Michele Emmer
Editor

Imagine Math 2

Between Culture and Mathematics

Springer

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Introduction

Imagine all the people
Sharing all the world ...
John Lennon

Imagine mathematics, imagine with the help of mathematics, imagine new worlds, new geometries, new forms. Imagine building mathematical models that make it possible to manage our world better, imagine solving great problems, imagine new problems never before thought of, imagine music, art, poetry, literature, architecture, theatre and cinema with mathematics. Imagine the unpredictable and sometimes irrational applications of mathematics in all areas of human endeavour.

Imagination and mathematics, imagination and culture, culture and mathematics. For some years now the world of mathematics has penetrated deeply into human culture, perhaps more deeply than ever before, even more than in the Renaissance. In theatre, stories of mathematicians are staged; in cinema Oscars are won by films about mathematicians; all over the world museums and science centres dedicated to mathematics are multiplying. Journals have been founded for relationships between mathematics and contemporary art and architecture. Exhibitions are mounted to present mathematics, art and mathematics, and images related to the world of mathematics.

The volumes in the series "Imagine Math" are intended to contribute to grasping how much that is interesting and new is happening in the relationships between mathematics, imagination and culture.

This second volume of the series begins with the connections between mathematics, numbers, poetry and music, with the latest opera by Italian composer Claudio Ambrosini. Literature and narrative also play an important role here. There is cinema too, with the "erotic" mathematics films by Edward Frenkel, and the new short "Arithmtique" by Munari and Rovazzani. The section on applications of mathematics features a study of ants, as well as the refined forms and surfaces generated by algorithms used in the performances by Adrien Mondot and Claire Bardainne. Last but not least, in honour of the hundredth anniversary of his birth, a mathematical, literary and theatrical homage to Alan Turing, one of the outstanding figures of the twentieth century, a great mathematician who was the victim of the prejudices of his day.

The topics are treated in a way that is rigorous but captivating, detailed but full of evocations. An all-embracing look at the world of mathematics and culture.

Michele Emmer

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Mathematics, Numbers and Music

The Fascination of Numbers, between Music and Poetry

Michele Emmer

I choose numbers because they are so constant, confined, and artistic.

Numbers are probably the only real discovery of mankind. A number of something is something else. It's not pure number and has other meanings.

Hanne Darboven, artist [1]

God made the natural numbers; all else is the work of man.
Leopold Kronecker, mathematician

Mathematics and numbers, mathematics and poetry

A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with *ideas*. A painter makes patterns with shapes and colours, a poet with words. A painting may embody an 'idea', but the idea is usually commonplace and unimportant. In poetry, ideas count for a good deal more; but, as Housman insisted, the importance of ideas in poetry is habitually exaggerated: 'I cannot satisfy myself that there are any such things as poetical ideas.… Poetry is not the thing said but a way of saying it.'

Not all the water in the rough rude sea
Can wash the balm from an anointed King.[1]

Could lines be better, and could ideas be at once more trite and more false? The poverty of the ideas seems hardly to affect the beauty of the verbal pattern. A mathematician, on the other hand, has no material to work with but ideas, and so his patterns are likely to last longer, since ideas wear less with time than words. The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics.

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[1] From Shakespeare, *Richard II*, act III, scene II.

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These are the words of the mathematician G. H. Hardy from the autobiographical *A Mathematician's Apology* [2]. Aside from the statements about the supremacy of mathematics over every other activity of the human spirit, Hardy neglects music, which is quite often and for good reason compared to the activity of the mathematician.

The ties between poetry and mathematics is the subject of a small book entitled *Poetry and Mathematics* published in 1929 by Scott Buchanan, when he was Assistant Director of the People's Institute in New York. It was reprinted with a new introduction by the author in 1962 [3]. Buchanan sets out to show the correspondence between the forms of poetry and those of mathematics, with particular regard to numbers, figures and equations: between counting in arithmetic and recounting in the narrative, between the transformation of figures in geometry and the development of characters in the tale.

Buchanan writes that he was able to arrive at simple conclusions about poetic and mathematical elements:

> The symbolic elements of poetry are words and the corresponding elements of mathematics are relations. It is rather easy to pass from the symbolic elements to the aspects of reality that they designate. Words stand for the qualities, ratios stand for the relations. Very simply, poetry and mathematics are two very successful attempts to deal with ideas. Both employ sets of symbols and systems of notations. In this respect they have very interesting and illuminating comparisons and contrasts. As they revolve through their lifecycles of fantasy, utility, culture, truth and falsity, they reveal what I shall call aspects of the mathematical and poetic object.

Luca Pietromarchi, professor of French literature at the Università di Roma Tre, observes:

> The poet is not a mathematician. And yet he is always counting. He counts the number of verses, the number of syllables, the number of strophes. He never ceases comparing word to number, assigning to poetry the function of stemming the infinite discourse of prose of the world, confining it within a numerical ratio [4].

These were the words that opened the conference entitled 'La poesia e i numeri' (Poetry and Numbers) organised by the Associazione Sigismondo Malatesta, which took place in the castle of Torre in Pietra (near Rome) in January 2010.

> It is the number in itself, the graphic representation that has been given to it over time, the form assumed from time to time, the absoluteness of its icastic manifestation, which has guided the selection of the works. Number: object and subject of paintings, sculptures, drawings, videos, films, photographs, installations. Aesthetic potential circumscribed by the configuration of number as abstract entity, self-sufficient, concluded in itself and thus absolute [5].

These instead are the words of Marco Pierini, co-curator with Lorenzo Fusi of the exhibition entitled *Numerica* mounted at the Palazzo delle Papesse in Siena in 2007. The Palazzo delle Papesse, once a centre for contemporary art, was closed immediately after the exhibition.

The exhibition opened with Balla's *Numeri innamorati*, which also appeared on the cover of the catalogue of the exhibition held in Stuttgart in 1997 entitled *Magie* *der Zahl* (The Magic of Number), a great supermarket of numbers in art: numbers, numbers, numbers without any criteria.

With Balla we are in 1924. Numbers appeared in the collages of the Cubists, in the works of Boccioni, hugely interested in the fourth dimension and the new geometries. Marinetti wrote that a love of precision and essential brevity had *naturally* provided him with a taste for numbers, which live and breathe like living beings in our new numerical sensitivity. Numbers are new though very ancient, symbols of modernity though immutable. The fascination of numbers.

Mel Bochner was present at the Siena exhibition with *Counting: 0-1- (#6).* He declared:

> Counting, the only thing that I could be sure to do well. [...] Numbers give me the freedom to think of other things, they are already invented and do not belong to anyone.

This exhibition was one that gave ample space to irony and the play of numbers.

There had to be a small part related to the golden section and the Fibonacci series. In one room of the historic palace decorated with the words *Utilità, ordine, prontezza* (Utility, order, alacrity), Mario Merz recalled those numbers that followed the crocodile along the spiral ramps of the Guggenheim in New York.

Pietromarchi added:

> That of poetry is an incessant comparison of the word with the infinity – metaphysical infinity, spatial infinity, but also the infinity of discourse – that it numbers, in order to dominate it, understand it, translate it. Poetry translates the infinite into number; every poetic text is formally enclosed in a numeric order. [...] The numbered word of poetry reabsorbs the infinite in the calculable, measurable, inhabitable. It exorcises the anxiety that numbers, by the very fact of their infinity, can suggest.

> *All is Number.*
>
> *Number is in all.*
>
> *Number is in the individual.*
>
> *Ecstasy is a Number.*
>
> Baudelaire

There is no need for us, poets and mathematicians, to trouble ourselves too much about what a number is, about what the first number is.

> That unit is not a number, but is good principle of each number, and it is that by means of every thing that is said to be one. And according to Severinus Boethius (475-525) in his music, each number is the unity in potential.
>
> Luca Pacioli, *Summa*, 1494

Many years later the mathematician Giuseppe Peano would write in his article “Sul concetto di numero” (On the Concept of Number) [6]:

> Number cannot be defined, since it is evident that however various words (symbols) are combined among themselves, there could never be an expression equivalent to number.

But poets and mathematicians have no reason to worry:

> Fortunately, the mathematician as such need not be concerned with the philosophical nature of the transition of collections of concrete objects to the abstract number concept. We shall therefore accept the natural numbers as given, together with the two fundamental operations, addition and multiplication, by which they may be combined.

This was written by Courant and Robbins in their famous book of 1941, *What is Mathematics?* [7]

Musicians needn't worry either: *5... 10... 20... 30... 36... 43...*

This is how *The Marriage of Figaro* begins, the opera by Wolfgang Amadeus Mozart, with a libretto by Lorenzo Da Ponte.
Numbers that become words that become music.

Leibniz observed, 'Music is the pleasure the human mind experiences from counting without being aware that it is counting'.

In his book on the history of mathematics, Morris Kline wrote:

> During the many years from the age of Pythagoras to the nineteenth century, mathematicians and musicians alike, Greek, Roman, Arabian and European, sought to understand the nature of musical sounds and to extend the relationship between mathematics and music. Systems of scales and theories of harmony and counterpoint were dissected and reconstructed.

We must not forget Fourier's theorem:

> Any periodic signal, under suitable mathematical conditions (which are always verified by physical signals), can be represented by means of the sum of a constant term and infinite sinusoidal functions, whose frequencies are whole multiples of that signal.
>
> The formula is a sum of simple terms of sines of the form a sin bx.

The theorem affirms that any musical sound, no matter how complex, is simply a combination of simple sounds such as those emitted by a diapason. It is theoretically possible to perform the entire Ninth Symphony of Beethoven with diapasons.

One of the most famous lyric poems of Catullus speaks of love and numbers:

To Lesbia
Give me a thousand kisses, a hundred more,
another thousand, and another hundred,
and, when we've counted up the many thousands,
confuse them so as not to know them all,
so that no enemy may cast an evil eye,
by knowing that there were so many kisses.

Words and numbers that become poetry, which in its turn becomes music. These become the leading characters in a quite recent musical work:

Wife:
And then, there is poetry also in numbers ...
Killer:
Catullus?
...

Wife:
Numbers are beautiful, you know?
There are long and short ones, and primes and infinites…

This is the dialogue between the leading character, the Word Killer, and his wife. His job is to strike from the dictionary all words that are by now out of use and substitute them with new words, but he can't do it, he can't make himself kill the words. His wife is an unscrupulous career woman, and she can't understand why her husband, the killer, is so blocked, so irresolute. It is necessary to go forward without hesitation, to kill that which no longer serves a purpose. And words don't serve; what serves is numbers, technology and numbers.

Killer di parole (Word Killer) is a lyric opera, indeed a 'playdrama', as Claudio Ambrosini, author of the libretto and the music, called it. Ambrosini is one of the most important contemporary musicians in Italy. The libretto was born of a story conceived by Ambrosini together with the French writer Daniel Pennac. The opera is contemporary, but reclaims and takes from the great music of the past centuries, and the experiences of today. It invents a music that resounds, recalls, echoes with ancient sounds and words but which is new, inventive, creative.

Humanity:
M...m(o)...m(o)...p(o)
M...P(ae)...m(ae)...sì
M(o)...m(ae)...no.
M.a è u i o, no
Sì, i a ü i è no, è sì a!

It is the beginning of the opera, the chorus intones these sounds, the first sounds of humanity. They are phonemes that, little by little, become words and music that unite into a background of primordial and ultramodern sounds that cast a spell on the listeners. What is being born in front of our very eyes is word, language, awareness.

The chorus of humanity is ever present in the background, the depository of the only truth. The sounds are crystalline. The Word Killer talks a lot. He wants to save all the words, all the sounds that have a history, and perhaps, e a future.

In the second act the failed Killer is supposed to cancel entire languages, cancel civilisations, which by now no longer has any reason to exist in a globalised world that has arrived at a single language, a single civilisation, a single culture. Obviously, he tries to record and save the many languages of the world.

The *ultimi parlanti*, last speakers, intone the vowels a i u o e , returning to the inarticulate sound heard at the opera's beginning. There is no hope.

The numbers return too, numbers that in the opera were the image of abstraction, aridity, evil technology.

Wife: ten, nine, eight, seven, six, five, four, three, two, one
Killer: zero!
Wife (with a hypocritical smile): Cheers ! and … good luck!

This is the end of the opera and of an intense musical and theatrical experience, thanks also to the set design and costumes of the second act. During the first part, that large red square that rotates in empty space while humanity tries to speak recalls the large squares at the beginning of the twentieth century, the red and black squares of Malevitch, El Lissitsky and Kandinsky.

Words, music, set design, emotions, participation and immense creativity, all were on stage at Venice's Teatro Fenice in early December 2010.

Morris Kline wrote in his *Mathematics in Western Culture* [8]:

> … the most abstract of the arts can be transcribed into the most abstract of the sciences, and the most reasoned of the arts is clearly recognized to be akin to the music of reason.

And the story, the story of numbers and poetry, continues, fortunately without end.

And he swore on the numbers

By the evens and the odds [9].

Translated from the Italian by Kim Williams

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The Solitude of Last Words

Claudio Ambrosini

A linguistic approach integrated with a mathematical type of procedure has become increasingly important over the last decades, ultimately arriving at being projected onto other areas of thought, communication and creativity. In music, for example, there has been an appearance of studies based on the idea of "generative grammar", derived from the theory by the same name developed by Noam Chomsky in the 1950s and applied in this case to the relationships between sounds. It is precisely around these two dimensions – language and science, or better, words and numbers – that revolve two lyric operas that I have worked on in recent years: *Big Bang Circus* (written in 2001 and performed in a world premiere at the *Biennale Musica* in Venice in 2002) and *Il killer di parole* ("The Word Killer"), which I began to compose in 2008 and was performed for the first time at Venice's Teatro La Fenice in 2010. In both of these works what at one time was viewed as two opposite approaches – one scientific, the other humanistic – were instead set face-to-face in order to discover the affinities that would make them appear to be synergistic.

Big Bang Circus

To support this assumption I will briefly quote from *Big Bang Circus* (subtitled *Piccola Storia dell'Universo*, "A Short History of the Universe"), an opera "in the form of a circus", which brings together the stories, legends and myths used by humans – whether the inhabitants of the Amazon forests, those of the frozen lands of the poles or those of the Pacific islands, or the Maya, Indians, Greeks, Romans or countless others – to describe the Origin, the apparently unexplainable moment in which the whole thing began. Difficult to explain but not unimaginable: in fact, the human imagination and the words of poetry have described this moment in innumerable ways, envisioning the most diverse scenarios: a dream attached to a thread, the energy of a scream, the breaking open of an egg, the fall of a drop of milk, the winding of the coils of a snake, a wave, a wind, a pearl, a sound that animates, and more.

What *Big Bang Circus* was intended to underline, in a musical tale where imaginary characters (who sing the mythical stories) alternate with historical figures like Aristarchus, Giordano Bruno, Galileo, Max Born and Einstein (who tell their own

Claudio Ambrosini
Composer, Venice (Italy).

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stories) is that when later men "of numbers" retrace the same steps, ask themselves the same questions, they end up formulating answers that, surprisingly, come to use the same terms, the same images and metaphors used much earlier by the men "of words".

One example might be seen in this passage of the libretto, which is based on the *Rig-Veda*:

Eterna, cosmica energia, danza sopra l'ignoranza nel cerchio di fiamma, aura di sapienza. Suona il tamburo, ruotando col fuoco nelle mani, Shiva!	Eternal, cosmic energy dances atop ignorance in the circle of fire, aura of wisdom. Plays the drum, spinning with fire in his hands, Shiva!

Indian depictions of this moment show the god Shiva performing a dance surrounded by a circle of small flames – we might say the irradiation of an explosive energy – trampling a monstrous being beneath his feet, which represents Ignorance. Shiva holds something in each hand: in his upper left hand a small flame – at once "the generating spark" and the symbol of death, a metaphor for the cycle of destruction and regeneration that characterises both this god and the life of the entire universe – and in his upper right an hourglass-shaped drum, which he uses to accompany his dance (Fig. 1).

In the reconstructions that today's scientists formulate, as they attempt to describe the same events, the images and terms that they employ are surprisingly similar: they in fact explain that the original nucleus of matter was so dense and compact that it was *stretched like the skin of a drum*. At this point the tension between the

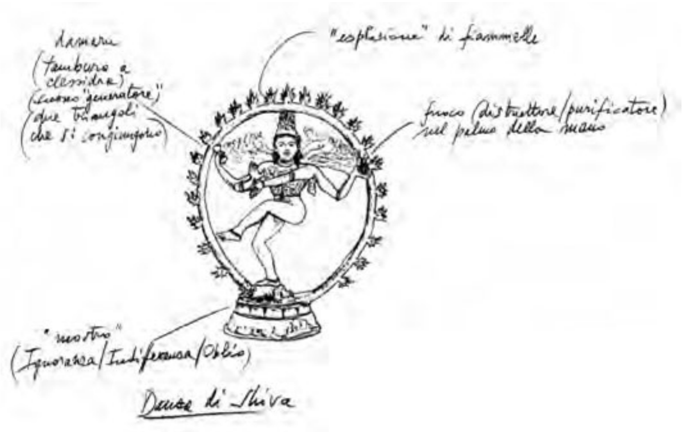

Fig. 1 Shiva dancing in the circle of fire. The small hourglass-shaped drum that he holds in his upper right hand is itself a symbol of the conjunction of two triangles which, in their turn, are abstract representations of the male and female reproductive organs. Their intersection thus represents the act of creation. This mimesis is depicted humanly each time there is contact between the upper and lower lips of the mouth (which are therefore also possible symbols of the sexual organs) in pronouncing the sacred syllable "Om", recalled here by the circular frame that surrounds the dancing god. Author's graphic representation and notes (© C. Ambrosini)

components of the nucleus was so great that they began to fibrillate, to *vibrate* like a sound wave,[1] with such *rhythm* and intensity that it produced a scission of the matter and gave rise to a flaming explosion that we call the *Big Bang*.

In other words, between humanistic culture and scientific culture, between thinking that tries to imagine and describe reality via symbols of intuition and one that faces the same reality by "re-reading" it in scientific terms, there are fewer difference and less distance than one would think.

"Il killer di parole"

Words and numbers are once again compared and contrasted, but to a greater extent, in the next opera of the cycle, *Il killer di parole* (The Word Killer),[2] which also aims to heighten the spectator's awareness of the question of the death of languages. Formally speaking, this is a *ludodramma*[3] in two acts, the first of which emphasises how (written) languages are continually subject to a process that is both evolutionary and degenerative, often caused by the borrowing of terms from other languages, in many cases absorbed indiscriminately.

In contrast to prime numbers, "prime" words – in the sense of new, recent – have the effect of conflicting with the language that receives them: while they are apparently an enrichment, in reality they often make it so that other words suddenly show their "age", and are unwittingly considered passé, gradually becoming "last" words. More than the appearance of neologisms, a symptom of creativity, it is the passive importation of terms coming from other languages that, rather than enriching the lexicon of the host language, often results in its impoverishment, a reduced variety in the use of shades of meaning, an atrophy of the capacity to connote.[4]

---

[1] Many other myths also speak about creation having taken place by means of a screaming or shrieking sound: the Word of divinity that rends the silence (*En arkè en o logos* . . .).

[2] Both of these works are part of a cycle of five whose first sketches date back to the 1980s and whose phases of composition and performance began in 1995 and concluded in 2012. The definitive order of the cycle comprises the following five works: *Big Bang Circus* (2001), *Il canto della pelle – Sex Unlimited* (2005); *Il killer di parole* (2008-2010); *Apocalypsis cum figuris* (2012) and *Il giudizio universale* (1996) (all concepts, music and librettos by Claudio Ambrosini, except for *Big Bang Circus*, on the libretto for which Sandro Cappelletto collaborated). The temporal arc spanned by this project therefore goes from the very first moment (*Big Bang Circus*) to the very last (*Il giudizio universale*). In the middle, an opera on Eros (*Il canto della pelle - Sex Unlimited* black), intended as attractive energy, present not only among living beings but also in magnetism, in the chemistry of the elements and so forth, the force capable of producing life, in "limitless ways and shapes", in the universe. This is followed by an opera about the languages of the world, capable of recounting all these things (*Il killer di parole*) and thus, of saving these local languages from the growth of more national and international languages (such as English or Chinese). The penultimate of the cycle is a foreshadowing of final judgment, *Apocalypsis cum figuris.*

[3] By *ludodramma* I mean a theatrical form that begins with the light tones of a playful opera and ends with a dark colours of a melodrama.

[4] In Italy, for example, in addition to the continual absorption of English words, even when there is a perfectly corresponding Italian word, there is also a widespread Italianisation of terms that often come from the field of computer science, with results that are silly if not downright misleading, such as *scannare* for "to scan" or *brifare* for "to brief". In some cases the adoption of one term has lead to the production of others, as in the case of *editare* (from "to edit"), related words have been produced – particularly regarding its variants of "editor", "edited by", etc. – amusing ambiguities with the pre-existing *editore*, the Italian word that referred, not to the person who edited or revised a text, but to the publisher or publishing house. We now await other such adoptions with bated breath. . .

The second act of *Il Killer di parole* is instead devoted to the constant tragedy that takes place in our world without our ever being aware of it: the repeated disappearance of languages (in this case spoken) that appertain to small communities, so isolated and underdeveloped that they have never developed a writing or recorded their cultural legacy for posterity. When the last person able to speak such a language dies, in that instant humanity loses not only "that language" but its entire cultural patrimony of tales, myths, proverbs, beliefs, songs and whatever else that the community that produced it had developed over the course of centuries.

The Killer, the Wife

When it came time to transform this premise into a lyric opera, it was necessary to create characters who could give a face and voice to these intentions, so at the heart of the opera there are two principal characters, a couple (baritone and coloratura soprano) who represent the two fundamental approaches to the situation just described: on one hand the Killer, who is immersed in the "abstract" dimension of the words, and on the other his Wife, the expression of a thought that is more concrete, based on numbers. In order to describe them in a stroke we need only say that they work for the same company (a large publishing house) but at very different jobs: the Killer is employed in the updating of dictionaries, while instead his wife has an administrative job.

Being, however, a person who loves words, and a poet in his own right, the "killer" is loathe to carry out his role as "expunger of obsolete words", that is, one assigned to remove from dictionaries the words that are seldom used to make way for terms recently introduced: he cannot accept "killing them"; indeed, he tries to defend them and to hinder the progress of reprinting. In the scene that opens the first act, we see him affectionately rocking his own son, just a few months old, as he tries to explain to him what words are:

IL KILLER DI PAROLE
(al bimbo, dondolando la culla):
Sono buone, le parole, sono belle,
sono loro la pelle delle idee, sono celle
per il miele dei pensieri, son giocattoli per la testa!
Sono tante le parole, sono tinte,
ben distinte, calibrate,
rare, nuove, sorpassate:
vecchie, ma buone! *(come per mangiarlo)* Am![5]

THE WORD KILLER
(to the child, rocking him in his cradle):
They are good, words, they are fine,
they are the skin of ideas, they are a hive
for the honey of thoughts, they are toys for the mind!
They are many the words, they are tinct,
quite distinct, calibrated,
rare, new, outdated
old, but good! *(pretending to bite)* Chomp!

Good, functional, perhaps adopted by the great writers or used in sublime poetry, and yet still destined for the sad fate that awaits the "last" words: dictionaries can't continue to grow infinitely, nor to weigh more and more beyond all reason, and so they must be eliminated to make room for the new. And this is precisely the job of the "Killer", a task that drives him to despair (Fig. 2 and 3).

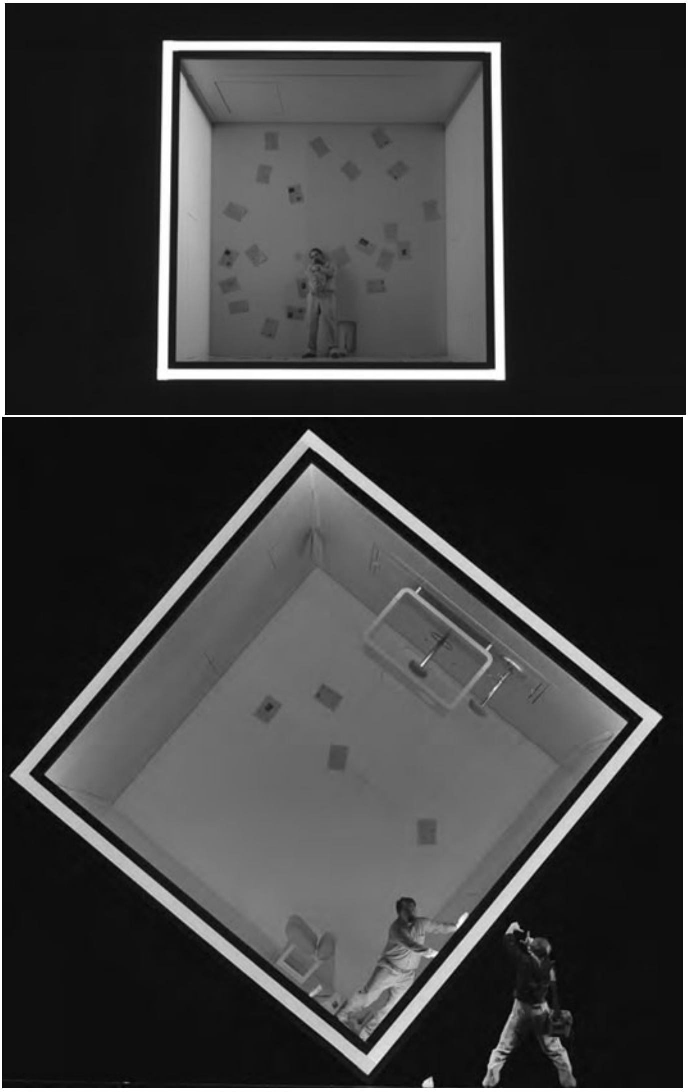

Fig. 4 - 3 C. Ambrosini, *Il killer di parole*, Act I, scenes 2 and 6. In the first act, the set design created by scenographer Nicholas Bovey situates the action in a cube suspended in the dark. As one scene follows another, the cube rotates, giving rise to a space that is both realistic and surreal (Reproduced with permission of Teatro La Fenice, Venice. Photo © Michele Crosera)

But the Wife too, talking to the child, is able in her turn to speak very affectionately of her "own" world, that of numbers:

LA MOGLIE
(Al bimbo, chinandosi sulla culla)
I numeri sono belli, sai?
Ce ne sono di lunghi e di corti e di primi e di infiniti…
Vedrai come ti piaceranno, perché sono divertenti! Molto!
(Lo prende in braccio e comincia ad allattarlo)
Bravo lui… Così.
(Il bimbo sugge il latte)
Dai, contiamo: un, due, tre:
m, m, m… Ancora. Un, due, tre:
m, m, m. Quattro, cinque, sei:
m, m, m, m, m, m, m, m, m, m, m, m, m, m, m…
e sono sedici!
(scherzosa, creando lì per lì un gioco di parole)
E se dici "sedici"… dici "se"! Capito? Se dici "sedici", dici… "se"![6]

THE WIFE
(to the baby, leaning over the crib)
Numbers are beautiful, you know?
There are long ones and short ones, and prime and infinite ones…
You'll see that you'll like them, because they are fun! Very!
(She picks him up and begins to nurse him)
Good boy … That's the way.
(The baby suckles)
Come on, let's count: one, two, three:
m, m, m… Again. One, two, three:
m, m, m… Four, five, six:
m, m, m, m, m, m, m, m, m, m, m, m, m, m, m… and that makes sixteen!
(playfully, making up a word game)
And if you say "sixteen" … you say "if" Get it? If you say "sixteen" … you say "if"![7]

Shortly after, to rock him to sleep with a lullaby (Fig. 4), she invents another word game, this time much more cryptic because it is based on the first syllable of the numbers, in succession from 1 to 10: *Uno*, *due*, *tre*, *quattro*, *cinque*, *sei*, *sette*, *otto*, *nove*, *dieci*.

Fig. 5 C. Ambrosini, *Il killer di parole*, Act I, scene 3. Roberto Abbondanza and Sonia Visentin as the Killer and the Wife, performing at the Teatro La Fenice in Venice (Reproduced with permission of Teatro La Fenice, Venice. Photo © Michele Crosera)

LA MOGLIE
(Cullando il bimbo)
Un due tre: qua-ci-sei!
Se- (h)o- no-di-… Un- do- tre-!
(Il bimbo si addormenta.
Più riflessiva, tra sé)
Spero davvero che ami i numeri
se vuol esser felice nella vita.

THE WIFE
(Rocking the baby)
Un due tre: qua-ci-sei!
Se- (h)o- no-di-… Un- do- tre-!
(The baby falls to sleep.
More thoughtfully, to herself)
I really do hope that he loves numbers
if he wants to be happy in life.

What she makes up is a surreal riddle which, rewritten in numbers, appears like this:

1, 2, 3: 4, 5, 6
7, 8, 9, 10… 1, 2, 3!

Later, the Wife addresses the Killer – who she had previously reproached, accusing him of losing himself in useless nitpicking – asking him to kiss her with the famous verses in Latin by Catullus:

LA MOGLIE
(per un attimo più dolce, con affetto e nostalgia)
Da mi basia mille, deinde centum,
dein mille altera, dein secunda centum,
deinde usque altera mille…

THE WIFE
(for a moment sweeter, with affection and nostalgia)
Da mi basia mille, deinde centum,
dein mille altera, dein secunda centum,
deinde usque altera mille… [8]

The rendering of this poem takes places in the form of a virtuoso aria distributed on different planes of sound, whose difficulty lies not so much in the traditional hail of notes as in the soprano's ability, as acrobatic as it is rare, to double or even triple her own voice, passing suddenly from very low notes to middle notes to acute and then down again and then up again and so on without any transitions, so that what is created is a kind of three-dimensional sound space (Fig. 5).

Fig. 6 C. Ambrosini, *Il killer di parole*, Act I, Scene 3, mss. 153-158. Three planes of sound that revolve around a B-flat that belongs to three different octaves (© Edizioni Ricordi 2010)

Saving the languages of the world

The second act takes to audience to another time (twenty-five years have passed since the first act) and poses new problems. Now the synergy of words and numbers is represented in another form, with numbers transformed directly into technology, into audio-visual recording instruments used in the attempt to save the languages of small communities in danger of extinction.

The characters are the same, except that in the meantime the Son (who in the first act was in his crib) has grown up and become a young attorney devoted, following in the footsteps of his father, to noble causes.[9] But many other things have also happened: the Wife has had an exceptional career, thanks to her administrative skills, and has been named general director of the entire company where both she and her husband work. The company itself has grown enormously and has gone from a simple publishing house to a huge multimedia production centre, with publications that go from books to disks, to videos and documentaries.

In contrast, the Killer's career has been unimpressive. Because of his hesitancy and excessive love for words, in the past twenty-five years he has only "killed" a few, and thus was not even able to complete the updating of the first dictionary he was assigned to. That had to be done, in his stead and in secret, by his wife. And it is precisely because she is now a powerful woman that the Killer has not been fired. In fact, a new job has been created just for him: recording the voices of the various "last speakers", the last people capable of speaking extremely rare languages, those belonging to communities that, upon the death of the last speakers, are destined to disappear without a trace.

In the course of the act, this time situated in an audio-visual studio, we thus see a parade of curious figures, coming from the most isolated places, who make the sounds of their languages, carefully recorded by Killer (Fig. 6).

Fig. 7 C. Ambrosini, *Il killer di parole*, Act II, scene 3. Recording of the "Last Speaker of the Windy Lands", an old man who recites an epic tale in a completely unknown language (Reproduced with permission of Teatro La Fenice, Venice. Photo © Michele Crosera)

[9] In reality the opera involves six singers, three of whom have a constant role: in addition to the Killer and the Wife, there is the Son (a tenor). The others alternate among several roles: the second soprano plays The Dead Word, the Photograph and the Last Speaking Woman in the Couple; the bass plays The Colleague, The Last Speaking Old Man, and the Last Speaking Man in the Couple. To these are added the chorus, which plays Humanity, The Last Speakers of the Coastlands, The Last Speakers of the Rocky Lands, and the Host of Last Speakers.

But all of this occurs in a climate increasingly filled with apprehension and suspense because of the pressing scansion of the numbers, or rather, of the tempo: we come to learn that this is by now the last day on which it will be possible to carry out this documentation. In fact, starting on the very next day the entire world will use only one currency, the Single Currency, and more importantly, there will be only one language, the Single Language. The "Killer", increasingly anguished over the impossibility of recording the hundreds, perhaps even thousands, of people who are still lined up outside his door, and feeling himself on the verge of failure once again, decides to allow all those waiting to come in at once (Fig. 8) and has them sing all together:

IL KILLER DI PAROLE
(con energia contagiosa, alla folla che si accalca)
Forza: raccontate, dite quello che volete, per salvarvi.
Mandate il vostro saluto al mondo che verrà:
univoco, vuoto e felice!
Parlate, sorridete, salutate …
E gridate: io sono! (Fig. 7)

THE WORD KILLER
(with contagious energy, to the host that crowds in)
Come on: tell your stories, tell what you want, to save yourselves.
Send your greetings to the world to come:
univocal, empty and happy!
Speak, smile, say hello …
And shout: I exist! (Fig. 7)

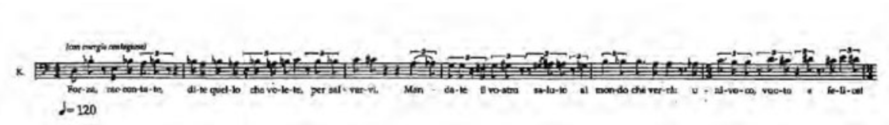

Fig. 8 C. Ambrosini, *Il killer di parole*, Act II, scene 9, mss. 29-34. The Killer encourages the host of Last Speakers with an ascending vocal line, almost like a sequence of pressing waves (© Edizioni Ricordi 2010)

But by now midnight is drawing inexorably closer, and the *ludodramma* takes on its darkest overtones. The enormous effort of the Killer will in fact turn out to be completely in vain. In the final moments, while outside the window are seen the luminous explosions of the fireworks celebrating the arrival of the new year, the Wife counts down the remaining seconds. The sequence of numbers that in the first act provided the material for a word game transformed into a lullaby, now reappear, reversed into a much more bitter game:

LA MOGLIE *(felice)* Dieci, nove, otto,
sette, sei, cinque, quattro, tre, due, uno!
IL KILLER DI PAROLE
(disperato, con rabbia) Zero!
(Tutti si interrompono di colpo e
rimangono immobili, a bocca aperta,
come in un fermo-immagine)
LA MOGLIE *(al pubblico,*
sollevando il calice, con un sorriso ipocrita)
Cin-cin e … auguri!

THE WIFE *(happy)* Ten, nine, eight,
seven, six, five, four, three, two, one!
THE WORD KILLER
(desperate, with anger) Zero!
(Everything is suddenly interrupted and all remain immobile, open-mouthed, as in a still image)
THE WIFE *(to the audience, raising a glass, with a hypocritical smile)*
Cheers and … best wishes!

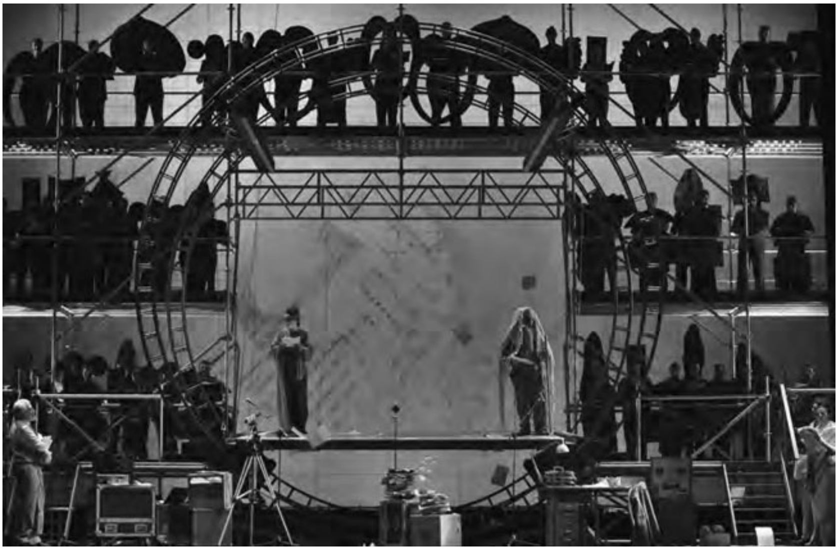

Fig. 9 C. Ambrosini, *Il killer di parole*, Act II, final scene. By now crowding into the recording studio there are dozens of "Last Speakers" coming from the most remote corners of the globe, and collectively engaged in a desperate farewell song, each in his or her own language, in the attempt to make themselves understood and communicate (Reproduced with permission of Teatro La Fenice, Venice. Photo © Michele Crosera)

These bitter good wishes must instead be taken up again with a certain amount of hope: the idea, apparently utopian, of recording all the endangered languages of the world, has instead been given very serious consideration: the National Geographic Society in the United States and the University of Cambridge in the United Kingdom have recently begun a program named "Enduring Voices" to record "talking dictionaries" of vanishing languages. The project will begin with 6,500 such languages.

The Word Killer would be happy to hear it.

Translated from the Italian by Kim Williams

Mathematics, Poetry and Literature

Gödel's Childhood and Other Algorithms

Vincenzo Della Mea

Computers and poetry

Computers and poetry seem two apparently distant topics, however poets are fascinated by the tool they often use for writing their own poems, either in terms of pure tool or for the underlying principles that animate it. Let's better analyze both aspects.

Nowadays, almost every writer is using a computer for the concrete production of his/her own texts, due to the vastly prevalent advantages over manual writing. However this is less true for poets, due to the smaller nature of the texts they produce, which still makes handwriting possible as a primary form of production, before the obviously needed translation into digital form.

The physical jest of writing itself has always been a subject for literature, in the form of images and metaphors. This can be found in the very first written trace of the Italian language, the famous "Indovinello veronese", or Veronese riddle:

Boves se pareba
alba pratalia araba
et albo versorio teneba
et negro semen seminaba.[1]

Now such a kind of self-reflection on the tool used for writing will need to be based on computers, their peripherals, software, and their respective pitfalls. In the years 1996-2001, the poet Valerio Magrelli edited a column in the journal *Teléma* [1], specifically devoted to the relationship between poetry and computers. In addition to his own interesting observations, he also hosted a number of the most important Italian poets, with their poems about the computer and related technologies. All poems can be traced to this first kind of relationship: computers as tools, not always

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[1] "In front of him he led oxen / White fields he plowed / A white plow he held / A black seed he sowed".

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DOI 10.1007/978-88-470-2889-0_4,

friendly and understandable. In the anthology *Verso i bit* [2], most of the poems are of the same kind, with a notable exception for Giuseppe Cornacchia, who wrote poems using the C++ programming language as basic syntax (now available in [3]).

On the other hand, computers and the science behind them may provide some deeper insight into the act of writing and thus some new expressive means; let's see how.

Like every art, writing poems involves abiding by, or at least recognizing, rules, which up to a certain point can be imagined as algorithms (though executing them is not sufficient to obtain poetically significant results). Even the shape assumed by written poems vaguely resembles that of computer program sources, with their characteristic line length smaller than page size. Those factors may have attracted the attention of some forward-looking *avant-garde* poets in the 1960s. In Italy, in 1962, Nanni Balestrini produced computer-generated poems using an algorithm – Tape Mark I – implemented in a IBM computer [4]. Italo Calvino was also fascinated by the possibility of generating literature in an automatic way, to diminish the role of the Author and put the Reader at the very centre [5]. In both cases, the basic underlying concept was combinatorics, which had also already fascinated OULIPO members [6], including Raymond Queneau with his *Cent mille milliards de poèmes* [7].

However, there is perhaps a third way of bridging computers and poetry, where computer science and technology in particular become sources of metaphors, suggestions and language. For me, the best example of this way can be found in Thomas Pynchon's novel *Vineland* [8]:

> If patterns of ones and zeroes were "like" patterns of human lives and deaths, if everything about an individual could be represented in a computer record by a long strings of ones and zeroes, then what kind of creature could be represented by a long string of lives and deaths? It would have to be up one level, at least – an angel, a minor god, something in a UFO. It would take eight human lives and deaths just to form one character in this being's name – its complete dossier might take up a considerable piece of history of the world. We are digits in God's computer, she not so much thought as hummed to herself to sort of a standard gospel tune. And the only thing we're good for, to be dead or to be living, is the only thing He sees. What we cry, what we contend for, in our world of toil and blood, it all lies beneath the notice of the hacker we call God.

The poems discussed in the rest of the present article were published in [9,10], and try to follow the last approach.

Gödel's Childhood and Algorithms

I have written poems for many years – few, but constantly. At a certain point, I felt the need for bridging my own computer science culture, on which I pleasantly based my job, with the other side of my life, more devoted to literature. This occurred just before I published the very first poetry book, *L'Infanzia di Gödel* (Gödel's Childhood) [9]: I wrote two poems in this direction, but I was somewhat puzzled by results. I was using concepts of computer science and technology in somewhat traditional poems – in terms of both metrics and content. I resolved my perplexities after discussing them with Mario Turello, a literary critic with a strong interest for science, who I knew from his book on Giulio Camillo, entitled *Theatre of memory* [11], in which he traced the origins of the concept of hypertext. He suggested that I read C.P. Snow's famous essay *The two Cultures* [12], as well as John Brockman's *Third Culture* [13], and encouraged me to pursue that sort of bridging. The two seminal poems were then included in my first book – and also gave it its title; one of them is presented in the next section.

The next book was decisively entitled *Algoritmi* (Algorithms) and included in a first part the two initial poems plus thirteen others. A second part, entitled *Nel mistero dell'interruttore* (In the mystery of the switch) was instead aimed at providing a reading guide for those not familiar with concepts of computer science. These were like notes, not only strictly related to the corresponding poem, but with digressions aimed at stimulating curiosity about the underlying topics. Poems are not didactic: they exploit computer science language and concepts for the universal aims of poetry. Instead, the second part may be considered didactic.

The following sections will present some of the poems, with a discussion on their originating ideas.

L'infanzia di Gödel
In Germania come in India o a Cuba,
i bimbi sono in fondo tutti uguali:
impilano giochi, rubano zucchero,
armeggiano con i coltelli affilati
dandosi agguati continui alle dita,
progettano dispetti e poi si pentono;
imparano intenti propri della specie
disperata che li ha radice e frutto.
Chi più, chi meno: Kurt alla madre
nega l'addebito – pure sorpreso –,
poi s'abbuia, ci ripensa e nel suo
mamma scusa ma questa è una bugia
l'innocenza ammazza il sogno ordinato
e brucia le radici – e il frutto è pigna.[2]

In 1931 Kurt Gödel proved two incompleteness theorems which, together with other results at that time, revolutionized mathematics and logic, and provided the foundations for computer science. In brief, the first theorem says that no formal logic system can be both consistent and complete. This also had consequences on positivist

[2] Gödel's childhood: "In Germany, like in India or Cuba, / children in the end are all equal: / they pile up toys, they steal sugar, / tinker with sharp knives / setting up continous ambushes on their own fingers, / design mischiefs and then regret; / they learn the intents of the desperate species / of which they are fruit and root. / Some more, some less: Kurt to his mother / – even surprised – denies the charge, / then becomes sad, rethinks it and in his / mama sorry but this is a lie / innocence kills the ordered dream / and burn the roots – and the fruit is a pinecone".

hopes ("the ordered dream" of the next to last line). However, knowing limitations also helps in managing them, and this is the pinecone of the last verse: a hard fruit that conceals something good inside, though difficult to get to.

The third to the last line refers, informally, to the principle on which the theorem demonstration is based: a variant of the Liar's Paradox.

A life
Nascoste bene dentro il disco rigido
ci stanno sette miliardi di lettere.
Meno di settecento è quant'è lunga
questa poesia, per breve che sia
non più di quel che serve per descrivere
il giorno medio di ozio e iterazione
di un normale funzionario, la cui vita
ariosamente dichiarata arriva
ai venti megabyte. Come dire
niente, ed ancora meno comprimendo
la ridondanza che ci fa uguali
nel ciclo standard dal parto alla morte,
escludendo quel bit che ci distingue
che ci fa valere un nome di file.[3]

Seven gigabytes, that is, the equivalent of seven billion alphabet letters, was the usual capacity of an hard disk in the years when I wrote this poem. Not much nowadays, but already sufficient to host about 17000 books of 200 pages each.

I was thinking of the hard disk like the memory of our life. If we were able to describe every day of life with a poem like this one, 20 Mbytes would be sufficient to store all of them (actually, this is true for the average Italian male, with a life expectancy of 78 years).

However, one poem a day may be excessive, given the repetitiveness of our lives. If such a poetized and then digitized life were compressed, the memory actually needed would be much less, since as a matter of fact, compression algorithms work by reducing redundancy and repetitions. In practice, this would mean that the 29,200 poems of such life could be synthetized in a compressed form, like "he slept, he worked, he fed himself for 29,200 times" plus some specific and unique details about the most important life events. As the Italian poet Umberto Piersanti wrote, "difficile fare della propria vita un'opera d'arte / e dei propri giorni i migliori dei nostri sonetti" ("it's hard to make a work of art from our own life / and from our own days, the best of our sonnets").

[3] A life: "Hidden deep in the hard disk / there is room for seven billion letters. / Less than seven hundred is the length / of this poem, which even so short / it is no more than what is needed to describe / the average day of idleness and iteration / of a generic office worker, whose life / expansively declared may go up / to twenty megabytes. Close to nothing, / and even less when compressing / the redundancy that makes us equal / in the standard cycle from birth to death, / excluding that bit that distinguishes us, / that makes us worth a file name".

Rumore

(a T.)

Informazione è la differenza
tra quel che sai e quello che so io.
È inutile quindi, amore mio,
che leggi con timore e ti preoccupi
se non trovi tue tracce in ciò che scrivo:
già lo sai benissimo, anche meglio
di me cosa a te mi lega e perché.
Se ne scrivessi, sarebbe rumore;
però tu non smettere mai di dirmi
l'ovvio bene che bene mi fa stare.[4]

In Shannon and Weaver's information theory [15], "information" refers to the degree of uncertainty present in a situation. On the other hand, when something is completely predictable, it is completely certain, thus it contains very little, if any, information. Redundancy, as data that does not add information to a message, is the opposite of information. Bateson delved into the topic by defining information as "a difference which makes a difference".

In this poem, dedicated to my wife, I interpreted the above mentioned definitions of information, by thinking of a message that does not contain information because its content is already known to the receiver: an elegant way to apologize for not having written many love poems.

In the third to the last line there is a scientific imprecision: redundancy is not noise in itself, i.e., an additional, unintended random signal that interferes with the meaning of a message. Nevertheless, redundancy might be considered "noise" in an intuitive sense, as a useless addition to the message. Apart from all considerations on scientific definitions, the sound of the word is well suited for that line, and thus finds in this its own justification. By the way, it has been reported that Charles Babbage, inventor of the Analytical Engine, ancestor of current computers, wrote to the young Alfred Tennyson after having read his poem "The vision of sin" [16]. The letter said:

> In your otherwise beautiful poem, one verse reads,
>
> Every moment dies a man
>
> Every moment one is born.
>
> It must be manifest that, if this were true, the population of the world would be at a standstill. In truth, the rate of birth is slightly in excess of that of death. I would suggest that in the next edition of your poem you have it read:
>
> Every moment dies a man,
>
> Every moment 1 1/16 is born.
>
> Strictly speaking this is not correct. The actual figure is a decimal so long I cannot get it into a line, but I believe the figure 1 1/16 will be sufficiently accurate for poetry.

---

[4] Noise: "Information is the difference/between what you know and what I know. / It is thus useless, my love, / to read with fear and worry / if you do not find traces of you in what I write: / you already know well, even better /than me, what ties me to you and why. / If I were to write about it, it would be noise; / but please never stop telling me / the obvious good that makes me feel good".

Oracolo
La macchina universale di Turing
se opportunamente caricata
con una descrizione minuziosa
della mia vita, per definizione
potrebbe raccontarmi in anticipo
cosa farò da grande, se farò
qualcosa; però se inerte raggiungo
il limite del nastro illimitato,
allora la macchina altro non può
che osservarmi con le sue transizioni,
lentamente, di stato in stato,
mentre anch'io l'osservo. Facendo niente.[5]

In 1936 Alan Turing invented a theoretical device aimed at solving the Hilbert decidability problem [17]: is there a well defined method to decide whether a given statement is provable from the axioms, using the rules of logic?

To do so, Turing studied how to delimit the concept of well defined method. The result is an abstraction of a mechanical device – the Turing Machine – composed of a limitless tape, divided into cells, a head able to read and write 0 or 1 into the cell on which is positioned, and the ability to move left and right. A table of instructions regarding what to write and how to move depending on what it was read completes this machine. The latter table corresponds to what now we call software: the instructions put into it provide the Turing machine with the possibility of being programmed to carry out different tasks.

Although the Turing machine seems really simple, it has been demonstrated that it is as powerful as any computer programming language.

Turing then developed a specific table to implement the so-called Universal Turing machine (UTM). This machine expects that the tape contains a description of another Turing machine, coded in terms of 0 and 1, the data it should process, and is able to execute it on those input data. In other words, he programmed a simulator of Turing machines directly on a Turing machine, and this is very similar to how computers function now: a single physical machine that, adequately programmed, does many different tasks.

The last step was to translate Hilbert's problem in terms of Turing machines: is the UTM able to decide whether another Turing machine always gives a result on any possible input data? He gave the UTM its own description as input, and with a demonstration similar to that of Gödel, demonstrated that the answer is no.

Following this recursive play of roles, the subject of the poem is observing him/herself in the act of observing – maybe locked in the act, instead of living.

[5] Oracle: "The Universal Turing Machine, / if opportunely loaded / with a meticolous description / of my life, by definition / could tell me in advance / what I will do when grown up, / if I will do anything; but if I idle reach / the limit of the limitless tape, / then the machine only will be able / to observe me with its transitions, / slowly, from state to state, / while I too observe it. Doing nothing".

Poeta
Tutto ciò che scrivo può generare
dall'applicazione ripetitiva
di regole che elaborano simboli,
entrambi presi da insiemi finiti.
Questo dice Chomsky, o buon lettore.
Così diventa mia consolazione
ricordare che la scimmia di Eddington
pestando a caso su una tastiera
potrebbe scrivere questa poesia.
Magari col tuo nome come autore.[6]

The last poem presented here is somewhat humorous, with a couple of suggestions.

