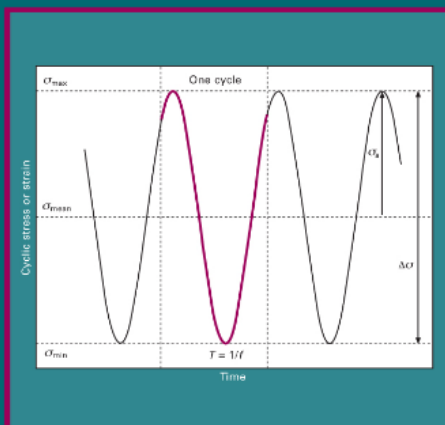




Fatigue life prediction of composites and composite structures

Edited by Anastasios P. Vassilopoulos

Fatigue life prediction of composites and composite structures

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Anastasios P. Vassilopoulos



CRC Press
Boca Raton Boston New York Washington, DC

WOODHEAD PUBLISHING LIMITED
Oxford Cambridge New Delhi

Published by Woodhead Publishing Limited, Abington Hall, Granta Park,
Great Abington, Cambridge CB21 6AH, UK
www.woodheadpublishing.com

Woodhead Publishing India Private Limited, G-2, Vardaan House, 7/28 Ansari Road,
Daryaganj, New Delhi – 110002, India
www.woodheadpublishingindia.com

Published in North America by CRC Press LLC, 6000 Broken Sound Parkway, NW,
Suite 300, Boca Raton, FL 33487, USA

First published 2010, Woodhead Publishing Limited and CRC Press LLC

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British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library.

Library of Congress Cataloging in Publication Data

A catalog record for this book is available from the Library of Congress.

Woodhead Publishing ISBN 978-1-84569-525-5 (book)

Woodhead Publishing ISBN 978-1-84569-979-6 (e-book)

CRC Press ISBN 978-1-4398-2789-5

CRC Press order number: N10159

The publishers' policy is to use permanent paper from mills that operate a sustainable forestry policy, and which has been manufactured from pulp which is processed using acid-free and elemental chlorine-free practices. Furthermore, the publishers ensure that the text paper and cover board used have met acceptable environmental accreditation standards.

Typeset by Replika Press Pvt Ltd, India

Printed by TJ International Limited, Padstow, Cornwall, UK

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Preface

Composite materials constitute the new challenge for several sectors of the scientific community, and their introduction in engineering and society generally represents an even greater challenge for researchers and academics. The use of composites in several sectors of industry seems inevitable, although in practice engineers are still hesitant when confronted with composites. The main reasons for this reluctance are the unawareness on the part of engineers concerning the advantages of composite materials, the complexity of classifying the behavior of composites in terms of rules and regulations, and the difficulty of developing a commonly accepted accurate method for the life prediction of these materials.

Despite this lack of enthusiasm, composites are (slowly but surely) being used for critical structural components and nowadays they are considered as being on the same footing as the traditionally used steel, aluminum or concrete in emerging structures. This development changes the common perception concerning the fatigue sensitivity of each structure.

Significant research on the fatigue behavior of composites has now been underway for over half a century. Numerous experimental programs were conducted for the characterization of the fatigue behavior of several structural composite materials of that time. As technology developed and new test frames and measuring devices were invented, it became easier to conduct complex fatigue experiments and measure properties and characteristics, something which some years earlier would not have been possible. As a result, almost all failure modes were identified and many theoretical models were established for modeling and eventually predicting the fatigue life of several different material systems.

This book attempts to address the problem of the fatigue life prediction of composites and composite structures and provides an update regarding current knowledge as well as on-going research. Engineers and scientists from different domains, including aeronautics, wind energy, and bridge construction, have contributed their experience for the compilation of this volume.

The conclusion is that the problem is well known and that solutions do exist. The way to reach them is to be 'open-minded' and work with a collaborative

spirit. Knowledge must be freely disseminated without limits, and efforts should be appropriately acknowledged following basic ethical rules.

I am considered as a young scientist, being under 40 years old and having only started my academic career in 2002. I became aware of fatigue in 1996, at the Department of Mechanical Engineering (later renamed the Department of Mechanical Engineering and Aeronautics) of the University of Patras in Greece, when I started working on a project for the construction of wind turbine rotor blades and was simultaneously doing my PhD on the development of a methodology for the fatigue life prediction of composite materials under complex stress states of variable amplitude. Woodhead Publishing offered me the opportunity to undertake the editing of this book and I have to admit, after two years of experience, that collaboration with the Woodhead Editorial Board is extremely agreeable. Rob Sitton invited me to join the project, thereafter Lucy Cornwell followed the project during the first year, and finally Laura Pugh, Francis Dodds and Nell Holden managed to 'push' the contributors (including myself) to meet the deadlines and complete the volume. I thank them all for their professionalism and kindness.

I am grateful also to the authors of the various chapters who devoted their valuable time to fulfill the requirements of this project and finally produced material of such high quality. It is very rewarding to see people from all over the world (we have contributions from America, Asia and Europe) sharing their expertise and working together towards a common goal. I knew some of the authors before working with them on this book, and, among others, my PhD supervisor (Theodore Philippidis) honored me by participating. I used to meet Wim Paepegem and Marino Quaresimin at the ECCM series conferences; I worked with Rogier Nijssen on a research project. I met Jack Lesko (co-author of Chapter 3) during one of his visits to the Ecole Polytechnique Fédérale de Lausanne, in Switzerland. The director of the Composite Construction Laboratory, where I currently work, Thomas Keller, also contributed a chapter, and my co-author in Chapter 5 (Stratos Georgopoulos) was my classmate during my years at high school, back in Kalamata, Greece! I consider all of them, as well as the other authors whom I had not previously known personally (I had only followed their research over the years, since they are some of the leading scientists in the field of fatigue of composites) good friends and hope that they will enjoy the book once it is completed.

And last but not least, I must acknowledge the patience, support and understanding of my wife Maria and our two little angels, Aggelos and Panayiota-Niki, who were always close to me and who with their monkey tricks contrived to keep me happy and fit during the two years that I have been working on this project.

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Introduction to the fatigue life prediction of composite materials and structures: past, present and future prospects

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Abstract: This chapter aims to provide an overview of the fatigue life prediction methods for composite materials and structures, recalling methods used in the past, discovering the present status and attempting to foresee future trends.

Key words: fatigue, composites, life prediction methods, constant amplitude loading, variable amplitude loading, S–N curves, residual strength, residual stiffness.

1.1 Introduction

One of the first fundamental facts of which human beings become aware is that nothing lasts forever. Life may come to a sudden end or last longer, but still for only a finite period. This latter case is normally supplemented by a reduction in efficiency, known as aging. This human life experience is directly reflected in materials science; under a high load a structure or a component can fail at once, whereas it can effectually sustain lower loads. On the other hand, the same structure or component can also fail under lower loads if they are applied over longer time frames in a constant (creep) or circular (fatigue) way. The phenomenon of the degradation of properties of a material due to the application of loads that fluctuate over time is called fatigue and the resulting failure is called fatigue failure.

Fatigue was identified as a critical loading pattern a long time ago by the scientific community. Already in 1829 the German mining engineer W. A. S. Albert was the first to carry out fatigue tests on metallic conveyor chains [1] and later to report his observations. Subsequently, numerous failures that could not be explained on the basis of known theory were attributed to fatigue loading. With the development of the railways, in the mid-nineteenth century, the failure of wagon axles was such a frequent occurrence that it attracted the attention of engineers. Between 1852 and 1870 a German engineer, August Wöhler, realized the first extended experimental program on the fatigue of metallic materials [1]. The program comprised full-scale

fatigue tests on wagon axles but also specimen tests under cyclic loading patterns of tensile, bending and torsional loads. Wöhler constructed a test rig on which he could test wagon axles under bending moments that were developed by loads suspended from the ends of the axles. The developed stresses were recorded together with the number of rotations up to failure. The results were drawn on the σ -N plane to formulate the first S-N curve, which, however, was restricted to the representation of experimental data, without proposing any mathematical formulation to describe this behavior.

These first attempts to analyze the fatigue behavior of materials and structures were based on experience with constructions operating under real loading conditions. Failures that could not be explained by existing theories were designated fatigue failures. As from 1850, engineers recognized fatigue as a critical loading pattern that could be the reason for a significant percentage of structural failures and it was thereafter widely accepted that fatigue should not be neglected. However, as mentioned in the work of Schütz [2], knowledge concerning certain methods was very advanced in one location, while a few kilometers away it was nonexistent. It was not until 1946, when the term fatigue was incorporated in the dictionary of the American Society for Testing and Materials (ASTM), when the E9 committee was founded to promote the development of fatigue test methods [3]. Today, it is documented that the majority of structural failures occur through a fatigue mechanism and, as mentioned in [4], after extensive study by the US National Institute of Standards and Technology, approximately 60% of 230 examined failures were associated with fatigue. This percentage was higher (between 80% and 90%) in another study carried out by the Battelle Institution [5].

During the following years, numerous experimental programs were conducted for the characterization of the fatigue behavior of several structural materials of that time. As technology developed and new test frames and measuring devices were invented, it became more and more straightforward to conduct complex fatigue experiments and measure properties and characteristics, something which some years earlier would not have been possible. As a result, almost all failure modes were identified and many theoretical models were established for modeling and eventually predicting the fatigue life of several different material systems.

Although composite materials are designated as fatigue insensitive, especially when compared to metallic ones, they suffer from fatigue loads as well. The introduction of composite materials in a wide range of applications obliged researchers to consider fatigue when investigating a composite material and obliged engineers to realize that fatigue is an important parameter that must be considered in calculations during design processes. Initially composites were used as replacements for previous 'conventional' materials such as steel, aluminum or wood, and later as 'advanced' materials that allow engineers to adopt a different approach to design problems, propose alternative design

concepts (based on the free formability and light weight characteristics of composites) and redesign structures. Unfortunately the situation regarding the fatigue behavior of composite materials is different from that of metallic ones. Therefore, the already developed, and validated, methods for the fatigue life modeling and prediction of 'conventional' materials cannot be directly applied to composite materials. Moreover, the large number of different material configurations resulting from the multitude of fibers, matrices, manufacturing methods, lamination stacking sequences, etc., makes the development of a commonly accepted method to cover all these variances difficult. As mentioned in [6], 'obviously, it is difficult to get a general approach of the fatigue behavior of composite materials including polymer matrix, metal matrix, ceramic matrix composites, elastomeric composites, glare, short fiber reinforced polymers and nano-composites'.

One way of dealing with the fatigue of composite materials is to undertake extended experimental programs and then develop analytical, mathematical, expressions in order to model fatigue life and be able to reproduce experimental results. Numerous experimental programs have been realized over the last three to four decades and very comprehensive databases have been constructed. Some of these are limited, refer to specific materials, and have been determined mainly in order to assist the development of a theoretical model, e.g. [7–12], but others, like [13] and [14], are more extensive and cover a wide range of materials for specific applications. Along with the aforementioned experimental work, a considerable number of theoretical models have been developed to model the fatigue behavior of the examined composites and consequently predict their behavior under unknown loading conditions, e.g. [7, 9, 15–19]. A literature search (www.scopus.com) with keywords 'fatigue' and 'composites' in the disciplines 'Engineering', 'Materials science', 'Energy', and 'Multidisciplinary' produces over 9500 research articles in the field, with more than 85% of them published after 1980, and around 400 articles per year after 1995.

Despite this explosive production of scientific publications in the field, countless unresolved topics exist in the domain of composite fatigue. Typical areas requiring further investigation concern the S–N and constant life diagram formulations for the interpretation of existing fatigue data, nonlinear damage accumulation rules that can take load sequence effects into account, cycle-counting methods that do not scramble the load sequence of the applied load time series, consideration of non-proportional stress components in a biaxial/multiaxial loading case, the exploitation of material behavior at very high cycle regimes, the development of methods that take into account the stochastic nature of the phenomenon, etc.

Researchers have attempted to address these topics in order to model or predict the fatigue behavior of composite materials of interest. However, the terms 'modeling' and 'prediction' have often been misused, usually by

adopting the term ‘prediction’ when a ‘modeling’ is performed with the aim of interpolating between known fatigue data. The term ‘prediction’ must be used when extrapolation is performed outside the existing database in terms of prediction of the behavior of the same material under new loading conditions, e.g. spectrum loading based on constant amplitude fatigue data, or extension of the modeling to low or high cycle fatigue regimes, when data exist in the range between 10^3 and 10^6 cycles, or even prediction of the behavior of other material systems based on models derived for a specific material.

Experimental work is the fundamental first step of any investigation aiming to describe the behavior of a composite material, model the failure mechanisms and predict its fatigue behavior under new, ‘unseen’, loading patterns. In the following section, the basic considerations for the design of an experimental program are discussed.

1.2 Experimental characterization of composite materials

1.2.1 Overview

Extensive experimental programs concerning composite materials and structures have been carried out over the last four decades and a significant amount of quasi-static and fatigue data has been gathered. The motivation for an experimental investigation is directly related to the aim of the experimental output. The aims of experimental programs can be classified as follows:

- *Specimen testing for research purposes #1* – usually refers to tests on standardized specimens to investigate material behavior and characterization of the damage development process. Representative examples can be found in [20–22] where the authors attempted to characterize laminate behavior and explain the macroscopically measured strength or stiffness degradation based on observation of failure surfaces, or in [23–25], where fractography was used in order to describe the Mode I interlaminar fracture toughness of multidirectional laminates [23], angle-ply carbon/nylon laminates [24], and unidirectional and angle-ply glass/polyester DCB specimens [25].
- *Specimen testing for research purposes #2* – usually refers to tests on standardized specimens in order to characterize the material and develop theoretical models for the description of its behavior. Unlike the previous category, this kind of testing program focuses mainly on macroscopic observations and data acquisition, e.g. measurement of stiffness degradation, residual strength or the derivation of S–N curves. Representative examples can be found in [7] and [11] where the authors created their own fatigue databases in order to develop multiaxial fatigue

failure criteria, or in [26] where the authors created their database in order to evaluate existing multiaxial fatigue failure theories.

- *Specimen testing for engineering purposes* – including tests on predetermined standardized specimens of different materials and/or different specimen configurations aimed at material selection and optimization of specimen configuration. Often, experimental programs of this class are realized within the framework of an industrial application, such as the wind turbine rotor blade industry for which a significant number of fatigue databases exist, e.g. [13] and [14], or the aerospace industry.
- *Component testing for research purposes* – aimed at the development of analytical models for the modeling and subsequent prediction of the fatigue life of the examined components. Representative works in this class include those on adhesively bonded and bolted joints that are used as structural components in a wide range of applications, e.g. [27–31].
- *Specimen/component/full-scale testing for design verification* – refers to experimental programs performed in order to validate the design of a structure, normally based on quasi-static load cases. The design verification is usually performed by applying representative constant amplitude fatigue loading, e.g. at the serviceability limit state, defined in [32] as the load that produces a maximum deformation equal to the span/600 for an FRP bridge deck, or by using accelerated testing for simulation of the long-term behavior of the examined materials. Full-scale testing is performed in order to validate the design of a prototype (verify that its lifetime is at least as long as expected) and measure the developed damage throughout this lifetime. Thanks to full-scale testing, such factors as size or free edge effects (introduced when shifting from the specimen to the full-scale application) are eliminated and credible results regarding the fatigue life of the final structure can be obtained. The high cost and the time limitations are the disadvantages of this type of test.

Depending on the selected class of experimental program, the researcher must determine the termination criterion. This can be a failure criterion referring to material rupture, e.g. [7, 11], or related to predetermined stiffness degradation, e.g. [33], or even a design objective, e.g. to exceed a certain number of cycles without failure, e.g. [32], or without obvious damage, e.g. any significant crack.

The recorded output is also related to the aim of each experimental program. Simple and relatively cheap measurements are required when the objective is the derivation of the S–N curves of the examined material, e.g. [7–9, 11]. However, more expensive configurations are needed when the output must provide information about strain development during the fatigue life (need

for strain gages or clip gages, e.g. [14]), crack propagation measurements using crack gages, e.g. [29, 34], and even more sophisticated measurements, such as acoustic emission as described in [35] and [36].

1.2.2 Fatigue test parameters

There are a number of steps that have to be followed for the design of a fatigue-testing program. Decisions must be taken concerning several parameters that affect the test results to a greater or lesser extent. Among these the following can be identified:

- *Loading pattern:* Although the majority of existing experimental databases contain data obtained from constant amplitude testing, experimental programs may contain more complicated loading patterns, such as block loading and variable amplitude loading, and are not necessarily limited to uniaxial loading. In fact in the case of anisotropic composite materials, multiaxial stress states can be developed even by the application of uniaxial loading on the specimens; see for example [37]. The necessity for these experimental programs is multiple. Block loading is used to investigate the sequence effect on the fatigue life of composite materials; see for example [38]. Variable amplitude loading is performed in order to investigate the material behavior under representative, realistic loading cases and provide reliable fatigue life prediction results, e.g. [39]. It also allows fatigue life prediction methodologies to be validated via the comparison of theoretical predictions with experimental data and occasionally serves as a mean for design verification. Multiaxial loading (due to its complexity, normally of constant amplitude) is used for the investigation of material behavior under complex stress states and for the development of models for the prediction of fatigue life under such conditions, e.g. [18] and [19].
- *Control mode:* In general a fatigue test can be performed under load or under displacement (or strain) control. The former case results in material failure after a number of applied load cycles. Since the load is kept constant, deformation is increased with load cycles due to the damage accumulated in the material. This type of control model is preferred for the derivation of standard S–N curves, e.g. [7–9, 11], to examine the sequence effects on fatigue life, e.g. [38], and also for the application of spectrum loading patterns on a specimen or structural component [39]. On the other hand, the selection of the displacement control mode leads to smoother damage development. The load is continuously decreased during fatigue loading under displacement control and therefore the examined material does not fail suddenly. In this case, other types of failure criteria must be established and serve as test termination criteria. This kind of control mode is preferred in fatigue fracture testing where

stable crack propagation that can be recorded during fatigue life is desirable, e.g. [40].

- *Stress ratio*: The type of the applied load, whether tensile or compressive or even a combination of both, can be easily determined by the stress ratio, i.e. the ratio of the minimum over the maximum applied cyclic stress ($R = \sigma_{\min}/\sigma_{\max}$). Composite materials behave differently under tension and under compression, since different mechanisms are developed under these loading patterns. When tensile loads are applied, fatigue failure is mainly fiber-dominated, whereas under compression the role of matrix, fiber misalignment, material defects, etc., is more pronounced. Therefore, the successful design of a testing program premises correct selection of the examined loading cases, in keeping with the application for which the material is intended.
- *Testing frequency/strain rate*: A limiting factor in the fatigue testing of composite materials is frequency. In contrast to metals, the fatigue life of composite materials is considerably affected by the testing frequency. Researchers in this subject agree that the dependence of fatigue life on loading frequency is due to the heating of the material at higher frequencies, or creep fatigue at lower frequencies, or the interaction of both [41–43]. Mechanical energy dissipated during each stress–strain hysteresis loop is transformed into heat and subsequently results in a greater localized temperature rise in the material. When the energy is such that it cannot be rejected into the environment, and produces temperatures close to or even higher than the glass transition temperature of the matrix, which is a viscoelastic material, fatigue life is considerably decreased. This is a common phenomenon observed at elevated test frequencies, although usually different for each material system, depending on the matrix material, fiber orientation in the laminate layers, geometry of the specimens, etc. Standards concerning the derivation of S–N curves for composite materials, e.g. ASTM D 3479–76 [44] or DIN 65 586 [45], refer to continuous loading, under the same constant amplitude load, until failure. As far as testing frequency is concerned, no directions are given, and the only prerequisite is that no significant changes in temperature must be noted. Keeping a constant frequency during a fatigue test results in different strain rates for different stress levels. On the other hand, to achieve a constant strain rate for all applied cyclic stress levels, the user must select multiple frequencies, i.e. lower frequencies for low stress levels and higher frequencies for higher stress levels. A procedure for defining the test frequency was established in [46]. It was proposed that the testing frequencies at a certain load level can be determined by using a constant energy approach (energy $\sim$ load $\times \epsilon^2$). However, real engineering structures made (entirely or partly) of polymeric matrix composite materials, like airplanes, helicopter or

wind turbine rotor blades, etc., do not operate continuously; they stop and ‘rest’ without any load being applied for a large percentage of their lifetime. Therefore, it is questionable as to whether fatigue design allowables used in design can be determined under continuous loading conditions or must be derived for conditions as similar as possible to the actual operational conditions [47].

- *Waveform*: The shape of the applied waveform can also affect the fatigue results. The sinusoidal waveform is the most commonly used since it can be easily generated, even in simple testing rigs, and can be assumed as being the more realistic one compared to other types of loading that represent sudden changes, like the triangular, step (square) and saw-tooth waveforms. For the same maximum applied load, waveforms that possess higher mean loads (like the square) and also allow for the application of the maximum load for a longer period (creep fatigue) are more damaging for the material, especially in low cycle fatigue regimes where the creep fatigue effect is more pronounced. A characteristic comparison between sinusoidal, triangular and square waveforms for the GFRP laminate is presented in Fig. 2.4 in [48].
- *Testing temperature*: The majority of testing programs in the literature refer to experimental results obtained under ambient environmental conditions. This is because this type of test is simpler and less expensive and provides basic information about material fatigue behavior. However, in reality, structures are subjected to combined thermomechanical loading patterns – see, e.g., [29, 48] – and therefore knowledge regarding their behavior under similar conditions becomes essential for credible design. In general, higher testing temperatures, especially in the range of the glass transition temperature of the composite, decrease the fatigue (and static) strength of composite materials. This is a very common phenomenon, although usually different for each material system, depending on the matrix material, fiber orientation in the laminate layers, specimen geometry, etc.

1.2.3 Fatigue nomenclature

Conventionally, in fatigue the following abbreviations are used:

CA: constant amplitude loading

H–L: high–low combination in a two-stage block loading pattern

L–H: low–high combination in a two-stage block loading pattern

L–H–L ...: multiple block loading patterns describing the load sequence

VA, irregular, spectrum, or random: refers to the loading under a variable amplitude fatigue spectrum

$\sigma_{\max}$ = maximum applied cyclic stress

$\sigma_{\min}$ = minimum applied cyclic stress

σ_m = mean stress

σ_a = cyclic stress amplitude

$\Delta\sigma$ = cyclic stress range

R -ratio: the ratio of minimum over maximum cyclic stress. This ratio defines the loading patterns that might be of:

T–T: tension–tension loading, when $0 \leq R < 1$

C–C: compression–compression loading, when $1 < R < +\infty$

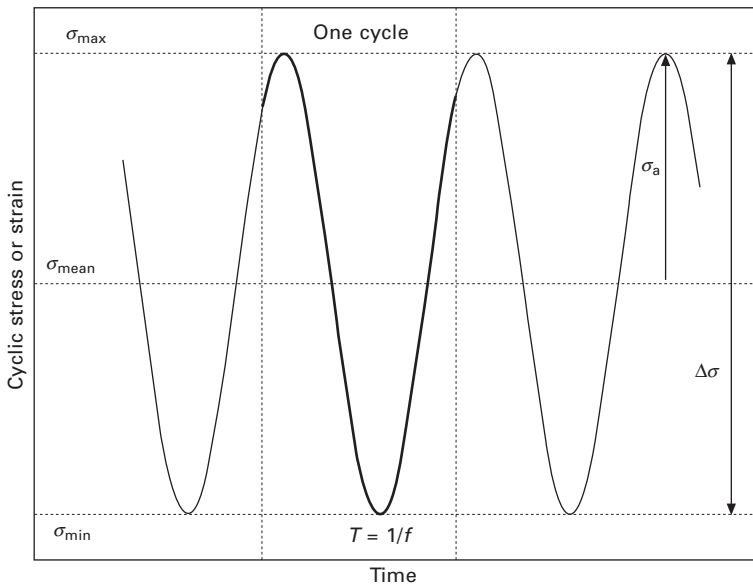
T–C or C–T: combined tension–compression loading when $-\infty < R < 0$

Special case: $R = -1$ when the compressive stress amplitude is the same as the tensile stress amplitude and the mean stress equals zero.

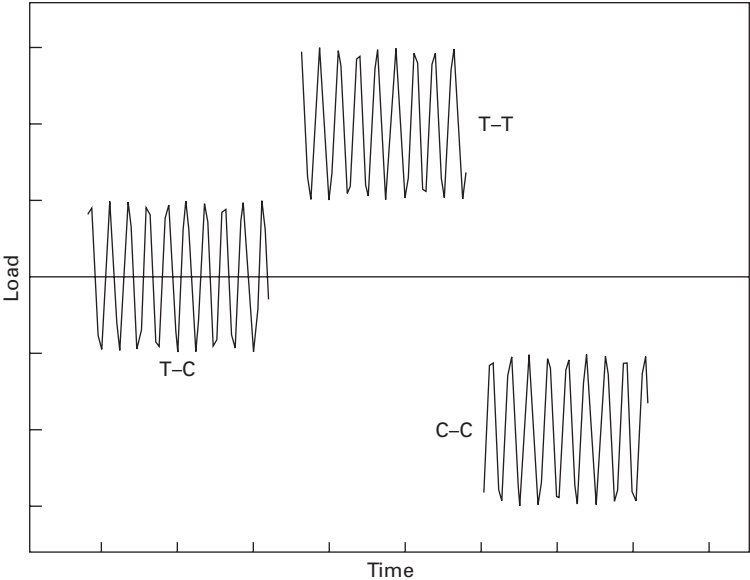
This is known as reversed loading

f : test frequency, measured in Hz, or loading cycles per second.

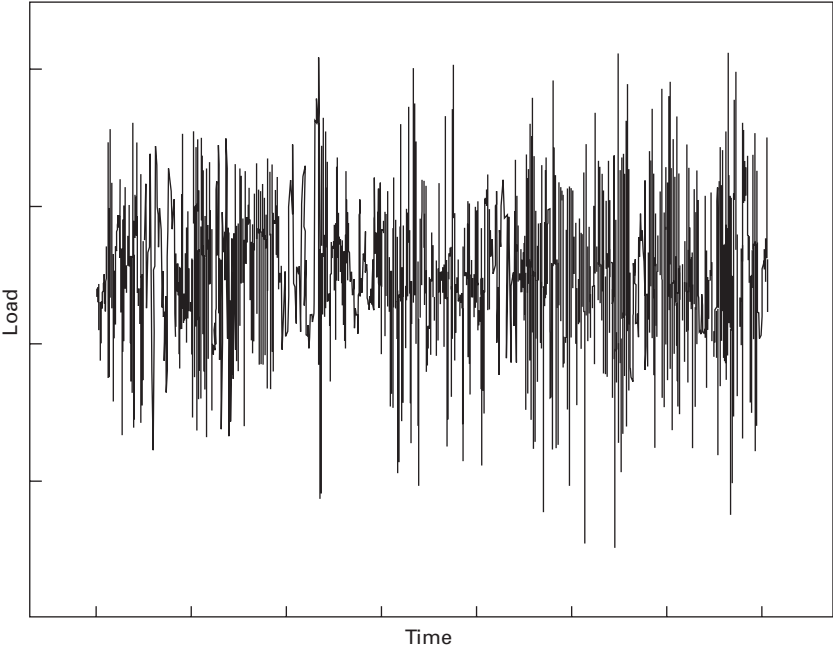
The basic fatigue terminology together with representative constant amplitude and variable amplitude loading patterns are schematically presented in Figs 1.1–1.3.



1.1 Basic fatigue terminology.



1.2 Representative constant amplitude loading patterns: T, tension; C, compression.



1.3 Example of an irregular fatigue time series.

1.3 Fatigue life prediction of composite materials and structures – past and present

1.3.1 Overview

The introduction of composite materials in a wide range of structural components attracted the attention of the scientific community and obliged engineers to reconsider fatigue as a primary loading pattern that dominates failure, even for structures where traditionally fatigue was not considered an issue. There is a long list of reasons for which accurate fatigue life prediction is critical for composite structural components or structures; some of them are listed below.

- Composites are used for critical structural components and nowadays they participate as an equal material candidate, compared to the traditionally used steel, aluminum or concrete, in emerging structures. This development changes the common perception concerning the sensitivity of each structure to fatigue. For example, a concrete bridge is not fatigue-sensitive since the dead loads are significantly higher than the life loads. However, fatigue becomes an issue for a lightweight composite bridge. On the other hand, the use of composites as the main structural material in emerging structures that must bear significant fatigue loads during operation, such as airplanes, means that accurate fatigue life modeling is essential.
- The hitherto followed practice for product development was based on an iterative process whereby a prototype was built and tested against real, or realistic, loading patterns. However, this process is costly and time-consuming. The ability to simulate the fatigue behavior of the material, structural component and/or structure reduces the cost and in addition allows the development of a wider range of products without the need for increasing the number of physical prototypes.
- The durability of composite structures is also an important factor. The danger of evaluating durability on the basis of static strength calculations is that the durability impact of cyclic loads is likely to be disregarded. The introduction of fatigue life prediction methodologies into durability simulation procedures allows the assessment of the durability performance early in the product development process and the establishment of clear recommendations for guiding major design choices.
- Unidirectional composite materials are in general brittle and behave linearly under load. Their failure is sudden, without any prior notice, and the accurate modeling of their behavior and prediction of their fatigue life are therefore of major importance.

1.3.2 Fatigue life prediction theories

Mathematical models were developed in order to describe fatigue damage analytically and finally predict the fatigue lifetime of composite materials. Based on previous experience of fatigue life prediction for metallic materials, similar measurable material characteristics were selected, according to the case studied, to form the fatigue damage metric. Material damage was then measured based on the degradation of that quantity with loading cycles. Different approaches have been adopted, based on different damage metrics for measuring fatigue damage accumulation. The aim of such studies was to establish a process requiring a minimum of experimental data, while reliably predicting the condition of the material.

Theoretical models can be classified into two major categories [49]. The first consists of those theories that are based on macroscopic failure criteria and theoretical formulations to predict life under constant or variable amplitude loading. Theories in this category do not take into account the experimental observation of the damage mechanisms and their development during fatigue loading. The characteristic of the second group of theories is that they are all based on actual damage measurements during fatigue life. Thus, a damage metric exists that is used as an indicator of damage accumulation. According to the damage metric, these theories can be further classified into sub-categories: strength degradation fatigue theories, where the damage metric is the residual strength after a cyclic program; stiffness degradation fatigue theories, where stiffness is conceived as the fatigue damage metric; and finally, actual damage mechanism fatigue theories based on the modeling of intrinsic defects in the matrix of the composite material that can be treated as matrix cracks.

The ideal fatigue theory is described by Sendekyj [49] as one based on a damage metric that accurately models the experimentally observed damage accumulation process, takes into account all pertinent material, test and environmental variables, correlates the data for a large class of materials, permits the accurate prediction of laminate fatigue behavior from lamina fatigue data, is readily extendable to two-stage and spectrum fatigue loading, and takes into account data scatter. These requirements cannot be met simultaneously for many reasons [49] and theoretical models that address only some of them have been introduced. For predicting the fatigue life of structural components made of composites, at least two alternative design concepts could be used: the *damage tolerant* (or *fail-safe*) and the *safe-life* design concepts. In the former it is assumed that a damage metric, such as crack length, delamination area, residual strength or stiffness, can be correlated to fatigue life via a valid criterion. The presence of damage is allowed as long as it is not critical – i.e. it cannot lead to sudden failure. In safe-life design situations, cyclic stress or strain is directly associated

with operational life via the S–N or ϵ –N curves. The structure is allowed to operate since no damage is observed, e.g. before the initiation of any measurable cracks.

Macroscopic failure theories – also designated empirical theories

This is the broadest group of theoretical models for the modeling and prediction of the fatigue behavior of composite materials. Mainly because of the simplicity of the necessary measurements during fatigue loading, this type of modeling was adopted far more rapidly than the others. August Wöhler, as far back as the 1850s, conceived the idea of representing cyclic stress against the numbers of cycles to failure in order to quantify the results of his experimental program. The only input required with regard to experimental data consists of pairs of numbers of cycles up to failure and the corresponding alternating stress or strain parameter. The S–N or ϵ –N curve of the material is then determined under the applied loading condition. However, another half-century went by before the introduction of the first mathematical model to describe this relationship. In 1910 Basquin stated that the lifetime of the material increases as a power law when the external load amplitude decreases. The value of the exponent, which denotes the slope of the S–N curve, is related to the examined material. It is now documented that for composite materials this exponent ranges between 0.04–0.15, being lower (less steep curve) for unidirectional carbon and higher (steeper curve) for multidirectional glass fiber composites. Although laws of this kind are empirical, and do not have any direct physical meaning, they follow the accumulation of the microscopic damage of the examined material which they finally describe [50].

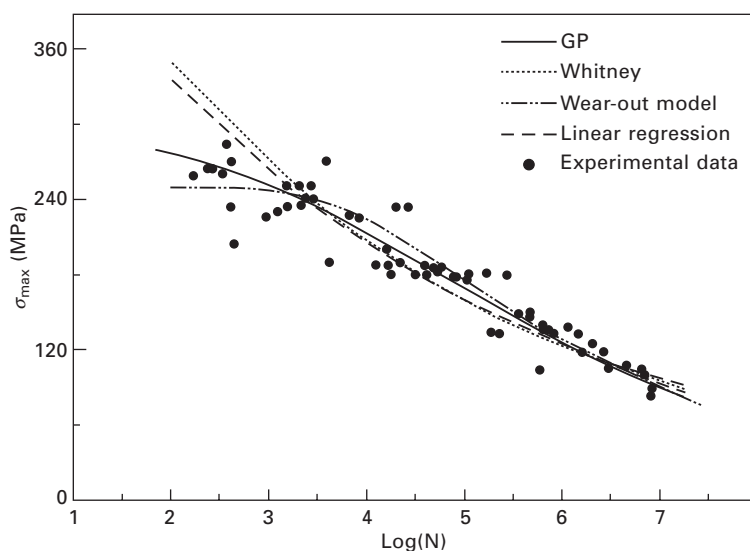
The choice of a particular (and appropriate) fatigue theory for composite materials is based on the material's behavior under the given loading pattern and the experience of the user. For uniaxial loading, the established S–N curves for metals were initially adopted for composites as well. Traditionally, the S–N data are fitted by a semi-logarithmic or logarithmic equation. Other types of S–N curve formulations are usually employed to take into account the statistical nature of fatigue data, e.g. [51, 52]. Recently, artificial intelligence methods and soft computational techniques have been introduced for the fatigue life modeling of composite materials. Artificial neural networks [53], adapted neuro-fuzzy inference systems [54] and genetic programming [55] have proved to be very powerful tools for modeling the nonlinear behavior of composite laminates subjected to cyclic, constant amplitude, loading. They can be used to model the fatigue life of several composite material systems, and compare favorably with other modeling techniques.

Finding the appropriate S–N curve type for the examined material is not simple, since there is no rule governing this selection process. The best S–N

curve type is the one that can best fit the available fatigue data. It is nowadays accepted that power curve-like equations can be better adapted than linear equations for composite material fatigue data [55]. Echtermeyer *et al.* [56] concluded that the fatigue data for glass fiber-reinforced composites can be well described by common strain-life (ϵ - N) fatigue standard curves, as long as some fibers in the material run in the load direction for reversed loading, flexural loading and tensile loading with similar R -ratios. These constraints, and the numerous assumptions imposed by the mathematical models, boosted the development of material-independent methods for the derivation of S-N curves, like the recently introduced computational techniques presented in [53–55]. The resulting S-N curves do not follow any specific mathematical form; they simply follow the trend of the available data, each time giving the best estimate of their behavior, without taking the mechanics of each material system into account [55].

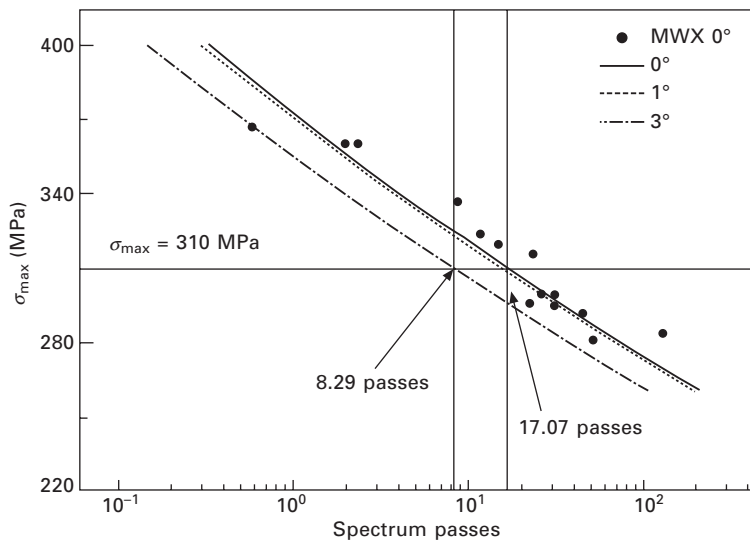
As presented in Fig. 1.4, the application of different methods for derivation of the S-N curve of the examined material leads to different types of curves. For the examined case, the model proposed by Sendeckyj (the wear-out model) [51] and the curve estimated using genetic programming (GP) [55] seem to be more accurate than the linear regression and Whitney [52] models.

The aforementioned fatigue theories are limited to uniaxial loading and do not take into account the effect of other stress components on fatigue life. This assumption seems reasonable for the highly anisotropic composite



1.4 Application of different S-N curve formulations for the description of the constant life fatigue behavior of a multidirectional glass-epoxy laminate with a stacking sequence $[(\pm 45/0)_4/\pm 45]_T$. [55]

materials and has even been adopted by the scientific community in the past. For example, a composite wind turbine rotor blade was treated by state-of-the-art design codes, e.g. [57, 58], as a typical beam-like structure in which fatigue life calculations are limited in that they consider only the action of the normal stress component in the beam axis direction. However, regardless of the effect of various model parameters on the accuracy of the theoretical predictions, a strong effect of the transverse normal and shear stress components on fatigue life has been recorded, e.g. [39]. The fatigue behavior of a multidirectional composite laminate was evaluated under multiaxial cyclic stress states [39]. For on- and off-axis loading, even a few degrees of misalignment yielded substantially different theoretical predictions. This phenomenon proves the significant role of transverse to the fiber normal and shear stress components on the fatigue life of the investigated laminate, since even slight deviations of the loading direction affect the values of the transverse normal stress σ_2 and shear stress σ_6 components in the principal material system, and drastically change the expected number of passes of the applied load spectrum. A misalignment of 3° , for example, from the ideal axial loading direction produces a plane stress state with $\sigma_1 = 99.7\%$ of σ_x , $\sigma_2 = 0.27\%$ of σ_x and $\sigma_6 = 5.23\%$ of σ_x . However, despite the low values of transverse normal and shear stress components, life prediction is apparently affected, as shown for example in Fig. 1.5 for specimens cut at 0° and tested under the applied spectrum. According to the example presented in Fig. 1.5, life is reduced by almost 50% due to 3° of off-axis loading. Similar



1.5 Effect of load misalignment on life prediction of specimens cut at 0° – 3° and tested under an irregular fatigue spectrum.

conclusions were drawn for CA loading [11] where a life reduction of the order of 25–40% was predicted, depending on the value of the stress ratio R , due to the presence of transverse normal and shear stresses.

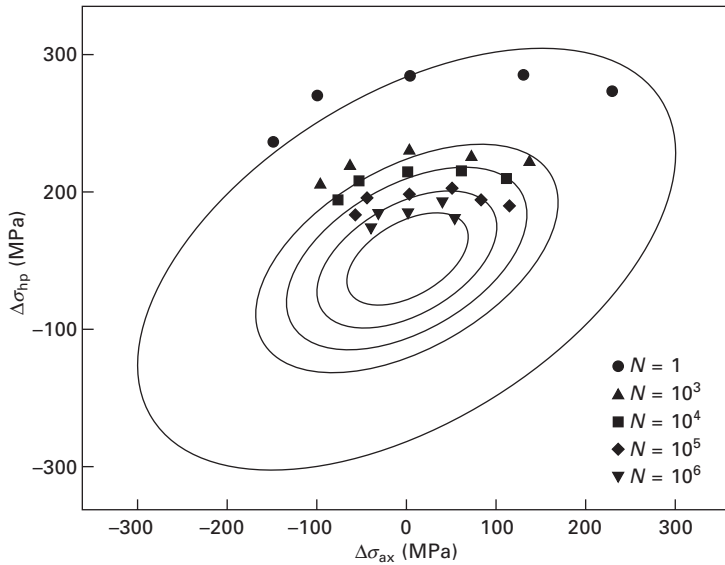
Under complex stress states, multiaxial limit state functions are introduced, usually being generalizations of static failure theories to take into account factors relevant to the fatigue life of the structure, i.e. number of cycles, stress ratio and loading frequency. Due to the fact that the damage-tolerant fatigue design of composite structures is still in its infancy, and that much more research is needed to establish reliable methodologies with general applicability, most of the industrial applications for this type of material lie in ‘safe life’ components.

Numerous attempts at generalizing a multiaxial static failure theory to take fatigue into account can be traced in the literature, e.g. [7, 15, 26, 59–61]. However, most of these studies treat the subject of life prediction only partially under an irregular plane state of stress. They concentrate mostly on the introduction and validation of fatigue strength criteria suitable for CA multiaxial proportional loading without addressing the question of life prediction under irregular load spectra. An exception to the above is presented in a series of publications [62–65] in which a complete life prediction methodology, for even a 3D state of stress, is established. Their approach is based on a ‘ply-to-laminate characterization’ scheme, dealing with damage accumulation under VA loading by adopting appropriate residual strength engineering models. Another paradigm of a complete life prediction methodology under irregular plane stress histories and an experimental database for a glass fiber-reinforced polyester (GFRP) $[0/\pm 45]_5$ laminate was presented in [18] and [19]. Fatigue strength-allowable values in the various material symmetry axes were derived based on the ‘direct characterization’ approach for a number of different CA loading cases. An experimental verification of the entire methodology is performed by means of VA complex stress tests in [39].

The application of multiaxial fatigue life prediction methodologies allows the efficient and reliable prediction of the lifetime of a composite material or structural component under complex stress states. For example, the fatigue failure loci of woven glass polyester cylinders loaded under biaxial hoop, σ_{hp} , and axial, σ_{ax} , stresses can be derived by using the failure tensor polynomial in fatigue (FTPF, [18]); see Fig. 1.6. As shown in this figure, the theoretical predictions are very well corroborated by the available experimental data (retrieved from [26]), proving the validity of the applied multiaxial fatigue strength theory.

Strength and stiffness degradation fatigue theories

Strength and stiffness degradation fatigue theories have been introduced in order to model and predict the fatigue life of composite materials by taking



1.6 FTFP predictions vs. experimental data from woven GRP cylindrical specimens loaded under hoop and axial stresses. [18]

into account the actual damage state, expressed by a representative damage metric of the material status. The damage metric is usually the residual strength or the residual stiffness. Failure occurs when one of these metrics decreases to such an extent that a certain limit is reached. Residual strength theories assume that failure occurs when the residual strength of the material reaches the maximum applied cyclic stress level. On the other hand, stiffness degradation theories are not linked to the macroscopic failure (rupture) of the examined material but rather to the prediction of its behavior in terms of stiffness degradation. Failure can be determined in various ways, e.g. when a predetermined critical stiffness degradation level is reached, or when stiffness degrades to a minimum stiffness designated by the design process in order to meet operational requirements for deformations, or even as a measure of the actual cyclic strains – e.g. failure occurs when the cyclic strain reaches the maximum static strain [66].

Both residual strength and residual stiffness methods have advantages and disadvantages. As mentioned in [67], residual strength prediction has been the subject of numerous investigations during recent decades for several good reasons: modeling the loss of strength of a material after cyclic loading can be a powerful tool in the development of life prediction schemes, especially when dealing with variable amplitude or spectrum loading, since it offers a physically meaningful alternative to empirical damage accumulation rules such as the Palmgren–Miner rule. In addition, knowledge regarding residual

strength in the case of composite structures designed and certified against an envelope of extreme static loads, also undergoing cyclic loading, enables the designer to certify their capacity to bear the design load during their operational life. According to state-of-the-art design practices, large safety factors take into account the loss of strength. Reliable prediction of static strength degradation would help to reduce such safety factors and ultimately achieve full use of the composite material. Residual strength fatigue theories can be successfully used in predicting the fatigue life of composite materials by means of a progressive damage modeling process, e.g. [68].

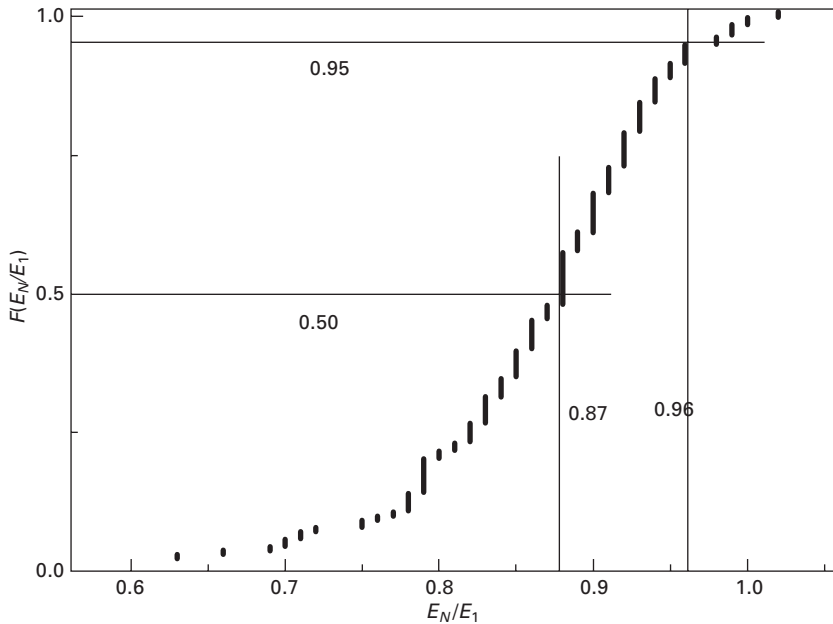
Residual stiffness fatigue theories were developed to overcome two of the major weaknesses of the residual strength fatigue theories: residual strength exhibits minimal decreases with the number of cycles until a stage close to the end of lifetime when it begins to change rapidly. Chou and Croman described this behavior as ‘sudden death’ in their work in 1979 [69]. On the other hand, stiffness exhibits greater changes during fatigue life. Another superior characteristic of the residual stiffness fatigue theories in comparison with their counterpart lies in their non-destructive inspection option of fatigue damage growth. However, while stiffness can be easily and continuously monitored, stiffness changes as a function of fatigue loading cannot be easily predicted [51] since this involves the accurate modeling of the damage accumulation process. In 2002 a stiffness degradation model was introduced by van Paeppegem *et al.* [70–72] in order to demonstrate qualitatively that cumulative damage rules are not needed when applying residual stiffness models in terms of damage growth rate equations to the problem of variable amplitude loading. Recent developments in this field are presented in Chapter 4.

Residual stiffness fatigue theories can also be implemented in order to derive fatigue design curves that do not correspond to failure but to a certain percentage of stiffness degradation. This concept was introduced by Salkind in 1972 [73], when he suggested drawing a family of S–N curves, being contours of a specified percentage of stiffness loss, to present fatigue data. The same concept was adopted later [33] in correlation with the statistical analysis of fatigue data to develop the so-called stiffness controlled curves (Sc–N).

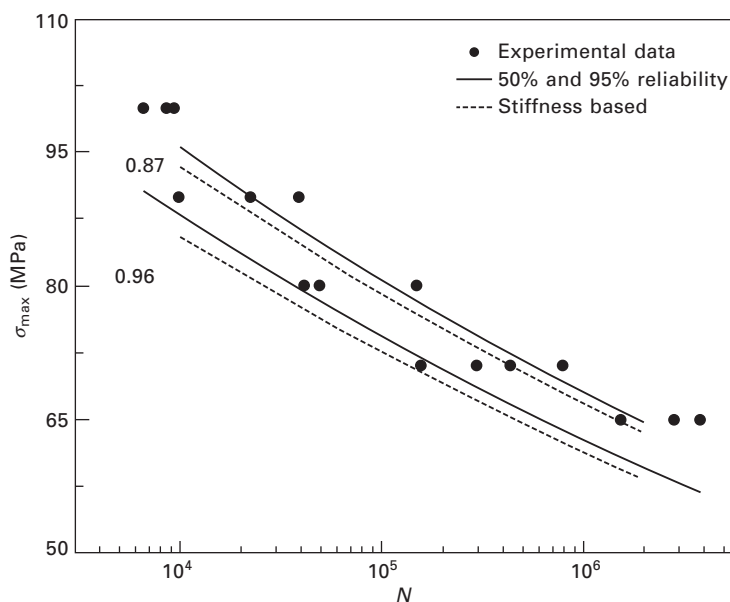
An empirical model was presented and validated in [33] for the determination of design curves that do not correspond to fatigue strength but to a predetermined value of stiffness reduction by using only a portion of the fatigue data, needed for the determination of a reliable S–N curve. Furthermore, S–N curves at any reliability level were determined after statistical analysis of each set of fatigue data. It was shown that these two kinds of design curves were comparable. A heuristic procedure was established to define the so-called Sc–N curves providing information on both the allowable stiffness degradation and the probability of survival. The process

is schematically presented in Figs 1.7 and 1.8. The authors derived both the Sc–N curves and the traditional S–N curves for different reliability levels. Comparing these two kinds of fatigue design curves it was concluded that they could be correlated in the following sense. To any survival probability level, $P_S(N)$, there corresponds a unique stiffness degradation value, E_N/E_1 , which can be determined from the cumulative distribution function, $F(E_N/E_1)$, of the respective data. It is this value of E_N/E_1 for which $F(E_N/E_1) = P_S(N)$, see Fig. 1.7. Observing the two different curves derived as stated above, it was concluded that they are similar for all cases considered in [33], with the Sc–N being generally slightly more conservative. Therefore an Sc–N curve providing information on both survival probability and residual stiffness can be used in the design process.

The derivation procedure of an Sc–N curve is schematically demonstrated in Figs 1.7 and 1.8 for a selected data set. In Fig. 1.8 both design curves, for 50% and 95% survival probability, are plotted together along with experimental failure data. It can indeed be observed that Sc–N and S–N curves from each set lie very close and that the former type of design curve is slightly more conservative. Using as design allowable the Sc–N at $E_N/E_1 = 0.96$, as derived from Fig. 1.7, a reliability level of at least 95% is guaranteed while stiffness reduction will be less than 5%.



1.7 Sampling distribution of stiffness degradation data, $R = 0.1$, specimen cut at 15° off-axis. [33]



1.8 Sc-N vs. S-N curves. $R = 0.1$, specimen cut at 15° off-axis. [33]

A significant drawback of residual strength and stiffness fatigue theories is their inability to take into account complex loading patterns and predict the lifetime of a composite material when multiaxial stress fields develop. A new theory to address this problem is presented in Chapter 11 of this book.

Diverse fatigue considerations

A common characteristic of all the aforementioned theoretical models is that they refer to deterministic material properties and use deterministic loading and material parameters. However, the phenomenon of fatigue is of a stochastic nature, and in general cases it is not possible to estimate the fatigue loading or material properties in a deterministic way. Probabilistic fatigue analysis is based on the hypothesis that every parameter affecting fatigue life is described by a distribution instead of a deterministic value. Therefore, the loading of the structure can follow a normal or a Weibull distribution, the strength of the material can follow a Weibull distribution, and so on. In contrast to the deterministic approach, in which each material property has a value, for example, and each S-N curve is normally derived for the mean or median lifetime, in probabilistic fatigue analysis the material properties have a value for a certain level of reliability and S-N curves for assumed reliability levels are used.

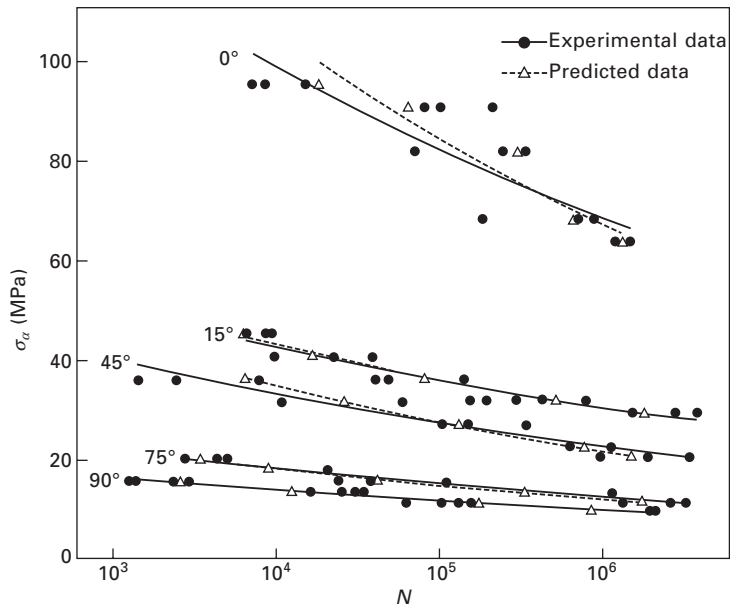
The material properties are the most common sources of uncertainty in a fatigue life calculation. This is because usually only a limited number of experimental data exist but also because the same material may encounter a wide range of environments and loading patterns during its lifetime. The test conditions in the laboratory cannot be made to reflect every possible combination of these two parameters.

The use of probabilistic models for the description of the fatigue behavior of composite materials and quantification of the uncertainties in fatigue life prediction form the subjects of Chapter 7 of this book. In addition to the difficulties encountered during the adoption of an uncertainty modeling technique that is required to address the stochastic nature of both the material properties and external loading, a main reason for the delay in the development of probabilistic fatigue failure theories is that a potentially accurate model would be computationally and experimentally extremely expensive: see Chapter 7.

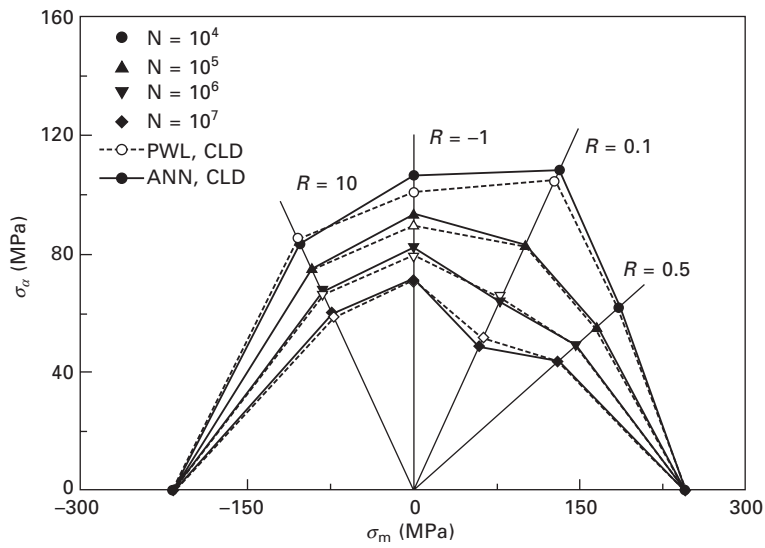
Assistance with this problem can be provided by the novel computational techniques that have emerged as one of the most powerful modeling tools in a number of scientific domains. In artificial intelligence, stochastic programs work by using probabilistic methods to solve or partially solve problems. In engineering, artificial neural networks and genetic programming have been used for optimization of design methods and manufacturing processes, e.g. [74]. The modeling of the fatigue life of composite materials and structures is a topic that has been addressed by this type of analytical tools only during recent years. Until recently, artificial neural networks constituted the only method used for the fatigue life modeling of composite materials and structures, e.g. [75] (see Fig. 1.9), used also in order to model the effect of mean stress on fatigue life and derive constant life diagrams (CLDs) for short fiber composite materials, e.g. [53, 76] (see Fig. 1.10). New tools like genetic programming and adaptive neuro-fuzzy inference systems have been applied lately [54, 55] with the same goal and were used in addition in order to predict the fatigue behavior of structural joints under thermomechanical loading patterns [74].

1.3.3 Fatigue life prediction under complex irregular loading

All the aforementioned fatigue theories were established mainly on the basis of relatively simple constant amplitude loading experiments. In reality, a procedure for the estimation of the fatigue life of a composite material or composite structure that operates in the ‘open air’ must comprise a number of steps, since the real loading is never (or almost never) of constant amplitude. Moreover, the real loading is not known in a deterministic way. Wind turbine rotor blades, for example, are *safe-life* designed composite structures.



1.9 Experimental data vs. ANN modeling. Tension–Tension fatigue, $R = 0.1$. Coupons cut at 0° , 15° , 45° , 75° and 90° ; 30% used for the training set, 70% for the test set. [75]



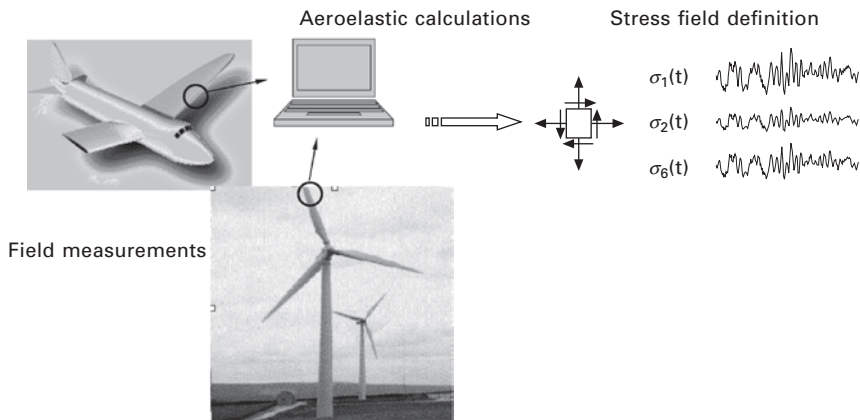
1.10 Piecewise linear constant life diagrams for a GFRP multidirectional laminate. [53]

The nominal lifetime for cost effectiveness of a rotor blade is 20–30 years during which the blade experiences in the order of 10^8 – 10^9 loading cycles of variable amplitude and mean value. The load spectrum of a rotor blade is composed of different stochastic and deterministic cases defined by strong loading fluctuations due to varying wind speed, atmospheric turbulence and topography effects. Operational peculiarities in a wind turbine such as stall, grid, control failure, etc., also give rise to additional strong excitations of the rotor blade.

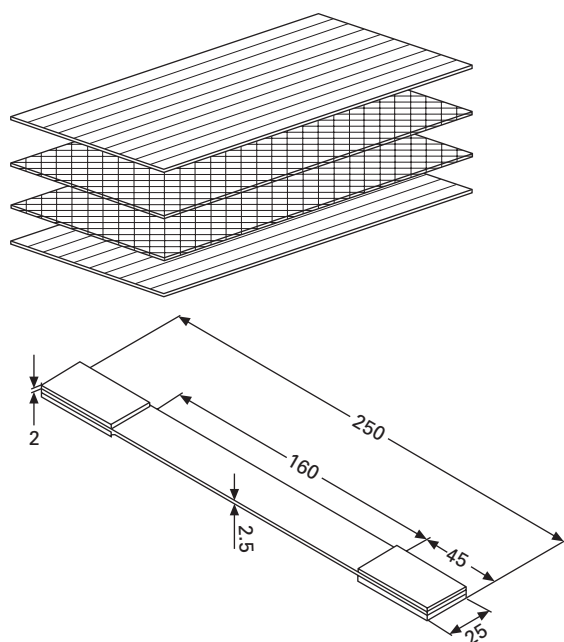
Although reliable computational tools to deal with such problems have been developed, e.g. [77], fatigue design calculations are even more difficult and of questionable validity for FRP composites, due mainly to the lack of a reliable nonlinear damage accumulation rule and limited knowledge concerning the fatigue properties of specific FRP laminates, especially at such high cycle loading levels. It is obvious therefore that the application of any fatigue life prediction methodology is founded on assumptions as from its first stage, the load case definition. In order to apply any fatigue life prediction methodology, the following steps must be implemented.

The derivation of the most representative load time series is the first step, normally based on field measurements and aeroelastic calculations: see Fig. 1.11. The resulting stress time series for each stress component can be proportional, or independent of each other (having different frequencies), depending on the excitation that causes the loads.

The derivation of the material fatigue properties is the second step, normally based on extensive experimental programs under quasi-static, constant, block and variable amplitude loading patterns. This stage includes material selection and preparation of the specimen (see Fig. 1.12). In the case of constant amplitude fatigue testing, the expected output is the number of



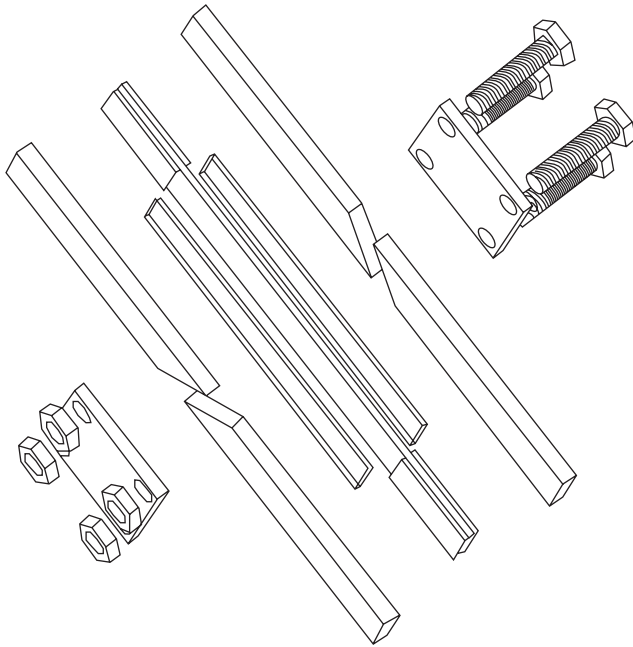
1.11 Definition of the load case.



1.12 Example of material architecture and specimen dimensions (mm).

cycles under each selected loading level and condition, e.g. application of tensile or compressive loads or even a combination of both. When compressive loads are present the risk of buckling of the usually thin specimens is high. Therefore, antibuckling devices like the one shown in Fig. 1.13 are used to protect the specimen from premature failure.

The adoption of methods for the interpolation (modeling) and extrapolation (prediction) of the static and fatigue data follows. During this step, deterministic or stochastic theoretical models can be employed. The use of the selected models (S–N curves, constant life diagrams, residual strength models, residual stiffness models, etc.) permits interpretation of the fatigue data and estimation of the fatigue life of the materials, theoretically, under any applied loading pattern. The method of estimating S–N curves and constructing CLDs based on constant amplitude fatigue data is schematically presented in Fig. 1.14. Three different tests are presented in Fig. 1.14(a), each corresponding to different stress levels and resulting in different number of cycles to failure. As expected, the lower the stress level, the longer the fatigue life of the examined material. Interpolation between the collected fatigue data results in the S–N curve of the material under the selected fatigue conditions, *R*-ratio, frequency, environment, etc. Data from several S–N curves obtained under different loading patterns (tensile, compressive and combinations of both) can be plotted together on the σ_m – σ_a plane (Fig. 1.14(b)) and form

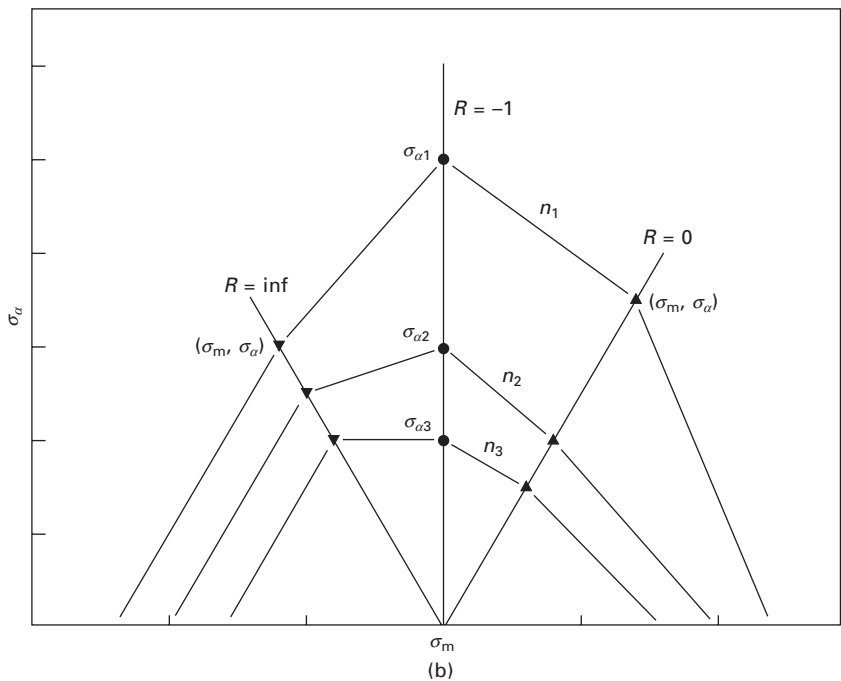
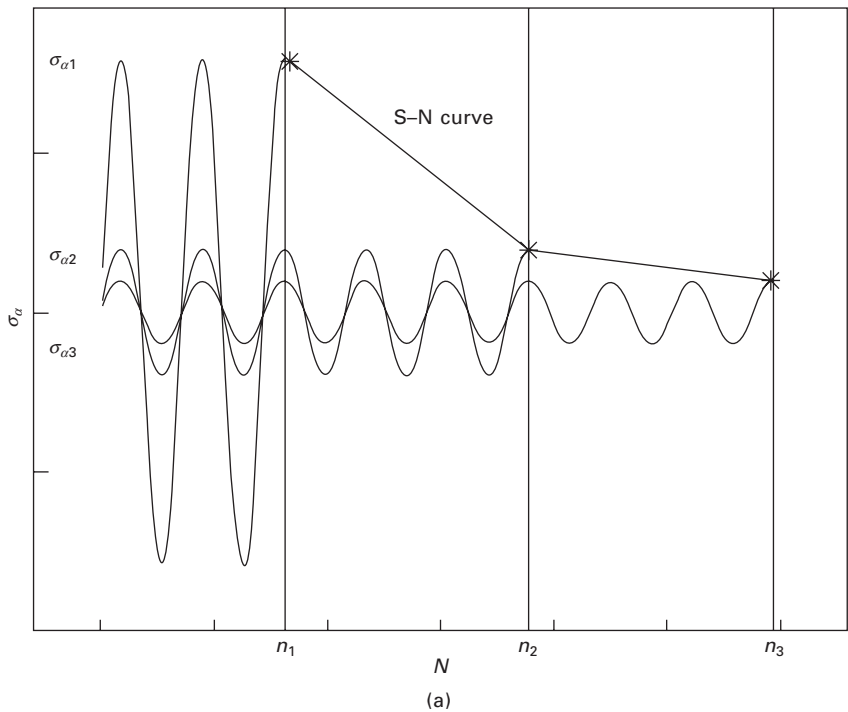


1.13 Antibuckling device according to DIN 65586. [78]

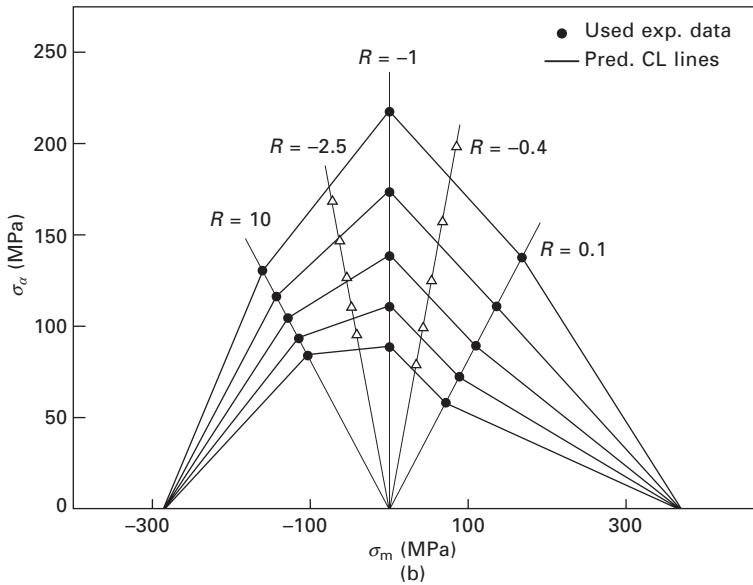
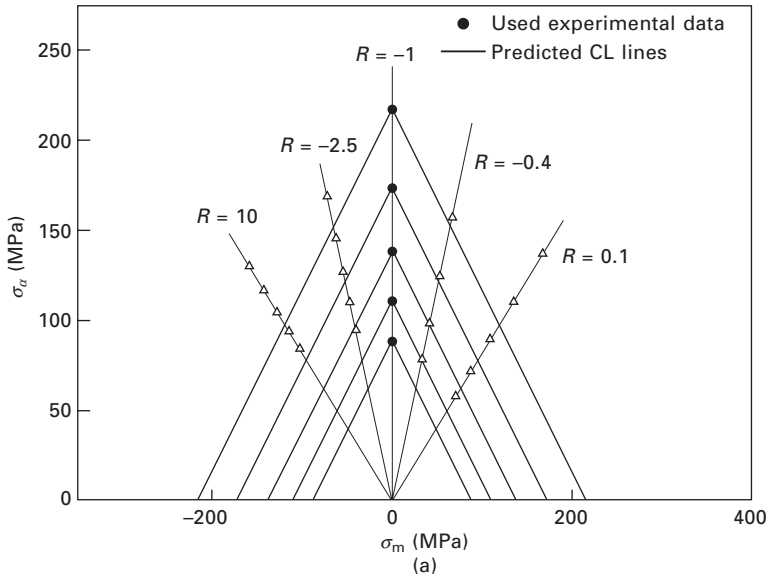
the so-called constant life diagram (CLD) in order to take into account the effect of mean stress on the fatigue life of the material. This diagram helps in the derivation of S–N curves under different R -ratios than those derived experimentally. Constant life (CL) lines connecting data from different S–N curves are then estimated. The CL lines can be linear or nonlinear depending on the selected CLD formulation. The most recent and commonly used CLDs are presented in [79]. Recently, a new formulation for predicting an asymmetric, piecewise nonlinear constant life diagram was introduced [80]. The development of the new model is based on the representation of the fatigue data on the stress ratio (R)–stress amplitude (σ_a) plane instead of the conventional representation on the mean stress (σ_m)–stress amplitude (σ_a) plane. Comparisons of data from different material systems proved that the new model offers a better performance than most conventional CLD models found in the literature [80].

The application of different formulations for the derivation of the constant life diagrams of multidirectional glass–polyester composite laminate [14] is presented in Fig. 1.15 for demonstration purposes. Details concerning the applied methods can be found in [79] and [80].

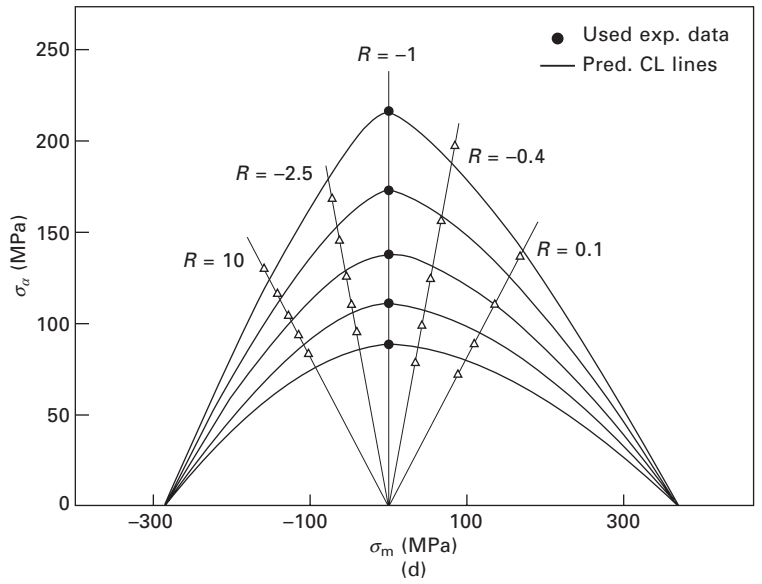
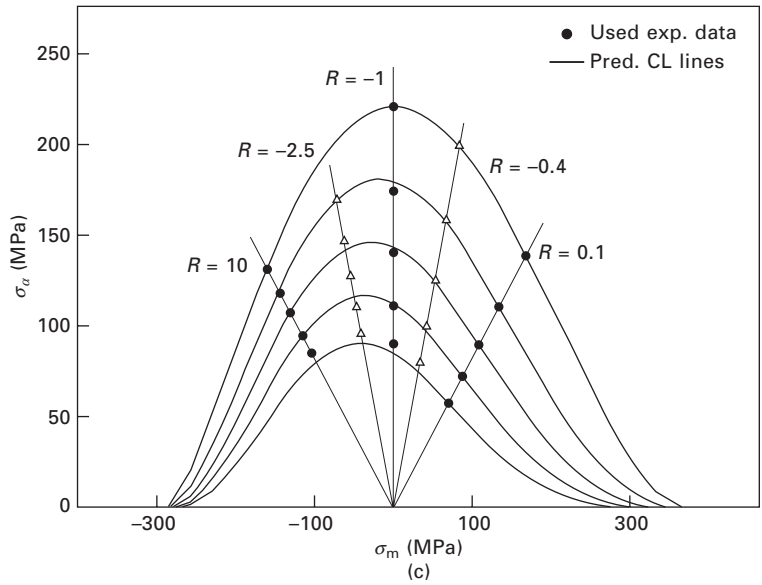
The selection of a cycle counting algorithm to summarize the irregular load vs. time histories and provide the number of occurrences of cycles of various sizes allows better management of fatigue data and theoretical



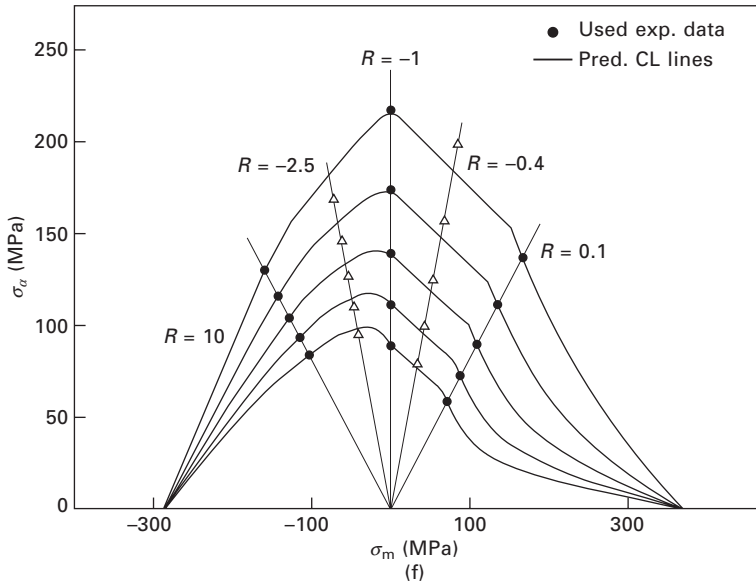
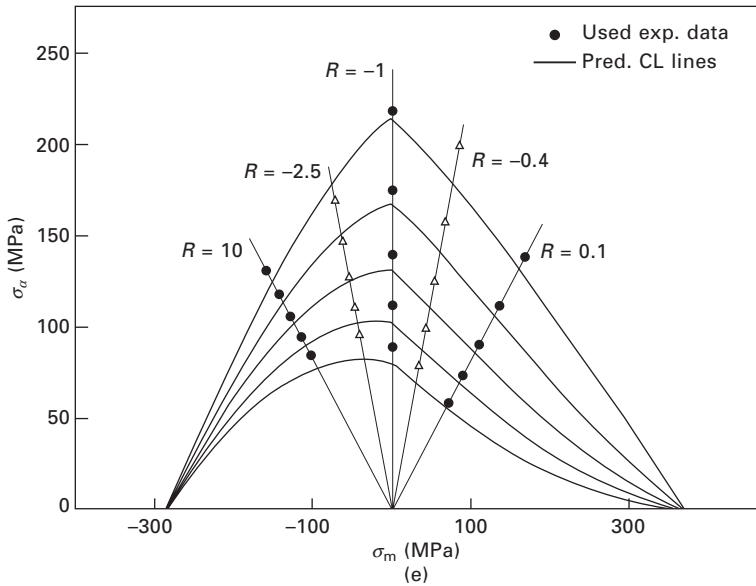
1.14 (a) Derivation of S-N curves; (b) construction of CLD.



1.15 Results of different CLD formulations for $N = 10^3$ – 10^7 : (a) linear; (b) piecewise linear; (c) Harris; (d) Kawai; (e) Boerstra; (f) piecewise nonlinear. [79, 80]



1.15 Continued



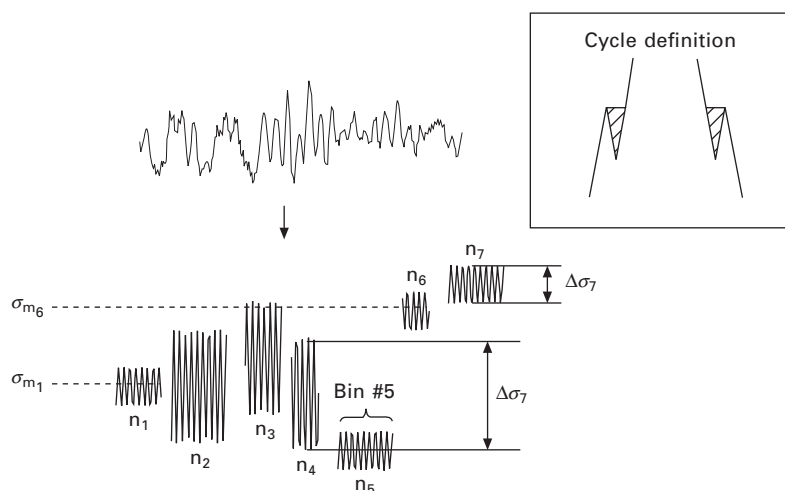
1.15 Continued

models. It is impossible to perform tests to cover all the potential loading cases that a structure may have to face during its operational lifetime. The conversion of the variable amplitude fatigue load cases to constant amplitude loads compensates for this deficiency. Simple methods exist, known as

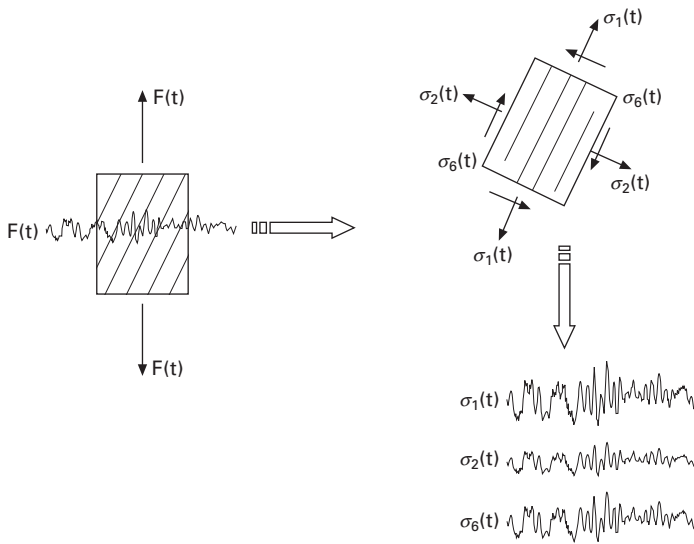
one-parameter techniques – e.g. level crossing counting or peak counting methods – but they are not applicable in the fatigue analysis of composite materials since they do not take into account the significant mean stress effect on lifetime. Rainflow counting, range-pair, and range-mean counting methods seem more appropriate for the analysis of composite material fatigue data, giving similar cycle counting results for most practical applications: see Chapter 9 of this book. However, they present a number of deficiencies: rainflow counting cannot be used for cycle-by-cycle analysis and therefore it is difficult to apply this method together with a residual strength fatigue theory. Moreover, rainflow counting algorithms categorize counted cycles, according to their range and mean values, without considering the location of the counted cycle in the time series, and put cycles of the same category in blocks (usually called bins). This process masks any possible load sequence effect and ‘imposes’ the use of the linear Miner damage summation rule. On the other hand, range-pair and range-mean counting mask the presence of large and damaging cycles. Based on the previous comments, it may be concluded that, according to the application and material, the appropriate cycle counting technique has to be very carefully selected. A comprehensive discussion regarding counting techniques can be found in [81]. A schematic representation of the rainflow counting method is presented in Fig. 1.16.

When the stress field is uniaxial, the situation is straightforward. However, in most real cases, a multiaxial stress field develops in the material’s principal coordinate system, even under uniaxial loading patterns, due to the anisotropy of the composite materials themselves (see Fig. 1.17).

Consider as an example the case of a 20-m wind turbine rotor blade made



1.16 Schematic representation of rainflow counting method.



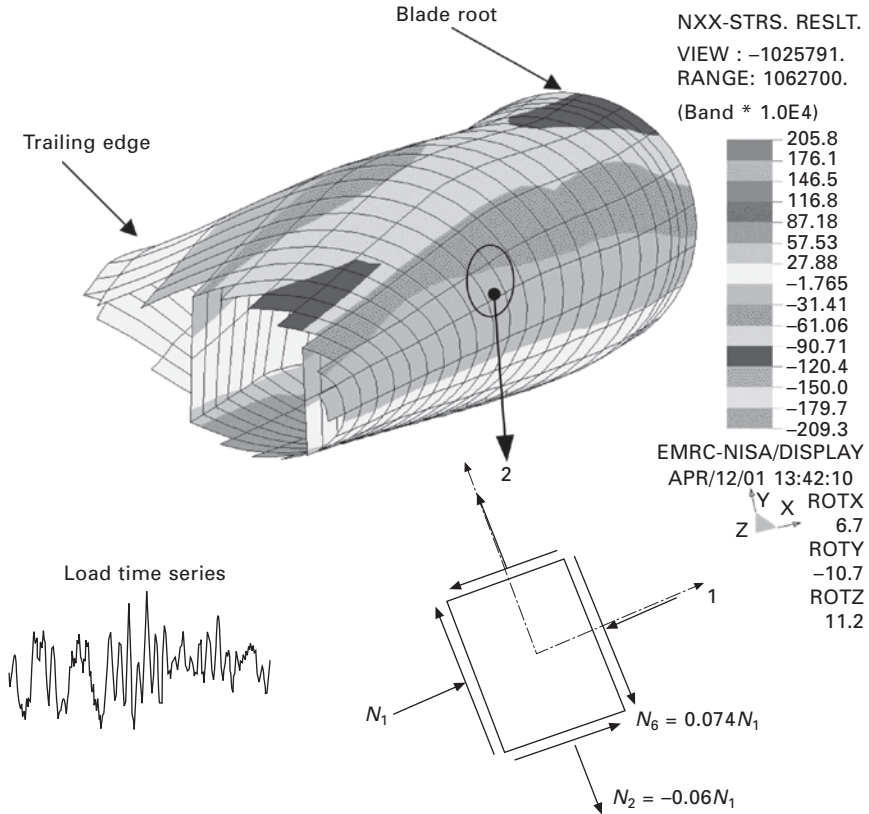
1.17 Complex stress state developed in principal coordinate system of an off-axis uniaxially loaded composite material.

of GRP. Under a realistic load case (power production at rated wind speed), the plane stress field developed in leading-edge inboard regions is an example of that presented in Fig. 1.18. The part of the finite element method (FEM) model shown corresponds to the cylindrical root and the transition region bridging to the aerodynamic section of the blade. Also shown is part of the inner structure, while grayscale shades correspond to different values of shear flow. Magnified is a typical element and the corresponding in-plane stress resultants, N_i ($i = 1, 2, 6$). It is indeed observed that the transverse normal and shear components are only small fractions of the axial stress resultant, albeit not negligible in fatigue life calculations as shown above (see Fig. 1.5).

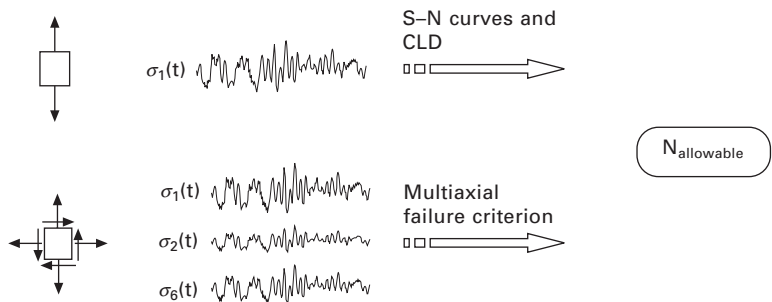
When a plane stress state, rather than only a stress component, is present, the calculation of the allowable number of cycles for each applied load case is based on the use of multiaxial fatigue failure criteria. This process is schematically presented in Fig. 1.19.

The final step of every fatigue life prediction methodology involves estimation of the fatigue life. This step can be based on damage rules used to quantify the damage accumulated during loading, or on residual strength or stiffness criteria that define failure based on the change of the selected damage metric.

To date there is no widely accepted theoretical model that can guarantee accurate lifetime estimation for any examined composite material system, composite component or composite structure under any variable amplitude



1.18 Detail from GRP rotor blade root. Typical state of in-plane stress resultants.



1.19 Derivation of allowable number of cycles for uniaxial and multiaxial stress states.

loading. The estimated lifetime can be accurate, conservative or non-conservative (usually called optimistic) according to the examined material, load complexity, available experimental data, selected solvers for each of the steps of the methodology, etc.

Unique solutions do not exist; there are more than five ‘widely accepted’ methods for deriving the S–N curves, all of them proved to be good methods for the modeling of a series of composite materials [55, 81]. There are more than three ‘widely accepted’ methods for derivation of the constant life diagrams [80]. There are a significant number of cycle counting methods [81] and fatigue failure criteria for composites under multiaxial stress states [7, 15, 17–19, 59–65]. Finally, there are a number of strength, e.g. [35, 67–69], and stiffness, e.g. [33, 66, 70–73], degradation theories and linear or nonlinear damage accumulation rules for the summation of partial damage and estimation of the fatigue life of the examined material. Nevertheless, no commonly acceptable universal fatigue life prediction methodology has yet been adopted.

1.4 Conclusions and future trends

The future of the topic looks prosperous, although very challenging. Several problems have been solved, whilst others still await a commonly accepted solution. Within the next few years the scientific community is expected to come up with a credible methodology for the life prediction of engineering FRP structures which will allow rapid and reliable prototyping without the need for the fabrication and testing of numerous physical prototypes. Therefore the commercialization of a wide range of reliable products will be feasible within shorter time periods and at reduced costs. This process is well assisted by developments in computer science, where computational power has been increased dramatically during the last decade. Although this increase is now slowed down, it is today feasible to use even personal computers to ‘run’ progressive damage models incorporated in finite element analysis software and estimate the fatigue life of an entire structure within hours.

Fatigue life prediction methodologies have been presented in the past in order to address the problem with regard to composite materials; however, they are all hampered by the same deficiency. At least one of their steps was inaccurate, thereby rendering the entire method inaccurate. For example, a simplified approach was presented by Nyman in 1996 [82] based on the reasonable assumption that the S–N curves for the laminae as part of the laminate can be different from the S–N curves for the layers themselves, due to interlaminar stresses. The author tried to establish a method for derivation of the fatigue curves by using a Tsai–Hill criterion. However, a semi-log representation of the constant amplitude fatigue data was used in the sequel, together with a Goodman diagram for the mean stress effect, but both methods subsequently proved to be inaccurate, e.g. [80, 81]. Other proposed methodologies, e.g. [39], are hindered by the use of the linear Miner’s damage rule for damage accumulation. It is therefore obvious that

certain reliable solutions have to be adopted for specific fatigue life prediction tasks, some of which are listed below.

- The debate on the representation of the constant amplitude fatigue data that began a century ago, with Basquin's equation, is still alive and well today. Yet in 2010 there is no universal theoretical model able to accurately describe the constant amplitude fatigue behavior of any composite material (in terms of S–N curve) under any thermomechanical loading condition. It is accepted that a log-log representation is sufficiently accurate for most commonly used composite laminates. However, other methods have been proposed to take into account the low cycle fatigue range as well, e.g. [51], and to consider the statistics of the data sample [51, 52, 83]. The use of quasi-static strength data for the derivation of fatigue curves (as fatigue data for 1 or $\frac{1}{4}$ cycle) is also arguable. No complete study on this subject exists. Previous publications, e.g. [84], showed that quasi-static data should not be a part of the S–N curve, especially when they have been acquired under strain rates much lower than those used in fatigue loading. The use of quasi-static data in the regression leads to incorrect slopes of the S–N curves as presented in [84]. On the other hand, excluding quasi-static data, although it improved the description of the fatigue data, introduced errors in the lifetime predictions when the low cycle regime is important, as for example for loading spectra with a few high-load cycles.
- As reported by Kawai in Chapter 6 of this book, the fatigue behavior of composites becomes more complicated when the influence of temperature is added. To the author's knowledge, there is no method in the literature that can address thermomechanical loads and predict the fatigue behavior for general thermomechanical loading patterns, although a lot of research efforts have been devoted to the characterization of the fatigue behavior of adhesively bonded composite joints and composite laminates under different temperature and humidity environments. A small number of modeling approaches have been published, e.g. [85, 86]. However, in order to accommodate a significant number of parameters that affect the fatigue life of FRP joints, or laminates, these phenomenological models adopt a great many assumptions. Therefore, their applicability could not be validated on the data of different material systems. Novel computational techniques seem able to address this problem, as presented in [74].
- The reduction of the irregular loading spectra to constant amplitude loading blocks is a critical step in a life prediction methodology. To date, among the proposed methods, the rainflow counting together with the range-mean and the range-pair counting methods have been proved the most appropriate for composite materials. However, as presented in Chapter 9 of this volume, they present both advantages and disadvantages.

Rainflow counting cannot be used for cycle-by-cycle analysis and it is therefore difficult to apply this method together with a residual strength fatigue theory. On the other hand, range-pair and range-mean counting mask the presence of large and damaging cycles. A new method presented in [81] is actually a mixture of both rainflow and range-mean counting. This method, known as ‘rainflow-equivalent-range-mean counting’, has been accurately used together with a strength degradation model in Chapter 9 to produce accurate fatigue life prediction results for a multidirectional composite laminate under variable amplitude loading. Further verification of the method on other types of composite materials and other loading spectra is needed.

- The mean stress effect on the fatigue life of composite materials and structures is very pronounced. As presented in a number of publications [87–89], this effect can be accurately modeled by means of the so-called constant life diagrams. The most recent CLD formulations were presented in 2007 by Kawai and Koizumi [88] and in 2010 by Vassilopoulos *et al.* [80]. Although from a theoretical point of view the classic representation of the CLD is rational, it presents a deficiency when seen from the engineering point of view. This deficiency is related to the region close to the horizontal axis, which represents loading under very low stress amplitude and high mean values with a culmination for zero stress amplitude ($R = 1$). The classic CLD formulations require that the constant life lines converge to the ultimate tensile stress (UTS) and the ultimate compressive stress (UCS), regardless of the number of loading cycles. However, this is an arbitrary simplification originating from the lack of information about the fatigue behavior of the material when no amplitude is applied. In fact, this type of loading cannot be considered fatigue loading, but rather creep of the material (constant static load over a short or long period). Although modifications have been introduced to take into account the time-dependent material strength, their integration into CLD formulations requires the adoption of additional assumptions, see e.g. [9, 87].
- Although the first S–N theoretical formulation was introduced by Basquin in 1910, it was Palmgren in 1924 and 1936 [83] who realized that ‘... the variation of fatigue test data is such that there is no pronounced accumulation of values around a mean value, and the minimum life is clearly not governed by laws to more than a trifling extent. Consequently, neither the average nor the minimum life provides a satisfactory basis for a definition.’ More than 80 years later the problem of the statistical analysis of composite material fatigue data still remains unsolved. New methods are proposed to address it, e.g. [90], where the authors proposed a stochastic damage accumulation model for symmetric composite laminates subjected to fatigue loading. The model is a hybrid strength/

stiffness degradation model which utilizes basic probability–stress–life information (P–S–N curves) in conjunction with the evolutionary distribution of stresses on the critical load-bearing plies of a laminate to determine the evolutionary probability of failure of a laminate. In Chapter 7 of this book, a general methodology for stochastic fatigue life prediction under variable amplitude loading is proposed, which combines a nonlinear fatigue damage accumulation rule with a stochastic S–N curve representation technique.

- Engineering structures like bridges or wind turbine rotor blades can experience between 10^8 and 10^9 loading cycles during their operational lifetime. On the other hand, the usual range of recorded fatigue data is between 10^3 and 10^6 – 10^7 loading cycles. It is obvious that a model to extrapolate material behavior is needed in order to estimate the fatigue life of materials in real loading environments. However, existing fatigue models usually fail to accurately model this effect since they have no information concerning this cycle regime. Accelerated testing is used in laboratories to compare the predicting ability of existing fatigue models to realistic loading, but this type of testing masks the effect of the existence of low cycles in real operational loads on fatigue life. Recently a number of researchers tried to perform high cycle fatigue tests [13, 91, 92]. The group of Mandell [91] performed tests on very small diameter impregnated strands with only sufficient fibers to be representative of the behavior of larger specimens. The load was applied by using low-frequency audio speakers (woofers) as actuators that can handle frequencies in the range of 300 Hz. Hosoi *et al.* [92] tested standardized quasi-isotropic carbon fiber-reinforced plastic laminates with a stacking sequence of $[45/0/-45/90]_S$ and reaching a frequency of 100 Hz for more than 10^8 loading cycles. According to the authors, the frequency does not affect fatigue behavior as long as the temperature remains well below the glass transition temperature of the examined material. These two publications prove that it is possible to perform high frequency tests and validate existing models (or develop new ones) for interpretation of the fatigue data at the high cycle fatigue regime as well.
- It is also a known fact that engineering structures used in open-air applications undergo more complicated loading spectra than the constant or block loading conditions usually applied in laboratories and used thereafter for the description of the fatigue behavior of the examined material. The behavior of composite materials is also known to be affected by load sequence [38], overloads [93, 94] and changes in loading. A considerable amount of time has been spent on addressing this problem and quantifying the effects of spectrum loading on the fatigue behavior of several material systems, e.g. [13, 14, 39], and structural components

[93, 94]. Other researchers worked on the development of methods for accelerated testing in order to simulate lengthy loading spectra, e.g. [95].

- Innovative measuring techniques can also assist in the health monitoring of a structure during its operational life in order to achieve fail-safe, cost-effective designs. Acoustic emission (AE) has been used in the past in combination with artificial neural networks to characterize damage in CFRP composite laminates, e.g. [96], or for the assessment of the normal and shear strength degradation in FRP composite materials during constant and variable amplitude fatigue loading, e.g. [35, 36]. Optical Fiber Bragg Grating (FBG) sensors have been used for the characterization of the residual stresses in single fiber composites [97] and the monitoring of hygrothermal aging effects in epoxy resins [98].
- As mentioned, a drawback of stiffness and strength degradation theories is their inability to take into account multiaxial property degradation. They focus on the properties of the material in the principal loading direction. However, efforts have recently been devoted to the development of models that also take other material elastic constants into account, such as the degradation of the Poisson's ratio, the transverse stiffness and the in-plane shear modulus (see Chapter 4), or to the use of strength degradation theories together with phenomenological modeling for estimation of the material properties during fatigue loading (see Chapter 11).
- The fatigue life prediction of composite materials can be characterized as an articulated procedure, since a number of sub-problems must be solved sequentially to produce the final result. However, dealing with each of the steps of such a methodology in depth is very demanding and time-consuming. On the other hand, the validity of the proposed solutions for each step could be effectively evaluated as part of the entire methodology. A limited number of commercial software solutions addressing the fatigue of composites are available, e.g. <http://www.ffatigue.net/index.php?task=home&l=en>, <http://www.fatigewizard.com/>, <http://www.prosig.com/dats/datsFatigue.html>, http://www.ncode.com/Product.aspx?Product_ID=90, but they mainly provide tools for work carried out in the enclosed environment of a large private company. In many cases, however, the needs of universities, research centers, and smaller firms are generally different from those that can be met by these software products. This implies customization, which is very difficult if not impossible from a commercial point of view. For the current state of engineering software packages, modifying or extending the code requires that users have intimate knowledge of data structures, their procedures and the extent to which they affect specific portions of the code. The possibility of reusing codes from other sources is also limited. This is

because data structures vary widely between programs. On the other hand, in-house software development, like [99–101], forces researchers to devote part of their research time to developing and testing software instead of concentrating on their main scientific tasks. The Internet has provided a robust real-time mechanism for communication and interaction. In the near future, the development and deployment of the next-generation Internet (<http://www.ipv6.org/>, previously on the website <http://www.ngi.gov/>), will establish the infrastructure for a worldwide communication network even faster and broader than most of today's local networks. Along the same lines as the next generation Internet, one of the major motivations to develop software for the fatigue life prediction of engineering structures is to provide an inexpensive, web-accessible tool for the interpretation of composite material fatigue data and the design of engineering structures that could be further modified by its own users in order to serve specific needs.

The problem is known and solutions do exist. The way to achieve them is to be 'open-minded' and work with a collaborative spirit. Knowledge must be spread freely, without limits, and efforts should be appropriately acknowledged in accordance with basic ethical rules. Experience has proved that Professor Tsai's dictum to 'Think composite' is, after 30 years, still appropriate. In addition to this, researchers should start thinking '*interdisciplinary*'. Novel interdisciplinary concepts for the development of new methodologies for the fatigue life prediction of composite materials and structures have to be introduced and adopted. Current computational power permits the development of methodologies that 10 years ago would have been inconceivable. Recent developments in artificial intelligence and evolutionary computational techniques allow the rapid establishment of models to solve or partially solve multi-parametric problems, such as the fatigue life prediction of composite materials. The scientific community already has mature knowledge regarding different topics in the field but the composition of a universal fatigue life prediction methodology, based on hybrid models, is yet to be accomplished.

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Phenomenological fatigue analysis and life modelling

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Abstract: Fatigue life prediction for realistic loading (variable amplitude loading) is typically based on constant amplitude fatigue data. This chapter details the steps between acquiring experimental data and calculating a life estimate, and explains how interpretation of data and choice of fatigue models can influence life prediction.

Key words: S–N curves, constant life diagram, fatigue experiments.

2.1 Introduction

For life prediction and fatigue analysis, a thorough knowledge is required on the fatigue behaviour of a material. This chapter addresses the description of fatigue behaviour of a material in terms of S–N curves and constant life diagrams (CLDs).

The fatigue analysis referred to in this chapter is based on experimental characterisation of structures and materials through repetitive loading until a macroscopically observable failure mode occurs: usually fracture resulting in the inability to carry the applied load. Hence the term ‘phenomenological’ in this chapter’s title.

A strong relation exists between the models described in this chapter and those in the next. First of all, S–N curves and CLDs are required for residual strength calculations and stiffness degradation models. These models add information on strength and stiffness that is not present in most S–N diagrams, but S–N information is required for their successful application, as stiffness and strength degradation is typically expressed as a function of fatigue life.

On a smaller scale, fracture mechanics and micromechanical modelling attempt to describe how stiffness and strength are affected by growing cracks, fibre–matrix debonds, delaminations, kinking and buckling of fibres, and the interaction of failure modes present in composites, until final structural failure occurs. Effectively, these models predict S–N curves with as input basic material properties. Genetic algorithms also predict S–N curves, albeit via another route.

This chapter focuses on constant amplitude fatigue life description in terms of S–N curves. Many parameters affect the measured fatigue characteristics of a composite, and tests are designed carefully to measure the fatigue behaviour that represents as closely as possible the fatigue behaviour of the material in a structure.

Once fatigue data are available, suitable mathematical treatment is required to use the data in design calculations. For design cases where the number of cycles exceeds by far the number of cycles attained in the experiments, appropriate extrapolation must be formulated.

The concepts described in this chapter are applicable to general composites and fatigue. However, the chapter is written from the background of wind turbine rotor blade materials research. These structures are subjected to a very large number of load cycles; low cycle fatigue will not be specifically addressed in this chapter. Furthermore, rotor blades operate under strongly varying loads. The S–N models and constant life diagrams discussed provide a basis for life prediction under load spectra containing fatigue loads of various types.

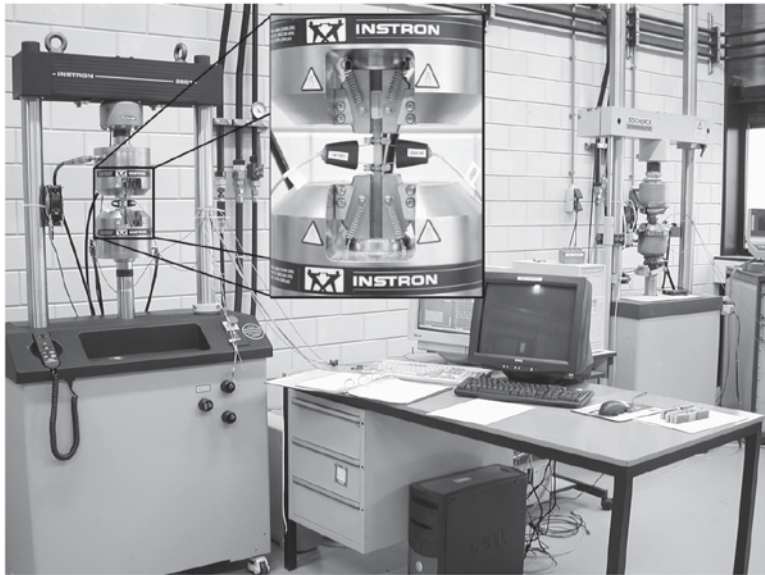
One of the main characteristics that are relevant for this type of material are that safe-life design is preferred (inspection and maintenance of wind turbine rotor blades should be minimised). Furthermore, the cost of raw material and manufacturing should be low. This typically results in relatively coarse material, and structures with some variation in properties within the structure and from product to product. It should be added that in recent years, high demand for rotor blades has led to considerable breakthroughs in production speed through automation, hence consistency is improving.

2.2 Fatigue experiments

Fatigue experiments are the basis for life prediction and validation of these predictions. In fact, most of the state-of-the-art design software is based on identifying the loads (strains, stresses) in a certain point on the structure, and quantifying the resulting damage through interpolation and extrapolation of the S–N curves and constant life diagrams described in this chapter. An example of commonly used fatigue test set-ups is shown in Fig. 2.1.

As any test result, the location of a point in the stress–life space should be viewed in the context of the many parameters affecting the fatigue life of a specimen. Choices made in the test set-up and procedure, and variations in test conditions, can significantly affect the results.

For static testing, test specimen dimensions, equipment, test conditions and measurements are described in detail in numerous standards or guidelines provided mostly by ISO, ASTM and many national institutes. In the case of fatigue testing of composites, standardisation is much more limited. Some guidelines exist for tension fatigue testing and tension–compression testing



2.1 Example of fatigue test set-up.

using mainly specimens and the methodology described in relevant static testing [1, 2]. An informal note on requirements for appropriate S–N curve testing is circulating around test laboratories [3].

The lack of standardisation of fatigue testing gives rise to issues when comparing fatigue strengths and lives under different experimental circumstances. A great number of variables affect fatigue life, most of which are free for the experimenter to manipulate at will, and a large part of which have not been considered in models. This makes extracting conclusions from different sets of fatigue data difficult. Some are discussed here.

2.2.1 Control mode

Most test frames can be set to load control or displacement control (cyclic loads or displacements are kept constant, respectively). In some cases, strain control is also possible. For laminates that feature a significant stiffness reduction during their fatigue life, the chosen control mode can have a big impact on test results. It is important to choose the relevant control mode for the intended use of the laminate.

In the case of load control, typically the grip displacement will increase during testing (decreasing specimen stiffness) and the specimen will fail destructively once it can no longer bear the applied load. This control mode is most commonly used in fatigue testing, especially for obtaining fatigue data for load-bearing laminates.

Displacement control or strain control can be very useful for monitoring crack growth or stiffness decrease in fatigue. It is possible that the specimen does not fail destructively, as decreasing stiffness leads to decreasing loads. The failure criterion should then be formulated in terms of, e.g., stiffness decrease or crack length. Displacement-controlled tests, however, have the disadvantage that damage or sliding in the grips can incorrectly be associated with stiffness degradation in the specimen. In cases where damage accumulates outside the measurement area during fatigue, strain control using the measurement signal of strain sensors at the gauge section can be employed.

Unless specifically mentioned in the description test set-up, reported strain–life data can usually be assumed to be actually obtained through load-controlled testing (converting stress to strain by using the modulus of elasticity). It is more correct in these cases to express the data in terms of initial strain versus life, as the stiffness of a laminate coupon can decrease significantly in fatigue, especially if it has a large off-axis fibre content.

2.2.2 Grips

Grips come in various sizes and fastening mechanisms, either mechanical (using bolts) or hydraulically operated clamps. The influence of the grips depends strongly on the planform and fibre content of the laminate: see Section 2.5. For dog-boned (waisted) specimens, grip influence is less significant than for prismatic specimens and specimens with a high fraction of fibres in the loading direction.

Most test frames allow one to set the overall hydraulic grip pressure. As a rule of thumb, grip pressure should be high enough to prevent sliding of the specimen from the grips, but not much higher. The higher the grip pressure, the more crushing load is introduced into the grip area.

Wear of grip faces should be checked periodically, as this might change the grip pressure distribution, thereby apparently lengthening the gauge length, and affecting fatigue life in loading types that include compression: see also Section 2.4.

2.2.3 System stiffness and alignment

After proper alignment of the grips, the stiffness of the test frame should be such that the grip alignment is not compromised by the application of high loads and possible asymmetry in the specimens. Poor alignment reduces the measured strengths because it lowers the specimen's resistance to buckling. Then the maximum load measured represents the buckling strength, not the material strength.

2.2.4 Loading rate

For static testing of load-bearing materials in most structures, high loading rates are typically avoided because they tend to yield an optimistic representation of strength and stiffness, while requiring high data acquisition rates. In fatigue, loading frequency should be low enough to prevent any significant influence on fatigue life, but as high as possible to reduce testing time: see Section 2.4.

2.2.5 Specimen geometry

Apart from its implications for specimen manufacturing, the geometry of the specimen has a significant influence on measured performance: see Section 2.5.

2.3 Measurements and sensors

Loads, displacements, strains and specimen surface temperature (or internal temperature if this can be measured in a non-invasive manner) should be measured continuously, and the number of fatigue cycles should be counted. Loads and displacements in most test frames are measured by a fixed load cell and an LVDT (linear variable displacement transducer) built into the servo-hydraulic actuator. Separate strain measurement and temperature measurement devices are typically used to document stiffness and check for temperature exceedance.

For strain measurements strain gauges are used, as well as strain-gauge based extensometers. A very relevant disadvantage of strain gauges is that they suffer from fatigue to a much larger extent than most composite laminates. This is understandable, since metal foil strain gauges also have an S–N curve, which is steeper than that of composites. Moreover, they experience considerable drift over the lifetime of the strain gauge (typically 1000–10,000 cycles). The drift, a gradual increase or decrease in measured strain, occurs as the strain gauge is damaged itself in fatigue, changing the electrical resistance of the strain gauge. Thus, they are really only appropriate for measurement of the initial modulus of the specimen.

Some solutions exist for long-term strain measurements. Strain-gauge based extensometers are much less sensitive to fatigue in the single-use strain gauges themselves. On the other hand, they often require a larger free gauge length and are much more expensive to buy. For this reason, additional measures are often taken to protect the extensometer after the specimen fails. In case explosive failure of the specimen is expected (for instance in specimens with a very long gauge length subjected to high loads), removal of the extensometer should be considered prior to failure. These measures

may preclude any potential stiffness changes at the end of fatigue life from being noticed.

Both for bonded strain gauges and for extensometers, some care should be taken in choosing the gauge length of the strain measurement device, in relation to the gauge length of the specimen and the microstructure of the material. For a short specimen gauge length, irregular strain fields can be expected, for example in the area near the grips. For short gauge lengths, this can lead to an unjustified ‘averaging’ of measured strain when using a large strain gauge length. On the other hand, the strain gauge should be large enough to average the measured strain in both the tows and matrix-rich areas between the tows, i.e., it should cover a couple of tows in the material’s structure.

Non-contact optical strain measurement techniques have become more popular. An example of full-field strain imaging is the Digital Image Correlation Technique (DICT), which uses postprocessing of video images of a specimen to calculate displacements (hence strains) in up to three dimensions. An advantage of this method is that non-uniform strain fields can be observed, and a better choice can be made over which area the strain should be averaged.

Optical fibres (glass fibres, with an optical grating etched into the material, reflecting light transmitted through this fibre) have the potential of measuring strains inside a composite material, as they can be embedded in the laminates during production. These sensors are highly durable and give reliable results, provided that temperature compensation is taken care of.

Measuring strains using the displacement readings from the test frames should be avoided, especially in fatigue experiments, as it might produce inaccurate results for various reasons. First of all, the displacement range during most tests is much smaller than the stroke of the test machine (i.e. the full scale range of the sensor). Thus, without modifications, the resolution is far less than the resolution of a strain gauge. More importantly, the displacement readings also include deformation of the specimen outside the gauge section. Strain calculation requires modelling of the specimen, which is not desirable and might not be possible to a sufficient extent.

Many specimens have non-uniform strain fields, so that the ‘strain’ measured using the actuator displacement sensor is an average value of the strain over the specimen. Furthermore, many specimens are equipped with tabs to protect the specimen surface from grip damage. Wear during the test usually causes part of the tab and gauge section to move with respect to the grips. The adhesive bond between tabs and specimen may partly disintegrate during the fatigue test, causing additional displacement in a load-controlled test, particularly in rectangular test specimens. Another downside is that the stiffness of the whole system, including the test machine, is measured when using the LVDT signal. The inaccuracies decrease as the stiffness of the

specimen becomes smaller with respect to the stiffness of the test frame, the specimens become longer, and the strain concentrations that lead to wear in the grips become smaller. Cycle counting is performed by software through peak detection algorithms, although mechanical cycle counters can be used as well.

During data acquisition for fatigue testing it is acceptable to record a summary of data at intervals, e.g. record the average maximum and range of the last x cycles, at intervals of $y > x$ cycles (provided that the absolute maxima and minima are reported if they exceed the average by a specified threshold value, typically 1–2%). The size of these intervals can be changed during fatigue tests to limit file size. It is advisable to also retain periodic records of full cycles (i.e. measured at a data acquisition rate of *ca.* 1/100th of the period), to enable checking of the applied loads and displacements, such as whether the specimen is still loaded using the sine-shaped load that is required, etc. Also, these records can be used for calculation of hysteretic damping and dynamic modulus, provided that there is no phase lag between measurements. Therefore, all sensors should preferably be fed through the same electrical circuit. For example, if the strain signal is directly fed into the acquisition system, whereas the load signal is looped through the control unit, their ‘electronic’ distance is different, causing a time delay which will show as hysteresis.

2.4 Test frequency

As indicated, the frequency of tests should be chosen as high as possible without influencing life significantly. For larger strain-ranges, a lower number of cycles and lower frequencies should typically be chosen (and vice versa). Testing frequency can influence fatigue life in the following three ways.

2.4.1 Creep/time at mean stress effects

Mishnaevsky and Brøndsted [4] reported that through deduction based on the kinetic concept of strength, fatigue life dependency on test frequency can be described. Longer fatigue lives are predicted for higher frequencies. In practice, heating will limit the test fatigue frequency considerably.

2.4.2 Frictional heating

As the sections of the specimens inside the grips and tabs are partly allowed to slide, some frictional heating can occur. This depends on how much the grip faces are allowed to follow the Poisson contraction/expansion during tensile/compressive loading. In addition, internally, frictional heat can be generated when delaminated layers move relative to each other or, on a

smaller scale, by fibres and matrix sliding with respect to each other after formation of fatigue damage.

2.4.3 Visco-elastic heating

Internally, a second mechanism can cause heating, i.e. visco-elastic hysteresis. (Internal) Frictional heating and visco-elastic heating can hardly be quantified separately in temperature measurements.

An appropriate frequency scheduling method was described in [5], where frequency scheduling was employed using the strain range as a criterion for frequency. First, a maximum allowable temperature rise was chosen (10°C above room temperature). Then, a reference test was run in the low-cycle, high-strain regime, monitoring surface temperature. The maximum frequency was adjusted so that the surface temperature rise was within acceptable limits. For other load levels, the frequency was proportional to the square of the change in strain, according to $f = f_{\text{ref}}(\epsilon_{\text{ref}}/\epsilon)^2$ [6]. This means that for different R -values, the frequencies for identical nominal fatigue lives were therefore different. The loading frequency for short fatigue lives (1000 cycles) was in the order of 1–2 Hz; for 1 million cycles nominal fatigue life, the frequency ranged between 3–8 Hz. In day-to-day laboratory practice, these frequencies limit the nominal target number of cycles to 1 million cycles. Choosing load levels such that 10 million cycles can be expected means that a test frame must be running for an average of ~1 month; the typical one decade of scatter means that a test frame can easily be unavailable for other tests twice as long.

2.5 Specimens

The overall controversy in choice of specimen is the representation of an undisturbed cut-out of the structure versus testability. The boundary conditions of a small region of the structure under investigation are very different from those in a specimen in a test machine; the small region in the blade is loaded by axial or biaxial remote loads, has no sides, is subjected to a limited loading frequency, and does not suffer from damage due to the load introduction, etc. Most specimens are loaded in the longitudinal direction, where the longitudinal stresses in the specimen are introduced through shear and friction, and the specimens have sides and – usually – no curvature. Measured material performance is affected significantly by the dimensions of the tested specimen.

The specimen shape can be tailored to specific shear, tensile or compression loading, meaning that the specimen is designed to fail by the maximum values for the required parameters. In quasi-static testing this is common practice, resulting in a large variety of specimen shapes, each standardised

and proven to be useful for obtaining specific material parameters, such as tensile, compressive and shear strengths and modulus. Comparison of performance of different specimen shapes does reveal differences caused by width or size effects, or the effects of stress/strain concentrations, or sensitivity to buckling (e.g. [7]).

For fatigue, the same specimens can be used as those used in quasi-static tests, tailored for a particular loading type. For instance, for tension-dominated fatigue, long slender specimens can be used; while short thick specimens can be employed for compression-dominated fatigue. For validating general fatigue models, however, it is recommended to define a single ‘universal’ specimen geometry and testing procedure in order to enable variable amplitude loading with both tension and compression loadings. This philosophy was followed in some exemplary wind turbine material research projects [5, 8, 9], as shown in Fig. 2.2. In that case, the choice of specimen dimensions is based on many conflicting arguments, some of which are indicated below.

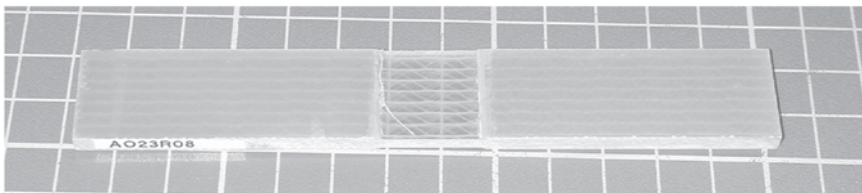
2.5.1 Manufacturability and batch size

Specimens should be manufacturable in as simple a manner as possible, requiring widely available tooling. This reduces the chance of batch-to-batch variation and allows for quick set-up of an experimental qualification programme. In practice, this implies preferred absence of notches and thickness changes.

It is recommended to obtain specimens from a single large batch of laminate, which helps to ensure that all specimens were manufactured under the same conditions. For large laminate plates, it is also advisable to retain the original position and orientation of the specimen within the plate, enabling possible research into effects of direction of infusion, mould time, etc.

2.5.2 Maximum load

The minimum cross-sectional area, fibre volume fraction, fibre orientation, and composite constituent ultimate strengths (and sometimes ambient test conditions) determine the maximum load required to break the specimen.



2.2 ‘Universal’ specimen used in [8] and [9].

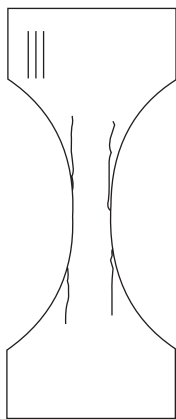
2.5.3 Tabs

The role of tabs is to protect the laminate surface, to increase friction between the specimen and the grips, and to make the overall specimen thickness larger, which allows for some measurement equipment to be inserted on the specimen side, and for improving end-loading. A large tab section area is recommended to gradually introduce the axial loads into the specimen, and to enable small grip pressures.

Tabs are usually bonded separately to the specimen. The tab thickness and bondline thickness should be well controlled to ensure that the tabs are parallel. However, tab thickness should be small, e.g. smaller than specimen thickness, to avoid overheating in the tabbed region during cyclic testing.

2.5.4 Planform

In isotropic materials, the centre of the specimen often has a reduced cross-sectional area to ensure localised failure and failure at the location of the strain gauges. Much of the composite structural material research is done on unidirectional laminates, which are not suited for reducing the cross-sectional area without significantly increasing the gauge section length or adding layers in other directions. Because of a non-uniform in-plane shear distribution, most in-plane ‘tailored’, ‘waisted’ or ‘dog-bone’ shaped specimens develop axial cracks starting at the sides of the minimum cross-section and growing towards the clamped region, essentially reducing them to a rectangular specimen during a fatigue test: see Fig. 2.3. Reducing the cross-sectional area through gradually building up the laminate towards the grips might be more useful, although delaminations are likely to occur. For large-scale production of test specimens this is also generally too labour-intensive.



2.3 Axial cracks in dog-bone specimen during fatigue.

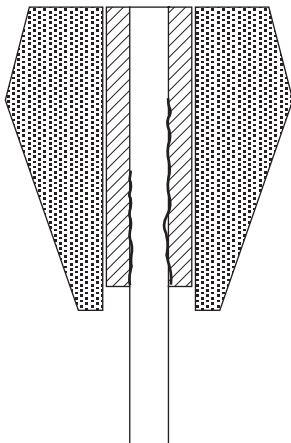
One of the most important implications of testing unidirectional laminates using a rectangular specimen is that the most likely failure location is in the grips. In fact, the combination of large strain gradients and Poisson contraction in the region where the tabs meet the gauge section typically results in a tab disbond gradually growing into the grip area, and ending with partial or complete fracture of the specimen in the tab (see Fig. 2.4). Damage progression in this region is usually associated with abrasion of the adhesive between tab and laminate, damage of the laminate surface, frictional heat generated in the disbonded area, redistribution of the grip loads, shear loads, and increased apparent gauge length. These mechanisms are particularly hard to measure and model, but it is suspected that grip failures are especially contributing to the difference between what the material is actually capable of and its measured fatigue performance.

2.5.5 Length and gauge length

Small length and width are generally desirable to obtain as many specimens as possible from a limited amount of available laminate. The specimen gauge length should be small enough to avoid buckling, but large enough to allow uniform strain distributions across most of the gauge section, and to allow application of strain and temperature sensors. The latter typically precludes the use of an anti-buckling guide.

2.5.6 Thickness

Increasing specimen thickness allows for a longer gauge length at constant Euler buckling sensitivity. Smaller thickness facilitates better uniform strain



2.4 Schematic of tabbed end of specimen in grips, showing axial cracks between tabs and laminate, developed during fatigue.

distributions and higher cyclic frequencies, since heat generated internally can be dissipated to the surface more quickly. In composites, the thickness of a specimen is a discrete function of the number of layers. For cost-effective composites, layer thickness is in the order of 1 mm (*cf.* 0.1 mm for aerospace grade composites).

2.5.7 Width

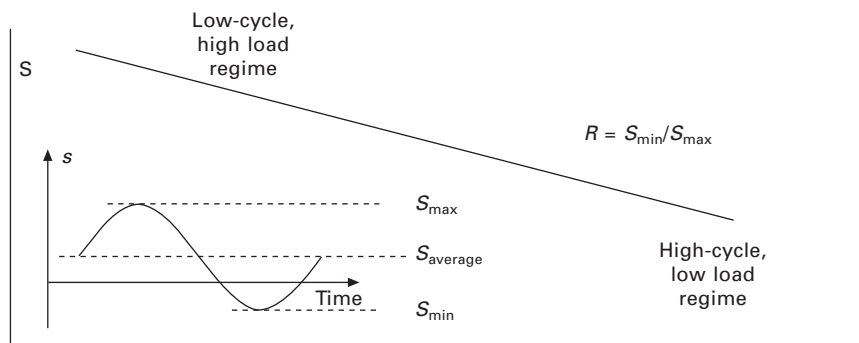
The width of a specimen should be significantly larger than the characteristic width of the laminate microstructure; in practice this is larger than four or five tows in a multi-layered laminate. Also, the width should be smaller than the gauge section length, e.g. one-third or smaller, although for compression loading or reversed loading the width/gauge length ratio can be ~ 1 .

2.6 S–N diagrams

The parameters ‘S’ and ‘N’ are used as generic terms. In the case of ‘S’ this means that the term can indicate cyclic loads, stresses and strains (within each of these categories a choice can be made between amplitude, mean, maximum, minimum or absolute values of maximum, or minimum, etc.). The generic term ‘N’ is usually number of cycles to failure, but can also be used to indicate number of cycles to a predefined stiffness change, or number of times a fixed load sequence has been repeated.

Since none of the above parameters for S completely defines the constant amplitude waveform, in general the S–N curves are usually plotted for a constant value of R . The definition of R is the ratio of minimum over maximum S, where compression has a negative sign. The value of R is a unique description of the loading type. See Fig. 2.5.

The first observation that should be made is that the S–N diagram, with S and N usually plotted on the ordinate and abscissa, respectively, is in



2.5 S–N diagram.

principle incorrectly oriented. As in all mathematical plots, the independent variable (the input into the specimen: S) should be plotted on the horizontal axis, whereas the dependent variable (output: N) should be plotted on the vertical axis. For unclear reasons, the S – N curve has been plotted the wrong way around, and for historical reasons it is acceptable to leave it like that.

In any case, in regression analyses performed for design, life should be treated as the dependent variable. Regression with the incorrect dependent variable will lead to an S – N curve that has a different slope and intercept from the correct curve, possibly leading to an overestimation or underestimation of performance, its severity depending on the degree of scatter and extrapolation.

In S – N curve definition and extrapolation, the life-axis can be divided into three regions: low cycle (corresponding to high strains and stresses), high cycle (corresponding to low strains and stresses), and the region in between. The corresponding number of cycles depends on the application, but for structures which can expect many load cycles during operation, the low cycle regime is up to 1000 cycles, and the high cycle regime starts at around 1 million cycles.

2.6.1 Statistical description of fatigue data

The S – N curves aim at characterising and predicting mean fatigue life at a given S . However, for design, it is important to know the fatigue life for which a certain percentile of the population can be expected to still be intact. This requires the characterisation of scatter, and the derivation of confidence limits.

On the life-axis, scatter in fatigue tests is 0.5–2 decades, depending mostly on loading type and material. In tensile fatigue, scatter is usually much smaller than in compression. In general, there seems to be a relation between the slope of the S – N curve and scatter: flat S – N curves show more scatter than steep ones. This opens possibilities for the theory that fatigue life is directly related to static strength; scatter on the strength-axis is related, via the slope of the S – N curve, to scatter on the life-axis.

Also, the scatter seems to be somehow proportional to the number of possible dominant damage (initiation) mechanisms in fatigue. This explains why composites exhibit more scatter than most metallic materials (multiple crack vs. single crack), and why tensile fatigue shows less scatter than compressive fatigue (final failure in compression fatigue is a combination of fatigue, and buckling/delamination). Consequently, this implies that very low scatter in the life coordinate could mean that a single damage mechanism is driving failure. In some cases, this could mean that the test is poorly designed; damage induced by severe grip misalignment or gripping abrasion has been known to result in S – N curves with very low scatter. So, although

low scatter is desirable for formulating design calculations, its origins need to be taken into account.

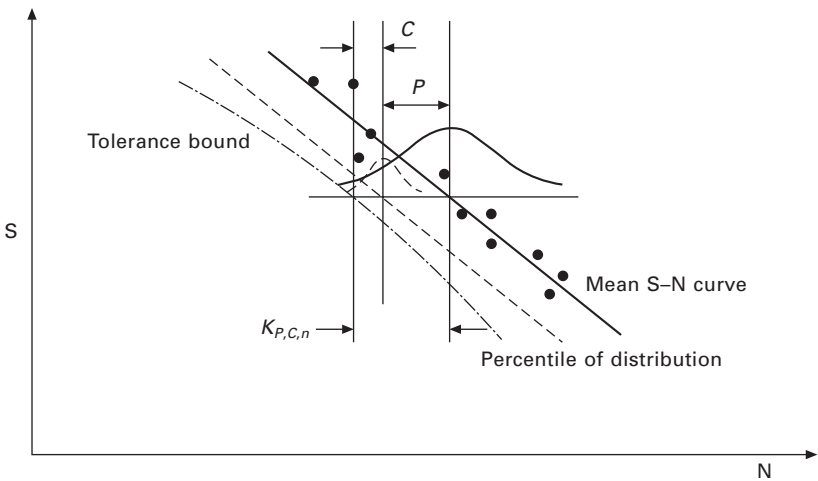
Characterisation of the scatter in fatigue data is the first step in deriving design data from experimental fatigue results. Typically, the designer utilises values of fatigue life, which are characterised by a probability of failure, and a confidence value. The probability of failure is associated with a value of N , above which a certain percentage of the specimens can be expected to fail. The confidence level is a measure of how accurate the probability of failure can be obtained from a sample of the population. A combination of failure probability and confidence is a confidence bound, or tolerance bound, which can also be associated with a value for N . A schematic of tolerance bounds is displayed in Fig. 2.6. The tolerance bound can be formulated as:

$$N_{\text{tolerance}} = N_{\text{average}} - K_{P,C,N}(S) \cdot s \tag{2.1}$$

where

- $N_{\text{tolerance}}$ = generic life (tolerance bound)
- N_{average} = generic life (average described by S–N formulation)
- K = factor depending on P , C and N
- P = probability of failure
- C = confidence level
- S = generic load input.

The K -factor can be found from Monte Carlo simulations or look-up tables [10, 11]. The K -factor depends largely on the distribution function that is chosen or derived to describe the scatter at a certain load level, as well as



2.6 Statistics of the S–N curve.

the number of specimens that are available at a load level or in the entire S–N data pool.

In principle, the uncertainty near the lower and upper bounds of the S–N curve is larger, hence the tolerance bounds should be located further away from the data there. Therefore, the K -factor depends also on N in the above equation. Often, tolerance bounds are typically formulated as bounds parallel to the median fatigue curve, dropping the dependency on N in K in the expression above.

2.6.2 Censoring and run-outs

In general, ‘censoring’ means leaving out test results. There can be several reasons for doing so. Data from invalid tests should be excluded from the dataset. Validity of the experiment can be lost for many reasons: tab failures, obvious defects in specimen, faulty test parameters, etc. In some cases it is wise to discard data if the influence of an aberration of test parameters on fatigue life is unclear.

It is possible that the test was nominally executed with correct conditions and no obvious defects. However, especially for long-running tests, it is possible that the test was interrupted and/or the specimen was taken out and reinserted into the machine. Limited reports are available on effects of interrupting fatigue tests on fatigue life, e.g. [12]. When reinserting a test specimen, small changes in specimen orientation may influence the rate of development of damage already present in the specimen. Therefore, reinsertion must depend on the number of cycles already tested. For a small number of cycles relative to the expected number of cycles, discarding a specimen is less of a problem (in those cases, not much machine time was invested anyway). For specimens tested in a modular fixture which can be taken out of the test frame and reinserted without changing the orientation of the specimen within the fixture, reinserting a specimen is less of a concern.

For practical reasons, a long-running fatigue test must sometimes be stopped. This can be done at a predetermined number of cycles or as machine availability dictates. Such a point in the S–N curve must be marked as a ‘run-out’. Treatment of run-outs in S–N curve determination should be evaluated from case to case.

2.6.3 Extrapolation of fatigue curves

As rules of thumb, different minimum requirements in numbers of cycles and cycle range apply to S–N curves. In most structural applications and laminates, fatigue life is most relevant from around 1000 cycles. The minimum number of cycles at the lowest load/strain levels should be 1,000,000 in most cases. For specific applications, more knowledge on fatigue behaviour

might be required. In the wind industry, design load cycles are in the range of 10 to 100 megacycles. However, only limited data are available in this cycle range.

Bach [13] presented high-cycle fatigue data obtained from wind turbine laminates at $R = -1$ and $R = 0.1$, up to $>10^7$ cycles. Other tests to even higher numbers of cycles have been done on wind turbine laminates to check the most appropriate S–N curve formulation, for instance by testing ordinary coupons in series for a time period of over a year [14]. For high-cycle, high-frequency testing up to the 1 billion cycle range, an effective and inventive solution was demonstrated [15], which used microphones as actuators to essentially perform constant displacement amplitude tests with strands.

Some recent high-cycle data on wind turbine blade laminate coupons are included in Fig. 2.7. For design, the strain levels in operation are often so low that they cannot reasonably be used in testing, because of time constraints.

As will be discussed, many S–N formulations can be used to describe S–N data within the load range of the experimental dataset, as is demonstrated in Fig. 2.8, which was published in extended form in [11]. All S–N formulations shown describe the data acceptably well. However, when extrapolation to the region of 10^7 – 10^8 cycles would be required, the prediction would depend severely on the choice of S–N curve and inherent extrapolation. This emphasises that the quality of S–N curve extrapolation is highly important.

2.7 S–N formulations

Various S–N formulations exist, of different types and with different backgrounds. An overview is given in this chapter. A summary is found at the end of the chapter.

2.7.1 Two-parameter S–N curve

The minimum number of parameters required to derive a curve through experimental data is two: a parameter describing the intercept with one of the axes, and a slope parameter. Classically, the logarithm of constant amplitude fatigue life N is assumed to be linearly dependent on the governing stress/strain S , or its logarithm.

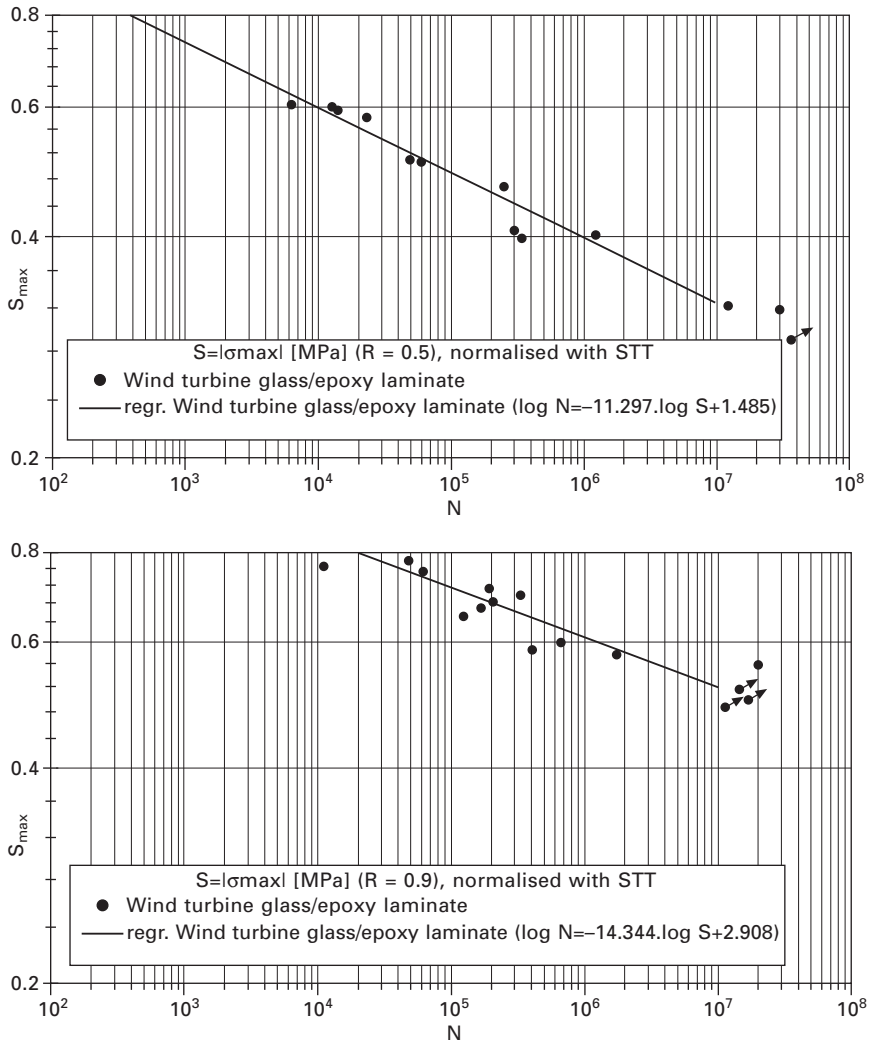
The two most used formulations of the S–N curve are:

$$\log N = a \log S + b, \text{ or, equivalently: } N = 10^b S^a \quad 2.2$$

(see Fig. 2.9) and

$$\log N = c + dS \quad 2.3$$

where



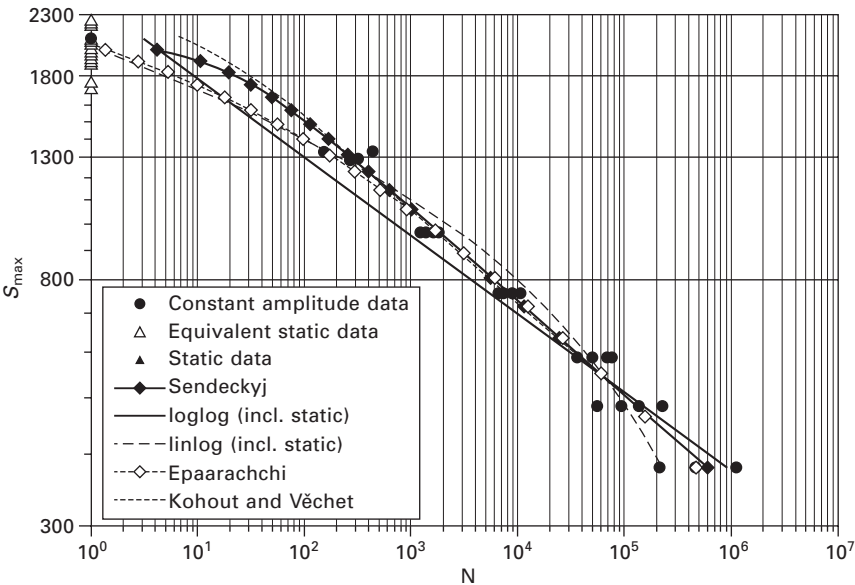
2.7 High cycle fatigue data for a wind turbine laminate at $R = 0.5$ and $R = 0.9$. [16]

N = generic life
 S = generic load input
 $a-d$ = fitting parameters.

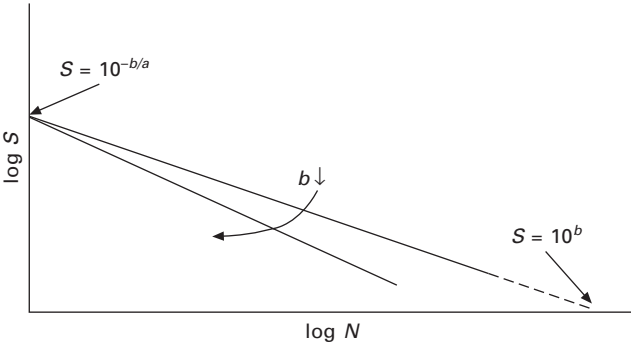
The parameters of the S–N curve are found by linear regression using the life variable as the dependent variable. Especially the first expression (log–log, power law) is used prevalently, as extrapolation to high-cycle fatigue is more accurate for many composites.

Both static and fatigue data can be used to calculate the parameters of the S–N curve. This means that static data are included in the regression analysis, taking static strength as the maximum load that leads to 1 cycle until failure.¹

For the above expressions, it is recommended to regard fatigue behaviour and static behaviour as unrelated, and limit the regression to the fatigue data.



2.8 Extrapolation of S–N curves.



2.9 Log–log S–N curve.

¹It can even be argued that this should be 0.25 cycles to failure, or 0.75 cycles to failure, if the load is sinusoidal starting with tension, and failure mode is tensile or compressive, respectively.

As a result, the above relations do not generally describe fatigue behaviour in the low-cycle region outside the actual data accurately, as demonstrated in [17]. In this reference, S–N curves were fitted through all data or only through fatigue data for $R = 0.1$, $R = -1$ and $R = 10$. In all cases, including static data led to a discrepancy between the resulting S–N curve and the data; on the other hand, excluding static data from the regression generally led to poor prediction of static strength based on fatigue only.

2.7.2 Adding parameters

Adding parameters allows for tailoring the S–N curve description for different cycle regions. In [18], two points were defined on the S–N curve. Between these points, the S–N curve is a log–log curve. Outside these points, their formulation allows for flattening or steepening of the curve, following an additional pair of parameters (Fig. 2.10):

$$S = \left[10^{-b} \frac{(N + B)C}{(N + C)} \right]^{\frac{1}{a}} \quad 2.4$$

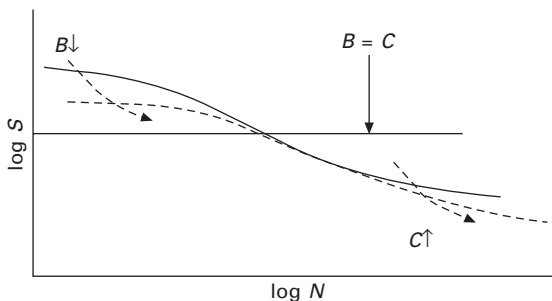
where

B, C = fitting parameters

a, b = fitting parameters from equation (2.2).

2.7.3 Strength-based S–N curve

Other S–N formulations exist which take into account low-cycle fatigue and/or a fatigue limit or slope change in the high-cycle region. An example is the formulation proposed by Sendekyj [19], who generates an S–N curve based on the concept of a direct correlation between strength and fatigue data (the ‘strength-life-equal-rank assumption’), and allows for the inclusion of residual strength data in the construction of an S–N curve, as in Fig.



2.10 Log–log S–N curve with additional parameters.

2.11. In this paper it was proposed to use the wear-out model, developed for description of residual strength by others earlier, as a means to describe residual strength and the S–N curve:

$$S_e = S_a[(S_r/S_a)^{1/D} + (N - 1)E]^D \quad 2.5$$

where

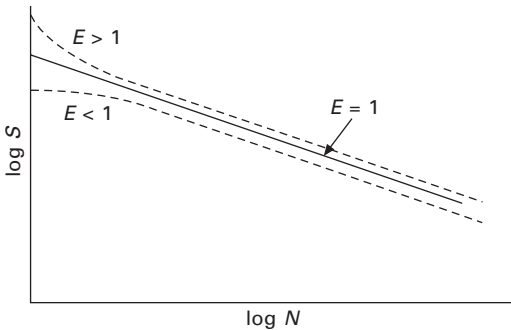
- S_e = equivalent static value of S
- S_a = amplitude of S
- S_r = residual S after N cycles
- D = fitting parameter for the high-cycle region
- E = fitting parameter for the low-cycle region.

For fatigue data only (no residual strength), and for $E = 1$, the formulation reduces to the log–log S–N curve. D is equivalent to the slope parameter in the log–log formulation.

Finding the parameters in the S–N description is a two-stage process. First, a set of equivalent static strengths is derived from each available fatigue and residual strength datum. The statistical description (Sendeckyj used Weibull parameters) is compared to the statistical description of actual static tests. Then, in an iterative process, D and E are adjusted until the equivalent strengths and the actual strengths match. The advantage is that this formulation also allows for a description of residual strength data. A disadvantage is that a numerical iteration/optimisation routine is necessary to determine the coefficients of the model, which means that the values can depend on the settings of the iterative routine.

2.7.4 Statistical formulations

Kensche [20] used the combination of a two-parameter Weibull distribution and a wear-out based S–N curve described in [19] with strain instead of stress:



2.11 Strength-based S–N curve.

$$S = \beta \cdot \frac{(-\ln[R(N)])^{1/\alpha}}{((N - A)E)^D} \quad 2.6$$

with

$$A = \frac{1 - E}{E} \quad 2.7$$

where

- $R(N)$ = probability of survival
- α, β = Weibull shape, scale parameter
- E, D = parameters from equation (2.5).

In earlier work [21], based on an S–N curve formulation from [22], a similar expression was derived via an alternative route:

$$S = [(-\ln R(N))^{-1/\alpha} (10^{-b} N)]^{1/2} \quad 2.8$$

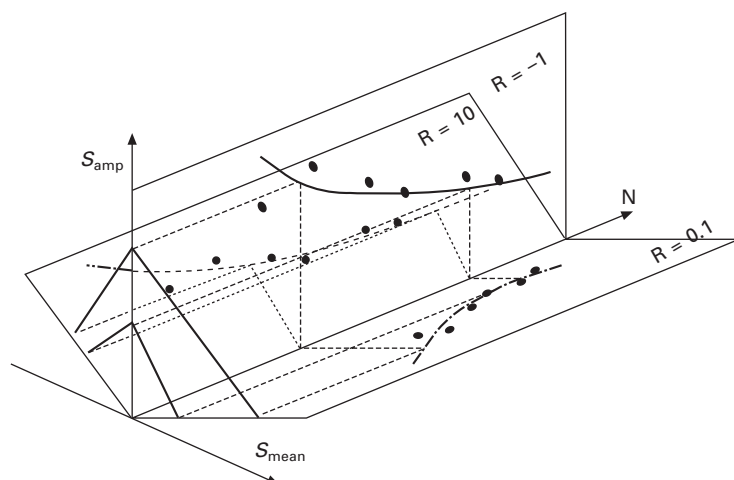
Once a probabilistic model has been chosen to describe the distribution of N for each S , this can be used in probabilistic structural models, where probability of failure of the material is combined with stochastic loading.

Liu and Mahadevan [23] describe a method to input the ply S–N data directly into a Miner summation to obtain an S–N curve for any laminate stacking sequence. This was extended to probabilistic analysis using Monte Carlo simulations, and for tension–tension fatigue the simulated S–N data tended to include the available experimental data. Regression lines through simulated data would disagree more with experimental data, as the simulations showed much more scatter than the experimental data in general.

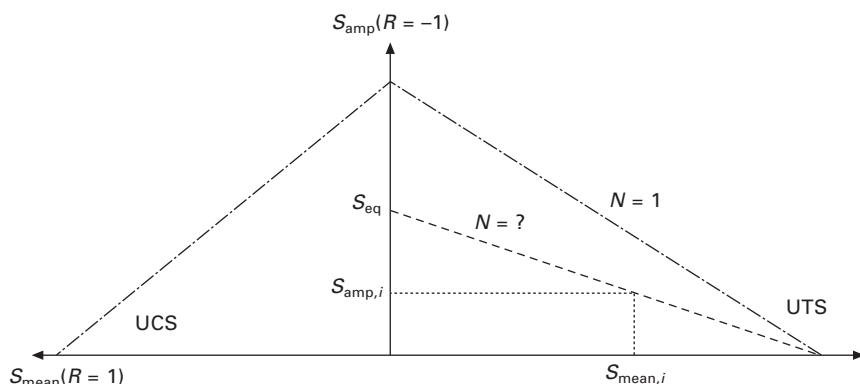
Various analytical models for probabilistic strength analysis of wind turbine blade structures were recently implemented in [24], employing material test data of UD laminates (with statistical descriptives), and first-ply-failure criteria, comparing analytical models to Monte Carlo simulations.

2.7.5 S–N curves that take into account the R -value

S–N curves are two-dimensional representations of fatigue behaviour for a single loading type, i.e. R -value. For describing the full fatigue behaviour of a material, knowledge of the fatigue behaviour at all other R -values is necessary as well. Often, a description of fatigue behaviour is available in the $S_{\text{amp}}, S_{\text{mean}}$ space. This is called the constant life diagram (CLD), as loading types leading to a particular fatigue life are indicated by lines of constant life or ‘iso-life curves’. The CLD and the S–N curve are frequently treated as separate life prediction tools. However, they can be quite interconnected. A schematic of the $S_{\text{amp}}, S_{\text{mean}}$ plane, with an additional axis in the third dimension representing life is shown in Fig. 2.12.



2.12 Relation between CLD and S-N curve.



2.13 Linear Goodman diagram.

The CLD is effectively a projection of the available S-N curves on the S_{amp} , S_{mean} plane. In the case of Fig. 2.12, the CLD contains piecewise linear iso-life curves, because it was generated from discrete S-N formulations at single R -values ($R = 0.1$, -1 and 10) with linear interpolations for R -values in between. Such a piecewise linear CLD can be constructed from virtually any type and combination of S-N curves. A well-known example is the linear Goodman diagram, made by linearly interpolating $R = -1$ data and failure stresses in the CLD: see Fig. 2.13.

In the literature, some S-N curves are formulated that include R -value influence, which can be related to the CLD also. To demonstrate the effect on the shape of the CLD, the S-N curve is shown together with a representative CLD for some of the expressions discussed below.

A simple modification was reported of the lin-log relation (see equation (2.3)) that Mandell proposed earlier [25]:

$$S = US(1 - F \log N) \quad 2.9$$

US = static strength:

ultimate tensile strength (UTS) if $|S_{\max}| > |S_{\min}|$,

ultimate compressive strength (UCS) if $|S_{\min}| > |S_{\max}|$

F = fitting parameter.

After analysing literature data, Appel and Olthoff [26] proposed a simple R -value dependency (Fig. 2.14):

$$F = 0.095 - 0.015R \quad 2.10$$

This expression works well for $-1 < R < 1$. For mean stresses below zero, the following adaptation is proposed here:

$$F = 0.095 - 0.015 \times (1/R) \quad 2.11$$

The resulting CLD shows a discontinuity at $R = -1$ that is proportional to the ratio of UCS to UTS. The CLD consists mostly of parallel lines (Fig. 2.15).

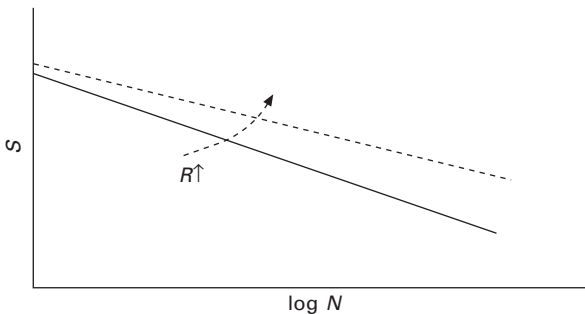
Another approach is to relate the fatigue behaviour at any R -value to a reference R -value. Based on [27] and [28], it was suggested in [29] to use a fatigue formulation which, although not explicitly mentioned in any of these references, reduces to a parallel-line CLD.

$$S_{\text{eq}} = \left(\frac{\sum n_i S_i^m}{\sum n_i} \right)^{1/m} \left(\frac{1}{M} \right)^{1/m} \quad 2.12$$

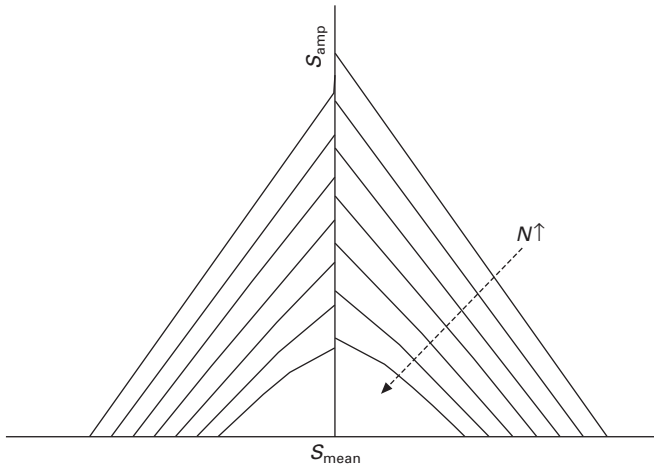
where

m, M = fitting parameters

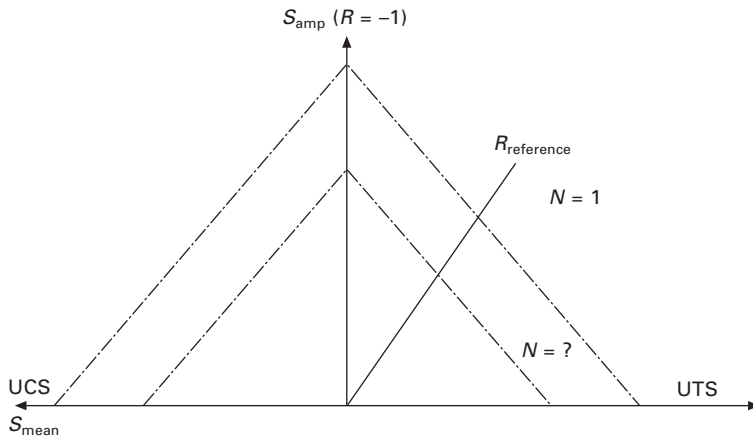
i = loading type.



2.14 Simple R -value dependency.



2.15 CLD for S-N curve of Fig. 2.14.

2.16 Equivalent S formulation CLD.

This formulation effectively calculates an equivalent S , which can be related to N via the reference S - N curve, usually at a reference R -value, e.g. $R = 0.1$. This CLD is shown in Fig. 2.16.

