APPLIED METHODS OF STRUCTURAL RELIABILITY

TOPICS IN SAFETY, RELIABILITY AND QUALITY

VOLUME 2

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How reliable?
How good is the quality?

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Applied Methods of Structural Reliability

by

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PREFACE

A quarter of the century has elapsed since I gave my first course in structural reliability to graduate students at the University of Waterloo in Canada. Since that time on I have given many courses and seminars to students, researchers, designers, and site engineers interested in reliability. I also participated in and was responsible for numerous projects where reliability solutions were required.

During that period, the scope of structural reliability gradually enlarged to become a substantial part of the general reliability theory. First, it is apparent that bearing structures should not be isolated objectives of interest, and, consequently, that *constructed facilities* should be studied. Second, a new engineering branch has emerged - *reliability engineering*. These two facts have highlighted new aspects and asked for new approaches to the theory and applications.

I always state in my lectures that the reliability theory is nothing more than *mathematized engineering judgment*. In fact, thanks mainly to probability and statistics, and also to computers, the empirical knowledge gained by Humankind's construction experience could have been transposed into a pattern of logic thinking, able to produce conclusions and to forecast the behavior of engineering entities. This manner of thinking has developed into an intricate network linked by certain rules, which, in a way, can be considered a type of *reliability grammar*. We can discern many grammatical concepts in the general structure of the reliability theory.

It has been my intention to outfit the reader with a system of principal concepts, rules, and techniques that can be used to understand many practical issues and unravel problems encountered in an engineer's life. I have tried to avoid repeating facts described elsewhere, and refer the reader to appropriate sources. This, of course, has been possible only to a certain degree; obviously, the basic techniques have to be mentioned in any monograph to provide a useful source book on reliability engineering. On the other hand, many important issues of the structural reliability theory could not be covered, as, for example, those of stochastic dynamics and stochastic finite elements.

I did not want to overburden the reader by a long list of references. Moreover, it has been difficult to choose papers and books that are the best for further reading. I have aimed at indicating publications where further information can be obtained and which can guide the reader and help as sources for more detailed studies. For this reason, the majority of references relates to publications that appeared in the last decade and that are currently accessible to western reader. There are many important publications in Polish, Russian and other languages that could not be included in the list. Fortunately, the information retrieval systems are now so well developed that they can comfortably supply any information needed. One only has to know what one needs.

I could not possibly have written this book without contact with reliability-minded colleagues in many countries; they have participated by discussions and criticisms in formulating my concepts. I am rather unhappy that I am not able to enumerate all of them here. However, I should definitely mention the creative atmosphere which I had enjoyed in the Civil Engineering Departments of the University of Waterloo, Canada, Chalmers Tekniska Högskola in Gothenburg, Sweden, and Politecnico di Milano, Italy; there I had passed short but fruitful periods of teaching and research. Of course, I cannot fail to remember my Alma Mater, the Czech Technical University in Prague, and particularly its Klokner Institute, where I have spent the main part of my career.

My special thanks have to be extended to my friend and long-time close collaborator, Prof. Miloš Vorlíček, who has affected my statistically untrained mind and who participated in creating the background to the book. In solving numerous theoretical and practical problems jointly with Miloš I have learned to be both cautious and audacious in introducing probability and statistics into the thinking system.

Thanks to Dr. Nigel Hollingworth and Ms. Mirjam van Eijsden from Kluwer Academic Press for editorial advice and for trying to debug my Central European English. Thanks also to Prof. E.G. Frankel from the Department of Ocean Engineering, MIT, for some language editing and encouragements.

Thanks to Ms. Kateřina Konířová for her meticulous drawings and also for her technical help.

Finally, I would like to give my warm thanks to my wife, Libuše, who was so patient when I passed most of my time working on the compuscript. She was reading and re-reading the text several times, trying to find out my fundamental blunders in English; she also helped in solving many linguistic problems. I am afraid we have not been so successful as we have wished.

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February 1993

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PRINCIPAL CONCEPTS

Key concepts in this chapter: *reliability system; constructed facility, CF; S-L-E system; structure; load; environment; life; uncertainty; indefiniteness; reliability parameters; design parameters; flaw; aberration; deterioration; defect; damage; deficiency; failure; fault; current use; ageing; state profile; limit state; serviceability limit state, SLS; ultimate limit state, ULS; string of limit states; progressive deterioration; progressive collapse; stages; design situations; design criterion; reliability requirement; design parameter; design format; codified design format.*

1.1 RELIABILITY SYSTEMS S-L-E AND CF

For the ensuing study of reliability requirements and probability-based design methods it is necessary to define properly the space which the respective requirements and methods refer to. *Entire systems,* not only their isolated components, have to be subjected to reliability analysis. The basic system usually investigated, for which the reliability requirements are formulated, is the STRUCTURE-LOAD-ENVIRONMENT system (S-L-E, Figure 1.1). It is assumed that the basic design parameters (for example, characteristic strengths, representative values of loads, reliability factors, cf. Section 1.4) are specified in the codes for the structural design, in the load codes, and in other documents.

However, if the *determination of design parameters* constitutes a part or even the main goal of the reliability analysis, a basic examination of an S-L-E system does not suffice, and a higher order system has to be examined - the CONSTRUCTED FACILITY (CF, Figure 1.1). In such a case the system S-L-E becomes a subsystem Sm_1 of the system CF, which obviously contains also further subsystems (Sm_2 , Sm_3 , ..., Sm_i , ..., Sm_n ; such systems are, for example, wiring, water supply, draining, HVAC). The subsystems Sm_2 through Sm_n , though they should always be considered, will not be discussed in this book. It should be mentioned, for completeness' sake, that only some of the subsystems are mutually *disjunctive* - cf., for example, in Figure 1.1 the pairs (Sm_1 , Sm_2) and (Sm_2 , Sm_3). The system reliability of vertical transport in a building can depend on horizontal displacements of the building under the action of wind. Two *conjunctive systems* are dealt with in this case. Three conjunctive systems are, for example, heating, water supply, and wiring.

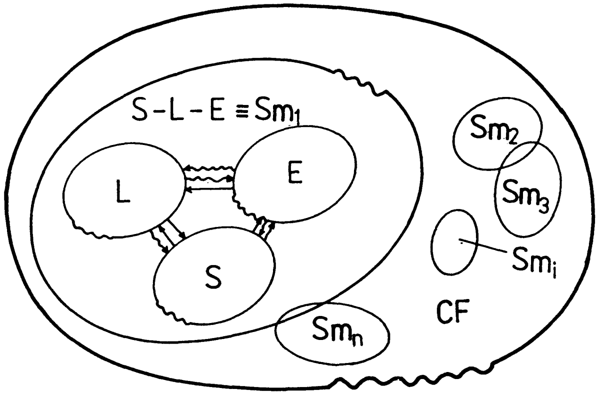

Fig. 1.1 - System "CONSTRUCTED FACILITY" (CF) and subsystem "STRUCTURE-LOAD-ENVIRONMENT" (S-L-E). Subsystems S , L , E , and subsystems Sm_i are disjunctive (for example, Sm_3 and S-L-E) or conjunctive (for example, Sm_2 and Sm_3). Undulations indicate uncertainties and indefiniteness.

A CF system is *fixed to a certain site* where it fulfills its function for a specified period called *life,* T_0. When, at a given point in time t , the system has reached a certain age $T_a < T_0$, the quantity

$$T_{res} = T_0 - T_a$$

is called the *residual life.* The time location of the CF system is defined by the point in time t_0 after which the system is ready to be used. This does not mean that the use really starts at this particular moment; it frequently happens that constructed facilities are not employed for long periods after their completion. As all subsystems of the CF system change during its life, the CF system itself constantly changes (Figure 1.2). - Note that the *fixation in space and time* is a typical feature of civil engineering systems. Systems encountered in mechanical engineering are expected to move in space and time.

The description of CF can never be perfect. As a rule, it is never free of larger or smaller *uncertainty* and *indefiniteness.* The distinction between these two concepts is important when reliability parameters and design parameters are to be established; this will be shown in Section 14.7.

Uncertainty refers to imprecise and incomplete information about the phenomenon investigated. For example, it is known that a structure will be subjected to wind load but the exact magnitudes of this load at specific moments of the life of the facility are unknown. Similarly, it can be expected that the grade of concrete will be, for example, C20, but it is not known what the values of the compression strength in particular cross-sections will be. The nature of uncertainties is mainly random. In a way, uncertainties can be considered *statistical regularities* (Ellingwood 1992).

Indefiniteness refers to the lack of unambiguous information whether the investigated phenomenon will occur or not. It is, for example, never known whether a designed building will be constructed; in many cases this indefiniteness is anticipated in the bidding designs. Further, it is never certain that the building will be used to conform with the assumptions made during the design. It is not even sure that the constructed building will be used at all! - Here, obviously, limit cases are introduced as examples but they are not unrealistic and we have to take them into account.

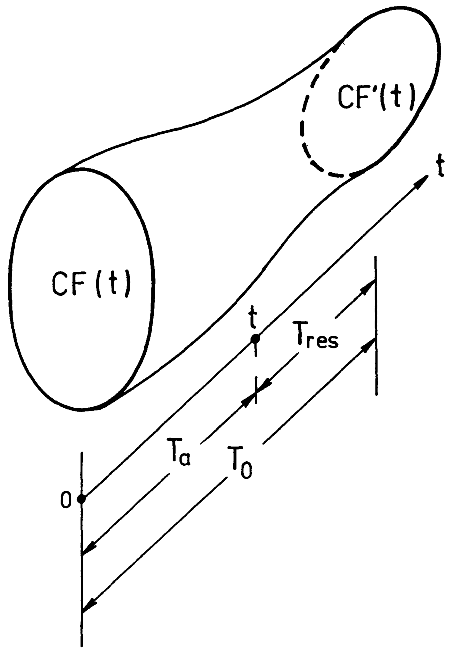

Fig. 1.2 - Time-dependent behavior of a "CONSTRUCTED FACILITY" system (t_0 = moment of erection of the facility, T_a = age of the facility at t , T_0 = life, T_{res} = residual life.

The uncertainty and indefiniteness can be identified not only in partial phenomena but also in subsystems of the S-L-E system and their components, or also in the *linkage of subsystems and components.* In Figure 1.1 this is indicating by undulated lines.

The uncertainties can either be *random* or *non-random.* This depends on the properties of the respective phenomenon. For example, the oscillation of wind velocity is mainly random, though in certain situations the wind velocity can be influenced by non-random phenomena (for example, by buildings in the neighborhood of the facility). In the main, indefiniteness are non-random, despite some randomness caused, for example, by random variations of socio-economic phenomena. It is generally true that uncertainties are *predictable,* and can be preconceived and expressed in unbiased terms, especially in terms of *mathematical statistics,* whereas indefiniteness is *unpredictable,* and can be expressed, in the best case, in terms of *engineering judgment.*

1.2 DEFECTOLOGIC CONCEPTS

In a reliability analysis we have to make clear what is the nature of the adverse effects of time, overloading, insufficient quality of material, poor workmanship, as well as of other effects of human imperfections. Any of these adverse phenomena can be experienced with construction projects of all kind and the task of the designer, contractor, client, etc., is to make the facility as reliable as possible against consequences of such effects. To classify the adverse effects, it is essential to define some basic defectologic concepts.

1.2.1 Flaw

From the onset of its existence, a constructed facility is endowed with *inherent flaws*. *Additional flaws* appear during the life of the CF system. Any component of CF and S-L-E reliability systems is subjected to potential flaws of both types. In general, we can define as flaw a deviation from the expected properties of the system or any of the subsystems. This deviation may be random or non-random.

We can say, though we are not used to doing so, that flaws occur in *load* if, for example, the structure is erroneously or willfully subjected to loads or other actions that were not assumed in design. Or, the structure's *environment* can become flawed, for example, due to corrosive media produced by some uncontrolled technology that virtually could not be envisaged at the time of design or construction. Obviously, load flaws as well as environmental flaws can be avoided by sufficient supervision of the respective facility. Similarly, the structural flaws can be eliminated by relevant quality control and inspection.

Nevertheless, for a thorough reliability analysis the problem of flaws is not so simple. First, we have to state that a flaw in the S-L-E system is only that deviation of subsystem properties that can damage the structure. There are many deviations that are favorable from the viewpoint of reliability of a CF system. Let us give a few simple examples of flaws with reference to the S-L-E system:

Structural flaws: imperfect weld in a steel frame, honeycomb concrete, incorrect positioning of rebars, undersizing of a critical cross-section, excessive amount of knots in wood, weakening of brickwork by an incision; *favorable deviations:* oversizing of a cross-section, higher grade of concrete than required by the designer, etc.

Load flaws: overloading of precast members during construction procedures, erroneous design loading pattern, neglecting loads due to temperature effects, misuse of a pedestrian bridge for a driveway; *favorable deviations:* smaller (or sometimes greater!) dead load due to pavements on floors, erroneous classifying of the site to a snow zone with higher nominal load, etc.

Environmental flaws: corrosive effects of anti-freeze chemicals, high air humidity; *favorable deviations:* wind shading of the facility by adjacent buildings, improvements of the atmosphere by ecological actions, etc.

Obviously, the ***origin of flaws*** can be diverse; we can distinguish:

- *flaws in design specifications,* consisting of incorrect assumptions on the use of the facility, life expectancy, location, geological situation, etc.;

♦ *flaws in codes and other regulatory documents,* consisting of imperfect or incomplete formulation of individual clauses, errors in data, misprints in documents, etc.;

♦ *flaws in design and execution documents,* that is, in calculations, software, structural drawings, shop drawings, etc.;

♦ *flaws in execution and workmanship,* including the material supply, waterproofing, insulation, fire protection, draining, etc.;

♦ *flaws in quality control and quality assurance,* including also flaws of load testing, acceptance procedures, etc.;

♦ *flaws in use,* caused by the user of the facility and other people; they consist in deviations from the expected load, erroneous adjustments of the structure, etc.;

♦ *flaws in maintenance* resulting from insufficient care or lack of care of the facility.

According to their *physical nature,* flaws can be classified as

♦ hidden and manifest;
♦ removable and irremovable;
♦ significant and insignificant.

Theoretically, we also have to differentiate between *flaws that have been discovered* and *flaws that have not been discovered.* A manifest flaw can remain undiscovered whereas a flaw hidden for a certain period can suddenly become apparent.

A flaw can be *permanent* or *transient.* A transient flaw disappears when the circumstances that have caused the flaw vanish. For example, an excessive deflection of a bridge structure can be caused by a temporary overload (a transient flaw in the LOAD subsystem). When the overload is withdrawn, the deflection will disappear or diminish. Obviously, this overload is a *reversible flaw.* If, however, the excessive deflection occurs under current load, it is due to insufficient stiffness of the structure (permanent flaw in the STRUCTURE subsystem) then the excessive deflection is permanent, and the stiffness flaw is *irreversible.*

It is impossible to find CF that remains flawless during its entire life, and thus it may be argued that flaws are unavoidable phenomena. Nevertheless, appropriate measures can always be taken to limit incidence of flaws, the possibilities and extent of such measures depending mainly upon the economical climate. If sufficient funds for *inspection* are available, structural flaws can be substantially reduced or even eliminated, or if they still occur they can be detected and removed. Similarly, load and environmental flaws can be limited by regular *supervision* of the constructed facility, by careful *maintenance,* etc. In general, it can be concluded, that *flaws are preventable phenomena that result from human activity or, on the contrary, non-activity.* Groups and individuals liable for flaws do not only originate from participants involved in a building project, but also from a wider range of people who are closely or remotely connected with CF.

Unfortunately, no official classification of structural flaws has been elaborated in any country, not to mention the international level. In this domain civil engineers get into dispute with lawyers who, as it is well known, have their own, at times rather surprising viewpoints. We should not forget to mention that beyond the *factual flaws,* which we are concerned with in this discussion, the concept of *legal flaws* (flaws in ownership, flaws in contracts, and others) exists. As a rule, legal flaws do not directly affect the structural reliability.

1.2.2 Aberration

During construction and use of a facility, phenomena can occur that could not be foreseen by the designer, contractor, client, and other involved people. As an example, assume that an earthquake occurs in a territory that has not been classified as a seismic zone in the respective code. No provisions for seismic load have been adopted in the design. Therefore, such a load, and also other unforeseen phenomena must be treated as *aberration* from the expected, supposedly current conditions, assumed in design, execution, use, and maintenance of the facility. Nobody can be made liable for aberrations. However, the unpredictability feature of a supposed aberration is often very difficult to prove, particularly in court.

Aberrations can be classified according to the same criteria as flaws, as far as the relationship to the S-L-E system (aberration in structure, load, and environment), or their physical nature (hidden or manifest, removable or irremovable, significant or insignificant, discovered or undiscovered) are concerned. In addition, we have to distinguish between *harmful* and *harmless* aberrations, as, contrary to flaws, some aberrations can have a favorable influence on reliability. In this text, however, the term "aberration" will always be understood as "harmful aberration."

The ***origin of aberrations*** cannot be identified so unambiguously as it is with flaws. Obviously, we cannot talk about an "aberration of the design," etc. The origin of aberrations is in phenomena that were not expected during the period of design. Let us give some simple examples of aberrations:

- ♦ At the time of the design of a highway bridge, heavy trucks had not existed. Therefore, at a later period the occurrence of such loads is an aberration.
- ♦ As a result of buildings erected in the neighborhood of the facility the air flow in the respective area has been changed. The resulting increase of wind load is an aberration.
- ♦ The time-dependent decrease in strength of concrete made with high-alumina cement is an aberration if the structure had been constructed before knowledge on this phenomenon had been collected.
- ♦ The corrosion of reinforcement and other signs of premature deterioration of a concrete structure, generated by carbonation of the surface layers of concrete, is an aberration, since until recently, the carbonation process had not been fully understood.

Observe, that the common feature of all these aberrations is their *unpredictability* and *unavoidability*. If the unpredictability is of random nature, a randomness is dealt with that is more significant than the current randomness covered by initial assumptions.

The difference between the two concepts, flaw and aberration, becomes obvious when *legal aspects* enter the considerations: a discovered flaw can be subjected to court or arbitration trial, even in cases when the flaw is insignificant. Conversely, in the case of an aberration no one can be made liable for its occurrence and consequences. Therefore, aberrations are only rarely subjected to trials (if this happens, this is due to the fact that the respective aberration is taken for a flaw before the situation has been clarified). Possible damages are carried by insurance and re-insurance companies, or by government, or simply by the owner of the facility.

In legalese, aberrations are usually classified as phenomena caused by "force majeure" or "inevitable circumstances." For engineers such terms sound rather rhetorical because they do not express the actual nature of aberrations.

1.2.3 Current use and ageing

While flaws and aberrations are phenomena that are not considered by designers (nevertheless, they are often considered and, in a way, examined by *code makers*), every design has to take into account the deterioration of the STRUCTURE system during current use of the facility, caused by wear and natural ageing of materials, members, and of bearing or non-bearing structural systems as a whole. These two phenomena have to be respected by the user, and an adequate maintenance and other means planned to avoid or limit consequences of the deterioration.

Until recently, no universal and generally valid criteria on deterioration during current use or ageing were available. It is important to observe that the rate of deterioration is approximately smooth, with an obvious acceleration towards the end of life of the structure.

1.2.4 Deterioration and damage

During the life of a constructed facility various time-dependent processes take place, which are caused either by accepted regular phenomena like use and natural ageing, or by phenomena that are always rejected from the beginning - flaws and aberrations. Some of these processes are favorable (for example, the time increase in strength of certain materials). Usually, however, they are adverse, because they degrade the system's reliability. The latter processes are manifest by a *physical deterioration* of the structure, load, and environment. The causes of adverse processes in structures are various, for example: current loading and overloading (stationary and repeated), corrosion of different nature, development of inherent flaws and occurrence of additional flaws, ageing of materials, rheological factors. Load does often not deteriorate but some examples can be given: ponding load, load due to accumulating industrial fall-out. The environment - this deteriorates without any discussion.

The consequence of the deterioration of the system's components is the *physical damage* to the structure. The difference between these two concepts, which are frequently interchanged, is simple: *deterioration is a time-dependent process* whereas *damage is a momentary attribute of the reliability system,* resulting from the deterioration.

In most cases, physical damage can be objectively measured and described, and in this way the deterioration can be assessed. Minor structural damage may be insignificant, and may even remain unobserved.

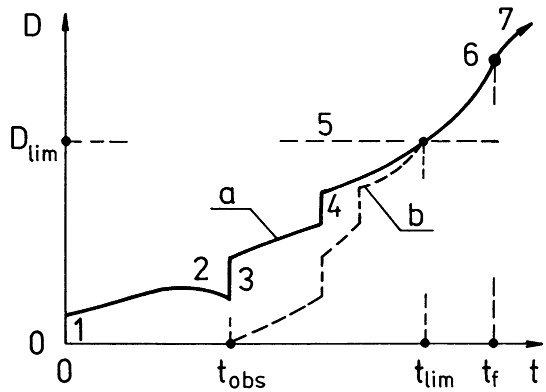

Fig. 1.3 - Development of the deterioration, D, of a reinforced concrete structure in dependence on time, t, up to the serviceability limit state (**a** - actual development of deterioration, **b** - observed development, first deterioration was identified at a flaw in use; **1** - initial deterioration caused by a flaw in design, **2** - reversible development of deterioration resulting from time-dependent increase of the concrete strength and modulus of elasticity, **3** - flaw in use, **4** - load aberration, **5** - first upset of users, **6** - serviceability failure, **7** - serviceability fault; t_{obs} = moment of the discovery of deterioration, t_{lim} = moment of first upset, t_f = moment of failure).

1.2.5 Defect

The deterioration of the S-L-E reliability system, which usually manifest by the deterioration of the structure, is an unbiased phenomenon, which may not be detected, and may not produce any concern to the public, users, owners, and other people having interest in the facility. At a particular point in time, t_{lim}, due to the accumulated deterioration, D_{lim} (Figure 1.3), an unacceptable state of the system is created, and, as a result, a *defect* is registered. Note that a flaw is the *input deficiency* while a defect is the *output deficiency* of the system. Unlike flaws, defects, similarly as deterioration, are non-stationary - they can diminish (*reversible defects*) or expand.

In general, the concept of defect is fuzzy since the level of damage at which a defect is observed depends, first, on the nature of the defect and, second, very much upon the attitudes of people who are either evaluating the state of the structure or who have some emotional, economical, and other kind of interest in CF.

The concept of defect is further complicated because the boundary between flaw and defect depends very much on the attitude of people involved. For example, an excessive deflection of a floor beam is considered a flaw by the owner of the building since he or she does not know the background to this deflection. The same deflection is considered a defect by an engineer who considers it as the result of some flaw, ageing, or aberration.

Defects are *fixed phenomena* related to structures, not to load or environment. The structural adversity levels of various defects are different. It can easily happen that a defect arousing great attention of laymen will not affect the overall system's reliability at all.

1.2.6 Failure

In the reliability analysis of an S-L-E system, only *defects of significant magnitude* are considered serious. This occurs when the following three criteria are fulfilled:

- ♦ the defect has been *observed,*
- ♦ the defect has *significantly changed the functional properties* of the S-L-E system,
- ♦ considering the future use of the CF system, the defect is *harmful.*

The *occurrence of a serious defect* is termed *failure.* Accepting the foregoing mode of thinking, we can say that failure is a *momentary phenomenon*, or in other words, an *event.*

When the above mentioned criteria are not satisfied we cannot talk about failure. Therefore, the moment of failure, t_f, depends on human perception and on the needs of the owners, users, and other people. Failure is always treated as an *adverse event.*

In general, failure is a complex concept. Various aspects of this concept can be demonstrated from the example of a highway bridge that is mostly used by people who have no civil engineering education:

Assume that the deflection of the bridge slowly increases due to bad workmanship. The individuals using the bridge become more and more disquieted, dependently on their psychological attitude to deflections and also on the way the bridge is used. At a certain deflection, the level of disquiet will not be equal amongst pedestrians, drivers, and persons observing the bridge from distance. Obviously, the *demarcation of failure* is fuzzy in this case. - Now, if the workmanship has been so bad that large cracks appear, the group of people noticing the cracks will clearly declare them as a serious defect, that is, a failure. - Finally, when the bridge has collapsed, this fact is accepted as failure not only by users but also by further individuals and groups.

The collapse of a highway bridge that is in everyday use is definitely considered a failure because this kind of defect is detrimental to transport. The collapse of a bridge that is no longer used and where no material damage and injuries are involved, is not a serious defect, and is not considered a failure. Nevertheless, this can change, as soon as the bridge is declared a *heritage structure.*

Following the development of the failure concept in this example, it can be seen that its nature changes from *high subjectivity* for deflections to *high objectivity* for collapse.

1.2.7 Fault

In civil engineering, no particular term for the "state after failure" has been used. In electrical engineering the term *fault* is used (IEV 191-1985), and it seems feasible to accept it in this specific meaning for our vocabulary, too.

Again, faults can either be *reversible* (if the system is able, without any change in the load-bearing structure, to return to the faultless, pre-failure state), or *irreversible* (if the structural consequences of the failure cannot disappear without substantial measures to be taken).

It has to be mentioned here that the term *distress* is often used for damage, or defect, or fault. We are reserving this term for other purposes, see Chapter 11.

1.3 STATES, STAGES, AND SITUATIONS

1.3.1 Limit states

In order to describe the level of deterioration of a constructed facility, some means must be defined for such purpose. For example, the deflection of a structure gives only limited information since the values of interest lie in only a narrow range of the loading process. A more general gage is required, which characterizes the deterioration process widely as possible.

One such gage is *damage* expressed in terms of *costs resulting from a deterioration of CF.* However, not only *effective costs* have to be considered but also all *potential costs* must be taken into account. Consequently, for any magnitude of load, costs C_D expressing damage occurring when that particular magnitude is reached are related. For example, when the structure collapses, damage is given by the value of the destroyed facility (including, of course, the value of the structure itself) and by all losses due to the fact that the facility is no longer of service. More concisely, it is better to express the costs C_D in terms of their ratio to the costs of realization of the respective CF. The relative costs C_D/C_{CF} can gain values in a very wide range. The potential damage can be often many times greater than the *initial costs of the constructed facility,* C_{CF}. For example, collapse of an electric tower can produce damage that is by several orders greater than the initial costs.

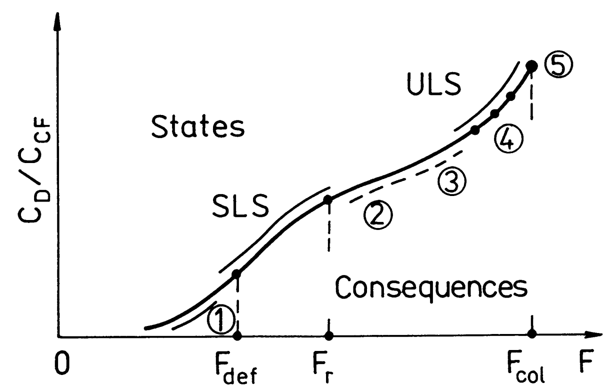

Fig. 1.4 - State profile of a CF system, referred to a given structure subjected to a given type of load, and placed in a given environment (**SLS** - serviceability limit states, **ULS** - ultimate limit states; **1** - maintenance, **2** - repairs, **3** - rehabilitation, **4** - evacuation, **5** - new facility; F_{def} = deformation limit, F_r = first-crack load, F_{col} = collapse load).

For a well defined facility with well defined properties, subjected to a well defined load, a relationship of the C_D/C_{CF} ratio to the load magnitude, F, can be plotted (Figure 1.4). Segments can be identified on this relationship that are characteristic from the viewpoint of the potential damage and also from the viewpoint of the consequences of defects. This relationship is called a ***state profile*** (it was proposed by H.A. Sawyer in 1964, and termed "failure-stage profile"). Economically it would be most effective to design each structure so that the sum of initial costs and all potential costs of maintenance, repairs, rehabilitation, and realization of new facility would be minimum. This is theoretically plausible, but reasonable solutions are not available - the calculation models would be too complex and unpractical, and the results achieved would not be worthwhile. Therefore, a simplified approach is used:

During loading, the load bearing structures pass continuously through *states of stress and deformation,* which we can mathematically describe using structural mechanics, strength of materials, etc. From these continuous states some are selected that are typical for certain levels of deterioration and that we are able to describe by relatively simple means: the state of collapse, the state of first signals of collapse, the state of crack occurrence, etc. Since these states define certain limits on the state profile, they are called the ***limit states.***

Groups of limit states

The bearing structure of CF has to fulfil *two principal requirements* during its entire life (including the periods of execution, transport, erection, etc.)

(a) *it must not collapse, or fail in a similar manner,* so that it should be demolished - this requirement does not refer only to the whole structure but also to its members and cross-sections; obviously, the ***ultimate capacity*** should not be achieved by the structure;

(b) *it should not halt, even temporarily, in fulfilling its functions* for which it has been designed and constructed, that is, it should not behave in a manner demanding limitations or eliminations of the use of CF; so, the structure must always be ***serviceable.***

The requirements of ultimate capacity and of serviceability are not conflicting, but they are not mutually interchangeable. A structure meeting the requirement of ultimate capacity need not meet the serviceability requirement and vice versa. In a general case, both requirements must be checked in the design, inspection, and maintenance. In many instances this is not necessary.

In the state profile, the first requirement is represented by the segment of ***ultimate limit states,*** ULSs, adjacent to the end point of the state profile. Several ultimate limit states can be identified in this segment, dependently on the definition of ultimate capacity. Therefore, we talk about the *group of ULSs.* Similarly, the serviceability requirement is represented by a segment of ***serviceability limit states,*** SLSs.

The principal differences between the two groups of limit states can be characterized, first, by the *nature of defects that appear after a limit state has been reached and exceeded.* When a certain ULS has been exceeded, the subsequent use of the structure is not possible, or only after a large repair or reconstruction. The faults that occur at an *ultimate failure* are, as a rule, *irreversible* and *irremovable.* On the other hand, if SLS has been exceeded, the structure can continue operating after de-loading without substantial measures taken. The defects related to a *serviceability failure* are, as a rule, *reversible and removable.*

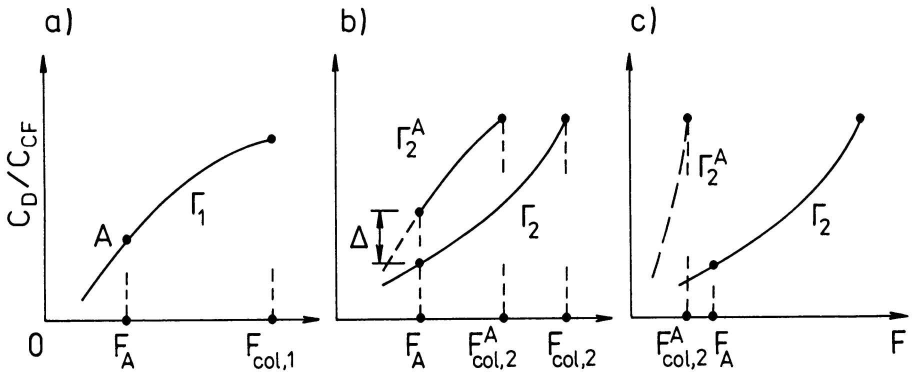

Fig. 1.5 - Two joint structural parts with strung limit states (Γ_1, Γ_2 - initial state profiles, Γ_2^A - state profile of part 2 after the state **A** had been reached in part 1; **a** - behavior of part 1, **b**, **c** - behavior of part 2, **b** - early collapse of part 2, **c** -sudden collapse of part 2 as soon as F_a has been reached).

A wide range of faults can be defined within both groups. In the SLS group, high subjectivity of fault observation prevails, and the definitions of corresponding failures or faults, respectively, are fuzzy. On the other hand, in the ULS group the subjectivity gradually disappears as the economical or moral damage resulting of a particular type of fault increase. Failure leading, for example, to a complete collapse of the structure does not arouse any doubts on its happening and the corresponding after-failure state is unanimously classified as a terminal fault by all individuals concerned.

The second principal difference is in the definitions of the individual limit states. While the ULSs are characterized by properties of the structure or by properties of the STRUCTURE-LOAD system, the SLSs are related to properties of the users of the facility, to properties of the technological equipment, properties of the environment, etc. These properties determine the values of governing variables (for example, deflections) in terms of which respective limit states are specified. The difference between the two groups of limit states affects the choice of the *reliability parameters* (see Chapter 10).

Limit states strings

Although in the past ultimate failures of structures were not rare, their consequences had not been so spectacular to encourage a detailed investigation and generalization of the deterioration processes, dependence of defects and developments of failures and faults. But several extensive catastrophes occurred during past decades attracted particular attention to the problems of behavior of structures as complex reliability systems. The most important conclusion is that the attainment of ultimate capacity is not a single phenomenon but a *result of a complex process.* This process can be continuous but it can be resolved in separate phases, analogously as we had specified groups of limit states on the state profile.

Let us examine two simple structures that are part of a larger system, and let us assume that their individual state profiles (Figure 1.5) when each of the structures is loaded separately are known. After joining the two parts, the state profile Γ_1 of the part 1 (Figure 1.5a) is obviously affecting the profile Γ_2 of part 2 and vice versa.

Suppose, to simplify the consideration, that Γ_2 changes only then when in Γ_2 a state A is reached. The new profile of part 2, Γ_2^A, is theoretically valid starting from zero value of the load F. Clearly, the relative damage in part 2 will suddenly increase by Δ - Figure 1.5b.

In an extreme case the state profile Γ_2 can, after the state A has been reached, change so much that the collapse load $F_{col,2}^A$ can be less than the original $F_{col,2}$, and, consequently, the structure will instantly collapse (Figure 1.5c). This example is very simplified but it is realistic, as can be seen from the following:

Under specific circumstances a *string of different limit states* can form; they can even belong to different groups. As a result, processes may develop that can be termed

- ***progressive deterioration,*** composed mainly of serviceability limit states;
- ***progressive collapse,*** composed mainly of ultimate limit states.

No general rule on the order of limit states in the string can be given. For example, the defect of a bridge support caused by vehicle impact can result in an excessive deflection of a main beam of the bridge. The word "mainly" implies that limit states can be mixed; in the first case some ULSs can be reached, and similarly, SLSs can participate in and even start progressive collapse.

1.3.2 Stages

During the life of CF the structure forming part of the S-L-E reliability system passes through a series of various stages. The stages differ by:

- *the arrangement and properties of the bearing system* (the system of supports, spans, critical lengths, etc., changes);
- *loads* (loading pattern, load combinations);
- *the age of the structure* (properties of materials change);
- *duration.*

For practical reasons, two groups of stages are distinguished, as a rule:

♦ ***execution stages*** (including production, erection, loading test, current repairs, restoration, dismantling); these stages usually take only a part of CF's life (about 1/50 to 1/20), in specific cases they can be even longer;

♦ ***utilization stages;*** they cover the major part of life; in the main, only one utilization stage is relevant.

The differences in stages affect the analysis and design on various levels: they have to be considered in the variables describing material properties (strength, elastic modulus, etc.) and also in the reliability parameters.

1.3.3 Design situations

Additionally to the different stages, different situations that the S-L-E system can experience during its life must be considered in the design. The individual situations are distinguished by structural patterns, types and arrangement of loads, environmental parameters, and also by reliability requirements and reliability levels. The principal criterion of difference between the individual situations is the probability of their occurrence (contrary to stages, which always must occur):

Permanent design situation. This has the period of duration $T_{sit,p}$ of the same order as the life of the facility, T_0. For example, it is the period between the realization to the change in use of the facility, or the period between two changes in use. The probability of occurrence of the permanent situation is $P_{sit,p} = 1$, because this situation must always arise.

Transient design situation. Its period of duration $T_{sit,t}$ is considerably shorter than T_0. For example, it is the period of execution, period of restoration, period of reconstruction, period of crossing a bridge by extra-heavy vehicles, etc. The probability of occurrence of the transient situation depends on the respective purpose. The execution must always take place, that is, $P_{sit,t} = 1$, whereas a future restoration of the facility is never a sure occurrence, $P_{sit,t} < 1$.

Accidental design situation. In specific conditions the structural system can suddenly change due to external, structure-independent phenomena, and, consequently, new loading patterns can arise. As a rule, this type of events occurs as a result of some accident. *This is the reason why the ensuing situation is termed accidental.* A significant feature of this situation is a very short period of duration, $T_{sit,a} \ll T_0$, and less severe performance requirements (for example, in an accidental situation nobody is really interested in deflections, crack width, etc.). The principal requirement is the possibility of performing rescue operations, evacuation, temporary supporting, and other related activities. The probability of occurrence of an accidental situation during the life of the facility is very small, say $P_{sit,a} = 0$ to 1.0E-6.

The concept of the accidental situation is often misunderstood. It is related erroneously to "accidental load." Structures are designed for accidental load in permanent as well as transient situations. Only when the load exceeds a certain magnitude, *without* *being necessarily classified as accidental,* or when an entirely unexpected load appears, then an accidental situation may or may not arise. Its occurrence can be induced also by other phenomena (for example, fire and explosion).

1.4 REQUIREMENTS, CRITERIA, AND PARAMETERS

Let us discuss a hypothetical case of a *fully defined S-L-E reliability system;* it does not contain any uncertainties and indefiniteness (see Section 1.1) and the properties of the three subsystems are perfectly known. When the reliability of this system is to be assessed, the ***relations*** in the system have to be described in such a way that it is possible to decide whether the system is reliable or not. These relations must be based on the physical description of phenomena entering the particular components of the system, and, therefore, they will be called ***physical reliability requirements.*** - The term "physical" is used here to stress the objectivity of the requirements and their independence on human decisions. Physical laws can of course describe phenomena that in their substance are of statistical nature (for example, the Boyle-Mariotte law).

In physical reliability requirements scalar variables and vectors of distinctive kind appear. The physical nature of these variables and vectors is denoted as "design criterion." A ***design criterion*** can be, for example, the axial force in a compressed member, deflection at mid-span, vibration frequency. It can even be a quantity that is not a load-effect: we can state a reliability requirement in terms of the cross-section area of a prestressing tendon, depth and width of a beam with a rectangular cross-section, width of a foundation strip, etc.

When properties of the system investigated are not exactly known, ***uncertainties*** and ***indefiniteness*** must be taken into account. The physical reliability requirements must either be adjusted by parameters covering the uncertainties and indefiniteness, or supplemented by further requirements. When the adjustments are based on experience, or also on theoretical considerations, but without regard to the randomness of phenomena, the respective requirements are called *deterministic.* If, however, the uncertainties of the S-L-E system are treated as random, they can be expressed in terms of the *probability of occurrence of adverse realizations of the respective phenomena.* Then, ***probabilistic reliability requirements*** can be formulated; this subject is elaborated in Sections 8.1 through 8.3.

The term "deterministic" is often used for physical and empirical formulas, for decision-based values of input variables, or simply for fixed physical constants. We will avoid it in this book at all; where necessary, the relevant quality of the formula, variable, and constant will be designated by the appropriate term.

If the random behavior of phenomena is expressed in the reliability requirements by purely mathematico-statistical procedures, without establishing the design parameters by means of probability concepts (the Hasofer-Lind reliability index method belongs into this family; see Sections 8.5 and 9.2) The respective requirements are called ***statistical;*** they are discussed in Section 8.5.

By synthesis of physical and probabilistic or statistical reliability requirements ***design requirements*** are obtained. These are contained in the design codes or can be, in particular cases, individually specified.

Quantities that, in design, govern the reliability level are called ***reliability parameters.*** Two principal reliability parameters must be considered in the reliability requirements of the present design methods: the *target failure probability,* P_{ft}, and the *target life,* T_{0t}, see Chapter 10. These are *primary parameters* based always on some decision. The older design methods were built-up on other primary parameters, such as the *safety factor* or, more exactly, *reliability factor.* The target life had not been taken into account.

As a rule, reliability parameters do not directly apply in the codified design requirements. They serve mainly code makers for the derivation of *design parameters* (partial reliability factors, characteristic strengths of materials, representative values of load, and others). Again, these parameters are subjected to decisions based on experience and calibration of codes, because the exact reliabilistic methods cannot be applied in general.

The set of concepts formed by particular design criteria, design requirements, and design parameters is usually termed the ***design format,*** which can be *theoretical* or *codified.*

2

TOOLS

Key concepts in this chapter: *probability; statistics; random phenomena; random events; prior and posterior probability; conditional probability; random variable; population; random sample; population parameters; sample characteristics; union of samples; standardized random variable; probability distribution; CDF, PDF, IDF; fractile; truncated distribution; joint probability distribution; estimation; hypotheses testing; statistical dependence; statistical dependence function; correlation coefficient; response function; random function; random sequence; random process; autocorrelation function; spectral density function; repeated events; mean return period; reliability systems, elements, and items; reliability connections; reserve; reliability function; failure rate; bath-tub curve; life; method of moments; quasi-parameters; Monte Carlo simulation; draw, trial, and realization; random number generator, RNG; histogram; seed number; execution time; ordering algorithm.*

2.1 PROBABILITY AND STATISTICS

Nearly all monographs on structural reliability contain one or more chapters on the theory of probability and mathematical statistics. Probability and statistics are fundamental tools of reliability theory; they are used extensively in a range of exercises, from solutions of sophisticated theoretical problems to everyday rules of quality control and assurance. Reliability theory relies on the theory of probability and statistics.

The amount of literature on probability and statistics is enormous. It is not intended to repeat here information that is readily accessible. Instead, we will concentrate on only some particularities that are either not simply available, or are a frequent source of misunderstanding. This affects the order of presentation of various concepts.

It is assumed in this book that the reader has some basic knowledge in probability and statistics; nonetheless, it is useful to give newcomers an indication on some of available monographs:

♦ ***general:*** Beaumont 1983, 1986, Hines and Montgomery 1990, *Wahrscheinlichkeitsrechnung* 1983;

♦ ***specialized, aimed at structural reliability problems:*** Ang and Tang I 1975, Augusti *et al.* 1984, Benjamin and Cornell 1970, Harr 1987, Madsen *et al.* 1986, Melchers 1987, Smith 1986.

2.1.1 General concepts

Phenomenon, event

Statistics deals with ***random events***, which are *realizations* of certain ***collective random or non-random phenomena.*** Phenomena will be designated by H or by the operation symbol Ph(.); events will be designated by E or, in operation notation, by Ev(.).

Mathematical description of random events is given by ***random variables, random functions,*** and ***random sequences.***

■ **Example 2.1.** The annual return of the winter period is a collective phenomenon, Ph(**winter**). This phenomenon is obviously *non-random* if the winter is defined astronomically. If, however, the occurrence of snow, Ev(**snow**), is taken as the criterion of the winter, than, for example, in the last 10,000 years the winter period definitely has been a non-random phenomenon in Luleå, Sweden, while it has been random in Chengdu, China.

To continue: if Ev(**snow**) arises, than Ev(**snow drift**) is possible (it may or may not happen). Observe that the latter event is possible only if Ev(**snow**) happened.

The weight of accumulated snow, s , producing snow load, is a random variable. The weight varies in time; it is time-dependent. It also varies in space. The time and space variations of snow load are described by random functions. ■

Probability

It is not necessary to elaborate the concept of probability here. Let us only reiterate some commonly known formulas which will be quoted later in the text.

Let the fact that the occurrence of an event Ev(X) is sure be expressed by P = 1; that it is impossible, by P = 0. The fact that under given conditions Ev(X) may or may not happen is expressed by $P \in [0; 1]$. P is termed the probability. - The operation symbol Pr(X) will be used for the probability of occurrence of Ev(X).

The value of probability can be defined in various ways. In simple cases we can write

$$P = \frac{m}{n} \tag{2.1}$$

where m = number of cases when Ev(X) *happens,* n = number of all cases when it *can happen.*

We will avoid the details and discussing the important concepts of the *prior probability* and the *posterior probability;* we will not need them in this book. Nevertheless, the reliability engineer should distinguish between

♦ ***prior probability*** (also called *subjective*) that is based on experience and believe; decisions on its value are made by speculation reflecting the engineering judgment; and

♦ ***posterior probability*** (also called *frequentist*) that is based on the analysis of measurements and observations; its values are obtained by calculations, not by judgment.

If, for example, Ph(**winter**) is defined astronomically, and at the given site Ev(**snow**) happened five times in 23 years of observation, than the estimated posterior probability of snow occurrence in the next winter period is $P = 5/23 = 0.217$. This, however, is only an estimate based on 23 data; the actual probability can be considerably either less or greater than 0.217.

When two or more random events are dealt with, it is necessary to know some formulas giving information on various kinds of their simultaneous or sequential happenings or non-happenings. Although the following formulas can be found in any monograph on probability and statistics, we give them here since most of them will be referred to later.

♦ Probability that Ev(X) will happen:

$$0 \le \mathrm{Pr}(X) \le 1 \tag{2.2}$$

♦ Probability that Ev(X) will never happen:

$$\mathrm{Pr}(\bar{X}) = 1 - \mathrm{Pr}(X) \tag{2.3}$$

♦ Probability that at least one of *mutually exclusive events* Ev(X) and Ev(Y), with respective Pr(X) and Pr(Y), will happen:

$$\mathrm{Pr}(X \cup Y) = \mathrm{Pr}(X) + \mathrm{Pr}(Y) \tag{2.4}$$

If in this case a calculation gives $\mathrm{Pr}(X \cup Y) > 1$, it is a sign that the two events are not exclusive or that the event probabilities were wrongly assessed.

♦ Probability that two *independent events*, Ev(X) and Ev(Y), will happen simultaneously or sequentially:

$$\mathrm{Pr}(X \cap Y) = \mathrm{Pr}(X) \cdot \mathrm{Pr}(Y) \tag{2.5}$$

♦ Probability that at least one of the two *mutually non-exclusive events*, Ev(X) and Ev(Y), will happen:

$$\mathrm{Pr}(X \cup Y) = \mathrm{Pr}(X) + \mathrm{Pr}(Y) - \mathrm{Pr}(X) \cdot \mathrm{Pr}(Y) \tag{2.6}$$

♦ Probability of Ev(X) *given* Ev(Y), where the two events are *mutually non-exclusive* (***conditional probability***):

$$\mathrm{Pr}(X|Y) = \frac{\mathrm{Pr}(X \cap Y)}{\mathrm{Pr}(Y)} \tag{2.7}$$

When Ev(X) and Ev(Y) are independent, it results from Equations (2.7) and (2.5)

$$\Pr(X|Y) = \frac{\Pr(X)\cdot\Pr(Y)}{\Pr(Y)} = \Pr(X) \tag{2.8}$$

The foregoing basic rules can be applied in the development of various formulas required in the solution of many problems. The following formulas can be useful (bars and overline indicate that the respective events will not happen; the events are considered *independent*):

♦ Ev(X) will happen, Ev(Y) will not happen:

$$\Pr(X \cap \bar{Y}) = \Pr(X)\cdot[1 - \Pr(Y)] \tag{2.9}$$

♦ neither Ev(X), nor Ev(Y) will happen:

$$\begin{aligned}\Pr(\bar{X} \cap \bar{Y}) &= [1 - \Pr(X)]\cdot[1 - \Pr(Y)] \\ &= 1 - \Pr(X) - \Pr(Y) - \Pr(X)\cdot\Pr(Y)\end{aligned} \tag{2.10}$$

♦ just one of the events Ev(X), Ev(Y) will happen:

$$\begin{aligned}\Pr[(X \cap \bar{Y}) \cup (\bar{X} \cap Y)] &= \Pr(X)\cdot[1 - \Pr(Y) \\ &\quad + [1 - \Pr(X)]\cdot\Pr(Y) \\ &= \Pr(X) + \Pr(Y) - 2\Pr(X)\cdot\Pr(Y)\end{aligned} \tag{2.11}$$

♦ maximum one of the events Ev(X), Ev(Y) will happen:

$$\Pr(\overline{X \cap Y}) = 1 - \Pr(X)\cdot\Pr(Y) \tag{2.12}$$

♦ minimum one of the events Ev(X), Ev(Y) will happen:

$$\begin{aligned}\Pr(\overline{\bar{X} \cap \bar{Y}}) &= 1 - [1 - \Pr(X) - \Pr(Y) + \Pr(X)\cdot\Pr(y)] \\ &= \Pr(X) + \Pr(Y) - \Pr(X)\cdot\Pr(Y) \equiv \Pr(X \cup Y)\end{aligned} \tag{2.13}$$

Population, random sample, and random variable

In the statistical analysis we deal with *collections*, that is, sets of events, or sets of data on events. Two types of statistical collections must be distinguished:

♦ **Population** is a set of all possible happenings of a random event. It can be either finite or infinite. As a rule, a population cannot be physically compiled, and therefore, its properties must be assessed by *estimation* based on one or more random samples.

♦ **Random sample** is a subset of happenings of the random event, which has been randomly obtained by measuring and observing a finite number of happenings of events that belong to a population. All happenings must have the same possibility of being included into the subset.

The concepts of population and random sample are often being confused. As a result, correct *estimates of population parameters* (see 2.1.7) are not established, and sample characteristics are introduced into calculations as population parameters without any adjustment. Serious errors can occur.

A random event is mathematically described by a ***random variable*** ξ; that is, a number x is assigned to each happening expressing the happening's magnitude. A set of such numbers, x_1 through x_n, gives information that can be statistically evaluated. - Greek letters are often used for random variables. Unfortunately, we are not able to keep to this convention throughout; symbols like f_y (for the yield stress of steel) and others are difficult or impossible to express in Greek characters. Therefore, Greek letters will be only used when necessary for clarity.

■ **Example 2.2.** All axle loads of vehicles that will act on a bridge structure during its life constitute a population. Values of axle loads observed by means of a scaling device at a measuring place during a specified period form a random sample. Yet, we have to consider whether the place and the period have been chosen in a random way. In other words, examining data obtained at a certain place during a certain period, we must ask what is the corresponding population. ■

2.1.2 Distributions, parameters, and characteristics

Functions

The behavior of random variables is described by ***probability distributions. Discrete variables*** appear only in very special cases of structural reliability problems (see, for example, Tichý and Vorlíček 1973 on variables in the evaluation of fatigue tests). Therefore, *continuous variables* will be considered in this book, except for Section 5.5 where a mixed *continuous-discrete* distribution of load magnitudes will be introduced.

Two typical functions are of practical importance in our considerations:

♦ ***Cumulative distribution function, CDF.*** We can consider it here basic (however, in the mathematico-statistical theory, the *moment-generating function* is usually considered primordial). For CDF it holds

$$0 \le \Phi(x) \le 1 \tag{2.14}$$

where

$$\Phi(x) = \Pr(\xi \le x)$$

♦ ***Probability density function, PDF.*** This function, $\varphi(x)$, is defined by

$$\Phi(x) = \int_{x_{inf}}^{x} \varphi(x)\mathrm{d}x \tag{2.15}$$

where x_{inf} = lower limit of the probability distribution. If there is no lower limit, then $x_{inf} \rightarrow -\infty$. - PDF gives a more graphic idea on the behavior of the random variable than CDF.

In many solutions, the ***inverse distribution function, IDF,*** is used. This function is defined through $\Phi(x)$ by

$$x = \Phi^{-1}(P) \tag{2.16}$$

where x = value of the random variable, ξ, P = given probability.

Parameters

In the mathematical description of probability distributions ***population parameters*** arise. As a rule, not more than four parameters apply in reliability solutions; the most common are:

- ♦ *mean,* μ;
- ♦ *variance,* σ^2;
- ♦ *coefficient of skewness,* α;
- ♦ *coefficient of excess,* ε.

In Section 2.3 the concept of ***quasi-parameters*** will be introduced, which is helpful in some calculations.

Instead of α and ε other parameters are frequently used; in this monograph we will keep mainly to μ, σ, and α. The coefficient of excess, ε, is, as a rule, only an auxiliary parameter.

Often, the *population coefficient of variation,* $\delta = \sigma/\mu$, is given as a measure of random variability. However, this derived parameter must be always considered with caution (see Sample characteristics below).

The lower and upper limits of a population are termed the *population infimum,* x_{inf}, and *supremum,* x_{sup}, respectively. These two parameters can sometimes be physically specified. For example, supremum of the randomly fluctuating water level in an open tank is defined simply by the brim.

Standardized random variable

The description of a random variable ξ is often simplified by introducing the standardized random variable

$$u = \frac{\xi - \mu}{\sigma} \tag{2.17}$$

Its mean is equal zero and the variance is one. The coefficients of skewness and excess are the same as for the non-transformed variable. The coefficient of variation is of course not defined.

Fractile

In many places in this monograph fractiles of random variables will be discussed and used as dominant quantities. In fact, fractile has many possibilities of application; it governs, first of all, the reliability requirement formulas at various levels. The concept of fractile is well known; therefore, let us give here only principal information on the notation that will be used throughout the following sections and chapters.

Consider a random variable ξ, whose first two parameters are the mean, μ, and the standard deviation, σ. A value x_κ is to be established for which

$$\Pr(\xi \leq x_\kappa) = \kappa \tag{2.18}$$

where κ = given value of the probability. The value x_κ is called the *κ-fractile of the random variable ξ.*

We are often interested in a fractile, x_κ, defined by

$$\Pr(\xi > x_\kappa) = \kappa \tag{2.19}$$

In the main, $\kappa < 0.5$, and so the fractiles defined by Equations (2.18) or (2.19) are situated at the left-hand or right-hand tail of PDF. We call them the *lower* and *upper fractile,* respectively.

The widely used expression for the κ-fractile of ξ is obtained from Equation (2.17):

$$x_\kappa = \mu + u_\kappa \sigma \tag{2.20}$$

where u_κ = κ-fractile of the standardized random variable u.

For the normal distribution, tables of CDF giving u_κ are presented in the majority of statistical monographs, and suitable programs can be found in any software library. Yet, for other distributions tables do not exist or have not been published. Then, it is necessary to calculate u_κ as the value of the inverse distribution function $\Phi^{-1}(P)$ for P = κ, or $P = 1 - \kappa$, whichever applies. As a rule, the problem has to be solved by approximation formulas or by iterations. For the three-parameter log-normal distribution values of u_κ are given in Appendix A. There, also values referring to the normal distribution can be found, taking simply $\alpha = 0$.

Selection of a probability distribution

The problem of selecting the appropriate probability distribution for a random variable is often considered crucial. Great attention is paid to which distribution should be used in probability modeling of a particular variable. In most cases such care is futile because an absolutely true description of random behavior can never be accomplished. Engineering judgment is necessary in finding the right distribution. The following steps are useful:

♦ Consider the possible *shape of PDF.* Is it bell-shaped? Symmetric? Asymmetric? Truncated?

♦ Consider *physical bounds* of the variable. Are there any?

♦ Estimate the *population parameters* (see 2.1.7).

♦ Plot the *probability density;* compare it visually with the histogram obtained from sample (if there is any). In doing so, consider whether the size of sample is sufficient enough to give a graphic histogram.

♦ Perform some statistical *goodness-of-fit tests.*

All books on structural reliability give basic information on several probability distributions; therefore, it will not be repeated here. Let us only give a survey of the most important distributions met in practice. Some are defined by two parameters (2P), others by three or even four parameters (3P, 4P).

♦ ***Rectangular distribution,*** 2P; symmetric; lower and upper bound. It can be used, for example, in modeling components of time processes when no better information is available (for example, in modeling the random duration of certain state).

♦ ***Exponential distribution,*** 2P; L-shaped; positively asymmetric; lower bound. The use is similar as that of the rectangular distribution. A J-shaped version can also be defined.

♦ ***Normal distribution,*** 2P; symmetric; bell-shaped; no bounds. The most common distribution, used in many practical problems.

♦ ***Log-normal distribution,*** 3P; bell-shaped; positively or negatively asymmetric, the symmetric form being identical with normal distribution; lower or upper bound. A *two-parameter* form of log-normal distribution with lower bound equal zero is commonly used; it is positively asymmetric. Yet, the more general *three-parameter* log-normal distribution is an effective tool for many problems where asymmetric variables are encountered. It is easily programmable. A detailed, though not exhaustive description of the three-parameter variant is given in Appendix A.

♦ ***Beta distribution,*** 4P; bell-shaped, J-shaped, L-shaped, U-shaped; lower and upper bound; rectangular distribution is a special case of the beta distribution. This is a very attractive distribution because of its lower and upper bounds and various shapes. It can be efficiently used in diverse problems. However, similarly as it is with other four-parameter distributions, its main drawback is that fitting to data is usually difficult when all four parameters are taken from observations. Some peculiar, unrealistic shapes of beta distribution (not displayed in available publications) can be obtained. It seems that the Nature does not like more than three parameters. A graphical plot of PDF is always recommended. - A practical *probability paper* can be based on a symmetric bell-shaped variant, see Appendix B.

♦ ***Distributions of extreme values,*** 3P in general; positively or negatively asymmetric; bell-shaped; lower or upper bound, or no bounds. This is a widely known family of distributions used whenever some extremes (minima or maxima) of phenomena enter the reliability calculation models.

Special technical questions arise when *direction-dependent data* have to be analyzed; these are encountered, for example, in the examination of wind load and sea waves load (see Mardia 1972).

Truncated distributions

On many occasions we have to deal with phenomena that have been in some way artificially confined so, that the lower or upper tail of the respective parent distribution or both have been cut off, Figure 2.1. Such distributions are called *truncated distributions.*

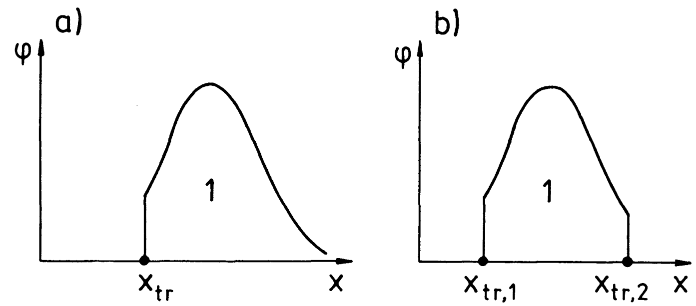

Fig. 2.1 - PDF of truncated probability distributions (**a** - left-hand-sided, **b** - two-sided).

Let us give here some useful formulas on the ***left-hand truncated distribution,*** Figure 2.2, which is met, for example, in the investigation of load magnitudes (see Section 5.4). For a right-hand truncated distribution the formulas are analogous.

Assume that a *parent distribution* exists, whose $\varphi(x)$ and $\Phi(x)$ are known. The point of truncation, x_{tr}, is, as a rule, well defined (for example, by a decision); let us establish $\Pr(\xi \leq x_{tr}) \equiv P_{tr} = \Phi(x_{tr})$. Since the area under the truncated PDF must be equal to one (Figure 2.2b), the *left-hand truncated PDF,* whose definition domain is $x_{tr} \leq \xi \leq x_{sup}$ (we can, of course, have $x_{sup} \to \infty$), is given by

$$\varphi^*(x) = \frac{1}{1 - P_{tr}} \varphi(x) \tag{2.21}$$

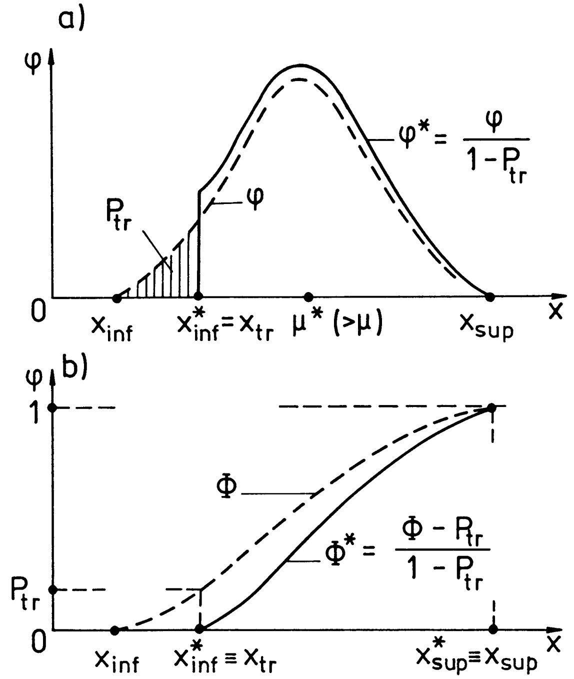

Fig. 2.2 - PDF and CDF of a left-hand truncated distribution (**a** - PDF, **b** - CDF; dashed line = parent distribution).

Analogously, the *truncated CDF* (Figure 2.2c) is

$$\Phi^*(x) = \frac{1}{1 - P_{tr}} [\Phi(x) - P_{tr}] \tag{2.22}$$

with $0 \leq \Phi^*(x) \leq 1$.

It is apparent that the left-hand truncated distribution has, additionally to those of the parent distribution, one supplemental parameter, P_{tr}, or x_{tr}. Thus, for example, in the case of a truncated three-parameter log-normal distribution, four parameters must be known.

Parameters of a truncated distribution can be calculated analytically only in simple cases (for example, for truncated normal distribution, where also tables exist). In most cases numerical integration has to be used. However, when samples are analyzed, we are usually not specifically interested in calculating the parameters, since we get their point estimates from the respective sample (see 2.1.7). Fractiles are usually the aim objective.

From a sample, the characteristics m, s, and a are obtained; the truncation point, x_{tr}, is defined. For the left-hand truncated distribution, it is $x_{tr} \equiv x^*_{inf}$. Given P, we look for

$$x = \Phi^{-1*}(P)$$

but the truncated IDF is, as a rule, not known. Therefore, setting in Equation (2.22) $\Phi^*(x) = P$, we get after rearrangement

$$x = \Phi^{-1}(P^*) \qquad (2.23)$$

where

$$P^* = P + (1 - P)P_{tr} \qquad (2.24)$$

Obviously, some assumption on the parent distribution with a known $\Phi^{-1}(.)$ must be accepted, and its parameters have to be settled. For the *truncated three-parameter log-normal distribution,* TLN(u_{inf}, α), the respective procedure is shown in Appendix A.

If the truncation of a phenomenon is not perfect, a certain part of realizations of ξ can be found beyond the truncation points. Then, instead of a truncated distribution, a *censored distribution* has to be considered.

Many other probability distributions could be mentioned here (for probability distributions of repeated events see 2.1.6). The interested reader is referred to specialized publications, particularly to Hahn and Shapiro 1967, Johnson and Kotz 1970a, 1970b, and 1972, and further also to *A Modern Course* 1974 and Cornell 1972. Nevertheless, advice from an experienced statistician or reliability engineer is useful. As a rule, the experts will suggest to use surprisingly simple distributions.

Fortunately, the results of probability solutions are, in reasonable limits, little sensitive upon the choice of probability model. *Parameters and their proper handling are more important.*

The Author's preference are the first four distributions mentioned above, with emphasis on the three-parameter log-normal one. For certain reliability techniques *special criteria* can affect the choice of probability distributions (see, for example, Lind and Chen 1987).

Multi-modal distributions

Any sample with a multi-modal histogram (or a multi-modal frequency curve) should be carefully examined to consider whether it does not consist of two or more independent samples, which have been merged into one. Several situations can lead to multi-modal frequency curves; for example:

- measurements of axle loads on highways show distinct bi-modal distributions that are caused by the two principal groups of vehicles: trucks and cars;
- wind velocities observed in coastal areas are often bi-modal since two types of wind are included into one sample: regular continental winds and cyclones; the random behavior of these winds is very different.

Multi-modal frequency curves are always suspicious. Whenever you obtain such a curve, be cautious, and try to find out what is behind it. A natural explanation is plausible in rare cases only.

Sample characteristics

Samples are described by *sample characteristics* which are quantities defined mainly by *ordinary* and *central moments* of data obtained through observations, measurements, etc., and also in other ways (dependent on the type of the characteristic). In Table 2.1 sample characteristics that can appear in current solutions are given; most of them are well known. However, formulas for a and e are not common.

It should be noted that *sample characteristics are random variables,* and so the relations between sample characteristics and population parameters are of random nature. Therefore, *population parameters* can be established from sample characteristics only by *estimation* with a certain amount of incertitude (see 2.1.7).

To obtain reliable information on sample characteristics that can be used in calculations, decisions, etc., a sample must be sufficiently large. The higher the order of the respective characteristic, the greater should be the sample size. According to experience, we need for

- mean: $n > 10$;
- standard deviation: $n > 20$;
- coefficient of skewness: $n > 100$.

To illustrate the problem, consider the normal distribution (that is, $\alpha = 0$). The sample coefficient of skewness has a distribution with $\mu_a = 0$ and

$$\sigma_a = \sqrt{\frac{6(n-2)}{(n+1)(n+3)}}$$

with n = sample size. Analysis shows that even for $n = 100$ we can obtain $-0.47 < \alpha < +0.47$ in 95 percent of cases!

Thus, when, for example, a sample of 30 data is available, the information on the coefficient of skewness is very poor. In that case it is better, when the phenomenon is for some reason considered "skew," to *assess the skewness by speculation.* And conversely, *when from a sample analysis* $a \neq 0$ *results, it does not mean that the true probability distribution is not symmetric.*

Union of two samples

On many occasions you can obtain characteristics of two or more samples that have been taken *from the same population.* It often happens that the data on observations are missing. Then, when characteristics m_1, s_1, a_1, m_2, s_2, and a_2 are given, characteristics of the *unified sample* can be calculated from

Table 2.1 - Sample characteristics; n = sample size; ϑ = corresponding population parameter

Sample characteristics	Description	ϑ
Mean, m	$m = \frac{1}{n}\sum_{i=1}^{n} x_i$	μ
Median, $\tilde{x}$	value dividing the ordered data in two equal parts	$\tilde{\mu}$
Mode, $\hat{x}$	value corresponding to the peak of the sample frequency curve	$\hat{\mu}$
Variance, s^2	$s^2 = \frac{n}{n-1} s_0^2; \quad s_0^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - m)^2$	σ^2
Standard deviation, s	$s = \sqrt{s^2}, \quad s > 0$	σ
Coefficient of variation, C_v	$C_v = s/m$	δ
Coefficient of skewness, a	$a = \frac{\sqrt{n(n-1)}}{n-2} a_0;$ $a_0 = \frac{1}{(s_0^2)^{3/2}} \cdot \frac{1}{n}\sum_{i=1}^{n}(x_i - m)^3$	α
Coefficient of excess, e	$e = \frac{n(n-1)}{(n-2)(n-3)}(e_0 + 3) - 3;$ $e_0 = \frac{1}{(s_0^2)^2} \cdot \frac{1}{n}\sum_{i=1}^{n}(x_i - m)^4 - 3$	ε
Minimum, x_{min}	$x_{min} = \min(x_1, x_2, \ldots, x_n)$	x_{inf}
Maximum, x_{max}	$x_{max} = \max(x_1, x_2, \ldots, x_n)$	x_{sup}
Range, R	$R = x_{max} - x_{min}$	-

$$m = \frac{n_1 m_1 + n_2 m_2}{n}$$

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n} + \frac{n_1 n_2}{n^2}(m_1 - m_2)^2$$

$$a = \frac{1}{s^3}\left[\frac{n_1 s_1^3 a_1 + n_2 s_2^3 a_2}{n} + \frac{3 n_1 n_2 (m_1 - m_2)(s_1^2 - s_2^2)}{n^2} - \frac{n_1 n_2 (n_1 - n_2)(m_1 - m_2)^3}{n^3}\right]$$

These formulas are useful in some Monte Carlo simulation exercises (Section 2.4).

Coefficient of variation

Here we should remark on the coefficient of variation, C_v, or also δ. It is rather a tricky characteristic, which can be either misused, or be a source of misinterpretation. We should always keep in mind that, given a constant standard deviation, the coefficient of variation increases hyperbolically with the decreasing mean. There are many phenomena whose statistical description leads to about zero mean, though their spread is small. The coefficient of variation tends to infinity in such cases, which can be misleading. *Readers are advised to assess carefully any information given in terms of coefficient of variation.*

■ **Example 2.3.** A ready-mixed concrete manufacturer, A, boasts that they are supplying concrete with a coefficient of variation of the compression strength only 0.05, while a competitive manufacturer B cannot achieve less than 0.08. Therefore, manufacturer B is considered worse.

Analysis of the data shows that the two numbers refer to concrete of different grade, with mean strength equal 40 N.mm^{-2} and 20 N.mm^{-2}, respectively. Since $\sigma = \delta \mu$, the standard deviation observed at A is 2 N.mm^{-2} and at B is 1.6 N.mm^{-2}. Obviously, company B is able to supply concrete of higher quality, as far as the spread of its compression strength is concerned. ■

The coefficient of variation should be taken only as an *auxiliary quantity*. We will try to avoid its use in this book, though on some occasions it is needed to simplify notation.

2.1.3 Multivariate problems

Random variables often appear in pairs, triplets, or n-tuplets, $(\xi_1, \xi_2, \ldots, \xi_n)$, forming samples and populations, similarly as single random variables. When collecting a *multivariate random sample,* the same rules must be observed as when collecting a random sample of one variable only (see 2.1.1).

For example, examining a sample of females, their body height and body weight can be measured and pairs $(\mathrm{FBH}_i, \mathrm{FBW}_i)$ can be collected. The sample must be homogeneous, that is, it must not involve any males and must be restricted to females of a specified age. - Similarly, we can collect daily information on snow load and wind load at a certain observation point and establish a sample of annual snow load and wind load maxima, $(s_{max,i}, w_{max,i})$. - Observe that there are substantial differences between these two samples:

♦ The two measured female body properties, FBH_i and FBW_i, have obviously much in common: using statistical terms we can say that $(\mathrm{FBH}_i, \mathrm{FBW}_i)$ are pairs of realizations of two mutually dependent random variables. They are dependent through the bodies on which measurements were taken. In general, the taller a woman, the greater her weight. This, however, is not always true; at times, our observations are quite opposite.

♦ The pairs $(s_{max,i}, w_{max,i})$ consist of observations that, as a rule, were not obtained simultaneously at one point in time during the yearly observation period. In the year i, maximum snow load might be observed on February 25, and maximum wind load on August 21. Their common attribute is the place of observation. As a rule, there is no dependence encountered between $s_{max,i}$ and $w_{max,i}$ measured at one observation point.

It is important to note that in both cases variables have been observed *jointly.* In the first example, this joint observation is embedded in the *nature of the phenomenon,* the female body, while in the other case the joint observation is the result of our *decision* to make a sample of annual maxima.

Thus, when evaluating samples of random n-tuplets and before making conclusions from such samples, we should always consider the background to observations, whether there are some decisions involved, what kind of measurements was applied, etc.

Extending the idea of random sample of n-tuplets (cf. 2.1.1, Population, random sample, and random variable) we can describe the random behavior of a *multivariate population* by a *multivariate probability distribution,* termed, as a rule, the ***joint probability distribution.*** CDFs and PDFs of joint probability distributions can be defined similarly as those of probability distributions of single variables. Sufficient information on joint probability distributions can be found in any textbook on statistics and probability (see the introductory suggestions in this Section).

Nevertheless, two important concepts related to joint probability distributions and used in further text must be mentioned here:

(1) ***Marginal probability distribution*** is a probability distribution obtained when a multivariate population is investigated from the aspect of only one of the variables. This variable, ξ^*, need not be necessarily identical with any of the variables ξ_1 through ξ_n

entering the n-tuplet. It can be defined, for example, by an arbitrary X^*-projection onto a straight line along which all realizations of the population are plotted (Figure 2.3). Evidently, the number of possible marginal distributions is not limited. Marginal distributions of ξ^* are not subjected to any conditions containing statements on the variables forming the n-tuplet.

(2) ***Conditional probability distribution*** is obtained when a variable is investigated taking into account some condition related to the remaining n-1 variables of the n-tuplet. For example, we can find the distribution of one of n variables, given particular values of the other variables.

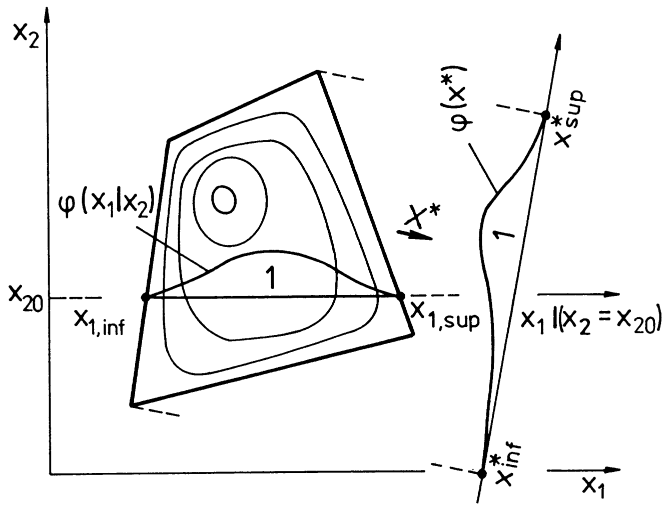

Fig. 2.3 - Bivariate joint PDF over a trapezium-shaped definition range ($\varphi(x_1|x_2)$ -conditional PDF of $\xi_1|(\xi_2 = x_{20})$, $\varphi(x^*)$ - marginal PDF of ξ^*).

■ **Example 2.4.** Consider a hat-shaped bivariate joint probability distribution with $\Phi(x_1,x_2)$ defined over a trapezium (Figure 2.3). The figure shows:

◆ a marginal PDF of ξ^*, obtained as a projection of all pairs (x_1, x_2) onto a straight line perpendicular to the projection direction X^*;

◆ a conditional PDF of $\xi_1|(\xi_2 = x_{20})$, obtained as an intersection of the probability density hat with a plane at $x_2 = x_{20}$ perpendicular to the coordinate system $[x_1, x_2]$.

Note that areas under both $\varphi(x^*)$ and $\varphi(x_1|x_2)$ must be equal to 1. ■

Analytical solutions of problems containing joint probability distributions are always difficult. In the main, they are not possible at all. The only exception is the *normal joint probability distribution,* whose analysis is well elaborated. It should be remembered that all marginal and all conditional distributions derived from a multivariate joint normal distribution are normal again; this simplifies many calculations.

When a joint probability distribution is defined, we can always make some statement on the statistical dependence of variables participating in the distribution. Two extreme cases can be met: *perfect dependence* and *perfect independence*.

A perfect dependence arises when a clear one-to-one physical dependence between the variables exists, while perfect independence indicates that no physical dependence can be expected. However, statistical dependence can be observed also where no apparent physical dependence can be identified. Dependence always indicates some physical relations between variables, though it can be concealed by factors that are not noticeable at first glance.

When the random variables ξ_1 and ξ_2 are perfectly dependent, then, in fact, only one variable is dealt with.

Correlation coefficient

The statistical dependence of random variables can be expressed by various means. The most frequently used is the *correlation relationship,* in which for a given value of one variable, $\xi = x_0$, the conditional mean $\mu_{\eta|x0}$ of the other variable, η, is established. At times, the correlation relationship is linear and we are able to express it in terms of the ***correlation coefficient,*** ϱ. In general, for multivariate distributions with linear dependence of variables *multiple correlation coefficients* can be defined. In current reliability analysis we meet, in the main, the bivariate correlation coefficient, ϱ. When a normal joint probability distribution is treated, the dependence is always linear; in the case of a bivariate normal distribution, ϱ is simply the fifth parameter, along with (μ_ξ, σ_ξ) and (μ_η, σ_η).

Similarly as in the case of other characteristics, the *population correlation coefficient,* ϱ, and the *sample correlation coefficient,* r, must be distinguished. For a *random sample of pairs* (x_i, y_i), $i = 1$ through n, r is obtained from

$$r = \frac{\sum_{i=1}^{n} (x_i - m_x)\cdot(y_i - m_y)}{\left[\sum_{i=1}^{n} (x_i - m_x)^2 \cdot \sum_{i=1}^{n} (y_i - m_y)^2\right]^{\frac{1}{2}}} \tag{2.25}$$

In practice, we may be supplied by a *grouped sample* of k pairs (x_i, y_i), each group consisting of n_i elements, $i = 1$ through k. When the widths of groups are equal, the sample correlation coefficient can be calculated from

$$r = \frac{\Sigma n_i x_i y_i - \Sigma n_i x_i \Sigma n_i y_i}{\{[n \Sigma n_i x_i^2 - (\Sigma n_i x_i)^2]\cdot[n \Sigma n_i y_i^2 - (\Sigma n_i y_i)^2]\}} \tag{2.26}$$

where $n = \Sigma n_i$, and Σ stands for "sum from $i = 1$ to $i = k$."

The values of r are always in [-1, +1]. The degree of dependence can be classified verbally. We can suggest:

Interval of $\|r\|$	Degree of dependence
0 to 0.3	low
0.3 to 0.5	medium
0.5 to 0.7	important
0.7 to 0.9	strong
0.9 to 1	very strong

Similarly as with the coefficient of variation, the reader must be warned on *misinterpretations of the correlation coefficient* that are often encountered in practice:

(1) We must keep in mind that the correlation coefficient describes only the *degree of linear dependence* between two variables. When the dependence is non-linear, the picture provided by the correlation coefficient can be confusing. For example, a perfect circular dependence gives $\varrho = 0$. Thus, a value of $r \neq 0$ conveys that there is *some dependence*, nothing more. This information can be improved if it is known that the partial dependence is linear; then, the above grading can be applied. When the dependence is non-linear, we can obtain that the degree of dependence is, say, medium, but in reality it can be strong. *A graphical plot of the random pairs is recommended.*

(2) Even when the dependence is linear, the information on r must be considered with caution when *small samples* are analyzed. The spread of sample correlation coefficient for such samples is very large. We can easily obtain, for example, medium negative dependence in a case where the actual dependence is positive and strong. Reliable information on the degree of dependence of two variables can be obtained through samples with about $n > 50$. The problem is similar to that of the coefficient of skewness.

(3) Statements on correlation coefficient can sometimes be completely wrong. This happens when the correlation coefficient have been calculated for a sample of pairs, where the values of one variable have been determined by decision. For example, observed values have been plotted on time axis in fixed intervals, and a "correlation coefficient" has been calculated using Equation (2.25). *Here, no correlation coefficient is dealt with.* Regression analysis shall be used in such cases.

■ **Example 2.5.** In an extensive research program 21,228 of pairs of cube strength of concrete, f_{cube}, and volume density, γ_c, were obtained. The collection consisted of a number of samples that could be classified as random. The size of samples was different - from 10 to about 4000. Correlation coefficients, $r_{f\gamma}$, were calculated for each sample; in Figure 2.4 $r_{f\gamma}$ are plotted against the size, n. Observe the wide spread of $r_{f\gamma}$ for small samples and diminishing spread with increasing n. The final estimate of the population correlation coefficient was $\varrho = 0.39$ (medium dependence). ■

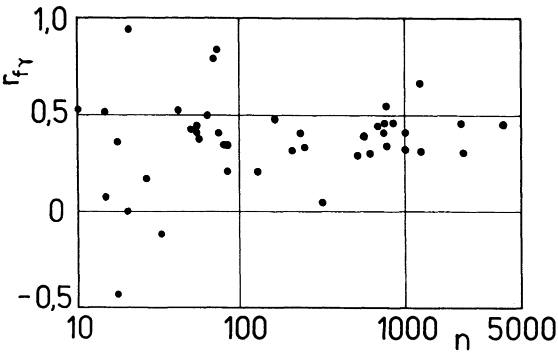

Fig. 2.4 - *Example* 2.5. Correlation coefficient $r_{f\gamma}$ of the cube strength, f_{cube}, and volume density, γ_c, of concrete (n = sample size).

Union of two bivariate samples

Let the characteristics of two bivariate samples taken *from the same population* be given:

$m_{1x}, s_{1x}, m_{1y}, s_{1y}, r_1$

$m_{2x}, s_{2x}, m_{2y}, s_{2y}, r_2$

When data on individual observations are missing, the coefficient of correlation of the *unified sample* can be calculated from

$$r = \frac{n_1 + n_2}{n_1 + n_2 - 1} \frac{s_{xy}}{s_x s_y}$$

$$s_{xy} = s_{1xy} + s_{2xy} + \frac{n_1 n_2}{n_1 + n_2} (m_{2x} - m_{1x})(m_{2y} - m_{1y})$$

$$s_{1xy} = \frac{n_1 - 1}{n_1} s_{1x} s_{1y} r_1, \quad s_{2xy} = \frac{n_2 - 1}{n_2} s_{2x} s_{2y} r_2$$

where s_x and s_y = unified standard deviations of x and y, respectively (see 2.1.2, Union of two samples).

Statistical dependence function

When some statistical dependence between variables is expected, correlation coefficient is habitually used by many researchers as an input parameter during investigation of structural reliability, that is, mainly in the calculation of failure characteristics, P_f and β^{HL} (see Chapter 9). Assumptions on values of ϱ are made, and ϱ is included into calculation models. This, however, substantially limits the generality of results. Moreover, values of correlation coefficients are usually not known; they can be only estimated. On the other hand, we can easily develop a statistical dependence formula in the following manner:

Consider a variable η that depends on other variable ξ in such a way that for individual pairs (x, y) the following physical function holds:

$$y = Cp\, x^{Cq} + Cr \tag{2.27}$$

where Cp, Cq, and Cr = assumed constants. Now, any one of these constants, or all, can be declared a random variable with the mean

$$\mu_p = Cp, \quad \mu_q = Cq, \quad \mu_r = Cr$$

and with additional parameters like σ_p, α_p, σ_q, etc. In this manner the non-statistical dependence has been changed to a statistical one, and the number of random variables has been increased from one to two, three, or four. Other types of equation (2.27) can be selected, of course.

According to the Author's experience, this technique of describing dependence is clearer and more flexible than techniques based on correlation coefficient. It facilitates the assessment of dependence; as a rule, an *a priori* decision on correlation coefficient, ϱ, is more difficult and subjected to more incertitude than decisions on random variability of "constants." Moreover, using Equation (2.27), we can easily treat non-linear dependencies.

In some cases, the coefficient of correlation, ϱ, of two variables, ξ and η, described by their respective parameters (μ_ξ, σ_ξ) and (μ_η, σ_η) is sufficiently known and it would be doubtlessly immoderate to discard this information. Then, the statistical dependence function can be expressed as

$$\eta = \varrho \frac{\sigma_\eta}{\sigma_\xi} \xi + \zeta \tag{2.28}$$

where ζ = random variable with

$$\mu_\zeta = \left(\mu_\eta - \varrho \frac{\sigma_\eta}{\sigma_\xi} \mu_\xi\right)$$
$$\sigma_\zeta = \sigma_\eta \sqrt{1 - \varrho^2} \tag{2.29}$$

2.1.4 Derived random variable

In diverse reliability exercises situations are very frequently met when a random variable ξ is transformed into a variable η through

$$\eta = \Xi(\xi) \tag{2.30}$$

where $\Xi(.)$ = *response function*, and η = derived random variable. Owing to the transformation $\xi \rightarrow \eta$ the CDF $\Phi_\xi(x)$ changes in $\Phi_\eta(y)$. It is evident that the parameters of the two distributions are different and also the sample characteristics cannot be identical. Nevertheless, when the response function is monotonic, it holds

$$\Phi_\xi(x_\kappa) = \Phi_\eta(y_\kappa)$$

where x_κ, y_κ = κ-fractiles. Set $y_i = \Xi(x_i)$; when x_i are ordered by magnitude, $x_1 \le x_2 \le \ldots \le x_n$, the corresponding y_i will also be ordered by magnitude. This is not true when $\Xi(.)$ is not monotonic.

■ **Example 2.6.** The wind pressure w (kN.m^{-2}) depends on the wind velocity v (m.s^{-1}) according to

$$w = 0.613\text{E}{-3} \cdot v^2 \tag{a}$$

Table 2.2 shows sample characteristics of a sample of annual maximum wind velocities and of the corresponding sample of maximum wind pressures that has been created through Equation (a).

Observe, that Equation (a) holds approximately only between means, but not between standard deviations.

Table 2.2 - *Example 2.6.* Sample characteristics of v and w

Sample characteristics	Wind velocity	Wind pressure
Size (number of years)	37	37
Mean	31.0 m.s^{-1}	0.59 kN.m^{-2}
Standard deviation	3.2 m.s^{-1}	0.13 kN.m^{-2}
Coefficient of skewness	1.7	2.0

■

The probability distributions of derived variables can be strongly affected by the non-linearity of the response function (Levi 1972). The distribution of the output random variable can differ very much from that of the input random variable. Moreover, the evaluation of derived distributions becomes difficult, since mathematically treatable models are not at hand.

In many structural reliability problems it is necessary to describe the random behavior of η for which a physical relationship exists,

$$\eta = \Xi(\xi_1, \xi_2, \ldots, \xi_n) \tag{2.31}$$

where ξ_1 through ξ_n = random variables. Response functions $\Xi(.)$ are usually complicated, often non-linear. Except for very simple cases, an analytic solution of this problem is almost impossible. Two methods of obtaining parameters of η exist - the *method of moments* and the *Monte Carlo simulation.* They are briefly outlined in Sections 2.3 and 2.4, respectively.

■ **Example 2.7.** A reinforced concrete simple beam is subjected to a random load F at mid-span. The distribution of F is rectangular, $F \in [0, F_{sup}]$, Figure 2.5a,b.

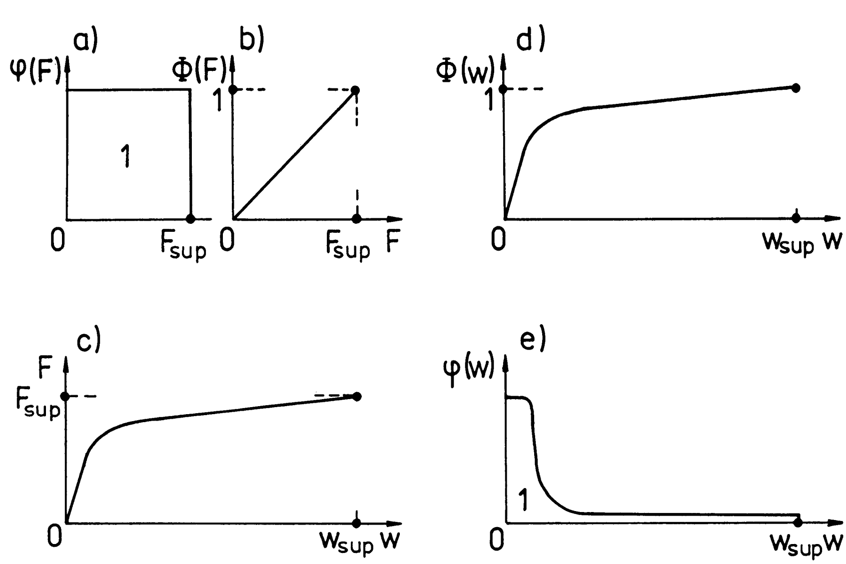

Fig. 2.5 - *Example 2.7.* Transformation of a rectangular probability distribution through a non-linear response function (F = random load, w = deflection due to random load).

Assume that the dependence between the load and the mid-span deflection, w, is non-linear (Figure 2.5c). CDF and PDF of w are plotted in Figures 2.5d,e. Observe how the non-linear part of the load-deflection relationship has influenced the probability distribution of w! ■

2.1.5 Random functions and sequences

If we manage to measure, discretely or continuously, the dependence of a random variable ξ upon a non-random argument t, we can describe this dependence by a function

$$x = f(t) \tag{2.31a}$$

or by a time-ordered n-tuplet

$$!(x_1 \| x_2 \| ... \| x_n) \qquad (2.31b)$$

When several independent observations are carried out, we can find that the developments of f(t)

- ♦ are identical at each observation, or
- ♦ change systematically, or
- ♦ change randomly.

In the first two cases we are able to predict x for any value of t, while in the latter case an exact prediction is impossible.

Obviously, a development of f(t) is an *event*. When the development is random, and when all developments are considered as a set, such a set is described by a *random function*

$$\xi = f_\xi(t) \qquad (2.32)$$

or, when time-ordered n-tuplets are investigated, a *random sequence*

$$\xi = \mathrm{seq}_\xi(x_i), \qquad i = 1, 2, ..., n \qquad (2.33)$$

A single development of ξ is called the *realization of a random function.*

The variability of realizations of ξ can be diverse. Figure 2.6 shows five typical examples of random functions and one example of random sequence; many others are possible.

The treatment of random sequence is analogous to that of random functions. Therefore, we will not pay a special attention to random sequences. For random functions and random sequences a summary term is used: *random processes.*

Parameters of a random function

If for any value of the argument t, values of ξ are collected, a set is obtained that can be described by a probability distribution. Establishing the mean, standard deviation and also possibly other parameters for each value of the argument, argument-dependent functions are obtained again.

In Figure 2.6, PDFs for a given t are plotted. It is apparent that they do not provide sufficient information on the behavior of the respective processes. Random functions (a) through (d), and the random sequence (f) have, say, identical PDF in the considered point; nevertheless, they differ considerably. Observe, further, that for function (e) not only the mean of the function but also the probability density changes. Therefore, also other parameters of the function will change along t. Two conclusions can be made:

- ♦ population parameters of a random function are functions of the argument, t;

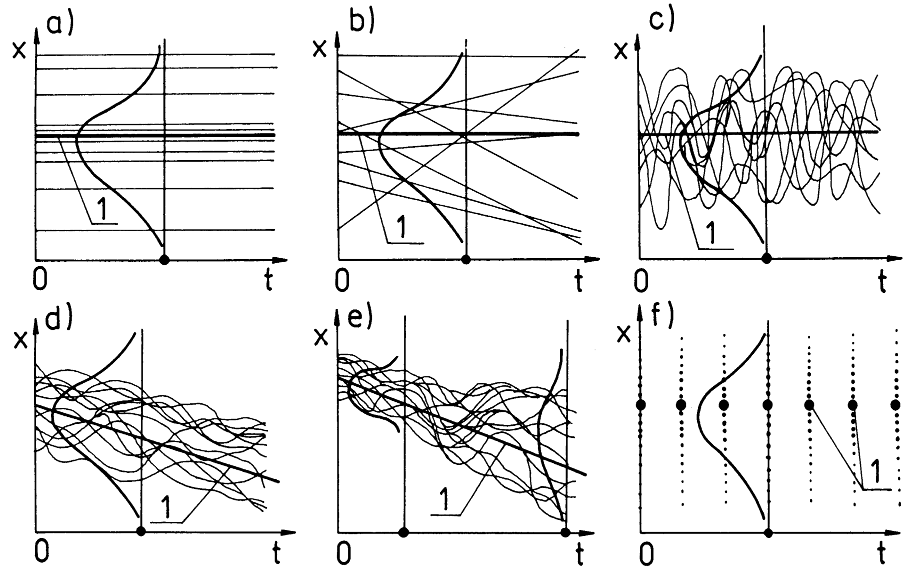

Fig. 2.6 - Some typical random functions (**1** - mean; **a**, **b**, **c** - stationary, **d**, **e** - non-stationary functions, **f** - stationary random sequence).

♦ the description of a random function, compared to that of a random variable, must be expanded by further argument-dependent parameters.

The ***mean of a random function*** can be expressed by

$$f_E(t) = f\{E[\xi(t)]\} \tag{2.34}$$

For illustration, means are plotted in Figure 2.6. Observe, that for cases (a) through (c) they are same, though the random functions are evidently different. Similarly, the ***variance of the random function*** can be established:

$$f_D(t) = f\{D[\xi(t)]\} \tag{2.35}$$

In conformity with the prevailing practice, symbols E[.] and D[.] are used for the mean (expectation) and variance (dispersion) of $\xi(t)$, respectively; some authors use D^2 for D. We could also write $\mu[.]$ and $\sigma^2[.]$.

The nature of the mean and variance of a random function is analogous to those of a random variable. If these parameters are independent of the argument, then a *stationary random function* is dealt with. For example, the monthly maxima of wind velocity can be taken as stationary, while the daily maxima are non-stationary (it is well known that a "strong" wind today will be, with a certain probability, followed by a "strong" wind tomorrow).

A parameter specific for random functions is the ***autocorrelation function***

$$K(t_1,t_2) = E(\{\xi(t_1) - E[\xi(t_1)]\}\{\xi(t_2) - E[\xi(t_2)]\})$$

which describes the linear random dependence of the values of ξ at t_2 on the values of ξ at t_1. The value of K is, in fact, the *covariance* of $\xi(t_1)$ and $\xi(t_2)$. Hence, the ***standardized autocorrelation function*** (Figure 2.7a),

$$\kappa(t_1,t_2) = \frac{K(t_1,t_2)}{D[\xi(t_1)] \cdot D[\xi(t_2)]} \tag{2.36}$$

equals the correlation coefficient of $\xi(t_1)$ and $\xi(t_2)$.

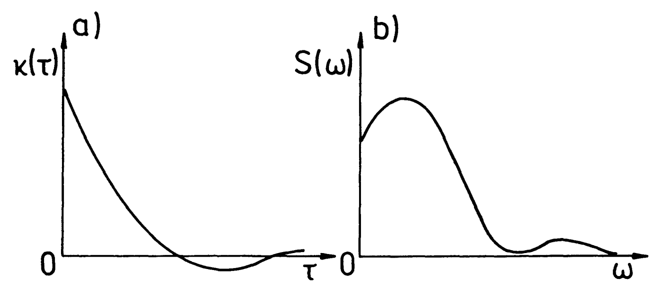

Fig. 2.7 - Standardized autocorrelation function $\kappa(\tau)$ and spectral density $s(\omega)$.

You may try to sketch the standardized autocorrelation functions for cases in Figure 2.6. Note that for $t_1 = t_2$

$$K(t_1,t_2) = D[\xi(t_1)]$$

or

$$\kappa(t_1,t_2) = 1$$

When a random function is stationary, $\kappa(t_1, t_2)$ depends only on the distance between t_1 and t_2, that is, on $\tau = t_2 - t_1$. The distance τ for which $\kappa(\tau) = \kappa_0$, where κ_0 is a defined value, close to or equal zero, is termed the *correlation distance.*

For completeness' sake, let us mention here also the ***spectral density function,*** defined, in standardized form, by

$$s(\omega) = \frac{2}{\pi}\int_0^\infty \kappa(\tau)\cos\omega\tau\, d\tau \tag{2.37}$$

where ω = frequency (Figure 2.7b). The spectral density function applies mainly in the investigation of reliability of structures subjected to dynamic load.

For random functions, we could introduce, similarly as in the case of random variables, the concepts of sampling, sample, sample characteristics, etc. Though a full theoretical basis connecting random functions samples and random function populations has not yet been elaborated, we can make much benefit of the *random function philosophy* in many structural reliability exercises. Its applications focus particularly on structural loads where, in practical solutions, it is necessary to describe the time or space-dependent behavior of load or both. The reader is referred to Wen 1990 where several typical random functions (random processes) are presented.