One is the so-called generative grammars studied by Noam Chomsky [18]. A generative grammar is a finite set of rules that can be applied to generate all those sentences that are grammatical in a given language, from a finite set of symbols.

The other suggestion comes from a paradoxical metaphor used in 1928 by the physicist Arthur Eddington to explain the concept of entropy [19]: "If an army of monkeys were strumming on typewriters, they might write all the books in the British Museum". The metaphor can also be reduced to the *monotasking* version, as in the poem.

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The Wild Number Problem: Math or Fiction?

Philibert Schogt

"Let's have a look then, shall we?" Dimitri said, as if he was a doctor about to examine a patient. I handed him the proof. He removed the paper clip and spread the pages out on his desk.

"Theorem: the set of wild numbers is infinite", he read the first line out loud. "My god, Isaac, why didn't you tell me you were working on this?".[1]

1 Escape from reality

In the above scene from my novel *The Wild Numbers*, the 35-year-old mathematician Isaac Swift believes he has found a solution to the famous Wild Number Problem and is showing it to his older colleague, the Russian mathematician Dimitri Arkanov. But what exactly is the Wild Number Problem, and why did I choose it as the theme for my novel, rather than some other mathematical problem?

Actually, my original plan was quite different.

It was 1992, and I had just moved into a new apartment in Amsterdam. A friend of mine, who is a mathematician, was helping me install the electricity. While we were working, I told him that I had finally managed to construct a good plot for what was to become my first novel. I was sure he would appreciate its theme and was looking forward to his reaction.

The main character was a troubled mathematician struggling with his mediocrity. For the past few years, his research has been heading absolutely nowhere. But then one day: eureka! In a sudden flash of insight he believes to have found the proof of Fermat's Last Theorem.

"Not a good idea," my friend remarked.

I was stunned. What I had just told him was only meant as an introduction. I hadn't even come to the main story-line or the most important characters. How could he be so quick to pass judgement?

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[1] Schogt P.: The Wild Numbers, p. 101.

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What he objected to was the improbability of a mediocre mathematician suddenly solving what was generally considered to be the most famous unsolved problem in mathematics.

"He *thinks* he has solved the problem", I corrected him.

This nuance failed to impress him. The mathematics needed to tackle Fermat's Last Theorem was so complex, he explained, that only the very best minds would ever be able to find its proof. Unless my hero had miraculously turned into a genius overnight, he would have to be a complete idiot to think he had the answer. This was not the sort of protagonist my friend could identify with. Or sympathise with.

"But the average reader might be able to", I protested.

"Perhaps. But the average mathematician will never take someone like that seriously, not even for a moment".

His commentary was devastating. My aim was to write a novel about a mathematician that "the average reader" could understand and enjoy, but I wanted the story to be sufficiently true to life to appeal to professional mathematicians as well.

It had taken me months to develop my plan. Now, the very first mathematician who heard about it coldly dismissed it in less than half a minute. I was furious with him but was not in a position to let it show. After all, he was helping me with my apartment. We worked on in silence. I sat on the floor trying to put together a socket. Cramming too many wires into too small a box seemed just the right punishment for the grand gestures with which I had presented my plan. Meanwhile, with a grimace that struck me as somewhat Satanic, my friend began drilling holes in the wall. The shrill sound of the drill and the fine red powder that came spewing out of the holes were painful reminders of the damage he had just done to my ego.

When we were done, I did my best to thank him for his help. The lights were working, but the idea for my novel lay in ruins.

But the next day, I was blessed with one of those flashes of insight that I had wanted to bestow on my main character. The solution to my problems was so obvious that I wondered why I hadn't come up with it earlier. Instead of writing a novel about a real mathematical problem such as Fermat's Last Theorem, as a writer of fiction I was perfectly free to make up a mathematical problem of my own. And so I created the French mathematician Anatole Millechamps de Beauregard, who in his turn invented the wild numbers in 1823.

As I developed my new idea further, I realised that there were other drawbacks to the original plan which could now be avoided.

I am extremely lazy when it comes to doing research. If I had written my novel about Fermat, I would have had to go to the mathematics library, talk to various number theorists, strain my mathematical capabilities to the utmost in order to understand the latest developments.

Moreover, I would have had to consider introducing true historical figures into the story (more research!), keeping them in line with biographical facts and thus limiting my freedom to shape their personalities.

And then there was the problem that writers of popular science always run into: how do you present a complicated subject to the general public without oversimplifying or even distorting the facts?

Now that I had entered the realm of pure fiction, I no longer had to worry about such matters. Thanks to Beauregard and the wild numbers, the time I spent on research was reduced to a blissful near-zero.

2 Fiction imitating math

This does not mean to say that the mathematics in my novel could just be any old nonsense. Quite the contrary. One of the greatest challenges that I was now faced with was to make the Wild Number Problem seem as real as possible, appealing to the imagination of the general public and professional mathematicians alike.

The first thing that I needed to do was to have my French mathematician Anatole Millechamps de Beauregard define the problem, that is to say, I had to create the illusion that something was being defined. So I let him play a gambling game with his friends. They each deposited a sum of money, he would pose a mathematical riddle, and the first person to solve it would win the jackpot. One of these riddles came to be known as the Wild Number Problem:

> Beauregard had defined a number of deceptively simple operations, which, when applied to a whole number, at first resulted in fractions. But if the same steps were repeated often enough, the eventual outcome was once again a whole number. Or, as Beauregard cheerfully observed: "In all numbers lurks a wild number, guaranteed to emerge when you provoke them long enough". 0 yielded the wild number 11, 1 brought forth 67, 2 itself, 3 suddenly manifested itself as 4769, 4, surprisingly, brought forth 67 again. Beauregard himself had found fifty different wild numbers. The money prize was now awarded to whoever found a new one. [2]

The trick here, of course, was that I didn't specify what Beauregard's "deceptively simple operations" were.

The next step was to provide the Wild Number Problem with a history. It had to be old enough to count as a famous unsolved problem, but at the same time, to maintain the illusion that we were dealing with real mathematics, it was better to avoid going into too much historical detail. 1823 seemed just about the right starting point, allowing for three, maybe four important developments:

> 1823: Anatole Millechamps de Beauregard poses the Wild Number Problem in its original form.
>
> 1830s: The problem is generalised: how many wild numbers are there? Do the same ones keep popping up, or are there infinitely many?
>
> 1907: Heinrich Riedel ends speculations that perhaps all numbers are wild by proving that the number 3 is not. Later he extends his proof to show that there are infinitely many of such non-wild, or "tame" numbers.
>
> early 1960s: Dimitri Arkanov sparks renewed interest in the almost forgotten problem by discovering a fundamental relationship between wild numbers and prime numbers.
>
> the present: Isaac Swift finds a solution.

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[2] The Wild Numbers, p. 34.

When my main character Isaac Swift explains what wild numbers are and recounts their history, he addresses the reader more or less directly. Here, I was able to imitate the tone of popular science books, intended to reassure readers that they can grasp the general idea without having to bother with the complexities of the subject. Only in this case, the existence of such complexities was entirely illusory.

But elsewhere in my novel, when Isaac is working on the problem or discussing it with colleagues, the tone had to resemble the discourse of professional mathematicians, too technical for a layperson to understand. Here, I had to introduce some make-believe jargon into the narrative, with which I could pretend to be taking steps in a mathematical proof. As with historical details, the key once again was not to overdo it. Too much pseudo-mathematical mumbo jumbo would scare off the average reader and have mathematicians shaking their heads. In the end, I managed to limit the number of nonsense terms to five. Apart from the wild numbers themselves and their counterparts, tame numbers, I introduced the term "calibrator set", a mathematical tool developed by Isaac's older colleague Dimitri Arkanov that was useful in establishing a set's "K-reducibility"[3], a deep number theoretic property. And Isaac himself constructs "pseudo-wild numbers" – a type of number with a somewhat weaker definition than wild numbers – hoping to use them as a stepping stone in his proof.

With the help of these terms, I could now let my main character argue with himself while frenziedly pacing back and forth in his apartment:

> *Assuming there is a set of pseudo-wild prime numbers Q_p that is infinite and K-reducible, find a correspondence between the elements q_p and w_p – wild primes – such that for every pseudo-wild prime there exists at least one wild prime...*
>
> "What do you mean *assume* Q_p is infinite and K-reducible? You are only shifting the problem!"[4].
>
> ...
>
> A calibrator set. A calibrator set. If only I found a suitable calibrator set![5]

As work on my novel progressed, I remember having crazy conversations with my mathematician friend, discussing various steps in the imaginary proof of a non-existent problem, and considering whether they were realistic enough. It was a bit like that famous scene in *Blow-up*, the film by Michelangelo Antonioni [6], where two people play tennis without a ball.

I don't think I would ever have had as much fun if I had stuck to my original plan and written a novel about Fermat's Last Theorem. In retrospect, I am deeply indebted to my friend, not only for helping me out with the electricity that day, but more importantly, for setting my imagination free.

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[3] I later found out that there really is such a thing as K-reducibility, albeit in a branch of mathematics far -removed from number theory. The term must have caught my eye at some point in the past, lingering in my mind without my being aware of it.

[4] The Wild Numbers, p. 82.

[5] Ibid, p. 85, cf. Shakespeare's *Richard III*: "A horse. A horse. My kingdom for a horse!".

[6] *Blow-up*, directed by M. Antonioni, Great Britain-Italy-USA, 1966.

3 Back to reality

When I completed the manuscript of *The Wild Numbers* in 1994, I left it lying on a shelf for quite some time before gathering the courage to send it to a literary agent. Several months later, he came with his diagnosis: great story, needs a sub-plot. More "human interest" was what he was looking for, fearing that the mathematics in my novel would scare off too many readers. Another agent responded in the same vein: she would have no trouble at all getting my novel published, if it were three times as long.

As an unpublished writer, I was inclined to take these professional opinions extremely seriously. But when I tried to follow up on the agents' suggestions, changing the narrative from first person to third person to allow for more non-mathematical detail, emphasising the love element, and so on, I felt that if anything, the story was weakened, not strengthened. The whole idea of my project had been to show that mathematics and its practitioners were interesting enough in themselves, and didn't need help from outside in the form of sub-plots to captivate the reader. And perhaps the brevity of my novel served a deeper purpose as well. One of the distinguishing features of mathematics is its aim to be clear and concise, to express everything in the simplest possible terms. Perhaps the considerations mentioned above - inventing a fictional problem to avoid the complexities of a real problem, limiting the amount of historical detail, keeping technical jargon to a minimum - could all be seen as an attempt to reduce the drama to its simplest form, thus reflecting the spirit of mathematics in a way that a thicker novel, fluffed up with interesting but superfluous detail, could not.

To make a long story short, I decided that my novel was fine the way it was. Unfortunately, this meant that my manuscript went back to gathering dust on a shelf.

Meanwhile, out in the real world, something truly amazing took place. When my original plan was still intact, I did wonder once or twice what would happen if Fermat's Last Theorem were actually proved while I was writing a story on the same subject. I had decided not to worry. The latest news back in 1992 was that some or other mathematician had published a proof, which, like so many others in the past, had turned out to be wrong. Considering the 350 years of failed attempts and modest steps in the right direction, surely it would be too much of a coincidence if a solution were found during the two or three years that I needed to write my novel.

And yet, this is exactly what happened.

In 1995, the English mathematician Andrew Wiles published his definitive proof of Fermat's Last Theorem. It was over 100 pages long, the result of seven years of hard work. If I had stuck to my old plan, I could have thrown my novel straight into the garbage can. Now that I had written about a fictional problem, at least my story was safe.

It was more than safe, actually. Partly thanks to Andrew Wiles' spectacular achievement, mathematics was becoming a hot topic in the media, and all sorts of books, films and plays were appearing on the market. Evidently, people were much more open to mathematics than the publishing industry had given them credit for. It was in this favourable climate that a Dutch editor read my manuscript and was willing to take the gamble. After I had translated the story into Dutch, it was finally published in 1998, under the title *De wilde getallen*. The original English version appeared in 2000. Since then, it has been translated into various other languages, including German, Greek and Italian.

4 Math imitating fiction

Whether I have succeeded in creating a credible and enjoyable piece of mathematical fiction is up to my readers to decide. But I consider it an encouraging sign that so many people have asked me if wild numbers really exist. Friends of mine, who took my book along on their vacation with another family, told me that they had debated this issue on the beach one day. Their teenage daughter settled the question, insisting that her math teacher had discussed wild numbers in class.

One literary critic, on the other hand, dismissed my novel because the mathematics in it was so evidently nonsensical. For a brief moment, I was dumbstruck. I had spent a great deal of time and effort making the wild numbers seem as real as possible. Where had I given myself away? Interestingly, the critic draws a correct conclusion from the wrong premises. His main objection was that the sequence of wild numbers mentioned in my book (11, 67, 2, 4769, 67) was too erratic to be realistic. In his opinion, I make matters worse by having my French mathematician Beauregard define his wild numbers with a "series of deceptively simple operations."

"I would certainly like to see those 'deceptively simple' operations!" the critic scoffs in his review.

But deceptive simplicity leading to erratic results is by no means peculiar to the wild numbers. In fact, this is a typical feature of many existing problems in number theory, one that inspired me to write my novel in the first place.

And I was happy to discover that *The Wild Numbers* in its turn appealed to a great number of mathematicians for the very same reason. Though seemingly fictional, the Wild Number Problem sparked a lively debate, centring on the issue of whether the wild numbers could be generated after all with a series of simple operations as described in my book, i.e. by operations which, when applied to an integer, would at first result in fractions, but upon sufficient iteration would once again produce an integer. Attempts to generate the exact sequence of numbers mentioned in my book were unsuccessful. But contrary to the literary critic's intuition, various mathematicians did come up with beautiful and indeed deceptively simple ways to produce similarly erratic integer sequences.

Here is one example, suggested by the Dutch mathematician Floor van Lamoen:

> For a rational number p/q let $f(p/q) = p * q$ divided by the sum of digits of p and q; $a(n)$ is obtained by iterating f, starting at $n/1$, until an integer is reached, or if no integer is ever reached then $a(n) = 0$.

For example, for $n = 2$:

$2/1 \rightarrow 2/3 \rightarrow 6/5 \rightarrow 30/11 \rightarrow 330/5 = 66.$

Here are the first 48 terms of the sequence:

0, 66, 66, 462, 180, 66, 31395, 714, 72, 9, 5, 15, 3, 36, 42, 39, 2, 9, 45, 462, 12, 12, 90, 3703207920, 1692600, 84, 234, 27, 3043425, 74613, 6, 7930296, 264, 4290, 510, 315, 315, 73302369360, 1155, 3, 8, 239872017, 6, 4386, 1989, 18, 17740866, 499954980.

To my delight, one outcome of these discussions was that the wild numbers were accepted as an entry in "The On-line Encyclopedia of Integer Sequences"[7] a huge data base developed by the American mathematician Neil Sloane, offering information on every thinkable kind of integer sequence. As an extra feature, all the sequences in the encyclopedia have been set to music, so I was even able to *listen* to the wild numbers! The various efforts of mathematicians to create Beauregard-like sequences also made it into the encyclopaedia, being dubbed "pseudo-wild numbers". [8]

But the story does not end there. In 2004, I received a phone call from Jeff Lagarias, an American mathematician who was in Holland to attend a math conference. He had read my novel and was eager to meet me, so we had dinner together in a Thai restaurant in the centre of town. As it turned out, Lagarias was one of the world's leading experts on the unsolved $3n+1$ problem, otherwise known as the Collatz conjecture. I was unfamiliar with the problem, but fortunately, he had no trouble at all explaining the principle to me, not because I am so intelligent, but because it is easy enough for a ten-year old to understand:

Starting with any integer n, if even, divide by two, if odd, multiply by 3 and add 1. Repeat this process indefinitely. The conjecture states that no matter which integer we start out with, ultimately, the sequence will reach 1.

For instance, starting with $n = 6$, we get the sequence:

6, 3, 10, 5, 16, 8, 4, 2, 1.

For $n = 11$, the sequence is:

11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1.

If we consider the sequence for $n = 27$ and see how the numbers keep rising and falling before finally dropping down to 1, it becomes clear why the conjecture has been nicknamed "the hailstone problem":

27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1.

When we came to talk about my book, Lagarias told me that the wild numbers had reminded him of his own work in so many ways, that he wanted to incorporate them into an article that he was planning on writing. I was greatly honoured, although I wasn't quite sure what to expect.

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[7] See Neil Sloane's On-line Encyclopedia of Integer Sequences™ (www.OEIS.org), entry A058883.

[8] Ibid, entries A058971, A058972, A058973, A058977, A058988, A059175.

In 2006, Lagarias published two articles on the $3n+1$ problem, one in the *American Mathematical Monthly*, the other, co-authored with David Applegate in the *Journal of Number Theory*, in which he introduces the term “wild number”, citing my novel as a source of inspiration. How Lagarias defines these wild numbers goes beyond the scope of our present discussion. For our purposes, it suffices to compare the following two statements:

Theorem. The set of wild numbers is infinite. - Isaac Swift, *The Wild Numbers*.

Theorem 3.1. The semigroup of wild integers contains infinitely many irreducible elements (i.e., there are infinitely many wild numbers).- Jeffrey C. Lagarias , *Wild and Wooley numbers*, American Mathematical Monthly 113 (2006), 9–108.

From now on, when readers ask me if wild numbers really exist, I will be able to tell them the good news.

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A Play at Dusk. Mathematics in Literature

Carlo Toffalori

1 Mann and Pessoa

> Since it deals only with dead numbers and empty formulas, mathematics can be perfectly logical, but the rest of science is no more than child's play at dusk [1, p. 132].

The total pessimism pervading the pages of *The Book of Disquiet* does not spare even science. According to Fernando Pessoa, human life has no meaning at all and thus every attempt of knowledge is just a *play at dusk*, as crazy as wishing that the clouds would stand still in the sky. If mathematics is spared from this absolute decree, it is only because it deals with nothing.

Some years before Pessoa, Thomas Mann had treated in his book, *Royal Highness* [2], a completely different matter, namely the polite courtship of Klaus Heinrich, a German prince of the nineteenth century, of his young American guest Imma Spoelmann – the only daughter of a wealthy father, and an enthusiastic student of Mathematics. Here the gallant prince does not neglect showing a kind interest in mathematics, with the following results:

> " And your course of study?" he asked, "May I ask about it? It's mathematics, I know. Don't you find it too much? Isn't it terribly brain wracking?
>
> Absolutely not", she said, "It's just splendid. It's like playing in the breezes, so to speak, in a dust free atmosphere. It's as cool there as in the Adirondacks [2, p. 188].

Here mathematics – only useless *play at dusk* according to Pessoa – becomes *playing in the breezes*, a symbol of fancy and lightness. Notably, this quotation of Mann is likely to have some autobiographic flavour. In fact, it has been said that the character of Imma Spoelmann was inspired by Mann's meeting with Katja Pringsheim, the daughter of a university professor of mathematics and physics in Munich, herself a student of mathematical sciences, and later Mann's wife, which supports the supposition that the fictional dialogue between Klaus Heinrich and Imma Spoelmann recalls an actual conversation between Mann and his future wife.

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These two quotations provide a useful introduction to the broad and subtle theme of the relationship between mathematics and literature. In fact both are drawn from literary classics and deal with mathematics, even if the former underlines its vacuity and the latter its surprising lightness. Indeed these two characteristics – vacuity and lightness – may coexist. Nevertheless, whereas Pessoa appears critical and negative, Mann sounds indulgent and well-disposed. Thus the comparison between these two authors also introduces the delicate question whether the connection between mathematics and literature, and more generally between scientific and humanistic culture, must necessarily be antithetical and discordant, or if, on the contrary, may be considered, to quote Manzoni, a betrothal.

Actually some underlying links between the two worlds are undeniable, such as, for instance, the mysterious common genesis of numbers and letters. Moreover, the classics of poetry, prose and thought are replete with quotations for and against mathematics. But our question is more profound, indeed too profound to be answered simply by one, two or even a hundred aphorisms. In fact, what we need to understand is the ultimate identity of what we call "art", which includes creativity, fancy and freedom from any predetermined scheme, but at the same time calculation, experience and rigor. Thus, if on the one hand the artistic attitude seems to disdain mathematics and its stifling aridness, on the other hand it often needs and even espouses it. In conclusion, how does it happen – to quote Robert Musil and his *Mathematical Man* [3] that "this intellect gobbles up everything around it, and as soon as it lays hold of the feelings, it becomes spirit" [3, p. 43]? A fascinating question, as we already underlined, and yet hard to solve in few pages.

2 A power play

It is then right to focus on some particular aspects of the problem. To do that, let us continue to consider those opening quotations of Pessoa and Mann and their common denominator, that is the idea of play – intended, at least in the latter case, as a pure mental pleasure. We wonder: why not also spend our pages just to "play", here and now, on the subtle border that both divides and joins numbers and letters? And, given the current world situation, why not imagine, just for fun and in the space of a few lines, Mathematics in power? We have experienced, in recent years, governments headed by entrepreneurs, bankers and professors – but not of Mathematics. Then why not imagine, just for a moment, a government of mathematicians?

Actually this idea is not new, and can indeed boast some acclaimed defenders. One such defender was Plato, who, after describing in Book VII of *The Republic* [4] the politicians he and many would like – men who are "rich not in silver gold, but in virtue and wisdom" [4: 521a] and who rule out of duty "but are not lovers of the task" [4: 521b] – explains how to bring up and to promote such pearls among men. To this purpose he lists the five pillars supporting his opinion of political wisdom, that is, arithmetic, plane and solid geometry, astronomy and harmony – all disciplines that are either explicitly or implicitly related to mathematics. What is more, Plato asserts that mathematics is the cornerstone of the education of not only politicians, but of all citizens. In fact "qualities of number lead to the apprehension of truth" [4: 525b], geometry "tends to facilitate the apprehension of the idea of good" [4: 526e] and, in general, mathematics has to be learnt by anyone wishing to be a man.

On the basis of this authoritative opinion, let us return to our proposal to consider the potentialities of mathematics in politics.

History suggests that we have never experienced this form of government. Thus let us turn to fiction and examine those literary classics – actually a good number – that, on the contrary, have represented Mathematics in power. Our search might help, if not to solve all global economic problems, then at least to know how the classics depict mathematics and compare their image with the real one.

3 From Aristophanes to Dickens

Plato's contemporaries themselves seem to harbour some serious doubts about a government of mathematicians. This can be said, for instance, of Aristophanes. In his comedy *The Birds* [5] he tells the story of two Athenians who, tired of their city's grey and quarrelsome life, decide to found a new town of licence and anarchy in the middle of the sky, among the birds. However, they immediately have to face plenty of cheats trying to interfere and make an easy profit from this novelty. Even Meton – a geometer, indeed a real figure of Aristophanes' times – is among them. He arrives to the city in the sky and, like a Minister of Public Works, proposes an advanced project of urban development, based on mathematical techniques for "measuring the air"; and yet, like the other braggarts before and after him, is swiftly beaten and chased away.

The opinion that clouds are the proper realm of mathematicians, as well as any other abstract thinkers, is largely agreed upon. For instance, this view is supported by another famous writer, Jonathan Swift, several centuries after Aristophanes. In fact, the third of *Gulliver's Travels* [6], the one following the adventures among Lilliputians and giants, takes place on an island of mathematicians. But this land is not where one would expect it to be, namely in the middle of the sea, but where it just suits mathematicians, that is, in the sky. Absent-mindedness is the main peculiarity of its inhabitants. In fact, this is the way Swift describes them: "dexterous enough upon a piece of paper in the management of the rule, the pencil, and the divider [nowadays we could add computers], yet in the common actions and behavior of life, clumsy, awkward, and unhandy… slow and perplexed in their conceptions upon all other subjects, except those of mathematics and music,… very bad reasoners, and vehemently given to opposition, unless when they happen to be of the right opinion, which is seldom their case". Thus they are incapable of producing any true novelty in politics, and are indeed inclined to ape the same behaviours and faults as the real governments of their times.

Even Edwin Abbott's *Flatland: A Romance of Many Dimensions* [7] introduces an imaginary fanciful mathematical world. But this time no human being lives in it. In fact Flatland, as the name itself suggests, is a planar, two-dimensional surface, and as such is intended for pure geometrical objects, like Segments, Polygons and Circles. In this way a clear hierarchic classification – "ordine geometrico demonstrata" [8] – is naturally established among its inhabitants, since in Flatland the more sides one has, the more authoritative one is. Accordingly Triangles are confined to the lowest level, while Circles occupy the highest one and hold complete sway. But this mathematical rigor is not sufficient to avoid wrongs and oppression, such as, just to name a few, the discrimination against women, and the suppression of any irregular polygon, thus resulting in the refusal of any diversity, or, more generally, the mortification of any imagination and utopia. Flatland is a flat land even and especially in its spirit and its moral values. Any longing for higher dimensions – such as the third, or the fourth, or higher – is banished and punished.

Geometry and politics are also intertwined in *The Inheritors* [9], an "extravagant story" written jointly by Joseph Conrad and Ford Madox Ford in 1901. It tells about a *femme fatale* coming from some mysterious Fourth Dimension and plotting to conquer the Earth. Many say that the tale by Conrad and Ford is sometimes charming, but its scientific contents are basically zero. Indeed the protagonist does show some features that could be defined "mathematical" – she is chilly and lucid in her reasoning, resolute in her purposes, incapable of sharing human feelings –, yet her fourth dimension, more than a revolutionary geometric intuition, looks one of the many alien universes – of space, time or mind – that fill the science fiction classics.

Some years before Abbott and Conrad and Ford, another popular writer, Charles Dickens, spoke out against mathematics and, above all, against the mathematics invading the political field. He did this in one of his more complex works, *Hard Times* [10].

Actually, the main target of that book's criticism was the philosophical current of utilitarianism, and mathematics was only indirectly involved, but it is still worth explaining how and why. First of all, let us briefly recall the foundations of utilitarian thought. It was Jeremy Bentham who conceived and developed it at the end of the seventeenth century, at the dawning of the British Industrial Revolution [11]. His praiseworthy intent was to regulate its growth and in this way obtain the "greatest happiness" for most people. However Bentham's belief was that happiness is measured by ***facts***, and not by sophisms, sentimentalisms and abstract principles, and facts show that happiness is produced by pleasure and destroyed by sorrow. Thus pleasures are to be searched for, and pains are to be removed, establishing the equation *happiness* = *pleasure*. Actually this emphasis on pleasure is not to be understood as simple hedonism or unbridled selfishness. In fact, in Bentham's opinion, there is a more mature, conscious and adult form of pleasure, that may even decide to sacrifice the myopic joy of the present in view of greater advantages in the future. Furthermore the welfare to promote is not only that partial to individuals, but the general welfare of an entire community. The *utility principle* just lies in reconciling them. Mathematics is of fundamental benefit in this perspective, not just because it might be expected to devise some unlikely magic formula securing happiness, but because of its bent for singling out essential things, hence for recognizing and classifying facts and for programming behaviours. For this reason Bentham supported the development and the use of a *moral arithmetic* and a *felicific calculus* to guide people in their choices. To achieve the utilitarian aims, Bentham also proposed a new system of education in which mathematics was promoted as an indispensable subject and liberal arts neglected as useless [12].

That is utilitarian thought in a nutshell. Needless to say, its risks are clear. Indeed the cult of facts and the search for collective pleasure and common advantage, in spite of the noble purpose behind them, may easily cause all sorts of distortion. In particular, they may support on the one hand the base interests of masters and owners and, on the other hand, for the working class, the adoption of social systems as enlightened and well-balanced in theory as they are enslaving and rigid in practice.

Now let us come back to Dickens and *Hard Times*. The setting of the novel is the city of Coketown – a "blur of soot and smoke" at the eyes of Dickens and common people and yet, according to the industrialists and the politicians who planned it, "a triumph of fact", where everything is perfectly arranged for the general welfare and strictly expressed in numbers. Dickens's sharp, biting criticism reproves the perverse effects of utilitarianism and consequently even the greyness, the shabbiness and the sameness of mathematics supporting it. We do not need facts, but imagination, Dickens says, we do not need programming, but freedom: "all those subtle essences of humanity… will elude the utmost cunning of algebra until the last trumpet ever to be sounded shall blow even algebra to wreck" [10, p. 136].

4 Dystopian Tales

Blokken (Blocks) is a short story written in 1931 by the Dutch author Ferdinand Bordewijk [13]. Using a peculiar style – dry and sober but at the same time thrilling and fascinating – it describes an imaginary state, totalitarian and distressing, that is in principle aimed at promoting "the most perfect order on the Earth", but in practice is directed to a systematic homologation and the repression of its citizens. Art and culture are banished. Individuals are seen as enemies. The governors themselves – the ten members of a Supreme Council – are continuously anguished by the fear of interpreting and enforcing orthodoxy in the right way. Then Bordewijk's world looks like the most dreadful nightmare, the one people would willingly erase from the innermost recesses of their memory. But the trouble is that Bordewijk's state uses mathematics as a basis and foundation, not only because in that oppressive systems individuals mislay even a name and end by becoming numbers – indeed there is a string of figures to label and identify each of them, just like a car number plate; but above all because it is geometry that symbolizes and scans that world. The *Blocks* of the title are the cubes, parallelepipeds, angles and edges squaring its streets, buildings, gardens and behaviours. A state cubism underlies and regulates everything; anyone who contradicts it and defends spheres, roundness, irrationality and imagination is declared mad and put away. However, at the end of the novel, it is precisely a curved arc wrinkling the stiff array of the army that foretells to the terrorized eyes of the Supreme Council members their state's impending downfall.

Some years before Bordewijk, the Russian writer Yevgeny Zamjatin published his novel *We* [14]. The time was that of Stalinism's coming to power and the story, although set in a future of pure fiction, is nevertheless explicit in its references to the Soviet reality of that period. It presents a State that is a "unique, powerful body of millions of cells" – the We in the title – and crushes and annihilates the I's of single individuals. A supreme Benefactor – clearly foreshadowing Orwell's Big Brother – is at the head of everything. However the most striking feature of this repressive State is that, again, it is openly based on mathematics. As in *Blocks*, citizens are reduced to strings of numbers, like addenda in a sum or, more precisely, infinitesimals in an integral. Moreover rigorous mathematical laws are established to assert and strengthen the superiority of the collective over individuals. This time art is admitted, but only to serve and support the regime. Thus it does not despise, and indeed it pursues mathematical ways. Consequently even a love poem becomes a hymn to mathematics' unbreakable perfection:

> Forever amorous two-times-two.
> Forever amalgamated in passionate four.
> The hottest lovers in the world.
> Inseparable two-times-two [14, p. 59].

We's protagonist and narrator is D-503, an engineer and mathematician. He has a passionate, "heretical" love story with a woman, who is perhaps a rebel spy infiltrating the State. But at the end of the novel, as cruel as in 1984, he does not hesitate to sacrifice his beloved to rescue himself and recover orthodoxy.

Therefore even in *We* mathematics and its " two times two makes four" become the emblem of an oppressive system, indeed one the most oppressive that ever existed on the Earth, that is, Stalinism. Consequently *We* worthily crowns our series of examples of mathematical experimentations in politics. But, as a conclusion, it could not be more distressing. At this point it is right to wonder if things are really so. Is mathematics really as repressive, dry and shabby as Zamjatin, Bordewijk, Abbott, Dickens and the others describe it? Or, on the contrary, is the mathematics they depict only that of bureaucrats and formalists?

5 The final revolution

Let us then at least sketch an attempt at defence. Indeed the literary references we have quoted give it some starting points. In fact it often happens in detective stories that the clues, that a myopic and biased prosecutor collected to accuse the supposed culprit, are reversed and become arguments to clear him and even exalt his innocence. Well, something like that occurs even about mathematics.

Let us consider, for instance, the case of Aristophanes, Meton and the other swindlers and parasites, mathematicians or not, going to *Birds*' rising city to obtain some illicit benefit. It is easy to contrast their avidity to what Hermann Broch writes in *The Unknown Quantity*, that mathematics may be useless but is surely upright, indeed "an island of decency" [15, p. 7]. In further support to this opinion one could quote Stendhal and those passages of his autobiography entitled *The Life of Henry Brulard* where the French writer recalls the bent he had as a student for mathematics: "I loved, and still do love, mathematics for itself as not allowing room for hypocrisy or vagueness, my two pet aversions" [16, p. 120].

Even Jonathan Swift is easy to refute. It suffices to notice that the ability for abstracting, in spite of the absent-mindedness it sometimes causes, is the natural requisite for concentration and understanding the essence of things and problems, the heart of matters, and in this way to start their solution.

In Abbot's *Flatland*, if the flat, mean society of Circles and Polygons sounds unquestionably mathematical, also, and perhaps more, mathematical is the longing to overcome its foolish bonds and the fancy for imagining exciting new dimensions. Similarly, it is the freedom of curves that enliven Bordewijk's world and contrast the dark rigor of the blocks. So a suspicion comes to mind: that real mathematics is not ineluctable bigotry, but uneasiness, getting past any pre-established schema, the opposite of the mortifying image Dickens gives it in *Hard Times*.

Finally, as for Zamjatin, the *We* society, which looks so rational and soaked in mathematics, and consequently, let us underline, so oppressive, there is the very strange case of the number i, that is the square root of -1, and the way the protagonist D-503 reacts when faced with it. In fact he sees i as a sign, not of order and common sense, but of discord and bewilderment: unreasonable, raving, terrible and alien. It is as though the number i as well as the mathematics introducing and supporting it exemplify apostasy and heresy much more than orthodoxy.

There is another intense, significant passage of *We* that illustrates these surprising features even better. It presents a dialogue between D-503 and his beloved. She is the one who speaks first [14, 30^{th} Entry]:

> My dear – you are a mathematician. More – you are a philosopher, a mathematical philosopher. Well, then: name me the final number.
>
> What do you mean? I... I do not understand: what final number?
>
> Well, the final, the ultimate, the largest.
>
> But that's preposterous! If the number of numbers is infinite, how can there be a final number?
>
> Then how can there be a final revolution? There is no final one; revolutions are infinite.

Zamjatin himself confirms the same opinion:

> When Lobačevskij crumbles with his books the walls of the millenary Euclidean world and opens a way in the incommensurable non-Euclidean space, that's revolution. Revolution is everywhere, is in all things, is infinite; a final revolution, a final number do not exist (quoted in [17], p. 89).

The image of mathematics we get in this way is radically different from the one – traditional – of a crabby, unpleasant science. On the contrary, as non-Euclidean geometries show, mathematics is imagination and not fact, sometimes burning, erupting magma, curiosity and then fancy, lightness, dream and jocosity. Actually this is the way Mann portrays it, and in some sense Pessoa, and his "play at dusk", repeat the same concept. So do innumerable other authors, including Carroll, Borges, Queneau and Calvino. Indeed it would be intriguing to investigate and compare how they approach mathematics. Unfortunately there is no space here to do that, but let us at least recall what Georg Cantor wrote in a paper of 1883 [18], that "the essence of mathematics lies in its freedom". This may sound odd or funny, and yet is authoritatively corroborated by all the writers we have just mentioned. To further support it let us also cite an idea of Imre Toth, which often recurs in his work and in particular is stated explicitly in his autobiographical notes *Matematica ed emozioni* [19]. It asserts that freedom is looking for essential things and not for caprices, doing what is necessary and not what is arbitrary, hence exercising responsibility and maturity; in short, choosing the best and then standing by one's choice. In this sense freedom is not too dissimilar from Kant's categorical imperative. Taking this premise as a starting point, it is possible to argue that freedom is mathematics. But Toth says even more. In fact he observes that choosing presupposes a comparison of at least two alternative options, and so presumes that there is a "yes" and a "no" from which to decide. Well, one can undoubtedly agree that mathematics does succeed in creating new unexpected imaginary worlds, contradicting all evidence and any tradition. The number *i* itself – but Toth calls it a "non-number" – testifies that. However, the most authoritative proof is given by non-Euclidean geometries. Toth points out that non-Euclidean echoes can be felt even in classical authors, not only Saccheri, but Aristotle and Nicholas of Cusa as well. Above all, he underlines the peculiar power of mathematics, which in non-Euclidean geometries reconciles the divine faculty of creating and the human tool of negating; more precisely, it disclaims the illusory perfection of the Euclidean "yes" and proposes the "no" of other models, alternative and yet equally consistent. It is on this surprising mathematical talent for creating by negating that, according to Toth, the supreme exercise of freedom can rest.

6 Two plus two

Two plus two, as well as two times two, always make four. This is what Zamjatin recalled in his love poem. Actually it would be another intriguing task to track down and compare all the literary references that just deal with two plus two, or two times two. Their analysis would supply us with an unbelievable diversity of opinions. To inaugurate it we might recall Dostoevsky's *Notes from Underground*, whose embittered protagonist, in his poisonous invective against everyone and everything, also protests that life cannot be only mathematics and "extracting square roots", adds that "Twice two makes four seems to me simply a piece of insolence. Twice two makes four is a pert coxcomb who stands with arms akimbo barring your path and spitting" [20, p. 31] and concludes that even the heresy of two times two equals five "is sometimes a very charming thing too".

On the other hand Dostoevsky's underground man might be confuted by Molière's *Don Juan* [21] who as in the certainty that two and two make four, so in the religion of arithmetic and facts, finds a complete and absolute justification of his vocation for libertinage and his intolerance of any human or divine law, constraint and authority. Alternatively, to involve democracy and politics again, one might oppose Dostoevsky's and Victor Hugo's wise opinion that eight, or ten, or one thousand votes agreeing that "two and two make five, that the straight line is the longest road, that the whole is less than its part" are not enough to overthrow the natural order of things and deny the truth.

However let us confine our attention to Zamjatin and to other novels similar to *We*. Indeed, let us at last mention the prince of dystopian tales of the twentieth century, that is, George Orwell's 1984 [22]. It is worth recalling what the protagonist of that novel, Winston Smith, writes in his notebook in the first part of the story, regarding arithmetical laws: "Freedom is the freedom to say that two plus two make four. If this is granted, all else follows" [22, Part 3, Chapter 2]. Thus, in Orwell's opinion, the boring triviality of two plus two becomes a sort of ultimate bulwark of free and independent minds against the regime's adulterated truths. Indeed at the end of the story the tyrant who requires from Winston a full submission to the Inner Party forces him to acknowledge, about two plus two, that "sometimes they are five. Sometimes they are three. Sometimes they are all of them at once", according to Big Brother's will [22, Part 3, Chapter 2]. By the way, it seems that this arithmetical metaphor was inspired to Orwell by a Stalinist slogan. As we read in Eugene Lyons's *Assignment in Utopia* [23], the motto "Two plus two equals five" was used in the Soviet Union in Stalin's time to celebrate the quick achievement of a five-year plan of development. This proves that, when facing despotic propaganda and rhetoric, even the quiet honesty of "two and two makes four" may become the ultimate stronghold of freedom.

7 Analogies

We began these notes by mentioning two kinds of playing, that of Pessoa "at dusk" and that of Mann "in the breezes". Let us conclude in the same way, with another kind of play. But this time let us refer to *The Glass Bead Game* by Hermann Hesse [24] – a tale that ignores any destroying dystopia and replaces it with the utopia of a new world founded on a superior wisdom and on a prophetic and Pythagorean community leading its development. In fact the "Glass Bead Game" in the title is not a prosaic, everyday hobby, but, on the contrary, the ascetic practice of this human sublimation. What is even more interesting for us, the foundations of this enlightened society are mathematics, music and meditation. Moreover, a new form of writing, symbolic and ideographic, so mathematical, is among its main tools and vehicles, eventually bringing to fruition the old wishes and intuitions of Leibniz and later of Frege. Unfortunately Hesse does not provide explicit details about his game and how it is played, but we can advance some hypotheses about its mathematical bases and conjecture that they correspond to what Stefan Banach asserts in a famous aphorism of his. Banach first says that the clever mathematicians are the ones who realize analogies between theorems, that is, recognize in seemingly different fields a common matrix, a hidden and yet solid relation. He then adds that the most brilliant mathematicians are the ones able to see analogies even between theories. He finally concludes that the mathematical geniuses are the ones who perceive analogies between analogies. Actually one might debate this opinion, and whether mathematical skill really consists in discerning analogies of increasing order. Nevertheless one should agree that the wisdom Hesse describes, and one of the main contributions of mathematics to human culture and progress, rely on the cleverness of grasping the heart of the matter and, as Hesse himself says, of "connecting between them the content and the results of almost all the sciences": a virtue that is granted to every true philosopher – etymologically understood as a lover of knowledge – but, in Hesse's opinion, is particularly designed for mathematicians and musicians.

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Mathematics according to Italo Calvino

Gabriele Lolli

Many and variegated are the relationships between mathematics and literature; it should be no surprise that two expression of the human spirit exhibit analogies and contact points.

Some of these relationships however are not very interesting, especially those that descend from the guidelines imposed by the cultural or publishing industry. Under the pressure of the market, as soon as a new or seemingly original idea appears, it becomes a fashion.

The easiest form of cross breeding is the borrowing of terms; nowadays the vocabulary of the sciences is ransacked and in the titles of novels there is an abundance of mathematical words:[1] from prime numbers to chaos theory to infinity,[2] even when the plot has negligible or null scientific content.

The same phenomenon can be seen in literary criticism, but here it is more justified: Sandro Veronesi for example calls "attractor" a literary work upon which converge the evolution of a social or cultural system.[3]

In the opposite direction, science sometimes but rarely absorbs common language words into its technical vocabulary: the most noble literary case is that of *quark*; at a higher level, it has been noted the use of rhetorical *tropoi* in scientific argumentation.[4]

The scientists' world can be chosen as context for personages and plots. Novels talking of scientists are praised and given awards,[5] as if they were some exceptional events; but already C. P. Snow had measured with this subject sixty years ago, in the

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[1] We refer mainly to examples taken from Italian publishing market.

[2] Giordano P.: La solitudine dei numeri primi. Mondadori, Milano (2008); Boero S.: Teoria del caos. Salani (2011); Banville J.: The infinities (2009), translated into Italian as Teoria degli infiniti. Guanda (2011), whereas the story is concerned with no theory.

[3] He is talking of Roth J.: Die Flucht ohne Ende, published in the series "I grandi della narrativa" published by La Repubblica (June 2011) in the series "I grandi della narrativa".

[4] Lolli G.: La retorica di Galileo. In: Bravo G.M., Ferrone V. (eds.): Il processo a Galileo Galilei e la questione galileiana. Edizioni di storia e letteratura, Rome, pp. 287-92 (2010) .

[5] See e.g. in Italy Arpaia B.: L'energia del vuoto. Guanda (2011), or Schogt P.: The Wild Numbers. Four Walls Eight Windows. New York (2000). See also the paper by P. Schogt, included in this volume, The Wild Number Problem: math or fiction? or S. Baluja: [The] Silicon Jungle (2011). Dedalo, Bari (2012).

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saga comprising *The New Men* and *The Masters*; in 1970 Iris Murdoch wrote, with *A fairly honourable defeat*, a novel about a living mathematician, who was even recognizable by his colleagues.

It is a fact that the frequency of mathematicians as protagonists has lately risen, thanks in particular to the allure of *quants*, these practitioners of mathematics in the economic life who uncritically accept its responses without mediating them with ethical, social or historical considerations;[6] however the ancient Greeks already represented philosophers (their scientists) on the stage (Aristophanes' *The Clouds*), and in the nineteenth century literature the positivistic scientist or philosopher is often a protagonist (from Dickens' educator Mr. Gradgrind to Conan Dolyle's positivistic physicians).[7]

The scientist and critic Carlo Rovelli,[8] in discussing the present rich production of books related to science, laments that they do not attain the height of Musil or Lucretius; if it is true, as Italo Calvino (1923-1985) claims, that "in every century and in every thought revolution it is science and philosphy that remodel the mythical dimension of imagination",[9] however the deep main thrust rarely shows up in the lesser works, especially if an education purposely aimed at cultivating the scientific imagination is lacking. We are far from Calvino's hope and aim:

> I would like to use the scientific datum as a propellent charge to evade from habits of imagination, and to live daily life in terms as far as possible from normal experience.[10]

Few have been able to follow this ideal, among them Primo Levi, to whom Calvino confessed: "Your fantastic mechanism which springs from a scientific-genetic starting point has a strong power of intellectual and poetical suggestion, the same that have for me the genetic and morphologic wanderings of Jean Rostand".[11]

Also in Calvino' work we find examples of products of fantasy that stem from some scientific suggestion, but Calvino has done something more important from a cultural point of view. He not only stated the thesis that:

> The scientific and poetic attitudes coincide: both are attitudes at the same time of research and design, of discovery and invention,[12]

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[6] In Coetzee J.M.: Diary of a Bad Year (2008), or Siti W.: Resistere non serve a niente. Rizzoli (2012).

[7] A separate treatment deserve detective or science fiction novels; among the first, those featuring mathematicians are classic (among the most recent ones T. Michaelides, K. Devlin, G. Lorden, The Numbers bewhind NUMBERS: Solving Crime with Mathematics. Plune, 2007; Pythagorean Crimes, 2006, or tv Numb3rs serial); *big science* is instead a perfect setting for spy stories and intrigues (see e.g. Gomez Cadenas J. J.: Materia strana. Dedalo, 2012).

[8] In *Il Sole24ore* of 3-4-2012, p. 25.

[9] "I buchi neri", in *Corriere della Sera*, 10-7- 1975, p. 3.

[10] Preface to *La memoria del mondo e altre storie cosmicomiche*, 1968. Calvino's quotation are translated by the author, but for those from *Six Memos for the Next Millennium*, taken from Patrick Creagh's translation for the Penguin 2009 edition.

[11] Letter of 1961 to Primo Levi, in Calvino I.: Lettere 1940-1985 (Baranelli L. ed.). Mondadori, Milano (2000), p. 695.

[12] La sfida del labirinto. Il Menabò, July 1962.

but he has also offered tools to verify it, in particular in his *Six Memos for the Next Millennium*.[13] An exploration of mathematical thought can follow the lead given by this book conceived for literary criticism.

If one accepts the claim of the coincidence of the capacities at work both in literary and mathematical creativity, and the coincidence of aesthetic qualities of their most successful specimens, then in *Six Memos* one recognizes a work of metaliterature that is also a work of metamathematics.[14]

Calvino chooses five qualities or values which have to be preserved in the forthcoming millennium: lightness quickness, exactitude, visibility, multiplicity.[15] To recognize the presence of these characteristics in mathematics poses no problem, and is a useful educational exercise. Calvino's appeal for their preservation is quite pertinent for mathematics. We have entered the new millennium. I cannot say for literature, but mathematics is rapidly changing, both in its content and in its image.

The above mentioned values are not at risk of disappearance from mathematics, they are its essence; what could happen in the near future is that creative and fantastic mathematics becomes a patrimony only of a few. The mathematics which will be learned by the new generations will be very different from that of the twentieth century, less conceptual and more similar to what was known as school mathematics in its concern with usefulness; functional rather than attractive; in a word, less amusing. Students will have hard time to recognize Aristotle's description:

> The philosophers who claim that the mathematical sciences make no room for either the beautiful or the good are surely mistaken [...] Beauty on the contrary is the principal object of reasoning in the sciences and in the proofs.[16]

Besides Aristotle's authority we could appeal for support to perception psychology: there is for example the widespread, although unsustained belief that the golden rectangle (they in which the sides are in the golden ratio) be the most pleasing image to the eyes, and for this reason often used in art. There is no denying the beauty of mathematical figures:

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[13] The title had been chosen by Calvino for the Charles Eliot Norton Lectures 1985-86. They have been published by Harvard Univ. Press in 1988, and in Penguin Classics in 2009. In Italian the title is *Lezioni americane. Sei proposte per il prossimo millennio* (1985), in Calvino I.: Saggi 1945-1985 (Barenghi M. ed.). Mondadori, Milano (1995), vol. 1, pp. 627-753.

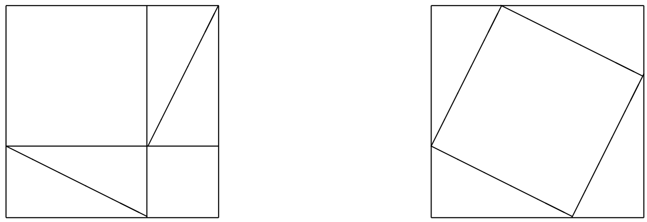
[14] An interpretation of this kind has been defended in Lolli G.: Discorso sulla matematica. Una rilettura delle "Lezioni americane" di Italo Calvino. Bollati Boringhieri, Torino (2011).

[15] They should have been six, including *consistency*, but the last lecture was never written.

[16] Quoted from Ayoub R.G. (ed.): Musings of the Masters. MAA (2004), p. 83.

not only fractals, but more unassuming hand made figures, those to which Calvino's description of "clear, incisive, memorable visual images" applies.[17]

Look at this proof without words of Pythagora's theorem from the *Chou pei suan ching* (200 B.C.):



Among mathematicins aesthetic judgements are frequent, und usually agreed upon by experts:

> I always try to combine the true with the beautiful, but when I have to choose one or the other, I usually choose the beautiful.[18]
>
> Beauty is the first test: there is no permanent place in this world for ugly mathematics.[19]
>
> The privileged unconsciuos phenomena, those susceptible to become conscious, are those which, directly or indirectly, affect most profoundly our emotional sensibility. It may be surprising to see emotional sensibility invoked à propos of mathematical demonstrations which, it would seem, can interest only the intellect. This would be to forget the feeling of mathematical beauty, of the harmony of numbers and forms, of geometric elegance.[20]

Aesthetic criteria however are seldom made explicit, so they remain mysterious to nonpractitioners. Geodfrey Hardy (1877-1947) in his *Mathematician's Apology* makes a list, with comments, of the characters of mathematical beauty he is partial to: generality, depth unexpectedness, inevitability, economy.[21]

A few of these overlap with Calvino's values, or integrate them. The latter are more utilizable having been didactically perused by Calvino to show how they enter in the composition of an art work and how they produce an aesthetic pleasure.