In [30], a rather comprehensive model was composed for the full description of fatigue life by integrating various models from the literature. Starting with the assumption that testing temperature is constant, and based on strength degradation of the specimen (assuming, furthermore, that the static strength was determined under the same strain rate as fatigue life), the following S - N formulation was found:

$$US - S_{\max} = \gamma G(R, S_{\max}, US) \frac{1}{f^{\delta}} (N^{\delta} - 1) \quad 2.13$$

where

- γ, δ = fitting parameters depending on material
- G = function of R , S and static strength
- f = cyclic loading frequency.

Based on [19] and [31], the following formulation for G was postulated:

$$G(R, S_{\max}, US) = US^{1-\lambda} S_{\max}^{\lambda} (1 - R)^{\lambda} \quad 2.14$$

Further, the exponent was related to R and fibre orientation:

$$\lambda = 1.6 - \psi \sin \vartheta \quad 2.15$$

where

- ψ = function of R :
 $1/R$ for $1 < R < \infty$ (compression–compression),
 R for $-\infty < R < 1$ (the remaining loading types)
- θ = smallest angle between fibres and loading direction.

Substituting, rearranging and lumping parameters yields:

$$US - S_{\max} = p[S_{\max}] \left(\frac{S_{\max}}{US} \right)^{k-1} (N^{\delta} - 1) \quad 2.16$$

$$k = 1.6 - \psi |\sin \theta|$$

$$p = \gamma(1 - \psi) k \frac{1}{f^{\delta}}$$

Lumping parameters even further, this can be rewritten into:

$$\frac{H}{\gamma} = N^{\delta} - 1 \quad 2.17$$

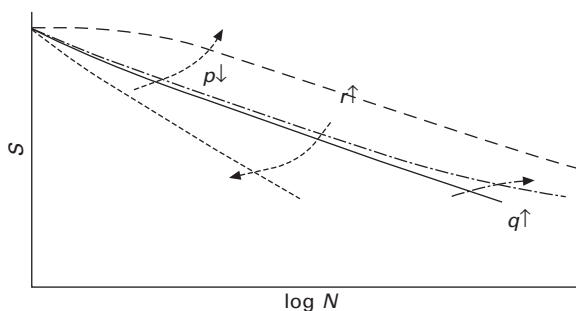
Then, a method that is described in [30] was used to determine the parameters γ and δ by linear regression.

In [15] and [32], equation (2.16) was simplified to:

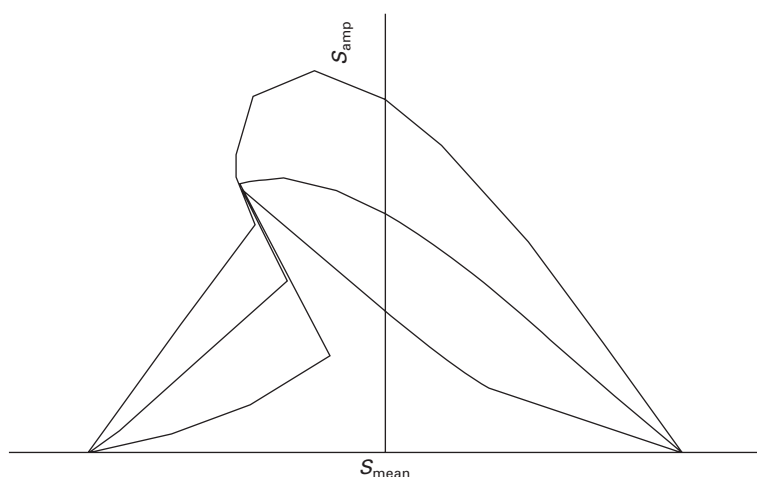
$$US - S_{\max} = p S_{\max} \left[\frac{S_{\max}}{US} \right]^q (N^r - 1) \quad 2.18$$

The parameters p and q were used there as fitting parameters, employing iterative routines to find the appropriate values for a particular dataset. The S–N model is illustrated in Fig. 2.17. Due to the formulation of ψ , the CLD shows a discontinuity around $R = \pm\infty$: see Fig. 2.18.

In [33], two-parameter log–log S–N curves were used, but the slope was made dependent on S_{mean} instead of R . This was named the ‘multislope’ model. An example of the associated CLD is shown in Fig. 2.19. The approach was developed separately from earlier work published in [34]. In



2.17 S-N model of equation (2.18).

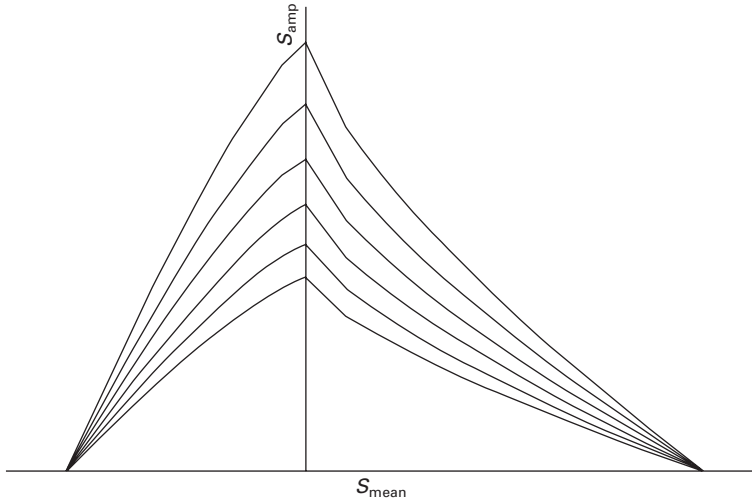


2.18 CLD from model of equation (2.16).

this work, the iso-life curves are continuous bell-shaped functions between tensile and compressive strength in the CLD (not necessarily symmetric).

For both models, the data pooling scheme for finding the model parameters was similar. All the experimental data are used in the regression defining the curve parameters. So, whereas in a piecewise continuous CLD the fatigue behaviour at a particular R would be determined only by the closest R -values for which data are available, in the Behesty/Boerstra models in principle all test results can work together to determine the fatigue behaviour at any point in the CLD. In Boerstra's model, the derivations of the CLD left and right of the $R = -1$ line are separated, which is more practical and makes sense, since fatigue mechanisms in tension and compression can be expected to be physically different and therefore should probably not be mixed when constructing the CLD.

Despite the potential in physically correctly representing fatigue behaviour,



2.19 Multislope CLD (S–N curves with slope that depends on S_{mean}).

there is still some lack of physical argumentation for the choice of some model parameters. For instance, the formulation shown in Fig. 2.19 is based on the ‘apex’ of the constant life diagram being located on the ‘mean stress equal to zero’ plane. Therefore it is continuous only in the tension or compression plane, and there is a discontinuity at $S_{\text{mean}} = 0$; furthermore, it has been observed for various materials that the symmetry line of the CLD is not on the $S_{\text{mean}} = 0$ axis.

An important implication of these methods is that a multi-parameter non-linear regression algorithm is required to determine the model parameters and the result is hence dependent on the initial and boundary conditions.

2.7.6 Strength-based S–N curves with R -value dependency

Recently, Kassapoglou [35] described formulations to predict S–N curves by using the statistical properties of static data. Essentially, different regions of the CLD were classified on the basis of R -value and associated with a single S–N formula. This formula describes life as a function of only the maximum stress and the properties of the static strength distribution.

$$N = \frac{1}{(S_{\text{max}}/\beta_{\text{T}})^{\alpha_{\text{T}}} + (S_{\text{min}}/\beta_{\text{c}})^{\alpha_{\text{c}}}} \quad \text{for } R < 0 \quad 2.19$$

$$N = \left(\frac{\beta_{\text{T}}}{S}\right)^{\alpha_{\text{T}}} \quad \text{for } 0 \leq R < 1, \quad N = \left(\frac{\beta_{\text{c}}}{S}\right)^{\alpha_{\text{c}}} \quad \text{for } R > 1 \quad 2.20$$

where

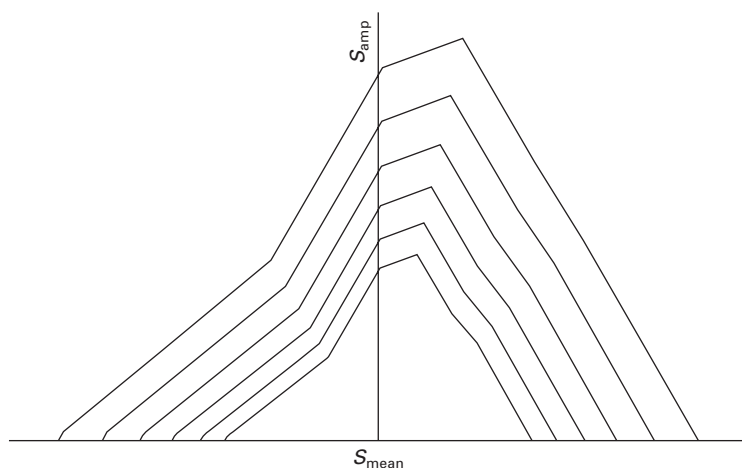
α, β = shape and scale parameters of the Weibull distribution of static strength

T, C = subscripts indicating tensile and compression strength, respectively.

To plot the CLD, some of the equations need to be solved iteratively to find the stress value as a function of life. In the original document, predictions were compared to S–N curves only in the constant R -value domain, but Fig. 2.20 is an example of what a complete CLD looks like using the proposed set of expressions. When static properties in tension and compression are equal, the CLD is symmetric with respect to $R = -1$.

2.7.7 Final notes on S–N curves and the CLD

- Some of the CLDs shown in the previous sections show parallel lines in a large part of the fatigue domain, rather than constant life lines that converge to static strengths. Already in [36] and [37], it was noted that predictions became much more accurate when based on a CLD with such parallel lines. In the region close to $R = 1$, where the fatigue loading resembles creep rather than fatigue (high mean, low amplitude), fatigue data and creep response could be related [15]. As more detailed CLDs have become available in recent years for unidirectional dominated wind turbine laminates [15, 38], the notion that constant life lines converging to UTS may not be realistic has become more obvious, at least in the tensile region.



2.20 CLD associated with reference [35].

- S–N curves and CLDs are usually experimentally determined and used in models for load-bearing laminates. The focus in this chapter has been on these unidirectional-dominated composites. As design becomes ‘leaner’, fatigue characteristics of other laminates become increasingly relevant. These CLDs can have a distinctly different shape or modelling requirements. For non-load-bearing transversely reinforced laminates, the top of the CLD has been reported to be located in the compression-dominated region. Publications of CLDs describing shear fatigue strengths are rare.
- An important aspect of evaluating and validating structural design is to take into account the failure mode. The structural failure mode cannot always be described by coupons. Therefore, full-scale validation of the structure remains necessary. Complementary to this, testing of subcomponents is advisable. Very generally, a subcomponent is a detail of the structure that is smaller in size for easier handling, and can be loaded in such a way that one or more failure modes are evoked, that are representative for the full-scale structure. The principles of S–N curve construction and CLD analysis are equally applicable to sub-scale or full-scale structures.
- Furthermore, additional experimental work is often necessary, assessing material or structural performance under variable amplitude loading, using load sequences or spectra that are representative for the application.
- As fatigue data become available at more R -values, better fatigue life predictions can be made [11, 32]. For calculating life for interpolated S–N curves, the method depends on the S–N curve formulation. For linear–logarithmic S–N curves, the fatigue life can be analytically determined at any point in the CLD from the bounding R -value S–N curves. For log–log S–N curves, an iterative procedure can be used. Both methods are detailed in [39]. Other authors have proposed a method for obtaining the prediction analytically [40].

2.8 Future trends

The state-of-the-art in life prediction on high-cycle composite fatigue was, until relatively recently, based on a limited description of fatigue behaviour of a laminate, like the linear Goodman diagram. It is currently shifting to more elaborate and realistic formulations of the constant life diagram, derived from tests performed in loading types other than zero-mean cyclic loads.

For load-bearing, predominantly unidirectional laminates, an increasingly large database is being created, allowing for such a detailed description of the fatigue behaviour. For other laminates and subcomponent structures, such as multi-axially loaded laminates, bondlines, sandwiches, spar structures, etc., accurate fatigue characterisation is becoming more important as constructions

become more structurally efficient. For these materials and subcomponents, detailed constant life diagrams are also required if the structure's design load contains strongly varying load components. More extensive standardisation of fatigue specimens and subcomponents is required to facilitate the maintenance of a consistent database, or interconsistent databases.

Experimentally deriving a constant life diagram from constant amplitude fatigue tests and fatigue models, nevertheless, is a procedure that is both time consuming and limited in the sense that it is only descriptive. It typically does not include any analysis of the physical mechanisms that drive fatigue. Current micro-mechanical models are not sufficiently mature or robust to reliably predict fatigue behaviour of a laminate or laminated structure.

Future efforts should therefore be aimed at expanding the description of constant amplitude fatigue behaviour, through extended experiments at more loading conditions including variable amplitude, more laminate types and orientations, and more diverse environmental conditions, as well as through validating life prediction methods using these data. Success of (numerical) prediction methods will be validated more and more on subcomponent structures, as a preliminary to full-scale structural verification. On the opposite length scale, micromechanical models have the potential of more efficiently focusing the experimental effort associated with fatigue testing.

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Residual strength fatigue theories for composite materials

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Abstract: This chapter discusses the most common residual strength-based failure theories for composite materials present in the literature. Particular attention is paid to the ability of the models to reproduce experimental observations for multi-stress level (spectrum) loading conditions. Comparisons are also introduced with the familiar Miner's rule results.

Key words: residual strength, fatigue lifetime prediction, composite materials, spectrum loading.

3.1 Introduction

The prediction of fatigue damage and fatigue life for composite materials has been the subject of many investigations during recent years. Hwang and Han (1986) suggested four requirements for a universal fatigue damage model:

1. It should explain fatigue phenomena at an applied stress level.
2. It should explain fatigue phenomena for an overall applied stress range:
 - During a cycle at a high applied stress level the material should be more damaged than that at a low applied stress level.
 - If it is true that failure occurs at each maximum applied stress level, then the final damage (damage just before failure) at a low applied stress level should be larger than that at a high applied stress level.
3. It should explain multi-stress level fatigue phenomena.
4. It is desirable to establish the fatigue damage model without an S–N curve.

An excellent review of work in this area of fatigue life predictions has been given by Liu and Lessard (1994). In this paper, they divided the models used to predict fatigue life into three classes: residual strength degradation, modulus degradation, and damage tolerance approaches. According to Huston (1994), most of the life prediction methods for polymeric composite materials are based on the residual strength degradation. However, he suggested that

theories for fatigue failure based on the reduction of stiffness have one significant advantage over the remaining strength theories: remaining life can be assessed by non-destructive techniques. Also, Huston suggested that less testing needs to be conducted for stiffness-degradation-based models. Despite these potential advantages, residual strength-based theories have found favor because of their relative simplicity, as well as one key physical argument: that in stress-controlled tests, the residual strength-based models provide a clear explanation for failure that the stiffness-based models do not – failure occurs when the instantaneous value of the strength is equal to the instantaneous value of the applied stress. In comparison with damage accumulation models, the residual strength models also have an advantage – because the damage is evaluated in terms of a physical quantity (the residual strength), the models can be fit or verified experimentally at intermediate points during the fatigue lifetime.

In general, residual strength models are based on the assumption that the residual strength is a monotonically decreasing function of cycles applied (Post *et al.*, 2008b). They also enforce that the initial strength is equal to the static strength, and that under constant amplitude fatigue loading, the residual strength at failure ($n = N$) is equal to the applied constant amplitude load. In addition, many residual strength models have been developed with the goal of modeling/predicting the distribution of residual strengths and fatigue lifetimes. Such predictions are most often developed by integrating some rate of degradation relationship into the statistical distributions of initial strength and ultimate lifetimes. The common method of connection is the assumption that there is a one-to-one correspondence between the probability rank of the initial strength distribution and the rank of fatigue lifetime. Such a relationship was termed the strength-life equal rank assumption (SLERA) by Chou and Croman (1978), and had originally been suggested by Hahn and Kim (1975). Because the residual strength evolution equations are monotonic, this implies that the residual strength curves do not cross. While such an argument makes intuitive sense, it cannot be verified experimentally, as both the static strength measurement and the fatigue lifetime measurement are destructive tests – there is no way to make the same measurement on the same composite article.

3.2 Major residual strength models from the literature

In this section, an overview of the major residual strength models in the literature is presented. A more recent review of fatigue lifetime approaches for composites materials has been presented by Post *et al.* (2008b). In it, the authors present a convenient means of describing many of the residual strength based failure theories has been presented by Sarkani *et al.* (2001):

$$\frac{d\sigma_r}{dn} = -B \frac{An^{A-1}}{C\sigma_r^{C-1}} \quad 3.1$$

where σ_r is the residual strength, n is the number of fatigue cycles, and A , B , C are ‘constants’ that may depend upon the applied fatigue stress level. Integration of Equation (3.1) for the case of constant amplitude fatigue, and enforcing that the residual strength is equal to the ultimate strength at zero cycles and the applied stress at failure, results in

$$\sigma_r^C = \sigma_{ult}^C - (\sigma_{ult}^C - \sigma_p^C) \left(\frac{n}{N} \right)^A \quad 3.2$$

where σ_{ult} is the ultimate strength, σ_p is the peak stress during the fatigue cycle, and N is the number of cycles to failure that corresponds to σ_u .

This integrated form will provide the basis for subsequent discussions, which are an abbreviated version of that which appeared in Post *et al.* (2008b); additional information may be found in the full discussion.

3.2.1 Broutman and Sahu

The earliest residual strength model in the literature of which the current authors are aware was that introduced by Broutman and Sahu (1972). In this work, Broutman and Sahu examined the tensile fatigue behavior of E-glass reinforced epoxy. Their experimental work consisted of 35 quasi-static tests, followed by 134 constant amplitude fatigue tests conducted at a fatigue R -ratio of 0.5 ($R = 0.05$). The results of these tests are summarized in Table 3.1. Subsequent to these tests, two-stress-level block fatigue loading tests were conducted. In these two-stress-level tests, samples were fatigued under constant amplitude loading conditions for a predetermined number of cycles. The stress amplitude was then adjusted to a second constant amplitude, and the samples cycled until failure occurred. Broutman and Sahu assumed (and then compared with limited experimental data) that the residual strength at a given fatigue stress amplitude varied linearly with the number of cycles, so that

Table 3.1 Broutman–Sahu constant amplitude fatigue data (E-glass/epoxy, $R = 0.05$)

σ_1 , MPa	$Fa = \sigma/\sigma_{ult}$	N (median)	95% confidence limit	No. specimens
386	0.862	493	420–570	35
338	0.754	2470	2170–2820	31
290	0.646	14,700	12,100–17,590	37
241	0.538	172,200	139,200–213,000	31

Source: Broutman and Sahu (1972).

$$\sigma_r = \sigma_{ult} - \sum_i (\sigma_{ult} - \sigma_p^i) \frac{n_i}{N_i} \quad 3.3$$

(This assumption corresponds to $A = 1$, $C = 1$ in Equation (3.1).) For the special case of two-stress-block loading, it is possible to show that Equation (3.3) results in

$$\left(\frac{1 - Fa_1}{1 - Fa_2} \right) \frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad 3.4$$

where $Fa_1 = \sigma_1/\sigma_{ult}$, $Fa_2 = \sigma_2/\sigma_{ult}$ and N_1 , N_2 are the number of cycles to failure corresponding to σ_1 and σ_2 , respectively.

The results of the Broutman–Sahu experimental study of two-stress-block loading are summarized in Table 3.2 for the case in which $A = 1$. The form of Equation (3.4) makes it easy to compare with the linear damage accumulation (Miner's rule) result (Miner, 1945)

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad 3.5$$

Thus, there are two key related features of the Broutman and Sahu analysis: (1) there is a clear sequence of loading effect on the predicted lifetimes, and (2) the results predicted using linear degradation in residual strength with fatigue cycles are *not* the same as those predicted by linear damage accumulation. In some cases, no experimental cycles occur when the stress level is changed. This result is observed experimentally for the cases in which the stress changes from 241 MPa to 386 MPa and when the stress level changes from 290 MPa to 386 MPa.

In Table 3.3, the Miner's sum calculations for the Broutman and Sahu data are presented. Broutman and Sahu pointed out that for high–low fatigue loading, the calculated Miner's sum could be greater than unity, and that for low–high loading, the calculated Miner's sum could be less than unity. However, in the Broutman–Sahu analysis, the residual strength at the end of the two-step loading process is independent of the order of loading. (This feature was not discussed in the Broutman–Sahu reference.)

There are some key observations that come out of the Broutman–Sahu work. First of all, there is a considerable amount of material characterization required to calibrate the model because of the statistical variation in the initial material strength (and according to the strength–life equal rank assumption, the resulting fatigue lifetime). Secondly, the residual strength approach can provide improved estimates of the fatigue lifetimes under the block loading condition compared with the Miner's rule approach *with no additional experimental characterization*. In essence this added accuracy for the block-loading case comes for free, as long as the variation of residual strength with

Table 3.2 Two-stress-level data of Broutman and Sahu (1972). The original Broutman–Sahu model is obtained from the $A = 1.00$ column of the table

σ_1 , MPa	σ_2 , MPa	Fa_1 = σ_1/σ_{ult}	Fa_2 = σ_2/σ_{ult}	n_1/N_1	n_2 (exper.)	n_2 (Miner's rule)	n_2 , residual strength A , Equation (3.1)				
							0.75	1.00	1.25	5	100,000
386	241	0.862	0.538	0.507	192,000	84,895	154,666	146,008	138,877	103,578	84,896
386	241	0.862	0.538	0.203	183,000	137,243	165,180	161,713	158,858	144,724	137,244
386	290	0.862	0.646	0.507	5840	7247	12,567	11,784	11,182	8522	7247
386	290	0.862	0.646	0.203	11,970	11,716	13,846	13,532	13,291	12,226	11,716
386	338	0.862	0.754	0.507	1250	1218	1889	1766	1680	1354	1218
386	338	0.862	0.754	0.203	1635	1969	2237	2188	2154	2023	1969
338	241	0.754	0.538	0.405	86,000	102,459	142,036	135,005	130,022	110,698	102,459
338	241	0.754	0.538	0.101	162,500	154,808	164,678	162,924	161,682	156,863	154,808
338	290	0.754	0.646	0.405	8670	8747	11,030	10,558	10,247	9163	8747
338	290	0.754	0.646	0.101	8000	13,215	13,785	13,667	13,589	13,319	13,215
290	241	0.646	0.538	0.680	96,500	55,104	90,036	82,426	77,527	61,164	55,104
290	241	0.646	0.538	0.136	110,800	148,781	155,767	154,245	153,265	149,993	148,781
241	290	0.538	0.646	0.290	3730	10437	8625	9140	9427	10,204	10,437
241	290	0.538	0.646	0.116	9490	12,995	12,270	12,476	12,591	12,902	12,995
241	338	0.538	0.754	0.290	931	1754	814	1127	1286	1658	1754
241	338	0.538	0.754	0.116	804	2183	1808	1933	1996	2145	2183
241	386	0.538	0.862	0.290	0	350	0	16	118	311	350
241	386	0.538	0.862	0.116	124	436	208	302	343	420	436
290	338	0.646	0.754	0.680	293	790	0	56	225	664	790
290	338	0.646	0.754	0.136	1290	2134	1925	1987	2021	2109	2134
290	386	0.646	0.862	0.680	0	158	0	0	0	89	158
290	386	0.646	0.862	0.136	355	426	259	322	351	412	426
338	386	0.754	0.862	0.405	297	293	63	138	177	269	293
338	386	0.754	0.862	0.101	503	443	386	404	414	437	443

Table 3.3 Miner’s sum calculations for the two-stress-level data of Broutman and Sahu (1972). The original Broutman–Sahu model is obtained from the $A = 1.00$ column of the table

$Fa_1 = \sigma_1/\sigma_{ult}$	$Fa_2 = \sigma_2/\sigma_{ult}$	n_1/N_1	Miner’s sum A , Equation (3.1)				
			0.75	1.00	1.25	5	100,000
0.862	0.538	0.507	1.405	1.355	1.313	1.108	1.000
0.862	0.538	0.203	1.162	1.142	1.126	1.043	1.000
0.862	0.646	0.507	1.362	1.309	1.268	1.087	1.000
0.862	0.646	0.203	1.145	1.124	1.107	1.035	1.000
0.862	0.754	0.507	1.272	1.222	1.187	1.055	1.000
0.862	0.754	0.203	1.109	1.089	1.075	1.022	1.000
0.754	0.538	0.405	1.230	1.189	1.160	1.048	1.000
0.754	0.538	0.101	1.057	1.047	1.040	1.012	1.000
0.754	0.646	0.405	1.155	1.123	1.102	1.028	1.000
0.754	0.646	0.101	1.039	1.031	1.025	1.007	1.000
0.646	0.538	0.680	1.203	1.159	1.130	1.035	1.000
0.646	0.538	0.136	1.041	1.032	1.026	1.007	1.000
0.538	0.646	0.290	0.877	0.912	0.931	0.984	1.000
0.538	0.646	0.116	0.951	0.965	0.973	0.994	1.000
0.538	0.754	0.290	0.619	0.746	0.810	0.961	1.000
0.538	0.754	0.116	0.848	0.899	0.924	0.984	1.000
0.538	0.862	0.290	0.290	0.323	0.530	0.921	1.000
0.538	0.862	0.116	0.538	0.729	0.812	0.968	1.000
0.646	0.754	0.680	0.680	0.703	0.771	0.949	1.000
0.646	0.754	0.136	0.915	0.941	0.954	0.990	1.000
0.646	0.862	0.680	0.680	0.680	0.680	0.860	1.000
0.646	0.862	0.136	0.661	0.788	0.848	0.972	1.000
0.754	0.862	0.405	0.533	0.685	0.763	0.951	1.000
0.754	0.862	0.101	0.883	0.921	0.941	0.988	1.000

fatigue cycles is approximately linear. Finally, while the predicted lifetime depends upon the order of loading for this two-block case, the residual strength is independent of the loading order.

3.2.2 Reifsnider and Stinchcomb

Reifsnider and Stinchcomb (1986) proposed a ‘critical element model’ for the residual strength of composites under fatigue loading. In this model, the residual strength is assumed to vary as

$$Fr(n) = 1 - \int_0^n (1 - Fa(n))A\left(\frac{n}{N}\right)^{A-1} d\left(\frac{n}{N}\right)$$

3.6

where Fr is the normalized residual strength for the critical element, and Fa is the corresponding value of the failure criterion. The critical element is that portion of the composite laminate whose failure directly implies failure of

the composite laminate. (For additional information on the implementation of this model, the reader is referred to Reifsnider and Case (2002) and Post *et al.* (2006).)

As a starting point, the case in which Equation (3.6) is applied at a laminate level using the maximum stress failure criterion is considered. In such a case Post *et al.* (2006) recast Equation (3.6) in the form of

$$Fr = 1 - \left[\sum_i (1 - Fa_i)^{1/A} \left(\frac{n_i}{N_i} \right) \right]^A \quad 3.7$$

Equation (3.7) is identical to Equation (3.2) if $C = 1$ and A is a constant for the case in which the entire composite is smeared together. This approach greatly simplifies the analysis, as load redistribution effects within the composite do not need to be considered. However, it also requires that laminates made of the same composite material but having different stacking sequences be characterized individually. Wahl and co-workers (Wahl *et al.*, 2001, 2002; Wahl and Mandell, 2001), Nijssen *et al.* (2005) and Post *et al.* (2008a) used Equation (3.7) to varying degrees of success in describing constant R -ratio spectrum fatigue loading. An alternative approach (Post *et al.*, 2006) used the change in elastic modulus recorded during fatigue testing to estimate the stress in the critical element.

Because there is often data available for the two-block loading case (such as the Broutman–Sahu data) it is of interest to examine the predictions of Equation (3.7) for the two-block loading case. This result is given by

$$\left(\frac{1 - Fa_1}{1 - Fa_2} \right)^{\frac{1}{A}} \frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \quad 3.8$$

There is an obvious similarity to the Broutman–Sahu result given in Equation (3.4) as well as the Miner's rule result of Equation (3.5). Indeed, for Miner's rule and the Reifsnider and Stinchcomb residual strength analysis to give consistent results, the value of the residual strength parameter A must approach infinity. Thus, the use of Miner's rule implies *sudden-death material failure* (at least in the context of residual-strength based theories). For all cases where $A > 0$, Equation (3.8) implies that for a low stress level followed by a high stress level, the lifetime is always less than the Miner's rule result, and for a high stress level followed by a low stress level, the resulting lifetime is always less than the Miner's rule result. (Cases where $A < 0$ correspond to physically unreasonable results where the strength increases with the number of fatigue cycles.)

3.2.3 Schaff and Davidson

Schaff and Davidson (1997a, 1997b) used an evolution equation for the residual strength in the form of

$$\sigma_r = \sigma_{ult} - (\sigma_{ult} - \sigma_a) \left(\frac{n}{N} \right)^A \quad 3.9$$

where A is assumed to vary with the magnitude of σ_a as well as with the R -ratio. To account for the observed statistical spreading of residual strength data, they applied a linear reduction to the Weibull shape parameter at each stress level. This method essentially results in a complex curve fit for the statistical strength distribution. A more mathematically consistent model has been proposed by Yang and Liu (1977).

In their first paper, Schaff and Davidson applied the model of Equation (3.9) to two-stress-level repeated block loading fatigue. They noted that in some cases, repeatedly changing the stress level seemed to cause more fatigue damage than applying the same loads in longer blocks. To account for this effect, which they termed the ‘cycle mix effect’, they introduced a *cycle mix factor*, CM, to reduce the residual strength whenever the magnitude of the mean stress increased. This cycle mix factor was given by

$$CM = C_m \sigma_{ult} \left[\frac{\Delta \sigma_m}{\sigma_r} \right] \left(\frac{\Delta \sigma_a}{\Delta \sigma_m} \right)^2 \quad 3.10$$

where C_m is a constant that must be determined from experimental data on a repeat block fatigue loading with different numbers of cycle mix events. Schaff and Davidson found this model appeared to fit their limited dataset well.

In their second paper, Schaff and Davidson applied their model to spectrum loading cases using two sets of data. In both cases, they performed curve fitting of the constant lifetime diagram and a linear interpolation to determine the value of A for each loading cycle. Because they did not have sufficient information to evaluate CM, they assumed that it did not have a significant impact. (Thus, it was not possible to evaluate the effectiveness of this CM parameter in describing spectrum loading behavior.) Nijssen *et al.* (2005) did evaluate the impact of treating A as a function of applied stress. They found that a large data set is required to develop an empirical functional form for this variable. Thus far, it has only been possible to develop the value for a limited number of materials at a limited number of stress levels, so it is not possible to draw general conclusions about the overall form the parameter takes. Nijssen *et al.* (2005) showed that improved predictions were possible for two-stress-level block loading when the values of A were found from

constant amplitude residual strength tests for each stress level. Such a result is not surprising, given that additional parameters have been introduced into the modeling.

3.2.4 Hahn and Kim

Hahn and Kim (1975) as well as Hashin (1985) and Whitney (1981) used residual strength equations equivalent to Equation (3.2) where $A = 1$ and C is an unknown parameter fitted to experimental data (and potentially a function of stress level), so that for the constant amplitude case, the result is

$$\sigma_r^C = \sigma_{ult}^C - (\sigma_{ult}^C - \sigma_p^C) \left(\frac{n}{N} \right) \quad 3.11$$

3.2.5 Other residual strength models

Other residual strength models have been presented in the literature. In particular, the discussion above has focused on deterministic representations of residual strength changes. Yang and Liu (1977) assumed that the initial strength distribution conformed to a two-parameter Weibull representation, and that the residual strength varied according to

$$\frac{d\sigma_r}{dn} = - \frac{H(\sigma_p, f, R)}{C \sigma_r(n)^{C-1}} \quad 3.12$$

where f is the fatigue frequency. The predicted distribution of fatigue lifetime is then given by

$$F_N(n) = 1 - \exp \left[- \left(\frac{n + s_p^C / H(\sigma_p)}{\sigma_0^C / H(\sigma_p)} \right)^{\alpha/C} \right] \quad 3.13$$

where σ_0 and α are the Weibull location parameter and the shape parameter for the initial strength distribution, respectively. Equation (3.13) is then in the form of a three-parameter Weibull distribution where the characteristic fatigue lifetime is given by

$$N_0 = \sigma_0^C / H(\sigma_p) \quad 3.14$$

A similar approach was employed by Sendekyj (1990).

3.3 Fitting of experimental data

In general, variable amplitude fatigue models require empirical fitting of parameters for a given material. Therefore, predictions made with that

model can only be applied to the laminate for which they were fitted. Thus, comparison of the models can take place where the data required for fitting all of the selected models is available, and at least one spectrum case that can be used for verification of the predictions is available on a given material system. Post *et al.* (2008b) analyzed data sets containing fatigue life under stress-controlled tests similar to those typically available for structural design applications. Four different material systems were considered, covering a wide range of E-glass/polymer matrix composites typical of those used for wind turbine blade and naval architecture. These data sets were considered because they each include a statistically significant number of measurements of the initial strength, constant amplitude fatigue to failure when subjected to various R -ratios, and residual strength at various fractions of lifetime during constant amplitude loading. Some of the data sets also contain various cases of block loading. Each data set contains several examples of spectrum loading that enable validation of the predictive capabilities of each model.

The first material data set, denoted DD16, is part of the DOE/MSU database (Mandell, 2004). The tests performed are detailed in Wahl and Mandell (2001), and various analyses of the data are provided in (Mandell *et al.* (2002, 2003), Nijssen *et al.* (2005), Wahl *et al.* (2001, 2002) and Wahl and Mandell (2001). Validation spectra available for the DD16 data set include fatigue to failure under the standard WISPERX spectrum and two modified versions of WISPERX where the valleys of each cycle were adjusted to enforce a constant R -ratio of $R = 0.1$. Following Post *et al.* (2008b), the Wahl and Mandell (2001) notation for these spectra is used: WISPK includes all WISPERX cycles peaks, but adjusts the valleys to require that $R = 0.1$, while WISXR01 includes only the tension–tension WISPERX cycles and forces resulting valleys to $R = 0.1$.

The second and third material data sets are MD2 (R0400 geometry) and UD2 (I1000 geometry). These are part of the Optimat Blades database, publicly available online in Excel format. Spectrum predictions were made for the WISPER and WISPERX standard spectra fatigue to failure experimental results also available in the data set.

The final data set was collected at Virginia Tech, with the details and test results available in Post (2008). This material consists of 10 layers of woven roving E-glass (Vetrotex 324) with a $[0/+45/90/-45/0]_s$ stacking sequence (denoted by the warp direction) in a rubber toughened vinyl ester matrix (Ashland Derakane 8084). This material system is characteristic of those used in US Navy ship construction. The variable amplitude fatigue loading data for this material system includes Rayleigh-distributed loading with 95% autocorrelation (a measure of the degree of load ordering: see Post *et al.* (2008c)) with a nominal value of the fatigue R -ratio, $R = -1$, and the same peaks with the following valleys forced to $R = 0.1$ called RAY95 and RAY95R01, respectively.

In the paragraphs that follow, approaches for fitting the experimental data to provide inputs to the residual strength models are discussed. Since the ultimate goal for practical loading situations is to be able to analyze spectrum loading, each of the residual strength models described above requires a separate empirical model for determining the total number of cycles to failure, N_i , under a constant amplitude stress equivalent to the current applied cycle in the spectrum characterized by the peak stress, σ_p , and the valley stress, σ_v . Wahl *et al.* (2002) and Wahl and Mandell (2001) examined the results of fatigue predictions for two residual strength models and Miner's rule. In doing so, they considered exponential and power law representations, both including and excluding the initial strength data in determining the fitting parameters. They found (not surprisingly) that there can be significant differences in the resulting spectrum fatigue predictions, because the shape of the extrapolated S–N curve in the longer lifetime region is different – often substantially so. While the selection of the S–N fits is important, particularly because of the practical need to extrapolate fatigue lives beyond those measured experimentally, the selection of fits should be on the basis of their ability to represent the constant amplitude data rather than as a ‘tuning’ method in the spectrum fatigue predictions, if the goal is *prediction* rather than *representation*. Sendekyj (1990) identifies a number of the equations used to represent S–N data as

$$\sigma_{\text{ult}}/\sigma_a = N^S \quad 3.15$$

$$\sigma_a = \sigma_{\text{ult}} - b \log N \quad 3.16$$

$$\sigma_{\text{range}} = a + b/N^x \quad 3.17$$

$$\sigma_{\text{range}} = a + b/N^x - c/A^y \quad 3.18$$

$$\sigma_a/\sigma_{\text{ult}} = a + b/(\log N)^x \quad 3.19$$

where

$$A = (1 - R)(1 + R) = \sigma_{\text{range}}/\sigma_{\text{mean}}$$

$$R = \sigma_{\text{min}}/\sigma_{\text{max}} \quad 3.20$$

and the subscripts a, range, mean, max, and min to σ are used to denote the stress amplitude, stress range, mean stress, maximum stress, and minimum stress, respectively. The remaining quantities in the equations are experimentally determined material constants. Equation (3.15) is the classical power-law representation that leads to a straight line on a log–log plot. Equation (3.16) has been used by Mandell (1990) to characterize short fiber glass-reinforced composite materials. (In his work, Mandell has suggested that $b \approx 0.1 \sigma_{\text{ult}}$.) Sendekyj (1990) points out that Equation (3.17) was used by Albrecht (1962) for a large number of metallic systems. Equation (3.18), an extension of Equation (3.17) that includes mean stress effects, was used

by Sims and Brogdon (1977) to represent glass/epoxy fatigue data. Finally, Reifsnider and Jen (1982) used Equation (3.19) to characterize a number of composite materials.

For example purposes, the constant amplitude fatigue data is fitted using Equation (3.15) cast in the form of

$$N = B \|\sigma_p\|^a \quad 3.21$$

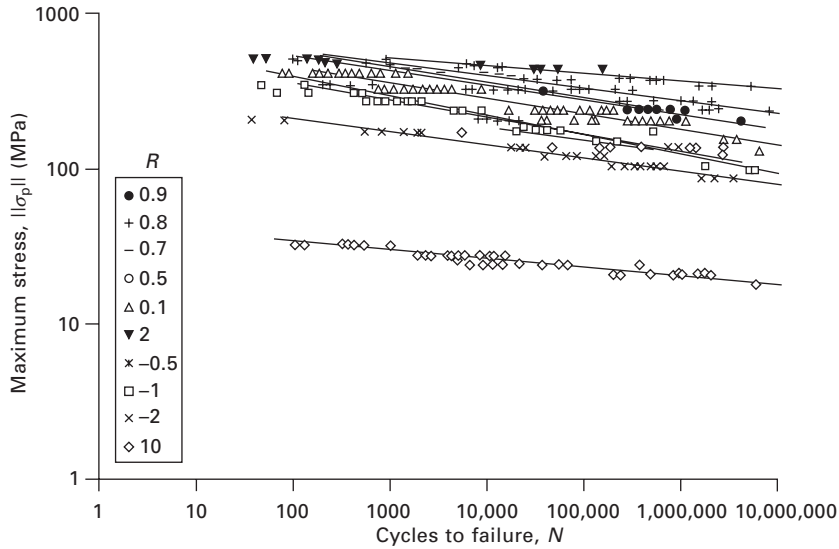
where B and a are fitting parameters, N is the number of cycles to failure, and σ_p is the maximum stress in the applied cyclic loading. Alternatively, Equation (3.21) may be expressed as

$$\log N = a \log \|\sigma_p\| + b \quad 3.22$$

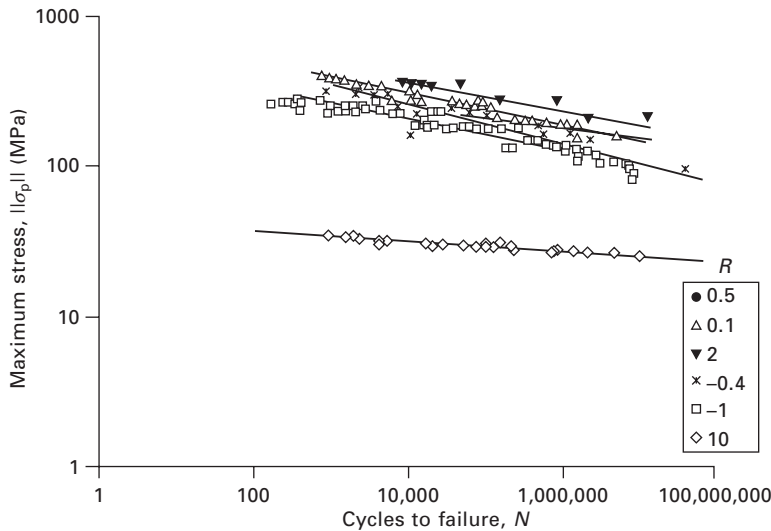
where a and $b = \log B$ are the slope and intercept of a log–log plot of stress versus cycles to failure. (In performing the fitting, we note that the number of cycles to failure, N , is the dependent variable with the stress, σ_p , as the independent variable rather than vice-versa.) Equation (3.22) can be fitted using standard linear regression tools. Johannesson *et al.* (2005) point out that this expression coincides with the maximum likelihood method assuming log-normally distributed errors in the fatigue lifetime data.

The typical literature approach is to use the $\{\|\sigma_p\|, \|\sigma_v\|\}$ for σ_p in Equation (3.22) and thus to calculate N in terms of the minimum stress σ_v for $R < 0$ and $R > 1$ (for cases in which the largest compressive stress magnitude is greater than the largest tensile stress magnitude). However, the absolute value of the maximum stress as indicated in Equation (3.22) may be used because it will simplify the mathematics of interpolation on the constant lifetime diagram. Thus, the values of b that are calculated will be different from those reported elsewhere, and the plotted S–N curves for $R > 1$ appear to be at much lower stresses. However, as long as a consistent approach is applied, this choice will not impact the outcome of the calculations. The S–N curves for each of the four material systems studied are given in Figs 3.1–3.4. In addition the resulting values for the parameters a and b are listed in Table 3.4.

To apply the models for loading conditions in which the R -ratio is not a value that has been characterized experimentally, it is necessary to use the values that have been measured with some scheme to arrive at values for other R -ratios. Some authors (e.g. Epaarachchi and Clausen (2003), Fatemi and Yang (1998), Schaff and Davidson (1997b)) have chosen to curve-fit an equation to the constant lifetime diagram, such as that illustrated in Figs 3.5–3.8. However, because of the shape of the constant lifetime plot, it is unlikely that any function will be able to fit the data available for all data sets. One possible solution is the artificial neural network approach suggested by Vassilopoulos *et al.* (2007). However, this is more complicated than may be desirable for many situations. Post *et al.* (2008b) have suggested

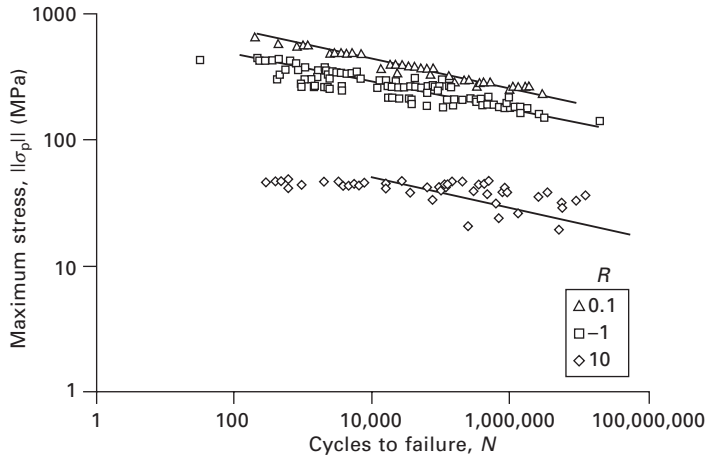


3.1 S–N data for the DOE/MSU DD16 material data set (Post *et al.*, 2008b).

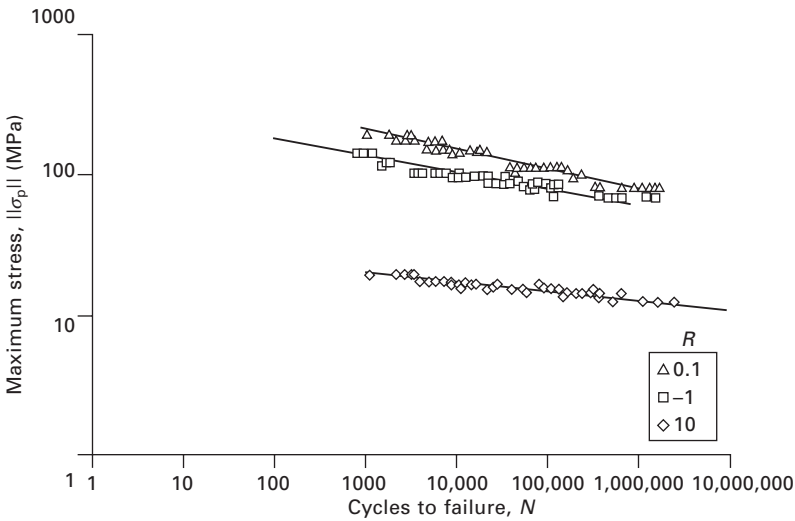


3.2 S–N curves for the Optimat MD2 material data set (Post *et al.*, 2008b).

using linear interpolation on the constant lifetime plot in order to determine lifetimes for a particular set of mean stress and stress amplitude values. This process is illustrated graphically in Fig. 3.9. Here R_1 and R_3 are R -ratios for which S–N data has been collected. The stress amplitude, σ_a , and the

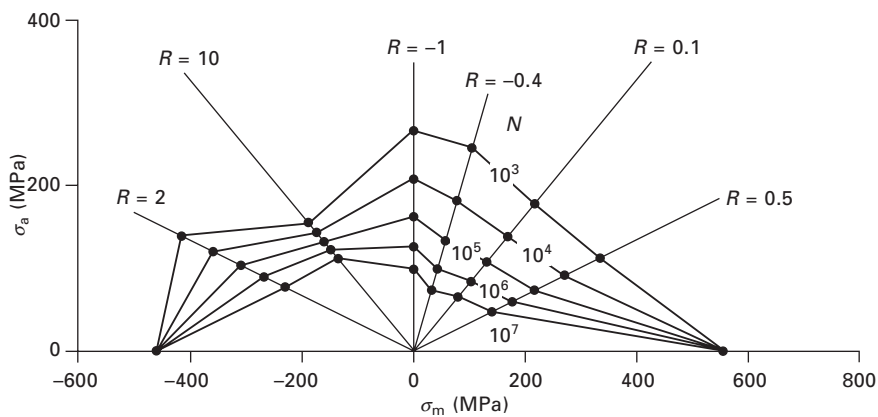


3.3 S–N curves for the Optimat UD2 material data set (Post *et al.*, 2008b).

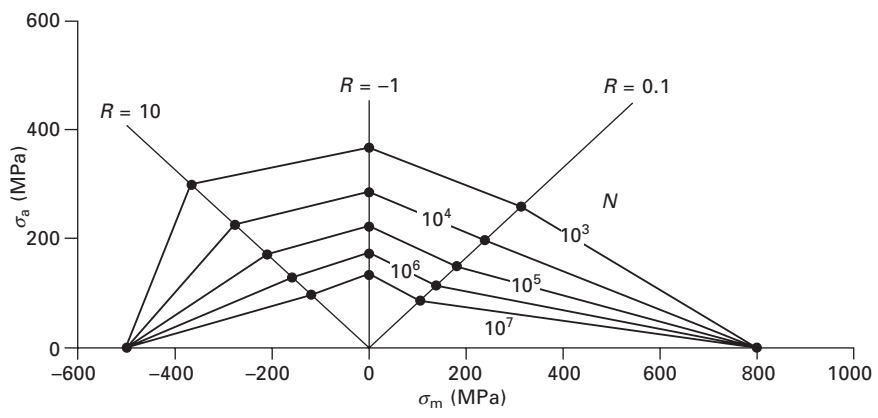


3.4 S–N curves for the Virginia Tech VT8084 material data set (Post *et al.*, 2008b).

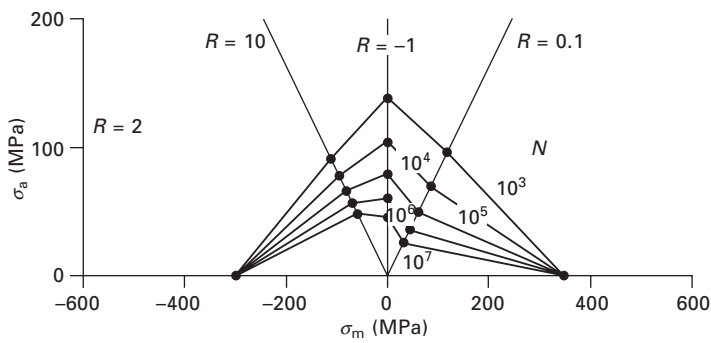
mean stress, σ_m , correspond to an R -ratio at which experimental data has not been collected. A line is then drawn which passes through the points on the line corresponding to R_1 and R_3 at a lifetime N , where N is the lifetime that corresponds to σ_a and σ_m . In their analysis, Post *et al.* solve numerically for the appropriate value of N using a trial-and-error process. Further details are available in Post *et al.* (2008b).



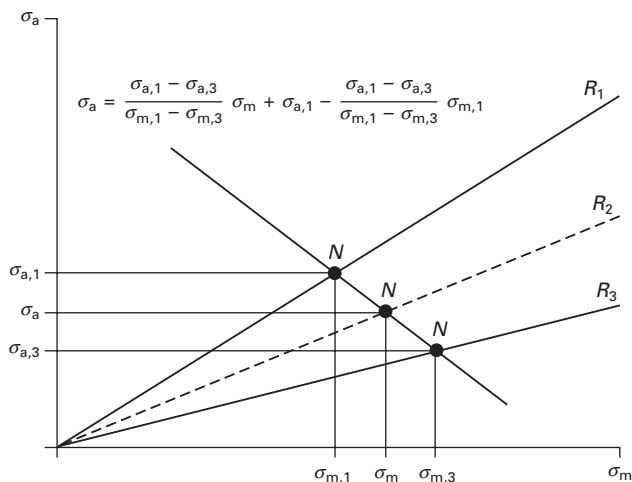
3.6 Constant lifetime plot for the Optimat MD2 material data set (Post *et al.*, 2008b).



3.7 Constant lifetime plot for the Optimat UD2 material data set (Post *et al.*, 2008b).



3.8 Constant lifetime plot for the Virginia Tech VT8084 material data set (Post *et al.*, 2008b).



3.9 Linear interpolation scheme used by Post *et al.* (2008b) on constant lifetime plots to determine lifetimes for R -ratios not tested experimentally.

Once the S–N data has been established, it is then necessary to evaluate the other model parameters that may appear, such as the A and C values in Equation (3.2). The easiest place to begin is with the Broutman–Sahu model given by Equation (3.3). This model requires only the initial strength data in addition to the fatigue lifetimes. The other residual strength models require additional parameter determinations. For example, the Post *et al.* model given by Equation (3.7) requires the fitting of the parameter A , and the Han and Kim model given by Equation (3.11) requires the fitting of C . For these types of models, the parameters A and C which impact the shape of the residual strength curve are fitted to residual strength data collected by interrupted constant amplitude fatigue tests where residual strength was measured. A challenge that arises in the experimental tests is that premature failures may occur. (Such failures are predicted by models of the type given by Equation (3.13).) Exclusion of these premature failures from the residual strength has the effect of biasing the calculated distribution to higher residual strengths. To avoid this bias, a two-parameter Weibull distribution may be fitted to the residual strength distribution at each residual strength measurement point using the approach described by Yao and Himmel (2000). This method considers premature failures in calculating the median rank of the surviving specimens' strength and thus estimates the entire residual strength distribution including the 'imaginary' residual strength that is calculated to be below the applied stress for those specimens that failed prematurely. Then the mean residual strength may be calculated from the Weibull distribution as

$$\sigma_r = \sigma_{r0} \Gamma \left(1 + \frac{1}{\alpha} \right) \quad 3.23$$

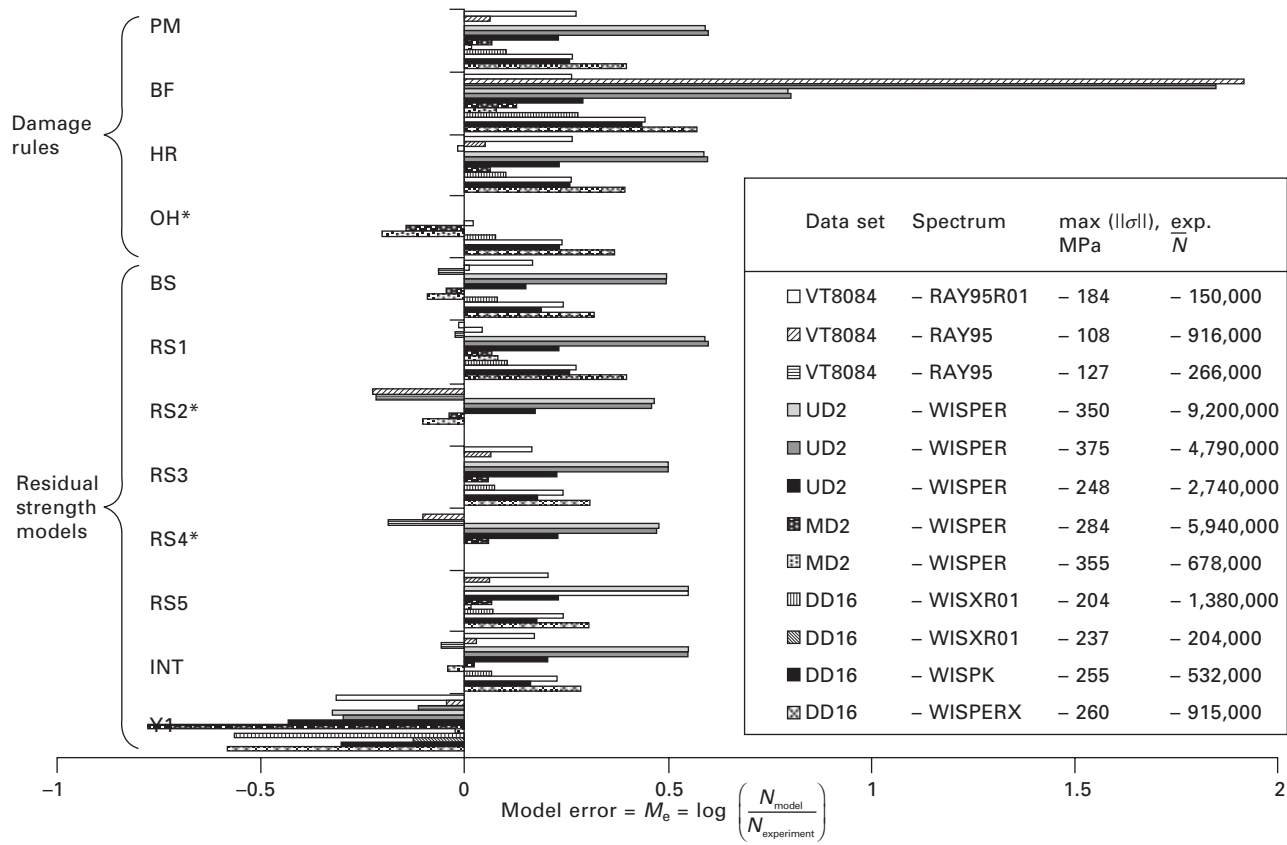
where Γ is the Euler gamma function and α and σ_{r0} are the shape and location parameters. The calculated mean strength is then used to fit the residual strength model parameters by minimizing the least-square error in residual strength between the model and experiment for the relevant mean residual strength points.

3.4 Prediction results

Using this approach, Post *et al.* (2008b) evaluated the performance of 12 lifetime prediction models using the available spectrum loading data for a combination of residual strength and damage accumulation models. These results are summarized in Fig. 3.10. Here we will focus on the results for the residual strength models. The simple residual strength model developed by Broutman and Sahu (1972) gave more conservative predictions of fatigue lifetime than Miner's rule (PM) in all cases because failure occurred at the highest load point in each spectrum. Since Miner's rule generally over-predicted fatigue lifetime, the Broutman–Sahu results were generally better. The Reifsnider and Stinchcomb model (RS1) performed better than Broutman–Sahu for the VT8084 material, particularly for tensile loading, but performed worse than Broutman–Sahu for the DD16, MD2, and UD2 materials under WISPER-type spectra. The Han and Kim model (RS3) performed similarly to RS1. Overall, the simplicity and relatively good accuracy of the Broutman–Sahu model suggest that it should be used in place of the Miner's rule result. It requires no additional characterization, and provides improved accuracy. It is more difficult to assess the performance of the remaining models – they require additional data to calibrate, and those additional data requirements do not appear to translate into improved prediction accuracy – at least not for the cases studied.

3.5 Conclusions and future trends

In this chapter, some of the basic residual strength model implementations have been discussed. In spite of the long history associated with these types of models, progress continues to be made, albeit somewhat slowly. As indicated, the application of fatigue models to real data sets without allowing the models to be 'adjusted' after the fact highlights many of the difficulties associated with them. The phenomenological nature of the models and the scatter inherent in all fatigue data, but particularly that of composite materials, limits general conclusions that can be drawn. At a very minimum, it appears that the Broutman–Sahu approach offers improved predictions over the



3.10 Comparison of fatigue lifetime prediction model performance for models evaluated by Post *et al.* (2008b).

Miner's rule approach without requiring additional characterization testing to be performed. However, because the predictions given by the Broutman–Sahu analysis lead to shorter lifetimes, there is a natural disincentive for designers to adopt it – if they are already struggling to meet safety margins that require different knockdowns for fatigue and statistical variation by themselves, they are not likely to be interested in approaches that require further knockdowns. For this reason, ongoing efforts are focused not only on improving the accuracy of the residual strength predictions, but also on providing designers with systematic calculations of knockdown factors that incorporate fatigue degradation and statistical variability of the material.

The long-term goal is to combine the understanding of environmental effects, fatigue and fatigue spectrum in a unified model to predict material behavior under realistic design conditions. Post *et al.* (2009) suggest that the use of such understandings could be reduced to a series of knockdown factors, k_i , or partial safety factors that could be used in an allowable strength design approach as suggested here, where the applied stresses have to be smaller than the reduced strength.

$$\sigma_{\text{app}} \leq \sigma_{\text{ult}} \prod_i k_i$$

$$\leq \sigma_{\text{ult}} (k_{\text{fatigue}} k_{\text{statistical}} k_{\text{moisture}} k_{\text{temperature}} k_{\text{aging}} k_{\text{UV}} k_{\text{weathering}} k_{\text{scale-up}} \dots) \quad 3.24$$

To examine a reasonably realistic condition, Post *et al.* examined a representative 30-year ship history spectrum, created by NSWCD (Speckart, 2008). In this spectrum, a five-year load spectrum is followed by 25 repetitions of a one-year load spectrum (so that the load levels in the five-year spectrum are greater in magnitude than those in the one-year spectrum), and applied it to the VT8084 material. To calculate a design knockdown factor, they assume that the initial strength distribution for the composite may be represented by a two-parameter Weibull distribution so that

$$P_f = 1 - \exp \left[- \left(\frac{\sigma}{\beta} \right)^\alpha \right] \quad 3.25$$

For a given specified (desired) probability of failure, they can solve for the initial compression strength at this specified probability of failure

$$X_{P_f} = \beta \left[\ln \left(\frac{1}{1 - P_f} \right) \right]^{\frac{1}{\alpha}} \quad 3.26$$

where it is noted that α and β may be determined with a required confidence level based upon the available experimental data. (For the E-glass/vinyl ester composite studied here, the values of α and β are 26.2 and 305 MPa, respectively.) A probability of failure of 10% corresponds to a value of 280 MPa for X_{P_f} . The residual strengths are then calculated as

$$\frac{\sigma_r}{X_{P_f}} = 1 - \left[\int \left(1 - \frac{\sigma_a}{X_{P_f}} \right)^{\frac{1}{j}} \frac{dn}{N} \right]^j \quad 3.27$$

where σ_a is the applied stress in each block of loading, and N is determined from

$$\log N = A \log (Fa) + B \quad 3.28$$

where

$$Fa = \sigma_{\text{peak}}/X_{t,c} \quad 3.29$$

with

$$Fa = \frac{\sigma_a}{X_{P_f}} \quad 3.30$$

The scaled values of the stresses in the spectrum are then adjusted so that the composite is predicted to just survive. This scaling factor is then a function of (1) the spectrum itself, (2) the properties of the composite, and (3) the allowed probability of failure. For the particular values studied here, the corresponding scaling (knockdown factor) is that the maximum stress is allowed to be 73% of the median compression strength for a probability of failure of 10%. This level is significantly higher than the current Navy design practice, and so perhaps offers some opportunity to encourage designers to adopt the approach.

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Fatigue damage modelling of composite materials with the phenomenological residual stiffness approach

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Abstract: In this chapter, the phenomenological residual stiffness models are explored. First, a general background of the framework is given, then some established residual stiffness models are briefly discussed and the numerical implementation of a representative model is explained in detail. Next, the advantages of residual stiffness models in modelling variable amplitude loading are discussed, and finally, the degradation of elastic properties other than longitudinal stiffness is illustrated.

Key words: composites, fatigue, modelling, residual stiffness.

4.1 Introduction

Fatigue damage can be modelled in various ways, and the approach of phenomenological residual stiffness models is one of them. A phenomenological model can be defined as ‘*a theory which expresses mathematically the results of observed phenomena without paying detailed attention to their fundamental significance*’ (Thewlis, 1973), or else ‘*a body of knowledge which relates several different empirical observations of phenomena to each other, in a way which is consistent with fundamental theory, but is not directly derived from theory*’ (Wikipedia, 2008).

The term stiffness degradation refers to the (gradual) degradation of elastic properties, and in particular the axial/longitudinal stiffness, during fatigue loading of fibre-reinforced composite structures. The residual stiffness is the remaining stiffness of the laminate after a certain number of fatigue loading cycles.

In this chapter, the phenomenological residual stiffness models are explored. First, a general background of the framework is given, then some established residual stiffness models are briefly discussed and the numerical implementation of a representative model is explained. Next, the advantages of residual stiffness models in modelling variable amplitude loading are discussed, and finally, the degradation of other elastic properties than longitudinal stiffness is illustrated.

4.2 What are phenomenological residual stiffness models?

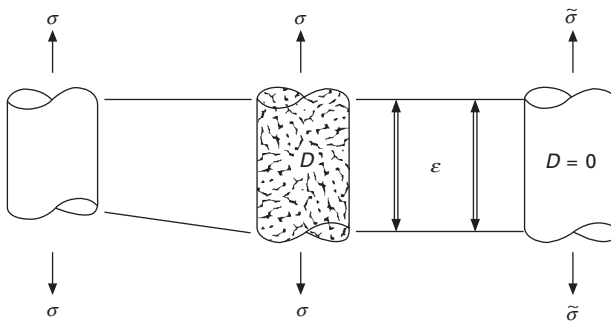
In this section, a general background of the phenomenological residual stiffness models will be given, in particular the basics of continuum damage mechanics theory. One of the major causes of stiffness degradation is distributed matrix cracking, and such type of progressive damage suggests the use of a continuum damage model to describe the material behaviour. Therefore, the vast majority of the phenomenological residual stiffness models are based on the continuum damage mechanics theory.

In 1958, Kachanov proposed to describe brittle creep rupture under uniaxial tension by a scalar field variable, the continuity ψ (Kachanov, 1958, 1986). To a completely defect-free material was ascribed the condition $\psi = 1$, whereas $\psi = 0$ was defined to characterize a completely destroyed material with no remaining load-carrying capacity. The continuity ψ may be said to quantify the absence of material deterioration. The complementary quantity $D = 1 - \psi$ is therefore a measure of the state of deterioration or damage. For a completely undamaged material $D = 0$, whereas $D = 1$ corresponds to a state of complete loss of integrity of the material structure. The symbol ω is also commonly used in the literature.

Later on, Lemaitre introduced the concept of strain equivalence, which states that a damaged volume of material under the nominal stress σ shows the same strain response as a comparable undamaged volume under the effective stress $\tilde{\sigma}$ (Lemaitre, 1971). Applying this principle to the elastic strain, the relation is (Krajcinovic and Lemaitre, 1987):

$$\epsilon_e = \frac{\tilde{\sigma}}{E_0} = \frac{\sigma}{E_0(1 - D)} \quad 4.1$$

where E_0 is the modulus of elasticity for the undamaged material. Figure 4.1 schematically shows the strain equivalence concept. As such, the damage variable D becomes a measure of stiffness degradation:



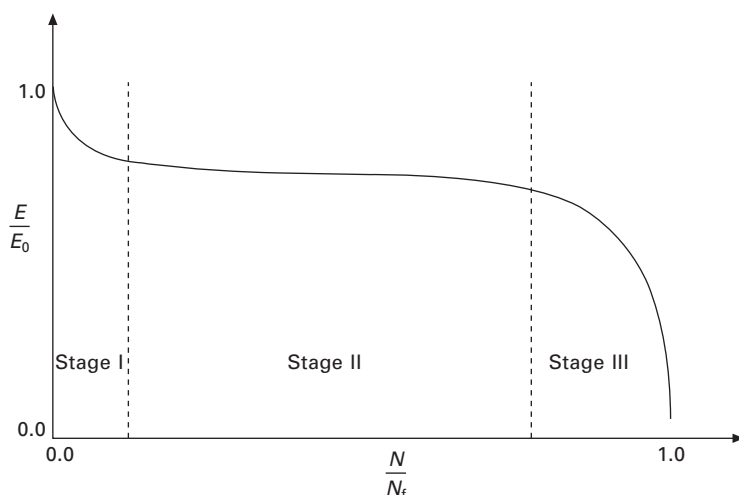
4.1 Schematic representation of the strain equivalence concept.

$$D = 1 - \frac{E}{E_0} \quad 4.2$$

Research was further elaborated, among others, by Sidoroff (1984), Krajcinovic (1985), Krajcinovic and Lemaitre (1987) and Chaboche (1988a, 1988b). These efforts evolved in the ‘Continuum Damage Mechanics’ theory, which can generally be defined as ‘... *mechanical and phenomenological models of the material degradation leading to failure and aimed at durability predictions and including mechanical weakening*’ (Sidoroff, 1984).

Early investigations on stiffness degradation were conducted by the research groups of Schulte (Schulte, 1984; Schulte *et al.*, 1985, 1987) and Reifsnider (O’Brien and Reifsnider, 1981; Highsmith and Reifsnider, 1982; Reifsnider, 1987). Schulte thoroughly studied the damage development of carbon/epoxy specimens with stacking sequence $[0^\circ/90^\circ/0^\circ/90^\circ]_{2s}$ during tension–tension fatigue ($R = 0.1$) (Schulte, 1984; Schulte *et al.*, 1985, 1987). Schulte distinguished three distinctive stages in the stiffness reduction curve for these specimens (for a schematic drawing see Fig. 4.2):

- The initial region (stage I) with a rapid stiffness reduction of 2–5%. The development of transverse matrix cracks dominates the stiffness reduction ascertained in this first stage.
- An intermediate region (stage II), in which an additional 1–5% stiffness reduction occurs in an approximately linear fashion with respect to the number of cycles. Predominant damage mechanisms are the development of edge delaminations and additional longitudinal cracks along the 0° fibres.



4.2 Typical stiffness degradation curve for a wide range of fibre-reinforced composite materials.

- A final region (stage III), in which stiffness reduction occurs in abrupt steps ending in specimen fracture. In stage III, a transition to local damage progression occurs, when the first initial fibre fractures lead to strand failures.

These three stages in the stiffness degradation curve can be distinguished for a wide variety of composite materials, although the extent of the individual stages can strongly depend on the layup of the composite laminate.

Summarized, residual stiffness models describe the degradation of the elastic properties during fatigue loading. To describe stiffness loss, the variable D is often used, which in the one-dimensional case is defined through the well-known relation $D = 1 - (E/E_0)$. It may be noticed that, although D is often referred to as a damage variable, the models are classified as phenomenological models and not as progressive damage models, when the damage growth rate dD/dN is expressed in terms of macroscopically observable properties, and is not directly based on the actual damage mechanisms.

Further, stiffness can be measured frequently or even continuously during fatigue experiments, and can be measured without further degrading the material (Highsmith and Reifsnider, 1982). The residual stiffness model may be deterministic (a single-valued stiffness property is predicted) or statistical (predictions are for stiffness distributions).

One of the important outcomes of all established fatigue models is the lifetime prediction. Residual stiffness models are dealing with different definitions of 'failure' and already in the early 1970s, Salkind suggested drawing a family of S–N curves, being contours of a specified percentage of stiffness loss, to present fatigue data (Salkind, 1972). In another approach, fatigue failure is assumed to occur when the modulus has degraded to a critical level which has been defined by many investigators. Hahn and Kim (1976) and O'Brien and Reifsnider (1981) state that fatigue failure occurs when the fatigue secant modulus degrades to the secant modulus at the moment of failure in a static test. According to Hwang and Han (1986b), fatigue failure occurs when the fatigue resultant strain reaches the static ultimate strain.

In the design stage, residual stiffness models can be applied to optimize the stacking sequence for a maximum allowed stiffness degradation over the lifetime. Of course, the residual stiffness models should then be developed on the level of one individual ply, not on the level of the complete laminate. Indeed, residual stiffness models are sometimes calibrated for the whole laminate, but as a consequence, the material constants must be determined again for every new stacking sequence. If the residual stiffness model has been built up for the individual ply, it can be used to assess the effect of different stacking sequences on the residual stiffness, and hence the lifetime, of the composite structure.

4.3 Literature review of some representative residual stiffness models

This section focuses on some representative *residual stiffness* models for the fatigue behaviour of fibre-reinforced polymers. For an exhaustive review of different fatigue modelling approaches, the reader is referred to other references (Goetchius, 1987; Reifsnider, 1990; Stinchcomb and Bakis, 1990; Sendekyj, 1990; Saunders and Clark, 1993; Degrieck and Van Paeppegem, 2001). The author has chosen to preserve the style of the equations (symbols, notations, etc.) as used by the researchers themselves, because the familiarity of the reader with the commonly known models could be lost when changing the symbols and notations of the equations.

One of the pioneering phenomenological residual stiffness models was proposed by Sidoroff and Subagio (1987). They developed the following model for the damage growth rate:

$$\frac{dD}{dN} = \begin{cases} \frac{A(\Delta\varepsilon)^c}{(1-D)^b} & \text{intension} \\ 0 & \text{in compression} \end{cases} \quad 4.3$$

where the variable $D = 1 - (E/E_0)$; A , b and c are three material constants to be identified from experiments, and $\Delta\varepsilon$ is the applied strain amplitude. The model was applied to the results from three-point bending tests on glass-epoxy unidirectional composites under fixed load amplitudes. The model of Sidoroff and Subagio (1987) was later adopted by other researchers, but often in terms of stress amplitude instead of strain amplitude, probably because most fatigue tests in laboratory conditions are load-controlled.

Vieilleveigne *et al.* (1997) defined the damage growth rate as:

$$\frac{dD}{dN} = K_d \frac{\sigma^m}{(1-D)^n} \quad 4.4$$

where σ is the local applied stress, m and n are fixed parameters, while K_d depends on the dispersion. In the compression regime, dD/dN was again assumed to be zero. The formula was applied to three-point bending tests.

Kawai (1999) modified the model for off-axis fatigue of unidirectional carbon fibre-reinforced composites:

$$\frac{d\omega}{dN} = \frac{K(\sigma_{\max}^*)^n}{(1-\omega)^k} \quad 4.5$$

where ω represents the damage D , while K , n and k are material constants and $\sigma_{\max}^*$ is a non-dimensional effective stress corresponding to a maximum fatigue stress.

Hwang and Han (1986b) introduced the concept of the ‘fatigue modulus F ’, which is defined as the slope of applied stress and resultant strain at a specific cycle. The fatigue modulus degradation rate is assumed to follow a power function of the number of fatigue cycles:

$$\frac{dF}{dn} = -Acn^{c-1} \quad 4.6$$

where A and c are material constants. Further they assumed that the applied stress σ_a varies linearly with the resultant strain in any arbitrary loading cycle, so that:

$$\sigma_a = F(n_i) \cdot \varepsilon(n_i) \quad 4.7$$

where $F(n_i)$ and $\varepsilon(n_i)$ are the fatigue modulus and strain at loading cycle n_i , respectively. After integration and introducing the strain failure criterion, the fatigue life N can be calculated as:

$$N = [B(1 - r)]^{1/c} \quad 4.8$$

where $r = \sigma_a/\sigma_u$ is the ratio of the applied cyclic stress to the ultimate static stress, and B and c are material constants.

Hwang and Han (1986a) proposed three cumulative damage models based on the fatigue modulus $F(n)$ and the resultant strain. These cumulative damage models have been used by Kam *et al.* (1997, 1998) to study the fatigue reliability of graphite/epoxy composite laminates under uniaxial spectrum stress using statistical methods.

Whitworth (1987) proposed a residual stiffness model for graphite/epoxy composites:

$$\left(\frac{E(N^*)}{E(0)} \right)^a = 1 - H \left(1 - \frac{S}{R(0)} \right)^a N^* \quad 4.9$$

where $N^* = n/N$ is the ratio of applied cycles n to the fatigue life N , S is the applied stress level, $R(0)$ is the static strength, $E(0)$ is the initial modulus, and a and H are parameters which are independent of the applied stress level.

Later, Whitworth (1998) proposed a new residual stiffness model, which follows the degradation law:

$$\frac{dE^*(n)}{dn} = \frac{-a}{(n+1)[E^*(n)]^{m-1}} \quad 4.10$$

where $E^*(n) = E(n)/E(N)$ is the ratio of the residual stiffness to the failure stiffness $E(N)$, n is the number of loading cycles, and a and m are parameters that depend on the applied stress, loading frequency, etc. By introducing the strain failure criterion, the residual stiffness $E(n)$ can be expressed in terms

of the static tensile strength S_u and a statistical distribution of the residual stiffness can then be obtained, assuming that the static ultimate strength can be represented by a two-parameter Weibull distribution. Similar models have been proposed by Yang *et al.* (1990, 1992) and Lee *et al.* (1996).

Hansen (1997, 1999) developed a fatigue damage model for impact-damaged woven fabric laminates, subjected to tension–tension fatigue:

$$\beta = A \int_0^N \left(\frac{\varepsilon_e}{\varepsilon_0} \right)^n dN \quad \beta \leq \beta_{\text{lim}} \quad 4.11$$

where N is the number of cycles, ε_e is the effective strain level and ε_0 the reference strain level, and A and n are constants. The damage variable β is related to the elastic properties by the relations:

$$\begin{aligned} E &= E_0(1 - \beta) \\ \nu &= \nu_0(1 - \beta) \end{aligned} \quad 4.12$$

The experiments revealed that the tension–tension fatigue behaviour of the woven composites was affected by the low-energy impact damage for high-cycle fatigue (moderate or low peak load levels), but not for low-cycle fatigue (high peak load levels). Infrared thermography, which monitors the heating by internal losses and friction within the damaged regions, was found to be very successful in detecting damage initiation and growth.

Brøndsted *et al.* (1997a, 1997b) extended stiffness reduction to the lifetime prediction of glass fibre-reinforced composites. The predictions were based on experimental observations from wind turbine materials subjected to constant amplitude loading, block loading and variable amplitude loading. The material is a four-layer fabric with a chopped strand mat on both sides. The stiffness change is calculated as:

$$\frac{d\left(\frac{E}{E_1}\right)}{dN} = -K \left(\frac{\sigma}{E_0} \right)^n \quad 4.13$$

where E is the cyclic modulus after N cycles, E_1 is the initial cyclic modulus, E_0 is the static modulus, σ is the maximum stress, and K and n are constants. This expression is based on their observed relationship between the stiffness and fatigue cycles in the second stage of the stiffness degradation curve:

$$\frac{E}{E_1} = AN + B \quad 4.14$$

where the stress dependence of the parameter A is assumed to be a power law relationship. The researchers supposed that the stiffness change is history independent. The model can then be utilized to predict the lifetime for variable amplitude loading conditions.

This limited overview gives the reader an impression of how phenomenological residual stiffness models can look, but the numerical implementation of these models is rarely discussed in detail. In the next section, this numerical implementation is discussed and some typical results are shown to highlight the features of residual stiffness modelling.

4.4 Numerical implementation of residual stiffness models

Most published residual stiffness models are only applied to uniaxial loading with constant stress amplitude (typically tension–tension fatigue). In that particular case, the numerical evaluation of the residual stiffness model is straightforward, because the stress amplitude is constant for all material points and there is no stress redistribution. Unfortunately this loading condition is very rare in real composite structures, and in general, stress and strain amplitude are not constant, neither in space nor in time. In the latter case, the numerical implementation of residual stiffness models is much more cumbersome, and calculation times can run up very high.

This section discusses the numerical implementation of a phenomenological residual stiffness model that is applied to displacement-controlled cantilever bending of a composite laminate. It should help the reader to understand the general framework of numerical implementation and the specific problems that arise when dealing with residual stiffness models. The particular loading case was chosen to compare later with experimental results.

4.4.1 General approach

The model of Sidoroff and Subagio (1987) (Equation (4.3)) will be used as the *generic, representative model* to study the numerical implementation of the class of phenomenological residual stiffness models for several reasons:

- The model has a straightforward correlation with residual stiffness through the relation $D = 1 - E/E_0$, which is one of the basic equations of continuum damage mechanics.
- The model has been applied to four-point bending fatigue experiments (Sidoroff and Subagio, 1987). Although the simulations were limited to analytical calculations with classical beam theory, the predictions were rather accurate.
- The model has also been used by other researchers, although they replaced the strain amplitude by (some measure of) the stress amplitude.

Following these other researchers, the author has chosen to define the stress amplitude as the main parameter in the differential equation for damage growth rate. The layout of the representative phenomenological residual stiffness model finally becomes:

$$\frac{dD}{dN} = \begin{cases} \frac{A \left(\frac{\Delta\sigma}{\sigma_{TS}} \right)^c}{(1-D)^b} & \text{in tension} \\ 0 & \text{in compression} \end{cases} \quad 4.15$$

where

D = local damage variable

N = number of cycles

$\Delta\sigma$ = amplitude of the applied cyclic loading

σ_{TS} = tensile strength

A , b and c are three material constants.

It is clear that such a damage model can only be applied when each local material region in the composite structure is subjected to either tension–tension fatigue or compression–compression fatigue. If certain material areas are loaded in tension–compression, the assumption of zero stiffness degradation in compression cannot hold.

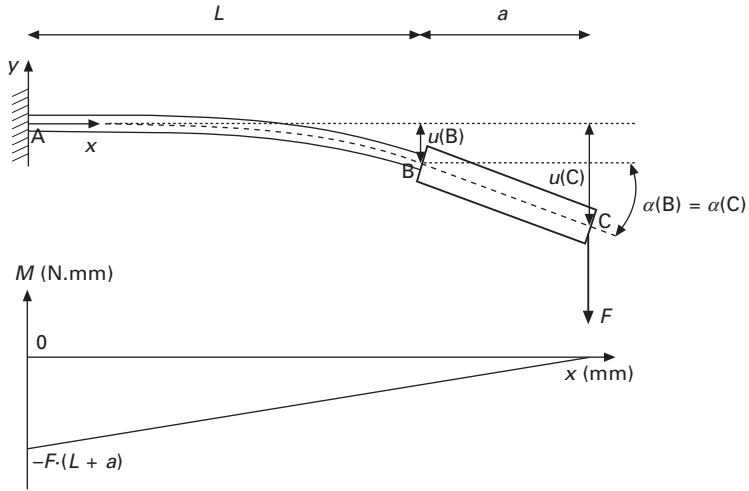
Besides the fatigue damage model, constitutive equations for stress and strain are needed to complete the modelling framework. The constitutive equations for stresses and strains are based on the classical linear beam theory (valid for small displacements). The composite specimen is considered as a bending cantilever beam, connected with a rigid rod (the lower clamp of the specimen) where a sinusoidally varying displacement $u(C)$ is applied. The force F is the necessary force to impose this bending displacement. Figure 4.3 illustrates the beam model and the bending moment distribution along the specimen length.

The stresses σ_{xx} and strains ϵ_{xx} in the composite beam are calculated for a two-dimensional grid of integration points in the (x, y) -plane. Sidoroff and Subagio (1987) have calculated that the stresses and strains for a bending beam with damage distribution $D(x, y)$ can be written as:

$$\begin{aligned} \epsilon_{xx}(x, y) &= \frac{-M(x)(y - y_0(x))}{EI(x)} \\ \sigma_{xx}(x, y) &= E_0(1 - D(x, y)) \epsilon_{xx}(x, y) \end{aligned} \quad 4.16$$

It is very important to take into account in Equation (4.16) a possible shift of the neutral axis $y_0(x)$. When the axial force is supposed to remain zero and when only a bending moment exists, the position of the neutral axis $y_0(x)$ at each moment of time is calculated as (Sidoroff and Subagio, 1987):

$$y_0(x) = \frac{\int_{-h/2}^{+h/2} [1 - D(x, y)] y \, dy}{\int_{-h/2}^{+h/2} 1 - D(x, y) \, dy} \quad 4.17$$



4.3 Bending of the cantilever composite beam following the conventions of the classical beam theory.

where

y = thickness-coordinate, with $y = 0$ at mid-thickness of the specimen
 h = total thickness of the specimen.

According to Sidoroff and Subagio (1987), the degraded bending stiffness $EI(x)$ becomes (with b the specimen width):

$$EI(x) = bE_0 \int_{-h/2}^{+h/2} [1 - D(x, y)]y^2 dy \quad 4.18$$

However, the author has shown that the expression (4.18) is not correct. The complete derivation can be found in Van Paepegem *et al.* (2005). Here only the correct expression for $EI(x)$ is recalled:

$$EI(x) = bE_0 \left[\int_{-h/2}^{h/2} (1 - D)y^2 dy - \frac{\left(\int_{-h/2}^{h/2} (1 - D)y dy \right)^2}{\int_{-h/2}^{h/2} (1 - D) dy} \right] \quad 4.19$$

It is obvious that the correct Equation (4.19) will be used for all subsequent calculations.

Since the bending fatigue experiments are displacement-controlled, the corresponding force F , necessary to impose the bending displacement, must be determined. The unknown force F can be solved from the equation (Van Paepegem *et al.*, 2005):

$$u_{\max} = u(C) = F \left(a \int_0^L \frac{L + a - x}{EI(x)} dx + \int_0^L dx \int_0^L \frac{L + a - x'}{EI(x')} dx' \right) \quad 4.20$$

where

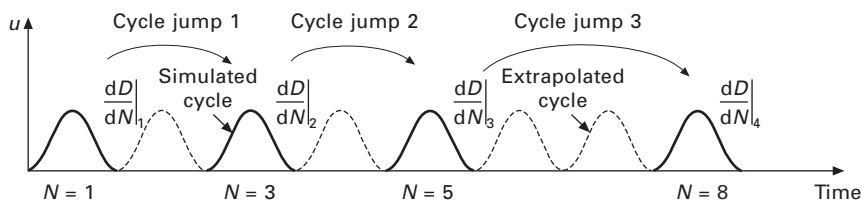
F = force measured by the strain gauges and acting on the hinge (point C in Fig. 4.3)

a = length of the moving clamp (Fig. 4.3).

This numerical model can be easily implemented in a mathematical software package such as Mathcad™. First, the distribution of the bending moment along the length of the specimen is determined. Secondly, the stresses and strains in each integration point can be calculated. The damage law is applied and a new cycle is evaluated. From Equation (4.20) the necessary force to impose the displacement with amplitude $u_{\max}$ can be calculated for each cycle. This algorithm must be solved for each calculated loading cycle, and as a consequence, the Mathcad™ worksheet must be called several times. The remaining problem is that the numerical implementation deals with two important, but contradictory demands:

- In order to correctly predict the damage and residual stiffness of the composite specimen after a certain number of cycles, the simulation should trace the complete path of successive damage states to keep track of the continuous stress redistribution.
- As it is impossible to simulate each of the hundreds of thousands of loading cycles for a real construction, or even a part of it, the numerical simulations should be fast and computationally efficient.

The widely used solution for this problem is that the computation is done for a certain set of loading cycles at deliberately chosen intervals, and the effect on the stiffness degradation of these loading cycles is extrapolated over the corresponding intervals in an appropriate manner (Van Paepegem *et al.*, 2001). Figure 4.4 illustrates the cycle jump principle: the cycles in continuous line are fully simulated, the damage growth rates dD/dN are evaluated, the cycle jump is determined, and the computation is restarted at the next simulated loading cycle with altered damage and stiffness properties.



4.4 Illustration of the cycle jump principle.

The value of the cycle jumps $NJUMP (= N_{i+1} - N_i)$ should not be *fixed* in advance, but should be determined by an automated criterion. It is clear that the evaluation of the cycle jump size is the possible Achilles heel of the concept: when the cycle jump size is determined very conservatively, the simulation will be rather accurate, because the differential equation is integrated with very small increments of the cycle number N . However, the computational effort will be huge, because almost every single cycle would be simulated. On the other hand, when the cycle jump $NJUMP$ is too large, the predicted damage values for the next loading cycle $N+NJUMP$ will be rather different from the exact solution of the differential equation.

To automate the choice of this set of cycle numbers where the damage growth law is evaluated for each integration point, a criterion could be imposed to the stress components, to the damage variable(s) or to some weighted combination of them. It appears now that the damage curves (damage evolution vs. number of cycles) have some favourable properties as compared to the stress curves (stress evolution vs. number of cycles). Indeed, although the damage curves of the integration points can be rather different in shape, they have two important advantages for extrapolation, compared to the stress curves:

- The value of the damage variable D is always lying between known values: *zero* (virgin state material) and *one* (complete failure of the material).
- The gradient of dD/dN has to be positive or zero. The curve can never decrease, because the damage state reached can no longer be reversed. On the other hand, depending on stress redistributions, stresses can increase or decrease without any foreknowledge. It is obvious that it would be very difficult to extrapolate the stress value beyond the current loading cycle for the stress curves. There exists no single master curve to fit the stress curve and allow simple extrapolation.

Therefore, the damage value D will be used as a measure for determining the cycle jumps. To implement the cycle jump concept in the numerical MathcadTM model, each integration point has been assigned – besides the damage variable D – a second state variable $NJUMP1$, which is the number of cycles that could be jumped over without losing accuracy on the piecewise integration of the fatigue damage law dD/dN (Equation (4.15)) for that particular integration point. This state variable $NJUMP1$ is called the *local* cycle jump, as opposed to the *global* cycle jump $NJUMP$ which will be applied to all integration points.

Now it is matter of defining the criterion for calculating the local cycle jumps $NJUMP1$. One possible and straightforward method to tackle the problem is to use the simple Euler explicit integration formula for evaluating the local increase of damage for each integration point:

$$D_{N+NJUMP1} = D_N + \left. \frac{dD}{dN} \right|_N \cdot NJUMP1 \quad 4.21$$

The value of the local cycle jump NJUMP1 can then be determined by imposing a maximum allowed increase to the damage variable D . For example, $D_{N+NJUMP1}$ can be limited to $D_N + 0.01$, when the D -values are in the range $[0, 1]$.

Based on the same philosophy, more accurate numerical techniques can be applied. For example, the damage D could be numerically extrapolated to $D + 0.1$, taking into account the full damage-cycle history information, instead of using only the last known damage value with the Euler method. However, this does not limit the applicability of the approach presented here, and more advanced extrapolation techniques are discussed in Van Paeppegem *et al.* (2005).

Since each fatigue loading cycle represents a physical amount of time t_f ($t_f = 1/\text{frequency}$), the size of the *global* cycle jump NJUMP must be the same for all simulated parts of the structure under fatigue, otherwise the next simulated loading cycle $N+NJUMP$ would not be the same for all simulated parts of the composite structure. Thus, the global cycle jump NJUMP for all integration points should of course be a single-valued property. The simplest approach is to define NJUMP as the minimum value of all NJUMP1 values, but this is not recommended, because normally at each moment in the fatigue life of the composite construction, there are integration points with a fast-increasing damage variable D . Hence the NJUMP1 value will be small. As a consequence, the global cycle jump NJUMP will always be small and the calculation will proceed too slowly. Therefore it is assumed that the global cycle jump NJUMP can best be taken as a small percentile of the cumulative relative frequency distribution of all NJUMP1 values. As the global cycle jump NJUMP is the same for all integration points of the finite element mesh, a small percentage of these integration points will have a larger cycle jump than the NJUMP1 value that was considered to be safe for these integration points. However, these integration points will just be the ones that are already seriously damaged and where extrapolation errors will be rather negligible.

Figure 4.5 shows a flowchart of the final numerical implementation in the MathcadTM/MathconnexTM environment.

4.4.2 Typical results

The cycle jump implementation has been applied to the bending fatigue test on a plain woven glass fabric/epoxy laminate with eight plies with 45° orientation ($[45^\circ]_8$), because these specimens show a very gradual degradation of the stiffness and the stresses are relatively small. As mentioned earlier,

Given: $u_{\max}$: prescribed displacement E_0 : Young's modulus
 L : free specimen length b : specimen width
 a : length moving clamp h : specimen height

$$\text{Solve } F \text{ from } u_{\max} = F \cdot \left(a \cdot \int_0^L \frac{L+a-x}{E_0 I} dx + \int_0^L dx \int_0^x \frac{L+a-x'}{E_0 I} dx' \right)$$

► $M(x, N) = F(N) \cdot [(L+a) - x]$
 $\forall$ integration points

$$\begin{cases} \varepsilon_{xx}(x, y) = \frac{M(x)(y - y_0(x))}{EI(x)} \\ \sigma_{xx}(x, y) = E_0(1 - D(x, y)) \varepsilon_{xx}(x, y) \\ \frac{dD(x, y)}{dN} = \begin{cases} A \cdot \left(\frac{\sigma_{xx}(x, y)}{\sigma_{TS}} \right)^c & \text{in tension} \\ 0 & \text{in compression} \end{cases} \end{cases}$$

Calculate local *cycle jump* NJUMP1

$$\text{NJUMP1} = \frac{D(x, y)_{N+N\text{JUMP1}} - D(x, y)_N}{\left. \frac{dD(x, y)}{dN} \right|_N} = \frac{\Delta D}{\left. \frac{dD(x, y)}{dN} \right|_N}$$

NJUMP = percentile frequency distribution (NJUMP1)

$\forall$ integration points

$$\begin{cases} \text{Update damage } D \\ \text{► } D(x, y)_{N+N\text{JUMP}} = D(x, y)_N + \left. \frac{dD(x, y)}{dN} \right|_N \cdot \text{NJUMP} \end{cases}$$

Calculate $y_0(x)$

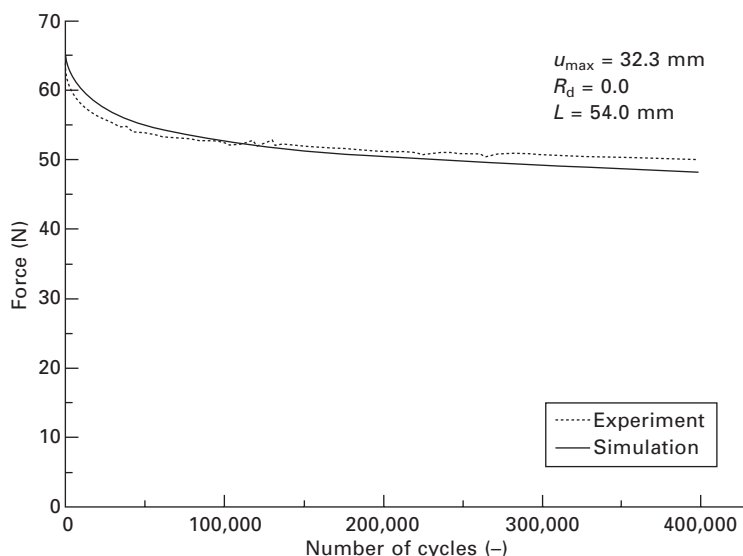
Calculate $EI(x)$

$$\text{Solve } F \text{ from } u_{\max} = F \cdot \left(a \cdot \int_0^L \frac{L+a-x}{EI(x)} dx + \int_0^L dx \int_0^x \frac{L+a-x'}{EI(x')} dx' \right)$$

4.5 Flowchart of the Mathcad™ implementation.

this implementation is a one-dimensional implementation and the $[\#45^\circ]_8$ specimens are only characterized by the longitudinal stiffness E_{xx} and the longitudinal static strength σ_{TS} .

Figure 4.6 shows the experimental and simulated results for the cantilever bending fatigue of the $[\#45^\circ]_8$ specimen Pr04_4. The prescribed displacement $u_{\max}$ is 32.3 mm, the displacement ratio R_d is zero (no fully reversed bending) and the length L is 54.0 mm. The measured force initially equals 65.3 N, the elapsed number of cycles in the experiment is 400,000 and the measured force then equals 50.1 N. With regard to the simulation, the composite specimen is modelled with a mesh of 400 integration points. The values of the constants A , b , c and σ_{TS} (see Equation (4.15)) are respectively 1.32×10^{-3} [1/cycle], 0.45 [–], 8.3 [–] and 201.2 [MPa]. The values of A , b and c are determined with a combined numerical–experimental technique. The



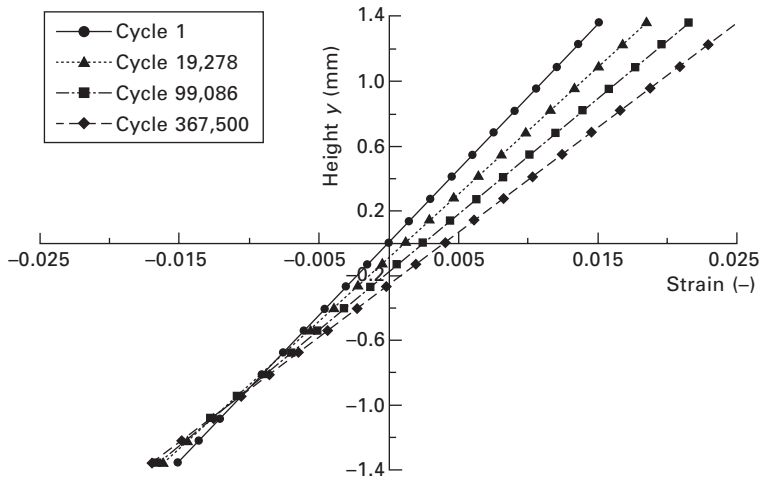
4.6 Degradation of the force versus the number of cycles for the $[\#45^\circ]_8$ specimen Pr04_4.

parameters are optimized with a non-linear optimization procedure to fit the experimental results of this experiment. The predicted force at 400,000 cycles is 48.2 N, which results in an error of 3.8% after 400,000 cycles.

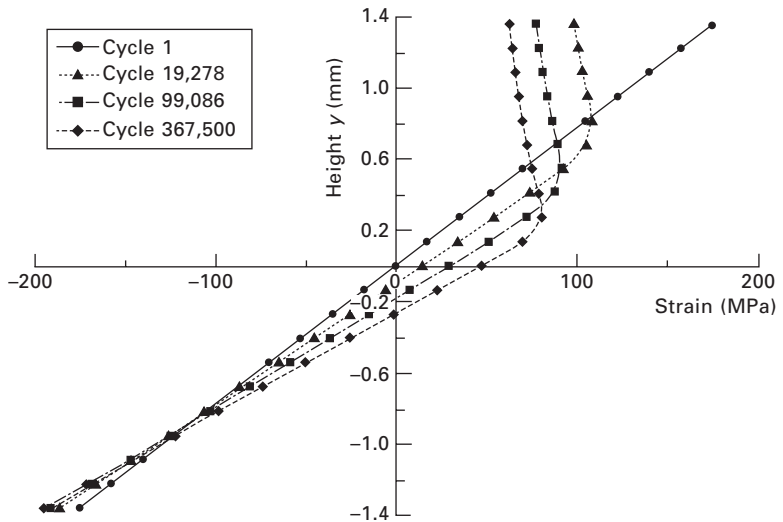
The results from this experiment can be used now for a more detailed discussion of the simulated strain, stress and damage distributions in the composite specimen during fatigue life.

Figure 4.7 shows the strain distribution over the cross-section near the clamped end at a few decisive stages in the fatigue life. The ordinate axis represents the height y (mm) in the cross-section. The specimens have a thickness of 2.72 mm, so the coordinates of the integration points are in the range $(-1.36 \text{ mm}, +1.36 \text{ mm})$.

At the first cycle, the tensile and compressive strain are equal and the strain is zero in the middle of the cross-section. When damage is growing, the neutral fibre is moving towards the compression side of the specimen. The influence of damage can be observed even more clearly when the distribution of the stresses is calculated, as shown in Fig. 4.8. At the first cycle, the stress distribution is linear and the normal stress is zero in the middle of the cross-section. When damage is initiating at the side that is loaded in tension, the tensile stresses in these layers are reduced, because the loading is displacement-controlled and the local stiffness has decreased. As a consequence, the load is transferred towards the inner layers. Because the damage law assumes that there is no damage growth at the compressive side (see Equation (4.15)), the peak tensile stresses are moving towards



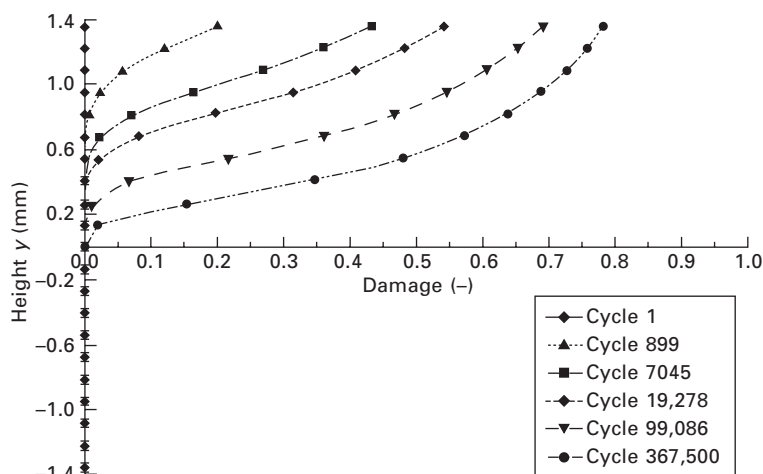
4.7 Strain distribution in the clamped cross-section.



4.8 Stress distribution in the clamped cross-section.

the compression side and as a consequence the neutral fibre is moving down.

Finally, Fig. 4.9 shows the damage growth in the integration points at the clamped cross-section. The value of the damage variable D is lying between zero (no damage) and one (final failure of that integration point). As the damage variable is a measure of the stiffness degradation, and not a direct measure of the physically perceived damage mechanisms (matrix cracking,



4.9 Damage distribution in the clamped cross-section.

fibre/matrix interface failure, fibre fracture), the predicted damage values can only be compared in a qualitative sense with the experimental observations. In this case, the damage distribution qualitatively corresponds with the experimental observations, where the outermost layers at the tension side are damaged while the compression side shows no observable damage. However, in quantitative terms, the damage values at the tensile side are too high. These values would indicate considerable damage ($D \approx 0.8$) of the whole tensile area, but such severe damage could not be observed experimentally for the $[\#45^\circ]_8$ specimens. Of course, this is due to the fact that a one-dimensional residual stiffness model is applied to a stacking sequence with only 45° plies, where the local stress state in each individual ply is not one-dimensional and strongly dominated by shear stresses. As a consequence the degradation of the shear modulus should also be taken into account.

4.5 Variable amplitude loading

In studying the fatigue behaviour of fibre-reinforced composites, it is of course best to simulate the in-service fatigue loading conditions as closely as possible. In the mechanical load–time history acting on a component in service, loading is virtually always of variable amplitude and only rarely of constant amplitude. Nevertheless, fatigue testing is still being carried out under constant amplitude, this choice being dictated largely by the expensive and time-consuming nature of the variable amplitude experiments, the limitations of standard fatigue testing facilities and the uncertainties about the in-service loading spectrum (Schutz, 1981). Moreover most in-service loading spectra

are stochastic in nature (e.g. wind spectrum) and not all loading spectra can be simulated in laboratory conditions.

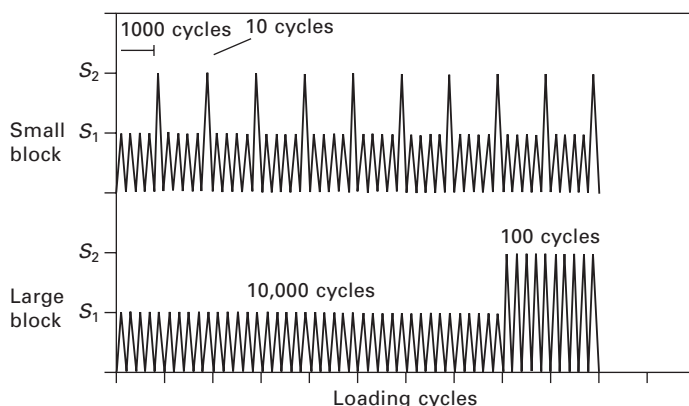
One of the most complicating factors in variable-amplitude loading is the experimentally observed 'load sequence effect': a different fatigue life of composite components under low-high and high-low load sequences. To investigate the load sequence effect, block loading experiments are the most commonly used experiments. Cycle blocks with constant load amplitude level are imposed and the effect of their sequence on the fatigue life of the composite component is investigated.

When studying the literature about this subject, there is only one general conclusion to be drawn: there is no agreement at all which load sequences have the worst effect on fatigue life. For example, in 1998, Bartley-Cho *et al.* wrote: '*For composites, these tests reveal a load sequence effect where a low-high loading sequence results in a shorter fatigue life than a high-low loading sequence.*' In 2000, Gamstedt and Sjögren claimed: '*In an experimental investigation, the interaction of these mechanisms has shown why a sequence of high-low amplitude level results in shorter life-times than a low-high order.*' From the literature review in Van Paepegem and Degrieck (2002d) about load sequence effects, it can be concluded that the opinions are strongly divided. Moreover it is very difficult to assess the generality of these experimental observations, because different materials, lay-ups and block loading conditions have been used in each experimental workplan.

Further, there is the damaging effect of frequent transitions from low to high mean stress. This 'cycle mix effect' was described in detail by Farrow (1989) and means that the residual strength and the fatigue life of composite laminates have been observed to decrease more rapidly when the loading sequence is repeatedly changed after only a few loading cycles (Fig. 4.10).

The cumulative damage under subsequent block loadings is usually evaluated using residual life theory or residual strength theory (Hashin, 1985). Hashin (1985) has shown that the residual life and residual strength cumulative damage theories are completely equivalent since an assumed functional form of residual strength curves determines a damage function and thus the residual life. Such cumulative damage theories are then used to assess the fatigue life under block loading and spectrum loading.

In the next subsection, the residual stiffness model developed by the author will be used to demonstrate qualitatively that cumulative damage rules are not needed when applying residual stiffness models in terms of damage growth rate equations for dD/dN to the problem of variable amplitude loading. Indeed, the damage growth rate equation for dD/dN is simply integrated over the various loading blocks. The description will be limited to the one-dimensional formulation.



4.10 Schematic representation of the 'cycle-mix effect' (Schaff and Davidson, 1997a).

4.5.1 Outline of phenomenological residual stiffness model by Van Paepegem and Degrieck (2002a, 2002b, 2002c)

As mentioned above, the three stages in the stiffness degradation curve can be distinguished for a wide variety of composite materials. It is therefore somewhat surprising that – to the author's knowledge – damage growth rate equations of residual stiffness models have rarely been expressed with a clear distinction between damage initiation and damage propagation. On the other hand, research on impact behaviour of composites at the author's department has shown that damage growth rate equations which discriminate between damage initiation and damage propagation lead to very good results (Seaman, 1989; Dechaene *et al.*, 2000; Leus, 1999; Verleysen, 1999). Therefore, the author proposed to establish a fatigue damage growth law of the generic form:

$$\frac{dD}{dN} = f_i(\sigma^*, D, \dots) + f_p(\sigma^*, D, \dots) \quad 4.22$$

where σ^* is some measure of the applied stress, and the damage variable D is a measure of the stiffness reduction in the considered material element due to matrix cracks, fibre/matrix debonding, fibre pull-out, etc. The growth rate dD/dN is the damage increment per cycle N , f_i is a function which describes the initial stage of damage initiation, and f_p is a function which describes the second and third stages of damage propagation and final failure.

The stress measure σ^* should represent, in each material point, the actually applied stress which can vary during fatigue life (for example, due to stress redistributions). Its relative importance depends on the value of the related

strength as well. To include the notion of strength into the fatigue damage model, the well-known Tsai–Wu failure criterion is used in a modified way. The stress σ is replaced by the effective stress $\tilde{\sigma}$ ($= \sigma/(1 - D)$), see Equation (4.1)) in the one-dimensional Tsai–Wu criterion, and the corresponding *fatigue* failure index $\Sigma(\sigma, D)$ is calculated from:

$$\left(\frac{\sigma}{\Sigma(1 - D)} \right)^2 \frac{1}{X_T |X_C|} + \frac{\sigma}{\Sigma(1 - D)} \left(\frac{1}{X_T} - \frac{1}{|X_C|} \right) - 1 = 0 \quad 4.23$$

It can be easily calculated that the roots of Equation (4.23) are $\Sigma = \sigma/[X_T(1 - D)]$ and $\Sigma = -\sigma/|X_C|(1 - D)$. Depending on the sign of the nominal stress σ , the *fatigue* failure index $\Sigma(\sigma, D)$ can be written as:

$$\Sigma(\sigma, D) = \frac{\tilde{\sigma}}{X} = \frac{\sigma}{1 - D} \stackrel{\text{Eq. (4.1)}}{=} \frac{E_0 \varepsilon}{X} \begin{cases} X = X_T & \text{if } \sigma \geq 0 \\ X = -|X_C| & \text{if } \sigma < 0 \end{cases} \quad 4.24$$

In fact, for one-dimensional loading, the fatigue failure index could be simply defined as the ratio of the effective stress to the static strength, without making any mention of the Tsai–Wu failure criterion. However, the Tsai–Wu criterion was also used to extend the definition of the fatigue failure index to multiaxial loading (Van Paepegem and Degrieck, 2002b).

Further, although the fatigue failure index $\Sigma(\sigma, D)$ has been derived from a stress-based failure criterion, it is basically a measure of the applied strain (see Equation (4.24)). Hence, this stress measure solves the stress–strain ambiguity and can be accepted as a suitable stress measure σ^* . The main characteristics of the fatigue failure index $\Sigma(\sigma, D)$ are:

- It is proportional to the effective stress $\tilde{\sigma}$, or equivalently to the applied strain ε .
- Its value is dimensionless and lies between zero ($\tilde{\sigma} = 0$) and one ($\tilde{\sigma} = X_T$ or $\tilde{\sigma} = -|X_C|$).
- Final failure under fatigue loading can be predicted by introducing the fatigue failure index into the equation for dD/dN .
- By using a static failure criterion, the extension of the fatigue failure index definition to multiaxial loading conditions is possible.
- The definition of the fatigue failure index does not require any new relations to be established between the residual tensile/compressive strength and the fatigue damage.

The detailed derivation of the damage growth rate equations for the one-dimensional fatigue loading can be found in Van Paepegem (2002) and Van Paepegem and Degrieck (2002a, 2002b). The definitive form of the damage initiation and propagation functions is the following:

$$\frac{dD}{dN} = \begin{cases} c_1 \Sigma \cdot \exp\left(-c_2 \frac{D}{\sqrt{\Sigma}}\right) + c_3 D \Sigma^2 [1 + \exp(c_5 (\Sigma - c_4))] & \text{if } \sigma \geq 0 \\ \left[c_1 \Sigma \cdot \exp\left(-c_2 \frac{D}{\sqrt{\Sigma}}\right) \right]^3 + c_3 D \Sigma^2 \left[1 + \exp\left(\frac{c_5}{3} (\Sigma - c_4)\right) \right] & \text{if } \sigma < 0 \end{cases} \quad 4.25$$

where c_i ($i = 1, \dots, 5$) are material constants.

Further, it is worth mentioning that this fatigue damage model obeys the generally accepted ‘strength–life equal rank assumption (SLERA)’. This assumption was first formulated by Hahn and Kim (1975) as follows: ‘A specimen of a certain rank in the fatigue life distribution is assumed to be equivalent in strength to the specimen of the same rank in the static strength distribution.’ A few years later, Chou and Croman (1978) first called this assumption the ‘strength–life equal rank assumption’, which implies that for a given specimen its rank in static strength is equal to its rank in fatigue life. This assumption applies well to the presented fatigue damage model, because if X_T for a certain specimen is smaller, the failure index $\Sigma(\sigma, D)$ will be larger (smaller reserve to failure) and the damage growth rate will be larger. Hence, the fatigue life will be reduced.

The one-dimensional fatigue damage model has since been extended for fully reversed loading and multiaxial loading conditions. The details can be found in Van Paepegem and Degrieck (2002c, 2002d, 2002e, 2002f, 2005).

4.5.2 Application to variable amplitude loading

Here, it will be shown qualitatively that the fatigue damage model can simulate the load sequence and cycle mix effects without any modification.

Although many researchers have used the experimental data of Broutman and Sahu (1972) to benchmark their models, these data could not be used here, because the results published in Broutman and Sahu’s paper are not adequate to determine the five material constants c_i ($i = 1, \dots, 5$) in Equation (4.25). Indeed, the present fatigue damage model is basically a residual stiffness model, and the cycle history of the stiffness should be available to estimate the material constants. Therefore the present model will be applied to the plain woven glass/epoxy material which was used in the author’s experiments. Naturally, the following results are intrinsic to the material used, due to the introduction of the five material constants c_i ($i = 1, \dots, 5$) in Equation (4.25). Nevertheless, the trend of the results is valuable for a broad range of fibre-reinforced composites, because the general layout of the damage growth rate equation for dD/dN is characteristic of many fibre-

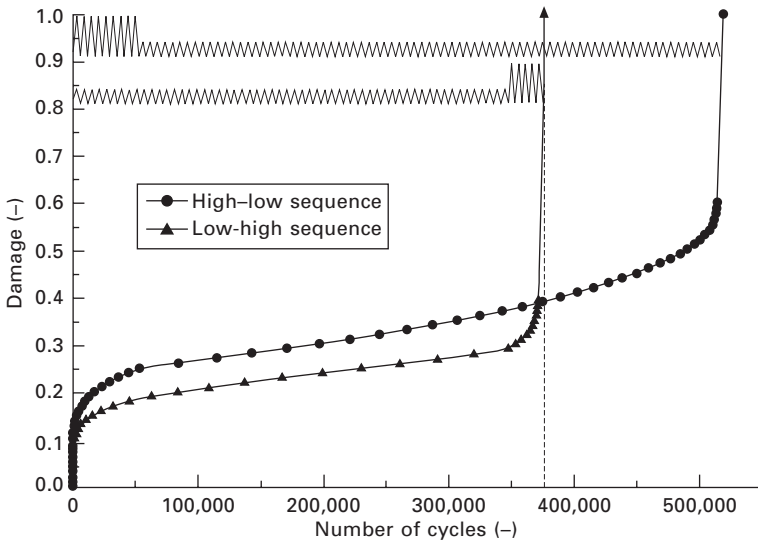
reinforced composite materials: a damage initiation phase (matrix cracks), followed by a stage of gradual propagation and finally ultimate failure.

Before simulating the block loading experiments, the fatigue life N_{fi} for certain constant amplitude stress levels σ_i (0.3, 0.4, 0.6 and $0.7 \times X_T$) is calculated by applying the residual stiffness model until the damage value reaches 1.0. In Table 4.1, the applied stress levels σ_i and the corresponding number of cycles to failure N_{fi} are listed.

Next a high–low and a low–high load sequence is simulated, where the second block is applied when the Palmgren–Miner’s sum of the first block equals 0.5. Thus, for the high–low load sequence, 53,663 cycles at stress level $0.6X_T$ are followed by a runout at stress level $0.4X_T$, while for the lowhigh load sequence, 347,610 cycles at stress level $0.4X_T$ are followed by a runout at stress level $0.6X_T$. The damage evolution for both load sequences is shown in Fig. 4.11.

Table 4.1 Fatigue life for different constant amplitude stress levels in zero-tension fatigue

$\frac{\sigma_i}{X_T}$ ($X_T = 390.7$ MPa)	Number of cycles to failure N_{fi}
0.3	1,533,400
0.4	695,220
0.6	107,326
0.7	22,220



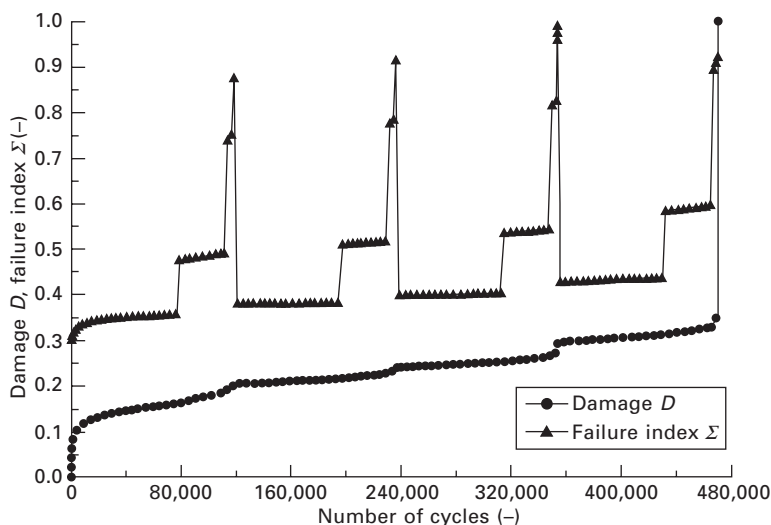
4.11 Load sequence effect on damage-cycle history for high–low and low–high load sequences.

Failure is predicted at 376,660 cycles for the low–high load sequence and at 541,253 cycles for the high–low load sequence. The corresponding Miner’s sum at failure equals 0.77 and 1.162, respectively.

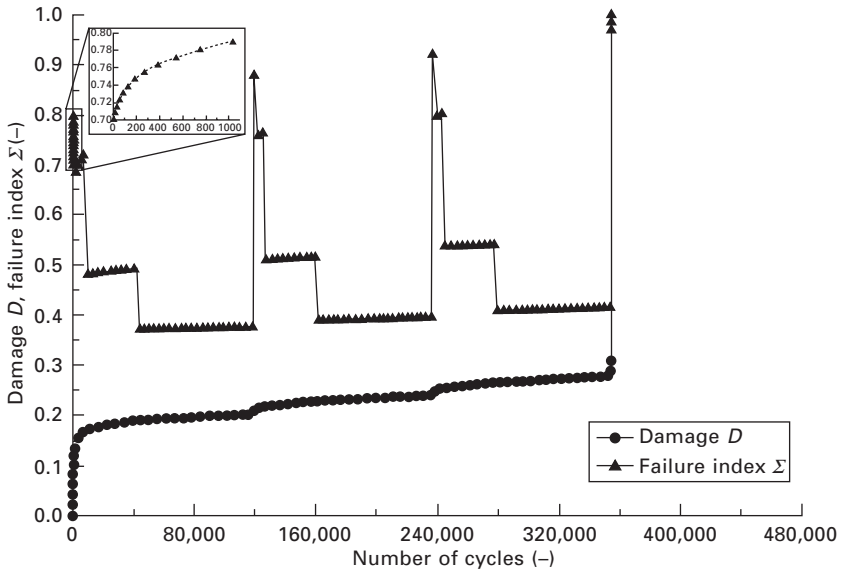
The transition from a low stress level to a high stress level is here more damaging than from high to low, because when damage is already present, the effective stress $\tilde{\sigma} (= \sigma/(1 - D))$ increases more than the applied nominal stress σ . For instance, after the low level block, the damage D equals 0.25, so when the nominal stress σ is raised from $0.4X_T$ to $0.6 X_T$, the failure index is increased not by 0.2, but by 0.266.

Next, the model is applied to the four-unit block loading scheme that was used by Gathercole *et al.* (1994) and Adam *et al.* (1994): four-unit blocks with respective stress levels of $0.3, 0.4, 0.6$ and $0.7 \times X_T$ are applied once in ascending order and once in descending order. The fractional life of each separate unit is 5% ($= n_i/N_{fi}$), so that the Palmgren–Miner’s sum of a complete four-unit block is 0.20. The data are summarized in Table 4.2.

Figures 4.12 and 4.13 show the simulated cycle history of damage and fatigue failure index for the low–high and high–low order of the four-unit block load sequence, respectively. As observed by Adam *et al.* (1994), the fatigue life for the high–low order ($N_f = 354,651$ cycles) is predicted to be considerably shorter than for the low–high order ($N_f = 470,522$ cycles), and in agreement with the observations by Adam *et al.*, failure occurs for both cases at the start of a new block with the highest stress level.



4.12 Cycle history of damage and fatigue failure index for low–high ascending order of the four-unit block load sequence.



4.13 Cycle history of damage and fatigue failure index for high-low descending order of the four-unit block load sequence.

Table 4.2 Four-unit block loading simulations

	Applied stress levels	Length cycle blocks	Miner's sum
	$\sigma_1 = 0.3$ $X_T = 117.2$ MPa	$n_1 = 76,670$ cycles	$\sum n_i = 117,908$ cycles $\sum \frac{n_i}{N_{fi}} = 4 \times 0.05 = 0.2$
	$\sigma_2 = 0.4$ $X_T = 156.3$ MPa	$n_2 = 34,761$ cycles	
	$\sigma_3 = 0.6$ $X_T = 234.4$ MPa	$n_3 = 5,366$ cycles	
	$\sigma_4 = 0.7$ $X_T = 273.5$ MPa	$n_4 = 1,111$ cycles	
	$\sigma_1 = 0.7$ $X_T = 273.5$ MPa	$n_1 = 1,111$ cycles	$\sum n_i = 117,908$ cycles $\sum \frac{n_i}{N_{fi}} = 4 \times 0.05 = 0.2$
	$\sigma_2 = 0.6$ $X_T = 234.4$ MPa	$n_2 = 5,366$ cycles	
	$\sigma_3 = 0.4$ $X_T = 156.3$ MPa	$n_3 = 34,761$ cycles	
	$\sigma_4 = 0.3$ $X_T = 117.2$ MPa	$n_4 = 76,670$ cycles	

In the author's opinion, it may therefore be concluded that there is no general statement that low-high load sequences are more or less damaging than high-low load sequences. It strongly depends on the amplitude difference between low and high stress level (compared to the static strength values), and the frequency with which these transitions occur during the fatigue life time ('cycle-mix effect'). The present simulations provide strong evidence that due to the stiffness degradation during fatigue life, the loading history should be fully simulated to correctly take into account all load sequence effects. Besides, once the fatigue damage model has been elaborated, no experimental damage accumulation rules or 'cycle-mix factors' should be

applied, since the damage evolution under each stress level is predicted by the damage growth rate equation for dD/dN .

4.6 Degradation of other elastic properties

Most one-dimensional damage models for fibre-reinforced composites only account for the effect of damage on the stiffness degradation (Hwang and Han, 1986b; Sidoroff and Subagio, 1987; Whitworth, 1987; Yang *et al.*, 1990; Vieilleigne *et al.*, 1997; Kawai, 1999; Brøndsted *et al.*, 1997a, 1997b). The degradation of the Poisson's ratio, transverse stiffness or in-plane shear modulus is neglected in most cases. Nevertheless this degradation has been frequently observed experimentally and is not negligible.

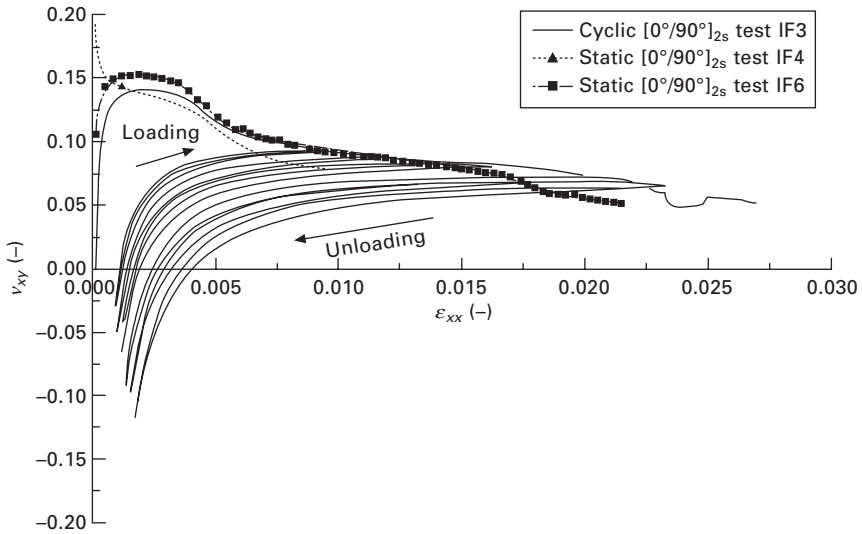
In this section experimental evidence is discussed for the degradation of the Poisson's ratio, the in-plane shear modulus and the degradation of elastic properties in biaxial fatigue loading. Efforts should be increased to include the degradation of these elastic properties in a more generalized form of the residual stiffness modelling approach.

4.6.1 Degradation of Poisson's ratio

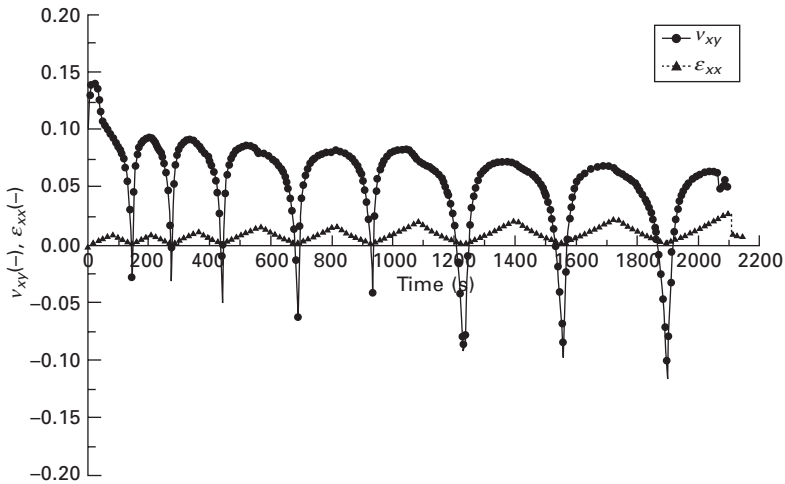
Poisson's ratio is a little like the ugly duckling. Interest in the degradation in fatigue of this elastic property is very small. Nevertheless Bandoh *et al.* (2001) showed that the Poisson's ratio of a carbon/epoxy UD laminate can drop by 50% under static tensile loading, while Pidaparti and Vogt (2002) proved that Poisson's ratio is a very sensitive parameter to monitor fatigue damage in human bone. Here, it is proved that the Poisson's ratio can be used as a sensitive indicator of damage in fibre-reinforced composites for both static, cyclic and fatigue loading. Just like the stiffness, it can be measured accurately and non-destructively. Further, it gives information about the damage state of the off-axis plies in a multi-directional composite laminate.

In a first step, quasi-static loading–unloading cycles are performed on a $[0^\circ/90^\circ]_{2s}$ glass/epoxy laminate. In Fig. 4.14, the evolution of the Poisson's ratio ν_{xy} is plotted against the longitudinal strain ϵ_{xx} , together with its evolution in the static tensile tests IF4 and IF6. It can be clearly seen that the maxima of the cyclic ν_{xy} curves follow the static curve very well for the ϵ_{xx} range $[0; 0.015]$. As the Poisson's ratio changes drastically during unloading, its value must be stress dependent because no further damage occurs during unloading.

Figure 4.15 shows the corresponding time history of the Poisson's ratio ν_{xy} . In the region of low forces (and thus low strains ϵ_{xx}), the Poisson's ratio ν_{xy} becomes negative, due to the slightly positive value of the transverse strain ϵ_{yy} for small loading values. Although this peculiar behaviour of the Poisson's ratio has been observed for several other specimens, it can be



4.14 Evolution of the Poisson's ratio ν_{xy} depending on the longitudinal strain ε_{xx} for the quasi-static cyclic test of $[0^\circ/90^\circ]_{2s}$ specimen IF3.



4.15 Time history of the Poisson's ratio ν_{xy} for the quasi-static cyclic test of $[0^\circ/90^\circ]_{2s}$ specimen IF3.

questioned whether this is an intrinsic material behaviour or an artefact of the measurement method, for example due to the effect of the multiple transverse cracks in the 90° plies on the bonding quality of the transverse strain gauge. Therefore three measurement methods have been compared: (i)

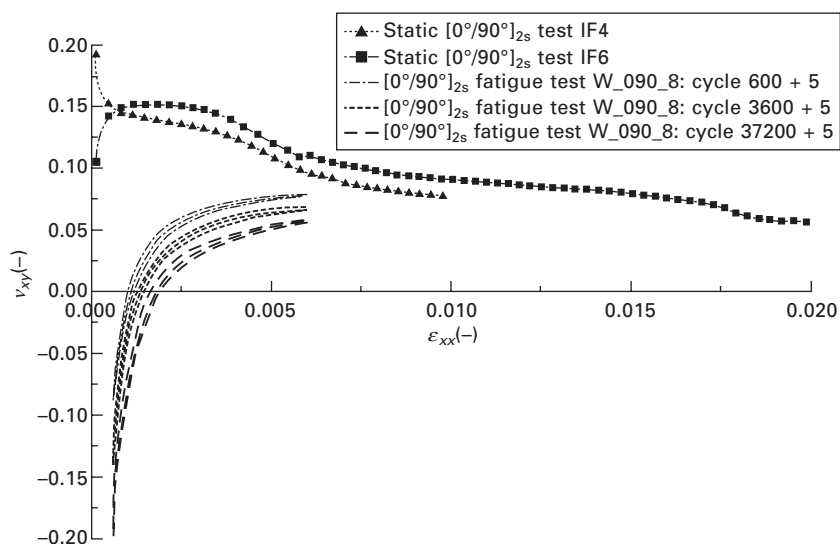
transverse strain gauge, (ii) transverse extensometer, and (iii) transversely mounted external optical fibre sensor. All measurement methods confirmed the strain measurement (Van Paepegem *et al.*, 2008).

Next, another specimen W_090_8 was tested for a larger number of cycles in tension-tension fatigue ($R = 0.1$). In Fig. 4.16, the evolution of the Poisson's ratio ν_{xy} is plotted against the longitudinal strain ϵ_{xx} for three sets of five consecutively measured cycles.

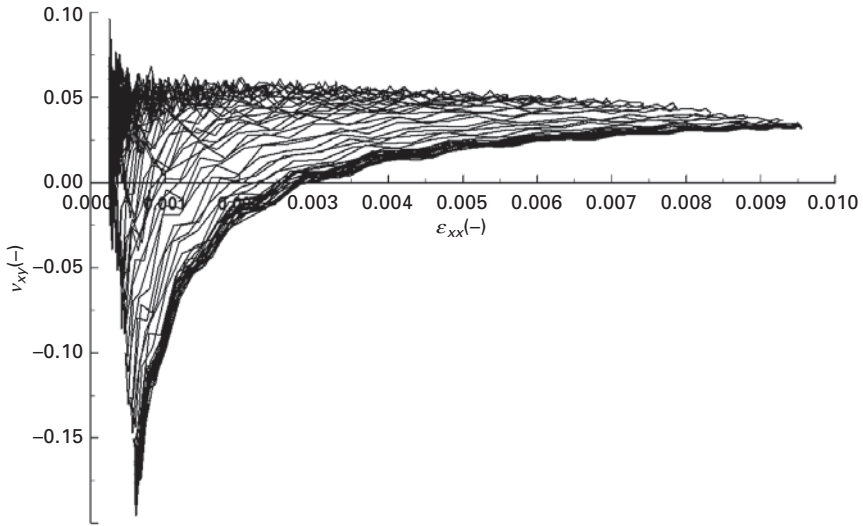
Finally, the observed behaviour is not specific for the $[0^\circ/90^\circ]_{2s}$ glass/epoxy laminate. Figure 4.17 shows the evolution of the Poisson's ratio ν_{xy} depending on the longitudinal strain ϵ_{xx} for the $[(0^\circ, 90^\circ)]_{4s}$ carbon/PPS specimen K6 at the running-in of the fatigue test. During running-in, the amplitude of the load-controlled fatigue cycles gradually builds up within the preset envelope time. During these first loading cycles, a gradual change of the value of the Poisson's ratio can be observed, starting from a steady value of about 0.05 towards more and more negative values.

4.6.2 Degradation of in-plane shear modulus

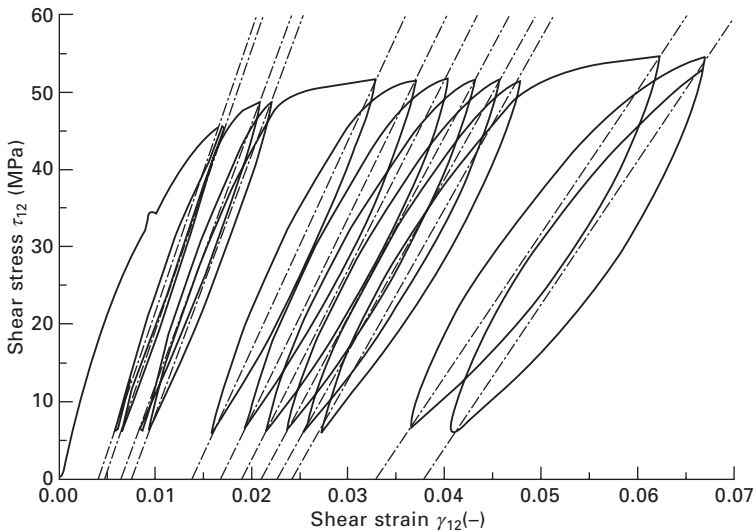
When the shear component in fatigue loading is substantial, degradation of the in-plane shear modulus can be very relevant, as well as accumulation of permanent shear strain. Although the latter is not an elastic property, it is very important to measure this strain, because the shear stress is only



4.16 Evolution of the Poisson's ratio ν_{xy} depending on the longitudinal strain ϵ_{xx} for the $[0^\circ/90^\circ]_{2s}$ specimen W_090_8 at three chosen intervals in the fatigue test.



4.17 Evolution of the Poisson's ratio ν_{xy} depending on the longitudinal strain ε_{xx} for the $[(0^\circ, 90^\circ)]_{4s}$ carbon/PPS specimen K6 during running-in of the fatigue test.



4.18 Shear stress–strain curve for the cyclic $[+45^\circ/-45^\circ]_{2s}$ tensile test IH6.

determined by the elastic part of the (total) shear strain. For example, Fig. 4.18 shows the shear stress–strain curve for cyclic loading–unloading tensile tests on a $[+45^\circ/-45^\circ]_{2s}$ glass/epoxy laminate. The displacement speed was

2 mm/min. It is clear from this figure that a considerable permanent shear strain is built up during the consecutive cycles.

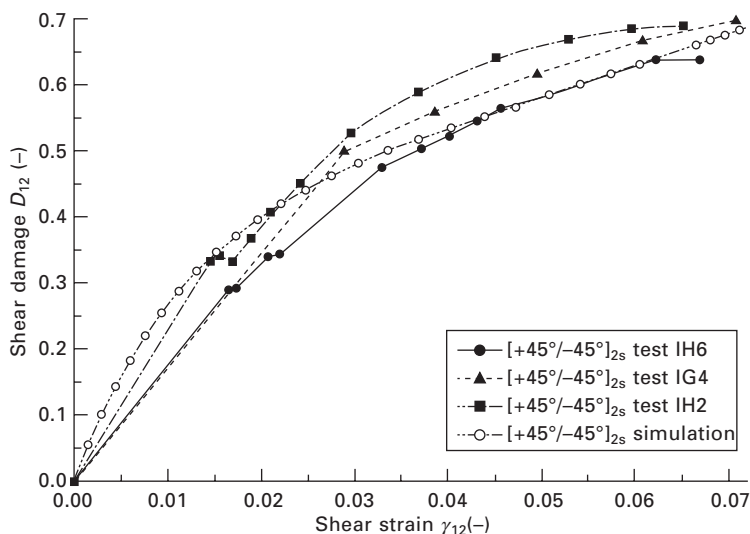
To characterize the degradation of the shear properties, two variables are introduced: the shear damage D_{12} and the permanent shear strain γ_{12}^p . They are defined by the following constitutive relations:

$$\begin{aligned}\tau_{12} &= G_{12}^0(1 - D_{12})(\gamma_{12} - \gamma_{12}^p) \\ D_{12} &= 1 - \frac{G_{12}^*}{G_{12}^0}\end{aligned}\quad 4.26$$

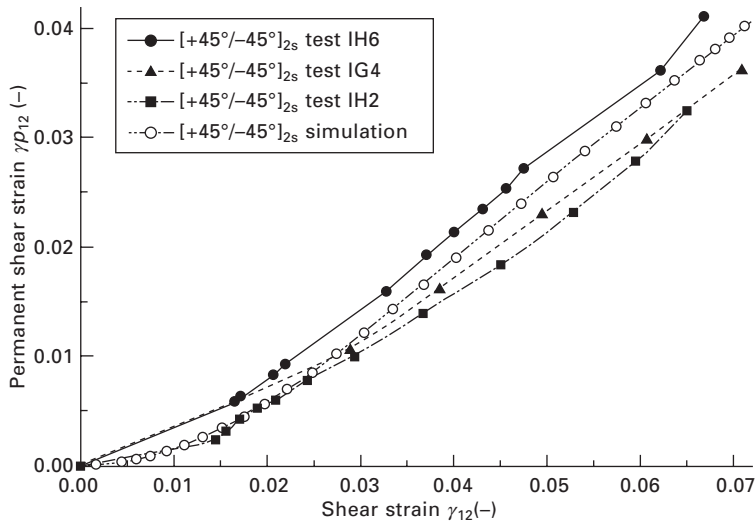
where τ_{12} is the shear stress, G_{12}^0 is the virgin shear modulus and G_{12}^* is the shear modulus of the damaged material. The shear modulus G_{12}^* is defined as the secant shear modulus for one loading–unloading cycle. This definition is in agreement with the definition used by Lafarie-Frenot and Touchard (1994) in their study of the inplane shear behaviour of long carbon-fibre composites.

With the definitions of shear damage D_{12} and permanent shear strain γ_{12}^p , the evolution of these variables as functions of the total shear strain γ_{12} is plotted in Figs 4.19 and Fig. 4.20 respectively for the three cyclic $[+45^\circ/-45^\circ]_{2s}$ tensile tests IH6, IG4 and IH2.

A phenomenological model for quasi-static loading–unloading tests, predicting the degradation of the in-plane shear modulus and the accumulation of permanent shear strain, can be found in Van Paepegem *et al.* (2006a, 2006b).



4.19 Evolution of the experimental and simulated shear damage D_{12} for three cyclic $[+45^\circ/-45^\circ]_{2s}$ tensile tests.



4.20 Evolution of the experimental and simulated permanent shear strain γ_{p12} for three cyclic $[+45^\circ/-45^\circ]_{2s}$ tensile tests.

4.6.3 Degradation of in-plane elastic properties in biaxial fatigue loading

In biaxial or multiaxial fatigue loading, the stress and strain fields are typically heterogeneous and varying over time. As a consequence, the degradation of the elastic properties varies from point to point and also their evolution in time can be different from point to point.

By measuring accurately the surface strain fields over time, it should be possible to determine the degradation of the *local* elastic properties, by using mixed experimental–numerical techniques and inverse methods. Lecompte *et al.* have already shown that the in-plane elastic properties of a cruciform specimen under biaxial loading can be identified through the combined use of digital image correlation and inverse methods (Lecompte *et al.*, 2007). In that paper, degradation of the elastic properties was not taken into account.

On the other hand, Ramault *et al.* (2008) have shown that digital image correlation (DIC) and electronic speckle pattern interferometry (ESPI) are valuable candidates for full-field strain imaging in biaxial fatigue testing of cruciform specimens. Very good correlation with 3D numerical simulations was obtained (Lamkanfi *et al.*, 2007).

4.7 Future trends and challenges

At least two major trends and challenges can be observed in the modelling of fatigue damage: (i) the trend towards multi-scale modelling of fatigue

damage in composites, and (ii) modelling the degradation of other elastic properties.

4.7.1 Towards multi-scale modelling

Currently, fatigue simulation tools exist to predict the crack density in one layer of unidirectional fibres (= one UD-ply) (Joffe and Varna, 1999; Allen *et al.*, 1987; Ogin *et al.*, 1985), or to predict the degradation of the homogenized elastic properties of a UD-based laminate (Whitworth, 1998; Yang *et al.*, 1990; Highsmith and Reifsnider, 1982), but none of these simulation tools bridges the gap between the micromechanical damage phenomena and the structural response of the damaged laminate. Moreover, very little work has been carried out on the fatigue behaviour of textile-based composites.

The promising approach to these problems is multi-level modelling, which allows the inclusion of mesoscopic behaviour features in macroscopic descriptions, without the need for an *a priori* postulated macroscopic constitutive law (Zako *et al.*, 2003; Carvelli and Poggi, 2001; Tang and Whitcomb, 2003; Edgren *et al.*, 2004). Macroscopic constitutive relations (material properties on the laminate level) are obtained from scaling up material modelling at lower (meso- and micro-) scales, where the detailed material structure with its specific material behaviour is represented. The multi-scale approach couples the advantages of a pure micro- or mesomechanical approach to those of a pure macroscopic modelling. The complex material behaviour is properly captured by the modelling at lower scales, while large-scale analyses at the macroscale remain numerically feasible.

4.7.2 Modelling the degradation of other elastic properties

As shown in Section 4.6, not only the longitudinal stiffness should be considered in modelling the degradation of the elastic properties of composite laminates under fatigue loading. Up till now, most models do not account for the degradation of other in-plane properties such as Poisson's ratio, shear modulus and transverse stiffness.

On the other hand, the development of such extended models requires a much broader input from the experimental measurements. For a long time, fatigue testing of composites was focused only on providing the S–N fatigue life data in uniaxial tension–tension fatigue. No efforts were made to gather additional data from the same test by using more advanced instrumentation methods. The development of methods such as digital image correlation (strain mapping) and optical fibre sensing allows for much better instrumentation, combined with traditional equipment such as extensometers, thermocouples and resistance measurement.

Validation with finite element simulations of the realistic boundary conditions and loading conditions in the experimental set-up must maximize the generated data from one single (multiaxial) fatigue test.

4.8 Sources of further information and advice

The textbook *Fatigue in Composites. Science and Technology of the Fatigue Response of Fibre-reinforced Plastics* edited by Bryan Harris (2003) is one of the best reference books for further information about fatigue of composite materials.

The three-yearly International Conference on Fatigue of Composites provides a forum for all researchers active in this field. The first was held in 1997 (Paris, France), followed by the next conferences in 2000 (Williamsburg, USA), 2004 (Kyoto, Japan) and 2007 (Kaiserslautern, Germany). The next conference will be held in China in 2010.

In a broader context, the two-yearly European and International Conference on Composite Materials (ECCM and ICCM respectively) are interesting fora for further information.

4.9 References

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Novel computational methods for fatigue life modeling of composite materials

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Abstract: Novel computational methods such as artificial neural networks, adaptive neuro-fuzzy inference systems and genetic programming are used in this chapter for the modeling of the nonlinear behavior of composite laminates subjected to constant amplitude loading. The examined computational methods are stochastic nonlinear regression tools, and can therefore be used to model the fatigue behavior of any material, provided that sufficient data are available for training. They are material-independent methods that simply follow the trend of the available data, in each case giving the best estimate of their behavior. Application on a wide range of experimental data gathered after fatigue testing glass/epoxy and glass/polyester laminates proved that their modeling ability compares favorably with, and is to some extent superior to, other modeling techniques.

Key words: fatigue, composites, artificial neural network, genetic programming, ANFIS, S–N curves.

5.1 Introduction

Computational engineering involves the design, development and application of computational systems for solving physical problems encountered during the study of science and engineering. These computational systems are used to obtain solutions of mathematical models that represent particular physical processes through the use of algorithms, software and other methods. Computational engineering has emerged as a fast-growing multidisciplinary area with connections to such scientific fields as engineering, mathematics and computer science and is bound to play an important role in new inventions and discoveries.

Computation is today regarded as an important tool for the advancement of scientific knowledge and engineering practice, in addition to the already existing tools of theoretical analysis and physical experimentation. Simulation techniques allow scientists to easily study complex natural phenomena and processes, which would otherwise be very difficult, dangerous or even impossible. The need for greater accuracy and detail in such simulations has created the need for faster and more efficient computer algorithms

and architectures. It is because of these advancements that scientists and engineers can solve highly complex problems that were once thought to be unsolvable.

The development of ‘thinking machines’ has been an objective of humanity for centuries. However, it was only in the twentieth century with the development of powerful computers that scientists began to build ‘intelligent’ machines based on recent discoveries in several scientific domains. Artificial intelligence is the branch of computer science that deals with the development of algorithms and techniques that can simulate or even recreate the capabilities of the human mind. Artificial intelligence (AI), methods such as artificial neural networks (ANNs), genetic algorithms (GAs), genetic programming (GP) and fuzzy logic (FL) techniques have been successfully used for years in different scientific fields for optimization, pattern recognition, data clustering and signal processing.

Over the last decade these novel computational methods have been introduced in the new sectors of engineering and material science. Neural networks and genetic algorithms have initially been used as tools for optimization of design methods, and damage characterization of composite materials [1–4]. These methods appear to offer a means of dealing with many multivariate problems for which an accurate analytical model does not exist or would be very difficult to develop. Computational intelligence techniques offer an advantage over conventional optimization techniques in that they could find maxima in multidirectional search spaces, and in that sense are very valuable for solving the multi-parametric optimization problems that arise in engineering, for example the optimization of laminate design against buckling [1], the optimization of a composite laminate to maximize its strength [2], the optimum design of bolted composite lap joints [3], or the characterization of damage in carbon/carbon composite laminates. Genetic algorithms and neural networks were used in conjunction with other analytical tools and experimental methods – finite element in [2], stress analysis in [3] and acoustic emission in [4] – to achieve the final objective.

The application of similar computer techniques for modeling material behavior under static and fatigue loading conditions, e.g. [5], went a step further. Subsequently, artificial neural networks (ANN), genetic programming (GP) and adaptive neuro-fuzzy inference systems (ANFIS) have been used for the interpretation of the fatigue data of composite materials. A limited number of relevant articles have been published concerning the fatigue of composite materials and structures, e.g. [6–19]. In [6], Lee and his co-workers tried to model fatigue lives of $[(\pm 45/0_2)_2]_S$ and unidirectional composite laminates under constant amplitude at different stress ratios (the ratio of minimum over maximum cyclic stress) and also under block loading. Comparison of their findings proved that, at least for the case of constant amplitude loading, ANN modeling is equivalent to, if not better than, the

modeling ability of other conventional modeling techniques. Two assets of ANN modeling were recognized: less computational effort and effective modeling, even for relatively small databases. However, even when ANNs are used for modeling, there is a lower limit to data set size. An ANN model can be considered as a black box and can be well trained to produce accurate output data, even for a very small amount of existing experimental data. However, cases like these should be treated with care since the use of limited data sets for training may lead to indeterminate and unstable systems as mentioned in [20]. Moreover, ANN does not offer any improvement in accuracy compared to simple regression analyses when simple relationships are modeled [21].

Al-Assaf and El Kadi [7–9] succeeded in modeling the fatigue life of unidirectional composite laminates by using different ANN paradigms and discussed the possibility of improving modeling efficiency by using other types of ANN, besides the classic feed-forward algorithm. The good modeling efficiency of ANN was also proved in a series of articles by Vassilopoulos *et al.* [10, 11] in which it was also pointed out that ANN could be used to reduce experimental effort and cost, as efficient modeling could be achieved even if only 50% of the available experimental data were used. ANN also proved a good tool for the construction of constant life diagrams by using a reduced data set [11–13]. ANNs have also been used to model the fatigue life of composite structures, e.g. [14] where an ANN was used for the prediction of the fatigue life of sandwich composites under flexural loading and [15] where the same method was employed for the modeling of the fatigue crack growth rate of bonded FRP–wood interfaces.

Other computational methods were also adapted to the analysis of the fatigue data of composite materials. A hybrid neuro-fuzzy method designated ANFIS (adaptive neuro-fuzzy inference system) has been used to model the fatigue life of unidirectional and multidirectional composite laminates. Results of its application to two material systems have been presented in [16] and [17]. ANFIS is a combination of ANN and fuzzy logic and combines the advantages of both techniques: the ability of ANNs in adapting and learning, together with the merit of approximate reasoning offered by fuzzy logic. In fatigue life modeling, ANFIS is based on linguistic rules that are dictated by experience, e.g. if developed stress is low and the off-axis angle is low then the life of a laminate is long. Despite the introduction of these fuzzy rules and the combination of the advantages of two techniques, no improvement in modeling was mentioned in [16] and [17] when compared to ANN results.

Genetic programming (GP) was also used in this field. GP has been successfully used as a tool for modeling the fatigue behavior of composite materials, as presented by Vassilopoulos *et al.* [18, 19]. This tool can be used to model the fatigue behavior of composite laminates subjected to

constant amplitude loading. Since it is a material-independent method, it can be used to model the fatigue behavior of any composite material, and compares favorably with conventional modeling techniques.

Interpretation of the fatigue data of composite materials seems to represent an ideal problem for the computational methods under consideration. The problem consists of the modeling of the material's behavior under the given environmental and mechanical loading conditions. The data available in fatigue databases frequently suffice for the adequate training of the tools. This is in any case the condition for accurate and reliable modeling using the described computational methods. With a reasonable amount of fatigue data, problems like the production of an indeterminate model (when input data are limited compared to the designed model parameters) or model overfitting (when the number of input parameters is relatively large compared to the input data) can be eliminated. In fatigue life modeling, between one and four input parameters are considered, depending on the existing ones, e.g. cyclic stress, stress ratio, testing temperature, off-axis angle, etc., and one single output parameter, the number of cycles to failure.

During the training and testing process, the structure, learning algorithm and other parameters of the neural network, genetic programming or ANFIS model should be optimized. The training process for an ANN involves minimizing the error between actual and predicted outputs, using the available training data, by continuously adjusting the connection weights. During the development of a genetic algorithm model or a genetic programming model, parameters such as population size, the genetic operator's probabilities, and the termination criteria should also be well defined in order to generate a well-structured model. A number of methods have been developed to address the problem of the optimization of model parameters, e.g. [22, 23]. A sufficiently optimal model, trained with the available experimental data, can subsequently generate accurate results for any new input data set.

Other, 'conventional' methods [19], in the form of deterministic or stochastic mathematical models, were developed for modeling the fatigue behavior of composite materials. However, the question 'Which type of S–N curve?' has not yet been satisfactorily answered. Extensive literature exists on this subject, a short, but comprehensive review of which is given in [24]. As pointed out, the S–N curve type selection, but also type of fitting (e.g. linear regression or use of Weibull statistics), could lead to extremely different inter/extrapolated results. The best choice depends mainly on the material. For example, it was shown in [25] that a log-log expression is the best function for fitting 0/±45 glass fiber-reinforced fatigue data but shows pure modeling results for another glass polyester material system – see Fig. 7 of [19]. Moreover, since engineering applications are designed for an extended time period, it is imperative to assume an S–N curve that is able to accurately estimate life at the high-cycle fatigue regime as well. In this

case S–N curves with different slopes in low-cycle and high-cycle fatigue regimes seem more adequate.

This chapter aims to explore the possibilities of novel computational methods and examine their ability to model/predict the fatigue life of composite laminates. Artificial neural networks, adaptive neuro-fuzzy inference systems and genetic programming will be considered. Their applicability in the modeling of the fatigue life of several different material systems will be demonstrated. The discussion will focus on the limitations of these methods, but also the advantages that they offer when compared to the conventional methods used by researchers and engineers over the last five decades.

5.2 Theoretical background

The computational techniques described in the following sections fall under the category of data-driven as opposed to model-driven approaches. These techniques build models based on available input–output mapping data, which is one of the main reasons for these techniques being so well suited to problems like the one examined here. AI methods work on available experimental data, in the form of input–output mappings that are separated to define two or three data sets usually, one designated ‘training set’, the second ‘validation set’ and the third, if one exists, the test set. Every data set contains the input and the corresponding output values of the physical or artificial system under investigation. The technique that is used (neural network, ANFIS or genetic programming) develops a model able to describe the relationship between the inputs and the outputs in the training data set (this phase is known as training). In a second phase, the model produced is evaluated using the validation data set in order to examine the generalization ability of the produced model. In other words, the model’s ability to accurately predict (with a small error) the behavior of new (‘unseen’) data, i.e. data not used in training, is validated. The training and validation procedure is usually an iterative one. This procedure is completed when certain termination criteria have been fulfilled, for example a maximum number of generations or training epochs have been reached. At this stage the final model has been generated. The performance of the final model can be further evaluated using new data which form the test (or applied) set, a set of new and ‘unseen’ data that do not belong in the training or validation sets and provide a measure of the real generalization ability of the produced model.

