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Complex Sports Biodynamics

With Practical Applications in Tennis

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Volume 2

Editors: Rüdiger Dillmann · Yoshihiko Nakamura · Stefan Schaal · David Vernon

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and Sasa Markovic

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Preface

What are motor abilities of Olympic champions? What are essential psychological characteristics of Mark Spitz, Carl Lewis and Roger Federer? How to discover and maximally develop motor intelligence? How to develop indomitable will power of Olympic champions? What are the secrets of selection for the future Olympic champions? Does for every sport exist a unique model of an Olympic champion? This book gives a modern scientific answers to the above questions. Its purpose is to give you the answer to everything you ever wanted to ask about sport champions, but didn't know who or how to ask.

In particular, the purpose of this book is to give you the answer to everything you ever wanted to ask about advanced tennis, but didn't know who or how to ask. Its aim is to dispel classical myths of a "biomechanically sound" serve, forehand, and backhand, as well as provide methods for developing *superior tennis weapons*, a *lightning–fast game*, and *unrivaled mental speed and strength* – essential qualities of a *future tennis champion.*

This book *does not* describe a method that was used by Sampras, or Borg, or any other great tennis champion from the past. Nor does it explain current tennis basics as so many other books do. This book takes a totally different perspective, it describes and explains the physical and mental abilities of a champion in future tennis. Weapons of a future tennis game will comprise of *whip–like* tennis serves and strokes, based on the *stretch–reflex*, and using the whole body in a *fluid* and integrated manner, thus manifesting a superb combination of speed and strength. To ensure that these weapons will perform consistently, and under all possible circumstances, phenomenal mental strength and speed are also needed.

Now, full appreciation should be given to the current world number one, Roger Federer. He *is* the present model of a champion (especially when combined with Nadal and Djokovic). Regarding the future tennis champion model that will be outlined in this book, this Federer–model will be taken as a basis: all his abilities, both physical and mental, both technical and tactical, even including his body height and weight. This Federer–model will just be empowered with tremendously–strong muscles and lightning–fast reflexes, giving him a 300+ km/h serve, a 240+ km/h forehand and a 200+ km/h backhand, together with a visual perception and complex reaction quick enough to anticipate and follow the bullet–like ball generated by the mentioned strokes, with Federer's concentration and anticipation of the opponent.

By combining ex–Russian sport science with today's American wealth and technology, future tennis world champions could easily be produced.

Think! Don't be constrained by anyone. Sport is a science not a religion. Learn the facts, apply the knowledge and believe in your unlimited potential and you can become a tennis champion. Producing a sport champion is a joy, satisfaction and fulfilment; not frustration and suffering. A *brain* also is needed to complete a tennis champion: a *strong & fast brain* would make *strong & fast muscles* invincible.

This myth–buster book gives modern scientific answers to all the questions that must have arisen in your head after reading the past few paragraphs. The book includes 12 chapters on various topics related to complex sports biodynamics, a strong list of references on sports science in general and tennis in particular, as well as a comprehensive index. To make the book more readable for the widest possible audience, the last Chapter on tennis has been written in a popular (non-rigorous) question & answer format.

Tijana Ivancevic, Ph.D. in Applied Mathematics and Master of Sports Biomechanics, is a co-author of 10 advanced, biomechanics–related, scientific monographs (seven of them published with Springer and three with World Scientific). She is currently working as a Senior Researcher in mathematical modelling in medicine at the University of South Australia. Previously, she developed breast–cancer classifiers based on a differential geometry of neural networks at the University of Adelaide. Tijana has also worked on various artificial/computational intelligence projects, as well as neural networks applications to sports science and biomechanics. Bojan Jovanovic is currently developing a biomechanical dynamics simulator at the University of Novi Sad, for sport games in general, handball in particular. Swetta Djukic has over thirty years of experience in competitive tennis and in 2006 was awarded as an undefeated senior tennis player at the famous Trinity Gardens Tennis Club. Milorad Djukic is an Associate Professor of Handball at the University of Novi Sad and the Chair of Technical Committee of of the handball club Vojvodina. Sasa Markovic is an Associate Professor of Handball at the University of Nis and the President of the Handball Coaches Association of Serbia.

Adelaide, October 2008

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Chapter 1
Introduction

Complex Sport Biodynamics (*CSB*, for short) is a *new kind of sport science*, the *Know–How to make sport champions. CSB* combines the essential principles of complex systems dynamics [II06b, B-Y97], biomechanics, [II05], biodynamics and sports physiology [II06a, Mar98, GM88], chaos theory [II07a], neurophysiology [II07b] and computational psychodynamics [IA07, II07c] – with a *unique goal*: the *SUPREME SPORT RESULT*.[1]

This unique goal is in *CSB* represented by the two *essential CSB tasks* (see Figure 1.1)

1. *Direct–training task*: given the set of empirically proclaimed *talents* $\longrightarrow$ develop the *champion model*
2. *Inverse–selection task*: given the *champion model* $\longrightarrow$ develop the *talent model*

In *CSB* all the champions are represented by the *champion model* for a particular discipline, and all the talents are represented by the *talent model* for the same discipline. In language of 'factor analysis' (see below), both the champion and the talent have *the same factor structure* – only it is fully developed in case of the champion and yet undeveloped in case of the talent. These general representatives are usually some heuristic combinations of *causal–system* models, *empirical–expert* models, and *statistical–factor* models.

Both essential problems are solvable by certain combination of the three basic *CSB–methods*:

1. mathematical modelling, control, and learning;
2. computer simulations and animations; and
3. mental concentration and meditation.

The direct problem is called *CSB–learning* or training process. The inverse problem is called *CSB–recognition* or selection process. Theoretically

[1] The goal of *CSB* is the *SUPREME SPORT RESULT* – the 'Olympic Gold', or the 'Grand Slam', without drugs, without anything 'unhealthy' and without any kind of 'cheating', which is all, unfortunately, so much present in a nowadays sport.

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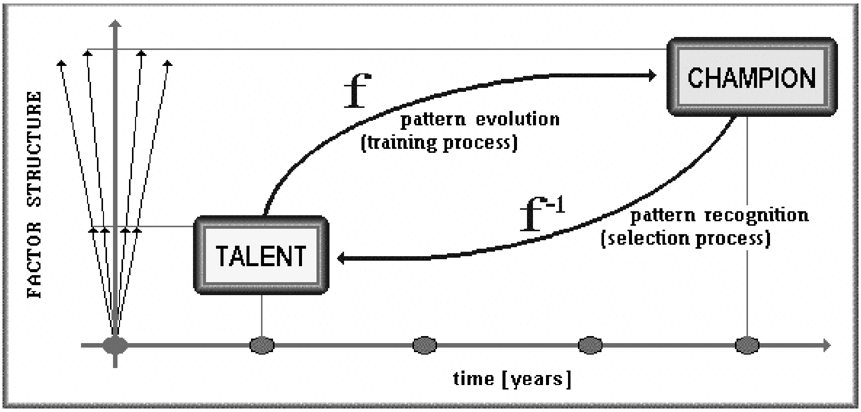

Fig. 1.1 Two main recursive $CSB-$tasks. A talent is a child with the same basic pattern–structure as a champion. Training dynamics is a pair of nonlinear transforms between patterns of talent and champion: (i) Direct, continuous training flow = evolution of the talent pattern into the champion pattern, and (ii) Inverse, discrete selection map = recognition of the champion pattern inside the talent pattern.

speaking, the inverse problem can be solved much easier, because the model of the talent (by definition) has all the components as the model of the champion, but in a non–developed form of potentials. And the direct problem is in fact how to use all CSB engines in developing the potentials of the talent, to make him the champion.

The *kernel* of the CSB–method is the mathematical structure called *mapping*, or *map* (Figure 1.2), i.e., a correspondence between two abstract sets (see [AM78]). It is said that the mapping f maps the *original set* X (domain) into the *image set* Y (range, or codomain), denoted by $f : X \to Y$, if there is a correspondence of elements $x_1, x_2, \ldots, x_n$ from X with elements $y_1, y_2, \ldots, y_m$ from Y.

For the *existence of the inverse mapping* f^{-1} (in which the set Y becomes the original one and the set X the image one) the *necessary and sufficient condition* is that the direct mapping f is *bijective*, i.e., (i) *surjective* (if each element from Y is the image of a certain element from X), and (ii) *injective* (if different elements from X are mapped into different elements from Y) simultaneously.

If X and Y represent sets of real numbers, a mapping $f : X \to Y$ is usually called a *function*. One–dimensional function is represented by a curve (Figure 1.3) in an Euclidean plane. Two–dimensional function is represented by a surface in $3D$ Euclidean space, and $N-$dimensional function is represented by a hypersurface in $N-$dimensional Euclidean space. For the *existence* of all the image elements $y_1, y_2, \ldots, y_m$ (distributed along the set Y) it is necessary that the curve (respectively surface, or hypersurface) f is *continuous*. And for the *uniqueness* of the image elements $y_1, y_2, \ldots, y_m$ it is necessary that f is *continuous and smooth*.

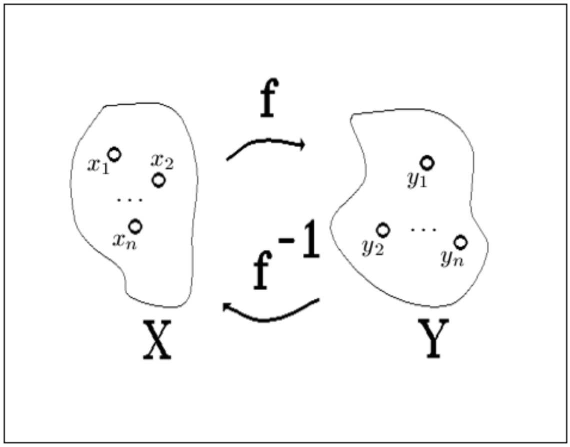

Fig. 1.2 The *kernel* of the CSB–method

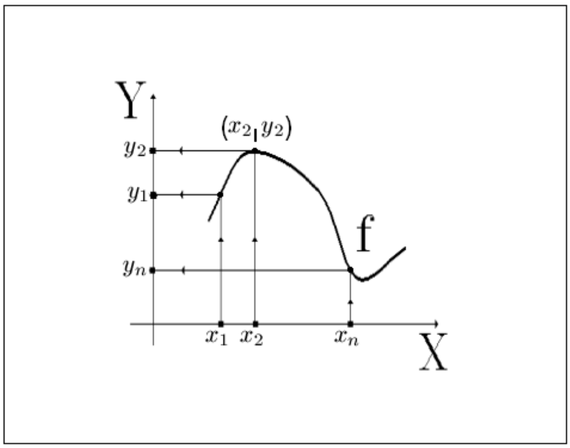

Fig. 1.3 One–dimensional function

In the system language, f is called the *feedforward* path, and f^{-1} is called the *feedback* path.

In the functional language, the $CSB-$goal is represented by the pair of mappings (f, f^{-1}), where f represents the direct problem of $CSB-$training, and f^{-1} represents the inverse problem of $CSB-$selection.

Thus, sports science is all about *training methods* (directed to make a champion) and *selection methods* (directed to finding talents). In sports science all the champions are represented by the *champion model* for a particular discipline (e.g., tennis), and all the talents are represented by the *talent model* for the same discipline (see Figure 1.1). In statistical language, both the champion and the talent have *the same factor structure* – only it is fully developed in case of the champion and yet undeveloped in case of the talent.

For example, on the current tennis circuit, Nadal, Roddick, Federer, and Djokovic had all been talents. However, so far only one of them has proved to be a real champion – Roger Federer, the man who apparently defies all tennis statistics.[2] Today, in our opinion, the highest chances to become future tennis champions have Nadal and Djokovic.

[2] For the *tennis performance criteria* we can use the 10 points of the standard tennis game statistics (in brackets are the current ranks of Roger Federer, the world number one, on October 22nd 2007, as given by *ATPtennis.com*):[3]

- *Service game:* (i) number of aces (4), (ii) 1st serve percentage (29), (iii) 1st serve points won (6), (iv) 2nd serve points won (1), (v) service games won (3), and (vi) break points saved (8).
- *Return of service:* (vii) points won returning 1st serve (4), (viii) points won returning 2nd serve (17), (ix) break points converted (36), and (x) return games won (10).

Chapter 2
CSB−Physics and Metaphysics

More then three centuries ago, more precisely in 1686, Sir Isaac Newton, one of the foundation–stones of human thought (see [AM78]), in his famous book 'Philosophiae Naturalis Principia Mathematica', stated the metaphysical and physical basis of modern sciences, including *CSB* (in spite of the influences of modern physics).

Methodology of all sciences tries to solve two main problems: *explanation* and *prediction*. The problem of explanation (or basic understanding of the structure and function of the object in consideration) has been (more or less successfully) solved by both natural and social sciences. But the problem of prediction has been (in a limited range) solved only by the so–called 'exact sciences' with developed mathematical, measurement, and computer simulation equipment.

Any form of prediction in science is based on Newton's *principle of causality*. We can even say that the human thought apparatus is based on this (meta)physical principle.

Newton's principle of causality (Figure 2.1) states:
If the initial state (in any chosen initial time) of any *CSB*–system is known (measured on the system $S-$axis), and if all the influences upon the system considered are known from the initial time on (measured on the time $t-$axis), then the future behavior of the system (its 'destiny') is completely determined.

More precisely Newton's causality principle can be formulated thus:
If the law (i.e., the balance) of forces acting upon the system is known together with its initial conditions, than the law of motion (or, generally, behavior) can be obtained exactly (by solving, either analytically or numerically, the system equations for the given initial conditions).

For the sake of mathematical formulation of the causality principle Newton invented (independently of a mathematician– philosopher G.F. Leibnitz) differential and integral calculus. The basic geometric idea of differential calculus consists of the limiting process which transforms *bilocally* (i.e., in two distinct space and/or time points) defined classic vector quantity (representing some

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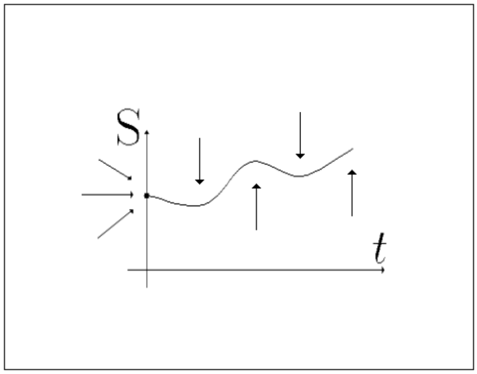

Fig. 2.1 Newton's causality principle

average force, velocity or acceleration) into *unilocally* defined tangent vector (representing instant force, velocity or acceleration). The process of obtaining the tangent vector in each point of a curve is called differentiation, while the inverse process is called integration. Differentiation of the time–dependent trajectory with respect to time gives the curve of velocity, and differentiation of the later gives the curve of acceleration. Geometric construction of the tangent vector in each point of the curve is the special case of construction of the tangent bundle on the smooth manifold (a smooth curve is a one-dimensional smooth manifold, surface is two–dimensional, and so on). The projection of the tangent bundle on the original manifold represents the process of (indefinite) integration.

Newton's crucial second law of motion (see [AM78]– [MR94]) says: the force acting on any CSB–system is proportional to the time rate of change of velocity of the system, and the proportionality constant is the measure of inertia of the system. Simplifying this statement, we have: the force acting upon the system is equal to the product of its mass and its acceleration. Formally: $F = ma = m\dot{v} = m\ddot{x}$, where overdot denotes the time derivative (i.e., tangent vector in the given point of the curve, or the time rate of change of the quantity considered), F represents the force, m – the mass, a – the acceleration, v – the velocity, and x – the position coordinate.

This equation implies some *frame of reference* with respect to which the acceleration $a = \dot{v} = \ddot{x}$ is measured. It is a fact of experience that Newton's law of motion expressed in this simple form gives results in close agreement with experience when, and only when, the coordinate axes are fixed relative to the average position of the 'fixed' stars moving with uniform linear velocity and without rotation relative to the stars. In either case the frame of reference is referred to as an inertial frame and corresponding coordinates as inertial coordinates.

Newton's causality principle can be now reformulated as: if the law of force $F = F(t)$ is known together with the initial conditions $x_0 = x(0)$ and $v_0 = v(0)$, then the solution of upper (differential) equation of motion gives the law of motion $x = x(t)$.

Now, let us say a few words about explanation, or basic understanding. In its customary meaning, the word 'to understand' means to form oneself a clear image or a diagram of an object or process. No matter how paradoxically this sounds, modern physics (predominantly quantum theory) cannot be understood in this way. One of its founders, P.A.M. Dirac, wrote in this respect [Dir67]:

"... The main object of physical science is not the provision of pictures, but is the formulation of laws governing phenomena and the applications of these laws to the discovery of new phenomena ..."

In the case of microscopic phenomena no picture can be expected to exist in the usual sense of the word 'picture', by which is meant a model functioning essentially on classical lines. One may, however, extend the meaning of the word 'picture' to include any way of looking at the fundamental laws which makes their self–consistency obvious. With this extension, one may gradually acquire a picture of microscopic phenomena by becoming familiar with the laws of modern physics. In *CSB* we are not dealing with microscopic phenomena, but the *logic of life* itself hides something similar to microscopic objects and processes.

2.1 Qualitative *CSB* and Standard Physical Theory

According to modern sports biomechanics (see [Zat98, Zat02]) as well as general biodynamics (see [II05, II06a, II06b]), a human moving subject carrying an accelerometer represents a 3D dynamical system governed by the Newton Second Law of Motion. Therefore, modern *dynamical systems theory* seems to be the most appropriate theoretical background for short-time motion data acquisition using 3–axial accelerometers. In the following text we give a 'plain–English' brief description of modern dynamical systems theory of Newtonian mechanics.

2.1.1 Poincaré's Qualitative Dynamics

2.1.2 Poincaré's Point of View: Phase–Portrait

Poincaré visualized a *dynamical system* as a *vector–field* (i.e., a field of vectors resembling those in electromagnetism) on the system's *phase–space*, in which a solution is a smooth curve tangent at each of its points to the vector based at that point. His *qualitative dynamics* is based on geometrical properties of the system's *phase–portrait*: the family of solution curves, which fill up the entire phase–space. For questions such as *stability*, it is necessary to study the entire phase–portrait, including the behavior of solutions for all values of the time parameter. Thus it was essential to consider the entire phase–space at once as a single *geometric object* [AM78, Arn89].

Doing so, Poincaré found the prevailing mathematical model for mechanics inadequate, for its underlying space was Euclidean (or, a domain of several real variables), whereas for a mechanical problem with angular variables or constraints, the phase–space might be a more general, nonlinear space, such as a generalized cylinder. Thus the global view in the qualitative dynamics led Poincaré to the notion of a *smooth manifold* (or, a *differentiable manifold*) as a mechanical phase–space.

In mechanical systems, this manifold always has a special geometric structure pertaining to the occurrence of phase variables (coordinates and momenta) in canonically conjugate pairs, called a *symplectic structure*. Thus the new mathematical model for mechanics consists of a *symplectic manifold*, together with a *Hamiltonian vector field*, or global system of first–order differential equations preserving the symplectic structure.

This model offers no natural system of coordinates. Indeed a manifold admits a coordinate system only locally, so it is most efficient to use *Cartan's intrinsic calculus* rather than conventional Newton's calculus in the analysis of this model. By suppressing unnecessary coordinates the full generality of the theory becomes evident.

Poincaré's Method: Differential Topology

The second characteristic of Poincaré's qualitative dynamics is the replacement of analytical methods by differential–topological ones in the study of the phase–portrait. For many questions, for example the stability of the solar system, one is interested finally in qualitative information about the phase–portrait. In earlier times, the only techniques available were analytical. By obtaining a complete or approximate quantitative solution, qualitative or geometric properties could be deduced. It was Poincaré's idea to proceed directly to qualitative information by qualitative, that is, geometric methods. Thus Poincaré, Birkhoff, Kolmogorov, Arnold and Moser show the existence of periodic solutions in the 3–body problem by applying differential–topological theorems to the phase–portraits in addition to analytical methods. No analytical description of these orbits has been given. In some cases the orbits have been plotted approximately by computers, but the computer cannot prove that these solutions are periodic [AM78].

Poincaré's Problem: Structural Stability

A third aspect of the qualitative dynamics is a new question that emerges in it, namely the problem of *structural stability*, the most comprehensive of many different notions of stability. This problem, first posed by Andronov–Pontriagin, asks: If a dynamical system X has a known phase portrait P, and is then perturbed to a slightly different system X' (for example, changing the coefficients in its differential equation slightly), then is the new phase portrait P' close to P in some topological sense? This problem is of obvious importance, since in practice the qualitative information obtained for P is to be applied not to X, but to some nearby system X', because the coefficients of the equation may be determined experimentally or by an approximate model and therefore approximately [AM78, Arn93].

2.1.3 Standard Description of a Physical Theory

Recall that the standard description of a physical theory, most clearly enunciated by [Duh54], consists of an *experimental domain*, a *mathematical model*, and a *conventional interpretation*. The model, being a mathematical system, embodies the logic, or axiomatization, of the theory. The interpretation is an agreement connecting the parameters and therefore the conclusions of the model and the observables in the experimental domain.

Traditionally, the philosopher–scientists judge the usefulness of a theory by the criterion of *adequacy*, that is, the verifiability of the predictions, or the quality of the agreement between the interpreted conclusions of the model and the data of the experimental domain. To this Duhem adds the criterion of *stability* [AM78]. This criterion refers to the structural stability or continuity of the predictions, or their adequacy, when the model is slightly perturbed. The general applicability of this type of criterion has been suggested by René Thom [Tho75].

This stability concerns variation of the model only, the interpretation and domain being fixed. Therefore, it concerns mainly the model, and is primarily a mathematical or logical question. Certainly all of the various notions of stability in qualitative mechanics and ordinary differential equations are special cases of this notion, including Laplace's problem of the stability of the solar system and structural stability, as well as Thom's stability of biological systems.

Chapter 3
CSB–Structure and Function

3.1 Basic Input–Output CSB–System

The essential CSB object, mapping, corresponds to unilateral logical operation *implication* (i.e., *If–Then* conditional), while the bilateral implication represents logical *equivalence* (i.e., biconditional, or bilateral equality). By introducing the *time factor* into the logical implication we obtain the Newtonian causal (i.e., *cause and effect*) process in which the effect necessarily follows the cause with some time delay. Behavior of all CSB–systems (with the exception of the highest level processes of learning) belongs to the category of causal processes.

The essential structure of all CSB–systems (see Figure 3.1) consists of the pair of mappings (f, f^{-1}) between the set X of *input signals* (stimuli, or excitations) and the set Y of *output signals* (responses, or reactions). Both the direct mapping f, and the inverse mapping f^{-1} must be continuous and smooth. The behavior of such a kernel–system represents a specific *flow of matter, energy and information*, taking place in space and time.

The beginning of the system approach in life–sciences is usually connected with the name of Canadian biologist L. von Bertalanffy [Ber73], who introduced the term 'open system' in 1932. This open system continually communicates (i.e., interchanges matter, energy and information) with its surroundings by means of its metabolism i.e., the totality of processes of anabolism (input assembling process) and catabolism (output disassembling process). In the same year Walter Cannon considered 'the wisdom of the body' as the ability of the organism to maintain the stability of its internal midst, while Claude Bernard (see [Ber73]) introduced the term homeostasis to signify the ability of the living organisms for automatic self–stabilization of its internal midst in spite of various perturbations of their surroundings. From this time, the homeostasis has been mainly concerned with processes of regulation associated with physical movement, energy flow and material concentrations in the living systems. The term 'feedback' was placed by Norbert Wiener [Wie48] in 1948 in the foundation of *cybernetics*, the general science of control and

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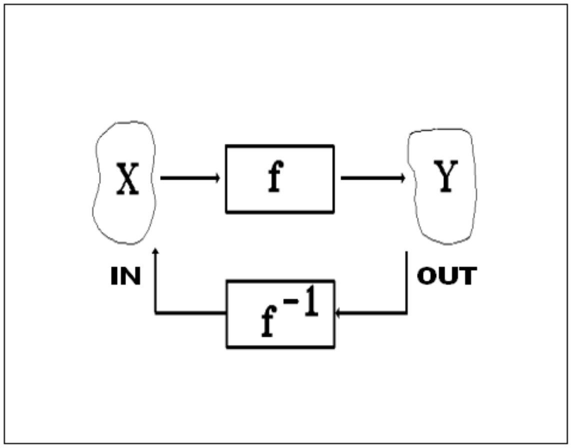

Fig. 3.1 Basic *CSB* input–output system

communication processes in biological and technical systems, based on the studies of plants or controlled subsystems (for example: biophysical processes, living beings, groups of people, machines, work organizations, sport teams or sport training process) subject to external disturbances, acquisition of data on processes inside the plants and generation of control actions ensuring an optimal state of the plants. Control theory grew from regulation theory. Automatic regulation is the maintaining of a specified quantity characterizing the process at a given level or variation of this quantity in accordance with a specified law by measuring the plant state or disturbances affecting it and by adjusting the controller of the plant. Control is a much more comprehensive concept. It is usually understood as automatic implementation of a group of actions selected from among a set of feasible actions on the basis of available data and intended to maintain or improve the functioning of the plant in keeping with the goal of control (for example, our fundamental *CSB* goal). A feedback control system is a system which tends to maintain a prescribed relationship of one system variable to another by comparing functions of these variables and using the difference as a means of control.

Now, let us define some elementary control system terms (based on [Wie48, Ber73]).

A (bio)system is an arrangement, set, or collection of (bio)physical components connected or related in such a manner as to form and/or act as an entire unit. The input is the stimulus or excitation applied to a system from an external energy source, usually in order to produce a specified response from the system. The output is the actual response obtained from a system. The control action is the quantity responsible for activating the system to produce the output. An open–loop system is one in which the control action is independent of the output. A closed–loop system is one in which the control action is somehow dependent on the output. Feedback is that property of closed–loop system (distinguishing them from open–loop systems) which permits the output to be compared with the input to the system so that the appropriate control action may be formed as some function of the output and input. More generally, feedback is said to exist in a system when a closed sequence of cause–and–effect relations (i.e., a causal chain) exists between system variables.

3.2 Example of a 'Pure $CSB-$System': Human Skeletal Muscle

The basic example of a CSB–system with single input and single output is a *skeletal muscle control system* [Wil56] (Figure 3.2. The direct mapping f here is represented by a causal electro–mechanical chain with a chemical transmission link.

A stimulus S in the form of a 'train' of nerve impulses of bio–electric nature (neural action potential represents the phenomenon of the flow of depolarization on the coaxial axon membrane, exactly described by the Nobel–awarded equations of A.L. Hodgkin and A.F. Huxley) [HH52] arrives along the motor neuron (axon) membrane to the neuro–muscular synapse. The effect of synaptic excitation is the release of a transmitter–mediator substance (most frequently acetyl–choline, rarely noradrenalin or serotonin) which stimulates the muscle fiber (belonging to the same motor unit). Muscular action potential (the muscular analog of the neural action potential) arises, causing the chain–effect in the muscle fiber:

1. Diffusion of the Ca^{++} ions towards the midst of the fiber where the contractile mechanism is located;

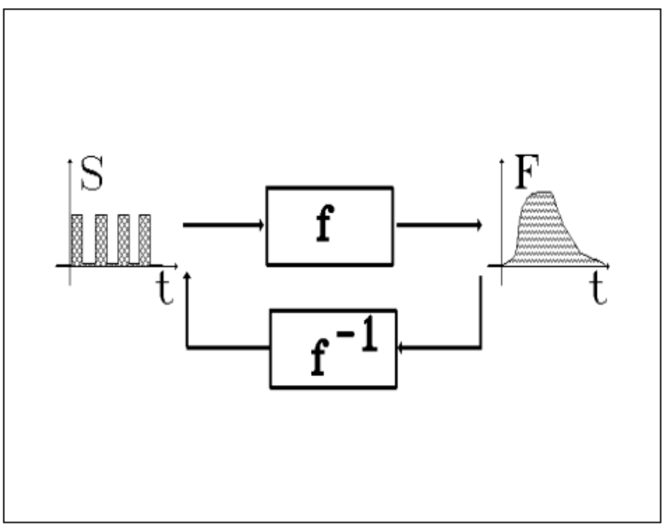

Fig. 3.2 The skeletal muscle input–output control system

2. Disassembling of ATP into ADP and free phosphor as a fuel for muscular contraction;
3. The contraction (shortening) of the fiber by the mechanism of the 'sliding filaments' of actin and myosin (discovered and mathematically formulated by Sir Andrew Huxley [Hux57]).

The final response of the skeletal muscle to the neural efferent electric excitation (either voluntary or external) is the genesis of muscular tension–force F, the phenomenon of pure mechanical nature.

Both electric stimulus S and mechanical response F are time dependent quantities, and thus the behavior of the system represents a causal process. The intensity of the muscle tension–force F depends on the intensity of the excitation S (as neural fibers are activated by the 'all or nothing' principle, the intensity of the stimulus depends only on the frequency of the stimulus, i.e., density of the impulse–bars, and not on its amplitude which is either a positive constant or zero), and the physico–chemical characteristics of the whole system f (cross–section area of the neural fiber determining the amplitude of the excitation S, capacity of the synapse determining the amount and flow of the transmitter, and several characteristics of the muscular fiber: its cross–section, amount and flow of Ca^{++} ions, amount of ATP, as well as the quantity and quality of the sliding–filaments contractile mechanism.

Generally speaking, all these causal components of muscle force can be trained both physically, using muscular work in anaerobic or in aerobic conditions, and mentally, by developing concentration. Depending on the duration and frequency of the excitation, possible muscular responses are in the form of twitch, tetanus or incomplete tetanus.

As a complete control system [Hat77a], the skeletal muscle has also the inverse mapping, i. e., feedback mechanism f^{-1}. As mechanical force is a regulated quantity, nature gave the muscle a special *motor servo* sub–system consisting of two 'autogenetic' reflex feedback mechanisms (Figure 3.3) for its control [Hou79]:

1. stretch (or myotatic) reflex increasing the output force (thus having the function of a positive feedback), and
2. Golgi's tendon reflex decreasing the output force (thus having the function of a negative–homeostatic feedback).

On the higher levels there is a hierarchy of reflex and voluntary control systems (kinaesthesia, cerebellar synergy and the highest cortical control) performing more complex motor control and enabling motor learning.

Now, let us explain the skeletal muscle control system in some more detail [Iva91]. An EFS-response f of a skeletal muscle (the muscle system $M-$response to an efferent functional stimulation from the CNS system N) can be stated in a form of a time–dependent mapping $f_t : N_t \to M_t$, where: t denotes stimulation time, while N and M correspond to the linear (vector) spaces of neural and muscular systems included. The

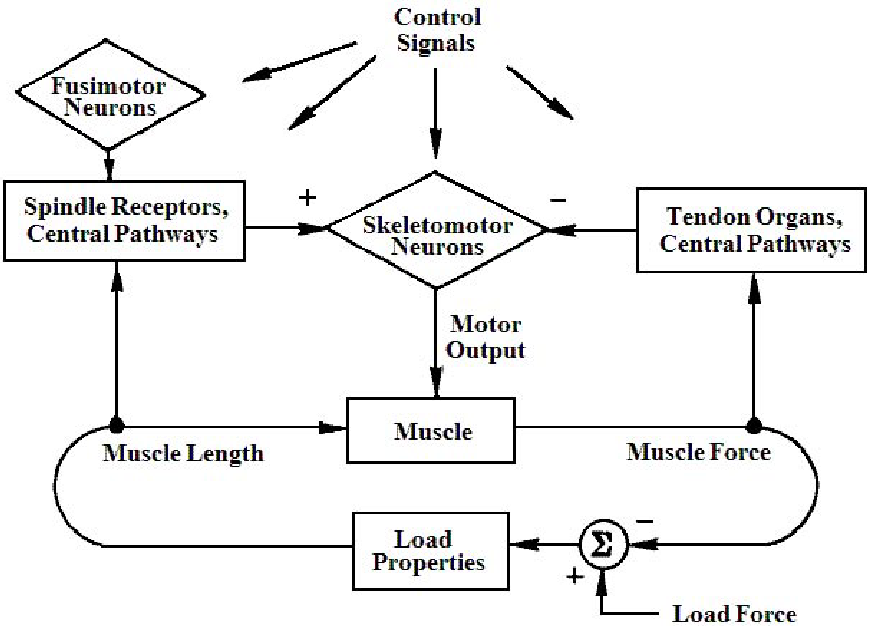

Fig. 3.3 The skeletal muscle 'motor servo'

mapping f can be considered as an effect of a fifth–order *transmission cascade* $(f_1 \to f_2 \to f_3 \to f_4 \to f_5)$:

1. Neural action potential,
2. Synaptic potential,
3. Muscular action potential,
4. Excitation-contraction coupling, and
5. Muscle tension generating.

All of these transmission signals $(f_1 \to f_2 \to f_3 \to f_4 \to f_5)$ can be considered as being some kind of diffusion processes, forming the fifth–order transmission flux cascade. For all included motor units in the particular muscle contraction, it can be physically described by the fifth order recurrent distributed parameter diffusion system, but for the sake of simplicity, it can be electro-physiologically represented by the lumped parameter RC–electric network.

The single muscle behavior in the lumped approximation form is given by the recurrent sum of its transient and weighting exponential terms.

Now, the feedback control, i.e. the inverse mapping f^{-1} of the skeletal muscle EFS–response mapping f is defined by muscular 'autogenetic reflexes', forming the muscular motor servo (Figure 3.3).

It is well known that EFS–responded contraction of a muscle (behavior of the system M) is reflexly excited (positive reflex feedback $+f^{-1}$) by responses of its spindle receptors to stretch and is reflexly inhibited (negative reflex feedback $-f^{-1}$) by responses of its Golgi tendon organs to contraction. Stretch and unloading reflexes are mediated by combined actions of several autogenetic neural pathways [Hou79].

'Autogenetic' means that the stimulus excites receptors located in the same muscle that is the target of the reflex response. The most important of these muscle receptors are the primary and secondary endings in muscle–spindles, sensitive to length change – positive length feedback $+f^{-1}$, and the Golgi tendon organs, sensitive to contractile force – negative force feedback $-f^{-1}$.

The gain G of length feedback $+f^{-1}$ can be expressed as the positional stiffness (the ratio $S = dF/dx$ of the EFS–responded force F change to the length x change) of the muscle system M. The greater the stiffness S, the less will the muscle be disturbed by a change in load and the more reliable will be the performance of the system M in executing controlled changes in length $+f^{-1}$. Positional stiffness S has been measured in soleus muscles as the slope of the stretch reflex curve, i.e., the increase in muscular force F which results from a unit increase in muscle length x. One component of this curve can be related to the well–known length–tension ($F - x$) characteristic (see below) of skeletal muscle, while the second one is probably unknown amount of inhibition from tendon organs (negative force feedback $+f^{-1}$), which would tend to decrease the extra force produced by stretch, and, hence, decrease the stiffness S as measured from a stretch reflex curve.

The autogenetic circuits $+f^{-1}$ and $-f^{-1}$ appear to function as servoregulatory loops that convey continuously graded amounts of excitation and inhibition to the large (alpha) skeletomotor neurons. Small (gamma) fusimotor neurons innervate the contractile poles of muscle spindles and function to modulate spindle–receptor discharge [Hou79].

The skeletal muscle is usually mechanically represented as a simple four–component rheological system (Figure 3.4), where [Iva91]:

- (CC) denotes contractile component = muscle force generator, which responds to the EFS with a force which intensity depends on the excitation frequency;
- (VC) denotes viscous component = linear damper, resisting to the length–change of a muscle with a force proportional to the velocity of this change; together with CC it leads to basic muscular relation force–velocity (see below), which means that the total force of contraction is:
  (a) equal to the pure CC–force in isometric contraction,
  (b) equal to the difference (CC−VC)–force in concentric contraction,
  and (c) equal to the sum (CC+VC)–force in eccentric contraction;
- (PEC) denotes parallel elastic component = linear spring corresponding to inner muscular elasticity; it is stretched during external muscular stretching and proportionally resists to the stretching; together with CC it leads to basic muscular force–length relation (see below);
- (SEC) denotes serial elastic component = linear spring corresponding to tendon elasticity; during muscular work it is stretched and shortened

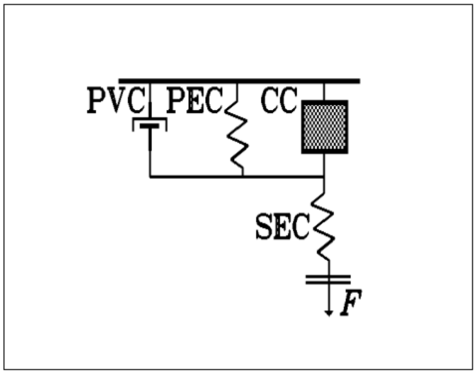

Fig. 3.4 Rheological model of the skeletal muscle

according to the force acting on it and it is capable to utilize energy of its elastic deformation more than PEC.

The force generated by muscular contraction is a function of time, muscle length and velocity of contraction. These are three essential mechanical characteristics of muscular contraction: force–time $(F - t)$, force–length $(F - x)$, and force–velocity $(F - v)$, having direct applications to the training of muscular strength [Iva91].

By the use of presented multicomponent rheological conceptual scheme, Nobel laureate A.V. Hill formulated in 1938 the first basic relation of muscular mechanics [Hil38], the so–called force–velocity $(F - v)$ curve (Figure 3.5).

The hyperbolic force–velocity relation, which is valid on all possible levels (from a single muscle fiber level to the global human musculo–skeletal

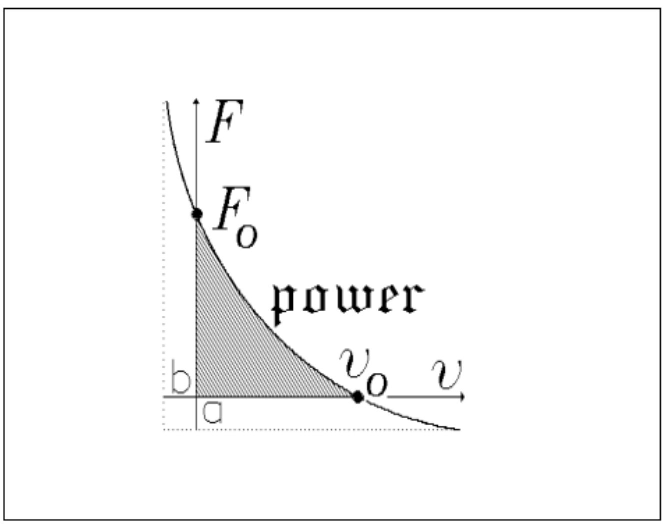

Fig. 3.5 Force–velocity curve of the skeletal muscle

apparatus) shows that a muscle can generate its maximum force in isometric conditions (without motion), and that it can contract with its maximum velocity (i.e., speed) in isotonic conditions (without external load). The area under the force–velocity hyperbole corresponds to the mechanical power (dimensionally force x velocity $[Nms^{-1}]$, exactly defined as $\int_{v_1}^{v_2} F dv$), of a single muscle contraction. Therefore, it tells us that we can have either force (strength) or velocity (speed) in our muscular contraction, but not both. If we want both of them, and it is so–called power, we must shift the curve up–and–right, which corresponds to the development of the muscle power. The asymptotes a and b of the curve, having respectively dimensions of force $[N]$ and velocity $[m/s]$, correspond to the amount of energy dissipated during the contraction and velocity of chemo-mechanical processes involved. Hill's mechanical as well as thermodynamic equation has the form: $(F+a)(v+b) = (Fo+a)b$. This is essential muscular characteristic for start acceleration in sprint running (see the next paragraph), and can be indirectly measured by varying loadings and performing maximal speed movements.

The parabolic force–length $(F-x)$ curve was discovered by Nobel laureate A.F. Huxley in 1957 as a fundamental microscopic relation of a contraction of basic muscular unit - sarcomere ('sliding filament theory' [Hux57]). It tells us that the muscle is strongest in the middle of its length (when it is half–shortened). Physically it represents the work (dimensionally force x length $[Nm]$, exactly defined as $\int_{x_1}^{x_2} F dx$), performed by a single muscle contraction, and has its physiological application in the motor servo.

The third basic relation of muscular mechanics is the exponential force–time $(F-t)$ curve and represents in fact the basic response of the skeletal muscle as a control system [Wil56, Hat77a] (see Figure 3.2). If we restrict ourselves to the muscle twitch, this curve has an impulse form (Figure 3.6) determined by four parameters:

1. Maximum muscle force F_0 (in the previous paragraph we saw that it is possible only in isometric conditions),
2. Twitch duration (which corresponds to the local muscular endurance in isometric conditions),
3. Time constant of the contraction, and
4. Time constant of the muscle relaxation.

The area under the force–time curve represents the total impulse (dimensionally force x time $[Ns]$, exactly defined as $\int_{t_1}^{t_2} F dt$). This is essential muscular ability for sprint running (after start acceleration till the end of the race); it can be directly measured on force–plates (tensiometric platforms), and obtained pattern of the total impulse represents a 'personal card' for each athlete, which should be used both for the selection and for the training process [Iva91].

It is obvious that both the power and the impulse are based on the maximal isometric muscle force. Therefore, this is the first muscle ability to be trained (in the preparation period of sports–training) and all other muscle abilities depend (more or less) on this basic capacity (see [Zat95]).

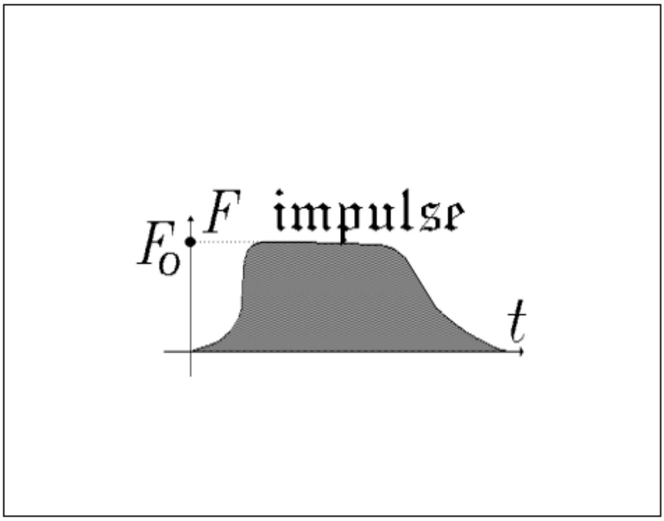

Fig. 3.6 Force–time curve of the skeletal muscle

The impulse itself depends on the time constants of contraction and relaxation. Therefore, it is applicable in all sports activities in which the generation of actual muscle force is timely–limited. Main examples are the jumps from a running start, where the running speed determines the take–off duration (in the high–jump the take–off lasts about $0.16s$, in the long–jump and the triple–jump it lasts about $0.12s$ and in gymnastic jumps it lasts about $0.10s$). Therefore, the total time constant of the muscular contractions included (for leg extensor muscles) is the most important capacity.

On the other hand, in the jumps from a standing start (the high–jump and the long–jump) as well as in the throws (the javelin, hammer, discus or shot), there is not time–limit, the motion can last arbitrarily long but the aim is to achieve the maximum final velocity of the body or the object in the jump or throwing. Therefore, the dominant muscular capacity herein is the power.

The difference between the two fundamental muscular capacities, the impulse and the power, is important to keep in mind both in testing the take–off abilities (the abilities for the take–off from a standing start have law correlation with the abilities for the take–off from a running start, and even this low connection is caused only by the common maximum isometric muscle force), and in the conception of the 'striking method' (like jumping up–and–down) of the strength training (performed, of course, after some reasonable level of maximum force has been achieved by weight–lifting): if the impulse is being trained the strike–duration must be limited (i.e., as short as possible), while if the power is being trained, no time limitation is imposed on the movement [Zat95].

As the third step in the specific strength development (performed, of course, after the weight–lifting and the striking method are maximally utilized), we propose the training method of 'jump in the depth' (for leg extensor muscles) and 'drop in the push–up' (for arm extensor muscles). The training of 'jump in the depth' should be performed once a week, jumping from the 3–4 meters high platform onto the plastic (not elastic) soft ground in a series of about 30 repetitions with enough (total) recoveries between jumps. This method develops all three basic muscle contractile capacities: maximum force, impulse and power – in the 'eccentric conditions' (on the left–side of the zero motion on the force–velocity curve, thus having the greater force–potential than the isometric maximum). If the amortization is limited (either by time or by joint–angle) the impulse is being predominantly developed; if there is not any restriction on the amortization, the power is being developed. But for this training method both muscles and joints should be previously adequately strengthened, and for this some proposed 'muscle force factor' (the weight which can be lifted in some exercise per kilogram of body weight) must be achieved (for example, in a 'back squat' factor 1 is a minimum for most athletes, factor 2 is desired for jumpers, while weight–lifters should have factor 3). Also, some time (at least a month) spent in the striking method is desirable (see [Zat95]).

3.3 Example of an 'Applied CSB–System': Sprint Velocity Curve

The first step in any serious sprint–training program is the construction of an apparatus called 'the velocimeter', a kind of speedometer designed for sprint training. The velocimeter permits a continuous registration of the running speed of an athlete over distances of up to $100m$ (see Figure 3.7).

When interpreting data from the velocimeter the coach must realize that the $100m$–dash consists of three phases. In the first phase, the athlete accelerates out of the blocks. There is a very steep increase of running speed during the first 10 meters. In the second phase, the athlete builds up his maximum speed. In the third phase, the athlete tries to maintain that speed until the finish. On the basis of the velocimeter data the running speed in each phase of the run can be analyzed.

The following parameters are selected to quantify the performance in the three phases:

1. The mean acceleration during the first ten meters – this is the speed on the $10m$ point divided by the $10m$ running time;
2. The maximum running speed v_0;
3. The percentage of speed loss during the last $40m$ of the $100m$–dash sprint (see Figure 3.7).

Both scientific research and field expertise indicate that the performance related factors differ from phase to phase [Zat95]. This means, for example, that

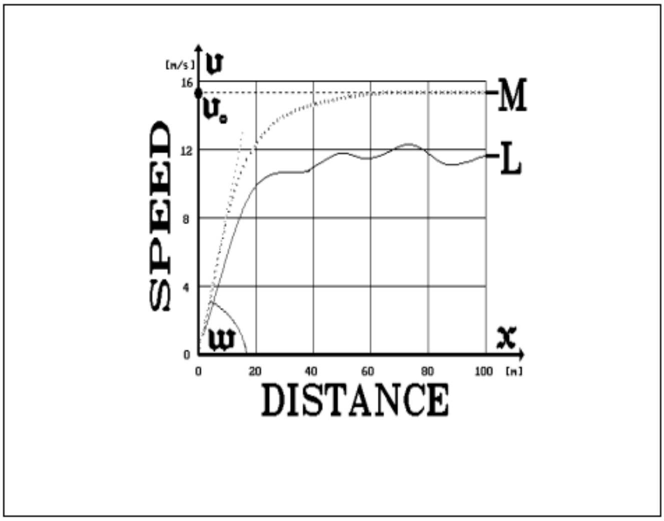

Fig. 3.7 Sprint velocity curve for $100m-$dash. Bottom line **L**: Carl Lewis (9.86s). Top line **M**: theoretical–model curve.

a good performance in the first phase of the run does not necessarily correspond with a high maximum running speed or a good speed endurance, and vice versa.

According to the real exponential velocity curve, the ideal mathematical model can be easily obtained as a solution–curve, $v(x) = v_0(1 - e - cx)$, of a simple, first order linear differential equation, $M\frac{dv}{dx} + Bv = E$, (i.e., the first order inertial element, like a RC electric circuit, with input E and output v), with a real initial condition: $v(0) = 0$.

Here v denotes velocity, x – distance, v_0 – maximal velocity or the maximum running speed (horizontal asymptote of the curve), c – velocity gradient (quantity related to acceleration), M – runner's body mass, B – total outer and inner viscous friction (inner–muscular viscosity is inverse of the muscular force–velocity characteristic), $E = E(x)$ – energy input to the running system (according to the simple dimensional analysis the resultant dragging force $F = F(x)$ generated along the distance is equal $F = ET^3L^{-1}$, therefore $F(x)$ is approximately equal to $10E(x)$). The maximal velocity (speed) of running is equal to $v_0 = E/B$, the velocity gradient is equal to $c = B/M$, and the slope of the curve is given by $\tan(w) = E/M$.

The following interpretations could be given to the model:

1. Any velocity curve of any real runner (including the world–record–holder) represents a particular fluctuation about the average–model curve.
2. Provided the runner is stable enough to prevent the speed fluctuations, the result depends exclusively on two parameters: maximal velocity v_0 and velocity gradient c.

3. As $v_0 = E/B$, $c = B/M$, and $tan(w) = E/M$, we can say that the result is better as the resultant muscle force $F = 10E(x)$ generated along the whole distance is greater, while the runner's body mass M and viscous friction B are smaller.
4. Viscous friction B is smaller if the ground contact in each stride is shorter, which (together with the greater force) demands greater muscular explosiveness; and force–velocity curve (of all included muscles) is shifted towards the up–and–right corner, which is used mainly in the start acceleration.

By least–squares–fitting of any real running speed-distance data, according to the solution-function: $v(x) = v_0(1 - e - cx)$, all mentioned parameters can be obtained for a particular runner. For example (see Figure 3.7), by fitting the speed–distance data of the famous Olympic champion Carl Lewis [Iva93], the former world–record–holder with $9.86s$ on $100m$–dash, the values of the three main parameters are: $v_0 = 11.9481ms^{-1}$, $c = 0.0696$ and $tan(w) = 0.832$ (or $w = 39.78$). The two derived parameters depend on the mass M: for $M = 80kg$, $F = 665.62N$ and $B = 5.57$; for $M = 70kg$, $F = 582.39N$ and $B = 4.87$; for $M = 60kg$, $F = 499.23N$ and $B = 4.18$. Here, it must be stressed that $v_0 = 11.9481ms^{-1}$ is not the maximum point of velocity curve (the maximum point is about $12.7ms^{-1}$), but rather the average stabilized speed.

In this way the sprinters can be compared and their development can be observed and controlled, because such a set of speed–distance parameters represent their 'kinematic personal cards'. Their 'dynamic personal cards' can be obtained in the similar manner by modelling and fitting their ground–reaction force–time characteristics (i.e., impulses), measured by the appropriate force–plates fixed along the track.

Chapter 4
CSB−Biomechanics: Structure and Function of Human Motion

4.1 History

Several attempts have been made in both biomechanical and robotic literature to create a general mechanical model/theory of human motion [Iva91]. The earliest systematic study of human and animal motion principles appears to be due to Muybridge [Muy99], who in 1877 invented a type of motion–picture camera which was used to obtain the first photographic record of the successive phases of a number of quadruped gaits. An overall hierarchical–control philosophy of the dynamics of human motion was proposed in 1947 by Bernstein [Ber47], the father of Russian school of biomechanics. He applied the Pavlov–Anohin physiological theory of *functional systems* to the analysis of human motion and formulated its basic building blocks in terms of ordinary linear differential equations.

Real dynamic analysis of human (loco)motion was first proposed by Chow and Jacobson in 1970 *via* optimal programming [CJ70]. They tried, using a criterion of minimum power, to achieve driving torques in the hip and knee and the appropriate 'optimal' trajectories of corresponding joints. To solve this optimisation problem they made various mechanical and mathematical simplifications. They employed a seven-segment model of the human body, restricted to the sagittal–plane gait in the two basic configurations. This was the first attempt to solve the inverse dynamic problem by the use of Lagrange's equations.

Bernstein's followers, Moreinis and Gritzenko, developed and investigated the mathematical and physical model of the human locomotor system in 1974 [MG74] in order to learn about the dynamic characteristics both of a healthy man and of one using prosthesis. They formulated model with nine degrees of freedom by the use of Lagrange's equations and solved the inverse dynamic problem: the joint kinematics are given experimentally and the objective is to find the forcing functions (joint torques and reaction forces) causing the desired locomotor task.

Vukobratovic, the father of Yugoslav school of robotics and biomechanics, finally solved, with his collaborators, the inverse problem of anthropomorphic

T.T. Ivancevic et al.: Complex Sports Biodynamics, COSMOS 2, pp. 23–53.
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locomotion. In a series of papers and monographs published between 1973 and 1980 they developed computer methods for automatic–setting mathematical models of anthropomorphic mechanisms [Vuk82]. This was based on d'Alembert's and kinetostatic principles and Newton–Euler, Lagrange and Gibbs–Appell equations. The programs were written in Fortran and later translated to C–language, adapted to all computer–platforms existing at that period. Vukobratovic also formulated the so–called 'half–inverse' method of gait synthesis, in which the dynamics of the motion of the legs was inversely solved, and simultaneously the compensatory dynamics of the upper body (with two degrees of freedom) was solved directly. Sophisticated methods were also developed for hierarchical adaptive control design including open and closed kinematic chains plus actuators in the form of DC drives, stability and sensitivity analysis.

After Bernstein, a few authors, both in robotics and biomechanics literature, have considered the forward dynamic problem: given the forcing functions (joint torques and reaction forces), the objective is to determine the resulting joint kinematics in various simple tasks of human motion. The formulation of highly nonlinear systems of differential equations of motion was a difficult task, especially with complicated kinematic chains with ten or more degrees of freedom. The solution of these equations for fixed or variable initial and/or boundary conditions was more difficult and the problem of control more difficult again. The best–known forward dynamical models in the biomechanical literature are those presented by Huston and Passerello [HP71], Aleshinsky and Zatsiorsky [AZ78] and Hemami and Wyman [HW79], all of which include muscular input torques in the form of Hill's model [Hil38] of muscular contraction.

Much of this research is summarised in the mid and late 1970s in the work of Hatze, the founder of the Austrian biomechanics school. Hatze attempted to develop

1. the general myocybernetic control model of skeletal muscle [Hat77a], and
2. a complete set of control equations for the human musculo–skeletal system.

His muscle–control model involves excitation dynamics of neuromuscular inputs (motor units) and contraction dynamics based on Huxley's sliding–filament theory of muscle contraction [Hux57]. On the basis of his muscle–control model, Hatze developed his 'hominoid dynamics' by the use of Lagrange's equations of the first and the second kind. He described the motion of the 17 major (interacting) segments of the human body, subjected to a variety of external and internal forces and torques, including the internal torques produced by the 46 major muscle groups of the body. His model had 42 degrees of freedom for three–dimensional motion and 21 for planar motion. The model was simulated on the supercomputer CDC Cyber 174 and successfully predicted the take–off phase of the long jump.

In describing and predicting human motion dynamics, Hatze's humanoid model was much more powerful than all other published models in both the

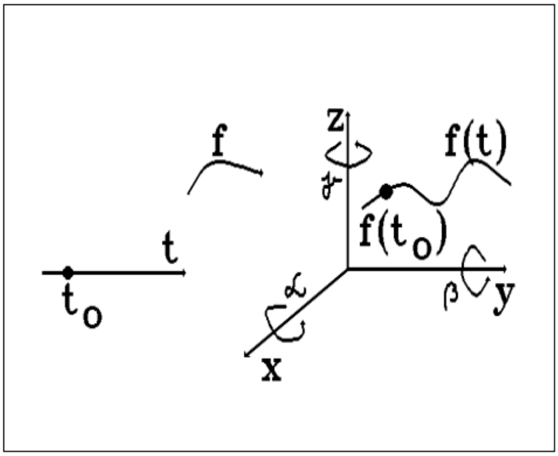

Fig. 4.1 The motion, as defined geometrically

robotic and the biomechanical literature, before and long after him. In one sense he solved the problem of the dynamics of human motion. However his muscular and humanoid models were both very complicated and included a large number of nonlinear differential equations. For example, for the sagittal–plane simulation of the long–jump take–off, he used a humanoid state–space vector of dimension 42 and a muscular state–space vector of dimension 230. His papers have rarely been cited in the literature and subsequent authors, both in robotics and in biomechanics, have made fresh attempts on the problems that Hatze had already solved in a more general way.

4.2 Group Dynamics

Now, mechanical definition of motion (see [AM78]– [MR94]) represents the exact foundation for any more complex form of behavior of CSB–systems. In a classical Euclidean frame of geometry and Newtonian frame of mechanics the motion is defined as a *homeomorphic immersion* (i.e., continually–deformable sinking) mapping $f : t \to E_3$ of the absolute time axis t into the absolute $3-$dimensional Euclidean space E_3 with an orthogonal Cartesian coordinate basis (Figure 4.1). The image of the mapping in E_3 represents the trajectory of motion and any time–point t_0 on the time axis t has its image $f(t_0)$ on this trajectory.

In the language of modern mathematics and physics we can say that in the linear Euclidean space, i.e., the space of real numbers, $E_3 = \mathbb{R}^3$, acts a *six–parameter Euclidean group of motions*, represented by the product of the group of translations $T(3)$ (along Cartesian coordinate axes) and the group of rotations $R(3)$ (around coordinate axes).

The term *group* means that geometric transformations (translations and rotations) are combined additively; there is a forward and an inverse transformation, and there is also a center, or unit transformation; these are characteristics of an *algebraic group*. Also, both translations and rotations can be (additively) combined in continuous and smooth manner, so they form continuous and smooth groups, or the so–called *Lie groups*. One additional characteristic of all Lie groups is that although they act on the space of real numbers – our absolute Euclidean space, they are analytically represented by complex numbers.

For example, the transformations of the three–dimensional rotation group $R(3)$ are products of three matrices of the form

$$R(\theta) = \begin{pmatrix} cos\theta \\ sin\theta\ 0 \\ - \\ sin\theta \\ cos\theta\ 0 \\ 0\quad 0\ 1 \end{pmatrix}.$$

Any of these can be diagonalized to give

$$R(\theta) = \begin{pmatrix} exp(i\theta)\quad 0\ 0 \\ 0 \\ exp(-i\theta)\ 0 \\ 0\qquad 0\ 1 \end{pmatrix},$$

with complex entries (here θ denotes one of the angles α, β, γ from Figure 4.1, and $i = \sqrt{-1}$ is the imaginary unit).

In a similar way, the transformations of the three–dimensional translation group $T(3)$ are products of three matrices with complex entries. Both translations and rotations (as well as any other group of geometrical transformations) need complex numbers.

The group of translations $T(3)$ *acts on* our absolute Euclidean space by means of three Newton's equations of motion along the Cartesian orthogonal coordinate axes of inertial frame, i.e.,

$$T(3) : \{F_x = m\ddot{x},\ F_y = m\ddot{y},\ F_z = m\ddot{z}\}$$

where F_x denotes the force along the x–axis, m – the mass of the body (concentrated in its center of mass), $\ddot{x}$ – the acceleration along the x–axis (and similarly for other two coordinates).

The group of rotations $R(3) \equiv SO(3)$ acts on our absolute Euclidean space by means of three angular equations (completely analogous to Newton's ones) around Cartesian coordinate axes of inertial frame, i.e.,

$$R(3) : \{T_x = I_x\ddot{\alpha},\ T_y = I_y\ddot{\beta},\ T_z = I_z\ddot{\gamma}\}$$

where T_x denotes the torque around the $x-$axis, I_x – the moment of inertia around the $x-$axis, and $\ddot{\alpha}$ – the angular acceleration around the $x-$axis (and similarly for other two angular coordinates).

The basic example for the group of translations is a single muscular contraction. The lumped–parameter muscle model (Figure 3.4) is used, while the motion of the external load is performed along a vertical translation coordinate.

The basic example for the group of rotations is the basic $R(2) \equiv SO(2)$ biomechanical unit: *uniaxial joint* (like knee, or elbow) with an antagonistic muscle pair as a motion actuator and regulator (Figure 4.2). Two translatory muscular mechanisms acting on the opposite sides of the free body–segment generate an actuating torque $\mathbf{T}$, which moves the free segment with respect to the fixed one. The motion is causal and performed according to the given equation and initial conditions (initial angle and angular velocity). The muscles are mutually controlled. Besides, all previously mentioned reflex mechanisms, as well as central control mechanisms are included in the basic biomechanical unit.

Briefly speaking, general motion patterns start from the Brodmann field no. 6 (premotor cortex) and descend through the extrapyramidal neural tract; specific motion patterns start from the Brodmann field no. 4 (motor cortex – gyrus precentralis) and descend through the pyramidal neural tract. It is believed that the motor learning firstly involves the field no. 4, and with the advance in the motor skill the starting impulse is transferred into the field no. 6. The pyramidal tract leads directly to the motor neurons in the front horns of medulla spinalis, innervating (across various motor units) the corresponding muscles. The extrapyramidal tract does not lead directly to the motor neurons, but across nuclei cerebri (having the roles of relay stations), and one part (across pons) leads to the cerebellum. At the beginning of the motor learning this system activates the so–called 'cortical feedback'.

4.3 Hamiltonian Biomechanics

The 'Newtonian universe' consists of particles moving around in a space which is subject to the laws of Euclid's geometry. The forces acting upon them determine the accelerations of these particles. With a specific law of force the Newtonian dynamical scheme translates to a precise and determinate system of dynamical equations. If the positions, velocities, and masses of the various particles are specified at one (so–called initial) time then their positions and velocities are mathematically determined for all later times, in accordance with the causality principle. The Newtonian scheme works very well when dealing with simple systems (mainly linear, with one degree of freedom).

However, when dealing with complex, nonlinear, coupled, many–degree–of–freedom systems, Newtonian scheme becomes insufficient.

After Newton, in the framework of classical mechanics, famous mathematical physicists D'Alembert, Euler, Lagrange and Hamilton formulated more powerful dynamical schemes. The last and the most sophisticated of them, so–called Hamiltonian scheme summarizes much of this work and represents (combined with the group theory) the most powerful framework for modern physics (see [AM78]– [MR94]). Accordingly, in its extended, dissipative and muscle–driven form, we consider the Hamiltonian scheme as a most suitable dynamical framework for biomechanics.

Within the Hamiltonian dynamics we must select the momenta of the particles rather than the velocities. (The momentum p of a particle is just its velocity multiplied by its mass, or otherwise, it is equal to the force acting on some time.) In the Hamiltonian formulation, we have two sets of first–order equations, instead of one set of Newtonian (or Lagrangian) second–order equations. One of these tells us how the momenta p of the various particle changes with time, and the other tells us how the positions q change with time.

Roughly speaking, the first set of Hamilton's equations states Newton's crucial second law of motion (rate of change of momentum = force), while the second set of equations is telling us what the momenta actually are, in terms of the velocities (in effect, rate of change of position = momentum/mass). The

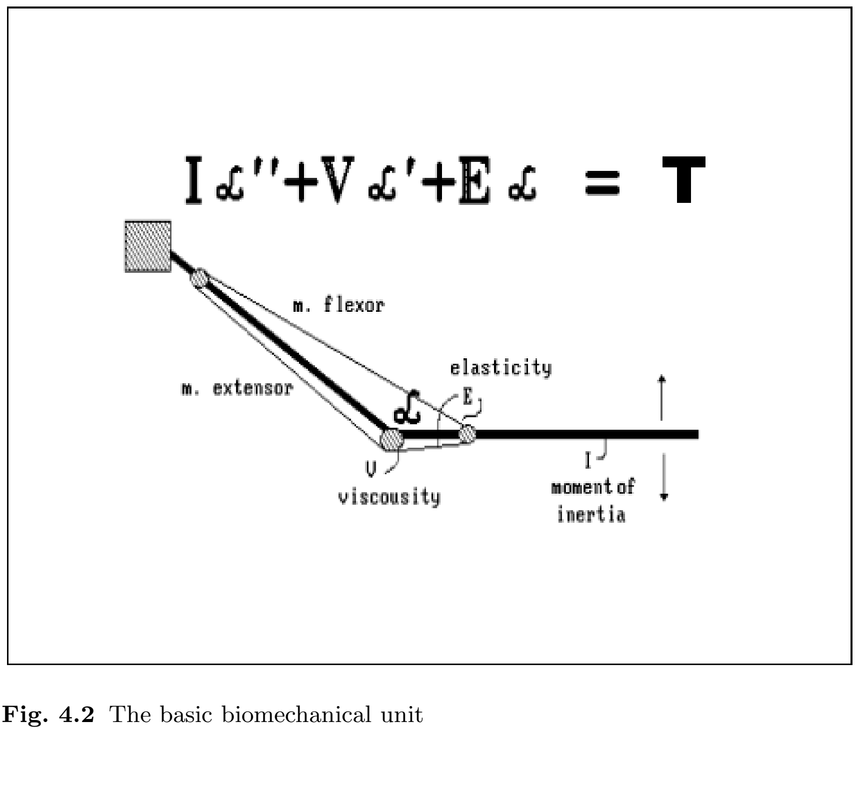

Fig. 4.2 The basic biomechanical unit

laws of motion of Euclid–Newton were described in terms of accelerations, i.e. rates of change of rates of change of position (i.e., 'second order' equations). Now, in the Hamiltonian dynamics we need only to talk about rates of change of things (i.e., 'first order' equations), rather than rates of change of rates of change of things. All these equations are derived from just one important quantity: the Hamiltonian function H, which is the expression for the total energy (the sum of the kinetic energy and the potential energy) of the system in terms of all the position q and momentum p variables.

The Hamiltonian formulation provides a very elegant and symmetrical description of mechanics, based on the first order, 'canonical' equations:

$$\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = \frac{\partial H}{\partial q}$$

where overdot denotes rate of change with respect to time (of momentum, in the first equation, and position, in the second), ∂ refers to rates of change of the Hamiltonian function H with respect to coordinates q, in the first case, and momenta p, in the second.

The coordinates q and momenta p are actually allowed to be more general things than just ordinary translational Cartesian coordinates (i.e., with the q being ordinary distances, measured off in three different directions at right angles) as used in Newtonian mechanics. For the sake of simplicity, in the human musculo–skeletal dynamics all movable (synovial) joints are considered as rotational: (i) one–degree–of–freedom hinge and pivot joints (like humero–ulnar and ulno–radial joints, respectively); (ii) two–degrees–of–freedom joints (like elbow); and (iii) three–degrees–of–freedom ball–and–socket joints (like shoulder).

Consequently, all the coordinates q are angles, while the corresponding momenta p are angular momenta, rather than translational momenta. Remarkably, the Hamilton's equations still hold in exactly the same covariant form. (Covariant means the same form of equations in both groups of Euclidean translations and Euclidean rotations). Set of all coordinates q of an $N-$degree–of–freedom biomechanical system represents its N–dimensional configuration space, while the set of all coordinates q and momenta p represents its $2N-$dimensional phase space.

Any pure Hamiltonian system is by definition conservative. Then, the Liouville's theorem implies inevitable diverging of the system trajectories in time. To prevent this 'phase–space spreading effect' biomechanics includes the II law of thermodynamics (law of entropy increase) in the pure mechanical observation of the system, in the form of dissipation function that stops entropy growth. Also, the system is muscle–driven, i.e. the non–conservative Hamiltonian motion is not self–generated (as in physics), but rather produced by internal, muscular forces and torques. Furthermore, it is controlled by neuro–muscular control.

4.4 Muscular Mechanics

4.4.1 Elements of Muscular Histology

Human skeletal and face muscles, accounting for more than 40% of the body weight in man, consist of bundles of elongated, cylindric cells called *muscle fibers*, 50 to 200 μ in diameter and often many centimeters long. Bundles of muscle fibers, each called *fasciculus*, are surrounded by a connective tissue covering, the *endomysium* (see, e.g., [Mou80, Mar98, II06a]).

A muscle consists of a number of fasciculi encased in a thick outer layer of connective tissue, the *perimysium*. At both ends of a muscle the connective tissue melds into a tendon by which the muscle is attached to the face or bony skeleton. In some muscles (*fusiform*), the muscle fibers run the whole length of muscle between the tendons, which form at opposite ends. In most muscles (*pennate*), one of the tendons penetrates through the center of the muscle; muscle fibers run at an angle to the axis of the whole muscle from the central tendon to the perimysium.

Like other cells, muscle cells are surrounded by a cell membrane, the *sarcolemma*. *Myofibrils*, the *contractile elements*, are numerous parallel, lengthwise threads 1 to 3 in diameter that fill most of the muscle fiber. The *cross striations*, seen in the skeletal and face muscles with electron microscope, are located in the myofibrils. Squeezed between the myofibrils and the sarcolemma is a small amount of cytoplasm, the *sarcoplasm*, in which are suspended multiple nuclei, numerous mitochondria, lysosomes, lipid droplets, glycogen granules, and other intracellular inclusions. The sarcoplasm contains glycogen, glycolytic enzymes, nucleotides, creatine phosphate, amino acids, and peptides.

Sarcoplasm also contains a well–developed endoplasmic reticulum, which in muscle is called *sarcoplasmic reticulum*. The sarcoplasmic reticulum forms an extensive hollow membranous system within the cytoplasm surrounding the myofibrils. Periodically, there are branching invaginations of the sarcolemma called *T tubules* or transverse tubules. The sarcoplasmic reticulum bulges out on either side of the T tubules to form large *lateral cisternae*. The T tubule and two sets of lateral cisternae constitute a *triad*. The triads play an important role in muscle excitation-contraction coupling (by release of Ca^{++} ions).

Two types of muscle fibers are found in human skeletal and face muscles: *red and white muscle fibers*, being histochemically and functionally distinctive. Many muscles are mixed, containing both types of fibers, which can be distinguished by various histochemical stains. In addition to muscle cells and fibroblasts in the connective tissue, a whole muscle contains fat cells and histocytes.

Each muscle fiber contains numerous contractile elements - *myofibrils* ($1 - 3\,\mu$ in diameter) which are biological machines that utilize chemical

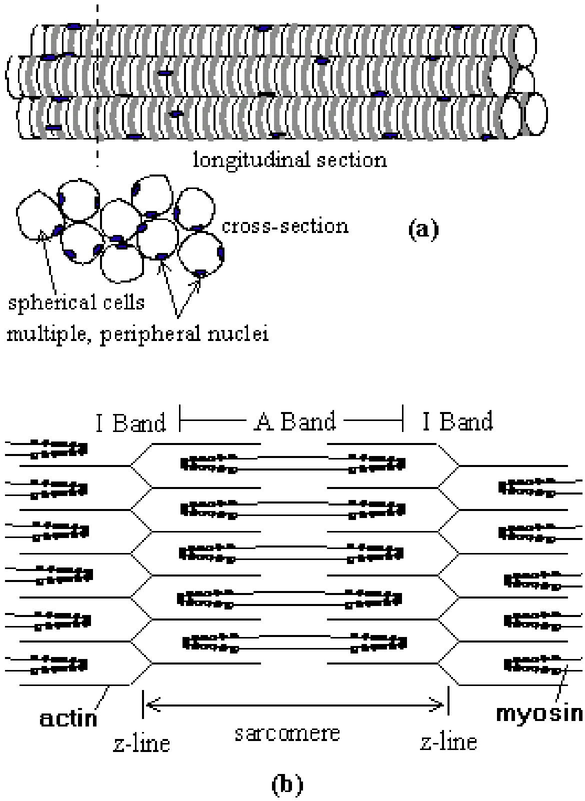

Fig. 4.3 Cellular structure of the voluntary (skeletal) human muscle: (a) Muscular fibers with their cross–sections; (b) Sarcomere with overlapping myofilaments

energy from metabolism of food in the form of *adenosine triphosphate*, ATP hydrolysis to produce mechanical work. An understanding of contractility and muscle function requires, thus, both histo-mechanical and bio-energetic insight.

Contractile machinery unit of the myofibril, *sarcomere* ($1.5 - 3.5\,\mu$ long; on electron microscope it is seen as bounded by two Z lines, with H zone in the middle of the A band) is constituted of a great number of longitudinal protein filaments of two kinds: thick, *myosin* filaments (about $120\,\dot{A}$ in diameter and about $1.8\,\mu$ long; they are located in the center of the sarcomere arranged in a hexagonal array about $450\,\dot{A}$ apart) and thin, *actin* filaments (about $80\,\dot{A}$ in diameter and about $1.0\,\mu$ long; they are anchored into the transverse filaments forming the Z line) (see Figure 4.3). Each myosin filament is surrounded by six actin filaments. Each myosin filament has two heads and two projections from opposite sides at about $143\,\dot{A}$ intervals along its length.

4.4.2 Huxley's Sliding–Filaments Dynamics

Essential for the contraction process are *cross bridges* (see Figure 4.3). They extend from myosin filaments to touch one of the adjacent actin filaments. Each thin filament receives cross bridges from the three adjacent thick filaments. During shortening the two sets of interdigitating filaments slide with respect to each other, cross and finally overlap each other. This process of muscle shortening involving progressive interdigitation of the two sets of protein filaments represents the *sliding filament mechanism*, discovered and mathematically formulated as a *microscopic theory of muscular contraction* in 1954–57 by A.F. Huxley [HN54, Hux57].

According to Huxley, the myosin heads and cross bridges are elastic elements with a mechanism for attaching themselves transiently to specific sites on the thin filaments. The following cyclic events take place during muscular contraction:

1. The cross bridges extend from myosin filaments and attach themselves to specific sites on actin filaments. The probability that attachment will occur is $f(x)$, where x is the instantaneous distance between the equilibrium position (0) and the maximum distance for attachment h along the myofibrillar axis.
2. The cross bridges detach with probability $g(x)$.

If we let N equal the density of cross bridges and n the fraction of cross bridges that are attached, then nN equals the density of attached cross bridges. Huxley's rate equation for cross–bridge attachment–detachment, i.e. the *sliding filament model* of muscular contraction is now given by:

$$\dot{n} = f(x)[1 - n(x,t)] - g(x)n(x,t) = f(x) - [f(x) + g(x)]n(x,t). \qquad (4.1)$$

Huxley's model (4.1) leads to expressions for the force developed by the cross bridges. For an *isometric steady–state contraction* the *contraction tension* or *contraction force* is given by:

$$F_0 = 0.5\, N\, h^2 \frac{kf}{f+g}, \qquad (4.2)$$

where $k = k(x)$ is the stiffness of the cross–bridge spring. For *isotonic* steady states it recovers the classical *Hill's force–velocity* relation (4.3). The *static force* expression says that the force (or tension) generated in the muscle is the function of the interfilamentar overlap, and its maximum is about the middle of the shortening, where the acto–myosin overlap is maximal. This is the so–called *parabolic length–tension curve* of muscular contraction.

4.4.3 Hill's Force–Velocity (Thermo)Dynamics

The *dynamic force–velocity relation* of muscular contraction is firstly discovered in 1938, by A.V. Hill [Hil38], in his thermodynamic studies of muscular work, and put into the basis of *macroscopic muscle–load dynamics*. Hill's famous *hyperbolic force–velocity curve* (recall Figure 12.4) has the equation:

$$(F + a)\, v = (F_0 + F)\, b, \tag{4.3}$$

and says that the muscle force is greatest in isometric conditions (without motion), while the velocity of shortening is maximal without external load; in other words, muscle is either 'strong' or 'fast', but no both. Constants a and b correspond respectively to the energy dissipated during the contraction and the velocity of the mechano–chemical processes.

Hill showed that energy change in muscle during contraction can be described by the following *thermodynamic relation*:

$$U = A + W + M, \tag{4.4}$$

where U is the total energy change associated with contraction, A is the *activation heat* (i.e., the heat production associated with the activation of the contractile elements), W is the mechanical work performed by the muscle by lifting a load, $\alpha \Delta x$ is the *shortening heat*, and M is the *maintenance heat* of contraction.

The activation heat begins and is almost completely liberated before any tension is developed, i.e. it is predominantly connected with the excitation–contraction coupling process, and corresponds in time to the *latency relaxation* of muscle. It is associated with the internal work required to transform the contractile elements from the resting to the active state. Part of the activation heat probably is associated with a change in the elastic properties of muscle, but about two thirds of it is associated with the release of Ca^{++} ions from the triads, its binding by troponin and the subsequent rearrangement of the thin filament proteins. The activation heat is greatest for the first twitch after a period rest and becomes smaller with succeeding twitches.

The maintenance heat begins at about the time tension begins and can be divided into two parts: the labile maintenance heat and the stable maintenance heat. For isometric contractions at shorter than rest length, both the labile and the stable heats diminish. For stretched muscle, the labile heat is approximately constant, whereas the stable heat diminishes with stretching and is roughly proportional to the degree of interfilamentar overlap. The stable heat has quite different values in functionally different muscles; it is law when the muscle maintains tension efficiently and vice versa.

The shortening heat is proportional mainly to the distance of shortening and does not depend greatly on the load, the speed of shortening, or the amount of work performed. Since mechanical work is $W = P\Delta x$, substituting this in the above thermodynamic relation (4.4) gives the *heat equation*:

$$U = A + (P + \alpha)\,\Delta x + M. \tag{4.5}$$

From the analogy of the term $(P + \alpha)$ in the heat equation (4.5) and the term $(P + a)$ in the force-velocity equation (4.3), Hill was able to show a rough equivalence between the coefficient of the shortening heat α and the force-velocity constant a. The shortening heat is greatest for the first twitch after a period of rest and is less for subsequent twitches.

Last, note should be made of *thermoelastic heat*. Generally speaking, resting muscle has rubber-like thermoelastic properties, whereas actively contracting muscle has springlike thermoelastic properties. During the development of tension the change in elastic properties is accompanied by an absorption of heat by the muscle. As tension falls during relaxation, an equivalent amount of heat is released by the muscle owing to its elastic properties. The various kinds of muscle heat must be corrected for the thermoelastic heat. However, for a complete cycle of contraction and relaxation, the net heat produced by thermoelastic mechanisms is zero.

In the same seminal paper [Hil38], Hill also proposed a three–element rheological model of the skeletal muscle–tendon complex. In this model the length–tension property of muscle is represented by an active contractile element (CE) in parallel with a passive elastic element. Total isometric muscle force is assumed to be the sum of muscle force when it is inactive (passive) and when it is maximally excited (active). The muscle is in series with tendon, which is represented by a nonlinear spring. Pennation angle (α) is the angle between tendon and muscle fibers. Tendon slack length is the length of tendon at which force initially develops during tendon stretch. The model was scaled to represent each muscle by specifying the muscle's peak force, optimal fiber length, tendon slack length, and pennation angle based on data collected in anatomical experiments.

Hill's muscle–tendon model has been widely applied in biomechanical musculo–skeletal modelling.

4.4.4 Basic Musculo–Skeletal Dynamics

Passive Joint Dynamics

Recall that all biological systems are *dissipative structures*, emphasizing *irreversible processes* inefficient energetically, but highly efficient in terms of information and control (see [NP77]). In case of biomechanics, we have the passive damping contribution to the joint torques, $T_i(t,\, q^i,\, p_i)$, which has the basic stabilizing effect to the complex human movement. This effect can be described by (q, p)-quadratic form of the *Rayleigh – Van der Pol's dissipation function* (see [II06b])

$$R = \frac{1}{2}\sum_{i=1}^{9} p_i^2\,[a_i \,+\, b_i(q^i)^2], \tag{4.6}$$

where a_i and b_i denote dissipation parameters. Its partial derivatives $\partial R/\partial p$ give rise to viscous forces in the joints which are linear in p_i and quadratic in q^i. It is based on the unforced Van der Pol's oscillator

$$\ddot{x} - \left(a + b\,x^2\right)\,\dot{x} + x = 0,$$

where the damping force $F^{dmp}(\dot{x}) = -\partial R/\partial \dot{x}$ is given by the Rayleigh's dissipation function $R = \frac{1}{2}\left(a + b\,x^2\right)\,\dot{x}^2$ – with the velocity term $\dot{x}$ replaced by our momentum term p^2.

Using (4.6) we obtain the *dissipative joint Hamiltonian biodynamics equations* (see [II05, II06a, II06b])

$$\dot{q}^i = \frac{\partial H(q,p)}{\partial p_i} + \frac{\partial R(q,p)}{\partial p_i}, \tag{4.7}$$
$$\dot{p}_i = -\frac{\partial H(q,p)}{\partial q^i} + \frac{\partial R(q,p)}{\partial q^i}, \qquad (i = 1, \dots, N),$$

which reduces to the gradient system in case $H = 0$ (as well as to the conservative system in case $R = 0$).

Active Muscular Dynamics

Muscular dynamics describes the internal *excitation* and *contraction* dynamics [Hat77b, Hat78, II06a, II05] of *equivalent muscular actuators*, anatomically represented by resulting action of *antagonistic muscle-pairs* for each uniaxial joint. We attempt herein to describe the equivalent muscular dynamics in the simplest possible way (for example, Hatze used 51 nonlinear differential equations (of the first order) to derive his, arguably most elaborate, myocybernetic model [Hat77b, Hat78]), and yet to include the main excitation and contraction relations.

The active muscular contribution to the joint torques, $T_i(t,\, q^i,\, p_i)$, should describe the internal *excitation* and *contraction* dynamics [II05, II06a, II06b] of *equivalent muscular actuators*, anatomically represented by resulting action of *antagonistic muscle-pairs* per each active degree-of-freedom.

(a) The *excitation muscular dynamics* can be described by impulse *torque-time* relation

$$T_i^{imp} = T_i^0(1 - \mathrm{e}^{-t/\tau_i}), \qquad \text{if stimulation } > 0$$
$$T_i^{imp} = T_i^0\mathrm{e}^{-t/\tau_i}, \qquad \text{if stimulation } = 0,$$

where F_i^0 denote the maximal isometric muscular torques applied at i-th joint, while τ_i denote the time characteristics of particular muscular actuators. This is a rotational-joint form of the solution of the Wilkie's *muscular active-state element equation* [Wil56]

$$\dot{x} + \beta x = \beta S A, \quad x(0) = 0, \quad 0 < S < 1,$$

where $x = x(t)$ represents the active state of the muscle, β denotes the element gain, A corresponds to the maximum tension the element can develop, and $S = S(r)$ is the 'desired' active state as a function of motor unit stimulus rate r.

(b) The *contraction muscular dynamics* has classically been described by the Hill's *hyperbolic force-velocity* relation [Hil38], which we propose here in the rotational (q, p)-form

$$T_i^{Hill} = \frac{(T_i^0 b_i - a_i p_i)}{(p_i - b_i)},$$

where a_i (having dimension of torque) and b_i (having dimension of momentum) denote the *rotational Hill's parameters* (see [II05,II06a]), corresponding to the energy dissipated during the contraction and the phosphagenic energy conversion rate, respectively.

Therefore, we can describe the excitation/contraction dynamics for the ith equivalent muscle-joint actuator, i.e., antagonistic muscle pair, by the simple impulse-hyperbolic product–relation

$$T_i(t, q, p) = T_i^{imp} \times T_i^{Hill}, \qquad (i = 1, \dots, N). \tag{4.8}$$

Using (4.8) we obtain the general formulation for the *forced dissipative Hamiltonian musculo-skeletal dynamics* (see [II05, II06a, II06b]),

$$\dot{q}^i = \frac{\partial H(q,p)}{\partial p_i} + \frac{\partial R(q,p)}{\partial p_i}, \qquad (i = 1, \dots, N), \tag{4.9}$$
$$\dot{p}_i = T_i(t, q, p) - \frac{\partial H(q,p)}{\partial q^i} + \frac{\partial R(q,p)}{\partial q^i}.$$

4.5 Stretch Reflex and Motor Servo

Recall from Chapter 2 that the myotatic stretch–reflex, also known as the monosynaptic reflex,[1] is the simplest reflex known: it involves only two neurons (one sensory, or afferent and one motor, or efferent), with one synapse between them. Therefore, it depends only on the monosynaptic connection between primary afferent fibers from *muscle spindles* and motor neurons innervating the same muscle.

[1] It is called a monosynaptic reflex because it depends only on the simple connection between primary afferent fibers from muscle spindles and motor neurons innervating the same muscle. In other spinal reflexes such as those produced by cutaneous stimuli, one or more interneurons may be interposed between the primary afferent fibers and the motor neurons.

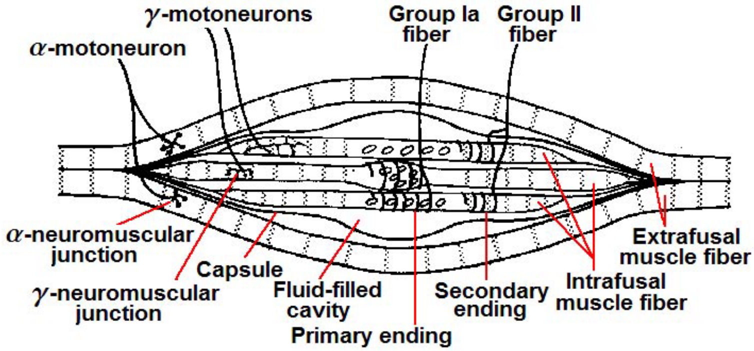

Fig. 4.4 Schematic of a muscular length–sensor, the muscle spindle

The muscle spindle (see Figure 4.4) contains specialized elements that sense muscle length and velocity of length change. The spindle is innervated by large myelinated afferent fibers known as type Ia afferent fibers. The cell bodies of these neurons are clustered near the spinal cord in the dorsal root ganglia. They are an example of a bipolar cell: one branch of the cell's axon goes out to the muscle and the other runs into the spinal cord. In the spinal cord the Ia afferent fibers make monosynaptic excitatory connections to alpha motor neurons innervating the same muscle from which they arise and motor neurons innervating synergistic muscles . They also inhibits motor neurons controlling antagonistic muscles through an inhibitory interneuron. Half the neurons in the brain are inhibitory. They release neurotransmitters that hyperpolarize the membrane potential of the postsynaptic cell, thus reducing the likelihood of firing. The muscle spindle is sensitive to stretch so that when the muscle is stretched the Ia afferent fibers increase the firing rate. This leads to the contraction of the same muscle and its synergists and relaxation of the antagonist muscle. The reflex therefore tends to counteract the stretch, enhancing the spring like properties of the muscles.

A physician tapping the reflex hammer against the patellar tendon traditionally elicits the special case of myotatic stretch–reflex, called the knee jerk reflex. The tap of the reflex hammer causes stimulation of the muscle spindles afferents and Golgi tendon organs, by altering the stretch and length of the muscle. The afferent neuron synapses in the anterior horn of the spinal cord with an ipsilateral efferent neuron. The efferent neuron leads to a neuromuscular junction with the respective effector muscle the quadriceps femoris. The stretch–reflex also plays a central role in the maintenance of balance.

The intensity of the stretch–reflex is modulated by excitatory and inhibitory supraspinal input. Damage to supraspinal input, affecting descending input, precipitates abnormally high gain of the stretch–reflex. The effect is an increase in muscle stiffness, also referred as hypertonus.

About three decades ago, James Houk pointed out in [Hou67, HSG70, Hou78, Hou79] that stretch and unloading reflexes were mediated by combined actions of several autogenetic neural pathways. In this context, "autogenetic" (or, autogenic) means that the stimulus excites receptors located in the same muscle that is the target of the reflex response. The most important of these muscle receptors are the primary and secondary endings in muscle spindles, sensitive to length change, and the Golgi tendon organs, sensitive to contractile force. The autogenetic circuits appear to function as servo-regulatory loops that convey continuously graded amounts of excitation and inhibition to the large (alpha) skeletomotor neurons. Small (gamma) fusimotor neurons innervate the contractile poles of muscle spindles and function to modulate spindle–receptor discharge. Houk's term "motor servo" [Hou78] has been used to refer to this entire control system, summarized by the block diagram in Figure 3.3.