In the lightness lecture for example Calvino takes Guido Cavalcanti (1258-1300) to typify the particular version of lightness represented by "a visual image of lightness that acquires emblematic value".[22] Calvino remembers the action of his getting free from obnoxiuos friends by means of an agile jump, as recounted in a Giovanni

[17] Calvino continues: "in Italian we have an adjective that doesn't exists in English, *icastico*".

[18] Hermann Weyl (1885-1955), personal communication to Freeman Dyson, quoted in F. Dyson, "Science on the Rampage", *New York Review of Books*, april 5, 2012.

[19] Hardy G.H.: A Mathematician's Apology. Cambridge Univ. Press (1996), p. 85.

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[21] Hardy G.: Apology of a mathematician, p. 103.

[22] "Una immagine figurale di leggerezza che assume un valore emblematico".

Boccaccio (1313-1375)'s novel, "sì come colui che leggerissimo era" [a man very light in body], and examples of his poetry inhabitated by sighs, luminous rays, impulse called spirits, characterized by so light words, always on the move and yet vectors of information.

> Va tu, leggera e piana / dritt'a la donna mia
> [Go, light and soft / straight to my lady]

or

> e bianca neve scender senza vento
> [and white snow falling without wind].

He compares this line with an almost equal one by Dante Alighieri (1265-1321): "come di neve in alpe sanza vento" [as snow falls in the mountains without wind]. He remarks how the adverb "as" used by Dante closes the scene in the framework of a metaphor, in which the scenery takes a concrete meaning, while in Cavalcanti's line the landscape fades away in an athmosphere of suspended abstraction.

When discussing exactitude, Calvino wonders whether Giacomo Leopardi (1798-1837) was right in his claim that "the more vague and imprecise language is, the more poetic it becomes".[23] Calvino reads then passages from Leopardi's *Zibaldone* trying to understand what he meant by saying that "the words *notte, notturno* [night, nocturnal] etc., descriptions of the night, etc., are highly poetic because, as night makes objects blurred, the mind receives only a vague, indistinct, incomplete image".[24]

In Leopardi's description of situations favorable to the spiritual attitude of the indefinite, Calvino understands that:

> [so] this is what Leopardi asks of us, that we may savor the beauty of the vague and indefinite! What he requires is a highly exact and meticulous attention to the composition of each image, to the minute definition of details, to the choice of objects, to the lighting and the atmosphere [...] The poet of vagueness can only be the poet of exactitude, who is able to grasp the subtlest sensations with eyes and ears and quick, unerring hands [...] the search for the indefinite becomes the observation of all that is multiple, teeming, composed of countless particles.[25]

Analyses as such are the substance of literary criticism; as when, always for Leopardi, one points to the use and function in his works of the *iperbaton*, or of archaic words. To traspose this kind of criticism to mathematics, one has to find suitable corresponding examples. We'll give only one, referring to our *Discorso sulla matematica* (see footnote 14) for a thorough treatment.

[23] "Il linguaggio è tanto più poetico quanto più è vago, impreciso".

[24] "Le parole notte, notturno ecc., descrizioni della notte ecc. sono poeticissime, perché la notte confondendo gli oggetti, l'animo non ne concepisce che un'immagine vaga, indistinta, incompleta [...]".

[25] "[Ecco] cosa richiede da noi Leopardi per farci gustare la bellezza dell'indeterminato e del vago. È una attenzione estremamente precisa e meticolosa che egli esige nella composizione d'ogni immagine, nella definizione minuziosa dei dettagli, nella scelta degli oggetti, dell'illuminazione, dell'atmosfera [...]. Il poeta del vago può essere solo il poeta della precisione, che sa cogliere la sensazione più sottile con occhio, orecchio, mano pronti e sicuri [...] la ricerca dell'indeterminato diventa l'osservazione del molteplice, del formicolante, del pulviscolare".

In the quickness lecture, a short complete text of a Charlemagne legend exemplifies the style of folk tales: “[a] bare résumé, in which everything is left to the imagination and the speed with which events follow one another conveys a feeling of the ineluctable”.[26] To keep together the chain of events there is a verbal liaison (the word “love” [amore], and a narrative one, a magical ring that “establishes a logical relationship of cause and effect between the various episodes”.[27] Through the comparison with other versions of the legend Calvino shows how much it is gained by eliminating what is inessential, thanks to the intentional austere poverty of expressive means; in Calvino’s version everything that is not essential to the conveying of the intended message is left to imagination.

In fairy and folk tales a functional criterion prevails: “It leaves out unnecessary details, but stresses repetition: for example when the tale consists of a series of the same obstacles to be overcome by different people”.[28] Fairy tales and folk tales are told with great expressive economy. If a king is sick, there is no need to telling his ailment. “But everything mentioned has a necessary function in the plot”.[29]

Calvino could have also observed that in Charlemagne’s story the phrases are all very regular, almost all atomic, that is subject-verb-complement, the subject is always esplicit, few pronouns, few conjunctions or other logical particles.

Those who are familiar with mathematics immediately perceive that Calvino’s description applies in a literal sense to mathematical proofs, in particular to formal ones. These are not the way mathematics is done, but the ideal form of expression that captures its essence. Formal proofs are defined as *sequences* of sentences, that is sequences of strings of symbols obeying syntactic formation rules; such sequences are produced by causal agents, namely the logical rules, applied to preceding sentences: “the secret of the story lies in its economy: the events, however long they last, become punctiform, connected by rectilinear segments, in a zigzag pattern that suggests incessant motion”.[30] The graphical representation of a formal proof is a diagram agreeing with Calvino’s description.

In a deduction the details are omitted, since nothing is said about the meaning of symbols; as for nonlogical symbols, everything is left to the imagination; there are no connective words, absorbed into the logical rules, but for the pleonastic “then”, or “hence”; the feeling of the ineluctable is given by the fact that rules are what they are, few (sometimes one), and it is not possible to avoid them; repetition is almost inevitable, since few are the logical inferences allowed. “[I]n prose narrative there are events that rhyme”.[31] In deductions, the repeated “hence” marking the effect of

[26] “Un racconto secco dove gli avvenimenti si susseguono veloci, come in uno scarno riassunto, dove tutto è lasciato all’immaginazione e la rapidità della successione dei fatti dà un senso d’ineluttabile”.

[27] “Stabilisce tra i vari episodi un rapporto di causa ed effetto”.

[28] “Si trascurano i dettagli. Al contrario si insiste sulle ripetizioni, per esempio quando la fiaba consiste in una serie di ostacoli da superare”.

[29] “Ma tutto ciò che è nominato ha una funzione necessaria nell’intreccio”.

[30] “Il segreto del piacere della storia sta nell’economia del racconto: gli avvenimenti, indipendentemente dalla loro durata [reale], diventano puntiformi, collegati da segmenti rettilinei, in un disegno a zig-zag che corrisponde a un movimento senza soste”.

[31] “Nelle narrazioni in prosa ci sono avvenimenti che rimano tra loro”.

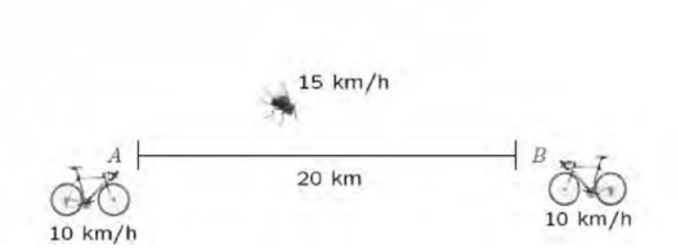
the *deus ex machina* rules give to the reading of the graph growth if not a rhyming almost a rap rhythm.

Obviously the rhythm of a story is something different from its invention. In mathematics, the quicker are the solutions, the lesser they resemble the long road of computations; but the length of a proof, or of the solution of a problem, has little to do with its discovery or invention.

Quickness does not consist in running fast along a road, or in mathematics in quickly doing long computations and deductions, but in finding the shortest route. There are mathematicians who love computations, but the majority is made of lazy people, so lazy that they are ready to the utmost exertions to find the solution which costs less effort. The praise of quickness is not meant to undervalue the pleasures of leisure.

The consideration of mental velocity makes us reverse the value of quickness conceived as going directly to the goal. Quickness of style and thought mean "agility, mobility, and ease, all qualities that go with writing where it is natural to digress, to jump from one subject to another, to lose the thread a hundred times and find it again after a hundred more twists and turns".[32]

In mathematics, the capacity to change perspective, hence also tools, to use lighter and more powerful ones, is the source of satisfaction and of aesthetic enjoyment. The maximum pleasure is reached when the answer is short and immediate, *a posteriori* obvious, banal, almost impossible to explain to those who do not see it; a common reaction is a good laugh. Many mathematical games exploit this possibility, as is shown by the fly puzzle:



The question is how much space the fly covers by flying back and forth from one bicycle to the other, while the two bicycles travel one towards the other, all at uniform velocity; the quick answer does not calculate the space, which is possible with an infinite series, as purportedly John von Neumann (1903-1957) was able to do in his mind on the spot, but the time (before the fly is smashed beween the two handles). The solution is easy if one changes perspective, the surprise lies in the use of a factor not mentioned in the question data.

[32] "Agilità, mobilità, disinvoltura, tutte qualità che s'accordano con una scrittura pronta alle divagazioni, a saltare da un argomento all'altro, a perdere il filo cento volte e a ritrovarlo dopo cento giravolte".

More serious and important cases are those in which the solution of a problem related to some knowledge domain uses concepts and results from another.

In the study of arithmetical congruences for example, to the segment of numbers between 0 and $p-1$ is associated the finite ring $\mathbb{Z}_p$ of residue classes modulo p. The numbers from 0 to $p-1$ are the representatives of these classes. The addition $\oplus$ is defined by: $x \oplus y$ = remainder of the division of $x+y$ by p. The elements different from 0 form a group, with multiplication defined by: $x \circ y$ = remainder of the division of $x \cdot y$ by p.

When p is prime, $\mathbb{Z}_p$ is a field.

The group properties of $\circ$ eliminate the heavy computations with congruences. A famous example is that of the theorem conjectured by Fermat in 1640, stating that if p is prime and $1 \leq k \leq p-1$ then

$$k^{p-1} - 1 \text{ is a multiple of } p.$$ [33]

A basic theorem of finite groups of order n assures that every element multiplied by itself n times is equal to the unity; since the order of $\mathbb{Z}_{p-1}$ is $p-1$, if k is such that $1 \leq k \leq p-1$ then

$$\underbrace{k \circ k \circ \ldots \circ k}_{p-1} = 1,$$

meaning that $k^{p-1} - 1$ is divisible by p.

The old purely arithmetic computational proofs are not even remembered.[34]

Translated from the Italian by Kim Williams

[33] Another example is Wilson's theorem, see Kac M., Ulam S.M.: Mathematics and Logic. Penguin Books (1971), pp. 70-72.

[34] To find what they looked like, one has to consult some old classic texbook of number theory, such as Dickson L.E.: Introduction to the Theory of Numbers. Dover, New York (1957).

The Mathematical Mind - Iconography of a Tension

Paolo Pagli

Definition

There is a concept, an idea I want to propose and to illustrate. I call it *the mathematical mind*. What do I mean by this?

The mathematical mind is the creative capacity, common to all human beings, to select, choose, recognize as well as propose and build, various regularities in the world; to search for, or work out, simple objects of reasonable uniformity which match a permanent structure, something which is stable within the variety of the real things

Such regularities are the straight lines (actually, line segments) and circles in place of (or with) irregular lines, the strange curves that we observe in trees, leaves, stones, mountains, and landscapes.

A search for and the construction of regularities, uniformities, in space, but also in time: first of all the stability of sets of objects at different moments.

The pictures, I hope, will clarify this concept.

Mathematics, of course, came from the mathematical mind, when and where mathematics emerged. This happened in all cultures with a higher degree of "complexity": in this case a closer examination and further inquiry into the products of the mathematical mind was needed for utilitarian purposes, and, as a secondary result, became interesting in their own right. But in every time and in every part of the globe the mathematical mind is still living as a part of our interpretation of the world.

Regularity

The Lascaux caves in southwest France (department of Dordogne) were discovered in 1940. They contain more than 1500 paintings, some 17,000 years old or even

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older. They mostly depict large animals present in the area at the time; only one human figure is present. Their purpose cannot be known for certain. The artistic level is very high.


Fig. 1 Caves of Lascaux (Montignac, France), Great Hall of the Bulls (© Ministère de la Culture - Médiathèque du Patrimoine, Dist. RMN-Grand Palais / Image IGN)

What is striking here is the "geometric" feature of the drawing under the foot of the big cow (Fig. 1), but of course this term is out of context. We don't know the meaning of these strange rectangles, but we have the proof of the existence, in very ancient times, of these space models (straight lines, right angles, parallel lines) which, after thousands of years, were to bring about a new world.

With a vertiginous leap but in my opinion not without an element of continuity (not in the history but in the tendency), I propose another situation.

At dawn on August 14, when the sun is rising, people meet in Valentano, a little town in the Italian region of Umbria, in a field 4 km from a little hill. With a couple of white oxen and an old plough a furrow is cut in the ground, with great precision (Fig. 2). When the sun is shining, little lamps are put in the furrow. This rite is very ancient.

This ceremony occurs at a time near the solstice, and is surely what remains of a more complex event whose meaning is lost. In any case we have here the realization of Euclid's first postulate: given two points there exists (people build!) a straight line passing for them. This did line not exist before and is not the picture of something already existing: we have a new object which is the first object of a new world.

But, as already mentioned, the search for regularity takes place not only in space, but also in time and in the abstract. Ishango is a village on the shores of Lake Edward, one of the sources of the Nile, in the Democratic Republic of the Congo. Here 20,000 years ago there were Mesolithic communities of hunters, gatherers and fishermen, later dispersed by a volcanic eruption. The "Ishango bone" (a fossilized baboon bone) was discovered here in 1960 (Fig. 3). It is the size of a pencil, and has a sharp piece of quartz inserted at one end. It has three series of tally marks running along its length.

As before, the purpose of the object is obscure. What was it used for? What's the meaning of the carved notches? Counting, perhaps, but of what having this struc-

Fig. 2 Valentano's furrow (from Giovanni Feo, *Geografia sacra*, Stampa Alternativa, Viterbo, 2006)

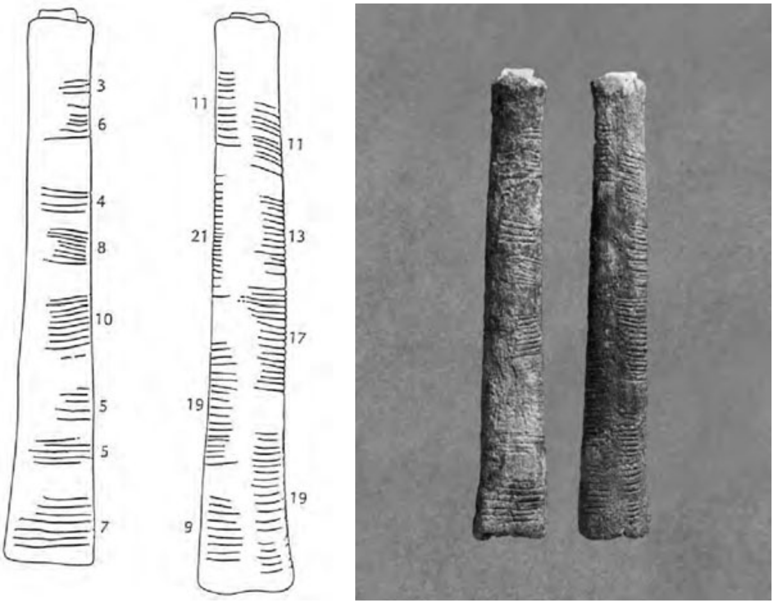

Fig. 3 Ishango Bone (11.000-8500 BC) (© The Royal Belgian Institute of Natural Sciences, Brussels)

ture? In any case regularities seem certain. We can "read" some operations in the first two series, and four prime numbers in the third. In Africa, about ten thousand years ago:

- $f_1(n) = 2n$; $f_2(n) = n/2$: $f_3(n) = n + 2$;
- 10 + 1; 20 +1; 20 - 1; 10 - 1;
- $\{$all the prime numbers x: $10 < x < 20\}$.

Regularization

The regularities just described are almost surely regularizations of phenomena or objects whose contexts have been lost. But in other cases the process and the aim are clear.

The *groma* is the fundamental (probably Etruscan) tool used by Roman surveyors (*gromatici*) for constructing orthogonal lines and alignments on the ground (Fig. 4). It was known from Latin authors, but then one example and some fragments were found in Pompeii excavations.

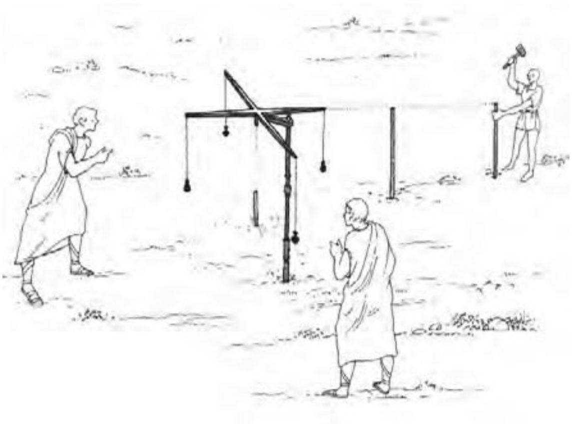

Fig. 4 Groma

Straight lines and right angles. Greek geometry was concerned with *defining* these objects:

> A straight line is a line which lies evenly with the points on itself … When a straight line set up on a straight line makes the adjacent angles equal to one another, each of the equal angles is right …
>
> Euclid, *Elements*, Book I, Definitions 4, 10

But the aim of the Roman surveyors, with the aid of their tools, was the organization of real space, the ground. Filling it with (ideal) squares or rectangles was a way of expressing their will, facilitating the description for measurements and the recording of property. The sides of the regularities became civil, political boundaries.

This process of regularization does not only concern things that are material or visible, but also those of our imagination.

Jeroen Anthoniszoon van Aken, or Hieronymous Bosch, as often he signed his paintings (ca. 1450-1516), was "the inventor of monsters and chimeras", the creator of "wondrous and strange fantasies", as some ancient critics judged him. In *The Flight to Heaven* (also known as *The Ascent of the Blessed*, Fig. 5) he is very simple

Fig. 5 Hieronymus Bosch, *The Flight to the Heaven* (1500-1504), Venice, Palazzo Ducale

and linear. The painting is part of a polyptych, originally part of a larger altarpiece of which only four panels remain.

The passage from the world of visible things to the other world is through a mathematical object, a cylinder. The Beyond cannot be described or represented (the Light in the background), but the transition is marked with a creation of the mathematical mind.

The helix-shaped trajectory of the souls (bodies!), assisted by the angels, is only hinted at. Their irregular external shapes (arms, legs, wings) are still naturalistic; instead the last space they pass through is symmetrical.

The final expression of the matter that precedes the absolute immaterial, the glorified matter, is the matter of the mathematical mind.

Towards the Abstract

In our time also the mathematical mind continues its rhapsodic epiphanies, not always consciously.

The history of Japanese gardens follows a specific pattern. In China the house and garden were in contrast: the house more formal ("Confucian"), the garden more natural ("Taoist"). In Japan the garden was always an extension of the house, both being elements in nature. But from the thirteenth century, for complex cultural reasons, Japanese gardens began to be regarded as objects of contemplation, not spaces to spend time in, and were associated with Zen temples. In the fifteenth century "stone gardens" (*karesansui*, "arid landscape") emerged: there were no longer trees, leaves or flowers, but only sand and rocks, representing the flowing water of a river or a lake and mountains. The only living element was moss. So the cycle of seasons no longer influenced them; they were timeless. The *Ryan-ji* in Kyoto, built in 1490, is one of the most famous gardens in Japan (Fig. 6). It is the size of a tennis court, has "waves" of sand, and fifteen rocks. These are very accurately spaced in five groups and from any given angle only fourteen of them can be seen.

Fig. 6 Ryan-ji (1490), Kyoto (Japan)

It seems difficult to interpret all the complex history just recounted as a deployment of the mathematical mind. But the whole process, though with different aims, follows the same route and the result, even in its materiality, is reminiscent of the products of this faculty. The *Ryan-ji* is not a garden at all, nor is it the picture of a garden: it is a new object in the world. It is abstract, not symbolic: it represents itself, a simplified (and therefore non-existent) object which we construct, like the products of the mathematical mind, but in this case – this case alone – materially and not simply in our minds. We add it to the real world: "regular" in a broad sense, not subject to change, and thus everlasting. And like natural numbers, men created it, but can never see (know) it completely.

Piet Mondrian is the author of very famous "sequence" of paintings of trees. We have here a product of the mathematical mind, and also a description of its way of operating now (more precisely: at the beginning of the twentieth century), in a world with very highly developed mathematics.

If we can still see *The Red Tree* (1909-10) as our (interpreted) reality, *The Gray Tree* (1911) exists in a world that does not exist (for example, the tree it is not inserted in a space, but he creates it). It is already a product of the mathematical mind, but is not a definite object like the Bosch cylinder: its articulated ramifications (which seem to continue indefinitely) recall and portray the features of a more complex mathematical object, like the real numbers. But the last picture is the very product of the mathematical mind: a pattern of abstract relations, joints, contacts, which can represent not just a single object, but a plurality of objects having a similar structure. Also a red tree, a grey tree, a (real) apple tree in flower, as in the title, all trees in the universe. A nonexistent object which assimilates, describes and summarizes a world of objects.

Other ways

Sometimes the way of the mathematical mind is inverted, sometimes (rarely) it is completely absent: these situations are very useful for a better understanding of the process itself. The most apparent example of the first case is our response to the vision of the stars in the sky.

In a meta-time which is still running, large mythical animals were going along and singing. Their moving was forming space, their songs called matter and life to existence. So the world was created in the land that many centuries later we called Australia. Big rock paintings record the histories of these ancient mythical ancestors.

In the Yrrkala painting shown in Fig. 7 a shark (the stars α e β of the Centaur constellation) chases a stingray, creating the Southern Cross.

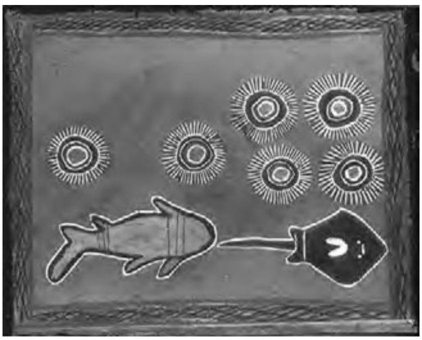

Fig. 7 Yrrkala painting (Arnhem Land). Adelaide, State Library of South Australia

This is a picture from a faraway culture which shows the two elements of the process we mentioned before: the beginning and our (fixed) reaction as human beings. We see the stars as points: they are moving, of course, as are the planets among them, with their more complicated flight. In this case the reality is dematerialized, already essential, geometric. How could the mathematical mind act on this scene? It cannot act, but a different process is set into motion: we fill our vision with matter, we construct shapes, forms, starting from these abstract lighting points. So animals, objects, human beings emerge from the stars and live and operate in the sky. They are less determined, less justifiable of all the products of the mathematical mind when it dematerializes and simplifies reality. And in this case our constructions are doing, acting: a drama, a tale begins in the sky.

The precision of Aboriginals in keeping the vision and the conclusion, the two moments, distinct, is higher than our own in the similar processes and representation.

But more radical than this reverse process is the total absence of mathematical mind.

In *The Waterfall on Mount Lu*, by the Chinese painter Shi Tao (1642-1707), we see masses, never volumes, neither dematerialized nor mathematized (Fig. 8). They are not realistic, but strongly real, overhanging. From one side they appear to bear the force of gravity, from the other they produce it. This force assails the water,

Fig. 8 Shi Tao, *The Waterfall on Mount Lu*, The Palace Museum, Beijing (reproduced with permission)

which is almost compressed. Nothing is idealized. Further, the "geometrical" element, the tree, which is the only living thing in the picture, is more a tension than a form: it can be neither the outcome nor the beginning of a process of regularization.

Perhaps, once the concept of mathematical mind has been singled out, it is also possible to glimpse its essence in pictures, like this one, in which it is not present at all.

Conclusion

More often unconscious, rhapsodic, everlasting, the mathematical mind does exist. Indeed, it gave birth to mathematics, but we find it still, alive and at work, throughout history. It represents a universal, essential part of our human condition, an eternal consequence of our intersection with the world.

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Mathematics and Film

Spatial Rhythms in Cinema between the Avant-Garde and Entertainment

Gian Piero Brunetta

to Lorenzo

> I think that the essence of cinema is defined by the rhythm with which the images follow one another and, as a rational and Cartesian thinker, I consider number to be the basis of everything, of art and of nature. The rhythm of film editing of *Film no. 4*, for example, was based on the Fibonacci numerical scale. Now unfortunately some of the frames are missing and their exact calculation does not exist anymore [...] It isn't a technique that I invented, because it was also used in *Battleship Potemkin*; in that film as well, all of the editing was constructed according to the Fibonacci scale. I used this method, which I have always used in my painting.[1]

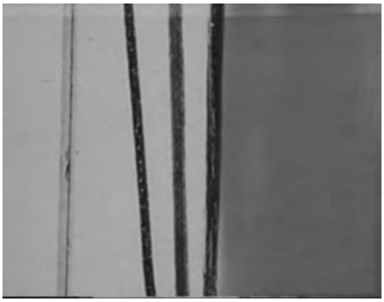

Fig. 1 *Film n. 4*, Luigi Veronesi, 1940

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[1] "Io penso che l'essenza del cinema sia data dal ritmo col quale si succedono le immagini e, quale razionale e cartesiano, considero il numero alla base di tutto, dell'arte e della natura. Il ritmo di montaggio del Film n. 4, per esempio, si basava sulla scala numerica di Fibonacci. Ora purtroppo mancano alcuni fotogrammi e non esiste più il loro calcolo esatto... Non è una tecnica che ho inventato io, perché è stata adoperata anche nell'Incrociatore Potemkin; anche in questo film tutto il montaggio è stato costruito secondo la scala di Fibonacci. Io ho usato questo metodo che ho sempre adoperato nella mia pittura." Veronesi L.: Del film astratto. Ferrania **2**(9) (1948), p. 23.

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I begin with a statement by Luigi Veronesi, painter and maker of about ten films, even though the number of his films arrives to the number 13, because, as he himself said, “my love for the cinema does not include the ten, the eleven and the twelve”. In addition, the number 13, a golden number, recurs continually in the structure and the editing of this last film: the sequences are formed of groups of thirteen or multiples of thirteen frames.

On other occasions Veronesi has confirmed the structural influence of mathematics on his cinematographic works:

> The abstract film is born [. . .] from an idealistic presupposition that, in the most rigorous cases, poses number and the laws of higher mathematics as the foundation of the work of art. Using only essential geometric elements, and thus exactly measurable and controllable in their proportions and ratios, and combining and developing those elements according to the laws of harmony and visual counterpoint, the makers of abstract films intend to create works in which the musicality is given by the visual elements alone: shapes and colours in motion and in ratio to each other.[2]

These statements – formulated in a text entitled “Del film astratto” (On abstract film) of 1948 – which embody the meaning of a part of the line of thought that I intend to cover here, already seem to have been exemplified in a kind of synthesis in the opening sequence to *An Optical Poem* by Oskar Fischinger, created for Metro Goldwyn Mayer ten years earlier, in 1938. A film seven minutes long, it is intended to be the translation of musical notes and rhythms of Liszt’s *Hungarian Rhapsody*, into corresponding visual rhythms of geometric shapes in motion that are formed in the mind. The short was conceived to be projected in front of a general audience along with one of the many popular films of the season. The cinema could become the ideal vehicle for popularising and making accessible the most advanced creative processes then ongoing in the various arts. A short time later, Fischinger himself collaborated with Walt Disney for the abstract sequence set to the music of Bach in *Fantasia.*

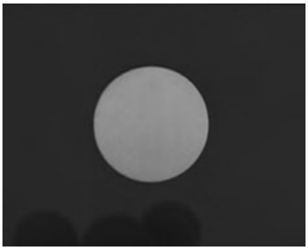

Fig. 2 *An Optical Poem*, Oskar Fishinger, 1938

[2] “Il film astratto nasce ... da un presupposto idealistico che, nei casi di massimo rigore, pone alla base dell’opera d’arte il numero e le leggi della matematica superiori. Usando soltanto elementi geometrici essenziali, perciò esattamente misurabili e controllabili nelle loro proporzioni e nei loro rapporti, e combinando e sviluppando tali elementi secondo leggi di armonia e contrappunto visivo, i realizzatori di film astratti intendono fare opere in cui la musicalità sia data dai soli elementi visuali: forme e colori in movimento e in rapporto tra loro.” *Ibid.*

> To most of us music suggests definite mental images of form and colour. The picture you are to see is a novel scientific experiment – its object is to convey these mental images in visual form.

I like seeing the roar of MGM's Lion, which is a guarantee of the quality of its entertainment product, associated with a type of work situated almost at the vertex of the parabola of an trajectory of vectors and dynamics of avant-garde and experimental cinema and of the kind of spectacle and entertainment in which there is an entire series of intersections, interferences, movements along lines that are now parallel and now converging or tangent. And that it is precisely in mathematics, physics and geometry that are found the factors and possibilities of identifying planes and surfaces, shared nodes and arches, and admissible solutions, never before recognised and examined for their productivity in the exploration of constantly expanding frontiers in the use of cinematographic space. Oskar Fischinger, Mary Ellen Bute, Busby Berkeley and Norman McLaren all constructed cinematographic models in which are found almost optimal solutions to the problems of perfectly translating and harmonising musical rhythms into visual rhythms.

From its very beginnings – from the first serpentine dance of Loie Fuller and other dancers, filmed in 1896-97, in which the human figures dissolves, creating an indefinite and constantly changing number of geometric figures – cinema has been able to transmit, together with movement, also the idea of movement, and to render materially visible that fourth dimension that Boccioni and the Futurists tried to represent in their works.[3]

The great mutation of the poetics of artistic ways and procedures of making of the early twentieth century is also due to the fact that, thanks to cinema, the movement and the dynamics of the body in space could be regulated by rhythms and scanned according to measurable tempos, something that at the time was known only to music. In cinema tempo is incorporated with images, ordering and organising them, the possibility of their use in space raised to the nth power.

At the turn of the twentieth century, many artists, beginning with the thinking of Wagner and Nietzsche and going through the Futurists manifestos, believed in *Gesamtkunstwerke*, the Total Work of Art: the cinema, the new kid on the block of entertainment and the arts, soon appeared to be a topological space, an ideal container and a point for connecting and translating all languages and all forms of art, as well as forms and theories of scientific thinking, from the simplest Euclidean geometry to the most complex geometries, giving the impression of almost being able to hear the echoes of the newborn theory of relativity and, later, that of quantum mechanics, not to mention psychoanalysis.

Following the end of the first world war the screen became the transfer of the collective desire of entire groups of artists and intellectuals who intended to aban-

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[3] In the vast body of literature about cinema, I will limit myself to noting the names of Mario Verdone and Giovanni Lista, the scholars who have contributed the most to reconstructing the history and restoring the central place and importance in the history of cinema of the avant-garde. Among the pioneering works of M. Verdone see at least *Cinema e letteratura del futurismo* (Edizioni di Bianco e Nero, Rome, 1968) and of G. Lista see *Le cinéma futuriste* (Paris Expérimental, Paris, 2008), which gathers and summarises forty years of research on the topic in the most efficient way.

don the obsolete forms of visual and literary culture practiced up to that time and launch themselves forward; to suppress the traditional space-time coordinates to live in the future, following the ways of abstraction, of the visual transcription of new languages. From the Futurists to the Surrealists, from the Dadaists to the Constructivists, to the Expressionists, discussions of the cinema and its potential appear above all to recognise in film the capacity to provide an answer to the long search for *Gesamtkunstwerke* and the power to meld, to a greater degree than any other form of artistic expression, tactile, visual, sonorous and olfactory sensations according to scientific rules and canons. Cinema was the “new arm”, the term which just a few years earlier Mario Morasso had used to celebrate the automobile.[4] It presented itself as the most extraordinary container and modifier of all forms of art, the new art form and new Muse that had long been prophesised and eagerly awaited.

In the first two decades there would be above all encounters between cinema and mathematics. With only a few exceptions, in the animated films, in the set designs by Enrico Prampolini for *Perfido Incanto*, a film directed in 1917 by Anton Giulio Bragaglia, the cinema progressively discovered the dimensions of space and time, inspired primarily by the models of the various schools of nineteenth-century painting. But from the first experiments in Dadaist paintings by Hans Richter in 1921, the motion picture camera began a very creative and conscious dialogue with mathematics, physics and different geometries, taking their principles as its own and beginning to use the screen as a blackboard on which to represent two- and many-dimensional space, roving among a most extensive range of possibilities, from the point at infinity.

In the early 1920s painters, writers, poets and intellectuals moved toward cinema, not to take it over and colonise it, but with humility, almost with an awareness of having by that time to recognise its absolute hegemony among the arts, as well as the possibilities for cross-pollination with all fields of knowledge. Cinema embodies the most modern form of the Renaissance polymath. It has a sense of ‘designed serendipity’ that unites more than a few statements by various artists who ventured into a new territory, exploring, day by day, its possibilities, trying to solve many problems with the help of scientific instruments.

The field of observing shows byways that split off into diverging, multiple directions, only to then converge and superimpose on one another. On one hand, there are the artists of the avant-garde, who aim to destroy narrative and the equivalence of images, things and objects and human figures and point to the de-realisation of reality, who find in cinema a correspondence to the rhythms of combustion engines of automobiles and the construction of a meaning that is born of pure juxtaposition and chance combination of elements or rhythmic measures. This is what happens in Léger’s *Ballet mécanique* of 1924, in which the human body becomes a thing and the objects, decontextualised and randomly combined with each other, provide new, unexpected meanings at every performance, and in which the music of George Antheil becomes a kind of chaotic magma that makes all harmony and counterpoint explode.

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[4] See Morasso M.: La nuova arma (la macchina). Centro Studi Piemontesi, Turin (1994).

On the other hand there are the productions of the cinema industry, which then proceeds along parallel planes, but quite soon intersects, metabolises and makes its own some of the formal, rhythmic and prosodic innovations of experimental cinema, which seem to favour the multiplication of the spectacular use of space and the language of cinematography. In certain cases, by Oskar Fischinger as we have seen, or by Walter Ruttmann, it is the artists themselves who build bridges between one plane and the other.

While there have been reflections on and not a few contributions on the presence and representations of science and scientists in this history of cinema, beginning with the work of Georges Méliès, inroads have not yet been made in a direction that will shed light on how some aspects of scientific thought were assimilated and metabolised by various directors, both in their becoming fundamental elements of their own artistic poetic, and in constructing the formal structures of their works, becoming thematic motives, highly meaningful narrative elements, apart from the process of abstraction and the abandonment of the intention to mimic and represent reality.

The points of contact or representation on the screen of mathematics, physics, geometry, in their most varied declinations, can be summed up in view of the future works in the following sets. Some have already been explored in exemplary fashion by mathematicians who are also lovers of the cinema, including, it must be acknowledged, with excellent results, Michele Emmer, the organiser of the conference series "Mathematics and Culture".[5]

- A first, very clearly defined line can be recognised in the genre "biopic", and is given by the subgenre that recounts the lives of the great scientists, from Galileo to Descartes, from Evariste Galois to Fermi, Einstein, Alan Turing and John Nash. As a whole, this is a set of a modest number of works in which the pedagogical intentions appear to prevail over the desire to monumentalise or enter into the everyday life of genius; mostly, they have prevailed over the reconstruction of its disorderliness. From the point of view of dramaturgy, generally speaking, the genius has in any case a low entertainment coefficient.
- A second line is cinema that uses and narrates science as an allotrope of magic, as an obscure force that tends to subvert the laws of nature, as Promethian hubris, destined to let loose powers that are occult and uncontrollable.
- A third set is comprised of films that show scenes in which mathematics or other elements of science acquire a dramatic role in themselves, where they help to define more clearly the personality traits of the characters and the environment, becoming magical aids in a positive sense, playing roles that solve dilemmas, sometimes not only planes that are strictly scientific.
- A fourth set is cinema that adopts mathematical procedures as linguistic or structural elements in order to reveal the potentials of its own prosody and metrics.

[5] Of Michele Emmer's books I note at least *Visibili armonie. Arte teatro cinema matematica* (Bollati Boringhieri, Turin, 2008), and the most recent *Numeri immaginari* (Bollati Boringhieri, Turin, 2011). See also Emmer M., Manaresi M. (eds.): Mathematics, Art, Technology and Cinema. Springer, Berlin (2003).

The theory behind the film editing of Eisenstein could still today constitute a fundamental point of reference.

- A fifth set is cinema that uses light like a paintbrush, making a leading character of plane and solid geometry as it ventures into the territories of metamorphosis and homeomorphism.
- The sixth set is cinema that makes use of the procedures and figures of geometry, mathematics and physics as visual forms and horizons that permit us to see through them transparently to philosophical and theological concerns, or cosmogonic and metaphysical reflections, etc.

In short, cinema represents a complex and highly articulated topological space that from its very beginnings has opened various possible roads to research, some of which are but little travelled, if at all.

On this present occasion we are interested in seeing how a series of artists working in different realms of the avant-garde movement, from Cubism to Futurism, from De Stijl to Suprematism, from Dadaism to Surrealism, have, at a certain moment in the course of their work, encountered the motion picture camera and used it to transcribe plane and solid geometric figures with light on the screen, in order to emit their shapes into the fourth dimension, to combine these shapes with ordered and rational rhythms.[6]

It is though a group of artists, taking the movie camera in hand to transfer elementary lines and shapes onto film and attempt to give them movement, were to consider those operations as algorithms for obtaining a determined visual rhythm that would correspond as closely and congruently as possible to pre-existing musical rhythms. It is an extraordinary experience that results more than once from the collaboration of many artists and film makers with mathematicians in flesh and blood, with theorems both classical and in the course of being developed in those years.

In fact, in the space of about twenty years tangent points were continuously created between research in avant-garde cinema and research more closely related to performance and entertainment. Research in one field progressively fuelled the discovery of the possibilities of representation of elements in space in directions that are not mimetic of the other. In some cases, such as those of Walter Ruttmann or Slavko Vorkapich, there is a continual back-and-forth and interaction between one plane and another.

Encountering some of these makers leads us naturally to come across, even in disorderly fashion, a great quantity of articulated elements that appear to come out of a textbook of differential geometry: curves, incident lines, parallel lines, hyperbolas, parabolas, ellipses, conics, etc. There is a moment in which a series of points of the cinematographic work of the marginal cinema of artists, painters, set designers, photographers who moved into film direction, form genuine polar lines that touch

[6] The theoretical writings about cinema of a good part of avant-garde artists who will be cited are excellently anthologised in Rondolino G.: L'occhio tagliato. Martano, Turin (1972); and Id.: Il cinema astratto. Testi e documenti. Editrice Tirrenia Stampatori, Turin (1977).

those that they might intersect like the conics of films of entertainment or full-length films.

For some of these artists, who explicitly declare the debt they owe to scientific thought, not only of mathematics but of physics, astronomy, biology as well, the issue is no longer only to enter into theorems and geometric functions, terrestrial and celestial mechanics, or to penetrate into the incessant motion of life, as much as it is to go beyond them, to attempt to dilate the vision and representation of space, up to perceiving the invisible, to translating the inexpressible and ineffable into shapes and figures.

Some of them appear to move along a spatial-temporal line that pushes them back towards recovering the earliest elementary shapes created by *homo sapiens*, while others project themselves into the future in the attempt to use the screen and the movie camera to measure and represent infinity.

In various cases they seem to use the movie camera to translate and write a kinematic geometry. It is as though the starting points for many directors were mathematical numbers or geometric figures, or the desire to apply the laws of mechanics or physics more generally. As though, at the end of the creative process, as if on the basis of an algorithm, an entire mass of geometric shapes and physical and mathematical laws were to find on the screen a perfect proof in another language and be brought together to form a lexicon, a prosody and a syntactical vision and attempt to initiate, although with the enormous effort, a new process of communication.

Such a phenomenon, in its turn, presents not a few difficulties.

Let us take two examples at random. In 1921 the painter Theo von Doesburg, leader of De Stijl, tells of having taken a film by Richter to project it in Paris at the Thèatre Michel: "An old gentleman looked with interest at the title *Il cinema è ritmo* [Cinema is rhythm] and began to carefully clean his glasses, and when he had put them back on his nose the film was finished".

Richter's *Rithmus 21*, for example, was never projected in public.

Some of the authors cited had been perfectly aware of these difficulties from the very beginning, which helps us to understand their procedures and sheds light on the founding elements of their artistic making and their poetic. Hans Richter recalls that at the basis of his first works there was:

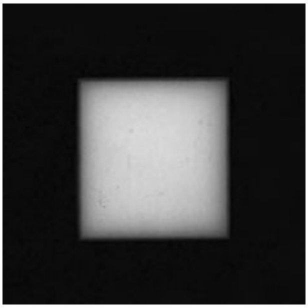

Fig. 3 Rithmus 21, Hans Richter, 1921

> A search for structural elements: Cezanne thought that all forms of nature could be traced back to the sphere, the square and the cone or pyramid. [...] After cubism came abstract painting. I spent two years from 1916 to 1918 groping to try to find the principles that constitute the rhythm of painting. I studied the principles of the counterpoint of Bach's fugues and preludes with Busoni's help and finally found valuable solutions in the negative-positive relationships with which I experimented in painting. In 1918 I met the Swedish painter Viking Eggeling, who had had similar experiences. With a variety of proportions, numbers, intensities, positions, etc., new contrasts and new analogies were born in perfect order until what was developed was a kind of functionality between the different formal units that gave the feeling of rhythm, of continuity [...] like the music of Bach".[7]

The two artists began to work on rolls on which they had glued pieces of paper in relationship to each other according to coherent ratios like those of mathematics: "Even though no roll contained more than eight or ten characteristic variations on a theme, it was evident to us that these rolls implied movement and that movement implied cinema ... Few arrived to cinema in such an unexpected way".

Given the difficulties of animating the rolls, Richter interrupted their creation and turned to animating "a series of squares of paper of all sizes from grey to white. In the square I had a simple form, which by its very nature established a ratio with the square of the screen. I made my squares of paper appear and disappear, jump and slide in time with a carefully controlled tempo according to a pre-established rhythm".

In reconstructing the course taken by the first avant-garde abstract cinema, Hans Richter – who would later be reproached by Anton Giulio Bragaglia for having neglected the firstborn of the entire Futurist experience from 1909 forward – immediately associated his own work to that of Eggeling as well as that of Ruttmann: "In 1922 Walter Ruttmann, who was also a modern painter who had been more subject to the influence of Kandinsky than that of the Cubists, presented in Berlin the first abstract film of a different genre ... While this film was less geometric than his other films, he also availed himself from time to time of geometric movements". Speaking of one of his later films of 1926, entitled *Filmstudie*, Richter recalls that he used "heads, wavy eyes like analogies of luminous circles, like moons, rising from the surface of the screen in delicate rufflings that explode into abstract waves, spirals and triangles. The wall that separates the world of objects from that of abstract forms is abolished".

In this phase the objects taken back to the purity of their forms are desemantized, so that they can then go on and form different associative and discursive chains.

[7] "La ricerca degli elementi strutturali: Cezanne pensava che tutte le forme della natura potessero essere ricondotte alla sfera, al quadrato al cono o alla piramide [...] Dopo il cubismo venne la pittura astrattista. Trascorsi due anni dal 1916 al '18 a cercare a tentoni i principi di quello che costituisce il ritmo nella pittura. Studiai i principi del contrappunto nelle fughe e nei preludi di Bach con l'aiuto di Busoni e trovai finalmente soluzioni preziose nel rapporto negativo-positivo con cui feci esperimenti nella pittura. Nel 1918 incontrai il pittore svedese Viking Eggeling, che aveva fatto esperienze simili. Con varietà di proporzioni, di numero, d'intensità, di posizione, ecc. nuovi contrasti e nuove analogie nascevano in ordine perfetto finché si sviluppava una specie di funzionalità tra le diverse unità formali che davano la sensazione di ritmo, di continuità [...] come la musica di Bach." Richter H.: Il film d'avanguardia in Germania. In: G. Rondolino G. (ed.): L'occhio tagliato, *op. cit.*, p. 94.

Cinema, the latest arrival on the art and entertainment scene, assumed in a stroke, thanks to these films and these experiences, the role of a guide to art.

In any case, beginning with these experiences and from those of films that are more well known, from the various *Opus* films by Ruttmann to Man Ray's *Retour à la raison*, from Léger's*Ballet mécanique* to *Emak–Bakia*, from Duchamp's *Anémic-Cinéma* to Richter's *Filmstudie*, circles, squares, rectangles, rhombuses, and other geometric figures unite, in a unified visual lexicon, in addition to the artists already cited, the work of painters, photographers, choreographers and directors such as Eggeling, Louis Delluc and Germaine Dulac, René Clair, Dziga Vertov, Kuleshov and Sergei Eisenstein, Ralph Steiner, Jay Leyda, Slavko Vorkapich, Busby Berkeley, Oskar Fischinger, Len Lye, Norman McLaren, Mary Ellen Bute, Douglas Crockwell, Harry Smith, Dwinell Grant, and many others.

Fig. 4 *Le Retourn à la raison*, Man Ray, 1923

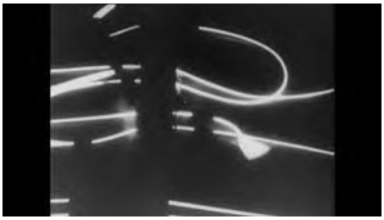

Fig. 5 *Donbass Symphony*, Dziga Vertov, 1931

Filippo Tommaso Marinetti as well, in 1926, would launch a *Manifesto del Cinema Puro e Astratto*, the influence of which can be seen in other works of Italian experimental cinema throughout the fifteen years that followed.[8] But the experience and course taken by avant-garde and experimental cinema often ceased to be coplanar and arrived, as we have said, at grafting itself in a very productive and leading way onto the cinema of fiction, dilating the linguistic and expressive possibilities and awareness.

[8] For more about Marinetti and his influence on later avant-garde cinema, see Strauven W.: Marinetti e il cinema. Campanotto, Udine (2006).

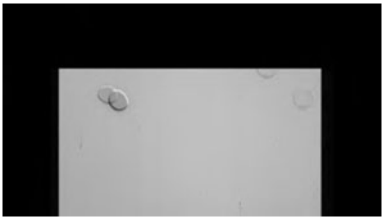

Fig. 6 *Scherzo*, Norman McLaren, 1938

One of the first to collaborate with performance cinema production was Ruttmann, who in 1923 worked with Fritz Lang on the first episode of the *Nibelungen (Siegfrieds Tod)*, creating the dream of Kriemhild, which features abstract geometrical figures that move within the character's mental space.

It is interesting to observe how in time the representation of dreams would be translated in several instances into plane or solid geometric shapes that are animated, from Vorkapich to Richter, with *Dreams that Money Can Buy* of 1948. Often too, when the motion picture camera crosses the threshold of the eye and moves into interior space, it encounters elementary geometrical shapes.

One of the first important attempts to hybridise the two lines of development took place via the modes of visual and symphonic orchestration of the metropolis.

From the 1920s, well before Lang's *Metropolis*, Ruttmann's *Berlin: Symphony of a Metropolis*, and Vertov's *Man with a Movie Camera*, the metropolises of the different continents, New York first of all, inspired a series of documentaries and films that used the form of visual symphonies to celebrate the new monuments to modernity, making skyscrapers the leading characters, or almost, and projecting their solid figures and geometric forms into space and on the plane of the sky, opening up new processes of signification that rapidly veered from the sense of exciting *ascensus* to the perception of the atomic isolation of the individual and the descent into an inferno, as happened to not a few Gold Diggers in the 1930s. I will limit myself to citing a brief list of titles of lesser known but crucial works: it begins with *Manhatta* by Charles Sheeler and Paul Strand of 1921, and goes to *24 Dollar Island* by Robert Flaherty of 1926, a *Skyscraper Symphony* by Robert Florey in 1929, to arrive to *Manhattan Medley* by Bonney Powell in 1931 and to *A Bronx Morning* by Jay Leyda, also of 1931, which marks a turning point, substituting at a stroke the pleasure of visually orchestrating the shapes of the skyscraper projected into space with a closer look at the people, at the poverty that serves as a counterpoint to the celebration of progress and the secular "gloria in excelsis" chanted by the skyscrapers.

Water is another element that offered more than one filmmakers in Europe and the United States an opportunity to move in and pass from the abstract dimensions to real ones. In 1929 Ralph Steiner, with H_20, dons the double vest of visual poet and consultant for hydrodynamics to explore the movement of water from points of view that are both dynamic and kinematic. In effect, he appears to want to define

Fig. 7 *Skyscraper Symphony*, Robert Florey, 1929

Fig. 8 H_2O, Ralph Steiner, 1929

new laws and systems of measurement of several water surfaces that are very close to fractal geometry.

Among all the directors who inserted episodes of commercial cinema along the open road of experimental cinema using bodies in space as applications of mathematics and geometry, Busby Berkeley is the one who succeeded in combining the happiest marriage of rigor and the search for perfection of the figures with the pleasure of playing with numbers and the shapes of geometry and challenging the laws of mechanics and physics.

Berkeley seems to have wanted to use the classic screen of the golden ratio of 1 : 1.333 as a reflective surface on which to represent the harmonic composing and decomposing of points, lines and curves; a space of two or three dimensions in which to translate visually, by means of bodies and objects that form circular ellipses, curves that generate orthogonal cylindrical surfaces, crowded with all sorts of curves, of polygonal arches formed of a series of points that can be continually decomposed and reformed into other shapes – ellipsoids, hyperboloids, paraboloids, ellipses – Berkeley directs harmonic motions in water and arrives at representations of systems that are continuously deformable but regulated by rigorous internal principles. In his numerous works such as *Footlight Parade, Wonder Bar* and *Gold Diggers of 1935,* he uses bodies as vectors, as geodesic polar coordinates, characterised by the same modules and capable of being oriented in several directions.

All of the vectors issue from the origin and it is possible to carry out and represent on the screen perfectly performed operations of addition or subtraction among the vectors.