5.2.1 Artificial neural networks

Artificial neural networks (ANNs) are mathematical or computational models that are based on biological neural networks and especially on the biological neural networks of the brain. ANNs are networks consisting of

many interconnected simple processing units, termed neurons or nodes, which process information using a connectionist approach to computation (in contrast to symbolic artificial intelligence). The connections between neurons are known as synapses and are characterized by a weight value. In most cases, an ANN is an adaptive system that changes its structure based on external or internal information that flows through the network during the learning phase. ANNs can be seen as nonlinear statistical data modeling tools and can be used to model complex relationships between inputs and outputs. ANNs possess many advantages that make them very suitable for real-life applications.

Generally an ANN can be considered as a nonlinear mapping $F_{\text{ANN}}: \mathcal{R}^I \rightarrow \mathcal{R}^K$ from $\mathcal{R}^I$ to $\mathcal{R}^K$, where I and K are respectively the dimensions of the input and desired output space. This function is usually a complex function of a set of nonlinear functions, one for each neuron in the ANN. The basic building block of an ANN is the artificial neuron (AN), which implements a nonlinear mapping $f_{\text{AN}}: \mathcal{R}^I \rightarrow [0, 1]$ or $[-1, 1]$ depending on the activation function used. Generally, every neuron calculates its weighted sum of input signals and produces an output using the following formula:

$$o_{\text{AN}} = f_{\text{AN}} \left(\sum_{i=0}^I w_i \cdot x_i \right) \quad 5.1$$

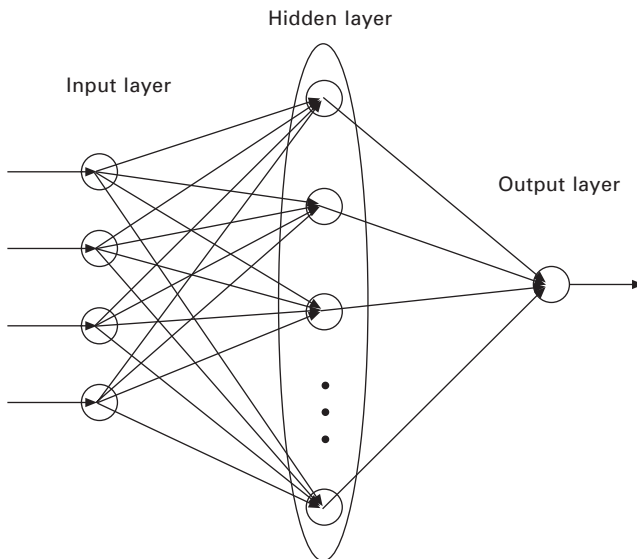
where o_{AN} is the output of the artificial neuron, f_{AN} is the activation function used, I is the number of its inputs, x_i is the value of the incoming signal from some other neuron i , w_i is the value of the weight of the synapse that connects the neuron i with the AN, and x_0 equals +1 or -1 depending on the use of a bias or a threshold term, respectively. A number of different activation functions can be used such as the linear, the step, the ramp, the sigmoid, the hyperbolic or the Gaussian [26].

The most important characteristic that makes ANNs so appealing is their ability to learn, that is to adjust their connection weight values in order to perform a specific task. There are three main types of learning:

- Supervised learning, where the ANN is provided with a data set consisting of the input vectors and target (desired) outputs associated with each input vector. The aim is to adjust the weight values of the ANN so that the error between the target and the network's actual output is minimized.
- Unsupervised learning, where the aim is to discover patterns or features in the input data without any external guidance or assistance. The unsupervised learning generally performs a clustering of the training patterns.
- Reinforcement learning, where the aim is to train not by giving the target outputs but by rewarding good performance and penalizing bad performance.

One of the best known ANN topologies, the multilayer feed-forward network (also called multilayer perceptron, MLP), is used in this chapter. The MLP is trained by a supervised learning method, the error back-propagation (EBP) algorithm, and organized into layers of neurons with forward connections only. Usually an MLP has an input layer, one or more hidden layers and an output layer. The first, or input, layer consists of a number of sensory neurons (neurons that do no processing but just sense the incoming signals and pass them to the next layer). The last layer (the one that produces the final output of the network) is known as the output layer and consists of a number of computational neurons. All the other layers between the input and output layers are called hidden layers and consist of computational neurons. In contrast to sensory neurons that do not do any processing, computational neurons process their incoming signals as described above. The input signal in an MLP propagates from the input to the output layer through the hidden layers in a forward direction. These networks are usually fully connected, which means that each neuron in any layer of the network is connected to all the neurons in the next layer. The typical example of an MLP shown in Fig. 5.1 consists of one input, one hidden and one output layer. Generally speaking, an MLP can have more than one hidden layer and more than one neuron in the output layer.

As explained above, the EBP algorithm is used to train the MLP. The EBP is the best known and widely used supervised learning algorithm and operates in two phases:



5.1 Typical ANN topology.

1. The feed-forward pass in which it calculates the output value(s) of the network (it propagates the input signal forward from the input layer to the output layer).
2. The backward pass in which it propagates an error signal backwards from the output layer towards the input layer. The weights of the MLP are adjusted in order to minimize this error signal, according to this back-propagated error signal.

These two phases constitute a learning iteration (one iteration is also known as an epoch). The training of an MLP using EBP includes a large number of such iterations until a satisfactory performance is attained. An in-depth description of the EBP algorithm can be found in [26] and [27] or any other neural network textbook.

For the successful application of MLP in modeling the fatigue life of FRP composite materials, the following aspects should be considered:

- The pre-processing of the application data; i.e. the normalization of the data, the creation of the training, validation and test set.
- The selection of the most appropriate MLP architecture for the problem at hand; i.e. how many inputs and hidden layers and neurons in each hidden layer will be used.
- The selection of the most suitable parameters for the EBP algorithm; i.e. termination criterion, learning rate and momentum.
- Scatter of the available experimental fatigue data is also a factor that has to be taken into account for modeling accuracy. However, data scatter would have a similar effect on any other conventional modeling method.

5.2.2 ANFIS: adaptive neuro-fuzzy inference system

A fuzzy logic system is unique in the sense that it is able to simultaneously handle numerical data and logic knowledge. It is a nonlinear mapping of an input data (feature) vector into a scalar output, i.e. it maps numbers into numbers. Fuzzy set theory and fuzzy logic establish the specifics of the nonlinear mapping. A fuzzy logic system can be expressed mathematically as a linear combination of fuzzy functions. It is a nonlinear universal function approximator, a property that it shares with feed-forward neural networks.

The ANFIS is a combination of ANN and fuzzy logic and combines the advantages of both methods. ANFIS was introduced by Takagi and Sugeno in 1985 [28] and further developed by Jang [29]. Zhang and Morris [30] discussed the architecture of adaptive neuro-fuzzy inference systems (ANFIS), which could construct a nonlinear input–output mapping, based mostly on human expertise and stipulated data pairs.

Fuzzy logic methods have been used to model various highly complex

and nonlinear systems based on a set of sample data and fuzzy ‘if–then rules’. A fuzzy inference system can model the qualitative aspects of human knowledge without employing any quantitative analyses. The following notation is common in fuzzy logic modeling and is adapted to serve the needs of the present study:

- *Linguistic variables*: these form the basic concept underlying fuzzy logic, i.e. a variable whose values are expressed in words rather than numbers. The input linguistic variables specified here for the specific problem of fatigue life modeling are the following: orientation angle (q), stress ratio (R), maximum cyclic stress ($s_{\max}$) and cyclic stress amplitude (s_a). The number of cycles to failure (N) is used as the only output variable.
- *Membership function (MF)*: this is the curve which defines the way each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. The membership function type can be any appropriate parameterized membership function such as triangular, Gaussian or bell-shaped.
- *Fuzzy sets*: unlike a classic set, a fuzzy set does not have a crisp boundary, i.e. the transition from the case of ‘belonging to a set’ to the case of ‘not belonging to a set’ is gradual. Normally this smooth transition is characterized by a *membership function* which gives flexibility to the fuzzy sets in commonly used modeling linguistic expressions. For the case studied here, a linguistic expression could be ‘fibre orientation angle (q) is close to zero’ or ‘stress ratio (R) is high’, etc.
- *Linguistic rules*: a set of linguistic ‘if–then’ rules applied to the defined linguistic variables. A single fuzzy ‘if–then’ rule assumes the form ‘If x is A then y is B ’, where A and B are linguistic values defined by fuzzy sets on the ranges X and Y , respectively. The if-part of the rule, ‘ x is A ’, is termed the *antecedent* or *premise*, while the then-part of the rule, ‘ y is B ’, is termed the *consequent* or *conclusion*. Fuzzy ‘if–then’ rules with multiple antecedents like the following are often used:
Rule: If the fiber orientation angle is near to zero, the stress ratio is low, the stress amplitude is low and the maximum stress is low, then specimen life is long.

The output resulting from the described fuzzy logic method has to be *defuzzified* or else converted to a crisp value by using any of the available defuzzification methods, such as the centre of gravity method. The membership functions used to represent linguistic variables may have a significant effect on modeling performance as the type of MF used determines when a given rule is to be activated (in fuzzy logic ‘the rule is fired’). Three types of membership functions – triangular, Gaussian and bell-shaped – have been used in this study to examine MF influence on the modeling efficiency of ANFIS.

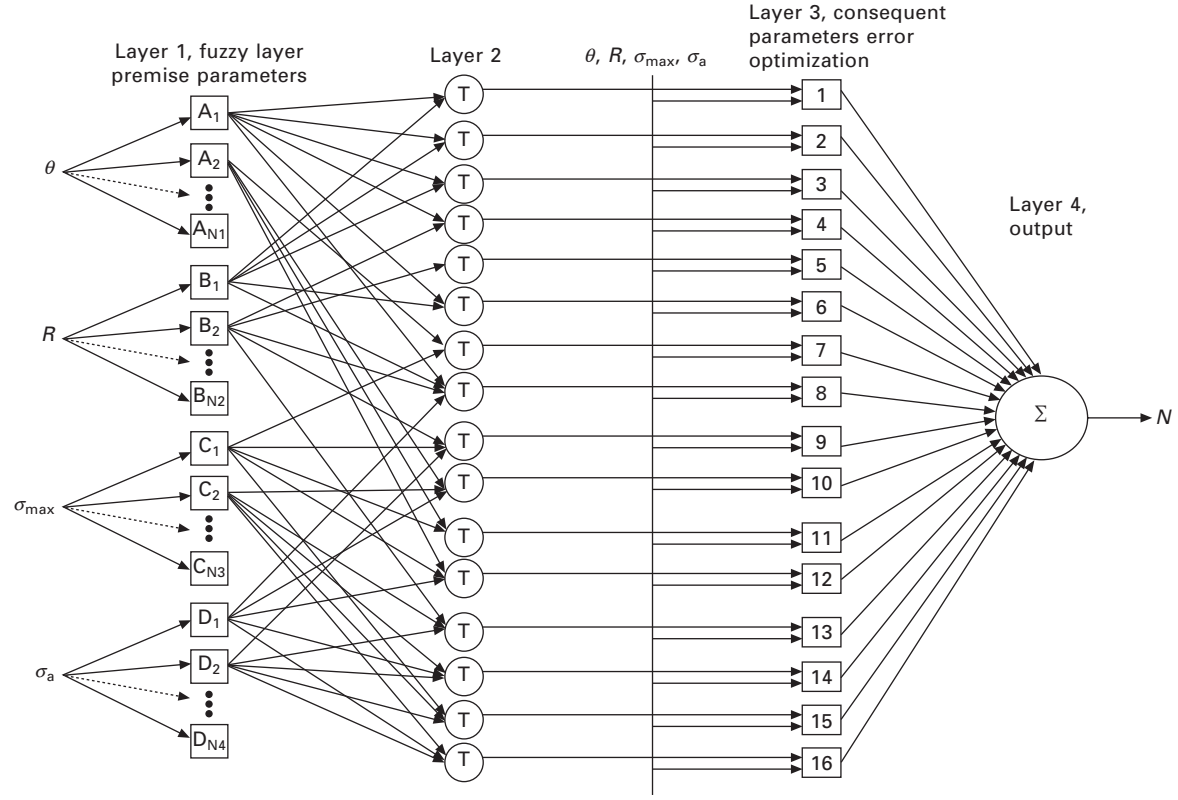
Although the fuzzy inference system has a structured knowledge representation in the form of fuzzy ‘if–then’ rules, it lacks the adaptability to deal with a changing external environment. Therefore neural network learning concepts have been incorporated into fuzzy inference systems, resulting in adaptive neuro-fuzzy modeling. The adaptive inference system is a network which consists of a number of interconnected nodes. Each node is characterized by a node function with fixed or adjustable parameters. The network ‘learns’ the behavior of the available data during the training phase by adjusting the parameters of the node functions to fit that data. The basic learning algorithm, the error back-propagation (EBP) algorithm, is also applied to minimize a set measure or a defined error, usually the sum of squared differences between desired and actual model outputs.

An ANFIS architecture based on the first-order Takagi–Sugeno model is schematically presented in Fig. 5.2. It is assumed that the desired output is a function of all the input parameters. The relationship between input and output parameters is dominated by linguistic rules. Moreover, the input parameters are defined by fuzzy sets rather than crisp sets. The fuzzy inference system shown in Fig. 5.2 is composed of four layers, each involving several nodes. The output signals from the nodes of the previous layer will be accepted as the input signals in the current layer. After manipulation by the node function in the current layer, the output will serve as input signals for the subsequent layer.

- *Layer 1:* The first layer of this architecture is the fuzzy layer. Each node of this layer makes the membership grade of a fuzzy set.
- *Layer 2:* Every node in layer 2 is a fixed node, indicated by a circle, whose output is the product of all the incoming signals i.e. T-norm operation: The output signal denotes the firing strength of the associated rule. The firing strength is also called the ‘degree of fulfillment’ of the fuzzy rule, and represents the degree to which the antecedent part of the rule is satisfied.
- *Layer 3:* Every node in layer 3 is an adaptive node, indicated by a square node. The consequent parameters in this layer will be adapted in order to minimize the error between the ANFIS outputs and experimental results.
- *Layer 4:* Every node in layer 4 is a fixed node, indicated by a circle node. The node function computes the overall output by summing all the incoming signals.

This ANFIS structure represents a four-dimensional space partitioned into $N_1 \times N_2 \times N_3 \times N_4$ regions, each governed by a fuzzy ‘if–then’ rule. In other words, the premise part of a rule defines the fuzzy region, while the consequent part specifies the output within the region.

A hybrid learning algorithm is used to adapt the parameters of the first



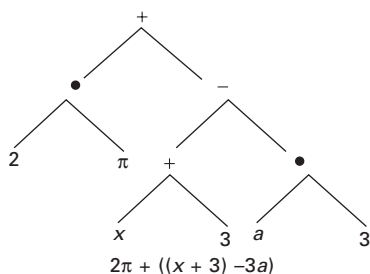
5.2 Takagi–Sugeno ANFIS model.

layer, known as premise or antecedent parameters, and the parameters of the third layer, referred to as consequent parameters, in order to optimize the network. The network uses a combination of back-propagation and the least-squares method to estimate membership function parameters. More specifically, in the forward pass of the hybrid learning algorithm, node outputs go forward as far as layer 3 and the consequent parameters are identified by the least-squares method. In the backward pass, error signals propagate backwards and the premise parameters are updated by a gradient descent method.

5.2.3 Genetic programming

Genetic programming (GP) is a domain-independent problem-solving technique in which computer programs are evolved to solve, or approximately solve, problems. Genetic programming is a member of the broad family of techniques called evolutionary algorithms. All these techniques are based on the Darwinian principle of reproduction and survival of the fittest and are similar to biological genetic operations such as crossover and mutation. Genetic programming addresses one of the central goals of computer science, namely automatic programming, which is to create, in an automated way, a computer program that enables a computer to solve a problem [31].

In genetic programming, the evolution operates on a population of computer programs of varying sizes and shapes. These programs are habitually represented as trees, as for example the one shown in Fig. 5.3, where the function $f(x) = 2\pi + ((x + 3) - 3a)$ is represented in tree format. The operations in the 'tree branches' are performed and the result is given at the 'tree root'. Genetic programming starts with an initial population of thousands or millions of randomly generated computer programs composed of the available programmatic ingredients and then applies the principles of biological evolution to create a new (and often improved) population of programs. This new population is generated in a domain-independent way using the Darwinian principle of survival of the fittest, an analogue of the



5.3 Tree representation in genetic programming.

naturally occurring genetic operation of crossover (sexual recombination), and occasional mutation [32]. The crossover operation is designed to create syntactically valid offspring programs (given closure among the set of programmatic ingredients). Genetic programming combines the expressive high-level symbolic representations of computer programs with the near-optimal efficiency of learning of Holland's genetic algorithm. A computer program that solves (or approximately solves) a given problem often emerges from this process [32].

Six major preparatory steps should be performed before applying genetic programming [32] in a given problem. These steps include preparation of data sets, setting-up of the model and design of the termination criteria, as explained in the following:

1. Determination of the *set of terminals*. The terminals can be seen as the inputs to the as-yet-undiscovered computer program. The set of terminals (or *Terminal Set T*, as it is often called) together with the set of functions are the ingredients from which genetic programming constructs a computer program to solve, or approximately solve, the problem.
2. Determination of the *set of primitive functions*. These functions will be used to generate the mathematical expression that attempts to fit the given finite sample of data. Each computer program is a combination of functions from the function set *F* and terminals from the terminal set *T*. The selected function and terminal sets should have the closure property so that any possible combination of functions and terminals produces a valid executable computer program (a valid model).
3. Determination of the *fitness measure* which drives the evolutionary process. Each individual computer program in the population is executed and then evaluated, using the fitness measure, to determine how well it performs in the particular problem environment. The nature of the fitness measure varies with the problem: e.g. for many problems, fitness is naturally measured by the discrepancy between the result produced by an individual candidate program and the desired result; the closer this error is to zero, the better the program. For some problems, it may be appropriate to use a multi-objective fitness measure incorporating a combination of factors such as correctness, parsimony (smallness of the evolved program), efficiency, etc.
4. Determination of the *parameters for controlling the run*. These parameters define the guidelines in accordance with which each GP model is evolved. The population size, that is the number of created computer programs, the maximum number of runs, i.e. of evolved program generations, and the values of the various genetic operators are included in the list of parameters.
5. Determination of the *method for designating a result*. A frequently used

method of result designation for a run is to appoint the best individual obtained in any generation of the population during the run (i.e. the *best-so-far individual*) as being the result of the run.

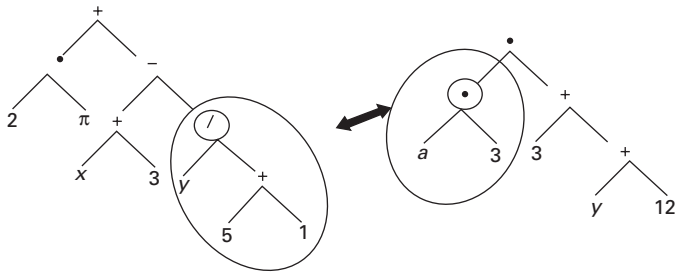
6. Determination of the *criterion for terminating a run*. The maximum number of generations, or the maximum number of successive generations for which no improvement is achieved, values that were determined in step 4, are usually considered as the termination criteria.

Genetic programming starts with an initial population (generation 0) of randomly generated computer programs composed of the given primitive functions and terminals. Typically, the size of each program is limited, for practical reasons, to a certain maximum number of points (i.e. total number of functions and terminals) or a maximum depth of the program tree.

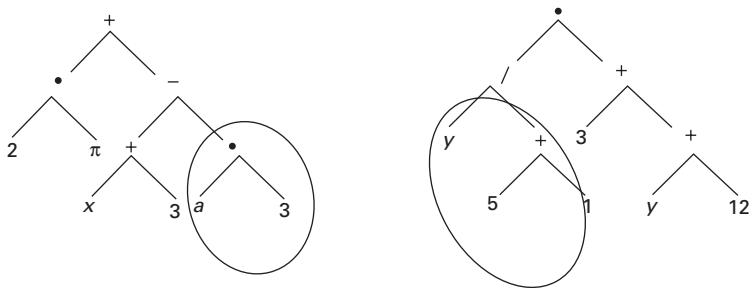
Typically, each computer program in the population is run over a number of different *fitness cases* so that its fitness is measured as a sum or an average over a variety of different representative situations. For example, the fitness of an individual computer program in the population may be measured in terms of the sum of the absolute value of the differences between the output produced by the program and the correct answer (desired output) to the problem (i.e. the Minkowski distance) or the square root of the sum of the squares (i.e. the Euclidean distance). These sums are taken over a sampling of different inputs (fitness cases) to the program. The fitness cases may be chosen at random or in some structured way, e.g. at regular intervals [32].

The computer programs in generation 0 (initial population) will almost always produce a very poor performance, although some individuals in the population will fit the input data better than others. These differences in performance are then exploited by genetic programming. The Darwinian principle of reproduction and survival of the fittest and the genetic operations of crossover and mutation are used to create a new offspring population of individual computer programs from the current population. The reproduction operation involves selecting a computer program from the current population of programs based on fitness (i.e. the better the fitness, the more likely the individual is to be selected) and allowing it to survive by copying it into the new population.

The crossover operation creates new offspring computer programs from two parental programs that are selected based on their fitness. The parental programs in genetic programming are usually of different sizes and shapes. The offspring programs are composed of sub-expressions from their parents. These offspring programs are usually of different sizes and shapes than their parents. For example, consider the two parental computer programs (models) represented as trees in Fig. 5.4. One crossover point is randomly and independently chosen in each parent. Consider that these crossover points are the division operator (/) in the first parent (the left one) and the



5.4 Crossover operation: the two parental programs in tree representation.



5.5 Crossover operation: the two offspring programs after the crossover operation.

multiplication operator ($\bullet$) in the second parent (the right one). These two crossover fragments correspond to the underlying sub-programs (sub-trees) in the two parents – the sub-trees circled in Fig. 5.4. The two offspring resulting from the crossover operation depicted in Fig. 5.5 have been created by swapping the two sub-trees between the two parents in Fig. 5.4. Thus, the crossover operation creates new computer programs using parts of existing parental programs. Since entire sub-trees are swapped, the crossover operation always produces syntactically and semantically valid programs as offspring, regardless of the choice of the two crossover points. Because programs are selected to participate in the crossover operation with a probability based on their fitness, crossover allocates future trials to regions of the search space whose programs contain parts from promising programs [32].

The mutation operation creates an offspring computer program from one parental program that is selected based on its fitness. One mutation point is randomly and independently chosen and the sub-tree occurring at that point deleted. A new sub-tree is then grown at that point using the same growth procedure as was originally used to create the initial random population (this is only one of the many different ways in which a mutation operation can be implemented) [32].

After the genetic operations have been performed on the current population, the new population of offspring (the new generation) replaces the old population (the old generation) and the generation index increases by one. Each individual in the new population is then measured for fitness, and the process is repeated over many generations until the termination criterion/criteria is/are satisfied.

5.3 Modeling examples

One of the fundamental ways of interpreting fatigue data and modeling the fatigue behavior of composite materials and structures is the so-called stress-based (or strain-based) method. The output of this technique is a phenomenological model that simply correlates the number of loading cycles that the tested material can sustain under a given cyclic (frequently sinusoidal) stress to certain loading parameters, such as stress ratio, testing frequency, developed stress level, etc. Test results from different specimens tested under various cyclic stress levels form the S–N curve. A mathematical model is then introduced in order to use these test results in any design process. Frequently, statistical models are implemented for the analysis of fatigue data and derivation of S–N curves for certain reliability levels; see, for example, [33–35].

A number of different types of fatigue models (or types of S–N curves) have been presented in the literature, the most ‘famous’ being the semi-logarithmic (also called lin–log) and the logarithmic (log–log) relationships. Based on these it is assumed that the logarithm of the loading cycles is linearly dependent on the cyclic stress parameter, or its logarithm. Fatigue models defined in this way do not take different stress ratios or frequencies into account, i.e. different model parameters should be determined for different loading conditions. A drawback of these methods is that they are case-sensitive, since they may provide very accurate modeling results for one material system but very poor ones for another. Other types of fatigue formulations that take the influence of stress ratio and/or frequency into account have also been reported [36, 37]. A unified fatigue function that permits the representation of fatigue data under different loading conditions (different stress ratios) in a single two-parameter fatigue curve is proposed by Adam *et al.* [36]. In another work by Epaarachchi *et al.* [37], an empirical model that takes the influence of stress ratio and loading frequency into account is presented and validated against experimental data for different glass fiber-reinforced plastic composites. Although these models seem promising, their empirical nature is a disadvantage as their predictive ability is strongly affected by the selection of a number of parameters that must be estimated or even, in some cases, assumed.

When the expected loading is well determined, especially when it comprises

constant amplitude patterns, the task is easy and relatively inexpensive. However, when loading patterns are of variable amplitude – and even more so when they are of a stochastic nature, as in most cases of real structures operating in the environment – things become much more complicated. This is due to the theoretically unlimited number of tests that must be conducted to characterize material behavior under all possible loading conditions. The so-called constant life diagrams (CLDs) are used to avoid this inconvenience. CLDs represent mean stress, σ_m , vs. stress amplitude, σ_a , for several loading conditions, i.e. for several different stress ratios. In the previous relationship $\sigma_{\min}$ and σ_m denote minimum and maximum cyclic stress levels. To determine a CLD, S–N curves for at least three different stress ratios along with static strengths in tension and compression must be determined experimentally. The S–N curves at $R = 0.1$ for tension–tension (T–T) loading, at $R = -1$ for reversed tension–compression (T–C) loading, and at $R = 10$ for compression–compression (C–C) loading are used in the majority of published works, e.g. [38–41], to describe all three regions of a CLD. Any other curve that needs to be determined is calculated by linear or another type of interpolation between the known curves, as proposed in, for example, [41].

The AI techniques were successfully implemented for representation of the fatigue data of composite materials and structures under constant amplitude and spectrum loading. The application of these techniques to a number of fatigue data and the discussion of the results is presented below.

5.4 Experimental data description

A considerable amount of fatigue data for composite materials (especially composite laminates) exists in the literature and existing databases contain data covering a significant number of loading cases. Some of these databases are relatively limited and refer to a specific material system, primarily aimed at the verification of new theoretical models, e.g. [42–44]. Other databases are more extensive, however, and were developed for the characterization of entire categories of materials primarily used in specific applications, such as databases DOE/MSU [45] and OptiDAT [46] for materials used in the wind turbine rotor blade industry. A new fatigue database has recently been released by Virginia Tech [40] containing experimental data from axial loading on pseudo-quasi-isotropic glass/vinyl ester specimens fabricated using the vacuum-assisted resin transfer molding (VARTM) technique.

Selected material data have been used in the following examples to demonstrate the applicability of computational techniques and their potential for the modeling of fatigue behavior.

5.4.1 Material #1, GFRP multidirectional laminate with stacking sequence $[0/(\pm 45)_2/0]_T$

A database created over the last few years by one of the authors [44] is used in this work. It refers to specimens cut at on-axis and several off-axis angles from a multidirectional GFRP composite laminate. The stacking sequence of the on-axis specimens was $[0/(\pm 45)_2/0]_T$ which is a typical material used in the wind turbine rotor blade industry. Seven different material configurations were tested as specimens were cut at seven different angles from the multidirectional laminate, namely 0° , 15° , 30° , 45° , 60° , 75° and 90° . Constant amplitude fatigue tests were performed at a frequency of 10 Hz using an MTS 810 servo hydraulic test rig of 250 kN capacity. In total 257 valid constant amplitude fatigue data points were collected. Tests were conducted under four different stress ratios ($R = \sigma_{\min}/\sigma_{\max}$), two corresponding to tension–tension loading (0.1 and 0.5), one corresponding to tension–compression loading (−1) and one to compression–compression loading (10). At least three specimens were tested at each of the four or five stress levels pre-assigned for the determination of each S–N curve. This resulted in the existence of at least 12 and up to 18 specimens for each S–N curve data set, in a range between 1000 to 5.3 million cycles. The experimental data produced were used for the determination of 17 S–N curves corresponding to various loading patterns and material configurations. Typical input parameters for the application of computational methods to this data set are cyclic stress level (amplitude, σ_a or maximum stress, $\sigma_{\max}$), on- or off-axis angle, θ , and stress ratio, R . The number of cycles to failure, N , can be considered as the single output.

5.4.2 Material #2, GFRP multidirectional laminate with stacking sequence $[90/0/\pm 45/0]_S$

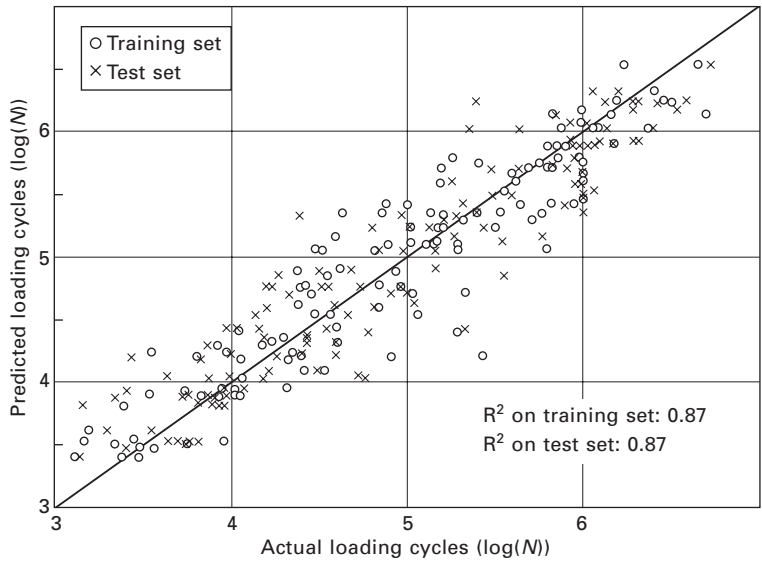
The second example is based on experimental fatigue data retrieved from the DOE/MSU database [45]. The material is a multidirectional laminate consisting of eight layers, six of the stitched unidirectional material D155 and two of the stitched, $\pm 45^\circ$, DB120. CoRezyn 63-AX-051 polyester, with the codename DD16 in the DOE/MSU database, was used as matrix material. For constant amplitude fatigue the material was tested under 12 stress ratios for a comprehensive representation of a constant life diagram. Reading counter-clockwise on the constant life diagram the following stress ratios can be identified: 0.9, 0.8, 0.7, 0.5, 0.1, −0.5, −1, −2, 10, 2, 1.43 and 1.1. The data set consists of 360 observations (valid fatigue test results) that were used for the derivation of the 12 S–N curves. The absolute maximum stress level during testing ranged between 85 MPa and 500 MPa, while the corresponding recorded cycles up to failure were between 37 and 30.4 million.

The stress ratio, R , and maximum cyclic stress, $\sigma_{\max}$, were considered as the input parameters. The number of cycles to failure, N , was also assigned as the only output parameter.

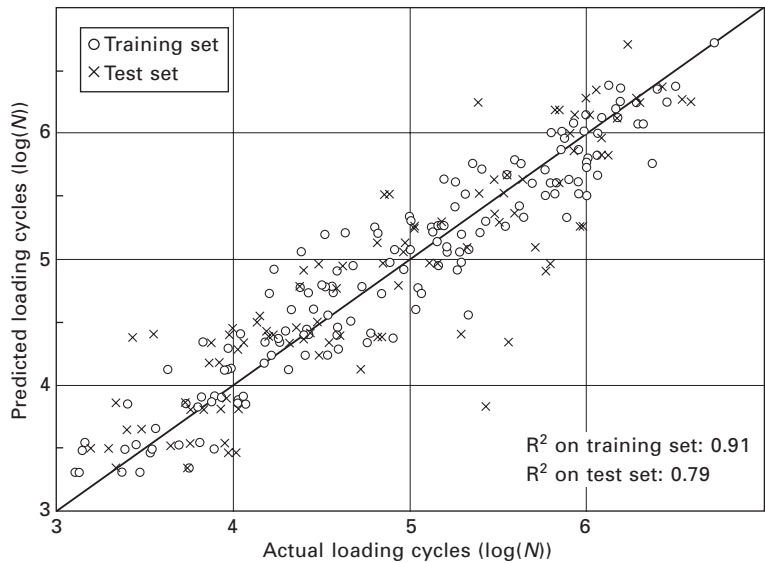
5.4.3 Material #3, multidirectional glass/epoxy laminate with stacking sequence $[(\pm 45/0)_4/\pm 45]_T$

The third material used is a multidirectional glass epoxy laminate consisting of nine plies, four with fibers in the 0° direction and five stitched layers with fibers in both the 45° and -45° directions. The stacking sequence of the laminate is $[(\pm 45/0)_4/\pm 45]_T$. Laminated plates were fabricated using the vacuum infusion method. Non-standard specimen geometry was used in order to also provide uniform specimens for tensile and compressive testing. The length of the rectangular specimens was 150 mm, the length of the tabs glued to both ends was 55 mm and consequently the free length of the specimens was 40 mm to avoid buckling during compressive loading. The thickness of the specimens of this type was 6.57 mm. A total of 147 valid fatigue data points were found in the OptiDAT database [46] for this material tested under three different constant amplitude conditions: 47 specimens for tension–tension loading at $R = 0.1$, 64 specimens for tension–compression loading at $R = -1$, and 36 specimens at compression–compression loading $R = 10$. In this data set, the maximum stress level obtained was between -350 MPa for compressive loading and 400 MPa for $R = 0.1$. Recorded lifetime was between 150 for low-cycle fatigue and 10.2 million cycles for longer lifetimes.

Previous results [10, 11, 17] have shown that 50–60% of available experimental data suffices for the derivation of reliable S–N curves and CLDs. As presented in Figs 5.6 and 5.7, an arbitrarily selected proportion of 50% of the data points proved sufficient for the derivation of reliable S–N curves by implementing an ANN model (Fig. 5.6) or an ANFIS model (Fig. 5.7). Moreover, as presented in [10], after a parametric study for the examined material and testing conditions, an increase in the data used in the training set does not seem to significantly influence the accuracy of the ANN model. The same applies for the application of the ANFIS modeling: see Fig. 5.8 [17]. Preprocessing of the fatigue data is frequently required, however. Generally, a simple normalization in order to obtain a data set in the range $[0, 1]$ is enough, although more advanced preprocessing techniques like data clustering have proved applicable in order to produce a concise representation of system behavior [17]. Data clustering can be very beneficial, for example, during the derivation of an ANFIS model, since it is a means of minimizing the linguistic rules needed to define the model [17].



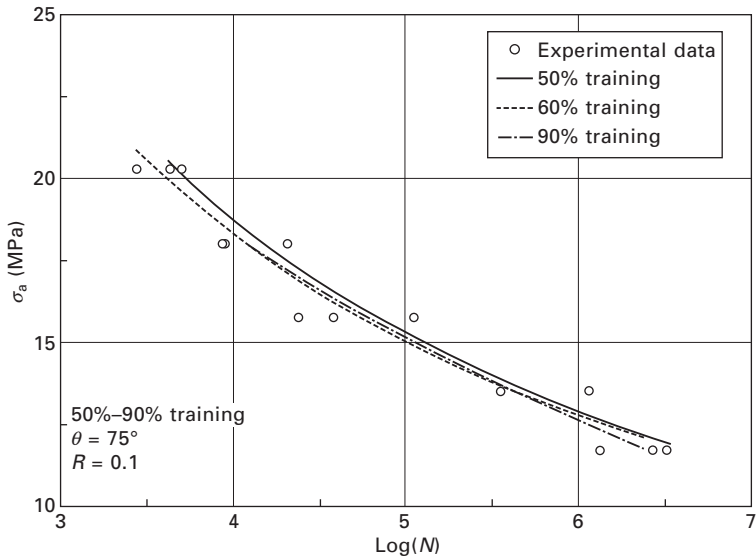
5.6 Modeling ability of ANN for 50% training set, 50% test set.



5.7 Modeling accuracy of ANFIS for 60% training set, 40% test set.

5.5 **Application of the methods**

The computational techniques introduced above were implemented on the three data sets. All data were handled as follows:



5.8 Modeling accuracy of ANFIS according to data points used to train network.

1. The available data sets were divided into training and validation sets using a randomization technique; approximately 50% of the data were used (randomly selected) for training, while the rest were used for model validation. The method of using training and validation sets is known as the cross-validation method and works as follows: the training set is used for the generation of a population of models of the system. These models are evaluated using the validation set and the one exhibiting the best performance is selected.
 - The training set contained the data that were used for the training phase of all methods. Where applicable, maximum stress values, on/off-axis angle and stress ratio were used as input parameters, while the corresponding cycles up to failure were considered as the desired output. The process can be characterized as a nonlinear stochastic regression analysis. During the training phase each computational tool established several relationships between the input and output variables. Using an iterative process the parameters of the established relationships were adjusted in order to minimize the difference (error) between desired and actual outputs.
 - The validation set contained data for the evaluation of the evolved models, after the training phase and should therefore contain patterns that were not used in the training set. It can thus appraise the generalization ability of the produced model, that is its ability to perform well with unknown data. It is also imperative that the

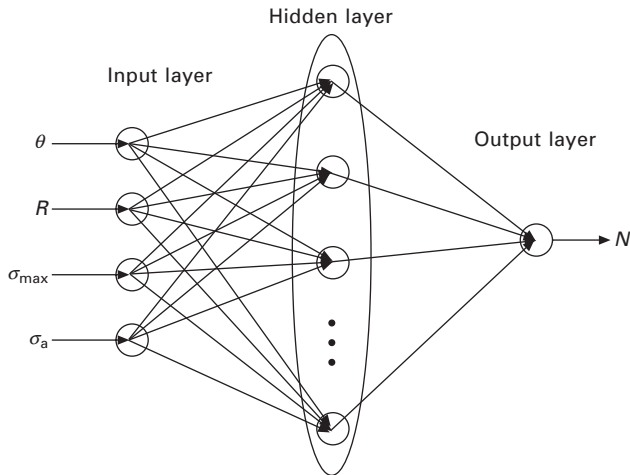
validation set should contain patterns comprising a good representative set of samples from the training domain (and generally the domain in which the model will operate).

2. A test, or applied, set was subsequently constructed, containing input data for which the output ought to be calculated by the selected model. Although these data were 'new' and had not been used for training or validation of the model, they should be in the range of the training set (the operational range of the model), since the ability of the described computational methods for extrapolation outside the training set has not yet been validated. For the case studied, the test sets were prepared in such a way that they covered the entire range from minimum to maximum cyclic stress levels. With the sets of input and output data, an entire S–N curve can be plotted without any fitting being required. Normally, there is no need to specify the output parameter since output values would be predicted by the models.
3. The same model can be stored and potentially used to predict other output values for a new applied input data set.

5.5.1 Artificial neural networks

The neural network used to model the fatigue life of the first material system was a multilayer feedforward network with four inputs (θ , R , $\sigma_{\min}$ and σ_a), one hidden layer with a variable number of computation neurons that use a sigmoid activation function, and a single output (N) with one computation neuron using a linear activation function. This neural network was trained by the error back-propagation (EBP) algorithm with the use of momentum (also known as the generalized delta rule) [26]. The target was to model the number of loading cycles until failure (N) as a function of the orientation angle of the fibers (θ), stress ratio (R), maximum stress applied to the specimen ($\sigma_{\max}$) and amplitude of the stress (σ_a). Only three of the finally used parameters are independent, since if maximum stress and stress ratio are known, stress amplitude is also known. The reason for using a dependent variable was to check whether or not a greater number of input parameters, even if they were interconnected, could affect the ANN model. For material #1 it was shown during calculations [11] that the ANN with four inputs (θ , R , σ_a , $\sigma_{\max}$) performs better than the one with three inputs (θ , R , $\sigma_{\max}$) with no significant increase in the complexity of the network. The general structure of the neural network used for material system #1 is shown in Fig. 5.9.

The data set used consists of 257 observations (fatigue life experiments). In order to make these experimental data suitable for processing by the neural network, the following preprocessing was carried out:



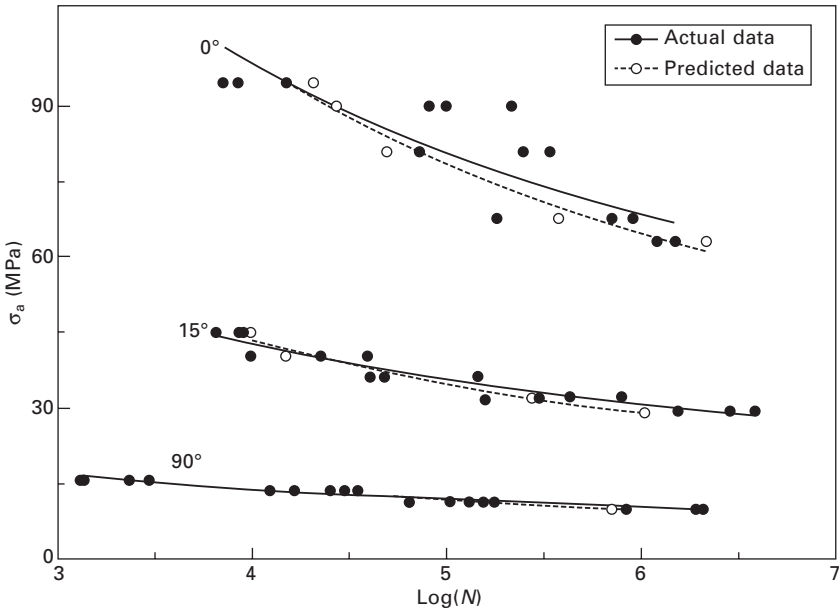
5.9 General structure of the ANN model used for material #1.

- The values for θ , $\sigma_{\max}$ and σ_a were normalized in $[0, 1]$, while R values were normalized over their maximum, 10.
- The logarithmic values of the number of cycles to failure (N) were calculated and used. This was done because N has values of between 1296 and 5,269,524 loading cycles. Training a neural network with such a wide range of values will result in extremely poor modeling performance.

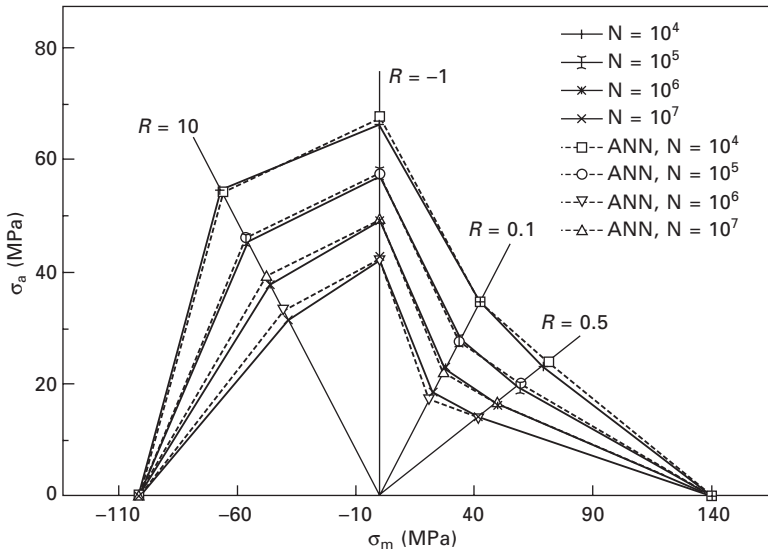
From the resulting data set, a training set was constructed by randomly selecting a proportion of between 30% and 50% of the available data. The remaining 50% to 70% was used as the testing set for validating the predictions. The training and test sets were selected using a simple uniform random sampling. A small proportion (10%), selected using uniform random sampling, of the training set was used as a validation set, in order to use the cross-validation method for assessing the performance of the neural networks. The results of the application of the neural network modeling technique to the experimental data concerning material #1 are given in Figs 10.10 and 5.11. It is shown that even for the extreme case when only 30% of the available fatigue data is used for model training, modeling efficiency may be very high: see Fig. 5.10. Moreover, as presented in Fig. 5.11, the ability of ANN to derive constant life diagrams of similar accuracy to those derived by using the conventional linear interpolation between known experimental data is instantly recognizable.

5.5.2 Adaptive neuro-fuzzy inference system

The ANFIS architecture used in the present study is based on the first-order Takagi–Sugeno model and is schematically presented in Fig. 5.2. It is



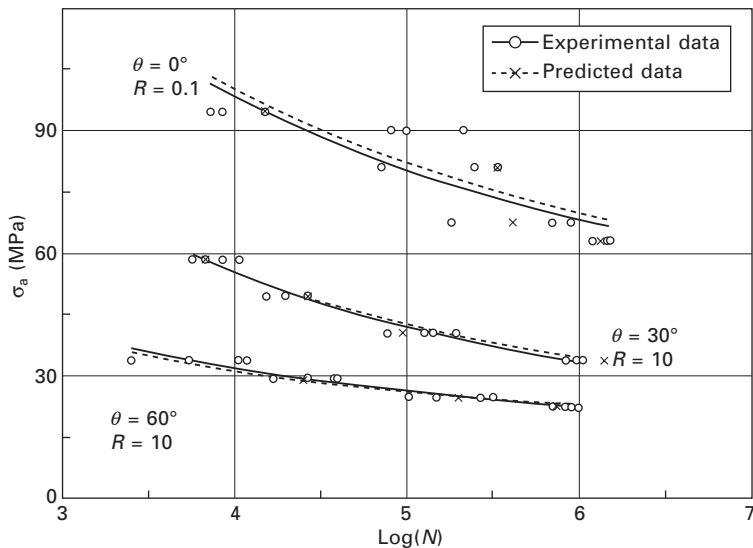
5.10 Experimental data vs. ANN modeling; tension–tension fatigue, $R = 0.1$; coupons cut at 0° , 15° , 45° , 75° and 90° ; 30% used for training set, 70% for test set.



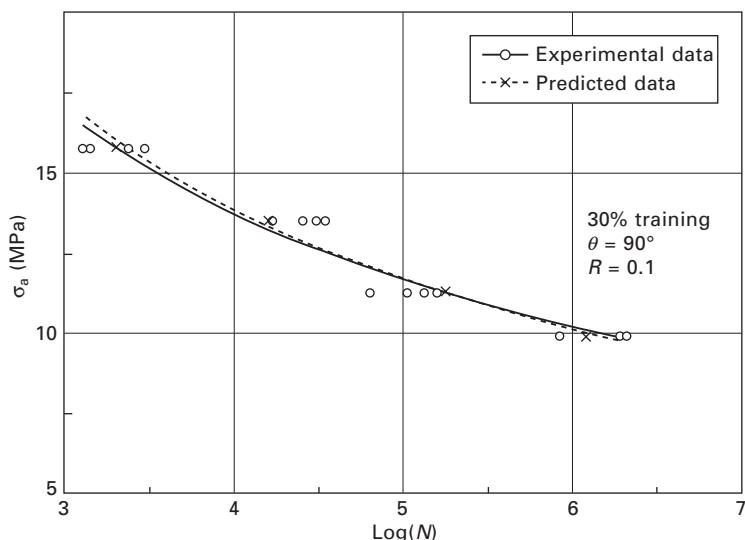
5.11 CLD for 45° off-axis coupons of material #1; ANN predictions vs. experimental data in range 10^4 – 10^7 loading cycles.

assumed that the number of cycles to failure (N) under fatigue loading is a function of the fiber orientation angle (θ), stress ratio (R), maximum stress ($\sigma_{\max}$) and stress amplitude (σ_a). Therefore θ , R , σ_a and $\sigma_{\max}$ were the input parameters as in the case of ANN, while the number of cycles corresponding to each combination of the four input parameters was the unique output of the ANFIS model. Data clustering was performed in order to improve the ANFIS modeling performance. The advantage of using data clustering in the proposed solution is the improvement of modeling accuracy by the development of a much simpler neuro-fuzzy model. ANFIS produced 15–17 fuzzy rules according to the size of the training data set when the fatigue data were clustered, and up to 1372 rules when no data clustering was performed. ‘If–then’ rules consist of the premise or antecedent (‘if’) part and the consequent (‘then’) part and work in the following way: if the premise is true, the consequent is also true. In fuzzy logic this simple representation is slightly different. A fuzzy rule indicates that if the premise is true to some degree of membership then the consequent is also true to the same degree of membership. In the examined case the antecedent of the fuzzy rules (the ‘if’ part) contains more than one condition for the parameters, θ , R , σ_a and $\sigma_{\max}$ respectively, which should be treated simultaneously. The consequent part of the rule contains only the number of cycles to failure.

The implementation of the ANFIS modeling technique derived accurate fatigue models for all the examined cases, as presented indicatively in Figs 5.12 and 5.13. As depicted in Fig. 5.12, when 60% or more of the available



5.12 ANFIS modeling accuracy for 50% training.

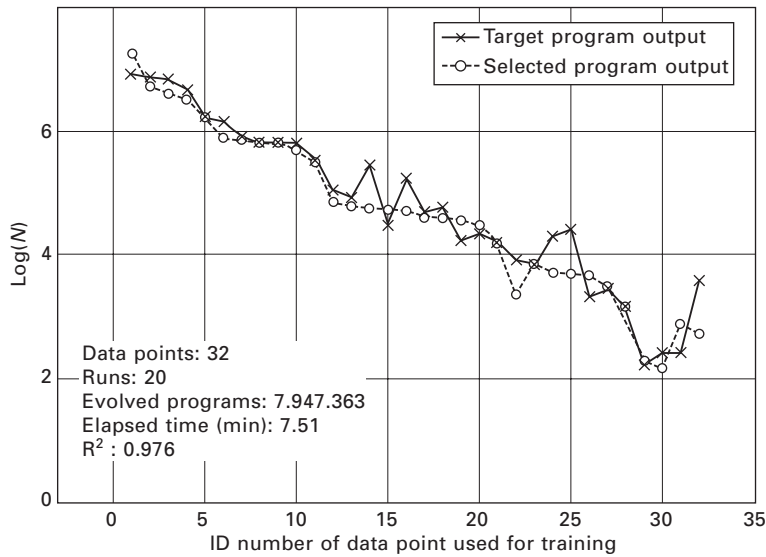


5.13 ANFIS modeling accuracy for 30% training.

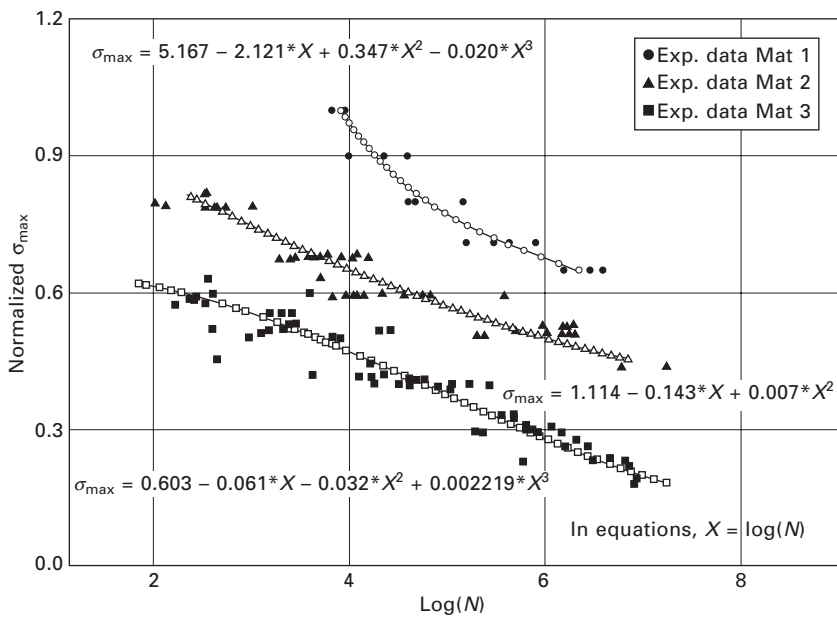
experimental data were used for model training, all ANFIS predictions were very well corroborated by the experimental data. The modeling accuracy was adequate even when only small proportions of the available experimental data were used for model training, e.g. 30%: see Fig. 5.13. However, this good ANFIS performance was not the rule when the training set was less than 50%.

5.5.3 Genetic programming

A genetic programming model was also developed for the representation of the fatigue life data of the treated material. The training efficiency of the GP tool was very good when 50% or more of the available data were used to form the training set as well. As shown in Fig. 5.14, where the target output is compared to the best program output after the training process, the coefficient of multiple determination (R^2) was 0.98 for the selected case of the reversed loading of material #3. For all the treated cases the training accuracy was very high, with R^2 values above 0.95. It should be noted that there is no relationship between the two symbols R^2 and R used in this text. The results of the application of the GP tool to the treated material are presented in Fig. 5.15, where selected S–N curves are plotted together with the experimental data for each material system on the S–N plane. For this demonstration, only one S–N curve from each data set was selected, namely that corresponding to specimens cut at 15° off-axis and tested under tension–tension, $R = 0.1$ loading, from the first data set, the S–N curve for



5.14 Training of GP model with experimental data for material #3.



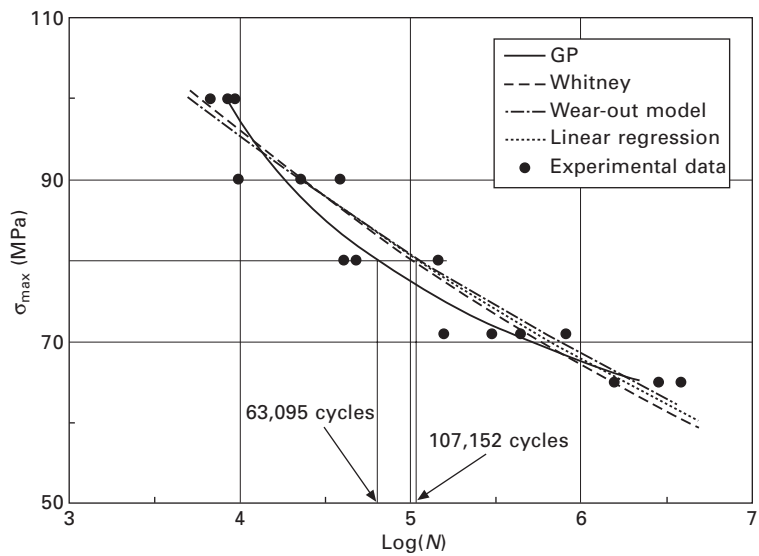
5.15 Fitted lines to predicted S–N curves together with experimental data. Predictions indicated by open symbols.

compression–compression loading under $R = 10$ for material #2, and the S–N curve for the reversed loading of material #3. In order to plot all curves on the same graph, stress data were normalized over their maxima: 100 MPa for material #1, 408.6 MPa for material #2 and 450.4 MPa for material #3. Fig. 5.15 shows that the modeling accuracy of genetic programming is excellent. In all the studied cases the produced curves follow the trend of the experimental data perfectly. It should be mentioned that the S–N curve predicted by the genetic programming tool is not of a predetermined type such as power curve, polynomial, semi-logarithmic, etc. The resulting curve consists of data pairs (input and output) that can be simply plotted on the S–N plane. Although such use of the model suffices for the subsequent analysis, it should be noted that output data, even if not necessary, can easily be fitted by a second- to fourth-order polynomial equation, as shown in the same figure.

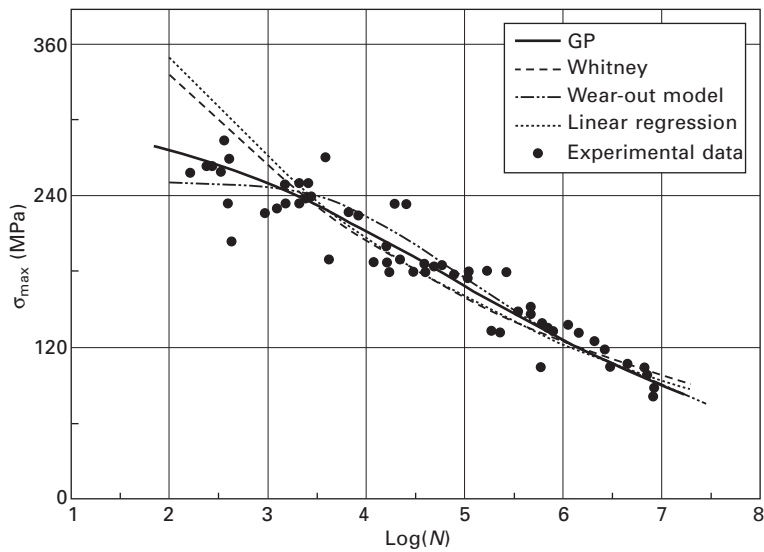
5.6 Comparison to conventional methods of fatigue life modeling

Fatigue models can also be derived using conventional methods, such as linear regression, Sendeckyj's wear-out model [33], Whitney's Weibull statistics [34], etc. A detailed comparison of these conventional methods and genetic programming was presented in [19]. Linear regression and ANFIS results were compared briefly in [17]. The S–N curves predicted using all the available methods are presented in Figs 5.16–5.18 for comparison. It may be concluded that, although based on different approaches, generally speaking all fatigue models were able to adequately represent the fatigue behavior of the selected experimental data, at least for the central part of the S–N curve, for $\log(N) = 3$ to $\log(N) = 6$. Figure 5.16, in which predictions for the data of material #1 (15° off-axis, $R = 0.1$) are presented, shows that the GP curve 'follows' the trend of the experimental data more closely than the other three fatigue models that produce a somewhat straight curve on the $\log(N)$ –S plane. For example, when examining the stress level of 80 MPa, the experimental average number of cycles could be calculated as 77,985 and the corresponding estimated numbers from the GP curve and other methods as 63,095 and 107,152 cycles respectively. Moreover, for the stress level of 71 MPa, the GP tool estimates 380,189 loading cycles and the other methods approximately 562,341 loading cycles, while the experimental average is 421,213 loading cycles. For both examined stress levels, the GP curve underestimates actual loading cycles by a factor of 9.7–19.1%, while the other curves overestimate the loading cycles by a factor of more than 33.5%.

The weakness of linear regression and Whitney's method is demonstrated in Fig. 5.17, where the S–N curves for the $[(\pm 45/0)_4/\pm 45]_T$ laminate, material

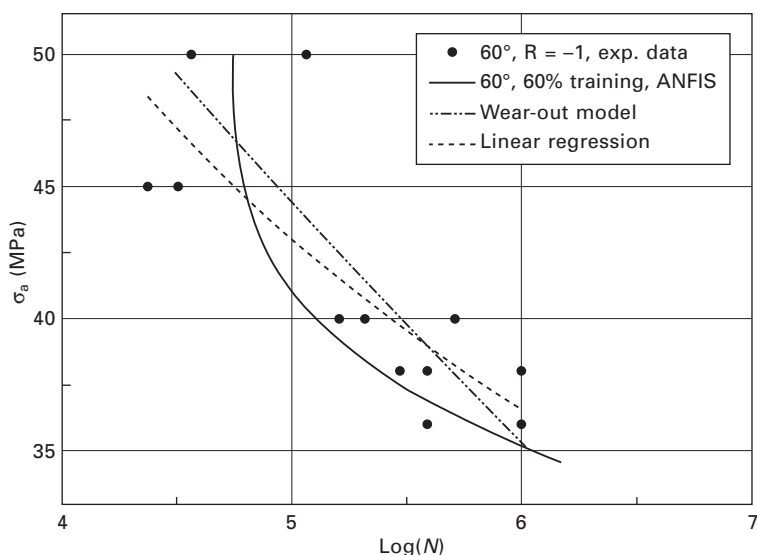


5.16 Comparison of different methods for S–N data interpretation with GP predictions, material #1.



5.17 Comparison of different methods for S–N data interpretation with GP predictions, material #3.

#3, are presented. When observed in detail, this figure shows that the GP and wear-out models are the most appropriate for interpretation of low-cycle fatigue data, $\log(N) < 3$, as they are capable of producing multi-slope curves.



5.18 Comparison of different methods for S–N data interpretation with ANFIS predictions, material #1.

On the other hand, linear regression analysis and the Whitney method are based on a power curve equation for the whole fatigue cycle range.

Generally speaking, GP predictions seem to compare favorably with those produced by fatigue models. In some of the examined cases GP proves superior as it can follow the real trend of the experimental data, without the constraints of a specific equation type. The superiority of the computational methods lies in their ability to adapt to each different data set by following its actual trend and without any constraints regarding equation types as shown in Fig. 5.18. In this figure, the fatigue model derived using the ANFIS tool is compared to that derived after linear regression analysis of the fatigue data from the first database (material #1, specimens cut at 60° off-axis and tested under reversed loading). In this case linear regression analysis is clearly not the appropriate method for interpretation of the given fatigue data. The model based on the wear-out model, although better than the linear regression, is still restricted by the predetermined power curve equation. On the other hand, the ANFIS model is obviously capable of following the trend of the fatigue data and deriving a reliable S–N curve.

The selected AI methods were proved (see also [19]) to be better than conventional techniques for modeling the fatigue behavior of a number of composite laminates. Previous results proved that, when enough experimental data exist, computational methods are very efficient. However, these methods should be utilized with caution since they do not offer a magic solution to

all modeling problems. A neural network or genetic programming model that produces exceptional training results is not necessarily well trained. The following considerations should be kept in mind.

When very few experimental data are available for model training, the risk of overfitting is high. Overfitting is what occurs when trying to fit more parameters than the available data pairs utilized for the training of the model. It is like trying to estimate the model parameters of a parabola passing through two points. In fact there are an infinite number of parabolic curves that can cross two points. Therefore, the training process gives a perfectly trained – although misleading – model since the predictive ability of an overfitted model is very weak. The model is unable to extrapolate any reliable predictions since it is trained within a narrow range of experimental data. Thus, the predictions provided by the model (compared to the test data set) become poorer as the training set diminishes.

An example of overfitting was presented in [17]. Training the ANFIS model with only 10% of the available experimental data resulted in the calculation of an R^2 error of 0.99 during training. However, this apparently perfect model was unable to generalize and therefore the R^2 error on the test data was 0.25.

The presented AI methods have a common characteristic: they are data-driven methods in the sense that they are not based on any assumptions concerning material behavior or the adoption of mathematical models that must be employed to simulate it. Although this independence with regard to the material is an asset for their application, it can also become a disadvantage, since modeling accuracy depends on the quality of the available experimental data and cannot be improved by any physical considerations.

5.7 Conclusions and future trends

Novel computational methods such as artificial neural networks, adaptive neuro-fuzzy inference systems and genetic programming have been shown to be very powerful modeling tools for the nonlinear behavior of composite laminates subjected to constant amplitude loading. They can be used to model the fatigue behavior of several different material systems. Their modeling ability compares favorably with, and is to some extent superior to, other modeling techniques.

In their present form, computational methods have been used as stochastic nonlinear regression analysis tools. Their advantages compared to other conventional methods can be summarized as follows.

Computational methods are stochastic nonlinear regression tools, and can therefore be used to model the fatigue behavior of any material, provided that sufficient data are available for training. Their stochastic nature is also of the utmost importance as different output is produced after every run

of the model for the same input data. Following this procedure, new data sets could be created based on a specific input in order to augment limited databases.

Modeling is not based on any assumptions, for example that the data follow a specific statistical distribution, or that the S–N curve is a power curve equation. Moreover, the process does not take the mechanics of each material system into account. Strictly speaking, the presented computational methods are material-independent data-driven methods that correlate input with output values in order to establish a model describing the relationship between them. In this context the proposed methods can easily be applied to any material, provided that an adequate amount of data exists.

The S–N curves derived do not follow any specific mathematical form. They simply follow the trend of the available data, in each case giving the best estimate of their behavior. However, as shown in previous works, output data can easily be fitted by simple second to fourth-order polynomial equations. Nevertheless, although the behavior of limited data sets can easily be simulated by artificial intelligence methods, such cases should be treated with caution since there is always the risk of overfitting.

Artificial intelligence methods have been successfully used for modeling the fatigue behavior of various metallic and composite material systems. However, there is to date no evidence that these methods include any predictive ability. Up until now, all these methods have clearly been proved to be very accurate tools for interpolating material behavior within a known database, but they cannot extrapolate any predictions outside the database, either for different loading conditions or for different materials. Future research should be focused in this direction.

It is also anticipated that the presented computational methods can be implemented in solving more complex modeling and predicting problems. Some potential applications of the computational methods are:

- Modeling of the behavior of composite materials under block and variable amplitude loading conditions.
- Prediction of the off-axis fatigue behavior of composite laminates. Computational methods can compete with existing failure criteria insofar as they could potentially simulate the off-axis behavior of the examined material using the minimum number of experimental data.
- Modeling of the stiffness degradation of composite materials. Stiffness degradation during fatigue frequently exhibits a sigmoid trend. For the simulation of stiffness degradation during fatigue computational methods can compete with existing models, which often fail to accurately interpret experimental data.

Initial results indicate that computational methods can be used for the derivation of models to simulate the thermomechanical behavior of composite

materials. To date, there is no commonly accepted phenomenological (or other type of) model for this task.

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Fatigue life prediction of composite materials under constant amplitude loading

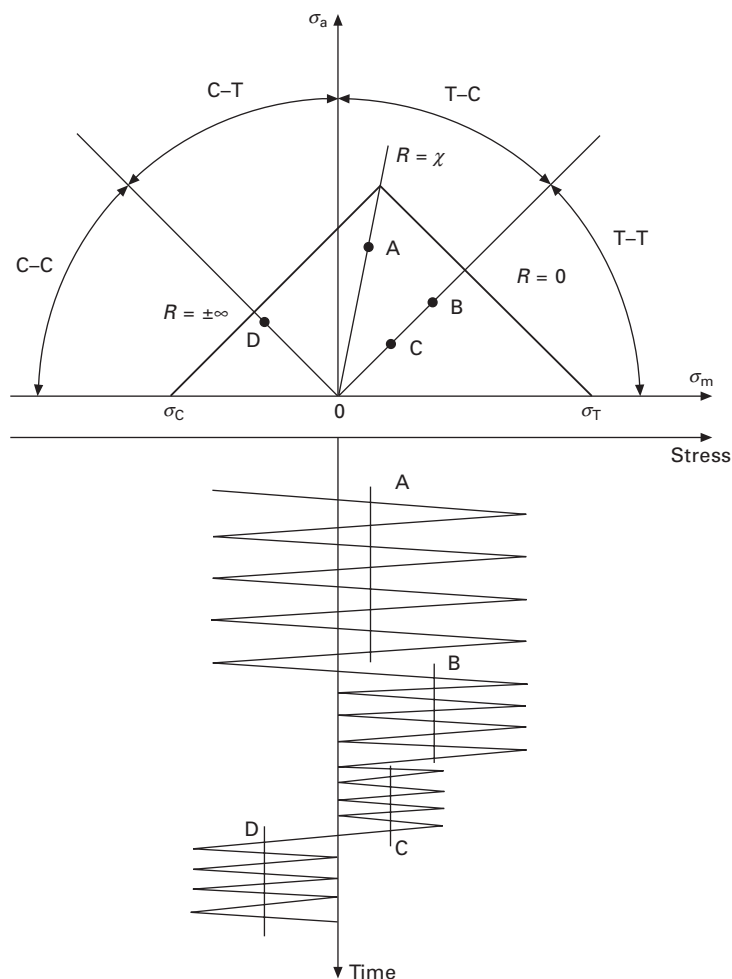
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Abstract: This chapter describes a practical method of efficiently predicting the fatigue lives of composite materials for different values of stress ratio. It focuses on the constant fatigue life (CFL) diagram to predict the S–N curves for composites under different constant amplitude fatigue loading conditions. A particular emphasis is placed on a most general CFL diagram, called the anisomorphic CFL diagram, that allows efficiently identifying the mean stress sensitivity in a non-Goodman type of fatigue behavior of composites. An extended version of the anisomorphic CFL diagram that shows higher flexibility and thus allows better prediction is also presented.

Key words: polymer matrix composites, S–N curve prediction, mean stress sensitivity, constant fatigue life diagram.

6.1 Introduction

Fatigue load that should be withstood by machines and structures varies in the alternating stress amplitude and mean stress. Furthermore, the shape and configuration of the stress–time pattern during service takes many different forms according to their actual operation (Harris, 2003). Therefore, for safely applying fiber-reinforced composite materials to structural components, especially in large-scale aircraft and wind turbine applications and axial flow fans in thermal power stations, since they should be designed to work throughout their specified lives which are finite, we need a reliable engineering technique by which the fatigue lives of composites subjected to variable loading can accurately be predicted. Development of an engineering method of accurately predicting the fatigue lives of composites under variable loading conditions requires the understanding and quantification not only of the effect of alternating stress and mean stress on the fatigue lives of composites under constant amplitude loading conditions, but also of the effect of variation in alternating and mean stresses on the fatigue lives: see Fig. 6.1. Evaluation of the effect of loading mode on the sensitivity to fatigue of composites needs a large amount of fatigue testing for various kinds of cyclic loading conditions, which consumes considerable time and cost. From a practical point of view, therefore, it is required to develop a time- and cost-saving procedure for identifying and coping with the loading mode dependence of



6.1 Schematic illustration of variable amplitude (VA) and variable R -ratio (VR) fatigue loading.

the fatigue lives of composites with reasonable accuracy on the basis of a limited number of experiments. This can only be achieved with the aid of theoretical models to predict the fatigue lives of composites under constant and variable cyclic loading conditions, respectively.

For development of a new fatigue life evaluation system for composites that is applicable to complicated service loading conditions, it is an essential prerequisite to establish a theoretical fatigue life calculation method for constant amplitude fatigue loading. Two approaches have been developed so far to meet the prerequisite: (1) the approach using a master S - N curve (Ellyin and El Kadi, 1990; D'Amore *et al.*, 1996; Caprino and D'Amore,

1998; Caprino and Giorleo, 1999; Kawai, 1999; Kawai *et al.*, 2000, 2001a, b; Kawai and Suda, 2004; Kawai and Taniguchi, 2006); and (2) the approach using a constant fatigue life (CFL) diagram. While it is an elegant solution, the master S–N curve approach relies on the quest for an effective fatigue strength parameter. It is not straightforward to reach such a general measure of fatigue strength. The CFL diagram approach, by contrast, allows easy accommodation to the mean stress sensitivity observed by experiment, suggesting that the CFL diagram approach is more flexible, and thus more fruitful for most engineers, than the master S–N curve approach. Therefore, the CFL diagram for a given composite is considered to be a most practical and efficient tool for predicting the S–N curves for any stress ratios.

The Goodman diagram (Goodman, 1899), which is the simplest graphical description of the mean stress sensitivity in fatigue, is the classic fatigue analysis tool for conventional materials. However, it is not always applicable in the constant amplitude fatigue behavior of composites (Salkind, 1972). Ramani and Williams (1977) have examined the fatigue behavior of $[0/\pm 30]_{3S}$ carbon/epoxy laminates at different values of mean stress. They observed that the CFL diagram becomes asymmetric about the alternating stress axis and the peak position of the CFL diagram is slightly shifted to the right of the alternating stress axis, while the CFL diagram as a plot of alternating stress versus mean stress can approximately be represented by straight lines, regardless of the given constant values of fatigue life. A similar tendency for the CFL diagrams plotted using the alternating and mean stress components of fatigue stress to become asymmetric can also be found in the experimental results reported by Ansell *et al.* (1993), Harris *et al.* (1990, 1997), Adam *et al.* (1989, 1992), Gathercole *et al.* (1994), and Beheshty *et al.* (1999). These experimental results also indicate that the alternating stress component of fatigue stress takes a maximum value at a particular stress ratio that is almost equal to the ratio of compressive strength to tensile one of the material considered. Philippidis and Vassilopoulos (2002a, b) have examined the fatigue behavior of $[0/(\pm 45)_2/0]_T$ glass/polyester laminates with different material orientations at four stress ratios ($R = 10, -1, 0.1, 0.5$), and demonstrated that the CFL points, i.e. the pairs of mean stress and alternating stress for different constant values of fatigue life which are calculated on the basis of the S–N curves for the four stress ratios, deviate from the Goodman straight lines.

More detailed and systematic investigation that deals with the shape of the CFL diagram has been carried out by Harris and coworkers (Harris *et al.*, 1990, 1997; Adam *et al.*, 1989, 1992; Gathercole *et al.*, 1994; Beheshty *et al.*, 1999) on different kinds of CFRP laminates over the whole range of stress ratio. They found that the CFL diagrams for the CFRP laminates can approximately be described using nested bell-shaped curves. A different nonlinear CFL diagram, called the anisomorphic CFL diagram, has recently

been proposed by Kawai (2006) and Kawai and Koizumi (2007), and it was shown to be valid for quasi-isotropic $[45/90/-45/0]_{2S}$ and $[0/60/-60]_{2S}$ carbon/epoxy laminates and for a cross-ply $[0/90]_{3S}$ carbon/epoxy laminate. The anisomorphic CFL diagram approach has a great advantage over existing methods in efficient identification of the mean stress sensitivity in fatigue of composites, and it can be built using only the static strengths in tension and compression and the reference S–N relationship associated with a particular stress ratio; this particular stress ratio has been called the critical stress ratio. This method has further been developed into a more general form that allows improved prediction of the CFL diagrams and S–N curves for different types of composite laminates (Kawai and Murata, 2008).

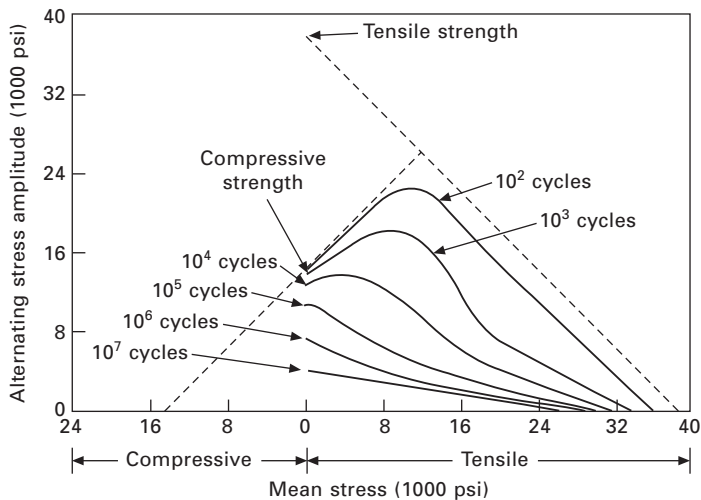
This chapter will focus on the CFL diagrams for composites that are viewed as useful tools to predict the S–N curves under different constant amplitude fatigue loading conditions. The recent progress in constant amplitude fatigue analysis techniques based on CFL diagrams is reviewed. The review is not intended to be comprehensive, but it aims to help update the fatigue life prediction methods that have been developed since publication of the practical encyclopedic textbook on the fatigue of composites (Harris, 2003). Only a phenomenological approach is examined in the present attempt. Furthermore, a particular emphasis is placed on a most general CFL diagram, called the anisomorphic CFL diagram (Kawai, 2006; Kawai and Koizumi, 2007; Kawai and Murata, 2008), that allows efficiently identifying the mean stress sensitivity in a non-Goodman type of fatigue behavior of composites. The anisomorphic CFL diagram approach is discussed in detail in its formulation and predictive accuracy. Validity of the method is evaluated for the fiber-dominated and matrix-dominated fatigue behaviors of multidirectional carbon/epoxy laminates. An extended version of the anisomorphic CFL diagram that shows higher flexibility and thus allows better prediction is also presented. Finally, the influence of factors such as temperature, moisture and loading rate on the CFL diagrams for composites is briefly described.

6.2 Constant fatigue life (CFL) diagram approach

A stress level below which fatigue life becomes infinite, i.e. a fatigue limit, cannot clearly be identified in the S–N curves for most continuous fiber composite laminates, especially for those in which some constituent plies are most favorably orientated in the direction of fatigue load. Of a given composite laminate subjected to constant amplitude fatigue loading, therefore, it is essential to evaluate the maximum stress level below which the fatigue life of the composite becomes longer than a specified number of cycles to failure. A most practical method for this purpose is to make use of a family of nested mean stress versus alternating stress ($\sigma_m - \sigma_a$) loci for different constant values of life for a given composite, which is called the

constant fatigue life (CFL) diagram. Once the CFL diagram is constructed for a given composite, it allows speedy evaluation of the maximum fatigue stress sustained by the composite for any given number of cycles to failure with the aid of the CFL diagram. In other words, the safe stress region can be identified in which the constant amplitude cyclic loading condition should lie so that the composite does not fail before a specified number of cycles (Hertzberg, 1989). In addition to quick identification of the fatigue strength for a given number of cycles to failure and of the associated safe stress region, efficient prediction of the S–N curves for a given composite under constant amplitude fatigue loading at any stress ratios can be made by means of the CFL diagram. This allows quick preparation of inputs to the fatigue life analysis of composites for any operational load spectra.

While they are useful for engineering fatigue analysis, the CFL diagrams for fibrous composites have not been given a standardized procedure to be followed in their construction. This is partly because of complications involved. The CFL diagrams for composites are not always accurately described by means of Goodman's linear relation (Goodman, 1899) or by Gerber's quadratic relation (Gerber, 1874), which was observed by Boller (1957, 1964), almost a decade after the birth of glass fiber reinforced composites, '*when they were continually being developed for structural use in aircraft and power plants and the factors affecting the fatigue strength were being desired by the designers*'. Figure 6.2 shows the CFL diagram that Boller constructed in his articles for a glass fabric composite tested at high temperature. Along with the fact that the experimental CFL envelopes



6.2 Effect of mean stress on alternating stress amplitude for a glass-fabric/polyester laminate. (Boller, 1964)

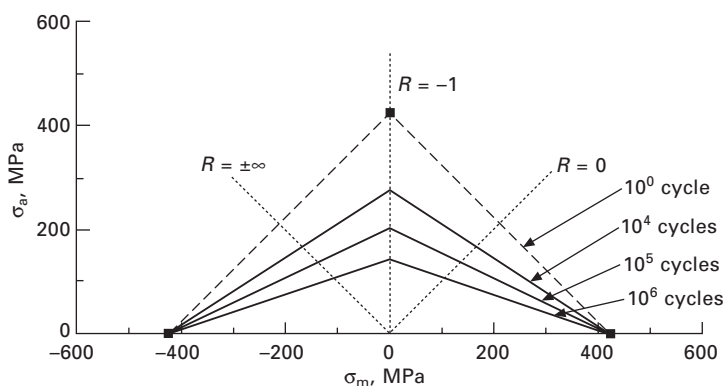
deviate from the Goodman lines and from the Gerber curves as well, it can also be found that (1) the glass fabric composite exhibits different strengths in tension and compression, and (2) the experimental CFL envelopes become asymmetric about the alternating stress axis, and higher alternating stress amplitudes can be sustained at low levels of mean stress than at zero mean stress, as clearly mentioned in Boller's article. Moreover, (3) the shape of CFL envelopes changes with increasing number of cycles to failure; this feature was noticed early on by Hahn (1979). All of these features that can be observed in the experimental CFL diagram plotted by Boller are the requirements that should be considered for accurate description of the CFL diagrams for composites. Recent progress in developing the procedure for constructing the CFL diagrams for composites has been made by taking into account the requirements mentioned above, and it is reviewed in the following section.

6.3 Linear constant fatigue life (CFL) diagrams

6.3.1 Symmetric and asymmetric Goodman diagrams

The linear CFL diagram for a material that is symmetric about the alternating stress axis is schematically illustrated in Fig. 6.3. The symmetric Goodman diagram can be described by means of the following piecewise-defined function in the $\sigma_m - \sigma_a$ stress plane:

$$-\frac{\sigma_a - \sigma_a^{R=-1}}{\sigma_a^{R=-1}} = \begin{cases} \frac{\sigma_m}{\sigma_T}, & 0 \leq \sigma_m \leq \sigma_T \\ -\frac{\sigma_m}{\sigma_T}, & -\sigma_T \leq \sigma_m < 0 \end{cases} \quad 6.1$$



6.3 Schematic illustration of symmetric Goodman diagram.

where σ_T is the ultimate tensile strength of the composite at a given temperature and loading rate. Another parameter, $\sigma_a^{R=-1}$, denotes the alternating stress amplitude for completely reversed tension-compression cyclic loading ($R = -1$), and it is a function of the number of cycles to failure N_f ; i.e. $\sigma_a^{R=-1} = \hat{\sigma}_a(N_f; R = -1)$. The fatigue strength function $\sigma_a^{R=-1}$ can be identified by fitting a function $\hat{\sigma}_a(N_f)$ to the S-N curve for the fatigue loading at zero mean stress ($R = -1$). Nasr *et al.* (2005) have examined the effect of mean stress on the torsional fatigue behavior of a glass/polyester laminate using thin-walled tubular specimens with two fiber orientations, $[\pm 45]$ and $[0/90]$. They observed that the Goodman relation accurately applies to the $[\pm 45]$ fiber orientation over the whole range of shear mean stress, and to the $[0/90]$ fiber orientation over a limited range in which the shear mean stress is larger than a certain value.

The piecewise-defined function for the symmetric Goodman diagram can be rearranged as

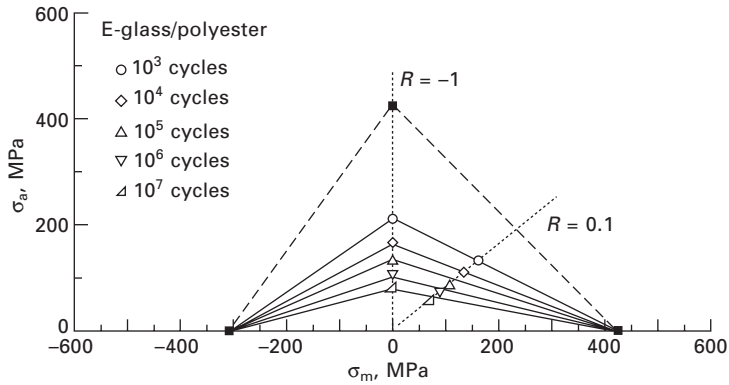
$$\frac{\sigma_a^{R=-1}}{\sigma_T} = \begin{cases} \frac{\frac{\sigma_a}{\sigma_T}}{1 - \frac{\sigma_m}{\sigma_T}}, & 0 \leq \sigma_m \leq \sigma_T \\ \frac{\frac{\sigma_a}{\sigma_T}}{1 + \frac{\sigma_m}{\sigma_T}}, & -\sigma_T \leq \sigma_m < 0 \end{cases} \quad 6.2$$

The left-hand side of each formula is a function of the number of cycles to failure only, suggesting that the right-hand sides of the formulas define the fatigue strength parameters that consider the effect of mean stress on the fatigue lives of composites. The fatigue strength parameter suggested by equation [6.2] for the range of non-negative mean stress was tested on unidirectional composites (Kawai, 2004) and was called the modified fatigue strength ratio.

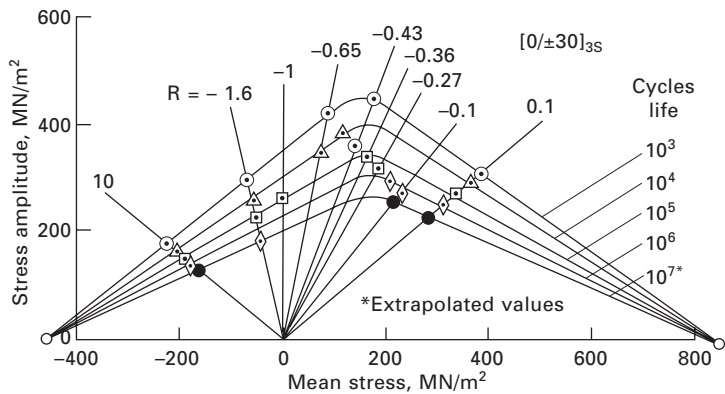
By taking into account different strengths in tension and compression, we can modify the symmetric Goodman diagram and obtain the following asymmetric form (Fig. 6.4):

$$-\frac{\sigma_a - \sigma_a^{R=-1}}{\sigma_a^{R=-1}} = \begin{cases} \frac{\sigma_m}{\sigma_T}, & 0 \leq \sigma_m \leq \sigma_T \\ \frac{\sigma_m}{\sigma_C}, & \sigma_C \leq \sigma_m < 0 \end{cases} \quad 6.3$$

where $\sigma_T(> 0)$ and $\sigma_C(< 0)$ are the tensile and compressive strengths of the composite, respectively. Note that the largest alternating stress amplitude is



6.4 Schematic illustration of asymmetric Goodman diagram.



6.5 Shifted Goodman diagram for a $[0/\pm30]_{3S}$ carbon/epoxy laminate. (Ramani and Williams, 1977)

sustained at zero mean stress in the asymmetric Goodman diagram as well, in line with the symmetric Goodman diagram mentioned above. Such an asymmetric form of linear CFL diagram is applicable to wood and polymer matrix composites (Ansell *et al.*, 1993; Bond and Ansell, 1998a, b; Bond, 1999) and to fiberglass composites (Sutherland and Mandell, 2004).

6.3.2 Shifted Goodman diagram

The highest alternating stress amplitude cannot always be sustained at zero mean stress, even if CFL envelopes can approximately be described using nested straight lines as in the symmetric and asymmetric Goodman diagrams. Ramani and Williams (1977) examined the fatigue behavior of $[0/\pm30]_{3S}$ carbon/epoxy laminates at different magnitudes of mean stress, and obtained the CFL diagram which is reproduced in Fig. 6.5. It demonstrates that while

the CFL envelope can approximately be represented by a straight line over the whole range of fatigue life, the symmetry axis of the CFL diagram is slightly shifted to the right of the alternating stress axis. The peak of the observed Goodman diagram appears at the midpoint σ_M of the mean stress interval $[\sigma_C, \sigma_T]$, i.e. $\sigma_M = (\sigma_T + \sigma_C)/2 = (\sigma_T - |\sigma_C|)/2$, regardless of the number of cycles to failure. This observation allows formulating a shifted Goodman diagram as

$$-\frac{\sigma_a - \sigma_A}{\sigma_A} = \begin{cases} \frac{\sigma_m - \sigma_M}{\sigma_T - \sigma_M}, & \sigma_M \leq \sigma_m \leq \sigma_T \\ \frac{\sigma_m - \sigma_M}{\sigma_C - \sigma_M}, & -\sigma_C \leq \sigma_m < \sigma_M \end{cases} \quad 6.4$$

where σ_A denotes the maximum alternating stress amplitude for a given number of cycles to failure, and it always appears at the constant value of mean stress σ_M in the shifted Goodman diagram. The maximum alternating stress amplitude σ_A is a function of the number of cycles to failure: $\sigma_A = \hat{\sigma}_A(N_f)$. Note that the peak points of the shifted Goodman diagram for different numbers of cycles to failure are related to different values of stress ratio: $R = (\sigma_M - \sigma_A)/(\sigma_M + \sigma_A)$.

6.3.3 Inclined Goodman diagram

In addition to the results reported by Ramani and Williams (1977), much experimental evidence can be found that supports the shift of the peak positions of the CFL envelopes for composites to the right or left of the alternating stress axis, e.g. Ansell *et al.* (1993); Harris *et al.* (1990, 1997); Adam *et al.* (1989, 1992); Gathercole *et al.* (1994); Beheshty *et al.* (1999); Phillips (1981); Kawai and Koizumi (2007); and Kawai and Murata (2008). The experimental results reported in those studies imply that the CFL diagrams for composites with different strengths in tension and compression tend to be shifted by the difference between the tensile and compressive strengths. In regard to the asymmetry in the CFL diagrams for composites, however, a question is raised about where the peak position of each envelope for a given number of cycles to failure should come. Although the CFL diagram plotted by Boller (1957, 1964) suggests an answer to this question, it was not clearly mentioned until recently. Observing, with this question in mind, Boller's plots of the constant amplitude fatigue data on glass fabric composites, we notice that the peaks of the CFL envelopes almost fall on a single radial line associated with a certain constant stress ratio. This observation suggests another variant of the Goodman diagram; here it is called an inclined Goodman diagram.

Assume that the peak points of CFL envelopes fall on the radial line with

the equation $\sigma_a/\sigma_m = (1 - \chi)/(1 + \chi)$ associated with a particular value of stress ratio $R = \chi$. Then, the inclined Goodman diagram can be described by means of the following piecewise-defined function:

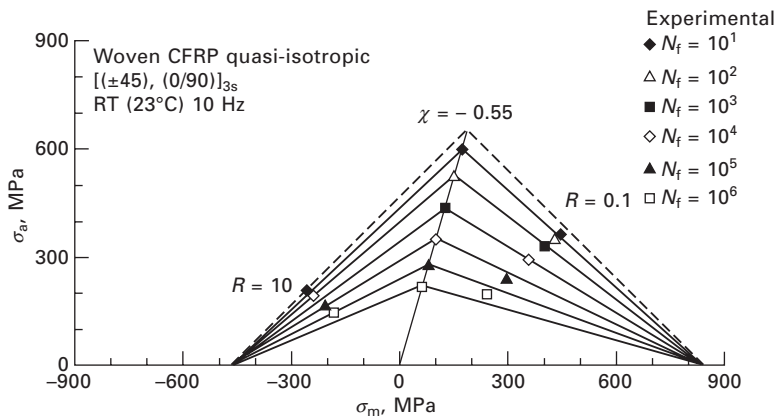
$$-\frac{\sigma_a - \sigma_a^\chi}{\sigma_a^\chi} = \begin{cases} \frac{\sigma_m - \sigma_m^\chi}{\sigma_T - \sigma_m^\chi}, & \sigma_m^\chi \leq \sigma_m \leq \sigma_T \\ \frac{\sigma_m - \sigma_m^\chi}{\sigma_C - \sigma_m^\chi}, & \sigma_C \leq \sigma_m < \sigma_m^\chi \end{cases} \quad 6.5$$

where σ_a^χ and σ_m^χ represent the coordinates of the peak point of the CFL envelope for a given constant value of life. They are the alternating and mean stress components of the maximum fatigue stress $\sigma_{\max}^\chi$ for the fatigue loading at the particular stress ratio $R = \chi$, and are given as

$$\sigma_a^\chi(N_f) = \frac{1}{2}(1 - \chi)\sigma_{\max}^\chi(N_f) \quad 6.6$$

$$\sigma_m^\chi(N_f) = \frac{1}{2}(1 + \chi)\sigma_{\max}^\chi(N_f) \quad 6.7$$

The superscript χ attached to these quantities is not an exponent, but a label to emphasize that they are associated with the fatigue loading at the particular stress ratio $R = \chi$. The inclined Goodman diagram predicts that the maximum value of the alternating stress component is sustained under the fatigue loading at the particular stress ratio $R = \chi$, as seen in the example shown in Fig. 6.6. The inclined Goodman diagram is utilized below as a basis for the development of a nonlinear CFL diagram.



6.6 Inclined Goodman diagram for a [(+45/-45), (0/90)]_{3s} carbon-fabric/epoxy laminate.

6.4 Nonlinear constant fatigue life (CFL) diagrams

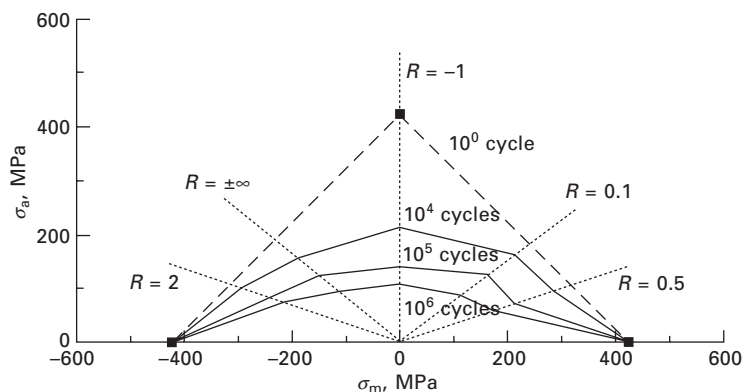
Nonlinearity in the CFL envelopes for composites is a fundamental deviation from the traditional Goodman diagram. Consideration of nonlinearity in the CFL envelopes for composites is essential not only for their better description but also for more accurate prediction of the S–N curves for any stress ratios using the constructed CFL diagrams. Boller (1957, 1964) has demonstrated the nonlinearity in the CFL envelopes for glass fabric composites, as mentioned above. Other examples of nonlinear CFL diagrams can be found in the literature for different kinds of composites: e.g. Ansell *et al.* (1993), Bond and Ansell (1998a,b) and Bonfield and Ansell (1991) for wood composites; Sutherland and Mandell (2004) for fiberglass composites; and Harris *et al.* (1990, 1997), Adam *et al.* (1989, 1992), Gathercole *et al.* (1994), Beheshty *et al.* (1999), Phillips (1981), Kawai and Koizumi (2007) and Kawai and Murata (2008) for carbon fiber composites.

6.4.1 Piecewise linear CFL diagram

In a primitive but effective engineering approach to construction of the nonlinear CFL envelopes for composites, it is natural to consider the description using piecewise-defined linear functions. Consider division of the entire domain of mean stress into a specified number of subdomains as $[\sigma_C, \sigma_T] = \bigcup [\sigma_m^{(i-1)}, \sigma_m^{(i)}] = \bigcup [\sigma_m^{\chi_{i-1}}, \sigma_m^{\chi_i}]$, in which the open subintervals $(\sigma_m^{\chi_{i-1}}, \sigma_m^{\chi_i})$ are disjoint, and the symbols χ_{i-1} and χ_i indicate the partition stress ratios associated with the left and right endpoints of the i th subinterval $[\sigma_m^{\chi_{i-1}}, \sigma_m^{\chi_i}]$; the i th partition stress ratio χ_i can be defined as $\chi_i = (\sigma_m^{(i)} - \sigma_a^{(i)})/(\sigma_m^{(i)} + \sigma_a^{(i)})$. Then, the linear interpolation of the CFL envelopes for the i th subinterval of mean stress $[\sigma_m^{\chi_{i-1}}, \sigma_m^{\chi_i}]$ can be described as

$$\frac{\sigma_a - \sigma_a^{\chi_i}}{\sigma_a^{\chi_i} - \sigma_a^{\chi_{i-1}}} = \frac{\sigma_m - \sigma_m^{\chi_i}}{\sigma_m^{\chi_i} - \sigma_m^{\chi_{i-1}}}, \quad \sigma_m^{\chi_{i-1}} \leq \sigma_m \leq \sigma_m^{\chi_i} \quad 6.8$$

This gives a CFL diagram just like half a spider's web that is shown schematically in Fig. 6.7. This type of CFL diagram was called a multiple R -value CFL diagram (Nijssen, 2006). Such piecewise linear approximation of nonlinear CFL envelopes has been tested by Harris *et al.* (1990). This approach has also been adopted in recent studies, e.g. Bond and Farrow (2000), Philippidis and Vassilopoulos (2004), and Sutherland and Mandell (2004). It is obvious that the piecewise linear approximation of the nonlinear CFL diagrams for composites requires the constant amplitude fatigue data for partition stress ratios $R = \chi_i$. Therefore, this approach is basically equivalent to the experimental method for constructing the CFL diagram for a given



6.7 Schematic illustration of a piecewise linear constant fatigue life diagram.

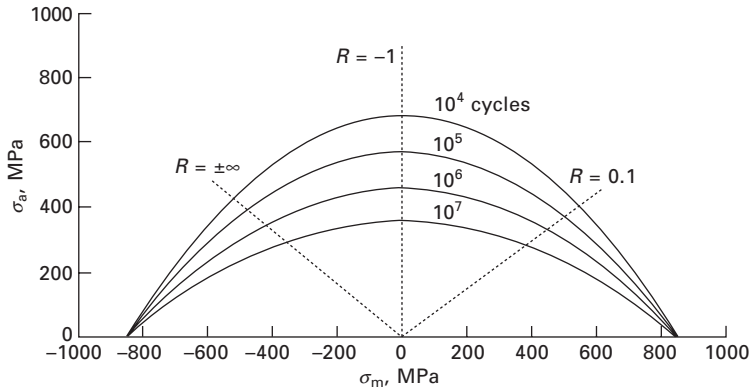
composite, and the equi-life data points obtained from constant amplitude fatigue tests at different stress ratios are connected by straight lines. Note that any asymmetric shape of CFL envelopes for composites can be dealt with by means of this approach, and a typical example can be found in Harris *et al.* (1990).

6.4.2 Symmetric and asymmetric Gerber diagrams

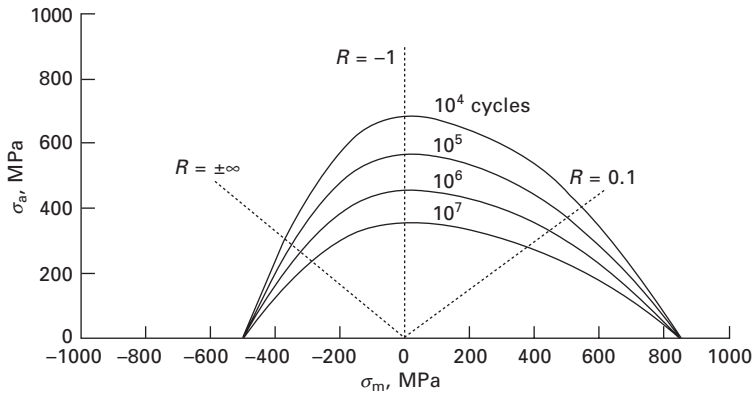
The piecewise linear approximation is simple in mathematical structure, and it is flexible enough to accommodate any complex shape of CFL diagram for a given composite. The accuracy of piecewise linear approximation of the nonlinear shape of CFL envelope for a composite increases as the number of mean stress partitions increases. However, there is no practical guideline for a choice of the number of nodes in the piecewise linear approximation. It has to be determined according to the complexity of the load spectra that should be considered in the fatigue analysis of composite structures.

In order to reduce the number of nodes associated with the endpoints of the partitioned mean stress intervals, nonlinear interpolation can be considered for each of the coarsely divided subintervals of the mean stress domain $[\sigma_C, \sigma_T]$. The Gerber diagram (Gerber, 1874) is an extreme example in which a single analytical function can be assumed, i.e. a parabolic function, for nonlinear interpolation over the entire domain of mean stress. The symmetric Gerber diagram and an asymmetric variant can be considered, in line with the variants of the Goodman diagram discussed above; see equations [6.1] and [6.3].

The symmetric Gerber diagram is schematically shown in Fig. 6.8. It consists of nested parabolas that have vertices on the alternating stress axis, open downward, and cross with the mean stress axis at σ_T and $\sigma_C = -\sigma_T$.



6.8 Schematic illustration of symmetric Gerber diagram.



6.9 Schematic illustration of asymmetric Gerber diagram.

It can be described by means of the following parabolic tent function:

$$-\frac{\sigma_a - \sigma_a^{R=-1}}{\sigma_a^{R=-1}} = \left(\frac{\sigma_m}{\sigma_T} \right)^2, \quad -\sigma_T \leq \sigma_m \leq \sigma_T \quad 6.9$$

where $\sigma_a^{R=-1}$ is the σ_a -coordinate of the vertex of the parabola for a given number of cycles to failure. Note that $\sigma_a^{R=-1}$ is a function of the number of cycles to failure, and it can be identified by fitting a certain function to the fatigue data for $R = -1$.

The asymmetric Gerber diagram that considers different strengths in tension and compression is schematically illustrated in Fig. 6.9. Mathematically, it can be described using the following piecewise-defined function:

$$-\frac{\sigma_a - \sigma_a^{R=-1}}{\sigma_a^{R=-1}} = \begin{cases} \left(\frac{\sigma_m}{\sigma_T}\right)^2, & 0 \leq \sigma_m \leq \sigma_T \\ \left(\frac{\sigma_m}{\sigma_C}\right)^2, & \sigma_C \leq \sigma_m < 0 \end{cases} \quad 6.10$$

6.4.3 Shifted asymmetric and symmetric Gerber diagrams

To predicting the parabolic CFL envelopes with vertices that appear at a constant magnitude of non-zero mean stress, we can apply a shifted asymmetric Gerber diagram that can be described by means of the following piecewise-defined function:

$$-\frac{\sigma_a - \sigma_A}{\sigma_A} = \begin{cases} \left(\frac{\sigma_m - \sigma_M}{\sigma_T - \sigma_M}\right)^2, & \sigma_M \leq \sigma_m \leq \sigma_T \\ \left(\frac{\sigma_m - \sigma_M}{\sigma_C - \sigma_M}\right)^2, & \sigma_C \leq \sigma_m < \sigma_M \end{cases} \quad 6.11$$

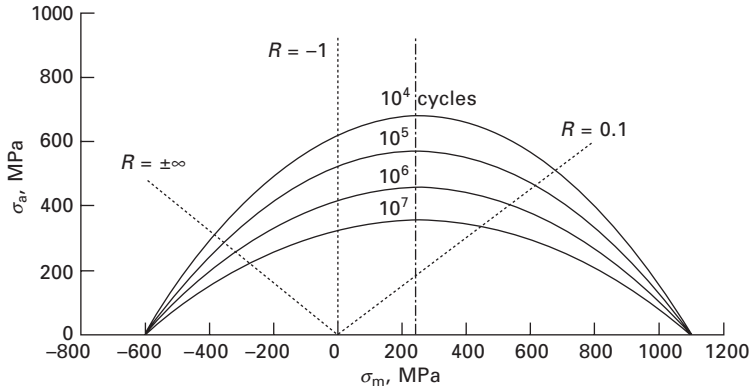
where σ_M and σ_A denote the coordinates of the vertices. Note that σ_M is constant in this case, while σ_A is a function of the number of cycles to failure. The graph of the shifted asymmetric Gerber diagram consists of two parabolas with different foci, and they are smoothly connected at σ_M .

In a particular case where $\sigma_M = (\sigma_T + \sigma_C)/2$, the shifted asymmetric Gerber diagram turns symmetric about the vertical line with the equation $\sigma_m = \sigma_M$, and the resulting shifted symmetric Gerber diagram can be described by means of a single quadratic function:

$$-\frac{\sigma_a - \sigma_A}{\sigma_A} = \left(\frac{\sigma_m - \sigma_M}{\sigma_T - \sigma_M}\right)^2, \quad \sigma_C \leq \sigma_m \leq \sigma_T \quad 6.12$$

The shifted symmetric Gerber diagram is schematically shown in Fig. 6.10, and it has been shown to be valid for carbon/Kevlar hybrid composites (Adam *et al.*, 1989) and carbon fiber composites (Harris *et al.*, 1990; Gathercole *et al.*, 1994). Note that the peak points for different numbers of cycles to failure in the shifted symmetric and asymmetric Gerber diagrams are related to different values of stress ratio: $R = (\sigma_M - \sigma_A)/(\sigma_M + \sigma_A)$.

Incidentally, it can easily be checked that the shifted symmetric Gerber diagram given by equation [6.12] can equivalently be expressed as



6.10 Schematic illustration of shifted Gerber diagram.

$$-\frac{\sigma_a}{\sigma_T} = f\left(1 - \frac{\sigma_m}{\sigma_T}\right)\left(\frac{\sigma_C}{\sigma_T} - \frac{\sigma_m}{\sigma_T}\right), \sigma_C \leq \sigma_m \leq \sigma_T \quad 6.13$$

where f is related to the σ_a -coordinate of the vertex of the parabola for a given number of cycles to failure; i.e.

$$f = \frac{4 \frac{\sigma_A}{\sigma_T}}{\left(1 + \frac{|\sigma_C|}{\sigma_T}\right)^2} \quad 6.14$$

Replacing σ_C (< 0) in equation [6.13] with $-|\sigma_C|$, we can recover exactly the same formula as in the article by Adam *et al.* (1989) and Harris *et al.* (1990):

$$\frac{\sigma_a}{\sigma_T} = f\left(1 - \frac{\sigma_m}{\sigma_T}\right)\left(\frac{|\sigma_C|}{\sigma_T} + \frac{\sigma_m}{\sigma_T}\right), -|\sigma_C| \leq \sigma_m \leq \sigma_T \quad 6.15$$

The last formula has been developed further into a more general nonlinear form that is now called the bell-shaped CFL diagram; it will also be reviewed later on.

6.4.4 Inclined Gerber diagram

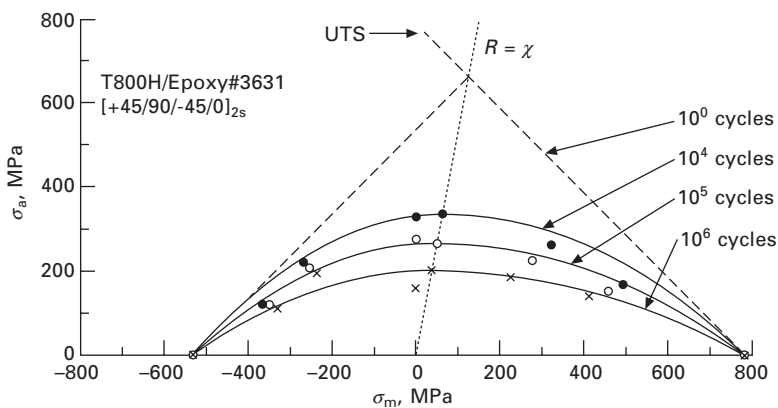
If the peak points in the shifted Gerber diagram for different numbers of cycles to failure are associated with a particular value of stress ratio $R = \chi$, in line with the shifted Goodman diagram, the shifted asymmetric Gerber diagram can be modified to the following form (Kawai *et al.*, 2008):

$$-\frac{\sigma_a - \sigma_a^\chi}{\sigma_a^\chi} = \begin{cases} \left(\frac{\sigma_m - \sigma_m^\chi}{\sigma_T - \sigma_m^\chi} \right)^2, & \sigma_m^\chi \leq \sigma_m \leq \sigma_T \\ \left(\frac{\sigma_m - \sigma_m^\chi}{\sigma_C - \sigma_m^\chi} \right)^2, & \sigma_C \leq \sigma_m < \sigma_m^\chi \end{cases} \quad 6.16$$

where σ_a^χ and σ_m^χ represent the coordinates of the peak points at which two different parabolas are smoothly connected. They are analytical functions of the number of cycles to failure, similar to the definitions given by equations [6.6] and [6.7]. The inclined Gerber diagram is a particular case of a more general nonlinear CFL diagram (Kawai, 2006; Kawai and Koizumi, 2007) that will be described in detail later on. Figure 6.11 shows that the inclined Gerber diagram is valid for the $[45/90/-45/0]_{2S}$ carbon/epoxy laminate in the regime of high cycle fatigue (Kawai and Koizumi, 2007).

6.4.5 Bell-shaped CFL diagram

A revolutionary departure from the traditional Goodman diagram has been made by Harris and coworkers (Adam *et al.*, 1989, 1992; Gathercole *et al.*, 1994; Harris *et al.*, 1997; Beheshty and Harris, 1998; Beheshty *et al.*, 1999). They attempted to accurately model the nonlinear shapes of the CFL diagrams for composite laminates, and developed the nonlinear CFL diagram that has been called the bell-shaped CFL diagram. Mathematically, the bell-shaped CFL diagram may be interpreted as an extension of the shifted symmetric Gerber diagram described using equation [6.13] which is



6.11 Inclined Gerber diagram for a T800H/3631 $[+45/90/-45/0]_{2S}$ carbon/epoxy laminate. (Kawai and Koizumi, 2007)

equivalent to equation [6.12]. The formula for the bell-shaped CFL diagram for composites can be expressed as

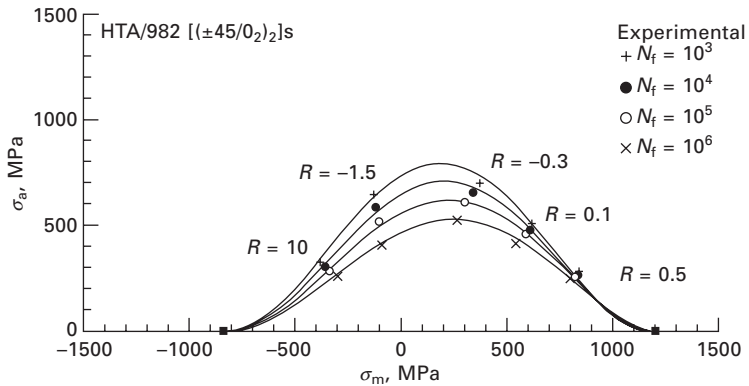
$$\frac{\sigma_a}{\sigma_T} = f \left(1 - \frac{\sigma_m}{\sigma_T} \right)^u \left(\frac{|\sigma_C|}{\sigma_T} + \frac{\sigma_m}{\sigma_T} \right)^v, \quad -|\sigma_C| \leq \sigma_m \leq \sigma_T \quad 6.17$$

where f , u and v are known to be functions of the number of cycles to failure; i.e. $f = \hat{f}(N_f, \dots)$, $u = \hat{u}(N_f, \dots)$, and $v = \hat{v}(N_f, \dots)$. These functions are identified by fitting them to constant amplitude fatigue data for different values of stress ratio. In view of the sign of the power function, the expression given by equation [6.17] is mathematically preferable in which only the positive arguments are involved: $\sigma_T - \sigma_m \geq 0$ and $|\sigma_C| + \sigma_m \geq 0$. The bell shape of the CFL diagram changes with increasing number of cycles to failure. How it changes with the increase in fatigue life depends on the material functions identified. An example of the bell-shaped CFL diagram for a HTA/982 $[\pm 45/0_2]_{2S}$ carbon/epoxy laminate (Beheshty and Harris, 1998) is shown in Fig. 6.12.

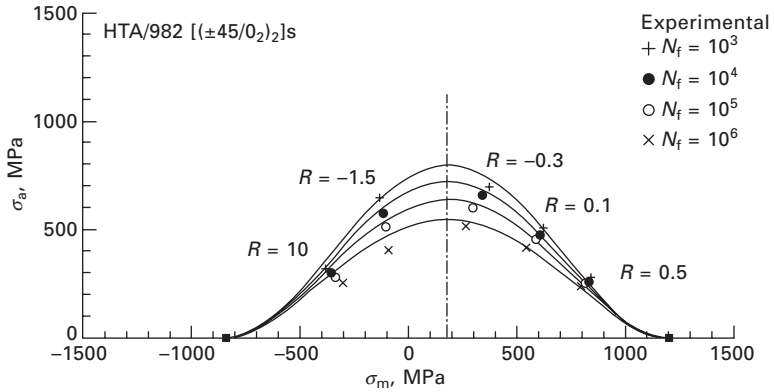
In a particular case of equation [6.17] with $u = v$, the equation reduces to the following form (Gathercole *et al.*, 1994):

$$\frac{\sigma_a}{\sigma_T} = f \left[\left(1 - \frac{\sigma_m}{\sigma_T} \right) \left(\frac{|\sigma_C|}{\sigma_T} + \frac{\sigma_m}{\sigma_T} \right) \right]^u, \quad -|\sigma_C| \leq \sigma_m \leq \sigma_T \quad 6.18$$

In this special case, the bell-shaped CFL diagram becomes symmetric, as schematically shown in Fig. 6.13. Note that the peak positions of the bell-shaped CFL envelopes for different constant values of life are associated with different values of stress ratio in general, except for the case in which $u = v$ and $\sigma_C = -\sigma_T$.



6.12 General bell-shaped constant fatigue life diagram for a HTA/982 $[\pm 45/0_2]_{2S}$ carbon/epoxy laminate. (Beheshty and Harris, 1998)



6.13 Symmetric bell-shaped constant fatigue life diagram. Data from Beheshty and Harris (1998).

The bell-shaped CFL diagram has been shown to be valid for various types of multidirectional carbon/epoxy laminates over the whole range of stress ratio; for example, see Harris *et al.* (1997). Shokrieh and Lessard (1997) have applied it to formulating the multiaxial fatigue behavior of unidirectional composites. Recently, Passipoularidis and Philippidis (2009) have used the bell-shaped CFL diagram to study the factors that affect the life prediction of composites subjected to spectrum loading.

For a fixed stress ratio $R = \chi$, the bell-shaped CFL diagram can be expressed as

$$\frac{\sigma_a^\chi}{\sigma_T} = f \left(1 - \frac{\sigma_m^\chi}{\sigma_T} \right)^u \left(\frac{|\sigma_C|}{\sigma_T} + \frac{\sigma_m^\chi}{\sigma_T} \right)^v \quad (6.19)$$

where $\sigma_a^\chi = \hat{\sigma}_a^\chi(N_f; R = \chi)$ and $\sigma_m^\chi = \hat{\sigma}_m^\chi(N_f; R = \chi)$. Dividing equation [6.17] on both sides by equation [6.19], we can obtain the following relation:

$$\frac{\sigma_a}{\sigma_a^\chi} = \left(\frac{\sigma_T - \sigma_m}{\sigma_T - \sigma_m^\chi} \right)^u \left(\frac{|\sigma_C| + \sigma_m}{|\sigma_C| + \sigma_m^\chi} \right)^v, \quad -|\sigma_C| \leq \sigma_m \leq \sigma_T \quad (6.20)$$

This expression of the bell-shaped CFL model suggests a different procedure for identifying the material functions involved. In the suggested procedure, the S–N relationship for the selected reference stress ratio $R = \chi$ is first approximated by means of a certain nonlinear function, e.g.

$$\sigma_{\max}^\chi = \hat{\sigma}_{\max}^\chi(N_f; R = \chi) \quad (6.21)$$

Then, the two remaining functions, $u = \hat{u}(N_f, \dots)$ and $v = \hat{v}(N_f, \dots)$, are determined by fitting equation [6.20], by means of nonlinear regression, to additional CFL data for different numbers of cycles to failure that should be obtained from constant amplitude fatigue tests at different values of stress ratio.

6.4.6 Anisomorphic CFL diagram

Kawai and coworkers (Kawai *et al.*, 2006, 2008; Kawai and Koizumi, 2007) have recently developed another challenging fatigue life prediction method for composites that is based on a nonlinear CFL diagram called the anisomorphic CFL diagram. All the requirements suggested by Boller (1957, 1964) have been taken into account in the formulation. In particular, the change in shape of the CFL envelope with an increasing number of cycles to failure has been more explicitly considered in the modeling than before. The anisomorphic CFL diagram can be built using only the static strengths in tension and compression and the reference S–N curve for a particular stress ratio that is called the critical stress ratio. Efficient construction of the CFL diagram for a given composite using a minimal amount of test data is a great advantage of the method.

Formulation

The anisomorphic CFL diagram is based on the following basic assumptions: (A1) the constant amplitude fatigue behavior of a given composite is characterized by the reference fatigue behavior at a particular stress ratio χ ; the characteristic stress ratio χ , called the critical stress ratio, is equal to the ratio of the compressive strength to the tensile strength of the composite; (A2) the alternating stress component σ_a of fatigue stress for a given constant value of fatigue life N_f becomes largest at the critical stress ratio χ ; and (A3) the shape of the CFL envelope progressively changes from a straight line to a parabola with an increasing number of cycles to failure.

A theoretical CFL envelope for a given constant value of fatigue life is composed of two smooth members associated with T–T and C–C fatigue failure modes, respectively, and they are smoothly connected with each other at a point on the radial straight line with the equation

$$\frac{\sigma_a}{\sigma_m} = \frac{1 - \chi}{1 + \chi} \quad 6.22$$

which is associated with the critical stress ratio χ . The theoretical CFL curve is described by means of a function that is defined by different formulas depending on the position of the mean stress σ_m in the domain $[\sigma_C, \sigma_T]$ as

$$-\frac{\sigma_a - \sigma_a^\chi}{\sigma_a^\chi} = \begin{cases} \left(\frac{\sigma_m - \sigma_m^\chi}{\sigma_T - \sigma_m^\chi} \right)^{2-\psi_\chi}, & \sigma_m^\chi \leq \sigma_m \leq \sigma_T \\ \left(\frac{\sigma_m - \sigma_m^\chi}{\sigma_C - \sigma_m^\chi} \right)^{2-\psi_\chi}, & \sigma_C \leq \sigma_m < \sigma_m^\chi \end{cases} \quad 6.23$$

where σ_C (< 0) and σ_T (> 0) are the compressive and tensile strengths of the composite, respectively, and σ_a^χ and σ_m^χ represent the alternating and mean stress components of the maximum fatigue stress $\sigma_{\max}^\chi$ for the fatigue loading at the critical stress ratio $\chi = \sigma_C/\sigma_T \in (-\infty, 0)$; see equations [6.6] and [6.7]:

$$\sigma_a^\chi = \frac{1}{2}(1 - \chi)\sigma_{\max}^\chi$$

$$\sigma_m^\chi = \frac{1}{2}(1 + \chi)\sigma_{\max}^\chi$$

In this formulation, the coordinates $(\sigma_m^\chi, \sigma_a^\chi)$ represent the peak positions of CFL envelopes. The above formulation ensures that the arguments of the power functions in the right-hand side of equation [6.23] are non-negative.

It is important to note that the exponent ψ_χ in equation [6.23] is a function of the number of cycles to failure. The variable exponent ψ_χ is the fatigue strength ratio for the fatigue loading at the critical stress ratio χ , and is defined as

$$\psi_\chi = \frac{\sigma_{\max}^\chi}{\sigma_B^\chi} \quad 6.24$$

where σ_B^χ (> 0) is a constant reference strength to be identified for the fatigue behavior at the critical stress ratio. The variable exponent ψ_χ is described as a monotonic continuous function of the number of cycles to failure N_f , and it can be identified by fitting a function of the form $\psi_\chi = f^{-1}(2N_f)$ to the fatigue data for the critical stress ratio $R = \chi$. The reference strength σ_B^χ normally has the value of σ_T , i.e. $\sigma_B^\chi = \sigma_T$. Since $0 \leq \psi_\chi \leq 1$, the range of the exponent $2 - \psi_\chi$ becomes $1 \leq 2 - \psi_\chi \leq 2$. Therefore, the proposed piecewise-defined CFL function produces nested parametric curves whose shape smoothly changes from a straight line to a parabola with the increase in fatigue life.

Procedure for constructing the anisomorphic CFL diagram

The anisomorphic CFL diagram for a given composite can be constructed using only the static strengths in tension and compression and the reference S–N

relationship $2N_f = f(\psi_\chi)$ for the critical stress ratio $R = \chi$. The construction procedure can be described as follows:

1. Evaluate the tensile strength $\sigma_T > 0$ and compressive strength $\sigma_C < 0$ of the composite.
2. Calculate the value of the critical stress ratio $\chi = \sigma_C/\sigma_T (< 0)$.
3. Perform constant amplitude tension–compression fatigue tests at the critical stress ratio $R = \chi$ to obtain the reference S–N data.
4. Identify the normalized reference S–N curve for the critical stress ratio by fitting a function $2N_f = f(\psi_\chi)$ to the reference S–N data obtained in the previous step. For example, use the following function:

$$2N_f = \frac{1}{K_\chi} \frac{1}{(\psi_\chi)^n} \frac{(1 - \psi_\chi)^a}{(\psi_\chi - \psi_\chi^L)^b} \quad 6.25$$

where ψ_χ^L is the fatigue strength ratio associated with a fatigue limit of the composite under a given fatigue loading condition, but it may be identified by matching equation [6.25] to the reference fatigue data, in line with determination of the material constants K_χ , a , b , and n .

5. Calculate the coordinates $(\sigma_m^\chi, \sigma_a^\chi)$ of the peak positions of the CFL envelopes for different constant values of fatigue life N_f . They are calculated using equations [6.6] and [6.7]:

$$\sigma_a^\chi(N_f) = \frac{1}{2}(1 - \chi)\sigma_{\max}^\chi(N_f) = \frac{1}{2}(1 - \chi)\psi_\chi(N_f)\sigma_B^\chi$$

$$\sigma_m^\chi(N_f) = \frac{1}{2}(1 + \chi)\sigma_{\max}^\chi(N_f) = \frac{1}{2}(1 + \chi)\psi_\chi(N_f)\sigma_B^\chi$$

6. Calculate a sufficient number of points (σ_m, σ_a) for each of the selected constant values of fatigue life using the piecewise-defined CFL functions given by equation [6.23].
7. Join the adjacent points (σ_m, σ_a) for the same number of cycles to failure by a straight line.

Procedure for predicting the S–N curves for any stress ratios with the help of the anisomorphic CFL diagram

Once the anisomorphic CFL diagram is constructed, it allows the prediction of S–N curves for the composite laminate at any stress ratios. S–N curves for any stress ratios can be predicted by solving the following system of nonlinear equations for each of the two domains of mean stress which are separated by the radial line with equation [6.22]:

$$f(\sigma_m, \sigma_a) = \begin{cases} \sigma_a - \sigma_a^\chi \left[1 - \left(\frac{\sigma_m - \sigma_m^\chi}{\sigma_T - \sigma_m^\chi} \right)^{2-\psi_\chi} \right] = 0, & \sigma_m^\chi \leq \sigma_m \leq \sigma_T \\ \sigma_a - \sigma_a^\chi \left[1 - \left(\frac{\sigma_m - \sigma_m^\chi}{\sigma_C - \sigma_m^\chi} \right)^{2-\psi_\chi} \right] = 0, & \sigma_C \leq \sigma_m < \sigma_m^\chi \end{cases} \quad 6.26$$

$$g(\sigma_m, \sigma_a) = \begin{cases} \sigma_a - \frac{1-R}{1+R} \sigma_m = 0, & \sigma_m^\chi \leq \sigma_m \leq \sigma_T \\ \sigma_a - \frac{1-R}{1+R} \sigma_m = 0, & \sigma_C \leq \sigma_m < \sigma_m^\chi \end{cases} \quad 6.27$$

Note that the solutions $(\sigma_m, \sigma_a) = (\sigma_m^R, \sigma_a^R)$ for different numbers of cycles to failure N_f under fatigue loading at a given stress ratio R allow us to obtain the S–N coordinates $(\sigma_{\max}^R = \sigma_m^R + \sigma_a^R, N_f)$ for the constant amplitude fatigue loading at the given stress ratio R .

Particular cases of the anisomorphic CFL diagram

Elimination of the exponent ψ_χ from the formulas for the anisomorphic CFL diagram yields the inclined Gerber diagram (equation [6.16]). Replacing the exponent ψ_χ in the formulas for the anisomorphic CFL diagram with a constant value of unity, we can reduce it to the inclined Goodman diagram (equation [6.5]). For a class of composites with the same strength level in tension and compression, the critical stress ratio has a value of -1 ($R = \chi = -1$), and thus $\sigma_m^\chi = \sigma_m^{R=-1} = 0$. In this particular case, the formulas for the anisomorphic CFL diagram can be reduced to the following single formula:

$$-\frac{\sigma_a - \sigma_a^{R=-1}}{\sigma_a^{R=-1}} = \left(\frac{|\sigma_m|}{\sigma_T} \right)^{2-\psi_{R=-1}}, \quad -\sigma_T \leq \sigma_m \leq \sigma_T \quad 6.28$$

Note that the nonlinear CFL diagram predicted by equation [6.28] becomes symmetric about the alternating stress axis. This nonlinear CFL diagram may be interpreted as another extension of the symmetric Gerber diagram, though it is no longer parabolic over a range of fatigue life.

6.5 Prediction of constant fatigue life (CFL) diagrams and S–N curves

This section is devoted to an evaluation of the anisomorphic CFL diagram approach. It is tested for capability to predict the full shape of the CFL

diagram and the S–N relationships at different stress ratios, not only for the fiber-dominated fatigue behavior (Kawai and Koizumi, 2007) but also for the matrix-dominated fatigue behavior (Kawai and Murata, 2008) of carbon/epoxy laminates.

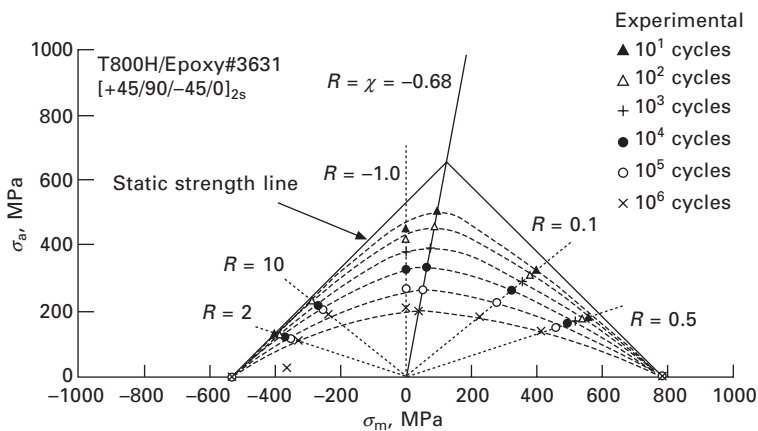
6.5.1 Application to the fiber-dominated fatigue behavior of composite laminates

The effectiveness of the anisomorphic CFL diagram is evaluated for the constant amplitude fatigue behavior of a quasi-isotropic [45/90/–45/0]_{2S} carbon/epoxy laminate at room temperature. The anisomorphic CFL diagram is constructed according to the procedure described above in Section 6.4.6. First, the tensile and compressive strengths of the laminate are evaluated. From the static tension and compression tests on the laminate, the values $\sigma_T = 781.9$ MPa and $\sigma_C = -532.4$ MPa were obtained. Using these static strengths, the value of the critical stress ratio can be calculated as $\chi = \sigma_C/\sigma_T = -0.68$. Second, the reference S–N data ($\sigma_{\max}^\chi$ versus $2N_f$) are derived from fatigue tests at the critical stress ratio $R = \chi$. The reference fatigue data are approximated by means of an analytical function $2N_f = f(\psi_\chi)$. Since fatigue limit was not clearly observed in the reference fatigue behavior of the [45/90/–45/0]_{2S} laminate at $R = \chi$, the following reduced form of equation [6.25] has been employed for this purpose:

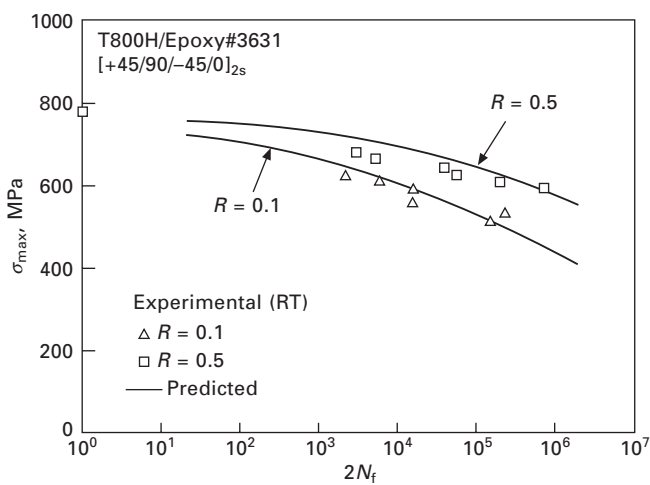
$$2N_f = f(\psi_\chi) = \frac{1}{K_\chi} \frac{(1 - \psi_\chi)^a}{(\psi_\chi)^n} \quad 6.29$$

Through curve fitting, the material constants involved in this function were determined as $K_\chi = 0.0015$, $n = 8.5$, and $a = 1$. It is emphasized that the reference S–N relationship for the critical stress ratio should be described by means of the function defined using ψ_χ , equation [6.29], for the [45/90/–45/0]_{2S} laminate or equation [6.25] in general, since the value of ψ_χ that varies with the number of cycles to failure is used as the variable exponent in the piecewise-defined functions for the anisomorphic CFL diagram; see equation [6.23]. This is all that is necessary for drawing the anisomorphic CFL envelopes (σ_m , σ_a) corresponding to different numbers of cycles to failure. Using the piecewise-defined functions for the anisomorphic CFL diagram, equation [6.23], the S–N relationship ($2N_f$, $\sigma_{\max}$) can readily be predicted for any constant amplitude fatigue loading.

Figure 6.14 shows comparison between theory and experiment for the [45/90/–45/0]_{2S} laminate; the dashed lines indicate the predicted anisomorphic CFL envelopes, and symbols designate the experimental CFL data. A good agreement between the predicted and observed CFL curves can be seen over the range of fatigue life. Figures 6.15, 6.16, and 6.17 show comparisons

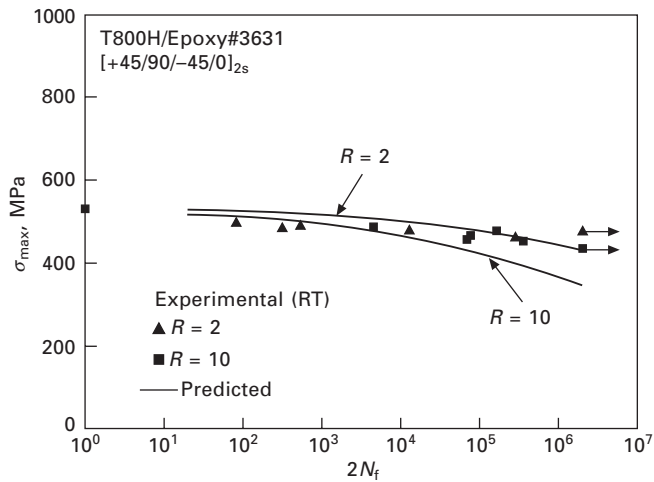


6.14 Anisomorphic constant fatigue life diagram for a $[+45/90/-45/0]_{2s}$ carbon/epoxy laminate. (Kawai and Koizumi, 2007)

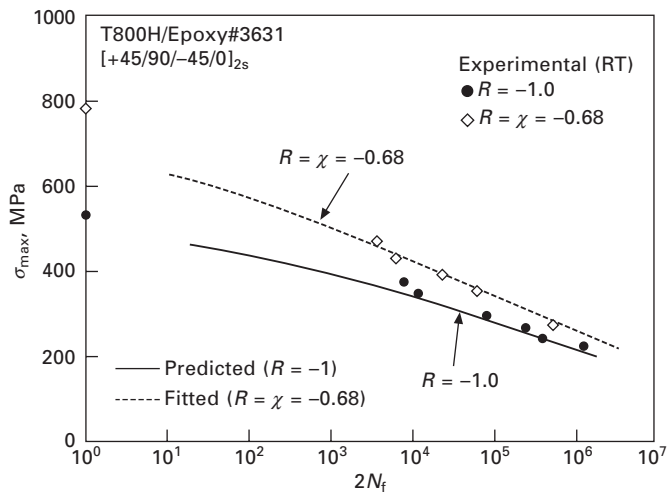


6.15 S–N relationships predicted using the anisomorphic constant fatigue life diagram for a $[+45/90/-45/0]_{2s}$ carbon/epoxy laminate subjected to tension–tension fatigue loading. (Kawai and Koizumi, 2007)

between the predicted and observed S–N relationships under tension–tension (T–T), compression–compression (C–C) and tension–compression (T–C) fatigue loading, respectively. The solid lines in these figures indicate the predictions, and the dashed line in Fig. 6.17 indicates the reference S–N curve identified by fitting equation [6.29] to the fatigue data for the critical stress ratio. It is seen that the mean stress dependence of the S–N relationship for the $[45/90/-45/0]_{2s}$ laminate is adequately predicted by means of the



6.16 S-N relationships predicted using the anisomorphic constant fatigue life diagram for a [+45/90/-45/0]_{2s} carbon/epoxy laminate subjected to compression-compression fatigue loading. (Kawai and Koizumi, 2007)



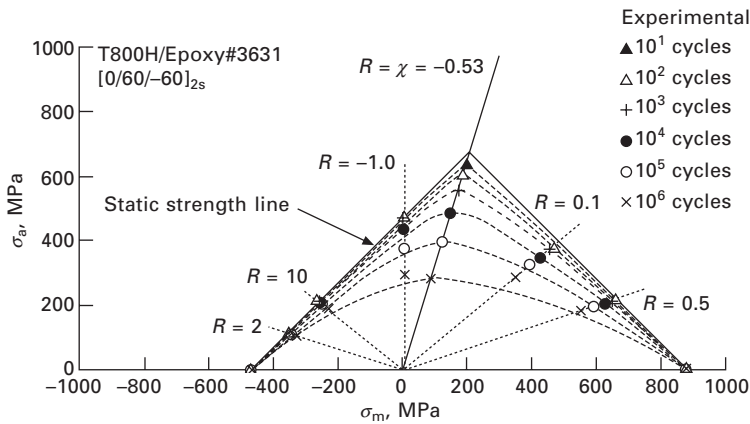
6.17 S-N relationships predicted using the anisomorphic constant fatigue life diagram for a [+45/90/-45/0]_{2s} carbon/epoxy laminate subjected to tension-compression fatigue loading. (Kawai and Koizumi, 2007)

anisomorphic CFL diagram. It is important to note that all the solid lines in Figs 6.15, 6.16, and 6.17 indicate predictions, since only the fatigue data for the critical stress ratio were used for construction of the anisomorphic CFL diagram.

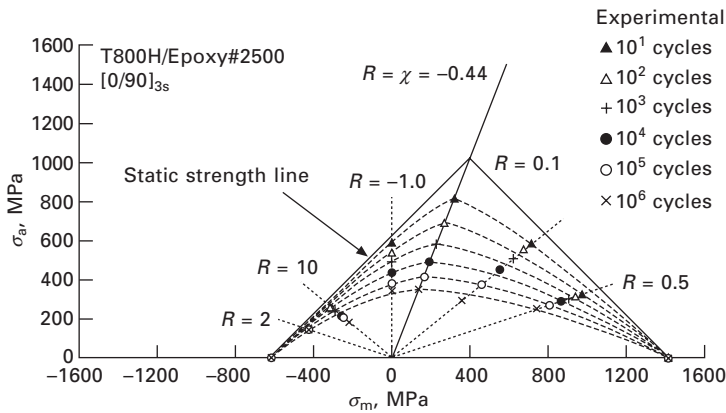
The anisomorphic CFL diagram approach was also successfully applied to different carbon/epoxy laminates of $[0/60/-60]_{2S}$ and $[0/90]_{3S}$ lay-ups. The predicted CFL diagrams for these laminates are presented in Figs 6.18 and 6.19.

6.5.2 Application to the matrix-dominated fatigue behavior of composite laminates

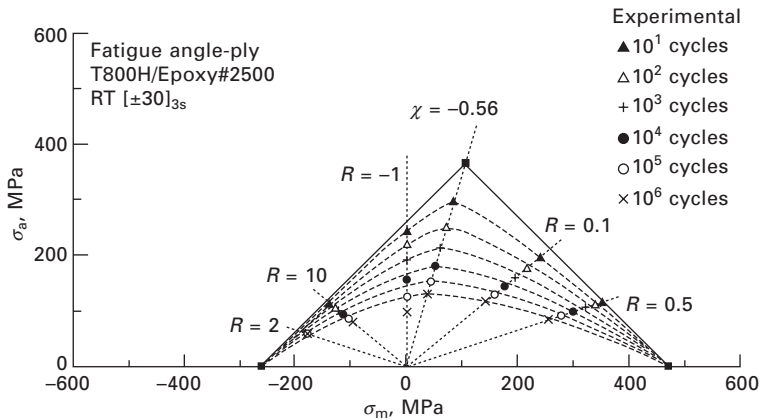
The anisomorphic CFL diagram is further tested for the capability in predicting the matrix-dominated fatigue behavior of angle-ply $[\pm\theta]_{3S}$ carbon/epoxy laminates. Figure 6.20 shows the anisomorphic CFL diagram for the $[\pm30]_{3S}$



6.18 Anisomorphic constant fatigue life diagram for a $[0/60/-60]_{2S}$ carbon/epoxy laminate. (Kawai and Koizumi, 2007)



6.19 Anisomorphic constant fatigue life diagram for a $[0/90]_{3S}$ carbon/epoxy laminate. (Kawai and Koizumi, 2007)

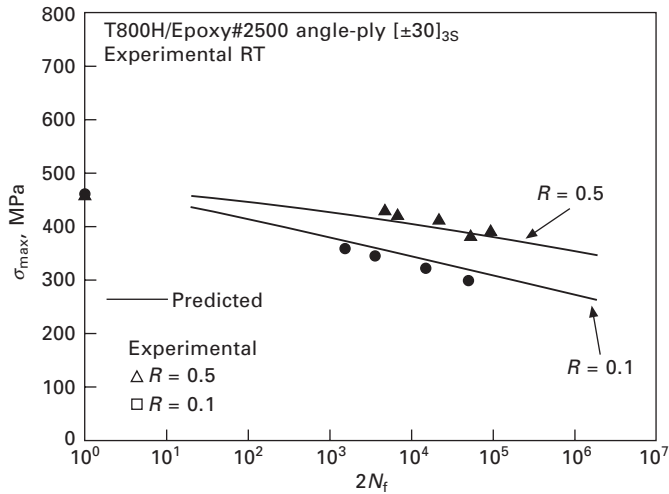


6.20 Anisomorphic constant fatigue life diagram for a $[\pm 30]_{3S}$ carbon/epoxy laminate. (Kawai and Murata, 2008)

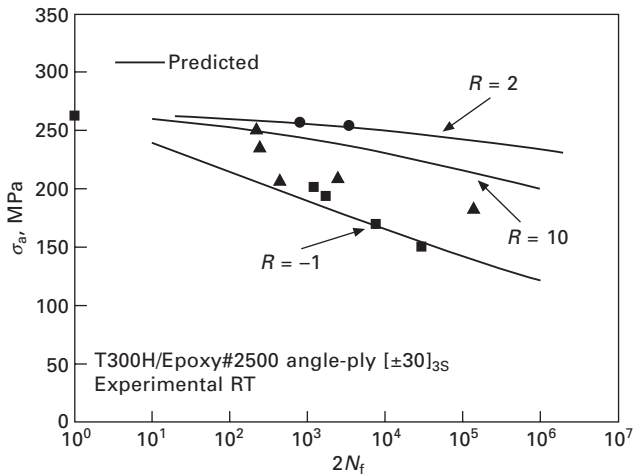
laminate with the critical stress ratio $\chi = -0.56$; the dashed lines indicate predictions, and symbols designate experimental results. It is seen that the predicted and observed CFL envelopes agree well with each other over the range of fatigue life. The successful application of the anisomorphic CFL diagram demonstrates that consideration of the asymmetry and variable nonlinearity in CFL envelopes is decisive for accurate description of the CFL diagram for the $[\pm 30]_{3S}$ laminate over the whole range of fatigue life. It also reveals that the traditional Goodman diagram cannot accurately be applied to description of the effect of mean stress on the fatigue life of the $[\pm 30]_{3S}$ laminate. Figures 6.21 and 6.22 show the predicted S–N relationships for the $[\pm 30]_{3S}$ laminate under T–T, C–C, and T–C fatigue loading. Reasonably good agreements between the predicted and observed S–N relationships have been achieved.

The anisomorphic CFL diagram approach was also successfully applied to a different angle-ply laminate of $[\pm 45]_{3S}$ lay-up. It is interesting that the CFL diagrams for the matrix-dominated $[\pm 30]_{3S}$ and $[\pm 45]_{3S}$ laminates are similar in asymmetry and variable nonlinearity to those for the fiber-dominated $[45/90/-45/0]_{2S}$, $[0/60/-60]_{2S}$, and $[0/90]_{3S}$ laminates.

Unlike the angle-ply laminates of $[\pm 30]_{3S}$ and $[\pm 45]_{3S}$ lay-ups examined above, the compressive strength of the $[\pm 60]_{3S}$ carbon/epoxy laminate was larger than the tensile strength. The larger strength in compression than in tension suggests that the anisomorphic CFL diagram inclines to the left of the alternating stress axis, which is demonstrated in Fig. 6.23. The dashed lines in Fig. 6.23 indicate the anisomorphic CFL envelopes of the $[\pm 60]_{3S}$ laminate for different numbers of cycles to failure. It is seen that the agreement between the predicted and experimental CFL envelopes for the $[\pm 60]_{3S}$ laminate is poor in the left segment partitioned by the radial line

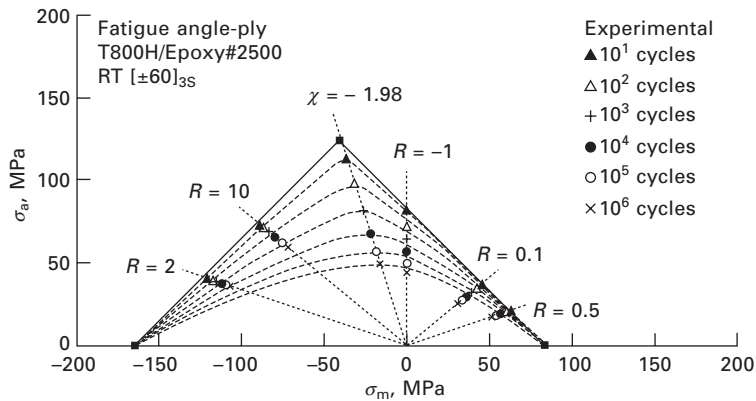


6.21 S–N relationships predicted using the anisomorphic constant fatigue life diagram for a $[\pm 30]_{3S}$ carbon/epoxy laminate subjected to tension–tension fatigue loading. (Kawai and Murata, 2008)



6.22 S–N relationships predicted using the anisomorphic constant fatigue life diagram for a $[\pm 30]_{3S}$ carbon/epoxy laminate subjected to compression–compression fatigue loading. (Kawai and Murata, 2008)

associated with the critical stress ratio $\chi = -2$. The discrepancy is ascribed to a significant change in mean stress sensitivity in fatigue for a range of stress ratios in the left neighborhood of the critical stress ratio. For composites in which such an appreciable change in the sensitivity to mean stress happens, it is not reasonable to assume that their fatigue performance is characterized by



6.23 Anisomorphic constant fatigue life diagram for a $[\pm 60]_{3S}$ carbon/epoxy laminate. (Kawai and Murata, 2008)

the representative fatigue behavior at a particular stress ratio (i.e. the critical stress ratio). In fact, similar distortion in the CFL diagram can be found in the experimental results for other composites, e.g. Schütz and Gerharz (1977) and Phillips (1981). Therefore, the significant change in the mean stress sensitivity in fatigue observed in the $[\pm 60]_{3S}$ laminate, as well as in the composites tested by Schütz and Gerharz (1977) and Phillips (1981), suggests that some extension of the anisomorphic CFL diagram should be made in order to allow for accommodating such an anomalous mean stress sensitivity in fatigue for a class of composites.

Incidentally, an idea of mapping a reference CFL curve for a representative number of cycles to failure that reflects the actual shape observed by experiment to the CFL curve for any given number of cycles to failure (Boerstra, 2007) provides a solution to the problem of accurately describing highly distorted CFL diagrams for a class of composites. While it is interesting, a method based on this idea requires a model by which the S–N relationships for any mean stresses can be predicted. Since focusing on development of a method that allows predicting the S–N relationships for any mean stresses in this study, we confine our attention to modeling the nested CFL envelopes that depend on the number of cycles to failure, and further seek a CFL diagram based solution to the problem. An attempt is made in the next section.

6.6 Extended anisomorphic constant fatigue life (CFL) diagram

This section is devoted to development of an extended anisomorphic CFL diagram approach which is furnished with enhanced capability to more accurately describe the nonlinear shape of the CFL diagram and thus with

more general applicability to a variety of composites with different mean stress sensitivities. The validity of the extended anisomorphic CFL diagram is demonstrated by comparing with experimental results.

The anisomorphic CFL diagram approach to prediction of fatigue lives of composites assumes that the S–N relationships for any stress ratios can be predicted on the basis of only the fatigue data for the critical stress ratio. This assumption was found to be valid for the fiber-dominated quasi-isotropic carbon/epoxy laminates of $[45/90/-45/0]_{2S}$ and $[0/60/-60]_{2S}$ lay-ups and the cross-ply laminate of $[0/90]_{3S}$ lay-up, and for the matrix-dominated angle-ply carbon/epoxy laminates of $[\pm 30]_{3S}$ and $[\pm 45]_{3S}$ lay-ups. However, it was too optimistic for the angle-ply $[\pm 60]_{3S}$ carbon/epoxy laminate, as observed above. The experimental CFL diagram for the $[\pm 60]_{3S}$ laminate showed a considerable change in mean stress sensitivity in the left neighborhood of the critical stress ratio, and accordingly it was not accurately described using the anisomorphic CFL diagram. This unsatisfactory result reveals that relying on only the S–N data for the critical stress ratio to construct the CFL diagram over a whole range of mean stresses leads to oversimplification, especially for a class of composite laminates that exhibit higher sensitivity to mean stress in a transitional segment between the T–T and C–C dominated segments in the σ_m – σ_a plane.

To cope with this problem, it was attempted to generalize the anisomorphic CFL diagram further without much loss of convenience (Kawai and Murata, 2008). If the mean stress sensitivity in fatigue becomes higher in the vicinity of the critical stress ratio, a transitional segment is assumed to appear between the two segments associated with T–T and C–C dominated fatigue failure. The transitional segment plays a role in accommodating a distortion in the CFL diagram due to a change in mean stress sensitivity, and it connects the two neighboring segments with the aid of linear interpolation. The three-segment version of the anisomorphic CFL diagram was called a connected anisomorphic CFL diagram (Kawai and Murata, 2008). In addition to the assumptions of the original anisomorphic CFL diagram, the following assumptions were added to formulate the connected anisomorphic CFL diagram: (B1) if an appreciable change in mean stress sensitivity is involved in the CFL diagram, a sub-critical stress ratio χ_s is additionally introduced to define the transitional segment in the CFL diagram which is bounded by the critical line $\sigma_a/\sigma_m = (1 - \chi)/(1 + \chi)$ and the sub-critical line $\sigma_a/\sigma_m = (1 - \chi_s)/(1 + \chi_s)$, the critical and sub-critical stress ratios dividing the CFL diagram into three segments; (B2) the CFL curves in the right and left segments that are partitioned by the critical and sub-critical lines, respectively, are drawn according to the procedure prescribed in the original formulation; and (B3) in the transitional segment, linear interpolation is assumed. Thus, the points of the same fatigue life located on the critical and sub-critical lines are connected by straight lines.

Mathematically, the connected anisomorphic CFL diagram is described by means of a function on the domain $[\sigma_C, \sigma_T]$. The function is prescribed by different formulas depending on the position of mean stress σ_m , which are given as follows:

I. Tension-dominated zone:

$$-\frac{\sigma_a - \sigma_a^\chi}{\sigma_a^\chi} = \left(\frac{\sigma_m - \sigma_m^\chi}{\sigma_T - \sigma_m^\chi} \right)^{2-\psi_\chi^{k_T}}, \quad \sigma_m^\chi \leq \sigma_m \leq \sigma_T \quad 6.30$$

II. Transitional zone:

$$-\frac{\sigma_a - \sigma_a^\chi}{\sigma_a^\chi - \sigma_a^{\chi_s}} = \frac{\sigma_m - \sigma_m^\chi}{\sigma_m^{\chi_s} - \sigma_m^\chi}, \quad \sigma_m^{\chi_s} \leq \sigma_m < \sigma_m^\chi \quad 6.31$$

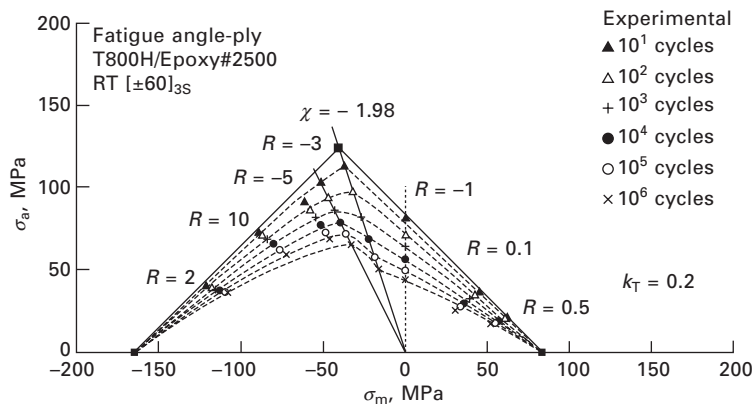
III. Compression-dominated zone:

$$-\frac{\sigma_a - \sigma_a^{\chi_s}}{\sigma_a^{\chi_s}} = \left(\frac{\sigma_m - \sigma_m^{\chi_s}}{\sigma_C - \sigma_m^{\chi_s}} \right)^{2-\psi_{\chi_s}^{k_C}}, \quad \sigma_C \leq \sigma_m < \sigma_m^{\chi_s} \quad 6.32$$

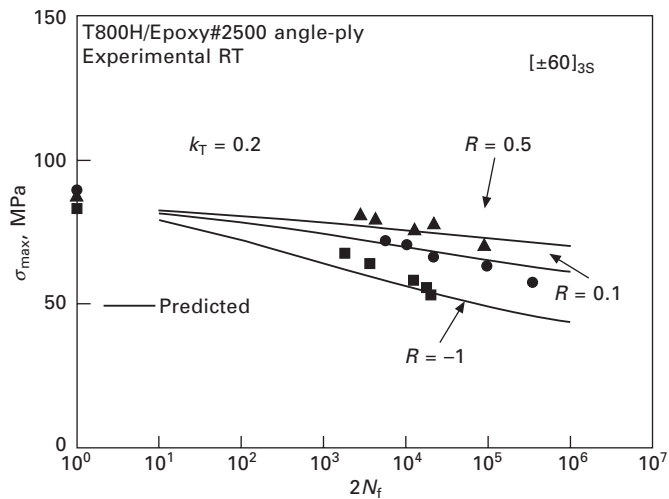
where σ_a^χ , σ_m^χ , $\sigma_a^{\chi_s}$, and $\sigma_m^{\chi_s}$ represent the alternating and mean stress components of the maximum fatigue stresses $\sigma_{\max}^\chi$ and $\sigma_{\max}^{\chi_s}$ which are associated with the fatigue loading at the critical and sub-critical stress ratios, χ ($= \sigma_C / \sigma_T$) and χ_s , respectively. The fatigue strength ratios ψ_χ and ψ_{χ_s} are associated with the critical and sub-critical stress ratios, χ and χ_s , respectively. The critical fatigue strength ratio ψ_χ is expressed as $\psi_\chi = \sigma_{\max}^\chi / \sigma_T$, while the sub-critical fatigue strength ratio ψ_{χ_s} is defined as $\psi_{\chi_s} = \sigma_{\max}^{\chi_s} / \sigma_T$ if $\chi \leq \chi_s \leq 0$, and $\psi_{\chi_s} = \sigma_{\min}^{\chi_s} / \sigma_C$ if $\chi_s < \chi$. They are described by means of the monotonic continuous functions of the same form as given by equation [6.25] (or [6.29]). Note that the exponents k_T and k_C , which are constant, are added to the constituent functions; they allow adjusting the transition from a straight line to a parabola, independently for the right and left halves of CFL curves, to obtain a better description of the nonlinear CFL diagram.