When conclusions and decisions based on a continuous or densely discrete record of observations are to be made, it is necessary to derive some parameters describing the phenomenon. This is usually performed by a suitable *discretization of records.* In this way random sequences are obtained, the statistical treatment of which is, as a rule, reasonably simple. Three methods of discretization will be shown in Section 5.4.

General information on random functions can be obtained from, for example, Hines and Montgomery 1990. Advanced information can be found in Cramér and Leadbetter 1967, Cinlar 1975, Vanmarcke 1983, Wong and Hajek 1985.

Random variables can of course depend on more arguments then one. Then, *random fields* are dealt with (see Vanmarcke 1983).

2.1.6 Repeated events

Let a random variable, ξ, be described by

$$\Phi_1(x) = \Pr(\xi \le x)$$

where x = specified value of ξ; the subscript 1 stands for single occurrence of ξ. Let n *independent occurrences of* x be expected. The probability $P_{x.n}$ that $\mathrm{Ev}(\xi \le x)$ takes place in all n repetitions of x is, according to Equation (2.5)

$$P_{x.n} \equiv \Pr(\xi \le x \,|\, n) = [\Pr(\xi \le x)]^n$$

Thus, obviously, ***CDF of a repeated event*** (Figure 2.8) is given by

$$P_{x.n} \equiv \Phi_n(x) = [\Phi_1(x)]^n \tag{2.38}$$

and, taking into account Equation (2.15), ***PDF of a repeated event*** is

$$\varphi_n(x) \equiv \frac{\mathrm{d}\Phi_n(x)}{\mathrm{d}x} = n[\Phi_1(x)]^{n-1} \cdot \varphi_1(x) \tag{2.39}$$

Equations (2.38) and (2.39) are valid only when ξ is *stationary,* that is, when $\Pr(\xi \le x)$ does not change during the repetitions.

An analytical treatment of $\Phi_n(x)$ and $\varphi_n(x)$ is only possible in particular cases (for example, for normal distribution and exponential distribution). The repetitions affect the shape of the probability distribution considerably. For example, when $\mathrm{Ev}(\xi \le x)$ is distributed normally with $\mu^{(1)} = 0$, $\sigma^{(1)} = 1$, and $\alpha^{(1)} = 0$, the event $\mathrm{Ev}(\xi \le x|1000)$ has a distribution with $\mu^{(1000)} = 3.24$, $\sigma^{(1000)} = 0.35$, and $\alpha^{(1000)} = 0.9$.

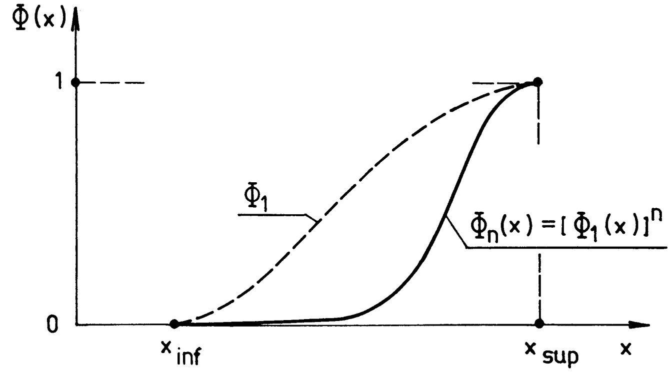

Fig. 2.8 - Development of CDF for a repeated event Ev($\xi \leq x$).

As far as fractiles are concerned, no difficulties arise. Suppose, for example, that $\Phi_1(x)$ is given and that a κ-fractile of $\Phi_n(x)$, $x_\kappa^{(n)}$, is required. Considering Equation (2.38) we can calculate

$$\kappa' = \kappa^{\frac{1}{n}} \tag{2.40}$$

and establish the κ'-fractile of $\Phi_1(x)$. It is $x_{\kappa'}^{(1)} \equiv x_\kappa^{(n)}$.

Mean return period

When dealing with time-dependent phenomena it is important to know the probability that an event will happen in a specified period, or to know how frequently will it happen, etc.

Let a random variable be observed over *consecutive observation periods*, T_{obs}. In each period one appropriately defined value x (for example, the maximum of all values observed over T_{obs}) was found. This value is also a random variable; let its random behavior be described by $\Phi_1(x)$.

Consider a *reference period* $T_{ref} > T_{obs}$. The number of observation periods during T_{ref} is $n = T_{ref}/T_{obs}$. Using Equation (2.38), we now can calculate the probability $P_{x.n}$ that a specified x will not be exceeded in n observation periods; conversely, given $P_{x.n}$, x can be found.

Next, we want to know what the *expected frequency of occurrence of* Ev($\xi > x$) *during* T_{ref} will be. In other words, we want to find the *mean return period* of Ev($\xi > x$).

The probability of Ev($\xi > x$) during T_{obs} is

$$\Pr(\xi > x) = 1 - \Phi_1(x) \tag{2.41}$$

The probability P^* that $\mathrm{Ev}(x_i > x)$ happens at least once in n successive intervals T_{obs} is given by the sum of probabilities $\Pr(\xi > x)$ [see Equation (2.4)], that is,

$$P^* = n \cdot \Pr(\xi > x)$$

Then, the number of intervals n^* during which $\mathrm{Ev}(\xi > x)$ *is expected to happen at least once* results from

$$P^* = 1$$

that is

$$n^* \cdot \Pr(\xi > x) = 1$$

Using Equation (2.41) yields

$$n^* = \frac{1}{1 - \Phi_1(x)} \qquad (2.42)$$

which, in other words, is the *expected number of independent repetitions* of ξ until $\mathrm{Ev}(\xi > x)$ happens.

The period during which $\mathrm{Ev}(\xi > x)$ is expected to happen is the required mean return period T_{ret} of $\mathrm{Ev}(\xi > x)$; it is given by

$$T_{ret} \equiv n^* \cdot T_{obs} = \frac{T_{obs}}{1 - \Phi_1(x)} \qquad (2.43)$$

Note the terms "expected frequency of occurrence," "expected number of repetitions," and "is expected to happen." These imply that the respective events may or may not happen, that is, the *effective return period can be less or greater than the* ***mean*** *return period,* T_{ret}. - The term "mean recurrence interval" is also used for T_{ret}.

■ **Example 2.8.** Let the probability of Ev(**the annual maximum of wind velocity,** v_{max}**, is greater then 30 m.s**$^{-1}$) be equal 0.01. The observation period is $T_{obs} = 1$ year. The number of intervals during which the event is expected to happen once is $n^* = 1/0.01 = 100$. Consequently, the mean return period of $\mathrm{Ev}(v_{max} > \mathbf{30\ m.s^{-1}})$ is $T_{ret} = 100 \times 1 = 100$ years. ■

Equation (2.43) gives

$$\Phi_1(x) = 1 - \frac{T_{obs}}{T_{ret}} \qquad (2.44)$$

Equations (2.38), (2.42), and (2.44) yield

$$P_{x.n} = \left(1 - \frac{T_{obs}}{T_{ret}}\right)^{\frac{T_{ref}}{T_{obs}}} \tag{2.45}$$

Because

$$\left(1 - \frac{1}{N}\right)^N \approx e^{-1}$$

we can write after arrangement

$$P_{x.n} \approx \exp(-T_{ref}/T_{ret}) \tag{2.46}$$

Introducing this approximation into Equation (2.45), we obtain the mean return period of Ev($\xi > x$)

$$T_{ret} \approx \frac{T_{ref}}{\ln(1/P_{x.n})} \tag{2.47}$$

Observe that $\Phi_1(x)$ does not appear in Equations (2.45) nor (2.47). Hence, T_{ret} *is independent of the probability distribution of the random variable* x, or, in other words, it is *distribution-free.*

When establishing, for example, design parameters of load, we are usually interested in the *probability that a certain value* x *will be exceeded during a reference period* T_{ref} (for example, during the life of the constructed facility). It is given by

$$P_x \equiv \Pr(\xi > x | T_{ref}) = 1 - P_{x.n} \tag{2.48}$$

When T_{ref} and T_{ret} are known, $P_{x.n}$ is established from Equation (2.46). Note that P_x should not be confused with $\Pr(\xi > x)$, which, in fact, is only an auxiliary quantity, expressing the mean return period of Ev($\xi > x$); it is

$$\Pr(\xi > x) = \frac{1}{T_{ret}} \tag{2.49}$$

■ **Example 2.9.** The design value of snow load, s_d, is established by means of a sample of annual maxima of snow load, s_{max}, as a value that occurs or is exceeded once in 100 years on the average. Thus, T_{ret} = 100 years, $\Pr(s_{max} > s_d) = 0.01$. Using Equations (2.46) and (2.48), the probability $P_s \equiv P_x$ of $\mathrm{Ev}[(s_{max} > s_d)|T_0]$, in dependence on the life, T_0 ($\equiv T_{ref}$), is

T_0 years	P_s
10	0.10
50	0.39
100	0.63
150	0.78

■

■ **Example 2.10.** Find, for a structure with target life expectancy T_{0t} ($\equiv$ T_{ref}), the design value of annual maximum wind velocity, v_d, which will not be exceeded with a probability $P_{x,n}$ = 0.4. Further, find also the mean return period, T_{ret}, between two consecutive exceedances of v_d. From the analysis of the annual wind velocity maxima, v_{max}, the population mean μ_v = 31.0 m.s^{-1} and standard deviation σ_v = 3.2 m.s^{-1} were obtained. - From Equation (2.47) $T_{ret}/T_{0t} \equiv T_{ret}/T_{ref}$ = 1.09, and so the mean return period is

$$T_{ret} = 1.09 \times 80 = 87.2 \text{ years}$$

Since the observation interval is T_{obs} = 1 year,

$$n^* \equiv T_{ret} = 87.2$$

From Equation (2.42) we get

$$\Phi_1(v_d) \equiv \frac{n^* - 1}{n^*} = 0.988532$$

The velocity v_d is defined as the κ-fractile of the annual maxima v_{max}, κ = 0.988532. Assuming that the distribution of v_{max} is three-parameter log-normal (see Appendix A), we get u_κ = 2.816, and so, using Equation (2.20),

$$v_d = \mu_v + 2.816\,\sigma_v = 40.0 \text{ m.s}^{-1}$$

Probability that v_d will be exceeded during any current year is

$$\Pr(v_{max} > v_d) = 1 - \Phi(v_d) = 0.011468$$ ■

2.1.7 Estimation and hypotheses testing

As it has been mentioned in 2.1.2, any quantities gained by sample analysis (for example, sample mean, standard deviation, median) are *random variables*. Thus, we can assign to them appropriate probability distributions. Parameters of these distributions can be established, and some important information can be drawn from them, helping to answer the following families of questions:

♦ What are, in terms of population parameters and other relevant quantities, properties of the population from which the particular sample has been taken?

♦ Do two or more samples gained by independent random sampling stem from the same population?

♦ Assuming that the population has a certain probability distribution, is this distribution the best fit to the random sample?

These questions, which can be developed or split into further detailed ones according to the intended purpose of analysis (confidence analysis, prediction analysis, statistical tolerance analysis), are answered with more or less success by a branch of mathematical statistics termed the *statistical inference.* Two specific areas of statistical inference can be discerned:

♦ ***estimation*** of population parameters; the results of estimation are *values* describing the investigated parameter;

♦ ***testing hypotheses*** on certain parameters, and also distributions; the result of testing hypotheses are statements whether a hypothesis formulated is true or false.

In the estimation, which refers to the first family of questions set above, two types of answers can be given:

(a) The population parameter, ϑ, is assumed to be just equal to a certain value calculated from the sample - the ***point estimate of*** ϑ. *No measure of uncertainty is accompanying this answer.* The only information on the quality of a point estimate is whether it is *biased* or *non-biased,* which depends upon the theoretical background to the point estimate. Sample characteristics given in Table 2.1 are, by definition, point estimates of population parameters.

(b) The population parameter, ϑ, is in a *confidence interval,* CI ≡ $[\vartheta_1, \vartheta_2]$, defined by the ***confidence level,*** λ. This number says that in $100 \times \lambda$ percent of cases ϑ will be in CI, and in $(1-\lambda) \times 100$ percent of cases ϑ will be beyond CI. This procedure is called the ***interval estimation.*** In reliability assurance also the concepts of *prediction intervals* and *tolerance intervals* can be met, their overall philosophy being similar to that of confidence intervals.

Again, answers in the hypotheses testing can be given in terms of biased or non-biased TRUE-FALSE statements, or in terms of statements that accept or reject the hypothesis with a specified confidence level, λ.

In the main, the *choice of the confidence level,* λ, is a matter of engineer's decision, governed by economic and engineering considerations. Some quality control regulatory documents specify confidence levels for particular procedures. In advanced reliability methods, the estimation of population parameters can be affected by the objectives of the general solution. For example, Lind and Chen 1987 introduced a *consistency principle approach* to avoid arbitrariness in the selection of confidence levels.

We will not deal with the particularities of estimation and hypotheses testing here. A statistically trained reader is well acquainted with both concepts. The newcomers can find general information in specialized monographs on probability and statistics (for example, Benjamin and Cornell 1970, Hines and Montgomery 1990) as well as in many university textbooks on mathematical statistics. Hahn and Meeker 1991 is an excellent monograph on interval estimations, aimed at engineers.

Information on estimation of μ , σ , and fractiles of the three-parameter log-normal distribution is given in Appendix A.

2.2 RELIABILITY THEORY

Similarly as in the foregoing section, only some principal concepts of the reliability theory, bound to structural problems, will be given here. Sufficient reliabilistic literature is available; there the reader can find an elaborated presentation and detailed information on numerous techniques applicable in structural reliability practice. A few tips for neophytes:

♦ ***general:*** Barlow and Proschan 1975, Gnédenko *et al.* 1972, *Handbook of Reliability Engineering* 1988, Kececiouglu 1990;

♦ ***specialized, aimed at structural reliability problems:*** Ang and Tang 1975/1984, Bolotin 1982, Harr 1987, Melchers 1987.

2.2.1 Principal concepts

It should be stated here, that the reliability theory is nothing more, nothing less than a ***mathematized engineering judgment,*** that is, long-term engineering experience collected during the development of Humankind, transformed into calculation models. This transformation would never be possible without

- ♦ *mathematical statistics* and *theory of probability;*
- ♦ thinking in terms of *systems;*
- ♦ consistent introduction of *time* as an additional dimension.

These three fundamental features of the reliability theory and its applications should be continually kept in mind by all who want to master the reliabilistic approach to the problems of constructed facilities.

Reliability systems, elements, items, and connections

The concept of the reliability system S-L-E introduced in Chapter 1 showed the general idea of interaction of components forming a constructed facility. Any of these components can be mathematically investigated as a specific *reliability system* consisting of one or more *reliability elements.* A reliability system need not be identical with the respective *structural system* and a reliability element need not be identical with a *structural member.* In general, we can say, that a structural system embodies several reliability systems (or, "subsystems") that, subsequently, can be split into reliability elements.

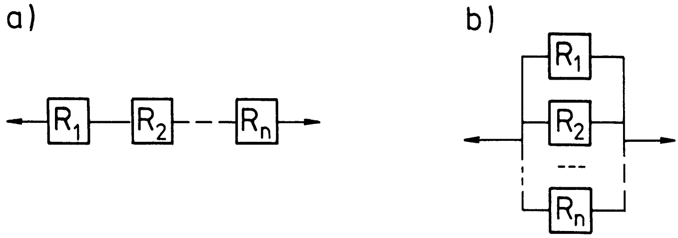

Fig. 2.9 - Principal reliability connections (**a** - serial connection, **b** - parallel connection).

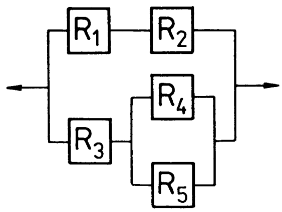

Fig. 2.10 - Combined reliability connection.

Systems and elements will be comprehensively termed ***items***. It will be understood that a system is composed of items, some of which can be lower level reliability systems, the others just reliability elements. The effect of scale must be taken into account: an item considered as element of a system, can be, by itself, a clearly defined reliability system. Thus, to get a comprehensible picture of a problem, it is necessary, in any reliability-based considerations, to *identify various levels of systems and elements*.

To describe correctly the behavior of a system on any level, ***connections*** between items forming the investigated system must be identified. Two basic types of connections have to be distinguished (Figure 2.9):

♦ ***serial connection*** where the failure of any item brings failure of the higher level system; a system where only serial connections are involved is called a *serial system;*

♦ ***parallel connection*** where the failure of a single item is not a sufficient condition for the general failure of the system (*parallel system*); in systems with n items connected in parallel, failure can be defined in various manner:

- by failure of minimum k items, with $k \le n$,
- by failure of only specified items, etc.

Further, we can consider that the capacity of an item participating in a parallel connection will be completely lost after certain value of load has been achieved, or we can consider that the item will not resist any further increase in load. Other possibilities exist. Here, the terms "capacity" and "load" must both be conceived in a wide sense. *Not only mechanical load and bearing capacity can govern a system analysis.* On the other hand, structural loads can be investigated as systems "loaded" in a specific manner.

Combined connections, consisting of items connected in series and in parallel (Figure 2.10), and ***conditional connections,*** that depend on the mode of failure of connected items, are frequently encountered in many engineering problems.

In the general reliability theory, the concept of *reserve* is of importance. Two basic modifications are met:

- ***active reserve*** that is simultaneously fulfilling the same function as the item covered by the reserve; active reserve can be *loaded* or *unloaded*;
- ***stand-by reserve*** that gets loaded only after the respective item has failed.

Again, take the term "loaded" in a very general sense.

An important property of any reliability item is whether it is ***repairable*** or ***non-repairable.*** A further property, that belongs to the same family of concepts, is the ***restorability*** or ***non-restorability*** of an item.

The foregoing concepts, which are only a small sample of the general theory of reliability vocabulary, are entering the structural reliability models. Unfortunately, the implementation is slow. It is increasingly recognized that various types of connections and reserves met in mechanical and electrical facilities exist also in constructed facilities. And, in addition, that such concepts can be efficiently used in many design solutions. Some examples of structural systems conceived as reliability systems are shown in Figure 2.11 (cf. Benjamin 1970). The reader is encouraged to elaborate, as an exercise, further examples, and to reveal applications of the above referred concepts in constructed facilities.

Reliability function

It is difficult to state which of the various mathematical functions applied in the reliability analysis is the most important one. Let us present here only the functions referred to in the next chapters of this monograph.

Let us investigate a collection of items whose size at the beginning of its service ($t = 0$), is N_0. At a moment t, N_s items have remained in service (*survived*), while N_f items have *failed.* The *time-dependent probability* $R(t)$ that any of N_0 items will survive till t is given by Equation (2.1), that is,

$$R(t) = \frac{N_f}{N_0} \tag{2.50}$$

This probability is termed the ***reliability function.*** As a rule, $R(t)$ is presented in a general form

$$R(t) = \exp\left(-\int_0^t \lambda(t)\,\mathrm{d}t \right) \tag{2.51}$$

where $\lambda(t)$ = ***failure rate,*** defined as the relative number of items failed per unit of time.

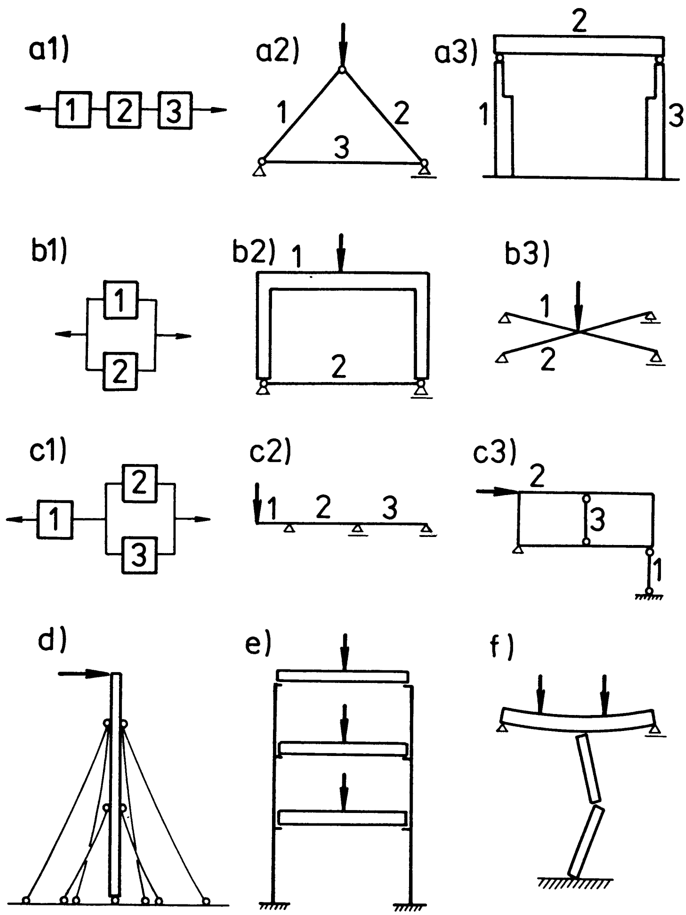

Fig. 2.11 - Structural reliability systems [**a** - serial systems, **b** - parallel systems, **c** - mixed systems, **d** - stand-by system, **e** - consequence system, **f** - multi-state system (deflection of the floor leads to collapse of a partition wall)].

When the failure rate is constant,

$$\lambda(t) = \lambda_c$$

the reliability function becomes

$$R_c(t) = e^{-\lambda_c t} \qquad (2.52)$$

Failure rate

It appears that the time-dependence of the failure rate, $\lambda(t)$, Figure 2.12, is, to a certain degree, the most instructive function in reliability thinking. - In a general case three distinct ranges can be identified on the respective curve, which is often termed, because of its shape, the *bath-tub curve*. In the majority of cases failures are more frequent in the beginning of service of a collection of items than at later stages. The items weakened by flaws drop out of service very early, and the failure rate decreases. This range of decreasing $\lambda(t)$ is usually termed the *early failure period* or also *burn-in period*.

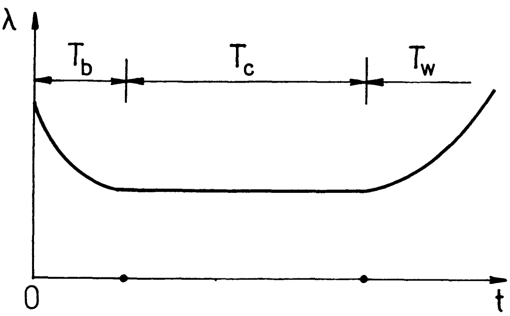

Fig. 2.12 - Failure rate *vs.* time ("bath-tub curve").

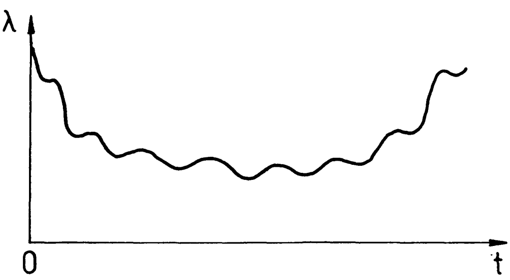

Fig. 2.13 - Failure-rate curve for a structural system.

After the number of failures has stabilized, the failure rate becomes constant for the *constant failure period* or *period of current failures* (also *useful life period*).

Finally, the items that have survived both foregoing periods, start ageing, and, as a result, the failure rate increases; this range of the bath-tub curve is termed the *wear-out period*.

The bath-tub curve features can be identified not only in the life of engineering systems, but also in the life of humans, philosophic, economic, and political systems, etc. You may even observe bath-tub curve properties in the behavior of structural reliability concepts and structural design codes!

As far as constructed facilities are concerned, the bath-tub curve is never smooth. We know that buildings, bridges, and other constructed facilities, are exposed to short-time and long-time fluctuation behavior of the S-L-E system. Let us mention here only the seasonal changes in snow load and temperature load. Further, some building materials improve their properties during a certain period before starting to age. Therefore, a true failure-rate curve for an S-L-E system is undulated more or less (Figure 2.13).

Life

The concept of life of a CF system will be discussed in Section 10.2. Here, some remarks are necessary to clarify this concept from the point of view of the reliability theory.

Assume again a collection of N_0 items. Some of them have failed over the early failure period, others during the constant failure period, and finally, the remaining items have failed during the wear-out period. The lives of individual items constitute a sample. This sample cannot be statistically analyzed as a whole, since its elements come from three different populations. Obviously, causes of failure in the first period substantially differ from those of failures occurring in the second and third periods. Thus, samples of item lives reached over respective periods have to be analyzed separately. This is often a difficult exercise, as distinct boundaries between the three periods of the failure-rate curve exist only in theory.

Obviously, the item life is a random variable. For practical reasons, we will not deal with the life referred to the burn-in period; it has no practical meaning in structural problems. - Let us first investigate the life referred to the constant failure period (Figure 2.14).

We can write

$$\Phi(t) = 1 - R(t) \tag{2.53}$$

where $\Phi(t)$ = CDF of the *time to failure.* Obviously, the meaning of t is the "random life." The PDF of t is

$$\varphi(t) \equiv \frac{\mathrm{d}\Phi(t)}{\mathrm{d}t} = -\frac{\mathrm{d}R(t)}{\mathrm{d}t}$$

Since $t_{inf} = 0$ and $t_{sup} \to \infty$, the mean of t follows from

$$\mu_t \equiv \int_0^\infty \varphi(t) t \,\mathrm{d}t = \int_0^\infty \left(-\frac{\mathrm{d}R(t)}{\mathrm{d}t}\right) t \,\mathrm{d}t$$

After integration and rearrangement the *mean time to failure,* or the *mean life,* μ_t , is obtained:

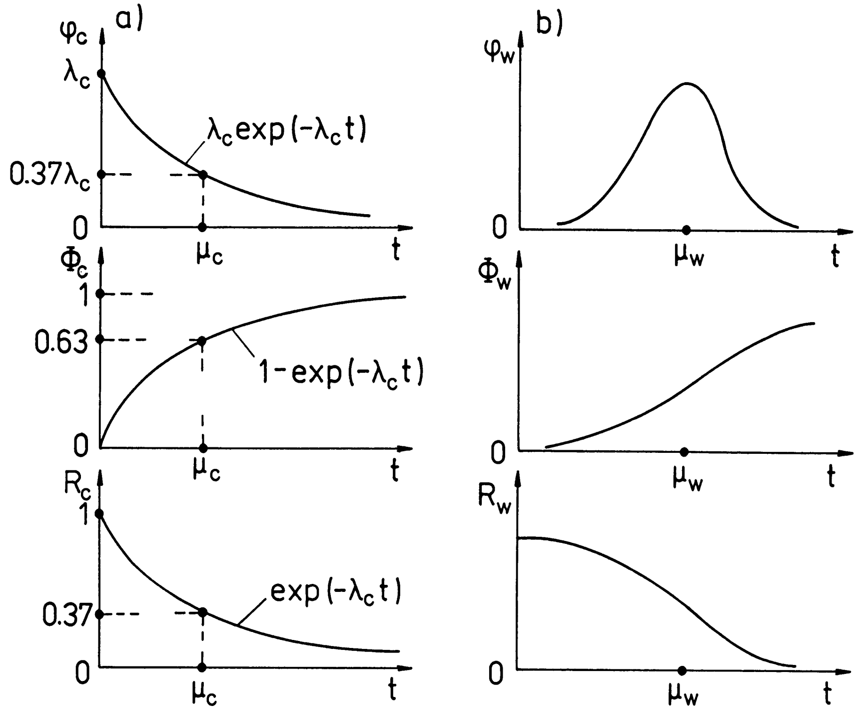

Fig. 2.14 - PDF and CDF of the life, t , and the respective reliability functions (**a** - constant failure period, **b** - wear-out period).

$$\mu_t = \int_0^{\infty} R(t)\,\mathrm{d}t \tag{2.54}$$

When the failure rate is constant, $\lambda = \lambda_c$, Equations (2.54) and (2.52) yield

$$\mu_{tc} = \frac{1}{\lambda_c}$$

Setting $t = \mu_{tc} = 1/\lambda_c$ into Equation (2.52), we obtain

$$R(\mu_{tc}) = \mathrm{e}^{-1} \approx 0.37$$

which means that for any item from the initial collection the probability of reaching μ_{tc} is equal 0.37, or, in other words, that at $t = \mu_{tc}$ only 37 percent of the collection will be surviving.

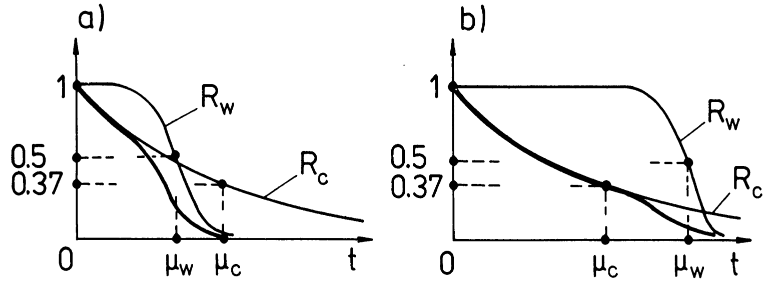

Fig. 2.15 - Reliability function in the wear-out period (a - $\mu_c > \mu_w$, b - $\mu_w > \mu_c$).

Now, let us pay attention to the wear-out range of the bath-tub curve. Assume that the lives of items that have failed because of ageing can be separated from the other item lives (this, in fact, is in some cases really possible when suitable criteria of ageing are set). Then, PDF, CDF, and the wear-out part of the reliability function, $R_w(t)$, can be estimated (Figure 2.14b).

The summary failure probability, taking into account failures that belong to both periods, and assuming that the two types of failure are *independent,* is given by Equation (2.6), that is,

$$P_f = P_{fc} + P_{fw} - P_{fc} \cdot P_{fw}$$

Setting

$$P_{fc} = 1 - R_c, \quad P_{fw} = 1 - R_w$$

where R_c, R_w = reliability functions corresponding to the constant failure period and to the wear-out period, respectively. After rearrangement it results

$$P_f = 1 - R_c R_w$$

and further, using Equation (2.3), the reliability function in the domain of simultaneous occurrence of current failures and wear-out failures is

$$R = R_c R_w$$

Figure 2.15 shows R for two cases: $\mu_c > \mu_w$ and $\mu_c < \mu_w$. It is assumed here that PDF of the wear-out life is symmetric, which definitely may not be so in practice.

On many occasions it is not important *how long* the service periods of an item were but *how many times* it was put in service. All formulas given in the foregoing paragraphs also remain valid for such situations. Obviously, they can be simply adjusted by substituting the time by the *number of service cycles*.

So, for example, Equation (2.51) will be

$$R(n) = \exp\left(-\int_0^n \lambda(n)\mathrm{d}n\right)$$

Similarly, Equation (2.53) can be written as

$$\Phi(n) = 1 - R(n)$$

where $\Phi(n)$ = CDF of the number of cycles to failure.

Also, instead of time and number of cycles, other arguments of the reliability function can be used. For example, the number of items consecutively produced, number of segments of a lifeline, and others.

2.2.2 Reliability function of a system

When a set of reliability items with independent *item reliability functions* R_1 through R_n is connected in a reliability system, we can establish the reliability function of this system, R_{sys}. - For simplicity, the argument, (t), is omitted in the following notation; it is understood that all R are time-dependent.

Consider first a ***serial system***. Taking into account again that the reliability function is, in fact, identical with the probability of survival we can determine the reliability function of the system as [see Equation (2.5)]

$$R_{sys}^{ser} = R_1 \cdot R_2 \cdot \dots \cdot R_n \tag{2.55}$$

When the item reliability functions are

$$R_i = e^{-\lambda_{ci} t}$$

with all λ_{ci} constant, Equation (2.55) yields

$$R_{sys}^{ser} = \exp\left(-\sum_{i=1}^{n} \lambda_{ci} \cdot t\right) \tag{2.56}$$

Hence, comparing Equations (2.56) and (2.52), the failure rate of a serial system is given by

$$\lambda_{sys}^{ser} = \sum_{i=1}^{n} \lambda_{ci}$$

An analogous solution can be applied to a ***parallel system*** composed of n items with independent reliability functions R_1 through R_n. The reliability function for such system, R_{sys}^{par}, can be derived from the failure probability of the system

$$P_{f,sys} = P_{f1} \cdot P_{f2} \cdot \ldots \cdot P_{fn}$$

that is

$$P_{f,sys} = (1 - R_1) \cdot (1 - R_2) \cdot \ldots \cdot (1 - R_n)$$

and so the system reliability function is

$$R_{sys}^{par} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

In reliability systems with items connected in parallel, the failure rate is often equal for all items, $\lambda_{ci} = \lambda_c$. Then, the failure rate of the system is given by

$$\lambda_{sys}^{par} = \frac{1}{\mu_c}$$

$$\mu_c = \frac{1}{\lambda} + \frac{1}{2\lambda} + \ldots + \frac{1}{n\lambda}$$

The reader should try to draw the reliability function and the failure rate for a system with the combined connection according to Figure 2.9.

Analogous calculations can be performed for systems with reserves of any kind. In specialized reliability monographs and handbooks numerous examples can be found.

2.3 METHOD OF MOMENTS

When computers were not available and, consequently, the Monte Carlo simulation technique (Section 2.4) could not be efficiently used, the method of moments provided a good tool to the analysis of derived random variables. At present, the method of moments still remains a practical instrument, but its objectives have changed:

- ♦ it supplies a quick overview on parameters of the derived random variable; behavior of the parameters can be simply assessed without performing any simulation calculations;
- ♦ it is used in establishing the first-order and also second-order members of probability distribution moments of the derived variable (reliability margin, as a rule) in the reliability investigation based on the Hasofer-Lind reliability index, β^{HL} (see Sections 8.5, 9.2, and 9.3).

We will not give mathematical developments of the moment method (they can be found in Tichý and Vorlíček 1972); only results important for the above mentioned aims will be introduced.

2.3.1 Parameters and quasi-parameters

First, let us define a derived random variable by

$$\eta = f_\eta(\xi_1, \xi_2, \dots, \xi_n) \tag{2.57}$$

where ξ_1 through ξ_n = random variables, each described by three population parameters, μ_i, σ_i, and α_i, respectively. Basically, no other information on the probability distributions of ξ_i is needed. Nevertheless, a general type of the distribution (say, three-parameter log-normal) must be assumed since in some formulas coefficients of excess, ε_i, appear.

Second, the following assumptions, valid in the respective definition domain, must be accepted:

- ♦ the random variables ξ_1 through ξ_n are continuous and independent; when some dependence is observed or is obvious, it can be dealt with by compiling an appropriate statistical dependence function as explained in 2.1.3;
- ♦ f_η (.) is continuous and differentiable up to the fourth derivative;
- ♦ the expansion of f_η(.) in a Taylor series is convergent in the domain of investigation; this assumption is very difficult to verify in advance.

To simplify, the subscript η at f_η will be omitted.

Expanding f(.) in a Taylor series, neglecting expansion members containing derivatives of order $n \geq 5$, and performing some analytical calculations, the approximation formulas for the mean, standard deviation, and coefficient of skewness of the derived variable, η, can be written (see Tichý and Vorlíček 1972):

$$\mu_\eta = f_0 + \frac{1}{2}\sum_i f_{ii}\sigma_i^2 + \frac{1}{6}\sum_i f_{iii}\sigma_i^2\alpha_i + \frac{1}{24}\sum_i f_{iiii}\sigma_i^4(\varepsilon_i + 3) + \frac{1}{4}\sum_{i<j} f_{iijj}\sigma_i^2\sigma_j^2 \tag{2.58}$$

$$\sigma_\eta^2 = \sum_i f_i^2\sigma_i^2 + \sum_i f_i f_{ii}\sigma_i^2\alpha_i + \frac{1}{4}\sum_i f_{ii}^2\sigma_i^4(\varepsilon_i + 2) + \frac{1}{3}\sum_i f_i f_{iii}\sigma_i^4(\varepsilon_i + 3) + \sum_{i<j}(f_{ij}^2 + f_j f_{ijj} + f_i f_{iij})\sigma_i^2\sigma_j^2 \tag{2.59}$$

$$\alpha_\eta = \frac{1}{\sigma_\eta^3}\left[\sum_i f_i^3\sigma_i^3\alpha_i + \frac{3}{2}\sum_i f_i^2 f_{ii}\sigma_i^4(\varepsilon_i + 2) + 6\sum_{i<j} f_i f_j f_{ij}\sigma_i^2\sigma_j^2\right] \tag{2.60}$$

where

$$f_i = \frac{\partial f(.)}{\partial \xi_i}, \quad f_{ij} = \frac{\partial^2 f(.)}{\partial \xi_i \partial \xi_j}, \quad f_{ii} = \frac{\partial^2 f(.)}{\partial \xi_i^2}, \quad \text{etc.}$$

the derivatives shall be taken about the point (μ_1, μ_2, ..., μ_n). Further, f_0 = value of f(.) obtained by setting $\eta_i = \mu_i$ for all i, and ε_i = coefficient of excess of the respective probability distribution. Since knowledge of only three independent parameters is assumed, ε_i has to be expressed in terms of these three parameters, the respective formula being dependent on the type of distribution chosen. For the three-parameter log-normal distribution, the coefficient of excess can be calculated from formulas given in Appendix A.

Equations (2.58) through (2.60) *are not expansions,* but formulas derived by expanding f(.) in Taylor series and, subsequently, performing necessary integrations to obtain mean, variance, and coefficient of skewness. Obviously, when f(.) is *linear,* only first members apply.

When f (.) is *non-linear,* the analytical solution of the above formulas becomes complicated. Moreover, some numerical investigations show that good estimates of the respective parameters are obtained only when simple, "well-behaved" functions are dealt with. Unfortunately, such functions are rarely encountered in practice, and they can be found, as a rule, in textbooks only. Further, some numerical solutions have shown that from Equations (2.58) through (2.60) reliable results are obtained only when the coefficients of variation, $\delta_i = \sigma_i / \mu_i$, of the input variables are small, that is, not greater than about 0.15. The actual values of δ_i often exceed this limit (for example, when time-dependent structural load is dealt with).

Now, first members of these formulas can be used efficiently for specific purposes. To simplify the phrasing, let the first members be called ***quasi-parameters,*** that is *quasi-mean, quasi-variance* (with quasi-standard deviation, or shortly: *quasi-sigma*), and quasi-skewness (*quasi-alpha*). Thus we obtain:

$$Q\mu_\eta = f_0 \tag{2.61}$$

$$Q\sigma_\eta^2 = \sum_i f_i^2 \sigma_i^2 \tag{2.62}$$

$$Q\alpha_\eta = \frac{1}{Q\sigma_\eta^3} \sum_i f_i^3 \sigma_i^3 \alpha_i \tag{2.63}$$

In Equation (2.63), the approximation $\sigma_\eta \approx Q\sigma_\eta$ has been used. Equation (2.62) can be further approximated to

$$Q\sigma_\eta^2 = Q\mu_\eta \sum_i \delta_i^2 \tag{2.63a}$$

where $\delta_i = \sigma_i/\mu_i$. This is acceptable only for algebraic functions and with δ_i small, that is, less than 0.10. Otherwise the formula is useless.

Obviously, for linear f(.) , it is

$$Q\mu_\eta = \mu_\eta, \quad Q\sigma_\eta^2 = \sigma_\eta^2, \quad Q\alpha_\eta = \alpha_\eta$$

In terms of standardized variables, u_i, with $\mu_{ui} = 0$, $\sigma_{ui} = 1$, and $\alpha_{ui} = \alpha_i$, Equation (2.57) can be transformed setting

$$\xi_i = \mu_i + u_i\sigma_i \tag{2.64}$$

and so

$$\eta^u = f^u(u_1,u_2,\dots,u_n) \tag{2.65}$$

Then, Equation (2.61) becomes

$$Q\mu_\eta^u = f_0^u$$

Since $\mu_{ui} = 0$ for all i , we obtain, considering Equation (2.64) with $u_i = \mu_{ui}$,

$$Q\mu_\eta^u \equiv Q\mu_\eta = f_0 \tag{2.66}$$

Finally,

$$Q\sigma_\eta^{u2} = \sum_i f_i^{u2} \tag{2.67}$$

$$Q\alpha_\eta^u = \frac{1}{Q\sigma_\eta^{u3}} \sum_i f_i^{u3}\alpha_i \tag{2.68}$$

2.3.2 Some simple functions

To create a rough picture on various typical functions formulas for parameters of a few simple derived random variables are given, derived by M. Vorlíček in 1961. The input variables are considered independent, their parameters being μ, σ, and α, with appropriate subscripts where relevant. - To simplify the notation, coefficients of variation, $\delta = \sigma/\mu$, are used in some formulas.

$$\eta = a\xi^2 + b\xi + c \tag{2.69}$$

$$\sigma_\eta^2 = (2a\mu + b)\cdot[2a(\mu + \sigma\alpha) + b]\,\sigma^2 + \frac{1}{2}a^2\sigma^4(4 + 3\alpha^2)$$

$$\alpha_\eta = \sigma_\eta^{-3}\sigma^3(2a\mu + b)^2\cdot[(2a\mu + b)\,\alpha + \frac{3}{2}a\,\sigma(4 + 3\alpha^2)]$$

$$\eta = a\xi^n \tag{2.70}$$

$$\mu_\eta = a\mu^n[1 + \frac{1}{2}n(n-1)\delta^2 + \frac{1}{6}n(n-1)(n-2)\delta^3\alpha$$
$$+ \frac{1}{16}n(n-1)(n-2)(n-3)(2 + \alpha^2)\delta^4]$$

$$\sigma_\eta^2 = (an\mu^n\delta^n)^2\cdot\mathrm{A}$$

$$\alpha_\eta = \mathrm{sign}\left(\frac{a}{n}\right)\frac{\mathrm{B}}{\mathrm{A}^{3/2}}$$

$$\mathrm{A} = 1 + (n-1)\,\delta\alpha + \frac{1}{2}(n-1)(3n-5)\delta^2$$
$$+ \frac{1}{8}(n-1)(7n-11)\delta^2\alpha^2$$

$$\mathrm{B} = \alpha + \frac{3}{4}(n-1)(4 + 3\alpha^2)\,\delta$$

$$\eta = \frac{a}{\xi} + b \tag{2.71}$$

$$\mu_\eta = \frac{a}{\mu}(1 + \delta^2 - \delta^3\alpha + 3\delta^4 + \frac{3}{2}\delta^4\alpha^2) + b$$

$$\sigma_\eta^2 = \frac{a^2}{\mu^2}\delta^2 \cdot \mathrm{A}$$

$$\alpha_\eta = \operatorname{sign} a \frac{\mathrm{B}}{\mathrm{A}^{3/2}}$$

$$\mathrm{A} = 1 - \delta\alpha + 8\delta^2 + \frac{9}{2}\delta^2\alpha^2$$

$$\mathrm{B} = 6\delta - \alpha + \frac{9}{2}\delta^2\alpha^2$$

$$\zeta = a\frac{\xi}{\eta} + b \tag{2.72}$$

$$\mu_\zeta = a\frac{\mu_\xi}{\mu_\eta}(1 + \delta_\eta{}^2 - \delta_\eta{}^3\alpha_\eta + 3\delta_\eta{}^4 + \frac{3}{2}\delta_\eta{}^4\alpha_\eta{}^2) + b$$

$$\sigma_\zeta^2 = a^2\left(\frac{\mu_\xi}{\mu_\eta}\right)^2 \cdot \mathrm{A}$$

$$\alpha_\zeta = \operatorname{sign} a \frac{\mathrm{B}}{\mathrm{A}^{3/2}}$$

$$\mathrm{A} = \delta_\xi{}^2 + \delta_\eta{}^2 - 2\delta_\eta{}^3\alpha_\eta + 8\delta_\eta{}^4 + 3\delta_\xi{}^2\delta_\eta{}^2 + \frac{9}{2}\delta_\eta{}^4\alpha_\eta^2$$

$$\mathrm{B} = \delta_\xi{}^3\alpha_\xi - \delta_\eta{}^3\alpha_\eta + 6\delta_\eta{}^4 + 6\delta_\xi{}^2\delta_\eta{}^2 + \frac{9}{2}\delta_\eta{}^4\alpha_\eta{}^2$$

$$\eta = \sum_{i=1}^{n} a_i \xi_i + b \tag{2.73}$$

$$\mu_\eta = \sum_{i=1}^{n} a_i \mu_i + b$$

$$\sigma_\eta^2 = \sum_{i=1}^{n} a_i^2 \sigma_i^2$$

$$\alpha_\eta = \sigma_\eta^{-3} \cdot \sum_{i=1}^{n} a_i^3 \sigma_i^3 \alpha_i$$

$$\eta = a \prod_{i=1}^{n} \xi_i + b \tag{2.74}$$

$$\mu_\eta = a \prod_{i=1}^{n} \alpha_i + b$$

$$\sigma_\eta^2 = (a \prod_{i=1}^{n} \mu_i)^2 \cdot \mathrm{A}$$

$$\alpha_\eta = \operatorname{sign} a \, \frac{\mathrm{B}}{\mathrm{A}^{3/2}}$$

$$\mathrm{A} = \sum_{i=1}^{n} \delta_i^2 + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \delta_i^2 \delta_j^2$$

$$\mathrm{B} = \sum_{i=1}^{n} \delta_i^3 \alpha_i + 6 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \delta_i^2 \delta_j^2$$

2.4 MONTE CARLO SIMULATION

The Monte Carlo simulation technique is a well known and widely used tool in structural reliability analysis. It will not be discussed in detail here. The following books are useful on this technique:

♦ ***general:*** Forsythe *et al.* 1977, Rubinstein 1981, as well as many books on numerical analysis;

♦ ***specialized, aimed at structural reliability problems:*** Ang and Tang II 1984, Augusti *et al.* 1984, Elishakoff 1983, Melchers 1987.

The present availability of computers and high processing speeds permit fast and comfortable applications of Monte Carlo techniques to a diverse range of problems. It can be expected that with further expansion of computing power, the Monte Carlo technique will continue to develop in the future. *We should keep in mind that without it many reliability problems would be simply insolvable.*

The principle of Monte Carlo technique is very simple. Consider Equation (2.57); assume that the probability distributions, or in other words, the populations of ξ_1 through ξ_n are known and defined. Now, let us *randomly* draw from each population i a value of ξ_i. An n-tuplet ($x_1^{(1)}$, $x_2^{(1)}$, ..., $x_n^{(1)}$) is obtained, and

$$y^{(1)} = f(x_1^{(1)}, x_2^{(1)}, ..., x_n^{(1)})$$

is calculated. The n-tuplet of *draws* is called a *trial,* and $y^{(1)}$ is a *realization.* The drawing of trials is repeated N-times, and so a *sample of realizations,* $y^{(1)}$ through $y^{(N)}$, is obtained. Obviously, $n \times N$ draws have to be performed. Then, the sample is subjected to further statistical and probabilistic treatment. Several derived random variables η_1, η_2, ..., η_n can be investigated simultaneously; realizations $y_1^{(k)}$, $y_2^{(k)}$, ..., $y_m^{(k)}$ are obtained with the trial k.

During recent years several adjustments of the Monte Carlo technique were proposed (for a good survey, see Bjerager 1991), all aimed at shortening the processing time. Most of these techniques introduce some *bounds to the sampling procedure* which results in a number of trials less than required in a "plain" Monte Carlo simulation, where the chance of input variables being included into the sample is governed solely by their respective probability distributions (see, for example, Bucher 1988, Florian 1992, and Schuëller *et al.* 1989).

In general, however, *any limitations decrease the amount of information supplied,* though with certain techniques estimates of the population mean and variance can be improved.

As a rule, sample reducing techniques provide good information about the population mean, population variance, distribution fractiles that are close to the mean, etc. Yet, when other parameters are needed (coefficient of skewness, fractiles in the domain of the distribution tails, and others), the quality of information decreases rapidly with the decreasing sample size whatever time-saving method is used. Since in many cases failure probabilities (see Sections 8.1 and 9.1) or fractiles corresponding to given target failure probabilities are required, large samples are necessary to attain a sufficiently reliable result whenever the expected or the intended probability is small.

In calculation of probabilities and fractiles a substantial reduction of processing time can often be achieved by *combining Monte Carlo simulation with estimation of the probability distribution* of the investigated variable (see the S-E technique in 9.1.1). Then, for satisfactory results, the number of trials needed is only 5,000 to 10,000.

Most of the published monographs limit the information on the Monte Carlo technique to its theoretical aspects, describing procedures, random number generators, etc. Here, a few ***practical hints*** are given that can be useful during the solution of problems:

(1) Contact the programming adviser or system engineer to obtain basic information on the *software concerning the Monte Carlo technique* available in your computer facility. You should be sure that the *random number generator,* RNG, has been proved and subjected to statistical tests. Fortunately, no substantial problems have occurred to the present in this area; since the beginning of the eighties software RNGs are sufficiently well tested.

(2) Study the problem you intend to solve from the simulation point of view. Consider its mathematical formulation and *try to visualize your problem as a complex natural phenomenon.* Remember that you are trying to *imitate Nature.* You may discover that the calculation model you want to subject to a Monte Carlo treatment is biased in some way. For example, constants appearing in the problem might be, in fact, random variables; their randomness was underestimated, neglected, or entirely unknown to the authors of formulas. The calculation model or a part of it can originate from times when no particular attention had been paid to the random variability of engineering phenomena. Therefore, try to identify its background whenever possible. Do not forget that "one" and "zero" are also constants that may stand for random variables with mean equal to one or zero, respectively. - See Example 2.11.

(3) It is important not to include *illogical realizations* into the sample generated; the sample size has to be *diminished by the number of such realizations;* this number should be recorded. If it is too high (more than about 20 percent of the intended sample size), the calculation model should be checked for correctness.

(4) Select *appropriate probability distributions* to describe the input variables. For example, when negative values of a variable are physically impossible, use a lower-bounded distribution. Log-normal distribution (see Appendix A), is recommended for variables with lower or upper bound. Yet, when both bounds are apparent, a single-bounded distribution will still suffice in most cases. The more important bound should govern the choice.

(5) Identify all possible *dependencies* among input variables and try to model them by appropriate statistical dependence functions (see 2.1.3). Again, do not forget that in the past the dependencies were not fully recognized, and may not be expressed in the functions investigated.

(6) In the beginning of the analysis, before performing a series of large sample solutions, make some *sensitivity tests* to understand influences that are to be dealt with. Perform some pilot tests and examine the behavior of the sample characteristics of the parameter studied in relation to the sample size and to other parameters of the problem.

(7) Do not hesitate to *plot histograms.* A study of the histogram can reveal irregularities and pitfalls of your calculation model. Multi-modality, humps and other aberrations in the empirical frequency curve signalize that the calculation model might be biased. Do not try to explain the aberrations by some sophisticated pondering until you have checked all simpler reasons. In the majority of cases, histograms converge to smooth curves.

Undulations, humps, local peaks of frequency curves are not typical for phenomena dealt with. However, possibility of "regular irregularities" cannot be utterly excluded.

(8) Always record the *seed number* and also the *sample size* used at each run. This is important for a possible repetition of the simulation with different input parameters. You or your colleague may for some reason return to the problem after a certain period and may like to compare new results with the previous solution. Yet, do not forget, that for different sample size, or even a different number of input variables, the same seed yields incomparable results. Do not forget that RNGs are often computer-dependent.

(9) It is often better and less time-consuming to *repeat* m-times a simulation with sample size n and to take, from these m runs, the mean value of the parameter investigated (see 2.1.2, Union of two samples), than to perform a single solution with sample size $n \times m$.

(10) It is practical to *monitor* the development of one or more simulated quantities (statistical characteristics, probability, and others) on the computer screen. You can stop the calculation as soon as the simulated values of the respective quantity become sufficiently stable.

(11) Some parameters require a higher *computing precision.* For example, when calculating the coefficient of skewness with a Fortran program, the DOUBLE PRECISION arithmetic must be used.

(12) Fluctuations of the *coefficient of skewness,* α, and of the *coefficient of excess,* ε, depend upon the coefficient of variation, δ; the smaller δ, the greater fluctuations. However, we are, as a rule, not interested in ε.

(13) Pay attention to the *processing time.* Make some time-sensitivity study whenever a program with expected repeated use is prepared. Always record the execution time.

(14) The *ordering algorithms,* needed chiefly in the analysis of fractiles, can have diverse properties as far as the execution time is concerned. Some algorithms are relatively fast for small samples, while being lazy for large samples, and conversely.

■ **Example 2.11.** The random variability of the active earth pressure factor, K_a, is to be analyzed. The input variables are shown in Figure 2.16. - To avoid confusion with notation used throughout this book, boldface Greek characters will be used in this example for quantities referred to the earth pressure problem.

For K_a the well known Coulomb formula, verified by experiments, is valid

$$K_a = \frac{\cos^2(\boldsymbol{\varphi} - \boldsymbol{\alpha})}{\cos^2\boldsymbol{\alpha} \cdot \cos(\boldsymbol{\delta} + \boldsymbol{\alpha})} \left\{ 1 + \left[\frac{\sin(\boldsymbol{\varphi} + \boldsymbol{\delta}) \cdot \sin(\boldsymbol{\varphi} - \boldsymbol{\beta})}{\cos(\boldsymbol{\delta} + \boldsymbol{\alpha}) \cdot \cos(\boldsymbol{\alpha} - \boldsymbol{\beta})} \right]^{\frac{1}{2}} \right\}^{-2}$$

In a current design we simply set for angles $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\delta}$, and $\boldsymbol{\varphi}$ values specified in regulations or resulting from some report, without considering various relations among them. These values reflect the logic of the case investigated.

Nevertheless, when simulating the random behavior of K_a, the dependence between some variables must be considered:

♦ First, the angle of internal friction, $\boldsymbol{\varphi}$, and the angle of friction of the earth against the wall, $\boldsymbol{\delta}$, are obviously dependent. It is often suggested to take

$$\boldsymbol{\delta} = \frac{2}{3}\boldsymbol{\varphi}, \quad \text{or} \quad \boldsymbol{\delta} = 0.5\boldsymbol{\varphi}$$

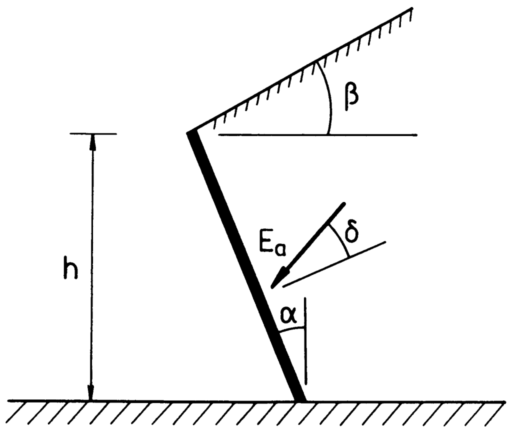

Fig. 2.16 - *Example 2.11.* Active earth pressure; notation of input variables.

which, however, represents only an "average" dependence. Considering the statistical viewpoint the dependence is more complicated. For example, we can write

$$\delta = \frac{2}{3}\varphi + \delta'$$

where δ' = random variable with the mean $\mu_{\delta'} = 0$ and variance $\sigma^2_{\delta'} > 0$.

Or, we can assume

$$\delta = \kappa\varphi + \delta'$$

where κ = random variable with $\mu_\kappa = \frac{2}{3}$ and $\sigma^2_\kappa \neq 0$.

Even when φ and δ were independent, we should always include into the respective algorithm the condition that for

$$\delta > \varphi \qquad \text{(a)}$$

the respective realization of K_a is not considered in the sample.

(b) Further, it is clear that the angle of the earth slope, β, must be less than the angle of internal friction, φ. Obviously, β and φ are independent; however, β can never be greater than δ, otherwise the slope could not be carried out. Thus, we must again include the condition that for

$$\beta > \varphi$$

the respective value of K_a is ignored. Even when $\mu_\beta \leq \mu_\varphi$, some random pairs (β , φ) could satisfy the above inequality, and the equation for K_a would not have any solution.

(c) The last limitation necessary in the given problem is that for

$\delta < 0$

the realization of K_a is again ignored. Actually, when $\delta < 0$, the passive earth pressure is dealt with, for which the K_a formula does not hold any more.

When, for example, Equation (a) is fulfilled, we cannot set $\delta = \varphi$ and include the calculated K_a into the sample. This would result in a hump or peak in the histogram. ■

Many applications of the M.C. simulation method can be found in various publications (for example: Floris and Mazzucchelli 1991, Fogli 1982, Strating and Vos 1973, Mirza and Skrabek 1992, and Van Breugel 1992). The reader is suggested to study one or two papers carefully to get acquainted with all the finesse of the technique.

2.5 FUZZY LOGIC

An important instrument of the structural reliability theory is the fuzzy logic, which is a particular branch of the *multi-valued logic* family. Its value system is either continuous or discrete, dependently on the type of the problem given and the calculation model used. The fuzzy logic is a formal basis for the *fuzzy set theory,* which can be used in diverse reliability problems to describe a particular type of indefiniteness (see Section 1.1) not covered by probabilistic models.

In general, a fuzzy set, $\dot{\mathrm{F}}$, consists of two groups of elements, **e**; one group unambiguously belongs to $\dot{\mathrm{F}}$, while the other group is composed of elements that are partially members of $\dot{\mathrm{F}}$ and partially members of the complementary set $\dot{\mathrm{F}}'$. It is

$$\dot{\mathrm{F}} \cup \dot{\mathrm{F}}' = \dot{\mathrm{U}}$$

where $\dot{\mathrm{U}}$ = universal set. Obviously, $\dot{\mathrm{F}}'$ and the boundary between $\dot{\mathrm{F}}$ and $\dot{\mathrm{F}}'$ are also fuzzy.

The association of **e** with either $\dot{\mathrm{F}}$ or $\dot{\mathrm{F}}'$ is expressed by the *membership function,* $\mu(\mathbf{e})$, whose values are in the interval [0, 1]. If $\mu(\mathbf{e}) = 1$, then $\mathbf{e} \in \dot{\mathrm{F}}$, if $\mu(\mathbf{e}) = 0$, then $\mathbf{e} \notin \dot{\mathrm{F}}$. Various forms of the membership function are possible.

The methodology of fuzzy sets was introduced in the structural reliability outfit by Blockley 1980, where the reader is primarily referred to. Since that time the fuzzy set concepts have been gradually implemented in advanced calculation models. In general, opinions on the benefits of the use of fuzzy set techniques has not yet stabilized. It is true that exercises where fuzzy logic is applied can also be solved by traditional procedures. However, on many occasions, the fuzzy set approach can highlight aspects of the problem that could not be recognized in other way. Fuzzy sets can be applied in various specific areas of reliability investigation, as, for example, in risk analysis, evaluation of tests, and serviceability limit states.

We will not use any fuzzy set solutions in this book; to readers interested, a few take-off publications can be recommended: *Applications* 1989, Bardossy and Bogardi 1989, Blockley 1987, Chou and Yuan 1992, Der Kiureghian 1989, Hadipriono 1986, Holický 1991, Munro 1987.

3

PHENOMENA, EVENTS, AND RELATIONS

Key concepts in this chapter: *phenomenon; event; absolutely adverse phenomenon; relatively adverse event; existential relation; sequential relation; relation formula; physical relation; statistical relation; combination; defined combination; arbitrary combination.*

3.1 PHENOMENA AND EVENTS

When investigating the reliability of constructed facilities, *certain* and *possible phenomena,* as well as *certain* and *possible events* have to be dealt with. Although each of these terms has its particular meaning, they are frequently confused. Let us discuss them in this chapter more closely.

A ***phenomenon*** consists of facts that can be perceived and that describe the *state of things as they are or as they appear to be.* A phenomenon is related either to *matter* (for example, strength of material in a general sense, gravity, temperature, explosion) or to *consciousness* (for example, human sensitivity to vibrations of the building, relation of the man to CF), or it can imply both (for example, reliability margin with respect to deflections defined by aesthetic criteria). - Each phenomenon has its *substance* expressing the entirety of its properties. For example, the substance of the phenomenon "strength of concrete" are properties of the aggregate, properties of cement, size of the specimen, and age. The substance of the phenomenon "reliability margin with respect to ultimate resistance" are material properties, geometry of a structure, properties of load, and others. - Again, phenomena will be designated by H and by the operation symbol Ph(.).

When a phenomenon can be repeated or when it can assume several different forms, then any of the occurrences and any of the forms is called a *realization.* A realization can be expressed by a value of a continuous or discrete variable, or by a value of a function. An occurrence of the realization of the respective phenomenon, or an occurrence of separate realizations of several phenomena, accompanied by a change in state of CF that can be observed by user, owner, or by other persons, or not observed at all, is called an ***event.*** Events will be designated by E and, in operation notation, by Ev(.).

A simple fact must be realized: a certain event E can happen only when the respective *parent phenomenon* H exists. Here is the source of confusion that we often encounter when talking about these two concepts. It frequently happens that no distinction is being made between them.

To clarify, Table 3.1 introduces two examples of phenomena, realizations, and events.

Table 3.1 - Examples of phenomena, realizations, and events

Example	**1**	**2**
Phenomenon, H	yield stress of the steel used in the structure	wind load
Realization of H	yield stress of steel in the cross-section	wind pressure in the given location
Event, E	insufficient yield stress of steel in the cross-section	excessive wind pressure in the given location

Relations among phenomena and among events

Among phenomena of a certain group, *four categories of relations* can be identified:

- existential relations;
- sequential relations;
- physical relations;
- statistical relations.

Some of these categories may not be interesting for structural reliability solutions, or their influence on reliability is negligible.

When a certain type of relation is found between several phenomena, an analogous relation must exist between the corresponding events. An analysis of relations that takes phenomena into account results in *qualitative information* necessary for the formulation of ***physical reliability requirements*** (see Chapter 7).

Considering events, the analysis of relations provides *quantitative information* needed for ***probabilistic reliability requirements*** (see Chapter 8) and for the derivation of *design parameters*. They are particularly important in solutions of diverse *combination problems* and in drawing *event trees* (also: fault trees, failure trees). Such trees are helpful in many reliability-based exercises where knowledge of *behavior history* of a system is needed (see, for example, Bruneau 1992, Hadipriono *et al.* 1986, Karamchandani *et al.* 1992, Karamchandani and Cornell 1992, Reed and Brown 1992, Schall *et al.* 1988, Whitman 1984).

3.2 EXISTENTIAL RELATIONS

3.2.1 Four fundamental relations

When a set $\dot{\mathrm{H}}$ of phenomena H_i ($i = 1, 2, \ldots, n$), created by Nature and by Humankind, is analyzed from the viewpoint of simultaneous occurrence of these phenomena in a certain place and at a certain point in time, the following *four types of existential relations* can be found between individual members of the set:

(a) ***existentially simultaneous phenomena;*** these are phenomena that cannot exist alone, and their simultaneous occurrence is a necessity;

(b) ***existentially independent phenomena;*** they may or may not occur separately, or they may or may not occur simultaneously;

(c) ***existentially positively dependent phenomena;*** these are formed by a group of *primary phenomena,* H^*, and by a group of *secondary phenomena,* H^{**}; phenomena H^* and H^{**} may or may not occur, but phenomena H^{**} can only occur when simultaneously phenomena $\mathbf{H}^*$ take place;

(d) ***existentially negatively dependent phenomena;*** they exclude each other, and so they can only occur individually, never simultaneously.

The phrase *"may or may not occur"* does not refer to the randomness of the occurrence of the phenomena. We are still in the non-random domain, and so the phenomena are discussed here without any reference to their random nature.

The principal importance of the existential relations consists in the determination in what ***existential combinations*** the phenomena subjected to analysis can occur. As an existential combination a *simultaneous occurrence of several phenomena* that belong to a set $\dot{\mathrm{H}}$ of n phenomena is considered. For completeness' sake, also the occurrence of a *single phenomenon* is regarded as a combination, supposing of course that this phenomenon belongs to $\dot{\mathrm{H}}$. Existential combinations will be denoted by

$$(\mathrm{H}_i, \mathrm{H}_k, \ldots, \mathrm{H}_l)_e$$

where i, k, ..., l = subscripts of some of the phenomena H_1 through H_n of the given set.

For "existential combination" the simpler term "combination" is often used (for example, combination of snow load and wind load).

3.2.2 Relation formulas

To describe various relations, simple and clear symbols are needed. The following notation will be used:

!(.)	necessity, simultaneity
(.)	independence

(.[.])	superiority, primarity (brackets indicate primary phenomena)
N(.)	impossibility

Periods in parentheses and brackets can denote sets of phenomena, that is, either single phenomena or groups of phenomena, or also a relation formula.

The significance of the relations between phenomena can be simply demonstrated by an example of *two phenomena* and *three phenomena*. It is of course possible to extend the discussion to larger phenomena sets but that would involve us into unnecessarily complicated elaborations. We only want to show the reader the general approach needed when the simultaneity of several phenomena is studied.

Two phenomena

Let us investigate existential relations between phenomena H_1 and H_2.

(1) Two existentially simultaneous phenomena have only one possibility of occurrence, that is, they can only appear in only a *single existential combination:*

♦ H_1 and H_2 simultaneously: $(H_1, H_2)_e$

Individual occurrence of any of these phenomena is excluded. The respective relation formula is:

$$!(H_1, H_2)_e$$

(2) *Three existential combinations* are possible when H_1 and H_2 are existentially independent:

♦ H_1 alone: $(H_1)_e$
♦ H_2 alone: $(H_2)_e$
♦ H_1 and H_2 simultaneously: $(H_1, H_2)_e$

The relation formula is:

$$(H_1,H_2)$$

(3) Let H_1 be the primary phenomenon (H^*) and H_2 the secondary phenomenon (H^{**}), the latter being independent of H_1. Then, only *two existential combinations* are possible:

- ♦ H_1 alone: $(H_1)_e$
- ♦ H_1 and H_2 simultaneously: $(H_1, H_2)_e$

Clearly, an isolated occurrence of the secondary phenomenon is impossible. The relation formula is:

$$(H_2 [H_1])$$

(4) Similarly, for two existentially negatively dependent phenomena only *two combinations* are possible:

- ♦ H_1 alone: $(H_1)_e$
- ♦ H_2 alone: $(H_2)_e$

A simultaneous occurrence of the two phenomena is impossible. The relation formula is:

$$N(H_1, H_2)$$

The possible existential combinations of two phenomena, H_1 and H_2, are summarized in Table 3.2.

Table 3.2 - Existential combinations of two phenomena (X -occurrence, 0 - non-occurrence of the respective combination)

Case	**(1)**	**(2)**	**(3)**	**(4)**
Relation	$!(H_1,H_2)$	(H_1,H_2)	$(H_1[H_2])$	$N(H_1,H_2)$
1: $(H_1)_e$	0	X	0	X
2: $(H_2)_e$	0	X	X	X
3: $(H_1,H_2)_e$	X	X	X	0
Number of existential combinations	1	3	2	2

Three phenomena

Whereas only four possible relations can be found for two phenomena [these relations are identical with the existential relations (a) through (d)], a much larger set of combination possibilities is offered by three phenomena, H_1, H_2, and H_3. Let us introduce here only some of them:

(5) All three phenomena are existentially simultaneous, and so the relation formula is:

$$!(H_1, H_2, H_3)$$

Again, only one existential combination can occur:

$$(H_1, H_2, H_3)_e$$

Table 3.3a - Possible existential combinations of three phenomena with some arrangement of the existential relations (X - occurrence, 0 - non-occurrence of the respective combination)

Case	**(5)**	**(6)**	**(7)**	**(8)**
Relation	$!(H_1,H_2,H_3)$	(H_1,H_2,H_3)	$(H_3[H_2[H_1]])$	$(H_3[H_1], N(H_1,H_2))$
1: $(H_1)_e$	0	X	X	X
2: $(H_2)_e$	0	X	0	X
3: $(H_3)_e$	0	X	0	0
4: $(H_1,H_2)_e$	0	X	X	0
5: $(H_1,H_3)_e$	0	X	0	X
6: $(H_2,H_3)_e$	0	X	0	0
7: $(H_1, H_2,H_3)_e$	X	X	X	0
Number of existential combinations	1	7	3	3

(6) All three phenomena are existentially independent; thus, seven combinations are possible (see Table 3.3a):

$$(H_1)_e, \quad (H_1, H_2)_e, \quad (H_1, H_2, H_3)_e$$

$$(H_2)_e, \quad (H_1, H_3)_e$$

The relation formula for this set of phenomena is:

$(H_3)_e$, $(H_2, H_3)_e$

(H_1, H_2, H_3)

(7) Let H_2 be positively dependent on H_1, and H_3 positively dependent on H_2. The existential combinations possible in this case are shown in Table 3.3a; the relation formula is:

$(H_3\,[H_1\,[H_1]])$

(8) Let phenomena H_1 and H_2 be negatively dependent, and let H_3 depend positively on H_1. Possible existential combinations are given in Table 3.3a, and the relation formula is now:

$(H_3\,[H_1],\ N(H_1, H_2))$

Table 3.3b - Possible existential combinations of three phenomena with some arrangement of existential relations (X - occurrence, 0 - non-occurrence of the respective combination)

Case	**(9)**	**(10)**	**(11)**
Relation	$(H_3, !(H_1, H_2))$	$N(H_1, H_2, H_3)$	$(H_3, [!(H_1, H_2)])$
1: $(H_1)_e$	0	X	X
2: $(H_2)_e$	0	X	X
3: $(H_3)_e$	X	X	0
4: $(H_1, H_2)_e$	X	X	0
5: $(H_1, H_3)_e$	0	X	X
6: $(H_2, H_3)_e$	0	X	X
7: $(H_1, H_2, H_3)_e$	X	0	0
Number of existential combinations	3	6	4

(9) Let H_1 and H_2 be existentially simultaneous, with H_3 existentially independent of H_1 and H_2 (see Table 3.3b). The relation formula is:

$(H_3, !(H_1, H_2))$

(10) The three phenomena cannot occur simultaneously (see Table 3.3b); the relation formula is:

$N(H_1, H_2, H_3)$

(11) Let H_3 be existentially dependent on either H_1, or on H_2, but H_1 and H_2 be mutually exclusive (see Table 3.3b). The relation formula is:

$(H_3 [N(H_1, H_2)])$

Observe some *facts*:

(a) the *number of existential combinations* depends on the nature of relations among phenomena,

(b) the *nature of existential combinations* differs according to the type of the relation formula; when considering combinations of two or more phenomena, *three types of existential combinations* can be distinguished (see Tables 3.3):

♦ *closed combinations,* where all phenomena are existentially simultaneous [for example, combination No. 3 in case (1), and combination No. 7 in case (5), combination No. 4 in case (9)];

♦ *fixed combinations,* where at least one phenomenon is existentially independent of the others and at least one phenomenon is primary [for example, combination No. 3 in case (3), combinations Nos. 4 and 7 in case (7), combination No. 5 in case (8), combination No. 7 in case (9)];

♦ *free combinations,* where none of the phenomena is bound to other phenomena [for example, combination No. 3 in case (2), combinations Nos. 4 through 7 in case (6)].

3.2.3 Number of existential combinations

The foregoing paragraphs show that the number of existential combinations, in which the phenomena can occur, depends on the type of the respective existential relations.

When existentially simultaneous phenomena are dealt with, only one existential combination is possible, that is, $m_e = 1$.

For a basic set H of existentially independent phenomena, $(H_1, H_2, ..., H_n)$, the number m_e of possible existential combinations of n phenomena is expressed by

$$m_e = \sum_{k=1}^{n} \binom{n}{k} \tag{3.1}$$

where k = number of phenomena in a combination.

The number of existential combinations referred to a set $\dot{\mathrm{H}}$ of n phenomena that are either positively or negatively existentially dependent is given by

$$m_e = \sum_{k=1}^{n} \binom{n}{k} - \bar{m}_e \tag{3.2}$$

where $\bar{m}_e$ = number of existential combinations that cannot occur. A general formula for $\bar{m}_e$ cannot be presented, since the diversity of relations is unlimited. Moreover, such a formula is without practical significance. It suffices to find $\bar{m}_e$ by simple judgment.

When in a set of n phenomena a group of r simultaneous phenomena appears, and the remaining $n-r$ phenomena are existentially independent, the group of simultaneous phenomena should be considered as a single phenomenon. The number of existential combinations becomes

$$m_e = \sum_{k=1}^{n-r+1} \binom{n-r+1}{k} \tag{3.3}$$

The number L_e of independent phenomena participating in a combination is called the ***order of the existential combination.*** For a combination of existentially independent phenomena, L_e equals k. However, for combinations that contain q sets of existentially simultaneous phenomena we get

$$L_e = k - \sum_{i=1}^{q} r_i + q$$

where r_i = number of simultaneous phenomena in the set i .

3.2.4 Examples of existential relations

■ **Example 3.1.** A reinforced concrete member cannot exist without a simultaneous occurrence of the following phenomena [for simplicity, the symbol Ph(.) is omitted]:

- ♦ strength of concrete, f_c ,
- ♦ yield stress of steel, f_y ,
- ♦ member geometry, G^* .

Here, Ph(strength) and Ph(yield stress) shall be understood as properties, not as quantities. - If any of these three phenomena are missing, the reinforced concrete member does not exist. Therefore, the relation formula in this case is

$$!(f_c, f_y, G^*)$$ ■

■ **Example 3.2.** Consider the phenomenon Ph(**load acting on a railway bridge**), Figure 3.1. The following partial phenomena can get involved:

- permanent load due to self-weight, G;
- imposed load created by cars, V_1, and by the locomotive, V_2, or by some auxiliary vehicles, V_3;
- wind load, acting separately on the bridge, w_1, and on vehicles, w_2;
- load due to temperature changes, F_{tem}.

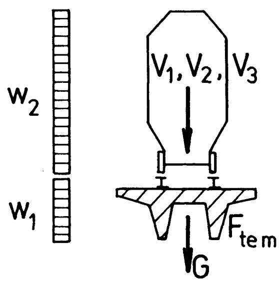

Fig. 3.1 - *Example 3.2.* Loads on a railway bridge.

Among these phenomena the following existential relations can be found:

(a) The imposed loads V_1, V_2, and V_3 cannot be simultaneously applied to a certain point of the bridge (they can, of course, appear simultaneously in different places along the bridge). Thus, the relation formula shall be:

$N(V_1, V_2, V_3)$

(b) Wind load w_2 can only affect vehicles if also the bridge is subjected to wind load w_1:

$(w_2[w_1])$

(c) Wind load affecting vehicles, w_2, can only take place if vehicles are present:

$(w_2\,[N(V_1, V_2, V_3)])$

that is, including the case according to (b):

$(w_2[!(w_1,N(V_1, V_2, V_3))])$

(d) Load produced by temperature changes may or may not occur simultaneously with the other imposed loads:

$(F_{tem},\ w_2[!(w_1,N(V_1, V_2, V_3))])$

(e) All the foregoing loads can only occur if the bridge exists, that is, if the permanent load, G, is present:

$$((F_{tem}, w_2[!(w_1, N(V_1, V_2, V_3))])[G])$$ ■

3.3 SEQUENTIAL RELATIONS

3.3.1 Seven fundamental relations

Let us now study a set $\dot{H}$ of time-dependent phenomena, H_1, H_2, ..., H_n, taking into consideration possibilities of their *successive occurrence*. The possibility of repeated occurrence of H_i will not be considered. Among the individual phenomena the following types of *sequential relations* can be distinguished:

(a) *sequentially necessary non-ordered phenomena* that all must follow one after the other in an arbitrary order;

(b) *sequentially free non-ordered phenomena* that may follow in an arbitrary order, and some of them may not occur;

(c) *sequentially necessary ordered phenomena* that all must follow in a specified order;

(d) *sequentially free ordered phenomena* that must follow in a certain order, but some may not appear;

(e) *sequentially excluding phenomena* that cannot follow one after the other;

(f) *sequentially* a posteriori *dependent phenomena* that are formed by a group of primary phenomena, H^*, and by a group of secondary phenomena, H^{**}; phenomena H^{**} can only appear when they are preceded by phenomena H^*; any group can have one or more members;

(g) *sequentially* a priori *dependent phenomena* that are analogous to the foregoing type: H^{**} can only appear if they are followed by H^*.

Similarly as in the case of existential relations, sequential relations determine what ***sequential combinations*** are possible in a particular case. A sequential combination is defined by *successive occurrence of several phenomena* belonging to the basic set $\dot{H}$ of n phenomena. Notation

$$(H_i, H_k, ..., H_l)_s$$

will be used for sequential combinations, where i, k, l = subscripts referring to the phenomena that appertain to the basic set, $\dot{H}$.

3.3.2 Relation formulas

Again, relation formulas can be written to describe sequential combinations. The following notation will be used:

!(.)	necessity
(.)	possibility
N(.)	impossibility
. ‖ .	orderliness
. ~ .	non-orderliness
. → .	dependence *a posteriori*
. ← .	dependence *a priori*

Three phenomena

The individual sequential relations and the way of their presentation can be exemplified by some cases of *three phenomena:*

(a) Three sequentially necessary non-ordered phenomena are denoted by

$!(H_1 \sim H_2 \sim H_3)$

Six sequential combinations exist:

$(H_1, H_2, H_3)_s, \quad (H_2, H_3, H_1)_s$

$(H_1, H_3, H_2)_s, \quad (H_3, H_1, H_2)_s$

$(H_2, H_1, H_3)_s, \quad (H_3, H_2, H_1)_s$

(b) For three sequentially possible non-ordered phenomena the following formula holds:

$(H_1 \sim H_2 \sim H_3)$

and 15 possibilities of successive occurrence can be found:

$(H_1)_s, \quad (H_1, H_2)_s, \quad (H_1, H_2, H_3)_s$

$(H_2)_s, \quad (H_1, H_3)_s, \quad (H_1, H_3, H_2)_s$

$(H_3)_s, \quad (H_2, H_1)_s, \quad (H_2, H_1, H_3)_s$

$(H_2, H_3)_s$, $(H_2, H_3, H_1)_s$

$(H_3, H_1)_s$, $(H_3, H_1, H_2)_s$

$(H_3, H_2)_s$, $(H_3, H_2, H_1)_s$

(c) For sequentially necessary ordered phenomena, that is, for

$!(H_1 \parallel H_2 \parallel H_3)$

one sequential combination is only possible:

$(H_1, H_2, H_3)_s$

(d) With sequentially free ordered phenomena

$(H_1 \parallel H_2 \parallel H_3)$

the following sequential combinations can be identified:

$(H_1)_s$, $(H_1, H_2)_s$, $(H_1, H_2, H_3)_s$

$(H_2)_s$, $(H_2, H_3)_s$

$(H_3)_s$, $(H_1, H_3)_s$

(e) For sequentially exclusive phenomena with the relation formula

$N(H_1 \sim H_2 \sim H_3)$

single phenomenon combinations can only take place

$(H_1)_s$, $(H_2)_s$, $(H_3)_s$

(f) When, for example, phenomena are sequentially *a posteriori* dependent according to

$((H_1) \rightarrow (H_2 \sim H_3))$

five possible combinations exist:

$(H_1)_s$, $(H_1, H_2)_s$, $(H_1, H_2, H_3)_s$
$(H_1, H_3)_s$, $(H_1, H_3, H_2)_s$

Observe that H_1 is the primary phenomenon, and $(H_2 \sim H_3)$ the secondary group.
Plainly, three phenomena can stand also in other *a posteriori* relations, as

for example

$$(\mathrm{N}(\mathrm{H}_1 \sim \mathrm{H}_2) \rightarrow \mathrm{H}_3)$$
$$(\mathrm{H}_1 \rightarrow !(\mathrm{H}_2 \parallel \mathrm{H}_3))$$

The possible sequential combinations can be easily determined.

(g) Similar conclusion is valid for an *a priori* dependence. For example, if

$$(\mathrm{H}_1 \leftarrow !(\mathrm{H}_2 \leftarrow \mathrm{H}_3))$$

the following sequential combinations are possible:

$$(\mathrm{H}_2, \mathrm{H}_3)_s, \quad (\mathrm{H}_1, \mathrm{H}_2, \mathrm{H}_3)_s$$

3.3.3 Number of sequential combinations

Previous paragraphs suggest that the number of sequential combinations depends again on the type of the relation. However, this number is, in general, greater than for existential combinations. For n sequentially free non-ordered phenomena, that is, for

$$(\mathrm{H}_1 \sim \mathrm{H}_2 \sim \dots \sim \mathrm{H}_n)$$

the number of sequential combinations, m_c, is mathematically given by

$$m_c = \sum_{k=1}^{n} \frac{n!}{(n-k)!} \tag{3.4}$$

The number m_c can be easily determined for the following sequential relations:

$!(\mathrm{H}_1 \sim \mathrm{H}_2 \sim \dots \sim \mathrm{H}_n)$: $m_c = n!$

$!(\mathrm{H}_1 \parallel \mathrm{H}_2 \parallel \dots \parallel \mathrm{H}_n)$: $m_c = 1$

$\mathrm{N}(\mathrm{H}_1 \sim \mathrm{H}_2 \sim \dots \sim \mathrm{H}_n)$: $m_c = n$

When phenomena are *a posteriori* or *a priori* dependent, the determination of m_c gets complicated, because also the order of phenomena must be respected.

3.3.4 Examples of sequential relations

■ **Example 3.3.** The phenomenon H_2 ≡ **Ph(collapse of a structure damaged by fire)** may or may not follow H_1 ≡ **Ph(fire in the building)**. Here we deal with an *a posteriori* dependence,

$$(H_1 \rightarrow H_2)$$ ■

■ **Example 3.4.** In a building vulnerability study, the possibility of a progressive collapse of floors, assuming that H_1 ≡ **Ph(collapse of floor 1)** will take place, is considered (Figure 3.2). The arrangement of sequential relations is determined by reliability margins related to the respective floors. When the margins are low, the following arrangement can take place:

$$(H_1 \rightarrow !(H_2 \mid H_3 \mid H_4))$$

that is, collapse of floor 1 suffices to cause collapse of floors 2, 3 and 4. When, however, reliability margins are higher, the floor 2 may prevent spread of the collapse situation. The relation formula is

$$(!(H_1 \mid H_2) \rightarrow !(H_3 \mid H_4))$$ ■

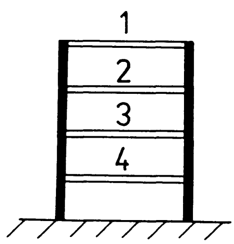

Fig. 3.2 - *Example 3.4.* Floor structures in a building.

■ **Example 3.5.** Seismic foreshocks, H_1, shocks, H_2, and aftershocks, H_3, are in the following sequential relation:

$$((H_1 \leftarrow H_2) \rightarrow H_3)$$

You can observe two levels of primary and secondary phenomena. The phenomenon H_2 is doubly primary - when no shock happens, there is no sense in talking about foreshocks and aftershocks. ■

3.3.5 Importance of sequential relations

Let us give some examples where, in reliability considerations, sequential relations can be of significance:

♦ in the analysis of geometrically and physically *non-linear S-L-E systems* (for example, when the load-bearing capacity of the structure depends on the loading history);

- in the reliability analysis of CFs where *successive failures* are possible (for example, in the case of string limit states);
- in the analysis of S-L-E systems with *time-dependent non-random behavior;*
- in the analysis considering *design situations* (for example, in the analysis of facilities in seismic areas);
- in the *vulnerability analysis* of buildings and structures exposed to detrimental phenomena (collisions, explosions, and others);
- in the *fire-risk analysis*, design of sprinklers, etc.;
- in the design and analysis of *smart structures.*

Sequential relations are not yet currently treated, neither in theoretical investigations nor in the design practice. We should, nevertheless, keep them in mind.

3.4 PHYSICAL RELATIONS

Whenever a set of phenomena governing the reliability of constructed facilities has one or more common sources, *physical relations* can be identified between individual phenomena. For example, wind velocity and snow, from which wind load and snow load are derived, have doubtlessly several common sources: changes in atmospheric pressure, temperature changes, and perhaps others. Their physical relations are very feeble, however, and as far as reliability requirements are concerned, they are without any significance.

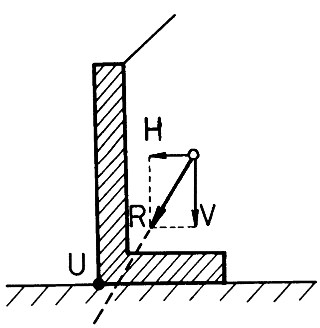

Fig. 3.3 - Loading pattern of an L-shaped retaining wall.

Another example of a physical relation is that between self-weight and ultimate resistance of a reinforced concrete beam. Here we can observe the relation between strength and volume density of concrete, or also dependence of the member's self-weight and ultimate resistance upon its dimensions. Nonetheless, these physical relations are being not respected in the reliability analysis of concrete structures.

When a physical relation between two or more phenomena is strong, such a set of phenomena can be substituted in reliability solutions by a single phenomenon. For example, the stabilizing and destabilizing effect of the earth pressure acting on a retaining wall (Figure 3.3) can be expressed in terms of only a moment about the rotation axis of the wall.

3.5 STATISTICAL RELATIONS

In the overwhelming majority of cases, phenomena that govern the reliability of CF are of *random nature,* and, consequently, must be described by *random variables* (for example, yield stress of steel) or *random functions* (for example, wind load). This is a well established fact, which is not necessary to elaborate on detail. Of course, reliability is also affected by *non-random phenomena* (for example, annual cycles of seasons) and also by phenomena the random variability of which is of minor significance, so that we can consider such phenomena non-random (for example, elastic modulus of steel).

Phenomena can be *statistically dependent* to a different degree (see 2.1.3). A statistical dependence is always present when a physical dependence between random phenomena exists. Nevertheless, we can meet statistical dependence also in cases where no physical dependence is known. Thus, the robustness of a statistical dependence corresponds to the robustness of the respective physical relation only partially. Whenever a strong physical relation between two phenomena is encountered, then also the statistical dependence is strong. When, however, the physical relation is weak, the statistical dependence can be either weak or strong. As an example, the relation between the wind velocity and the snow intensity can be mentioned.

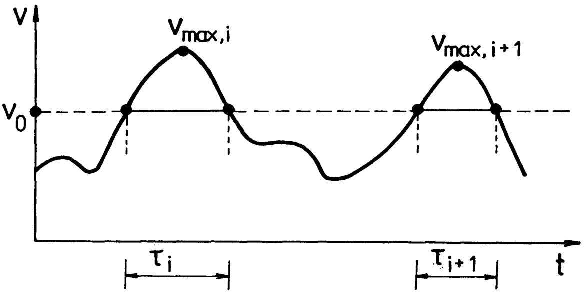

Fig. 3.4 - *Example 3.6.* Record of wind velocity measurement.

■ **Example 3.6.** Let us study two phenomena: Ph(**maximum wind velocity,** v_{max}**, during an out-crossing of a specified level,** v_0) and Ph(**duration of the out-crossing,** τ); see Figure 3.4. Obviously, two existentially simultaneous phenomena are dealt with, because none of them cannot exist alone. Their physical dependence is non-existent or rather unknown; nevertheless, a significant statistical dependence is apparent. For example, from the analysis of the random sequence of daily maxima of wind velocity observed at a certain point, values of the correlation coefficient $r(v_{max}, \tau)$ were found for different crossing levels v_0, Table 3.4. Observe that with increasing v_0 the correlation coefficient $r(v_{max}, \tau)$ diminishes. ■

Table 3.4 - *Example 3.6.* Correlation coefficient between daily maximum wind velocity, v_{max}, and the duration, τ, of crossing a level v_0 (based on 47 years of observations in Prague, Czechia)

v_0 (m.s^{-1})	$r(v_{max}, \tau)$
9.1	0.62
13.3	0.46
15.4	0.36
23.1	0.04

3.6 FAVORABLE AND ADVERSE PHENOMENA

Consider a phenomenon H $\equiv$ Ph(x) expressed in terms of a non-random variable x. When with the increasing value of x the reliability of CF is deteriorating, H is considered *absolutely adverse.* On the contrary, if the reliability is improving with growth of x, the phenomenon H is *absolutely favorable.* For example, Ph(**wind load**) is absolutely adverse, while Ph(**material strength**) and Ph(**size of the cross-section**) are absolutely favorable in most cases.

As soon as H is assumed to be random, also the corresponding variable, ξ, must be taken as random. Consequently, "favorableness" and "adverseness" become *relative concepts.* The boundary between the two concepts is, in general, *fuzzy,* since some realizations of H can be clearly adverse, some rather neutral, and some favorable. At certain phenomena, the favorable realizations may be completely missing. Analogously, adverse realizations can be absent at other phenomena.

■ **Example 3.7.** The Ph(**air movement**) is described, aside from other variables, by *wind velocity.* From the reliability viewpoint, a velocity of 35 m.s^{-1} and greater is plainly adverse; the velocity of, say, 3 m.s^{-1} and less can be considered neutral. The velocity of about 10 m.s^{-1} can excite vibrations of some structural members and must be considered adverse for such members. Observe that wind velocity can be both favorable and adverse when snow-load on roofs is considered. Wind transports snow off the roofs but, on the other hand, it creates snow drifts. ■

■ **Example 3.8.** The *weight of material* is absolutely adverse in regard to many members, as it consumes a certain amount of their bearing capacity. Therefore, its high values are adverse in such a case; low values, less then, say, the mean value, are neutral. There are no favorable realizations of self-weight at such members. In other situations self-weight can have a stabilizing effect. We can thus distinguish both favorable and adverse realizations of the self-weight. ■

The problems of fuzzy boundaries between favorable, neutral, and adverse realizations of phenomena are studied by the *fuzzy set theory* (see Section 2.5). For further discussions we only need to define mathematically the boundary from which on realizations of a particular phenomenon are supposed to be adverse. Let us show the procedure on a simple case.

■ **Example 3.9.** Consider the random phenomenon Ph(self-weight load) described by the magnitude, G. Clearly, the magnitude G is a random variable, the behavior of which can be expressed in terms of the probability density $\varphi(G)$ (cf. Figure 3.5). It is well known that magnitudes of G greater than an *in the extreme acceptable magnitude*, G_{exm}, are not annoying. Therefore, the event Ev$(G > G_{exm})$ is considered *relatively adverse*, E_{adv}. On the other hand, no attitude is usually taken to Ev$(G \le G_{exm})$, which can be designated as *relatively neutral*, E_{ntr}.

The self-weight load has been purposefully used in this example, since an *inversion of adversity* can take place. Obviously, when Ph(self-weight load) has a favorable effect on reliability, the event Ev$(G \le G'_{exm})$ becomes relatively adverse, and Ev$(G > G'_{exm})$ relatively neutral. Here G'_{exm} = another extremely acceptable value of load, different from G_{exm}. ■

Analogous considerations hold also for other random phenomena participating in the structure's reliability. We can try to generalize:

(a) In a set $\dot{H}$ containing all possible realizations of a phenomenon H, described by a random variable ξ, a subset $\dot{E}_{adv}$ of *relatively adverse realizations* E_{adv} can be identified. The difference between the set $\dot{H}$ and the subset $\dot{E}_{adv}$ is a set $\dot{E}_{ntr}$ of *relatively neutral realizations,* E_{ntr}. We can write

$$\dot{E}_{adv} \subseteq \dot{H}$$

$$\dot{E}_{ntr} = \dot{H} \setminus \dot{E}_{adv}$$

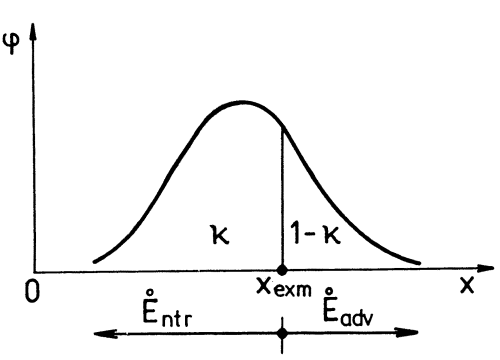

Fig. 3.5 - PDF of a random variable ξ; definition of x_{exm}.

(b) The boundary between $\dot{E}_{adv}$ and $\dot{E}_{ntr}$ is defined by the *extremely acceptable value* (shortly: extreme value) x_{exm} of the variable ξ, see Figure 3.5.

(c) When H is absolutely adverse, the event

$$E_{adv} \equiv Ev(\xi > x_{exm})$$

is considered relatively adverse.

(d) When H is absolutely favorable, the relatively adverse event is

$$E_{adv} \equiv Ev(\xi \leq x_{exm})$$

A "relatively favorable event," E_{fav}, can be defined in a similar way as E_{adv}. Such a concept is without significance for further elaborations, though it cannot be excluded that E_{fav} might get important on some other occasion.

Let us now show the nature of the relationship between the probability of occurrence of a relatively adverse event, $P_E = Pr(E_{adv})$, and the extremely acceptable value, x_{exm}, of the random variable ξ. The boundary x_{exm} between the set $\dot{E}_{adv}$ of relatively adverse events and the set $\dot{E}_{ntr}$ of relatively neutral events shall be defined by the *κ-fractile of the random variable* ξ (see 2.1.2 and Figure 3.5). The probability κ is given by

♦ when H is *absolutely favorable:*

$$\kappa = 1 - P_E \tag{3.5}$$

♦ when H is *absolutely adverse:*

$$\kappa = P_E \tag{3.6}$$

Discussing the probabilities κ and P_E we must not forget that some phenomena *may or may not happen* in the space investigated and during the period considered. Among such phenomena belongs, for example, $H_1 \equiv$ Ph(**snow load in Venice, Italy, in March**). On the contrary, some phenomena *must happen,* for example, $H_2 \equiv$ Ph(**self-weight load**). While for H_1 the probability of occurrence $P_{occ,H1} \equiv$ Pr(**occurrence of** H_1) is less than one, for H_2 the respective probability is $P_{occ,H2} = 1$. The probability $P_E = Pr(E_{adv})$ of occurrence of the adverse event, E_{adv}, is given, for phenomena having $P_{occ,H} < 1$, by the *conditional probability* [cf. Equation (2.8)]

$$P_E \equiv Pr(x_{adv} \mid H) = \frac{Pr(x_{adv} \cap H)}{Pr(H)}$$

where, for simplicity, c_{adv} denotes an adverse realization of ξ, that is, $x_{adv} > x_{exm}$, or $x_{adv} \leq x_{exm}$. Because the magnitude of ξ on condition that H has taken place does not depend on the phenomenon itself, the event Ev(x_{adv}) and event Ev(**occurrence of** H) are statistically independent. Therefore:

$$P_E = \Pr(x_{adv}) \cdot \Pr(H) \tag{3.7}$$

In general, Equations (3.5) through (3.7) hold for any phenomena H, certain or uncertain. However, calculating the κ-fractile, we must employ such a probability distribution that can express the nature of H (see 2.1.2) as close as possible.