Prior to a study by Matthews [Mat69], it was widely assumed that secondary endings belong to the mixed population of "flexor reflex afferents," so called because their activation provokes the flexor reflex pattern − excitation of flexor motoneurons and inhibition of extensor motoneurons. Matthews' results indicated that some category of muscle stretch receptor other than the primary ending provides important excitation to extensor muscles, and he argued forcefully that it must be the secondary ending.

The primary and secondary muscle spindle afferent fibers both arise from a specialized structure within the muscle, the *muscle spindle*, a fusiform structure 4–7 mm long and 80–200 μ in diameter. The spindles are located deep within the muscle mass, scattered widely through the muscle body, and attached to the tendon, the endomysium or the perimysium, so as to be in parallel with the extrafusal or regular muscle fibers. Although spindles are scattered widely in muscles, they are not found throughout. Muscle spindle (see Figure 4.4) contains two types of intrafusal muscle fibers (intrafusal means inside the fusiform spindle): the nuclear bag fibers and the nuclear chain fibers. The nuclear bag fibers are thicker and longer than the nuclear chain fibers, and they receive their name from the accumulation of their nuclei in the expanded bag-like equatorial region-the nuclear bag. The nuclear chain fibers have no equatorial bulge; rather their nuclei are lined up in the equatorial region-the nuclear chain. A typical spindle contains two nuclear bag fibers and 4-5 nuclear chain fibers.

The pathways from primary and secondary endings are treated commonly by Houk in Figure 3.3, since both receptors are sensitive to muscle length and both provoke reflex excitation. However, primary endings show an additional sensitivity to the dynamic phase of length change, called dynamic responsiveness, and they also show a much–enhanced sensitivity to small changes in muscle length [Mat72].

The motor servo comprises three closed circuits (Figure 3.3), two neural feedback pathways, and one circuit representing the mechanical interaction between a muscle and its load. One of the feedback pathways, that from spindle receptors, conveys information concerning muscle length, and it follows that this loop will act to keep muscle length constant. The other feedback pathway, that from tendon organs, conveys information concerning muscle force, and it acts to keep force constant.

In general, it is physically impossible to maintain both muscle length and force constant when external loads vary; in this situation the action of the two feedback loops will oppose each other. For example, an increased load force will lengthen the muscle and cause muscular force to increase as the muscle is stretched out on its length-tension curve. The increased length will lead to excitation of motoneurons, whereas the increased force will lead to inhibition. It follows that the net regulatory action conveyed by skeletomotor output will depend on some relationship between force change and length change and on the strength of the feedback from muscle spindles and tendon organs. A simple mathematical derivation [NH76] demonstrates that the change in skeletomotor output, the error signal of the motor servo, Should be proportional to the difference between a regulated stiffness and the actual stiffness provided by the mechanical properties of the muscle, where stiffness has the units of force change divided by length change. The regulated stiffness is determined by the ratio of the gain of length to force feedback.

It follows that the combination of spindle receptor and tendon organ feedback will tend to maintain the stiffness of the neuromuscular apparatus at some regulated level. If this level is high, due to a high gain of length feedback and a low gain of force feedback, one could simply forget about force feedback and treat muscle length as the regulated variable of the system. However, if the regulated level of stiffness is intermediate in value, i.e. not appreciably different from the average stiffness arising from muscle mechanical properties in the absence of reflex actions, one would conclude that stiffness, or its inverse, compliance, is the regulated property of the motor servo.

In this way, the autogenetic reflex motor servo provides the local, reflex feedback loops for individual muscular contractions. A voluntary contraction force F of human skeletal muscle is reflexly excited (positive feedback $+F^{-1}$) by the responses of its *spindle receptors* to stretch and is reflexly inhibited (negative feedback $-F^{-1}$) by the responses of its *Golgi tendon organs* to contraction. Stretch and unloading reflexes are mediated by combined actions of several autogenetic neural pathways, forming the *motor servo* (see [II05,II06a,II06b]).

In other words, branches of the afferent fibers also synapse with with interneurons that inhibit motor neurons controlling the antagonistic muscles – *reciprocal inhibition*. Consequently, the stretch stimulus causes the antagonists to relax so that they cannot resists the shortening of the stretched muscle caused by the main reflex arc. Similarly, firing of the Golgi tendon receptors causes inhibition of the muscle contracting too strong and simultaneous *reciprocal activation* of its antagonist.

4.6 Cerebellar Movement Control

When someone compares learning a new skill to learning how to ride a bike they imply that once mastered, the task seems imbedded in our brain forever.

Well, imbedded in the cerebellum to be exact. This brain structure is the commander of coordinated movement and possibly even some forms of cognitive learning. Damage to this area leads to motor or movement difficulties.

A part of a human brain that is devoted to the sensory-motor control of human movement, that is motor coordination and learning, as well as equilibrium and posture, is the cerebellum (which in Latin means “little brain”). It performs integration of sensory perception and motor output. Many neural pathways link the cerebellum with the motor cortex, which sends information to the muscles causing them to move, and the spino–cerebellar tract, which provides proprioception, or feedback on the position of the body in space. The cerebellum integrates these pathways, using the constant feedback on body position to fine–tune motor movements [Ito84].

The human cerebellum has 7–14 million Purkinje cells. Each receives about 200,000 synapses, most onto dendritic splines. Granule cell axons form the *parallel fibers*. They make excitatory synapses onto Purkinje cell dendrites. Each parallel fibre synapses on about 200 Purkinje cells. They create a strip of excitation along the cerebellar folia.

Mossy fibers are one of two main sources of input to the cerebellar cortex (see Figure 4.5). A mossy fibre is an axon terminal that ends in a large, bulbous swelling. These mossy fibers enter the granule cell layer and synapse on the dendrites of granule cells; in fact the granule cells reach out with little ‘claws’ to grasp the terminals. The granule cells then send their axons up to the molecular layer, where they end in a T and run parallel to the surface. For this reason these axons are called *parallel fibers*. The parallel fibers synapse on the huge dendritic arrays of the Purkinje cells. However, the individual parallel fibers are not a strong drive to the Purkinje cells. The Purkinje cell dendrites fan out within a plane, like the splayed fingers of one hand. If we were to turn a Purkinje cell to the side, it would have almost no width at all. The parallel fibers run perpendicular to the Purkinje cells, so that they only make contact once as they pass through the dendrites.

Unless firing in bursts, parallel fibre EPSPs do not fire Purkinje cells. Parallel fibers provide excitation to all of the Purkinje cells they encounter. Thus, granule cell activity results in a strip of activated Purkinje cells.

Mossy fibers arise from the spinal cord and brainstem. They synapse onto granule cells and deep cerebellar nuclei. The Purkinje cell makes an inhibitory synapse (GABA) to the deep nuclei. Mossy fibre input goes to both cerebellar cortex and deep nuclei. When the Purkinje cell fires, it inhibits output from the deep nuclei.

The *climbing fibre* arises from the inferior olive. It makes about 300 excitatory synapses onto one Purkinje cell. This powerful input can fire the Purkinje cell.

The parallel fibre synapses are plastic—that is, they can be modified by experience. When parallel fibre activity and climbing fibre activity converge on the same Purkinje cell, the parallel fibre synapses become weaker (EPSPs are smaller). This is called long-term depression. Weakened parallel fibre

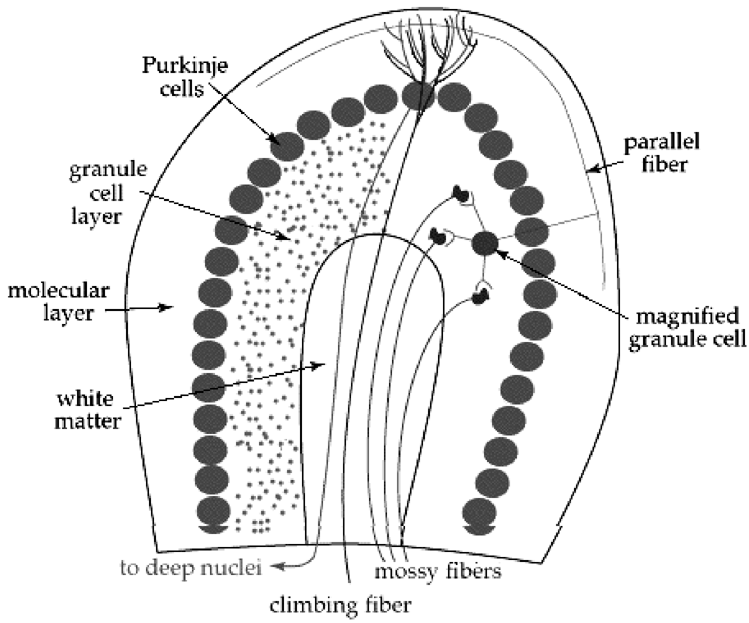

Fig. 4.5 Stereotypical ways throughout the cerebellum

synapses result in less Purkinje cell activity and less inhibition to the deep nuclei, resulting in facilitated deep nuclei output. Consequently, the mossy fibre collaterals control the deep nuclei.

The *basket cell* is activated by parallel fibers afferents. It makes inhibitory synapses onto Purkinje cells. It provides lateral inhibition to Purkinje cells. Basket cells inhibit Purkinje cells lateral to the active beam.

Golgi cells receive input from parallel fibers, mossy fibers, and climbing fibers. They inhibit granule cells. Golgi cells provide feedback inhibition to granule cells as well as feedforward inhibition to granule cells. Golgi cells create a brief burst of granule cell activity.

Although each parallel fibre touches each Purkinje cell only once, the thousands of parallel fibers working together can drive the Purkinje cells to fire like mad.

The second main type of input to the folium is the *climbing fibre*. The climbing fibers go straight to the Purkinje cell layer and snake up the Purkinje dendrites, like ivy climbing a trellis. Each climbing fibre associates with only one Purkinje cell, but when the climbing fibre fires, it provokes a large response in the Purkinje cell.

The Purkinje cell compares and processes the varying inputs it gets, and finally sends its own axons out through the white matter and down to the *deep nuclei*. Although the inhibitory Purkinje cells are the main output of the cerebellar cortex, the output from the cerebellum as a whole comes from the deep nuclei. The three deep nuclei are responsible for sending excitatory output back to the thalamus, as well as to postural and vestibular centers.

There are a few other cell types in cerebellar cortex, which can all be lumped into the category of inhibitory interneuron. The *Golgi cell* is found among the granule cells. The *stellate* and *basket cells* live in the molecular layer. The basket cell (right) drops axon branches down into the Purkinje cell layer where the branches wrap around the cell bodies like baskets.

The cerebellum operates in 3's: there are 3 highways leading in and out of the cerebellum, there are 3 main inputs, and there are 3 main outputs from 3 deep nuclei. They are:

The 3 highways are the *peduncles*. There are 3 pairs (see [Mol97, Har97, Mar98]):

1. The *inferior cerebellar peduncle* (restiform body) contains the dorsal spinocerebellar tract (DSCT) fibers. These fibers arise from cells in the ipsilateral Clarke's column in the spinal cord (C8–L3). This peduncle contains the cuneo–cerebellar tract (CCT) fibers. These fibers arise from the ipsilateral accessory cuneate nucleus. The largest component of the inferior cerebellar peduncle consists of the olivo–cerebellar tract (OCT) fibers. These fibers arise from the contralateral inferior olive. Finally, vestibulo–cerebellar tract (VCT) fibers arise from cells in both the vestibular ganglion and the vestibular nuclei and pass in the inferior cerebellar peduncle to reach the cerebellum.
2. The *middle cerebellar peduncle* (brachium pontis) contains the pontocerebellar tract (PCT) fibers. These fibers arise from the contralateral pontine grey.
3. The *superior cerebellar peduncle* (brachium conjunctivum) is the primary efferent (out of the cerebellum) peduncle of the cerebellum. It contains fibers that arise from several deep cerebellar nuclei. These fibers pass ipsilaterally for a while and then cross at the level of the inferior colliculus to form the decussation of the superior cerebellar peduncle. These fibers then continue ipsilaterally to terminate in the red nucleus ('ruber–duber') and the motor nuclei of the thalamus (VA, VL).

The 3 inputs are: *mossy fibers* from the *spinocerebellar* pathways, climbing fibers from the *inferior olive*, and more mossy fibers from the *pons*, which are carrying information from *cerebral cortex* (see Figure 4.6). The mossy fibers from the spinal cord have come up ipsilaterally, so they do not need to cross. The fibers coming down from cerebral cortex, however, do need to cross (as the cerebrum is concerned with the opposite side of the body, unlike the cerebellum). These fibers synapse in the pons (hence the huge block of fibers in the cerebral peduncles labelled 'cortico–pontine'), cross, and enter the cerebellum as mossy fibers.

The 3 deep nuclei are the *fastigial*, *interposed*, and *dentate nuclei.* The fastigial nucleus is primarily concerned with balance, and sends information mainly to vestibular and reticular nuclei. The dentate and interposed nuclei

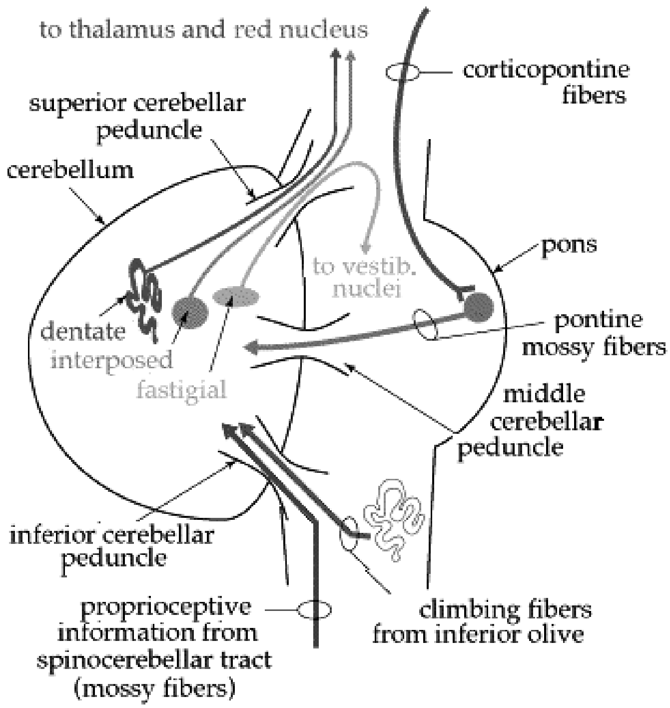

Fig. 4.6 Inputs and outputs of the cerebellum

are concerned more with voluntary movement, and send axons mainly to thalamus and the red nucleus.

The main function of the cerebellum as a motor controller is depicted in Figure 4.7. A coordinated movement is easy to recognize, but we know little about how it is achieved. In search of the neural basis of coordination, a model of spinocerebellar interactions was recently presented in [AG05], in which the structure-functional organizing principle is a division of the cerebellum into discrete micro–complexes. Each micro–complex is the recipient of a specific motor error signal - that is, a signal that conveys information about an inappropriate movement. These signals are encoded by spinal reflex circuits and conveyed to the cerebellar cortex through climbing fibre afferents. This organization reveals salient features of cerebellar information processing, but also highlights the importance of systems level analysis for a fuller understanding of the neural mechanisms that underlie behavior.

The authors of [AG05] reviewed anatomical and physiological foundations of cerebellar information processing. The cerebellum is crucial for the coordination of movement. The authors presented a model of the cerebellar paravermis, a region concerned with the control of voluntary limb movements through its interconnections with the spinal cord. They particularly focused on the olivo-cerebellar climbing fibre system.

Climbing fibres are proposed to convey motor error signals (signals that convey information about inappropriate movements) related to elementary limb movements that result from the contraction of single muscles. The actual encoding of motor error signals is suggested to depend on sensorimotor transformations carried out by spinal modules that mediate nociceptive withdrawal reflexes.

The termination of the climbing fibre system in the cerebellar cortex subdivides the paravermis into distinct microzones. Functionally similar but spatially separate microzones converge onto a common group of cerebellar nuclear neurons. The processing units formed as a consequence are termed 'multizonal micro-complexes' (MZMCs), and are each related to a specific spinal reflex module.

The distributed nature of microzones that belong to a given MZMC is proposed to enable similar climbing fibre inputs to integrate with mossy fibre inputs that arise from different sources. Anatomical results consistent with this notion have been obtained.

Within an individual MZMC, the skin receptive fields of climbing fibres, mossy fibres and cerebellar cortical inhibitory interneurons appear to be similar. This indicates that the inhibitory receptive fields of Purkinje cells within a particular MZMC result from the activation of inhibitory interneurons by local granule cells.

On the other hand, the parallel fibre–mediated excitatory receptive fields of the Purkinje cells in the same MZMC differ from all of the other receptive fields, but are similar to those of mossy fibres in another MZMC. This indicates that the excitatory input to Purkinje cells in a given MZMC originates in non–local granule cells and is mediated over some distance by parallel fibres.

The output from individual MZMCs often involves two or three segments of the ipsilateral limb, indicative of control of multi–joint muscle synergies. The distal–most muscle in this synergy seems to have a roughly antagonistic action to the muscle associated with the climbing fibre input to the MZMC.

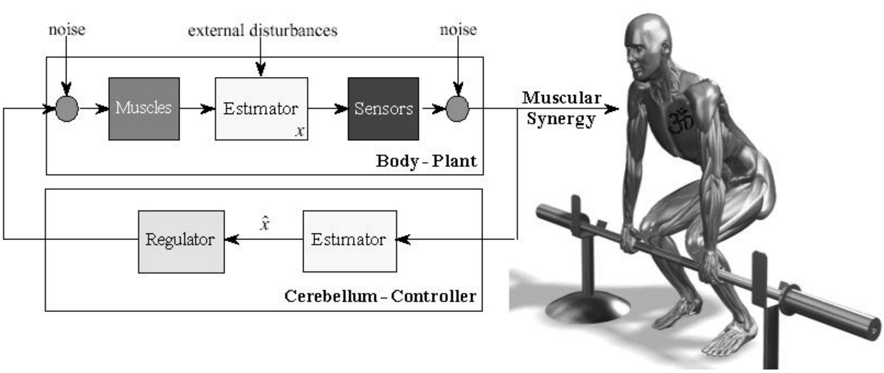

Fig. 4.7 The cerebellum as a motor controller

The model proposed in [AG05] indicates that the cerebellar paravermis system could provide the control of both single– and multi–joint movements. Agonist-antagonist activity associated with single–joint movements might be controlled within a particular MZMC, whereas coordination across multiple joints might be governed by interactions between MZMCs, mediated by parallel fibres.

Two main theories address the function of the cerebellum, both dealing with motor coordination. One claims that the cerebellum functions as a regulator of the "timing of movements." This has emerged from studies of patients whose timed movements are disrupted [IKD88].

The second, "Tensor Network Theory" provides a mathematical model of transformation of sensory (covariant) space-time coordinates into motor (contravariant) coordinates by cerebellar neuronal networks [PL80, PL82, PL85].

Studies of motor learning in the vestibulo–ocular reflex and eye-blink conditioning demonstrate that the timing and amplitude of learned movements are encoded by the cerebellum [BK04]. Many synaptic plasticity mechanisms have been found throughout the cerebellum. The *Marr–Albus model* mostly attributes motor learning to a single plasticity mechanism: the long-term depression of parallel fiber synapses. The Tensor Network Theory of sensory–motor transformations by the cerebellum has also been experimentally supported [GZ86].

4.7 Closing the (Bio)Mechanical Circle

The number of independent quantities which must be specified in order to define uniquely the position and orientation in space of any (bio)mechanical system is called the number of *degrees of freedom*, DOF, for short. Any N quantities $q^i \quad (i = 1, ..., N)$, which completely define the position and orientation in space of a system with N DOF are called the *generalized coordinates* of the system. Their first time derivatives $\dot{q}^i$ (i.e., rates of change with respect to time) are called the *generalized velocities*, and their second time derivatives $\ddot{q}^i$ (i.e., rates of change of rates of change with respect to time) are called the *generalized accelerations* (see [AM78]– [MR94]).

The *inverse* of the Newton's causality principle states: If all the coordinates q^i and velocities $\dot{q}^i$ are given in some instant, the accelerations $\ddot{q}^i$ are uniquely defined.

The motion, of any *conservative*[2] N DOF (bio)mechanical system is generally governed by *Hamilton's principle of least action*. The quantity that describes such a system is the *Lagrangian function* $L = L(q, \dot{q}, t)$, representing the difference between the system's *kinetic* and *potential* energies.

Given the Lagrangian function L, any conservative (bio)mechanical motion can be described in the following way. Let our system has the configuration

[2] Conservative system is an isolated system that does not exchange energy with its surrounding, i.e., system without inputs and outputs.

q^i_{start} at the instant t_{start} (the start of motion), and the configuration q^i_{end} at the instant t_{end} (the end of motion). The Hamilton's principle states that our system moves between the start and the end in such a way that the integral called *Action* takes the least possible value

$$\text{Action} = \int_{t_{start}}^{t_{end}} L(q, \dot{q}, t)\, dt \to \min$$

From the calculus of variations it is known that the Hamilton's principle is satisfied if the system's motion is described by the set of N second–order *Euler–Lagrange's equations*

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^i}\right) = \frac{\partial L}{\partial q^i}$$

where the partial derivative ∂ denotes the *space rate of change.*

The term in bracket (including the derivatives of the Lagrangian with respect to the generalized velocities) represents the *generalized momenta*

$$p_i = \frac{\partial L}{\partial \dot{q}^i}$$

while the term on the right–hand side (including the derivatives of the Lagrangian with respect to the generalized coordinates) represents the *generalized forces*

$$F_i = \frac{\partial L}{\partial q^i}$$

In this way, we can *close the (bio)mechanical circle* and obtain the *generalized Newton's equations of motion*

$$\dot{p}_i = F_i$$

Alternatively, we can define the *Hamiltonian function* $H = H(q, p, t)$, either directly, as the system's total energy, i.e., the sum of the kinetic energy E_K and the potential energy E_P

$$H = E_K(p) + E_P(q)$$

or indirectly, from the Lagrangian

$$H = \sum_i p_i q^i - L$$

In this way, we arrive to the set of $2N$ *Hamilton's canonical equations*

$$\dot{q}^i = \frac{\partial H}{\partial p_i}, \qquad \dot{p}_i = \frac{\partial H}{\partial q^i}$$

which can be formally derived from the Hamiltonian version of the Hamilton's principle of least action

$$\text{Action} = \int_{t_{start}}^{t_{end}} (\sum_i p_i q^i - H)\, dt \to \min$$

In particular, if the Hamiltonian does not depend explicitly on time, as in the case $H(q,p) = E_K(p) + E_P(q)$ – in that case our (bio)mechanical system is called *autonomous*, then we have the *law of conservation of energy*

$$\dot{H} = 0$$

The first of Hamilton's equations is called the *velocity equation*, defining the generalized velocities in most general way – in terms of the canonical momenta.

The second Hamilton's equation is called the *force equation*, defining the generalized forces – again representing the fundamental Newton's law of motion and closing the (bio)mechanical circle

$$\dot{p}_i = \frac{\partial L}{\partial q^i} = \frac{\partial H}{\partial q^i} = F_i$$

4.8 Biomechanical Chain

Biomechanical chain represents the motion of several biomechanical units connected into the unified movement. Mathematical model for the kinematic chain represents the system of coupled equations of motion for all included joints. In this way it is possible (at least in principle), joint by joint, with possible branching, and with known initial conditions, to define causally any form of human locomotion. This is classical, analytic approach to modelling of human locomotion, used for years by robotics, formally based on Lagrange's, Hamilton's, Euler's, or Appel's equations of motion.

A classical example of the biomechanical chain model in extended Lagrangian form is given by the set of equations of motion

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}^i}\right) - \frac{\partial E_K}{\partial q^i} + \frac{\partial E_P}{\partial q^i} + \frac{\partial E_D}{\partial \dot{q}^i} = T_i$$

where ∂ denotes partial derivative (space rate of change with respect to the $i-$th coordinate), E_K – kinetic energy, E_P – potential energy, E_D – dissipation energy, q^i, $(i = 1, ..., DOF)$ – joint angles, DOF – number of degrees of freedom, i.e, active joints, $\dot{q}^i$ – joint angular velocities and T_i – joint torques exerted by antagonistic muscle pairs in all active joints.

In a modern treatment the biomechanical chain model of general musculo–skeletal dynamics can be separated into 3 parts [Iva91]

1. Rigid skeleton dynamics, defining conservative motion of body segments influenced by inertial and gravitational forces,
2. Muscular (force–velocity–time) biodynamics, defining synergy of included skeletal muscles' excitation and contraction dynamics, and
3. Joint biodynamics, defining anatomical–synovial dissipation in qua–silinear terms.

This can be presented in the form of the following conceptual equation:

$$\boxed{biomechanics = skeleton\ dynamics + joints/muscles\ dynamics}$$

with the corresponding *Hamiltonian dynamical system*, which in vector–matrix form yields

$$\dot{q} = \partial H/\partial p + \partial R/\partial p, \qquad (*)$$

$$\dot{p} = -\partial H/\partial q + \partial R/\partial q + T(t, q, p)$$

where $q = q^i(t)$ are joint angles, $p = p_i(t)$ are associate joint angular momenta, $H = H(q, p, t)$ is the Hamiltonian (total energy) function, $R = R(q, p)$ is Rayleigh–Van der Pol's dissipation function and $T(t, q, p)$ denote active muscular torques as functions of time, coordinates and momenta. For $T = 0$ and $R = 0$ this reduces to a conservative Hamiltonian system; for $T = 0$ and $H = 0$ this becomes a gradient system.

Now, the modern, synthetic approach to modelling and computer simulations / animations of human locomotion (as well all other CSB–systems), uses differential–geometric apparatus of tensorial fields and their flows on the smooth configuration manifolds with Lie groups of motion acting on them [Iva91, II06b]. In this approach, instead of analytic picture of motion of n joints in a linear 3D Euclidean space, an abstract and synthetic picture of nonlinear motion of a representative point for the whole kinetic chain is used in a n–dimensional Riemannian configuration manifold. Instead of Newton's absolute space and matter, herein is applied Einstein's general relativity rule: 'Space tells matter how to move, matter tells space how to curve ... Physics is simple only when analyzed locally.'

Let us consider the simple mechanical example of a double (hinged) pendulum swinging in the vertical plane. Denote the point where the pendulum is attached by O, the hinge by O', and the end of the pendulum by O''. Each position of the system is given by the direction of the rod $00'$ and of the rod $O'O''$, or by the pair of angles (φ, ψ) varying independently in the intervals $0 \leq \varphi < 2\pi$, $0 \leq \psi < 2\pi$. The *configuration manifold* of the given system is thus the Cartesian product of two circumferences $S^1 \times S^2$, i.e., the 2–dimensional torus T^2. The generalization is obvious: for a n–segmental pendulum (i.e., an open kinematic chain), the configuration manifold is the Cartesian product of n circumferences $S^1 \times S^2 \times ... \times S^n$, i.e., the n–dimensional torus T^n.

More complicated configuration manifolds [AM78]– [MR94], i.e., smooth (hyper) surfaces, emerge in the study of more complex biomechanical systems.

These conditions are usually given in the form of equations to which the coordinates of all the point masses should satisfy (i.e., geometric relations). It is the set of geometric relations that determines a smooth manifold immersed in the real Euclidean space R^{3n}, where n is the number of point masses. An ordered set of coordinates in R^{3n} of n point masses determines a position of a mechanical system in the configuration manifold.

In this way, each point of a smooth configuration manifold corresponds to a certain position of the biomechanical system in consideration [Iva91]. Under the action of muscular and external forces, the biomechanical system alters its position. The *representative point* of the configuration manifold corresponding to it moves describing a certain trajectory, viz., a path on the manifold. An important characteristic of this type of motion is the velocity which changes with time. The state of a biomechanical system at each given moment of time is defined as a Hamiltonian angle–momentum pair (q, p). The collection of all states of the biomechanical system is called the *biomechanical phase–space*. The state of the system varies with time in accordance with the laws of biomechanics, describing the path called the *phase trajectory* (the *phase orbit*, in the case of closed trajectories) of the point $q = q^i(t)$. This process is causal, which means that the state of the system is determined uniquely, in the future and in the past, by its present state. Such processes are described by their dynamical groups (i.e., *translational and rotational Lie groups*), and their infinitesimal generators (i.e., associate *Lie algebras*), representing the biomechanical law in consideration.

4.9 Estimation of Musculo–Skeletal Parameters

To obtain the set of relevant parameters, as well as input–output data for the Hamiltonian dynamical system (*) we need to establish the valid experimental methodology and equipment. Here we present one of the possible solutions.

4.9.1 Measurement of Muscular Input Torqes

- A multichannel integrated EMG can be used on–line for exact timing of muscular contractions and approximate estimating of the torques exerted during various moving patterns.
- ‘Cybex’ isokinetic dynamometers can be used off–line for dynamical measuring submaximal muscular torques (maximal torques are possible only in static conditions) in the following moving patterns:

1. Shoulder:
   –adduction/abduction
   –flexion/extension
   –internal/external rotation
   –horizontal adduction/abduction;

2. Elbow:
   –flexion/extension;
3. Forearm/wrist:
   –pronation/supination
   –flexion/extension
4. Hip:
   –flexion/extension
   –adduction/abduction
   –internal/external rotation
5. Knee:
   –tibial internal/external rotation
6. Ankle:
   –plantar/dorsi flexion
   –inversion/eversion

- Muscular data fitting (nonlinear least squares fitting using the sophisticated Levenberg–Marquardt (LM) nonlinear regression algorithm[3]) by the use of impulse force–time (torque–time) and force–velocity (torque–momentum) curves.

4.9.2 Measurement of Skeleton and Joint Parameters

- The following parameters of the body segments included in the model can be estimated by usual ergonomics measurements:
  1. segmental lengths;
  2. segmental center–of–mass positions;
  3. segmental masses;
  4. segmental moments of inertia.
- Estimation / calculation of linear and quadratic damping–elastic joint parameters.

4.9.3 Testing of Model Outputs

- Cinematographic analysis of model–output trajectories, using:
  1. Opto–electronic system can be used on–line for exact experimental tracking of joint orbits, trajectories of segmental centers of masses as well as other interesting points, which can be compared with simulated trajectories.

[3] The LM algorithm combines the steepest–descent method and a Taylor series based method to obtain a fast, reliable technique for nonlinear optimization; the steepest descent method works best far away from the minimum, and the Taylor series method works best close to the minimum. The LM algorithm allows for a smooth transition between these two methods as the iteration proceeds.

2. High–speed video or cine–film system can be used off–line for approximate estimating of joint orbits, trajectories of segmental centers of masses as well as other interesting points, which can be roughly compared with simulated trajectories.

- Force–plate testing of ground–reaction forces as predicted by the model. The Piezoelectric force platforms can be used on–line for measuring of ground–reaction forces occurring in a single– and a double–leg take–off, running, walking, somersault and other ground–reaction movements. Such obtained force–time and torque–time signals can be further FFT–filtered or least–squares–fitted to get the closed–form mathematical functions, suitable for comparison with the model predicted ground–reaction forces.

4.9.4 Further Analysis of Model Outputs

- Transient analysis of joint coordinates $q = q(t)$ and momenta $p = p(t)$ – trajectories. Numerically differentiate them to obtain joint angular velocities $\dot{q} = \dot{q}(t)$ and torques $\dot{p} = \dot{p}(t)$.
- Phase–plane analysis, giving deeper insight into nonlinear (possibly chaotic) dynamics of joint movements based on plotting $p - q$ trajectories in the phase–plane. Search for singular points, bifurcations, attractors and repellors.
- Spectrum and cross–correlations analysis using advanced, FFT–based frequency analysis of trajectories of special sport–technique related interest, as well as mutual coordination of several joints.

4.10 Stochastic Forces

To make our deterministic Hamiltonian biomechanics more realistic, we have to include *stochastic forces*, which are of two kinds

1. continuous diffusion (Wiener) fluctuations, and
2. discontinuous ('Master') jumps.

Description of the stochastic forces is based on the concept of *Markov process*, which represents the probabilistic analogue to the deterministic dynamics. The Markov process is characterized by a *lack of memory*, i.e., the statistical properties of the immediate future are uniquely determined by the present, regardless of the past (see [Iva91]).

For example, a *random walk* is *Markov chain*, i.e., a discrete–time Markov process, such that the motion of the system in consideration is viewed as a sequence of states, in which the transition from one state to another depends only on the preceding one, or the probability of the system being in state k depends only on the previous state $k - 1$. The property of a Markov chain of prime importance in biomechanics is the existence of an *invariant* *distribution of states*: we start with an initial state x_0 whose absolute probability is 1. Ultimately the states should be distributed according to a specified distribution.

Between pure deterministic (in which all degrees of freedom of the system in consideration are explicitly taken into account, leading to classical dynamical equations like Hamilton's) and pure stochastic dynamics (Markov process), there is so called *hybrid dynamics*, particularly *Brownian dynamics*, in which some of degrees of freedom are represented only through their *stochastic influence* on others. As an example, suppose a system of particles interacts with a viscous medium. Instead of specifying a detailed interaction of each particle with the particles of the viscous medium, we represent the medium as a *stochastic force* acting on the particle. The stochastic force *reduces the dimensionally* of the dynamics.

The Brownian dynamics represents the phase–space trajectories of a collection of particles that individually obey *Langevin* equations in the field of force (i.e., the particles interact with each other via some deterministic force). For one free particle the Langevin equation of motion is given by

$$m\dot{v} \,=\, R(t) \,-\, \beta v,$$

where m denotes the mass of the particle and v its velocity. The right–hand side represent the coupling to a *heat bath*; the effect of the random force $R(t)$ is to heat the particle. This is exactly what happens in human motion dynamics in the heating of Hill's *muscular actuators* [Hil38]. To balance overheating (on the average), the particle is subjected to *friction* β. In human motion dynamics this is performed with the Rayleigh – Van der Pol's *joint dissipation*.

The stochastic properties of the Brownian dynamics depend significantly on the stochastic properties of the random force $R(t)$. The random force $R(t)$ is Gaussian distributed. Then the problem boils down to finding the solution to the Langevin stochastic differential equation with the supplementary condition (mean zero and variance)

$$< R(t) >= 0, \quad < R(t)\, R(0) >= 2\beta k_B T \delta(t),$$

where $< . >$ denotes mean value, T is temperature, k_B−*equipartition* (i.e uniform distribution of energy) coefficient, $\delta(t)$ – Dirac 'delta'–function.

Algorithm for computer simulation of the Brownian dynamics (for a single particle) can be written as [Iva91]:

1. Assign an initial position and velocity.
2. Draw a random number from a Gaussian distribution with mean zero and variance.
3. Integrate the velocity to obtain v^{n+1}.
4. Add the random component to the velocity.

Another approach to taking account the coupling of the system to a heat bath is to subject the particles to collisions with *virtual* particles [Iva91]. Such collisions are imagined to affect only momenta of the particles, hence they affect the kinetic energy and introduce fluctuations in the total energy. Each stochastic collision is assumed to be an instantaneous event affecting only one particle.

Chapter 5
CSB–System

5.1 Linear CSB

Generalizing the basic CSB–system, with one input and one output (Figure 3.1), and including the *principle of superposition* for the system inputs and outputs, we come to the R.E. Kalman's [Kal60] modular concept of linear multivariable system (Figure 5.1)

$$\dot{x} = Ax + Bu, \qquad y = Cx, \qquad u = Dy$$

with the *space of input signals* (stimuli, excitations) $U = \{u\}$, *state space* $X = \{x, \dot{x}\}$ and *space of output signals* (reactions, responses) $Y = \{y\}$. (A linear, or vector, space is an ordered set whose elements can be added to each other, previously multiplied by a constant.)

It is obvious that the linear system in the vector state–space is an idealized case, because in nature nothing is linear, but it is the unique existing model which always works well in various cybernetic fields, can be immediately used for understanding and prediction of complex multivariable systems, and its linear *feedback gains* are actually smoothing the natural *nonlinearities*.

To introduce the subject, we first have to make a distinction between (bio)*physical* and *abstract* objects. A (bio)physical object is an object perceived by our senses whose time behavior we wish to describe, and its abstraction is the mathematical relationships that give some expression for its behavior (according to the principle of causality). This distinction is made because, in making an abstraction, it is possible to lose some of the relationships that make the abstraction behave similar to the (bio)physical object, and also, not all mathematical relationships can be realized by a (bio)physical object.

The Kalman's concept of *state* relates to those (bio)physical objects whose behavior can change with time, and to which a stimulus can be applied and the response observed. To predict the future behavior of the (bio)physical object under any input, a series of experiments could be performed by

T.T. Ivancevic et al.: Complex Sports Biodynamics, COSMOS 2, pp. 55–67.
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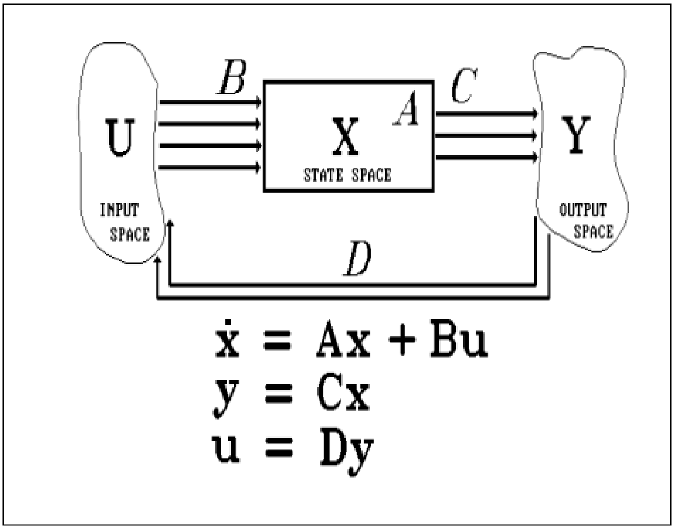

Fig. 5.1 The generic linear CSB–system

applying stimuli, or inputs, and observing the responses, or outputs. From these experiments we could obtain a listing of these inputs and their corresponding observed outputs, i.e., a list of input–output pairs. An *input–output pair* is an ordered pair of real time functions defined for all $t > t_0$, where t_0 is the time the input is first applied.

The state of a (bio)physical object is any property of the object which relates input to output such that knowledge of the input time–function for $t > t_0$ and state at time $t = t_0$ completely (causally) determines a unique output for $t > t_0$.

An abstract object is the totality of input–output pairs that describe the behavior of a (bio)physical object. We can often summarize the abstract object by using the mathematical equations representing (bio)physical laws.

The *state of an abstract object* is a collection of numbers which together with the input $u(t)$ for all $t > t_0$ uniquely determines the output $y(t)$ for all $t > t_0$. In essence the state parameterizes the listing of input–output pairs. The state is the answer to the question 'Given $u(t)$ for $t > t_0$ and the mathematical relationships of the abstract object, what additional information is needed to completely specify $y(t)$ for $t > t_0$?'

A *state variable*, denoted by the vector $x(t)$, is the time function whose value at any specified time is the state of the abstract object at time. The state space, denoted by X, is the set of all $x(t)$.

The state representation is not unique. There can be many different ways of expressing the relationship of input to output.

As an example, let us imagine a typical sport–physiological experiment: a controlled system or plant is an athlete running on a treadmill with a respiratory–mask on his face and ECG–electrodes on his chest. The controlled inputs to the system are velocity and inclination of the treadmill. The continually measured outputs are: ECG and maximal relative oxygen uptake, while the discontinuously measured output is concentration of lactates in the athlete's blood. The state–space of the system is the athlete's organism with its most significant physico–chemical reactions.

The linearity hypothesis states that the inputs are mutually summable (which quite agrees with the nature of the experiment). This input–linearity hypothesis implies less obvious (or less natural) conclusions that both the system outputs and the states are also linear (i.e., mutually summable). ECG represents the controlled output variable from which the two feedback paths lead towards the system input (if the heart–frequency is too high, or some extra–systole or some heart–noise occurs, the input loading should be decreased; if the heart–frequency is too low for achieving the maximum lactate concentration or maximal relative oxygen uptake, i.e., significantly below 200 bits per minute, the input should be increased, either by the velocity or by the inclination).

Matrix A corresponds to the state operator (i.e., multilinear mapping) of the system time evolution, thus $A : X_t \to X_{t+1}$; B is input operator, thus $B : U \to X$; C is output operator, thus $C : X \to Y$; D is feedback operator, thus $D : Y \to U$ (see Figure 4.2). The first equation, solved for the state vector $\dot{x} \in X$ is the state equation; the second, solved for the output vector $y \in Y$, is the output equation, and the third, solved for the input vector $u \in U$, is the control (or feedback) equation. The last equation (the feedback operator D) is usually connected to the sophisticated optimization tool, Kalman's regulator (based on the minimization of some energy–functional, and mathematically obtained by the solution of matrix Riccati equation).

If the initial values $x_0 = x(0)$ of the chosen state variables $x \in X$ (representing some most significant physiological processes in the athlete's organism) are known, and the matrices (A, B, C, D) are identified for each particular athlete (statistical methodology of parameter identification has been classically based on the concept of cross–correlation and various autoregressive estimation procedures with moving averages, like AR, ARMA, ARIMA, ARMAX models – for linear systems; NARMAX models and Wiener's white–noise based functional expansion methodology were recently developed for nonlinear system identification; modern recursive estimation techniques include filtration concepts, like Kalman's filter; herein, we propose *factor analysis methods* for discipline–specific identification of the matrices (A, B, C, D) – see below), the future behavior of the system considered (i.e., particular athlete tested) can be more or less exactly predicted according to Newton's causality principle. Although analytic methods of solution of the state equation exist (via transition matrix and its convolution integral), much easier and efficient is numerical approximate solution by computer simulation (either by writing the state, input and output equation, together with initial conditions, into some mathematical software–package (like 'Matlab', with

'Control Toolbox'), or by coding them into some of the higher programming languages (like 'Visual C^{++}'). It is obvious that the presented cybernetic apparatus works as well for the arbitrarily great dimensions of the system (just for the sake of simplicity of explanation we used the example with 2 inputs, 3 outputs and 2 feedback paths).

The presented model of the Kalman's multilinear system behavior in its hypothetical state space and associated Kalman's regulator (together with an adequate mathematical software–package for numerical solution of this initial value problem, as well as a sophisticated statistical procedure for identification of the system matrices (A, B, C, D)) – represents the first serious computer–simulation apparatus for the optimal control of the sport training process.

This model is also the linear and deterministic basis for some more sophisticated models of the training process, like nonlinear dynamics (bifurcations and fractal attractors in the system's phase–space), stochastic dynamics (Fokker–Planck's equation for the system's probability–density and related Ito's calculus) and filtration theory (Wiener's and Kalman's filters), which are all applied in the modern concept of learning (training) of neural networks (see below).

5.2 Functional *CSB*

The general frame for modelling of the complex physiological structures and functions of human organism gives the theory of functional systems of P. K. Anohin (see [Iva93]). Its operation architectonics (see Figure 5.2) includes the following links:

1. Functional adaptation result as the leading element and the fundamental factor around which the functional system is formed;
2. Receptors of the result;
3. Feedback afferentation upon output results;
4. Central architecture;
5. Somatic, vegetative and endocrine components included into the organization of the functional (or useful for adaptation) behavior.

Anohin's model includes four distinct groups of activity results of particular subsystems defining normal conditions for the flow of metabolic processes in human organism. The first group consists of internal (homeostatic) constants which determine the normal metabolism by their interactions. The second group consists of adaptation results of interactions between organism and its surroundings, directed to satisfaction of internal biological needs and survival of specie. The third group consists of results of human group activities directed to satisfaction of their biological needs. The fourth group consists of results of human social activities. The vertical hierarchy is expressive between the groups: transition onto the higher level is caused by previous satisfaction of the lower one.

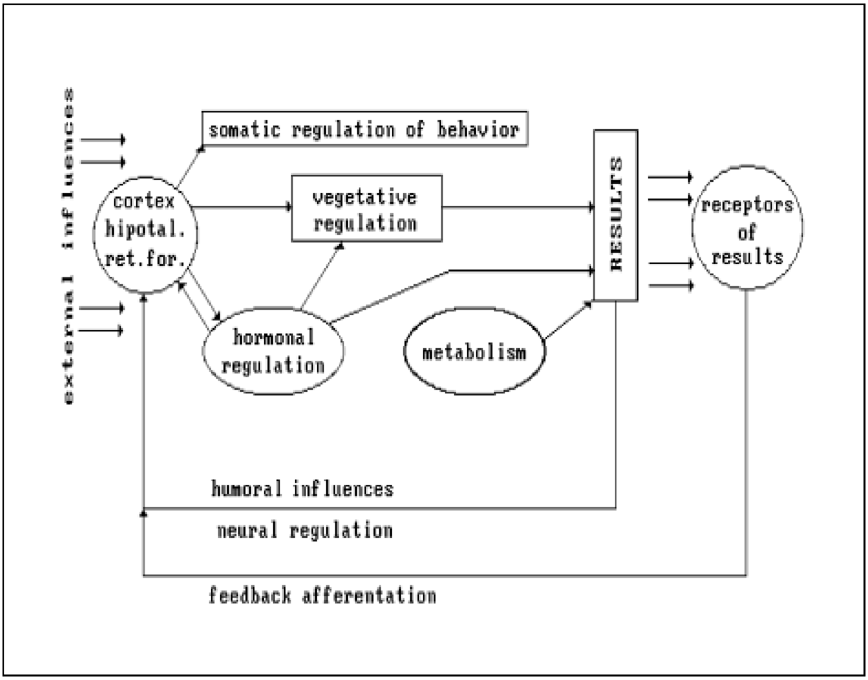

Fig. 5.2 Anohin's model of the functional systems

Anohin's rule of the 'golden norm' says that the factors returning the organism into its homeostatic state are always stronger then disturbances (both external and internal) from its normal metabolism dynamics. This rule is the bases of an automatic auto–regulative process which activates immediately upon any disturbance generated on some of the higher levels. And the bigger the difference between particular result and the level of the normal metabolism, the bigger is the reaction of mechanisms directed to its stabilization.

Now, we can state the fundamental principle of the sports training process: optimal dosage (by intensity and volume of the training loading, see next paragraph) of external disturbances upon the homeostasis, and careful control of the factors returning the organism into its homeostatic state, in order to achieve homeostasis on the desired, higher level. Therefore, in the training process we artificially, in controlled conditions, impose certain disturbances upon the homeostasis and develop the natural capacity of its returning factors, for improving its stability and maintaining it on the higher level of homeostatic constants.

This fundamental principle of the sports training can be also defined as the *overloading principle*: It is limiting (the greatest possible) disturbance of homeostasis to which the organism responds by positive adaptation (if this limit is crossed, the organism goes out of balanced homeostatic dynamic – homeokinaesis – and pathological processes begin). So, it is necessary to choose such training methods which most effectively disturb organism's homeokinaesis, and yet do not lead to pathology. Here we should emphasize that the amount of work applied in the training process and externally manifested effect are not in linear connection. The organism, in the process of natural adaptation, tolerates (does not respond to them) all loadings below certain threshold (these are so called 'subliminal loadings') and responds by desired adaptations (i. e., structural and functional transformations of physical and chemical mechanisms and processes of the organism) only when the loadings are higher then the threshold of the training excitation ('supraliminal loadings').

5.3 Nonlinear *CSB*

The generalized Hamiltonian model for the sport training process is depicted in Figure 5.3. It requires system identification using neural networks (see below). Here, the *champion pattern structure* is estimated by means of exploratory factor analysis. After spanning its phase–space with orthogonal factors, the pattern structure is identified in the form of dissipative, driven, stochastic Hamiltonian dynamical system.

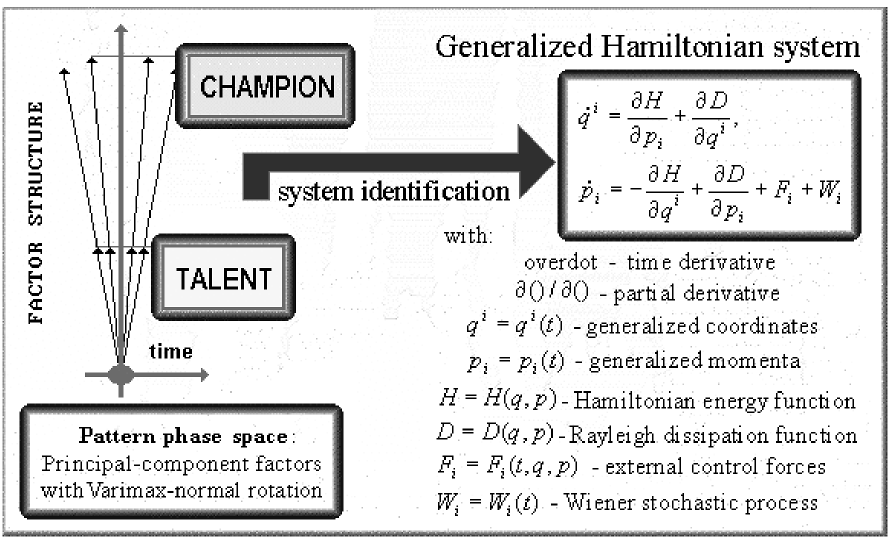

Fig. 5.3 The generalized Hamiltonian system of the sport training process. Evolution of the unique pattern-structure, including: (i) Functional evolution, or local training-flow; and (ii) Parameter evolution, or global phase-transitions – is the purpose of the training process.

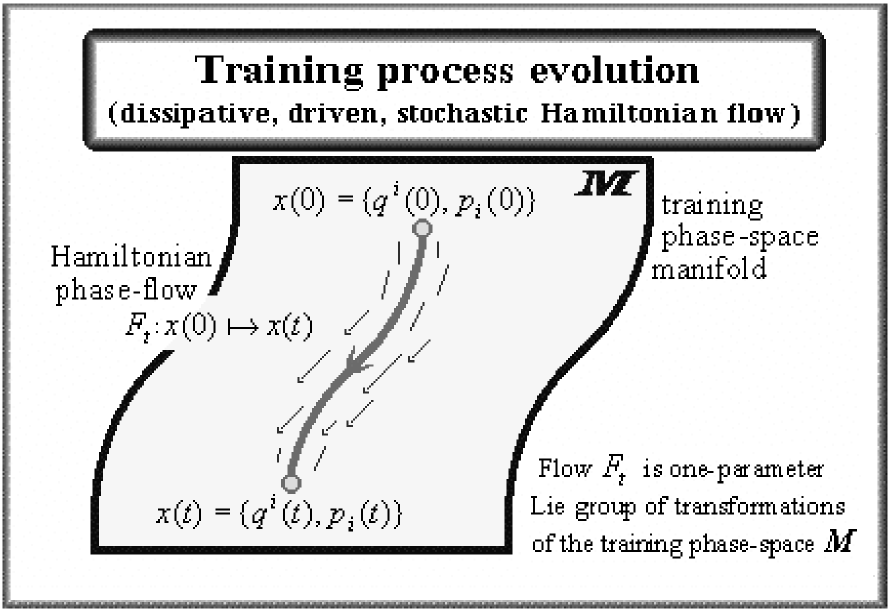

Fig. 5.4 Dissipative, driven, stochastic Hamiltonian system, or Hamiltonian vector-field dx/dt, with specified initial state $x(0)$, uniquely defines the training phase–space flow $x(t)$

Now, the general, nonlinear, *sport–training flow* is obtained by numerical solution of general *Hamiltonian training equations* (see Figure 5.4). Initial state $x(0)$ corresponds to the talent–phase of the training dynamics (see [II06b]).

Finally, a few technical words about *stability* [Iva91]. The output of a linear system, as a function of time, can be broken down into two parts, *steady state* terms which are directly related to the input, and *transient* terms which are either exponential of oscillatory with an envelope of exponential form. If the exponentials (oscillations) decay to zero, the system is said to be *stable*. If any exponential (oscillation) increases, the system is said to be *unstable*.

Instability is commonly an unfortunate product of closed loop control. The higher the open loop–gain, the more likely is instability; thus, high precision closed loop control and good stability are incompatible. An unstable system does not require an input to start oscillation; the act of closing the loop or small noise signals are sufficient.

Stability is one particular feature which is universally considered necessary for satisfactory performance, but the dynamic nature of the system's components often makes it difficult to achieve. Negative (i.e., homeostatic) feedback is a necessary technique by which the long–term stability in complex systems like human organism can be obtained, but its presence is not a sufficient condition for the stability.

5.4 *CSB*−Cognition

Recall that the term *cognition* (Latin: *cognoscere = to know*) is used in several loosely–related ways to refer to a facility for the human like processing of information, applying knowledge and changing preferences.[1]

Cognition, (consisting of a set of cognitive processes) can be natural or artificial, conscious or subconscious [II07b]. Therefore, it can be analyzed from different perspectives and in different contexts, in neurology, psychology, philosophy, system science and computer science. The concept of cognition is closely related to such abstract concepts as, mind, reasoning, perception,

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[1] For example, according to the so–called *top–down object–based goal-oriented approach* (TOGA) standard, the Information–Preferences–Knowledge *cognitive architecture* consists of:

Data: everything what is/can be processed/transformed in computational and mental processes. Concept data is included in the ontology of 'elaborators', such as developers of methods, programmers and other computation service people. In this sense, data is a relative term and exists only in the couple (data, processing).

Information: data which represent a specific property of the domain of human or artificial agent's activity (such as: addresses, tel. numbers, encyclopedic data, various lists of names and results of measurements). Every information has always a source domain. It is a relative concept. Information is a concept from the ontology of modeler/problem–solver/decision–maker.

Knowledge: every abstract property of human/artificial agent which has ability to process/transform a quantitative/qualitative information into other information, or into another knowledge. It includes: instructions, emergency procedures, exploitation/user manuals, scientific materials, models and theories. Every knowledge has its reference domain where it is applicable. It has to include the source domain of the processed information. It is a relative concept.

Preference: an ordered relation among two properties of the domain of activity of a *cognitive agent*, it indicates a property with higher utility. Preference relations serve to establish an intervention goal of an agent. Cognitive preferences are relative. A preference agent which manages preferences of an intelligent agent can be external or its internal part.

Goal: a hypothetical state of the domain of activity which has maximal utility in a current situation. Goal serves to the choice and activate proper knowledge which process new information.

Document: a passive carrier of knowledge, information and/or preferences (with different structures), comprehensive for humans, and it has to be recognized as valid and useful by one or more human organizations, it can be physical or electronic.

Computer Program: (i) from the modelers and decision-makers perspective: an active carrier of different structures of knowledge expressed in computer languages and usually focused on the realization of predefined objectives (a design-goal). It may include build-in preferences and information and/or request specific IPK as data. (ii) from the software engineers perspective: a data-processing tool (more precise technical def. you may find on the Web).

intelligence, learning, and many others, which describe numerous capabilities of human mind and expected properties of artificial, or synthetic intelligence. Cognition is an abstract property of advanced living organisms, so it is studied as a direct property of a brain, or of an abstract mind on symbolic and sub–symbolic levels.

In psychology and AI, it is used to refer to the mental functions, mental processes and states of intelligent entities (humans, human organizations, highly autonomous robots), with a particular focus toward the study of such mental processes as, comprehension, inferencing, decision-making, planning and learning (see also cognitive science and cognitivism). Recently, advanced cognitive researchers are especially focused on the capacities of abstraction, generalization, concretization/specialization and meta-reasoning which descriptions involve such concepts as, beliefs, knowledge, desires, preferences and intentions of intelligent individuals/objects/agents/systems.

The term ‘cognition’ is also used in a wider sense to mean the act of knowing or knowledge, and may be interpreted in a social or cultural sense to describe the emergent development of knowledge and concepts within a group that culminate in both thought and action.

On the other hand, recall that Teuvo Kohonen’s *associative memory* (AM), also called the *content–addressed memory*[2] [Koh82, Koh88], or *associative storage*, or *associative array*, represents a high–speed–searching memory organization in which the *memory is accessed by its content*, which is opposed to an explicit address by the memory location within the certain storage medium. Thus, reference clues are ‘associated’ with actual memory contents until a desirable match (or set of matches) is found. For example, AM stands as the most likely model for various cognitive memories. At the same time, obvious examples of systems that employ such a memory are *production systems*, tools used by AI (especially within the *expert systems*), consisting of:

1. A database of rules;
2. A working memory;
3. A pattern matcher; and
4. A procedure that resolves conflicts between the rules.

Naturally, humans retrieve information best when it can be *associated* (or linked) to other related information. This *association–map*, or more generally, *association–functor* (see below) is fast, direct, many–to–many, and homomorphic.

In modern computational intelligence (see [II07b]), unlike standard computer memory (random access memory, or RAM) in which the user supplies a memory address and the RAM returns the data word stored at that address, an AM is designed such that the user supplies a data word and the AM searches its entire memory to see if that data word is stored anywhere in it. If the data word is found, the AM returns a list of one or more storage addresses where the word was found (and in some architectures, it also

[2] Note that conventional shortcut is CAM, not AM, which we use for simplicity.

returns the data word, or other associated pieces of data). Thus, an AM is the hardware embodiment of what in software terms would be called an associative array. As an AM is designed to search its entire memory in a single operation, it is much faster than RAM in virtually all search applications.[3]

AM is often used in *computer networking devices*. For example, when a network switch receives a packet from one of its ports, it updates an internal table with the packet's source *MAC address*[4] and the port it was received on. It then looks up the destination MAC address in the table to determine what port the packet needs to be forwarded to, and sends it out that port. The MAC address table is usually implemented with a binary AM so the destination port can be found very quickly, reducing the switch's latency.

Ternary AMs are used in network routers, where each address has two parts: the network address, which can vary in size depending on the *sub–net configuration*, and the host address, which occupies the remaining bits.[5]

[3] Note, however, that there are cost disadvantages to AM. Unlike a RAM chip, which has simple storage cells, each individual memory bit in a fully parallel AM must have its own associated comparison circuit to detect a match between the stored bit and the input bit. Additionally, match outputs from each cell in the data word must be combined to yield a complete data word match signal. The additional circuitry increases the physical size of the AM chip which increases manufacturing cost. The extra circuitry also increases power dissipation since every comparison circuit is active on every clock cycle. Consequently, AM is only used in specialized applications where searching speed cannot be accomplished using a less costly method. To achieve a different balance between speed, memory size and cost, some implementations emulate the function of AM by implementing standard tree search or hashing designs in hardware, using hardware tricks like replication or pipelining to speed up effective performance. These designs are often used in routers.

[4] In computer networking a Media Access Control address (MAC address) is a unique identifier attached to most forms of networking equipment. Most layer 2 network protocols use one of three numbering spaces managed by the IEEE: MAC-48, EUI-48, and EUI-64, which are designed to be globally unique. Not all communications protocols use MAC addresses, and not all protocols which do require such globally unique identifiers. The IEEE claims trademarks on the names 'EUI-48' and 'EUI-64'. ARP/RARP is commonly used to map the layer 2 MAC address to an address in a layer 3 protocol such as Internet Protocol (IP). On broadcast networks such as Ethernet the MAC address allows each host to be uniquely identified and allows frames to be marked for specific hosts. It thus forms the basis of most of the layer 2 networking upon which higher OSI Layer protocols are built to produce complex, functioning networks.

[5] Each sub–net has a network mask that specifies which bits of the address are the network address and which bits are the host address. Routing is done by consulting a routing table maintained by the router which contains each known destination network address, the associated network mask, and the information needed to route packets to that destination. Without AM, the router compares the destination address of the packet to be routed with each entry in the routing table, performing a logical AND with the network mask and comparing it with

In software engineering, an *associative array* (also called *dictionary*, *finite map*, *lookup table*,[6] and in query–processing an *index file*) is an *abstract data type*[7] composed of a collection of keys and a collection of values, where each key is associated with one value. The operation of finding the value associated with a key is called a lookup or indexing, and this is the most important operation supported by an associative array. The relationship between a key and its value is sometimes called a *mapping* or *binding*. Associative arrays are very closely related to the mathematical concept of a *function with a finite domain*. As a consequence, a common and important use of associative arrays is in *memoization*.[8] From the perspective of a programmer using an associative array, it can be viewed as a *generalization of an array*: While a regular array maps integers to arbitrarily typed objects, an associative array maps arbitrarily typed objects to arbitrarily typed objects. The operations that are usually defined for an associative array are:

(i) Add: Bind a new key to a new value;
(ii) Reassign: Bind an old key to a new value;

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the network address. If they are equal, the corresponding routing information is used to forward the packet. Using a ternary AM for the routing table makes the lookup process very efficient. The addresses are stored using 'don't care' for the host part of the address, so looking up the destination address in the AM immediately retrieves the correct routing entry; both the masking and comparison are done by the AM–hardware.

[6] Recall that in computer science, a lookup table is a data structure, usually an array or associative array, used to replace a runtime computation with a simpler lookup operation. The speed gain can be significant, since retrieving a value from memory is often faster than undergoing an expensive computation.

[7] Recall that in computing, an abstract data type (ADT) is a specification of a set of data and the set of operations that can be performed on the data. Such a data type is abstract in the sense that it is independent of various concrete implementations. The definition can be mathematical, or it can be programmed as an interface. The interface provides a constructor, which returns an abstract handle to new data, and several operations, which are functions accepting the abstract handle as an argument.

[8] Recall that Memoization is a technique used to speed up computer programs by storing the results of functions for later reuse, rather than recomputing them. Memoization is a characteristic of *dynamic programming*. Functions can only be memoized if they are referentially transparent – that is, if they will always return the same result given the same arguments. Operations which are not referentially transparent, but whose results are not likely to change rapidly, can still be cached with methods more complicated than memoization. In general, memoized results are not expired or invalidated later, while caches generally are. In imperative languages, both memoization and more general caching are typically implemented using some form of associative array. In a *functional programming language* it is possible to construct a higher-order function memoize which will create a memoized function for any referentially transparent function. In languages without higher-order functions, memoization must be implemented separately in each function that is to benefit from it.

(iii) Remove: Unbind a key from a value and remove it from the key set; and
(iv) Lookup: Find the value (if any) that is bound to a key.

All these machines, most notably *artificial neural networks* (ANNs, see section below), have capability of storing, recognizing and classifying *spatio–temporal patterns*. A *pattern* can be stored in ANN by associating (through an appropriate choice of the connections among the neurons of a network) the configurations of the nervous activity elicited by the presentation of the pattern itself with a steady–state, or *attractor*, of the network. Many physical systems can operate as AMs, once their dynamical evolutions converge towards their attractors (from arbitrary initial conditions).

Recall that above mentioned *attractor* is a set of system's states (i.e., points in the system's phase–space), invariant under the dynamics, towards which neighboring states in a given *basin of attraction* asymptotically approach in the course of dynamic evolution.[9] An attractor is defined as the smallest unit which cannot be itself decomposed into two or more attractors with distinct basins of attraction. This restriction is necessary since a dynamical system may have multiple attractors, each with its own basin of attraction. Conservative systems do not have attractors, since the motion is periodic. For dissipative dynamical systems, however, volumes shrink exponentially, so attractors have 0 volume in nD phase–space. In particular, a stable *fixed–point* surrounded by a dissipative region is an attractor known as a *map sink*.[10] Regular attractors (corresponding to 0 *Lyapunov exponents*) act as *limit cycles*, in which trajectories circle around a limiting trajectory which they asymptotically approach, but never reach. The so–called *strange attractors*[11] are bounded regions of phase–space (corresponding to positive Lyapunov characteristic exponents) having zero measure in the embedding phase–space and a *fractal dimension*. Trajectories within a strange attractor appear to skip around randomly.

In psychology, there is an AM–related term *trans–derivational search* (TDS), meaning when a search is being conducted for a fuzzy match across a broad field. In computing the equivalent function can be performed using AM. Unlike usual searches, which look for literal (i.e., exact, logical, or regular expression) matches, a trans–derivational search is a search for a possible meaning or possible match as part of communication, and without which an incoming communication cannot be made any sense of whatsoever. It is thus an integral part of processing language, and of attaching meaning to

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[9] A *basin of attraction* is a set of points in the system's phase–space, such that initial conditions chosen in this set dynamically evolve to a particular attractor.

[10] A *map sink* is a stable fixed–point of a map which, in a dissipative dynamical system, is an attractor.

[11] A strange attractor is an attracting set that has zero measure in the embedding phase–space and has fractal dimension. Trajectories within a strange attractor appear to skip around randomly.

communication. An example of TDS use is in *Ericksonian hypnotherapy*,[12] where vague suggestions are used that the patient must process intensely to find their own meanings for, thus ensuring that the practitioner does not intrude his own beliefs into the subject's inner world.

[12] Hypnotherapy is the application of hypnosis as a form of treatment, usually for relieving pain or conditions related to one's state of mind. Practitioners believe that when a client enters, or believes he has entered, a state of trance, the patient is more receptive to suggestion and other therapy. The most common use of hypnotherapy is to remedy maladies like obesity, smoking, pain, ego, anxiety, stress, amnesia, phobias, and performance but many others can also be treated by hypnosis, including functional disorders like Irritable Bowel Syndrome. American psychiatrist M. H. Erickson is noted for his often unconventional approach to psychotherapy, such as described in the book 'Uncommon Therapy' by Jay Haley and the book 'Hypnotherapy: An Exploratory Casebook' by Milton H. Erickson and Ernest L. Rossi. New York, Irvington Publishers, Inc. (1979).

Chapter 6
CSB–Synergetics: Escape from Chaos

6.1 Biomechanical Chaos

The so–called *deterministic chaos* represents *irregular and unpredictable* time evolution of many simple nonlinear deterministic systems [GOY87, II07a], like a single–joint musculo–skeletal movement. Its central characteristics is that the system *does not repeat its past behavior* (even approximately). If we now the forcing amplitude A and the frequency w of a muscle–actuator, as well as the linear damping b in the uniaxial joint, then the motion of the flexion–extension type can be described in canonical $q-p$–coordinates as

1. as a linear harmonic oscillator
$$\dot{q} = p, \qquad \dot{p} + b\,p + q = Acos(w\,t)$$
or,
2. as a nonlinear Hamilton's oscillator
$$\dot{q} = p, \qquad \dot{p} + b\,p + sinq = Acos(w\,t)$$

The difference between these two muscle–joint models is just in the q versus $sinq$ term, but the produced dynamics is qualitatively different. However, for a very short time, or a small change of the joint angle q, the difference could be neglected, and nonlinear model could be satisfactorily approximated by a linear one.

The unique character of chaotic dynamics may be seen most clearly by imagining the system to be started twice, but from slightly different initial conditions (initial joint angle and momentum). We can think of this small initial difference as resulting from measurement error. For non–chaotic systems this uncertainty leads only to an error in prediction that *grows linearly* with time. For chaotic systems, on the other hand, the error grows *exponentially* in time, so that the state of the system is essentially unknown after

T.T. Ivancevic et al.: Complex Sports Biodynamics, COSMOS 2, pp. 69–78.
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very short time. This phenomenon, firstly recognized by Poincaré in 1913, which occurs only when the governing equations are nonlinear, is known as *sensitivity to initial conditions*. Another type of sensitivity of chaotic systems is *sensitivity to parameters*: a small variation of system parameters (like segment mass, length and moment of inertia) results in great change of system output [GOY87].

If prediction becomes impossible, it is evident that a chaotic system can resemble a stochastic system (like randomly driven joint–movement). However, the source of the irregularity is quite different. For chaos, the irregularity is part of the intrinsic dynamics of the system, not unpredictable outside influences (like random muscular torques).

The *necessary condition for existence of chaos* satisfies any temporal (or spatio–temporal) continuous system, i.e., vector field, with three independent nonlinear variables and nonlinear coupling terms (like $SO(2)$–musculo–skeletal unit, determined by joint angle and momentum and muscle torque frequency). Whether the behavior of any such system will be actually chaotic depends upon the values of its parameters (and/or initial conditions). Usually, for some values of involved parameters the system behavior is oscillating, and for other values of the parameters the behavior is chaotic [GOY87].

Chaos theory has developed special mathematical procedures to *understand* irregularity and unpredictability of nonlinear systems, including Poincaré sections, bifurcation diagrams, power spectra, Lyapunov exponents, period doubling, fractal dimension, stretching and folding, special identification–estimation techniques, etc. [GOY87] On the other hand, the most powerful scientific tool for *escape from chaos* has been developed outside of the chaos theory, with intention to deal with much more complex, many–dimensional, highly nonlinear and coupled, stochastic, hierarchical systems, in the realm of Haken's *synergetics* [Hak83].

6.2 Basic Principles of Synergetics

The aim of synergetics (see [Hak83]) is to describe processes of *spontaneous self-organization and cooperation* in complex systems built from many subsystems which themselves can be complicated nonlinear objects, like musculo–skeletal components of human motion. General properties of the subsystems are their nonlinear–chaotic dynamics as well as their nonlinear–chaotic interactions. Furthermore, the systems of synergetics are regarded as *open*. The influence from outside is measured by a certain set of *control parameters* $\{\sigma\}$ (like amplitudes, frequencies and time characteristics of neuro–muscular driving forces). Processes of self–organization, or *coordination* in human motion dynamics are observed as temporal macroscopic patterns. They are described by a small set of *order parameters* $\{q\}$, similar to those in Landau's *phase–transition theory* of physical systems in *thermal equilibrium*.

On the lowest, *microscopic hierarchical level of organization*, the human motion dynamics can be described by dissipative, driven Hamiltonian system (in vector–matrix form)

$$\dot{q} = \partial H/\partial p + \partial R/\partial p, \qquad \dot{p} = -\partial H/\partial q + \partial R/\partial q + T(t, q, p, \sigma),$$

describing the human motion dynamics on the lowest, *microscopic hierarchical level of organization*, including n joints as musculo–skeletal components of interest, – *order parameter equations of macroscopic synergetics* can be either exactly derived along the lines of *mezoscopic synergetics*, or phenomenologically stated by the use of certain biophysical analogies and expert–knowledge based system identification and control.

Let us start with a quantity called *entropy*, which represents a*measu–re for the degree of disorder* in the system. The (phenomenologically derived) laws of thermodynamics state that in a closed system (i.e., a system with no contacts to the outer world) the entropy ever increases to its maximal value, i.e., to the total disorder of the system.

On the other hand, when we manipulate a system from the outside, by the use of certain control parameters $\{\sigma\}$, we can change its degree of order. Consider for example water vapor. At elevated temperature its molecules move freely without mutual correlation. When temperature is lowered, a liquid drop is formed, the molecules now keep a mean distance between each other. Their motion is thus highly correlated. Finally, at still lower temperature, at the freezing point, water is transformed into ice crystals. The transitions between the different aggregate states, also called phases, are quite abrupt. Though the same kind of molecules are involved all the time, the macroscopic features of the three phases differ drastically.

Similar type of ordering, but not related to the thermal equilibrium conditions, occurs in quantum lasers. Lasers are certain types of lamps which are capable of emitting coherent light. A typical laser consists of a crystal rod filled with gas, with the following important features (from the synergetics point of view): when the atoms the laser material consists of are excited or 'pumped' from the outside, they emit light waves. So, the pump power, or pump rate represents the control parameter σ. At low pump power, the waves are entirely uncorrelated as in a usual lamp. Could we hear light, it would sound like noise to us.

When we increase the pump rate to a critical value σ_c, the noise disappears and is replaced by a pure tone. This means that the atoms emit a pure sinusoidal light wave which in turn means that the individual atoms act in a perfectly correlated way – they become self–organized. When the pump rate is increased beyond a second critical value, the laser may periodically emit very intense and short pulses. In this way the following instability sequence occurs:

$$\text{noise} \mapsto \{\text{coherent oscillation at freq. } \omega_1\} \mapsto$$

$$\{\text{periodic pulses at freq. } \omega_2 \text{ modulating oscillation at freq. } \omega_1\}$$

i.e.,

$$\text{no oscillation} \mapsto 1\,\text{freq.} \mapsto 2\,\text{freq.}$$

Under different conditions the light emission may become chaotic or even 'turbulent'. The frequency spectrum becomes broadband.

The laser played a crucial role in the development of synergetics for various reasons. In particular, it allowed detailed theoretical and experimental study of the phenomena occurring within the transition region: *lamp* ↔ *laser*, where a surprising and far–reaching analogy with phase transitions of systems in thermal equilibrium was discovered. This analogy includes a symmetry breaking instability, critical slowing down and hysteresis effect.

6.3 Phase Transitions

An example for phase transitions (see Figure 6.1) in thermal equilibrium is water vapor. Another typical example is a *ferromagnet*. When a ferromagnet is heated, it suddenly loses its magnetization. When temperature is lowered, the magnet suddenly regains its magnetization. What happens on a microscopic, atomic level, is this: We may visualize the magnet as being composed of many, elementary (atomic) magnets (called spins). At elevated temperature, the elementary magnets point in random directions. Their magnetic moments, when added up, cancel each other and no macroscopic magnetization results. Below a critical value of temperature T_c, the elementary magnets are lined up, giving rise to a macroscopic magnetization. Thus the *order on the microscopic level* is a cause of a *new feature* of the material *on the macroscopic level*. The change of one phase to the other one is called *phase transition*.

Similar *disorder* ↦ *order* transitions occur also in various nonequilibrium systems of physics, chemistry, biology, psychology, sociology, as well as in biomechanics and CSB in general. The analogy is subsumed in Table 1.

Table 6.1 Phase transition analogy

System in thermal equilibrium	Nonequilibrium system
Free energy potential $\mathcal{F}$	Generalized potential V
Order parameters q_i	Order parameters q_i
$\dot{q}_i = -\frac{\partial \mathcal{F}}{\partial q_i}$	$\dot{q}_i = -\frac{\partial V}{\partial q_i}$
Temperature T	Control input u
Entropy S	System output y
Specific Heat c	System efficiency e

In the case of human motion dynamics, control inputs u_i are muscular torques T_i, system outputs y_i are joint momenta p_i, while system efficiencies e_i represent the changes of joint momenta with changes of corresponding muscular torques for the i-th joint

$$e_i = T_i \frac{\partial p_i}{\partial T_i}$$

Order parameters q_i represent certain important qualities of the human motion system, depending on muscular torques as control inputs, similar to *magnetization*, and usually defined by equations of the form

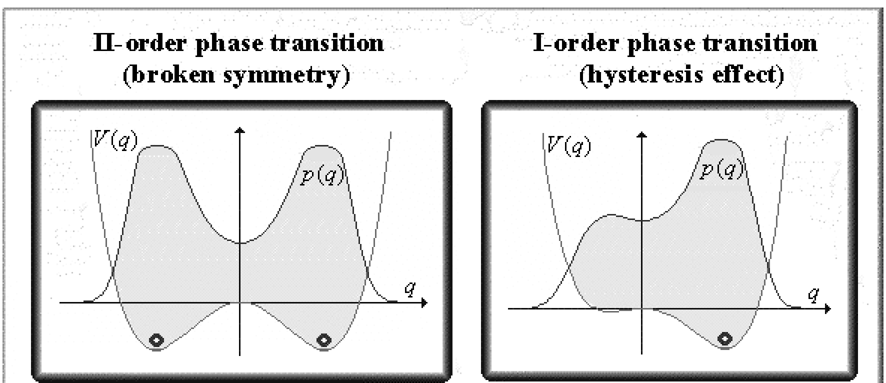
$$\dot{q} = -\alpha q - \beta q^3$$

or

$$\dot{q} = -\alpha q - \gamma q^2 - \beta q^3$$

with nonnegative parameters α, β, γ, and corresponding to the second and first order phase transitions, respectively. The choice of actual order parameters is a matter of *expert knowledge* and *purpose* of *macroscopic system modelling*.

Both in individual and in collective sports, the nonlinear training dynamics naturally passes through the series of self–organizing transitions, from lower, to higher degree of order. Either 'Soft', I-order or 'hard', II-order transitions (see Figure 6.1) in the training process are analogous to the phase–transitions of water molecules manipulated by temperature: from non–correlated phase in vapor, across highly correlated phase in liquid, to crystallized phase in solid ice. Similarly, the same individual, or team, in the training process, naturally passes through the series of phases, the first of them representing a talent–phase, and the last one, a champion–phase.



$$V(q) = \frac{\alpha}{2} q^2 + \frac{\beta}{4} q^4 \quad \longleftarrow \text{potential} \longrightarrow \quad V(q) = \frac{\alpha}{2} q^2 + \frac{\gamma}{3} q^3 + \frac{\beta}{4} q^4$$

$$\dot{q} = -\alpha q - \beta q^3 + W(t) \quad \leftarrow \text{order parameter equation} \rightarrow \quad \dot{q} = -\alpha q - \gamma q^2 - \beta q^3 + W(t)$$

$$\alpha < 0,\ \beta > 0 \qquad \text{probability density} \longrightarrow p(q) \approx e^{-V(q)} \qquad \alpha < 0,\ \beta, \gamma > 0$$

Fig. 6.1 Nonequilibrium phase transitions (see text for explanation)

6.4 Order Parameters

Equation (*) is in general quite complicated and cannot be solved completely in the whole phase–space Γ spanned by the set of possible joint vectors $\{q(t), p(t)\}$. We therefore have to restrict ourselves to local concepts for analyzing the behavior of our musculo–skeletal system. To this end we shall consider a reference musculo–skeletal state $\{q_0(t), p_0(t)\}$ and its neighborhood. Following Haken [Hak83], we assume that the reference state has the properties of an attractor and is a comparably low dimensional object in Γ. In order to explore the behavior of our movement system (dependent on the set of control parameters $\{\sigma\}$) in the neighborhood of $\{q_0(t), p_0(t)\}$ we look for the time development of small deviations from the reference state

$$q(t) = q_0(t) + \delta q(t), \qquad p(t) = p_0(t) + \delta p(t)$$

and consider $\delta q(t)$ and $\delta p(t)$ as small entities. As a result we may linearize the equations of δq and δp in the vicinity of the reference state $\{q_0(t), p_0(t)\}$. We now summarize the results by discussing the following time-evolution formulas for joint coordinates $q(t)$ and momenta $p(t)$

$$q(t) = q_0(t) + \xi_j(t)\, l^{(j)}(t), \qquad p(t) = p_0(t) + \zeta_j(t)\, k^{(j)}(t)$$

which describes the time dependence of the phase vectors $q(t)$ and $p(t)$ through the evolution of the collective patterns. Obviously, the reference musculo–skeletal state $\{q_0(t), p_0(t)\}$ can be called stable when all the possible excitations $\{\xi_j(t), \zeta_j(t)\}$ decay during the curse of time. When we now change the control parameters $\{\sigma\}$ some of the $\{\xi_j(t), \zeta_j(t)\}$ can become unstable and start to grow in time. The border between decay and growth in parameter space is called a *critical region*. Haken has shown that the few unstable amplitudes, denoted by u_q and u_p, change very slowly in the vicinity of a critical region, whereas the damped amplitudes, denoted by s_q and s_p, quickly decay to values which are completely prescribed by the unstable modes. This fact is expressed as the *slaving principle* of synergetics, in our case reading as

$$s_q = s_q(u_q), \qquad s_p = s_p(u_p)$$

These *order parameter equations* then completely rule the behavior of our microscopic n–dimensional musculo–skeletal system on macroscopic scales near an instability.

The fundamental result of synergetics consists in the observation that on macroscopic scales new laws can be discovered which exist in their own right [Hak83]. These laws which are expressed by the order parameter equations turn out to be independent of the detailed nature of the subsystems and their interactions. As a consequence this allows us to introduce the concept of normal forms as a method to discus instabilities and qualitative dynamics in the neighborhood of the critical regions. This method of phenomenological synergetics allows us to start qualitative analysis from purely macroscopic considerations.

Using the so–called 'adiabatic elimination of fast variables', one tries to identify macroscopic quantities related to global musculo–skeletal dynamics (similar but different from the 'mean–field' center–of–mass dynamics) – from experience and classifies them according to time–scale arguments. The slowest variables are usually identified with the control parameters which are assumed to be quasi static quantities. The slow macroscopic dynamics of the system has to be attributed to the order parameters. Very quickly relaxing variables have to be considered as enslaved modes.

6.5 Macroscopic Biomechanics

The microscopic level of human neuro–musculo–skeletal dynamics, given previously by dissipative, muscle-driven Hamiltonian vector field (*) (and simulated by corresponding Hamiltonian phase–flow for the given initial joint coordinates and momenta), can be viewed from the *macroscopic center–of–mass level* of dynamics as a simple Hamilton oscillator [Iva91], physically representing the damped, sinusoidally driven pendulum of the unit mass and length l

$$l^2 \frac{d^2q}{dt^2} + \gamma \frac{dq}{dt} + lg \sin q = A \cos(p_D t).$$

This equation expresses Newton's second law of motion with the various terms on the left representing acceleration, damping, and gravitation. The angular momentum of the forcing p_D, may be different from the natural frequency of the pendulum. In order to minimize the number of adjustable parameters the equation may be rewritten in dimensionless form as

$$\frac{d^2q}{dt^2} + (1/\nu) \frac{dq}{dt} + \sin q = \epsilon \cos(p_D t),$$

where ν is the damping or quality parameter, ϵ is the forcing amplitude, and p_D is the drive frequency. The low–amplitude natural angular frequency of the pendulum is unity, and time is regarded as dimensionless. This equation satisfies the necessary conditions for chaos when it is written as an extended Hamiltonian system

$$\dot{q} = p, \qquad \dot{p} = -(1/\nu)p - \sin q + \epsilon \cos \phi, \qquad \dot{\phi} = p_D.$$

The variable ϕ is introduced as the phase of the drive term. Three variables are evident and also two nonlinear coupling terms. Whether the motion is chaotic depends upon the values of the three parameters: damping, forcing amplitude and drive frequency. For some values the pendulum locks onto the driving force, oscillating in a periodic motion whose frequency is the driving frequency, possibly with some harmonics or sub–harmonics. But for other choices of the parameters the pendulum motion is chaotic. One may view the chaos as resulting from a subtle interplay between the tendency of the pendulum to oscillate at its 'natural' frequency and the action of the forcing term. The transitions between non–chaotic and chaotic states, due to changes in the parameters, occur in several ways and depend delicately upon the values of the parameters.

To include the muscle excitation–contraction dynamics, and thus make the damped, driven Hamilton oscillator a more realistic macroscopic model for human motion dynamics, we assume that the time–dependent forcing amplitude $\epsilon = \epsilon(t)$ has the form of a low pass filter, a characteristic feature of biological systems, given by first–order transfer function $\frac{K}{Ts+1}$. Here K denotes gain of the filter and T its time constant.

Therefore, macroscopic mechanical model of human motion dynamics obtains the fully–functional form [Iva91]

$$\frac{d^2q}{dt^2} + (1/\nu)\frac{dq}{dt} + \sin q = K(1 - \mathrm{e}^{-t/T})\cos(p_D t),$$

which, written as an extended Hamiltonian system, yields

$$\dot{q} = p, \quad \dot{p} = -(1/\nu)p - \sin q + K(1 - \mathrm{e}^{-t/T})\cos\phi, \quad \dot{\phi} = p_D. \qquad (**)$$

6.6 Control of the Biomechanical Chaos

To control the macroscopic mechanical model (**), let us first define the *steady–state movement error* $e = e(t)$ as a difference between *desired* and *obtained* values of orbits $q = q(t)$, $p = p(t)$ and $\phi = \phi(t)$, i.e.

$$e = \{q(t)^*, p(t)^*, \phi(t)^*\} - \{q(t), p(t), \phi(t)\},$$

where * denotes desired value.

Now, to obtain quick command response to the movement error e, we design a robust, *fuzzy–logic controller* [Lee90]. Fuzzy control is like following what a person says by language (*fuzzy set*), utilizing the basic knowledge about musculo–skeletal dynamics. It correlates the magnitude of the movement error e to the gain K and time constant T of the muscular excitation–contraction filter, following the *fuzzy–implication* rules:

If e is large, then K is large. If e is small, then K is small.

If e is large, then T is small. If e is small, then T is large.

For these rules we can assign appropriate nonlinear membership functions $T(e)$ and $K(e)$ in Gaussian distribution form and 'symmetrical to Gaussian form', respectively

$$T(e) = \frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{1}{2}(\frac{e}{\sigma})^2), \qquad K(e) = \exp(-T(e)),$$

with parameter σ called in statistics 'standard deviation'.

Another useful technique for distinguishing chaotic and nonchaotic behavior is the calculation of *Lyapunov exponents*. In the $2n-$dimensional phase–space of human motion dynamics, sensitivity to initial conditions [GOY87], and therefore chaotic time evolution, can be measured by Lyapunov exponents λ_i, $(i = 1, \dots, 2n)$, which give the average rates of divergence of neighboring phase trajectories $\{q^i(t),\, p_i(t)\}$ where the average must be taken over many initial conditions $\{q^i(0),\, p_i(0)\}$ spread over the trajectories. If all of λ_i are negative, then slightly separated trajectories converge and the evolution is not chaotic. If the sum of all exponents is negative, but at least one of the exponents is positive, nearby trajectories diverge; the evolution is sensitive to initial conditions and therefore chaotic.

The direction of maximum divergence is a changing local property on the *chaotic attractor*. The motion must be monitored at each point along the trajectory. Therefore, a small sphere is defined whose center is a given point on the attractor and whose surface consists of phase points from nearby trajectories. As the center of the sphere and its surface points evolve in time, the sphere becomes an ellipsoid, with principal axes in the directions of contraction and expansion. The average rates of expansion or contraction along the principal axes are the Lyapunov exponents. For the ith principal axis, the corresponding exponent is defined as

$$\lambda_i = \lim_{t\to\infty}\{(1/t)\ln[L_i(t)/L_i(0)]\},$$

where $L_i(t)$ is the radius of the ellipsoid along the ith principal axis at time t.

An initial volume V_0 of phase-space develops on average as

$$V(t) = V_0 \mathrm{e}^{(\lambda_1+\lambda_2+\dots+\lambda_{2n})t}$$

and therefore the rate of change of $V(t)$ is simply

$$dV/dt = \sum_{i=1}^{2n} \lambda_i V(t).$$

In the case of two–dimensional phase area A, evolving as $A(t) = A_0 \mathrm{e}^{(\lambda_1+\lambda_2)t}$, a *Lyapunov dimension* d_L is defined as

$$d_L = \lim_{\epsilon\to 0}\left[\frac{d(\ln(N(\epsilon)))}{d(\ln(1/\epsilon))}\right],$$

where $N(\epsilon)$ is the number of squares with sides of length ϵ required to cover $A(t)$, and d represents an ordinary *capacity dimension*,

$$d = \lim_{\epsilon\to 0}\left(\frac{\ln N}{\ln(1/\epsilon)}\right).$$

Lyapunov dimension can be extended to the case of n–dimensional phase–space by means of *generalized Kaplan–Yorke relation* as

$$d_L = j + \frac{\lambda_1 + \lambda_2 + \cdots + \lambda_j}{|\lambda_{j+1}|},$$

where the λ_i are ordered (λ_1 being the largest) and j is the index of the smallest nonnegative Lyapunov exponent.

Now, the simplest way to 'control the chaos' is to adjust the parameters of a system to a regime of periodic rather than chaotic oscillation. This can be quite easily done by means of fuzzy–logic robust control. We can use fuzzy–implication rules, like

If d_L is large, then σ_i is large and/or σ_j is small.

If d_L is small, then σ_i is small and/or σ_j is large.

Here $\sigma_{i,j}$ denote two sample control parameters (for example segment mass and length). For these rules we can assign appropriate nonlinear membership functions $\sigma_{i,j}(d_L)$.

The other, geometric approach to control the chaos [GOY88] is to modify the actual dynamics in such a way that an unstable periodic orbit becomes stable, so that the motion (if it is in the basin of attraction of this periodic orbit) becomes periodic.