The connected anisomorphic CFL diagram for the $[\pm 60]_{3S}$ laminate is shown in Fig. 6.24, along with the experimental CFL data. Good agreement between the predicted and observed CFL curves for all the stress ratios over the range of fatigue life can be achieved. Comparisons between the predicted and observed S-N relationships for T-T, C-C, and T-C fatigue loading are shown in Figs 6.25 and 6.26, respectively. It is seen that the S-N relationships for the $[\pm 60]_{3S}$ laminate at different mean stress levels have been accurately predicted by means of the connected anisomorphic CFL diagram.

These results demonstrate that insertion of a transitional zone into the anisomorphic CFL diagram greatly improves the accuracy of prediction of the

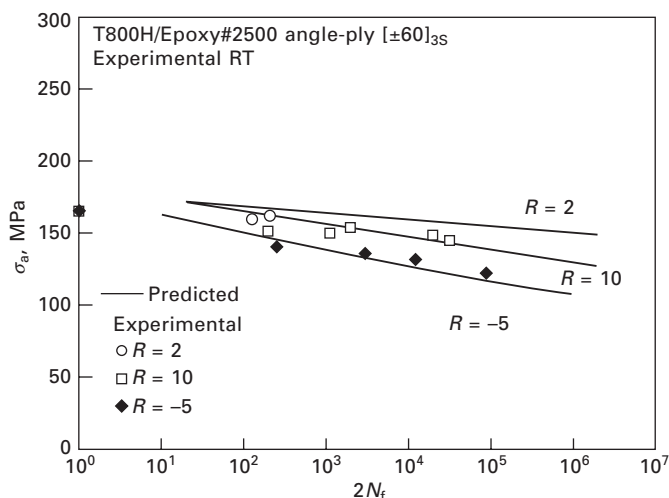


6.24 Extended anisomorphic constant fatigue life diagram for a $[\pm 60]_{3S}$ carbon/epoxy laminate. (Kawai and Murata, 2008)



6.25 S-N relationships predicted using the extended anisomorphic constant fatigue life diagram for a $[\pm 60]_{3S}$ carbon/epoxy laminate subjected to tension-tension fatigue loading. (Kawai and Murata, 2008)

full shape of the nonlinear CFL diagram for a given composite, and allows application to a greater variety of composites with different mean stress sensitivity. The connected anisomorphic CFL diagram requires additional fatigue data for the second critical stress ratio, i.e. the sub-critical stress ratio, which impairs the great simplicity in the original two-segment anisomorphic CFL diagram. However, it still carries significant advantages not only in construction of nonlinear CFL envelopes in an efficient manner and with a



6.26 S–N relationships predicted using the extended anisomorphic constant fatigue life diagram for a $[\pm 60]_{3S}$ carbon/epoxy laminate subjected to compression–compression fatigue loading. (Kawai and Murata, 2008)

small amount of fatigue data, but also in its enhanced capability to describe a local distortion in the CFL diagram due to a significant change in mean stress sensitivity in fatigue of composites.

6.7 Conclusions

Accurate prediction of the constant amplitude fatigue lives of composites at any amplitude levels for any stress ratios is a vital prerequisite to the successful fatigue life analysis of composite structures subjected to complicated service loading. In order to meet the prerequisite, two approaches have been developed so far: (1) the approach using a master S–N relationship; and (2) the approach using a CFL diagram. The CFL diagram approach easily accommodates itself to the mean stress sensitivity observed by experiment, suggesting that the CFL diagram approach is more flexible, and thus more fruitful for most engineers, than the master S–N curve approach, especially when dealing with a non-Goodman type of fatigue behavior of composites. This chapter, therefore, focused on the CFL diagram approach and reviewed the linear and nonlinear CFL diagrams which were developed so far to account for the effect of mean stress on the fatigue lives of composites in a systematic manner. A particular emphasis has been placed on the recent progress in the CFL diagram approach that has been made by taking into account the requirements suggested by Boller (1957, 1964) for accurate description of the CFL diagrams for fiber-reinforced composites, and on

the smooth link to the latest model, called the anisomorphic CFL diagram (Kawai, 2006).

The anisomorphic CFL diagram is one of the most general theoretical tools to date for predicting the mean stress sensitivity in fatigue of composites, and it has been formulated by taking into account all the requirements suggested by Boller (1957, 1964): (1) the asymmetry in CFL envelopes about the alternating stress axis; (2) the nonlinearity in CFL envelopes; and (3) the gradual change in shape of CFL envelopes with increasing number of cycles to failure. For a given composite, the anisomorphic CFL diagram can be constructed using only a limited amount of experimental data: (i) the static strengths in tension and compression; and (ii) the fatigue data for a particular stress ratio, called the critical stress ratio, which is equal to the ratio of the compressive strength to the tensile strength. The ease of drawing CFL envelopes with a minimal amount of experimental data is an inherent advantage of the method.

The validity of the fatigue life prediction method based on the anisomorphic CFL diagram has been evaluated for the fiber-dominated and matrix-dominated fatigue behaviors of carbon/epoxy laminates. For the fiber-dominated fatigue behaviors of the $[45/90/-45/0]_{2S}$, $[0/60/-60]_{2S}$, and $[0/90]_{3S}$ laminates, it was demonstrated that the CFL envelopes and S–N curves predicted using the anisomorphic CFL model agree well with the experimental results, regardless of the type of laminate. The anisomorphic CFL diagram was also shown to be valid for the matrix-dominated fatigue behavior of the $[\pm 30]_{3S}$ and $[\pm 45]_{3S}$ laminates. However, it failed to accurately predict the mean stress sensitivity in the fatigue of the $[\pm 60]_{3S}$ laminate. The failure was due to a significant change in mean stress sensitivity in fatigue life of the laminate, and it happened at stress ratios in a narrow range that are smaller than the critical stress ratio.

To overcome the above problem, an extension of the anisomorphic CFL diagram has been attempted. The extended anisomorphic CFL diagram, which is called the connected anisomorphic CFL diagram, consists of three segments: the T–T and C–C dominated segments, and a transitional segment in between. It was demonstrated that the extended (connected) anisomorphic CFL diagram can successfully be applied to describing the CFL diagram for the $[\pm 60]_{3S}$ laminate as well, and thus the S–N curves for constant amplitude fatigue loading at any stress ratios can accurately be predicted for all of the fiber-dominated and matrix-dominated carbon/epoxy laminates tested in this chapter. The extended anisomorphic CFL diagram approach is also applicable to the carbon/epoxy laminates examined by Schütz and Gerharz (1977) and Phillips (1981). The extended method requires additional fatigue data for another reference stress ratio, called the sub-critical stress ratio, to define the transitional mean stress interval bounded by the critical and sub-critical stress ratios. This slightly impairs the efficiency of the original

method, but the slight increase in inefficiency is almost cancelled by the enhanced flexibility of the extended method. The extended anisomorphic CFL diagram approach allows describing a distortion of CFL envelopes for a class of composites that is caused by a significant change in mean stress sensitivity in fatigue in a transitional mean stress interval. Once the CFL diagram for a given composite has accurately been identified over a range of fatigue life by means of the proposed method, it allows predicting the S–N curves of the composite for any constant amplitude fatigue loading, and using them in conjunction with a damage accumulation rule for evaluation of the fatigue lives of the composite for any operational load spectra (Kawai *et al.*, 2008).

6.8 Future trends

The spectra of fatigue load sustained by composite structures during service often involve a small number of large-amplitude cycles, and the maximum fatigue stress during the large-amplitude cycles may accidentally reach a high level of fraction of the static strength of the composite material employed. Phillips (1981) has reported that the fatigue lives of carbon/epoxy laminates under spectrum loading are sensitive to the high load cycles involved and should not be truncated in spectrum fatigue life analysis. This explains why it is required to predict the fatigue lives of composites in a short range as well under large-amplitude cyclic loading. Therefore, the accuracy of prediction of the CFL envelopes for composites should be taken into account not only for a typical range of fatigue life $N_f = 10^4$ – 10^7 but also for a short life range $N_f = 10^0$ – 10^3 .

Another concern that has not fully been discussed so far is to evaluate the accuracy of prediction using CFL models for longer lives beyond the fatigue life that can be observed by experiment. If the anisomorphic CFL diagram approach is valid for a given composite in the long life range at low stress levels, it allows prediction of the fatigue lives of composites in the long life range on the basis of the long life fatigue data only at the critical stress ratio. No fatigue testing at any other stress ratio in the long life range is required. Such an efficient implementation of fatigue analysis would be of great significance for applications in which a long life fatigue design of components subjected to variable cyclic loading becomes a critical issue. Therefore, it is worth pursuing further the applicability of the method in a range of long fatigue life.

On the other hand, the anisomorphic CFL diagram for composites is affected by factors such as temperature, moisture, loading rate, and damage that change the static strengths in tension and compression and the reference S–N relationship for tension–compression fatigue loading.

The increase in test temperature of composites often results in decrease

in their tensile and compressive strengths (Schulte and Stinchcomb, 1989; Kawai *et al.*, 2001a; Kawai and Sagawa, 2008). The degree of reduction in compressive strength due to temperature rise is different from that in tensile strength, and the former is often more significant (Schulte and Stinchcomb, 1989; Kawai *et al.*, 2009). This suggests that the absolute value of the critical stress ratio, which is given by the ratio of the compressive strength to the tensile strength, tends to decrease as temperature increases, and thus the anisomorphic CFL diagram shrinks and the peak position in the σ_m – σ_a plane moves to the right with increasing temperature (Matsuda *et al.*, 2008; Kawai *et al.*, 2009). Incidentally, the reduction in the in-plane compressive strengths of composites due to impact damage (Swanson *et al.*, 1993) leads to similar changes in their CFL diagrams (Beheshty and Harris, 1998).

The fatigue behavior of composites becomes more complicated with the influence of temperature added in. It has been reported that the fatigue strengths of plain weave carbon/epoxy fabric laminates at 100°C are lower than those at room temperature, not only in the fiber direction but also in off-axis directions (Kawai and Taniguchi, 2006). The reduction in fatigue strength in the fiber direction at 100°C that was observed in the study was reflected by the increase in the slope of the S–N relationship. For the fatigue performance of the cross-ply carbon/epoxy laminate in the fiber direction, however, no significant difference was found in the results obtained at room temperature and 100°C (Kawai and Maki, 2006). In contrast, the slope of the S–N relationship for unidirectional carbon/epoxy laminates slightly increased with increasing temperature (Kawai *et al.*, 2001a). It is well known that the properties of the constituents of composites have significant influences on their fatigue performance (Konur and Matthews, 1989; Kawai *et al.*, 1996). Thus, the temperature dependence of the fatigue behavior of composites reflects the change in properties of the matrix and fiber/matrix interfaces due to temperature. Khan *et al.* (2002) have demonstrated that the thermal degradation of matrix resins at high temperature changes the temperature dependence of the fatigue resistance of composite laminates and results in even more complicated fatigue behavior.

Great care is needed when dealing with the static and fatigue strengths of composites that are exposed to hygrothermal environments (Jones *et al.*, 1984; Selzer and Friedrich, 1997). It has been observed for two kinds of carbon/epoxy laminates, $[\pm 45/0_3/\pm 45/0]_S$ and $[0/\pm 45/0_2/\pm 45/0]_S$, that the tensile strengths increase with increasing moisture content in contrast to a consistent reduction in the compressive strengths (Kellas *et al.*, 1990a, b). This observation suggests that the absolute value of the critical stress ratio decreases with increasing moisture content and accordingly the anisomorphic CFL diagram inclines rightward, similar to the change with increasing temperature. According to Asp (1998), on the other hand, moisture content and temperature produce a significant effect on the interlaminar delamination

toughness, i.e. the critical strain energy release rate decreases with moisture content in mode II and mixed mode loading, and with temperature in mode II loading, whereas it slightly increases with increasing temperature in mode I loading. These experimental results imply that the growths of damage in composite laminates in hygrothermal environments are differently observed depending on a competition between moisture and temperature, especially under mode II loading condition. Shan and Liao (2001) have compared the fatigue behaviors of unidirectional glass fiber reinforced and glass–carbon fiber reinforced epoxy matrix composites in wet and dry environments at 25°C, respectively. They found that while both systems are more sensitive to a wet environment, especially at low stress levels, the hybrid system containing 25% of carbon fibers shows better resistance to fatigue in water than the all-glass fiber system in water over the range up to 10^7 cycles. The former observation is consistent with the reduction in interlaminar toughness due to the uptake of water, and the latter corresponds to the observation of the moderate degradation of fatigue performance in the wet-conditioned cross-ply $[0/90]_S$ and angle-ply $[\pm 45]_S$ carbon/epoxy laminates (Sala, 2000). These experimental results suggest that the shape of the CFL curve for a given fatigue life and its deformation with increasing number of cycles to failure in a wet environment may differ from those in a dry environment.

A higher frequency of cyclic loading has been reported to have a more significant degrading effect on the fatigue performance of carbon fiber reinforced composites. Curtis *et al.* (1988) examined the fatigue behaviors of quasi-isotropic $[-45/0/45/90]_{2S}$ and angle-ply $[\pm 45]_{4S}$ APC-2/AS4 laminates at different loading frequencies (0.5 Hz, 5 Hz), and demonstrated that the fatigue strength at 5 Hz is lower than that at 0.5 Hz, regardless of the stacking sequence of laminates. A similar reduction in fatigue performance was observed for a plain weave carbon/epoxy fabric laminate (Kawai and Taniguchi, 2006). It is considered that the reduction in fatigue strength of composites with the increase in loading frequency is caused by the change in the properties of the matrix and the matrix–fiber interface due to the temperature rise in specimens during fatigue loading, although the frequency-dependent reduction in fatigue strength is not always ascribed to the reduction in static strength due to temperature rise (Curtis *et al.*, 1988).

The strength of fiber-reinforced composites degrades with time (Dillard *et al.*, 1982; Raghavan and Meshii, 1997). Such stress rupture (or creep rupture) behavior becomes more significant in matrix-dominated laminates at higher temperatures (Brinson, 1999; Kawai *et al.*, 2006; Kawai and Sagawa, 2008). In constructing the anisomorphic CFL diagrams for composites, therefore, the creep strengths in tension and compression for a given total time to fracture which is equivalent to the duration of a given constant number of cycles to failure should be used in place of their initial static strengths if the reduction in strength due to creep becomes significant (Mallick and Zhou,

2004). The necessity of considering the creep rupture strength of composites in identifying their CFL diagrams had already been pointed out by Boller (1957, 1964).

Consequently, when building the CFL diagrams for composites, we need to take into account the changes in their strengths that are caused by temperature, impact damage, water uptake, and time. Almost all the factors that influence the static and fatigue strengths of continuous fiber-reinforced polymer matrix composites have been reviewed by Schulte and Stinchcomb (1989) and Agarwal and Broutman (1990), respectively. The information from these articles allows us to qualitatively understand how the anisomorphic CFL diagram for a given composite is affected by the factors, but further efforts are necessary to quantify the effect of the factors on the mean stress sensitivity in fatigue of composites through experiment and to assess the validity of the theoretical methods for constructing the CFL diagrams for composites.

6.9 Source of further information and advice

The engineering methods for predicting the S–N relationships for composites under constant amplitude fatigue loading at any stress ratios have been described in the book *Fatigue in Composites*, edited by Harris (2003), that covers the most important aspects of the fatigue of composites. Progress that has been made since the publication of this encyclopedic book will be found in the present volume. So, these two books would be excellent aids in the continued journey of developing realistic fatigue life prediction methods suitable for composites. It is also helpful to revisit pioneering articles on the subject discussed in this chapter, e.g. Boller (1957, 1964), and to learn the history of the early development of the CFL diagram that has been reviewed by Sendekyj (2001). In addition to the master S–N curve and CFL diagram approaches, the cultivation of the fatigue failure criteria based on the principal residual strengths, e.g. Hashin and Rotem (1973), Sims and Brogdon (1977), Hashin (1981), Sendekyj (1990), Kawai *et al.* (2001a), and Liu and Mahadevan (2007), may inspire a different approach to constant amplitude fatigue life prediction. The guidelines for the design of wind turbines (Risø, 2002) and the SAE Fatigue Design Handbook (Rice, 1997), which describe the current technologies and procedures for fatigue design of industrial products, although the latter is intended mainly for conventional materials, will also help to clearly understand the current state of the fatigue life prediction methods for composites and to further elaborate them. In regard to the factors that should be considered for establishing a more accurate fatigue life prediction method based on a CFL diagram, the reader can obtain access to distributed information with the aid of the two main reference sources, along with the additional references provided in the previous section.

6.10 Acknowledgments

The research on which this chapter is based has been carried out with my students at the University of Tsukuba, mainly with the financial support of the University of Tsukuba and the Ministry of Education, Culture, Sports, Science and Technology of Japan. The author is grateful to all the members in my lab who have contributed to the work. This chapter has been written in the course of the research work supported in part by the Ministry of Education, Culture, Sports, Science and Technology of Japan under a Grant-in-Aid for Scientific Research (No. 20360050).

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Probabilistic fatigue life prediction of composite materials

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Abstract: This chapter discusses the use of probabilistic models to describe the fatigue behavior of composite materials and to quantify the uncertainties in fatigue life prediction. The chapter first reviews existing methods for probabilistic fatigue life prediction and then discusses how to quantify the uncertainties in applied loading and material properties. Following this, a random process approach using the Karhunen–Loeve expansion technique is discussed to quantify the input uncertainties as well as their correlations. Probabilistic methods are presented to solve the time-dependent reliability problem. The probabilistic life prediction method is demonstrated under variable amplitude loading for several example problems.

Key words: fatigue, probabilistic method, life prediction, reliability, composite materials, variable amplitude loading.

7.1 Introduction

The fatigue process of structural components under service loading is stochastic in nature. Life prediction and reliability evaluation is still a challenging problem despite extensive progress made in the past decades. A comprehensive review of early developments can be found in Yao *et al.* (1986). Compared to fatigue under constant amplitude loading, fatigue modeling under variable amplitude loading becomes more difficult from both deterministic and probabilistic points of view. An accurate deterministic damage accumulation rule is required first, since the frequently used linear Palmgren–Miner rule may not be sufficient to describe the physics (Fatemi and Yang 1998). Second, an appropriate uncertainty modeling technique is required to include the stochasticity in both material properties and external loadings, which should accurately represent the randomness of the input variables and their covariance structures. In addition to the above difficulties, such a model also needs to be computationally and experimentally inexpensive. The last characteristic is the main reason for the popularity of simpler models despite their inadequacies.

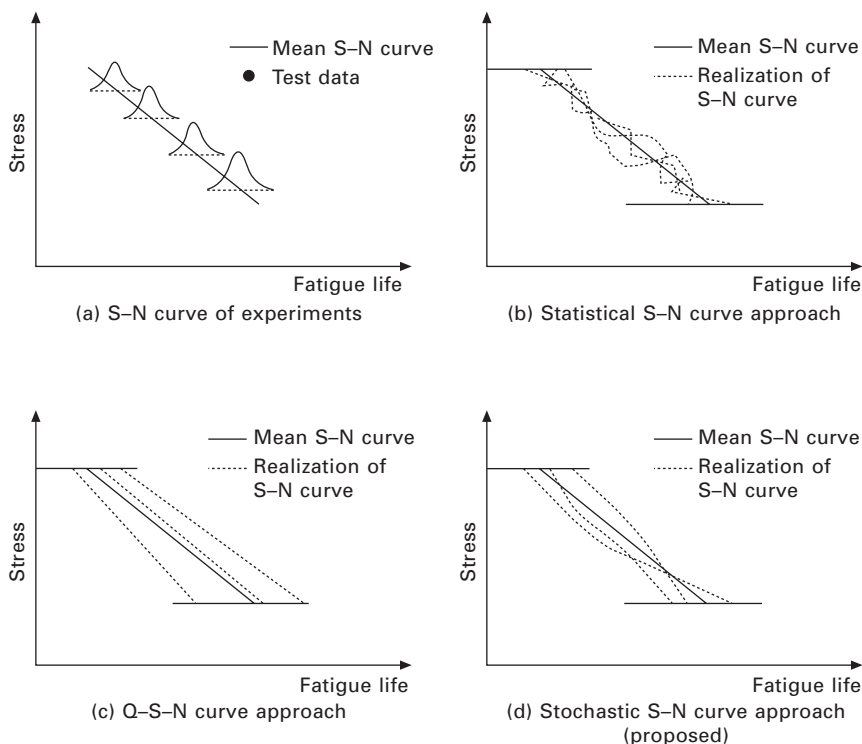
In classical fatigue life analysis, a fatigue damage accumulation rule together with the material properties measured under constant amplitude

loading is used to predict the fatigue life under variable amplitude loading. Several probabilistic methods have been proposed to describe the statistics observed under constant amplitude fatigue tests and to evaluate the reliability under variable amplitude loading. Depending on the method to handle the randomness in constant amplitude tests, previously available probabilistic methods can be grouped into two categories.

One type is to treat the fatigue lives at different stress levels as independent random variables. The statistics of the random variables are described using a statistical distribution function, such as Weibull or lognormal. In this chapter, we name this type of approach the statistical S–N curve approach. Most of the statistical S–N curve approach assumes linear damage accumulation rule (LDAR) lognormal distribution of random variables (Liao *et al.* 1995; Kam *et al.* 1998; Le and Peterson 1999; Shen *et al.* 2000). A perturbation-based stochastic finite element method is proposed (Kaminski 2002) for fatigue analysis of composites, in which the restrictions on the input random variables describing fatigue life are low, since only the moments of the individual random variables are required in the computational methodology, such as mean and variance. The other widely used approach is to use a family of fully correlated S–N curves corresponding to different survival probabilities of the material (Shimokawa and Tanaka 1980; Rowatt and Spanos 1998; Pascual and Meeker 1999; Ni and Zhang 2000; Zheng and Wei 2005). This approach may be referred to as the quantile or percentile S–N curve approach (referred to as the Q–S–N curve later in this chapter). A similar approach named the intrinsic fatigue curve (IFC) was proposed for fatigue life prediction (Kopnov 1993, 1997), which is another format of the quantile S–N curve approach. The difference between the quantile S–N curve and IFC is that Q–S–N uses a set of deterministic S–N curves and each represents a different survival probability level, whereas IFC uses a single random function in which the realizations of the random function are the same as Q–S–N.

From a statistical point of view, both the statistical S–N curve and the Q–S–N curve have implicit assumptions in representing the set of random variables. The statistical S–N curve approach assumes that the covariance function of these variables is zero, and the quantile S–N curve approach assumes the covariance function as unity. Either assumption can be barely achieved in the realistic condition. A more appropriate approach is to propose an S–N curve representation technique which can include the covariance structure of the constant amplitude fatigue lives. A schematic comparison of the various methods for representing the S–N curves is shown in Fig. 7.1.

In this chapter, a general methodology for stochastic fatigue life prediction under variable amplitude loading is proposed, which combines a nonlinear fatigue damage accumulation rule and a stochastic S–N curve representation technique. An efficient nonlinear damage accumulation is discussed first. Next, the uncertainty modeling is discussed and a stochastic S–N curve approach



7.1 Schematic comparisons of different approaches in representing fatigue S-N curves.

using the Karhunen–Loeve expansion technique is discussed to represent the randomness and covariance observed in the experimental data. Following this, numerical methods are introduced to solve the time dependent fatigue reliability problem and to calculate the probabilistic fatigue life. The methods are demonstrated using several numerical examples.

The methodology discussed in the following sections offers two advantages over existing approaches. First, it uses a nonlinear fatigue damage accumulation rule, which improves the accuracy of the LDAR by considering the load dependence effect of the fatigue damage. It is shown in the numerical example that LDAR gives very non-conservative prediction for composite materials. Unlike most of the previous nonlinear fatigue damage accumulation models, the proposed model does not require cycle-by-cycle calculation, which significantly reduces the calculation effort, especially for the reliability evaluation. Second, a stochastic S-N curve approach can capture the covariance structure of the fatigue damage process under different stress levels (which is usually ignored by other models), and thus makes the reliability evaluation more accurate compared to the existing models.

7.2 Fatigue damage accumulation

7.2.1 Existing models

Fatigue damage increases with applied loading cycles in both constant amplitude loading and variable amplitude loading. However, the characteristics of damage accumulation under different loadings are different. For more than 80 years, researchers have tried to find the best rule to describe the fatigue damage accumulation behavior. A comprehensive review is not the objective of this paper and can be found in Fatemi and Yang (1998). Only a few damage accumulation rules are briefly described below.

Among all the fatigue damage accumulation rules, the LDAR (linear damage accumulation rule), also known as the Palmgren–Miner rule, is probably the most commonly used. The LDAR (Miner 1945) describes the fatigue damage accumulation under variable amplitude load as

$$D = \sum_{i=1}^k \frac{n_i}{N_i} \quad 7.1$$

where D is the fatigue damage of the material, n_i is the number of applied loading cycles corresponding to the i th load level, and N_i is the number of cycles to failure at the i th load level, from constant amplitude experiments. Equation 7.1 implies that fatigue damage accumulates in a linear manner.

If LDAR is used for fatigue life prediction, it is usually assumed that the material fails when the damage D reaches unity. However, it has been shown that LDAR produces a large scatter in the fatigue life prediction of both metal and composites (Shimokawa and Tanaka 1980; Kawai and Hachinohe 2002). Also, LDAR cannot explain the load level dependence of fatigue damage observed in the experiments (Halford 1997). Despite all those deficiencies, LDAR is still frequently used due to its simplicity.

In order to improve the accuracy of LDAR, nonlinear functions have been proposed to describe the damage accumulation. For example, a nonlinear damage accumulation rule was proposed as (Marco and Starkey 1954)

$$D = \sum_{i=1}^k \left(\frac{n_i}{N_i} \right)^{C_i} \quad 7.2$$

where C_i is a material parameter related to the i th loading level. A similar formula named the damage curve approach has been proposed in Manson and Halford (1981). Equation 7.2 can reflect the load-level dependence and load-sequence dependence effects of the fatigue damage accumulation. It

is shown that the Miner's sum $\sum_{i=1}^k \frac{n_i}{N_i} > 1$ for low–high load sequences and $\sum_{i=1}^k \frac{n_i}{N_i} < 1$ for high–low sequences (Fatemi and Yang 1998). As pointed out

by Van Paepegem and Degrieck (2002), this conclusion cannot be applied to all materials in the existing experimental database in the literature.

Due to the nonlinearity of Eq. 7.2, the fatigue damage under service loading needs to be computed in a cycle-by-cycle manner, which requires a large amount of computational effort. This disadvantage can be circumvented by approximating the nonlinear function by double linear functions (Halford 1997). In each stage, a linear damage accumulation rule is applied. For two-block loading, the double linear damage model is easy to implement. For the multi-block loading or spectrum loading, the determination of the parameters in the model becomes complicated (Halford 1997; Goodin *et al.* 2004).

Several more complex fatigue damage accumulation functions have been proposed for increased accuracy. Halford and Manson (1985) proposed a double damage curve approach, which combines the accurate parts of both the double linear damage approach and the damage curve approach. A similar result was obtained by using the fatigue crack growth concept (Vasek and Polak 1991). A more recent approach for fatigue damage accumulation is to use a nonlinear continuum damage mechanics model (Cheng and Plumtree 1998; Fatemi and Yang 1998; Shang and Yao 1999). In all these damage functions, the basic idea is to calculate the fatigue damage in an evolutionary manner using a scalar damage variable. The main differences lie in the number and characteristics of the parameters used in the model, in the requirements for additional experiments, and in their applicability (Fatemi and Yang 1998).

From the brief discussion above, it is found that most of the nonlinear fatigue damage models improve the deficiencies within LDAR by considering additional loading effects. However, they are usually computationally expensive compared to LDAR, especially when the applied loading is repeated block loading or spectrum loading, since most of them require cycle-by-cycle calculation. This disadvantage makes it difficult to perform simulation-based reliability evaluation. The following few sections present a nonlinear damage accumulation model, which improves the deficiency in the linear damage accumulation rule but still maintains its computational simplicity. Since the major deficiency of LDAR is that it is independent of applied load levels, this method attempts to modify the LDAR to make it load level dependent and yet preserve the linear summation form to make the calculation easier.

7.2.2 Fatigue damage accumulation under stationary loading

Fatigue cyclic loading is stochastic in nature. In this section, we first discuss the stationary loading process. The stationary loading refers to the loading history which does not alter its statistical distribution corresponding to time,

i.e. the loading is stationary corresponding to time. The repeated multi-block loading can be treated as a stationary loading case. Within each block, the loading is not stationary. But the assumption of a stationary process holds when the material experiences many blocks before it fails (i.e. high-cycle fatigue problem). Under the stationary assumption, the distribution of applied loading cycles is adequate to describe the loading process.

To make the discussion easier for fatigue damage accumulation under stationary loading, let us consider a fatigue problem under a repeated two-block loading first. N_f is the total number of cycles to failure. If the linear damage accumulation rule is used, we obtain:

$$\begin{cases} \frac{n_1}{N_1} + \frac{n_2}{N_2} = 1 \\ n_1 + n_2 = N_f \end{cases} \quad 7.3$$

Equation 7.3 can be rewritten as

$$\begin{cases} \frac{n_1}{N_1} + A_1 \frac{N_f - n_1}{N_1} = 1 \\ A_1 = \frac{N_1}{N_2} \end{cases} \quad 7.4$$

From Eq. 7.4 we can express the cycle ratio $\frac{n_1}{N_1}$ as a function of cycle distribution $\omega_1 = \frac{n_1}{N_f}$ as

$$\frac{n_1}{N_1} = \frac{1}{\frac{A_1}{\omega_1} + 1 - A_1} \quad 7.5$$

If the fatigue S–N curve under constant amplitude loading (s) is expressed as

$$N = N(s) \quad 7.6$$

then A_1 in Eq. 7.5 is a material parameter depending on the two stress levels and equals $\frac{N(s_1)}{N(s_2)}$. Similarly, the cycle ratio of the second stress level can be expressed as a function of the cycle distribution as

$$\frac{n_2}{N_2} = \frac{1}{\frac{A_2}{\omega_2} + 1 - A_2} \quad 7.7$$

where A_2 is a material parameter depending on the two stress levels and equals $\frac{N(s_2)}{N(s_1)}$.

Substituting Eq. 7.5 and Eq. 7.7 into Eq. 7.4, we obtain

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} = \sum_{i=1}^2 \frac{1}{\frac{A_i}{\omega_i} + 1 - A_i} \quad 7.8$$

The above derivation is under the assumption of a linear damage accumulation rule. For the materials that follow this rule, the right-hand side of Eq. 7.8 equals unity. For materials that do not follow the linear rule, A_i cannot be determined only by constant amplitude experiments. They depend on the material properties and loading conditions. This parameter can be calibrated using one additional fatigue experiment under variable amplitude loading. In the method below, we plot the cycle ratio and cycle distribution together for each stress level. Then we compute the coefficients A_i in Eq. 7.5 through least-squares regression. Based on an earlier study (Liu and Mahadevan 2007), the following empirical function is used to calculate A_i :

$$A_i = \alpha (s_i / \bar{s})^\beta \quad 7.9$$

where α and β are material parameters; s_i is the current stress level and $\bar{s}$ is the mean value of all the stress amplitudes in each block. Notice that the modified LDAR (Eq. 7.8) thus becomes stress dependent.

Equation 7.8 is extended for repeated multi-block loading as

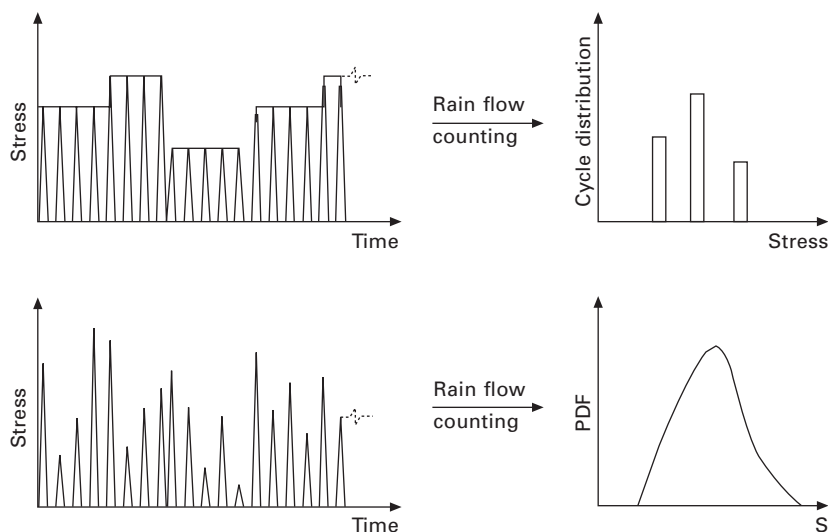
$$\sum_{i=1}^k \frac{n_i}{N_i} = \sum_{i=1}^k \frac{1}{\frac{A_i}{\omega_i} + 1 - A_i} \quad 7.10$$

For continuous stationary spectrum loading, Eq. 7.10 is expressed as

$$\int \frac{n(s)}{N(s)} ds = \int \frac{1}{\frac{A(s)}{f(s)} + 1 - A(s)} ds \quad 7.11$$

where the cycle distribution ω_i (probability description for block loading) becomes the probability density function (PDF) $f(s)$ of the applied continuous random loading (see Fig. 7.2). Equations 7.10 and 7.11 constitute the proposed fatigue damage accumulation model under stationary loading. Compared with the linear damage rule, the proposed model includes the effect of the stress levels. The Miner's sum is not a constant but depends on the cycle distribution of the applied loadings.

When using Eq. 7.10 (or 7.11) for fatigue life prediction, the right-hand side of Eq. 7.10 (or 7.11) is first calculated. For repeated multi-block



7.2 Schematic illustration of cycle distribution using rain flow counting method.

loading, the cycle distribution of the different stress levels at failure can be approximated using the cycle distribution value in a single block. For high-cycle fatigue, this is a reasonable approximation. Then the fatigue life prediction is performed in the same way as the classical procedure using the linear damage accumulation rule.

From Eq. 7.10 (or 7.11), it is seen that this method still maintains the simplicity of the linear damage rule. It can directly use the cycle counting results and does not require cycle-by-cycle calculation. The method includes the load level and load content effects, which improve the deficiencies within the LDAR model. In later sections, it is shown that this method gives a more accurate prediction with similar calculation effort when compared to the LDAR model for composite materials collected in the current investigation.

7.2.3 Fatigue damage accumulation under non-stationary loading

Fatigue damage accumulation under non-stationary loading is complicated compared to that under stationary loading. The model described in Section 7.2.2 is only applicable to stationary loading as it only considers the cycle distribution of the applied loading. For non-stationary applied loading, the cycle distribution is not sufficient to describe the loading process. Step loading is one type of non-stationary loading and is used to demonstrate the proposed methodology. Detailed study under real variable amplitude loading needs further consideration. For step loading, the material is first pre-cycled under

one or several stress levels. Then the material is cycled till failure at a certain stress level. This type of loading is non-stationary as the mean value and variance of the applied loading change with time. For this type of loading, the load sequence effect in fatigue damage accumulation is observed for some materials (Fatemi and Yang 1998). The high–low and the low–high loading sequences result in different Miner’s sums. In the current study, several step loading experimental data are collected. The model shown in Section 7.2.2 is modified to include the load sequence effect for step loadings.

The model coefficients A_i expressed in Eq. 7.9 are modified as (Liu and Mahadevan 2007)

$$\log(A_i) = \alpha + \beta \log\left(\frac{s_i}{\bar{s}}\right) + \gamma \log\left(\frac{\bar{s}}{s_k}\right) \quad 7.12$$

where γ is a material parameter to describe the load sequence effect of the material, and s_k is the stress amplitude at the final step. The third term in Eq. 7.12 represents the load sequence effect on the final fatigue damage of the material. When the material does not experience the load sequence effect ($\gamma = 0$) or the applied loading is stationary ($\log(\bar{s}/s_k) = 0$), Eq. 7.12 reduces to Eq. 7.9. The material parameters in Eq. 7.12 can be calibrated using the high–low and the low–high step loading experiments following the same procedure for repeated block loading.

Equations 7.10 and 7.12 are used together for fatigue life prediction under step loadings. For non-stationary loading, the cycle distribution at failure is not known before hand. Therefore a trial and error method can be used to find the solution of Eq. 7.10. The initial values for cycle distribution can be computed using the LDAR model. It is found that usually a few iterations are enough for convergence.

7.3 Uncertainty modeling

The large uncertainties within the fatigue problem come from several sources. Among them, external applied loading and material fatigue resistance are the most important and thus are discussed in this section.

7.3.1 Uncertainty modeling of external loading

Two approaches are commonly used to describe the scatter in the random applied loading. One is in the frequency domain and uses power spectral density methods. The other is in the time domain and uses cycle counting techniques. The major advantages of the frequency domain approach are that it is more efficient and can obtain an analytical solution under some assumptions of the applied loading process, such as Gaussian process, stationary and narrow banded. This of course limits the applicability of

the frequency domain approach to some real problems (Jiao 1995; Tovo 2000). Also, most of the frequency domain approaches assume the linear fatigue damage accumulation rule (Fu and Cebon 2000; Banvillet *et al.* 2004; Benasciutti and Tovo 2005), due to the loss of loading sequence information during the computation of the power spectral density function from the loading history.

The time domain approach is used in this chapter. Among many different cycle counting techniques, rain flow counting is predominantly used and is adopted in the proposed methodology. A detailed description of the rain flow counting method can be found in Downing and Socie (1982) and ASTM (1985).

In the nonlinear fatigue damage accumulation method described in the last section, the cycle distribution is required for fatigue life prediction. This information can be obtained by performing rain flow counting of the loading history. A schematic explanation is shown in Fig. 7.2 for two different loading histories. The upper example is for the block loading and the discrete probability distribution of the load amplitude. The lower example is for the random variable loading and the continuous probability density distribution of the load amplitude.

7.3.2 Uncertainty modeling of material properties

Section 7.1 described different approaches in modeling the material property variability. In this section, the fatigue lives N under different constant amplitude tests are treated as random fields/processes with respect to different stress levels s . Stochastic expansion techniques are very successful in describing the variation in the corresponding random field/process. Several methods are available, such as the spectral representation method (Shinozuka and Deodatis 1991; Grigoriu 1993), the Karhunen–Loeve (KL) expansion method (Loeve 1970), polynomial chaos expansion (Ghanem and Spanos 1991; Ghanem 1999), etc. In this chapter, the KL expansion technique is used and a new stochastic S–N curve method is proposed based on the KL expansion technique.

The fatigue lives under constant amplitude loading are assumed to follow the lognormal distribution for the sake of illustration. As mentioned earlier in the introduction, both lognormal and Weibull distributions are commonly used in the literature. The lognormal assumption makes $\log(N(s))$ a Gaussian process with mean value process of $\log(\bar{N}(s))$ and standard deviation of $\sigma_{\log(N)}(s)$, where $\log(\bar{N}(s))$ is the mean S–N curve obtained by regression analysis. It needs to be pointed out that the Gaussian assumption is not a requirement in the proposed methodology. Non-Gaussian methods for random field representation are available and can be applied to the problem without difficulty.

It has been shown that the variance is not a constant but a function of stress level s (Pascual and Meeker 1999). The $\sigma_{\log(N)}(s)$ represents the scatter in the data and can be obtained by classical statistical analysis. Based on the above assumption, the process

$$Z(s) = \frac{(\log N(s)) - \log(\bar{N}(s))}{\sigma_{\log(N)}(s)} \quad 7.13$$

is a normal Gaussian process with zero mean and unit variance.

From a physical standpoint, the autocovariance function of the fatigue lives should decrease as the difference between stress levels increases. An exponential decay function is proposed for the covariance function $C(s_1, s_2)$ of $Z(s)$ as

$$C(s_1, s_2) = e^{-\mu|s_1 - s_2|} \quad 7.14$$

where μ is a measure of the correlation distance of $Z(s)$ and depends on the material. In classical S–N fatigue experiments, the specimen is tested until failure or runout at a specified stress level and cannot be tested at the other stress levels. Due to the non-repeatable nature of fatigue tests, the covariance function cannot be determined by constant amplitude fatigue experimental data alone. Since in the proposed methodology, the nonlinear fatigue damage accumulation model also needs one additional variable amplitude loading fatigue test to calibrate the model parameters α , β and/or γ , μ can be calibrated by the same variable amplitude loading fatigue test data as well.

In the KL expansion, the random process/field $Z(s)$ can be expressed as a function of a set of standard random variables, or, in other words, expressed as a combination of several random functions. Generally, the expansion takes the form

$$Z(s) = \sum_{i=1}^{\infty} \sqrt{\lambda_i} \xi_i(\theta) f_i(s) \quad 7.15$$

where $\xi_i(\theta)$ is a set of independent random variables, satisfying

$$\begin{cases} E(\xi_i(\theta)) = 0 \\ E(\xi_i(\theta)\xi_j(\theta)) = \delta_{ij} \end{cases} \quad 7.16$$

where E denotes the mathematical expectation operator, and δ_{ij} is the Kronecker-delta function.

In Eq. 7.15, $\sqrt{\lambda_i}$ and $f_i(x)$ are the i th eigenvalues and eigenfunctions of the covariance function $C(s_1, s_2)$, evaluated by solving the homogeneous Fredholm integral equation analytically or numerically:

$$\int_D C(s_1, s_2) f_i(s_2) = \lambda_i f_i(s_1) \quad 7.17$$

In practical calculation, only a truncated number of terms in Eq. 7.15 is required to achieve the satisfied accuracy. If a Gaussian assumption is used, 10–20 terms are adequate to get very precise results. The detailed computational procedure for the KL expansion can be found elsewhere (Huang *et al.* 2001; Phoon *et al.* 2002).

From Eqs 7.13–7.17, we obtain

$$\log(N(s)) = \sigma_{\log(N)}(s) \sum_{i=1}^{\infty} \sqrt{\lambda_i} \xi_i(\theta) f_i(s) + \log(\bar{N}(s)) \quad 7.18$$

Substituting Eq. 7.18 into Eq. 7.10 (or 7.11), we can solve for the fatigue life under variable amplitude loading. Different methods for solving this problem are discussed in the Section 7.4. The proposed uncertainty modeling method in this section includes the covariance structure in the fatigue analysis. The importance of the covariance structure on the final reliability evaluation can be illustrated using the example below.

Consider a two-block loading case under the linear damage accumulation assumption. The covariance functions for the statistical S–N approach, Q–S–N and the proposed stochastic S–N approach can be expressed as

$$\begin{cases} C(s_1, s_2) = 0 & \text{statistical S–N curve} \\ C(s_1, s_2) = 1 & \text{Q–S–N} \\ C(s_1, s_2) = e^{-\mu|s_1 - s_2|} & \text{stochastic S–N curve} \end{cases} \quad 7.19$$

For fatigue damage accumulation

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} = D_1 + D_2 \quad 7.20$$

the mean value of the fatigue damage is

$$E(D) = E(D_1) + E(D_2) \quad 7.21$$

and the variance of the fatigue damage is

$$\text{Var}(D) = \text{Var}(D_1) + \text{Var}(D_2) + 2\rho\sqrt{\text{Var}(D_1)\text{Var}(D_2)} \quad 7.22$$

where ρ is the correlation coefficient of the random variables D_1 and D_2 . It is seen that the different approaches have no effect on the mean value of the fatigue damage but have effect on the variance. Thus,

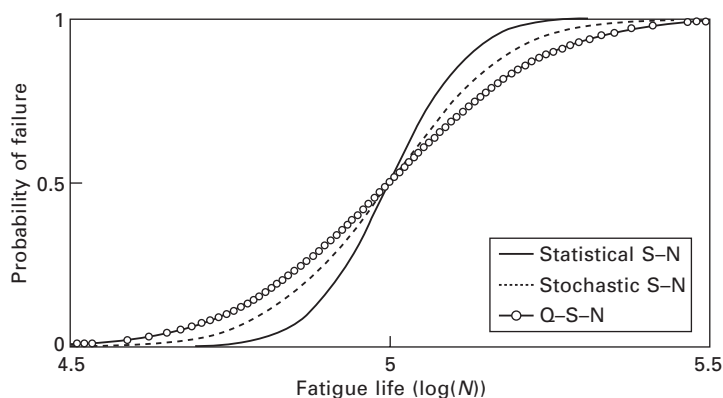
$$\text{Var}(\text{statistical S–N}) \leq \text{Var}(\text{stochastic S–N}) \leq \text{Var}(\text{Q–S–N}) \quad 7.23$$

A schematic representation of the failure probability with respect to time from the three methods is shown in Fig. 7.3. The mean value of the fatigue life is 5 (log scale) and the standard deviation is 0.1, 0.15 and 0.2 (log scale) for statistical S–N, stochastic S–N and Q–S–N, respectively. Thus different approaches give different fatigue reliability estimates. The difference is especially significant around the tail region. For design and maintenance against fatigue, it is usually required that the mechanical component stay at a very low failure probability (i.e. less than 0.1%). At this stage, the difference among the three approaches is around 0.16 in log scale, which is about 45% difference in real life cycles. Zheng and Wei (2005) used the Q–S–N approach and observed that the standard deviation of the predicted fatigue life of 45 steel notched elements under variable amplitude loading is larger than that of test results. The authors stated that the reason behind this finding should be further investigated. Equations 7.19–7.23 give a possible explanation for this phenomenon, i.e., the effect of the correlation structure. Since the fully uncorrelated and fully correlated cases can be barely found in reality, the standard deviation of the experimental results should lie between those predicted by the statistical S–N approach and the Q–S–N approach.

It is interesting to note that the statistical S–N approach and Q–S–N are two special cases of the proposed method. If μ in Eq. 7.19 approaches $+\infty$, the covariance function reduces to zero, giving the statistical S–N method. If μ in Eq. 7.19 approaches zero, the covariance function reduces to 1, giving the Q–S–N method.

7.4 Methods for probabilistic fatigue life prediction

Using the uncertainty quantification technique and damage accumulation rule described in the last section, the reliability can be calculated by numerical



7.3 Failure probability predictions by different approaches.

simulations, such as the Monte Carlo method. While Monte Carlo simulation is powerful in solving the reliability problem, the computational effort prohibits its application. This section discusses two accurate and simple time-dependent reliability calculation methods considering the stochastic damage accumulation. The methods are based on the random process representation of the material S–N curve (Section 7.3) in which the correlation parameter is taken into account. The formulation of the proposed methods is shown below.

7.4.1 Fatigue reliability under stationary loading

First, consider a material under stationary variable amplitude loading. The material S–N curve $N(s)$ is described using Eq. 7.18 as a random process whose covariance function is expressed by Eq. 7.14. The fatigue damage caused in a single cycle at the stress level s can be expressed as a fraction of the total number of cycles to failure:

$$D(s) = \frac{1}{N(s)} \quad 7.24$$

Equation 7.24 shows that the damage in a single cycle can also be expressed as a random process when considering multiple stress levels. The covariance function $C(s_1, s_2)$ of $D(s)$ is assumed to be an exponential decay function as

$$C(s_1, s_2) = \sigma_{D(s_1)} \sigma_{D(s_2)} e^{-\lambda |s_1 - s_2|} \quad 7.25$$

where $\sigma_{D(s)}$ is the standard deviation of $D(s)$ at the stress level s . λ is a measure of the correlation distance of $D(s)$ and depends on the material.

At any arbitrary time T , the accumulated damage $D_{T,i}$ at the i th stress level s can be expressed as

$$D_{T,i} = \frac{n_i(s)}{N_i(s)} = n_i(s) D_i(s) \quad 7.26$$

Under the stationary assumption, the number of applied loading cycles $n_i(s)$ corresponding to the i th load level can be expressed as

$$n_i(s) = T f_i(s) \quad 7.27$$

where $f_i(s)$ is the probability density at the i th load level obtained from the rain flow counting results. Using the damage accumulation rule described in Section 7.2, the total damage at time T considering all the stress levels is the summation of damage at each stress level:

$$D_T = \sum_{i=1}^k D_{T,i} = \sum_{i=1}^k \frac{n_i(s)}{N_i(s)} = \sum_{i=1}^k n_i(s) D_i(s) = T \sum_{i=1}^k \omega_i D_i(s) \quad 7.28$$

For continuous stationary spectrum loading, Eq. 7.11 is expressed as

$$D_T = \int_0^\infty \frac{n(s)}{N(s)} ds = \int_0^\infty n(s) D(s) ds = T \int_0^\infty f(s) D(s) ds \quad 7.29$$

Equation 7.28 (or 7.29) is the probabilistic damage growth function of the material under cyclic fatigue loading. It is shown that the damage of the material depends on material properties $D(s)$, applied loading $f(s)$ and time T . The right-hand side of Eq. 7.29 is an integral of a random process. At a fixed time instant, it becomes a random variable, which is the damage at time T . Under arbitrary external loading, the integral of the random process is not amenable to an analytical solution. Thus, numerical approximation methods are required to calculate the time-dependent reliability.

Once the time evolution of fatigue damage is known (Eqs 7.28 and 7.29), it is convenient to express the limit state function of failure G by combining Eqs 7.10 and 7.28 for discrete load spectrum as

$$G = \psi - D_T = \sum_{i=1}^k \frac{1}{\frac{A_i}{\omega_i} + 1 - A_i} - T \sum_{i=1}^k \omega_i D_i(s) \quad 7.30$$

Similarly, the limit state function for the continuous load spectrum case can be expressed by combining Eqs 7.11 and 7.29 as

$$G = \psi - D_T = \int_0^\infty \frac{ds}{\frac{A(s)}{f(s)} + 1 - A(s)} - T \int_0^\infty f(s) D(s) ds \quad 7.31$$

where ψ is the critical value for the damage at failure, which can be calculated using Eqs 7.10 and 7.11 discussed in Section 7.2. If G is less than zero, fatigue failure will occur.

7.4.2 Method I: moments matching approach

Although the analytical calculation of Eq. 7.29 is usually very difficult or impossible, the first two central moments of the fatigue damage can be obtained. For continuous loading, the mean and variance of fatigue damage can be expressed as

$$\begin{cases} \mu_{D_T} = E\left(T \int_0^\infty f(s) D(s) ds\right) = T \int_0^\infty f(s) \mu_{D(s)} ds \\ \sigma_{D_T}^2 = \text{Cov}\left(T \int_0^\infty f(s) D(s) ds\right) \\ \quad = T^2 \int_0^\infty \int_0^\infty f(s_1) f(s_2) \sigma_{D(s_1)} \sigma_{D(s_2)} e^{-\lambda |s_1 - s_2|} ds_1 ds_2 \end{cases} \quad 7.32$$

For discrete loading, the mean and variance of damage can be expressed as

$$\begin{cases} \mu_{D_T} = T \sum_{i=1}^{\infty} \omega_i \mu_{D(s_i)} \\ \sigma_{D_T}^2 = T^2 \left[\sum_{i=1}^{\infty} \omega_i^2 \sigma_{D(s_i)}^2 + 2 \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \omega_i \omega_j \sigma_{D(s_i)} \sigma_{D(s_j)} e^{-\lambda |s_i - s_j|} \right] \end{cases} \quad 7.33$$

where $\mu_{D(s)}$ and $\sigma_{D(s)}$ in Eqs 7.32 and 7.33 are the mean and standard deviation of damage in a single cycle at stress level s , which are obtained from constant amplitude loading tests.

In order to calculate the time-dependent fatigue reliability, we need to assume the probability distribution of the fatigue damage D_T since only the first two central moments are available. Equations 7.28 and 7.29 can be treated as a summation of a set of random variables. It is well known that a summation of Gaussian random variables is a Gaussian random variable. However, the distribution of the summation of non-Gaussian random variables is usually unknown. Studies for some special cases of summation of non-Gaussian random variables have been reported. Fenton (1960) proposed a method to approximate the summation of a set of correlated lognormal random variables as a single lognormal random variable. The method matches the mean and variance of the lognormal sum to the target random variable. It has been shown that this method is very accurate at the tail region, which is usually of the most interest for the reliability analysis. Once the distribution type of D_T is known or assumed, the reliability can be directly calculated. For example, if D_T follows the lognormal distribution, $\ln(D_T)$ follows the normal distribution with the mean and variance determined by

$$\begin{cases} \bar{\mu}_{D_T} = 2 \ln(\mu_{D_T}) - \frac{1}{2} \ln(\mu_{D_T}^2 + \sigma_{D_T}^2) \\ \bar{\sigma}_{D_T}^2 = -2 \ln(\mu_{D_T}) + \ln(\mu_{D_T}^2 + \sigma_{D_T}^2) \end{cases} \quad 7.34$$

The limit state function is defined as shown in Eq. 7.30 (or 7.31). The failure probability P_f is the damage exceedance probability, i.e.

$$P_f = P\left(\frac{D_T}{\Psi} \geq 1\right) = \Phi\left(\frac{\bar{\mu}_{D_T} - \ln(\Psi)}{\bar{\sigma}_{D_T}}\right) \quad 7.35$$

Following the lognormal assumption of the fatigue damage, the time-dependent reliability can be expressed as

$$\text{Reliability} = 1 - P_f = 1 - \Phi\left(\frac{\bar{\mu}_{D_T} - \ln(\Psi)}{\bar{\sigma}_{D_T}}\right) \quad 7.36$$

where Φ is the cumulative density function (CDF) of the standard Gaussian variable. $\bar{\mu}_{D_T}$ and $\bar{\sigma}_{D_T}$ have been determined by Eqs 7.32 and 7.33. ψ is the critical damage value determined by Eqs 7.10 and 7.11. Since the variable T is explicitly included in the mean and variance of the fatigue damage, the reliability calculated by Eqs 7.34–7.36 is time-dependent.

7.4.3 Method II: FORM approach

The proposed moments matching method described above needs to assume the type of probability distribution of the accumulated fatigue damage. In order to calculate the time dependent reliability without assuming the fatigue damage distribution, another approximation method is proposed based on the first-order reliability method (FORM). The limit state function is defined in Eq. 7.30. Therefore, the probability of failure P_f is defined through a multi-dimensional integral

$$P_f = \int \cdots \int_{g < 0} f_D(D_1(s), D_2(s), \dots, D_i(s)) dD_1(s) dD_2(s) \dots dD_i(s) \quad 7.37$$

where $f_D(D_1(s), D_2(s), \dots, D_i(s))$ is the joint probability density function for the basic random variables $D_1(s), D_2(s), \dots, D_i(s)$ and the integral is performed over the failure region, that is, $G < 0$.

In general, the multi-dimensional integral is difficult to evaluate. Various analytical methods have been developed to estimate the value of integral in Eq. 7.37. The FORM approach transforms all the random variables to an uncorrelated standard normal space, finds the minimum distance from the limit state to the origin, and estimates the failure probability based on the minimum distance. The minimum distance point on the limit state is also called the most probable point (MPP). The first-order failure probability estimate is computed as

$$P_f = \Phi(-\beta) \quad 7.38$$

where β , referred to as the reliability index, is the minimum distance from the origin to the MPP, and Φ is the cumulative distribution function of a standard normal variable. Various techniques can be used to find the most probable point (MPP). This study uses the recursive formula proposed by Rackwitz and Fiessler (1978) to search for the MPP. The FORM method is well established and details can be found in textbooks (Haldar and Mahadevan 2000).

The limit state function in Eq. 7.30 includes the variable T and thus is time-dependent. At each time instant, the failure probability can be computed using Eq. 7.38.

The moments matching method (Method I) assumes the probability

distribution of the accumulated fatigue damage D_T under variable amplitude loading and directly calculates the fatigue reliability. The use of this assumption makes the calculation very fast, which is a major advantage. The disadvantage is also introduced by this assumption. The damage quantity used in fatigue analysis is empirical and is hard to verify by experimental data. The FORM method does not assume the probability distribution of the accumulated fatigue damage D_T under variable amplitude loading. It only uses the statistics of the basic variables and calculates the joint failure probability. The advantage of the FORM approach is that it is more general and has fewer assumptions. Thus it can accommodate other types of distributions. The computational expense of the FORM approach is greater than that of the moments matching approach, because the FORM computation is iterative. For numerical fatigue reliability calculation under the continuous loading spectra, the external loading can be divided into many small segments, which results in many random variables. Under this condition, it is expected that the FORM approach will not be as efficient as the moments matching approach.

7.4.4 Fatigue reliability under non-stationary loading

The above discussion is only applicable to stationary loading since it only considers the cycle distribution of the applied loading. Estimation of fatigue damage accumulation under non-stationary loading is complicated compared to that under stationary loading. For non-stationary applied loading, the cycle distribution changes corresponding to time T . A general expression for the number of applied loading cycles $n_i(s)$ corresponding to the i th load level can be given as

$$n_i(s) = T f_i(s, T) \quad 7.39$$

If the non-stationary description of the applied loading (i.e., $f(s, T)$) is known, then the first two central moments of the fatigue damage D_T can be calculated. For continuous loading, the mean value and variance of the fatigue damage can be expressed as

$$\begin{cases} \mu_{D_T} = E\left(T \int_0^\infty f(s, T) D(s) ds\right) = T \int_0^\infty f(s, T) \mu_{D(s)} ds \\ \sigma_{D_T}^2 = \text{Cov}\left(T \int_0^\infty f(s, T) D(s) ds\right) \\ \quad = T^2 \int_0^\infty \int_0^\infty f(s_1, T) f(s_2, T) \sigma_{D(s_1)} \sigma_{D(s_2)} e^{-\lambda|s_1 - s_2|} ds_1 ds_2 \end{cases} \quad 7.40$$

For discrete loading, the mean value and variance of D_T can be expressed as

$$\left\{ \begin{array}{l} \mu_{D_T} = T \sum_{i=1}^{\infty} f(s_i, T) \mu_{D(s_i)} \\ \sigma_{D_T}^2 = T^2 \left[\sum_{i=1}^{\infty} f^2(s_i, T) \sigma_{D(s_i)}^2 + 2 \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} f(s_i, T) f(s_j, T) \sigma_{D(s_i)} \sigma_{D(s_j)} e^{-\lambda |s_i - s_j|} \right] \end{array} \right. \quad 7.41$$

For example, two-step loading is commonly used for variable loading tests under laboratory conditions. The material is first pre-cycled under stress level S_a for T_a cycles. Then the material is cycled till failure at another stress level S_b . The cycle distribution function $f(s, T)$ can be expressed for the two-step loading as

$$f(s, T) = \left\{ \begin{array}{ll} T \leq T_a & \begin{cases} 1 & s = S_a \\ 0 & s = S_b \end{cases} \\ T > T_a & \begin{cases} \frac{T_a}{T} & s = S_a \\ \frac{T - T_a}{T} & s = S_b \end{cases} \end{array} \right. \quad 7.42$$

For the moments matching approach, the first two central moments of the fatigue damage D_T can be expressed as

$$\left\{ \begin{array}{ll} \mu_{D_T} = \begin{cases} T \mu_{D(s_a)} & T \leq T_a \\ T_a \mu_{D(s_a)} + (T - T_a) \mu_{D(s_b)} & T > T_a \end{cases} \\ \sigma_{D_T}^2 = \begin{cases} T^2 \sigma_{D(s_a)}^2 & T \leq T_a \\ T_a^2 \sigma_{D(s_a)}^2 + (T - T_a)^2 \sigma_{D(s_b)}^2 + 2T_a(T - T_a) \sigma_{D(s_a)} \sigma_{D(s_b)} e^{-\lambda |s_a - s_b|} & T > T_a \end{cases} \end{array} \right. \quad 7.43$$

For the FORM approach, the limit state function can be expressed as

$$G = \left\{ \begin{array}{ll} \psi - TD(s_a) & T < T_a \\ \psi - T_a D(s_a) - (T - T_a) D(s_b) & T > T_a \end{array} \right. \quad 7.44$$

The time-dependent fatigue reliability can now be calculated following the same procedure described for stationary loading.

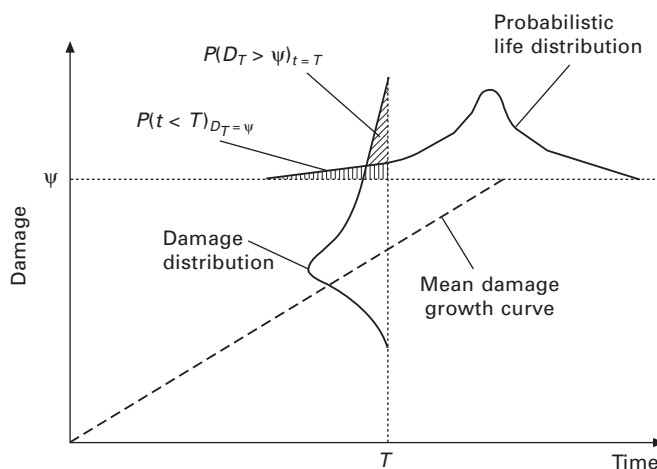
7.4.5 Time-dependent fatigue reliability and probabilistic life distribution

The proposed approximation methods are simple formulations for time-dependent fatigue reliability analysis. Using these methods, the reliability at time T can be calculated. Similarly, for a given reliability level (or probability of failure), the corresponding fatigue life of the material (i.e., time T) can also be calculated. Thus, the current formulation can also be used for probabilistic fatigue life prediction. The probability of fatigue damage being larger than a critical damage amount ψ at the time instant T is equal to the probability of fatigue life being less than at the time instant T when the fatigue damage is ψ . The relationship of time-dependent failure probability and the probabilistic fatigue life distribution is shown in Fig. 7.4 schematically. Mathematically, this relation is expressed as

$$P(D_T > \Psi)_{t=T} = P(t < T)_{D_T=\Psi} \quad 7.43$$

7.5 Demonstration examples

In this section, the discussed probabilistic methodologies above are demonstrated using both numerical and experimental examples. The first example is a simple numerical example under two-block loading. Parametric study is performed to investigate the effects of different approaches, and the model predictions are verified with direct Monte Carlo simulation results. The second considers continuous random external loading and demonstrates



7.4 Schematic illustration of probabilistic fatigue life distribution and time-dependent fatigue reliability.

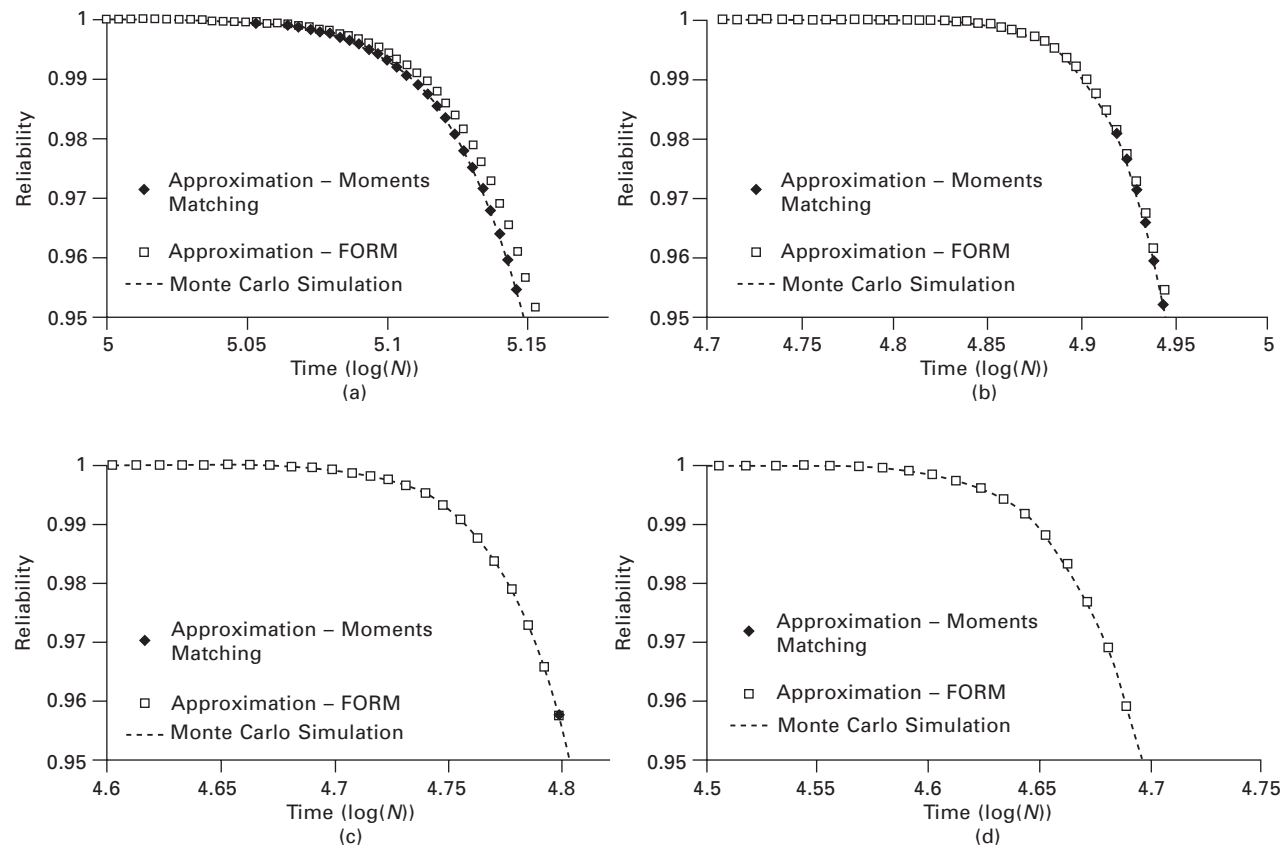
the application of the proposed methodology to a realistic situation. The third example is a validation example using measured fatigue failure data of composite laminates.

7.5.1 Numerical example 1

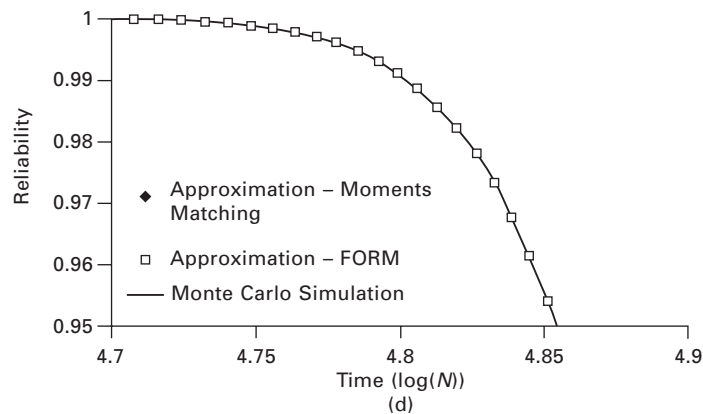
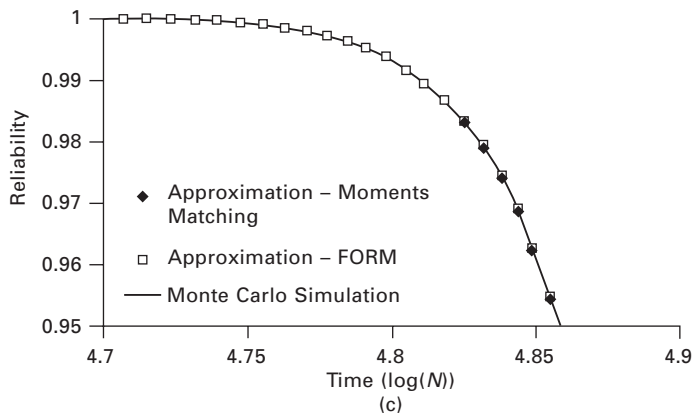
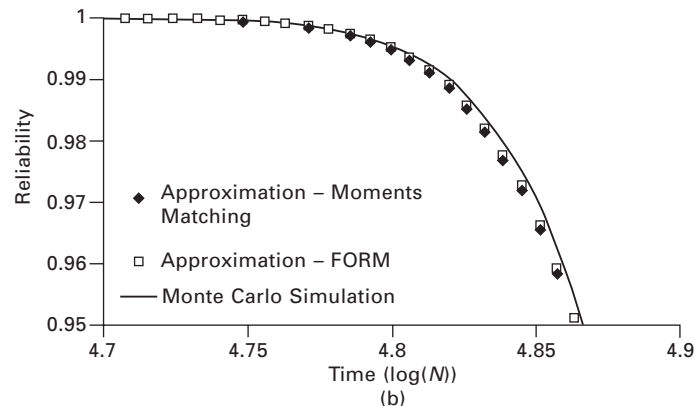
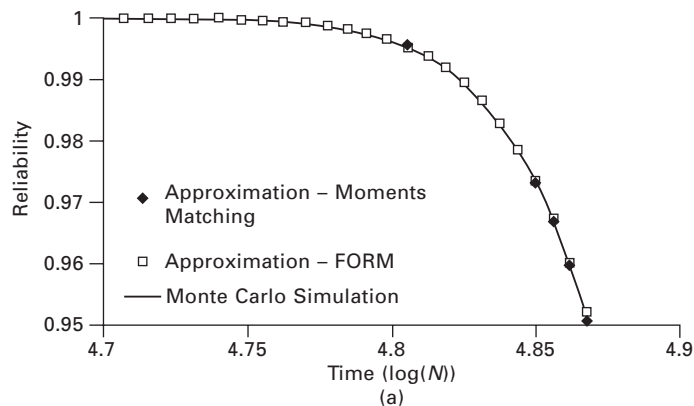
A numerical example is calculated and compared with direct Monte Carlo simulation to show the accuracy of the discussed moments matching approach and FORM method. Consider a two-block variable amplitude loading ($S_1 = 666$ MPa and $S_2 = 478$ MPa). The means of single cycle damage at the two stress levels are $\text{Mean}(D(S_1)) = 1.89\text{E-}05$ and $\text{Mean}(D(S_2)) = 2.44\text{E-}06$ using constant amplitude loading at each individual stress level. The standard deviations of single cycle damage at the two stress levels are $\text{Std}(D(S_1)) = 3.16\text{E-}06$ and $\text{Std}(D(S_2)) = 5.72\text{E-}07$. The Monte Carlo simulation uses 10^6 samples at each time instant and is assumed to be the exact solution. Both the moments matching method and the FORM method are used to calculate the fatigue reliability and are compared with the direct MC simulation. Parametric studies are performed to investigate the effect of the cycle fraction at each stress level and the correlation coefficient between single cycle damage at the two stress levels. The cycle fraction effects are compared in Fig. 7.5 for four different cycle fractions of S_1 with the correlation coefficient fixed at zero. The correlation effects are compared in Fig. 7.6 for four different correlation coefficients with the cycle fraction fixed at 0.5. In this numerical example, the input random variables are assumed to follow a lognormal distribution. It is shown that the approximation for the lognormal distribution is very accurate. Overall, the moments matching method and the FORM method give very good approximations.

7.5.2 Numerical example 2

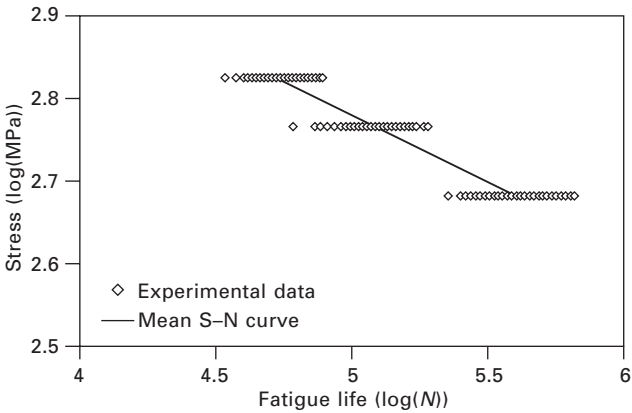
A numerical example is shown here to demonstrate the applicability of the proposed method to a continuous random load spectrum. Fatigue S–N testing under constant amplitude load is shown in Fig. 7.7. The statistics of measured fatigue life are listed in Table 7.1. Suppose the material is under stationary continuous random load spectrum. Rain-flow counting is used to get the cycle distribution of the external loading. Here it is assumed to follow the lognormal distribution with a mean value of 600 MPa and a standard deviation of 30 MPa. In the numerical calculation, the continuous cycle distribution is divided into 30 equal segments. The cycle distribution is plotted in Fig. 7.8(a). The prediction results using the moments matching approach, the FORM approach and the direct Monte Carlo simulation approach are plotted together in Fig. 7.8(b). The results of all three methods are in very close agreement. The computational time for the moments matching approach, the



7.5 Effects of cycle distribution using the moments matching approach. Cycle distribution at the first stress level: (a) 0.2; (b) 0.4; (c) 0.6; (d) 0.8.



7.6 Effects of correlation using FORM. Correlation coefficient between the two stress levels: (a) 0.0; (b) 0.4; (c) 0.6; (d) 1.0.



7.7 Constant amplitude S–N curve data for the numerical example.

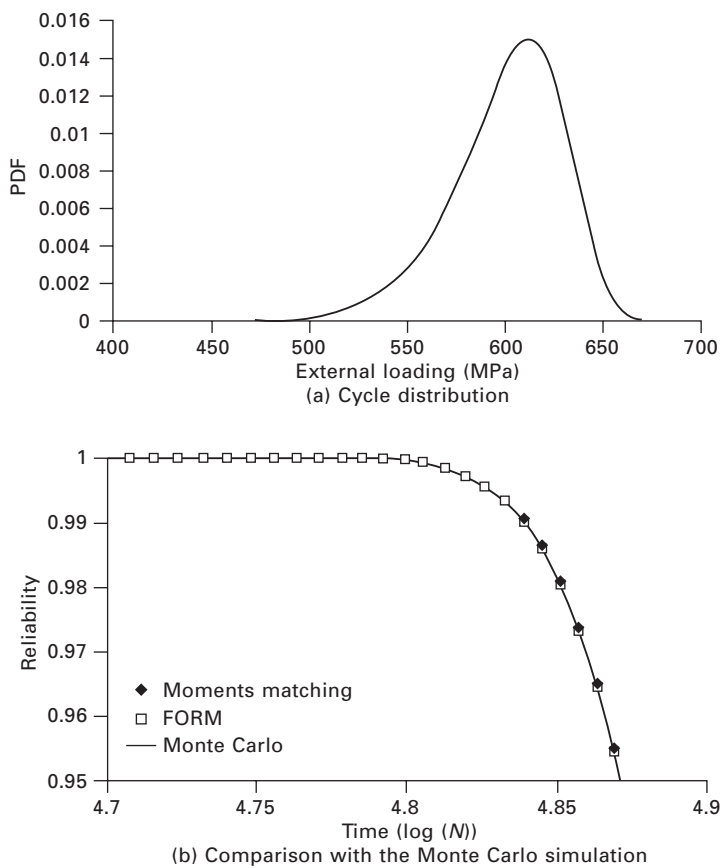
Table 7.1 Statistics of constant amplitude S–N curve data – numerical example

Stress amplitude (MPa)	Statistics of single cycle fatigue damage (1/N)		
	Mean	Std.	Distribution
478	2.44E-06	5.72E-07	Lognormal
583	8.32E-06	1.79E-06	Lognormal
666	1.89E-05	3.16E-06	Lognormal

FORM approach and the direct Monte Carlo simulation approach are 0.3 s, 1.4 s and 425 s, respectively. The computer has 3 GB memory and a 2.4 GHz dual-core processor. The operating system is Windows XP Pro and the computation algorithm was performed in Matlab 2007(a).

7.5.3 Experimental validation

In this section, the discussed probabilistic method is applied to composite materials for fatigue reliability calculation. Since both the moments matching method and the FORM method yield similar solutions, only the solution using the FORM method is discussed here. The material used is a fiberglass composite laminate (Mandell and Samborsky 2003). Constant amplitude fatigue S–N data and their statistics are shown in Fig. 7.9 and Table 7.2, respectively. Time-dependent reliability under variable loading (two-block) is plotted together with empirical cumulative density function of experimental data in Fig. 7.10. It is observed that the discussed probabilistic method gives a very good prediction of probabilistic life distribution of the composite laminate. In Section 7.2, we have discussed the use of the nonlinear damage accumulation rule. The predicted Miner’s sums under variable amplitude

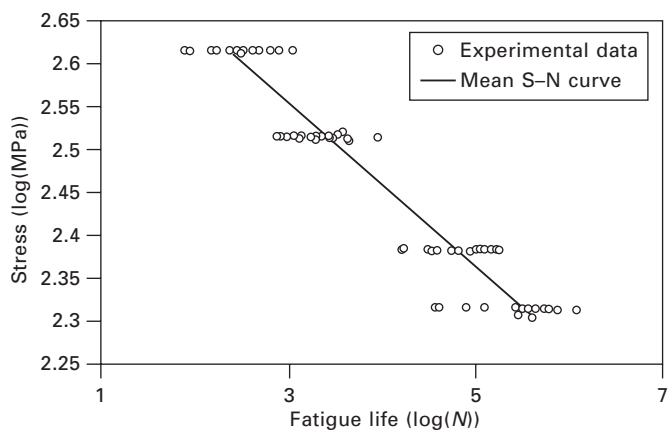


7.8 Comparison between the moments matching approach and the FORM approach for continuous loading.

loading are compared with experimental results for the composite material in Fig. 7.11. From Fig. 7.11, it is shown that the proposed method gives a better prediction compared to the LDAR for two loading cases. If the LDAR is used, very non-conservative results will be obtained since the Miner’s sum is well below unity under some loading conditions.

7.6 Conclusion

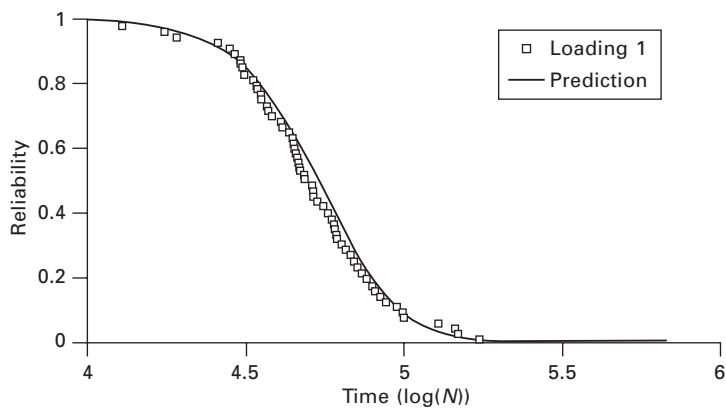
Two efficient fatigue reliability calculation methods are discussed in this chapter and applied to composite material fatigue analysis. They are based on a stochastic process representation of the material properties under constant amplitude loading and a non-linear damage accumulation rule. In the moments matching approach, the fatigue damage under variable amplitude



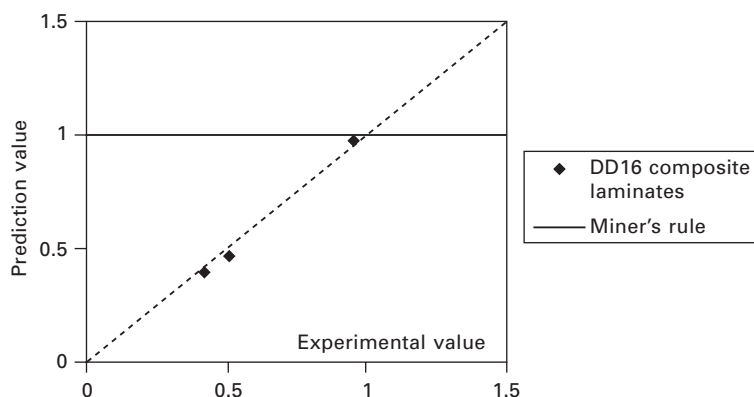
7.9 Constant amplitude S–N curve data for DD16 composite laminates.

Table 7.2 Statistics of constant amplitude S–N curve data – DD16 composite laminates

Stress amplitude (MPa)	Statistics of single cycle fatigue damage (1/N)		
	Mean	Std.	Distribution
206	5.48E-06	7.17E-06	Lognormal
241	1.97E-05	1.97E-05	Lognormal
328	0.000615	0.000362	Lognormal
414	0.004569	0.003278	Lognormal



7.10 Time-dependent reliability variation comparisons between prediction and experimental results.



7.11 Comparisons between predicted and experimental Miner's sum for different materials.

loading is assumed to follow the lognormal distribution, and the first two central moments are determined analytically without approximation. This results in a simple analytical solution for either the probability distribution of the service time to failure (fatigue life) or the probability distribution of the amount of damage at any service time. In the FORM approach, no assumption is made for the damage distribution under variable amplitude loading and the statistics of the basic variables are used together with the first-order reliability method.

The methods discussed above are very efficient in calculating the time-dependent reliability variation under cyclic fatigue loading compared to the simulation-based approaches. The methods also include the correlation effect of the damage accumulation under variable amplitude loading, which has been mostly ignored in previous models. Currently available models in the literature are shown to be two special cases of the proposed approach, i.e. independent random variables and fully correlated random variables. The discussed methodology has been demonstrated using numerical examples and validated using experimental data under deterministic variable amplitude loading. Due to the general format and the simplicity of the calculation, the discussed probabilistic methodology may be conveniently applied to composite structures and materials.

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Fatigue life prediction of composite materials based on progressive damage modeling

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Abstract: The main goal of this research is the fatigue life prediction of cross-ply laminated composites under cyclic tension–tension loading conditions. For this purpose, a progressive fatigue damage model is developed based on two main assumptions. First, it is assumed that off-axis plies of a cross-ply laminate are responsible for stress redistribution into the laminate. Secondly, it is assumed that on-axis plies of a cross-ply laminate are responsible for strength reduction and final failure of the laminate. Results predicted by the model show a very good correlation with provided experimental data in this research.

Key words: progressive fatigue damage modeling, carbon/epoxy, cross-ply composites, stiffness degradation, strength degradation.

8.1 Introduction

Questions about the reliability and lifetime performance of advanced polymeric composites should be answered comprehensively in order to enable their extensive use as primary structural materials. In isotropic materials, failure under static and fatigue loading condition is usually governed by initiation and growth of a single dominated crack. However, in composite materials many cracks in different plies are responsible for fatigue failure. Also, damage is initiated with matrix cracking and proceeds with other modes of failure such as longitudinal cracks, delamination and fiber breakage. All these modes of failure should be considered in fatigue damage modeling. By considering these complexities, a large number of fatigue models are presented in the literature to simulate the fatigue behavior of composites, e.g. Sendekyj (1990) and Degrieck and Van Paepegem (2001). The wide variation of analysis methods used to evaluate the reliability and life of these materials demonstrates the complexity of damage and failure mechanisms. Available fatigue models for composite materials are usually classified in three major categories: fatigue life models which use S–N curves or Goodman-type diagrams; phenomenological residual stiffness and strength models; and finally, progressive damage models which consider one or more

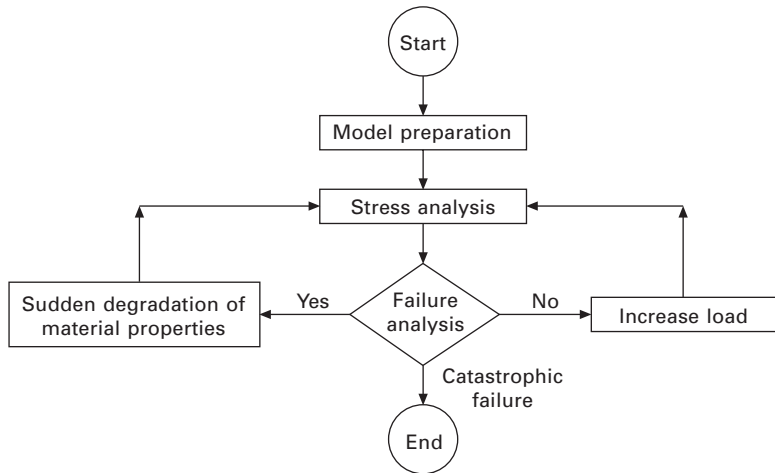
measurable physical damage variables such as transverse matrix crack density and interlaminar delamination size.

The calculation of the fatigue life of the entire laminate as a function of the number of cycles is the purpose of progressive fatigue damage modeling of laminated composites. None of the mentioned models can be used separately to predict the fatigue life. For this reason, an appropriate progressive algorithm that uses the static and fatigue properties of a lamina as a building block of the laminate must be developed. The model should be able to predict the damage growth inside the laminate in a ply-by-ply way. To describe progressive fatigue damage modeling, it is helpful to explain the traditional progressive static damage modeling technique used in failure analysis of composite laminates under static loading.

8.2 Progressive damage modeling under static loading

The idea of progressive damage modeling of laminated composites under static loading conditions was initially proposed by Chou *et al.* (1976). Chou *et al.* did not utilize a set of failure criteria capable of distinguishing between different failure modes. Therefore, after failure detection, all material properties of the failed region were diminished. The idea of detection of different failure modes by a set of discrete failure criteria was initially proposed by Hashin (1980b), and was successfully utilized by other investigators in the progressive damage modeling approach of composite laminates under static loading conditions, e.g. Chang and Chang (1987) and Lessard and Shokrieh (1995).

In general, a progressive damage model is a combination of stress analysis, failure analysis and material degradation rules. The model can be clearly described by means of the flowchart shown in Fig. (8.1). Initially, the finite element model is prepared and material properties, appropriate boundary conditions and a load increment is defined. Then, a stress analysis at a selected load increment is performed and on-axis stresses for each ply are calculated and used for failure analysis. By performing failure analysis, the existence of sudden modes of failure for all plies is checked. If the failure criteria are not met, there is no failure at this load increment. The load is subsequently increased and the stress and failure analyses are performed for all plies. If there is failure in a certain ply, the material properties of this ply are changed by using 'sudden death' material property degradation rules (Lessard and Shokrieh, 1991). After degrading the material properties, a new stiffness matrix is established for the finite element model. Thereafter, the stress analysis step is repeated, and the entire loop in the algorithm is continued. Therefore, if a lamina undergoes failure, not only are all appropriate material properties reduced, but the stress state is also relaxed and recalculated for the entire



8.1 Flowchart of progressive damage model used for static loading conditions.

model. As the load level increases, the laminated composite cannot tolerate any more load increment and the final failure load and failure mechanism can be estimated.

8.3 Progressive fatigue damage modeling

The major difference between progressive fatigue damage modeling and the traditional progressive damage modeling is the existence of the gradual material property degradation which occurs during the fatigue loading. Under fatigue loading conditions, the material is loaded by a stress state which is less than the maximum strength of the material; therefore, there is no static mode of failure. However, by increasing the number of cycles, the material properties degrade, and eventually catastrophic failure occurs. The idea of using static failure criteria to predict the life of a composite ply under fatigue loading has been utilized by many investigators (Rotem and Hashin, 1976; Rotem and Nelson, 1981; Rotem, 1982; Hashin, 1981a, 1981b; Sims and Brogdon, 1977; Hahn, 1979; Ellyin and El-Kadi, 1990; Kawai, 2004). The residual strength as a function of number of cycles has been used in the denominators of failure criteria instead of the static strength of the material. The application of their models, in practice, has some restrictions as explained in the following.

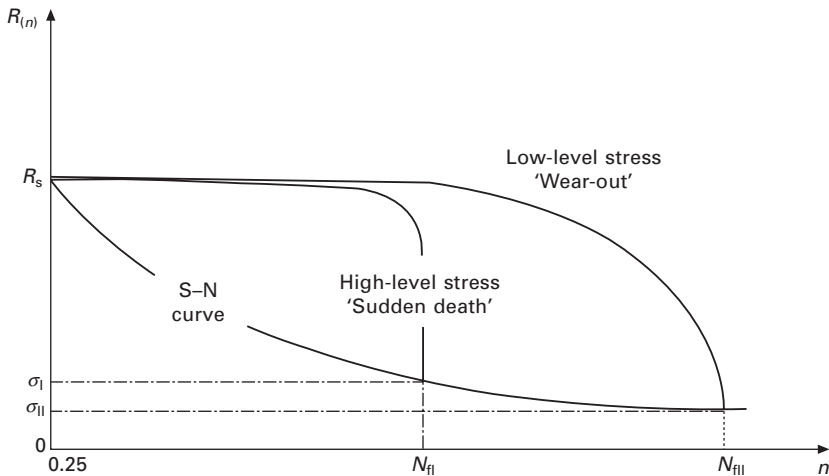
To show the restriction of application of fatigue failure criteria in traditional forms, consider the maximum stress criterion modified for fatigue loading conditions as the following:

$$\frac{\sigma_{xx}}{X_t(R, \sigma, n)} = g_f \text{ (if } g_f > 1, \text{ then failure occurs)} \quad 8.1$$

where σ_{xx} is the applied stress, and $X_t(R, \sigma, n)$ is the residual longitudinal tensile strength of a unidirectional ply under uniaxial fatigue loading, and is a function of n , σ and R , which are number of cycles, stress state and stress ratio, respectively.

The fatigue behavior of a composite lamina varies under different stress states. For instance, under a high stress level, the residual strength as a function of the number of cycles is nearly constant and decreases rapidly near the end of fatigue life as in the sudden death model proposed by Chou and Croman (1978, 1979). However, under low stress levels, the residual strength of the lamina degrades gradually with the number of cycles according to the wear-out model of Halpin *et al.* (1973). Figure 8.2 shows strength degradation as a function of number of cycles under two different stress states. In Fig. 8.2 σ_I and σ_{II} are stress levels, N_{fI} and N_{fII} are fatigue lives related to the stress levels, and $R_{(n)}$ and R_s are residual and static strength respectively.

Therefore, the residual tensile strength of a unidirectional ply, $X_t(R, \sigma, n)$, must be fully characterized under different stress levels and stress ratios, in order to apply Eq. 8.1. This requires a large number of experiments just to predict the fiber failure in tension fatigue of a unidirectional ply under simple uniaxial fatigue loading conditions. By considering the other modes of failure and the multiaxial states of stress which are encountered in the real fatigue design of composite structures, the proposed method presents severe difficulties which make its application possible only for specific conditions.



8.2 Strength degradation under different states of stress.

To overcome these difficulties many investigators (Rotem and Hashin, 1976; Rotem and Nelson, 1981; Rotem, 1982; Hashin, 1981a, 1981b; Sims and Brogdon, 1977; Hahn, 1979; Ellyin and El-Kadi, 1990) focused their research on specific stress ratios. This assumption is too restrictive for general cases. For example, in the analysis of a fatigue loaded composite laminate under fatigue loading, failure occurs in a region due to material property degradation. Therefore, the stress ratio and the stress state are not constant at different points. This means that in practice, stresses redistribute during the fatigue loading.

Reifsnider and coworkers (1986a, 1986b, 1990, 1993) proposed the approach of the representative volume concept which is further divided into critical and sub-critical elements. The critical elements in the representative volume are responsible for final failure and control the strength of the laminate. In the sub-critical elements damage initiation and propagation are modeled on a micromechanical level and the local stress fields are calculated.

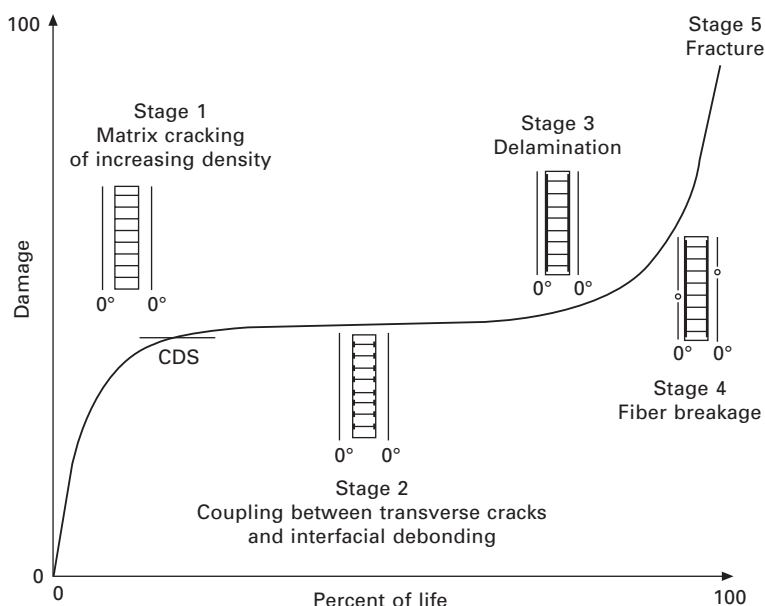
Hashin-type static failure criteria were developed to predict the fatigue failure in different damage modes in a progressive fatigue damage model proposed by Shokrieh and Lessard (1997a, 1997b, 2000a, 2000b). The so-called generalized residual material property degradation model to simulate the material property (stiffness and strength) degradation in material directions (in-plane and out-of-plane) of a unidirectional ply under a multiaxial state of stress and arbitrary stress ratio was also introduced.

Shokrieh's model is a general one and is capable of life prediction of various laminates under general loading conditions. However, it needs a large amount of experimental data to predict the fatigue life of a laminate. In order to limit the number of experiments needed for modeling, a new strategy is presented in the following section.

8.4 Problem statement and solution strategy

It has been observed from experiments (Charewicz and Daniel 1986; Reifsnider, 1990) that fatigue damage growth in cross-ply laminates consists of (a) transverse matrix cracking followed by longitudinal matrix cracks (stages 1 and 2), (b) local delamination between adjacent layers (stage 3), and (c) fiber breakage of the 0° plies (stages 4 and 5) as schematically shown in Fig. 8.3. The local stress state in each lamina changes continuously due to damage accumulation. Especially, matrix cracking of 90° plies reduces the stiffness and consequently load is redistributed and transferred to the 0° plies. Such changes of material properties and redistributions of local stresses have to be addressed in order to represent the physical process involved in fatigue failure, and to predict the fatigue life of a composite laminate.

The overall objective of the present research is the development of a progressive fatigue damage model to simulate stiffness degradation and



8.3 Fatigue damage growth of cross-ply laminates (Reifsnider, 1990).

prediction of the fatigue life of cross-ply laminates under cyclic tension–tension loading conditions. The model is developed based on two main assumptions which were initially proposed by Reifsnider and Stinchcomb (1986b). Firstly, it is assumed that the 90° plies of a cross-ply laminate are responsible for stress redistribution. Secondly, it is assumed that the 0° plies of a cross-ply laminate are responsible for strength reduction and final failure of the laminate. The developed model comprises three fundamental parts: stress analysis, gradual stiffness and strength degradation, and failure analysis.

Many investigators have shown that the intralaminar crack density increases with number of fatigue cycles or static load up to a certain limit, which is called the characteristic damage state (CDS) (Reifsnider and Masters, 1982). In this research a simple shear–lag analysis is integrated into the model to find out the magnitude of stiffness of 90° plies related to the CDS level. The gradual stiffness degradation of this phase with number of cycles is calculated by using a semi-empirical model. During the second phase, as described above, delamination initiated between layers and the entire stiffness of the laminate is reduced linearly. During this phase, it is assumed that the 90° plies cannot carry any load, so a sudden death rule is used to reduce the stiffness of the 90° plies to zero. After this point, stiffness degradation is related to longitudinal cracks and fiber breakage which occurs in the on-axis (0°) plies. It should be mentioned that the stiffness degradation which is related to the longitudinal cracks and fiber breakage in on-axis plies will

not change the stress state inside the laminate because the entire laminate is loaded externally under a cyclic constant amplitude loading. Therefore, stiffness degradation of on-axis plies increases the strain of the laminate instead of stress. Strength degradation of 0° plies is calculated by using a normalized model, which is capable of predicting the strength degradation of the unidirectional laminates under various stress states. The model considers all conceivable failure modes in the loading direction, such as longitudinal cracks and fiber breakage to simulate gradual degradation of strength. Hence, stiffness degradation due to longitudinal cracks and fiber breakage is indirectly considered in this approach. In the following sections, the essential elements of the progressive fatigue damage modeling strategy are explained in detail.

8.5 Gradual material property degradation

The scenario of material property degradation of a unidirectional ply under static and fatigue loading conditions is different before occurrence of sudden failure. In a laminated composite under static loading conditions the load is increased monotonically and at a certain load level failure initiation in a ply of the laminate is detected by the static failure criteria. At this stage, the mechanical properties of the failed ply of the laminate must be changed to almost zero based on sudden death rules (Lessard and Shokrieh, 1991). However, in a laminated composite, the strength of the plies can be higher than the stress state in the first cycle under fatigue loading conditions. Thus, during the first cycles, the proposed fatigue failure criteria (as Eq. (8.1)) do not detect any failure mode. However, by increasing the number of cycles, the material properties of each ply are degraded. This type of degradation is called gradual material property degradation. By further increasing the number of cycles, the mechanical properties of the plies eventually reach a level where different modes of failure can be detected by the proposed fatigue failure criteria (as Eq. (8.1)). At this stage, the mechanical properties of the failed material are changed by sudden material property degradation rules.

To apply the fatigue failure criteria (as Eq. 8.1), the residual material properties of a unidirectional ply (strength in the longitudinal direction and stiffness in the transverse direction) must be modeled under an arbitrary multiaxial state of fatigue stress and stress ratio. For this purpose, gradual strength degradation of on-axis plies and gradual stiffness degradation of off-axis plies will be explained hereafter.

8.5.1 Gradual strength degradation model

As explained previously, the on-axis plies control the state of the material and final failure in a cross-ply laminate. Knowing the gradual strength degradation

of these plies when they are subjected to fatigue loading is a necessary step in life prediction. In order to predict the gradual strength degradation of a certain laminate under various stress ratios and stress states, an interaction model is proposed by Adam *et al.* (1986) as follows:

$$t^x + r^y = 1 \quad 8.2$$

where the exponents x and y are parameters with no clear physical interpretation and are derived by fitting Eq. 8.2 to the residual strength experimental data. In order to have a trustworthy fit, residual strength tests must be performed at a variety of life fractions.

Parameters t and r are normalized quantities varying from 0 to 1 and are defined by the following equations:

$$r = \frac{R - \sigma}{R_s - \sigma} \quad 8.3$$

$$t = \frac{\log n - \log 0.5}{\log N_f - \log 0.5} \quad 8.4$$

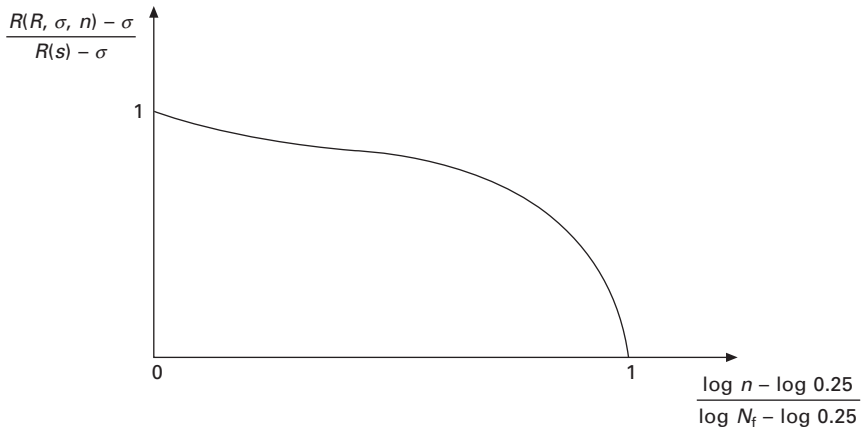
where R = residual strength, R_s = static strength, σ = maximum applied stress, n denotes the number of applied cycles and N_f is the fatigue life at the applied stress. The proposed model requires a relatively small experimental effort for the determination of the two parameters, once the static strength (R_s) and the S–N curve of the material are known.

Shokrieh and Lessard (1997a, 1997b) developed this model to predict the gradual strength degradation in material directions of a unidirectional lamina under various stress ratios. The following relation was established:

$$R(R, \sigma, n) = \left[1 - \left(\frac{\log n - \log 0.25}{\log N_f - \log 0.25} \right)^\beta \right]^{\frac{1}{\alpha}} (R_s - \sigma) + \sigma \quad 8.5$$

where $R(R, \sigma, n)$ is the residual strength in fiber, matrix and in-plane shear directions, and β and α are curve-fitting parameters. By using a normalized residual strength degradation model such as Eq. 8.5, all the different curves under various stress states and stress ratios collapse to a single curve as schematically shown in Fig. 8.4.

It is worth mentioning that Eq. 8.2 basically was developed for a composite laminate under a uniaxial state of stress. In other words, Eq. 8.2 is capable of predicting the residual strength of a certain laminate under tension–tension loading for each combination of arbitrary stress state and stress ratio, whereas Eq. 8.5 is an application of Eq. 8.2 for a lamina under uniaxial stress states. For this reason, the applied stress (σ) in this equation is the stress component in the loading direction, whereas in practice each lamina of the laminate is



8.4 Gradual strength degradation in material directions.

under a multiaxial state of stress. To overcome this problem, Shokrieh and Lessard (1997a, 1997b) combined a Hashin-type failure criterion along with a complete set of gradual strength degradation rules for all material directions to find out residual strength and fatigue life. These kinds of models are very complex and expensive because of the large amount of experimental data they need to find out the constant parameters of the models.

In other words, it is obvious that the amount of load carried by the off-axis plies (90°) is much less than the load carried by the on-axis plies (0°) in general cross-ply laminates. For this reason, even by assuming strength degradation for the transverse direction, its effect on life prediction is negligible. In fact, considering strength degradation in the transverse and in-plane directions in comparison with the longitudinal direction is not useful for cross-ply laminates. For this reason, the established model in this research considers stiffness degradation in the transverse direction and strength degradation in the longitudinal direction. Equation 8.5 cannot be used in its present form to predict the strength degradation in the longitudinal direction because of the existence of a multiaxial state of stress in each layer. An alternative equation is therefore proposed using the normalized stress components as:

$$Fr = \left[1 - \left(\frac{\log n - \log 0.25}{\log N_f - \log 0.25} \right)^\beta \right]^{\frac{1}{\alpha}} (1 - Fa) + Fa \quad 8.6$$

where $Fr = \frac{R(R, \sigma, n)}{R_s}$ is the dimensionless residual strength and $Fa = \frac{\sigma}{R_s}$ is a local failure function.

The local failure function Fa can be calculated by the energy-based static failure criterion proposed by Sandhu *et al.* (1982) as follows:

$$Fa = Fa(\sigma_i, \epsilon_i) = \frac{\sigma_{1\max} \epsilon_{1\max}}{X \epsilon_{1u}} + \frac{\sigma_{2\max} \epsilon_{2\max}}{Y \epsilon_{2u}} + \frac{\sigma_{6\max} \epsilon_{6\max}}{S \epsilon_{6u}}, i=1, 2, 6 \quad 8.7$$

where $\sigma_{1\max}$, $\sigma_{2\max}$, $\sigma_{6\max}$, $\epsilon_{1\max}$, $\epsilon_{2\max}$ and $\epsilon_{6\max}$ are applied stress and strain components, and X , Y , S , ϵ_{1u} , ϵ_{2u} and ϵ_{6u} are the ultimate static stress and strain components in material directions.

In Eq. 8.6, the number of cycles to failure (N_f) is a function of the state of stress (σ) and the stress ratio ($R = \sigma_{\min}/\sigma_{\max}$). Therefore, a suitable relationship between the fatigue life (N_f), state of stress and stress ratio is needed to simulate the residual strength of a unidirectional ply under a general fatigue loading (arbitrary state of stress and stress ratio). Some investigators (Adam *et al.*, 1986, 1994; Shokrieh and Lessard (1997a, 1997b) employed Goodman-type diagrams to add this ability to their models. In the current research, another method, called the unified fatigue life model, is developed that needs less experimental data and has more accuracy to simulate strength degradation in higher stress states. This model is explained in the next section.

8.5.2 Unified fatigue life model

Constant life (Goodman-type) diagrams are used widely to take into account the effect of mean stress on the fatigue life of isotropic and composite materials (Boller, 1957). An analytical method has been proposed by Adam *et al.* (1989) to present all data from a constant life diagram to a single two-parameter fatigue curve, which can reduce the number of needed experiments drastically. Later, Adam *et al.* (1992) and Gathercole *et al.* (1992) introduced an empirical power law model that produces a bell-shaped curve, which corresponds more closely to the material behavior under fatigue loading conditions as follows:

$$a = f(1 - q)^u (c + q)^v \quad 8.8$$

where $a = \frac{\sigma_a}{\sigma_t}$, $q = \frac{\sigma_m}{\sigma_t}$, $c = \frac{\sigma_c}{\sigma_t}$, $\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$ = alternating stress, $\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$ = mean stress; f , u and v are curve fitting constants; σ_c is compressive strength, σ_t is tensile strength and $\sigma_{\max}$, $\sigma_{\min}$ = maximum and minimum of applied stress.

In a paper by Gathercole *et al.* (1994), it was shown that the exponents u and v determine the shapes of the left and the right wings of the bell-shaped curve. They assumed u and v are equal and are linear functions of the logarithm of fatigue life (N_f):

$$u = v = A + B \log N_f \quad 8.9$$

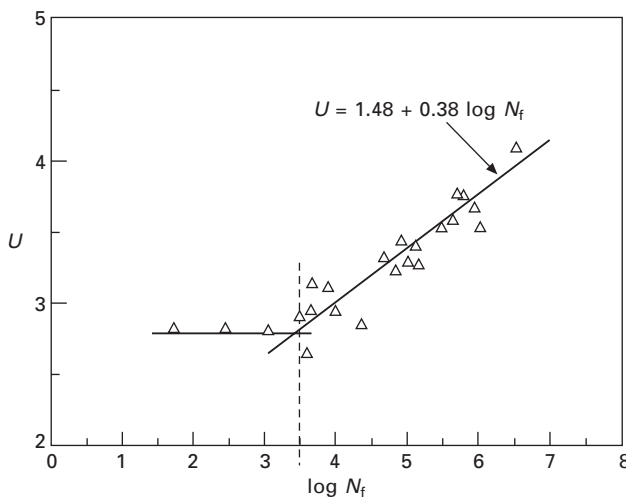
where A and B are the curve-fitting constants. Then by substituting Eq. 8.9 into Eq. 8.8 and rearranging it, the following equation is derived:

$$\frac{\ln(a/f)}{\ln[(1-q)(c+q)]} = A + B \log N_f \quad 8.10$$

It should be mentioned that Eq. 8.10 is fundamentally based on some important assumptions, including that (a) u and v are equal and are linear functions of the logarithm of the fatigue life, $\log(N_f)$, (b) parameter f is set to be 1.06 based on available experimental data on certain materials and lay-ups, (c) the model is developed only for tension–tension fatigue conditions, and (d) Eq. 8.10 is not valid for lives less than almost 2000 fatigue cycles, based on the authors' experimental data on $[(\pm 45/0_2)_2]_s$ of T800/5245 carbon/epoxy laminates. The relation between $u = \ln(a/f)/\ln [1 - q)(c + q)]$ versus $\log N_f$ is shown in Fig. 8.5, from which A and B are found (Adam *et al.*, 1989).

Shokrieh and Lessard (2000a, 2000b) employed Eq. 8.10 in their progressive damage model to predict the fatigue life for each arbitrary stress state and stress ratio. They performed comprehensive experimental work to find constant parameters such as A , B and f for all material directions under general loading conditions. But in this research, because of the above-mentioned shortcomings of Eq. 8.10 and the large amount of experimental work needed to obtain constant parameters, another life prediction method is used as explained in the following section.

When S–N curves are employed to predict the fatigue life, maximum stresses or amplitudes of stresses are selected as fatigue measures and plotted



8.5 Predicting parameter U for $[(\pm 45/0_2)_2]_s$ of T800/5245 carbon/epoxy laminates (Adam *et al.*, 1992).

versus fatigue life. It has been shown by many researchers, e.g. Hashin and Rotem (1973) and Awerbuch and Hahn (1981), that selecting a suitable fatigue measure can strongly decrease the number of tests needed to predict the fatigue life.

In research presented by Shokrieh and Taheri-Behrooz (2006), the energy-based static failure criterion developed by Sandhu *et al.* (1982) for composite materials is extended for fatigue loading and introduced as a new fatigue measure to predict the fatigue life of a lamina under various stress ratios. The new fatigue measure in the on-axis coordinate system can be written as follows:

$$\Delta W^* = \Delta W_I^* + \Delta W_{II}^* + \Delta W_{III}^* = \frac{\Delta \sigma_1 \Delta \epsilon_1}{X \epsilon_{u1}} + \frac{\Delta \sigma_2 \Delta \epsilon_2}{Y \epsilon_{u2}} + \frac{\Delta \sigma_6 \Delta \epsilon_6}{S \epsilon_{u6}} \quad 8.11$$

where Δ before a symbol indicates its range and $\sigma_1, \sigma_2, \sigma_6$ and $\epsilon_1, \epsilon_2, \epsilon_6$ are stress and strain components in the material directions.

In the new fatigue model ΔW^* represents a sum of strain energy densities contributed by all stress components in material directions. It should be mentioned that in tension–tension loading $X = X_t, Y = Y_t$ and in compression–compression loading $X = X_c, Y = Y_c$. Total strain energy under cyclic loading can be shown by Eq. 8.12:

$$\Delta W = \frac{1}{2} (\Delta \sigma_1 \Delta \epsilon_1 + \Delta \sigma_2 \Delta \epsilon_2 + \Delta \sigma_6 \Delta \epsilon_6) \quad 8.12$$

The normalized strain energy density associated with the longitudinal direction may be written as Eq. 8.13:

$$\Delta W_I^* = \frac{1}{X \epsilon_{1u}} (\sigma_{1\max} \epsilon_{1\max} - \sigma_{1\min} \epsilon_{1\min}) \quad 8.13$$

The nonlinear responses of the unidirectional composites in the transverse direction and under shear loading are ignored in this model. Therefore, the stress–strain response is assumed to be linear in the material directions.

$$X = E_1 \epsilon_{1u} \rightarrow \epsilon_{1u} = \frac{X}{E_1}, \sigma_{1\max} = E_1 \epsilon_{1\max} \rightarrow \epsilon_{1\max} = \frac{\sigma_{1\max}}{E_1} \quad 8.14$$

By substituting Eq. 8.14 into Eq. 8.13:

$$\Delta W_I^* = \frac{1}{X^2} (\sigma_{1\max}^2 - \sigma_{1\min}^2) = \frac{1}{X^2} \frac{1+R}{1-R} (\Delta \sigma)^2 \quad 8.15$$

The on-axis stress components are found by transforming the off-axis stresses into the on-axis coordinates using a suitable transformation rule (Tsai, 1980).

For the uniaxial case $\sigma_x \neq 0$ and $\sigma_y = \sigma_s = 0$, stress components are obtained

as follows:

$$\begin{aligned}\sigma_{1\max} &= \sigma_{x\max} \cos^2 \theta \\ \sigma_{2\max} &= \sigma_{x\max} \sin^2 \theta \\ \sigma_{6\max} &= -\sigma_{x\max} \cos \theta \sin \theta\end{aligned}\tag{8.16}$$

By substituting Eq. 8.16 into Eq. 8.15:

$$\Delta W_I^* = \frac{1}{X^2} \frac{1+R}{1-R} (\Delta \sigma_x)^2 \cos^4 \theta\tag{8.17}$$

where θ is the fiber angle. Similarly the strain energy densities in the transverse direction and under shear loading are:

$$\begin{aligned}\Delta W_{II}^* &= \frac{1}{Y^2} \frac{1+R}{1-R} (\Delta \sigma_x)^2 \sin^4 \theta, \\ \Delta W_{III}^* &= \frac{1}{S^2} \frac{1+R}{1-R} (\Delta \sigma_x)^2 \sin^2 \theta \cos^2 \theta\end{aligned}\tag{8.18}$$

By substituting Eqs 8.17 and 8.18 into Eq. 8.11, a complete form of the new fatigue model is obtained as follow:

$$\begin{aligned}\Delta W^* &= \Delta W_I^* + \Delta W_{II}^* + \Delta W_{III}^* \\ &= \frac{1+R}{1-R} (\Delta \sigma_x)^2 \left(\frac{\cos^4 \theta}{X^2} + \frac{\sin^4 \theta}{Y^2} + \frac{\sin^2 \theta \cos^2 \theta}{S^2} \right)\end{aligned}\tag{8.19}$$

It should be mentioned that Eq. (8.19) is developed for different fiber load angles under positive stress ratios ($R \geq 0$) for both tension–tension and compression–compression cyclic loadings. This new fatigue model is capable of taking into account the effects of all the different modes of failure of an orthotropic lamina to predict the fatigue life.

The new fatigue measure ΔW^* is related to the fatigue life (N_f) by using the relation utilized by Ellyin and El-Kadi (1990) for composite materials as follows:

$$\Delta W^* = k N_f^\alpha\tag{8.20}$$

where k and α are material constants and are independent of the stress ratio and fiber orientation. These two constants are obtained for each material by using one set of the fatigue test data in an arbitrary stress ratio and a fiber orientation.

Instead of the normalized fatigue life model (Eq. 8.10) used by Shokrieh and Lessard (2000a, 2000b) and Adam *et al.* (1992, 1989), the unified

fatigue life model (Eqs 8.19 and 8.20) is used in this research to calculate the fatigue life for each combination of the stress ratio and the stress state. The unified fatigue life model is implemented in the progressive fatigue damage algorithm developed in this research to predict the fatigue life of cross-ply laminates under general loading conditions.

8.5.3 Gradual stiffness degradation model

The residual stiffness of the material is also a function of the state of stress and the number of cycles and degrades cycle-by-cycle as explained for the strength. The idea of normalizing the residual stiffness curves and establishing a master curve has been used by many authors (Boller, 1957; Shimokawa and Hamaguchi, 1983; Stinchcomb and Bakis, 1990).

To present the residual stiffness as a function of number of cycles in a normalized form, Eq. 8.5 was developed by Shokrieh and Lessard (1997a, 1997b), based on a similar idea as that used for residual strength in the previous section. The relation between the stiffness component degradation with the stress state, the number of cycles and the fatigue life is:

$$E(R, \sigma, n) = \left[1 - \left(\frac{\log n - \log 0.25}{\log N_f - \log 0.25} \right)^\lambda \right]^{\frac{1}{\gamma}} \left(E_s - \frac{\sigma}{\varepsilon_f} \right) + \frac{\sigma}{\varepsilon_f} \quad 8.21$$

where $E(R, \sigma, n)$ is the residual stiffness, E_s is the static stiffness, σ is the magnitude of the applied maximum stress, ε_f is the average strain to failure, and λ and γ are experimental curve-fitting parameters.

By using this normalization technique, all the different curves for different states of stress collapse to a single curve as explained for the strength (Fig. 8.4). N_f and ε_f are two critical parameters in Eq. 8.21. Both are measured experimentally; the former is obtained by using Eq. 8.10 and the latter is obtained by a static test in the transverse direction. But the calculated parameter u (Eq. 8.10) in the material direction is not valid for fatigue life $N < 2000$ (Adam *et al.*, 1986, 1989). This is more important where crack accumulation that leads to leakage is considered as the main failure mode of the structure, such as in pressure vessels. This limitation is solved in this research by using the unified fatigue life model, Eq. 8.20, in place of Eq. 8.10.

Another problem arises when Eq. 8.21 is used to obtain the stiffness degradation of a cross-ply laminate under high stress states. For instance, the tensile strength of [0/90₄]_s AS4/3501-6 carbon/epoxy laminate is 456 MPa (Lee *et al.*, 1990). According to classical lamination theory and the Tsai–Wu criterion, first-ply failure occurs at 208 MPa, which is only 45% of the external load. For aerospace applications, where weight is a critical

matter, components are designed according to their maximum capacity in terms of both the strength and the fatigue life.

Equation 8.21 simulates the stiffness reduction only if stress and strain in the transverse directions are less than the strength and the ultimate strain of the relevant lamina, respectively. In other words, this model reduces to the ply discount model if the stress in the transverse direction is more than its strength. But there is considerable information in the literature (Lee *et al.*, 1990; Wharmby *et al.*, 2003; Charewicz and Daniel, 1986) which shows that even under loads greater than the first-ply failure load, there is a gradual degradation in stiffness. In addition, strain in 90° plies becomes more than its ultimate static magnitude. To overcome this weakness, a shear-lag analysis is integrated in Eq. 8.21 under high stress state conditions.

Modification of the stiffness degradation model for high stress state conditions

In a cross-ply laminate under external load, transverse cracks are formed in 90° plies when the stress reaches the strength limit of these layers. By increasing the external load in a static loading condition and the number of cycles in a fatigue loading condition, crack spacing gets smaller up to a certain lower limit that is called the characteristic damage state (CDS). After this point, by adding more cycles or increasing the magnitude of the static load, new cracks will not develop any more. The CDS seems to be a laminate property. It depends only on the properties and thickness of the plies and their stacking sequence. In addition, it is similar for fatigue and static loading (Highsmith and Reifsnider, 1982). According to the simplest case of shear-lag analysis, stress, strain and stiffness are calculated for a damaged laminate as follows (Diao *et al.*, 1997; Wharmby *et al.*, 2003):

$$\sigma_{x1} = \frac{E_1}{E_0} \left\{ 1 + \frac{E_2 d \cosh(\zeta x)}{E_1 b \cosh(\zeta l/2)} \right\} \sigma_a \quad 8.22$$

$$\sigma_{x2} = \frac{E_2}{E_0} \left\{ 1 - \frac{\cosh(\zeta x)}{\cosh(\zeta l/2)} \right\} \sigma_a \quad 8.23$$

$$\epsilon_a = \frac{1}{l} \int_0^l \frac{\sigma_{x1}}{E_1} dx = \frac{\sigma_a}{E_0} \left(1 + \frac{2E_2 d}{\zeta l E_1 b} \tanh(\zeta l/2) \right) \quad 8.24$$

$$E_x = 1 + \frac{E_0}{1 + \frac{2E_2 d}{\zeta l E_1 b} \tanh(\zeta l/2)} \quad 8.25$$

$$E_{90} = \frac{1 - \frac{2}{\zeta l} \tanh(\zeta l/2)}{1 + \frac{2E_2 d}{\zeta l E_1 b} \tanh(\zeta l/2)} \quad 8.26$$

$$E_0 = \frac{b}{b+d} E_1 + \frac{d}{b+d} E_{21}, \quad \zeta^2 = \frac{3(d+b)E_0 G_{12} G_{23}}{bd E_1 E_2 (dG_{12} + bG_{23})} \quad 8.27$$

where σ_a is the applied laminate stress, ϵ_a is the average axial strain of the entire laminate, l is the matrix cracking space, E_1 and E_2 are the stiffness of zero and 90° plies, b and d are the half-thickness of 0° and 90° plies, E_0 is the stiffness of undamaged laminate, G_{12} and G_{23} are the shear moduli in the x - y and y - z planes, ζ is the shear-lag parameter, and E_{90} is the reduced stiffness in 90° plies.

Stiffness degradation versus crack density or crack density variation with static load is well studied for cross-ply laminates by various investigators both theoretically and experimentally (Highsmith and Reifsnider, 1982; Berthelot, 2003), whereas obtaining similar parameters for fatigue conditions requires a large quantity of tests for various stress states and stress ratios (Berthelot, 1999).

The present research uses two stages to simulate the entire stiffness reduction with number of cycles under high stress state conditions. In the first stage, classical lamination theory is used to calculate the stress distribution inside the laminate. Then, by using Eqs 8.22–8.27 rapid stiffness reduction and stress redistribution are calculated. In the second stage, a modified form of Eq. 8.21 is used to find the stiffness reduction during the fatigue life. In the modified form of Eq. 8.21, the fatigue life of a lamina is calculated by using Eq. 8.20 and stiffness is gradually reduced by using Eq. 8.28 to the strain value related to the CDS level. After this level a sudden death rule is used to reduce the stiffness and the strength of off-axis plies to nearly zero. The modified form of Eq. 8.21 has the following form:

$$E(R, \sigma, n) = \left[1 - \left(\frac{\log n - \log 0.25}{\log N_f - \log 0.25} \right)^\lambda \right]^{\frac{1}{\gamma}} \left(E_s - \frac{\sigma}{\epsilon_{CDS}} \right) + \frac{\sigma}{\epsilon_{CDS}} \quad 8.28$$

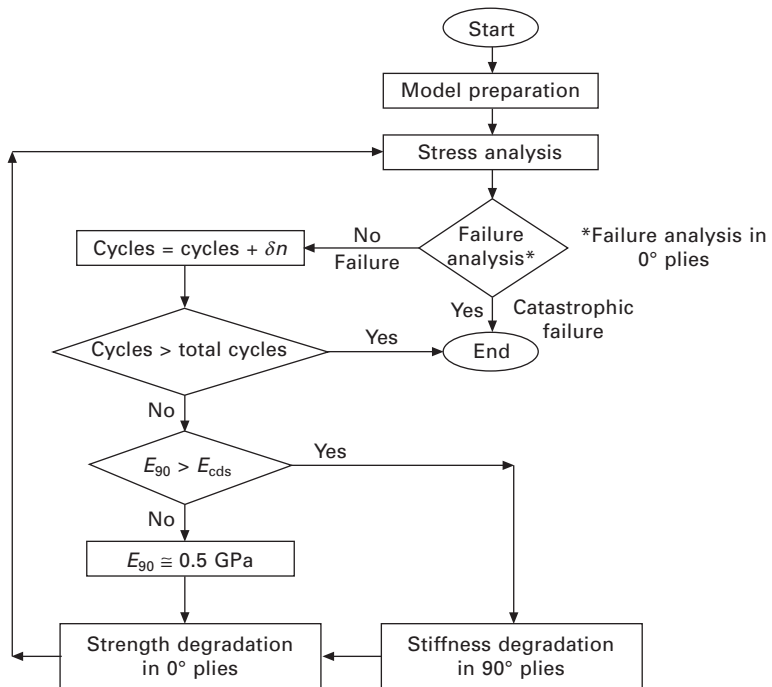
where $\epsilon_{CDS} = \frac{\sigma_{x2}}{E_{90}}$ is the strain related to the characteristic damage state.

Up to this point, a gradual strength degradation model, a gradual stiffness degradation model and a unified fatigue life measure, which are the three fundamental elements of progressive fatigue damage modeling of composites, have been explained in detail. Equations (8.6, 8.20 and 8.28) are integrated in a progressive damage algorithm to predict the residual strength and the fatigue life of a cross-ply laminate.

8.6 Framework of progressive fatigue damage modeling of cross-ply laminates

In this section, the framework of the progressive fatigue damage model of cross-ply laminates is established and explained in detail. The model is capable of simulating the residual strength, fatigue life and final failure mechanism of cross-ply composite laminates with arbitrary stress state, stress ratio and stacking sequence under tension–tension fatigue loading conditions using the results of uniaxial fatigue experiments on the longitudinal and transverse directions of unidirectional plies. A user-friendly computer code has been developed based on the algorithm that is presented in the flowchart of Fig. 8.6. The computer code is able to simulate the cycle-by-cycle behavior of a cross-ply laminate composite under tension–tension fatigue loading conditions and predicts the residual strength and the fatigue life.

The model preparation part, material properties, maximum and minimum fatigue load, maximum number of cycles, incremental number of cycles, etc., are defined. Then the stress analysis, based on the maximum and minimum fatigue load, is performed. Consequently, the maximum and minimum induced on-axis stresses of all layers are calculated and the stress ratio for



8.6 Flowchart of progressive fatigue damage modeling of cross-ply laminates.

each layer is determined. In the next step, failure analysis is performed and the maximum stresses are examined by Eq. 8.1 for on-axis plies. If there is a sudden mode of failure, the program reports the catastrophic failure and fatigue life of the laminate is reached. In this step, if there is no sudden mode of failure, an incremental number of cycles are applied (e.g., $\delta n = 100$). If the number of cycles is greater than a preset total number of cycles, the computer program stops. Otherwise, transverse stiffness of off-axis plies and longitudinal strength of on-axis plies are changed according to gradual material property degradation rules using Eqs 8.6–8.28. Then, stress analysis is performed again and the above loop is repeated until catastrophic failure occurs, or the maximum number of cycles (predefined by the user) is reached. The fatigue life of the laminate under examination is estimated when catastrophic failure is reached. On the other hand, the residual strength of the examined material can be estimated by using a suitable strength degradation model of the on-axis plies, if the program is finished before failure of the laminate. The model is validated with experimental data in the following paragraphs.

8.7 Required experiments

In order to use the progressive fatigue damage model for studying the fatigue behavior of composite materials, a complete set of material properties (stiffness, strength and fatigue life) of a unidirectional ply under static and fatigue loading conditions is required. The experiments are designed to determine the experimental parameters needed by the model explained in previous sections. As a first step, all material properties of a unidirectional ply (stiffness and strength) under tensile static loading conditions are measured. Then, from the experiments conducted in fatigue, the residual strength in the longitudinal direction of on-axis plies, the residual stiffness in the transverse direction of off-axis plies, and the fatigue life of the unidirectional plies in the longitudinal and transverse directions under tension–tension cyclic loading are fully determined.

In addition, the characteristic damage state (CDS) of off-axis plies of $[0/90_s/0]$ cross-ply laminates is investigated experimentally by using an optical microscope. Eventually, the fatigue life and stiffness degradation simulation capabilities of the model are evaluated experimentally for cross-ply laminates under static and fatigue loading. In the following, each set of experiments is explained in detail.

8.8 Specimen fabrication

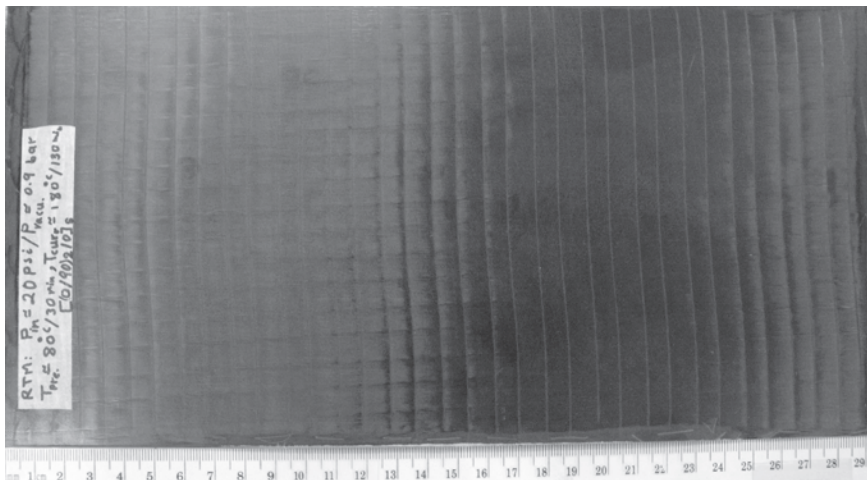
All required test specimens were fabricated from Carbon type T700 continuous fiber and Cycom 890 RTM epoxy resin at McGill University. The RTM

technique was assisted with vacuum to fabricate specimens with less void content. The resin injection pressure was about 138 kPa and full vacuum was applied to reduce void content inside the specimens. Epoxy resin was preheated up to 80°C and degassed for 30 minutes under full vacuum before injection. During injection, the mold temperature was held around 120°C by using two heating plates, whereas injection equipment such as the cylinder, piston and pipes were held at 80°C. After injection the mold was held at 180°C for 2 hours to cure the samples. The fiber volume fraction of manufactured plates varied between 48 and 51%.

All composite plates were fabricated to dimensions of 16 × 28 cm by using a two-part mold, then cut into coupons with a diamond saw. The edges were polished by using fine sandpaper to reduce surface edge effects. As described by Kitano *et al.* (1993), removing edge defects delays the onset of transverse cracking. All specimens were equipped with carbon/epoxy tabs to reduce the gripping effects. Figure 8.7 shows a sample of manufactured plates in this research.

8.9 Experimental set-up and testing procedures

Tests were performed using a 250 kN capacity MTS testing machine, equipped with hydraulic grips, under ambient laboratory conditions. A computer was connected to the testing machine for data acquisition. Displacement, load and strain were recorded for static experiments. During fatigue tests, maximum and minimum displacement, load and strain as well as number of cycles were measured and monitored. A 50 mm clip gauge extensometer was mounted on



8.7 T700/Cycom 890 plate manufactured by VARTM technique.

the surface of the specimen with elastic bands to measure strain. Static tests were performed under displacement control, while fatigue tests were carried out under load control conditions. The fatigue load was applied in a sine wave profile with a frequency equal to 6 Hz to minimize the generation of hysteretic heat in the specimens, which could degrade the material properties. All fatigue tests were performed at a stress ratio of $R = 0.1$.

8.10 Longitudinal tensile tests

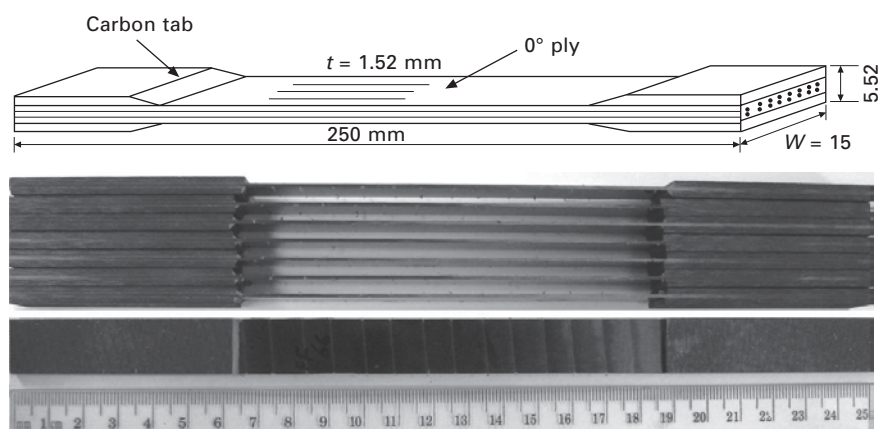
In this section, the results of static and fatigue experiments for characterizing the material properties of a unidirectional ply $[0]_{10}$ in the longitudinal direction under tensile loading are summarized. The specimens for fiber in tension tests were fabricated based on ASTM D 3039 M (2000) and ASTM D 3479 (2002) standards. Specimens and their dimensions are shown in Fig. 8.8.

8.10.1 Static stiffness and strength tests

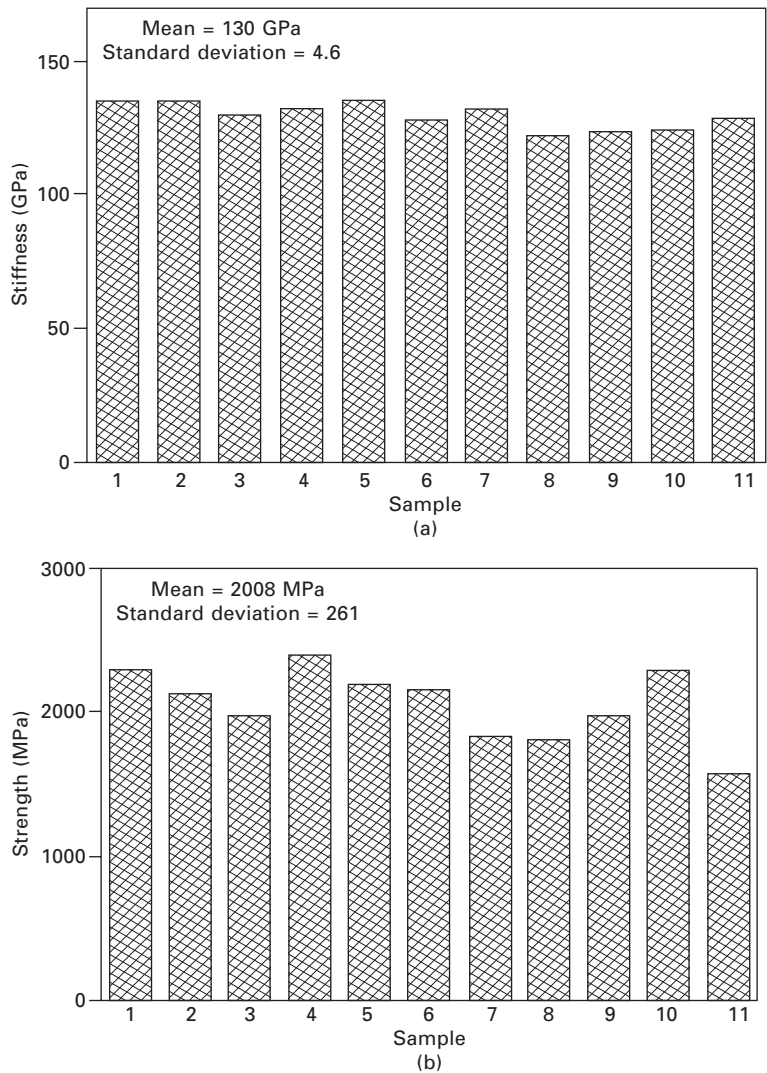
Typical test results of the static stiffness and strength of the unidirectional 0° ply under longitudinal tensile loading conditions (average values and standard deviations) are shown in Fig. 8.9 (a) and (b), respectively. Test crosshead speed was set to 1 mm/min and 11 samples were tested under monotonic tensile loading.

8.10.2 Normalized residual strength in fiber direction

To measure the residual strength of unidirectional 0° plies under tension–tension fatigue loading, 11 samples were tested at the stress state equal to 65% of

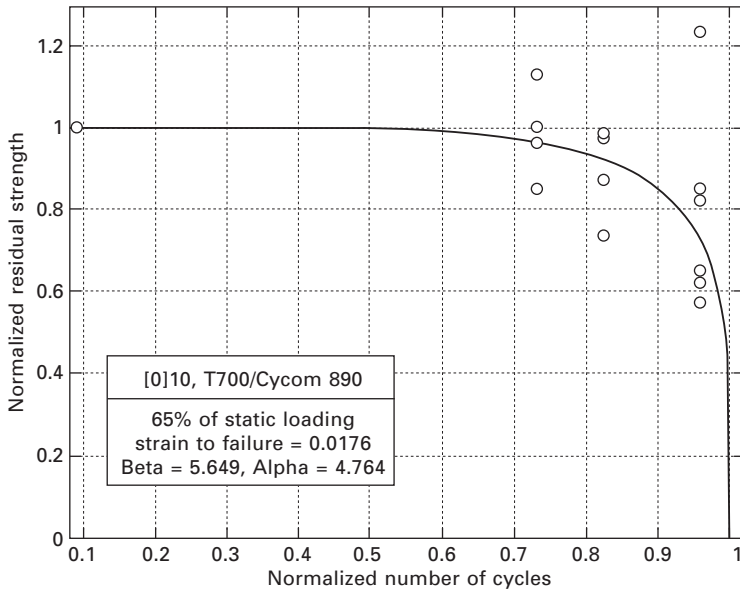


8.8 Typical test specimens of unidirectional $[0]_{10}$ laminates and their dimensions.



8.9 (a) Stiffness and (b) strength of T700/Cycom 890 RTM loaded in longitudinal direction.

the longitudinal tensile strength. Specimens were divided in three groups and fatigued up to 8000, 30,000 and 200,000 cycles, and then cyclic tests were stopped and static tensile tests were performed on fatigued samples to measure the residual strengths. The test results of residual strength are shown in Fig. 8.10. By using the least-squares method a curve was fitted to the experimental data and the curve-fitting parameters were also calculated by



8.10 Normalized residual strength of a unidirectional 0° ply under longitudinal tension-tension test.

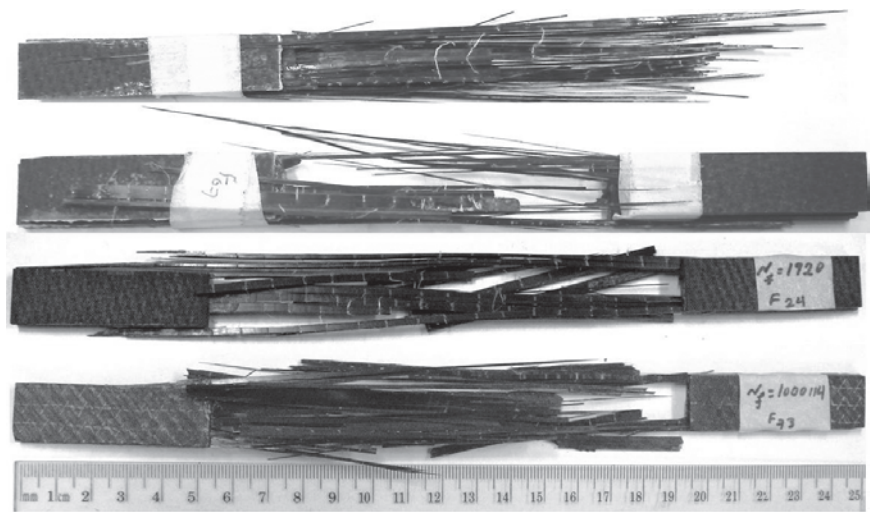
using Eq. 8.6 and mentioned in Fig. 8.10. The curve fitted to the experimental data is in the following form:

$$R(R, \sigma, n) = \left[1 - \left(\frac{\log n - \log 0.25}{\log 360,000 - \log 0.25} \right)^{5.649} \right]^{\frac{1}{4.764}} (700) + 1300 \quad 8.29$$

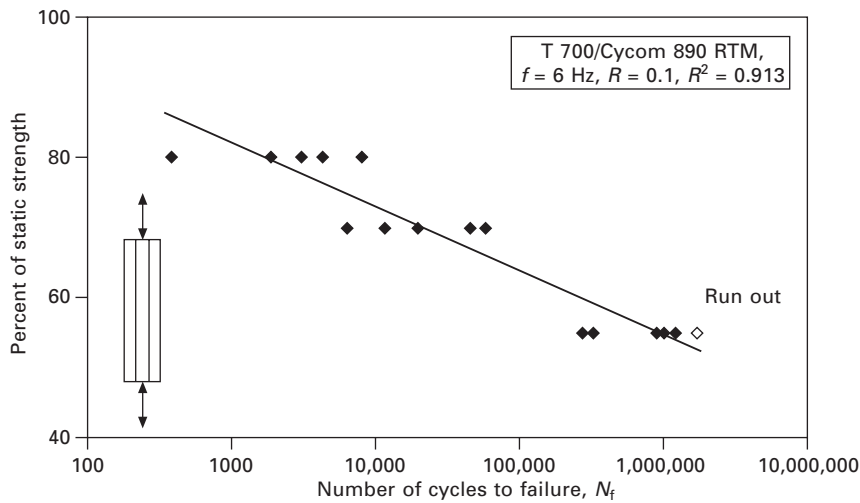
Photographs of unidirectional 0° plies failed under tension-tension uniaxial fatigue loading conditions are shown in Fig. 8.11.

8.10.3 S-N curve in fiber direction

Longitudinal tension-tension fatigue tests were performed on 20 samples under three different stress states of 55, 65 and 80% of static strength to find the fatigue life of a unidirectional 0° ply. The results obtained are presented as an S-N curve in Fig. 8.12, in a semi-log scale coordinate. Open diamonds are related to the samples broken after 1,500,000 cycles or run-out tests, and the solid line shows the line fitted to the experimental data (solid diamonds) by using the least-squares method.



8.11 Typical fatigue failure mode in fiber direction of unidirectional $[0_{10}]$ laminates.



8.12 S-N curve of unidirectional $[0_{10}]$ laminates in longitudinal direction.

8.11 Transverse tensile tests

In the following, the results of static and fatigue experiments for characterizing the material properties of a unidirectional ply $[90]_{10}$ in the transverse direction under tensile loading are summarized. The specimens for matrix

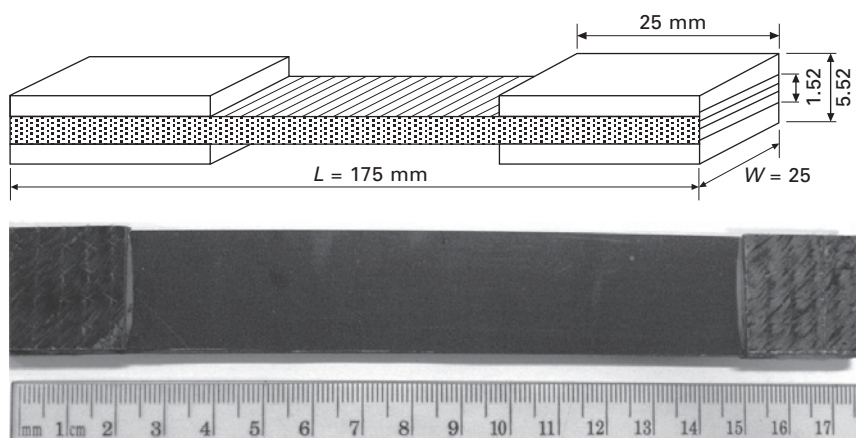
in tension tests were manufactured based on ASTM D 3039 M (2000) and ASTM D 3479 (2002) standards. Figure 8.13 shows the specimens and their dimensions.

8.11.1 Static stiffness and strength tests

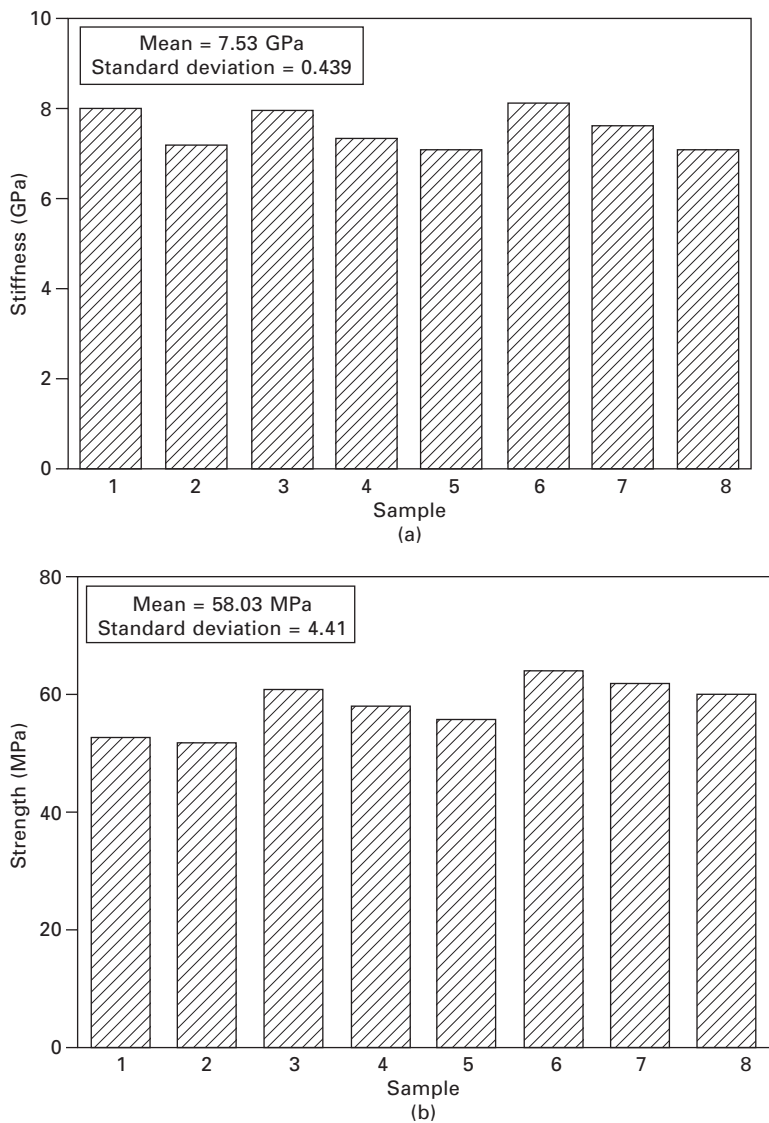
Typical test results for the static stiffness and strength of the unidirectional 90° ply under longitudinal tensile loading conditions (average values and standard deviations) are shown in Fig. 8.14 (a) and (b), respectively. The test speed was set to 0.6 mm/min and eight samples were tested under monotonic tensile loading. Because of less scatter in the test results for the transverse direction in comparison to the longitudinal direction, only eight specimens were tested in this direction.

8.11.2 Normalized residual stiffness in transverse direction

To measure the stiffness degradation of a unidirectional 90° ply under tension–tension fatigue, specimens were fatigued to a given number of cycles and then the fatigue tests were stopped and a static test was performed to measure the residual stiffness, then the fatigue test on the sample resumed until final failure occurred. Fatigue tests were performed on eight specimens at states of stress that were, 70 and 55% of the transverse tensile static strength. By selecting these two different states of stress, high and low stress levels were applied. Test results of a unidirectional 90° ply under tension–tension

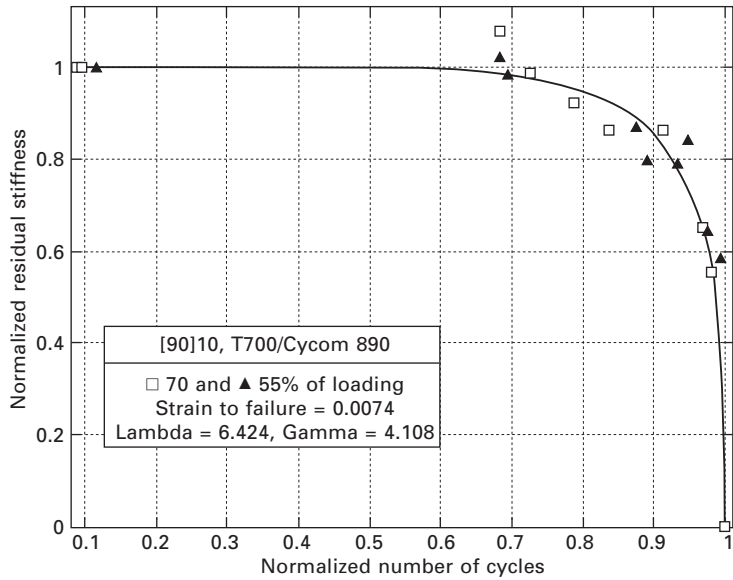


8.13 Typical test specimens of unidirectional $[90_{10}]$ laminates and their dimensions.

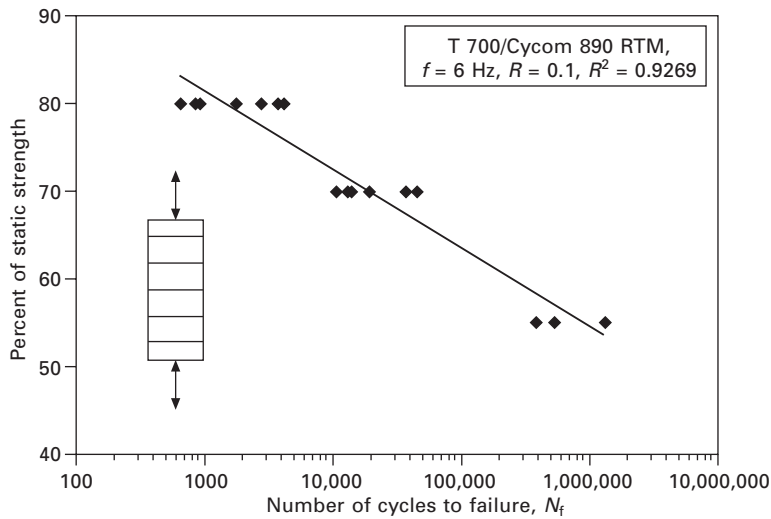


8.14 (a) Stiffness and (b) strength of T700/Cycom 890 RTM loaded in transverse direction.

fatigue, are depicted in a semi-log scale coordinate in Fig. 8.15. Squares and triangles in Fig. 8.15 show the test results for 70 and 55% of static strength respectively, and the solid line depicts the curve fitted to the test results by the least-squares method.



8.15 Residual stiffness in transverse direction.



8.16 S–N curve of unidirectional [90₁₀] laminates in transverse direction.

8.11.3 S–N curve in transverse direction

In order to obtain the fatigue life of a unidirectional 90° ply, 16 samples were tested at stress states of 55, 70 and 80% of static strength under

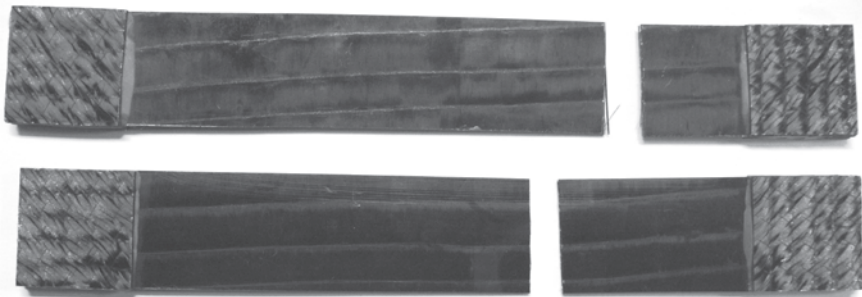
longitudinal tension–tension loading. Experimental results in the transverse direction were depicted as an S–N curve in Fig. 8.16, in a semi-log scale coordinate. Diamonds show the experimental data and the solid line presents the line fitted to the experimental data by using the least-squares method. Photographs of unidirectional 90° plies failed under fatigue loading in the transverse direction are shown in Fig. 8.17.