Fig. 9 *Wonder Bar*, Busby Berkeley, 1934

In every film he seems to pose problems of differential geometry and try to solve them in an infinite number of ways. He continually carries out the decomposition of polygons, almost as though he also wanted to solve on the screen the classic theorems of homeomorphism. With their bodies the female swimmers create circles and triangles with their arms and legs, or numerical series along the various circular platforms. He invented a kind of dimension that is suspended between matter and abstraction.

Let's go back to pure abstraction. Mary Ellen Bute worked with the musician and mathematician Joseph Schillinger starting in 1932 and in 1934 made *Rhythm in Light,* which was paired with *Becky Sharp* by Rouben Mamoulian. Four other short films of hers were shown in the same theatre, for a week, in front of an audience that numbered a total of about one hundred thousand spectators. Bute created her films to delight the little old ladies of New Jersey who periodically ventured to the heart of New York to enjoy a new film.

Fig. 10 *Rithm in light*, Mary Ellen Bute, 1934

Together with Rutherford Boyd, Mary Ellen Bute with John Nemeth made *Parabola* in 1937. Boyd had begun to study the problem with a series of drawings and sculptures entitled *Parabolas descending*, completing this project in 1939 with *Slanted Parabola*. Combining his interest with the possibilities of representing the parabola artistically, at a certain moment Boyd became fascinated by light and the possibility of using it as a paintbrush to create a genuine film. He was helped to carry out this passage by Mary Ellen Bute, who, together with Fischinger, tried to build bridges between experimental cinema and entertainment cinema. The point of fusion, as we said, occurred in the second half of the 1930s. Fischinger's films, like those of Bute, were shown along with full-length feature films and projected in the temples of cinematography, like New York's Radio City Music Hall, the largest movie theatre in the world, becoming gratifyingly successful.

Fig. 11 *Parabola*, Rutherford Boyd and Mary Ellen Bute, 1937

In Fischinger's experiments, the kinematics of the points and figures in space is always described in relation to the musical rhythms and measures.

So too in Mary Ellen Bute, music impresses acceleration on the bodies, determining movement in space.

In *Escape*, her masterpiece, she develops a genuine narrative, following the dynamics of a little triangle of space on the surface of the earth, and thus finite, towards an infinite space. In contemporary with the representation of its movements, the maker tells us about an evolution and dilation of meaning, from a simple geometric figure to a figure of great allegorical and symbolical capacity.

Mary Elle Bute gives the impression, within these processes of signifying that interbreed two different systems of cinematography, of having identified the problem as well as her objective, and to have succeeded in translating one formal language into another, going so far as to create a kine-mathematic algorithm that provides, among the admissible solutions, one that is metaphysical as well.

Translated from the Italian by Kim Williams

Lessons in Mathematics, at the Cinema

Michele Emmer

Mathematics is the art of the possible.
J. & E. Coen, *A Serious man* [1]

Before you retired, what did you teach?
Mathematics.
I hate mathematics.
Xiaolu Guo *, She, A Chinese* [2]

The teaching of mathematics, knowledge of mathematics, knowledge of the sciences are some of the parameters used to evaluate the ability of a country to improve its capacity for production and creativity. The spread of scientific knowledge, and especially of mathematics, is an essential factor. It is precisely those countries that were once defined as 'emerging' and are now among the most industrialised and powerful on earth that have invested in basic training, in excellent scientific training, and in mathematics in particular. Obviously these include China, India and Korea. Hence the importance of teaching, and of mathematics teaching.

Morris Kline wrote:

> Mathematics has determined the direction and content of philosophical thought, has destroyed and rebuilt religious doctrine, has supplied substance to economics and political theories, has fashioned major painting, musical, architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of man and his universe. [...] As an incomparably fine human achievement mathematics offers satisfactions and aesthetic values at least equal to those offered by any other branch of our culture [3].

And what might be the role of the mathematics teacher in the cinema, of the lessons of mathematics on the screen? Here are a few examples [4].

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Emmer M. (Ed.): Imagine Math 2. Between Culture and Mathematics
DOI 10.1007/978-88-470-2889-0_10,

Lessons in mathematics

In 1948 Leonardo Sinisgalli and Virgilio Sabel made a documentary called *La lezione di geometria* ("The Geometry Lesson") [5]. Sinisgalli's interest in cinema and in documentary was profound, as was that for mathematics. It is clear that the topic was the result of his studies in mathematics at the mathematical institute at the University of Rome under the guidance of teachers such as Severi, Fantappiè and Levi Civita.

Sinisgalli won an award for *La lezione di geometria* from the Ufficio Centrale di Cinematografia at the Venice Film Festival for the best Italian short film. The brief but important experiment was realised in collaboration with the director Virgilio Sabel, photographed by Mario Bava, the sound track written by Goffredo Petrassi, and produced by Carlo Ponti. A team like no other.

Given that the film is not easy to find and watch, I transcribed the text of its dialogue in one of my books, taking it from a VHS copy made from a positive copy of the original film owned by the Cineteca Nazionale di Roma [6]. The copy being in terrible condition, in order to transcribe it I had to watch it on a screen and copy it word by word. It is impossible to duplicate the original on DVD. Some images from Sinisgalli's film can be found in a copy of the magazine *Comunità* of 1949.

This is a film in which the appeal of geometric forms and mathematical surfaces drove Sinisgalli to speak about art, mathematics and poetry. The entire text of the documentary is read by a narrator off-screen. In the film appear plaster models of mathematical surfaces that were mainly produced in Germany at the end of the nineteenth century by request of the great German mathematicians Riemann, and above all Klein. The models were intended to serve a aids to help students grasp the new surfaces discovered by the mathematicians of that period. Widely circulated among all the mathematical institutes of the world, they arrived even to Japan, where many years later the Japanese artist Hiroshi Sugimoto photographed them, making even newer models and obtaining effects that are particularly fascinating.

Some years after Sinisgalli's documentary, the French writer Raymond Queneau made another short film, seven minutes long, about mathematics. Queneau himself wrote the screenplay. The film appeared with the name *Arithmétique*, directed by Pierre Kast. It was supposed to be the first of a series, a kind of cinematographic encyclopaedia, but the undertaking was never carried through. The complete title of the film was *Encyclopédie filmée – lettre A (première partie): Arithmétique* [7]. It was a lesson in arithmetic given by Queneau himself. With a very serious expression on his face, the French writer makes a series of extremely banal statements about numbers: additions and subtractions of match sticks, firecrackers, pieces of ice, bicyclers and other things. In order to subtract, he simply throws things out the window. The lesson is interrupted by sudden bursts of music and images. It is one of the finest lessons in surreal mathematics ever seen at the cinema: mathematics between irony and dream, an homage to the magic of mathematics, a rebuke to teachers and professors of mathematics who aren't able to make the discipline they teach interesting and fascinating. It is a ferocious criticism of academia in all its guises.

There was a film by Walt Disney that I watched at school, *Donald in Mathmagic Land* [8], made in the 1950s. Here Donald Duck ventures in the land of Mathmagic (in the footsteps of *Alice in Wonderland*, another Disney film which is explicitly cited in some scenes; both were directed by the same director), where he encounters, among other things, the golden section and the harmony of nature and attempts to apply their laws to his own duck body. Naturally, Donald meets the Pythagoreans and measures the golden proportions of the Parthenon, under the influence (not Donald, but the film's screenplay writers and scientific consultants) of the book by Matila Ghyka of twenty years earlier, and perhaps of the ideas of Le Corbusier at the end of the 1940s. The Donald Duck film includes banal images which could have been avoided, such as when geometric shapes are shown in the playing fields of various sports – the rectangle of soccer, the diamond of baseball – when the workings of various machines are shown with geometric elements, or when mathematics is linked to games. While chess is strongly tied to mathematics, for the other games it is only a question of more or less simple geometric shapes.

Donald in Mathmagic Land, directed by Hamilton Luske, was made in 1959. It was obvious that the film would capture the imagination of many students, myself included. Anyone who was forced to sit through classes in algebra and geometry in Italian middles schools of some years ago can understand how surprised I was to discover that it is possible to watch mathematics with imagination and a sense of wonder, and have fun. Among other films, Luske directed *Cinderella* (1940), *Alice in Wonderland* (1951), *Lady and the Tramp* (1955) and *101 Dalmatians* (1961).

The little that I was able to find out about the film was that the screenplay was written by Milt Banta, Bill Berg and Heinz Haber. It was made at the time that Walt Disney was producing educational segments for his television series. Some of these programs talked about space. Haber was one of the series' consultants, and Disney probably put him to work on the mathematics film as well. Thus the film began as one of the episodes of the television series. But those at Disney who sent me the information think that, since the work turned out so well, Walt Disney decided to use it first of all as a movie for the big screen. His aim was not to make educational films, but rather entertainment films that could stimulate interest in the subject, in this case mathematics.

The film proved so interesting that at the time it was nominated for an Oscar. Even after many years it can still be shown at a cinema and be a great success.

A few years later there appeared another lesson in mathematics for children. In 1965 appeared a film which featured a character who is criticised with the words, 'But he is a mathematician!'. The film starred James Stewart, and the mathematician in question is a boy who is in love with mathematics and with Brigitte Bardot, hence the title, *Dear Brigitte* [9].

In the 1960s the French actress Brigitte Bardot was at the peak of her success, and no one had to ask who the Brigitte of the title was. Bardot played herself. She is the object of the dreams of the boy who is Stewart's son in the film. Stewart played a poet named Robert leaf, an English literature professor in an American university. He is in perennial conflict with the scientists at his university, and thinks that scientific culture, and especially mathematics, is of little importance in education.

One day tragedy strikes at home (obviously this is only a manner of speaking, since the film was a comedy for family viewing). The son, an elementary school student, is a mathematical genius. Or better, he has a great capacity for performing calculations in his head. This is not at all what mathematicians do, since they usually do not work with numbers and their elementary operations, but it is completely normal in common parlance, and thus in cinema, that the words ‘mathematical genius’ are used to indicate a boy who can perform calculations really fast. His teacher, who discovers the boy’s talent by accident, rushes happily to find his parents. On hearing the teacher say that his son is a prodigy in mathematics (of elementary mathematics at best), Jimmy Stewart, becoming pale, puts a hand on the shoulder of the boy’s mother to comfort her. Then, after the teacher leaves, he begins to speak to the boy, begging him to not tell anyone of this talent of his, the source of so many problems, above all of the fact that as he walks down the street people will shout in his direction, ‘He is a mathematician!’, a phrase that Stewart pronounces with disgust, commenting, ‘We would never want something like that to happen!’

The one who comes off looking badly is the mathematics teacher. When she writes the multiplication that the students are to solve on the blackboard, and the boy answers immediately, she rebukes him saying, ‘But you looked in the book!’. The boy responds that he doesn’t have the book, to which the teacher, looking at him with suspicion, replies, ‘Alright, let’s try another one’. She writes the multiplication of two larger numbers on the board, which he again solves instantly. She stands stock still with the chalk in her hand, and says, ‘You mean you did that in your head?’ ‘I don’t know’, he answers. A light American comedy, Stewart is perfect in the role, and the film would have gone down quickly without his presence on the screen.

Anyway, when the boy is taken for a check-up, to the doctor’s question about what he is interested in, he replies without hesitation, ‘Brigitte Bardot’. And when finally at the end of the film his dream comes true and he sees Miss Bardot in person, he is rendered speechless, paralysed and blushing in front of his idol. So much for numbers!

> Teacher: I hope that you like mathematics, and that we can work well together even as we try continually to solve problems. … Yes, we are getting to know each other … if there is anyone who has a question, let him ask it without fear.
>
> Student: Well, I speak in the name of an interdisciplinary group that has been formed to study the relationships between science, art and literature. You see, we want to ask a question about the magic square shown in the etching by Albrecht Dürer, *Melencholia I*.
>
> Teacher: Yes, *Melencholia*, I remember it.
>
> Student: It appears that in the Renaissance they were convinced that the magic square of order four could drive away feelings like melancholy and sadness.
>
> Teacher: Ah, yes! Interesting. And so …
>
> Student: You seen, the professor told us that Dürer had set at the bottom the date of the etching, which was in fact composed in 1514. Do you follow me?
>
> Teacher: Yes.
>
> Student: So, you see, we want to know how to always come up with 34 by adding every line and column and diagonal.

> Teacher: Always with 34...
>
> Student: Yes, if you can show us how to do it.
>
> Teacher: Well! This seems to me to be a little outside what we are supposed to do, and it might not interest everyone
>
> Students: Yes, yes, we are interested! Yes, explain it, explain it!
>
> Teacher: You are interested ... ahem. Since it's the first day of school, wouldn't it be better to settle in a little ...
>
> Student: But look, professor, you don't have to
>
> Teacher: Yes, sure ... ahem! (He stands up and goes to the blackboard, remains unsure what to do, and is saved by the bell that brings the class to a close).

The dialogue between the new mathematics teacher named Michele and the students is taken from the 1983 film *Bianca* by Nanni Moretti [10]. Moretti plays a teacher grappling with the difficulty of how to treat mathematics in a way that is less pedantic and boring, and in particular of addressing the possible relationships between mathematics and art. He is a teacher who suffers from the lack of order and harmony that predominates in man's world, and wants everything to work as it does in mathematics, precisely, rationally. In consequence, he even goes so far as to murder his friends so as to reconstruct the order that he feels is missing. He is the mathematician who aspires to a rational world in which everything takes place according to determined rules, a supreme harmony whose laws cannot not be mathematical. In short, he is the mathematician as an idealist who can even arrive at murder to keep from seeing his own dream of the harmony of the world disturbed.

The lead character of the film *Stand and Deliver* [11], directed by Ramon Menendez, is a mathematician who believes blindly in the educational power of mathematics. He decides to leave a steady job as a computer engineer to teach in school. His name is Jaime Escalante, the name of the person on whom the film, a true story, is based (the real Escalante died on 31 March 2010). The story is set in a Hispanic neighbourhood.

Escalante moves to Los Angeles and begins to teach in a high school infamous for its low results. The environment is that of teens, who live in near poverty and moral decadence, some of whom are genuine hooligans. Escalante aspires to try to redeem them both humanly and culturally. The school where he is to teach computer sciences doesn't have a single computer, its teachers are decidedly not up to the task assigned to them, and it is headed by a principal who is honest but doesn't have the will to change the situation.

Escalante is supposed to teach mathematics to a class whose undisputed leader is a kind of hooligan named Angel. He immediately tries to come into contact with the students by adapting himself to their ways of speaking and behaving, trying little by little to draw them into his world to begin to teach them, starting from their scant knowledge of the subject. He approaches the first elements of algebra and analysis using games. He then begins to dig into the hearts of the rebel students, asking himself how he can get past the street bully part of their behaviour. After many difficulties, thanks to his force of will, he acquires their friendship and trust. At that point he sets objectives that are even more ambitious, believing that anything can be obtained if you want it badly enough. He pushes them to study so much that they complete the program and arrive to the final exams well prepared. He also risks his life in this great striving for moral and cultural redemptions: the energy and effort that he puts into achieving his goal cause him to have a heart attack.

The examination they have to pass is national, and thus pits the various schools against one another. The results of the test are so good that it appears that the teacher has been able to manipulate them. They are thus held to be invalid, to the great disappointment of Escalante and the students. An inspector is sent to examine the case, played by Andy Garcia in one of his first appearances on the screen. Escalante protests, asking to see proof that the tests were manipulated. The test is repeated, and the results are even more brilliant than before, bringing the school where Escalante teaches to national attention. Eighteen students in the class are given scholarships to continue their studies at university, when usually no recognition is ever given to any students of that school. In fact, the method used by the real Escalante has become a paradigm for teaching mathematics in difficult classes, such as those in schools in Hispanic communities. The film was awarded the 'Independent Spirit Award' for best directing. It was also nominated for a Golden Globe award, and Edward James Olmos, who played Escalante and is well-known for his role as Martin Castillo in the television series *Miami Vice*, was nominated for an Oscar as best actor.

In the Chinese film *Not One Less* by the director Zhang Yimou [12], a young girl must substitute the teacher in the school of a small remote village. One day one of the schoolchildren runs away to the city and the girl must find the money to pay for the bus to go get him and bring him back,. And thus it is that, instead of the usual exercises in algebra that normally appear when an elementary mathematics lesson is needed in a film, the young girl (that is, the screenwriters writing for her) invents a lesson in applied mathematics: 'A ticket for the city costs 20 yuan and a half. How much money do we need to go and come back, paying the ticket for Zhang, the boy who has run away. Now we have 9 yuan. Who knows the answer?'. She asks Dong Dong, and this account is written on the blackboard: 20.5 x 3 = 61.5. 'Very good. And so we still need 52 yuan and a half. Yesterday the manager of the factory told me that for moving 10,000 pieces [the work done by the children] he will pay us 40 yuan. Given that we need 52 yuan and a half, who many pieces do we have to move?' She asks Jiao Jie, who goes to the blackboard. He makes a mistake and the whole class corrects him because he has put one too many zeroes. Then all of the children begin to recite, 'For 10,000 pieces they give us 40 yuan. Multiply 52.5 by 10,000 and divide by 40'. On the blackboard appears 13,000. '13,000 pieces? That is a lot of work for us.' The teacher proposes that they make an exact calculation of how much time will be needed for each of them, and they conclude that they will need two days of work apiece.

This is an emotional and involving way to teach mathematics in a classroom in the middle of nowhere, obviously without any teaching equipment. Even the classroom is an improvised school.

When the mayor becomes aware of the children's enthusiasm he says happily that the teacher is good because she knows how to teach mathematics. She will go far. Mathematics teaching is reincarnated as a mission, a mission that can succeed in emancipating, can succeed in involving, that makes people more active and better. In short, the fascination of mathematics strikes again.

Another lesson in a completely different kind of film, a Hollywood comedy, not particularly sophisticated. But now the lead character is a man.

> Mathematician: Notice the elegance of the proof. It's beautiful! lt reminds me of a quote from Socrates: "lf measure and symmetry are absent from any composition, the ruin awaits both the ingredients and the composition." Measure and symmetry are beauty and virtue the world over.
>
> Female student 1: He's cute. Do you think he's straight?
>
> Female student 2: Yeah, he's too boring to be gay.
>
> Mathematician: I'm releasing you a half hour early today. l have ... an appointment. I'm giving a lecture on my new book tonight. You are all welcome.
>
> (None of the students stay)

This is the dialogue of the opening scene of the film *The Mirror Has Two Faces* [13], directed by Barbra Streisand, with Jeff Bridges, George Segal and Lauren Bacall, who was nominated for an Oscar for best supporting actress. Bridges is a mathematics professor at Columbia University in New York, awkward and incapable of communicating. Only a few students attend his classes, and those who do are bored to death. The mathematician learns how to teach mathematics from Streisand, a crazy literature professor who can captivate hundreds of students. He is a mathematician who is neither bad nor good, a nitwit whose mathematical abilities (if he has any, they remain in the shadows) have to be intuited. Bridges loves classical music, can't remember a thing, and can't do anything practical: he is kind of mentally handicapped; in short, the nutty mathematician. We don't even understand if he knows how to do mathematics, given that of mathematics, except for a few bland examples, does not appear. While the mathematician played by Bridges is innocuous and gains our sympathy, we know that mathematics teachers, as we have already heard, are often hated by their students, and thus also by their parents. Many times we have to admit that they are right.

The mathematics teacher can also be the victim of a murder, if he comes to be seen as a nightmare for the murderer's children. We can thus understand why a wild (and fatter) Kathleen Turner murders, running over him several times with a car, her son's mathematics teacher, guilty of having said that the boy was obsessed with horror movies and of suggesting that there were probably problems in the family. The mother played by Turner, who shows up at the meeting with the teacher carrying a cake, like every good American mother should, simply eliminates physically the problems that involve her son without taking lessons from anyone, least of all from a mathematics teacher. The film is *Serial Mom* [14]. Little by little during the course of the meeting with the teacher, her expression becomes increasingly angry, finally becoming vicious. She waits for the teacher outside the school, in the car park, and as soon as the right moment arrives, she runs over him with the car. He survives, so she runs him over again and again until she kills him. The film was based on the 1996 book by Martyn Bedford, *Acts of Revision* [15]. Its chapter 4, entitled 'Mathematics' begins with these words:

> **mathematics**. 1a. science of numbers, their operations, interrelations and combinations. 1b. science of space configurations and their structure, measurements, etc. 2. mathematics or mathematical operations involved in a particular problem field of study, etc.
>
> Mr. Teja was a Paki. . . .
>
> When I told Mr. Teja that I didn't see the point of maths, he explained the difference between pure and applied mathematics. Pure mathematics is the abstract science of space, number and quantity, he said, a theoretical science. Applied mathematics is theory put into practical use, especially the application of the general principles to solve definite problems.
>
> Mathematics is mental gymnastics.
>
> When I told him I didn't understand maths he said, What is there to understand, it is only numbers, it all comes down to numbers.
>
> There is no knowledge without calculation. . . . Numbers, he said, are integral to understanding the world we inhabit. . . . Those who know most about existence are not the poets, or clerics or philosophers, but those who can compute. The mathematicians.
>
> Numbers are creation."

In the book Professor Teja is not murdered, because by the time the serial killer finds his house, years after he had left school, he is already dead. How many teachers like Teja there are!

Kathleen Turner co-stars in another film, *The Virgin Suicides* di Sofia Coppola [16], who became famous for the Oscar she won for the screenplay of the film *Lost in Translation* (a film that was highly overrated). *The Virgin Suicides* is based on the story of five girls, all sisters, who killed themselves one after the other in the 1970s. Turner plays their mother, a bigot of low intelligence, incapable of understanding. The father, played by James Woods, is a mathematics teacher who has all the worst defects: dry, repetitive, incapable of communicating with anyone, let along his own daughters, whose disturbances and problems he doesn't understand. He is even incapable of teaching mathematics to his students. In his lessons he talks about unions and intersections of sets while drawing on the blackboard with both hands separately. None of the students follow him; all are bored. In the hallway he talks to the plants: 'Have we photosynthesized our breakfast today?'.

To be sure not everyone was so lucky as to have a mathematics teacher like that.

The overview presented here by means of only a few of the many examples that could be given, has shown that mathematics teachers are portrayed in many different ways: arid, incapable of understanding, unable to help their students. Above all it is the mathematics teachers in elementary, middle and high schools who are seen like his. As we know, recollections of school tend to leave indelible traces, and evidently screenwriters and directors have had their share of dramatic experiences at school. Instead, university professors are geniuses, or at least parodies of genius, with the tics and the distractions of mathematical geniuses are imagined to have. The mathematician who lives outside the real world, in a world all of his own. Who knows what he does the whole blessed day? On the other hand we are not in the least interested in trying to understand what he does. Hence the weightlessness, the fragile connection to reality, the incapacity to dialogue with others. Genius and its parody.

Translated from the Italian by Kim Williams

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Mathematics, Love, and Tattoos[1]

Edward Frenkel

The lights were dimmed... After a few long seconds of silence the movie theater went dark. Then the giant screen lit up, and black letters appeared on the white background:

Red Fave Productions

in association with Sycomore Films

with support of

Fondation Sciences Mathématiques de Paris

present

Rites of Love and Math

The 400-strong capacity crowd was watching intently. I'd seen it countless times in the editing studio, on my computer, on TV... But watching it for the first time on a panoramic screen was a special moment which brought up memories from the year before.

I was in Paris as the recipient of the first *Chaire d'Excellence* awarded by Fondation Sciences Mathématiques de Paris, invited to spend a year in Paris doing research and lecturing about it.

Paris is one of the world's centers of mathematics, but also a capital of cinema. Being there, I felt inspired to make a movie about math. In popular films, mathematicians are usually portrayed as weirdos and social misfits on the verge of mental illness, reinforcing the stereotype of mathematics as a boring and irrelevant subject,

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[1] Parts of this article are borrowed from my book *Love and Math*, which will be published by Basic Books in the Fall of 2013. For more information about the film, visit http://ritesofloveandmath.com.

Emmer M. (Ed.): Imagine Math 2. Between Culture and Mathematics
DOI 10.1007/978-88-470-2889-0_11,

far removed from reality. Would young people want a career in math or science after watching these movies? I thought something had to be done to confront this stereotype.

My friend, mathematician Pierre Schapira, introduced me to a young talented film director, Reine Graves. A former fashion model, she had previously directed several original, bold short films (one of which won the Pasolini Prize at the Festival of Censored Films in Paris). At a lunch meeting arranged by Pierre, she and I hit it off right away. I suggested we work together on a film about math, and she liked the idea. Months later, asked about this, she said that she felt mathematics was one of the last remaining areas where there was genuine passion.

As we started to brainstorm what our film would be about, I showed Reine a couple of photographs I had made, in which I painted tattoos of mathematical formulas on human bodies. We decided we would try to make a film involving the tattoo of a formula.

Tattoo, as an art form, originated in Japan. I had visited Japan a dozen times, was fascinated by the Japanese culture. We turned to the Japanese cinema for inspiration; in particular, to a film by the great Japanese writer Yukio Mishima *Rite of Love and Death*, based on his short story *Yûkoku* (or *Patriotism*). Mishima himself directed and starred in it.

This film made a profound impression on me. It was as though I was possessed by a powerful force.

Rite of Love and Death is black-and-white; it unfolds on the austere stylized stage of the Japanese Noh Theater. No dialogue, with music from Wagner's opera *Tristan and Isolde* playing in the background. There are two characters: a young officer of the Imperial Guard, Lieutenant Takeyama, and his wife, Reiko. The officer's friends stage an unsuccessful *coup d'etat* (here the film refers to actual events of February 1936, which Mishima thought had a dramatic effect on Japanese history). The Lieutenant is given the order to execute the perpetrators of the *coup*, which he cannot do – they are close friends. But neither can he disobey the order of the Emperor. The only way out is ritual suicide, *seppuku* (or *harakiri*). When he tells Reiko, she says she will follow him to the better world. After they make love, the Lieutenant commits *seppuku* (this is shown in graphic detail). Then Reiko kills herself by driving a knife in her throat... At the end of the film we see them both lying dead in final embrace on the beautifully groomed pebbles of a traditional Zen garden.

The 29 minutes of film touched me deeply. I could sense the vigor and clarity of Mishima's vision. His presentation was forceful, raw, unapologetic. You may disagree with his ideas (and in fact his vision of the intimate link between love and death does not appeal to me), but you have to respect the author for being so strong and uncompromising.

Mishima's film went against the usual conventions of cinema: it was silent, with written text between the "chapters" of the movie to explain what's going to happen next. It was theatrical; scenes carefully staged, with little movement. But I was captivated by the undercurrent of emotion underneath. (I did not know yet the details of Mishima's own death, its eerie resemblance to what happened in his film – and this was probably for the best.)

Perhaps, the film resonated with me so much in part because Reine and I were also trying to create an unconventional film, to talk about mathematics the way no one had talked about it before. I felt that Mishima had created the aesthetic framework and language that we were looking for.

I recount what happened next in my forthcoming book *Love and Math*. I called Reine and told her that we should make a film just like Mishima's. "But what will our film be about?" she asked. Suddenly, words started coming out of my mouth. Everything was crystal clear.

"A mathematician creates a formula of love", I said, "but then discovers the flip side of the formula: it can be used for Evil as well as for Good. He realizes he has to hide the formula to protect it from falling into the wrong hands. And he decides to tattoo it on the body of the woman he loves".

We decided to call our film *Rites of Love and Math*. We envisioned it as an allegory, showing that a mathematical formula can be beautiful like a poem, a painting, or a piece of music. The idea was to appeal not to the cerebral, but to the intuitive, visceral. Let the viewers *feel* rather than *understand* it first. We thought that this would make mathematics more human, inspire viewer's curiosity about it.

We also wanted to show the passion involved in mathematical research. People tend to think of mathematicians as sterile, cold. But the truth is that our work is full of passion and emotion. And the formulas you discover really do get *under your skin* – that was the intended meaning of the tattooing in the film.

In the film, the Mathematician discovers the "formula of love." Of course, this is a metaphor: We are always trying to reach for complete understanding, ultimate clarity, want to know everything. In the real world, we have to settle for partial knowledge and understanding. But what if someone were able to find the ultimate Truth; what if it could be expressed by a mathematical formula? This would be the "formula of love".

Such a formula, being so powerful, must also have a flip side: it could also be used for evil. This is a reference to the dangers of modern science. Think of a group of theoretical physicists trying to understand the structure of the atom. What they thought was pure scientific research inadvertently led them to the discovery of atomic energy. It brought us a lot of good, but also destruction and death. Likewise, a mathematical formula discovered as part of our quest for knowledge could potentially lead to disastrous consequences.

So our protagonist, the Mathematician, hides his formula by tattooing it on the body of the woman he loves. It's his gift of love, the product of his creation, passion, imagination. But who is she? In the framework of the mythical world we envisioned, she is the incarnation of Mathematics, Truth itself (hence her name Mariko, "truth" in Japanese; and that's why the word *ISTINA*, "truth" in Russian, is calligraphed on the painting hanging on the wall). The Mathematician's love for her is meant to represent his love for Mathematics and Truth, for which he sacrifices himself. But she survives and carries his formula, as she would their child. The Truth is eternal.

As Reine and I were getting more excited about our project, so did people around us. Soon, a crew of about 30 people was working on the film. Raphaël Fernandez, a talented musician, composed original music. We ordered a kimono and a painting. An artist was working on the decor. The film was taking life of its own.

The shooting took three days, in July of 2009. Those were some of the most exciting, and exhausting, days of my life. I wore several hats: co-director (with Reine), producer, actor... All of this was new to me, and I was learning on the job. It was an amazing journey, a wonderful collaboration with Reine and other filmmakers and artists helping us to fulfill our dream.

The central scene of the movie is the making of the tattoo. We shot it on the last day. Since I never had a tattoo, I had to learn about the process. These days tattoos are made with a machine, which would be an anachronism on the stage of Noh theater. In the past, tattoos were engraved with a bamboo stick – a longer, more painful process. I've been told it's still possible to find tattoo parlors in Japan which use this old technique. This is how we presented it in the film.

Oriane Giraud, our special effects artist, asked me a few days before the shooting to give her the formula that would be tattooed, so she could create the blueprint. Which formula should play the role of "formula of love"? A big question! It had to be sufficiently complicated (it's a formula of love, after all), aesthetically pleasing. We wanted to convey that a mathematical formula could be beautiful in content as well as form. And I wanted it to be *my* formula.

Doing "casting" for the formula of love, I stumbled on this:

$$\int_{\mathbb{P}^1} \omega F(qz,\overline{qz}) = \sum_{m,\overline{m}=0}^{\infty} \int_{|z|<\varepsilon^{-1}} \omega_{z\overline{z}}\, z^m \overline{z}^{\overline{m}} dz d\overline{z} \cdot \frac{q^m \overline{q}^{\overline{m}}}{m!\overline{m}!} \partial_z^m \partial_{\overline{z}}^{\overline{m}} F \Big|_{z=0}$$

$$+ q\overline{q} \sum_{m,\overline{m}=0}^{\infty} \frac{q^m \overline{q}^{\overline{m}}}{m!\overline{m}!} \partial_w^m \partial_{\overline{w}}^{\overline{m}} \omega_{w\overline{w}} \Big|_{w=0} \cdot \int_{|w|<q^{-1}\varepsilon^{-1}} F\, w^m \overline{w}^{\overline{m}} dw d\overline{w}.$$

It appears as formula (5.7) in a 100-page paper [1], *Instantons Beyond Topological Theory I*, which I wrote with my two good friends, Andrey Losev and Nikita Nekrasov, in 2006.[2]

When we show the film, people always ask: What does this formula mean? Which is exactly what we were hoping for. If we had made a film in which I wrote this formula on a blackboard and tried to explain its meaning, how many people would care for it? But seeing it in the form of a tattoo elicited a totally different reaction. It really got under everyone's skin.

So *what does it mean*? This work was the first installment in a series of papers we wrote about a new approach to quantum field theories with "instantons" – special solutions of the theory minimizing the "action" . Quantum field theory is a mathematical formalism for describing the behavior and interaction of elementary particles. Though it has been successful in accurately predicting a wide range of phenomena, there are many fundamental issues that are still poorly understood. Let me recall that all atoms consists of protons, neutrons, and electrons. Protons and neutrons, in turn, consist of smaller particles, called quarks. And those quarks are confined there – they cannot be separated. A proper theoretical explanation of this phenomenon is still lacking.

In the conventional (so-called, perturbative) approach to quantum field theory the starting point is the so-called "free theory" describing idealized non-interacting particles. Then we "turn on" the interaction between them. The problem is that in this approach the contribution of each instanton appears to be negligible, even though altogether they may "conspire" to create a powerful effect. Many physicists believe that the difficulties of the conventional formalism with taking the instantons into account could be the reason we don't have a satisfactory explanation of the confinement of quarks and other similar effects.

In our paper, we proposed a new approach, in which the starting point is not a free theory, but an idealized *interacting* theory in which the instantons are present from the beginning. The advantage of our theory is that the main quantities – the so-called "correlations functions" – are expressed by *finite-dimensional* integrals (in contrast to the conventional formalism, in which the integrals are infinite-dimensional and hence poorly defined). Therefore, our theory is in principle completely solvable.

The above formula expresses the identity between two ways to compute correlation functions in our theory. We discovered it when we were working on this project three years earlier, in April of 2006, also in Paris.

Our world is four-dimensional (if we include both space and time dimensions), but four-dimensional theories are very complex. To simplify matters, we looked at the analogous two-dimensional, and then one-dimensional, models. That is to say, there is only time dimension. Such models, commonly referred to as "quantum mechanics," describe a single particle moving in a particular space (which could be of any dimension). Despite the simplifications, these one-dimensional models possess the salient features of the more realistic, four-dimensional, models. That's why it is useful to study them.

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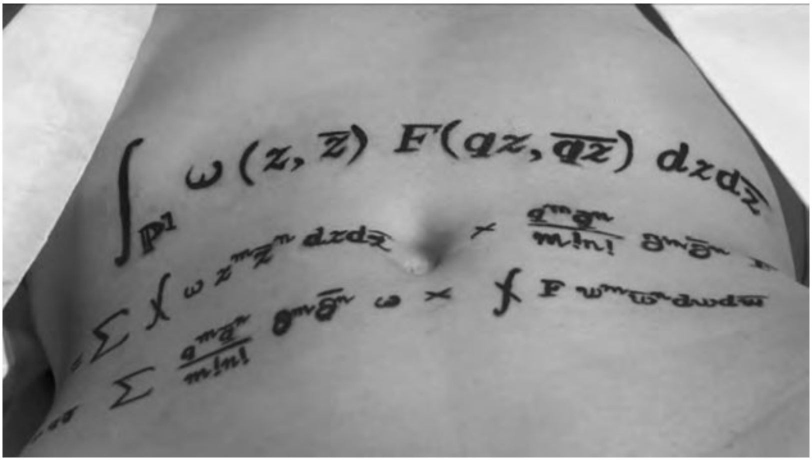
[2] In the version published in *Journal de l'Institut de Mathématiques de Jussieu*, there is a footnote explaining that it played the role of "formula of love" in *Rites of Love and Math*.

We considered the theory in which a particle was moving on the sphere, also known in mathematics as the “complex projective line,” denoted $\mathbb{P}^1$ ($\mathbb{P}$ stands for “projective,” and 1 for one-dimensional, as a complex manifold). You can see this notation under the integral sign on the left hand side of the formula.

We wanted to compute the simplest correlation function in this theory, involving two “observables,” denoted by F and ω in our formula. On the one hand, the answer is given by an integral over the sphere; this is the left hand side of the formula. On the other hand, in our new theory we were getting a different answer: a sum over the “intermediate states,” appearing on the right hand side. This answer is surprising, and so is the equality between the two expressions. If our new approach were correct, the two sides would have to be equal to each other. And indeed they are.

In other words, our formula says that two ways of computing the correlation functions – the old and the new – give the same answer. Little did we know at the time we discovered it that it would soon be slated to play the role of formula of love.

Oriane liked the formula, but said it was too involved for a tattoo. Could I simplify it? I slighly changed the notation, and here’s how it appears in our film:



The work on the tattoo scene took us many hours. It was psychologically and physically draining both for me and for Kayshonne Insixieng May, the actress playing Mariko. We finished shooting close to midnight. It was an emotional moment for all of us on the set, after everything we had been through together.

Two months of post-production followed, at *Sycomore Films*, a French production company, with our multi-talented editor Thomas Bertay and visual effects magician Pierre Borde. And finally, the film finished, it was time to organize the premiere. Folks from Fondation Sciences Mathématiques de Paris, which generously supported our film, agreed to sponsor the premiere. We picked a wonderful venue: *Max Linder Panorama*. An old movie theater with a huge screen and modern video and sound systems, it is one of best theaters in Paris.

The crowd that gathered at the theater on April 14, 2010 was diverse: mathematicians, artists, filmmakers... Most of our crew was there, including Reine, Kayshonne, and our Director of Photography *extraordinaire* Daniel Barrau. On the big screen the picture was sharp and crisp, the colors vivid (the decision to use the most expensive Sony camera had paid off).

The first articles about the film started to appear. *Le Monde* called *Rites of Love and Math* "a stunning short film" that "offers an unusual romantic vision of mathematicians". And the *New Scientist* wrote:

> It is beautiful to look at... If Frenkel's goal was to bring more people to maths, he can congratulate himself on a job well done. The formula of love, which is actually a simplified version of an equation he published in a 2006 paper on quantum field theory entitled "Instantons beyond topological theory I", will probably soon have been seen – if not understood – by a far larger audience than it would otherwise ever have reached.

Since then, the film has been shown at international film festivals in France, Spain, and in Berkeley, California. There have been more showings in Paris, Kyoto, Madrid, Santa Barbara, Bilbao, Venice... The screenings and the ensuing publicity gave me the opportunity to meet many people and hear different opinions. At first, this came as culture shock. Some of my mathematical works can be fully understood only by a small number of people; sometimes, no more than a dozen in the whole world at first. This was a film intended for a wide audience: hundreds were being exposed to it. And of course, they all interpreted it in their own ways.

In mathematics there is only one truth, and only one path to reach that truth. My mathematical work is perceived and interpreted in essentially the same way by everybody who reads it. Not so in cinema and in the arts in general. First, there isn't a single truth, and second, there are so many different paths to express the truth. And the viewer is always part of an artistic project: at the end of the day, it's all in the eye of the beholder. I have no influence over their perception. Coming to terms with this was a challenge to me, but gradually I came to embrace it. We get enriched when others share their views and insights. What matters the most for a work of art is that it touches people in some way, does not leave them indifferent.

When we show the film, people invariably ask: "Do you know the formula of love?" My response: "Every formula we create is a formula of love". Doing mathematics is a creative pursuit that requires passion, just like painting, music, and poetry. In order to discover something new and eternal about the world, you have to be in love with what you do.

Acknowledgements I thank Michele Emmer for the invitation to show our film at the annual Symposium "Matematica e Cultura" in Venice and to contribute to the Symposium Proceedings. I am grateful to Thomas Farber for his comments on a draft of this article.

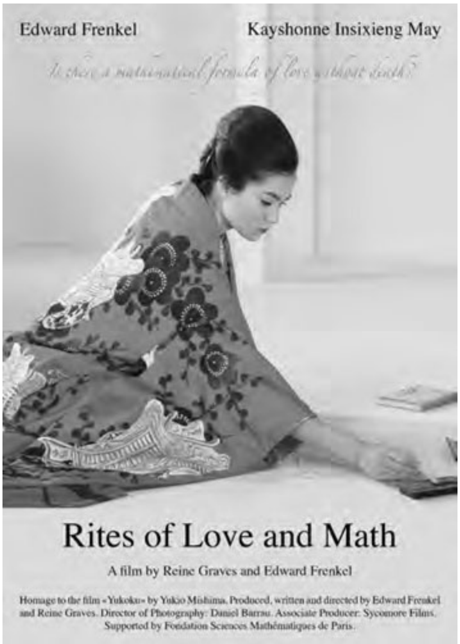


Fig. 1 Film poster

Film Credits

Rites of Love and Math, 2010. 26 minutes, in color.
Written, produced, and directed by Edward Frenkel and Reine Graves.
Associate Producer *Sycomore Films*.
Cinematography by Daniel Barrau.
Music by Richard Wagner and Raphaël Fernandez.
With Edward Frenkel and Kayshonne Insixieng May.
More information: http://ritesofloveandmath.com

Rite of Love and Death (*Yûkoku*), 1965. 29 minutes, in black-and-white.
Written, produced, and directed by Yukio Mishima.
Associate producer Haraoki Fujii.
Associate director Masaki Domoto.
Cinematography by Kimio Watanabe.
Music by Richard Wagner.
With Yukio Mishima and Yushiko Tsuruoka.

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Arithmétique

Giovanni Munari

In an old-fashioned Normandy country home, Guillaume, a badly behaved young boy, refuses to do his Math homework until it surprisingly takes shape and becomes an old man supported by a chorus of numbers. *Arithmétique* is a short film utilising traditional animation with digital support, inspired by *L'enfant et les sortilèges*, a masterpiece of music history.

Arithmétique takes place in 1925, whereas the Opera's was composed during World War I, when the Opéra Paris director Jacques Rouché asked Sidonie-Gabrielle Colette to provide the text for a fairy ballet. Colette wrote the story in only eight days, under the title *Divertissements pour ma fille*, and after refusing several composers she finally became enthused by the prospect of working with Maurice Ravel. The opera was sent to him in 1916 while he was still serving in the war; however, the mailed script was lost. In 1917, Ravel finally received a copy and agreed to complete the score, humorously replying to Collette, "I would like to compose this, but I have no daughter." He eventually began working on the opera in 1920 and it was completed in 1924. Colette, who had believed that the work would never be complete, expressed her extreme pleasure, and accepted the title of *L'enfant et les Sortilèges*.

The first performance took place on March 21, 1925 in Monte Carlo, conducted by Victor de Sabata with ballet sequences choreographed by George Balanchine. Ravel said of the premiere: "Our work requires an extraordinary production: the roles are numerous, and the phantasmagoria is constant." For such a small opera, a truly sumptuous production is necessary and this is probably the main reason for its rarity in theatre programs nowadays. Quite simply, without a great sum of money *L'enfant* was impossible to bring on stage. But there is one medium that is able to bring this wonderful opera to life: animation. In 1939, one year after Ravel's death, his brother happened to watch Disney's first animated feature *Snow White and the Seven Dwarfs* première. "This is the way *L'enfant* should be presented" he said. And he was right. A few years later, Lotte Reiniger, a German filmmaker, tried to make an animated version of it using her famous silhouette technique, but her attempt didn't make it past the preproduction stage due to copyright issues.

Luckily, Dalila and I found that the composer's rights had recently expired so we only had to fund the recording and, thanks to Centro Sperientale di Cine-

Giovanni Munari
Director, Italy.

Emmer M. (Ed.): Imagine Math 2. Between Culture and Mathematics
DOI 10.1007/978-88-470-2889-0_12,

matografia, we began working on our short film. After just one listen of an aria called *Arithmétique*, a polka in a 2/4 “presto andantissimo” we knew we had found the perfect score for our film and it slowly became the backbone and soundtrack to the story. In our animation, Guillame is bored by his homework and, through his lack of concentration and carelessness, he drops some ink on to his notebook. From the ink, the ancient, black Daemon of Math escapes, scaring the child and pulling him in to his world. Mathematics exists within a world “apart” in *Arithmétique*; A dimension where time and space don’t exist and everything is pure and incorruptible; a world that must appear mysterious to a child’s eyes. Guillame’s mystery turns to fear when things get bigger (literally) and numbers and cubes surround him, scaring him until he finally wakes up, relieved. Unfortunately, his relief is short-lived and his room is soon invaded by the black hands of the Daemon, grabbing him and pulling him back to the nightmare; a nightmare that ends only after he decides to close the Math book.

In creating *Arithmétique*, dedicated to our nieces and nephews and to our math professors, we wanted to draw two different and almost opposite worlds but both visually inspired by the Twenties; the “real” world inspired by the golden age of childrens’ books (Arthur Rackham, Sir John Tenniel, Antonio Rubino) containing a mixture of engravings and watercolours, while the more perfect “math” world was inspired by the abstractions of Viking Eggeling, El Lissitzky and Russian constructivism, Leger’s *ballet mécanique* which was popular then, while the labyrinth of cubes is a subtle homage to the Berlin-based monument to the Holocaust memory. The film starts with a clock not only to stress the anxiety and the sense of boredom, but also because it was an obsession of Ravel and features as one of the characters in the original opera (along with the cat and the Chinese teapot).

Our aim was to mix together very different languages in order to create a very solid whole: to embrace the medium of cinema and a passion for classical music, to combine theatre and animation, music and pictures, description and abstraction. We were influenced by a huge number of sources from the worlds of animation and art history including Gianini and Luzzati, Lotte Reininger, Hans Rickter, Edward Gorey and Anke Feuchtenberger.

And now for some numbers. The film consists of more than six thousand drawings, four hands, two brains, eight months of production, more than twenty backgrounds and three characters. The puppet of the Daemon is composed of almost 40 pieces.

Mathematics is a daemon that can carry you somewhere new and absolute - you don’t have to fear it.


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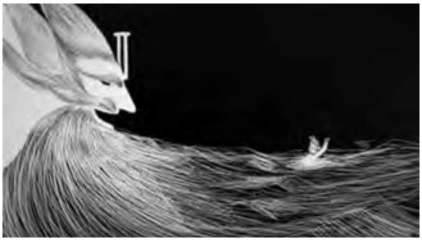

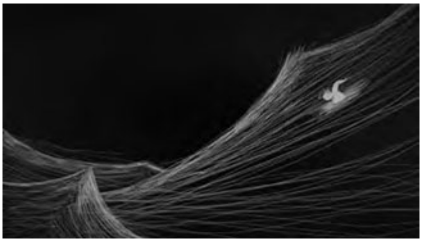


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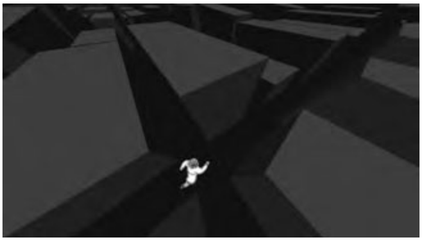

Mathematics and Art

L'art du Trait est l'attrait de l'Art[1]

Sophie Skaf

Every artwork starts with the simple action of drawing a line. Straight, curved or mixed, the line has always been, for me, a source of fascination, because the magic of the line reflects an imprint that has layers of hidden meanings beyond the simple act of drawing.

The Art of the line lies in the drafting of technical drawings, of plans as well as in the art of drawing the line itself. Moreover, the Art of the line could be defined as a *descriptive geometry* which has as its basic structure, the module. At the essence of all drawings, the elementary geometric forms appear. Only a play of these - the square, the elongated square, the circle, and the triangle- could create the most intriguing figures. But most importantly, the fundamentals of these forms rest on the golden number, the divine proportions. The universe, spherical in reality, is flat in drawings, represented by a simple line. This flatness, long discussed by astrophysicists, describes the geometry of the Universe. 'To say that the Universe is flat signifies that its spatial curvature is null or very small. This indicates that at the scale of the Universe, light travels in a straight line', explained Francois – Xavier Désert, a scientist from the astrophysics laboratory of the science of the Universe Observatory of Grenoble, in France [1].

In my work, however, the art of the line cannot be separated from the element of 'water', which will be the connecting thread in my presentation.

It is as important to learn to appreciate the value of water, since before the line, water existed as a source of balance and harmony, as a vital element that cannot be separated from the act of Creation, in all its aspects. Water has a primordial existence and role that even, beyond the mystery of conception and birth, our balance is found in water. Since birth, a child is embraced with love, observed with attention, awaiting every new gesture that is a result of his ongoing growth; his first smile, his first word, and most of all his first steps. Yet, what is still a mystery to most is that a child would only be able to have his first steps the instant the water in his ears have reached the threshold of balance. In our internal ear, three semi circular canals help us stand in balance, according to the X, Y, and Z axes. These three axes intersect at a right angle: one parallel to the ground, one parallel to the sides of the head, one

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Beyrouth (Liban).

[1] This paper was originally written in French language.

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parallel to the forehead or the face. These three canals are responsible for depicting the movements of the head in all three dimensions. This is done through a liquid in the canals, and sensory cilia linked to receptor cells that transmit the information to the brain. A divine mystery!

Not only that, but it would have never been possible to build cathedrals without water! In an era where the laser did not exist, the 'water level' was the only way to mark the horizontal level, the line. Used since Roman times, this simple method, far from being revolutionary, was widely efficient and is still used nowadays.

Terrains are scarcely ever horizontal, which makes the positioning of the scaffolding quite a long and tedious job. Let's see how the water level works:

The main tools for this are a transparent plastic tube open on both ends, and water. The plastic tube is to be held giving a U shape form. Water is to be poured with a funnel from one of the openings. What is peculiar to notice is that the water will fill both branches of the U tube, and will stabilize at the same level, thus giving a straight horizontal line. Water will keep its stable level, no matter how the tube is deformed. This method is used at the beginning of a construction on an empty lot, using posts and a wire, it is as well used in the further steps of the construction, however then, using the walls instead of the posts. This procedure requires two individuals. Assuming that we are in a room with four walls, the first person marks a height of 1m on one wall. This will be the height reference. The tube is to be put with the water level at this 1m reference height, the other end of the tube put against another wall by the second person. It is only after some time that the water will stabilize indicating the 1m height on this wall. This is to be repeated on different locations on the walls, resulting in drawing several points. Connecting these points will give a straight line that will be the reference line at 1m used in construction. And so, using water, builders are able to get a straight horizontal line without which, they could not construct.

Moreover, in almost all religions, water, symbol of life, has an additional dimension, a divine and sacred one. In the Christian religion, its importance is manifested in the baptism.

The symbolism of water conveys a serene state that almost reduces life to a simple straight line. This fascinating simplicity is attained with a straightforward, noble gesture, in a society that is relentlessly transforming everything into a spectacle. This spectacle is in reality a whirlwind of lines, when a simple 'nothing' would be enough to celebrate the emancipation of life.

Birth, Equilibrium. Water as a construction tool

How is 'liquid architecture' able to create emotional creations? A covert story or an intentional architectural aim? Here is an illustrated example that translates the necessity of water in the perfection of the line, this line that becomes a synonym of the horizon, of horizontality.