An important property of *unstable fixed points* is that they are *saddle points*: there are both attracting and repelling directions. These directions lie along the *stable* W^S and *unstable* W^U *manifolds*, respectively. The manifolds W^S and W^U are simply the trajectories that approach and depart most quickly from the unstable equilibrium. Once the local geometry is determined, one of the system parameters is chosen as the control parameter σ. The effect on the fixed point coordinates of varying σ is determined by simulation with σ slightly displaced from its value σ_0 at the fixed point.

The control algorithm [GOY88] uses slight changes $\delta\sigma$ in the control parameter σ_i to force the orbit toward thee direction of contraction (the stable manifold) near the fixed point. The natural contraction then drives the orbit toward the fixed point. However, a small error will cause the orbit to miss the fixed point, so the adjustment process must be carried out repeatitively to achieve stabilization.

Chapter 7
$CSB-$Subsystems: Energy and Information Flows

7.1 $CSB-$Energy Flows

Human energy production spans the range of human movements from those requiring large bursts of energy over short periods of time (as in sprint running) – to those activities requiring small but sustained energy production (as in marathon running). Even within the same activity, the energy requirements change from one moment to the next [Mar98, GM88].

7.1.1 The Immediate Energy Source

Adenosine triphosphate (ATP) is the immediately usable form of chemical energy for muscular activity. It is stored in most cells, particularly muscle cells. Other forms of chemical energy, such as that available from the foods we eat, must be transferred into the ATP form before they can be utilized by the muscle cells.

The chemical structure of ATP is complicated, but for our purposes it can be simplified, saying that ATP consists of a large complex of molecules called adenosine and three simpler components called phosphate groups. The last two phosphate groups represent 'high energy bonds'. In other words, they store a high level of potential chemical energy. When the terminal phosphate bond is broken, energy is released, enabling the cell to perform work. The kind of work performed by the cell depends on the cell type. for example, mechanical work (contraction) is performed by muscle cells (skeletal, smooth and heart muscle), nerve conduction by nerve cells, secretion by secretory cells (e.g., endocrine cells), and so on. All 'biological' work performed by any cell requires the immediate energy derived from the breakdown of ATP.

7.1.2 The Principle of Coupled Reactions

Since energy is released when ATP is broken down, it is not too surprising that energy is required to rebuild or resynthesize ATP. The building

T.T. Ivancevic et al.: Complex Sports Biodynamics, COSMOS 2, pp. 79–86.
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blocks for ATP synthesis are the by-products of its breakdown, adenosine diphosphate (ADP) and inorganic phosphate (P_i). Therefore, we have two essential formulas, the direct: $ATP \to ADP + P_i + energy$, and the inverse one: $ADP + P_i \to ATP$. The energy for ATP resynthesis comes from three different series of chemical reactions that take place within the body. Two of the three depend upon the food we eat, whereas the other depends upon a chemical compound called phosphocreatine. The energy released from any one of these series reactions is coupled with the energy needs of the reaction that resynthesizes ATP. In other words, the separate reactions are functionally linked together in such a way that the energy released by the one is always used by the other. Biochemists refer to these functional links as coupled reactions, and it has been shown that such coupling is the fundamental principle involved in the metabolic production of ATP.

7.1.3 ATP − PC: The Phosphagen System

PC is an abbreviation for phosphocreatine, another one of those 'energy–rich' phosphate compounds closely related to ATP. For example, PC, like ATP, is stored in muscle cells, and when it is broken down (i.e., when its phosphate group is removed), a large amount of energy is released. The released energy, of course, is coupled to the energy requirement necessary for the resynthesis of ATP. In other words, as rapidly as ATP is broken down during muscular work, it is continuously reformed from ADP and P_i by the energy liberated during the breakdown of the stored PC: $ADP + P_i \to ATP$. For every mole of PC broken down, one mole of ATP is resinthesized.

The total muscular stores of both ATP and PC (collectively referred to as phosphagens) are very small – only about $0.3mole$ in females and $0.6mole$ in males. Thus, the amount of energy obtainable through this system is limited. In fact, if you were to run $100m$ as fast as you could, the phosphagen stores in the working muscles would probably be empty by the end of the sprint. However, the usefulness of the $ATP-PC$ system lies in the rapid availability of energy rather than in the quantity. For example, activities such as sprinting, jumping, swinging and other similar skills requiring only a few seconds to complete are all dependent upon the stored phosphagens for their primary energy source.

7.1.4 The Lactic Acid System

This system is also known as 'anaerobic glycolysis'. 'Glycolysis' refers to the breakdown of sugar; 'anaerobic' means without oxygen. In this system, the breakdown of sugar (a carbohydrate, one of the foodstuffs) supplies the necessary energy from which ATP is manufactured. When the sugar is only partially broken down, one of the end products is lactic acid (hence the name lactic acid system).

When lactic acid accumulates in the muscles and blood reaches very high levels, temporary muscular fatigue results. This is a very definite limitation, and is the main cause of the 'early' fatigue. Another limitation of the lactic acid system that relates to its anaerobic quality is that only a few moles of ATP can be resynthesized from the breakdown of sugar as compared to the yield possible when oxygen is present. For example, only three moles of ATP can be manufactured from the anaerobic breakdown of $180gr$ of glycogen (glycogen is the storage from glucose or sugar in the muscle). As we will soon see, the aerobic breakdown of $180gr$ of glycogen results in enough energy to resynthesize $39moles$ of ATP!

The lactic acid system, like the $ATP-PC$ system, is extremely important to us, primarily because it too provides for a rapid supply of ATP energy. For example, exercises that are performed at maximum rates for between $1min$ and $3min$, such as sprinting $400m$ and $800m$, depend heavily upon the lactic acid system for ATP energy. Also, in some performances, such as running $1500m$ of a mile, the lactic acid system is used predominantly for the 'kick' at the end of the race.

7.1.5 The Oxygen, or Aerobic, System

In the presence of oxygen, the complete breakdown of $180gr$ of glycogen to carbon dioxide (CO_2) and water (H_2O) yields enough energy to manufacture $39moles$ of ATP. This series of reactions, like the anaerobic series, takes place within the muscle cell, but is confined to specialized subcellular compartments called mitochondria. Mitochondria are slipper–shaped cell bodies often referred to as the 'powerhouse' of the cell because they are the seat of the aerobic manufacture of ATP energy. Muscle cells are rich with mitochondria.

The aerobic breakdown of carbohydrates, fats, and even proteins provides energy for ATP resynthesis: $ADP + P_i \to ATP$. Since abundant ATP can be manufactured without yielding fatiguing by–product, the aerobic system is most suited for endurance activities. The carbon dioxide that is produced diffuses freely from the muscle cell into the blood and is carried to the lung, where it is exhaled. The water that is formed is useful within the cell itself, since the largest constituent of the cell is, in fact, water.

Another feature of the aerobic system that should be noticed is that concerned with the type of foodstuff required for breakdown. Not only glycogen but fats and proteins as well can be aerobically broken down to carbon dioxide and water, with energy released for ATP synthesis. For example, the breakdown of $256gr$ of fat will yield $130moles$ of ATP. During exercise, both glycogen and fats, but not protein, are important sources of ATP–yielding energy.

The amount of oxygen we need to consume from the environment in order to synthesize one mole of ATP is approximately $3.5l$ if glycogen is the food fuel and about $4.0l$ with fat. At rest, most of us consume between $0.2l$ and $0.3l$ of oxygen per minute. In other words, a mole of ATP is aerobically manufactured every $12min$ to $20min$ under normal resting conditions. During maximal exercise, a mole of ATP can be aerobically supplied to the working muscles every minute by most of us. For the highly trained endurance athlete, more than $1.5moles$ of ATP can be aerobically synthesized and supplied to the muscles every minute during maximal effort.

In summary, then, the aerobic system is capable of utilizing both fats and glycogen for resynthesizing large amounts of ATP without simultaneously generating fatiguing by–products. Therefore, the aerobic system is particularly suited for manufacturing ATP during prolonged, endurance–type activities. For example, during marathon running, approximately $150moles$ of ATP are required over $2.5h$ the race takes. Such a large, sustained output of ATP energy is possible only because early fatigue can be avoided and large amounts of food (glycogen and fats) and oxygen are readily available.

7.1.6 The Energy Continuum Concept

Therefore, ATP is immediate form of muscular energy, and is supplied in three ways:

1. by the stored phosphagens, ATP and PC;
2. by the lactic acid system (anaerobic glycolysis); and
3. by the oxygen, or aerobic, system. And ability of each system to supply the major portion of the ATP required in any given activity is related to the specific kind of activity performed.

For example, in short–term, high–intensity types of activities, such as the $100m$ dash, most of the ATP is supplied by the readily available phosphagen system. In contrast, longer–term, lower intensity types of activities, such as the $42.2km$ marathon, are supported almost entirely by oxygen, or aerobic, system.

In the middle range of activities are those tasks that rely heavily on the lactic acid system for ATP production – e.g., the $400m$ and $800m$ dashes. Also in the middle are those activities that require a blend of both anaerobic and aerobic metabolism – e.g., the $1500m$ and mile runs. In these latter activities, the anaerobic systems supply the major portion of ATP during the sprint at both the start and finish of the race, with the aerobic system predominating during the middle or steady–state period of the run.

Just from these examples it can be seen that what is at work in physical activities is an energy continuum, a continuum relating the way in which ATP is made available and the type of physical activity performed.

7.2 *CSB*−Information Flows

7.2.1 *CSB*−Motor Learning

The *CSB*–motor learning is generally performed according to the following *commutative triangle* (two mapping paths with the same start and end) [Iva91]:

$$\begin{array}{ccc} MOTOR \;\; PROGRAM & \longrightarrow & MUSCULAR \;\; SYNERGY \\ & \searrow \qquad \swarrow & \\ & \begin{array}{c} LOCOMOTOR \\ COORDINATION \end{array} & \end{array}$$

To create the first link in this diagram, the *motor program*, i.e., dynamic stereotype, we have to improve the certain motor–information abilities of the athlete, according to the general formula:

$$\text{new value} \;=\; \text{old value} \;+\; \text{innovation}$$

(see below, adaptive filtration, and also learning of neural networks), and Newton's causality principle, starting with some initial level of these abilities.

The motor–information abilities, as obtained by general multivariate *factor analysis* on a set of motor tests, are:

1. Ability for general structuring of the motor program;
2. Ability for special–situation structuring of the motor program;
3. Ability for quick reprogramming of the motor program; and
4. Ability for selective control of excitation and inhibition of afferent and efferent neuro–muscular pathways.

These abilities are in the process of sports training usually developed through the three stages:

1. The *general training stage* (adoption of basic locomotion structure and creation of observable and controllable mechanisms; with the beginners the most important is to establish the audio-visual channels);
2. The *directed training stage* (connection of basic motor components in flexible logic units, synchronization of space–time locomotion characteristics, establishing of probabilistic observable and controllable mechanisms; locomotion becomes more flexible, economic and stable);
3. The *specific–situation training stage* (increase in efficiency of locomotion related to the specific–situation conditions of competition; development of abilities for quick reprogramming of locomotion stereotypes and corresponding recursive algorithms according to the needs of the competition, or motoric criteria).

In each of these stages the training process goes through two phases: information (i.e., cognitive) and motoric. In the cognitive phase various sources of information are used (movements demonstrations by the coach, using of audio–visual equipment, explanations of the motion flow, emphasizing of the most important demand in the realization of the main locomotion phase) about the movements to be produced. All these informations are received through visual and auditive channels and used for the comparative analysis on the cortical level, of the relations between the realized locomotion and its model (the model of the champion). The result of these analyses should be the correction of locomotion, which means more efficient functioning of observable and controllable mechanisms.

In the motoric phase of the training process the channels for locomotor proprioception and kinaesthesis are opening and the model for control of locomotion is created which exclusively uses this sort of informations. Adaptive filtration (i.e., increasing the signal and decreasing the noise by adjusting certain system parameters) of the error signal and its time derivatives is used for this task. In the beginning of the training process the control is based on the amplitude of the error signal, later on it advances towards the control based on the more subtle components of velocity, acceleration, and jerk.

7.2.2 *CSB−Adaptive Filtration*

All mathematical models that have been presented till now are covered by Newton's causality principle. However, the 'logic of life', to quote the Nobel laureate F. Jacob[1], cannot be satisfactorily described in such a manner. The logic of life is 'acausal', to quote another Nobel laureate J. Monod[2], (Jacob's colleague from the Paris' Pasteur Institute, in discovering 'operon', the operator–repressor gene regulation system in bacteria E. Coli), in the sense that it is able to respond even before it is externally stimulated. All living systems essentially have this characteristic. They have the ability to predict, because they are pre–excited.

The simplest technical model of the *acausal system* is *low–pass filter* (Figure 7.1), whose time characteristics can be mathematically described by a special function *sine–integral*, and characterized by its amplitude and three time phases (left and right sinusoidal and the middle–sigmoid) [Iva91].

Low–pass filter (i.e., sine–integral) (see [Wie48]) can be considered as optimal model for a wide range of biosystems: a single neuron, a single muscle fiber, a single endocrine gland, ... Therefore, any physiological system can be technically described as a network of low–pass filters. And, also, the first and the most important information process in biological communication–networks is *filtration*, i.e., increasing the signal and reducing the noise (which is always present). We can even say that the filtration is the *essence of*

[1] Francois Jacob, *La Logique de Vivant*, Gallimard, Paris, 1970.
[2] Jacoues Monod, *L'Hasard et L'Necesite*, Gallimard, Paris, 1972.

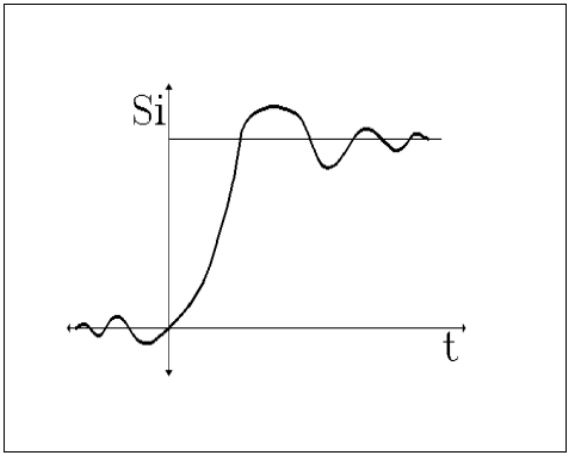

Fig. 7.1 Acausal time characteristics of a low–pass filter

biological adaptation. The filtration with on–line adjustment of characteristics of the filters included into the communication network is called *adaptive filtration*.

The filtration theory is in the very basis of cybernetics. It was mainly developed by two persons, N. Wiener and R. Kalman, and today both Wiener's and Kalman's filters are most frequently used for both interpolation and extrapolation of stochastic processes, as well as identification of systems in real–noisy conditions. Kalman's filter, or recursive state–estimator, is based on Kalman's regulator and is considered today as the best tool for the identification of linear systems.

The concept of adaptive filtration is useful for modelling of the sports training process (Figure 7.2). The training process can be described as tracking of the athlete's behavior (represented by $base\,signal + noise$), performed by the coach (represented by $tracking\,signal$) [Iva91]. On each training lesson the coach performs adaptive filtration of the $base\,signal + noise$ of the athlete. If this filtration is well done, by the assumptions of the filtration theory, the coach is able to do two things more:

1. *interpolation* of the previous behavior of the athlete; and
2. *extrapolation* (i.e., prediction) of the future behavior of the athlete.

Therefore, the coach is able to track and control the behavior of the athlete on the training.

Now, if each training lesson is so time–adjusted that its filtration coincides with the extrapolation of the previous one, and such time–adjustment is iteratively repeated, then the training process *converges*. At every lesson the training process is described by equation:

$$\text{new value} = \text{old value} + \text{innovation}$$

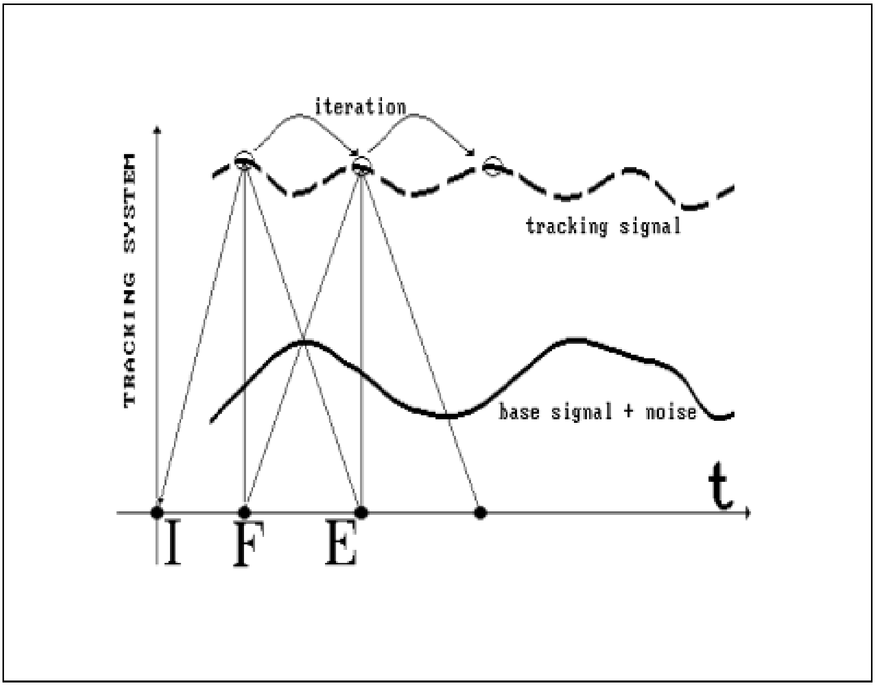

Fig. 7.2 Adaptive filtration in the sports training process

where

$$\text{innovation} = \text{filter} \cdot (\text{desired state} - \text{actual state})$$

and filter = signal / nois.

The *speed of convergence*, i.e., the number of steps required for convergence (for obtaining the desired sport result), depends on innovation. From the formula, it is quicker if the ratio *signal / noise* is greater (i.e., filtration better), as well as the difference (*desired state* − *actual state*). The first component (*filtration*) is the capacity of the coach. The second component (*innovation step*) is the capacity of the athlete. This innovation step probably represents the best possible *criterion for selection*, i.e., recognition of the talents (see Figure 1.1).

Chapter 8
Neuro–CSB: Artificial Neural Networks

8.1 Introduction

Artificial neural networks (ANNs, see [Hay94, Kos92]) are models of the brain's cognitive process. In contrast with conventional single–processor computers, the brain has a multiprocessor architecture that is highly inter–connected. This architecture has been described as parallel distributed processing (PDP). PDP has many advantages over single–processor models for many difficult computer science problems. It is one of the reasons that the brain, with its slow, error–prone individual hardware units (neurons) so much outperforms even supercomputers in some areas.

The neuron is the basic processor in ANNs (Figure 8.1). Each neuron has one output, which is generally related to the state of the neuron – its activation – and which may fan out to several other neurons. Each neuron receives several inputs over these connections, called synapses. The inputs are the activations of the incoming neurons multiplied by the weights of the synapses. The activation of the neuron is computed by applying a threshold function to this product.

All of the 'knowledge' that a neural network possesses is stored in the 'synapses', the weights of the connections between the neurons (see Figure 8.2). The synapses between two layers of neurons are usually represented as matrices.

Once the knowledge is present in the synaptic weights of the network, presenting a pattern for input to the network will produce the correct output. However, how does the network acquire that knowledge? This happens during 'learning' (or training). Pattern associations are presented to the network in sequence, and the weight are adjusted to capture this knowledge. The weight adjustment scheme is known as the learning law.

An, unsupervised learning method is one in which weight adjustments are not made based on comparison with some target output. There is no 'teaching signal' feed into the weight adjustments. This property is also known as self–organization.

T.T. Ivancevic et al.: Complex Sports Biodynamics, COSMOS 2, pp. 87–103.
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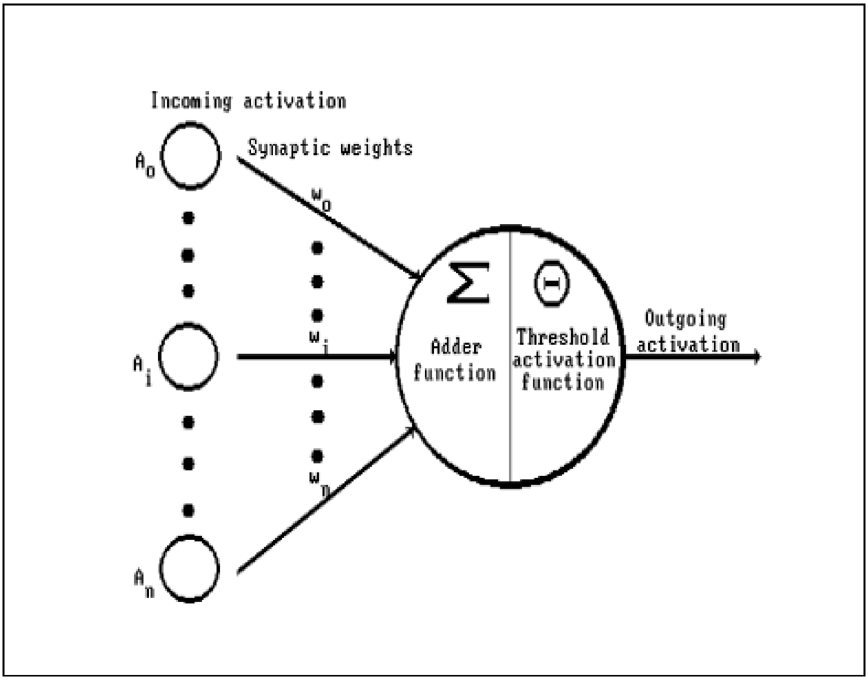

Fig. 8.1 Schematic of an artificial neuron

In many ANNs learning takes the form of supervised training. We present input patterns one after the other to the neural net and observe the recalled output pattern in comparison with our desired result. We then need some way of adjusting the weights that takes into account any error in the output pattern. It is called error correction law.

Computer applications that use PDP, or incorporate neural networks, allow problems that were once very difficult to solve on a computer to be attacked with relative ease. So, we consider ANNs the best existing tool for the optimal control of the sport training process. By the use of ANNs and PDP, on the basis of Newton's causality principle and Anohin's concept of functional systems, we can design a real 'coach–assistant' apparatus.

8.2 History

Neural networks originated as a model of how the brain works (see [Hay94, Kos92]). Cognitive theories of William James and other nineteenth–century psychologists laid the groundwork of ideas that was to give birth to early neural network research.

In 1943 McCulloch and Pitts formulated the first neural network model. It featured digital neurons but no ability to learn. The work of another

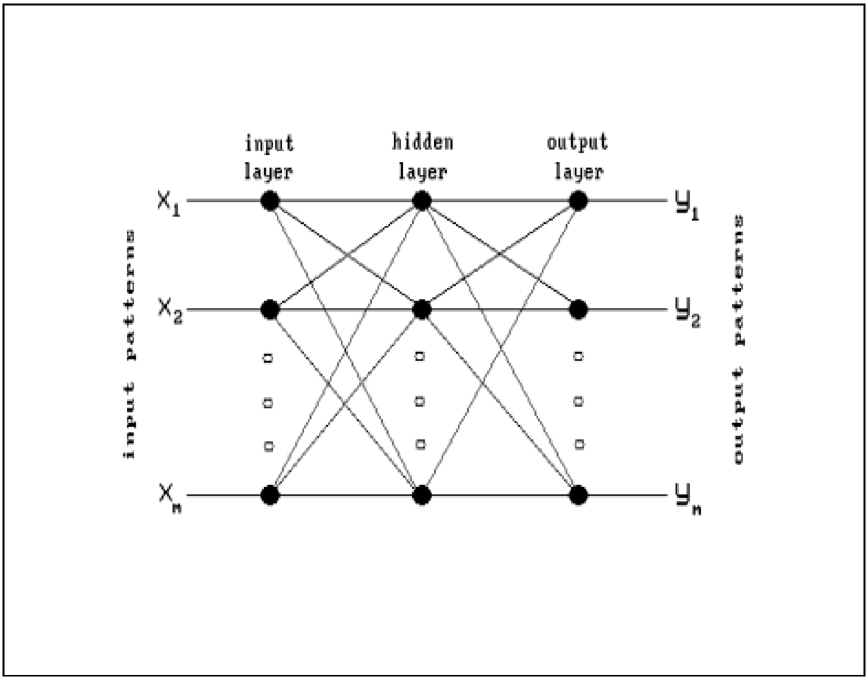

Fig. 8.2 Schematic of an artificial neural network

psychologist, Donald Hebb, introduced in 1949 the idea of Hebbian learning, which states that changes in synaptic strengths (connections between neurons) are proportional to the activations of the neurons. This was a formal basis for the creation of neural networks with the ability to learn.

The theory of Hebbian learning described a rule for updating synaptic strengths in two–layer networks, enabling these networks to learn. In 1957 Frank Rosenblatt incorporated this idea of learning into a two–layer network, calling the result a perceptron. Rosenblatt formulated a learning rule based on weights adjusted in proportion to the error between output neurons and target outputs. His perceptron convergence theorem proved that this would result in a desired set of weights. Rosenblatt also formulated a three–layer perceptron and attempted to incorporate learning into it. However, he was unable to come up with a provably sound method of training the weights between the input and hidden–layer neurons.

Still, many problems cannot be solved with two-layer networks. The lack of a mathematically rigorous procedure to allow learning in multilayer networks was a major stumbling block to the development of *ANNs* solutions. In 1969 Minsky and Papert mentioned this problem in 'Perceptrons'. Minsky correctly pointed out the limitations of the perceptron for many problems and acknowledged the possibility that multilayer networks would overcome these limitations. But he did not see any way to enable these machines to learn, and so he felt that multilayer networks were a dead end.

Some interesting work continued to be performed within the bounds of two–layer networks. In 1984 J. Hopfield, and in 1984 T. Kohonen, used two–layer network to build a content addressable memory, in which no separate index must be given to retrieve a particular item; the item itself is the index. That is, instead of making a memory reference as $a[i]$, where a is the array and i is the index, to get to the contents c, we refer to the memory location as $a[c]$. Hopfield developed the physical basis for the self–organizing neural networks, using Hamiltonian formalism. Kohonen referred to this as associative memory. Associative memory is based on an unsupervised learning algorithm, where the weights are adjusted purely on the basis of the presented patterns, without regard to some desired output. An extension of this concept was developed in 1988 by Bart Kosko and is known as bidirectional associative memory (BAM). Kohonen later developed another unsupervised learning–based model known as the learning vector quantizer (LVQ). The LVQ uses competitive learning to allow input neurons to activate one and only one output neuron.

Since the mid 1960s Stephen Grossberg has been developing mathematical models of the brain's function. The Center for Adaptive Systems is oriented to psychological and biological research. However, this research has resulted in several useful and unique ANNs models, which are characterized by the ability to do training on–line, the capacity for self–organization, and the ability to form compact representations of complex phenomena. In addition, the 'on–line training' aspect has allowed many of Grossberg's models (most prominently ART) to handle data that changes fundamentally over time. Specific models include the additive and multiplicative Grossberg models, the competitive learning concept just mentioned, adaptive resonance theory (ART), and various ANNs–based minimal encoding methods, such as out-star encoder. The out–star encoder was combined by Hecht-Nielsen with the Kohonen LVQ to form the counterpropagation model.

Despite all the useful work occurring with two–layer systems, the independent discovery of backpropagation (BP) by Werbos and Parker was the next major step in the advancement in ANNs, after Rosenblatt's perceptron. BP allows the training of multilayer networks. Thus the original objection that multilayer networks were a dead end because they could not be trained was removed. Werbos discovered it in 1974 while working on his doctoral thesis in statistics, and he called the algorithm dynamic feedback. Parker discovered it independently in 1982 while doing graduate work on Stanford, and he called the algorithm learning logic. Rumelhalt, Hinton, and Williams exploited BP in 1986 in their work in simulating cognitive process.

Since then BP has been exploited in a number of fields having nothing to do with studying or simulating cognitive processes. It is a powerful and practical tool for solving problems that would be quite difficult using conventional computer science techniques. These problems range from image processing to speech recognition to character recognition to forecasting to optimization.

8.3 Backpropagation of Error

The BP model (Figure 8.3) has three layers of neurons: an input layer, a hidden layer, and an output layer (see [Hay94, Kos92]). There are two layers of synaptic weights. There is a learning rate term, a, in the subsequent formulas, indicating how much of the weight change to effect on each pass. This is typically a number between 0 and 1. There is a momentum term, Q, indicating how much a previous weight change should influence the current weight change. There is also a term indicating within what tolerance we can accept an output as 'good'. The BP model training consists of encoding and recall.

8.3.1 Encoding

Assign random values between −1 and +1 to the weights between the input and hidden layers, the weights between the hidden and output layers, and

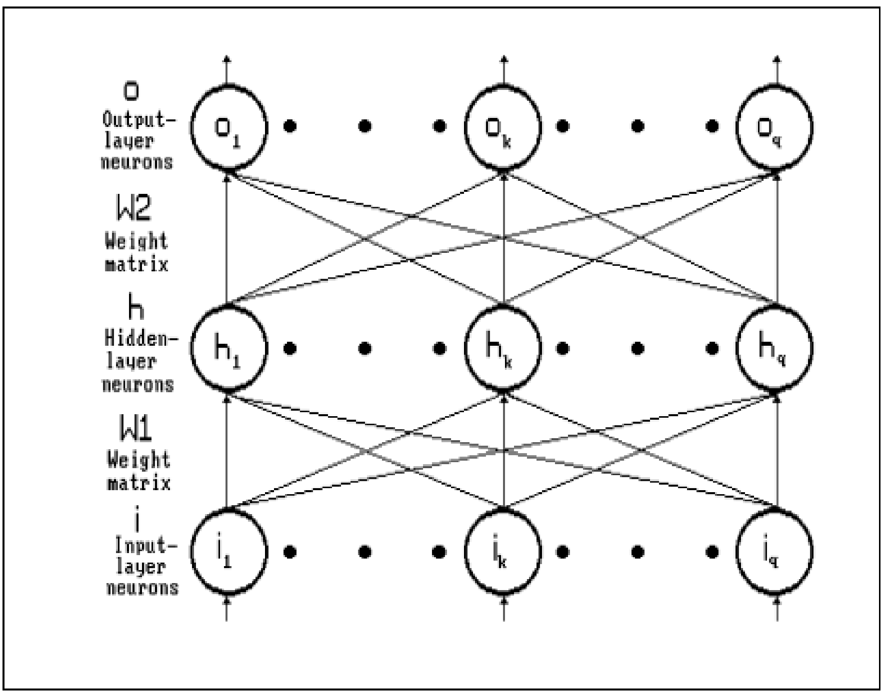

Fig. 8.3 Schematic of the backpropagation neural network

the thresholds for the hidden–layer and output–layer neurons. Train network by performing the following procedure for all pattern pairs.

Forward Pass

1. Compute the hidden–layer neuron activations: $h = F(iW_1)$, where h is the vector of hidden–layer neurons, i the vector of input–layer neurons, W_1 the weight matrix between the input and hidden layers, and $F()$ is a sigmoid ($tanh$) activation function.
2. Compute the output–layer neuron activations: $o = F(hW_2)$, where o represents the output layer, and W_2 the matrix of synapses connecting the hidden and output layers.

Backward Pass

1. Compute the output–layer error (the difference between the target and the observed output): $d = o(1 - o)(o - t)$, where d is the vector of errors for each output neuron, and t is the target (correct) activation of the output layer.
2. Compute the hidden–layer error: $e = h(1 - h)W_2d$, where e is the vector of errors for each hidden–layer neuron.
3. Adjust the weights for the second layer of synapses: $W_2 = W_2 + DW_2$, where DW_2 is a matrix representing the change in matrix W_2. It is computed as follows: $DW_2^t = ahd + QDW_2^{t-1}$.
4. Adjust the weights for the first layer of synapses: $W_1 = W_1 + W_1^t$, where: $W_1^t = aie + QDW_1^{t-1}$.

Repeat both the forward and the backward passes on all pattern pairs, until the output–layer error (vector d) is within the specified tolerance for each pattern and for each neuron.

8.3.2 Recall – Test

Present the input pattern to the input layer of neurons of our BP net:

1. Compute the hidden-layer activation: $h = F(W_1i)$;
2. Compute the output layer: $o = F(W_2h)$.

The vector o is our recalled pattern.

The BP network has the ability to learn any arbitrarily complex nonlinear mapping. This is due to the introduction to the hidden layer. It also has a capacity much greater than the dimensionality of its input and output layers. However, BP can involve very long training times. If we have strong relationships between inputs and outputs and we are willing to accept results within a relatively broad tolerance, our training time may be reasonable.

Hardware support for this ANNs algorithm (BP chips) should help matters quite a bit. Many components of the algorithm are highly parallel. The adjustments of the weight matrices and neuron activation vectors are all parallel procedures. The feed from one layer to the next for both encoding and recall must be performed sequentially (the hidden–layer neurons must be computed before the output–layer neurons can be). So most of BP can be sped up with parallel hardware.

A feedforward BP network can be used for Hamiltonian identification (see Figure 8.4). Talent selection can be formalized as identification of driving forces, Hamiltonian and dissipation functions. As Hamiltonian relations are *a priori* determined, the selection–identification map is reduced to estimation of included parameters. The estimation map is realized by feedforward multilayer perceptron with 'gray' output layer (known relations, unknown parameter values) and BP (or, fast LM) learning algorithm.

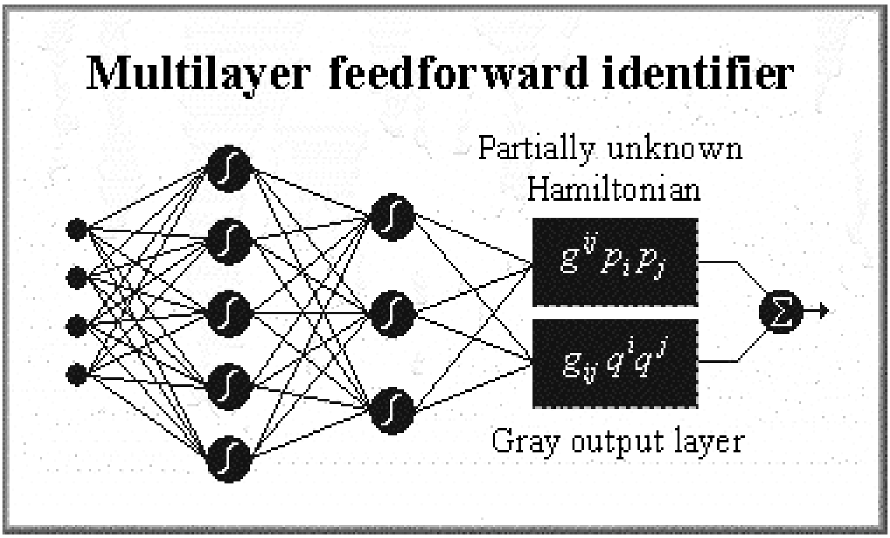

Fig. 8.4 Identification of the training Hamiltonian using a multilayer perceptron. In the gray output layer the predefined system relations of the Hamiltonian function, dissipation and driving forces are used instead of classic (sigmoid) activation functions.

8.4 Hopfield Neural Network

The paradigm for the unsupervised, self–organizing ANN is the discrete Hopfield network (see [Hay94, Kos92]). Hopfield gives a collection of simple threshold automata, called *formal neurons* by McCulloch & Pitts: two-state, 'all-or-none', firing or nonfiring units that can be modelled by *Ising spins* (uniaxial magnets) $\{S_i\}$ such that $S_i = \pm 1$ (where the label of the neuron is i and ranges between 1 and the size of the network N). The neurons are connected by synapses J_{ij}.

A (firing) *patterns* $\{\xi_i^\mu\}$ represent specific S_i*–spin configurations* (where the label of the pattern is μ and ranges between 1 and q).

Using random patterns $\xi_i^\mu = \pm 1$ with equal probability 1/2, we have the *synaptic efficacy* J_{ij} of jth neuron operating on ith neuron given by

$$J_{ij} = N^{-1} \sum_{\mu=1}^{q} \xi_i^\mu \xi_j^\mu \equiv N^{-1} \xi_{\mathbf{i}} \cdot \xi_{\mathbf{j}}.$$

Postsynaptic potential (PSP) represents an *internal local field*

$$h_i(t) = \sum_{j=1}^{N} J_{ij} S_j(t).$$

Now, the *sequential (threshold) dynamics* is defined in the form of discrete equation

$$S_i(t + \Delta t) = \mathrm{sgn}[h_i(t)].$$

The sequential dynamics is equivalent to the rule that the state of a neuron is changed, or a spin is flipped if and only if the total network *energy*, given by *Ising Hamiltonian*

$$H_N = -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} J_{ij} S_i S_j$$

is lowered. Therefore, the Ising Hamiltonian H_N represents the monotonically decreasing *Lyapunov function* for the sequential dynamics, which converges to a local minimum or ground state of H_N. This holds for any *symmetric coupling* $J_{ij} = J_{ji}$ with $J_{ii} = 0$ and if spin-updating in dynamics is asynchronous. In this case the patterns $\{\xi_i^\mu\}$ after convergence become identical, or very near to, ground states of H_N, each of them at the bottom of the valley.

(On the other hand, *asymmetric* synapses $J_{ij} \neq J_{ji}$, are a necessary prerequisite for temporal associations such as would be involved in reciting a poem or a tune, in counting, or in the control of rhythmic motion. Synaptic interconnections in biological neural networks are in general asymmetric.)

Data are *stored* in the neural net if, by a suitable choice of the J_{ij}, several specific patterns $\{\xi_i^\mu\}$ are made local minima of H_N. If this can be achieved, the neural net will function as *content-addressable* or *auto–associative memory*. A network state which 'somehow resembles one of the stored prototypes corresponds to a location in the energy landscape which is close enough to the minimum representing that prototype to lie in its *basin of attraction*. By spontaneously moving downhill, or relaxing to the energy minimum, the network *recalls* the data or reconstructs the prototype.

Suppose that we have somehow stored several (stationary) patterns $\{\xi_i^\mu\}$ in the J_{ij} and that the system is offered a noisy version of one of them. If the noise was not too strong, the system remains in the valley associated with that pattern and under its natural dynamics it will relax *by itself* to the energy minimum were the stored patterns live. That is, the system has recalled the pattern.

Following the sequential dynamics, a network with symmetric synapses would eventually settle in one of the energy valleys associated with the Ising Hamiltonian H_N, and moderate amounts of external noise (allowing, with small probability, for uphill motion in the energy landscape created by H_N) would not help it to escape.

In statistical mechanics, one is usually given the synapses J_{ij} and one of the first tasks consists in finding the minima of the Ising Hamiltonian H_N. In the theory of neural networks, however, one is given the patterns $\{\xi_i^\mu\}$ and one is asked to solve the *inverse problem*: finding synapses J_{ij} such that the patterns $\{\xi_i^\mu\}$ are minima of the Hamiltonian H_N.

To see why the Hopfield model has patterns $\{\xi_i^\mu\}$ as *attractors* of the sequential dynamics, note that the sequential dynamical law embodies a two-step process, the evolution of the local field (PSP), which is a *linear* operation, and a *nonlinear* decision process. Assuming that the number q of stored patterns is small ($q/N \to 0$), we find that the synapses (1) give rise to a local field of the form

$$h_i(t) = \sum_{\mu=1}^{q} \xi_i^\mu m_\mu(t),$$

where

$$m_\mu(t) = N^{-1} \sum_{i=1}^{N} \xi_i^\mu S_i(t)$$

is the *overlap* of the network state $\{S_i(t)\}$ with the pattern $\{\xi_i^\mu\}$[1] – that measures the proximity between them. We can see that $m_\mu = 1$ if $\{S_i(t)\}$ and $\{\xi_i^\mu\}$ are identical patterns, $m_\mu = -1$ if they are each other's complement, and $m_\mu = \mathrm{O}(1/\sqrt{N})$ if they are uncorrelated with each other. Overlaps m_μ are related to the *Hamming distance* d_μ between the patterns (the fraction of spins which differ) by $d_\mu = \frac{1}{2}(1 - m_\mu)$. The similarity between two patterns is measured by their overlap. For similar patterns the overlap is close to unity whereas for uncorrelated patterns it is random variable with zero mean and small $(1/\sqrt{N})$ variance. Overlaps give the *global* information about the network and hence are good *order parameters*.

[1] In other words, this is *mutual overlap* or *correlation matrix* between two patterns ξ_i^μ and ξ_i^ν equal $C^{\mu\nu} = N^{-1}\sum_{i=1}^{N} \xi_i^\mu \xi_i^\nu$).

Using overlaps, the Ising Hamiltonian becomes

$$H_N = -\frac{1}{2}N\sum_{\mu=1}^{q} m_\mu^2.$$

The similarity between two different patterns ξ_i^μ and ξ_i^ν is measured by their *mutual overlap* or *cross–overlap* $m_{\mu\nu}$ (or *Karhunen–Loeve matrix*, which extracts the principal components from a data set)[2], equal

$$m_{\mu\nu} = N^{-1}\sum_{i=1}^{N}\xi_i^\mu \xi_i^\nu.$$

For similar patterns the cross–overlap is close to unity whereas for uncorrelated patterns it is random variable with zero mean and small $(1/\sqrt{N})$ variance.

For continuous overlaps $m(t)$ we have the *learning dynamics* given by differential equation

$$\dot{m}(t) = -m(t) + f(m(t)),$$

where $f(m(t))$ represents the sigmoid innovation function.

For the evaluation of the athlete's performance in the framework of the Hopfield ANN, we propose two different approaches:

1. The *manifest pattern approach*: if we represent the performance of the ith athlete in the μth sports–test as a Hopfield pattern ξ_i^μ (like row–data matrix), then the auto–overlap m_μ resembles an auto–regressive structure in the manifest space, while the cross–overlap $m_{\mu\nu}$ resembles a correlation matrix in the manifest space, both representing objective measures of the athlete's general performance in the manifest space.
2. The *latent pattern approach*: if we represent factor scores (previously evaluated by means of linear oblique factor analysis) of the ith athlete on the μth factor as Hopfield pattern ξ_i^μ, then the auto–overlap m_μ resembles an auto–regressive structure in the latent space, while the cross–overlap $m_{\mu\nu}$ resembles a factor–inter–correlation matrix, both representing objective measures of the athlete's general performance in the latent space.

Both the supervised BP and the unsupervised Hopfield ANNs could be considered as optimal tools for the solution of both main CSB–tasks (see Figure 1.1): the direct mapping f (transforming the model of the talent into the model of the champion), and the inverse mapping f^{-1} (recognizing the model of the champion inside the model of the talent).

[2] Resembling the cross–correlation function of two time–series, with several distinct peaks, indicating that the two series are very similar at each point in time where the peaks occur.

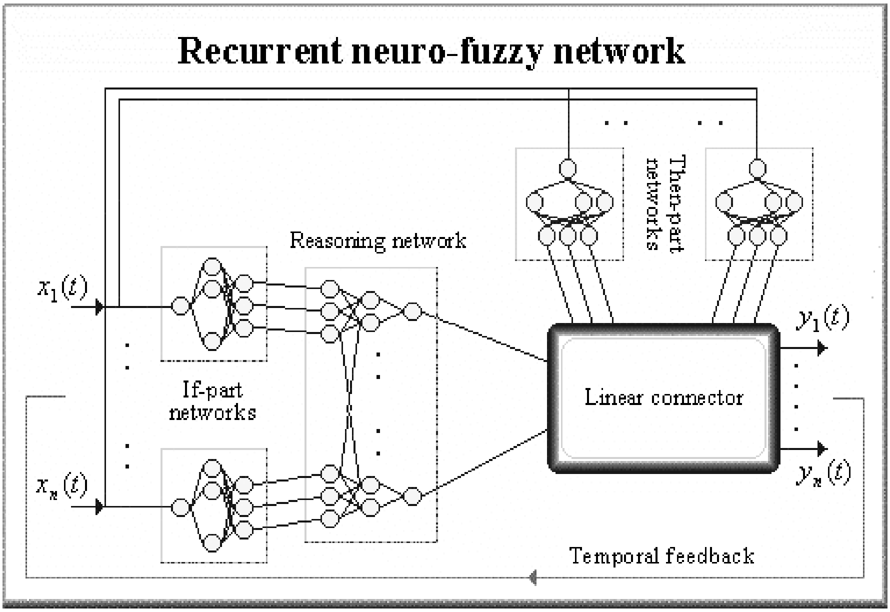

Fig. 8.5 The recurrent neuro–fuzzy network. Here, fuzzy control is like following what a person says by language. Artificial neural–network control is like following what a person does by data. Intelligent controller is a recurrent neuro–fuzzy network designed in the feedback–loop of the training system.

A more sophisticated model of the recurrent neuro–fuzzy network (see [II07b]) is presented in Figure 8.5. This *complex adaptive system* represents a real 'coach–assistant', which can help coaches to develop the *champion model* for any particular sport discipline, out of available empirical data. The *intelligent training control* is performed in three steps:

1. Optimal training flow and phase–transitions are empirically defined.
2. Actual training flow and phase–transitions are simulated.
3. Nonlinear feedback control is designed in the form of recurrent neuro–fuzzy network.

8.5 *CSB*−Neurodynamics: The Cerebellum

When someone compares learning a new skill to learning how to ride a bike they imply that once mastered, the task seems imbedded in our brain forever. Well, imbedded in the cerebellum to be exact. This brain structure is the commander of coordinated movement and possibly even some forms of cognitive learning. Damage to this area leads to motor or movement difficulties. Some scientists have discovered cognitive problems as well (see [Mar98, II06a]).

Neuro–scientists believe the structure coordinates movement of muscles and joints by synthesizing data from the brain stem, the spinal cord, and another the cerebral cortex along with sensory input from muscles and other areas. The brain stem and spinal cord provide information on body positioning and the cerebral cortex is responsible for all conscious experience, including perception, emotion and planning.

Some scientists suspect that there are two main information pathways in the cerebellum that interact to synthesize incoming information. One carries a large amount of data from different brain and body areas and contains memory cells. The other originates in the brain stem and interacts with the first pathway to learn new patterns of movement based on incoming information. New skills are learned by trial and error and then coded into the cerebellar memory. Clinical observations suggest that mental activities also are coordinated in the cerebellum in a similar manner.

The cerebellum is a kind of a 'little brain', or a 'biocomputer', involved in the coordination of human movement. A simple way to look at its purpose is that it compares what we thought we were going to do (according to motor cortex) with what is actually happening down in the limbs (according to *proprioceptive feedback*), and corrects the movement if there is a problem. The cerebellum is also partly responsible for motor learning, such as riding a bicycle. Unlike the cerebrum, which works entirely on a *contralateral* basis, the cerebellum works *ipsilaterally.*

The cerebellum has convolutions similar to those of cerebral cortex, only the folds are much smaller. Like the cerebrum, the cerebellum has an outer cortex, an inner white matter, and deep nuclei below the white matter.

If we enlarge a single fold of cerebellum, or a *folium*, we can begin to see the organization of cell types. The outermost layer of the cortex is called the *molecular layer*, and is nearly cell–free. Instead it is occupied mostly by axons and dendrites. The layer below that is a monolayer of large cells called *Purkinje cells*, central players in the circuitry of the cerebellum. Below the Purkinje cells is a dense layer of tiny neurons called *granule cells*. Finally, in the center of each folium is the white matter, all of the axons travelling into and out of the folia. These cell types are hooked together in stereotypical ways throughout the cerebellum.

The human cerebellum has 7–14 million Purkinje cells. Each receives about 200,000 synapses, most onto dendritic splines. Granule cell axons form the *parallel fibers.* They make excitatory synapses onto Purkinje cell dendrites. Each parallel fibre synapses on about 200 Purkinje cells. They create a strip of excitation along the cerebellar folia.

Mossy fibers are one of two main sources of input to the cerebellar cortex (see Figure 8.6). A mossy fibre is an axon terminal that ends in a large, bulbous swelling. These mossy fibers enter the granule cell layer and synapse on the dendrites of granule cells; in fact the granule cells reach out with little 'claws' to grasp the terminals. The granule cells then send their axons up to the molecular layer, where they end in a T and run parallel to the surface. For this reason these axons are called *parallel fibers.* The parallel fibers synapse on the huge dendritic arrays of the Purkinje cells. However, the individual

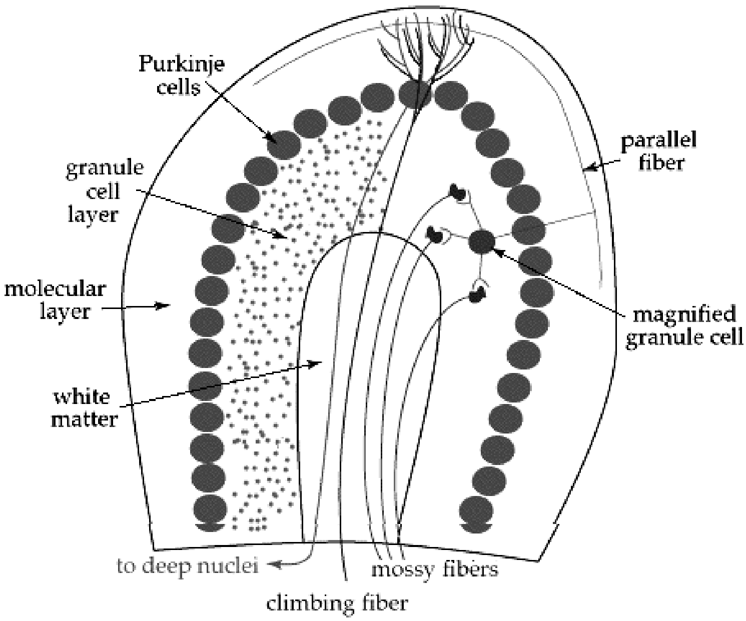

Fig. 8.6 Stereotypical ways throughout the cerebellum

parallel fibers are not a strong drive to the Purkinje cells. The Purkinje cell dendrites fan out within a plane, like the splayed fingers of one hand. If we were to turn a Purkinje cell to the side, it would have almost no width at all. The parallel fibers run perpendicular to the Purkinje cells, so that they only make contact once as they pass through the dendrites.

Unless firing in bursts, parallel fibre EPSPs do not fire Purkinje cells. Parallel fibers provide excitation to all of the Purkinje cells they encounter. Thus, granule cell activity results in a strip of activated Purkinje cells.

Mossy fibers arise from the spinal cord and brainstem. They synapse onto granule cells and deep cerebellar nuclei. The Purkinje cell makes an inhibitory synapse (GABA) to the deep nuclei. Mossy fibre input goes to both cerebellar cortex and deep nuclei. When the Purkinje cell fires, it inhibits output from the deep nuclei.

The *climbing fibre* arises from the inferior olive. It makes about 300 excitatory synapses onto one Purkinje cell. This powerful input can fire the Purkinje cell.

The parallel fibre synapses are plastic—that is, they can be modified by experience. When parallel fibre activity and climbing fibre activity converge on the same Purkinje cell, the parallel fibre synapses become weaker (EPSPs are smaller). This is called long-term depression. Weakened parallel fibre synapses result in less Purkinje cell activity and less inhibition to the deep nuclei, resulting in facilitated deep nuclei output. Consequently, the mossy fibre collaterals control the deep nuclei.

The *basket cell* is activated by parallel fibers afferents. It makes inhibitory synapses onto Purkinje cells. It provides lateral inhibition to Purkinje cells. Basket cells inhibit Purkinje cells lateral to the active beam.

Golgi cells receive input from parallel fibers, mossy fibers, and climbing fibers. They inhibit granule cells. Golgi cells provide feedback inhibition to granule cells as well as feedforward inhibition to granule cells. Golgi cells create a brief burst of granule cell activity.

Although each parallel fibre touches each Purkinje cell only once, the thousands of parallel fibers working together can drive the Purkinje cells to fire like mad.

The second main type of input to the folium is the *climbing fibre*. The climbing fibers go straight to the Purkinje cell layer and snake up the Purkinje dendrites, like ivy climbing a trellis. Each climbing fibre associates with only one Purkinje cell, but when the climbing fibre fires, it provokes a large response in the Purkinje cell.

The Purkinje cell compares and processes the varying inputs it gets, and finally sends its own axons out through the white matter and down to the *deep nuclei*. Although the inhibitory Purkinje cells are the main output of the cerebellar cortex, the output from the cerebellum as a whole comes from the deep nuclei. The three deep nuclei are responsible for sending excitatory output back to the thalamus, as well as to postural and vestibular centers.

There are a few other cell types in cerebellar cortex, which can all be lumped into the category of inhibitory interneuron. The *Golgi cell* is found among the granule cells. The *stellate* and *basket cells* live in the molecular layer. The basket cell (right) drops axon branches down into the Purkinje cell layer where the branches wrap around the cell bodies like baskets.

The cerebellum operates in 3's: there are 3 highways leading in and out of the cerebellum, there are 3 main inputs, and there are 3 main outputs from 3 deep nuclei. They are:

The 3 highways are the *peduncles*. There are 3 pairs (see [Mar98, II06a]):

1. The *inferior cerebellar peduncle* (restiform body) contains the dorsal spinocerebellar tract (DSCT) fibers. These fibers arise from cells in the ipsilateral Clarke's column in the spinal cord (C8–L3). This peduncle contains the cuneocerebellar tract (CCT) fibers. These fibers arise from the ipsilateral accessory cuneate nucleus. The largest component of the inferior cerebellar peduncle consists of the olivocerebellar tract (OCT) fibers. These fibers arise from the contralateral inferior olive. Finally, vestibulocerebellar tract (VCT) fibers arise from cells in both the vestibular ganglion and the vestibular nuclei and pass in the inferior cerebellar peduncle to reach the cerebellum.
2. The *middle cerebellar peduncle* (brachium pontis) contains the pontocerebellar tract (PCT) fibers. These fibers arise from the contralateral pontine grey.

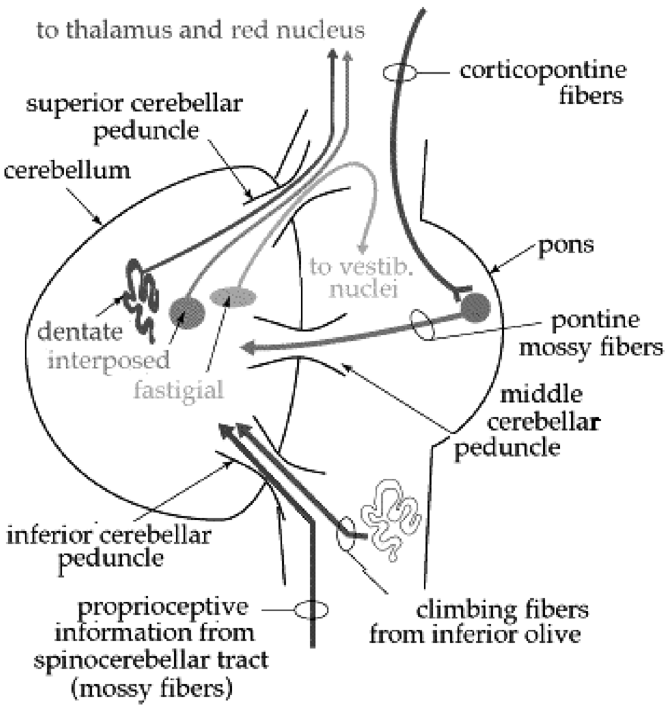

Fig. 8.7 Inputs and outputs of the cerebellum

3. The *superior cerebellar peduncle* (brachium conjunctivum) is the primary efferent (out of the cerebellum) peduncle of the cerebellum. It contains fibers that arise from several deep cerebellar nuclei. These fibers pass ipsilaterally for a while and then cross at the level of the inferior colliculus to form the decussation of the superior cerebellar peduncle. These fibers then continue ipsilaterally to terminate in the red nucleus ('ruber–duber') and the motor nuclei of the thalamus (VA, VL).

The 3 inputs are: *mossy fibers* from the *spinocerebellar* pathways, climbing fibers from the *inferior olive*, and more mossy fibers from the *pons*, which are carrying information from *cerebral cortex* (see Figure 8.7). The mossy fibers from the spinal cord have come up ipsilaterally, so they do not need to cross. The fibers coming down from cerebral cortex, however, do need to cross (as the cerebrum is concerned with the opposite side of the body, unlike the cerebellum). These fibers synapse in the pons (hence the huge block of fibers in the cerebral peduncles labelled 'corticopontine'), cross, and enter the cerebellum as mossy fibers.

The 3 deep nuclei are the *fastigial*, *interposed*, and *dentate nuclei*. The fastigial nucleus is primarily concerned with balance, and sends information mainly to vestibular and reticular nuclei. The dentate and interposed nuclei are concerned more with voluntary movement, and send axons mainly to thalamus and the red nucleus.

The main function of the cerebellum as a motor controller is depicted in Figure 4.7 above. A coordinated movement is easy to recognize, but we know little about how it is achieved. In search of the neural basis of coordination, a model of spinocerebellar interactions was recently presented in [AG05], in which the structure-functional organizing principle is a division of the cerebellum into discrete micro–complexes. Each micro–complex is the recipient of a specific motor error signal - that is, a signal that conveys information about an inappropriate movement. These signals are encoded by spinal reflex circuits and conveyed to the cerebellar cortex through climbing fibre afferents. This organization reveals salient features of cerebellar information processing, but also highlights the importance of systems level analysis for a fuller understanding of the neural mechanisms that underlie behavior.

The authors of [AG05] reviewed anatomical and physiological foundations of cerebellar information processing. The cerebellum is crucial for the coordination of movement. The authors presented a model of the cerebellar paravermis, a region concerned with the control of voluntary limb movements through its interconnections with the spinal cord. They particularly focused on the olivo-cerebellar climbing fibre system.

Climbing fibres are proposed to convey motor error signals (signals that convey information about inappropriate movements) related to elementary limb movements that result from the contraction of single muscles. The actual encoding of motor error signals is suggested to depend on sensorimotor transformations carried out by spinal modules that mediate nociceptive withdrawal reflexes.

The termination of the climbing fibre system in the cerebellar cortex subdivides the paravermis into distinct microzones. Functionally similar but spatially separate microzones converge onto a common group of cerebellar nuclear neurons. The processing units formed as a consequence are termed 'multizonal microcomplexes' (MZMCs), and are each related to a specific spinal reflex module.

The distributed nature of microzones that belong to a given MZMC is proposed to enable similar climbing fibre inputs to integrate with mossy fibre inputs that arise from different sources. Anatomical results consistent with this notion have been obtained.

Within an individual MZMC, the skin receptive fields of climbing fibres, mossy fibres and cerebellar cortical inhibitory interneurons appear to be similar. This indicates that the inhibitory receptive fields of Purkinje cells within a particular MZMC result from the activation of inhibitory interneurons by local granule cells.

On the other hand, the parallel fibre–mediated excitatory receptive fields of the Purkinje cells in the same MZMC differ from all of the other receptive fields, but are similar to those of mossy fibres in another MZMC. This indicates that the excitatory input to Purkinje cells in a given MZMC originates in non–local granule cells and is mediated over some distance by parallel fibres.

The output from individual MZMCs often involves two or three segments of the ipsilateral limb, indicative of control of multi–joint muscle synergies. The distal–most muscle in this synergy seems to have a roughly antagonistic action to the muscle associated with the climbing fibre input to the MZMC.

The model proposed in [AG05] indicates that the cerebellar paravermis system could provide the control of both single– and multi–joint movements. Agonist-antagonist activity associated with single–joint movements might be controlled within a particular MZMC, whereas coordination across multiple joints might be governed by interactions between MZMCs, mediated by parallel fibres.

Chapter 9
CSB–Intelligence

If you watched a Grand Slam tennis final in last 3–4 years, you would see that at least one half of the abilities of Roger Federer and Rafael Nadal, world number 1 and 2 players, is *intelligence.* In complex sport games, like tennis, intelligence is simply a dominant quality of a champion. That is the reason for this Chapter.

9.1 Human Mind

Recall that the word *intelligence* (plural *intelligences*) comes from Latin *intellegentia*.[1] It is a property of *human mind* that encompasses many related *mental abilities*, such as the capacities to *reason*, *plan*, solve problems, think abstractly, comprehend ideas and *language*, and learn. Although many regard the concept of intelligence as having a much broader scope, for example in *cognitive science* and *computer science*, in some schools of *psychology*,[2] the

[1] *Intellegentia* is a combination of Latin *inter* = *between* and *legere* = *choose, pick out, read.* Inter–lege–nt–ia, literally means 'choosing between.'

Also, note that there is a is a scientific journal titled 'Intelligence', dealing with intelligence and psychometrics. It was founded in 1977 by Douglas K. Detterman of Case Western Reserve University. It is currently published by Elsevier and is the official journal of the International Society for Intelligence Research.

[2] Recall that *psychology* is an academic and applied field involving the study of the human mind, brain, and behavior. Psychology also refers to the application of such knowledge to various spheres of human activity, including problems of individuals' daily lives and the treatment of mental illness.

Psychology differs from anthropology, economics, political science, and sociology in seeking to explain the mental processes and behavior of individuals.

T.T. Ivancevic et al.: Complex Sports Biodynamics, COSMOS 2, pp. 105–214.
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study of intelligence generally regards this trait as distinct from *creativity*, *personality*, *character*, or *wisdom*.

Briefly, the word *intelligence* has five common meanings:

1. Capacity of human mind, especially to understand principles, truths, concepts, facts or meanings, acquire knowledge, and apply it to practise; the ability to learn and comprehend.
2. A form of life that has such capacities.
3. Information, usually secret, about the enemy or about hostile activities.
4. A political or military department, agency or unit designed to gather such information.
5. Biological intelligent behavior represents animal's ability to make productive decisions for a specific task, given a root objective; this decision is based on learning which requires the ability to hold onto results from previous tasks, as well as being able to analyze the situation; the root objective for living organisms is simply survival; the 'specific task' could be a choice of food, i.e., one that provides long steady supply of energy as it could be a long while before the next mealtime; this is in perfect harmony with the root biological objective – survival.

According to Enciclopaedia Britannica, *intelligence* is the *ability to adapt effectively to the environment, either by making a change in oneself or by changing the environment or finding a new one.* Different investigators have emphasized different aspects of intelligence in their definitions. For example, in a 1921 symposium on the definition of intelligence, the American psychologist Lewis Terman emphasized the *ability to think abstractly*, while another American psychologist, Edward Thorndike, emphasized *learning* and the ability to give good responses to questions. In a similar 1986 symposium, however, psychologists generally agreed on the importance of adaptation to the environment as the key to understanding both what intelligence is and what it does. Such adaptation may occur in a variety of environmental situations. For example, a student in school learns the material that is required to pass or do well in a course; a physician treating a patient with an unfamiliar disease adapts by learning about the disease; an artist reworks a painting in order to make it convey a more harmonious impression. For the most part, adapting involves making a change in oneself in order to cope more effectively,

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Psychology differs from biology and neuroscience in that it is primarily concerned with the interaction of mental processes and behavior, and of the overall processes of a system, and not simply the biological or neural processes themselves, though the subfield of neuropsychology combines the study of the actual neural processes with the study of the mental effects they have subjectively produced.

The word psychology comes from the ancient Greek 'psyche', which means 'soul' or 'mind' and 'ology', which means 'study'.

but sometimes effective adaptation involves either changing the environment or finding a new environment altogether. Effective adaptation draws upon a number of cognitive processes, such as perception, learning, memory, reasoning, and problem solving. The main trend in defining intelligence, then, is that it is not itself a cognitive or mental process, but rather a selective combination of these processes purposively directed toward effective adaptation to the environment. For example, the physician noted above learning about a new disease adapts by perceiving material on the disease in medical literature, learning what the material contains, remembering crucial aspects of it that are needed to treat the patient, and then reasoning to solve the problem of how to apply the information to the needs of the patient. Intelligence, in sum, has come to be regarded as not a single ability but an effective drawing together of many abilities. This has not always been obvious to investigators of the subject, however, and, indeed, much of the history of the field revolves around arguments regarding the nature and abilities that constitute intelligence.

Now, let us quickly reflect on the above general *intelligence–related keywords.*

Reason

Recall that in the philosophy of arguments, *reason* is the ability of the human mind to form and operate on concepts in abstraction, in varied accordance with rationality and logic —terms with which reason shares heritage. Reason is thus a very important word in Western intellectual history, to describe a type or aspect of mental thought which has traditionally been claimed as distinctly human, and not to be found elsewhere in the animal world. Discussion and debate about the nature, limits and causes of reason could almost be said to define the main lines of historical philosophical discussion and debate. Discussion about reason especially concerns:

(a) its relationship to several other related concepts: language, logic, consciousness etc,
(b) its ability to help people decide what is true, and
(c) its origin.

The concept of reason is connected to the concept of language, as reflected in the meanings of the Greek word ‘logos’, later to be translated by Latin ‘ratio’ and then French ‘raison’, from which the English word derived. As reason, rationality, and logic are all associated with the ability of the human mind to predict effects as based upon presumed causes, the word ‘reason’ also denotes a ground or basis for a particular argument, and hence is used synonymously with the word ‘cause’.

It is sometimes said that the contrast between reason and logic extends back to the time of Plato[3] and Aristotle[4]. Indeed, although they had no

[3] Plato (c. 427 — c. 347 BC) was an immensely influential ancient Greek philosopher, a student of Socrates, writer of philosophical dialogues, and founder of the Academy in Athens where Aristotle studied. Plato lectured extensively at the Academy, and wrote on many philosophical issues, dealing especially in politics, ethics, metaphysics, and epistemology. The most important writings of Plato are his dialogues, although some letters have come down to us under his name. It is believed that all of Plato's authentic dialogues survive. However, some dialogues ascribed to Plato by the Greeks are now considered by the consensus of scholars to be either suspect (e.g., First Alcibiades, Clitophon) or probably spurious (such as Demodocus, or the Second Alcibiades). The letters are all considered to probably be spurious, with the possible exception of the Seventh Letter. Socrates is often a character in Plato's dialogues. How much of the content and argument of any given dialogue is Socrates' point of view, and how much of it is Plato's, is heavily disputed, since Socrates himself did not write anything; this is often referred to as the 'Socratic problem'. However, Plato was doubtless strongly influenced by Socrates' teachings.

Platonism has traditionally been interpreted as a form of metaphysical dualism, sometimes referred to as Platonic realism, and is regarded as one of the earlier representatives of metaphysical objective idealism. According to this reading, Plato's metaphysics divides the world into two distinct aspects: the *intelligible world* of 'forms', and the *perceptual world* we see around us. The perceptual world consists of imperfect copies of the intelligible forms or ideas. These forms are unchangeable and perfect, and are only comprehensible by the use of the intellect or understanding, that is, a capacity of the mind that does not include sense-perception or imagination. This division can also be found in Zoroastrian philosophy, in which the dichotomy is referenced as the *Minu* (intelligence) and *Giti* (perceptual) worlds. Currently, in the domain of mathematical physics, this view has been adopted by Sir Roger Penrose.

[4] Aristotle (384 BC — March 7, 322 BC) was an ancient Greek philosopher, a student of Plato and teacher of Alexander the Great. He wrote books on divers subjects, including physics, poetry, zoology, logic, rhetoric, government, and biology, none of which survive in their entirety. Aristotle, along with Plato and Socrates, is generally considered one of the most influential of ancient Greek philosophers. They transformed Presocratic Greek philosophy into the foundations of Western philosophy as we know it. The writings of Plato and Aristotle founded two of the most important schools of Ancient philosophy.

Aristotle valued knowledge gained from the senses and in modern terms would be classed among the modern empiricists. He also achieved a 'grounding' of dialectic in the Topics by allowing interlocutors to begin from commonly held beliefs (Endoxa), with his frequent aim being to progress from 'what is known to us' towards 'what is known in itself' (Physics). He set the stage for what would eventually develop into the empirical scientific method some two millennia later. Although he wrote dialogues early in his career, no more than fragments of these have survived. The works of Aristotle that still exist today are in treatise form and were, for the most part, unpublished texts. These were probably lecture notes or texts used by his students, and were almost certainly revised repeatedly separate Greek word for logic as opposed to language and reason, Aristotle's

over the course of years. As a result, these works tend to be eclectic, dense and difficult to read. Among the most important ones are Physics, Metaphysics (or Ontology), Nicomachean Ethics, Politics, De Anima (On the Soul) and Poetics. These works, although connected in many fundamental ways, are very different in both style and substance.

Aristotle is known for being one of the few figures in history who studied almost every subject possible at the time, probably being one of the first polymaths. In science, Aristotle studied anatomy, astronomy, economics, embryology, geography, geology, meteorology, physics, and zoology. In philosophy, Aristotle wrote on aesthetics, ethics, government, metaphysics, politics, psychology, rhetoric and theology. He also dealt with education, foreign customs, literature and poetry. His combined works practically constitute an encyclopedia of Greek knowledge.

According to Aristotel, everything is made out of the five basic elements:

1. Earth, which is cold and dry;
2. Water, which is cold and wet;
3. Fire, which is hot and dry;
4. Air, which is hot and wet; and
5. Aether, which is the divine substance that makes up the heavenly spheres and heavenly bodies (stars and planets).

Aristotle defines his philosophy in terms of essence, saying that philosophy is 'the science of the universal essence of that which is actual'. Plato had defined it as the 'science of the idea', meaning by idea what we should call the unconditional basis of phenomena. Both pupil and master regard philosophy as concerned with the universal; Aristotle, however, finds the universal in particular things, and called it the essence of things, while Plato finds that the universal exists apart from particular things, and is related to them as their prototype or exemplar. For Aristotle, therefore, philosophic method implies the ascent from the study of particular phenomena to the knowledge of essences, while for Plato philosophic method means the descent from a knowledge of universal ideas to a contemplation of particular imitations of those ideas. In a certain sense, Aristotle's method is both inductive and deductive, while Plato's is essentially deductive from a priori principles.

In the larger sense of the word, Aristotle makes philosophy coextensive with reasoning, which he also called 'science'. Note, however, that his use of the term science carries a different meaning than that which is covered by the scientific method. "All science (dianoia) is either practical, poetical or theoretical." By practical science he understands ethics and politics; by poetical, he means the study of poetry and the other fine arts; while by theoretical philosophy he means physics, mathematics, and metaphysics.

Aristotle's conception of logic was the dominant form of logic up until the advances in mathematical logic in the 19th century. Kant himself thought that Aristotle had done everything possible in terms of logic. The Organon is the name given by Aristotle's followers, the Peripatetics, for the standard collection of six of his works on logic. The system of logic described in two of these works, namely On Interpretation and the Prior Analytics, is often called Aristotelian logic.

syllogism (Greek 'syllogismos') identified logic clearly for the first time as a distinct field of study: the most peculiarly reasonable ('logikê') part of reasoning, so to speak.

No philosopher of any note has ever argued that logic is the same as reason. They are generally thought to be distinct, although logic is one important aspect of reason. But the tendency to the preference for 'hard logic', or 'solid logic', in modern times has incorrectly led to the two terms occasionally being seen as essentially synonymous or perhaps more often logic is seen as the defining and pure form of reason.

However machines and animals can unconsciously perform logical operations, and many animals (including humans) can unconsciously, associate different perceptions as causes and effects and then make decisions or even plans. Therefore, to have any distinct meaning at all, 'reason' must be the type of thinking which links language, consciousness and logic, and at this time, only humans are known to combine these things.

However, note that reasoning is defined very differently depending on the context of the understanding of reason as a form of knowledge. The logical definition is the act of using reason to derive a conclusion from certain premises using a given methodology, and the two most commonly used explicit methods to reach a conclusion are deductive reasoning and inductive reasoning. However, within idealist philosophical contexts, reasoning is the mental process which informs our imagination, perceptions, thoughts, and feelings with whatever intelligibility these appear to contain; and thus links our experience with universal meaning. The specifics of the methods of reasoning are of interest to such disciplines as philosophy, logic, psychology, and artificial intelligence.

In deductive reasoning, given true premises, the conclusion must follow and it cannot be false. In this type of reasoning, the conclusion is inherent in the premises. Deductive reasoning therefore does not increase one's knowledge base and is said to be non–ampliative. Classic examples of deductive reasoning are found in such syllogisms as the following:

1. One must exist/live to perform the act of thinking.
2. I think.
3. Therefore, I am.

In inductive reasoning, on the other hand, when the premises are true, then the conclusion follows with some degree of *probability*.[5] This method of

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Aristotle was the creator of syllogisms with modalities (modal logic). The word modal refers to the word 'modes', explaining the fact that modal logic deals with the modes of truth. Aristotle introduced the qualification of 'necessary' and 'possible' premises. He constructed a logic which helped in the evaluation of truth but which was difficult to interpret.

[5] Recall that the the word *probability* derives from the Latin 'probare' (to prove, or to test). Informally, probable is one of several words applied to uncertain events or knowledge, being closely related in meaning to likely, risky, hazardous, and

reasoning is ampliative, as it gives more information than what was contained in the premises themselves. A classical example comes from David Hume:[6]

1. The sun rose in the east every morning up until now.
2. Therefore the sun will also rise in the east tomorrow.

A third method of reasoning is called abductive reasoning, or inference to the best explanation. This method is more complex in its structure and can involve both inductive and deductive arguments. The main characteristic of abduction is that it is an attempt to favor one conclusion above others by

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doubtful. Chance, odds, and bet are other words expressing similar notions. Just as the theory of mechanics assigns precise definitions to such everyday terms as work and force, the theory of probability attempts to quantify the notion of probable.

The scientific study of probability is a modern development. Gambling shows that there has been an interest in quantifying the ideas of probability for millennia, but exact mathematical descriptions of use in those problems only arose much later. The doctrine of probabilities dates to the correspondence of Pierre de Fermat and Blaise Pascal (1654). Christiaan Huygens (1657) gave the earliest known scientific treatment of the subject. Jakob Bernoulli's 'Ars Conjectandi' (posthumous, 1713) and Abraham de Moivre's 'Doctrine of Chances' (1718) treated the subject as a branch of mathematics.

Pierre–Simon Laplace (1774) made the first attempt to deduce a rule for the combination of observations from the principles of the theory of probabilities. He represented the law of probability of errors by a curve $y = \varphi(x)$, x being any error and y its probability, and laid down three properties of this curve: (i) it is symmetric as to the $y-$axis; (ii) the $x-$axis is an asymptote, the probability of the error being 0; (iii) the area enclosed is 1, it being certain that an error exists. He deduced a formula for the *mean* of three observations. He also gave (1781) a formula for the law of facility of error (a term due to Lagrange, 1774), but one which led to unmanageable equations. Daniel Bernoulli (1778) introduced the principle of the maximum product of the probabilities of a system of concurrent errors.

The *method of least squares* is due to Adrien–Marie Legendre (1805), who introduced it in his 'Nouvelles méthodes pour la détermination des orbites des comètes' (New Methods for Determining the Orbits of Comets). In ignorance of Legendre's contribution, an Irish–American writer, Robert Adrain, editor of 'The Analyst' (1808), first deduced the law of facility of error,

$$\phi(x) = ce^{-h^2x^2}$$

where c and h are constants depending on precision of observation. He gave two proofs, the second being essentially the same as John Herschel's (1850). Carl Friedrich Gauss gave the first proof which seems to have been known in Europe (the third after Adrain's) in 1809. Further proofs were given by Laplace (1810, 1812), Gauss (1823), James Ivory (1825, 1826), Hagen (1837), Friedrich Bessel (1838), W. F. Donkin (1844, 1856), and Morgan Crofton (1870).

[6] David Hume (April 26, 1711 – August 25, 1776) was a Scottish philosopher, economist, and historian, as well as an important figure of Western philosophy and of the Scottish Enlightenment.

either attempting to falsify alternative explanations, or showing the likelihood of the favored conclusion given a set of more or less disputable assumptions.

A fourth method of reasoning is analogy. Reasoning by analogy goes from a particular to another particular. The conclusion of an analogy is only plausible. Analogical reasoning is very frequent in common sense, science, philosophy and the humanities, but sometimes it is accepted only as an auxiliary method. A refined approach is *case–based reasoning*.

Plan

Recall that a *plan* represents a proposed or intended method of getting from one set of circumstances to another. They are often used to move from the present situation, towards the achievement of one or more objectives or goals.

Informal or ad–hoc plans are created by individual humans in all of their pursuits. Structured and formal plans, used by multiple people, are more likely to occur in projects, diplomacy, careers, economic development, military campaigns, combat, or in the conduct of other business.

It is common for less formal plans to be created as abstract ideas, and remain in that form as they are maintained and put to use. More formal plans as used for business and military purposes, while initially created with and as an abstract thought, are likely to be written down, drawn up or otherwise stored in a form that is accessible to multiple people across time and space. This allows more reliable collaboration in the execution of the plan.

The term planning implies the working out of sub–components in some degree of detail. Broader–brush enunciations of objectives may qualify as metaphorical road–maps.

Planning literally just means the creation of a plan; it can be as simple as making a list. It has acquired a technical meaning, however, to cover the area of government legislation and regulations related to the use of resources.

Planning can refer to the planned use of any and all resources, as for example, in the succession of Five–Year Plans through which the government of the Soviet Union sought to develop the country. However, the term is most frequently used in relation to planning for the use of land and related resources, for example in urban planning, transportation planning, and so forth.

Problem Solving

The *problem solving* forms part of thinking. Considered the most complex of all intellectual functions, problem solving has been defined as higher–order cognitive process that requires the modulation and control of more routine or fundamental skills. It occurs if an organism or an artificial intelligence system does not know how to proceed from a given state to a desired goal state. It is part of the larger problem process that includes problem finding and problem shaping.

The nature of human problem solving has been studied by psychologists over the past hundred years. There are several methods of studying problem solving, including: *introspection*,[7] *behaviorism*,[8] computer simulation and experimental methods.

Beginning with the early experimental work of the Gestaltists in Germany (see e.g., [Dun35], and continuing through the 1960s and early 1970s, research on problem solving typically conducted relatively simple, laboratory tasks that appeared novel to participants (see e.g. [May92]). Various reasons account for the choice of simple novel tasks: they had clearly defined optimal solutions, they were solvable within a relatively short time frame, researchers could trace participants' problem–solving steps, and so on. The researchers made the underlying assumption, of course, that simple tasks such as the Tower of Hanoi captured the main properties of 'real world' problems, and that the cognitive processes underlying participants' attempts to solve simple problems were representative of the processes engaged in when solving 'real world' problems. Thus researchers used simple problems for reasons of convenience, and thought generalizations to more complex problems would become possible. (See more on problem solving below.)

Learning

Recall that learning is the process of acquiring knowledge, skills, attitudes, or values, through study, experience, or teaching, that causes a change of behavior that is persistent, measurable, and specified or allows an individual to formulate a new mental construct or revise a prior mental construct

[7] Introspection is contemplation on one's self, as opposed to extrospection, the observation of things external to one's self. Introspection may be used synonymously with self–reflection and used in a similar way. Cognitive psychology accepts the use of the scientific method, but rejects introspection as a valid method of investigation. It should be noted that Herbert Simon and Allen Newell identified the 'thinking–aloud' protocol, in which investigators view a subject engaged in introspection, and who speaks his thoughts aloud, thus allowing study of his introspection.

Introspection was once an acceptable means of gaining insight into psychological phenomena. Introspection was used by German physiologist Wilhelm Wundt in the experimental psychology laboratory he had founded in Leipzig in 1879. Wundt believed that by using introspection in his experiments he would gather information into how the subject's minds were working, thus he wanted to examine the mind into its basic elements. Wundt did not invent this way of looking into an individual's mind through their experiences; rather, it can be dated back to Socrates. Wundt's distinctive contribution was to take this method into the experimental arena and thus into the newly formed field of psychology.

[8] Behaviorism is an approach to psychology based on the proposition that behavior can be studied and explained scientifically without recourse to internal mental states. A similar approach to political science may be found in Behavioralism. The behaviorist school of thought ran concurrent with the psychoanalysis movement in psychology in the 20th century. Its main influences were Ivan Pavlov, who investigated classical conditioning, John B. Watson who rejected introspective methods and sought to restrict psychology to experimental methods, and B.F. Skinner who conducted research on operant conditioning.

(conceptual knowledge such as attitudes or values). It is a process that depends on experience and leads to long–term changes in behavior potential. Behavior potential describes the possible behavior of an individual (not actual behavior) in a given situation in order to achieve a goal. But potential is not enough; if individual learning is not periodically reinforced, it becomes shallower and shallower, and eventually will be lost in that individual.

Short term changes in behavior potential, such as fatigue, do not constitute learning. Some long–term changes in behavior potential result from aging and development, rather than learning.

Education is the conscious attempt to promote learning in others. The primary function of 'teaching' is to create a safe, viable, productive learning environment. Management of the total learning environment to promote, enhance and motivate learning is a *paradigm shift*[9] from a focus on teaching to a focus on learning.

The stronger the stimulation for the brain, the deeper the impression that is left in the neuronal network. Therefore a repeated, very intensive experience perceived through all of the senses (audition, sight, smell) of an individual will remain longer and prevail over other experiences. The complex interactions of neurons that have formed a network in the brain determine the direction of flow of the micro–voltage electricity that flows through the brain when a person thinks. The characteristics of the neuronal network shaped by previous impressions is what we call the person's 'character'.

The most basic learning process is *imitation*, one's personal repetition of an observed process, such as a smile. Thus an imitation will take one's time

[9] Recall that an *epistemological paradigm shift* was called a *scientific revolution* by epistemologist and historian of science Thomas Kuhn in his 1962 book 'The Structure of Scientific Revolutions', to describe a change in basic assumptions within the ruling theory of science. It has since become widely applied to many other realms of human experience as well.

A scientific revolution occurs, according to Kuhn, when scientists encounter anomalies which cannot be explained by the universally accepted paradigm within which scientific progress has thereto been made. The paradigm, in Kuhn's view, is not simply the current theory, but the entire worldview in which it exists, and all of the implications which come with it. There are anomalies for all paradigms, Kuhn maintained, that are brushed away as acceptable levels of error, or simply ignored and not dealt with (a principal argument Kuhn uses to reject Karl Popper's model of falsifiability as the key force involved in scientific change). Rather, according to Kuhn, anomalies have various levels of significance to the practitioners of science at the time. To put it in the context of early 20th century physics, some scientists found the problems with calculating Mercury's perihelion more troubling than the Michelson–Morley experiment results, and some the other way around. Kuhn's model of scientific change differs here, and in many places, from that of the logical positivists in that it puts an enhanced emphasis on the individual humans involved as scientists, rather than abstracting science into a purely logical or philosophical venture. When enough significant anomalies have accrued against a current paradigm, the scientific discipline is thrown (attention to the details), space (a location for learning), skills (or practice), and other resources (for example, a protected area). Through copying, most infants learn how to hunt (i.e., direct one's attention), feed and perform most basic tasks necessary for survival.

The so–called *Bloom's Taxonomy*[10] divides the learning process into a six–level hierarchy, where knowledge is the lowest order of cognition and evaluation the highest [Blo80]:

1. Knowledge is the memory of previously–learnt materials such as facts, terms, basic concepts and answers.
2. Comprehension is the understanding of facts and ideas by organization, comparison, translation, interpretation, and description.
3. Application is the use of new knowledge to solve problems.
4. Analysis is the examination and division of information into parts by identifying motives or causes. A person can analyze by making inferences and finding evidence to support generalizations.
5. Synthesis is the compilation of information in a new way by combining elements into patterns or proposing alternative solutions.
6. Evaluation is the presentation and defense of opinions by making judgments about information, validity of ideas or quality of work based on the following set of criteria:

into a state of crisis, according to Kuhn. During this crisis, new ideas, perhaps ones previously discarded, are tried. Eventually a new paradigm is formed, which gains its own new followers, and an intellectual 'battle' takes place between the followers of the new paradigm and the hold–outs of the old paradigm. Again, for early 20th century physics, the transition between the Maxwellian electromagnetic worldview and the Einsteinian Relativistic worldview was not instantaneous nor calm, and instead involved a protracted set of 'attacks', both with empirical data as well as rhetorical or philosophical arguments, by both sides, with the Einsteinian theory winning out in the long–run. Again, the weighing of evidence and importance of new data was fit through the human sieve: some scientists found the simplicity of Einstein's equations to be most compelling, while some found them more complicated than the notion of Maxwell's aether which they banished. Some found Eddington's photographs of light bending around the sun to be compelling, some questioned their accuracy and meaning. Sometimes the convincing force is just time itself and the human toll it takes, Kuhn pointed out, using a quote from Max Planck: "A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it." After a given discipline has changed from one paradigm to another, this is called, in Kuhn's terminology, a scientific revolution or a paradigm shift. It is often this final conclusion, the result of the long process, that is meant when the term paradigm shift is used colloquially: simply the (often radical) change of worldview, without reference to the specificities of Kuhn's historical argument.

[10] Benjamin Bloom (21 February 1913 – September 13, 1999) was an American educational psychologist who made significant contributions to the classification of educational objectives and the theory of mastery learning.

- *Attention* – the cognitive process of selectively concentrating on one thing while ignoring other things. Examples include listening carefully to what someone is saying while ignoring other conversations in the room. Attention can also be split, as when a person drives a car and talks on a cell phone at the same time. Sometimes our attention shifts to matters unrelated to the external environment, this is referred to as mind-wandering or 'spontaneous thought'. Attention is one of the most intensely studied topics within psychology and cognitive neuroscience. Of the many cognitive processes associated with the human mind (decision–making, memory, emotion, etc), attention is considered the most concrete because it is tied so closely to perception. As such, it is a gateway to the rest of cognition. The most famous definition of attention was provided by one of the first major psychologists, William James[11] in his 1890 book 'Principles of Psychology': "Everyone knows

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Bloom's classification of educational objectives, known as Bloom's Taxonomy, incorporates cognitive, psychomotor, and affective domains of knowledge. While working at the University of Chicago in the 1950s and '60s, he wrote two important books, Stability and Change in Human Characteristics and Taxonomy of Educational Objectives (1956). Bloom's taxonomy provides structure in which to categorize test questions. This taxonomy helps teachers pose questions in such a way to determine the level of understanding that a student possesses. For example, based upon the type of question asked, a teacher can determine that a student is competent in content knowledge, comprehension, application, analysis, synthesis and/or evaluation. This taxonomy is organized in a hierarchal way to organize information from basic factual recall to higher order thinking. This data table below is from the article written by W. Huitt titled, "Bloom *et al.*'s Taxonomy of the Cognitive Domain". The table below describes the levels of Bloom's Taxonomy, beginning with the lowest level of basic factual recall. Each level in the table is defined, gives descriptive verbs that would foster each level of learning, and describes sample behaviors of that level. Bloom's taxonomy helps teachers better prepare questions that would foster basic knowledge recall all the way to questioning styles that foster synthesis and evaluation. By structuring the questioning format, teachers will be able to better understand what a child's weaknesses and strengths are and determine ways to help students think at a higher–level.

[11] William James (January 11, 1842 — August 26, 1910) was a pioneering American psychologist and philosopher. He wrote influential books on the young science of psychology, educational psychology, psychology of religious experience and mysticism, and the philosophy of pragmatism. He gained widespread recognition with his monumental Principles of 'Psychology' (1890), fourteen hundred pages in two volumes which took ten years to complete. 'Psychology: The Briefer Course', was an 1892 abridgement designed as a less rigorous introduction to the field. These works criticized both the English associationist school and the Hegelianism of his day as competing dogmatisms of little explanatory value, and sought to re–conceive of the human mind as inherently purposive and selective.

