,M_{y.Sd} / M_{pl.y} = (169{,}5 / 2063) + (0{,}97 \times 459{,}8 / 515)$ — *Appendix D.3.4*

$= \quad 0{,}082 + 0{,}866$

$= \quad 0{,}948 \quad < 1\ \therefore\ OK$

4.4 Buckling between intermediate restraints

The checks are as Appendix D.3.4 or D.4, as for columns or rafters in the plastic portal worked example

4.5 Buckling between torsional restraints

The checks are as Appendix D.3.4 or D.4, as for columns or rafters in the plastic portal worked example

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5. HAUNCH

The checks are as in the plastic portal worked example

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