I have taken interest in construction work since 2006. It was for me a needed step to be able to better translate the architectural design of my own projects. Thus, in one of my architectural creations, an important experimentation could not have been achieved without the intervention of water that was the only tool capable of defining the line; hence, I find it quite interesting telling the story. The experiment consisted in assembling a *water mirror* in stainless steel (Fig.1). After the execution of the rectangle - 16m long, 1.4m wide, 1m deep - and despite the usage of laser, progressive and slow oeuvre, state of the art, modified and completed, with a know-how based on the principle of the *descriptive geometry*, the 'science of the line' and the art of working with stainless steel, it was crucial to fill the *water mirror*, to cancel the weight effect, leaving the whole to stabilize over several weeks before obtaining a perfect straight line. In a constant search for perfection, it was a must to keep the water mirror filled the whole time, to insure a constant overflow, to observe, to depict imperfections, to examine the meticulousness of the straightness of the line, to file the inaccuracies that disrupted the regularity of the overflow. For any construction to stand correctly, the rules in question ought to be applied. And so, the construction of the *water mirror* and the perfecting of its structure relied on the deduction and application of this specific rigorous method.

On the other hand, the line of the horizon, hardly seen with the naked eye, is in reality curved due to the laws of attraction. It follows the curvature of the earth; circumference of the earth which is equal to the radius – hardly measurable.

No one drowns in water because of a lack of talent, but because of failing to learn a proper method of swimming. It is the same when it comes to innovating: this individual and collective competence does not rely on talent, contrarily to an idea sadly spread. Innovating relies on a method.

Like breathing is not a one time process, the articulation between divergence and convergence needs to be repeated several times to bear fruit. We distinguish in particular three vital instances, which have each its own successive importance.

This approach of three instances allows presenting the intermediate stages of a project, guaranteeing along the way that all the actors accompany its progression in each stage. They also can validate separately and one after the other the problematic presented at the first phase, the expertise explored in the second and the action plan resulting from the third.

The Art of the line - recipe or procedure?

The elementary geometric traces, the *geometric ornament*, is one of the primitive manifestations of art. It began by examining the oeuvre as the sum of the traces where the lines of the geometric network of the body organically interlaced one with the other converge. A whole totally geometric which was only made possible by the usage systematically of preliminary drawings.

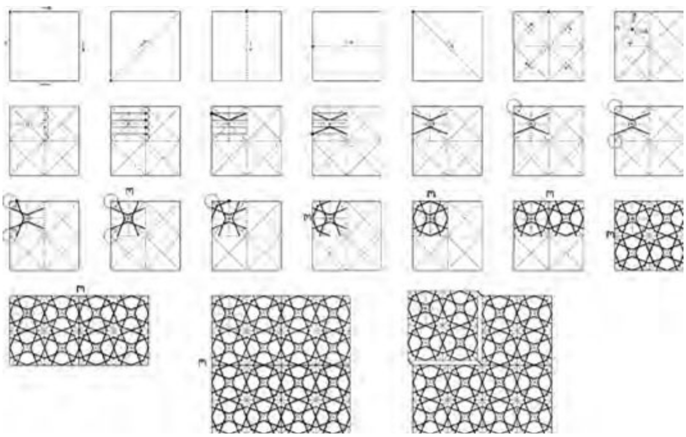
This constructional drawing presents the trace of a *geometric ornament* based on a colored cement tile motif from the book entitled *20x20.Beyrouth.Paris.tunis.Barcelone* [2][2] based on the principles of the horizontal and vertical projections and the sections and mirroring of diverse parts of a construction. The application of these *geometric traces* is highly advanced with the Orientals and the Egyptians since long back (Fig. 2).

Water at the service of the line, is also a theme we can observe in the fabrication of colored cement tiles, where it plays a primordial role. Water is an indispensable element without which the motifs could not be obtained. Actually, the viscosity of the clay paste is a main factor in the removal of the mould successfully, preventing the mixing of the colors.

Looking at the fabrication process of the cement tiles, a process that hasn't changed since the XIX century, allows one to value the importance of water without which the delicate, subtle and precise line could not be obtained, an element that is of great importance in motifs that obey a strict geometry, similar to the cement tiles that I have had the pleasure to identify and inventory in Beirut, Paris and Tunis.

In these three cities, it is not rare to find geometrical motifs, based on straight lines. These motifs are the result of the assemblage of four to sixteen tiles, obtained with the same mould, however, chosen to be colored similarly or in different ways.

[2] The book was an award winner at the "Dubai International Print Award", 2011 More than 300 motifs, *geometrically ornamented* explored in four cities observed at ground level, over a journey where the decorated cement tiles still thrive, their moment of glory having been the years between 1910 and 1940. They are a true "work of art", which would interest equally well lovers of art books as persons who appreciate decorative arts, architecture, design and arts and crafts, or who quite simply have a nostalgic curiosity for such works. These motifs are presented in their historical context and reproduced in drawing with their construction imprint.



The complexity of the motif widely depends on the coloring of its different components. Noticeably, the more varied the colors used in the same motifs, the more complex the drawing of the motif becomes. In opposition, the less varied the colors used, the simpler the drawing of the motif becomes, to a point where the tile can be simple monochromatic if all components are colored in a uniform manner.

And so, it is possible to obtain with the same mould, countless possibilities of motifs. Additionally, the variety of motifs obtained, as we will see later, not only depends on the coloring of the components, but also on the way the motifs are juxtaposed and laid, in the same manner or in different directions.

To obtain such a wide selection of motifs, coming from the same mould, renders this technique rich and economical. This strategy which can be qualified as French, has been personalized and used in several ways that have turned these tiles into an element that characterizes the different artistic eras that have animated the end of the XIX century and the first half of the XX century: stylized flowers of the Art Nouveau, geometric composition of the Art Deco, optical illusions of the Kinetic Art, Celtic interlaces, Moorish arabesque.

Moreover, while the production of the cement tiles reflects an interpretation of the crafts and the craftsman; its motifs were often a mixture of different movements. This gave birth to surfaces that narrate stories, where the tiles play the role of mediator between various influences, and where the ornamental language, when it doesn’t borrow from existing figures, submits to the imagination of the craftsman. To be able to translate his creations, the artisan bases himself on the elementary geometrical drawings or on the figures of the linear drawings like the point, the line, the axis of symmetry, the centre of gravity, as seen in the previous demonstration.

Conversely, the tiles we find in Barcelona hide a different story. In these, the straight line disappears, leaving place to the curved line. The motifs are organic with nature as their protagonist. Furthermore, they often need more than just one mould for their production.

In one of his masterpieces, the Casa Mila, of 1906, Gaudi chose for the floor cladding to stray from the squared tile and make way to the hexagonal tile, to waive the polychromic in favor of the monochromic, to move from the smooth drawing to the drawing in relief, from the mat finishing to the polished one.

However, these differences between the Barcelonan tiles and the French tiles can only be clarified if one was to understand the approach behind them:

The French method was mainly an economical one, following a strategy of distribution that aimed at spreading its know-how outside its own borders, making it reachable to whoever was keen. The goal was to make it available to the greater number, its designs, its motifs, its moulds. Taking advantage of France's international relations with other countries, this technique travelled across borders, leaving France sometimes as a cement tile, or simply as know-how to be produced in the countries of destination, according to European or Oriental motifs imported from Italy.

On the other hand, the Barcelonian approach was far from a mass production. Designing a motif became a task only entrusted to well-known architects that had them executed by the famous Escofet. It was no longer the product of the imagination of an anonymous craftsman, but, of an artist and industrial tandem that took over the production of the designed cement tiles, giving a new course to the development of this material.

Going back to the French cement tiles, it is very peculiar to highlight the method of laying them, and the diversities they offer. Generally, the motif in question appears after having laid four cement tiles of 20x20. However, depending on the direction in which the same motif is placed, the whole design could be changed entirely, going from a completely static one to a completely dynamic one.

Having continuously been revisited and readapted through the years, the cement tiles have always been subject to change. Yet, no matter what the inspiration is, the art of the line will always be strongly connected to "l'attrait de l'Art" (Fig. 3) [3].

This evolution becomes more explicit and justified when recalling that the elements linking the line to the Art as well in the *descriptive geometry* as in the geometric ornaments are profound and numerous. At the base of every work of art, a structural study evolves and develops, shaping, creating, making. Both cannot be dissociated. And it would not be an exageration to state to state that structural instances could not be concretized without 'water'.

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Mathematics and Morphology

Morphogenesis and Dynamical Systems
A View Instantiated by a Performative Design Approach

Sara Franceschelli

1 Introduction

Following the Scottish morphologist Wentworth D'Arcy Thompson, the study of living morphogenesis should take into account the effects of mechanical forces acting on the growing processes.

> Cell and tissue, shell and bone, leaf and flower, are so many portions of matter, and it is in obedience to the laws of physics that their particles have been moved, moulded and conformed. [...] Their problems of form are in the first instance mathematical problems, their problems of growth are essentially physical problems, and the morphologist is, *ipso facto*, a student of physical science [1, pp. 7-8].

One of the well known thesis that Thompson defends in his *On Growth and Form* is that, in order to understand the genesis of natural forms, natural selection is not enough, whereas the roles played by mechanics and physics are underestimated. Thompson does not advocate an abandon of Darwinism, however he indicates that structural transformations, determined by forces, have to be taken into account in order to explain the genesis of these forms.

This book has inspired generations of researchers, from the most various fields, not only in the most evident field of theoretical biology, but also in art, architecture, and design (on these developments one can report to [2] and [3]). One of the common aspects characterizing the heirs of D'Arcy Thompson concerns their interest in biomimicry - an interest which is rather based on the idea of grasping and reproducing the generative principles of the natural growing processes, than on the direct imitation of natural forms.

Both in the field of theoretical biology and in the field of design (including architecture) the efforts of the researchers on morphogenesis produced a renewal in the way of considering this dynamic aspect, driven by two main motors of change: on one side, the development of dynamical systems theory and of the mathemati-

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cal treatment of non-linear systems, on the other, the use of computer as a tool for calculus and for digital morphogenesis.

In this chapter, we present a teaching and research approach developed under our direction in a school of applied arts, *Ecole nationale supérieure des Arts Décoratifs* (EnsAD, Paris), that proposes the performative design of morphogenetic devices.

The first paragraph concerns questions on generative principles of morphogenesis emerging from the history and more contemporary practice of theoretical biology; the second paragraph considers contemporary practices in digital morphogenesis and performative design. In the third paragraph we illustrate how some spatial and dynamical devices, conceived in the framework of our teaching and research experience, offer a reactivation of some of the questions emerging from the epistemological consideration of the subtle relations between morphogenesis and dynamical systems.

2 Looking for generative principles

As far as the domain of theoretical biology itself, the theoreticians that tried to follow D'Arcy Thompson perspective in the study of morphogenesis - as for example Conrad Hal Waddington, Alan Turing, René Thom, Brian Goodwin, Peter Saunders – understood, everyone in a characteristic manner, that the renewal, initially internal to classical physics, of the study of the behaviour of non-linear systems, could and should be considered as pertinent for the study of the processes of formation of patterns and shapes in the living world. One of the first students, even if not the only one, that explored the mathematical properties of non-linear systems to understand pattern formation has been Alan Turing. A reader of D'Arcy Thompson, he renewed the study of morphogenesis by its contribution of 1952: *The Chemical Basis of Morphogenesis* [4]. In this paper Turing mathematically shows how the instabilities in a process of reaction-diffusion of two interacting chemical substances, expressed by a differential system, can produce spatial patterns because of symmetry breaking. He inserts himself in the structural perspective of his mentor, but he proposes a new theory, based on the properties of his model of equations of reaction-diffusion, to understand the dynamics of structural transformations that generate forms. As most of Alan Turing's works, the one on morphogenesis is very original: despite the fact that Turing has been one of the founders of informatics, and that the use of metaphors coming from computer science was already practiced at his time[1], there is no notion of genetic program in Turing's works. His theory is based on the study of the symmetry breakings of a system, *i.e.*its bifurcations, leading the system from a morphological state to a different one. This can be considered the minimal generative principle needed to understand the production of different forms on the basis of purely physico-chemical laws.

[1] See for example Erwin Schrödinger's *What is life?*, of 1944, in which the physicst described genes as a codescript.

In this perspective - also shared, for example, by Brian Goodwin [5] and Peter Saunders [4,7] - criticizing the dominant neo-darwinian paradigm, natural selection can choose among existing forms, generated on the basis of physico-chemical principles. If a natural form exists, it has certainly been selected. However, prior to this, it has to have been growing as a generic natural form.

In this structural picture, the choice and regulation among these different possibilities is driven by the genes, but the genes themselves are not seen as the only determinants of the deploying forms.

If we switch from the problem of understanding natural forms to the problem of designing artificial forms, Turing model can perhaps be seen as a prototype of parametric design: genes serve to define the parameters in the equations describing the potentially bifurcating systems and can thus determine whether or not bifurcations occur.

3 Digital morphogenesis and performativity

Notions and paradigmatic ways of reasoning from contemporary biology nourish the contemporary research work on morphogenetic design, also informed, on another side, by the advances in computational tools.

One of the leaders of contemporary research in morphogenetic design, Achim Menges, uses the metaphor of the passage genotype-phenotype to qualify the type of work done thanks to computational morphogenesis:

> In computational morphogenesis the genotypic definition unfolds a performative phenotypic material system [8, pp. 55-56]

What do the expressions "performative design/architecture", or "performance oriented design/architecture" mean? If considered from the point of view of the designed products - at different scales: from the object, to the building, to the urban plan – the adjectives "performative" and "performance-based" point to the performances of these products with respect to the environment or its users. The architect, or the designer - it is generally claimed in this perspective - has to offer appropriated responses to location and use. How designers' intentions which respects ecological, ergonomic, cultural and social aspects, can be realized?

The main idea emerging form literature on performance-oriented design [9], or performance-based design [10] or performative architecture [11] is that design processes have to be guided by the desired performance and that building performance has to be engaged with form generation at a quite early stage in design process.

Achim Menges stresses that this morphogenetic research is not interested only in a system shape, but also in its behaviour - and on the relations between shapes and behaviors. In this spirit, the designer, or the architect, is not seen as the conceiver of a particular form that tries to imitate nature, leaving to the engineers the work of calculating the conditions of stability and robustness of the designed object. The idea is to grasp the natural principles that allow for a certain form generation, and to let them do, in a sense, the work of the designer.

Achim Menges stresses that:

> In computational morphogenesis form is not defined through a sequence of drawing or modelling procedures but generated through algorithmic, rule-based process [8, p. 51].

He qualifies the performative design processes as follows:

> Material and morphologic characteristics are derived through iterative feed-back loops, which continually process the material system interaction with statistics, thermodynamics, light, and so on [8, p.48].

In order to stress the novelty introduced by morphogenetic performative design, Menges contrasts it with CAD (Computer Assisted Design), claiming that CAD has not really changed the way architects design, since it transfers in the digital realm drawing and modeling techniques previously executed manually, employing "the computer as a helpful extension of established design processes" [12, pp. 015003-015004].

If we fully agree that in CAD case the designer is in the classical position of form-making, whereas morphogenetic design concerns form-finding in the next paragraph, we are going to ask whether how one can experience morphogenetic and performative design research, without using the digital tool.

Is it possible to explore this kind of design working directly in the three-dimensional space, by analogical experimentation?

4 Spatial dynamical devices as forms producers

In this chapter we suggest a positive answer, based on a teaching and research program ("dynlan - dynamic landscapes") on performative design of spatial and dynamical devices in a school of applied art (EnsAD), that we run in parallel of our epistemological work on morphogenesis and dynamical systems. The idea was also to develop a way to communicate the interest of these questions on morphogenetic conception to several groups of design and art students working with us. We needed to develop a discourse around these questions, both as pertinent as possible, and not expressed in technical terms. With this idea in mind, we largely used the visual composite metaphor coming from the development of embryology and theoretical biology: the epigenetic landscape, introduced by Conrad Hal Waddington.

Several experiences of non-digital morphogenetic design are presented in [13, 14]. The idea was to start from an epistemological analysis of morphogenesis, in the structuralist – albeit dynamical perspective – from contemporary theoretical biology, briefly evoked in the first paragraph of this chapter. Focusing on the paradigmatic change carried by the generative aspects that have been indicated, *i.e.* in structural properties that can parametrically produce bifurcating behaviors, we decided to conceive parametric structures that can underlies different morphological states.

We share the perspective defended by Achim Menges about digital morphogenesis research: the point is form finding, not form making. However, by the design of dynamical devices that can parametrically underpin different forms, we want to provide a field of possible forms directly in the three-dimensional world, without passing through the phase of writing programs in order to launch a digital production of virtual morphologies.

In our first experiences, the generative principles, as well as the performative *desiderata*, have been extracted from a structural interpretation of the figure of the epigenetic landscape: images of epigenetic landscapes can be found in [15, pp. 29, 36]. We briefly remind that the composite metaphor of the epigenetic landscape is visually represented by an undulated surface on which lies a ball ready to roll down, accompanied by an underlying part, defined by a network of pegs interconnected by guy-ropes, some of which directly connected with the undulated surface. Conrad Hall Waddington uses this metaphor as a mental image to think at his theory of morphogenesis in embryology, in particular at the problem of cellular differentiation, but also at the canalization of development and at the necessity of considering together embryological development and complex genetic interactions. René Thom's interpretation of these images in terms of catastrophes theory has been also inspiring ([16]; for an excellent introduction to catastrophes theory, by a specialist of morphogenetic thinking, Jean Petitot, one can see [17]). Even if Waddington and Thom did not agree on a mathematical interpretation of the figure of landscape in terms of catastrophes theory, as the analysis of their correspondence shows [18], their divergences open questions on the role and the nature of *equilibria*, on the role of structural stability in morphogenesis, and on the specificity of time in the study of living systems. By fixing an agenda for the behavior of the *equilibria* and for the robustness properties one would like to observe in the dynamics of the designed devices, we also defined our particular acceptation of performative design: here it means for us to be able to conceive parametric structures underpinning particular dynamic states of the systems. From a practical point of view, we tried to conceive some dynamical devices reproducing, at least partially, some of the narratives defined by the series of these dynamic states.

The device "Paysages sensibles et dynamiques" has been realized by a group of first year students of EnsAD in 2008, in the framework of a 4 weeks workshop (co-directed with the EnsAD colleague and architect, specialized on tensed membranes, Yves Mahieu), which has been inspired by the notion of deployment arising from the analysis of the Waddington-Thom correspondence ([18], see also [13]). Here we focused on the notion of singularity, represented by dynamic *equilibria*, local *minima* and *maxima* involved in a dynamic underpinning the deployment of a complex surface (Figg.1, 2). We worked on the calibration of the parameters of the dynamics to obtain a periodic deployment of the surface itself.

The device, *Paysage magnétique* (Fig. 3) has been realized by two 2^{nd} year students, Ferdinand Dervieux and Maia d'Abboville in the framework of the workshop "Dynamics of a landscape", run during 2 weeks in February 2010, in the framework of a interdisciplinary teaching experience in collaboration with the research program "dynlan – dynamic landscapes". As in the first example, we started with a

Fig. 1 "Paysages sensibles et dynamiques". The device during its unfolding (© EnsAD)

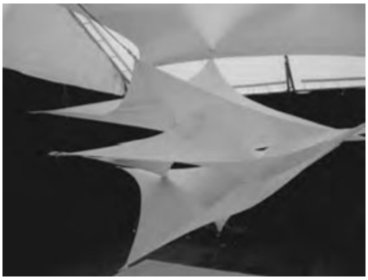

Fig. 2 "Paysages sensibles et dynamiques". Another moment of the unfolding (© EnsAD)

workshop presenting the image of the epigenetic landscape and, more in general, the figure of landscape in theoretical biology. The idea for the students, this time, was to render dynamic every element of the image of the epigenetic landscape, ball included. The device has been built around the dynamic properties of a magnetic fluid parametrized both by mechanical and by magnetic actions. The behavior of the conceived device, in response to the user stimuli, raised the following questions: Are there recurrent morphologies in function of external stresses? Can we recognize recurrent histories? And try and obtain them again? Under the variation of the action of the control parameters, one can observe different spatio-temporal "scenarios". The retained definition of scenario is the one coming from the study of transition to chaos in dynamical systems: a scenario is defined by the series of bifurcation a system undergoes under the variation of its parameters.

Since the last decades of the 19^{th} century, thanks to the seminal work of Henri Poincaré, it is well known that simple deterministic systems, as defined, for exam-

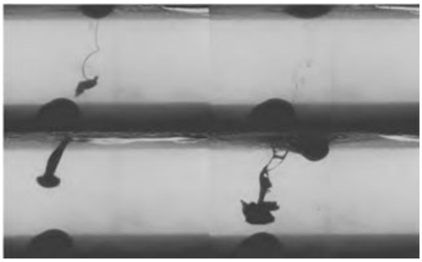

Fig. 3 "Paysage magnétique" (EnsAD students: Maya D'Abboville, Ferdinand Dervieux). Different morphologies in function of external actions on the device define different scenarios (© ensadlab dynlan – dynamic landscapes, EnsAD)

ple, by three interacting planets, can produce unpredictable behaviors, and that the qualitative theory of differential equations – equally founded by Poincaré himself – offers a perspective to topologically characterize them. Breaking with the laplacian vision, predictive power of equations – even if deterministic - is not guaranteed. One can say that, in order to make some predictions on the dynamics of this kind of systems, the question of establishing a good representation of the phenomena thanks to the equations is not enough: one needs to know the history of the system, its behavior under the effect of the variation of some control parameters. What does the system perform spontaneously in time, and under the effect of its parameters variations? Is it possible to recognize generic series of bifurcations defining generic scenarios? Thanks to our experimentation, this set of questions, associated to a dynamical systems perspective, have been re-actualized and explored by observing the dynamic behaviors of the *Paysage magnétique*, in response to the user actions.

The third device discussed here, "Green shots" (Fig. 4) is an inflatable device, constructed in textile, enabling a production of dynamic shapes which can evolve under the effect of several control parameters (e.g. air flows, retracting threads and strings, and many others to invent...). The project invites the user to interact with its dynamic process, in which strict repeatability is not guaranteed (Fig. 5)! Further developments could include different actions and interfaces such as interactive dance and musical compositions.

Fig. 4 "Green shots" is a collaborative project by Anne Ferrer (artist, professor Fashion Design, EnsAD) with Sara Franceschelli (dynlan - dynamic landscapes), in the framework of the "Ateliers de morphologie EHESS- EnsAD - Morphogenèse et dynamiques urbaines" held on the 2-3 avril 2012 at EnsAD. The "s" in the title points to the variety of the possible morphologies that the device underpins (© Anne Ferrer)

Fig. 5 A particular morphology produced by the actions on strings by a user inside the device (©Anne Ferrer)

It has been conceived in the framework of the *Ateliers de morphologie EHESS-EnsAD - Morphogenèse et dynamiques urbaines* held on the 2-3 avril 2012 at EnsAD. We organised at that occasion a workshop of performative design, following the conferences on morphogenesis of six invited speakers[2]. Here the idea was of conceiving devices grasping generic dynamical features issues from the *atelier* discussions. The enabled dynamics are meant as instances of generic morphogenetical behaviours.

"Green shots" is a collaborative project conceived in a workshop run by the artist (and EnsAD professor) Anne Ferrer - who has a practice in inflatable installations (www.anneferrer.com) - and myself, for the program "dynlan – dynamic landscapes", by a group of undergraduate students and researchers. We wanted to realize a material instance of a bifurcating path, which leaded us towards the choice of the global form. In the spirit of the "dynlan" program, we wanted not to conceive a single, particular object, but a family of virtual objects that the interaction with users could actualize. We realized this intention thanks to the possibility, for a user, to enter inside the inflatable structure and to act (to pull) on several strings fixed on the tissue from the interior (Fig. 6). If the device has been conceived on the basis of its performative properties (pragmatics), inspired by an evident analogy with the underlying part of an epigenetic landscape thought as a dynamical device, the interaction with the users - with their histories and their questions - is intended to eventually endow it with multiple semantics. The device "Green shots" has been exposed at Château de la Roche Guyon on June, 21-22th, 2012, in occasion of *Ateliers de morphologie EHESS- EnsAD - Morphogenèse et dynamiques urbaines*, together with other morphological objects carried by the participants.

Fig. 6 Detail of the device: the possibility to enter inside (©Anne Ferrer)

[2] Invited speakers: Giuseppe Longo (CNRS & ENS), Nadine Peyriéras (CNRS), Bénédicte Grosjean (ENSAP Lille), Alberto Magnaghi (Università di Firenze), Luc Gwiazdzinski (Université de Grenoble), Mark Burry (RMIT Melbourne).

Discussion

As in the digital morphogenesis, we dealt with the design of dynamical processes but, in our case, despite the differences in material realization (tensed membranes, magnetic fluids. . .) of the various devices of this experience, the designed processes unfold directly in the three-dimensional, material space, rather than on a computer screen.

The morphogenetic narratives we considered - shared by images of landscape (thought as processes) and material devices - can be supposed to be interesting on the basis of their genericity. In our design experiences we worked thus on pragmatics, with the idea in mind to eventually come back to semantics, carrying out questions and insights from the observed dynamic behaviors. This second phase (back to semantics), in our research work should be realized thanks to the discussion with experts in different fields. It is not the object of this chapter, in which we focus on morphodynamics narratives and generic questions, compatible with the behavior of the designed material devices. We are thus conscious that our acceptation of performative design, because of this abstraction from semantics, is a useful simplification we adopted in our experimentation.

The experiences discussed here question the possibility of an experimental epistemology of complex and dynamical systems from a practice of spatial and dynamic design. Circularly, the idea is to enlighten the morphogenetic properties of morphogenetic design from an epistemological perspective. We hope that this circularity is, at least partially, of the order of the virtuous circle, at least from two points of view:

- If our epistemological work informs and propels the design work with the students and young researchers working with us, some of the developments of the latter turned out to go further the initial input, and in some cases new questions arose, able to inform, in turn, our epistemological work.
- The interaction between the two aspects (epistemology and design) allowed us to present our ideas to a much larger and diversified set of interlocutors than our pairs in traditionally academic research, and *via* a different *medium* with respect the traditional written paper.

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Empirical Evidence that the World Is Not a Computer

James W. McAllister

1 Introduction

In this chapter, I assess the hypothesis that the world is a computing device. According to this hypothesis, programs or algorithms running on this device generate the numerical values of physical parameters, including the empirical data sets that we collect in the course of observations and measurements in science. I shall argue that this hypothesis entails a testable prediction, namely that empirical data sets form algorithmically compressible strings. The discovery of empirical data sets that are algorithmically incompressible would therefore refute the hypothesis. I shall argue that we already have good evidence that some empirical data sets are algorithmically incompressible, from which I conclude that we can rule out that the world is a computing device.

2 The Hypothesis that the World Is a Computer

The hypothesis that the world is a computing device has at least two intellectual antecedents. The first is the long-standing tradition in philosophy and in science that conceives the world as a machine, such as a clockwork mechanism [1]. The second antecedent is the view that the function and value of laws of nature are to give a concise summary of the results of observations and measurements. Ernst Mach was one of the founders of this view: expressions such as Snell's law of refraction were, he said, "rules for the reconstruction of great numbers of facts" [2, p. 582]. Writing at the beginning of the computer age, Ray J. Solomonoff similarly conceived laws of nature as algorithms capable of generating empirical data: "The laws of science that have been discovered can be viewed as summaries of large amounts of empirical data about the universe. In the present context, each such law can be transformed

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into a method of compactly coding the empirical data that gave rise to the law." [3, pp. 16–17]. The view that laws of nature constituted a compression of empirical data sets motivated Solomonoff's contribution to algorithmic information theory, to which we shall return below.

Building in part on these approaches, several thinkers since the 1960s have suggested that the world is a computing device of some kind. They have conjectured that programs or algorithms running on this device create the numerical values of all physical parameters, including those that make up our perceptual experience and those that we gather in observations and measurements in science. In consequence, the evolution of the universe is a digital information process analogous to that which takes place in a computing device.

There are two main conjectures about the nature of the computing device. Some writers, like Jürgen Schmidhuber, have suggested that the world is a universal Turing machine [4]. Laws of nature of the traditional form could play the role of the programs running on such a machine, since laws are mathematical equations that generate the values of dependent variables from those of independent variables. Laws of nature would then amount to "the software of the universe", in the phrase of Mauro Dorato [5, pp. 36–44]. Other advocates of the hypothesis that the world is a computing device, however, have had in mind a more specific device: a cellular automaton. Konrad Zuse and Edward Fredkin have endorsed the view that the best explanation of all phenomena in the universe is that they are produced by cellular automata [6, 7]. In this case, laws of nature of the traditional form would be less fundamental descriptions of nature: the deep description of the world would require principles of a new form, as Stephen Wolfram has proposed [8]. These principles would be approximately as concise as the laws of nature with which we are familiar. Regardless of the precise form of the device postulated, thus, the hypothesis that the world is a computing device is an attempt to derive the great complexity of observable physical phenomena from highly simple rules [9, 10].

Yet other writers have viewed the hypothesis that the world is a computing device not as our best estimate of the structure of the universe, but as a means to investigate whether our perceptual experience is a reliable guide to that structure. If our perceptual experience were the product of a computer program, perhaps created by a deceiving agent, would we be able to detect that fact? The "brain in a vat" thought experiment of Hilary Putnam established the scenario, although Putnam himself believed that semantic externalism precluded systematic reference failure [11, pp. 1–21]. Nick Bostrom has argued to the contrary that our perceptual experience is indistinguishable from a computer simulation and furthermore that it is highly probable that we inhabit such a simulation [12].

3 Empirical Tests of the Hypothesis

The hypothesis that the world is a computing device has a testable consequence: if the hypothesis were true, the empirical data sets that are the results of our observations and measurements would form algorithmically highly compressible strings. Let us elucidate this consequence with the help of algorithmic information theory.

Any infinite string of digits can be generated by many different algorithms provided as input to a universal Turing machine. Let us consider the shortest algorithm capable of generating a given string. In the case of some strings, this shortest algorithm is identical to the string itself: this is the case if a string shows no regularities or patterns that can be exploited to form a shorter representation of the string, so the string constitutes its own shortest description. Such strings are said to have maximal algorithmic complexity and to be algorithmically incompressible. Other strings have high redundancy, which permits an algorithm much shorter than the string itself to encode the information contained in the string. Such strings have lower algorithmic complexity and are algorithmically compressible [13].

Some implications of the hypothesis that the universe is a computing device now become clearer. This hypothesis suggests that an algorithm of approximately the length of a law of nature generates the numerical values of physical parameters. Scientists ascertain the values of physical parameters in observations and measurements, and collect them in empirical data sets. Thus, the hypothesis that the universe is a computing device entails that empirical data sets constitute algorithmically highly compressible strings. We are therefore capable of testing the hypothesis by checking whether empirical data sets are algorithmically compressible. A positive finding would be compatible with the hypothesis, although it would not establish its truth conclusively; the finding that empirical data sets are algorithmically incompressible, by contrast, would refute the hypothesis that the world is a computing device.

Most strings are incompressible: this follows from the fact that strings of a given length are much more numerous than strings of shorter lengths. However, it is not possible to prove that a given string is incompressible. In informal terms, the existence of a short program that proved that a certain string was incompressible would entail that the string was compressible: hence, no such program can be constructed [13, pp. 3–4]. For this reason, we can demand at most fallible evidence that a certain string is incompressible. There are two ways of gathering such evidence. First, we can make resourceful and persistent attempts to produce a shorter description of a string. If these attempts fail, this failure is evidence suggesting that the string is incompressible. Second, we can examine whether a string passes statistical tests of randomness.

Are the strings constituted by empirical data sets algorithmically compressible? The available evidence, while not conclusive, indicates that they are not. Let us begin with attempts to produce shorter descriptions of empirical data sets. If science is indeed devoted to summarizing empirical data, as Mach, Solomonoff, and others have suggested, we should expect the compression of empirical data sets to be a routine achievement in the history of science. In fact, scientists have had little success in this regard. The most telling cases are those in which scientists have single-mindedly attempted to fit a curve to a data set by all available mathematical techniques, with no regard for any *a priori* commitments, such as to an explanatory model or to a certain form of equation. Historians of science tell us that several past scientists developed laws of nature by means of such pure data-fitting exercises: this is how Johannes Kepler formulated his first law of planetary motions of 1609, Johann Balmer his law of the wavelengths of the emission lines of the hydrogen spectrum of 1885, and Max Planck his law of blackbody radiation of 1900, for example. These laws are the likeliest candidates for compressions of empirical data sets. In fact, of course, these laws do not precisely generate either the empirical data available at the time of their formulation or data gathered subsequently. For example, Balmer's law deviated from the data available to him by one part in 40,000 [14]. While these laws are impressive achievements on other grounds, thus, they do not amount to an algorithmic compression of a data set.

Further reflection on these laws of nature suggests that their most important function is not to compress the empirical data in their entirety, but to model an additive component of the data that we regard as significant or as informative about the structure of the world. This amounts to capturing a pattern that is exhibited in a data set together with a noise term. We shall return to this insight below.

Scientists are engaged in some further research programmes that test whether sets of measurement readings are algorithmically compressible. The programme known as search for extraterrestrial intelligence (SETI) is based on the expectation that an extraterrestrial civilization would emit signals in electromagnetic radiation and that such signals would be distinguishable from naturally produced radiation by their regularities [15]. SETI thus consists in monitoring electromagnetic radiation from astronomical sources and testing whether it is algorithmically compressible. So far, SETI has reported no success. SETI is limited in various respects, admittedly: the detectors monitor only a narrow wavelength band, for example, and the software is capable of detecting regularities of certain kinds only. Nonetheless, SETI is a research programme of precisely the sort required to test the hypothesis that empirical data sets are algorithmically compressible, and the negative result to date is evidence—however partial—that they are not.

Some further opportunities for testing whether empirical data sets are algorithmically compressible are available. If a short algorithm produced the values of physical parameters, we might under certain circumstances detect defects or distortions in empirical data caused by the discrete rendering of continuous phenomena. For example, our visual experience of the world might show temporal aliasing, such as the wagon-wheel effect, in which a spoked wheel appears to rotate at an anomalous rate [16, p. 118]. The occurrence of aliasing effects in our visual experience would tend to confirm that a computing device had produced the values of physical parameters, whereas the absence of these effects constitutes tentative evidence that this is not so. There is no indication that effects of this kind occur in our visual experience of the world.

4 Statistical Randomness Tests

The second way of gathering evidence that a certain string is incompressible is to examine whether it passes statistical tests of randomness. Strings of maximal algorithmic complexity pass all such tests; however, some strings with low algorithmic complexity also pass them. For example, although several simple algorithms can generate the decimal expansion of π, statistical surveys of initial sequences of this expansion have found no departures from randomness [13, p. 49]. Let us consider whether typical empirical data sets satisfy two such tests.

One randomness test is whether every digit in the counting base has an equal chance of occupying each position in the string. In a string that passes this test, the frequency of each digit tends to the same value as the length of the string grows to infinity. The outcome of a fair coin-tossing trial, expressed as a binary sequence, passes this test; it seems initially, by contrast, that most empirical data sets collected in science do not. The first significant digit in the measured value of a physical parameter is more likely to be small than large: the probability that it is 1 is approximately 0.301, for example, whereas the probability that it is 9 is approximately 0.046. A similar, though less marked, probability distribution holds for subsequent digits. This regularity is dubbed Benford's law. However, it is not clear that this probability distribution is a consequence of the structure of the world: the most widely endorsed explanation of Benford's law is that it is an artefact of measurement systems, arising from the requirement that the statistical distribution of the magnitudes of physical parameters be independent of the units of measurement in which these magnitudes are expressed [17, pp. 253–264].

A second randomness test is whether there are no correlations among the digits of a string. A string passes this test if the probability that a certain digit occupies a certain position in the string is independent of the preceding digits, so the string exhibits no memory effects. The outcome of a fair coin-tossing trial satisfies this test too. At first glance it might appear that empirical data sets do not: it seems intuitively obvious that the outcomes of two subsequent measurements of a physical parameter are correlated with one another. However, the strength of the correlation between successive measurements of a variable depends on the zero point of the scale used. Many important physical parameters, including energy, have an arbitrary zero point. The strength of the correlation between successive measurements of such a parameter is therefore also arbitrary, as we may show by shifting the zero point. For example, if the measured values of a parameter are expressed as 999, 1000, and 1001 units, it appears that there is a strong correlation among the measurement outcomes: they lie within 0.2 percent of one another. By shifting the zero point, however, we can express these values as –1, 0, and +1 units. The correlation between the measurement outcomes now appears weak.

Are empirical data sets collected in science statistically random? A definitive verdict would clearly require much more systematic testing. However, we can state the following tentative conclusions. First, while different digits have unequal probabilities of occurring at a given position in an empirical data set, this seems to be due to a consequence of the requirement of scale invariance. Second, while the digits in an empirical data set appear to show correlations with one another, the strength of these correlations depends on the zero point, and is in many interesting cases arbitrary. It therefore appears that empirical data sets do not obviously fail simple statistical randomness tests.

This completes the evidence for thinking that at least some empirical data sets are algorithmically incompressible. Previous debates have addressed further aspects of the question whether empirical data sets are algorithmically random and different forms that algorithmic compression can take [18, 19, 20].

5 Replies to Objections

In this section, we reply to three possible objections against one of the premises used in the previous section: the conditional claim that, if empirical data sets are algorithmically incompressible, then they cannot be the output of a computing device.

The first objection is that a computing device would be able to produce an algorithmically incompressible string if it ran an algorithm that generated random numbers. This objection is easily parried: since any algorithmically produced string exhibits regularities, any such algorithm would be, at best, a pseudo-random number generator. Use of such an algorithm would thus not make the output of a computing device algorithmically incompressible.

The second objection is that a computing device would be able to produce an algorithmically incompressible string if it incorporated quantum mechanical processes [21]. Allowing the computing device to incorporate quantum mechanical processes, however, would water down the content of the hypothesis that the world is a computing device. The computing device would come to satisfy to a greater degree our traditional account of the physical universe, and the hypothesis that the universe is a computing device would no longer offer an ontology alternative to that of standard physics. For example, our inability to rule out that the universe consists of a computing device that incorporates quantum mechanical processes would pose an only mild scepticist challenge to scientific knowledge, since we already believe on other grounds that the universe contains such processes.

The third objection is that a computing device would be able to produce an incompressible string if there were no limit on the length of the input to the device. We can arrange for a device to generate an even infinitely long incompressible string if we are allowed to use as input a string identical to the desired output: the device can than be programmed, trivially, to reproduce the input string. Two replies are available. First, this possibility would not meet the expectation that the algorithms responsible for generating the values of physical parameters are short, and of approximately the length of laws of nature. The project of accounting for complex outcomes by appeal to simple rules would be frustrated. Second, the question would arise what reason there would be to attribute to the universe the structure of a computing device to generate the numerical values of physical parameters, if the universe already contained a list of such values to be used as input.

6 Laws of Nature and Patterns in Empirical Data

If empirical data sets are algorithmically incompressible, what can we salvage of the intuition of Mach, Solomonoff, and others that science aims at producing compressions of empirical data?

A string can be described in terms of patterns even if it is itself patternless, or contains no regularities or redundancies that permit algorithmic compression of the string. Any empirical data set can be described as the sum of two components: a pattern and a noise term. This is a consequence of the fact that any mathematical function, however irregular, can be expressed as the sum of a simple and regular function and a second term that accounts for the difference. In the following example, $F(x)$ is an empirical data set, the harmonic function, $a \sin \omega x + b \cos \omega x$, is a pattern that a scientist might discern in the data set, and $R(x)$ is the mathematical difference between these two terms, or "noise" in the sense of classical information theory:

$$F(x) = a \sin \omega x + b \cos \omega x + R(x)\,. \tag{1}$$

If empirical data sets are algorithmically incompressible, then the function $F(x)$ has maximal algorithmic complexity. Regardless of the degree of complexity of $F(x)$, however, it will always be possible to identify a simple component in it, such as the harmonic function picked out here. The sole condition for doing so is that we accept a noise term, $R(x)$, which also has maximal algorithmic complexity.

This decomposition of an empirical data set into two components, pattern and noise, differs from algorithmic compression because it will be necessary to provide a value of the noise term, $R(x)$, for every data point in the data set, $F(x)$. This ensures that the right-hand side of equation (1) is as long as the left hand-side, or—in more technical terms—has the same degree of algorithmic complexity. If the empirical data set, $F(x)$, is algorithmically incompressible, then the right-hand side of equation (1) is incompressible too.

We may thus say that science aims to identify and account for trends underlying the data, which are exhibited in the data with non-zero noise. In other words, a law of nature provides an algorithmic compression not of a data set in its entirety, as Mach, Solomonoff and others believed, but only of a regularity that constitutes a component of the data set and which the scientist picks out in the data. The remainder of the data set, which is algorithmically incompressible, is regarded as noise.

The picture is complicated by the fact that empirical data sets exhibit different patterns at different noise levels. Climatology offers a good illustration. A data set on atmospheric temperatures may be described as showing any one of many different patterns, together with a certain noise term. A weather forecaster will tune in to patterns with a period of a few days, which are exhibited with a low noise level. A meteorologist studying seasonal temperature variations will pick out patterns with an annual period, which are displayed with an intermediate noise level. A climatologist studying climate change, finally, will home in on patterns with a period of hundreds of years, exhibited with a high noise level. The fact that an empirical data set exhibits a multiplicity of patterns at different noise levels has far-reaching implications for metaphysics and epistemology [22, 23].

7 Conclusions

The hypothesis that the universe is a computing device is an intriguing metaphysical theory. To its merit, this hypothesis, unlike many others in the domain of metaphysics, admits empirical test. It predicts that the numerical values of physical parameters form algorithmically compressible strings—indeed, if the algorithms were as concise as the laws of nature with which we are familiar, the strings would show a very high degree of compressibility. However, there is good evidence that at least some empirical data sets, which contain the results of observations and measurements of the values of physical parameters, are algorithmically incompressible. This evidence takes two forms: the failure of resourceful and persistent attempts by scientists to produce algorithmic compressions of empirical data sets, and the finding that empirical data sets do not obviously fail statistical randomness tests. If, as this evidence suggests, at least some empirical data sets are algorithmically incompressible, then it follows that the universe cannot be a computing device.

Empirical data bear the fingerprints of their origin: we can discover a lot about the nature of the world from statistical features of data. In this case, the discovery that the results of our observations and measurements are not algorithmically compressible shows that the entity that caused them does not have the structure of a computing device.

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Mathematics, Art and Music

Numeri Malefici (Evil Numbers): Homage to Fabio Mauri

Michele Emmer

Fig. 1 Fabio Mauri (photo: Claudio Martinez)

Fabio Mauri (1926-2009):

> I pondered the numbers of death, the equation that could provide a mathematical representation. What I was doing was *mathematism*, but I knew that [...] I considered the separation equation of two events $s_2 = t_2 - x_2 - y_2 - z_2$ [...] reflecting that, first I had to completely understand whether death was an event in itself, as psychology led me to believe, or if it simply constituted a cessation. Of an event that is unique and one-of-a-kind, that is, life. Thus I could drape it in the mathematics that can be attributed to life, that is, all of it. But it constituted *one* point. That was an interesting aspect. Perhaps I had found in physical death a point that was mathematical, metaphysical, not infinite [1, p. 31].

From his early youth Mauri was in close contact with writers and intellectuals with whom he shared ties of friendship and collaboration. His early years were marked by events of the war and Fascism, which he experienced in part through psychic crises, religious experiences and an intense social commitment. In the three decades 1944-1974 he worked for the publisher Bompiani, directing its Roman headquarters from 1958. He was one of the founders of the magazine *Quindici* (1967), along with Umberto Eco, Edoardo Sanguineti and others, and of the magazine for art and criticism called *La città di Riga* (1976). Availing himself of media that went from writing to sculpture, from installations to performance, he addressed the theme of communication and its manipulation in terms that were sociological and ideological. Here is a look at some of his best known works, finally arriving to the installation entitled *I numeri malefici* (Evil Numbers).

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Che cosa è il fascismo, 1971

Mauri has written that with the performance artwork entitled *Che cosa è il fascismo* (What Fascism Is), he wanted to reconstruct what he and his friend Pier Paolo Pasolini had seen and lived through: the rally in Florence in 1938 of the *Gioventù Italiana del Littorio* (G.I.L.) and the *Hitlerjungen*, on the occasion of a visit by Hitler. This was a gathering dedicated to *Ludi juveniles* games for young people of all kinds, athletes, intellectuals, artists as was usual in Fascist and Nazi youth organisations. Mauri observed that the two dictatorships were played out against great mass stage settings, adding that first and even more than being the dangerous and deadly politicians, Hitler and Mussolini were scenographers with no limits on what they could spend. 2 April 1971: a celebration in honour of General Ernst von Hussel, who was passing through Rome. This was the fictional occasion which Mauri created in order to reconstruct a reality made of memories, to turn a personal experience into an experience for everyone. He recreated one of the usual situations that were distinguishing features of the Fascist and Nazi period (and of all totalitarianisms) and reproduced it philologically, imposing a kind of full immersion in an event that was strongly ideological. This was the artist's intention.

> I wanted to understand how the beauty of sport could coexist with lies, and to discover the reason for the situation young people were in. At one stroke I saw this situation of young people turned on its head, and those who were a little older than me go to the front, in Russia, and to the concentration camps. That is how *Che cosa è il fascismo* was born because, strangely, at that time no one was talking about it anymore, it seemed a case already closed, and instead an incredible Fascism was spreading. [...] By constantly asking myself what they were doing I was able to create a performance that was astonishing because it seemed all too true [2].

The audience was invited to take their seats in the six platforms divided according to social groups. In the middle was a large rectangular carpet with the symbol of the Nazi swastika, the scene of the action. On one end was the Podium of Command, with behind it a screen on which was written *The End*. On the other end there were two more platforms with the symbol of the Star of David, destined for Jewish men and women, behind which was a scaffolding from which would be projected a documentary made by the Istituto Luce.[1]

> Young Italians and Young Fascists perform before General von Hussel [a wax manikin] and Consul Eritreo [an actor]. The harmonious actions of the *Bella Gioventù* will create an enchanting spell broken at the end by the roar of a hailstorm of bombs. The bombardment at the end will give a sonorous body to the subtle anxiety generated by the lapidary writing *The End*, a presage of a final collapse [3, p. 68].

This performance took place for the first time in 1971 with second- and third-year students of the "Silvio d'Amico" National Academy of Dramatic Arts in the Safa Palatino cinematographic studios in Rome, at the end of a seminar entitled *Gesto e*

[1] The Istituto Luce (an acronym for "L'Unione Cinematografica Educativa") was dedicated at the distribution of films for purposes of education and information. Founded in Rome in 1924, it became a powerful tool for propaganda under the Fascists.

comportamento nell'arte, oggi (Gesture and behaviour in art, today), as part of the courses taught by the director Giorgio Pressburger. It was also performed in 1974 at the Venice Biennale, at the Performing Garage in New York in 1979, and in 1993 at the Museo Pecci in Prato on the occasion of the exhibition entitled *Inside out*.

When it was performed in 1971 a mimeograph sheet was distributed, containing, among other things, the following:

> Here can be felt for a short time, false ideology, the abyss of institutionalised Superficiality, the Tautology of Absolute Power, the intimate malignancy of the Lie concealed in Order, the shame of cultural confusion, the irresponsibility of those who assume for themselves the liberty of collective judgment, the deception of youth which sets grace and trust as a prelude to all massacres [3, pp. 82-83].

Mauri concluded:

> *Che cosa è il fascismo* is for me above all an image, a physical and sonorous form, in which a certain number of juxtaposed meanings maintain the image itself in equilibrium with its critical significance, like a spine. Something like a poem [1, p. 96].

Fig. 2 - 3 *Che cosa è il fascismo*, 1971. Safa Palatino cinematographic studios, Rome (photo: Marcella Galassi)

Fig. 4 *Che cosa è il fascismo*, 1979. Performing Garage, New York (photo: Elisabetta Catalano)

Ebrea, 1971

> With satanic joy in his face, the black-haired Jewish youth lurks in wait for the unsuspecting girl whom he defiles with his blood, thus stealing her from her people. With every means he tries to destroy the racial foundations of the people he has set out to subjugate. Just as he himself systematically ruins women and girls, he does not shrink back from pulling down the blood barriers for others, even on a large scale. It was and it is Jews who bring the Negroes into the Rhineland, always with the same secret thought and clear aim of ruining the hated white race by the necessarily resulting bastardization, throwing it down from its cultural and political height, and himself rising to be its master [Adolf Hitler, *Mein Kampf*, Book I, Chap. 11].

Mauri wrote:

> I have often dealt with evil; it is interesting to me to understand how certain evil is possible. With Ebrea [Jewess] I hypothesised that something horrible had happened; this exhibition also has something didactic in it, because to have this natural, vile invention, I remade the soapbars of the Jews with modern soapbars and remade some objects that could not be made with human skin. [. . .] In order to achieve the effects that are the works, my study really did seem like a concentration camp, because I tried many things, some of which I discarded. In fact, there are about ten works in *Ebrea* that are usually exhibited, but I actually made twenty-five works, fifteen of which I never exhibit. I wanted to give truth to this operation, that is, its metaphorical, artificial side is clear. I could allow myself those more showy things, if I sort of proposed to myself again to make things that the Germans had evidently made [2, p. 23].

Ebrea became a symbol of anti-racism and of tolerance at all levels.

Bartolomeo Pietromarchi wrote:

> For Mauri history has a precise aesthetic and conceptual dimension, and it is that European, biographical history that nullified the dialectic power of language, abdicating to that single thought which found its highest expression in Fascism and Nazism. For Mauri, *language is war*, to the degree that it has the power to affect reality, with the instruments of demogogy and ideology, and thus to condition the course of history. That apparatus of distortion of reality is always lying in ambush if the correct social value is not given to critical thought, to dissent, to dialectic, and if these are not put into practice. This is the responsibility of art and culture, to investigate and remain vigilant, to make visible the perils and the tendencies of language and the instruments which it uses to amplify its power: the media [4, p. 122].