Particular cases

In some particular cases *the concept "absolutely adverse phenomenon" merges with "relatively adverse event."* This happens when any realization of a phenomenon is adverse. A typical case is Ph(**fire in the building**); it can be described by one or more variables (for example, extent, duration, maximum temperatures reached), but it is always, from the point of view of reliability, an adverse phenomenon. *Talking about fractiles of fire is meaningless.*

3.7 COMBINATIONS OF EVENTS

Let us study now a *set* $\dot{E}$ *of statistically independent events* E_1, E_2, ..., E_n, that can take place with probabilities P_1, P_2, ..., P_n, respectively; to simplify the notation, subscript E is omitted. Existential relations are not considered for the time being. Then, the *probability that* E_1 *through* E_2 *will all occur simultaneously (or will all follow without regard to the order of appearance)* is given by Equation (2.5), that is,

$$^{n}P_{12\ldots n} = \prod_{i=1}^{n} P_i \tag{3.8}$$

In a general case, events E_1 through E_n can take place in various combinations of L-th order, $L \le n$. For example, when $n \ge 7$ we can have

(E_1, E_2), (E_4, E_7), (E_1, E_3, E_6, E_7), etc.

The probability of simultaneous and sequential occurrence of a combination of L events with subscripts λ_1, λ_2, ..., λ_L is given by

$$^{L}P_C \equiv {}^{L}P_{\lambda 1, \lambda 2, \ldots, \lambda L} = \prod_{i=1}^{L} P_{\lambda i} \tag{3.9}$$

Frequently, combinations of L *specified events* taken from a larger set of n events are studied. For example, only combination (E_2, E_4, E_7) is of interest. In such a case a ***defined combination*** is dealt with. Should the *probability of the defined combination of order* L *(and no other) be calculated,* Equation (3.9) must be supplemented by probabilities that the events belonging to $\dot{E}$ but not contained in the defined combination will not happen. We get:

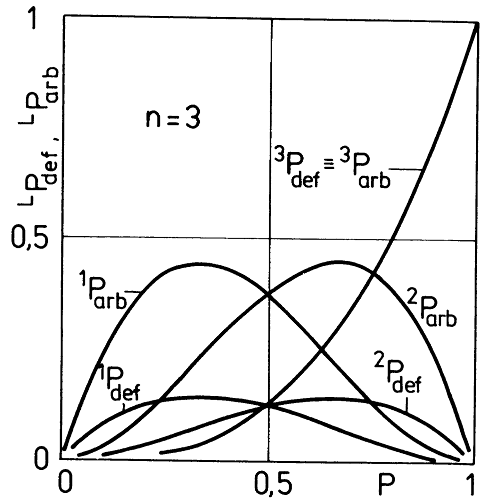

Fig. 3.6 - Relations among combination probabilities.

$$^{L}P_{def} = \prod_{i=1}^{L} P_{\lambda i} \cdot \prod_{j=1}^{n-L} (1 - P_{\varrho j}) \tag{3.10}$$

where $\varrho 1$, $\varrho 2$, ..., $\varrho [n - L]$ = subscripts of events not included in the respective combination.

Equation (3.10) can be rearranged to

$$^{L}P_{def} = \prod_{i=1}^{L} \frac{P_{\lambda i}}{1 - P_{\lambda i}} \cdot \prod_{j=1}^{n} (1 - P_j) \tag{3.11}$$

A simple analysis of Equation (3.11) shows:

♦ the probability of occurrence of a defined combination of L-th order, $^{L}P_{def}$, is always less than the smallest of probabilities referring to events that form the combination;

♦ if the probabilities of occurrence of all separate events equal P, then for $P < 0.5$:

$$^{1}P_{def} > {}^{2}P_{def} > \dots > {}^{n}P_{def}$$

for $P = 0.5$:

$$^{1}P_{def} = {}^{2}P_{def} = \dots = {}^{n}P_{def}$$

and for $P > 0.5$:

$$^{1}P_{def} < {}^{2}P_{def} < \dots < {}^{n}P_{def}$$

To illustrate the problem, the dependence of $^{L}P_{def}$ on P for the case of three events is shown in Figure 3.6. Observe the range $P \in [0.5; 1]$: with increasing P, values of $^{1}P_{def}$ diminish because the isolated occurrence of a phenomenon alone becomes less and less likely. At the same time, $^{3}P_{def}$ markedly increases to one. Finally, it is apparent that for $P = 1$ all three events must occur simultaneously.

Equation (3.11) is true for any particular combination of events, if, however, such a combination is possible. The possibility of combination can be established by analysis of *existential* and *sequential relations* between separate events.

We are sometimes interested in the *probability of occurrence of an* ***arbitrary combination*** *taken from a set of possible combinations of* L*-th order;* let us denote it by $^{L}P_{arb}$. According to the probability theory, $^{L}P_{arb}$ shall be established as the sum of probabilities of occurrence of all possible defined combinations of L-th order, that is

$$^{L}P_{arb} = \sum_{k}^{m(L)} {}^{L}P_{def,k} \tag{3.12}$$

where $m(L)$ = number of possible defined combinations of L-th order, $^{L}P_{def,k}$ = probability of occurrence of the k-th combination of L-th order according to Equation (3.11). Using Equation (3.11) we obtain

$$^{L}P_{arb} = \prod_{j=1}^{n}(1 - P_j) \cdot \sum_{k=1}^{m(L)} \Big(\prod_{i=1}^{L} \frac{P_{\lambda i}}{1 - P_{\lambda i}} \Big)_k \tag{3.13}$$

For *three phenomena* it results

$$^{1}P_{arb} = P_1 + P_2 + P_3 - 2(P_1P_2 + P_1P_3 + P_2P_3) + 3P_1P_2P_3$$

$$^{2}P_{arb} = P_1P_2 + P_1P_3 + P_2P_3 - 3P_1P_2P_3$$

$$^{3}P_{arb} \equiv {}^{3}P_{def} = P_1P_2P_3$$

The development of $^{L}P_{arb}$ for $P_1 = P_2 = P_3 = P$ is shown in Figure (3.6).

Consider the difference between a *combination of events* and a *combination of phenomena*. Equations (3.8) through (3.13) are, without any adjustments, *qualitatively* *valid* also for combinations of phenomena; now, the probability P_i expresses the probability of occurrence of the respective phenomenon, $P_{occ,Hi}$. *Quantitatively,* however, great differences are obtained. When, for example, n certain phenomena are combined, the probability of occurrence of H_1 through H_n [that is, probability of events Ev(**occurrence of** H_i), $i = 1$ through n] equals one, whereas the probability of the combination of corresponding adverse events can be considerably less.

4

STRUCTURE

Key concepts in this chapter: *material properties; grade of the material; members; cross-sections; geometry of the structure; nominal dimension; dimensional deviations; boundary conditions; model uncertainty; structural resistance; multi-component; resistance; failure modes; structural stiffness; axial stiffness; bending stiffness.*

Though all components of S-L-E systems are equally important, STRUCTURE is doubtlessly always the parent subsystem. In the following, we will consider as structure the entire bearing system including soil, or a member of such a system, or only a cross-section of a member. Nevertheless, these three grades of bearing systems will be distinguished only when necessary.

The ability of a structure to carry diverse loads, to resist environmental effects, or to fulfil user's special requirements is described by two principal sets of quantities:

♦ ***Structural resistance,*** specifying the structure's capacity to resist static and dynamic *stress load-effects* (forces, moments, stresses) caused by short-time, long-time, and repeated load. The term "resistance" includes ultimate load-bearing capacities of cross-sections, members, soils, and of the structure as a whole, than also first-crack load (chiefly in case of masonry and concrete structures).

♦ ***Structural stiffness,*** describing the structure's deformation abilities and the magnitude of static and dynamic *strain load-effects*. In the analysis, stiffness of cross-sections, members, soils, as well as entire bearing systems applies, according to what kind of strain load-effects is investigated. As a rule, stiffness is expressed in terms of force, moment, and stress producing unit values of deformation and displacement. However, in current design and testing, stiffness is verified indirectly by checking the relevant deformation or slenderness of members.

4.1 ELEMENTARY PROPERTIES

To describe the resistance and stiffness we need to know:

♦ ***geometry of the structure:*** shape and dimensions of cross-sections, members, and systems (see 4.1.1);

♦ ***boundary conditions:*** arrangement of the bearing system, supports, considering static as well as dynamic functions of the system (see 4.1.2);

♦ ***material properties:*** strength, elastic modulus, stress-strain diagram, etc. (see 4.1.3);

- ***life expectancy*** (see Section 10.2);
- ***age of the constructed facility*** at the point in time investigated.

Except for the age, all the foregoing properties are random and time-dependent. Nevertheless, the time-dependence can be also identified in case of the life expectancy: during the service of CF, deterioration can bring changes in the expected life.

4.1.1 Geometry

Cross-section

The actual dimensions of structural members differ from those assumed in design documents. The reasons for these differences are mainly *technological,* but the significant deviations are caused by *human error,* for example by incorrect reading of drawings. The latter group is, as a rule, not directly considered in reliability analyses. Nevertheless, great attention is paid to them in the *quality control* and *quality assurance process.*

The magnitude of technological deviations is limited by regulations giving *tolerance limits.* These limitations are never perfectly effective, and therefore, deviations larger than prescribed must be also expected.

Geometry is a typically human-controlled phenomenon, and no randomness should be expected in it. However, when *large collections of dimensional deviations* are investigated, the random behavior of deviations is emergent and can be described by suitable probability distributions. When *small samples* are examined, we can observe that, for the greater part, geometry deviations are *systematic.* The background to this phenomenon is very simple: the deviation caused by a certain technological equipment is constant or increases in time up to the next periodical checking and adjustment of the equipment. Thus, the shape of all products coming from a certain period is biased in a similar manner. In very large samples (we can call them "national samples") these systematic deviations get covered by other systematic deviations and so the samples can be examined as random.

We should also note that dimensions can be time-dependent. For example, the highway concrete pavement is exposed to abrasive effects of vehicles. Thus, the thickness diminishes with time, and so the life of the pavement is limited. This fact is respected in design of pavements, as a rule. We should also mention the loss of dimension due to corrosion effects. This is observed, for example, in marine structures exposed to sea level fluctuations; we can encounter a catastrophic size reduction of rebars in reinforced concrete structures in chemical plants, etc.

The cross-sectional deviations of dimensions affect *structural properties* (resistance, stiffness) and the *self-weight load.* Both influences can be important. As a rule, absolute deviations do not substantially depend upon the nominal size of the respective dimension, and so when the size decreases, *relative deviations* increase and attain considerable values. The influence on the self-weight load is of secondary importance. For example, when the thickness d = 100 mm of a reinforced concrete slab is reduced by 20 mm, the self-weight load will decrease by 20 percent, while the moment of inertia will be reduced by 49 percent, and so the deflection due to self-weight of the slab will increase by 57 percent and the deflection due to variable load by 96 percent.

In practice, the adverse effects of deviations from nominal dimensions on structural properties are considered in different ways specified in diverse regulatory documents:

- ♦ deviation is *included into the random behavior of material strength;* this technique is the less suitable, since the true nature of the phenomenon is hidden;
- ♦ material strength is adjusted by a *partial reliability factor* including the effect of deviations, see Section 14.7;
- ♦ deviation is *expressed directly* by an appropriate reduction of the nominal size;
- ♦ deviation is covered by *partial reliability factors for load* (which definitely is not the best solution);
- ♦ deviations are covered by special *adjustment factors* reducing values of the relevant comprehensive structural property (for example, bending stiffness);
- ♦ random behavior of deviations is included into a *general calculation model* when higher level design methods are used.

■ **Example 4.1.** From an extensive research program in former Czechoslovakia and Hungary, in which about 60,000 dimensions of various concrete members, precast and in-situ, were checked, the following population parameters of the deviation from nominal dimensions, given by drawings,

$$y = x_{eff} - x_{nom}$$

have been found:

$$\mu_y = 0.25 + 0.003x_{nom}, \quad \mu_y \le 3\,\text{mm}$$

$$\sigma_y = 4 + 0.006x_{nom}, \quad \sigma_y \le 10\,\text{mm}$$

$$\alpha_y = 0.23 + 0.007x_{nom}, \quad \alpha_y \le 1$$

The probability distribution of y can be assumed as three-parameter log-normal (see Appendix A). Of course, the above formulas are subjected to residual variance. Yet, they can be used as guidance in reliability solutions. ■

Structural systems

Inherent longitudinal deviations have no particular influence upon the structural behavior. Their absolute values are approximately equal to those observed at cross-sections, and consequently, they are of no concern in the majority of situations. Nevertheless, longitudinal deviations can cause *angular deviations* of frame system members, with adverse effect on columns.

Time-dependent longitudinal deviations, caused by temperature fluctuations or by shrinkage and creep of materials can affect the structural behavior substantially. This phenomenon is usually treated in the overall determination of load-effects, and is not included into the family of geometry issues.

Transverse deviations are of greater importance than longitudinal ones. They are caused by *shape imperfections* (random or non-random curvature of members, angular deviations) and also by *cross-sectional non-homogeneity* (non-homogeneity of the elastic modulus, influence of cable ducts, openings, etc.). Transverse deviations affect the load-effects significantly; they are particularly important when instability problems, physical as well as geometrical, are dealt with.

The influence of transverse deviations is respected, for example,

- by an *additional eccentricity of axial forces;* this eccentricity is often conceived of as random;
- by *2nd order analysis* of structural systems.

The random behavior of the system geometry deviations is essentially analogous to that of cross-sectional deviations. However, little statistical information is available. More detailed information on geometry deviations can be found in Casciati *et al.* 1991 and Tichý 1979.

The variability of dimensions is an important issue in the analysis of dimensional accuracy of structures, establishment of tolerance intervals, and in other exercises related to assembly of structural members on site. See Vorlíček and Holický 1989.

4.1.2 Boundary conditions

In concrete situations, it is only rarely possible to ensure the boundary conditions as they were assumed in design. Not too much effort is given to imitate exactly the design assumptions during the execution of the structure. In design, boundary conditions are very simplified, and so the real distribution of load-effects over the structure can substantially deviate from that which has been assumed (cf. 4.3.2, Example 4.2). This phenomenon is ranged under a common set of problems designated as the *model uncertainties problem.* However, the latter problem is more general, since it includes also other differences between design assumptions and reality.

Considering the effect of boundary conditions, it is often stated that they are not, except for very special structures, a significant reliability element. This statement is, however, a virtue of necessity. Though the existence of deviations is admitted, their statistical treatment is, at present, beyond technical achievement. Therefore, the variability of boundary conditions is usually covered by *model uncertainty factors,* included in the prevailing system of design parameters. When evaluating an existing facility, the designer and the reliability engineer should always pay attention to boundary conditions, particularly to those that were not considered in the design at all. Do not forget that *boundary conditions are time-dependent;* for example, an effective support can occur at a place where no support has been envisaged. Or, owing to construction activities unforeseen at the time when the respective structure was designed, unexpected soil settlement can take place. Many examples of catastrophic structural as well as non-structural damage caused by unforeseen change in boundary conditions can be found in literature. Using the computer makes a sensitivity study of various arrangement of boundary conditions very easy. For minimum cost, large loss due to decisions based on a simplified approach can be avoided.

4.1.3 Materials

Materials are, without any doubt, the most explored reliability item. Attention has been paid to strength of materials since the beginning of Humankind's construction activities, and in the later periods also to other properties: elastic modulus, stress-strain diagram, rheological behavior. Abundant data are available, particularly on strength and on modulus of elasticity, where also statistical investigation of dependencies was performed.

In general, materials are heterogeneous, being composed of many components; components are assembled *systematically* (masonry, laminated glass) or *randomly* (concrete, wood). No building material is known that can be denoted as perfectly homogeneous, since even materials that are apparently such are in their very nature heterogenous again. The heterogeneity is the main cause of the random behavior of material properties. In the description of random behavior of a material we deal, in the majority of cases, with only a single random variable. In special cases time-dependent behavior of a material should be expressed in terms of a non-stationary random function, but sufficient data and techniques are lacking for such description. Therefore, description of materials is simplified as much as possible. The diverse side-effects, which are beyond our theoretical possibilities, are covered by partial reliability factors again (see Section 14.7, Examples 14.3 and 14.4).

The *grade of the material* is defined in various ways, depending on traditions related to each material. However, common today is the definition of the material grade in term of the *characteristic strength,* f_k. As a rule, f_k is specified by

$$\Pr(f \leq f_k) = 0.05 \tag{4.1}$$

where f = random variable strength.

The random behavior of material properties observed on material specimens subjected to testing cannot be considered as a random behavior of the material proper. In testing the specimens many influences are involved that affect the randomness of results.

Random variability of material properties can be expressed by a *bell-shaped probability distribution.* The variance usually does not substantially change in dependence upon the mean; therefore, the coefficient of variation increases with diminishing mean. Two facts are important in the selection of the probability distribution for material strength:

- strength can never be less then zero;
- strength cannot be higher than a certain physical limit; while "zero" is always fixed, the upper limit can be only estimated.

These two limits affect the shape of the probability distributions. When dealing with a lower grade material, the distribution of strength is dominated more by the lower limit and the respective coefficient of skewness is positive. For a high grade material, the upper limits becomes important and the coefficient of skewness is negative. Thus, the coefficient of skewness is usually $\alpha \in (-1, +1)$. The three-parameter log-normal distribution can be recommended again (Appendix A); distributions with both bounds are difficult to get adjusted to the observation results.

A survey of the existing knowledge on random behavior of materials can be obtained from Schuëller 1987.

4.1.4 Prestress

The stress and deformation state of various structures can be favorably adjusted by artificially introduced forces. These forces create an additional stress state, the prestress, that is superimposed on the stress state due to load (including loads from support settlement, temperature changes, and others). The conceptual treatment of prestressing forces is not unified. Two approaches exist:

- ♦ prestressing forces are considered as *external load;*
- ♦ prestress is a *property of the structure.*

These conceptions are not contradictory; nevertheless, they have to be discriminated when reliability requirements are formulated. Further, prestress can affect variability of resistance and stiffness.

Some information on the variability of prestress can be found in Tichý and Vorlíček 1972; a detailed reliabilistic analysis of prestress has been presented by Mathieu 1991.

4.2 RESISTANCE

The description of resistance is governed by

- ♦ physical and geometrical properties of the structure, including time-dependence aspects,
- ♦ properties of the stress load-effect; multi-component load-effects, such as combined bending and axial force, require multi-component description space of the resistance; the dimension of the description must be always equal or greater to that of the load-effect;
- ♦ possible modes of failure of the structure (that is, structure as a whole, member, or cross-section).

Thus, in a general case, a *resistance vector* has to be dealt with,

$$\boldsymbol{R} \equiv (R_1, R_2, \ldots, R_n) \tag{4.2}$$

where R_1 through R_n = partial resistances, given by

$$f_{Ri}(\varrho_1, \varrho_2, \ldots, \varrho_{n\varrho}; X_1, X_2, \ldots, X_m) = 0, \quad i = 1, 2, \ldots, n \tag{4.3}$$

where ϱ_1 through $\varrho_{n\varrho}$ = random variables describing the physical properties of the structure, and X_1 through X_m = coordinates of the m-dimensional space defined by the load-effect. Obviously, $f_{Ri}(.)$ is a non-stationary random function in the respective space, related to R_i. In general, no explicit formula can be given for $\boldsymbol{R}$. The value of resistance is usually expressed in terms of one of the coordinates, X_k, setting the other coordinates equal to fixed values, c. Then

$$X_k = \min(R_1, R_2, \ldots, R_n \mid X_1 = c_1, X_2 = c_2, \ldots, X_m = c_m, \sim X_k)$$

Similarly as $\mathbf{f}_R$, the resistance vector is random again. During the evaluation of X_k we have to keep in mind that some of elementary variables, ϱ_k, can appear in the physical description of several partial resistances, R_i. Therefore, partial resistances must be treated as *statistically dependent.*

The problem of resistance has been investigated since the beginning of probability-based design. We will not repeat the existing solutions. For basic information the reader is referred to Tichý and Vorlíček 1972. At present, the resistance problem can be easily treated by Monte Carlo simulation (see Section 2.4). When handling resistance in advanced reliability investigations, various techniques can be applied, depending on what principal approach to the reliability investigation has been selected in the particular case (see, for example, Bjerager 1991, see also Section 13.3).

4.3 STIFFNESS

4.3.1 Cross-sections

There are two basic types of cross-section stiffness to be distinguished: ***axial stiffness,*** B_a, and ***bending stiffness,*** B_b.

The axial stiffness is defined by

$$B_a = \frac{N}{\Delta l / l}$$

where N = axial force, l = length of the member, and Δl = elongation of the member (positive or negative). Thus, B_a represents a *notional axial force,* N^*, that elongates the member to $2l$, or compresses it to zero.

Similarly, the bending stiffness is defined by

$$B_b = \frac{M}{(1/r)}$$

where M = bending moment, $1/r$ = curvature of the bending line, r = radius of curvature. Again, B_b is a *notional bending moment,* M^*, that would produce a radius of curvature equal to 1 in terms of the length unit applied in the expression for the bending moment.

We could define also *other types of stiffness:* shear stiffness, torsional stiffness, bending stiffness of two-way slabs, and others. These usually do not appear in current problems, and moreover, the general approach to the solution would be the same as for B_a and B_b.

Obviously, the character of $B_a = N^*$ and $B_b = M^*$ must be analogous to that of the resistance. Therefore, considerations made in Section 4.2 can be applied for stiffness too. In theory, multi-component stiffness (for example, combined axial and bending) could be also considered.

Cross-section stiffness is not explicitly used as design criterion, and it does not enter any reliability requirements. On the other hand, in the determination of stress and strain load-effects a description of the random behavior of stiffness is often necessary. As a rule, parameters μ_B, σ_B, and α_B and also possibly information on the probability distribution of B are required. Since no statistical data on stiffness are available, Monte Carlo simulation has to be used.

It should be mentioned that the probability distribution of stiffness is basically asymmetric. For example, when the bending stiffness formula for an elastic and homogeneous cross-section is analyzed,

$$B_b = EI \tag{4.4}$$

where E = elastic modulus, and I = moment of inertia, the non-linear dependence of I upon the cross-section depth leads to positive skewness of the probability distribution of B_b. Three-parameter log-normal distribution can be used for stiffness again.

4.3.2 Members and systems

Similarly as in the case of cross-section stiffness, the member stiffness can be expressed in terms of the load that produces a unit deformation of the member. Since many deformation variables are involved and also load affecting members is diverse, no general member stiffness formula, analogous to, say, Equation (4.4), can be defined.

■ **Example 4.2.** Examine the mid-span deflection, $f \equiv w_{mid}$, of a simple beam with nominally constant cross-section, subjected to concentrated load at mid-span, F. The deflection of the beam at any point x_i is given by

$$w_i = \int_0^l M(x) \cdot m_i(x) \cdot \frac{1}{B_b(x)} \mathrm{d}x \tag{4.5}$$

where $M(x)$ = bending moment due to F, $m_i(x)$ = bending moment due to unit force acting at i, and $B_b(x)$ = bending stiffness at x (Figure 4.1a).

Assuming that the bending stiffness is nominally constant along the beam, $B_b(x) = B_{b0}$, and setting for the bending moment, Equation (4.5) gives after arrangement the well known formula for mid-span deflection

$$w_{mid} = \frac{1}{48} \frac{Fl^3}{B_{b0}} \tag{4.6}$$

We are interested in the statistical parameters of w_{mid}. We naturally could, knowing or estimating the random behavior of the elementary variables F, l, and B_{b0}, subject Equation (4.6) to a Monte Carlo simulation or to the moment method analysis. However, *using simply Equation (4.6) for this purpose would be a mistake.* We know that

♦ except for laboratory conditions, the hinge and gliding supports of simple beams are never perfect; some partial fixed-end effect always exists and friction hinders free longitudinal movement; thus, supplemental moments and axial forces act in the beam;

♦ the position of load is never exactly fixed;

♦ the bending stiffness is never constant along the member.

When an *a priori* solution is dealt with, the deviations in support properties, load position, and bending stiffness (Figure 4.1b) can be considered random. Thus, $M(x)$, $m_i(x)$, and $B_b(x)$ are *random functions*, $M(x)$ and $m_i(x)$ being dependent on the development of $B_b(x)$.

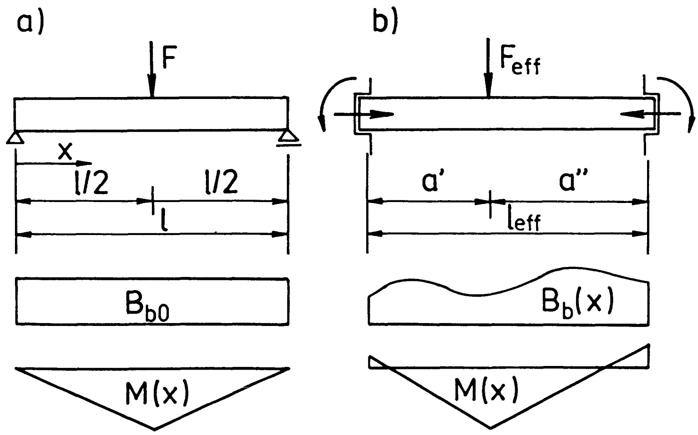

Fig. 4.1 - *Example 4.2.* Simple beam subjected to concentrated load at mid-span (**a** - theoretical state, **b** - possible effective state).

Owing to many uncertainties involved, a straight analysis of this problem, based on random functions with ordinate x as argument, is difficult. The situation gets even more complex at statically indeterminate systems, with non-linear materials, crack occurrence, etc., and particularly when time-dependencies enter the calculation model. Of course, with a certain computational effort, all difficulties can be overcome, but the economic importance of such solutions is questionable. ■

In general, probability distributions of deflection and other strain load-effects are not normal. Nevertheless, since no dependable data on deformation variability exist, the probability distributions are considered normal and parameters are assessed on an *a priori* basis. While the mean is taken as quasi-mean (which means, in the foregoing example, that random function features of the bending stiffness are ignored), the standard deviation of the deformation can be taken as proportional to the mean.

5

LOAD

Key concepts in this chapter: *LOAD system; load/structure relations; load/load relations; load occurrence; load magnitude; load duration; load repetition; load presence and absence periods; amplitude analysis of load; comprehensive load analysis; discretization of observations; floating-level method, FLOLEV; fixed-interval method, FIXINT; fixed level method, FIXLEV; one-variable model of load; loading history; load path; load combinations; combination rules; combination formulas; combination sequence.*

Prima facie, the studies of structural load appear to consist of a set of simple operations: a set of data is collected, and, subsequently, load values needed for design are found.

However, the actual situation is substantially different: problems concerning structural load are more sophisticated. They cannot be treated without sufficient knowledge of the matter. Any overestimation of load can induce increased consumption of materials, labor, and energy, or can demand special construction techniques to be used, and bring extra outlays to the client. On the other hand, underestimating or neglecting load, and also misunderstanding its nature, can lead to diverse types of structural failure, and cause financial loss again. A simple "data approach" to problems of structural load does not reflect the needs of the theory of reliability of constructed facilities. Delicate solutions based, first, on *theoretical analysis of the loading phenomena,* and, second, on *engineering judgment* must not be refrained.

Though the problems of particular loads have been the subject of extensive theoretical and experimental research, and a wealth of important research papers exists, the comprehensive information on loads is sparse. Aside from general publications on structural reliability where often detailed chapters on load can be found (see particularly Madsen *et al.* 1986, Melchers 1987, Schuëller 1981) only one specialized monograph can be quoted, Wen 1990, that treats the theoretical aspects of the load problem with particular emphasis on load combinations. A *Handbook on Structural Load,* aimed at designers, has been published (Tichý *et al.* 1987) but, unfortunately, in Czech language only. The problems of structural loads are currently studied by various international and national bodies (for example, CIB Commission W81 *Actions on Structures*; *BSI Report on a new approach* 1990).

5.1 LOAD/STRUCTURE RELATIONS

5.1.1 Sources of load

Structural load is produced by phenomena that can be basically divided into two main groups:

- ♦ ***natural phenomena,*** produced by the Nature; for example: gravity field and mass, changes in atmospheric pressure, climatic changes in temperature;
- ♦ ***technological and social phenomena,*** resulting from human activity; for example: acceleration fields, technological changes in temperature.

Some load arises only from the first or the second group; nevertheless, very often the two groups interact. For example: wind load is produced by air flow (natural phenomenon), and its magnitude depends on the shape of the building (technological phenomenon); live load in residential buildings (natural phenomenon) depends upon social situation in the particular country (social phenomenon; see Andam 1990).

The amount of influence occurring in the generation of a load is usually large. For example, the snow load is produced by solar activity, wind, gravity, etc., and is affected by human decisions: selection of the roof shape, surface and insulation properties of the roof covering, conditions of use of the building, and others. Several classifications of load exist, which depend mainly on objectives followed. As a rule, structural codes classify loads according to their duration (permanent, variable, accidental loads, etc.). A detailed classification can be found in Mathieu 1980.

Most phenomena in both groups fluctuate randomly, and so load magnitudes have to be expressed in terms of random variables and random functions. Essentially, all loads should be studied as *multi-argument random functions*; at present this is almost impossible due to lack of data and suitable mathematical models. Good results are achieved with some simple one-argument random functions. For example, Poisson rectangular pulse process can be comfortably treated. See Wen 1990.

It is sometimes necessary to distinguish between the *load* and the *load magnitude.* In practical cases, "load" frequently stands for "load magnitude"; this simplification can cause misunderstandings.

Note also that we have to distinguish between *load* and *load-effect.* The latter term covers stress and strain phenomena produced by load in a structure (forces, moments, stress, and others; deflections, curvatures, strain, displacements, and others). Thus, load-effects, which can be divided into *stress load-effects* and *strain load-effects,* are obviously properties of the LOAD-STRUCTURE system.

5.1.2 Basic features of load

Considering the S-L-E system approach, loads are components of the LOAD system. Some basic facts on loads should be kept in mind:

(1) *The existence of a load is given by the existence of the structure.* On the other hand, a structure can exist without being subjected to any load. Thus the relation between load and structure is *one-way existential.*

(2) *Each component of the LOAD system has some existential relation to the remaining components* (if there are any).

(3) *A full description of any load is necessarily multi-dimensional.* A set of variables, numerical or logical, must be established whenever a load is to be thoroughly examined in the framework of an S-L-E system.

(4) *Properties of load depend upon general properties of CF* (building, bridge, etc.). In Figure 5.1 a building is subjected to wind load. Obviously, the wind pressure, as well as its distribution over the surface of the building, does not depend on the bracing structure inside the building. It depends on the shape of the building and properties of its surface.

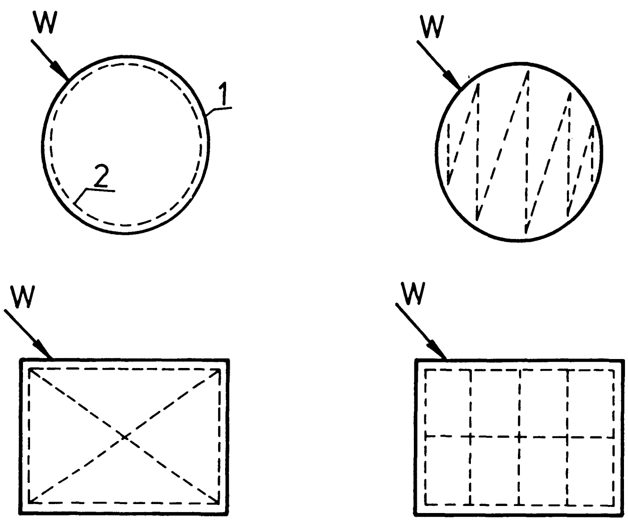

Fig. 5.1 - Dependence of wind load, ***W***, on the shape of the building and its independence from the bearing structure (**1** - sheathing, **2** - bearing structure).

(5) *Properties of the load do not depend on properties of the structure.* This statement is regularly exposed to objections: in engineer's thinking, load is often substituted by load-effects; this happens namely in the problems of *structural dynamics.* If ever a load is seemingly changed by properties of the structure, load-effects themselves are, in fact, dealt with. Consider, for example, a vehicle moving along a bridge: the primary kinematic and dynamic properties of the vehicle are independent of the bridge structure. As the vehicle enters the bridge, the movement of the "bridge-vehicle" system produces load-effects in the bridge and also in the vehicle itself. Obviously, the *effect* of the same vehicle on an adjacent highway pavement will be different from that on the bridge.

5.2 LOAD/LOAD RELATIONS

When solving reliability problems where several types of load apply, it must be kept in mind that certain relations exist between separate loads, and load magnitudes. Four types of relations can be found:

(1) ***Physical relations*** can be identified between some loads. For example, wind load and snow load come from common sources, the principal source being the solar activity and geographic factors. The two loads are indubitably interdependent. However, as far as the basic values of snow load are concerned, this dependence is of no practical importance. Of course, when load due to snow accumulation caused by wind on roofs behind edge beams and edge walls, etc., is considered, the effect of wind cannot be ignored.

(2) ***Statistical relations*** arise when there is a physical dependence between loads. However, this dependence is either very weak, and so no statistical relation can be observed, or it is so strong that the loads are examined as a single case. Consider again snow and wind (actually, the pair "snow load-wind load" has many typical features that can be generalized to other cases, and therefore we will discuss it several times from different aspects): so large is the number of influences affecting wind and snow on their way to the structure, that any relation virtually vanishes.

On the other hand, considering wind load on a building, we know that, at a given point in time, turbulence effects make the dependence between wind pressure and wind suction highly random. Anyway, statistical aspects are neglected and full dependence is assumed in design; in relevant loading patterns, pressure and suction are considered simultaneous.

(3) ***Existential relations.*** All four types of existential relations outlined in Section 3.2 can be found among structural loads. These relations are extremely important when solving load combination problems.

(4) ***Sequential relations.*** Again, most of the relation patterns discussed in Section 3.3 can be encountered. They are significant, for example, in the reliability analysis of non-linear systems and in vulnerability studies.

5.3 RANDOM BEHAVIOR OF LOAD

To obtain a mathematically treatable description of a load, *four primary load properties* must be considered:

♦ **OCCURRENCE;** a load is either present or absent; the value of OCCURRENCE is YES or NO. When OCCURRENCE = YES, we talk about a *physical component of load,* when OCCURRENCE = NO, we talk about a *zero component of load.*

♦ **MAGNITUDE;** when OCCURRENCE = YES, then the load is physically present, and the magnitude of its physical component is expressed by a variable, F, where $F > 0$ or $F \leq 0$. Note that $F = 0$ does not mean the same as OCCURRENCE = NO; in the latter case we should say that F identically equals zero, $F \equiv 0$.

♦ **DURATION;** when OCCURRENCE = YES, the duration of *presence of load* is $T_i \le T_{ref}$, where the subscript i stands for the i -th occurrence of load and T_{ref} is the reference period, which can be equal to the life of the facility, T_0, and to other defined period; it is

$$\sum_{i=1}^{m} T_i \le T_{ref}$$

where m = number of load occurrences.

When OCCURRENCE = NO, the duration of *absence of load* is $\bar{T}_i < T_{ref}$; it is

$$\sum_{i=1}^{m} \bar{T}_i < T_{ref}$$

♦ **REPETITION;** this property is characterized by the number of occurrences, m, of the physical component of the load; separate occurrences can either be contiguous, with no absence periods, or there can be occurrences of zero component between the presence periods; the latter feature is denoted as *pulse process.*

Further load properties could be introduced here, for example, *direction* (in case of wind load, sea and earthquake wave loads, etc.), *acceleration, velocity of movement* (in case of dynamic load).

The four properties are *random functions of space and time.* Since in the design of structures simple rules and parameters are necessary, corresponding random functions are simplified according to the character of the property considered.

The *load occurrence* can be represented through the *probability of occurrence of load* at a point in time and space (for simplicity, we will be discussing the time-dependence only; all considerations can be expanded to space also). The probability of load occurrence is given by

$$P_{occ} = \frac{\sum_{i=1}^{m} T_i}{T_{ref}} \tag{5.1}$$

where the reference period, T_{ref}, usually equals the total observation period, T_{tot}, given by the sum of all partial observation periods, T_{obs} (cf. 2.1.6, Mean return period).

When reduced to a random variable, the *load magnitude, F*, must be described according to the nature of load investigated. A variety of probability distributions can be applied; no general rules can be given on the selection of appropriate distributions. Suggestions given in 2.1.2 are advised. We have to keep in mind not only the actual behavior of load but also the way of establishing values subjected to probability modeling.

Similarly, no general rules on probability distributions of the *presence* or *absence periods* can be given. These distributions can be estimated during analysis of load observations (see Section 5.4). Their importance is often underrated; we need them, nevertheless, whenever a load process is modeled.

Though the *number of load repetitions,* m, is also a random variable, it is usually considered fixed. For long reference periods, the number of repetitions is less important. However, this is not true when *fatigue* and *rheological behavior of structures* is studied. Then, attention has to be paid to m.

5.4 ANALYSIS OF LOAD DATA

A *continuous observation* of a random load is rather an exception. In most cases data on actions are collected by means of *discrete measurements* (for example, live load in buildings); in other cases, continuous observations are intentionally discretized to simplify analysis. Finally, some loads are discrete due to their inherent nature (traffic loads).

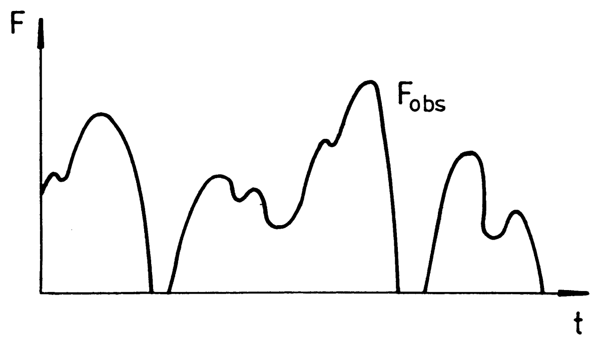

Fig. 5.2 - Record of observed load magnitudes, F_{obs}.

Methods of observing a particular load vary from country to country. It is regrettable that virtually no unified approach has been achieved in this sphere yet. Nevertheless, the general type of data sets is almost identical everywhere. For time-dependent load, *random sequences of successive magnitudes* are, as a rule, extracted from continuous or discrete observations. Unfortunately, the methods of analysis of such sequences also differ.

The main purpose of the *magnitude analysis* is to provide satisfactory statistical information on the random behavior of the load examined in order to acquire a sufficient basis for the derivation of design parameters. Sequences observed are usually autocorrelated, which creates some difficulties in evaluation if a random function solution is not used. The magnitude analysis must either respect the autocorrelation or it must eliminate it as far as possible.

When a continuous measurement of load magnitudes has been performed and a record is available (Figure 5.2), two basic types of analysis are possible.

Amplitude analysis

The family of amplitude analysis is mainly concerned with *peak magnitudes* and sometimes also with their *duration*. Three goals of the amplitude analysis can be distinguished:

♦ *discretization of a continuous observation record* (if there is any), or adjustment of an observed sequence record; the discretization yields *filtered random sequence*, which is then subjected to further treatment;

♦ determination of *random samples of defined maximum load magnitudes* and also, if possible, of random samples of *defined durations of maxima*, and of other useful variables;

♦ estimate of *probability distributions* to be used in further reliability analysis.

The following techniques of amplitude analysis can be distinguished:

(1) ***Floating-level method***, FLOLEV. This method takes into account *successive peaks*, F_{max}, of the record (Figure 5.3a). Further, samples are formed of *successive minima* (= floating levels), F_{min}, *periods between minima*, T, periods of load non-occurrence, $\bar{T}$, and *exceedance areas*, A, over secants connecting two successive minima (see the pointed line in Figure 5.3a). Clearly, the sample size is equal to the number of peaks. In this form, the analysis is purely statistical because no decision-based parameters are involved.

However, the random sequence of peaks embeds a certain autocorrelation; looking at a graphical record, we often feel that some of adjacent peaks are not independent. In order to eliminate this local bias, a refinement of FLOLEV can be introduced. The screening can be adjusted in such a way that likely local deviations of the process are discarded. Two decision parameters affecting the filtration degree are involved: the *filtration distance*, d_{fil}, and the *relative filtration deviation*, α_{fil}. These parameters must be estimated by the reliability engineer in order to obtain a sample corresponding to the nature of the phenomenon studied. No explicit rules on filtration parameters can be given. For example, peak No. 5 would not be included into the sample.

We can assume that the period between two minima represents the *duration* of the corresponding maximum. There is always a statistical relation between the maximum and the underlying duration.

(2) ***Fixed-interval method***, FIXINT. This method is used very often, though it is definitely not the best. The total observation period, T_{tot}, is divided into *intervals of constant length*, T_{obs}, Figure 5.3b. Then the record over T_{obs} is screened, and maximum load magnitude is identified. Unfortunately, information supplied by FIXINT is very poor.

First, there always exists some possibility of autocorrelation that cannot be screened off. Evidently, maxima Nos. 1 and 2, and Nos. 6 and 7 belong to the same segment of the record, yet, in FIXINT, they are treated as two independent outcomes. The pitfall of autocorrelation practically disappears when T_{obs} is large enough.

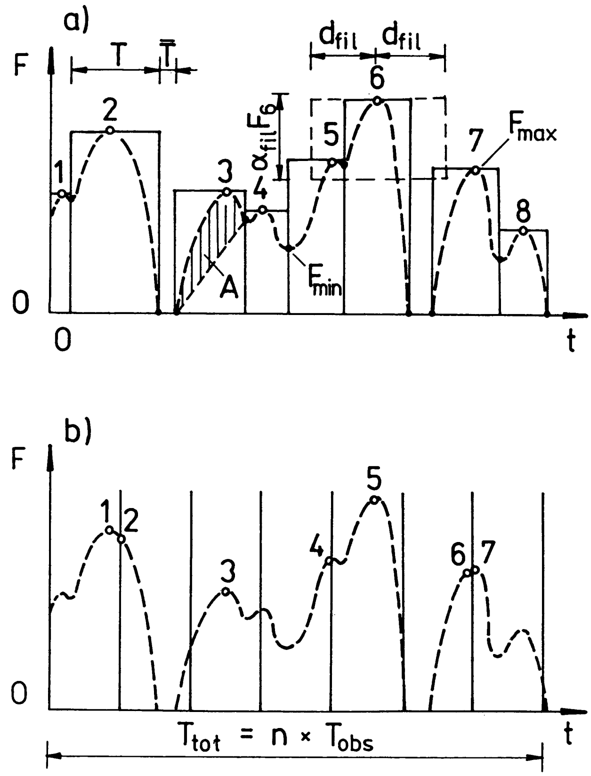

Fig. 5.3 - Floating-level (a) and fixed-interval (b) evaluation of the F_{obs} record in Figure 5.2.

Second, some important peaks may escape the count. Note the maximum load magnitude in the interval with peak No. 7; it has not been included into the sample.

Third, no information on durations of maxima results from FIXINT.

Finally, the sample size is a decision-based value, given by T_{tot} / T_{obs}.

Again, no universal rule can be given on the width of intervals. A general picture of the time-dependence of the respective phenomenon must be taken and analyzed. For example, when analyzing wind data, we can find that the wind situation changes with a mean period of seven days. This, however, can be true only for a certain region; at other site this period can be longer or shorter.

(3) ***Fixed level method,*** FIXLEV. It consists in choosing a certain screening level of load, F_i, and finding out maxima related to each outcrossing, Figure 5.4. Simultaneously, periods of exceedance of this level, T, periods between up-crossings, $\bar{T}$, and exceedance areas, A, are registered and evaluated, if necessary.

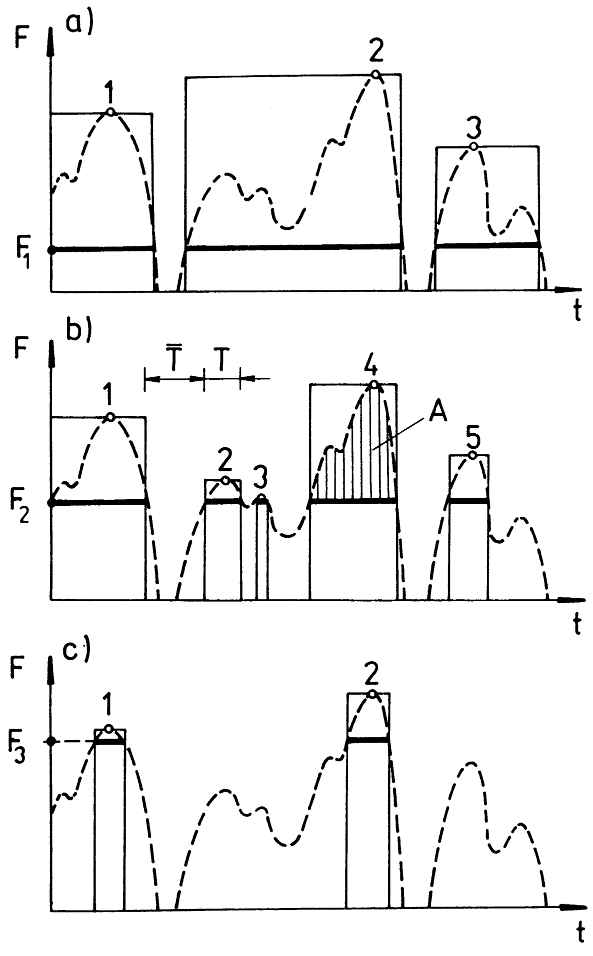

Fig. 5.4 - Fixed-level evaluation of the record in Figure 5.2 (**a**, **b**, **c** - evaluation for three different levels).

The quality of information obtained through FIXLEV depends very much on the selection of the level, F_i, which is thus a decision-based parameter. The autocorrelation of the obtained random sequence of maxima is usually low. The sample size depends upon the chosen level.

Though the source of the amplitude analysis is the same for all three techniques, samples obtained differ. This can be demonstrated by plotting PDF curves of probability distributions related to variables defined through FLOLEV, FIXINT, and FIXLEV (Figure 5.5).

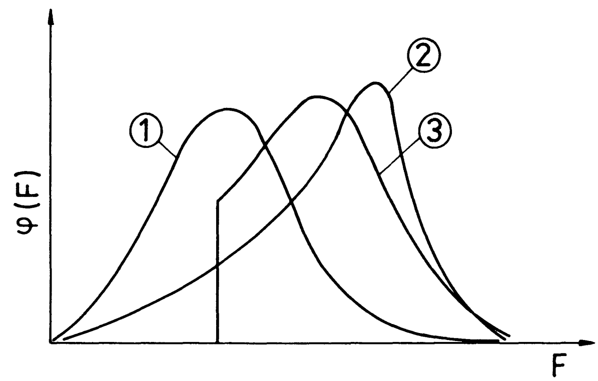

Fig. 5.5 - PDFs obtained by various methods of load process evaluation (1 - fixed-interval, 2 - floating-level, 3 - fixed-level evaluation).

■ **Example 5.1.** The random sequence of daily maximum wind velocities measured in Prague, Czechia, in 1925-1971, was subjected to analysis by FLOLEV, FIXINT, and FIXLEV, using several variants of the decision-based screening parameters. The results of evaluation are summarized in Table 5.1, where n = sample size, m_v, s_v, and a_v = sample mean, standard deviation and coefficient of skewness, respectively, $k_v(1)$ = correlation coefficient of two successive magnitudes of the obtained random sequence [that is, the value of standardized sample autocorrelation function for $\tau = 1$, cf. Equation (2.36)]. Other variables are not given here. ■

Note the differences between the three techniques and note also the dependence of sample characteristics on the decision-based parameters, that is, the filtration degree, width of the interval, T_{obs}, and screening level. These differences make difficulties in interpreting and comparing results obtained in different countries and by different researchers. Therefore, the way how certain sample characteristics supplied to the reliability engineer were determined, should be always ascertained in order to understand the meaning of the sample correctly.

Comprehensive analysis

While the amplitude analysis deals with the peak values of load magnitudes, the comprehensive analysis covers *all values recorded.* Two principal methods exist:

- *statistical summary,*
- *rain-flow analysis.*

The method of statistical summary, proposed by Mathieu 1974, is very efficient and offers very good information on the load properties that are not captured by the foregoing amplitude analysis techniques. The statistical summary analysis is important for structures sensitive to fatigue and rheology phenomena as it gives information on duration and repetition of load at various levels. Unfortunately, it is not widely used, mainly because large sequences of observations are necessary to obtain a reliable picture of the load magnitude behavior. We will not describe its details here; the reader is referred to the paper by Mathieu.

The rain-flow method (see Frýba 1993) is mainly used in the dynamic analysis of structures exposed to random loading processes and to fatigue analysis of structures subjected to high-cyclic repeated loading.

Table 5.1 - *Example 5.1.* FLOLEV, FIXINT, and FIXLEV evaluation of the random sequence of daily maxima of wind velocity, v ($m.s^{-1}$), in Prague, 1925-1971

Evaluation criterion	n	m_v	s_v	a_v	$k_v(1)$
Filtration degree			FLOLEV		
None	2498	12.0	4.6	0.79	0.28
Weak	1973	12.2	4.7	0.77	0.15
Strong	1359	12.4	4.9	0.72	-0.03
T_{obs} **(days)**			FIXINT		
1	8729	9.1	4.2	0.97	0.46
2	4364	10.7	4.4	0.85	0.38
5	1745	13.2	4.5	0.73	0.24
10	872	15.3	4.5	0.63	0.18
20	436	17.6	4.3	0.46	0.19
Level ($m.s^{-1}$)			FIXLEV		
9.1	1413	13.8	4.1	1.22	0.07
15.4	497	19.0	3.1	1.20	0.07
23.1	56	25.5	1.9	1.16	-0.07

5.5 ONE-VARIABLE MODEL OF LOAD

For many loads their dependence on space and time is small and of little importance, and can be, therefore, either neglected at all, or the time-dependent random behavior of load can be reduced to single variables. Such simplification can be extended also to loads with expressive time dependence. A one-variable probabilistic model for a load can be formulated in such a way that it can reflect almost all properties of the load that are of interest in the reliability analysis.

Let us assume that the random behavior of the ***physical component*** of the load, F, is described by $\Phi(F)$, and the ***zero component*** is defined by P_{occ}, Equation (5.1). Let $\Phi(F)$ include also possible repetitions according to 2.1.6. We will try to find CDF covering both the absence and presence of load, Φ_{gen}.

The *probability of load absence* is

$$P_{non} = 1 - P_{occ} \tag{5.2}$$

where P_{occ} = probability of load occurrence defined by Equation (5.1).

Consider a value F of the random variable F_{rnd}. Since the events Ev(**load magnitude** $F_{rnd} \leq F$) and Ev(**presence of load**) are independent, the probability of their simultaneous happening is

$$\Pr[(F_{rnd} \leq F) \cap (\text{pres}\, F)] = \Phi(F) \cdot Pr(\text{pres}\, F)$$

Then, the distribution function taking into account the intermittence of load is given by the probability that either Ev(**absence of load**) or Ev[($F_{rnd} \leq F$) $\cup$ (**presence of load**)] will happen. Since both events are mutually exclusive, it is, according to Equation (2.4),

$$\Phi_{gen}(F) = P_{non} + P_{occ} \cdot \Phi(F)$$

or, setting for P_{non} from Equation (5.2)

$$\Phi_{gen}(F) = 1 - P_{occ} \cdot [1 - \Phi(F)] \tag{5.3}$$

which is valid for $F \geq 0$.

When the distribution of F has an infimum $F_{inf} > 0$ (that is, for $F \leq F_{inf}$ it is $\Phi(F) = 0$), it holds for $F \in [0, F_{inf}]$

$$\Phi_{gen}(F) = 1 - P_{occ} \tag{5.4}$$

Observe, that for $\Pr(F_{rnd} \leq F) < P_{non}$, the corresponding fractile of F identically equals zero.

With some load also magnitudes $F < 0$ are possible, that is, $F_{inf} < 0$; in such a case it is for $F \leq 0$

$$\Phi_{gen}(F) = P_{occ} \cdot \Phi(F) \tag{5.5}$$

while for $F \geq 0$ Equation (5.3) holds again. At $F = 0$, $\Phi_{gen}(F)$ is discontinuous.

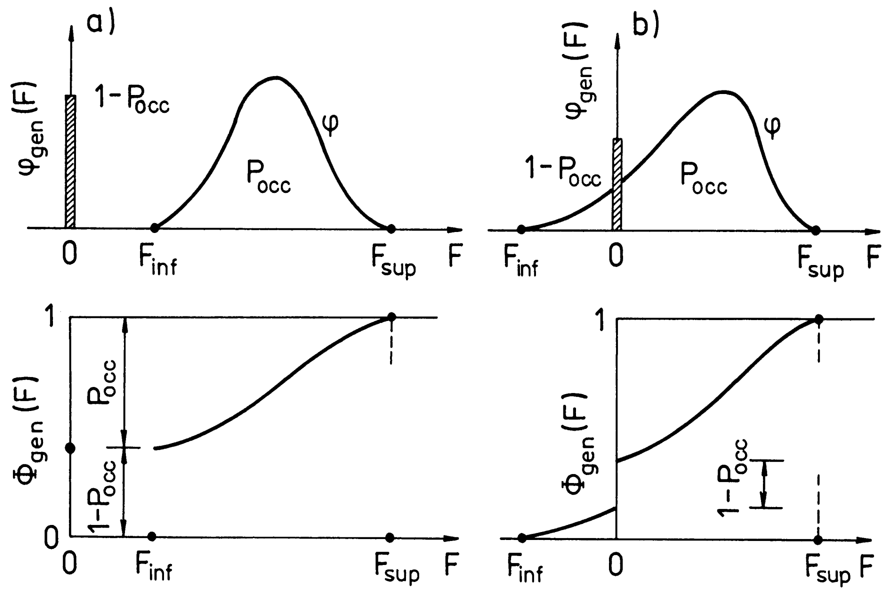

Fig. 5.6 - General probability distribution of a scalar load magnitude, taking into account the probability of occurrence, P_{occ} , of physical load magnitudes, F (a - $F_{inf} > 0$, b - $F_{inf} < 0$).

Figure 5.6 shows PDFs and CDFs for the two cases of F_{inf}. Obviously, a ***mixed probability distribution*** is dealt with, consisting of a discrete part for $F \equiv 0$, and a continuous part between F_{inf} and F_{sup}. We can have, of course, $F_{inf} \rightarrow -\infty$ or $F_{sup} \rightarrow \infty$ or both.

5.6 LOADING HISTORY

The term "loading history" embodies

♦ *time development of a separate load,* that is, the increase or decrease of its magnitude, and also its absence during certain period;

♦ *order in which several loads are applied to the structural system.*

As a rule, the loading history can be expressed by a time-dependent function, which is, to a certain degree, non-stationary random, but which also depends on decisions connected with the use of CF. Load codes do not give any guidance on loading history; designers are only advised not to forget its possible influence on the behavior of the structure. Sometimes loading history is specified for particular structures but this is more an exception than a rule. Therefore, in the design of structures and also in the evaluation of existing structures the loading history must be estimated *a posteriori* or *a priori*, respectively. For example, in the re-design of structures subjected to rehabilitation the loading history has to be established through a study of the past use of the facility. Also, the time development of load should be known whenever the *load path,* that is, the stress and strain state, can change after each loading and de-loading of a structure.

There is no need to consider the loading history when the following two conditions are satisfied:

- ♦ the *law of superposition* is valid for both stress and strain load-effects (structural mechanics aspect);
- ♦ all loads occur *simultaneously* (reliability aspect).

Although these two conditions are almost never fulfilled (particularly the second one), the loading history does not significantly affect, in the majority of cases, the behavior of the structure as far as the mechanical properties and design criteria are concerned. Nevertheless, loading history should not be ignored whenever *non-linear phenomena* are involved. The non-linearity can be of different nature (physical, or geometrical; time-dependent, or not) and also its consequences can be diverse (reversible or irreversible deformations, etc.). Therefore, sequential relations among interacting phenomena should be carefully assessed in such cases (see Section 3.3).

The reliability aspect of the loading history becomes mainly important in the assessment of existing structures. This problem will be briefly discussed in 15.5.4.

5.7 LOAD COMBINATIONS

The problem of load combinations is probably the most exciting chapter of the theory of structural loads. It is steadily attracting attention of researchers, since it is offering broad possibilities for various sophisticated exercises. Nevertheless, we can now say that *the load combination problem has been solved.* At present, several solutions are known, each of them being correct in a certain domain but exposed to criticisms coming from the other domains. Although differences between particularities of load combination solutions are significant, it is typical, that differences in terms of resulting load-effects are very small.

The load combination research has been oriented in two directions:

- ♦ comprehensive evaluation of the reliability of a structure subjected to combined loads;
- ♦ search for appropriate combination rules, combination formulas and load combination factors.

While the first group of research activities forms part of the general reliability solutions (see, for example, Wen 1989), the second group deals with details of the general problem.

Whenever approaching a load-combination problem, we must keep in mind that, in principle, *combination of two and more random load processes* should be analyzed. Computer programs that make such an analysis possible can be easily written or are available in soft-ware libraries. In the main, nevertheless, simplified approach is used, taking into account only the magnitude of loads and disregarding the repetition and duration of individual realizations.

In this book, load combination problem is dealt separately in the framework of the three families of probability-based design methods discussed in Chapters 12 through 14. Therefore, we will introduce here only its main features.

Some basic concepts will be used:

♦ ***Load combination*** is a set of loads that act *simultaneously* on a structure. When loads with a mixed probability distribution (see Section 5.5) are involved, only their *physical components can become members of a combination.* For example, a triplet of loads with magnitudes $F_1 > 0$, $F_2 > 0$, and $F_3 \equiv 0$ (that is, the third load enters the set with its zero component), constitutes a combination of order 2, not of order 3. To include F_3 into the combination is senseless. See below, Combination of load sequences.

♦ ***Combination formula*** defines the load-effect as a function of combined loads.

♦ ***Selection formula*** screens the load-effects resulting from the set of combination formulas; the most adverse load-effect is to be considered in sizing and checking.

♦ ***Combination rule*** consists of a set of combination formulas and the selection formula. Frequently, distinction between combination formula and combination rule is not recognized, and combination formulas are often called combination rules.

♦ ***Load combination factor*** is a design parameter whose role in combination formulas is to express the lower probability of occurrence of adverse magnitudes of combined loads in comparison with the probability of occurrence of adverse magnitudes of loads considered separately. We will deal with it in Section 14.3.

Combination of load sequences

Consider for simplicity only two intermittent loads, F_1 and F_2. An amplitude analysis (for example, FIXLEV) has yielded two random sequences of refined maxima and presence and absence periods. Assume that these loads produce a scalar load-effect, that is, $S(F_1)$ and $S(F_2)$, and plot the two sequences of $S(F_1)$ and $S(F_2)$ along the time axis (Figure 5.7).

At any point in time, t, a combined load-effect is

$$S_c = S(F_1) + S(F_2)$$

which can be subjected to statistical treatment.

In the statistical evaluation of the S_c sequence the above definition of load combination must be respected, that is, only S_c values resulting from physical components of F_1 and F_2 can be introduced into the analysis. Thus

$$S_c = S(F_1) + S(F_2 \equiv 0)$$
$$S_c = S(F_1 \equiv 0) + S(F_2)$$

should not be taken into the S_c sample. Otherwise, a tri-modal histogram and frequency curve of S_c would be obtained, with one mode referring to F_1, another to F_2 and the third to the combination of both. Of course, when $\mu_{F1} = \mu_{F2}$ and both distributions are bell-shaped, only bi-modal curve might be expected (cf. 2.1.2, Multi-modal distributions).

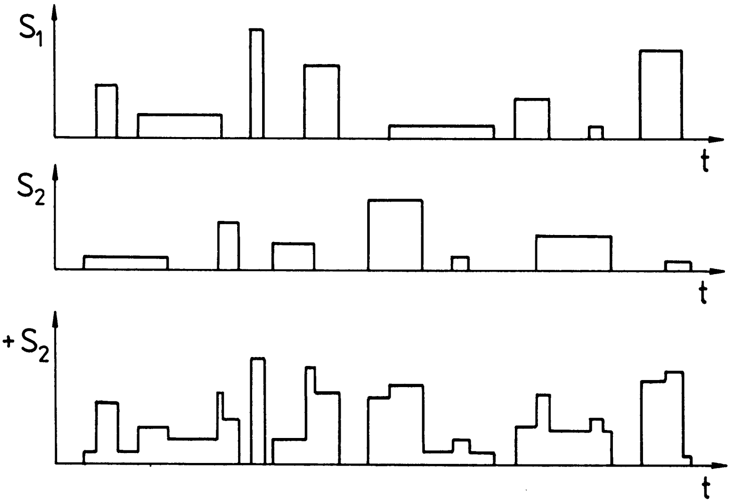

Fig. 5.7 - Combination of two idealized load-effect processes, linear structural response.

Given data on load maxima, and on the presence and absence periods, a random sequence can be easily simulated and analyzed by Monte Carlo technique.

6

ENVIRONMENT

Key concepts in this chapter: *elements of environment; environmental parameters; constraints; human factor.*

ENVIRONMENT, the third component of any S-L-E system, has been very little studied from the point of view of structural reliability. We can even say that it has been neglected. Only little attention has been paid to the environmental aspects of constructed facilities. The result is a poor level or entire absence of clauses concerning environmental concepts in structural design codes and other documents. However, the economical importance of the environmental factors entering design is often significant and determining the design solutions.

Elements of environment

Let the concept of the ENVIRONMENT system include *all that surrounds a constructed facility or is a part of it, or is in some connection with it.* Then, the elements of environment are, *for example,*

- ♦ solar radiation;
- ♦ atmosphere (outdoor, indoor; wind), and all particles carried by the air movement;
- ♦ water (retained and moving water; surface and underground water; rain, snow, ice, and icing) and all particles carried by water;
- ♦ soils and rocks;
- ♦ stored materials (materials in silos, gas and fluids in tanks);
- ♦ non-bearing structures (partition walls, window frames, roofing, waterproofing, insulation);
- ♦ building equipment (HVAC, wiring, water supply, gas supply, draining);
- ♦ technological equipment (machinery, electrical and electronic devices);
- ♦ transport means of various kind (automobiles, railway cars, elevators);
- ♦ animals;
- ♦ humans.

The relations between environment elements and a constructed facility can be

- ♦ *mechanical, biological, physical,* and *chemical,* controlling the durability and performance of the CF system; mechanical relations are represented by load;

Table 6.1 - Environment-active and environment-passive relations between environmental elements and a constructed facility, CF; only typical examples are given; loading effects are not included

Environmental element	Relation of the environmental element to CF	
	active	**passive**
Solar radiation	♦ deterioration of material due to temperature changes and radiation effects	
Atmosphere	♦ corrosive effects of gaseous atmosphere elements; ♦ damage by flying objects; ♦ wind abrasion	♦ leakage of air containing disinfection gas out of grain silos through cracks and other tightness defects
Water	♦ corrosive effects; ♦ deterioration of materials due to humidity and freezing	♦ leaks through cracks of stored material and contamination of underground water
Soils and rocks	♦ corrosive effects	
Stored materials	♦ corrosive effects; ♦ abrasive effects	♦ leakage of stored materials
Non-bearing structures		♦ deterioration due to deformations of the structure
Building equipment		♦ restriction in use due to static and dynamic deformations
Technological equipment	♦ abrasive effects due to movement of vehicles; ♦ corrosion by chemicals, oil, etc.	♦ failures of normal function due to deformations and vibrations of bearing structures
Humans and animals	♦ corrosive effects; ♦ general deterioration of CF	♦ alarm feelings due to deformations, vibrations, and cracks

♦ *physiological, psychological,* and *aesthetic,* determining the attitudes of people getting in contact with a completed CF.

The non-mechanical relations can be *environment-active* (CF is affected by environment), or *environment-passive* (environment is affected by CF). Table 6.1 shows some typical relations, active and passive. Only adverse relations are shown in the table. Frequently, nevertheless, the effects of environment can be positive. For example, cracks in water tanks get tightened and water leaks disappear because of bacteria and other microorganisms settling in cracks, and because of physical and chemical effects of water acting on the hardened cement paste.

At present, no theoretical description of environmental properties exists, and it is doubtful whether such a description, sufficiently general, is possible at all. However, in individual cases, specific solutions evaluating environmental aspects can be applied. If human factor is involved, *psychometric methods* can be efficiently used to evaluate attitudes towards environmental hazards. As a rule, environmental properties affect mainly values of ***constraints*** that are included into the reliability requirements (see Section 10.5).

Human factor

Human factor is doubtlessly one of the governing components of the ENVIRONMENT system. Its role is many-sided, and its discussion could take several pages. In fact, an extensive monograph could be written on this issue. The reader is referred to Melchers 1987, where a good survey of the problem is given with several data on available research studies. In a more general setting, important information on human factor can be found in Blockley 1980, Brown and Yin 1988, *Engineering Safety 1992,* and Kuhlmann 1986.

7

PHYSICAL RELIABILITY REQUIREMENTS

Key concepts in this chapter: ***attack; barrier; reliability requirement, RelReq; formative RelReq; global RelReq; elementary RelReq; global variables; formative variables; elementary variables; basic variables; physical RelReq; reliability margin; global reliability factor; equivalent reliability margin.***

♦ In this chapter, all phenomena and variables are considered **non-random.**

♦ Since in the text the expression "reliability requirement" is frequently used, the abbreviation **RelReq** is introduced to simplify the reading.

7.1 FORMATIVE REQUIREMENT

Let a physical demonstration of an S-L-E system be called the ***attack,*** $\mathbf{A}$. During a reference time, T_{ref}, the attack is subjected to *non-random changes.* At a particular *point in time,* t, the attack is described by a set of phenomena, $\dot{\mathbf{A}}$, in an n-dimensional space, where n is the number of components necessary for a full description of the attack.

The ability of an S-L-E system to resist an attack will be termed the ***barrier,*** $\mathbf{B}$. Similarly as the attack, also the barrier can change in systematic manner (for example, the material of a structure can be exposed to corrosion). In many cases, these changes depend on the respective attack (for instance, repeated loadings diminish the strength of the material). Then, at a particular moment t the barrier is described by a set of phenomena, $\dot{\mathbf{B}}$, in the same n-dimensional space as $\dot{\mathbf{A}}$.

Finally, let a set $\dot{\mathbf{B}}^*$, confined by $\dot{\mathbf{B}}$ (Figure 7.1), be specified. Then, a ***physical reliability requirement*** can be formulated:

$$\forall t \in T_{ref}: \ \dot{\mathbf{A}} \subseteq \dot{\mathbf{B}}^* \tag{7.1}$$

which can verbally be expressed as follows:

An S-L-E system is considered reliable if at any point in time, t, during a reference period, T_{ref}, the *attack set* $\dot{\mathbf{A}}$, is a subset of the *barrier set,* $\dot{\mathbf{B}}$. - If RelReq (7.1) is not fulfilled, the S-L-E system has reached its limit state, or, in other words, failure has occurred.

The terms "attack" and "barrier" have been chosen to emphasize the generality of the reliability requirement. The two sides of the requirement are sometimes called "demand" and "capacity," "load-effect" and "resistance," etc.

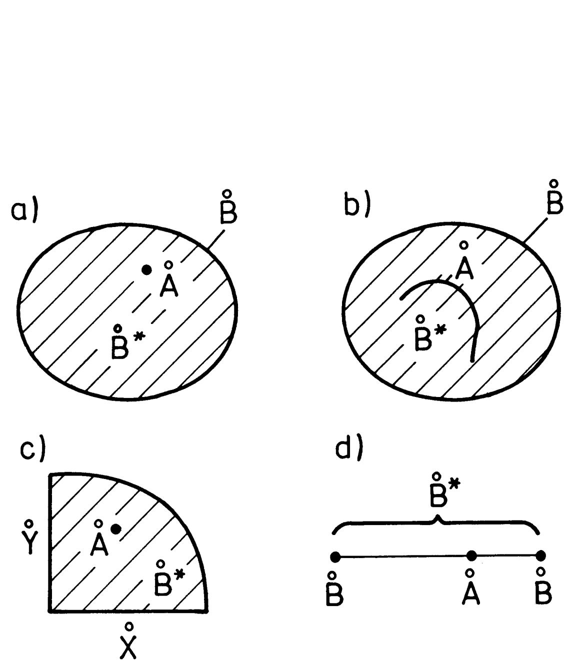

Fig. 7.1 - Graphical interpretation of the sets $\mathring{A}$, $\mathring{B}$, and $\mathring{B}^*$, and of RelReq (7.1) (**a** - attack is represented by a point, **b** - attack is represented by a curve, **c** - the set $\mathring{B}^*$ is confined by $\mathring{B}$ and by the sets $\mathring{X}$ and $\mathring{Y}$, **d** - one-component case with $\mathring{A}$ represented by a point; RelReq is fulfilled in all shown cases).

The sets $\mathring{A}$ and $\mathring{B}$ are called the *formative sets,* because they describe two phenomena, Ph(**attack**) and Ph(**barrier**), forming RelReq. Simultaneous existence of both these phenomena is necessary, otherwise RelReq cannot be written. Therefore, RelReqs according to Equation (7.1) will be called *formative physical reliability requirements.* - Figure 7.1 shows graphically the significance of $\mathring{A}$, $\mathring{B}$, and $\mathring{B}^*$ in one-dimensional and two-dimensional cases.

RelReq (7.1) is valid for a fully defined S-L-E system situated at a *defined point* or in an *area of space* during a *defined* T_{ref}. This is in conformity with the fact that, in the main, constructed facilities are *fixed, non-moveable artifacts* (see Section 16.1). Nevertheless, it must be taken into account that certain fully defined facilities, such as, for example, standardized buildings, high voltage masts, can exist in different places of space, which, actually, can change its properties in non-random way (exposure to wind, intensity of snow fall, and others). For this type of cases the definition domain of RelReq (7.1) must be specified to the particular space considered (for example, the territory of Southern Italy). For simplicity, this generalization will not be considered in the following, though it should be always kept in mind.

The mathematical description of the attack and the barrier depends on the nature of the problem studied. In general, a barrier can be represented by a *hypersurface in an* n*-dimensional space* (that is, for example, in the two-component case, by a curve); an attack can be expressed by a *point,* or by a *hypersurface* as well.

The formative arrangement of RelReq (7.1) is the most frequent type of RelReq used in structural design; we may call it "reliability axiom." It is, however, a source of unclearness in cases where a *physical relationship between the attack,* **A**, *and the barrier,* **B**, exists. - In Table 7.1 some examples of attack and barrier are given.

Table 7.1 - Examples of attacks and barriers

No.	Attack	Barrier
1	bending moment and axial force in a cross-section of an eccentrically loaded member	ultimate limit state function expressed, for example, as an interaction diagram
2	bending moment, M, in a prestressed concrete beam due to external load	first-crack limit, described by the first-crack moment M_r
3	tensile stress in extreme fibers of a steel member subjected to bending, σ	yield stress of steel, f_y
4	crack width, w, in an R.C. tank	limit value of crack width, w_{lim}
5	eigenfrequency of a pedestrian bridge, f	frequency bounds, f_i and f_s, defining an acceptable frequency range
6	effective cross-section area of a steel bar, A_{eff}	cross-section area, A_{nec}, necessary to bear the load-effect
7	effective diameter, d_{eff}, of the bar in row 6	necessary diameter, d_{nec}
8	moment M_{act} affecting a retaining wall	resisting moment, M_{pas}, acting against a possible rotation of the wall
9	effective width of a retaining wall, b_{eff}	width of the wall, b_{nec}, necessary for the wall equilibrium
10	effective life of a CF system, $T_{0,eff}$, resulting from physical properties of CF (see Section 10.2)	target life, T_{0t}, resulting from socio-economic requirements

7.2 GLOBAL REQUIREMENT

7.2.1 Reliability requirement

RelReq (7.1) does not offer any quantitative information on the reliability of the investigated system. Let us try to find a suitable gage that might be used to compare various cases. A geometric interpretation of RelReq (7.1) can be used for this purpose. Consider three possible forms of attack, A, and barrier, B:

(1) Let the attack and barrier be described in terms of *scalar variables* A and B. They can be illustrated as points on the X-axis (Figure 7.2a). In this case, RelReq (7.1) has a simple form:

$$\forall t \in T_{ref}: \ A \leq B \tag{7.2}$$

Since on both sides scalars appear, the requirement will be called *scalar RelReq,* and A and B denoted ***formative variables.*** It is apparent that a limit state is attained when the points A and B merge. Therefore, for a reliability gage the distance Z between these two points can be taken. The greater Z, the higher the reliability.

(2) Consider a *two-component case:* let the attack be expressed by a *vector* (that is, an ordered pair of numbers), described by a point, (X_A, Y_A); let the barrier be expressed by a *function* $f(X_B, Y_B) = 0$, represented by a curve in the coordinate system $[X, Y]$, see Figure 7.2b. The limit state is reached when the point A and the curve B merge. Again, for a reliability gage the minimum distance, Z, from A to B can be taken, defined as the radius of circle K centered in A, having a common tangent with B at a point of contact L. This obviously holds also for a case when a multi-component attack is dealt with, and so the barrier is a function of more than two variables.

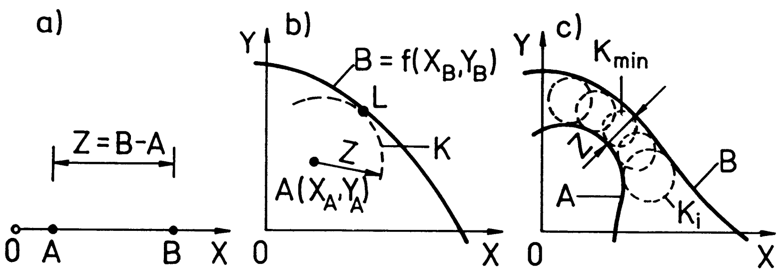

Fig. 7.2 - Graphical interpretation of RelReq (7.2) (**a** - one-component case, **b** - two-component case, the attack is described by a vector, the barrier is a function, **c** - two-component case, both the attack and barrier are functions).

(3) Finally, let A and B be expressed by *functions,* $f(X_A, Y_A) = 0$ and $f(X_B, Y_B) = 0$, described by curves A and B (Figure 7.2c). The limit state is reached when A and B contact. Therefore again, the minimum distance between the two curves

can be taken as the reliability gage. This distance is defined as the diameter of the smallest of circles K_i touching simultaneously A and B in the investigated domain. The smallest circle, K_{min}, touches A and B at points where the two curves have a common normal line. Again, the foregoing is also valid, with appropriate generalization, for functions of several variables.

In particular cases, the definition of minimum distance Z based on circles K, or K_i, is not possible. This happens when the definition domain, Ω_{def}, of the respective RelReq is limited. Figure 7.3a shows a case when a circle drawn around A has no common tangent with B, and similarly, Figure 7.3b shows a case when the curves A and B have no common normal. Nevertheless, a minimum distance between A and B can be defined again, as it appears from the figure.

On certain occasions we are not interested in the absolute value of the distance between A and B, and only the relative information whether RelReq (7.1) is satisfied is sufficient. In such cases the distance between A and B can be measured in an arbitrary direction. However, when comparing two or more cases this direction must not be changed. A simple drawing will usually help to understand the particular problem solved and to identify its possible pitfalls.

We can conclude that a RelReq common to all three cases (including cases where Ω_{def} is confined and cases where for some reason the shortest distance between A and B cannot be evaluated) is given by

$$\forall t \in T_{ref}: \quad Z \geq 0 \tag{7.3}$$

Because of its general meaning, this RelReq can be considered the *parent physical RelReq,* from which all other physical RelReqs descend.

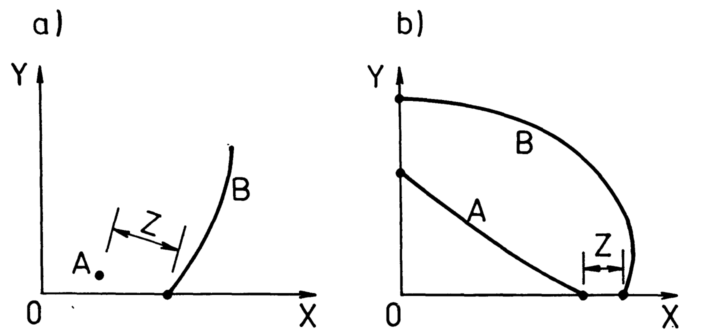

Fig. 7.3 - Determination of the reliability margin when Ω_{def} is confined (**a** - attack is a vector, **b** - attack is a function, barrier being a function in both cases).

7.2.2 Reliability margin

The minimum distance Z between an attack A and the corresponding barrier **B** is always a scalar. It is called ***reliability margin*** and it is one of the principal quantities analyzed in theoretical investigations of reliability of constructed facilities. It is a *global variable* because it describes a *global phenomenon* Ph(**properties of an S-L-E system**). Consequently, RelReq (7.3) will be called ***global physical RelReq.***

Since, during a period T_{ref}, the attack and the barrier change, the reliability margin is, in general, also a time-dependent variable. For instance, the magnitude of a load can increase or decrease, or the resistance of a cross-section can diminish under repeated loadings. Moreover, a barrier can depend on the development of the corresponding attack (cf., for instance, the influence of loading history on the load-bearing capacity of structures with non-linear behavior). Therefore, *reliability margin must be defined, in general, for any specified point in time, t* . However, to simplify subsequent formulas, a definition domain $\forall t \in T_{ref}$ will be indicated only when necessary.

In a general case, units of a reliability margin cannot be unambiguously defined, since the units of components X, Y, or others are usually of different kind (for example, kN and kN.m).

As a rule, a reliability margin, Z, can be simply established by an analytical formula only in one-component cases. For multi-component cases numerical solutions are often necessary. Moreover, since Z is not a dimensionless quantity, it can be generalized only with difficulties. For this reason, RelReq (7.3) is almost never considered an initial RelReq in current design, but it can be used to demonstrate some theoretical procedures. Nevertheless, when the coordinate system is normalized, a reliability margin can be expressed non-dimensionally; see 8.5.3 and Section 9.2.

Case (1) is met, for example, in the design of a cross-section subjected to bending moment. Case (2) is typical for a cross-section subjected to combined bending and axial load. Case (3) can be encountered when a complex stress state is dealt with (Example 7.1).

■ **Example 7.1.** At a point in time, t , a reinforced concrete member with a constant cross-section is subjected to bending moments M_x and M_y acting in two mutually perpendicular planes. The development of moments along the member is shown in Figure 7.4. The ultimate capacity diagram of the cross-section, and consequently, of the whole member, is given by a curve, Π . From the development of M_x and M_y, a curve describing the attack, Ψ , results. The location and shape of Ψ change in dependence on the changing load. At a certain point in time t , the reliability margin of the member, Z, is given by the minimum distance between Ψ and Π . ■

Assume that A is time-dependent. In dependence on A and B . When A and B are independent, then only Z changes with changing A . However, when A and B are mutually dependent, then also the barrier changes, and diverse developments of A can bring diverse developments of B , even if the final magnitude of A is identical in all development (see Figure 7.5). Therefore, a reliability margin must always be investigated for only a specified point in time and, in cases where the dependence between attack and barrier is significant, also for a specified time-dependent development of the attack (in many practical cases the ***problem of loading history,*** see Section 5.6, is met).

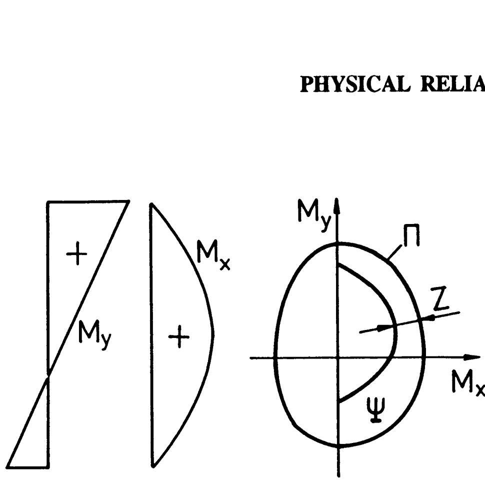

Fig. 7.4 - Reliability margin of a bar subjected to a two-way bending (Π - ultimate capacity curve, Ψ - stress load-effect curve).

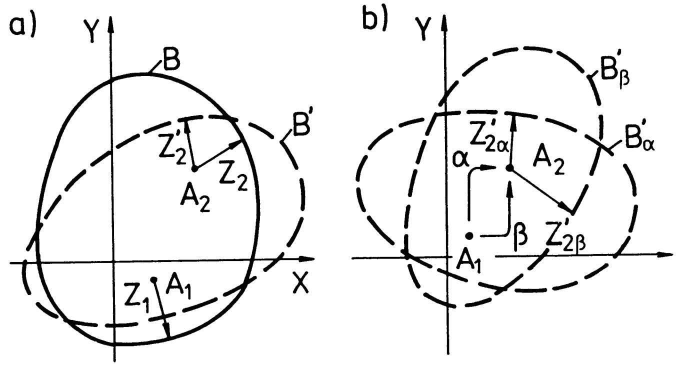

Fig. 7.5 - Changes in the reliability margin when the attack varies (**a** - barrier depends on the attack, **b** - barrier depends on the attack and on the attack history; A_1 , A_2 - two successive attacks, **B** - independent barrier, B' - dependent barrier, B'_α , B'_β - attack-dependent barriers for two different developments, α and β , of the attack).

RelReq (7.3) is often given in the following form:

$$g(x_1, x_2, \ldots, x_n) \geq 0 \tag{7.4}$$

without defining the physical meaning of $g(.)$, called the *limit state function* or *failure function*. Frequently, the failure function is formulated in terms of a load-effect. Nevertheless, when a general definition of the attack and the barrier is considered, the failure function can be of diverse physical meaning. In Table 7.2 reliability margins are given for cases of A and B shown in Table 7.1. Obviously, in *one-component cases,* such as those in rows 2 through 10, the reliability margin is expressed simply by

$$Z = B - A \tag{7.4a}$$

Table 7.2 - Reliability margins, cf. Table 7.1

No.	Reliability margin	Unit
1	multi-component case	-
2	$Z\|M = M_r - M$	N.m
3	$Z\|\sigma = f_y - \sigma$	$N.m^{-2}$
4	$Z\|w = w_{lim} - w$	m
5	$Z_i\|f = f - f_{lim,i}$ $Z_s\|f = f_{lim,s} - f$	s^{-1}
6	$Z\|A = A_{nec} - A_{eff}$	m^2
7	$Z\|d = d_{nec} - d_{eff}$	m
8	$Z\|M = M_{pas} - M_{act}$	N.m
9	$Z\|b = b_{nec} - b_{eff}$	m
10	$Z\|T_0 = T_{0,eff} - T_{0*}$	year

Margins $Z\,|\,.$ in rows 2 through 5 refer each to a separate problem. However, the margins given in rows 6 and 7, and similarly the margins in rows 8 and 9 of Table 7.2 are closely related. In fact, the same RelReqs are dealt with, but the respective design criteria are different. We can ask whether the paired RelReqs

$$Z\,|\,A \geq 0, \quad Z\,|\,d \geq 0$$

or

$$Z\,|\,M \geq 0, \quad Z\,|\,b \geq 0$$

are *equivalent*. In other words, we can ask whether RelReq for $Z\,|\,d$ is fulfilled when RelReq for $Z\,|\,A$ is fulfilled. And conversely, when RelReq for $Z\,|\,d$ is fulfilled, is also RelReq for $Z\,|\,A$ satisfied? These *practical questions* lead to a *theoretical problem:*

For a given design criterion, let RelReq (7.4) be fulfilled. Assume that $n \geq 2$ (otherwise the problem would be meaningless). Answers to the following three questions are required:

(a) When considering RelReq (7.4), is it possible to find other RelReqs that can be equivalently used in the assessment of reliability?
(b) If the answer to (a) is YES, what is the relation between the respective reliability margins?
(c) Can any one of the reliability margins defined for the equivalent RelReqs be used in the reliability assessment of the investigated system?

Equivalent reliability margin

Consider RelReq (7.4) and denote

$$Z_0 = g(x_1, x_2, ..., x_n) \tag{7.5}$$

Let us call Z_0 the ***initial reliability margin.*** Variables x_i describe partial phenomena H_i, which cannot be further decomposed, or which are assumed as such. These phenomena and also the respective variables x_i will be termed *elementary.* To simplify and to take into account the actual development of phenomena, it will be assumed that all variables are continuous.

Assume now that Z_0 is *monotonic in* Ω_{def} *with respect to any elementary variable.* When with growing x_i the reliability of the system improves, the respective phenomenon H_i is assumed *absolutely favorable,* in the opposite case it is *absolutely adverse* (cf. Section 3.6).

For further derivations the ***influence function*** is introduced:

$$\Lambda_k = \operatorname{sign} \frac{\partial Z_0}{\partial x_k} \tag{7.6}$$

which assumes values +1 or -1 according to whether H_i is absolutely favorable or absolutely adverse, respectively. For non-linear g(.) the assumption on monotoneity cannot be fulfilled in the entire definition domain, and so Λ_k can change its value.

By an *equivalent rearrangement* of g(.) , RelReq (7.4) can be changed to

$$g^*(x_1, x_2, ..., x_n) \geq 0 \tag{7.7}$$

The quantity

$$Z^* = g^*(x_1, x_2, ..., x_n) \tag{7.8}$$

is called the ***equivalent reliability margin.*** - A rearrangement g(.) → g*(.) is considered equivalent if for any ordered n-tuplet (x_1 , x_2 , ... , x_n) the following holds in Ω_{def} :

if g(.) > 0, then g*(.) > 0
if g(.) ≤ 0, then g*(.) ≤ 0

Further, let an ordered n -tuplet of elementary variables, $(x_1, x_2, ..., x_n)$, be given. For this n -tuplet margins Z_0 and Z^* assume certain values, $Z_0 \neq Z^*$. When, however, $g(.) = 0$ for a certain n -tuplet $(x_1, x_2, ..., x_n)_0$, it is also $g^*(.) = 0$ and, consequently,

$$Z_0 \equiv Z^* = 0$$

Thus, as far as the information on reliability is concerned, the initial and derived reliability margins are equivalent.

In general, we are able to define, for a given Z_0 , an infinite number of derived reliability margins Z^* . Considering the set Z_{eqv} of all equivalent reliability margins referred to the particular system, any member of this set can be taken as initial and all other members can be considered derived. Thus, the separate equivalent reliability margins form a *formal system with one axiom* observing the *rule* "equivalent rearrangement." Hence, the answer to question (a) is obviously positive.

Since Z_0 and Z^* are functions of the same set of elementary variables x_i , they are mutually perfectly dependent. The law of dependence is given by the expressions $g(.)$ and $g^*(.)$. Details of this law are not interesting, but the following conclusion is important:

When, for a given design criterion, an expression for reliability margin Z^* is derived from Z_0 by means of an equivalent rearrangement of RelReq (7.4), then, for any ordered n -tuplet of elementary variables, $(x_1, x_2, ..., x_n)$, the following relationship between reliability margins holds :

$$\operatorname{sign} Z^* = \operatorname{sign} Z_0 \tag{7.9}$$

In other words, *when, for a given design criterion, a RelReq formulated for any of the reliability margins* $Z \in Z_{eqv}$ *is satisfied, then also RelReqs written for other equivalent reliability margins are fulfilled.*

Thus, also the questions (b) and (c) have been answered.

In practical problems the reliability margin

$$Z \mid x_k = \Lambda_k[x_k - h(x_1, x_2, ..., x_n; \sim x_k)] \tag{7.10}$$

is important. Here $Z \mid x_k$ = reliability margin referred to the elementary variable x_k , Λ_k = value of the influence function defined by Equation (7.6), $h(.)$ = function of elementary variables; symbol $\sim x_k$ denotes that x_k is not contained in $h(.)$.

Reliability margins $Z \mid x_k$ are currently used in proportioning and checking of structures. Assume, for example, that x_k refers to a cross-section dimension or to a material property. Since these are, as a rule, absolutely favorable, that is, $\Lambda_k = +1$, Equation (7.10) with RelReq (7.3) leads to RelReq

$$x_k \geq h(x_1, x_2, ..., x_n; \sim x_k) \tag{7.11}$$

In general, we can relate a reliability margin to any elementary variable x_1 through x_n . In practical cases, however, this is not always possible because some elementary variables cannot be explicitly expressed, and consequently, the respective Equation (7.11) cannot be written.

■ **Example 7.2.** Find ultimate limit states design formulas for a STRUCTURE-LOAD system consisting of a steel bar of circular cross-section having a diameter d (STRUCTURE subsystem). The bar is subjected to an axial load $N > 0$ (LOAD). Yield stress of steel is f_y. The definition domain, Ω_{def}, is given by extreme possible values of elementary variables. - This very simple system can help illustrate individual rules of compiling a formal system of reliability margins. We will use it also in several other examples.

The initial RelReq can be written, for example, in terms of load

$$N \leq \pi d^2 f_y / 4$$

Obviously, a triplet of elementary variables, (d, N, f_y), is dealt with here, and so three design reliability margins can be formulated:

$$Z\,|\,d = d - 2[N/(\pi f_y)]^{-2}$$

$$Z\,|\,f_y = f_y - 4N/(\pi d^2)$$
$$Z\,|\,N = \pi d^2 f_y/4 - N$$

or also

$$Z\,|\,A = A - N/f_y$$

where $A = \pi d^2/4$. Note that influence function values are $\Lambda_d = 1$, $\Lambda_{fy} = 1$, $\Lambda_N = -1$, $\Lambda_A = 1$, respectively.

Thus, the bar can be *sized* with respect to the diameter d (or cross-section area, A) and strength f_y, and it can be *checked* with respect to the force N. ■

7.2.3 Reliability factor

The reliability requirements based on only a qualitative assessment of the relation between the attack and the barrier do not provide any general information on *how reliable a fully defined S-L-E system with a specified pair* (A, B) *is*. For this purpose, the margin Z should be *normalized* in some way, otherwise various particular cases of S-L-E systems could not be compared,neither qualitatively nor quantitatively.

A reliability margin can be directly normalized only then, when the attack and the barrier are scalars. In such a case (cf. Figure 7.1a) we can write

$$Z = B - A$$

Setting for Z into Equation (7.3) we obtain

$$B - A \geq 0$$

Now, let us normalize this RelReq with respect to A, that is,

$$\frac{B}{A} - 1 \geq 0$$

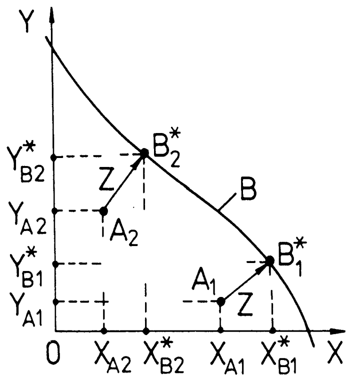

Fig. 7.6 - Simplification of the global RelReq; the problem is reduced to one-component cases.

and so the physical RelReq becomes:

$$\theta \equiv \frac{B}{A} \geq 1 \tag{7.12}$$

The ratio θ is the *global reliability factor,* commonly known as the "safety factor." - RelReq (7.12) is, of course, a physical requirement again. Note that we are still discussing a non-random S-L-E system!

A generalization of the reliability factor for multi-component cases is impossible. Let this fact be demonstrated on a two-component case, see Figure 7.6. Assume a barrier, **B**, and two different, independent attacks, A_1 and A_2. The nearest point on **B** with respect to A_1 is denoted by B_1^*; an analogous point with respect to A_2 is B_2^*. Let the margins Z_1 and Z_2 be equal, and so, for both attacks, RelReq (7.3) is complied with to the "same degree."

Let us now try to find reliability factors referred to both attacks. The two components, X and Y, have the following values at A_1, A_2, B_1^*, and B_2^*

$$(X_{A1}, Y_{A1}), \quad (X_{A2}, Y_{A2}), \quad (X^*_{B1}, Y^*_{B1}), \quad (X^*_{B2}, Y^*_{B2})$$

For a reliability factor, ratios of mutually corresponding components could be taken. We can obtain

$$\theta_{X1} = \frac{X^*_{B1}}{X_{A1}}, \quad \theta_{Y1} = \frac{Y^*_{B1}}{Y_{A1}}, \quad \theta_{X2} = \frac{X^*_{B2}}{X_{A2}}, \quad \theta_{Y2} = \frac{Y^*_{B2}}{Y_{A2}}$$

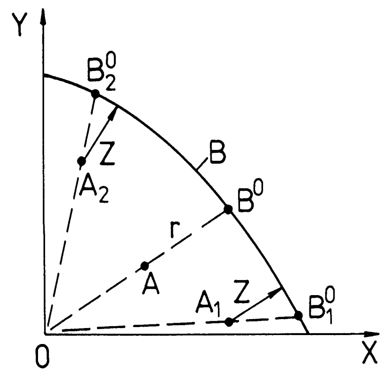

Fig. 7.7 - Two-component case, polar form of the global reliability factor.

where

$$\theta_{X1} \neq \theta_{Y1}, \quad \theta_{X2} \neq \theta_{Y2}$$

and further

$$\theta_{X1} < \theta_{X2}, \quad \theta_{Y1} > \theta_{Y2}$$

although the reliability margin is the same for both cases!

Various manipulations with the global reliability factor are possible. Frequently, a *global reliability factor in polar form* is used (Figure 7.7). A half line, **r** , is led through the point A referred to the attack; the half line cuts the curve B at B^0 . Then, the global reliability factor is defined by

$$\theta_r = \frac{\overline{OB^0}}{\overline{OA}}$$

where $\overline{OA}$, $\overline{OB^0}$ = distances according to Figure 7.7. It is obviously

$$\theta_r \equiv \frac{X_B^0}{X_A} = \frac{Y_B^0}{Y_A} \tag{7.13}$$

Although this expression has certain advantage in some solutions, the principal drawback of the reliability factor remains. For two attacks with equal reliability margins, $Z = Z_1 = Z_2$, two different values of the reliability factor, $\theta_{r1} \neq \theta_{r2}$, can be obtained again.

Unfortunately, the foregoing simple facts on the global reliability factor are still not fully understood by many engineers.