James defined *true beliefs* as those that prove useful to the believer. Truth, he said, is that which works in the way of belief. "True ideas lead us into useful verbal

what attention is. It is the taking possession by the mind in clear and vivid form, of one out of what seem several simultaneously possible objects or trains of thought... It implies withdrawal from some things in order to deal effectively with others." Most experiments show that one neural correlate of attention is enhanced firing. Say a neuron has a certain response to a stimulus when the animal is not attending to that stimulus. When the animal attends to the stimulus, even if the physical characteristic of the stimulus remains the same the neurons response is enhanced. A strict criterion, in this paradigm of testing attention, is that the physical stimulus available to the subject must be the same, and only the mental state is allowed to change. In this manner, any differences in neuronal firing may be attributed to a mental state (attention) rather than differences in the stimulus itself.
- *Habituation* – an example of non–associative learning in which there is a progressive diminution of behavioral response probability with repetition of a stimulus. It is another form of integration. An animal first responds to a sensory stimulus, but if it is neither rewarding nor harmful the animal learns to suppress its response through repeated encounters. One example of this can be seen in small song birds – if a stuffed owl (or similar predator) is introduced into the cage, the birds react to it as though it were a real predator, but soon realise that it is not and so become habituated to it. If another stuffed owl is introduced (or the same one removed and re–introduced), the birds react to it as though it were a predator, showing that it is only a very specific stimulus that is being ignored (namely, one particular unmoving owl in one place). This learned suppression of response is habituation. Habituation is stimulus specific. It does not cause a general decline in responsiveness. It functions like an average weighted history wavelet interference filter reducing the responsiveness of the organism to a particular stimulus. Frequently one can see opponent processes after the stimulus is removed. Habituation is connected to associational reciprocal inhibition phenomenon, opponent process, motion after effect, color constancy, size constancy, and negative image after effect. Habituation is frequently used in testing psychological phenomena. Both infants and adults look less and less as a result of consistent exposure to a particular stimulus. The amount of

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and conceptual quarters as well as directly up to useful sensible termini. They lead to consistency, stability and flowing human intercourse" but "all true processes must lead to the face of directly verifying sensible experiences somewhere," he wrote.

Pragmatism as a view of the meaning of truth is considered obsolete by many in contemporary philosophy, because the predominant trend of thinking in the years since James' death in 1910 has been toward non–epistemic definitions of truth, i.e., definitions that don't make truth dependent upon the warrant of a belief. A contemporary philosopher or logician will often be found explaining that the statement 'the book is on the table' is true iff the book is on the table.

time spent looking to a presented alternate stimulus (after habituation to the initial stimulus) is indicative of the strength of the remembered percept of the previous stimulus. It is also used to discover the resolution of perceptual systems, for example, by habituating a subject to one stimulus, and then observing responses to similar ones, one can detect the smallest degree of difference that is detectable by the subject.

Closely related to habituation is *neural adaptation* or *sensory adaptation* – a change over time in the responsiveness of the sensory system to a constant stimulus. It is usually experienced as a change in the stimulus. For example, if one rests one's hand on a table, one immediately feels the table's surface on one's skin. Within a few seconds, however, one ceases to feel the table's surface. The sensory neurons stimulated by the table's surface respond immediately, but then respond less and less until they may not respond at all; this is neural adaptation. More generally, neural adaptation refers to a temporary change of the neural response to a stimulus as the result of preceding stimulation. It is usually distinguished from memory, which is thought to involve a more permanent change in neural responsiveness. Some people use adaptation as an umbrella term that encompasses the neural correlates of priming and habituation. In most cases, adaptation results in a response decrease, but response facilitation does also occur. Some adaptation may result from simple fatigue, but some may result from an active re–calibration of the responses of neurons to ensure optimal sensitivity. Adaptation is considered to be the cause of perceptual phenomena like afterimages and the motion aftereffect. In the absence of fixational eye movements, visual perception may fade out or disappear due to neural adaptation.

- *Sensitization* – an example of non–associative learning in which the progressive amplification of a response follows repeated administrations of a stimulus [BHB95]. For example, electrical or chemical stimulation of the rat hippocampus causes strengthening of synaptic signals, a process known as long–term potentiation (LTP). LTP is thought to underlie memory and learning in the human brain. A different type of sensitization is that of kindling, where repeated stimulation of hippocampal or amygdaloid neurons eventually leads to seizures. Thus, kindling has been suggested as a model for temporal lobe epilepsy. A third type is central sensitization, where nociceptive neurons in the dorsal horns of the spinal cord become sensitized by peripheral tissue damage or inflammation. These various types indicate that sensitization may underlie both pathological and adaptive functions in the organism, but whether they also share the same physiological and molecular properties is not yet established.
- *Classical Pavlovian conditioning* – a type of associative learning. Ivan Pavlov described the learning of conditioned behavior as being formed by pairing two stimuli to condition an animal into giving a certain response. The simplest form of classical conditioning is reminiscent of

what Aristotle would have called the law of contiguity, which states that: ‘When two things commonly occur together, the appearance of one will bring the other to mind.’ Classical conditioning focuses on reflexive behavior or involuntary behavior. Any reflex can be conditioned to respond to a formerly neutral stimulus. The typical paradigm for classical conditioning involves repeatedly pairing a neutral stimulus with an unconditioned stimulus. An unconditioned reflex is formed by an unconditioned stimulus, a stimulus that elicits a response–known as an unconditioned response–that is automatic and requires no learning and are usually apparent in all species. The relationship between the unconditioned stimulus and unconditioned response is known as the unconditioned reflex. The conditioned stimulus, is an initially neutral stimulus that elicits a response–known as a conditioned response–that is acquired through learning and can vary greatly amongst individuals. Conditioned stimuli are associated psychologically with conditions such as anticipation, satisfaction (both immediate and prolonged), and fear. The relationship between the conditioned stimulus and conditioned response is known as the conditioned (or conditional) reflex. In classical conditioning, when the unconditioned stimulus is repeatedly or strongly paired with a neutral stimulus the neutral stimulus becomes a conditioned stimulus and elicits a conditioned response.

- *Operant conditioning* – the use of consequences to modify the occurrence and form of behavior. Operant conditioning is distinguished from Pavlovian conditioning in that operant conditioning deals with the modification of voluntary behavior through the use of consequences, while Pavlovian conditioning deals with the conditioning of involuntary reflexive behavior so that it occurs under new antecedent conditions. Unlike reflexes, which are biologically fixed in form, the form of an operant response is modifiable by its consequences. Operant conditioning, sometimes called instrumental conditioning or instrumental learning, was first extensively studied by Edward Thorndike,[12] who observed

[12] Edward Lee Thorndike (August 31, 1874 - August 9, 1949) was an American psychologist who spent nearly his entire career at Teachers College, Columbia University. His work on animal behavior and the learning process led to the theory of connectionism.

Among Thorndike’s most famous contributions were his research on how cats learned to escape from puzzle boxes, and his related formulation of the law of effect. The law of effect states that responses which are closely followed by satisfying consequences are associated with the situation, and are more likely to reoccur when the situation is subsequently encountered. Conversely, if the responses are followed by aversive consequences, associations to the situation become weaker. The puzzle box experiments were motivated in part by Thorndike’s dislike for statements that animals made use of extraordinary faculties such as insight in their problem solving: ”In the first place, most of the books do not give us a psychology, but rather a eulogy of animals. They have all been about animal intelligence, never about animal stupidity.” (Animal Intelligence, 1911).

the behavior of cats trying to escape from home-made puzzle boxes. When first constrained in the boxes, the cats took a long time to escape. With experience, ineffective responses occurred less frequently and successful responses occurred more frequently, enabling the cats to escape in less time over successive trials. In his Law of Effect, Thorndike theorized that successful responses, those producing satisfying consequences, were 'stamped in' by the experience and thus occurred more frequently. Unsuccessful responses, those producing annoying consequences, were stamped out and subsequently occurred less frequently. In short, some consequences strengthened behavior and some consequences weakened behavior. Burrhus Skinner[13] built upon Thorndike's ideas to construct a more detailed theory of operant conditioning based on:

Thorndike meant to distinguish clearly whether or not cats escaping from puzzle boxes were using insight. Thorndike's instruments in answering this question were 'learning curves' revealed by plotting the time it took for an animal to escape the box each time it was in the box. He reasoned that if the animals were showing 'insight,' then their time to escape would suddenly drop to a negligible period, which would also be shown in the learning curve as an abrupt drop; while animals using a more ordinary method of trial and error would show gradual curves. His finding was that cats consistently showeds gradual learning.

Thorndike interpreted the findings in terms of associations. He asserted that the connection between the box and the motions the cat used to escape was 'strengthened' by each escape. A similar, though radically reworked idea was taken up by B.F. Skinner in his formulation of Operant Conditioning, and the associative analysis went on to figure largely in behavioral work through mid-century, now evident in some modern work in behavior as well as modern *connectionism*.

[13] Burrhus Frederic Skinner (March 20, 1904 – August 18, 1990) was an American psychologist and author. He conducted pioneering work on experimental psychology and advocated behaviorism, which seeks to understand behavior as a function of environmental histories of experiencing consequences. He also wrote a number of controversial works in which he proposed the widespread use of psychological behavior modification techniques, primarily operant conditioning, in order to improve society and increase human happiness; and as a form of social engineering.

Skinner was born in rural Susquehanna, Pennsylvania. He attended Hamilton College in New York with the intention of becoming a writer and received a B.A. in English literature in 1926. After graduation, he spent a year in Greenwich Village attempting to become a writer of fiction, but he soon became disillusioned with his literary skills and concluded that he had little world experience, and no strong personal perspective from which to write. During this time, which Skinner later called 'the dark year,' he chanced upon a copy of Bertrand Russell's book 'An Outline of Philosophy', in which Russell discusses the behaviorist philosophy of psychologist John B. Watson. At the time, Skinner had begun to

(a) *reinforcement* (a consequence that causes a behavior to occur with greater frequency), (b) *punishment* (a consequence that causes a behavior to occur with less frequency), and (c) *extinction* (the lack of any consequence following a response). There are four contexts of operant conditioning:

(i) *Positive reinforcement* occurs when a behavior (response) is followed by a favorable stimulus (commonly seen as pleasant) that increases the frequency of that behavior. In the Skinner box experiment, a stimulus such as food or sugar solution can be delivered when the rat engages in a target behavior, such as pressing a lever.

(ii) *Negative reinforcement* occurs when a behavior (response) is followed by the removal of an aversive stimulus (commonly seen as unpleasant) thereby increasing that behavior's frequency. In the Skinner box experiment, negative reinforcement can be a loud noise continuously sounding inside the rat's cage until it engages in the target behavior, such as pressing a lever, upon which the loud noise is removed.

(iii) *Positive punishment* (also called 'Punishment by contingent stimulation') occurs when a behavior (response) is followed by an aversive stimulus, such as introducing a shock or loud noise, resulting in a decrease in that behavior.

(iv) *Negative punishment* (also called 'Punishment by contingent withdrawal') occurs when a behavior (response) is followed by the removal of a favorable stimulus, such as taking away a child's toy following an undesired behavior, resulting in a decrease in that behavior.

- *Observational (or social) learning* – learning that occurs as a function of observing, retaining and replicating behavior observed in others. It is most associated with the work of psychologist Albert Bandura,[14] who implemented some of the seminal studies in the area and initiated social learning theory. Although observational learning can take place at any

take more interest in the actions and behaviors of those around him, and some of his short stories had taken a 'psychological' slant. He decided to abandon literature and seek admission as a graduate student in psychology at Harvard University (which at the time was not regarded as a leading institution in that field).

[14] Albert Bandura (born December 4, 1925 in Mundare, Alberta) is a Canadian psychologist most famous for his work on social learning theory (or Social Cognitivism) and self efficacy. He is particularly noted for the Bobo doll experiment.

stage in life, it is thought to be particularly important during childhood, particularly as authority becomes important. Because of this, social learning theory has influenced debates on the effect of television violence and parental role models. Bandura's Bobo doll experiment is widely cited in psychology as a demonstration of observational learning and demonstrated that children are more likely to engage in violent play with a life size rebounding doll after watching an adult do the same. Observational learning allows for learning without any change in behavior and has therefore been used as an argument against strict behaviorism which argued that behavior change must occur for new behaviors to be acquired. Bandura called the process of social learning modelling and gave four conditions required for a person to successfully model the behavior of someone else: (i) attention to the model (a person must first pay attention to a person engaging in a certain behavior – the model); (ii) retention of details (once attending to the observed behavior, the observer must be able to effectively remember what the model has done); (iii) motor reproduction (the observer must be able to replicate the behavior being observed; e.g., juggling cannot be effectively learned by observing a model juggler if the observer does not already have the ability to perform the component actions, i.e., throwing and catching a ball); (iv) motivation and opportunity (the observer must be motivated to carry out the action they have observed and remembered, and must have the opportunity to do so; e.g., a suitably skilled person must want to replicate the behavior of a model juggler, and needs to have an appropriate number of items to juggle to hand). Social learning may affect behavior in the following ways: (i) teaches new behaviors; (ii) increases or decreases the frequency with which previously learned behaviors are carried out; (iii) can encourage previously forbidden behaviors; (iv) can increase or decrease similar behaviors (e.g., observing a model excelling in piano playing may encourage an observer to excel in playing the saxophone).

- *Communication* – the process of symbolic activity, sometimes via a language. Specialized fields focus on various aspects of communication, and include: (i) *mass communication* (academic study of various means by which individuals and entities relay information to large segments of the population all at once through mass media); (ii) *communication studies* (academic discipline that studies communication; subdisciplines include argumentation, speech communication, rhetoric, communication theory, performance studies, group communication, information theory, intercultural communication, interpersonal communication, intrapersonal communication, marketing, organizational communication, persuasion, propaganda, public affairs, public relations and telecommunication); (iii) *organizational communication* (the study of how people communicate within an organizational context, or the influence of, or interaction with organizational structures in communicating/organizing), (iv) *conversation analysis* (commonly abbreviated as CA, is the study of talk in interaction; CA generally attempts to describe the orderliness, structure and sequential patterns of interaction, whether this is institutional, in the school, doctor's surgery, courts or elsewhere, or casual conversation); (v) *linguistics* (scientific study of human language and speech; usually is conducted along two major axes: theoretical vs. applied, and autonomous vs. contextual); (vi) *cognitive linguistics* (commonly abbreviated as CL, refers to the school of linguistics that views the important essence of language as innately based in evolutionary–developed and speciated faculties, and seeks explanations that advance or fit well into the current understandings of the human mind); (vii) *sociolinguistics* (the study of the effect of any and all aspects of society, including cultural norms, expectations, and context, on the way language is used); (viii) *pragmatics* (concerned with bridging the explanatory gap between sentence meaning and speaker's meaning – how context influences the interpretation is crucial); (ix) *semiotics* (the study of signs, both individually and grouped in sign systems; it includes the study of how meaning is made and understood); and (x) *discourse analysis* (a general term for a number of approaches to analyzing written, spoken or signed language use; includes: discourse grammar, rhetoric and stylistics). Communication as a named and unified discipline has a history of contestation that goes back to the Socratic dialogues, in many ways making it the first and most contestatory of all early sciences and philosophies. Seeking to define 'communication' as a static word or unified discipline may not be as important as understanding communication as a family of resemblances with a plurality of definitions as Ludwig Wittgenstein[15] had put forth. Some definitions are broad, recognizing that animals can communicate, and some are more narrow, only including human beings within the parameters of human symbolic interaction. Nonetheless, communication is usually described along three major dimensions: content, form, and destination. In the advent of 'noise' (internal psychological noise and/or physical

[15] Ludwig Josef Johann Wittgenstein (April 26, 1889 – April 29, 1951) was an Austrian philosopher who contributed several ground-breaking works to contemporary philosophy, primarily on the foundations of logic, the philosophy of mathematics, the philosophy of language, and the philosophy of mind. He is widely regarded as one of the most influential philosophers of the 20th century.

realities) these three components of communication often become skewed and inaccurate. (between parties, communication content include acts that declare knowledge and experiences, give advice and commands, and ask questions. These acts may take many forms, including gestures (nonverbal communication, sign language and body language), writing, or verbal speaking. The form depends on the symbol systems used. Together, communication content and form make messages that are sent towards a destination. The target can be oneself, another person (in interpersonal communication), or another entity (such as a corporation or group). There are many theories of communication, and a commonly held assumption is that communication must be directed towards another person or entity. This essentially ignores intrapersonal communication (note intra–, not inter–) via diaries or self–talk. Interpersonal conversation can occur in dyads and groups of various sizes, and the size of the group impacts the nature of the talk. Small–group communication takes place in settings of between three and 12 individuals, and differs from large group interaction in companies or communities. This form of communication formed by a dyad and larger is sometimes referred to as the psychological model of communication where in a message is sent by a sender through channel to a receiver. At the largest level, mass communication describes messages sent to huge numbers of individuals through mass media, although there is debate if this is an interpersonal conversation.

Language

Recall that a language is a system of signals, such as voice sounds, gestures or written symbols that encode or decode information.

Human spoken and written languages can be described as a system of symbols (sometimes known as lexemes) and the grammars (rules) by which the symbols are manipulated. The word 'language' is also used to refer to common properties of languages.

Language learning is normal in human childhood. Most human languages use patterns of sound or gesture for symbols which enable communication with others around them. There are thousands of human languages, and these seem to share certain properties, even though many shared properties have exceptions.

Languages are not just sets of symbols. They also often conform to a rough grammar, or system of rules, used to manipulate the symbols. While a set of symbols may be used for expression or communication, it is primitive and relatively unexpressive, because there are no clear or regular relationships between the symbols.

Human languages are usually referred to as natural languages, and the science of studying them is *linguistics*, with Ferdinand de Saussure[16] and Noam Chomsky[17] as the most influential figures.

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[16] Ferdinand de Saussure (November 26, 1857 – February 22, 1913) was a Geneva–born Swiss linguist whose ideas laid the foundation for many of the significant developments in linguistics in the 20th century. He is widely considered the 'father' of 20th–century linguistics.

Saussure's most influential work, 'Course in General Linguistics', was published posthumously in 1916 by former students Charles Bally and Albert Sechehaye on the basis of notes taken from Saussure's lectures at the University of Geneva. The Cours became one of the seminal linguistics works of the 20th century, not primarily for the content (many of the ideas had been anticipated in the works of other 19th century linguists), but rather for the innovative approach that Saussure applied in discussing linguistic phenomena. Its central notion is that language may be analyzed as a formal system of differential elements, apart from the messy dialectics of realtime production and comprehension.

Saussure's famous quotes are:

"A sign is the basic unit of language (a given language at a given time). Every language is a complete system of signs. Parole (the speech of an individual) is an external manifestation of language."

"A linguistic system is a series of differences of sound combined with a series of differences of ideas."

[17] Noam Avram Chomsky (born December 7, 1928) is the Institute Professor Emeritus of linguistics at the MIT. Chomsky is credited with the creation of the theory of generative grammar, considered to be one of the most significant contributions to the field of theoretical linguistics made in the 20th century. He also helped spark the cognitive revolution in psychology through his review of B.F. Skinner's 'Verbal Behavior', in which he challenged the behaviorist approach to the study of mind and language dominant in the 1950s. His naturalistic approach to the study of language has also affected the philosophy of language and mind. He is also credited with the establishment of the so–called *Chomsky hierarchy*, a classification of formal languages in terms of their generative power.

'Syntactic Structures' was a distillation of Chomsky's book 'Logical Structure of Linguistic Theory' (1955) in which he introduces transformational grammars. The theory takes utterances (sequences of words) to have a syntax which can be (largely) characterised by a formal grammar; in particular, a *context–free grammar* extended with transformational rules. Children are hypothesised to have an innate knowledge of the basic grammatical structure common to all human languages (i.e. they assume that any language which they encounter is of a certain restricted kind). This innate knowledge is often referred to as universal grammar. It is argued that modelling knowledge of language using a formal grammar accounts for the 'productivity' of language: with a limited set of grammar rules and a finite set of terms, humans are able to produce an infinite number of sentences, including sentences no one has previously said.

Chomsky's ideas have had a strong influence on researchers investigating the acquisition of language in children, though some researchers who work in this area today do not support Chomsky's theories, often advocating emergentist or

Humans and computer programs have also constructed other languages, including constructed languages such as Esperanto, Ido, Interlingua, Klingon, programming languages, and various mathematical formalisms. These languages are not necessarily restricted to the properties shared by human languages.

Some of the areas of the human brain involved in language processing are: Broca's area, Wernicke's area, Supramarginal gyrus, Angular gyrus, Primary Auditory Cortex.

Mathematics and computer science use artificial entities called *formal languages* (including programming languages and markup languages, but also some that are far more theoretical in nature). These often take the form of character strings, produced by some combination of formal grammar and semantics of arbitrary complexity.

The classification of natural languages can be performed on the basis of different underlying principles (different closeness notions, respecting different properties and relations between languages); important directions of present classifications are:

1. Paying attention to the historical evolution of languages results in a genetic classification of languages—which is based on genetic relatedness of languages;
2. Paying attention to the internal structure of languages (grammar) results in a typological classification of languages—which is based on similarity of one or more components of the language's grammar across languages; and
3. Respecting geographical closeness and contacts between language-speaking communities results in areal groupings of languages.
4. The different classifications do not match each other and are not expected to, but the correlation between them is an important point for many linguistic research works. (Note that there is a parallel to the classification of species in biological phylogenetics here: consider monophyletic vs. polyphyletic groups of species.)

The task of genetic classification belongs to the field of historical–comparative linguistics, of typological—to linguistic typology. The world's

connectionist theories reducing language to an instance of general processing mechanisms in the brain.

Chomsky's work in linguistics has had major implications for modern psychology. For Chomsky linguistics is a branch of cognitive psychology; genuine insights in linguistics imply concomitant understandings of aspects of mental processing and human nature. His theory of a universal grammar was seen by many as a direct challenge to the established behaviorist theories of the time and had major consequences for understanding how language is learned by children and what, exactly, is the ability to use language. Many of the more basic principles of this theory (though not necessarily the stronger claims made by the principles and parameters approach described above) are now generally accepted in some circles.

languages have been grouped into families of languages that are believed to have common ancestors. Some of the major families are the Indo–European languages, the Afro–Asiatic languages, the Austronesian languages, and the Sino–Tibetan languages. The shared features of languages from one family can be due to shared ancestry.

An example of a typological classification is the classification of languages on the basis of the basic order of the verb, the subject and the object in a sentence into several types: SVO, SOV, VSO, and so on, languages. (English, for instance, belongs to the SVO language type.)

The shared features of languages of one type (= from one typological class) may have arisen completely independently. (Compare with analogy in biology.) Their cooccurence might be due to the universal laws governing the structure of natural languages—language universals.

The following language groupings can serve as some linguistically significant examples of areal linguistic units, or sprachbunds: Balkan linguistic union, or the bigger group of European languages; Caucasian languages. Although the members of each group are not closely genetically related, there is a reason for them to share similar features, namely: their speakers have been in contact for a long time within a common community and the languages converged in the course of the history. These are called ‘areal features’.

Mathematics and computer science use artificial entities called formal languages (including programming languages and markup languages, but also some that are far more theoretical in nature). These often take the form of character strings, produced by some combination of formal grammar and semantics of arbitrary complexity.

Abstraction

Recall that *abstraction* is the process of reducing the information content of a concept, typically in order to retain only information which is relevant for a particular purpose. For example, abstracting a leather soccer ball to a ball retains only the information on general ball attributes and behavior. Similarly, abstracting an emotional state to happiness reduces the amount of information conveyed about the emotional state.

Abstraction typically results in complexity reduction leading to a simpler conceptualization of a domain in order to facilitate processing or understanding of many specific scenarios in a generic way.

In philosophical terminology, abstraction is the thought process wherein ideas are distanced from objects.

Abstraction uses a strategy of simplification, wherein formerly concrete details are left ambiguous, vague, or undefined; thus effective communication about things in the abstract requires an intuitive or common experience between the communicator and the communication recipient.

Abstractions sometimes have ambiguous referents; for example, ‘happiness’ (when used as an abstraction) can refer to as many things as there are people and events or states of being which make them happy. Likewise, ‘architecture’ refers not only to the design of safe, functional buildings, but also to elements of creation and innovation which aim at elegant solutions to construction problems, to the use of space, and at its best, to the attempt to evoke an emotional response in the builders, owners, viewers and users of the building.

Abstraction in philosophy is the process (or, to some, the alleged process) in concept–formation of recognizing some set of common features in individuals, and on that basis forming a concept of that feature. The notion of abstraction is important to understanding some philosophical controversies surrounding empiricism and the problem of universals. It has also recently become popular in formal logic under predicate abstraction.

Some research into the human brain suggests that the left and right hemispheres differ in their handling of abstraction. One side handles collections of examples (e.g., examples of a tree) whereas the other handles the concept itself.

Abstraction in mathematics is the process of extracting the underlying essence of a mathematical concept, removing any dependence on real world objects with which it might originally have been connected, and generalizing it so that it has wider applications.

Many areas of mathematics began with the study of real world problems, before the underlying rules and concepts were identified and defined as abstract structures. For example, geometry has its origins in the calculation of distances and areas in the real world; statistics has its origins in the calculation of probabilities in gambling; and algebra started with methods of solving problems in arithmetic.

Abstraction is an ongoing process in mathematics and the historical development of many mathematical topics exhibits a progression from the concrete to the abstract. Take the historical development of geometry as an example; the first steps in the abstraction of geometry were made by the ancient Greeks, with Euclid being the first person (as far as we know) to document the axioms of plane geometry. In the 17th century Descartes introduced Cartesian coordinates which allowed the development of analytic geometry. Further steps in abstraction were taken by Lobachevsky, Bolyai and Gauss[18] who generalized the concepts of geometry to develop non–Euclidean

[18] *Gauss–Bolyai–Lobachevsky space* is a non–Euclidean space with a negative Gaussian curvature, that is, a *hyperbolic geometry*. The main topic of conversation involving Gauss–Bolyai–Lobachevsky space involves the impossible process (at least in Euclidean geometry) of squaring the circle. The space is named after Carl Gauss, János Bolyai, and Nikolai Lobachevsky.

Carl Friedrich Gauss (30 April 1777 – 23 February 1855) was a German mathematician and scientist of profound genius who contributed significantly to many fields, including number theory, analysis, differential geometry, geodesy, magnetism, astronomy and optics. Sometimes known as 'the prince of mathematicians' and 'greatest mathematician since antiquity', Gauss had a remarkable influence in many fields of mathematics and science and is ranked among one of history's most influential mathematicians. Gauss was a child prodigy, of whom geometries. Later in the 19th century mathematicians generalised geometry even further, developing such areas as geometry in n dimensions, projective geometry, affine geometry, finite geometry and differential geometry. Finally Felix Klein's 'Erlangen program'[19] identified the underlying theme of all of these geometries, defining each of them as the study of properties invariant under a given group of symmetries. This level of abstraction revealed deep connections between geometry and abstract algebra.

The advantages of abstraction are:

(i) It reveals deep connections between different areas of mathematics;
(ii) Known results in one area can suggest conjectures in a related area; and
(iii) Techniques and methods from one area can be applied to prove results in a related area.

An abstract structure is a formal object that is defined by a set of laws, properties, and relationships in a way that is logically if not always historically independent of the structure of contingent experiences, for example, those involving physical objects. Abstract structures are studied not only in logic and mathematics but in the fields that apply them, as computer science, and in the studies that reflect on them, as philosophy and especially the philosophy of mathematics. Indeed, modern mathematics has been defined

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there are many anecdotes pertaining to his astounding precocity while a mere toddler, and made his first ground–breaking mathematical discoveries while still a teenager. He completed Disquisitiones Arithmeticae, his magnum opus, at the age of twenty–one (1798), though it would not be published until 1801. This work was fundamental in consolidating number theory as a discipline and has shaped the field to the present day. One of his most importatn results is his 'Theorema Egregrium', establishing an important property of the notion of curvature as a foundation of differential geometry.

János Bolyai (December 15, 1802–January 27, 1860) was a Hungarian mathematician. Between 1820 and 1823 he prepared a treatise on a complete system of non–Euclidean geometry. Bolyai's work was published in 1832 as an appendix to a mathematics textbook by his father. Gauss, on reading the Appendix, wrote to a friend saying "I regard this young geometer Bolyai as a genius of the first order." In 1848 Bolyai discovered not only that Lobachevsky had published a similar piece of work in 1829, but also a generalisation of this theory.

Nikolai Ivanovich Lobachevsky (December 1, 1792–February 24, 1856 (N.S.) was a Russian mathematician. Lobachevsky's main achievement is the development (independently from János Bolyai) of non–Euclidean geometry. Before him, mathematicians were trying to deduce Euclid's fifth postulate from other axioms. Lobachevsky would instead develop a geometry in which the fifth postulate was not true.

[19] Felix Christian Klein (April 25, 1849, Düsseldorf, Germany – June 22, 1925, Göttingen) was a German mathematician, known for his work in group theory, function theory, non-Euclidean geometry, and on the connections between geometry and group theory. His 1872 Erlangen Program, classifying geometries by their underlying symmetry groups, was a hugely influential synthesis of much of the mathematics of the day.

in a very general sense as the study of abstract structures by the *Bourbaki* group.[20]

The main disadvantage of abstraction is that highly abstract concepts are more difficult to learn, and require a degree of mathematical maturity and experience before they can be assimilated.

In computer science, abstraction is a mechanism and practice to reduce and factor out details so that one can focus on a few concepts at a time.

The concept is by analogy with abstraction in mathematics. The mathematical technique of abstraction begins with mathematical definitions; this has the fortunate effect of finessing some of the vexing philosophical issues of abstraction. For example, in both computing and in mathematics, numbers are concepts in the programming languages, as founded in mathematics. Implementation details depend on the hardware and software, but this is not a restriction because the computing concept of number is still based on the mathematical concept.

Roughly speaking, abstraction can be either that of control or data. Control abstraction is the abstraction of actions while data abstraction is that of data structures. For example, control abstraction in structured programming is the use of subprograms and formatted control flows. Data abstraction is to allow for handling data bits in meaningful manners. For example, it is

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[20] Nicolas Bourbaki is the collective allonym under which a group of (mainly French) 20th-century mathematicians wrote a series of books presenting an exposition of modern advanced mathematics, beginning in 1935. With the goal of founding all of mathematics on set theory, the group strove for utmost rigour and generality, creating some new terminology and concepts along the way.

While Nicolas Bourbaki is an invented personage, the Bourbaki group is officially known as the Association des collaborateurs de Nicolas Bourbaki ('association of collaborators of Nicolas Bourbaki'), which has an office at the École Normale Supérieure in Paris.

The emphasis on rigour may be seen as a reaction to the work of Jules–Henri Poincaré, who stressed the importance of free–flowing mathematical intuition, at a cost in completeness (i.e., proof) in presentation. The impact of Bourbaki's work initially was great on many active research mathematicians world–wide.

Notations introduced by Bourbaki include: the symbol $\varnothing$ for the *empty set*, and the terms *injective*, *surjective*, and *bijective*.

Aiming at a completely self-contained treatment of most of modern mathematics based on set theory, the group produced the following volumes (with the original French titles in parentheses):

I Set theory (Théorie des ensembles);
II Algebra (Algèbre);
III General Topology (Topologie générale);
IV Functions of one real variable (Fonctions d'une variable réelle);
V Topological vector spaces (Espaces vectoriels topologiques);
VI Integration (Intégration);
VII Commutative algebra (Algèbre commutative); and
VIII Lie groups and algebras (Groupes et algèbres de Lie).

the basic motivation behind data–type. Object–oriented programming can be seen as an attempt to abstract both data and code.

Creativity

Now, recall that *creativity* is a mental process involving the generation of new ideas or concepts, or new associations between existing ideas or concepts. From a scientific point of view, the products of creative thought (sometimes referred to as divergent thought) are usually considered to have both originality and appropriateness. An alternative, more everyday conception of creativity is that it is simply the act of making something new. Although intuitively a simple phenomenon, it is in fact quite complex. It has been studied from the perspectives of behavioral psychology, social psychology, psychometrics, cognitive science, artificial intelligence, philosophy, history, economics, design research, business, and management, among others. The studies have covered everyday creativity, exceptional creativity and even artificial creativity. Unlike many phenomena in science, there is no single, authoritative perspective or definition of creativity. Unlike many phenomena in psychology, there is no standardized measurement technique.

Creativity has been attributed variously to divine intervention, cognitive processes, the social environment, personality traits, and chance (‘accident’, ‘serendipity’). It has been associated with genius, mental illness and humor. Some say it is a trait we are born with; others say it can be taught with the application of simple techniques. Although popularly associated with art and literature, it is also an essential part of innovation and invention and is important in professions such as business, economics, architecture, industrial design, science and engineering.

Despite, or perhaps because of, the ambiguity and multi–dimensional nature of creativity, entire industries have been spawned from the pursuit of creative ideas and the development of creativity techniques. This mysterious phenomenon, though undeniably important and constantly visible, seems to lie tantalizingly beyond the grasp of scientific investigation.

More than 60 different definitions of creativity can be found in the psychological literature (see [Tay88]). The etymological root of the word in English and most other European languages comes from the Latin ‘creatus’, which literally means ‘to have grown’. Perhaps the most widespread conception of creativity in the scholarly literature is that creativity is manifested in the production of a creative work (for example, a new work of art or a scientific hypothesis) that is both novel and useful. Colloquial definitions of creativity are typically descriptive of activity that results in producing or bringing about something partly or wholly new; in investing an existing object with new properties or characteristics; in imagining new possibilities that were not conceived of before; and in seeing or performing something in a manner different from what was thought possible or normal previously.

A useful distinction has been made by [Rho61], between the creative person, the creative product, the creative process, and the creative ’press’ or environment. Each of these factors are usually present in creative activity. This has been elaborated by [Joh72], who suggested that creative activity may exhibit several dimensions including sensitivity to problems on the part of the creative agent, originality, ingenuity, unusualness, usefulness, and appropriateness in relation to the creative product, and intellectual leadership on the part of the *creative agent*.

Boden [Bod04] noted that it is important to distinguish between ideas which are psychologically creative (which are novel to the individual mind which had the idea), and those which are historically creative (which are novel with respect to the whole of human history). Drawing on ideas from artificial intelligence, she defines psychologically creative ideas as those which cannot be produced by the same set of generative rules as other, familiar ideas.

Often implied in the notion of creativity is a concomitant presence of inspiration, cognitive leaps, or intuitive insight as a part of creative thought and action [Koe64]. Popular psychology sometimes associates creativity with right or forehead brain activity or even specifically with lateral thinking. Some students of creativity have emphasized an element of chance in the creative process. Linus Pauling,[21] asked at a public lecture how one creates scientific theories, replied that one must endeavor to come up with many ideas — then discard the useless ones.

The formal starting point of the scientific study of creativity is sometimes considered to be J. Joy Guilford's[22] address to the American Psychological

[21] Linus Carl Pauling (February 28, 1901 – August 19, 1994) was an American quantum chemist and biochemist, widely regarded as the premier chemist of the twentieth century. Pauling was a pioneer in the application of quantum mechanics to chemistry (quantum mechanics can, in principle, describe all of chemistry and molecular biology), and in 1954 was awarded the Nobel Prize in chemistry for his work describing the nature of chemical bonds. He also made important contributions to crystal and protein structure determination, and was one of the founders of molecular biology. Pauling is noted as a versatile scholar for his expertise in inorganic chemistry, organic chemistry, metallurgy, immunology, anesthesiology, psychology, debate, radioactive decay, and the aftermath of nuclear weapons, in addition to quantum mechanics and molecular biology.

Pauling received the Nobel Peace Prize in 1962 for his campaign against above-ground nuclear testing, becoming the only person in history to individually receive two Nobel Prizes (Marie Curie won Nobel Prizes in physics and chemistry, but shared the former and won the latter individually; John Bardeen won two Nobel Prizes in the field of physics, but both were shared; Frederick Sanger won two Nobel Prizes in chemistry, but one was shared).

Later in life, he became an advocate for regular consumption of massive doses of vitamin C, which is still regarded as unorthodox by conventional medicine.

[22] Joy Paul Guilford (1897–1988) was a US psychologist, best remembered for his psychometric study of human intelligence.

He graduated from the University of Nebraska before studying under Edward Titchener at Cornell. He then held a number of posts at Nebraska and briefly at the University of Southern California before becoming Director of Psychological

Association in 1950, which helped to popularize the topic (see [SL99]). Since then, researchers from a variety of fields have studied the nature of creativity from a scientific point of view. Others have taken a more pragmatic approach, teaching practical creativity techniques. Three of the best–known are Alex Osborn's[23] *brainstorming* techniques, Genrikh Altshuller's[24] 'Theory of Inventive Problem Solving' (TIPS), and Edward de Bono's[25] *lateral thinking* (1960s to present).

The *neurology of creativity* has been discussed by F. Balzac in [Bal06]. The study found that creative innovation requires *coactivation and communication between regions of the brain that ordinarily are not strongly connected.* Highly creative people who excel at creative innovation tend to differ from others in three ways: they have a high level of specialized knowledge, they are capable of divergent thinking mediated by the frontal lobe, and they are able to modulate neurotransmitters such as norepinephrine in their frontal lobe. Thus, the frontal lobe appears to be the part of the cortex that is most important for creativity. The study also explored the links between creativity and sleep, mood and addiction disorders, and depression.

Research at Santa Ana Army Air Base in 1941. There he worked on the selection and ranking of air–crew trainees.

Developing the views of L. L. Thurstone, Guilford rejected Charles Spearman's view that intelligence could be characterized in a single numerical parameter and proposed that three dimensions were necessary for accurate description: (i) content, (ii) operations, and (iii) productions. He made the important distinction between convergent and divergent production.

[23] Alex Faickney Osborn (May 24, 1888 – May 4, 1966) was an advertising manager and the author of the creativity technique named *brainstorming*.

[24] Genrikh Saulovich Altshuller (October 15, 1926 - September 24, 1998), created the Theory of Inventive Problem Solving (TIPS). Working as a clerk in a patent office, Altshuller embarked on finding some generic rules that would explain creation of new, inventive, patentable ideas.

[25] Edward de Bono (born May 19, 1933) is a psychologist and physician. De Bono writes prolifically on subjects of lateral thinking, a concept he is believed to have pioneered and now holds training seminars in. Dr. de Bono is also a world-famous consultant who has worked with companies like Coca-cola and Ericsson. In 1979 he co–founded the School of Thinking with Dr Michael Hewitt–Gleeson.

De Bono has detailed a range of 'deliberate thinking methods' – applications emphasizing thinking as a deliberate act rather than a reactive one. His writing style is simple and clear, though often criticized for being dry and repetitive. Avoiding academic terminology, he has advanced applied psychology by making theories about creativity and perception into usable tools. A distinctive feature of De Bono's books is that he never acknowledges or credits the ideas of other authors or researchers in the field of creativity.

De Bono's work has become particularly popular in the sphere of business – perhaps because of the perceived need to restructure corporations, to allow more flexible working practices and to innovate in products and services. The methods have migrated into corporate training courses designed to help employees and executives 'think out of the box' / 'think outside the box'.

J. Guilford's group developed the so–called 'Torrance Tests of Creative Thinking'. They involved simple tests of divergent thinking and other problem–solving skills, which were scored on [Gui67]:

1. Fluency: the total number of interpretable, meaningful, and relevant ideas generated in response to the stimulus;
2. Flexibility: the number of different categories of relevant responses;
3. Originality: the statistical rarity of the responses among the test subjects; and
4. Elaboration: the amount of detail in the responses.

Personality

On the other hand, *personality* is a *collection of emotional, thought and behavioral patterns unique to a person* that is consistent over time. Personality psychology is a branch of psychology which studies personality and individual different processes – that which makes us into a person. One emphasis is on trying to create a coherent picture of a person and all his or her major psychological processes. Another emphasis views it as the study of individual differences. These two views work together in practice. Personality psychologists are interested in broad view of the individual. This often leads to an interest in the most salient individual differences among people.

The word *personality* originates from the Latin *persona*, which means 'mask'.[26] In the History of theater of the ancient Latin world, the mask was not used as a plot device to disguise the identity of a character, but rather was a convention employed to represent, or typify that character.

There are several theoretical perspectives on personality in psychology, which involve different ideas about the relationship between personality and other psychological constructs, as well as different theories about the way personality develops. Most theories can be grouped into one of the following classes.

Generally the opponents to personality theories claim that personality is 'plastic' in time, places, moods and situations. Changing personality may in fact resulting from diet (or lack of), medical effects, historical or subsequent events, or learning. Stage managers (of many types) are especially skilled in changing a person's resulting 'personality'. Most personality theories will not cover such flexible nor unusual people situations. Therefore, although personality theories do not define personality as 'plastic' over time like their opponents, they do imply a drastic change in personality is highly unusual.

[26] A *persona*, in the word's everyday usage, is a social role, or a character played by an actor. The word derives from the Latin for 'mask' or 'character', derived from the Etruscan word 'phersu', with the same meaning.

For instance, in Dostoevsky's novel, Notes from Underground (generally considered to be the first existentialist novel), the narrator ought not to be conflated with Dostoevsky himself, despite the fact that Dostoevsky and his narrator may or may not have shared much in common. In this sense, the persona is basically a mouthpiece for a particular world–view.

According to the Diagnostic and Statistical Manual of Mental Disorders of the American Psychiatric Association, personality traits are 'prominent aspects of personality that are exhibited in a wide range of important social and personal contexts.' In other words, persons have certain characteristics which partly determine their behavior. According to the theory, a friendly person is likely to act friendly in any situation because of the traits in his personality. One criticism of trait models of personality as a whole is that they lead professionals in clinical psychology and lay–people alike to accept classifications, or worse offer advice, based on superficial analysis of one's profile.

The most common models of traits incorporate four or five broad dimensions or factors. The least controversial dimension, observed as far back as the ancient Greeks, is simply extraversion vs. introversion (outgoing and physical–stimulation–oriented vs. quiet and physical–stimulation–averse).

Gordon Allport[27] delineated different kinds of traits, which he also called dispositions. Central traits are basic to an individual's personality, while secondary traits are more peripheral. Common traits are those recognized within a culture and thus may vary from culture to culture. Cardinal traits are those by which an individual may be strongly recognized.

Raymond Cattell's[28] research propagated a two–tiered personality structure with sixteen 'primary factors' (16 Personality Factors) and five 'secondary factors' (see Table 9.1). Cattell referred to these 16 factors as *primary factors*, as opposed to the so–called 'Big Five' factors which he considered

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[27] Gordon Willard Allport (November 11, 1897 - October 9, 1967) was an American psychologist. He was born in Montezuma, Indiana, the youngest of four brothers. One of his older brothers, Floyd Henry Allport, was an important and influential psychologist as well. Gordon W. Allport was a long time and influential member of the faculty at Harvard University from 1930-1967. His works include Becoming, Pattern and Growth in Personality, The Individual and his Religion, and perhaps his most influential book The Nature of Prejudice.

Allport was one of the first psychologists to focus on the study of the personality, and is often referred to as one of the fathers of personality psychology. Characteristically for this ecletic and pluralistic thinker, he was also an important contributor to social psychology as well. He rejected both a psychoanalytic approach to personality, which he thought often went too deep, and a behavioral approach, which he thought often did not go deep enough. He emphasized the uniqueness of each individual, and the importance of the present context, as opposed to past history, for understanding the personality.

[28] Raymond Bernard Cattell (20 March 1905 – 2 February 1998) was a British and American psychologist who theorized the existence of fluid and crystallized intelligences to explain human cognitive ability. He was famously productive throughout his 92 years, and ultimately was able to claim a combined authorship and co–authorship of 55 books and some 500 journal articles in addition to at least 30 standardized tests. His legacy includes not just that intellectual production, but also a spirit of scientific rigor brought to an otherwise soft science and kept burning by his students and co–researchers whom he was survived by.

global factors. All of the primary factors correlate with global factors and could therefore be considered subfactors within them.

A different model was proposed by Hans Eysenck,[29] who believed that just three traits: *extroversion*, *neuroticism* and *psychoticism* – were sufficient to describe human personality. Eysenck was one of the first psychologists to study personality with the method of *factor analysis*, a statistical technique introduced by Charles Spearman[30] and expanded by Raymond Cattell.

In keeping with his devotion to rigorous scientific method, Cattell was an early proponent of the application in psychology of factor analytical methods, in place of what he called mere 'verbal theorizing.' One of the most important results of Cattell's application of factor analysis was the derivation of 16 factors underlying human personality. He called these 16 factors source traits because he believed that they provide the underlying source for the surface behaviors that we think of as personality. ('Psychology and Life, 7 ed.' by Richard Gerrig and Philip Zimbardo.) This theory of 16 personality factors and the instruments used to measure them are known collectively as the 16 Personality Factors.

[29] Hans Jürgen Eysenck (March 4, 1916 – September 4, 1997) was an eminent psychologist, most remembered for his work on intelligence and personality, though he worked in a wide range of areas. At the time of his death, Eysenck was the living psychologist most frequently cited in science journals.

Hans Eysenck was born in Germany, but moved to England as a young man in the 1930s because of his opposition to the Nazi party. Eysenck was the founding editor of the journal Personality and Individual Differences, and authored over 50 books and over 900 academic articles. He aroused intense debate with his controversial dealing with variation in IQ among racial groups.

[30] Charles Edward Spearman (September 10, 1863 - September 7, 1945) was an English psychologist known for work in statistics, as a pioneer of factor analysis, and for Spearman's rank correlation coefficient. He also did seminal work on models for human intelligence, including his theory that disparate cognitive test scores reflect a single general factor and coining the term g factor. Spearman had an unusual background for a psychologist. After 15 years as an officer in the British Army he resigned to study for a PhD in experimental psychology. In Britain psychology was generally seen as a branch of philosophy and Spearman chose to study in Leipzig under Wilhelm Wundt. Besides Spearman had no conventional qualifications and Leipzig had liberal entrance requirements. He started in 1897 and after some interruption (he was recalled to the army during the South African War) he obtained his degree in 1906. He had already published his seminal paper on the factor analysis of intelligence (1904). Spearman met and impressed the psychologist William McDougall who arranged for Spearman to replace him when he left his position at University College London. Spearman stayed at University College until he retired in 1931. Initially he was Reader and head of the small psychological laboratory. In 1911 he was promoted to the Grote professorship of the Philosophy of Mind and Logic. His title changed to Professor of Psychology in 1928 when a separate Department of Psychology was created. When Spearman was elected to the Royal Society in 1924 the citation read "Dr. Spearman has made many researches in experimental psychology. His many published papers cover a wide field, but he is especially distinguished by his pioneer work in the application of mathematical methods to the analysis of

Table 9.1 Cattell's 16 Personality Factors

Descriptors of Low Range	Primary Factor	Descriptors of High Range
Impersonal, distant, cool, reserved, detached, formal, aloof (Sizothymia)	Warmth	Warm, outgoing, attentive to others, kindly, easy going, participating, likes people (Affectothymia)
Concrete thinking, lower general mental capacity, less intelligent, unable to handle abstract problems (Lower Scholastic Mental Capacity)	Reasoning	Abstract–thinking, more intelligent, bright, higher general mental capacity, fast learner (Higher Scholastic Mental Capacity)
Reactive emotionally, changeable, affected by feelings, emotionally less stable, easily upset (Lower Ego Strength)	Emotional Stability	Emotionally stable, adaptive, mature, faces reality calm (Higher Ego Strength)
Deferential, cooperative, avoids conflict, submissive, humble, obedient, easily led, docile, accommodating (Submissiveness)	Dominance	Dominant, forceful, assertive, aggressive, competitive, stubborn, bossy (Dominance)
Serious, restrained, prudent, taciturn, introspective, silent (Desurgency)	Liveliness	Lively, animated, spontaneous, enthusiastic, happy go lucky, cheerful, expressive, impulsive (Surgency)
Expedient, nonconforming, disregards rules, self indulgent (Low Super Ego Strength)	Rule–Consciousness	Rule–conscious, dutiful, conscientious, conforming, moralistic, staid, rule bound (High Super Ego Strength)
Shy, threat–sensitive, timid, hesitant, intimidated (Threctia)	Social Boldness	Socially bold, venturesome, thick skinned, uninhibited (Parmia)
Utilitarian, objective, unsentimental, tough minded, self–reliant, no–nonsense, rough (Harria)	Sensitivity	Sensitive, aesthetic, sentimental, tender minded, intuitive, refined (Premsia)
Trusting, unsuspecting, accepting, unconditional, easy (Alaxia)	Vigilance	Vigilant, suspicious, skeptical, distrustful, oppositional (Protension)
Grounded, practical, prosaic, solution oriented, steady, conventional (Praxernia)	Abstractedness	Abstract, imaginative, absent minded, impractical, absorbed in ideas (Autia)
Forthright, genuine, artless, open, guileless, naive, unpretentious, involved (Artlessness)	Privateness	Private, discreet, nondisclosing, shrewd, polished, worldly, astute, diplomatic (Shrewdness)
Self–Assured, unworried, complacent, secure, free of guilt, confident, self satisfied (Untroubled)	Apprehension	Apprehensive, self doubting, worried, guilt prone, insecure, worrying, self blaming (Guilt Proneness)
Traditional, attached to familiar, conservative, respecting traditional ideas (Conservatism)	Openness to Change	Open to change, experimental, liberal, analytical, critical, free thinking, flexibility (Radicalism)
Group–oriented, affiliative, a joiner and follower dependent (Group Adherence)	Self–Reliance	Self–reliant, solitary, resourceful, individualistic, self sufficient (Self-Sufficiency)
Tolerated disorder, unexacting, flexible, undisciplined, lax, self-conflict, impulsive, careless of social rules, uncontrolled (Low Integration)	Perfectionism	Perfectionistic, organized, compulsive, self-disciplined, socially precise, exacting will power, control, self-sentimental (High Self–Concept Control)
Relaxed, placid, tranquil, torpid, patient, composed low drive (Low Ergic Tension)	Tension	Tense, high energy, impatient, driven, frustrated, over wrought, time driven. (High Ergic Tension)

Eysenck's results suggested two main personality factors [Eys92a, Eys92b]. The first factor was the tendency to experience negative emotions, and Eysenck referred to it as 'neuroticism'. The second factor was the tendency enjoy positive events, especially social events, and Eysenck named it 'extraversion'. The two personality dimensions were described in his 1947 book 'Dimensions of Personality'. It is common practice in personality psychology to refer to the dimensions by the first letters, E and N. E and N provided a 2–dimensional space to describe individual differences in behavior. An analogy can be made to how latitude and longitude describe a point on the face of the earth. Also, Eysenck noted how these two dimensions were similar to the four personality types first proposed by the ancient Greek physician Galen[31]:

1. High N and High E = Choleric type;
2. High N and Low E = Melancholic type;

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the human mind, and his original studies of correlation in this sphere. He has inspired and directed research work by many pupils."

Spearman was strongly influenced by the work of Francis Galton. Galton did pioneering work in psychology and developed correlation, the main statistical tool used by Spearman. Spearman developed rank correlation (1904) and the widely used correction for attenuation (1907). His statistical work was not appreciated by his University College colleague Karl Pearson and there was long feud between them. Although Spearman achieved most recognition for his statistical work, he regarded this work as subordinate to his quest for the fundamental laws of psychology (see [WZZ03] for details).

[31] Galen, (Latin: Claudius Galenus of Pergamum) was an ancient Greek physician. The forename 'Claudius' is absent in Greek texts; it was first documented in texts from the Renaissance. Galen's views dominated European medicine for over a thousand years.

Galen transmitted Hippocratic medicine all the way to the Renaissance. His On the Elements According to Hippocrates describes the philosopher's system of four bodily humours, blood, yellow bile, black bile and phlegm, which were identified with the four classical elements, and in turn with the seasons. He created his own theories from those principles, and much of Galen's work can be seen as building on the Hippocratic theories of the body, rather than being purely innovative. In turn, he mainly ignored Latin writings of Celsus, but accepted that the ancient works of Asclepiades had sound theory.

Galen's own theories, in accord with Plato's, emphasized purposeful creation by a single Creator ('Nature' – Greek 'phusis') – a major reason why later Christian and Muslim scholars could accept his views. His fundamental principle of life was pneuma (air, breath) that later writers connected with the soul. These writings on philosophy were a product of Galen's well rounded education, and throughout his life Galen was keen to emphasize the philosophical element to medicine. Pneuma physicon (animal spirit) in the brain took care of movement, perception, and senses. Pneuma zoticon (vital spirit) in the heart controlled blood and body temperature. 'Natural spirit' in the liver handled nutrition and metabolism. However, he did not agree with the Pneumatist theory that air passed through the veins rather than blood.

3. Low N and High E = Sanguine type; and
4. Low N and Low E = Phlegmatic type.

The third dimension, 'psychoticism', was added to the model in the late 1970s, based upon collaborations between Eysenck and his wife, Sybil B.G. Eysenck, the current editor of Personality and Individual Differences (see [Eys69, Eys76]).

The major strength of Eysenck's model was to provide detailed theory of the causes of personality (see his 1985 book 'Decline and Fall of the Freudian Empire'). For example, Eysenck proposed that extraversion was caused by variability in cortical arousal; "introverts are characterized by higher levels of activity than extraverts and so are chronically more cortically aroused than extraverts'. While it seems counterintuitive to suppose that introverts are more aroused than extraverts, the putative effect this has on behavior is such that the introvert seeks lower levels of stimulation. Conversely, the extravert seeks to heighten their arousal to a more optimal level (as predicted by the *Yerkes–Dodson Law*) by increased activity, social engagement and other stimulation–seeking behaviours.

Differences between Cattell and Eysenck emerged due to preferences for different forms of factor analysis, with Cattell using oblique, Eysenck orthogonal, rotation to analyze the factors that emerged when personality questionnaires were subject to statistical analysis. Today, the Big Five factors have the weight of a considerable amount of empirical research behind them. Building on the work of Cattell and others, Lewis Goldberg[32] proposed a five–dimensional personality model, nicknamed the 'Big Five' personality traits:

Extroversion (i.e., 'extroversion vs. introversion' above; outgoing and physical–stimulation–oriented vs. quiet and physical–stimulation–averse);

1. Neuroticism (i.e., emotional stability; calm, unperturbable, optimistic vs. emotionally reactive, prone to negative emotions);
2. Agreeableness (i.e., affable, friendly, conciliatory vs. aggression aggressive, dominant, disagreeable);
3. Conscientiousness (i.e., dutiful, planful, and orderly vs. spontaneous, flexible, and unreliable); and
4. Openness to experience (i.e., open to new ideas and change vs. traditional and staid).

Character

A *character structure* is a system of relatively permanent motivational and other traits that are manifested in the characteristic ways that an individual relates to others and reacts to various kinds of challenges. The word 'structure' indicates that these several characteristics and/or learned patterns of

[32] Lewis R. Goldberg is an American personality psychologist and a professor emeritus at the University of Oregon. Among his other accomplishments, Goldberg is closely associated with the Big Five taxonomy of personality. He has published well over 100 research articles and has been active on editorial boards.

behavior are linked in such a way as to produce a state that can be highly resistant to change. The idea has its roots in the work of Sigmund Freud[33] and several of his followers, the most important of whom (in this respect) is Erich Fromm.[34] Among other important participants in the establishment of this concept must surely be counted Erik Erikson.[35]

Among the earliest factors that determine an individual's eventual character structure are his or her genetic characteristics and early childhood nurture and education. A child who is well nurtured and taught in a relatively benign and consistent environment by loving adults who intend that the child should learn how to make objective appraisals regarding the environment will be likely to form a normal or productive character structure. On the other hand, a child whose nurture and/or education are not ideal, living in a treacherous environment and interacting with adults who do not take the long–term interests of the child to heart will be more likely to form a pattern of behavior that suits the child to avoid the challenges put forth by a malign social environment. The means that the child invents to make the best of a hostile environment. Although this may serve the child well while in that bad environment, it may also cause the child to react in inappropriate ways, ways damaging to his or her own interests, when interacting with people in a more ideal social context. Major trauma that occurs later in life, even in adulthood, can sometimes have a profound effect. However, character may also develop in a positive way according to how the individual meets the psychosocial challenges of the life cycle (Erikson).

[33] Sigmund Freud (May 6, 1856–September 23, 1939) was an Austrian neurologist and the founder of the psychoanalytic school of psychology. Freud is best known for his studies of sexual desire, repression, and the unconscious mind. He is commonly referred to as 'the father of psychoanalysis' and his work has been tremendously influential in the popular imagination–popularizing such notions as the unconscious, defence mechanisms, Freudian slips and dream symbolism – while also making a long-lasting impact on fields as diverse as literature, film, marxist and feminist theories, literary criticism, philosophy, and of course, psychology.

[34] Erich Pinchas Fromm (March 23, 1900 – March 18, 1980) was an internationally renowned German-American psychologist and humanistic philosopher. He is associated with what became known as the Frankfurt School of critical thinkers.

Central to Fromm's world view was his interpretation of the Talmud, which he began studying as a young man under Rabbi J. Horowitz and later studied under Rabbi Salman Baruch Rabinkow while working towards his doctorate in sociology at the University of Heidelberg and under Nehemia Nobel and Ludwig Krause while studying in Frankfurt. Fromm's grandfather and two great grandfathers on his father's side were rabbis, and a great uncle on his mother's side was a noted Talmudic scholar. However, Fromm turned away from orthodox Judaism in 1926 and turned towards secular interpretations of scriptural ideals.

[35] Erik Homburger Erikson (June 15, 1902 – May 12, 1994) was a developmental psychologist and psychoanalyst known for his theory on social development of human beings, and for coining the phrase identity crisis.

Freud's first paper on character described the anal character consisting of stubbornness, stinginess and extreme neatness. He saw this as a reaction formation to the child's having to give up pleasure in anal eroticism.The positive version of this character is the conscientious, inner directed obsessive. Freud also described the erotic character as both loving and dependent. And the narcissistic character as the natural leader, aggressive and independent because of not internalizing a strong super ego.

For Erich Fromm, character develops as the way in which an individual structures modes of assimilation and relatedness. The character types are almost identical to Freud's but Fromm gives them different names, receptive, hoarding, exploitative. Fromm adds the marketing type as the person who continually adapts the self to succeed in the new service economy. For Fromm, character types can be productive or unproductive. Fromm notes that character structures develop in each individual to enable him or her to interact successfully within a given society, to adapt to its mode of production and social norms may be very counter–productive when used in a different society.

Wisdom

On the other hand, *wisdom* is the ability, developed through experience, insight and reflection, to discern truth and exercise good judgment. It is sometimes conceptualized as an especially well developed form of common sense. Most psychologists regard wisdom as distinct from the cognitive abilities measured by standardized intelligence tests. Wisdom is often considered to be a trait that can be developed by experience, but not taught. When applied to practical matters, the term wisdom is synonymous with prudence. Some see wisdom as a quality that even a child, otherwise immature, may possess independent of experience or complete knowledge. The status of wisdom or prudence as a virtue is recognized in cultural, philosophical and religious sources. Some define wisdom in a utilitarian sense, as foreseeing consequences and acting to maximize the long–term common good.

A standard philosophical definition says that wisdom consists of making the best use of available knowledge. As with all decisions, a wise decision may be made with incomplete information. The technical philosophical term for the opposite of wisdom is folly. For example, in his *Metaphysics*, Aristotle defines wisdom as knowledge of causes: why things exist in a particular fashion.

Beyond the simple expedient of experience (which may be considered the most difficult way to gain wisdom as through the 'school of hard knocks'), there are a variety of other avenues to gaining wisdom which vary according to different philosophies. For example, the so–called *freethinkers*[36] believe that wisdom may come from pure reason and perhaps experience. Recall

[36] Freethought is a philosophical doctrine that holds that beliefs should be formed on the basis of science and logical principles and not be comprised by authority, tradition or any other dogmatic or otherwise fallacious belief system that

that *freethought* is a philosophical doctrine that holds that beliefs should be formed on the basis of science and logical principles and not be comprised by authority, tradition or any other dogmatic or otherwise fallacious belief system that restricts logical reasoning. The cognitive application of freethought is known as *freethinking*, and practitioners of freethought are known as freethinkers. Freethought holds that individuals should neither accept nor reject ideas proposed as truth without recourse to knowledge and reason. Thus, freethinkers strive to build their beliefs on the basis of facts, scientific inquiry, and logical principles, independent of the factual/logical fallacies and intellectually-limiting effects of authority, cognitive bias, conventional wisdom, popular culture, prejudice, sectarianism, tradition, urban legend and all other dogmatic or otherwise fallacious principles. When applied to religion, the philosophy of freethought holds that, given presently–known facts, established scientific theories, and logical principles, there is insufficient evidence to support the existence of supernatural phenomena. A line from 'Clifford's Credo' by the 19th Century British mathematician and philosopher William Clifford[37] perhaps best describes the premise of freethought: "It is wrong always, everywhere, and for anyone, to believe anything upon insufficient evidence." Since many popular beliefs are based on dogmas, freethinkers' opinions are often at odds with commonly–established views.

On the other hand, there is also a common belief that wisdom comes from *intuition* or, 'superlogic', as it is called by Tony Buzan,[38] inventor of *mind*

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restricts logical reasoning. The cognitive application of freethought is known as freethinking, and practitioners of freethought are known as freethinkers.

[37] William Kingdon Clifford, FRS (May 4, 1845 – March 3, 1879) was an English mathematician who also wrote a fair bit on philosophy. Along with Hermann Grassmann, he invented what is now termed *geometric algebra*, a special case being the *Clifford algebras* named in his honour, which play a role in contemporary mathematical physics. He was the first to suggest that gravitation might be a manifestation of an underlying geometry. His philosophical writings coined the phrase 'mind–stuff'.

Influenced by Riemann and Lobachevsky, Clifford studied non–Euclidean geometry. In 1870, he wrote On the space theory of matter, arguing that energy and matter are simply different types of curvature of space. These ideas later played a fundamental role in Albert Einstein's general theory of relativity. Yet Clifford is now best remembered for his eponymous Clifford algebras, a type of associative algebra that generalizes the complex numbers and William Rowan Hamilton's *quaternions*. The latter resulted in the octonions (biquaternions), which he employed to study motion in non–Euclidean spaces and on certain surfaces, now known as Klein–Clifford spaces. He showed that spaces of constant curvature could differ in topological structure. He also proved that a Riemann surface is topologically equivalent to a box with holes in it. On Clifford algebras, quaternions, and their role in contemporary mathematical physics.

[38] Tony Buzan (1942–) is the originator of mind mapping and coined the term mental literacy. He was born in London and received double Honours in psychology, English, mathematics and the General Sciences from the University of British

maps. For example, *holists* believe that wise people sense, work with and align themselves and others to life. In this view, wise people help others appreciate the fundamental interconnectedness of life. Also, some religions hold that wisdom may be given as a gift from God. For example, *Buddha* taught that a wise person is endowed with good bodily conduct, good verbal conduct and good mental conduct and a wise person does actions that are unpleasant to do but give good results and doesn't do actions that are pleasant to do but give bad results; this is called *karma*. According to *Hindu scriptures*, spiritual wisdom – *jnana* alone can lead to liberation. *Confucius* stated that wisdom can be learned by three methods: (i) *reflection* (the noblest), (ii) *imitation* (the easiest) and (iii) *experience* (the bitterest).

9.2 Human Intelligence

At least two major 'consensus' definitions of intelligence have been proposed. First, from 'Intelligence: Knowns and Unknowns', a report of a task force convened by the *American Psychological Association*[39] in 1995 (see [APS98]): Individuals differ from one another in their ability to understand complex ideas, to adapt effectively to the environment, to learn from experience, to engage in various forms of reasoning, to overcome obstacles by taking thought. Although these individual differences can be substantial, they are never entirely consistent: a given person's intellectual performance will vary on different occasions, in different domains, as judged by different criteria. Concepts of 'intelligence' are attempts to clarify and organize this complex set of phenomena.

A second definition of intelligence comes from the 'Mainstream Science on Intelligence', which was signed by 52 intelligence researchers in 1994 (also see [APS98]): Intelligence is a very general mental capability that, among other things, involves the ability to reason, plan, solve problems, think abstractly,

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Columbia in 1964. He is probably best known for his book, Use Your Head, his promotion of mnemonic systems and his mind–mapping techniques. Following his 1970s series for the BBC, many of his ideas have been set into his series of five books: Use Your Memory, Master Your Memory, Use Your Head, The Speed Reading Book and The Mind Map Book.

In essence, Buzan teaches "Learn how your brain learns rapidly and naturally." His work is partly based on the explosion of brain research that has taken place since the late 1950s, and the work on the left and right brain by psychologist Robert Ornstein and Nobel Laureate Roger Wolcott Sperry.

[39] The American Psychological Association (APA) is a professional organization representing psychology in the US. It has around 150,000 members and an annual budget of around $70m. The APA mission statement is to "advance psychology as a science and profession and as a means of promoting health, education, and human welfare." The APA was founded in July 1892 at Clark University by a group of 26 men. Its first president was G. Stanley Hall. There are currently 54 professional divisions in the APA. It is affiliated with 58 state and territorial and Canadian provincial associations.

comprehend complex ideas, learn quickly and learn from experience. It is not merely book learning, a narrow academic skill, or test–taking smarts. Rather, it reflects a broader and deeper capability for comprehending our surroundings, i.e., 'catching on', 'making sense' of things, or 'figuring out' what to do.

Individual intelligence experts have offered a number of similar definitions:

(i) David Wechsler:[40] " ... the aggregate or global capacity of the individual to act purposefully, to think rationally, and to deal effectively with his environment."

(ii) Cyril Burt:[41] " ...innate general cognitive ability."

(iii) Howard Gardner:[42] " To my mind, a human intellectual competence must entail a set of skills of problem solving, enabling the individual to resolve genuine problems or difficulties that he or she encounters and, when appropriate, to create an effective product, and must also entail the potential for finding or creating problems, and thereby laying the groundwork for the acquisition of new knowledge."

(iv) Richard Herrnstein[43] and Charles Murray: " ...cognitive ability."

(v) Robert Sternberg:[44] "... goal–directed adaptive behavior."

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[40] David Wechsler (January 12, 1896, Lespedi, Romania – May 2, 1981, New York, New York) was a leading Romanian-American psychologist. He developed well–known intelligence scales, such as the Wechsler Adult Intelligence Scale (WAIS) and the Wechsler Intelligence Scale for Children (WISC).

[41] Sir Cyril Lodowic Burt (March 3, 1883 — October 10, 1971) was a prominent British educational psychologist. He was a member of the London School of Differential Psychology. Some of his work was controversial for its conclusions that genetics substantially influence mental and behavioral traits. After his death, he was famously accused of scientific fraud.

[42] Howard Gardner (born in Scranton, Pennsylvania, USA in 1943) is a psychologist based at Harvard University best known for his theory of multiple intelligences. In 1981 he was awarded a MacArthur Prize Fellowship.

[43] Richard J. Herrnstein (May 20, 1930 – September 13, 1994) was a prominent researcher in comparative psychology who did pioneering work on pigeon intelligence employing the Experimental Analysis of Behavior and formulated the 'Matching Law' in the 1960s, a breakthrough in understanding how reinforcement and behavior are linked. He was the Edgar Pierce Professor of psychology at Harvard University and worked with B. F. Skinner in the Harvard pigeon lab, where he did research on choice and other topics in behavioral psychology. Herrnstein became more broadly known for his work on the correlation between race and intelligence, first in the 1970s, then with Charles Murray, discussed in their controversial best–selling 1994 book, The Bell Curve. Herrnstein described the behavior of hyperbolic discounting, in which people will choose smaller payoffs sooner instead of larger payoffs later. He developed a type of non–parametric statistics that he dubbed ρ.

[44] Robert J. Sternberg (borne 8 December 1949) is a psychologist and psychometrician and the Dean of Arts and Sciences at Tufts University. He was formerly IBM Professor of Psychology and Education at Yale University and the President of the American Psychological Association. Sternberg currently sits on the

9.2.1 Psychometric Definition of Intelligence

Despite the variety of concepts of intelligence, the most influential approach to understanding intelligence (i.e., with the most supporters and the most published research over the longest period of time) is based on *psychometric testing*,[45] which regards intelligence as *cognitive ability*.

Recall that *psychometrics* is the field of study concerned with the theory and technique of psychological measurement, which includes the measurement of knowledge, abilities, attitudes, and personality traits. The field is primarily concerned with the study of differences between individuals. It involves two major research tasks, namely:

(i) the construction of instruments and procedures for measurement; and

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editorial board of Intelligence. Sternberg has proposed the so–called *Triarchic theory of intelligence* and a triangular theory of love. He is the creator (with Todd Lubart) of the investment theory of creativity, which states that creative people buy low and sell high in the world of ideas, and a propulsion theory of creative contributions, which states that creativity is a form of leadership.

[45] Psychometrics is the field of study concerned with the theory and technique of psychological measurement, which includes the measurement of knowledge, abilities, attitudes, and personality traits. The field is primarily concerned with the study of differences between individuals. It involves two major research tasks, namely: (i) the construction of instruments and procedures for measurement; and (ii) the development and refinement of theoretical approaches to measurement. Much of the early theoretical and applied work in psychometrics was undertaken in an attempt to measure intelligence. The origin of psychometrics has connections to the related field of psychophysics. Charles Spearman, a pioneer in psychometrics who developed approaches to the measurement of intelligence, studied under Wilhelm Wundt and was trained in psychophysics. The psychometrician L.L. Thurstone later developed and applied a theoretical approach to the measurement referred to as the law of comparative judgment, an approach which has close connections to the psychophysical theory developed by Ernst Heinrich Weber and Gustav Fechner. In addition, Spearman and Thurstone both made important contributions to the theory and application of factor analysis, a statistical method that has been used extensively in psychometrics. More recently, psychometric theory has been applied in the measurement of personality, attitudes and beliefs, academic achievement, and in health-related fields. Measurement of these unobservable phenomena is difficult, and much of the research and accumulated art in this discipline has been developed in an attempt to properly define and quantify such phenomena. Critics, including practitioners in the physical sciences and social activists, have argued that such definition and quantification is impossibly difficult, and that such measurements are often misused. Proponents of psychometric techniques can reply, though, that their critics often misuse data by not applying psychometric criteria, and also that various quantitative phenomena in the physical sciences, such as heat and forces, cannot be observed directly but must be inferred from their manifestations. Figures who made significant contributions to psychometrics include Karl Pearson, L. L. Thurstone, Georg Rasch and Arthur Jensen.

(ii) the development and refinement of theoretical approaches to measurement. Much of the early theoretical and applied work in psychometrics was undertaken in an attempt to measure intelligence.

The origin of psychometrics has connections to the related field of psychophysics. Charles Spearman, a pioneer in psychometrics who developed approaches to the measurement of intelligence, studied under Wilhelm Wundt[46] and was trained in psychophysics. The psychometrician Louis Thurstone[47] later developed and applied a theoretical approach to the measurement

[46] Wilhelm Maximilian Wundt (August 16, 1832–August 31, 1920) was a German physiologist and psychologist. He is generally acknowledged as a founder of experimental psychology and cognitive psychology. He is less commonly recognised as a founding figure in social psychology, however, the later years of Wundt's life were spent working on Völkerpsychologie which he understood as a study into the social basis of higher mental functioning.

Wundt combined philosophical introspection with techniques and laboratory apparatuses brought over from his physiological studies with Helmholtz, as well as many of his own design. This experimental introspection was in contrast to what had been called psychology until then, a branch of philosophy where people introspected themselves. Wundt argued in his 1904 book 'Principles of Physiological Psychology' that "we learn little about our minds from casual, haphazard self–observation...It is essential that observations be made by trained observers under carefully specified conditions for the purpose of answering a well–defined question."

The methods Wundt used are still used in modern psychophysical work, where reactions to systematic presentations of well–defined external stimuli are measured in some way–reaction time, reactions, comparison with graded colors or sounds, and so forth. His chief method of investigation was called *introspection* in the terminology of the time, though *observation* may be a better translation.

Wundt subscribed to a 'psychophysical parallelism' (which entirely excludes the possibility of a mind–body/cause–effect relationship), which was supposed to stand above both materialism and idealism. His epistemology was an eclectic mixture of the ideas of Spinoza, Leibniz, Kant, and Hegel.

[47] Louis Leon Thurstone (29 May 1887–29 September 1955) was a U.S. pioneer in the fields of psychometrics and psychophysics. He conceived the approach to measurement known as the law of comparative judgment, and is well known for his contributions to *factor analysis*. He is responsible for the standardized mean and standard deviation of IQ scores used today, as opposed to the Intelligence Test system originally used by Alfred Binet. He is also known for the development of the Thurstone scale.

Thurstone's work in factor analysis led him to formulate a model of intelligence center around 'Primary Mental Abilities' (PMAs), which were independent group factors of intelligence that different individuals possessed in varying degrees. He opposed the notion of a singular general intelligence that factored into the scores of all psychometric tests and was expressed as a mental age. This idea was unpopular at the time due to its obvious conflicts with Spearman's 'mental energy' referred to as the law of comparative judgment, an approach which has close connections to the psychophysical theory developed by Ernst Weber and Gustav Fechner (see below). In addition, Spearman and Thurstone both made important contributions to the theory and application of factor analysis, a statistical method that has been used extensively in psychometrics. More recently, psychometric theory has been applied in the measurement of personality, attitudes and beliefs, academic achievement, and in health–related fields. Measurement of these unobservable phenomena is difficult, and much of the research and accumulated art in this discipline has been developed in an attempt to properly define and quantify such phenomena. Critics, including practitioners in the physical sciences and social activists, have argued that such definition and quantification is impossibly difficult, and that such measurements are often misused. Proponents of psychometric techniques can reply, though, that their critics often misuse data by not applying psychometric criteria, and also that various quantitative phenomena in the physical sciences, such as heat and forces, cannot be observed directly but must be inferred from their manifestations. Figures who made significant contributions to psychometrics include Karl Pearson, Louis Thurstone, Georg Rasch and Arthur Jensen.

Intelligence, narrowly defined by psychometrics, can be measured by intelligence tests, also called *intelligence quotient* (IQ)[48] tests. Such intelligence

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model, and is today still largely discredited. Nonetheless, Thurstone's contributions to methods of factor analysis have proved invaluable in establishing and verifying later psychometric factor structures, and has influenced the hierarchical models of intelligence in use in intelligence tests such as WAIS and the modern Stanford–Binet IQ test.

The *seven primary mental abilities* in Thurstone's model were *verbal comprehension, word fluency, number facility, spatial visualization, associative memory, perceptual speed and reasoning.*

[48] An intelligence quotient or IQ is a score derived from a set of standardized tests of intelligence. Intelligence tests come in many forms, and some tests use a single type of item or question. Most tests yield both an overall score and individual sub–tests scores. Regardless of design, all IQ tests measure the same general intelligence. Component tests are generally designed and chosen because they are found to be predictable of later intellectual development, such as educational achievement. IQ also correlates with job performance, socioeconomic advancement, and 'social pathologies'. Recent work has demonstrated links between IQ and health, longevity, and functional literacy. However, IQ tests do not measure all meanings of 'intelligence', such as creativity. IQ scores are relative (like placement in a race), not absolute (like the measurement of a ruler). The average IQ scores for many populations were rising during the 20th century: a phenomenon called the *Flynn effect*. It is not known whether these changes in scores reflect real changes in intellectual abilities. On average, IQ scores are stable over a person's lifetime, but some individuals undergo large changes. For example, scores can be affected by the presence of learning disabilities.

tests take many forms, but the common tests (*Stanford–Binet*,[49] *Raven's Progressive Matrices*,[50] *Wechsler Adult Intelligence Scale*,[51] *Wechsler–Bellevue*

[49] The modern field of intelligence testing began with the Stanford-Binet IQ test. The Stanford-Binet itself started with the French psychologist Alfred Binet who was charged by the French government with developing a method of identifying intellectually deficient children for placement in special education programs. As Binet indicated, case studies may be more detailed and at times more helpful, but the time required to test large numbers of people would be huge. Unfortunately, the tests he and his assistant Victor Henri developed in 1896 were largely disappointing [Fan85].

[50] Raven's Progressive Matrices are widely used non–verbal intelligence tests. In each test item, one is asked to find the missing part required to complete a pattern. Each Set of items gets progressively harder, requiring greater cognitive capacity to encode and analyze. The test is considered by many intelligence experts to be one of the most **g**–loaded in existence. The matrices are offered in three different forms for different ability levels, and for age ranges from five through adult: (i) Colored Progressive Matrices (younger children and special groups); (ii) Standard Progressive Matrices (average 6 to 80 year olds); and (iii) Advanced Progressive Matrices (above average adolescents and adults). According to their author, Raven's Progressive Matrices and Vocabulary tests measure the two main components of general intelligence (originally identified by Spearman): the ability to think clearly and make sense of complexity, which is known as eductive ability (from the Latin root 'educere', meaning 'to draw out'; and the ability to store and reproduce information, known as reproductive ability. Adequate standardization, ease of use (without written or complex instructions), and minimal cost per person tested are the main reasons for its widespread international use in most countries of the world. It appears to measure a type of *reasoning ability* which is fundamental to making sense out of the 'booming buzzing confusion' in all walks of life. Thus, it has among the highest predictive validities of any test in most occupational groups and, even more importantly, in predicting social mobility... the level of job a person will attain and retain. Although it is sometimes criticized for being costly, this is based on a failure to calculate cost per person tested with re–usable test booklets that can be used up to 50 times each. The authors of the Manual recommend that, when used in selection, RPM scores are set in the context of information relating to Raven's framework for the assessment of Competence. Some of the most fundamental research in cognitive psychology has been carried out with the RPM. The tests have been shown to work–scale–measure the same thing – in a vast variety of cultural groups. There is no truth in the assertion that the low mean scores obtained in some groups arise from a general lack of familiarity with the way of thought measured by the test. Two remarkable, and relatively recent, findings are that, on the one hand, the actual scores obtained by people living in most countries with a tradition of literacy – from China, Russia, and India through Europe to Kuwait – are very similar at any point in time. On the other hand, in all countries, the scores have increased dramatically over time... such that 50% of our grandparents would be assigned to special education classes if they were judged against today's norms. Yet none of the common explanations (e.g., access to television, changes in education, changes in family size etc.) hold up. The

I,[52] and others) all measure the same dominant form of intelligence, **g** or 'general intelligence factor'. The abstraction of **g** stems from the observation that scores on all forms of cognitive tests *positively correlate* with one another. **g** can be derived as the *principal intelligence factor* from *cognitive test scores* using the *multivariate correlation statistical method* of *factor analysis* (FA).

9.2.2 Correlation and Factor Analysis

Recall that *correlation*, also called *correlation coefficient*, indicates the strength and direction of a linear relationship between two random variables (see Figure 9.1). In other words, correlation is a measure of the relation between two or more statistical variables. In general statistical usage, correlation (or, co–rrelation) refers to the departure of two variables from independence, although correlation does not imply their *functional causal relation*. In this broad sense there are several coefficients, measuring the degree of correlation, adapted to the nature of data. A number of different coefficients are used for different situations. Correlation coefficients can range from -1.00 to $+1.00$. The value of -1.00 represents a perfect negative correlation while a value of $+1.00$ represents a perfect positive correlation. The perfect correlation indicates an existence of functional relation between two statistical variables. A value of 0.00 represents a lack of correlation. Geometrically, the correlation coefficient can also be viewed as the cosine of the angle between the two vectors of samples drawn from the two random variables (see, e.g. [Har75]).

The most widely–used type of correlation simple linear coefficient is Pearson r, also called *linear* or *product–moment* correlation, which assumes that

explanation seems to have more in common with those put forward to explain the parallel increase in life expectancy... which has doubled over the same period of time.

[51] Wechsler Adult Intelligence Scale or WAIS is a general IQ test, published in February 1955 as a revision of the Wechsler–Bellevue test (1939), standardized for use with adults over the age of 16. In this test intelligence is quantified as the global capacity of the individual to act purposefully, to think rationally, and to deal effectively with the environment.

[52] David Wechsler (January 12, 1896, Lespedi, Romania – May 2, 1981, New York, New York) was a leading Romanian–American psychologist. He developed well–known intelligence scales, such as the Wechsler Adult Intelligence Scale (WAIS) and the Wechsler Intelligence Scale for Children (WISC). The Wechsler Adult Intelligence Scale (WAIS) was developed first in 1939 and then called the Wechsler–Bellevue Intelligence Test. From these he derived the Wechsler Intelligence Scale for Children (WISC) in 1949 and the Wechsler Preschool and Primary Scale of Intelligence (WPPSI) in 1967. Wechsler originally created these tests to find out more about his patients at the Bellevue clinic and he found the then–current *Binet IQ test* unsatisfactory. The tests are still based on his philosophy that intelligence is "the global capacity to act purposefully, to think rationally, and to deal effectively with (one's) environment."

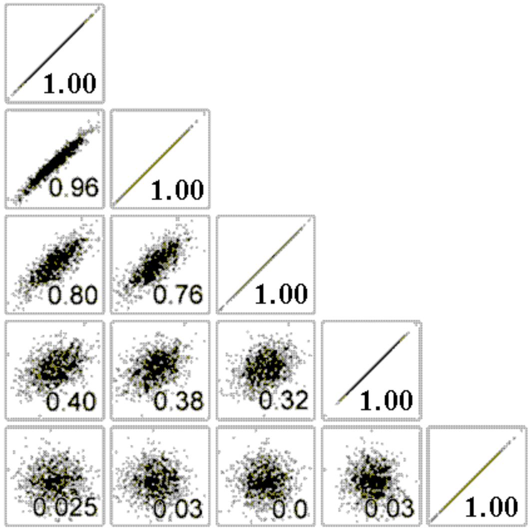

Fig. 9.1 Example of positive linear correlations between 1000 pairs of numbers. Note that each set of points correlates maximally with itself, as shown on the diagonal. Also, note that we have not plot the upper part of the correlation matrix as it is symmetrical.

the two variables are measured on at least interval scales, and it determines the extent to which values of the two variables are 'proportional' to each other. The value of correlation coefficient does not depend on the specific measurement units used. Proportional means linearly related using regression line or least squares line. If the correlation coefficient is squared, then the resulting value (r^2, the *coefficient of determination*) will represent the proportion of common variation in the two variables (i.e., the 'strength' or 'magnitude' of the relationship). In order to evaluate the correlation between variables, it is important to know this 'magnitude' or 'strength' as well as the significance of the correlation.

The significance level calculated for each correlation is a primary source of information about the reliability of the correlation. The significance of correlation coefficient of particular magnitude will change depending on the size of the sample from which it was computed. The test of significance is based on the assumption that each of the two variables is normally distributed and that their bivariate ('combined') distribution is normal (which can be tested by examining the 3D bivariate distribution histogram). However, *Monte–Carlo* studies suggest that meeting those assumptions (especially the second one) is not absolutely crucial if our sample size is not very small and when the departure from normality is not very large. It is impossible to formulate precise recommendations based on those Monte–Carlo results, but many researchers follow a rule of thumb that if our sample size is 50 or more then serious biases are unlikely, and if our sample size is over 100 then you should not be concerned at all with the normality assumptions.

Recall that the *normal distribution*, also called *Gaussian distribution*, is an extremely important probability distribution in many fields. It is a family of distributions of the same general form, differing in their location and scale parameters: the *mean* ('average') μ and *standard deviation* ('variability') σ, respectively. The *standard normal distribution* is the normal distribution with a mean of zero and a standard deviation of one. It is often called the *bell curve* because the graph of its *probability density function pdf*, given by the *Gaussian function*

$$pdf = \frac{1}{\sigma\sqrt{2\pi}} exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

resembles a bell shape (here, $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ is the *pdf* for the standard normal distribution). The corresponding *cumulative distribution function cdf* is defined as the probability that a variable X has a value less than or equal to x, and it is expressed in terms of the *pdf* as

$$cdf = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} exp\left(-\frac{(u-\mu)^2}{2\sigma^2}\right) du.$$

Now, the correlation $r_{X,Y}$ between two *normally distributed random variables* X and Y with expected values μX and μY and standard deviations σX and σY is defined as:

$$r_{XY} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E((X-\mu_X)(Y-\mu_Y))}{\sigma_X \sigma_Y},$$

where E denotes the expected value of the variable and cov means covariance. Since $\mu X = E(X)$, $\sigma_X^2 = E(X^2) - E^2(X)$ and similarly for Y, we can write (see, e.g., [CCW03])

$$r_{XY} = \frac{E(XY) - E(X)E(Y)}{\sqrt{E(X^2) - E^2(X)}\,\sqrt{E(Y^2) - E^2(Y)}}.$$

Assume that we have a data matrix $\mathbf{X} = \{x_{i\alpha}\}$ formed out of the *sample* $\{\mathbf{x_i}\}$ of n normally distributed simulator tests called observable–vectors or *manifest variables*, defined on the sample $\{\alpha = 1, \dots, N\}$ of pilot (for the statistical significance the practical user's criterion is $N \geq 5n$). The *maximum likelihood estimator* of the Pearson correlation coefficient r_{ik} between any two manifest variables $\mathbf{x_i}$ and $\mathbf{x_k}$ is defined as[53]

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[53] A time–dependent generalization $C_{\alpha\beta} = C_{\alpha\beta}(t)$ of the correlation coefficient r_{XY} is the *correlation function*, defined as follows. For the two time–series, $x_\alpha(t_i)$ and $x_\beta(t_i)$ of the same length $(i = 1, ..., T)$, one defines the correlation function by

$$C_{\alpha\beta} = \frac{\sum_i (x_\alpha(t_i) - \bar{x}_\alpha)(x_\beta(t_i) - \bar{x}_\beta)}{\sqrt{\sum_i (x_\alpha(t_i) - \bar{x}_\alpha)^2 \sum_j (x_\beta(t_j) - \bar{x}_\beta)^2}},$$

where $\bar{x}$ denotes a time average over the period studied. For two sets of N time–series $x_\alpha(t_i)$ each $(\alpha, \beta = 1, ..., N)$ all combinations of the elements $C_{\alpha\beta}$ can be

$$r_{ik} = \frac{\sum_{\alpha=1}^{N}(x_{i\alpha} - \mu_i)(x_{k\alpha} - \mu_k)}{\sqrt{\sum_{\alpha=1}^{N}(x_{i\alpha} - \mu_i)^2}\sqrt{\sum_{\alpha=1}^{N}(x_{k\alpha} - \mu_k)^2}},$$

where

$$\mu_i = \frac{1}{N}\sum_{\alpha=1}^{N} x_{i\alpha}$$

is the arithmetic mean of the variable $\mathbf{x_i}$.[54] Correlation matrix $\mathbf{R}$ is the matrix $\mathbf{R} \equiv \mathbf{R}_{ik} = \{r_{ik}\}$ including $n \times n$ Pearson correlation coefficients r_{ik} calculated between n manifest variables $\{\mathbf{x_i}\}$. Therefore, $\mathbf{R}$ is symmetrical matrix with ones on the main diagonal. The correlation matrix $\mathbf{R}$ represents the total variability of all included manifest variables. In other words it stores all information about all simulator tests and all pilot. Now, if the number of included simulator tests is small, this information is meaningful for the human mind. But if we perform one hundred tests (on five hundred pilot), then

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used as entries of the $N \times N$ correlation matrix $\mathbf{C}$. By diagonalizing $\mathbf{C}$, i.e., solving the eigenvalue problem:

$$\mathbf{C}\mathbf{v}^k = \lambda_k \mathbf{v}^k,$$

one gets the eigenvalues λ_k $(k = 1, ..., N)$ and the corresponding eigenvectors $\mathbf{v}^k = \{v_\alpha^k\}$.