Talking about both Ebrea and *Che cosa è il fascismo* Pietromarchi adds:

> They are two performances with which the artist puts the spectator in front of his recent past, interrogates him in the deepest of his convictions and conventions, forcing him to turn his gaze to the *roots of evil* and not forget [4, p. 156].

With regard to *Ebrea* he underlines that:

> we are faced with an action that is much more dramatic, a single and intimate action, in which a young, naked woman seated in front of a mirror cuts her hair. With slow movements she attaches them to the mirror, drawing what we little by little discover to be a Star of David [. . .] What appeared to be unimagineable to us manifests itself in its most naked truth, in a testimony made still more grotesque and frightening since it is presented with the trappings of an attractive aesthetic: the nude female body and objects of an accurate and refined design, within a reassuring family dimension that is everyday and intimate [4, p. 157].

At the opening of *Ebrea* in 1971, at the La Salita art gallery in Rome, Mauri wrote:

> I patiently recreate, with my own hands, the experience of the shameful. I explore the mental possibilities of it. Extending the act, I invent new objects *made of* new men. I impede in passing the secular security of contemporary design that is so trusting of *progress* [3, p. 64].

Paola Montenero (1971)?

Fig. 5 *Ebrea*, 1971 (photo: Studio Fabio Mauri)

Fig. 6 *Ebrea*, 1971. Galleria Barozzi, Venice (photo: Elisabetta Catalano)

Fig. 7 *Saponi* (*Soap bars*), from *Ebrea*, 1971; wood, soap, paper (32x70x7 cm)

A book: *Le piccole provinciali di M. de P.*

> Of an event that is unique and one-of-a-kind, that is, life. Thus I could drape it in the mathematics that can be attributed to life, that is, all of it. But it constituted *one* point. That was an interesting aspect. Perhaps I had found in physical death *a* point that was mathematical, metaphysical, not infinite [1, p. 31].

This quotation, like the one cited at the beginning of this homage to Mauri, appeared in a short epistolary novel which was never published, a reinvention of *Lettres écrites par Louis de Montalte à un Provincial de ses amis et aux R.R. Pères Jésuites* by Blaise Pascal (1623-1662).

Pascal's *Provincial Letters* deal with theological and moral questions, with salvation and grace, as they were discussed by the Jansenists and the Jesuits, in the guise of an exchange between two fictional characters, Luis de Montalte and a friend who lives far from Paris and the environs of the Sorbonne. Pascal also dealt with mathematics and physics, so much so that he was permitted to attend the meetings of the circle of Marin Marsenne, who was in his turn in contact with many of the most important scientists of the day.

In Mauri's book he invents different characters some who actually existed, others who were fictional who exchange letters. Many things are talked about, but there are two principal themes: art and the existence of God. In search of a formula, a mathematical law that makes true knowledge possible.

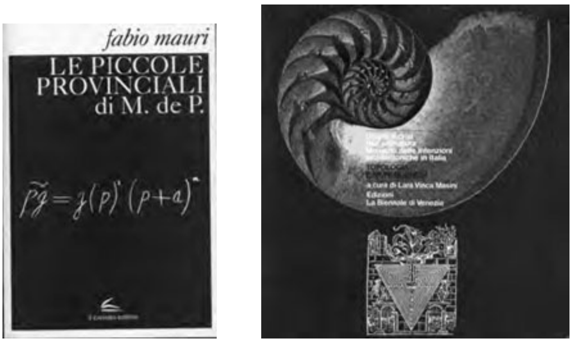


Fig. 8 Cover of *Le piccole Provinciali di M. de P.* by Fabio Mauri; cover of the exhibition catalogue of *Topologia e morfogenesi*, held during the 1978 Venice Biennale

> Inside my head there is a debate on the thesis *quid est veritas*, that is, both what the truth is rationally outside of ourselves, and why is it that we have this power to err, and err profoundly? [...] an ant does not make mistakes, but man does, it is the thing that most distinguishes him. Man can make a calculation and err. If you think about it, it is a clamorous liberty and potential [5].

Mauri recalls that he was thinking of this when he created, for the 1978 Venice Biennale, the installation entitled I numeri malefici (Evil Numbers): "a work constructed in the same way that addends are put into an equation" [5, p. 96].

The general theme of the Biennale that year was *Dalla natura all'arte, dall'arte alla natura* (From Nature to Art, from Art to Nature) [6]. The Committee Members for the Italian Pavilion were Luigi Carluccio, Enrico Crispolti and Lara Vinca Masini. Masini defined the theme that she was concerned with as *Topology and Morphogenesis*. It was in the context of this section that Mauri's installation I numeri malefici was exhibited. In the general catalogue of that year's Biennale, in a polemic with the committee members who had chosen the general theme of the Biennale, Masini wrote:

> My position regarding the theme proposed for the 1978 Biennale is clearly problematic and plainly critical and dissenting: I believe to be definitively outdated and anachronistic both the positivistic conception and the idealist conception of the relationship art/nature in a situation such as our contemporary one, which makes increasingly evident the crisis in and precariousness of natural equilibriums and existential conditions [6, p. 146].

Masini favoured the theme of:

> Topological investigation as the possible identification of a mental territory that is different and alternative; an investigation that necessarily results in an ideological framework and in a critical reconsideration of history. What ensues is the recovery of anthropological memory and the practice of citation.

This is the section where Mauri's installation was inserted.

In particular, in the catalogue of the exhibition *Topologia e morfogenesi*, which was part of the exhibition *Utopia e crisi dell'antinatura* (Utopia and the Crisis of the Anti-Nature), also at the 1978 Biennale, Masini explains:

> Topology and morphogenesis are further re-examined as a taking of position in the face of a theme that is too generic and all-embracing, as well as being outdated in its literal statement, as a search for different angles from which to analyse the *territory* of art in its continual aggressiveness with respect to an actual social-political, economic and in any case existential context, albeit one that is increasingly less commensurate and conform with the life of mankind [7, p. 6-10].

In particular, Masini's interpretation of the term *Topology* includes:

> In intuitive terms, that position of the impossibility of identifying an alternative topos in reality, of ascertaining the conditions of *anti-nature* in which man's panorama takes shape, today, both in the private realm as well as in his condition as a user of the city and the territory, in the increasingly evident state of alienation [7, p. 6-10].

[Remember, we're in the 1970s]

Mauri had also spoken of topology with regard to *I numeri malefici*: "In keeping with the prerogative of art, the formula is hidden with evidence in the same place as its topological identity" [2, p. 23].

Masini reminds us that this leads to the search for territories that are mental, utopian, not controlled and discriminated by *aesthetic systems*, and "implicates ideology, the resort to memory and the analysis of history, implicates every position that can be recognised in non-acceptance of the canons imposed". Thus it is necessary to abandon positions of desecration and refusal, such as those of Burri and Manzoni (*Merda d'artista*, 1961). Because of all of this, Fabio Mauri could not help but appear among the artists present, precisely with his *I numeri malefici*.

In his "Saggio senza parole" (Essay without Words) of that same year, 1978, Mauri wrote:

> No one has ever considered the largest deposit of human history, the error of calculation and judgment, as a unique and primary subject of interpretation of man and of history. It is the theme of the exhibit hall [of the 1978 Venice Bienniale], composed of elements that are apparently disparate, of a search which is instead compact in this sense [1, pp. 101-102].

Fig. 9 - 10 - 11 *I numeri malefici*, 1978. Installation with blackboard, photograph, iron cage, sound equipment, fragment of a fresco by Giotto. Italian Pavillion, XXXVIII Biennale di Venezia (photo: Elisabetta Catalano)

On one wall of the hall is a mural-size photograph of Goebels inaugurating the exhibition entitled *Entartete Kunst* (*Degenerate Art*) with the smug smile of a man who knows what quality and art are.

Facing that, on a wall, is a slate blackboard with a formula written in chalk in the middle of it.

Below, a dais, or only a metal structure, composed on a square module, whose logical formalisation is easily understood, as Euclidean as it appears. From this structure, through a microphone, come samples of contemporary thought. Anyone who goes close to it expresses his own personal critical idea of nature, which is shown here, by way of comparison, to be strongly unknown.

On the floor, in the centre of the room, an iron frame surrounds a fresco by Giotto (perhaps by Giottino): *The Mystical Marriage of Saint Catherine of Alexandria*, from the Bonacolsi Chapel in Mantua's Torre della Gabbia]. It is art. It stays concretely in place of the X, which is the nature of art, as theory implies. Its damaged state (*the image is incomplete: part of the fresco, two angels, are housed in New York's Metropolitan Museum*) does not spoil the integrity of the fragment.

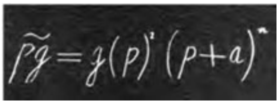
In 1870 the artist Bortolo Bosio discovered the medieval chapel in Mantua. That same year two compositions were detached: a Crucifixion and the Madonna Enthroned with Saints, which was divided into four fragments. *The Mystical Marriage of Saint Catherine and Deacon* and *Two Angels* are believed to be by the School of Giotto. The detached fragments are now housed in different locations, including the Metropolitan Museum of New York and the J. F. Willumsens Museum in Frederikssund (Denmark).

Hung on the wall opposite the blackboard is a photograph which shows the German minister Goebels accompanied by Professor Ziegler at the opening of *Entartete Kunst*, the exhibition of degenerate art that took place in Munich in 1937. The photograph shows canvasses by Emile Nolde and Ernst Ludwig Kirchner.

> It is necessary to go back to the formula written on the blackboard that faces the institutional photograph. It is a formula composed by the author in the course of research on the principle of intellectual error. In an almost conclusive way it is part of an essential theme on the mind, the world and error, on the possibility of calculating its effect, of theoretically preventing it, as a useful remainder of an idea of a universe skewed by a similar structural error [1, pp. 101-102].

Further,

> Nature, art and critical ideology are represented here as centres of attraction of identical wefts, involved and conditioned by a single law. The answers they provide, however, are different. In what? the room asks. The terms of the proposition, installed as they are in an incongruous way, or drawn close to each other by an infinitesimal affinity, if visited by privately determined paths, that is, other than that proposed by the order of the room (which is crossed through down the middle by coincidence of the exhibition space) opens more than perspective of *truth*. Understanding the formula would facilitate such a collocation in the world and in the mind in a new combination. But, according to the prerogative of art, it is not given [1, pp. 101-102].



In private conversation, Achille Mauri suggested to me that it is quite probable that the formula came to Fabio Mauri's mind inspired either by that of Verlhust's dynamics:

$$x_n = (1+r)^n x_0$$

or by that which generates the Mandelbrot set. The idea of a formula for the geometry of the universe was also due to Fabio Mauri's profound religious faith. This too was suggested by Achille Mauri.

$$x_{n+1} = f_c(x_n) = x_n^2 + c\,.$$

Fabio Mauri adds:

> Just as effectively, it is possible for the mind of the common, that is, cultured, observer to understand the formula, when [that mind] is open to the perception of the pre-arranged symbols as in every location of the expression, where faith in the phenomenon of exchange comes to be re-established, recreated as a rite that is essential to a idea of the universe that is correct, that is, simply, less fanciful, more imaginative, real.

In the catalogue of the exhibition dedicated to Mauri which took place in Milan's Palazzo Reale in 2012, following the section dedicated to *I numeri malefici*, we read:

Mathematics in infinite in two directions. Is this bad?

In the "Saggio senza parole", Mauri maintains that understanding that formula written on the blackboard:

> Composed in the course of research on the principle of intellectual error in calculation and in judgment, would have permitted the collocation of the world and of the mind in a new combination. But, as said, the prerogative of art, the rule is not revealed. It is ciphered in a form that is correctly symbolic [8, p. 85].

It is precisely this formula, "the evil numbers", which was featured on the cover of the book inspired by Pascal.

The book speaks about art, about the "quality that is necessary for a work of art to be just that".

> What is quality? What entity are we dealing with? Fixed? Variable? Discrete? One of the characters asks, "Is it quality that can make me understand what quality is? [...] FORM! There is nothing else! IT'S THAT, ART! If Form is Art we are home free [9].

When it turns to culture or religion, the discussion is ironic, amusing, refined. The mathematician Lipinsky is asked, "Is quality discrete or continuous?" He answers with a mathematism: quality is "a finite set in which the whole is never equal to the sum of the parts", at which the one who was questioning him thinks,

> "Education sometimes plays dirty tricks. He said yes, but he rushed into the void". Mathematics, in effect, provides no answers. Its *relationship to reality is notoriously uncertain*. The mathematician cannot help but observe: "You suffer from intellectual claustrophobia". On the other hand, "There are certain arguments in which it is mathematical that the world wants to deceive itself [10].

And the formula, "the evil numbers"?

> What do you say? Could it not be reduced more simply? M. de P. replies that he has tried. But it is "grammatically undoable. Is it a good thing to unite what is logically, not essentially, divided? Perhaps it is. But there are substantially two other things: it is not good to reduce them to unity without reducing the meaning to 0 [11].

Every phrase, every idea, every word is involving, striking, amusing. Words on heresy and relativity (in the sense of theory) of art and form always in a tone that was profound and light. What is spoken of is present day, the time in which the book was written, but also of today.

> You propose that quality in itself is a mathematical structure?" "Not exactly. Rather, I think that not a single number exists in nature. If it had, it would already have disappeared, as the concept would disappear after such an assertion, if it were mine! For a moment I will consider mathematical language as a proper and organic language.

The last three letters, the author says, were left out because they were "completely scientific, technically difficult ... unrelated to art, which is the only thing that mattered to us".

The book, Mauri tells, was not ever published because first Valentino Bompiani and then the publishing house Adelphi asked him for a final letter with the numbers and ciphers.

"And I answered that, the numbers, I did not have them".

Translated from the Italian by Kim Williams

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From Pollock's *Summertime* to Jacksontime

Davide Amodio

Fig. 1 J. Pollock, *Summertime: Number 9A* (1948) (©Tate, London 2012)

Beginning with *Summertime* by Jackson Pollock and arriving at the musical composition "Jacksontime" by means of mathematics involved an operation of *translation* in the Latin sense of *traducere*, "carry from one place to another".

Transcribing is not a mechanical or neutral action; in artistic fields, in order to be fruitful every passage needs two fundamental ingredients: interpretation and creativity.

Interpreting is being an intermediary in a communication between individuals who do not possess the elements necessary to comprehend each other but who have the potential and need to understand and transmit their thoughts. Thus, the interpreter[1] becomes the instrument for that communication. In art this always has to occur through the sum and integration of several "creators", so that a message that is complex and symbolic can arrive at its destination with the same vitality and strength that characterised the original. In any case, this was a "heavy" translation, a long-distance "transportation" from painting to music. For this reason it was necessary to rely on the sure contribution of geometry.

Art and science: two fields in which I have always sought (starting from the former and looking towards the latter) common points of work.

By seeking in history those occasions that recognised interdisciplinarianism as an effective means for broadening knowledge, I became aware of the absolute importance of uniting the research efforts of not only the so-called sister arts, but also of uniting them with those of scientific areas as well.

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[1] Etymologically speaking, from *Inter,* mediation, and *pretium*, price, in the sense of a mediator who does not deceive the parties but gives to each his due [3].

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As is well known, at one time music was part of the *Quadrivium*, together with arithmetic, geometry and astronomy; equally well known is the idea that mathematics contains a strong artistic-aesthetic component.

In today's world, professional specialisation is an absolute requirement, and yet collaboration between fields of knowledge that are very far apart has maintained its precious validity throughout the centuries.

Already during my doctoral research my work concerned relationships between poetry, music and mathematics, ranging through the marvels of the music of the thirteenth century, in which improvising poets were at once both scientists and musicians [2].

Even though it is impossible to go back to the level of "interknowledge" of past times, it is still necessary to expand as far as possible the "duty trade zone" for the sharing of knowledge, where experts in a single field can interact with and exchange ideas with others. But let's go back to Pollock.

Some years ago, standing in front of Jackson Pollock's *Summertime* at the Tate Modern in London, I was invaded by sounds that passed silently from the canvas into my head. It was this incredible experience that led me to imagine the possibility of creating an interpretation of those sounds, one not merely inspired by the canvas, but rather based on the precise data of its proportions.[2]

It was thanks to the willingness to help and the precision of Chiara de Fabritiis that I was able to access those very valuable data, allowing me to construct a close relationship between the music and the image of the painting.[3]

Chomsky's generative grammar has demonstrated, especially in light of recent neurological investigations,[4] that human language (and its organisation) is not a uniquely cultural aspect of man's activities, but owes its very existence to the particular and unique conformation of the human brain.

In particular, syntax is a complex organisation that obeys, we might say, natural rules of human physiology, instead of simply abstract rules that are learned by rote. The organisation of different elements into a tree structure, as Chomsky was the first to demonstrate in his by now famous book *Syntactic Structures* [4] of 1957, can be "exported" into artistic languages such as music, and even into that of painting, especially abstract painting.

For this reason we can say that the purpose of the experiment we carried out, Chiara de Fabritiis (Università Politecnica delle Marche) and me, was to make evident, by means of mathematics, the syntactical rules of Pollock's painting *Summertime* and to then translate them into another idiom, that of music.

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[2] For other mathematical analyses of Pollock's work, see [1], [7], [11], [13], [14], [15].

[3] For a mathematical investigation of the painting, see [5].

[4] [10], especially chap. 4, section 2, "La lingua nel cervello", p. 261: "The phases make no sense in themselves, but acquire meaning because our brain is constructed to decodify them, just as our eyes are constructed to analyse light, but as Moro explains, comparing language and light, we do not see light, but only the effects that it has on objects. Our language functions the same way; words do not have an intrinsic content, but when they arrive to someone's attentive (and competent) ear, they become something, they exist" (taken from the review by Leonardo Caffo).

Although this operation was purely authorial, the primary interest was in carrying out an experiment: to bring the intent into focus by arranging determined elements and finally being able to demonstrate the validity and veracity of the initial assumptions.

The thesis we began with was that Pollock's canvas is not dominated by chaos, as detractors of the time accused, and that, to the contrary, it contained a discernable structure that could be exported and utilised in another medium, in this case music. The construction of a musical composition with the same geometrical characteristics of Pollock's painting made it possible to render the structure audible, thus making it doubly perceivable.

In 1950, Pollock sent a telegram to *Time* magazine in response to the article by the Italian critic Bruno Alfieri, according to whom the work was characterised above all by chaos. Pollock wrote, "No chaos damn it. Damned busy painting as you can see ...".

Very eloquent phrases appear in several autograph notes of Pollock's that were published posthumously: *total control* ... *denial of* ... *states of order* ... *organic intensity*..... (and above all two extremely important thoughts) *energy and motion made visible* ... *memories arrested in space* [6].

These phrases clearly show how strongly the idea of plasticity of motion is united to the way of painting. Uniting energy and motion and making them visible means that the action that records the time of the action itself can be traced. Energy made visible simply indicates the variation of intensity of the motion, the complex "alphabet" of signs; it thus already embodies in itself an entire and complete indication of the structure and the "voices" that move within it.

The "memories arrested in space" identify further a delimited space that contains an action to be seen within a determined time.

Although in vision it is possible to go back, the form of the painting follows the linear direction of a film, and therefore, of a piece of music.

In fact, the special characteristic of Pollock's famous method of painting consists in executing the brush strokes with a motion of the entire body. This means that the marks on the canvas are principally "recordings" of his motions. In this way each mark has a duration in time, just as musical notes indicate, in addition to pitch, the duration of each sound. In addition, each mark on the canvas also indicates the intensity, the ratio of proportion and strength that each has with the other marks of different shapes and colours.

So, in *Summertime* Pollock executes a weave of rhythmic motion, gestures, organised movements that illustrate the dance inherent in number (number in the sense it is used in Latin, *numerus* as *rythmus*) by means of a "generative grammar" of signs, legible in sound.

The chromatic mark acquires the value of the motion, is its semantic, that is, its detailed stenographic. In fact, the colours express different weights through tonal variations, as do their positions within the space of the canvas.[5]

[5] See the general remarks in Kandinsky's *Point and Line to Plane* [8].

Fig. 2 Jackson Pollock at work

The "text" in Pollock's painting is at once visible and legible, that is, capable of interpretation, in one execution that is only mental and in another that is a genuine execution of sound, comparable to a contrapuntal score. Pollock's superimpositions of successive layers embody the polyphonic agility of interdependent voices. The long rectangular shape of the painting itself invites a Western left-to-right reading; the visible structure on which the splatters of colour (there are seven red, twenty-eight blue, twenty-eight yellow and sixty black splatters) are clearly arrayed as a function of dynamic and melodic expressiveness, suggesting a clarity of organisation in the inertia of the physicality of the body and in the dynamic of emotion (the word intended in the sense of *emovere*, carry outside), which can be likened to a kind of Flemish rigour and even, by association to counterpoint, Beethoven's touch.

Interpreting, translating into sound a text that is not evidently musical, besieges and interferes with the arbitrariness to which the executor/interpreter is always prey: how faithful to be? how creative? And in this particular case, faithful to which arrangement, to which marks?

The material difference between Pollock's *Summertime* (shared by many others of his works) and a musical score consists in the independence of the painting from any interpretation whatsoever: the mark of the author, his gesture, is accessible to any silent observer; the work is concluded in the final act of its exposition, sufficient unto itself, and thus an interpretation (here too in the sense of the etymological term *inter* and *pretium*) is not necessary. However, the musical score, whose very nature requires it to become sonorous, could in the same way be sufficient unto itself, not finding in a *raison d'etre* in the execution when someone reading silently completes in his mind, as in the reading of a play, the part implied, since the sonorous weave is already there, set down in conventional signs.

So, if sound is not necessary for an expert reader to "hear" the written music, it is not necessary for *Summertime*, but if we do want to hear it, to share in a dynamic and sonorous interpretation, filtered through the sensitivity and taste of the interpreter, then in Pollock it is possible to hear the sounds that his marks give rise to, even if they were not aimed at such an auditory condensation.

However, the passage from seeing to hearing requires a geometric decodification, a precise computation of the components of the painting, an analytical exemplification of it that predisposes the organisation of another text in a new set of conventional symbols, that can now be read with a minimal margin of interpretation with respect to the image that was the point of departure.

The language of metrics and space (not of imagery, which Pollock's paintings only rarely evoke) must be transposed into a language of rhythm and sound. This is possible thanks to mathematical metrics, an interpretation that is scientific but equally valid in artistic/interpretive terms.

The contribution of the variables to the analysis and the understanding of the superimposed layers, expressed in order of carrying capacity (and thus not of importance or hierarchy) guides and orders the simple unwinding in space of numerical information, exact ciphers that will be later translated into another kind of ciphers that are just as exact: musical ciphers. This passage is not without difficult choices, but here the language of arrival, musical idiom, becomes the guide, imposing its own requirements for corresponding in order to replicate the proportions and dimensions, which are completely congruent.

Another interpretation will then be necessary of the final sign which is strictly musical (the performance) but by now the shore has been reached and the ford has been passed, since what are left are minor differences with respect to the great initial translation.

I will now briefly illustrate the musical correspondences.

With regard to the duration and distribution of time I have made the musical tempo correspond to a metronome speed of 60 for every quarter note, which means that every beat corresponds to a second. (In fact, there are 289 bars, of which some last two, others three and the majority five seconds for a total of exactly 1440 seconds/ 24 minutes, which was the overall duration that we had established at the beginning.)

In the book *Concerning the Spiritual in Art* Kandinsky proposed a relationship between colours and musical instruments; for example, yellow was compared to the sound of a trumpet, and blue to a flute.

My choice was principally polyphonic, and thus a colour corresponds not only to an instrument but to a group of instruments which form a distinct unit, as in imitation of the superimposed layers of *Summertime.* The twenty-eight blue splatters are realised with a quartet of woodwinds: flute, clarinet, bass clarinet and bassoon.

Splatter number 1 is identified in the music by position and duration, but only the first one has no variation of orientation. Thus the form is approached by supporting it: the musical figuration is inscribed within the shape of the splatter almost as though understanding it as a sort of neume (Figg. 3 and 4).

Fig. 3 The entry at 26″ of the quartet of woodwinds: beginning with the **bassoon**, then the two clarinets and the flute

Fig. 4 *Summertime*, detail: the first blue splatter on the lower left

From the second splatter on, the variations were created by orienting the four cardinal points to the four voices of the score (see Fig. 5).

Northwest North Northeast

Southwest South Southeast

Fig. 5 The entry at 26″ of the quartet of woodwinds with the orientation to cardinal points

Establishing this orientation with regard to the directions in order to organize the score was not very complicated. Instead, it was much more complicated to work in the two directions **West** and **East**; falling as they do on the horizontal axis, they had an influence on the temporal dimension, which was instead blocked according to the preset duration. Thus the variation had to be understood within an rectangular image whose dimensions were given instance by instance, whose interior was filled or emptied according to the measures indicated, but with the same system to both right and left. Thus a variation that led to a lengthening towards West or East demanded that the central voices began or finished before the others.

The red marks in Fig. 6 show how the two intermediate voices extend to West and East without exceeding the overall duration of eighteen seconds determined by splatter number 2.

Fig. 6 The entry of the woodwind quartet corresponding to the second blue splatter

The shape variations thus added up to those of duration and position, but soon an ulterior variable was added: that of the intersection of splatters, sometimes two or three superimposed. This made it necessary to follow a contrapuntal logic by sector: first among the four voices of each splatter, and then among the voices of all the splatters together; almost like sets of containers, one inside the other. This was very laborious, but also very exciting.

In inserting the series of yellow splatters the counterpoint sometimes began to transform itself through an amplification of certain voices and not just a mere addition, in part because in the achievement of pictorial balance within the tangle of marks and splatters, the weave became an interdependence between the proportions and the nearness of colours which, even though separate, added up to form a single shade.

The yellow splatters are represented by a quartet of brass instruments: trumpet, French horn, trombone, and bass tuba.

The series of seven red splatters, as their scarce appearance and number suggest, is represented by simple strikes of a triangle, which, always respecting the duration, intervene to give light and emphasis at determined points.

The inner space of thick black structural splatters is created by the couple double bass-vibraphone, with a rhythm that either supports and reinforces the voices of the colours that it crosses, or becomes more rhythmic in the points that it occupies where there are no colours.

Finally, the multitude of interior marks – tiny points or curved lines – are created with percussion instruments playing either multiple sounds or sliding.

It is worthwhile noting that I often use the term "counterpoint" only in the direct sense (note against note) and not with all of the correlated features and numerously varied rules that musical tradition teaches us. I have naturally followed the rules of consonance, of illusion of tonality, but these are only rules of aesthetics and practice that, according to Schoenberg, force a composer to choose one note rather than another to satisfy his own personal and absolute necessity.

Translated from the italian by Kim Williams

Fig. 7 Presentation and concert at Guggenheim museum of Venice. Photos by Andrea Sarti

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From Canvas to Music: Mathematics as a Tool for the Composition of Jacksontime

Chiara de Fabritiis

1 Introduction

The creation of "Jackson time" is a project which involves a composer, Davide Amodio, and a mathematician, Chiara de Fabritiis. Our common aim was to to "translate" a painting by Jackson Pollock, Summertime n. 9, into a piece of music, making use of different mathematical tools to detect the quantities needed for the composition. We were inspired by the idea that the painting itself contained some kind of inner–music, due to the fact that Pollock's moves during the dripping on the canvas had a sort of rhythm, indeed they were often described by witnesses as a dance. This paper describes the mathematical background, in particular it illustrates both the analysis of the painting which was carried out by the two of us and the choice of the mathematical techniques applied to compute the parameters needed for the composition, which is due to the author. The reader will find a more detailed report on the composition itself in Davide Amodio's contribution [2].

2 How to translate a painting into a piece of music?

In recent times, several examples of "objects" turned into music appeared. In 2011, Michael Blake produced a video entitled "What π sounds like", which was posted on YouTube on March, 14 (π day). Blake's idea was to associate each digit to a musical note and to perform them in the order given by the decimal expansion of π up to the thirty-first one, also the tempo was suggested by π, since he decided to play 157 beats per minute (157 being 314 divided by 2). Lars Erickson, another composer who had a similar idea in 1992, claimed Blake violated his copyright and sued him in front of the District Court of Nebraska (USA). In 2012 Judge M. H. Simon dis-

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missed the case, writing "π is a non–copyrightable fact, and the transcription of π to music is a non–copyrightable idea".

There are also examples of so–called DNA music: roughly, bases are turned into notes and a musical piece is produced by playing them in the order they appear in the DNA molecule. There are several works of this kind, mainly coming from the collaboration between Peter Gena and Charles Strom (Gena composed several pieces of music inspired by DNA, some of which are obtained as a mere conversion of the sequences, while others come from more sophisticated techniques, like Chopin's catarrh, see [4] and [5]). Also Antonella Prisco, a researcher at CNR, Naples, produced some pieces of music which were played during visits at CNR laboratories for the aim of scientific divulgation, see [9] and [10].

The common feature of these works is the fact that in a certain sense they are not "compositions", indeed they are obtained as a mere translation of a given sequence into music. After the identification of either the figures (in the case of π) or the basis (when DNA is involved) has been done, the piece comes out automatically, we could even say that the music is already contained in the object.

Our research has a different approach: the piece is not a mere "mathematical reproduction" of the object because there is not an apriori-given sequence to translate into music. Our work is the overlap of three different levels of discretional interpretation:

1. Analysis of the graphical structure in search of the forms that appear in the painting and of their spatial organization.
2. Choice of the mathematical techniques to be used for the study of each of the structures detected in 1, computation of the parameters given by the different parts of the painting.
3. Draft of the score and performance of the work on the basis of the parameter computed in 2.

Each of these levels entrains an arbitrariness of choice by the authors, also in the mathematical part of the investigation. On the other hand, once choices have been made, the computations are performed following mathematical techniques. In the present paper we will describe the content of the first two points, while the development of the third one can be found in Davide Amodio's essay [2].

We recall that a completely different kind of mathematical investigation on Pollock's works was carried out by Prof. R. P. Taylor (Phys. Dept., University of Oregon), see [11] and [12] among many others, who published several papers devoted to the study of Pollock's works, where he looked for geometrical (especially fractal) patterns in Pollock's paintings. In particular, Taylor's studies were used as a tool in order to decide about the possible attribution to Pollock of pictures found by Alex Matter in Wainscott (NY) during 2003, see also [6].

3 Analysis of the Painting

3.1 Forms and Colours in the Picture

The structure of the painting appears to be a stratification of different layers, as can be seen by the images obtained with oblique light.

Looking to colours, shapes and features of the paints and to the techniques used to apply them, we can identify the following list of different geometrical forms:

1. Blue regions.
2. Yellow regions.
3. Red regions.
4. Black and grey “patches” .
5. Black and grey “thick” structure.
6. Black and grey “thin” structure.
7. Coloured dots.
8. Short curves (or “long” points).

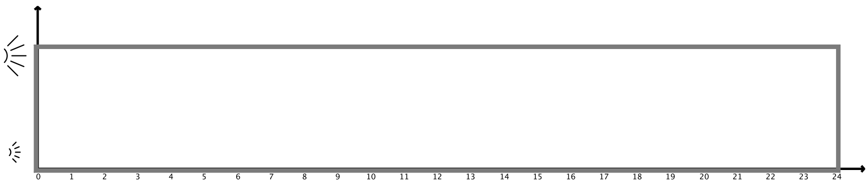
Indeed these forms are obtained by distinct pictorial material and techniques which produce a big difference in the rendering. Black and grey patches are wall paints (water distemper) dripped on the canvas which was laid horizontally on the floor of Pollock’s studio. Blue, yellow and red regions were obtained by brushing oil colours on the areas delimited by the dripping of grey and black paint. Coloured dots and short curves are strokes of the brush with oil colours, while black and grey structures (both thick and thin) arise form the dripping of the liquid: the first ones are produced when the brush starts moving after a large drop has fallen creating a black or grey patch; as the dye continues to trickle, the structure becomes thinner and thinner, and looks like it is made of threads.

3.2 A Reference Frame on the Picture

To measure up quantities which will be needed for the composition, we choose to put on the painting a Cartesian reference frame in which the x-axis coincides with the horizontal lower side and the y-axis coincides with the vertical left side.

The form of Summertime n 9, which is much longer than higher, suggests an analogy with a stave, hence it is quite natural to identify the variable on the horizontal axis as time. The unit of measurement is settled so that the whole painting is 24 minutes long. The determination of this number is due to several reasons: first of all we have to take into account that the work is very dense, so a relatively large duration is needed in order to have enough time to avoid unpleasant superpositions of sound, we estimated that some 20–30 minutes would be a reasonable length; a second reason is that 24 is an integer which has a lot of divisors (in particular 24 is abundant, *i.e.* the sum of all its divisors different from 24 itself, exceeds 24), then it is easy to divide the picture into slots of time which are integer submultiples of the duration.

The choice of the variable on the vertical axis requires deeper considerations: the parallel with the stave would lead us to measure pitch on the vertical axis, but with this choice we could have no freedom in the modulation of sounds, because composition would have been in a certain sense contained in the picture as we should be forced to produce a specific sound according to the position of each of the coloured regions. Consequently we decided to put loudness on the vertical axis. Human perception of volume range is quite wide, but usually the variation within the same piece does not exceed the ratio 1 : 3, so we decided that points on the upper horizontal side of the picture would sound three times louder than points on the lower horizontal side.



In this way we associated to each point in the painting a couple of coordinates which are given in the form $(a;b)$, where $a \in [0,24]$ is a measure of the time elapsed from the beginning and $b \in [1,3]$ is a measure of the loudness of the sound to which the point will be associated in the musical composition.

4 Choice of the Mathematical Techniques and Computation of the Parameters

Now that we have a reference frame on the painting, we have to take the needs of the composition into account; in particular we should interpret the feeling the picture evokes by means of different mathematical techniques. For instance, the occurrence of patches of the same colour in different positions suggest the idea of a theme that is played repeatedly at certain times, being modified according to suitable rules which keep track of the modifications of the patch. In this section we analyse the different methods applied to the distinct types of forms present in the picture and we compute the parameters needed for the writing of the piece.

The main tools needed for the computations of the compositional parameters are a good reproduction of the painting, that the Tate Gallery kindly supplied us for a symbolic fee, and a drawing program: Inkscape©, which works with SVG (Scalable Vector Graphics) standards and allows users to transform and manipulate shapes, moreover it can also be used to measure quantities (like width, height of figures, angles between lines and so on).

Importing the image into Inkscape gives us the possibility to compute duration and loudness of sounds simply by linearly rescaling the duration of the picture to 24 minutes and the loudness to the range $1-3$.

4.1 Techniques and Computations for the Black and Grey Patches

The black and grey patches are the first objects that appear on the canvas during Pollock's creation of the painting; they are obtained by immersing a large circular brush in the wall paint and letting the liquid fall vertically on the canvas laid horizontally on the floor. For this reason, these patches have the form of large circular or elliptic drops; moreover, since they are perceived almost exactly in the same way by the observers' eyes, we treat them together. First of all we number these forms increasingly from left to right, then we draw the ellipse which better suits each drop; in order to get a better visual approximation we only draw its contour, using a transparent filling; Inkscape also draws the rectangle which bounds the selected ellipse.

To establish the position of each of the ellipses and to deduce from it the parameters for the composition, we need to take into account several quantities: we measure X and Y, the horizontal and vertical coordinates of the lower left vertex of the rectangle which bound the ellipse; W and H, the width and the height of this rectangle; then we rotate the ellipse so that its major axis becomes horizontal and we keep trace of the angle of rotation α and of the width and the height, W' and H', of the rectangle tangent to the rotated ellipse.

From X and W we compute the starting time of each patch and its duration, using the scaling factor 0.3627 which is the ratio between $60 \cdot 24 = 1440$ seconds of the duration and the width of the picture in the Inkscape coordinates; from Y and H we compute the loudness, choosing for each patch the ordinate of its center $Y+H/2$; finally we estimate the eccentricity of the ellipse by taking the ratio H'/W' of the two axes of the rotated ellipse. All measured data are recorded in an Excel file and processed as described above to obtain the parameters used for the composition.

4.2 Techniques and Computation for the Coloured Patches

Now we turn to the coloured patches: first of all we choose to consider each colour (yellow, blue, red) separately; then we associate to the first region of each colour a theme (see [2] for a detailed description of its composition); at last all remaining patches are assigned a theme which is obtained from the original one according to the modification of the coloured region.

The mathematical problem is to find a suitable theory which describes the parameters which allow us to follow the variations in the shape of the patches. Our idea was to look to the painting as a subset of the complex plane and to use holomorphic maps to compute the parameters we need to describe the modifications in the patches.

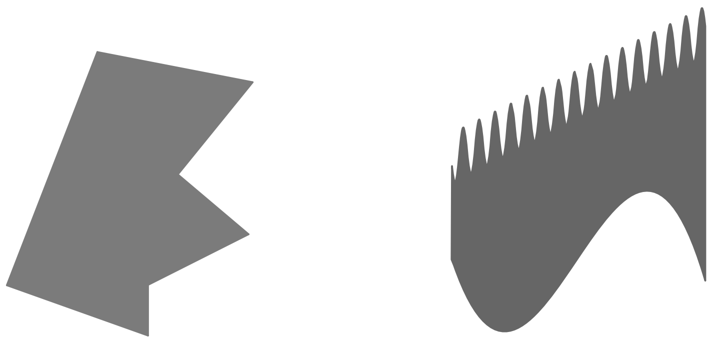
Definition 18.1. Let $D \subseteq \mathbb{C}$ be a domain (that is an open connected subset of $\mathbb{C}$). A function $f : D \to \mathbb{C}$ is *holomorphic* if it is differentiable and $f_x + i f_y = 0$. A *biholomorphism* between two domains is a holomorphic bijection whose inverse is also holomorphic.

Holomorphic functions can be characterized as the ones which can be written locally as a sum of a convergent power series in the complex variable z, *i.e.* f is holomorphic on D if and only if for all $z_0 \in D$ there exist $a_n \in \mathbb{C}$ for $n \in \mathbb{N}$ such that $f(z) = \sum_{n\in\mathbb{N}} a_n (z - z_0)^n$ on a suitable disk centered in z_0. In particular, holomorphic functions are conformal mappings, that is they preserve angles between curves in the complex plane. For a self-contained exposition on the theory of holomorphic function see [7] and [8].

A very important result in the theory of domains in $\mathbb{C}$ is the following

Theorem 18.1 (Riemann mapping theorem–simplified version). *A simply connected domain in $\mathbb{C}$ which does not coincide with $\mathbb{C}$ is biholomorphic to disk.*

As a consequence, any simply connected (that is, without holes) bounded domain contained in $\mathbb{C}$ is biholomorphic to a disk. As being biholomorphic is an equivalence relation we obtain that any two bounded simply connected regions in the complex plane are biholomorphic. In particular the two domains in the figure are biholomorphic.



Since we are interested in computing some parameters which describe the modification of a shape into another, we first of all focus our attention on the practical problem of describing a biholomorphism from the disk Δ of center 0 and radius 1 and a given domain $D \subset \mathbb{C}$. The achievement of our goal is simplified by the following result:

Theorem 18.2 (Morera). *A holomorphic map from Δ to $\mathbb{C}$ is globally the sum of a power series in z.*

In particular, if we want to describe the way a bounded simply connected domain D is mapped to a bounded simply connected domain D', we could write the coefficients of a biholomorphism $f : \Delta \to D$, the coefficients of a biholomorphism $g : \Delta \to D'$ and from these coefficients we would compute the ones for $g \circ f^{-1}$ which is a biholomorphism from D to D'.

A first remark is that the bihomorphism f from Δ to D is almost uniquely determined, since a holomorphic automorphism of the unit disk which fixes the origin and has derivative equal to 1 at the origin coincides with the identity map of Δ. So if we translate D so that it contains the origin, it is well known there exists a unique biholomorphism from Δ to D which fixes the origin and such that its derivative is a real positive number.

Our aim is to describe the transformation of a coloured patch contained in the complex plane in order to obtain some coefficients which allow us to modify the theme associated to the first patch of the same colour, so that we can obtain the themes associated to the consecutive patches.

How many coefficients do we need and how can we compute them? In principle, one could compute *all* the coefficients of the power series which gives a biholomorphism $f : \Delta \to D$, the same could be done for a biholomorphism $g : \Delta \to D'$ and from them we could get *all* the coefficients of the power series at 0 of the biholomorphism $g \circ f^{-1} : D \to D'$.

However, for compositional reasons, 8 coefficients are enough; if we choose a reference system in which the domains are both contained in the unit disk, the most important coefficients are the ones of the lowest degree terms.

The problem now is to obtain an effective computation of the first coefficient of the biholomorphism. This is more a matter of interest for applied mathematicians and engineers than for pure mathematicians. Luckily, some fifty years ago a method for the computation of coefficients of biholomorphism from the unit disk to a bounded simply connected domain $D \subset \mathbb{C}$ was developed by Filchakov. After translating the domain D so that $0 \in D$, the key idea is considering $4n$ points equispaced on the unit circle and the rays connecting the origin with these points. We can approximate the biholomorphism by a recursive method; indeed if we have a Fourier-style polynomial at step n, we take the images of the $4n$ points and we modify the coefficients so that the images of $2n$ points lay outside D while the other $2n$ lay inside D.

This technique inspires us the method we use to compute the parameters starting from the images of the patches. We start from numbering all patches of the same colour, then we circumscribe a rectangle with sides parallel to the coordinate axes to each patch and we take the coordinates X and Y of the lower left vertex of the rectangle, its width W and height H. The width of these rectangles give us the duration of the theme, while the X coordinate indicates when it starts; the Y coordinate gives a measure of how loud it sounds.

Note that the patches are more than just simply connected, they are indeed, with a few minor exceptions, starlike with respect to the center of the rectangle where they are inscribed. Therefore the intersection of a half line coming through the center with the patch is a segment, in particular we consider the half lines in the direction of the vertical and horizontal half axes (denoted by W(est), E(ast), N(orth), S(outh)) and of the diagonal of the rectangle (SE (SouthEast), SW (SouthWest), NE (NorthEast), NW (NorthWest)). Now we measure the length of these segments and the length of the segments obtained from the intersection of the rectangle with the half lines in the same directions. To get an estimate of the form of a region, for each of the above directions we compute the the ratio between the length of the segment obtained from the patch with the length of the segment obtained from the rectangle, such a ratio measures the rate of “occupation” of the patch, that is how far the patch is from being a rectangle. The idea is that a ratio which is near 1 means that in the given direction the patch “fills” the rectangle, while a ratio near 0 implies that in the given direction the patch leaves a large part of the rectangle “uncovered” .

The parameters we need for the composition should keep track of the variations of each region with respect to the first one: the easiest way to do so is to take, in each of the eight directions, the ratio between the occupation of the given patch and the occupation of the first one. We recorded all the data for a patch into a file and the parameters were computed by means of simple mathematical formulas, the way they are used as compositional parameters is detailed in [2].

4.3 Techniques and Computation for Percussions

The remaining parts of the painting are used to obtain the background accompaniment of the piece of music which will be realized by percussions. Since these instruments cannot play different notes, a simpler analysis, based on a frequency principle, will be enough for the compositional purposes.

We recall that in the picture there are coloured dots and short curves, which are obtained by small touches of the brush dampened in oil colours, a “thick” structure and a “thin” structure, which are both obtained from the dripping of the grey and black paint; they arise from the fact that when the liquid starts dripping on the canvas, first it produces the large drops, then a strip of colour which becomes a thread as the dripping goes on and on.

These different pieces will all be treated with the same technique, with minor adaptations to the different situations: the idea is to consider a measure given by some kind of density; indeed it appears natural that a richer covering of the canvas in the picture entails a denser musical tissue. In the composition each of these forms will be associated to a percussion instrument and for each of them one by one we will “follow the instructions” given by the painting to modify their presence as time goes on.

We then divide the painting into twelve vertical strips, lasting 2 minutes each. As for the coloured dots and the short curves, we simply count their number in each strip and compute the ratio between the number of the objects in the given strip and the number of objects in the first one. In this way we obtain a parameter which evaluates how much the associated instrument should increase or decrease its presence: since the ratio between the number of coloured dots in the second strip and the number of coloured dots in the first strip is approximately 1.5, then the instrument associated to coloured dots will produce 50% more sound between minutes 3 and 4 than in the two first minutes of the piece. In the case of short curves we also provide additional information, by specifying how many of them are placed at the bottom, at the center and at the top of the strip.

The thick lines present in the structure are "weighed" by counting their number and taking into account their area, which is computed by approximating them with suitable rectangles or parallelograms. Futhermore, in order to obtain a parameter which tells us how much we have to increase or decrease the presence of sounds produced by the instruments associated to the thick structure, we compute the ratio between the measure obtained in each strip and the measure of the first one, Finally, lines of the thin structure are counted by taking a close grid of parallel lines at constant pace and counting the number of intersections they create with the parallels. Since the directions of the thin lines of the painting are random, the number of intersections with the lines of the grid does not on depend on the orientation of the grid itself and it estimates both the number of the thread of the structure and their length.

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Mathematics and Applications

Extracting Information from Chaos: a Case in Climatological Analysis

Francesco Bonghi, Roberto Ferretti

1 Introduction

In a way, the analysis and simulation of atmospheric models is a paradigm of large-scale scientific computing problems. First, it deals with a system in which relatively simple physical mechanisms are combined to produce a complex behaviour. Second, it presents a deep interaction between the mathematical model and the approximation techniques used, and this interaction strongly affects the efficiency (and even the feasibility) of any computational approach.

With respect to the former point, it is well-known that atmospheric models are *chaotic*. Without defining in a rigorous way deterministic chaos, we just recall that the key feature of chaotic models is to have solutions with a critical dependence on perturbations. In particular, similar initial conditions may lead, in relatively short times, to completely different evolutions. Since errors and perturbations appear intrinsically when measuring the initial conditions of the physical system, as well as when performing numerical approximations, it becomes clear that after a certain critical time no hope exists of closely reproducing the evolution of the system (and even of the mathematical model).

Depending on the framework, chaos is approached with different tools and purposes. In particular, despite using similar models, *Meteorology* and *Climatology* apply two different and complementary strategies:

- *Meteorology* is typically interested in small time, deterministic analysis/forecast. In the time horizon of interest in Numerical Weather Prediction (up to about 10 days) the attempt is to reproduce as close as possible the evolution of atmosphere, and chaotic behaviour represents a limit to enlarge this time interval.
- *Climatology* is typically interested in the evolution of atmosphere for long times (years or decades) and at planetary scale. Here, chaos becomes an inherent fea-

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ture of the problem, and the tools used in the analysis are mainly of statistical nature.

The basic set of equations describing the physical phenomena taking place in the atmosphere is termed as *system of primitive equations*. This system incorporates at least conservation of mass, conservation of momentum, conservation of energy and ideal gas law, but can also account for a more complex physics. Due to its intrinsic complexity, and depending on the scales and phenomena of interest, the system of primitive equations has generated an extensive hierarchy of models of use in meteorology and climatology:

- *Full models* incorporate as many physical mechanisms as possible to obtain a more accurate description.
- *Intermediate complexity models* extract only the most significant features of the full system so as to obtain a more tractable model.
- *Toy models* are oversimplified and formulated in low dimension, but retain some particular feature of the original system and may be used to obtain qualitative indications on a specific physical mechanism.

The most famous among toy models is probably the *Lorenz system*, proposed in 1963 by E. Lorenz in a paper [2] which turned out to be of the greatest impact on the scientific community, by initiating the study of deterministic chaos. We will examine more in detail some features of the Lorenz system in the next section.

2 Climatic regimes

A point of great interest in climatology is the study of *regimes*. A climatic regime is a configuration which tends to be kept for some time. A classical example is given by the so-called *Azores High*, an anticyclonic area which sets up with some periodicity over the Azores region in north Atlantic (see Fig. 1).

Clearly, a regime is not associated to a precise climatic configuration, but rather to a set (also termed as a *cluster*) of similar states. In more mathematical terms, it corresponds to a neighbourhood of a given (average) point in the space of configurations of the system. The points associated to different regimes act as some sort of metastable equilibria, around which the systems may evolve for a while.

At a very global level, the chaoticity of the system might be interpreted as a series of transitions (with unpredictable times) between different regimes. This approach, which has a well-established tradition started in the late 80s (see [5]) aims at characterizing climatic regimes as some sort of Markov chain. We will soon try to clarify this approach by examining the behaviour of the Lorenz model.

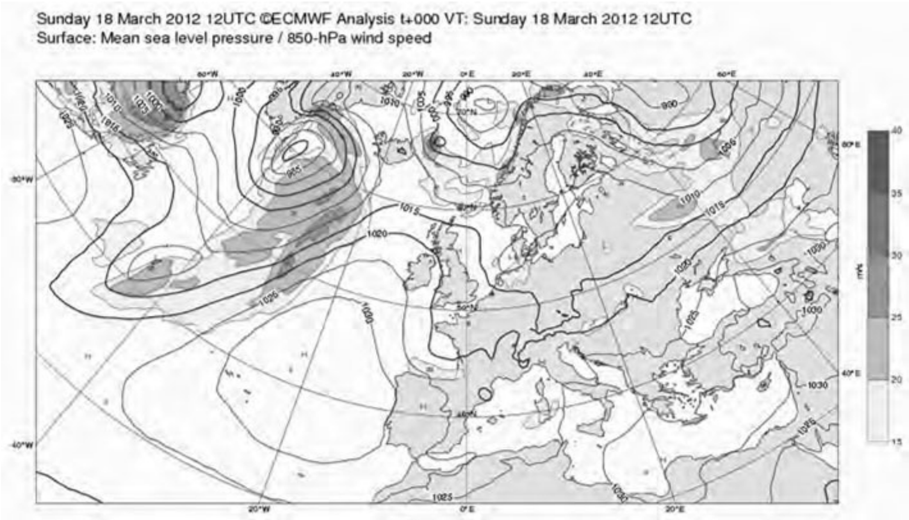

Fig. 1 A typical climatic regime: the Azores High (courtesy of ECMWF)

2.1 A toy model for regime transitions

The Lorenz model originates from the so-called *Rayleigh–Bénard experiment* (see Fig. 2), in which a fluid is heated from below and cooled down from above, thus causing the appearance of convective cells.