7.3 ELEMENTARY REQUIREMENTS

The two formative phenomena Ph(**attack**) and Ph(**barrier**) result from a series of elementary phenomena: strength of materials, structure's geometry, load, temperature, time and others.

■ **Example 7.3.** When designing a simply reinforced concrete cross-section according to the ultimate limit state, the following RelReq must be fulfilled:

$$S \leq R$$

where S = load-effect, R = respective structural resistance. Here, Ph(**attack**) and Ph(**barrier**) are specified by S and R, respectively. - Clearly, the load-effect in the cross-section results from several loads acting simultaneously on a structure with given dimensions. In this way phenomena Ph(**load**) and Ph(**geometry of the structure**) apply in Ph(**attack**). Similarly, the cross-section resistance consists of Ph(**concrete strength**), Ph(**steel strength**), Ph(**cross-section geometry**), Ph(**time**), or also other phenomena. ■

The elementary phenomena can be investigated one by one, directly or indirectly. For example, variable loads in residential buildings are directly measured by scaling furniture and occupants, whereas the wind load is obtained indirectly through measurements of wind velocity. - The elementary phenomena are expressed by ***elementary variables*** a_1, a_2, ..., a_{na} [referred to Ph(**attack**)] and b_1, b_2, ..., b_{nb} [referred to Ph(**barrier**)]. Elementary variables are, for example: load magnitude, strength of material, acceptable deflection.

The concept of elementary variables is almost identical with that of *basic variables* used by many authors in formulations of RelReqs and calculation models.

When a one-component case is dealt with, we can write:

$$A = \mathrm{f}_A(a_1, a_2, ..., a_{na})$$
$$B = \mathrm{f}_B(b_1, b_2, ..., b_{nb})$$

where $\mathrm{f}_A(.)$, $\mathrm{f}_B(.)$ = functions describing the attack and the barrier, respectively.

Consequently, a physical RelReq can be written with regard to the elementary variables either in the *formative form,*

$$A(a_1, a_2, ..., a_{na}) \leq B(b_1, b_2, ..., b_{nb}) \tag{7.14}$$

or in the *global form:*

$$Z(a_1, a_2, ..., a_{na}; b_1, b_2, ..., b_{nb}) \geq 0 \tag{7.15}$$

In particular cases, RelReq (7.14) can be transformed in such a way that the attack and barrier become *isomorphic,* that is, that to each a_i a certain b_j corresponds. Thus, the number of elementary variables is identical at both sides of the requirement, n_a = n_b = n. As a result, such RelReq can be substituted by a *system of elementary RelReqs*

$$
\begin{aligned}
a_1 &\le b_1 \\
a_2 &\le b_2 \\
\dots & \quad \dots \\
a_n &\le b_n
\end{aligned}
\tag{7.16}
$$

which must be complied with simultaneously. The inequality symbol $\le$ will apply in RelReqs (7.16) only when Z decreases with increasing a_i (that is, when $\Lambda_{ai} = -1$ in the domain investigated). If the opposite is true, symbol $\ge$ has to be used.

An important drawback of the reliability assessment based on RelReqs (7.16) is the *loss of information on the reliability margin,* Z. We could substitute RelReq (7.15) by a system of requirements

$$
\begin{aligned}
z_1 &\ge 0 \\
z_2 &\ge 0 \\
\dots & \quad \dots \\
z_n &\ge 0
\end{aligned}
\tag{7.17}
$$

without any significant practical gain, however. The reliability margin could be, in such a case, expressed in terms of a *vector* $Z(z_1, z_2, \dots, z_n)$ *of elementary reliability margins* z_i. The greater the number of inequalities in RelReqs (7.16), or (7.17), the more uncertain the quantitative information on reliability that can be drawn from the vector. Nevertheless, the qualitative information (in terms of YES or NO) obtained from RelReqs (7.16) or (7.17) is the same as that resulting from RelReq (7.14).

Historically, isomorphic solutions precede higher level concepts. The *method of working stress* has been based on assumed isomorphic relation between the stress state of a cross-section under service load and the stress state under ultimate load, both stress states being described by the elastic theory. Developments in structural mechanics have made possible an elasto-plastic description of the stress state under ultimate load, which obviously is not isomorphic with that under service load. This fact has resulted in RelReqs based on the *limit state approach.*

Since, in general, the description of limit states does not allow any isomorphization of the attack and barrier, an *anisomorphic RelReq in the formative form* is used (for a two-component case):

$$
\begin{aligned}
& A(a_{1,exm}, a_{2,exm}, \dots, a_{na,exm}) \\
& \qquad \le B(b_{1,exm}, b_{2,exm}, \dots, b_{nb,exm})
\end{aligned}
\tag{7.18}
$$

Here, $a_{i,exm}$, and $b_{j,exm}$ are defined limits of elementary variables a_i and b_j ($i = 1$, 2, ..., n_a; j = 1, 2, ..., n_b) while no association exists between, say, a_k and b_k, or between $a_{k,exm}$ and $b_{k,exm}$; in general, $n_a \neq n_b$.

In other words, RelReq (7.18) requires that ($n_a + n_b$) *elementary RelReqs must be* simultaneously *satisfied:*

$$
\begin{array}{ll}
a_1 = \text{fav}(a_{1,exm}), & b_1 = \text{fav}(b_{1,exm}) \\
a_2 = \text{fav}(a_{2,exm}), & b_2 = \text{fav}(b_{2,exm}) \\
\ldots \quad \ldots & \ldots \quad \ldots \\
a_{na} = \text{fav}(a_{na,exm}), & b_{nb} = \text{fav}(b_{nb,exm})
\end{array}
\tag{7.19}
$$

where fav(x_{exm}) = value of x on the favorable side with respect to a defined x_{exm}. When, for example, a_1 refers to an absolutely favorable phenomenon (see Section 3.6), it must be

$$a_1 \leq a_{1,exm}$$

and when it refers to an absolutely adverse phenomenon:

$$a_1 > a_{1,exm}$$

Note that the global, formative, and elementary RelReqs differ by the number of phenomena, or variables, which is 1, 2, and $n_a + n_b$, *respectively.*

8

PROBABILISTIC AND STATISTICAL RELIABILITY REQUIREMENTS

Key concepts in this chapter: *reliability requirement, RelReq; probabilistic RelReq; statistical RelReq; physical RelReq; user's RelReq; owner's RelReq; target values; failure characteristics; failure probability; rho-measure; global RelReq; formative RelReqs; elementary RelReqs; probabilistic design methods; direct method, DM; method of extreme functions, MEF; method of extreme values, MEV; level 1 through 4 methods; time factor; reliability index; invariance of* P_f *and* β^{HL}.

Since, in general, the elementary phenomena forming the attack and the barrier are random, any investigation of the reliability of a CF system must consider this fact. This can be done in two basic ways: either *probabilities* of occurrence of relatively unfavorable phenomena, or *population parameters* of the attack and the barrier govern the solutions.

In the first case ***probabilistic reliability requirements*** are specified, on the basis of which necessary design parameters are derived. The principal failure characteristic involved is the failure probability, P_f, that can have several forms, as it will be shown. In the second case ***statistical reliability requirements*** can be formulated.

Again, the abbreviation *RelReq* will be used for the "reliability requirement," to simplify the reading.

8.1 GLOBAL PROBABILISTIC RELIABILITY REQUIREMENT

8.1.1 Two principal types of probabilistic requirements

As a global probabilistic RelReq, the *relationship between the actual and an extremely acceptable failure probability* can be considered. Before assessing this relationship for a given constructed facility (or for a class of facilities) we have to consider the respective "quality of the reliability." According to the user's attitude to CF, *two basic types of probabilistic RelReqs* can be distinguished. - Assume again that the definition domain, Ω_{def}, of probabilistic requirements is given (for example, the territory of Greece, or the territory of the snow zone No. 2), and therefore Ω_{def} will not be emphasized any more.

The "extremely acceptable" values of failure probability and of other probabilistic quantities, which will be discussed later, will be termed, in conformity with the current practice, the "target values."

Subjective probabilistic RelReqs

A specific circle of users of CFs (let us call them ***internal users***) is not interested in the life expectancy of the particular facilities at all. As a rule, an internal user is implicitly interested not to enjoy any failures and to have an approximately constant *perception of safety* during the whole period when he or she is using the facility. In the main, this perception is subconscious. Sometimes an individual is willing to accept a certain likelihood of failure but he or she is usually not able to assign to it any specific number in terms of probability. This interest refers to all facilities that will be utilized by the user during his or her personal life, T_u. Assume, for example, that a person has lived in an apartment house whose life expectancy is $T_0 = 70$ years, and than has moved to another building with a life expectancy of 120 years. Yet, this person's attitude to the reliability of the two buildings is not affected by the buildings' life expectancy. Neither are such users interested whether a certain part of the facility life has been already consumed, or what its residual life is. Thus, for internal users, the governing quantity is the *failure rate,* λ (see 2.2.1). Their global RelReq can be mathematically expressed as

$$\forall t \in T_u\colon \; (\lambda \le \lambda_t) \tag{8.1}$$

where λ_t = *target failure rate*. The requirement is valid in the specified domain. Since this requirement refers to the subjective personal attitudes of an individual, or a group of individuals, we will call it the *subjective reliability requirement.*

For many practical reasons it is better to take as a basic quantity the ***annual failure probability*** $\dot{P}_f$, which is defined by

$$\dot{P}_f = \int_t^{t+1} \lambda(t)\,\mathrm{d}t \tag{8.2}$$

where t is given in years. The value of $\dot{P}_f$ depends on the period $t - t_0$, where t_0 is the point in time when the facility is put into operation. As a rule, however, $\dot{P}_f$ is related to the *constant failure period* (see 2.2.1), where $\lambda(t)$ is supposed constant, so that $\dot{P}_f$ is also constant. Then, the subjective RelReq becomes

$$\forall t \in T_u\colon \; (\dot{P}_f \le \dot{P}_{ft}) \tag{8.3}$$

with $\dot{P}_{ft}$ = target value of the annual failure probability (see Section 10.3).

Typical facilities that are governed by subjective RelReqs are *residential buildings, school and kindergarten buildings,* and similar works. The individuals and groups of individuals concerned are persons using the facility during their everyday life, or people emotionally attached to such persons, such as fathers and mothers.

Facility reliability requirement

Another group of facility users takes the *facility life,* T_0, as the governing quantity. These users are mainly institutions, various public bodies, or simply society as a whole; let us call them ***external users.*** They are interested in avoiding any failure during T_0, expressed, as a rule, in years. In some cases, external users are willing to accept a certain value of the failure probability referred to T_0.

The life of a CF system can be considered a well defined period with respect to which a global RelReq

$$\forall t \in T_0: \ (\bar{P}_f \leq \bar{P}_{ft}) \tag{8.4}$$

can be written. Here $\bar{P}_f$ = *comprehensive probability that the failure will happen during* T_0, P_{ft} = target value of P_f. Since RelReq (8.4) relates to the facility, it will be termed the *facility reliability requirement.* Facility RelReqs should be used for, for example, *TV towers, storehouses, water tanks, grain silos, some agricultural buildings, and utility lines.*

There are, nevertheless, many CFs where both types of requirements can apply: *bridges, public buildings, and other important facilities.* Thus, the reliability engineer has to decide which type of RelReq is appropriate in a specific case.

8.1.2 Relationship between the annual and comprehensive probabilities

The following simple formulas are valid for the relationship between a constant annual probability $\dot{P}$ of occurrence of a random event E and the respective comprehensive probability $\bar{P}$ referred to the period of n years:

$$\bar{P} = 1 - (1 - \dot{P})^n \tag{8.5}$$

$$\dot{P} = 1 - (1 - \bar{P})^{\frac{1}{n}} \tag{8.6}$$

These formulas can be derived from Equations (2.3) and (2.5).

Thus, the subjective RelReq (8.3) can be written for the comprehensive failure probability referred either to the user's life, T_u, that is,

$$\forall t \in T_u: \ (\bar{P}_f \leq \bar{P}_{ft}) \tag{8.7}$$

or to the facility life, T_0:

$$\forall t \in T_0: \ (\bar{P}_f \leq \bar{P}_{ft}) \tag{8.8}$$

where $\bar{P}_f$ can be established from Equation (8.5) with $n = T_u$ or $n = T_0$. In general, the respective P values in RelReq (8.7) are not equal to P values in RelReq (8.8).

Analogously, the facility RelReq (8.4) can be expressed by

$$\forall t \in T_0: \ (\dot{P}_f \leq \dot{P}_{ft}) \tag{8.9}$$

where $\dot{P}_f$ follows from Equation (8.6) with $n = T_0$.

Probabilistic RelReqs as axioms

Similarly as with RelReq (7.1), evident, though non-provable requirements are expressed by RelReqs (8.1) or (8.3), and (8.4). Therefore, they are taken as axioms. All further RelReqs discussed in Sections 8.2 and 8.3 are derived from them.

It will be shown later that the synthesis of physical and probabilistic RelReqs results in ***design requirements,*** specified by structural design codes.

8.1.3 Effective failure probability and its estimate

The failure probability P_f (probabilities $\dot{P}_f$ and $\bar{P}_f$ will not be distinguished in the following text, and the time quantifier will be omitted) entering RelReqs can be defined by

$$P_f = \Pr(Z \leq 0)$$

or, equivalently, by

$$P_f = \Pr(\Theta \leq 1)$$

where Z = reliability margin (see 7.2.2), Θ = global reliability factor (see 7.2.3). Therefore, only a ***single phenomenon*** has to be investigated in the analysis of RelReqs (8.3) or (8.4), which can be written in a more general form

$$P_f \leq P_{ft} \tag{8.10}$$

The reliability margin is a global phenomenon including all factors that affect the reliability of an S-L-E system. Therefore again, the probabilistic RelReqs (8.1), (8.3), (8.4), etc. are denoted as ***global*** [cf. RelReq (7.3)].

Taking into account the discussion on equivalent reliability margins in 7.2.3, it can be easily proved that, for a certain system, the failure probability is invariant with respect to the reliability margin formula. This statement is obvious, since the failure probability is an objective characteristic of an S-L-E system that cannot depend on the calculation model used. However, in all probability-based investigations we must keep in mind that *two variants* of the failure probability P_f (that is, $\dot{P}_f$ or $\bar{P}_f$) are dealt with:

♦ the ***effective failure probability,*** $P_{f,eff}$, that describes a certain existing "probability state" of the reliability system S-L-E; its exact value cannot be established by available calculation means, and we may doubt whether it can be ever established at all;

♦ the ***estimate failure probability,*** $P_{f,est}$, which is a value that we can find by performing theoretical solutions (see Sections 9.1 and 9.3) if random properties of elements and components of the S-L-E system are known or assumed, and if appropriate calculation models are used; this probability is obviously subjective; when for a particular case solutions according to several authors are used, the values of $P_{f,est}$ can differ by orders (see Grimmelt and Schuëller 1982).

The calculation model for $P_{f,est}$ should be always such that

$$P_{f,est} \geq P_{f,eff}$$

so that with

$$P_{f,est} \leq P_{ft}$$

also the requirement

$$P_{f,eff} \leq P_{ft}$$

would be fulfilled.

We will not investigate the difference between $P_{f,eff}$ and $P_{f,est}$, nor will we consider it in the following discussions any more. We will also avoid using attributes like *notional, formal, operative,* etc., met in papers and documents on probability-based design. Only the plain term "failure probability P_f" (that is, $\dot{P}_f$ or $\bar{P}_f$) will be used. Nevertheless, the reader should keep in mind its double features, and should not forget that P_f obtained by analysis based on an accepted calculation model always is only an estimate.

8.1.4 Logarithmic measure of reliability

The theoretical merits of P_f as a governing quantity in the reliability studies surpass all arguments. Unfortunately, however, P_f does not convey clear information on the reliability to an ordinary engineer.

In probability-based design procedures, that is, in proportioning and checking, any value of $P_f \in [0, 1]$ can be met. An engineer with a four to five years' education in civil engineering, including, in a favorable case, a one-term course of mathematical statistics and probability theory, has practically no feeling for probability values ranging between, say, 1.0E-8 and 0.6. We can, of course, try to explain to him or her that with P_f = 1.0E-6 ten times less structures of that type designed by him or her might collapse than with P_f = 1.0E-5, etc. Similarly, we can suggest that with P_f = 0.1 every tenth floor beam might show deflection greater than is acceptable to deflection-sensitive people. But an engineer does not wish nor expect any collapse and any unacceptable deflections of his or her structures! Moreover, are we really able to claim that a structure with P_f = 1.0E-6 is "ten times more reliable" than another with P_f = 1.0E-5 ? We know that in the domain of low values (less, for example, than 1.0E-3) the failure probability P_f is very sensitive to quite small changes in basic variables, whereas the available experience does not confirm this theoretical observation. Under such circumstances (others might be introduced here), the designer becomes rather reluctant to appreciate the finesse of probability-based design, as well as all benefits coming from it. For most non-research engineers the failure probability remains a mystic quantity beyond understanding.

In 1966 V.V. Bolotin proposed for the reliability measure a simple *logarithmic transform* of P_f:

$$\varrho = -\log P_f \tag{8.10a}$$

Since ϱ (let us call it simply the ***rho-measure***) is perfectly dependent on P_f, it provides all technical functions of a reliability measure. In addition, however, it possesses the above-stated psychological qualities.

Using the logarithmic transform of P_f, we can write RelReq (8.10) as

$$\varrho \geq \varrho_t \tag{8.10b}$$

where ϱ_t = target value of the rho-measure.

The logarithmic transform of the failure probability seems to be a natural move towards better understanding the reliability concepts, as *people think, due to psychological phenomena, in logarithms* when dealing with quantities having values expressed by numbers differing by orders. This was shown for large numbers (> 1) by Hofstadter 1986 who says: "Logarithmic thinking happens when you perceive only a linear increase even if the thing itself doubles in size." Indeed, we use the logarithmic scale when measuring the level of sound intensity (in decibels), the piano keyboard is logarithmic, etc. There is no reason why we should not extend this way of thinking to very small numbers describing reliability.

It has to be stressed that no failure characteristics, be it the failure probability, rho-measure, or the reliability index (see Section 8.5), can solve the general problem of measuring structural reliability. It is obvious that the structural reliability is a vectorial property, *the failure characteristics being just one of its components. It is very likely that further components, analogous to those applied in the domain of mechanical and electrical engineering (see also Section 16.1), will be added to the description of structural reliability in the future.*

8.2 FORMATIVE PROBABILISTIC RELIABILITY REQUIREMENTS

The formal difficulties met in calculations of the failure probability and also uncertainties caused by imperfect calculation models can be diminished when the assessment of the reliability of a facility is based on two formative probabilistic RelReqs:

$$P_A \leq P_{At} \tag{8.11}$$

$$P_B \leq P_{Bt} \tag{8.12}$$

which refer to the formative phenomena Ph(A) and Ph(B), respectively. Here

$$P_A = \Pr(A_{adv}), \quad P_B = \Pr(B_{adv})$$

are probabilities of occurrence of events Ev(A_{adv}) and Ev(B_{adv}), that is, probabilities of relatively adverse realizations of the attack, A, and of the barrier, B. Then, P_{At}, P_{Bt} = target values of P_A and P_B, respectively. - Note that RelReqs (8.11) and (8.15) are the corollary of the physical RelReqs (7.1), or (7.2).

The separation of RelReqs with respect to formative phenomena simplifies the probability-based solution because values of P_A or P_B can be calculated with less effort and with greater accuracy than the value of P_f. On the other hand, *two target values,* P_{At} and P_{Bt}, are needed (see 13.1.1).

8.3 ELEMENTARY PROBABILISTIC RELIABILITY REQUIREMENTS

Further simplification is achieved by separating elementary phenomena in conformity to RelReq (7.19), so that *elementary probabilistic RelReqs*

$$P_{ai} \le P_{ait} \quad (\text{for } i = 1, 2, ..., n_a) \tag{8.13}$$

$$P_{bi} \le P_{bit} \quad (\text{for } i = 1, 2, ..., n_b) \tag{8.14}$$

should be simultaneously satisfied. Here

$$P_{ai} = \Pr(a_{i,adv}), \quad P_{bj} = \Pr(b_{j,adv})$$

are the probabilities of occurrence of relatively adverse elementary phenomena Ph($a_{i,adv}$), Ph($b_{i,adv}$), and P_{ait}, P_{bjt} = respective target probabilities.

Boundaries between phenomena Ph($a_{i,adv}$) and phenomena Ph($a_{i,ntr}$), and between Ph($b_{i,adv}$) and Ph($b_{j,ntr}$), are mathematically defined by extreme values $a_{i,exm}$, $b_{j,exm}$ (cf. Section 3.6).

Since only isolated input phenomena entering RelReqs (for example, material strengths, load magnitudes, limit deflections) are subjected to analysis, the above system of elementary requirements yields the most simple solution. It is necessary, however, to make (n_a + n_b) *decisions on target probabilities* associated with Ph($a_{i,adv}$) and Ph($b_{j,adv}$). This problem is discussed in Section 14.1.

8.4 CLASSIFICATION OF THE PROBABILISTIC DESIGN METHODS

In the current design practice, it is almost impossible to calculate and assess probabilities required for the evaluation of probabilistic RelReqs. The level of engineering education has not yet so developed, courses on structural reliability theory and its applications being regularly given only at few universities, and therefore a wider implementation of reliabilistic knowledge cannot be expected in the nearest future. For this reason, sophisticated probabilistic RelReqs are being substituted by *design RelReqs* (see Chapters 12 through 14) in order to obtain the relationships among the design input variables transparent as much as possible.

According to the way of implementation of probabilistic considerations into design three principal probabilistic design methods can be distinguished (see Tichý and Vorlíček 1972):

(a) ***direct method,*** DM, based on the analysis of the *reliability margin,* see Chapter 12;

(b) ***method of extreme functions,*** MEF, where the *formative variables* are subjected to investigation, see Chapter 13;

(c) ***method of extreme values,*** MEV, analyzing the *elementary variables,* see Chapter 14.

When we consider these three principal methods as a whole, it becomes apparent that the simpler the formulation of the method (called *design format,* as a rule) the more troublesome its practical application. It must be emphasized that each of the above mentioned methods has its particular structure. Therefore, it would be a mistake to expect that in a specific, well defined case of an S-L-E system the probabilistic design according to these methods will lead to identical results. It also is erroneous to transplant results obtained from the theoretical analysis based on one method into another method, which is often being suggested. Of course, comparisons of solutions using different methods can help in calibrating the design parameters.

We should mention here that the above classification, based on the *depth of probabilization of RelReqs,* is close to the "level classification" (see, for example, PROBAN 1991). This classification is based on the *extent of information about the structural reliability problems:*

Level 1: random variables are represented by *characteristic values* and a system of *partial reliability factors* is used (see Section 14.6);

Level 2: random variables are represented by *population means and variances;* correlation between variables is considered;

Level 3: *joint probability distributions* of random variables are introduced in the analysis;

Level 4: *economic analyses* are supplemented to level 3.

8.5 STATISTICAL RELIABILITY REQUIREMENTS

8.5.1 Reliability index

The global, formal, and elementary RelReqs could also be written in terms of population parameters of the respective variables. For example, we could require

$$\mu_Z \leq \mu_{Zt}$$

$$\sigma_Z \leq \sigma_{Zt}$$

$$\alpha_Z \le \alpha_{Zt}, \quad \text{or} \quad \alpha_Z > \alpha_{Zt}$$

where μ_Z, σ_Z, α_Z = population mean, standard deviation, and coefficient of skewness, respectively, and μ_{Zt}, σ_{Zt}, α_{Zt} = corresponding target values. Analogous RelReqs might be formulated for the formative and elementary levels as well. It is obvious that RelReqs of this kind cannot be generalized, and are not good for practical use. As a rule, *non-dimensional failure characteristics*, which can be codified, are required.

Such a characteristic is the ***reliability index*** defined, in general, by

$$\beta_Z = \frac{\mu_Z}{\sigma_Z} \tag{8.15}$$

Then, the ***statistical RelReq*** is

$$\forall t \in T_{ref}: \; (\beta_Z \ge \beta_{Zt}) \tag{8.16}$$

where β_{Zt} = target value of the reliability index, T_{ref} = reference period. Obviously, β_Z is simply the inverse of the variation coefficient, δ_Z.

RelReq (8.16) is designated as statistical because it does not involve any probability and is governed only by two population parameters, μ_Z *and* σ_Z. We can also call it a *distribution free RelReq.* The target value, β_{Zt}, is directly established by decisions (though some authors derive it from the target failure probability, P_{ft} (see Section 10.3). Obviously, RelReq (8.16) is a *global requirement,* analogous to RelReqs (8.1), (8.3), (8.4), and (8.7) to (8.9).

In like manner we can write two *formative requirements:*

$$\begin{aligned} &\forall t \in T_{ref}: \; (\beta_A \ge \beta_{At}) \\ &\forall t \in T_{ref}: \; (\beta_B \ge \beta_{Bt}) \end{aligned} \tag{8.17}$$

where β_A, β_B = *partial reliability indices* defined by

$$\beta_A = \frac{\mu_A}{\sigma_A}, \quad \beta_B = \frac{\mu_B}{\sigma_B} \tag{8.18}$$

and β_{At}, β_{Bt} = respective target values, μ_A, σ_A, μ_B, σ_B = population parameters of the attack and the barrier, respectively.

Finally, the *elementary RelReqs* could be written:

$$\forall t \in T_{ref}: \; (\beta_{ai} \ge \beta_{ait}), \; i = 1,2, ..., n_a \tag{8.19}$$

$$\forall t \in T_{ref}: \; (\beta_{bj} \ge \beta_{bjt}), \; i = 1,2, ..., n_b \tag{8.20}$$

where the partial reliability indices and their respective target values refer to elementary variables a_i, b_j. It is:

$$\forall i: \ \beta_{ai} = \frac{\mu_{ai}}{\sigma_{ai}} \equiv \frac{1}{\delta_{ai}}, \qquad \forall j: \ \beta_{bj} = \frac{\mu_{bj}}{\sigma_{bj}} \equiv \frac{1}{\delta_{bj}}$$

where μ , σ , δ = mean, standard deviation, and variation coefficient of the respective random variable.

Owing to the deceptive properties of the variation coefficient (see 2.1.2, Coefficient of variation), the method of reliability indices cannot be used at the level of elementary RelReqs. - Observe an important fact: while probabilistic RelReqs have a clear meaning at all levels, the applicability of statistical RelReqs, based on β , declines when descending from the global to the elementary level.

Another fact is important: in the relationships for β_Z, β_A, β_B, β_{ai}, and β_{bj}, only parameters μ and σ appear; no other population parameters (as, for example, the coefficient of skewness, α) are concerned. Thus a certain amount of information on the random behavior of the variables is lost (when it is available, of course).

The reference period, T_{ref}, is included in the statistical RelReqs through μ and σ , which depend on T_{ref} when Z , A and B , or a_i and b_j are time-dependent. Similarly as it is with the probabilities we have to distinguish β referred to T_{ref} = 1 year, and β referred to T_{ref} = n years. This distinction is often neglected, and, consequently, it becomes unclear which period the target values, β_t, are referred to.

As we already know (see 7.2.2), *equivalent rearrangements* of any reliability margin are possible. However, each of the equivalent Z will give a different value of β_Z. In other words, *the index β_Z is not invariant with respect to the form of the reliability margin,* whereas the failure probability P_f is. This conclusion concerns also the partial reliability indices β_A and β_B related to the formative variables, A and B . For elementary variables this discussion is, of course, meaningless. First, nobody would dare to base proportioning of structures on variation coefficients of elementary variables, and, second, an equivalent rearrangement of a single elementary variable is impossible.

In the following, let us introduce two important reliability indices that can be considered as specific forms of β_Z; for simplicity, subscript Z will be omitted. All conclusions on these reliability indices can be extended also to the partial indices β_A or β_B.

8.5.2 Cornell's index

In 1969 a reliability index has been proposed by C.A. Cornell:

$$\beta^C = Q\mu_Z \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial Z}{\partial \xi_i} \frac{\partial Z}{\partial \xi_j} \sigma_{ij} \right)^{-\frac{1}{2}} \tag{8.21}$$

where $Q\mu_Z$ = quasi-mean of the margin Z [see Equation (2.61)], σ_{ij} = covariance of ξ_i and ξ_j, n = number of elementary variables. For mutually independent elementary variables ξ_i and ξ_j (when for $i \neq j$ all σ_{ij} are zero, and for $i = j$ we have σ_{ij} =

σ_i) the Cornell's index has the form

$$\beta^{C} = Q\mu_Z(\Sigma\delta_i^2)^{-\frac{1}{2}}, \quad i = 1, 2, ..., n \tag{8.22}$$

where δ_i = coefficient of variation of ξ_i.

Because $Q\mu_Z$ depends on the expression for the reliability margin, Z, the Cornell's index is not invariant with respect to $Z \in \dot{Z}_{eqv}$. Under certain circumstances β^C can be reasonably used in the practical design problems; this, however, is without practical importance. We have introduced it here for its historical importance; it has been often referred to in publications on structural reliability.

8.5.3 Hasofer-Lind index

An exceptional contribution to the general development of the probabilistic design was brought by Hasofer and Lind 1974. They formulated a reliability index, now commonly called the ***Hasofer-Lind reliability index,*** or simply *HL-index,* β^{HL}, in the following way.

Let

$$Z = g(\xi_1, \xi_2, ..., \xi_n) \tag{8.22a}$$

be the reliability margin of an S-L-E system. Then, the limit state function

$$g(\xi_1, \xi_2, ..., \xi_n) = 0$$

describes a random *limit state hypersurface* Γ in the coordinate system $[x_1, x_2, ..., x_n]$ and in the *definition domain* Ω_{def} (Figure 8.1). Assume that the population means, μ_i, and standard deviations, σ_i, of the respective elementary random variables ξ_i are known and, further, that the variables are mutually fully independent (it will be shown in 9.2.6 that the β^{HL} method can be employed also for dependent random variables).

Let us standardize the elementary variables ξ_i by means of Equation (2.17):

$$u_i = \frac{\xi_i - \mu_i}{\sigma_i}, \quad i = 1, 2, ..., n \tag{8.23}$$

Setting

$$\xi_i = \mu_i + u_i\sigma_i, \quad i = 1, 2, ..., n \tag{8.24}$$

the function $g(.) = 0$ is transformed to

$$g^u(u_1, u_2, ..., u_n) = 0 \tag{8.25}$$

which in the coordinate system $[u_1, u_2, ..., u_n]$ describes a transformed limit state hypersurface G^u, defined on Ω^u_{def}. The hypersphere G^u divides Ω^u_{def} into two domains: the ***survival domain,*** Ω_s, where $Z^u > 0$, and the ***failure domain,*** Ω_f, with $Z^u \leq 0$.

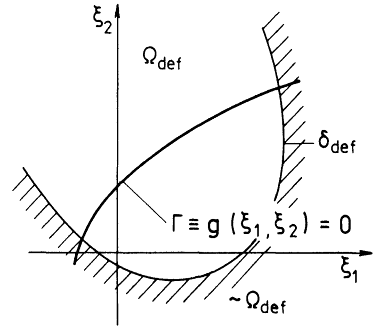

Fig. 8.1 - Random hypersurface Γ (two-component case) and the definition domain Ω_{def}; δ_{def} - boundary of the definition domain.

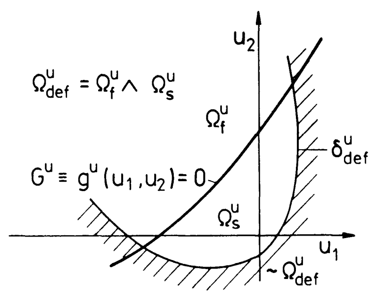

Fig. 8.2 - Transformed function $g^u(.)$ (two-component case) and the definition domain Ω^u_{def}; δ^u_{def} - boundary of the definition domain.

Let only two variables, ξ_1 and ξ_2, be considered; the reliability margin is

$$Z = g(\xi_1, \xi_2) \tag{8.26}$$

and the transformation yields

$$Z^u = g^u(u_1, u_2) \tag{8.27}$$

with $g^u(.) = 0$ being represented by the curve G^u in $[u_1, u_2]$, see Figure 8.2.

Let us now look for the reliability measure which could be used in the assessment of the reliability of a particular system, whose properties are random, described by two population parameters, μ_i and σ_i. The exact values of elementary variables are not known. Nevertheless, we can estimate that all properties of the system are "average," or, in mathematical terms, that each elementary variable is represented by its population mean,

$$\xi_1 = \mu_1, \quad \xi_2 = \mu_2$$

so that

$$u_1 = 0, \quad u_2 = 0$$

Thus, the "average system" belongs to the origin of $[u_1, u_2]$, and the corresponding "average reliability margin," Z^u_{av}, is given by

$$Z^u_{av} = g(\mu_1, \mu_2)$$

that is

$$Z^u_{av} = g^u(u_1 = 0, u_2 = 0) \equiv g(\mu_1, \mu_2)$$

Assume for the moment that the "average system" is in the survival domain, Ω_s.

On the other hand, all notional "failure systems," which can happen in the case investigated, are represented by $Z^u \leq 0$. The most dangerous of these "failure systems" is the one closest to the "average system." The closer these two systems, the greater the danger. The level of danger can be expressed in terms of distance between the points in $[u_1, u_2]$ representing the two systems. In mathematical terms, *the reliability measure of the system investigated is the minimum distance between the origin* O *and the transformed limit state curve* G^u. *This particular distance is the Hasofer-Lind reliability index,* β^{HL} (Figure 8.3). The point on G^u that is nearest to the origin, O, is called ***design point,*** D.

The foregoing consideration can be extended also to the case when the "average system" is in the failure domain, Ω_f. However, the danger grows with increasing distance of the "average system" from the design point. Therefore, similarly as β_Z according to Equation (8.15), also β^{HL} can assume values ≤ 0 or > 0. Assume that the phenomenon H_1 is absolutely favorable [its influence function is $\Lambda_1 = 1$, see Equation (7.6)], and H_2 absolutely adverse ($\Lambda_2 = -1$). The value of β^{HL} becomes zero when G^u passes through the origin of coordinates (Figure 8.4), that is, when $Z^u_{av} = 0$. So, if the origin of coordinates is in the survival region Ω^u_s, then $\beta^{HL} > 0$, if the origin is in the failure region, Ω^u_f, then $\beta^{HL} < 0$, . Observe that *when the sign of* β^{HL} *changes, signs of all coordinates,* u_{id}, *of the design point,* D, *change, too.* The design point, however, must be in the definition domain Ω^u_{def}, otherwise the solution would be meaningless.

These conclusions can be evidently extended to a case where the limit state function depends on n elementary variables. It holds:

$$[g(\mu_1, \mu_2, ..., \mu_n) = 0] \rightarrow (\beta^{HL} = 0)$$

$$[g(\mu_1, \mu_2, ..., \mu_n) > 0] \rightarrow (\beta^{HL} > 0)$$

$$[g(\mu_1, \mu_2, ..., \mu_n) < 0] \rightarrow (\beta^{HL} < 0)$$

The latter two conditions can also be written as

$$[\mathrm{sign}\,(u_{id}) = -\Lambda_i] \rightarrow (\beta^{HL} > 0), \quad i = 1, 2, ..., n$$

$$[\mathrm{sign}\,(u_{id}) = \Lambda_i] \rightarrow (\beta^{HL} < 0), \quad i = 1, 2, ..., n$$

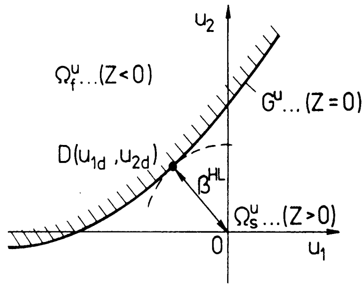

Fig. 8.3 - Definition of the Hasofer-Lind reliability index, β^{HL} , and of the design point, D , in a two-component case.

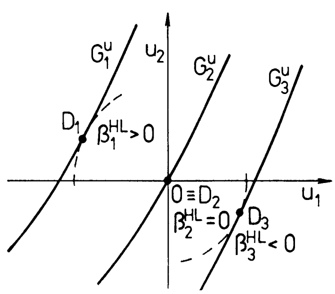

Fig. 8.4 - Hypersurface G^u for β^{HL} greater, equal, or less than zero.

or, in general

$$\operatorname{sign} \beta^{\mathrm{HL}} = \Lambda_i \cdot \operatorname{sign}(u_{id}), \quad i = 1, 2, ..., n \tag{8.28}$$

Obviously, on basis of the foregoing considerations, it is necessary to generalize the definition of β^{HL} stating that β^{HL} is the ***minimum oriented distance from the origin of coordinates to the transformed limit state hypersurface*** G^u .

Note that β^{HL} is identical with $\beta_Z = \mu_Z / \sigma_Z$ and with β^{C} if and only if $g(.)$ is linear. In such a case the hypersurface G^u becomes hyperplane G^u_{hp} , and β^{HL} is equal to the distance from the origin to G^u_{hp} , with the appropriate sign.

9

CALCULATION OF THE FAILURE CHARACTERISTICS

Key concepts in this chapter: *failure characteristics; time factor; dependent variables; invariancy of the failure characteristics; failure probability calculation; simulation-and-estimation technique, S-E; moment-and-estimation technique, M-E; HL-index calculation; hypersphere method, HSM; directional cosines method, DCM; successive approach method, SAM; difficulties with HL-index; first-order second-moment method, FOSM; first-order third-moment method, FOTM; FORM/SORM methods.*

9.1 CALCULATION OF THE FAILURE PROBABILITY

9.1.1 Principal techniques

In the description of the equivalent reliability margins $Z \in \dot{\mathbf{Z}}_{eqv}$, discussed in 7.2.2, the ordered n-tuplet of non-random variables $(x_1, x_2, \ldots, x_n)$ appeared. Let us now assume that all elementary variables entering the expression for Z are *independent random variables,* $\xi_1, \xi_2, \ldots, \xi_n$, which occur in *ordered random n-tuplets* $(x_1, x_2, \ldots, x_n)$ where x_1 through x_n are random outcomes of ξ_1 through ξ_n. In the following, the independence of the elementary variables will be automatically assumed and will not be emphasized any more. For the dependence problems see 2.1.3.

We can write:

$$Z = g(\xi_1, \xi_2, \ldots, \xi_n) \tag{9.1}$$

where again $g(.)$ = limit state function; it is one of the functions that belong to the set of equivalent reliability margins, $\dot{\mathbf{Z}}_{eqv}$.

Since ξ_1 through ξ_n are random, the reliability margin Z is also random (strictly, Z should be read as "uppercase zeta"). Let us suppose that the random behavior of each elementary variable is sufficiently well described by a suitable probability distribution with CDF $\Phi_i(\xi_i)$. For a realization of the elementary variables ξ_i, described by an ordered n-tuplet $(x_1, x_2, \ldots, n)$, the reliability margin Z is either $Z \leq 0$ or $Z > 0$.

As we have shown in 7.2.2, for any two reliability margins Z_i, Z_k that belong to $\dot{\mathbf{Z}}_{eqv}$, Equation (7.9) is valid. From that equation, a simple conclusion results: whenever a sample of m ordered random n-tuplets is analyzed, the number m_{neg} of events

$\mathrm{Ev}(Z < 0)$ is the same for all reliability margins $Z \in \dot{Z}_{eqv}$. This is also true for the number m_0 of events $\mathrm{Ev}(Z = 0)$ and for the number m_{pos} of events $\mathrm{Ev}(Z > 0)$.

Assuming $\mathrm{Ev}(Z \leq 0)$ to be identical with Ev(**failure**), the *failure probability* P_f can be defined by Equation (2.1), that is,

$$P_f \equiv \Pr(Z \leq 0) = \frac{m_{neg} + m_0}{m} \tag{9.2}$$

Evidently, the theoretical failure probability P_f obtained from a mathematico-statistical analysis of the reliability margin does not depend on which of the reliability margins belonging to $\dot{Z}_{eqv}$ is subjected to investigation. In other words, the *probability* P_f *is invariant to the form of the reliability margin.* This fully conforms with the fact that the value of the failure probability P_f is, for a specified system, ***an objective value existing independently of our decisions,*** that is, independently of the way it is established. Consequently, all failure characteristics based on P_f are invariant to Z. In other words, if $Z \in \dot{Z}_{eqv}$, it is not important which formula for Z is used when calculating P_f. Similarly, when using in the design of an S-L-E reliability system one of the equivalent reliability margins referred to a specified random variable, it is not important which of ξ_1 through ξ_n has been taken as the reference variable. Any of the reliability margins of $\dot{Z}_{eqv}$ can be chosen and always the same probabilistic RelReq

$$\forall t \in T_{ref}\colon \quad P_f \leq P_{ft} \tag{9.3}$$

is to be verified. Here P_{ft} = the *target value of the failure probability,* t = point in time, and T_{ref} = reference period during which RelReq has to be complied with. RelReq (9.3) stands for any of RelReqs (8.3), (8.4), and (8.7) through (8.9).

That what holds for the theoretical failure probability does not refer to the *population parameters* of the random variable Z, for example, to the mean μ_Z, standard deviation σ_Z, coefficient of skewness α_Z, or others. Parameters μ_{Zi}, σ_{Zi}, α_{Zi} belong only to $Z_i \in \dot{Z}_{eqv}$; they are not identical with the respective parameters of Z_k, that is,

$$\mu_{Zi} \neq \mu_{Zk}, \quad \sigma_{Zi} \neq \sigma_{Zk} \neq \sigma_{Zk}, \quad \alpha_{Zi} \neq \alpha_{Zk}, \quad \text{etc.}$$

Thus, since the margins differ by their population parameters, they have ***different probability distributions.***

Using Equation (9.2) we can calculated P_f by *Monte Carlo simulation.* The number of trials necessary to get a reasonably accurate result depends on the value of P_f. Observe that the simulated P_f is, in fact, a pseudo-random function of the number of trials, N. As N increases, P_f becomes stable; *monitoring* P_f *during the calculation is helpful.* Various methods reducing processing time are available (see Section 2.4).

With a good accuracy, P_f can be established using an ***estimated probability distribution*** in the following manner:

Let us write the expression for the *standardized reliability margin*

$$u = \frac{Z - \mu_Z}{\sigma_Z} \tag{9.4}$$

Setting for Z from Equation (9.4) into RelReq (7.3) yields

$$\mu_Z + u\,\sigma_Z \geq 0$$

Since

$$\Pr(Z<0) = \Pr(u<0)$$

we can establish, supposing the probability distribution of Z is known or assumed, the failure probability from (see Figure 9.1):

$$P_f = \Phi\left(-\frac{\mu_Z}{\sigma_Z}\right) \tag{9.4a}$$

where $\Phi(.)$ = CDF of the respective probability distribution. The population parameters μ_Z and σ_Z, as well as other possible parameters required for the description of $\Phi(.)$ (coefficient of skewness α_Z, as a rule) can be estimated using either a Monte Carlo simulation (see Section 2.4) or the moment method (see Section 2.3). These procedures can be called ***S-E technique*** (that is, *Simulation-and-Estimation*) and ***M-E technique*** (*Moment-and-Estimation*), respectively. The type of the distribution of the reliability margin has always to be estimated. It is recommended to use the three-parameter log-normal distribution, see Appendix A.

Monte Carlo simulation can be efficiently joint with the HL-index analysis described in Section 9.2 (see, for example, Puppo and Bertero 1992, Sweeting and Finn 1992). *A combination of the two principal approaches can save processing time and improve accuracy.*

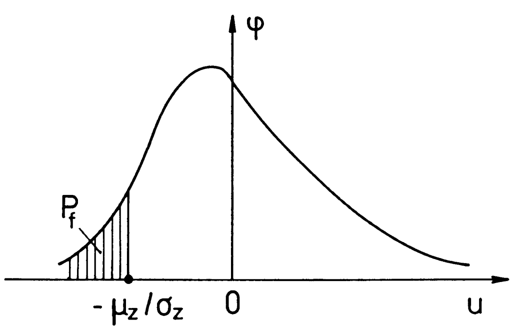

Fig. 9.1 - Determination of the failure probability.

The efficiency of various methods of P_f estimation can be compared using the *efficiency index*

$$\tau = \left|\frac{\varrho_{est}}{\varrho_{sim}} - 1\right| \tag{9.4b}$$

where ϱ_{est} = rho-measure (see Equation 8.10a) related to the estimated failure probability $P_{f,est}$, ϱ_{sim} = rho-measure obtained by plain Monte-Carlo simulation. The closer τ to zero the better the estimate of the failure probability.

The failure probability P_f can of course be found by ***analytical*** or ***numerical integration*** of expressions derived from the multivariate probability distribution of the reliability margin (see, for example, Augusti *et al.* 1984, Ferry Borges and Castanheta 1985, Melchers 1987, Schuëller 1981, Spaethe 1987). However, in practical cases such calculations are complicated and virtually inapplicable.

9.1.2 Time factor in P_f

Time-dependent phenomena must always be respected when the failure probability P_f is calculated. The ways of expressing time relationships depend on the *number of phenomena,* on the *simultaneity of their occurrence,* and finally on the amount of *statistical information* available on each of them.

Let us assume first that *only a single time-dependent phenomenon,* $\mathrm{H}(t)$, occurs in the particular S-L-E system. Let this phenomenon be described by a random variable $\xi(t)$ (for example, the maximum water depth in a reservoir). If, for example, the probability distribution for the interval maxima, x_{max}, of the variable $\xi(t)$, obtained in defined observation periods, T_{obs}, are known (see Section 5.4), and if T_{obs} is not identical with the reference period, T_{ref} ($T_{obs} < T_{ref}$), it is necessary to *derive the probability distribution referred to T_{ref} from the probability distribution based on intervals T_{obs}.*

During T_{ref}, n periods T_{obs} occur (obviously, $n = T_{ref}/T_{obs}$); therefore, to calculate P_f, the CDF

$$\Phi_{ref}(.) = [\Phi_{obs}(.)]^n$$

has to be used (see 2.1.6), whose parameters are $\mu^{(n)}$, $\sigma^{(n)}$, etc. In short: a transformed distribution of T_{ref}-related maxima (or minima) instead of the distribution of T_{obs}-related maxima (or minima) has to be applied in the solution. It suffices, in a simplified solution, to choose for $\Phi_{ref}(.)$ some of three-parameter distributions with $\mu^{(n)}$, $\sigma^{(n)}$, and $\alpha^{(n)}$.

■ **Example 9.1.** Evaluate the failure probability of the system "steel bar & axial load" discussed in Example 7.2. Assume the target life expectancy T_{0t} = 50 years. The parameters of the distributions of elementary variables entering the problem are shown in Table 9.1. Assume further that the distributions of N, d, and f_y are log-normal, LN(α_x), the distribution of N being referred to 50-year maxima. The problem shall be solved by the S-E technique.

For the reliability margins $Z\,|\,d$, $Z\,|\,N$, and $Z\,|\,f_y$ values of sample characteristics m_Z, s_Z, and a_Z were established by a Monte Carlo simulation with N = 10,000. Then, the S-E technique estimates of the failure probability, $\bar{P}_{f,est}$, were found supposing that the reliability margins are log-normally distributed with LN(a_Z). The results are shown in Table 9.2. Also, the probability $\bar{P}_{f,sim}$ according to Equation (9.2) with 100,000 trials was calculated (its values are equal for all $Z|.$). ■

Table 9.1 - *Example 9.1.* Population parameters of elementary variables N, d, and f_y

Variable	μ	σ	α
Axial load, N	90 kN	3 kN	-1.5
Diameter, d	18 mm	0.4 mm	0.5
Yield stress, f_y	0.400 kN.mm^{-2}	0.02 kN.mm^{-2}	1.0

Table 9.2 - *Example 9.1.* Sample characteristics of the equivalent reliability margins, Z, and comprehensive failure probabilities, $\bar{P}_f$, obtained by the S-E technique (N = 10,000) and by plain Monte Carlo simulation (N = 100,000)

Z	m_Z	s_Z	a_Z	$-\frac{m_Z}{s_Z}$	$\bar{P}_{f,est}$ × 10	$\bar{P}_{f,sim}$ × 10
$Z\|d$	1.10	0.64	0.43	1.74	0.273	
$Z\|N$	12.30	7.45	0.61	1.65	0.270	0.271
$Z\|f_y$	0.048	0.03	0.50	1.71	0.264	

When in the solution *several time-dependent phenomena* $H_i(t)$ with various durations and various periods of non-occurrence apply, the problem becomes more complicated. A correct solution cannot be performed without modeling the time-dependent phenomena by random functions or random sequences. This, however, is numerically not difficult.

Some simplification is achieved when probabilities of occurrence, $P_{occ,Hi}$, of the individual phenomena $H_i(t)$ at an arbitrary point in time $t \in T_{ref}$ are known. Then again the distributions of T_{ref}-maxima are employed for $H_i(t)$. In addition, however, the possibility of simultaneous occurrence of the phenomena involved must be taken into account. For this purpose [cf. Equation (2.5)] the probability

$$P_{occ,12\ldots n} \equiv \Pr(H_1 \cap H_2 \cap \ldots \cap H_n) = \prod_{i=1}^{n} P_{occ,Hi} \tag{9.5}$$

is to be applied, with n = number of phenomena. It is assumed that H_i are independent.

The failure probability, P_f, is then established as the probability that the following two events occur simultaneously:

$E_f \equiv$ **Ev[failure of the facility assuming that all phenomena $H_i(t)$ occur simultaneously],**

$E_H \equiv$ **Ev[simultaneous occurrence of all phenomena $H_i(t)$].**

The probability of E_f can be calculated in the same way like the failure probability in a problem with time-independent phenomena. Appropriate distributions of maxima (or minima) referred to T_{ref} must be considered. As for E_H, it is, in this case, identical with $Ph(H_1, H_2, \dots, H_n)$, and thus its probability of occurrence is

$$Pr(E_H) = P_{occ,12...n}$$

Since E_f and E_H are independent, Equation (2.5) yields:

$$P_f = P_{occ,12...n} \cdot Pr(E_f)$$

The meaning of P_f depends on the reference period, T_{ref}. For $T_{ref} = 1$ year, we have $P_f \equiv \dot{P}_f$; for $T_{ref} > 1$, $P_f \equiv P_f$. The probability $P_{occ,12...n}$ does not depend on the duration of the reference period because it is a quantity referred to any point in time, that is, also to the moment of failure. Thus the resulting P_f means either $\dot{P}_f$ or P_f.

9.1.3 Multi-modal failure in P_f

On many practical occasions, M simultaneous possibilities of failure have to be considered, and so M ***partial reliability margins*** can be defined

$$Z^{(k)} = g_{(k)}(x_1, x_2, \dots, x_n), \quad k = 1,2,\dots,M$$

where x_1 through x_n = elementary variables. Note that for certain (k) some of x_i can equal zero, that is, they may not appear in $g_{(k)}(.)$. The investigation of such cases would be easy and straightforward if the elementary variables involved were not random. Then, we could write

$$Z = \min(Z^{(1)}, Z^{2)}, \dots, Z^{(M)})$$

where the superscripts refer to the particular types of failure.

However, this solution can be generalized also to random variable partial reliability margins. When establishing P_f from Equation (9.2) by a Monte Carlo simulation, values of $Z^{(1)}$ through $Z^{(M)}$ are repeatedly calculated. At each trial the least value is found and assigned to Z. The governing failure modes can be different in successive trials. It is not important which of the failure modes (1), (2), ..., (M) are contained in the number $m_{neg} + m_0$ of $Ev(Z \leq 0)$.

For computational difficulties the *M-E technique* cannot be used in multi-modal exercises. The main problem with M-E is in the partial dependencies among the reliability margins $Z^{(1)}$ through $Z^{(M)}$; it can be overcome only with approximations. See Chou *et al.* 1983.

Using the *S-E technique,* we should always plot a histogram of Z when multi-modal problems are solved. It is necessary to check the distribution of Z; the multi-modality of failure can produce an unexpected shape of the frequency curve, and the use of a routine probability distribution could lead to errors in the estimation of P_f.

The identification of all possible failure modes can be a complicated task. Engineering judgment supported by good knowledge of the structural material assumed and loads expected is necessary; various failure modes must be considered to eliminate unlikely ones. In particular, well defined cases, theoretical, probability-based identification is feasible (see, for example, Garson 1980, Rashedi and Moses 1988, Reed and Brown 1992, Thoft-Christensen 1987, Zimmermann *et al.* 1992).

9.2 CALCULATION OF THE HASOFER-LIND RELIABILITY INDEX

We must keep in mind that the transformed limit state function is linear only in exceptional cases. Non-linear problems are frequently dealt with, and the establishing of β^{HL} by "manual means" becomes a tedious and boring task, subjected to calculation errors. At present, three methods are available, which give quick solutions, if programmed appropriately; they will be discussed in the next paragraphs.

9.2.1 Hypersphere method, HSM

The transformed limit state hypersurface G^u is expressed by

$$g^u(u_1, u_2, ..., u_n) = 0 \tag{9.6}$$

Then, a hypersphere S^u_β osculating G^u in the design point, D, can be found, with its center in the origin of coordinates. It is obvious that the point D, whose coordinates are $(u_{1d}, u_{2d}, \ldots, u_{nd})$, must be situated simultaneously on G^u and on S^u_β (Figure 9.2). Then, the radius r_β of S^u_β defines the minimum distance of the hypersurface from the origin of coordinates. According to the definition of β^{HL}, it is

$$r_\beta = |\beta^{\mathrm{HL}}|$$

the sign of β^{HL} being not yet known. A general hypersphere S^u crossing the hypersurface G^u is defined by

$$u_1^2 + u_2^2 + \ldots + u_n^2 = r^2 \tag{9.7}$$

where r = radius of S^u; it is obviously $r \geq r_\beta$.

We are looking for the minimum of r and, simultaneously, for the point D where

$S^u \equiv S^u_\beta$ osculates the hypersurface G^u. The following requirements for the minimum must be satisfied:

$$\frac{\partial r}{\partial u_i} = 0, \quad i = 1, 2, ..., n \tag{9.8}$$

Since the radius r is an absolute quantity, it is possible, for calculation convenience, to search for the minimum of r^2, instead of r :

$$\frac{\partial (r^2)}{\partial u_i} = 0, \quad i = 1, 2, ..., n \tag{9.9}$$

taking Equation (9.6) as the constraint condition.

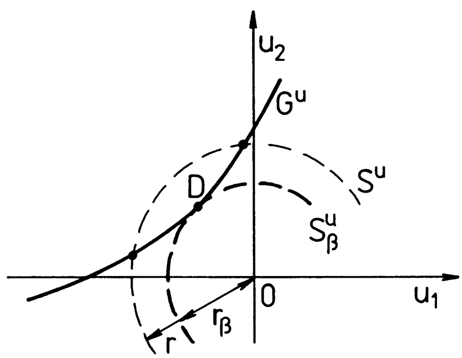

Fig. 9.2 - Determination of β^{HL} by means of the hypersphere method (two-component case); S^u - general hypersphere, S^u_β - hypersphere osculating the hypersurface G^u at D .

Let us assume that one of the variables, u_k, can be explicitly expressed from Equation (9.6). It has to be emphasized that this assumption is only auxiliary. The expression g^u(.) is often such that no explicit formula can be written for any of the elementary variables.

Differentiate Equation (9.7) with respect to u_i; the following relationships are obtained for $\partial(r^2)/\partial u_i$:

$$\frac{\partial (r^2)}{\partial u_i} = 2u_i + 2u_k \frac{\partial u_k}{\partial u_i}, \quad i = 1, 2, ..., n \tag{9.10}$$

so that the requirements for the minimum are:

$$u_i + u_k \frac{\partial u_k}{\partial u_i} = 0, \quad i = 1, 2, ..., n \tag{9.11}$$

When in the environment of the design point, D , the *partial derivatives of* g^u (.) *are continuous* and the *function* g^u (.) *is differentiable,* it is, according to the rules valid for derivatives of implicit functions,

$$\frac{\partial u_k}{\partial u_i} = -\frac{\partial g^u / \partial u_i}{\partial g^u / \partial u_k}, \quad i = 1, 2, ..., n \tag{9.12}$$

Now, substituting for $\partial u_k / \partial u_i$ into Equations (9.10) we get

$$u_i - u_k \frac{\partial g^u / \partial u_i}{\partial g^u / \partial u_k} = 0, \quad i = 1, 2, ..., n \tag{9.13}$$

Obviously, for $i = k$ the identity is obtained, and therefore only n-1 Equations (9.13) for n unknowns are available. In order to establish $|\beta^{HL}| \equiv r$ a system of n equations, comprising Equations (9.13) and Equation (9.6), must be solved for unknowns u_1 through u_n; as a rule, iteration solutions must be applied. Solving this system, we obtain the *coordinates of the design point,* D , that is, u_{1d} through u_{nd}. Then r_β follows from Equation (9.7) with $u_i = u_{id}$. Observe that it appears from Equation (9.13) that no variable need be explicitly expressible. Theoretically, it is not important which of the elementary variables u_k is considered explicitly expressible. It can happen, nevertheless, that when selecting different u_k as governing variables, different results, referred to different *local extremes* of r , are obtained.

It remains to establish the *sign of* β^{HL}. As it was already explained in 8.5.3, the sign is governed by the position of the origin O with respect to the transformed limit state hypersurface G^u. When the origin is situated in the survival domain, Ω_s^u, then sign β^{HL} = +1, when it is in the failure domain, Ω_f^u, sign β^{HL} = -1. In order to find the position of the origin, the value of the quasi-mean of the transformed reliability margin, Z^u, has to be calculated. It is [see Equation (2.66)]

$$Q\mu_Z^u = Q\mu_Z \tag{9.14}$$

where $Q\mu_Z$ results from Equation (8.22a) after setting $\xi_i = \mu_i$ for all i . It is then:

- for $Q\mu_Z > 0$: sign $\beta^{HL} = +1$
- for $Q\mu_Z < 0$: sign $\beta^{HL} = -1$

Finally, the equation for β^{HL} is:

$$\beta^{HL} = \text{sign} Q\mu_Z \left(\sum_{i=1}^{n} u_{id}^2 \right)^{\frac{1}{2}} \tag{9.15}$$

When for some particular reason also the position of the design point, **D**, shall be determined, it is necessary to verify the signs of the respective u_{id} coordinates. For any of the coordinates, the following equation must be true:

$$\operatorname{sign} u_{id} = A_i \operatorname{sign} Q \mu_Z \tag{9.16}$$

where A_i = influence function defined by Equation (7.6).

It suffices to verify Equation (9.16) for only one of the coordinates. When it is satisfied, the signs of all coordinates are correct, otherwise they all have to be changed.

9.2.2 Directional cosines method, DCM

This method has been used for calculation of β^{HL} from the very beginnings. It was thoroughly described several times in numerous publications (see, for example, Ang and Tang, vol. II 1984, Madsen *et al.* 1986, *Rationalisation CIRIA* 1977, Schuëller 1981, Thoft-Christensen and Baker 1982, Smith 1986, Spaethe 1987) and it will not be discussed in detail here. Let us only show its principal idea.

In DCM n values of directional cosines $a_i = \cos \alpha_i$ and simultaneously the value of β^{HL} are established from a system of $n + 1$ equations

$$a_i = \frac{\partial g^u}{\partial u_i} \cdot \left[\sum \left(\frac{\partial g^u}{\partial u_i}\right)^2 \right]^{-\frac{1}{2}}, \quad i = 1, 2, ..., n$$
$$g^u(u_1, u_2, ..., u_n) = 0$$

where $u_i = -a_i \beta^{HL}$. Whereas in HSM β^{HL} is calculated separately from Equation (9.15), and thus the number of equations in the system is not increased to $n + 1$, in DCM the index becomes an additional unknown of the equation system.

As in HSM, the sign of β^{HL} is found from

$$\operatorname{sign} \beta^{HL} = \operatorname{sign} Q \mu_Z \tag{9.17}$$

For the *signs of coordinates of the design point* Equation (9.16) is valid again.

9.2.3 Successive approach method, SAM

The method, defined by Fiessler 1979, consists in successive approaching to the design point, **D**, starting from an arbitrary point $\mathbf{p}_0$ described by an ordered n-tuplet (u_{10}, u_{20}, ..., u_{n0}). For a transformed two-component limit state function, $g^u(u_1, u_2) = 0$, the idea of SAM is geometrically shown in Figure 9.3:

♦ A value $Z^u = g^u(u_1, u_2)$ refers to each point **p** in $[u_1, u_2]$. When Z^u is plotted at every point **p**, a surface Z^u is obtained (not shown in the figure).

♦ An arbitrary starting point p_0 is chosen, and a curve $G_0^u \equiv g^u(u_{10}, u_{20}) = Z_0^u$ passing through p_0 is found (Figure 9.3a); u_{10}, u_{20} = coordinates of p_0.

♦ A normal line, N_0, is drawn to G_0^u, and a planar section through the surface Z^u, passing N_0, is determined (Figure 9.3b).

♦ At the point where the ordinate of the Z^u surface is equal Z_0^u, a tangent, T_0, is drawn. This tangent intersects N_0 at p_0' at a distance Δu from the starting point, p_0.

♦ Turn now back to Figure 9.3a. A straight line, t_0, perpendicular to N_0 is constructed in p_0', and a point, p_0'', on t_0 that is *nearest to the origin of coordinates* is found. Obviously, t_0 is, for the time being, an *assumed* linear approximation of $g^u(u_1, u_2) = 0$ at p_0''.

♦ Using p_0'' as a new starting point, p_0^{new}, the next p_0'' is found in the same manner as above. The procedure is repeated until $Z_{0''}^u = 0$ is achieved with some acceptable error (or, in other words, until $p_0' \equiv p_0'' \equiv D$).

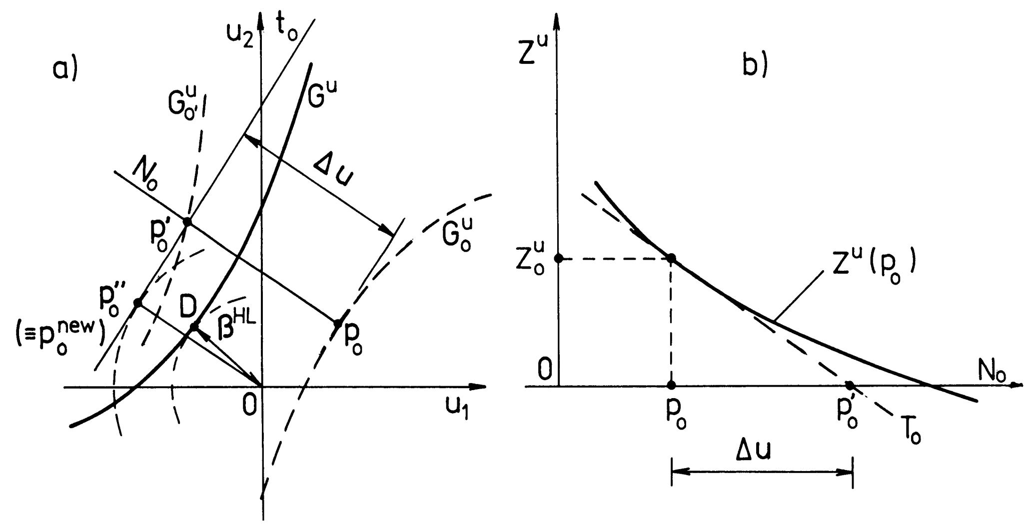

Fig. 9.3 - Iteration procedure in the determination of β^{HL} by means of SAM (**a** - iteration steps, **b** - development of Z^u along the normal, N).

No solution of a system of equations is necessary in SAM; we only have to solve $n+1$ *separate equations:*

$$u_i = -\frac{\partial g^u}{\partial u_i} \cdot \frac{1}{Q\sigma_Z^u} \left(\frac{g^u(.)}{Q\sigma_Z^u} + \beta^{HL} \right), \quad i = 1, 2, ..., n$$

$$\beta^{\mathrm{HL}} = \operatorname{sign} Q\mu_Z \left(\sum_{i=1}^{n} u_i^2 \right)^{\frac{1}{2}}$$

where $Q\mu_Z$ = quasi-mean of the reliability margin, Z, and $Q\sigma_Z^u$ = quasi-sigma given by Equation (2.62). With $\sigma_{u1} = \sigma_{u2} = \dots = \sigma_{un} = 1$ it is

$$Q\sigma_Z^u = \left[\sum_{i=1}^{n} \left(\frac{\partial g^u}{\partial u_i} \right)^2 \right]^{\frac{1}{2}} \tag{9.18}$$

During the iteration, the distance Δu is approaching β^{HL} and $Z^u \equiv g^u(.) \to 0$. Since $\mu_{ui} = 0$, the partial derivatives have to be taken at successive starting points, $\mathbf{p}_0$.

9.2.4 Difficulties with calculation of β^{HL}

Computer solutions must be applied as soon as the expression for the reliability margin is only a little complicated; this happens in the absolute majority of practical cases. Manual calculations are unworkable and extremely demanding even in very simple, textbook cases. Computer programs for β^{HL} or for the design point coordinates are available, though they can have diverse shortcomings. The most frequent source of problems is the rooted idea that always $\beta^{\mathrm{HL}} > 0$, because the failure probabilities are expected to be very small. Negative values of β^{HL} are mentioned in the available publications only exceptionally (Leira and Langen 1981, Casciati and Faravelli 1991). As a rule, values $\beta^{\mathrm{HL}} > 1.5$ are expected. But, in design, we can *currently encounter* $\beta^{\mathrm{HL}} \le 0$

- when trying to find a correct variant of the solution,
- in the ***assessment of an existing system,*** which can often be undersized,
- in the design according to ***serviceability limit states,*** where the target failure probabilities can be sometimes greater than 0.5.

Because for $\beta^{\mathrm{HL}} = 0$ the failure probability is $P_f \approx 0.5$ (when $g(.)$ is linear and the distributions of the elementary variables are symmetrical, then $P_f = 0.5$ exactly). Therefore, it cannot be stated, in general, that for $\beta^{\mathrm{HL}} = 0$ the system is fully unreliable.

It can thus easily happen that an insufficiently tested computer program gives $\beta^{\mathrm{HL}} > 0$, though the effective value is negative. Therefore, it is always necessary to verify whether the result complies with the logic of the particular case.

Further, programs based on HSM or DCM must contain *subroutines for the solution of a system of non-linear equations.* Such subroutines are found in the software libraries of any computer, but they can be based on various principles. The iteration procedures are intricate, as a rule, and can lead to untrue results, even when the initial estimate of u_{id} or β^{HL} is very close to the exact solution. In general, the calculation of β^{HL} can finish in any of the following possible ways:

(a) β^{HL} obtained appears to be *logical* and is *correct;*

(b) β^{HL} obtained appears *logical*, but it is *incorrect*, as the iteration leads to a local extreme of the transformed limit state function;
(c) β^{HL} obtained is clearly *illogical;*
(d) solution is *singular;*
(e) solution *does not converge.*

There is no need to stress that an outcome type (b) is extremely deceitful. Unfortunately, no rules on what result is or is not logical cannot be suggested. The decision on this issue must be founded only on the designer's judgment.

■ **Example 9.2.** Consider the reliability margin

$$Z = 1 - x^2 - y$$

where the elementary variables have population parameters

$$\mu_x = 0, \qquad \sigma_x = 0.2$$
$$\mu_y \in [0,\ 1.4], \qquad \sigma_y = 0.2$$

The definition domain Ω^u_{def} is for any μ_y bounded by

$$u_x \geq -1, \quad u_y \geq -2.5$$

Transforming according to Equation (8.24), we obtain

$$\sigma_x^2 u_x^2 + 2\,\mu_x \sigma_x u_x + \sigma_y u_y + \mu_x^2 + \mu_y - 1 = 0$$

Geometrically, this represents a parabola whose position can be shifted by adjusting μ_y (Figure 9.4).

Performing some analysis, it can be shown that for $\mu_y < 0.5$ the index β^{HL} is described by a skew radius vector, whereas for $\mu_y \geq 0.5$, β^{HL} is given by the distance of the parabola vertex from the origin of coordinates. *When using HSM and DCM with a particular iteration algorithm for different initial estimates of the variables, all five, that is, (a) through (e), outcomes mentioned above were obtained.* An unfavorable outcome, that is, (b) to (e), was reached only exceptionally, but with both methods. For example, at $\mu_y < 0.5$, solution using DCM supplied, as the result, the distance vertex-origin, because at vertex a local extreme of the distance between the origin and the transformed limit state curve $\mathbf{G}^u$ exists. For $\mu_y = 0.4$ the correct value of β^{HL} is 2.9 (skew radius vector), but we obtained $\beta^{HL} = 3.0$; using HSM the correct result was reached.

The same case solved by SAM ended in either (a) or (e), since, owing to the approach procedure, the outcomes type (b) and (c) are less frequent, and the outcome (d) is not possible at all, no system of equations being solved in SAM. ■

The following ***recommendations*** based on practical experience can be given for the calculation of the β^{HL} index:

(1) use *all three methods* simultaneously;
(2) repeat solution for *different initial estimates of the input variables;*

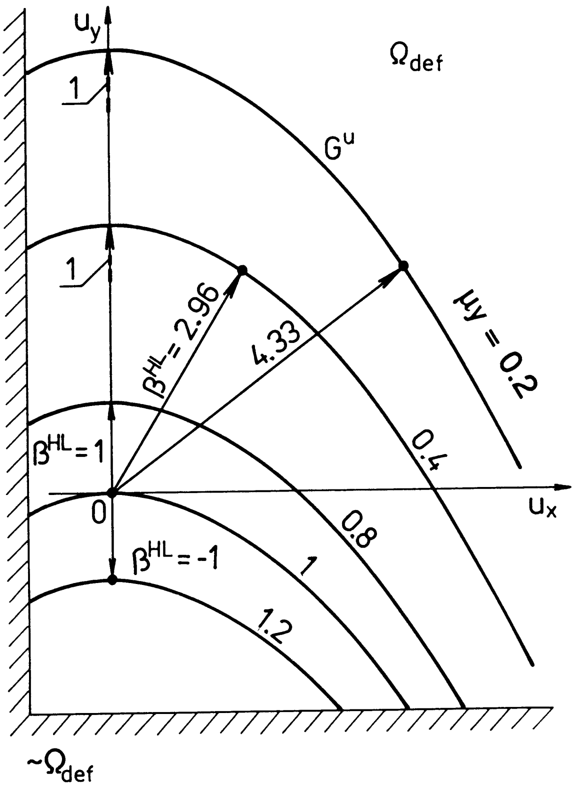

Fig. 9.4 - *Example 9.2.* Determination of β^{HL} (1 - possible but incorrect results).

(3) verify the *logic of the results:*

♦ check whether the values of the *influence functions,* A_i, related to the design point correspond with the assumption on the influence of the individual elementary variables,

♦ check whether the *design point,* D, is in the definition domain Ω^u_{def}.

With a good iteration algorithm, the three methods demand only a small number of iteration steps. Usually, the least number is needed for HSM, the greatest for SAM, but this is not a common rule. It is quite easy to verify several variants of initial estimates. However, the number of iterations is not important in most cases.

Writing a computer program for the calculation of β^{HL} is a relatively simple exercise. Nevertheless, you have to keep in mind all the above peculiarities (and perhaps also others, not yet encountered) in order to avoid possible pit-falls. At present, good software is commercially available [for example, COSSAN (see Bucher *et al.* 1989), PROBAN 1991], which supplies unambiguous and correct solutions. The users of any software should, however, get always acquainted with its theoretical background and of course with the principles of the HL-index method in general.

9.2.5 Time factor in β^{HL}

Time-dependent phenomena can be treated during calculation of β^{HL} in a quite analogous way as in the calculation of P_f.

Population parameters $\mu^{(n)}$ and $\sigma^{(n)}$ referred to the assumed number, n, of occurrences of an event E_i belonging to $H_i(t)$ during the reference period T_{ref} shall be introduced into Equations (8.24). Because in the calculation of β^{HL} the coefficient of skewness does not apply, a certain amount of existing information on the elementary variables is not used, however.

The solution becomes practically unworkable for a large number of time-dependent phenomena $H_i(t)$. When the value of the reliability index is needed for the evaluation of a statistical RelReq, the failure probability P_f must be first established, and then the corresponding ***generalized reliability index*** (also *Ditlevsen index;* see Ditlevsen 1979) defined, in principle, by

$$\beta^G = -\Phi_N^{-1}(P_f)$$

shall be calculated. It is then assumed that $\beta^{HL} \approx \beta^G$. Here, Φ_N^{-1} = IDF of the normal distribution.

Provided we are able to calculate the probability of simultaneous occurrence of all phenomena $H_i(t)$, that is, $P_{occ,\ 12 \dots n}$ [see Equation (9.5)], we can establish β^{HL} approximately from

$$\beta^{HL} \approx -\Phi_N^{-1}[P_{occ,\ 12 \dots n} \cdot \Phi_N(-\beta^{HL*})]$$

where β^{HL*} = index calculated from the assumption that all phenomena occur simultaneously (that is, disregarding their intermittency and covering their time-dependent behavior by respective population parameters). It is apparent that in time-dependent problems probability-based concepts cannot be escaped.

9.2.6 Dependent variables in β^{HL}

Any dependence of variables entering the reliability margin formula can be easily treated by considering a corresponding statistical dependence function [see 2.1.3, Equation (2.27)]. Solutions using this approach are simple and more general than solutions based on known or assumed covariances of multivariate distributions.

9.2.7 Multi-modal failure in β^{HL}

While the multi-modality of failure does not bring substantial difficulties when the failure probability, P_f, is calculated by Monte Carlo simulation, problems encountered in the calculation of β^{HL} are serious.

A straight solution would be possible only for perfectly independent failure modes. In such a case the reliability index would be simply

$$\beta^{HL} = \min_{k=1}^{M}(\beta^{(k)})$$

with $\beta^{(k)}$ = HL-index calculated for mode (k), M = number of failure modes.

Unfortunately, such an ideal situation is virtually never met; the failure modes are dependent (see 9.1.3). Therefore, when information on the reliability of an S-L-E system is to be obtained in terms of reliability index, the *generalized reliability index* must be used again (see 9.2.5; Madsen *et al.* 1986).

9.3 ESTIMATE OF P_f BASED ON β^{HL}

9.3.1 First-order second-moment method

We are interested in the relationship between various beta indices and the failure probability of the respective system. When the value of a β_Z index according to Equation (8.15) is known, the failure probability is given by [cf. Equation (9.4a)]

$$P_f = \Phi(-\beta_Z) \tag{9.19}$$

where $\Phi(.)$ = CDF of the standardized probability distribution referred to the considered reliability margin Z. For individual equivalent reliability margins, which belong to $\dot{Z}_{eqv}$, the probability distributions are different (and also β_Z are different) but P_f does not change (see 9.1.1). As a rule, however, functions $\Phi(.)$ are approximations, and so for different $Z \in \dot{Z}_{eqv}$ we often obtain slightly differing values of P_f.

When

(a) the reliability margin is *linear* with respect to its elementary variables,

and

(b) the distributions of all elementary variables are *jointly normal,*

then

$$P_f = \Phi_N(-\beta_Z) \tag{9.20}$$

or, since in this case $\beta_Z \equiv \beta^{HL}$,

$$P_f = \Phi_N(-\beta^{HL}) \tag{9.21}$$

where $\Phi_N(.)$ = CDF of the standardized normal distribution.

Owing to the lack of invariance of β_Z, Equation (9.20) cannot be applied if any of the assumptions (a) or (b) is not satisfied. However, because β^{HL} is invariant to $Z \in \dot{Z}_{eqv}$, Equation (9.21) is considered "acceptably good"; thus, if (a) and (b) are "almost fulfilled," it can be written

$$P_f \approx \Phi_N(-\beta^{HL}) \tag{9.22}$$

In general, we can say that $P_f \in [P_{f1}, P_{f2}]$ where P_{f1} = lower bound, P_{f2} = upper bound of the failure probability. These bounds depend on properties of the limit state function $g^u(.)$ and on the random behavior of the elementary variables. Let us show here the range of P_{f1} and P_{f2} for the case when the distributions of all elementary variables are normal:

(1) When for $\beta^{HL} > 0$ the limit state hypersurface, G^u, is ***convex*** with respect to the origin of coordinates, it holds

$$P_{f1} = 0, \quad P_{f2} = \Phi_N(-\beta^{HL})$$

(2) when for $\beta^{HL} > 0$ the hypersurface G^u is ***concave*** with respect to the origin, it is

$$P_{f1} = \Phi_N(-\beta^{HL}), \quad P_{f2} = 1 - \Phi_{\chi^2(n)}[(\beta^{HL})^2]$$

where $\Phi_{\chi^2(n)}$ = CDF of the chi-square distribution with n degrees of freedom. It appears from P_{f2} for the concave case that, in general, P_f depends, among others, on the number n of the elementary variables.

The method of estimation of P_f based solely on Equation (9.22) is referred to as "first-order second-moment method," FOSM, since the first-order members of the Taylor series expansion, and the first and second moments of the probability distribution of Z^u apply in the calculation of the reliability index. FOSM is considered "distribution-free" because probability distributions of elementary variables do not appear in the solution.

9.3.2 First-order third-moment method

In practical design, *asymmetric variables* are frequently encountered. For example, the probability distribution of the yield stress of structural steel often has a coefficient of skewness, α, greater than 0.5; distributions of load maxima referred to life expectancy often have α much smaller than zero, etc. Considerable errors can be committed by neglecting the asymmetry of the respective variables. The HL-index method can be easily adjusted to such variables as well.

Let us assume that, additionally to μ_i and σ_i, the *coefficients of skewness,* α_i, of the respective probability distributions Φ_i of variables ξ_i are known. It is, nevertheless, not necessary to know these distributions in all details. Then, using Equations (2.62) and (2.63) with $\sigma_{ui} = 1$ and taking into account the fact that $\alpha_{ui} = \alpha_i$, the *quasi-alpha of the transformed reliability margin,*

$$Q\alpha_Z^u = (Q\sigma_Z^u)^{-3}\sum_i g_i^{u3}\alpha_i \tag{9.23}$$

can be calculated. Here, $Q\sigma_Z^u$ = quasi-sigma of Z_u, see Equation (9.18). The first partial derivatives of $g^u(.)$ shall be referred to the design point, D , that is to the point ($u_i = u_{id}$) for all i . This results from the substitution of the limit state function in D by a tangent hyperplane.

Now, we can develop the P_f -estimate according to Equation (9.22) writing

$$P_f \approx \Phi_a(-\beta^{HL})$$

where Φ_a = CDF of an asymmetric probability distribution, having the coefficient of skewness, α , as the third parameter. In such a distribution $Q\alpha_Z^u$ from Equation (9.23) is approximately taken for α .

Good results have been obtained when using the three-parameter log-normal distribution, LN(α), see Appendix A. That is

$$P_f \approx \Phi_{LN}(-\beta^{HL}) \tag{9.24}$$

where Φ_{LN} = CDF of LN($Q\alpha_Z^u$). Of course, any other "reasonably shaped" three-parameter probability distribution can be employed.

It can be proved that, similarly as β^{HL} , also ***the quasi-alpha is invariant to the reliability margin*** $Z \in \mathring{Z}_{eqv}$.

Because the first, second, and third moments of the probability distribution of Z^u enter the solution, the procedure using the quasi-alpha is termed "first-order third-moment," FOTM.

■ **Example 9.3.** Evaluate the failure probability of the "steel bar & axial load" system discussed in Example 7.2. Assume the target life expectancy T_{0t} = 50 years. The distributions of elementary variables entering the problem have population parameters shown in Table 9.1. Assume further that the distributions of N, d, and f are log-normal, LN(α_x), the distribution of N being referred to 50-year maxima. The problem should be solved using the FOTM method.

Based on the foregoing procedures and formulas, calculations have yielded the following results:

$$\bar{\beta}^{HL} = 1.623, \quad Q\alpha_Z^u = 0.542$$

and further, using the FOSM method:

$$\bar{P}_f \approx \Phi_N(-\bar{\beta}^{HL}) = 0.0523$$

and the FOTM method:

$$\bar{P}_f \approx \Phi_{LN}(-\bar{\beta}^{HL}) = 0.0326$$

From a Monte Carlo simulation we obtained $\bar{P}_{f,sim}$ = 0.0271 , see Table 9.2.

In terms of the rho-measure [see Equation (8.10a)] we get

$$\varrho_{FOSM}^{HL} = 1.282, \quad \varrho_{FOTM}^{HL} = 1.486, \quad \varrho_{sim} = 1.567$$

and the efficiency indices according to Equation (9.4b) are

$$\tau_{FOSM} = 0.182, \quad \tau_{FOTM} = 0.052$$

Evidently, the FOTM estimate of $\bar{P}_f$ is closer to the result obtained by simulation than the FOSM estimate, based solely on normal distribution. In an extensive testing of FOTM, no case of $\tau_{FOTM} > \tau_{FOSM}$ was found. ■

Again, FOTM can be used for statistically dependent variables with a statistical dependence function introduced into the calculation model (see 2.1.3).

9.3.3 FORM/SORM methods

While FOTM does not affect the principles of the calculation of the HL-index, there is a large family of methods treating the non-normality of the input elementary variables by specific transformations of the respective distributions. These methods are known as ***FORM*** (*first-order reliability methods*) or ***SORM*** (*second-order reliability methods*), or jointly FORM/SORM. Survey of these methods has been given, for example, by Ayyub and Haldar 1984, Bjerager 1991, and Shinozuka 1983. Therefore, let us introduce here only their main principles.

The common feature of FORM and SORM consists in transforming the probability distributions of non-normal (symmetric or asymmetric) elementary variables into normal ones by appropriate transformation patterns. Then, the means and standard deviations of the transformed distributions are found and the calculation described in Section 9.2 is performed.

This technique results, in fact, in mapping $g^u(.)$ onto a transformed system $[u_1, u_2, \ldots, u_n]^*$. The transformed $[g^u(.)]^*$ can have a shape that substantially differs from $g^u(.)$.

The most simple transformation is based on the following technique. Instead of using Equation (8.23), $g^u(.)$ is established through

$$u_i = \Phi^{-1}[\Phi_{non}(\xi_i)] \tag{9.25}$$

where $\Phi_{non}(\xi_i)$ = CDF of the non-normal elementary variable ξ_i, $\Phi_N^{-1}[.]$ = IDF of the standardized normal distribution.

This principle can be extended to dependent variables; in such a case the ***Rosenblatt transformation*** (see Rosenblatt 1952) can be used when the joint probability distribution of the set of random variables is known. Conditional probability distributions apply in the solutions which means that the procedure depends on the order in which the transformations are performed (see, for example, Casciati and Faravelli 1991). Many particular techniques have been developed from this approach.

Finally, let us mention here the ***Rackwitz-Fiessler algorithm*** (Rackwitz and Fiessler 1976). It consists in the substitution of all non-normal variables ξ_i by normal variables ξ_i^{N} in such a way that at the design point, D, the following two requirements are satisfied:

$$\varphi(x_{id}) = \varphi_{\mathrm{N}}(x_{id}), \quad i = 1, 2, \ldots, n \tag{9.26}$$

$$\Phi(x_{id}) = \Phi_{\mathrm{N}}(x_{id}), \quad i = 1, 2, \ldots, n \tag{9.27}$$

Here, x_{id} = coordinates of D; $\varphi(.)$ and $\Phi(.)$ = original PDF and CDF; $\varphi_{\mathrm{N}}(.)$ and $\Phi_{\mathrm{N}}(.)$ = substitute normal PDF and CDF; n = number of variables. For those ξ_i that are normally distributed, the distribution adjustment is irrelevant.

Then, in the calculation of β^{HL}, μ_i^{N} and σ_i^{N} of the *substitute normally distributed variable,* ξ_i^{N} are repeatedly established in the successive iteration steps. These parameters are used in the mapping according to Equation (8.24); it is set:

$$\xi_i^{\mathrm{N}} = \mu_i^{\mathrm{N}} + u_i \sigma_i^{\mathrm{N}}, \quad i = 1, 2, \ldots, n$$

The distribution adjustment according to Equations (9.26) and (9.27) is based on the assumption that $\varphi(x_{id}) > 0$ and $\Phi(x_{id}) \in (0, 1)$. For large $|\beta^{\mathrm{HL}}|$, when with some probability distributions we can have $\varphi(x_{id}) = 0$, $\Phi(x_{id}) = 0$, or $\Phi(x_{id}) = 1$, this transformation may not yield results.

10

RELIABILITY PARAMETERS

Key concepts in this chapter: *reference period; life of CF; life expectancy; target failure probability; values of CF; tangible values; intangible values; losses; target reliability index; cost function; reliability differentiation of CF; differentiation multiplier; importance factor; constraint; deflection; crack width; vibrations; strain load-effect.*

In the reliability requirements outlined in Section 8 two principal parameters determining the reliability level of an S-L-E system appear:

♦ the ***target reference period,*** $T_{ref,t}$, which is usually taken as the value of the target life of the respective CF, T_{0t};

♦ the ***target failure probability,*** P_{ft}, in its annual form, $\dot{P}_{ft}$, or comprehensive form, P_{ft}.

The values of $T_{ref,t}$ and P_{ft} cannot be derived from the physical properties of the S-L-E system. They have to be determined by *decisions* based on opinions and on needs of individuals, groups, or social entities, supported by economic analyses and, particularly, backed by experience gained with similar facilities. Obviously, we deal here with autonomous ***primary quantities,*** which have in the structural reliability theory the significance comparable to that of the *Prime Rate* and *Money Supply* in free market economies. By deciding on the target life and target failure probability, the society - represented by qualified groups of experts - takes on *responsibilities for the amount and consequences of possible failures.* Such decisions are not simple since many aspects have to be pondered. The principal aspect is, without any doubt, the *importance of the facility for an individual or the society.*

10.1 VALUES OF CONSTRUCTED FACILITIES

10.1.1 Two systems of values

Values that determine the importance of CFs can be classified into two basic groups:

- *tangible values,*
- *intangible values.*

(1) The background to ***tangible values*** is economic. In the main, we are first interested

- in the *initial value,* V_0, which is, as a rule, specified by *initial costs,* C_0, spent on the materialization of the facility, and further,
- in the *utility value,* V_{ut}, which, as a rule, reflects a *general economic,* not emotional, *attitude of individuals or groups to the facility.*

These tangible values are summarily expressed by the *market price,* which is based on the initial and utility values. Price of CF can be easily termed in *monetary units.* It deviates from the initial value, since the utility value can strongly affect the result according to the actual state of *demand and offer,* including short-run or long-run regional influences, and also according to the physical condition of the facility itself. Market prices of buildings and structures are based mainly on *open market values, depreciated replacement cost values,* and *revenue-based values,* taking into account general economic and political situation (inflation, recession, etc.). Price includes also the *influence of intangible values,* which creates the main difficulties in the importance assessment of CFs. Though on many cases tangible values dominate, the principles of *property valuation* cannot be applied in the importance assessment.

(2) On many occasions ***intangible values*** are the only governing criteria of importance of CFs. A large set of various values and their modifications can be found among intangible values . As a rule, the following basic values are considered:

- *psycho-physical value,* reflecting the biological attitude of an individual to a facility; this value might be also called *gratification value,* as it expresses the amount of contentment, pleasure, satisfaction, or other positive feeling of an individual with regard to the facility (for example, feeling of family members toward their house);
- *emotional value,* based on the relation of an individual or a group of individuals to a facility (for example, relation to a national monument);
- *moral value,* originating in community feeling toward a facility (for example, the relation of people to a nursery school building);
- *cultural and political value,* expressing the attitude of large social groups (nations, humankind) to CFs of artistic, historic, or political importance (cf. buildings in Venice, Italy); obviously, a particular class of moral values is dealt with;
- *strategic value,* expressing the importance of a facility for national economy and defense (cf. bridges, large dams, electricity plants);

♦ *aesthetic value,* which is a special form of the cultural and political value.

None of these values can be expressed in monetary units, but some *psychometric evaluation* is possible.

The intangible values are *invaluable;* they do not stem from the inner structure and utility features of a facility, but from the fact that the facility has become carrier of certain social and economic bonds. So, the intangible values are very difficult to be measured, more often they are simply not measurable at all. Except some special cases, methods of evaluation are not available in the construction domain. Another problem, specific for intangible values, is their *non-uniform distribution* - a certain facility may have diverse levels of cultural, moral and other values for different individuals, social groups, or even nations.

The factor that substantially influences all values, tangible or intangible, is the *time,* in two main aspects:

(a) *Properties of a facility are changing* - the facility is ageing, and, as a result, particular values either increase or decrease. The change may not be proportional at all intangible values; for some values it can be even contrary-going.

(b) *Properties of the social subject are changing* - the social situation fluctuates (for example, economic conditions), and also attitudes of individuals or groups are not constant. Under extreme conditions (natural catastrophe, war) an *extinction of values* is possible, though the facility can survive such events.

Values often depend upon the *type of possible fault;* this becomes apparent when *consequences of the facility failure* are considered. This also refers to intangible values - a minor fault can leave emotional and moral values of a facility intact. Many intangible values depend upon the *appraisers.* For example, a one-family house has emotional value only for its owners or users. This follows from the fact that intangible values are usually based on some relation between the facility and the appraiser. In a sample of "facility-appraisers," *random deviations* of values can be expected.

Until now, efforts to find a *comprehensive value of CF* have usually failed for two reasons:

♦ sufficient data necessary for the appraisal have not been available (not even for tangible values);

♦ the ways of evaluating *human life* or its loss have not yet been established.

Two extreme views can be met:

♦ human life has a certain, limited economic value, and

♦ human life stands beyond any valuation and cannot be included in any value analysis.

It is remarkable, though not surprising, that in discussions on this issue "young engineers" (up to the age of about 50) support the first opinion, whereas the "elderly" favor the second. The latter is based on general principles of humanity but, unfortunately, it does not correspond to the actual social facts. Human lives have been always subjected to valuations at different levels and from diverse viewpoints.

The efforts in finding a value of an individual or group of individuals concentrate on economic aspects of the problem. This happens in the area of various types of insurance, in the assessment of training costs, etc. The principles of assessment are not uniform, they depend on many factors and change from country to country. As far as the CF systems are concerned, some basic investigations have been already started (see, for example, Lind 1991, Lind *et al.* 1991, Needleman 1982).

In the assessment of values we also have to take into account the ***losses***, L, that arise as an effect of a fault. Some losses are related to respective values. Some are, yet, independent and their magnitude is a function of many complex factors (for example, function of the legal system that prevails in the area where the facility is situated).

10.1.2 Cost function

Neglecting the intangible values or intangible losses a *cost function* can be written for any CF:

$$\mathrm{C} = \mathrm{C}_0 + \mathrm{C}_m + \sum_k \bar{P}_f^{(k)} \mathrm{C}_D^{(k)} \tag{10.1}$$

where C = comprehensive costs joined with the existence of the facility and referred to its life T_0, C_0 = initial costs of the facility, C_m = costs of maintenance and repairs expected during T_0, $\mathrm{C}_D^{(k)}$ = costs ensuing from a possible failure k, $\bar{P}_f^{(k)}$ = comprehensive failure probability referred to the failure k during T_0. The costs $\mathrm{C}_D^{(k)}$ can be expressed by

$$\mathrm{C}_D^{(k)} = \alpha_0^{(k)} \mathrm{C}_0 + \alpha_{ut}^{(k)} \mathrm{V}_{ut} + \sum_j \mathrm{L}_j^{(k)} \tag{10.2}$$

where $\alpha_0^{(k)}$, $\alpha_{ut}^{(k)}$ = coefficients describing material consequences of failure k, referred to values $\mathrm{V}_0 \equiv \mathrm{C}_0$ and V_{ut}, respectively; $\mathrm{L}_j^{(k)}$ = economical losses caused by the failure k that are independent of $\mathrm{V}_0 \equiv \mathrm{C}_0$ and V_{ut}. No information is available on α_0 and α_{ut}. For serviceability failures we may, for example, have $\alpha_0 \approx 0$, for ultimate failures $\alpha_0 \geq 1$, etc.

Observe that, except C_0, all quantities appearing in Equations (10.1) and (10.2) are *time-dependent.* Their development is practically impossible to forecast. Therefore, efforts to derive target failure probabilities by means of cost functions bring no results, though at the outset of probability-based design much was promised.

10.2 TARGET LIFE

The life of a constructed facility can be defined as a distance between two points in time: the moment of *erection* of the facility, $t = 0$, and the moment of its *demolition,* $t = t_{dem}$, Though this definition seems clear enough, it is not sufficient since t_{dem} depends upon various factors, and, therefore, must be specified in more detail.

The demolition of a facility can be caused by various circumstances. Basically, two types of demolition can be distinguished: foreseeable and unforeseeable. A *foreseeable demolition* is expected by both the designer and the owner, and it is, in some implicit way, contained in the design, economic assessment, etc. On the other hand, *unforeseeable demolitions* are, as a rule, not considered at all, although it is commonly known that such demolitions can prevail over the foreseeable ones. Five principal reasons for demolition can be identified, and accordingly, five variants of the ***effective life***, $T_{0,eff}$, can be defined, Table 10.1. Although in general all factors governing the particular life variants can be considered random (even human decisions are subjected to randomness), only $T_{0,mt}$, $T_{0,ph}$, and $T_{0,bd}$ can be treated as random variables.

The ***effective life*** of a facility is obviously given by the minimum of the life variants shown in Table 10.1,

$$T_{0,eff} = \min(T_{0,mt},\ T_{0,ph},\ T_{0,ut};\ T_{0,bd}, T_{0,sc})$$

In an effort to find the value of the *target life,* T_{0t}, this formula is not too helpful since each of the life variants is governed by substantially differing factors, as shown in Table 10.1.

It is well known that the actual failure rate of buildings and structures (see 2.2.1), or in other words, the *incidence of random demolitions,* is very small. Therefore, $T_{0,mt}$ does not appear in the designer's or owner's considerations (it is, however, not neglected in the mechanical or electrical reliability engineering). Similarly, owners and designers do not assume, in their decisions, any *unforeseeable demolitions,* defining either $T_{0,bd}$ or $T_{0,sc}$. Thus only the physical life, $T_{0,ph}$, and the utility life, $T_{0,ut}$, remain as a basis for determining T_{0t}.