8.12 In-plane static shear tests

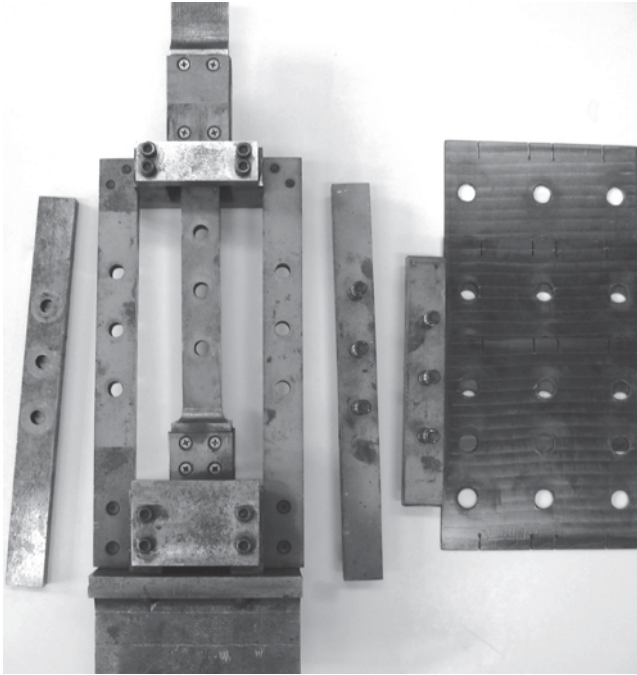
According to the classical lamination theory, in-plane shear strength is the same for all laminations such as $[0]_n$, $[90]_n$ and $[0_s/90_m]_n$. But based on the available experimental data in the literature (Chang and Chen, 1987), in this research $[(0/90)_n]_s$ lamination is used to measure the strength and stiffness of the material. Figure 8.18 shows three rail shear test fixture and manufactured samples based on ASTM D 4255 (2001). A 12.7 mm extensometer was mounted on the samples at 45° to the loading direction to measure strain during testing. Specimens were tested under static tensile loading, the test speed was selected to be 1 mm/min, and four samples were tested. The average shear strength and stiffness were 104 MPa and 4.8 GPa respectively. A typical nonlinear shear stress–displacement curve and a specimen after failure are shown in Fig. 8.19 (a) and (b) respectively. The mechanical properties of T700/Cycom 890 RTM carbon/epoxy unidirectional composites manufactured by VARTM techniques under static loading conditions are summarized in Table 8.1.

Because of the transversely isotropic material property assumption, the out-of-plane modulus is not independent and can be calculated by using the following equation:

$$E_{yz} = \frac{E_{yy}}{2(1 + \nu_{yz})} \quad 8.30$$



8.17 Fatigue failure mode in transverse direction under tension–tension loading of $[90]_{10}$ laminates.



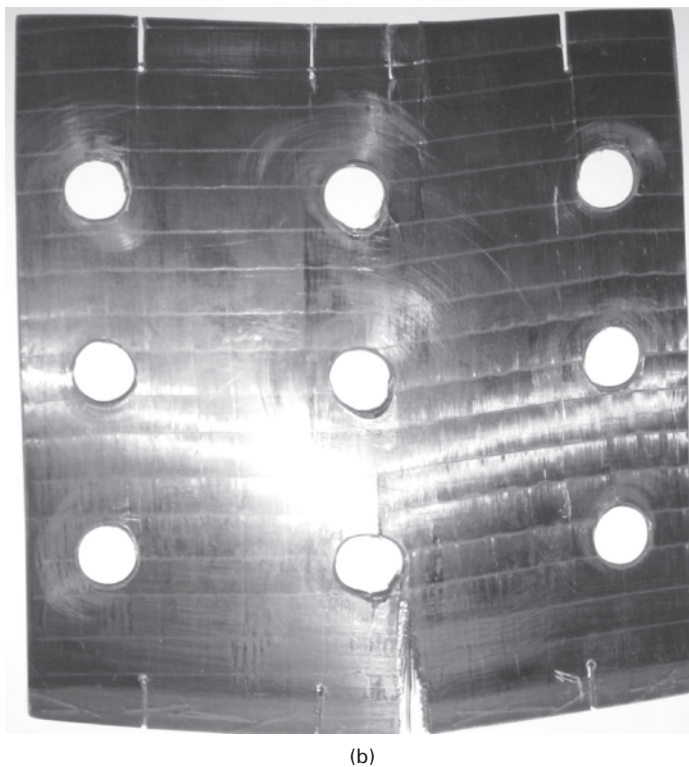
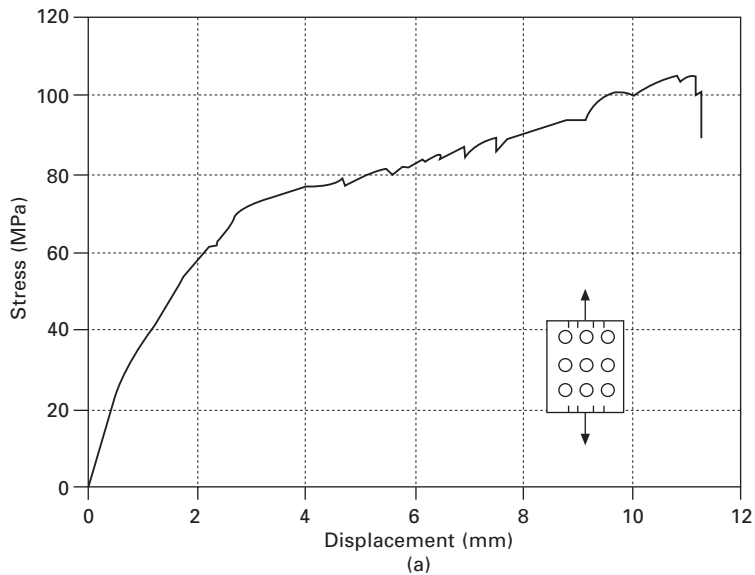
8.18 Three-rail test fixture and shear test samples.

The major Poisson's ratio ν_{xy} is obtained during the tensile test, and the out-of-plane Poisson's ratio ν_{xy} is obtained by using Christensen's equation (Christensen, 1988) in the following form:

$$\nu_{yz} = \frac{\nu_{xy} \left(1 - \frac{\nu_{xy} E_{yy}}{E_{xx}} \right)}{1 - \nu_{xy}} \quad 8.31$$

8.13 Experimental evaluation of the model

In the previous section, the material properties of the unidirectional T700/Cycom 890 under static and fatigue loading conditions were fully characterized experimentally. In this section, before final evaluation of the progressive fatigue damage model, its basic components such as the unified fatigue life model and strength degradation are verified experimentally under variable amplitude loading. Then the fatigue life prediction and stiffness degradation simulation capabilities of the model are experimentally evaluated by performing static and fatigue tests on cross-ply laminates. In the following, each set of experiments is explained in detail.



8.19 (a) Shear stress-displacement curve; (b) shear sample after failure.

Table 8.1 Static test results for unidirectional T700/Cycom 890 RTM carbon/epoxy laminates manufactured by the VARTM technique

Property	Magnitude (mean)	Standard deviation
E_{xx} (GPa)	130	4.6
$E_{yy} = E_{zz}$ (GPa)	7.53	0.439
$E_{xy} = E_{xz}$ (GPa)	4.8	–
E_{yz} (GPa)	2.72	–
$\nu_{xy} = \nu_{xz}$	0.28	–
ν_{yz}	0.382	–
X_t (MPa)	2008	261
Y_t (MPa)	58.3	4.41
$S_{xy} = S_{xz}$ (MPa)	103	6.2
ϵ_{uf}	0.0176	–
ϵ_{um}	0.0074	–
Ply thickness (mm)	0.152	–
Fiber volume fraction (%)	52	–

8.13.1 Evaluation of unified fatigue life model

A unified fatigue life model which is explained in detail elsewhere (Shokrieh and Taheri-Behrooz, 2006) is used in the progressive fatigue damage model to predict the fatigue life of unidirectional composite laminates under various stress ratios and cyclic loading.

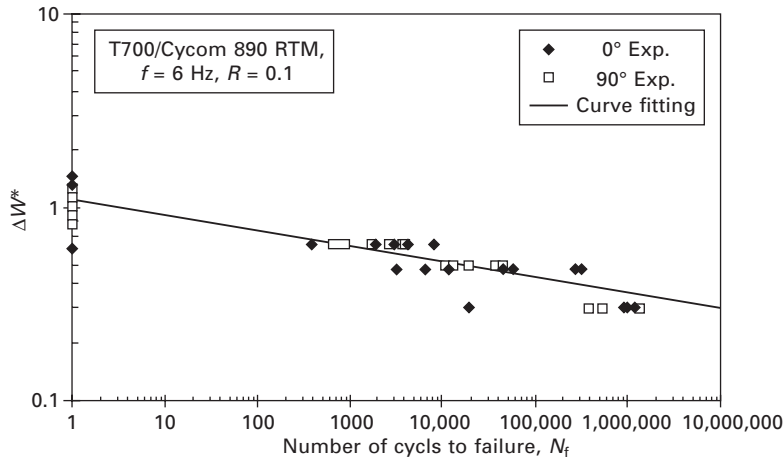
In this section, the test results obtained for static and fatigue loading in Table 8.1 and Figs 8.12 and 8.16 are used to calculate the normalized strain energy density based on Eq. 8.19. Then the normalized strain energy density (ΔW^*) is plotted against fatigue life (N_f) on log–log scale coordinates to obtain curve-fitting parameters, k and α . Variation of normalized strain energy density with fatigue life is depicted in Fig. 8.20, diamonds and squares showing the experimental results in the longitudinal and transverse directions, respectively. By using the linear least-squares method a line is fitted to the experimental data in the following form:

$$W^* = 1.088(N_f)^{-0.0825} \quad 8.32$$

8.13.2 Evaluation of residual fatigue life prediction

In the progressive fatigue damage model developed in this research, on-axis plies are considered as critical elements which are responsible for final failure. In addition, even under constant amplitude fatigue loading, each ply in the laminate experiences variable amplitude fatigue loading because of damage growth and stress redistribution. For this reason, the capability of the proposed strength degradation model, in life prediction under variable amplitude loading, must be evaluated experimentally.

The Palmgren–Miner rule (Miner and Calif, 1945) is a traditional method of



8.20. Variation of normalized strain energy density with fatigue life under stress ratio of 0.1.

Table 8.2 Residual fatigue life of $[0_{10}]$ unidirectional laminates, measured by experiments and theory

Loading ratio sequence (%)	Applied cycles (n_1, n_2, n_3)	Predicted cycles, n_3 (Miner's rule)	Predicted cycles, n_3 (Eq. 8.6)	Predicted cycles, n_3 (experiment)
60–70–80	100,000–10,000– n_3	581	160	275
80–70–60	300–10,000– n_3	528,489	1,181,590	–

predicting residual life under variable fatigue loading. However, experimental evidence (Hashin and Rotem, 1978; Hashin, 1980a; Golos and Ellyin, 1987; Buch, 1988) reveals that that Palmgren–Miner rule is not able to predict residual life in general. Also, it does not distinguish between the sequences of applying stress levels which are important in life prediction.

To evaluate Eq. 8.6 under variable amplitude loading, three unidirectional specimens, $[0_{10}]$, are tested under load levels of 80, 70 and 60% of the maximum static strength. Experimental results and predictions by Miner's rule and Eq. 8.6 are summarized in Table 8.2. In Table 8.2, n_1 , n_2 and n_3 are the number of cycles under various amplitude fatigue loadings. All load ratios and number of cycles (n_1 and n_2) are given, whereas the number of cycles related to the last loading sequence (n_3) is calculated by Miner's rule, Eq. 8.6 and experimentally.

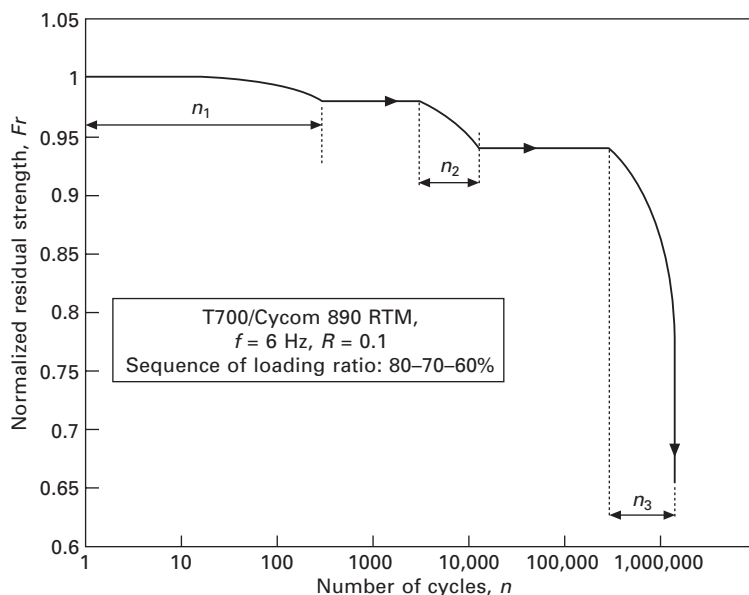
The average residual fatigue life for three specimens tested at 60, 70 and 80% of static strength is 275 cycles. As shown in Table 8.2, Miner's rule and Eq. 8.6 predictions in comparison with the experimental data show 112%

and 41% error, respectively. The acceptable percentage of error produced by Eq. 8.6 shows a fairly good agreement between the simulation and the experiments. Gradual strength degradation simulated by Eq. 8.6 is plotted in Fig. 8.21 for loading ratios of 80, 70 and 60% of static strength. As shown in Table 8.2, the residual life predicted by the model is 1,181,590 cycles against 528,489 cycles predicted by Miner's rule.

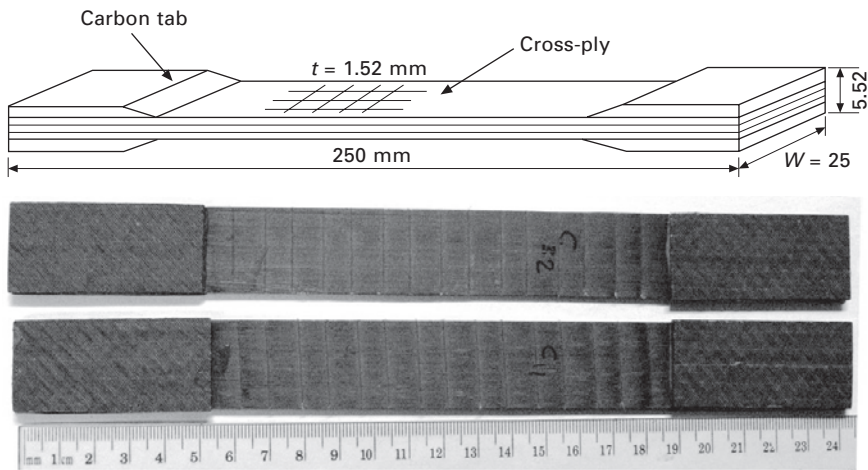
8.13.3 Experimental evaluation of cross-ply composites [0/90₅/0], [0/90₄]_s

For obtaining strength and stiffness of cross-ply laminates, [0/90₅/0] and [0/90₄]_s, specimens were cut to size in accordance with ASTM D 3039 M (2000) from manufactured plates by the VARTM method, as shown in Fig. 8.22. Table 8.3 presents the average results for tensile strength and stiffness values of five specimens tested for each laminate family. A typical stress–strain curve is presented in Fig. 8.23; FPF in the figure refers to the first-ply failure where the strain of off-axis plies reaches their allowable values.

Theoretical magnitudes of stiffness are calculated based on classical lamination theory and experimental magnitudes are considered in accordance with the first linear part of the bilinear stress–strain curve. Disagreement between theoretical and experimental results is associated with a small



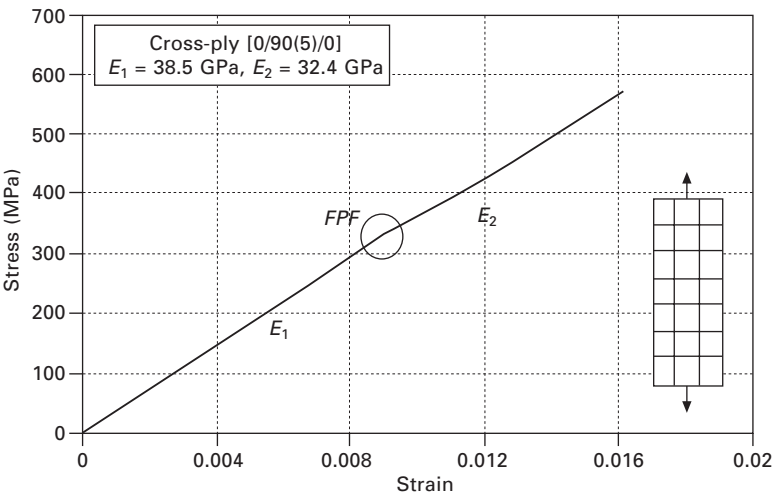
8.21 Residual strength under fatigue loading sequence of 80, 70 and 60% of static strength.



8.22 Test specimen and dimensions of cross-ply specimens.

Table 8.3 Stiffness and strength of cross-ply laminates

Property	[0/90 ₅ /0]		[0/90 ₄]s	
	Experiment	Theory	Experiment	Theory
Tensile strength (MPa)	585.6	614.8	411	446
Tensile modulus (GPa)	39.3	42.6	29.1	32.12
Stress of FPF point (MPa)	—	330.82	—	240



8.23 Typical stress–strain curve of [0/90₅/0] laminate under uniaxial tensile loading.

variation of fiber volume fraction during specimen fabrication and micro-cracks, which occur during testing.

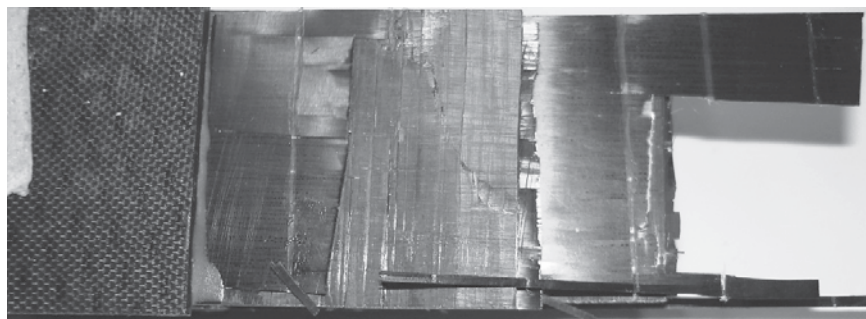
The Tsai–Wu failure criterion is employed to obtain the strength of the laminates theoretically. For simplicity, once the FPF occurs in off-axis plies, the stiffness and strength of these plies reduce to nearly zero in strength calculations. Figure 8.24 shows the static failure mode of $[0/90_5/0]$ cross-ply laminate under uniaxial tensile loading.

8.13.4 Evaluation of stiffness degradation in $[0/90_5/0]$ cross-ply composites

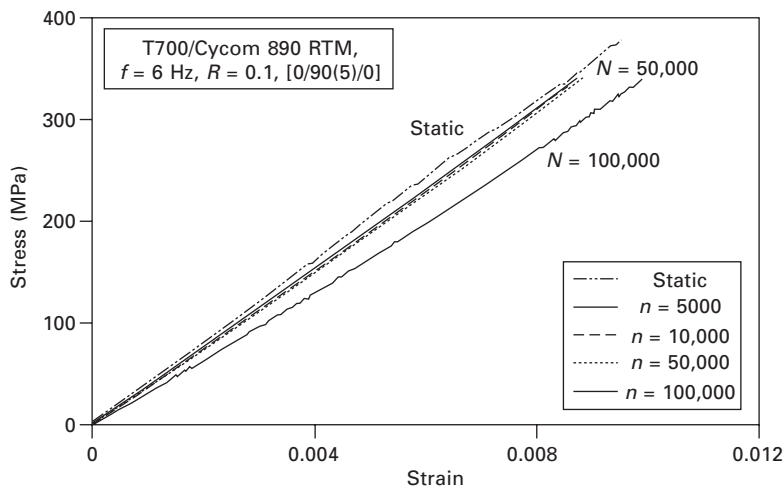
In order to simulate stiffness degradation in the off-axis plies of $[0/90_5/0]$ cross-ply composites, the magnitude of stiffness and strain of the characteristic damage state (CDS) in the off-axis plies must be characterized. For this reason, five samples were cut to size based on ASTM D 3039 M (2000) from manufactured cross-ply $[0/90_5/0]$ plates, and were tested under tensile fatigue loading at a stress of 65% of laminate static strength.

In order to monitor stiffness degradation during the course of fatigue tests, the cyclic loading was intermittently stopped and the stiffness was measured by performing a static test. A typical stress–strain curve for different numbers of cycles is depicted in Fig. 8.25.

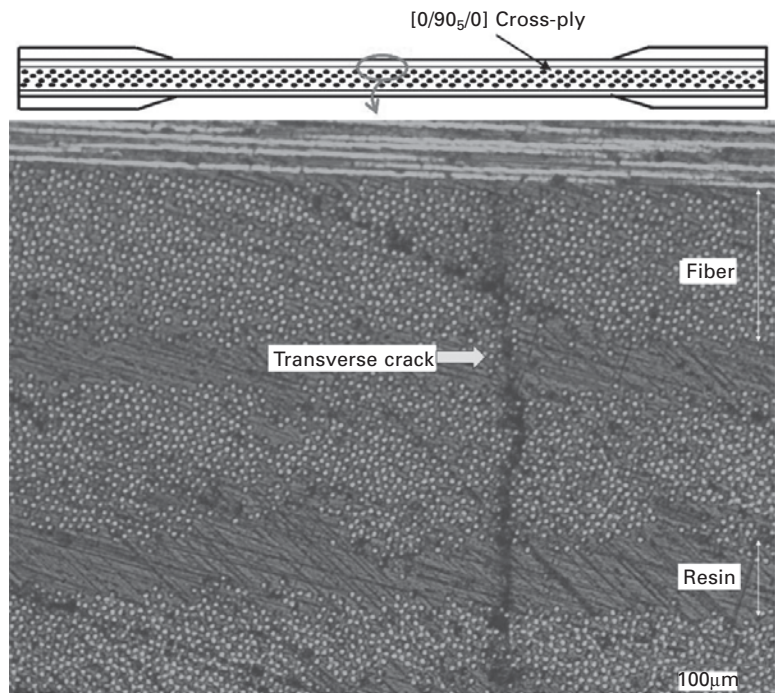
The specimens were observed under an optical microscope along their edges to obtain the CDS of $[0/90_5/0]$ cross-ply specimens subjected to static and fatigue tests. A close-up view of crack trajectories in 90° plies of the laminate is presented in Fig. 8.26. In Fig. 8.26 white points refer to the off-axis plies (90°) and white lines in the upper side show the on-axis (0°) plies. To better clarify the crack pattern in the specimens, only three off-axis plies and one on-axis ply are shown in the picture. Five samples were studied for this case and an average crack spacing of 2.02 mm was obtained.



8.24 Failure mode of cross-ply $[0/90_5/0]$ laminates under static loading.



8.25 Typical stress–strain curve of [0/90_s/0] laminate under cyclic loading.

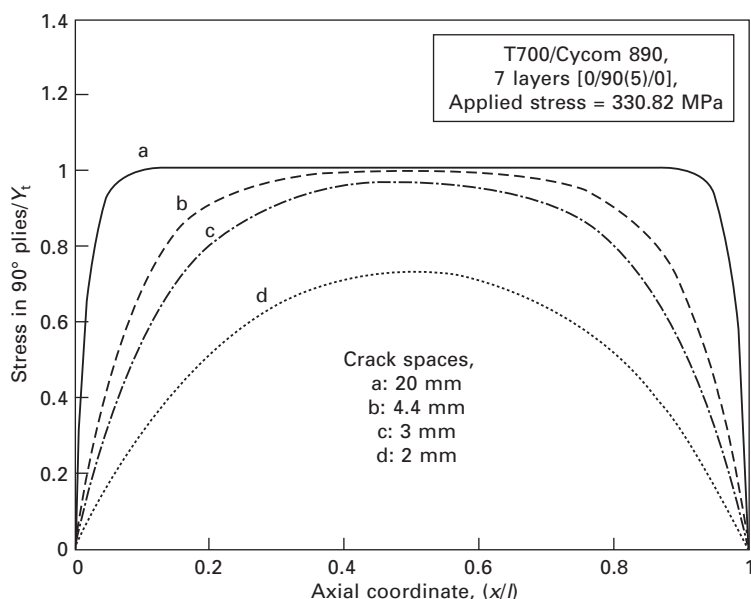


8.26 Optical micrographs showing layered microstructure of the composites.

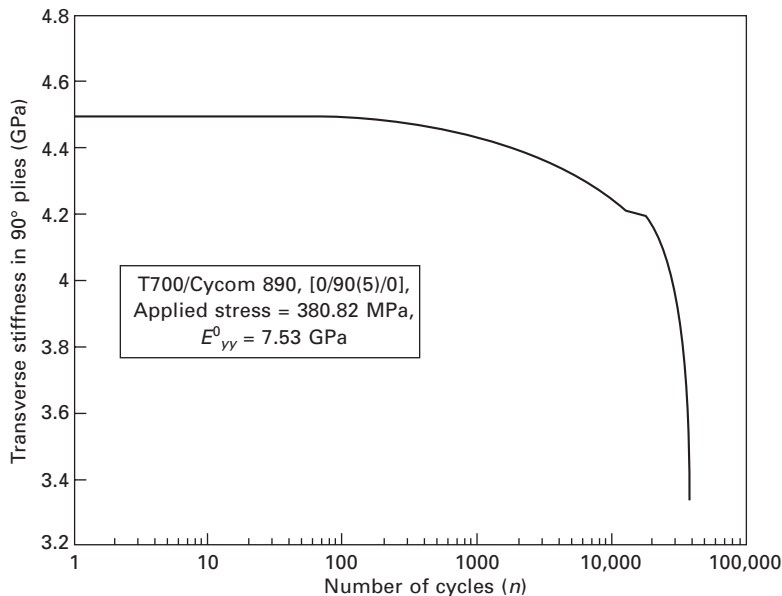
Equations 8.22–8.27 together with measured experimental data in Table 8.1 are used to calculate the characteristic damage state (CDS) theoretically. Figure 8.27 shows the variation of stress distribution between two transverse cracks with different crack spacing. Finally, crack spacing associated with 480 MPa static load was considered as the CDS level of $[0/90_5/0]$ cross-ply laminates. Then, by using Eqs 8.24 and 8.26 strain and stiffness related to the characteristic damage state of 90° plies were found to be 1.22% and 4.5 GPa respectively.

By using Eq. 8.26 the stiffness degradation of 90° plies associated with matrix cracking against the number of cycles is simulated as shown in Fig. 8.28. As the cyclic applied load exceeds the first-ply failure load during the first fatigue cycle, some cracks are created in the 90° plies and their stiffness is reduced suddenly from their static value, 7.53 GPa, to 4.5 GPa. After this point, increasing the number of cycles causes gradual stiffness degradation of 90° plies in agreement with Eq. 8.28. Stiffness degradation in 90° plies with the number of cycles continued to the stiffness value of 3.4 GPa, which is the stiffness of the CDS level. After the CDS point, the stiffness of 90° plies is suddenly reduced to almost zero. Figure 8.29 shows the stiffness degradation of 90° plies from first cycle to delamination initiation.

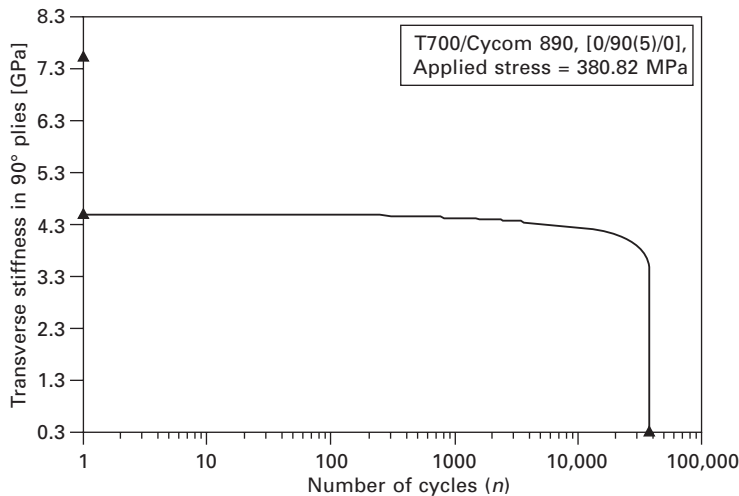
The relation between the effective engineering constant (in-plane longitudinal stiffness) E_1^0 and the compliance component a_{11} of the laminate is governed by Eq. 8.33:



8.27 Stress distribution between two transverse cracks in 90° plies of $[0/90_5/0]$ laminate.



8.28 Stiffness degradation of 90° plies from first cycle to CDS level.



8.29 Stiffness degradation of 90° plies from first cycle to delamination initiation.

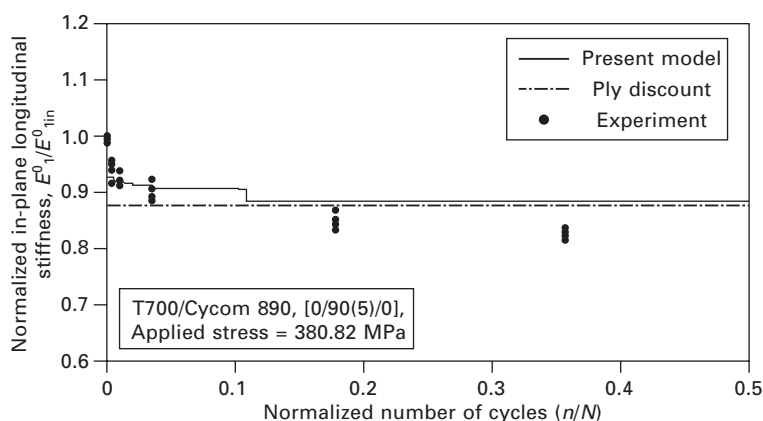
$$E_l^0 = \frac{1}{a_{11}h} \quad 8.33$$

where h is the laminate thickness. By substituting Eq. 8.33 into Eq. 8.28, in-plane longitudinal stiffness reduction is obtained for the given laminate

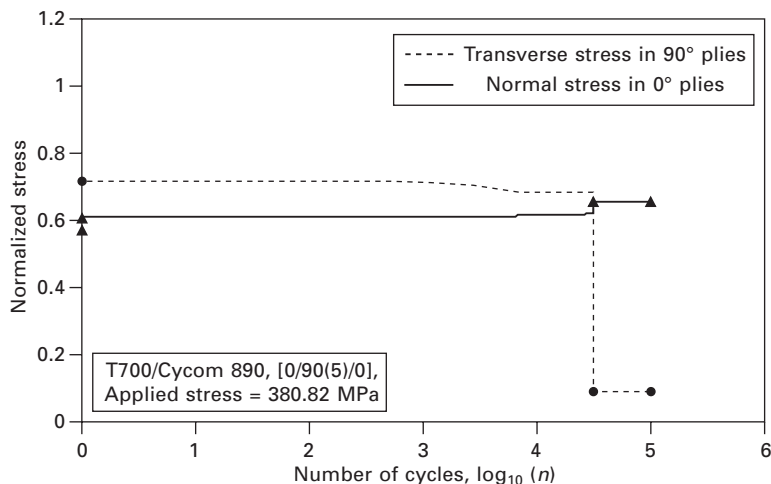
Fig. 8.30. Results predicted by the modified model are compared with the experimental data provided in this research, and ply discount results. The stiffness degradation due to the matrix cracking causes in-plane longitudinal stiffness degradation until the life ratio is equal to $n/N = 0.18$. After this point, as Fig. 8.30 shows, the values of experimental data lie below the ply discount result, which indicates the initiation of other failure modes such as delamination, longitudinal cracks and fiber breakage in the laminate. This subject is studied by Taheri-Behrooz *et al.* (2008) in more detail elsewhere.

It is worth mentioning that there is a large deviation from experimental data after the life ratio n/N is equal to 0.18. However, this deviation does not affect the fatigue life predicted by the technique developed in this research. The reason is that other damage mechanisms such as longitudinal cracks and fiber breakage normally occur near the end of fatigue life over a short time interval, so one cannot consider redistribution of internal stress of the laminate. The stiffness degradation model developed in this research focuses on predicting stiffness reduction with number of cycles until the crack saturation level, and uses shear-lag analysis along with classical lamination theory to simulate stress redistribution with number of cycles under cyclic loading. Results predicted by this model are used along with a normalized residual strength model, which considers inherently the effect of longitudinal cracks and fiber breakage, to predict the fatigue life of cross-ply laminates with more accuracy and less requirement for experimental data.

Figure 8.31 shows the stress history in 0° and 90° plies under constant amplitude fatigue loading. It shows that in the first cycle failure occurs in 90° plies, causing a rapid stress reduction in 90° plies and a rapid stress



8.30 In-plane longitudinal stiffness degradation of cross-ply $[0/90_5/0]$ laminate under fatigue loading.



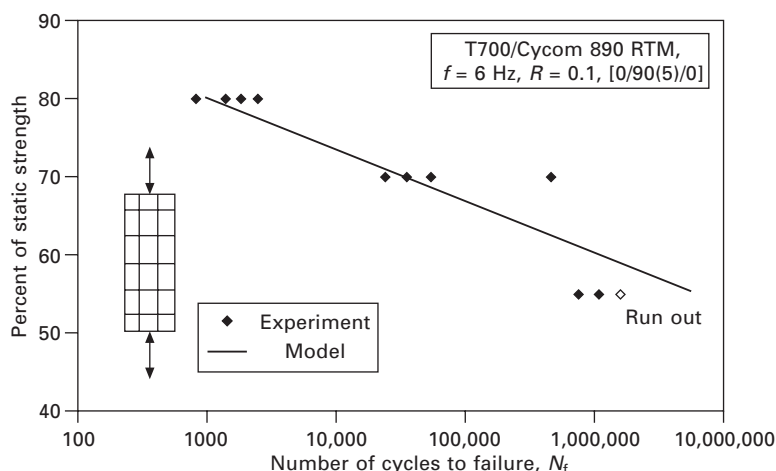
8.31 Stress variations with number of cycles in 0° and 90° plies, under fatigue loading.

growth in 0° plies that is followed by a nearly gradual stress redistribution in both layers.

8.13.5 Evaluation of progressive fatigue damage model

In this section, the capability of the progressive fatigue damage model to simulate the fatigue life of composite laminates is examined by performing a series of fatigue tests on [0/90₅/0] laminates. All tests are performed at a load ratio ($F_{\min}/F_{\max}$) of 0.1 and a frequency of 6 Hz. It is obvious that although the load ratio is kept constant, the stress ratio for different layers is not constant because of damage growth in off-axis plies and consequently stress redistribution with number of cycles. Based on the maximum static strength of the laminate, presented in Table 8.2, 11 samples were tested at three different stress states of 55, 70 and 80% of the static strength to obtain the S–N curve experimentally.

As Fig. 8.32 shows, there is good agreement between results predicted by the progressive fatigue damage model and experimental data for stress states more than 65%. However, the model overestimated in low stress states such as 55. The reason is that during fatigue tests in low stress states for more than 1,200,000 cycles the test is stopped and reported as a run-out test. It is more likely that, by performing more tests and allowing enough time to obtain real fatigue life, disagreement between the model and the experiment would become lower at this stress state.

8.32 Fatigue life prediction of cross-ply $[0/90_5/0]$ laminate.

8.14 Conclusion

The main goal of this research is the fatigue life prediction of cross-ply laminated composites under cyclic tension–tension loading conditions. Fatigue life prediction of cross-ply laminates is investigated in terms of gradual stiffness degradation and fatigue life prediction. A progressive fatigue damage model is developed based on two main assumptions: firstly, that off-axis plies of a cross-ply laminate are responsible for stress redistribution into the laminate, and secondly, that on-axis plies of a cross-ply laminate are responsible for strength reduction and final failure of the laminate. The developed model consists of three fundamental parts: stress analysis, gradual stiffness degradation and strength, and failure analysis. It is able to predict the fatigue life of a wide range of cross-ply composite laminates under different loading conditions.

In the first step of this research, a new unified fatigue life model based on the energy method is presented for unidirectional polymer composites subjected to constant amplitude, tension–tension or compression–compression fatigue loadings. By employing the static failure criterion of Sandhu, the proposed model is capable of predicting the fatigue life of unidirectional composite laminates over the range of positive stress ratios with various fiber orientation angles. One set of experimental data obtained in current research is used to verify the model. The model predictions are in good agreement with the experimental data from unidirectional composites.

A stiffness degradation model is developed by using the new unified fatigue life model and shear-lag analysis to improve the correlation with experimental data from low fatigue cycles. The developed model is integrated

into a progressive approach to predict the transverse stiffness degradation of cross-ply laminates under both constant and variable amplitude cyclic tensile loadings. Results predicted by the model show good correlation with the experimental data available in the literature and experiments performed in this research on [0/90₅/0] lay-ups made of T700/Cycom 890 RTM carbon/epoxy composites.

To predict the strength degradation of on-axis plies, a model developed by Shokrieh and Lessard is modified by integrating a new failure function and the unified fatigue life criterion. The first modification is employed to consider all stress components in a single equation. The second modification is used to take into account the fatigue life and residual strength under various stress ratios and states of stresses for a unidirectional ply by a single equation. In addition, the progressive fatigue damage model developed in this research, in contrast to the traditional damage accumulation rules such as Palmgren–Miner's, is sensitive to the loading history and capable of distinguishing between high–low and low–high loading sequences. This capability enables the model to predict the fatigue life and residual strength of cross-ply laminates under variable amplitude fatigue loading conditions. Model predictions were verified by performing experiments on the fiber direction of unidirectional T700/Cycom 890 RTM carbon/epoxy composites.

For evaluation of the progressive fatigue damage model, experiments were performed on a cross-ply [0/90₅/0] lay-up of T700/Cycom 890 RTM carbon/epoxy composites under three different states of stresses and a stress ratio of $R = 0.1$. Results predicted by the model show good correlation with experimental data obtained in this research.

8.15 References

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Fatigue life prediction of composite materials under realistic loading conditions (variable amplitude loading)

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Abstract: Two of the most widely used methods for the fatigue life prediction of composite materials under variable amplitude (VA) loading patterns are presented in this chapter. The first method is based on the theoretical formulation and use of a damage summation rule to predict life under VA loading without recourse to experimental observation of the damage accumulation process. An alternative to this classic fatigue life prediction methodology are the residual strength fatigue theories, where residual strength is used as the damage metric. Comparison of the remaining strength of the material to the static strength allows the estimation of the fatigue cycles until failure. The basic fatigue modeling introduced in previous chapters of this book for interpretation of the fatigue data (Chapter 2), residual strength theories (Chapter 3), and constant life diagrams (Chapter 6) is combined here to establish fatigue life prediction methodologies.

Key words: S–N curves, life prediction, variable amplitude fatigue, cycle counting, constant life diagrams.

9.1 Introduction

The majority of engineering structures comprise parts that are subjected to cyclic loading patterns. In fact, most structural failures occur due to mechanisms driven by fatigue loading, whereas purely static failure is rarely observed in open-air applications [1]. Fatigue mainly affects the weak links of structures, usually laminates and joints that are used to transfer loads from one part of the structure to another. The structural integrity of these components is therefore of great importance for the viability of the entire system. The durability of a structure is also affected by environmental loads, mainly extreme temperature and humidity differences during lifetime, and a structure generally undergoes a complex thermomechanical cyclic loading profile throughout its lifetime.

A thorough knowledge of the fatigue behavior and property degradation

under the applied thermomechanical loading profile results in appropriate design. However, although an estimation of the potential loading patterns can be achieved, the random behavior of the excitation (e.g. air or traffic loads, impact loads, etc.) cannot be accurately modeled [2]. On the other hand, the vast number of loading cases that could be designated after the aeroelastic calculations for the determination of the loading renders experimental investigation impossible.

Thus theoretical approaches have been developed in the past, especially over the last 40 years, for the modeling of the fatigue behavior and prediction of the fatigue life of composite materials and structures under block loading and complex, irregular loading spectra, e.g. [3–8].

The fatigue behavior of a material under variable amplitude (VA) loading can be validated when experimental work under this loading is performed. However, the result will be useful and available only for the examined loading scenario and the tests should be repeated for a new applied variable amplitude spectrum. It is therefore obvious that theoretical models should be established based on simple experiments and should have the ability to generalize modeling and predictions for more complicated cases such as block and variable loading situations.

One widely used approach is based on the theoretical formulation and use of a damage summation rule to predict life under VA loading without recourse to experimental observation of the damage accumulation process. The most popular and best known example of this category, which does not always lead to accurate results, however, is the linear Palmgren–Miner rule. Other summation rules were also proposed as alternatives to the Palmgren–Miner rule in order to evaluate and quantify damage accumulation and accurately predict the fatigue lifetime of glass and carbon-fiber reinforced plastic composites loaded under block or VA loading patterns, e.g. [9, 10].

Modeling of the material behavior is based on phenomenological models of constant amplitude fatigue test results. These models rely on the cyclic stress or strain vs. life relationship of the examined material, the effect of mean stress on fatigue life and also estimation of fatigue strength under complex loading patterns. Numerous methods have been proposed for the solution of each step of the aforementioned classic methodology, some of which are described in Chapters 2 and 6 of this volume.

As an alternative to the classic fatigue life prediction methodology, actual damage measurements during fatigue life can be employed to establish another type of fatigue theory. A damage metric is used as an indicator of damage accumulation. According to the damage metric, these theories can be further classified into sub-categories: strength degradation fatigue theories, where the damage metric is the residual strength after a cyclic program, e.g. [11, 12], stiffness degradation fatigue theories, where stiffness is conceived as the fatigue damage metric e.g. [13–16], and finally, actual damage mechanism–

fatigue theories based on the modeling of intrinsic defects in the matrix of the composite material that can be considered as matrix cracks. The way in which these cracks propagate in composite materials can be estimated by means of linear fracture mechanics calculations.

This chapter aims to provide an overview of fatigue life prediction methodologies for composite materials under variable amplitude fatigue loading. The effect of the parameters that dominate the life prediction results will be discussed. The classic life prediction methodology and a residual strength-based methodology will be implemented on fatigue data from the literature to demonstrate their applicability. The advantages and disadvantages of each method will be thoroughly debated and future trends in this field will be discussed in conclusion.

9.2 Theoretical background 1: classic fatigue life prediction methodology

The classic fatigue life prediction methodology that leads to the calculation of the Miner's damage coefficient and the residual strength fatigue analysis are presented in this section. Classic fatigue life prediction methodology can be considered as an articulated method, since a number of sub-problems must be solved sequentially to produce the final result. Four to five basic steps can be identified in this method:

- Cycle counting
- Modeling of the experimental constant amplitude fatigue behavior
- Interpretation of fatigue behavior for assessment of the mean stress effect
- Adoption of the fatigue failure criterion
- Damage summation.

Modeling constant amplitude fatigue behavior involves determination of the S–N curves (plot of cyclic stress vs. life, typically by grouping data at a single R -value ($R = \sigma_{\min}/\sigma_{\max}$) (incidentally, depending on the R -value, an S–N curve can be constructed from data obtained at varying mean and amplitude). Interpretation of the fatigue behavior for the assessment of the mean stress effect results in the construction of the constant life diagram (CLD). These two processes can be treated as separate steps, but are related in the sense that the CLD is constructed from the available S–N curves, and new S–N curves could be extracted from this CLD.

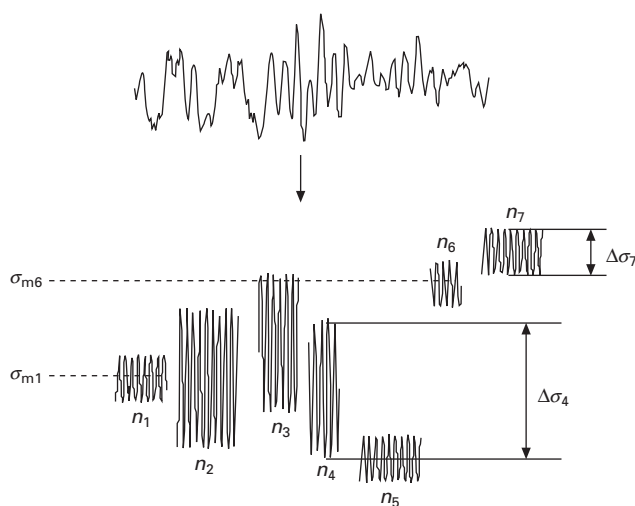
9.2.1 Cycle counting

Cycle counting is used to summarize (often lengthy) irregular load-versus-time histories by providing the number of occurrences of cycles of various

sizes, see Fig. 9.1. The definition of a cycle varies with the method of cycle counting. A significant number of cycle-counting techniques have been proposed over the last 30 years, e.g. [17], with rainflow counting being the most widely used. Cycle-counting methods known as one-parameter techniques, e.g. level-crossing counting or peak-counting methods, are not applicable for the fatigue analysis of composite materials since they do not consider the significant mean stress effect on lifetime.

A comprehensive description of the available cycle-counting methods for the analysis of spectra applied on composite materials is given in [18]. As mentioned in [18] the history of cycle-counting methods goes back to the 1950s and 1960s when only simple range-counting or range-mean-counting methods were used. The drawback of these methods is their inability to take into account the stress-strain history to which the material is subjected, and consequently their tendency to miss the largest overall load cycle in a sequence. The rainflow counting and related methods were introduced to address this problem [19–21]. Based on these algorithms stress-strain hysteresis loops are counted rather than stress range and mean values.

Rainflow-counting, range-pair, and range-mean methods seem to be the most appropriate for the analysis of composite material fatigue data, giving similar cycle counting results for most practical applications [22]. However they present a number of deficiencies: rainflow counting cannot be used for cycle-by-cycle analysis and therefore it is difficult to apply this method in combination with a residual strength fatigue theory. On the other hand, range-pair and range-mean counting mask the presence of large and damaging



9.1 Schematic representation of the application of a cycle counting method.

cycles. Based on the previous comments, it is concluded that – according to the application and material used – the appropriate cycle-counting technique should be selected very carefully.

In this work rainflow counting, range–mean and range–pair-counting methods are implemented in the fatigue life prediction methodologies. The influence of the selection of the cyclic counting method is assessed via the predictive ability of the entire methodology.

9.2.2 Representation of constant amplitude fatigue data

Traditionally, the S–N data are fitted by a semi-logarithmic or a logarithmic equation. In the first case, it is assumed that the logarithm of the number of cycles is linearly proportional to the stress parameter, while in the second, the logarithm of the number of cycles depends linearly on the logarithm of the stress parameter. The stress parameter S could refer to any cyclic stress definition, $\sigma_{\max}$ (maximum stress), σ_a (stress amplitude), or even $\Delta\sigma$ (stress range). The mathematical expression of the aforementioned statement is given in Eqs 9.1 and 9.2:

$$\text{Log}(N) = a + bS \quad 9.1$$

$$\text{Log}(N) = c + d\text{Log}(S) \quad 9.2$$

Based on their mathematical expressions, the first model, Eq. 9.1, is called ‘lin–log’ or ‘exponential’ and the second one, Eq. 9.2, ‘log–log’ or ‘power law’ as it is equivalent to a power curve of the form:

$$N = KS^d \quad 9.3$$

In the previous equations, a , b , c , d and K are material parameters that must be determined, in pairs, by fitting one of the aforementioned fatigue models to existing experimental data. Since the stress parameter S alone cannot define the loading pattern, another parameter must always be used in the fatigue model – either another stress parameter, or the stress ratio, $R = \sigma_{\min}/\sigma_{\max}$. Long-life testing has demonstrated that between Eqs 9.1 and 9.2, Eq. 9.2 is the preferred formulation for extrapolation to lower levels of stress, outside the range of experimental data [23, 24].

The easiest way to estimate the material parameters is via linear regression analysis (with N as the dependent variable), which can be performed even by hand calculations. The resulting S–N curve yields an estimate of the mean time to failure as a function of the corresponding stress parameter.

Other types of S–N curve formulations are usually employed to take into account the statistical nature of fatigue data, e.g. [25, 26]. Recently, artificial intelligence methods and soft computational techniques have been introduced for the fatigue life modeling of composite materials. Artificial neural networks

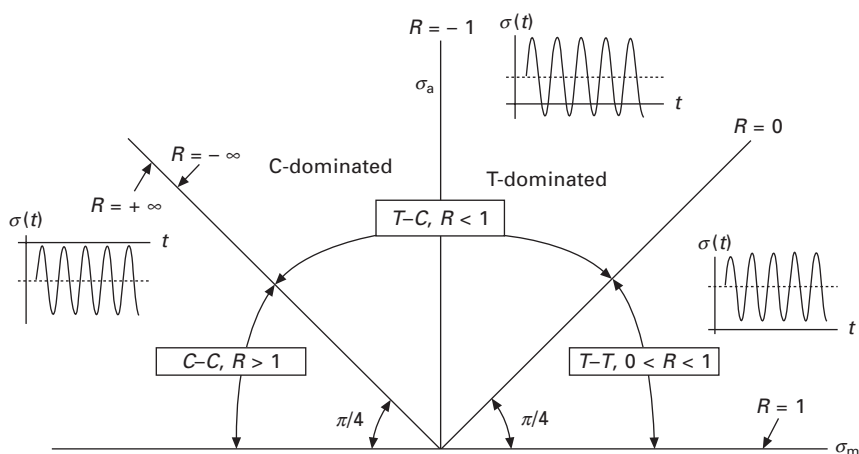
[27], adapted neuro-fuzzy inference systems [28] and genetic programming [29] have proved very powerful tools for modeling the non-linear behavior of composite laminates subjected to constant amplitude loading. They can be used to model the fatigue life of several composite material systems, and can be favorably compared with other modeling techniques.

A comprehensive review of the available methods is presented in Chapter 2 of this book. A description of the new computational methods is presented in Chapter 5. A model for the stochastic interpretation of the fatigue data is presented in Chapter 7.

9.2.3 Assessment of the mean stress effect

The effect of the mean stress on the constant amplitude fatigue life of the material is assessed using constant life diagrams. Constant life diagrams offer a predicting tool for estimation of the fatigue life of the material under loading patterns for which no experimental data exist. The main parameters that define a CLD are the mean cyclic stress, σ_m , the cyclic stress amplitude, σ_a , and fatigue life. The diagram connects combinations of σ_m and σ_a which lead to the same number of cycles to failure. Lines of constant R are straight lines emanating from the origin of this σ_m - σ_a diagram. A typical CLD is presented in Fig. 9.2.

As shown, the positive σ_m - σ_a half-plane is divided into three sectors, the central one comprising combined tensile and compression loading. The tension-tension (T-T) sector is bounded by radial lines representing the S-N curves at $R = 1$ and $R = 0$, the former corresponding to static fatigue (creep) and the latter to tensile cycling with $\sigma_{\min} = 0$. S-N curves belonging to this sector have positive R -values of less than unity. Similar comments



9.2 Annotation for σ_m - σ_a plane [30].

regarding the other sectors can be derived from the annotations shown in Fig. 9.2. To every radial line with $0 < R < 1$, i.e. in the T–T sector, corresponds its symmetric line with respect to the σ_a -axis, which lies in the compression–compression (C–C) sector and whose R -value is the inverse of the tensile value, e.g. $R = 0.1$ and $R = 10$.

Radial lines emanating from the origin are expressed by:

$$\sigma_a = \left(\frac{1-R}{1+R} \right) \sigma_m \quad 9.4$$

and usually represent a single S–N curve. Points along these lines are points of the S–N curve for that particular stress ratio. Constant life diagrams are formed by joining points of consecutive radial lines, all corresponding to a certain value of cycles.

Although from a theoretical point of view the aforementioned representation of the CLD is rational, it presents a ‘grey area’ when seen from the engineering point of view. This grey area is related to the region close to the horizontal axis which represents loading under very low stress amplitude and high mean values with a culmination for zero stress amplitude ($R = 1$). The classic CLD formulations require that the constant life lines converge to the ultimate tensile stress (UTS) and the ultimate compressive stress (UCS) independent of the loading cycles. However, this is a simplification originating from the lack of information concerning the fatigue behavior of the material at high-mean, low-amplitude loading, and creep loading, where no amplitude is applied. Although modifications to take the time-dependent material strength into account have been introduced, their integration into the CLD formulations requires the adoption of additional assumptions, see e.g. [31, 32].

Several different models have been presented in the literature [30, 31, 34–44]. Starting with the basic idea of the symmetric and linear Goodman diagram and the non-linear Gerber equation, different modifications were proposed in order to cover the peculiarities of the behavior of composite materials. Linear interpolation between different S–N curves in a modified Goodman diagram concept was used in several cases [30, 31, 33]. Analytical expressions for the theoretical derivation of any desired S–N curve were developed based on this idea [5]. Other proposed models tried to use a minimum of experimental data, comparable to the simple Goodman diagram, and simultaneously accommodate the particular characteristics of composites. To this end, a model proposed in [34] was introduced for the first time for composite materials by [35] and [36]. An alternative semi-empirical formulation was proposed in a series of papers by Harris’s group [37–40]. The solution was based on fitting the entire set of experimental data with a non-linear equation to form a continuous bell-shaped line from the ultimate tensile stress to the ultimate compressive stress of the examined material.

The drawback of this idea was the need to adjust a number of parameters based on experience and existing fatigue data.

Kawai [41, 42] proposed the so-called anisomorphic CLD which can be derived by using only one 'critical' S–N curve. The critical R -ratio equals the ratio of the ultimate compressive over the ultimate tensile stress of the examined material. The obvious drawback of this model is the need for experimental data for this specific S–N curve and therefore, theoretically, it cannot be applied to existing fatigue databases. However, the minimum amount of data required is an asset of the proposed methodology.

Based on the Gerber line, another formulation of the CLD was proposed in [43]. This formulation provides a simple method for the lifetime prediction of laminate structures subjected to fatigue load with continuously varying mean stress and abandons any classification of the fatigue data in R -values. The drawback of this method is the complicated optimization process with five variables that must be followed in order to derive the CLD model, a method which had been independently investigated previously in [40].

A new model was proposed recently by Kassapoglou [44] based on the assumption that the probability of failure during any fatigue cycle is constant and equal to the probability of failure under static loading. According to this assumption, S–N curves under any loading pattern can be derived by using only tensile and compressive static strength data. However, the restricted use of static data masks the different damage mechanisms that develop during fatigue loading and frequently leads to erroneous results, e.g. [32].

Novel computational methods have also been employed during the last decade for modeling the fatigue behavior of composite materials and derivation of constant life diagrams based on limited amounts of experimental data, e.g. [27, 45]. These methods offer a means of representing the fatigue behavior of the examined composite materials that is not biased by any damage mechanisms and not constrained by any mathematical model description.

An overview of available methods for the construction of the CLD for composite materials is presented in Chapter 2, and [46]. A description of a method for the construction of a CLD based on only one S–N curve and the static strengths of the examined material is presented in Chapter 6 of this book.

9.2.4 Fatigue failure criterion

When appropriate modeling has been achieved, the determination of the fatigue life of a composite material is the next step. This task is relatively simple when uniaxial loading is applied to a material and a uniaxial stress state develops as a result. In this case, the fatigue life can be estimated by means of reliable S–N curves and the assistance of the constant life diagrams.

However, when complex stress states develop due to the application of either multiaxial fatigue loading patterns or uniaxial loading patterns that produce complex stress states along the material's principal system, e.g. off-axis fatigue of unidirectional laminates, a multiaxial fatigue failure criterion must be introduced to take all the stress components present into account. As mentioned in [30] the presence of a transverse normal, σ_2 , and shear stress, σ_6 , components of only 0.25% and 5.00% of the applied stress, σ_x , leads to a life reduction of 35%, due to the significantly lower strength of the material under shear loading.

A number of interactive fatigue failure criteria have been introduced, e.g. [47–51]. Most of these criteria are based on the modification of corresponding static failure criteria to take fatigue into account. Usually the static strength components of the static criteria were replaced by the S–N curves or the residual strength and thus transformed into fatigue failure criteria. The applicability of this type of criteria has been proved by comparison to experimental data relating to the off-axis fatigue life prediction of unidirectional and multidirectional laminates [6, 47–49] or prediction of the fatigue life of composite laminates under biaxial loading, tension–bending or tension–torsion, e.g. [52].

9.2.5 Damage summation

The last module in the life prediction algorithm is the accumulation of damage, which is carried out according to the linear Palmgren–Miner rule. The number of operating cycles, n , of each bin, derived from rainflow counting, is divided by the allowable number of cycles, N_f (derived directly using the S–N curve equation when a uniaxial fatigue stress state develops, or by the solution of the fatigue failure criterion for N_f when multiaxial fatigue stress fields are present) to form a partial damage coefficient. The summation of all partial damage coefficients and comparison with unity indicate whether the material will survive the application of the VA loading under examination.

Various non-linear damage accumulation models have been proposed as alternatives to the Palmgren–Miner rule to improve the life prediction of anisotropic composites under VA loading. However, these ‘rules’, developed mainly for two-stage and/or multistage block loading tests, cannot be used for spectrum loading, especially in the case of anisotropic composites.

Non-linear ‘rules’ yield more accurate life predictions than the Palmgren–Miner rule, simply because they are fitted to VA experimental data. They cannot be used for design purposes where numerous different load cases, composed of different spectra, must be examined. In this respect, they are not really damage accumulation rules.

Owen and Howe [9] developed a stress-independent non-linear cumulative damage model for chopped strand mat GRP, given by:

$$D = \sum_{i=1}^k \left[A \left(\frac{n_{oi}}{N_{fi}} \right) + B \left(\frac{n_{oi}}{N_{fi}} \right)^2 \right] \quad 9.5$$

A modification of this model with the quadratic exponent replaced by another variable parameter was presented in [7]:

$$D = \sum_{i=1}^k \left[A \left(\frac{n_{oi}}{N_{fi}} \right) + B \left(\frac{n_{oi}}{N_{fi}} \right)^c \right] \quad 9.6$$

In the above equations, n_{oi} and N_{fi} are the numbers of operating cycles and cycles to failure respectively of the i th bin, and in [7] the three parameters A , B and c were calculated using an iterative procedure aiming to fit Eq. 9.6 to the experimental data. The same model, Eq. 9.6, was also used for the life prediction of CFRP angle-ply and pseudo-isotropic laminates tested under the FALSTAFF (Fighter Aircraft Loading STandard For Fatigue) spectrum [8] and to predict the variable amplitude fatigue behavior of glass-fiber reinforced polyester laminates used for the construction of wind turbine rotor blades [5].

9.3 Theoretical background 2: strength degradation models

As indicated in Section 9.2.1, a detailed and accurate constant life diagram contributes quite significantly to accurate life prediction, and in many cases a simple (non-linear) damage rule suffices for an appropriate prediction, as indicated in many sources, e.g. [18], [46] and [53].

In many applications, life prediction based solely on the knowledge of the number of load cycles to failure does not suffice for a successful design. In these cases, the actual degradation of properties due to fatigue damage can be a critical design parameter.

9.3.1 Strength degradation in literature

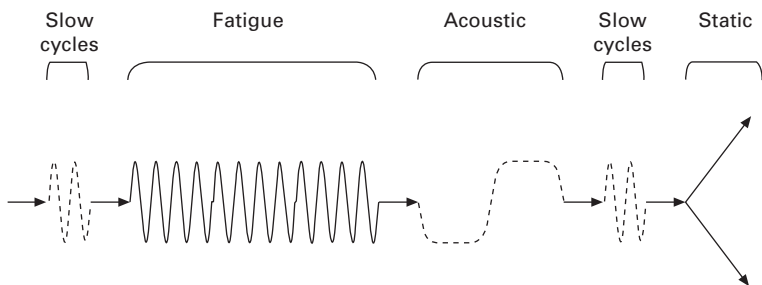
Historically, strength degradation in fatigue was described according to the concept of damage tolerance, focusing on notched specimens. A good example is the residual strength of composites with a hole, subjected to fatigue loads, the description of which is very relevant in aerospace design. On this basis, extensive research has been carried out in the field of describing the strength and stiffness degradation of aerospace composites, e.g. [11] and [54]. For unnotched wind turbine rotor blade composites, [12], [55] and [56] suggested using a strength degradation approach.

9.3.2 Acquiring strength degradation data

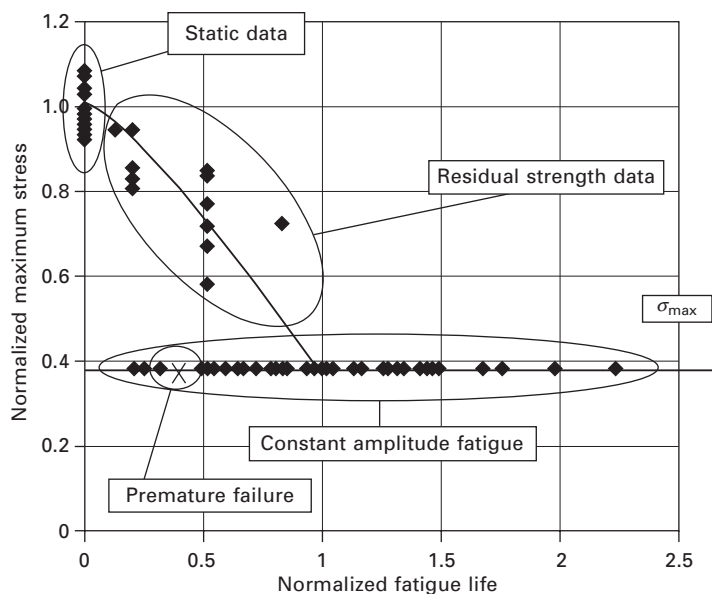
Strength degradation after fatigue can be described using residual strength tests. In these experiments, a specimen is subjected to fatigue loading and tested destructively in tension or compression at a series of predefined percentages of nominal fatigue life. Depending on the data acquisition system and condition monitoring equipment used, the fatigue test may be preceded or interspersed by a slow cycle or cycles to obtain a quasi-static value of the stiffness, or to apply acoustic methods for damage detection (e.g. [57]). Thus, the load pattern to which the specimen may be subjected can be schematically represented as in Fig. 9.3.

Experimental data and a line representing a commonly used one-parameter strength degradation model are shown in Fig. 9.4, which shows the following:

- Static strength data. These are shown as a vertical band of data on the ordinate.
- Constant amplitude fatigue data, shown as a horizontal band of data, plotted at the cyclic maximum stress $\sigma_{\max}$.
- Residual strength data. In this case, tensile strength tests were carried out at 20%, 50% and 80% of average fatigue life; hence the residual strength data are shown as vertical bands of data at these nominal life fractions.
- A premature failure. This test was intended to be a residual strength test at a certain life fraction, but failed before the intended life fraction was attained. This result cannot be considered as part of the constant amplitude fatigue population because multiple premature failures in the dataset would then bias average fatigue life to a lower value than that resulting from the constant amplitude fatigue results without premature failures.
- A strength degradation curve, connecting the average static strength to the average fatigue life, $\sigma_{\max}$. Strength degradation for this dataset is almost linear. In all strength degradation models, strength degrades



9.3 Typical load pattern for a residual strength test.



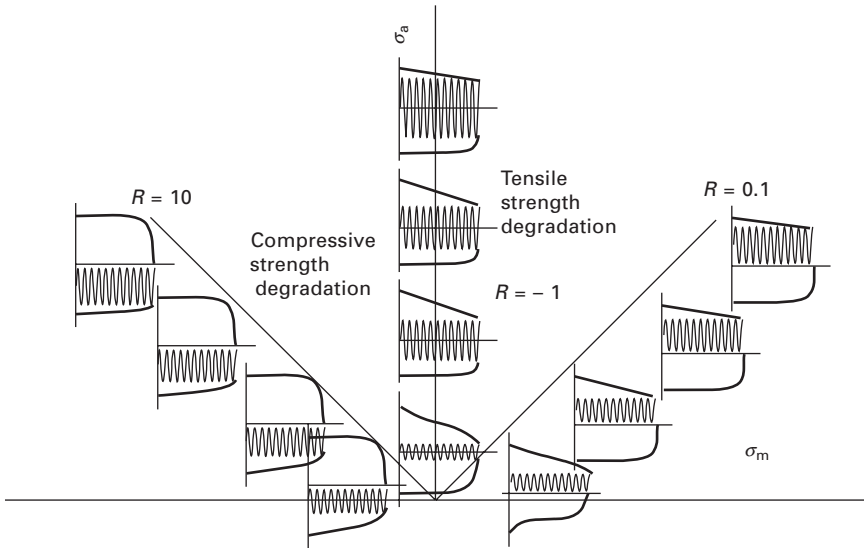
9.4 Experimental strength degradation data and one-parameter fatigue model.

monotonously from either tensile or compressive strength down to the maximum (of absolute values) applied cyclic load.

More extensive residual strength data for wind turbine UD-dominated laminates is included in [12]; for a recent wind turbine blade laminate fatigue research project, the Optimat project, detailed results are plotted in [58], and results from both the DOE/MSU and OptiDAT database are summarized in [18], describing degradation patterns for tension and compression strength for three R -values and three load levels for each R -value. These patterns are reproduced in Fig. 9.5. This figure shows sinusoidal loading schematically, at three different load levels, where the highest load level is for 1000 cycles nominal fatigue life and the lowest level for 1,000,000 cycles. Three loading types are shown: $R = 0.1$, -1 , and 10 . Strength degradation starts at either tensile strength or compressive strength and degrades until the maximum applied stress at the nominal number of cycles is reached.

9.3.3 Life prediction using strength degradation

A life prediction method using strength degradation can be briefly described as follows. The strength degradation model ‘tracks’ residual strength during fatigue loading, as it is affected by each subsequent loading cycle, and checks whether the following load peak does not produce stresses that exceed this



9.5 Strength degradation patterns for a wind turbine rotor blade laminate.

residual strength (note that both tensile strength and compressive residual strength need to be checked against load peaks and valleys for most realistic loadings). If the residual strength is exceeded by the developed stresses, the structure fails, and this is recorded as end of fatigue life; see Fig. 9.6.

9.3.4 Modeling strength degradation

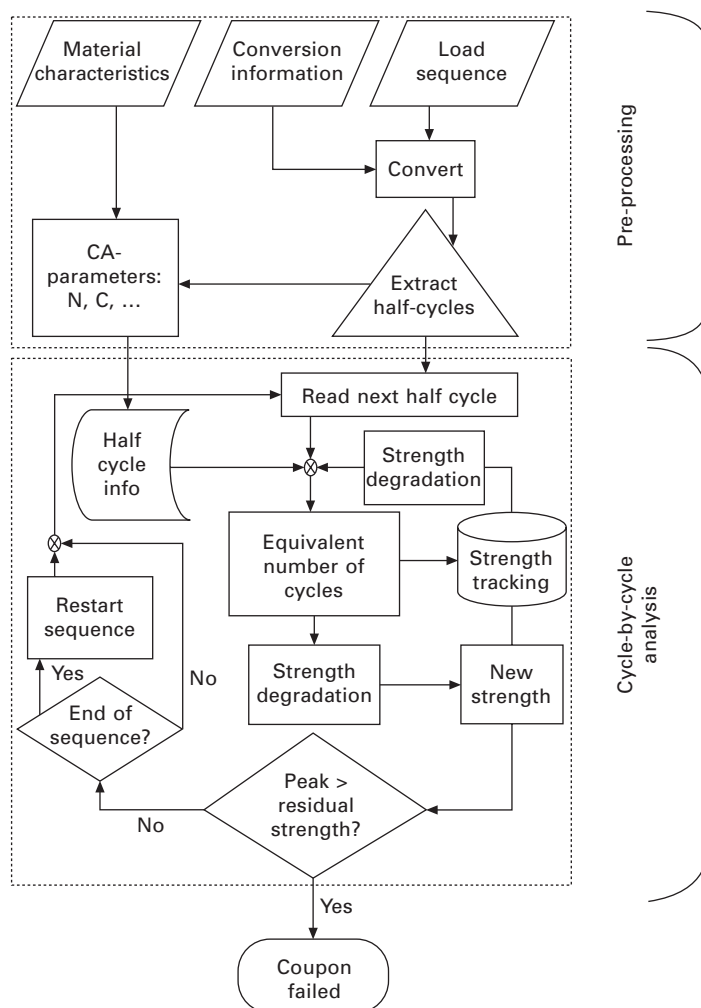
A typical formulation for strength degradation that can be used in such a model (and will be used later in this chapter) is:

$$\sigma_{r_i} = \sigma_0 - (\sigma_0 - \sigma_{\max_i}) \left(\frac{n + n_{\text{eq}}}{N} \right)_i^{C_i} \quad 9.7$$

where

- i = current cycle type
- σ_{r_i} = residual strength after n_i cycles at $S_{\max_i}$
- σ_0 = initial strength
- $\sigma_{\max}$ = maximum load applied in fatigue
- n_i = number of cycles with $S_{\max}$
- n_{eq} = number of cycles at $S_{\max}$ which would have led to strength $S_{r,i-1}$
- N = number of cycles at $S_{\max}$ which would lead to failure.

The parameter C describes the nature of strength degradation: linear degradation, early degradation, or ‘sudden death’. See also Fig. 9.7.

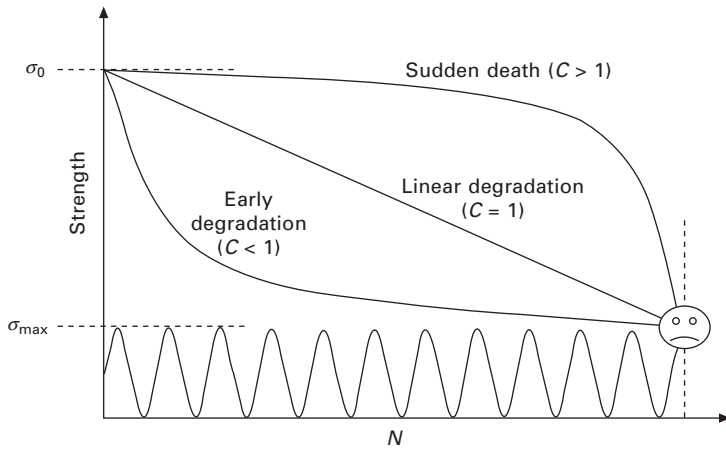


9.6 Schematic of life prediction using residual strength.

9.3.5 Load sequence effects

On a related – though more fundamental – level, strength degradation models can be used to assess the effects on the (remaining) fatigue life of the order of the load cycles. At this point, the definition of ‘sequence effect’ should be investigated to some extent. Various definitions are conceivable and are used both implicitly and explicitly in the literature.

A life prediction consists of linearly and independently summing the damage contributions of constant amplitude parts in a sequence, assuming failure occurs at the Miner’s sum of 1. If the result of a Miner summation



9.7 Schematic representation of single-parameter strength degradation model, typical types of behavior.

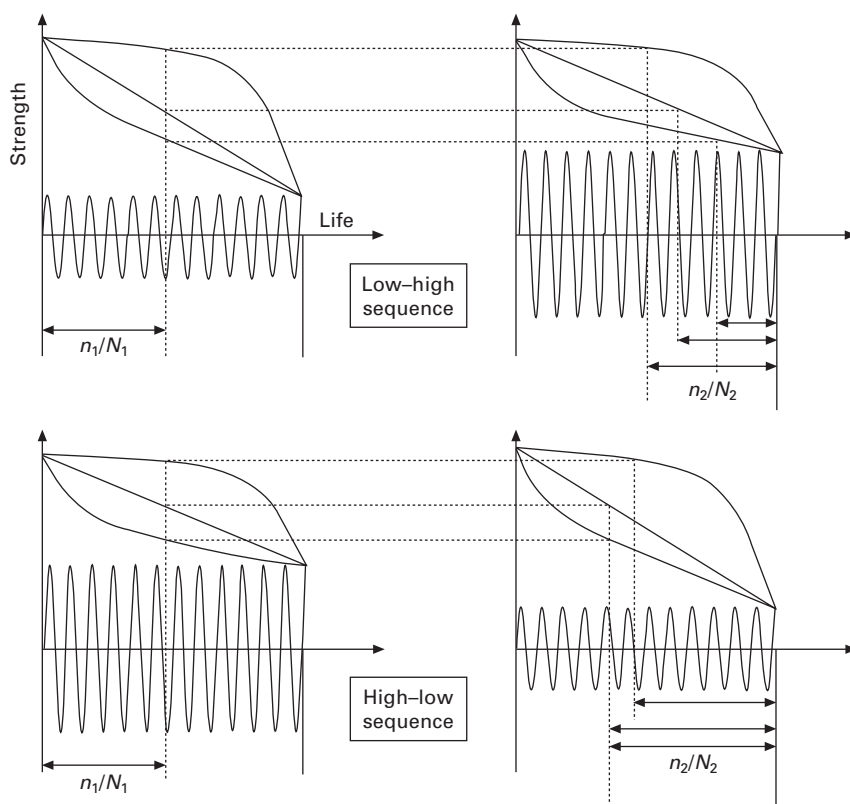
for a load sequence experiment is not equal to 1, this is usually attributed to some kind of ‘sequence effect’. It is seldom specified what the consequence of this effect is, but in such a case, the inability to predict failure accurately is typically attributed to the Miner summation, which is based on the premise that damage accumulation in two constant amplitude blocks of the sequence is not mutually influenced.

Typical cases for steel, where the fatigue limit and notch root plasticity, for example, are significant in determining damage growth in load cycles preceded by other load cycles, are described in [59]. For example, a two-block loading sequence with the first block loads lower than the fatigue limit does not initiate cracks, hence the second block, with loads above this limit, will follow the S–N curve. Contrarily, if the first block is above the fatigue limit, cracks will be initiated and, depending on the load in the second block, may grow, even if the load in the second block is below the fatigue limit. In VA life predictions for steel, this is taken into account by removing the fatigue limit from the S–N curve, and replacing it by a branch of the S–N curve with a larger slope, quantifying to some extent the change in S–N curve behavior of the composite. Another example dictates that very high loads in the first block may cause a plastic zone around the crack tip, which will delay crack growth in a subsequent lower loading block; if the loading sequence is reversed, notch root plasticity may not have this crack growth retardation effect. The author describes the variation in damage accumulation in different subsequent loading types as ‘interaction effects’, and considers these responsible for ‘sequence effects’.

Note that sequence effects can be predicted without interaction effects. Strength degradation models can predict a Miner’s sum different from unity

(sequence effect) while the damage accumulation, i.e. strength degradation pattern, in two loading blocks is not (mutually) influenced; in other words, the interaction effect is absent. This can be evaluated using an example of two-block variable amplitude loadings. If a two-stage load sequence starts with the high block, ends with the low block, and strength degradation is linear, different Miner's sums are predicted for a high–low and a low–high sequence, as illustrated in Fig. 9.8. This prediction is fundamentally different from the similar examples obtained for steel. In the current example, neither the strength degradation type (linear degradation) nor the nominal life for either constant amplitude block changes.

Note that if the strength degradation in the example were of the ‘sudden-death’ type (strength does not degrade throughout most of the fatigue life), the Miner's sums predicted for both sequences would be very close to 1, as demonstrated in [18] and [60]. Strength degradation patterns have been shown to be sudden-death for wind turbine laminates in compression fatigue



9.8 Miner's sum for two-block tests, linear strength degradation model.

by [18] and [61]. This implies that for a spectrum with loads following sudden death strength behavior the added value of a strength degradation model is very limited, as the life prediction will be identical to a much less time-consuming Miner's sum, and the additional strength information is obviously trivial.

In the fatigue of composites, the search for interaction effects and sequence effects is hampered by large scatter (typically one decade in fatigue life), implying that a very large number of fatigue tests and block tests is required to find sequence effects that may be marginally smaller than the scatter in fatigue. In addition, different constituent materials and laminate lay-ups may result in quite different VA fatigue behavior.

9.3.6 Multiple block loadings

The aforementioned two-block tests, in which each block occurs only once, are very useful to demonstrate and investigate the effect of sequence on life and strength. However, block loading is not close enough to a more realistic loading spectrum.

The effect of sequence for repeated blocks was illustrated by Wahl [12]. The author investigated the effect of variable amplitude for wind turbine rotor blade composites thoroughly. He compared sequences consisting of a repeated pattern of a high load and a low load block. The high load block had a nominal life of ~10,000 cycles; the low load block had a nominal life of ~1,000,000 cycles. Three different patterns were used:

- 10 high followed by 1000 low
- 10 high, 100 low (10 times)
- Blocks of 10 high loads randomly interspersed in the 1000 cycles.

The patterns were repeated until failure. No statistically significant difference in cycles to failure was found and thus there was no 'load sequence effect'.

On the other hand, 95% of the tests had a Miner's sum at failure of lower than 1, averaging 0.1. This means that although the exact sequence within the repeated patterns did not have an effect on life, actually mixing the different cycles did not yield the total fatigue life that would have been expected based on the Miner's sum. Thus, the order of load cycles within a pattern did not have an effect on life, whereas alternating between loads did.

Schaff and Davidson [62, 63] attributed the mismatch with the expected Miner's sum to the mixing of cycles of different types and proposed a 'mix' factor to quantify the effect of mixing rather than sequence. In [12] the 'effect of load interaction' is also mentioned. Note that regarding the abovementioned effect of sudden-death strength degradation, compression-repeated two-block tests performed in [12] did show that the load interaction effect diminished in comparison to the tensile cases.

9.3.7 Advantages and disadvantages of strength degradation compared to classic methodology

Knowledge of the actual strength degradation in composites is not necessary for calculating fatigue life. Nevertheless, it may be useful to use a strength degradation model for fatigue life prediction in realistic loading conditions.

The main weakness of the ‘classic’ approach, which includes the previously mentioned ‘damage’ rules, is that no information regarding the actual damage exhibited by the composite is actually included in the life prediction. The prediction, even if it can be considered accurate, yields a binary result: either the structure failed, or it did not. In contrast, a model that describes property degradation may not describe the nature of the damage (location, size, or orientation of cracks), but it does quantify the symptoms of the damage – a reduction in strength, for example.

The ability to quantify strength reduction at any point during realistic loading applied to the structure constitutes the major asset of this method for life prediction, but at the same time is the source of its main drawbacks. Obviously, if the residual strength after fatigue loading can be quantified, in many applications, combining this knowledge with load forecasting and load control/alleviation is potentially useful for profitable operation.

As an example, wind turbine blades are not designed according to a site-specific load description, but for a ‘wind class’, a site category describing the typical expected loading. This implies that any turbine might be over- or under-designed for a more specific site or when site conditions deviate from the expected loading. Recording blade loads during operation, and deriving blade residual strength, can provide an estimate of the blade’s structural reserves and allow condition-based operation (at the time of writing, such a strategy, sometimes referred to as the ‘life-odometer’, is still only a concept).

One of the drawbacks of strength degradation modeling is that the experimental determination of strength degradation is associated with a very large number of experiments. Scatter along both the life and strength axes requires a relatively large number of test duplicates. Residual strength must be measured in both tension and compression, and for various loading types. However, as it can only be measured destructively, the experimental effort required for describing strength degradation after fatigue for a representative number of load types is quite considerable.

Once a number of strength degradation results are available, extraction of the actual strength degradation parameter(s) from residual strength experiments is not straightforward, as censoring of high life-fraction residual strength data should be taken into account (including premature failures), and assumptions regarding the number of parameters have to be made based on the shape of the strength degradation patterns.

Another drawback is that prediction of life and strength using strength degradation models is performed on a cycle-by-cycle basis, or rather on a peak-by-peak basis, which means that for the entire sequence, regardless of whether it is composed of sub-patterns, all peaks and valleys need to be evaluated one by one, and if the sequence is repeated, the counting must be repeated. This is obviously more time-consuming than using the Miner's sum, which can be based on a summary of the load sequence from a single cycle-counting process. On a more detailed level, the direct application of cycle-counting routines which do not necessarily process all cycles in order of occurrence, such as rainflow counting, is not possible for residual strength models. A method of dealing with this to some extent was proposed in [18]. This report suggested transforming an existing sequence by reordering the cycles in such a manner that when the cycles were counted cycle-by-cycle, the counting results would be similar to the results of a rainflow counting of the original sequence (see below).

Constant amplitude fatigue behavior is described for all possible combinations of mean and cyclic amplitude as was illustrated in Chapter 2 by interpolating between known values of fatigue stress parameters and static strengths. The same holds true for strength degradation information; it is directly available only from experiments for selected load cases. For other load cases, the strength degradation parameters can only be obtained by interpolation, or by assuming certain values for strength degradation parameters per region of the constant life diagram.

9.4 Experimental data

A considerable amount of fatigue data for composite materials (especially composite laminates) can be found in the literature and existing databases contain data covering a significant number of loading cases. Some of these databases are relatively limited and refer to a specific material system, primarily aimed at the verification of new theoretical models, e.g. [30, 48, 64]. Other databases are more extensive, however, and were developed for the characterization of entire categories of materials primarily used in specific applications, such as the DOE/MSU [29] and OptiDAT [65] databases for materials used in the wind turbine rotor blade industry. A new fatigue database has recently been released by Virginia Tech [66] containing experimental data from axial loading on pseudo-quasi-isotropic glass/vinyl ester specimens fabricated using the vacuum-assisted resin transfer molding (VARTM) technique. Data from the OptiDAT database [65] will be used here for the demonstration of the described methods. Complementary data from [5] and [30] will also be used for the application of classic fatigue life prediction methodology.

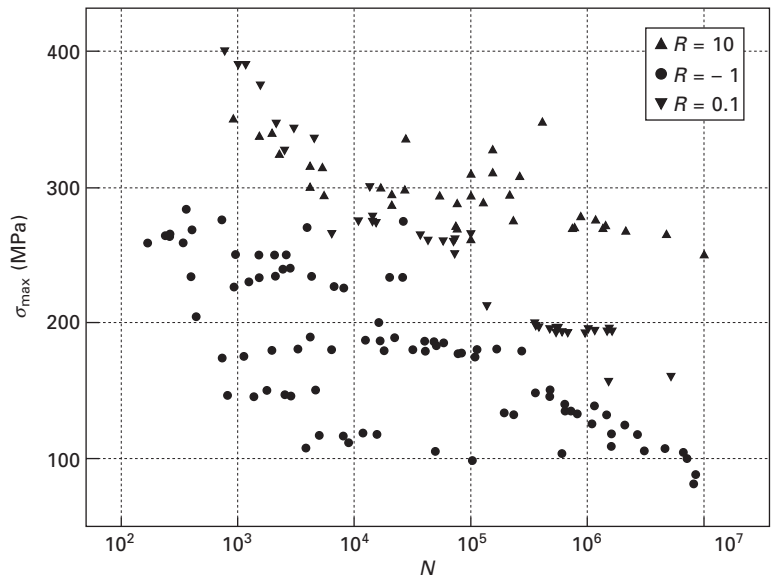
9.4.1 Multidirectional glass/epoxy laminate with stacking sequence $[(\pm 45/0)_4/\pm 45]_T$

The fatigue results from experiments on the MD2 material (OptiDAT nomenclature) have been used in this chapter. Static strength values, constant amplitude fatigue data, residual strength data and fatigue data under variable amplitude spectra are available in [65] for this material system. The MD2 material is a multidirectional glass epoxy laminate consisting of nine plies, four with fibers in the 0° direction and five stitched layers with fibers in both the 45° and -45° directions. The stacking sequence of the laminate is $[(\pm 45/0)_4/\pm 45]_T$. Laminated plates were fabricated by LM GlasFiber using the vacuum infusion technique. Non-standard specimen geometry was used in order to provide uniform specimens for tensile and compressive testing. The length of the rectangular specimens was 150 mm, the length of the tabs glued to both ends was 55 mm and consequently the free length of the specimens was 40 mm to avoid buckling during compressive loading. The thickness of the specimens of this type was 6.57 mm, and the width 25 mm.

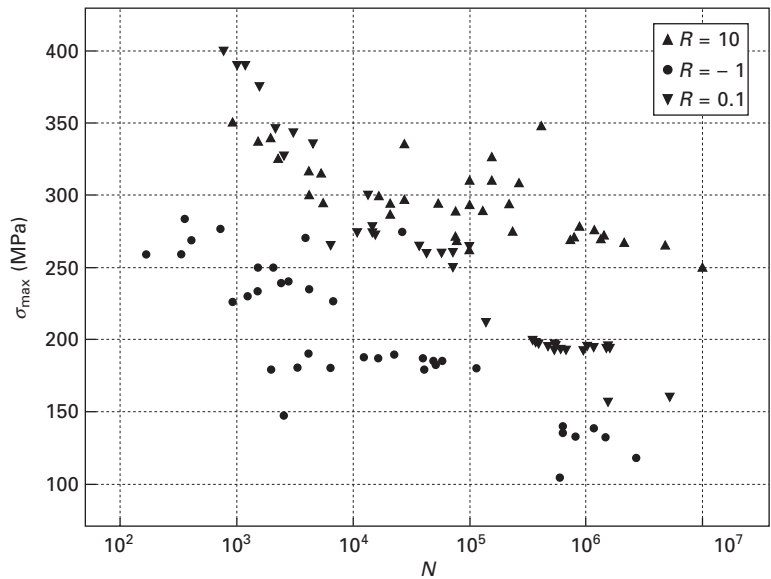
The ultimate tensile stress (UTS) and the ultimate compressive stress (UCS) of the material have been determined. The values are $UTS = 555.88 \pm 63.87$ MPa, and $UCS = -471.39 \pm 46.90$ MPa. All data found in the OptiDAT database were used in the calculations. Although a number of data points can be excluded, by following some censoring rules, the authors did not do so in order to avoid introducing another parameter into the analyses. All available data were used for the demonstration of the methods. This applies to the constant and variable amplitude fatigue data as well.

A total of 239 constant amplitude fatigue data points were retrieved from the OptiDAT database [65] for this material tested under seven different stress ratios, namely $R = 10, 2, -2.5, -1, -0.4, 0.1$ and 0.5 . For some of the aforementioned stress ratios, e.g. $R = 2$, the number of fatigue data is limited, though sufficient for the derivation of models for representation of fatigue life by means of an S–N curve. Selected S–N curve data for the cases ($R = 10, R = -1$ and $R = 0.1$) are presented in Fig. 9.9. Plotted results indicate that the material behaves similarly under tensile and compressive loading at low cycle regimes, until reaching 10^5 cycles. Beyond this lifetime, the material seems to be more vulnerable to tensile loading patterns. The high scatter that is presented in Fig. 9.9, especially for the reversed loading, is due to the fact that all recorded fatigue data were plotted, including those for which a high temperature was recorded during loading, tests under high loading rate, those characterized as tab failures, etc. Data-censoring techniques can be used to refine the exported data. The S–N data after censoring are presented in Fig. 9.10. The entire data set with all test details can be found in [65].

Residual tensile and compressive strength results were also recorded. Specimens were tested quasi-statically under tensile or compressive loading



9.9 Selected S–N curve data for the MD2 material [65].



9.10 Censored S–N curve data for the MD2 material [65].

after different fractions of the expected fatigue life (i.e. average life of constant amplitude tests carried out at the same load level), namely at 20%, 50% and 80% of the nominal average lifetime. To obtain a good image

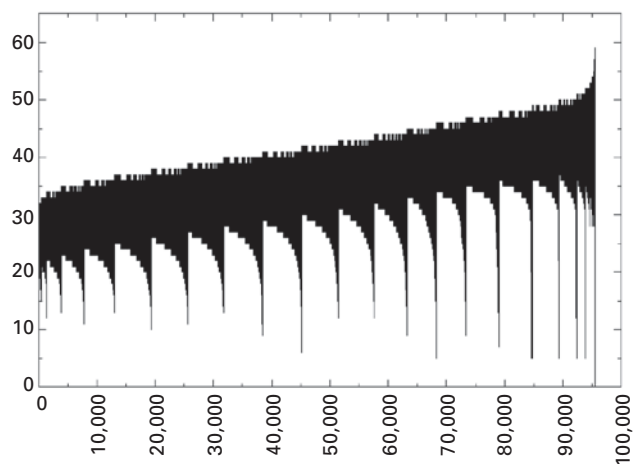
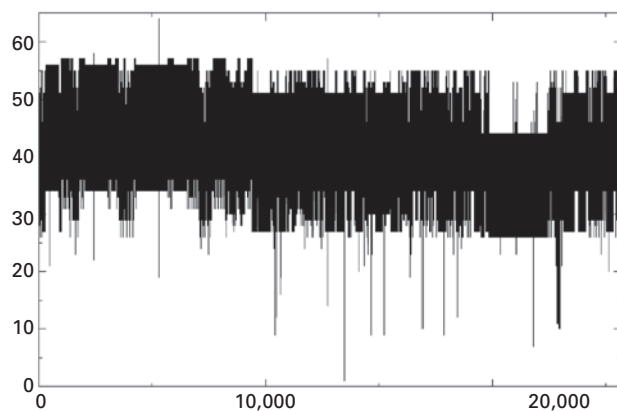
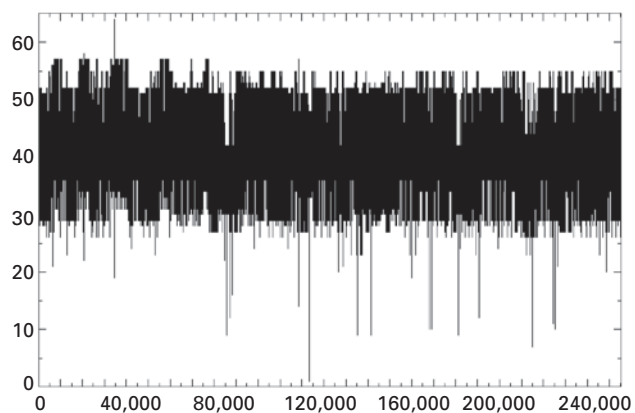
of the possible variation of strength degradation as a function of constant amplitude fatigue load level and loading type, residual strength tests were carried out at $R = 0.1$, -1 and 10 . For each of these R -values, three distinct load levels were chosen, with nominal target fatigue lives of 100, 50,000 and 1,000,000 cycles. A sufficient number of constant amplitude fatigue tests were performed to obtain a reliable estimate of average fatigue life at this load level. Subsequently, residual strength tests were performed in both tension and compression at each of the three levels. As each residual strength test was repeated in quadruplicate, and especially at the higher target life fractions of 50% and 80%, significant premature failures occurred, some 600 CA and residual strength tests were required to obtain a schematic overview of the MD2 strength degradation behavior.

Nominally identical specimens were also tested under variable amplitude loading spectra. The WISPER (W) and WISPERX (WX) spectra [67], well known in the wind turbine rotor blade industry, but also a new WISPER (NW) spectrum [68] were used for this purpose during the Optimat Blades project. A summary of these load sequences is shown in Fig. 9.11 and Table 9.1. The load spectra are 'dead' sequences of integers. Figure 9.11 shows the value of the integers versus peak/valley number. The line at level 25 indicates zero stress level. Table 9.1 shows some more detailed statistics.

The experimental results of the W and WX spectra are presented in Fig. 9.12 and will be used in this chapter. In Fig. 9.12, $\sigma_{\max}$ denotes the stress corresponding to the maximum load in each time series. Note that the W and WX tests show that, although the number of cycles is quite different, there is no significant difference in damage between the two sequences for this dataset and in the stress range shown. Nevertheless, at lower maximum stresses, WISPERX yields longer lives than WISPER. Note that in [69] the WISPERX spectrum was found to be less damaging than the WISPER spectrum over a wider range of maximum stresses.

9.4.2 Multidirectional glass/polyester laminate with stacking sequence $[0/(\pm 45)_2/0]_T$

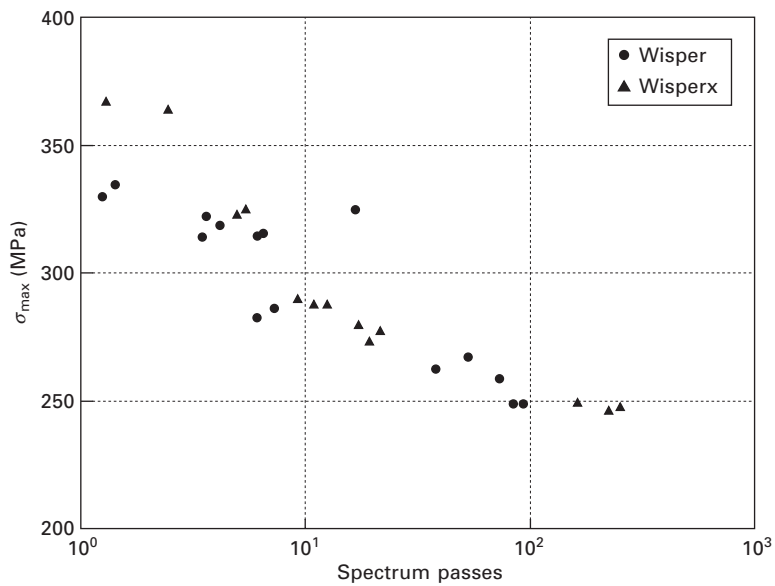
A database created during recent years by one of the authors [5, 30] is also used in this work. It refers to specimens cut at on-axis and several off-axis angles from a multidirectional GFRP composite laminate. The stacking sequence of the on-axis specimens was $[0/(\pm 45)_2/0]_T$ which is a typical material used in the wind turbine rotor blade industry. Seven different material configurations were tested as specimens were cut at seven different angles from the multidirectional laminate, namely 0° , 15° , 30° , 45° , 60° , 75° and 90° . Constant amplitude fatigue tests were performed at a frequency of 10 Hz using an MTS 810 servo-hydraulic test rig of 250 kN capacity. In total



9.11 From top to bottom: the WISPER, WISPERX and NWISPER VA time series.

Table 9.1 Statistics of the VA spectra

	Spectrum		
	WISPER	WISPERX	NEW WISPER
Integers			
Minimum	1	1	5
Maximum	64	64	59
Zero stress	25	25	22
Number of cycles	132,711	12,831	47,735
Average <i>R</i> -value	0.394	0.248	0.213
Levels			
Cumulative levels	3,612,010	487,864	1,397,142
Average segment length	14	19	15
Average level	14	41	34
Maximum at segment	34,482	5298	95,459
Minimum at segment	123,303	13,482	1

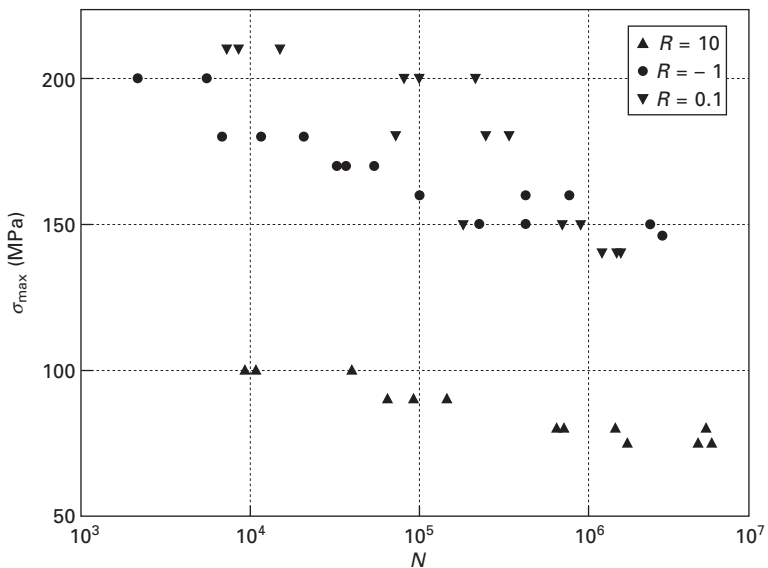


9.12 Lifetime of MD2 laminate under WISPER and WISPERX spectra.
The entire dataset can be retrieved from the OptiDAT database [65].

257 valid constant amplitude fatigue data points were collected. Tests were conducted under four different stress ratios ($R = \sigma_{\min}/\sigma_{\max}$), two corresponding to tension-tension loading ($R = 0.1$ and 0.5), one corresponding to tension-compression loading ($R = -1$) and one to compression-compression loading ($R = 10$). At least three specimens were tested at each of the four or five

stress levels preassigned for the derivation of each S–N curve. This resulted in the existence of at least 12 and up to 18 specimens for each S–N curve data set, in a range between 1000 and 5.3 million cycles. The experimental data produced were used for the derivation of 17 S–N curves corresponding to various loading patterns and material configurations. Additionally, variable amplitude fatigue tests were performed. A modified version of the WISPERX spectrum, which contained only tensile fatigue cycles, was used in order to exclude buckling from the possible failure modes and designated MWISPERX. Specimens cut at on- and off-axis angles were tested under this spectrum. The on-axis results will be used here for the demonstration of the applicability of classic fatigue life prediction methodology.

Typical S–N data for selected cases ($R = 10$, $R = -1$, $R = 0.1$) are presented in Fig. 9.13. Results prove the similar behavior of the material under tensile and compressive loading patterns, with the tensile being slightly superior. This behavior is reflected in the static strength data as well, as the UTS of the examined material equals 244.84 MPa and the UCS equals 216.68 MPa. However, results [5] proved that there was a significant influence of the loading rate on these values. The UTS of the material was 1.7 times higher than the reported value when a high rate (40 kN/s load control) was applied.

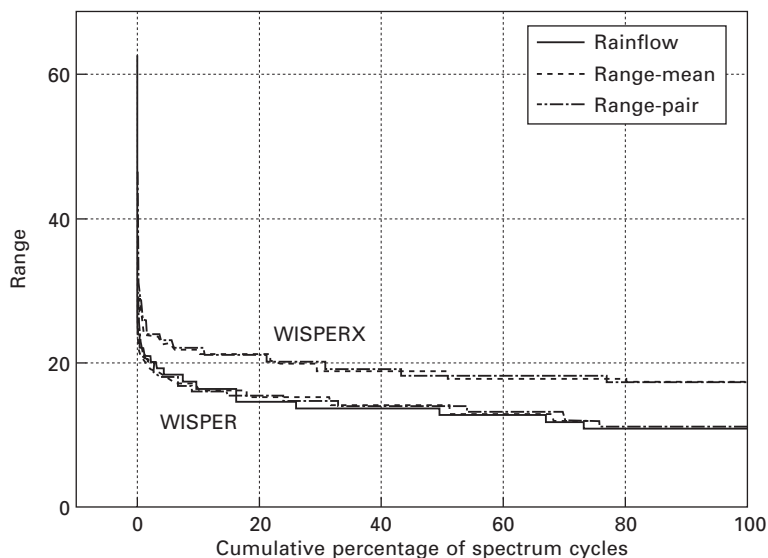


9.13 Selected S–N data for the examined glass/polyester laminate.

9.5 Life prediction examples – discussion

9.5.1 Cycle counting

An introduction to the different methods for counting the load reversals in a fatigue spectrum can be found in Section 2.1. A more detailed description of the methods and their advantages and disadvantages can be found in [18]. Different methods have been presented, and three of the most commonly used, rainflow counting, range–mean and range–pair counting, will be implemented in the framework of the presented fatigue life prediction methodologies. As presented in Fig. 9.14, the cumulative spectra that result from all methods are similar for both examined variable amplitude time series. However, both the range–pair and range–mean methods count more cycles than does rainflow counting. Looking at the two spectra, 12,831 cycles can be counted (rainflow) for the WISPERX, with no cycle with a range of less than 30% of the maximum range value in the spectrum. On the other hand, WISPER contains 132,711 cycles (rainflow) and the minimum counted range is approximately 7% of its maximum value. The modified WISPERX used for the second material of this study presents the same characteristics as the original spectrum. As shown in Fig. 9.14, and in [18], the difference in the results from each counting method depends partly on the regularity and organization (e.g. autocorrelation) of the load sequence itself.



9.14 Cumulative spectra of WISPER and WISPERX time series.

9.5.2 Interpretation of fatigue data – S–N curves

Four common methods for the interpretation of available fatigue data and the derivation of the S–N curves will be implemented here. Two are based on a linear regression on a lin–log or a log–log scale, while the other two (Whitney [26] and Sendekyj [25]) use Weibull statistics for the data analysis in order to take into account the statistical nature of the fatigue data as well. The method proposed by Sendekyj [25] is fundamentally different from the linear regression and Whitney’s method in the sense that it makes use of the SLERA (Strength Life Equal Rank Assumption), thus assuming that fatigue life is related to initial static strength and residual strength. Simply put, the method assumes that the stronger specimen under quasi-static testing should be the one with the longer fatigue life or the higher residual strength after a certain number of fatigue cycles. Although this assumption cannot be validated experimentally, it allows all available data, i.e. static strength, constant amplitude fatigue and residual strength, to be used for derivation of the S–N curve.

Normally, constant amplitude fatigue data are collected in the range between 100 and 10^7 loading cycles. Therefore, a regression is necessary for the modeling of the fatigue life of the examined material in the very low-cycle fatigue region. The use of the static strength data can, theoretically, assist this regression, since they can be considered as the fatigue strength for one cycle until failure.¹ However, it is not certain that this assumption should be adopted, for reasons such as the following:

- The static strength data was, in most cases, obtained at strain rates much lower than the maximum strain rates in fatigue loading.
- Even in cases when the static strength data was collected under strain rates comparable to the corresponding rates in fatigue loading, the assumption that the static strength data should be part of the S–N determination process has not been validated, e.g. [69]. As mentioned in [69], an increase in static strength of around 35–40% was observed when the quasi-static tests were performed under a ‘fatigue strain rate’. This was higher than that ‘predicted’ by a lin–log regression line excluding static data, but considerably lower than that suggested by extrapolating a log–log regression line excluding static data to $N = 1$.
- Although the use of static strength data, especially if data at a ‘fatigue strain rate’ exist, can improve modeling at the low-cycle fatigue range, it affects the entire S–N curve and therefore the regression for the high-cycle fatigue range without any justification.

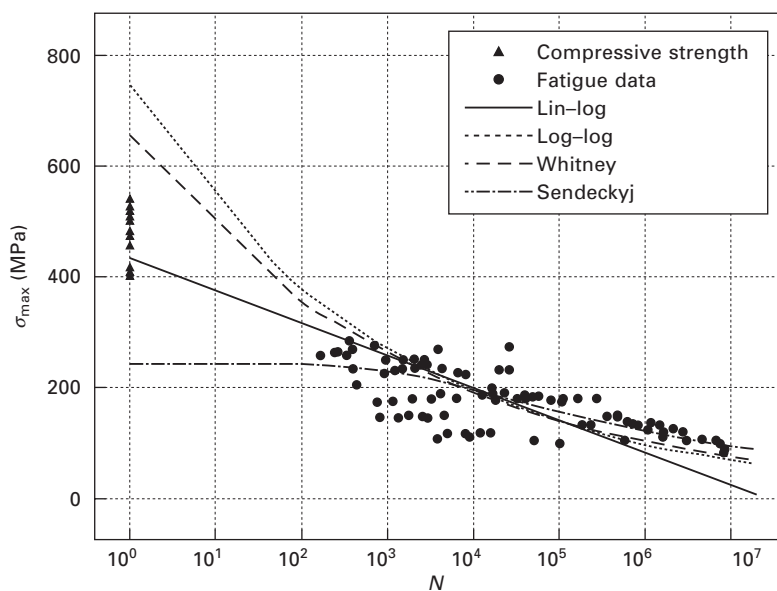
¹ It can even be argued that this should be 0.25 cycles to failure, or 0.75 cycles to failure, if the load is sinusoidal starting with tension, and the failure mode is tensile or compressive, respectively.

- The failure modes under quasi-static loading are in general different from the fatigue failure mode and therefore the static strength data cannot be considered as valid fatigue data for $N = 1$.

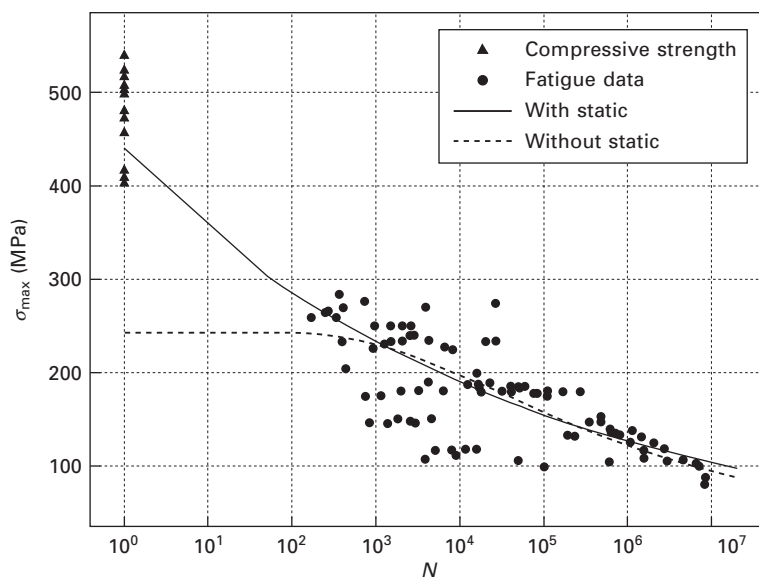
In principle, three of the four used S–N curve formulations can include static strength data. Whitney does not include data of this type by definition [26]. As presented in Fig. 9.15, all curves are able to model the fatigue data reasonably well in the range between 10^2 and 10^6 cycles, with the lin–log formulation being more conservative than the others in the high-cycle range. The method proposed by Whitney is based on a linear regression of the characteristic number of cycles per stress level versus the corresponding stress level on a log–log scale. Therefore it leads to S–N curves similar to those produced by the log–log formulation.

The inclusion of static strength data can considerably affect the shape of the S–N curve. As shown in Fig. 9.16, use of the compressive strength data in the formulation derives an S–N curve that is less conservative at both the low- and high-cycle fatigue regimes for the examined material.

For the demonstration of the fatigue life prediction methodology and estimation of the damage index in the following paragraphs, the log–log formulation without considering static strength data as part of the fatigue data pool will be used. Calculations based on the Sendeckyj model with static strength data will also be presented for comparison.



9.15 S–N curves of the glass/epoxy material [65] for reversed loading. No static data was included in the calculations.



9.16 S–N formulation based on Sendeckyj [25] for the glass/epoxy laminate [65] under reversed loading.

9.5.3 Constant life diagrams

A detailed description of the available constant life diagram (CLD) formulations is presented in Section 2.3. One of the models (Kawai) is analytically described and validated for a wide range of materials in Chapter 6 of this book. As pointed out in [46] the selection of an accurate CLD formulation is essential for the overall accuracy of a fatigue life prediction methodology. As shown, the ‘wrong’ choice can produce very conservative or very optimistic S–N curves, which is directly reflected in the corresponding life assessment.

Four criteria were considered in [46] in order to evaluate the applicability of the examined CLD models and assess their influence on the fatigue life prediction of the examined composite materials:

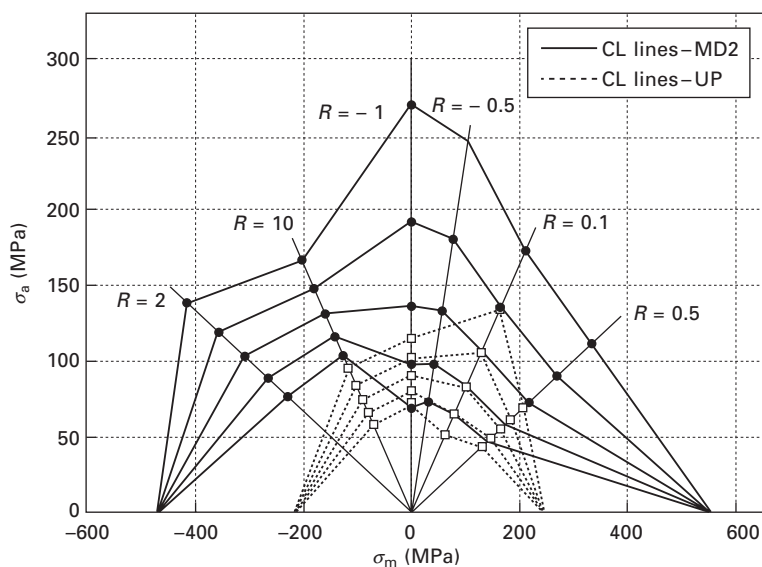
- Accuracy of predictions: quantified by the accuracy of predicting new S–N curves
- Need for experimental data: quantified by the number of S–N curves required to form each CLD model
- Difficulty of application: qualitative criterion
- Implemented assumptions: qualitative criterion.

Comparison of the commonly used CLDs revealed that the piecewise linear is more stable than the others since it is not based on any assumption. It is constructed by linear interpolation over the available fatigue data and therefore accurately depicts their behavior. The other three diagrams examined were

very sensitive to the selection of input data and the estimation of model parameters.

All methods entail the problem of mixing static and fatigue data. Their accuracy is reduced when curves close to $R = +1$ (in tension or compression) have to be predicted. Moreover, the same applies for the derivation of accurate S–N curves to describe the very low-cycle fatigue regime, i.e. $N < 100$. The unified equation that is used in Harris's model to describe fatigue behavior under both tension and compression loading also considers the influence of the damage mechanisms that developed under different loading patterns. All other models work separately for tension and compression loading, i.e. they are based on different equations for the description of different parts of the constant life diagram.

Application of the piecewise linear model to the examined fatigue data [65] and [30] leads to the derivation of the CLDs presented in Fig. 9.17. All available fatigue data were used for the construction of both diagrams, i.e. constant amplitude fatigue data under $R = 10, 2, -1, -0.4, 0.1$ and 0.5 for the MD2 material from the OptiDAT database [65] and $R = 10, -1$ and 0.1 for the glass/polyester laminate (UP) from [30]. The influence of the resin and method of fabrication on material strength is obvious. Although both laminates consist of glass fiber layered with similar stacking sequences, the MD2 laminates were fabricated by using the vacuum infusion technique and



9.17 Piecewise linear constant life diagram for both examined materials. Constant life (CL) lines for cycles between 10^2 and 10^7 are presented.

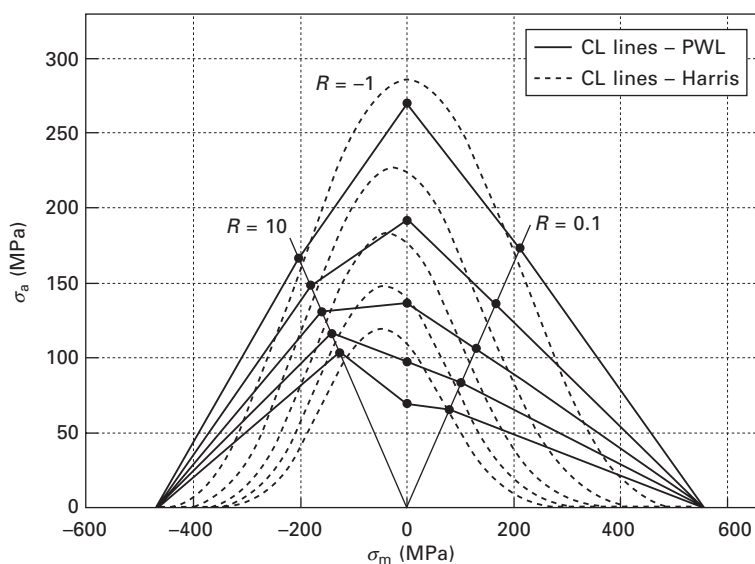
epoxy resin, while the UP laminates were fabricated using the hand-lay-up technique, with polyester resin cured at ambient temperature.

A comparison of two of the most commonly used constant life diagram formulations is shown in Fig. 9.18 for the MD2 material. The derived CLDs were based on the experimental data under the three basic R -ratios, $R = 10$, -1 and 0.1 . As shown in Fig. 9.18, the PWL diagram is more accurate as it is formulated based on an interpolation of available experimental data. However, the Harris diagram seems to address the problem of the CLDs close to $R = +1$ more efficiently.

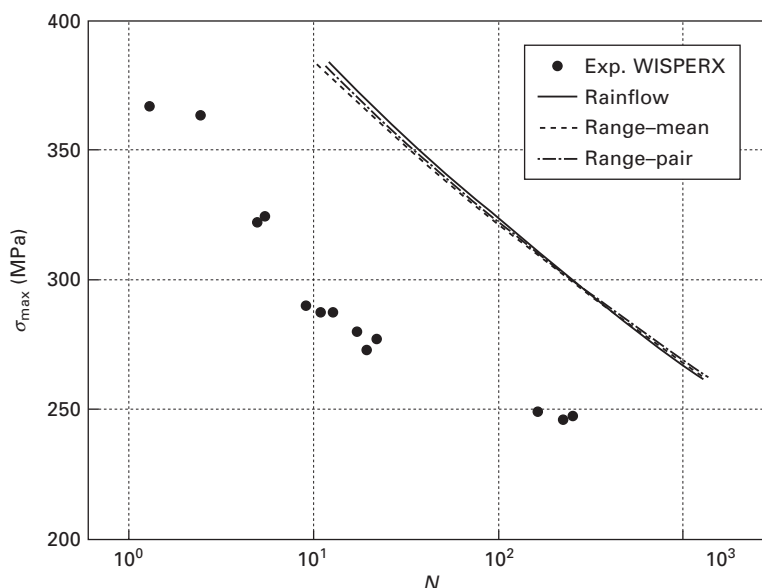
9.5.4 Lifetime predictions

In a classic fatigue life prediction methodology, the failure index is represented by the Miner sum. Other damage summation rules can be used for this purpose to address several parameters that cannot be dealt with by the simple, linear, Miner damage rule, such as the load sequence effect. However, a number of variable amplitude fatigue data are necessary for the application of any other rule except Miner in order to estimate the model parameters.

The Miner rule is used here for the demonstration of the classic life prediction methodology, and the effect of the selection of the method from among several possibilities for the solution of each one of the steps of the process is discussed. As shown in Fig. 9.19, the selection of the cycle-



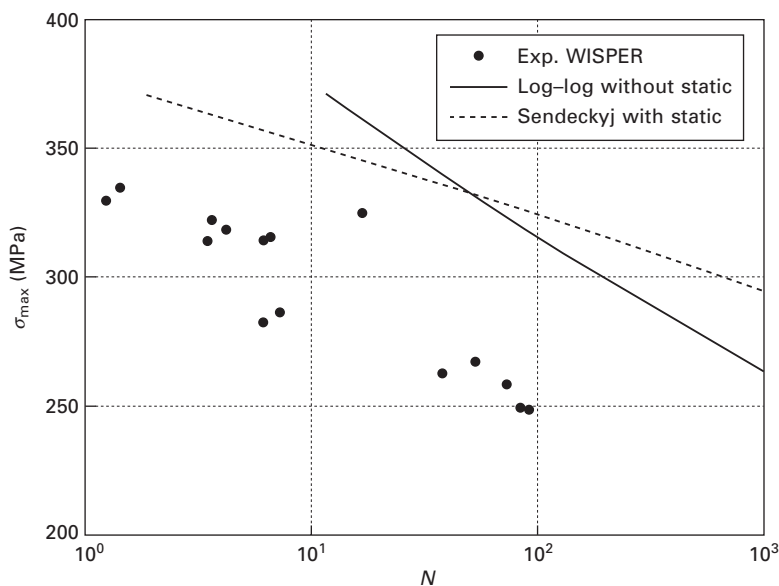
9.18 Piecewise linear vs. Harris constant life diagram for the MD2 material. CL lines for cycles between 10^2 and 10^7 are presented.



9.19 Lifetime prediction – cycle counting: rainflow counting/range-mean/range-pair counting; S–N curve: log–log without static; CLD: PWL; damage summation: linear Miner. Experimental data: WISPERX on MD2 laminate.

counting technique – rainflow, range-mean or range-pair counting – cannot significantly affect the damage calculation. This result was expected since all three methods produced similar cumulative spectra as shown in Fig. 9.14. When the static strength data are included in the process for the derivation of the S–N curve, the life prediction results can be significantly different (see Fig. 9.20). Use of the static strength data together with the Sendecyk S–N formulation accurately estimates lifetime at higher stress levels, but becomes less accurate at lower stress levels. This result reflects the behavior presented in Fig. 9.16 where use of the static strength data leads to the derivation of a less conservative S–N curve for the high-cycle fatigue regime.

In Fig. 9.21, S–N-curve/CLD-based life predictions are compared with the WISPER data on MD2. The same S–N formulation was used in all plots (log–log formulation), and the CLD formulation was varied between a ‘single R -value’ linear Goodman diagram, a three R -value PWL constant life diagram (using $R = 0.1, -1$ and 10), and a CLD using all seven R -values available for the MD2 laminate. Finally, the strength degradation model was used to predict fatigue life on a cycle-by-cycle basis, leading to the most conservative prediction. This prediction also used the multiple R -value CLD formulation to obtain the number of cycles to failure for each load type, and

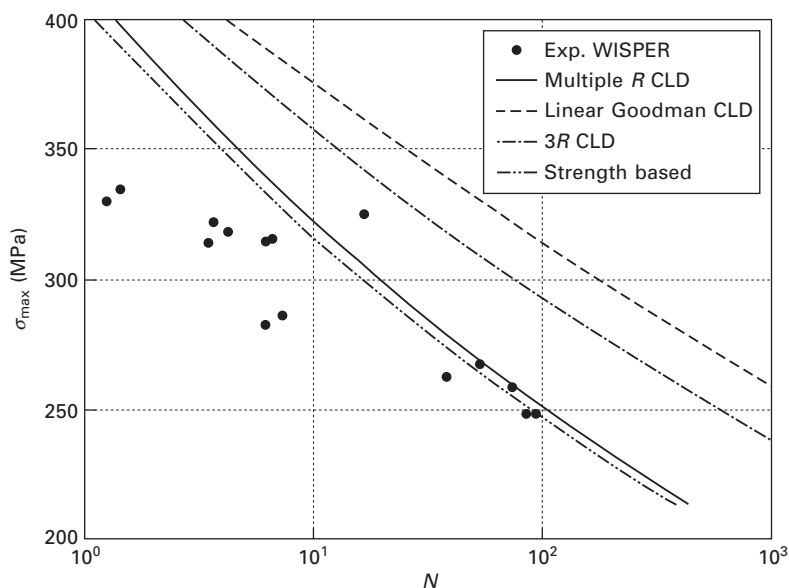


9.20 Lifetime prediction – cycle counting; rainflow counting; S–N curve: log–log without static/Sendeckyj with static; CLD: PWL; damage summation: linear Miner. Experimental data: WISPER on MD2 laminate.

as Fig. 9.21 illustrates, employing a strength-based life prediction does not produce significantly different results from the Miner's sum.

It should be noted, incidentally, that the counting method employed in Fig. 9.21, designated the 'rainflow-equivalent range–mean' method, is not a standard method. This method, described in [18], attempts to overcome the disadvantage of the rainflow counting method (not suitable for cycle-by-cycle analysis, as the largest cycle is considered last during a count and can thus be counted much later than it actually occurs). The method consists of rainflow counting a sequence, then creating a new, transformed, sequence from the results, and range–mean counting the results (range–mean counting is suitable for cycle-by-cycle analysis).

It is also noted that each load cycle obtained from the load sequence should be allocated its own set of strength degradation parameters, as mentioned earlier. However, strength degradation data are available for this material only at $R = 0.1$, -1 and 10 . For all other R -values, the strength degradation parameters can be obtained by interpolation, in much the same way as a PWL constant life diagram gives information on fatigue life through interpolation. This process was suggested in [63]. In the case of Fig. 9.21, a further simplification was made in that the strength degradation parameters chosen were identical for all load cycles. This is justified by the nature of the

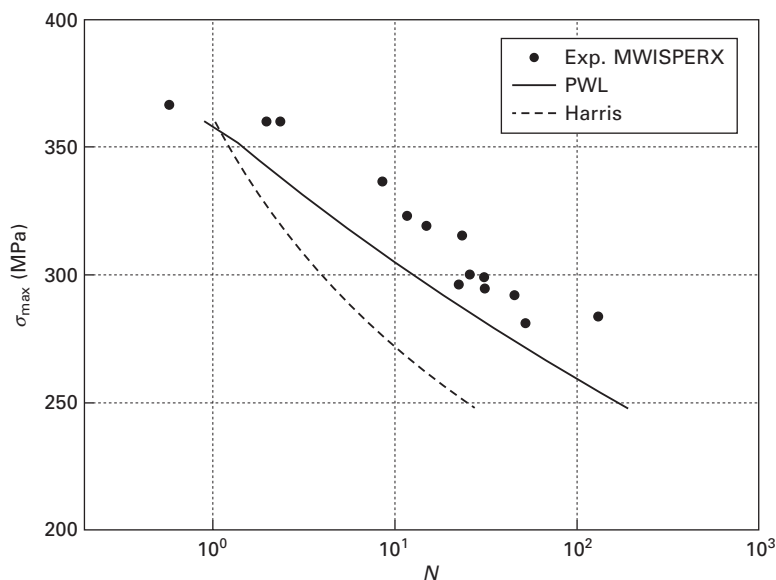


9.21 Lifetime prediction – cycle counting: rainflow-equivalent range–mean counting; S–N curve: log–log without static; CLD: PWL; damage summation: linear Miner; strength degradation model on transformed sequence, with tensile strength degradation parameter set to unity (linear strength degradation) and compression strength degradation parameter set to 10 (sudden death). Experimental data: WISPER on MD2 laminate.

loadings in the WISPER sequences (dominated by tension), and the scatter in the strength degradation data, which hampers detailed determination of the strength degradation parameters from experimental data. Tensile strength was assumed to degrade linearly (strength degradation parameter equal to 1) and compression strength was assumed to degrade according to a sudden-death type pattern (the value of the strength degradation parameter used was 10).

The CLD formulation is also critical for life prediction, since significantly different S–N curves can be derived from the application of different CLD formulations as shown in Fig. 9.18. In fact, Harris’s CLD is more conservative for higher numbers of cycles (also true for the UP laminate) and this is reflected in the fatigue lifetime calculations in Fig. 9.22. Both CLDs give the same lifetime predictions for higher stress levels but PWL is more accurate, although still conservative, when lower stress levels are present.

Lifetime predictions for three different variable amplitude load spectra applied on two different material systems are presented in Figs 9.19–9.22. Based on the same process, significantly conservative, or on the contrary, non-conservative, ‘optimistic’ theoretical results can be derived. Although



9.22 Lifetime prediction – cycle counting; rainflow counting; S–N curve: log–log without static; CLD: PWL/Harris; damage summation: linear Miner. Experimental data: MWISPERX on UP laminate. High rate static strength used for the CLD.

differences were found when different methods were employed for the solution of each of the steps of the classic fatigue life prediction methodology, it is difficult to quantify them and get a clear idea regarding the criticality of each step, since there is a direct connection between them. For example, the decision concerning the S–N curve type also affects the shape of the constant life diagram and subsequently the lifetime calculation.

9.6 Conclusion and future trends

The fatigue life prediction for a composite material or a composite structure under spectrum loading is a very complicated and difficult task since it is based on the results of different procedures that should be performed one after the other. The scatter of the experimental data, the assumptions, found especially in the cycle counting process, and the uncertainty in the modeling of the fatigue data introduce more ambiguity in the final results.

The standard procedure that results in the calculation of a damage coefficient that is often called the ‘Miner sum’ may be accurate or not, depending on the examined material and quality of the available fatigue data. However, very low (e.g. 0.1) and very high values (e.g. 10) of damage coefficient may result for different cases. The adoption of such values can lead to a

conservative or a highly non-conservative design, which in any case is not at all reliable.

Looking at each step of the procedure, the following conclusions can be drawn:

- The selection of the cycle-counting routine is not that critical in the entire process. Most common methods produce similar counting results and do not significantly affect the damage index.
- The type of S–N curve used for interpretation of the available constant amplitude fatigue data is critical. Use of the ‘proper’ S–N formulation that can adequately model the constant amplitude fatigue behavior of the examined material is essential, especially when extrapolation to lifetimes beyond the laboratory data is concerned. When adequate static strength data exist, e.g. data recorded under conditions similar to those existing for the fatigue testing, the method proposed by Sendeckyj can be used to accommodate these data too. However, the validity of this assumption has not yet been verified. In the case examined, use of the static strength data improved the modeling at the low-cycle regime, but not at the high-cycle fatigue regime.
- The same comments apply to the selection of the constant life diagram for quantification of the effect of mean stress on the life of the examined composite material. Although complicated methods exist, there is no evidence that their use is justified. The ‘simple’ piecewise linear diagram seems sufficiently accurate without the need to solve any sophisticated equation or perform any optimization for derivation of the model parameters as is the case for other CLD formulations.
- The linear Miner’s rule used for the damage summation is not accurate for the composite materials examined. However, any other damage summation rule that leads to non-linear damage summation and can potentially consider any load sequence effects needs extra experimental data, under the applied variable amplitude spectrum, for the estimation of model parameters. On the other hand, the use of non-linear rules to consider load sequence effects may be disputable since the load non-linearity is lost as from the first step of the process, the cycle counting. All methods that are used here derive a result that does not consider the effect of order of load cycles in the load sequence.
- The shortcomings of the abovementioned ‘non-linear’ rules could be overcome by strength degradation models. Strength degradation models can be used in life predictions, allowing the influence of the order of the load cycles to be retained. This comes, however, at significantly increased computational cost, and at the expense of collecting extensive strength degradation data.
- The improvement in life prediction with respect to the Miner’s summation

of the residual strength model depends on the extent of the sequence effects.

- The extent of the sequence effects depends on strength degradation behavior. ‘Sudden death’ behavior results in minimal sequence effects, whereas linear strength degradation and ‘early failure’ behavior result in more pronounced sequence effects. Thus, for cases with dominant sudden death behavior, the benefit of using a strength degradation model for life prediction is minimal.
- Compared to the influence of the constant life diagram formulation, the influence of the formulation of damage rule on the accuracy of the predictions is relatively small.

This chapter has emphasized the importance of creating an appropriate formulation for the constant life diagram. Although the piece-wise linear constant life diagram with a sufficient number of R -values is reliable, improved fatigue models are needed to reduce the experimental backbone required for creating this diagram. This still necessitates extensive experimental research and better understanding of failure mechanisms.

It can be argued that the question remains as to whether the added computational and experimental burden of strength degradation models outweighs the advantages in terms of life prediction. A significant advantage of modeling strength degradation, which has only been rudimentarily discussed in this chapter, however, is its strength prediction potential. Many contemporary engineering structures are designed for a particular ‘mission’. This mission is defined by a series of load cases, the exact magnitude of which is a statistical entity, and cannot be known exactly *a priori*. Thus, it is possible that the actual load sequence may be quite different from the design loads. This could lead either to a situation where the ‘operational life’ is spent well before the design life, or to an unexpected excess of residual life. In this case, using cycle-by-cycle strength predictions, simultaneously with the loading of the structure, can assist in the definition of condition-based operation.

The focus here has been on life and strength in ‘bulk’ materials. Most material research is related to final applications and structures. It is, at least, open for discussion whether a structural design can be based solely on the (uniaxial) behavior of, essentially, a material specimen. Research into the – possibly very different – failure mechanisms in representative structural parts such as connections and repairs is now emerging in the composite field.

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Fatigue of fiber reinforced composites under multiaxial loading

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Abstract: The chapter provides an overview on the multiaxial fatigue behavior of short fiber and continuous fiber reinforced composite materials based on recent experimental results as well as on those coming from an extensive literature investigation. Life prediction criteria available in the literature are illustrated and the reliability of their estimations is discussed. Eventually, indications for future developments on the subjects are provided.

Key words: continuous fiber composites, short fiber composites, multiaxial fatigue, life assesement.

10.1 Introduction

The response of short fiber composites as well as that of composite laminates reinforced with continuous unidirectional fibers under uniaxial cyclic stress states has been studied quite extensively for the purpose of developing methodologies for safe fatigue assessments. Although these methodologies are not fully developed yet, the evolution of the fatigue damage under uniaxial loadings has been reasonably understood and clarified in many cases, as illustrated in several chapters of this book. On the other hand, the fatigue behavior under multiaxial or variable amplitude loadings has been far less investigated, despite its importance in the design of structural components for the complexity of the real load histories and the variability of loading directions and intensity.

In the attempt to provide a contribution to this area and to draw the lines of future research activities, this chapter presents an overview on the multiaxial fatigue behavior of short fiber and continuous fiber reinforced composite materials with polymeric matrix. The analysis is based on recent experimental results as well as on those coming from an extensive, up-to-date literature investigation. Some of the life prediction criteria available in the literature and applicable to the two classes of materials are illustrated and the reliability of their estimations is discussed and quantified through validation against the experimental results.

Clearly, the difference in the intrinsic material morphology of short fiber composites and continuous unidirectional fiber/woven fabric reinforced

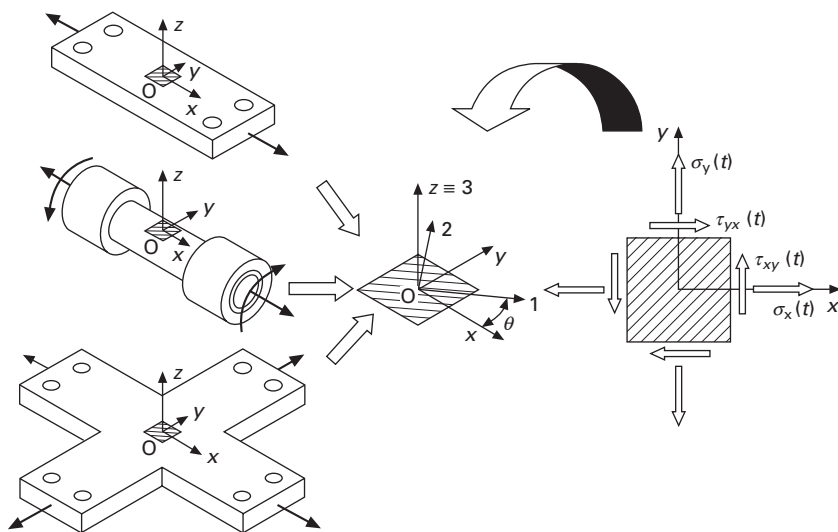
laminates induces different material response and the fatigue mechanisms in the two cases are also different. The chapter will therefore be divided into two separate sections pertaining to short fiber composites and to continuous fiber reinforced laminates.

10.1.1 Frames of reference and stress parameters

Before starting the discussion and the analysis of the results available, it can be useful to define the frame of reference of the problem and the stress parameters which will be used later for quantifying the degree of multiaxiality and for describing and classifying the fatigue data. Figure 10.1 shows three possible external, cyclic loading conditions: the uniaxial tension loading of a plate, the tension–torsion loading of a tubular sample and the tension–tension loading of a cruciform specimen. In the case of a layered or anisotropic material, the applied loads result in a plane stress condition which can be resolved, at local level, either in the structure (geometrical) coordinate system ($Oxyz$) or in the local (material) coordinate system ($O123$).

In general, fatigue results are generated under sinusoidal loading conditions, where the time variation of the three stress components is given by

$$\begin{aligned}\sigma_x(t) &= \sigma_{x,m} + \sigma_{x,a} \sin(\omega t) \\ \sigma_y(t) &= \sigma_{y,m} + \sigma_{y,a} \sin(\omega t - \delta_{y,x}) \\ \tau_{xy}(t) &= \tau_{xy,m} + \tau_{xy,a} \sin(\omega t - \delta_{xy,x})\end{aligned}\tag{10.1}$$



10.1 Loading conditions, local (material) and structure (geometrical) coordinate system.

where ω is the angular velocity, and $\delta_{y,x}$ and $\delta_{xy,x}$ are the phase shifts between $\sigma_y(t)$ and $\sigma_x(t)$ and between $\tau_{xy}(t)$ and $\sigma_x(t)$, respectively. After a suitable tensorial transformation, the stresses can be expressed in the material coordinate system as follows:

$$\begin{aligned}\sigma_1(t) &= \sigma_{1,m} + \sigma_{1,a} \sin(\omega t) \\ \sigma_2(t) &= \sigma_{2,m} + \sigma_{2,a} \sin(\omega t - \delta_{2,1}) \\ \sigma_6(t) &= \sigma_{6,m} + \sigma_{6,a} \sin(\omega t - \delta_{6,1})\end{aligned}\tag{10.2}$$

with analogous meaning of the symbols.

Now, to measure the degree of multiaxiality of the external loads one can introduce the biaxiality ratios calculated from the amplitudes of the geometrical stress components:

$$\begin{aligned}\lambda_C &= \frac{\sigma_{y,a}}{\sigma_{x,a}} \\ \lambda_T &= \frac{\tau_{xy,a}}{\sigma_{x,a}}\end{aligned}\tag{10.3}$$

These parameters, however, are not fully representative of the degree of multiaxiality of the stress field acting at local level, which instead can more efficiently be quantified by the biaxiality ratios calculated in terms of the stress amplitudes $\sigma_{1,a}$, $\sigma_{2,a}$ and $\sigma_{6,a}$:

$$\begin{aligned}\lambda_1 &= \frac{\sigma_{2,a}}{\sigma_{1,a}} \\ \lambda_2 &= \frac{\sigma_{6,a}}{\sigma_{1,a}} \\ \lambda_{12} &= \frac{\sigma_{6,a}}{\sigma_{2,a}}\end{aligned}\tag{10.4}$$

10.2 Fatigue behavior of short fiber composites under multiaxial loading

10.2.1 Fatigue behavior of short fiber composites under uniaxial loading

The investigations available in the literature on the fatigue behavior of short fiber reinforced composites (SFRC) under uniaxial loading are helpful for identifying the design parameters of major influence and to explain how structure and morphology can ‘drive’ the fatigue damage evolution. For comprehensive reviews on this subject, the reader is referred to references [1]

and [2]. Here it is worth surveying some of the works available to highlight important features of the fatigue behavior of SFRC.

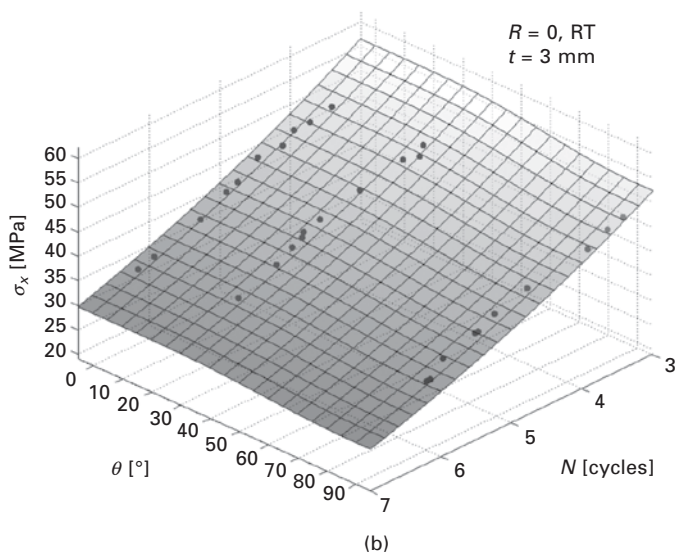
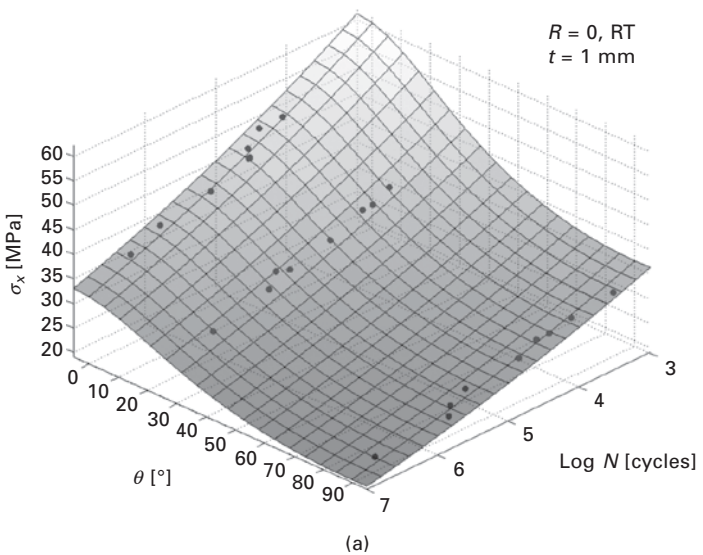
Since short fiber composite parts are mostly made by injection molding, it is important to address the through-the-thickness morphology of the part as it has a significant influence on the material behavior and damage evolution under loading. A layered structure can usually be recognized in the injected parts due to the interaction between the flow of the melt material and the mold walls. This layered structure can be described by a skin-shell-core-shell-skin architecture or, alternatively, by a shell-core-shell structure and is characterized by a non-uniform fiber orientation distribution through the thickness [3]. Near the mold walls the fiber orientation is mainly parallel to the flow direction whereas it is almost perpendicular to it in the inner core. The degree of fiber alignment and the induced anisotropic properties of the material can vary significantly depending on the local thickness [4, 5]: thin parts are characterized by highly oriented fiber distribution (with a reduced core thickness) and consequently by a strong anisotropy, while in thick parts the core layer is well developed, resulting in a reduced degree of anisotropy.

Static and fatigue behavior of SFRC at different orientations of the applied load with respect to the nominal fiber direction (called off-axis behavior) was investigated in [4]–[6]. Variation in off-axis static and fatigue strength was accurately described by using the Tsai–Hill criterion, in its conventional form [7], or in a modified version [8] suitable to account for cyclic loading.

The capability of the criterion to correctly describe off-axis data can be observed in Fig. 10.2 [5] where off-axis fatigue data for a polyamide 6,6 reinforced with 35% short glass fibers (PA6.6-GF35) are reported. A significant decrease in material anisotropy can also be observed as the sample thickness increases and is justified on the basis of the reduced degree of fiber alignment observed in thicker samples [5]. The possibility of describing the off-axis fatigue behavior with a modified ‘cyclic’ version of the Tsai–Hill criterion is quite important in view of the modeling of multiaxial fatigue data.

It is noted first that the behavior of composites under uniaxial loading can be viewed as a particular case of more general multiaxial loading. Due to the intrinsic material anisotropy, an external uniaxial loading in any direction generally induces a multiaxial local stress field described in the material coordinate system. This suggests that two different multiaxial conditions should be considered: the local (inherent) multiaxiality generated by the material anisotropy and the global (external) multiaxiality due to the externally imposed loading. On this basis it can be expected that the modified Tsai–Hill criterion can also be a useful tool for describing the fatigue behavior under external multiaxial cyclic loading conditions.

Another important process-induced effect is the presence of weld lines in



10.2 Tension–tension off-axis fatigue data for PA6.6-GF35 at room temperature fitted with the modified Tsai–Hill criterion (grid surface): (a) 1 mm thick samples, (b) 3 mm thick samples [5].

the injected part due to the meeting of two or more melting flows in the mold cavity. Their deleterious effect on fatigue strength was investigated in [9] for short glass fiber polycarbonate and in [10] for short glass fiber polyamide. The presence of weld lines was found to cause a significant reduction (up to 50%) in the high cycle fatigue strength of the material with respect to the properties measured in the flow direction.

The effect of the load ratio on the fatigue strength of short E-glass fiber reinforced PA 6,6 composite was investigated by Mallick and Zhou [11]. A significant influence of the load ratio was demonstrated and described by using a Gerber-like equation modified to account also for creep effect. Results reported in references [10] and [12] allow us to quantify the significant influence of holes and stress concentrations on the fatigue behavior of short glass fiber reinforced (SGFR) polyamide.

Under uniaxial loading, the fatigue behavior of SFRP in the presence of notches was successfully described by averaging the local strain energy density (SED) over a control volume [13]. By using this approach, room temperature fatigue data for PA6.6-GF35 specimens with five different notch radii ρ (80, 5, 1, 0.5 and 0.2 mm) and two load ratios ($R = 0$ and $R = -1$) taken from reference [12] were summarized in a single scatter band in terms of the average SED range.

To conclude this section, it is worth mentioning the work by Zago and Springer [14], who reported the results of an extensive experimental program on short fiber reinforced copolyamide aimed at investigating the influence of fiber content and orientation on the tension–tension and tension–compression fatigue. Their results were then compared to data reported in the literature for a wide range of thermoplastic materials reinforced by short fibers of glass and carbon, described by a normalized Mandell-like master curve.

10.2.2 Experimental results on the fatigue behavior of short fiber composites under multiaxial loading

A recent review published by the authors on the multiaxial fatigue of polymer composites [15] indicates that the number of publications in the literature dealing with this subject is very limited. Moreover, all of the results available were obtained by testing advanced composites (i.e. continuous unidirectional fiber or woven fabric reinforced laminates) and will be discussed in the second part of this chapter.

For short fiber reinforced composites, to the best of the authors' knowledge, no works or data on the multiaxial fatigue behavior have been published so far, apart from those produced in the frame of the doctoral thesis of Dr Matthias De Monte [16], sponsored by Robert Bosch GmbH (Germany) and defended in March 2008. The results obtained are collected in references [4], [5] and [17–21] and the discussion which follows here is based mainly

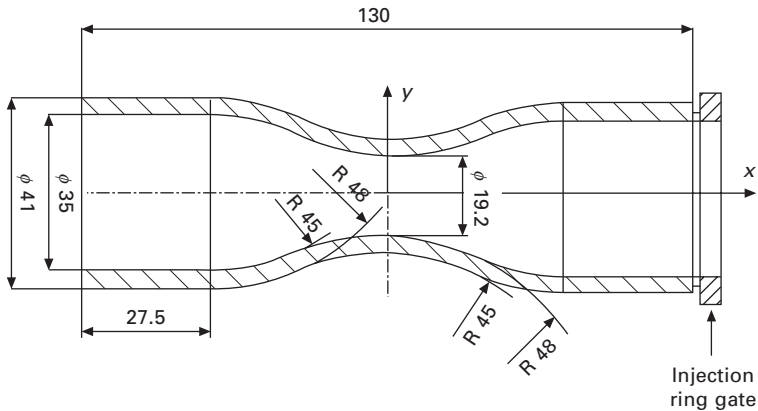
on these publications. Since the experimental research program was quite extensive and comprehensive, it is possible to assess and quantify the influence of several design parameters on the multiaxial fatigue behavior of short fiber reinforced composites and to draw conclusions of reasonably general validity.

Tension–torsion fatigue tests on plain samples

The material under investigation was a short fiber reinforced polyamide 6,6, containing 35 wt% glass fibers (designated as PA6.6-GF35). Hollow tubes were manufactured by injection molding according to the geometry and dimensions reported in Fig. 10.3. Further details on material, manufacturing process and sample morphology are available in [4] and [19]. Samples were injected through a ring gate resulting in a nominal fiber orientation following the longitudinal profile of the tube. It is important to note that on the central cross-section of the samples, where the stress components are calculated, the nominal fiber orientation is parallel to the longitudinal axis. This implies that material and geometrical frames of reference coincide, and the same holds true for the stress components ($\sigma_1 = \sigma_x$ and $\sigma_6 = \tau_{xy}$).

Fatigue experiments included pure tension, pure torsion, and combined tension–torsion at different biaxiality ratios λ_2 and phase-shifting angles δ between the stress components. Tests were carried out under load ratios $R = 0$ and -1 , at room temperature (RT) as well as at 130°C .

Fatigue data produced within the program were reanalyzed by using the stress-life approach, under the hypothesis of log-normal distribution of the number of cycles to failure (intended as complete separation of the samples). Fatigue lines are described by the following equation:



10.3 Geometry and dimensions (in mm) of the tubular samples [19].

$$\sigma_i(N) = \sigma_{i,A} \left(\frac{N_A}{N_f} \right)^{\frac{1}{k_i}} \quad (i = 1, 2, 6) \quad 10.5$$

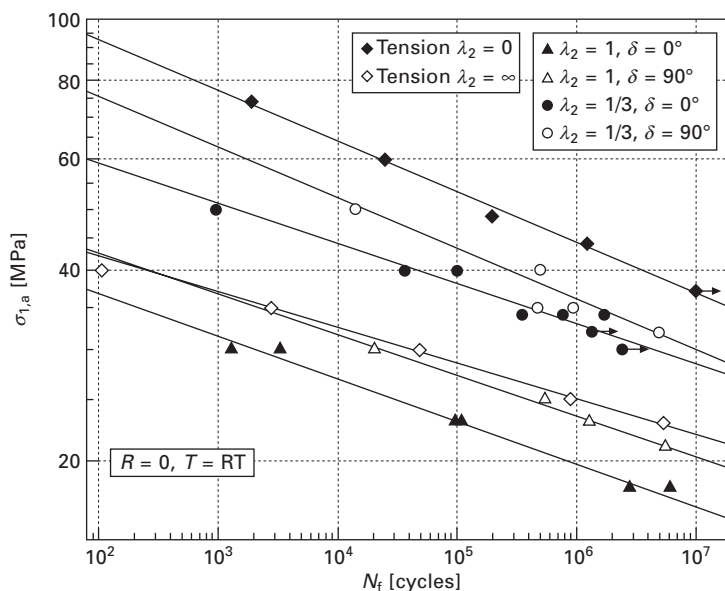
where the fitting parameters $\sigma_{i,A}$ and k_i represent the average stress calculated at $N_f = 2 \times 10^6$ cycles and the inverse slope of the fatigue curve, respectively. For each curve, the associated scatter index T_σ , defined as the ratio of the reference stress values at $N_A = 2 \times 10^6$ cycles and for 10% and 90% probability of survival, was also calculated.

Table 10.1 summarizes the main characteristics and the parameters of fatigue curves for the 20 series investigated; the relevant fatigue data are plotted in Figs 10.4–10.7 in terms of nominal stress amplitudes versus the number of cycles to failure N_f . All the series are characterized by a narrow scatter band, as shown by the plots and quantified by the low value of the T_σ ratio. Hence, the assessment of a reliable fatigue curve is possible based on few experimental points only. This condition was quite helpful in view of the very low frequencies that had been used to avoid hysteretic heating of the samples during fatigue testing.

Table 10.1 Summary of the main characteristics and parameters of fatigue curves for the multiaxial fatigue tests on hollow tubular samples [19]

Series	T (°C)	R	λ_2	δ (°)	$\sigma_{1,A} (2 \times 10^6)$ (MPa)	k	T_σ
1	RT	0	0	–	42.0	12.45	1.073
2	RT	–1	0	–	60.7	12.07	1.063
3	RT	0	∞	–	24.1 ^a	17.66	1.026
4	RT	–1	∞	–	27.7 ^a	13.97	1.003
5	RT	0	1	0	18.9	14.87	1.078
6	RT	–1	1	0	24.3	15.84	1.046
7	RT	0	1	90	22.5	15.56	1.045
8	RT	–1	1	90	25.4	15.39	1.070
9	RT	0	$\frac{1}{3}$	0	29.7	15.79	1.122
10	RT	–1	$\frac{1}{3}$	0	49.5	15.63	1.054
11	RT	0	$\frac{1}{3}$	90	34.2	12.49	1.116
12	RT	–1	$\frac{1}{3}$	90	43.7	10.60	1.143
13	130	0	0	–	21.4	15.25	1.043
14	130	–1	0	–	27.4	15.58	1.094
15	130	0	∞	–	11.2 ^a	16.74	1.085
16	130	–1	∞	–	10.7 ^a	13.63	1.003
17	130	0	1	0	9.6	14.97	1.069
18	130	–1	1	0	11.0	15.18	1.107
19	130	0	1	90	9.8	16.17	1.031
20	130	–1	1	90	11.3	17.86	1.018

^a Values reported here are the shear stress amplitude $\sigma_{6,A}$ at $N = 2 \times 10^6$ cycles.



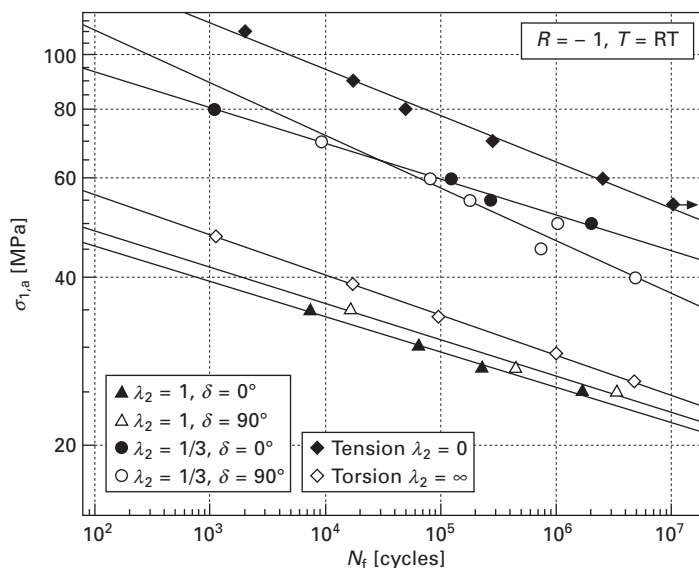
10.4 Fatigue curves for the multiaxial fatigue testing of PA6.6-GF35 on tubular specimens at load ratio $R = 0$ and room temperature [19]. (Fatigue data are plotted in terms of axial stress amplitudes apart from the series under pure torsion, plotted in shear stress amplitudes.)