Describing the physics of a single cell in terms of a Fourier series, and truncating to the first terms, Lorenz obtained the differential system

$$\begin{cases} \dot{x} = \sigma(y-x) \\ \dot{y} = \rho x - xz - y \\ \dot{z} = xy - \beta z \end{cases}$$

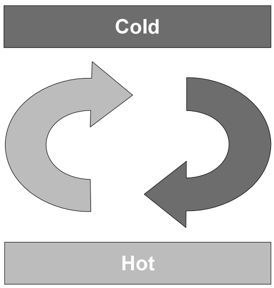

Fig. 2 A sketch of the Rayleigh–Benard experiment

in which, depending on the value of the constants σ, ρ and β, a chaotic behaviour may appear. The upper plot of Fig. 4 shows a typical trajectory of the Lorenz system on the famous butterfly-shaped attractor.

Despite the very rough mathematical formulation, the variable x still keeps a physical meaning: it represents the main component of vorticity – in other words, its sign says whether the cell is rotating clockwise or counterclockwise. The lower plot of Fig. 4 shows the evolution of the x component along the trajectory of the upper plot. Since the two wings of the Lorenz attractor correspond to positive and negative values of x, a transition of the trajectory between a wing and the other is recognized by a change in the sign of $x(t)$ and has the meaning of a change of direction in the rotation of the cell.

Looking again at the lower plot of Fig. 3, it should be apparent that the system swaps between two different states ($x(t) > 0$ and $x(t) < 0$), without any fixed periodicity. This mechanism is therefore an example (most likely, the simplest example) of regimes in a chaotic evolution.

This situation is represented as a Markov chain in Fig. 4. The chain switches between two regimes, which have two average holding times T_1 and T_2. In the specific case of the Lorenz system, these two holding times coincide.

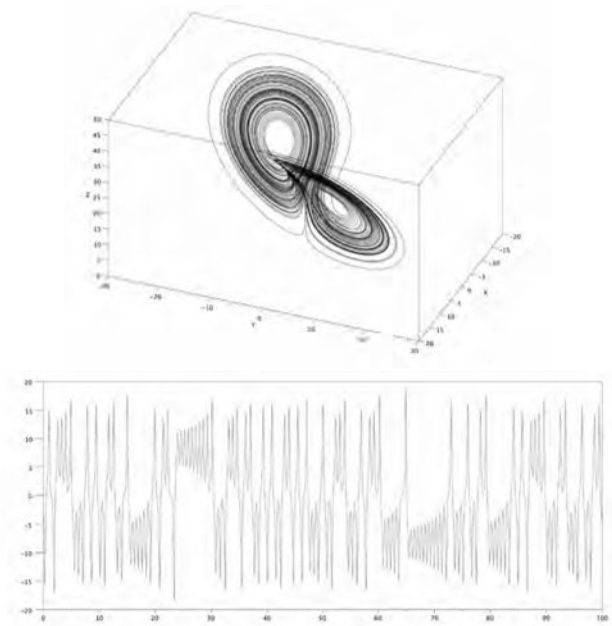

Fig. 3 A trajectory on the Lorenz attractor and its x-component as a function of time

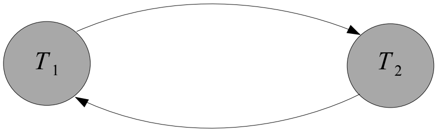

Fig. 4 Markov chain corresponding to the Lorenz system

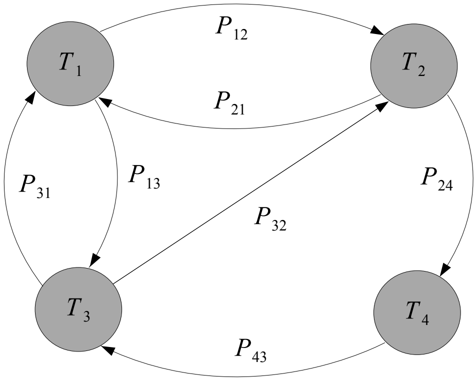

Fig. 5 Markov chain in a more general case of multiple regimes

2.2 *The general case*

In the general case of real climatic regimes, the situation is clearly more complex. The endpoint of the Markov chain approach could be in this case a graph like the one depicted in Fig. 5, in which the i-th regime is characterized by its holding time T_i, as well as by the probabilities P_{ij} of transition to other regimes (as usual in Markov chains, we should assume that $\sum_j P_{ij} = 1$). Although a complete representation of regime transitions is presently out of reach, still this approach represents a promising direction of research, in particular concerning the study of physical conditions which lead to regime transitions.

3 Numerical aspects

Long-term simulations, as the ones involved in climatological analysis, require some special care. Since a chaotic evolution basically appears as a stochastic process, it should be clear that, as soon as chaotic model are under consideration, the accuracy of numerical approximation is no longer the main issue. Rather, it would be more important that the discretized system could preserve as much as possible the statistical properties of the original model.

In nondissipative (in particular, Hamiltonian) dynamical systems a key role is played by the *invariants* of the system. To fix ideas, we can focus on the total energy. It turns out that, even for systems for which the total energy is constant, energy may or may not be preserved by numerical schemes, regardless of the accuracy of approximation.

A simple example can be carried out with the equation of the undamped harmonic oscillator, that is

$$\ddot{x}(t) + x(t) = 0,$$

whose solution is of the form

$$x(t) = A\sin(t - \phi).$$

The oscillator has been simulated on the interval $[0, 200]$, with $A = 1$ and 1000 time steps, by means of two classical approximation schemes, the Crank–Nicolson method and the second-order Runge–Kutta method – while these two schemes have a comparable accuracy in computing the solution, the latter does not conserve energy. Fig. 6 shows the histograms of the positions of the exact solution (upper plot) versus the Crank–Nicolson (center plot) and the Runge–Kutta (lower plot) methods. Due to the expansion of oscillations, an increase of energy appears in the Runge–Kutta approximation, and this results in a deformation of the histogram. Therefore, the statistical properties of the solutions are also changed, and the change is even more dramatic on longer time intervals.

The message could be summarized by saying that, especially in long-term simulations, it is crucial to avoid undue introduction or dissipation of energy in the numerical approximation. In the class of Hamiltonian systems, this leads to the use

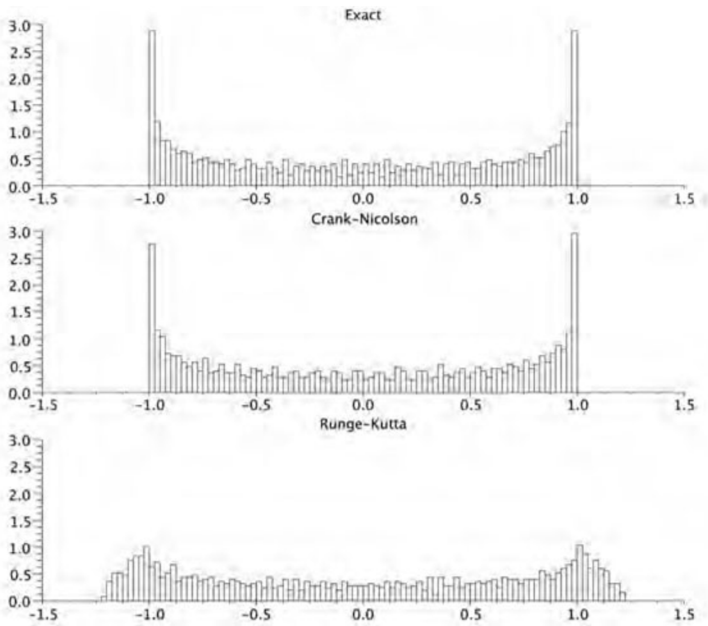

Fig. 6 Histograms for the harmonic oscillator and two different numerical approximations

of the so-called *symplectic approximations*, which have the property of retaining at discrete level the geometric structure of the exact dynamical system.

4 A test case

The ideas discussed so far go in the direction of

- Constructing an approximation for primitive equations, which could retain a certain number of qualitative features of the original model (e.g, conservation of energy).
- Recognizing recurrent or metastable cluster of states in the space of configurations of the approximate model.

We show some preliminary results obtained with the procedure outlined above. The starting point is a large-scale model of the atmosphere, the *barotropic quasi-geostrophic equation* (see [3, 4, 6]). The construction of a discrete approximation has been been carried out with methods which conserve energy (and, in fact, preserve the Hamiltonian structure) in the discretization of both space derivatives [7] and time derivatives [1]. Although the model could lend itself to relatively accurate planetary scale simulations, we start from a very coarse discretization obtained by truncating to a low number of harmonic components, so that only the largest scales of the solution can be recognized (in some sense, this is still a toy model).

Fig. 7 shows a typical evolution of the model, simulated on a time interval of about 1800 days. According to the results of the simulation, the space of configu-

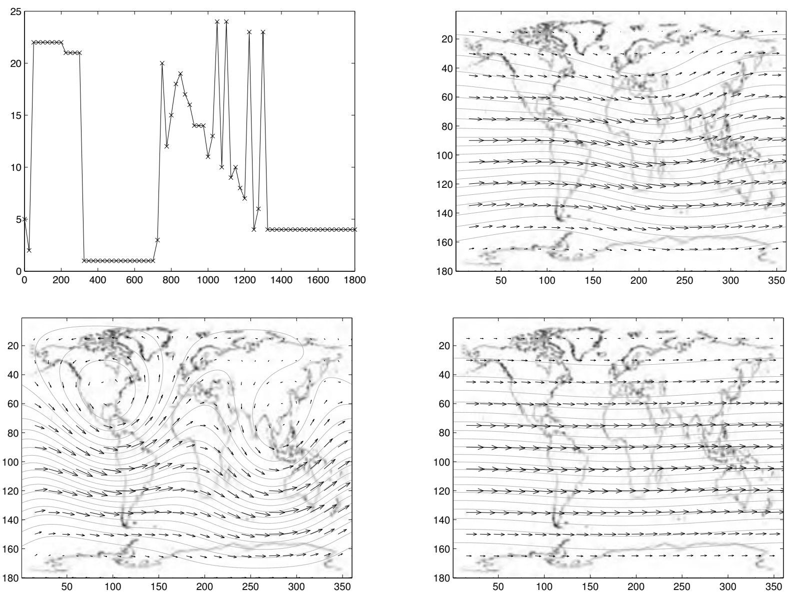

Fig. 7 Clustering diagram and three different regimes in a simplified model on the sphere

rations has been partitioned in a certain number of clusters, and the number of the current cluster versus time is plotted in the upper left graph. It can be recognized that at least three regimes appear in the time interval under consideration: the first is attained near the time range $[50,200]$, the second near $[300,700]$ and the third for $t > 1300$. The three regimes are displayed in the other plots, which show the atmospheric velocity field versus the earth surface. Clearly, at such a low resolution the model does not provide an accurate description of the physical behaviour, but nevertheless succeeds in reproducing the onset of simple climatic regimes.

5 Conclusions

The simulation of the atmospheric system is a typical problem of scientific computing, in which the interaction of the mathematical model with its approximation needs special caution to preserve the features of interest for the specific case. In particular, the approach of climatology requires to produce statistically significant long-term simulations, and this urges the use of energy preserving numerical approximations. Despite the chaotic nature of both the system and the model, we have shown how the experimental observation of its physical behaviour (in particular, the emergence of climatic regimes), as well as the study of simplified models, suggests a global technique to extract significant and concise informations on its evolution.

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On the Tangible Boundary between Real and Virtual

Andrien Mondot

Adrien M / Claire B is an art practice investigating digital and living arts founded in 2004. The practice creates performances ranging from shows to exhibits, mixing the real and virtual realms, thanks to their own customized digital tools. They focus on the human dimension and body, which they place at the centre of the technological and artistic challenges, using contemporary tools to create timeless poetry, and to build and bring into play a visual language based on amusement and delight, eliciting imagination. The artistic project is currently led by Adrien Mondot and Claire Bardainne.

The practice is located in a research and creation atelier, in the centre of Lyon.

A first work about juggling led to the creation in 2003 of a small piece entitled *Fausses Notes et Chutes de Balles* (Sour notes and Ball Falls). This minimalist duo for a juggler and an accordionist is a light show, suitable for any event. The artists are freed from the constrictions of the stage and can therefore go out into public space to take part in street festivals and white nights, or perform in market places or during informal evenings.

Convergence 1.0, created in 2005 was the founding act of the Adrien M practice and focussed on the research themes of living arts and digital art. This show was the manifesto of the transdisciplinary approach of the practice. It was the result not out only of the desire to mix juggling and computer science, but also of the wish to make these themes, which are deeply rooted in Adrien Mondot's universe, accessible to artistic creation.

Pursuing the research on juggling and digital arts that started with *Convergence 1.0*, *reTime* was a short show exploring new visual matters. In order to develop the mix of disciplines and digital experiments, the practice organized various workshops. Procedures and participants might have varied, but the goal was always the same: to provide a work and research place for artistic encounters and investigations on hybrid matters. The show *Cinématique* (Kinematics), an abstract inquiry about movement created in January 2010, was related to this field of research, half way between dance, juggling and motion design. For three seasons (from 2005 to 2008) the company worked in partnership with the *Manège de Reims*, a French national theatre company, and from 2009 to the end of 2011 they were engaged in new re-

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Andrien Mondot
Claire Bardainne, Lyon (France).

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search with the French theatre company *Hexagone-Scène Nationale* of the city of Meylan.

Being at the point of intersection between the arts of the circus and the digital arts, the work of Adrien Mondot is also connected to the atelier “Arts-Sciences” supported by the cultural departments and the *Comissariat à l’Energie Atomique*.

In 2011 the practice became Adrien M / Claire B and from that time Adrien Mondot’s creations have been co-authored with Claire Bardainne. They mix movement and graphics, digital design and construction of space, intuitive approach and dramatic scripts, always driven by the search for a living digital art. One of the main goals rekindling the company is the wish to transcend the space of the stage and the duration of the show.

The exhibition *XYZT, Les paysages abstraits* (XYZT, Abstract landscapes) is a visual and sensorial experience which enlaces the visitor’s body, thus placing him in a realm between visual arts and living arts. In December 2011 they co-authored the conference-show *Un point c’est tout* (Once and for all[1]) at the *Hexagone-Scène Nationale* theatre in Meylan, in which the bases for the research are revealed, the very substance that shapes the actual company program.

Adrien Mondot, who founded the company in 2004, is a multidisciplinary artist, a computer scientist and a juggler. His work explores and challenges movement, forming a crossroads between juggling and information technology.

At first a computer scientist, he worked for three years at the Grenoble *Institut National de Recherche en Informatique et Automatique* where he studies and creates new tools for graphic design that go beyond realism. In the meantime he also develops, for various cultural institutions, programs helping to control the complex anamorphic problems of image projection.

Invited by the choreographer Yvann Alexandre to be part of the collective creation *Oz*, he discovers dance in 2003. The opening of his own practice gave him the opportunity to mix digital art with sound, juggling and movement, and investigate the relationship between technological innovation and artistic creation. In his performances, thanks to his own customised tools, he goes beyond the laws of gravity and time, he scrambles the tracks, challenges the circus art and the computer science in a play of magic, choreographic and poetic illusion. He also developed collaborations with other artists, namely Kitsou Dubois, Stéphanie Aubin, Ez3kiel and others, within the multidisciplinary workshops that he regularly organizes to promote his research. He also participated in the latest show by Wajdi Mouawad: *Ciels* (Skys) created in Avignon in 2009.

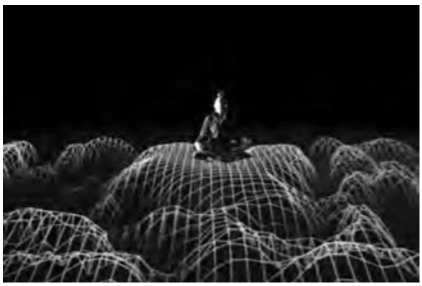
In 2004, his project *Convergence 1.0* was awarded the Prize *Jeunes Talents du Cirque*. He was supported by the *Société des Auteurs et Compositeurs Dramatiques* (Society of Dramatic Authors and Composers) in the “new shows” category for the creation of the circus performance *Kronoscop*, which was inspired by *reTime*. With *Cinématique*, the Adrien M company won the Grand Jury First Prize of the international competition *Da se et Nouvelles Technologie*s organized by the *Bains Numériques* Festival in Enghien-les-Bains, in June 2009. He met Claire Bardainne

[1] Translator’s note: The title of the show, if translated literally, would be: One dot and that’s all. There is an untranslatable play on words in this French expression that the artists have used.

in 2010 during the workshop *Labo#5*. In 2011 they decided to become partners and the company became Adrien M / Claire B. Still oriented towards the search for a living digital art, the creations are now co-authored. Together they created, in August 2011, the digital scenography for the show *Grand Fracas issu de rien* (Big crash from nothing), a group performance directed by Pierre Guillois.

They co-signed the exhibit *XYZT, Les paysages abstraits* in September 2011, and the show *Un point c'est tout* in December 2011 at the *Hexagone* theatre in Meylan.

Claire Bardainne is a visual artist, scenographer and graphic designer who graduated from the *Ecole Estienne* and the *Ecole Natonale Supérieure des Arts Décoratifs* of Paris. Her visual research focuses on the connections between sign, space and path, investigating the back and forth between real and imaginary realms. Between 2001 and 2005 she worked on several projects related to urban mobility with the *Atelier Ici Même Paris*, and is also part of the project *Troll*, lead by the architectural practice AWP, which consisted of a series of workshops that concluded with an evening performance in Rome in 2005 with the practice Stalker. In 2004, with Olivier Waissmann, she opened the practice BW whose activities focussed on the design of visual identity systems, exhibition signage, and multi-media graphic art, especially for culture and architecture. In 2007 she was a fellow-in-residence at the McLuhan Program in Culture and Technology of the University of Toronto, where she started a project entitled *Wicklow* that mixes drawing, micro edition and performance. From 2007, as a visual artist, with her graphic works and image creations, she has provided complements to the theoretical works of the researchers in sociological imaginary from the *Centre d'Études sur l'Actuel et le Quotidien* of the Sorbonne in Paris, a research unit focusing on the new social forms and contemporary imaginary. She is a contributing editor of the yearly journal *Cahiers européens de l'imaginaire* published by the CNRS (*Centre National de la Recherche Scientifique*), and, with Vincenzo Susca, she co-authored an art book entitled *Récréations. Galaxies de l'imaginaire postmoderne*, published by the CNRS Editions of Paris in 2009.



She met Adrien Mondot in February 2010 while participating in the workshop *Labo#5*. They co-authored the interactive digital show entitled *Sens dessus dessous* (Upside down) released at the Poitiers Auditorium during the 2010-2011 season.

In 2011 they decided to become partners and the company became Adrien M / Claire B. From that time on, the works created by the company, oriented towards the search for a living digital art, are co-authored. In August 2011 they created the digital scenography for the show *Grand Fracas issu de rien* (a collective performance directed by Pierre Guillois) played at the *Théâtre du Peuple* in Bussang. They created, in September 2011, the exhibit *XYZT, Les paysages abstraits*, and in December 2011 the show *Un point c'est tout* performed at the *Hexagone* theatre in Meylan, and the dance performance *Hakanaï*, performed at the *Festival Temps d'Images* in October 2012.

Translated from the French by Sylvie Duvernoy

Fort Marghera and the French and Austrian Plans of Defence

Mauro Scroccaro

Post-Unity Italy: Fort Marghera's beginnings as an entrenched camp

Upon the annexation of the Veneto to the Kingdom of Italy, the Italian Army's Defence Commission immediately promoted studies to evaluate the new defensive requirements dictated by the changes in the geographic situation. The first studies of the Venetian fortifications, based in part on the experiences of 1848-49, revealed the weakness of the defensive system towards the mainland centred around Fort Marghera (Fig. 1).

Fig. 1 Fort Marghera: the French barracks

Fort Marghera showed serious structural defects that rendered it vulnerable to bombardment. The studies also spoke of a project to construct a bridgehead in Mestre to "support the operations of the army acting in defence of the Venetien

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provinces"; the redoubt of such an entrenchment would have been Fort Marghera. The plan proposed six works: three large forts (one on the Malcontenta drainage channel, one "in front of Mestre", and one at Campalto) and "three small, closed forts or batteries" (Dogaletto, Zelarino and Carpenedo). In the following years the entrenched camp of Mestre was developed on the basis of these first ideas. Successively, in 1881, a specific project to modernise the armaments of the Venice fortifications assigned a very important role to Fort Marghera, in spite of its serious defects. It was believed necessary to augment the strength of the weaponry all around, since an offensive action might possibly have attacked all of its sides. During World War I, neither Fort Marghera nor any of the other forts in Mestre formed part of Italy's strategic lines of defence, and the weaponry was dismantled and moved to the front. The fortress was used primarily as a deposit for munitions and a workshop for the reparation of weapons, a role it subsequently maintained until recent times. It ceased to function as a military base on 30 June 1996.

Fort Marghera: Description of the fortification

The construction of the fort was started by the Austrians in 1805, but above all by French during the second period of their domination of Italy (1805-1814), Fort Marghera was erected on flat, marshy terrain close to a fenland of the lagoon criss-crossed by a web of small canals and waterways. The area was inhospitable, its climate humid and malaria-prone, harmful to men and to cultivation. The fortifications took the place of the small village of Marghera, or Malghera as it was sometimes called, a stopover and warehousing district for trade with Venice, connected to the village of Mestre by the 'Cava Gradeniga', or Salso Canal, and by a road that flanked it.

The fort was positioned on the edge of the lagoon, fewer than four kilometres from Cannaregio, at the point closest to Venice, that is, where it was supposed that enemy forces could push forward unhindered to lay siege to and bombard the city on the lagoon. It became the largest of the numerous military forts in the Venetian lagoon, functioning as a base and safe haven for armed forces operating on the mainland. It was also served as a garrison for the canal of Mestre, one of the most important communication routes between Venice and the cities on the mainland. The fort exploited the military advantages of being situated in an area that was isolated and unpopulated; of being in the midst of marshes and lands prone to flooding by the Osellino River; of not being vulnerable to attack on the side of Venice, with which, in case of siege, it could continue to communicate for transportation of troops, munitions and supplies. Were it ever to be overtaken, it would not have been of any use to the enemy; the lagoon would have been defended by a numerous armed vessels and by the second line of batteries that surrounded Venice.

Fort Marghera has a pentagonal plan. It is composed of an external wall consisting of four bastions forming tenailles around a central redoubt which also has four bastions, whose sides are protected in turn by two counterguards; an external wall and a central body are surrounded by a double moat, one inside and one outside, that draw water from the Salso Canal, thus from the lagoon. Towards Mestre, on the so-called side of attack, there are three lunettes separated from the central body by a few hundred metres, conceived to augment the fort's defensive capability, and above all to make it possible to attack the enemy. While the entrance to the fort today is through the road that comes from Mestre, originally supplies and troops were transported to the fort via boats from Venice, from the small, oval-shaped port that is still found today on the side where the fort was entered. The fort was situated astride the Salso Canal, whose bed originally corresponded to its central axis. The waterway was deviated into the external moats. The fort's construction included the excavation of a military canal that ran from the rear of the fort directly to the lagoon. The bastions are formed of embankments constructed of the dirt extracted during the digging of the canals that surround the fort; only the bases of the escarpments are dressed in stone to prevent their being eroded by the water. On top of the embankments of the bastions and the curtains, which are some five metres above ground level, the artillery was placed in the open, an arrangement known as "barbette", each piece separated from the other by earthworks. The arrangement was partially modernised at the beginning of the twentieth century, on the northeast curtain of the outer wall, with a battery of eight cannons housed in concrete wells. The most significant buildings still today are the two barracks that date from the French period (1805-1814), located on the side where the fort was entered near the small port. Conceived to house troops, each of these could hold about 150 to 200 men. They served as a final defence in case the fort had to be evacuated. They are two-storey constructions with elements in Istrian stone and a fireproof roof. In the central redoubt there are also two powder houses, located in the rear spaces of the bastions facing Mestre. These have bomb-proof vaulted roofs, and here again Istrian stone was used to dress them. One of the powder houses is French, while the other was completed by the Austrians. Worthy of note is a building constructed on top of a sixteenth-century bridge, three original arches of which are visible, the only surviving elements from the original village of Marghera.

Corresponding to each bastion, the inside escarpment, there is a powder deposit with stone doorways. There are also four small "Italian" barracks, built in the 1880s, with very thick walls, reinforced roofs and a rounded side towards the bastion with slits for artillery. Alongside the barracks used today as an artillery museum, there is a small cemetery where the bodies of those who fell in the siege of 1848-49 are buried. The very large space of the fort is also featured by many other less significant buildings used until recently for storage and housing. There are also many large trees, not especially valuable, some planted by the military to supply wood for the fort, some growing spontaneously. Fort Marghera is an example of the kind of polygonal bastioned fortification that developed from the sixteenth century until the mid-nineteenth century in response to the discovery of gunpowder and the progressive refinements in ballistic techniques that had rendered medieval masonry curtain wall fortifications ineffective. Still visible in the fort is the conception of the bastioned wall, comprising a polygonal central body surrounded by moats and counterguards, but also evident is the evolution underway at the end of the eighteenth century and

Fig. 2 Plan of Fort Marghera

the beginning of the nineteenth, when the increased range of artillery fire made it necessary to keep enemy cannons far away by means of permanent defensive works outside of the principal perimeter, structured as strongholds which also served for purposes of offence. In Fort Marghera these are represented by the three lunettes on the side of attack. The fort is the result of the French legacy, and such elements are in fact also found in other fortifications of the Napoleonic period, such as the forts in Peschiera, Ronco dell'Adige and Palmanova, all of which were designed under the direction of General Chasseloup. Still today the original structure of the fort is intact, that is, it has not been covered over by later structural work. All of the defensive bastions, casemates and munitions deposits are constructed of "bomb-proof" earthworks with floors reinforced with dirt constituting the only element of protection. The arrangement of the batteries, which were situated in "barbette" in the bastions, has remained in its original state. In addition to the necessity to restore the variable buildings, such as the French barracks, the powder houses and the casemates of the Italian period, also in need of attention are the external lunettes, in a sad state of decay, especially those on the sides, which are in danger of falling apart altogether. Also requiring restoration is Fort Manin, one of the detached fortified elements which, as we have seen, was an integral part of Marghera's system of fortifications.

Fig. 3 Plan of Marghera

Why there was an entrenched camp in Mestre

20 September 1870. After the liberation of Rome the new Italian state had to create its own military organisation, replace the old weaponry, fit out a genuine navy, and define the strategic positions to fortify. Among the great topics of discussion, those of permanent defence played a fundamental role. The initial work of the Generals of the Commission of Staff was meticulous and analytical. It took one hundred and eleven sittings to produce the first projects, where, in order to make no mistakes, it was thought to fortify everything. The Italian peninsula was to be filled with forts, fortresses and entrenched camps: the military servitude that derived from all of these constructions could have saved the nation from the disaster of post-World War II rebuilding! In the first series of meetings the Commission, presided over by Luigi Mezzacapo, examined in detail the defence of the northeast borders. The first line of defence was made to coincide with the bed of the Piave River, while the second line was established along the Adige River. In the middle, to complete the reinforcement of Venice's maritime stronghold and as a manoeuvring point for an armed force, the construction of several forts at Mestre was proposed.

The importance of Venice derived from the weakness of the line of defence offered by the Tagliamento and Piave Rivers. Then, in those very years a large railway centre was developing in Mestre, which could have played a significant strategic role. Finally, fear that Austria might invade Italy, starting at Pola and taking advantage of Italian waterways (above all the Po and Adige Rivers), convinced the generals of the need to consider the Venetian stronghold among the most important in Italy, along with Genoa, La Spezia and Messina. The Commission, which took up their work again in February of 1882, thus proposed a very extensive system of mainland and coastal fortifications, in part to offer safe harbours for the fleet and to protect the most important Venetian military structure, the Arsenal. The high cost of the plan induced the government to ask for a reduced plan that could be carried out more quickly and at less expense, and thus a significant reduction of the fortification throughout Italy was decided on by General Ricotti, a firm opponent of permanent defence structures. The operation began with the reduced project: bear in mind that at that time the government was above all wary of French aggression and therefore gave special consideration to the defence of Rome and the western shore, while the alliance with Austria (the Treaty of Triple Alliance) made it an embarrassment to continue to fortify the northeast.

The Venice stronghold. Initially the intention was not just to reinforce the defence of Venice, a city that enjoyed natural barriers and was difficult to isolate during a siege, but rather to create a military stronghold for the Italian army which, with the quadrilateral of Verona reachable by navigating the Adige River, would have formed a system with characteristics that were mainly offensive and capable of controlling all of the Veneto.

The mainland front. With the defences it had Venice was only capable of sustaining an attack from the mainland for a short time. The city was too interesting from a strategic point of view: it was almost impossible to block it off completely because of its connections to both the sea and the network of waterways. This made possible the continuous flow of fresh troops and supplies, meaning that it could remain a constant threat to the flank of an enemy army.

Fig. 4 An aerial view of Fort Tron

The solution was found in the construction of an entrenched camp at Mestre and the creation of a fleet to impede outflanking on the side of the lagoon. What kind of entrenched camp was to be built at Mestre? The decision to build a bridgehead in Mestre gave rise to an almost endless debate about the kind of works to undertake, and thus on the number and locations of the forts. The complete plan called for a bridgehead with three separate forts and intermediate batteries capable of forming a system with Marghera and protecting Fusina, a very important strategic location, but the cost of the plan was ten million lire. The reduced plan called for a single fort at Campalto at an expected cost of three million lire, including the works on the maritime front. After some seesawing, the idea of a military camp to support army operations ongoing in the provinces of the Veneto was abandoned. Building such a camp for forty or fifty thousand men would have proved to be almost impossible: the particular characteristics of the territory would have required a immense tract of land, while the usable area of terrain available for the use of troops measured just slightly more than a triangle with a base length of twelve kilometres long between Fusina and Campalto, and a height of five kilometres and a half, in low-lying, unhealthy ground that was meant to be flooded should need arise. Taken as a whole, these considerations led to the opinion that the Venice stronghold was unsuitable to house troops. It was thus decided to create an entrenched camp consisting of five forts.

Fig. 5 Map of Austria ca. 1900

The definitive project

From the general plan of the camp can be deduced an ample bridgehead with a length of 18,568 metres, at a distance from Venice that varied from a minimum of 7,000 metres to a maximum of 10,400. The outer left-hand side lay on the edges of the lagoon in the delta formed by the Bondante canal and the Malcontenta drainage channel, from which the fort took its name. The outer right-hand side lay between the blocks of buildings of Pezzana, Terzo and Tessera. Forts Malcontenta and Tron are separated by a distance of 4,868 metres; Tron and Brendole by 4,619 metres; Brendole and Carpenedo by 3,966 metres; Carpenedo and Tessera by 5,119 metres. Fort Marghera is located at a distance from each that varies between 4,500 and 7,000 metres. Suitably modernised, it constitutes a redoubt of fundamental importance.

The "first generation" forts. The works of the Mestre camp were constructed on the model of the forts designed by the Austrian colonel Andreas Tunkler (they were also called the Prussian type after the state that built a large number of them), although in 1910 they underwent several modifications in a vain attempt to update their weaponry in line with progress made in modern artillery.

Fig. 6 Plan of Fort Brendole (1883)

Description of the Tunkler type of fort. This kind of construction takes the form of a six-sided polygon completely covered by masses of earth and surrounded by a deep moat that was as many as forty metres wide on the side where the fort was entered. The interior is structured so as to leave undressed masonry only on the sides of the buildings that are exposed to the defensive face. Almost all of the rooms are located along the perimeter of the building, and are linked internally by a narrow corridor that passes along the entire length of the fort, and by a series of small spiral staircases that permit access to the batteries of the ramparts. In the middle of the interior courtyard rose a transversal structure that was at least 130 metres long, in which were located the command post, the officers' quarters, the infirmary and the latrines. The classic weaponry used in this type of fortification consisted in fourteen artillery posts located on the main side (ten 149G medium-calibre defensive cannons on carriages, and four 210G howitzers) and three on each side (120G cannons), all located on the platforms on the ramparts. The four small 87B cannons located on the side of the entrance and the ten two-barrel machine-guns set up in the caponiers completed the defence against attack from both far and near.

Fort Brendole (Gazzera, 1883). Constructed in the village of Gazzera between the Marzenego River and the Rio Dosa, Fort Brendole was the first to be built and was considered an archetype for the ones that followed. The area of the fort, which covers some 15.5 hectares overall, lies in a zone that at the time was rich in waterways, and densely populated due to the mill named Mulino Da Lio, with several great mansions with chapels annexed to them, a dozen farmer's houses and four *casoni*, typical buildings with interior structures in wood, mud walls and thatched roofs.

Forte Carpenedo (1887). Located in the centre of a wooded area by the same name, composed of distantly spaced, small trees.

Fort Tron (1887). The twin of Carpenedo, it was built in Sabbioni, between Oriago and Marghera, on land comprised by the townships of Gambarare, Oriago and

Chirignago. The characteristics of the buildings found in the area serving it lead us to think of a place in the open countryside: at the time there were nineteen farmer's houses, sixteen *casoni*, and not a single public building.

Forts Tessera and Malcontenta. Long believed to be indispensable and of decisive importance, not least for the connections to batteries of the lagoon and the sea front, these were not then constructed because of the delicate strategic nature of their locations and for a series of problems related to costs.

Forte Brondolo. Even though this was not directly part of the entrenched camp of Mestre, the fort at Brondolo was very important in the context of the conception of land-based defence of the Venetian stronghold. It was begun by the Austrians and later completed by the French in the early years of the 1800s, and closed the far southern end of the lagoon, near the mouths of the Bacchiglione and Gorzone Canals and the Canal di Valle. Its form was an irregular quadrilateral and it covered an area of some 800 square metres.

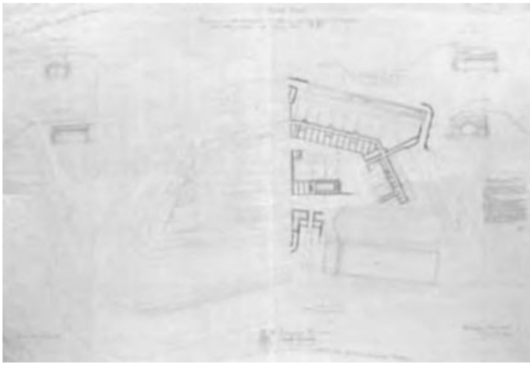

Fig. 7 Forte Tron (1887)

The "second-generation" forts. At the beginning of the 1900s international tensions, the tendency to create a diversion to internal political and social struggles, territorial claims and the fight to acquire new markets gave rise in Europe to an arms race, which ultimately reached its climax in the first world war. For its part, Italy decided to take up again and complete the projects to fortify its north-eastern front. Construction was planned along Italy's borders, from the plains in Friuli to the Ortler, of about forty new armoured batteries whose function was to hold back the enemy in the early days of a war, giving the Italian army time to arrive and set up operations. Mestre was included in the plan, which in addition to the completion of the entrenched camp, also called for adaptations to be made to the Italian system of defence in keeping with technological progress made in artillery. It had also been decided to intervene in some way in old forts, placing the primary armaments on crosspieces and utilising the six 149G cannons in installations on a central mainstay, in open emplacements, to be converted at a future time with metal coverings; the predetermined secondary armament consisted in four 74A cannons and six machine-guns. The new, definitive structure of the field was the following: 1) an external line of five forts: to the north laid out along the Dese River, from the confluence of the Zero River (Fort Pepe) to the Treviso railway (Forts Consenz and Mezzacapo); to the west and south, on the canal known as the Taglio Novissimo, a diversion of the Brenta River, to Spinea (Fort Sirtori) and Ponte Damo (Fort Poerio); 2) a second line of works comprised of Fort Tessera, in construction, and the already existing Forts Carpenedo, Brendole and Tron; 3) Fort Marghera, in a third line, was to have constituted the redoubt of the perimeter defence, a direct protection of the railway bridge that leads to Venice and of the powder houses to be established in Forts Rizzardi and Manin, while it would have immediately had under its control the railway station of Mestre, the maritime terminal of San Giuliano and the planned, extremely important intermediate port of call of Bottenighi foreseen by the regulatory plan of the port of Venice. In 1910 some of the central crosspieces of the old forts were demolished with the aim of building new structures in concrete for the six 149G cannons. The operation was almost completely useless: the Japanese had already successfully used their mobile 280mm siege mortars against the Russians at Port Arthur, and ten years later the Germans would pulverize the Belgian system of defence with "portable" 420mm cannons. Even the six newly conceived works were outdated within the space of just a few years; it suffices to think that just a few short years later, in 1911, the Italians themselves would make use of the first airplanes used in war during the bombing of Libya, and would at the same time begin production of anti-aircraft weapons, which were completely lacking in the forts, while in 1915 asphyxiating gas was used for the first time.

In any case, let us see where the forts were located and what their principal characteristics were.

Fort Giuglielmo Pepe (Pagliaga, 1909). This was the advance point of the array of Mestre. Located on the edges of the Pagliaga Valley, the fort was built on low-lying, marshy terrain, unprotected, enclosed within the meander that the Dese River makes south of Altino before it reaches the canal called dell'Osellino. The structure of the work recalls that of the central crosspiece of a Tunkler fort, with the six cannons in a line (149 A cannons installed in Armstrong cupolas with rotating barrels), dominating the sloping land in front of it, which, on the side of attack, goes down to the moat. Four retracting machine-gun turrets are located in the corners, the two front ones encased in the ground could be raised by their metallic structures about eighty centimetres when they had to fire at the inclined earthwork and moat.

Fort Carlo Mezzacapo (Marocco, 1911). Practically identical to Fort Pepe, Mezzacapo is located south of the Dese River in the village of Gatta. Its position allowed it to control the tract of road known as the Terraglio, the railway to Treviso and the road that goes from Marocco to Martellago.

Fig. 8 Fort Guglielmo Pepe (1909)

Fort Poerio (Ponte Damo, 1910). Located in the village of Ponte Damo, just north of Gambarare and close to the Brenta Canal, this fort was the definitive replacement of that of Malcontenta, and terminated the southern front of the entrenched camp. The initial project of 1908 was a faithful reproduction of the corresponding one to the north, Fort Tessera-Rossarol, and so had a two-storey structure more than nine metres high; but the following year it was decided to follow the model of Forts Pepe and Mezzacapo, with six cannons aligned on the side of attack, batteries of two 75A cannons for the flanking sides, dormitories for ninety artillery and fifty infantry troops.

Fort Giuseppe Sirtori (Spinea, 1911). Located south of the central area of the Venetian provinces called the Miranese, between Chirignago and Spinea, Fort Sirtori controls the railway to Padua, along with nearby Fort Tron. The structure is somewhat smaller than that of the other forts described above, although its basic layout is the same. The fort is seventy-three metres long, with four 149A cannons set ten metres apart. The flanks have the usual batteries of small-calibre (75A) cannons and, oddly enough, there are no machine-guns positions.

Fort Enrico Cosenz (Dese, 1911). The twin of Fort Sirtori, Fort Cosenz is located on the meanders of the Dese River just north of Favaro Veneto.

Fort Cesare Rossarol (Tessera, 1907). Intended, according to the reduced plan of 1880, to be the only fort on the mainland of Mestre, it was finally built in March 1907. Absolutely atypical, it is the most original of the new forts. Constructed on two floors up to a height of nine metres, its appearance is somewhat science fiction-like, thanks to the four machine gun wells that pop up in the middle of the slope of the front earthworks. These retracting positions could be accessed by a tunnel fourteen metres long, which sloped down gently two metres from the body of the fort. On the lower floor were located the dormitories for the troops, the deposits for food supplies and munitions, the ventilation station and the electric generator. The second floor was entirely devoted to the four positions for the cannons which, aligned horizontally, were flanked by the battery observatory and command station. Two machine-gun positions, on retracting turrets, protected the access bridge and the moat (2.6 metres deep) on what was the back side of the fort.

The conclusion of the works. At least five years of swift work were required to complete work on the entrenched camp of Mestre: the order was to finish everything within 1913. Shortly before the outbreak of the war, after more than thirty years, the stronghold of Venice was finally completed: the fortifications on land and on sea, the command posts, deposits and observatories were ready for genuine wartime manoeuvres. But the reality outstripped all forecasts: no one could have expected the appalling slaughter that involved the Veneto during the first war of the modern era.

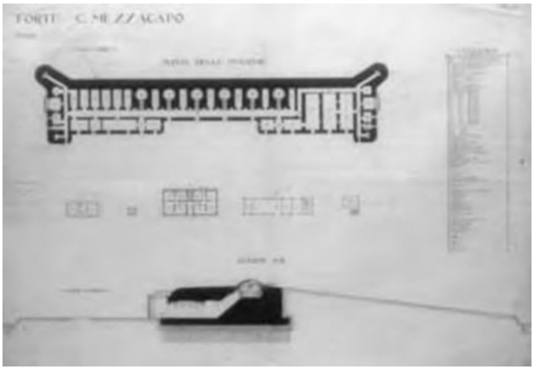

Fig. 9 Fort Mezzacapo (1911)

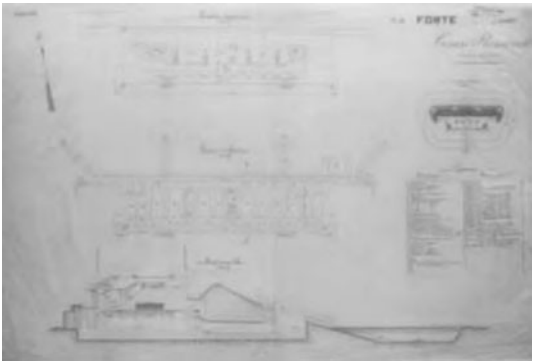

Fig. 10 Fort Rossarol (1907)

The entrenched camp at war

World War I. The Italian declaration of war only partly surprised the Austrian army deployed along the north-eastern border of Italy. For some time espionage at the service of Emperor Franz Josef had been in possession of detailed maps showing the locations and plans of the Italian forts. Thus, to deter a possible attack by Italian Alpine troops and as an offensive support for the invasion of the Po valley, starting in 1911 Field Marshall Franz Conrad von Hötzendorff established a fortified belt in the South Tyrol, exactly in front of the new Italian batteries. The shot fired from the cannon of Fort Verena at 3:00 am on 24 May 1915 signalled the beginning of the great artillery duel between the Italian batteries of Verena, Campolongo, Campomolon and Punta Corbin, and the Austrian forts of Verle, Lusern, Cherle and Vezzena (at 1908 metres above sea level), from which, on clear days, it was possible to see Venice, reachable in three days of steady marching. In the first four days of war, Italian artillery of the highlands of Asiago pummelled the Austrians, holed up in their concrete refuges, with all the gun power they had. At first the greater range of the Italian 149 cannons presented a serious threat to the enemy, who saw the cupolas of their howitzers fly away like tumbleweeds. For the first time the resistance of concrete, steel and human courage were put to the test:

> All the men in the fort have realised that something extraordinary has taken place. The effects of the explosion were felt in the most remote corners, had torn away doors and brought down objects like an earthquake. There is no more light. Short circuit; all the lights are out. In the machine room, fumbling in the dark, they unplugged the automatic circuits to prevent fire in the battery room. 'Light! Light!' And then even sharper yelling, 'Nurses! Nurses!' Trampling, pushing, cursing in the dark corridor and on the stairs. The ringing of the alarm adds to the confusion, because everyone now tries to reach his post, tries to find his rifle and equipment. Only a few maintain their composure. Finally the first portable lamps are lit and nurses with their faces covered by gas masks come running into the corridor that leads to the battery. Almost all the artillery men have taken refuge in the munitions deposit to escape the poisonous smoke of the explosion. Before going up to the second howitzer there is a heap of rubble and twisted iron that indicates that the grenade had exploded here. And above, on the turret, a fearsome spectacle of destruction unfolds before the eyes of the rescuers: the armour has been pierced and the light of day filters through the circular opening. The piece is bent over, the platform broken, the horribly mutilated bodies of the gunners lie among the pieces of steel, broken staffs and detritus. No one is left alive.

These words of Lieutenant Fritz Weber, an artillery man in the Austrian fort of Verle, aptly express the feelings of horror, stupor and rage that were surely felt by the Italian soldiers as well, when, in their turn, they were subjected to the hammering bombardment of the enemy. Once the first moments of disorientation had passed, and after having seen with relief that the Italian troops did not dare to attack to the end, the Austrians reorganised, bringing their most modern cannons into position. On 12 June 1915 a shot from a 305 mm Skoda mortar, fired from the area of Vezzena, struck the very centre of Fort Verena, killing Commander Trucchetti and forty other men of the garrison. Enemy strikes were devastating precisely because the fortified works of the highlands, of the same kind of those of Mestre, were principally equipped to receive frontal strikes, not mortar strikes, whose parabolic trajectories allowed them to fall from the skies almost perpendicularly to strike the weakest part of the defensive barriers. The destruction of Verena, and more generally, the difficulty of procuring cannons for the front zones, convinced the high command to order an extensive disarming of the forts. In September 1915 the Minister of War decided to remove seventy-two 149-calibre cannons from the armoured cupolas in the upper and lower regions of the Tagliamento River and from the entrenched camp at Mestre, which, far from the zone of operations and inadequate for the kind of battles taking place, was shut down. The war ended at Vittorio Veneto a short time later, leaving Venice feeling that it had miraculously escaped incumbent danger. Following this experience, the Italian Navy would decide to transfer the productive activity of Venice to the arsenals of La Spezia and Taranto, with the aim of preserving Venice's historic city centre from further dangers. The Venice arsenal was notably reduced, its military activities almost completely eliminated, and the structures began a slow decline along with the forts that had been constructed to defend them.

World War II. During the second world war the forts, not suitable for use in any other way, were assigned as powder houses. Defended by new anti-aircraft positions, the deposits were the target of American bombing raids more than once, but were never damaged. In the last fifty years the forts have continued to serve their role as deposits. Only in the most recent times have they been progressively decommissioned and demilitarised. Fort Tron has been left in a state of total abandon, prey to vandals and the curious in search of relics. Other forts, however, are scheduled to be surveyed by the "Coordination for the restoration of forts", an association of volunteers who propose useful adaptations for these noble structures.

Translated from the Italian by Kim Williams

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Mathematics and Ants

Myrmedrome: Simulating the Life of an Ant Colony

Simone Cacace, Emiliano Cristiani, Dario D'Eustacchio

1 Some basic facts about ants

Ants are a Family of insects that includes about 12,200 different species, counting only those studied so far. Although they all appear similar, these social insects show an enormous *biological diversity*: from their anatomical characteristics to their reproductive behaviour, from the population densities of colonies to the types of nests, from what they eat to the many kinds of interactions that many species of ants establish with different species of living organisms. Even though it is difficult to generalise, let us describe the 'typical' biology of these extremely evolved creatures.

1.1 Life cycle

Let's take the moment in which a queen founds a new colony as our point of departure. The queen is a winged ant whose ability to fly is limited, but nevertheless permits it to mate while flying. After mating with one or more males, also winged, it sheds its by now useless wings and begins the search for a suitable location for a new colony. Once the site for the nest is found, the queen may either lay its eggs immediately, or wait as long as several months, depending on climatic conditions. Usually during this period the queen remains inactive and does not eat. The first workers born will leave the nest and search for food, while the queen itself never leaves the nest and will continue to be cared for and fed by its offspring until its

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death. The queens with the longest life spans may live up to fifteen or twenty years after the colony is founded, and lay a number of eggs, depending on the species, that ranges from a few dozen to several million, without mating again. The eggs develop into individuals who are either fertile (males and queens) or are sterile (workers). Fertile individuals are winged, never leave the nest until it is time for them to mate, and are nourished by the workers. In their turn the workers, sterile females, carry out all the work necessary for the colony: foraging for and storing food, defending their colony, defending their territory, building and maintaining the nest, caring for and protecting the broods. Thus it is workers that we usually see travelling around.

Once a year, usually in spring or autumn depending on the species, the unmated males and queens leave the nest *en masse* to mate in flight, making what is known as 'nuptial flights'. Shortly after mating the males die, and only a very small percentage of newly-mated queens are successful in founding new colonies.

1.2 The usefulness of sterility

The predominance of workers within a colony would appear to contradict the theory of evolution: thousands of sterile individuals whose sole purpose is to serve other individuals! The contradiction is only apparent; the principles of the theory of evolution must be sought in the kinship that bind members of the colony. In fact, all ants that belong to the same colony are the offspring of a single queen and thus share part of their gene. To be more precise, because of a peculiar form of vital cycle called *haplodiploidy*, a worker shares 3/4 of its gene with its 'sisters', while having in common only half with its mother. By this same principle, any given worker (if it could reproduce) would have only half of its gene in common with its offspring. Thus, cooperating to increase the number of sisters means allowing 3/4 of itself to live on, while reproducing itself would mean renouncing 1/4 of its own gene. This is one of the most accepted theories for explaining the 'altruistic' behaviour of these insects.

1.3 Division of labour

The workers in a colony are generally divided according to their role: there are ants who take care of the nest and the brood, others who forage for food, and still others, usually larger, with a larger head and above all, larger jaws, who defend the territory, patrol the paths that lead to food, and break the larger prey into bits. In the species in which there is an accented polymorphism, that is, noticeable differences in size among workers, the division of labour may be related to their size. In some species the difference in size is such that the largest workers can be considered as a separate caste, the soldier caste. In any case, flexibility of behaviour is one of the great strengths of ants: the division of labour is never hard and fast, so that roles are interchangeable and no function that is vital for the colony is left untended in case of an unforeseen event.

1.4 The search for food

Foraging for food is a very expensive activity for a colony, because workers can easily be lost, killed by predators or other causes. In many species there is a small number of workers, called explorers, who are used for this purpose. When an explorer identifies a source of food, it returns to the nest, leaving a trial marked by an odorous substance secreted by glandular structures. This odor trail quickly leads other workers to the food, thus creating the long lines of ants that we are used to seeing. In turn, foraging ants reinforces the trail until they find food or let it evaporate when the food vanishes. This is an extremely effective and secure technique which, thanks to the evaporation of the odor trail, makes it possible to determine the shortest trajectory between food and nest. Let's suppose that there are two pathways, A and B, that lead to the same source of food, A being shorter than B. Since the initial intensity of the odor trails along the two pathways is the same, both paths are followed by the same number of ants. However, over time, path A, being shorter, will have a greater concentration of odor than path B, and will thus attract a greater number of ants. Within a short time, all the ants will choose path A.