At the time of design, the value of $T_{0,ph}$ is unknown and it must be estimated from various properties of CF and its environment (material properties, load properties, corrosive ambience, etc.). The value of $T_{0,ut}$ can often be specified more or less exactly but it is frequently exceeded, because of several reasons. Then, two viewpoints can be held in establishing the value of the target life:

- ♦ the *owner* should specify the ***required life*** of the facility, $T_{0,req}$, based mainly on $T_{0,ut}$;
- ♦ the *designer* should assume an ***expected life*** (or also *life expectancy*) $T_{0,exp}$, based either on $T_{0,ph}$ or on $T_{0,ut}$; if the designer relies on the owner's $T_{0,req}$, he or she usually takes $T_{0,exp} > T_{0,req}$ with a certain safety margin for the magnitude of which, however, no guidance exists up to now.

During the use of CF the circumstances considered by the owner or designer can change, and so it finally becomes

$$T_{0,eff} \le \text{ or } > T_{0,req}, \qquad T_{0,eff} \le \text{ or } > T_{0,exp}$$

Table 10.1 - Lives of constructed facilities

Type of demolition	Reason for demolition	Lives	Definition of T_0 or t_{dem}
Foreseeable	***Random irreversible foreseeable failure*** of CF	*Mathematical life,* $T_{0,mt}$	$T_{0,mt}$ = inverse of the failure rate, λ, cf. 2.2.1
	Physical wear of CF	*Physical life,* $T_{0,ph}$	t_{dem} = point in time when maintenance and rehabilitation costs exceed an acceptable level
	Economic wear of CF (its further existence is not necessary)	*Utility life,* $T_{0,ut}$	t_{dem} is decision-based
Unforeseeable	***Random or non random irreversible unforeseeable failure*** of CF caused by critical flaws or aberrations in the structure, load, and environment, and resulting in breakdown of CF	*Break-down life,* $T_{0,bd}$	t_{dem} is given by the breakdown situation
	Social wear of CF	*Social life,* $T_{0,sc}$	t_{dem} is given by general economic situation, urban planning, political decisions, etc.

Of course, the influence of random phenomena can result in

$$T_{0,mt} < T_{0,req}, \quad T_{0,mt} < T_{0,exp}$$

When establishing values of the target life, T_{0t}, expert opinion, as well as economic considerations, must be used; such an approach was used by the Author. During an investigation of the problem, 46 outstanding civil engineers from different fields of construction in former Czechoslovakia gave their estimates of $T_{0,ph}$ and $T_{0,sc}$ for various types of CFs. Some results of the inquiry are shown in Table 10.2. No economy experts were involved at this stage, and so it can be said that values of $T_{0,ph}$ are, in fact, close to the designer's $T_{0,exp}$. For simplicity, the table shows round-off sample means only. The sample range of opinions, however, was surprisingly narrow for most types of facilities.

In the next step of the solution, economic criteria were taken into account, considering the depreciation periods T_{dep} specified for buildings and structures in a Czech legal document. Assuming that the owner's $T_{0,req}$ should be by 20 to 30 percent greater

Table 10.2 - The values of life (years) obtained from the opinions of civil engineers

Constructed facility	Material	Life	
		$T_{0,ph}$	$T_{0,sc}$
Residential buildings	Masonry	110	60
	Concrete	120	70
Single-story industrial buildings	Concrete	90	45
	Steel	60	35
Highway bridges	Concrete	110	55
	Steel	80	40
Gravity dams	Concrete	260	300
	Earth	220	200
Grain silos	Concrete	100	80
	Steel	50	70
Tanks	Concrete	85	70
Chimney stacks	Masonry	85	80
	Concrete	90	70
	Steel	30	45
Cooling towers	Concrete	90	45
	Steel	40	20
Weekend chalets	-	55	30

Table 10.3 - Guidance values of the target life, T_{0t}, specified in the Czech code ČSN 73 0031-88 (years)

Constructed facilities	T_{0t}
Buildings	
residential	100
industrial	60
mining	50
power plants	30
agricultural	50
hydrotechnics	80
temporary	15
Structures	
towers	40
tanks, bunkers	80
bridges	100
highways, general structure	100
rigid surface	25
non-rigid surface	15
railroads, general	120
bed	40
dams	120
tunnels, underground facilities	120

than the respective depreciation period, and using also the expert inquiry results, values of target life, T_{0t}, were finally established, Table 10.3.

The life expectancy is only slowly getting embedded in the regulatory documents. At present, 50 years are often taken as a reference period in many calculations, though the actual life expectancy can be shorter or longer. A detailed analysis of the life expectancy problem is contained in the Draft British Standard *Guide to Life Expectancy* 1988. Important information on life issue can be found in Bennett 1989, Bolotin 1984(1989), De Kraker *et al.* 1982, Hognestad 1991, Sentler 1987, *The Design Life* 1991.

10.3 TARGET FAILURE PROBABILITY

The second principal reliability parameter, often considered basic, is the *target failure probability,* P_{ft}. Much attention has been paid to values of P_{ft}, but results have been rather poor until now. Tables of suggested values of P_{ft} are shown in various general codification documents, but a common consensus on P_{ft} has not yet been reached. Evidently, the problem of the target failure probability is more intricate than it looks, and it is definitely more complicated than that of the target life. Whereas T_{0t} is a meaningful, independent quantity, which, in a way, is *testable* and can be *verified by experience,* P_{ft} is a value that is difficult to conceive and check by common designers, contractors, or many other participants of the construction process. Further, *whenever a value of* P_{ft} *is given, the calculation model, or rather a complex system of calculation models, supplying the failure probability* P_f *must be defined simultaneously.* It has been graphically shown (see Grimmelt and Schuëller 1982) that even for very simple, textbook structures, with clearly defined properties, subjected to clearly defined actions, a wide spectrum of P_f values can be obtained by using different calculation methods. The reason for the observed discrepancies is obvious: *any calculation model proposed at present is only a very rough approximation of the actual behavior of the system.* - On the other hand, since no connection between T_{0t} and the calculation model exists, the T_{0t} is never model-dependent.

All considerations of this section can be extended also to the target rho-measure, ϱ_t.

Among code makers, there exists a natural psychological reluctance to give definite values of P_{ft} and to accept the idea that a certain number of CFs will fail. This reluctance to fix P_{ft} strengthens with the growing potential damage consequence of a failure.

For these reasons, any recommended values of P_{ft} must be viewed with utmost caution and always within the context of the complete set of factors affecting the reliability. This fact, nevertheless, should not prevent us from discussing briefly several ways of arriving at P_{ft} values.

It is recalled that *two levels of target failure probabilities* need to be specified: one for *serviceability failures,* and the other for *ultimate failures.* The P_{ft} values for these levels can differ by many orders of magnitude. This is obviously due to the well known attitudes of the public, regarding the two types of failure. As a guidance it can be said that for serviceability limit states the summary value P_{fts}, referred, for example, to T_{0t} = 70 years, is between 1.0E-1 to 1.0E-3, whereas for ultimate limit states it should be considerably less, say, $P_{ftu} \in$ (1.0E-5; 1.0E-8). However, it must be kept in mind that very small probabilities, that is, 1.0E-6 and less, are very, very vague, intestable numbers (cf. on this issue, though in a substantially different environment, Feynman 1989 on investigating the space shuttle Challenger disaster).

Four fundamental methods of fixing P_{ft} can be distinguished.

Recalculation method

The recalculation method consists in the analysis of an existing system by means of an established probability-based method. Then, the value of the failure probability, P_f, resulting from this solution is considered just equal to P_{ft} for that particular system. If a large set of systems is subjected to such analyses, a set of P_{ft} values is obtained and, after some considerations, the most acceptable value of P_{ft} is applied in the design of future facilities, or in the derivation of design parameters for codes. Clearly, this method is based on ***experience*** with CFs that have already been in current use, and, in this way, P_{ft} depends upon these facilities themselves. The method can be applied for both the ultimate and the serviceability limit states. Unfortunately, it is obviously model-dependent.

The principles of the recalculation approach have been widely used in the ***calibration of design parameters*** (partial reliability factors for load and material, and others) when new codified design formats have been introduced. Detailed information can be found in Augusti *et al.* 1984, Ghosn and Moses 1986, Lind 1971, Madsen *et al.* 1986, Murzewski 1988, and Melchers 1987.

Analogy method

The analogy method is based on the *evaluation of other phenomena of a catastrophic nature* (see, for example, Hooper 1978, Kinchin 1978). Therefore, the ultimate failure probability, P_{fu}, is studied by this method. It is suggested that the target value for ultimate limit states, P_{fut}, should be derived, for example, from the comprehensive probability (that is, referred to the human life expectancy) that an individual would be accidentally killed on his or her way home from the work place. The comprehensive probability that a person will die because of a railway accident is about 1.0E-7, that he or she will be killed during a highway accident is 1.0E-2, that he or she will be killed on his or her way home from work 1.0E-7, the annual probability that a building will be damaged by fire is 2.1×1.0E-4. Then, it is recommended, for example, to base the target ultimate failure probability on the accidental death probability during a railway journey, which is accepted by the population without any knowledge of its actual value. Using this or similar approach, different authors arrive at target values of the comprehensive ultimate failure probability, P_{ft}, of the order between 1.0E-2 and 1.0E-6, or even 1.0E-9. Now, when we want to generalize this way of thinking, we discover that there is no basis for an analogous analysis referred to the serviceability failure probabilities. For example, a target failure probability related to the occurrence of cracks in prestressed concrete members cannot be derived with such an approach.

A mutation of the analogy method is the establishment of P_{ft} or also of the partial probabilities referred to individual phenomena in such a way that the respective event - collapse, crack occurrence - *should never occur during the life of CF.* This approach can be used, for example, in connection with fatigue problems. The reciprocal value of the failure probability at the end of life of the structure should be greater than the expected number of loading cycles. With a similar concept, target values P_{ait}, P_{bit} in the design method based on extreme values are established. For example, it is required that a strength of concrete less than the design strength shall "practically never" develop. According to some opinions, it is sufficient to take P_c = 1.0E-3.

Many other possibilities have been offered (see, for example, Bennett 1989, Kuhlmann 1985). Several drawbacks of the analogy method can be shown, but, on the other hand, the method gives enough space for engineering judgment.

Discomfort method

Any defect or fault in CF creates uneasy feelings amongst users and owners. An appropriate analysis of the attitudes of individuals or groups in assumed or real failure situations can lead to reasonably well-founded, model-independent target failure probabilities. Methods of *attitude evaluation* are elaborated by applied psychology, but their use in establishing the target failure probabilities is rather uncommon.

The discomfort method becomes particularly suitable when the definition of the respective limit state is fuzzy (deflection limit state, crack-width limit state, and others). It is not of too much help, however, in the domain of ultimate limit states.

■ **Example 10.1.** Consider a hypothetical building with 1000 rooms. The building is used by 1000 persons, each person being allocated to one room at random. Assume that one of the persons is sensitive to any crack in the ceiling, while no cracks are ever registered by any of the remaining persons. Obviously the event Ev(**sensitive person in a particular room**) is considered. Assume further that also Ev(**occurrence of cracks in a particular ceiling**) is a random event.

The floor slabs have been designed exactly so that the comprehensive value of probability of first crack occurrence in a slab during the life of the building is $\bar{P}_{rt}$ = 1.0E-3. Obviously, $\bar{P}_{rt}$ is the target probability of occurrence of an adverse attack, $\bar{P}_{At} \equiv \bar{P}_{rt}$ (though here the attack is not expressed in numerical terms).

Now, the probability that a particular room will shelter the crack-sensitive person is

$$\bar{P} = \frac{1}{1000} \equiv 1.0\text{E-}3$$

which, in fact, is the probability of occurrence of the adverse barrier (the random variable barrier is defined by the sensitivity of persons), that is, $\bar{P}_B \equiv \bar{P}$.

Since in this case A and B are independent discrete random variables that can only have either YES or NO value, the serviceability failure probability is

$$\bar{P}_{f1} = \bar{P}_{At} \cdot \bar{P}_B = 1.0\text{E-}6$$

Assume now that next to the building there is an entrance to the subway. The reinforced concrete frame is visible and it has been designed for the same first-crack probability, 1.0E-3. All 1000 users of the building, including the sensitive one, walk through the subway entrance. If a crack in the frame occurs, it is surely registered by the sensitive person, and so $\bar{P}_B$ = 1. Thus, the failure probability is

$$\bar{P}_{f2} = 1.0\text{E-}3 \times 1 \equiv 1.0\text{E-}3$$

The discomfort of the public is substantially different in both cases. In the building only a single user will feel uneasy because of the crack, whereas all people passing through the entrance will get aroused with a probability 1.0E-3 (the sensitive one will tell the colleagues about it) with a probability 1.0E-3. Consequently, if for the two facilities the same level of reliability should be achieved, the concrete frame should be designed for $\bar{P}_{n}$ = 1.0E-6. ■

At CFs used by public the discomfort is greater, and levels of reliability higher than those for facilities used by individuals or small groups have to be applied.

Optimization methods

The optimization methods are the most exact of the methods aiming at the target failure probabilities: they usually deal with economic analysis of a CF system in time. The costs of design, execution, quality control, and maintenance of CF, and also damage caused by possible failures of the facility, are combined into an appropriate ***objective function*** where separate P_{ft} values for possible modes of failure must appear as variables. Then, as objectives of the solution, the values of P_{ft} can be found by *minimizing total costs,* under defined constraints. The objective function can also be written in terms of energy, or material consumption, or in terms of losses due to failures only, but the optimization principle remains *economic* (see Needleman in *Technological Risk* 1982).

The optimization idea is simple: a properly formulated cost function (see 10.1.2) is taken as a basis of the analysis. The concept is clear from the theoretical viewpoint but the practical solution is too complicated and it can be hardly accomplished with our present possibilities. Besides, it is model-dependent. Therefore, the method can be used only for particular facilities, its use for entire classes of structures being still in theoretical space.

The merits of the method are rather eclipsed by the simple fact that human lives, in civilized societies, and under normal conditions, cannot be subjected to optimization (see 10.1.1). Whenever ultimate failures that may involve loss of lives are captured by the objective function, the whole solution becomes doubtful. This problem is dealt with by proposals for minimizing the *mortality rate* due to structural failures and determining P_{ft} under such an approach (Rüsch and Rackwitz 1973). It is evident that this technique cannot be used for the serviceability failures.

A generalization of the economic and mortality optimization approach is the *minimization of risk* resulting from the use of the facility (Rosenblueth 1987). Also this method can be applied in special situations only, as, for example, in design of nuclear facilities, off-shore structures, and similar unique and well defined cases.

The use of optimization analyses for common structures can be considered difficult and economically little efficient. Until now the respective methods have not escaped bounds of textbooks. The main trouble is not in the optimization procedures, which are now attainable with present software, but in calculation models.

Considering all possible methods, we could reach to a wide spectrum of $\dot{P}_{ft}$ or $\bar{P}_{ft}$. Fortunately, *the sensitivity of design parameters to P_{ft} values is small,* and so large imprecisions are not too harmful. For ULSs values of $\dot{P}_{ft}$ in the range 1.0E-5 to 1.0E-7 are given, for SLSs from 5 × 1.0E-3 to 2 × 1.0E-4 (see, for example, *Grundlagen zur festlegung* 1981). Some authors give even larger ranges. It must be emphasized that those are *guidance values for code makers* who will use them for developing the respective design parameters needed in current calculations.

We always must be cautious when using any suggested value of target failure probability. The reader should be warned to rely on values where no time reference is given and where the calculation model properties are not outlined.

10.4 RELIABILITY DIFFERENTIATION

10.4.1 Differentiation possibilities

In the design of constructed facilities, structures, or members it is often required to differentiate reliability according to various criteria. In the main, the following *differentiation categories* are suggested:

(a) differentiation between the two limit states groups (see 1.3.1). This differentiation is already implicitly contained in the input reliability parameters, P_{ft} or β_t^{HL};

(b) differentiation among limit states of one group. For example, we want to distinguish the limit states of brittle and ductile fracture;

(c) differentiation of bearing members according to their significance for the stability and robustness of the entire structure;

(d) differentiation according to the level of design elaboration, according to the quality of calculation models, etc.;

(e) differentiation according to the level of workmanship and inspection during the execution of the structure, and according to the expected level of maintenance;

(f) differentiation according to accessibility and repairability of bearing structures;

(g) differentiation according to the design situations (see 1.3.3), or also according to the stages of construction or of use;

(h) differentiation according to social and economic importance of CFs.

Although these differentiation categories are evidently diverse, no particular differences can be found among them as far as the technique of determining the design parameters is concerned. Therefore, we will study only the category (g), which is often subjected to discussion among experts and engineering public as well.

10.4.2 Differentiation of constructed facilities

Because CFs have different tangible and intangible values, and defects and faults of facilities are a source of loss, different importance should be attributed to different facilities. This idea has been well known for long time and nothing is basically new in it. Yet, the problem of the *importance quantification* is relatively recent. Essentially, two possibilities exist:

♦ *verbal classification of facilities* into several classes with assigned reliability levels and design parameters,

♦ *vector description of importance* based on the analysis of losses, **L**, mentioned in 10.1.1, with a functional assignment of reliability levels and design parameters.

In current practice only the first technique is used because no methods have been formulated for the vector description; until now no data are available. In the main, verbal classifications are based on classification of governing factors into *two groups: tangible* and *intangible,* in conformity with the pattern of values attributed to CF (see 10.1.1). In each group *subclasses*, detailed more or less, are defined. Then, the importance of a facility is expressed by a combination of two subclasses, and the facility is included into the respective *reliability class.* - At present, verbal classification is specified in most basic documents that treat the importance problem (*General principles JCSS* 1982, *Grundlagen zur Festlegung* 1981, Otstavnov *et al.* 1981, and others). We can say that the situation is, from the practical viewpoint, stabilized. The following classification prevails:

(a) Subclasses according to tangible values

(a1) ***Limited economical loss;*** for example: single floor buildings, greenhouses, farm silos, communication poles, fences, open or partially closed stores of raw materials.

(a2) ***Large economical loss;*** for example: individual apartment houses, industrial buildings, stores of products and equipment, tall chimney stacks, buildings of railway stations, railway and highway bridges, large capacity silos.

(a3) ***Very large economical loss;*** for example: standardized apartment houses, main buildings of industrial entities, TV towers, main utility networks and associated structures, bridges on main communications, subway structures, facilities with particular equipment, grand stands of large open stadia.

(b) Subclasses according to intangible values

(b1) ***Human life is only exceptionally in danger during a failure of the facility;*** for example: greenhouses, underground silos, electrical towers, communication poles, stores of shipping and delivering facilities, transport structures in industrial facilities, aerial masts outside residential areas.

(b2) ***Human life is currently in danger;*** for example: apartment houses, industrial buildings with permanent staff, TV towers in residential areas.

(b3) ***Many lives are in danger;*** for example: grand stands, theaters, dancing halls, supermarkets, railway stations, subway facilities, schools, bridges, nuclear plants, dams.

Classification of facilities into subclasses is highly subjective and it is not possible to avoid overlapping, indefiniteness, and ambiguities. In general, there are nine combinations of subclasses (a) and (b). Taking into account the differentiation objectives, some of these resulting classes are equivalent from the reliability quantification viewpoint, and, therefore, the number of classes is usually confined to four according to the pattern in Table 10.4.

Table 10.4 - Classification of constructed facilities based on differentiation according to tangible and intangible values

Subclasses according to tangible values	Subclasses according to intangible values		
	b1	b2	b3
a1	L	M	N
a2	M	M	N
a3	N	N	H

Requirements on facility's reliability:

H - high, N - normal, M - medium, L - low

Considering Table 10.4 in more detail, we can find that *the importance of CF depends on the purpose which the facility is expected to serve.* This can be shown on many examples. The dependence of importance upon purpose is particularly distinct in situations where CF was changing its use during past periods. We might give examples of many heritage buildings and structures that went through several stages of importance, even through the stage of *negative importance,* when the facility faced demolition.

To each class of CFs, classified according to importance or purpose, target values of annual or comprehensive failure probabilities, $\dot{P}_{ft}$, $\bar{P}_{ft}$, can be assigned (dependent on the type of RelReq considered, that is, subjective RelReq or facility RelReq, see 8.1.1). This is possible to base on failure probability P_{ft}^{R} related to a *reference class,* R, for example, the class of most frequent facilities, and to write

$$P_{ft}^{\mathrm{K}} = m^{\mathrm{K}} P_{ft}^{\mathrm{R}} \tag{10.3}$$

where m^{K} = ***differentiation multiplier.*** We will see later (in 13.4.2) how this multiplier can be used in the derivation of design parameters. Clearly, the multiplier m^{K} must have different values for $\dot{P}_{ft}^{\mathrm{K}}$ and $\bar{P}_{ft}^{\mathrm{K}}$, that is, $\dot{m}^{\mathrm{K}}$ and $\bar{m}^{\mathrm{K}}$. If, for the description of reliability, the rho-measure (see 8.1.4) were used, then of course the differentiation multiplier would be transformed to *differentiation supplement* $\Delta\varrho = \log m$. From the theoretical viewpoint the solution is of course identical.

Table 10.5 - Differentiation multipliers $\dot{m}^{K}$, $\bar{m}^{K}$ (example of values; reference class R ≡ N)

Class	Combination of subclasses	$\dot{m}^{K}$		$\bar{m}^{K}$ 1)	
		ULS	SLS2)	ULS	SLS2)
H	a3 & b3	0.1	0.5	0.1	0.5
N	a1 & b3 a2 & b3 a3 & b1 a3 & b2	1	1	1	1
M	a1 & b2 a1 & b1 a3 & b2	10	2	9.96	1.45
L	a1 & b1	100	5	96.15	1.81

1) Values of $\bar{m}^{K}$ were obtained from $\dot{m}^{K}$ assuming that

$$\dot{P}^{N}_{ft,\mathrm{ULS}} = 1.0\mathrm{E}\text{-}5, \quad \dot{P}^{N}_{ft,\mathrm{SLS}} = 1.0\mathrm{E}\text{-}2$$

and the life expectancy T_{0t} = 80 years.

2) See the remark on differentiation problem in Section 10.5.

When using a verbal classification of facilities, a table of differentiation multipliers can be developed. Of course, multipliers cannot be equal for both groups of limit states, ULSs and SLSs, as it must always be $P^{K}_{ft} < 1$. A possible set of differentiation multipliers is shown in Table 10.5.

At present, many differentiation solutions based on verbal classifications are available. Their nature is analogous, the individual solutions differ in formal aspects, especially in the way of treating the differentiation in design. In the majority of present structural design codes the *importance factor,* γ_n, is employed (Murzewski 1985a, 1985b, Tichý 1985). It is associated with either loads or resistances. To avoid unsafe or uneconomical design, the technical use of γ_n must be always thoroughly described.

Let us present some typical *regulatory documents* covering the facility differentiation. Obviously, it is not possible to introduce all codes; there are now many available, covering the differentiation problem.

ANSI A58.1-1982: The importance of facilities is respected in the calculation of wind load, snow load, and seismic load; four classes are distinguished according to the facility's purpose. The importance factor, γ_n, is considered from 0.8 to 1.5; it is used to multiply stress load-effects.

BS 5502:1980: Four importance classes of facilities are distinguished, the criterion being life expectancy, number of people working in the buildings, and also danger to third persons or properties. The distance of the facility from residential areas and public roads governs the latter criterion. Values of γ_n from 0.85 to 1.0 are assigned to the four classes.

ČSN 73 0031-88: Four purpose classes are distinguished, and a detailed classification of facilities is given. Rules for introducing γ_n (values from 0.8 to $\geq$ 1, the upper limit being not defined) into calculations are specified. Stress load-effects are factored.

Eurocodes: The possibility of differentiation is suggested, but no detailed rules or values are given.

Grundlagen DIN 1981: Three classes of facilities with different values of target failure probabilities, P_{ft}, or Hasofer-Lind reliability indices, β_t^{HL}, are given; γ_n values are derived for these classes in the range of 0.9 to 1.2.

Guidelines of NKB 1987: Three importance classes and three *inspection classes* are defined; γ_n ranges from 0.9 to 1.1.

SNiP II-50-44 1975: Four importance classes are specified. The γ_n factor (1.1 to 1.25) is used to divide a calculated ultimate capacity.

It has to be noted that *a facility differentiation lacks sense when the importance or the purpose of the facility is already covered by design parameters.* For example, in the verification of deflections of floors in residential buildings no differentiation should be used because it is already embedded in the limit deflection f_{lim}.

The technique of the differentiation multiplier can be efficiently applied also in the other differentiation problems [except the problem (a), where the differentiation has already been included in current design procedures for a long time]. The differentiation multiplier makes a *clear comparison of diverse cases* possible, taking one of the cases as a reference case.

10.5 CONSTRAINTS

When the barrier, B, appearing in RelReqs (7.1), (7.2), or others, represents environmental properties, or expresses, in some way, relation between the STRUCTURE and the ENVIRONMENT systems, it is usually defined in terms of a specific reliability parameter called ***constraint,*** C. Consequently, RelReq (7.2) can be written as

$$\forall t \in T_{ref}:\ A \leq C \tag{10.4}$$

As a rule, constraints relate to *strain load-effects;* that is, the attack, A, is given in terms of strain, deflection, slope, rotation, width of cracks, etc. Vibration parameters can also be included into this family; see below.

Table 10.6 - Examples of static deformations evaluated in design

Deformations	Aspects (examples)
Deflection	aesthetic aspects; operation of technological equipment
Sway	integrity of partition walls in frame structures subjected to horizontal forces
Slope of the bending line	movement of cranes on gantry beams; draining of water from flat roofs; comfortable movement of vehicles along highway bridges; lateral stability of partition walls
Axial deformations	reliable function of building equipment (elevators, piping, wiring)
Rotation of adjacent cross-sections	assembling of precast structures
Curvature of bending line or of bending surface	integrity of ceilings
Strain	integrity of cladding and tiling

In the majority of cases, constraints are scalars, specified by *fixed, decision-based values*. Constraint values that have been established in existing design codes have been derived in various ways. At onset of codified design, most of C were based on traditions; nobody could offer any scientific justification for the respective magnitudes. Now, the situation has been slowly changing, since statistical and probability concepts, the system of reliabilistic thinking, and last but not least, practical needs have brought new ideas into the constraint issue. Therefore, as for constraints, modern codes become open-minded, and allow or even encourage the designer to adjust values given in the respective code clauses whenever it is advantageous. Thus, occasions when designers themselves are compelled to specify a constraint value, are getting more and more recurrent. It then happens that the designer, having reached the conclusion that a constraint RelReq should be verified in the particular situation, founds the available design code unsatisfactory, as for the information given. Then, the designer has to answer *two questions:*

♦ What should be the physical meaning of the constraint C, or in other words, what design criterion should govern the RelReq?

♦ What should be the value of C?

Let us illustrate the main features of the general constraint problem on the case of *static deformations*. Table 10.6 shows some types of deformation that are frequently verified in design. It is well known that in a general case several RelReqs (10.4), written for *various deformation criteria,* have to be checked. Only in simple cases, such as floor beams, floor slabs, etc., a single deflection check is sufficient. Note that the list given in Table 10.6 is far from complete!

In an SLS design, we must not forget that deformations shall be checked for *various stages of the construction process,* not only for the stage of current use. Further, we must keep in mind the *time-dependencies* involved: first, those related to load, then those related to material (including soil), and finally also the time-dependence of the constraints themselves. The latter is usually underestimated; actually, some constraint can govern the design RelReq in initial periods of the existence of the facility, and can be entirely ignored later.

It is now acknowledged that constraints are, in general, random variables, or more exactly, that they can be established by statistical analysis of aspects that determine their values. This can be shown by Example 10.2.

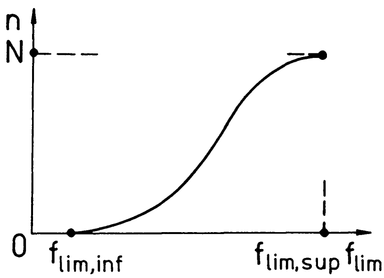

Fig. 10.1 - *Example 10.2.* Number of alarmed visitors *vs.* deflection of floor in a lecture hall.

■ **Example 10.2.** A lecture hall is regularly visited by a group of N individuals. Owing to time-dependent properties of the bearing structure, the deformation of the floor grows with time. Let us take the mid-span deflection, f, as deformation criterion. At a certain value of f one of the regular visitors becomes disturbed and begins to be suspicious about the safety of the structure. Obviously, the respective value of f is the visitor's *personal constraint,* f_{lim}. When the deformation continues to grow, the number of alarmed visitors, n, increases (Figure 10.1). At each lecture, additional Δn visitors will observe the dangerous deflection (let us assume that sensitive visitors' worries are not transferrable). The alarm process is discrete, though the deformation can increase continuously; however, the periods when lectures are given are discrete. Probability that a randomly selected visitor will get annoyed by $f \leq f_{lim}$ is

$$P = \frac{n}{N} \qquad \text{(a)}$$

probability that a randomly selected visitor will get annoyed just when f_{lim} has been achieved is

$$p = \frac{\Delta n}{N} \qquad \text{(b)}$$

Obviously, each individual has a *personal barrier,* whose exceedance arouses his or her discomfort. As psychological and emotional properties of humans are random, the limit deflection, f_{lim}, is also a random variable. Considering a very large population of individuals, Equations (a) and (b) can be written as

$$P = \Phi(f_{lim})$$
$$p = \varphi(f_{lim})$$

See Figure 10.2. Consequently, if the probability distribution of f_{lim} were known, we could find for an intended probability P_{lim} the value of admissible deflection, f_{adm}, from

$$\Pr(f_{lim} \le f_{adm}) = P_{lim}$$ ■

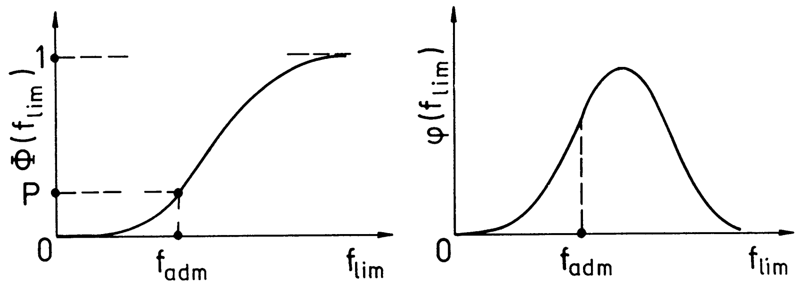

Fig. 10.2 - *Example 10.2.* CDF and PDF of limit deflections related to different persons.

Unfortunately, experimental information on random behavior of constraints is still very scarce or nil. This fact compels to establish values of constraints, called often "admissible deflections," "admissible crack width," etc., on empirical considerations. When no guidance on constraints can be found in codes or other documents, the designer should ask qualified people acquainted with the problem area for advice. *For example,* we can get

♦ from *civil and structural engineers:* admissible displacements or deformations with regard to bearing or non-bearing structures that are adjacent to the structure designed;

♦ from *mechanical engineers:* admissible displacements of machines, elevators, piping, etc., that will not impair safe function of the equipment;

♦ from *electronics engineers:* admissible displacements of a TV aerial (larger displacements can cause transmission trouble);

♦ from *chemical engineers:* admissible vibrations that are not harmful to certain chemical process;

♦ from *agricultural engineers:* admissible deflections and vibrations that do not scare stalled animals. Etc.

However, data supplied shall be always checked for consistency, and their background should be known. It happens that we are offered by our engineering colleagues either exaggerated, or, on the contrary, understated data on admissible deformations or displacements.

In the assessment of obtained data we have to take into account also the *variability of the attack* (for example, in case of the TV aerial the deflection of the TV tower due to wind load fluctuations, then also the daily changes in deflection caused by thermal effects of solar radiation, etc.) and the possibilities of rectifying the displacements (for example, adjustment of the position of the aerial at the top of the TV tower). Attention has been paid to the problem of admissible deformations and several practically oriented general documents have been produced: ACI 435.3R-68, *Déformations admissibles* 1980, ISO/DIS 4356:1976.

As it has been already mentioned, *no reliability differentiation problem arises with constraints.* As a rule, values of constraints given in codes already include the importance of the facility because they are closely connected to the purpose of the building or structure. Nevertheless, the differentiation multiplier for SLSs does not loose its meaning; it can apply, for example, in the design according to the first-crack limit of prestressed concrete members.

Crack width

Cracks are a phenomenon encountered in all materials. However, only concrete and masonry structures are subjected to serviceability RelReqs based on the occurrence or width of cracks. Considering the crack width as a criterion, we have to take into account that cracks, for example, can

- ♦ be a starting factor in *material corrosion;*
- ♦ be the principal cause of *untightness* of tanks for fluids, gases, or loose materials;
- ♦ deteriorate the *sound-proofing* and also *odor-proofing* of partition walls;
- ♦ cause *annoyance of the users* of CF.

Similarly as in the case of deflections, a sensitivity threshold can be found both for the structures and for people involved (see an interesting study by Díaz Padilla and Robles 1971). This threshold can be expressed simply in terms of a limit crack width, w_{lim}, which again is a random variable. Its admissible value, w_{adm}, can be found in the same manner as that of the admissible deflection, f_{lim}.

We should mention here that the crack width need not be the only governing quantity. When, for example, gas-tight structures are concerned, the *summary area of cracks* is of importance. Or, individuals never evaluate the crack width in terms of a physical distance of two opposite faces of the cracked body; their attitude to a cracked structure depends on many factors: *length, shape,* and *density of cracks.* It happens, that a crack of considerable width, say 3 mm, escapes any attention of users, or even inspection engineers.

Vibrations

When vibration effects of dynamic load are evaluated in terms of vibration parameters, *range RelReqs* can be of interest:

$$C_i \leq A \leq C_s$$

where C_i, C_s = lower and upper constraints, respectively, related to the design criterion entering the RelReq. The latter can be of diverse physical nature: eigenfrequency, acceleration, velocity of vibrations. It depends on the particularities of the problem solved.

As for vibration parameters, not only engineers are source of decisions on constraints. In case of buildings, admissible vibration parameters are, as a rule, specified by *hygienic regulations*. Many designers are unhappy with such regulations, which are often based on concepts different from those built-up in the structural reliability area. Mutual understanding of engineers and hygienists is often needed.

11

PROBABILITY-BASED OPTIMIZATION

Key concepts in this chapter: *probability-based optimization; distress; maximum distress; minimum distress requirement; maximum distress probability; decomposition of target probabilities; transposition of target probabilities; ranking of target probabilities; determinate problem; overdeterminate problem.*

11.1 PROBLEM STATEMENT

For design methods based on formative or elementary reliability requirements, the *target probabilities of occurrence of adverse events,* that is, values P_{At} and P_{Bt} (Section 8.2), or values P_{ait} and P_{bjt} (Section 8.3), have to be found. *Two approaches* can be employed:

(a) the individual target probabilities can be *derived from a given target failure probability,* P_{ft}; when this approach is used, the resulting lower level target probabilities, either formative or elementary, are, at the respective level, *mathematically dependent;*

(b) the lower level target probabilities can be established *independently,* one by one, applying the same type of approaches as those used for the target failure probability P_{ft} (see Section 10.3).

The larger the number of independent estimates entering the establishing of design parameters, the less stable and comparable the results of design. To a certain degree, *the comprehensive uncertainty of the design expands with the number of independently established parameters.* Therefore, *it is desirable to limit the number of independent input target probabilities as much as possible.* Preference should be given to the approach (a).

When looking for the formative or elementary target probabilities (P_{At}, P_{Bt}; P_{ait}, P_{bjt}) we must first keep in mind that there already exists a ***stabilized state of the codified design format.*** It is not possible to change this state suddenly. Efforts to find target probabilities for the lower level design methods from values of P_{ft} established by some of methods shown in Section 10.3, are often fruitless. A simultaneous complete change in the existing, deeply rooted codified design format is never possible. As a rule, the design parameters found in this way are numerically inconsistent with the stabilized design routine. So, we have to look for procedures that would avoid problems of this kind, that is, *we must attempt to find procedures that do not distort the present state of the codified design format abruptly.*

It has to be underlined here that the probability-based optimization method, ***outlined in this chapter and developed in Chapters 13 and 14, is not supposed to be a substitute for the established probability-based methods nor to be an everyday design tool. Its main objective is to give guidance in determining values of design parameters needed either for various regulatory documents, or in solving issues arising when rational design of specific structures subjected to specific loads is contemplated.***

11.2 MAXIMUM DISTRESS PROBABILITY

In Section 3.6 the concept of relatively adverse event was defined. Now we can say that an *occurrence of a relatively adverse event,* that is, a non-compliance with any RelReq specified in Sections 8.1, 8.2, and 8.3, causes a ***distress*** to the client, designer, contractor, owner, and user of CF, or also to other persons not directly involved (for example, code makers). The distress can be *distinct,* occurring simultaneously with the occurrence of an adverse event, or *dormant,* materializing only under specific circumstances. No distinction will be made between the *receiver of distress* (code maker, client, designer, contractor; parents, teachers; planning board, law maker, government authorities, etc.) and no distinction will be made among various *kinds of distress* as far as their importance is concerned, either. Sometimes an event producing only a "feeble distress" can lead to heavy material and other consequences and vice versa. Obviously, distress is distress. On the other hand, we know that distress accumulates, increasing the "size" of discomfort feeling.

The concept "distress" is identical with the concept "failure" (see 1.2.6) if and only if the reliability is assessed by RelReq (7.3) joint with RelReq (8.10) or (8.16), that is, when only one relatively adverse event, Ev($Z \le 0$), is considered. In other cases of RelReqs, when several events enter the assessment, the incidence of a single relatively adverse event may not (but can) result in failure. This refers to RelReqs (7.1) and (7.2) joined with RelReqs (8.11) and (8.12) [or with (8.17)], or to RelReqs (7.16) or (7.17) joined with (8.13) and (8.14). The greater the number of relatively adverse events, the greater also the size of distress to people involved, and also *the likelier becomes failure of the S-L-E system.* It can be assumed that *failure is inevitable* when all relatively adverse events happen simultaneously, that is, when the *maximum of distress* takes place (of course, failure can occur at an even smaller number of adverse events). The probabilistic relationship between maximum distress and failure will be discussed in 13.1.2 and 14.1.2 [see Equations (13.11) and (14.10)].

Let us assume that all phenomena entering RelReqs, H_1 through H_n, are statistically independent. Then, the probability of simultaneous occurrence of all adverse events, called the *probability of maximum distress,* P_d, is given by

$$P_d = \prod_{i=1}^{n} P_i \tag{11.1}$$

where

$$P_i = \Pr(E_{adv,i})$$

is the probability of occurrence of the i -th relatively adverse event.

Similarly as it is with the failure probability (see 8.1.3) the *effective and estimate values of the maximum distress probability,* $P_{d,eff}$ and $P_{d,est}$, respectively, are distinguished. However, in all following considerations the difference between these two values will be ignored, and so only one quantity, P_d, will be discussed. As far as the time factor is concerned, the *annual value*, $\dot{P}_d$, and the *comprehensive value*, $\bar{P}_d$, will be distinguished when necessary, similarly as it is with P_f. For the relationship between $\dot{P}_d$ and $\bar{P}_d$, Equations (8.5) and (8.6) hold true again.

It is now possible to write a *RelReq in terms of the maximum distress probability*

$$P_d \leq P_{dt} \tag{11.2}$$

where P_{dt} (that is, $\dot{P}_{dt}$ or $\bar{P}_{dt}$) = ***target maximum distress probability*** (see Section 11.4).

Again, RelReq (11.2) can be investigated as a *subjective* or *facility RelReq* (cf. Section 8.1.1), in dependence on the attitudes of the respective user, who can be internal, external, or mixed. - RelReq (11.2) can be used in both design problems: *proportioning and checking of an S-L-E system.*

If only one phenomenon, the reliability margin, is considered, RelReq (11.2) is identical with the global RelReq (8.10). According to the foregoing exposition, incidence of the distress is, in this case, identical with failure, since only one possible distress (≡ failure) is dealt with. It therefore holds

$$P_d \equiv P_f, \quad P_{dt} \equiv P_{ft} \tag{11.3}$$

In the early days of probability-based design, P_d given by Equation (11.1) had been erroneously regarded as failure probability, and so RelReq (11.2) had been considered as the governing RelReq. This of course had proved to be wrong after some deeper implementation of statistical and probabilistic thinking into reliability problems. We certainly do not intend to return to that period. RelReq (11.2) has been used here as an instrument of further development.

Let us first assume that the reliability of an *existing facility* is to be assessed. If the random behavior of all n input phenomena is sufficiently described, and the probabilities of occurrence of relatively adverse events, P_1, P_2, ..., P_n, are calculated, then P_d is obtained from Equation (11.1), and finally RelReq (11.2) is checked. The solution is obviously simple, *each phenomenon being examined separately;* no sophisticated calculation of the failure probability or the reliability index is necessary, etc.

However, in this ***checking problem,*** the following situation can arise: if any of the input probabilities P_i is equal to zero, then also P_d becomes zero, and so RelReq (11.2) is automatically fulfilled. Thus, to make a system apparently reliable, the designer could declare one of the constants appearing in the calculation model for random variable, "calculate" the respective zero probability of adverse realization of that variable, and in this easy way arrive at zero probability of maximum distress! *Without any doubt, this would be a definitely wrong evaluation strategy.* It has to be emphasized that a relatively adverse event with zero probability of occurrence cannot be taken into consideration (just because it cannot occur) and *RelReq (11.2) should be evaluated for only those adverse events whose occurrence probabilities are greater than zero.*

From the practical point of view, the checking problem is of no interest. The checking problem can be treated by assessing separate probabilistic RelReqs (8.11) and (8.12), or (8.13) and (8.14), or the design RelReqs derived from these.

What is important, is the ***proportioning problem,*** since it is the starting point for the *determination of design parameters* required in design RelReqs. Again, let the solution of the proportioning problem be based on RelReq (11.2). We now have to handle the following difficulty: setting $P_d = P_{dt}$ into Equation (11.1), the target probabilities P_{1t} through P_{nt} can be arbitrarily chosen so as to obtain

$$\prod_{i=1}^{n} P_{it} = P_{dt} \tag{11.4}$$

that is, *for example,* in this option:

$$\prod_{i=1}^{n-1} P_{it} = P_{dt}, \quad P_{nt} = 1$$

or, *for example,*

$$P_{1t} = P_{2t} = \dots = P_{n-1,t} = 1, \quad P_{nt} = P_{dt}$$

Such options, of course, would be absurd. The first one might lead to uneconomical design and possible failure, while the second would almost surely result in failure. Obviously, the same kind of difficulty as that met with zero probability of an adverse event in the checking problem is encountered here.

It is then clear that, in order to remove arbitrariness, an *additional condition is needed* to arrive at a definite solution, not at a dubious set of target probabilities that can be lavishly adjusted without any conceptual framework.

11.3 MINIMUM DISTRESS REQUIREMENT

The supplemental condition for determining the target probabilities $P_{1t}, P_{2t}, \dots, P_{nt}$, which enter Equation (11.4), can be formulated using the *probability-based optimization approach,* founded on the following requirement:

♦ *Internal and external users of CFs, and also code makers, designers, contractors, and other participants in a construction project are interested in minimizing any dormant or distinct distress that might occur during the respective reference period.*

We have intentionally avoided talking here about *minimization of losses;* this belongs to another category of attitudes with corresponding optimization objectives.

The minimization of distress is, in a way, compatible with feelings of people involved. When a construction setback comes out, the perception of *psychic tort* usually arrives first. The feeling of an *economical loss* develops only later. The ***minimum distress requirement,*** which will be later analyzed in separate problems, is evidently closely bound to psychological viewpoints applied in establishing the target probabilities P_{ft} or P_{dt}.

The detailed mathematical treatment of the minimum distress requirement depends upon the type of the problem solved. We will see (Chapters 13 and 14) that in the determination of design parameters *three types of problems* are met; they differ by objectives:

(1) ***Decomposition of target probabilities*** is carried out when lower level target probabilities are derived from a "superior" target probability. For example, values P_{At} and P_{Bt} can be obtained from P_{dt} (see 13.1.1), values P_{ait} can be derived from P_{At}, values P_{bjt} from P_{Bt} (Section 14.1). Other features of the decomposition problem exist.

(2) ***Transposition of target probabilities*** is met in the differentiation of CFs, when for a class of facilities target probabilities are established using the already known and proven values valid for a class considered as reference class (see Sections 13.4 and 14.5).

(3) ***Ranking of target probabilities*** is applied in cases when a phenomenon can occur in one or more combinations of different order with other phenomena (see 14.2.1).

In transposition and ranking *two modifications of the problem* are discerned:

(a) If only a single target probability (for example, P_{dt}) enters the solution as a starting quantity, then a ***determinate problem*** is dealt with.

(b) If it is necessary to base the solution on *existing values of some target probabilities* (that is, on a certain existing "state of probabilities"), we talk about an ***overdeterminate problem.***

A more detailed discussion of the determinate and overdeterminate problems will be found in Sections 13.4, 14.2, and 14.5.

In all problems mentioned, that is, (1), (2a), (2b), (3a), and (3b), the same solution approach will be applied - the *minimization of distress*. Input and output quantities, and the objective function appearing in the solutions are

♦ the *known and sought target probabilities of occurrence of relatively adverse events;*

♦ the *probability of at least one of possible adverse events,* the particular meaning of which depends on the type of problem.

This is the reason why the respective optimization method introduced here is called *probability-based.*

The probability-based optimization has to be related to the *reference period* for which the respective target probability has been specified. Most often again, the human life expectancy, T_u, or the facility life expectancy, T_0, are used as reference period.

11.4 TARGET VALUES $\dot{P}_{dt}$ AND $\bar{P}_{dt}$

Considering the qualitative viewpoint, the target probability of maximum distress, P_{dt}, is analogous to the target failure probability, P_{ft}. As for the direct probabilistic design method (Section 8.4 and further Chapter 12), the two target probabilities are identical [cf. Section 11.2 and Equation (11.3)]. Therefore, when determining P_{dt}, procedures similar to those used in the determination of P_{ft} (that is, the recalculation method, analogy method, discomfort method, or some optimization method; see Section 10.3) can be employed.

It will be later shown that, under certain conditions, in the design based on the method of extreme functions (see 13.1.2) or on the method of extreme values (see 14.1.2), the relationship

$$P_f \geq P_{dt}$$

is valid, P_f = failure probability [see later Equation (13.11), or Equation (14.10)]. Therefore, when looking for the target maximum distress probability, P_{dt}, we should theoretically arrive at values smaller than the corresponding values of P_{ft}. Some calibration is necessary, based on the knowledge of P_{ft} and of the calculation model.

The relationships between the annual and comprehensive target maximum distress probabilities are given by Equations (8.5) and (8.6), that is

$$\bar{P}_{dt} = 1 - (1 - \dot{P}_{dt})^n \tag{11.5}$$

$$\dot{P}_{dt} = 1 - (1 - \bar{P}_{dt})^{\frac{1}{n}} \tag{11.6}$$

Let us finally mention that the target probabilities P_{dt} can be differentiated according to the *importance of facilities* in the same way as it is done with the target failure probabilities, P_{ft} (see 10.4.2).

12

DIRECT METHOD

Key concepts in this chapter: *direct method, DM; proportioning based on direct method; codified design format based on direct method; intestability of the reliability margin.*

12.1 PRINCIPLES

Theoretically, the general RelReq (8.10) is satisfied if the structure is designed in such a way as to obtain

$$Z_{min} \geq 0 \tag{12.1}$$

where Z_{min} = P_{ft}-fractile of the probability distribution of the random variable reliability margin

$$Z = f_Z(\xi_1, \xi_2, ..., \xi_n)$$

and ξ_1 through ξ_2 = elementary random variables. The fractile is found from

$$\Pr(Z \leq Z_{min}) = P_{ft} \tag{12.2}$$

using, for example, Equation (2.20).

Analogously, design can be based on RelReq

$$\Theta_{min} \geq 1 \tag{12.3}$$

where Θ_{min} = P_{ft}-fractile of the probability distribution of the global reliability factor

$$\Theta = f_\Theta(\xi_1, \xi_2, ..., \xi_n)$$

RelReqs (12.1) and (12.3) are the *design requirements.* They are a synthesis of the physical RelReq (7.3), or (7.12), and of the probabilistic RelReq (8.10). At present, RelReqs operating with Θ are mainly used in geotechnical engineering (see, for example, Matsuo and Suzuki 1985, Whitman 1984).

12.2 PROPORTIONING BASED ON THE DIRECT METHOD

12.2.1 Probabilistic method

Generally, the proportioning (that is, establishing of one or more parameters of an elementary random variable ξ_k for a specified target failure probability, P_{ft}), based on the *probabilistic direct method* must be performed step-by-step. Several sets of population parameters of the investigated variable are chosen, probabilities P_f corresponding to each set are calculated, and, by interpolation, the respective parameters are found for $P_{ft} = P_f$. The procedure becomes easy when only one population parameter is looked for. The population mean of an elementary variable is usually the target parameter. Then, a discrete series of solutions can be executed, and the interpolation (using, for example, the probability paper technique; see Appendix B) leads quickly to a result.

Proportioning by direct method can be considerably facilitated when the following three assumptions are satisfied:

(a) the investigated elementary variable, say ξ_k, can be explicitly expressed as a function of the remaining elementary variables entering the problem; that is, we can write Equation (7.10);

(b) it can be assumed that the second and third order population parameters, for example, the standard deviation, σ_k, and the coefficient of skewness, α_k, do not depend upon the population mean, μ_k;

(c) population parameters σ_k and α_k are known or estimated.

Then, the value of the mean μ_{xk} is looked for to meet RelReq (12.1). We can use the following technique:

(1) Set

$$\xi_k = \mu_k + \Delta\xi_k$$

where $\Delta\xi_k$ = random variable with population parameters

$$\mu_{\Delta k} = 0, \quad \sigma_{\Delta k} = \sigma_k, \quad \alpha_{\Delta k} = \alpha_k$$

(2) Using Equation (7.10), express the reliability margin

$$Z \mid \xi_k = \Lambda_k[\mu_k + \Delta\xi_k - \mathrm{h}(.)] \tag{12.4}$$

where $\mathrm{h}(.) = \mathrm{h}(\xi_1, \xi_2, \ldots, \xi_n; \sim\xi_k)$ and Λ_k = value of the influence function, Equation (7.6).

(3) Introduce an auxiliary random variable

$$\eta = \mathrm{h}(.) - \Delta\xi_k \tag{12.5}$$

(4) Estimate the population parameters of η, that is, μ_η, σ_η, and α_η, using either Monte Carlo simulation technique (Section 2.4; sample size $n = 1{,}000$ to 5,000 is fully sufficient), or - in very simple cases - the moment method (Section 2.3).

(5) Choose an appropriate probability distribution of η, with IDF $\Phi^{-1}(.)$. The three-parameter log-normal distribution is good in most cases.

(6) Calculate the κ-fractile, y_κ, of the probability distribution of η (Figure 12.1), where

$$\kappa = \frac{\Lambda_k + |\Lambda_k|}{2} - \Lambda_k P_{ft} \tag{12.6a}$$

This fractile is equal to the required μ_k, that is,

$$y_\kappa \equiv \mu_k = \mu_\eta + u_{k\kappa}\sigma_\eta \tag{12.7}$$

where

$$u_{k\kappa} = \Phi^{-1}(\kappa)$$

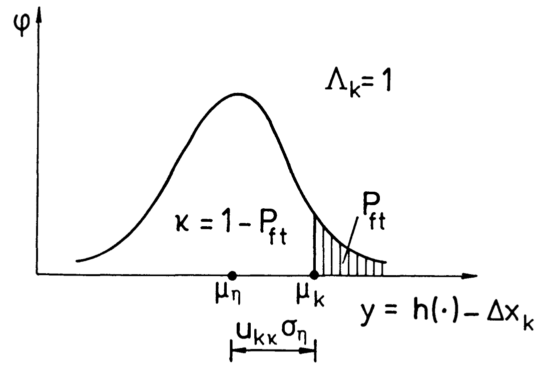

Fig. 12.1 - Determination of the population mean, μ_k, in the DM proportioning; H_k is absolutely favorable in the domain considered.

Obviously, *no iterations are needed!* - In most cases, the assumption (a) holds true, and also the assumption (b) is valid in a reasonable range of μ_k.

■ **Example 12.1.** The system "**steel bar & axial load,**" which was already studied in Examples 7.2 and 9.1, is to be proportioned with regard to the bar diameter d. The comprehensive target failure probability is $\bar{P}_{ft}$ = 1.0E-3 and the target life expectancy is T_{0t}. Find the mean of the bar diameter, μ_d.

Evidently, the reliability margin

$$Z|d = d - 2(N/\pi R)^{\frac{1}{2}}, \quad \Lambda_d = +1$$

shall be investigated. The random variable η [see Equation (12.5)] has the following form:

$$\eta = 2(N/\pi R)^{\frac{1}{2}} - \Delta d$$

The population parameters of N and f_y are taken from by Table 9.1, and the parameters of Δd are

$$\mu_{\Delta d} = 0, \quad \sigma_{\Delta d} = 0.4\,\text{mm}, \quad \alpha_{\Delta d} = 0.5$$

Applying Monte Carlo simulation to Equation (a) (sample size $n = 10{,}000$ is lavish) the sample characteristics of η were obtained:

$$m_\eta = 16.9\,\text{mm}, \quad s_\eta = 0.64\,\text{mm}, \quad a_\eta = -0.42$$

They can be taken for the population parameters μ_η, σ_η, and α_η. Assume that the probability distribution of η is log-normal, LN(-0.42). Since $\Lambda_d = +1$, we have

$$\kappa = 1 - P_{ft} = 0.999$$

Table A.1 in Appendix A gives $u_{d\kappa} = 2.55$, and so the required mean of the diameter is [Equation (12.7)]

$$d_{req} \equiv \mu_d = 16.9 + 2.55 \times 0.64 = 18.5\,\text{mm}$$

12.2.2 Statistical method

The proportioning procedure based on HL-index is analogous to that of the probability-based proportioning. In an *iterative solution* several sets of population parameters of the respective variable, ξ_k, are chosen, β^{HL} is calculated for each set, and by interpolation parameters giving $\beta^{\text{HL}} = \beta_t^{\text{HL}}$ are found.

When, however, the assumptions (a) through (c) in 12.2.1 are valid, proportioning can be simplified, using Equation (12.6) again. First, take Equation (8.15) for the reliability index

$$\beta_Z \equiv \mu_Z / \sigma_Z = \Lambda_k(\mu_k - \mu_\eta)/\sigma_\eta$$

Since Equation (12.6) is linear, we can set $\beta_Z = \beta_t^{\text{HL}}$, and further, as $\Lambda_k = 1/\Lambda_k$,

$$\mu_k = \mu_\eta + \Lambda_k \beta_t^{\text{HL}} \sigma_\eta \tag{12.8}$$

which is parallel to Equation (12.7).

According to Equations (2.61) and (2.73) it holds in the first approximation:

$$\mu_\eta \approx \text{Q}\mu_\eta \tag{12.9}$$

$$\sigma_\eta^2 \approx \sigma_{\Delta k}^2 + \sigma_h^2 \tag{12.10}$$

where $\text{Q}\mu_\eta$ = quasi-mean of the random variable η, $\sigma_{\Delta k}^2$ = variance of ξ_k, σ_h^2 =

variance of the term $\mathbf{h}(.)$. As $\mu_{\Delta k} = 0$, it is

$$Q\mu_{\eta} = Q\mu_{h}$$

where $Q\mu_h$ = quasi-mean of $\mathbf{h}(.)$.

The variance σ_h^2 can be approximated by the quasi-variance $Q\sigma_h^2$ using Equation (2.62) or even Equation (2.63a). In non-linear cases, it is almost impossible to estimate the coefficient of skewness or the quasi-alpha. Equations (2.69) through (2.74) can be used if suitable.

12.3 CODIFIED DESIGN FORMAT

Though a wide-scale practical utilization of the direct method cannot be expected in the coming decades, there is no reason why to eliminate it from codification strategies. It seems that there is not too much to be codified; nevertheless, a good direct-method code will be a rather complicated document. We now know sufficiently well what should be contained in such a code:

(1) *Statistical and probabilistic information* giving adequate support to the reliability evaluation, in particular:

- ♦ general assumptions needed for calculation of the failure probability P_f (or of the HL-index, β^{HL}); detailed provisions are difficult to be given due to large variety of approaches;
- ♦ target values of P_{ft}, in terms of $\dot{P}_{ft}$ or $\bar{P}_{ft}$, for different classes of CFs (or target values of the HL-indices); for designer's convenience, also basic techniques of establishing these target values should be specified by the code;
- ♦ target values of life expectancy, T_{0t}, or other relevant reference periods;
- ♦ statements on general properties of probability models of input elementary variables, covering time-dependence, load intermittence and other important features;
- ♦ data on probability models of those variables that occur frequently in design.

(2) *Physical calculation models,* analogous to those given today in codes based on the method of extreme values, but with additional information on the respective model uncertainties.

Principles of a direct-method-based code, probabilistic or statistical or both, are now already given in very general documents (*Grundlagen zur Festlegung* 1981, *Guidelines* 1987, ISO 2394:1986). In these documents only target values of the failure probability are specified, the designer is free to perform the reliability analysis according to his or her engineering judgment. At present (1993), the ISO Technical Committee ISO/TC98 is specifying the DM design in a new version of ISO 2394.

12.4 MERITS AND DRAWBACKS

Merits of DM:

♦ only one random phenomenon is analyzed;

♦ consequently, it is not necessary to perform any decomposition of target probabilities;

♦ combinations of phenomena or events, that is, in particular, combinations of loads, are not directly investigated;

♦ through suitable adjustment of values of the respective reliability parameters, differentiation of CFs is directly obtained;

♦ the method is good for design of special, or unique facilities as, for example, off-shore structures, containments of nuclear reactors, important bridges.

Drawbacks of DM:

♦ the principal drawback is the intestability of the method; it is not possible to test whole structures subjected to random loads;

♦ as a consequence, population parameters of the reliability margin, Z, cannot be experimentally evaluated;

♦ further, due to the two foregoing features, a standardization of DM at a general level is almost impossible; the necessary parallel codes for quality control, inspection, and maintenance, based on testing of the reliability margin, cannot be developed; code clauses that specify the direct method can be limited to a very general phrasing, because in practical applications the method must be always newly formulated for a given S-L-E system with well defined properties of all its components;

♦ the number of RelReqs (8.10) to be checked is given by the number of possible load combinations; it can never be told in advance whether this or that combination is relevant or not, which makes the solution complicated.

13

METHOD OF EXTREME FUNCTIONS

Key concepts in this chapter: method of extreme functions, MEF; minimum distress; magnitude of distress; lower and upper bound of failure probability; load-effects in MEF; resistances in MEF; theory of interaction diagrams; differentiation problem in MEF; determinate problem; overdeterminate problem; partial importance factor; partial differentiation multiplier; capacity reduction factor; testability of resistance; intestability of combined loads.

13.1 PRINCIPLES

13.1.1 Decomposition of P_{dt}

The method of extreme functions, MEF, is based on the formative probabilistic RelReqs (8.11) and (8.12). The principles of the method will be explained on a one-component case, where both the attack and the barrier, A and B, are expressed as scalars. It has to be reiterated that the attack and barrier must be *independent;* if they are not, the solution would become fallacious or meaningless.

The RelReqs are met if the design requirement

$$A_{max} \leq B_{min} \tag{13.1}$$

is fulfilled. Here, A_{max} and B_{min} = fractiles of the random variables

$$A = f_A(a_1, a_2, ..., a_{na})$$

$$B = f_B(b_1, b_2, ..., b_{nb})$$

obtained from

$$\Pr(A \leq A_{max}) = 1 - P_{At} \tag{13.2}$$

$$\Pr(B \leq B_{min}) = P_{Bt} \tag{13.3}$$

The design RelReq (13.1) is a synthesis of the physical RelReq (7.1) and the probabilistic RelReqs (8.11) and (8.12).

The target probabilities P_{At} and P_{Bt} can be chosen independently of each other, using decisions based on some of the methods referring to the target failure probability, P_{ft} (see Section 10.3). This, however, is an undesired approach, adding needless uncertainties to the set of design parameters. We will show how the two target probabilities, P_{At} and P_{Bt}, can be derived from the *target value of the maximum distress probability,* P_{dt} (Section 11.2), by applying the *decomposition procedure.*

According to Equation (11.4) it must be

$$P_{At}P_{Bt} = P_{dt} \tag{13.4}$$

Obviously, this equation can be complied with by an *infinite number of pairs* (P_{At}, P_{Bt}), and therefore the *additional requirement of minimum distress* (Section 11.3) must be formulated.

In conformity with Chapter 11 we can say that a distress is equivalent to the occurrence of a relatively adverse event. The danger of *possible distress* associated with a relatively adverse event is obviously proportional to the probability of occurrence of such an event. If the summary magnitude of all possible distress is to be minimized, then the S-L-E system must be designed in such a manner that *the relatively adverse events happen with minimum probability.* Thus, the minimized function should be the *probability of occurrence **of at least one** of the possible relatively adverse events* Ev(A_{adv}) and Ev(B_{adv}). This probability, P_{adv}, is given by Equation 2.6, that is,

$$P_{adv} = P_{At} + P_{Bt} - P_{At}P_{Bt} \tag{13.5}$$

As far as the design RelReqs are concerned, P_{adv} is without any importance; it only describes the *measure of occurrence of relatively adverse events,* which is to be minimized. Nothing more. Therefore, P_{adv} is just an *auxiliary quantity,* having the role of objective function in the minimization algorithm.

The values of P_{At} and P_{Bt} that minimize P_{adv} are easily found by solving

$$\frac{\partial P_{adv}}{\partial P_{At}} = 0, \quad \frac{\partial P_{adv}}{\partial P_{Bt}} = 0, \quad P_{At}P_{Bt} = P_{dt} \tag{13.6}$$

We obtain:

$$P_{At} = P_{Bt} \tag{13.7}$$

and further

$$P_{At} = \sqrt{P_{dt}}, \quad P_{Bt} = \sqrt{P_{dt}} \tag{13.8}$$

For later convenience, Equation (13.7) will be written as

$$P_{At} - P_{Bt} = 0 \tag{13.9}$$

It is interesting to notice (Figure 13.1) what values of P_{adv} were possible when P_{At} and P_{Bt} were chosen so as to comply only with Equation (13.4), that is, without minimizing P_{adv}. Taking, for example, $P_{At} = 1$, $P_{Bt} = P_{dt}$, or $P_{At} = P_{dt}$, $P_{Bt} = 1$, we get $P_{adv} = 1$. Obviously, for the first choice, all values of the attack, A, and for the second choice, all values of the barrier, B, would belong to the respective sets of adverse events, Ev(A_{adv}) and Ev(B_{adv}), respectively. Thus, in terms of the foregoing definition, *distress would become certain.*

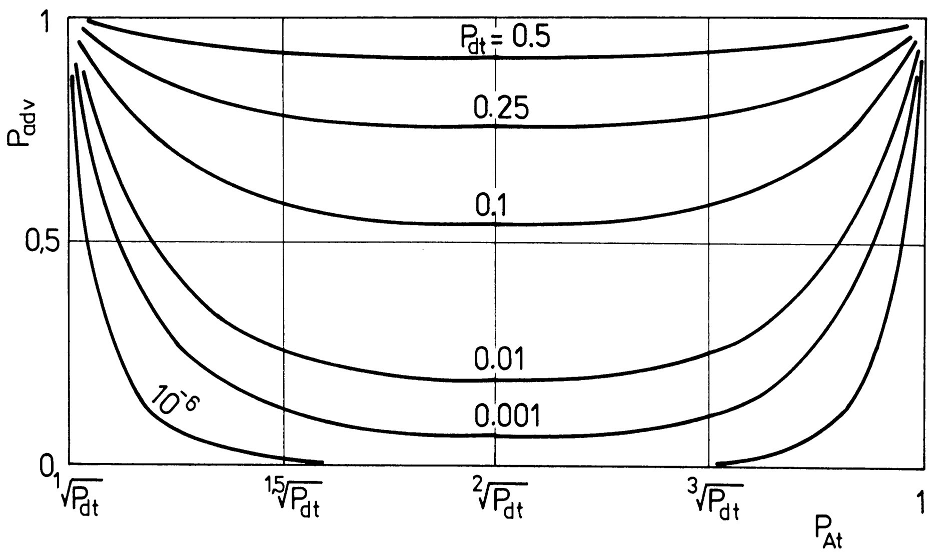

Fig. 13.1 - Probability P_{adv} *vs.* P_{At} when $P_{At} \cdot P_{Bt} = P_{dt}$ (for different values of P_{dt}).

The probability-based minimization of relatively adverse events does not take into account the fact that consequences of these events change in dependence upon the variable c. This complies with our approach: *it is not important how adverse the distress situations are; it is important how many distress situations can annoy the persons involved.* In other words, it is irrelevant whether the adversity of distress is strong or weak. *The mere fact of adversity is relevant.*

In all operations on probabilities the ***time factor*** must not be forgotten. When, for example, fractiles A_{max} and B_{min} are to be determined for P_{At} and P_{Bt}, respectively, probability distributions of A and B related to the reference period considered must be applied. - On the other hand, when determining the target probabilities with respect to the reference period in question (one year, T_u, or T_0), it shall always be kept in mind that some of the adverse events can be time-independent. Let us show this in the following Example.

■ **Example 13.1.** Find, for a residential building (the subjective reliability requirement applies, Section 8.1) with a target life expectancy $T_0 = 50$ years, the target probabilities $\bar{P}_{At}^{(50)}$ and $\bar{P}_{Bt}^{(50)}$. Assume that the target probability of maximum distress is $\bar{P}_{dt}^{(70)} = 2 \times 1.0\text{E-}6$ (referred to the user's life T_u =

70 years). The attack (for example, a variable load) is time-dependent, the barrier (a resistance) is supposed to be time-independent.

Equation (13.8) yields

$$\bar{P}_{At}^{(70)} = 1.414\,\mathrm{E}{-3}, \quad \bar{P}_{Bt}^{(70)} = 1.414\,\mathrm{E}{-3}$$

and, further, Equation (11.6) gives

$$\dot{P}_{At} = 20.22\,\mathrm{E}{-6}$$

Therefore, for $T_0 = 50$ years we obtain from Equation (11.5)

$$\bar{P}_{At}^{(50)} = 1.010\,\mathrm{E}{-3}$$

Since the barrier is time-independent, the respective target probability, P_{Bt}, is also time-independent, either. Therefore, it must be

$$\dot{P}_{Bt} \equiv \bar{P}_{Bt}^{(50)} \equiv \bar{P}_{Bt}^{(70)} = 1.414\,\mathrm{E}{-3}$$ ■

Constraint RelReq

When the right-hand side of RelReq (13.1) is a fixed, non-random value, C (cf. Section 10.5), the problem gets simplified. In fact, it can be formulated in terms of the direct method, including C into the reliability margin, Z (see also 13.1.3).

13.1.2 Interval of the failure probability

Let us assume that

(a) the design based on RelReq (13.1) is *economical,* that is, inequality becomes equality;

(b) *no indefiniteness* is contained in functions describing the attack and barrier.

It is then possible to draw from P_{At} and P_{Bt} a certain amount of information about the expected failure probability, P_f, of the system designed.

The *lower bound* of P_f is identical with the probability of simultaneous occurrence of the two adverse events; thus

$$P_f \geq P_{At} P_{Bt} \tag{13.10}$$

A lower value of P_f, when assumptions (a) and (b) are met, is not possible. If the assumptions are not complied with, we may of course have $P_f < P_{At} P_{Bt}$. For example, if the resistance of a structure is much greater than the load, failure may physically not be able to happen, and so $P_f = 0$. A similar situation can take place when in design some decision-based, non-probabilistic reliability factors apply (as, for example, γ_A, γ_B).

However, we must maintain that an event Ev(**failure**) cannot be limited to simultaneous occurrence of relatively adverse events. Failure can also occur *when only one of the formative quantities, A or B, attains an adverse value* (we should better say "a value adverse enough"). Then obviously, P_f is greater than $P_{At}P_{Bt}$. It can easily happen, depending on physical and random properties of the S-L-E system, that the effect of variability of one of the two formative variables will be suppressed, and so RelReq will be governed by only the other variable. This refers to both variables. Therefore, the *upper bound* of P_f is given by the greater of the two probabilities; thus

$$P_f \le \max(P_{At}, P_{Bt})$$

Again, reliability factors can diminish the upper bound.

Thus, the *interval* on which the failure probability can be found (when supplementary decision-based reliability factors are ignored) is

$$P_{At}P_{Bt} \le P_f \le \max(P_{At}, P_{Bt}) \tag{13.11}$$

In the *optimized case,* when values of P_{At} and P_{Bt} are derived from Equations (13.8), it is

$$P_{dt} \le P_f \le \sqrt{P_{dt}}$$

It is necessary to stress that *this interval refers to the method,* not to the given S-L-E system or a group of systems, where it can be notably narrower.

Though at first glance MEF seems to be formally very simple, its practical use is complicated. The greatest hindrance to its general use are difficulties with mathematico-statistical analysis of functions $f_A(.)$ and $f_B(.)$. Diverse techniques can be applied: Monte Carlo simulation, moment method (Sections 2.3 and 2.4); all are demanding on the analysis or processing time or both.

As far as *existential combinations of events* are concerned, no particular difficulties are encountered with MEF. Again, RelReq (13.1) has to be assessed for all combinations that are likely to occur. The concepts of closed, fixed, and free combinations (see 3.2.2) have no probabilistic meaning here; nonetheless, they are important for our *decisions on which combinations shall be considered* in the particular problem.

13.1.3 Generalized reliability margin

The concept of the reliability margin Z can be extended also to lower levels of RelReqs. We can define a *generalized reliability margin* as the *distance between two specified values of a variable.*

Consider, for example, the maximum attack in RelReq (13.1), A_{max}. Assume that A_{max} is the value of attack that should not be exceeded; then, we can define a generalized reliability margin as

$$Z_A = A_{max} - A \tag{13.12}$$

and similarly

$$Z_B = B - B_{min} \qquad (13.13)$$

where A and B are random.

All operations that are possible on $Z = B - A$ are also possible on Z_A or Z_B. For example, we can specify A_{max} by decision, or by setting $A_{max} = B_{min}$ (where B_{min} has been established probabilistically), or by setting $A_{max} = C$ (where the constraint, C, has been specified by decision). Then, we can treat RelReq

$$Z_A \equiv A_{max} - A \geq 0$$

in the same manner as RelReq (12.1) in Chapter 12. Analogous operation can be performed on Z_B. Target probabilities P_{At}, P_{Bt}, or target reliability indices β_{At}^{HL}, β_{Bt}^{HL} should be applied in the solution.

The concept of generalized reliability margin can be efficiently used in the analysis of design parameters for resistance and loads (for example, Ayyub and White 1987).

13.2 LOAD AND LOAD-EFFECTS

13.2.1 One-component case

Except for a few special cases (members subjected to simple load), the LOAD system in MEF is represented in terms of *load-effects,* S. Solution is simple for *one-component load-effect conditions.* Let us write a combination formula

$$S = v_1 F_1 + v_2 F_2 + \dots + v_n F_n \qquad (13.14)$$

where F_i = load magnitudes, v_i = influence coefficients depending upon the properties of the structural system and, in general, also upon loads entering Equation (13.14). It can be

$$v_i = f_{vi}(F_1, F_2, \dots, F_n)$$

for any i ; in such a case the load-effect is non-linear. Other possibilities of non-linearity can be shown. When all v_i are load-independent, a linear load-effect is dealt with.

In the *proportioning based on MEF* we want to find S_{max} such that

$$\Pr(S > S_{max}) = P_{St} \qquad (13.15)$$

where P_{St} = target value. The solution is obviously simple. A Monte Carlo simulation is the most comfortable technique.

Equation (13.15) has to be investigated for each possible load combination taking into account prevailing existential relations among loads appearing in the problem. The design load-effect of combined loads is found from the selection formula

$$S_{Cd} = \operatorname*{adv}_{k=1}^{m} (S_{max}^{(k)})$$

where $S_{max}^{(k)}$ = load-effect established according to Equation (13.14) from the combination formula k, adv(.) = operation symbol indicating the most adverse load-effect obtained.

13.2.2 Multi-component case

In *multi-component conditions,* when a load-effect is expressed in terms of an ordered n-tuplet of variables, or also in terms of a function, the solution becomes more intricate. Consider, for example, a two-component load-effect affecting a cross-section. Let the components be denoted S_x, S_y (they can represent, for example, bending moment and axial load). The cross-section be subjected to the load-effects due to two existentially independent loads, F_1 and F_2. Three existential combinations are possible:

$$(F_1)_e,\ (F_2)_e,\ (F_1,F_2)_e$$

The individual random outcomes of the *one-member existential combination* $(F_1)_e$, or $(F_2)_e$, are described by *points in the coordinate system* $[S_x, S_y]$. According to the type of the *structural response* - linear or non-linear, these points are *distributed along a straight or curved line,* Figure 13.2. The position of points is random, dependent on the probability distributions of F_1 or F_2. The distribution of outcomes can be described by marginal density function $\varphi(S_x)$ or $\varphi(S_y)$. If the response is linear, the probability distribution can be plotted and evaluated along the respective straight line. - Observe that, in the case of a one-member combination, a one-to-one correspondence between the two marginal distributions exists.

The *two-member combination* $(F_1,F_2)_e$ is described in $[S_x, S_y]$ by *points in a specified area defined by the definition domains of the two loads.* It is assumed here, for clarity, that the probability distributions of F_1 and F_2 are both-side bounded. The random behavior of the population of outcomes (S_x, S_y) can be expressed in terms of a bivariate joint probability distribution (see 2.1.3), whose properties depend on the distributions of the respective loads and on the response of the structure (see Ferry Borges and Castanheta 1985, where a more detailed analysis of this problem is presented).

To evaluate the reliability requirement, the *fractile* S_{max} *of the load-effect* must be known. When the *one-member "combination"* is investigated, the solution is simple: it suffices to know the distribution of F_1 or any of the *marginal distributions.* For the given target probability, P_{St}, any of the marginal fractiles $S_{y,max}$ defines unambiguously a *fractile point* S_{max}. This holds for linear and non-linear response as well.

Now, let us turn to the two-member load combination, $(F_1,F_2)_e$. For simplicity, a case when the dependence between S_x and S_y remains linear with changing F_1 or F_2 is considered. The point S^* (Figure 13.3) resulting from the vector sum of S_{max} found for isolated loads cannot be assumed as a design point because it does not correspond to P_{St}. Instead, a *fractile curve* S_{max} must be found in the following way:

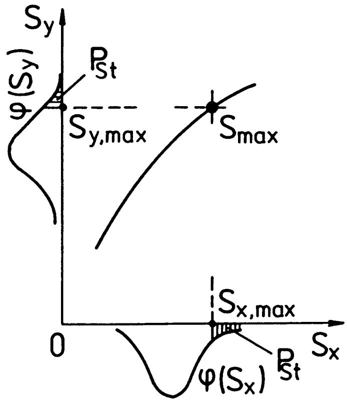

Fig. 13.2 - *Method of extreme functions.* Two-component load-effect in a structure with non-linear behavior.

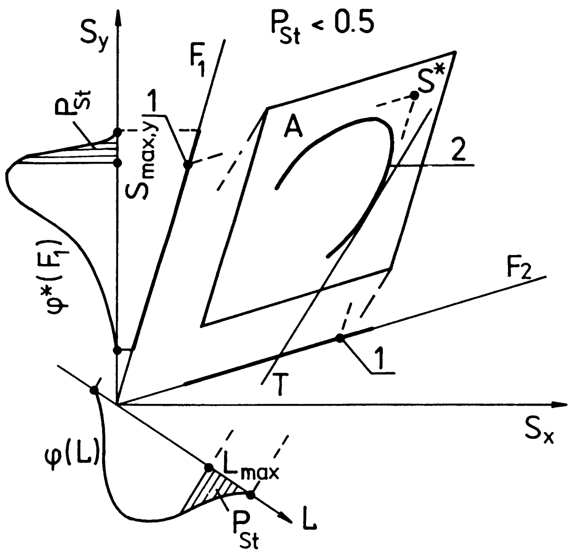

Fig. 13.3 - *Method of extreme functions.* Two-component load-effect, two existentially independent, upper and lower bounded loads, linear response (A - region of (F_1,F_2) outcomes, 1 - fractile points S_{max} for combinations $(F_1)_e$ and $(F_2)_e$, 2 - fractile curve S_{max} for combination $(F_1,F_2)_e$). Figure shows the way of determining S_{max} when P_{St} < 0.5.

Consider a marginal PDF obtained as a projection of the joint probability distribution onto an arbitrary straight line, L, passing through the origin of coordinates (Figure 13.3; however, any point in the coordinate system can be taken for origin of L). For this projection, marginal distribution of a random variable L is found, where L = abscissa referred to the outcome (F_1,F_2) measured on L from the origin of coordinates. For known or assumed $\Phi(L)$ the fractile L_{max} is established from

$$\Pr(L > L_{max}) = P_{St}$$

Let us now draw a straight line T perpendicular to L and passing through L_{max}. This straight line separates a part P_{St} of the unit volume under the joint PDF.

Rotating L about the origin of coordinates, straight lines T associated with each position of L define an envelope S_{max}. In this way a *system of* S_{max} *curves* for diverse P_{St} can be established. Note that S_{max} curves are not identical with curves of equal probability density.

Thus, the design load-effect is, in this particular case of two existentially independent loads, defined by

- two fractile points: $S_{max}(F_1)$ and $S_{max}(F_2)$,
- and one fractile curve: $S_{max}(F_1, F_2)$,

obtained for P_{St}.

Figure 13.3 was drawn for $P_{St} < 0.5$. For $P_{St} > 0.5$ (such values of P_{St} *are* possible when treating, for example, a serviceability limit state), the S_{max} curve becomes convex with respect to the origin of coordinates (Figure 13.4).

The described solution can be extended to an *arbitrary number of loads*. If, for example, the cross-section is subjected to the effects of three existentially independent loads, F_1, F_2, and F_3, it has to be designed for the following existential combinations:

$$(F_1)_e,\ (F_1,F_2)_e,\ (F_1,F_2,F_3)_e$$

$$(F_2)_e,\ (F_1,F_3)_e$$

$$(F_3)_e,\ (F_2,F_3)_e$$

Similarly as in the previous case, we must find (Figure 13.5):

- three single-load fractile points: $S_{max}(F_1)$, $S_{max}(F_2)$, and $S_{max}(F_3)$,
- three double-load fractile curves: $S_{max}(F_1,F_2)$, $S_{max}(F_1,F_3)$, and $S_{max}(F_2,F_3)$
- and one three-load fractile curve: $S_{max}(F_1,F_2,F_3)$.

Consequently, we have to assess the cross-section for the single-load fractile points and the envelope $S_{max,c}$ that follows the relevant sectors of the four partial curves. It is evident that neither all points S_{max} nor all curves S_{max} must be necessarily involved in the design - some can be overlapped by others (Figure 13.5).

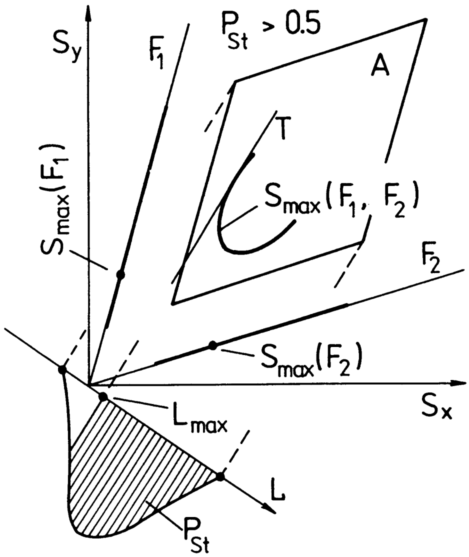

Fig. 13.4 - *Ditto* as Figure 13.3 but with $P_{St} > 0.5$.

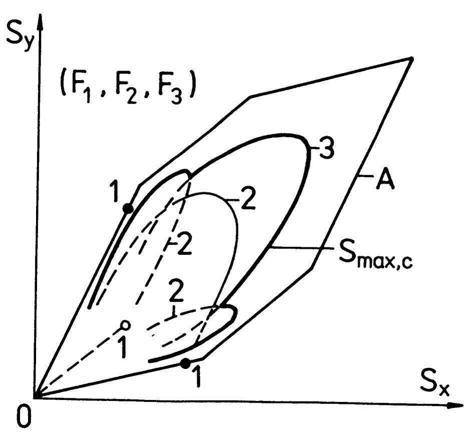

Fig. 13.5 - *Method of extreme functions.* Two-component load-effect, three existentially independent, upper and lower bounded loads, linear response (**A** - region of outcomes of F_1, F_2, and F_3, **1** - fractile points S_{max} related to the combinations of order 1, **2** - fractile curves S_{max} related to the combinations of order 2, **3** - fractile curve S_{max} related to the combination of order 3).

13.3 RESISTANCE

The problem of resistance in MEF has been already solved. Actually, we can say that the principles of this method were formulated just in relationship to resistance of reinforced and prestressed concrete structures (Tichý and Vorlíček 1972). Let us summarize here only the *main results.*

When a one-component load-effect is dealt with, a *one-component resistance,* R, has to be investigated:

$$R = f_R(r_1, r_2, ..., r_n)$$

where r_i = resistance variables, that is, mechanical and geometric properties of a cross-section, member, or a whole structure. By a mathematico-statistical analysis of the function $f_R(.)$ the P_{Rt}-fractile, R_{min}, can be found from

$$R_{min} = \mu_R + u_{min}\,\sigma_R$$

where μ_R = population mean of the probability distribution of R, σ_R = population standard deviation, and u_{min} = value of the standardized variable established for the given target probability P_{Rt}.

In the codified design, R_{min} should be expressed by a non-dimensional design parameter, the *resistance factor*

$$\varphi_R = \frac{R_{min}}{\mu_R}$$

which is close to the *strength reduction factor* in, for example, ACI 318-89.

The case when resistance is defined by a *function* has been solved by means of the *"theory of interaction diagrams"* (Tichý and Vorlíček 1972). The principles of the solution can be simply explained by a two-component problem (Figure 13.6).

The fractile R_{min} is expressed by a continuous curve in the coordinate system $[S_x, S_y]$. It can be established setting

$$R_x = f_{Rx}(S_y^*; r_1, r_2, ..., r_n) \tag{13.16}$$

or

$$R_y = f_{Ry}(S_x^*; r_1, r_2, ..., r_n)$$

or also

$$\sqrt{R_x^2 + R_y^2} = f_{Rxy}[(\frac{S_x}{S_y})^*; r_1, r_2, ..., r_n]$$

The asterisk denotes fixed values of the respective components.

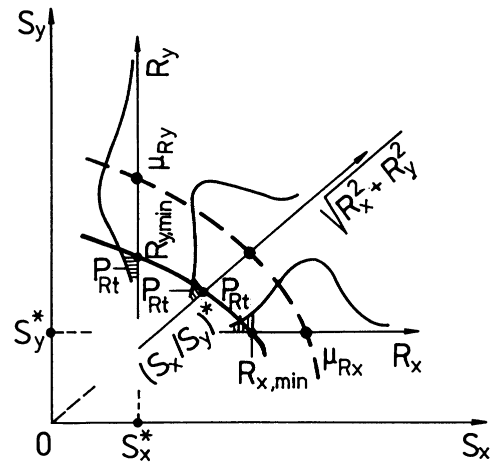

Fig. 13.6 - *Method of extreme functions.* Two-component load-effect. Curve of minimum resistance, R_{min}.

Analyzing, for example, Equation (13.16), we can determine, for the given target probability P_{Rt} and an assumed probability distribution, a fractile $R_{x,min}$ in the same way as in a one-component case. A curve R_{min} is defined as a locus of all points describing R_{min} for different levels of S_y^*. Certain rules must be observed in the analysis; we will not repeat them here (see Tichý and Vorlíček 1972). - However, *the resistance factor depends on the level of the fixed component,* which makes a simple codification of MEF difficult.

When ***multi-modal resistance*** is dealt with, the idea of the approach described in 9.1.3 for calculation of failure probability should be used. Again, Monte Carlo simulation is the most comfortable technique of obtaining the R_{min} curve. In each simulation trial the resistance is defined by

$$R = \min(R^{(1)}, R^{(2)}, ..., R^{(M)})$$

where $R^{(i)}$ = realizations of partial resistances (1) through (M), M = number of resistance modes.

13.4 DIFFERENTIATION PROBLEM

MEF allows to differentiate constructed facilities according to their socio-economic importance, or to differentiate bearing structures and members according to their participation in the reliability of the system. In dependence on the input reliability parameters two differentiation problems are distinguished: the *determinate problem* and the *overdeterminate problem.*

13.4.1 Determinate problem

Let us investigate a set of *classes of CF* (or structures, or bearing members). A *target maximum distress probability,* $\bar{P}_{dt}^{K}$, is associated with facility class K, referred to an appropriate reference period, $T_{ref} = n$ years (for example, the target life expectancy, T_{0t}). By the decomposition technique shown in Section 13.1 we obtain for class K:

$$\bar{P}_{At}^{K} = \sqrt{\bar{P}_{dt}^{K}}, \quad \bar{P}_{Bt}^{K} = \sqrt{\bar{P}_{dt}^{K}}$$

and so the design RelReq (13.1), written for class K, will be (assuming a one-component case)

$$A_{max}^{K} \le B_{min}^{K} \tag{13.17}$$

The fractiles A_{max}^{K}, B_{min}^{K} are determined from

$$\Pr(A \le A_{max}^{K}) = 1 - \bar{P}_{dt}^{K}$$

$$\Pr(B \le B_{min}^{K}) = \bar{P}_{dt}^{K}$$

Now, let a specified facility class be taken as a *reference class,* R. A target probability $\bar{P}_{dt}^{R}$ and also the two lower level target probabilities, $\bar{P}_{At}^{R}$ and $\bar{P}_{Bt}^{R}$, relate to this class. Again, fractiles A_{max}^{R} and B_{min}^{R} (*reference fractiles*) are determined from Equations (13.2) and (13.3). The ratios

$$\gamma_{nA}^{K} = \frac{A_{max}^{K}}{A_{max}^{R}}, \quad \gamma_{nB}^{K} = \frac{B_{min}^{K}}{B_{min}^{R}} \tag{13.18}$$

define *partial importance factors.* Using Equation (2.20) we can write after arrangement

$$\gamma_{nA}^{K} = \frac{1 + u_{At}^{K}\delta_A}{1 + u_{At}^{R}\delta_A}, \quad \gamma_{nB}^{K} = \frac{1 + u_{Bt}^{K}\delta_B}{1 + u_{Bt}^{R}\delta_B}$$

where δ_A, δ_B = variation coefficients of the attack and barrier, respectively, u_{At}, u_{Bt} = values of the standardized random variables determined for $\bar{P}_{At}^{K}$ and $\bar{P}_{Bt}^{K}$, and for $\bar{P}_{At}^{R}$ and $\bar{P}_{Bt}^{R}$ (with corresponding superscripts).

We now can write the design RelReq (13.17) as

$$\gamma_{nA}^{K} A_{max}^{R} \le \gamma_{nB}^{K} B_{min}^{R} \tag{13.19}$$

In practical design it is more convenient to handle only one *global importance factor*, γ_n, associated with either the attack or the barrier (see 10.4.2). Denote

$$\frac{\gamma_{nA}^{K}}{\gamma_{nB}^{K}} = \gamma_n^{K}$$

Then, in practical design, either the reference fractile of the attack, A_{max}^{R}, is multiplied, or the reference fractile of the barrier, B_{min}^{R}, is divided by γ_n^{K}. The design RelReq for the facility class K becomes

$$\gamma_n^{K} A_{max}^{R} \leq B_{min}^{R} \tag{13.20}$$

or also

$$A_{max}^{R} \leq \frac{1}{\gamma_n^{K}} B_{min}^{R} \tag{13.21}$$

The difference between RelReqs (13.20) and (13.21) seems to be only formal, but it plays a role in non-linear and also multi-component cases.

It is evident that a global factor γ_n does not agree with the principles of MEF that are based on independent investigation of attack and barrier.

In the differentiation of target probabilities, distinction between facilities with *subjective RelReqs* and facilities with *facility RelReqs* has to be taken into account. In the first case, the target probability $\dot{P}_{dt}$, in the other case, the target probability P_{dt} shall be used in the decomposition solution.

■ **Example 13.2.** Find the importance factors γ_{nS} (referred to loads), γ_{nR} (referred to resistances), and the global factor γ_n for three classes of facilities, H, N, and M, with high, normal, and medium socio-economic importance, respectively. All facilities are supposed to be subjected to the facility RelReq. The following comprehensive target probabilities, with $T_{ref} = T_0$, were selected for these classes:

$$\bar{P}_{dt}^{H} = 1.0E-6, \quad \bar{P}_{dt}^{N} = 1.0E-5, \quad \bar{P}_{dt}^{M} = 1.0E-4$$

Hence, the number of class M facilities supposed to fail during the life T_0 is expected to be 10 times greater than in class N and 100 times greater than in class H. In calculations, the population parameters were assumed according to Table 13.1, three-parameter log-normal distributions were considered as probability models. The solution and results are shown in Table 13.2. ■

13.4.2 Overdeterminate problem

The method of extreme functions can be used also in cases (very frequent ones) when the *state of design is stabilized* and characterized by *empirical values of target probabilities* $P_{At,emp}$ and $P_{Bt,emp}$. These values, notional more or less, can be found, for example, by analysis

Table 13.1 - *Example 13.2.* Population parameters of the load-effect, S, and of the resistance, R

Variable x	Coefficient of variation, δ_x	Coefficient of skewness, α_x
Load-effect, S	0.5	-0.5
Resistance, R	0.1	+0.8

of available regulations or by evaluation of existing structures or both. Of course, probability distributions of the attack and barrier and calculation models must be sufficiently well known for this purpose. In this case, *the decomposition of probabilities had been empirically settled in the past*, independently of our present decisions or wishes, without, of course, any optimization criteria specified.

As a rule, the two empirical target probabilities differ substantially. For example, empirical values related to certain time-dependent loads (for example, wind load) are between 0.1 and 0.5 (referred to the life expectancy of the facility), while the probabilities of occurrence of adverse resistances are of the order 1.0E-6 or even less.

Assume that for a facility class K importance factors γ_{nA}^{K} and γ_{nB}^{K}, or a global factor γ_n^{K}, shall be determined. We have to establish the target probabilities P_{At}^{K} and P_{Bt}^{K} from the probabilities

$$\bar{P}_{At,emp}^{R} \neq \bar{P}_{Bt,emp}^{R}$$

referred to the reference class R and to the reference period T_{ref}. - Obviously, a *transposition of known, empirically fixed target probabilities from class* R *to class* K has to be carried out.

Again, the probability-based optimization technique can be used. We start from the empirical target probability of maximum distress for class R facilities given by

$$\bar{P}_{dt,emp}^{R} = \bar{P}_{At,emp}^{R} \cdot \bar{P}_{Bt,emp}^{R}$$

and associated with T_{ref}.

Table 13.2 - *Example 13.2.* Determining the importance factor in the method of extreme values. Class H is the reference class

Quantity		**Class of facilities**		
		H	**N**	**M**
Target maximum distress probability, P_{dt}		1.0E-6	1.0E-5	1.0E-4
Target probability $P_{St} = P_{Rt}$		1.0E-3	3.16E-3	1.0E-2
Standardized random variables	u_S	2.457	2.238	1.976
	u_R	-2.155	-1.993	-1.792
Partial importance factors	γ_{nS}	1	0.95	0.89
	γ_{nR}	1	1.02	1.05
Global importance factor, γ_n		1	0.93	0.85

When facilities where the *internal user's attitude* prevails are dealt with, the annual value of the target probability $\dot{P}^{R}_{dt}$ has to be calculated using Equation (11.6). Then, for class K the target annual probability is found

$$\dot{P}^{K}_{dt} = m^{K}\dot{P}^{R}_{dt}$$

where m^{K} = differentiation multiplier (it must be $m^{K} < 1/\dot{P}^{R}_{dt}$). Using Equation (11.5) the comprehensive target maximum distress probability, $\bar{P}^{K}_{dt}$, is calculated from $\dot{P}^{K}_{dt}$. -

Observe that

$$\bar{P}^{K}_{dt} \neq m^{K}\bar{P}^{R}_{dt}$$

When, on the other hand, the *external user's attitude* governs, the comprehensive target probability follows directly from

$$\bar{P}^{K}_{dt} = m^{K}\bar{P}^{R}_{dt}$$

where, in general, the differentiation multiplier should be different from m^{K} in the internal user's case.

Theoretically, splitting P^{K}_{dt} in P^{K}_{At} and P^{K}_{Bt} we could apply Equations (13.8). That, however, would lead to *unacceptable changes in the design parameters*. Therefore, in the overdeterminate case, the transposition of target probabilities from class R to class K should always meet the following three requirements:

(a) when $m^{K} > 1$, it should be for both Ph(A) and Ph(B)

$$P^{K}_{At} \geq P^{R}_{At,emp}, \quad P^{K}_{Bt} \geq P^{R}_{Bt,emp}$$

when $m^{K} < 1$:

$$P^{K}_{At} \leq P^{R}_{At,emp}, \quad P^{K}_{Bt} \leq P^{R}_{Bt,emp}$$

(b) if, for example,

$$P^{R}_{At,emp} > P^{R}_{Bt,emp}$$

it should be also

$$P^{K}_{At} > P^{K}_{Bt}$$

(c) "relations" between P^{K}_{At} and P^{K}_{Bt}, resulting from transposition, should not differ "too much" from "relations" between the two existing empirical probabilities associated with class R.

It is not possible to define exactly the concepts "relation" and "too much," mainly because the target probabilities are empirical by themselves. An exact definition being not important, let us keep these concepts vague as they are.

The above requirements can be simply adjusted also to the case when $m^K < 1$.