[54] The following algorithm (in pseudocode) estimates bivariate correlation coefficient with good numerical stability:

```
Begin
  sum_sq_x = 0;
  sum_sq_y = 0;
  sum_coproduct = 0;
  mean_x = x[1];
  mean_y = y[1];
  for i in 2 to N:
    sweep = (i - 1.0) / i;
    delta_x = x[i] - mean_x;
    delta_y = y[i] - mean_y;
    sum_sq_x += delta_x * delta_x * sweep;
    sum_sq_y += delta_y * delta_y * sweep;
    sum_coproduct += delta_x * delta_y * sweep;
    mean_x += delta_x / i;
    mean_y += delta_y / i ;
  end_for;
  pop_sd_x = sqrt( sum_sq_x / N );
  pop_sd_y = sqrt( sum_sq_y / N );
  cov_x_y = sum_coproduct / N;
  correlation = cov_x_y / (pop_sd_x * pop_sd_y);
End.
```

the correlation matrix contains ten thousand Pearson correlation coefficients. This is the reason for seeking the 'latent' factor structure, underlying the whole co–variability contained in the correlation matrix.

Therefore, the correlation is defined only if both of the standard deviations are finite and both of them are nonzero. It is a corollary of the *Cauchy–Schwarz inequality*[55] that the correlation cannot exceed 1 in absolute value. The correlation is 1 in the case of an increasing linear relationship, -1 in the case of a decreasing linear relationship, and some value in between in all other cases, indicating the degree of linear dependence between the variables. The closer the coefficient is to either -1 or 1, the stronger the correlation between the variables (see Figure 9.1). If the variables are independent then the correlation is 0, but the converse is not true because the correlation coefficient detects only linear dependencies between two variables. For example, suppose the random variable X is uniformly distributed on the interval from -1 to 1, and $Y = X^2$. Then Y is completely determined by X, so that X and Y are dependent, but their correlation is zero; this means that they are uncorrelated. The correlation matrix of n random variables $X_1, ..., X_n$ is the $n \times n$ matrix whose ij entry is $r_{X_i X_j}$. If the measures of correlation used are product–moment coefficients, the correlation matrix is the same as the covariance matrix of the standardized random variables X_i/σ_{X_i} (for $i = 1, ..., n$). Consequently it is necessarily a non–negative definite matrix. The correlation

[55] The Cauchy–Schwarz inequality, named after Augustin Louis Cauchy (the father of complex analysis) and Hermann Amandus Schwarz, is a useful inequality encountered in many different settings, such as linear algebra applied to vectors, in analysis applied to infinite series and integration of products, and in probability theory, applied to variances and covariances. The Cauchy–Schwarz inequality states that if x and y are elements of real or complex inner product spaces then

$$|\langle x, y\rangle|^2 \leq \langle x, x\rangle \cdot \langle y, y\rangle.$$

The two sides are equal iff x and y are linearly dependent (or in geometrical sense they are parallel). This contrasts with a property that the inner product of two vectors is zero if they are orthogonal (or perpendicular) to each other. The inequality hence confers the notion of *the angle between the two vectors* to an inner product, where concepts of *Euclidean geometry* may not have meaningful sense, and justifies that the notion that inner product spaces are generalizations of *Euclidean space*.

An important consequence of the Cauchy–Schwarz inequality is that the inner product is a continuous function.

Another form of the Cauchy–Schwarz inequality is given using the notation of norm, as explained under norms on inner product spaces, as

$$|\langle x, y\rangle| \leq \|x\| \cdot \|y\|.$$

matrix is symmetrical (the correlation between X_i and X_j is the same as the correlation between X_j and X_i).

As a higher derivation of the correlation matrix analysis and its eigenvectors, the so–called principal components, the *factor analysis* (FA) is a multivariate statistical technique used to explain variability among a large set of observed random variables in terms of fewer unobserved random 'latent' variables, called *factors*. The observed, or 'manifested' variables are modelled as linear combinations of the factors, plus 'error terms'. According to FA, classical bivariate correlation analysis is an artificial extraction from a rial multivariate world, especially in human sciences. FA originated in psychometrics, and is used in social sciences, marketing, product management, operations research, and other applied sciences that deal with large multivariate quantities of data.

For example,[56] suppose a psychologist proposes a theory that there are two kinds of intelligence, 'verbal intelligence' and 'mathematical intelligence'. Note that these are inherently unobservable. Evidence for the theory is sought in the examination scores of 1000 students in each of 10 different academic fields. If a student is chosen randomly from a large population, then the student's 10 scores are random variables. The psychologist's theory may say that the average score in each of the 10 subjects for students with a particular level of verbal intelligence and a particular level of mathematical intelligence is a certain number times the level of verbal intelligence plus a certain number times the level of mathematical intelligence, i.e., it is a linear combination of those two 'factors'. The numbers by which the two 'intelligences' are multiplied are posited by the theory to be the same for all students, and are called 'factor loadings'. For example, the theory may hold that the average student's aptitude in the field of amphibology is

{ 10 × the student's verbal intelligence } + { 6 × the student's mathematical intelligence }.

The numbers 10 and 6 are the factor loadings associated with amphibology. Other academic subjects may have different factor loadings. Two students having identical degrees of verbal intelligence and identical degrees of mathematical intelligence may have different aptitudes in amphibology because individual aptitudes differ from average aptitudes. That difference is called the 'error' — an unfortunate misnomer in statistics that means the amount by which an individual differs from what is average. The observable data that go into factor analysis would be 10 scores of each of the 1000 students, a total of 10,000 numbers. The factor loadings and levels of the two kinds of intelligence of each student must be inferred from the data. Even the number of factors (two, in this example) must be inferred from the data.

[56] This oversimplified example should not be taken to be realistic. Usually we are dealing with many factors.

In the example above, for $i = 1, ..., 1,000$ the ith student's scores are

$$\begin{array}{ccccccccc} x_{1,i} & = & \mu_1 & + & \ell_{1,1} v_i & + & \ell_{1,2} m_i & + & \varepsilon_{1,i} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ x_{10,i} & = & \mu_{10} & + & \ell_{10,1} v_i & + & \ell_{10,2} m_i & + & \varepsilon_{10,i} \end{array}$$

where $x_{k,i}$ is the ith student's score for the kth subject, μ_k is the mean of the students' scores for the kth subject, ν_i is the ith student's 'verbal intelligence', m_i is the ith student's 'mathematical intelligence', $\ell_{k,j}$ are the factor loadings for the kth subject, for $j = 1, 2$; $\epsilon_{k,i}$ is the difference between the ith student's score in the kth subject and the average score in the kth subject of all students whose levels of verbal and mathematical intelligence are the same as those of the ith student. In matrix notation, we have

$$X = \mu + LF + \epsilon,$$

where X is a $10 \times 1,000$ matrix of observable random variables, μ is a 10×1 column vector of unobservable constants (in this case constants are quantities not differing from one individual student to the next; and random variables are those assigned to individual students; the randomness arises from the random way in which the students are chosen), L is a 10×2 matrix of factor loadings (unobservable constants), F is a $2 \times 1,000$ matrix of unobservable random variables, ϵ is a $10 \times\ 1,000$ matrix of unobservable random variables.

Observe that by doubling the scale on which 'verbal intelligence', the first component in each column of F, is measured, and simultaneously halving the factor loadings for verbal intelligence makes no difference to the model. Thus, no generality is lost by assuming that the standard deviation of verbal intelligence is 1. Likewise for 'mathematical intelligence'. Moreover, for similar reasons, no generality is lost by assuming the two factors are uncorrelated with each other. The 'errors' ϵ are taken to be independent of each other. The variances of the 'errors' associated with the 10 different subjects are not assumed to be equal.

Mathematical basis of FA is *principal components analysis* (PCA), which is a technique for simplifying a dataset, by reducing multidimensional datasets to lower dimensions for analysis. Technically speaking, PCA is a *linear transformation*[57] that transforms the data to a new coordinate system such that

[57] Recall that a *linear transformation* (also called *linear map* or *linear operator*) is a function between two vector spaces that preserves the operations of vector addition and scalar multiplication. In the language of abstract algebra, a linear transformation is a *homomorphism of vector spaces*, or a *morphism* in the category of vector spaces over a given field.

Let V and W be vector spaces over the same field K. A function (operator) $f : V \to W$ is said to be a *linear transformation* if for any two vectors $x, y \in V$ and any scalar $a \in K$, the following two conditions are satisfied:

$$\text{additivity} : f(x + y) = f(x) + f(y), \qquad \text{and}$$
$$\text{homogeneity} : f(ax) = af(x).$$

the greatest variance by any projection of the data comes to lie on the first coordinate (called the first principal component), the second greatest variance on the second coordinate, and so on. PCA can be used for *dimensionality reduction*[58] in a dataset while retaining those characteristics of the dataset that contribute most to its variance, by keeping lower–order principal components and ignoring higher–order ones. Such low–order components often contain the 'most important' aspects of the data. PCA is also called the (discrete) *Karhunen–Loève transform* (or KLT, named after Kari Karhunen and Michel Loève) or the *Hotelling transform* (in honor of Harold Hotelling[59]). PCA has the distinction of being the optimal linear transformation for keeping the subspace that has largest variance. This advantage, however, comes at the price of greater computational requirement if compared, for example, to the discrete cosine transform. Unlike other linear transforms, the PCA does not have a fixed set of basis vectors. Its basis vectors depend on the data set.

Assuming zero *empirical mean* (the empirical mean of the distribution has been subtracted from the data set), the principal component $\mathbf{w}_1$ of a dataset $\mathbf{x}$ can be defined as

$$\mathbf{w}_1 = \arg \max_{\|\mathbf{w}\|=1} E\left\{ \left(\mathbf{w}^T \mathbf{x}\right)^2 \right\}.$$

With the first $k-1$ components, the kth component can be found by subtracting the first $k-1$ principal components from $\mathbf{x}$,

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This is equivalent to requiring that for any vectors $x_1, ..., x_m$ and scalars $a_1, ..., a_m$, the following equality holds:

$$f(a_1 x_1 + \cdots + a_m x_m) = a_1 f(x_1) + \cdots + a_m f(x_m).$$

[58] Dimensionality reduction in statistics can be divided into two categories: *feature selection* and *feature extraction*.

Feature selection approaches try to find a subset of the original features. Two strategies are *filter* (e.g., information gain) and *wrapper* (e.g., genetic algorithm) approaches. It is sometimes the case that data analysis such as regression or classification can be carried out in the reduced space more accurately than in the original space. On the other hand, feature extraction is applying a mapping of the multidimensional space into a space of fewer dimensions. This means that the original feature space is transformed by applying e.g., a linear transformation via a *principal components analysis*.

Dimensionality reduction is also a phenomenon discussed widely in physics, whereby a physical system exists in three dimensions, but its properties behave like those of a lower–dimensional system.

[59] Harold Hotelling (Fulda, Minnesota, September 29, 1895 - December 26, 1973) was a mathematical statistician. His name is known to all statisticians because of *Hotelling's T–square distribution* and its use in statistical hypothesis testing and confidence regions. He also introduced canonical correlation analysis, and is the eponym of *Hotelling's law*, *Hotelling's lemma*, and *Hotelling's rule* in economics.

$$\hat{\mathbf{x}}_{k-1} = \mathbf{x} - \sum_{i=1}^{k-1} \mathbf{w}_i \mathbf{w}_i^T \mathbf{x},$$

and by substituting this as the new dataset to find a principal component in

$$\mathbf{w}_k = \arg \max_{\|\mathbf{w}\|=1} E \left\{ \left(\mathbf{w}^T \hat{\mathbf{x}}_{k-1} \right)^2 \right\}.$$

Thefore, the Karhunen–Loève transform is equivalent to finding the *singular value decomposition*[60] of the data matrix $\mathbf{X}$,

$$\mathbf{X} = \mathbf{W\Sigma V}^T,$$

and then obtaining the reduced–space data matrix $\mathbf{Y}$ by projecting $\mathbf{X}$ down into the reduced space defined by only the first L singular vectors $\mathbf{W_L}$,

$$\mathbf{Y} = \mathbf{W_L}^T \mathbf{X} = \mathbf{\Sigma_L V_L}^T.$$

The matrix $\mathbf{W}$ of singular vectors of $\mathbf{X}$ is equivalently the matrix $\mathbf{W}$ of eigenvectors of the matrix of observed covariances $\mathbf{C} = \mathbf{XX}^T$,

$$\mathbf{XX}^T = \mathbf{W\Sigma}^2\mathbf{W}^T.$$

The eigenvectors with the largest eigenvalues correspond to the dimensions that have the strongest correlation in the dataset.

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[60] Recall that in linear algebra, the *singular value decomposition* (SVD) is an important factorization of a rectangular real or complex matrix, with several applications in signal processing and statistics. The SVD can be seen as a generalization of the *spectral theorem*, which says that normal matrices can be unitarily diagonalized using a basis of eigenvectors, to arbitrary, not necessarily square, matrices.

Suppose M is an $m \times n$ matrix whose entries come from the field K, which is either the field of real numbers, or the field of complex numbers. Then there exists a factorization of the form:

$$M = U \Sigma V^*,$$

where U is an $m \times m$ unitary matrix over K, the matrix Σ is $m \times n$ with nonnegative numbers on the diagonal and zeros off the diagonal, and V^* denotes the conjugate transpose of V, an $n \times n$ unitary matrix over K. Such a factorization is called a singular–value decomposition of M.

The matrix V thus contains a set of orthonormal 'input' or 'analyzing' basis vector directions for M. The matrix U contains a set of orthonormal 'output' basis vector directions for M. The matrix Σ contains the singular values, which can be thought of as scalar 'gain controls' by which each corresponding input is multiplied to give a corresponding output. A common convention is to order the values Σ_{ii} in non–increasing fashion. In this case, the diagonal matrix Σ is uniquely determined by M (although the matrices U and V are not).

Now, FA is performed as PCA[61] with subsequent orthogonal (non–correlated) or oblique (correlated) *factor rotation* for the simplest possible interpretation (see, e.g., [KM78a]).

FA is used to uncover the latent structure (dimensions) of a set of variables. It reduces attribute space from a larger number of variables to a smaller number of factors and as such is a 'non–dependent' procedure (that is, it does not assume a dependent variable is specified). Factor analysis could be used for any of the following purposes:

1. To reduce a large number of variables to a smaller number of factors for modelling purposes, where the large number of variables precludes

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[61] The alternative FA approach is the so–called *principal factor analysis* (PFA, also called *principal axis factoring*, PAF, and *common factor analysis*, PFA). PFA is a form of factor analysis which seeks the least number of factors which can account for the common variance (correlation) of a set of variables, whereas the more common principal components analysis (PCA) in its full form seeks the set of factors which can account for all the common and unique (specific plus error) variance in a set of variables. PFA uses a PCA strategy but applies it to a correlation matrix in which the diagonal elements are not 1's, as in PCA, but iteratively–derived estimates of the *communalities*.

In addition to PCA and PFA, there are other less–used extraction methods:

1. Image factoring: based on the correlation matrix of predicted variables rather than actual variables, where each variable is predicted from the others using multiple regression.
2. Maximum likelihood factoring: based on a linear combination of variables to form factors, where the parameter estimates are those most likely to have resulted in the observed correlation matrix, using MLE methods and assuming multivariate normality. Correlations are weighted by each variable's uniqueness. (As discussed below, uniqueness is the variability of a variable minus its communality.) MLF generates a *chi–square goodness–of–fit test*. The researcher can increase the number of factors one at a time until a satisfactory goodness of fit is obtained. Warning: for large samples, even very small improvements in explaining variance can be significant by the goodness-of-fit test and thus lead the researcher to select too many factors.
3. Alpha factoring: based on maximizing the reliability of factors, assuming variables are randomly sampled from a universe of variables. All other methods assume cases to be sampled and variables fixed.
4. Unweighted least squares (ULS) factoring: based on minimizing the sum of squared differences between observed and estimated correlation matrices, not counting the diagonal.
5. Generalized least squares (GLS) factoring: based on adjusting ULS by weighting the correlations inversely according to their uniqueness (more unique variables are weighted less). Like MLF, GLS also generates a chi–square goodness–of–fit test. The researcher can increase the number of factors one at a time until a satisfactory goodness of fit is obtained.

modelling all the measures individually. As such, factor analysis is integrated in *structural equation modelling* (SEM),[62] helping create the latent

[62] Structural equation modelling (SEM) grows out of and serves purposes similar to multiple regression, but in a more powerful way which takes into account the modelling of interactions, nonlinearities, correlated independents, measurement error, correlated error terms, multiple latent independents each measured by multiple indicators, and one or more latent dependents also each with multiple indicators. SEM may be used as a more powerful alternative to multiple regression, path analysis, factor analysis, time series analysis, and analysis of covariance. That is, these procedures may be seen as special cases of SEM, or, to put it another way, SEM is an extension of the general linear model (GLM) of which multiple regression is a part.

SEM is usually viewed as a confirmatory rather than exploratory procedure, using one of three approaches:

a. Strictly confirmatory approach: A model is tested using SEM goodness–of–fit tests to determine if the pattern of variances and covariances in the data is consistent with a structural (path) model specified by the researcher. However as other unexamined models may fit the data as well or better, an accepted model is only a not–disconfirmed model.
b. Alternative models approach: One may test two or more causal models to determine which has the best fit. There are many goodness–of–fit measures, reflecting different considerations, and usually three or four are reported by the researcher. Although desirable in principle, this AM approach runs into the real-world problem that in most specific research topic areas, the researcher does not find in the literature two well-developed alternative models to test.
c. Model development approach: In practice, much SEM research combines confirmatory and exploratory purposes: a model is tested using SEM procedures, found to be deficient, and an alternative model is then tested based on changes suggested by SEM modification indexes. This is the most common approach found in the literature. The problem with the model development approach is that models confirmed in this manner are post–hoc ones which may not be stable (may not fit new data, having been created based on the uniqueness of an initial dataset). Researchers may attempt to overcome this problem by using a cross–validation strategy under which the model is developed using a calibration data sample and then confirmed using an independent validation sample.

Regardless of approach, SEM cannot itself draw causal arrows in models or resolve causal ambiguities. Theoretical insight and judgment by the researcher is still of utmost importance.

The SEM process centers around two steps: validating the measurement model and fitting the structural model. The former is accomplished primarily through confirmatory factor analysis, while the latter is accomplished primarily through path analysis with latent variables. One starts by specifying a model on the basis of theory. Each variable in the model is conceptualized as a latent one, measured by multiple indicators. Several indicators are developed for each model, with a view to winding up with at least three per latent variable after confirmatory factor analysis. Based on a large ($n > 100$) representative sample, factor analysis (common factor analysis or principal axis factoring, not principle components

variables modeled by SEM. However, factor analysis can be and is often used on a standalone basis for similar purposes.
2. To select a subset of variables from a larger set, based on which original variables have the highest correlations with the principal component factors.
3. To create a set of factors to be treated as uncorrelated variables as one approach to handling multi–collinearity in such procedures as multiple regression
4. To validate a scale or index by demonstrating that its constituent items load on the same factor, and to drop proposed scale items which cross–load on more than one factor.
5. To establish that multiple tests measure the same factor, thereby giving justification for administering fewer tests.
6. To identify clusters of cases and/or outliers.
7. To determine network groups by determining which sets of people cluster together.

The so–called *exploratory factor analysis* (EFA) seeks to uncover the underlying structure of a relatively large set of variables. The researcher's à priori assumption is that any indicator may be associated with any factor. This is the most common form of factor analysis. There is no prior theory and one uses factor loadings to intuit the factor structure of the data.

On the other hand, the so–called *confirmatory factor analysis* (CFA) seeks to determine if the number of factors and the loadings of measured (indicator) variables on them conform to what is expected on the basis of pre–established theory. Indicator variables are selected on the basis of prior theory and factor analysis is used to see if they load as predicted on the expected number of factors. The researcher's à priori assumption is that each factor (the number

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analysis) is used to establish that indicators seem to measure the corresponding latent variables, represented by the factors. The researcher proceeds only when the measurement model has been validated. Two or more alternative models (one of which may be the null model) are then compared in terms of *model fit*, which measures the extent to which the covariances predicted by the model correspond to the observed covariances in the data. The so–called modification indices and other coefficients may be used by the researcher to alter one or more models to improve fit.

Advantages of SEM compared to multiple regression include more flexible assumptions (particularly allowing interpretation even in the face of multicollinearity), use of confirmatory factor analysis to reduce measurement error by having multiple indicators per latent variable, the attraction of SEM's graphical modelling interface, the desirability of testing models overall rather than coefficients individually, the ability to test models with multiple dependents, the ability to model mediating variables, the ability to model error terms, the ability to test coefficients across multiple between–subjects groups, and ability to handle difficult data (time series with autocorrelated error, non–normal data, incomplete data).

and labels of which may be specified à priori) is associated with a specified subset of indicator variables. A minimum requirement of confirmatory factor analysis is that one hypothesize beforehand the number of factors in the model, but usually also the researcher will posit expectations about which variables will load on which factors (see, e.g., [KM78b]). The researcher seeks to determine, for instance, if measures created to represent a latent variable really belong together.

The *factor loadings*, also called component loadings in PCA, are the correlation coefficients between the variables (rows) and factors (columns) in the *factor matrix*. Analogous to Pearson's r, the squared factor loading is the percent of variance in that variable explained by the factor. To get the percent of variance in all the variables accounted for by each factor, add the sum of the squared factor loadings for that factor (column) and divide by the number of variables (note that the number of variables equals the sum of their variances as the variance of a standardized variable is 1). This is the same as dividing the factor's eigenvalue by the number of variables.

The *factor scores*, also called component scores in PCA, factor scores are the scores of each case (row) on each factor (column). To compute the factor score for a given case for a given factor, one takes the case's standardized score on each variable, multiplies by the corresponding factor loading of the variable for the given factor, and sums these products. Computing factor scores allows one to look for factor outliers. Also, factor scores may be used as variables in subsequent modelling.

Rotation serves to make the output more understandable and is usually necessary to facilitate the interpretation of factors. The sum of eigenvalues is not affected by rotation, but rotation will alter the eigenvalues (and percent of variance explained) of particular factors and will change the factor loadings. Since alternative rotations may explain the same variance (have the same total eigenvalue) but have different factor loadings, and since factor loadings are used to intuit the meaning of factors, this means that different meanings may be ascribed to the factors depending on the rotation – a problem some cite as a drawback to factor analysis. If factor analysis is used, the researcher may wish to experiment with alternative rotation methods to see which leads to the most interpretable factor structure.

Varimax rotation is an orthogonal rotation of the factor axes to maximize the variance of the squared loadings of a factor (column) on all the variables (rows) in a factor matrix, which has the effect of differentiating the original variables by extracted factor. Each factor will tend to have either large or small loadings of any particular variable. A varimax solution yields results which make it as easy as possible to identify each variable with a single factor. This is the most common rotation option.

The oblique rotations allow the factors to be correlated, and so a factor correlation matrix is generated when oblique is requested. Two most common oblique rotation methods are:

Direct oblimin rotation – the standard method when one wishes a non–orthogonal solution , that is, one in which the factors are allowed to be correlated; this will result in higher eigenvalues but diminished interpretability of the factors; and

Promax rotation rotation – an alternative non–orthogonal rotation method which is computationally faster than the direct oblimin method and therefore is sometimes used for very large datasets.

FA advantages are:

1. Offers a much more objective method of testing intelligence in humans;
2. Allows for a satisfactory comparison between the results of intelligence tests; and
3. Provides support for theories that would be difficult to prove otherwise.

Charles Spearman pioneered the use of factor analysis in the field of psychology and is sometimes credited with the invention of factor analysis. He discovered that schoolchildren's scores on a wide variety of seemingly unrelated subjects were positively correlated, which led him to postulate that a general mental ability, or g, underlies and shapes human cognitive performance. His postulate now enjoys broad support in the field of intelligence research, where it is known as the g theory.

Raymond Cattell expanded on Spearman's idea of a two–factor theory of intelligence after performing his own tests and factor analysis. He used a multi–factor theory to explain intelligence. Cattell's theory addressed alternate factors in intellectual development, including motivation and psychology. Cattell also developed several mathematical methods for adjusting psychometric graphs, such as his 'scree' test and similarity coefficients. His research lead to the development of his theory of fluid and crystallized intelligence. Cattell was a strong advocate of factor analysis and psychometrics. He believed that all theory should be derived from research, which supports the continued use of empirical observation and objective testing to study human intelligence.

Factor Structure and Rotation

Starting with the correlation matrix **R** including the number of significant correlations, the goal of exploratory factor analysis (FA) is to detect latent underlying dimensions (i.e., the factor structure) among the set of all manifest variables. Instead of the correlation matrix, the factor analysis can start from the covariance matrix (see Figure 4), which is the symmetrical matrix with variances of all manifest variables on the main diagonal and their covariances in other matrix cells. For the purpose of the present project the correlation matrix is far more meaningful starting point. Three main applications of factor analytic techniques are (see [CL71, And96, Har75]):

1. to *reduce* the number of manifest variables,
2. to *classify* manifest variables, and
3. to *score* each individual soldier on the latent factor structure.

Factor analysis model expands each of the manifest variables $\mathbf{x}_i$ with the means μ_i from the data matrix $\mathbf{X} = \{x_{i\alpha}\}$ as a linear vector–function

$$\mathbf{x}_i = \mu_i + \mathbf{L}_{ij}\,\mathbf{f}_j + \mathbf{e}_i, \quad (i = 1, \dots, n;\ j = 1, \dots, m) \tag{9.1}$$

where n and m denote the numbers of manifest and latent variables, respectively, $\mathbf{f}_j$ denotes the jth common–factor vector (with zero mean and unity–matrix covariance), $\mathbf{L} = \mathbf{L}_{ij}$ is the matrix of factor loadings l_{ij}, and $\mathbf{e}_i$ corresponds to the ith specific–factor vector (specific variance not explained by the common factors, with zero mean and diagonal–matrix covariance).

That portion of the variance of the ith manifest variable $\mathbf{x}_i$ contributed by the m common factors $\mathbf{f}_j$, the sum of squares of the loadings l_{ij}, is called the ith communality.

Now, in the correlation matrix $\mathbf{R}$ the variances of all variables are equal to 1.0. Therefore, the total variance in that matrix is equal to the number of variables. Extraction of factors is based on the solution of eigenvalue problem, i.e., characteristic equation for the correlation matrix $\mathbf{R}$,

$$\mathbf{R}\mathbf{x}_i = \lambda_i \mathbf{x}_i,$$

where λ_i are eigenvalues of $\mathbf{R}$, representing the variances extracted by the factors, and $\mathbf{x}_i$ now represent the corresponding eigenvectors, representing principal components or factors. The question then is, how many factors do we want to extract? Note that as we extract consecutive factors, they account for less and less variability. The decision of when to stop extracting factors basically depends on when there is only very little 'random' variability left. According to the widely used Kaiser criterion we can retain only factors with eigenvalues greater than 1. In essence this is like saying that, unless a factor extracts at least as much as the equivalent of one original variable, we drop it. The proportion of variance of a particular item that is due to common factors (shared with other items) is called communality. Therefore, an additional task facing us when applying this model is to estimate the communalities for each variable, that is, the proportion of variance that each item has in common with other items. The proportion of variance that is unique to each item is then the respective item's total variance minus the communality. A common starting point is to use the squared multiple correlation of an item with all other items as an estimate of the communality. The correlations between the manifest variables and the principal components are called factor loadings. The first factor is generally more highly correlated with the variables than the second, third and other factors, as these factors are extracted successively and will account for less and less variance overall.

Therefore, the principal component factor analysis of the sample correlation matrix $\mathbf{R}$ is specified in terms of its $m < n$ eigenvalue–eigenvector pairs $(\lambda_j, \mathbf{x}_j)$ where $\lambda_j \geq \lambda_{j+1}$. The matrix of estimated factor loadings l_{ij} is given by

$$\mathbf{L} = \left[\sqrt{\lambda_1}\mathbf{x}_1|\sqrt{\lambda_2}\mathbf{x}_2|\dots|\sqrt{\lambda_m}\mathbf{x}_m\right].$$

Factor extraction can be performed also by other methods, collectively called *principal factors*, including: (i) Maximum likelihood factors, (ii) Principal axis method, (iii) Centroid method, (iv) Multiple R^2-communalities, and (v) Iterated Minres communalities. However, we shall stick on the principal components because of their obvious eigen–structure.

In any case, matrix of factor loadings $\mathbf{L}$ is determined only up to an orthogonal matrix $\mathbf{O}$. The communalities, given by the diagonal elements of $\mathbf{LL}^T$ are also unaffected by the choice of $\mathbf{O}$. This ambiguity provides the rationale for 'factor rotation', since orthogonal matrices correspond to 'coordinate' rotations.

We could plot, theoretically, the factor loadings in a $m-$dimensional scatter–plot. In that plot, each variable is represented as a point. In this plot we could rotate the axes in any direction without changing the relative locations of the points to each other; however, the actual coordinates of the points, that is, the factor loadings would of course change. There are various rotational strategies that have been proposed. The goal of all of these strategies is to get a clear pattern of loadings, that is, factors that are somehow clearly marked by high loadings for some variables and low loadings for others. This general pattern is also sometimes referred to as simple structure (a more formalized definition can be found in most standard textbooks). Typical rotational strategies are Varimax, Quartimax, and Equimax (see [And96]). Basically, the extraction of principal components amounts to a variance maximizing Varimax–rotation of the original space of manifest–variables. We want to get a pattern of loadings on each factor that is as diverse as possible, lending itself to easier interpretation. After we have found the line on which the variance is maximal, there remains some variability around this line. In principal components analysis, after the first factor has been extracted, that is, after the first line has been drawn through the data, we continue and define another line that maximizes the remaining variability, and so on. In this manner, consecutive factors are extracted. Because each consecutive factor is defined to maximize the variability that is not captured by the preceding factor, consecutive factors are independent of each other. Put another way, consecutive factors are uncorrelated or orthogonal to each other.

Basically, the rotation of the matrix of the factor loadings $\mathbf{L}$ represents its post–multiplication, i.e. $\mathbf{L}^* = \mathbf{LO}$ by the rotation matrix $\mathbf{O}$, which itself resembles one of the matrices included in the classical rotational Lie groups $SO(m)$ (containing the specific $m-$fold combination of sinuses and cosinuses. The linear factor equation (9.1) represents the orthogonal factor model, provided that vectors $\mathbf{f}_j$ and $\mathbf{e}_i$ are independent (orthogonal to each other, i.e., having zero covariance).

The most frequently used Kaiser's Normal Varimax rotation procedure selects the orthogonal transformation $\mathbf{T}$ that 'spreads out' the squares of the loadings on each factor as much as possible, i.e., maximizes the total 'squared' variance

$$V = \frac{1}{n}\sum_{j=1}^{m}\left[\sum_{i=1}^{n}(l_{ij}^{*})^4 - \frac{1}{n}\left(\sum_{i=1}^{n}(l_{ij}^{*})^2\right)^2\right],$$

where l_{ij}^{*} denote the rotated factor loadings from the rotated factor matrix $\mathbf{L}^{*}$.

Besides orthogonal rotation, there is another concept of oblique (non-orthogonal, or correlated) factors, which could help to achieve more interpretable simple structure. Specifically, computational strategies have been developed to rotate factors so as to best represent clusters of manifest variables, without the constraint of orthogonality of factors. Oblique rotation produces the factor structure made from the smaller set of mutually correlated factors. An oblique rotation to the simple structure corresponds to *nonrigid* rotation of the factor-axes (i.e., principal components) in the factor space such that the rotated axes $\mathbf{l}_{\mathbf{j}}^{*} = \mathbf{L}_{\mathbf{obl}}^{*}$ (no longer perpendicular) pass (nearly) through the clusters of manifest variables. Although the purest mathematical background does not exist for the non–orthogonal factor rotation, the *parsimony principle*: "explain the maximum of the common variability of the data matrix $\mathbf{X} = \{x_{i\alpha}\}$ with the minimum number of factors", is fully developed only in this form of factor analysis, and the factor–correlation matrix $\mathbf{L}_{\mathbf{obl}}^{*}$ resembles the correlation matrix between manifest variables in the latent, factor space with double–reduced number of observables.

The linear factor equation (9.1) becomes now the *oblique factor model*

$$\mathbf{x}_i = \mu_i + \mathbf{L}_{\mathbf{obl}}^{*}\,\mathbf{f}_j + \mathbf{e}_i, \quad (i = 1, \dots, n;\ j = 1, \dots, m),$$

where the vectors $\mathbf{f}_j$ and $\mathbf{e}_i$ are interdependent (correlated to each other). With oblique rotation, using common procedures, like Kaiser–Harris Orthoblique, Oblimin, Oblimax, Quartimin, Promax (see [And96]), we could

1. perform a hierarchical (iterated) factor analysis, obtaining second–order factors, third–order factors, etc., finishing with a single general factor (for example using principal component analysis of the factor–correlation matrix $\mathbf{L}_{\mathbf{obl}}^{*}$); and
2. develop the so–called 'cybernetic models': when two factors in the factor–correlation matrix $\mathbf{L}_{\mathbf{obl}}^{*}$ are highly correlated we can assume a linear functional link between them; connecting all correlated factors on the certain hierarchical level, we can make a block–diagram out of them depicting a linear system; this is the real point of the *exploratory* factor analysis.

The factor scores $S_{j\alpha}$ (where j labels factors and α labels individual pilot) are incidental parameters that characterize general performance of the individuals (see [CL71, And96, Har75]). Factor scores with zero mean and unity-matrix covariance are usually automatically evaluated in principal–component, orthogonal and oblique factor analysis, according to the formula:

$$S_{j\alpha} = (\mathbf{L}^T\mathbf{L})^{-1}\mathbf{L}^T(x_{j\alpha} - \bar{x}_{j\alpha}),$$

and replacing $\mathbf{L}$ by $\mathbf{L}^*$, and by $\mathbf{L}^*_{\mathbf{obl}}$, respectively. They represent an objective measure of the general performance of pilot on the battery of psycho–tests.[63]

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[63] Here is the Mathematica algorithm for calculating the basic factor structure:
$Mean[x\_] := \frac{Plus@@x}{Length[x]};$
$Variance[x\_] := \frac{Plus@@(mean[x]-x)^2}{Length[x]};$
$StDev[x\_] := \sqrt{Variance[x]};$
$Covar[x1\_, x2\_] := \frac{Plus@@((mean[x1]-x1)((mean[x2]-x2))}{Length[x1]};$
$Corr[x1\_, x2\_] := \frac{Covar[x1,x2]}{StDev[x1]\,StDev[x2]};$
$CorrMat[X\_] := Table[Corr[X[[1,j]], X[[1,i]]]//N, \{i,m\}, \{j,m\}];$
Generate random data–matrix (m variables $\times$ n cases):
$NoVars = 10;\ NoCases = 50;\ m = NoVars;\ n = NoCases;$
$data = Array[x, \{NoCases, NoVars\}]//MatrixForm;$
$Table[x[i,j] = Random[Integer, \{1,5\}], \{i, NoCases\}, \{j, NoVars\}];$
Print[“data = ”,data//MatrixForm];
Calculate correlation matrix:
$R = CorrMat[data];\ Print[$”R=”,$R//MatrixForm]$
Calculate eigenvalues of the correlation matrix:
$\lambda = Eigenvalues[R]//MatrixForm$
Corresponding eigenvectors:
$vec = Eigenvectors[R];\ Print[vec//Transpose//MatrixForm]$
Determine significant principal components
according to the criterion $\lambda \geq 2$:
$Print[$“PRINCIPAL COMPONENTS”
$\to \{vec[[1]], vec[[2]]\}//Transpose//MatrixForm]$
Define operator matrix:
$NoFact = 2;\ P = Array[p, NoVars, NoFact];$
$Table[p[i,j] = 1, \{i, NoVars\}, \{j, NoFact\}];$
$Table[p[i,j] = 0, \{i, 2, NoVars, 2\}, \{j, 2, NoFact, 2\}];$
$Table[p[i,j] = 0, \{i, 1, NoVars, 2\}, \{j, 1, NoFact, 2\}];$
$Print[“P = ”, P//MatrixForm];$
Perform oblique rotation:
$Q = Transpose[P];\ S = R.P;\ G = Q.S;$
$Do[k = \frac{1}{\sqrt{G\|i,i\|}}, \{i, NoFact\}];$
$F = Sk;\ Z = kG;\ C = Zk;$
$L = Inverse[C];\ \Phi = F.L;$
Factor structure matrix:
$Print[“F = ”, F//MatrixForm]$
Inter-factor correlation matrix:
$Print[“C = ”, C//MatrixForm]$
Factor projection matrix:
$Print[“\Phi = ”, \Phi//MatrixForm]$
Calculate factor scores for individual pilot:
$var[x\_] := x - mean[x];$
$Table[v[i] = var[X[[i]]//N], \{i, n\}];$

The factor scores can be used further for multivariate regression in the latent space (instead in the original manifest space) for reducing the number of predictors in the general regression analysis (see [CL71]).

Quantum–Like Correlation and Factor Dynamics

To develop correlation and factor dynamics model, we are using geometrical analogy with *nonrelativistic quantum mechanics* (see, e.g. [II08]). A time dependent state of a quantum system is determined by a normalized (complex), time–dependent, wave psi–function $\psi = \psi(t)$, i.e. a unit Dirac's 'ket' vector $|\psi(t)\rangle$, an element of the Hilbert space $L^2(\psi)$ with a coordinate basis (q^i), under the action of the Hermitian operators, obtained by the procedure of quantization of classical mechanical quantities, for which real eigenvalues are measured. The state–vector $|\psi(t)\rangle$, describing the motion of de Broglie's waves, has a statistical interpretation as the probability amplitude of the quantum system, for the square of its magnitude determines the density of the probability of the system detected at various points of space. The summation over the entire space must yield unity and this is the normalization condition for the psi–function, determining the unit length of the state vector $|\psi(t)\rangle$.

In the coordinate q–representation and the Schrödinger S–picture we consider an action of an evolution operator (in normal units Planck constant $\hbar = 1$)

$$\hat{S} \equiv \hat{S}(t, t_0) = exp[-i\hat{H}(t - t_0)],$$

i.e., a one–parameter Lie–group of unitary transformations evolving a quantum system. The action represents an exponential map of the system's total energy operator – Hamiltonian $\hat{H} = \hat{H}(t)$. It moves the quantum system from one instant of time, t_0, to some future time t, on the state–vector $|\psi(t)\rangle$, rotating it: $|\psi(t)\rangle = \hat{S}(t, t_0)|\psi(t_0)\rangle$. In this case the Hilbert coordinate basis (q^i) is fixed, so the system operators do not evolve in time, and the system evolution is determined exclusively by the time–dependent Schrödinger equation

$$i\partial_t|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle, \qquad (\partial_t = \partial/\partial t), \tag{9.2}$$

with initial condition given at one instant of time t_0 as $|\psi(t_0)\rangle = |\psi\rangle$.

If the Hamiltonian $\hat{H} = \hat{H}(t)$ does not explicitly depend on time (which is the case with the absence of variables of macroscopic fields), the state vector reduces to the exponential of the system energy:

$$|\psi(t)\rangle = exp(-iE(t - t_0)|\psi\rangle,$$

satisfying the time–independent (i.e., stationary) Schrödinger equation

$$\hat{H}|\psi\rangle = E|\psi\rangle, \tag{9.3}$$

$T_F = Transpose[F]$; $F_F = Inverse[T_F.F].T_F$;
$Table[F_F.v[i], \{i, n\}]//MatrixForm.$

which represents the characteristic equation for the Hamiltonian operator $\hat{H}$ and gives its real eigenvalues (stationary energy states) E_n and corresponding orthonormal eigenfunctions (i.e., probability amplitudes) $|\psi_n\rangle$.

To model the correlation and factor dynamics we start with the characteristic equation for the correlation matrix

$$\mathbf{R}\mathbf{x} = \lambda\mathbf{x},$$

making heuristic analogy with the stationary Schrödinger equation (9.3). This analogy allows a 'physical' interpretation of the correlation matrix $\mathbf{R}$ as an operator of the 'total correlation or covariation energy' of the statistical system (the simulator–test data matrix $\mathbf{X} = \{x_{i\alpha}\}$), eigenvalues λ_n corresponding to the 'stationary energy states', and eigenvectors $\mathbf{x}_n$ resembling 'probability amplitudes' of the system.

So far we have considered one instant of time t_0. Including the time–flow into the stationary Schrödinger equation (9.3) we get the time–dependent Schrödinger equation (9.2) and returning back with our heuristic analogy, we get the basic equation of the n–dimensional correlation dynamics

$$\partial_t\mathbf{x}(t) = \mathbf{R}(t)\,\mathbf{x}_k(t), \tag{9.4}$$

with initial condition at time t_0 given as a stationary manifest–vectors $\mathbf{x}_k(t_0) = \mathbf{x}_k$ $(k = 1, \dots, n)$.

In more realistic case of 'many' observables (i.e., very big n), instead of the correlation dynamics (9.4), we can use the reduced–dimension factor dynamics, represented by analogous equation in the factor space spanned by the extracted (oblique) factors $\mathbf{F} = \mathbf{f}_i$, with inter–factor–correlation matrix $\mathbf{C} = c_{ij}$ $(i, j = 1, \dots, \text{no. of factors})$

$$\partial_t\mathbf{f}_i(t) = \mathbf{C}(t)\,\mathbf{f}_i(t), \tag{9.5}$$

subject to initial condition at time t_0 given as stationary vectors $\mathbf{f}_i(t_0) = \mathbf{f}_i$.

Now, according to the fundamental existence and uniqueness theorem for linear autonomous ordinary differential equations, if $A = A(t)$ is an $n \times n$ real matrix, then the initial value problem

$$\partial_t\mathbf{x}(t) = A\mathbf{x}(t), \qquad \mathbf{x}(0) = \mathbf{x}_0 \in \mathbb{R}^n,$$

has the unique solution

$$\mathbf{x}(t) = \mathbf{x}_0 \mathrm{e}^{tA}, \qquad \text{for all } t \in \mathbb{R}.$$

Therefore, analytical solutions of our correlation and factor–correlation dynamics equations (9.4) and (9.5) are given respectively by exponential maps

$$\mathbf{x}_k(t) = \mathbf{x}_k$$
$$exp[t\,\mathbf{R}],$$
$$\mathbf{f}_i(t) = \mathbf{f}_i$$
$$exp[t\,\mathbf{C}].$$

Thus, for each $t \in \mathbb{R}$, the matrix $\mathbf{x}$
$exp[t\,\mathbf{R}]$, respectively the matrix $\mathbf{f}$
$exp[t\,\mathbf{C}]$, maps

$$\mathbf{x}_k \mapsto \mathbf{x}_k exp[t\,\mathbf{R}], \qquad \text{respectively} \qquad \mathbf{f}_i \mapsto \mathbf{f}_i exp[t\,\mathbf{C}].$$

The sets $g^t_{corr} = \{$
$exp[t\,\mathbf{R}]\}_{t\in\mathbb{R}}$ and $g^t_{fact} = \{$
$exp[t\,\mathbf{C}]\}_{t\in\mathbb{R}}$ are 1–parameter families (groups) of linear maps of $\mathbb{R}^n$ into $\mathbb{R}^n$, representing the *correlation flow*, respectively the *factor–correlation flow* of simulator–tests. The linear flows g^t (representing both g^t_{corr} and g^t_{fact}) have two essential properties:

1. identity map: $g^0 = I$, and
2. composition: $g^{t_1+t_2} = g^{t_1} \circ g^{t_2}$.

They partition the state space $\mathbb{R}^n$ into subsets that we call 'correlation orbits', respectively 'factor–correlation orbits', through the initial states $\mathbf{x}_k$, and $\mathbf{f}_i$, of simulator tests, defined respectively by

$$\gamma(\mathbf{x}_k) = \{\mathbf{x}_k g^t | t \in \mathbb{R}\} \qquad \text{and} \qquad \gamma(\mathbf{f}_i) = \{\mathbf{f}_i g^t | t \in \mathbb{R}\}.$$

The correlation orbits can be classified as:

1. If $g^t\mathbf{x}_k = \mathbf{x}_k$ for all $t \in \mathbb{R}$, then $\gamma(\mathbf{x}_k) = \{\mathbf{x}_k\}$ and it is called a *point orbit*. Point orbits correspond to equilibrium points in the manifest and the factor space, respectively.
2. If there exists a $T > 0$ such that $g^T\mathbf{x}_k = \mathbf{x}_k$, then $\gamma(\mathbf{x}_k)$ is called a *periodic orbit*. Periodic orbits describe a system that evolves periodically in time in the manifest and the factor space, respectively.
3. If $g^t\mathbf{x}_k \neq \mathbf{x}_k$ for all $t \neq 0$, then $\gamma(\mathbf{x}_k)$ is called a *non–periodic orbit*.

Analogously, the factor–correlation orbits can be classified as:

1. If $g^t\mathbf{f}_i = \mathbf{f}_i$ for all $t \in \mathbb{R}$, then $\gamma(\mathbf{f}_i) = \{\mathbf{f}_i\}$ and it is called a point orbit. Point orbits correspond to equilibrium points in the manifest and the factor space, respectively.
2. If there exists a $T > 0$ such that $g^T\mathbf{f}_i = \mathbf{f}_i$, then $\gamma(\mathbf{f}_i)$ is called a periodic orbit. Periodic orbits describe a system that evolves periodically in time in the manifest and the factor space, respectively.
3. If $g^t\mathbf{f}_i \neq \mathbf{f}_i$ for all $t \neq 0$, then $\gamma(\mathbf{f}_i)$ is called a non–periodic orbit.

Now, to interpret properly the meaning of (really discrete) time in the correlation matrix $\mathbf{R} = \mathbf{R}(t)$ and factor–correlation matrix $\mathbf{C} = \mathbf{C}(t)$, we can perform a successive time–series $\{t, t + \Delta t, t + 2\Delta t, t + k\Delta t, \cdots\}$ of simulator tests (and subsequent correlation and factor analysis), and discretize our correlation (respectively, factor–correlation) dynamics, to get

$$\mathbf{x}_k(t + \Delta t) = \mathbf{x}_k(0) + \mathbf{R}(t)\,\mathbf{x}_k(t)\Delta t, \qquad \text{and}$$
$$\mathbf{f}_i(t + \Delta t) = \mathbf{f}_i(0) + \mathbf{C}(t)\,\mathbf{f}_i(t)\Delta t,$$

respectively. Finally we can represent the discrete correlation and factor–correlation dynamics in the form of the (computationally applicable) *three–point iterative dynamics equation*, respectively in the manifest space

$$\mathbf{x}_k^{s+1} = \mathbf{x}_k^{s-1} + \mathbf{R}_k^s\,\mathbf{x}_k^s,$$

and in the factor space

$$\mathbf{f}_i^{s+1} = \mathbf{f}_i^{s-1} + \mathbf{C}_i^s\,\mathbf{f}_i^s,$$

in which the time–iteration variable s labels the time occurrence of the simulator tests (and subsequent correlation and factor analysis), starting with the initial state, labelled $s = 0$.

FA Definition of Intelligence

In the psychometric view, the concept of intelligence is most closely identified with Spearman's **g**, or Gf ('fluid **g**'). However, psychometricians can measure a wide range of abilities, which are distinct yet correlated. One common view is that these abilities are hierarchically arranged with **g** at the vertex (or top, overlaying all other cognitive abilities).[64]

On the other hand, critics of the psychometric approach, such as Robert Sternberg from Yale, point out that people in the general population have a somewhat different conception of intelligence than most experts. In turn, they argue that the psychometric approach measures only a part of what is commonly understood as intelligence. Other critics, such as Arthur

[64] Intelligence, IQ, and **g** are distinct terms. As already said above, intelligence is the term used in ordinary discourse to refer to cognitive ability. However, it is generally regarded as too imprecise to be useful for a scientific treatment of the subject. The intelligence quotient (IQ) is an index calculated from the scores on test items judged by experts to encompass the abilities covered by the term intelligence. IQ measures a multidimensional quantity: it is an amalgam of different kinds of abilities, the proportions of which may differ between IQ tests. The dimensionality of IQ scores can be studied by factor analysis, which reveals a single dominant factor underlying the scores on all IQ tests. This factor, which is a hypothetical construct, is called **g**. Variation in **g** corresponds closely to the intuitive notion of intelligence, and thus **g** is sometimes called *general cognitive ability* or *general intelligence*.

Eddington,[65] argue that the equipment used in an experiment often determines the results and that proving that e.g., intelligence exists does not prove that current equipment measure it correctly. Sceptics often argue that so much scientific knowledge about the brain is still to be discovered that claiming the conventional IQ test methodology to be infallible is just a small step forward from claiming that *craniometry*[66] was the infallible method for measuring intelligence (which had scientific merits based on knowledge available in the nineteenth century).

A more fundamental criticism is that both the psychometric model used in these studies and the conceptualization of cognitive ability itself are fundamentally off beam. These views were expressed by none other than Charles Spearman, the ‘discoverer’ of **g** – himself. Thus he wrote: “Every normal man, woman, and child is a genius at something. It remains to discover at what. This must be a most difficult matter, owing to the very fact that it occurs in only a minute proportion of all possible abilities. It certainly cannot be detected by any of the testing procedures at present in current usage. But these procedures are capable, I believe, of vast improvement.” In this context he noted that it is more important to ask ‘What does this person think about?’ than ‘How well can he or she think?’ Spearman went on to observe that the tests from which his **g** had emerged had no place in schools since they did not reflect the diverse talents of the children and thus deflected teachers from their fundamental educational role, which is to nurture and recognize these diverse talents.

He also noted, as paraphrased here, that the so–called ‘cognitive ability’ is not primarily cognitive but affective and conative. In constructing meaning out of confusion (Spearman’s eductive ability) one first follows feelings that beckon or attract. One then has to engage in ‘experimental interactions with the environment’ to check out those, largely non–verbal, ‘hunches’. This requires determination and persistence — *conation*. Now, all of these are difficult and demanding activities which will only be undertaken whilst one is undertaking activities one cares about. So the first question is: ‘What kinds of activity is this person strongly motivated to undertake’ (and the kinds of activity which people may be strongly motivated to undertake are legion and mostly unrelated to those assessed in conventional ‘intelligence’ tests). And the second question is: ‘How many of the cumulative and substitutable components of competence required to carry out these activities effectively

[65] Sir Arthur Stanley Eddington, OM (December 28, 1882 — November 22, 1944) was an astrophysicist of the early 20th century. The Eddington limit, the natural limit to the luminosity that can be radiated by accretion onto a compact object, is named in his honor. He is famous for his work regarding the Theory of Relativity. Eddington wrote an article in 1919, Report on the relativity theory of gravitation, which announced Einstein’s theory of general relativity to the English–speaking world.

[66] Craniometry is the technique of measuring the bones of the skull. Craniometry was once intensively practiced in anthropology/ethnology.

does this person display whilst carrying out that activity?' So one cannot, in reality, assess a person's intelligence, or even their eductive ability, except in relation to activities they care about. What one sees in e.g., the Raven Progressive Matrices is the cumulative effect of how well they do all these things in relation to a certain sort of task. The problem is that this is not — and cannot be — 'cognitive ability' in any general sense of the word but only in relation to this kind of task. As Roger Sperry[67] has observed, what is neurologically localized is not 'cognitive ability' in any general sense but the emotional predisposition to 'think' about a particular kind of thing (for more details, see e.g., papers of John Raven[68] [Rav02]).

Most experts accept the concept of a single dominant factor of intelligence, general mental ability or **g**, while others argue that intelligence consists of a set of relatively independent abilities [APS98]. The evidence for **g** comes from factor analysis of tests of cognitive abilities. The methods of factor analysis do not guarantee a single dominant factor will be discovered. Other *psychological tests*, which do not measure cognitive ability, such as *personality tests*, generate multiple factors.

Proponents of *multiple–intelligence theories* often claim that **g** is, at best, a measure of academic ability. Other types of intelligence, they claim, might be just as important outside of a school setting. Robert Sternberg has proposed a 'Triarchic Theory of Intelligence'. Howard Gardner's theory of multiple intelligences breaks intelligence down into at least eight different components: logical, linguistic, spatial, musical, kinesthetic, naturalist, intra–personal and inter–personal intelligences. Daniel Goleman and several other researchers

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[67] Roger Wolcott Sperry (August 20, 1913 – April 17, 1994) was a neuropsychologist who, together with David Hunter Hubel and Torsten Nils Wiesel, won the 1981 Nobel Prize in Medicine for his work with *split–brain* research. Before Sperry's experiments, some research evidence seemed to indicate that areas of the brain were largely undifferentiated and interchangeable. In his early experiments Sperry challenged this view by showing that after early development circuits of the brain are largely hardwired. In his Nobel–winning work, Sperry separated the *corpus callosum*, the area of the brain used to transfer signals between the right and left hemispheres, to treat epileptics. Sperry and his colleagues then tested these patients with tasks that were known to be dependent on specific hemispheres of the brain and demonstrated that the two halves of the brain may each contain consciousness. In his words, each hemisphere is "... indeed a conscious system in its own right, perceiving, thinking, remembering, reasoning, willing, and emoting, all at a characteristically human level, and . . . both the left and the right hemisphere may be conscious simultaneously in different, even in mutually conflicting, mental experiences that run along in parallel." This research contributed greatly to understanding the lateralization of brain functions. In 1989, Sperry also received National Medal of Science.

[68] John Carlyle Raven first published his Progressive Matrices in the United Kingdom in 1938. His three sons established Scotland–based test publisher JC Raven Ltd. in 1972. In 2004, Harcourt Assessment, Inc. a division of Harcourt Education acquired JC Raven Ltd.

have developed the concept of *emotional intelligence* and claim it is at least as important as more traditional sorts of intelligence. These theories grew from observations of human development and of brain injury victims who demonstrate an acute loss of a particular cognitive function (e.g., the ability to think numerically, or the ability to understand written language), without showing any loss in other cognitive areas.

In response, **g** theorists have pointed out that **g**'s *predictive validity*[69] has been repeatedly demonstrated, for example in predicting important non–academic outcomes such as job performance, while no multiple–intelligences theory has shown comparable validity. Meanwhile, they argue, the relevance, and even the existence, of multiple intelligences have not been borne out when actually tested [Hun01]. Furthermore, **g** theorists contend that proponents of multiple–intelligences (see, e.g., [Ste95]) have not disproved the existence of a general factor of intelligence [Kli00]. The fundamental argument for a general factor is that test scores on a wide range of seemingly unrelated cognitive ability tests (such as sentence completion, arithmetic, and memorization) are positively correlated: people who score highly on one test tend to score highly on all of them, and g thus emerges in a factor analysis. This suggests that the tests are not unrelated, but that they all tap a common factor.

9.2.3 Cognitive Versus Not–Cognitive Intelligence

Clearly, biologically realized 'cognitive intelligence' is the most complex property of human mind and can be perceived only by itself. Our problem is what we call or may call cognitive intelligence. From the formal, computational perspective, cognitive intelligence is one of ill defined concepts. Its definitions are immersed in numerous scientific contexts and mirrors their historical evolutions, as well as, different 'interests' of researchers. Its weakness is usually based on its abstract multifaces image and, on the other hand, a universal utility character.

[69] In psychometrics, *predictive validity* is the extent to which a scale predicts scores on some criterion measure. For example, the validity of a cognitive test for job performance is the correlation between test scores and, say, supervisor performance ratings. Such a cognitive test would have predictive validity if the observed correlation were statistically significant. Predictive validity shares similarities with concurrent validity in that both are generally measured as correlations between a test and some criterion measure. In a study of concurrent validity the test is administered at the same time as the criterion is collected. This is a common method of developing validity evidence for employment tests: A test is administered to incumbent employees, then a rating of those employees' job performance is obtained (often, as noted above, in the form of a supervisor rating). Note the possibility for restriction of range both in test scores and performance scores: The incumbent employees are likely to be a more homogeneous and higher performing group than the applicant pool at large.

The classical behavioral/biologists definition of intelligence reads: "Intelligence is the ability to adapt to new conditions and to successfully cope with life situations." This definition seems to be the best, but 'intelligence' here depends on available physical tools and specific life experience (individual hidden knowledge, preferences and access to information), therefore it is not enough selective to be measured, compared or designed. In general, cognitive intelligence is a human–like intelligence. Unfortunately there are many opinions what human–like intelligence means. For example, (i) cognitive intelligence uses a human mental introspective experience for the modelling of intelligent system thinking; and (ii) cognitive intelligence may use brain models to extract brain's intelligence property.

Therefore, cognitive intelligence can be seen as a product of human self–conscious recognition of efficient mental processes, defined a'priori as intelligent. In order to get a consensus on the notion of cognitive intelligence is useful to have an agreement on which intelligence is not cognitive. A not–cognitive intelligence could be considered as an intelligence being developed using not human analogies; e.g., it is possible to construct very different models of flying objects starting from the observation of storks, balloons, beetles or clouds – maybe this observation can be useful.

The difference between human and artificial intelligence theories is similar to the difference between a birds theory of fly and the airplanes fly theory, the both can lead to a more general theory of fly but this last needs a goal–oriented and a higher abstraction level of the conceptualization/ontology.

According to the *TOGA meta–theory paradigms*,[70] for scientific and practical modelling purposes, it is reasonable to separate conceptually the following five concepts: *information, knowledge, preferences, intelligence and emotions.* If properly defined, all of them can be independently identified and designed.

Such conceptual modularity should enable to construct: *emotional intelligence, social intelligence, skill intelligence, organizational intelligence,* and many other X–intelligences, where X denotes a type of knowledge, preferences or a carrier system involved.

For example, business intelligence and emotional intelligence, rather are applications of intelligence either for business activities or for the second, under emotional/(not conscious) constrains and 'biological requests'.

[70] According to the *top–down object–based goal–oriented approach* (TOGA) standard, the Information–Preferences–Knowledge *cognitive architecture* consists of:

Data: everything what is/can be processed/transformed in computational and mental processes. Concept data is included in the ontology of 'elaborators', such as developers of methods, programmers and other computation service people. In this sense, data is a relative term and exists only in the couple (data, processing).

Information: data which represent a specific property of the domain of human or artificial agent's activity (such as: addresses, tel. numbers, encyclopedic data, various lists of names and results of measurements). Every information has always a source domain. It is a relative concept. Information is a concept from the ontology of modeler/problem–solver/decision–maker.

In the above context, an *abstract intelligent agent* can be considered as a functional kernel of any natural or artificial intelligent system.

9.2.4 Intelligence and Cognitive Development

Although there is no general *theory of cognitive development*, the most historically influential theory was developed by Jean Piaget.[71] *Piaget theory*

Knowledge: every abstract property of human/artificial agent which has ability to process/transform a quantitative/qualitative information into other information, or into another knowledge. It includes: instructions, emergency procedures, exploitation/user manuals, scientific materials, models and theories. Every knowledge has its reference domain where it is applicable. It has to include the source domain of the processed information. It is a relative concept.

Preference: an ordered relation among two properties of the domain of activity of a *cognitive agent*, it indicates a property with higher utility. Preference relations serve to establish an intervention goal of an agent. Cognitive preferences are relative. A preference agent which manages preferences of an intelligent agent can be external or its internal part.

Goal: a hypothetical state of the domain of activity which has maximal utility in a current situation. Goal serves to the choice and activate proper knowledge which process new information.

Document: a passive carrier of knowledge, information and/or preferences (with different structures), comprehensive for humans, and it has to be recognized as valid and useful by one or more human organizations, it can be physical or electronic.

Computer Program: (i) from the modelers and decision-makers perspective: an active carrier of different structures of knowledge expressed in computer languages and usually focused on the realization of predefined objectives (a design-goal). It may include build-in preferences and information and/or request specific IPK as data. (ii) from the software engineers perspective: a data-processing tool (more precise technical def. you may find on the Web).

[71] Jean Piaget (August 9, 1896 – September 16, 1980) was a Swiss natural scientist and developmental psychologist, well known for his work studying children and his theory of cognitive development. Piaget served as professor of psychology at the University of Geneva from 1929 to 1975 and is best known for reorganizing cognitive development theory into a series of stages, expanding on earlier work from James Baldwin: four levels of development corresponding roughly to (1) infancy, (2) pre–school, (3) childhood, and (4) adolescence. Each stage is characterized by a general cognitive structure that affects all of the child's thinking (a structuralist view influenced by philosopher Immanuel Kant). Each stage represents the child's understanding of reality during that period, and each but the last is an inadequate approximation of reality. Development from one stage to the next is thus caused by the accumulation of errors in the child's understanding of the environment; this accumulation eventually causes such a degree of cognitive disequilibrium that thought structures require reorganising. For his development of the theory, Piaget was awarded the Erasmus Prize.

provided many central concepts in the field of developmental psychology. His theory concerned the growth of intelligence, which for Piaget meant the ability to more accurately represent the world, and perform logical operations on representations of concepts grounded in the world. His theory concerns the emergence and acquisition of schemata, schemes of how one perceives the world, in 'developmental stages', times when children are acquiring new ways of mentally representing information. Piaget theory is considered 'constructivist, meaning that, unlike nativist theories (which describe cognitive development as the unfolding of innate knowledge and abilities) or empiricist theories (which describe cognitive development as the gradual acquisition of knowledge through experience), asserts that we construct our cognitive abilities through self–motivated action in the world.

Piaget divided schemes that children use to understand the world through four main stages, roughly correlated with and becoming increasingly sophisticated with age:

The four development stages are described in Piaget's theory as:

1. Sensorimotor stage: from birth to age 2 years (children experience the world through movement and senses)
2. Preoperational stage: from ages 2 to 7(acquisition of motor skills)
3. Concrete operational stage: from ages 7 to 11 (children begin to think logically about concrete events)
4. Formal Operational stage: after age 11 (development of abstract reasoning).

These chronological periods are approximate, and in light of the fact that studies have demonstrated great variation between children, cannot be seen as rigid norms. Furthermore, these stages occur at different ages, depending upon the domain of knowledge under consideration. The ages normally given for the stages, then, reflect when each stage tends to predominate, even though one might elicit examples of two, three, or even all four stages of thinking at the same time from one individual, depending upon the domain of knowledge and the means used to elicit it. Despite this, though, the principle holds that within a domain of knowledge, the stages usually occur in the same chronological order. Thus, there is a somewhat subtler reality behind the normal characterization of the stages as described above. The reason for the invariability of sequence derives from the idea that knowledge is not simply acquired from outside the individual, but it is constructed from within. This idea has been extremely influential in pedagogy, and is usually termed constructivism. Once knowledge is constructed internally, it is then tested against reality the same way a scientist tests the validity of hypotheses. Like a scientist, the individual learner may discard, modify, or reconstruct knowledge based on its utility in the real world. Much of this construction (and later reconstruction) is in fact done subconsciously. Therefore, Piaget's four stages actually reflect four types of thought structures. The chronological sequence is inevitable, then, because one structure may be necessary in order to construct the next level, which is simpler, more generalizable, and more powerful. It's a little like saying that you need to form metal into parts in order to build machines, and then coordinate machines in order to build a factory.

1. Sensorimotor stage (years 0–2),
2. Preoperational stage (years 2–7),
3. Concrete operational stage (years 7–11), and
4. Formal operational stage (years 11–adulthood).

Sensorimotor Stage

Infants are born with a set of congenital reflexes, according to Piaget, as well as a drive to explore their world. Their initial schemas are formed through differentiation of the congenital reflexes (see assimilation and accommodation, below).

The sensorimotor stage is the first of the four stages. According to Piaget, this stage marks the development of essential spatial abilities and understanding of the world in six sub–stages:

1. The first sub–stage occurs from birth to six weeks and is associated primarily with the development of reflexes. Three primary reflexes are described by Piaget: sucking of objects in the mouth, following moving or interesting objects with the eyes, and closing of the hand when an object makes contact with the palm (palmar grasp). Over these first six weeks of life, these reflexes begin to become voluntary actions; for example, the palmar reflex becomes intentional grasping.
2. The second sub–stage occurs from six weeks to four months and is associated primarily with the development of habits. Primary circular reactions or repeating of an action involving only ones own body begin. An example of this type of reaction would involve something like an infant repeating the motion of passing their hand before their face. Also at this phase, passive reactions, caused by classical or operant conditioning, can begin.
3. The third sub–stage occurs from four to nine months and is associated primarily with the development of coordination between vision and prehension. Three new abilities occur at this stage: intentional grasping for a desired object, secondary circular reactions, and differentiations between ends and means. At this stage, infants will intentionally grasp the air in the direction of a desired object, often to the amusement of friends and family. Secondary circular reactions, or the repetition of an action involving an external object begin; for example, moving a switch to turn on a light repeatedly. The differentiation between means also occurs. This is perhaps one of the most important stages of a child's growth as it signifies the dawn of logic. Towards the late part of this sub–stage infants begin to have a sense of object permanence, passing the A–not–B error test.
4. The fourth sub-stage occurs from nine to twelve months and is associated primarily with the development of logic and the coordination between means and ends. This is an extremely important stage of development, holding what Piaget calls the 'first proper intelligence'. Also, this stage marks the beginning of goal orientation, the deliberate planning of steps to meet an objective.

5. The fifth sub–stage occurs from twelve to eighteen months and is associated primarily with the discovery of new means to meet goals. Piaget describes the child at this juncture as the 'young scientist', conducting pseudo–experiments to discover new methods of meeting challenges.
6. The sixth sub–stage is associated primarily with the beginnings of insight, or true creativity. This marks the passage into the preoperational stage.

Preoperational Stage
The Preoperational stage is the second of four stages of cognitive development. By observing sequences of play, Piaget was able to demonstrate that towards the end of the second year a qualitatively quite new kind of psychological functioning occurs. Operation in Piagetian theory is any procedure for mentally acting on objects. The hallmark of the preoperational stage is sparse and logically inadequate mental operations.

According to Piaget, the Sensorimotor stage of development is followed by this stage (2–7 years), which includes the following five processes:

1. Symbolic functioning, which is characterised by the use of mental symbols words or pictures which the child uses to represent something which is not physically present.
2. Centration, which is characterized by a child focusing or attending to only one aspect of a stimulus or situation. For example, in pouring a quantity of liquid from a narrow beaker into a shallow dish, a preschool child might judge the quantity of liquid to have decreased, because it is 'lower', that is, the child attends to the height of the water, but not to the compensating increase in the diameter of the container.
3. Intuitive thought, which occurs when the child is able to believe in something without knowing why she or he believes it.
4. Egocentrism, which is a version of centration, this denotes a tendency of child to only think from own point of view.
5. Inability to Conserve; Through Piaget's conservation experiments (conservation of mass, volume and number) Piaget concluded that children in the preoperational stage lack perception of conservation of mass, volume, and number after the original form has changed. For example, a child in this phase will believe that a string of beads set up in a 'O–O–O–O–O' pattern will have the same number of beads as a string which has a 'O–O–O–O–O' pattern, because they are the same length, or that a tall, thin 8-ounce cup has more liquid in it than a wide, fat 8–ounce cup.

Concrete Operational Stage
The concrete operational stage is the third of four stages of cognitive development in Piaget's theory. This stage, which follows the Preoperational stage and occurs from the ages of 7 to 11, is characterized by the appropriate use of logic. The six important processes during this stage are:

1. Decentering, where the child takes into account multiple aspects of a problem to solve it. For example, the child will no longer perceive an

exceptionally wide but short cup to contain less than a normally-wide, taller cup.
2. Reversibility, where the child understands that numbers or objects can be changed, then returned to their original state. For this reason, a child will be able to rapidly determine that $4 + 4$ which they can answer to be 8, minus 4 will equal four, the original quantity.
3. Conservation: understanding that quantity, length or number of items is unrelated to the arrangement or appearance of the object or items. For instance, when a child is presented with two equally–sized, full cups they will be able to discern that if water is transferred to a pitcher it will conserve the quantity and be equal to the other filled cup.
4. Serialisation: the ability to arrange objects in an order according to size, shape, or any other characteristic. For example, if given different–shaded objects they may make a color gradient.
5. Classification: the ability to name and identify sets of objects according to appearance, size or other characteristic, including the idea that one set of objects can include another. A child is no longer subject to the illogical limitations of animism (the belief that all objects are animals and therefore have feelings).
6. Elimination of Egocentrism: the ability to view things from another's perspective (even if they think incorrectly). For instance, show a child a comic in which Jane puts a doll under a box, leaves the room, and then Jill moves the doll to a drawer, and Jane comes back; a child in this stage will not say that Jane will think the doll is in the drawer.

Formal Operational Stage

The formal operational stage is the fourth and final of the stages of cognitive development of Piaget's theory. This stage, which follows the Concrete Operational stage, commences at around 11 years of age (puberty) and continues into adulthood. It is characterized by acquisition of the ability to think abstractly and draw conclusions from the information available. During this stage the young adult functions in a cognitively normal manner and therefore is able to understand such things as love, 'shades of gray', and values. Lucidly, biological factors may be traced to this stage as it occurs during puberty and marks the entering into adulthood in physiologically, cognitive, moral (Kohlberg), psychosexual (Freud), and social development (Erikson). Many people do not successfully complete this stage, but mostly remain in concrete operations.

9.2.5 Psychophysics

Recall that *psychophysics* is a subdiscipline of psychology, founded in 1860 by Gustav Fechner[72] with the publication of 'Elemente der Psychophysik',

[72] Gustav Theodor Fechner (April 19, 1801 – November 28, 1887), was a German experimentle psychologist. A pioneer in experimental psychology.

dealing with the relationship between physical stimuli and their subjective correlates, or percepts. Fechner described research relating physical stimuli with how they are perceived and set out the philosophical foundations of the field. Fechner wanted to develop a theory that could relate matter to the mind, by describing the relationship between the world and the way it is perceived (Snodgrass, 1975). Fechner's work formed the basis of psychology as a science. Wilhelm Wundt, the founder of the first laboratory for psychological research, built upon Fechner's work.

The *Weber–Fechner law* attempts to describe the relationship between the physical magnitudes of stimuli and the perceived intensity of the stimuli. Ernst Weber[73] was one of the first people to approach the study of the human response to a physical stimulus in a quantitative fashion. Gustav Fechner later offered an elaborate theoretical interpretation of Weber's findings, which

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Fechner's epoch–faking work was his Elemente der Psychophysik (1860). He starts from the Spinozistic thought that bodily facts and conscious facts, though not reducible one to the other, are different sides of one reality. His originality lies in trying to discover an exact mathematical relation between them. The most famous outcome of his inquiries is the law known as *Weber–Fechner law* which may be expressed as follows: "In order that the intensity of a sensation may increase in arithmetical progression, the stimulus must increase in geometrical progression." Though holding good within certain limits only, the law has been found immensely useful. Unfortunately, from the tenable theory that the intensity of a sensation increases by definite additions of stimulus, Fechner was led on to postulate a unit of sensation, so that any sensations might be regarded as composed of n units. Sensations, he argued, thus being representable by numbers, psychology may become an 'exact' science, susceptible of mathematical treatment.

His general formula for getting at the number of units in any sensation is $S = c \log R$, where S stands for the sensation, R for the stimulus numerically estimated, and c for a constant that must be separately determined by experiment in each particular order of sensibility. This reasoning of Fechner's has given rise to a great mass of controversy, but the fundamental mistake in it is simple. Though stimuli are composite, sensations are not. "Every sensation," says William James, "presents itself as an indivisible unit; and it is quite impossible to read any clear meaning into the notion that they are masses of units combined." Still, the idea of the exact measurement of sensation has been a fruitful one, and mainly through his influence on Wilhelm Wundt, Fechner was the father of that 'new' psychology of laboratories which investigates human faculties with the aid of exact scientific apparatus.

[73] Ernst Heinrich Weber (Wittenberg, June 24, 1795 – January 26, 1878) was a German physician who is considered a founder of experimental psychology. Weber studied medicine at Wittenberg University. In 1818 he was appointed Associate Professor of comparative anatomy at Leipzig University, where he was made a Fellow Professor of anatomy and physiology in 1821.

Around 1860 Weber worked with Gustav Fechner on psychophysics, during which time he formulated Weber's Law. In 1866 Weber retired as professor of physiology and also as professor of anatomy in 1871. Around this time he and his brother, Eduard Weber, descovered the inhibitory power of the vagus nerve.

he called simply Weber's law, though his admirers made the law's name a hyphenate. Fechner believed that Weber had discovered the fundamental principle of mind/body interaction, a mathematical analog of the function Rene Descartes once assigned to the pineal gland.

In one of his classic experiments, Weber gradually increased the weight that a blindfolded man was holding and asked him to respond when he first felt the increase. Weber found that the response was proportional to a relative increase in the weight. That is to say, if the weight is 1 kg, an increase of a few grams will not be noticed. Rather, when the mass is increased by a certain factor, an increase in weight is perceived. If the mass is doubled, the threshold is also doubled. This kind of relationship can be described by a linear ordinary differential equation as,

$$dp = k\frac{dS}{S},$$

where dp is the differential change in perception, dS is the differential increase in the stimulus and S is the stimulus at the instant. A constant factor k is to be determined experimentally. Integrating the above equation gives: $p = k \ln S + C$, where C is the constant of integration. To determine C, we can put $p = 0$, which means no perception; then we get, $C = -k \ln S_0$,where S_0 is that threshold of stimulus below which it is not perceived at all. In this way, we get the solution

$$p = k \ln \frac{S}{S_0}.$$

Therefore, the relationship between stimulus and perception is logarithmic. This logarithmic relationship means that if a stimulus varies as a geometric progression (i.e. multiplied by a fixed factor), the corresponding perception is altered in an arithmetic progression (i.e. in additive constant amounts). For example, if a stimulus is tripled in strength (i.e, 3×1), the corresponding perception may be two times as strong as its original value (i.e., $1 + 1$). If the stimulus is again tripled in strength (i.e., $3 \times 3 \times 1$), the corresponding perception will be three times as strong as its original value (i.e., $1 + 1 + 1$). Hence, for multiplications in stimulus strength, the strength of perception only adds. This logarithmic relationship is valid, not just for the sensation of weight, but for other stimuli and our sensory perceptions as well.

In case of vision, we have that the eye senses brightness logarithmically. Hence stellar magnitude is measured on a logarithmic scale. This magnitude scale was invented by the ancient Greek astronomer Hipparchus in about 150 B.C. He ranked the stars he could see in terms of their brightness, with 1 representing the brightest down to 6 representing the faintest, though now the scale has been extended beyond these limits. An increase in 5 magnitudes corresponds to a decrease in brightness by a factor 100.

In case of sound, we have still another logarithmic scale is the decibel scale of sound intensity. And yet another is pitch, which, however, differs from the other cases in that the physical quantity involved is not a 'strength'. In the case of perception of pitch, humans hear pitch in a logarithmic or geometric ratio–based fashion: For notes spaced equally apart to the human ear, the frequencies are related by a multiplicative factor. For instance, the frequency of corresponding notes of adjacent octaves differ by a factor of 2. Similarly, the perceived difference in pitch between 100 Hz and 150 Hz is the same as between 1000 Hz and 1500 Hz. Musical scales are always based on geometric relationships for this reason. Notation and theory about music often refers to pitch intervals in an additive way, which makes sense if one considers the logarithms of the frequencies, as $\log(a \times b) = \log a + \log b$.

Psychophysicists usually employ experimental stimuli that can be objectively measured, such as pure tones varying in intensity, or lights varying in luminance. All the senses have been studied:vision, hearing, touch (including skin and enteric perception), taste, smell, and the sense of time. Regardless of the sensory domain, there are three main topics in the psychophysical classification scheme: absolute thresholds, discrimination thresholds, and scaling.

The most common use of psychophysics is in producing scales of human experience of various aspects of physical stimuli. Take for an example the physical stimulus of frequency of sound. Frequency of a sound is measured in Hertz (Hz), cycles per second. But human experience of the frequencies of sound is not the same as the frequencies. For one thing, there is a frequency below which no sounds can be heard, no matter how intense they are (around 20 Hz depending on the individual) and there is a frequency above which no sounds can be heard, no matter how intense they are (around 20,000 Hz, again depending on the individual). For another, doubling the frequency of a sound (e.g., from 100 Hz to 200 Hz) does not lead to a doubling of experience. The perceptual experience of the frequency of sound is called pitch, and it is measured by psychophysicists in mels.

More analytical approaches allow the use of psychophysical methods to study neurophysiological properties and sensory processing mechanisms. This is of particular importance in human research, where other (more invasive) methods are not used due to ethical reasons. Areas of investigation include sensory thresholds, methods of measurement of sensitivity, and signal detection theory.

Perception is the process of acquiring, interpreting, selecting, and organizing sensory information. Methods of studying perception range from essentially biological or physiological approaches, through psychological approaches to the often abstract 'thought–experiments' of mental philosophy.

Experiments in psychophysics seek to determine whether the subject can detect a stimulus, identify it, differentiate between it and another stimulus, and describe the magnitude or nature of this difference [Sno75]. Often, the classic methods of experimentation are argued to be inefficient. This is because a lot of sampling and data has to be collected at points of the psychometric function that is known (the tails). Staircase procedures can be used to quickly estimate threshold. However, the cost of this efficiency, is that we do not get the same amount of information regarding the *psychometric function* as we can through classical methods; e.g., we cannot extract an estimate of the slope (derivative) of the function.

A psychometric function describes the relationship between a parameter of a physical stimulus and the responses of a person who has to decide about a certain aspect of that stimulus. The psychometric function usually resembles a sigmoid function with the percentage of correct responses (or a similar value) displayed on the ordinate and the physical parameter on the abscissa. If the stimulus parameter is very far towards one end of its possible range, the person will always be able to respond correctly. Towards the other end of the range, the person never perceives the stimulus properly and therefore the probability of correct responses is at chance level. In between, there is a transition range where the subject has an above–chance rate of correct responses, but does not always respond correctly. The inflection point of the sigmoid function or the point at which the function reaches the middle between the chance level and 100% is usually taken as sensory threshold. A common example is visual acuity testing with an eye chart. The person sees symbols of different sizes (the size is the relevant physical stimulus parameter) and has to decide which symbol it is. Usually, there is one line on the chart where a subject can identify some, but not all, symbols. This is equal to the transition range of the psychometric function and the sensory threshold corresponds to visual acuity.

On the other hand, a *sensory threshold* is a theoretical concept which states: "A stimulus that is less intense than the sensory threshold will not elicit any sensation." Whilst the concept can be applied to all senses, it is most commonly applied to the detection and perception of flavours and aromas. Several different sensory thresholds have been defined:

1. Absolute threshold: the lowest level at which a stimulus can be detected.
2. Recognition threshold: the level at which a stimulus can not only be detected but also recognised.
3. Differential threshold: the level at which an increase in a detected stimulus can be perceived.
4. Terminal threshold: the level beyond which a stimulus is no longer detected.

In other words, a threshold is the point of intensity at which the participant can just detect the presence of, or difference in, a stimulus. Stimuli with intensities below the threshold are considered not detectable, however stimuli at values close to threshold will often be detectable some proportion of the time. Due to this, a threshold is considered to be the point at which a stimulus, or change in a stimulus, is detected some proportion p of the time. An absolute threshold is the level of intensity of a stimulus at which the subject is able to detect the presence of the stimulus some proportion of the time (a p level of 50% is often used). An example of an absolute threshold is the number of hairs on the back of one's hand that must be touched before it can be felt, a participant may be unable to feel a single hair being touched, but may be able to feel two or three as this exceeds the threshold. A difference threshold is the magnitude of the difference between two stimuli of differing intensities that the participant is able to detect some proportion of the time (again, 50% is often used). To test this threshold, several difference methods are used. The subject may be asked to adjust one stimulus until it is perceived as the same as the other, may be asked to describe the magnitude of the difference between two stimuli, or may be asked to detect a stimulus against a background. Absolute and difference thresholds are sometimes considered similar because there is always background noise interfering with our ability to detect stimuli, however study of difference thresholds still occurs, for example in pitch discrimination tasks (see [Sno75]).

The *sensory analysis* applies principles of experimental design and statistical analysis to the use of human senses (sight, smell, taste, touch and hearing) for the purposes of evaluating consumer products. The discipline requires panels of human assessors, on whom the products are tested, and recording the responses made by them. By applying statistical techniques to the results it is possible to make inferences and insights about the products under test. Most large consumer goods companies have departments dedicated to sensory analysis. Sensory Analysis can generally be broken down into three sub–sections:

1. Effective Testing (dealing with objective facts about products);
2. Affective Testing (dealing with subjective facts such as preferences); and
3. Perception (the biochemical and psychological aspects of sensation).

The *signal detection theory* (SDT) is a means to quantify the ability to discern between signal and noise. It has applications in many fields such as quality control, telecommunications, and psychology (see [Abd06]). The concept is similar to the signal to noise ratio used in the sciences, and it is also usable in alarm management, where it is important to separate important events from background noise. According to the theory, there are a number of psychological determiners of how we will detect a signal, and where our threshold levels will be. Experience, expectations, physiological state (e.g, fatigue) and other factors affect thresholds. For instance, a sentry in wartime will likely detect fainter stimuli than the same sentry in peacetime. SDT is used when psychologists want to measure the way we make decisions under conditions of uncertainty, such as how we would perceive distances in foggy conditions. SDT assumes that 'the decision maker is not a passive receiver of information, but an active decision–maker who makes difficult perceptual judgements under conditions of uncertainty'. In foggy circumstances, we are forced to decide how far an object is away from us based solely upon visual stimulus which is impaired by the fog. Since the brightness of the object, such as a traffic light, is used by the brain to discriminate the distance of an object, and the fog reduces the brightness of objects, we perceive the object to be much further away than it actually is. To apply signal detection theory to a data set where stimuli were either present or absent, and the observer categorized each trial as having the stimulus present or absent, the trials are sorted into one of four categories, depending upon the stimulus and response:

	Respond 'Absent'	Respond 'Present'
Stimulus Present	Miss	Hit
Stimulus Absent	Correct Rejection	False Alarm

9.2.6 Human Problem Solving

Beginning in the 1970s, researchers became increasingly convinced that empirical findings and theoretical concepts derived from simple laboratory tasks did not necessarily generalize to more complex, real–life problems. Even worse, it appeared that the processes underlying creative problem solving in different domains differed from each other [Ste95]. These realizations have led to rather different responses in North America and in Europe.

In particular, George Pólya's 1945 book 'How to Solve It' [Pol45], is a small volume describing methods of problem–solving. It suggests the following steps when solving a mathematical problem:

1. First, you have to understand the problem.
2. After understanding, then make a plan.
3. Carry out the plan.
4. Look back on your work. How could it be better?

If this technique fails, Polya advises: "If you cannot solve a problem, then there is an easier problem you can solve: find it." Or, "If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem?"

His small book contains a dictionary–style set of heuristics, many of which have to do with generating a more accessible problem, like the ones given in the table on next page.

The technique 'have I used everything' is perhaps most applicable to formal educational examinations (e.g., n men digging m ditches, see footnote below) problems. The book has achieved 'classic' status because of its considerable influence. Marvin Minsky[74] said in his influential paper 'Steps Toward Artificial Intelligence': "And everyone should know the work of George Polya on how to solve problems." Polya's book has had a large influence on mathematics textbooks. Most formulations of a problem solving framework in U.S. textbooks attribute some relationship to Polya's problem solving stages. Other books on problem solving are often related to less concrete and more creative

[74] Marvin Lee Minsky (born August 9, 1927), sometimes affectionately known as 'Old Man Minsky', is an American cognitive scientist in the field of artificial intelligence (AI), co–founder of MIT's AI laboratory, and author of several texts on AI and philosophy.

Heuristic	**Informal Description**	**Formal analogue**
Analogy	can you find a problem analogous to your problem and solve that?	Map
Generalization	can you find a problem more general than your problem...?	Generalization
Induction	can you solve your problem by deriving a generalization from some examples?	Induction
Variation of the Problem	can you vary or change your problem to create a new problem (or set of problems) whose solution(s) will help you solve your original problem?	Search
Auxiliary Problem	can you find a subproblem or side problem whose solution will help you solve your problem?	Subgoal
Here is a problem related to yours and solved before	can you find a problem related to yours that has already been solved and use that to solve your problem?	Pattern recognition Pattern matching
Specialization	can you find a problem more specialized?	Specialization
Decomposing and Recombining	can you decompose the problem and "recombine its elements in some new manner"?	Divide and conquer
Working backward	can you start with the goal and work backwards to something you already know?	Backward chaining
Draw a Figure	can you draw a picture of the problem?	Diagrammatic Reasoning
Auxiliary Elements	can you add some new element to your problem to get closer to a solution?	Extension

techniques, like e.g., lateral thinking, mind mapping and brainstorming (see below).

To sum up, researchers' realization that problem–solving processes differ across knowledge domains and across levels of expertise (see, e.g. [Ste95]) and that, consequently, findings obtained in the laboratory cannot necessarily generalize to problem–solving situations outside the laboratory, has during the past two decades led to an emphasis on real–world problem solving. This emphasis has been expressed quite differently in North America and Europe, however. Whereas North American research has typically concentrated on studying problem solving in separate, natural knowledge domains, much of the European research has focused on novel, complex problems, and has been performed with computerized scenarios (see [Fun95], for an overview).

Characteristics of Difficult Problems

As elucidated by Dietrich Dorner and later expanded upon by Joachim Funke, difficult problems have some typical characteristics. Recategorized and somewhat reformulated from these original works, these characteristics can be summarized as follows:

Intransparency (lack of clarity of the situation), including commencement opacity and continuation opacity;

Polytely (multiple goals), including inexpressiveness, opposition and transience;

Complexity (large numbers of items, interrelations, and decisions), including enumerability, connectivity (hierarchy relation, communication relation, allocation relation), and heterogeneity;

Dynamism (time considerations), including temporal constraints, temporal sensitivity, phase effects, and dynamic unpredictability.

The resolution of difficult problems requires a direct attack on each of these characteristics that are encountered.

Some *standard problem–solving techniques*, also known as creativity techniques, include:

1. Trial–and–error;[75]

[75] Trial and error (also known in computer science literature as generate and test and as 'guess and check' when solving equations in elementary algebra) is a method of problem solving for obtaining knowledge, both propositional knowledge and know-how.

This approach can be seen as one of the two basic approaches to problem solving and is contrasted with an approach using insight and theory.

In trial and error, one selects (or, generates) a possible answer, applies it to the problem and, if it is not successful, selects (or generates) another possibility that is subsequently tried. The process ends when a possibility yields a solution.

In some versions of trial and error, the option that is a priori viewed as the most likely one should be tried first, followed by the next most likely, and so on until a solution is found, or all the options are exhausted. In other versions, options are simply tried at random.

This approach is most successful with simple problems and in games, and is often resorted to when no apparent rule applies. This does not mean that the approach need be careless, for an individual can be methodical in manipulating the variables in an effort to sort through possibilities that may result in success. Nevertheless, this method is often used by people who have little knowledge in the problem area.

Trial and error has a number of features:

solution-oriented: trial and error makes no attempt to discover why a solution works, merely that it is a solution.

problem-specific: trial and error makes no attempt to generalize a solution to other problems.

non-optimal: trial and error is an attempt to find a solution, not all solutions, and not the best solution.

2. Brainstorming;[76]

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needs little knowledge: trial and error can proceed where there is little or no knowledge of the subject.

For example, trial and error has traditionally been the main method of finding new drugs, such as antibiotics. Chemists simply try chemicals at random until they find one with the desired effect.