Representative examples of the fracture paths measured on the samples during fatigue testing are shown in Fig. 10.8: it can be easily observed that fatigue crack paths are mainly driven by the external loading conditions. An extensive discussion on the mechanics of damage evolution is reported in ref. [19].

From the analysis of the results presented in Table 10.1 and in Figs 10.4–10.7 the following preliminary conclusions can be drawn.

Degree of multiaxiality

The presence of combined loading induces a significant reduction in the fatigue strength with respect to the pure tensile or pure torsional loading conditions. Assuming the biaxiality ratio λ_2 as an indicator of the degree of multiaxiality, its influence on fatigue behavior at RT can be quantified from Figs 10.4 and 10.5: as the biaxiality ratio (and thus the shear stress component) increases from 0 to 1 a remarkable decrease in fatigue strength can be noticed. Similar trends can be observed independently from the load ratio. At room temperature, the greatest reductions with respect to the pure tensile loading are for the cases $\lambda_2 = 1$, $\delta = 0$, with ratio $\frac{\sigma_{1,A}(\lambda_2=1, \delta=0)}{\sigma_{1,A}(\lambda_2=0, \delta=0)}$



10.5 Fatigue curves for the multiaxial fatigue testing of PA6.6-GF35 on tubular specimens at load ratio $R = -1$ and room temperature [19]. (Fatigue data are plotted in terms of axial stress amplitudes apart from the series under pure torsion, plotted in shear stress amplitudes.)

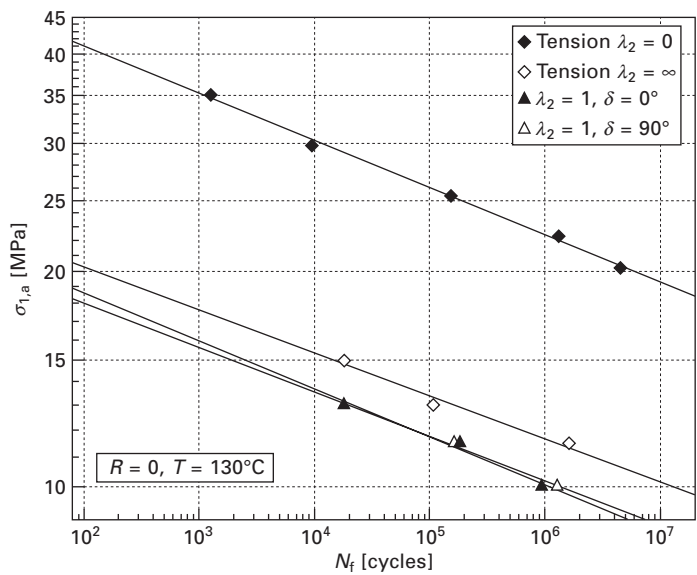
equal to 0.45 and 0.4 for $R = 0$ and $R = -1$, respectively. Similar relative values can be calculated from the results at $T = 130^\circ\text{C}$, suggesting that the biaxiality ratio effect is weakly dependent on temperature.

Phase shifting between load components

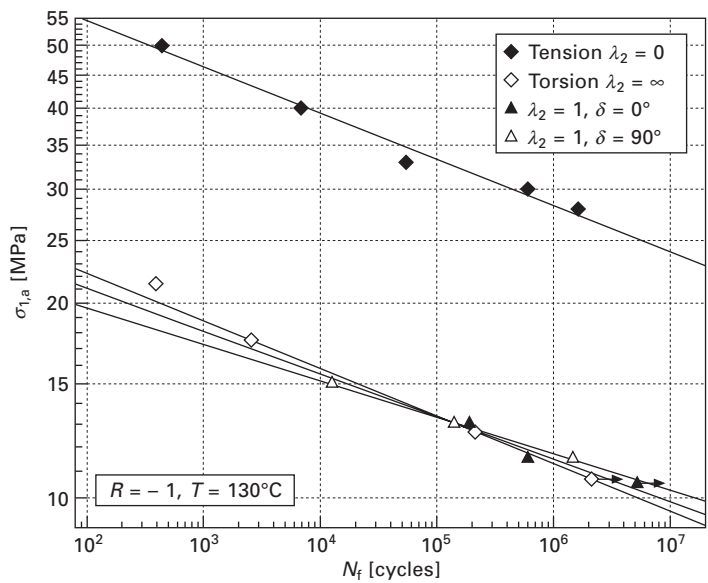
A slightly positive influence of the out-of-phase loading seems to be present at room temperature, but for design purposes it would be safer to overlook it.

Load ratio

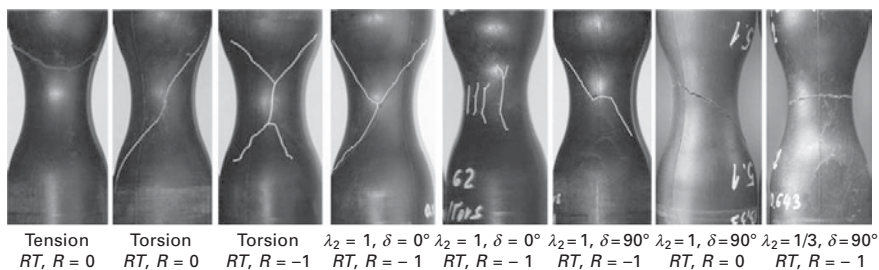
The effect of the load ratio is more pronounced for tensile than for torsional loading. The ratio $\frac{\sigma_{1,A(R=0)}}{\sigma_{1,A(R=-1)}}$ varies between 0.59 ($\lambda_2 = 1/3$, $\delta = 0$) and 0.86 (pure torsion) at RT and between 0.78 (pure tension) and 1.03 (pure torsion) at $T = 130^\circ\text{C}$. It is interesting to note that at temperatures higher than the glass transition temperature the material seems to be insensitive to mean stresses.



10.6 Fatigue curves for the multiaxial fatigue testing of PA6.6-GF35 on tubular specimens at load ratio $R = 0$ and $T = 130^\circ\text{C}$ [19]. (Fatigue data are plotted in terms of axial stress amplitudes apart from the series under pure torsion, plotted in shear stress amplitudes.)



10.7 Fatigue curves for the multiaxial fatigue testing of PA6.6-GF35 on tubular specimens at load ratio $R = -1$ and $T = 130^\circ\text{C}$ [19]. (Fatigue data are plotted in terms of axial stress amplitudes apart from the series under pure torsion, plotted in shear stress amplitudes.)



10.8 Fracture paths for different multiaxial loading conditions [19].

Temperature

A significant reduction in fatigue strength can be observed for the high temperature series: the ratio $\frac{\sigma_{1,A(130)}}{\sigma_{1,A(RT)}}$ varies between 0.43 and 0.51 when load ratio $R = 0$ and between 0.39 and 0.45 when load ratio $R = -1$. As shown in Figs 10.7 and 10.8, at high temperature the fatigue behavior under combined loading is governed by the shear stress.

As a general comment, it can also be observed that in view of the scatter the inverse slope of fatigue is not significantly influenced by the parameters and loading conditions investigated (i.e., biaxiality ratio, phase shift angle, load ratio and temperature).

Another important design variable to account for is the part thickness, for the significant influence it can have on the part morphology, as previously illustrated. This parameter was not directly incorporated in the multiaxial testing program reported in references [16]–[21]. However, one reasonable approach could be to apply to the case of external multiaxial loading the results obtained under inherently multiaxial off-axis fatigue testing [5], i.e., an increase from very thin to medium thicknesses induce a change in the overall morphology and a less anisotropic behavior of the material, which turns out therefore to be less sensitive to the nominal fiber orientation.

Influence of notches

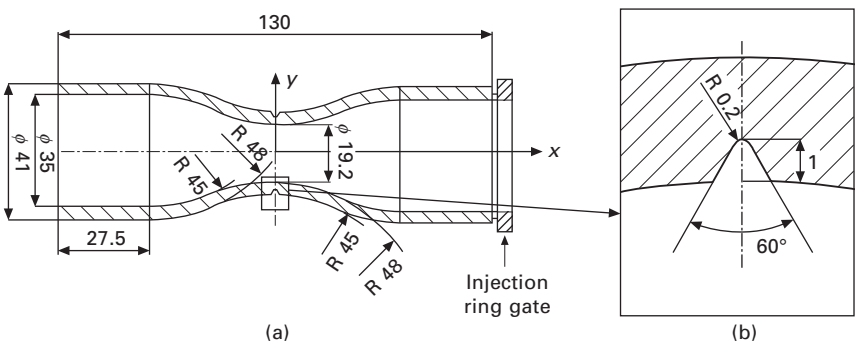
The presence of notches and cutouts cannot, in general, be avoided when designing structural parts due to their functional requirements. At the same time, the stress concentrations due to notches can significantly harm the material response, inducing fatigue strength reductions. The importance of this parameter is therefore significant from a design point of view.

The influence of notches under uniaxial fatigue loading of SFRC has been investigated quite extensively (see, as examples, references [10] and

[12]) and fatigue design procedures were proposed [12, 13]. No exhaustive studies are indeed available in the case of multiaxial fatigue loading and only a preliminary investigation was included in [16]. Results available have been published in [20] and are briefly summarized below.

Fatigue tests were carried out on the hollow tubular samples with a circumferential V-notch according to the geometry shown in Fig. 10.9. For a proper representation of the actual components it is important to note that the notch was produced directly during the injection molding of the sample and not by machining. The strong reduction in the local section due to the presence of the notch has in fact a significant influence also on the sample morphology, producing locally a far higher degree of orientation of the fibers in the flow direction [20].

Pure tension, pure torsion and two in-phase combined loading conditions were investigated at room temperature. Table 10.2 summarizes the main



10.9 (a) Geometry of the notched tubular sample; (b) details of the V-notch (dimensions in mm) [20].

Table 10.2 Summary of the main characteristics and parameters of fatigue curves for the multiaxial fatigue tests on V-notched hollow tubular samples [20] (room temperature, phase shift $\delta = 0$ for all the series)

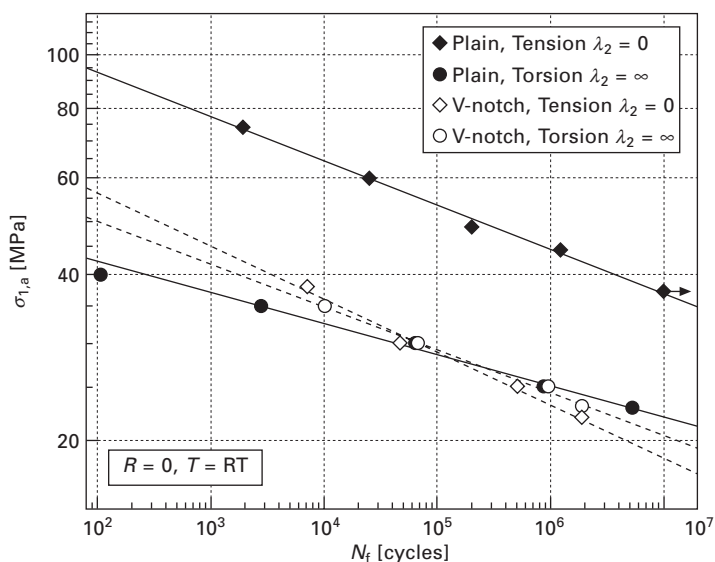
Series	R	λ_2	$\sigma_{1,A} (2 \times 10^6)$ (MPa)	k	T_σ
1	0	0	21.7	10.42	1.080
2	-1	0	27.6	8.00	1.211
3	0	∞	23.1 ^a	12.89	1.034
4	-1	∞	21.1 ^a	7.16	1.212
5	0	1	14.7	10.81	1.045
6	-1	1	20.6	10.47	1.059
7	0	$\frac{1}{3}$	19.7	9.72	1.091

^aStress values reported here are the shear stress amplitude $\sigma_{6,A}$ at $N = 2 \times 10^6$ cycles.

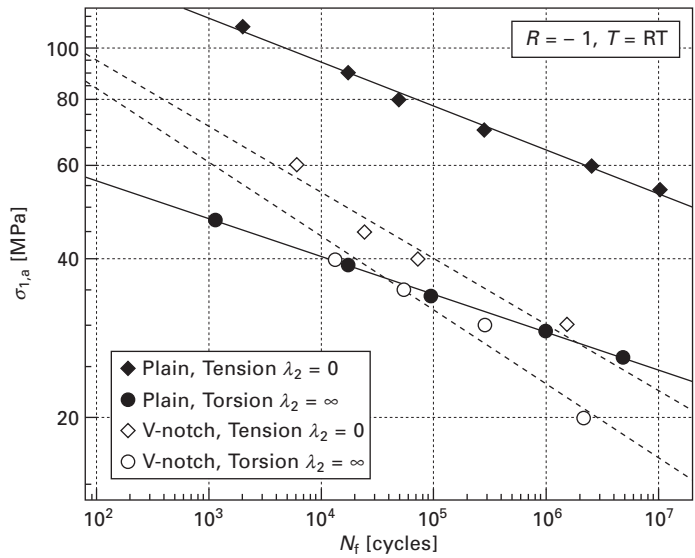
characteristics and the parameters of fatigue curves for the seven conditions analyzed and a comparison between fatigue curves for plain and notched samples is presented in Figs 10.10–10.13.

Despite the reduced amount of data available, it can be clearly observed from these figures and from a quantitative comparison of data in Tables 10.1 and 10.2 that the notch sensitivity is higher in the case of pure tension and very reduced, if not negligible, in the case of pure torsion. Accordingly, in the case of combined loading the notch sensitivity decreases as the shear stress component, and thus the biaxiality ratio λ_2 , increases.

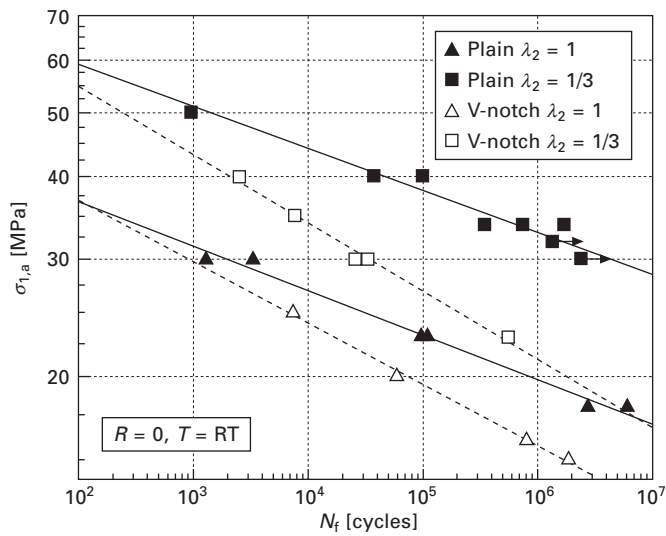
A possible explanation, to be confirmed by further and more extensive experimental investigations, is that the apparent insensitivity observed for the SFRP is related to a balance of two opposing factors: the detrimental effect of the stress concentration and the shear strength increase due to the local modified morphology. To verify the possible influence of the material morphology on the notch sensitivity, a few tests were performed on tubular samples after machining the notch instead of producing it during molding. A radial hole of 1 mm radius was drilled on the plain tubular samples (in this case plain and notched samples have the same local morphology). Fatigue curves for the different conditions are compared in Fig. 10.14 where one can easily observe that the samples with the drilled hole experienced indeed a significant reduction in fatigue strength with respect to the plain ones, thus



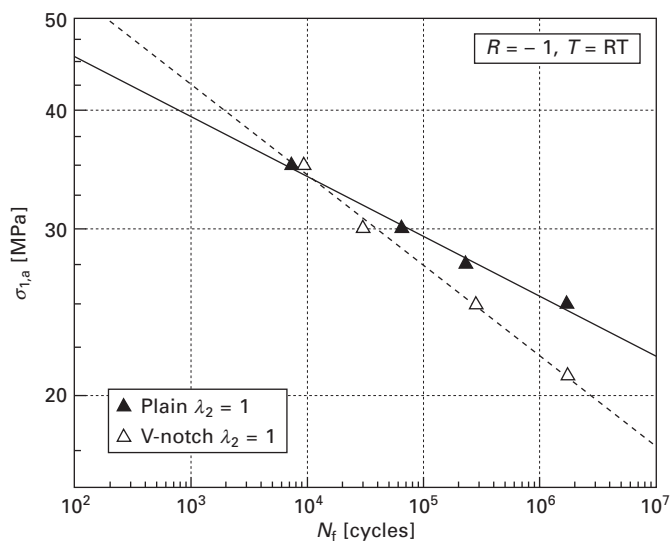
10.10 Comparison of pure tension and pure torsion fatigue curves for plain and V-notched tubular samples ($R = 0$, room temperature and phase shift $\delta = 0$) [20].



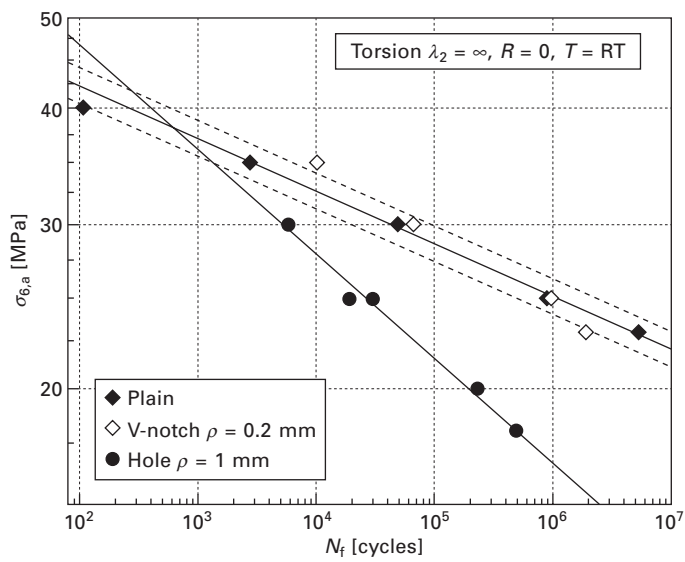
10.11 Comparison of pure tension and pure torsion fatigue curves for plain and V-notched tubular samples ($R = -1$, room temperature and phase shift $\delta = 0$) [20].



10.12 Comparison of multiaxial fatigue curves for plain and V-notched tubular samples ($R = 0$, room temperature and phase shift $\delta = 0$) [20].



10.13 Comparison of multiaxial fatigue curves for plain and V-notched tubular samples ($R = -1$, room temperature and phase shift $\delta = 0$) [20].



10.14 Comparison of pure torsion fatigue curves for plain, molded V-notch and drilled hole tubular samples ($R = 0$, room temperature and phase shift $\delta = 0$) [20].

confirming the notch sensitivity of this material when the results are not biased by other factors, like morphology.

The reduced amount of results available do not allow drawing conclusions of general validity, though they are clear enough to suggest that an efficient investigation on the effect of notches on the fatigue behavior of SFRP must account for the effects of local morphology (in terms of concentration and orientation of reinforcing fibers).

10.2.3 Multiaxial fatigue ratio for short fiber composites

The multiaxial fatigue ratio was proposed by the authors [15, 22] as a preliminary design tool useful for macroscopically correlating the multiaxial degree due to the external loading with the resulting fatigue strength. It is defined as the ratio between the reference fatigue strength under specified multiaxial conditions and the corresponding static strength of the material, both referred to the fiber direction (1) in the material coordinate system:

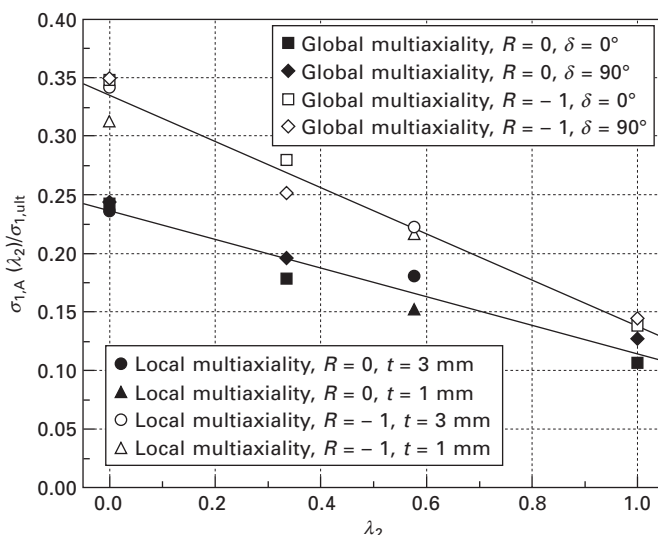
$$\phi_M = \frac{\sigma_{1,A(\lambda_2)}}{\sigma_{1,ult}} \quad 10.6$$

This ratio was found to be mainly influenced by the shear stress component, represented by the biaxiality ratio λ_2 , for both global and local multiaxial loading. Although not providing any indication about damage onset or evolution, the multiaxial fatigue ratio turns out to be useful for quickly quantifying the loss of strength due to fatigue damage under specified multiaxial conditions. If we now reanalyze the off-axis fatigue data on plain samples presented in references [5] and [17] and the multiaxial fatigue data summarized here and described extensively in [18] and [19], we can directly compare the effect of inherent and external multiaxial loading conditions. This is done in Fig. 10.15 where the non-dimensional multiaxial fatigue ratio is plotted versus the biaxiality ratio λ_2 .

The first indication from the analysis of Fig. 10.15 is that the sensitivity of the material to the biaxiality ratio λ_2 (and to the presence of a shear stress component) is as follows: by keeping the load ratio as the only variable, the inherent and external data can be described by the same linear trend with respect to the biaxiality ratio λ_2 .

This is important for modeling of multiaxial fatigue data: if the fatigue strength reduction is the same under inherent and external multiaxial loading, then the calibration of life prediction models under external multiaxial loading can be made on the basis of data on uniaxial off-axis loading, which are far easier to obtain experimentally.

It is worth noting that this equivalence in terms of fatigue damage under inherent and external multiaxial loading has not been identified for composite



10.15 Comparison of multiaxial fatigue ratio for local (inherent) [5–17] and global (external) multiaxial loading conditions [18, 19] (room temperature data).

laminates reinforced with unidirectional or woven continuous fibers, as discussed in reference [15] and later in the chapter.

The identified linear relationship between the multiaxial fatigue ratio and the biaxiality ratio λ_2 provides also a very useful design tool suitable for extrapolating the effect of the multiaxial fatigue ratio, thus reducing the need for experimental testing.

As a final comment, it can be noted that the influence of the load ratio decreases with the increase of the applied shear stress component.

10.2.4 Life prediction and modeling of multiaxial fatigue data

In the introduction of this section it has already been said that no results are available in the literature on the multiaxial fatigue behavior of short fiber composites. It is therefore not surprising that an extensive literature search indicated that not even failure criteria under multiaxial loading have been specifically formulated so far for this class of material.

However, the polynomial Tsai–Hill criterion, modified to account for cycling loading [8], has been shown to be effective in describing off-axis SFRC fatigue data [5, 6, 17]. It has also been mentioned that off-axis fatigue results in a local, inherently multiaxial, stress state and that the strength reduction due to fatigue damage is approximately the same under inherent

and external multiaxial loading at the same biaxiality ratio. It is therefore reasonable to expect that the modified Tsai–Hill criterion could work even for multiaxial fatigue of SFRC under externally applied multiaxial loading. Being a criterion based on the energy associated with a specific critical stress state, it is attractive also from a physical point of view.

The polynomial form of the criterion can be written as

$$\left[\frac{\sigma_{1,a}}{K_1(N_f)} \right]^2 + \left[\frac{\sigma_{2,a}}{K_2(N_f)} \right]^2 - \frac{\sigma_{1,a}\sigma_{2,a}}{K_1^2(N_f)} + \left[\frac{\sigma_{6,a}}{K_6(N_f)} \right]^2 = 1 \quad 10.7$$

where

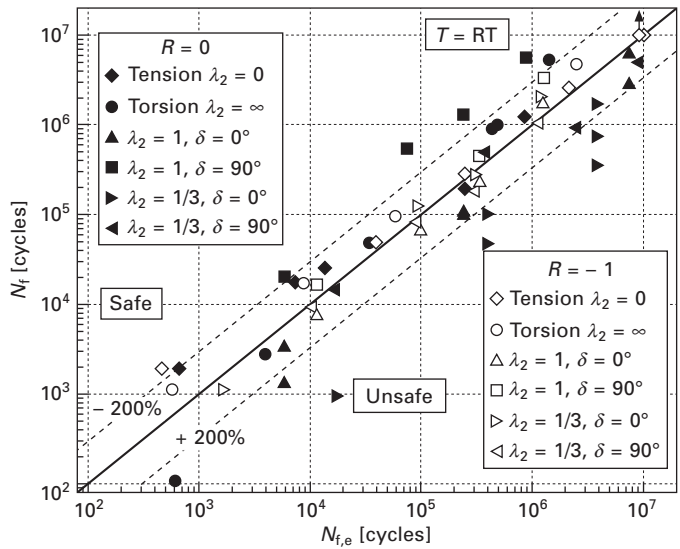
$$K_i(N_f) = \sigma_{i,A} \left(\frac{N_A}{N_f} \right)^{\frac{1}{k_i}} \quad (i = 1, 2, 6) \quad 10.8$$

are material functions or, better, the fatigue curves obtained under pure longitudinal, transverse and shear loading for the condition of interest in terms of temperature and load ratio. As suggested in references [18] and [21], the material functions $K_i(N_f)$ ($i = 1, 2, 6$) can be conveniently derived from the fatigue curves calculated for the uniaxial off-axis fatigue data [5].

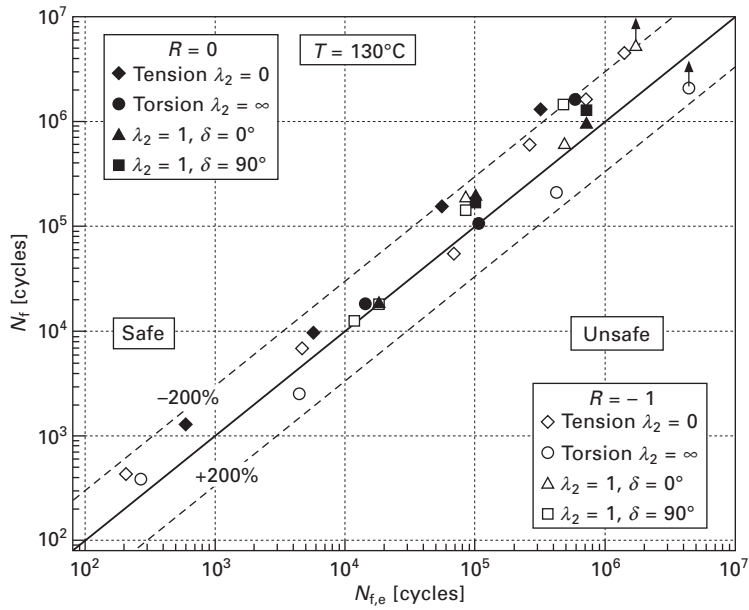
Having these material functions to hand, the polynomial model has been applied to the fatigue data obtained on plain samples listed in Table 10.1 to verify its accuracy. The application of the model required first to group the 20 fatigue data series in homogeneous subsets depending on load ratio ($R = 0$ or $R = -1$) and temperature (room temperature or 130°C). Different model calibration parameters were derived for each group. To reduce the number of parameters involved, some assumptions were made, based on the available results and engineering considerations [18, 21]:

- Constant value of the inverse slope of fatigue curves under different loading conditions (in terms of load ratio, biaxiality ratios and temperature), $k_1 = k_2 = k_{12} = 14$.
- Equivalence of tensile and compressive stresses in terms of fatigue damage caused (in the life prediction model).

A straightforward assessment of a life prediction model is given by comparison with the experimental data. This is done in Figs 10.16 and 10.17 for all the experimental fatigue data summarized in Table 10.1. Upper and lower bounds of $\pm 200\%$ error in life assessment are also plotted in the figures. The comparison shows a good agreement in some, but not all, cases between fatigue lives estimated by the modified Tsai–Hill criterion and the experimental fatigue data. Additionally, it is noted that in spite of the wide range of testing conditions investigated almost all the results reported fall within the $\pm 200\%$ life error band.



10.16 Accuracy of modified Tsai–Hill criterion in estimating fatigue lifetime under multiaxial loading (room temperature data) [18, 21].



10.17 Accuracy of modified Tsai–Hill criterion in estimating fatigue lifetime under multiaxial loading (130°C data) [18, 21].

It is worth pointing out that for obtaining the results shown in Figs 10.16 and 17, the model was applied by using a '*time independent approach*' [18, 21], which means considering the maximum value assumed by each stress component within one fatigue loading cycle. This approach results in an easy application of the model; nevertheless, it does not allow the designer to consider the possible effect of the phase shift between load components because the maximum of each stress component is calculated separately in the loading cycle. An alternative option would be to use an '*instantaneous approach*' [18, 21], where the stress components to be included in eq. 10.1 are calculated at every instant within one load cycle. However, the instantaneous approach turned out to be more complicated and, although theoretically more precise, it provided slightly less conservative life predictions compared to the time independent approach, which is therefore suggested as a safer design approach.

10.3 Fatigue behavior of continuous fiber composites under multiaxial loading

As discussed in the first part of the chapter, the technical literature is rather lacking in terms of data and analysis on short fiber composites. The situation for continuous unidirectional fiber or woven fabric reinforced laminates is not as dramatic; however, considering the importance of the problem in the design, experimental studies in this area are still limited. In recent years [15, 22] we have progressively updated the collection of literature papers dealing with experimental investigation of advanced continuous fiber reinforced composites subjected to external, multiaxial cyclic loadings (containing either experimental fatigue results, information on damage evolution or review on published data). The papers available so far [23–72] represent at least a basis to discuss the influence of the main design parameters on the fatigue strength of advanced composites, and to validate and quantify the reliability of life prediction criteria, as well as to identify the areas to be further investigated.

Methods to achieve a biaxial stress state in a composite laminate are discussed in the reviews by Found [73] on multiaxial fatigue testing and by Chen and Matthews on experimental results [48]. These are (1) axial loading and internal pressure on tubular specimens, (2) tension/torsion, tension/bending or bending/torsion loading on tubular or bar specimens, (3) tension/tension loading on cruciform specimens, and (4) biaxial bending on plates. Among these, tests on cruciform and tubular specimens are the most common in the published results, also with a predominance of papers using cylindrical samples. In fact, in spite of being preferable for the nominal constancy of the through-the-thickness strain, cruciform specimen testing is complicated to perform and samples are difficult to design and manufacture [69, 74–76].

Moreover, by testing cruciform samples it is impossible to induce global shear stresses; to do this one can only apply torsion loading to cylindrical specimens. These reasons explain the limited number of experimental programs carried out on cruciform specimens [35, 36, 38, 45, 59, 69].

As a general comment on the works published so far, it is important to point out that there is a total lack of detailed information about damage evolution. Also, information reported is mainly based on macroscopic parameters, and damage mechanisms at microscopic level are rarely investigated. When this is done, damage growth is usually not quantified and related to fatigue life. This is a quite significant limitation in view of the development of predictive criteria of general validity which needs, as an essential starting base, systematic studies of damage mechanisms [77]. A discussion on damage mechanics is presented at the end of the chapter together with some indications for future developments.

It is also important to note that the damage mechanisms responsible for fatigue failure occur locally and are driven by the local stress fields. The same external multiaxial loads will generally produce different local stress states depending on the laminate stacking sequence. Thus, fatigue failure criteria formulated on the basis of global (average) stresses have little chance of succeeding in predicting fatigue life for other than the particular laminate under consideration.

This was already illustrated by Owen and Griffiths, in one of their pioneering works [25], when comparing the predicted failure loci based on a number of static failure theories, mainly polynomial, with multiaxial fatigue data. ten years of research and thousands of multiaxial fatigue tests were still not enough to identify a suitable failure theory for thin-walled composite tube, as clearly stated by the authors: 'It is still surprisingly difficult to draw firm conclusions from the results' [26]. Serious doubts about the possibility of extending to cyclic loading failure criteria developed for quasi-static loading conditions were further expressed by Amijima *et al.* [41].

However, several papers have been published on the development of polynomial criteria for fatigue failure or dealing with the extension of existing static criteria to life prediction for local as well as global multiaxial cyclic loadings. Examples are reported in refs [8, 30, 39, 40, 43, 44, 46, 54, 55, 57, 78–86] and some of them are briefly discussed below.

Sims and Brogdon [8] were the first to propose the extension of the polynomial static failure criterion due to Tsai–Hill [7] to cyclic loading by replacing the strength properties with suitable fatigue curves. Francis and co-workers [30] extended Hill's anisotropic yielding criterion [87] to fatigue for $[\pm 45]$ graphite/epoxy tubes. The Tsai–Wu criterion [88] was used by Fujii and Lin [55] to calculate failure envelopes at constant fatigue lives for tension–torsion data on glass/polyester tubes. Aboul Wafa *et al.* [57] investigated the extension of several polynomial failure criteria and reported

significant difficulties in describing constant life diagrams for bending/torsion data from $[\pm 45]$ glass/polyester tubes.

As explained before in Section 10.2.1, even the uniaxial loading of an off-axis plate generates a local (inherent) multiaxial stress state. It is therefore worth mentioning the investigations by Kawai and co-workers for the description of the off-axis fatigue behavior of UD and woven reinforced laminates [80, 89, 90] and their fatigue damage mechanics model [80]. The model is based on the non-dimensional effective stress concept, which is the square root of the Tsai–Hill polynomial.

The Tsai–Wu criterion, modified for cyclic loadings, was applied by Philippidis and Vassilopoulos [84] for predicting the fatigue strength of multidirectional laminates tested under different loading directions [85, 86]. The Hashin model [78, 79] was adapted by Shokrieh and Lessard [81–83], including material property degradation, for the assessment of the fatigue strength under a multiaxial stress state and an arbitrary stress ratio. The model was validated on fatigue data for off-axis, cross-ply and pin-loaded laminates. Strain energy based models [91, 92] were also used to describe the off-axis fatigue behavior of composite laminates.

After an extensive experimental test campaign on glass/polyester cruciform specimens [38], Smith and Pascoe made an attempt to develop a multiaxial fatigue failure criterion based on the main damage mechanisms observed during the tests: rectilinear cracking, shear failure, and a mixed-mode failure. A quadratic polynomial function (to account for the strain energy associated with cracking) was combined with a maximum shear stress criterion (to account for shear failure) to describe the mixed-mode failure [93].

Another model explicitly developed for life prediction under multiaxial loading is that by Fawaz and Ellyin [94–96]: the actual multiaxial loading conditions and load ratio are considered through modification of a reference fatigue curve.

As a macroscopic indicator of the damage evolution during fatigue life, stiffness degradation was used by Fujii and co-workers to formulate a model for multiaxial life predictions based on continuum damage mechanics [56]. The ratios of modulus decay in tension and shear were used as indicators for damage variables.

Stiffness degradation has also been considered by Adden and Horst in their recent model for the ply-by-ply description of the fatigue damage evolution under multiaxial loadings [97].

10.3.1 Fatigue behavior of continuous fiber reinforced composites under multiaxial loading

The amount of data generated under global (external) multiaxial loading can be used as reference to discuss the influence of the main design parameters on

the fatigue strength of continuous fiber reinforced composites, thus providing information which may be useful in the preliminary design of components. Representative series of local (inherent) multiaxial fatigue data obtained by testing off-axis samples under uniaxial cyclic loading [78–80, 89, 90, 92, 98] are also included in the discussion. However, when using the information derived from this analysis for design purposes, it must be kept in mind that papers report, in general, fatigue data in terms of stress versus number of cycles to complete failure without indications about stiffness degradation and damage growth.

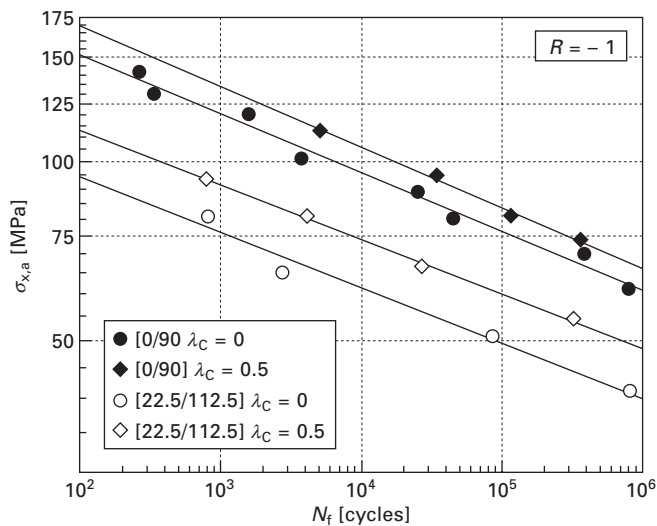
As indicated by the investigations already published by the authors, several design parameters can have a significant influence on the multiaxial fatigue strength of continuous fiber reinforced composites. On the basis of data available, the effects of stacking sequence and off-axis angle, degree of multiaxiality, load phase shift and stress concentration were identified and discussed [15, 22]. The continuous update of the papers and data collection allows refinement and further improvement of the previous analyses.

As a general approach, fatigue data available in the original sources were reanalyzed in terms of geometrical or material stresses under the assumption of log-normal distribution of the number of cycles to failure. The comparison was then made in terms of the fatigue curves (see eq. 10.8 in Section 10.2.4) pertaining to different sets of data and different testing conditions. The stress level calculated at 2 million cycles was assumed as the reference fatigue strength $\sigma_{i,A}$.

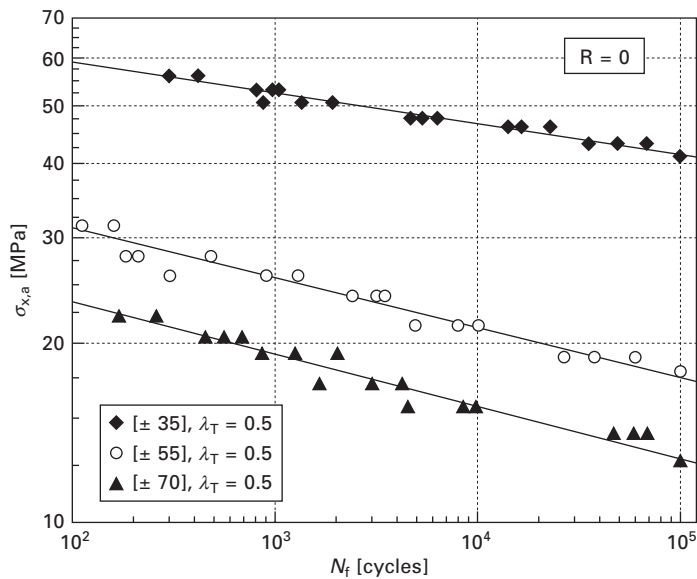
We can start with the analysis of the influence of stacking sequence and off-axis angle: it is quite clear that these two parameters in the case of cross-ply and angle-ply laminates have the same meaning, depending only on the coordinate system taken as reference. Figure 10.18 shows the results produced by Smith and Pascoe [38] testing woven glass/polyester cruciform specimens under tension–tension cyclic loading. Data, represented in terms of geometrical stresses, indicate a clear influence of the off-axis angle as one would expect from the behavior under uniaxial loading: independently of the degree of multiaxiality the fatigue strength of [22.5/112.5] samples is lower than that of those with lay-up [0/90].

The same situation, even more evident, can be observed in Fig. 10.19 for filament-wound glass/epoxy tubes tested under pulsating tension/torsion loading: an increase in the off-axis angle induces a decrease in the fatigue strength; alternate tension/torsion loading was reported to produce a comparable reduction [70].

Geometrical stresses, however, are not fully representative of the degree of multiaxiality acting at local level, which instead can be quantified, more efficiently, by stresses in the material coordinate system. Indeed, the variation of the off-axis angle can be seen even as a variation of the local degree of multiaxiality represented by the biaxiality ratios λ_1 and λ_2 .

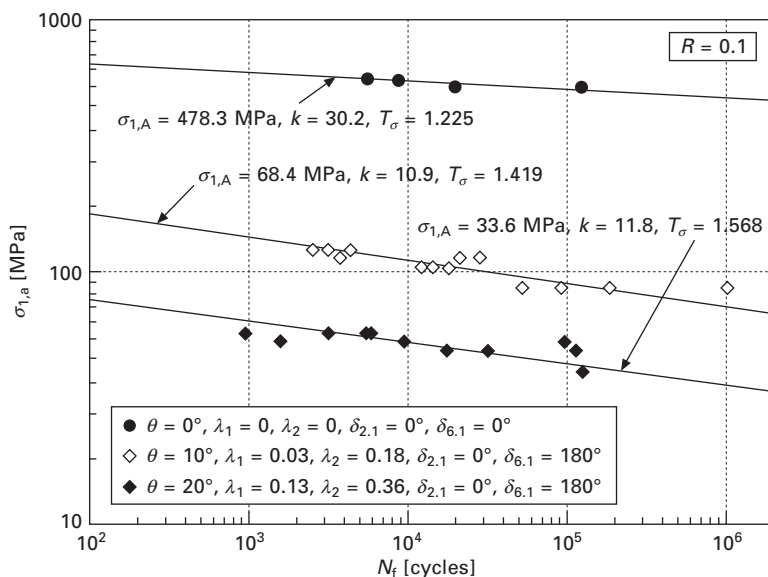


10.18 Influence of off-axis angle on the fatigue strength of glass/polyester cruciform specimens under tension-tension loading [38].



10.19 Influence of off-axis angle on the fatigue strength of glass/epoxy tubes under pulsating tension-torsion loading [70].

As an example of this, in the case of local multiaxiality, Fig. 10.20 shows some of the results reported by Awerbuch and Hahn [98] for uniaxial off-axis fatigue of graphite/epoxy laminates, plotted in terms of material stresses. The

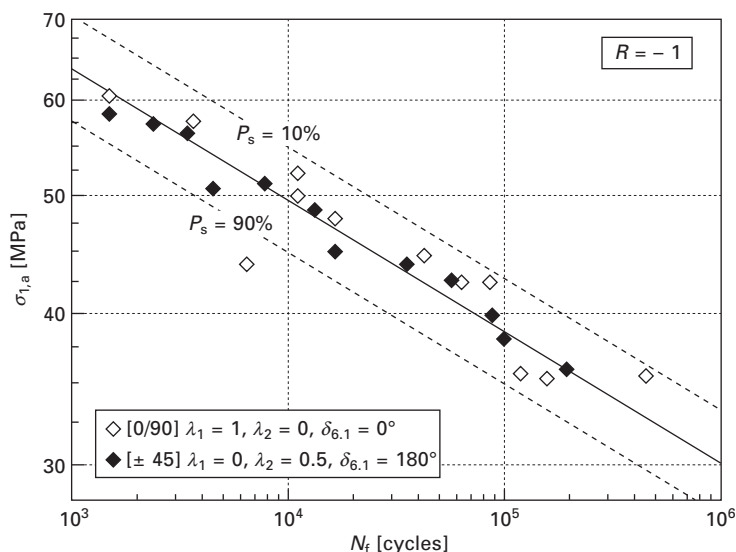


10.20 Uniaxial off-axis fatigue data for UD graphite/epoxy specimens [98].

expected reduction in the fatigue strength with the increase of the off-axis angle can now be seen as the effect of the increased biaxiality ratios λ_1 and λ_2 . When passing from $\theta = 0^\circ$ to $\theta = 10^\circ$ the increment of λ_1 is negligible, whereas λ_2 increases up to 0.18. This is enough to drop the corresponding reference fatigue strength, $\sigma_{1,A}$, from 478.3 MPa down to 68.4 MPa, with the clear indication that, even in the presence of a limited shear stress component, the decrease in fatigue properties can be significant.

Biaxiality ratios in terms of material stress components turn out to be useful also in explaining the results reported in Fig. 10.21 for [0/90] and [± 45] glass/polyester tube tested under combined in-phase bending and torsion cyclic loading [57]: it can be shown that the different values for λ_1 and λ_2 ratios for the two series correspond to the same local stress state. In spite of the different lay-ups, when the local multiaxial stress state is the same the material response to cyclic loading does not change and thus the fatigue strength remains the same. This confirms what was said in the introduction to this section that damage mechanisms responsible for fatigue failure occur locally and are driven by local stress fields.

Once it is proved that the biaxiality ratios λ_1 and λ_2 are efficient parameters for describing the effects of the local stress field on the fatigue strength of composite materials under multiaxial stress states, it is worth investigating the influence of their variation. The easiest way to clarify the effects of the ratio λ_1 is by using data derived from cruciform specimens, where this

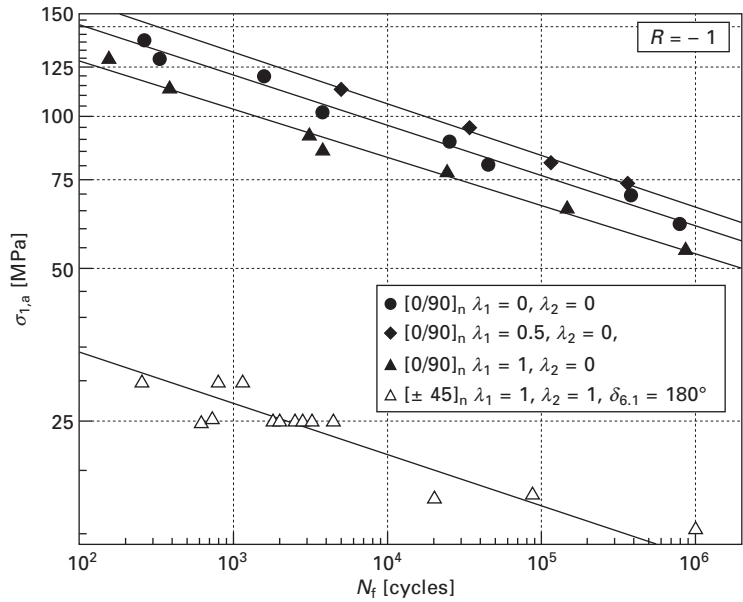


10.21 Results for glass/polyester tubes under bending and torsion with different lay-ups and identical local stress state [57].

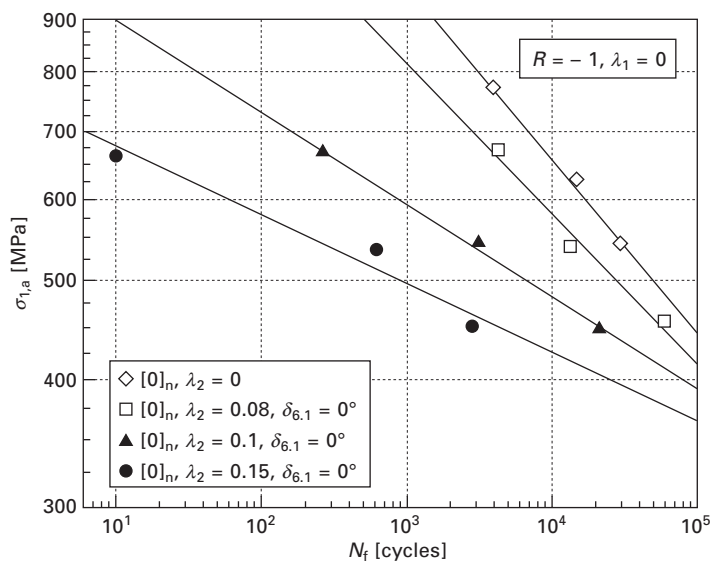
parameter can be varied in a controlled manner. Hence, data again from the extensive investigation by Smith and Pascoe [38] on woven glass/polyester cruciform specimens are taken as reference and presented in Fig. 10.22. From the three series with [0/90] lay-up it is quite evident that there is a very limited influence of λ_1 . On the other hand, a dramatic reduction in the fatigue strength can be observed for the $[\pm 45]$ series, when the λ_2 ratio rises from 0 to 1, due to the presence of a shear stress component of the same amplitude of the normal stress.

The detrimental effect of the shear stress component and thus of the associated biaxiality ratio λ_2 is illustrated also in Figs 10.23–10.25. Figure 10.23 shows fatigue data obtained from in-phase bending–torsion tests on glass/epoxy filament wound unidirectional bars [44]: it is clearly evident that even a reduced presence of a shear stress induces a significant reduction in the low-cycle fatigue strength. Results for glass/polyester [0/90]_n tubes tested under combined in-phase tension and torsion loading [41, 49, 56] presented in Figs 10.24 and 10.25 provide the same indication: an increase in the biaxiality ratio λ_2 is always associated with a decrease in the fatigue strength. From the analysis of the results it turns out quite clearly that the influence of the λ_2 ratio on the fatigue strength is significantly higher than that of λ_1 .

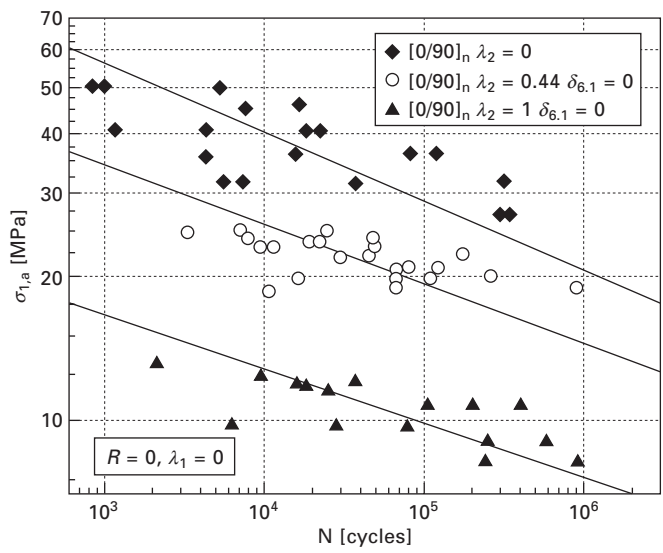
Figures 10.24 and 10.25 illustrate fatigue data for pulsating tension–torsion loading; however, from the design point of view, the combination of compressive and shear loading could be even more interesting. Due to the



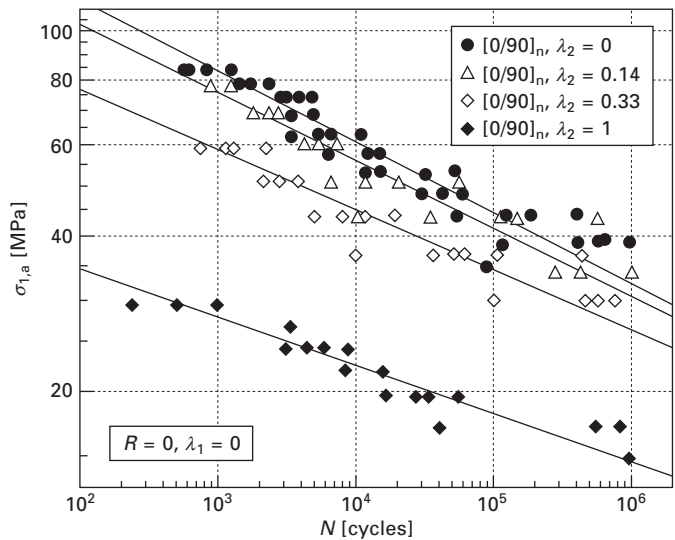
10.22 Influence of biaxiality ratios λ_1 and λ_2 on the fatigue strength of glass/polyester cruciform specimens subjected to combined tension-tension loading [38].



10.23 Influence of biaxiality ratio λ_2 on the fatigue strength of glass/epoxy $[0]_n$ bars under combined bending and torsion loading [44].



10.24 Influence of biaxiality ratio λ_2 on the fatigue strength of glass/polyester $[0/90]_n$ tubes under combined tension and torsion loading [41].



10.25 Influence of biaxiality ratio λ_2 on the fatigue strength of glass/polyester $[0/90]_n$ tubes under combined tension and torsion loading [49, 56].

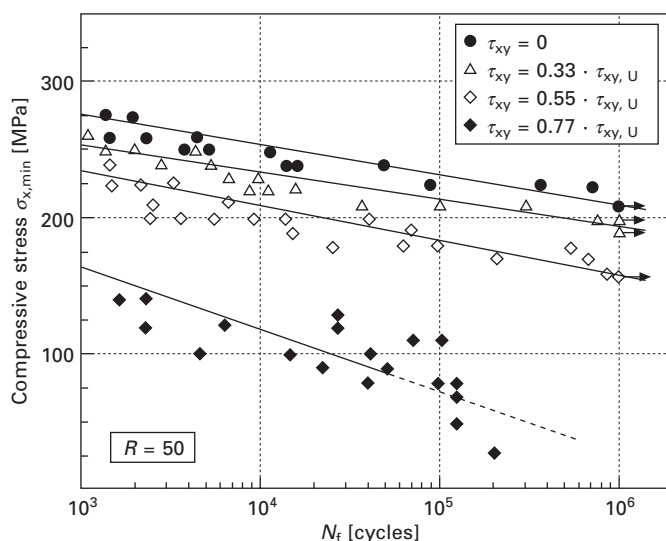
difficulty of applying compression to thin tubes without causing buckling, very few works have been published for this specific condition.

The fatigue behavior for $[0]_{10}$ woven glass/epoxy tubes under combined

static shear and cyclic compressive loadings was investigated by Limonov *et al.* [47]. Results, presented in Fig. 10.26, indicate a significant fatigue strength reduction with the increase in the shear stress: when the applied shear stress is 55% of the strength value, the reference fatigue strength at 2×10^6 cycles decreases by about 25%. For higher shear stress levels, the authors attributed the most probable cause of failure to creep effects.

Anderson *et al.* [40] studied the effects of combined cyclic tension or compression loading and torsional loading on $[\pm 30/90]_S$ aramid/epoxy tubes. They reported that in the case of predominantly axial loading the combination with a small torsion component slightly improves the fatigue strength in the case of tension and decreases it for compression. Similarly, when the predominant applied load is torsion, a small amount of axial loading has a slightly beneficial effect when it is in tension and a detrimental effect in the case of compression.

Further indications on the influence of combined compression and shear can be taken, indirectly, also from the results by Qi and Cheng [70] for cyclic tension–torsion loading on filament-wound glass/epoxy tubes. Fatigue strength of tubes tested at $\lambda_T = 0.5$ under alternate loading ($R = -1$) was significantly lower than that under pulsating loading ($R = 0$) (about 50%, 25% and 10% reduction for $[\pm 35]$, $[\pm 50]$ and $[\pm 70]$ tubes, respectively). This trend was confirmed by Fujii *et al.* [51] for woven glass/polyester tubes tested under pulsating tension combined with pulsating or alternate torsion. The normalized fatigue strength at $\lambda_T = 0.33$ in the case of alternate torsion



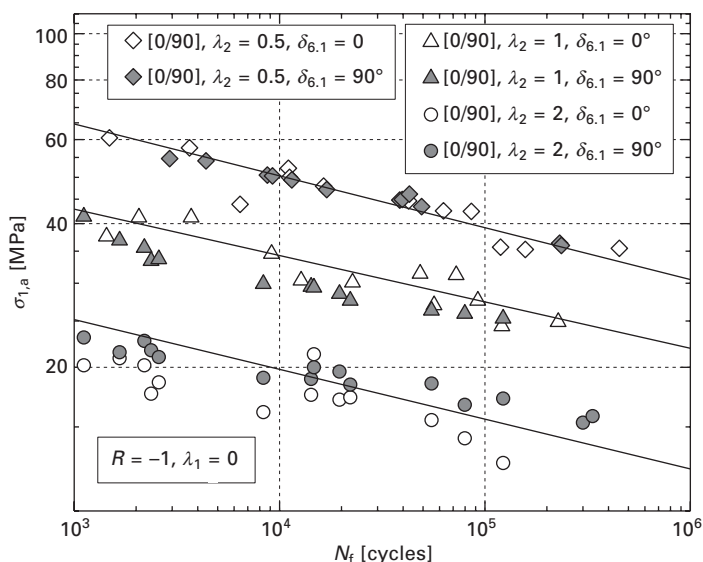
10.26 Influence of a static shear component on the compressive fatigue strength of $[0]_{10}$ woven glass/epoxy tubes [47].

loading was found to be about 40% lower than that under pulsating torsion loading. These examples clearly indicate a synergistic detrimental effect of the combination of compression and shear.

To conclude the analysis of the influence of biaxiality ratios, it is worth mentioning the work by Perevozchikov *et al.* [39]. They investigated the fatigue behavior of unidirectional glass/epoxy tubular samples under the combined effect of shear stresses and transverse tension or compression stresses and reported the negative effect of cyclic shear stress components in the case of both tensile and compressive transversal stress. It was also shown that the increase of λ_{12} ratio induced a reduction in the fatigue strength.

When planning the experimental campaign for investigating the multiaxial fatigue behavior of a new material, parameters to be defined are the loading path, the load waveform and the degree of non-proportionality of the loading conditions, quantified by the phase shift between the applied load waves. Loading path and load waveform were found to have only a negligible influence on the multiaxial fatigue strength, as extensively reported in refs. [56, 60, 62]. Results available in the literature concerning the influence of the phase lag between load components are instead limited and contradictory.

As indicated in a previous analysis [15], some sources clearly indicate a negligible influence of the non-proportionality of the applied loads on the fatigue strength. As an example consider the results reported by Aboul Wafa *et al.* for [0/90] glass/polyester tubes under combined bending and torsion loading [57] and shown in Fig. 10.27: the presence of a phase shift $\delta_{6,1}$ of



10.27 Influence of phase lag on the fatigue strength of [0/90] glass/polyester tubes under combined bending and torsion loading [57].

90° does not affect the fatigue response of the tubes. From these results can also be observed, once again, the significant and clear influence of the biaxiality ratio λ_2 . Other evidence of a limited or negligible influence of the phase shift can be found from the analysis of the fatigue data obtained by Smith and Pascoe on cruciform specimens [38].

Opposite indications come from the work of Anderson *et al.* [46], reporting the results of combined tension or compression and torsion cyclic loading on $[\pm 30/90]_S$ aramid/epoxy tubes. Under combined tension and shear loading, a phase shift $\delta_{xy,x}$ equal to 180° induces a fatigue strength reduction of about 25%, and the strength reduction reaches about 50% in the case of combined cyclic compression and shear. Other results supporting the negative influence of load phase shift have been reported by Ohlson [34] for ± 45 or UD carbon/epoxy samples under bending and torsion and by Susuki [45] for $[0/45/0/45]_S$ glass/epoxy cruciform specimens.

On the basis of the results discussed here, it is impossible to give any conclusive judgment on the influence of the load non-proportionality: indeed it may depend on the material system and the stacking sequence and possibly on the loading conditions.

In spite of the importance that notches and stress concentrations can have in the design of structural parts, their influence on multiaxial fatigue strength has rarely been investigated [30, 35, 36, 49, 52, 53]. It is difficult to draw general design indications from the few data available; however, from at least one of the papers cited, some interesting conclusions can be obtained.

Fujii *et al.* [49] reported the results of a comprehensive investigation on the fatigue behavior under pulsating tension–torsion loading of $[0/90]_n$ woven glass/polyester tubes. By comparing the fatigue curves for plain and notched samples under pure tension, pure torsion and three intermediate biaxiality ratios λ_T , they calculated the fatigue notch factor β which was seen to decrease when the shear stress component increased. Values of β (the ratio between plain and notched fatigue strength values) calculated at 2 million cycles are reported in Table 10.3.

The trend is similar to that discussed on page 345 for short fiber composites, but at the same time it is opposite to the general behavior of plain samples of both short fiber and continuous fiber composites, where the fatigue strength decreases when the biaxiality ratio λ_2 increases. However, on the basis of the limited amount of data available it is impossible to justify the experimental evidence and further researches are indeed needed to clarify the mechanisms responsible for this peculiar behavior. Results on $[0/\pm 45]_{2S}$, $[0/\pm 45/90]_S$ and $[0_2/\pm 45]_S$ graphite/epoxy cruciform laminates reported by Jones *et al.* [36] are less clear than those just discussed: in some cases increasing the biaxiality ratio λ_1 decreased the fatigue life, for others the life was increased.

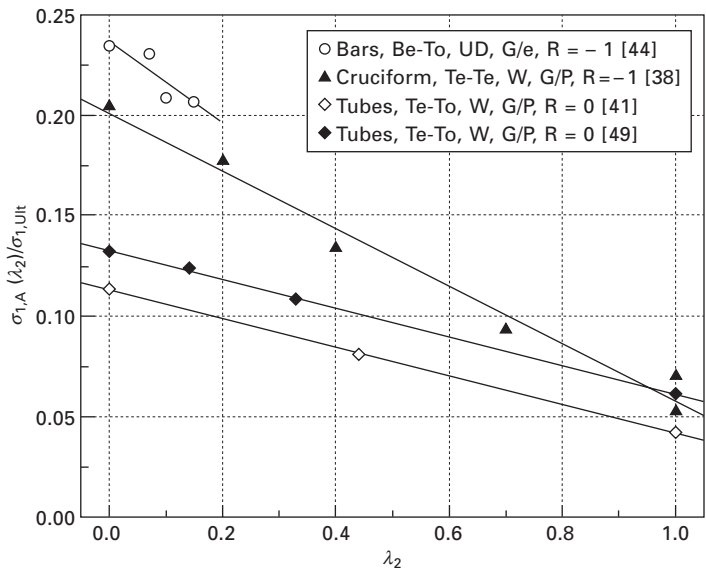
Table 10.3 Summary of the parameters of fatigue curves and fatigue notch factors for $[0/90]_n$ woven glass/polyester tubes under pulsating tension–torsion loading [58]

λ_2	Plain tubes			Tubes with circular hole			
	k	$\sigma_{1,A} (2 \times 10^6)$ (MPa)	T_σ	k	$\sigma_{1,A} (2 \times 10^6)$ (MPa)	T_σ	β
0	7.13	58.23	1.399	7.17	23.94	1.445	2.43
0.14	7.61	55.82	1.543	8.63	25.1	1.481	2.22
0.33	8.51	48.48	1.443	8.54	24.18	1.233	2.00
1	10.38	27.13	1.362	8.86	16.8	1.411	1.61
∞	13.9	33.9	1.265	10	26.7	1.324	1.27

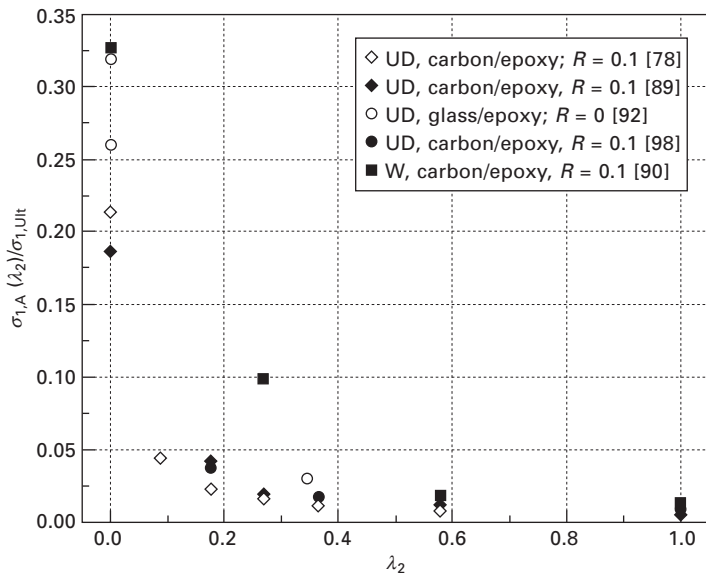
10.3.2 Multiaxial fatigue ratio

In the previous paragraphs it was clearly illustrated that, among the parameters of interest for the design of structural parts, the biaxiality ratio λ_2 is probably that with the greatest influence on the multiaxial fatigue strength even for continuous fiber composites. Therefore, it may be useful here to correlate its value with that of the multiaxial fatigue ratio for some cases of interest, as already done in the previous investigations [15, 22] and in Section 10.2.3 for short fiber composites. According to eq. 10.6 in Section 10.2.3, the multiaxial fatigue ratio ϕ_M is defined as the ratio between the reference fatigue strength (at 2 million cycles) under specific multiaxial conditions and the relevant static strength of the material, both resolved into direction (1) of the material coordinate system. Figure 10.28 shows the trend of the multiaxial fatigue ratio for fatigue data obtained under external multiaxial loadings (global multiaxiality), while data reported in Fig. 10.29 refer to uniaxial fatigue tests on off-axis samples, suitable to generate a local (inherent) multiaxial stress state. As reported for short fiber composites in Section 10.2.3, in the case of global multiaxiality an almost linear reduction in the fatigue strength can be observed with the increase of the biaxiality ratio λ_2 . On the other hand, in the case of local multiaxiality, the influence of λ_2 is far stronger.

The clear difference, for the same biaxiality ratio, in the fatigue strength reduction under global and local multiaxial loading conditions suggests attention when deriving a model for life prediction under multiaxial fatigue loading: if the calibration parameters are taken from uniaxial fatigue tests (far easier to carry out) they could be not representative enough of the actual global multiaxial conditions. To clarify this situation a comparison between the damage evolution under global and local multiaxiality would be required. It is worth noting, however, that being defined as a macroscopic parameter, the multiaxial fatigue ratio itself cannot provide any indication about damage onset and growth. Dedicated experimental investigations are therefore required in this area. However, in spite of the limitations illustrated,



10.28 Influence of biaxiality ratio λ_2 on the multiaxial fatigue ratio φ_M in the case of global multiaxiality [15].



10.29 Influence of biaxiality ratio λ_2 on the multiaxial fatigue ratio φ_M in the case of local multiaxiality [15].

the multiaxial fatigue ratio represents indeed a useful design tool for the preliminary assessment of the loss in the fatigue strength under specified multiaxial loading conditions.

10.3.3 Fatigue life prediction criteria

The most common approaches and criteria available for life prediction of composite laminates under multiaxial fatigue have been briefly presented in the introduction to this section dedicated to continuous fiber reinforced composites. Among them, the criteria formulated by Fawaz and Ellyin [94, 95] and Smith and Pascoe [93], and a polynomial formulation based on the Tsai–Hill criterion [7], have already been discussed and analyzed in the recent review proposed by the authors [15]. As mentioned in the introduction, the model by Fawaz and Ellyin was explicitly formulated for life prediction under multiaxial loadings, that proposed by Smith and Pascoe is claimed to be physically based with general applicability and, eventually, the extension to cyclic loading of a polynomial function approach is a very popular approach when attempting to predict the fatigue life of composite parts.

For these reasons we decided to consider these criteria with the aim of assessing their reliability in terms of life estimation, by comparing their predictions with some of the experimental results taken from the extensive database available. This would be of help in obtaining information useful for design purposes, like strengths and weaknesses of each criterion, and, at the same time, in further clarifying the directions and the need for the development of life prediction models of general applicability.

The main features of the criteria will be discussed hereafter, together with the essential information for their validation. For a thorough description of each criterion the reader is referred, obviously, to the original sources and to the critical analysis of their performance reported in [15]. Only a selection of the available papers was taken as reference for the validation phase. The main information about the different groups is summarized in Table 10.4 (for global multiaxial data) and in Table 10.5 (for local multiaxial data). The reader is referred, again, to the original sources for a complete illustration of the characteristics of each series of tests. As a general approach, fatigue data taken into consideration are those referring to the final failure.

Fawaz and Ellyin criterion

The criterion [94, 95] defines a fatigue design curve for the case of interest by modifying a reference fatigue curve according to the actual multiaxial loading conditions and load ratio. The generic expression of the fatigue curve is taken in the form of

Table 10.4 Selected experimental results for global multiaxial cyclic loading (external biaxial loading)

Group	Reference	Matrix	Fiber ^a	V_f (%)	Lay-up	Specimen geometry ^b	Applied loading ^c	R	Test frequency [Hz] ^d	No. of data	$\sigma_{1,ult}$ (MPa) ^d	$\sigma_{6,ult}$ (MPa) ^d
A	[41]	Polyester	Glass (W)	27±3	[0/90] _n	Tu	Te–To	0	2 (10)	77	164.5	71.5
B	[57]	Polyester	Glass (W)	60±4	[0/90] _n , [±45] _n	Tu	Be–To	–1	16.7	224	n.a.	n.a.
C	[49, 56]	Polyester	Glass (W)	35.7	[0/90] _n	Tu	Te–To	0	2	139	224.3	73.2
D	[38]	Polyester	Glass (W)	46	[0/90] ₁₃ , [22.5/ 112.5] ₁₃ , [±45] ₁₃	Cr	Te–Te	–1	0.1–0.6	69	238	82.5
E	[70]	Epoxy	Glass (W)	64	[±35], [±55], [±70]	Tu	Te–To	0/–1	2	129	980	70
F	[44]	Epoxy	Glass (U)	64	[0] _n	Ba	Be–To	–1	n.a.	12	1300	177

^aFibers: W = woven fabric, U = unidirectional.

^bSpecimen geometry: Tu = tubular, Cr = cruciform, Ba = bar.

^cLoading condition: Be = bending, Te = tension, To = torsion.

^dn.a. = not available.

Table 10.5 Selected experimental results for local multiaxial cyclic loading (uniaxial off-axis loading)

Group	Ref.	Matrix	Fiber ^a	V_f (%)	Off-axis angle θ (°)	R	No. of data	$\sigma_{1,ult}$ (MPa)	$\sigma_{6,ult}$ (MPa)
G	[98]	Epoxy	Graphite (U)	70	0, 10, 20, 30, 45, 60, 90	0.1	60	1836	n.a.
H	[92]	Epoxy	Glass (U)	n.a.	0, 19, 45, 71, 90	0.5, 0, -1	89	800	n.a.
I	[78]	Epoxy	Glass (U)	60	0, 5, 10, 15, 20, 30, 60	0.1	148	1236	n.a.
K	[90]	Epoxy	Carbon (W)	n.a.	0, 15, 30, 45, 90	0.1	26	599.5	112
L	[89]	Epoxy	Carbon (U)	64	0, 10, 15, 30, 45, 90	0.5, 0.1, -1	85	2472	60
M	[80]	Epoxy	Carbon (U)	64	0, 10, 15, 30, 45, 90	0.1	37	1934	77.8

^aFibers: W = woven fabric, U = unidirectional.

$$\sigma_{i,\max} = b_i + m_i \log(N_f) \quad 10.9$$

where b_i and m_i are the material parameters pertaining to the direction i (which may coincide with the direction x in Fig. 10.30) and N_f is the number of cycles to failure. The reference fatigue curve has the same form:

$$\sigma_{r,\max} = b_r + m_r \log(N_f) \quad 10.10$$

where the subscript r indicates parameters related to the direction r taken as reference (see again Fig. 10.30). The correlation between the parameters of the two curves is made as follows:

$$m_i = f(\lambda_C, \lambda_T, \theta) \cdot g(R) \cdot m_r \quad 10.11$$

$$b_i = f(\lambda_C, \lambda_T, \theta) \cdot b_r \quad 10.12$$

Substitution of eqs 10.11 and 10.12 into eq. 10.9 brings us to the final form:

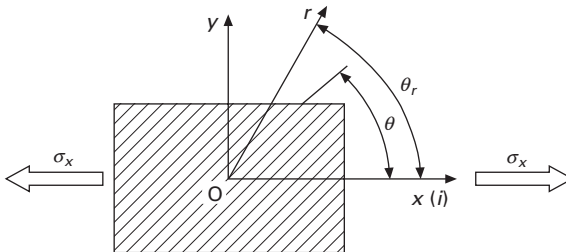
$$\sigma_{i,\max} = f(\lambda_C, \lambda_T, \theta) [b_r + g(R) \cdot m_r \log(N_f)] \quad 10.13$$

The correction of the reference line parameters to account for the actual degree of multiaxiality (represented here by the biaxiality ratios λ_C and λ_T) and for the off-axis angle θ is made through the function f which is calculated as the ratio between the static properties along the x axis (direction i) determined under the actual loading parameters and under the reference loading parameters:

$$f(\lambda_C, \lambda_T, \theta) = \frac{\sigma_{x,\text{ult}}(\lambda_C, \lambda_T, \theta)}{\sigma_{x,\text{ult}}(\lambda_{C,r}, \lambda_{T,r}, \theta_r)} \quad 10.14$$

The function g accounts, instead, for the different load ratio between the actual and the reference condition and is defined as:

$$\begin{aligned} g(R) &= 1 && \text{if } R = R_r \text{ or } R \leq 0 \\ g(R) &= \frac{1-R}{1-R_r} && \text{if } R > 0 \end{aligned} \quad 10.15$$

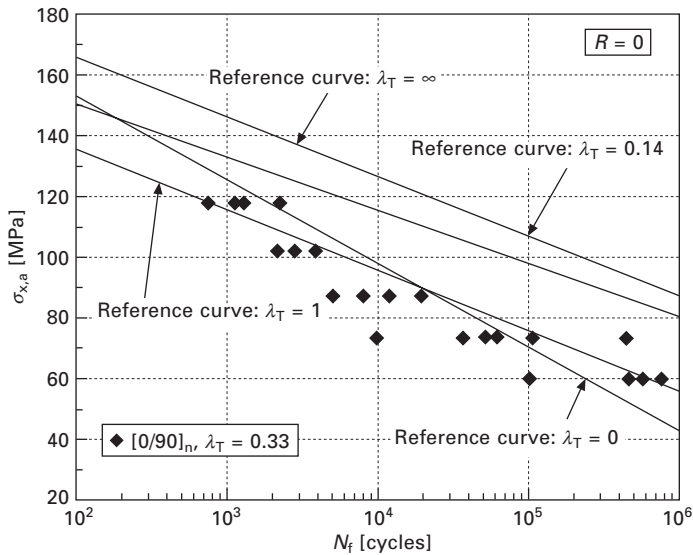


10.30 Actual and reference directions for Fawaz and Ellyin's model.

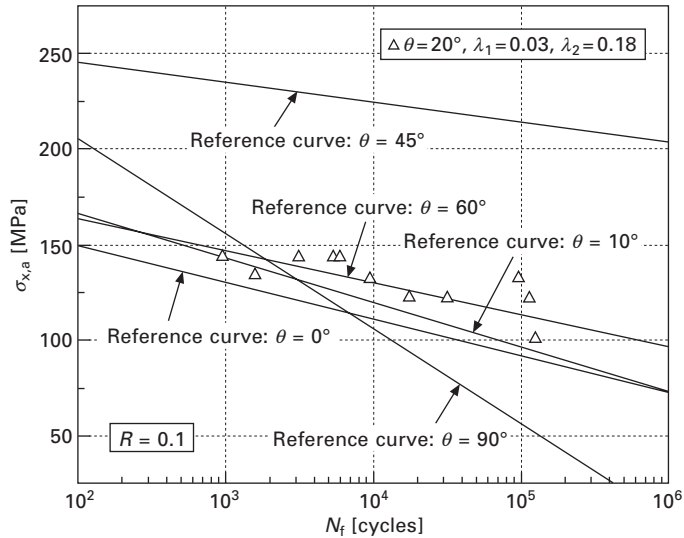
A further correction factor κ is suggested in [94] for modifying the slope of the design curve depending on the fiber orientation of the reference line. The corrected slope is

$$m_i = \kappa \cdot f(\lambda_C, \lambda_T, \theta) \cdot g(R) \cdot m_r \quad 10.16$$

From this brief presentation it is quite evident that the criterion is rather easy to apply, requiring the knowledge of one single fatigue curve, that in the reference direction, together with the static properties in the reference direction and in the multiaxial condition of interest. However, one of the main limitations of the approach proposed, already pointed out in [86, 99], is its high sensitivity to the choice of the calibration curve, as shown in the examples reported in Figs 10.31 and 10.32. Figure 10.31 shows global multiaxial data by Fujii *et al.* [49, 56] for [0/90] glass/polyester tubes under combined tension and torsion loading with $\lambda_T = 0.33$. The predictions made by taking as reference series with $\lambda_T = 0$ (pure tension) or 1 are in good agreement with the experimental data; however, when the reference curves are those with $\lambda_T = \infty$ (pure torsion) or 0.14 the predicted curves are overestimated and thus non-conservative. The same situation can be observed in Fig. 10.32 for the case of local multiaxiality. Data by Awerbuch and Hahn [98] referring to 20° off-axis tensile fatigue on graphite/epoxy samples are compared with the estimated fatigue curves, obtained by taking as reference



10.31 Fawaz and Ellyin’s model: sensitivity to the reference curve in the case of data for [0/90] glass/polyester tubes under combined tension and torsion loading with $\lambda_T = 0.33$ [49, 56].



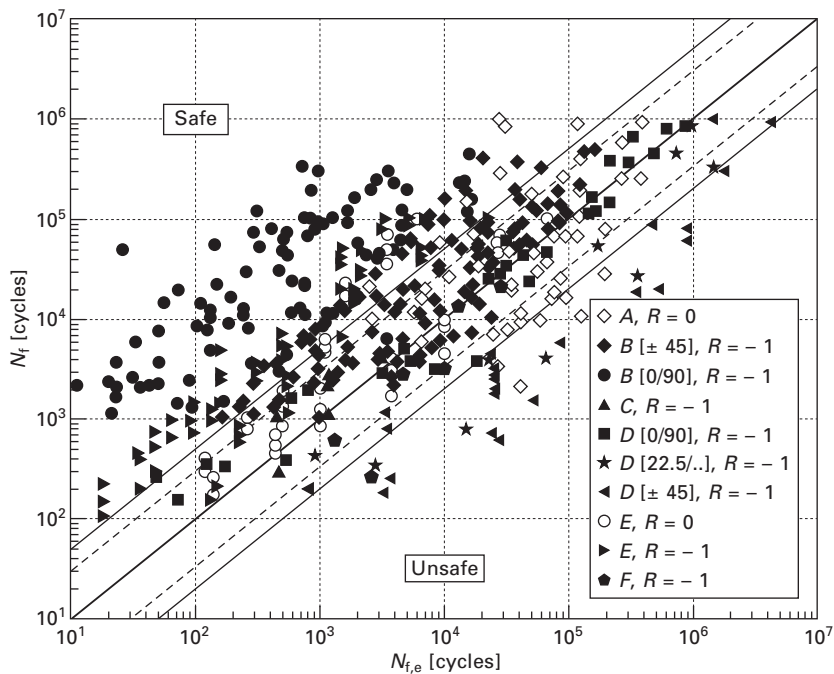
10.32 Fawaz and Ellyin’s model: sensitivity to the reference curve in the case of uniaxial 20° off-axis data for graphite/epoxy samples [98].

Table 10.6 Fatigue curves taken as reference for the calibration of Fawaz and Ellyin’s criterion [15]

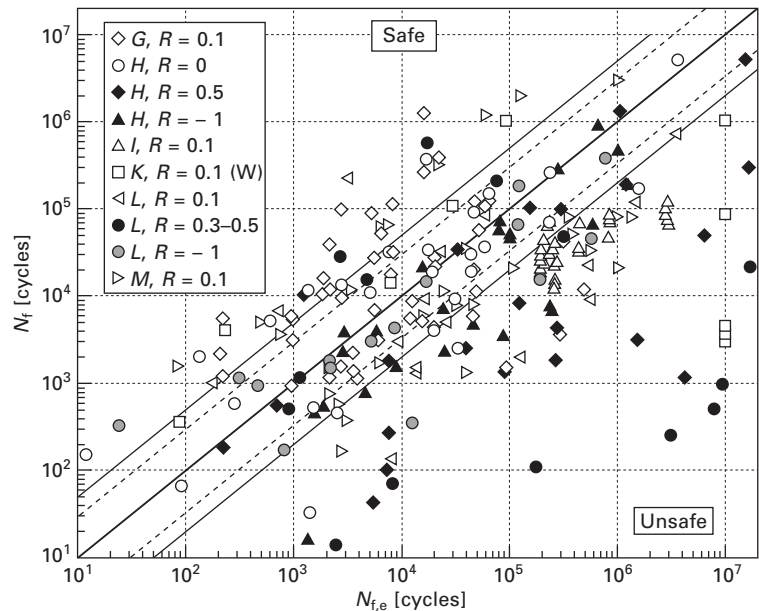
Group	Reference	Ref. curve	R	κ
A	[41]	$\lambda_T = 0$	0	–
B	[57]	$[0/90] \lambda_T = 0$	–1	–
C	[49, 56]	$\lambda_T = 0$	–1	–
D	[38]	$[22.5/112.5] \lambda_C = 0$	–1	–
E	[70]	$[\pm 35] \lambda_T = 0.5$	–1	–
F	[44]	$\lambda_T = 0$	–1	–
G	[98]	$\theta = 10^\circ$	0.1	0.69
H	[92]	$\theta = 45^\circ$	0	0.42
I	[78]	$\theta = 15^\circ$	0.1	0.64
K	[90]	$\theta = 90^\circ$	0.1	–
L	[89]	$\theta = 15^\circ$	0.1	3.25
M	[80]	$\theta = 10^\circ$	0.1	0.67

curves other series of the same group, with θ equal to 0°, 10°, 45°, 60° and 90°, respectively: it is quite clear again that completely different estimations can be obtained by changing the reference curve. In spite of this limitation, the model was applied to the global and local multiaxial data listed in Table 10.4 and 10.5. The series taken as reference for the different group of data are listed in Table 10.6.

To verify the accuracy in the life prediction, the experimental fatigue lives are compared with those predicted by the criterion, for a given set of data. Figures 10.33 and 10.34 show the results for global and local multiaxial



10.33 Accuracy of Fawaz and Ellyin's criterion in the life estimation for global multiaxial data [15].



10.34 Accuracy of Fawaz and Ellyin's criterion in the life estimation for local multiaxial data [15].

data, respectively. Upper and lower bounds representative of $\pm 200\%$ and $\pm 400\%$ error in life assessment are plotted together with the data for an easier interpretation of the results.

From the analysis of Figs 10.33 and 10.34 it can be concluded that the accuracy of the criterion is not very high and also depends on the type of data: slightly more conservative for global multiaxial data, frequently non-conservative for local multiaxial data. Moreover, as illustrated above, the choice of the reference fatigue curve can play an important role in increasing or decreasing this accuracy of the criterion. Unfortunately, the ‘correct’ reference curve being unknown *a priori*, it is possible to obtain highly unreliable estimations of the fatigue life.

Smith and Pascoe criterion

This multiaxial fatigue criterion [93] was developed by the authors on the basis of the main damage mechanisms observed during an extensive campaign on glass/polyester cruciform specimens [38]. According to the experimental evidence the following three main mechanisms were responsible for the fatigue failure of cruciform specimens under tension–tension loading: rectilinear cracking and fiber failure, shear deformation along the fiber plane and combined rectilinear cracking/matrix shear deformation. The model was developed by an independent modeling of the contribution to fatigue failure of each of the damage mechanisms.

To describe rectilinear cracking and fiber failure the authors proposed a strain energy criterion expressed as

$$U_{F,a} = \frac{1}{2} \left\{ \frac{\sigma_{1,a}^2}{E_1} - \left(\frac{\nu_{12}}{E_1} + \frac{\nu_{21}}{E_2} \right) \sigma_{1,a} \sigma_{2,a} + \frac{\sigma_{2,a}^2}{E_2} \right\} = U_{F,A} \left(\frac{N_A}{N_f} \right)^{\frac{1}{k_1}} \quad 10.17$$

where $U_{F,a}$ is the amplitude of the strain energy density, assumed to be a decreasing function of the cycles to failure. For application purposes, eq. 10.17 can also be rewritten as:

$$U_{F,a} = K_{SE}(N_f) = \frac{1}{2} \frac{\sigma_{SE,a}^2(N_f)}{E_1} \quad 10.18$$

The contribution of the shear deformation was related to the applied shear stress by the following expression:

$$\sigma_{6,a} = K_6(N_f) = \sigma_{6,A} \left(\frac{N_A}{N_f} \right)^{\frac{1}{k_6}} \quad 10.19$$

For the simultaneous contribution of rectilinear cracking/fiber failure

and shear deformation, the authors proposed the following interaction model:

$$\frac{1}{\sigma_{1,a}^2(N_f)} = \frac{1}{\sigma_{SE,a}^2(N_f)} + \frac{1}{\sigma_{6,a}^2(N_f)} \tag{10.20}$$

For a more practical application of the model, by taking advantage of the biaxiality ratios, eq. 10.20 was reformulated as follows [15]:

$$\sigma_{1,a}^2 \left\{ \frac{\frac{1}{E_1} - 2 \frac{\nu_{12}}{E_1} \lambda_1 + \frac{\lambda_1^2}{E_2}}{2 K_{SE}(N_f)} + \left[\frac{\lambda_2}{K_6(N_f)} \right]^2 \right\} = 1 \tag{10.21}$$

where the first term between brackets quantifies the effects of fiber failure and rectilinear cracking in terms of fatigue damage and the second one accounts for those of the shear deformation.

In this form the criterion can be clearly seen as a polynomial function of the applied stress components. Therefore in spite of being derived from experimental observation of the damage evolution, the model does not eventually incorporate any parameter suitable for describing the growth and accumulation of damage along the fatigue life.

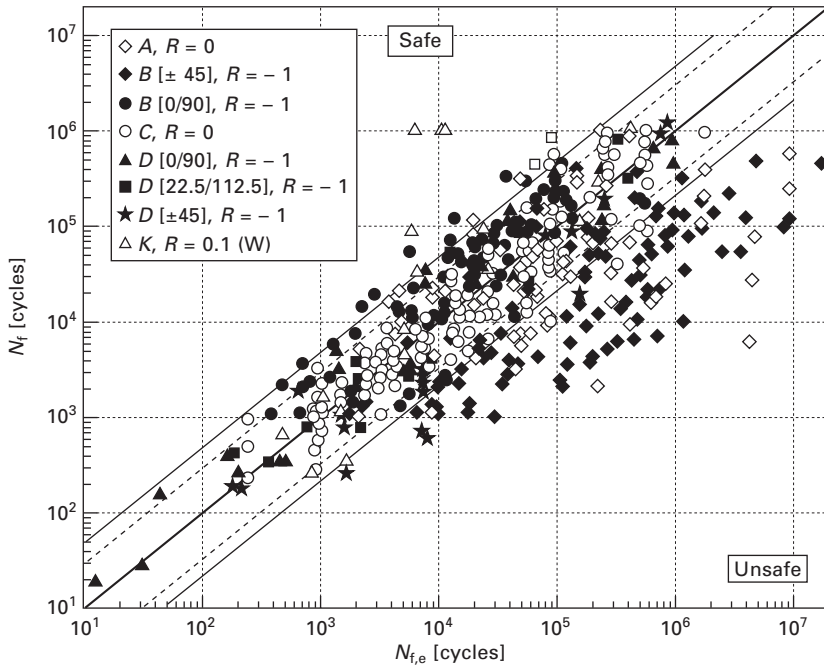
The accuracy of the model in the lifetime prediction was verified even in this case by comparison between model estimations and experimental fatigue lives. The parameters of the calibration fatigue curves together with the elastic properties of the laminates investigated are listed in Table 10.7. In some cases, elastic and static strength properties not available in the original sources were estimated from constituent properties or taken from literature.

Results of the validation process for the groups of data A, B, C, D and K listed in Tables 10.4 and 10.5 are presented in Fig. 10.35. It can be seen that the criterion produces reasonably accurate assessments apart from two cases, for group A [41] and for the series [±45] of group B [57], when non-

Table 10.7 Parameters for the application of Smith and Pascoe’s criterion [15]

Code	$U_{F,A}$ (J/m ³)	k_1	$\sigma_{6,A}$ (MPa)	k_6	E_1 (GPa)	E_2 (GPa)	ν_{12}	ν_{21}
A	4.75×10^{-2}	3.5	25.6	12.1	14.5 ^a	14.5 ^a	–	–
B	1.63×10^{-2}	2.9	24.3	12.9	21.7 ^a	21.7 ^a	0.17 ^a	0.17 ^a
C	9.50×10^{-2}	3.7	33.8	15.0	17.0 ^a	17.0 ^a	–	–
D	9.43×10^{-2}	5.0	13.7	8.6	17.1	18.3	0.16	0.17
K	0.36×10^{-2}	13.4	7.4	6.3	52.9	55.6	0.03	0.03

^aEstimated values.



10.35 Accuracy of Smith and Pascoe's criterion in the life estimation [15].

conservative predictions were obtained. However, we did not find information in the original sources suitable to explain this apparently anomalous behavior. On the other hand, the way the criterion is formulated is also of no help.

The criterion is, in general, rather easy to apply, since it requires only two fatigue curves for its calibration; this is of great help in overcoming the difficulty in considering, explicitly, variations of the load ratio. A limitation, however, is that the model, in the present form, cannot be applied to life prediction of unidirectional laminates due to their anisotropic response resulting in different limits for the strain energy density in the fiber direction and normal to it.

Polynomial criterion

The extension of the polynomial formulation of the Tsai–Hill criterion [7] to cyclic loading has already been discussed in Section 10.2.4 for short fiber composites. Recalling the formulation, the criterion can be expressed as

$$\left[\frac{\sigma_{1,a}}{K_1(N_f)} \right]^2 + \left[\frac{\sigma_{2,a}}{K_2(N_f)} \right]^2 - \frac{\sigma_{1,a}\sigma_{2,a}}{K_1^2(N_f)} + \left[\frac{\sigma_{6,a}}{K_6(N_f)} \right]^2 = 1 \quad 10.22$$

where

$$K_i(N_f) = \sigma_{i,A} \left(\frac{N_A}{N_f} \right)^{\frac{1}{k_i}} \quad (i = 1, 2, 6) \tag{10.23}$$

are the fatigue curves obtained under longitudinal, transverse and shear loading for the load ratio of interest. The criterion, in fact, does not allow explicitly accounting for variations of the load ratio R . It is evident that the validation of the criterion is quite demanding and requires at least three fatigue curves. Only some of the groups listed in Table 10.3 had all the necessary data to calibrate the criterion; for them the calibration parameters are listed in Table 10.8 and the results of the validation are presented in Fig. 10.36. The accuracy in predicting the lifetime for global multiaxiality conditions is again reasonably good and comparable with that obtained by using the criterion proposed by Smith and Pascoe. However, even in this case, fatigue lives for data of group A [41] and for the series $[\pm 45]$ of group B [57] were significantly overestimated and again it was impossible to find a justification.

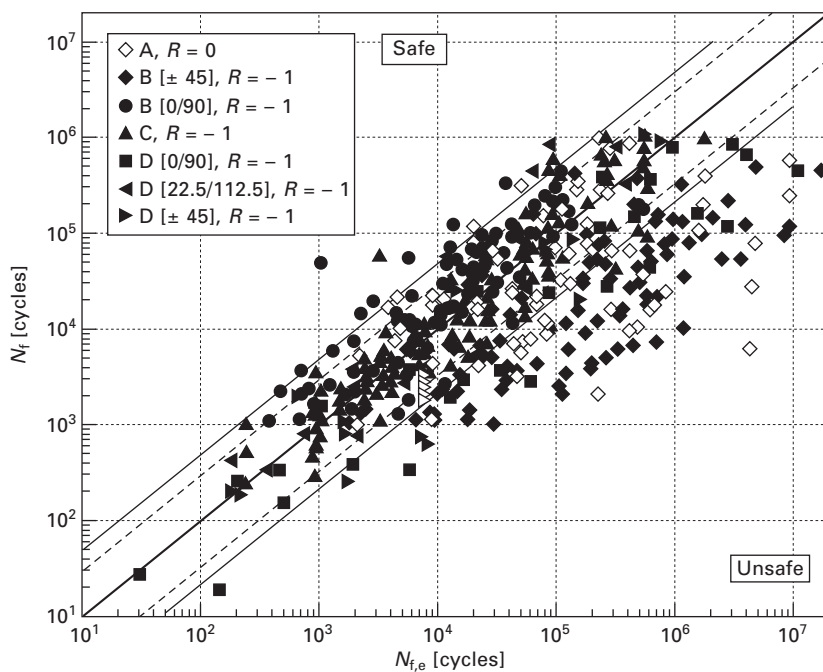
To conclude, the calibration parameters for the validation of the criterion in the case of local multiaxiality conditions are listed again in Table 10.8, and Fig. 10.37 compares experimental and estimated data. Although the number of series and fatigue data analyzed is rather limited, accuracy in life assessment is reasonable; in some cases predictions are, however, even too conservative.

10.3.4 Comments on life prediction criteria and damage mechanics

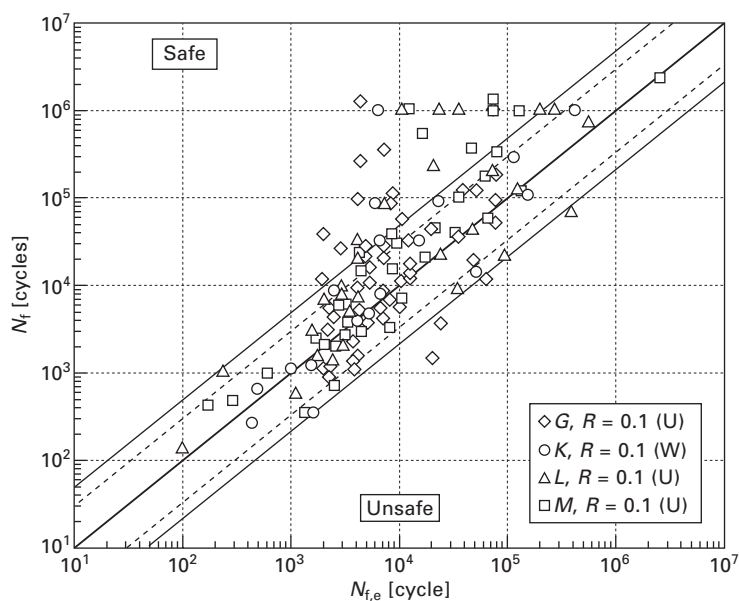
The overview of the behavior of continuous fiber reinforced composites under multiaxial cyclic loading presented in Section 10.3.2 and the assessment of

Table 10.8 Parameters of the material functions $K_i(N_f)$ in the polynomial criterion [15]

Code	Reference	$\sigma_{1,A}$ (MPa)	k_1	$\sigma_{2,A}$ (MPa)	k_2	$\sigma_{6,A}$ (MPa)	k_6	R
A	[41]	37.3	6.9	37.3	6.9	25.6	12.1	0.0
B	[57]	26.9	5.9	26.9	5.9	24.4	13.0	-1.0
C	[49, 56]	29.5	7.3	29.5	7.3	16.9	15.0	0.0
D	[38]	56.8	10.1	56.8	10.1	13.7	8.6	-1.0
G	[98]	478.3	30.2	4.1	4.4	12.1	10.8	0.1
K	[90]	196.0	26.6	196.0	26.6	7.4	6.3	0.1
L	[89]	461.9	17.1	6.1	8.3	18.1	28.8	0.1
M	[80]	504.8	24.8	6.2	7.2	16.3	14.9	0.1



10.36 Accuracy of the modified Tsai–Hill criterion in the life estimation for global multiaxial data [15].



10.37 Accuracy of the modified Tsai–Hill criterion in the life estimation for local multiaxial data [15].

certain predictive criteria reported in Section 10.3.3 provide indeed some support for the design of structural composite parts subjected to in-service loadings. On the other hand, they also display the complexity of the problem exacerbated by the large number of variables which influence the material response and, consequently, limit the present criteria for life assessment.

Focusing on the last point, available criteria can provide in some cases reasonably accurate life predictions, but reliability of the predictions seems to still be lacking. None of the criteria evaluated here incorporate in its formulation description of the fatigue damage mechanisms controlling final failure and, as far as the authors are aware, no such models are available in the literature. Faced with this situation, it is very difficult, if not impossible, to improve the largely unsafe or too conservative predictions by the criteria discussed above. Furthermore, it is difficult by using this approach to describe the property degradation due to damage evolution under fatigue loading, as well as to discriminate between damage onset and final failure, all aspects of great importance in the design of a structural part.

The way to resolve the problem is quite clear, although not very straightforward. In fact, a predictive criterion of general validity and wide applicability can be developed only on the basis of an accurate description and understanding of the damage mechanisms under multiaxial loading, incorporating these quantitatively into the model formulation.

A recent review by the authors [15] presents several works on fatigue damage under multiaxial loading. Unfortunately, most of these describe damage evolution through macroscopic parameters and only a few illustrate damage mechanisms at the microscopic level. Quantitative correlations between damage variables and fatigue life are also missing. A good example in this direction, however, is represented by the work from Adden and Horst [66] on NCF glass/epoxy tubes under cyclic tension/torsion, where crack density distributions under several biaxial loading conditions are reported. For a more extensive treatment on this subject the reader is referred to the present authors' review [15] and the references quoted therein. In the same review an outline has been presented for a mechanisms-based approach to the characterization of fatigue damage in composite laminates under cyclic multiaxial loading.

Very briefly, the analysis at microscopic level of the damage aims to identify several basic mechanisms: (1) fiber failure, (2) fiber-bridged matrix cracking, (3) matrix cracking and fiber/matrix debonding, (4) fiber microbuckling, etc. These, of course, are the same basic mechanisms acting even in the case of uniaxial loading. However, depending on the stress magnitudes and the actual degree of multiaxiality, different combinations of mechanisms can occur and interactions between them can develop. As essential requirement to understand the phenomenon, the characterization of the basic mechanisms pertaining to the mutual interaction between stress components is needed.

Great attention is therefore to be paid when planning experimental activities. Testing hoop-wound tubes under combined tension–torsion loading is a good solution, for instance, to investigate the influence of the shear stress on the transverse fatigue strength (and vice versa) and the associated damage mechanisms of matrix cracking and fiber/matrix debonding. Similarly, tubes with fibers aligned along the longitudinal axis tested under tension–torsion loading provide information about the influence of the shear stress on the on-axis fatigue strength and the associated mechanisms. Simply changing the load from cyclic tensile to cyclic compressive on these last samples makes it possible to investigate the enhancing contribution of cyclic shear to the fiber microbuckling in compression. Tension–tension loading on unidirectional cruciform specimens can clarify the mutual influence of normal and transverse stresses, and so on. Furthermore, basic test configurations can also be identified for the analysis of damage mechanisms as well as to study their interaction in the case of more complicated, multidirectional ply stacking sequences.

After careful understanding and characterization of the damage mechanisms, it would be possible to develop a description of the property degradation due to the damage evolution under fatigue. The capability for describing damage evolution under general cyclic loading conditions is seen as the essential key for the development of a general criterion of wide applicability. In the attempt to provide a practical contribution in this area, a dedicated experimental and modeling research program aimed at implementing the strategy just described is already underway and involves research groups within Europe and the USA.

10.4 Conclusions

An overview of the fatigue behavior of short fiber composites as well as of advanced continuous fiber reinforced composites has been presented. Experimental data along with assessments of life prediction criteria have been discussed.

The results presented in the chapter help to identify the influence of the main design parameters on the multiaxial fatigue behavior of these materials and to evaluate the reliability of the present predictive criteria for life assessment, thus providing a useful basis for the design of structural applications. At the same time, deficiencies remain in the applicability of these criteria.

In particular, it is clear that there is a significant need for further research in this field for better characterization and understanding of damage mechanisms acting under cyclic multiaxial loading and for their incorporation into reliable predictive models of general validity. This is the direction the scientific and research community should focus on in the coming years.

10.5 Acknowledgments

The authors wish to acknowledge two colleagues who significantly contributed to the research presented in this chapter: Prof. Luca Susmel, University of Ferrara-Italy, for the analysis on continuous fiber composite data, and Dr Matthias De Monte, former PhD student of Marino Quaresimin and now Project Manager at Corporate Sector Research and Advance Engineering – CR/APP2, Robert Bosch GmbH, Waiblingen, Germany, for the experimental and modeling research activity on short fiber composites. The financial support of Robert Bosch GmbH to the PhD research of Dr Matthias De Monte is also acknowledged.

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10.7 List of symbols

$E_1, E_2, G_{12}, \nu_{12}$	in-plane elastic parameters
k_i	negative inverse slope of the fatigue curve ($i = 1, 2, 6$)
N	number of fatigue cycles
N_f	number of cycles to failure
$N_{f,e}$	estimated number of cycles to failure
N_A	reference number of cycles to failure ($N_A = 2 \times 10^6$ cycles)
O_{xyz}	structure (geometrical) frame of reference
O_{123}	local (material) frame of reference
P_S	probability of survival
r	reference direction
R	load ratio $\sigma_{i,\min}/\sigma_{i,\max}$ ($i = 1, 2, 6$)
T_σ	scatter index $\sigma_{i,A}(P_S = 10\%)/\sigma_{i,A}(P_S = 90\%)$
$U_F = K_{SE}$	strain energy density
$U_{F,A}$	reference amplitude of the strain energy density at 2×10^6 cycles to failure
b_i, m_i	material parameters in Fawaz and Ellyin's criterion
b_r, m_r	reference material parameters in Fawaz and Ellyin's criterion
f	multiaxial degree correction function in Fawaz and Ellyin's criterion
g	load ratio correction function in Fawaz and Ellyin's criterion
κ	off-axis/on-axis constant in Fawaz and Ellyin's criterion
K	material function in polynomial criterion
β	fatigue notch factor
δ	phase shift angle
θ	off-axis angle
θ_r	reference angle
λ_C	$\sigma_{y,a}/\sigma_{x,a}$ (cruciform specimen)
λ_T	$\tau_{xy,a}/\sigma_{x,a}$ (tubular specimen)
λ_1	$\sigma_{2,a}/\sigma_{1,a}$
λ_2	$\sigma_{6,a}/\sigma_{1,a}$
λ_{12}	$\sigma_{6,a}/\sigma_{2,a}$
$\sigma_{i,A}$	reference stress amplitude ($P_S = 50\%$) at 2×10^6 cycles to failure ($i = 1, 2, 6$)
$\sigma_{i,ult}$	ultimate static strength ($i = 1, 2, 6$)
σ_{SE}	uniaxial stress associated with strain energy density
σ_1, σ_2	normal stresses calculated in the material frame of reference (in plane stress condition)
σ_6	shear stress calculated in the material frame of reference (in plane stress condition)

σ_x, σ_y	geometrical normal stresses (in plane stress condition)
τ_{xy}	geometrical shear stress (in plane stress condition)
ϕ_M	multiaxial fatigue ratio $\sigma_{1,A(\lambda)}/\sigma_{1,ult}$
NCF	non crimp fabric
sFRC	short fiber reinforced composite

Stress subscripts

a	amplitude
A	reference amplitude
max,m	maximum value

A progressive damage mechanics algorithm for life prediction of composite materials under cyclic complex stress

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Abstract: A non-linear ply-to-laminate approach to analyze failure onset and damage propagation in generic laminates under multiaxial cyclic loading is presented. The FADAS (FAtigue DAmage Simulator) algorithm implements on the one hand simple phenomenological models to describe strength and stiffness loss at each ply due to fatigue and on the other hand adequate failure criteria to predict damage progression triggered by different failure mechanisms. The numerical model is suitable for predicting strength and stiffness of the laminate after arbitrary cyclic loading. Validation of theoretical results was performed by comparing with constant and variable amplitude fatigue data and residual static strength after fatigue from multidirectional glass/epoxy laminates. The agreement was satisfactory with most of the test results.

Key words: progressive damage, stiffness degradation, failure modes, glass/epoxy composites, constant (CA) and variable-amplitude (VA) fatigue, residual strength, life prediction.

11.1 Introduction

When complex stress fields are developed in composite structures operating under cyclic load, life prediction becomes a formidable task, especially in cases of irregular spectrum loading. The implementation of the numerical procedure for fatigue analysis consists of a number of distinct calculation modules, related to the main theme of life prediction. Some of them are purely conjectural or of a semi-empirical nature, e.g. the failure criteria, while others rely heavily on experimental data, e.g. S–N curves and constant life diagrams (CLDs).

In cases of composite laminates under uniaxial loading, leading to uniform axial stress fields, the situation might be substantially simplified since almost all relevant procedures could be implemented by experiment. On the other hand, for complex stress states, the laminated material is considered as being a homogeneous orthotropic medium and its experimental characterization, i.e. static and fatigue strength, is performed for both material principal directions and in-plane shear. The relevant stress parameters taken into account when

comparing with strength in the failure criteria are the stress resultants (N_x , N_y , N_s), as defined in classical plate theory.

Such a *laminates approach* is a straightforward one, in predicting fatigue strength under plane stress conditions, avoiding the consideration of damage modelling, interaction effects between the plies and stress redistribution. The experimental data set required to implement the procedure, to cover variable amplitude (VA) loading for structures such as wind turbine rotor blades, consists of a number of S–N curves at various values of the stress ratio, R . Not less than three values are taken, usually $R = 10$, -1 and 0.1 . These characteristics must be derived for both principal material directions and in-plane shear, resulting in a total of at least seven S–N curves by assuming that in-plane shear fatigue strength is independent of stress ratio, R . The experimentally defined fatigue property set is unique for each laminate. The approach was implemented by Philippidis and Vassilopoulos (2002a, 2002b, 2003, 2004) for a glass/polyester multidirectional laminate of $[0/\pm 45]$ stacking sequence and was shown to yield satisfactory predictions for fatigue strength under complex stress conditions for both constant amplitude (CA) and VA loading.

When the number of different stacking sequences in a structural element is limited, the *laminates approach* is a reliable alternative to a life prediction task. However, with large composite structures, composed of different components of varying structural details, numerous laminates of different stacking sequences are usually in order. The huge amount of test effort to characterize fatigue strength of all different lay-ups prevents one from implementing a *laminates approach* methodology. In these cases, a *ply-to-laminates approach*, in which experimental characterization is performed at the basic constitutive ply level and then fatigue strength of whatever laminate configuration is numerically derived, is certainly a more effective procedure.

Nevertheless, with such an option a number of additional calculation modules need to be developed, requiring both theoretical and experimental implementation. These are related to how damage is modelled in each lamina, the implications in local stress fields, stress redistribution in neighbouring plies, and finally, how damage propagates as a function of loading cycles.

Relatively few works have been published on the subject, most of them in the last 15 years. Based on the *internal state variable approach* of Lee *et al.* (1989) to describe stiffness degradation of a material element due to distributed damage, Coats and Harris (1995) and Lo *et al.* (1996) presented the first contributions in the field. Input data for damage progression were derived experimentally and consisted of crack surface area and crack density measurements by means of edge replicas and X-rays. Model predictions were possible only for tension–tension loading, while experimental verification

so far has been provided only for stiffness degradation as a function of the applied number of cycles.

The approach followed by Harris and co-workers is of the 'ply-to-laminate' type in which all constitutive formulation takes place at the ply level. Prediction of life, strength or stiffness for a laminate of any stacking sequence, composed of the building ply, is in general possible. One of the most complete works of that type of approach was published by Shokrieh and Lessard (2000a, 2000b). They have developed a method that could be used as a design tool, predicting life, residual stiffness and strength of a laminate based on ply properties. Linear stress analysis was performed, although nonlinear effects for the shear stresses were included in their failure criteria.

Van Paepegem and Degrieck (2001, 2002a, 2002b, 2003) and Van Paepegem *et al.* (2001, 2005) have developed a stiffness degradation-based damage mechanics model, using material properties of a cross-ply laminate and not of the UD ply. So, the model does not predict failure modes of the ply but the macroscopic failure of the cross-ply laminate. It includes a number of parameters, fitted by experiments on a specific load case (single side displacement-controlled bending) which probably depend on the stacking sequence and the load case.

Sihn and Park (2008) have presented an integrated design module for predicting strength and life of composite structures. Their analysis was based on micromechanics of failure by considering separately the composite constituents. Viscoelastic behaviour of the polymeric matrix was also taken into account. No experimental evidence on the validity of their method was presented. Although this type of approach, based on micromechanics, seems promising for the future, it also has serious disadvantages related especially to the mechanical characterization of fibre and matrix interaction and the description of damage evolution laws. Further, from an engineering point of view, composite material properties are certified at the ply level and thus before a life prediction method based on micromechanics could be used, wide agreement on characterization procedures and test methods should be sought.

In this chapter, a continuum damage mechanics method is implemented in a *ply-to-laminate* life prediction scheme for composite laminates under cyclic CA or VA loading. According to theoretical foundations of distributed damage, e.g. as in Lee *et al.* (1989), Ladeveze (1992), Renard *et al.* (1993) and Maire and Chaboche (1997), instead of considering the geometric description of a type of defect induced by local failure, a set of appropriate stiffness degradation rules is applied, resulting in a modified stiffness tensor, i.e. an equivalent, homogeneous, continuum description, such that either the resulting strain field or the strain energy density under the same load is similar to that of the damaged medium.

This effective medium description requires, besides *sudden stiffness* degradation due to failure onset driven by the stress at a point, also strength and stiffness degradation of a progressive character due to cycling, expressed as a function of the number of load cycles, n . Hence, residual strength and stiffness after cycling become of importance for this *progressive damage mechanics* approach and certainly require great experimental effort, besides efficient modelling, to cover the various loading conditions, e.g. tension–tension (T–T), tension–compression (T–C), etc., at various stress ratio values and material principal directions.

To assess conditions for incipient failure in a specific mode, compatible with certain defect types and their respective stiffness degradation strategies, appropriate failure criteria considering separately the various failure modes, such as those introduced by Puck and Schürmann (1998, 2002), Puck *et al.* (2002) or Chang and Lessard (1991), were implemented in the numerical procedure. The material model consists also of the detailed description of fatigue strength in each principal material direction and in-plane shear, always for the basic building ply, for several R values to ease the implementation of CLD formulations.

A detailed step-by-step load simulation of each cycle is foreseen in the realization of the algorithm; however, in practice, albeit more accurate, this could be extremely time-consuming, especially when the routine is implemented in finite element formulations. Alternatively, calculations are performed in steps for a complete cycle and then after a block, Δn , of cycles again in steps for a detailed complete cycle and so forth. The size of cycle jump is defined by the user.

Non-linear response of the unidirectional (UD) ply, especially under static in-plane shear and normal loading transverse to the fibres, is also taken into account, introducing appropriate models derived by fitting experimental data. In the numerical analysis, non-linearity is modelled by implementing an incremental stress–strain constitutive law.

An extensive comparison of life prediction numerical results with experimental data from constant (CA) or variable amplitude (VA) tensile cyclic testing of a $[\pm 45]_S$ plate and loading at various R -ratios of a multidirectional (MD) glass/epoxy laminate $[(\pm 45/0)_4/\pm 45]_T$ was presented. Coupons cut from the MD plate were loaded either on-axis, where fibre-dominated failure modes were observed, or off-axis in various directions where matrix damage also made a significant contribution to the observed failure. Predictions of tensile residual strength after cyclic loading of $[\pm 45]_S$ coupons were also compared with test results.

11.2 Constitutive laws

The progressive damage simulator for life prediction under cyclic complex stress presented in this work was initially devised for glass/epoxy composites

used in the wind turbine rotor blade industry. It relies on material data from a huge experimental effort in the frame of an EC-funded research project that resulted in a comprehensive material property database with test results from static, cyclic and residual strength experiments under axial and multiaxial loading conditions. All data is free for download from the official Optimat Blades site (<http://www.wmc.eu/optimatblades.php>) along with the relevant reports and publications.

11.2.1 Ply response under quasi-static monotonic loading

The basic building block of all laminates considered is the UD ply, hereafter called OB_UD glass/epoxy. Besides information on mechanical property characterization that was reported in the OptiDAT database as indicated in the above, most of the data were also published by Philippidis and Antoniou (2010). In-plane mechanical properties of the UD ply were obtained through an extensive experimental program consisting of static tests, both parallel and transverse to the fibres and also in shear. The unique specimen geometry used in the characterization procedure has replaced the numerous ISO geometry coupons in all kinds of tests, static, fatigue and residual strength. Experimental data compare very well with those produced with ISO specimens, except in the case of compression along the fibres where the so-called OB-coupon suffers from buckling (ISO strength is adopted). Shear properties were still obtained through the ISO procedure – see also Philippidis and Assimakopoulou (2008).

UD material performs linearly in the fibre direction as expected; however, transversely to the fibres, mainly in compression and under in-plane shear, the material behaviour is highly non-linear. To take into account this observation, incremental stress–strain theory is implemented, retaining the validity of the generalized Hooke law for each individual interval:

$$\begin{aligned}
 d\sigma_1 &= \frac{E_1}{1 - \frac{E_{2t}}{E_1} \nu_{12}^2} d\varepsilon_1 + \frac{\nu_{12}E_{2t}}{1 - \frac{E_{2t}}{E_1} \nu_{12}^2} d\varepsilon_2 \\
 d\sigma_2 &= \frac{\nu_{12}E_{2t}}{1 - \frac{E_{2t}}{E_1} \nu_{12}^2} d\varepsilon_1 + \frac{E_{2t}}{1 - \frac{E_{2t}}{E_1} \nu_{12}^2} d\varepsilon_2 \\
 d\sigma_6 &= G_{12t} d\varepsilon_6
 \end{aligned}
 \tag{11.1}$$

In the above equations, E_1 and ν_{12} were considered constant throughout the static loading up to failure, while the tangential elastic moduli were given by the nonlinear constitutive relation introduced by Richard and Blacklock (1969):

$$\sigma_i = \frac{E_{oi}\varepsilon_i}{\left[1 + \left(\frac{E_{oi}\varepsilon_i}{\sigma_{oi}}\right)^{n_i}\right]^{\frac{1}{n_i}}}, i = 2, 6 \quad 11.2$$

E_{oi} , n_i and σ_{oi} are model parameters defined by fitting the experimental data. Tangential elastic moduli were derived using the following relations:

$$\begin{aligned} E_{2t} &= \frac{d\sigma_2}{d\varepsilon_2} = E_{o2} \left[1 - \left(\frac{\sigma_2}{\sigma_{o2}} \right)^{n_2} \right]^{\frac{1}{n_2}+1} \\ G_{12t} &= \frac{d\sigma_6}{d\varepsilon_6} = G_{o12} \left[1 - \left(\frac{\sigma_6}{\sigma_{o6}} \right)^{n_6} \right]^{\frac{1}{n_6}+1} \end{aligned} \quad 11.3$$

The parameters E_{o2} , σ_{o2} and n_2 were found to be different in tension and compression. Numerical values for all the above constants are summarized in Table 11.1. The predicted stress–strain curves compare favourably to the experimental data as seen in Fig. 11.1 and Fig. 11.2.

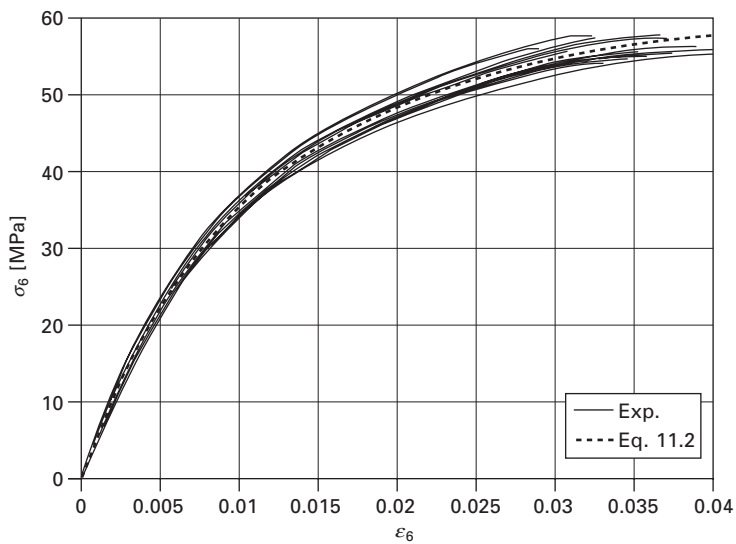
Mean values of the ply in-plane failure stresses are given in Table 11.2. The respective strengths in the fibre direction, transversely and in-plane shear are denoted by X , Y and S . Average ply thickness from measurements on uniaxial UD [0] coupons was found to be 0.94 mm, while when stitched fabric [± 45] was used for multi-directional lay-ups, ply thickness was taken to be 0.33 mm. Therefore, attention should be paid to the definition of the MD lay-up, [$\pm 45/0$]₄/ ± 45]_T, in which the thickness of the [0] ply is about three times that of the [45] layers.

11.2.2 Loading–unloading–reloading (L–U–R)

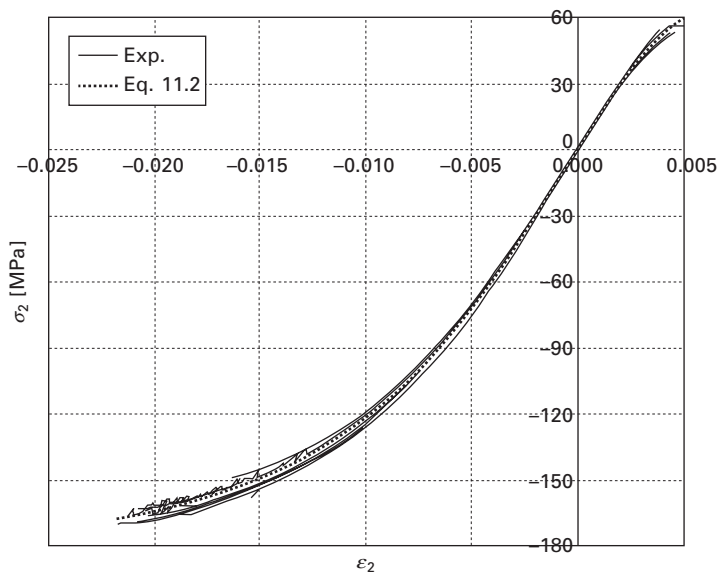
Engineering elastic constants appearing in the constitutive relations, Eq. 11.1, are valid for monotonic loading conditions. Upon unloading, the stiffness changes and must be again defined experimentally. It was further observed that stiffness decreases upon repeated L–U–R cycles, depending on the stress level previously reached. As compiled by Philippidis *et al.* (2007),

Table 11.1 Elastic constants (in MPa), OB_UD glass/epoxy
($E_1 = 37,950$; $\nu_{12} = 0.28$)

	E_{oi}	σ_{oi}	n_i
$E_{2t}^{(T)}$	15,035	75	3
$E_{2t}^{(C)}$	15,262	188	2.18
G_{12t}	5,000	67	1.3



11.1 In-plane shear stress–strain behaviour of OB_UD glass/epoxy.

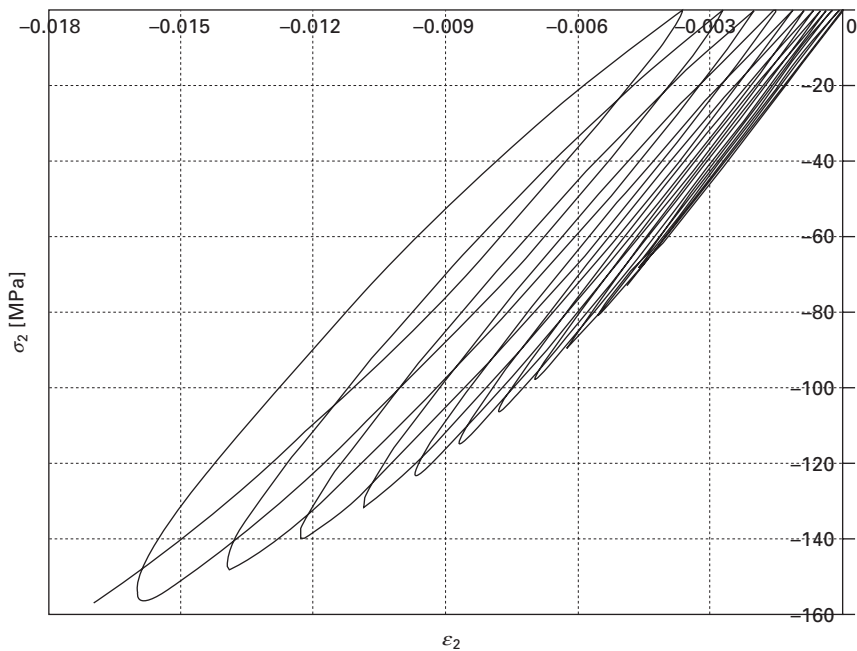


11.2 Transverse tension–compression response of OB_UD glass/epoxy.

the stiffness reduction is more severe for matrix-dominated response, e.g. in-plane shear and transverse loading to the fibres: see Fig. 11.3. In fact, no stiffness reduction due to L–U–R cycles was foreseen under loading parallel to the fibres.

Table 11.2 Strength values (in MPa), OB_UD glass/epoxy

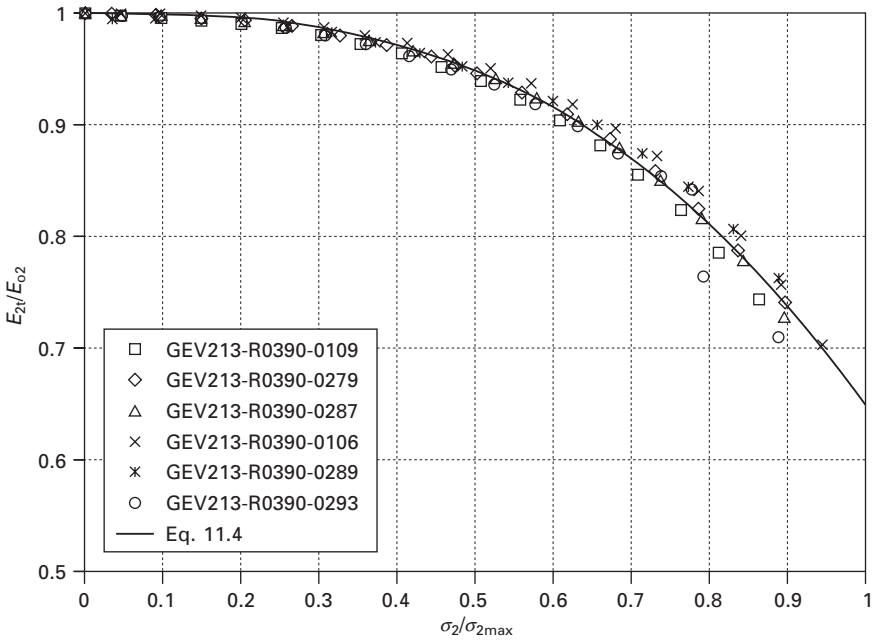
X_T	X_C	Y_T	Y_C	S
776	686	54	165	80



11.3 Typical stress–strain cycles in static L–U–R compression transverse to the fibres.

This type of stiffness degradation, although probably due to micro-cracking of the polymeric matrix possibly in the interface region with the fibres, and individual fibre breaks, was considered as a constitutive tensor property of the lamina that was derived by means of dedicated experiments. Strain was recorded during the L–U–R tests using strain gauges. The elastic modulus was determined as the slope of the linear regression model of each stress–strain loop. The values from a test were normalized with respect to the modulus of the first cycle and plotted against the normalized (with respect to the maximum stress of each test) stress level: see Fig. 11.4.

The stiffness degradation models were determined with nonlinear regression applied on the normalized stiffness–stress data from all tests and given by:



11.4 Modulus degradation transverse to the fibres due to compressive L–U–R cycles.

$$\frac{E_{2t}}{E_{o2}} = 1 - (1 - a_2) \left(\frac{\sigma_{2G_{\max}}}{Y_T} \right)^{b_2}$$

$$\frac{G_{12t}}{G_{o12}} = 1 - (1 - a_6) \left(\frac{\sigma_{6G_{\max}}}{S} \right)^{b_6} \quad 11.4$$

The global maximum stress reached during cycling is denoted by $\sigma_{iG_{\max}}$, while Y_T and S stand for the tensile strength transversely to the fibres and the in-plane shear strength respectively. Since values of E_{o2} or G_{o12} presented slight variations for the different coupon tests, the respective values from Table 11.1 were implemented along with Eq. 11.4. The parameters a_2 and b_2 were found to be different in tension and compression. Numerical values for all the above constants are summarized in Table 11.3.

When the first equation of Eqs 11.4 is used to determine the compressive elastic modulus transverse to the fibres the tensile strength, Y_T , should be replaced by the corresponding compressive strengths, Y_C .

The elastic modulus as determined by Eq. 11.4 is implemented for the reloading branch. Considering a slightly greater unloading modulus, e.g. multiplying by a factor greater than unity, enables the model to take into account the permanent strains as well. The inclusion of this type of stiffness

Table 11.3 Parameter values for L–U–R stiffness degradation models, Eq. (11.4)

	a_i	b_i
$E_{2t}^{(T)}$	0.88	1.60
$E_{2t}^{(C)}$	0.65	2.77
G_{12t}	0.38	1.40

reduction in the constitutive material model is expected to affect numerical predictions, especially in cases of VA loading.

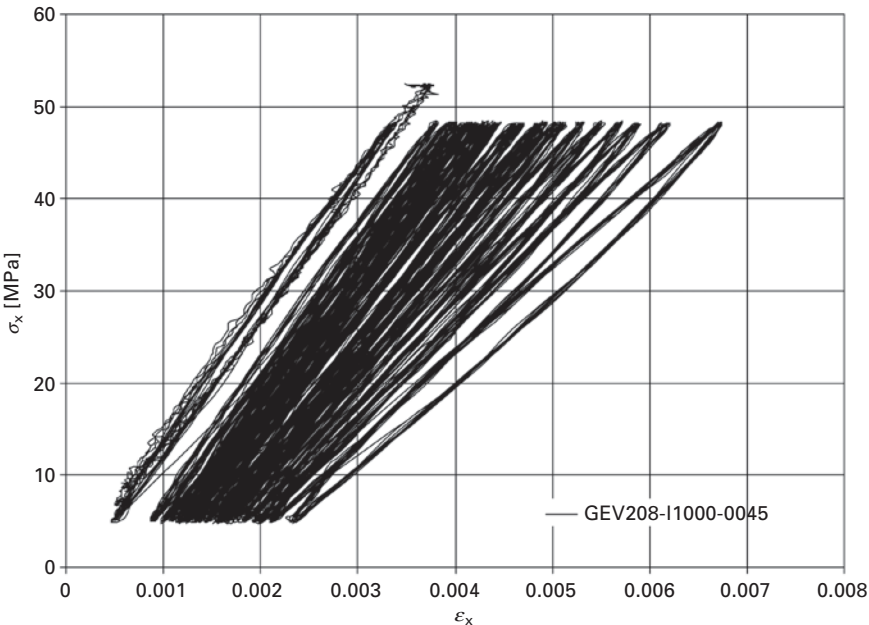
11.2.3 Stiffness degradation

In-plane stiffness of the lamina degrades for several reasons, e.g. sudden stiffness reduction due to some kind of failure occurrence or progressive stiffness reduction due to cycling. In general, the latter is nonlinear and several formulations have been proposed in the literature to describe it.

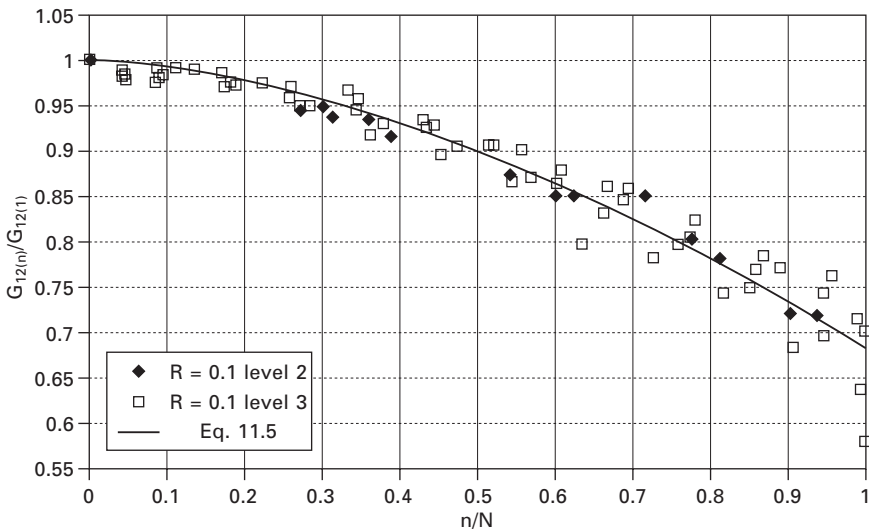
As presented by Philippidis *et al.* (2007), during the fatigue tests, load–displacement data were recorded periodically and were transformed to respective stress–strain data, e.g. see Fig. 11.5 where experimental data from tensile cyclic loading (T–T) of a $[\pm 45]_S$ coupon at a stress ratio $R = 0.1$ are shown. The stiffness of the coupon at each cycle was determined as the slope of the linear regression model of the respective stress–strain loop. These stiffness values were normalized with respect to the stiffness of the first cycle and plotted with the normalized number of cycles with respect to the number of cycles at failure. The stiffness degradation models were determined with nonlinear regression applied to the normalized stiffness–cycle number data from all coupon tests, at various loading levels. An example for the degradation of the in-plane shear modulus G_{12} of the OB_UD glass/epoxy ply is shown in Fig. 11.6. The experimental data were derived from T–T cyclic tests at $R = 0.1$.

In the present FADAS implementation the regression models depend only on the fatigue life fraction, i.e. the ratio of the applied cycles versus the nominal fatigue life at the current stress level. In this way, the modulus degradation depends implicitly also on the stress ratio, R , and the maximum applied cyclic stress, $\sigma_{\max}$. The following functional forms were fitted to the experimental data:

$$\begin{aligned} \frac{E_2(n)}{E_2(1)} &= 1 - (1 - c_2) \left(\frac{n}{N} \right)^{d_2} \\ \frac{G_{12}(n)}{G_{12}(1)} &= 1 - (1 - c_6) \left(\frac{n}{N} \right)^{d_6} \end{aligned} \quad 11.5$$



11.5 Stress-strain loops under CA fatigue of an ISO 14129 [± 45]_S coupon.



11.6 In-plane shear modulus degradation data for the OB_UD glass/epoxy.

The parameters c_2 and d_2 were found to be different in tension and compression. Numerical values for all the above constants are presented in Table 11.4. Since the modulus values at the first cycle from the different coupons tested are different, $E_2(1)$ and $G_{12}(1)$ in Eq. 11.5 were substituted by the respective reloading stiffness values corresponding to $\sigma_{G\max}$, as given by Eq. 11.4.

Progressive stiffness degradation for any cyclic loading type parallel to the fibres was not important and thus it was neglected in the numerical model.

Pre-failure material models

In case no failure was detected, the simulated ply response and especially the description of stiffness evolution for VA cyclic loading were expressed by combining the constitutive relations presented in the above. Each stress tensor component at the k th loading step is examined to see whether it corresponds to loading, i.e. $|\sigma_{i(k)}| \geq |\sigma_{i(k-1)}|$, $i = 1, 2, 6$ or else to unloading.

In the former case, if $\sigma_{i(k)}$ is higher than the global maximum stress, $\sigma_{iG\max}$, or lower than the global minimum stress, $\sigma_{iG\min}$, the initial material behaviour under quasi-static loading, presented in Section 11.2.1, is assumed, that is, a constant modulus E_1 and Poisson ratio ν_{12} while E_2 and G_{12} are functions of $\sigma_{2(k)}$ and $\sigma_{6(k)}$ as expressed by Eq. (11.3). If $\sigma_{i(k)}$ lies between the global minimum and maximum stress, the reload elastic properties are used, Eq. (11.4), calculated at the global maximum or minimum stresses, degraded according to the stiffness degradation models due to cycling, i.e. Eq. (11.5).

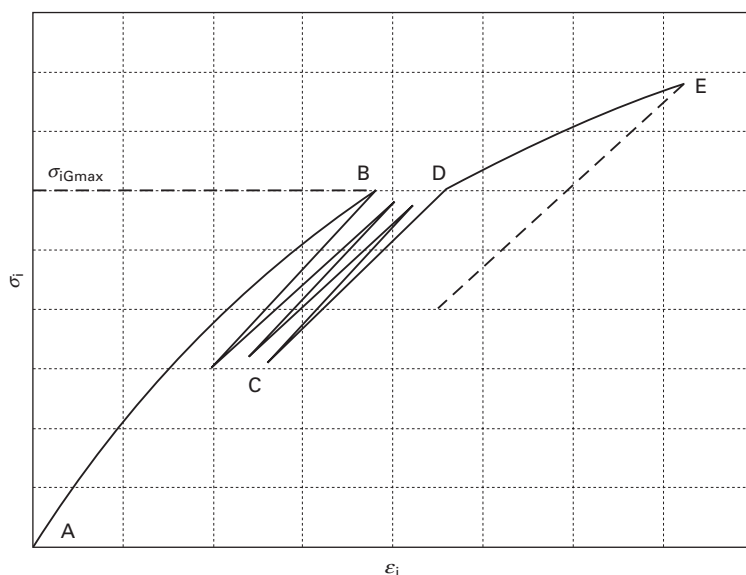
In the case of unloading, elastic properties slightly higher than in the case of reloading are used to introduce an increasing permanent strain due to cyclic loading. In the routine this is realized by multiplying by 1.00002 the reloading stiffness values for E_2 and G_{12} .

The above is illustrated in Fig. 11.7:

- A–B: Initial loading. Stress is always greater than its previous global maximum value, so the non-linear material behaviour under quasistatic loading is used.
- B–C–D: Stress cycling under CA or VA. Stress values remain between their global minimum and maximum values, 0 and $\sigma_{iG\max}$ respectively,

Table 11.4 Parameter values for the cyclic stiffness degradation models, Eq. (11.5)

	c_i	d_i
$E_2^{(T)}$	0.75	3.17
$E_2^{(C)}$	0.95	0.62
G_{12}	0.68	1.65



11.7 Pre-failure material model for the OB_UD glass/epoxy.

so the reload and unload elastic properties are used, gradually degrading with increasing number of cycles.

- D–E: Stress becomes greater than its previous global maximum value σ_{IGmax} , so the initial material behaviour is assumed, etc.

As seen in Fig. 11.7, the behaviour of the material under cyclic stress is assumed to be linear elastic, its stiffness depending on the global maximum stresses reached so far and also on the applied number of cycles. However, when the applied stress level exceeds previous maxima, nonlinear response is again recalled.

Post-failure material models

Upon failure onset in some loading step, the stiffness degrades, depending on the failure mode observed, and the changes apply for the next loading step. In fibre failure (FF) under either tensile or compressive stresses the three engineering elastic constants, E_1 , E_2 and G_{12} , drop to zero. If matrix damage modes occur, also called inter-fibre failure (IFF) (see Section 11.3 for a detailed description), only E_2 and G_{12} drop to zero.

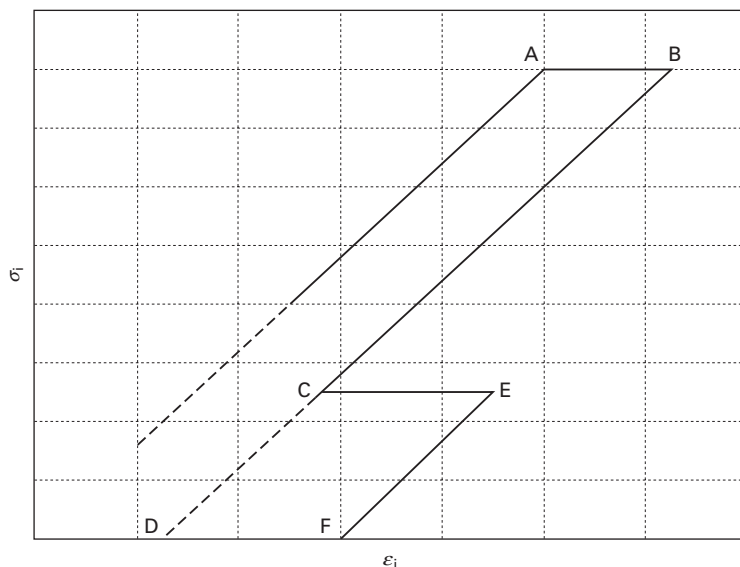
After fibre failure (FF), the unload behaviour for all three stress tensor components remains as in the virgin material, i.e. degraded reload values for E_2 and G_{12} multiplied by the appropriate factors mentioned earlier to take into account residual strains. If reloading occurs before any stress

tensor component has changed sign, the respective modulus, i.e. E_1 , E_2 or G_{12} , drops to zero. If the stress has changed sign once, the corresponding modulus remains always at zero. The above is illustrated in Fig. 11.8:

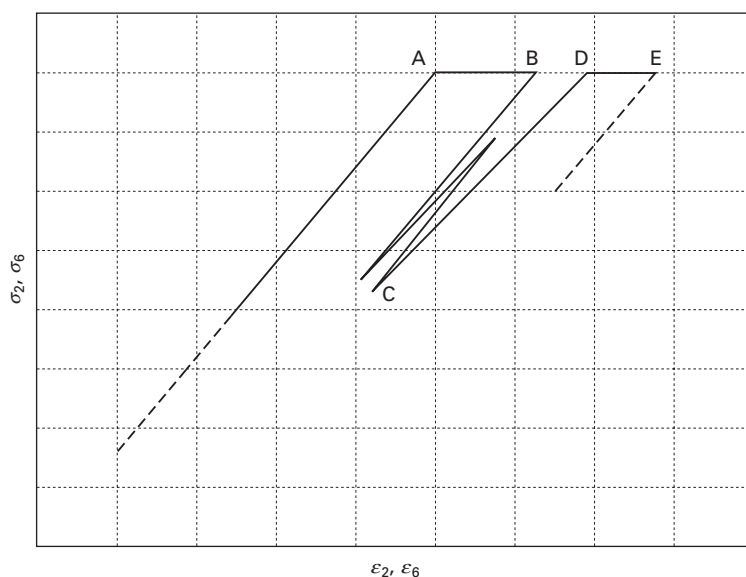
- A: Stress level at which FF mode was detected.
- A–B: If $\sigma_{i(k)}$ stands for loading, the corresponding engineering elastic constant drops to zero.
- B–C, C–D, E–F: Unloading using the unloading elastic properties.
- C–E: If reloading is encountered before stress has changed sign, the elastic property drop to zero.
- D, F: Following unloading, a stress tensor component changes sign. The corresponding elastic property drops and remains henceforth at zero.

In case of matrix failure (IFF damage modes), E_1 remains unaffected and only the normal stress transverse to the fibres and the in-plane shear component are taken into account in the stiffness degradation model.

Once IFF is detected, both unload and reload properties remain as for the virgin material models presented in Sections 11.2.2 and 11.2.3, unless IFF is detected again; both engineering elastic constants E_2 and G_{12} drop to zero. If only the value of the normal stress transverse to the fibres σ_2 or the in-plane shear stress σ_6 exceeds its value for which IFF has been predicted last time, the respective elastic property (E_2 or G_{12}) drops to zero and the process is continued. With respect to Fig. 11.9, illustrating the above, the following characteristics can be noted.



11.8 Post-FF material model for the OB_UD glass/epoxy.



11.9 Post-IFF material model for the OB_UD glass/epoxy.

- A: Stress level at which IFF was first detected.
- A–B: Loading is continued; both E_2 and G_{12} drop to zero.
- B–C–D: No IFF is predicted again. The stress component remains lower than its value at failure. The reload and unload elastic properties of the virgin material are used, gradually degraded with the number of cycles.
- D–E: IFF is predicted once more or stress σ_2 or σ_6 becomes equal to or greater than its value when IFF was predicted. The corresponding elastic property drops to zero.

11.3 Failure onset conditions

Predicting laminate strength under cyclic complex stress states is conceptually different from predicting failure onset under monotonic loading. The latter is a first ply failure (FPF) approach directly implementing ply stresses in a suitable limit condition and finally suggesting the layer with the maximum risk of failure. On the other hand, fatigue strength prediction is a last ply failure (LPF) procedure involving modelling of progressive damage, e.g. consideration of strength and stiffness degradation due to cyclic stresses when a ply-to-laminate approach is implemented, as by Shokrieh and Lessard (2000a, 2000b).

When the laminate is considered macroscopically as the anisotropic material (and not the ply), its static and fatigue strength need to be adequately

characterized experimentally; then fatigue strength criteria such as those developed for example by Fawaz and Ellyin (1994) or Philippidis and Vassilopoulos (1999) can be used.

The methodology used in FADAS is of the ply-to-laminate type with progressive damage modelling. In such an approach it is sufficient to use a static limit condition at the ply level where, however, material strength parameters are replaced by the corresponding residual strength values, which are in general functions of the number of cycles and the type of loading. For the cases studied in this work, numerical results were derived by implementing the Puck criterion in the FADAS routine.

Based on the concepts first introduced by Hashin (1980) for different damage mechanisms in composite materials and the Mohr–Coulomb hypothesis for brittle materials that fracture is exclusively triggered by stresses acting on the fracture plane, the criterion of Puck and Schürmann (1998, 2002) describes several failure modes using different equations. For 2D plane stress analysis five types of damage were assumed, two related to fibre fracture (FF), one in tension and another in compression, and the other three concerning matrix failure (inter-fibre fracture or IFF). Under tension ($\sigma_2 \geq 0$), cracks open transverse to the applied normal stresses ($\theta_{fp} = 0^\circ$, the angle subtended by the fracture plane and the vertical one to the layer plane), described as mode A. In compression, either closed cracks are formed transverse to the applied normal stresses ($\theta_{fp} = 0^\circ$), described as mode B, or an oblique rupture occurs with the fracture plane forming an angle θ_{fp} between $\pm 45^\circ$ and $\pm 55^\circ$, described as mode C.

Fibre and matrix failure effort or stress exposure factor, $f_{E(FF)}$ and $f_{E(IFF)}$ respectively, can be calculated as follows. For fibre failure (FF) under tensile loading:

$$f_{E(FF)}^T = \frac{1}{X_T} \left[\sigma_1 + \left(\frac{E_1}{E_{f1}} \nu_{f12} m_{\sigma f} - \nu_{12} \right) \sigma_2 \right] \leq 1 \text{ if } [\dots] \geq 0 \quad 11.6$$

Respectively, for fibre failure (FF) under compressive loading:

$$f_{E(FF)}^C = \frac{1}{X_C} \left[\left[\sigma_1 + \left(\frac{E_1}{E_{f1}} \nu_{f12} m_{\sigma f} - \nu_{12} \right) \sigma_2 \right] \right] \leq 1 \text{ if } [\dots] < 0 \quad 11.7$$

For Mode A, IFF condition, for which the fracture plane is vertical to the layer plane ($\theta_{fp} = 0^\circ$):

$$f_{E(IFF)}^A = \sqrt{\left(\frac{\sigma_6}{S} \right)^2 + \left(1 - p_{\perp\parallel}^{(+)} \frac{Y_T}{S} \right)^2 \left(\frac{\sigma_2}{Y_T} \right)^2} + p_{\perp\parallel}^{(+)} \frac{\sigma_2}{S} + (0.9 f_{E(FF)})^6 \leq 1 \text{ if } \sigma_2 \geq 0 \quad 11.8$$

For transverse compression and moderate in-plane shear, Mode B, IFF condition, for which again the fracture plane is vertical to the layer plane ($\theta_{fp} = 0^\circ$):

$$f_{E(IFF)}^B = \frac{1}{S} \left[\sqrt{(\sigma_6)^2 + (p_{\perp\perp}^{(-)} \sigma_2)^2} + p_{\perp\perp}^{(-)} \sigma_2 \right] + (0.9f_{E(FF)})^6 \leq 1 \quad (11.9)$$

if $\sigma_2 < 0$ and $0 \leq \left| \frac{\sigma_2}{\sigma_6} \right| \leq \frac{R_{\perp\perp}^A}{|\sigma_{6c}|}$

Finally, for the explosive mode C, IFF condition, the fracture plane forms an oblique angle with the vertical to the ply plane ($\theta_{fp} \neq 0^\circ$):

$$f_{E(IFF)}^C = \left[\left(\frac{\sigma_6}{2(1 + p_{\perp\perp}^{(-)} S)} \right)^2 + \left(\frac{\sigma_2}{Y_c} \right)^2 \right] \left(\frac{Y_c}{(-\sigma_2)} \right) + (0.9f_{E(FF)})^6 \leq 1 \quad (11.10)$$

if $\sigma_2 < 0$ and $0 \leq \left| \frac{\sigma_6}{\sigma_2} \right| \leq \frac{|\sigma_{6c}|}{R_{\perp\perp}^A}$

E_{f1} and ν_{f12} are the elastic modulus and the Poisson ratio of the fibres. The term $m_{\sigma f}$ accounts for a stress magnification effect caused by the different moduli of fibres and matrix which leads to an uneven distribution of the transverse stress σ_2 from a micromechanical point of view; in the fibres it is slightly higher than in the matrix. For the variety of parameters implemented in the above relations, guidelines and typical values were explicitly presented by Puck *et al.* (2002). The values used in the present version are given by:

$$E_{f1} = 72.45 \text{ GPa}, \nu_{f12} = 0.22, m_{\sigma f} = 1.3, p_{\perp\perp}^{(+)} = 0.3, p_{\perp\perp}^{(-)} = 0.25$$

$$p_{\perp\perp}^{(-)} = \frac{1}{2} \sqrt{1 + 2p_{\perp\perp}^{(-)} \frac{Y_c}{S}} - 1, \sigma_{6c} = S \sqrt{1 + 2p_{\perp\perp}^{(-)}}, R_{\perp\perp}^A = \frac{Y_c}{2(1 + p_{\perp\perp}^{(-)})} \quad (11.11)$$

When the criterion is used for cyclic stresses, the lamina strength values X_T , X_C , Y_T , Y_C and S given in Table 11.2 must be replaced by the corresponding residual strength values: see Section 11.4.

11.4 Strength degradation due to cyclic loading

Static strength degradation or residual strength after fatigue in composites has been intensively investigated during the last 30 years. Numerous research groups have developed a variety of models; an appraisal of their effectiveness has been recently presented by Philippidis and Passipoularidis (2007). The majority of the work concerns modelling of residual tensile strength in the

laminate level, under axial loading and in most cases at a single R -ratio, usually in the tension–tension region. Very limited are the experimental data sets concerning complex stress states. The model introduced by Shokrieh and Lessard (1997a, 1997b) is perhaps one of the first phenomenological approaches to examine damage evolution and failure due to multiaxial fatigue in a composite laminate in terms of the strength and stiffness degradation of the building ply.

On the other hand, only a few tests under limited loading conditions have been performed for model evaluation, without any investigation on more complex issues, e.g. residual tensile strength after compression–compression fatigue. In general, the lack of detailed experimental data, under various fatigue conditions and for a single material, has restricted the study of residual strength-based models to limited loading conditions and to specific lay-ups.

To link existing knowledge and promote the modelling of static strength degradation due to stress cycling, a comprehensive experimental program was undertaken in the frame of the European research project Optimat Blades in order to study, amongst other things, the static strength, fatigue life and residual strength behaviour of wind turbine rotor blade materials. In particular, these properties have been studied in the symmetry directions of an orthotropic UD glass/epoxy material (OB_UD), i.e. along and transverse to the fibres and under in-plane shear.

Regarding residual strength, both the tensile and compressive residual strength were studied in detail – for the first time – under different stress ratios of fatigue loading, for all the principal directions of the specific lamina, using a single coupon geometry for tensile and compressive tests under either static or fatigue loads, in an effort to keep results unbiased by different coupon geometries, use of anti-buckling devices, etc. The investigation considered stress ratios in the range of those experienced at different points of the blades during operation, i.e. tension–tension and reversed loading as well as compression–compression fatigue.

From the processing of the experimental data, the main conclusions of Philippidis and Passipoularidis (2006) were derived and formulated as guidelines for further development. First, the residual strength in both principal material directions is not affected when cyclic stress of the opposite sign is applied, i.e. tensile strength is not reduced under purely compressive cycles and vice versa. A similar trend was also observed by Nijssen (2006) from tests in the fibre-dominated direction of a $[(\pm 45/0)_4/\pm 45]_T$ laminate, made also of the OB_UD glass/epoxy, under various loading conditions (R -ratios).

The tensile and the in-plane shear static strength experienced degradation of up to 40% when tested at a nominal life fraction of 80%. The compressive residual strength, on the other hand, did not show significant degradation in all types of loading and material directions. Concerning the many theoretical models considered in this investigation, only a few corroborated satisfactorily

with the majority of the experimental data (Passipoularidis and Philippidis, 2009a). It was demonstrated in a clear manner that the complexity of a model is not related to the accuracy of its predictions. In addition, it was also proved by Passipoularidis and Philippidis (2009b) that life prediction results under VA loading are not very sensitive to the particular residual strength model, of those few validated, of course, that is used as damage metric.

Therefore, two models are used herein to describe the phenomenon. For the modelling of tensile residual strength along the principal material directions, under T–T or T–C cyclic loading, as well as under in-plane shear, the linear degradation model proposed by Broutman and Sahu (1972) was implemented. Besides being the simplest one available, it requires no residual strength testing while at the same time it has been proven by Philippidis and Passipoularidis (2007) to produce always safe residual strength predictions under various stress conditions and lay-ups. It is described respectively by the following equations:

$$\begin{aligned} X_{T_r} &= X_T - (X_T - \sigma_{1\max}) \left(\frac{n}{N_1} \right) \\ Y_{T_r} &= Y_T - (Y_T - \sigma_{2\max}) \left(\frac{n}{N_2} \right) \\ S_r &= S - (S - \sigma_{6\max}) \left(\frac{n}{N_6} \right) \end{aligned} \quad 11.12$$

X_{T_r} and Y_{T_r} are the tensile residual strength parallel and transverse to the fibres respectively, while S_r is the residual shear strength. $\sigma_{1\max}$, $\sigma_{2\max}$ and $\sigma_{6\max}$ are the maximum cyclic stresses applied for n cycles, and N_i , $i = 1, 2, 6$, are the corresponding fatigue lives at the specific stress level. Although the three relations of Eq. 11.12 seem to depend only on the applied stress level, they also depend on the stress ratio through the fatigue life N_i obtained for a specific stress ratio through the constant life diagram (CLD) used: see Section 11.5. The model can be implemented once the static strength and fatigue S–N curves at arbitrary R -ratios are known.