Requirements (a) through (c) are easy to satisfy when *differences between the respective target probabilities*, that is,

$$\Delta P^K_{At} = P^K_{At} - \bar{P}^R_{At,emp}$$
$$\Delta P^K_{Bt} = P^K_{Bt} - \bar{P}^R_{Bt,emp}$$

are subjected to optimization. That is, the expression

$$\Delta P_{adv} = \Delta P^K_{At} + \Delta P^K_{Bt} - \Delta P^K_{At} \Delta P^K_{Bt}$$

analogous to Equation (13.5), is minimized. To solve the system

$$\frac{\partial \Delta P_{adv}}{\partial \Delta P^K_{At}} = 0, \quad \frac{\partial \Delta P_{adv}}{\partial \Delta P^K_{Bt}} = 0$$

a supplemental equation is needed. The relationship

$$(P^R_{At,emp} + \Delta P^K_{At})(P^R_{Bt,emp} + \Delta P^K_{Bt}) = m^K P^R_{At,emp} \tag{13.22}$$

does not lead to a solution meeting the requirements (a) through (c) mentioned above; it can result, in some cases, even in negative values of ΔP_{adv}. However, introducing [cf. Equation (13.9)]

$$\Delta P^K_{At} - \Delta P^K_{Bt} = 0 \tag{13.23}$$

and solving with Equation (13.22) for ΔP^K_{At}, we obtain

$$\Delta P^K_{At} = \frac{1}{2}\left\{[Q^2 + 4(m^K - 1)P^R_{dt,emp}]^{\frac{1}{2}} - Q\right\} \tag{13.24}$$

where

$$Q = P^R_{At,emp} + P^R_{Bt,emp}$$

Let us denote:

$$\frac{P^K_{At}}{P^R_{At,emp}} = m^K_A, \quad \frac{P^K_{Bt}}{P^R_{Bt,emp}} = m^K_B$$

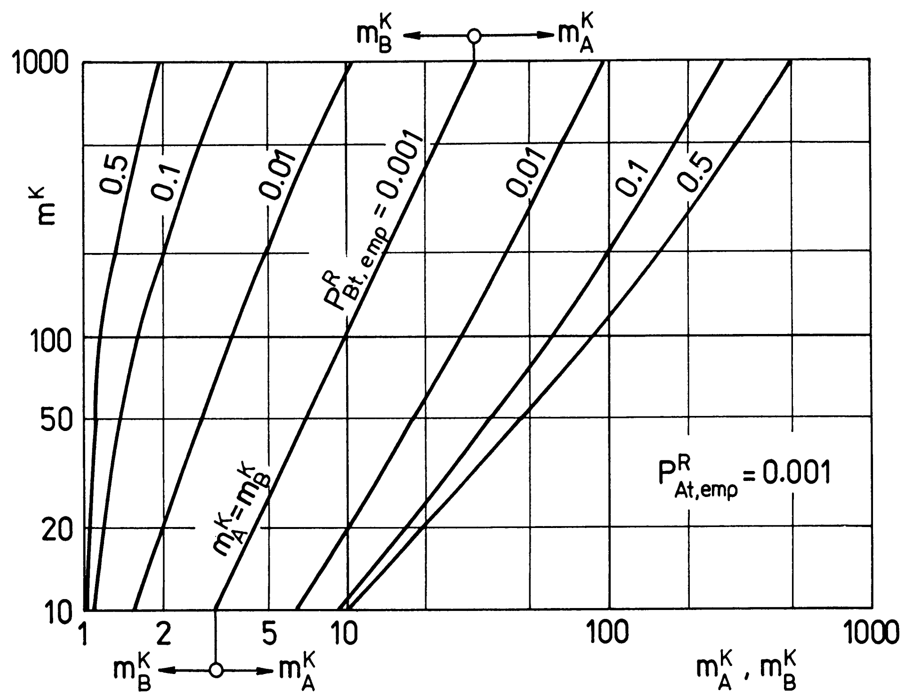

Fig. 13.7 - Partial differentiation multipliers.

Figure 13.7 shows, for $P^R_{At,emp} = 0.001$ and $P^R_{Bt,emp} \in [0.001; 0.5]$, the *partial differentiation multipliers* m_A^K and m_B^K dependent on the global differentiation multiplier m^K. Observe that in this case always $m_A^K > m_B^K$, and that the change in smaller probabilities is more notable. For example, when $m^K = 10$, $P^R_{At,emp} = 0.001$, and $P^R_{Bt,emp} = 0.5$, then P^K_{At} is almost ten times greater than the original value while P^K_{Bt} is only slightly greater. Thus the solution based on Equations (13.22) and (13.23) delivers results that satisfy requirements (a) through (c).

The described transposition procedure *minimizes distress at two levels:* first, relations between the initial target probabilities are not substantially modified (any greater change in design parameters would bring difficulties in practical design), and, second, *only distress resulting from additional adverse events is minimized.* This is true also for $m^K < 1$.

Notice that in the overdeterminate problem only possible distress caused by new decisions *is subjected to minimization! The* status quo *of design is not touched, it is implied that all participants have been happy with it.*

13.5 CODIFIED DESIGN FORMAT

Since in MEF the two parts of RelReq are clearly separated, the codification should be theoretically simple. Nonetheless, MEF is not widely codified, although from the viewpoint of codification it stands much better then the direct method.

Design parameters referred to resistances can be easily established for particular types of structures, but an economically efficient system of resistance factors, φ_R, would be clumsy. Further, owing to large variety of loads, the treatment of the load part of RelReq is also difficult. A combination of MEF with the method of extreme values (see Chapter 14) is possible: resistance is considered as one formative variable, while loads are investigated as elementary variables one-by-one. This approach has been used in the North American limit states design codes; for example ACI 318-89 is combined with ANSI A58.1-1982. In ACI 318-89 resistance factors, called "strength reduction factors," are specified for six types of stress-state: bending, axial compression, axial tension, shear, torsion, and bearing on concrete. However, the present values of strength reduction factors have not yet been established by probabilistic and statistical procedures.

The only problem met in MEF concerns cases where the left-hand side and right-hand side of the respective RelReq are dependent. Then, a statistically correct MEF solution is impossible and the direct method should be used instead.

13.6 MERITS AND DRAWBACKS

Merits of MEF:

♦ only two phenomena are examined;

♦ splitting RelReq in two parts enables separate, independent analyses of the attack and barrier;

♦ analysis of combinations of phenomena is contained in the analysis of the respective formative variable;

♦ the resistance part of MEF is physically testable;

♦ design based on MEF can be codified;

♦ MEF is a good basis for development of proof-testing techniques, quality control of simple prefabricated members, etc.

Drawbacks of MEF:

♦ information on failure probability is given in terms of an interval, not by one number;

♦ a complex system of resistance factor values would be necessary to obtain an economically efficient code;

♦ handling the LOAD system in general is difficult, almost impossible;

♦ the load part of MEF is not testable;

♦ exact handling cases where the attack and barrier are dependent is not possible.

14

METHOD OF EXTREME VALUES

Key concepts in this chapter: *method of extreme values, MEV; progressive decomposition of target probabilities; two cases of overdeterminacy; empirical target probabilities; notional probability of successive occurrence; notional minimization; problem of closed combination; snow and wind load combination; approximate formulas; differentiation problem in MEV; theoretical design format; load parameters; resistance parameters; central value; variability factor; infimum variability factor; adjustment factor; codified design format; semi-probabilistic design; characteristic value; design value; load combination factor; resistance combination factor; partial importance factors; partial reliability factors; composition formulas; decomposition of* γ *factors.*

14.1 PRINCIPLES

14.1.1 Decomposition of target probabilities

The method of extreme values, MEV, is based on the elementary probabilistic RelReqs (8.13) and (8.14). Its principles will be explained on a one-component case; both the attack and the barrier are expressed as scalar quantities. Similarly as in the previous chapter on MEF, the attack and barrier will be assumed *statistically independent.* The same assumption is made on elementary random variables a_i and b_j (as for the dependence problem, see 2.1.3).

RelReqs of MEV are satisfied if the design RelReq (7.18), that is,

$$A(a_{1,exm}, a_{2,exm}, ..., a_{na,exm}) \\ \leq B(b_{1,exm}, b_{2,exm}, ..., b_{nb,exm}) \tag{14.1}$$

is fulfilled; here $a_{i,exm}$, $b_{j,exm}$ = fractiles of the random elementary variables a_i, b_j (i = 1, 2, ..., n_a; j = 1, 2, ..., n_b), defined by

$$\Pr(a_{i,adv}) = P_{ait}, \quad i = 1, 2, ..., n_a \tag{14.2}$$

$$\Pr(b_{j,adv}) = P_{bjt}, \quad j = 1, 2, ..., n_b \tag{14.3}$$

The design RelReq (14.1) is a synthesis of the physical RelReqs (7.14) or (7.15), or (7.16), and of the two sets of probabilistic RelReqs (8.13) and (8.14).

To simplify, we will assume that the design RelReq (14.1) does not contain any *partial reliability factors* covering uncertainties and indefiniteness of random or non-random nature. - For ease in identification, let us denote

$$A(a_{1,exm}, a_{2,exm}, ..., a_{na,exm}) = A_d$$

$$B(b_{1,exm}, b_{2,exm}, ..., b_{nb,exm}) = B_d$$

Quantities A_d and B_d will be called the "design attack" and the "design barrier," respectively.

The target probabilities P_{ait} and P_{bjt} can be established by independent decisions based on some of the methods indicated in Section 10.3 for the target failure probability, P_{ft}. A more exact determination is based on the principles explained in Chapter 11; *decomposition* of target probabilities is employed.

First, P_{At} and P_{Bt} are found by decomposing the target maximum distress probability, P_{dt}, in the same manner as in the case of MEF, that is, using Equations (13.4) through (13.8). Then, further decomposition of P_{At} and P_{Bt} is to be carried out for P_{ait} and for P_{bjt}, respectively. The decomposition pattern is shown in Figure 14.1.

In MEV, the minimum distress requirement leads to the minimization of probabilities

$$P_{adv,A} = 1 - \prod_{i=1}^{n_a} (1 - P_{ait}) \tag{14.4}$$

$$P_{adv,B} = 1 - \prod_{j=1}^{n_b} (1 - P_{bjt}) \tag{14.5}$$

To find the target probabilities P_{ait}, the following system of equations is used:

$$\frac{\partial P_{adv,A}}{\partial P_{ait}} = 0, \qquad i = 1, 2, ..., n_a \tag{14.6}$$

$$\prod_{i=1}^{n_a} P_{ait} = P_{At} \tag{14.7}$$

where P_{At} = the target probability of adverse attack, established either from P_{dt} by Equations (13.8), or by decision.

Equations (14.6) and (14.7) give

$$P_{a1t} = P_{a2t} = ... = P_{a,na,t} = \sqrt[n_a]{P_{At}}$$

Similarly, a solution for P_{bjt} yields

$$P_{b1t} = P_{b2t} = ... = P_{b,nb,t} = \sqrt[n_b]{P_{Bt}}$$

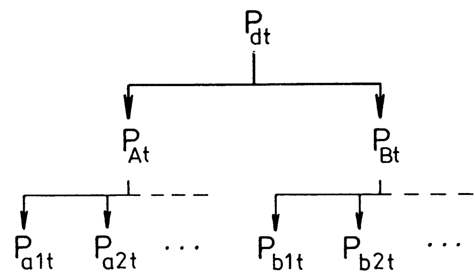

Fig. 14.1 - Decomposition of target probabilities.

Thus it must be

$$P_{ait} - P_{a,i+1,t} = 0, \quad i = 1, 2, ..., n_a - 1 \qquad (14.8)$$

$$P_{bjt} - P_{b,j+1,t} = 0, \quad j = 1, 2, ..., n_b - 1 \qquad (14.9)$$

Details of the decomposition technique will be demonstrated in 14.3.2 for loads, and in Section 14.4 for resistances.

Constraint RelReq

When the right-hand side of RelReq (14.1) is a fixed, non-random value (cf. Section 10.5), the principles of MEV do not change. As the constraint RelReqs are usually based on deformation criteria of various kind, the attack embodies load, resistance, and geometry variables in one function, which is non-linear, as a rule. The general routine used for RelReqs with both sides random remains valid.

14.1.2 Interval of the failure probability

When a design based on RelReq (14.1) is economical and without indefiniteness (that is, no decision-based design parameters, such as partial reliability factors, γ, are involved), the interval of the failure probability P_f is defined by

$$\prod_{i=1}^{n_a} P_{ait} \cdot \prod_{j=1}^{n_b} P_{bjt} \leq P_f \leq \max_{i,j}(P_{ait}, P_{bjt}) \qquad (14.10)$$

We can obtain it by reasoning analogous to that in 13.1.2. This interval is valid also for cases when the barrier is a constraint, and all random variables participating in RelReq are contained in the attack, A.

14.2 COMBINATIONS OF ADVERSE EVENTS

14.2.1 Problem statement

While in DM and MEF the problem of combinations of phenomena constituting RelReqs is solved implicitly, it must be paid a special attention in MEV. Actually, in design RelReqs, we are concerned about expressing the fact that the probability of simultaneous occurrence of adverse events is always smaller than the probability of their isolated occurrence (Section 3.7). In the higher level methods this is done automatically in the analysis of the respective variables.

The following basic requirement can be stated:

The level of reliability provided by all design RelReqs should be equal, independent of the number of phenomena involved.

In other words, the target probability of a single relatively adverse event in an existential combination of order $\nu > 1$ shall be greater than the respective target probability associated with an isolated phenomenon. Or, considering a phenomenon H_i combined with further $\nu - 1$ phenomena, the set ${}^{\nu}\dot{E}_{i,adv}$ of its relatively adverse occurrences is a subset of ${}^{\nu+1}\dot{E}_{i,adv}$. Therefore, we would like to have

$$ {}^{1}\dot{E}_{i,adv} \subset {}^{2}\dot{E}_{i,adv} \subset \dots \subset {}^{n}\dot{E}_{i,adv}, \qquad i = 1, 2, \dots, n $$

where n = number of all events that can be physically combined.

The set $\dot{E}_{i,adv}$ of relatively adverse events $E_{i,adv}$ is defined by the bound $x_{i,exm}$ defined as the κ-fractile of the random variable ξ_i describing H_i (see Section 3.6, Figure 3.5). The probability κ is given by Equation (3.5) or (3.6). Thus, to each set ${}^{\nu}\dot{E}_{i,adv}$ a different probability κ corresponds, that is, a different target probability of occurrence of the relatively adverse event, $\Pr({}^{\nu}E_{i,adv})$. The greater ν, the less the target probability. For the individual values of ν, the target probabilities are ranked

$$ \Pr({}^{1}E_{i,adv}) < \Pr({}^{2}E_{i,adv}) < \dots < \Pr({}^{n}E_{i,adv}) $$

Thus, the solution of the combination problem in MEV consists in ***ranking of target probabilities.***

Similarly as in the transposition problem (Section 13.4), two variants of the ranking problem are distinguished: *determinate* and *overdeterminate*. It is also necessary to take into account the quality of the respective existential combination.

Notation

Let in any existential combination of order ν *the relatively adverse events be consecutively numbered* 1 *through* ν. In general, this numbering need not be identical with the numbering of members of the original set of phenomena H_1 through H_n.

The following notation will be used for target probabilities:

${}^{v}P_{it}$ = target probability of occurrence of the relatively adverse event ${}^{v}E_{i,adv}$ that is in existential combination of order v with relatively adverse events No. 1 through i -1 and No. i +1 through v. This symbol will be used in general solutions.

In specific cases, the target probabilities will be denoted in a graphic manner:

$P_{2(13)t}$ = target probability of the relatively adverse event ${}^{3}E_{2,adv}$ in the existential combination of order 3 with events ${}^{3}E_{1,adv}$ and ${}^{3}E_{3,adv}$;

$P_{s(w)t}$ = target probability of Ev(**adverse snow load**) in combination with Ev(**adverse wind load**), and similarly,

$P_{w(s)t}$ = target probability of Ev(**adverse wind load**) in combination with Ev(**adverse snow load**).

14.2.2 Determinate problem

In the ranking of probabilities the determinate problem is specified as follows:

Let the target probability P_t of occurrence of adverse realizations of a formative phenomenon (that is, P_{At} or P_{Bt}) be given. Let the respective formative phenomenon comprise elementary phenomena H_1, H_2, ..., H_n. The target probabilities ${}^{v}P_{it}$ of relatively adverse events ${}^{v}E_{i,adv}$ (i = 1 through v) for individual existential combinations of order v shall be established.

Obviously, the problem is marked as "determinate" because the target probabilities of ${}^{v}E_{i,adv}$ in various combinations are derived from a given single P_t. *No other reliability parameters enter the solution.*

The probabilistic RelReqs (8.11) and (8.12) should be met for any existential combination k, and so the target probabilities of occurrence of all individual combinations of relatively adverse events must equal P_t:

$$\forall k: \ {}^{v}P_{Ckt} = P_t, \qquad v = 1, 2, \ldots, n \tag{14.11}$$

where n = number of phenomena.

The probability P_t (that is, P_{At} or P_{Bt}) is the probability of maximum distress associated with the formative phenomenon investigated. Therefore, we can subject P_t to operations similar to those related with P_{dt} in Section 14.1. By analogy with Equation (11.4) we can write

$$\prod_{i=1}^{v} {}^{v}P_{it} = P_t \tag{14.12}$$

which is valid for any existential combination of order v.

Again, values of ${}^{v}P_{it}$ are established by *minimizing the probability of occurrence of at least one of possible relatively adverse events,* ${}^{v}P_{adv}$, given by

$${}^{v}P_{adv} = 1 - \prod_{i=1}^{v} (1 - {}^{v}P_{it}) \tag{14.13}$$

Table 14.1 - Values of ranked target probabilities of occurrence of relatively adverse events, ${}^{v}P_{it}$ (Roman), and the respective values of P_{adv} *(Italics)*, in dependence on the target probability of maximum distress, P_{dt}, and on the order of existential combination, v

Target probability of maximum distress, P_{dt}	**Order of combination, v**		
	2	**4**	**6**
0.5	0.707	0.841	0.891
	0.914	*0.999*	*1.000*
0.25	0.5	0.707	0.794
	0.750	*0.993*	*1.000*
0.1	0.316	0.562	0.681
	0.532	*0.963*	*0.999*
0.01	0.1	0.316	0.464
	0.190	*0.781*	*0.976*
0.001	0.032	0.178	0.316
	0.062	*0.543*	*0.898*
1.0E-6	1.0E-3	0.032	0.1
	0.002	*0.121*	*0.469*
1.0E-9	3.2E-5	5.6E-3	0.032
	6.3E-5	*0.022*	*0.175*

The equation system

$$\frac{\partial\,{}^{v}P_{adv}}{\partial\,{}^{v}P_{it}} = 0, \qquad i = 1, 2, \ldots, v \tag{14.14}$$

and Equation (14.12) yield

$${}^{v}P_{it} = \sqrt[v]{P_t}, \qquad i = 1, 2, \ldots, v \tag{14.15}$$

and further

$$^{\nu}P_{it} - {}^{\nu}P_{i+1,t} = 0, \qquad i = 1, 2, \ldots, \nu - 1 \tag{14.16}$$

Solving for all ν, a sequence of *ranked target probabilities of occurrence of relatively adverse events,* $E_{i,adv}$, is obtained:

$$^{1}P_{it} < {}^{2}P_{it} < \ldots < {}^{\mu}P_{it}$$

where μ = highest order of combination in which the phenomenon H_i occurs.

Table 14.1 gives values of $^{\nu}P_{it}$ in dependence on the combination order ν and on the target probability P_t. For illustration, the corresponding values of $^{\nu}P_{adv}$ are also shown. Probabilities $^{\nu}P_{it}$ and probabilities $^{\nu}P_{adv}$ grow with the increasing number of phenomena involved. *Do not forget that* P_{adv} *is only an auxiliary quantity!*

Observe that

$$({}^{\nu}P_{it})^{\nu} - {}^{1}P_{it} = 0, \qquad i = 1, 2, \ldots, n_a \tag{14.17}$$

14.2.3 Overdeterminate problem

The ranking problem becomes overdeterminate when for n isolated phenomena the target probabilities of occurrence of adverse events have already been *fixed by decisions* as a result of past experience, tradition, established practice, etc. Denoting

$$\Pr({}^{1}E_{i,adv}) = {}^{1}P'_{it} \tag{14.18}$$

we have, in general,

$$|{}^{1}P'_{it} - {}^{1}P'_{i+1,t}| > 0 \tag{14.19}$$

at least for one of $i = 1, 2, \ldots, n-1$.

The problem is also overdeterminate when *relationships* among the target probabilities $^{1}P'_{it}$ have been fixed by some rule. For example, the ratios

$$\alpha_i = \frac{{}^{1}P'_{it}}{{}^{1}P'_{1t}}, \qquad i = 2, 3, \ldots, n \tag{14.20}$$

can be given so that

$$\alpha_i \neq 1 \tag{14.21}$$

for at least one of $i = 2, \ldots, n$.

From the viewpoint of the probability ranking both cases of the overdeterminacy are qualitatively equal, but their origins are different.

Here, all *primed symbols* indicate values reached without any reliability analysis.

Tradition-based overdeterminacy

The overdeterminacy specified by Equations (14.18) and (14.19) is met in the current design methods using *"traditional," mainly empirical design values* x'_{it} of input variables ξ_i. Values x'_{it} have been fixed as a result of long-time experience, step-by-step developments of design codes, etc. Applying appropriate probability distributions, empirical, though never perceived, target probabilities ${}^1P'_{it}$ of events Ev($\xi_i > x'_{it}$) or Ev($\xi_i \leq x'_{it}$) can be calculated.

It can be concluded from Section 14.1 that an inequality of probabilities ${}^1P'_{it}$ is not, from the viewpoint of a theoretical minimization of distress, advantageous. Yet, as a rule, it is not possible to change the values of input quantities embedded in the present design codified format of MEV suddenly and substantially (see Section 14.6). Drifts of *practical difficulties* would be created not only in the area of design, but also in testing, quality control of materials and precast members, evaluation of loading tests, and finally also in the execution of structures. Therefore, in ranking the target probabilities, we will stick to similar principles as those presented in 13.4.2 in the transposition process:

(a) for any $E_{i,adv}$ it should be

$$ {}^1P'_{it} < {}^vP_{it}, \quad v = 1, 2, ..., n $$

(b) if, for example,

$$ {}^1P'_{1t} > {}^1P'_{2t} > ... > {}^1P'_{nt} $$

it should also be

$$ {}^vP_{1t} > {}^vP_{2t} > ... > {}^vP_{nt} $$

(c) "relations" among ranked probabilities ${}^vP_{it}$, should not differ "too much" from "relations" among the existing empirical probabilities ${}^1P'_{it}$.

These requirements are fulfilled if the solution is based on *minimization of the sum of relative increases of ranked probabilities* ${}^vP_{it}$ *with respect to the outset probabilities* ${}^1P'_{it}$. We can benefit of knowledge gained in 14.2.2 on the determinate problem; there the *notional order of magnitude of the target probability decreases with the increasing order of the adverse events combination.* Therefore, we will minimize

$$ \pi = \sum_{i=1}^{v} \frac{{}^vP^v_{it} - {}^1P'_{it}}{{}^1P'_{it}} \qquad (14.22) $$

by solving the equation system

$$\frac{\partial \pi}{\partial {}^{v}P_{it}} = 0, \qquad i = 1, 2, ..., v$$

$${}^{v}P_{1t} \cdot {}^{v}P_{2t} \cdot ... \cdot {}^{v}P_{vt} = P_t$$

It results:

$$\forall i: \ {}^{v}P_{it} = \sqrt[v]{P_t} \left[\frac{({}^{1}P'_{it})^{v}}{\prod_{j=1}^{v} {}^{1}P'_{jt}} \right]^{\frac{1}{v^2}} \tag{14.23}$$

It now remains to decide what should be the target probability P_t occurring in Equation (14.23). It is obvious that its value has to be in some relation to the existing values of the target probabilities ${}^{1}P'_{it}$, found from x'_i and related to the events entering the combination investigated.

A ***notional minimization*** should start with the supplemental equation

$$\prod_{i=1}^{v} {}^{1}P_{it,thr} = P_{thr} \tag{14.24}$$

where P_{thr} = theoretical probability that the events ${}^{1}E_{i,adv}$ (with $i = 1, 2, ..., v$) will follow in an arbitrary sequence. As a result, we should have [cf. Equation (14.16)]

$${}^{1}P_{it,thr} - {}^{1}P_{i+1,t,thr} = 0, \quad i = 1, 2, ..., v-1$$

and

$${}^{1}P_{it,thr} = \sqrt[v]{P_{thr}}, \quad i = 1, 2, ..., v \tag{14.25}$$

This is in conformity with the requirement that for all combinations of any order (that is, also for combinations of order 1) the target probabilities P_t should be equal, or

$$P_t = {}^{1}P_{it,thr}, \quad i = 1, 2, ..., v \tag{14.26}$$

We can assume that the present diversity of target probabilities ${}^1P'_{it}$ arose from a ***primordial minimization*** of isolated occurrences of v adverse phenomena. The existing "state of probabilities" is, in general,

$$ {}^1P'_{1t} \neq {}^1P'_{2t} \neq \dots \neq {}^1P'_{vt} $$

with a *notional empirical probability,* P_{emp}, *that events* ${}^1\mathrm{E}_{1,adv}$ through ${}^1\mathrm{E}_{v,adv}$ *follow in an arbitrary sequence,*

$$ P_{emp} = \prod_{i=1}^{v} {}^1P'_{it} \qquad (14.27) $$

Note that P_{emp} refers only to the set of phenomena H_1 through H_v, forming the particular combination. For other sets, different P_{emp} values would be obtained.

From Equations (14.25) and (14.26) we get

$$ P_t = \sqrt[v]{P_{thr}} $$

By analogy we can set

$$ P'_t = \sqrt[v]{P_{emp}} \qquad (14.28) $$

Now, introducing

$$ P_t = P'_t $$

into Equation (14.23) and applying Equations (14.27) and (14.28), we obtain

$$ {}^vP_{it} = \sqrt[v]{{}^1P'_{it}}, \quad i = 1, 2, \dots, v \qquad (14.29) $$

This equation meets the requirements (a) and (b), Figure 14.2. The requirement (c) is obviously also satisfied; we have:

$$ \frac{\log {}^vP_{it}}{\log {}^1P'_{it}} = \frac{1}{v}, \qquad i = 1, 2, \dots, v $$

The ratio of logarithms of the ranked target probabilities is, for combinations of order ν, constant for all phenomena H_1 through H_ν.

It is necessary to say that the assumption on the primordial minimization of adverse events is not always complied with. To minimize the number of adverse events, the causes of random structural failures should be about uniformly distributed among subsystems of the S-L-E system. However, the reality is rather different; we know that random failures due to erroneous estimate of load parameters or environmental parameters have been more frequent than failures due to incorrect estimate of resistance parameters. This fact is not surprising, because the behavior of structures is much better explored than the behavior of loads and environment. However, there is also another reason for discrepancies in the empirical target probabilities. Since the early beginnings of construction activities, constructors had been cautious to design and construct buildings, bridges, dams and other facilities so, as to avoid getting blamed for possible structural failures. It had been always less risky to attribute the failure to some force majeure, a god, goddess or just to the Nature than to the responsible constructor.

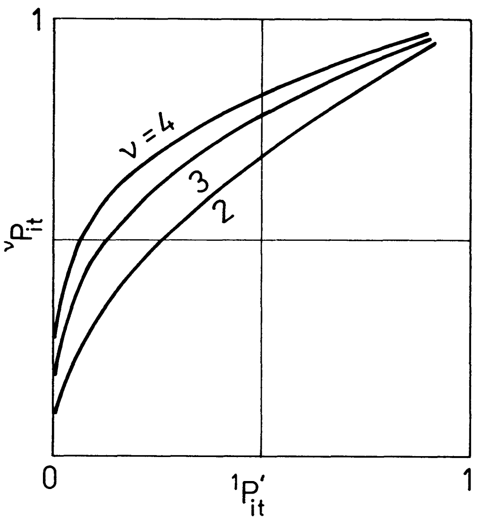

Fig. 14.2 - Graphical interpretation of Equation (14.29).

Decision-based overdeterminacy

The overdeterminacy according to Equations (14.20) and (14.21) arises when in the decomposition of target probabilities one or several events ${}^1E_{i,adv}$ are supposed to be more important than others, that is, the minimum distress requirement is intentionally abandoned. For example, failure of a reinforced concrete member by crushing of concrete is usually considered more dangerous than failure due to yield of reinforcement. - The difference in importance can be expressed by the multiplier α_i. For example, the target probability of occurrence of an adverse yield stress of reinforcement, P_{st}, can be taken ten times greater than the target probability P_{ct} assigned to adverse compression strength of concrete.

Then, if a value of the target probability ${}^1P_{t1}$ is accepted, we can write

$$
\begin{aligned}
{}^1P_{t1}^{\prime} &= {}^1P_{t1} \\
{}^1P_{t2}^{\prime} &= \alpha_2\,{}^1P_{t1} \\
\dots &\quad \dots \\
P_{tn}^{\prime} &= \alpha_n\,{}^1P_{t1}
\end{aligned}
$$

and then proceed as in the first case of overdeterminacy.

14.2.4 Problem of closed combinations

Consider an existential combination of v phenomena where, for example, phenomenon H_1^* is primary; the presence of H_1^* is indispensable for secondary phenomena H_2^{**} through H_v^{**} in the respective combination. In such a case, the relations among combined phenomena are *not equivalent;* this fact must be regarded in the solution. *Two conditions* have to be investigated:

Condition 1: All realizations of H_1^* are considered adverse, taking H_1^* as superior to the remaining combined phenomena. Thus we have

$$\dot{E}_{1,adv} = \dot{H}_1^*$$

For combination of any order, the target probability of occurrence of a relatively adverse primary event $E_{1,adv}^*$ is

$$P_{1t}^* \equiv \Pr(E_{1,adv}^*) = 1$$

Then, for an existential combination of secondary events ${}^vE_{i,adv}^{**}$ with the primary event $E_{1,adv}^*$ the relationship

$$P_{1t}^* \cdot {}^vP_{2t}^{**} \cdot \dots \cdot {}^vP_{vt}^{**} = P_t$$

must be considered. However, the target probability P_{1t}^* has been fixed, $P_{1t}^* = 1$, and so the decomposition technique has to be applied only to the remaining (v -1) target probabilities, ${}^vP_{it}^{**}$. That is, a combination of order (v -1) is examined; it results

$${}^vP_{it}^{**} = \sqrt[v-1]{P_t}, \quad i = 2, \dots, v$$

For absolutely adverse phenomena the target probability $P^*_{1t} = 1$ is associated with the *infimum,* $x_{1,inf}$, of the random variable ξ_1 describing the respective phenomenon. For absolutely favorable phenomena, the *supremum,* $x_{1,sup}$, refers to $P^*_{1t} = 1$. However, probability distributions used for the description of random variables often have

$$x_{1,inf} \to -\infty, \quad x_{1,sup} \to \infty$$

Such values are obviously not acceptable for design. It is therefore suggested to take

$$P^*_{1t} = 0.1\,{}^1P_{1t}, \qquad P^*_{1t} = 1.0\text{E}-4$$

whichever is lesser. Obviously, ${}^1P_{1t} = P_t$ in the determinate case, ${}^1P_{1t} = {}^1P'_{1t}$ in the overdeterminate case.

Condition 2: It is not important, from the viewpoint of separate combinations, whether a phenomenon is primary or not; the probabilistic RelReqs (8.13) and (8.14) must be complied with for all combined phenomena, and so

$${}^{v}P_{1t} \cdot {}^{v}P_{2t} \cdot \ldots \cdot {}^{v}P_{vt} = P_t$$

The optimization gives

$${}^{v}P_{it} = \sqrt[v]{P_d}$$

Let us show the significance of these two conditions considering the following simple case:

Assume that a one-component attack A is a function of two variables, a_1 and a_2, describing absolutely adverse phenomena H_1, H_2, that are in an existential relation (H_2 [H_1]). Both variables are random with PDFs in Figure 14.3a,b. RelReqs corresponding to the two foregoing conditions must be fulfilled for two design values of the attack:

- condition 1:

$$A_d = A(a_{1,inf}, {}^1a_{2,exm})$$

- condition 2:

$$A_d = A({}^2a_{1,exm}, {}^2a_{2,exm})$$

where $a_{1,inf}$ = infimum of the probability distribution of the random variable a_1; the extreme values can be found from (Figure 14.3b,c,d)

$$P^{**}_{a2(a1)t} \equiv \Pr(a_2 > {}^1a_{2,exm}) = P_{At}$$

$$P_{a1(a2)t} \equiv \Pr(a_1 > {}^2a_{1,exm}) = \sqrt{P_{At}}$$

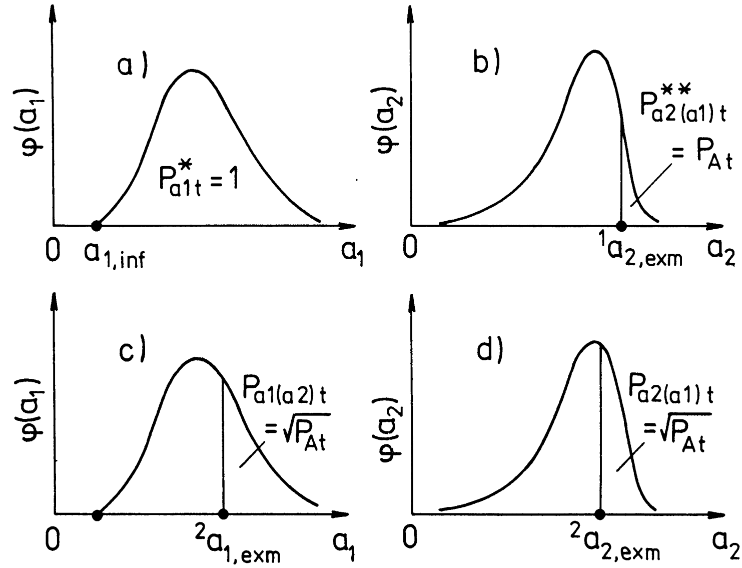

Fig. 14.3 - Determination of extreme values for two absolutely adverse phenomena in an existential relation $(H_2[H_1])$.

$$P_{a2(a1)t} \equiv \Pr(a_2 > {}^2a_{2,exm}) = \sqrt{P_{at}}$$

Similar technique is to be used if any of the phenomena is absolutely favorable.

14.3 LOAD AND LOAD-EFFECTS

14.3.1 Design format

The following considerations refer to the *theoretical design format* based on MEV. It is not identical with the codified format, known as the *semi-probabilistic limit states design format,* which is now specified in many national and international codes; we will discuss it in Section 14.6. However, the two formats are very close.

The theoretical MEV design format includes:

♦ *central value of load*

$$F_{cnt} = \mu_F \tag{14.30}$$

♦ *extreme value of single load*

$$^{1}F_{exm} = \mu_F + {}^{1}u\,\sigma_F$$

♦ *variability factor for load*

$$\Gamma_F = \frac{^{1}F_{exm}}{F_{cnt}} \tag{14.31}$$

♦ *load combination factor*

$$^{\nu}\psi_0 = \frac{^{\nu}F_{exm}}{^{1}F_{exm}} \tag{14.32}$$

In the above Equations μ_F, σ_F = population mean and standard deviation of the load F, $^{1}F_{exm}$, $^{\nu}F_{exm}$ = extreme values of F when the load is isolated (that is, in "combination" of order 1), or combined in a load combination of order $\nu > 1$, respectively, ^{1}u = value of standardized random variable specified for given $\Pr(F > {}^{1}F_{exm})$.

The design parameters $F_{cnt} = \mu_F$, $^{1}F_{exm}$, and $^{\nu}\psi_0$ are established in probabilistic way using data and decisions taken beyond the design format. They can be considered *principal.*

The variability factor Γ_F, being derived from the two principal parameters, is an *auxiliary quantity* giving a non-dimensional picture of the random behavior of the respective load. It involves not only the dispersion but also the asymmetry of the probability distribution. To avoid misunderstandings, it must be remarked here that in the codified design format the partial reliability factors γ are non-statistical and non-probabilistic, expressing such features of load and material that cannot be treated by statistical and probabilistic analysis (see Section 14.6).

It results from the discussion in 14.2.3 that for loads that can become primary also an *infimum factor,* Γ_{inf}, defined by

$$\Gamma_{inf} = \frac{F_{inf}}{F_{cnt}} \tag{14.33}$$

or also an *infimum combination factor*

$$\psi_{0,inf} = \frac{F_{inf}}{^{1}F_{exm}} \tag{14.34}$$

independent of the number of combined loads, ν, should be specified. Here, F_{inf} = infimum value of the respective load, either effective, or notional, defined for P^{*}_{1t}. The latter two factors are not embedded in the codified design format.

Observe that the distinction between design parameters related to an isolated primary load and those of a primary load combined with secondary loads is expressed in ANSI

A58.1-1982. There, dead load is factored by 1.1, or by 1.0, when acting alone, or when combined with other loads, respectively.

We should keep in mind that the above theoretical *load parameters* are based only on the statistical description of the random behavior of elementary variables. Thus, only a specific part of uncertainties is expressed, while load-related indefiniteness (see Section 1.1) and model uncertainties are not embedded in these parameters. The set of design parameters should be supplemented by *adjustments factors* covering, where appropriate, non-random and also random deviations from calculation models.

14.3.2 Load combinations

Although a transition from codified to theoretical design format cannot be expected in the nearest future (we may ask whether it can be expected at all, see Section 16.2), some solutions based on the latter can already be used in the investigation of load combinations. As it has been mentioned in Chapter 12, the load combination problem does not exist in the direct method, and it has specific features in the method of extreme functions (Section 13.2).

At present, we can assert that *the load combination problem has been already solved,* though several solution concepts exist. The various approaches available do not differ in principles. The main differences are in the depth of the probabilistic reasoning and in approximations and simplifications used. Surprisingly, in terms of design load-effects, the results do not substantially differ. Good information on various aspects of the load combination problem can be gained from Wen 1990.

There are two main concepts that must be treated in the load combination problem:

(a) combination rules;
(b) load combination factors.

Assume that a structure is subjected to n loads; a combination of v loads, $v \leq n$, is to be investigated. Assume also that for load-effects the law of superposition is valid. The structure has to be designed for a scalar load-effect, ${}^{v}S_{Cd}$, taking into account the load combination properties, that is, in particular, their probability aspects.

For load-effects a *combination rule,* consisting of m combination formulas and the selection formula, can be formulated:

$$\begin{aligned} S_C^{(k)} &= {}^{v}\psi_{01}^{(k)} S_{1d} + {}^{v}\psi_{02}^{(k)} S_{2d} + \dots + {}^{v}\psi_{0n}^{(k)} S_{nd} \\ {}^{v}S_{Cd} &= \operatorname*{adv}_{k}(S_C^{(k)}), \qquad k = 1, 2, \dots, m \end{aligned} \tag{14.35}$$

where S_{id} = load-effect due to design load F_{id}, ${}^{v}\psi_{0i}^{(k)}$ = load combination factor associated with S_{id} combined with (v-1) loads in the combination formula k, adv(.) = most adverse of load-effects obtained by combination formulas. For simplicity, the numbering of loads is independent in each combination.

In general, the individual ${}^{v}\psi_{0i}^{(k)}$ factors are not mutually equal. Further, by definition,

$$0 < {}^{v}\psi_0 \leq 1 \tag{14.36}$$

We can never have ${}^{v}\psi_0 \leq 0$, since in such a case the respective load would drop out of the combination; the order of combination would decrease by one, and the respective formula should be considered in another combination rule.

Theoretically, the number of combination formulas, m, can be infinite; this of course is technically impossible. Therefore, in codes one or two combination formulas are specified in a combination rule. Designers always fight for minimizing the number of combination rules and formulas while many load experts insist that the number of codified rules and formulas is insufficient. Many combination rules exist in codes and literature (for example, the *Turkstra's rule,* see Wen 1990).

All existing combination rules, for both scalar and vector load combinations, can be expressed in terms of Equations (14.35).

The problem becomes complicated in cases where the superposition law is not valid. Then, a higher level solution must be employed (for example, a MEF solution) and the problem of load combination factors disappears.

It is obvious that ${}^{v}\psi_0$ values are formula-dependent. When a particular combination formula is accepted, it is necessary to find the value of one or more ${}^{v}\psi_{0i}$ factors so as to obtain

$$\Pr\left(\sum_{i=1}^{v} S_i \geq \sum_{i=1}^{v} {}^{v}\psi_{0i} S_{id}\right) = P_t \tag{14.37}$$

where S_i = load-effect due to random load F_i, P_t = intended target probability. Two extreme possibilities exist in fixing ${}^{v}\psi_{0i}$ factors for the formula:

(a) v-1 values are fixed by decision, the value of only one factor has to be found from Equation (14.37);

(b) it is accepted that

$$\forall i: \quad {}^{v}\psi_{0i} = {}^{v}\psi_0$$

and so again only one value, ${}^{v}\psi_0$, has to be determined.

Neither of these two solutions is good, nor is there any particular reason for using the former or the latter. Therefore, let us try to base the solution on MEV concepts stated in Section 14.2.

Using Equation (2.20) we can write

$$F_{exm} = \mu_F + u\,\sigma_F$$

where μ_F = population mean of the load magnitude F, σ_F = standard deviation, and u = value of the standardized random variable obtained for given target probability P_{Ft} of occurrence of relatively adverse load magnitudes, F_{adv}. Then, Equation (14.32) can be written as

$$ {}^{v}\psi_0 = \frac{1 + {}^{v}u\,\delta_F}{1 + {}^{1}u\,\delta_F} \tag{14.38} $$

where ${}^{1}u$ and ${}^{v}u$ refer to the probabilities ${}^{1}P_{Ft}$ and ${}^{v}P_{Ft}$, respectively, found by the technique explained in Section 14.2.

The probability distribution of the load magnitude is defined by Equations (5.3) through (5.5) where we have to set for P_{occ} the *probability of occurrence of the respective load combination* (that is, combination of *any values* of combined loads), $P_{occ,C}$. It is given by

$$ P_{occ,C} = \prod_{i=1}^{v} P_{occ,i} \tag{14.39} $$

where $P_{occ,i}$ = probability of occurrence of the load i , determined from Equation (5.1). When

$$ \sqrt[v]{P_{Ft}} > P_{occ,C} $$

the respective load cannot apply in design, and so the investigated design combination of adverse load magnitudes does not exist. This does not mean, of course, that no combination of arbitrary values of the corresponding loads (that is, also values less than the respective fractiles) is possible.

Similarly, when calculating $\Phi(F)$ from Equation (5.3), a value $\Phi(F) < 0$ can be obtained. Since $\Phi(F) \in [0;1]$, the negative result signifies again that the respective load combination is meaningless. The same happens when the calculated load combination factor becomes $\psi_0 < 0$.

■ **Example 14.1.** Find the combination factor $\psi_{w(s)}$ referred to the wind load, w, combined with snow load, s , and also the load combination factor $\psi_{s(w)}$ referred to the snow load combined with wind load. The input parameters of the solution are given in Table 14.2.

Log-normal probability distribution of annual maxima is assumed for both loads. The periods T_0, T^*, and T^*_{occ} are considered non-random; the value of T^{ret} has been fixed by previous long-time experience.

The target annual probability of adverse single load value, $F > {}^{1}F_{exm}$, follows from Equation (2.49):

$$ \dot{P}_{Ft} = \frac{1}{T^{ret}} $$

that is,

$$ \dot{P}'_{st} = \frac{1}{100}, \quad \dot{P}'_{wt} = \frac{1}{80} $$

Table 14.2 - *Example 14.1.* Input data

Parameter	Wind	Snow
Variation coefficient, δ_F	0.4	0.6
Coefficient of skewness, α_F	1.0	1.0
Period of Ph(load) occurrence, T^*_{occ}**, per year**	0.5 y	0.08 y or 0.30 y
Target return period, T^{ret}**, of relatively adverse load values, that is, of** $Ev(F > {}^1F_{exm})$	80 ys	100 ys
Value of standardized random variable for isolated load	2.87	3.03
Target life of the facility, T_0		80 ys
Annual duration of the winter period (that is, the period when the snow load can occur), T^*		0.4 y

Thus, for target life T_0 = 80 years the empirical target probabilities of adverse snow load, $\bar{P}'_{st}$, and adverse wind load, $\bar{P}'_{wt}$, are according to Equation (8.5):

$$\bar{P}'_{st} = 1 - (1 - \frac{1}{100})^{80} = 0.552$$

$$\bar{P}'_{wt} = 1 - (1 - \frac{1}{80})^{80} = 0.634$$

A *tradition-based overdeterminate problem* is obviously dealt with; the notional probability of occurrence of an adverse load combination is established from Equations (14.27) and (14.28)

$$\bar{P}_t \equiv \bar{P}'_t = \sqrt{\bar{P}'_{st} \cdot \bar{P}'_{wt}} = 0.592$$

The target probabilities of occurrence of combined relatively adverse load values are obtained from Equation (14.23), or, in this case, directly from Equation (14.29):

$$\bar{P}_{w(s)t} = \sqrt{\bar{P}'_{wt}} = 0.797$$

$$\bar{P}_{s(w)t} = \sqrt{\bar{P}'_{st}} = 0.743$$

The probability of occurrence of the wind load (that is, of wind load value exceeding a specified level) at any point in time of the winter period is, assuming that the random process of wind velocity is stationary, according to Equation (5.1)

$$P_{occ,w} \equiv \frac{T^*_{occ,w}}{T^*} = 0.5$$

and for snow load, for $T^*_{occ,s} = 0.08$ y:

$$P_{occ,s} \equiv \frac{T^*_{occ,s}}{T^*} = 0.20$$

For $T^*_{occ,s} = 0.3$ y it is $P_{occ,s} = 0.75$; in the next text of this Example, values referred to the latter occurrence period are given in parentheses. - According to Equation (14.39) the probability of occurrence of the (**snow load & wind load**) combination is

$$P_{occ,ws} \equiv P_{occ,s} \cdot P_{occ,w} = 0.100 \quad (0.375)$$

Let us first find the *load combination factor for wind load combined with snow load,* $\psi_{w(s)}$. Assume that in the relevant geographic region the annual maximum of wind velocity can occur at any day during a calendar year (this is not a general rule; in many regions annual peak wind velocities appear systematically during particular seasons only). Consequently, when treating wind load, we have to consider only that period when the adverse combination (**snow load & wind load**) is apt to occur, that is, the total length of all winter periods of the specified target life. It is

$$T = T^* \cdot 80 = 32 \ ys$$

Equation (8.6) can be written as

$$\dot{P}_{Ft} = 1 - (1 - \bar{P}_{Ft})^{\frac{1}{T}}$$

Introducing

$$\dot{P}_{Ft} = \frac{1}{T^{ret}_{w(s)}}, \quad \bar{P}_{Ft} = \bar{P}_{w(s)t}$$

the expression for the mean return period, $T^{ret}_{w(s)}$, of relatively adverse wind load values, $w > w_{s,exm}$, combined with relatively adverse snow load, $s > s_{exm}$, is obtained after rearrangement

$$T^{ret}_{w(s)} \equiv \frac{1}{1 - \left[1 - (\bar{P}_{w(s)t})^{1/T}\right]} = 20.6 \ \ \text{ys}$$

From $T^{ret}_{w(s)}$ the probability of Ev($w \le w_{s,exm}$) can be found:

$$\Pr(w \le w_{s,exm}) \equiv \Phi_{gen,w}(w_{s,exm}) \equiv 1 - \frac{1}{T^{ret}_{w(s)}} = 0.951$$

With $P_{occ} = P_{occ,ws}$, we obtain from Equation (5.3):

$$\Phi_w(w_{s,exm}) \equiv 1 - \frac{1 - \Phi_{gen,w}(w_{s,exm})}{P_{occ,ws}} = 0.510 \quad (0.869)$$

As the coefficient of skewness of the annual wind load maxima is $\alpha_w = 1.0$, the value of the standardized random variable is (see Appendix A)

$$u_{w(s)} = -0.13 \quad (1.10)$$

and the load combination factor follows from Equation (14.38):

$$\psi_{w(s)} = \frac{1 + u_{w(s)}}{1 + {}^1u_w \, \delta_w} = 0.44 \quad (0.67)$$

Table 14.3 - *Example 14.1.* Values of quantities depending upon duration of the snow occurrence period. Roman: wind related values, *italics:* snow related values

Quantity	Annual period of snow occurrence	
	0.08 y	**0.3 y**
Φ	0.510	0.869
	0.830	*0.955*
u	-0.13	1.10
	0.88	*1.92*
ψ	0.44	0.67
	0.54	*0.76*

By definition, the annual snow load maximum occurs during every winter period. Therefore, when calculating the *load combination factor for snow load combined with wind load,* $\psi_{s(w)}$, we must take

$$T \equiv T_0 = 80 \ \text{ys}$$

Following the same procedure as has been used for snow load, we obtain

$$\Phi_{gen,s}(s_{w,exm}) = 0.983$$

$$T^{ret}_{s(w)} = 59.4 \ \text{ys}$$

and further the values of quantities that depend upon duration of the snow occurrence period (see Table 14.3). ■

The Example shows that both combination factors depend upon the lengths of the winter period and of the period when the adverse load values can appear. In the regions where snow cover duration is long, load combination factors are greater than in the regions with short snow cover period. It can easily happen that in regions of the latter type one of the two combination factors becomes zero; in such a case, the **(snow load & wind load)** combination may not be considered in design.

In general, we can conclude that *in the theoretical design format a specific load combination factor should be assigned to each load participating in an investigated load combination.* The value of this factor depends upon

(a) the number of combined loads, v ;
(b) random behavior of the investigated load and also of the companion loads, time-dependent or not;
(c) reference period for which the load combination factor is determined, T_{ref};
(d) duration of individual combined loads, T_{Fi} ;
(e) period of possible occurrence of the load combination, $T_{occ,C}$;
(f) target probability of occurrence of adverse load values, P_{Ft} , related to the relevant reference period, T_{ref} .

14.4 RESISTANCE VARIABLES

14.4.1 Design format

In the theoretical design format, we can define, similarly as in the case of loads (14.3.1), *central values of resistance variables,* r_{cnt} , *variability factors for resistance variables,* Γ_r , and also *resistance combination factors,* ψ_r . These quantities can be shortly called *resistance parameters.* Again, to express indefiniteness and model uncertainties, the set of resistance parameters should be supplemented by *adjustment factors.*

Resistance combination factor

Parameters r_{cnt} and Γ_r have a meaning close to that of analogous parameters in present regulations. Yet, no resistance combination factors have been used until now, though their theoretical possibility has been already stated. Some influences affecting the reliability, connected with the smaller probability of simultaneous occurrence of adverse values of variables entering resistance formulas, are expressed by other means than by a resistance combination factor. They are, in fact, covered by reliability parameters for material properties. This happens without intermediary of any combination analysis.

The substance of the resistance combination factor is the same as that of the load combination factor: when only a single variable, r_k, governs a resistance, its extreme value is calculated from

$$\Pr(r_k \leq r_{k,exm}) = P_{kt} \equiv P_{Rt}$$

where P_{Rt} = target probability of occurrence of a relatively adverse event Ev(R_{adv}). However, if the same resistance variable occurs in formulas accompanied by one or more further resistance variables (that is, it participates in an existential combination), the target probability P_{kt} should be established with regard to this fact. It will be $P_{kt} \geq P_{Rt}$.

Consider, for example, concrete members. The ultimate capacity of *plain concrete members* originates from phenomenon Ph(**strength of concrete**), and therefore, a certain extreme value of strength, $f_{c,exm,1}$, can be defined as an appropriate fractile of the probability distribution of the respective strength. When *reinforced concrete members* are dealt with, the ultimate capacity is produced jointly by Ph(**strength of concrete**) and Ph(**yield stress of steel**). The extreme strength of concrete, $f_{c,exm,2}$, for a reinforced member should obviously be greater than that for a plain concrete. This fact is reflected in structural concrete codes though it is treated in another way or even differently explained. As a rule, differences between the strain behavior of concrete in unreinforced and reinforced members, the brittleness, or, on the contrary, the plasticity of concrete are given as background to the difference in design strength of concrete; *probabilistic aspects are usually suppressed or ignored.*

The resistance is a typical phenomenon where *existential simultaneity of partial phenomena* applies. Consider, for example, the design RelReq

$$M_S(g, v, w, s, ...) \leq M_u(f_y, f_c, b, d, ...)$$

where M_S = bending moment in a cross-section of a reinforced concrete structure and M_u = ultimate moment. While M_S can be a scalar function of one or several loads (self-weight, g, variable load, v, wind load, w, snow load, s, and others) with a single load sufficient for existence of M_S, the ultimate moment M_u must include all relevant variables - strength of concrete, f_c, yield stress of steel f_y, width of cross-section, b, effective depth, d, etc. Clearly, phenomena participating in Ph(**ultimate moment**) are existentially simultaneous.

14.4.2 Problem of approximate formulas

The calculation of resistance, R, is often unnecessarily complicated, and so various approximate formulas are used in practice. In the main, simplifications consist in *reduced number of resistance variables.* Most often, simplifications are written for *mean values of some resistance variables,* and based on prediction formulas resulting from tests, etc. Approximate formulas obtained in this way are valid in specified range of input and output variables, and it is accepted that in this range the departure of the approximate result from the exact result ("exact" being a very relative term, of course),

$$\Delta = \frac{R' - R}{R}$$

is in an admissible interval, $\Delta \in [\Delta_1, \Delta_2]$. The effect of simplification on the reliability level is usually overlooked, and it is expected that the approximate formula will give adequate results even for probabilistic extreme values or other non-central values of the resistance variables. This presumption can be entirely wrong, particularly when non-linear problems are dealt with.

Assume that Ph(**resistance**) is produced by n random phenomena characterized by variables r_1 through r_n; the Ph(**resistance**) is expressed by a physical formula

$$R = f_R(r_1, r_2, ..., r_n) \tag{14.40}$$

Let us now simplify this formula so that only the first v variables remain, $v < n$,

$$R' = f'_R(r_1, r_2, ..., r_v) \tag{14.41}$$

Thus only v phenomena are considered, the participation of the discarded phenomena being implicitly embedded in constants of the approximate formula.

Now, when a target probability of maximum distress, P_{Rt}, is specified for R, its value must be, to meet the respective RelReq, taken also for the approximate resistance R'. Applying the probability-based optimization technique, target probabilities for the exact formula calculation are established:

$$P_{r1t} = P_{r2t} = ... = P_{rnt} = \sqrt[n]{P_{Rt}} \tag{14.42}$$

and similarly for the approximate formula calculation:

$$P'_{r1t} = P'_{r2t} = ... = P'_{rvt} = \sqrt[v]{P_{Rt}}, \quad v < n \tag{14.43}$$

It is:

$$P'_{rit} < P_{rit}, \quad i = 1, 2, ..., v$$

When P_{rit} and P'_{rit} have been established, extreme values of the respective resistance variables $r_{i,exm}$ $(i = 1, 2, ..., n)$ and $r'_{i,exm}$ $(i = 1, 2, ..., v)$ can be found. Subsequently, Equations (14.40) and (14.41) yield the two design values of the resistance, R_d and R'_d.

We are interested in the relation between these two design resistances and also in their relation to the population means of the exact and approximate solutions, μ_R and μ'_R, respectively. For simplification, quasi-means (see 2.3.1) can be considered, that is

$$Q\mu_R = f_R(\mu_{r1}, \mu_{r2}, ..., \mu_{rn})$$

$$Q\mu_R' = f_R'(\mu_{r1}, \mu_{r2}, ..., \mu_{rv})$$

In general, it holds:

$$Q\mu_R \neq Q\mu_R'$$

and similarly

$$R_d \neq R_d'$$

Therefore, for the evaluation of an approximate design formula its relative deviation from the exact one,

$$\Delta_d = \frac{R_d' - R_d}{R_d}$$

must be found and further also ratios

$$\vartheta_R = \frac{Q\mu_R}{R_d}, \quad \vartheta_R' = \frac{Q\mu_R'}{R_d'}$$

In order to accept an approximate formula, it should satisfy, in the interval of R_d investigated, two requirements:

$$\Delta_d \in [\Delta_{d1}, \Delta_{d2}] \tag{14.44}$$

$$\frac{\vartheta_R'}{\vartheta_R} \geq 1 \tag{14.45}$$

On some occasions, attention must be drawn to the fact that an approximate formula had been derived taking into account also *random phenomena not contained in the formula.* We should regard this trait in establishing target probabilities P_{rit}' and in evaluation of RelReq (14.45). When it is known that during drafting of an approximate formula (based, for example, on experimental research) all n participating phenomena had been considered (though not mathematically expressed), we can determine values $r_{i,exm}'$ using probabilities P_{rit} according to Equation (14.42), instead of P_{rit}' according to Equation (14.43).

When in the derivation of an approximate formula the random behavior of some phenomena had been neglected, we must treat the formula like an exact one, or, in other words, like a *formula without an exact parallel.* This reflects the reality, when many formulas are assumed to be exact up to the moment when they are supplemented by more exact ones (which may not be necessarily better!). Obviously, the exactness of any formula must always be conceived as a relative concept.

■ **Example 14.2.** Investigate the ultimate moment of a singly reinforced rectangular R.C. cross-section. The member is to be used in a structure that belongs to facility class N with P_{Rt} = 1.0E-3. Material and geometric properties are normally distributed with variation coefficients given in Table 14.4. All phenomena are assumed time-independent.

Assume that the exact physical formula for the ultimate moment (exact with certain concession) is

$$M_u = A_s f_y d \left(1 - \frac{1}{2} \cdot \frac{A_s}{bd} \cdot \frac{f_y}{f_c}\right) \tag{a}$$

where A_s = area of reinforcement, b, d = width and effective depth of the cross-section, respectively, f_c = compression strength of concrete, f_y = yield stress of steel. The formula already comprises some approximations: the stress diagram of concrete is considered rectangular, a perfect bond is assumed, etc.

Equation (a) shows that phenomena Ph(b), Ph(d), and Ph(f_c) are absolutely favorable, while phenomena Ph(A_s) and Ph(f_y) can be either absolutely favorable or absolutely adverse, in dependence on the value of M_u.

According to Equation (2.20) the extreme value of a resistance variable is

$$r_{i,exm} = \mu_{ri}(1 + u\,\delta_{ri})$$

where μ_{Ri}, δ_{ri} = population mean and coefficient of variation, respectively, of the random variable r_i, u = value of the standardized variable for P_{rit}. The variability factor for r_i is given by

$$\Gamma_{ri} \equiv \frac{\mu_{ri}}{r_{i,exm}} = \frac{1}{1 + u\,\delta_{ri}}$$

When for the resistance variables in Equation (a) the population means are set, quasi-mean of M_u results [cf. Equation (2.61)]:

$$\mathrm{Q}\mu_{Mu} = \mu_{As}\mu_{fy}\mu_d(1 - \tfrac{1}{2}\bar{\varrho})$$

where

$$\bar{\varrho} = \frac{\mu_{As}}{\mu_b \mu_d} \cdot \frac{\mu_{fy}}{\mu_{fc}}$$

Similarly, we can write for the design ultimate moment:

$$M_{ud} = \frac{\mu_{As}\mu_{fy}\mu_d}{\Gamma_{As}\Gamma_{fy}\Gamma_d}(1 - \tfrac{1}{2}\bar{\varrho}_d)$$

where

$$\bar{\varrho}_d = \bar{\varrho}\,\frac{\Gamma_b \Gamma_d \Gamma_{fc}}{\Gamma_{As}\Gamma_{fy}}$$

Table 14.4 - *Example 14.2.* Variability factors of resistance variables, Γ_{ri} (Roman), and the respective resistance combination factors, ψ_{ri} *(Italics)*. For A_s and f_y also variability factors derived under assumption that the two phenomena are absolutely favorable, are given (bottom rows)

Resistance variable, r_i	Variation coefficient, δ_{ri}	Number of phenomena, ν: 1	3	5
		Probability ${}^{\nu}P_{rit}$		
		0.001	0.100	0.251
		Standardized variable, u		
		3.091	1.282	0.671
b	0.03	1.09	1.04	1.02
d		-	*0.95*	*0.93*
f_c	0.12	1.38	1.15	1.08
		-	*0.84*	*0.79*
A_s	0.03	1.09	1.04	1.02
		-	*0.95*	*0.93*
		0.91	0.96	0.98
f_y	0.08	1.25	1.10	1.05
		-	*0.88*	*0.84*
		0.75	0.90	0.95

Table 14.5 - *Example 14.2.* Cross-section reliability factor, φ_M, for the cross-section and the ratio $\bar{\varrho}$ calculated for facility class N with P_{Rt} = 1.0E-3 (Roman) and facility class H with P_{Rt} = 3.16E-4 *(Italics)*; in the last row the partial importance factor γ_{nR}^{N} is shown, cf. Section 14.5

Row	Solution	$\bar{\varrho}$		
		0.1	**0.3**	**0.5**
1	φ_M according to Equation (b), for all $\Gamma_{ri} > 1$	0.91 *0.89*	0.90 *0.88*	0.90 *0.90*
2	*ditto*, but $\Gamma_{As} < 1$, $\Gamma_{fc} < 1$	1.04 *1.05*	1.02 *1.02*	0.98 *0.98*
3	φ'_M according to Equation (c), all $\Gamma_{ri} > 1$		0.83 *0.82*	
4	φ'_M / φ_M, φ_M according to Row 1	1.10 *1.09*	1.10 *1.08*	1.09 *1.07*
5	partial importance factor, γ_{nR}^{N}	0.98	0.98	0.97

It is convenient to refer the solution to a non-dimensional parameter, which can be called a *cross-section reliability factor* (analogous to the resistance factor in Section 13.3):

$$\varphi_M = \frac{M_{ud}}{Q\mu_{Mu}}$$

Setting for M_{ud} and $Q\mu_{Mu}$, we get

$$\varphi_M = \frac{1 - \frac{1}{2}\bar{\varrho}}{\Gamma_{As}\Gamma_{fy}\Gamma_d} \tag{b}$$

Then, for $\nu = 5$, the target probability of occurrence of a relatively adverse value of the *resistance variable* is

$$^5P_{rit} = \sqrt[5]{P_{Rt}} = 0.251, \qquad i = 1, 2, ..., 5$$

Assuming that all variables are normally distributed, we have for $^5P_{rit}$

$$u = \pm 0.67$$

Table 14.4 shows Γ_{ri} used for calculation of Table 14.5. Also values of Γ_{ri} for $\nu = 1$ and $\nu = 3$ are given; we need them for further part of the example.

In Row 1 of Table 14.5 values φ_M in dependence on $\bar{\varrho}$ are given for all $\Gamma_{ri} > 1$ (that is, for the case when phenomena A_s and f_y are considered absolutely favorable); and further in Row 2, values φ_M resulting for $\Gamma_{As} < 1$ and $\Gamma_{fy} < 1$. The case according to Row 1 governs, since φ_M is smaller here than in Row 2.

For the ultimate moment, the approximate formula

$$M'_u = 0.9 A_s f_y d$$

is frequently used. It does not involve Ph(f_c) and Ph(b); their effect is covered by the constant 0.9. The quasi-mean of the approximate ultimate moment is given by

$$Q\mu'_{Mu} = 0.9\,\mu_{As}\mu_{fy}\mu_d$$

and the design value is

$$M'_{ud} = 0.9\frac{\mu_{As}\mu_{fy}\mu_d}{\Gamma_{As}\Gamma_{fy}\Gamma_d}$$

The cross-section reliability factor for the approximate formula, φ'_M, follows from

$$\varphi'_M \equiv \frac{M'_{ud}}{Q\mu'_{ud}} = \frac{1}{\Gamma_{As}\Gamma_{fy}\Gamma_d} \tag{c}$$

The target probabilities of occurrence of adverse values of the three resistance variables are, for $\nu = 3$,

$$ {}^{3}P_{rit} = \sqrt[3]{P_{Rt}} = 0.1, \qquad i = 1,2,3 $$

and the corresponding value of the standardized normal variable is

$$ u = \pm 1.28 $$

In Row 3 of Table 14.5 values φ'_M and also ratios φ'_M / φ_M are given. Observe that φ'_M does not depend upon $\bar{\varrho}$ any more.

To illustrate all features of the resistance problem, values of *resistance combination factor* are given in Table 14.4; this factor is defined by

$$ {}^{\nu}\psi_{ri} = \frac{{}^{1}\Gamma_{ri}}{{}^{\nu}\Gamma_{ri}} $$

Obviously, the variability factors Γ_{ri} should be multiplied by their respective ψ_{ri}, which would increase the cross-section reliability factor φ_M. However, such a procedure is meaningless since ψ_{ri} can be directly included into γ^*_{ri}. ■

No general conclusions should be drawn from Example 14.2. It mainly serves demonstrating the seemingly tricky and clumsy solution technique based on MEV, in comparison with the solutions based on the direct method or method of extreme functions, and indicating the possibilities of the probability-based optimization method.

14.5 DIFFERENTIATION PROBLEM

In MEV, similarly as in the method of extreme functions, it is also possible to differentiate structures according to the importance of CFs. Again, the number of partial importance factors is given by the number of phenomena participating in the relevant RelReq.

Let us write relationships for partial importance factors for facility class K referred to class R:

$$ \gamma^{\mathrm{K}}_{nai} = \frac{a^{\mathrm{K}}_{i,exm}}{a^{\mathrm{R}}_{i,exm}}, \qquad i = 1,2,\ldots,n_a $$

$$ \gamma^{\mathrm{K}}_{nbj} = \frac{b^{\mathrm{K}}_{j,exm}}{b^{\mathrm{R}}_{j,exm}}, \qquad j = 1,2,\ldots,n_b $$

These relationships are analogous to Equations (13.18). Using Equation (2.20), we can write

$$\gamma_{nai}^{\mathrm{K}} = \frac{1 + u_{ait}^{\mathrm{K}}\,\delta_{ai}}{1 + u_{ait}^{\mathrm{R}}\,\delta_{ai}}, \qquad \gamma_{nbj}^{\mathrm{K}} = \frac{1 + u_{bjt}^{\mathrm{K}}\,\delta_{bj}}{1 + u_{bjt}^{\mathrm{R}}\,\delta_{bj}}$$

The meaning of symbols should be clear from the previous text.

At variance with the method of extreme functions, no global importance factor, γ_n, can be defined through partial factors γ_{nai}, γ_{nbj}. However, writing RelReqs for facility classes K and R,

$$A_d^{\mathrm{K}} \le B_d^{\mathrm{K}}$$

$$A_d^{\mathrm{R}} \le B_d^{\mathrm{R}}$$

where A_d and B_d are design values of one-component attack and barrier, respectively (see 14.1.1), we get by comparing both RelReqs

$$\gamma_{nA}^{\mathrm{K}} = \frac{A_d^{\mathrm{K}}}{A_d^{\mathrm{R}}}, \qquad \gamma_{nB}^{\mathrm{K}} = \frac{B_d^{\mathrm{K}}}{B_d^{\mathrm{R}}}$$

Numerical values of these factors will obviously be different from those of factors according to Equations (13.14). Then, the global importance factor can be defined by

$$\gamma_n^{\mathrm{K}} = \frac{\gamma_{nA}^{\mathrm{K}}}{\gamma_{nB}^{\mathrm{K}}}$$

It should be used as multiplier of the design attack, or as divisor of the design barrier, both referred to class R.

From the theoretical viewpoint, a single importance factor in MEV is not a correct solution. Nonetheless, it is used in practice and it is specified by various regulatory documents (see 10.4.2). Theoretically, the MEV design format should contain a large family of partial importance factors. Though such an approach seems to be rather discouraging, it is currently used. Let us mention, for example, the differentiation of design strengths according to the type of member (beams, columns), and type of CF (buildings, bridges). Such differentiations, based mainly on engineering judgment, exist in all modern codes, though they are often not expressively stated.

The *overdeterminate problem* can also be encountered in MEV. This happens when target probabilities of adverse phenomena in the reference class, R, are established on long-time design experience. In deriving importance factors related to class K, the same approach is used as in the method of extreme functions (see 13.4.2). The solution is founded on optimization relationships

$$|\Delta P^{\mathrm{K}}_{ait} - \Delta P^{\mathrm{K}}_{a,i+1,t}| = 0, \qquad i = 1, 2, ..., n_a - 1$$

$$|\Delta P^{\mathrm{K}}_{bjt} - \Delta P^{\mathrm{K}}_{b,j+1,t}| = 0, \qquad j = 1, 2, ..., n_b - 1$$

where

$$\Delta P^{\mathrm{K}}_{ait} = P^{\mathrm{K}}_{ait} - P^{\mathrm{R}}_{ait,emp}$$

$$\Delta P^{\mathrm{K}}_{bjt} = P^{\mathrm{K}}_{bjt} - P^{\mathrm{R}}_{bjt,emp}$$

and $P^{\mathrm{R}}_{ait,emp}$, $P^{\mathrm{R}}_{bjt,emp}$ = empirical target probabilities related to class R.

For illustration, in Table 14.5, Row 5, class N resistance importance factor, γ^{N}_{nR}, is shown. A determinate case was considered, with class H (P_{Rt} = 3.16E-4) as the reference class.

14.6 CODIFIED DESIGN FORMAT

In the majority of present design codes the design format is very close to the theoretical design format mentioned in 14.3.1 and 14.4.1. Nevertheless, the *philosophy of the codified design format,* often called *semi-probabilistic,* is from many aspects *empirical.* This format, now widely used, has developed from older design methods: *working stress design* and *safety factor design.* Though its origins format can be traced back to Max Mayer in 1926, its practical applications did not start earlier than in the late forties. For a good survey of the actual design codes see Galambos 1992.

Most readers are well acquainted with the current state of the semi-probabilistic design format. Therefore, we will give here only general information on features common to all its existing mutations, pointing out differences between the codified and theoretical format.

The following *principal and auxiliary design parameters* related to an elementary random variable, ξ, dominate the format (see also Table 14.6):

Characteristic values, x_k, are defined, as a rule, by

$$\Pr(\xi > x_k) = 0.05$$

or

$$\Pr(\xi \le x_k) = 0.05$$

according to whether the phenomenon examined, Ph(c), is absolutely adverse or favorable, respectively. Obviously, characteristic values are *distribution-dependent design parameters.* As far as material properties are concerned, the definition of an appropriate x_k value is clear and straightforward, if of course a probability distribution of the particular phenomenon is available. In the case of load, however, the situation is often complex, particularly when *time-dependent load* is dealt with. Probability distribution of such a load depends upon the reference period considered (see 2.1.6).

Though x_k is probabilistically well defined, it often happens that its value is strongly affected by non-probabilistic decisions. For example, *Eurocodes* 1992 admit that characteristic values can be specified, for a particular project, by the client or designer. Thus, x_k can become a *decision-based parameter.* - Since characteristic values are established beyond the framework of the code, they should be considered *principal parameters.*

Design values, x_d, are *auxiliary parameters,* being derived as factored values

$$x_d = \gamma^{\Pi} \cdot x_k \tag{14.46}$$

where γ = partial reliability factor (see below), Π = 1 for loads, Π = -1 for material properties. Notice that unlike Λ according to Equation (7.6), Π is given by straight definition, not by calculation.

Partial reliability factors (often called "partial *safety* factors") are *probability-free parameters* (though some authors object to this statement, see Section 14.7); their values are established by decisions based on experience and socio-economic considerations. They belong to the group of principal parameters, their role being essentially the same as that of adjustment factors in the theoretical design format (see 14.3.1 and 14.4.1). The partial reliability factors for material properties are $\geq$ 1, while for loads they can be both $\leq$ 1, or $\geq$ 1, in dependence on whether the load is absolutely favorable or adverse, respectively. This is because Π in Equation (14.46) depends on the general quality of the phenomenon considered, not on its absolute adversity or non-adversity at the respective point of the definition domain.

The background to partial reliability factors is not sufficiently stated; we will, therefore, discuss them more closely in Section 14.7.

Combination factors should be established probabilistically for given, *decision-based combination rules.* At present, their values are often determined by consensus in code-making committees, similarly as those of partial reliability factors.

Many structural design codes and their companion codes on execution, testing, and quality control are built-up on the semi-probabilistic design format. Today, codes of various countries or regional groups do not differ in principles though differences in details exist. The *Eurocode system,* which is now being introduced in countries of the European Community and of the European Free Trade Association, is fully founded on the semi-probabilistic design format.

Constraints, C, are principal design parameters, expressed, as a rule, as limit values of deformation parameters (deflections, rotations, etc.; vibration parameters; crack widths; and others). These are, in the main, decision-based (see Section 10.5).

Table 14.6 - Design parameters in the theoretical and codified (semi-probabilistic) design format, based on MEV

Theoretical format	**Codified format**	
	Load variables	**Resistance variables**
Central values (principal, probabilistic)	Characteristic values (principal, probabilistic)	
Extreme values (principal, probabilistic)	Design values = factored characteristic values (auxiliary)	
Variability factors (auxiliary) Adjustment factors (principal, decision-based)	Partial reliability factor for load (principal, decision-based)	Partial reliability factor for material properties (principal, decision-based)
Load combination factor (principal, probabilistic)	Load combination factor (principal, probabilistic)	None
-	Frequent-load factor, Quasi-permanent-load factor	Fatigue factor
Importance factor (principal, probabilistic)	Importance factor (principal, decision-based)	-
Constraints (principal, decision-based)	Constraints (principal, decision-based)	

14.7 PARTIAL RELIABILITY FACTORS

In a way, the partial reliability factors, γ, govern consumption of material and the level of overall reliability of CFs, affecting thereby the level of risk and variables of economic and social character. In the early days of the probability-based design, γ factors were fixed by using only engineering judgment. Sufficient statistical information had not yet been collected at that time. However, the general situation was conceptually different from that of today.

Values of design parameters for *known loads and well established materials* had been looked for, to enable writing of codes based on the new design philosophy. The reasons for discussing partial reliability factors have now changed. Two groups of issues can be distinguished.

First, we now frequently want to find γ values for *new loads* occurring with new technologies and new ways of life, and for *new materials* that have not been used in construction up to now, or for traditional materials to be employed under *new environmental conditions*. No "theory of γ factors" is available, yet reliability engineers are obliged to offer some solution whenever new loading or material conditions arise.

Second, it is often felt that, when *new knowledge on the physical and random behavior of load or material* has been collected, it can be used as a basis for improving values of design parameters, particularly values of γ factors.

These two main problem areas lead to the following concepts:

The concept of the *probabilization of γ factors* is based on the assumption that statistical knowledge about the behavior of loads or materials will increase with time. It is also anticipated that the level of human errors will decrease owing to better technological equipment, surveillance of workmanship, and other measures. Interesting studies have been already put forward for discussion (Östlund 1991). The probabilization concept is very close to the concept of variability factors Γ_F and Γ_r in the theoretical design format of MEV, outlined in 14.3.1 and 14.4.1.

Another approach to the γ problem is founded on the *analysis of the actual γ values with subsequent extrapolation of the results to other loads or materials, or adjustment to other conditions.* In a way, this concept is turning back to the earliest efforts to formulate the semi-probabilistic limit states design format. Let us go more into detail.

In general, a γ factor is defined as a parameter that should cover *possible adverse departures of loads or material properties (strength in particular) from their expected values.* For a given phenomenon, load or material property, values of γ can differ in dependence on the limit state investigated and the design situation considered (see 1.3.3). Life expectancy of CF should also be taken into account whenever a time-dependent phenomenon is dealt with. It seems that, to a certain degree, γ factors depend upon the definition of the respective characteristic values.

The investigation of γ factors should rely on the following principles:

- ♦ γ factors are *"probability-free,"* that is, no statistical data is used in their establishment; if such data were available, it should be employed in the probabilistic component of the design parameters, that is, in the characteristic values;
- ♦ all γ factors in RelReq are *independent;*
- ♦ γ factors reflect the amount of *non-statistical information* available in the case of a particular phenomenon;

♦ γ factors cover *uncertainties* and *indefiniteness* that cannot be encompassed by statistical models, that is that lie beyond the "statistical field";

♦ γ factors express *engineering judgment* and experience existing in a certain period of development, or in a certain economic climate.

Thus, when statistical data abound, we can dismantle those components of the respective γ referring to items on which statistical information has been enriched and, perhaps, *redefine the characteristic value in general,* establishing it as, for example, 0.01-fractile of the probability distribution (instead of 0.05-fractile). Of course, the 0.05 (or, for loads, 0.95) fractile could be kept and γ could be diminished anyway. On the other hand, γ factors can be adjusted whenever conditions on which they were established have changed. For example, raising the quality control level for material and products, the material factor can be reduced.

No manipulation with the γ factor is possible, however, without *understanding its structure.* To create some idea on this, we must specify the main items that are expressed by the factor. Knowing the particular items that compose γ we must find the *composition scheme*. Let us assume, for example, that all items are uniformly important; then, *two ways of composition* of γ are possible:

(a) ***additive composition:*** γ is expressed by

$$\gamma = 1 + \sum_{i=1}^{n} \Delta_i \qquad (14.47)$$

where Δ_i = supplement to γ due to item No. i , n = number of relevant items;

(b) ***multiplicative composition:***

$$\gamma = \prod_{i=0}^{n} \gamma_i^* \qquad (14.48)$$

where γ_i^* = component of γ due to item i , n = number of relevant items; clearly γ_0^* = 1.

Let us illustrate the decomposition technique by Example 14.3.

■ **Example 14.3.** The partial reliability factors for concrete strength, γ_{cc} and γ_{ct} (in compression and tension, respectively), are to be analyzed. First, we have to identify the main items covered by the γ_c factor:

1 - the imperfect, or even defective description of random properties of concrete (it may be argued that this item is non-probabilistic; we can view it in both ways, probabilistic or non-probabilistic);

2 - danger that during the production of concrete a gross error undiscovered by inspections has taken place;

3 - undiscovered flaws occurring during the casting of concrete; even when they are discovered, and corrective measures are taken, the local quality of the structure is impaired;

4 - deterioration of concrete because of ageing, corrosion, abrasion, and other effects;

5 - unfavorable ratio between tension and compression properties of concrete;

6 - danger of sudden rupture without advanced warning.

Table 14.7 - *Example 14.3.* Verification of partial reliability factors for concrete subjected to compression or tension

Partial reliability factors	Composition	
	additive	multiplicative
γ_{cc} derived from γ_{ct}	1.45	1.48
γ_{ct} derived from γ_{cc}	1.33	1.31

Let us assume that the list is complete; doubtlessly further items could be added. Thus we have $n = 6$ (or rather $n > 6$) items to be comprised by the respective γ_c.

For *compression properties* of concrete only items 1 through 4 are important, whereas items 5 and 6 must always be considered when we discuss *tension concrete.* For concrete under compression, values of γ_{cc} from 1.3 to 1.5 are fixed in various codes, for concrete under tension, γ_{ct} are 1.5 to 1.7. Using the shown composition formulas, we can calculate γ_{cc} from γ_{ct} or vice versa. Taking $\gamma_{cc} = 1.3$ and $\gamma_{ct} = 1.5$, we arrive at values shown in Table 14.7.

As there are no great differences between the respective cross-results, we can conclude that the two tentative types of composition formulas can be considered satisfactory. However, this may not be true in general! ■

Let us now show an example where the composition technique is used for a new material and where the importance of items is not uniform.

■ **Example 14.4.** Partial reliability factor for *laminated glass,* γ_g, employed as bearing material for cladding, roofing, and other structural purposes in Czechia was to be established. The producer needed the factor to be included in a catalog containing information on design parameters of glass properties.

First of all, it was necessary to identify the uncertainties and indefiniteness of the laminated glass that was subjected to investigation:

1 - imperfect description of the random behavior of strength; some statistical information must be available, of course, otherwise the problem could not be solved at all;
2 - mechanical flaws, flaws in connecting the glass layers;
3 - flaws created during cutting and further treatment of glass panels (notches, etc.);
4 - imperfect setting and assembly;
5 - temperature effects that cannot be expressed in the analysis;

6 - unexpected damage caused by hard flying objects;
7 - danger of sudden failure;
8 - size effects.

Some of these items can be eliminated or neglected owing to their minor importance:

2: glass with detectable mechanical flaws, delaminated, or damaged during transport is never fixed in a structure;

3: flaws produced by cutting have been already expressed in the results of tests;

4 and 8: these items can be combined; the greater the glass panel the more care is needed in its installation;

6: winds carrying hard flying objects are extremely rare in Central Europe;

7: though sudden failure is typical for glass, the interlayers provide some protection for people nearby.

Thus, only items 1, 4, 5, and 8 must be considered; items 4 and 8 can be combined. The composition Equation (14.48) was used. The component value γ_g^* was taken equal to that obtained for concrete, that is, equal to 1.070, which gave $\gamma_g = 1.225$, after rounding up, $\gamma_g = 1.25$.

It must be emphasized that design parameters for laminated glass suggested to producers for inclusion in catalogs did not rely only on the foregoing way of thinking. We carried out thorough analysis of the problem, taking into account experience of designers and of mounting crews, so that we could obtain a detailed picture of possible situations that can occur during life of buildings. ■

It is one of the golden rules of the reliability engineering that several approaches should be always used to solve a problem.

14.8 MERITS AND DRAWBACKS

Merits of MEV:

- each elementary phenomenon is investigated separately;
- separate, independent analysis of the attack and barrier is enabled;
- design based on the MEV philosophy can be very easily codified;
- information on the random behavior of elementary variables can be obtained by simple means, because many of the variables involved are testable and can be established experimentally or by observation;
- evaluation parameters (see Section 15.4) can be correlated to the design parameters;
- load combination analysis makes possible to eliminate verification of some trivial RelReqs.

Drawbacks of MEV:

- ♦ information on failure probability is given by a too broad interval;
- ♦ an exact handling of cases where the attack and barrier are mutually dependent is impossible;
- ♦ solutions taking into account all aspects of MEV are cumbersome.

15

RELIABILITY ENGINEERING

Key concepts in this chapter: *reliability engineering; reliability engineer; reliability assurance; reliability control; quality assurance; quality control; engineering factors; operational factors; economic factors; economic climate; legal factors; consulting firm; codes; prescriptive codes; performance codes; code systems; code revisions; testing; evaluation format; existing constructed facilities; re-design of existing structures; reliability engineer's report.*

In the foregoing chapters some concepts, methods, and techniques of the theoretical reliability analysis have been displayed that can be applied in the assessment of structural systems. We are aware of the fact that the presented information is definitely not exhaustive and that many details could be added as far as theoretical calculation models are concerned.

In the opinion of many people the whole job of the reliability assessment consists in performing sophisticated probabilistic calculations, composed of determination of appropriate failure characteristics and of evaluation of one or more reliability requirements. Yet, no theoretical analysis and no design RelReqs can encompass the manifold features of constructed facilities and of all situations that can appear. Theoretical solutions are only one part of problems solved in *reliability assessment.*

15.1 RELIABILITY ENGINEER

The gradual implementation of reliability-based methods into design, execution, inspection, and maintenance of CFs shows that a particular profession is shaped: the reliability engineer.

The tasks of reliability engineers are definitely not limited to reliability calculations. Their activities are much wider, as we will see in the following sections.

Let us first try, however, to outline what technical background and knowledge the reader who intends to be professionally engaged in reliability assessment of CFs should possess:

(1) Good theoretical and practical experience with one *structural material,* be it steel, concrete, wood, masonry, or soil is definitely necessary. You should master the principles of design, execution, inspection, and maintenance of a particular type of structures. It is virtually impossible to master all materials; understanding one, however, generates a good feeling for the behavior of CFs whatever materials are they made of. For example, good understanding of the ways of deterioration of masonry can surprisingly help comprehend deterioration of, say, waterproofing sheets. When you encounter a problem

that is beyond your own professional experience, you should always seek assistance of the respective expert. Nonetheless, good knowledge of one specific structural material makes formulation of your questions and understanding the expert's answers easier.

(2) Basic training in *probability and statistics* is a must. When solving reliability problems, the importance of statistical knowledge is often underrated; many civil engineers think that some rudimentary erudition is sufficient. It is surely not enough to know only formulas for mean, standard deviation, and correlation coefficient. In the main, an elementary course in probability and statistics is necessary. If a certain time has elapsed since you took such a course on probability and statistics, do not hesitate to repeat it. It will pay. Nonetheless, do not expect, that a trained civil or structural engineer is able to cover the whole domain of probability and statistics. The most precious advantage of your simplified statistical training should be *good feeling for your bounds*. Whenever a problem beyond these bounds arises, remember that your knowledge is narrow, and avoid solving the problem without competent aid of a fully trained statistician. Engineering oriented experts should be preferred.

(3) Finally, a certain level of *economic training* is needed. For example, sound economic thinking is a necessary condition for successful risk analysis. Reliability engineers should know what are the socio-economic implications of failures of whatever kind, and also what the consequences of their reliability-based recommendations can be. The concept of life expectancy should be thoroughly understood; otherwise, mistaken attitudes towards reliability and design parameters can be adopted. Try to look at all parameters taking into account their economical significance. This becomes particularly important when time-dependent phenomena are dealt with.

15.2 RELIABILITY ASSURANCE

The assurance of the reliability of CF is a decision process that begins with the first intention of building the facility for the defined purpose, in the defined space and time, and in the assumed environment. This decision process (the ***reliability assurance process,*** RAP) ends in dismantling, or demolition, of the facility. The reliability engineer should get acquainted with the particular issues of RAP as deep as possible. The general conception of RAP is not yet fully profiled, but many professional reliability engineers, construction managers, developers, etc. have already conceived the importance of RAP (see, for example, Matousek 1992).

Four principal groups of aspects govern RAP and the decisions taken during its development:

- engineering factors,
- operational factors,
- economic factors,
- legal factors.

Each group can be discussed and investigated separately but their effects upon the reliability of CF are dependent. This fact is very often neglected during the education of civil engineers; in university courses in reliability, when they ever exist, emphasis is usually laid on the engineering factors; the other three groups remain in the shadow of the first one. This particularly refers to the legal factors where, on the whole, engineering education is usually surprisingly poor.

15.2.1 Engineering factors

The group of engineering factors consists, in the first place, of theoretical and empirical knowledge, which both have prominent positions in RAP. ***Theoretical knowledge*** is either *general,* and as such it is covered by university curricula, or *particular.* The latter is not currently taught in engineering courses and, consequently, must be gained from specialized literature, by individual consultations, etc. It happens that such knowledge, when needed in a specific practical case, is not easily available or does not exist at all, and therefore, theoretical or experimental research has to be carried out. Theoretical knowledge is often overemphasized in the engineering education as a result of the traditionally academic approach to training programs. However, demand for practically minded and design experienced teachers has grown in recent years almost everywhere.

The engineer's ***empirical knowledge*** comes from two sources: *general experience,* obtained as a continuous heritage of the past. It is usually conveyed to engineering undergraduates by means of lectures and tutorials, and to practicing engineers by means of technical literature and codes (see Section 15.3), and by contact with professional colleagues in specialized seminars, conferences, etc. The other source is *personal experience* resulting from an individual's activities during a professional career. Bad experience, as may be guessed, is about five times more useful than good experience. Experience in reliability assurance is extremely important and it is always a gross mistake to underestimate its value.

As soon as sufficient information on a particular phenomenon becomes available, empirical knowledge treating such a phenomenon can be substituted by theoretical knowledge. For example, it is generally known that in Central Europe the depth of the foundation level should never be less than 1.5 m, to avoid the unfavorable effects of frozen soil on the foundation properties. We are now able to calculate exactly the value of the safe depth, by means of sophisticated soil theories, weather data, statistical and probability-based analysis, etc. Obviously some other value of the minimum depth might be reached by these means, say 1.45 m or 1.57 m. Nevertheless, nobody would perform such calculations, or accept the new numbers, since experience on this particular point is very strong, and more powerful than any exact study. Moreover, any sophistication in such a case would be a waste of time and money because the results would not justify the research expense, in economic terms.

In the discussion of engineering knowledge ***tradition*** must not be omitted. This is a particular branch of experience, fossilized it may be said, which can sometimes have a retrograde effect on RAP. Many cases are known where sticking to tradition had resulted in bad solutions, diminishing the reliability of the system. On the other hand, good workmanship is always based on tradition!

15.2.2 Operational factors

Evidently, RAP follows the *general construction process;* this consists of the following main sectors:

- ♦ *planning,* which covers fundamental decisions on the purpose, location, size, and cost of a planned CF; this sector is usually managed by non-engineering bodies, such as owners, public authorities, etc.;
- ♦ *design* refers not only to the elaboration of design documents, but also to surveying, choice of the type of bearing structures, choice of other systems that are part of CF; here a consulting firm, architectural or structural, is the main performer;
- ♦ *execution,* supplied by contractors and subcontractors; it also includes transport of material and structural elements;
- ♦ *quality control, inspection,* and *testing,* where additional subcontractors, or also independent agencies, enter the process;
- ♦ *maintenance* and *use,* responsibilities for which are carried by owners or users, according to the type of facility and possible lease agreements.

The decision procedures that govern the mutual relationship between the foregoing sectors, and also internal arrangements within the sectors, can be called the *operational factors of RAP.* Thus, the aim of RAP in the operational field is to seek for optimum solutions for the procedures in each sector and to obtain good link-ups between the multitude of separate, simultaneous or successive, activities involved.

Operational defaults can have a very adverse effect on the reliability of any CF; many examples of various types of failure due to such defaults can be given:

- ♦ insufficient geotechnical surveying can give biased information for the foundation design, with ensuing consequences;
- ♦ poor organization of design activities can cause overloading on designers who, consequently, are not able to analyze the complex reliability features of the particular system thoroughly;
- ♦ loss of designer's control over execution of the design can lead to serious failure-producing mistakes;
- ♦ defaults in material supply can lead to a deterioration in material properties;
- ♦ inadequate maintenance can lead to early corrosion.

It becomes evident that a spectrum of individuals and bodies must actively participate in RAP. In the majority of cases their contacts are very weak and, because of this, interface problems arise. Transitions between successive sectors are always subjected to various types of decision.

For example, in the selection of a consulting firm the following evaluation criteria are frequently used (see Gipe 1989, *Selection by Ability,* 1991);

- ♦ relevant experience, technical competence;
- ♦ completed projects;
- ♦ qualifications of firm and staff, managerial abilities of the team;

- special expertise;
- qualifications of sub-consultants;
- availability of financial and manpower resources;
- professional independence and integrity;
- fairness of fee structure;
- quality assurance system;
- size of staff;
- current and projected workloads;
- geographic location.

Any decision taken in a certain sector of RAP can influence the reliability of CF *at any later moment of its service life.* A similar observation can be made concerning *non-decisions,* which must be expected whenever a certain aspect of the reliability is neglected, or even unknown. It often happens, for example, that no regular maintenance is prescribed by design documents. This can result, after a couple of years, in poor behavior of materials, deterioration, loss of durability, etc. Therefore, all sectors of RAP must be thoroughly linked-up, as close as possible. The leading role of the designer is evident here, though his or her position is usually very unfavorable. Whereas the consulting firm is selected by means of the criteria mentioned above, the contractor is chosen on the lowest-bid principle, as a rule, assuming that all conditions stated in the contract documents are met. Therefore, in the majority of cases, the designer does not usually know who will win the bidding in a particular case. This is a well known weak link in RAP, though it is economically fully justified, without any doubt. Whenever an outstanding facility (chemical process plant, power plants, sporting stadium) is planned, *design-construct firms,* covering two or more sectors of RAP and supplying full *construction management,* are preferred.

The decision making, at any point of RAP, is subjected to external influences that cannot be predicted (or sometimes even expected) by decision-makers. This results in a specific *randomness of RAP,* the nature of which, unfortunately, has not yet been subjected to research. Though knowledge is minimum here, at present, it would be a gross mistake to neglect the random features of RAP.

15.2.3 Economic factors

A general rule (perhaps platitudinous) can be stated: the more money is allocated to RAP, the higher the level of reliability of the particular CF is achieved. On the other hand, another rule rings true: the lower the reliability level, the greater the possible costs involved with failures, repairs, re-design, litigation, etc. Obviously, a certain balance should be reached, in some way, so that the potential "failure cost" should equal "waste costs." Such a balance is the desired objective of any RAP, and, consequently, it can be ascertained that *it is mainly economic factors that govern RAP.* However, the economic problems of RAP are far from being so straightforward as it first appears because not only monetary categories enter the considerations, but also some issues cannot be treated in economic terms at all. Therefore, whenever economic features of RAP are discussed a largely general meaning must be assigned to the concept of costs. It must be primarily understood that costs are equal to the benefits given up when a certain construction procedure is accepted. In this philosophy costs express:

- financial costs;
- loss of life and limb;
- various general psychologistic values (emotional, moral, etc.);
- labor productivity.

It is hardly possible to unify all these cost branches in one scalar variable. Theoretically, it can be assumed that a certain unit of satisfaction, a "util" can be defined, on the base of which a *cost-benefit analysis* might be built-up, and the *opportunity cost* of a particular construction process covering RAP evaluated. This is, at present, obviously not possible to do in general.

It should be noted that deviations from the two rules specified at the beginning of this paragraph can be observed whenever a simplified and less expensive design or construction procedure eliminates possible sources of human errors by reducing the complexity of particular operations (see, for example, Stewart 1991). Unfortunately, this fact is not generally understood and very little benefits are drawn from it. - On the other hand, it must be kept in mind that simplifications to calculation models or inspection procedures, being intentionally set on the safe side as a rule, lead to oversizing of members, that is, to higher construction costs.

There are two principal areas where economic factors control RAP. The first one refers to *decisions on reliability levels,* made in terms of variables affecting the system of design and reliability parameters; the other is represented by the *economic climate* of the particular country and period.

Economic climate

It can be ascertained that in RAP considerations the owner-designer-contractor chain appears, affecting the economy of the particular construction process in detail, and consequently affecting RAP. Any savings achieved by one of the members of this chain can typically bring loss to himself or herself, and also to other members. Thus, RAP is subjected at this level, to *short-run economic considerations* dominated by the current economic situation of the respective environment.

In the particular sectors of RAP some rudimentary cost-benefit analyses can be made by the designer. - He or she should consider, for example, the opportunity cost of changing a certain value of the target failure probability, P_{ft1} , to another value, P_{ft2} . Or, the contractor can choose some patent procedures that are supposed to decrease the outlays of the bid and, analogously, the owner can abandon some maintenance action, accepting the risk of an earlier decay of the facility. Many examples of good and bad decisions can be given here.

Now, the economic problem of RAP extends beyond the owner-designer-contractor chain because the dynamics of the national, or even world economy can exert considerable influence upon RAP and the resulting reliability level, in many ways. In the first place, any economic restrictions imposed on the construction industry by *recession,* by political decisions, or by other investment-unfriendly factors are accompanied by tendencies to keep down bid budgets, that is, to diminish costs. As a rule, quality assurance and control is affected initially, then material quality, maintenance procedures, etc. Further discussion of these phenomena would be superfluous here.