The *scientific method* can be regarded as containing an element of trial and error in its formulation and testing of hypotheses. Also compare *genetic algorithms*, *simulated annealing* and *reinforcement learning* – all varieties of search which apply the basic idea of trial and error.

Biological Evolution is also a form of trial and error. Random mutations and sexual genetic variations can be viewed as trials and poor reproductive fitness as the error. Thus after a long time 'knowledge' of well–adapted genomes accumulates simply by virtue of them being able to reproduce.

Bogosort can be viewed as a trial and error approach to sorting a list.

In mathematics the method of trial and error can be used to solve formulae – it is a slower, less precise method than algebra, but is easier to understand.

[76] Brainstorming is a creativity technique of generating ideas to solve a problem. The main result of a brainstorm session may be a complete solution to the problem, a list of ideas for an approach to a subsequent solution, or a list of ideas resulting in a plan to find a solution. Brainstorming was originated in 1953 in the book 'Applied Imagination' by Alex Osborn, an advertising executive. Other methods of generating ideas are individual ideation and the morphological analysis approach.

Brainstorming has many applications but it is most often used in:

New product development – obtaining ideas for new products and improving existing products

Advertising – developing ideas for advertising campaigns

Problem solving – issues, root causes, alternative solutions, impact analysis, evaluation

Process management – finding ways of improving business and production processes

Project Management – identifying client objectives, risks, deliverables, work packages, resources, roles and responsibilities, tasks, issues

Team building – generates sharing and discussion of ideas while stimulating participants to think

Business planning – develop and improve the product idea.

Trial preparation by attorneys.

Brainstorming can be done either individually or in a group. In group brainstorming, the participants are encouraged, and often expected, to share their ideas with one another as soon as they are generated. Complex problems or brainstorm sessions with a diversity of people may be prepared by a chairman. The chairman is the leader and facilitator of the brainstorm session.

The key to brainstorming is to not interrupt the thought process. As ideas come to mind, they are captured and stimulate the development of better ideas. Thus a group brainstorm session is best conducted in a moderate–sized room, and participants sit so that they can all look at each–other. A flip chart, blackboard,

3. Morphological box;[77]
4. Method of focal objects;[78]
5. Lateral thinking;[79]

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or overhead projector is placed in a prominent location. The room is free of telephones, clocks, or any other distractions.

[77] Morphological analysis was designed for multi-dimensional, non-quantifiable problems where causal modelling and simulation do not function well or at all. Fritz Zwicky developed this approach to seemingly non-reducible complexity [Zwi69]. Using the technique of cross consistency assessment (CCA) [Rit02], the system however does allow for reduction, not by reducing the number of variables involved, but by reducing the number of possible solutions through the elimination of the illogical solution combinations in a grid box.

[78] The technique of *focal objects* for problem solving involves synthesizing the seemingly non–matching characteristics of different objects into something new.

For example, to generate new solutions to gardening take some ideas at random, such swimming and a couch, and invent ways for them to merge. Swimming might be used with the idea of gardening to create a plant oxygen tank for underwater divers. A couch might be used with the idea of gardening to invent new genes that would grow plants into the shape of a couch. The larger the number of diverse objects included, the greater the opportunity for inventive solutions.

Another way to think of focal objects is as a memory cue: if you're trying to find all the different ways to use a brick, give yourself some random 'objects' (situations, concepts, etc.) and see if you can find a use. Given 'blender', for example, I would try to think of all the ways a brick could be used with a blender (as a lid?). Another concept for the brick game: find patterns in your solutions, and then break those patterns. If you keep finding ways to build things with bricks, think of ways to use bricks that don't involve construction. Pattern–breaking, combined with focal object cues, can lead to very divergent solutions.

[79] Lateral thinking is a term coined by Edward de Bono [Bon73], a Maltese psychologist, physician, and writer, although it may have been an idea whose time was ready; the notion of lateral truth is discussed by Robert M. Pirsig in Zen and the Art of Motorcycle Maintenance. de Bono defines Lateral Thinking as methods of thinking concerned with changing concepts and perception. For example:

It took two hours for two men to dig a hole five feet deep. How deep would it have been if ten men had dug the hole for two hours?

The answer appears to be 25 feet deep. This answer assumes that the thinker has followed a simple mathematical relationship suggested by the description given, but we can generate some lateral thinking ideas about what affects the size of the hole which may lead to different answers:

A hole may need to be of a certain size or shape so digging might stop early at a required depth.

The deeper a hole is, the more effort is required to dig it, since waste soil needs to be lifted higher to the ground level. There is a limit to how deep a hole can be dug by manpower without use of ladders or hoists for soil removal, and 25 feet is beyond this limit.

Deeper soil layers may be harder to dig out, or we may hit bedrock or the water table.

Each man digging needs space to use a shovel.

6. Mind mapping;[80]

It is possible that with more people working on a project, each person may become less efficient due to increased opportunity for distraction, the assumption he can slack off, more people to talk to, etc.

More men could work in shifts to dig faster for longer.

There are more men but are there more shovels?

The two hours dug by ten men may be under different weather conditions than the two hours dug by two men.

Rain could flood the hole to prevent digging.

Temperature conditions may freeze the men before they finish.

Would we rather have 5 holes each 5 feet deep?

The two men may be an engineering crew with digging machinery.

What if one man in each group is a manager who will not actually dig?

The extra eight men might not be strong enough to dig, or much stronger than the first two.

The most useful ideas listed above are outside the simple mathematics implied by the question. Lateral thinking is about reasoning that is not immediately obvious and about ideas that may not be obtainable by using only traditional step–by–step logic.

Techniques that apply lateral thinking to problems are characterized by the shifting of thinking patterns away from entrenched or predictable thinking to new or unexpected ideas. A new idea that is the result of lateral thinking is not always a helpful one, but when a good idea is discovered in this way it is usually obvious in hindsight, which is a feature lateral thinking shares with a joke.

Lateral thinking can be contrasted with critical thinking, which is primarily concerned with judging the truth value of statements and seeking error. Lateral Thinking is more concerned with the movement value of statements and ideas, how to move from them to other statements and ideas.

For example the statement 'cars should have square wheels' when considered with critical thinking would be evaluated as a poor suggestion, as there are many engineering problems with square wheels. The Lateral Thinking treatment of the same statement would be to see where it leads. Square wheels would produce predictable bumps. If bumps can be predicted then suspension can be designed to compensate. Another way to predict bumps would be a laser or sonar on the front of the car examining the road surface ahead. This leads to the idea of active suspension with a sensor on the car that has normal wheels. The initial statement has been left behind.

[80] Recall that a *mind map* is a diagram used to represent words, ideas, tasks or other items linked to and arranged radially around a central key word or idea. It is used to generate, visualize, structure and classify ideas, and as an aid in study, organization, problem solving, and decision making.

It is an image–centered diagram that represents semantic or other connections between portions of information. By presenting these connections in a radial, nonlinear graphical manner, it encourages a brainstorming approach to any given organizational task, eliminating the hurdle of initially establishing an intrinsically appropriate or relevant conceptual framework to work within.

A mind map is similar to a semantic network or cognitive map but there are no formal restrictions on the kinds of links used.

7. Analogy with similar problems;[81] and

Most often the map involves images, words, and lines. The elements are arranged intuitively according to the importance of the concepts and they are organized into groupings, branches, or areas. The uniform graphic formulation of the semantic structure of information on the method of gathering knowledge, may aid recall of existing memories.

People have been using image centered radial graphic organization techniques referred to variably as mental or generic mind maps for centuries in areas such as engineering, psychology, and education, although the claim to the origin of the mind map has been made by a British popular psychology author, Tony Buzan.

The mind map continues to be used in various forms, and for various applications including learning and education (where it is often taught as 'Webs' or 'Webbing'), planning and in engineering diagramming.

When compared with the earlier original concept map (which was developed by learning experts in the 1960s) the structure of a mind map is a similar, but simplified, radial by having one central key word.

Mind maps have many applications in personal, family, educational, and business situations, including note-taking, brainstorming (wherein ideas are inserted into the map radially around the center node, without the implicit prioritization that comes from hierarchy or sequential arrangements, and wherein grouping and organizing is reserved for later stages), summarizing, revising and general clarifying of thoughts. For example, one could listen to a lecture and take down notes using mind maps for the most important points or keywords. One can also use mind maps as a mnemonic technique or to sort out a complicated idea. Mind maps are also promoted as a way to collaborate in color pen creativity sessions.

[81] Recall that analogy is either the cognitive process of transferring information from a particular subject (the analogue or source) to another particular subject (the target), or a linguistic expression corresponding to such a process. In a narrower sense, analogy is an inference or an argument from a particular to another particular, as opposed to deduction, induction, and abduction, where at least one of the premises or the conclusion is general. The word analogy can also refer to the relation between the source and the target themselves, which is often, though not necessarily, a similarity, as in the biological notion of analogy.

Niels Bohr's model of the atom made an analogy between the atom and the solar system.Analogy plays a significant role in problem solving, decision making, perception, memory, creativity, emotion, explanation and communication. It lies behind basic tasks such as the identification of places, objects and people, for example, in face perception and facial recognition systems. It has been argued that analogy is 'the core of cognition'. Specifically analogical language comprises exemplification, comparisons, metaphors, similes, allegories, and parables, but not metonymy. Phrases like and so on, and the like, as if, and the very word like also rely on an analogical understanding by the receiver of a message including them. Analogy is important not only in ordinary language and common sense, where proverbs and idioms give many examples of its application, but also in science, philosophy and the humanities. The concepts of association, comparison, correspondence, homomorphism, iconicity, isomorphism, mathematical homology, metaphor, morphological homology, resemblance, and similarity are closely

8. Research;[82]

9.2.7 Human Mind

Recll that the word *mind* commonly refers to the collective aspects of *intellect* and *consciousness* which are manifest in some combination of *thought*, *perception*, *emotion*, *will*, *memory*, and *imagination*.

related to analogy. In cognitive linguistics, the notion of conceptual metaphor may be equivalent to that of analogy.

Analogy has been studied and discussed since classical antiquity by philosophers, scientists and lawyers. The last few decades have shown a renewed interest in analogy, most notable in cognitive science.

With respect to the terms source and target, there are two distinct traditions of usage:

The logical and mathematical tradition speaks of an arrow, homomorphism, mapping, or morphism from what is typically the more complex domain or source to what is typically the less complex codomain or target, using all of these words in the sense of mathematical category theory.

The tradition that appears to be more common in cognitive psychology, literary theory, and specializations within philosophy outside of logic, speaks of a mapping from what is typically the more familiar area of experience, the source, to what is typically the more problematic area of experience, the target.

[82] Research is often described as an active, diligent, and systematic process of inquiry aimed at discovering, interpreting, and revising facts. This intellectual investigation produces a greater understanding of events, behaviors, or theories, and makes practical applications through laws and theories. The term research is also used to describe a collection of information about a particular subject, and is usually associated with science and the scientific method.

The word research derives from Middle French; its literal meaning is 'to investigate thoroughly'.

Thomas Kuhn, in his book 'The Structure of Scientific Revolutions', traces an interesting history and analysis of the enterprize of research.

Basic research (also called fundamental or pure research) has as its primary objective the advancement of knowledge and the theoretical understanding of the relations among variables. It is exploratory and often driven by the researcher's curiosity, interest, or hunch. It is conducted without any practical end in mind, although it may have unexpected results pointing to practical applications. The terms "basic" or "fundamental" indicate that, through theory generation, basic research provides the foundation for further, sometimes applied research. As there is no guarantee of short-term practical gain, researchers often find it difficult to get funding for basic research. Research is a subset of invention.

Applied research is done to solve specific, practical questions; its primary aim is not to gain knowledge for its own sake. It can be exploratory, but is usually descriptive. It is almost always done on the basis of basic research. Applied research can be carried out by academic or industrial institutions. Often, an academic institution such as a university will have a specific applied research program funded by an industrial partner interested in that program. Common areas of applied research include electronics, informatics, computer science, material science, process engineering, drug design...

There are many theories of what the mind is and how it works, dating back to Plato, Aristotle and other Ancient Greek philosophers. Modern theories, based on a scientific understanding of the brain, see the mind as a phenomenon of psychology, and the term is often used more or less synonymously with *consciousness*.

The question of which human attributes make up the mind is also much debated. Some argue that only the 'higher' intellectual functions constitute mind: particularly reason and memory. In this view the emotions – love, hate, fear, joy – are more 'primitive' or subjective in nature and should be seen as different in nature or origin to the mind. Others argue that the rational and the emotional sides of the human person cannot be separated, that they are of the same nature and origin, and that they should all be considered as part of the individual mind.

In popular usage *mind* is frequently synonymous with *thought*: It is that private conversation with ourselves that we carry on 'inside our heads' during every waking moment of our lives. Thus we 'make up our minds,' or 'change our minds' or are 'of two minds' about something. One of the key attributes of the mind in this sense is that it is a private sphere. No–one else can 'know our mind.' They can only know what we communicate.

Both philosophers and psychologists remain divided about the nature of the mind. Some take what is known as the substantial view, and argue that the mind is a single entity, perhaps having its base in the brain but distinct from it and having an autonomous existence. This view ultimately derives from Plato, and was absorbed from him into Christian thought. In its most extreme form, the substantial view merges with the theological view that the mind is an entity wholly separate from the body, in fact a manifestation of the soul, which will survive the body's death and return to God, its creator.

Others take what is known as the functional view, ultimately derived from Aristotle, which holds that the mind is a term of convenience for a variety of mental functions which have little in common except that humans are conscious of their existence. Functionalists tend to argue that the attributes which we collectively call the mind are closely related to the functions of the brain and can have no autonomous existence beyond the brain, nor can they survive its death. In this view mind is a subjective manifestation of consciousness: the human brain's ability to be aware of its own existence. The concept of the mind is therefore a means by which the conscious brain understands its own operations.

A leading exponent of the *substantial view* at the mind was George Berkeley, an 18th century Anglican bishop and philosopher. Berkeley argued that there is no such thing as matter and what humans see as the material world is nothing but an idea in God's mind, and that therefore the human mind is purely a manifestation of the soul or spirit. This type of belief is also common in certain types of spiritual non–dualistic belief, but outside this field few philosophers take an extreme view today. However, the view that the human mind is of a nature or essence somehow different from, and higher than, the mere operations of the brain, continues to be widely held.

Berkeley's views were attacked, and in the eyes of many philosophers demolished, by T.H. Huxley,[83] a 19th century biologist and disciple of Charles Darwin,[84] who agreed that the phenomena of the mind were of a unique order, but argued that they can only be explained in reference to events in the brain. Huxley drew on a tradition of materialist thought in British philosophy dating to Thomas Hobbes,[85] who argued in the 17th century that mental events were ultimately physical in nature, although with the biological knowledge of his day he could not say what their physical basis was. Huxley blended Hobbes with Darwin to produce the modern *functional view*.

[83] Thomas Henry Huxley, FRS (4 May 1825 – 29 June 1895) was an English biologist, known as 'Darwin's Bulldog' for his defence of Charles Darwin's theory of evolution. His scientific debates against Richard Owen demonstrated that there were close similarities between the cerebral anatomy of humans and gorillas. Huxley did not accept many of Darwin's ideas, such as gradualism and was more interested in advocating a materialist professional science than in defending natural selection.

A talented populariser of science, he coined the term 'agnosticism' to describe his stance on religious belief. He is credited with inventing the concept of 'biogenesis', a theory stating that all cells arise from other cells and also 'abiogenesis', describing the generation of life from non–living matter.

[84] Charles Robert Darwin (12 February 1809 – 19 April 1882) was an English naturalist who achieved lasting fame by producing considerable evidence that species originated through evolutionary change, at the same time proposing the scientific theory that natural selection is the mechanism by which such change occurs. This theory is now considered a cornerstone of biology.

Darwin developed an interest in natural history while studying first medicine, then theology, at university. Darwin's observations on his five-year voyage on the Beagle brought him eminence as a geologist and fame as a popular author. His biological finds led him to study the transmutation of species and in 1838 he conceived his theory of natural selection. Fully aware that others had been severely punished for such 'heretical' ideas, he confided only in his closest friends and continued his research to meet anticipated objections. However, in 1858 the information that Alfred Wallace had developed a similar theory forced an early joint publication of the theory.

His 1859 book 'On the Origin of Species by Means of Natural Selection' established evolution by common descent as the dominant scientific explanation of diversification in nature.

[85] Thomas Hobbes (April 5, 1588–December 4, 1679) was an English philosopher, whose famous 1651 book Leviathan set the agenda for nearly all subsequent Western political philosophy. Although Hobbes is today best remembered for his work on *political philosophy*, he contributed to a diverse array of fields, including history, geometry, ethics, general philosophy and what would now be called political science. Additionally, Hobbes's account of human nature as self–interested cooperation has proved to be an enduring theory in the field of philosophical anthropology.

Huxley's view was reinforced by the steady expansion of knowledge about the functions of the human brain. In the 19th century it was not possible to say with certainty how the brain carried out such functions as memory, emotion, perception and reason. This left the field open for substantialists to argue for an autonomous mind, or for a metaphysical theory of the mind. But each advance in the study of the brain during the 20th century made this harder, since it became more and more apparent that all the components of the mind have their origins in the functioning of the brain. Huxley's rationalism, was disturbed in the early 20th century by Freudian a theory of the unconscious mind, and argued that those mental processes of which humans are subjectively aware are only a small part of their total mental activity.

More recently, Douglas Hofstadter's[86] 1979 Pulitzer Prize–winning book 'Gödel, Escher, Bach – an eternal Golden Braid', is a *tour de force* on the subject of mind, and how it might arise from the neurology of the brain. Amongst other biological and cybernetic phenomena, Hofstadter places tangled loops and recursion at the center of self, self–awareness, and perception of oneself, and thus at the heart of mind and thinking. Likewise philosopher Ken Wilber posits that Mind is the interior dimension of the brain holon, i.e., mind is what a brain looks like internally, when it looks at itself.

Quantum physicist David Bohm[87] had a theory of mind that is most comparable to Neo–Platonic theories. "Thought runs you. Thought, however, gives false info that you are running it, that you are the one who controls

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[86] Douglas Richard Hofstadter (born February 15, 1945 in New York, New York) is an American academic, the son of Nobel Prize–winning physicist Robert Hofstadter. He is probably best known for his book Gödel, Escher, Bach: an Eternal Golden Braid (abbreviated as GEB) which was published in 1979, and won the 1980 Pulitzer Prize for general non-fiction. This book is commonly considered to have inspired many students to begin careers in computing and artificial intelligence, and attracted substantial notice outside its central artificial intelligence readership owing to its drawing on themes from such diverse disciplines as high-energy physics, music, the visual arts, molecular biology, and literature.

[87] David Joseph Bohm (born December 20, 1917 in Wilkes–Barre, Pennsylvania, died October 27, 1992 in London) was an American–born quantum physicist, who made significant contributions in the fields of theoretical physics, philosophy and neuropsychology, and to the Manhattan Project.

Bohm made a number of significant contributions to physics, particularly in the area of quantum mechanics and relativity theory. While still a post-graduate at Berkeley, he developed a theory of plasmas, discovering the electron phenomenon now known as Bohm–diffusion. His first book, Quantum Theory published in 1951, was well–received by Einstein, among others. However, Bohm became dissatisfied with the orthodox approach to quantum theory, which he had written about in that book, and began to develop his own approach (Bohm interpretation), a non–local hidden variable deterministic theory whose predictions agree perfectly with the nondeterministic quantum theory. His work and the EPR argument became the major factor motivating John Bell's inequality, whose consequences are still being investigated.

thought. Whereas actually thought is the one which controls each one of us...” [Boh92].

The debate about the nature of the mind is relevant to the development of artificial intelligence (see next section). If the mind is indeed a thing separate from or higher than the functioning of the brain, then presumably it will not be possible for any machine, no matter how sophisticated, to duplicate it. If on the other hand the mind is no more than the aggregated functions of the brain, then it will be possible, at least in theory, to create a machine with a mind.

Bohm also made significant theoretical contributions to neuropsychology and the development of the so–called *holonomic brain model*. In collaboration with Stanford neuroscientist Karl Pribram, Bohm helped establish the foundation for Pribram's theory that the brain operates in a manner similar to a hologram, in accordance with quantum mathematical principles and the characteristics of wave patterns. These wave forms may compose hologram–like organizations, Bohm suggested, basing this concept on his application of *Fourier analysis*, a mathematical method for decomposing complex waves into component sine waves. The holonomic brain model developed by Pribram and Bohm posits a lens defined world view, much like the textured prismatic effect of sunlight refracted by the churning mists of a rainbow, a view which is quite different from the more conventional ‘objective’ approach. Pribram believes that if psychology means to understand the conditions that produce the world of appearances, it must look to the thinking of physicists like Bohm.

Bohm proposes thus in his book ‘Thought as a System’ a pervasive, systematic nature of thought: “What I mean by ‘thought’ is the whole thing – thought, ‘felt’, the body, the whole society sharing thoughts – it's all one process. It is essential for me not to break that up, because it's all one process; somebody else's thoughts becomes my thoughts, and vice versa. Therefore it would be wrong and misleading to break it up into my thoughts, your thoughts, my feelings, these feelings, those feelings... I would say that thought makes what is often called in modern language a system. A system means a set of connected things or parts. But the way people commonly use the word nowadays it means something all of whose parts are mutually interdependent – not only for their mutual action, but for their meaning and for their existence. A corporation is organized as a system – it has this department, that department, that department. They do not have any meaning separately; they only can function together. And also the body is a system. Society is a system in some sense. And so on. Similarly, thought is a system. That system not only includes thoughts and feelings, but it includes the state of the body; it includes the whole of society – as thought is passing back and forth between people in a process by which thought evolved from ancient times. A system is constantly engaged in a process of development, change, evolution and structure changes... although there are certain features of the system which become relatively fixed. We call this the structure... Thought has been constantly evolving and we can't say when that structure began. But with the growth of civilization it has developed a great deal. It was probably very simple thought before civilization, and now it has become very complex and ramified and has much more incoherence than before...

Currently, the Mind/Brain/Behavior Interfaculty Initiative (MBB) at Harvard University aims to elucidate the structure, function, evolution, development, and pathology of the nervous system in relation to human behavior and mental life. It draws on the departments of psychology, neurobiology, neurology, molecular and cellular biology, radiology, psychiatry, organismic and evolutionary biology, history of science, and linguistics.

On the other hand, human brain has been considered (by E.M. Izhikevich, Editor of the new Encyclopedia of Computational Neuroscience) as a *weakly–connected neural network*, with possibly *chaotic behavior* [Izh99b], consisting of n quasi–periodic cortical oscillators $X_1, ..., X_n$ forced by the thalamic input X_0 (see Figure 9.2)

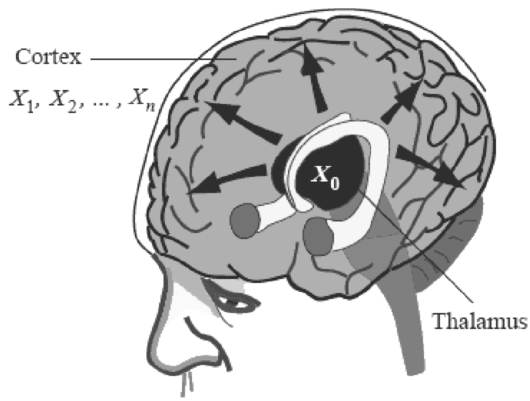

Fig. 9.2 A *possibly chaotic* 1–to–many relation: *Thalamus* ⇒ *Cortex* in the human brain (with permission from E. Izhikevich)

9.2.8 The Mind–Body Problem

The *mind–body problem* is essentially the problem of explaining the relationship between minds, or mental processes, and bodily states or processes (see, e.g., [Kim95a]). Our perceptual experiences depend on stimuli which arrive at our various sensory organs from the external world and that these stimuli cause changes in the states of our brain, ultimately causing us to feel a sensation which may be pleasant or unpleasant. Someone's desire for a slice of pizza will tend to cause that person to move their body in a certain manner in a certain direction in an effort to get what they want. But how is it possible that conscious experiences can arise out of an inert lump of gray matter endowed with electrochemical properties? [Kim95b]. How does someone's desire cause that individual's neurons to fire and his muscles to contract in exactly the right manner? These are some of the essential puzzles that have confronted philosophers of mind at least from the time of René Descartes.[88]

[88] René Descartes (March 31, 1596 – February 11, 1650), also known as Cartesius, was a noted French philosopher, mathematician, and scientist. Dubbed the 'Founder of Modern Philosophy' and the 'Father of Modern Mathematics', he

Dualism

Recall that *dualism* is a set of views about the relationship between mind and matter, which begins with the claim that mental phenomena are, in some respects, non–physical [Har96]. One of the earliest known formulations of mind–body dualism existed in the eastern *Sankhya school* of Hindu philosophy (c. 650 BCE) which divided the world into *Purusha* (mind/spirit) and *Prakrti* (material substance). In the Western philosophical tradition, we first encounter similar ideas with the writings of Plato and Aristotle, who maintained, for different reasons, that man's *intelligence* could not be identified with, or explained in terms of, his physical body (see, e.g., [Rob95]). However, the best–known version of dualism is due to René Descartes (1641), and holds that the mind is a non–physical substance [Des91]. Descartes was the first to clearly identify the mind with consciousness and self–awareness and to distinguish this from the brain, which was the seat of intelligence. Hence, he was the first to formulate the mind–body problem in the form in which it still exists today.

The main argument in favor of dualism is simply that it appeals to the common–sense intuition of the vast majority of non–philosophically–trained people. If asked what the mind is, the average person will usually respond by identifying it with their self, their personality, their soul, or some other such entity, and they will almost certainly deny that the mind simply is the brain or vice–versa, finding the idea that there is just one ontological entity at play to be too mechanistic or simply unintelligible [Har96]. The majority of modern philosophers of mind reject dualism, suggesting that these intuitions, like many others, are probably misleading. We should use our critical faculties, as well as empirical evidence from the sciences, to examine these assumptions and determine if there is any real basis to them [Har96] Another very important, more modern, argument in favor of dualism consists

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ranks as one of the most important and influential thinkers of modern times. Much of subsequent western philosophy is a reaction to his writings, which have been closely studied from his time down to the present day. Descartes was one of the key thinkers of the Scientific Revolution in the Western World. He is also honoured by having the *Cartesian coordinate system* used in plane geometry and algebra named after him.

Descartes was a major figure in 17th century continental rationalism, later advocated by Baruch Spinoza and Gottfried Leibniz, and opposed by the empiricist school of thought, consisting of Hobbes, Locke, Berkeley, and Hume. Leibniz, Spinoza and Descartes were all versed in mathematics as well as philosophy, and Descartes and Leibniz contributed greatly to science as well. As the inventor of the Cartesian coordinate system, Descartes founded analytic geometry, that bridge between algebra and geometry crucial to the invention of the calculus and analysis. Descartes' reflections on mind and mechanism began the strain of western thought that much later, impelled by the invention of the electronic computer and by the possibility of machine intelligence, blossomed into, e.g., the Turing test. His most famous statement is "Cogito ergo sum" (I think, therefore I am).

in the idea that the mental and the physical seem to have quite different and perhaps irreconcilable properties [Jac82]. Mental events have a certain subjective quality to them, whereas physical events obviously do not. For example, what does a burned finger feel like? What does blue sky look like? What does nice music sound like? Philosophers of mind call the subjective aspects of mental events qualia (or raw feels). There is something that it is like to feel pain, to see a familiar shade of blue, and so on; there are qualia involved in these mental events. And the claim is that qualia seem particularly difficult to reduce to anything physical [Nag74].

Interactionist dualism, or simply *interactionism*, is the particular form of dualism first espoused by Descartes in the 'Meditations' [Des91]. In the 20th century, its major defenders have been Karl Popper[89] and John

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[89] Sir Karl Raimund Popper (July 28, 1902 – September 17, 1994), was an Austrian and British philosopher and a professor at the London School of Economics. He is counted among the most influential philosophers of science of the 20th century, and also wrote extensively on social and political philosophy. Popper is perhaps best known for repudiating the classical observationalist–inductivist account of scientific method by advancing empirical falsifiability as the criterion for distinguishing scientific theory from non–science; and for his vigorous defense of liberal democracy and the principles of social criticism which he took to make the flourishing of the 'open society' possible. In 1934 he published his first book, 'The Logic of Scientific Discovery', in which he criticized psychologism, naturalism, inductionism, and logical positivism, and put forth his theory of potential falsifiability being the criterion for what should be considered science.

Popper coined the term *critical rationalism* to describe his philosophy. This designation is significant, and indicates his rejection of classical empiricism, and of the observationalist-inductivist account of science that had grown out of it. Popper argued strongly against the latter, holding that scientific theories are universal in nature, and can be tested only indirectly, by reference to their implications. He also held that scientific theory, and human knowledge generally, is irreducibly conjectural or hypothetical, and is generated by the creative imagination in order to solve problems that have arisen in specific historico–cultural settings. Logically, no number of positive outcomes at the level of experimental testing can confirm a scientific theory, but a single genuine counterexample is logically decisive: it shows the theory, from which the implication is derived, to be false. Popper's account of the logical asymmetry between verification and falsification lies at the heart of his philosophy of science. It also inspired him to take falsifiability as his criterion of demarcation between what is and is not genuinely scientific: a theory should be considered scientific if and only if it is falsifiable. This led him to attack the claims of both psychoanalysis and contemporary Marxism to scientific status, on the basis that the theories enshrined by them are not falsifiable. His scientific work was influenced by his study of quantum mechanics (he has written extensively against the famous Copenhagen interpretation) and by Albert Einstein's approach to scientific theories.

In his book 'All Life is Problem Solving' (1999), Popper sought to explain the apparent progress of scientific knowledge, how it is that our understanding of the universe seems to improve over time. This problem arises from his position that

Eccles [90] (see [PE02]). It is the view that mental states, such as beliefs and desires, causally interact with physical states [Har96]. Descartes' famous

the truth content of our theories, even the best of them, cannot be verified by scientific testing, but can only be falsified. If so, then how is it that the growth of science appears to result in a growth in knowledge? In Popper's view, the advance of scientific knowledge is an evolutionary process characterised by his formula:

$$PS_1 \to TT_1 \to EE_1 \to PS_2 \, .$$

In response to a given problem situation, $PS1$, a number of competing conjectures, or tentative theories, TT, are systematically subjected to the most rigorous attempts at falsification possible. This process, error elimination, EE, performs a similar function for science that natural selection performs for biological evolution. Theories that better survive the process of refutation are not more true, but rather, more 'fit', in other words, more applicable to the problem situation at hand, $PS1$. Consequently, just as a species' 'biological fit' does not predict continued survival, neither does rigorous testing protect a scientific theory from refutation in the future. Yet, as it appears that the engine of biological evolution has produced, over time, adaptive traits equipped to deal with more and more complex problems of survival, likewise, the evolution of theories through the scientific method may, in Popper's view, reflect a certain type of progress: toward more and more interesting problems, $PS2$. For Popper, it is in the interplay between the tentative theories (conjectures) and error elimination (refutation) that scientific knowledge advances toward greater and greater problems; in a process very much akin to the interplay between genetic variation and natural selection.

As early as 1934 Popper wrote of the *search for truth* as one of the "strongest motives for scientific discovery." Still, he describes in 'Objective Knowledge' (1972) early concerns about the much–criticised notion of *truth as correspondence*. Then came the *semantic theory of truth* formulated by the logician Alfred Tarski. Popper writes of learning in 1935 of the consequences of Tarski's theory, to his intense joy. The theory met critical objections to truth as correspondence and thereby rehabilitated it. The theory also seemed to Popper to support metaphysical realism and the regulative idea of a search for truth.

Among his contributions to philosophy is his answer to David Hume's 'Problem of Induction'. Hume stated that just because the sun has risen every day for as long as anyone can remember, doesn't mean that there is any rational reason to believe it will come up tomorrow. There is no rational way to prove that a pattern will continue on just because it has before. Popper's reply is characteristic, and ties in with his *criterion of falsifiability*. He states that while there is no way to prove that the sun will come up, we can theorize that it will. If it does not come up, then it will be disproven, but since right now it seems to be consistent with our theory, the theory is not disproven. Thus, Popper's demarcation between science and non–science serves as an answer to an old logical problem as well. This approach was criticised by Peter Singer for masking the role induction plays in empirical discovery.

[90] Sir John Carew Eccles (January 27, 1903 – May 2, 1997) was an Australian neurophysiologist who won the 1963 Nobel Prize in Physiology or Medicine for his work on the synapse. He shared the prize together with Andrew Fielding Huxley and Alan Lloyd Hodgkin.

argument for this position can be summarized as follows: Fred has a clear and distinct idea of his mind as a thinking thing which has no spatial extension (i.e., it cannot be measured in terms of length, weight, height, and so on) and he also has a clear and distinct idea of his body as something that is spatially extended, subject to quantification and not able to think. It follows that mind and body are not identical because they have radically different properties, according to Descartes [Des91]. At the same time, however, it is clear that Fred's mental states (desires, beliefs, etc.) have causal effects on his body and vice-versa: a child touches a hot stove (physical event) which causes pain (mental event) and makes him yell (physical event) which provokes a sense of fear and protectiveness in the mother (mental event) and so on. Descartes' argument obviously depends on the crucial premise that what Fred believes to be 'clear and distinct' ideas in his mind are necessarily true. Most modern philosophers doubt the validity of such an assumption, since it has been shown in modern times by Freud (a third–person psychologically–trained observer can understand a person's unconscious motivations better than she does), by Pierre Duhem[91] (a third–person philosopher of science can know a person's methods of discovery better than she does), by

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In the early 1950s, Eccles and his colleagues performed the key experiments that would win Eccles the Nobel Prize. To study synapses in the peripheral nervous system, Eccles and colleagues used the stretch reflex as a model. This reflex is easily studied because it consists of only two neurons: a sensory neuron (the muscle spindle fiber) and the motor neuron. The sensory neuron synapses onto the motor neuron in the spinal cord. When Eccles passed a current into the sensory neuron in the quadriceps, the motor neuron innervating the quadriceps produced a small excitatory postsynaptic potential (EPSP). When he passed the same current through the hamstring, the opposing muscle to the quadriceps, he saw an inhibitory postsynaptic potential (IPSP) in the quadriceps motor neuron. Although a single EPSP was not enough to fire an action potential in the motor neuron, the sum of several EPSPs from multiple sensory neurons synapsing onto the motor neuron could cause the motor neuron to fire, thus contracting the quadriceps. On the other hand, IPSPs could subtract from this sum of EPSPs, preventing the motor neuron from firing.

Apart from these seminal experiments, Eccles was key to a number of important developments in neuroscience. Until around 1949, Eccles believed that synaptic transmission was primarily electrical rather than chemical. Although he was wrong in this hypothesis, his arguments led himself and others to perform some of the experiments which proved chemical synaptic transmission. Bernard Katz and Eccles worked together on some of the experiments which elucidated the role of acetylcholine as a neurotransmitter.

[91] Pierre Maurice Marie Duhem (10 June 1861 – 14 September 1916) French physicist and philosopher of science. Duhem's sophisticated views on the philosophy of science are explicated in 'The aim and structure of physical theory' (foreword by Prince Louis de Broglie). In this work he refuted the inductivist untruth that Newton's laws can de deduced from Kepler, *et al.* (a selection was published as Medieval cosmology: theories of infinity, place, time, void, and the plurality of worlds. He gave his name to the Quine-Duhem thesis, which holds that for any

Bronisław Malinowski[92] (an anthropologist can know a person's customs and habits better than he does), and by theorists of perception (experiments can make one see things that are not there and scientists can describe a person's perceptions better than he can), that such an idea of privileged and perfect access to one's own ideas is dubious at best.

Other important forms of dualism which arose as reactions to, or attempts to salvage, the Cartesian version are:

(i) Psycho–physical parallelism, or simply parallelism, is the view that mind and body, while having distinct ontological statuses, do not causally influence one another, but run along parallel paths (mind events causally interact with mind events and brain events causally interact with brain events) and only seem to influence each other [Rob95]. This view was most prominently defended by Gottfried Leibniz.[93] Although Leibniz was actually an ontological monist who believed that only one fundamental substance, monads, exists in the universe and everything else is reducible to it, he nonetheless maintained that there was an important distinction between 'the mental' and 'the physical' in terms of causation. He held that God had arranged things in advance so that minds and bodies would be in harmony with each other. This is known as the doctrine of pre–established harmony [Lei714].

(ii) Occasionalism is the view espoused by Nicholas Malebranche which asserts that all supposedly causal relations between physical events

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given set of observations there are an innumerably large number of explanations. Thus empirical evidence cannot force the revision of a theory.

92 Bronisław Kasper Malinowski (April 7, 1884 – May 16, 1942) was a Polish anthropologist widely considered to be one of the most important anthropologists of the twentieth century because of his pioneering work on ethnographic fieldwork, the study of reciprocity, and his detailed contribution to the study of Melanesia.

93 Gottfried Wilhelm Leibniz (July 1 (June 21 Old Style) 1646 – November 14, 1716) was a German polymath. Educated in law and philosophy, Leibniz played a major role in the European politics and diplomacy of his day. He occupies an equally large place in both the history of philosophy and the history of mathematics. He invented *calculus* independently of Newton, and his notation is the one in general use since. He also invented the *binary system*, foundation of virtually all modern computer architectures. In philosophy, he is most remembered for *optimism*, i.e., his conclusion that our universe is, in a restricted sense, the best possible one God could have made. He was, along with René Descartes and Baruch Spinoza, one of the three great 17th century rationalists, but his philosophy also both looks back to the *Scholastic tradition* and anticipates logic and analysis. Leibniz also made major contributions to physics and technology, and anticipated notions that surfaced much later in biology, medicine, geology, probability theory, psychology, knowledge engineering, and information science. He also wrote on politics, law, ethics, theology, history, and philology, even occasional verse. His contributions to this vast array of subjects are scattered in journals and in tens of thousands of letters and unpublished manuscripts. To date, there is no complete edition of Leibniz's writings, and a complete account of his accomplishments is not yet possible.

or between physical and mental events are not really causal at all. While body and mind are still different substances on this view, causes (whether mental or physical) are related to their effects by an act of God's intervention on each specific occasion [Sch02].

(iii) Epiphenomenalism is a doctrine first formulated by Thomas Huxley [Hux898]. Fundamentally, it consists in the view that mental phenomena are causally inefficacious. Physical events can cause other physical events and physical events can cause mental events, but mental events cannot cause anything, since they are just causally inert by-products (i.e. epiphenomena) of the physical world [Rob95] The view has been defended most strongly in recent times by Frank Jackson [Jac82].

(iv) Property dualism asserts that when matter is organized in the appropriate way (i.e., in the way that living human bodies are organized), mental properties emerge. Hence, it is a sub–branch of emergent materialism [Har96]. These emergent properties have an independent ontological status and cannot be reduced to, or explained in terms of, the physical substrate from which they emerge. This position is espoused by David Chalmers and has undergone something of a renaissance in recent years [Cha97].

Monism

In contrast to dualism, *monism* states that there is only one fundamental substance. Monism, first proposed in the West by Parmenides[94] and in modern times by Baruch Spinoza,[95] maintains that there is only one substance; in the East, rough parallels might be the Hindu concept of *Brahman* or the *Tao* of Lao Tzu [Spi670]. Today the most common forms of monism in Western philosophy are physicalistic [Kim95b]. Physicalistic monism asserts that the only existing substance is physical, in some sense of that term to be clarified by our best science [Sto05]. Another form of monism is that which states that the only existing substance is mental. Such idealistic monism is currently somewhat uncommon in the West [Kim95b].

[94] Parmenides of Elea (early 5th century BC) was an ancient Greek philosopher born in Elea, a Hellenic city on the southern coast of Italy. Parmenides was a student of Ameinias and the founder of the School of Elea, which also included Zeno of Elea and Melissus of Samos.

[95] Benedictus de Spinoza (November 24, 1632 – February 21, 1677), named Baruch Spinoza by his synagogue elders, was a Jewish–Dutch philosopher. He is considered one of the great rationalists of 17th–century philosophy and, by virtue of his magnum opus the 'Ethics', one of the definitive ethicists. His writings, like those of his fellow rationalists, reveal considerable mathematical training and facility. Spinoza was a lens crafter by trade, an exciting engineering field at the time because of great discoveries being made by telescopes. The full impact of his work only took effect some time after his death and after the publication of his 'Opera Posthuma'. He is now seen as having prepared the way for the 18th century Enlightenment, and as a founder of modern biblical criticism. 20th century philosopher, Gilles Deleuze (1990), referred to Spinoza as "The absolute philosopher, whose Ethics is the foremost book on concepts."

Phenomenalism, the theory that all that exists are the representations (or sense data) of external objects in our minds and not the objects themselves, was adopted by Bertrand Russell[96] and many of the logical positivists during

[96] Bertrand Arthur William Russell, (3rd Earl Russell, 18 May 1872 – 2 February 1970), was a British philosopher, logician, and mathematician, working mostly in the 20th century. A prolific writer, Bertrand Russell was also a populariser of philosophy and a commentator on a large variety of topics, ranging from very serious issues to the mundane. Continuing a family tradition in political affairs, he was a prominent liberal as well as a socialist and anti–war activist for most of his long life. Millions looked up to Russell as a prophet of the creative and rational life; at the same time, his stances on many topics were extremely controversial.

Russell was born at the height of Britain's economic and political ascendancy. He died of influenza nearly a century later, at a time when the British Empire had all but vanished, its power dissipated by two debilitating world wars. As one of the world's best–known intellectuals, Russell's voice carried great moral authority, even into his early 90s. Among his political activities, Russell was a vigorous proponent of nuclear disarmament and an outspoken critic of the American war in Vietnam.

In 1950, Russell was made a Nobel Laureate in Literature, "in recognition of his varied and significant writings in which he champions humanitarian ideals and freedom of thought."

Russell is generally recognized as one of the founders of *analytical philosophy*, even of its several branches. At the beginning of the 20th century, alongside G.E. Moore, Russell was largely responsible for the British 'revolt against Idealism', a philosophy greatly influenced by Georg Hegel. This revolt was echoed 30 years later in Vienna by the logical positivists' 'revolt against metaphysics'. Russell was particularly appalled by the idealist doctrine of internal relations, which held that in order to know any particular thing, we must know all of its relations. Russell showed that this would make space, time, science and the concept of number unintelligible. Russell's logical work with Alfred Whitehead continued this project.

Russell had great influence on modern mathematical logic. His first mathematical book, An Essay on the Foundations of Geometry, was published in 1897. This work was heavily influenced by Immanuel Kant. Russell soon realised that the conception it laid out would have made Albert Einstein's schema of space-time impossible, which he understood to be superior to his own system. Thenceforth, he rejected the entire Kantian program as it related to mathematics and geometry, and he maintained that his own earliest work on the subject was nearly without value. Russell discovered that Gottlob Frege had independently arrived at equivalent definitions for 0, successor, and number, and the definition of number is now usually referred to as the *Frege–Russell definition*. It was largely Russell who brought Frege to the attention of the English-speaking world. He did this in 1903, when he published 'The Principles of Mathematics', in which the concept of class is inextricably tied to the definition of number. The appendix to this work detailed a paradox arising in Frege's application of second– and higher–order functions which took first-order functions as their arguments, and he offered his first effort to resolve what would henceforth come to be known as the *Russell Paradox*, which he later developed into a complete theory, the

the early 20th century [Rus18]. It lasted for only a very brief period of time. A third possibility is to accept the existence of a basic substance which is neither physical nor mental. The mental and physical would both be properties of this neutral substance. Such a position was adopted by Baruch Spinoza [Spi670] and popularized by Ernst Mach[97] [Mac59] in the 19th century. This neutral monism, as it is called, resembles property dualism.

Behaviorism

Behaviorism dominated philosophy of mind for much of the 20th century, especially the first half [Kim95b]. In psychology, *behaviorism* developed as a reaction to the inadequacies of introspectionism. Introspective reports on one's own interior mental life are not subject to careful examination for accuracy and are not generalizable. Without generalizability and the possibility of third-person examination, the behaviorists argued, science is simply not possible [Sto05]. The way out for psychology was to eliminate the idea of an interior mental life (and hence an ontologically independent mind) altogether and focus instead on the description of observable behavior [Ski72].

Parallel to these developments in psychology, a philosophical behaviorism (sometimes called logical behaviorism) was developed [Sto05]. This is characterized by a strong verificationism, which generally considers unverifiable statements about interior mental life senseless. But what are mental states if

Theory of types. Aside from exposing a major inconsistency in naive set theory, Russell's work led directly to the creation of modern axiomatic set theory. It also crippled Frege's project of reducing arithmetic to logic. The Theory of Types and much of Russell's subsequent work have also found practical applications with computer science and information technology.

Russell continued to defend *logicism*, the view that mathematics is in some important sense reducible to logic, and along with his former teacher, Alfred Whitehead, wrote the monumental 'Principia Mathematica', an *axiomatic system* on which all of mathematics can be built. The first volume of the Principia was published in 1910, and is largely ascribed to Russell. More than any other single work, it established the specialty of mathematical or symbolic logic. Two more volumes were published, but their original plan to incorporate geometry in a fourth volume was never realised, and Russell never felt up to improving the original works, though he referenced new developments and problems in his preface to the second edition. Upon completing the Principia, three volumes of extraordinarily abstract and complex reasoning, Russell was exhausted, and he never felt his intellectual faculties fully recovered from the effort. Although the Principia did not fall prey to the paradoxes in Frege's approach, it was later proven by Kurt Gödel that neither Principia Mathematica, nor any other consistent system of primitive recursive arithmetic, could, within that system, determine that every proposition that could be formulated within that system was decidable, i.e., could decide whether that proposition or its negation was provable within the system (*Gödel's incompleteness theorem*).

[97] Ernst Mach (February 18, 1838 – February 19, 1916) was an Austrian–Czech physicist and philosopher and is the namesake for the 'Mach number" (aka Mach speed) and the optical illusion known as Mach bands.

they are not interior states on which one can make introspective reports? The answer of the behaviorist is that mental states do not exist but are actually just descriptions of behavior and/or dispositions to behave made by external third parties in order to explain and predict others' behavior [Ryl49]. Philosophical behaviorism is considered by most modern philosophers of mind to be outdated [Kim95a]. Apart from other problems, behaviorism implausibly maintains, for example, that someone is talking about behavior if she reports that she has a wracking headache.

Continental Philosophy of Mind

In contrast to Anglo–American *analytic philosophy*[98] there are other schools of thought which are sometimes subsumed under the broad label of *continental philosophy*. These schools tend to differ from the analytic school in that they focus less on language and logical analysis and more on directly understanding human existence and experience. With reference specifically to the discussion of the mind, this tends to translate into attempts to grasp the concepts of thought and perceptual experience in some direct sense that does not involve the analysis of linguistic forms [Dum01]. In particular, in his

[98] Analytic philosophy is the dominant academic philosophical movement in English–speaking countries and in the Nordic countries. It is distinguished from Continental Philosophy which pertains to most non–English speaking countries. Its main founders were the Cambridge philosophers G.E. Moore and Bertrand Russell. However, both were heavily influenced by the German philosopher and mathematician Gottlob Frege and many of analytic philosophy's leading proponents, such as Ludwig Wittgenstein, Rudolf Carnap, Kurt Gödel, Karl Popper, Hans Reichenbach, Herbert Feigl, Otto Neurath, and Carl Hempel have come from Germany and Austria. In Britain, Russell and Moore were succeeded by C. D. Broad, L. Stebbing, Gilbert Ryle, A. J. Ayer, R. B. Braithwaite, Paul Grice, John Wisdom, R. M. Hare, J. L. Austin, P. F. Strawson, William Kneale, G. E. M. Anscombe, and Peter Geach. In America, the movement was led by many of the above-named European emigres as well as Max Black, Ernest Nagel, C. L. Stevenson, Norman Malcolm, W. V. Quine, Wilfrid Sellars, and Nelson Goodman, while A. N. Prior, John Passmore, and J. J. C. Smart were prominent in Australasia.

Logic and philosophy of language were central strands of analytic philosophy from the beginning, although this dominance has diminished greatly. Several lines of thought originate from the early, language-and-logic part of this analytic philosophy tradition. These include: logical positivism, logical empiricism, logical atomism, logicism and ordinary language philosophy. Subsequent analytic philosophy includes extensive work in ethics (such as Philippa Foot, R. M. Hare, and J. L. Mackie), political philosophy (John Rawls, Robert Nozick), aesthetics (Monroe Beardsley, Richard Wollheim, Arthur Danto), philosophy of religion (Alvin Plantinga, Richard Swinburne), philosophy of language (David Kaplan, Saul Kripke, Richard Montague, Hilary Putnam, W.V.O. Quine, Nathan Salmon, John Searle), and philosophy of mind (Daniel Dennett, David Chalmers, Putnam). Analytic metaphysics has also recently come into its own (Kripke, David Lewis, Salmon, Peter van Inwagen, P.F. Strawson).

'Phenomenology of Mind', G.W. F. Hegel[99] discusses three distinct types of mind: the subjective mind, the mind of an individual; the objective mind, the mind of society and of the State; and the Absolute mind, a unity of all concepts. In modern times, the two main schools that have developed in response or opposition to this Hegelian tradition are *phenomenology* and *existentialism*. Phenomenology, founded by Edmund Husserl,[100] focuses on the contents

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[99] Georg Wilhelm Friedrich Hegel (August 27, 1770 – November 14, 1831) was a German philosopher born in Stuttgart, Württemberg, in present-day southwest Germany. His influence has been widespread on writers of widely varying positions, including both his admirers (F.H. Bradley, J.P. Sartre, Hans Küng, Bruno Bauer), and his detractors (Kierkegaard, Schopenhauer, Heidegger, Schelling). His great achievement was to introduce for the first time in philosophy the idea that History and the concrete are important in getting out of the circle of philosophia perennis, i.e., the perennial problems of philosophy. Also, for the first time in the history of philosophy he realised the importance of the Other in the coming to be of self–consciousness, see slave–master dialectic.

Some of Hegel's writing was intended for those with advanced knowledge of philosophy, although his 'Encyclopedia' was intended as a textbook in a university course. Nevertheless, like many philosophers, Hegel assumed that his readers would be well–versed in Western philosophy, up to and including Descartes, Spinoza, Hume, Kant, Fichte, and Schelling. For those wishing to read his work without this background, introductions to Hegel and commentaries about Hegel may suffice. However, even this is hotly debated since the reader must choose from multiple interpretations of Hegel's writings from incompatible schools of philosophy. Presumably, reading Hegel directly would be the best method of understanding him, but this task has historically proved to be beyond the average reader of philosophy.[citation needed] This difficulty may be the most urgent problem with respect to the legacy of Hegel.

One especially difficult aspect of Hegel's work is his innovation in logic. In response to Immanuel Kant's challenge to the limits of Pure Reason, Hegel developed a radically new form of logic, which he called speculation, and which is today popularly called *dialectics*. The difficulty in reading Hegel was perceived in Hegel's own day, and persists into the 21st century. To understand Hegel fully requires paying attention to his critique of standard logic, such as the *law of contradiction* and the *law of the excluded middle*, and, whether one accepts or rejects it, at least taking it seriously. Many philosophers who came after Hegel and were influenced by him, whether adopting or rejecting his ideas, did so without fully absorbing his new speculative or dialectical logic.

[100] Edmund Gustav Albrecht Husserl (April 8, 1859, Prostějov – April 26, 1938, Freiburg) was a German philosopher, known as the father of phenomenology. Husserl was born into a Jewish family in Prostějov (Prossnitz), Moravia, Czech Republic (then part of the Austrian Empire). A pupil of Franz Brentano and Carl Stumpf, Husserl came to influence, among others, Edith Stein (St. Teresa Benedicta of the Cross), Eugen Fink, Martin Heidegger, Jean–Paul Sartre, and Maurice Merleau–Ponty; in addition, Hermann Weyl's interest in *intuitionistic logic* and impredicativity appear to have resulted from contacts with Husserl. Rudolf Carnap was also influenced by Husserl, not only concerning Husserl's notion of essential insight that Carnap used in his Der Raum, but also his notion

of the human mind and how phenomenological processes shape our experiences. Existentialism, a school of thought led by Jean–Paul Sartre,[101] focuses on the content of experiences and how the mind deals with such experiences [Fly04].

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of *formation rules* and *transformation rules* is founded on Husserl's philosophy of logic. In 1887 Husserl converted to Christianity and joined the Lutheran Church. He taught philosophy at Halle as a tutor (Privatdozent) from 1887, then at Göttingen as professor from 1901, and at Freiburg im Breisgau from 1916 until he retired in 1928. After this, he continued his research and writing by using the library at Freiburg, until barred therefrom because of his Jewish heritage under the rectorship of his former pupil and intended protege, Martin Heidegger.

Husserl held the belief that *truth–in–itself* has as ontological correlate *being–in–itself*, just as meaning categories have formal–ontological categories as correlates. The discipline of logic is a formal theory of judgment, that studies the formal a priori relations among judgments using meaning categories. Mathematics, on the other hand, is formal ontology, it studies all the possible forms of being (of objects). So, in both of these disciplines, formal categories, in their different forms, are their object of study, not the sensible objects themselves. The problem with the psychological approach to mathematics and logic is that it fails to account for the fact that it is about formal categories, not abstractions from sensibility alone. The reason why we do not deal with sensible objects in mathematics is because of another faculty of understanding called *categorial abstraction*. Through this faculty we are able to get rid of sensible components of judgments, and just focus on formal categories themselves. Thanks to 'eidetic (or essential) intuition', we are able to grasp the possibility, impossibility, necessity and contingency among concepts or among formal categories. Categorial intuition, along with categorial abstraction and eidetic intuition, are the basis for logical and mathematical knowledge.

101 Jean–Paul Charles Aymard Sartre (June 21, 1905 – April 15, 1980), was a French existentialist philosopher, dramatist and screenwriter, novelist and critic.

The basis of Sartre's existentialism is found in his 'The Transcendence of the Ego'. To begin with, the thing–in–itself is infinite and overflowing. Any direct consciousness of the thing–in–itself, Sartre refers to as a 'pre–reflective consciousness'. Any attempt to describe, understand, historicize etc. the thing–in–itself, Sartre calls 'reflective consciousness'. There is no way for the reflective consciousness to subsume the pre–reflective, and so reflection is fated to a form of anxiety, i.e., the human condition. The reflective consciousness in all its forms, (scientific, artistic or otherwise) can only limit the thing–in–itself by virtue of its attempt to understand or describe it. It follows therefore that any attempt at self–knowledge (self–consciousness) is a construct that fails no matter how often it is attempted. (self-consciousness is a reflective consciousness of an overflowing infinite) In Sartre's words "Conciousness is consciousness of itself insofar as it is consciousness of a transcendent object." The same holds true about knowledge of the 'Other' (being), which is a construct of reflective consciousness. One must be careful to understand this more as a form of warning than as an ontological statement. However, there is an implication of Solipsism here that Sartre considers fundamental to any coherent description of the human condition.

Neurobiology

On the other hand, within the tangible field of *neurobiology*, there are many subdisciplines which are concerned with the relations between mental and physical states and processes [Bea95]:

1. Sensory neurophysiology investigates the relation between the processes of perception and stimulation [Pine97].
2. Cognitive neuroscience studies the correlations between mental processes and neural processes [Pine97].
3. Neuropsychology describes the dependence of mental faculties on specific anatomical regions of the brain [Pine97].
4. Lastly, evolutionary biology studies the origins and development of the human nervous system and, in as much as this is the basis of the mind, also describes the ontogenetic and phylogenetic development of mental phenomena beginning from their most primitive stages [Pink97].

Since the 1980's, sophisticated neuroimaging procedures, such as fMRI, have furnished increasing knowledge about the workings of the human brain, shedding light on ancient philosophical problems.The methodological breakthroughs of the neurosciences, in particular the introduction of high–tech neuroimaging procedures, has propelled scientists toward the elaboration of increasingly ambitious research programs: one of the main goals is to describe and comprehend the neural processes which correspond to mental functions [Bea95]. A very small number of neurobiologists, such as Emil Reymond[102] and John Eccles have denied the possibility of a 'reduction' of mental phenomena to cerebral processes (see [PE02]). However, the contemporary neurobiologist and philosopher Gerhard Roth continues to defend a form of 'non–reductive materialism' [Rot01].

9.2.9 Analytical Psychology

Recall that *analytical psychology* (AP) is part of the *Jungian psychology movement* started by Carl G. Jung[103] and his followers. Although considered

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[102] Emil du Bois–Reymond (November 7, 1818, Berlin, Germany – November 26, 1896), was a German physician and physiologist, discoverer of the nerve action potential and the father of experimental electrophysiology.

[103] Carl Gustav Jung (July 26, 1875 – June 6, 1961) was a Swiss psychiatrist and founder of *analytical psychology.*

Jung's unique and broadly influential approach to psychology emphasized understanding the *psyche* through exploring the worlds of dreams, art, mythology, world religion and philosophy. Though not the first to analyze dreams, he has become perhaps the best–known pioneer in the field of *dream analysis*. Although he was a theoretical psychologist and practicing clinician for most of his life, much of his life's work was spent exploring other realms: Eastern vs. Western philosophy, alchemy, astrology, sociology, as well as literature and the arts.

Jung also emphasized the importance of balance. He cautioned that modern humans rely too heavily on science and logic and would benefit from integrating spirituality and appreciation of the unconscious realm. Interestingly, Jungian ideas are not typically included in curriculum of most major universities' psychology departments, but are occasionally explored in humanities departments. Many pioneering psychological concepts were originally proposed by Jung. Some of these are: (i) *archetype*, (ii) *collective unconscious*, (iii) *unconscious complex*, and (iv) *synchronicity*. In addition, the popular career test currently offered by high school and college career centers, the *Myers–Briggs Type Indicator*, is strongly influenced by Jung's theories.

The overarching goal of Jung's work was the reconciliation of the life of the individual with the world of the *supra–personal archetypes*. He came to see the individual's encounter with the unconscious as central to this process. The human experiences the unconscious through symbols encountered in all aspects of life: in dreams, art, religion, and the symbolic dramas we enact in our relationships and life pursuits. Essential to the encounter with the unconscious, and the reconciliation of the individual's consciousness with this broader world, is learning this symbolic language. Only through attention and openness to this world (which is quite foreign to the modern Western mind) are individuals able to harmonize their lives with these supra–personal archetypal forces. In order to undergo the individuation process, the individual must be open to the parts of oneself beyond one's own ego. In order to do this, the modern individual must pay attention to dreams, explore the world of religion and spirituality, and question the assumptions of the operant societal world–view (rather than just blindly living life in accordance with dominant norms and assumptions).

The collective unconscious could be thought of as the DNA of the human psyche. Just as all humans share a common physical heritage and predisposition towards specific physical forms (like having two legs, a heart, etc.) so do all humans have a common psychological predisposition. However, unlike the quantifiable information that composes DNA (in the form of coded sequences of nucleotides), the collective unconscious is composed of archetypes. In contrast to the objective material world, the subjective realm of archetypes can not be fully plumbed through quantitative modes of research. Instead it can be revealed more fully through an examination of the symbolic communications of the human psyche — in art, dreams, religion, myth, and the themes of human relational/behavioral patterns. Devoting his life to the task of exploring and understanding the collective unconscious, Jung theorized that certain symbolic themes exist across all cultures, all epochs, and in every individual.

The *shadow* is an *unconscious complex* that is defined as the diametrical opposite of the conscious self, the ego. The shadow represents unknown attributes and qualities of the ego. There are constructive and destructive types of shadow. On the destructive side, it often represents everything that the conscious person does not wish to acknowledge within themselves. For instance, someone who identifies as being kind has a shadow that is harsh or unkind. Conversely, an individual who is brutal has a kind shadow. The shadow of persons who are convinced that they are ugly appears to be beautiful. On the constructive side, the shadow may represent hidden positive influences. Jung points to the story of Moses and Al–Khidr in the 18th Book of the Koran as an example. Jung to be a part of *psychoanalysis*, it is distinct from *Freudian psychoanalysis*.[104] While Freudian psychoanalysis assumes that the repressed material hidden in the unconscious is given by repressed sexual instincts, analytical psychology has a more general approach. There is no preconceived assumption about the unconscious material. The unconscious, for Jungian analysts, may contain repressed sexual drives, but also aspirations, fears, etc.

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emphasized the importance of being aware of shadow material and incorporating it into conscious awareness, lest one project these attributes on others. The shadow in dreams is often represented by dark figures of the same gender as the dreamer. According to Jung the human being deals with the reality of the shadow in four ways: denial, projection, integration and/or transmutation.

Jung identified the *anima* as being the unconscious feminine component of men and the *animus* as the unconscious masculine component in women. However, this is rarely taken as a literal definition: many modern–day Jungian practitioners believe that every person has both an anima and an animus. Jung stated that the anima and animus act as guides to the unconscious unified *Self*, and that forming an awareness and a connection with the anima or animus is one of the most difficult and rewarding steps in psychological growth. Jung reported that he identified his anima as she spoke to him, as an inner voice, unexpectedly one day. Oftentimes, when people ignore the anima or animus complexes, the anima or animus vies for attention by projecting itself on others. This explains, according to Jung, why we are sometimes immediately attracted to certain strangers: we see our anima or animus in them. Love at first sight is an example of anima and animus projection. Moreover, people who strongly identify with their gender role (e.g., a man who acts aggressively and never cries) have not actively recognized or engaged their anima or animus. Jung attributes human rational thought to be the male nature, while the irrational aspect is considered to be natural female. Consequently, irrationality is the male anima shadow and rationality is the female animus shadow.

There are four primary modes of experiencing the world in Jung's *extrovert/introvert model*: two rational functions: *thinking* and *feeling*, and two perceptive functions: *sensation* and *intuition*. Sensation is the perception of facts. Intuition is the perception of the unseen. Thinking is analytical, deductive cognition. Feeling is synthetic, all–inclusive cognition. In any person, the degree of introversion/extroversion of one function can be quite different to that of another function. Broadly speaking, we tend to work from our most developed function, while we need to widen our personality by developing the others. Related to this, Jung noted that the unconscious often tends to reveal itself most easily through a person's least developed function. The encounter with the unconscious and development of the underdeveloped function(s) thus tend to progress together.

Jung had a profesional relationship with the Nobel lauret physicist Wolfgang Pauli. Their work has been published in the books [PJ55, PJ01] as well as in Jung's famous 'Alchemy'.

[104] For a period of some 6 years, Carl Jung was a close friend and collaborator of Sigmund Freud. However after Jung published his 'Wandlungen und Symbole der Libido' (The Psychology of the Unconscious) in 1913, their theoretical ideas had diverged sharply.

The aim of AP is the personal experience of the deep forces and motivations underlying human behavior. It is related to the so–called *depth psychology* and *archetypal psychology*. Its basic assumption is that the personal unconscious is a potent part, probably the more active part, of the normal human psyche. Reliable communication between the conscious and unconscious parts of the psyche is necessary for wholeness. Also crucial is the belief that *dreams* show ideas, beliefs, and feelings of which individuals may not be readily aware, but need to be, and that such material is expressed in a personalized vocabulary of visual metaphors. Things 'known but unknown' are contained in the unconscious, and dreams are one of the main vehicles for the unconscious to express them.

AP distinguishes between a *personal and* a *collective unconscious*. The collective unconscious contains *archetypes* common to all human beings. That is, individuation may bring to surface symbols that do not relate to the life experiences of a single person. This content is more easily viewed as answers to the more fundamental questions of humanity: life, death, meaning, happiness, fear. Among these more spiritual concepts may arise and be integrated into the personality.

AP distinguishes two main psychological types or temperaments: (i) *extrovert*, and (ii) *introvert*.[105]The attitude type could be thought of as the

[105] In the context of *personality psychology*, *extroverts and introverts* differ in how they get or lose energy as a function of their immediate social context. In particular, extroverts feel an increase of perceived energy when interacting with large group of people, but a decrease of energy when left alone. Conversely, introverts feel an increase of energy when alone, but a decrease of energy when surrounded by large group of people.

Extroverts tend to be energetic when surrounded by people and depressive when not. To induce human interactions, extroverts tend to be enthusiastic, talkative, and assertive. Extroverts enjoy doing activities that involve other people, such as taking part in community activities and involving in business, religious, political, and scientific affairs; their affinity to large groups allow them to enjoy large social gatherings including parties and marches. As such, an extroverted person is likely to enjoy time spent with people and find less reward in time spent alone.

On the other hand, introverts are 'geared to inspect' rather than to act in social settings. In a large social setting, introverts tend to be quiet, low–key, deliberate, and engaged in non–social activities. Conversely, introverts gain energy when alone performing solitary activities. Thus they tend to enjoy reading, writing, watching movies at home, inventing, and designing - and doing these activities in quiet, minimally socially interactive environment such as home, library, labs, and quiet coffee shops. While introverts avoid social situations with large numbers of people, they tend to enjoy intense, one–to–one or one–to–few social interactions. They tend to have small circle of very close friends, compared to the extroverts' typically larger circle of less–close friends.

While most people view being either introverted or extroverted as a question with only two answers, levels of extraversion in fact fall in a normally distributed bell curve, with most people falling in between. The term *ambivert* was coined to *energy flow* of *libido*, or *psychic energy* (*ch'i* in Roman–Chinese and 'ki' in Roman–Japanese).[106] The introvert's energy flow is inward to the subject and away from the object, i.e., external relations. The extrovert's energy flow is outward toward the object, ie. towards external relations and away from the inner, subjective world. Extroverts desire breadth, while introverts seek depth. The introversion/extroversion attitude type may also influence mental breakdown. Introverts may be more inclined to catatonic type schizophrenia and extroverts towards manic depression.

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denote people who fall more or less directly in the middle and exhibit tendencies of both groups. An ambivert is normally comfortable with groups and enjoys social interaction, but also relishes time alone and away from the crowd.

[106] Freud introduced the term *libido* as the instinctual energy or force that can come into conflict with the conventions of civilized behavior. It is the need to conform to society and control the libido, contained in what Freud defined as the Id, that leads to tension and disturbance in both society and the individual. This disturbance Freud labelled neurosis. Thus, libido has to be transformed into socially useful energy, according to Freud, through the process of 'sublimation'.

Ch'i (or *qi,* or *ki*) is a fundamental concept of traditional Chinese culture. Ch'i is believed to be part of everything that exists, as in 'life force' or 'life energy', something like the 'force' in Lucas' Star Wars. It is most often translated as 'energy flow,' or literally as 'air' or 'breath'.

The nature of ch'i is a matter of controversy among those who accept it as a valid concept, while those who dismiss its very existence ignore it, except for purposes of discussion with its adherents. Disputing the nature of qi is an old controversy in Chinese philosophy. Among some traditional Chinese medicine practitioners, qi is sometimes thought of as a metaphor for biological processes similar to the Western concept of energy flow for *homeostatic balance* in biological regulations. Others argue that qi involves some new physics or biology. Attempts to directly connect qi with some scientific phenonomena have been attempted since the mid–nineteenth century. *Ch'i* is a central concept in many martial arts; e.g., in the Japanese arts, Ki is developed in Aikido and given special emphasis in Ki–Aikido (a classic combat story concerns two opponents who held each others hands before a fight, while doing so each felt the others *ch'i* and the one with the weaker *ch'i* resigned without a blow being struck).

The concept of *quantum tunneling* in modern physics where physical matters can 'tunnel' through energy barriers using quantum mechanics captured some of the similar concepts of *ch'i* (which allows one to transcend normal physical forces in nature). The seemingly impossibility of tunneling through energy barriers (walls) is only limited by the conceptual framework of classical mechanics, but can easily be resolved by the *wave–particle duality* in modern physics. By the same token, this duality is similar to the metaphorical duality of *yin* and *yang*, which is governed by the flow of energy *ch'i*. Examples of quantum tunneling can be found as a mechanism in biology used by enzymes to speed up reactions in lifeforms to millions of times their normal speed [MRJ06]. Other examples of quantum tunneling are found in semiconductor and superconductors, such as field emission used in flash memory and major source of current leakage in *very–large–scale integration* (VLSI) electronics draining power in mobile phones and computers.

Samuels [Sam95] has distinguished three schools of 'post–Jungian' psychotherapy: the classical, the developmental and the archetypal. The classical school is that which tries to remain faithful to what Jung himself proposed and taught in person and in his 20–plus volumes of work. The developmental school, associated with M. Fordham, B. Feldman etc., can be considered a bridge between Jungian psychoanalysis and M. Klein's *object relations theory*. The archetypal school (sometimes called 'the imaginal school'), with different views associated with the *mythopoeticists*, such as J. Hillman in his intellectual theoretical view of archetypal psychology, C.P. Estés, in her view that ethnic and Aboriginal people are the originators of archetypal psychology and have long carried the maps to the journey of the soul in their songs, tales, dream–telling, art and rituals; M. Woodman who proposes a feminist viewpoint regarding archetypal psychology, and other Jungians like T. Moore and R. Moore, as well. Most mythopoeticists/archetypal psychology innovators either imagine the *Self* not to be the main archetype of the collective unconscious as Jung thought, but rather assign each archetype equal value... Others, who are modern progenitors of archetypal psychology (such as Estés), think of the *Self* as that which contains and yet is suffused by all the other archetypes, each giving life to the other.

Chapter 10
Smart $CSB-$Agents for Games Modelling

10.1 $CSB-$Agents

Recall that the *agent theory* concerns the definition of the so–called *belief–desire–intention agents* (BDI–agents, for short), as well as multi–agent systems, properties, architectures, communication, cooperation and coordination capabilities (see [RG98, II07b]).

A common definition of an agent reads: An agent is a computer system that is situated in some environment, and that is capable of autonomous action in this environment in order to meet its design requirements [Woo00].

Practical side of the agent theory concerns the agent languages and platforms for programming and experimenting with agents. According to [Fer99], a BDI–agent is a physical or virtual entity which:

1. *is capable of limited perceiving its environment* (see Figure 10.1),
2. *has only a partial representation of its environment,*
3. *is capable of acting in an environment,*
4. *can communicate directly with other agents,*
5. *is driven by a set of tendencies,*[1]
6. *possesses resources of its own,*
7. *possesses some skills and can offer services,*
8. *may be able to reproduce itself,*
9. *whose behavior tends towards satisfying its objectives,*

– taking into account the resources and skills available to it and depending on its perception, its representation and the communications it receives. Agents' actions affect the environment which, in turn, affects future decisions of agents. The *multi–agent systems* have been successfully applied in numerous fields (see [Fer99] for the review).

Agents embody a new software development paradigm that attempts to merge some of the theories developed in artificial intelligence research with

[1] In the form of individual objectives or of a satisfaction/survival function which it tries to optimize.

T.T. Ivancevic et al.: Complex Sports Biodynamics, COSMOS 2, pp. 215–228.
springerlink.com

Fig. 10.1 A basic agent–environment loop (modified from [Woo00])

computer science. The power of agents comes from their intelligence and also their ability to communicate with each other. A simple mapping of agent technology compared to relevant technologies is illustrated in Figure 10.2.

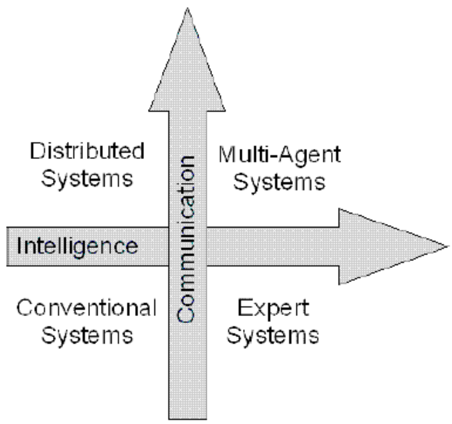

Fig. 10.2 Agent technology compared to relevant technologies

Agents can be considered as the successors of *object–oriented programming* techniques, applied to certain problem domains. However, the additional layer of implementation in agents provides some key functionalities and deliberately creates a separation between the implementation of an agent from the application being developed. This is done in order to achieve one of the core properties of agents, autonomy. Objects are able to assert a certain amount of control over themselves via private variables and methods, and other objects via public variables and methods. Consequently, a particular object is able to directly change public variables of other objects and also execute public methods of other objects. Hence, objects have no control over the values of public variables and who and when executes their public methods. Conversely, agents are explicitly separated, and can only request from each other to perform a particular task. Furthermore, it cannot be assumed that after a particular agent makes a request, another agent will do it. This is because performing a particular action may not be in the best interests of the other agent, in which case it would not comply [Woo00].

10.2 Types of $CSB-$Agents

Here we give a general overview of different types of agents and groups them into several intuitive categories based on the method that they perform their reasoning [Woo00, II07b].

10.2.1 Deliberate Agents

Deliberate agents are agents that perform rational reasoning, take actions that are rational after deliberating using their *knowledge base* (KB), carefully considering the possible effects of different actions available to them. There are two subtypes of deliberate agents: *deductive reasoning agents* and *production–rule agents*.

1. Deductive reasoning agents are built using expert systems theory, they operate using an internal symbolic KB of the environment. Desired behavior is achieved by manipulating the environment and updating the KB accordingly. A utility function is implemented that provides an indication on how good a particular state is compared on what the agent should achieve. An example of the idea behind these type of agents is an agent that explores

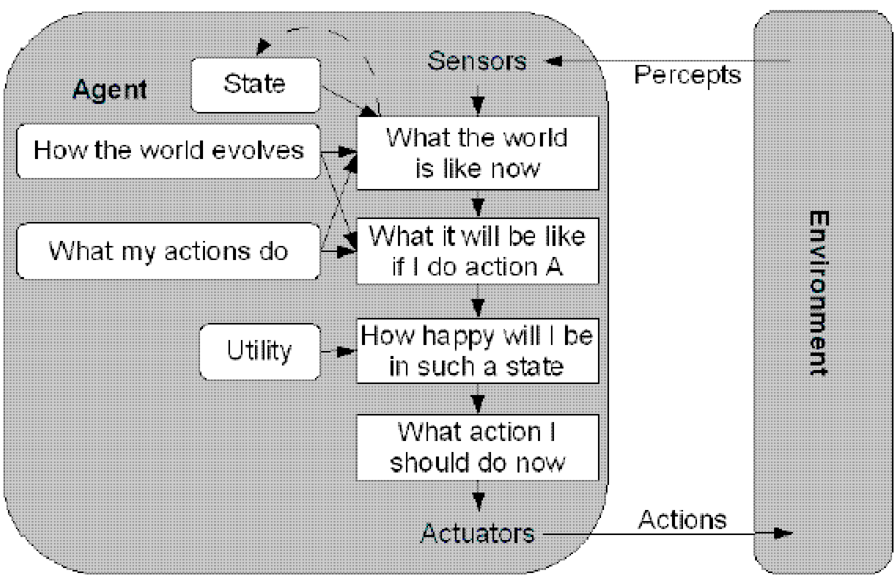

Fig. 10.3 A concept of deductive reasoning agents (modified from [RN03])

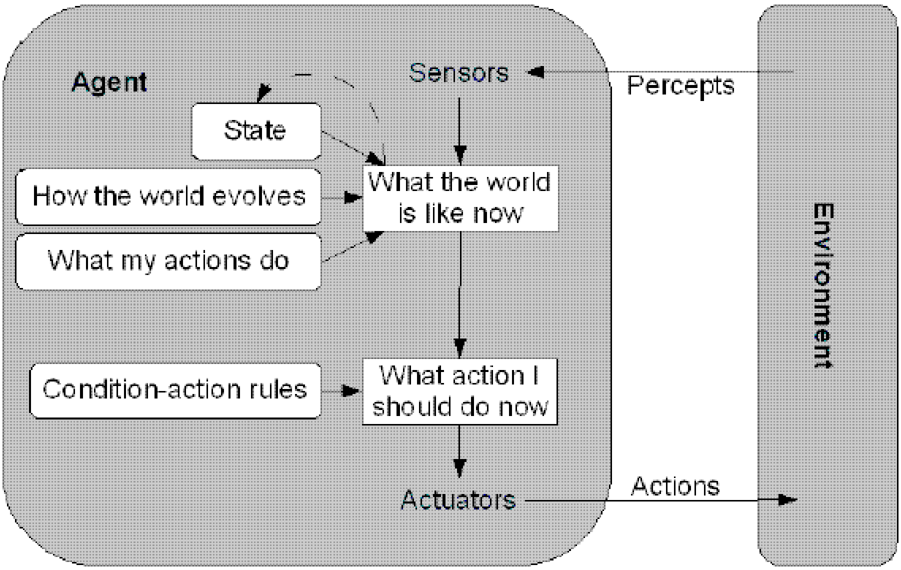

Fig. 10.4 A concept of production–rule agents (modified from [RN03])

a building. It has the ability to move around and it uses a video camera, the video signal is processed and translated to some symbolic representation. As the agent explores the world it maintains a data structure of what it has explored. The internal structure of deductive reasoning agents is illustrated in Figure 10.3. There are two key problems encountered when trying to build deductive reasoning agents. Firstly, the transduction problem is the problem of translating the real world into an accurate, symbolic description in time for it to be useful. Secondly, the representation or reasoning problem is the problem of representing acquired information symbolically and getting agents to manipulate/reason with it [Woo00].

2. Production systems are also an extension of expert systems. However they place more emphasis how decisions are made based on the state of the KB. The general structure of production system agents is illustrated in Figure 10.4. The KB is called working memory and is aimed to resemble short term memory. They also allow a designer to create a large set of condition-action rules called productions that resemble long term memory. When a production is executed it is able cause changes to the environment or directly change the working memory. This in turn possibly activates other productions. Production systems typically contain a small working memory, and a large number of rules that can be executed so fast that production systems are able to operate in real time with thousands of rules [RN03]. An example of a production–rule agent development environment is called SOAR (State, Operator And Result). SOAR uses a KB as a problem space and production rules to look for solutions in a problem. IT has a powerful problem solving mechanism whereby every time that it is faced with more than one choice of productions (via a lack of knowledge about what is the best way to proceed) it creates an impasse that results in branching of the paths that it takes through the problem space. The impasse asserts subgoals that force the creation of sub–states of problem solving behavior with the aim to resolve the super–state impasse [Sio05, II07b].

10.2.2 Reactive Agents

Deliberate agents were originally developed using traditional software engineering techniques. Such techniques define pre-conditions required for operation and post–conditions that define the required output after operation. Some agents however, cannot be easily developed using this method because they maintain a constant interaction with a dynamic environment, hence they are called reactive agents. Reactive agents as especially suited for real–time applications where there are strict time constraints (i.e., milliseconds) on choosing actions.

Reactive systems are studied by behavioral means where researchers have tried to use entirely new approaches that reject any symbolic representation and decision making. Instead, they argue that intelligent and rational behavior emerges from the interaction of various simpler behaviors and is directly linked to the environment that the agent occupies [Woo00]. The general structure of reactive agents is illustrated in Figure 10.5 The main contributor of reactive agent research is Rod Brooks from MIT, with his *subsumption architecture*, where decision making is realized through a set of task–accomplishing behaviors. Behaviors are arranged into layers where lower layers have a higher priority and are able to inhibit higher layers that represent more abstract behaviors [Bro86]. A simple example of the subsumption architecture is a multi–agent system used to collect a specific type of rock scattered in a particular area on a distant planet. Agents are able to move around, collect rocks and return to the mother–ship. Due to obstacles on the surface of the planet, agents are not able to communicate directly, however they can carry special radioactive crumb that they drop on the ground for other agents to detect. The crumbs are used to leave a trail for other agents to follow. Additionally, a powerful locater signal is transmitted from the mother–ship, agents can find the ship by moving towards a stronger signal. A possible behavior architecture for this scenario are the following set of *heuristic IF–THEN rules*:

1. IF detect an obstacle THEN change direction (this rule ensures that the agent avoids obstacles when moving);
2. IF carrying samples and at the base THEN drop samples (this rule allows agent to drop samples in the mother–ship);

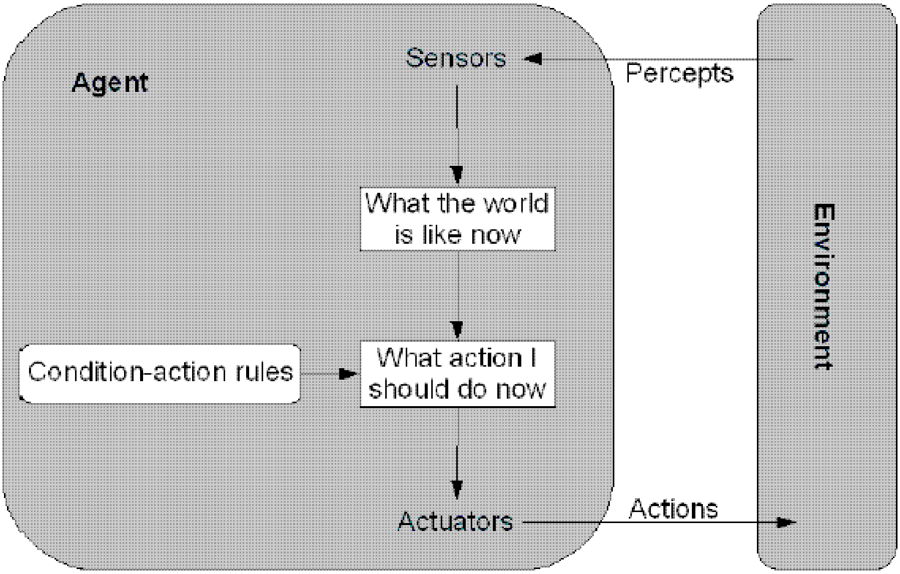

Fig. 10.5 A concept of reactive agents (modified from [RN03])

3. IF carrying samples and not at the base THEN drop 2 crumbs and travel up signal strength (this rule either reinforces a previous trail or creates a new one);
4. IF detect a sample THEN pick sample up (this rule collects samples);
5. IF sense crumbs THEN pick up 1 crumb and travel away from signal strength (this rule follows a crumb trail that should end at a mineral deposit; crumbs are picked up to weaken the trail such that it disappears when the mineral deposit has depleted);
6. IF true THEN move randomly (this rule explores the area until it stumbles upon a mineral deposit or a crumb trail).

10.2.3 Hybrid Agents

Hybrid agents are capable of expressing both reactive and pro-active behavior. They do this by breaking reactive and proactive behavior into different subsystems called layers. The lowest layer is the reactive layer and it provides immediate responses to changes for the environment, similarly to the subsumption architecture. The middle layer is the planning layer that is responsible for telling the agent what to do by reviewing internal plans, and selecting a particular plan that would be suitable for achieving a goal. The highest layer is the modelling layer that manages goals. A major issue encountered when developing solutions with hybrid reasoning agents is that agents must be able to balance the time spent between thinking and acting. This

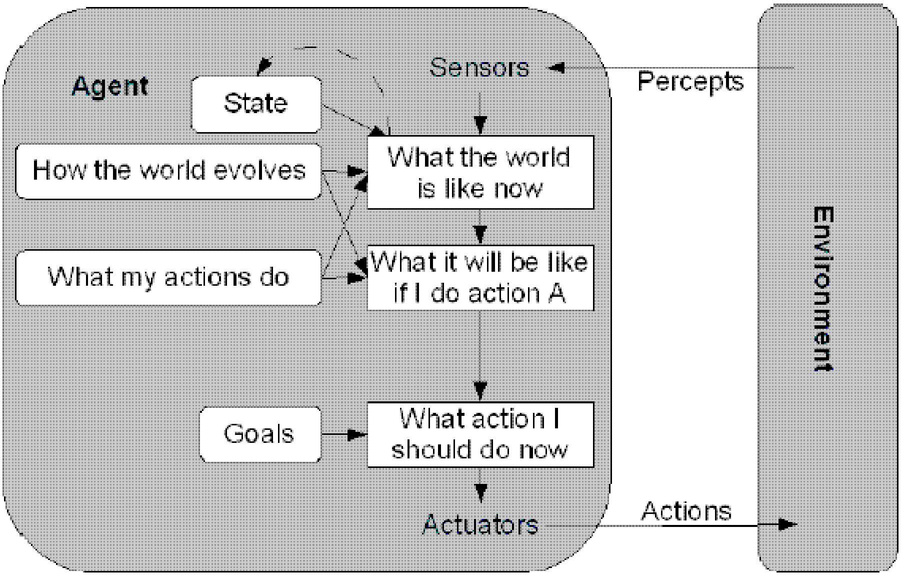

Fig. 10.6 A concept of hybrid, goal–directed agents (modified from [RN03])

includes being able to stop planning at some point and commit to goal, even if that goal is not optimal [Woo00]. The general structure of hybrid agents is illustrated in Figure 10.6.

10.3 *CSB*–Agents' Environments

Agent technology has been applied to many different application areas, each focusing on a specific aspect of agents that is applicable to the domain at hand. The role that BDI–agents play in their environment distinctly depends on the application domain. The agent research community is very active and environments are mostly viewed as test–beds for developing new features in agents and showing how they are successfully used to solve a particular problem. Fortunately, in most cases this is a two–sided process, by understanding, developing and improving new agent technologies it becomes possible to solve similar real life problems. Consequently, as the underlying foundation of agent software matures, new publications describe how agents are being applied successfully in increasingly complex application domains [Sio05, II07b].

The BDI–agent is usually understood to be a *decision–maker* and anything that it interacts with, comprising everything outside the agent itself, is referred to as the *environment*. The environment has a number of features and generates *sensations* that contain some information about the features. A *situation* is commonly understood as a complete snapshot of the environment for a particular instance in time.[2] Hence, if an agent is able to get or deduce the situation of its environment it would know everything about the environment at that time. A *state* is here defined as a snapshot of the agent's beliefs corresponding to its limited understanding of the environment. This means that the state may or may not be a complete or accurate representation of the situation. This distinction supports research being conducted on improving the agent's *situation awareness* (SA), whereby SA measures how similar the state is as opposed to the situation.

The agent and the environment interact continually, the agent selects actions and the environment responds to the actions by presenting new sensations to the agent [SB98]. The interaction is normally segmented in a sequence of discrete time steps, whereby, at a particular time step the agent receives data from the environment and on that basis selects an action. In the next time step, the agent finds itself in a new state (see Figure 10.1).

Various properties of environments have been classified into six categories [RN03]:

1. *Fully observable or partially observable.* A fully observable environment provides the agent with complete, accurate and up–to–date information of the entire situation. However, as the complexity of environments increases, they become less and less observable. The physical world is considered a partially observable environment because it is not possible to know everything that happens in it [Woo00]. On the other hand, depending on the application, the environment should not be expected to be completely observable (e.g., if an agent is playing a card game it should not be expected to know the cards of every other player). Hence, in this case, even though there is hidden information in the environment and this information would be useful if the agent knew it, is not necessary for making rational decisions [SB98]. An extension of this property is when sensations received from the environment are able to summarize past sensations in a compact way such that all relevant information from the situation can be deduced. This requires that the agent maintains a history of all past sensations. When sensations succeeds in retaining all relevant information, they are said to have the Markov property. An example of a Markov sensation for a game of checkers is the current configuration of the pieces on the board, this is because it summarizes the complete sequence of sensations that led to it. Even though much of the information about the sequence is lost, all important information about the future of the game is retained. A difficulty encountered when dealing with partially observable environments is when the agent is fooled to perceiving two or more different situations as the same state, this problem is known as perceptual aliasing. If the same action is required for the different situations then aliasing is a desirable

[2] In a number of references, the term state is used with the same meaning. In this section a clear distinction is made between the two terms, a situation is defined as a complete snapshot of the real environment.

effect, and can be considered a core part of the agent's design, this technique is commonly called state generalization [SB98].