The compressive strength, both parallel and transverse to the fibres, has been shown not to degrade significantly due to fatigue, especially when the specimens were subjected to tensile cyclic stress. Nevertheless, in modelling the compressive residual strength under C–C or T–C cyclic loading, a degradation equation simulating constant strength throughout the life with a sudden drop near failure (sudden death) of the following form was implemented:

$$\begin{aligned} X_{c_r} &= X_c - (X_c - |\sigma_{1\min}|) \left(\frac{n}{N_1} \right)^k \\ Y_{c_r} &= Y_c - (Y_c - |\sigma_{2\min}|) \left(\frac{n}{N_2} \right)^k \end{aligned} \quad 11.13$$

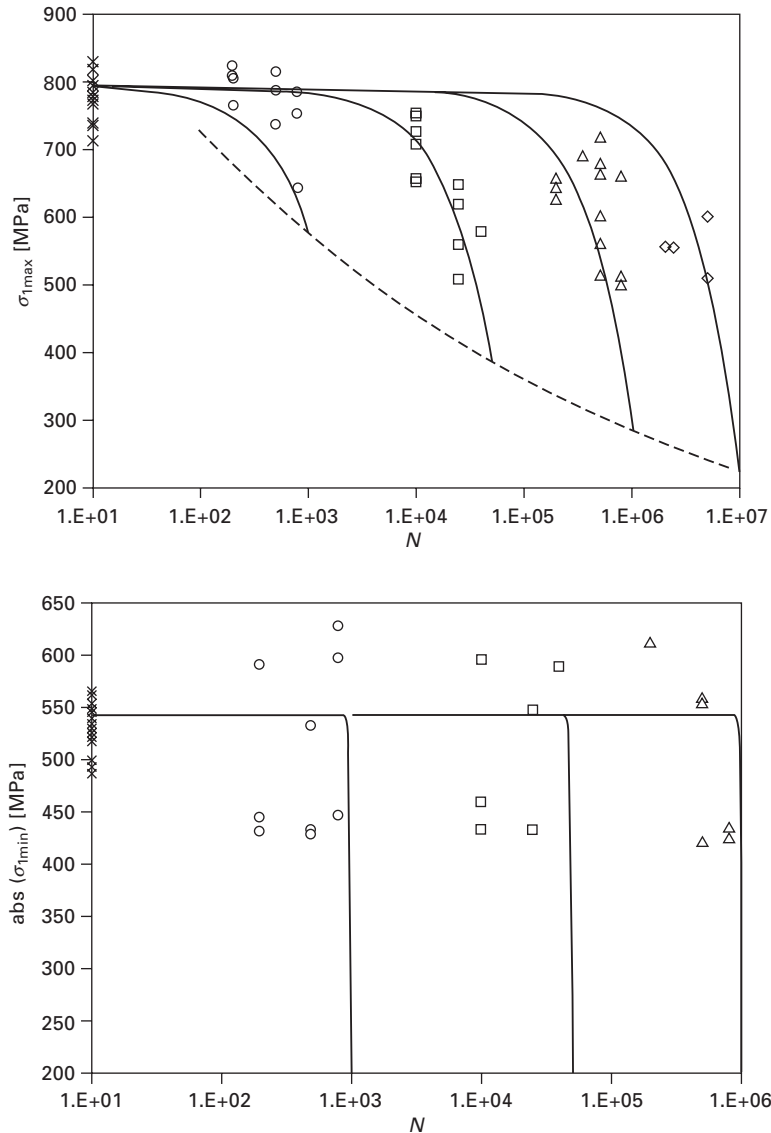
A summary of experimental evidence on the effectiveness of Eqs 11.12 and 11.13 in modelling the residual strength behaviour in the principal material directions of the OB_UD glass/epoxy is presented in Figs. 11.10 to 11.15. In each of these figures, static strength data, tensile, compressive or in-plane shear, are plotted on the ordinate axis which was moved to $N = 10$ or 100 for increased resolution. The corresponding S–N curve is also shown as a dashed line, where appropriate, e.g. under T – T loading at $R = 0.1$ it appears in the picture for the residual tensile strength whereas for compressive $R = 10$ loading it is plotted along with the residual compressive strength.

The solid lines appearing in Figs 11.10 to 11.15 are theoretical predictions from Eq. 11.12 or Eq. 11.13 for the residual compressive strength. The exponent k for the latter case was set equal to 50. Different data point sets were also displayed that correspond to three different stress levels, while for each set the data correspond to coupons cycled up to 20%, 50% or 80% of their nominal life. Details of all these tests were reported by Philippidis and Passipoularidis (2006).

Initially tests were planned for three stress ratios, both parallel and transverse to the fibres. Nevertheless, compressive tests at $R = 10$ in the fibre direction were skipped due to the very flat S–N curve derived at this stress ratio, which made the definition of stress levels for specific fatigue lives very sensitive to even slight variations of applied load. That has also introduced uncertainty on the quality of the results and also caused many premature failures. The residual shear strength tests were performed using the ISO 14129 standard $[\pm 45]_S$ tensile coupon. For this reason only cyclic tests at $R = 0.1$ were possible.

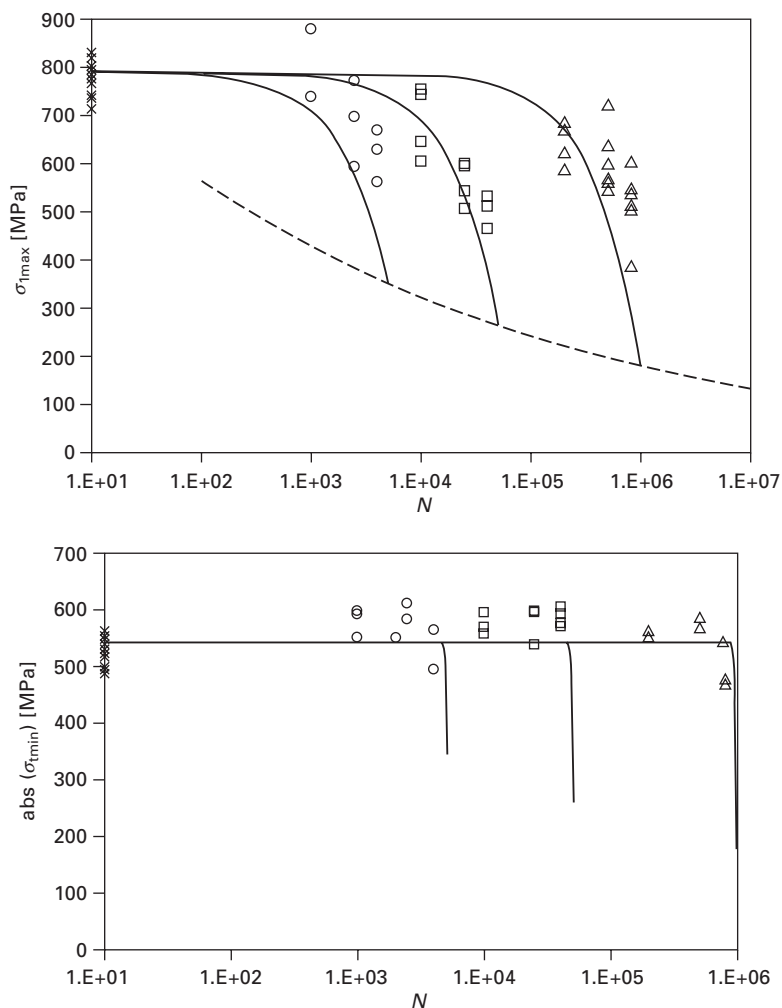
Summarizing the above, the residual strength model in the symmetry directions of the unidirectional glass/epoxy layer, and also for in-plane shear, due to cyclic loading is given by a different set of equations, depending on the value of the cyclic stress ratio, R . As is well known from the representation of constant life diagrams (CLD) in the $(\sigma_a - \sigma_m)$ plane of the alternating (σ_a) , and mean (σ_m) stress characteristics of the cyclic loading, radial lines emanating from the origin of the coordinate system correspond to stress states with constant R values. Then, with respect to Fig. 11.16, the following sets of equations are valid for purely tensile (T – T) cyclic loading:

$$\begin{aligned} X_{T_r} &= X_T - (X_T - \sigma_{1\max}) \left(\frac{n}{N_1} \right), X_{C_r} = X_C \\ Y_{T_r} &= Y_T - (Y_T - \sigma_{2\max}) \left(\frac{n}{N_2} \right), Y_{C_r} = Y_C \\ S_r &= S - (S - \sigma_{6\max}) \left(\frac{n}{N_6} \right) \end{aligned} \quad 0 \leq R < 1 \quad 11.14$$



11.10 Strength degradation of OB_UD in the fibre direction under $R = 0.1$ (tensile residual strength: top; compressive residual strength: bottom).

For negative R values, corresponding to compressive $\sigma_{\min}$ and tensile $\sigma_{\max}$ values where, however, the tensile stresses are of greater magnitude (T–C):

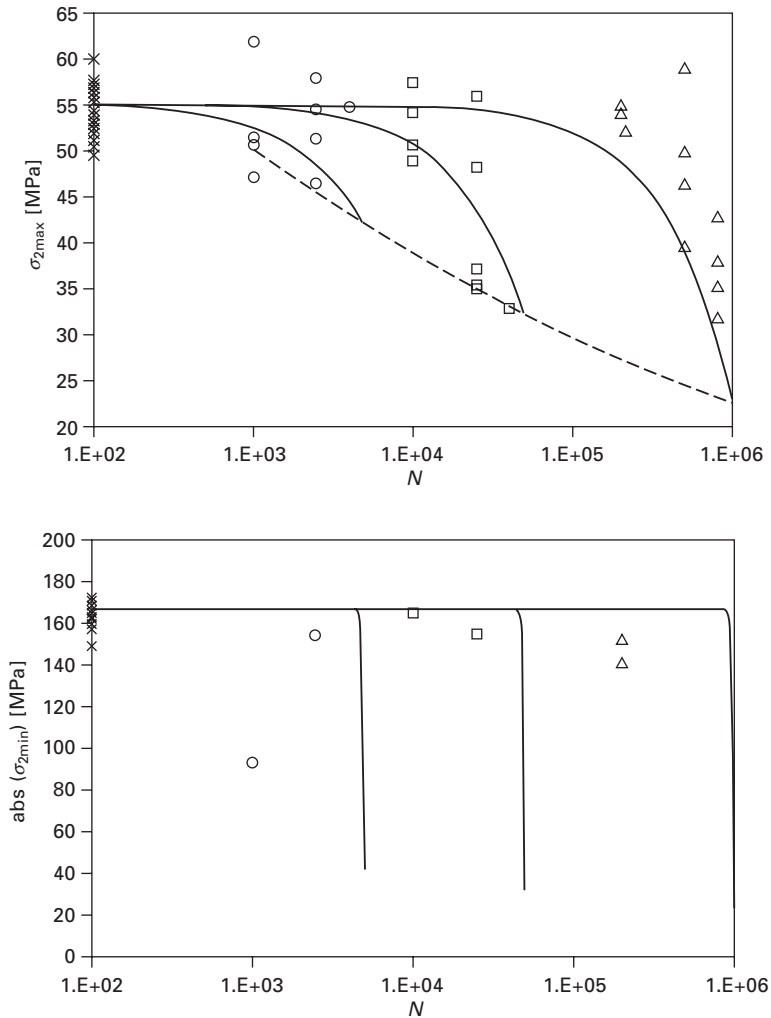


11.11 Strength degradation of OB_UD in the fibre direction under $R = -1$ (tensile residual strength: top; compressive residual strength: bottom).

$$X_{\text{Tr}} = X_{\text{T}} - (X_{\text{T}} - \sigma_{1\max}) \left(\frac{n}{N_1} \right), \quad X_{\text{Cr}} = X_{\text{C}} - (X_{\text{C}} - |\sigma_{1\min}|) \left(\frac{n}{N_1} \right)^k$$

$$Y_{\text{Tr}} = Y_{\text{T}} - (Y_{\text{T}} - \sigma_{2\max}) \left(\frac{n}{N_2} \right), \quad Y_{\text{Cr}} = Y_{\text{C}} - (Y_{\text{C}} - |\sigma_{2\min}|) \left(\frac{n}{N_2} \right)^k \quad -1 \leq R < 0$$

$$S_{\text{r}} = S - (S - |\sigma_{6\max}|) \left(\frac{n}{N_6} \right)$$

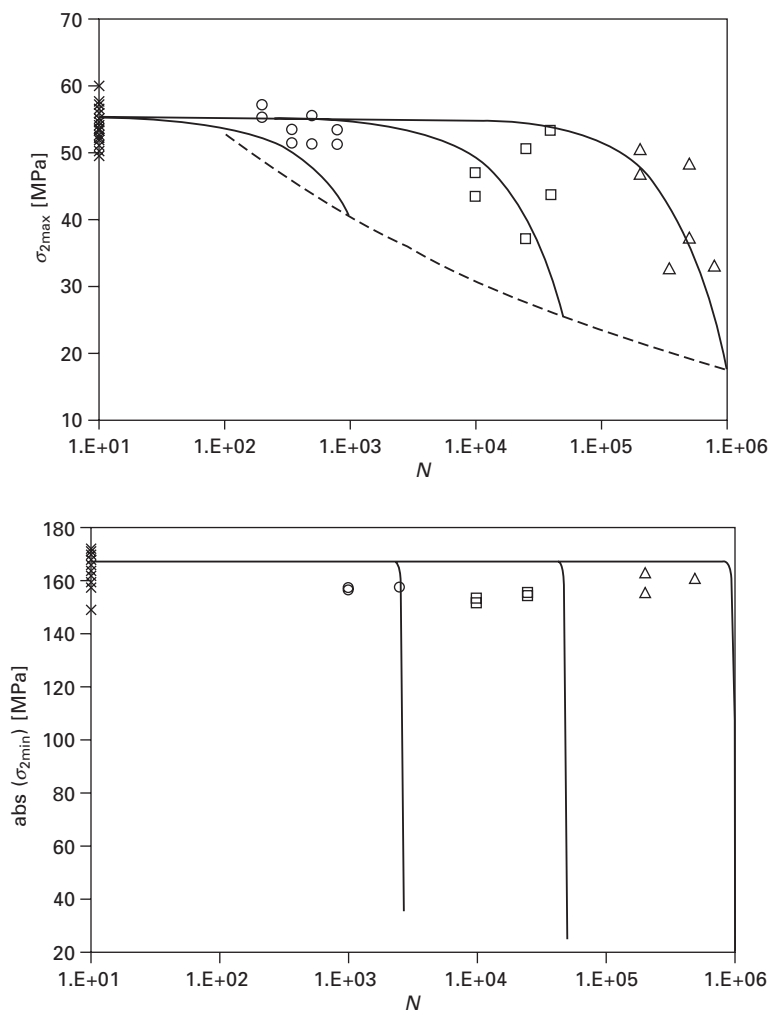


11.12 Strength degradation of OB_UD transversely to the fibres, $R = 0.1$ (tensile residual strength: top; compressive residual strength: bottom).

For compression dominated cyclic loading (C–T) in the range of negative R values, the same set of Eq. 11.15 is valid with the exception of the relation for the residual shear strength which is now given by:

$$S_r = S - (S - |\sigma_{6min}|) \left(\frac{n}{N_6} \right) \tag{11.16}$$

Finally, the residual strength equations for purely compressive cyclic loading (C–C) are given by:

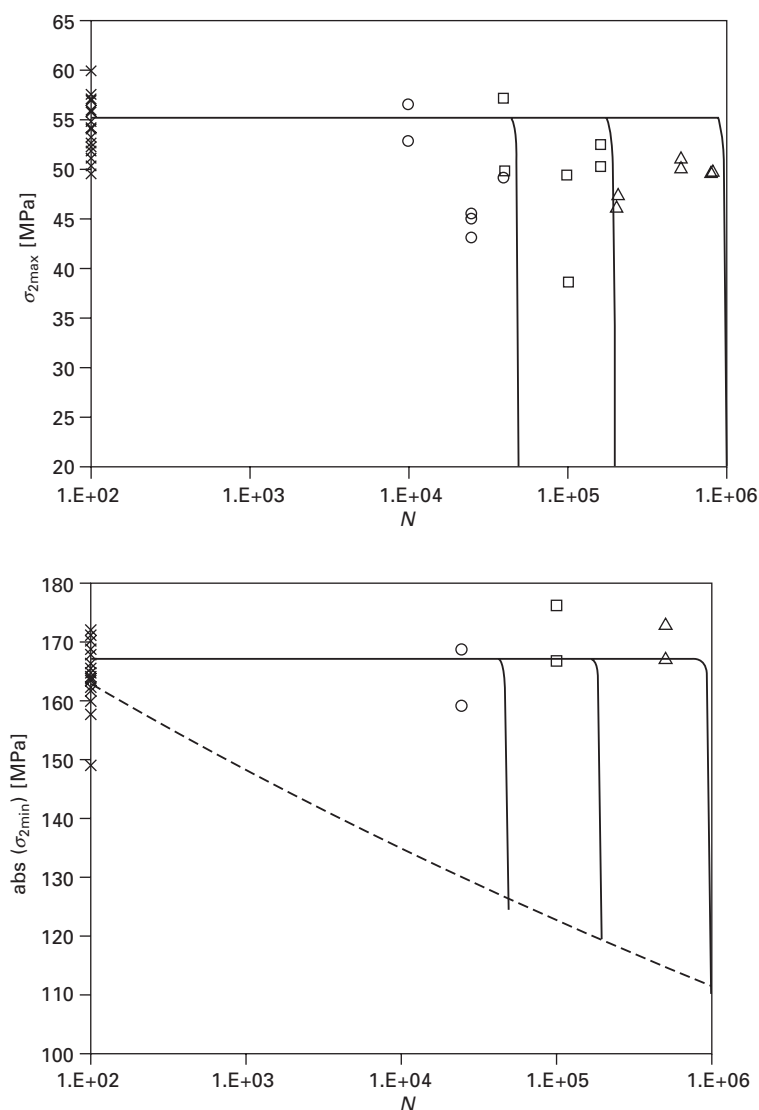


11.13 Strength degradation of OB_UD transversely to the fibres, $R = -1$ (tensile residual strength: top; compressive residual strength: bottom).

$$X_{T_r} = X_T, X_{C_r} = X_C - (X_C - |\sigma_{1\min}|) \left(\frac{n}{N_1} \right)^k$$

$$Y_{T_r} = Y_T, Y_{C_r} = Y_C - (Y_C - |\sigma_{2\min}|) \left(\frac{n}{N_2} \right)^k \quad 11.17$$

$$S_r = S - (S - |\sigma_{6\min}|) \left(\frac{n}{N_6} \right)$$

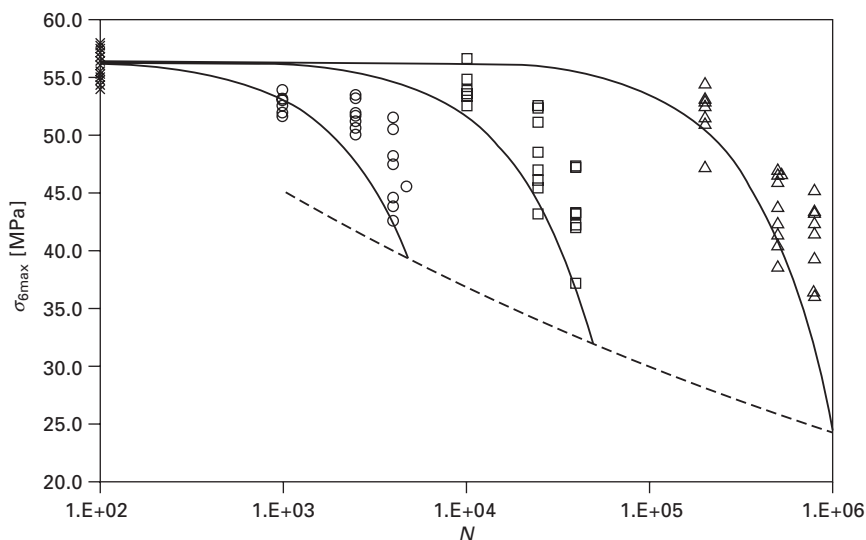


11.14 Strength degradation of OB_UD transversely to the fibres, $R = 10$ (tensile residual strength: top; compressive residual strength: bottom).

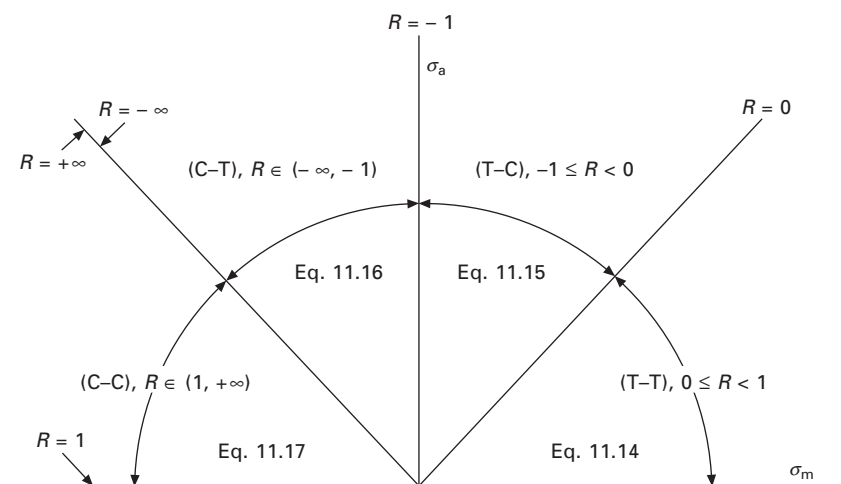
In all the above equations for compressive residual strength, expressed by the ‘sudden death’ relation, the exponent k assumes a very high value, *ca.* 50.

11.5 Constant life diagrams and S–N curves

The life prediction methodology presented herein, although it can be used for any glass/epoxy laminate under any type of cyclic loading, was initially



11.15 In-plane shear strength degradation of OB_UD, $R = 0.1$.



11.16 (σ_a, σ_m) -plane notation and region of validity for residual strength models.

introduced for wind turbine rotor blade applications where VA fatigue under various R -ratios spans the entire (σ_a, σ_m) -plane: see Fig. 11.16. Therefore, fatigue behaviour must be characterized in the principal coordinate system of the orthotropic UD ply for several R -values, and then by using an appropriate interpolation scheme we need to define the ‘constant life diagram’ or else define the number of cycles to failure, N , for every pair (σ_a, σ_m) .

A great number of CA cyclic tests have been performed and the respective S–N curves parallel, transverse to the fibre and in shear, at three stress ratios R , have been obtained. All test details and results can be found at the Optimat Blades site, cited earlier, either in the OptiDAT database or in reports by Philippidis *et al.* The R -ratios for which tests were performed are $R = 0.1$, -1 and 10 which, apart from being proposed by wind turbine rotor blade certification bodies as GL or DNV, cover a minimum range of fatigue conditions, in both tension and compression. For in-plane shear, fatigue strength tests were performed only for $R = 0.1$, and the common assumption that shear strength in the principal material system does not depend on the sign of the shear stress along with the Goodman approach led to a symmetric CLD.

Experimental results from CA tests parallel and transverse to the fibres and under in-plane shear are presented in Figs 11.17, 11.18 and 11.19 respectively. The S–N curves drawn in these figures are given by:

$$\sigma_a = \sigma_o N^{\left(\frac{1}{k}\right)} \quad 11.18$$

In the above relation, σ_a stands for the alternating component of the cyclic stress, and N for the number of cycles to failure, while constants σ_o and k , depending on the R -ratio and the stress component, are given in Table 11.5.

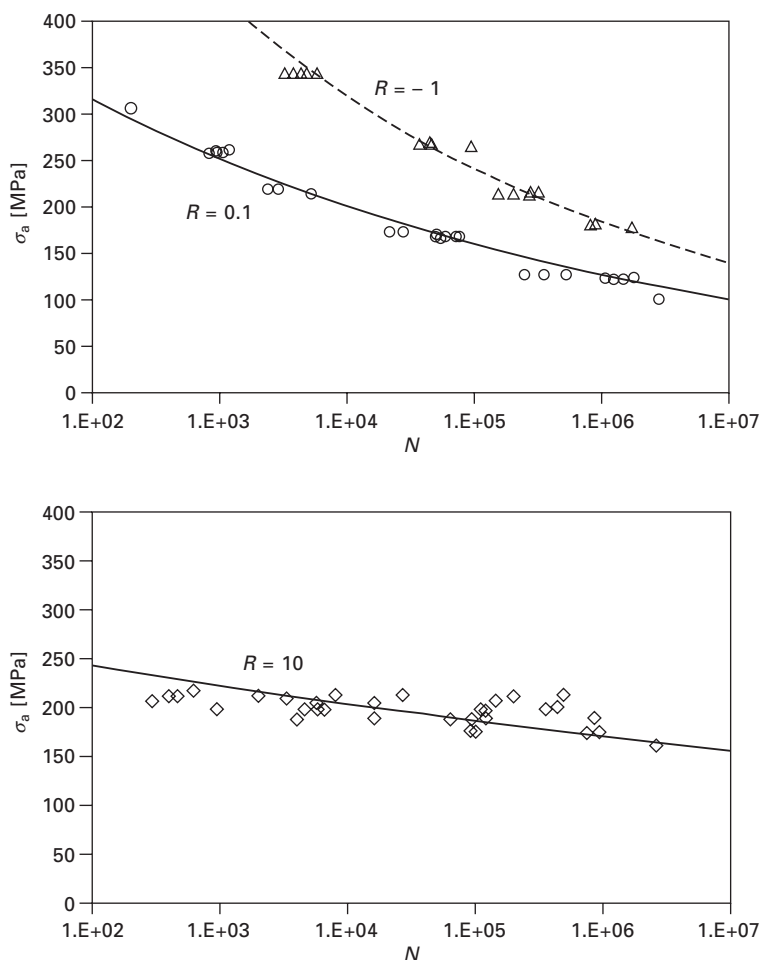
Fatigue behaviour at different stress ratios was obtained by linear interpolation between the already known S–N curves as described by Philippidis and Vassilopoulos (2004). Alternative formulations, e.g. the interpolation model by Harris (2003), were already developed for the OB_UD glass/epoxy material (see Passipoularidis and Philippidis, 2009b) but not used in deriving the results presented herein.

The constant life diagrams for axial loading parallel or transverse to the fibres and in-plane shear are presented in Figs 11.20, 11.21 and 11.22 respectively.

11.6 FAtigue DAmage Simulator (FADAS)

The FADAS methodology developed for life prediction of composite shell structures under VA complex cyclic loading, e.g. wind turbine rotor blades, considers a basic orthotropic UD lamina to be the constitutive element of the multidirectional (MD) lay-up. Application of the model requires in-plane mechanical properties of the ply, i.e. on-axis, transverse to the fibre and in shear, to be experimentally derived.

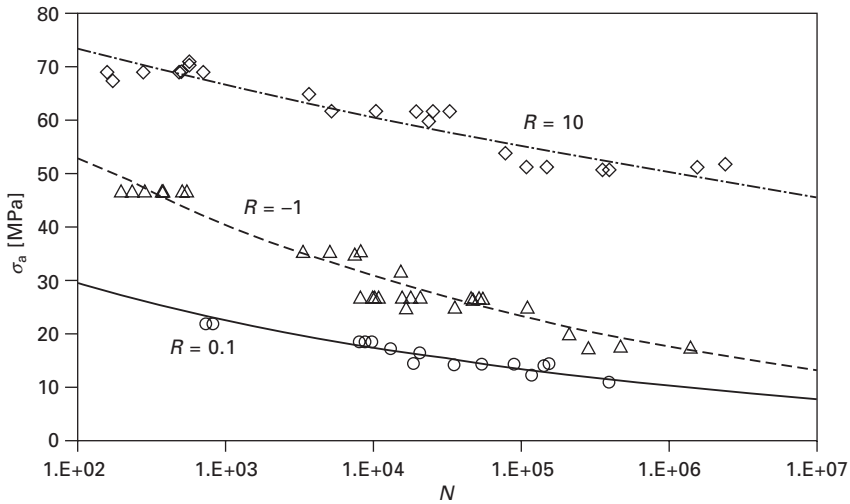
The elastic behaviour is assumed to be nonlinear, as already presented in Section 11.2.1. Elasticity is subject to a gradual degradation due to the



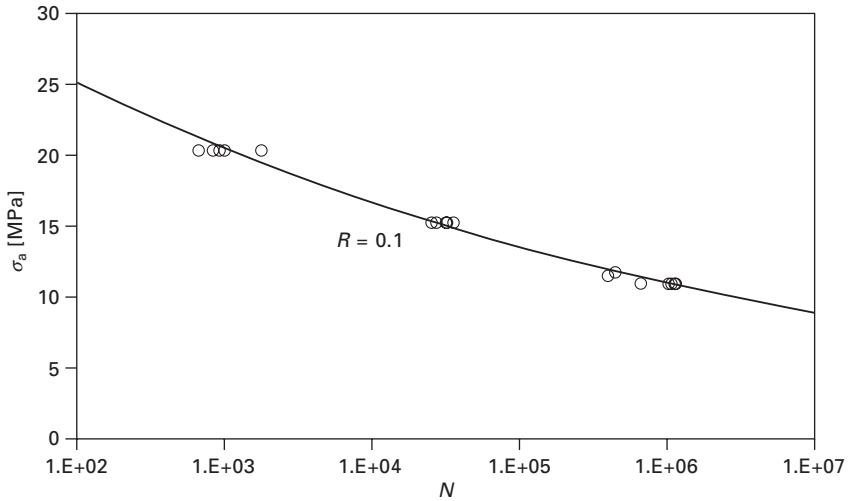
11.17 S-N curves for the OB_UD glass/epoxy in the fibre direction.

fatigue loading, which is accounted for through the simple models presented in Section 11.2.3. Adequate formulations for degradation of tensile, compressive and shear static strength due to fatigue, in both tension and compression, were also taken into account. Residual static strength is used as the damage accumulation metric. Complex cyclic stress states are combined by means of the failure criterion to predict the life of the component.

Once all properties and constitutive behaviour are implemented, the algorithm proceeds by means of CLT assumptions for the current external load segment (peak–trough pair), and stress–strain components are calculated at the principal coordinate system of each ply. An alternative approach for the stress analysis using a Reissner–Mindlin shell FEM formulation was



11.18 S–N curves for the OB_UD glass/epoxy transverse to the fibres.

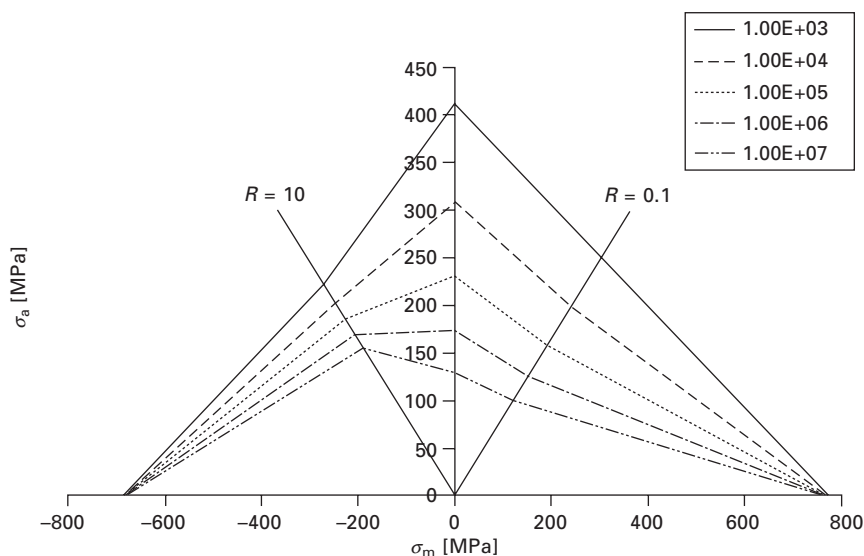


11.19 S–N curve under in-plane shear for the OB_UD glass/epoxy.

also developed by Eliopoulos (2007) and Philippidis *et al.* (2007) while an earlier linear version of the method, also based on CLT assumptions, was presented by Passipoularidis and Philippidis (2007). With stresses known, the failure criterion is applied. If failure occurs in some ply, its stiffness is modified as presented on page 404. After failure events are accounted for, the algorithm progresses with calculation of the gradual strength and stiffness degradation due to fatigue: The stress amplitude and R -ratio of each stress

Table 11.5 S-N curve parameters for the OB_UD glass/epoxy

R	σ_1		σ_2		σ_6	
	σ_o (MPa)	k	σ_o (MPa)	k	σ_o (MPa)	k
0.1	500.8	10.03	50.2	8.63	38.1	11.06
-1	972.2	8.05	87.5	8.43	N/A	N/A
10	289.5	26.08	88.5	24.32	N/A	N/A



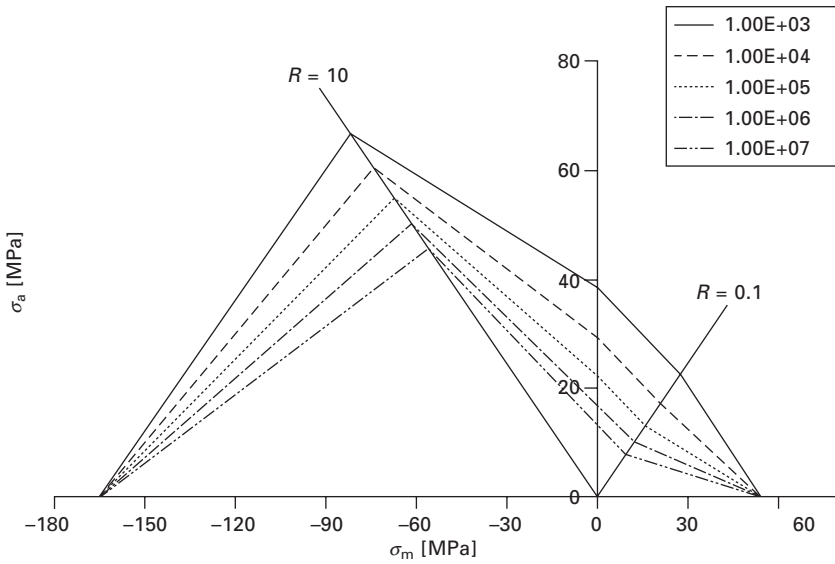
11.20 CLD for the OB_UD glass/epoxy parallel to the fibres.

component are defined and the corresponding fatigue life, N_f is derived using appropriate CLD data. A general flowchart of the algorithm is shown in Fig. 11.23. Further details on the operation of the various modules shown in the diagram are given below.

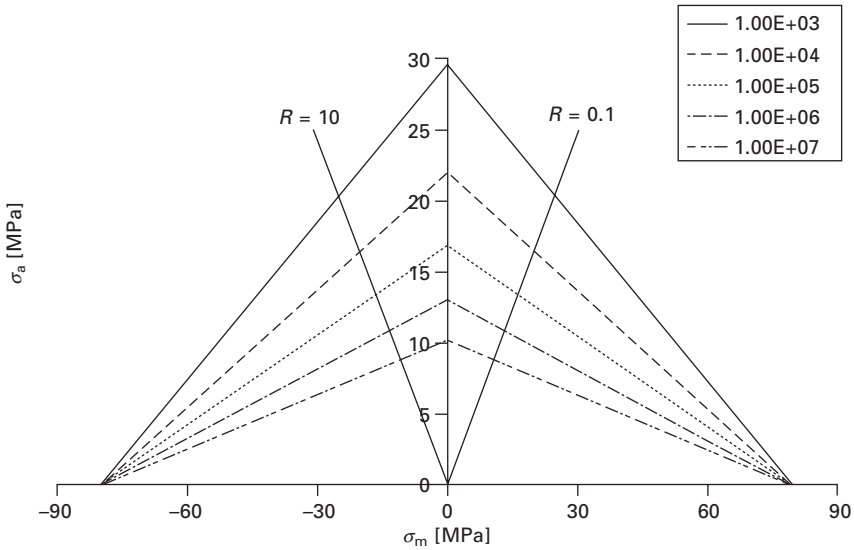
11.6.1 Calculations under VA cyclic stresses

The model simulates any load sequence presented as input cycle by cycle, or more specifically segment by segment with a number of load steps per segment. Even external loading of constant amplitude causes cyclic stress components of variable amplitude at each ply due to nonlinearity, reload stiffness and stiffness degradation. The latter, being driven by cyclic loading and local failures, causes stress redistribution.

While a load segment is incrementally applied, the maximum and minimum values of each component of stress at each ply are determined. When a local



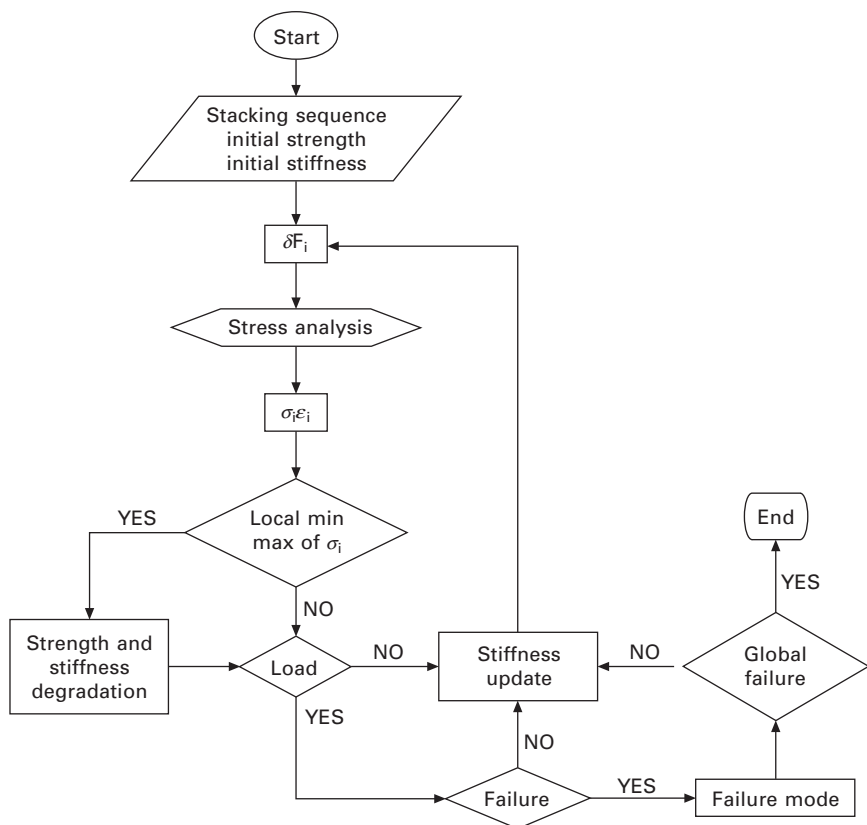
11.21 CLD for the OB_UD glass/epoxy transverse to the fibres.



11.22 CLD under in-plane shear for the OB_UD glass/epoxy.

minimum or maximum stress component σ_i is detected, the corresponding stress ratio R_i and stress amplitude σ_{ia} can be calculated by:

$$R_i = \frac{\sigma_{i_{\min}}}{\sigma_{i_{\max}}}, i = 1, 2, 6 \quad 11.18$$



11.23 Flowchart of the FADAS algorithm.

$$\sigma_{ia} = \frac{\sigma_{i\max} - \sigma_{i\min}}{2}, i = 1, 2, 6 \quad 11.19$$

For the calculated R_i , the stress amplitude values σ_{ia} for number of cycles to failure, N , equal to 10^3 , 10^4 , 10^5 and 10^6 are calculated as described in Section 11.5 by linear interpolation from the CLD corresponding to each stress component. Then, with linear regression on these four data points on a double logarithmic scale, the S–N curve is derived for the specific stress ratio, and thus finally the number of cycles to failure for the developed stress range is calculated.

The obtained N_i is required for the calculation of the cumulative life fraction for each stress component that is used in the stiffness degradation models:

$$\sum \left. \frac{n_i}{N_i} \right|_{(\lambda+1)} = \sum \left. \frac{n_i}{N_i} \right|_{(\lambda)} + \frac{0.5}{N_i}, i = 1, 2, 6 \quad 11.21$$

Indices in parentheses in the above relation, e.g. (λ) , $(\lambda + 1)$, denote the stress segment number, hence the term 0.5 in the second term of the right-hand side denoting half a cycle, i.e. a stress segment. In the stiffness degradation models of Eq. 11.5 presented in Section 11.2.3, the ratio n/N for the case of VA cyclic stress has to be replaced by the cumulative live fraction given by Eq. 11.21.

The calculated N_i , $i = 1, 2, 6$, are also used with the strength degradation models presented in Section 11.4. When a local minimum or maximum value for the σ_1 stress component is detected, the residual tensile strength in the fibre direction $X_{T_r(\lambda+1)}$ is reduced by using an adapted form for VA cyclic stress of the Broutman–Sahu model, Eq. 11.12, only if the maximum stress is positive and lower than the previous residual strength $X_{T_r(\lambda)}$, otherwise it remains unchanged.

The residual strength at the $(\lambda + 1)$ segment of magnitude $\sigma_{1\max}$ can be readily proven to be given by:

$$X_{T_r(\lambda+1)} = X_T - (X_T - \sigma_{1\max}) \left[\frac{X_T - X_{T_r(\lambda)}}{X_T - \sigma_{1\max}} + \frac{0.5}{N_1} \right] \quad 11.22$$

The first term in the brackets added in $0.5/N_1$ is the equivalent number of cycles, n_{eq}/N_1 , which is the CA number of cycles at the actual stress level of $\sigma_{1\max}$ that would reduce the static strength down to the value of $X_{T_r(\lambda)}$: see Schaff and Davidson (1997).

For the compressive residual strength in the fibre direction, the use of the sudden death model given by Eq. 11.13 might be prohibited (numerical precision problems) for high values of the exponent of the equation due to very small values of the ratio n/N when used for VA stress where $n = 0.5$. In such cases an alternative formulation based on a modified linear model (Eliopoulos, 2007) is implemented:

$$X_{C_r(\lambda+1)} = X_C - \beta(X_C - |\sigma_{1\min}|) \left[\frac{X_C - X_{C_r(\lambda)}}{\beta(X_C - |\sigma_{1\min}|)} + \frac{0.5}{N_1} \right] \quad 11.23$$

The constant β is a small positive number, e.g. 10^{-3} , 10^{-6} . Its value is not of importance provided that the term $(1 - \beta) X_C$ is greater than the largest $|\sigma_{1\min}|$ in the time series. Nevertheless, the values suggested in the above were efficiently used for the verification examples presented in this work. The degradation model is supplemented by the following logical statement:

$$\text{If } X_{C_r(\lambda+1)} \leq (1 - \beta)X_C + \beta|\sigma_{1\min}| \text{ then } X_{C_r(\lambda+1)} = |\sigma_{1\min}| \quad 11.24$$

The degradation scheme for the compressive residual strength formed of the two Eqs 11.23 and 11.24 has a numerically stable sudden death response.

Therefore, under C–C and T–C cyclic loading and if the value of $|\sigma_{\min}|$ is lower than the residual strength of the previous segment, $X_{Cr(\lambda)}$, the compressive strength degradation is calculated by means of Eqs 11.23 and 11.24, otherwise it remains unchanged.

Implementation in the routine of the residual strength models for the direction transverse to the fibres and in-plane shear is performed in a similar way as for the fibre direction.

11.6.2 Computational procedure

Results from the FADAS routine presented herein were derived by a MATLAB implementation of the method. The code was initially developed as a user routine for a commercial finite element (FE) program; see Philippidis *et al.* (2007). Nevertheless, its adaptation as a stand-alone tool, based on CLT stress analysis assumptions, was straightforward without major modifications. For homogeneous stress fields, as it represents approximately the case of test coupons under axial loading or tubular specimens under combined torsion and axial load, the use of the present implementation results in significant CPU time savings compared to the use of a finite element model.

As was mentioned in previous sections, an incremental stress–strain analysis was performed and to that end external loading was also applied incrementally. The load series presented as input must be in the form of peaks and troughs, so previous treatment of continuous time series might be necessary. For the examples of MD laminates made of the OB_UD glass/epoxy ply included herein, the range between successive peak and trough values was divided into 20 load steps. The first two are very small, e.g. $1\text{E}-03$ times the respective peak or trough value, so the detection of any change of the loading condition, loading or unloading, was performed at an early stage with negligible stress and strain variation. Eighteen equal loading steps were used to span the remainder of the stress range.

With this resolution, the application of an irregular loading spectrum with $1\text{E}+05$ load reversal points results in $2\text{E}+06$ load steps for just one pass. For the simulation of the fatigue behaviour of the MD laminates treated in this work an average CPU time was 0.056 s per cycle. More specifically, for the MD laminate, $[(0/\pm 45)_4/\pm 45]$ with 14 plies, it took 0.082 s per cycle while it took only 0.041 s for the bidirectional (BD) $[\pm 45]_S$ coupon (four layers). Then, for life prediction of the MD laminate at the stress level of $1\text{E}+06$ cycles a nominal duration of 22.8 h was encountered, while half of that time was needed for the $[\pm 45]_S$ coupon. The above were derived by evaluating the performance of this FADAS version in an Intel Core 2 Quad CPU Q6600 at 2.4 GHz and 4 GB of RAM.

Final failure

While ply failure in one or more different modes might occur even from the first load steps of the first cycle, depending on the stress level, the routine continues the calculations without stopping until the stiffness matrix of the laminate loses its positive definiteness, which practically might not occur. However, near the final laminate failure a drastic stiffness loss is observed and for this reason the coupon is considered to have failed when the axial strain of its middle plane in the loading direction ε_x^o exceeds in a disproportional manner its previous values.

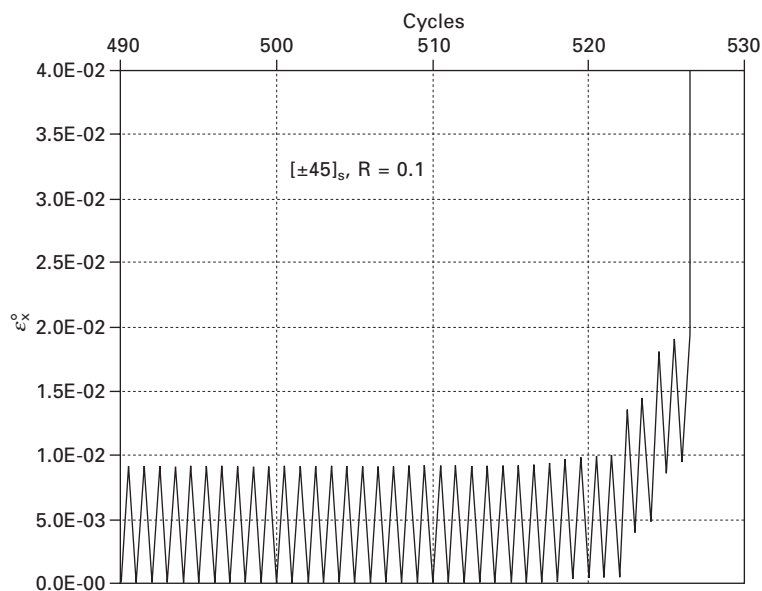
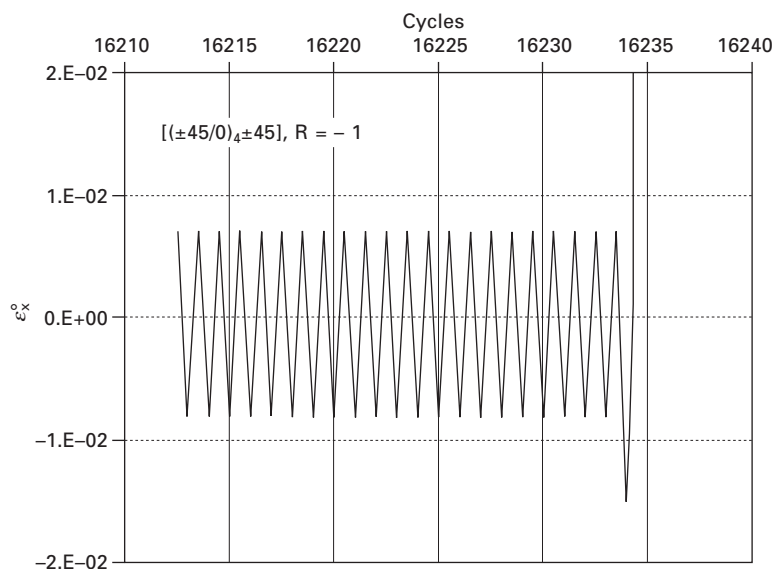
As an example, the last cycles near final failure for a $[(0/\pm 45)_4/\pm 45]$ coupon under $R = -1$ reversed loading and for a $[\pm 45]_S$ under $T-T$, $R = 0.1$, are presented in Fig. 11.24. The cases shown correspond to low-cycle fatigue stress levels, nevertheless the same trend was clearly also observed for all maximum stress values. Note that the real strain value at failure, for both graphs of Fig. 11.24, was considerably decreased to enhance resolution. If plotted with its real value then the oscillating part of the curve appears as a bold horizontal line. In fact, these strain values were 0.246 for the MD laminate while values several orders of magnitude greater were reached by the BD lay-up.

11.6.3 Validation of numerical predictions: experimental data

The present MATLAB version of FADAS has been implemented with ply properties derived for the OB_UD glass/epoxy composite. For the verification of the numerical predictions, extensive comparison with experimental data was performed. Multidirectional laminates made of the basic ply material were subjected to various loading conditions consisting of CA cyclic loading of different stress ratios, irregular spectrum loading, and finally static residual strength tests on coupons previously subjected to cyclic loading at various stress levels and life fractions. All tests were on prismatic specimens and consisted of axial loading.

Details of the experimental procedure, measurements and results discussed in this section can be found in the OptiDAT database and dedicated reports in the Optimat Blades site. More specifically, details and results were reported by Philippidis *et al.* (2004) for the CA cyclic tests on $[\pm 45]_S$ coupons, by Krause and Kensche (2006) for the MD laminate $[(\pm 45/0)_4/\pm 45]_T$ on-axis loaded at various R -ratios, and by Philippidis *et al.* (2006a) for the 10° and 60° off-axis loaded cases.

For the residual strength tests on the $[\pm 45]_S$ coupons, details were given by Philippidis *et al.* (2006b). Three sets of specimens prior to monotonic static loading to failure were subjected to tensile cyclic loading, $R = 0.1$, at three different stress levels, namely 78, 64 and 48 MPa corresponding to expected



11.24 Axial strain in the loading direction as a function of the applied number of cycles.

life of $5E+03$, $5E+04$ and $1E+06$ cycles, respectively. Three new subsets were formed from the members of each of the above sets for which fatigue tests were stopped at 20, 50 or 80% of the expected life, respectively.

Description of the experimental procedure and results from cyclic tests of VA performed on the $[\pm 45]_5$ coupons was given by Philippidis *et al.* (2006c). A modified NEW WISPER (NW) sequence with lower load value at zero, thus composed of purely tensile cycles, was used. The NW spectrum (Bulder *et al.*, 2005) was developed in the frame of the Optimat Blades project and consists of 95,472 load reversals. Tests were performed at three different $\sigma_{\max}$ levels equal to 90, 96 and 102 MPa respectively.

11.6.4 Validation of numerical predictions: results and discussion

Viscoelastic effects on matrix-dominated properties of the UD ply and especially dependence on the applied strain rate were not modelled in this version of the FADAS routine. Therefore, simulation of either cyclic or static tests was performed in the same way; the load series were presented in an input file where two consecutive entries were interpreted as load reversal points corresponding to a half cycle for CA loading or a peak-trough segment for spectrum loading or finally to a segment of a monotonic load sequence for static tests. In all cases, calculations were performed in 20 steps from one load reversal point to another.

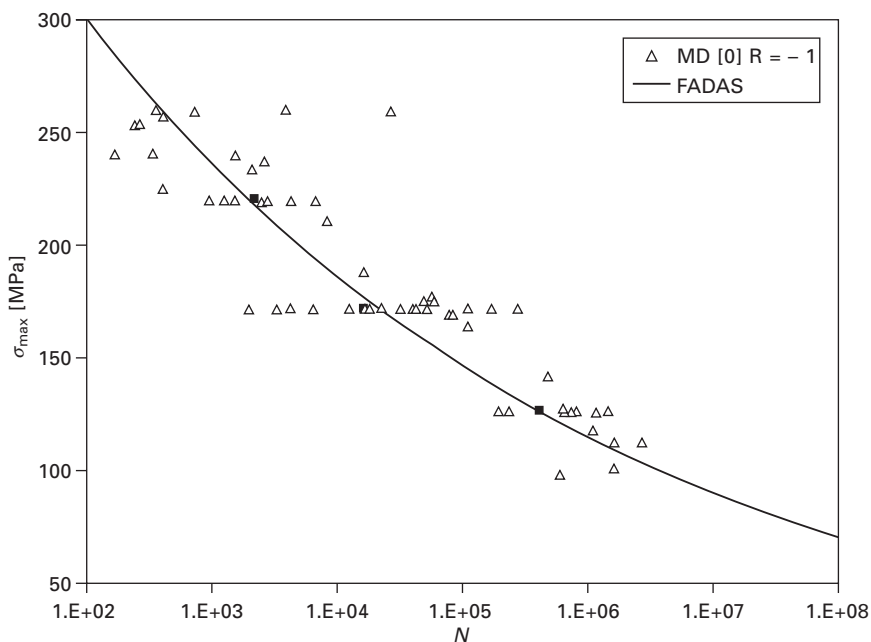
Mechanical properties used for the UD ply were as presented in Section 11.2. Its thickness was set equal to 0.94 mm except for the $+45^\circ$ and -45° layers of the $[(\pm 45/0)_4/\pm 45]_T$ laminate where a thickness of 0.33 mm was used. The reason is that for this laminate a bidirectional stitched fabric of ± 45 orientations was used instead of discrete layers from the basic OB_UD composite.

Constant amplitude fatigue of MD laminate loaded on- and off-axis

For the $[(\pm 45/0)_4/\pm 45]_T$ MD laminate under CA reversed loading, $R = -1$, calculations were performed for three $\sigma_{\max}$ levels, namely 220, 172 and 126 MPa corresponding to an expected number of cycles equal to 5×10^3 , 5×10^4 and 1×10^6 respectively. With 20 load steps per segment, the computational effort for the third stress level, for example, resulted in 4×10^7 steps.

The calculated S-N curve, derived by means of linear regression on a log-log scale of the three simulated stress levels, shown by solid square symbols, is presented in Fig. 11.25 as a solid line. As mentioned on page 424, coupon failure was indicated by the computational procedure itself, questioning for steep variation the value of the in-plane normal strain of the middle plane of the plate.

The first impression from the visual comparison of the numerical predictions and the test data is that a very good agreement was achieved. Nevertheless, when looking at the discrete values, it can be said that the predictions are



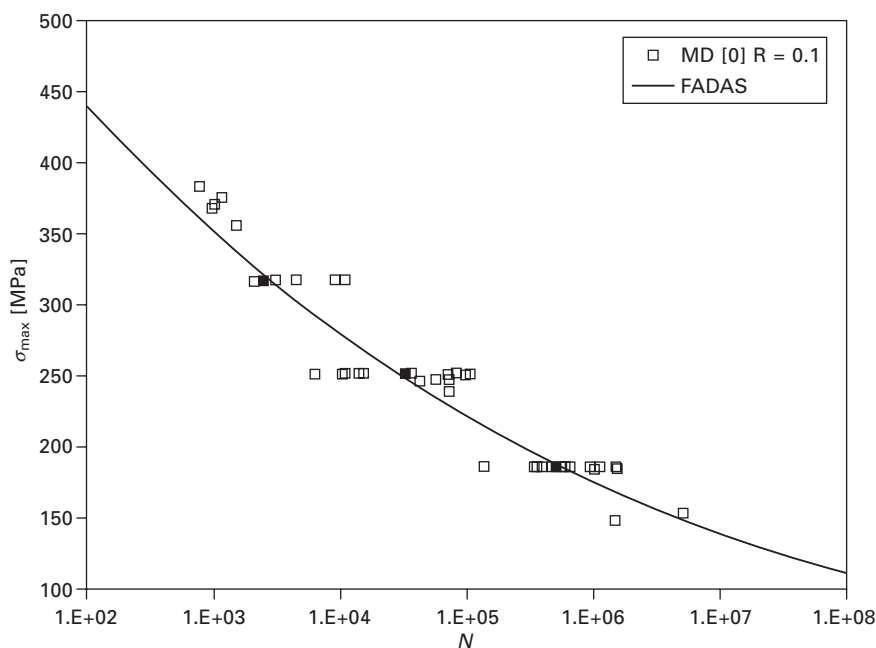
11.25 Comparison of FADAS predictions and experimental data; OB MD GI/Ep under $R = -1$.

conservative. For example, the predicted number of cycles to failure for the aforementioned three stress levels were 2216, 16,240 and 414,466 respectively. It should be noticed that in Fig. 11.25 to 11.27 all available data points from the OptiDAT database are shown, not only the earlier test results from which the S–N curve definitions were derived and used for the calculation of the expected number of cycles at some stress level. Nevertheless, the fact that the numerical predictions lie on the safe side is advantageous for design purposes.

The same trend of an overall satisfactory agreement, the numerical predictions being moderately conservative, was also observed for the cases of pure tensile, $R = 0.1$ and pure compressive, $R = 10$, CA loading. Comparison of theoretical and experimental results is presented in Fig. 11.26 and Fig. 11.27 respectively.

The calculations for all the loading cases were for the same expected number of cycles to failure. For $R = 0.1$, stress levels ($\sigma_{\max}$) of 317, 252 and 186 MPa respectively were simulated. The corresponding results were equal to 2480, 32,430 and 514,096 cycles (solid square symbols).

The stress levels for which simulations were performed for the compressive $R = 10$ case were for $\sigma_{\max} = 300$, 277 and 250 MPa with predicted number



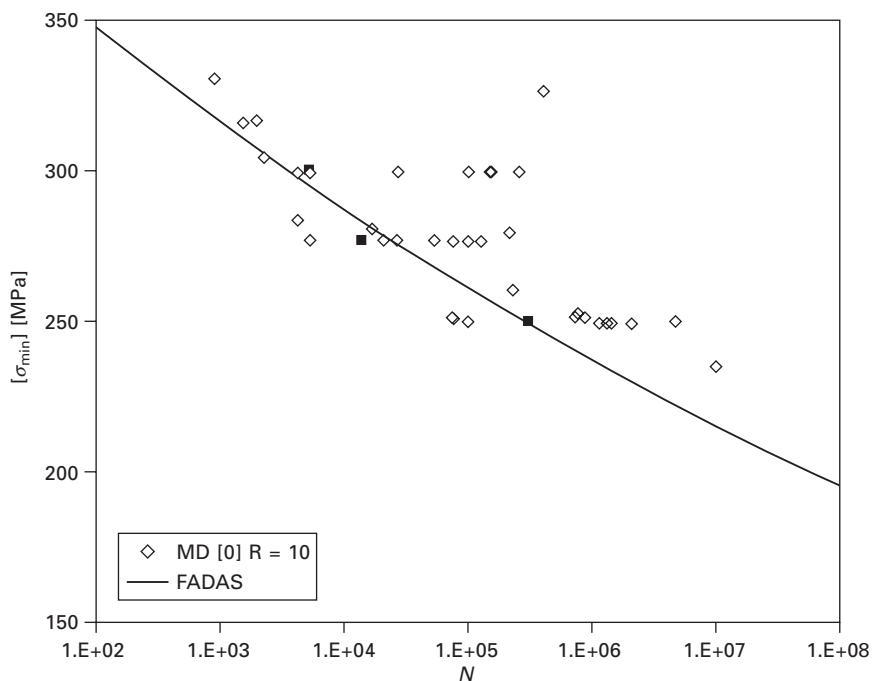
11.26 Comparison of FADAS predictions and experimental data; OB MD GI/Ep under $R = 0.1$.

of cycles to failure $N = 5221$, $14,095$ and $312,892$ respectively (solid square symbols).

For the on-axis CA cyclic tests of the MD laminate, matrix failure modes, IFF, were observed after the first few cycles in the ± 45 layers. The final failure mode was always FF in the $[0]$ ply either in tension for $R = 0.1$ or in tension/compression for $R = -1$ and in compression for $R = 10$.

Another two data sets were used from CA cyclic testing of coupons cut off-axis at 10° and 60° from the $[(\pm 45/0)_4/\pm 45]_T$ laminate in order to produce different complex stress states in the layers of the MD lay-up and to investigate the effect on numerical life prediction of lay-ups without fibres in the load direction. The stacking sequence of the two off-axis coupons is given by $[(35/-55/-10)_4/35/-55]_T$ and $[(-15/75/-60)_4/-15/75]_T$ respectively.

CA cyclic tests were performed at two stress levels, high and low, under reversed loading, $R = -1$. Experimental results along with FADAS numerical predictions for both lay-ups are presented in Fig. 11.28. For the $[(35/-55/-10)_4/35/-55]_T$ coupons, numerical simulations were performed for $\sigma_{\max} = 230$, 172 and 114 MPa, while $\sigma_{\max} = 105$, 87 and 54 MPa, were the corresponding levels for the MD $[60]$ -off-axis coupons. Theoretical and experimental results were in fair agreement for both lay-ups, this time the



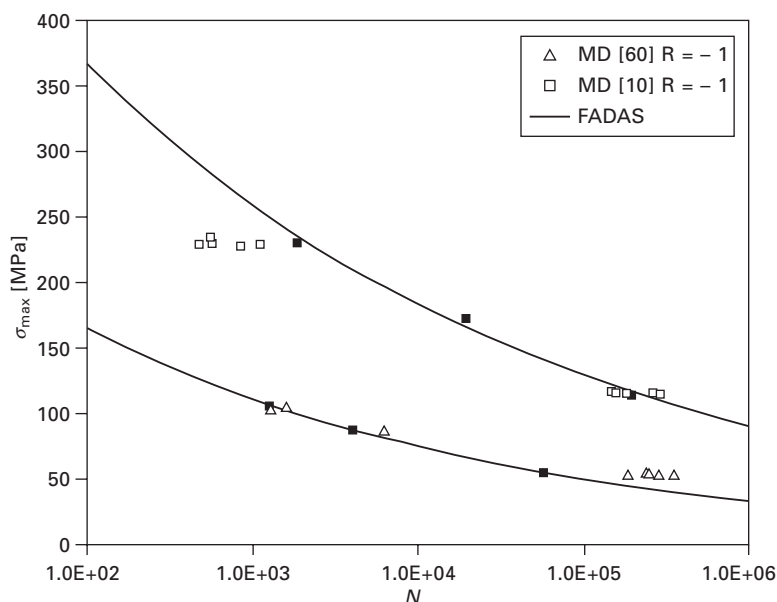
11.27 Comparison of FADAS predictions and experimental data; OB MD GI/Ep under $R = 10$.

numerical predictions being slightly optimistic for the high stress level of the $[(35/-55/-10)_4/35/-55]_T$ laminate.

Final failure was dominated in both cases by FF of the [10] or [15] plies of the 10° and 60° off-axis laminates, while their [55] and [75] plies respectively failed very early following IFF damage mode A. Therefore, for the MD lay-up loaded either on- or off-axis, the final failure was always dominated by FF, tensile or compressive depending on the load stress ratio. Life predictions by FADAS were found in general to be in good agreement with the test data.

Life prediction of $[\pm 45]_S$ specimens under CA and spectrum loading

A great number of cyclic stress tests under either CA or spectrum loading as well as initial and residual static strength (RS) tests after fatigue were performed as suggested by the ISO 14129 standard to derive the in-plane shear behaviour, i.e. modulus and strength of the building ply. On the other hand all these test results could be simulated by means of FADAS, thus validating its performance in predicting either fatigue or static strength of a

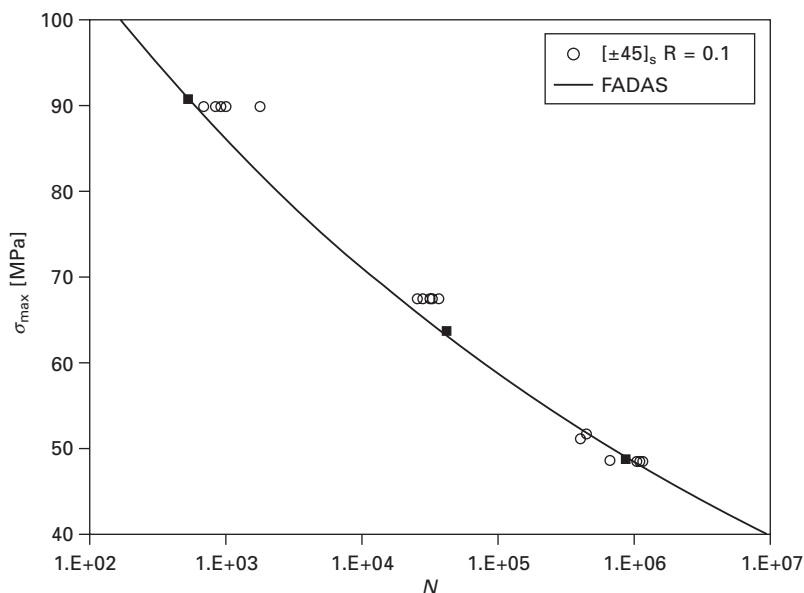


11.28 CA test data and model predictions for MD laminate off-axis loaded under $R = -1$.

multidirectional laminate in which initial and final failure were dominated by IFF modes without fibre fractures at all.

Concerning CA loading, T-T tests on the GI/Ep $[\pm 45]_S$ coupons were performed at a stress ratio $R = 0.1$, while calculations with FADAS were performed at three stress levels (σ_{max}) of 91, 64 and 49 MPa with corresponding expected number of cycles to failure equal to 1×10^3 , 5×10^4 and 1×10^6 respectively. The predicted cycles to failure were equal to 528, 41,298 and 875,780 cycles (solid square symbols). Numerical and experimental results were compared in Fig. 11.29 and as seen the overall agreement is satisfactory. FADAS predictions for low cycle fatigue were more conservative than for a greater number of cycles to failure. Initial and final predicted failure, IFF mode A, were the same for the $[\pm 45]_S$ lay-up and were verified by the experimental findings.

To simulate the experimental response of the $[\pm 45]_S$ coupons under the modified (purely tensile) NW spectrum, calculations were performed for the same three σ_{max} levels used in the experimental part. The comparison of the numerical and test results is presented in Fig. 11.30. Their agreement is satisfactory while the same trend as for the CA loading case was preserved; numerical predictions were more conservative for low than for high cycle fatigue, always being on the safe side. This is in general advantageous for the FADAS routine for high cycle fatigue applications, as in the case of



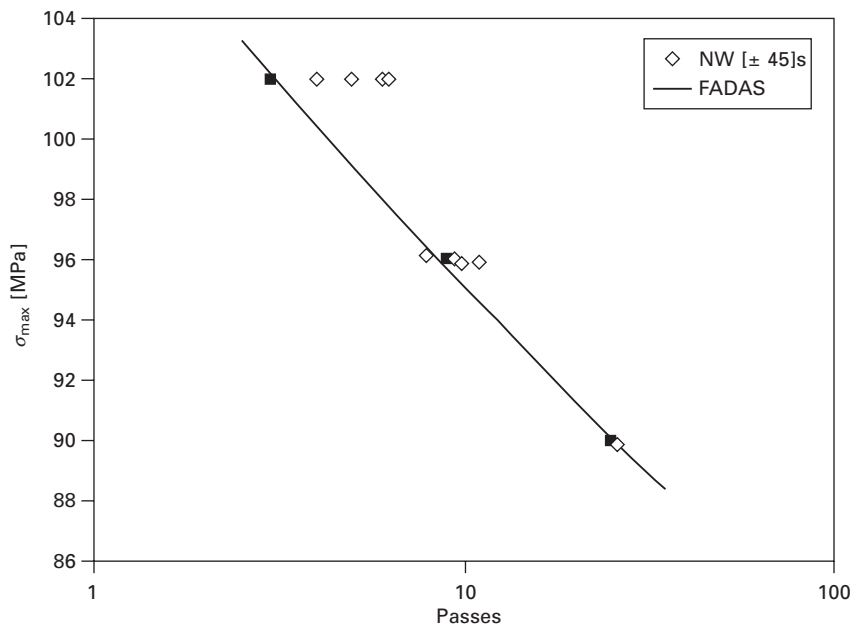
11.29 Comparison of FADAS predictions and experimental data; $[\pm 45]_s$ G/Ep under $R = 0.1$.

wind turbine rotor blades, which during their operational life of 20 years are subjected to load cycles of the order of 1×10^9 .

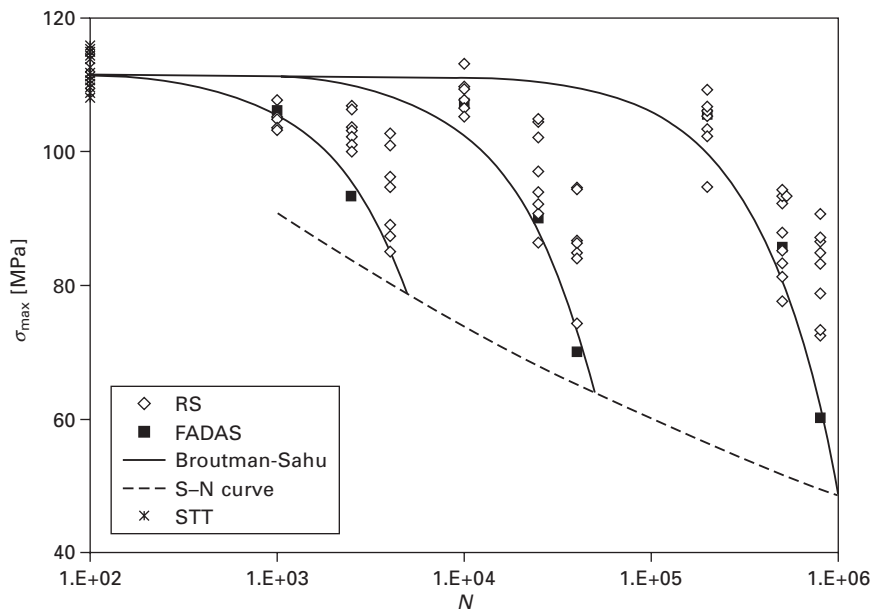
In investigating the effect of various theoretical models, when residual strength is used as a damage metric in life prediction schemes under VA loading, Passipoularidis and Philippidis (2006) have used the same experimental data set to validate numerical life predictions derived by a macroscopic laminate approach. It is interesting to note that the present FADAS predictions shown in Fig. 11.30 are as good as and even more accurate than the latter, which nevertheless were derived from a simpler theoretical model that relied on static and fatigue experimental data of the MD laminate under study.

Residual strength of $[\pm 45]_s$ coupons

As already mentioned, $[\pm 45]_s$ coupons were cycled at three life fractions of 20, 50 and 80% of the expected number of cycles to failure at three different stress levels. After fatigue, coupons were subjected to monotonic static loading to failure. Test results (RS) are presented in Fig. 11.31 along with the respective FADAS predictions. In the same figure the experimental S–N curve and static strength (STT) of the $[\pm 45]_s$ laminate along with macroscopic linear residual strength model predictions are shown to enhance comparisons.



11.30 Prediction vs. test data from $[\pm 45]_s$ GI/Ep under the NEW WISPER (NW) spectrum.



11.31 Predictions vs. residual strength test data after $R = 0.1$ cyclic loading; $[\pm 45]_s$ GI/Ep.

For each stress level, $\sigma_{\max} = 78, 64$ and 48 MPa, the RS tests were simulated using FADAS by applying first CA loading for 20, 50 or 80% of the expected life cycles and then unloading to zero. Finally, monotonic loading was applied with 20 steps resolution per 10 MPa. For example, the calculations to predict residual strength of the coupon cycled to 80% life fraction at the stress level of 48 MPa were performed in *ca.* 32×10^6 load steps.

The FADAS predictions, shown as solid squares in Fig. 11.31, agree well with the experimental data and the macroscopic predictions of the Broutman–Sahu model. They are even less conservative than the latter for higher cycle fatigue. It is interesting to note that for the stress level of 78 MPa, the simulation datum for the residual strength was not displayed since the FADAS routine predicted failure during the CA loading suite. Nevertheless, for lower stress levels of CA loading, the predictions were very good.

11.7 Conclusions

An anisotropic nonlinear constitutive model implementing progressive damage concepts to predict the residual strength/stiffness and life of composite laminates subjected to static or cyclic multiaxial loading was presented. In-plane mechanical properties of the material were fully characterized at the ply level while static or fatigue strength of any multidirectional stacking sequence was predicted.

The computational implementation of the theoretical model in a MATLAB routine, simulating fatigue damage progression in a composite laminate by means of a ply-to-laminate approach, was presented as well. The in-plane residual strength of the UD layer was used as fatigue damage metric. Strength and stiffness degradation were modelled using simple and cost-effective schemes, while the failure criterion of Puck along with post-failure behaviour of the material has been implemented. The model was set up for a glass/epoxy material typical of the wind turbine rotor blade industry and has been verified through a series of constant and variable amplitude fatigue tests on different lay-ups, simulating a variety of plane stress combinations and failure modes. Results from residual static strength prediction of $[\pm 45]_S$ coupons, previously loaded under cyclic tensile stress, $R = 0.1$, at various life fractions, were also validated by the experimental data.

These preliminary results indicate that the FADAS algorithm actually predicts satisfactorily static and fatigue strength under CA or spectrum loading of prismatic specimens under axial loads. The numerical predictions were slightly conservative; further improvement of their accuracy can be expected through implementation of more refined material models and exhaustive comparison with test results from multiaxial key experiments, e.g. from cyclic loading of thin-walled tubes under combined axial force and torque or cruciform specimens under biaxial normal loading.

11.8 Acknowledgements

Research was funded in part by the European Commission in the framework of the research programmes ‘Reliable Optimal Use of Materials for Wind Turbine Rotor Blades’ (Optimat Blades), contract no. ENK6-CT-2001-00552, and ‘Integrated Wind Turbine Design (UPWIND)’, contract no. 019945, SES6. Partial funding was also provided by the General Secretariat of Research and Technology (GSRT) of the Greek Ministry of Development, contracts F.K. 6660 and C037.

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Fatigue life prediction of bonded joints in composite structures

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Abstract: This chapter presents results from studies on the fatigue life modeling of adhesively-bonded double-lap joints (DLJs) composed of pultruded glass fiber-reinforced polymer (GFRP) profiles and on the fatigue behavior of full-scale adhesively-bonded pultruded GFRP bridge deck to steel girder connections. It was found that the fatigue life of the examined DLJs can be adequately modeled either based on stiffness degradation or by means of fracture mechanics. The deck to steel girder connections showed no sensitivity to fatigue loading in the longitudinal bridge direction. The effects of uplift fatigue loading on the ultimate load in the transversal bridge direction, however, were not negligible.

Key words: fatigue life modeling, adhesive joints, stiffness degradation, fracture mechanics.

12.1 Introduction

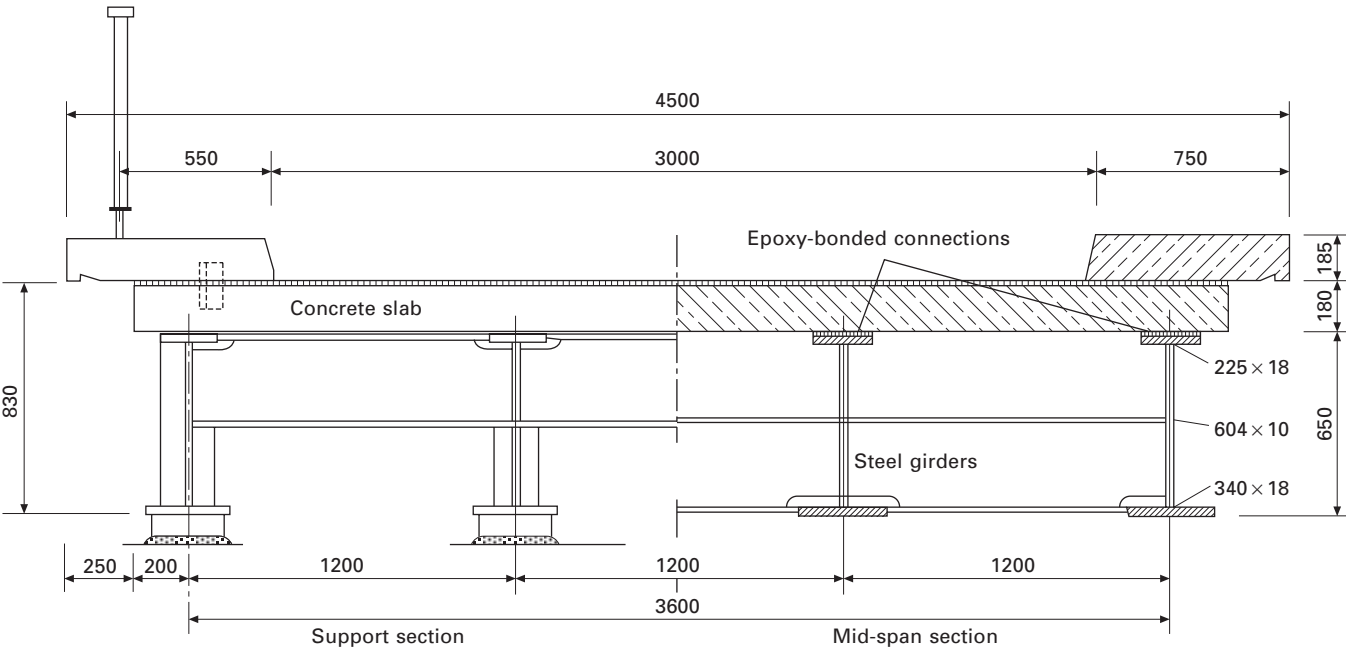
Fiber-reinforced polymer (FRP) structural profiles are being increasingly used for civil infrastructure applications. Due to limitations on FRP profile size and the requirements of transportation and handling, however, connections between FRP structural profiles are inevitable. Therefore, the need for efficient and reliable load-bearing joints that can endure various types of loading – including long-term fatigue and environmental loading – has become apparent. The advantageous properties of FRP composites could be overshadowed if the characteristics of the associated joints are not properly understood and designed.

Mechanical fastening, adhesive bonding and the combination thereof have been used for FRP composite joints [1]. Each method has its advantages and disadvantages. Since no cut-outs for bolts are required in a bonded joint, the reinforcing fibers remain continuous, stress concentrations are reduced and a refined joint geometry is obtained, thus giving it potentially better fatigue and durability performance. Furthermore, when connecting two or more components, a bonded connection weighs less than a mechanically fastened joint. The advantage of a lighter joint is relevant in the development of advanced lightweight composite structures.

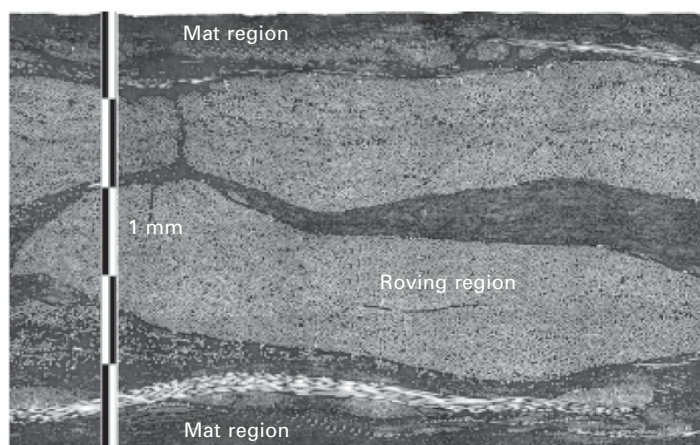
Joining via adhesive bonding has been widely used for connecting FRP composites in aerospace and aircraft structures [2]. In civil infrastructure, the application of adhesive bonding between steel and concrete structures has existed since the 1960s [3]. It is reported that, from 1968 to 1992, seven bridges spanning between 15 m and 32 m were constructed using epoxy adhesive bonding to connect the prefabricated concrete decks and the steel girders. A typical cross-section of these bridges is shown in Fig. 12.1 [3]. The thickness of the adhesive bonding ranged from 5 mm to 15 mm. The main objectives of using adhesive bonding were to save time and to improve the shear force transfer in order to ensure the composite action between the concrete deck and the steel girder. These bridges are still in service today and have experienced more than 40 years of exposure to dynamic traffic and environmental loading. Field experiences from these applications have shown that adhesive bonding can be a viable connection technique for connecting bridge decks and bridge girders. Connection through adhesive bonding has also been applied in FRP composite bridge structures [1]. However, concerns about the fatigue and long-term durability of bonded FRP connections have hindered their widespread application.

The fatigue behavior of adhesively-bonded FRP composite joints is influenced by many factors: the adhesives, FRP adherends, joint geometry, environmental conditions, loading and the quality of the joint fabrication process. Considerable research has been conducted to investigate the fatigue performance of adhesively-bonded composite joints for aerospace and aircraft applications, mostly on carbon fiber-reinforced polymer (CFRP) composite joints, e.g. [4]. Bonded FRP composite connections for civil infrastructure, however, have essential differences when compared with the FRP composite joints used in aerospace and aircraft applications. The differences include bond geometries (adhesive and adherend thicknesses), the types of materials and fabrication processes, loading and the type of environmental conditions. The adhesive layers for aerospace/aircraft structures are usually thin (0.1 to 1 mm), while in bridge and building structures adherends and adhesive layers are usually much thicker (1 to 20 mm). Furthermore, design requirements and service conditions of FRP structures for civil infrastructure differ from those employed in aerospace/aircraft structures. The service life of a bridge or building is much longer than that of an aircraft. In many countries 70 years or more is sought as the expected service life for bridges.

In civil infrastructure, pultruded glass fiber-reinforced polymer (GFRP) composites are the most widely used polymeric composite material. The reinforcements, matrix materials, lay-up scheme and fabrication process of a pultruded component are different from those of laminated composites employed in aerospace/aircraft structures. The cross-section of a 5-mm thick pultruded GFRP laminate is shown in Fig. 12.2. Typically in pultruded GFRP elements, rovings in forms of fiber bundles and stitched engineered



12.1 Cross-section of Mühlgraben Bridge at Görlitz, Germany, constructed in 1973 (dimensions in mm) [4].



12.2 Cross-sectional view of 5-mm-thick laminate using optical microscopy.

fabrics are used as reinforcements. Mats of woven or non-woven fibers are usually used as the outside layers to provide off-axis stiffness and strength. Accordingly, the layer-wise responses of pultruded FRP components differ from those of laid-up components.

Efforts have been made to study the fatigue behavior and model the fatigue life of pultruded GFRP profiles and adhesively-bonded joints composed of pultruded laminates. The fatigue behavior of bolted joints, bonded joints and combined bolted/bonded joints of pultruded GFRP laminates was studied in [5] and [6]. Comparisons of the performance of bolted connections and bonded/bolted connections showed an improved behavior of the bonded/bolted configuration compared to the bolted configuration [5]. It was suggested that more research is performed to understand the bonded joint behavior and to improve the joint design [6].

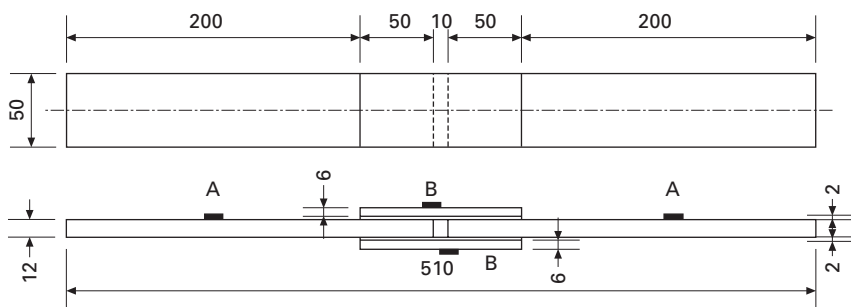
Several research projects have been carried out in the Composite Construction Laboratory at the Ecole Polytechnique Fédérale de Lausanne (EPFL) to better understand the fatigue behavior of adhesively-bonded joints composed of pultruded GFRP components, to model the fatigue life of such structural elements, and to explore the feasibility of using bonded GFRP connections for large-scale bridge and building structures [7–16]. One major part of the projects focused on basic research on the fatigue behavior of pultruded GFRP laminates and adhesively-bonded lap joints [7–13]. A second part concentrated on applied research, on the behavior of system-level adhesively-bonded connections, that is, adhesive connections between large-scale GFRP bridge decks and steel girders [14–16]. This chapter focuses on the fatigue behavior of component-level adhesively-bonded double-lap joints composed of GFRP laminates and system-level adhesively-bonded connections between GFRP bridge decks and steel main girders.

12.2 Fatigue behavior of adhesively-bonded double-lap joints

12.2.1 Experimental investigation

Balanced double-lap joints (DLJs) composed of pultruded GFRP laminates of 50 mm width and 6 or 12 mm thickness were investigated. The adhesive layer was 2 mm thick. Full-scale dimensions were chosen to exclude size effects. The pultruded GFRP laminates (supplied by Fiberline A/S, Denmark) were composed of E-glass fibers embedded in an isophthalic polyester resin. The glass fiber volume fractions were 43.6% and 48.5% respectively for 6-mm and 12-mm thick GFRP laminates. The fiber architecture of the two laminates was similar as they both comprised mat layers on the outside and rovings in the core region. The 12-mm thick laminate comprised two mat layers on each side, while the 6-mm thick laminate contained only one mat layer. A mat layer consisted of a chopped strand mat (CSM) and a woven mat $0^\circ/90^\circ$, stitched together. A polyester surface veil of 40 g/m^2 was also applied on the outer surfaces of the pultruded laminates. Due to the higher fiber content, the mean tensile strength and Young's modulus of the thicker laminate (355 MPa and 34.4 GPa) were higher than those of the thinner laminate (283 MPa and 31.4 GPa). A two-component epoxy adhesive was used as bonding material (SikaDur 330[®], Sika AG, Switzerland). The adhesive showed an almost elastic behavior and a brittle failure. The mean tensile strength was 38.1 MPa, and the mean Young's modulus was 4.55 GPa.

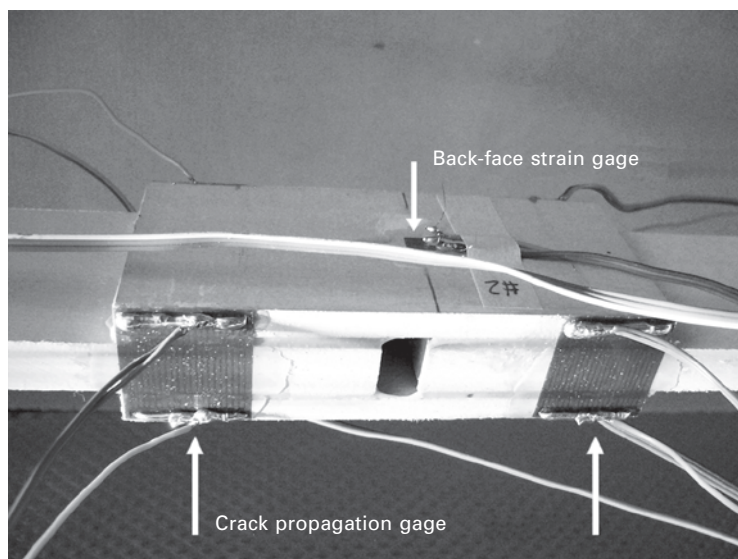
The dimensions of the specimens are shown in Fig. 12.3. All specimens were manufactured in ambient laboratory conditions and subsequently cured for 10 days. All surfaces subjected to bonding were mechanically abraded with sandblasting paper using a hand grinder and then chemically degreased using acetone before manufacture. Steel spacers of 2 mm thickness were used to support the GFRP laminates in order to keep the adhesive thickness constant.



12.3 Double-lap joint geometry (dimensions in mm).

All specimens were instrumented with strain gages, while one specimen per load level was additionally instrumented with four crack gages to detect crack initiation and record crack length during propagation, as shown in Fig. 12.4. The crack gages (HBM/RSD20) consisted of 20 wires spaced at 1.15-mm intervals perpendicular to the adhesive layer, and covered almost half of the overlap length where stable crack propagation was expected to occur. As the crack propagated, the wires were progressively broken, thus increasing the electrical resistance of the gage. There were four possible locations at the overlap edges where cracks could initiate and propagate. Therefore, the total crack length, measured with the four crack gages, was taken into account to describe crack propagation, rather than the crack length of one single crack. Two of the total of four strain gages per specimen were placed on the laminates, 100 mm from the joint edge (location A in Fig. 12.3), to measure axial strains outside the joint where stresses were expected to remain uniformly distributed across the laminate width and depth. The other two strain gages, designated back-face strain gages, were placed above locations sensitive to crack initiation (B in Fig. 12.3) to identify crack initiation. It was anticipated that a sudden change in strain would be recorded due to stress redistribution after crack initiation.

Experiments were carried out on an Instron 8800 universal testing rig of 100-kN capacity under load control and laboratory conditions ($23 \pm 2^\circ\text{C}$, RH $50 \pm 5\%$). The frequency was kept constant at 10 Hz for all specimens, while the load ratio ($R = F_{\min}/F_{\max}$) was 0.1, corresponding to a tension–tension



12.4 Instrumentation of DLJs (same configuration of crack gages on rear side).

fatigue loading. Experiments were continued until ultimate failure of the joint or 10^7 cycles, whichever occurred first. An HBM/Spider8 was used to record the measured fatigue data.

Four different nominal load levels were selected for each joint type in order to determine the load–cycle or F – N curve. Load levels were selected at 45%, 55%, 65% and 80% of the ultimate tensile load of each joint configuration in order to obtain representative experimental data in the range between 10^2 and 10^7 cycles. Each F – N curve was derived from 12 specimens, three at each of the four load levels. The specimens were labeled accordingly, with DLJ4501 signifying double-lap joint specimen 01 at 45% load level, for example.

12.2.2 Experimental results

A set of experimental data was collected for each type of structural joint. The data measured by the machine comprised load level and amplitude, as well as cycles to crack initiation (N_i), crack propagation ($N_f - N_i$) and failure (N_f), see Table 12.1. Furthermore, strains from strain gages and resistances from crack gages were recorded.

Failure modes

The dominant failure mode was a fiber-tear failure as shown in Fig. 12.5, similar to that observed under quasi-static loads. In the latter case, however, it was not possible to visually observe the damage development, since failure occurred very suddenly.

Failure initiation occurred in the adhesive–adherend interface at one joint end. The crack then rapidly penetrated into the 12-mm laminate over a length of approximately 1 mm and propagated in the outer mat layers. Other, less damaging cracks developed simultaneously from the opposite joint end. The initial dominant crack developed with fatigue loading and propagated until the load transfer area was too small to sustain the applied load and final failure occurred in a brittle manner.

F–N curves

F – N curves derived experimentally are shown in Fig. 12.6. Normalized load data was used, as maximum applied load, F , was divided by the corresponding ultimate tensile load, F_u .

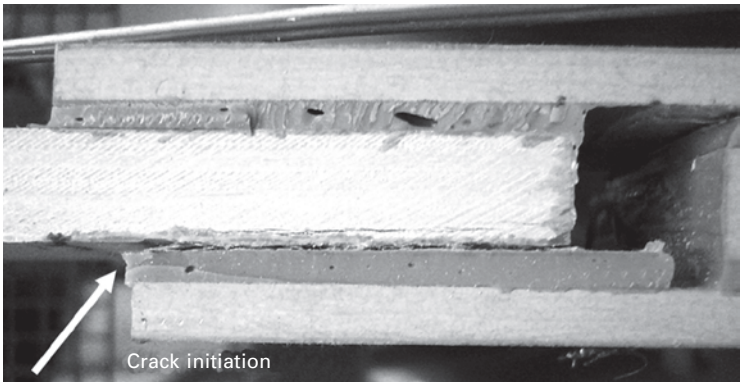
Stiffness degradation

Structural stiffness changes during fatigue life were recorded. Failure occurred after slight stiffness degradation, less than 5–7%, irrespective of load level,

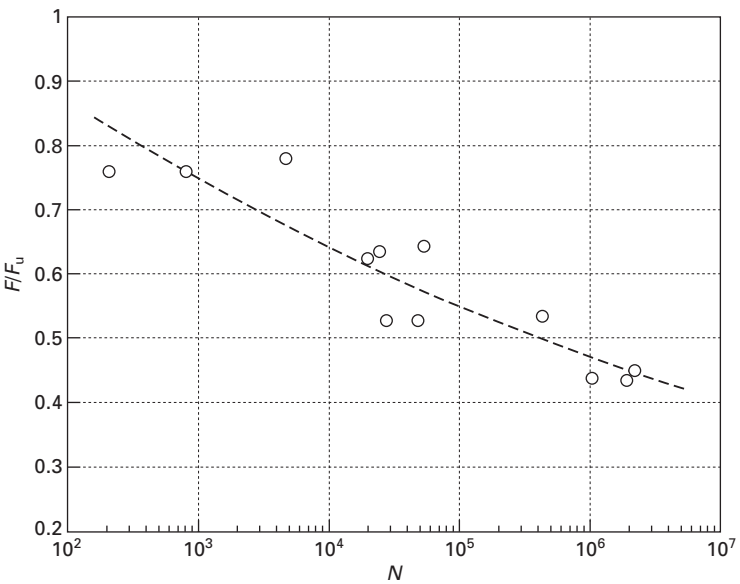
Table 12.1 Recorded fatigue data for double-lap joints

Nominal load level (% UTL)	Specimen ID	Maximum cyclic load, $F_{\max}$ (kN)	Cycles to failure, N_f	Cycles to crack initiation, N_i	Cycles for crack propagation, $N_f - N_i$	Cycle ratio to crack initiation, N_i/N_f (%)	Cycle ratio for crack propagation, $(N_f - N_i)/N_f$ (%)
45	DLJ4501 [#]	20.5	2,215,704	161,100	2,054,604	7.3	92.7
	DLJ4502	20.0	1,030,600	990,835	39,765	96.1	3.9
	DLJ4503	19.9	1,927,124	1,340,300	586,824	69.5	30.5
55	DLJ5501 [#]	24.4	432,844	6888	425,956	1.6	98.4
	DLJ5502	24.0	48,180	22,421	25,759	46.5	53.5
	DLJ5503	24.1	27,647	5391	22,256	19.5	80.5
65	DLJ6501 [#]	28.5	19,788	2880	16,908	14.6	85.4
	DLJ6502	29.0	24,306	—	—	—	—
	DLJ6503	29.4	53,664	37,178	16,486	69.3	30.7
80	DLJ8001 [#]	34.7	823	35	788	4.3	95.7
	DLJ8002	34.6	207	—	—	—	—
	DLJ8003	35.5	4731	3724	1007	78.7	21.3

[#]With four crack propagation gages.

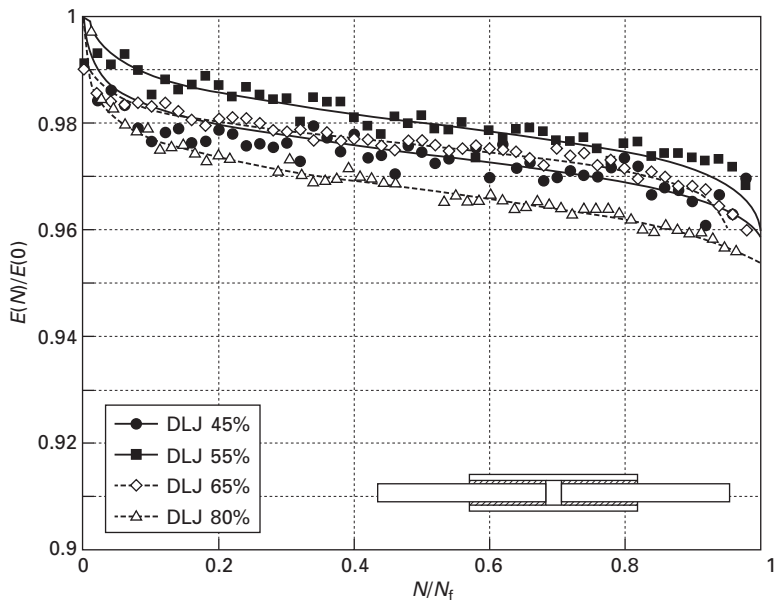


12.5 Fiber-tear failure mode of specimen DLJ5503.

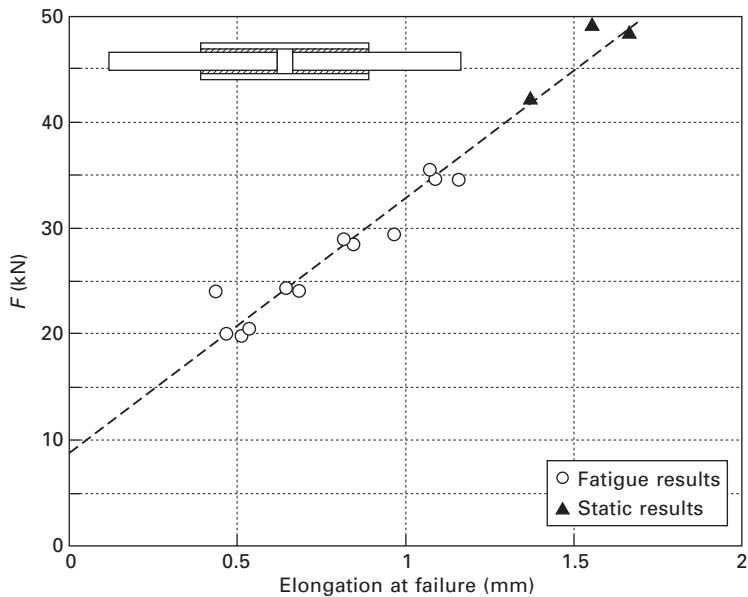


12.6 F – N curves of DLJs.

as indicated by the experimental results in Fig. 12.7. Here, the structural stiffness of the joint during the N th cycle, $E(N)$, was normalized over the value measured during the first loading cycle, $E(0)$. The number of loading cycles, N , was normalized over the life at failure, N_f . Almost linear stiffness degradation was observed during most of the fatigue process, i.e. in the $0.05 < N/N_f < 0.95$ range. Close to the final fatigue failure, when crack propagation was unstable, the stiffness degradation rate increased significantly. A linear relation is established if the maximum cyclic load is plotted against elongation at failure: see Fig. 12.8. In addition to fatigue data, this figure also shows the



12.7 Stiffness degradation of DLJs during fatigue.



12.8 Maximum elongation of DLJs at failure in static and fatigue experiments.

static ultimate loads and corresponding elongation. In this case, the residual stiffness can be considered as a critical structural stiffness, the same for all load levels, and once it is reached, final failure occurs. This critical stiffness value was established as being 32.2 ± 1.6 kN/mm.

Crack initiation and propagation

In contrast to the quasi-static results, the back-face strain gages were not sensitive to crack initiation; no significant changes in strains were exhibited during fatigue life. This difference in relation to the static loading was attributed to the limits of the measuring set-up. Continuous measurements at a frequency of 800 Hz could be performed during the static loading, while only periodic measurements could be recorded during fatigue loading due to the large amount of data to be collected.

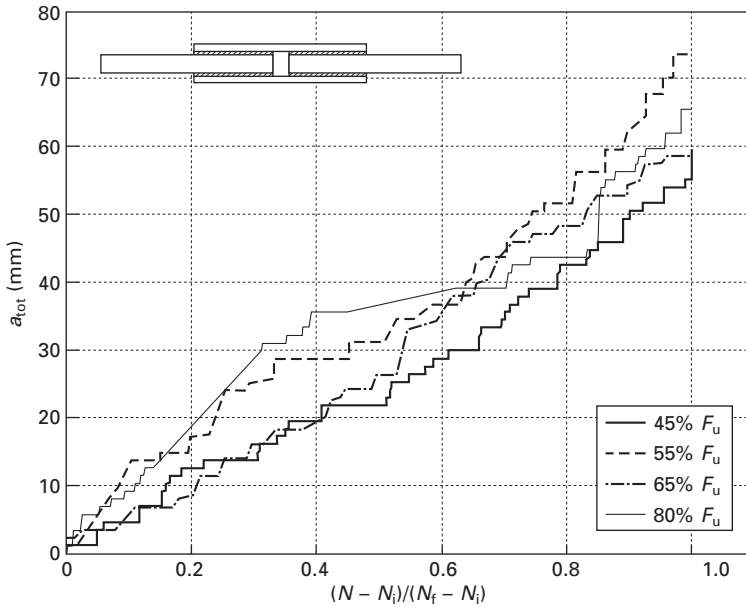
The present work concentrates on the crack propagation phase because life to crack initiation showed a high degree of variation, reaching from 0.7% to 96.7% of total life: see Table 12.1. This variation can be explained by the fact that the examined structural joints contained no pre-crack and no special care was taken to precisely shape the adhesive edge (no fillet was formed) during fabrication. Therefore, the crack could initiate at different locations in the joint, caused by existing defects, possible small asymmetries due to fabrication and/or loading misalignment. In contrast to aerospace applications, it is often not possible in civil engineering to precisely shape and control details such as adhesive fillet geometry for economic reasons. To remain on the safe side, it is therefore assumed that crack initiation already occurs at the beginning of life and that only crack propagation should be taken into account for design purposes.

Figure 12.9 shows the total crack length (sum of the four crack gages) measured at different load levels. The total crack length increased almost linearly and reached a value of 60–75 mm at the end of the fatigue life.

12.3 Stiffness-based modeling of fatigue life

12.3.1 Empirical model

With the exception of the initial and final periods, the stiffness exhibited a constant degradation rate until final fatigue failure: see Fig. 12.7. Therefore, a linear model was selected to describe this behavior and the subsequent prediction of fatigue life. The degree of damage was evaluated by measuring stiffness degradation $E(N)/E(0)$. The dynamic stiffness $E(0)$ at the first cycle is, in general, different from the one measured under quasi-static loading due to the different loading rate for fatigue and static tests. The following model was established:



12.9 Total crack length vs. normalized number of cycles for crack propagation for DLJs.

$$\frac{E(N)}{E(0)} = 1 - k_1 \left(\frac{F}{F_u} \right)^{k_2} N \quad 12.1$$

where F denotes the applied load level and could correspond to its amplitude, maximum value, or a normalized value of it. Model parameters k_1 and k_2 are dependent on available experimental data for stiffness degradation and it is assumed that they depend on the number of stress cycles and level of the applied load.

Equation 12.1 also establishes a stiffness-based design criterion, since for a preset value of stiffness degradation, $E(N)/E(0) = p$, N can be solved to obtain an alternative form of the F – N curve, corresponding not to material failure but to a specific stiffness degradation percentage:

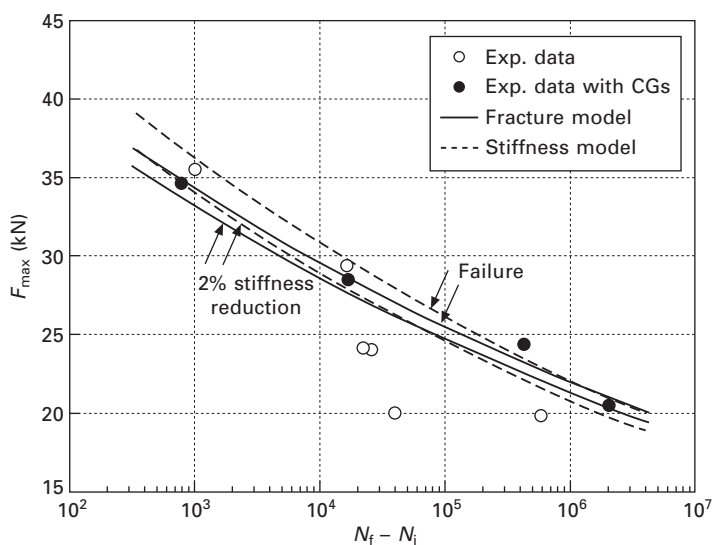
$$N = \frac{E(N) - E(0)}{E(0)k_1 \left(\frac{F}{F_u} \right)^{k_2}} \quad 12.2$$

12.3.2 Modeling results and discussion

Model parameters are estimated by plotting the stiffness degradation rate against the load levels for all the available experimental data. The resulting

estimations of parameters k_1 and k_2 are $k_1 = -0.00126$ and $k_2 = 14.176$. After derivation of the model parameters k_1 and k_2 the expected $F-N$ behavior can be extrapolated using Eq. 12.2. The results are presented in Fig. 12.10 and are compared to the experimentally determined $F-N$ data. Only fatigue behavior in the crack propagation stage was taken into account. As shown in Fig. 12.10 the linear model can produce theoretical predictions that compare well with experimental data. The slight overestimation of fatigue life is attributed to the ignorance of the initial and final periods of stiffness degradation. However, the effect of these two periods on the entire life is almost negligible. In addition to the $F-N$ curves, curves corresponding to predetermined stiffness reduction and not to failure data can be plotted and used as design allowables. Since the total stiffness degradation at failure was less than 7%, a curve for 2% decrease of stiffness is shown in Fig. 12.10.

Use of stiffness reduction as a damage metric offers several potential applications in the civil engineering field. It is a non-destructive detection method and the stiffness of the structural component is a parameter that can easily be monitored. The stiffness-based life prediction method can predict the residual life of structures regardless of their loading history. This is one of the advantages compared to the general stress-based life prediction methods. Another advantage of this method is its conformity to design code designations, since the allowable stiffness reduction is a widely used criterion. Derivation of stiffness-based curves corresponding to stiffness reduction and



12.10 $F-N$ curves obtained from fracture and stiffness models for DLJs (CG = crack gage), design allowable corresponding to 2% stiffness reduction.

not final failure provides a useful input for designing structures based on such criteria.

12.4 Fracture mechanics-based modeling of fatigue life

12.4.1 Overview

Two categories of fracture mechanics data can be distinguished: the first contains data that can be directly measured and recorded, such as crack length under specific load for a certain number of loading cycles. The second category contains data to be calculated using the recorded data, such as crack propagation rate, system compliance and its fluctuations during fatigue life and strain energy release rate as a function of crack length and/or crack propagation rate.

If the crack length can be measured during loading, the life of the examined structure can be modeled directly by integration of the crack propagation rate between two different crack lengths:

$$N - N_i = \int_{N_i}^N dN = \int_{a_i}^a \frac{1}{(da/dN)} da \quad 12.3$$

where N_i denotes number of cycles for crack initiation, and a_i denotes initial crack length. $N - N_i$ corresponds to the number of cycles for crack propagation between crack lengths a_i and a . The application of Eq. 12.3 is straightforward, since only a method for the calculation of the crack propagation rate, da/dN , has to be selected.

This application is directly linked to the specific experimental data and does not take into account the materials and/or geometry of the structural element. Based on fracture mechanics data, however, predictive methods can be developed that combine the experimental evidence obtained from one type of structural element and analytical or numerical solutions for other types of structural elements, made from the same materials, in order to predict the strength or fatigue life of the latter. One parameter that is often used for this purpose is the strain energy release rate, G . For an isotropic or anisotropic plate of constant thickness, B , and a crack with length, a , the strain energy release rate is a function of the applied load, F , and the rate of compliance change, dC/da :

$$G = \frac{F^2}{2B} \frac{dC}{da} \quad 12.4$$

In the case of cyclic loading, the maximum value of the strain energy release rate during one fatigue cycle can be deduced accordingly:

$$G_{\max} = \frac{F_{\max}^2}{2B} \frac{dC}{da} \quad 12.5$$

where $F_{\max}$ is the maximum cyclic load during the fatigue cycle. If $G_{\max}$ is plotted against the crack propagation rate da/dN using logarithmic axes to derive fatigue crack growth (FCG) graphs, the major part of the relationship is linear and can be fitted by the following equation:

$$\frac{da}{dN} = D(G_{\max})^m \quad 12.6$$

where D and m are empirical constants for the given loading ratio, frequency of testing and environment. In previous studies, D and m were considered as material constants that do not depend on joint configuration. This assumption allowed their estimation from standardized double-cantilever beam (DCB) tests and the subsequent application of the same values to estimate life for more complex joint configurations. Despite the fact that this argument seems rational, it was not supported by experimental evidence. As shown in [13], the adoption of this assumption led to an overprediction of the fatigue life of carbon fibre composite joints bonded with an epoxy adhesive.

By substituting da/dN with its equivalent from Eq. 12.6, Eq. 12.3 becomes:

$$N - N_i = \int_{a_i}^a \frac{1}{D(G_{\max})^m} da \quad 12.7$$

Depending on the values of $G_{\max}$ and the corresponding limits of the integration, Eq. 12.7 allows the calculation of conservative or non-conservative design allowables in line with a damage tolerance design philosophy (e.g. estimation of the number of cycles required to attain a specific crack length under a specific applied load).

12.4.2 System compliance

Load, displacement and crack length were recorded for the DLJs. To calculate the strain energy release rate, the only undefined term in Eq. 12.5 is the differential of system compliance with respect to crack length, dC/da . The system compliance is determined as the ratio of the axial joint displacement amplitude, $\delta_{\max} - \delta_{\min}$, to the applied load amplitude, $F_{\max} - F_{\min}$, in one cycle. Compliance increase (or stiffness degradation) results from crack propagation and the degradation of laminate stiffness. However, previous experimental work on similar adherends proved that degradation of laminate stiffness is insignificant at the low load levels applied here [7] and, therefore, degradation is entirely attributed to crack propagation. Since compliance

increase is caused by crack propagation, it is assumed that its relationship to crack length is independent of load level. A unique dC/da can therefore be calculated by using all available test data. The corresponding compliance vs. crack length relationship exhibits a similar linear trend to that observed in Fig. 12.9 for the crack length vs. number of cycles relationship and for the stiffness degradation shown in Fig. 12.7.

To derive dC/da , the experimental data was fitted using three methods: the Berry method (a power curve), a polynomial fitting, and, based on the experimental results, a linear fitting. Comparison of the different methods with experimental results revealed that the Berry method did not produce accurate results (R^2 -value of around 0.80). The fitting using polynomial and linear curves was more accurate with R^2 -values in the range between 0.92 and 0.93. The third-order polynomial method, however, led in this case to a subsequent calculation of negative values of $G_{\max}$. The linear method was therefore selected as being the best fitting method for the compliance of the DLJs.

12.4.3 Maximum strain energy release rate

The maximum strain energy release rate per cycle can be calculated by incorporating the derivative dC/da into Eq. 12.5. Based on the selected fitting methods, $G_{\max}$ results from:

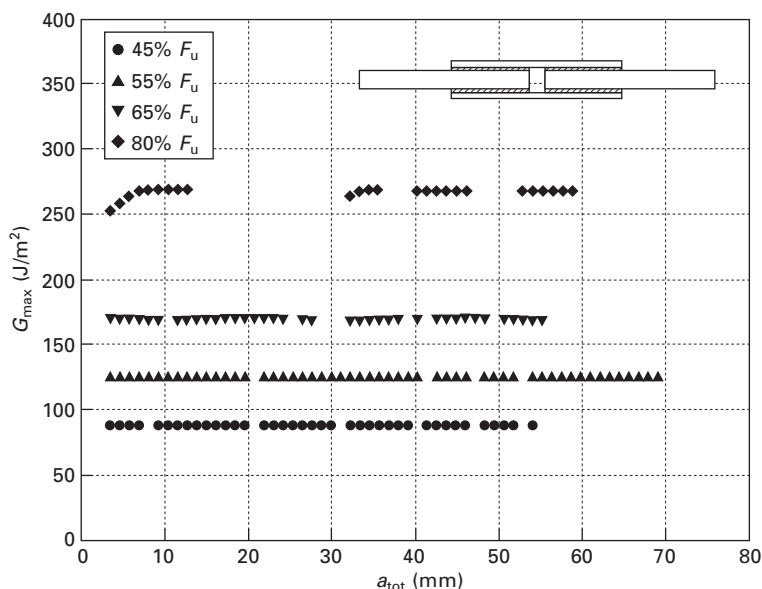
$$G_{\max} = \frac{F_{\max}^2}{2B} \cdot 1.05 \times 10^{-5} \quad 12.8$$

The maximum strain energy release rate vs. the crack length is shown in Fig. 12.11. $G_{\max}$ is almost constant during fatigue, showing increasing values for increasing load levels.

12.4.4 Crack propagation rate

The secant method and the incremental polynomial fitting (according to ASTM E647-99) were used to calculate the crack propagation rate. The incremental polynomial method better represented the experimental data. The five-point version ($m = 2$) was employed to not exclude information concerning the crack propagation rate in the initial and final stages of life. The crack propagation rate was almost constant for DLJs, in accordance with Fig. 12.9 and the stiffness degradation pattern (Fig. 12.7).

For the following analysis of fatigue life, an initial crack length (subsequent to crack nucleation) was assumed as being 4.6 mm for DLJs, corresponding to the failure of four wires of the crack propagation gages, one for each potential crack initiation location.

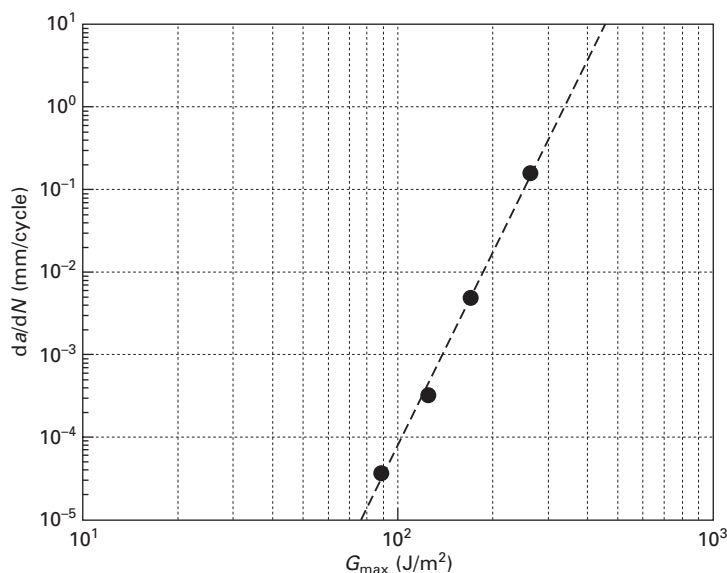


12.11 Maximum strain energy release rate vs. total crack length for DLJs.

12.4.5 FCG curves and design allowables

The FCG curves were derived by fitting Eq. 12.4 to the experimental data and determining the empirical constants D and m . For the calculations, the mean values of da/dN and G_{max} were considered for each load level, as presented in Fig. 12.12. Higher cyclic loads led to higher crack propagation rates and higher G_{max} values. The constants $D = 2.18 \times 10^{-20}$ and $m = 7.78$ were obtained, resulting in an R^2 -value of 0.99.

When D and m are known, the fatigue life for crack propagation can be calculated based on Eq. 12.5. F - N curves that correspond to failure or a predetermined crack length can then be easily calculated, thus establishing a method for the determination of damage-tolerant design allowables. Corresponding curves are presented in Fig. 12.10. Curves corresponding to joint failure agree well with the experimental data (data from specimens with crack gages are designated CG). In addition, design allowables corresponding to predetermined crack lengths were derived and compared to design allowables derived from stiffness degradation measurements where a stiffness degradation of 2% was considered. The crack length corresponding to a 2% stiffness degradation was estimated as being 20 mm. The resulting F - N curve derived from the fracture model is shown in Fig. 12.10 together with the corresponding curves obtained from the stiffness degradation model (see Section 12.3). The stiffness model shows a slightly steeper F - N curve



12.12 Crack propagation rate vs. maximum strain energy release rate for DLJs.

that tends to give more conservative results, especially towards the high cycle fatigue region. However, results from both models seem reasonable and accurate, proving their potential use for the derivation of reliable design allowables.

12.5 Structural joints: bridge deck-to-girder connections

12.5.1 Stress states in adhesive deck-to-girder connections

The fatigue behavior of full-scale system-level adhesively-bonded connections is more complicated than that of double-lap joint specimens presented in Section 12.2. Adhesive deck-to-girder connections of bridges, for example, are loaded differently in the longitudinal and transverse directions. The longitudinal direction refers to the main girder direction of the bridge, while the transverse direction runs perpendicular to the main girders in the horizontal plane.

In the longitudinal direction, the adhesive connections are subjected mainly to shear stresses due to composite action between deck and girders; Through-thickness tensile stresses theoretically arise only at the ends of the girders; however, these stresses are not critical because the bond layers

are subjected to high transverse compression stress due to the compression diagonal in the web in the support regions.

In the transverse direction, the adhesive connections are subjected to two critical loading types that can provoke significant through-thickness tensile stresses: uplift loading due to one-sided or unsymmetrical service loads, and bending moment loading due to possible deck fixation on torsion-resistant steel girders. The webs of steel girders with open cross-sections (I-sections), however, are normally sufficiently flexible (almost free to warp) to enable almost free deck rotation [16]. In most of the cases, therefore, the adhesive connections are only subjected to uplift loading in the transverse direction.

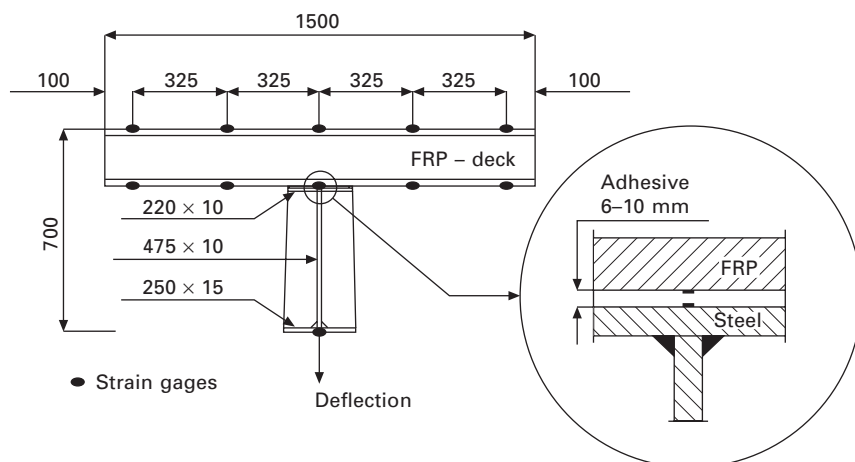
In order to consider these different longitudinal and transverse stress states in adhesive deck-to-girder connections, experiments were performed on full-scale specimens designed to simulate conditions in actual bridges. Two different types of pultruded deck-girder systems were first investigated to ultimate failure under quasi-static loading to study the quasi-static behavior of the adhesive connections. Fatigue experiments were then conducted to study the stiffness and strength characteristics of the full-scale specimens due to fatigue loading.

12.5.2 Fatigue behavior in longitudinal direction

To study the quasi-static and fatigue behavior of adhesively-bonded connections in the longitudinal direction, four adhesively-bonded hybrid girders (referred to as Fix 1–4) were fabricated. Each hybrid girder consisted of a welded steel girder and adhesively connected pultruded GFRP bridge deck panels, which acted as part of the top chord of the GFRP-steel section. DuraSpan® deck panels were used for girders Fix 1 and Fix 2, and ASSET® deck panels were used for girders Fix 3 and Fix 4 [14, 15]. Each of the four hybrid girders was 7.90 m long and spanned 7.50 m. The steel girders were 500 mm deep and the GFRP deck panels had a width of 1.50 m. These and other dimensions are shown in Fig. 12.13.

The width of the adhesive layers between the GFRP deck panels and the top flanges of the steel girders was 196 mm for the four hybrid girders. With a hard and soft shim arrangement, out-of-plane tolerances of the top steel flanges of up to 4 mm could be compensated. The resulting minimum thickness of the adhesive layer was 6 mm and the maximum thickness was 10 mm. The adhesive used for the deck-to-girder connections was the same as that used for the DLJs described in Section 12.2 (SikaDur 330®). The hybrid girders were loaded under a four-point bending configuration as shown in Fig. 12.14.

The above-described girder dimensions resulted from the preliminary dimensioning of a reference bridge with a 15 m span and 2.7 m girder



12.13 Cross-section of hybrid FRP-steel bridge girders with instrumentation (dimensions in mm).



12.14 Set-up of hybrid FRP-steel bridge girders with adhesive connections (longitudinal set-up).

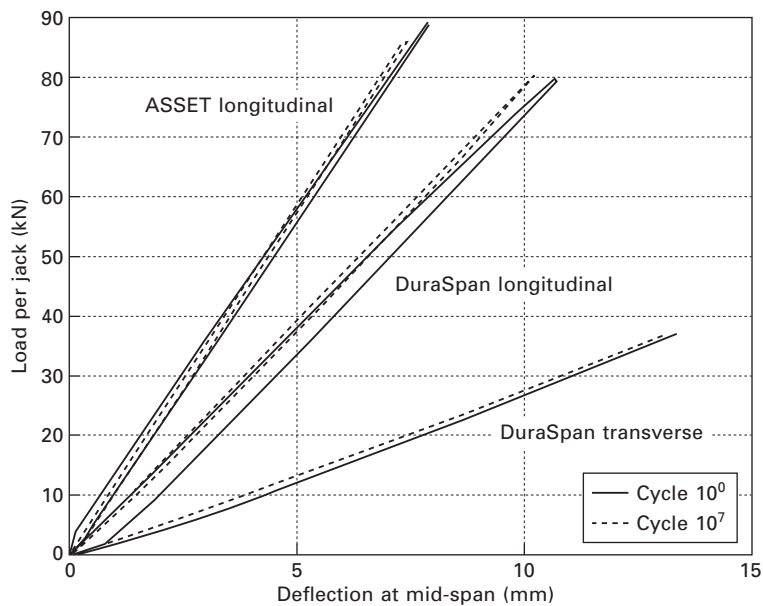
spacing, subjected to traffic loads according to Eurocode 1. The goal when dimensioning the girders was to attain similar shear stress magnitudes in the adhesively-bonded girder-to-deck connections of the experimental girders under four-point bending as in the reference bridge girders under the Eurocode loading. A short span of 15 m was chosen for the reference bridge, since

the shorter the span the higher the shear stresses in the adhesive connection, thus providing a worst-case scenario for the experimental studies.

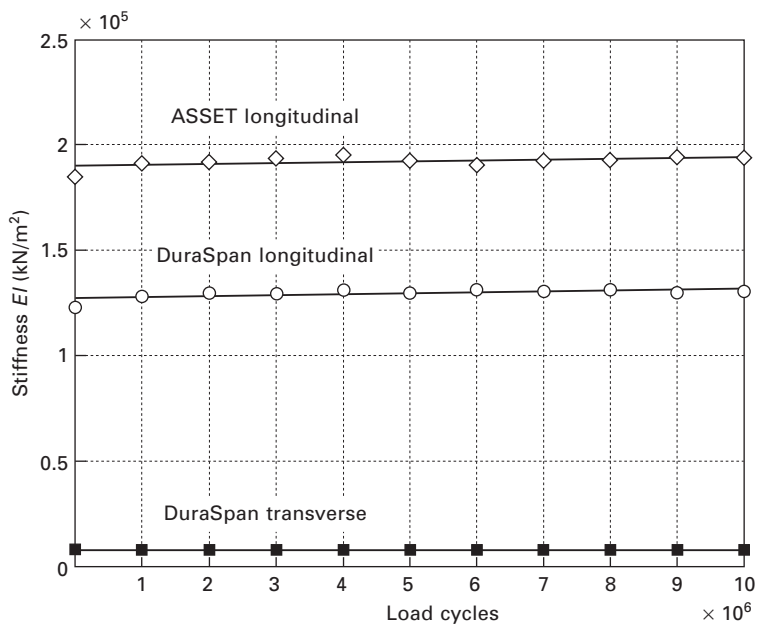
Girders Fix 1 and Fix 3 were quasi-statically loaded in three cycles: once to the service limit state (SLS) load, a second time to the ultimate limit state (ULS) load and a third cycle up to ultimate failure. All loadings were carried out under displacement control at a rate of 1 mm/min. Girders Fix 2 and Fix 4 were subjected to 10 million fatigue load cycles between 2×10 kN and 2×40 kN at a frequency of 2.0 Hz, and then loaded to ultimate failure. The applied sinusoidal fatigue load provoked a shear stress amplitude of 0.15 MPa in the adhesive bond, which corresponded to the shear stress amplitude in the girders of the reference bridge subjected to Eurocode fatigue loads. The 10 million load cycles correspond to approximately 600 truck passes per day and per lane during 250 days per year and during 70 years of service life. During the fatigue experiments, girders Fix 2 and Fix 4 were quasi-statically loaded to SLS loads at intervals of one million cycles. After 10 million cyclic fatigue loads, girders Fix 2 and Fix 4 were loaded to ultimate failure, similar to the ultimate failure loading performed on girders Fix 1 and Fix 3. From the quasi-static experiments it was found that the adhesive bond always provided full composite action between FRP decks and steel girders up to failure. Failure always occurred in the FRP decks during yielding of the bottom steel flanges.

Figure 12.15 shows the comparison between the mid-span deflections under SLS loading during the first cycle and after 10 million cycles for girders Fix 2 (DuraSpan longitudinal curve) and Fix 4 (ASSET longitudinal curve). The load–deflection responses of each girder type were very similar and neither a significant increase in the hysteresis loop area nor a significant loss in stiffness or a lateral shift of the loops due to cyclic loading was observed. Figure 12.16 shows that the bending stiffness of girders Fix 2 (DuraSpan longitudinal curve) and Fix 4 (ASSET longitudinal curve), calculated from the SLS load tests performed at intervals of one million cycles, also showed no significant decrease due to the fatigue loading. Furthermore, Fig. 12.17 shows the mid-span deflection responses of the failure experiments of all four girders. Comparing the responses of the girders with and without fatigue loading, no significant influence caused by fatigue loading could be detected. Failure in all four hybrid girders occurred in the FRP decks during yielding of the bottom steel flanges, while the deck-to-girder adhesive connection layers remained undamaged.

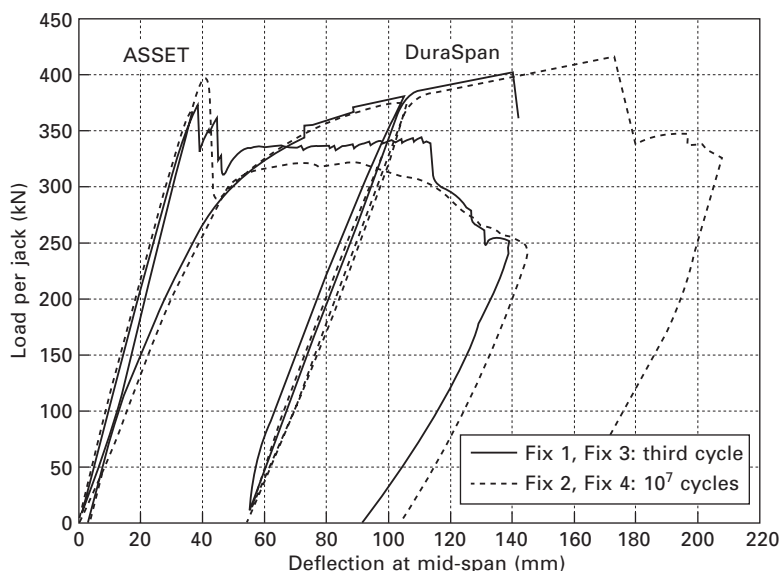
It could be concluded that the fatigue experiments performed on girders Fix 2 and Fix 4 resulted in no degradation or damage to the adhesive bonds. Significant changes due to fatigue loading in the deflection, stiffness, strain, ultimate failure load, and local and global failure behavior were not observed.



12.15 Load–deflection responses at first cycle and after 10 million cycles (from longitudinal and transverse set-up).



12.16 Girder and adhesively bonded deck stiffness degradation during 10 million fatigue cycles (from longitudinal and transverse set-up).



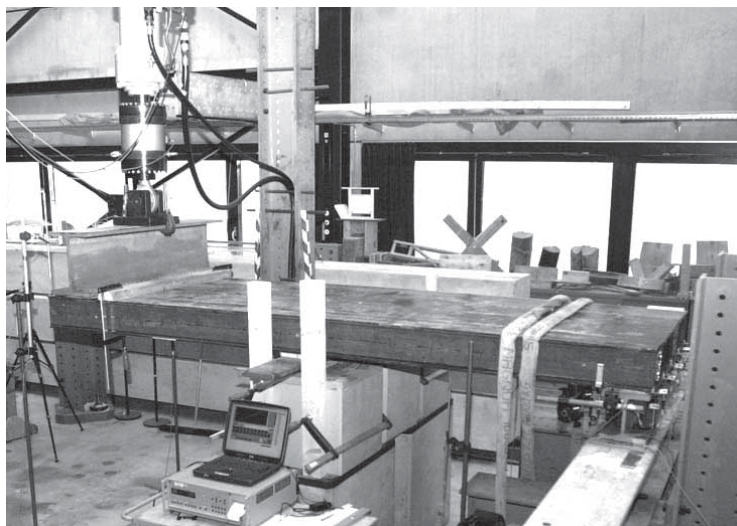
12.17 Load-deflection responses of girders at mid-span at failure cycle (longitudinal set-up).

12.5.3 Fatigue behavior in transverse direction

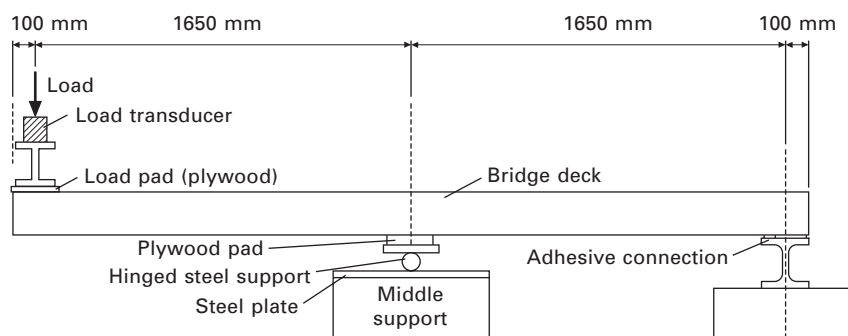
Figures 12.18 and 12.19 show the experimental set-up for the study of the fatigue behavior of adhesively-bonded connections in the transverse direction. Three identical specimens were prepared and tested for this purpose. In each experiment, a 3.5 m long and 1.2 m wide DuraSpan[®] deck panel was adhesively bonded to a 200 mm deep steel beam (HEM200 according to EN 10025), whose bottom flange was fixed. The adhesive surface was 1200 × 182 mm and the adhesive thickness was between 8 and 10 mm. The same two-component epoxy adhesive as for the previously described DLJ specimens and longitudinal girder-deck system experiments was used.

The adhesively-bonded connection was located at one edge of the deck panel and ran along the entire deck panel edge-length. At mid-width the panel was supported by a roller support. The resulting cantilever span and the adjacent span were 1.65 m. The load was applied by two hydraulic jacks acting on a load-distributing steel beam at the free cantilever edge. The flexibility of the steel girder webs and the roller support enabled free rotation of the deck panels at these locations. From this test set-up configuration, primarily uplift forces were achieved in the adhesive joint that were similar in magnitude to those expected in the reference bridge.

Three deck panels were quasi-statically loaded in three cycles (to SLS load, to ULS load and a third cycle to ultimate failure). The fourth panel



12.18 Uplift experiments on FRP decks adhesively connected to steel girders (transverse set-up).



12.19 Details of transverse set-up.

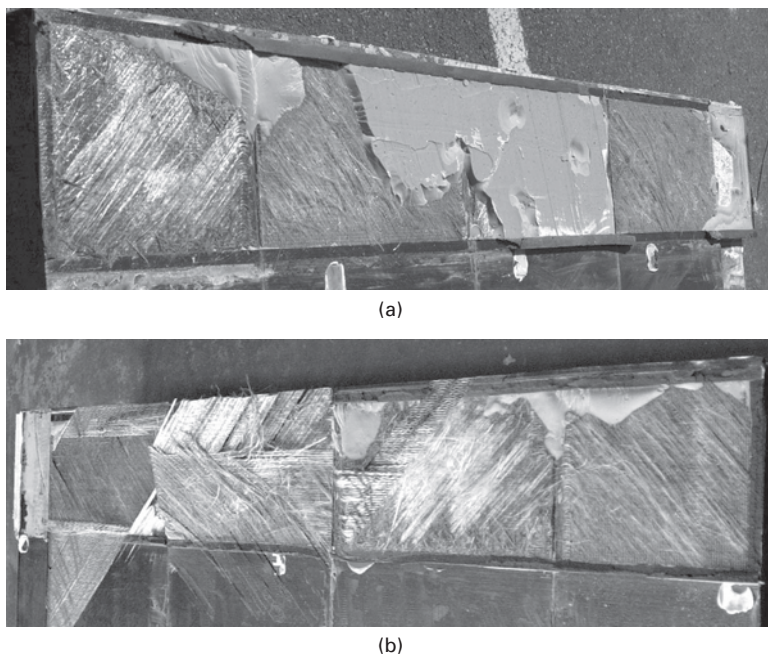
was first subjected to 10 million sinusoidal fatigue cycles at a frequency of 1.7 Hz. The load amplitude was constant at 11.7 kN, with the load ranging from 4.0 kN to 15.7 kN, and corresponded to 34% of the 34 kN SLS load. The resulting average through-thickness tensile stress amplitude was 0.05 MPa. To study the possible change in stiffness due to fatigue loading, the panel was loaded quasi-statically to the SLS load at intervals of one million cycles. After the 10 million fatigue cycles, the panel was loaded to failure under displacement control at a rate of 2 mm/min, in identical manner to the ultimate failure loading of other two panels.

As was reported for the hybrid girder experiments in Section 12.3.2 and as shown by the 'DuraSpan transverse' load-deflection curves in Fig.

12.15, neither an increase in the hysteresis loop area nor a decrease in slope or lateral shift of the loops could be observed. Meanwhile, the 'DuraSpan transverse' curve in Fig. 12.16 shows that the stiffness of the bonded panel remained constant during cyclic loading.

The load–deflection responses of all four panels were linear-elastic up to ultimate failure. Although the four panels exhibited identical stiffness, the ultimate failure load of the fatigue-loaded panel was 26% and 28% lower than the ultimate failure load of the three panels not subjected to fatigue loading. Based on the experimental studies, a total safety factor was calculated from the ratio of the ultimate failure load and the SLS load. A total safety factor of 4.1 was obtained for the fatigue-loaded panel and a total safety factor of 5.6 for the panels not subjected to fatigue loading. In a previous study [17], the DuraSpan[®] deck exhibited a total safety factor of 4.8 when considering bending moment failure due to traffic loading according to Eurocode. The fatigue-loaded panel–girder connection therefore showed an approximately 15% lower safety level than the deck system.

Figure 12.20 shows the failure surfaces of the fatigue-loaded panel and a quasi-statically loaded panel. It is seen that the failure mode of the fatigue-loaded panel was complex and different from that of the panels subjected only to quasi-static loading. For the fatigue-loaded panel, in the middle part failure



12.20 Failure modes of (a) fatigue-loaded panel and (b) quasi-statically loaded panel.

occurred between the steel flange and the adhesive; in the adjacent regions, fiber-tear failure was observed and at one edge failure occurred partially in the adhesive (cohesive failure). In the panels subjected to quasi-static loading, mainly fiber-tear failure occurred. Only in small regions (<5% of the bonded surface) did failure occur inside the adhesive layer. The different failure modes and the lower ultimate failure load of the fatigue-loaded panel could be explained by a steel/adhesive bond, which was more sensitive to uplift fatigue loading than the other interface layers in the connection and was damaged during the fatigue process. This conclusion, however, needs confirmation through further experimental work.

12.6 Conclusions and future trends

The fatigue performance of adhesively-bonded DLJs composed of pultruded GFRP profiles and adhesive connections between pultruded GFRP bridge decks and steel main girders was studied. The following conclusions were drawn:

1. DLJs exhibit a critical stiffness at which failure occurs independently of load level. Furthermore, DLJs show almost linear stiffness degradation during fatigue life, which, however, remained low at only around 5–7%. The constant rate was independent of the applied cyclic load level. A linear model well represented the residual stiffness.
2. A linearly increasing trend of compliance was observed during the fatigue life of the DLJs in agreement with the linear development of the crack length and stiffness degradation. The crack propagation rate remained almost constant throughout the fatigue life.
3. Based on measured stiffness degradation and fatigue crack growth curves derived from fracture mechanics data, the fatigue life of the DLJs was modeled and reliable design allowables were derived.
4. Adhesively bonded FRP bridge deck-to-steel girder connections showed no sensitivity to fatigue loading in the longitudinal direction of the bridge. However, sensitivity to uplift loading in the transverse direction may not be negligible.

Most of the efforts to date have been restricted to the characterization of the fatigue behavior of the adhesively-bonded joints and the modeling of their fatigue behavior under constant amplitude loading patterns. However, real structures are subjected to thermomechanical variable amplitude loading. To date no reliable fatigue life prediction methodology able to take into account the application of realistic loading patterns (variable amplitude, spectrum loading) to this type of structural connections exists. Furthermore, the quantification of the relationship between Mode I and Mode II fractures in different joint configurations as a function of the developed thermomechanical stress fields

would allow modeling the fatigue behavior of various joint configurations and, based on baseline fatigue data, the establishment of a reliable fatigue life prediction methodology for any type of joint.

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Health monitoring of composite structures based on acoustic emission measurements

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Abstract: Assessment of strength degradation in composite laminates, e.g. due to fatigue, is used in remaining life prediction through renowned engineering models. Herein, residual strength of pre-fatigued unidirectional fibre-reinforced composites is estimated using acoustic emission measurements. Two empirical schemes are presented, established on specimens under $R = 0.1$ constant-amplitude loading at several stress level and life fraction combinations. With matrix cracking being the dominant failure mechanism for the proof-loading magnitudes used, the methods are also considered applicable in all cases bearing the particular damage mode. Indeed, the schemes prove valid for various materials, specimen configurations and loading conditions, including different stress ratios as well as variable-amplitude loading.

Key words: GI/Ep composites, constant and variable-amplitude fatigue loading, acoustic emission, AE, residual strength and life prediction, matrix cracking.

13.1 Introduction

The aim of this research was to correlate fatigue-induced damage of unidirectional GI/Ep composites, expressed through their residual strength value, with parameters measured during acoustic emission monitoring. Acoustic emission (AE) is a non-destructive technique, associating the damage state of a component and the transient stress waves recorded during application on the structure of an appropriate loading. The source of the stress waves, ‘emitted’ during this ‘proof-loading’, is the micro-progression of one or more failure mechanisms.

Sinusoidal loading was applied at various stress ratios, except for a set of specimens where a variable-amplitude spectrum was simulated. The experimental database comprised specimens subjected to several stress level and life fraction combinations. These pre-fatigued specimens were then tested in static tension or compression until failure, while acoustic emission was being recorded.

Specimens used in this research were cut from $[\pm 45]_S$ and $[90]_7$ laminates. In both cases and for the proof-load magnitudes used, matrix cracking was the

dominant damage mechanism. The origin of the acoustic emission was thus assigned to matrix failure, one of the major damage mechanisms encountered in FRP composites during service. With this implementation requirement, two reliable AE-based schemes for residual strength assessment are presented. Prediction of remaining life is then possible, through dedicated engineering models.

13.2 Acoustic emission (AE) monitoring of composite structures

Acoustic emission (AE) is a well-established method for non-destructive inspection (NDI) of structural components. Released from micro-structural changes in the material, due to external triggering, AE is used for on-line inspection of components during operation or in maintenance intervals. Load application is mostly used to generate acoustic emission, although other causes, e.g. impacts, friction, curing and phase transformation, should also be included.

In the acoustic emission technique, transient stress waves resulting from damage progression are recorded using dedicated equipment. With proper sensor spacing, AE monitoring allows initiation and/or signal detection from the entire volume of the structure. However, inspection procedure demands experience and knowledge to ensure reliable results, optimum equipment selection and appropriate parameter adjustment.

13.2.1 Using the AE data

During the earliest stages of development and due to constraints in computer resources, AE inspection was performed through the cumulative or frequency distributions of particular signal parameters, e.g. amplitude (AMP), energy (EN), rise time (RT), counts (CNT), duration (DUR) and counts-to-peak (CNP) (Pollock, 1989). Signal parameters are called ‘descriptors’. Through appropriate AE descriptors, the damage state of a component can thus be expressed.

Although use of signal descriptors is indeed practical, an exact description of the acoustic signal, i.e. an integrated imprint of material properties and microstructure, is not provided. To exploit more information from the AE waveforms, a succeeding trend, modal acoustic emission (MAE), developed parallel to the conventional acoustic emission technique.

Interpretation of the gathered information, e.g. relating AE events to possible sources, is a non-trivial task. In this frame, for instance, pattern recognition (PR) techniques were used to separate the recorded AE waveforms into clusters of ‘similar’ members: the aim was to assign these clusters or ‘classes’ to particular failure mechanisms.

However, the properties of the propagation medium and the sensor/equipment characteristics can mask AE signatures. A problem of this nature, in the implementation of AE schemes, is to distinguish overlapping failure mechanisms, e.g. when various damage modes generate similar AE signals or when the produced classes demonstrate bimodal amplitude distributions (Philippidis *et al.*, 1999). As argued by Anastassopoulos and Philippidis (1995), this issue can be investigated through dedicated tests, producing a template class for the AE corresponding to each failure mechanism. For instance, tension in single fibres or fibre bundles should produce an indicative class for fibre failure, while tension in pure resin is expected to simulate matrix cracking. Issues discussed herein involve matrix cracking induced due to fatigue loading.

13.2.2 Work on AE-based fatigue damage assessment

Among damage mechanisms, matrix cracking is observed earliest and, although considered sub-critical, promotes the formation of delaminations and leads to fibre failure (Reifsnider, 1990; Gamstedt and Sjogren, 1999). Regarding transverse matrix cracks in particular, Tang and Henneke (1989) and Toyama *et al.* (2005) examined acoustic signals, captured in specimens under tensile static or fatigue loading, in terms of modal characteristics. Although focused on stiffness degradation, rather than strength, these works demonstrated that particular modes of wave propagation are indeed sensitive to damage due to matrix cracking.

A successful failure criterion for a CFRP prosthetic foot, subjected to cyclic loading, was presented by Unnthorsson *et al.* (2008). The method, intended to provide notice of impending failure rather than perform actual life prediction or damage assessment, relied on a large experimental effort and was validated through comparison with a conventional displacement criterion.

Life prediction schemes of a more quantitative character were developed by Bhat *et al.* (1994). AE data was recorded throughout tensile fatigue loading, at one particular stress level. A PR algorithm was then used to cluster the data into three classes. These classes were shown to correspond to the three main failure mechanisms, each being dominant at a particular stage of fatigue life. Although the class presumed to consist of matrix-crack-induced AE events bore no useful conclusions, classes representing interface debonding and fibre failure were correlated with 'consumed' fatigue life.

Nkrumah *et al.* (2005) used a static proof-loading to estimate remaining life after fatigue on a limited experimental data set. A more elaborate approach is encountered in the work of Caprino *et al.* (2005), using specimens subjected to constant-amplitude (CA) loading at one particular stress level. Except for low-cycle fatigue, a conventional AE parameter (CNT) was well-correlated

with residual strength. The same experimental data was processed using a neural networks approach by Leone *et al.* (2006), producing more accurate residual strength predictions.

Philippidis and Assimakopoulou (2008a) introduced a robust and reliable engineering model predicting degradation of in-plane shear strength of a UD composite, due to fatigue. The model, presented in two versions, was established and validated on an adequate population of 87 ISO standard 250-mm [± 45]_S GI/Ep specimens that had undergone CA sinusoidal loading at stress ratio $R = 0.1$. To investigate the influence of various loading configurations on damage accumulation, 10 stress level and life fraction combinations were accommodated. The scheme was based on AE monitoring of pre-fatigued specimens during a static proof-loading. Residual strength assessment was performed through a conventional AE descriptor, CNT, although other AE parameters also proved applicable. The procedure, however heuristic, was reliable in residual strength estimation of composite specimens featuring fatigue-induced matrix damage.

As mentioned above, the residual strength predictive model was established on CA fatigue tests. Two variations of the same model were introduced. The former scheme, 'model M_1 ', required *a priori* information on maximum loading stress. Using this information, a master curve valid for all stress levels and life fractions was obtained. The model was thus suggested for applications where the maximum stress encountered during cyclic loading was available, i.e. either fixed or recorded on-line. In cases of unknown loading histories a second scheme was recommended, 'model M_2 ', demanding no information on previous loading.

Two additional datasets were used to further validate the method. Validation sets again comprised ISO 250-mm [± 45]_S GI/Ep specimens: one consisted of specimens identical to the training set, subjected to tensile spectrum loading, and the second set of specimens made of another resin matrix, under $R = 0.1$ CA loading. Models were indeed applicable in both cases. Thus, the methods proved capable of predicting the residual static strength of specimens undergone constant- or variable-amplitude (VA) fatigue, at all stress levels and loading durations.

Although concerning tensile loading of the [± 45]_S specimen, recommended in international standards for shear testing of the respective unidirectional material, the work suggested indicative guidelines for implementation in other laminates and loading configurations. Of course, in a generalized laminate where damage is not matrix-dominated, all possible failure mechanisms should also be considered and a dedicated database might be required for each application.

As argued above, the proposed models prove applicable in cases where matrix cracking is the dominant damage mode, at least up to the proof-loading magnitude. In the work of Philippidis and Assimakopoulou (2008b),

the models were applied on an additional configuration, 145-mm long $[90]_7$ specimens, made of the same material as the original 87-specimen $[\pm 45]_5$ dataset. Results from $R = 0.1$, $R = -1$ and $R = 10$ CA loading were presented, suggesting the AE models to be applicable in tensile, reversed and compressive fatigue, although the nature and extent of damage varies depending on R -ratio. For instance, in the works of Gamstedt and Sjogren (1999) and El Kadi and Ellyin (1994), $R = -1$ is shown to be more detrimental than $R = 0.1$. Nonetheless, in contrast to the concluding remark of Maier *et al.* (1986), experiments at different R -ratios could indeed be compared. To be fair, however, one should mention that the work of Maier *et al.* (1986) was conducted on $[0_2/\pm 45/0_2/\pm 45/90]_5$ carbon fibre-reinforced polyimide specimens and also that from the fatigue tests at $R = 0.1$ and $R = -1$ presented, $R = -1$ specimens failed in compression tests due to buckling, in contrast to Philippidis and Assimakopoulou (2008b) where the corresponding failure mode was tensile.

Another major accomplishment, presented by Philippidis and Assimakopoulou (2008b), was using the same model for *compressive* residual strength estimation. Indeed, for $R = -1$, both tensile and compressive residual strength tests were available: failure, in both tension and compression, was matrix-dominated. Although the respective failure modes are distinguished as ‘mode A’ and ‘mode C’ in current damage-mode-based failure criteria (e.g. Puck and Schürmann, 1998), the model response for both cases seems to follow the same trend.

13.3 Materials and specimens

The main experimental series was performed on specimens of the same GI/Ep material, i.e. same glass reinforcement and resin matrix, named ‘reference’ material (Jacobsen, 2003). The reinforcement of the reference material was a 1150 g/m^2 non-woven unidirectional glass roving, stitched together with a chopped-strand-mat (CSM) layer of 50 g/m^2 with 50 g/m^2 of off-axis polyester (PES) yarn stitches. Total weight was 1258 g/m^2 and glass fibre density 2590 kg/m^3 . The resin used was Prime 20 from SP Systems, mixed with slow hardener. The cured resin had a density of 1145 kg/m^3 . Laminates were post-cured at 80°C for 4 hours, resulting in a fibre content of $73 \pm 3\%$ by weight and $55 \pm 3\%$ by volume. The nominal average thickness of each layer was 0.88 mm. A peel-ply was used during fabrication, producing a somewhat rough resin-rich surface finish to promote stable placement of extensometers. A smaller number of tests was conducted on an ‘alternative’ material. The matrix of the alternative material was from a resin batch other than that of the main set, leading to variations in material behaviour.

Damage in FRP components, due to fatigue, was simulated on two configurations: $250 \times 25 [\pm 45]_5$ and $145 \times 25 [90]_7$ specimens, dimensions

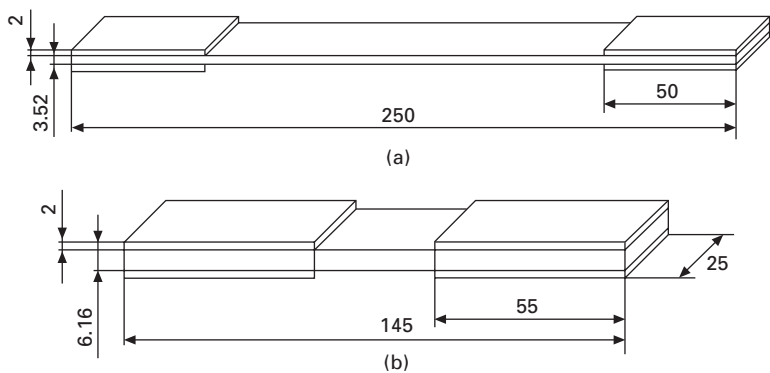
being given in mm. Specimens were produced and provided by LM Glasfiber. The manufacturing method was vacuum infusion. All specimens were made of the reference material except for a small number of $[\pm 45]_S$ ones made of the alternative material.