Conversely, an economic *expansion* can produce very similar effects upon the reliability of CFs, to those brought up by recession. Since the availability of resources, that is, of labor, entrepreneurship, capital, and natural resources becomes limited at expansion periods, the construction industry cannot follow the development, and, as a rule, the shortage of resources is supposed to be counterbalanced by pressures on labor availability. Under such circumstances, not too much attention is paid to the education of laborers as a mean of improving the resource situation. The subsequent negative impact on reliability is obvious again.

Similar phenomena affecting reliability during recession or expansion can also be observed during any *inflation period,* which can string along with both the recession or the expansion. Time factors apply here, since construction projects are to be completed in the shortest time and for the least initial cost (see Carper in *Forensic Engineering* 1989), which tends to escalate failure rates.

The economic factors of RAP are essentially the same in all market economy systems, as well as in command systems. Whenever basic economic laws are distorted by non-economic aberrations, the reliability of CFs suffers. *Many examples of spectacular failures, with an economic background, can be given from various countries.*

15.2.4 Legal factors

The principles of structural mechanics are identical worldwide; the rules of execution and workmanship deviate slightly from country to country. Economic rules differ according to historical and political situations in the particular region but in framework of various economic systems they still follow analogous principles. In contrast, the laws people have imposed upon themselves show a very diversified pattern. Owing to this diversity, legal factors of reliability assurance cannot be uniform. Nevertheless, *two basic effects of the legal climate* on the reliability of CFs can be identified in any country, and in any socio-economic system:

- regulatory effects,
- deterrent effects.

In the first case more or less complicated assemblies of laws, rules, codes, etc., formulate *relations between the participants of the construction process,* and their *relations to the society,* usually represented by authorities on various levels (national, provincial, cantonal, municipal, local, and others). Further, other regulations specify *performance characteristics* of construction products, buildings, bridges, and other facilities. Design and execution codes, which have already been mentioned here, also belong to this family though they are primarily conceived as tools of mutual understanding and unification. When all the particular regulations are carefully followed, everything runs smoothly, no legal problems arise and reliability is supposed to be well under control.

The spectrum of the regulations that are to be respected is very wide. It contains, first, laws that have, at first glance, nothing in common with engineering reliability - penal, civil, labor, tort law, maybe also others, and second, many regulations that refer directly to CFs and that treat fire protection, occupational safety, and other factors of good performance.

Most of these documents contain *sanction clauses* saying what punishment is presumed when the regulatory clauses are not respected. A direct deterrent effect of regulations thus enters RAP. However, the main deterrent effects of a legal character are direct.

The threat of *litigation on construction issues* has expanded during recent decades. The reasons for this tendency are perhaps known to lawyers and politicians but they often are not fully clear to engineers. What, however, *is* known is that fear from being involved in a court trial has a favorable influence upon the quality assurance and control with the designer, and with the contractor as well. Both these parties are usually well aware of problems following a structural failure that had been subjected to attention of public during the investigative period and, then, during litigation. The subsequent *loss of credibility* is often more expensive than the damage paid by the respective liable party. Moreover, firms that are not found responsible also suffer owing to the fact that the public becomes suspicious to all who have been involved, in some way, in a failure case. Whenever possible, the decisions of a private judge or arbitrator are preferred to court trials.

It can thus be maintained that the juridical factors of the construction process, and also of the post-construction period, have a definitely positive bearing on the level of reliability of CFs. There is an exception to every rule, however. It happens that new design procedures, and new technologies, are stubbornly resisted by the profession, for a long period, due to insufficient experience with the new techniques. Managers are afraid of getting sued for potential losses caused by some burn-in errors committed by personnel when using a new solution (cf. problems arising in the United States with limit state design code for steel structures, as described by Burns and Rosenbaum 1989). Note the favorable by-effects of juridical factors in discussions among engineers on responsibilities for possible failures (Becker 1986).

15.2.5 Reliability control

In various areas of civil and structural engineering, systems exist that contain one or more *observational feedback subsystem* (also called *damage supervising systems*). Such a subsystem delivers, in the course of construction, information about the main bearing system. Visual and instrumental observations are evaluated and consequent decisions and measures are taken to achieve the intended reliability level.

■ **Example 15.1.** In the "New Austrian Tunnelling Method" (NATM; see Rabcewicz 1973) the general reliability system consists of several subsystems:

- parent rock;
- rock arch;
- outer shotcrete lining;
- monitoring and evaluation of the tunnel deformations;
- supplement anchors and temporary supports;
- inner lining.

The last two subsystems are being continuously adjusted to the results of observations. ■

■ **Example 15.2.** Owing to thermal effects in hardening concrete, cracks occur in walls of large fluid and gas tanks. This is a regular phenomenon (see, for example, Kimura and Ono 1988), which must be considered in the design. After the concrete had hardened, the surface of walls is inspected and cracks, which can be easily distinguished by trained eye, identified. To eliminate or reduce leakage of fluid or gas, cracks should be sealed by appropriate means. The identification of cracks, sealing and the ensuing permeability test are subsystems of the general reliability system "tank." ■

The reliability control approach shall not be confined to the construction period of RAP. It is also important to design measures to be taken during service life of CFs. Many good suggestions on *maintenance engineering* can be found in other engineering branches (see Anderson and Neri 1990). Guidance on reliability control and maintenance problems can be obtained from *reliability assurance and control codes* that are available in some countries (for example, BS 5760:1986, BS 8210:1986).

15.3 STRUCTURAL CODES

15.3.1 Objectives of codes

In many cases the theoretical and empirical understanding of a certain phenomenon is not uniform among engineers and engineering experts. Where the respective non-uniformity can affect RAP in an adverse manner, the diversified knowledge, theoretical or empirical, must be unified by means of codes, standards, and similar regulations. Thus, calculation models, reliability factors, target failure probabilities, material testing procedures, load test evaluation rules, construction methods, and other processes are subjected to codified unification.

Law experts say that codes are legal documents "sui generis," that is, of specific nature, because they do not treat social relationships (relationships between individuals or bodies or between both), but *relationships of humans to the natural and engineering phenomena,* and *ways of manipulating such phenomena in technology.* Codes form a part of the legal system of any developed country, and consequently the actual status of the codes depends very much on the intrinsic nature of the legal system; a large variety of concepts exists. In many countries, *design codes* are *mandatory documents,* and the designer and contractor are obliged to follow them, while in other countries codes are *optional.* The difference is more or less formal since optional codes are used in the same manner as the mandatory ones. As a rule, clients, local authorities, insurance companies, and others insist on the use of a specific design code, even when it is of an optional character. The code-users on various levels expect to get "how-to-do-it" guidance, which is contained in *prescriptive codes.* "What-to-do" recommendations are less appreciated, and therefore, the idea of *performance codes,* originating in the seventies, did not take up.

The influence of the optional and economical factors is now systematically subjected to harmonization procedures among groups of countries with the aim of achieving a uniform, consistent system. This harmonization is driven by purely economical needs; by no means is it conditioned by the nobel wishes of scientists who, as individuals, are often not completely satisfied with some of its results and consequences. A very typical and most recent example of a successful code harmonization is the *Eurocode system* of structural design codes worked out by the members of the European Community. It is based on agreement of countries fixed by *Directive* 1989. The Eurocodes have not yet been completed but it is now certain that they will form a viable code system.

Two principal aims of any design code, including load codes, can easily be discerned:

- harmonization of calculation models,
- fixing of reliability levels.

The *harmonization of calculation models* is important from various viewpoints. First, it is needed to support the compatibility of different design solutions. Second, it further helps the designer to avoid difficulties in selecting the respective analysis procedure. Third, it reduces, in a way, the level of designer's responsibilities. New ideas reach the designer frequently just by the intermediation of a code. It is sometimes necessary to specify, for a particular problem, one single calculation model, but in some other problems it may be desirable to advise the designer that he or she is free to use any model that he or she may think suitable (and, of course, logical) for his or her problem. It is noted here that *decisions* of bodies (for example, code committees) or individuals govern the choice of the calculation models.

As it was explained in the preceding chapter, the reliability of structures designed according to a certain design code is described by design requirements, which contain the *design parameters*. Again, both are established by a group or individual decisions. It has to be mentioned that the values of the design parameters depend partly upon the properties of the respective calculation model. Whenever a calculation model is not based on a clear physical description of the phenomenon under consideration and consequently, simplifications, or empirical constants or functions are entered into that model, this fact must necessarily be reflected by the design parameters. There are cases where the dependence of the design parameters upon the calculation model is very strong; this occurs particularly in non-linear problems of analysis (such as instability of axially loaded members, deflection of non-homogeneous beams, etc.). Then, when the calculation model is changed for some reason, for example, from a biased model to an unbiased one, the model-dependent design parameters have also to be adjusted in order to achieve the same reliability level that pertained the changes.

The role of codes in the context of reliability of CFs has been recently discussed by Allen 1992.

15.3.2 Code revisions

Constant and continuous developments in the construction industry, efforts to save materials, energy and labor, and also the need to introduce new structural systems lead to a demand for periodical *revisions of codes*. The aim of a revision procedure is always to improve the actual code statements, to make them more general, or more exact, and to add clauses covering new knowledge gained during the period elapsed from the last issue of the code. If the new knowledge cannot be implemented without substantial changes in the design method then the design format is also updated during the revision.

Nobody enjoys extensive code revisions. Any change in the design format is usually a disagreeable intervention into the design concepts and its impact on practical design can have wide economical effects. Some structures, designed according to the old code, consume more material than when the design is based on a new code, and conversely. It is, however, not in the interest of society to increase, through acceptance of updated design methods, consumption of materials and energies in general; local adverse deviations from this rule must be balanced by savings in other areas covered by the code, or code system.

Whenever a design code is changed, many side effects, some very important, must be expected. Any single change in the design parameters and reliability requirements affects the *evaluation parameters* (see Section 15.4), and consequently the testing and quality control codes must be thoroughly revised, and a new *evaluation format* has to be formulated. Here is a large problem area, which has not yet been fully investigated; research is being currently directed at these problems, however.

It is usually not sufficient to simply compare the results obtained by means of the old and new code, and to adjust the design parameters. The *code calibration* must be made more sophisticated, to match extensions to the code system. In a scientific calibration, defined classes of structures are subjected to investigation as stochastic entities, and the respective design parameters are determined by *optimization;* this would ensure that structures belonging to the respective class are reliable and economical. At a *first step* the principle is usually followed that *the average result of the design should not be substantially changed by the new codified design format.* This, of course, means that some structures must be needlessly oversized by the new code, and therefore, at the *second step,* parameters are further adjusted, using *past experience.*

15.3.3 Code systems

Today, every developed country benefits from a system of codes that is concerned with the engineering component of RAP. The legally binding detailed features of such code systems are not comparable, from country to country, but their general outlines are almost identical. As already mentioned above, the following principal areas are dealt with by code systems:

- design,
- execution and workmanship,
- testing and quality control, and
- maintenance and use.

The first two areas are usually given prominence, whilst the remaining areas, particularly the last one, are neglected in many code systems. Nonetheless, all four are equally important in RAP and all influence each other, up to a certain degree. Therefore, revision of a code related to any area must be projected into the others. Such a fact is only rarely recognized during revisions. This brings subsequent difficulties to code makers, and to code users as well.

It should be remarked that code systems and also individual codes are, in fact, reliability systems themselves! When they are considered from this angle, all main features of regular reliability systems become apparent: *components* (= individual codes), *burn-in period* (= the period after the first publication and before the first revision), *constant failure period, wear-out period* (= the period during which the codes become gradually obsolete, notwithstanding revisions, and must be completely abandoned and rewritten), *serial* and *parallel subsystems* arrangement, etc. - Therefore, whenever a system of codes is subjected to changes, improvements, enlargements, etc., its reliability features must not be neglected and proper steps must be taken to avoid its deterioration.

At present, the importance of code systems in economies is well appreciated and much effort is attached to bringing the existing systems to perfection. This is an endless task, mainly because the potential code users are almost never happy with any change. On the other hand, the actual economic climate calls for unification in various fields of engineering activity. It is not necessary to explain why. Codes are now commonly considered a very powerful tool to achieving unification goals by successive steps. In Europe code *harmonization efforts* date back to the late forties. In the construction industry they started somewhat later, in the early sixties. It must be noted that a similar course has been followed in the United States, where an *interstate unification of codes* is requested by economists and engineers, their motives being the same as those of their colleagues in Europe.

15.4 QUALITY CONTROL AND QUALITY ASSURANCE

An important operational component of RAP is the *quality assurance and control process,* QACP, which is usually presented in terms of two sub-components,

- *quality assurance,* QA, and
- *quality control,* QC.

The line between QC and QA is not clearly defined (see Kagan 1989 where several interesting practical ideas on QC and QA are presented). The two sub-components either overlap, or their contents can be interchanged. Similarly to RAP, the respective QACP has to start at the very beginning of the general construction process, and continues till the end of the use of CF.

The difference between the two concepts, RAP and QACP, can be easily demonstrated on the partial reliability factors for material, γ_m, see Section 14.7. During code-making, values of γ_m are established and based, theoretically or empirically, on a large set of assumptions. Now, the task of the QACP is to control and check conditions ensuring the validity of all those assumptions during the expected life of the facility. Obviously both the establishing of γ_m factors, and the control and checking of assumptions belong to RAP. Whereas γ_m factors are usually settled beyond the respective construction process, QACP runs through the whole life of the facility, T_0. Maintenance and periodic inspections must be considered as a part of QACP. Reliability theory can provide theoretical basis for such actions (see, for example, Attwood *et al.* 1991).

It can be observed, in the long run, that QACP costs have increased to about 1 to 3 percent of the project cost, on the average. This is a *definitely reasonable investment* since in countries where the importance of QACP is not understood and appreciated, various reworking costs can reach a 5 percent level, or even higher. The merits of a systematic QACP do not refer to catastrophic failures only. It is equally important to prevent or remove minor mistakes that can eventually bring major trouble to the owner or that can simply delay the execution, so that the time schedule is not complied with. Delays are always costly, and contractors happen to be sued by owners for money lost because of a late start of operations of the particular facility. Here, insurance companies can participate in the construction process, but they always check whether the reliability is sufficiently covered.

A constantly growing level of attention is now being paid to construction quality problems (see, for example, Bannister 1991, Ferry Borges 1988, *Qualitätssicherung* 1991, *Quality Control of Concrete Structures* 1979). Nonetheless, general concepts have not yet been stabilized. Most industrial countries have elaborate systems of QAQC regulations (for example, BS 5750:1979. At present, a system of QAQC codes is being prepared by the International Organization for Standardization: ISO 9000:1987 and associated codes on *quality systems, quality management,* etc. Many codes on statistical techniques of the QAQC have also been published by ISO.

It happens that, under QACP, full reliability assurance is conceived, and the role of RAP as a decision process is not correctly understood, or recognized at all. Of course, much progress has been made as a result of computerization. There are many specialized software programs available that treat the operational factors of RAP, though subjective phenomena still affect most of the RAP sectors. The results of a computerized decision analysis must always be verified by independent means, based on engineering judgement, and possibly adjusted to the actual situation.

Though many theoretical questions are to be solved in QACP, we can happily observe that the QA concepts have already entered common practice. You can find clauses on QA in many contract documents, as the importance of such clauses is acknowledged by all participants of the construction process. Sometimes such clauses are very simple (for example, a clause obliging the contractor to keep a copy of the appropriate execution code on site), nevertheless, they are a good start for further development.

Evaluation format

Let us consider concepts that are important in the quality control of structures and prefabricated members, as well as other occasions where testing is employed: the evaluation format, evaluation criterion, evaluation requirement, and evaluation parameters. These concepts are parallel to those of design format, design requirement, etc., outlined in Section 1.4.

Consider a simple precast reinforced concrete floor beam. To cover all adverse loading cases, the beam is designed for uniform load, for concentrated load at mid-span, or also for concentrated loads near the ends. Assume that RelReqs for the bending moment at mid-span, M, and for shear at the supports, T, are written. Further, deflection at mid-span, f, and maximum crack width, w, are calculated. Thus, four RelReqs, related to four design criteria, must be verified:

$$M_d \le M_{ud}, \qquad T_d \le T_{ud},$$

$$f_d \le f_{lim}, \qquad w_d \le w_{lim}$$

The meaning of (M_d, M_{ud}), (T_d, T_{ud}), (f_d, f_{lim}), and (w_d, w_{lim}) depends on the probability-based method selected. For example, when MEV is used as design format, characteristic values, partial reliability factors, and combination factors enter expressions for M_d, M_{ud}, T_d, T_{ud}, f_d, and w_d, while f_{lim} and w_{lim} are constraints specified by decision of code makers.

Now, when the beam is supposed to be mass-produced, *prototype tests* are performed to verify the calculation model (see, for example, Leicester 1984), and later, *control tests* are routinely carried out to check the production (for example, one of 100 elements is tested). Or, *proof-testing* can be designed as a part of the production process (Fujino and Lind 1977, Grigoriu and Hall 1984, Jiao and Moan 1990, Hong and Lind 1991).

To simplify, consider only the control testing. A uniformly distributed load is usually costly and unreliable, and so test beams are subjected, say, to two concentrated loads at the span thirds. These loads are not random, since only the quality of beams is tested, not their reliability under random loading. During the control testing, deflection at mid-span at certain levels of testing load, dependent on the design RelReqs and also on the expected ultimate load F_u, are observed. Then, ***evaluation requirements*** have to be specified in the *quality control plan:*

$$f(\delta_1 F_{ref}) \le \delta_2 f_{lim}, \qquad w(\delta_5 F_{ref}) \le \delta_3 w_{lim}$$

$$f(\delta_6 F_{ref}) \le \delta_4 f_d, \qquad F_u \ge \delta_7 F_{ud}$$

etc.

where F_{ref} = reference load, the value of which is usually close to service load.

Obviously, three ***evaluation criteria:*** load, mid-span deflection, and crack width were used in the above evaluation requirements. Factors δ_i are the ***evaluation factors.*** F_{ref} and δ_i are ***evaluation parameters;*** the system of evaluation requirements and parameters is called the ***evaluation format.*** Deflections and crack widths can be checked several times after de-loading and re-loading the beam again. Also residual deflections after de-loading are often verified. Thus, the number of possible evaluation requirements can be very large, theoretically unlimited.

The evaluation criteria, requirements, and factors have to be fixed in such a manner that we obtain precast beams meeting RelReqs of the adopted design format. Clearly, the evaluation format, and the evaluation factors in particular, depend:

- ♦ upon the *size of samples, frequency of tests,* and *confidence levels,* which can be summarily called *evaluation parameters;*
- ♦ upon all *RelReqs and design parameters governing the background design format* (for example, partial reliability factors, importance factor);
- ♦ upon the *arrangement of test, number of loadings,* etc.

At present, no general method of transforming a design format into respective evaluation format exists. Studies are under way, yet only results concerning specialized problems like *proof-testing* (Ayyub and McCuen 1990, Fujino and Lind 1977, Grigoriu and Hall 1984, Hong and Lind 1991), *load-testing* (Menzies 1979), and *design-by-testing* have been presented.

15.5 RELIABILITY ASSESSMENT

Reliability assessment consists in taking several steps to answer questions formulated by the reliability engineer's client concerning the reliability of a facility, a particular design solution, maintenance plan, etc. Questions set can be multifarious and goals of assessment can be diverse. Nevertheless, four principal areas can be distinguished where a complex reliability assessment is applied. Virtually identical philosophy is used in all four areas, except for differences resulting from particular goals.

In general, reliability cannot be simply described by a single figure. We can state that, for example, *the investigated facility is reliable or not,* but we usually cannot give a simple quantitative answer. Of course, we are able to calculate, for example, the failure probability P_f (in terms of $\dot{P}_f$ or P_f), but this gives only partial information, which usually is not of too much help to the client. As a rule, a "vector answer" is to be given, specifying not only P_f and the residual life, T_{res}, but also conditions on which P_f has been determined, that is, conditions of use, inspection, maintenance, etc., which should be satisfied during T_{res}.

15.5.1 Defect and failure assessment

The a posteriori *assessment of defects and failures* is a frequent reliability engineer's exercise. When a defect or a failure of CF occurs, be it collapse or "only" excessive deflections, immediate consequences are the primary issue. After a short period, the attention of the participating parties, possibly including government agencies or other authorities, is paid to the background to distress. In this situation, which, in the case of spectacular failures can be messy, experienced people should enter the failure theater to determine causes of the incident or to determine the background to and possible development of apparent defects. A reliability engineer should always be in the investigation team, since through his ways of thinking important aspects of the event can be detected. Obviously, reliability engineering is close to *forensic engineering*. For good information on forensic engineering methods see *Forensic Engineering* 1990.

The role of the reliability engineer in the defect or failure assessment consists in *identification of technical background to the event,* not in identification of responsible persons. It very often happens, that after finding the origins of flaws and faults, that is, stating that they are in design documents, shop drawings, bores, or elsewhere, we are asked to say who has been responsible. Do not answer. Keep to technical aspects.

Most defects and failures submitted to reliability engineers relate to serviceability limit states. Ultimate failures are less frequent; they are the issues of only about 10 percent of disputes, not more. Many simple ultimate failures happen during construction and thus they are covered up to escape publicity and the ensuing loss of credibility, which can be disastrous to the designer, contractor, and sometimes also to the client, whatever their individual responsibilities are, if there are any. However, these "hidden failures" can affect the time behavior of the CF system in an unexpected way.

Information on defects and failures of any kind is extremely precious, and the reliability engineer should regularly get acquainted with available publications (for example, Kaminetzky 1991). Reports on defects and failures are currently published in engineering journals, some of which are specialized in this particular topic (for example, the ASCE *Journal of Performance of Constructed Facilities*). Civil engineering agencies of several countries publish concise information on defects and failures (for example, the *Building Research Establishment* in the United Kingdom).

15.5.2 Risk assessment

The a priori *assessment of risk* is becoming one of the main problems subjected to the attention of reliability engineers. The probability-based risk assessment, PRA, has become an important component of decision-making related to industrial facilities, large structures, utilities, etc. Unfortunately, the concept of risk has not yet been fully defined, though many people pretend to be clear on what the term "risk" means. When several, more or less advanced studies of the risk problems are compared, various qualitative as well as quantitative definitions of risk can be encountered. For example, among structural reliability engineers the opinion is still frequently met that "risk" is simply identical with the "failure probability." Accepting such a statement, economic factors of risk are lost or fully misunderstood, and practical conclusions can be hardly drawn from risk studies based on similar simplifications.

It appears that the methodically clearest description of risk, Rs, formulated by Kaplan and Garrick 1981, is given by a set of triplets

$$\mathrm{Rs}_i \equiv (\mathrm{Sc}_i, \bar{P}_i, D_i), \qquad i = 1, 2, \ldots, n$$

where Sc_i = specific *hazard scenario* (Schneider 1985) of what can happen, $\bar{P}_i$ = *probability of occurrence* of that particular scenario during the reference period, T_{ref} (for example, the target life T_{0t}), and D_i = *measure of damage* associated with Sc_i, related to T_{ref} again. In a general case, n scenarios can be found, so that n triplets have to be studied and a weighted summarized risk has to be evaluated, setting

$$\mathrm{Rs} = \sum_{i=1}^{n} D_i \bar{P}_i$$

This formula is good only when hazard scenarios are independent. If not, it does not hold, and the dependencies have to be accounted for.

Therefore, all investigations of risk consist in finding answers to three basic questions:

- *What can go wrong?*
- *How likely is that to happen?*
- *If it does happen, what are the consequences?*

These questions, which became already classic, are expected to be answered by a reliability engineer. Obviously, the third question implies economic analyses. We should not forget, however, that D_i is a general term, which can be expressed in terms of money or human lives or in terms of other relevant units. Fortunately, reliability engineers are usually asked only whether there is a risk of certain adverse event, and just a YES or NO answer is required. Clients are rarely interested in a probabilistic answer; as a rule, they are not sufficiently qualified to appreciate replies phrased in probability-based terms.

Growing *risk perception* by the public has highlighted problems of risk identification and assessment. Instigating information for reliability engineers can be found in Allen *et al.* 1992, Aven 1992, Benjamin 1983, *Bridge Rehabilitation* 1992, *Engineering Safety* 1992, *Evaluation of Risk* 1991, Harms-Ringdahl 1993, Lind *et al.* 1991, Matousek 1988, *Risk, Structural Engineering and Human Error* 1984, *Technological Risk* 1982, and also in several collections of papers published at international inter-disciplinary conferences (for example, *Safety and Reliability* 1992, *Probabilistic Safety Assessment and Management,* 1991). The latter are particularly important as many valuable thoughts can be drawn from other engineering and even non-engineering branches. A good guidance can be obtained from CAN/CSA-Q634-91 (which is a document giving full outlines of the risk analysis; further Canadian documents on risk evaluation, risk control, and others concerning the risk management problems, are supposed to be published). A quarterly information retrieval journal, *Risk Abstracts*, has been being published since 1984.

15.5.3 Assessment of technologies and products

When a new construction technology is being developed, or a well known technology is to be used under conditions different from those originally presumed, a complex reliability assessment is of utmost importance. The change in conditions can refer not only to technical aspects (for example, subsurface ground conditions) but also to *socio-economic situation* (for example, lower standard of workmanship). This does not refer only to large or to many times repeated systems but also to individual products that should be used in construction.

Technologies and products that are not covered by any codified regulations or that differ significantly from existing ones should be subjected to assessment by reliability engineers. Then, technical approvals are granted by government agencies or other independent bodies, based on reliability assessment (cf. the European Technical Approval issued according to *Directive* 1989). No generally valid procedures can be given; the approach usually depends upon particular features of the technology or product assessed. In case of mass products and mass-applied technologies *systems of assessment* can be compiled. A good reliability-based assessment system has been introduced in Germany (see *Grundlagen zur Beurteilung* 1986).

15.5.4 Assessment of existing constructed facilities

During recent years the number of CFs subjected to rehabilitation has been constantly growing. This fact has brought a variety of problems connected with the assessment of CFs and, in particular, of structures that were already used in the past. In the main, cases are encountered, when a structure has been damaged either by unexpected events or by use or simply by age. Cases of immaculate structures are unique.

In the main, reliability of an existing structure is investigated on the following occasions:

♦ *rehabilitation* of an existing CF, during which new structures or members are added to the existing load-bearing system;

♦ *repair* of an existing structure that has been deteriorated due to time dependent environmental effects, current or abnormal use, etc.;

♦ adequacy checking in case of *change in use* with the objective to establish whether the existing structure can resist new types or higher (or lesser) magnitudes of load associated with the anticipated change;

♦ adequacy checking in case of *doubts on the CF's reliability,* often motivated by deterioration of material, corrosion, deflections, etc.;

♦ *legal processing* at transfer of property during which the technical condition of CF have to be verified.

Analysis and design during the assessment of an existing structure should be based on *general principles valid for structures of new facilities.* Earlier codes and design specifications valid in the period when the original structure was designed, based on abandoned theoretical principles, should be used only as *guidance documents.* We often encounter tendency of using old codes without sufficient understanding their background, without taking into account the fact that building materials have changed, etc.

As a rule, *analysis* need not be performed for those parts of an existing structure that will not be affected by structural changes, rehabilitation, repair, or that has not been damaged. A complete analysis is necessary:

♦ in *re-design for rehabilitation;*

♦ in *re-design for repair,* if the load arrangement or the structural system or both are supposed to get altered during the repair;

♦ in any *adequacy checking when the use of the building is changed* so that codified values of future loads increase (or also decrease) unfavorably in comparison with codified values corresponding with the original use;

♦ when *deterioration or defects reducing reliability* have been observed in CF.

When only the use of the facility is subjected to changes and when *no deterioration or defects diminishing the reliability* of the bearing structure have been observed, the verification and proportioning of members and cross-sections in the case can be simplified. Only a *comparison of stress load-effects* (bending moments, shear forces, stress, etc.) due to loads acting on the structure and its part before and after the change in use is sufficient. In the parts of the structure in which the change in use would produce stress load-effects higher than those before this change, a complete analysis and re-design must be carried out. This of course refers also to cases where alterations can cause *instability problems.*

For RelReqs ***values of elementary variables*** shall be taken as follows:

♦ *dimensions:* with their prevailing *values ascertained by site measurements;* when original design documents are available and no change in dimensions has taken place in the course of previous use of CF (for example, due to abrasion, corrosion, adjustments), *nominal dimensions* given in the documents should be used in analysis; these dimensions should be verified by site examination to an adequate extent;

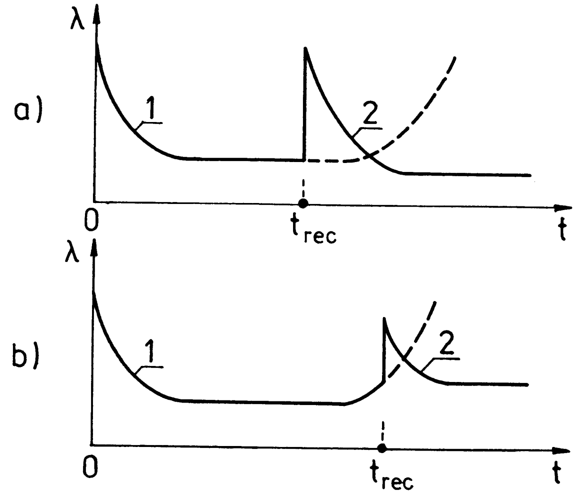

Fig. 15.1 - Failure-rate curves *vs.* time for two existing structures after rehabilitation (**a** - worsened reliability, **b** - improved reliability, **1** - failure-rate curve before rehabilitation, **2** - after rehabilitation).

♦ *loads* should be introduced with values corresponding with the actual situation; when overloading in the past occurred or when some load has been relieved, codified magnitudes of load can be appropriately reduced;

♦ *material properties* should be considered according to the actual state of the structure; to avoid bias effected by local damage, corrosion, etc., tests should be evaluated statistically with caution; in many cases, engineering judgment is necessary and deemed to be sufficient when no random sample can be collected.

Prestress and also ***restraints*** should be considered according to the actual situation, taking into account possible defects. For example, it is not sensible to assume a reinforced concrete beam as fully fixed-ended when cracks at fixed ends have been noticed.

When giving re-design recommendations based on the assessment of an existing CF, the reliability engineer should consider what will happen to the original bath-tub curve (see 2.2.1) related to the facility. After the rehabilitation, the curve is supposed to acquire the shape according to Figure 15.1. We can identify a *renewed burn-in period,* due to some initial flaws and defects. Examples of buildings can be given when structures collapsed just after the rehabilitation had been completed! The failure rate in the constant λ period can be greater or less than for the original facility.

Great attention has been paid to existing structures in recent years; various viewpoints have been considered. Some publications: Allen 1991, Bartlett and Sexsmith 1991, Bea 1987, *Bridge Rehabilitation* 1992, Chou and Yuan 1992, Ciampoli *et al.* 1990, Delbecq and Sacchi 1983, Yao 1980. Guides and manuals where many practical advice can be found are also available (for example, *Manual for Maintenance Inspection* 1984, Ventolo 1990). A national code on re-design of existing structures is used in Czechia: ČSN 73 0038-1986; a similar one is being worked out in Canada. Manuals on evaluation of buildings and bridges are available in some countries: *Appraisal* 1980, *Load Capacity Evaluation* 1987, *Strength Evaluation* 1987.

15.5.5 Some suggestions

Actions

Before starting a reliability assessment, always ask your client to *specify his or her instructions* by formulating questions in writing. Though clear questions are important not only to the reliability engineer, but also to the client, many clients will be surprised by such a request. When appointing a reliability engineer, clients are often not explicit enough on the goals of the assessment. - If your client is not able or not willing to submit any questions, try to prepare them yourself and ask him or her for endorsement. Do not accept any questions concerning responsibilities and liabilities, particularly in trial and arbitration disputes (see also 15.5.1). Such questions have to be answered by judge or arbitrator.

When *partial goals of the assessment* are not specifically stated in the instructions, it is necessary to outline such goals. This is an important help in solving the problem. During the solution you can adjust the objectives according to intermediate results and conclusions.

Now, when questions have been formulated and partial or final objectives specified, you should take following steps (not all may be relevant, and on the other hand the list can be enlarged):

♦ Identify *parties involved.* This should be done always, that is, also when no court trial or arbitration are pending.

♦ Gather all possible *documents:* contract documents, structural drawings, shop drawings, transmittal letters, boring logs, site records, etc.

♦ Gather *photographs and videotapes,* if available.

♦ Gather information and records on *loading tests* if there were any carried out.

♦ When existing structures are assessed, do not confine yourself to only one site visit. It pays to make *two or more successive examinations;* the second examination should follow after you have already created partial assumptions, got acquainted with drawings and contract documents, and formulated a preliminary opinion of the problem.

♦ Identify all *reliability systems and subsystems.* Start with the system to be investigated and extend it to branch systems. In structural problems, do not concentrate only on load bearing systems. Accessorial systems can be equally important as the bearing ones.

♦ Identify *reliability connections* (serial, parallel, combined, etc.) between reliability items and study their physical meaning.

♦ Identify *possible failure modes.*

♦ Consider any possible *irregularities* in the system behavior.

♦ Gather information on the *past behavior* of the assessed system (failures, defects, loading history) if relevant, and also information on analogous systems, if available.

♦ Identify behavior of *joints* (rigid, semi-rigid, sway), *structural connections* and *connectors*.

♦ Identify all relevant *time-dependent factors*.

♦ Identify all *physical variables* affecting systems' behavior, their random properties, their statistical dependencies.

♦ Check the *subsurface ground conditions*, standing-water level, etc., even when your problem has apparently nothing in common with ground.

♦ In case that design or execution faults are obvious, consider also *why the structure did not collapse*. It can help understanding the structure's behavior.

♦ Define *variables entering calculation and evaluation models*. Obviously, their number can be less than that of all variables involved.

♦ Gather information on the *random properties of variables*, including information on their statistical dependencies and auto-dependencies (when space and time-dependent variables are dealt with).

♦ Try to compile *relation formulas* for phenomena involved; in other words, determine possible combinations of phenomena and events.

♦ Consider *exposure to hazards* of the CF system investigated, and the associate properties: vulnerability, fragility, robustness, integrity. Identify possible *hazard scenarios*.

♦ Specify and evaluate *performance criteria* and *reliability requirements*. Do not forget time dependencies, and also size effects.

♦ Study *contract documents* and try to translate the legalese hereinafters and hereinbelows into engineering parlance. Contract documents often contain important material, particularly when structural defects and failures are analyzed.

♦ Check *codes, regulations*, etc. When an existing structure is assessed, the earlier codes can give a good guidance in understanding the original design.

♦ Gather information on *adjacent buildings and structures*. Their behavior might be important in your assessment.

♦ All your statements on the issue should be *in writing*. Do not give any conclusive opinions in oral form except when heard as expert witness in front of court.

♦ Check your statements for ambiguity. Avoid any vagueness in your replies. When you are not able to give clear answer, state just this fact.

Report

A reliability engineer's report, submitted to the client, should be written considering always the possibility that third persons entitled could for some reason be interested to study it. Further, reports can be examined and re-examined after a certain time has already elapsed by people who are only superficially acquainted with the problem and who are for some reason obligated to study it. Therefore, we should be meticulous in giving all details of findings and in being clear in the conclusions.

A general outline of a report is suggested:

♦ *Basic data* (CF examined, situation and site, client, designer, contractor, authorities and agencies involved, etc.).

♦ *Instructions and assignment specification* (questions set by your client, backgrounds to the problem).

♦ *Documents used* (codes, design documents, contract documents, boring logs, transmittal letters, time schedules, minutes of meetings, etc.). Indicate also documents which have not been accessible (lost, stolen, damaged, withheld, classified, etc.).

♦ *Technical data* (observation and measurements records, photographs, videotapes).

♦ *Examinations carried out;* participants to examinations should also be listed.

♦ Laboratory and in-situ *tests* performed by the reliability engineer.

♦ *Facts found* (facility description, condition, execution problems, documents missing, etc.; just simple statements on facts, with no analysis, should be given here).

♦ *Analysis of facts* (evaluation of data, theoretical analysis, calculation models used, etc.).

♦ *Findings* (which should be, in the main, a critical summary of the analysis).

♦ *Answers to questions.*

♦ *Recommendations* if relevant (for example, on measures to be taken, on facts found that are not relevant to the assignment but that can be important for health and occupational safety, etc.).

16

THE FUTURE

Key concepts in this chapter: *general theory of reliability, GTR; structural reliability theory; constructed vs. mechanical facilities; limitations of the reliability-based design; data problem; Joint Committee on Structural Safety, JCSS; future tasks.*

In the introducing chapters of most monographs a survey of work done, a state-of-the-art report, or the story of the respective discipline is presented. Let us try to avoid this routine pattern and let us pay attention to the future. Forecasting, however, is always difficult; this is particularly true as far as the science and technology are concerned. Just consider one typical phenomenon: leafing through science-fiction stories of the thirties and forties, we can hardly trace any mention on computer technology and its superb service. Yet, we can say without exaggeration that we are really living in a sci-fi engineering climate, which would be considered, a couple of decades ago, a mere product of fantasy. Therefore, any prognostication can be fraudulent; this refers also to the reliability-based aspects of the construction process.

16.1 POSITION OF THE STRUCTURAL RELIABILITY THEORY

The general theory of reliability, GTR, exists as an excellent tool to description and analysis of many phenomena of the world we live in. The laws of GTR are valid not only in all *engineering branches* (mechanical, civil, electrical, nuclear, and others) but also in *other fields of human activity.* You can easily fix a bath-tub curve to reliability features of human life, of parts of your own biological entity, of social systems, urban areas, and also, to turn back to our profession, of structural design codes... All concepts of GTR can be applied in the analysis of natural, psychological, and any engineering phenomena. Unfortunately, the large complex of GTR has not yet been presented as a whole; available monographs and handbooks were prepared by specialized engineers (mechanical, electrical, structural) for their respective colleagues, and therefore, they never cover the field of GTR in entirety.

Of course, not all concepts of GTR are applicable in all branches, though a constant development and convergence is now evident (this can be observed on international meetings; see, e.g., *Probabilistic Methods* 1985, *Probabilistic Safety Assessment* 1991). - For example, a possibility of spare parts and reserve systems for components of human body was unimaginable in the past. At present, using mechanical, electrical, and biological spare parts, reserves, etc. (prostheses, pace-makers, blood transfusion, transplants) has become an everyday fact in health care.

Table 16.1 - Constructed *vs.* mechanical facilities; their principal features

Aspect	Constructed facilities	Mechanical facilities
Technological (with regard to higher level entities)	♦ principal purpose: bearing (for example, floor), covering (roof), flattening (road), retention (dam), etc.	♦ principal purpose: production (steel mill), transport (cars), information (clock)
	♦ stationary facilities, their movement is strongly unwanted (except relocatable buildings, special bridges)	♦ movable facilities; movement is the mean or objective of the activity
	♦ power consumers	♦ power producers or transmitters
Technical (with regard to their behavior)	♦ resist natural and technological loads	♦ resist mainly technological loads
	♦ in the main, loads have a small number of repetitions	♦ loads are many times repeated
	♦ physical and geometric non-linearity must be often considered	♦ linear stress domain, non-linear kinematic behavior
Economical	♦ life expectancy is about 50 to 100 years	♦ life expectancy is about 5 to 20 years
	♦ replacement of the facility is usually enforced by deterioration	♦ replacement of the facility is enforced by modernization and innovation
	♦ replacement of elements is impossible or very exacting	♦ replacement of elements is a current practice
	♦ reserve systems and elements are economically unacceptable	♦ reserve systems and elements are a normal solution
	♦ substitution of a system after ultimate failure is difficult	♦ substitution is in most cases very simple in a short time
Social	♦ failures of elements and systems have a strong impact on public, even when their effect is small	♦ public is often indifferent to failures
	♦ some facilities get monumentalized or are planned as monuments from the beginning	♦ monumentalization is exceptional

Yet, *differences between various groups of systems* (let us now discuss only the engineering systems) are substantial, and, as a result, each engineering branch is drawing from the GTR pool only a certain amount of knowledge.

Consider two classic groups of systems: constructed facilities and mechanical facilities. Table 16.1 lists the principal features of the two groups and graphically shows that we have to distinguish between the *theory of reliability of constructed facilities*, or shortly *structural reliability theory*, SRT, and the *theory of reliability of mechanical facilities*. The differences, from various view-points, are doubtlessly substantial.

We have to bear in mind that, for ages, not too many *basic systems* had developed in the constructed family. *Houses, bridges, dams,* and *roads* have been built since the existence of humankind. As a result, beams and cantilevers, columns, arches, retaining walls, and compact soil are typical systems that constantly appear in all CFs. We are well acquainted with them and we use them without any trouble, as though they were the elements of some super-Lego set. Conversely, any complicated structural system, such as a space frame, space truss, can be reduced, with more or less effort, to a simply analyzable system. This is not a general rule, of course, but it may be reckoned that, historically, the engineering profession avoided systems that could not be easily simplified. Computers have brought completely new concepts into engineer's thinking but the drive towards simplification still prevails in the structural design philosophy.

In mechanical engineering basic elements are also simple, e.g., *wedge, wheel, lever, bearing, shaft,* and *screw,* but the resulting systems are multifarious. The endless list can start with *simple toys* and conclude, for the time being, with sophisticated *space shuttles* (which, however, involve also constructed, electrical, and electronic systems). Requirements on the level of reliability parameters are extremely broad; parameters additional to those governing the reliability of CFs (see Section 10) enter reliability models: *operating time, time to failure, down time, probability of restoring operability*, and many others. - Analogous considerations could be extended to electrical and electronic systems, which are conceptually still more removed from those forming constructed and mechanical facilities.

The basic nature of the engineering branches cannot change. While, for example, a telephone exchange system, which in the early twenties required a huge building, can now be housed in a suit-case-sized box, a residential house can never be compressed. Thus, the discrimination of the special reliability theories will persist.

It can be maintained that the *philosophy of reliability of CFs* is well defined; no revolutionary changes can be expected. Obviously,

♦ *mathematical models* of the reliability theory are sufficiently developed, and so practically all problems connected with existing and new structural systems and materials can be solved; naturally, the depth and quality of solutions depends naturally upon knowledge of calculation models covering the time-dependent physical behavior of the examined system;

♦ wide *practical application* of the theoretical results achieved in past decades is under way; modern structural codes are based on probabilistic methods; new reliability concepts enter codes: vulnerability, inspectability, maintainability, and others;

♦ methods of *reliability control and reliability assurance* in design, construction, and maintenance are quickly developed as an extension of the quality control and assurance methods.

As a result, the actual interest of the reliability researchers is getting concentrated more on practical applications than on development of sophisticated theories.

16.2 LIMITATIONS

There are no theoretical limits to the use of GTR in the domain of CFs. Virtually all mathematical models, techniques, and procedures used in mechanical and electrical reliability problems can be implemented into the body of SRT. Nonetheless, many models are of no specific importance in structural reliability, many concepts are superfluous. Consequently, practical application of GTR results is confined by the *inherent nature of CFs,* as outlined in Table 16.1. But we must not be misled by this assertion. What seems to be unfitting today can be of great help tomorrow.

For example, it seems to be useless to implant the concept of spare parts into present general design rules for structures, although in particular cases such a concept might be economically very useful (for example, replaceable elements in highway sound barriers). The situation develops quickly. We have already started using proof-testing methods in construction industry, and design-by-testing methods, which both would be hardly passable a couple of years ago. These are methods effectively applied in our colleague professions, and, moreover, their principles had been subconsciously used in the antiquity of construction as fundamental methods of building-up empirical experience.

Thus, from the viewpoint of CFs there are no theoretical limits that would prevent benefitting of theoretical knowledge accumulated in the GTR space. *The only limitations here are economical.*

The absolutely greatest part of CFs are "normal" buildings, bridges, roads, waterways, etc. where about 99 percent of funds allocated to construction are spent. For these facilities rules for design, construction, inspection, and maintenance that simplify concepts and avoid using complicated calculations must be developed; such rules must be sufficiently general. However, any such rule, though simplified enough, arouse opposition and distrust as soon as it is offered to the profession. In fact, it is not possible to convince designers of using, say, probability-based limit states design solely by advertising it as "a fine, advanced, scientifically founded method, reflecting the actual behavior of loads and structures." The new method can get implemented either under duress (by simply withdrawing the old regulations and ordering the use of new ones; this way should be condemned) or when some economical benefit to the designer, contractor, and client is evident. At first glance, these limitations are psychological, which is true, however, only during the period before economical merits of the new method are acknowledged. Then, psychological barriers are dropped. Many examples could be given from the Author's own professional experience.

The remaining 1 percent (or less) of construction-oriented funds is consumed by special, sophisticated facilities, where advanced reliability solutions are economical, and so fully justified.

A valuable *critique of structural reliability theory* was presented by Elms and Turkstra 1992.

Data problem

When examining the possibility of using advanced reliability-based calculation models in a particular situation, we frequently hit a "no-data barrier." This barrier can have either technical or economical background or both. Note the principal reasons why data are missing:

- nobody has expected a possible use of particular material or technology, or structural system; therefore, nobody has cared for data on respective strengths and moduli, load values, etc., on base of which statistical populations could be estimated;
- the required data are time-dependent, and their assembling has started only recently (this refers to new materials with rheologic or fatigue behavior, to materials exposed to new environment, to climatic loads in regions where no observations were carried out in the past, etc.);
- compiling data is technically difficult or even impossible;
- compiling data is economically unacceptable.

The less data we have, the more simplifications we must accept, and, consequently, the more costly is the result of the construction activities (here, "costly" must be referred to comprehensive costs, that is, costs including also potential failures originating from simplifications).

Nevertheless, an experienced reliability engineer can overcome the lack of data on almost any phenomenon. You can, for example, make a reasonable and safe guess on the random behavior of some material properties; it is necessary to be acquainted with the behavior of similar material, or to have some information on the physical nature of the material in question. Of course, there always is a danger of surprising effects under unusual conditions; this must be taken into account in the evaluation of risks involved.

Probabilization levels

The state of probability-based design has been stabilized during about past ten years. Methods of calculation are now sufficiently clearly specified, though innovations are always appearing. On the whole, major changes are not likely in the nearest future (say, in the coming 20 years).

Experience shows that a probability-based design method must meet the following requirements:

- *simplicity* even when high-tech computers are available;
- aptitude to *generalization* through structural design codes and the supporting codes on quality control;
- *low sensitivity* of results of design to various solution techniques;
- simple *testability* of the input parameters of the design.

Considering these requirements, it can be judged that the present *semi-probabilistic design format*, as specified, for example, by *Eurocodes,* will, in the coming two or three decades, prevail upon other, more advanced formats. The use of fully probabilistic solutions based on the direct method (see Chapter 12) will remain restricted to special structures, mass-produced elements, and to code making.

Topical issues

In 1986, the *Joint Committee on Structural Safety* compiled a list of main topics that should be examined in the coming years. After seven years, the list is still up-to-date, and it is presented here with only editorial adjustments. Observe, that overall theoretical issues are not contained, and that the main interest is paid to practical problems associated with the implementation of the reliability-based methods:

- risk identification and evaluation;
- load combinations;
- serviceability criteria (displacement and deformation limits, deterioration, evaluation of various reliability concepts);
- design by testing;
- deterioration and maintenance (classification of environment, maintenance cost, consequences of failures);
- reliability assessment of existing CFs (inspection, evaluation of residual life);
- reliability-based design (time-dependent phenomena, calculation models);
- material properties (probability modeling, quality control);
- geometry, imperfections, tolerances;
- fatigue of materials and structures;
- robustness and integrity of structural systems;
- geotechnical structures (interaction of structures and soils, statistical modeling of soil);
- verification of design assumptions (verification during and after execution, load tests, monitoring of the facility);
- design formats (rules of probability-based proportioning, reliability requirements);
- optimization problems (economical criteria, differentiation of safety and serviceability, system analysis of the construction process, optimization criteria);
- modeling of errors, occurrence of errors in the construction process;
- decision process (codes, contracts, warranty specifications, insurance);
- professional ethics in the construction process (liability of the designer and contractor towards client, public, and the profession);
- quality assurance education.

It seems likely that all features of the reliability assurance and control are covered by this list. There is hardly anything substantial to be added.

Appendix A

LOG-NORMAL DISTRIBUTION *

As a rule, *two-parameter log-normal distribution,* LN, is used by many authors in the solution of various structural reliability problems. It is fully described by the population mean, μ, the population standard deviation, σ and the respective CDF. It is always positively asymmetric with the coefficient of skewness, α, given by

$$\alpha = \delta(3 + \delta^2)$$

where δ = coefficient of variation, σ/μ. Its lower bound is $x_{inf} = 0$, no upper bound exists, that is, $x_{sup} \rightarrow \infty$.

However, experience shows that the two-parameter LN, being anchored to $x = 0$, is not flexible enough, and it does not fit in with the description of random behavior of many of the phenomena that it is applied to. Some authors are using LN even for phenomena with a technically clear negative skewness! It appears, however, that the ***three-parameter log-normal distribution***, LN(α), is more effective than the LN, since

♦ it can be either lower or upper-bounded by the *infimum,* x_{inf}, or *supremum,* x_{sup}, respectively;

♦ the coefficient of skewness, α, can be either positive or negative, respectively;

♦ for $\alpha = 0$, LN(α) becomes normal, N.

The use of LN(α) is well justified. In a specified range any continuous function

$$y = f(x_1, x_2, \ldots, x_n)$$

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* This Appendix has been prepared in close cooperation with Dr. Miloš Vorlíček, Klokner Institute, Czech Technical University, Prague.

can be approximated by

$$y = \prod_{i=1}^{n} x_i^{r_i}$$

where r_i = exponents found for the given range by regression analysis. Logarithming gives

$$\log y = \sum_{i=1}^{n} r_i \log x_i$$

Assume that x_i are random variables ξ_i with arbitrary probability distributions, than y becomes a random variable, η, also. When $n \to \infty$, then according to the Central Limit Theorem the probability distributions of $\log \eta$ tends to normal distribution and, consequently, the distribution of η tends to a log-normal one.

Though it has been only rarely employed and rather unusual in the prevailing structural reliability practice the three-parameter log-normal distribution has now been known and used in other science and engineering branches for many decades (see, for example, Cramér 1945, Finney 1941, Johnson 1949). However, until now, no detailed tables were prepared, and, in particular, the problem of estimating its parameters was not solved. Tables presented in this appendix are, very likely, the first contribution facilitating a more wide application of LN(α) in various practical exercises.

We should mention here also the *four-parameter log-normal distribution*, LN(p_1, p_2) having both the lower and upper bound (some analysis was performed by Aitchinson and Brown 1957). A study of its properties has proved that *it does not match with the actual performance of phenomena that are in the domain of a civil engineer's interest.* The particular drawback is its very pronounced peakedness.

Further information on log-normal distribution can be found in Calitz 1973, Cohen and Whitten 1980, Harter 1966, Johnson and Kotz 1970.

A.1 DESCRIPTION OF LN(α)

A.1.1 General

The probability distribution of the standardized random variable

$$u = \frac{\xi - \mu}{\sigma} \tag{A.1}$$

is log-normal when the distribution of

$$u^* = \ln |u - u_0| \tag{A.2}$$

is normal. Here, ξ = random variable, μ, σ = population mean and standard deviation of ξ x, respectively, and u_0 = standardized bound of the distribution.

Mathematically, the LN(α) probability distribution is a logarithmic (or rather antilogarithmic) transform of the normal distribution, N. Therefore, for calculation of its parameters, relationships valid for N can be applied.

A.1.2 Functions and parameters of LN(α)

Coefficient of skewness

The decision on the coefficient of skewness [in fact, it is a decision on the type of LN(α)] must be based on previous experience with the particular phenomenon or on subjective judgment taking into account all factors that may influence the random behavior of the variable investigated. For example, we know that the yield stress of steel of a specific grade cannot be technically less than a certain value. Naturally, we are never sure where the real bound is, but some decision can always be taken. A little "statistical experience" is necessary. Some support for the decision on α can be gained from random samples. However, it must be kept in mind that large samples are needed to assess α correctly.

Lower and upper bound

In the following formulas describing various parameters of LN(α), the parameter u_0 appears; it defines, in dependence on the sign of the *coefficient of skewness,* the infimum, u_{inf}, or supremum, u_{sup}, of LN(α). As a rule, the value of u_0 cannot be established from experiments or observations; however, we derive it from the *coefficient of skewness,* α .

The relationship between the bound and the coefficient of skewness is given by

$$\alpha = -\frac{1}{u_0}\left(3 + \frac{1}{u_0^2}\right) \tag{A.3}$$

Equation (A.3) yields

$$u_0 = -\frac{1}{\alpha}\left(|a + b|^{\frac{1}{3}} + |a - b|^{\frac{1}{3}} + 1\right) \tag{A.4}$$

where

$$a = -\frac{1}{\alpha}\left(\frac{1}{\alpha^2} + \frac{1}{2}\right), \quad b = \frac{1}{2\alpha^2}(4 + \alpha^2)^{\frac{1}{2}}$$

Note that the sign of u_0 is opposite to that of α . The dependence of $|u_0|$ upon $|\alpha|$ is shown in Figure A.1.

On the other hand, u_0 can be defined, in specific cases, by the *nature of the phenomenon.*

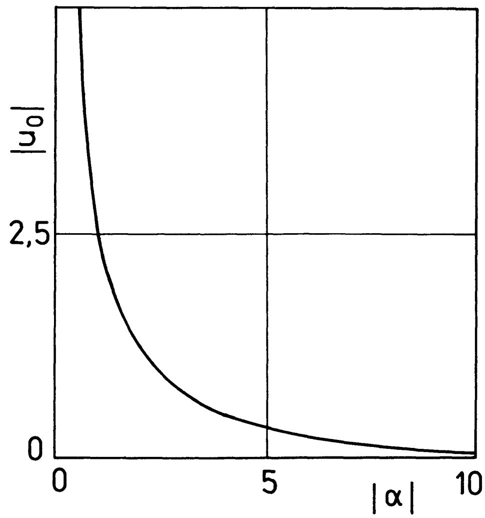

Fig. A.1 - *Log-normal distribution.* Bound, u_0, *vs.* coefficient of skewness, α.

In the subsequent formulas the following notation is used:

$$A = 1 + \frac{1}{u_0^2}$$

$$B = \left[\ln\left(A^{\frac{1}{2}}\frac{|u-u_0|}{|u_0|}\right)\right]^2 \cdot (2\ln A)^{-1}$$

Probability density function, PDF (in the definition domain)

$$\varphi_{LN} = \frac{1}{|u-u_0|}\, e^{-B}\,(2\pi \ln A)^{-\frac{1}{2}} \qquad \text{(A.5)}$$

For some values of α, PDFs are shown in Figure A.2.

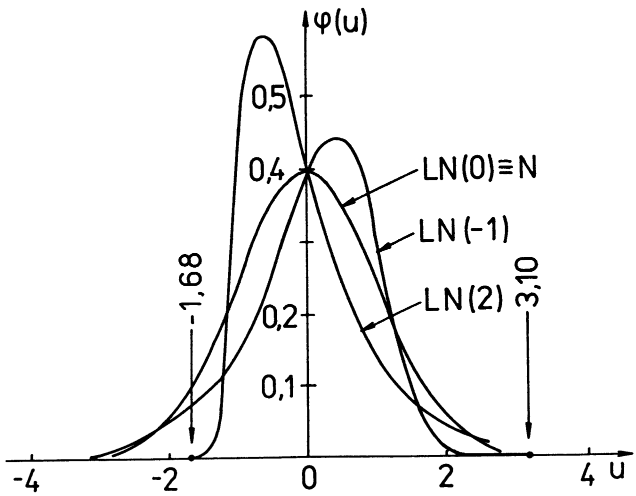

Fig. A.2 - *Log-normal distribution.* Examples of probability density function.

Cumulative distribution function, CDF (in the definition domain)

♦ For $\alpha > 0$:

$$\Phi_{LN}(u) = \int_{u_0}^{u} \varphi_{LN}\, du$$

♦ For $\alpha \leq 0$:

$$\Phi_{LN}(u) = \int_{-\infty}^{u} \varphi_{LN}\, du$$

The indefinite integral cannot be explicitly expressed. Therefore, either a numerical analysis is necessary, or, which is more simple, the equivalent fractile u^*, Equation (A.2), of the standardized normal distribution, N, is established. Then, the respective value of CDF of N is found for u^*. It holds

$$\Phi_{LN}(u) = \Phi_N(u^*) \qquad \text{(A.6)}$$

Values of CDF can be obtained from Table A.1.

Inverse distribution function, IDF

When calculating a value of the random variable u from a given value of $\Phi_{LN}(u)$ the same technique is used as in the case of CDF. We set

$$\Phi_N(u^*) = \Phi_{LN}(u)$$

and from $\Phi_N(u^*)$ the value of u^* (that is, for the normal distribution) is found. Then, u is calculated from

$$u = u_0\left\{1 - A^{-\frac{1}{2}} \exp\left[\operatorname{sign} \alpha \cdot u^*(\ln A)^{\frac{1}{2}}\right]\right\} \qquad (A.7)$$

Values of IDF are given in Table A.1.

Coefficient of excess

$$\varepsilon = \frac{1}{u_0^8}(16u_0^6 + 15u_0^4 + 6u_0^2 + 1) \qquad (A.8)$$

Mode

$$\hat{u} = u_0(1 - A^{-\frac{3}{2}}) \qquad (A.9)$$

A.2 ESTIMATION OF THE POPULATION PARAMETERS

In the evaluation of data the knowledge of LN(α) is assumed. That is, we define LN(α) by either selecting α or u_0 (see A.1.2).

A.2.1 Interval estimates for μ and σ

For interval estimates of the population mean, μ, and of the population variance, σ^2, procedures analogous to those used for normal distribution can be used. Values of the respective estimation coefficients must be different of course. Let us only introduce the estimate interval formulas and typical tables of the respective estimation parameters.

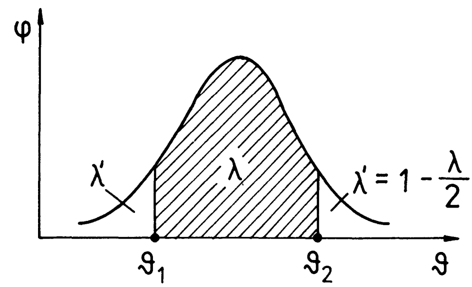

Fig. A.3 - *Log-normal distribution.* Interval estimate of a population parameter ϑ .

Confidence level, λ , must be specified in any interval estimation. It is given by the probability of occurrence of values of the parameter estimated, ϑ , confined by the interval bounds, ϑ_1 and ϑ_2 , Figure A.3.

According to the *amount of information available,* two cases of estimation must be distinguished:

- ♦ μ **unknown,** σ **known,**
- ♦ μ **unknown,** σ **unknown.**

Theoretically, the case " μ **known,** σ **unknown**" is also possible, but it is without practical significance in structural reliability problems.

Interval estimate for the population mean, μ

♦ For σ **known:**

$$m - q_2 \sigma \ < \ \mu \ < \ m - q_1 \sigma \qquad (A.10)$$

Values of estimation coefficients q_1 (lower lines) and q_2 (upper lines) are given in Tables A.2 in dependence on the intended confidence level, λ , coefficient of skewness, α , and on the size of the random sample, n .

♦ For σ **unknown:**

$$m - q_2^* s \ < \ \mu \ < \ m - q_1^* s \qquad (A.11)$$

Values of estimation coefficients q_1^* (lower lines) and q_2^* (upper lines) are given in Tables A.3.

Interval estimate for the population variance, σ^2

The confidence interval is

$$\frac{s^2}{r_2} < \sigma^2 < \frac{s^2}{r_1} \qquad \text{(A.12)}$$

Values of r_1 (lower lines) and r_2 (upper lines) are given in Tables A.4.

Point estimates of μ and σ

By definition, the point estimates for the mean, μ, and standard deviation, σ, are simply given by the respective sample characteristics, m and s; see formulas given in 2.1.2, Table 2.1.

A.2.2 Estimates of fractiles

Again, two types of fractile estimates can be used in practical problems: the ***interval estimate*** (with an intended confidence level, λ) and the ***point estimate.***

One-sided interval estimates

Since in structural reliability problems the extreme *adverse* values are sought for, only one-sided interval estimates are considered here. Let the interval estimate, $x_{\kappa,est}$, for a κ-fractile be defined as the respective bound of the left-hand-sided or right-hand-sided confidence interval,

$$x_\kappa^* = m^* + u_\kappa s^* \qquad \text{(A.13)}$$

Here, m^* = either the sample mean, m, or the population mean, μ; s^* = either the sample standard deviation, s, or the population standard deviation, σ. Which of the two associated characteristics is to be considered will be shown later.

The bound $x_{\kappa,est}$ is defined in the following manner:

- for $\kappa < 0.5$:

$$\Pr(x_\kappa > x_{\kappa,est}) = \lambda$$

- for $\kappa > 0.5$:

$$\Pr(x_\kappa \le x_{\kappa,est}) = \lambda$$

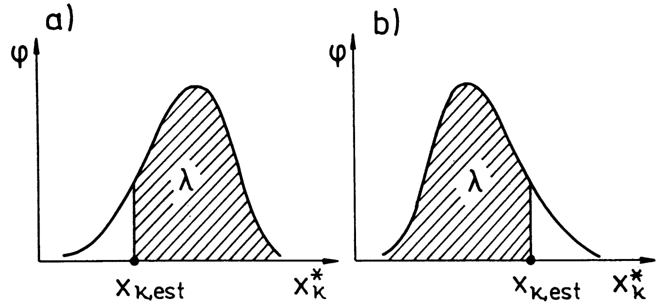

Fig. A.4 - One-sided interval estimate of fractile $x_{\kappa,est}$ (**a** - left-handed, **b** - right-handed).

where λ = intended confidence level, that is, the probability of occurrence of values x_κ being *more favorable* than the bound $x_{\kappa,est}$, see Figure A.4.

In general, $x_{\kappa,est}$ follows from:

$$x_{\kappa,est} = m^* + k_i s^* \tag{A.14}$$

where k_i = relevant interval estimate coefficient.
Taking into account information on σ, we have

♦ with σ **known:**

$$x_{\kappa,est} = m + k_1 \sigma \tag{A.15}$$

♦ with σ **unknown:**

$$x_{\kappa,est} = m + k_2 s \tag{A.16}$$

k_1 and k_2 are given in Tables A.5 and A.6.

Point estimate

The mean of the distribution of the random variable x_κ^*, given by Equation (A.13), can be considered a good *point estimate* of the κ-fractile, $x_{\kappa,est}$. Then the value of $x_{\kappa,est}$ is

$$x'_{\kappa,est} = m^* + k'_i s^* \tag{A.17}$$

where k'_i = point estimate coefficient.

Similarly, as for the interval estimate, two cases are important:

♦ with σ **known:**

$$x'_{\kappa,est} = m + k'_1 \sigma \tag{A.18}$$

♦ with σ **unknown:**

$$x'_{\kappa,est} = m + k'_2 s \tag{A.19}$$

Values of k'_1 are identical with $\Phi^{-1}_{LN}(\kappa)$ or $\Phi^{-1}_{LN}(1 - \kappa)$; see Table A.1. Values of k'_2 are given in Tables A.7.

A.2.3 Tables of estimation coefficients

Values of estimation coefficients, q , q^* , r , k , and k' , can be analytically determined only for some distributions (see, for example, Guenther *et al.* 1976 for exponential distribution). For the log-normal distribution LN($\alpha \neq 0$) no explicit formulas are known. Therefore, the respective values were calculated:

♦ by means of approximate formulas (q , q^* , r ; see Vorlíček 1991a, 1991b);
♦ by means of Monte Carlo simulation technique (k, k').

Values for $\alpha = 0$ were taken from existing estimation tables for normal distribution.

Use of tables

The following steps have to be taken in any estimate calculations:

(1) Sample characteristics, m and s are calculated.
(2) Coefficient of skewness, α , is determined (see A.2.1).
(3) Type of the estimation is decided.
(4) If relevant, confidence level is selected.
(5) The amount of information available is assessed. If the approximate value of the population standard deviation is known (for example, from quality control testing), use the corresponding estimation coefficients.
(6) The estimation coefficient is established from the pertinent table, and the corresponding estimation formula is used.

Tables of q, q^*, and r

The sets of Tables A.3 and A.4 give interval estimate coefficients for μ and σ^2 for $\alpha \in [0; 2]$, $n \in [3; 30]$, and $\lambda \in [0.90; 0.99]$. The negative and positive values of q and q^* refer to the upper and lower bound, respectively. When $\alpha < 0$, the signs at table values must be changed. The smaller value of r refers to the lower bound of the interval estimate.

Tables of k and k'

The sets of Tables A.5 and A.6 of *interval estimate coefficients for fractiles* are established for $\alpha \in [0; 2]$, $n \in [3; 30]$, and $\lambda \in [0.95; 0.50]$. The negative values of k_i refer to $\kappa \in [0.001; 0.250]$, and the positive values refer to $\kappa \in [0.750; 0.999]$. Values of κ and λ are given in the heading of each table.

When $\alpha < 0$, the sign of table values must be changed.

For intermediate values of κ and λ (if relevant), the estimate coefficients can be determined by non-linear interpolation, using a semi-logarithmic or beta-4 probability paper (see Appendix B). For intermediate values of α and n, linear interpolation can be used.

Tables A.7 of k_3' are arranged in a similar manner as those of interval estimate coefficients. However, the confidence level, λ, does not apply here.

■ **Example A.1.** A sample of six values of the compression strength of concrete, f_c, obtained by tests, is to be analyzed. Interval estimates of the population mean and variance, and also the left-hand-sided interval estimate of the characteristic strength, that is, of the 0.05-fractile, is to be established. Confidence level $\lambda = 0.90$ should be assumed for all estimations. The population standard deviation is not known.

The following values of strength (N.mm^{-2}) were observed:

22.1; 19.7; 20.2; 20.6; 23.5; 24.2

The sample characteristics are:

- $m_{fc} = 21.7$ N.mm^{-2}
- $s_{fc} = 1.84$ N.mm^{-2}
- $a_{fc} = 0.51$ (for illustration only)

Since the sample size is small, the information on skewness based on a_{fc} cannot be used. According to experience, the coefficient of skewness of the compression strength in the obtained range is about $\alpha_{fc} = 0.4$.

(a) **Interval estimation of the population mean.** Equation (A.11) is used. From Table A.3.1 values $q_1^* = 0.85$ and $q_2^* = -0.80$ are found. The respective confidence interval is

$$21.7 - 0.85 \times 1.84 < \mu_{fc} < 21.7 + 0.80 \times 1.84$$

that is, $\mu_{fc} \in$ **[20.1; 23.2] N.mm^{-2}.**

(b) **Interval estimation of the population variance.** Equation (A.12) is used. From Table A.4 values $r_1 = 2.31$, $r_2 = 0.22$ are found. The respective confidence interval is

$$\frac{1.84^2}{2.31} < \sigma_{fc}^2 < \frac{1.84^2}{0.22}$$

that is, $\sigma_{fc}^2 \in$ **[1.47; 15.4] (N.mm^{-2})^2**, or, in terms of the standard deviation: $\sigma_{fc} \in$ **[1.22; 3.92] N.mm^{-2}.**

(c) **Interval estimation of the characteristic strength.** Equation (A.16) is used. From Table A.6.10 $k_2 = -2.73$. The interval estimate value of the characteristic strength is:

$$f_{ck,est} \equiv f_{c,0.05,est} = 21.7 - 2.73 \times 1.84 = \mathbf{16.6\ N.mm^{-2}}$$

By analogous procedure, the value of the **fractile point estimate** is obtained, Equation (A.19); with $k_2' = -1.79$ from Table A.7.3, it is $f_{ck,est}' = 18.4$ N.mm^{-2}. ■

■ **Example A.2.** The right-hand-sided interval estimate of the 0.9875-fractile of the annual snow load maxima, s_{max}, is to be established with confidence level 0.90. Only nine successive anual maxima are available (in kN.m^{-2}):

0.440	0.360
0.050	0.120
0.200	0.056
0.350	0.057
0.140	

Let us assume further that the observations are statistically independent (long-run climatic changes are not considered). Let us assume that the probability distribution of snow load maxima is log-normal. An extreme values distribution would be theoretically more appropriate; however, taking into account the objectives of analysis (determination of the design value of the snow load), the differences in results are not significant. - The following sample characteristics have been calculated:

- $m_s = 0.197$ kN.m^{-2}
- $s_s = 0.065$ kN.m^{-2} (for illustration only)
- $a_s = 0.62$ (for illustration only)

The evaluation of data obtained at the neighboring observation stations yielded $\sigma_s = 0.100$ kN.m^{-2} and $\alpha_s = 1.0$.

No table gives estimate coefficients just for $\kappa = 0.9875$ (nor for 0.0125). Therefore, some interpolation had to be performed, which yielded $k_1 = 3.28$. So, using Equation (A.15), the interval estimate is

$$s_{\kappa,est} = 0.197 + 0.100 \cdot 3.28 = \mathbf{0.525\ kN.m^{-2}}$$

For the purpose of this example we have used only the first nine values from a sequence of 35 years observations of snow load maxima. The 0.9875-fractile estimate based on the full sequence is 0.443 kN.m^{-2}. ■

A.3 TRUNCATED LOG-NORMAL DISTRIBUTION, TLN

A truncated log-normal distribution is governed by four parameters (see 2.1.2). Therefore, in standardized form, where $\mu_u = 0$, $\sigma_u = 1$, two further parameters are necessary. According to experience, the coefficient of skewness, α^*, and the standardized infimum, u^*_{inf}, are a reasonable choice. The latter parameter is defined by

$$u^*_{inf} = \frac{x_{tr} - \mu^*}{\sigma^*} \tag{A.20}$$

where μ^*, σ^* = mean and standard deviation of TLN(u^*_{inf}, α^*); these parameters can be taken equal to corresponding sample characteristics (that is, as point estimates of population parameters) obtained from the analyzed random sample, while α^* has to be selected according to the behavior of the phenomenon, with a possible support by sampling results.

For the selected α^* and known u^*_{inf} the coefficient of skewness, α, and the standardized truncation point, u_{tr}, of the parent probability distribution can be obtained from Figure A.5 and further also P_{tr} from $P_{tr} = \Phi^{-1}_{LN}(u_{tr})$.

We can write

$$u_{tr} = \frac{x_{tr} - \mu}{\sigma} \tag{A.21}$$

where μ, σ = parameters of the parent distribution; u_{tr} and x_{tr} are known.

Obviously, a further equation is needed to calculate μ and σ. The point estimate of the median, $\tilde{\mu}^*$, of TLN(u^*_{inf}, α^*) will be used for this purpose; it can be easily established as the sample median, $\tilde{x}^*$.

Setting $\Phi^*(x) = 0.5$ in Equation (2.18) we get after rearrangement

$$\Phi(\tilde{x}^*) = \frac{1}{2}(1 + P_{tr})$$

Then

$$u' = \Phi^{-1}[\tfrac{1}{2}(1 + P_{tr})] \tag{A.22}$$

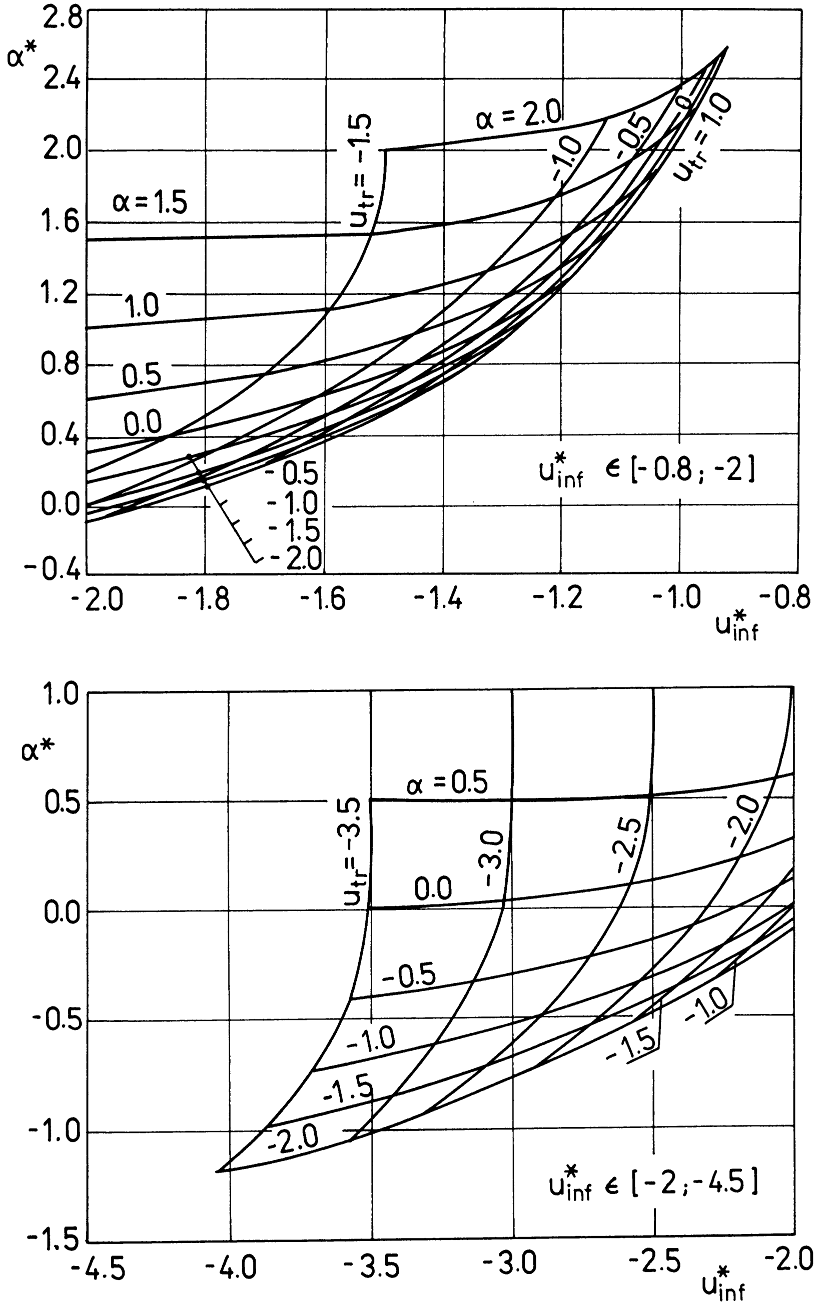

Fig. A.5 - *Truncated log-normal distribution.* Diagrams for the determination of the coefficient of skewness, α , of the parent log-normal distribution, and of the standardized truncation point, u_{tr} . The diagrams were calculated by Monte Carlo simulation.

which refers to $\tilde{x}^*$. Thus, the second equation for μ and σ can be set

$$u' = \frac{\tilde{x}^* - \mu}{\sigma} \tag{A.23}$$

Equations (A.21) and (A.23) yield

$$\mu = x_{tr} - u_{tr}\sigma$$
$$\sigma = \frac{\tilde{x}^* - x_{tr}}{u' - u_{tr}} \tag{A.24}$$

Sufficiently large samples with $n \geq 50$ must be available to employ the above technique.

■ **Example A.3.** A sample of 72 measurements of wind pressure is to be analyzed. The truncation point is $x_{tr} = 0.8$ kN.m^{-2}, and the sample characteristics are $m^* = 1.27$ kN.m^{-2}, $s^* = 0.18$ kN.m^{-2}, the coefficient of skewness has been selected by $\alpha^* = 0.8$. The sample median is $\tilde{x}^* = 1.2$ kN.m^{-2}. Find the parameters of the parent distribution, assumed three-parameter log-normal.

First, calculate

$$u^*_{inf} = \frac{0.8 - m^*}{s^*} = -2.6$$

Using Figure A.5, we determine for given α^* and u^*_{inf} parameters of the parent distribution:

$$u_{tr} \approx -1.3$$
$$\alpha \approx 0.5$$

and further

$$P_{tr} = \Phi^{-1}_{\mathrm{LN}}(u_{tr}) = 0.082066$$

Then, for $\tilde{x}^*$, u' is found from Equation (A.22), $u' = 1.00$. Finally, from Equations (A.24) the two parameters of the parent distribution are calculated:

$$\mu = 1.03 \ \ \mathrm{kN.m^{-2}}$$
$$\sigma = 0.17 \ \ \mathrm{kN.m^{-2}}$$

■

Table A.1 - *Log-normal distribution.* Inverse distribution function

$\Phi_{LN}(u)$	$\alpha \geq 0$												×
	0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**	**3.0**	
1.0E-6	-4.75	-4.10	-3.55	-3.10	-2.74	-2.44	-2.20	-2.00	-1.83	-1.69	-1.57	-1.19	**1-1.0E-6**
1.0E-5	-4.26	-3.73	-3.28	-2.91	-2.59	-2.33	-2.11	-1.93	-1.78	-1.65	-1.54	-1.18	**1-1.0E-5**
1.0E-4	-3.72	-3.32	-2.97	-2.66	-2.41	-2.19	-2.00	-1.84	-1.71	-1.59	-1.49	-1.16	**1-1.0E-4**
5.0E-4	-3.29	-2.98	-2.70	-2.45	-2.24	-2.05	-1.89	-1.76	-1.64	-1.53	-1.44	-1.13	**1-5.0E-4**
0.001	-3.09	-2.82	-2.57	-2.35	-2.16	-1.99	-1.84	-1.71	-1.60	-1.50	-1.42	-1.12	**0.999**
0.002	-2.88	-2.64	-2.43	-2.24	-2.06	-1.91	-1.77	-1.66	-1.55	-1.46	-1.38	-1.10	**0.998**
0.004	-2.65	-2.46	-2.27	-2.11	-1.96	-1.82	-1.70	-1.59	-1.50	-1.42	-1.34	-1.08	**0.996**
0.006	-2.51	-2.34	-2.18	-2.03	-1.89	-1.76	-1.65	-1.55	-1.46	-1.39	-1.32	-1.07	**0.994**
0.008	-2.41	-2.25	-2.10	-1.96	-1.84	-1.72	-1.61	-1.52	-1.48	-1.36	-1.30	-1.05	**0.992**
0.01	-2.33	-2.18	-2.04	-1.91	-1.79	-1.68	-1.58	-1.49	-1.41	-1.34	-1.28	-1.04	**0.99**
0.02	-2.05	-1.95	-1.84	-1.74	-1.64	-1.56	-1.47	-1.40	-1.33	-1.27	-1.21	-1.01	**0.98**
0.04	-1.75	-1.68	-1.61	-1.54	-1.47	-1.40	-1.34	-1.28	-1.22	-1.17	-1.13	-0.95	**0.96**
0.05	-1.64	-1.59	-1.53	-1.46	-1.40	-1.34	-1.29	-1.23	-1.18	-1.14	-1.10	-0.93	**0.95**
0.06	-1.55	-1.51	-1.45	-1.40	-1.35	-1.29	-1.24	-1.19	-1.15	-1.11	-1.07	-0.91	**0.94**
0.08	-1.41	-1.37	-1.33	-1.29	-1.25	-1.20	-1.16	-1.12	-1.08	-1.05	-1.01	-0.88	**0.92**
0.10	-1.28	-1.26	-1.23	-1.20	-1.16	-1.13	-1.09	-1.06	-1.03	-1.00	-0.97	-0.85	**0.90**
×	**0**	**-0.2**	**-0.4**	**-0.6**	**-0.8**	**-1.0**	**-1.2**	**-1.4**	**-1.6**	**-1.8**	**-2.0**	**-3.0**	$\Phi_{LN}(u)$
	$\alpha \leq 0$												

Continued

Table A.1 - Continued

$\Phi_{LN}(u)$	α ≥ 0												×
	0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**	**3.0**	
0.10	-1.28	-1.26	-1.23	-1.20	-1.16	-1.13	-1.09	-1.06	-1.03	-1.00	-0.97	-0.85	**0.90**
0.15	-1.04	-1.03	-1.02	-1.01	-0.99	-0.97	-0.95	-0.93	-0.91	-0.89	-0.86	-0.77	**0.85**
0.20	-0.84	-0.85	-0.85	-0.85	-0.84	-0.84	-0.83	-0.81	-0.80	-0.79	-0.77	-0.70	**0.80**
0.25	-0.67	-0.69	-0.70	-0.71	-0.71	-0.71	-0.71	-0.71	-0.70	-0.69	-0.68	-0.64	**0.75**
0.30	-0.52	-0.55	-0.57	-0.58	-0.59	-0.60	-0.60	-0.60	-0.60	-0.60	-0.60	-0.57	**0.70**
0.35	-0.39	-0.41	-0.44	-0.46	-0.47	-0.49	-0.50	-0.50	-0.51	-0.51	-0.51	-0.50	**0.65**
0.40	-0.25	-0.28	-0.31	-0.34	-0.36	-0.38	-0.39	-0.40	-0.41	-0.42	-0.42	-0.43	**0.60**
0.45	-0.13	-0.16	-0.19	-0.22	-0.24	-0.26	-0.28	-0.30	-0.31	-0.32	-0.33	-0.36	**0.55**
0.50	0.00	-0.03	-0.07	-0.10	-0.12	-0.15	-0.17	-0.19	-0.21	-0.22	-0.24	-0.28	**0.50**
0.55	0.13	0.09	0.06	0.03	-0.03	-0.03	-0.06	-0.08	-0.10	-0.12	-0.13	-0.19	**0.45**
0.60	0.25	0.22	0.19	0.16	0.13	0.10	0.07	0.04	0.02	-0.02	-0.02	-0.09	**0.40**
0.65	0.30	0.36	0.32	0.29	0.26	0.23	0.20	0.17	0.15	0.13	0.10	0.02	**0.35**
0.70	0.52	0.50	0.47	0.44	0.41	0.38	0.35	0.32	0.29	0.27	0.25	0.15	**0.30**
0.75	0.67	0.65	0.63	0.60	0.58	0.55	0.52	0.49	0.46	0.44	0.41	0.31	**0.25**
0.80	0.84	0.83	0.81	0.79	0.77	0.75	0.72	0.69	0.67	0.64	0.61	0.51	**0.20**
0.85	1.04	1.04	1.03	1.02	1.01	0.99	0.97	0.95	0.92	0.90	0.87	0.77	**0.15**
0.90	1.28	1.30	1.31	1.32	1.32	1.32	1.31	1.30	1.28	1.26	1.24	1.15	**0.10**
×	**0**	**-0.2**	**-0.4**	**-0.6**	**-0.8**	**-1.0**	**-1.2**	**-1.4**	**-1.6**	**-1.8**	**-2.0**	**-3.0**	$\Phi_{LN}(u)$
	α ≤ 0												

Continued

Table A.1 - Continued

$\Phi_{LN}(u)$	$\alpha \geq 0$												×
	0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.2**	**1.4**	**1.6**	**1.8**	**2.0**	**3.0**	
0.90	1.28	1.30	1.31	1.32	1.32	1.32	1.31	1.30	1.28	1.26	1.24	1.15	**0.10**
0.92	1.41	1.44	1.46	1.48	1.49	1.49	1.49	1.49	1.48	1.46	1.45	1.37	**0.08**
0.94	1.55	1.60	1.64	1.67	1.70	1.71	1.72	1.73	1.73	1.73	1.72	1.66	**0.06**
0.95	1.64	1.70	1.75	1.79	1.82	1.85	1.87	1.88	1.89	1.89	1.89	1.85	**0.05**
0.96	1.75	1.82	1.88	1.93	1.98	2.02	2.05	2.07	2.10	2.10	2.11	2.09	**0.04**
0.98	2.05	2.16	2.26	2.36	2.45	2.53	2.60	2.66	2.72	2.76	2.80	2.89	**0.02**
0.99	2.33	2.48	2.62	2.77	2.91	3.03	3.15	3.26	3.36	3.44	3.52	3.78	**0.01**
0.992	2.41	2.57	2.73	2.90	3.05	3.20	3.33	3.46	3.57	3.67	3.76	4.01	**0.008**
0.994	2.51	2.69	2.88	3.06	3.23	3.40	3.56	3.71	3.85	3.97	4.08	4.49	**0.006**
0.996	2.65	2.86	3.07	3.28	3.49	3.70	3.89	4.07	4.24	4.40	4.54	5.09	**0.004**
0.998	2.88	3.13	3.39	3.66	3.93	4.20	4.46	4.70	4.94	5.16	5.37	6.20	**0.002**
0.999	3.09	3.39	3.70	4.03	4.36	4.70	5.03	5.35	5.66	5.96	6.24	7.42	**0.001**
1-5.0E-4	3.29	3.63	4.00	4.39	4.80	5.21	5.62	6.02	6.42	6.80	7.17	8.75	**5.0E-4**
1-1.0E-4	3.72	4.17	4.67	5.22	5.80	6.40	7.03	7.66	8.29	8.91	9.52	12.33	**1.0E-4**
1-1.0E-5	4.26	4.88	5.59	6.38	7.24	8.18	9.17	10.21	11.28	12.36	13.45	18.80	**1.0E-5**
1-1.0E-6	4.75	5.54	6.46	7.52	8.72	10.04	11.49	13.08	14.66	16.36	18.11	27.12	**1.0E-6**
×	**0**	**-0.2**	**-0.4**	**-0.6**	**-0.8**	**-1.0**	**-1.2**	**-1.4**	**-1.6**	**-1.8**	**-2.0**	**-3.0**	$\Phi_{LN}(u)$
	$\alpha \leq 0$												

End

Table A.2.1

q

$\lambda = 0.90$

n	α							
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.5**	**2.0**
3	-0.95	-0.93	-0.91	-0.89	-0.87	-0.85	-0.80	-0.76
	0.95	0.97	0.99	1.00	1.02	1.03	1.06	1.08
4	-0.82	-0.81	-0.79	-0.78	-0.76	-0.75	-0.71	-0.67
	0.82	0.84	0.85	0.86	0.87	0.88	0.91	0.92
5	-0.74	-0.72	-0.71	-0.70	-0.69	-0.68	-0.64	-0.61
	0.74	0.75	0.76	0.77	0.78	0.79	0.81	0.82
6	-0.67	-0.66	-0.65	-0.64	-0.63	-0.62	-0.60	-0.57
	0.67	0.68	0.69	0.70	0.71	0.71	0.73	0.75
9	-0.55	-0.54	-0.54	-0.53	-0.52	-0.52	-0.50	-0.48
	0.55	0.55	0.56	0.57	0.57	0.58	0.59	0.60
12	-0.47	-0.47	-0.47	-0.46	-0.46	-0.45	-0.44	-0.42
	0.47	0.48	0.48	0.49	0.49	0.50	0.51	0.52
15	-0.42	-0.42	-0.42	-0.41	-0.41	-0.41	-0.40	-0.38
	0.42	0.43	0.43	0.44	0.44	0.44	0.45	0.46
20	-0.37	-0.36	-0.36	-0.36	-0.36	-0.35	-0.35	-0.34
	0.37	0.37	0.37	0.38	0.38	0.38	0.39	0.39
30	-0.30	-0.30	-0.30	-0.29	-0.29	-0.29	-0.29	-0.28
	0.30	0.30	0.30	0.31	0.31	0.31	0.31	0.32

Table A.2.2

q

$\lambda = 0.95$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.13	-1.11	-1.07	-1.04	-1.01	-0.98	-0.90	-0.85
	1.13	1.16	1.19	1.22	1.25	1.28	1.34	1.38
4	-0.98	-0.96	-0.93	-0.91	-0.88	-0.86	-0.81	-0.75
	0.98	1.00	1.03	1.05	1.07	1.09	1.14	1.18
5	-0.88	-0.86	-0.84	-0.82	-0.80	-0.78	-0.74	-0.67
	0.88	0.90	0.91	0.93	0.95	0.97	1.01	1.04
6	-0.80	-0.78	-0.77	-0.75	-0.74	-0.72	-0.68	-0.64
	0.80	0.82	0.83	0.85	0.86	0.88	0.91	0.94
9	-0.65	-0.64	-0.63	-0.62	-0.61	-0.60	-0.57	-0.55
	0.65	0.66	0.67	0.68	0.69	0.70	0.73	0.75
12	-0.57	-0.56	-0.55	-0.54	-0.53	-0.53	-0.51	-0.49
	0.57	0.57	0.58	0.59	0.60	0.60	0.62	0.64
15	-0.51	-0.50	-0.49	-0.49	-0.48	-0.47	-0.46	-0.44
	0.51	0.51	0.52	0.52	0.53	0.54	0.55	0.56
20	-0.44	-0.43	-0.43	-0.43	-0.42	-0.41	-0.40	-0.39
	0.44	0.44	0.45	0.45	0.46	0.46	0.47	0.48
30	-0.36	-0.35	-0.35	-0.35	-0.35	-0.34	-0.33	-0.32
	0.36	0.36	0.36	0.37	0.37	0.37	0.38	0.39

Table A.2.3

q

λ = 0.99

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.49	-1.42	-1.37	-1.31	-1.25	-1.20	-1.08	-1.00
	1.49	1.55	1.62	1.68	1.75	1.81	1.97	2.08
4	-1.29	-1.24	-1.20	-1.15	-1.11	-1.07	-0.98	-0.89
	1.29	1.34	1.38	1.43	1.48	1.53	1.66	1.77
5	-1.15	-1.11	-1.08	-1.04	-1.01	-0.98	-0.90	-0.83
	1.15	1.19	1.23	1.27	1.31	1.35	1.44	1.54
6	-1.05	-1.02	-0.99	-0.96	-0.93	-0.90	-0.84	-0.78
	1.05	1.08	1.12	1.15	1.18	1.21	1.29	1.37
9	-0.86	-0.84	-0.82	-0.80	-0.78	-0.76	-0.71	-0.67
	0.86	0.88	0.90	0.92	0.94	0.97	1.02	1.07
12	-0.74	-0.73	-0.71	-0.70	-0.68	-0.67	-0.63	-0.60
	0.74	0.76	0.78	0.79	0.81	0.82	0.86	0.91
15	-0.67	-0.65	-0.64	-0.63	-0.62	-0.60	-0.58	-0.55
	0.67	0.68	0.69	0.70	0.72	0.73	0.74	0.79
20	-0.58	-0.57	-0.56	-0.55	-0.54	-0.53	-0.51	-0.49
	0.58	0.58	0.59	0.60	0.61	0.62	0.65	0.67
30	-0.47	-0.46	-0.46	-0.45	-0.45	-0.44	-0.42	-0.41
	0.47	0.48	0.48	0.49	0.50	0.50	0.52	0.54

Table A.3.1

q^*

$\lambda = 0.90$

n	α					
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
3	-1.69	-1.65	-1.62	-1.58	-1.54	-1.51
	1.69	1.72	1.75	1.78	1.81	1.83
4	-1.18	-1.16	-1.13	-1.11	-1.09	-1.07
	1.18	1.20	1.22	1.23	1.25	1.27
5	-0.95	-0.94	-0.92	-0.91	-0.89	-0.87
	0.95	0.97	0.98	1.00	1.01	1.02
6	-0.82	-0.81	-0.80	-0.79	-0.77	-0.76
	0.82	0.83	0.85	0.86	0.87	0.88
9	-0.62	-0.61	-0.60	-0.60	-0.59	-0.58
	0.62	0.63	0.63	0.64	0.65	0.65
12	-0.52	-0.51	-0.51	-0.50	-0.50	-0.49
	0.52	0.52	0.53	0.53	0.54	0.54
15	-0.45	-0.45	-0.45	-0.44	-0.44	-0.43
	0.45	0.46	0.46	0.47	0.47	0.47
20	-0.39	-0.38	-0.38	-0.38	-0.37	-0.37
	0.39	0.39	0.39	0.40	0.40	0.40
30	-0.31	-0.31	-0.31	-0.30	-0.30	-0.30
	0.31	0.31	0.31	0.32	0.32	0.32

Table A.3.2

q^*

$\lambda = 0.95$

n	α					
	0.0	0.2	0.4	0.6	0.8	1.0
3	-2.49	-2.42	-2.35	-2.27	-2.21	-2.14
	2.49	2.55	2.62	2.69	2.75	2.81
4	-1.77	-1.73	-1.68	-1.64	-1.60	-1.56
	1.77	1.81	1.86	1.90	1.93	1.97
5	-1.24	-1.22	-1.19	-1.16	-1.13	-1.11
	1.24	1.27	1.29	1.32	1.35	1.37
6	-1.05	-1.03	-1.01	-0.99	-0.97	-0.95
	1.05	1.07	1.09	1.11	1.13	1.15
9	-0.77	-0.76	-0.74	-0.73	-0.72	-0.71
	0.77	0.78	0.79	0.80	0.82	0.83
12	-0.64	-0.63	-0.62	-0.61	-0.60	-0.59
	0.64	0.64	0.65	0.66	0.67	0.68
15	-0.55	-0.55	-0.54	-0.53	-0.53	-0.52
	0.55	0.56	0.57	0.57	0.58	0.59
20	-0.47	-0.46	-0.46	-0.45	-0.45	-0.44
	0.47	0.47	0.48	0.48	0.49	0.49
30	-0.37	-0.37	-0.37	-0.36	-0.36	-0.36
	0.37	0.38	0.38	0.38	0.39	0.39

Table A.3.3

q^*

$\lambda = 0.99$

n	α 0.0	0.2	0.4	0.6	0.8	1.0
3	-5.73	-5.49	-5.26	-5.04	-4.83	-4.63
	5.73	5.98	6.22	6.47	6.73	6.98
4	-2.92	-2.82	-2.71	-2.61	-2.52	-2.43
	2.92	3.03	3.14	3.25	3.36	3.47
5	-2.06	-1.99	-1.93	-1.86	-1.80	-1.74
	2.06	2.13	2.19	2.26	2.33	2.41
6	-1.65	-1.60	-1.55	-1.50	-1.46	-1.42
	1.65	1.70	1.75	1.80	1.85	1.90
9	-1.12	-1.09	-1.07	-1.04	-1.01	-0.99
	1.12	1.15	1.17	1.20	1.23	1.26
12	-0.90	-0.88	-0.86	-0.84	-0.82	-0.81
	0.90	0.92	0.93	0.95	0.97	0.99
15	-0.77	-0.75	-0.74	-0.73	-0.71	-0.70
	0.77	0.78	0.80	0.81	0.83	0.84
20	-0.64	-0.63	-0.62	-0.61	-0.60	-0.59
	0.64	0.65	0.66	0.67	0.68	0.69
30	-0.50	-0.50	-0.49	-0.48	-0.48	-0.47
	0.50	0.51	0.52	0.52	0.53	0.54