2. *Deterministic or stochastic.* Deterministic is the property when actions in the environment have a single guaranteed effect. In other words, if the same action is performed from the same situation, the result is always the same. A useful consequence of a deterministic environment is the ability to predict what will happen before an action is taken, giving rise to the possibility of evaluating multiple actions depending on their predicted effects. The physical world is classified as a stochastic environment as stated by [Woo00]. However, if an environment is partially observable it may appear to be stochastic because not all changes are observed and understood [RN03], if more detailed observations are made, including additional information, the environment becomes increasingly deterministic.
3. *Episodic or sequential.* Within an episodic environment, the situations generated are dependent on a number of distinct episodes, and there is no direct association between situations of different episodes. Episodic environments are simpler for agent development because the reasoning of the agent is based only on the current episode, there is no reason to consider future episodes [Woo00]. An important assumption made when designing agents for episodic environments, is that all episodes eventually terminate no matter what actions are selected [SB98]. This is particularly true when using learning techniques that only operate on the completion of an episode through using a captured history of situations that occurred within the episode. Actions made in sequential environments, on the other hand, affect all future decisions. Chess is an example of a sequential environment because short–term actions have long–term consequences.
4. *Static or dynamic.* A static environment is one that remains unchanged unless the agent explicitly causes changes through actions taken. A dynamic environment is one that contains other entities that cause changes in ways beyond the agents control. The physical world continuously changes with external means and is therefore considered a highly dynamic environment [Woo00]. An example of a static environment, is an agent finding its way though a 2D maze. In this case all changes are caused by the same agent. An advantage of static environments is that the agent does not need to continuously observe the environment while its deciding the next action. It can take as much time as it needs to make a decision and the environment will be the same as when previously observed [RN03].
5. *Discrete or continuous.* An environment is discrete if there is a fixed, finite number of actions and situations in it [Woo00]. Simulations and computer games are examples of discrete environments because they involve capturing actions performed by entities, processing the changes caused by the actions and providing an updated situation. Sometimes however, this process is so quick that the simulation appears to be running continuously. An example of a continuous environment is taxi driving, because the speed and location of the taxi and other cars changes smoothly over time [RN03].

6. *Single–agent or multi–agent.* Although the distinction between single and multi–agent environments may seem trivial, recent research has surfaced some interesting issues. These arise from the question of what in the environment may be viewed as another agent [RN03]. For example, does a taxi driver agent need to treat another car as an agent? What about a traffic light or a road sign? An extension to this question is when humans are included as part of the design of the system, giving rise to the new research area called human–agent teaming [Sio05, II07b].

10.4 $CSB-$Agents' Reasoning and Learning

The environments described above illustrate the need for *adaptation* when agent systems are required to interact with complex environments. Here we sill review how agents and humans are understood to perform reasoning and learning when they are faced with a particular environment.

Reasoning is understood as the thinking process that occurs within an agent that needs to make a particular decision. This topic has been tackled via two parallel directions with two different schools of thought. The first school of thought focuses on how agents can perform rational reasoning where the decisions made are a direct reflection of knowledge. The advantage of this approach is that decisions made by an agent can be understood simply by looking within its internal data structures, as the agent only makes decisions based on what it knows. This process includes maintaining the agent's knowledge base such that it contains accurate information about its environment, by performing operations in order to keep all knowledge consistent. Decisions are made through a collection of rules applied on the knowledge base that define what should occur as knowledge changes [Sio05, II07b].

Another school of thought is concerned with the way that humans perform reasoning and apply any concepts developed to agent technology. Humans are known to perform practical reasoning every day, their decisions are based on their desires and their understanding in regards to how to go about achieving them. The process that takes place between observing the world, considering desires and taking actions can be broken up into four main stages, each of which consists of a number of smaller components. Through learning, it also becomes possible to create agents that are able to change the way that they were originally programmed to behave. This can be advantageous when an agent is faced with a situation that it does not know how to proceed. Furthermore, it is useful when an agent is required to improve its performance with experience.

10.4.1 Reasoning and Behavior

Research on artificial reasoning and behavior has been tackled from different angles that can be categorized along two main dimensions (see Figure

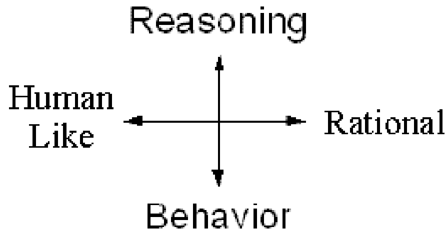

Fig. 10.7 Reasoning dimensions (modified from [RN03])

10.7). The vertical dimension illustrates the opposing nature of reasoning and behavior that correspond to thinking versus acting respectively. This is an important feature concept in every application using AI techniques. Great emphasis is given to the balance between processing time for making better decisions, and the required speed of operation. Approaches falling to the left side are based on how humans reason and behave while approaches falling on the right side are concerned with building systems that are rational, meaning that they are required to think and act as best they can, given their limited knowledge [RN03].

10.4.2 Rational Reasoning

Representation and search. Recall that the way that information is represented and used for intelligent problem solving forms a number of important but difficult challenges that lie within the core of AI research. Knowledge representation is concerned with the principles of correct reasoning. This involves two parallel topics of research. One side is concerned with the development of formal representation languages with the ability to maintain consistent knowledge about the world, the other side is concerned with the development of reasoning processes that bring the knowledge to life. The output of both of these areas results in a Knowledge Base (KB) system. KBs try to create a model of the real world via the collection of a number of sentences. An agent is normally able to add new sentences to the knowledge base as well as query the KB for information. Both of these tasks may require the KB to perform inference on its knowledge, where an inference is defined as the process of deriving new sentences from known information. An additional requirement of KBs is that when an agent queries the KB, the answer should be inferred from information previously added to the KB and not from unknown facts. The most important part of a KB is the logic in which the its sentences are represented. This is because all sentences in a KB are in fact expressed according to the syntax and semantics of the logic's representation language. The syntax of the logic is required for implementing well formed sentences while the semantics define the truth of each sentence with respect to a model of the environment being represented [RN03].

Problem solving using KBs involves the use of search algorithms that are able to search for solutions between different states of information within the

KB. Searching involves starting from an initial state and expanding across different successor state possibilities until a solution is found. When a search algorithm is faced with a choice of possibilities to consider, each possibility is thoroughly searched before moving to the next possibility. Search however has a number of issues, including [Lug02]:
(i) Guarantee of a solution being available; (ii) Termination of the search algorithm; (iii) The optimality of a particular solution found; and (iv) The complexity of the search algorithm with respect to the time and memory usage.

State space analysis is done with the use of graphs. A graph is a set of nodes with arcs that connect them, each node can have a label to distinguish it from another node and arcs can have directions to indicate the direction of movement between the nodes. A path in the graph connects a sequence of nodes with arcs and the root is a node that has a path to all other nodes in the graph.

There are two ways to search a state space, the first way is to use data-driven search by which the search starts by a given set of facts and rules for changing states. The search proceeds until it generates a path that leads to the goal condition. Data driven search is more appropriate for problems in which the initial problem state is well defined, or there are a large number of potential goals and only a few facts to start with, or the goal state is unclear [Lug02].

The second way is to use goal-driven search by which the search starts by taking the goal state and determining what conditions must be true to move into the goal state. These conditions are then treated as subgoals to be searched. The search then continues backwards through the subgoals until it reaches the initial facts of the problem. Goal driven search is more appropriate for problems in which the goal state is well defined, or there are a large number of initial facts making it impractical to prefer data driven search, or the initial data is not given and must be acquired by the system [Lug02].

The choice of which of the options to expand first is defined by the algorithm's search strategy. Two well known search strategies are: Breadth-first, where all successors of a given depth are expanded first before any nodes at the next level. Depth-first search involves expanding the deepest node for a particular option before moving to the next option. There are also strategies that include both elements, for example defining a depth limit for searching in a tree. It is also possible to use heuristics to help with choosing branches that are more likely to lead to an acceptable solution. Heuristics are usually applied when a problem does not have an exact solution or the computational cost to find an exact solution is too big. They reduce the state space by following the more promising paths through the state space [RN03].

An additional layer of complexity in knowledge representation and search is due to the fact that agents almost never have access a truly observable environment. Which means that agents are required to act under *uncertainty*. There are two techniques that have been used for reasoning in uncertain situations. The first involves the use of probability theory in assigning a value that represents a degree of belief in facts in the KB. The second method involves the use of *fuzzy sets* (see below) for representing how well a particular object satisfies a vague description [RN03].

Expert systems. Recall that knowledge–based reasoning systems are commonly called *expert systems* because they work by accumulating knowledge extracted from different sources, and use different strategies on the knowledge in order to solve problems. Simply put, expert systems try to replicate what a human expert would do if faced with the same problem. They can be classified into different categories depending on the type of problem they are used to solve [Lug02]:

- *interpretation*: making conclusions or descriptions from collections of raw data;
- *prediction/forecasting*: predicting the consequences of given situations;
- *diagnosis*: finding the cause of malfunctions based on the symptoms observed;
- *design*: finding a configuration of components that best meets performance goals when considering several design constraints;
- *planning*: finding a sequence of actions to achieve some given goals using specific starting conditions and run–time constraints;
- *monitoring*: observing a system's behavior and comparing it to its expected behavior at run–time;
- *debugging*: finding problems and repairing caused malfunctions; and
- *control*: controlling how a complex system behaves.

A common way to represent data in a expert systems is using first–order predicate calculus formulae. For example, the sentence 'If a bird is a crow then it is black' is represented as:

$$\forall X(\mathit{crow}(X) \Longrightarrow \mathit{black}(X)).$$

Production systems are based on a model of computation that uses search algorithms and models human problem solving. Production systems consist of production rules and a working memory. Production rules are predefined rules that describe a single segment of problem–solving knowledge. They are represented by a condition that determines when the production is applicable to be executed, and an action which defines what to do when executed. The working memory is an integrated KB that contains an ever–changing state of the world.

The operation of production systems generally follows a *recognize–act cycle* (see Figure 10.8). Working memory is initialized with data from the initial problem description and is subsequently updated with new information. At every step of operation, the state presented by the working memory is continuously captured as patterns and applied to conditions of productions. If a pattern is recognized against a condition, the associated production is added to a conflict set. A conflict resolution operation chooses between all enabled

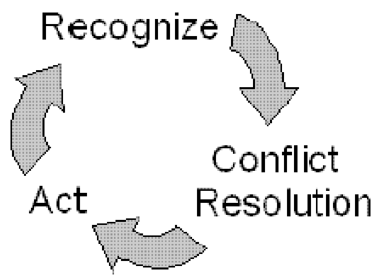

Fig. 10.8 A recognize–act operation cycle of production systems (modified from [Lug02])

productions and the chosen production is fired by executing its associated action. The actions executed can have two effects. Firstly, they can cause changes to the agent's environment which indirectly changes the working memory. Secondly, they can explicitly cause changes in the working memory. The cycle then restarts using the modified working memory until a situation when no subsequent productions are enabled. Some production systems also contain the means to do backtracking when there are no further enabled productions but the goal of the system has still not been reached. Backtracking allows the system to work backwards and try some different options in order to achieve its goal [Lug02].

Chapter 11
Psycho−*CSB*: Mental Concentration in Sport

11.1 Introduction

According to the classical electro–physiological research of human psychodynamics, in a grown–up man in a state of rest with eyes closed and free thought–flow, the most expressible EEG–component is the '$\alpha - rhythm$' – quite uniform wave pattern of amplitude about $50nV$ (registered from the surface of the skull in the parieto–occipital region) and frequency of $7 - 14Hz$. In a totally awakened state the waves of lower voltage and higher frequency of $14 - 30Hz$ predominate, the so–called '$\beta - rhythm$'. In a state of sleep the waves of higher voltage and lower frequency of $4 - 7Hz$ dominate, i.e., the '$\theta - rhythm$', while the state of deep sleep is characterized by the '$\delta - rhythm$' – big and slow wave pattern of frequency lower than $4Hz$.

Modern electro–physiological research, beginning with the work of Nobel laureate R. Sperry, completes this basic psycho–dynamic picture with the two new characteristics:

1. independent and disjunct activity of left and right hemisphere of cerebral cortex, and
2. developmental electro–physiology of psycho–dynamics.

According to this research, the leading role in a modern man's life (a right–handed person) is performed by his left brain hemisphere with its precise, sequential and linear analytical function, while his right, intuitive–synthetic cortical hemisphere is rarely used in an ordinary life, and is poorly known by analytical and linear science because of its extremely complex and nonlinear nature.

On the other hand, developmental electro–physiology of the human psycho–dynamics can be represented by hypothetical and popular '$Y - model$' (Figure 11.1) in which an age of a child (up till 20) approximately corresponds to an average daily EEG–frequency in a vigilant state [Iva91]. So, the pedestal point corresponds to child's birth and the EEG–frequency of $0Hz$, the branch–point approximately corresponds to the age of 10 years and the EEG–frequency of

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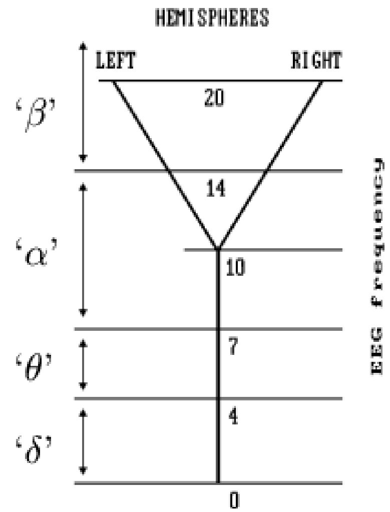

Fig. 11.1 The '$Y - model$' of human psycho–dynamics

$10Hz$, while the tops of the letter 'Y' approximately correspond to the age of 20 years and the EEG–frequency of $20Hz$. Therefore, the vertical line of the letter 'Y' covers the 'δ', the 'θ' and a half of the 'α' phases, and all this in an integral activity of the whole cerebral cortex. From the tenth year up separate functions of left and right hemispheres are being developed, going through the second half of the 'α' and remaining till the rest of life in the 'β' phase, with a pathological tendency of increasing the EEG–frequency above $30Hz$ (denoting neurotic, psychotic or epileptic pre–signs).

Psycho–CSB now states the problem of possibility of integral, synchronous and creative functioning of both brain hemispheres, with application to the *SUPREME SPORT CREATIVITY*. First, we have to activate consciously and willingly our unconscious right brain hemisphere and then release the full capacity of synchronous and integral action of the whole cerebral cortex, as is the case in a child.

The answer to this problem provides sports concentration and meditation as has been used by Olympic champion Carl Lewis and his teammates in 'Santa Monica Club' (see [Chi90]). And this answer has both theoretically and practically boundless possibilities for the transcendence of today's world records and so–called 'ending psycho–physical limits of man', which are genetically prescribed.

According to the famous sport–concentration and meditation teacher Sri Chinmoy[1], in meditation we gradually learn to control our thoughts. We can therefore become aware of what we really need in sport, whether it be:

1. improved health and fitness;
2. good feelings during sport;
3. being together with other athletes (people);
4. acceptance by other people; or
5. the challenge of growing into new areas of achievement and experience.

Through the combination of sport and meditation one can try to introduce the inner quiet of meditation, a state of 'being oneself', into the dynamism of sport. Unlike in mental training, the motivation for sport does not come from thinking, but from the psychological and spiritual qualities and capacities hidden within a person. For the athlete these can be categorized as follows:

- · · concentration;
- · · inspiration, joy, enthusiasm;
- · · discipline, determination, courage, progress;
- · · self-confidence, faith, meditation as a state of being relaxed;
- · · meditation as the training of inner abilities.

11.2 Concentration in Sport: Experiences of Top Athletes

To be able to concentrate is an ability which allows a person to learn things more consciously and more quickly in all fields of life. Whatever is done with concentration, whether it be at work or in sport, one is less absent–minded and distracted in the process. When looking at a flower or listening to a quiet piece of music, through concentrated looking or concentrated listening one's visual and auditory perceptions can be deepened. The same applies to the training stimuli in sport, which can be absorbed more consciously through concentration.

In addition to mastering techniques, choosing a healthy diet and a balanced lifestyle, concentration belongs to the most important abilities of an athlete. Concentration is imperative for success in decisive competition; in training it brings faster progress.

Carl Lewis, the long–jumper and sprinter, was famous for his great ability for concentration. He was probably the most successful athlete of all time, with nine Olympic golds (and one silver), and at the same time the great fighter against the use of drugs in sport. At the Olympic Games in Los Angeles he was four times gold–medallist, like the legendary Jesse Owens.

[1] Sri Chinmoy, the famous writer, composer, artist, philosopher and athlete, founder of the 'World Peace Run', promoted in 1990 the '20th Century First Global Man,' was the concentration and meditation teacher of may top athletes, among them Muhammed Ali and Carl Lewis.

In over 60 long–jump competitions between 1981 and 1989 he was never beaten. He says: ‘The most important thing for me was to focus inside all the time. During the race preparation it is always important to focus within yourself. At that time I concentrate completely on the track. The track is the place, where everything counts’ [Chi90].

11.3 Concentration Exercises for Training and Competition

These top athletes obviously already have the ability to concentrate. If, however, one still has difficulty with concentration, there are a few simple exercises to help. And because, above all in sport, progress is made through practice, we will begin with them straight away. At the beginning these exercises are best done at home, in the morning after getting up or whenever one has the time, but regular practice is also important. Otherwise, it is ideal to concentrate for an hour or half an hour before a competition, or immediately before a training session. The following two concentration exercises can also be incorporated very well before or after stretching, for example.

Exercise 1 (Sri Chinmoy): ‘Concentrate on only one object. If you want to concentrate on the tip of your thumb, start with imagination. Imagine that your only possession is your thumb. There is nothing else which you can claim as your own. The rest of the body does not belong to you - only the thumb. If you want to concentrate on the tip of your nose, feel that you are the possessor of only your nose; you are not the possessor of your eyes, your ears, your mouth, your limbs. If you begin to think of something else, feel that you are entering into foreign territory. In this way you will develop your power of concentration’.

Exercise 2 (Sri Chinmoy): ‘Take your right hand and with your index and middle finger touch the middle of your forehead a little above your eyebrows. Then repeat ‘Concentration’ a few times out loud and clearly (or inwardly and consciously). Then in the same manner repeat: ‘I am now concentrating’.’

11.4 Inspiration and Enthusiasm, Discipline and Progress

Inspiration is the enthusiasm we feel for sport. Not much can be achieved without it. Like positive thinking, it is often regarded as the basis of success and joy. Indeed, successful coaches also try to develop inspiration in athletes through talks and tips. One’s personal attitude, the correct choice of sport, the training–structure and the environment of the person doing sport are all important. If one has enough inspiration and enthusiasm for one’s sport, then it is much easier to have discipline, i.e. to organize one’s life in such a way that one is continuously making progress in the sport. In this way one can also maintain one's joy for the sport. For many athletes progress does not necessarily mean more outer success, but it can also be a better attitude, which leads to more satisfaction with oneself and with the sport.

Boredom with training and nervousness before a competition are signs indicating a lack of inspiration. Training can be developed most successfully if one is as relaxed as possible – with an 'empty mind' – and if one feels joy in the process.

At competitions, in spite of a good mental attitude, many athletes have that unpleasant feeling in the pit of the stomach. If this feeling is too strong, then it comes from excessive nervousness. It is then a combination of fear, restlessness and unnecessary worry about one's own performance. This nervousness can best be eased by quiet breathing. But the following breathing exercise is also recommended as preparation for a relaxed training session after a hectic day. Sri Chinmoy says that each person has over 80,000 subtle nerves and that they can be strengthened by calm, noiseless breathing.

Exercise 3 (Sri Chinmoy): 'Each time you breathe in, try to feel that you are bringing into your body peace, infinite peace. The opposite of peace is restlessness. When you breathe out, try to feel that you are expelling the restlessness within you and also the restlessness that you see all around you. When you breathe this way, you will find restlessness leaving you. After practicing this for a few times, please try to feel that you are breathing in power from the universe. And when you exhale, feel that all your fear is coming out of your body. After doing this a few times, try to feel that what you are breathing in is joy, infinite joy, and what you are breathing out is sorrow, suffering and melancholy'.

To be able to feel the positive energy of Exercise 3 during sport as well, it is good to be conscious of one's breathing.

The following athletes were successful over the years due to their inspiration, enthusiasm and discipline and because they could improve themselves again and again through these qualities.

Calvin Smith (World champion 200 m '83, '87; '83 world record 100 m 9.93; w.r. 4 x 100 m '83). 'As long as I give my best I am happy. If someone else is not happy with it, then that's his problem.

There is something in my running which I enjoy. If you enjoy something, if you enjoy running and if you are successful at the same time, then you're simply happy!

It is difficult to describe the feeling you have when you achieve a high point, for example an important win or a world record...You simply feel happy. You simply feel good. You feel that all the hard work which you put in over the many years has finally been worth it'.

Carl Lewis: 'What other people call pressure, I call inspiration'.

'For inspiration and determination I use the challenge of perfecting the best in myself. Of course, in athletics that just comes from repeated practice. The way you're going to get faster is to just keep working hard at it and wanting to do it. So many people have the capacity to be fast, but they all fall short because they do not want to work hard enough. But you also have to make sure that you do not overwork in your running. And if you ever feel tired, take a rest. If you ever feel you're not as excited on a certain day, just take a break – because there's always a reason'.

Exercise for Enthusiasm and Inspiration

Exercise 4 (Sri Chinmoy): 'While doing sport, do not think of yourself as 30, 40 or 60 years old. Only think of yourself as being 6 or 7 years old. At the age of 6 or 7, a child does not sit; he just runs here and there. So imagine the inspiration and the enthusiasm of a young child and identify yourself not with this child, but with the source of his inspiration and his enthusiasm'.

Exercise for Providing Energy and for Overcoming a Crisis

For endurance competitions in a team (long–distance runs, cycle–racing, cross–country skiing), in a crisis one can try to identify oneself with the athletes in front. In matches like tennis or football one can try to identify oneself with the 10 to 20 best players one knows.

Exercise 5 (Sri Chinmoy): 'A secret in sport is to identify yourself with ten or more of the best players you know. Only imagine the way they are breathing in and breathing out. Then, while you are inhaling, feel that you are breathing in their own breath and that the energy of the ten players is entering into you. Then, while you are exhaling, feel that all ten players are breathing out your tiredness and lack of enthusiasm.

During sport, it is difficult for you to feel that cosmic energy is entering into you. So, secretly you will breathe in the breath of ten players at a time. This energy which you get, which is nothing but enthusiasm, will carry you further, but you have to remember that you are breathing in their breath, their inspiration and determination, and not their tiredness. You are taking in the spiritual energy that is all around and inside them, just as it is inside you. But because they are better, you are more conscious of it in them'.

Chapter 12
Tennis Champion of the Future

12.1 Introduction

As the tennis game of today becomes increasingly involved with modern technology, the way that it is played changes rapidly. Modern tennis is more active than the game that was played a few decades ago, both physically and mentally. Innovative technology has revolutionized tennis into an extremely fast and dynamically efficient sport. Almost every point begins with an extremely fast serve followed by an extremely fast return. Every shot is an attempted winner, from any imaginable body position, and from any location on the court. It is quite simply a non–violent version of a deadly duel between two samurai.

Naturally, future tennis players would have to adapt in a similar way to that of the game. They would have to become stronger and faster, both physically and mentally, to succeed in the sport. Tennis balls of the future would be hit much harder by faster moving and thinking players than those today. It has been evident, even in these past few years, that balls hit by today's leading tennis players are flying through the air too fast for even the umpire's eyes[1].

Traditional tennis techniques would also have to change to accommodate a faster moving game of the future. To match a demand for more speed and strength, more efficient techniques would have to be devised. Exaggerated ballet–like swings in ground-strokes, and multiple–loops in serves, will inevitably be replaced by more efficient whip–like movements, biomechanically called "kinetic chains," or "segmental interaction", which may or may not include minor swings and loops. When a whip is cracked, a bending wave

[1] On the 14th of October, 2005, the International Tennis Federation announced that Hawk-Eye, an electronic line–calling system, had met the standards to be used for reviewing decisions made by on-court officials. Hawk-Eye was first put to use at the 18th Hyundai Hopman Cup in Perth, Australia and worked extremely well. It quickly became a very popular addition to the tennis game for players, officials, the media and the general public.

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travels along it, transferring both kinetic energy and momentum from handle to tip. These whip–like movements would have to be composed and trained as sequences, or chains of muscular stretch–reflexes (see Section 3.3, *Muscular Slingshots*), while using the whole body in a fluid and integrated manner.

This Chapter provides a method of forming a future tennis champion. It demonstrates and explains how to generate an athlete that could move, hit, anticipate, and react at a lightning–fast speed. With a 300+ km/h serve based on a Roddick–like technique, an explosive 240+ km/h forehand and a long–reaching, single–handed 200+ km/h backhand, both based on a Federer–like technique, such a tennis player would be unstoppable.[2] This Chapter also provides an approach to training a lightning–fast (complex) reaction, that would provide an efficient means to return an otherwise untouchable serve.

There is currently a kind of voting in progress for or against the possible use of bigger balls (6% larger in diameter)[3], which would slow-down all tennis shots (especially the serve), and increase rally lengths by an average of 10%. The effect that this possible change would have on the contents of this Chapter is two–fold. Firstly, if something like this were to actually happen and bigger balls were put to official use in Grand Slam and ATP–masters tournaments (which is very unlikely), tennis players with superior speed and strength would have an even greater advantage. Just imagine what such a player would be able to do with to a slow–moving tennis ball. Secondly, in the long run, this whole idea of slowing down tennis for the enjoyment of spectators is simply *ridiculous*. Today's world is one in which everything is increasing in speed. It can be seen that video games, movies, cars, trains, other sports, ... *everything* is speeding up! Is it reasonable to believe that tennis will slow down, contrary to everything else?

A future tennis champion may (or, may not) begin playing tennis at an early age, that is at the age of 3 – 4, as some of the current top players did. There is no harm in an early start, provided that it is for only half-an-hour a week. They should, however, at the same time, also start a training in gymnastics, handball and blitz–chess, all for about a half-an-hour a week. Therefore, yes an early introduction to tennis could be beneficial as long as it is not only tennis for three hours a day. At an age of about 6 – 7, a future tennis champion should still be practicing all of these activities, in parallel, only for longer periods of time (about 1 – 2 hours a week). By following the steps in this Chapter, at the tender age of 9 they should already be able to duplicate Roddick's serve and Federer's forehand and backhand.

[2] The mentioned speeds describe the magnitude of the ball's initial velocity when hit by such a stroke; e.g., if the forehand is hit from the baseline to land near the opposite baseline with an initial speed of 145 km/h, its speed at the net would be about 119 km/h and its velocity just before it bounces near the opposite baseline would be about 98 km/h (the ball's velocity would decrease because of air resistance).

[3] Current ball diameter specification last changed in 1966 (from 2.575 in to 2.675 in ($\approx$ 6.541 – 6.795 cm)).

However, unlike the current champions, at the same time, they should be able to do a somersault on the court, halfway through a tennis game – just to entertain the audience by showing what real athleticism is; in the change room, while waiting for the next game, they would be able to quickly solve a checkmate–in–3 problem; and, for fun, they would play super–fast video games, spontaneously developing a high–speed visual perception. Besides, at home, they would start developing visualization skills.

At about the age of 10 would come a shift: instead of gymnastics and handball, this entertaining tennis talent would shift to some *serious* track-and-field javelin training. Why? To begin crafting their tennis weapons. A 60+ m throw of an official (off–balance) javelin would give a 300+ km/h serve with the use of a technique like Roddick's, as well as 240+ km/h forehand and a 200+ km/h (single–handed) backhand with roughly Federer–like techniques. The real purpose of this serious javelin training, in parallel to an intense tennis training (including competing at various tennis tournaments), would be to make every tennis serve and stroke an efficient whip–like movement[4]. Each of these whip–like tennis movements would naturally be composed as a cascade of muscular stretch–reflexes (see subsection *Muscular Slingshots*).

This kind of training would be essential to make both the serve and groundstrokes really efficient weapons. If the trainee were to continue consistently in this way, that is, practicing an intense tennis and javelin training, while still doing visualization exercises and participating in minor bullet–chess tournaments, by the age of 18 they would be a tennis champion. Not only that—there is no need to begin the training at an age as early as 3–4 to create a champion by the age of 18. Starting at the age of 6–7 would be sufficient. However, even this is not essential—what is really crucial is that they develop into a champion at or near the beginning of their maturing. Such a player would be able to stay a champion for as long as the next decade.

To make the Chapter more readable for the widest possible audience, it is written in a popular (non-rigorous) question & answer format. For further reading, see the bibliography.

12.2 Contemporary Tennis Science

12.2.1 Tennis Muscles

Q. What really is the "muscle memory," so popular buzzword of coaches and players?

A. From the scientific perspective, the common term "muscle memory," so popular among coaches and players, is *shear nonsense.* If neural motor

[4] Recall the famous words of Bruce Lee, comparing a karate punch (that is, a loop–swing based tennis stroke) with a kung-fu punch (that is, a whip–like tennis stroke): "A karate punch is like an iron bar. A kung-fu punch is like an iron chain with an iron ball attached to the end." It is clear which one is more efficient.

pathways are damaged there is no any "muscle memory" left. This means that all the motor memory (containing all acquired motor skills) is in the neural system, not in the muscles. Muscles have their structure and function (of generating muscular force), they can be exercised and trained in strength, speed, endurance and flexibility, but they are still only dumb effectors (just like excretory glands). They only respond to neural command impulses. To understand the process of training motor skills we need to know the basics of neural motor control. Within the motor control muscles are force generators, working in antagonistic pairs. They are controlled by neural reflex feedbacks and/or voluntary motor inputs. If any of these neural pathways are damaged, muscles are dead flesh without any memory left (see section 2.5. Motor Control in Tennis).

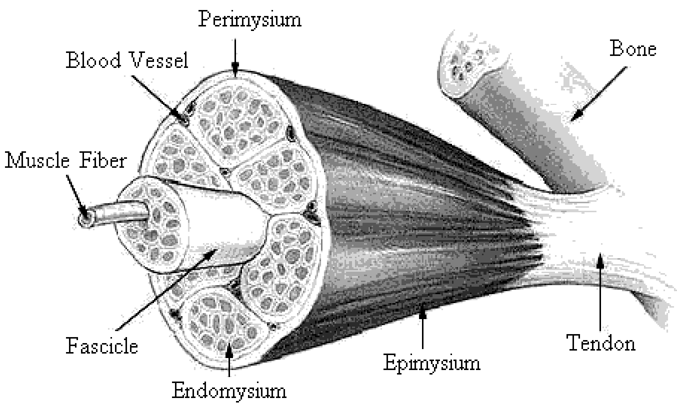

Fig. 12.1 Structure of human skeletal muscles. Epimysium consists of collagen fibers that surround muscle tissue. Perimysium is a connective tissue that separates adjacent fasciculi (small bundles of muscle fibers) in a skeletal muscle. Endomysium surrounds individual fibers. Epimysium and perimysium contain blood vessels and nerves.

Q. What is the basic structure and function of skeletal muscles?

A. Skeletal muscles are composed of muscle fibers wrapped by fascicles that also enveloped blood vessels. Muscles are connected to adjacent bones via elastic tendons, which are made up of epimysium, perimysium and endomysium (see Figure 12.1). Each muscle fiber (or muscle cell) is made up of many myofibrils. Each myofibril contains a series of contractile units called sarcomeres, made up from two types of protein filaments: thin actin filaments and thick myosin filaments (see Figure 12.2).

The muscles are "slaves" that contract (shorten) only when their "masters", called *motor units*, fire (see Figure 12.3), otherwise they relax. Therefore, muscles can only pull (contract), never push. They usually work in mutually antagonistic pairs (see Biomechanics section below).

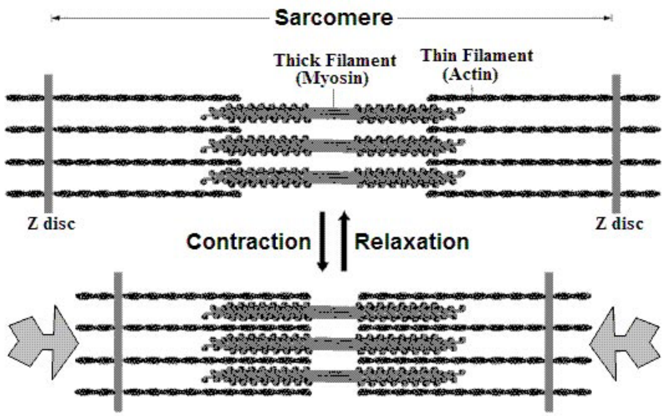

Fig. 12.2 Basic diagram of sarcomere shortening

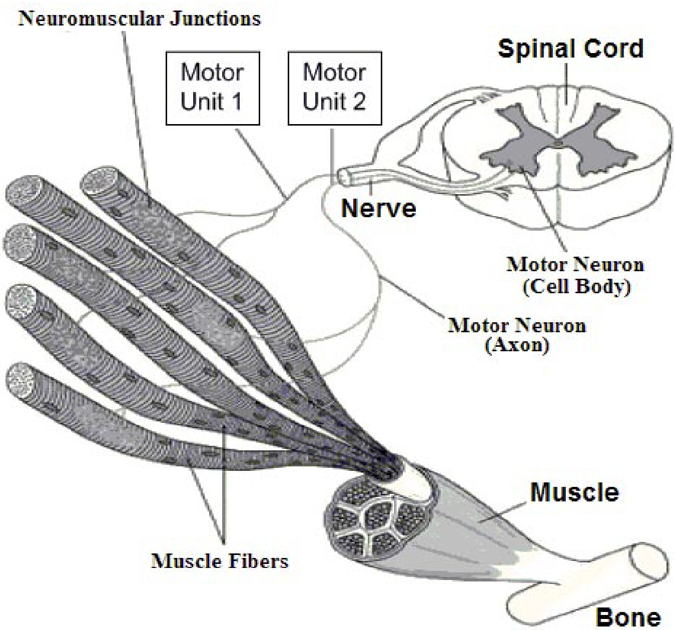

Fig. 12.3 One motor nerve/neuron and the muscle fibers that it controls via two motor units. Motor units fire fully or not at all (all or none Law).

All muscles have the following three basic properties:

1. Contractility (shortening while generating a force);
2. Excitability (responding to neural stimuli);
3. Elasticity (recoiling to original resting length after being stretched).

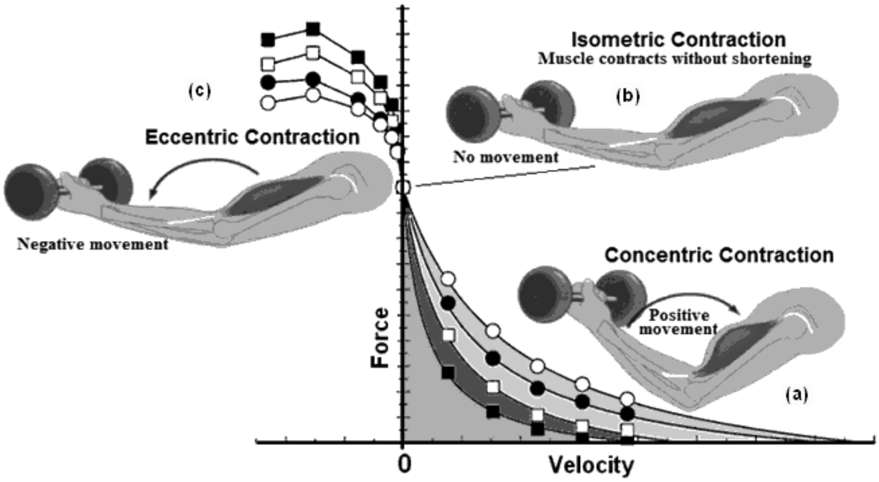

Fig. 12.4 Muscular force–velocity curve related to three types of muscular contractions: (a) concentric contraction (gives the positive movement), (b) isometric contraction (no movement), and (c) eccentric contraction (negative movement). The graph shows movements performed with four different speeds. The shaded area under the curve represents *muscular power* (explained in the section 12.2.3 below).

There are three basic types of muscular contractions, closely related to the fundamental *force–velocity curve* (see Figure 12.4):[5]

1. Concentric contraction, in which muscle shortens and makes positive movement – its generated force is greater than the loading force;
2. Isometric contraction, in which muscle generates force without shortening (there is no visible movement) – its generated force is equal to the loading force;
3. Eccentric contraction, in which muscle generates force, yet the negative movement occurs because the loading force is greater than the muscle–generated force.

1. When a muscle is activated and required to lift a load which is less than the maximum tetanic tension it can generate, the muscle begins to shorten. Contractions that permit the muscle to shorten are referred to as concentric contractions. An example of a concentric contraction in the raising of a weight during a bicep curl. In concentric contractions, the force generated by the muscle is always less than the muscle's maximum. As the load that the muscle is required to lift decreases, contraction velocity increases. This

[5] The force–velocity curve (Figure 12.4) shows the relationship between muscle tension (or, generated force) and the velocity of its shortening or lengthening. This fundamental muscular curve is used to analyze the effects of the speed–strength training, as well as to identify muscle fibre types used in different physical activities.

occurs until the muscle finally reaches its maximum contraction velocity. By performing a series of constant velocity shortening contractions, a force-velocity relationship can be determined.

2. In isometric contraction the muscle is activated, but instead of being allowed to lengthen or shorten, it is held at a constant length. An example of an isometric contraction would be carrying an object in front of you. The weight of the object would be pulling downward, but your hands and arms would be opposing the motion with equal force going upwards. Since your arms are neither raising or lowering, your biceps will be isometrically contracting. The force generated during an isometric contraction is wholly dependant on the length of the muscle while contracting.

3. During normal activity, muscles are often active while they are lengthening. Classic examples of this are walking, when the quadriceps (knee extensors) are active just after heel strike while the knee flexes, or setting an object down gently (the arm flexors must be active to control the fall of the object). As the load on the muscle increases, it finally reaches a point where the external force on the muscle is greater than the force that the muscle can generate. Thus even though the muscle may be fully activated, it is forced to lengthen due to the high external load. This is referred to as an eccentric contraction (please remember that contraction in this context does not necessarily imply shortening). There are two main features to note regarding eccentric contractions. First, the absolute tensions achieved are very high relative to the muscle's maximum tetanic tension generating capacity (you can set down a much heavier object than you can lift). Second, the absolute tension is relatively independent of lengthening velocity. This suggests that skeletal muscles are very resistant to lengthening.

Q. Are there specialized slow and fast muscles?

A. Yes, some muscles are more suited for explosive speed–strength type activities, while others are adapted for endurance exercises. More precisely, there are three basic types of human skeletal muscles:

- Type I - slow twitch muscles, which are dark–red in color (under electronic microscope; they are predominately used in the aerobic energy system; they have high concentration of myoglobin, lots of capillaries and mitochondria.)
- Type IIA - intermediate fast twitch, which are reddish–white in color; their contraction is moderately fast; they have moderate myoglobin concentration; they are predominately used in anaerobic glycolysis energy system.
- Type IIB - ultra fast twitch, which are white in color; their contraction speed is the fastest; they have low myoglobin concentration; they are predominately used in anaerobic alactatic energy system.

Q. Which type of muscles is essential for the success in tennis?

A. None. What is crucial for the success in tennis is neuro–motor control, of which the most important is the stretch–reflex (we will talk about it later).

12.2.2 Tennis Anatomy

Q. Can you give us a general anatomical description of a tennis serve?

A. Classical tennis serve has three stages:[6] (i) the ball toss, (ii) the jump, and (iii) the finishing smash.

(i) In the case of a right–handed player (like Federer), the ball toss is thrown with the left arm. The feet are apart, and the ball toss is performed with the contractions of the left deltoideus, the biceps and the palmar flexors muscles. This movement is done simultaneously with two other preparatory actions.

The first one of these preparatory actions is raising the right arm, "loading". The muscles used to carry this out are the right deltoideus, supraspinatus (a muscle going over the shoulder blade) and the biceps brachii. The second action is bending the knees, and thus preparing for the second stage of the serve (the jump). There are no flexor muscles used to bend the knees, for the bending of knees is accomplished by gravity alone (actually, leg–extensors are used in eccentric fashion).

(ii) The first–serve jump is performed high and forward. It is achieved by instantaneous actions of all the leg extensor muscles; left and right soleus, quadriceps femoris and gluteus maximus muscles. Jumping is the second part of "loading" in the serve. At the same time as the player lifts off, the racquet is placed behind the body, in a "back–scratching" position, and the right shoulder's rotation towards the ball begins. This movement involves the right biceps brachii and wrist extensor muscles. While in the air, the feet naturally join together (with Federer, the feet join in the air, not on the ground).

(iii) The finishing smash takes place in the air, before the player returns to the ground. To end the serve, the shoulders are rotated and the ball is hit simultaneously. By then, the shoulders should have been fully rotated and the feet prepared for landing. The internal and external obliques abdominal muscles complete the shoulder rotation. Hitting the ball is performed by the latissimus dorsi, then pectoralis major and finally triceps brachii muscles. To add a bit of spin or slice to the serve, the wrist is flicked slightly at the end, using the palmar flexors.

Q. Can you give us a general anatomical description of a forehand?

A. Standard tennis forehand, in the case of a right–handed player (like Federer), basically has two phases: (i) preparation, or "loading", and (ii) hitting the ball.

(i) Preparation for the forehand includes two simultaneous actions. One is stepping into the right position, with the left leg forward. The other is the first half of the loop movement, lifting the racquet above the shoulders in a curved c-shaped movement. (This does not need to be too far back, like in

[6] We skip the notorious preparation.

Hewitt's forehand, but can be more to the side.) This is accomplished by the right deltoideus and biceps brachii muscles.

(ii) Hitting the ball includes four main movements. The first of these four is a right hip rotation towards the ball, while the feet are still on the ground. The right gluteus maximus and medius muscles carry out this action.

Secondly, a leap into the air is necessary to be able to hit the ball from a higher body position, so as not to hit the net. This is performed by all the leg extensor muscles: left and right soleus, quadriceps femoris and gluteus maximus – working together.

Thirdly, an arm swing of the racquet, the second part of the loop action. The right pectoralis major, deltoideus and biceps brachii muscles complete this.

Lastly, to create topspin; a slight twist of the wrist to just brush over the ball. This is done by the right palmar flexors.

Q. Can you give us a general anatomical description of a two-handed backhand?

A. A common right–handed player's two-handed backhand (e.g., M. Safin's) basically has two stages: (i) preparation, or "loading", and (ii) hitting the ball.

(i) Preparation for the double-handed backhand includes two phases. One is stepping into the right position, with the right leg forward. The other is lifting the racquet in a loop movement similar to the forehand loop, only this time the racquet tends not to go further than about the shoulder–level. This is accomplished by both the left and right deltoideus and biceps brachii muscles.

(ii) Hitting the ball includes four main movements. The first of these four is a left hip rotation towards the ball. The left gluteus maximus and medius muscles carry out this action, helped by the left knee extension, which is performed by the quadriceps femoris.

Secondly, a rotation of the left shoulder towards the ball, achieved by both the internal and external obliques abdominal muscles.

Thirdly, the arm swing of the racquet is performed by pulling of the right arm and pushing of the left arm. The pulling action of the right arm is accomplished by the right latissimus dorsi, pectoralis major and triceps brachii muscles. The pushing action of the left arm is accomplished by the left deltoideus, pectoralis major and biceps brachii muscles.

Lastly, to create topspin; a slight twist of both wrists. This is done by the left palmar flexors and right dorsal flexors.

12.2.3 Tennis Energetics

Q. What are the sources of energy for muscular work in tennis?

A. There are three different energetic resources for any kind of muscular work, including tennis:

1. ATP–CP (or, anaerobic[7] alactatic) system, which lasts 10–15 seconds, uses stored ATP[8] and creatine phosphate (CP), with no by–products. This energy source is related to speed and strength. It is essential for the serve and winning shots in tennis.
2. Glycolysis (or, anaerobic lactic) system, which lasts 15 seconds – 3 minutes, uses blood glucose and muscular glycogen to make ATP; its by–product is lactic acid. This energy source represents anaerobic endurance. It is essential in long, exhausting tennis relays.
3. Aerobic system,[9] which lasts from 2–3 minutes to several hours, uses glucose, glycogen, fats, and proteins to make ATP within the aerobic energy pathway; its by–products are carbon dioxide and water. This energy source represents aerobic endurance. It is essential in the fourth and fifth set of any serious tennis match.

Q. What is VO_2−max?

A. VO_2−max, or maximal oxygen consumption, is the maximum volume of oxygen (O_2) that the human body can consume by breathing air at sea level during an intense whole–body exercise. This volume is expressed as a rate in milliliters per kg bodyweight per minute (ml/kg/min). As O_2−consumption is linearly related to energy expenditure, when we measure O_2−consumption, we are indirectly measuring an individual's maximal capacity for aerobic work.

Every cell consumes O_2 in order to convert food energy to usable ATP for cellular work (see Figure 12.5). However, it is muscle that has the greatest range in oxygen O_2−consumption (at rest, muscle uses little energy; however, contracting muscle cells have high demands for ATP, proportionally to the work intensity). Endurance athletes have developed a strong cardiovascular system, as well as a strong oxidative capacity in their skeletal muscles. To receive this O_2 and use it to make ATP for muscular contraction, our muscle fibers depend on 2 things: (i) an external delivery system to bring O_2 from the atmosphere to the working muscle cells, and (ii) mitochondria in the muscle cells to carry out the process of aerobic energy transfer. In other words, we need a big and efficient heart–pump to deliver O_2−rich blood to the muscles, and we also need mitochondria–rich muscles to use the O_2 and support high rates of exercise.

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7 "Anaerobic" means without the use of oxygen, that is, none of its metabolic activity will involve 0_2.

8 Adenosine–Triphosphate (ATP) is the fundamental energy source for muscular work, made in the mitochondria of muscular cells. It gives energy for muscular contraction according to the equation:

$$ATP \to ADP + P_i + Energy$$

where ADP is Adenosine–Diphosphate and P_i is inorganic phosphate. ATP is resynthesized both aerobically and anaerobically.

9 "Aerobic" means in the presence of oxygen (0_2), that is, all of its metabolic activity will involve 0_2.

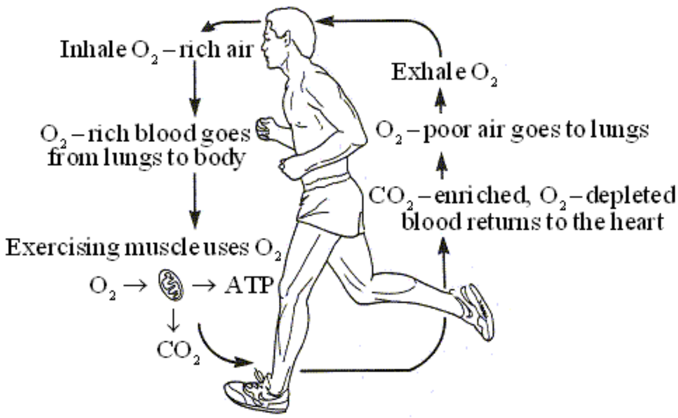

Fig. 12.5 O_2–consumption by skeletal muscles in mitochondrias

Q. What is lactic acid?

A. The carbohydrates you consume consist of several different sugar molecules; sucrose, fructose, glucose, etc. However, by the time the liver does it's job, all of these sugars are converted to glucose which can be taken up by all cells. Muscle fibers take up glucose and either use it immediately, or store it in the form of long glucose chains (polymers) called glycogen. During exercise, glycogen is broken back down to glucose which then goes through a sequence of enzymatic reactions that do *not* require oxygen to proceed. All of these reactions occur out in the cell fluid, or cytosol. These reactions proceed very rapidly and yield some energy for muscle work in the process. This glycogen/glucose breakdown pathway is called the *anaerobic glycolysis (or, glucose breakdown without oxygen) pathway*. Every single glucose molecule must go through this sequence of reactions for useful energy to be withdrawn and converted to ATP, the energy molecule that fuels muscle contraction, and all other cellular energy dependant functions.

In a single contracting muscle fiber the frequency and duration of contractions will determine ATP demand. ATP demand will be met by breaking down a combination of two energy sources: fatty acids and glucose molecules (ignoring the small contribution of protein for now). As ATP demand increases, the rate of glucose flux through glycolytic pathway increases. Therefore at high workloads within the single fiber, the rate of pyruvic acid production will be very high. If the muscle fiber is packed with lots of mitochondria, pyruvate will tend to be converted to Acetyl CoA and move into the mitochondria, with relatively little lactate production. Additionally, fatty acid metabolism will account for a higher percentage of the ATP need. Fat metabolism does not produce lactate, ever! If lactate is produced from glucose breakdown, it will tend to be transported from the area of high concentration inside the muscle cell to lower concentration out of the muscle fiber and into extra-cellular fluid, then into the capillaries.

Now let's look at an entire muscle, say the quadriceps muscle group during cycling. At a low workload, glycolytic flux is low (fatty acid breakdown is relatively high at low intensities) and the pyruvate produced is primarily shuttled into the mitochondria for oxidative breakdown. Since the intensity is low, primarily slow twitch muscle fibers are active. These fibers have high mitochondrial volume. As workload increases, more fibers are recruited and already recruited fibers have higher duty cycles (more work and less rest). Now ATP demand has increased in the previously active fibers, resulting in higher rates of pyruvic acid production. A greater proportion of this production is converted to lactic acid rather than entering the mitochondria, due to competition between the two enzymes LDH and PDH. Meanwhile, some fast twitch motor units are starting to be recruited. This will add to the lactate produced in and transported out from the working muscle due to the lower mitochondrial volume of these fibers. The rate of lactate appearance in the blood stream increases.

The quadriceps is just one of several muscles that are very active in cycling. With increasing intensity, increased muscle mass is called on to meet the force production requirements. All of these muscles are contributing more or less lactic acid to the extra-cellular space and blood volume, depending on their fiber type composition, training status and activity level. However, the body is not just producing lactate, but also consuming it. The heart, liver, kidneys, and inactive muscles are all locations where lactic acid can be taken up from the blood and either converted back to pyruvic acid and metabolized in the mitochondria or used as a building block to re-synthesize glucose (in the liver). These sites have low intracellular lactate concentration, so lactic acid is transported into these cells from the circulatory system. If the rate of uptake, or disappearance, of lactate equals the rate of production, or appearance, in the blood, then blood lactate concentration stays nearly constant. But, when the rate of lactate production exceeds the rate of uptake, lactic acid accumulates in the blood volume, then we see the onset of blood lactate accumulation. This is the traditional "lactate threshold" (LT).

The following five factors influence the rate of lactate accumulation:

1. Exercise intensity;
2. Training status of active muscles (higher mitochondrial volume improves capacity for oxidative metabolism at high glycolytic flux rates);
3. Fiber type composition (slow twitch fibers produce less lactate at a given workload than fast twitch fibers, independent of training status);
4. Workload distribution (a large muscle mass working at a moderate intensity will develop less lactate than a small muscle mass working at a high intensity); and
5. Rate of blood lactate clearance (with training, blood flow to organs such as the liver and kidneys decreases less at any given exercise workload, due to decreased sympathetic stimulation; this results in increased lactate removal from the circulatory system by these organs).

Q. What is physiological efficiency?

A. Recall from above that high level endurance performance depends on two factors: (i) a high VO_2-max, and (ii) a high lactate threshold. Your VO_2-max sets the upper limit for your sustainable work potential. On the other hand, the lactate threshold tells us something about how much of the cardiovascular capacity you can take advantage of in a sustained effort; it is determined by skeletal muscle characteristics and training adaptations. Multiplying VO_2-max with LT gives us a measure of the effective size of your endurance engine. Now we come to efficiency. What does efficiency have to do with endurance performance?

Physiological efficiency is defined as the percentage of energy expended by the body that is converted to mechanical work:

Physiol. efficiency = Mechan. work/Chem. energy expended

We can measure the mechanical work performed using an ergometer, like a bicycle ergometer, or rowing machine. We can measure the energy expended by the body indirectly via its oxygen consumption at sub maximal workloads. With some basic biochemistry we can convert the oxygen consumption we measure during exercise to a standard measure of energy like kJoules, or Calories. And, we can do the same for the work we measure on the ergometer. Work/time = Power. Power is measured in watts and is a measure of the intensity of work. Intensity (watts) times exercise duration (minutes) gives us total work, again measured in kJoules or Calories.

Q. What is a proper warm-up for a tennis game or a training?

A. Whilst the warm up for participation in any sporting or exercise activity is accepted as being essential for minimizing injuries and improving performance, the methods by which many sports attempt to achieve this are less than ideal. The warm up method used by many tennis players usually includes an initial jog around the field or court, followed by 10–15 minutes of static stretching. This is then followed by a few drills, and the players then begin their training session or game. Whilst the basis behind these methods may appear to be sound applications of current training principles, a closer analysis reveals major limitations with this method of preparing a player for tennis training.

The main physiological reason for a warm up include: to increase core temperature (an increase in rectal temperature of a least one to two degree Celsius appears to be sufficient); to increase heart rate and blood flow to skeletal tissues, which improves the efficiency of oxygen uptake and transport, carbon dioxide removal, and removal and breakdown of anaerobic byproducts (lactate); to increase the activation of the Central Nervous System (CNS; therefore increasing co-ordination, skill accuracy and reaction time); to increase the rate and force of muscle contraction and contractile mechanical efficiency (through increased muscle temperature); and to increase the suppleness of connective tissue (resulting in less incidence of musculo–tendinous injuries).

The major criticism against the “typical warm-up” is that it does not adequately prepare the athletes for the demands placed upon them in the ensuring session. Generally the initial jog is at a pace that has a minimal effect upon body temperature, and usually consists of jogging forwards, and in a straight line.

Similarly, the stretching performed is usually that of static stretching, with most stretches performed slowly and with the athletes either standing still or sitting on the ground. This method of stretching has been shown to be beneficial for the increase in limb range of motion, and aims to relax the muscles so that they are less resistant to passive stress for stretching. But this type of stretching does not prepare the muscle and connective tissue for the active contraction - relaxation process that will occur with any running, jumping or hitting movements as required in a tennis game situation.

During this stretching period (typically from 5-20 minutes), the body is very efficient in removing excess body heat, so the small increase in body temperature from the initial jog is quickly lost if the athlete does nothing but statically stretch for this time. This is even more prevalent in cold climates or cold seasons. In general, many injuries occur at the beginning of a competition due largely to an inadequate preparation for the activity. Also, inadequate warming-up can lead to less than optimal speed and skill levels that could result in quick scoring by the opponent or individual early in the game leading then to opponents having to catch-up placing more pressure on the player(s) involved.

The proper warm-up should be the complete physical and mental preparation for the dynamic actions to follow. The players should be able to begin the game or training session totally ready to perform at maximal intensity if required.

The initial jog is now replaced with a more dynamic series of running exercises that include regular alternation of running forwards, backwards, sideways, high knee drills, butt flicks, crossovers, bounding, jumps and progressive sprints. This component will only take 2-4 minutes depending on the climate. It is expected that the athletes are breathing quite heavily at the end of this short series of exercises.

With the stretching component, static stretching can still be included in the program, as many athletes still feel they need some static stretching to really prepare themselves. However, the dynamic stretching component is very important for the specific preparation of the musculature to dynamic movements. Dynamic stretching is defined as repetitive contractions of an agonist (prime mover) muscle to produce quick stretches of the antagonistic muscle, so any active callisthenic movement can be classified as dynamic stretching (jumping, body rotations, bending, etc).

Q. What about nutrition and hydration?

A. Tennis matches can last much longer than bouts in many other sports, so good nutrition and hydration are very important. Because of the amount of muscle exertion and stamina needed for tennis, the most important part of a player's diet is complex carbohydrates. Professional players eat up to eight servings of pasta, rice and bread every day. This is because the energy provided by carbohydrates is released slowly, compared with simple carbohydrates like sugars which provide a quick burst in energy.

Tennis players can lose up to 2.5 liters of fluid an hour through perspiration, so it is important to replace these fluids by drinking when you play. This means hydrating (drinking) before, during and after playing. Some drinks are better at hydrating you than others, and some like caffeinated drinks actually dehydrate you. The best things to drink when you're playing tennis, or any other sport, are energy drinks, water, and fruit juice. Because of their sugar content, soft drinks give a short-term energy boost for up to half an hour. Sports energy drinks have two big advantages over water. Firstly, they contain carbohydrates, which are very important for maintaining energy levels. Secondly, sports drinks replace electrolytes, such as sodium, which are lost through perspiration.

12.2.4 Tennis Biomechanics

Q. What is the mechanics behind topspin and backspin shots?

A. A topspin shot is hit by sliding the racquet up and over the ball as it is struck. By dragging the racquet over the ball, the friction between the racquet's strings and the ball is used to make the ball spin forward, towards the opponent. The shot dips down after impact and also bounces at an angle lower to the ground than a shot hit with no topspin. As a ball travels towards a player after bouncing, it has natural topspin that is caused by the friction of the tennis court. When hitting a topspin shot, the player is reversing the spin of the ball, which requires more energy.

A backspin shot is hit in the opposite manner, by sliding the racquet underneath the ball as it is struck. This causes the ball to spin towards the player who just hit it as it travels away. Generating slice, or backspin, requires only about half the racket head speed compared to hitting topspin, because the player is not required to change the direction in which the ball is spinning. The oncoming ball bounces off the court with topspin, spinning from top to bottom as it comes toward the player. When a player returns the ball with a slice shot the direction in which the ball spins around the axis of rotation is maintained. The direction of the shot changes, but the ball continues to spin from top to bottom, from the player's perspective as it moves away from the player.

Q. So why do we really need biomechanics in tennis?

A. In tennis, we transfer the energy from our body to the ball via a tennis racket to generate speed and spin of the ball. Energy can be either potential (stored energy) or kinetic energy (energy of movement). A specific type of potential energy is elastic energy (that is, the energy which causes, or is released by, the elastic distortion of a solid or a fluid). An example of elastic energy is the energy stored in a spring under tension. The human equivalent would be energy stored in muscles and their tendons under tension. On the other hand, kinetic energy specifically refers to the work required to accelerate the ball from a resting position to a desired velocity.

Let's examine how the body transfers the necessary energy to the ball in a tennis stroke. Here, we think of the body as a series, or a chain, of linkages connected to one another and affecting each other in a specific sequence. For example, the foot is a link, which is connected to the leg by the ankle joint, which is in turn connected to the thigh by the knee joint and so on. During the initiation of a forehand ground stroke the feet are oriented for either an open stance or close stance position. The shoulder and torso are turned approximately 45 degrees, which in turn causes a "coiling" of the abdomen and pelvis, which in turn produce a slight knee bend. With the current forehand the racket is held fairly high at about head level. In this position there exists a great deal of potential energy, both in the form of gravity with the racket head up high and the form of elastic stored energy in the tensed muscles that are stretched in the coiled position (both internal and external abdominal obliques muscles, pectorals major, forearm muscles, hip girdle musculature, quadriceps femoris). This energy is released and sequence and there is an overlap in the sequence of linkages. As the racket starts to drop and begin an oval path (loop) the hips start to uncoil. The hips and knees begin to straighten. In sequence with the uncoiling of the hips the next event is the uncoiling of the torso and then the shoulders as the racket is brought forward to contact the ball. At the same time, the back leg is fully extended to powerfully drive the body up and forward. In fact, many professional players actually leave the ground during this point. At ball contact only medium grip pressure is required to guide and stabilize the racket. This is because the forward momentum will carry the racquet through the ball without much effort. After contact the shoulder and torso and hips naturally rotate towards the non-dominant side following the path of the racquet resulting in a stretch of the opposite side musculature which decelerates the racquet.

Naturally, all of this occurs in one fluid motion with precise timing so that maximum energy (and momentum) transfer occurs from loading to releasing. And, for maximum racket-head speed, some body segments may be slowed down to increase the speed of the racket, as in cracking a whip. Thus, we see that not only we do need some basic biomechanics at all levels of our tennis maturity, but as we advance in tennis, we even need a special biomechanics of whip–like movements, which is crucial to make every serve, forehand and backhand – efficient weapons (we will talk about this in detail later).

Q. What is the basic biomechanical unit?

A. The basic biomechanical unit consists of a pair of mutually antagonistic muscles producing a common muscular torque, T_{Mus}, in the same joint, around the same axes. The most obvious example is the biceps–triceps pair (see Figure 12.6). Note that in the normal vertical position, the triceps

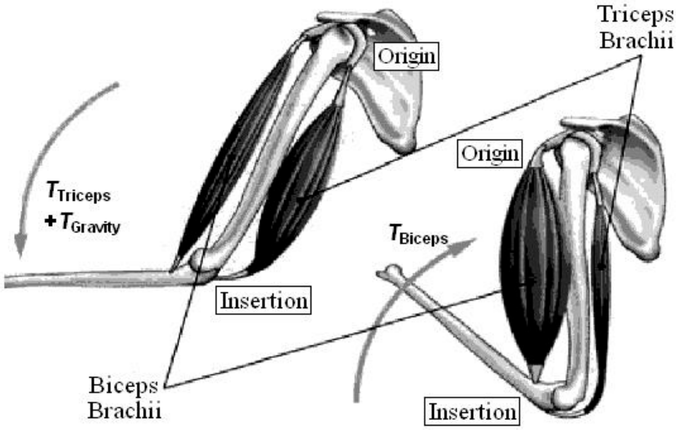

Fig. 12.6 Basic biomechanical unit: left – triceps torque T_{Triceps}; right – biceps torque T_{Biceps}

downward action is supported by gravity, that is the torque due to the weight of the forearm and the hand (with the possible load in it).

Q. What are the so–called degrees–of–freedom in human joints?

A. Some joints are only slightly movable, formed by two bones held together by cartilage, without joint cavity (e.g., an intervertebral joint in the spine consists of two vertebras and an intervertebral disc between them). On the other hand, major joints involved in human movement, like shoulder, hip, elbow and knee, are composed of several bones separated by a joint cavity, lubricated by synovial fluid and enclosed in a fibrous joint capsule. Different joints have different degrees–of–freedom (DOF) of movement: hinge joints have 1 DOF, gliding and saddle joints have 2 DOF, while ball-and-socket joints have 3 DOF (see Figure 12.7).

Q. Can you give us here the minimum of dynamics necessary for tennis biomechanics?

A. Briefly, dynamics of human motion is governed by the Newton-Euler equations of a rigid body motion in our 3D space (see Figure 12.8 depicting a tennis ball modelled as a free rigid body[10]). A rigid body freely moving in space has 6 degrees-of-freedom: 3 translations (along the X,Y,Z-axes) and 3 rotations (around the X,Y,Z-axes). Translational motion is defined by 3 Newton's equations of motion of the type:

[10] It is obvious that this is only an approximation, as a tennis ball is not rigid, but rather a soft body. However, soft–body dynamics is much more complicated, so let's stick to our rigid–body approximation. Even true rigid–body dynamics is more complicated.

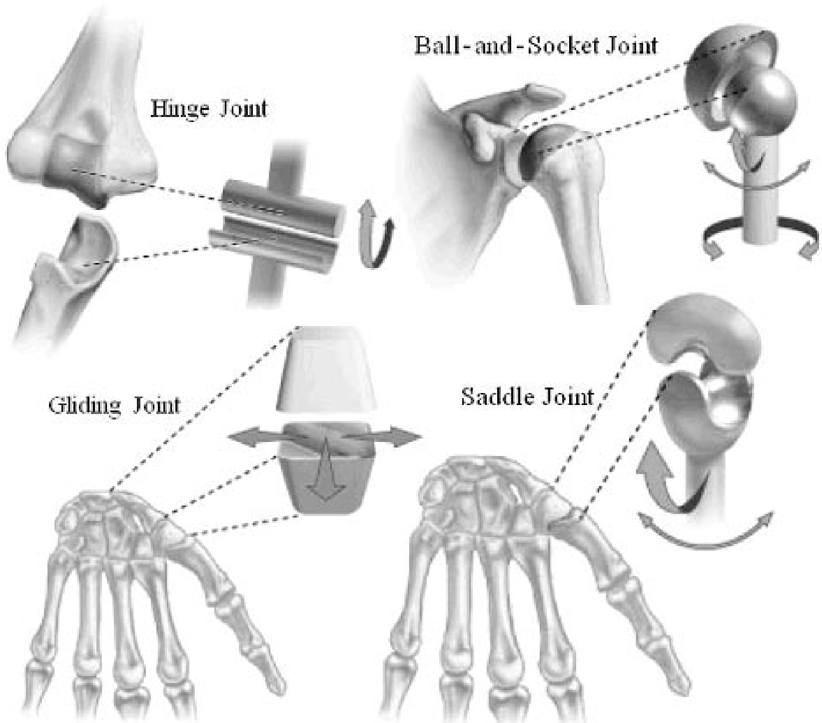

Fig. 12.7 Degrees-of-freedom in main human joints

$$\text{force } (\mathbf{F}) = \text{mass } (\mathbf{m}) \times \text{acceleration } (\mathbf{a})$$

Similarly, rotational motion (labelled by superscript R) is governed by 3 Euler's equations of motion:

$$\text{torque } (\mathbf{T}) = \text{inertia-moment } (\mathbf{I}) \times \text{rotational-acceleration } (\mathbf{a}^R).$$

In Figure 12.8, with each axis (denoted by index $\mathbf{x, y, z}$) there is associated a translational $\mathbf{F}$-equation and a rotational $\mathbf{T}$-equation; $\mathbf{v}$ denotes velocity, while dot over a quantity denotes it's rate-of-change (with respect to time).

Also,

$$\text{inertia-moment } (\mathbf{I}) = \text{mass } (\mathbf{m}) \times \text{moment-arm squared } (\mathbf{l}^2).$$

Finally,

$$\text{linear momentum } (\mathbf{p}) = \text{mass } (\mathbf{m}) \times \text{velocity } (\mathbf{v}),$$
$$\text{force } (\mathbf{F}) = \text{rate-of-change of linear momentum } (\dot{\mathbf{p}});$$

and similarly,

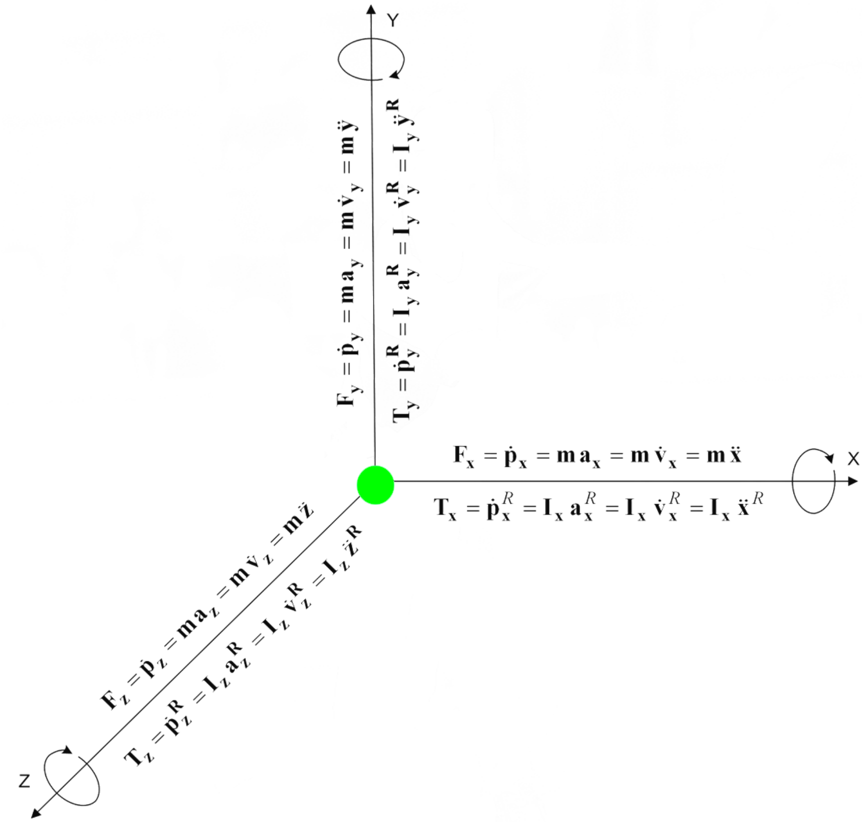

Fig. 12.8 Newton–Euler dynamics of a tennis ball modelled as a free rigid body

$$\text{angular momentum } (\mathbf{p}^R) = \text{inertia-moment } (\mathbf{I}) \times \text{angular velocity } (\mathbf{v}^R),$$
$$\text{torque } (\mathbf{T}) = \text{rate-of-change of angular momentum } (\dot{\mathbf{p}}^R).$$

Newton's Causality Principle states: a force is a cause of acceleration, which is a cause of velocity, which is a cause of linear motion. Similarly, a torque is a cause of angular acceleration, which is a cause of angular velocity, which is a cause of angular motion.

In biomechanics, the only active force is muscular force. In every major human joint an antagonistic pair of muscles generates a driving torque. Any kind of human motion is a result of driving torques in major human joints.

From muscular training perspective, the most important is *power*, which incorporates both strength and speed, mechanically defined as:

$$\text{power } (\mathbf{P}) = \text{force } (\mathbf{F}) \times \text{velocity } (\mathbf{v}).$$

Physiologically, it corresponds to the area under the force–velocity curve (see Figure 12.4). Muscular power is the key element of all *power sports*, including future tennis.

Q. What is the difference between *moment* and *momentum*?
A. To clarify this language ambiguities, we give here the main definitions:

- I: *Moment of Inertia:*

  $I \sim mr^2$: mass $\times$ moment–arm^2 $\quad$ ($\sim$ means "proportional to")

- p^R: *Angular Momentum (or, Moment of Momentum):*

  $p^R = Iv^R$: moment of inertia $\times$ angular velocity, analogous to: $p = mv$
  $p^R = p \times r$: angular momentum = linear momentum $\times$ moment–arm

- T: *Torque (or, Moment of Force)*

  $T = \dot{p}^R = I\dot{v}^R$: torque = derivative of angular momentum; this is analogous to: $F = \dot{p} = m\dot{v}$: force = derivative of linear momentum
  $T = F \times r$: torque = force $\times$ moment–arm

Note that in anatomical literature our *moment–arm* is called *lever–arm.*

Q. Why can an elite tennis player whirl around suddenly without falling down?
A. The above Newton-Euler dynamics (depicted in Figure 12.8) has its neural sensors. The *vestibular organ* in the inner ear helps to maintain equilibrium by sending the brain information about the motion (both linear and angular) and position of the head. The vestibular organs consist of three membranous semi-circular canals (SCC), and two large sacs, the utricle and saccule. All the vestibular organs share a common type of receptor cell, the hair cell.

The SCC within the vestibular organ of each ear contain fluid and hair receptor cells encased inside a fragile membrane called the cupula. The cupula is located in a widened area of each canal called the ampulla. When you move your head, the fluid in the ampulla lags behind, pushing the cupula a very tiny bit which causes the hairs to also bend a very tiny bit. The bending hairs stimulate the hair cells, which in turn trigger sensory impulses in the vestibular nerve going to the brain to "report" the movement. Hair cells are amazingly sensitive. For example, a cupula movement of even a thousandth of an inch is detected by the brain as a big stimulus.

The SCC are positioned roughly at right angles to one another in the three planes of space. Thus, the canals react separately and in combination to detect different types of angular head movement. They detect when we nod in an up and down motion (pitch), when we tilt our head to the side towards our shoulder (roll), and when we shake our head “no” in a side-to-side motion (yaw). The SCC are responsible for detecting any kind of rotational motion in the head, thus effectively sensing the Euler’s dynamics (Figure 12.8).

Two other vestibular organs are located in membranous sacs called the utricle and the saccule. On the inside walls of both the utricle and the saccule is a bed (a macula) of several thousand hair cells covered by small flat piles of calcium carbonate crystals which look like sand, imbedded in a gel-like substance. The crystals are called otoliths, a word which literally means “ear stones.” In fact, the utricle and the saccule are often called the *otolith organs*.

When a person’s head is in the normal erect position, the hair cells in the utricle lie approximately in a horizontal plane. When the head is tilted to one side, the stones want to slide “downhill.” This moves the gel just enough to bend the sensory hairs. The bending hairs stimulate the hair cells, which in turn send a signal to the brain about the amount of head tilt. The stones also move if the person is accelerated forward and back, or side to side. Similarly, the hair cells in the saccule are oriented in somewhat of a vertical position when the head is erect. When a person tilts their head, or is accelerated up and down (as in an elevator), or moved forward and back, the otoliths move and a signal is sent to the brain. The signals from the otoliths in the saccule and the utricle complement each other and give us an integrated signal about our movement. The otolith organs are primarily responsible for detecting any degree of linear motion of the head, thus effectively sensing the Newton’s dynamics (Figure 12.8).

12.2.5 Motor Control in Tennis

Q. How are tennis movements controlled by the brain?

A. All of the body’s voluntary movements are controlled by the brain. One of the brain areas most involved in controlling these voluntary movements is the motor cortex (see Figure 12.9).

In particular, to carry out goal–directed tennis movements, your motor cortex must first receive various kinds of information from the various lobes of the brain: information about the body’s position in space, from the the parietal lobe; about the goal to be attained and an appropriate strategy for attaining it, from the anterior portion of the frontal lobe; about memories of past strategies, from the temporal lobe; and so on.

There is a popular “motor map”, a pictorial representation of the primary motor cortex (or, Brodman’s area 4), called *Penfield’s motor homunculus*[11] (see Figure 12.10). The most striking aspect of this motor map is that the areas assigned to various body parts on the cortex are proportional not to their size, but rather to the complexity of the movements that they can perform. Hence, the areas for the hand and face are especially large compared

[11] Note that there is a similar “sensory map”, with a similar complexity–related distribution, called *Penfield’s sensory homunculus.*

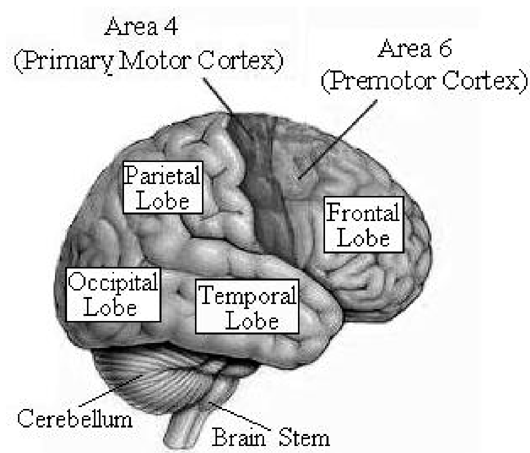

Fig. 12.9 Main lobes (parts) of the brain, including motor cortex

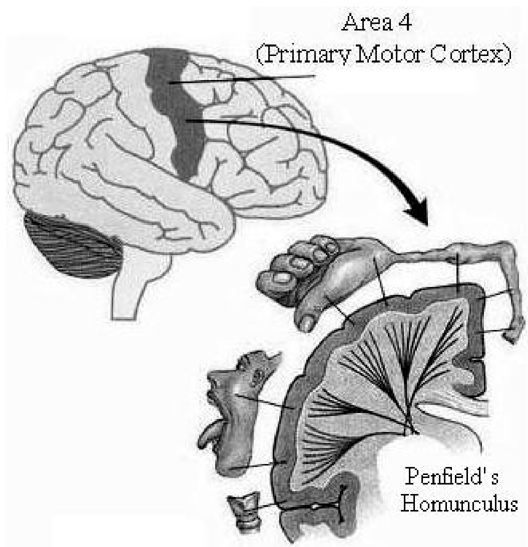

Fig. 12.10 Primary motor cortex and its motor map, the Penfield's homunculus

with those for the rest of the body. This is no surprise, because the speed and dexterity of human hand and mouth movements are precisely what give us two of our most distinctly human faculties: the ability to use tools and the ability to speak. Also, stimulations applied to the precentral gyrus trigger highly localized muscle contractions on the contralateral side of the body. Because of this *crossed control*, this motor center normally controls the voluntary movements on the opposite side of the body.

Planning for any tennis movement is done mainly in the forward portion of the frontal lobe of the brain (see Figure 12.11). This part of the cortex receives information about the player's current position from several other

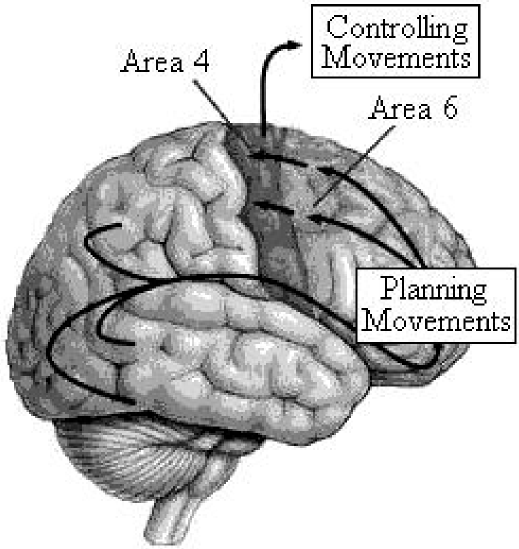

Fig. 12.11 Planning and control of movements by the brain

parts. Then, like the ship's captain, it issues its commands, to Brodman's area 6, the premotor cortex. Area 6 acts like the ship's lieutenants. It decides which set of muscles to contract to achieve the required tennis movement, then issues the corresponding orders to the primary motor cortex (Area 4). This area in turn activates specific muscles or groups of muscles via the motor neurons in the spinal cord.

Q. But, there must be something between brain and muscles?

A. Sure. Between brain (that plans the movements) and muscles (that execute the movements), the most important link is the *cerebellum* (see Figure 12.9). For you to perform even so simple a gesture as touching the tip of your nose, it is not enough for your brain to simply command your hand and arm muscles to contract. To make the various segments of your hand and arm deploy smoothly, you need an internal "clock" that can precisely regulate the sequence and duration of the elementary movements of each of these segments. That clock is the cerebellum.

The cerebellum performs this fine coordination of movement in the following way. First it receives information about the intended movement from the sensory and motor cortices. Then it sends information back to the motor cortex about the required direction, force, and duration of this movement (see Figure 12.12). It acts like an air traffic controller who gathers an unbelievable amount of information at every moment, including (to return to our original example) the position of your hand, your arm, and your nose, the speed of their movements, and the effects of potential obstacles in their path, so that your finger can achieve a "soft landing" on the tip of your nose.

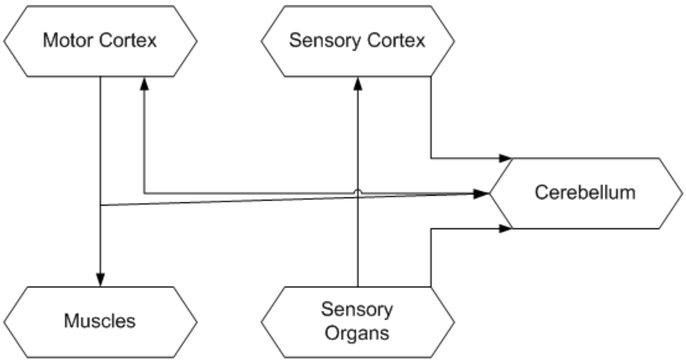

Fig. 12.12 The cerebellum loop for movement coordination

Therefore, to ensure efficiency of tennis movements, i.e., that all the movements are fast, precise, and well coordinated, the nervous system must constantly receive sensory information from the outside world and use this information to adjust and correct the hand's trajectory. The nervous system achieves these adjustments chiefly by means of the cerebellum, which receives information about the positions in space of the joints and the body from the proprioceptors. Even for a movement as simple as picking up a glass of water, one can scarcely imagine trying to consciously specify the sequence, force, amplitude, and speed of the contractions of every muscle concerned. Therefore, the cerebellum clearly does a very important job in tennis.

Q. What is the reflex arc?
A. A reflex is a spinal neural feedback, the simplest functional unit of a sensory–motor control. Its *reflex arc* involves five basic components (see Figure 12.13): (1) sensory receptor, (2) sensory (afferent) neuron, (3) spinal interneuron, (4) motor (efferent) neuron, and (5) effector organ – skeletal muscle. Its purpose is the quickest possible reaction to the potential threat.

Q. So does the cerebellum directly control the muscles?
A. No. While the motor cortex plans the movements (top control level), the cerebellum makes them efficient (middle control level), at the most basic level, movement is controlled by the *spinal cord* alone, with no help from the brain. The neurons of the spinal cord thus take charge of the *reflex movements* as well as the rhythmic movements involved in walking. We will talk about reflexes later. For the lightning–speed of the future tennis game, they are the most important neural part.

12.2.6 Tennis Psychology

Q. What is sports psychology about?
A. The main topics of sports psychology are: (i) motor (and general) learning; (ii) behavioral patterns (e.g., fight-or-flight); (iii) visualization; (iv)

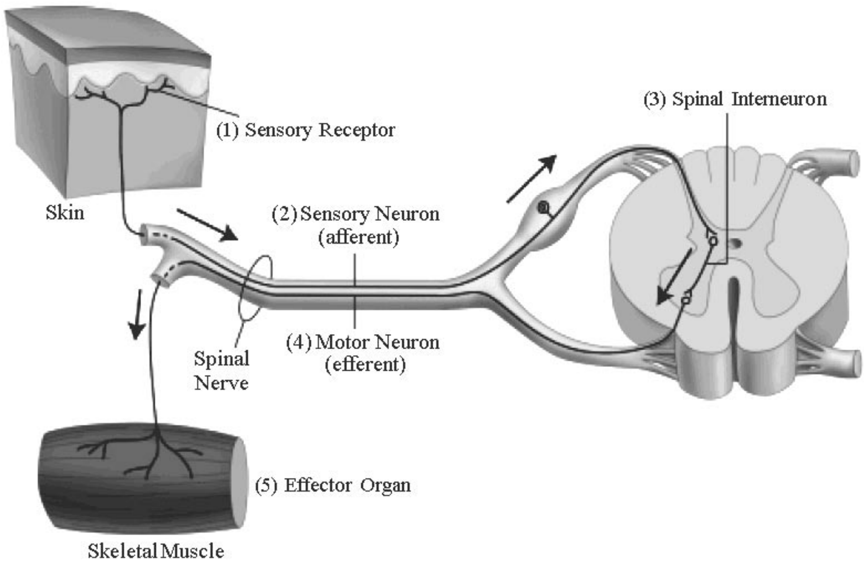

Fig. 12.13 Five main components of a reflex arc: (1) sensory receptor, (2) sensory (afferent) neuron, (3) spinal interneuron, (4) motor (efferent) neuron, and (5) effector organ – skeletal muscle

concentration; (v) relaxation strategies; (vi) self talk (introspective thought) strategies; (vii) arousal strategies; and (viii) stress management. We will talk about each of them in some detail in this Chapter.

Q. What actually is learning?

A. Learning is a relatively permanent change in behavior that marks an increase in knowledge, skills, or understanding thanks to recorded memories. A memory[12] is the fruit of this learning process, the concrete trace of it that is left in your neural networks.

More precisely, learning is a process that lets us retain acquired information, affective states, and impressions that can influence our behavior. Learning is the main activity of the brain, in which this organ continuously modifies its own structure to better reflect the experiences that we have had. Learning can also be equated with encoding, the first step in the process of memorization. Its result, memory, is the persistence both of autobiographical data and of general knowledge.

But memory is not entirely faithful. When you perceive an object, groups of neurons in different parts of your brain process the information about its shape, color, smell, sound, and so on. Your brain then draws connections among these different groups of neurons, and these relationships constitute

[12] "The purpose of memory is not to let us recall the past, but to let us anticipate the future. Memory is a tool for prediction" – Alain Berthoz.

your perception of the object. Subsequently, whenever you want to remember the object, you must reconstruct these relationships. The parallel processing that your cortex does for this purpose, however, can alter your memory of the object.

Also, in your brain's memory systems, isolated pieces of information are memorized less effectively than those associated with existing knowledge. The more associations between the new information and things that you already know, the better you will learn it.

If you show a chess grand master a chessboard on which a game is in progress, he can memorize the exact positions of all the pieces in just a few seconds. But if you take the same number of pieces, distribute them at random positions on the chessboard, then ask him to memorize them, he will do no better than you or I. Why? Because in the first case, he uses his excellent knowledge of the rules of the game to quickly eliminate any positions that are impossible, and his numerous memories of past games to draw analogies with the current situation on the board.

Psychologists have identified a number of factors that can influence how effectively memory functions, including:

1. Degree of vigilance, alertness, attentiveness,[13] and concentration.
2. Interest, strength of motivation,[14] and need or necessity.
3. Affective values associated with the material to be memorized, and the individuals mood and intensity of emotion.[15]
4. Location, light, sounds, smells..., in short, the entire context in which the memorizing takes place is recorded along with the information being memorizes.[16]

Forgetting is another important aspect of memorization phenomena. Forgetting lets you get rid of the tremendous amount of information that you process every day but that your brain decides it will not need in future.

Q. What exactly is memory?

A. Human memory is fundamentally associative. You can remember a new piece of information better if you can associate it with previously acquired knowledge that is already firmly anchored in your memory. And the more

[13] Attentiveness is often said to be the tool that engraves information into memory.

[14] It is easier to learn when the subject fascinates you. Thus, motivation is a factor that enhances memory.

[15] Your emotional state when an event occurs can greatly influence your memory of it. Thus, if an event is very upsetting, you will form an especially vivid memory of it. The processing of emotionally-charged events in memory involves norepinephrine, a neurotransmitter that is released in larger amounts when we are excited or tense. As Voltaire put it, "That which touches the heart is engraved in the memory."

[16] Our memory systems are thus contextual. Consequently, when you have trouble remembering a particular fact, you may be able to retrieve it by recollecting where you learnt it or the book from which you learnt it.

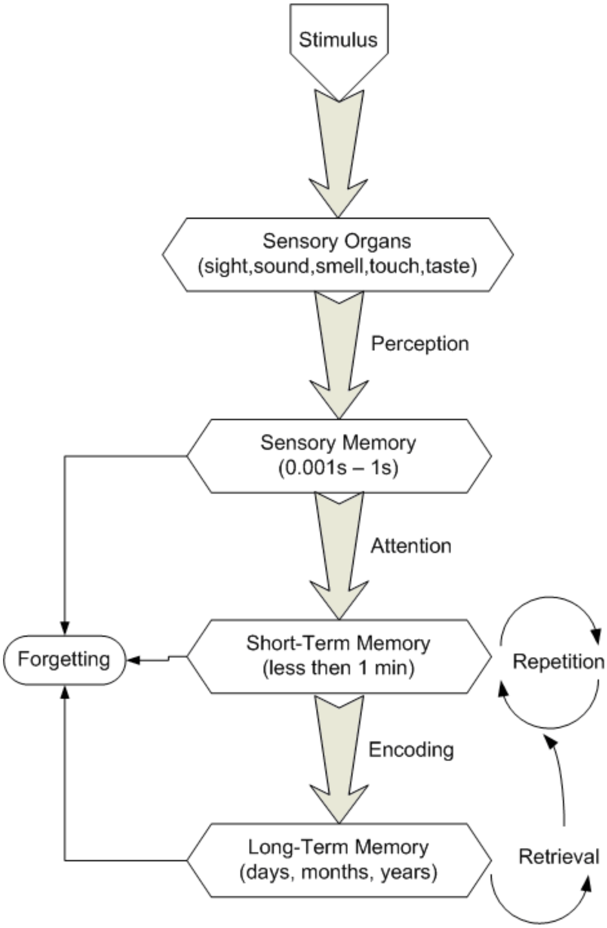

Fig. 12.14 Human cognitive memory

meaningful the association is to you personally, the more effectively it will help you to remember. Memory has three main types: sensory, short–term and long–term (see Figure 12.14).