Being 250 mm long, $[\pm 45]_S$ specimens were intended for tensile loading (see Fig. 13.1(a)). The particular laminate and loading configuration is recommended in the ISO 14129:1997(E) standard for the investigation of the in-plane shear properties of the unidirectional material. This method prevailed amongst others in the relevant review of Lee and Munro (1986) and is the one used by Yang and Jones (1978) and Philippidis and Passipoularidis (2007) on shear strength degradation due to CA loading.

It was important to design a specimen suitable for universal testing, i.e. to be used in both tension and compression, static and fatigue loading (see Wedel-Heinen *et al.*, 2006). The short 145-mm $[90]_7$ specimen, illustrated in Fig. 13.1(b), was the outcome of such an investigation for the transverse loading direction. Mechanical properties derived using this specimen were in perfect agreement with the respective ISO ones (see Philippidis, 2005). Thus, albeit failure planes were near the tab area, the experimental results were considered valid and the failure modes acceptable.

13.4 Material characterization

A comprehensive experimental program was performed in order to characterize the composites under consideration in terms of static mechanical properties and fatigue behaviour. This stage, although scrupulous, was a prerequisite for subsequent residual strength testing. All experimental results are reported in OPTIDAT (2006).



13.1 Experimental specimens: (a) $[\pm 45]_S$ ISO 14129:1997(E); (b) proposed $[90]_7$ coupon (dimensions in mm).

13.4.1 $[\pm 45]_S$ specimens

Static tensile tests

Ultimate tensile stress, UTS or X , tensile Young's modulus in the axial direction, E_x , and Poisson's ratio, ν_{xy} , of the reference material were determined through 26 static tensile tests (STT), to derive a reliable statistical static shear strength distribution of the stochastic behaviour of the Gl/Ep composite. Experiments, outlined by Philippidis *et al.* (2004b), were performed in displacement control mode, at a crosshead speed of 2 mm/min, in accordance with ISO 14129:1997(E) specifications. The 25-kN set-up of a 250-kN hydraulic MTS test machine was used. Each specimen was equipped with a 6-mm strain gauge rosette on one side and a single 6-mm strain gauge on the opposite side. All strain gauges were HBM, with a nominal electrical resistance of 350 ohms. Strains and load-displacement readings from the test machine were recorded via an HBM Spider8 data acquisition device.

The mechanical properties of the alternative material were also determined, through five similar tensile static tests (Philippidis *et al.*, 2006a). The average tensile strength of the alternative material was found to be about 85% of that of the reference material.

Mechanical properties of both composites are listed in Table 13.1. The UTS of the reference material could be modelled using a biparametric Weibull distribution of shape parameter a and scale parameter b , as shown in Fig. 13.2. UTS results from the static tensile tests are illustrated in Fig. 13.3.

Constant-amplitude fatigue tests

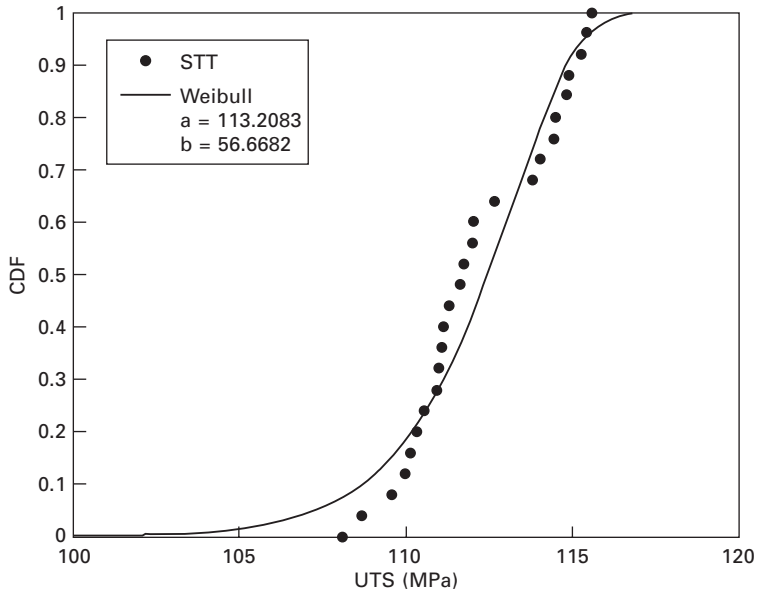
A set of 17 reference-material specimens was tested in CA fatigue, at stress ratio $R = 0.1$, in load control mode and until separation. The determined S-N curve, presented in Fig. 13.3, is given by:

$$S_{\max} = 169.16N^{-\frac{1}{11.06}} \text{ (MPa)} \quad 13.1$$

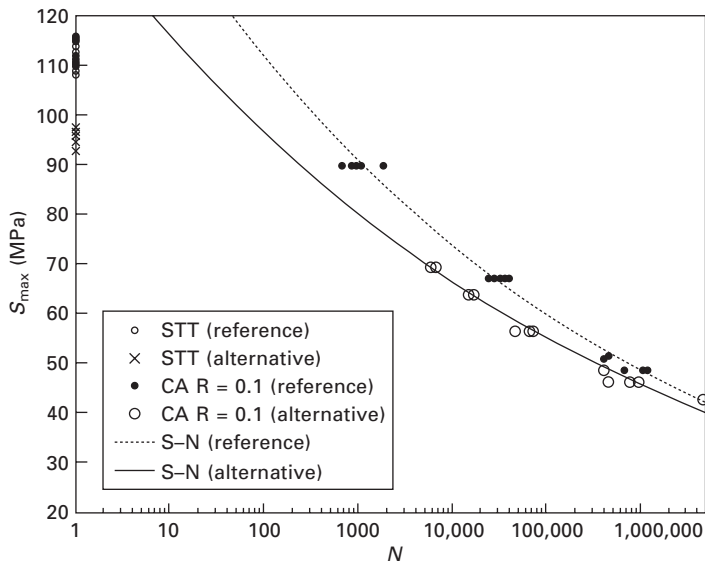
$S_{\max}$ is used to denote maximum fatigue stress and N the corresponding number of cycles to failure. Fatigue behaviour of the alternative material, inferior to that of the reference material, was also investigated through 14

Table 13.1 Mechanical properties, $[\pm 45]_S$ specimen

Material	E_x (GPa)	ν_{xy}	G_{12} (GPa)	UTS (MPa)
Reference	14.13	0.5383	4.232	112.14
Alternative	13.01	0.6041	3.937	95.30



13.2 Tensile static strength and respective biparametric Weibull distribution, $[\pm 45]_S$ specimen, reference material.



13.3 Tensile static and fatigue tests and S-N curves at $R = 0.1$ for the reference and the alternative material, $[\pm 45]_S$ specimen.

similar CA tests at $R = 0.1$. Results are demonstrated in Fig. 13.3. The corresponding S–N curve equation was:

$$S_{\max(\text{alt})} = 139.64N^{-\frac{1}{12.34}} \text{ (MPa)} \tag{13.2}$$

Fatigue data was treated according to the statistical method introduced by Whitney (1981), to provide reliability bounds for the determined S–N curves. The probabilistic formulation is expressed as:

$$S_{\max} = S_0 \{ [-\ln(P_S(N))]^{1/a_f k_f} \} N^{-1/k_f} \tag{13.3}$$

Equation 13.3 describes the S–N curve at a desired probability of survival, $P_S(N)$. Constants S_0 , a_f and k_f were found to be equal to 173.0 MPa, 4.4 and 10.9 for the reference material and 144.4 MPa, 5.8 and 11.9 for the alternative material.

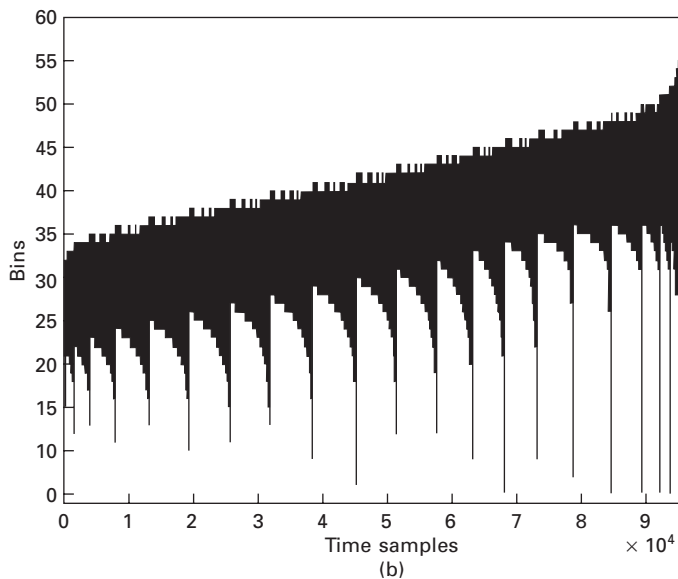
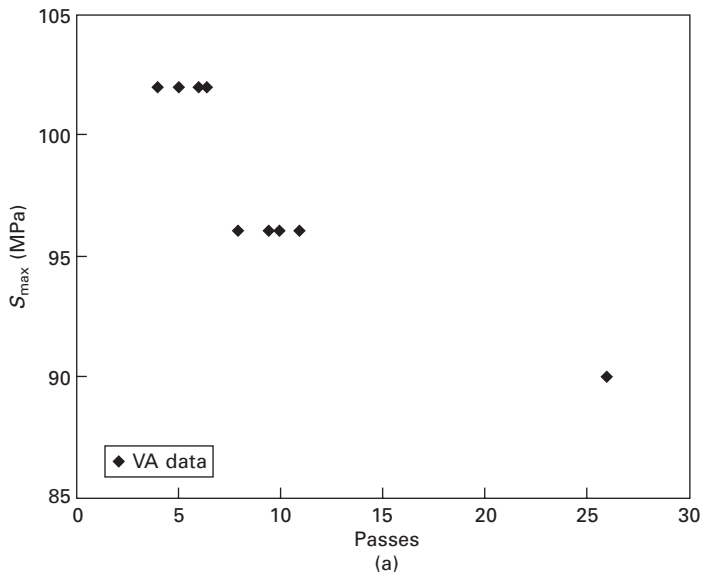
The stress-level definitions produced during CA fatigue tests, for subsequent use in residual strength testing, are listed in Table 13.2. Test frequencies, f , varied depending on stress level, so as to keep constant dissipated energy per cycle (Krause, 2002). Indeed, temperature on the specimen surface during all fatigue tests was maintained below 35°C. Lab air-conditioning and the use of a cooling fan were thus required. Temperature measurements were performed using Pt100 thermo-resistances, placed on the side of the lower-grip tab area with thermo-conducting glue.

Variable-amplitude fatigue tests

A set of 10 variable-amplitude fatigue tests were also performed on the reference material: see Fig. 13.4(a). Two maximum stress levels were investigated. Experiments are described by Philippidis *et al.* (2006b). Specimens were subjected to the ‘NEW WISPER’ spectrum, introduced by Soker *et al.* (2004). The NEW WISPER spectrum is demonstrated in Fig. 13.4(b). This is the general form of the spectrum, expressed in bins valued from 5 to 59. Level 59 corresponds to the ‘normalized’ $S_{\max}$ value.

Table 13.2 Stress-level definitions for the reference and the alternative material, [± 45]_S specimen

N (cycles)	$S_{\max}$ (MPa)		f (Hz)
	Reference	Alternative	
5000	78.31	70.01	1.31
50,000	63.60	58.09	1.99
220,000	55.62		2.60
1,000,000	48.50	45.57	3.42



13.4 (a) Variable-amplitude fatigue tests, reference material, $[\pm 45]_s$ specimen; (b) NEW WISPER spectrum, used in variable-amplitude fatigue loading.

NEW WISPER was produced following the concept of the WISPER spectrum, WInd turbine reference SPECTRum, established on measurements conducted on nine wind turbines, with diameters ranging from 11.7 to 100 m. Rotor blades were made of steel, GRP or wood. The sequence comprised

132,711 load cycles, representing two months of operation. NEW WISPER, on the other hand, was based on measurements performed on seven wind turbines, with diameters ranging from 37 to 80 m. Rotor blades were made of GFRP or CFRP. The series consisted of 95,472 load cycles and represented two months of operation (Soker *et al.*, 2004).

13.4.2 [90]₇ specimens

Static tests

Ultimate tensile stress (UTS), ultimate compressive stress (UCS) and elastic properties of the UD material, in the transverse direction, were determined through 25 tensile (STT) and 26 compressive (STC) static tests, conducted in displacement control mode on a 100-kN servo-hydraulic Dennison–Mayes DH 100S test rig, equipped with a 407 MTS controller.

Tensile experiments were performed at ISO-recommended strain rate (see EN ISO 527-5:1997(E)), resulting in a crosshead speed of 0.25 mm/min. For better resolution in load application, a configuration calibrated for a maximum load of 25 kN was used. Each specimen was instrumented with a 6-mm strain gauge rosette on one side and a single 6-mm strain gauge on the opposite side. The UTS was thus found to be equal to 53.95 MPa, the tensile Young's modulus in the axial direction (i.e. transverse to the fibres), E_{2T} , was 14.08 GPa, and the minor Poisson's ratio, ν_{21} , was 0.095.

Compressive tests were conducted at 1 mm/min crosshead speed, in accordance with ISO 14126:1999(E). Specimens were equipped with a 6-mm single strain gauge on each side. The UCS was –165.02 MPa and the compressive Young's modulus, E_{2C} , was 15.00 GPa. Again, strains and load-displacement readings from the test machine were recorded using an HBM Spider8 data acquisition device. All static experiments were outlined by Philippidis *et al.* (2004a).

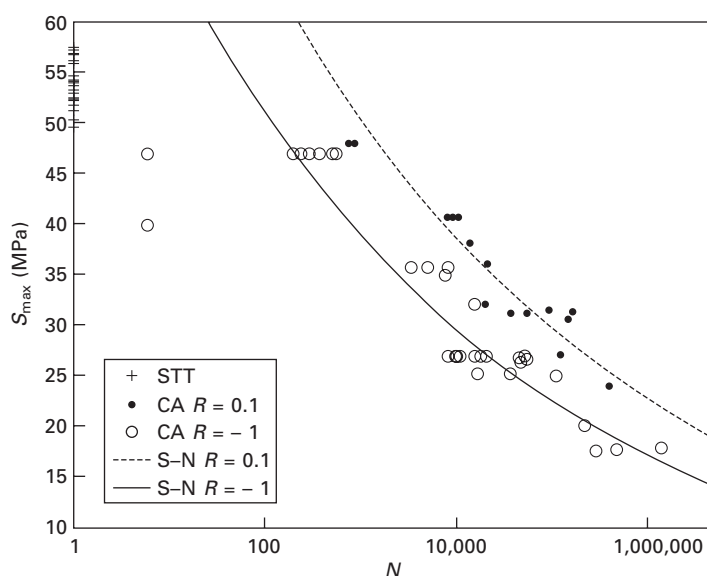
Constant-amplitude fatigue tests

Fatigue tests were performed on the 25-kN set-up of a 250-kN MTS test machine, in load control and until specimen separation. A set of 15 specimens was tested at stress ratio $R = 0.1$, 32 experiments were performed at $R = -1$, and 24 at $R = 10$. Fatigue response was reported by Philippidis *et al.* (2005a). Stress-level definitions, produced during CA fatigue tests for subsequent use in residual strength testing, are listed in Table 13.3. Experimental results are summarized in Figs 13.5 and 13.6 along with the respective S–N curves, given by:

$$S_{\max} = S_0 N^{\frac{1}{k}} \text{ (MPa)} \quad 13.4$$

Table 13.3 Stress-level definitions for the CA $R = 0.1$, $R = -1$ and $R = 10$, $[90]_7$ specimen

N (cycles)	$S_{\max}$ (MPa)			f (Hz)		
	$R = 0.1$	$R = -1$	$R = 10$	$R = 0.1$	$R = -1$	$R = 10$
1000		38.58			1.24	
5000	41.59	31.87	138.63	1.78	1.81	3.34
50,000	31.85	24.26	126.11	3.03	3.13	4.04
200,000			119.12			4.53
1,000,000	22.51	17.00	111.49	6.07	6.37	5.17



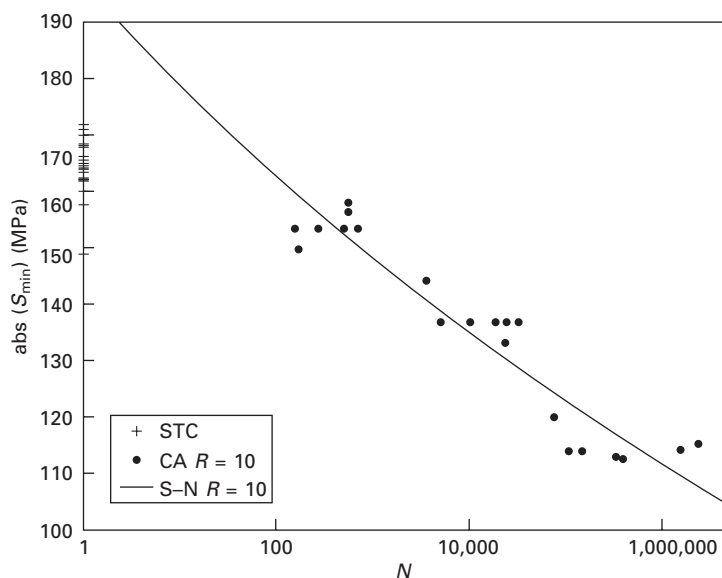
13.5 Static tensile tests, fatigue data and S-N curves at $R = 0.1$ and $R = -1$, $[90]_7$ specimen.

For $R = 0.1$, S_0 and k are equal to 111.6 MPa and 8.6, while for $R = -1$ the respective values are 87.5 MPa and 8.4. For $R = 10$, $S_{\max}$ represents the absolute $S_{\min}$ value, and S_0 and k are equal to 196.8 MPa and 24.3.

13.5 Residual strength degradation

13.5.1 $[\pm 45]_S$ specimens

Specimens were subjected to tensile constant-amplitude ($R = 0.1$) or variable-amplitude cyclic loading. The main experimental program comprised a set of 87 reference-material specimens. For each specimen, one out of four



13.6 Static compressive tests, fatigue data and S-N curve at $R = 10$, $[90]_7$ specimen.

specific fatigue stress levels was used, corresponding to 43%, 50%, 57% and 70% of the nominal reference material ultimate stress, equal to 112.14 MPa. Respective expected fatigue lives, N , corresponding to $P_S(N) = 50\%$ were 1,000,000, 220,000, 50,000 and 5000 cycles. Duration of the CA loading reached up to 20%, 50% or 80% of the estimated life.

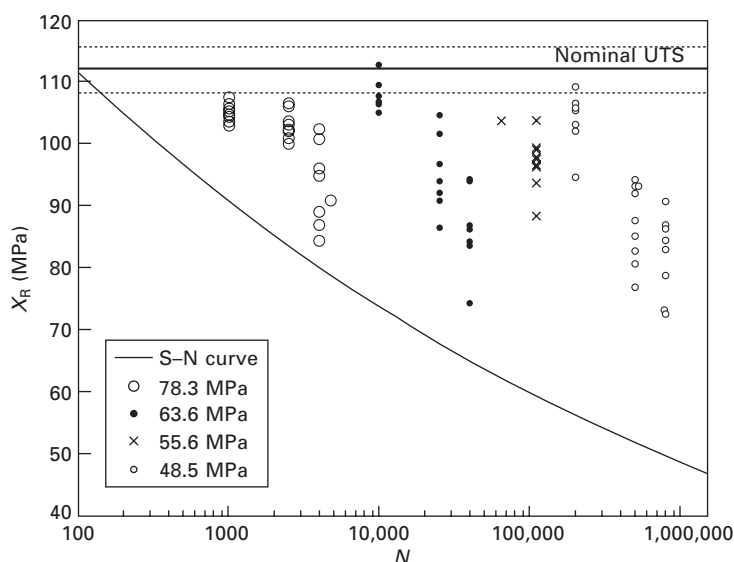
Two independent experimental sets were used to validate the proposed schemes. One was a small set of nine reference-material specimens, subjected to variable-amplitude loading. VA specimens were fatigued using the NEW WISPER spectrum until half of the expected passes to failure (50% life fraction). The other set consisted of 16 alternative-material specimens, subjected to CA $R = 0.1$ fatigue at three stress levels and for 20%, 50% or 80% life fraction. The number of tests corresponding to each configuration is listed in Table 13.4.

Residual strength experiments were performed at a constant crosshead speed of 2 mm/min while acoustic emission was being recorded. Experimental results, reported by Philippidis *et al.* (2005b), are presented in Fig. 13.7 for the reference and in Fig. 13.8 for the alternative material. As also discussed below, at each stress level there is clear strength degradation of up to 40% of the UTS, as fatigue cycles increase (see Philippidis and Passipoularidis, 2007).

Horizontal reference lines in Figs 13.7 and 13.8 are used to indicate the mean value (solid lines) and scatter band (dashed lines) of the UTS.

Table 13.4 Number of $[\pm 45]_S$ coupons for each loading configuration

Material	Loading	$S_{\max}$ (MPa)	N (cycles)	n/N		
				20%	50%	80%
Reference	CA	78.3	5000	8	8	8
		63.6	50,000	8	8	9
		55.6	220,000		13	
		48.5	1,000,000	8	9	8
	VA	96.0	10 passes		5	
		102.0	4 passes		4	
Alternative	CA	70.0	5000	2	2	
		58.1	50,000	2	2	2
		45.6	1,000,000	2	2	2

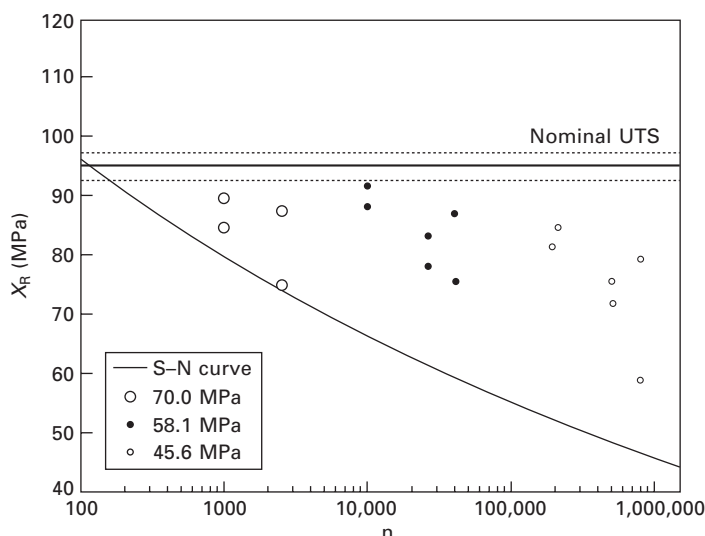


13.7 Tensile residual strength test results and S-N curve at $R = 0.1$, $[\pm 45]_S$ specimen, reference material.

determined in the respective static tests for material characterization. Note that while N is used to denote cycles to failure, n represents the number of cycles that a specimen is subjected to, at life fraction n/N . Residual strength is denoted as X_R .

13.5.2 $[90]_7$ specimens

For the $[90]_7$ specimens, residual strength tests were performed at a crosshead speed of 0.25 mm/min in tension and 1 mm/min in compression. For better



13.8 Tensile residual strength test results and S-N curve at $R = 0.1$, $[\pm 45]_S$ specimen, alternative material.

resolution in tensile load application, a test machine configuration calibrated for a maximum load of 25 kN was used. Experiments are reported in detail by Philippidis *et al.* (2005c).

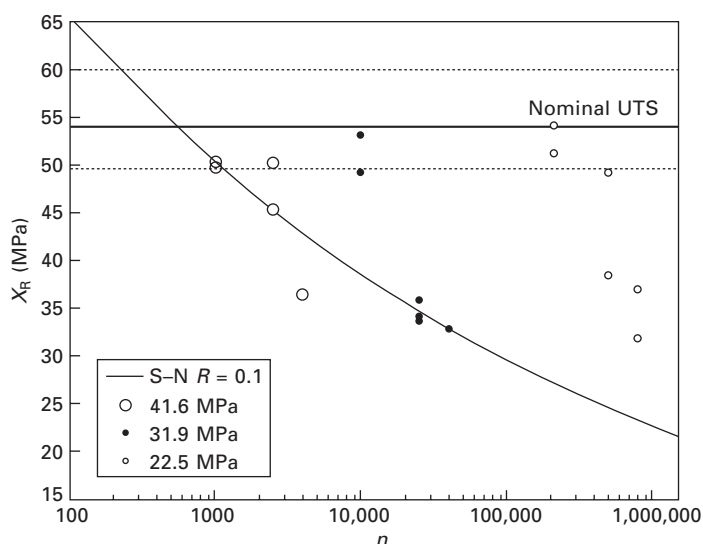
Static tests to failure were performed on specimens pre-fatigued at three stress ratios, $R = 0.1$, $R = -1$ and $R = 10$. During residual strength tests, acoustic emission was being recorded. For each specimen, one out of five specific fatigue stress levels was used, corresponding to expected fatigue lives, N , of 1,000,000, 200,000, 50,000, 5000 or 1000 cycles. Duration of the CA loading reached up to 20, 35, 50 or 80% of the estimated life. The respective stress levels, for each stress ratio, are given in Table 13.5. For $R = 10$, $S_{\max}$ corresponds to the absolute $S_{\min}$ value.

The number of tests corresponding to each loading configuration is also listed in Table 13.5. At $R = 0.1$, a total of 17 $[90]_7$ specimens was used, whereas 19 $[90]_7$ specimens were tested at $R = 10$. Residual strength tests of the respective specimens were tensile. However, at $R = -1$, a total of 31 experiments was performed, with 20 specimens tested for residual strength in tension and 11 in compression.

Residual strength test results are presented in Figs 13.9 to 13.12. Horizontal reference lines denote the mean UTS or UCS value (solid lines) and corresponding experimental scatter band (dashed lines). As mentioned above, UTS was found to be equal to 53.95 MPa and UCS to -165.02 MPa. Although tensile strength degradation was again considerable, all measured values of compressive residual strength fell in the static UCS scatter band, thus exhibiting 'sudden death' behaviour.

Table 13.5 Number of [90]₇ coupons for each loading configuration

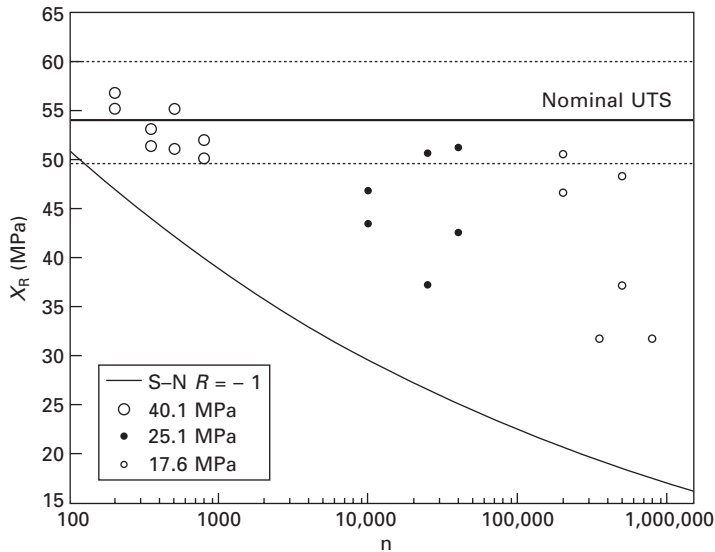
<i>R</i>	<i>S</i> _{max} (MPa)	<i>N</i> (cycles)	<i>n</i> / <i>N</i> (tensile RS)				<i>n</i> / <i>N</i> (compressive RS)	
			20%	35%	50%	80%	20%	50%
0.1	41.6	5000	2		2	1		
	31.9	50,000	2		3	1		
	22.5	1,000,000	2		2	2		
-1	40.1	1000	2	2	2	2		
	35.7	5000					2	2
	25.1	50,000	2		2	2	2	2
	17.6	1,000,000	2	1	2	1	2	1
10	138.6	5000			1			
	126.1	50,000	2		2	2		
	119.1	200,000	2		2	2		
	111.5	1,000,000	2		2	2		



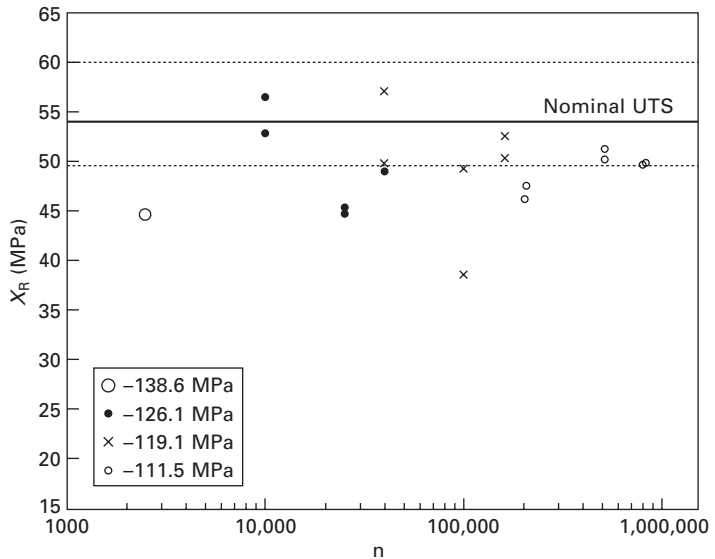
13.9 Tensile residual strength test results and S-N curve at *R* = 0.1, [90]₇ specimen.

13.6 Acoustic emission (AE) schemes

A PAC two-channel MISTRAS 2001 custom board (10–2000 kHz) was used to extract and record conventional AE descriptors from the transient stress waves. One broadband, 200–750 kHz, miniature PAC transducer (5.08 mm in diameter), model ‘Pico’, placed in the middle of the gauge length, was mounted on the specimens using a metal jig (see Fig. 13.13) and grease as coupling agent. The pre-amplifier was PAC model 1220A, with a 40-dB gain

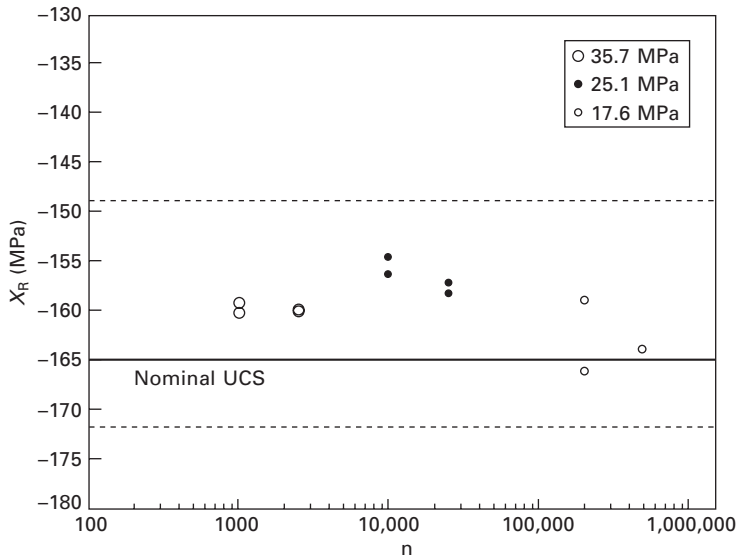


13.10 Tensile residual strength test results and S-N curve at $R = -1$, $[90]_7$ specimen.

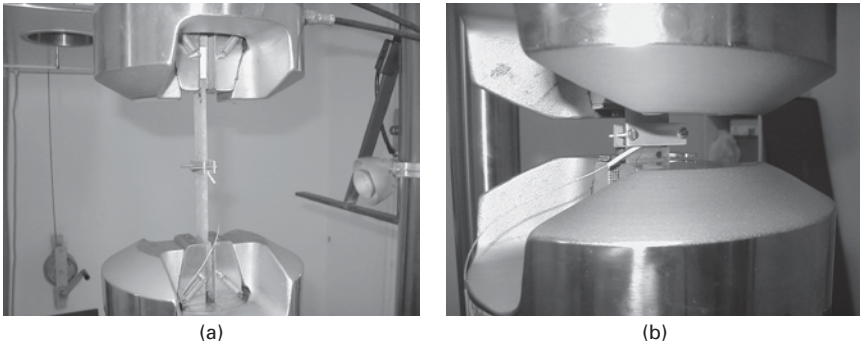


13.11 Tensile residual strength test results after CA $R = 10$ fatigue loading, $[90]_7$ specimen.

and no filters. Residual strength was recorded via both an HBM Spider8 device and the AE board. System timing parameters PDT (peak definition time), HDT (hit definition time) and HLT (hit lock-out time) were set to



13.12 Compressive residual strength test results after CA $R = -1$ fatigue loading, $[90]_7$ specimen.



13.13 Sensor mounted on specimens for AE monitoring: (a) $[\pm 45]_S$, (b) $[90]_7$ specimen.

50, 150 and 300 μ s respectively. The threshold was 40 dB, fixed, since the receiver was broadband and specimen dimensions were small.

Several lead breaks (Hsu pencil source excitation: see, e.g., ASTM E 976-94) ensured good sensor coupling. This procedure, conducted before each AE measurement, also ascertained that the performance of the transducers remained stable throughout the entire testing period. Indeed, variations in the sensor response due to possible damage or ageing were not observed.

Acoustic emission measurements proved sensitive to damage accumulation, although the proof-loading values used in this work were low enough to

prevent entering critical damage stages, e.g. the formation of delaminations in the case of $[\pm 45]_S$ specimens. Indeed, delaminations were not observed until failure. In addition, since there was no bridging of the 45° -oriented fibres from one specimen end to the other, fibre breakage was not expected. Acoustic emission was thus attributed to matrix cracking.

Matrix cracking was also the exclusive failure mode for the $[90]_7$ specimens, in all loading configurations. In the presence of other damage modes, however, the recorded AE signals should be assigned to particular mechanisms, as in, e.g., the work of Bhat *et al.* (1994), Anastassopoulos and Philippidis (1995) and Godin *et al.* (2004, 2005) prior to model implementation. Then, data corresponding to matrix cracking could be treated as proposed herein while other schemes should be developed for the rest.

13.6.1 $[\pm 45]_S$ specimens

Model development

As shown by Philippidis and Passipoularidis (2007), conventional non-linear residual strength models, e.g. the one introduced by Reifsnider and Stinchcomb (1986) and Schaff and Davidson (1997), proved capable of modelling the residual strength data, reported by Philippidis *et al.* (2005b). Residual strength, X_R , could thus be modelled using a generic equation of the form:

$$X_R = X - (X - S_{\max}) \left(\frac{n}{N} \right)^k \quad 13.5$$

In Eq. 13.5, n is again the number of cycles to which the specimen is subjected at maximum stress $S_{\max}$ and N the number of cycles to failure, corresponding to $S_{\max}$. Parameter k can either assume constant values (Reifsnider and Stinchcomb, 1986; Schaff and Davidson, 1997) or be a function of loading variables such as the stress level or life fraction (Philippidis and Passipoularidis, 2007).

The particular residual strength model was herein adapted to accommodate AE data from the main 87-specimen CA dataset. The modified model was:

$$X_R = X - (X - S_{\max}) \left(\frac{n_{\text{AE}}}{N_{\text{AE}}} \right)^m \quad 13.6$$

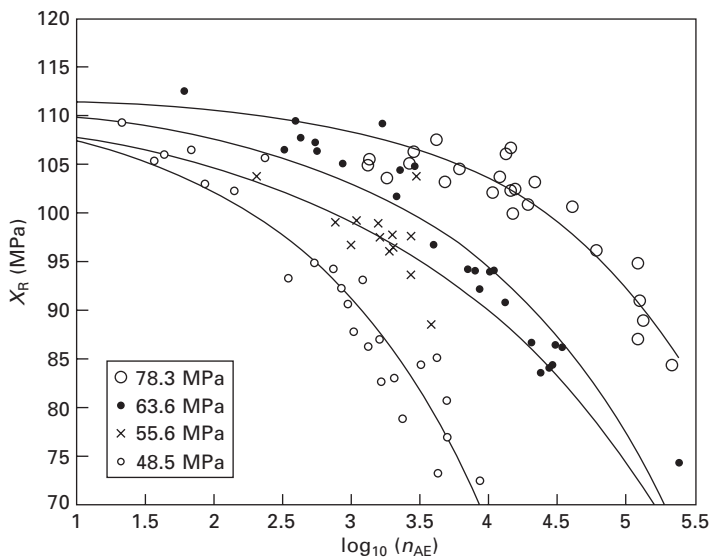
In Eq. 13.6, cumulative CNT recorded up to a particular stress value (proof-load) is denoted as n_{AE} . The unknown parameters of the model, henceforth denoted as ‘model RS_{mod}’, were N_{AE} and m .

To define the n_{AE} parameter, an appropriate proof-loading magnitude was required. According to common practice in pressure vessels as well as previous works performed on acoustic emission proof-testing of wind turbine rotor blades, an empirical stress value of 110% of S_{max} was selected. Thus, although specimens were tested in tension until failure and AE monitoring was continuous, n_{AE} was extracted from signals recorded up to a proof-load 10% higher than S_{max} . Using this proof load, n_{AE} was defined as the cumulative AE counts up to 110% of S_{max} . Since the particular proof-loading value could produce a considerable amount of acoustic emission without causing severe damage to the specimens, a fair and convenient descriptor was produced.

Correlation of the specific AE descriptor to residual strength is shown in Fig. 13.14. Unknown parameters N_{AE} and m were calculated via the commercial MATLAB[®] function *lsqcurvefit*, used to solve non-linear data-fitting problems in the least-squares sense. Appropriate initial values N_0 and m_0 were required to ensure convergence of the algorithm, for each specific stress level. Herein, N_0 was taken to be equal to the average n_{AE} value from all specimens of that particular stress level, while $m_0 = 1$. Calculated values for N_{AE} and m , for each stress level, are given in Table 13.6.

For completeness purposes, it should be stated that the linear version of modified model RS_{mod} (i.e. for constant $k = m = 1$), proposed by Broutman and Sahu (1972), exhibited bad performance.

It becomes clear that CNT is a function of both fatigue cycles, n , and



13.14 Tensile residual strength vs. $\log_{10}(n_{AE})$, [± 45]_S specimen, reference material. Unknown parameters: N_{AE} , m .

Table 13.6 Calculated values for RS_{mod} model parameters N_{AE} and m , for each stress level

S_{max} (MPa)	N_{AE}	m
78.3	443,250	0.3579
63.6	304,290	0.2951
55.6	562,210	0.2320
48.5	30,561	0.3246

S_{max} . Assuming that CNT is correlated with the general term ‘damage’, then damage could be quantified (n) and qualified (S_{max}). For example, two specimens presenting the same residual strength would produce different AE if not fatigued at the same stress level: each stress level follows a distinct trend. Life fraction does not seem to influence the slope of the trend of the data points, but rather their position on the alleged curve. Thus, besides being correlated with AE descriptors (e.g. CNT), fatigue damage should also relate to S_{max} . This is also stressed by Kim and Zhang (2001), where the evolution of matrix crack density is shown to depend on S_{max} . In accordance with Eq. 13.6, damage D could be expressed as:

$$D = \frac{X - S_{max}}{X} \left(\frac{n_{AE}}{N_{AE}} \right)^n \quad 13.7$$

In Fig. 13.15, residual strength is plotted vs. calculated damage, defined through Eq. 13.7. AE data from specimens fatigued at various stress levels seem to converge into the same scatter band.

However, the master curve derived from model RS_{mod} cannot be generalized for a new stress level, unless enough data is available to estimate parameters m and N_{AE} corresponding to the new S_{max} . A practical solution to the problem of correlating residual strength to acoustic emission descriptors is not therefore provided.

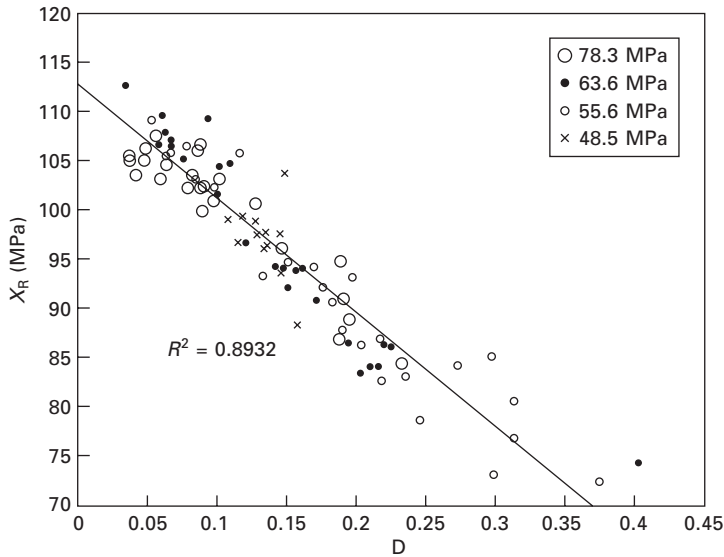
As observed in Fig. 13.14, each stress level follows a distinct trend. Thus, the slope of model RS_{mod} could be a function of S_{max} . In a logarithmic scale, this slope is equal to the m parameter. With m given through

$$m = \frac{S_{max}}{X} \quad 13.8$$

An interesting new damage parameter was obtained, henceforth denoted as ‘descriptor AE’:

$$AE_1 = \log_{10}(n^m) \quad 13.9$$

Parameter m is given through Eq. 13.8. The resulting correlation of descriptor



13.15 Tensile residual strength vs. calculated damage, $[\pm 45]_S$ specimens, reference material (model RS_{mod}).

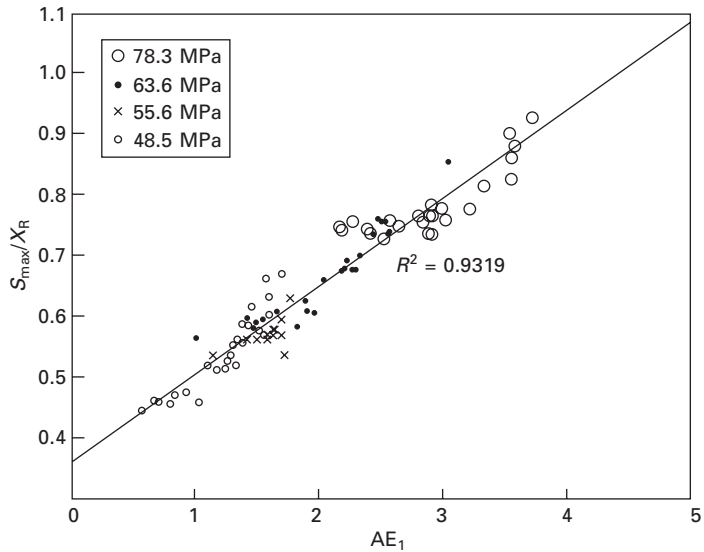
AE_1 to the ratio of S_{max} over residual strength is presented in Fig. 13.16. A linear model, henceforth denoted as ‘model M'_1 ’, was thus introduced:

$$\frac{S_{max}}{X_R} = 0.14AE_1 + 0.36 \quad 13.10$$

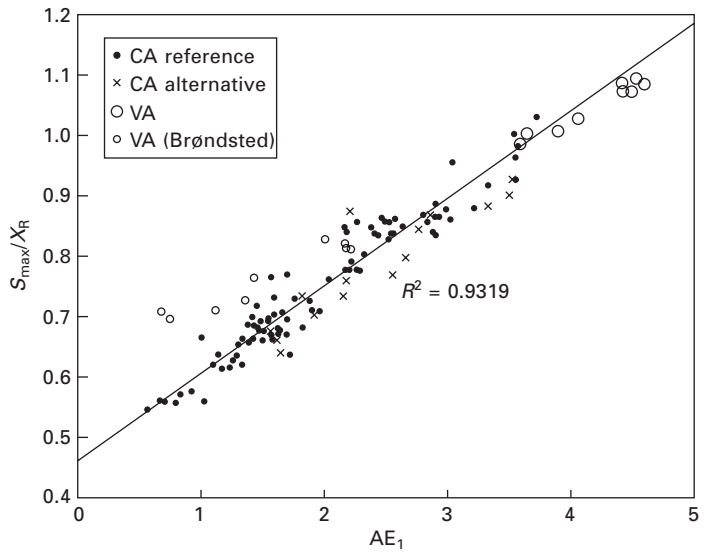
Since numerous structures are subjected to a fixed constant–amplitude loading, *a priori* information on fatigue S_{max} , combined with the conventional CNT descriptor, could therefore lead to reliable residual strength prediction.

To investigate whether the master curve of Fig. 13.16 could be considered universal, the same procedure was also implemented on the VA and alternative-material datasets. For the alternative-material data, the respective UTS value was used.

Two methods were used to derive the appropriate cumulative CNT amounts, in the VA data case. One engaged 110% of an equivalent S_{max} , calculated as proposed by Brønsted *et al.* (1997), and the other the maximum stress of the load spectrum. The reason why S_{max} was used in the second case, instead of 110% of S_{max} , was that 110% of S_{max} was considered excessive for a proof-loading purpose. Perhaps due to the inherent weakness in the assessment of a representative equivalent S_{max} , performance of the former method was mediocre. On the other hand, and being closer to the concept of proof-loading, the second method performed well. Residual strength of the alternative-material specimens was also predicted with considerable precision. Results are illustrated in Fig. 13.17.



13.16 $S_{\max}$ over residual strength vs. descriptor AE_1 for CA tests, $[\pm 45]_S$ specimen, reference material.



13.17 $S_{\max}$ over residual strength vs. descriptor AE_1 for CA tests, $[\pm 45]_S$ specimen.

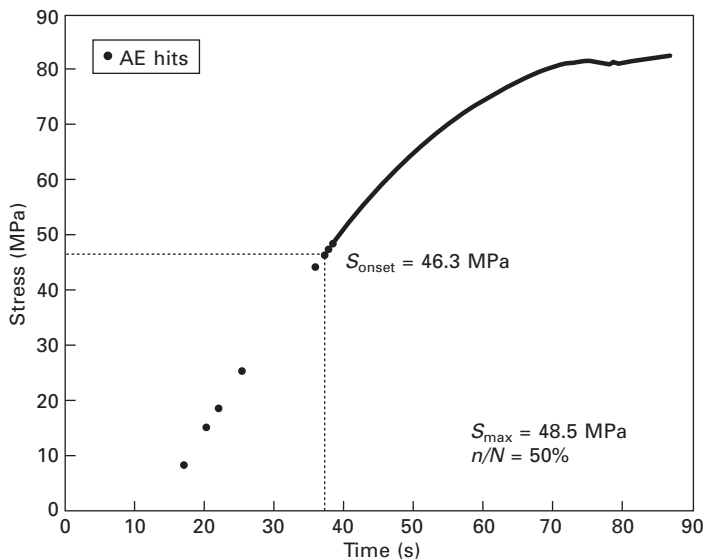
The $S_{\max}$ value was required in order to implement model M_1 on the AE data. Remarkable residual strength predictions were then obtained. Another option, although providing somewhat less precise residual strength

predictions, was through exclusive use of the acoustic emission data, with no information on preceding fatigue loading being required. In this stand-alone method, the load at which AE was released, henceforth referred to as S_{onset} , was used instead of S_{max} . As a convention, S_{onset} was defined as the stress above which at least 10 consecutive hits were emitted at smaller than 2-MPa intervals. In practice, this approaches the actual acoustic emission onset. A characteristic example from a specimen subjected to $S_{\text{max}} = 48.5$ MPa, $n/N = 50\%$, is illustrated in Fig. 13.18. S_{onset} depends on both S_{max} , see Fig. 13.19, and fatigue life fraction n/N , as shown in Fig. 13.20.

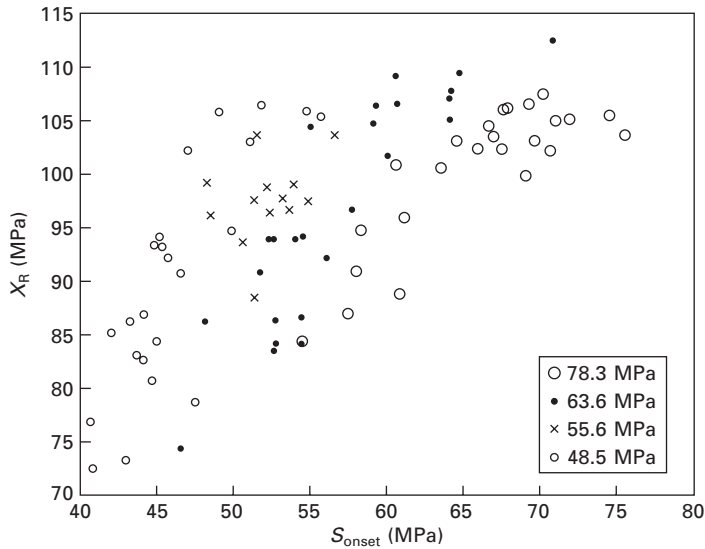
With S_{onset} being smaller than S_{max} in general, the felicity ratio (F_R) could be used to describe experimental AE behaviour, as proposed by, e.g., Gorman (1991). F_R is a macroscopic parameter indicating damage in composites, defined in this case as the ratio of S_{onset} over S_{max} . F_R values are in the 0–1 range, ‘0’ representing a state of absolute damage and ‘1’ the undamaged condition. Although use of F_R is common, e.g. as an empirical accept/reject criterion for filament-wound FRP pressure vessels, a more appropriate scheme was proposed herein. The relevant damage parameter, AE_2 , was:

$$AE_2 = \log_{10}(n_{\text{onset}}^{m_{\text{onset}}}) \quad 13.11$$

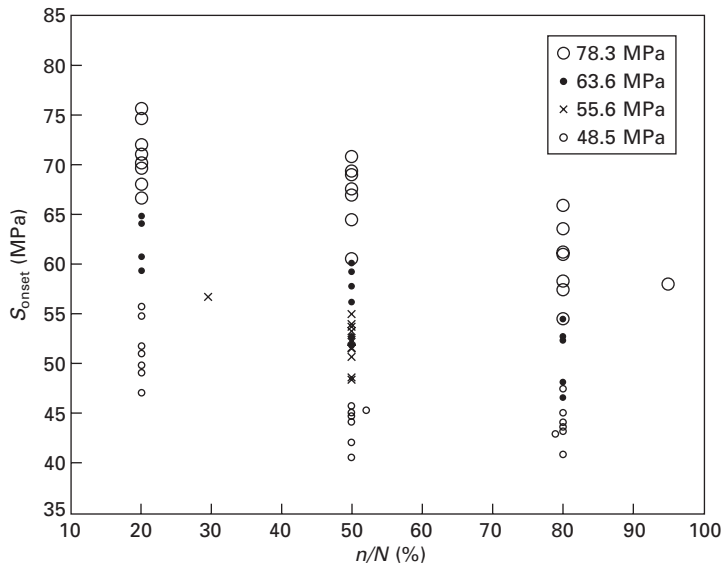
In Eq. 13.11, S_{onset} is used to substitute for S_{max} in the calculation of the m parameter of Eq. 13.8, resulting in the behaviour of Fig. 13.21. Cumulative



13.18 Acoustic emission from a characteristic specimen, showing definition of S_{onset} .

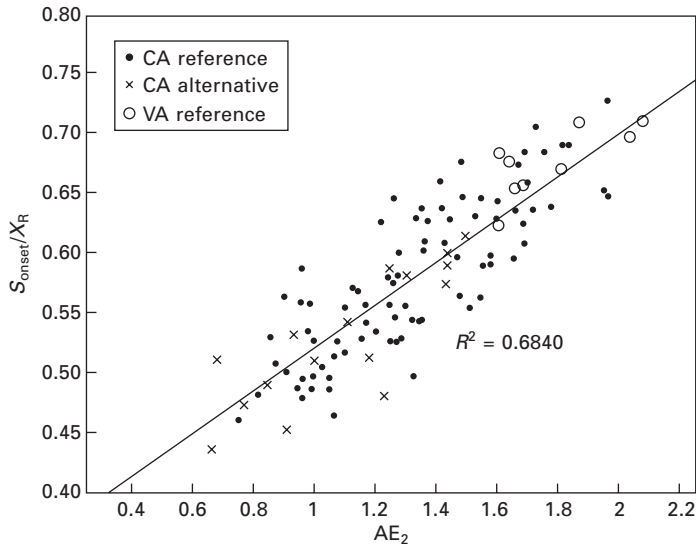


13.19 Residual strength vs. onset of acoustic emission.



13.20 Onset of acoustic emission vs. fatigue life fraction.

CNT recorded up to 110% of the S_{onset} was used, named n_{onset} . Selection of a proof-loading magnitude of 110% of S_{onset} was made according to the practice followed in the previous models and since S_{onset} was close to S_{max} (see Fig. 13.18). In Fig. 13.21, VA and alternative-material data is also illustrated.



13.21 S_{onset} over residual strength vs. descriptor AE_2 , [± 45]_S specimen.

Table 13.7 Correlation of proof-loading magnitude with model scatter

Model	Proof-load	R^2 (all coupons)	R^2 (excluding failures below 120% of S_{max})
M_1	S_{max}	0.8421	0.8004
	110% of S_{max}	0.9319	0.9132
	120% of S_{max}	0.9241	0.9234
M_2	S_{onset}	0.4658	
	110% of S_{onset}	0.6840	
	120% of S_{onset}	0.7057	

Note: five specimens in the M_1 case failed below 120% of S_{max} .

Due to an error in data acquisition, two out of the 87 AE recordings were not available. Thus, experimental results presented in Fig. 13.21 are from 85 specimens. The master curve, shown as a solid line, is again a linear regression model, named ‘model M_2 ’:

$$\frac{S_{\text{onset}}}{X_R} = 0.18AE_2 + 0.34 \quad 13.12$$

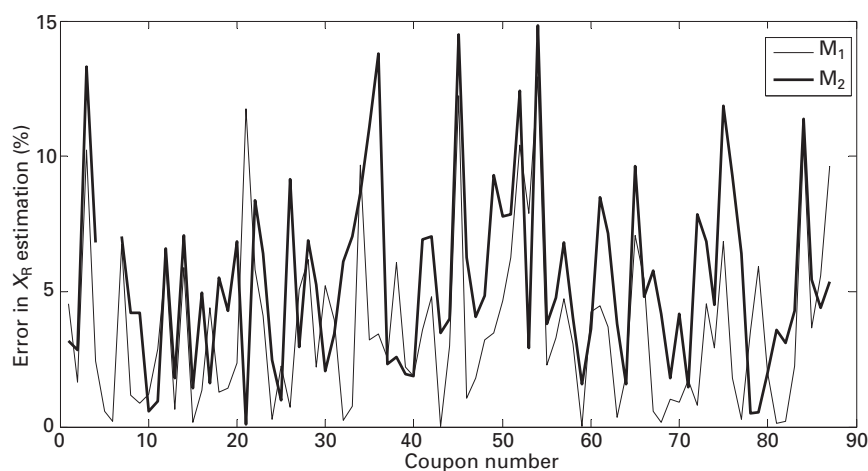
As in the case of model M_1 , the scheme becomes more reliable with increase of the proof-loading value: see Table 13.7. However, high proof-loads lead to an increasing number of specimen failures, as, e.g., for 120% of S_{max} . Indeed, although no rupture was observed during proof-loading at S_{max} , there

were five specimens failing below 120% of $S_{\max}$. Model performance for 120% of $S_{\max}$, estimated using all specimens, thus seemed inferior to that corresponding to the 110% of $S_{\max}$ proof-loading. To unmask the proof-loading value effect, the particular five specimens were excluded from all levels, i.e. $S_{\max}$, 110% of $S_{\max}$, and 120% of $S_{\max}$. The trend is presented in Table 13.7.

A compromise between model performance and preservation of the non-destructive nature of the method is nonetheless required. Using 110% of $S_{\max}$ in model M_1 was indeed a safe practice. However, a proof-load of 110% of S_{onset} implies an important advantage: the material self-dictates an appropriate magnitude for safe proof-loading, i.e. S_{onset} . Thus, information on $S_{\max}$ is no longer needed. In addition, model M_2 remains insensitive to previous load history and is therefore expected to perform well on specimens subjected to stochastic loading sequences.

Model validation

To validate models M_1 and M_2 for the set of 87 CA residual strength tests on the $[\pm 45]_S$ reference-material specimens (85 for M_2 , due to an error in AE data acquisition), modelling the experimental data with a regression line was repeated $k = 87$ (or 85) times, each time using $(k - 1)$ points to generate the model (the ‘leave-one-out’ method). Performance was interrogated with residual strength prediction of the remaining k th specimen, not used in model implementation. Error in residual strength estimation is given in Fig. 13.22.



13.22 Error in residual strength prediction of $[\pm 45]_S$ reference-material CA coupons, using AE models M_1 and M_2 ('leave-one-out' method).

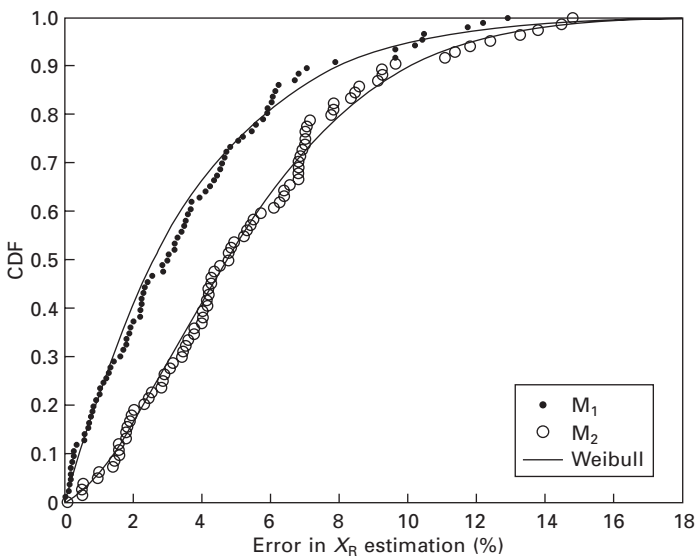
For model M_1 , the average error for the set of 87 specimens is listed in Table 13.8. The corresponding cumulative probability distribution (CDF) of the error in residual strength prediction is described through a biparametric Weibull distribution: see Fig. 13.23. It is seen that the bulk of the data points corresponds to a prediction error of less than 6.24%, while it can be estimated that at a reliability level of 95% the error does not exceed 10.20%.

In the case of model M_2 , the CDF of the error in residual strength prediction was again described by a Weibull distribution, see Fig. 13.23. The bulk of the data points corresponds to a prediction error of less than 7.15%. At a 95% reliability level, the error did not exceed 11.80%.

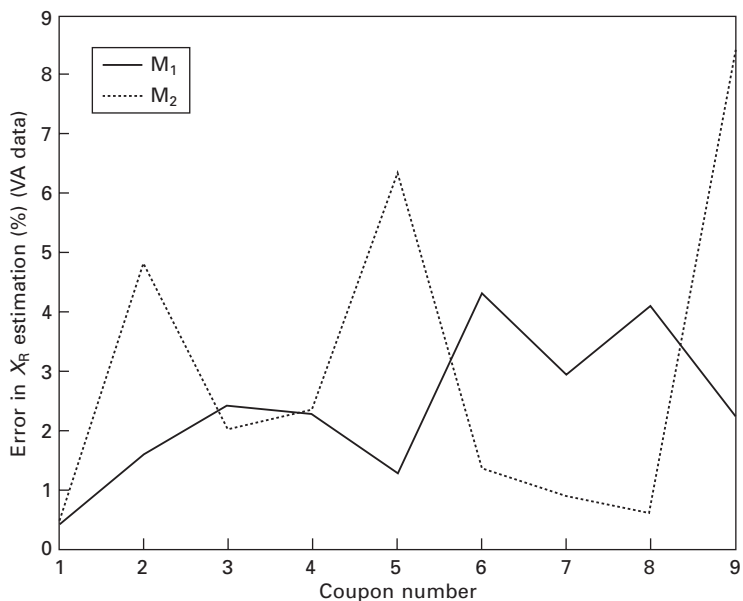
As stated above, the residual strength of the VA and the alternative-material dataset was also predicted. Model predictions are presented in Fig. 13.17 for model M_1 and in Fig. 13.21 for M_2 . Error in residual strength prediction is demonstrated in Fig. 13.24 for VA data and in Fig. 13.25 for the alternative

Table 13.8 Performance of acoustic emission models, reference material ('leave-one-out' method)

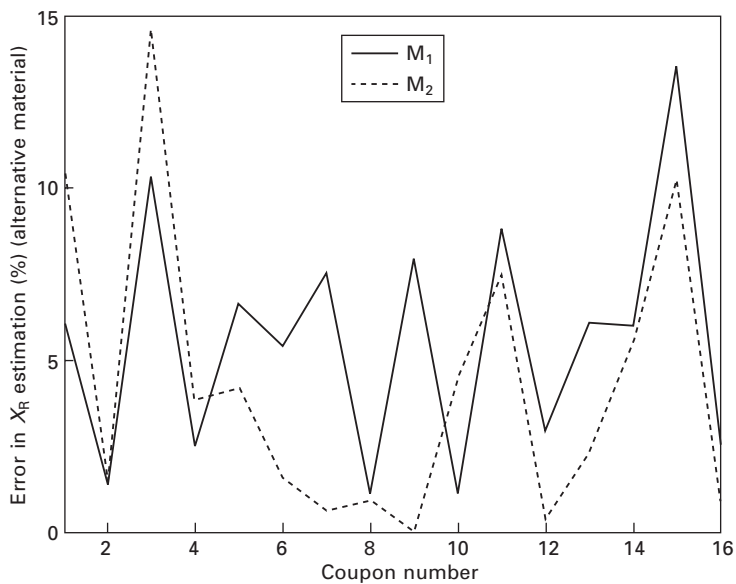
Model	Error _{mean} (%)	Error _{min} – Error _{max} (%)	Weibull distribution parameters	
			Shape	Scale
M_1	3.62	0.01–12.93	1.0734	3.7076
M_2	5.38	0.09–14.82	1.5964	5.9885



13.23 Probability distribution of error in residual strength prediction using models M_1 and M_2 , $[\pm 45]_S$ reference-material specimens.



13.24 Error in residual strength prediction of VA tests, using acoustic emission models M_1 and M_2 .



13.25 Error in residual strength prediction of alternative-material CA tests, using acoustic emission models M_1 and M_2 .

material. Some error statistics are given in Table 13.9. Predictions are excellent.

Pre-processing of AE data

All AE data presented herein was pre-processed before use, to eliminate outliers and false recordings, perhaps due to electromagnetic interference (EMI) (Philippidis *et al.*, 1999). Records with average frequency (AvF) greater than 500 kHz, zero signal energy, counts-to-peak greater than counts and rise time greater than duration were rejected. Major reduction of more than 50% of the available acoustic emission data was thus performed before processing, although a negligible amount of records with $CNT < CNP$ were encountered and there were no hits with $DUR < RT$. Peculiarly enough, this procedure had a minor influence on model performance: see Fig. 13.26.

This observation can be explained while considering the nature of the rejected data. Some indicative statistics (mean descriptor values) for a 'strong' and a 'weak' specimen, up to 110% of S_{max} , are presented in Table 13.10. Indeed, rejected data originated from quite weak signals that were not expected to contribute to damage assessment. As seen in Table 13.10, the average CNT number of the rejected signals is much lower than the respective CNT of the accepted ones. Thus, the cumulative number of AE counts, used in model implementation, is barely influenced.

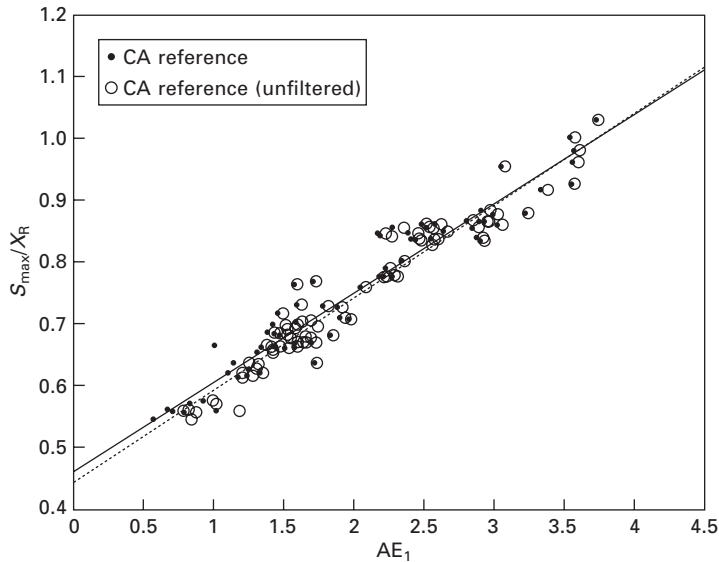
13.6.2 Model implementation on $[90]_7$ specimens

As expressed above, implementation of the proposed AE models for residual strength assessment after fatigue is restricted to cases where matrix cracking is the dominant damage mode. To validate this statement, the developed M_1 and M_2 methodologies were applied to the $[90]_7$ specimens, that were also expected to fail due to matrix cracking. Again, an extensive experimental series was conducted in order to characterize the particular specimen configuration.

The corresponding residual strength tests were performed at three stress ratios, $R = 0.1$, $R = -1$ and $R = 10$. Specimens pre-fatigued at $R = 0.1$ and

Table 13.9 Performance of AE models on VA and alternative-material validation sets, $[\pm 45]_S$ specimen

Model	Validation set	Error _{mean} (%)	Error _{min} – Error _{max} (%)
M_1	VA	2.39	0.41–4.31
M_2		3.04	0.45–8.46
M_1	Alternative	5.62	1.09–13.54
M_2		4.31	0.01–14.61



13.26 $S_{\max}$ over residual strength vs. descriptor AE_1 for CA tests, $[\pm 45]_S$ specimen, reference material: standard model M_1 implementation vs. implementation using unfiltered data.

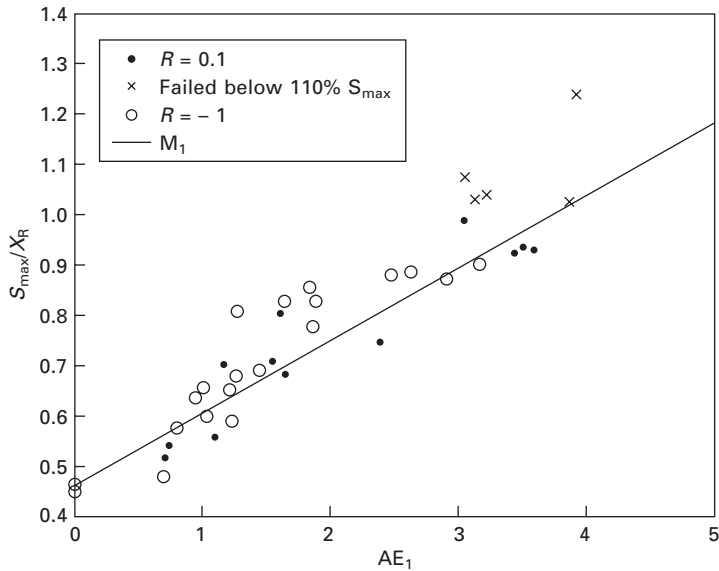
Table 13.10 Indicative statistics (mean values) of the accepted and the rejected (filtered-out) AE data

		RT	CNP	CNT	EN	DUR	AMP
Strong	Rejected	4.7	1.3	1.9	0.0	12.5	43.2
	Weak	4.1	1.7	4.4	0.0	18.6	46.9
Strong	Accepted	13.5	3.8	19.6	2.9	132.1	60.8
	Weak	8.9	4.0	31.4	8.3	269.8	65.5

$R = 10$ were tested for tensile residual strength, whereas, for the specimens subjected to $R = -1$ loading, both tensile and compressive tests to failure were performed.

Implementation of model M_1 (Eq. 13.10) on the $[90]_7$ $R = 0.1$ and $R = -1$ AE data, both failing in tension, is presented in Fig. 13.27. The UTS value of the $[90]_7$ specimen was used in descriptor AE_1 estimation. Proof-loading values, albeit 10% higher than $S_{\max}$, were in most cases quite lower than the residual strength. In some $R = 0.1$ cases, however, specimens failed below the proof-loading value (see $\times$ -symbols in Fig. 13.27).

There were several tests where almost no acoustic emission was recorded during proof-loading. Indeed, for $R = 0.1$ specimens, cumulative AE counts ranged from 51 to 148,208, while the respective range for $R = -1$ was 0 to 17,913. Although this could be attributed to data acquisition parameters,



13.27 $S_{\max}$ over tensile residual strength vs. descriptor AE_1 , $[90]_7$ specimens at $R = 0.1$ and $R = -1$.

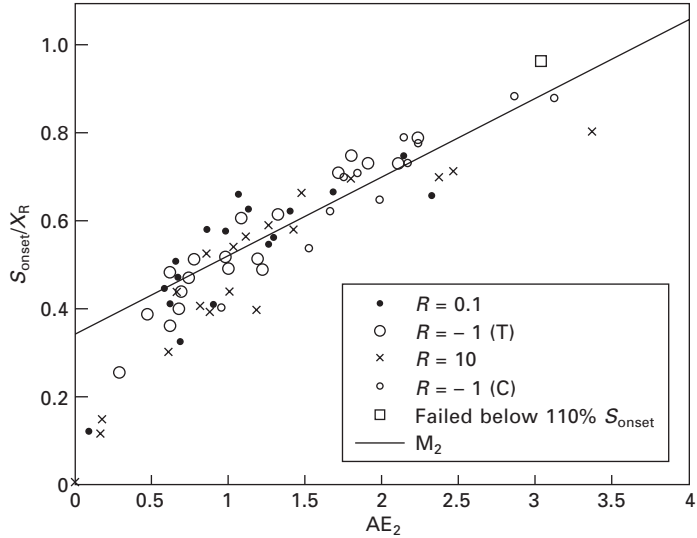
e.g. threshold, limited acoustic emission is expected in cases where a low proof-loading value is used and/or specimens retain most of their strength.

Although M_1 performs well in the $R = 0.1$ and $R = -1$ tensile residual strength case, there are certain considerations/limitations to be discussed. In M_1 implementation, knowledge of $S_{\max}$ is required. Therefore, in pure tensile loading, e.g. $R = 0.1$, a proof-loading of 110% of the $S_{\max}$ value would be used for tensile residual strength prediction. Likewise, for $R = 10$, a negative proof-load of 110% of $S_{\min}$ would serve for compressive residual strength assessment. It is obvious, however, that meaningful proof-loading values for compressive residual strength prediction after tensile fatigue or for tensile strength prediction after compressive fatigue cannot be defined. Model M_1 could thus be used either for tensile or for compressive residual strength assessment, depending on the sign of preceding fatigue loading. In an ideal case of $R = -1$, provided that $UTS = IUCSI$, M_1 could perhaps be implemented for both directions at the same time. However, even for $R = -1$, when e.g. UCS is greater than UTS, as for the $[90]_7$ specimen, and with the proof-loading value being 110% of $S_{\max}$ for tension and -110% of $S_{\max}$ for compression, compressive residual strength prediction might not be feasible. Indeed, for $[90]_7$ specimens, no AE events were recorded at compressive proof loads of -110% of $S_{\max}$.

Model M_2 , on the other hand, uses an alternative stress value, S_{onset} , to substitute for $S_{\max}$. Stress S_{onset} is defined as the stress above which at

least 10 consecutive AE hits are recorded at smaller than 2-MPa intervals. Using M_2 , information on $S_{\max}$ is no longer needed and the model becomes insensitive to previous loading histories. This is illustrated in Fig. 13.28. The performance of model M_2 (Eq. 13.12) for each stress ratio, for the $[90]_7$ specimen, is presented in Table 13.11. Therefore, by performing a tensile and then a compressive proof-loading on the same specimen, both tensile and compressive residual strength could be predicted.

In addition, using M_2 requires a proof-load of 110% of S_{onset} , which in general is lower than 110% of $S_{\max}$. Thus, in Fig. 13.28 there is one specimen failing below 110% of S_{onset} instead of five in the M_1 case (Fig. 13.27). Note that while M_1 failures were in the $R = 0.1$ stress ratio, the M_2 failure corresponded to a specimen pre-fatigued under $R = -1$.



13.28 S_{onset} over residual strength vs. descriptor AE_2 , $[90]_7$ specimens at $R = 0.1$, $R = -1$ and $R = 10$, tensile and compressive tests to failure.

Table 13.11 Model performance in tensile or compressive residual strength estimation, $[90]_7$ specimen

R	RS test	Model M_1		Model M_2	
		Error _{mean} (%)	Error _{min} –Error _{max} (%)	Error _{mean} (%)	Error _{min} –Error _{max} (%)
0.1	Tensile	8.31	2.21–18.87	10.51	0.48–30.39
–1		8.85	0.61–29.82	8.10	0.19–20.36
10				11.74	2.48–33.15
–1	Compressive			6.89	0.48–21.61

There are five data points diverging from the general trend of model M_2 (see lower left corner of Fig. 13.28). Acoustic emission in these cases began at low loads, i.e. below 10 MPa. Residual strength of these specimens cannot be well-predicted using M_2 , though the model provides a safe estimate of the actual residual strength.

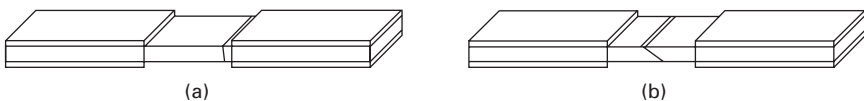
13.7 Failure modes: discussion

As shown in Fig. 13.27 and discussed by Philippidis and Assimakopoulou (2008a, 2008b), tensile residual strength data for $R = 0.1$ and $R = -1$ stress ratios, all stress levels and life fractions, can be described using model M_1 , for both the $[\pm 45]_S$ and the $[90]_7$ specimens. Two materials were examined, both performing well under the same regime. In addition to CA loading, $[\pm 45]_S$ specimens subjected to a variable-amplitude spectrum were also investigated.

All these cases bear the same failure mode, classed by Puck and Schürmann (1998) as ‘mode A’ and referring to matrix cracks due to shear and/or transverse tensile stresses. The discriminating characteristic of mode A is the crack plane being perpendicular to the plies and running parallel to the fibre direction. This work indicates that although M_1 was established on $[\pm 45]_S$ specimens, failing mostly due to shear stresses, it could also be applied on the $[90]_7$ specimens, failing due to tension transverse to the fibres. As mentioned earlier, both cases are characterized as failure mode A.

On the other hand, all mode A cases, including tension after $R = 10$ fatigue, complied with a single model, M_2 : see Fig. 13.28. In addition, compressive residual strength after $R = -1$ loading could also be predicted using M_2 although the corresponding damage mode, distinguished as ‘mode C’ by Puck and Schürmann (1998), is a matrix failure mode due to shear and perhaps also transverse compressive stresses. The difference between failure modes A and C is depicted in Fig. 13.29. The mode C crack plane, still parallel to the fibres, forms an oblique angle with the plane running normal to the plies.

Therefore, model M_2 is applicable independently of the failure mode and loading conditions, such as stress ratio, R . An obvious explanation for the former is that AE behaviour from matrix failure in general, including, e.g., transverse cracking and fibre–matrix debonding, is similar for impeding A



13.29 Characteristic failure modes for the $[90]_7$ specimen: (a) mode A (tensile), (b) mode C (compressive).

and C failure modes: note that AE data used in model implementation are acquired well before specimen rupture. Nevertheless, although the universal trend is supported, more research on this particular issue is required.

13.8 Conclusions

An extensive experimental program was performed to investigate the strength degradation of a $[\pm 45]_S$ and a $[90]_7$ GI/Ep composite, due to constant- or variable-amplitude fatigue. Strength degradation, a function of stress ratio, maximum stress and life fraction spent under cyclic loading, was matrix-dominated. Correlation of tensile and compressive strength degradation with characteristic parameters, measured via acoustic emission, was accomplished.

In the case of the $[\pm 45]_S$ specimen, all loading was tensile. Specimens were subjected to CA $R = 0.1$ or tensile VA (spectrum) loading, resulting in considerable tensile strength degradation of up to 40%. Specimens made of an alternative material were also tested. $[90]_7$ specimens, on the other hand, were subjected to constant-amplitude loading of either $R = 0.1$, $R = -1$ or $R = 10$ stress ratio. Static tests to failure indicated tensile strength degradation of up to 41%, although there was no considerable degradation in compression. The drop in tensile strength, in particular, was more intense under $R = 0.1$ and $R = -1$, while being almost non-existent for $R = 10$.

Simple and reliable engineering models, suitable for design considerations, provided excellent residual strength predictions. Two of the proposed AE descriptors, AE_1 and AE_2 , prevailed in residual strength assessment of composite specimens, featuring fatigue damage. The corresponding models, M_1 and M_2 , were validated with remarkable success. Model M_1 is more accurate, though it requires a known fatigue maximum load. Using this information, a master curve valid for all stress levels and life fractions is obtained. M_1 can thus be implemented in applications where cyclic loading is either fixed or recorded on-line. Model M_2 , on the other hand, presents adequate performance with no implementation requirements. In both cases, the proof-loading is low enough to prevent further material damage, e.g. delaminations.

The proposed models were established on 87 CA fatigue tests on the $[\pm 45]_S$ reference material. Two independent validation sets were also used to test the schemes: one contained data from $[\pm 45]_S$ reference-material VA (spectrum) tests and the other from $[\pm 45]_S$ specimens of an alternative resin matrix, subjected to $R = 0.1$ CA loading. Again, performance of both models was remarkable.

The schemes also provided excellent residual strength predictions for the $[90]_7$ stacking sequence, for $R = 0.1$, $R = -1$ and $R = 10$ CA loading, in both tension and compression. Although $[90]_7$ specimens present disparate

failure behaviour in tension and compression, distinguished as mode A and mode C respectively, all cases were well described using the same model.

It could therefore be presumed that the method is valid for all similar materials, stress ratios, life fractions, stress levels, constant- or variable-amplitude loading, and tensile or compressive residual strength test. Provided that prior application to an adequate number of samples is performed, to establish a robust database, the introduced methodologies are expected to perform well in residual strength estimation of new members of the population. However, further research on more materials, specimens and loading configurations would still be interesting, as well as an evaluation of the effectiveness of the model using alternative equipment and software settings.

This work correlated strength degradation with characteristic parameters measured via acoustic emission, a method suited to health monitoring of operating structures. However, being validated up to now only on small test specimens, the method is still in need of development. For example, since damage in a generalized laminate is not matrix-dominated, as in the $[\pm 45]_S$ and $[90]_7$ cases, it is imperative to consider all possible failure mechanisms as well.

For structures where failure is fibre-dominated, enhancement accounting for all expected damage mechanisms, e.g. fibre breakage and delaminations, is required. By assigning the recorded data to particular damage modes, e.g. using a pattern recognition algorithm, perhaps respective models could be introduced. Again, a dedicated database should be established for each particular application.

In large structures, however, signal attenuation should be considered. Furthermore, in waveguides such as plate and shell structures, e.g. wind turbine rotor blades, geometrical dispersion should also be accounted for. To compensate for the alteration of wave propagation characteristics due to such phenomena, all emitted signals should be recorded and studied. Then, developed schemes should be re-evaluated through full-scale testing.

An alternative approach to overcome scale issues is through, e.g., a zonal location technique: using a more refined sensor mesh on areas more susceptible to damage and with the neighbourhood of each sensor being of similar size as the test specimens, localized condition assessment could indeed be accomplished. The shear webs of the spar beam and the trailing edge sandwich panels of a wind turbine rotor blade are such locations. Most interestingly, those particular areas lack axial fibre-reinforcement, rendering the proposed model directly applicable.

The technique could be further developed so that periodic measurements of appropriate AE descriptors during service, e.g. using embedded sensors, provide information on the damage state of the component. The goal is to establish smart maintenance schedules to replace conservative prevention-

aimed repair routines. Monitoring of wind turbine rotor blades, using their own weight as loading, is a promising possible application.

13.9 Acknowledgements

This work was conducted in the frame of EC-funded research project 'OPTIMAT BLADES: Reliable Optimal Use of Materials for Wind Turbine Rotor Blades', ENK6-CT-2001-00552. Partial funding was provided by the Greek Secretariat for Research and Technology, F.K. 6660. Besides information recorded during acoustic emission, all data is free for download from the official OPTIMAT BLADES site (<http://www.wmc.eu/optimatblades.php>) along with the relevant reports and publications.

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Fatigue life prediction of wind turbine rotor blades manufactured from composites

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Abstract: Blades of modern wind turbines, which are commonly fabricated of composites, play an important role in capturing the kinematic energy of wind flow. The random nature of wind flow renders these blades as fatigue critical elements. They are exposed to different loads during their lifetime and they are expected to sustain their mission for at least 20 years. Therefore, reliable fatigue damage progress models need to be developed. Due to the random nature of the applied loading patterns, investigation of fatigue damage progress using stochastic simulation is inevitable. Furthermore, these models should be capable of considering various material properties. Prior to the establishment of the proper fatigue models, accurate characterization of different load cases is a vital task.

Key words: wind turbine blade, fatigue, stochastic analysis, composites.

14.1 Introduction

The blade is the most vulnerable structural component in a wind turbine, which nowadays is designed carefully to capture the maximum energy from the wind flow. Thanks to utilization of composite materials in blades of wind turbines, complex design constraints such as lower weight and proper stiffness are satisfied, while providing good resistance to the static and fatigue loading (Harrison *et al.*, 2000). Generally, wind turbines are fatigue critical machines and the design of many of their components (especially blades) is dictated by fatigue considerations. Long and flexible structure, intrinsic randomness in wind flow and continuous operation under different conditions are some factors exposing wind turbine blades to the fatigue phenomena (Spera, 1994). A wind turbine blade is expected to sustain its mission for about 20 to 30 years.

Consequently, design constraints for wind turbine structures fall into either extreme load or fatigue categories. For the case of extreme load design, the load estimation problem is limited to finding a single maximum load level that the structure can tolerate. For design against fatigue, however, loads must be defined for all input conditions and then summed over the distribution of input conditions weighted by the relative frequency of occurrence (Manuel *et al.*, 2001).

Sutherland (1999) reviewed comprehensively the development in fatigue analysis of wind turbine components, focusing on US technology. Kensché (2006) has also reviewed different aspects of fatigue in composite materials for wind turbine blade application.

Ronold *et al.* (1999) presented a probabilistic model for analysis of the safety of a wind turbine blade against fatigue failure in flap-wise bending. The model is constructed using Miner's rule. The probability of fatigue failure in flap-wise bending of the blade is calculated by means of a first-order reliability method when a 20-year design lifetime is considered. The authors used the results of reliability analysis to calibrate partial safety factors for load and resistance for use in conventional deterministic fatigue design. A 500 kW wind turbine with 19.5 m long rotor blades was selected for a case study in that research. The work was later extended by Ronold and Christensen (2001) to cover a whole class of wind turbines, sites and materials.

Philippidis and Vassilopoulos (2004) developed a methodology to predict the lifetime of composite laminates subjected to plane stress states and spectrum loading. They employed both the WISPERX load spectrum (Ten Have, 1988a, b) and realistic loadings obtained from aeroelastic simulations. Their method consists of four stages: rainflow counting of the variable amplitude spectra, formulation of the constant life diagram, derivation of allowable numbers of cycles using the multiaxial fatigue failure criterion called FTFP (Failure Tensor Polynomial in Fatigue) which had been developed previously by the same authors (Philippidis and Vassilopoulos, 1999, 2002), and finally, damage accumulation using Miner's rule.

Kong *et al.* (2005, 2006) employed the S–N linear damage equation, load spectrum and Spera's empirical formula (Spera, 1994) to predict the lifetime of the blade of a 750 kW wind turbine.

Marín *et al.* (2009) studied damage detected in the blades of 300 kW wind turbines qualitatively and quantitatively. The qualitative investigation is performed using visual inspection on the damaged region to find out the cause of damage progress, while the quantitative analysis is performed using a simplified model that is proposed in the Germanischer Lloyd (GL) rules and regulations.

Typically, different stages of fatigue investigation on wind turbine blades involve the following steps:

1. Determination of wind flow using a Weibull function (Gasch and Tvele, 2002)
2. Calculation or measurement of load spectra for different working conditions and estimation of the developed stress fields
3. Derivation of S–N curves for the specific case of study
4. Employing the linear Miner's rule to estimate the life (Miner, 1945; Palmgren, 1924)

5. Employing appropriate multiaxial failure criteria in the case of multiaxial stress states (in that case the use of simple S–N curve equations is not appropriate and the fourth step is replaced with this step).

Load spectra characterizations can be performed either experimentally or analytically. Due to the random nature of wind flow, wind turbines experience random loadings, so analytical characterization of loadings is quite difficult and usually applicable for deterministic spectra (Veers, 1983; Rizzotis and Voutsinas, 2000). Typically, it is preferable to use a representative sample of the loads imposed on the turbine during each state and then weight them by their expected rate of occurrence. This representative sample is obtained through statistical processing of real measured data.

According to the IEC-61400 standard, digital sampling of loads on a wind turbine blade is accomplished using a 10-minute record database. Since these records are just representative of 10 minutes of operation, they should be expanded to the lifetime of the turbine by appropriate methods reflecting representative sampling for long-term data. A rainflow counting algorithm (Rice *et al.*, 1998; Downing and Socie, 1982; Sutherland and Schluter, 1990) is employed to analyze the time series data obtained from the representative sample. Two standardized load spectra entitled WISPER and WISPERX are also available (Ten Have, 1988a, b). They are often used to characterize the composite materials employed in the structure of wind turbine blades allowing variable amplitude testing. Unlike almost all available fatigue data obtained on the basis of constant amplitude experiments, these two standard spectra present a load spectrum with variable amplitude, in fact the load sequence repeatedly applied during variable amplitude testing. Researchers have compared the methods they have developed with these spectra. The original WISPER is based on measurements on nine wind turbines with rotor blades made of steel, GRP and wood and with diameters ranging from 11.7 m to 100 m. Today's wind turbines differ from those wind turbines of the mid-eighties in terms of considerably larger average capacities with typically three bladed rotors operating with full span pitch control, having much lighter blades. The OPTIMAT blades project was carried out to support the development of new optimal design materials and procedures for modern wind turbines. The general motivation of the project was to ensure that the modern wind turbine loadings are truly reflected in a new WISPER spectrum.

Most ongoing research on fatigue phenomena uses load spectra obtained by digital sampling of a specific configuration of strain gages which read the strain at a specific location near the root of the blade. Then, the representative sample of each load spectrum is weighted by its rate of occurrence that can be obtained from the statistical study of the wind pattern. Finally, all the weighted load spectra are summed and the total load spectrum is derived. Identification of a proper place to install the strain gages in order to extract the load spectrum is also questionable.

Using a massive and high-cost material fatigue database (MSU/DOE or FACT) to obtain S–N curves is another problem with these methods. The MSU/DOE database developed by Mandel and Samborsky (1997) and the FACT database developed by de Smet and Bach (1994) present test results on glass/epoxy and glass/polyester composites with different configurations. They force researchers to characterize each configuration of the lay-up separately. Application of Miner's rule needs S–N curves for the laminated composites as a whole and is not capable of performing ply-by-ply analysis. Therefore, a complete set of data should be available for each lay-up configuration. The main reason for publishing new versions of these databases each year is the introduction of new lay-up configurations of the blade structure by industry.

Finally, the load spectrum obtained is utilized to estimate fatigue damage in the blade using Miner's rule. One of the main shortcomings of this method is the linear nature of Miner's rule. Not only is Miner's rule an improper rule for fatigue consideration in metals, but also it has been proven that this rule is not suitable for composites (Mandel *et al.*, 1999). Another problem with using Miner's rule is the weakness of this model to simulate the load sequence and history of load events. For instance, Sutherland (1999) reported different lifetimes for the same blades with two orders of magnitude. The load cases applied were exactly the same except for the load sequence.

One of the most important drawbacks of the investigations performed on fatigue phenomena in a wind turbine blade is that they most often suffer from the deterministic approach employed, while the real governing situation is a stochastic one. Since wind flow is random in its nature, the wind turbine blade will experience each load case randomly. While deterministic analysis of fatigue damage progress is limited to investigating the failure associated with the most critical load pattern, stochastic implementation captures the whole events that a wind turbine will encounter. Deterministic approaches capture just one load pattern as a compromise, which is not necessarily representative of the real conditions imposed on a wind turbine. In stochastic simulation of fatigue phenomena, both the frequency of load occurrences and their sequence play a significant role. Moreover, the probability of a particular event to which a wind turbine will be exposed is addressed carefully in stochastic analyses. Therefore, the contribution of a specific load is not taken into account identically over the whole span of working years but will be treated randomly as in real working conditions.

14.2 Framework of the developed modeling technique

Fatigue phenomena have been investigated for wind turbine composite blades in order to overcome the shortcomings described in the previous section.

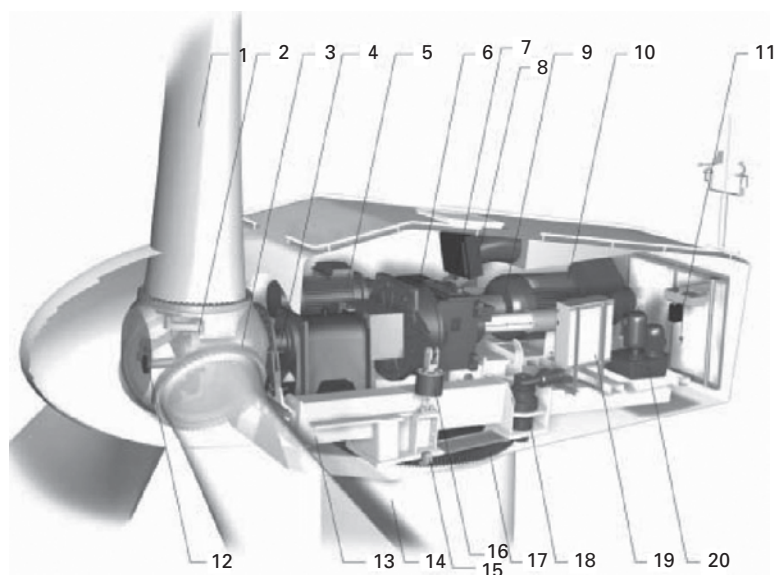
Load analysis and identification of different load cases based on the real working conditions of a wind turbine have been carefully studied. A finite element analysis is employed to simulate the response of the structure under various load cases to which a wind turbine blade will be exposed during its service lifetime. As a consequence, the most critical zone where catastrophic fatigue failure initiates is determined.

The accumulated fatigue damage model is employed as the damage estimation technique based on the generalized material property degradation rule. The method is rearranged and simplified based on the CLT (classical lamination theory) to study the fatigue failure in wind turbine blades. The model consists of three main parts: stress analysis, failure analysis and material degradation rules.

Since wind flow is a random parameter, statistical investigation is performed on the wind regime of a selected wind farm. Moreover, the probability density function of the wind flow is extracted to be implemented in a random analysis. Then, each load case is weighted in accordance with the frequency of occurrence obtained.

A virtually random wind scheme is produced in accordance with the governing wind pattern to simulate the real operational conditions on the investigated wind turbine blade. Investigation of fatigue failure is conducted on the wind turbine blade based on a full stochastic approach instead of a deterministic one. A computer code entitled 'STOFAT' has been developed for this purpose. This program comprises three main steps: wind pattern generation, load distribution analysis and fatigue damage assessment. The main advantage of this method is its ability to simulate the load sequence and load history. Moreover, based on the capability of the method for fatigue modeling, performing a large quantity of experiments in order to characterize complete fatigue behavior of materials is avoided.

As a case study, a 23 m blade of a V47-660 wind turbine, manufactured by the Vestas company, is selected. The investigated wind turbine is a three-bladed horizontal axis wind turbine which is installed upwind. The rotor consists of a set of blades and a nose cone connected to a main shaft. The rotor blades capture the energy of wind flow and transmit it to the gearbox with 1:52 ratio via the main shaft. An asynchronous generator with four poles is driven by the output shaft of the gearbox and produces 660 kW of energy. The whole assembly is placed inside a nacelle which is mounted on the top of a 42 m tower. This equipment is depicted in Fig. 14.1. The turbine is equipped with an active yaw control system and always stays in front of the wind flow by rotating the whole nacelle around the tower. In addition to activating the yaw control system, the blades are also equipped with a pitch control system which rotates all blades independently around their longitudinal axis up to 89°. By virtue of the power control system, the rotational speed of the rotor will not change with variation of wind flow



1. Blade	11. Service crane
2. Blade hub	12. Pitch cylinder
3. Blade bearing	13. Machine foundation
4. Main shaft	14. Tower
5. Secondary generator	15. Yaw control
6. Gearbox	16. Gear tie rod
7. Disc brake	17. Yaw ring
8. Oil cooler	18. Yaw gear
9. Carden shaft	19. VMP top (control unit)
10. Generator	20. Hydraulic unit

14.1 V47-660 kW wind turbine and its components (courtesy of Vestas Wind Systems A/S).

speed. Hence, the turbine is categorized as of the constant speed type. The detailed specifications of this wind turbine and its blade are shown in Table 14.1.

14.3 Loading

A wind turbine blade is subjected to different load cases arising from a wide variety of sources. Germanischer Lloyd (GL) rules have provided a clear insight to the classification of different loading conditions associated with experiencing natural and operational events. The load cases are determined by the combination of specific operating and external conditions. Operating and external conditions are assumed to be statistically independent. Specific combinations of different load cases are selected for different strength, static, dynamic and fatigue analyses.

Table 14.1 Technical specification of investigated wind turbine and its blade

General specification of the wind turbine		Specification of the blade	
Nominal power	660 kW	Length	22,900 mm
Rotor diameter	47 m	Minimum chord	282.5 mm
Swept area	1735 m ²	Maximum chord	2087 mm
Cut-in wind speed	4 m/s	Station of max. chord	R4500
Cut-out wind speed	25 m/s	Twist	15.17°
Rated power wind speed	15 m/s	Station of CG	R1800
Revolving speed of rotor	28.5 rpm	Weight of the blade	1250 kg
Elapsed time to reach 28.5 rpm from stationary status	30 s	Airfoil cross-section types	FFA-W3, NACA-63,mix
Yaw movement speed	0.5°/s	Surface area	28 m ²
Temperature range (°C)	-20° to + 40°	Tip to tower distance	4.5 m

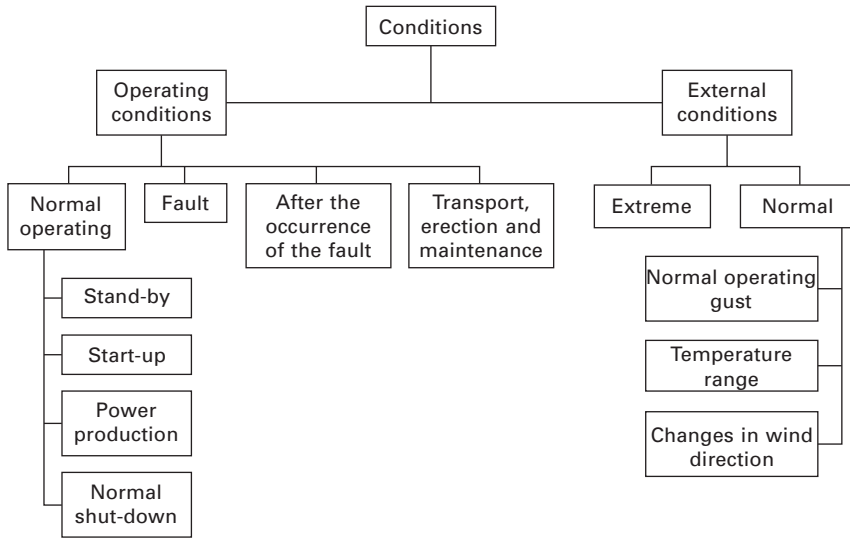
External conditions are divided into normal and extreme conditions. Normal conditions are expected to occur at least once during one working year, and extreme conditions are identified as those events that are not expected to occur more than once during 50 years of operation (50 years gust).

Operating conditions are divided into four classes: normal operating conditions, fault conditions, conditions after occurrence of a fault, and transport, erection and maintenance. According to the terms of the GL rules, a combination of normal external condition and normal operating condition has to be taken into account for fatigue investigation. Due to their rare occurrence, the other cases and/or combinations can be neglected because of their insignificant influence on the fatigue damage of wind turbine blades. A general view of different load cases and those subgroups involved in fatigue analysis is depicted in Fig. 14.2.

The fatigue investigations can be performed using either a load spectrum obtained through real measurement and digital sampling, or computer simulation. A new method proposed here will calculate each load case deterministically and convert them into a stochastic phenomenon using their weight of occurrence. This procedure is clearly explained in Section 14.7.

14.3.1 External conditions

External conditions describe the interactions between a wind turbine and its surrounding environment. They can be categorized as normal gust, changes in wind direction and temperature ranges. The acceleration of a normal gust is assumed to be 5 m/s² and as a consequence the normal gust factor is selected as 5/3. It is necessary to consider sudden unpredictable changes in wind direction up to $\pm 30^\circ$ relative to the dominant direction of wind flow (Germanischer Lloyd rules and regulations, 1993). Variation of ambient temperature can cause additional internal stresses on different layers of the



14.2 General view of operating and external conditions of a wind turbine.

composite structure of a blade due to different thermal expansion coefficient and should also be taken into account.

14.3.2 Normal operating conditions

It is assumed that in all subgroups comprising this main category, a wind turbine blade is working in a normal situation. The stand-by condition is defined as the situation in which there is neither a fault/shut-down condition nor is the wind speed outside the range in which power is generated. A start-up event takes place when the turbine is experiencing a transient either from the stand-by to the power production situation or from a lower wind speed to a higher value during power production. In the power production condition, a turbine generates power within cut-in and cut-out wind speeds. Normal shut-down implies a transient situation from power production to stand-by conditions or from a higher wind speed to a lower one.

14.3.3 Involved load cases

A specific combination of different load cases acting on a blade must account for each of the aforementioned normal operating conditions in accordance with the following pattern (Germanischer Lloyd rules and regulations, 1993):

- Stand-by: Aerodynamic loads, weight, annual gust and changes in wind direction

- Start-up: Aerodynamic loads, weight, annual gust and angular acceleration effect
- Power production: Aerodynamic loads, weight, centrifugal forces arising from angular velocity of a blade, gyroscopic effects arising from the yaw movement of the wind turbine, annual gust and changes in wind direction
- Normal shut-down: Aerodynamic loads, weight, activation of mechanical brake, effect of angular acceleration of a blade, annual gust.

14.3.4 Calculating loads

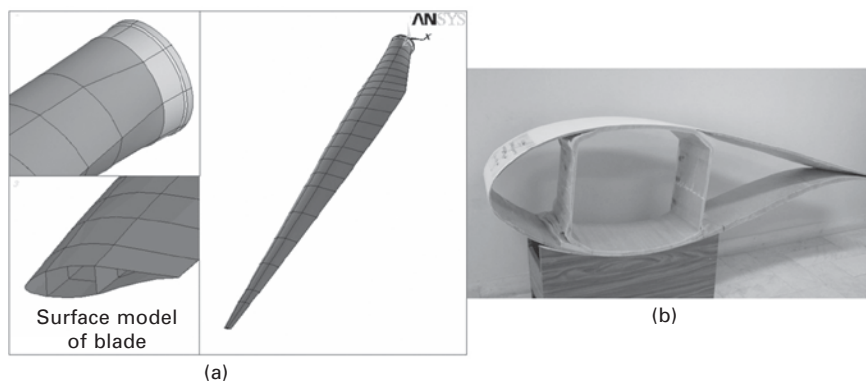
One of the main contributors to load cases is aerodynamic loads on a blade. Much research has been devoted to developing appropriate methods of load extraction using advanced aerodynamic theories and simulation (e.g. Stewart, 1976; Lupo, 1982; Riziotis and Voutsinas, 2000; Snel, 2003; Hasegawa *et al.*, 2004), but for simplicity the simple routine presented by GL is employed here. The details of the calculations for each load case can be found in the GL rules and regulations.

14.4 Static analysis

Prior to establishing a fatigue damage model, it is essential to obtain a clear understanding of structural response against different load cases. This will lead us to identify and extract the critical damage zone where catastrophic fatigue failure initiates. Further investigation of fatigue damage progress simulation will be focused on the critical zone obtained. So, accurate and careful accomplishment of this step plays an important role in the final results, obtained.

Blades of large, modern wind turbines consist of three different parts. shell, spar and root joint (see the technical brochure of Vestas Wind System A/S, 2002). The shell is responsible for helping to create the required pressure distribution on the blade. The cross-section of the shell is airfoil-shaped. The spar has to withstand loads on the blade arising from different sources. The root joint is the only metallic part in the current blade that connects the whole blade structure to the hub by screws. This metallic joint is covered by composite laminates internally and externally. It is worth mentioning that in newly designed wind turbine blades, this metallic part has also been replaced by a fully composite one. Figure 14.3 provides an overall view of the whole blade and its cross-section.

The cross-sections of the shell are different airfoils categorized under different class of NACA, FFA-W3 or mix one (Dahl and Fuglsang, 1998) dictated by refined aerodynamic theories to improve the efficiency in capturing wind energy, and the cross-section of the spar is of a box-shaped type. Due



14.3 (a) Geometrical model of a wind turbine blade; (b) cut section of a real wind turbine showing upper and lower shells and spar.

to aerodynamic and structural considerations (Burton *et al.*, 2001; Manwell *et al.*, 2001; Hansen, 2008), wind turbine blades not only have a tapered shape but also twist from root to tip (for the blade investigated the twist angle is about 15° from root to tip). Due to the aforementioned complex governing geometrical and structural configurations, a finite element method is employed to perform static analysis.

First of all, a geometrical frame model of the blade is constructed using the airfoil cross-sections at different stations from the root, then the geometrical model is completed by creating surfaces using the loft method. Loft creates a surface by specifying a series of cross-sections which define the profile of the resulting surface. The loft method is used to draw a surface in the space between the cross-sections (see the Auto-CAD user manual). The finite element model is built using second-order shell elements to increase the accuracy of modeling in ANSYS commercial software. Convergence analysis is necessary to obtain sufficient mesh density to optimize both the number of elements and the accuracy of modeling. Lay-up configurations of the blade, including both lay-up sequence and lay-up orientation, are fed into the software carefully. The wind turbine industry most often employs pre-impregnated materials to increase the efficiency of the production process from both quantity and quality aspects. These materials are available in three different forms as unidirectional, biaxial and triaxial plies called *U-D*, *Biax* and *Triax*, respectively. Biaxial and triaxial plies contain two or three of the same unidirectional fabrics, respectively, which are stitched together (not woven). The main reason for not using woven forms of composites is their poor fatigue performance. In the woven form, due to the out-of-plane curvature of fabrics, stress concentrations happen and consequently the fatigue performance of these materials decreases dramatically, as addressed by Mandel *et al.* (2002).

Triaxial and biaxial fabrics are used in the shell structure, and unidirectional and biaxial fabrics are used in the spar structure. The configuration of the biaxial lamina is $[0/90]_T$ and the configuration of the triaxial laminate is $[0/+45/-45]_T$. In order to construct the sandwich panel, two kinds of foam, polyvinyl chloride (PVC) and polymethacrylimide (PMI), are used at the spar and shell locations, respectively. For the required mechanical properties of the materials mentioned, such as longitudinal and transverse moduli, major Poisson's ratio and shear modulus, refer to Shokrieh and Rafiee (2006).

In addition to the convergence criterion, the finite element model has been checked using free vibration analysis to ensure the compatibility of the model with the real structure from different aspects of dimensions, material properties and lay-up configurations (Rafiee, 2004). Furthermore, buckling phenomena were also studied linearly and non-linearly and the results obtained indicated that the structure is in a safe condition. All of the aforementioned analyses established confidence for further static analysis which will be used as an input to the fatigue simulation procedure. The structure is also studied under isolated thermal loads. It is observed that the temperature range does not play any significant role in comparison with other load cases, so the temperature load can be neglected.

The load cases presented above associated with different working conditions are carefully applied to the finite element model and the response of the structure is investigated accordingly. Since the blades of modern wind turbines are very long structures, it is expected that non-linearity will be observed in the response of the structure, and the source of non-linear behavior can be found in the large rotation of the structure. Therefore, in several cases, performing non-linear analysis is unavoidable, since a linear analysis overestimates both distributed stresses and tip deflection of the structure.

The static analysis of the blade can be categorized in the different situations of stand-by, start-up, power production and shut-down, as previously described. A stand-by event covers wind speeds of less than 4 m/s and the most critical case in this situation will occur when the wind speed is 4 m/s and the effects of gust load and changes in wind direction are also considered. The results of this analysis are summarized in Table 14.2.

A start-up situation takes place when the speed of wind flow reaches 4 m/s and the most critical situation of this subgroup occurs when it is combined with gust load, changes in wind direction, and centrifugal force resulting from angular acceleration of the blade starting to rotate. The results of this event are presented in Table 14.2.

Power production situations are defined in the range of wind speed between 4 and 25 m/s. For a stall-controlled wind turbine the investigation is much easier than for a pitch-controlled one, since in the former case the wind turbine blade is always fixed at its mounted position and the most critical

Table 14.2 Static analysis of blade under different events

Event	Tip deflection (m)	Maximum longitudinal stress (MPa) (safety factor)	Maximum transverse stress (MPa) (safety factor)	Maximum in-plane shear stress (MPa) (safety factor)
Standby	1.23	520.12 – (2.22)	13.61 – (3.1)	21.33 – (3.77)
Start-up	1.67	650.3 – (1.79)	12.94 – (2.81)	32.5 – (2.5)
Power production	4.21	725 – (1.60)	15.5 – (2.3)	18.12 – (4.4)
Shut-down	4.33	764 – (1.52)	19.02 – (1.9)	21.27 – (3.81)

case will take place when the wind flow passes the rotor by its maximum speed. But for pitch-controlled blades, the effect of pitch angle associated with wind flow speed should be studied. It is not clear which wind speed will impose the most critical situations on the blade, since the critical zone is not fixed and changes its position due to rotation of the blade along its longitudinal axis, which is dictated by the pitch control mechanism. Therefore, for analysis, the wind speed intervals between 4 and 25 m/s are broken down into intervals with 1 m/s increments. The blade is subjected to a combination of appropriate load cases such as aerodynamic load, gust load, changes in wind direction and centrifugal force arising from angular velocity of the blade in each and every single interval. The results associated with wind flow of 25 m/s are given in Table 14.2, just to draw an overall picture and not to imply a most critical situation in the power production case.

Shut-down occurs when the wind speed exceeds 25 m/s. The most critical situation of this subgroup will happen when the blade is subjected to a combination of aerodynamic load, gust, changes in wind direction, and centrifugal load resulting from angular acceleration of the blade which is stopping. The results of this event are mentioned in Table 14.2. Knowing that the tip-to-tower distance of the current wind turbine is 4.5 m, these results show that the blade is in a safe condition from both tip deflection and strength aspects.

After performing the full range of static analyses, the most critical zone where fatigue failure begins is identified using the Tsai–Wu failure criterion. As was stated before, because the wind turbine investigated is powered with a pitch-control system, the critical zones predicted by the Tsai–Wu criterion vary in accordance with variation of wind speed. The static analyses associated with power production indicated that the critical zones can be divided into five groups for 4–8, 8–12, 12–16, 16–20 and 20–25 m/s wind speed intervals. In every interval mentioned, the location of the critical zone is fixed and the stress level is changed. The most probable wind speed range that has the highest accumulated wind speed probability addressing the most critical zone has to be identified based on the statistical investigations of the

wind pattern corresponding to the location of the wind turbine installation. Following this procedure, the most critical zone of the blade investigated is located on the upper flange of the spars which has 11 layers of 0° and one layer of $[+45/-45]$ biaxial laminate. It can be clearly seen that the critical zone is not placed on the shell, which is assumed in the literature.

Finally, a comparison between normalized longitudinal (N_X), transverse (N_Y) and shear forces (N_{XY}) to the length of the selected critical zone, proves that the magnitude of N_Y and N_{XY} compared with N_X is negligible. Also, considering normalized moments (M_X , M_Y and M_{XY}) will not change the amount of effective stresses significantly. So it can be concluded that the uniaxial fatigue mode is dominant for the critical region.

14.5 Fatigue damage criterion

Different methods have been presented for estimation of fatigue failure in composite materials relying on empirical equations, failure mechanism, stiffness degradation or strength degradation. Shokrieh and Lessard (2000) presented a 'generalized material property degradation technique' by integrating normalized residual strength, normalized residual stiffness and normalized residual fatigue life. They have developed a model to simulate fatigue failure in laminated composites subjected to a generalized condition (loading, geometry, etc.) using results from different uniaxial fatigue failures in unidirectional plies. The main advantages of this method in comparison with other developed methods can be summarized as follows:

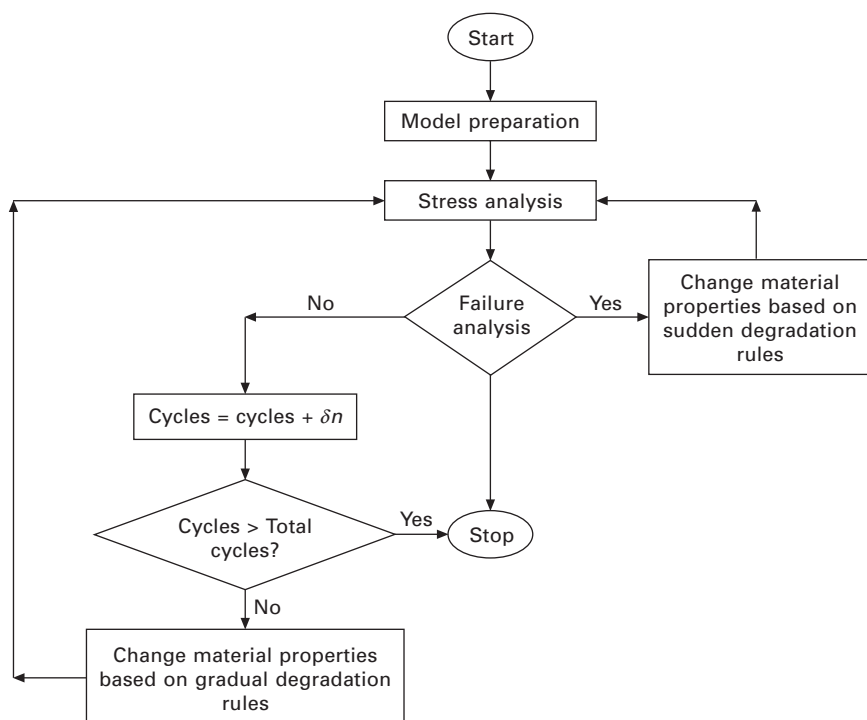
- The model investigates damage progress from the very first stage up to catastrophic final failure.
- The model is capable of taking into account load history and load sequence.
- The required material properties of any desired lay-up configuration as an input to the model will be obtained using the properties of the component unidirectional plies. Hence, full characterization of any lay-up configuration is avoided.

The method developed is able to simulate fatigue progress in unidirectional plies subjected to multiaxial stress and the desired stress ratio. The detailed description of this model can be found in Shokrieh and Lessard (2000). The model requires expensive hardware facilities, especially when it is applied to complex load cases of a wind turbine blade. Consequently, in this research, the method is simplified to simulate the behavior of laminated composite subjected to uniaxial fatigue loading. The simplified model is able to estimate the damage status in each part of a laminated composite for any desired stress level and number of load cycles from the beginning of loading to final sudden failure, and predicts the corresponding final life.

In order to accomplish these aims, an accumulated fatigue damage model is suggested based on classical lamination theory (CLT). The model consists of three main parts: stress analysis, damage estimation and material properties degradation. A flowchart of the algorithm employed for accumulated fatigue damage modeling is shown in Fig. 14.4.

14.5.1 Stress analysis

As depicted in Fig. 14.4, firstly an appropriate model to analyze the stress field has to be constructed. At this stage, material properties, maximum and minimum fatigue load, maximum number of cycles and incremental number of cycles are defined. Then stress analysis is performed based on the loading conditions and CLT theory (Hull, 1981). CLT is not able to capture the edge effect, but since the extracted critical zone is located in a confined region, no edge effect will be experienced and therefore CLT is appropriate enough for this purpose. The critical zone investigated consists of different lay-up configurations and mechanical properties. Also, after the start of loading and degradation of material properties, the material properties of the critical



14.4 Flowchart of accumulated fatigue damage modeling.

zone will change. Therefore, the stress analysis module should be able to analyze the hybrid laminates.

14.5.2 Failure analysis

The next step is devoted to failure analysis. If sudden failure happens, sudden degradation rules are employed, otherwise gradual stiffness degradation rules are applied. In other words, the gradual stiffness degradation procedure always exists but sudden degradation is checked prior to analysis and will be applied if sudden failure requirements are met. In this step, if there is no sudden mode of failure, an incremental number of cycles is applied. If the number of cycles is greater than a preset total number of cycles, then the procedure stops. Otherwise, the properties of all plies are reduced according to the gradual material property degradation and damage rules.

14.5.3 Gradual degradation rules

Different methods have been developed by several authors to express strength or stiffness degradation as a single criterion or combinations of stiffness and strength degradation as a combined criterion. In this study, for simplicity and to reduce computational costs, the stiffness degradation technique is employed. Most stiffness degradation techniques, like Ye's model (1989), employ damage level as a function of applied stress level. The main drawback of these models is the direct use of stress, hence, by changing the stress level all equations have to be rearranged. Limiting the model to a specific stress level will lead to improper results in general cases. In order to overcome this shortcoming, Shokrieh and Zakeri (2007) developed a stiffness degradation rule using a normalized model of damage estimation and a normalized model of residual stiffness. The residual stiffness of a unidirectional ply under a desired uniaxial state of stress and stress ratio in each load step is calculated using the following equation:

$$E(n, \sigma, \kappa) = \left(1 - \frac{\tilde{D}}{f(\sigma, \sigma_{ult})}\right) E_0 \quad 14.1$$

where E_0 , $E(n, \sigma, \kappa)$ and $\tilde{D}$ represent initial stiffness before start of fatigue loading, residual stiffness (as a function of number of cycles, applied stress and stress ratio) and the normalized damage parameter, respectively. Because the magnitude of $\tilde{D}$ does not depend on the applied stress level, in order to include the role of stress in the amount of damage $f(\sigma, \sigma_{ult})$ was used. A complete form of $f(\sigma, \sigma_{ult})$ is presented by Shokrieh and Lessard (2000) based on available experimental data for carbon/epoxy (commercially called AS4/3501-6).

A damage estimation method developed by Shokrieh and Zakeri (2007) is used to calculate $\tilde{D}$, describing a relation between $\tilde{D}$ and $\tilde{N}$, which is called the normalized number of cycles. Available experimental data show that normalized damage increases linearly from start of loading till $\tilde{N} = 0.67$ and after that the rate of damage increases non-linearly until final failure ($\tilde{N} = 1$). So the relation between $\tilde{D}$ and $\tilde{N}$ can be divided into two phases. All equations that describe the linear and non-linear relations between $\tilde{D}$ and $\tilde{N}$ are fully constructed for both phases by Shokrieh and Zakeri (2007).

14.5.4 Sudden degradation rules

Using gradual degradation rules for laminated composites subjected to fatigue loading, the material properties of the component will reach a specified level at which the component cannot tolerate any loading, in other words, the failure occurs. Failure criteria cannot be used to identify failure initiation in the method employed, since failure criteria rely on strength and stiffness, while in the method employed just stiffness is gradually degraded and the strength is reduced suddenly, not gradually. Therefore, in this method, the instantaneous stiffness is compared with the intact stiffness (static stiffness), and whenever in the longitudinal and transverse directions the instantaneous stiffness to static stiffness ratio (E/E_0) is equal to the stress to strength ratio, failure is met. For shear loading, this ratio varies between 0.6 to 0.76 and the average value of 0.68 is taken into account.

Consequently, if a sudden failure model occurs, the material properties of the failed plies are modified in accordance with corresponding sudden material property degradation rules. These rules were fully developed by Shokrieh and Lessard (2000) for a unidirectional ply subjected to multiaxial fatigue loading for seven different failure modes, i.e. tension and compression of fiber, tension and compression of matrix, normal tension and compression, and in-plane shear. According to these rules, tension and compression fiber failure modes are classified as catastrophic failure modes. In this case the failed materials cannot withstand any load and all mechanical and material properties will be set to zero. In other cases, some specific properties are set to zero in accordance with the failure mode. Since CLT theory is employed here, the aforementioned seven rules decrease to the following four rules:

- Fiber failure under tension/compression: All mechanical and strength properties are degraded nearly to zero.
- In-plane shear failure in fiber-resin: Transverse modulus of elasticity, major Poisson's ratio and in-plane shear strength are degraded nearly to zero.
- Matrix cracking under tension: Transverse modulus of elasticity, minor Poisson's ratio and tensile strength in the transverse direction are degraded nearly to zero,

- Matrix cracking under compression: Transverse modulus of elasticity, minor Poisson's ratio and compressive strength in the transverse direction are degraded nearly to zero.

After modifying the material properties using either gradual or sudden degradation rules, the stiffness matrix of the finite element model is reconstructed and the stress and failure analysis are performed again. The above loop is repeated until catastrophic failure occurs, or the maximum number of cycles (predefined by the user) is reached. It is worth mentioning that after modifying material properties in each increment, the total strain is checked and if its value is greater than static failure strain, the loop is stopped. In this case the final failure has happened and the final life will be reported. The final failure maybe also happens when total strain is still less than static failure strain; in these cases, the stress and strength levels will be checked and if their values are very small, the loop is stopped.

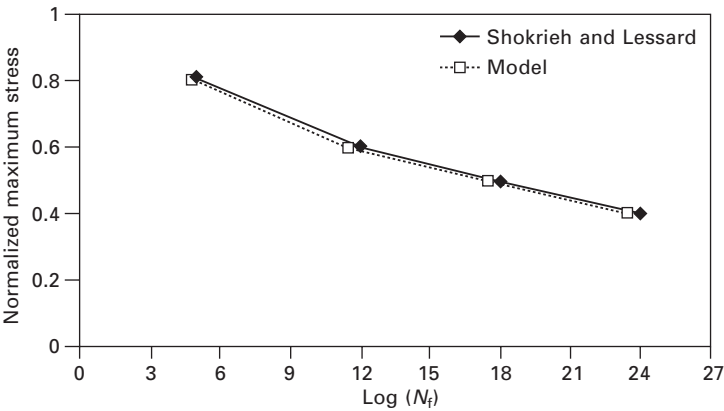
14.5.5 Evaluation of accumulated fatigue damage modeling

It is necessary to evaluate the accumulated fatigue damage modeling techniques using some simple cases to assure the accuracy and efficiency of the developed method prior to utilizing it for investigation of fatigue failure in a wind turbine blade, including much more complicated loading pattern and lay-up configurations. The method is applied to a 0° unidirectional ply of carbon/epoxy under tensile longitudinal fatigue loading, a 90° unidirectional ply of carbon epoxy under tensile transverse fatigue loading and a cross-ply of carbon/epoxy.

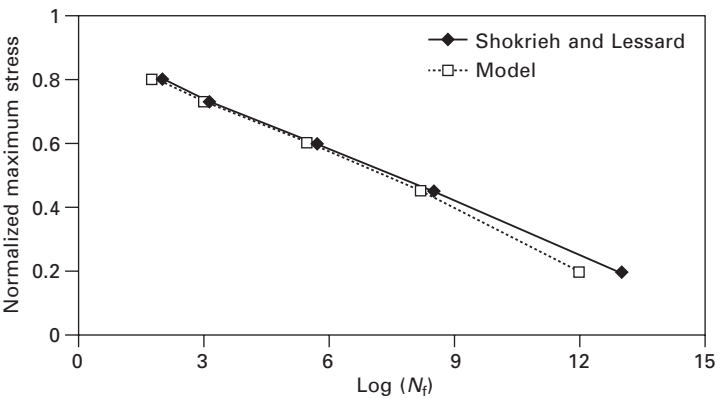
The results obtained for unidirectional plies are compared with experimental results reported by Shokrieh and Lessard (2000), and the results for cross-ply are compared with Charewics and Daniel (1986) and Lee *et al.* (1989). The comparisons are shown in Figs 14.5, 14.6 and 14.7 and show very good agreement between simulated results and experimental data.

All equations and relations have been developed based on available experimental data for carbon/epoxy composites. Since normalized life curves for glass/epoxy were not available, a generic behavior was estimated based on the pattern of decreasing mechanical properties and strengths of carbon/epoxy. The only available criteria for evaluation of the aforementioned generic fatigue behavior process are the mentioned criteria in the MSU/DOE fatigue database for wind turbine blade application. In MSU/DOE, the behavior of the composite materials involved in blade structures is defined by the following equation:

$$\sigma/\sigma_0 = 1 - b \log N \quad 14.2$$



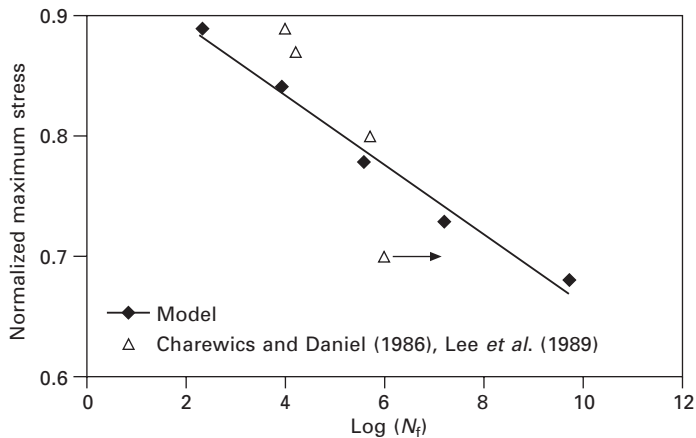
14.5 Experimental data and results of computer code for a 0° ply subjected to longitudinal loading.



14.6 Experimental data and results of computer code for a 90° ply subjected to transverse loading.

where σ is the maximum applied stress, σ_0 is the corresponding static strength in the same direction of applied stress, and b is a parameter which varies for the different materials. In MSU/DOE two magnitudes of b were presented which describe two boundaries of all curves. The upper bound refers to good material that behaves perfectly against fatigue and the lower bound refers to poor material that behaves weakly. The values of b for different fatigue loading situations are extracted from the MSU/DOE database and are given in Table 14.3.

Implementing the accumulated fatigue damage model with the aforementioned generic methods, the results obtained approach the bound of good materials (for instance, tensile mode results are shown in Fig. 14.8).



14.7 Experimental data and results of computer code for a cross ply subjected to tensile fatigue loading ($R = 0.1$).

Table 14.3 Values of 'b' for two bounds of good and poor materials (S-N data)

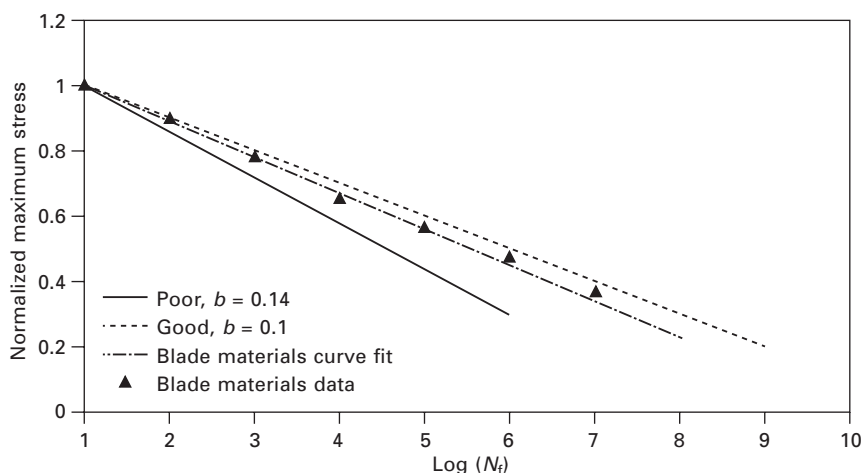
Type of data	Extremes of normalized S-N fatigue data for fiberglass laminates		
Tensile fatigue data ($R = \sigma_{\min}/\sigma_{\max} = 0.1$)	Good materials $b = 0.1$	Poor materials $b = 0.14$	Normalization UTS ^a
Compressive fatigue data ($R = 10$)	Good materials $b = 0.07$	Poor materials $b = 0.11$	Normalization UCS ^b
Reversal fatigue data ($R = -1$)	Good materials $b = 0.12$	Poor materials $b = 0.18$	Normalization UTS ^a

^aUltimate Tensile Strength.

^bUltimate Compressive Strength.

Since good materials are classified as high quality materials containing non-woven fabrics with an equally distributed volume of resin at all points, the pre-impregnated materials are known as materials that fulfill the aforementioned requirements. Therefore, an appropriate modeling technique can be applied on the basis of generic data. It is worth mentioning that the generic method can be interpreted as a proper modeling technique. However, a more accurate treatment would involve extracting the normalized life curve and other required parameters associated with the specific type of materials of which the wind turbine is made. However, for this study where the development and presentation of a methodology is much more important than the particular case study, it would be sufficient to use the generic method.

One of the most important advantages of using accumulated fatigue damage modeling can be found in its independence of the lay-up configuration. Knowing the behavior of the unidirectional plies involved, the model can



14.8 Normalized S–N tensile fatigue data ($R = 0.1$) obtained from the computer code based on used generic method in comparison with good and poor material data from MSU/DOE database.

be used for any configuration of fabrics and full characterization of each configuration is not necessary.

14.6 Stochastic characterization of the wind flow

Characterization of the wind flow in the vicinity of the wind farm is a vital task in the process of electricity generation. This is a two-fold issue which is an important process in wind potential assessment and subsequent macro- and micro-siting procedures, since the power of wind energy is proportional to the cubic power of the wind velocity. On the other hand, obtaining a crystal-clear picture of the wind pattern is essential for weighting different events for fatigue analysis. The wind speed is not a steady parameter due to its random nature. Therefore, different loadings will be imposed on the wind turbine blade during its lifetime which should be weighted in accordance with the corresponding rate of occurrence. Generally, working situations of a wind turbine can be divided into three main classes:

- The situations describing loading on the wind turbine during its normal operation and normal conditions
- The situations describing loading on the wind turbine during start-up and shut-down phases, covering both normal and emergency stops
- The situations describing loading on the wind turbine while the rotor is not rotating but the blades are exposed to high wind velocity or annual gusts.

Those load cases categorized under the second group of the above classification

will be weighted based on their frequency of occurrence and the other groups are weighted on the basis of wind speed pattern. These cases take place at certain wind speeds and, for any wind speed distribution scheme, their proportions will be derived accordingly. For instance, if one knows the load cases associated with a wind speed varying between 5 and 9 m/s, the rate of occurrence of this class of load case during one year of operation will be the same as the frequency of occurrence of that wind speed interval. This means that the proportion of time devoted to each and every situation is the same as the relative duration of the corresponding wind speed flow during the service lifetime of the turbine. Therefore, characterization of the governing wind speed distribution is an essential task, especially for fatigue consideration of a wind turbine blade.

The wind speed is never steady at any site, but is influenced by the weather system, the local terrain and the height above the ground surface. Therefore, annual mean speed needs to be averaged over 10 or more years. Such a long-term average raises confidence in wind speed distribution. However, long-term measurements are expensive, and designers cannot wait that long. In such situations, the short-term, say one year, data is compared with a nearby site having available long-term data to predict the long-term annual wind speed at the site under consideration. This is known as the ‘measure, correlate and predict (MCP)’ technique. Since wind is driven by the sun and the seasons, the wind pattern generally repeats itself over a period of one year. The wind site is usually described by the speed data averaged over a calendar year. The variation in wind speed v over the period can be described by a probability distribution function in the form of a Weibull function using the following equation (Gasch and Tvele, 2002):

$$h(v) = \left(\frac{k}{c} \right) \left(\frac{v}{c} \right)^{(k-1)} e^{-\left(\frac{v}{c} \right)^k} \quad 14.3$$

where c and k are scale and shape parameters, respectively.

Among wind characteristics those which are important in statistical investigation are wind speed, frequency and direction. Since all modern wind turbines are equipped with an active yaw system keeping the rotor up to wind, the wind direction does not play an important role in fatigue investigation, and the other two parameters will be described using a Weibull function.

The wind speed is usually measured using digital sampling and is averaged using one of the methods below:

1. The wind speed is measured at each second, and after 10 minutes, 100 digits are registered and their average, reflecting 10-minute average of the wind speed and the consequent wind speed average in one hour, can be obtained.

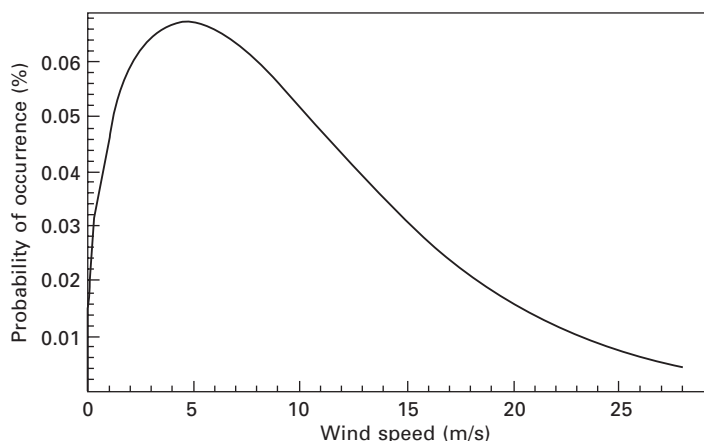
2. The wind speed is measured after each minute and is averaged over one year.

The frequency of occurrence for each wind speed is also logged and is used together with the value of wind speed to obtain a probabilistic distribution of a wind regime. Using meteorological data from Manjil (a city in the north of Iran), the following Weibull function is derived as a case study of this research:

$$h(v) = \left(\frac{1.425}{9.3206} \right) \left(\frac{v}{9.3206} \right)^{0.425} e^{-\left(\frac{v}{9.3206} \right)^{1.425}} \quad 14.4$$

where v is a wind speed and $h(v)$ is the corresponding probability of occurrence. Figure 14.9 shows this distribution.

As can be seen from Fig. 14.9, the dominant wind speed at the investigated wind farm is 5 m/s and the average wind speed is 8.24 m/s. The mean speed in that region is 7.3 m/s, i.e. the probability of the wind flow being below this amount is equal to the probability of the wind flow being above it. Obtaining a scale parameter k between 1 and 2 implies an appropriate location for wind turbine installation; k was determined as 1.425 for the investigated region. Furthermore, 9.32 m/s as the value of the shape parameter c pushed the curve to the right-hand side (upper wind speeds), showing a higher probability of wind flow at upper wind speeds. The accumulated probability of wind flow between 4 and 25 m/s, the operating wind speed range of the investigated wind turbine, is 72%. It is worth mentioning that the investigation performed is sufficient for fatigue considerations. However, for wind potential assessment



14.9 Weibull distribution function from Manjil city, Iran.

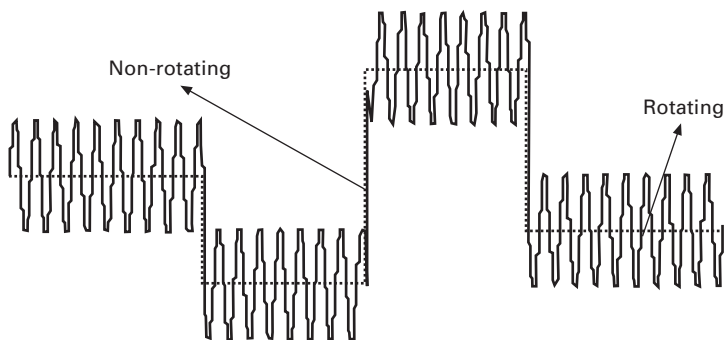
activities, a deeper study should be undertaken, which is beyond the scope of this chapter.

14.7 Stochastic implementation on fatigue modeling

The random nature of wind flow and the rotation of the wind turbine will inherently cause cyclic loading on the wind turbine blades during their service lifetime. In a very simple case, the main source of load variations can be found in changes of the weight vector direction toward the local position of the blade when the wind velocity which passes through the rotor is fixed. In this case, the maximum and minimum of the applied load are constant, and consequently the amplitude and the mean of the cyclic load are also constant. If the wind speed also varies, the amplitude and mean of cyclic loading will be changed. If a fixed blade is subjected to a variable wind inflow, the applied load graph to the structure will change with variation of wind speed, but if the wind speed does not fluctuate, the load level will not change at all; the main reason for load variation is changes in wind speed. But when a blade rotates, in addition to the aforementioned reason, changes of the weight vector toward the local position of the blade will cause a load variation between its maximum and minimum values with the same frequency as the rotor rotation. This phenomenon is depicted in Fig. 14.10.

It can be assumed that the time history of applied load to the wind turbine blade comprises three cyclic loadings:

- Type I: Variation between minimum and maximum values during one full rotation of the rotor
- Type II: Variation between minimum and maximum values during a significant change in wind speed



14.10 Difference of loading during rotation and stationary status of the blade.

- Type III: Variation between minimum and maximum values during a period of operation from start-up till shut-down.

In such a simple model, the number of fatigue cycles during a specified period of time is the duration of operation divided by the rotational speed of the rotor (for type I) plus changes in wind speeds (for type II) and one cycle of type III.

In order to investigate the fatigue phenomena of a wind turbine blade, different events experienced by the blade depending on the operational conditions and sources of variation have to be characterized and weighted based on their rate of occurrence. Therefore, it is preferred that working conditions of the wind turbine are divided into a series of independent operational conditions expressing different fatigue loadings. IEC-61400 presented the following classification which is widely used by different researchers:

1. A series of operational conditions in which the loading on the wind turbine is described as being under normal circumstances. In a conventional formulation, these situations depend on wind speed whose variations range from cut-in to cut-out.
2. A series of governing situations expressing loads in the two distinct cases of start-up and shut-down, the latter covering both normal and emergency shut-down.
3. A series of buffeting situations occurring when the rotor is in its stationary position, but the blades and other components are subjected to buffeting by wind flow.
4. Any other cases of exposure to the turbine that have a strong influence on fatigue loading but are not covered by the preceding three groups.

As was investigated in the preceding sections, loadings associated with wind speeds lower than 4 m/s do not contribute significantly to the fatigue damage process, so they can be neglected. It was also found that thermal loading is negligible in comparison with other loads. So the load cases classified under groups 1 and 2 will be considered, which are representative of power production and the transient events of start-up and shut-down. It is worth noting that according to the GL rules and regulations, consideration of normal operational conditions and normal external conditions is sufficient for fatigue investigation.

14.7.1 Sources of cyclic loadings

One of the most important steps in fatigue investigations of a wind turbine blade is to carefully determine the sources of load variations. As previously mentioned, the sources of load variations on a wind turbine blade are divided into two main categories. The first group involves those sources that change the total amount of load, while the second group consists of

those sources producing fluctuating load with constant amplitude and mean values. The former can be summarized as changes in wind speed, changes in wind direction and changes in pitch angle of the blade, and the latter are variation of blade weight vector toward its local position, annual gust and shear effect. Changes in wind speed are natural effects which have to be taken into account for fatigue investigation using a probability density function of the surrounding wind regime. Also changes in pitch angle of the blade are included intrinsically in changes of wind speed based on the approach employed, i.e., the static analyses described in Section 13.4. For each wind speed a new position of the blade dictated by the pitch control mechanism is considered in finite element analysis.

Changes in wind direction were also considered in Section 14.3. This source can be neglected owing to the short period of time over which it is effective. Almost all large modern wind turbines are equipped with an active yaw control system. This system will activate when the wind direction deviation exceeds 10° . For the turbine investigated the system will rotate the whole nacelle around the tower at a speed of 0.5° per second. So in the most critical case, when the wind direction changes by 45° , it takes 90 seconds to adopt the new position. Although this load case was applied during static analysis, it can be neglected for fatigue investigation due to the very short duration of its effectiveness compared with other load cases. Changes in direction of the weight vector toward the local position of the blade produce fluctuating load with a frequency identical to the rotor rotation frequency.

The wind shear effect arises from the difference in wind speed profile at different heights. The wind speed is usually measured at the center of the rotor hub, while in a large wind turbine the wind speed changes along the long blade from its root to its tip and a constant value can be assumed for the wind speed only when the blade is horizontal. The wind speed at different heights can be obtained using the following equation:

$$v(h) = v^* \left\{ \frac{h}{h^*} \right\}^\alpha \quad 14.5$$

where v^* is the wind speed at reference height h^* and α is an empirical parameter depending on the topological data. Noda and Fley (1999) proved that the wind shear effect acts in phase and will change with the weight vector and does not have any significant influence on fatigue damage, so this effect is not considered further.

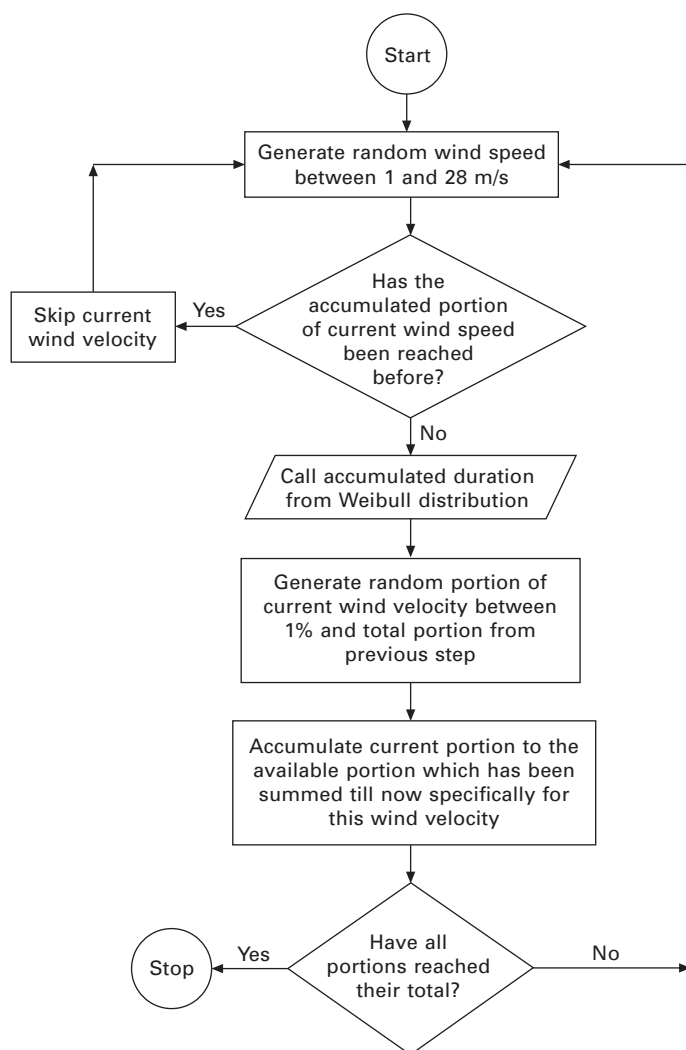
A sudden increase or decrease in the wind speed due to turbulence is defined as gust occurrence. Gust occurrence causes changes in the total amount of load due to changes in wind speed and fluctuation in constant amplitude load, while the latter is more important due to the short time of its occurrence.

During gust occurrence, the blade is excited and vibrated by the linear combination of its mode shapes because of the gust impact effect. Eggleston and Stoddard (1978) suggest that the higher-order frequencies (>2 Hz) contribute around 15% to the overall displacements. Therefore, considering first mode shape excitation will be conservative enough. Therefore, cyclic loading arising from gust occurrence is simulated in a proper manner for the considered wind speed. The maximum and minimum values of cyclic loadings are obtained from changes in the weight vector direction toward the local position of the blade, and the frequency of the cyclic loading is selected as the first mode shape vibration of the blade. Between flap-wise and chord-wise frequencies of the free vibrations, the lower amount causing higher cycles is selected in order to stay in a conservative region by applying a harsher situation. It is quite obvious that these two modes of frequencies cannot be isolated from each other during real operation, but for the sake of simplicity the one leading to the more conservative situation is selected accordingly. So, after defining cyclic loading sources, all of the corresponding applied stresses are derived from full range static analyses covering all events and will be fed as a database into the fatigue simulation code which will be explained in the next section.

14.7.2 Stochastic analysis

Implementation of stochastic loading is considered using the Monte Carlo simulation technique here (Kleiber and Hien, 1992). First of all, it is necessary to generate a wind pattern in a virtual space. The wind pattern should be a stochastic scheme in both wind speed values and corresponding duration of flow as the two major sources of randomness, so the approach employed is a stochastic–stochastic analysis. The wind pattern generated should be completely compatible with the Weibull function obtained. The algorithm for generating a stochastic wind pattern is depicted in Fig. 14.11.

A computer code is established in this research. This computer program is named STOFAT (STOchastic FATigue). At first STOFAT produces a 30-year wind pattern in accordance with the flowchart shown in Fig. 14.11. Namely, after generation of each wind speed, its duration is randomly selected between zero and the total predefined portion from the Weibull probability distribution function. This process continuously generates wind speed and its portion until the accumulated portion of each wind speed reaches its predefined amount. This code is also capable of considering an annual gust in wind pattern randomly in both wind speed and its position in the wind pattern. The duration of gust is considered to be one hour as dictated by GL rules and regulations. After generation of wind patterns for 30 years, in each event, STOFAT uses the ‘accumulated fatigue damage model’ as a subroutine in order to perform fatigue calculations and utilizes the corresponding applied stresses. The stresses have been previously supplied



14.11 Flowchart of stochastic wind pattern generation.

as the database of the code and obtained from a full range of static analyses. STOFAT is constructed on the Mathematica platform (Wolfram, 1999).

14.7.3 Results and discussion

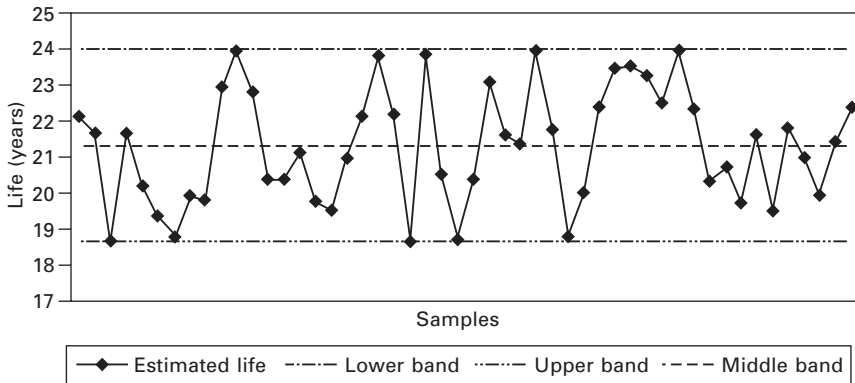
The STOFAT code is executed 50 times in order to obtain a sufficient range of results. The run-time of the code is about 2.5 hours in a computer with a 2.4 GHz processor which is powered by 512 MB of RAM. The critical region obtained towards the end of Section 14.4 with a configuration of 11

plies of 0° and one laminate of $\pm 45^\circ$ is analyzed based on its stress states for various wind speed values ranging from 1 to 25 m/s. The stress states are obtained using the static analyses explained in Section 14.4. Since the investigated wind turbine is of a constant speed type, due to the existence of a pitch control system, its rotor rotates at a constant 28.5 rpm. Therefore, in each year of operation, it rotates approximately 15×10^6 cycles. In order to ensure the appropriate performance of the developed code, one of the results of the code's execution is randomly selected and the progress of fatigue failure is traced step-by-step and shown in Table 14.4.

As can be seen from Table 14.4, the evolution of fatigue damage is logical, which shows the proper applicability of STOFAT code for the purpose of lifetime prediction of wind turbine blades. The results demonstrate that plies with $\pm 45^\circ$ configuration meet fatigue failure prior to other unidirectional plies. It can also be found that after nearly 50% damage progression in the laminate, the rate of damage evolution increases. The first failure happened after 9.3 years of continuous operation. During this period of time the turbine started/stopped 816 times. Figure 14.12 shows the results obtained by STOFAT for all 50 times of execution. One can observe that the results are bounded by 18.66 and 24 years as the lower and upper limits, while the average is 21.33 years. In order to investigate the validity of these results, the standard deviation (Montgomery and Runger, 1994) of the results is calculated as 1.59 years. Since wind patterns in the investigated region and consequently the applied load on the blade are stochastic, a proper scatter in the results obtained implies a proper fatigue modeling. Since no real measurement is available for the fatigue failure of the wind turbine blade under real loading conditions, a rough approach to evaluating the developed methodology executed by the STOFAT code is employed. As stated above,

Table 14.4 Fatigue failure trend for one sample result from STOFAT code

Layer no.	Lay-up	Year of failure
1	+45	9.30
2	-45	10.60
3	0	13.13
4	0	15.60
5	0	16.30
6	0	17.20
7	0	18.06
8	0	18.66
9	0	18.73
10	0	18.80
11	0	19.00
12	0	19.20
13	0	19.30



14.12 Predicted lifetime of blade by STOFAT computer code for 50 runs.

the investigated critical region consists of 11 layers of unidirectional and one laminate of biaxial ply. Since the dominant configuration of the critical region is unidirectional ply, most of the applied load will be tolerated by 0° plies. The lifetime of one unidirectional ply subjected to longitudinal loading is calculated by using equations developed in the preceding section. The lifetime obtained for this ply should be less than the lower bound of reported lifetime for the wind turbine blade computed by STOFAT (18.66 years). Under this condition, a unidirectional ply is exposed to a very harsh situation. This reflects exposure of the blade to the most critical loadings during its whole lifetime. In the most critical case, the ratio of maximum tensile stress to fiber strength is 1.74 and the blade lasted for 7.44 years to final failure.

14.8 Summary and conclusion

In this research, an investigation is performed to study the fatigue phenomenon of composite wind turbine blades. First, a brief review on conventional and on-going methods of fatigue modeling is presented together with shortcomings of these methods. A 23-meter full composite V47-660 turbine blade from the Vestas company is selected for investigation in this research. A full 3-D finite element model of this blade is constructed using ANSYS software and several checks from different structural viewpoints are performed to ensure the accuracy of simulation and the reliability of the reported results.

Different load cases of wind turbine rotor blades are obtained, calculated and evaluated. Moreover, those load cases that are negligible and have no significant influence on the occurrence of fatigue failure are ignored. The finite element model is analyzed from different static, vibration, thermal and buckling aspects in order to understand the response of the structure to

different conditions and events imposed on the blade during its operation. In some cases, performing nonlinear static and buckling analysis is necessary. Consequently, the critical zone where catastrophic fatigue failure initiates is determined. The accumulated fatigue damage model is employed as the damage estimation technique based on generalized material property degradation rules. The method is rearranged and simplified based on classical lamination theory, to study the fatigue failure in the wind turbine blade. The fundamentals of this model are constructed based on the progressive fatigue damage modeling developed by Shokrieh and Lessard. The model consists of three main parts: stress analysis, failure analysis and material degradation rules. The method employed for fatigue modeling is first evaluated for some simple lay-up sequences and orientations.

Because wind flow does not follow a deterministic pattern, a stochastic analysis is employed instead of a deterministic one. Therefore, statistical investigation is performed on the wind regime of the wind farm location, and a probability density function of wind flow is extracted to implement a random analysis. A virtually random wind scheme is produced in accordance with the governing wind pattern to expose the real operational conditions on the investigated wind turbine blade.

The main sources of cyclic loads on a wind turbine blade are identified and categorized as variations of wind speed, changes in wind direction, annual gust, variation of weight vector direction toward the local position of the blade, blade pitch movement and wind shear. The loading conditions associated with these sources are derived by finite element analyses and each event is weighted based on its frequency of occurrence. Investigation of fatigue failure is conducted on the wind turbine blade based on a full stochastic approach. A computer code entitled 'STOFAT' was developed for this purpose. This program comprises three main steps: wind pattern generation, load distribution analysis and fatigue damage assessment. The STOFAT computer code was executed 50 times and the results obtained are bounded between 18.66 years and 24 years as lower and upper limits. Moreover, the standard deviation of 1.59 years shows a small range of scatter in the range of results obtained. These results show that accumulated fatigue damage model presented and the stochastic method employed are able to simulate the progress of fatigue damage of a wind turbine composite blade. Considering the conservative nature of the technique employed, the blade will give 18.66 years of service in the worst situation and 24 years in the best situation.

14.8.1 Main contributions

The main contributions of the fatigue modeling technique for a wind turbine blade which is developed and employed in this chapter can be summarized as follows:

- A stochastic approach instead of a deterministic one
- Taking into account all load cases experienced and weighting them based on their rate of occurrence instead of using only the most critical load case
- Employing finite element modeling analysis to identify the critical zone from a fatigue viewpoint instead of using spectrum loading obtained from strain gages
- Developing a simple fatigue damage model based on CLT as an accumulated fatigue damage model to decrease the run-time of the simulation
- Taking into account the load sequence in the load history
- Extracting the required properties of the laminates with any desired configurations using the properties of their component unidirectional plies
- Full stochastic generation of wind flow pattern instead of block loading.

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