Table A.4.1

r

λ = 0.90

n	*α*					
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
3	0.05	0.05	0.05	0.04	0.04	0.03
	3.00	3.05	3.14	3.24	3.33	3.44
4	0.12	0.12	0.12	0.11	0.10	0.09
	2.60	2.64	2.72	2.81	2.90	3.00
5	0.18	0.17	0.17	0.16	0.15	0.14
	2.37	2.42	2.48	2.54	2.62	2.73
6	0.23	0.22	0.22	0.21	0.20	0.18
	2.21	2.25	2.31	2.39	2.47	2.55
9	0.34	0.34	0.33	0.32	0.30	0.28
	1.94	1.98	2.03	2.09	2.16	2.26
12	0.42	0.41	0.40	0.39	0.37	0.35
	1.79	1.82	1.86	1.91	1.98	2.08
15	0.47	0.46	0.45	0.43	0.41	0.39
	1.69	1.72	1.76	1.82	1.89	1.97
20	0.53	0.52	0.51	0.50	0.48	0.46
	1.59	1.61	1.63	1.68	1.74	1.82
30	0.61	0.60	0.59	0.57	0.55	0.54
	1.47	1.48	1.50	1.54	1.60	1.67

Table A.4.2

r

$\lambda = 0.95$

n	α					
	0.0	0.2	0.4	0.6	0.8	1.0
3	0.03	0.03	0.03	0.02	0.02	0.02
	3.69	3.85	4.03	4.20	4.37	4.58
4	0.07	0.07	0.07	0.06	0.06	0.05
	3.12	3.23	3.38	3.56	3.72	3.90
5	0.12	0.11	0.11	0.10	0.10	0.09
	2.79	2.87	2.99	3.13	3.30	3.49
6	0.17	0.16	0.16	0.15	0.14	0.13
	2.57	2.64	2.74	2.87	3.02	3.20
9	0.27	0.26	0.25	0.24	0.24	0.23
	2.19	2.25	2.34	2.44	2.56	2.71
12	0.35	0.34	0.33	0.31	0.30	0.29
	1.99	2.05	2.13	2.23	2.35	2.47
15	0.40	0.39	0.38	0.36	0.34	0.33
	1.87	1.92	1.99	2.08	2.20	2.32
20	0.47	0.45	0.44	0.42	0.40	0.38
	1.73	1.77	1.83	1.91	2.00	2.10
30	0.55	0.54	0.53	0.51	0.50	0.48
	1.58	1.60	1.64	1.70	1.77	1.87

Table A.4.3

r

$\lambda = 0.99$

n	α					
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
3	0.01	0.00	0.00	0.00	0.00	0.00
	5.30	5.56	5.92	6.25	6.64	7.13
4	0.02	0.02	0.02	0.01	0.01	0.01
	4.28	4.53	4.88	5.24	5.60	6.06
5	0.05	0.05	0.05	0.04	0.04	0.04
	3.72	3.91	4.18	4.50	4.87	5.34
6	0.08	0.08	0.08	0.07	0.07	0.07
	3.35	3.51	3.71	3.98	4.30	4.68
9	0.17	0.16	0.16	0.15	0.15	0.14
	2.74	2.93	3.14	3.37	3.60	3.97
12	0.24	0.23	0.23	0.22	0.21	0.20
	2.43	2.58	2.74	2.92	3.11	3.37
15	0.29	0.28	0.27	0.26	0.25	0.23
	2.24	2.38	2.53	2.70	2.86	3.05
20	0.36	0.35	0.34	0.32	0.31	0.29
	3.03	2.14	2.27	2.40	2.55	2.74
30	0.45	0.42	0.41	0.40	0.39	0.38
	1.80	1.85	1.93	2.04	2.19	2.37

Table A.5.1

k_1

$\kappa = 0.001$ $\lambda = 0.95$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-4.04	-3.76	-3.56	-3.34	-3.15	-3.02	-2.70	-2.48
	4.04	4.31	4.61	4.90	5.22	5.55	6.31	6.98
4	-3.91	-3.65	-3.42	-3.22	-3.03	-2.90	-2.56	-2.34
	3.91	4.21	4.49	4.79	5.11	5.45	6.21	6.92
5	-3.83	-3.55	-3.32	-3.10	-2.94	-2.79	-2.46	-2.24
	3.83	4.11	4.42	4.72	5.06	5.37	6.16	6.86
6	-3.76	-3.49	-3.26	-3.05	-2.86	-2.70	-2.39	-2.16
	3.67	4.04	4.35	4.67	5.00	5.32	6.12	6.82
9	-3.64	-3.38	-3.14	-2.93	-2.73	-2.55	-2.26	-2.01
	3.64	3.93	4.25	4.57	4.88	5.22	6.00	6.73
12	-3.57	-3.29	-3.05	-2.84	-2.66	-2.48	-2.17	-1.93
	3.57	3.86	4.17	4.49	4.82	5.16	5.95	6.66
15	-3.51	-3.25	-3.01	-2.77	-2.60	-2.43	-2.11	-1.87
	3.51	3.81	4.13	4.44	4.78	5.10	5.91	6.63
20	-3.46	-3.19	-2.94	-2.71	-2.54	-2.37	-2.03	-1.81
	3.46	3.77	4.07	4.39	4.73	5.05	5.85	6.58
30	-3.39	-3.13	-2.87	-2.65	-2.47	-2.29	-1.96	-1.73
	3.39	3.70	4.00	4.33	4.66	4.99	5.80	6.52

Table A.5.2

k_1

κ = 0.001 λ = 0.90

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.83	-3.57	-3.33	-3.10	-2.91	-2.74	-2.39	-2.15
	3.83	4.11	4.44	4.74	5.06	5.40	6.18	6.88
4	-3.73	-3.48	-3.22	-3.00	-2.81	-2.66	-2.31	-2.06
	3.73	4.04	4.33	4.63	4.97	5.32	6.10	6.81
5	-3.66	-3.40	-3.15	-2.94	-2.75	-2.58	-2.25	-2.00
	3.66	3.96	4.27	4.57	4.91	5.24	6.04	6.76
6	-3.61	-3.35	-3.09	-2.88	-2.69	-2.52	-2.20	-1.96
	3.61	3.90	4.22	4.54	4.87	5.20	6.00	6.72
9	-3.52	-3.26	-3.00	-2.79	-2.60	-2.41	-2.10	-1.86
	3.52	3.81	4.14	4.45	4.78	5.11	5.91	6.64
12	-3.46	-3.19	-2.94	-2.73	-2.53	-2.36	-2.03	-1.80
	3.46	3.75	4.07	4.39	4.74	5.07	5.87	6.59
15	-3.42	-3.15	-2.90	-2.69	-2.49	-2.31	-1.99	-1.76
	3.42	3.72	4.03	4.35	4.69	5.02	5.83	6.56
20	-3.38	-3.11	-2.86	-2.63	-2.45	-2.28	-1.94	-1.71
	3.38	3.68	3.99	4.31	4.65	4.97	5.79	6.52
30	-3.32	-3.06	-2.81	-2.58	-2.39	-2.22	-1.89	-1.65
	3.32	3.62	3.93	4.27	4.59	4.93	5.75	6.46

Table A.5.3

k_1

$\kappa = 0.001$ $\lambda = 0.80$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.58	-3.30	-3.05	-2.81	-2.61	-2.43	-2.08	-1.81
	3.58	3.87	4.20	4.51	4.85	5.19	6.00	6.72
4	-3.51	-3.25	-2.98	-2.76	-2.56	-2.39	-2.05	-1.79
	3.51	3.82	4.12	4.44	4.78	5.13	5.93	6.65
5	-3.47	-3.20	-2.94	-2.72	-2.53	-2.34	-2.00	-1.76
	3.47	3.77	4.08	4.40	4.74	5.08	5.89	6.61
6	-3.43	-3.17	-2.91	-2.68	-2.49	-2.31	-1.97	-1.74
	3.43	3.72	4.05	4.38	4.71	5.05	5.85	6.59
9	-3.37	-3.11	-2.84	-2.64	-2.44	-2.25	-1.92	-1.68
	3.37	3.66	3.99	4.31	4.65	4.99	5.79	6.53
12	-3.33	-3.06	-2.80	-2.60	-2.39	-2.22	-1.88	-1.64
	3.33	3.63	3.94	4.27	4.62	4.95	5.75	6.48
15	-3.31	-3.03	-2.78	-2.57	-2.37	-2.20	-1.86	-1.61
	3.31	3.60	3.92	4.26	4.59	4.92	5.73	6.46
20	-3.28	-3.00	-2.76	-2.53	-2.43	-2.17	-1.83	-1.59
	3.28	3.58	3.88	4.21	4.55	4.88	5.70	6.43
30	-3.24	-2.98	-2.72	-2.50	-2.31	-2.14	-1.80	-1.56
	3.24	3.55	3.85	4.18	4.52	4.86	5.67	6.40

Table A.5.4

k_1

$\kappa = 0.001$ $\lambda = 0.50$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.09	-2.81	-2.54	-2.32	-2.13	-1.92	-1.58	-1.31
	3.09	3.39	3.74	4.06	4.40	4.76	5.58	6.35
4	-3.09	-2.81	-2.55	-2.33	-2.13	-1.95	-1.59	-1.34
	3.09	3.39	3.73	4.05	4.39	4.75	5.57	6.32
5	-3.09	-2.81	-2.55	-2.33	-2.13	-1.95	-1.60	-1.36
	3.09	3.39	3.72	4.04	4.39	4.74	5.56	6.30
6	-3.09	-2.81	-2.55	-2.34	-2.14	-1.95	-1.61	-1.37
	3.09	3.39	3.72	4.04	4.38	4.73	5.55	6.28
9	-3.09	-2.81	-2.56	-2.34	-2.15	-1.97	-1.63	-1.39
	3.09	3.39	3.71	4.04	4.37	4.72	5.54	6.28
12	-3.09	-2.82	-2.56	-2.34	-2.15	-1.96	-1.64	-1.40
	3.09	3.39	3.71	4.04	4.37	4.71	5.53	6.27
15	-3.09	-2.82	-2.57	-2.34	-2.15	-1.98	-1.64	-1.40
	3.09	3.39	3.71	4.04	4.37	4.70	5.53	6.26
20	-3.09	-2.82	-2.57	-2.34	-2.15	-1.98	-1.64	-1.41
	3.09	3.39	3.70	4.04	4.37	4.70	5.52	6.25
30	-3.09	-2.82	-2.57	-2.34	-2.15	-1.98	-1.64	-1.41
	3.09	3.39	3.70	4.04	4.37	4.70	5.52	6.25

Table A.5.5

k_1

$\kappa = 0.010$ $\lambda = 0.95$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.28	-3.14	-3.02	-2.91	-2.80	-2.72	-2.54	-2.40
	3.28	3.41	3.53	3.67	3.79	3.90	4.11	4.28
4	-3.15	-3.01	-2.88	-2.76	-2.66	-2.56	-2.35	-2.21
	3.15	3.29	3.42	3.55	3.68	3.78	4.02	4.19
5	-3.06	-2.94	-2.81	-2.68	-2.57	-2.48	-2.27	-2.10
	3.06	3.20	3.33	3.46	3.60	3.71	3.94	4.13
6	-3.00	-2.86	-2.73	-2.61	-2.50	-2.40	-2.22	-2.06
	3.00	3.13	3.28	3.40	3.54	3.66	3.90	4.08
9	-2.87	-2.73	-2.60	-2.48	-2.37	-2.27	-2.05	-1.88
	2.87	3.01	3.16	3.30	3.42	3.55	3.08	4.00
12	-2.80	-2.66	-2.52	-2.40	-2.29	-2.19	-1.96	-1.79
	2.80	2.94	3.09	3.23	3.35	3.49	3.75	3.95
15	-2.75	-2.61	-2.47	-2.34	-2.23	-2.12	-1.90	-1.74
	2.75	2.90	3.04	3.18	3.31	3.44	3.71	3.90
20	-2.69	-2.54	-2.42	-2.29	-2.18	-2.07	-1.84	-1.67
	2.69	2.84	2.98	3.12	3.26	3.38	3.66	3.86
30	-2.63	-2.47	-2.34	-2.22	-2.11	-2.00	-1.77	-1.61
	2.63	2.77	2.92	3.06	3.20	3.32	3.60	3.81

Table A.5.6

k_1

κ = 0.010 λ = 0.90

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.07	-2.92	-2.79	-2.67	-2.54	-2.46	-2.21	-2.06
	3.07	3.21	3.35	3.48	3.60	3.72	3.98	4.16
4	-2.97	-2.83	-2.70	-2.56	-2.44	-2.35	-2.12	-1.95
	2.97	3.11	3.25	3.38	3.51	3.63	3.89	4.09
5	-2.90	-2.78	-2.63	-2.49	-2.37	-2.27	-2.05	-1.86
	2.90	3.05	3.19	3.32	3.42	3.57	3.84	4.03
6	-2.85	-2.72	-2.58	-2.45	-2.33	-2.23	-2.01	-1.82
	2.85	2.99	3.14	3.28	3.42	3.54	3.80	3.99
9	-2.75	-2.61	-2.48	-2.35	-2.23	-2.13	-1.88	-1.71
	2.75	2.90	3.04	3.18	3.32	3.44	3.81	3.91
12	-2.70	-2.55	-2.42	-2.29	-2.17	-2.07	-1.83	-1.66
	2.70	2.85	2.99	3.13	3.27	3.40	3.67	3.87
15	-2.66	-2.51	-2.38	-2.25	-2.13	-2.02	-1.79	-1.61
	2.66	2.80	2.95	3.09	3.23	3.36	3.63	3.83
20	-2.61	-2.47	-2.33	-2.20	-2.08	-1.97	-1.74	-1.57
	2.61	2.76	2.91	3.05	3.19	3.31	3.59	3.79
30	-2.56	-2.41	-2.27	-2.15	-2.03	-1.92	-1.70	-1.52
	2.56	2.71	2.86	3.00	3.14	3.26	3.54	3.75

Table A.5.7

k_1

$\kappa = 0.010$ $\lambda = 0.80$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.81	-2.66	-2.52	-2.38	-2.25	-2.14	-1.89	-1.72
	2.81	2.95	3.11	3.25	3.38	3.51	3.80	4.01
4	-2.75	-2.61	-2.47	-2.31	-2.19	-2.09	-1.84	-1.67
	2.75	2.89	3.04	3.19	3.33	3.45	3.73	3.95
5	-2.70	-2.57	-2.43	-2.27	-2.16	-2.05	-1.81	-1.63
	2.70	2.84	3.00	3.14	3.28	3.40	3.68	3.89
6	-2.67	-2.54	-2.39	-2.25	-2.12	-2.03	-1.79	-1.60
	2.67	2.81	2.96	3.10	3.24	3.37	3.65	3.85
9	-2.61	-2.46	-2.33	-2.19	-2.06	-1.96	-1.72	-1.54
	2.61	2.76	2.90	3.05	3.19	3.32	3.59	3.79
12	-2.57	-2.42	-2.28	-2.15	-2.03	-1.92	-1.69	-1.51
	2.57	2.72	2.87	3.02	3.15	3.29	3.55	3.76
15	-2.54	-2.40	-2.26	-2.13	-2.01	-1.90	-1.66	-1.49
	2.54	2.69	2.84	2.99	3.12	3.25	3.53	3.74
20	-2.51	-2.37	-2.23	-2.10	-1.98	-1.87	-1.63	-1.46
	2.51	2.66	2.81	2.96	3.10	3.22	3.50	3.71
30	-2.48	-2.33	-2.19	-2.07	-1.94	-1.84	-1.61	-1.44
	2.48	2.63	2.78	2.92	3.06	3.18	3.46	3.67

Table A.5.8

k_1

$\kappa = 0.010$ $\lambda = 0.50$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.33	-2.17	-2.02	-1.88	-1.75	-1.64	-1.39	-1.18
	2.33	2.49	2.65	2.80	2.96	3.10	3.38	3.61
4	-2.33	-2.17	-2.02	-1.88	-1.75	-1.64	-1.39	-1.19
	2.33	2.48	2.63	2.80	2.94	3.07	3.37	3.59
5	-2.33	-2.17	-2.03	-1.89	-1.76	-1.65	-1.40	-1.22
	2.33	2.48	2.63	2.79	2.93	3.06	3.36	3.58
6	-2.33	-2.17	-2.03	-1.89	-1.76	-1.65	-1.41	-1.23
	2.33	2.48	2.63	2.79	2.93	3.06	3.35	3.57
9	-2.33	-2.18	-2.04	-1.90	-1.78	-1.66	-1.42	-1.25
	2.33	2.48	2.63	2.78	2.92	3.05	3.34	3.55
12	-2.33	-2.18	-2.04	-1.90	-1.70	-1.67	-1.43	-1.26
	2.33	2.48	2.63	2.78	2.91	3.05	3.33	3.54
15	-2.33	-2.18	-2.04	-1.91	-1.79	-1.68	-1.44	-1.26
	2.33	2.48	2.63	2.78	2.91	3.04	3.32	3.54
20	-2.33	-2.18	-2.04	-1.91	-1.79	-1.68	-1.45	-1.26
	2.33	2.48	2.63	2.77	2.91	3.04	3.31	3.54
30	-2.33	-2.18	-2.04	-1.91	-1.79	-1.68	-1.45	-1.27
	2.33	2.48	2.63	2.77	2.91	3.04	3.31	3.53

Table A.5.9

k_1

$\kappa = 0.050$ $\lambda = 0.95$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.59	-2.57	-2.53	-2.48	-2.42	-2.36	-2.29	-2.23
	2.59	2.62	2.64	2.67	2.68	2.68	2.67	2.65
4	-2.47	-2.43	-2.37	-2.32	-2.26	-2.22	-2.12	-2.04
	2.47	2.52	2.55	2.58	2.61	2.60	2.59	2.56
5	-2.38	-2.34	-2.29	-2.23	-2.18	-2.14	-2.02	-1.91
	2.38	2.43	2.47	2.49	2.52	2.53	2.53	2.51
6	-2.32	-2.26	-2.22	-2.17	-2.12	-2.06	-1.93	-1.83
	2.32	2.37	2.40	2.43	2.46	2.47	2.48	2.46
9	-2.19	-2.14	-2.08	-2.03	-1.97	-1.91	-1.80	-1.71
	2.19	2.25	2.28	2.32	2.35	2.37	2.38	2.37
12	-2.12	-2.06	-2.01	-1.94	-1.89	-1.83	-1.73	-1.62
	2.12	2.18	2.21	2.24	2.28	2.30	2.32	2.31
15	-2.07	-2.02	-1.96	-1.89	-1.84	-1.78	-1.66	-1.55
	2.07	2.12	2.16	2.20	2.23	2.25	2.27	2.28
20	-2.01	-1.96	-1.90	-1.84	-1.78	-1.73	-1.60	-1.48
	2.01	2.07	2.11	2.16	2.19	2.20	2.23	2.22
30	-1.95	-1.89	-1.82	-1.77	-1.71	-1.65	-1.52	-1.41
	1.95	2.01	2.05	2.08	2.12	2.14	2.18	2.17

Table A.5.10

k_1

κ = 0.050 λ = 0.90

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.38	-2.33	-2.28	-2.23	-2.18	-2.12	-2.01	-1.90
	2.38	2.43	2.47	2.51	2.54	2.55	2.55	2.52
4	-2.29	-2.23	-2.17	-2.11	-2.05	-1.99	-1.86	-1.75
	2.29	2.34	2.38	2.43	2.46	2.47	2.47	2.46
5	-2.22	-2.16	-2.10	-2.04	-1.99	-1.94	-1.79	-1.68
	2.22	2.27	2.31	2.34	2.38	2.40	2.43	2.42
6	-2.17	-2.10	-2.05	-1.99	-1.93	-1.88	-1.74	-1.62
	2.17	2.23	2.27	2.30	2.33	2.35	2.38	2.36
9	-2.07	-2.00	-1.95	-1.89	-1.84	-1.76	-1.64	-1.53
	2.07	2.12	2.17	2.21	2.25	2.27	2.29	2.28
12	-2.01	-1.95	-1.89	-1.83	-1.78	-1.72	-1.59	-1.48
	2.01	2.07	2.11	2.15	2.19	2.20	2.23	2.23
15	-1.98	-1.92	-1.86	-1.79	-1.74	-1.68	-1.54	-1.43
	1.98	2.03	2.08	2.11	2.15	2.17	2.21	2.21
20	-1.93	-1.87	-1.81	-1.75	-1.69	-1.63	-1.50	-1.38
	1.93	1.99	2.03	2.08	2.11	2.13	2.16	2.16
30	-1.88	-1.82	-1.76	-1.70	-1.64	-1.58	-1.45	-1.34
	1.88	1.94	1.98	2.02	2.05	2.08	2.11	2.12

Table A.5.11

k_1

$\kappa = 0.050$ $\lambda = 0.80$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.13	-2.07	-2.02	-1.93	-1.87	-1.81	-1.68	-1.55
	2.13	2.20	2.24	2.28	2.32	2.34	2.37	2.35
4	-2.07	-2.01	-1.96	-1.87	-1.80	-1.75	-1.59	-1.47
	2.07	2.13	2.18	2.22	2.26	2.29	2.32	2.30
5	-2.02	-1.96	-1.90	-1.83	-1.77	-1.71	-1.55	-1.43
	2.02	2.08	2.12	2.16	2.20	2.23	2.27	2.26
6	-1.99	-1.93	-1.86	-1.80	-1.74	-1.69	-1.53	-1.40
	1.99	2.05	2.08	2.13	2.17	2.19	2.24	2.23
9	-1.93	-1.86	-1.80	-1.74	-1.68	-1.62	-1.47	-1.35
	1.93	1.99	2.03	2.07	2.12	2.14	2.17	2.17
12	-1.89	-1.82	-1.76	-1.70	-1.64	-1.58	-1.44	-1.32
	1.89	1.94	1.99	2.03	2.07	2.09	2.13	2.14
15	-1.86	-1.79	-1.74	-1.68	-1.62	-1.56	-1.41	-1.30
	1.86	1.91	1.96	2.01	2.04	2.07	2.12	2.12
20	-1.83	-1.77	-1.72	-1.65	-1.60	-1.53	-1.40	-1.27
	1.83	1.89	1.94	1.98	2.01	2.04	2.07	2.07
30	-1.80	-1.74	-1.68	-1.62	-1.55	-1.49	-1.36	-1.25
	1.80	1.85	1.90	1.94	1.98	2.01	2.05	2.05

Table A.5.12

k_1

κ = 0.050 λ = 0.50

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.64	-1.57	-1.51	-1.43	-1.35	-1.29	-1.14	-1.02
	1.64	1.72	1.77	1.82	1.87	1.90	1.95	1.97
4	-1.64	-1.58	-1.52	-1.43	-1.35	-1.31	-1.15	-1.08
	1.64	1.70	1.76	1.83	1.88	1.89	1.96	1.97
5	-1.64	-1.58	-1.52	-1.44	-1.37	-1.32	-1.16	-1.04
	1.64	1.71	1.76	1.80	1.85	1.88	1.94	1.95
6	-1.64	-1.58	-1.52	-1.44	-1.38	-1.32	-1.17	-1.05
	1.64	1.71	1.75	1.81	1.84	1.87	1.92	1.94
9	-1.64	-1.58	-1.52	-1.46	-1.38	-1.32	-1.18	-1.06
	1.64	1.70	1.75	1.79	1.84	1.87	1.92	1.94
12	-1.64	-1.58	-1.52	-1.46	-1.39	-1.32	-1.20	-1.08
	1.64	1.71	1.75	1.80	1.84	1.87	1.90	1.91
15	-1.64	-1.59	-1.52	-1.46	-1.39	-1.33	-1.20	-1.08
	1.64	1.70	1.75	1.80	1.83	1.86	1.91	1.92
20	-1.64	-1.59	-1.52	-1.46	-1.40	-1.33	-1.20	1.09
	1.64	1.70	1.75	1.80	1.82	1.86	1.90	1.90
30	-1.64	-1.59	-1.52	-1.47	-1.40	-1.33	-1.20	-1.09
	1.64	1.70	1.75	1.79	1.83	1.86	1.90	1.90

Table A.5.13

k_1

κ = 0.100 λ = 0.95

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.23	-2.22	-2.21	-2.19	-2.17	-2.16	-2.11	-2.07
	2.23	2.23	2.23	2.22	2.19	2.16	2.09	2.01
4	-2.10	-2.08	-2.07	-2.05	-2.04	-2.02	-1.96	-1.90
	2.10	2.11	2.12	2.11	2.10	2.08	2.00	1.92
5	-2.02	-2.01	-1.99	-1.97	-1.95	-1.93	-1.87	-1.87
	2.02	2.03	2.03	2.02	2.01	2.00	1.93	1.85
6	-1.95	-1.93	-1.91	-1.90	-1.88	-1.85	-1.76	-1.69
	1.95	1.96	1.97	1.96	1.95	1.93	1.88	1.82
9	-1.83	-1.81	-1.80	-1.78	-1.74	-1.71	-1.63	-1.54
	1.83	1.84	1.84	1.85	1.84	1.83	1.80	1.73
12	-1.76	-1.74	-1.72	-1.69	-1.66	-1.64	-1.55	-1.47
	1.76	1.77	1.78	1.79	1.78	1.76	1.72	1.66
15	-1.71	-1.69	-1.66	-1.63	-1.61	-1.58	-1.50	-1.41
	1.71	1.72	1.73	1.73	1.73	1.72	1.68	1.63
20	-1.65	-1.63	-1.60	-1.57	-1.54	-1.51	-1.43	-1.35
	1.65	1.66	1.67	1.68	1.68	1.67	1.63	1.58
30	-1.58	-1.56	-1.54	-1.50	-1.47	-1.44	-1.37	-1.28
	1.58	1.59	1.61	1.61	1.61	1.61	1.57	1.53

Table A.5.14

k_1

$\kappa = 0.100$ $\lambda = 0.90$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.02	-2.01	-1.98	-1.95	-1.92	-1.87	-1.81	-1.74
	2.02	2.03	2.03	2.04	2.03	2.02	1.96	1.88
4	-1.92	-1.90	-1.87	-1.84	-1.81	-1.78	-1.71	-1.63
	1.92	1.94	1.94	1.94	1.94	1.93	1.88	1.81
5	-1.86	-1.84	-1.81	-1.78	-1.75	-1.73	-1.64	-1.55
	1.86	1.88	1.88	1.88	1.88	1.87	1.83	1.75
6	-1.81	-1.78	-1.76	-1.73	-1.70	-1.67	-1.58	-1.50
	1.81	1.82	1.82	1.82	1.82	1.82	1.77	1.71
9	-1.71	-1.69	-1.66	-1.62	-1.59	-1.56	-1.48	-1.38
	1.71	1.73	1.74	1.74	1.74	1.74	1.70	1.63
12	-1.65	-1.64	-1.61	-1.57	-1.54	-1.51	-1.43	-1.33
	1.65	1.68	1.69	1.69	1.69	1.67	1.63	1.59
15	-1.61	-1.59	-1.56	-1.53	-1.50	-1.47	-1.39	-1.31
	1.61	1.63	1.63	1.64	1.64	1.64	1.61	1.56
20	-1.57	-1.55	-1.51	-1.48	-1.45	-1.42	-1.34	-1.25
	1.57	1.58	1.59	1.60	1.60	1.60	1.57	1.52
30	-1.52	-1.49	-1.47	-1.43	-1.40	-1.37	-1.28	-1.21
	1.52	1.53	1.54	1.55	1.54	1.54	1.52	1.47

Table A.5.15

k_1

$\kappa = 0.100$ $\lambda = 0.80$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.77	-1.75	-1.71	-1.67	-1.63	-1.58	-1.48	-1.39
	1.77	1.80	1.81	1.81	1.81	1.80	1.77	1.72
4	-1.70	-1.67	-1.64	-1.60	-1.56	-1.52	-1.44	-1.34
	1.70	1.72	1.73	1.74	1.75	1.74	1.71	1.65
5	-1.66	-1.64	-1.61	-1.57	-1.54	-1.50	-1.41	-1.32
	1.66	1.68	1.69	1.69	1.70	1.70	1.67	1.59
6	-1.63	-1.60	-1.57	-1.54	-1.50	-1.46	-1.37	-1.28
	1.63	1.65	1.65	1.66	1.66	1.66	1.63	1.58
9	-1.56	-1.54	-1.51	-1.48	-1.44	-1.41	-1.31	-1.22
	1.56	1.59	1.60	1.60	1.60	1.60	1.57	1.53
12	-1.52	-1.50	-1.47	-1.44	-1.40	-1.37	-1.28	-1.19
	1.52	1.55	1.56	1.57	1.57	1.56	1.53	1.49
15	-1.50	-1.48	-1.44	-1.41	-1.37	-1.34	-1.26	-1.17
	1.50	1.52	1.53	1.54	1.54	1.53	1.50	1.46
20	-1.47	-1.44	-1.41	-1.38	-1.35	-1.31	-1.23	-1.14
	1.47	1.49	1.51	1.51	1.51	1.51	1.48	1.43
30	-1.44	-1.41	-1.38	-1.35	-1.32	-1.28	-1.19	-1.12
	1.44	1.46	1.47	1.48	1.48	1.47	1.45	1.40

Table A.5.16

k_1

$\kappa = 0.100$ $\lambda = 0.50$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.28	-1.25	-1.22	-1.17	-1.13	-1.08	-0.96	-0.88
	1.28	1.31	1.33	1.35	1.36	1.36	1.36	1.33
4	-1.28	-1.25	-1.21	-1.18	-1.12	-1.09	-0.98	-0.89
	1.28	1.31	1.33	1.34	1.36	1.35	1.35	1.32
5	-1.28	-1.25	-1.22	-1.17	-1.14	-1.10	-1.01	-0.92
	1.28	1.30	1.32	1.34	1.35	1.34	1.32	1.29
6	-1.28	-1.25	-1.23	-1.18	-1.15	-1.10	-1.00	-0.91
	1.28	1.31	1.33	1.34	1.34	1.34	1.33	1.30
9	-1.28	-1.25	-1.23	-1.19	-1.15	-1.11	-1.01	-0.94
	1.28	1.30	1.31	1.32	1.33	1.33	1.32	1.27
12	-1.28	-1.25	-1.22	-1.18	-1.15	-1.12	-1.02	-0.94
	1.28	1.31	1.32	1.33	1.33	1.32	1.31	1.27
15	-1.28	-1.26	-1.23	-1.19	-1.16	-1.12	-1.03	-0.95
	1.28	1.30	1.32	1.33	1.33	1.32	1.30	1.26
20	-1.28	-1.26	-1.22	-1.19	-1.16	-1.12	-1.03	-0.95
	1.28	1.30	1.32	1.33	1.33	1.33	1.30	1.26
30	-1.28	-1.26	-1.23	-1.19	-1.16	-1.12	-1.03	-0.96
	1.28	1.30	1.31	1.32	1.32	1.32	1.30	1.25

Table A.5.17

k_1

κ = 0.250 λ = 0.95

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.62	-1.65	-1.69	-1.71	-1.73	-1.74	-1.76	-1.73
	1.62	1.58	1.54	1.49	1.44	1.39	1.27	1.16
4	-1.50	-1.53	-1.56	-1.58	-1.60	-1.61	-1.61	-1.58
	1.50	1.46	1.43	1.38	1.33	1.29	1.19	1.09
5	-1.41	-1.44	-1.46	-1.48	-1.50	-1.51	-1.51	-1.51
	1.41	1.38	1.35	1.33	1.27	1.22	1.12	1.04
6	-1.35	-1.37	-1.39	-1.41	-1.43	-1.43	-1.42	-1.41
	1.35	1.32	1.29	1.24	1.20	1.17	1.07	0.97
9	-1.22	-1.24	-1.26	-1.28	-1.29	-1.30	-1.30	-1.26
	1.22	1.20	1.17	1.14	1.10	1.05	0.96	0.89
12	-1.15	-1.17	-1.19	-1.21	-1.22	-1.22	-1.22	-1.21
	1.15	1.12	1.09	1.06	1.02	0.99	0.92	0.83
15	-1.10	-1.12	-1.13	-1.14	-1.15	-1.16	-1.15	-1.15
	1.10	1.08	1.05	1.02	0.98	0.95	0.87	0.79
20	-1.04	-1.05	-1.07	-1.08	-1.09	-1.09	-1.10	-1.08
	1.04	1.02	0.99	0.97	0.93	0.91	0.83	0.76
30	-0.98	-0.99	-1.00	-1.01	-1.01	-1.02	-1.01	-0.99
	0.98	0.95	0.92	0.89	0.86	0.83	0.76	0.70

Table A.5.18

k_1

$\kappa = 0.250$ $\lambda = 0.90$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.42	-1.44	-1.46	-1.47	-1.47	-1.48	-1.47	-1.37
	1.42	1.39	1.35	1.31	1.28	1.23	1.14	1.05
4	-1.32	-1.35	-1.36	-1.38	-1.39	-1.38	-1.35	-1.33
	1.32	1.29	1.26	1.23	1.19	1.16	1.07	0.98
5	-1.25	-1.27	-1.29	-1.30	-1.30	-1.31	-1.29	-1.29
	1.25	1.22	1.19	1.16	1.13	1.09	1.01	0.93
6	-1.20	-1.22	-1.23	-1.25	-1.25	-1.25	-1.23	-1.22
	1.20	1.17	1.14	1.12	1.08	1.05	0.96	0.87
9	-1.10	-1.12	-1.13	-1.14	-1.15	-1.15	-1.14	-1.11
	1.10	1.08	1.05	1.03	1.00	0.97	0.88	0.81
12	-1.05	-1.05	-1.07	-1.09	-1.09	-1.09	-1.08	-1.07
	1.05	1.02	0.99	0.96	0.93	0.91	0.83	0.76
15	-1.01	-1.02	-1.04	-1.04	-1.05	-1.05	-1.04	-1.03
	1.01	0.99	0.96	0.93	0.90	0.87	0.78	0.72
20	-0.96	-0.98	-0.99	-1.00	-1.00	-1.00	-1.00	-0.99
	0.96	0.94	0.92	0.89	0.86	0.83	0.76	0.70
30	-0.91	-0.92	-0.94	-0.95	-0.95	-0.95	-0.94	-0.91
	0.91	0.88	0.86	0.83	0.80	0.78	0.70	0.64

Table A.5.19

k_1

$\kappa = 0.250$ $\lambda = 0.80$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.16	-1.16	-1.18	-1.19	-1.19	-1.19	-1.15	-1.06
	1.16	1.14	1.11	1.08	1.06	1.03	0.95	0.89
4	-1.10	-1.11	-1.12	-1.13	-1.13	-1.12	-1.09	-1.06
	1.10	1.08	1.06	1.04	1.01	0.98	0.91	0.83
5	-1.05	-1.06	-1.07	-1.08	-1.08	-1.08	-1.05	-1.04
	1.05	1.03	1.00	0.98	0.96	0.93	0.86	0.78
6	-1.02	-1.03	-1.04	-1.05	-1.05	-1.04	-1.01	-0.99
	1.02	0.99	0.97	0.95	0.92	0.90	0.83	0.74
9	-0.96	-0.96	-0.98	-0.99	-0.99	-0.98	-0.97	-0.94
	0.96	0.94	0.92	0.89	0.86	0.84	0.76	0.70
12	-0.92	-0.92	-0.94	-0.96	-0.96	-0.96	-0.94	-0.92
	0.92	0.90	0.87	0.85	0.81	0.79	0.72	0.65
15	-0.89	-0.90	-0.92	-0.92	-0.93	-0.93	-0.91	-0.90
	0.89	0.87	0.84	0.82	0.80	0.76	0.69	0.63
20	-0.86	-0.88	-0.89	-0.89	-0.90	-0.90	-0.88	-0.87
	0.86	0.84	0.82	0.79	0.77	0.73	0.67	0.61
30	-0.83	-0.84	-0.86	-0.86	-0.86	-0.86	-0.85	-0.83
	0.83	0.80	0.78	0.76	0.73	0.70	0.63	0.57

Table A.5.20

k_1

$\kappa = 0.250$ $\lambda = 0.50$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-0.67	-0.67	-0.68	-0.68	-0.67	-0.67	-0.63	-0.58
	0.67	0.67	0.66	0.64	0.62	0.59	0.55	0.52
4	-0.67	-0.67	-0.68	-0.68	-0.69	-0.68	-0.64	-0.62
	0.67	0.65	0.64	0.63	0.60	0.58	0.54	0.48
5	-0.67	-0.69	-0.70	-0.70	-0.69	-0.69	-0.65	-0.63
	0.67	0.65	0.63	0.62	0.61	0.58	0.53	0.47
6	-0.67	-0.69	-0.69	-0.70	-0.69	-0.69	-0.66	-0.64
	0.67	0.66	0.65	0.62	0.60	0.57	0.52	0.46
9	-0.67	-0.68	-0.69	-0.70	-0.70	-0.69	-0.67	-0.64
	0.67	0.66	0.64	0.61	0.59	0.57	0.51	0.45
12	-0.67	-0.69	-0.70	-0.70	-0.71	-0.70	-0.69	-0.67
	0.67	0.66	0.63	0.61	0.58	0.56	0.49	0.43
15	-0.67	-0.69	-0.70	-0.70	-0.70	-0.71	-0.69	-0.67
	0.67	0.65	0.63	0.61	0.59	0.55	0.49	0.43
20	-0.67	-0.69	-0.70	-0.70	-0.71	-0.70	-0.69	-0.67
	0.67	0.66	0.63	0.61	0.58	0.56	0.49	0.43
30	-0.67	-0.69	-0.70	-0.71	-0.71	-0.71	-0.69	-0.67
	0.67	0.65	0.63	0.60	0.58	0.55	0.49	0.43

Table A.6.1

k_2

$\kappa = 0.001$ $\lambda = 0.95$

n	α							
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.5**	**2.0**
3	-13.86	-12.36	-11.34	-10.35	-9.45	-8.96	-7.62	-6.71
	13.86	15.40	17.27	19.40	22.25	25.40	32.95	41.28
4	-9.21	-8.34	-7.64	-6.86	-6.36	-5.84	-4.96	-4.37
	9.21	10.36	11.60	13.11	14.79	16.10	20.89	27.24
5	-7.50	-6.69	-6.06	-5.62	-5.24	-4.80	-3.93	-6.69
	7.50	8.22	9.36	10.30	11.84	13.20	17.00	21.90
6	-6.61	-5.84	-5.29	-4.85	-4.55	-4.25	-3.53	-3.22
	6.61	7.32	8.17	9.16	10.44	11.60	15.18	18.98
9	-5.41	-4.85	-4.51	-4.11	-3.93	-3.50	-2.97	-2.66
	5.41	6.07	6.82	7.59	8.50	9.67	12.08	14.97
12	-4.90	-4.40	-4.01	-3.67	-3.41	-3.14	-2.73	-2.41
	4.90	5.44	6.08	6.83	7.59	8.43	10.80	13.30
15	-4.61	-4.19	-3.78	-3.50	-3.22	-3.02	-1.55	-1.27
	4.61	5.09	5.70	6.37	7.02	7.82	9.88	12.12
20	-4.32	-3.95	-3.57	-3.22	-2.96	-2.78	-2.39	-2.12
	4.32	3.83	5.37	5.90	6.49	7.16	9.05	11.00
30	-4.02	-3.62	-3.34	-3.05	-2.78	-2.59	-2.20	-1.96
	4.02	4.46	4.94	5.45	5.95	6.65	8.20	9.93

Table A.6.2

k_2

$\kappa = 0.001$ $\lambda = 0.90$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-9.65	-8.70	-7.83	-7.12	-6.65	-6.25	-5.26	-4.60
	9.65	10.78	11.88	13.15	15.17	17.27	22.03	27.56
4	-7.13	-6.42	-5.84	-5.28	-4.83	-4.45	-3.85	-3.49
	7.13	8.05	8.94	10.05	11.20	12.30	16.08	20.74
5	-6.11	-5.45	-4.97	-4.55	-4.20	-3.88	-3.30	-3.02
	6.11	6.82	7.68	8.49	9.50	10.75	13.69	17.55
6	-5.56	-4.94	-4.46	-4.14	-3.80	-3.56	-3.06	-2.75
	5.56	6.02	6.92	7.69	8.60	9.60	12.47	15.54
9	-4.77	-4.28	-3.91	-3.61	-3.32	-3.12	-2.64	-2.37
	4.77	5.29	5.95	6.65	7.43	8.18	10.43	12.86
12	-4.42	-4.00	-3.60	-3.34	-3.09	-2.85	-2.48	-2.18
	4.42	4.93	5.48	6.17	6.75	7.46	9.47	11.55
15	-4.22	-3.82	-3.45	-3.21	-2.95	-2.72	-2.34	-2.09
	4.22	4.69	5.19	5.70	6.37	7.00	8.90	10.81
20	-4.01	-3.63	-3.32	-3.02	-2.78	-2.61	-2.24	-1.16
	4.01	4.44	4.91	5.47	5.98	6.64	8.25	10.04
30	-3.79	-3.43	-3.14	-2.88	-2.65	-2.45	-2.09	-1.86
	3.79	4.19	4.64	5.18	5.61	6.19	7.61	9.12

Table A.6.3

k_2

$\kappa = 0.001$ $\lambda = 0.80$

n	α							
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**	**1.5**	**2.0**
3	-6.62	-5.95	-5.38	-4.95	-4.55	-4.18	-3.62	-3.16
	6.62	7.35	7.40	9.21	10.38	11.65	15.18	18.40
4	-5.43	-4.94	-4.50	-4.10	-3.74	-3.46	-2.94	-2.66
	5.43	6.09	6.80	7.63	8.53	9.40	12.00	15.44
5	-4.87	-4.44	-4.06	-3.69	-3.41	-3.18	-2.70	-2.41
	4.87	5.41	6.15	6.71	7.60	8.50	10.75	13.43
6	-4.50	-4.10	-3.71	-3.45	-3.17	-2.97	-2.55	-2.27
	4.50	5.00	5.62	6.27	6.99	7.80	9.96	12.35
9	-4.16	-3.77	-3.42	-3.12	-2.89	-2.70	-2.31	-2.08
	4.16	4.62	5.15	5.70	6.28	6.95	8.74	10.67
12	-3.90	-3.57	-3.26	-2.97	-2.73	-2.57	-2.20	-1.95
	3.90	4.38	4.84	5.37	5.89	6.52	8.15	9.85
15	-3.80	-3.44	-3.14	-2.88	-2.67	-2.48	-2.12	-1.89
	3.80	4.21	4.64	5.16	2.70	6.28	7.83	9.45
20	-3.69	-3.34	-3.05	-2.78	-2.56	-2.39	-2.04	-1.81
	3.69	4.07	4.49	4.97	5.43	5.99	7.37	8.88
30	-3.56	-3.22	-2.95	-2.70	-2.48	-2.30	-1.96	-1.78
	3.56	3.91	4.33	4.76	5.17	5.69	6.95	8.27

Table A.6.4

k_2

$\kappa = 0.001$ $\lambda = 0.50$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.68	-3.34	-3.09	-2.83	-2.61	-2.43	-2.12	-1.87
	3.68	4.09	4.57	5.05	5.63	6.19	7.94	9.50
4	-3.46	-3.14	-2.89	-2.65	-2.47	-2.30	-1.98	-1.78
	3.46	3.81	4.25	4.70	5.21	5.74	7.10	8.69
5	-3.36	-3.06	-2.80	-2.56	-2.40	-2.25	-1.92	-1.73
	3.36	3.70	4.12	4.51	5.04	5.51	6.63	8.28
6	-3.29	-3.01	-2.73	-2.51	-2.34	-2.19	-1.88	-1.70
	3.29	3.62	3.80	4.40	4.88	5.39	6.74	8.03
9	-3.22	-2.94	-2.67	-2.46	-2.28	-2.13	-1.84	-1.62
	3.22	3.53	3.91	4.32	4.72	5.15	6.29	7.50
12	-3.18	-2.90	-2.64	-2.43	-2.25	-2.09	-1.79	-1.59
	3.18	3.51	3.85	4.24	4.59	5.03	6.11	7.22
15	-3.15	-2.88	-2.62	-2.41	-2.23	-2.07	-1.77	-1.57
	3.15	3.48	3.80	4.19	4.58	4.97	6.01	7.04
20	-3.14	-2.86	-2.61	-2.40	-2.20	-2.05	-1.74	-1.54
	3.14	3.45	3.79	4.15	4.51	4.95	5.92	6.88
30	-3.10	-2.85	-2.60	-2.40	-2.19	-2.04	-1.72	-1.50
	3.10	3.43	3.75	4.10	4.47	4.85	5.79	6.70

Table A.6.5

k_2

κ = 0.010 λ = 0.95

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-10.55	-9.55	-8.90	-8.35	-7.82	-7.32	-6.40	-5.63
	10.55	11.50	12.50	13.75	15.20	16.50	21.10	25.32
4	-7.04	-6.40	-5.95	-5.58	-5.29	-4.98	-4.29	-3.73
	7.04	7.69	8.25	9.12	9.93	11.00	13.34	15.64
5	-5.74	-5.30	-4.88	-4.55	-4.25	-4.20	-3.52	-3.21
	5.74	6.39	6.85	7.55	8.08	8.75	10.97	12.91
6	-5.06	-4.71	-4.34	-3.97	-3.71	-3.54	-3.15	-2.83
	5.06	5.51	5.97	6.55	7.03	7.74	9.42	11.53
9	-4.14	-3.82	-3.55	-3.31	-3.08	-2.93	-2.59	-2.28
	4.14	4.50	4.90	5.31	5.80	6.24	7.52	8.65
12	-3.75	-3.48	-3.21	-3.00	-2.82	-2.70	-2.35	-2.11
	3.75	4.10	4.45	4.75	5.21	5.64	6.71	7.81
15	-3.52	-3.25	-3.03	-2.82	-2.68	-2.51	-2.20	-1.97
	3.52	3.77	4.10	4.44	4.87	5.29	6.19	7.09
20	-3.30	-3.04	-2.86	-2.67	-2.51	-2.38	-2.06	-1.86
	3.30	3.57	3.84	4.18	4.49	4.74	5.63	6.42
30	-3.06	-2.81	-2.65	-2.48	-2.34	-2.21	-1.92	-1.73
	3.06	3.30	3.52	3.85	4.10	4.41	5.13	5.85

Table A.6.6

k_2

$\kappa = 0.010$ $\lambda = 0.90$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-7.34	-6.54	-6.15	-5.94	-5.50	-5.20	-4.58	-3.96
	7.34	7.75	8.50	9.50	10.46	11.43	14.40	16.90
4	-5.44	-4.86	-4.62	-4.30	-4.05	-3.82	-3.37	-2.97
	5.44	5.84	6.31	7.00	7.62	8.40	10.13	10.91
5	-4.67	-4.34	-4.06	-3.77	-3.55	-3.35	-2.92	-2.60
	4.67	5.15	5.54	6.05	6.50	7.01	8.77	10.25
6	-4.24	-3.85	-3.62	-3.38	-3.17	-3.00	-2.68	-2.38
	4.24	4.60	4.98	5.42	5.80	6.35	7.68	9.15
9	-3.64	-3.36	-3.13	-2.93	-2.76	-2.61	-2.28	-2.02
	3.64	3.96	4.34	4.61	5.04	5.42	6.45	7.52
12	-3.37	-3.13	-2.91	-2.73	-2.55	-2.41	-2.12	-1.90
	3.37	3.66	3.95	4.23	4.65	5.00	5.86	6.81
15	-3.22	-2.97	-2.78	-2.61	-2.46	-2.32	-2.02	-1.81
	3.22	3.44	3.69	4.03	4.35	4.76	5.49	6.25
20	-3.05	-2.84	-2.65	-2.47	-2.34	-2.19	-1.92	-1.71
	3.05	3.29	3.56	3.82	4.10	4.35	5.09	5.73
30	-2.88	-2.67	-2.51	-2.35	-2.20	-2.08	-1.82	-1.62
	2.88	3.08	3.33	3.58	3.85	4.09	4.70	5.31

Table A.6.7

k_2

$\kappa = 0.010$ $\lambda = 0.80$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-4.85	-4.63	-4.29	-4.00	-3.75	-3.57	-3.13	-2.78
	4.85	5.30	5.85	6.45	7.15	7.77	9.55	11.25
4	-4.05	-3.76	-3.54	-3.27	-3.05	-2.90	-2.59	-2.33
	4.05	4.49	4.88	5.33	5.73	6.20	7.35	8.76
5	-3.64	-3.43	-3.20	-2.98	-2.79	-2.61	-2.33	-2.15
	3.64	4.00	4.39	4.75	5.11	5.53	6.71	7.81
6	-3.40	-3.19	-2.99	-2.78	-2.60	-2.47	-2.23	-2.04
	3.40	3.75	4.08	4.40	4.76	5.11	6.08	7.17
9	-3.15	-2.91	-2.73	-2.55	-2.40	-2.26	-2.30	-1.84
	3.15	3.40	3.71	3.98	4.28	4.56	5.41	6.14
12	-3.00	-2.78	-2.59	-2.43	-2.28	-2.15	-1.91	-1.74
	3.00	3.22	3.50	3.74	4.01	4.30	5.01	5.70
15	-2.88	-2.67	-2.51	-2.36	-2.20	-2.10	-1.85	-1.69
	2.88	3.11	3.35	3.62	3.86	4.16	4.76	5.36
20	-2.80	-2.61	-2.44	-2.27	-2.14	-2.01	-1.79	-1.63
	2.80	2.99	3.22	3.44	3.67	3.91	4.48	5.00
30	-2.68	-2.50	-2.35	-2.20	-2.06	-1.95	-1.73	-1.55
	2.60	2.87	3.09	3.30	3.51	3.74	4.25	4.78

Table A.6.8

k_2

$\kappa = 0.010$ $\lambda = 0.50$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.75	-2.60	-2.40	-2.30	-2.15	-2.05	-1.84	-1.67
	2.75	3.00	3.23	3.50	3.72	4.09	4.70	5.38
4	-2.61	-2.45	-2.27	-2.16	-2.04	-1.94	-1.73	-1.58
	2.61	2.85	3.03	3.22	3.45	3.73	4.36	4.96
5	-2.54	-2.38	-2.21	-2.09	-1.96	-1.89	-1.68	-1.54
	2.54	2.73	2.89	3.12	3.32	3.59	4.13	4.66
6	-2.49	-2.34	-2.18	-2.05	-1.92	-1.84	-1.65	-1.51
	2.49	2.66	2.82	3.05	3.25	3.51	4.00	4.43
9	-2.41	-2.28	-2.13	-2.20	-1.88	-1.80	-1.59	-1.45
	2.41	2.60	2.76	2.95	3.12	3.36	3.81	4.21
12	-2.39	-2.25	-2.11	-1.98	-1.86	-1.77	-1.56	-1.42
	2.39	2.55	2.72	2.89	3.07	3.25	3.67	4.04
15	-2.38	-2.24	-2.09	-1.96	-1.85	-1.75	-1.54	-1.40
	2.38	2.55	2.71	2.85	3.03	3.21	3.59	3.95
20	-2.37	-2.23	-2.07	-1.95	-1.83	-1.74	-1.52	-1.39
	2.37	2.53	2.68	2.83	3.01	3.19	3.54	3.88
30	-2.36	-2.22	-2.07	-1.94	-1.82	-1.71	-1.50	-1.37
	2.36	2.51	2.68	2.82	2.98	3.14	3.46	3.80

Table A.6.9

k_2

κ = 0.050 λ = 0.95

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-7.66	-7.25	-6.87	-6.49	-6.15	-5.88	-5.25	-4.78
	7.66	8.41	9.05	9.57	10.32	10.90	12.20	13.80
4	-5.14	-4.85	-4.58	-4.32	-4.08	-3.91	-3.42	-3.09
	5.14	5.50	5.90	6.22	6.61	7.00	8.05	9.30
5	-4.20	-3.95	-3.73	-3.56	-3.38	-3.18	-2.84	-2.76
	4.20	4.48	4.77	5.12	5.46	5.83	6.65	7.75
6	-3.71	-3.50	-3.32	-3.12	-2.97	-2.82	-2.54	-2.34
	3.71	3.92	4.18	4.40	4.76	5.03	5.84	6.61
9	-3.03	-2.84	-2.67	-2.53	-2.42	-2.33	-2.17	-2.03
	3.03	3.19	3.40	3.62	3.88	4.04	4.72	5.24
12	-2.74	-2.60	-2.44	-2.37	-2.23	-2.17	-1.98	-1.85
	2.74	2.96	3.08	3.24	3.40	3.61	4.05	4.57
15	-2.57	-2.44	-2.32	-2.21	-2.10	-2.02	-1.84	-1.71
	2.57	2.70	2.85	3.00	3.20	3.36	3.73	4.08
20	-2.40	-2.26	-2.16	-2.06	-1.97	-1.88	-1.74	-1.63
	2.40	2.51	2.61	2.77	2.90	3.05	3.38	3.65
30	-2.22	-2.11	-2.02	-1.92	-1.85	-1.77	-1.62	-1.51
	2.22	2.34	2.42	2.56	2.68	2.79	3.01	3.29

Table A.6.10

k_2

$\kappa = 0.050$ $\lambda = 0.90$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-5.31	-4.90	-4.60	-4.37	-4.17	-3.98	-3.62	-3.32
	5.31	5.70	6.10	6.50	6.94	7.30	8.54	9.83
4	-3.96	-3.70	-3.50	-3.32	-3.16	-3.00	-2.72	-2.53
	3.96	4.20	4.45	4.75	5.07	5.37	6.14	7.21
5	-3.40	-3.19	-3.02	-2.89	-2.76	-2.62	-2.37	-2.21
	3.40	3.63	3.87	4.10	4.38	4.64	5.27	6.04
6	-3.09	-2.92	-2.73	-2.60	-2.48	-2.37	-2.20	-2.04
	3.09	3.26	3.48	3.69	3.93	4.15	4.72	5.35
9	-2.65	-2.50	-2.37	-2.26	-2.17	-2.07	-1.90	-1.79
	2.65	2.79	2.95	3.12	3.31	3.52	4.01	4.40
12	-2.45	-2.34	-2.22	-2.12	-2.01	-1.94	-1.79	-1.68
	2.45	2.63	2.73	2.86	3.00	3.16	3.54	3.87
15	-2.33	-2.22	-2.11	-2.02	-1.93	-1.86	-1.70	-1.58
	2.33	2.45	2.58	2.68	2.84	2.97	3.30	3.61
20	-2.21	-2.10	-2.00	-1.93	-1.83	-1.76	-1.62	-1.52
	2.21	2.32	2.44	2.53	2.63	2.76	3.00	3.28
30	-2.08	-1.98	-1.89	-1.81	-1.72	-1.67	-1.54	-1.42
	2.08	2.17	2.27	2.38	2.47	2.58	2.76	2.98

Table A.6.11

k_2

$\kappa = 0.050$ $\lambda = 0.80$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.64	-3.37	-3.18	-3.02	-2.89	-2.74	-2.50	-2.34
	3.64	3.88	4.14	4.41	4.66	4.91	5.72	6.48
4	-2.96	-2.81	-2.67	-2.53	-2.43	-2.31	-2.07	-1.95
	2.96	3.14	3.34	3.55	3.78	4.00	4.53	5.19
5	-2.70	-2.55	-2.41	-2.28	-2.19	-2.11	-1.91	-1.80
	2.70	2.86	3.04	3.20	3.36	3.55	4.00	4.49
6	-2.51	-2.38	-2.26	-2.16	-2.06	-1.99	-1.82	-1.72
	2.51	2.66	2.83	2.97	3.12	3.28	3.66	4.15
9	-2.24	-2.15	-2.05	-1.95	-1.88	-1.81	-1.66	-1.57
	2.24	2.36	2.50	2.64	2.78	2.90	3.21	3.52
12	-2.15	-2.05	-1.95	-1.86	-1.79	-1.72	-1.59	-1.49
	2.15	2.25	2.37	2.48	2.60	2.71	2.85	3.22
15	-2.08	-1.98	-1.90	-1.82	-1.74	-1.68	-1.53	-1.43
	2.08	2.18	2.27	2.38	2.49	2.58	2.82	3.05
20	-2.01	-1.92	-1.83	-1.75	-1.68	-1.62	-1.49	-1.39
	2.01	2.09	2.19	2.28	2.37	2.46	2.67	2.88
30	-1.93	-1.84	-1.76	-1.69	-1.63	-1.56	-1.43	-1.33
	1.93	2.00	2.08	2.17	2.25	2.33	2.50	2.60

Table A.6.12

k_2

$\kappa = 0.050$ $\lambda = 0.50$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.95	-1.84	-1.75	-1.70	-1.67	-1.56	-1.44	-1.38
	1.95	2.04	2.13	2.24	2.34	2.43	2.73	2.97
4	-1.81	-1.75	-1.67	-1.62	-1.55	-1.50	-1.39	-1.32
	1.81	1.91	1.99	2.08	2.18	2.27	2.49	2.72
5	-1.77	-1.71	-1.63	-1.58	-1.52	-1.46	-1.36	-1.29
	1.77	1.86	1.94	2.02	2.10	2.19	2.38	2.53
6	-1.74	-1.69	-1.61	-1.56	-1.50	-1.44	-1.34	-1.28
	1.74	1.82	1.90	1.97	2.05	2.11	2.27	2.43
9	-1.71	-1.65	-1.58	-1.51	-1.46	-1.41	-1.31	-1.24
	1.71	1.78	1.85	1.91	1.97	2.03	2.17	2.27
12	-1.68	-1.63	-1.57	-1.50	-1.45	-1.40	-1.29	-1.21
	1.68	1.75	1.82	1.88	1.93	1.98	2.09	2.18
15	-1.68	-1.62	-1.56	-1.50	-1.44	-1.39	-1.28	-1.19
	1.68	1.75	1.81	1.86	1.92	1.96	2.06	2.14
20	-1.68	-1.61	-1.55	-1.49	-1.43	-1.38	-1.27	-1.18
	1.68	1.73	1.80	1.85	1.89	1.93	2.02	2.10
30	-1.67	-1.60	-1.54	-1.49	-1.43	-1.37	-1.25	-1.16
	1.67	1.72	1.78	1.82	1.86	1.90	1.98	2.04

Table A.6.13

k_2

$\kappa = 0.100$ $\lambda = 0.95$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-6.16	-5.90	-5.62	-5.41	-5.18	-4.97	-4.52	-4.08
	6.16	6.57	7.02	7.60	8.18	8.61	9.27	10.14
4	-4.16	-3.95	-3.77	-3.60	-3.44	-3.30	-2.98	-2.69
	4.16	4.35	4.63	4.94	5.15	5.38	6.00	6.51
5	-3.41	-3.25	-3.08	-2.94	-2.81	-2.66	-2.48	-2.32
	3.41	3.67	3.84	4.11	4.28	4.51	5.01	5.46
6	-3.01	-2.86	-2.76	-2.61	-2.51	-2.38	-2.23	-2.11
	3.01	3.19	3.35	3.55	3.72	3.84	4.23	4.81
9	-2.45	-2.34	-2.42	-2.14	-2.08	-2.02	-1.87	-1.78
	2.45	2.58	2.69	2.78	2.96	3.12	3.38	3.58
12	-2.21	-2.12	-2.02	-1.95	-1.90	-1.83	-1.68	-1.60
	2.21	2.30	2.42	2.53	2.61	2.68	2.95	3.15
15	-2.07	-2.01	-1.91	-1.84	-1.79	-1.74	-1.60	-1.50
	2.07	2.13	2.26	2.32	2.41	2.51	2.72	2.90
20	-1.93	-1.85	-1.78	-1.71	-1.67	-1.60	-1.50	-1.43
	1.93	2.01	2.06	2.15	2.22	2.29	2.46	2.60
30	-1.78	-1.72	-1.67	-1.62	-1.56	-1.51	-1.40	-1.34
	1.78	1.82	1.89	1.96	1.99	2.04	2.17	2.29

Table A.6.14

k_2

$\kappa = 0.100$ $\lambda = 0.90$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-4.26	-3.99	-3.81	-3.65	-3.45	-3.27	-2.98	-2.84
	4.26	4.55	4.86	5.12	5.36	5.67	6.45	6.95
4	-3.19	-2.99	-2.86	-2.76	-2.58	-2.53	-2.34	-2.42
	3.19	3.36	3.57	3.72	3.90	4.11	4.59	5.05
5	-2.74	-2.59	-2.48	-2.37	-2.31	-2.19	-2.03	-1.93
	2.74	2.91	3.08	3.24	3.49	3.55	3.98	4.30
6	-2.49	-2.37	-2.28	-2.20	-2.12	-2.02	-1.89	-1.78
	2.49	2.61	2.73	2.84	3.00	3.10	3.47	3.77
9	-2.13	-2.03	-1.94	-1.87	-1.83	-1.78	-1.65	-1.56
	2.13	2.22	2.32	2.39	2.52	2.63	2.85	3.02
12	-1.97	-1.89	-1.81	-1.75	-1.70	-1.64	-1.53	-1.46
	1.97	2.02	2.13	2.21	2.28	2.35	2.53	2.67
15	-1.87	-1.81	-1.74	-1.67	-1.64	-1.58	-1.47	-1.39
	1.87	1.92	1.99	2.07	2.15	2.22	2.35	2.50
20	-1.77	-1.80	-1.64	-1.58	-1.55	-1.49	-1.40	-1.32
	1.77	1.83	1.88	1.96	1.99	2.06	2.18	2.28
30	-1.66	-1.60	-1.55	-1.51	-1.47	-1.42	-1.32	-1.26
	1.66	1.69	1.75	1.82	1.85	1.88	1.97	2.05

Table A.6.15

k_2

$\kappa = 0.100$ $\lambda = 0.80$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.89	-2.72	-2.63	-2.51	-2.41	-2.30	-2.12	-2.01
	2.89	3.04	3.20	3.35	3.52	3.71	4.10	4.62
4	-2.39	-2.25	-2.13	-2.05	-1.97	-1.93	-1.81	-1.70
	2.39	2.49	2.60	2.72	2.81	2.94	3.21	3.54
5	-2.17	-2.06	-1.98	-1.91	-1.83	-1.78	-1.66	-1.57
	2.17	2.27	2.36	2.48	2.58	2.64	2.94	3.18
6	-2.02	-1.93	-1.86	-1.80	-1.72	-1.68	-1.57	-1.49
	2.02	2.10	2.18	2.26	2.34	2.46	2.66	2.93
9	-1.83	-1.75	-1.67	-1.63	-1.59	-1.55	-1.45	-1.36
	1.83	1.87	1.95	2.02	2.09	2.17	2.29	2.40
12	-1.71	-1.65	-1.59	-1.55	-1.50	-1.46	-1.37	-1.29
	1.71	1.75	1.83	1.89	1.93	1.98	2.08	2.20
15	-1.65	-1.60	-1.55	-1.49	-1.45	-1.41	-1.33	-1.25
	1.65	1.61	1.75	1.80	1.85	1.89	1.99	2.09
20	-1.59	-1.55	-1.50	-1.45	-1.40	-1.37	-1.28	-1.22
	1.59	1.65	1.68	1.73	1.76	1.79	1.88	1.96
30	-1.52	-1.48	-1.44	-1.40	-1.36	-1.32	-1.23	-1.18
	1.52	1.56	1.59	1.64	1.67	1.69	1.74	1.78

Table A.6.16

k_2

κ = 0.100 λ = 0.50

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.51	-1.44	-1.40	-1.37	-1.33	-1.29	-1.23	-1.17
	1.51	1.56	1.60	1.64	1.69	1.74	1.86	1.96
4	-1.41	-1.37	-1.34	-1.30	-1.27	-1.24	-1.17	-1.14
	1.41	1.46	1.50	1.55	1.59	1.60	1.71	1.80
5	-1.38	-1.35	-1.32	-1.28	-1.24	-1.22	-1.16	-1.12
	1.38	1.42	1.46	1.50	1.53	1.56	1.63	1.70
6	-1.36	-1.33	-1.30	-1.27	-1.24	-1.21	-1.14	-1.09
	1.36	1.39	1.42	1.45	1.47	1.50	1.57	1.64
9	-1.34	-1.30	-1.27	-1.25	-1.22	-1.20	-1.13	-1.06
	1.34	1.36	1.39	1.42	1.44	1.46	1.49	1.51
12	-1.32	-1.29	-1.27	-1.24	-1.21	-1.18	-1.11	-1.05
	1.32	1.35	1.37	1.39	1.41	1.43	1.44	1.45
15	-1.31	-1.29	-1.26	-1.22	-1.20	-1.17	-1.10	-1.04
	1.31	1.33	1.35	1.37	1.39	1.40	1.41	1.41
20	-1.30	-1.28	-1.25	-1.22	-1.19	-1.16	-1.09	-1.03
	1.30	1.32	1.34	1.36	1.37	1.38	1.38	1.37
30	-1.29	-1.27	-1.24	-1.21	-1.18	-1.15	-1.08	-1.02
	1.29	1.31	1.33	1.34	1.35	1.35	1.34	1.34

Table A.6.17

k_2

$\kappa = 0.250$ $\lambda = 0.95$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.81	-3.69	-3.55	-3.39	-3.27	-3.16	-2.93	-2.69
	3.81	3.95	4.09	4.25	4.40	4.54	4.84	5.16
4	-2.62	-2.56	-2.48	-2.39	-2.29	-2.21	-2.10	-1.90
	2.62	2.72	2.83	2.93	3.04	3.12	3.36	3.51
5	-2.15	-2.13	-2.08	-2.02	-1.94	-1.89	-1.78	-1.67
	2.15	2.22	2.29	2.37	2.44	2.50	2.69	2.86
6	-1.90	-1.84	-1.80	-1.74	-1.68	-1.61	-1.55	-1.48
	1.90	1.96	2.04	2.10	2.15	2.20	2.34	2.45
9	-1.53	-1.52	-1.48	-1.46	-1.42	-1.38	-1.31	-1.28
	1.53	1.57	1.61	1.63	1.66	1.70	1.81	1.85
12	-1.37	-1.34	-1.32	-1.29	-1.27	-1.26	-1.20	-1.17
	1.37	1.39	1.41	1.43	1.47	1.48	1.53	1.50
15	-1.27	-1.25	-1.22	-1.20	-1.17	-1.17	-1.16	-1.13
	1.27	1.29	1.31	1.33	1.35	1.36	1.39	1.39
20	-1.17	-1.15	-1.14	-1.12	-1.11	-1.10	-1.07	-1.05
	1.17	1.17	1.17	1.19	1.19	1.20	1.22	1.20
30	-1.06	-1.05	-1.04	-1.04	-1.04	-1.01	-1.00	-0.97
	1.06	1.07	1.07	1.08	1.05	1.04	1.04	1.02

Table A.6.18

k_2

$\kappa = 0.250$ $\lambda = 0.90$

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.60	-2.51	-2.42	-2.34	-2.27	-2.20	-2.07	-1.88
	2.60	2.75	2.85	3.00	3.16	3.18	3.40	3.66
4	-1.97	-1.90	-1.84	-1.78	-1.73	-1.70	-1.64	-1.52
	1.97	2.02	2.09	2.15	2.23	2.28	2.40	2.53
5	-1.70	-1.67	-1.63	-1.59	-1.54	-1.52	-1.42	-1.38
	1.70	1.76	1.81	1.86	1.91	1.96	2.06	2.18
6	-1.54	-1.50	-1.46	-1.42	-1.40	-1.36	-1.31	-1.29
	1.54	1.57	1.61	1.66	1.68	1.72	1.82	1.86
9	-1.30	-1.28	-1.25	-1.24	-1.21	-1.20	-1.15	-1.14
	1.30	1.33	1.36	1.38	1.39	1.41	1.45	1.49
12	-1.19	-1.18	-1.16	-1.14	-1.13	-1.11	-1.08	-1.06
	1.19	1.21	1.21	1.22	1.23	1.26	1.27	1.27
15	-1.12	-1.11	-1.09	-1.08	-1.07	-1.06	-1.04	-1.01
	1.12	1.12	1.13	1.13	1.13	1.13	1.16	1.14
20	-1.05	-1.04	-1.03	-1.02	-1.01	-1.00	-0.98	-0.96
	1.05	1.04	1.04	1.04	1.05	1.04	1.04	1.02
30	-0.97	-0.97	-0.97	-0.96	-0.96	-0.95	-0.93	-0.91
	0.97	0.97	0.97	0.97	0.95	0.93	0.92	0.87

Table A.6.19

k_2

$\kappa = 0.250$ $\lambda = 0.80$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.70	-1.65	-1.60	-1.56	-1.53	-1.51	-1.43	-1.32
	1.70	1.76	1.82	1.89	1.94	1.97	2.05	2.13
4	-1.41	-1.39	-1.37	-1.34	-1.31	-1.29	-1.26	-1.19
	1.41	1.43	1.47	1.51	1.55	1.57	1.64	1.68
5	-1.29	-1.29	-1.27	-1.24	-1.21	-1.19	-1.14	-1.11
	1.29	1.31	1.34	1.37	1.39	1.41	1.44	1.47
6	-1.20	-1.18	-1.16	-1.14	-1.12	-1.11	-1.09	-1.06
	1.20	1.21	1.22	1.25	1.28	1.28	1.32	1.33
9	-1.06	-1.05	-1.04	-1.03	-1.02	-1.01	-0.99	-0.98
	1.06	1.07	1.08	1.09	1.09	1.10	1.11	1.09
12	-1.00	-1.00	-1.00	-0.99	-0.98	-0.97	-0.95	-0.93
	1.00	1.01	1.00	1.00	0.99	0.98	0.97	0.95
15	-0.96	-0.95	-0.95	-0.94	-0.94	-0.94	-0.92	-0.90
	0.96	0.96	0.95	0.95	0.94	0.93	0.91	0.87
20	-0.91	-0.91	-0.91	-0.91	-0.90	-0.90	-0.89	-0.87
	0.91	0.90	0.90	0.89	0.88	0.87	0.85	0.80
30	-0.87	-0.87	-0.87	-0.87	-0.87	-0.87	-0.85	-0.83
	0.87	0.85	0.84	0.83	0.81	0.79	0.75	0.70

Table A.6.20

k_2

$\kappa = 0.250$ $\lambda = 0.50$

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-0.77	-0.78	-0.78	-0.78	-0.79	-0.79	-0.79	-0.77
	0.77	0.77	0.78	0.77	0.77	0.77	0.74	0.73
4	-0.74	-0.74	-0.75	-0.76	-0.76	-0.76	-0.78	-0.76
	0.74	0.73	0.73	0.71	0.70	0.69	0.65	0.62
5	-0.72	-0.74	-0.75	-0.75	-0.75	-0.76	-0.76	-0.75
	0.72	0.71	0.71	0.69	0.68	0.66	0.62	0.59
6	-0.70	-0.71	-0.71	-0.72	-0.73	-0.74	-0.74	-0.74
	0.70	0.70	0.69	0.68	0.67	0.65	0.61	0.57
9	-0.70	-0.71	-0.72	-0.73	-0.73	-0.73	-0.73	-0.74
	0.70	0.68	0.67	0.64	0.63	0.61	0.57	0.53
12	-0.69	-0.71	-0.72	-0.72	-0.72	-0.72	-0.73	-0.73
	0.69	0.68	0.65	0.64	0.62	0.61	0.54	0.50
15	-0.68	-0.70	-0.72	-0.72	-0.73	-0.73	-0.72	-0.72
	0.68	0.67	0.65	0.63	0.61	0.58	0.53	0.49
20	-0.69	-0.70	-0.72	-0.72	-0.72	-0.72	-0.72	-0.72
	0.69	0.66	0.64	0.63	0.60	0.58	0.54	0.46
30	-0.68	-0.69	-0.71	-0.71	-0.72	-0.72	-0.72	-0.71
	0.68	0.66	0.64	0.62	0.59	0.57	0.50	0.45

Table A.7.1

k_2'

κ = 0.001 ●

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-5.47	-4.92	-4.45	-4.12	-3.84	-3.60	-3.10	-2.75
	5.47	6.05	6.72	7.55	8.48	9.58	12.23	14.80
4	-4.27	-3.91	-3.55	-3.24	-2.99	-2.75	-2.37	-2.15
	4.27	4.76	5.25	5.84	6.48	7.12	8.97	11.19
5	-3.84	-3.50	-3.21	-2.96	-2.74	-2.56	-2.17	-1.95
	3.84	4.25	4.74	5.25	5.85	6.45	8.00	9.86
6	-3.61	-3.31	-3.03	-2.08	-2.59	-2.41	-2.06	-1.86
	3.61	4.03	4.45	4.95	5.48	6.02	7.55	9.22
9	-3.42	-3.12	-2.84	-2.62	-2.42	-2.25	-1.92	-1.71
	3.42	3.78	4.17	4.60	5.05	5.54	6.78	8.16
12	-3.30	-3.02	-2.76	-2.53	-2.34	-2.17	-1.86	-1.63
	3.30	3.66	4.04	4.44	4.84	5.28	6.48	7.66
15	-3.26	-2.97	-2.71	-2.49	-2.30	-2.14	-1.81	-1.60
	3.26	3.60	3.94	4.35	4.74	5.16	6.28	7.43
20	-3.22	-2.93	-2.68	-2.45	-2.25	-2.10	-1.78	-1.56
	3.22	3.54	3.88	4.24	4.62	5.06	4.10	7.14
30	-3.18	-2.89	-2.64	-2.42	-2.22	-2.06	-1.73	-1.51
	3.18	3.48	3.82	4.13	4.54	4.92	5.89	6.83

Table A.7.2

k_2'

$\kappa = 0.010$ ●

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-3.98	-3.76	-3.53	-3.32	-3.14	-2.98	-2.64	-2.37
	3.98	4.33	4.79	5.28	5.76	6.24	7.38	8.56
4	-3.19	-2.98	-2.78	-2.62	-2.46	-2.34	-2.11	-1.88
	3.19	3.49	3.76	4.03	4.36	4.76	5.56	6.37
5	-2.88	-2.71	-2.54	-2.39	-2.24	-2.12	-1.89	-1.74
	2.88	3.17	3.39	3.64	3.92	4.22	4.92	5.70
6	-2.77	-2.59	-2.42	-2.26	-2.13	-2.03	-1.80	-1.65
	2.77	2.99	3.19	3.42	3.64	3.90	4.55	5.22
9	-2.58	-2.41	-2.26	-2.12	-1.99	-1.89	-1.68	-1.52
	2.58	2.77	2.96	3.06	3.38	3.60	4.11	4.62
12	-2.50	-2.35	-2.20	-2.06	-1.94	-1.84	-1.62	-1.46
	2.50	2.67	2.87	3.03	3.24	3.43	3.94	4.36
15	-2.46	-2.29	-2.16	-2.03	-1.91	-1.81	-1.59	-1.44
	2.46	2.63	2.80	2.99	3.17	3.37	3.78	4.19
20	-2.43	-2.27	-2.13	-2.00	-1.88	-1.77	-1.56	-1.40
	2.43	2.59	2.76	2.93	3.09	3.26	3.66	4.04
30	-2.39	-2.23	-2.09	-1.97	-1.85	-1.74	-1.52	-1.37
	2.39	2.53	2.70	2.87	3.02	3.18	3.55	3.90

Table A.7.3

k_2'

$\kappa = 0.050$ ●

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.88	-2.77	-2.67	-2.55	-2.44	-2.33	-2.10	-1.95
	2.88	3.10	3.30	3.50	3.69	3.88	4.25	4.85
4	-2.26	-2.15	-2.06	-1.97	-1.91	-1.84	-1.68	-1.57
	2.26	2.38	2.51	2.65	2.78	2.94	3.26	3.61
5	-2.08	-1.98	-1.89	-1.81	-1.73	-1.67	-1.54	-1.45
	2.08	2.18	2.27	2.36	2.48	2.59	2.87	3.14
6	-1.96	-1.88	-1.79	-1.72	-1.65	-1.59	-1.47	-1.39
	1.96	2.03	2.14	2.22	2.33	2.41	2.66	2.91
9	-1.82	-1.75	-1.68	-1.62	-1.56	-1.51	-1.38	-1.30
	1.82	1.88	1.98	2.06	2.12	2.20	2.40	2.58
12	-1.76	-1.70	-1.64	-1.57	-1.51	-1.46	-1.33	-1.25
	1.76	1.82	1.91	1.98	2.04	2.10	2.25	2.38
15	-1.74	-1.68	-1.61	-1.55	-1.48	-1.43	-1.31	-1.22
	1.74	1.80	1.87	1.93	1.99	2.06	2.18	2.29
20	-1.72	-1.65	-1.58	-1.52	-1.46	-1.41	-1.29	-1.20
	1.72	1.77	1.84	1.88	1.95	2.01	2.11	2.21
30	-1.70	-1.62	-1.56	-1.50	-1.44	-1.39	-1.26	-1.17
	1.70	1.75	1.80	1.86	1.92	1.96	2.04	2.10

Table A.7.4

k_2'

κ = 0.100 ●

n	α							
	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-2.27	-2.16	-2.08	-2.00	-1.94	-1.88	-1.76	-1.66
	2.27	2.41	2.52	2.62	2.73	2.85	3.16	3.40
4	-1.76	-1.72	-1.65	-1.60	-1.55	-1.50	-1.41	-1.35
	1.76	1.84	1.91	1.98	2.05	2.12	2.28	2.42
5	-1.62	-1.56	-1.51	-1.46	-1.42	-1.38	-1.30	-1.25
	1.62	1.67	1.73	1.78	1.83	1.88	2.03	2.16
6	-1.53	-1.49	-1.45	-1.40	-1.36	-1.32	-1.25	-1.19
	1.53	1.58	1.61	1.66	1.71	1.74	1.87	1.98
9	-1.43	-1.39	-1.35	-1.32	-1.29	-1.26	-1.19	-1.13
	1.43	1.45	1.49	1.53	1.56	1.60	1.66	1.69
12	-1.39	-1.35	-1.31	-1.28	-1.24	-1.21	-1.14	-1.09
	1.39	1.41	1.45	1.47	1.49	1.51	1.55	1.59
15	-1.36	-1.33	-1.30	-1.26	-1.23	-1.20	-1.13	-1.06
	1.36	1.38	1.41	1.43	1.45	1.47	1.50	1.52
20	-1.34	-1.31	-1.28	-1.24	-1.21	-1.18	-1.11	-1.05
	1.34	1.37	1.39	1.41	1.42	1.43	1.45	1.45
30	-1.32	-1.29	-1.26	-1.23	-1.19	-1.16	-1.09	-1.03
	1.32	1.34	1.36	1.37	1.38	1.39	1.39	1.39

Table A.7.5

k_2'

$\kappa = 0.250$ ●

n	α 0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
3	-1.17	-1.15	-1.13	-1.12	-1.10	-1.08	-1.03	-0.97
	1.17	1.18	1.22	1.24	1.29	1.33	1.39	1.47
4	-0.93	-0.93	-0.92	-0.92	-0.91	-0.90	-0.88	-0.85
	0.93	0.93	0.94	0.95	0.96	0.97	0.98	0.99
5	-0.85	-0.85	-0.85	-0.84	-0.84	-0.83	-0.82	-0.80
	0.85	0.85	0.86	0.86	0.86	0.86	0.86	0.87
6	-0.80	-0.80	-0.80	-0.79	-0.79	-0.79	-0.79	-0.79
	0.80	0.81	0.81	0.81	0.80	0.79	0.79	0.78
9	-0.75	-0.75	-0.76	-0.76	-0.76	-0.76	-0.76	-0.76
	0.75	0.75	0.74	0.73	0.72	0.70	0.68	0.65
12	-0.73	-0.74	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75
	0.73	0.72	0.70	0.68	0.67	0.65	0.61	0.59
15	-0.71	-0.72	-0.73	-0.74	-0.74	-0.74	-0.74	-0.74
	0.71	0.70	0.69	0.67	0.65	0.63	0.59	0.55
20	-0.70	-0.72	-0.73	-0.73	-0.73	-0.73	-0.73	-0.73
	0.70	0.68	0.67	0.65	0.63	0.61	0.57	0.52
30	-0.69	-0.71	-0.72	-0.72	-0.73	-0.73	-0.73	-0.72
	0.69	0.67	0.66	0.64	0.62	0.59	0.53	0.48

Appendix B

BETA-4 PROBABILITY PAPER

A probability paper is often a very helpful tool for simple evaluation of data, interpolations in tables, and graphical solution of iteration problems. In the vast majority, *normal probability paper,* commonly available on the stationery market, is used. However, the normal paper is only good for cases where no bounds of the probability distribution exist or can be expected. According to the Author's experience, gained in evaluation of numerous samples of various kind and also during various probability-based calculations, a probability paper based on *standardized symmetric beta-distribution with the lower bound* u_{inf} = -4 *and the upper bound* u_{sup} = +4, BT4, is a better help than the normal paper. Figure B.1 shows PDF of BT4 and of normal distribution, N. It should be noted that PDF of BT6, that is, with $|u_{inf}| = u_{sup} = 6$, is closer to the normal PDF; nonetheless, BT4 proved to serve better to various purpose.

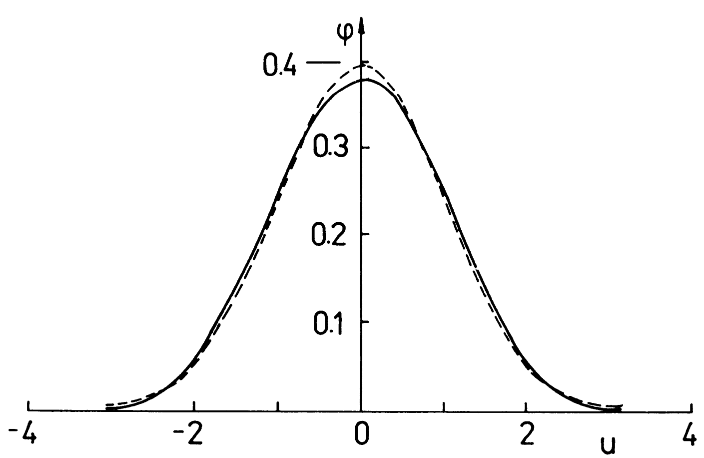

Fig. B.1 - Standardized PDFs of the beta distribution with u_{inf} = -4 and u_{sup} = +4 (full line) and of the normal distribution (dashed line).

The beta probability distribution, BT, has four parameters (see 2.1.2; for more information on BT, see References to this Appendix). In standard form, its PDF is

$$\varphi_{BT} = \frac{1}{B(a,b)} \frac{(ac+u)^{a-1}(bc-u)^{b-1}}{[(a+b)c]^{a+b-1}}$$

and CDF:

$$\Phi_{BT} = \int_{u_{inf}}^{u} \varphi_{BT}(u)\,du$$

where u = standardized random variable,

$$B(a,b) = \frac{\Gamma(a)\,\Gamma(b)}{\Gamma(a+b)}, \qquad c = \left(\frac{a+b+1}{a\,b}\right)^{\frac{1}{2}}$$

and $\Gamma(.)$ = gamma function.

Since, in standardized form, $\mu = 0$ and $\sigma = 1$, a, b are the remaining two parameters of BT.

For given bounds of the standardized random variable, u_{inf} and u_{sup}, the population parameters a and b can be established from

$$a = \frac{u_{inf}(u_{inf}u_{sup}+1)}{u_{sup}-u_{inf}}, \qquad b = -\frac{u_{sup}(u_{inf}u_{sup}+1)}{u_{sup}-u_{inf}}$$

More details on the beta distribution can be found in Cramér 1959, Hahn and Shapiro 1967, Johnson and Kotz 1967, Zelen and Severo 1965.

The BT4 probability paper is constructed for the beta distribution with

$$u_{inf} = -4, \quad u_{sup} = 4$$

that is with

$$a = b = 7.5$$

The paper grid is defined by values of the inverse distribution function $\Phi^{-1}(\kappa)$ and the values of standardized random variable, u_κ; see Figure B.2 and Table B.1.

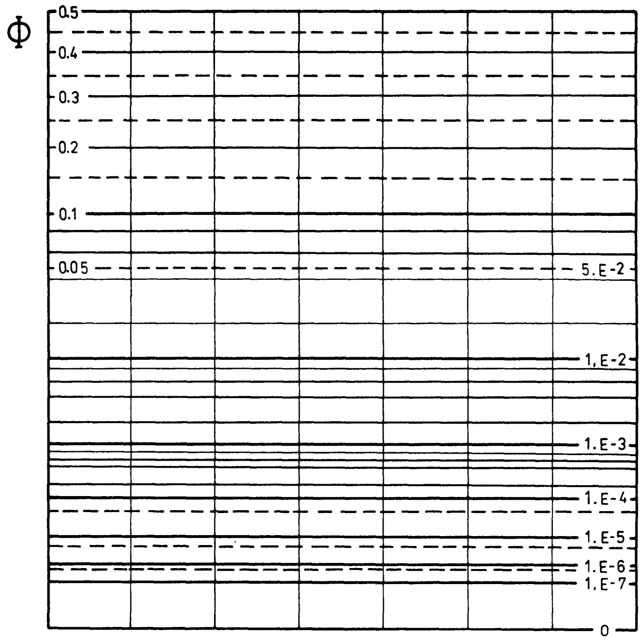

Fig. B.2 - BT4 probability paper for $\Phi_{BT} \in [0, 0.5]$.

Table B.1 - Inverse distribution function u_κ of the standardized symmetric beta distribution with $|u_{inf}| = u_{sup} = 4$ *vs.* probability κ

κ	u_κ	κ	u_κ
0.5	0	0.96	1.745
0.55	0.132	0.98	2.008
0.60	0.266	0.99	2.231
0.65	0.404	0.992	2.296
0.70	0.548	0.994	2.374
0.75	0.703	0.996	2.478
0.80	0.873	0.998	2.637
0.85	1.068	0.999	2.776
0.90	1.308	0.9995	2.899
0.92	1.423	0.9999	3.133
0.94	1.567	0.99999	3.378
0.95	1.649	1	4

Appendix C

SUMMARY OF NOTATIONS AND ABBREVIATIONS

BASIC SYMBOLS

Latin letters

A	attack; cross-section area; exceedance area
a	elementary random variable; sample coefficient of skewness
B	stiffness
b	elementary variable
C	constraint
C	cost
C_v	sample coefficient of variation
D	deterioration degree
d	diameter
E	elastic modulus
E	event
e	sample coefficient of excess
F	load magnitude
f_c	compression strength of concrete
f_y	yield stress of steel
G	self weight
H	phenomenon
I	moment of inertia
k	correlation coefficient (random sequence)
l	length
M	bending moment; number
m	differentiation multiplier; sample mean; number
N	axial force
n	number
P	probability
Q	quasi-
R	reliability function; resistance; sample range
r	radius of curvature; sample correlation coefficient; value of resistance variable
S	load-effect
s	sample standard deviation; snow load
T	period
T_0	facility life
t	time
u	standardized random variable
V	imposed load
v	wind velocity
w	deflection; wind load
x	value of random variable ξ
y	value of random variable η
Z	reliability margin

Greek letters

α	coefficient of skewness (population)
β	reliability index
β^{HL}	Hasofer-Lind index
Γ	variability factor
γ	partial reliability factor; volume density
γ_n	importance factor
δ	coefficient of variation (population)
ε	coefficient of excess (population)
η	random variable
τ	duration of out-crossing; efficiency index
Θ	comprehensive reliability factor
ϑ	population parameter in general; random variable
κ	probability
Λ	influence function
λ	confidence level; failure rate
μ	mean (population)
ξ	random variable
Φ	cumulative distribution function, CDF
Φ^{-1}	inverse distribution function, IDF
φ	cross-section reliability factor
ϕ	probability density function, PDF
ϱ	correlation coefficient (population); elementary variable (resistance); reliability measure
σ	standard deviation (population)
ψ	combination factor
ψ_0	load combination factor
Ω	domain

SUBSCRIPTS AND SUPERSCRIPTS

$\square_{a}$	age; axial
$\square_{act}$	active
$\square_{adm}$	admissible
$\square_{adv}$	adverse
$\square_{arb}$	arbitrary
$\square_{b}$	bending
$\square_{c}$	combination
$\square_{cnt}$	central
$\square_{col}$	collapse
$\square_{d}$	design point; design value; distress
$\square_{def}$	defined; definition; deformation
$\square_{dem}$	demolition
$\square_{e}$	existential
$\square_{eff}$	effective
$\square_{emp}$	empirical
$\square_{eqv}$	equivalent
$\square_{est}$	estimated
$\square_{exm}$	extreme
$\square_{exp}$	expected
$\square_{f}$	failure
$\square_{fav}$	favorable
$\square_{fil}$	filtered; filtering
$\square_{gen}$	general
$\square_{i}$	lower bound
$\square_{inf}$	infimum
$\square^{K}$	K-class
$\square_{k}$	characteristic
$\square_{LN}$	log-normal distribution
$\square_{mid}$	mid-span
$\square_{N}$	normal distribution
$\square_{nec}$	necessary
$\square_{neg}$	negative
$\square_{nom}$	nominal
$\square_{ntr}$	neutral
$\square_{obs}$	observation; observed
$\square_{occ}$	occurrence
$\square^{par}$	parallel
$\square_{p}$	permanent
$\square_{pas}$	passive
$\square_{pos}$	positive
$\square^{R}$	reference class
$\square_{r}$	resistance
$\square_{rec}$	reconstruction

$\square_{ref}$	reference
$\square_{req}$	required
$\square_{res}$	residual
$\square_{ret}$	return
$\square^{ret}$	return
$\square_{rnd}$	random
$\square_{s}$	reinforcement; sequential; serviceability; snow; survival; upper bound
$\square^{ser}$	serial
$\square_{sim}$	simulated
$\square_{sit}$	situation
$\square_{sup}$	supremum
$\square_{sys}$	system
$\square_{t}$	target value; transient
$\square_{tem}$	temperature
$\square_{thr}$	theoretical
$\square_{tot}$	total
$\square_{tr}$	truncated
$\square_{u}$	ultimate; user
$\square^{u}$	mapped onto the system of standardized variables
$\square_{ut}$	utility
$\square_{w}$	wind

DIACRITICALS

$\dot{\square}$	set
$\square'$	approximate; empirical, etc.
$\square^{*}$	analogous; derived; primary, etc.
$\square^{**}$	secondary
$\underline{\dot{\square}}$	annual value
$\square$	comprehensive value (related to a reference period)

OPERATION SYMBOLS

adv(.)	adverse value
Ev(.)	event
fav(.)	favorable
max(.)	maximum of values
Ph(.)	phenomenon
Pr(.)	probability

ABBREVIATIONS

ACI	American Concrete Institute
AIPC	*see* IABSE
ANSI	American National Standards Institute
ARTS	Advances in Reliability Technology Symposium
ASCE	American Society of Civil Engineers
ASME	American Society of Mechanical Engineers
BS	British Standard
BSI	British Standards Institution
BT	beta distribution
CDF	cumulative distribution function
CEB	Comité Euro-International du Béton
CEC	Commission of European Communities

CEN	European Committee for Standardization
CF	constructed facility
CIB	Conseil International du Bâtiment pour la Recherche, l'Étude et la Construction
CIRIA	Construction Industry Research and Information Association, United Kingdom
ÇOSSAN	a structural reliability software
ČSN	Czech National Standard
DBV	Deutscher Beton-Verein
DDR	former German Democratic Republic
DIN	Deutsches Institut für Normung
EEC	European Economic Community
ESRC	European Safety and Reliability Conference
FIDIC	Fédération Internationale des Ingenieurs-Conseils
FOSM	first-order second-moment
FORM	first-order reliability method
FOTM	first-order third-moment
GTR	general theory of reliability
HVAC	heating, ventilation, and air conditioning
IABSE	International Association for Bridge and Structural Engineering
IASSAR	International Association for Structural Safety and Reliability
ICAPS	International Conference on Applications of Statistics and Probability in Soil and Structural Engineering
ICE	International Electrotechnical Commission
ICOSSAR	International Conference on Structural Safety and Reliability
IDF	inverse distribution function
IEV	International Electrotechnical Vocabulary
ISO	International Organization for Standardization
I.T.B.T.P	Institut Technique du Bâtiment et des Travaux Publics
IUTAM	International Union of Theoretical and Applied Mechanics
IVBH	*see* IABSE
JCSS	Joint Committee for Structural Safety
JSCE	Japan Society of Civil Engineers
LN	log-normal distribution
M-E	moment and estimation
MIT	Massachusetts Institute of Technology
N	normal distribution
NKB	Nordic Committee on Building Regulations
PDF	probability density function
PRA	probabilistic risk assessment
PROBAN	a structural reliability software
PSA	probabilistic safety assessment
QAQC	quality assurance and quality control
QACP	quality assurance and control process
RAP	reliability assurance process
R.C.	reinforced concrete
RelReq	reliability requirement

RILEM	Réunion Internationale des Laboratoires d'Essais et de Recherches sur les Matériaux et les Constructions
RNG	random number generator
SBI	Statens Byggeforskningsinstitut, Denmark
S-E	simulation and estimation
SFB 96	Sonderforschungsbereich 96 *Zuverlässigkeitstheorie der Bauwerke,* Germany
S-L-E	structure-load-environment
SLS	serviceability limit states
SNiP	Russian Building Standard
SORM	second-order reliability method
SSRT	structural reliability theory
TU	Technical University
ULS	ultimate limit states
VDI	Verein Deutscher Ingenieure
VEB	Volkseigener Betrieb (former DDR)

Appendix D

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Actions on Structures. Irregular CIB Reports.

Durability of Building Materials. Published by Elsevier (since 1983).

Engineering Systems. Published by E. & F.N. Spon, London (since 1984).

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JCSS Working Documents. Published irregularly by IABSE.

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Risk Abstracts. A quarterly journal of abstracts, reviews, and references. Published by the Institute of Risk Research, University of Waterloo, Waterloo, Ontario (since 1984).

Structural Safety. Published by Elsevier, Amsterdam (since 1982).

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The list given below should be only considered a general guidance. When approaching a reliability problem, information retrieval system can supply detailed lists of publications related to the particular topic. For ease in identification, the following symbols have been used:

★ books on structural and general reliability methods, and related topics; proceedings of conferences

♦ books on probability and statistics, general risk analysis, etc.

§ codes, recommendations, and other regulatory documents

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