The *sensory memory* is the memory that results from our perceptions automatically and generally disappears in less than a second.

The *short–term memory* depends on the attention paid to the elements of sensory memory. Short–term memory lets you retain a piece of information for less than a minute and retrieve it during this time.

The *working memory* is a novel extension of the concept of short–term memory; it is used to perform cognitive processes (like reasoning) on the items that are temporarily stored in it. It has several components: a control system, a central processor, and a certain number of auxiliary "slave" systems.

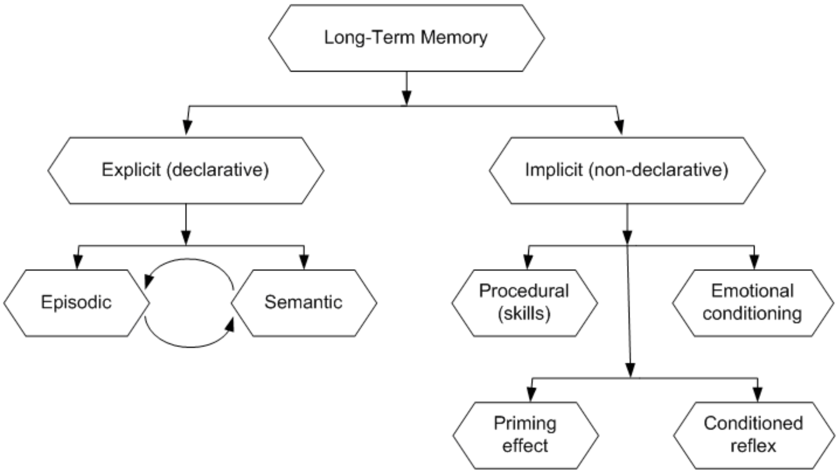

Fig. 12.15 Long–term memory

The *long–term memory* includes both our memory of recent facts, which is often quite fragile, as well as our memory of older facts, which has become more consolidated (see Figure 12.15). It consists of three main processes that take place consecutively: encoding, storage, and retrieval (recall) of information. The purpose of *encoding* is to assign a meaning to the information to be memorized. *Storage* can be regarded as the active process of consolidation that makes memories less vulnerable to being forgotten. Lastly, retrieval (recall) of memories, whether voluntary or not, involves active mechanisms that make use of encoding indexes. In this process, information is temporarily copied from long-term memory into working memory, so that it can be used there.

Retrieval of information encoded in long-term memory is traditionally divided into two categories: recall and recognition. Recall involves actively reconstructing the information, whereas recognition only requires a decision as to whether one thing among others has been encountered before. Recall is more difficult, because it requires the activation of all the neurons involved in the memory in question. In contrast, in recognition, even if a part of an object initially activates only a part of the neural network concerned, that may then suffice to activate the entire network.

Long–term memory can be further divided into explicit memory (which involves the subjects' conscious recollection of things and facts) and implicit memory (from which things can be recalled automatically, without the conscious effort needed to recall things from explicit memory, see Figure 12.15). Episodic (or, autobiographic) memory lets you remember events that you personally experienced at a specific time and place. Semantic memory is the system that you use to store your knowledge of the world; its content is thus abstract and relational and is associated with the meaning of verbal symbols.

Procedural memory, which is unconscious, enables people to acquire motor skills and gradually improve them. Implicit memory is also where many of our conditioned reflexes and conditioned emotional responses are stored. It can take place without the intervention of the conscious mind.

Q. Is an individual behavior genetically coded?

A. Whatever behavior we initiate, be it drinking, playing, reading, making strategic alliances, or making eyes at someone, it is always because we are subjectively feeling certain needs. A specific situation leads to a specific neural activity pattern in a person's brain that in turn leads to a specific behavior.

Each individual's genes activate a unique program for the development of that person's nervous system. But how this nervous system actually develops depends on each person's interactions with the environment, that is on unique personal experiences. The behaviors that this person is capable of are determined by the unique activity patterns of his/her nervous system, some of which are experienced as thoughts, emotions, memories, etc. Any given behavior by this person results from the interaction between his/her neural activity at that specific moment and his/her perception of this specific behavior.

Firstly, our most primitive behaviors (like our reflexes) are concerned with the present. These are incapable of adaptation. They make us react to external or internal stimuli automatically. Neurophysiologically speaking, these behaviors represent the activation of the "reptilian" structures of the brain, bringing the *hypothalamus* and the *brain stem* into play.

Secondly, our learned behaviors add our past experience to our present actions. These more sophisticated behaviors involve remembering pleasant or unpleasant sensations that we experienced in the past and the actions that caused us to experience them at the time. These behaviors represent most of the social and cultural knowledge that we acquire. In connection with these behaviors emotions arise, the awareness of the cardiovascular adjustments necessary for action. Neurophysiologically speaking, these behaviors represent the activation of the "mammalian" structures of the brain, bringing the *limbic system* into play.

Thirdly, our imagined behaviors respond to the present through past experience, by anticipation of the future result. They involve more elaborate planning. They call on the *imagination*, and hence on the *associative cortex*, to develop strategies for ensuring that our actions will be gratifying rather than painful. They represent the creative and innovative abilities of the human mind. Neurophysiologically speaking, these behaviors represent the activation of the "neocortical" structures of the brain, bringing the associative areas of the cerebral cortex into play.

Q. What is the so–called fight-or-flight reaction?

A. A *behavior* is a set of movements coordinated by the nervous system to preserve the structure of the organism. The basic behavior is therefore

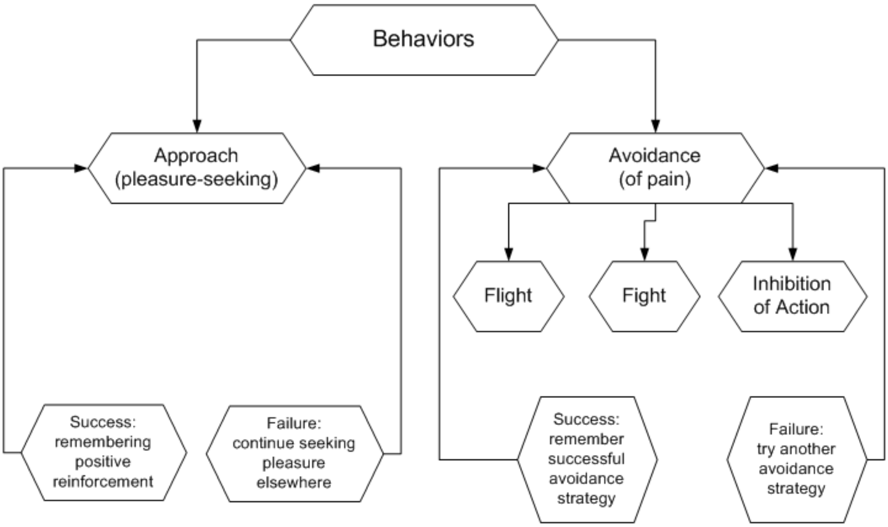

Fig. 12.16 Behavior is memorized if it succeeds and discarded for an alternative strategy if it fails

to approach or explore the resources available in the environment. When an action to acquire one of these resources is rewarded, this gratifying behavior is positively reinforced, and the strategy through which the need was satisfied is memorized.

The other main basic behavior is to avoid pain, and hence to avoid situations that might lead to the organism's premature death. Fight, flight, and inhibition of behavior are the three possible behavioral responses to a nociceptive stimulus. Just like gratifying behaviors, the response of fighting, fleeing, or inhibiting behavior in response to a threatening stimulus can be either effective or ineffective. The associated behavior is then memorized as either a winning or a losing strategy (see Figure 12.16).

Our environment is full of potentially gratifying objects that stimulate our approach behaviors. But it is also full of other people who also want to use these resources to ensure their own well-being. Each individual must therefore learn to decode other peoples intentions in order to choose the attitude that will make his or her own actions the most effective.

For example, if you resisted somebody who was trying to take some resource away from you, and you ended up not only losing the resource but also getting injured in the struggle, then the next time, the memory of this failure might very well cause you to simply run away.

And if this person were someone from whom you could not flee (for example, because you were economically dependent on them), then you would learn that the best thing to do would be to inhibit your own behavior and accept your subordinate status. This is the way how social hierarchies are established.

The three reactions that let you avoid pain trigger hormonal and vasomotor adjustments that are controlled by the sympathetic nervous system. The activation of this system provides increased motor autonomy, mainly by increasing the oxygenation of the skeletal muscles. But these changes cannot last indefinitely, or they would interfere with the healthy functioning of those parts of the body, such as the internal organs, that were left temporarily short of blood. Once the source of the threat has disappeared, the body's equilibrium must therefore be rapidly restored.

Q. How does visualization help in motor learning?

A. Mental imagery plays an important role in the learning of sport movements and the improvement of motor performance. The latest research is attempting to identify the determinants of efficient mental imaging in sport. Studies have already shown that people who are better at generating mental images make faster progress in motor learning, as do people who practice a movement physically before imagining it mentally. Research on mental imagery has shown that the processes for actually producing a movement and for representing it mentally are identical. Various experimental approaches have been used to show, for example, that the mental representation of an action seems to be based on the same mechanisms as the motor preparation for it.

With mental chronometry, for example, it has been shown that visual mental images preserve the spatial and structural characteristics of the object or scene that they represent. For example, it has been shown that the visual travel time between two points in a mental image of an object is proportional to the distance between these two points on the actual object.

Researchers have also found some physiological indicators whose activation resulted solely from mental imaging of actions. For example, a group of researchers measured how physical training and training by mental imaging affected the strength of finger muscles. Physical training increased finger muscle strength by 30%, but mental imaging alone still increased it by 22%. Since the subjects did not make any muscle contractions during their mental imagery training, the observed changes did not come from the peripheral motor system but actually came from the activation of circuits in the central motor system.

Here is yet another example. When subjects were asked to imagine walking or running at various speeds on a treadmill, their heart rate and total ventilation increased in proportion to the speed imagined in the course of this mental exercise, even though their oxygen consumption remained steady.

According to the theory for which Swedish neurobiologist David Ingvar coined the clever term "future memory", the *parietal cortex* is capable of producing internal models of movements to be performed, prior to any processing in the premotor and motor cortices. According to this theory, the brain is constantly simulating movements, only some of which are eventually externalized. This theory could provide a conceptual foundation for the mental training done by athletes and musicians, as well as for re-education through motor imagery.

The concept of motor imagery extends to the sensory modalities as well. When someone has any given sensory experience, then re-imagines its later, the brain activity in the two cases will be similar in both location and intensity. Because brain activity continuously influences the body, and vice versa, any experience has a given effect on the body, and re–visualizing it would generate a similar pattern of brain activity with similar effects on the endocrine system, the immune system, and so on. Speaking very generally, visualization can therefore be regarded as a form of autosuggestion or self–hypnosis which, by generating emotions, may have a beneficial physiological effect on the body.

Q. Do you think that top tennis players, like Federer, Nadal, Djokovic, and Roddick, can improve their performance with an appropriate sport–science knowledge?

A. Absolutely! Modern sports science can improve performance of any athlete! Very briefly, Federer needs only *superior tennis weapons* (explained below). Nadal needs the same plus Federer's tennis knowledge (a few kilos less would make him faster on the court). Roddick mostly needs Federer's anticipation and mental strength (and as in the case of Nadal, a few kilos less would also make him faster on the court). Djokovic needs superior tennis weapons plus Federer's tennis knowledge.

12.3 Tennis Science of the Future

12.3.1 High Performance in Tennis

Q. What are current tennis performance criteria?

A. The overriding principle governing *general sports performance* is the attempt of an individual, or a group of individuals, to perform a given task "in the best possible way." In this chapter we will focus on biomechanical and physiological principles of the *performance optimization* in the future tennis game. For the *tennis performance criteria* we can use the 10 points of the standard tennis game statistics (in brackets are the current ranks of Roger Federer, the world number one, on October 22nd 2007, as given by *ATPtennis.com*):[17]

- *Service game:* (i) number of aces (4), (ii) 1st serve percentage (29), (iii) 1st serve points won (6), (iv) 2nd serve points won (1), (v) service games won (3), and (vi) break points saved (8).

[17] Although we can immediately see that the standard statistics does not give us the full picture of the current tennis, as Federer is here ranked (1) only once, while, e.g., Ivo Karlovic, currently world number 25, is here ranked (1) four times.

- *Return of service:* (vii) points won returning 1st serve (4), (viii) points won returning 2nd serve (17), (ix) break points converted (36), and (x) return games won (10).

Q. What is sports science really about?
A. The very purpose of sports science is to provide solutions for two essential sport problems:

1. A *direct–training* problem: given the set of empirically proclaimed *talents*, develop the *champion model*.
2. An *inverse–selection* problem: given the *champion model*, develop the *talent model*.

Thus, sports science is all about *training methods* (directed to make a champion) and *selection methods* (directed to finding talents). In sports science all the champions are represented by the *champion model* for a particular discipline (e.g., tennis), and all the talents are represented by the *talent model* for the same discipline (see Figure 1.1 above). In statistical language, both the champion and the talent have *the same factor structure* – only it is fully developed in case of the champion and yet undeveloped in case of the talent. For example, Nadal, Roddick, Federer, and Djokovic had all been talents. However, so far only one of them has proved to be a real champion – Roger Federer, the man who apparently defies all tennis statistics. Today, in our opinion, the highest chances to become future tennis champions have Nadal and Djokovic.

12.3.2 Athleticism in Tennis

Q. How can a tennis player run faster on the court?
A. Running fast is the direct result of the athletes stride rate and stride length. Now, the question is how do we maximize both of these to achieve top–level performances in the sprints, or fastest runs on the court. We cannot have a maximum stride length and stride rate and be our fastest; what is needed is a maximum stride rate with an optimal stride length. Maximal stride rate is how fast we can produce one stride, or about 10 of them in 20 meters. Stride rate is dependent upon a number of factors including, strength and mechanics. In order to produce greater stride rates one must be able to execute the correct stride cycle as fast as possible and with optimal length. Optimal stride length is one that allows the athlete to execute the correct stride pattern in as short a time frame as possible.

On the other hand, ground time is the largest contributor to stride rate. It is known that almost all athletes spend approximately the same amount of time in the air during the sprint stride. The big difference comes in the amount of time spent on the ground. The goal of all sprinters and fast-legged tennis players should be to spend as little time on the ground as possible. In order to achieve this, they need the necessary plyometric strength (explained later) to get them through the correct cycle.

Also, during the short sprints on the court, at each leg joint the musculo-tendinous units absorb force by stretching (eccentric) just before they shorten (concentric) to generate the take–off force.

Q. How can a tennis player improve their jumping ability?

A. Jumping ability is defined as *leg power*. Recall that the muscular power is the product of muscular force (**F**) and movement velocity (**v**). It is represented by the area under the force–velocity curve (see Figure 12.4 above). In other words, it is the ability to generate muscular force quickly.

If you look at the force–velocity curve, you will see that high levels of power occur in the mid–range of either force or velocity. If an athlete develops greater power, this, in turn, enhances his ability to generate both force (strength) and velocity (speed). This amalgam of speed and strength may be more useful for athletic performance than strength alone.

Therefore, the leg power is the expression of the leg strength at speed. In the future tennis game, players at all levels will require certain levels of leg power to be successful, the higher the level, the higher the power needed.

The velocity (speed) component of the jumping ability is defined as the *stretch–shortening cycle* (to be explained later). Before we focus on the stretch–reflex and its derivative, stretch–shortening cycle, we need to know how to develop the strength (force) component of the jumping ability. There are several common methods for this, including:

1. General leg–strength exercises. These exercises are necessary to develop the force component of power. Squat exercises such as full squats, front squats, half squats, and split squats, develop the jumping musculature to a large degree. Very few, if any, successful high level field event athletes would not have good squatting ability. Even in novice athletes, some form of squatting could be performed with medicine balls or shot puts to challenge the jumping musculature. Although there is great benefit in performing general strength exercises such as squats and its variants, one must be careful not to use the squat as an end in itself as these exercises are slow in nature thus do not replicate the exact demands of the power events.
2. Special leg–strength exercises. These exercises attempt to convert general strength to specific jumping strength. Some examples may include Olympic lifts and their variants and jump squats. Some researchers suggest that the Olympic lifts are very similar in structure to a vertical jump. As jumping and throwing events have some component in the vertical direction the Olympic lifts are excellent exercises for these events.
3. Plyometric leg–strength exercises. These exercises are necessary to develop the velocity component of power and attempt to provide power improvement in a way which is specific to the required technique of an athlete.

Examples of such exercises would include bounding and hopping, weighted bounding and hopping in addition to single and double legged box jumps.

Q. Can we do traditional strength exercises in a more dynamical way?

A. Yes. Take for example the popular bench–press exercise. You can perform it in a traditional, bodybuilding–type form, where you remain tight throughout your entire body, and throughout the entire repetition. Alternatively, you can do it in a Russian plyometric form, which is more relaxed. This later one involves a faster negative movement (eccentric contraction), and a ballistic pressing action (concentric contraction), in reality more like a push press than a bench press (this is technically termed the stretch–shortening cycle). It is clear that the traditional exercise gives you a slow strength only, while the plyometric one gives you both strength and speed, that is the dynamical power. And this dynamical power is what we really need for the future tennis.

In simple mechanical words, given that force ($\mathbf{F}$) equals mass ($\mathbf{m}$) $\times$ acceleration ($\mathbf{a}$), and mass is basically always constant in tennis, what we really need is a huge acceleration. This huge acceleration will produce a huge racket-head speed, which will generate the lightning–speed of the ball.

Q. Can you give a general example of a true athleticism?

A. In contemporary tennis, the most prominent example of *athleticism* is Rafael Nadal. Other examples include the current champion, Roger Federer, and American James Blake.

Notwithstanding the common public sympathy towards athleticism of the popular players, we need to remark that every serious 9-year-old gymnast can do a somersault on the street (or, on the tennis court during the game). This shows that the very idea of "athleticism" has historically been a bit misunderstood within the tennis community.

Let us have a look at the real athleticism of a person with almost the same body posture and weight as Federer's. We are naturally talking about the track-and-field hero, Jan Zelezny the javelin world record holder, winner of three Olympic gold medals (Barcelona, Atlanta, Sydney) and three World Championships gold medals (Stuttgart, Gothenburg, Edmonton). He has five world javelin records, including the actual one of 98.48 m (set in 1996, Germany), as well as 34 performances over 90.00 m (which is more than all other javelin throwers combined).

This guy is not "a big boy". He does not have over 2 m and over 130 kg (as some "experts" might suggest, by considering his super–human throws). He does not look at all like Arnold Schwarzenegger, but rather as a younger and bigger brother of Bruce Lee. Zelezny's body height is only 1.86 m, (that is exactly between Roddick's and Federer's), while his body weight (at the time of his world-record throw) was only 80 kg. This is the same weight as Federer and the same as 1.87 m tall Novak Djokovic. At the same time, this is 5 kg less than

Nadal, or 8 kg less than Roddick, or 8 kg less than Tommy Haas, even one kg less then 1.82 m tall Fernando Gonzalez. That is to say, except for Federer and Djokovic, all these current elite tennis players are simply *too heavy.* Therefore, you do not need to be heavy and bulky to be able to hit hard! Tennis is not shot–put! The mass of the tennis racket is half the mass of the javelin. Therefore, common sense would say that the best javelin thrower of all times had to be heavier than top tennis players. Yet, at the time of his fantastic world record (as well as his other 33 throws over 90.00 m), Zelezny looked pretty skinny. What he had, though, were muscles made of steel and lightning–fast reflexes. That is, Zelezny's body was an efficient slingshot!

In other words, a man with a body very similar to the current champion, Roger Federer, was able to throw an 800 g javelin (purposely reconstructed that it cannot fly far), almost 100 meters. Could you imagine the speed of a tennis serve or forehand with muscles similar to Zelezny's. With muscles trained (for years) in utilizing the stretch–reflex. Zelezny would not only break the strings, if he attempted the tennis serve, but would also brake the *frame* of the "modern" tennis racket. A totally new racket technology would be required for players with muscles similar to Zelezny's. This example shows the real *athleticism:* after years of proper stretch–reflex training, the body becomes a big slingshot, composed of a number of small slingshots: legs, hips, torso, shoulders, elbow and wrist – all working as a kinetic chain, in a dynamical power–sequence.

Now, we do not suggest that a tennis player will ever need to be able to throw a javelin close to 100 meters, like Zelezny. No, a 70 m throw of the current (reconstructed) javelin, quite achievable for junior javelin throwers, would give a 300+ serve with a technique similar to Roddick's, as well as 240+ forehand and a 200+ (single–handed) backhand with a technique similar to Federer's.

What we are suggesting is that the future tennis champion will have something similar to Zelezny's body height and weight, but more importantly, Zelezny-like slingshot muscles. It is just a matter of proper stretch–reflex based muscular conditioning. This conditioning would give the future tennis champion the ability to serve consistently over 300 km/h,[18] forehand consistently over 240 km/h and backhand consistently over 200 km/h. Just imagine the current top tennis players playing against someone with these three superior tennis weapons. Would they be able to win a single game?

Q. What is the efficiency of a sport technique?

A. Basically, for dealing with any sport–science issue, we have two possible approaches: sport physiology and sport biomechanics.

[18] You don't have to be 9 feet tall nor does your combined height including your extended arm, racquet, and jump height need to be 9 feet or more. This is because both gravity and aerodynamic drag act on the ball during its flight. Gravity accelerates the ball downwards while drag (air resistance) creates a retarding force slowing the ball's forward motion. This causes a curved trajectory.

Sport and exercise physiology gives us a valid description of the energy systems involved in any kind of cyclic sport activity (like running, cycling, swimming, rowing. . .) of either short– , middle– or long–distance.

On the other hand, the current popular scientific discipline designed for analyzing sport techniques is biomechanics. Anatomical biomechanics describes various techniques using mostly methods from functional anatomy, while *Newton–Euler dynamics* (see section 2.4. Tennis Biomechanics) provides their mathematical modelling (and associated computer simulations), for the purpose of answering the What–If questions (by varying the athlete's parameters). Biomechanics can confidently do two things: (i) give us all Newtonian mechanical principles as guidance for development of any sport technique, and (ii) roughly describe the technique of a current champion (using the concept of bio-kinetic chains).

Both sport physiology and biomechanics are legitimate scientific tools for understanding standard sport activities and/or human movements. However, neither of them can give us the most precious answers to the crucial question: What is the *most efficient technique* for any particular human movement, both cyclic and non-cyclic? When it comes to understanding, prediction and control of the most efficient sport movements as well as training methods, both sport physiology and biomechanics fail.

For example, neither of them can help us understand the fastest ever tennis serve of American Andy Roddick (153 m/h), or the fastest tennis forehand of French Gael Monfils (120 m/h). These two guys are neither the tallest (which would give them a huge leverage), nor the strongest (which would give them a huge force), nor the fastest (which would give them a huge speed– arguably, Rafael Nadal is both stronger and faster then both of them) in the game. Neither of these two guys is the world number 1, which, as we all know, is Roger Federer, meaning that having one weapon does not win the match (and we will return later to the *Federer phenomenon*, while speaking about the tennis champion of the future). However, they are record holders for the serve and the forehand, and it wouldn't hurt even Federer to have a stronger serve and forehand – he would not need to go through so many tie–breaks.

To answer this, in our opinion, the most significant question in elite sport is the question that explains the secret technique of champions and to do this we need a completely different scientific approach. What is the secret behind Roddick's serve and Monfils' forehand? The secret is the *stretch–reflex*. Moreover, the stretch–reflex is the unique secret behind all highly–efficient movements in sport, including all athletic throws, jumps, sprints, all weight lifts, fast gymnastic movements and so on.

For his fastest serve, Roddick used the stretch–reflex in (the prime mover muscles of) all major joints: right shoulder, right elbow, right wrist, both hips, both knees and both ankles. For his record forehand, Monfils used the stretch–reflex in the same joints. The two techniques look completely different, but the neuro-physiological basis is the same; the stretch–reflex utilized in (the prime mover muscles of) the same joints.

The underlying similarity between these two apparently different sport movements also teaches us another important lesson: the "monkey see, monkey do" approach of copying others' technique (without even an attempt to understand what's really happening), used by almost all coaches and athletes, sometimes even sport scientists – is blind. It shows only trivial and superficial similarities, which are good enough only for kindergarten sport, without any ability to understand, predict or control the movement. Surely, sport science should be able to do better than just "monkey see, monkey do." It should be able to provide the means for understanding, predicting, controlling and developing the most efficient human movements.

The stretch–reflex is the most efficient feedback-control mechanism in the human body. Therefore, any human movement that uses it simultaneously in several major joints – is immediately highly efficient.

The stretch–reflex recipe reads: make efficient slingshots out of all your major muscles: quadriceps, gluteus, soleus, pectoralis, deltoideus, biceps, triceps, palmar flexors, and all abdominal muscles. Make your major joints flexible and all your major muscles at the same time strong, fast and elastic, so that you can safely stretch and fire them like slingshots. Train them so that you make them efficient slingshots. That is, *base your strength and speed training on utilizing the stretch–reflex of all major muscles*, previously conditioned by the flexibility training. At the same time, base your sport technique (e.g., serve, forehand, backhand, etc) on the stretch–reflex, understanding that your muscles truly are natural slingshots ready to fire. Therefore, you need to develop your body full of slingshots, and your tennis weapons (e.g., serve, forehand, backhand, etc) need to be based on those slingshots, so that in a real game situation you can efficiently fire them. That is all! It is as simple as that!

You can easily see that popular plyometrics training is a part of this new science, but only a minor part. The concept of plyometrics training lacks the underpinning knowledge of the stretch–reflex, so naturally it proposes only a very limited number of plyometric exercises. Once we fully understand the applied stretch–reflex, we can design any number of plyometric exercises for any part of the body. We just need to make our body full of slingshots, like Bruce Lee did three and a half decades ago, and like all athletics jumpers, throwers, sprinters and weightlifters do today.

12.3.3 Muscular Slingshots

Q. Are classical (and current) loops and swings in tennis strokes biomechanically sound?

A. The basic idea of coaches and players, as well as the simplistic biomechanics of loops and swings in tennis shots (serve, forehand and backhand) are derived from the concept of a simple physical pendulum. An arm with a

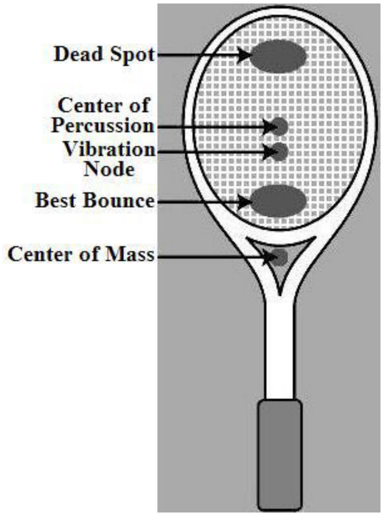

Fig. 12.17 Important points on a tennis racket. The common "sweet spot" is technically called the center of percussion (the point along the racquet's length where an impact produces no impulse reaction at the axis of rotation). Just below it is the vibration node. Note that the center of mass is below the racket–head.

hand holding a racket has been seen as a more-or-less single rigid body, with no more than 3 *degrees of freedom* (DOF). And really, if you have a robotic arm with only 3 DOF, and you want to hit a ball with it, then you need to have a loop; and, even more, if you want to hit the ball hard, you need to have a big swing. That is absolutely true. In a language of modern biomechanics, "its phase space is a simple circle."[19] That is how we originally got our current loops – to play "nice tennis", and swings – to be able to "hit hard". In particular, you need a big swing as you want to use all the potential energy of the racket's weight, so, as a preparation for the shot (say, forehand), you lift the racket–head as high as possible along the circle (that is, above your head). This is the common picture behind all loops and swings in tennis. Although simple to understand, it is not as easily implemented, which is where the many expensive lessons on the court come in. This system has produced thousands of young tennis players, hitting the ball in virtually the same way, with the same distinguished tennis movements: loops and swings.

However, this picture is wrong! Our reality is much more complex than this simplistic model of a loop (circle) and a swing (potential energy along the circle), which produces only "tennis ballet".

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[19] For technical details on modern geometrical biomechanics with hundreds of degrees of freedom, see one of our advanced scientific books: Human–like Biomechanics (Springer), Natural Biodynamics (World Scientific), Geometrical Dynamics of Complex Systems (Springer), High–Dimensional Chaotic and Attractor Systems (Springer), Neuro–Fuzzy Associative Machinery for Comprehensive Brain and Cognition Modelling (Springer), Applied Differential Geometry (World Scientific).

Firstly, the human arm has 9 DOF (not including fingers). Just the shoulder, elbow and wrist together have 9 DOF. This is a *redundant system*, because the racket itself has only 6 DOF (three translations and three rotations).[20] This means that there are infinite number of possible ways to hit the ball, and we can choose the best way in that situation. Therefore, a real "phase space" for the human arm with a racket is much more complicated than the simplistic circle–loop model. Besides, every coach advises, quite correctly, not to use the arm only, but the whole body. So we actually have several hundred DOF at our disposal to perform a tennis stroke.[21] Do you still believe that a simple circle and its associated loop is an appropriate model for the serve and ground-strokes?

Secondly, the question of utilizing the racket's own potential energy, that is, rising a racket–head up high as a preparation for the shot. This would make sense only for very weak players and very heavy rackets (in the same way as a weak player needs a long swing to accelerate the massive racket). Otherwise, if we have an athlete, and every elite tennis player has to be an athlete, the current 400 gram racket is simply not heavy enough to make this potential–energy contribution to the shot significant. It would be significant with a 4 kg racket, but not with the current 400 gram one.

Put simply, loops are totally useless: it does not matter at all how you get into the position for the shot. Swings are almost useless. They could be used say in the fifth hour of a five–set tennis match, to compensate for the lack of strength in shots, but the same fatigue would make it hard to raise the racket–head high above your head.

Also, we see that simplistic (bio)mechanics does not work with the human body. Robotics has already learned this lesson in the last three decades. Tennis has yet to learn it.

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[20] Common robotic–tennis arm would have exactly 6 DOF – to match the racquet's own 6 DOF; it wouldn't have any excessive DOF; because of this 6–6 correspondence, a robotic arm would perform every racquet movement using the exactly prescribed trajectories. However, human arm (and human body as a whole) is a highly redundant system; this *mechanical redundancy* allows every tennis player to execute any movement in an infinite number of ways and to choose the one which is *optimal* with respect to racket-head speed, or spin, or energy efficiency.

[21] In the wooden era, the player really needed the long, flowing swing to accelerate the old, heavy racket up to hitting speed, unless he was very strong and fast. Remember, muscular force needs to overcome inertial one, or $force\ =\ mass\ times\ acceleration$, and *acceleration* is the cause of the racket–head speed. Therefore, if we have a massive racket and a weak muscular force, we need a long movement to gradually accelerate and eventually get some racket–head speed. In addition, this gradual acceleration gave the player much more control of the racket head and allowed the player to hit the ball at approximately the same location on the racket face each time it was swung, because the "sweet spot" was small.

The only efficient type of movements in tennis shots (serve, forehand and backhand) are whip–like movements,[22] based on muscular slingshots.[23]

Q. What is a "muscular slingshot"?

A. The stretch–reflex causes a stretched muscle to contract stronger and at the same time inhibits the antagonist muscle from contracting (that is, slowing the movement). Because this is an involuntary reflex response the rate of contraction is significantly (several times) faster and more powerful than a completely voluntary muscular contraction. In fact, the faster the muscle is stretched eccentrically, the greater the force will be on the following concentric contraction.

Closely related to the muscular stretch–reflex is the stretch–shortening cycle, which occurs when elastic loading, through an eccentric muscular contraction, is immediately followed by an explosive concentric muscular contraction. The tension developed in the musculo-tendinous junction by the eccentric loading of the muscle causes it to act in a similar manner to a rubber band. When this stored energy is released, it helps to increase the strength of the following concentric contraction. These neuromuscular considerations have huge ramifications for both the composition and training of the tennis serve, forehand and backhand.

When we train our skeletal muscles so that they efficiently utilize both the stretch–reflex and the related stretch–shortening cycle, then we effectively make powerful slingshots out of them.

The prototype of such a muscular slingshot is the javelin throw.

Q. So what exactly is the stretch–reflex?

A. The stretch–reflex (or, myotatic reflex) is the *secret* behind both speed and strength in sport. It is the quickest reflex (see Figure 12.13) in the human body, in which the *reflex arc* is a closed loop: both the receptor and the effector are in the same skeletal muscle (see Figure 12.18).

When a person walks and accidentally steps into a hole, the stretch–reflex of the stepping knee extensor (quadriceps muscle) is activated, generating additional force for knee extension, to re–establish the lost balance. What happens is that a muscle spindle (a length sensor within the quadriceps muscle) is stretched, causing a reflex arc to fire, generating a higher force than the voluntary muscular contraction alone. If the muscle is sharply stretched prior to its contraction, it generates a stronger force than without previous stretching. That is, the muscle behaves like a nonlinear spring (or, "elastic

[22] With the whip–like movement it is more difficult to hit the ball at exactly the same location on the racket–head each time. However, due to the characteristics of the modern racket and the heavy topspin strokes used, the resulting ball trajectory is much less sensitive to the exact location of the ball impact on the strings. New rackets appear to have larger "sweet spots", technically called the centers of percussion, see Figure 12.17.

[23] General experience from all other hitting, kicking and throwing sports or games.

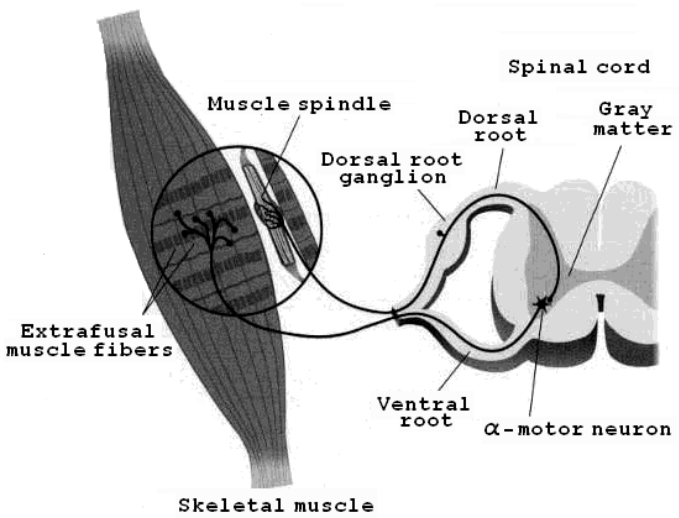

Fig. 12.18 Reflex arc of a myotatic stretch–reflex

muscular component"). As an anatomical consequence, the attached body segment moves faster.

The stretch–reflex is the basis for all fast cyclic movements (such as sprinting) as well as for any combination of fast individual movements. It is also the basis for efficient weight lifting and athletic throws. In other words, speed and strength might have different muscular characteristics, but they have the same neural support: the stretch–reflex. Finally, the stretch–reflex is the only proper physiological basis behind the popular *plyometrics* muscular training.

Biomechanically speaking, the stretch–reflex (or, if you insist, the stretch–shortening cycle) involves a sharp eccentric contraction immediately followed by a strong reflex concentric contraction. We have the stretch–reflex in action in all, mostly cyclic, fast movements in full joint amplitude (e.g., in sprinting, we have a maximal knee flexion immediately followed by a sharp knee extension, followed by a maximal knee flexion. . .). Whenever one of the two antagonistic muscles is stretched, either by his antagonist, or by gravity, it responds by generating a stronger force than it would be able to generate voluntarily without the previous stretching. As an anatomical result, we also have the faster movement of the attached body segment (a calf in the sprinting case).

Q. And, what exactly is plyometrics?

A. The modern day method for developing the jumping ability called "plyometrics" is actually an almost half a century old Russian explosive strength method called the "shock method," firstly proposed in 1959 by Professor Yuri Verkhoshansky (see bibliography at the end of this book). Plyometrics is generally used for improving the explosive strength and the muscular reactive capacity. It is a type of exercise that utilizes a rapid eccentric movement, followed by a short amortization phase, and then followed by an explosive concentric movement, which enables the synergistic muscles to engage in the (myotatic) stretch–reflex during the stretch–shortening cycle. Plyometric exercises use explosive movements to develop muscular power, the ability to generate a large amount of force quickly. Plyometric training acts on both the musculotendinous and neurological levels to increase an athlete's power output without necessarily increasing their maximum strength output. Plyometrics are used to increase the speed or force of muscular contractions, often with goals of increasing the height of a jump or speed of a punch or throw.

Recall that for a muscle to cause movement, it must shorten; this is known as a concentric contraction. There is a maximum amount of force with which a certain muscle can concentrically contract. However, if the muscle is lengthened (eccentric contraction) just prior to the contraction, it will produce greater force through the storage of elastic energy. This effect requires that the transition time between eccentric contraction and concentric contraction (amortization phase) be very short. This energy dissipates rapidly, so the following concentric contraction must follow the eccentric stretch before this occurs. The process is frequently referred to as the *stretch–shortening cycle*, and is one of the underlying mechanisms of plyometric training.

In addition to the elastic–recoil of the musculo–tendonous system there is a neurological component. The stretch–shortening cycle affects the sensory response of the muscle spindles (stretch–reflex) and golgi tendon organs (GTO). It is believed that during plyometric exercise, the excitatory threshold of the GTO's is increased, meaning they become less likely to send signals to limit force production when the muscle has increased tension. This facilitates greater contraction force than normal strength or power exercise, and thus greater training ability.

The muscle spindles are involved in the stretch–reflex and are triggered by rapid lengthening of the muscle as well as absolute length. At the end of the rapid eccentric contraction, the muscle has reached a great length at a high velocity. This may cause the muscle spindle to enact a powerful stretch–reflex, further enhancing the power of the following concentric contraction. The muscle spindle's sensitivity to velocity is another reason why the amortization phase must be brief for a plyometric effect.

A longer term neurological component involves training the muscles to contract more quickly and powerfully by altering the timing and firing rates of the motor units. During a normal contraction, motor units peak in a desynchronized fashion until tetanus is reached. Plyometric training conditions the neurons to contract with a single powerful surge rather than several disorganized contractions. The result is a stronger, faster contraction allowing a heavy load (such as the body) to be moved quickly and forcefully.

Some animals also take advantage of this effect; one is the kangaroo. If a kangaroo needed to use 100% new energy to contract its leg muscles every time it jumped, it would not be able to jump very far consistently. However, because of the muscles' ability to store energy from its previous jump before like a spring, the kangaroo only needs to use a fraction of the total energy in the jump.

In everyday sport, plyometrics commonly refers to jumps and similar movements that involve eccentric (stretching) muscular contraction, immediately and rapidly followed by a concentric (shortening) contraction, that is the stretch–shortening cycle. The phase between these two contractions is referred to as the amortization phase. Energy stored during the eccentric phase is partially recovered during the concentric phase. In order to best use this stored energy the eccentric phase must be rapidly followed by the concentric.

One plyometric exercise that is very useful for tennis serve, involves catching and tossing a medicine ball to an assistant while the exerciser lies on their back. The triceps and chest muscles work both while they are lengthening (catch phase) and while contracting (toss phase).

As another example, a typical sprinter's contact time on the ground is in the region of 0.01 of a second, while typical high jumper may spend as little as 0.12 seconds on the ground at take-off. A great deal of muscular force must be generated in this brief period of time. Through the correct use of plyometric exercises, this rate of force development can be enhanced. Note that high volume plyometric workouts will not enhance speed development. Therefore, by using plyometric jumping exercises we are trying to accomplish the following: (i) to shorten the time spent in the amortization phase, and (ii) to decrease the time spent on the ground yet generate maximum force.

Conventional wisdom dictates that in order to begin plyometric training there are prerequisite strength levels which are necessary. To begin to incorporate plyometric training in a program the prime concern is strength in the stabilizing muscles in order to prevent injury. The next concern after stabilization strength is eccentric strength. Eccentric strength is the limiting factor especially in more complex high volume and high intensity plyometric training. Without adequate levels of eccentric strength, rapid switching from eccentric to concentric work becomes very inefficient.

The following table gives several examples of plyometric jumping exercises:

Example	**Stress**	**Recovery time**
jump rope or ankle bounds or low amplitude jumps	very low	few hours
tuck jump or similar	low	one day
stair jumps or other similar short jumps	moderate	one to two days
hops or bounds for distance or similar	high	two days
depth jumps or other similar shock-type jumps	very high	three days

Proper execution of the exercises must be continually stressed regardless of the proficiency level. For the beginner, it is especially important to establish a sound technical base upon which to build the higher intensity work. Jumping is a constant interchange between force production and force reduction leading to a summation of forces utilizing all three joints of the lower body:

the hip, knee, and ankle. The timing and coordination of all limb segments will yield a positive ground reaction force which results in a high rate of force production.

A key element in the execution of a proper technique is the landing. The shock of landing is not absorbed exclusively with the foot, rather it is a combination of the ankle, knee, and hip joints working together to absorb the shock of landing and then to transfer that force. The proper utilization of all three joints will allow the body to use the elasticity of the muscles to absorb the force of landing and then utilize that force in the subsequent movement.

Thus, plyometric exercises promote high movement speed, fast twitch fibre recruitment and elastic tendon energy release. They always involve: (i) an eccentric contraction; (ii) a brief amortization phase (with no change in muscle length); and (iii) a short concentric contraction delivering maximum force in a short period of time.

Q. What is the general role of flexibility in sport?

A. The capacity of muscles, tendons, ligaments and fascia to stretch, the range of motion in the joints and the ability of the muscles to contract and coordinate all define how we move. Unlike strength, speed and other motor abilities, flexibility develops and determines efficient fluid movement. Heavy training and competition schedules place great stresses on our capacity for movement. We require a systematic approach to training for full recovery both mentally and physically. The implementation of a proper flexibility program is imperative for this recovery, but more importantly for an increase in performance.

Flexibility training should be integrated on a continual basis within a yearly training plan. When practiced regularly it provides immediate relief from fatigue and muscle soreness. This is very important in accommodating increases in training volume and intensity as the year progresses. Additionally, the accumulation of flexibility training attained by the athlete over an extended period of time will allow for an increased capacity to maintain these gains with less work.

Stretching should never be forced, but should be done with special care. It is important that the athlete/player focuses on the muscle group that is being stretched. This fosters greater body awareness, an overlooked attribute of a champion. Flexibility has to be both sequential and rhythmical in order to accommodate the effects of high training loads that have placed a greater load on the CNS. The constant stimulation of the nerve cells, whose high working capacity cannot be maintained for long, affects muscular action and ultimately athletic performance. When the period of competition and training has very high fuel consumption, fatigue sets in. Since blood glucose is depleted from the system, the CNS becomes fatigued. Properly applied stretching techniques promote enhanced blood circulation within the athlete. With this increase in circulation, there is removal of metabolic waste (lactic acid) products, as well as an increase in the transport of oxygen and nutrient to the muscle and tendon regions.

A properly designed flexibility program does not cause injury to the tissues but aids in their recovery and regeneration. Its greatest influence is at the myo-tendon junction, the transition zone between the muscle and tendon. The function of the tendon is to transmit the mechanical impulses that derive from muscular contraction to the joints. The myo-tendon junction must adapt each time to the functional needs of the musculoskeletal system. This region also helps in cushioning abrupt and violent movement. Most injuries occurring in this region are a result of micro-tears, referred to as micro-injuries. Micro-tears result in the development of scar tissue. As time progresses and these injuries go untreated, a common result is chronic pain. Associated with chronic pain at the physical level are muscle imbalances and compensation shifts. If an athlete does not adhere to a proper flexibility program they may slowly develop into an involuntary contraction machine (due to a stretch-reflex overload).[24] Their higher muscle tone, a result of greater tightness, affects the nervous system. A possible outcome of this neural fatigue may be muscle atrophy, as well as poor muscle coordination. This affects the development of power (that is, both strength and speed).

Q. How can we prevent serious injuries?

A. Firstly, it is very important for an athlete to be both physically and mentally fit for the competition and/or training. Besides, an athlete should be able to identify the presence of a minor injury and distinguish it from fatigue. Here is a list of simple ways by which you can identify injury: (i) pain is sustained steadily and does not subside; (ii) an extent of tightness can be felt that restricts full–range motion; (iii) you feel light–headed or nauseous; (iv) what initially appeared to be a minor injury does not heal promptly.

Highly motivated athletes always strive to push beyond their limits, both in training and in the competitions. Unfortunately, if an athlete does not take sufficient rest when needed, their muscles can actually be injured, rather than become stronger. Such injuries include: cramps (muscles become excessively tight in contractions), contusion (internal bleeding, swelling, pain and stiffness are caused by a serious bruise), sprains (a ligament has been overstretched and torn) and strains (a muscle or tendon attachment has been over-stretched or torn).

For example, when lifting weights in weight–training, the body's natural response to the heavy weights is have microscopic tears in the muscles' connective tissues. This explains the soreness that one would usually feel after each weight–training exercise. It is essential for an athlete to rest after weight–training. During the duration which the athlete rests, the microscopic tears are repaired. In the repair process, muscles become larger and stronger.

[24] This is an example of possible misuse of the stretch reflex if muscles are not flexible enough.

However, if an athlete fails to obtain adequate rest after weight–training, the repair process may be delayed or a true injury may result. This is because if the athlete continues to strenuously use the muscles without allowing rest for repair, the microscopic tears may become larger tears.

12.3.4 The Biomechanics of Whip–Like Movements

Q. What is a kinetic chain?

A. Biomechanical term for the whip–like movement is the *kinetic chain*: the sequential flow of energy and momentum from bigger segments to smaller ones. Tennis requires sequenced activation of muscles and movement of bones and joints to achieve the motions, positions, and velocities seen in a player. This sequencing is known as the kinetic chain. Kinetic energy and momentum, as well as muscular power, are developed from the legs, hips and trunk muscles and transferred to the arm muscles. This allows the energy, momentum and power to be transferred efficiently to the hands, moving the racquet-head with maximum speed to the ball.

More precisely, to achieve the highly–efficient technique of the tennis serve, forehand and backhand, a proper sequencing of muscle stretch–reflex based actions must take place. Two movement strategies are critical in this respect: (i) proximal-to-distal firing patterns, and (ii) active acceleration–deceleration of body segments.

Research evidence suggests that a proximal-to-distal firing pattern is the most effective for increasing the racket-head speed in the serve, forehand and backhand. In such a sequencing pattern, the stronger more heavily muscled proximal (close to the torso) joints should become activated before the weaker but faster distal joints. This firing pattern has proven the most efficient due to the fact that it takes advantage of each joints' linear and angular momentum generating characteristics. It suggests that power (that is, both speed and strength combined) for the serve, forehand and backhand is primarily generated with: (i) leg extension, (ii) hip rotation, and (iii) trunk rotation and flexion – before the actual arm action. The actions of these proximal joints account for more than 50% of the total forces in the serve, forehand and backhand.

A second characteristic of efficient movement coordination in the serve, forehand and backhand is consecutive acceleration and deceleration of the main body segments. When done well this permits the player to achieve racket-head speeds far greater than they would if they did not use an optimal acceleration-deceleration coordination pattern. The mechanism for this benefit is the transfer of both linear and angular momentum. This movement strategy aids in the transfer of momentum from the lower extremity to the upper, and from the upper extremity to the racket-head.

These two kinetic-chain patterns combined, generate a whip–like motion: when the upper leg and trunk musculature are the first to contract, greater separation is developed between the shoulders and hips which results in a whip effect as the hips are decelerated and the shoulders accelerate as they uncoil and the shot is released.

As already said, each muscular contraction in a chain can be either voluntary or reflex. We already know that the stretch–reflex based contraction is several times more efficient (that is, both faster and stronger) then the voluntary one.

12.3.5 Superior *Tennis Weapons*

Each effective tennis shot, be it a serve, a forehand or a backhand, is a whip–like movement performed by a complex coordination of all the body's segments working to place the racquet in the correct position at the right time and apply the maximal summed force to the tennis ball.

As we already said several times, the best power (strength + speed) exercise for both the serve and the forehand (and even for the single-handed backhand) is the javelin throw. Not only that, but all speed and strength exercises practiced by elite javelin throwers are perfectly suited for future champion tennis players.

In short, a *superior serve, forehand and backhand are whip–like movements, each composed as a cascade of stretch–reflexes in all major joints, starting from the feet and ending with the hitting hand.*

Q. What are the main characteristics of Andy Roddick's first serve?

A. A former world number one, and currently ATP–ranked 5, Andy Roddick, holds the record for the world's fastest tennis serve: 153 m/h (or, 246 km/h) fired at Queen's Club, UK, in 2004. When he first met Patrick McEnroe, his Davis Cup coach, he said: "Whatever you do, don't say anything to me about my serve. If I think about it, I'm in trouble." Why? Because it is all reflex, more precisely stretch–reflex. If you think about something that is performed reflexively, you simply mess it up. Therefore, it is crucial that the elite player develops a fully reflex–based technique. This will generate the highest possible racket–head speed of an elite athlete and thus maximize their performance/efficiency.[25]

However, coaches and sports scientists should analyze the most efficient movements to be able to teach the model techniques. For example, Professor Bruce Elliott from the University of Western Australia, has extrapolated the contributions of the body segments to racket-head speed using 3D video– and computer analysis. "These contributions vary from person to person, Elliott says, "but the data shows the clear importance of the trunk, shoulder internal rotation and wrist flexion in the swing to impact."

[25] Assuming that the player is already capable of consistently getting the serve in the square, keep their serve deep, able to serve to the opponent's backhand, body and/or forehand at will, and effectively use slice and/or topspin kick.

Q. What is the best strength exercise for the serve?

A. Apart from the javelin throw, the best strength exercise for the serve is the axe chop/sledgehammer. Its purpose is to simultaneously develop strength, speed and flexibility in the shoulder girdle, pectoral and upper back musculature. This exercise requires a sledgehammer or lumber axe, approximately of the mass and handle–length of the tennis racket, as well as a stable knee–height hitting surface such as a tractor tire, mound of dirt, or a large log. The athlete stands in front of a knee–height hitting surface. After a proper warm-up, the axe / hammer is brought over the head and swung violently down onto the hitting surface. The key for this exercise is to let the weight of the axe or hammer pull the arms back so that a stretch is felt through the shoulders and upper back. This will initiate a stretch–reflex contraction, while developing strength, speed and flexibility for throwing. It is important that: (i) the athlete initiate the movement with the whole body rather than just the arm, which will create a whip–like effect on the axe or hammer; and (ii) to imitate the serve movement as close as possible.

Other strength exercises include various overhead throws, like medicine ball throw and weighted ball throw. In particular, catching and tossing a medicine ball to an assistant while the exerciser lies on their back.

Q. What is the best speed exercise for the serve?

A. The best speed exercise for the serve is simulating the serve movement (without the ball) with a badminton racket instead of a tennis racket. This should be performed in a double series: 10 repetitions of a shadow serve with a tennis racket, then 10 repetitions with a badminton racket, then a 2 minute pause; in such a way to make 100 of each, performed with a maximal speed (after a proper warm-up).

Q. What are common technical misconceptions about the tennis forehand?

A. The wooden racket era, characterized by the heavy rackets and the weaker players, was dominated by the following 5 classical postulates for the proper technique for the tennis forehand:

1. You must have a loop in the stroke, either big or small, but the bigger the better;
2. If you want to "hit hard," you need to have a huge swing;
3. You need to rotate hips and shoulders together, that is simultaneously;
4. With a series of little steps you need to put yourself into the proper position for the effective forehand; and
5. The wrist must be fixed totally rigid.

The well–known result of this 5-point approach to the forehand technique was two–fold: (a) nice and slow "ballet on the court" and (b) tennis elbow injury. Today, with the light-weight metal rackets and stronger players, the 5th postulate has been dropped: we can see now plenty of wrist slaps.

However, the other 4 postulates are still assumed valid. Well, from the perspective of high efficiency/maximum performance in tennis of the future – all four postulates are wrong. They are all pure "cosmetics" without any substance.

To dispel this "myth of a proper tennis forehand," in this section we give a biomechanical description of a whip–like tennis forehand movement, composed as a series of stretch–reflexes in all major joints.

Q. What is a "complex reaction" in tennis?

A. A complex reaction has the highest importance in every sport's duel, including tennis. In general, a complex reaction has two components: smart anticipation and lightning–fast reaction. You first anticipate the opponent's movement and then you react by making your own movement.

Good anticipation of the opponent's actions is the essential characteristic of a master in any sports duel. The best anticipation is actually called "mind reading". It is the difference between master and disciple. It can be learned, and it should be learned if you have high expectations from tennis. Of the current top tennis players, Federer, Nadal and Djokovic have the best anticipation – and coincidentally they are world number one, two and three. Before them, Sampras and Agasi had the best anticipation, and so on. Apart from getting the experience from many tennis tournaments (at different competition levels), the best tool for developing anticipation would be the bullet/blitz chess (see section 3.7 Tennis Chess).

On the other hand, the basic speed of reaction is largely genetically predetermined and cannot be significantly improved. The best means for its developing is both technique and training based on the stretch–reflex.

Q. What are the main characteristics of the good return?

A. For the current first serve in the range of about 200 $km/h - 220\, km/h$, the available time budget for the return player is approximately 0.7 s on the slowest clay courts, approximately 0.6 s on faster hard courts, and approximately 0.5 s on the fastest lawn courts. These are average numbers, with about 10% deviations due to differences in the conditions of ball flight and individual returning strategies (e.g., the position of the return player on the court etc.).

The whole return can be roughly divided into three phases: (i) anticipating the ball, (ii) movement regulation and reprogramming, and (iii) hitting the ball. During the time of movement regulation the players adapt their racket movement to the demands of the approaching ball. The sequencing of maximum segment velocities (i.e., first hips, then shoulder, then elbow, and finally wrist), which is a common feature of both the ground-strokes and serve, cannot be clearly seen during the return. This is due to: (a) the unpredictable nature of the return and the high time pressure under which players are placed; and (b) the very high demand of precision on the return. Currently, on the 1st serve return, precision–orientation is dominant, while on the 2nd serve return, ball–speed generation is dominant (the racket–head speed at impact is significantly lower during 1st serve returns). Also, the ability to reprogram an incorrect anticipation/decision (e.g., shifting from backhand to forehand return) within a fraction of a second is a very important factor on the return.

Therefore, it is highly recommended to: (i) train anticipatory abilities; (ii) train returns by varying the time pressure imposed on the players in order to improve their time management; and (iii) offer tasks for reprogramming.

12.3.6 Mental Training in Tennis

Q. What is the best preparation for the mental speed in tennis?

A. The best preparation for the mental speed in tennis is the so–called *lightning chess*, or *bullet chess*. It is the faster version of the *blitz chess* game, where each side has less than 3 minutes to complete all of their moves. Often bullet chess is so fast that tactics and skill are secondary to quick moves. Under United States Chess Federation (USCF) rules, bullet games are considered blitz. Every year, an over-the-board lightning chess tournament is held in Apeldoorn in the Netherlands. The time control is 2 minutes per player per game. It is the only official championship in bullet chess. The winner can claim the title Open Dutch Champion in Lightning Chess.

Q. What is the most effective visualization exercise for tennis?

A. The most effective visualization exercise for tennis champions has the following seven phases:

1. You need to decide what level in tennis you want to achieve (e.g., "world number one," or "in the top 10," etc.). This needs to be something that you believe that you can do and that you deserve.
2. Imagine yourself as a winner at that level.
3. Visualize that your every first serve results in an ace.
4. Visualize that you can efficiently return every serve.
5. Visualize that your every ground-stroke is a winner.
6. Visualize that you are full of energy.
7. Visualize the enjoyment of all your aces and winners.

Q. What are common problems associated with a player's mental game and how can they improve their mental game?

A. The common problems are: (i) confusion about strategy; (ii) trying to do too many things at once; (iii) being easily distracted; (iv) too much concern about winning and losing; (v) perfectionism; (vi) complacency; (vii) having no plan or clear goal; (viii) too much spontaneity and creativity; (ix) lack of humility; (x) inappropriate reaction to errors; and (xi) negative self talk (self–criticism).

To improve your mental game you need to: (i) set clear goals; (ii) create action steps that will take you closer to these goals (learn to visualize: try to "see" in your mind what you want to achieve); (iii) use positive self–talk (self–criticism); (iv) practice yoga breathing techniques; (v) always put forth

100% effort; (vi) be process oriented; (vii) stay detached; (viii) no future tripping, no past tripping – stay in the present; (ix) be non–reactive to the opinions of others; (x) tolerate your inability to be perfect; (xi) do what it takes to have fun and smile; (xii) define winning in a way that includes more than just the final score.

Q. How can I focus on the ball and at the same time be aware of the opponent, the court and the rest?

A. Yes, this is a great problem for most players, and apparently Federer solves it consistently and routinely. Aside from a great tolerance to competitive pressure, focus is one of the key characteristics of a champion. You really need to "fix your eyes on the ball," as every coach advises, and yet to have a full amount of the so–called *Situation Awareness* (SA), which is defined in modern psychology as "the perception of the elements in the environment within a volume of time and space, the comprehension of their meaning, and projection of their status in the near future."[26] For example, SA is a key factor in the training of fighter pilots and *Formula* 1 racing drivers. The outcome of any complex situation critically depends on SA. Therefore, you need to develop: (i) strong focus on the moving tennis ball; (ii) ability for strong SA, as defined above; and (iii) to have both focus on the ball and SA at the same time. This is similar to learning to play a hard piece on a piano: first you learn the right–hand part, then you learn the left–hand part, then you learn how to combine them together. Clearly, both physical and mental fatigue (combined with other disturbances, like e.g., rain, wind, noisy audience, big overhead screen, etc.) will gradually reduce both focus and SA. Therefore, besides physical speed and strength, you obviously need mental strength and endurance, to be able to win.

Q. What will be the mental profile of the future tennis champion?

A. The future tennis champion will be a rare combination of talent, hard work and the right mental profile. Even in today's tennis, often the difference between the good and the elite players is their mental qualities. In this respect, most important are the following three psychological characteristics: (i) confidence, (ii) anxiety, and (iii) motivation.

(i) Confidence is an emotion or state of mind commonly associated with athletic success. Indeed, the following quote from the former number one Jimmy Connors provides great insight into the confidence level of an elite athlete:

> The whole thing is never to get negative about yourself. Sure, it's possible that the other guy you're playing is tough, and that he may have beaten you the last time you played, and okay, maybe you haven't been playing all that well yourself. But the minute you start thinking about

[26] Endsley, M.R., 1995, Towards a Theory of Situation Awareness. Human Factors, **37**, 32–64.

> these things you're dead. I go out to every match convinced that I'm going to win. That is all there is to it.

In general, elite athletes tend to have very high levels of confidence and feel that these high levels are needed for the performances that they are looking for. About 90% of all elite athletes have a very high level of self-confidence. Confidence is usually a result of an athlete anticipating success in their upcoming event. An athlete's anticipated outcome is the greatest indicator of confidence. This expectation for success can be based on an athlete's confidence in themselves, emotional readiness, physical ability, knowledge of the opponent, goals, strategies, physical condition, or on the coach. To reach the very pinnacle of sport, an athlete must have high confidence in their abilities; and getting to that elite level and all the preceding successes that it took to get to that level must surely build the confidence levels of an athlete.

(ii) The link between anxiety and poor performance in sport has been known for a long time. Stories abound of athletes or teams that performed poorly because they underestimated their opponent (below optimum anxiety levels) or worried themselves out of the game (above optimum anxiety levels). Dealing with anxiety successfully is an important characteristic of the future tennis champion. The ability to cope with pressure and anxiety will be an integral part of the champion.

One of the earliest models that attempted to explain the relationship between arousal/anxiety and performance was the so–called *inverted–U hypothesis*, which stated that as arousal increased, performance would increase as well; but if arousal became too great performance would deteriorate. In other words, as stress began to build an individual still felt confident in their ability to control it and performance would improve. However, once a stressor became so great that the individual started to doubt their ability to cope with it, performance would decline.

An individualistic approach was added to this hypothesis when the concept of individualized zones of optimal functioning, or IZOFs, was developed. According to this theory, each individual has an optimal level of pre-performance anxiety. If the athlete is in this "zone," peak performances will be the result. However, if anxiety levels are too high or too low, the athlete will not see optimum results. IZOFs can be determined by repeatedly measuring anxiety and performance or through athlete's recall of anxiety levels prior to peak performances.

Depending on the individual, anxiety levels can have a variety of effects on athletic performance. Today it is well-known that anxiety can be reduced through mental imagery, relaxation, and cognitive intervention. These methods not only aim at reducing stress and anxiety levels but also aim to improve confidence levels. The goal is to help the athlete enter his or her IZOF.

(iii) To become a champion in any sport requires many hours, days and years of training. Often this training is rigorous, painful, or exhausting. However, the athletes who have reached the pinnacle of their sport have more than likely put in their time to get to achieve that high level of success. To do this, these athletes must have something that motivates them to continually push their bodies, and come back from whatever struggles or setbacks they may experience along the way. This motivation may come intrinsically or extrinsically. Intrinsic motivation is an athletes' personal drive to achieve their goal. This may be setting a school record, winning a race, or defeating a particular opponent. Extrinsic motivation is the resulting motivation from an outside source such as parents, coaches, or teammates.

There are many players who have the talent to succeed but very few who have the motivational drive to do what it takes to become a champion. In light of this, it appears that intrinsic motivation may be the greater determinant of achieving success in sports. To achieve at an elite level in sport, an athlete must have the motivation to train hard on a daily basis and to overcome any obstacles or setbacks that they might face in reaching or maintaining that level of performance.

Overall, it seems that the following traits would be common among elite athletes: extreme self-confidence, low performance anxiety, and high motivation. These three things are very closely related and would seem to form a cyclic pattern, positively influencing one another.

Q. How can I use visualization effectively?

A. In the 1980s and 90s, Dr. Denis Waitley implemented what he called "Visual Motor Rehearsal" (VMH) into the U.S. Olympics program. He and his researchers had found that when an athlete competed in an event only in their mind, the same nervous reaction in the body occurred as when they did their event in real life. This was another proof: The mind cannot tell the difference between an actual, "real–life" event and a vividly imagined one.[27] In this way, the VMH has shown that by merely thinking and visualizing their event, athletes can enhance their performance.[28]

How to apply the VMH to tennis? Before a tennis match, sit back, relax, close your eyes and play the game in your mind. Do the VMH every time you have an important match. The more you practice the more effective VMH will be for you. Do the same with learning a new technique: first watch the chosen ideal (a champion) and then visualize the same movement as performed by yourself. The more you visualize the less you need to practice physically.

[27] See "The Psychology of Winning," a famous tape program for self–improvement, by Dr. Denis Waitley.

[28] More technically speaking, relationships between sensory and motor events can be learned, although the events may have no prior association. For example, although most drivers know how to respond to a traffic signal, the sensory–motor rules governing these relationships require learning, because no intrinsic association exists between a traffic signal color or its spatial position and the appropriate movements to modify the speed of an automobile. These arbitrary cue–response associations become learned through experience, commonly by trial and error.

Q. Is it possible to be happy and think clearly all the time during big tennis tournaments?

A. Yes, we believe one can be happy and still think clearly in big tournaments. We would like to quote from the book *Born to Believe*, by Andrew Newberg and Mark Waldman:

> The brain is very happy when you are focused on what you love doing. The more you focus on what you truly love and desire, the volume gets turned down in those parts of the limbic system where the destructive emotions of fear, anger, depression and anxiety are controlled. This allows you to think more clearly.
>
> You also turn up the volume in other parts of the limbic system that generate positive emotions. When this happens, you get a release of dopamine, endorphins, and a variety of stress-reducing hormones and neurotransmitters, which enhances clear thinking. The more you focus on what you truly love, the healthier you are likely to be, and the more you feel the positive effects of those stress-reducing neuro-chemicals in your body and mind.
>
> You can have a decrease in negative emotions and an increase in positive emotions when you align yourself with what you believe is most important to you.

Q. Is it better to have realistic goals or just a wish to be a champion?

A. A focused desire to become a champion is a major driving mental force. Again, we quote from the book *Born to Believe*:

> When we focus on the big questions, the really big questions, we are challenging our brains to think outside the box, and this causes the structure of our neurons to change, particularly in our frontal lobes, that part of the brain that controls logic, reason, language, consciousness, and compassion.
>
> New axons grow, reaching out to new dendrites to communicate in ways that our brains have never done before. When contemplating the *big* questions we use our frontal lobes to alter the function of other parts of our brain.

12.3.7 Tennis Chess

Like in any other sport game, the important part of the tennis game is tactics. Current tennis tactics will be slightly simplified in the future tennis game, as, due to the highly–increased speed of the ball, long rallies will rarely exist. Therefore, this "tennis chess" will consist of up to 3–4 movements at any one time.

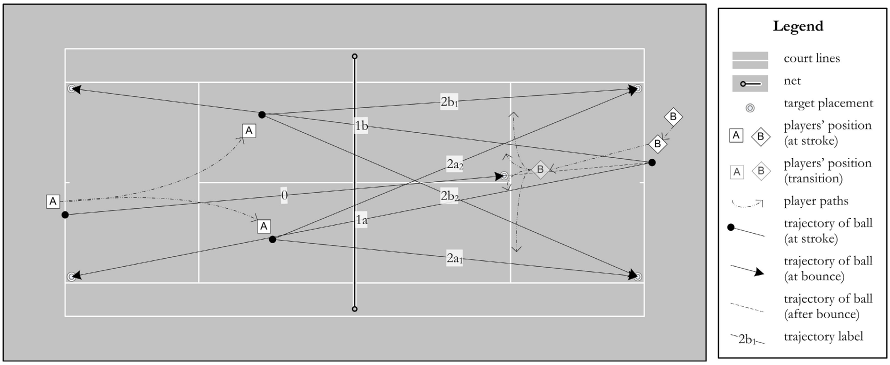

Fig. 12.19 Right–handed player targeting "T", serving to the right–handed player's backhand

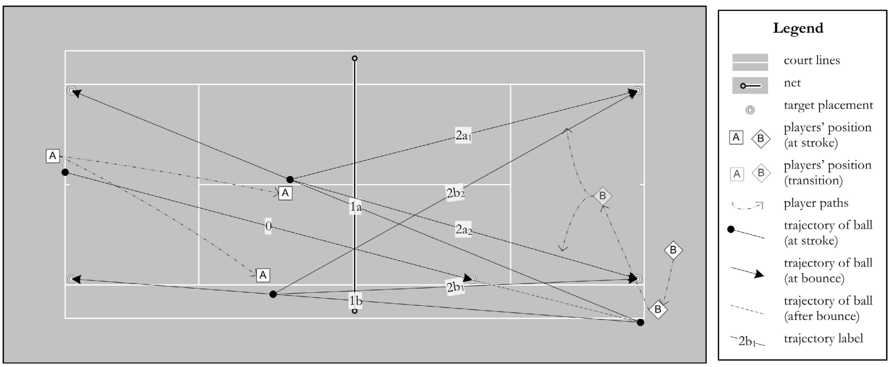

Fig. 12.20 Right–handed player targeting wide, serving to the left–handed player's backhand

Q. Assuming that both players are right-handed players, could you show us the optimal serve tactics?

A1. If you serve from the right side of the baseline centermark, then Figure 12.19 shows the optimal serve tactics.

A2. If you serve from the left side of the centermark, then Figure 12.20 shows the optimal serve tactics.

Q. Assuming that the right–handed player is serving and left–handed player receiving, could you show us the optimal serve tactics?

A1. If you serve from the right side of the centermark, then Figure 12.21 shows the optimal serve tactics.

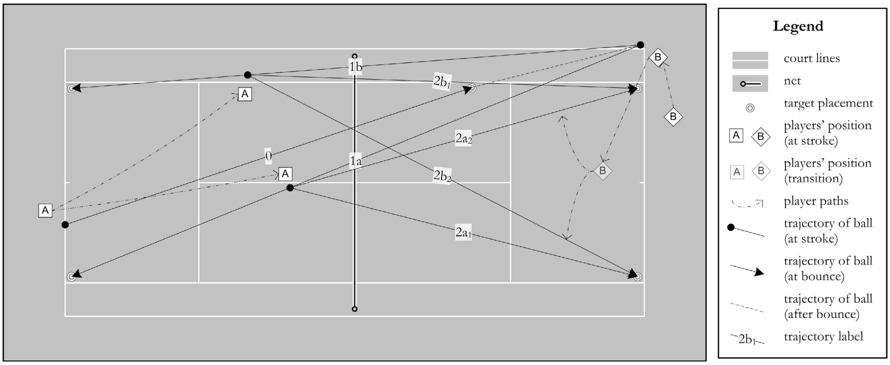

Fig. 12.21 Right–handed player targeting wide, serving to the right–handed player's backhand

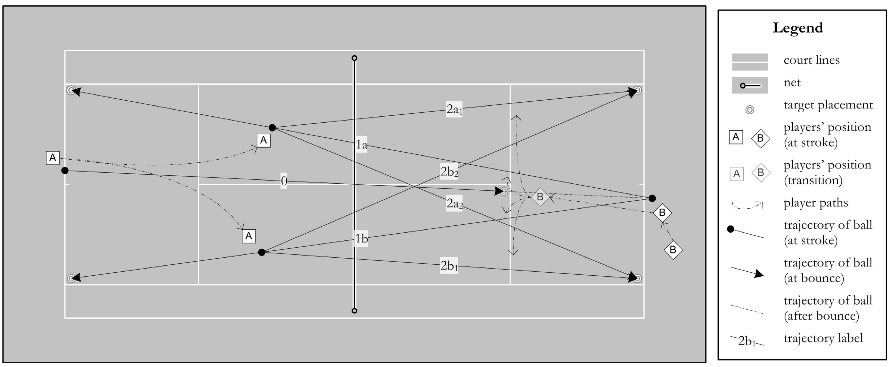

Fig. 12.22 Right–handed player targeting "T", serving to the left–handed player's backhand

A2. If you serve from the left side of the centermark, then Figure 12.22 shows the optimal serve tactics.

12.3.8 The Tennis Champion of the Future

Q. Finally, can you briefly describe the *tennis champion of the future*?

A. The tennis champion of the future is Roger Federer's hypothetical younger brother, who knows and can do everything that Roger knows and can do. In addition, he is both physically and mentally stronger and faster. Because of this addition, he will generally be much more efficient in the future tennis game, which will be much faster than today's tennis, due to improved racquet technology and players' improved psychophysical abilities.

The *superior tennis weapons* of the future champion will demonstrate their general efficiency in all game situations:

1. An aggressive 300+ km/h serve, targeting either the “T”–spot, or the weaker side of the opponent; the second serve is the same as the first 300+ km/h serve;
2. An aggressive, passing return, based on the ‘mind/body–reading’ *anticipation* and lightning–fast reaction (based on the stretch–reflex)
3. An aggressive, penetrating 240+ km/h forehand and single–handed 200+ km/h backhand (or, a two-handed 220+ km/h backhand), hit consistently from any body configuration and from any position on the court;
4. In addition, a serve–volley game can add-up to the general aggressive attitude.

A quick and accurate decision–making is crucial: both for the serve and for the aggressive return, as well as for the most appropriate shot selection in any other game situation. It is based on efficiently reading the ball and accurately predicting the opponent’s next move. In two words, this is what we call the *mental speed*. It will be trained (for years) by means of blitz and lightning–chess.

Besides the superior tennis weapons, the most obvious characteristic of the future champion will be *strong concentration*, consistent throughout the whole match/tournament. In two words, this is what we call the *mental strength*. It will be trained (for years) by means of *visualization* exercises.

In general, *high efficiency* means: “When the opportunity comes – *finish* the point, or game, or set, or match!”

High ability of *learning on the spot* means adjusting/changing both technique and tactics in the most appropriate way according to the situation on the court. (For example, if playing against a baseline player – being able to come to the net and efficiently finish the point with the winning volley.)

The ten main characteristics of the future tennis champion are:

- Natural talent;
- Commitment (to regular and consistent training/competition);
- Passion (desire to train, compete and win);
- Self–determination (responsibility for their performance);
- Self–belief (I can do it, self–confidence in their own capacity);
- Planned approach (clear goals; Federer himself admits that “scheduling has been a very important factor in my success – it helps me to heal from injuries and mentally get away”);
- Quick adaptation (to the changing situation on the court);
- Mind–body balance;
- Competitive toughness (like Federer, Nadal, Hennan); and
- Perseverance (persistence);

The three most prominent fitness characteristics of the future tennis champion are the three ‘A’s:

1. *Acceleration* (fast sprinting up to 15 m, from any starting body configuration);
2. *Agility* (quick changing of directions in both sprinting and jumping); and

3. *Athleticism*, which has two components:
   (i) ability to throw the current (reconstructed) official javelin 60 m;[29] and
   (ii) ability to perform a somersault on a court.[30]

Optimal body height (posture) for the future male tennis champion will be around 1.85–1.87 m, (like Federer, Nadal, Roddick, Djokovic), while his optimal body–weight will be 80 kg (like Federer and Djokovic). Optimal age for a male champion will be 22–27 (+/-2) years old.

If we would like to make a hypothetical future champion out of the current players, then it would be a combination of Roger Federer with Andy Roddick's power–serve and Gael Monfils' power–forehand.

In addition, every champion is a perfectionist: "If you can't do it properly, don't do it at all."

12.4 A Fuzzy–Logic Tennis Simulator

In this section we formulate a fuzzy–logic,[31] *attack* (AT) and *counter–attack* (CA) model for the *tennis game*. For technical details.

[29] This means doing several years of parallel javelin training during the teenage years; this would give a 300+ km/h serve.

[30] We do not suggest that the future tennis players will actually have to perform somersaults on the court. Rather, any gymnastics–trained kid that can perform a somersault on the street with a safe landing on his feet obviously have all the abilities commonly covered by the umbrella of *athleticism: superior jump take–off, superior coordination and superior balance.* Every elite tennis player can have this true athleticism if in his youth he had several years of parallel gymnastics training.

[31] Recall that fuzzy logic is a powerful problem–solving methodology with a myriad of applications in embedded control and information processing (e.g., train controllers, aircraft autopilots, air conditioning, control of nuclear reactors, etc). Fuzzy provides a remarkably simple way to draw definite conclusions from vague, ambiguous or imprecise information. In a sense, fuzzy logic resembles human decision making with its ability to work from approximate data and find precise solutions. Fuzzy logic usually works as a set of fuzzy IF–THEN rules. For example, in case of a heater controller with two inputs (temperature and humidity) and one output (fan speed) we have the following set of fuzzy rules:

IF temperature IS cold AND humidity IS high THEN fan-spd IS high
IF temperature IS cool AND humidity IS high THEN fan-spd IS medium
IF temperature IS warm AND humidity IS high THEN fan-spd IS low
IF temperature IS hot AND humidity IS high THEN fan-spd IS zero

IF temperature IS cold AND humidity IS med THEN fan-spd IS medium
IF temperature IS cool AND humidity IS med THEN fan-spd IS low IF
temperature IS war m AND humidity IS med THEN fan-spd IS zero
IF temperature IS hot AND humidity IS med THEN fan-spd IS zero

Attack Model: Tennis Serve

A. Simple Attack: Serve Only. The simple AT–dynamics is represented by a single fuzzy associative memory (FAM) map

$$\underset{CAT}{TARGET} \xrightarrow[FAM]{\mathcal{F}^{AT}} \underset{CAT}{ATTACK}$$

In the case of simple tennis serve, this AT–scenario reads

$$\underset{OPPONENT-IN}{O \ni o_m} \xrightarrow{\mathcal{F}^{AT}} \underset{SERVE-OUT}{SR \ni sr_n}$$

where the two $n-$categories, $O_{\dim=2} \ni o_m$ and $SR_{\dim=3} \ni sr_n$, contain the temporal fuzzy variables $\{o_m = o_m(t)\}$ and $\{sr_n = sr_n(t)\}$, respectively opponent–related (target information) and serve–related, partitioned by overlapping Gaussians, $\mu(z) =$
$exp\left[\frac{-(z-m)^2}{2\sigma^2}\right]$, and defined as:

$$\underset{OPPONENT-IN}{O} : \begin{array}{l} o_1 = Opp.Posit.Left.Right : (center, medium, wide), \\ o_2 = Opp.Antcp.Lft.Rght : (runCenter, stay, runWide), \end{array}$$

$$\underset{SERVE-OUT}{SR} : \begin{array}{l} sr_1 = 1.Serve.Speed : (low, medium, high) \\ sr_2 = 2.Serve.Spin : (low, medium, high) \\ sr_3 = 3.Serve.Placement : (center, medium, wide) \end{array}$$

In the fuzzy–matrix form this simple serve reads

$$\overset{O:\,OPPONENT-IN}{\begin{bmatrix} o_1 = Opp.Posit.Left.Right \\ o_2 = Opp.Anticip.Left.Right \end{bmatrix}} \xrightarrow{\mathcal{F}^{AT}} \overset{SR:\,SERVE-OUT}{\begin{bmatrix} sr_1 = 1.Serve.Speed \\ sr_2 = 2.Serve.Spin \\ sr_3 = 3.Serve.Place \end{bmatrix}}$$

B. Attack–Maneuver: Serve–Volley. The generic advanced AT–dynamics is given by a composition of FAM functors

$$\underset{CAT}{TARGET} \xrightarrow[FAM]{\mathcal{F}^{AT}} \underset{CAT}{ATTACK} \xrightarrow[FAM]{\mathcal{G}^{AT}} \underset{CAT}{MANEUVER}$$

In the case of advanced tennis serve, this AT–scenario reads

$$\underset{OPPONENT-IN}{O \ni o_m} \xrightarrow{\mathcal{F}^{AT}} \underset{SERVE-OUT}{SR \ni sr_n} \xrightarrow{\mathcal{G}^{AT}} \underset{RUN-VOLEY}{RV \ni rv_p}$$

IF temperature IS cold AND humidity IS low THEN fan-spd IS medium
IF temperature IS cool AND humidity IS low THEN fan-spd IS low
IF temperature IS warm AND humidity IS low THEN fan-spd IS zero
IF temperature IS hot AND humidity IS low THEN fan-spd IS zero.

where the new $n-$category, $RV_{\mathrm{dim}=2} \ni rv_p$, contains the opponent–anticipation driven volley–maneuver, expressed by fuzzy variables $\{rv_p = rv_p(t)\}$, partitioned by overlapping Gaussians and given by:

$$\underset{RUN-VOLEY}{RV} : \begin{array}{l} rv_1 = RV.For : (baseLine, center, netClose) \\ rv_2 = RV.L.R. : (left, center, right) \end{array}$$

In the fuzzy–matrix form this advanced serve reads

$$\overset{O:\,OPPONENT-IN}{\begin{bmatrix} o_1 = Opp.Posit.L.R. \\ o_2 = Opp.Anticip.L.R. \end{bmatrix}} \xrightarrow{\mathcal{F}^{AT}} \overset{SR:\,SERVE-OUT}{\begin{bmatrix} sr_1 = 1.Serve.Speed \\ sr_2 = 2.Serve.Spin \\ sr_3 = 3.Serve.Place \end{bmatrix}} \xrightarrow{\mathcal{G}^{AT}} \overset{RV:\,RUN-VOLEY}{\begin{bmatrix} rv_1 = RV.For \\ rv_2 = RV.L.R \end{bmatrix}}$$

Counter–Attack Model: Tennis Return

A. Simple Return. The simple CA–dynamics reads:

$$\underset{CAT}{ATTACK} \xrightarrow[FAM]{\mathcal{F}^{CA}} \underset{CAT}{MANEUVER} \xrightarrow[FAM]{\mathcal{G}^{CA}} \underset{CAT}{RESPONSE}$$

In the case of simple tennis return, this CA–scenario consists purely of conditioned–reflex reaction, no decision process is involved, so it reads:

$$\underset{BALL-IN}{B \ni b_{\mathcal{K}}} \xrightarrow{\mathcal{F}^{CA}} \underset{RUNNING}{R \ni r_{\mathcal{J}}} \xrightarrow{\mathcal{G}^{CA}} \underset{SHOT-OUT}{S \ni s_k}$$

where the $n-$categories $B_{\mathrm{dim}=5} \ni b_{\mathcal{K}}$, $R_{\mathrm{dim}=3} \ni r_{\mathcal{J}}$, $S_{\mathrm{dim}=4} \ni s_k$, contain the fuzzy variables $\{b_{\mathcal{K}} = b_{\mathcal{K}}(t)\}$, $\{r_{\mathcal{J}} = r_{\mathcal{J}}(t)\}$ and $\{s_k = s_k(t)\}$, respectively defining the ball inputs, our player's running maneuver and his shot–response, K.e.,

$$\overset{B:\,BALL-IN}{\begin{bmatrix} b_1 = Dist.L.R. \\ b_2 = Dist.\mathcal{F}.B. \\ b_3 = Dist.Vert \\ b_4 = Speed \\ b_5 = Spin \end{bmatrix}} \xrightarrow{\mathcal{F}^{CA}} \overset{R:\,RUNNING}{\begin{bmatrix} r_1 = Run.L.R. \\ r_2 = Run.\mathcal{F}.B. \\ r_3 = Run.Vert \end{bmatrix}} \xrightarrow{\mathcal{G}^{CA}} \overset{S:\,SHOT-OUT}{\begin{bmatrix} s_1 = Backhand \\ s_2 = Forehand \\ s_3 = Voley \\ s_4 = Smash \end{bmatrix}}$$

Here, the existence of efficient weapons within the $\underset{SHOT-OUT}{S}$ arsenal–space, namely $s_k(t) : s_1 = Backhand, s_2 = Forehand, s_3 = Voley$ and $s_4 = Smash$, is assumed.

The universes of discourse for the fuzzy variables $\{b_{\mathcal{K}}(t)\}$, $\{r_{\mathcal{J}}(t)\}$ and $\{s_k(t)\}$, partitioned by overlapping Gaussians, are defined respectively as:

$$\underset{BALL-IN}{B} : \begin{array}{l} b_1 = Dist.L.R. : (veryLeft, left, center, right, veryRight), \\ b_2 = Dist.\mathcal{F}.B. : (baseLine, center, netClose), \\ b_3 = Dist.Vert : (low, medium, high), \\ b_4 = Speed : (low, medium, high), \\ b_5 = Spin : (highTopSpin, lowTopSpin, flat, \\ \qquad\qquad\qquad lowBackSpin, highBackSpin). \end{array}$$

$$\underset{RUNNING}{R} :$$
$$r_1 = Run.L.R. : (veryLeft, left, center, right, veryRight),$$
$$r_2 = Run.\mathcal{F}.B. : (closeFront, front, center, back, farBack),$$
$$r_3 = Run.Vert : (squat, normal, jump).$$

$$\underset{SHOT-OUT}{S} : \begin{array}{l} s_1 = Backhand : (low, medium, high), \\ s_2 = Forehand : (low, medium, high), \\ s_3 = Voley : (backhand, block, forehand), \\ s_4 = Smash : (low, medium, high). \end{array}$$

B. Advanced Return. The advanced CA–dynamics includes both the information about the opponent and (either conscious or subconscious) decision making. This generic CA–scenario is formulated as the following composition + fusion of FAM functors:

$$\begin{array}{ccccccc} \underset{CAT}{ATTACK} & \xrightarrow[FAM]{\mathcal{F}^{CA}} & \underset{CAT}{MANEUV} & \xrightarrow[FAM]{\mathcal{G}^{CA}} & \underset{CAT}{DECISION} & \xrightarrow[FAM]{\mathcal{H}^{CA}} & \underset{CAT}{RESP} \\ & & & & \mathcal{K}^{CA} \Big\uparrow FAM & & \\ & & & & \underset{CAT}{TARGET} & & \end{array}$$

where we have added two new $n-$categories, $\underset{CAT}{TARGET}$ and $\underset{CAT}{DECISION}$, respectively containing information about the opponent as a target, as well as our own aiming decision processes. In the case of advanced tennis return, this reads:

$$\begin{array}{ccccccc} \underset{BALL-IN}{B \ni b_{\mathcal{K}}} & \xrightarrow{\mathcal{F}^{CA}} & \underset{RUNNING}{R \ni r_{\mathcal{J}}} & \xrightarrow{\mathcal{G}^{CA}} & \underset{DECISION}{D \ni d_l} & \xrightarrow{\mathcal{H}^{CA}} & \underset{SHOT-OUT}{S \ni s_k} \\ & & & & \mathcal{K}^{CA} \Big\uparrow & & \\ & & & & \underset{OPPONENT-IN}{O \ni o_m} & & \end{array}$$

where the two additional $n-$categories, $O_{\mathrm{dim}=4} \ni o_m$ and $D_{\mathrm{dim}=5} \ni d_l$, contain the fuzzy variables $\{o_m = o_m(t)\}$ and $\{d_l = d_l(t)\}$, respectively defining the opponent–related target information and the aim–related decision processes, both partitioned by overlapping Gaussians and defined as:

$$\underset{OPPONENT-IN}{O}: \begin{array}{l} o_1 = Opp.Posit.L.R. : (left, center, right), \\ o_2 = Opp.Posit.\mathcal{F}.B. : (netClose, center, baseLine), \\ o_3 = Opp.Anticip.L.R. : (runLeft, stay, runRight), \\ o_4 = Opp.Anticip.\mathcal{F}.B. : (runNet, stay, runBase). \end{array}$$

$$\underset{DECISION}{D}: \begin{array}{l} d_1 = Aim.L.R. : (left, center, right), \\ d_2 = Aim.\mathcal{F}.B. : (netClose, center, baseLine), \\ d_3 = Aim.Vert : (low, medium, high), \\ d_4 = Aim.Speed : (low, medium, high), \\ d_5 = Aim.Spin : (highTopSpin, lowTopSpin, noSpin, \\ \qquad lowBackSpin, highBackSpin). \end{array}$$

The corresponding fuzzy–matrices read:

$$\overset{B:\,BALL-IN}{\begin{bmatrix} b_1 = Dist.L.R. \\ b_2 = Dist.\mathcal{F}.B. \\ b_3 = Dist.Vert \\ b_4 = Speed \\ b_5 = Spin \end{bmatrix}}, \quad \overset{R:\,RUNNING}{\begin{bmatrix} r_1 = Run.L.R. \\ r_2 = Run.\mathcal{F}.B. \\ r_3 = Run.Vert \end{bmatrix}}, \quad \overset{D:\,DECISION}{\begin{bmatrix} d_1 = Aim.L.R. \\ d_2 = Aim.\mathcal{F}.B. \\ d_3 = Aim.Vert \\ d_4 = Aim.Speed \\ d_5 = Aim.Spin \end{bmatrix}},$$

$$\begin{bmatrix} O{:}\,OPPONENT-IN \\ o_1 = Opp.Posit.L.R. \\ o_2 = Opp.Posit.\mathcal{F}.B. \\ o_3 = Opp.Anticip.L.R. \\ o_4 = Opp.Anticip.\mathcal{F}.B. \end{bmatrix}, \quad \overset{S:\,SHOT-OUT}{\begin{bmatrix} s_1 = Backhand \\ s_2 = Forehand \\ s_3 = Voley \\ s_4 = Smash \end{bmatrix}}.$$

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