This amazing property has fascinated many mathematicians and computer scientists, inspiring the creation of optimisation algorithms in which virtual agents deposit virtual odors in order, for example, to minimise the time it takes data to travel through communication networks (telephones, Internet, etc.).

A possible experiment
If you come across a line of ants, you can try to eliminate the odor in a small part of the path, for example, rubbing it repeatedly with your finger. How do the ants respond when they reach the interruption? How much time does it take them to restore the trail's continuity?

1.5 The "social stomach"

Ants have a stomach and a 'pre-stomach' (ingluvies), which in many species performs a fundamental role in nourishing the colony. The food ingested fills the ingluvies, but only a part of it goes on into the ant's stomach where it is digested. Such "social stomach" serves the purpose of a deposit, and its contents can be regurgitated for other ants who request it (a form of feeding known as trophallaxis). In this way is formed a food chain which is capable of feeding the entire colony in a short time, in particular those ants who do not participated directly in the gathering of food.

1.6 The division of territories

Ants are anything but harmless; they are genuine machines for war, capable of literally tearing 'to bits' anyone or anything that bothers them or threatens their food and offspring. As already mentioned, in some species the soldier caste is dedicated to defending territory and the odor trails. In particular, they patrol the paths that lead to food, attacking possible predators and delimiting the borders of the territory used as a hunting ground, intimidating soldiers and workers from neighbouring colonies. They are capable of remaining upright on their back legs and immobile for hours, as if to say 'Go no further!'. Competition for food is often intense, and it never happens that two paths leading to food but marked by different colonies cross. This leads to a genuine division of territories.

How can you get rid of ants in your house?
It is useless to mount a mass attack: the queen, safe in its nest, will continue to replace the ants that die by giving birth to new ones. It is often useless as well to block the holes where ants come inside, since they are sure to find another way! The best tactic is to eliminate the odor trails with a wet cloth, and above all, to remove the source of food. This way the ants will find an alternative source of food and leave you alone.

2 Self-organisation and the computer

Are ants 'intelligent'? Are they 'aware' of their actions? Who decides how tasks within the colony are assigned? One of the great discoveries that made the ant world famous is that ants are not intelligent, or at least not intelligent enough to understand the consequences of their actions. For example, when an explorer returns to the nest leaving behind an odor trail, it is not in the least aware that in a very short this action will produce a line of ants. The complex organisation of the ant colony is simply the fruit of small, local and unseeing actions of single ants. In other words, the colony is self-organised. It has been amply demonstrated that an individual ant does not have the capacity to 'reason', that is, to solve problems. For example, as discussed in §1.4, 'knowing' how to find the shortest path between food and nest is not a capacity possessed by an individual ant (if anything, it is possessed by the entire colony), but is instead the overall result of simple individual actions, such as scenting an intense odor. This sheds light on the great power of natural selection. Allowing a personification, we might say that Evolution, in order to achieve a certain result, has two possibilities: it either positively selects single individuals capable of producing that result, or it positively selects a community of social beings who interact in such a way that the overall effect of their actions is the desired result.

The human brain as well appears to be organised like an ant colony: a great number of elementary units (neurons) carry out simple actions (activation/deactivation) whose overall result is the capacity to reason. Further, on large space-time scales, humans themselves, seen as social animals, are not altogether alien to the concept of self-organisation. Something similar to self-organisation might be found in the great migrations of the past, where men were the cause of something which they did not entirely know or control.

Self-organisation, which might also be called ‘unaware organisation’, is a phenomenon of great interest to mathematicians and computer scientists. Mathematicians are attracted to it simply because they are fascinated by all beautiful ideas. Instead, computer scientists see in this concept an analogy with computers, instruments designed to carry out billions of simple arithmetic operations per second, genuine electronic ant colonies capable of solving extremely complex problems. In the majority of programs, the organisation of resources for calculation is imposed from above, that is, by the programmer. However, there exists a class of algorithms based on the behaviour of ants, not coincidentally called *ant colony algorithms*, in which organisation emerges spontaneously.

3 The software

Myrmedrome (from the Greek *myrmex*, ant, and *dromos*, path) is a free software program written in C++ using the libraries *SDL* and *OpenGL*.[1] In particular, SDL makes it possible to manage the window of the program, the keyboard and the mouse, while OpenGL makes it possible to produce 2D and 3D graphics in real time, taking advantage of the computer’s video card.

Myrmedrome is a middle ground between a simulator and a videogame, and was conceived as an educational tool for those who want to learn about the ant world. The user can both tune different parameters and observe the evolution of the life of one or more colonies, and interact in real time with the virtual environment by means of numerous tools, modifying in consequence the course of the simulation. The heart of the about 10,000 lines of Myrmedrome’s code consists of three principal blocks: the *biological engine*, which manages the simulation, the *user interface*, which manages the interactive tools, and the *graphic engine*, which manages the graphics.

3.1 The biological engine

How can a digital representation of an actual ant be created in a computer? To answer this question we have to adopt the terminology proper to languages of *object-oriented programming* (like C++) and thus to construct a *class*, that is, an *abstract* description of the concept of ant. To be more precise, it is necessary first of all to identify the properties, or better, the attributes, needed to characterise the class it-

[1] www.libsdl.org, www.opengl.org.

self. In the case of the class *ant*, the possible attributes are, for example, the x, y coordinates of the *position*, or the integer numbers that quantify hunger and age, or the logical variables (with true or false values) that provide the ant's status, such as *it is alive, it is eating, it is fighting, it is following an odor trail*, and so forth. The more detailed and specific the list of attributes, the more faithful the digital representation of an actual ant. Just to give an idea, the ants of Myrmedrome possess 36 attributes.

At this point, however, the class *ant* is still unusable; what also needs to be specified is *what* it *knows how to do*, or better, its *methods*. For example, methods can be created for *exploring territory, following an odor trail, returning to the nest*, and so on.

Beginning with one class, it is possible to use the concept of *inheritance* to construct classes that are increasingly *specialised*, classes which in fact *inherit* all the attributes and methods of the parent class while adding new ones. For example, to divide the ants into castes, it is possible to construct the classes *workers* and *soldiers*, each of which is child of the class *ant*, but which has, as can easily be imagined, different added attributes and methods, described as follows:

Worker

- Explores the hunting territory in search of food; once food is found, it eats and fills its pre-stomach (ingluvies).
- Returns to the nest, leaving an odor trail that evaporates with time and attracts the other ants of the colony.
- Once arrived back to the nest, it deposits the food, which is used to feed the queen and the larvae.
- If its ingluvies is full, it regurgitates the contents into the mouths of other ants of the same colony (trophallaxis), so that resources are distributed.
- If an enemy ant is encountered, it can either fight to the death or return to its home nest, leaving an odor trail of alarm pheromones to alert the soldiers and guide them to the battle zone.
- If live prey is encountered, it climbs on top of it and tries to kill it.

Soldier

- Patrols the entrance to the nest.
- Receives nourishment via trophallaxis from workers and from other soldiers.
- If it comes across an odor trail of alarm pheromones, it hurries to the battle zone; once it encounters the enemy, it attacks and fights until death; if it wins, it returns to the nest.

The class *ant colony* is created in a way that is analogous. Among its attributes is a list of worker ants and a list of soldier ants, each of which in its turn contains a certain number of *objects* (that is, *concrete realisations* of abstract classes), of worker and soldier types respectively. The class *ant colony* also has further attributes to describe the *odor* and *alarm* trails, the *position of the nest*, the amount of *food* present in the nest, and so forth. Its methods are outlined as follows:

Ant colony

- Manages the workers.
- Manages the soldiers.
- Manages the births and deaths of the ants.

It is further possible to construct a very simple class *prey*, which describes a particular kind of moving *food*. Each prey is represented graphically by a small worm of variable length, which wanders around the hunting territory and whose sole purpose is to be attacked and killed.

Finally we arrive to the class *biological engine*, whose function is to manage the entire simulation. Among its attributes is a list of ant colonies and a list of prey, while its methods are as follows:

Biological engine

- Makes time go by.
- Receives instructions from the user interface (explained in the following section) to make modifications to the simulation.
- Manages the food present in the territory.
- Manages the prey.
- Manages the ant colonies and makes the odor and alarm trail evaporate.

It should be pointed out that the simulation is never rigid: each event always has a certain probability of not being correctly carried out. For example, an ant might lose an odor trail and become lost, or it might be unseated by a prey. This behaviour reproduces what really happens in nature, and makes the entire system extremely flexible and fault tolerant.

3.2 The user interface

The purpose of the class *user interface* is to provide the biological engine with instructions about how to modify the current setup of the simulation, on the basis of the commands given by the user (by means of graphic components such as buttons or sliders).

Before beginning the simulation, the user can decide the number of enemy colonies (up to 5) and specify, for each colony, the position of the nest and the number of workers and soldiers (up to 1,500 and 1,000 respectively). Once the simulation has begun, it is possible to change the value of numerous parameters: the speed of the simulation, the rate of evaporation of the odor trails, the quantity of food required to nourish a larva, the time needed for the development of a larva, the maximum age and hunger of an ant, the duration of its meal and the speed at which a battle is fought. On the other hand, there are various tools available for interacting with the virtual environment (Fig. 1), as described in what follows.

Fig. 1 Interactive tools in the user interface

Interactive tools

- *Lens*: enlarges and reduces a selected portion of territory.
- *Hand*: if the scene has been enlarged, makes it possible to scroll the selected portion of territory.
- *Nose*: activates and deactivates the visualisation of odor trails that indicate the presence of food.
- *Warning sign*: activates and deactivates the visualisation of alarm trails that indicate the presence of enemy ants.
- *Fingers forming frame*: makes it possible to select a colony or an individual ant, opening a window of information regarding the object selected.
- *Flip-flops*: If the scene has been enlarged, makes it possible to follow the ant selected, positioning it in the centre of the screen.
- *Fingers picking up*: makes it possible to move an ant.
- *Finger pressing down*: makes it possible to kill ants and eliminate odor and alarm trails.
- *Plate of spaghetti*: makes it possible to add or subtract food.
- *Worm*: makes it possible to add a prey in the hunting territory.

3.3 The graphic engine

The purpose of the class *graphic engine* is to render on the screen each element of the simulator and to update in a fluid manner (typically 60 frames per second) the animations of all ants and all prey.

Generally the user recognises graphics, especially if well done, as a strong point of a software, while often remaining unaware of or taking for granted what lies hidden behind it, that is, the thousands of lines of code necessary to implement techniques of visualisation that are constantly evolving. In the case of Myrmedrome, which has a simple classic graphic style like the videogames of the 1980s, the graphic engine still occupies more than two-thirds of the entire code. This is because the OpenGL graphic library used, while extremely efficient in terms of rendering speed and suitable for any need that arises, is a very low-level library, in the sense that it provides a very limited set of graphic primitives (essentially points, lines and triangles), leaving to the programmer the chore of creating the more complex elements from scratch. This is the price that must be paid in order to achieve a personalised program that doesn't feature the classic buttons and standard windows that typify all operating systems.

One of the main techniques used in the production of the graphics is the so-called *texture mapping*. Using a program for retouching photographs (such as the excellent and free *GIMP*[2]), an image is prepared containing all the elements to be visualised. This is taken forward via code, loading the image in memory and generating from it many small images (textures), one for each element. Then suitable rectangles are drawn on the screen in the desired position, applying (or better, mapping) on each rectangle the corresponding texture, and thus constructing the scene as a collage of textures. In the case of a button, at least three textures are necessary, one of which represents the button 'at rest', one to be used when the mouse moves over the button, and a third to use when the button is activated. Animations can be created in a similar fashion, creating a texture for each of the frames of the animation and then mapping them sequentially (and periodically) on the same rectangle, moving it and rotating it in time, possibly re-scaling it, so as to create the illusion of motion typical of an animated cartoon. Fig. 2 shows all the textures associated with the movement of an ant, and one particular mapping. Fig. 3 shows some details of the simulator in action.

Myrmedrome for Linux, Mac and Windows can be downloaded for free from the website *www.not-equal.eu/myrmedrome*.

Translated from the Italian by Kim Williams

Fig. 2 Animation of an ant by means of texture mapping

[2] www.gimp.org.

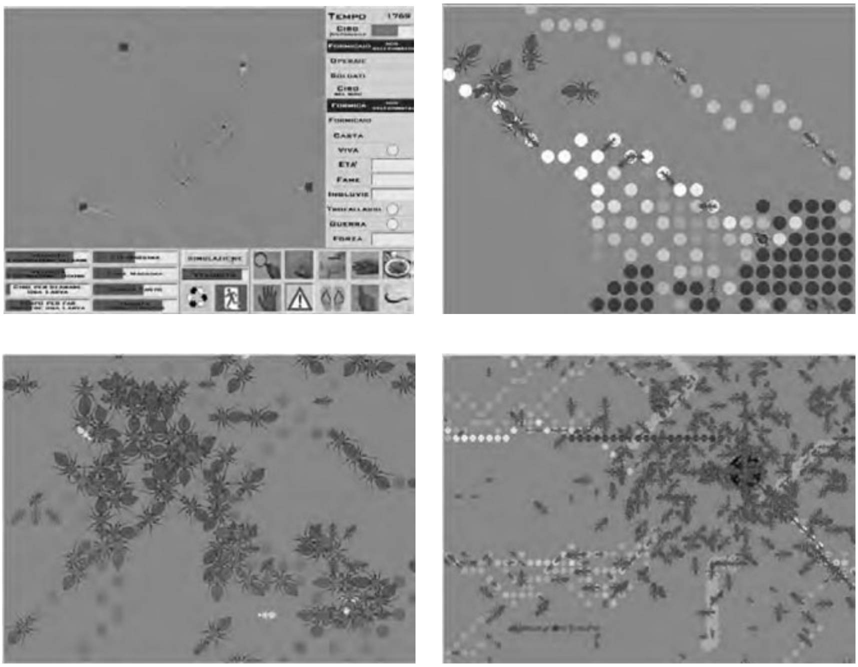

Fig. 3 Still images of a simulation

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Ants Searching for a Minimum

Maurizio Falcone

1 Looking for a minimum in a deterministic way

We are solving optimization problems every day. For example, we look for the best investments for our money, the best path to get back home, the best restaurant according to our wishes/budget. Clearly, the notion of "optimality" is very subjective and varies a lot from one person to another (have you ever tried to organize a party with some friends?). In a more abstract framework, we can say that we are looking for the best solution within a set of admissible solutions (the constraints which limit our decision). If we denote by S the set of admissible solutions, two different subjects can judge "optimal" two completely different options $x^*$and y^* satisfying the constraint, i.e. $x^*, y^* \in S$. This is not surprizing since every one has his/her own criterium to optimize and it can be very difficult to define it properly (do you remember the motivations your friend gave to have pizza instead of burgers at your graduation party?). From the mathematical point of view we need to have a clear definition of the priorities and we assume that they are represented by an *objective function* $f : S \to \mathbb{R}$. This allow us to determine if a solution x in better than y, because for every $x \in S$, the functions f gives a value $f(x)$ that we can compare with other values.

Let us give an example. We can assign to every path p going from a point A to a point B the time which is necessary to complete the path $t(p)$. Then we decide to compare two paths just in terms of their corresponding travel times so that a path p is better than a q if and only if $t(p) \leq t(q)$. Clearly, we could also include other costs in the criteria to be optimized, e.g. the cost for the fuel, the quality or length of the path etc. The more popular way to solve this kind of problem is to use deterministic methods and algorithms which can produce an approximate solution either in a finite number of steps or in the limit. The important point is to have always an estimate on the error of the approximation and on the global complexity of the method. A class of deterministic method which is widely used in many applications is the class of

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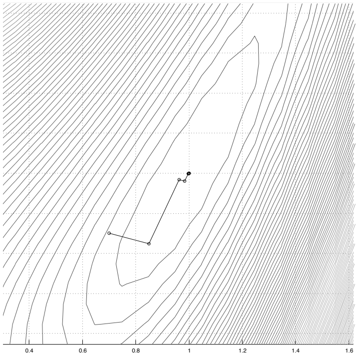

Fig. 1 The gradient method in action: along the path the values of f are decreasing

"descent methods" (see e.g. the monograph [7] for more details and more numerical methods). Assuming that we want to obtain the minimum of the objective function we start from an initial guess x^0 and we try to decrease the value of f along a sequence of approximate points x^n, $n \in N$. Clearly, one can try to obtain the best descent direction at avery point x^k instead of a generic "descent direction" and this is the idea behind the so-called "gradient method" which is the most famous descent method. It works as follows:

Start at x_0
Compute the direction indicated by the gradient vector $-\nabla f(x^0)$
Follow that direction until the function starts to increase
(this determines the new point x^1)
Replace x^0 *by* x^1 *and restart.*

Note that at every iteration, indexed by n, we compute the minimum of f along the line indicated by $-\nabla f(x^k)$ determining the next point x^{n+1} of the sequence. The iterative formula corresponding to the gradient method is:

$$x^{n+1} = x^n - \alpha_n \nabla f(x^n), n \in \mathbb{N}$$

where α_n is a real number which defines the length of our linear path connecting x^0 to x^1, α_n is called the step of the method. Clearly the method stops when the gradient is 0 (note that numerically it will stop when the gradient is "very" small).

2 Ant colony methods

Despite the simplicity of the gradient method, it has some limitations. In particular, it is important to note that we need to compute the gradient at every iteration, and this requires some regularity assumptions on f. Moreover, by construction, the method will stop where the gradient is equal to 0, so it will be able to compute only local minima. Other methods have been proposed, for example some stochastic method where the search is done at random according to new rules (processes). Among probabilistic methods a particular class is inspired by the behaviour of a *ant colony*. The first to propose a method of this kind has been an Italian researcher at the Politecnico di Milano, Mario Dorigo in the PhD thesis [2] (now Dorigo has a position at the Universitè Libre de Bruxelles). In fact, we know that ants belonging to a colony look for the food in a rather cooperative way, they go around at random but once they have found the food they start going back an forth to transfer the food to the colony and doing this they form a very visible path on the ground.

The idea which is behind "ant colony" algorithms probably came looking at the results of two experiments done at the beginning of the 90's. These experiments have shown that an ant colony has a collective intelligent behavior (although the single ant does not seem to be intelligent). Let us consider the problem of finding an optimal path from a starting point A (the nest where the ant colony lives) and a point B (where the food is located). In the first experiment, there are two paths of equal length connecting A to B (Fig. 3, left). The ant are going around trying the two paths and at the end they splits into two groups: the first follows the upper path whereas the second is following the lower path. In the second experiment, there are two paths of different length connecting A and B (Fig. 3, right) and the ants start going through both of them but after a while the ant queue through the short path starts growing and only few ant remain in the long path. This is due to the effect of the trace of pherhormone that the ant are leaving going back to the nest. This "smell" is used by the ants to detect which path has been more successful and as far as many ants are passing through that path the "smell" increases attracting other

Fig. 2 Dorigo and the cover of his book with Stützle *Ant Colony Optimization*

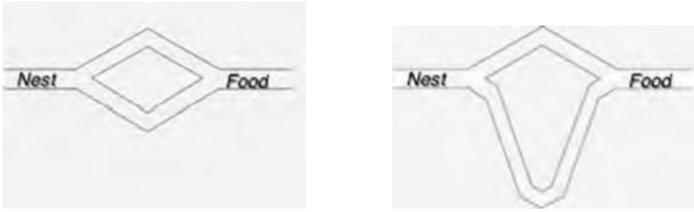

Fig. 3 Deneubourg et alia (1990) experiment, Goss et alia (1989) experiment

ants. This is possible due to the huge number of ants looking for the food and at the end only the short path becomes stable.

2.1 Ant Colony algorithms

In the method proposed by Dorigo, the main idea is to mimic the behavior of the ant colony and to exploit the informations coming from a huge number of individuals. As we said there are two main features which have to be taken into account, they become the two main steps of the algorithm,

Step 1: Explorator ants move at random looking for the food.
Step 2: Ants come back to the nest leaving a trace of pherhormone.

In this way incoming ants are guided to follow the same path by the smell of pherhormone and the queue starts. The problem is that the smells is not permanent since it is subject to a rapid evaporation. This is a crucial point to make the method successfull: infact if there are many other paths which originally have been indicated by the exploratory ants, after a while only the "best path" will remain active since that is the one where many ants are passing and the smell is stronger. All other path will disappear since they have been abandoned and, consequently, the pherhormone has vanished. Evaporation is the key to make the selection between different paths and to stabilize only the "best path".

Let us examine more in detail every step from a mathematical point of view. Trying need to mimic the random search of ants, the movement is free in every direction. We can introduce a *transition probability* p^k_{xy} (i.e. a real number between 0 and 1) which gives the probability for the k-th ant to go from x to y. Clearly, this probability also depends on a measure of *attractivity* of that path and on the trace of pherhormone on the path. We will denote by $\eta_{xy}(\alpha,k)$ the attractivity of the path α connecting x to y and by $\tau_{xy}(\alpha)$ its *trace of pherhormone*. Then, the transition probability can be written as

$$p_{xy}(\alpha,k) = \frac{\eta_{xy}(\alpha,k)\tau_{xy}(\alpha)}{\sum\limits_{\alpha}\eta_{xy}(\alpha,k)\tau_{xy}(\alpha)}.$$

The sum (Σ) appearing in the denominator is done other all the possible paths α connecting x to y so that every transition probability for the k-th ant verifies $p_{xy}(\alpha,k) \in [0,1]$. Moreover, the probability increases if the trace increases. Now we have to describe the second step of the method and give a mathematical rule to update the trace of pherohormone. After all the ants have moved according to the previous rule, we have to redistribute the pherhormone on the paths connecting the points. This will give us a new distribution of the "mathematical pherhormone" which will drive the method to the optimal solution. Doing this we must take into account the previous trace (τ_{xy}^n) and the evaporation rate, ρ. Assuming that the ants move at discrete times $t_n = n\Delta t$, the time between two moves is always Δt, so we get the following rule for the matematical pherhormone,

$$\tau_{xy}^{n+1}(\alpha) = (1-\rho)\tau_{xy}^n(\alpha) + \Delta\tau_{xy}^n(\alpha)$$

where ρ is assumed to be constant to simplify this presentation. However, the rate of evaporation can also depend on the time n. For example it can be increased as if the path started to be heated by the sun rays, so that $\rho^n > \rho^{n-1}$. The new trace τ^{n+1} associated to the path α is then obtained decreasing the previous trace by a factor $(1-\rho)$ and increasing it by a factor which is proportional to τ^n. Once we have the new traces associated to every path, every ant can make a new move driven by the new transition probability which are computed according to the previous rule. A pseudo-code for the Ant-Colony Optimization (ACO) algoritm is:

```
procedure ACO_MetaHeuristic
    while(not_termination)
       generateSolutions()
       daemonActions()
       pheromoneUpdate()
    end while
end procedure
```

One can also download some codes based on ACO since free software is available at [8, 9].

It is interesting to note that the ACO algorithm has solved the famous *traveling salesman problem*: "A traveling salesman has to visit N towns to sell his products and he would like to organize his journey minimizing the length of his journey. What is the minimal path connecting all the towns he has to visit?" Clearly, he wants to minimize the global length of its journey, which is given by the sum of all possible paths connecting two towns, x and y.

For this problem the ACO algorithm gives the optimal solution, as illustrated in Fig. 4. It starts from an initial path connecting all the towns (1), than computes all the possible connections on the network (2) selecting along its iterations the optimal path (3) which finally becomes stable (4) (see [3] for more details).

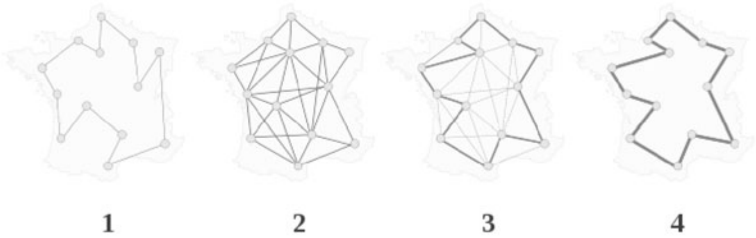

Fig. 4 The iterative solution of the traveling salesman problem

3 Animal behavior, swarms and intelligent robots

Ant colonies are not the only source of inspiration for scientists. Also other animal behaviors have been examined looking for new solutions to rather complicated problems where many individuals/robots have to interact and cooperate to reach a common goal. In fact, bird flocks and animal herds have shown to behave in an intelligent way when they need to perform a task which is crucial for their survival. Some of these groups of animals "follow a leader" creating a particular geometric configuration of the herd/swarm, this is the case of elephants herds walking in a single queue when they look for water or humid areas. Others groups of animals choose a particular geometric configuration just looking and interacting with their neighbors, this is the case of a flying duck flock and of a school of fishes (see Fig. 5). The

Fig. 5 Various animal behaviors have inspired scientist for the solution of their problems

analysis and the mathematical modeling of this natural behaviors is now a research area of growing importance (see the books [1] and [6]). In particular, scientist are trying to mimic herds animal behavior to manage large groups of *intelligent robots*. These robots are equipped with sensors and cameras so they able to detect the space around them. Moreover, they can exchange informations and simple instructions be-

Fig. 6 A group of "intelligent robots" and the cover of a book on swarm intelligence

tween them, they can move, climb or fly and do simple operations like bringing a piece from one point to another in a complex environment (e.g. an apartment with several rooms and corridors or a country field with some holes and obstacles). One of the main problems scientist have to solve is how to manage them so that they can cooperate and collectively solve a given task. Every single robot has a specialized action: one can fly and create a picture of the environment, another can climb but can not run, others can run and bring the climber with them and so on. Only by a coordinated collective behavior, these robots can perform the task, as it as been done in some advanced robot labs where the "swarm intelligence" has already produced some amazing products. In the future, we will probably see them in our houses.

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Mathematics and Marco

Exotic Spheres and John Milnor

Marco Abate

There are mathematicians able to solve incredibly difficult problems devising amazingly new ideas. There are mathematicians with a sure grasp of entire subjects, able to single out the more promising research directions. There are mathematicians with a crystal clear vision, able to explain and clarify any subject they talk about. And then there is John Milnor. He is all three: he solves, understands and explains, at an exceptional level on all counts. And he is a nice guy too.

In this short note, after a brief bibliographical sketch, I shall try and describe one of the most famous theorems proved by Milnor. We shall not go very much beyond explaining what the statement means, but I hope it would be enough for giving at least an idea of the brilliance of Milnor's mathematics.

1 John Willard Milnor

John Willard Milnor (known as Jack by his friends and colleagues) was born on 20 February 1931 in Orange, New Jersey, USA. He published his first mathematical paper [1] when he was only 19 years old. And it was not just any paper: he solved a 20 years old problem on the differential geometry of knots in 3-dimensional space (see [2] for a very short introduction to knots), proving a theorem, the Milnor-Fáry theorem on the total curvature of knots (Fáry was a Hungarian mathematician who independently proved the same theorem at the same time as Milnor; see [3]), that has become such a classic result to be included in several introductory texts on differential geometry (see, e.g., [4]).

Milnor enrolled as a Math major in Princeton University, where he received his A.B. in 1951, and his Ph.D. in Mathematics in 1954, under the supervision of Ralph Fox. His research work was so brilliant that he got a position in Princeton's math faculty in 1953, even before completing his doctorate. He was promoted to professor in 1960, staying in Princeton until 1967. He then moved first to the University of California, Los Angeles, and then to the Massachusetts Institute of Technology, before joining the faculty of the Institute for Advanced Study at Princeton in 1970.

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Finally, in 1989 he was appointed Director of the Institute for Mathematical Sciences at Stony Brook University in New York; he is still there, as Co-Director.

He has received all the major mathematical awards. He was a recipient of the Fields Medal in 1962 (when he was 31 years old, well before the age limit of 40), of the Wolf Prize in 1989, and of the Abel Prize in 2011. He was also awarded all three Steele prizes from the American Mathematical Society: for Seminal Contribution to Research in 1982, for Mathematical Exposition in 2004, and for Lifetime Achievement in 2011.

In the next section I shall discuss Milnor's research; but as suggested above Milnor's work goes well beyond proving theorems. He is a very gifted mathematical writer; his books are masterpieces of mathematical exposition, read and enjoyed by students and professors alike. Let me just mention *Topology from a differential viewpoint* [5], originally published in 1965 and still one of the best entrance points to the field of differential topology, and *Dynamics in one complex variable* [6], which is the book (of the many published on this subject) that I always suggest to students willing to learn this beautiful topic; but all his books have been and still are very influential. Mind you, they are not easy books; the reader has to work his/her way through them. But Milnor has the uncanny ability of precisely identifying the heart of the subject, and of explaining very clearly why things go as they do. A common (and frustrating) experience when reading mathematical papers is going through a proof, confirming the validity of each single step of the argument, and arriving at the end not having the least idea why that result should hold, what is its real significance. And this means we are missing the main point: as scientists, our aim is not just list a few scattered facts about the (mathematical) world, but to understand why it works as it does. Milnor's writings do exactly this: they aim (and succeed) to bring the reader to a full understanding of the subject matter. As written in the Abel Prize Commitee's statement: "Milnor is a wonderfully gifted expositor of sophisticated mathematics. He has often tackled difficult, cutting-edge subjects, where no account in book form existed. Adding novel insights, he produced a stream of timely yet lasting works of masterly lucidity. Like an inspired musical composer who is also a charismatic performer, John Milnor is both a discoverer and an expositor" [7].

And Jack also is a good guy to be around. He does not try to impress, or to show how good he is; he is more interested in understanding what you are saying, and is always willing to answer your questions. In talks, you can immediately see when something catches his attention. His blue eyes focus on the speaker, and if he asks a question it is always right on the mark but never threatening; his aim is not to point out a fallacy in the presentation, but to fully understand what the speaker is saying. And sometimes I have the impression that still now he looks at life with the beautiful wonder of a child. I remember his willingness to try a particularly exotic kind of Japanese cuisine in Kyoto; or his amusement in launching himself in a funny folk dance in Cuernavaca, Mexico. But you do not need to accept my word for this; just look at his smile (see Fig. 1; he is the tall guy on the left) and you will understand what I mean.

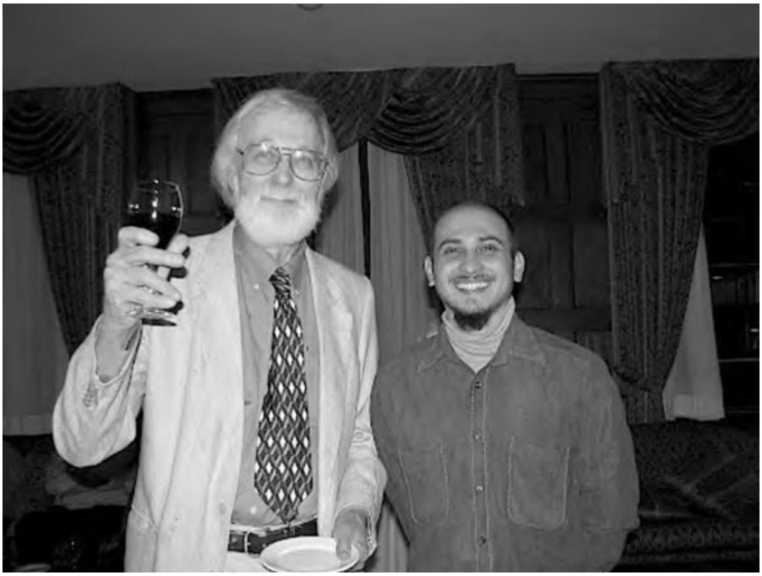


Fig. 1 John Milnor (left) and Enrico Le Donne (courtesy of Enrico Le Donne)

2 Exotic spheres

The result I would like to talk about is contained in the famous paper *On manifolds homeomorphic to the 7-sphere* [8], that by itself started a completely new mathematical subject, differential topology. Let me begin by trying to explain the words in the title.

What is a sphere? To answer this (apparently innocuous) question, we start from the 1-dimensional sphere, that is the circumference. Geometrically, the circumference of radius one centered at the origin O in the plane is the set of points at distance 1 from O. Introducing Cartesian coordinates centered at O, we can represent each point of the plane with a pair (x,y) of real numbers, and Pythagoras' theorem implies that the points of the circumference are exactly the points of the plane represented by pairs (x,y) satisfying the equation

$$x^2+y^2=1\,.$$

Let us then consider the unit sphere in 3-dimensional space, that is the set of points in the space at distance 1 from the origin. Introducing again Cartesian coordinates, each point in space is represented by a triple (x,y,z) of real number, and the points in the unit sphere satisfy the equation

$$x^2+y^2+z^2=1\,.$$

The mathematical symbol most often used to denote the unit sphere in 3-dimensional space is S^2 , where S stands for sphere, and the superscript 2 recalls that the sphere is a surface, that is a 2-dimensional object. Analogously, it is customary to denote the unit circumference by the symbol S^1, to recall that it is a 1-dimensional sphere.

The introduction of Cartesian coordinates allowed us to identify points in the plane with pairs of real numbers, and points in 3-space with triple of real numbers; and the equations describing S^1 and S^2 in coordinates clearly had the same structure. Mathematicians cannot help following a pattern, it is part of their nature; and so they immediately define the "3-dimensional sphere S^3 in a 4-dimensional space" as the set of quadruple (x,y,z,w) of real numbers satisfying the equation

$$x^2+y^2+z^2+w^2=1\,.$$

And why stop here? We can as easily define a 4-dimensional sphere in a 5-dimensional space, or a 7-dimensional sphere in a 8-dimensional space; in general, the n-dimensional sphere S^n in a $(n+1)$-dimensional space is the set of $(n+1)$-uples $(x_0,\dots,x_n)$ of real numbers satisfying the equation

$$x_0^2+\dots+x_n^2=1\,.$$

This is not only a fatuous trick; the use of coordinates in this way is a basic step for representing and understanding the geometry of the n-dimensional analogue of the usual surfaces, called (n-dimensional) *manifolds* (and so we have introduced another word of the title).

Let me pause a second to justify why it is useful to consider n-dimensional objects even if we are only interested in understanding our comfortable 3-dimensional world. Consider a particle in space: to describe its position we just need 3 real numbers, its 3 Cartesian coordinates as described before. But if the particle starts to move, we have to consider its velocity, that can again be represented by using 3 real numbers (the coordinates of the velocity vector in a Cartesian system centered at the position of the particle). Thus to describe a moving particle in space we need 6 real numbers — that is we need a 6-dimensional space. And if we want to describe a soccer team of moving particles we need $6\times 11=66$ real numbers, and we end up in a 66-dimensional space. And if we want to describe an actual soccer team composed by human beings, we need an N-dimensional space with N very large: indeed, in the human body there are a lot of parts that can be moved independently, and thus for each player we need at least 6 numbers for each such parts: 6 for each foot, 6 for each knee, 6 for the pelvis, 6 for the torso, 6 for the each hand (not counting the fingers), and so on... But not all N-tuples of real numbers can represent a soccer team: there are conditions to be satisfied. For instance, the distance between the left foot and the left knee is constant (it is equal to to the length of the lower left leg); this means that the numbers representing the positions of the left foot and of the left knee must satisfy an equation similar to the ones written above for the spheres. And exactly as happened for the sphere, most of these equations (not all, but this is a technical point not worth elaborating on here) define a subset of the N-dimensional space which is a high-dimensional analogue of a surface in 3-space.

So, in conclusion, to study your favorite soccer team you are forced to use high-dimensional manifolds. Think about this next time you turn on the TV to enjoy a soccer match...

We still have one word in the title left to explain: homeomorphic. Roughly speaking, it means "to have the same shape", where here "the same shape" has to be intended in the topological sense (and not in a more geometrical sense that we shall introduce below). Let me explain what it means for the case of surfaces in 3-spaces; very similar things can be said about manifolds in N-dimensional space. The idea is that two surfaces in 3-space are *homeomorphic* if we can deform one into the other *continuosly*, that is without cutting and without self-intersections. [1] For instance, a sphere and (the surface of) a rugby ball are homeomorphic: it suffices to inflate the rugby ball (without blowing it up) until it becomes spherical. By the same token, a cube and a sphere are homeomorphic too: again, it suffices to inflate the cube until it loses its edges and becomes round.

However, from another point of view the cube and the sphere geometrically do not have the same shape: the cube has edges, whereas the sphere is smooth everywhere. Being smooth is a identifying characteristic of all manifolds; in particular, the cube is not a manifold (is not a smooth surface). The rugby ball, on the other hand, is; so one can wonder whether the rugby ball and the sphere still have the same shape as smooth manifolds, where we say that two smooth surfaces have the same shape, or, in technical jargon, are *diffeomorphic* (where "diffeo" here comes from "differentiable", to recall that the smoothness can be expressed in terms of derivatives), if we can deform one onto the other without cutting, self-intersecting and in such a way that all the intermediate surfaces in the deformation are still smooth.[2]

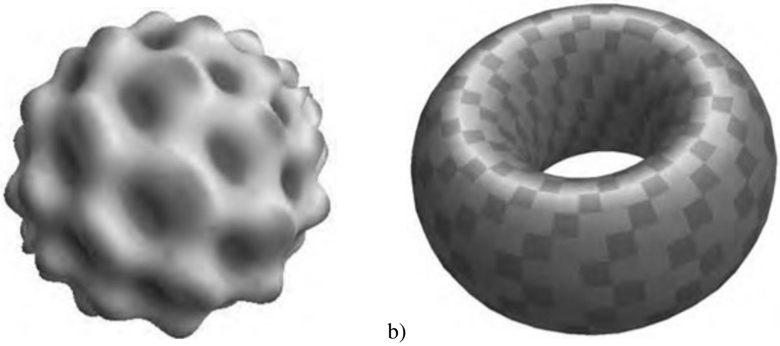
Under this definition, the rugby ball and the sphere are diffeomorphic, because all the intermediate shapes in the inflating process are still smooth. A similar argument also works for much more complicated surfaces, like the one in Fig. 2a. On the other hand, the doughnut (or, in mathematical terms, the torus) depicted in Fig. 2b, though smooth, cannot be diffeomorphic to the sphere. Indeed, it is not even homeomorphic to the sphere: there is no way to get rid of that hole without cutting the surface.

On the other hand, one can be hard pressed in finding a smooth surface homeomorphic but not diffeomorphic to the sphere; and indeed it does not exist.

To explain why, let us try and see what can go wrong (and why it does not). Consider the circumference S^1 in the plane. An ellipse (like the rugby ball in the 3-space) is clearly diffeomorphic to S^1, again by inflating. However, this is not the only way to continuously deform an ellipse into a circumference. Indeed, we might first deform the ellipse into a rectangle; then stretch the rectangle until it becomes a square; and then inflate the square until it becomes a circle. If this was the only way we knew for passing from the ellipse to the circumference, we would not be able to conclude that the two are diffeormorphic, because some of the intermediate curves in the deformation were not smooth; the transformation created corners. So

[1] To be more precise, two surfaces are *homeomorphic* if there is a continuous bijection between them having a continuous inverse; but allow me to use a slightly imprecise language here for the sake of clarity.

[2] Again, the technical definition is more complicated than this: two smooth surfaces are *diffeomorphic* if there is a differentiable bijection between them having a differentiable inverse.



a) b)

Fig. 2 a) A smooth surface diffeomorphic to the sphere; b) a smooth surface not homeomorphic to the sphere

Fig. 3 Is this curve diffeomorphic to a circumference?

the question is: might there exist a smooth curve so complicated (like the one in Fig. 3) to be homeomorphic but not diffeomorphic to a circumference?

Well, no: we can always smooth corners. Indeed, if we have a corner in a curve, we can change the curve just the tiniest bit replacing the corner by a smooth piece; an hyperbola is usually enough (see Fig. 4).

So, if during our deformation a corner appears, we can slightly change the deformation by smoothing the corner out just an instant before it appears, obtaining a deformation where all intermediate curves are smooth; and thus any smooth curve homeomorphic to a circumference also is diffeomorphic to the circumference. [3]

The situation is more or less the same for the sphere in 3-space. If deforming a smooth surface (homeomorphic to a sphere) a corner or an edge appears, then we can iron it out, like ironing out creases in a shirt, allowing us to smoothly and safely complete the deformation up to the sphere. In other words, any smooth surface homeomorphic to S^2 is diffeomorphic to it.

[3] Of course, this is not a proof; however, a formal proof is not that difficult, and can be understood by third-year math undergraduate students.

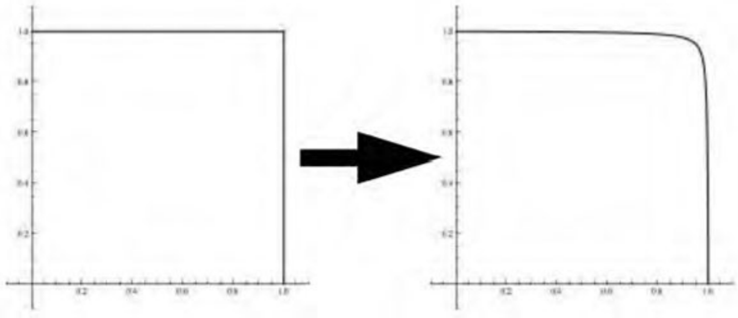


Fig. 4 How to smooth out a corner

It is then natural to conjecture that this holds in any dimension: any smooth n-dimensional manifold homeomorphic to S^n is diffeomorphic to it, where n is any positive dimension. This statement is now known as the *smooth Poincaré conjecture*, because similar in nature to the much more famous *topological Poincaré conjecture* stating that any n-manifold homotopically equivalent to S^n is homeomorphic to it, where (very roughly speaking) two manifolds are homotopically equivalent if one can be deformed onto the other without cutting but allowing some identification; for instance, a cylinder is homotopically equivalent to a circumference (just squeeze the cylinder onto a base circumference; this operation does not change the number and nature of the holes, which is what homotopy measures). The topological Poincaré conjecture has been recently settled in the positive by Perelman (see [9] for a popularizing presentation of Perelman's results): an n-manifold homotopically equivalent to S^n *always* is homeomorphic to it.

In 1956, when Milnor was working on his paper, the topological Poincaré conjecture was widely open; on the other hand, the smooth Poincaré conjecture was not really a conjecture. Most mathematicians took it for granted, without even realizing it; after all, what else can happen? So Milnor's paper came as a complete shock: he showed that a lot more can happen. In dimension 7, there is a 7-manifold which is homeomorphic but *not* diffeomorphic to a sphere: it has creases that cannot be ironed out, no matter how hard you try (the nightmare of any housewife — or, even more, of any househusband). It is called an *exotic sphere*; at first glance it looks like a sphere, but becomes positively strange under closer examination. This shows that the topological structure does not completely determine the differentiable (smooth) structure: and so the field of differential topology, the study of the relationships between topology and differentiability, was born.

Milnor's discovery of an exotic 7-sphere is a geometrical masterpiece, based on the most advanced geometrical tools available at the time. Actually, the most difficult part is not building the sphere, but proving that it is exotic. In very rough terms, the construction goes as follow: Milnor divided the standard 7-sphere in two emispheres, a north emisphere and a south emisphere, cutting along the equator. The border of each emisphere is a sphere of one dimension less (think of the usual equator of the standard 2-sphere: it is a circumference, that is a sphere of one less dimension). If we glue together the two emispheres along the respective borders (that is, we identify each point of the border of the north emisphere with exactly one point of the border of the south emisphere and conversely), it is not difficult to see (using a tool called Morse theory; see [10]) that we obtain a manifold still homeomorphic to the 7-sphere. If the identification is well-behaved (for instance, we just rotated one emisphere before gluing it back to the other one), the manifold we obtain is also diffeomorphic to the 7-sphere; but Milnor devised a really wild identification, and when he tried to check whether this new manifold M was still diffeomorphic to the 7-sphere he did not succeed (see [11] for Milnor's recollection of the events). So he decided to try and prove that M was not diffeomorphic to S^7. A characteristic of the usual 7-sphere is that it is the boundary of a 8-dimensional ball (exactly as the usual 2-sphere is the boundary of the usual unit 3-dimensional ball, the set of points at distance strictly less than 1 from the origin in 3-dimensional space). Starting from this observation, Milnor managed (and this is the deepest part of his work) to associate to any 7-dimensional manifold a number somehow measuring how far the manifold was from being the boundary of an 8-dimensional manifold; and the association was such that the same number would be assigned to two diffeomorphic manifolds. Since S^7 is the boundary of the 8-dimensional unit ball, its associated number is 0. However, the number associated to M is *not* zero; and hence the only possible conclusion is that M is not diffeomorphic to S^7, it is an exotic sphere.

This is only the beginning of the story. The mathematical community, as soon as it started to recover from the shock of discovering that something given for granted was actually false, realized the potentialities and the importance of the new field of differential topology, and started asking questions. For instance, now that we know that there might be smooth n-manifolds homeomorphic but not diffeomorphic to the n-sphere, is it possible to count them? That is, is it possible to exactly say how many n-manifolds not diffeomorphic to each other but all homeomorphic to the n-sphere there are for each dimension n?

Let us called a_n this number. In 1963, Milnor and Kervaire [12] showed that $a_7 = 28$, that is that there are exactly 28 different 7-manifolds homeomorphic to the 7-sphere. They can be obtained as the set of 5-tuples $(z_0, z_1, z_2, z_3, z_4)$ of complex numbers satisfying the following equations

$$|z_0|^2 + |z_1|^2 + |z_2|^2 + |z_3|^2 + |z_4|^2 = 1\,, \quad (z_0)^3 + (z_1)^{6k-1} + (z_2)^2 + (z_3)^2 + (z_4)^2 = 0$$

for k going from 1 to 28.

The quest for solving this problem for all n has been going on since then, and it is not finished yet. Thanks to the work of many people (let me just quote Browder [13] and Kervaire-Hill-Ravenel [14], but many more names should be added; see [15, 16] for recent surveys) ways to compute a_n, at least in principle, have been devised. In particular, a_n is now known for all small values of n (at least for $n \leqslant 64$; see Fig. 5 for a table containing its first few values), with one surprising (and frustrating) exception: $n = 4$. Indeed, the 4-dimensional case is completely open, to the point that it is conjectured that there are infinitely many exotic 4-spheres, but till now nobody has been able to prove the existence of a single one! So after more than 55 years Milnor's discovery has not yet finished to reveal all its intricacies.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a_n	1	1	1	?	1	1	28	2	8	6	992	1	3	2	16436	4	16	16	261632	24

Fig. 5 The first few values of a_n

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Cellular Automata: the Game of Life

Gian Marco Todesco

Introduction

The *Game of Life* (also known as *GoL* or simply *Life*) is a solitaire pastime invented by the British mathematician John H. Conway, in 1970. It is a game in a rather special way: there are no winners or losers. Actually it does not require any player at all: its evolution is entirely determined by the initial conditions. *Life* is a sort of virtual construction toy: one interacts with it by creating an initial shape and observing its evolution. The challenge is to build interesting configurations and, in this regard, the game has very much to offer: with few simple rules in a very simple environment, it allows to simulate a sort of primordial soup in which ordered and chaotic patterns develop together, competing with each other for the available space.

Life is a cellular automaton (or *CA*), that is a system made of a regular grid of cells and a set of rules that control their evolution. Each cell has a discrete state (e.g. *on* or *off*) and all cells are updated simultaneously at each *generation* or *move* of the game. The rules are the same for every cell and do not change over the generations. The new state of a cell depends only on its current state and on the state of its neighboring cells.

In *Life* the grid is an infinite square lattice, and cells have two states only: alive or dead. Visually, cells can be colored in two different colors, or they can be occupied by a counter or be empty.

The following picture shows the evolution of a simple configuration. The pattern repeats itself after eight steps.

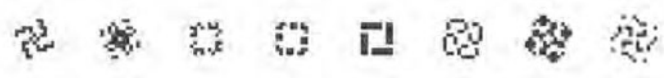

Fig. 1 The "Kok's galaxy": a period-8 oscillator pattern in Game of Life

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Digital Video S.r.l., Rome (Italy).

Emmer M. (Ed.): Imagine Math 2. Between Culture and Mathematics
DOI 10.1007/978-88-470-2889-0_25,

This pleasant symmetrical pattern is also fragile: some small changes, e.g, putting a live cell in the center at the fifth step, can affect all the following steps, with differences getting larger and larger, until – eventually – all the cells die.

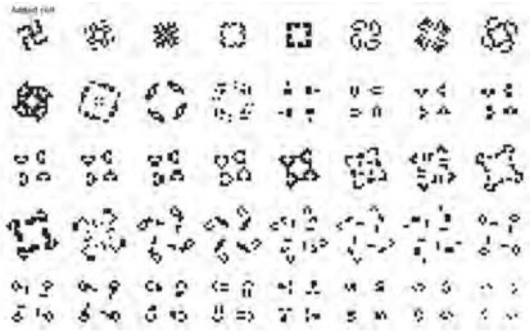

Fig. 2 A change in a cell can destroy the pattern

In the following pages, we will explore this fascinating game and discover even more complex behaviors.

section*The Belousov-Zhabotinsky reaction

There are countless different types of cellular automata. They can differ for the shape of the grid, the definition of the cells neighborhood, the number of cells states and of course for the rules that control the evolution. For instance, A.K. Dewdney proposed a model in 1988, where each cell could have up to *N* different states (visualized by different colors). The states could represent the different stages of an infection. The rules control how the healthy cells get infected by neighbors, and how the illness and the healing progress.

If we start with a random distribution of cells (i.e. the state of each cell is selected randomly between 0 and N-1), then interesting patterns eventually emerge: rotating spirals grow and cover all the available space.

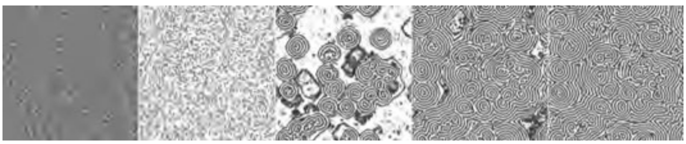

Fig. 3 A cellular automaton simulates the Belousov-Zhabotinsky reaction. (a) initial random state (b) after 5 steps (c) after 300 steps (d) after 600 steps (e) after 900 steps

The positions and number of the spirals are casual and depend on the initial state, but the shapes of the spirals are always the same.

Remarkably, the final stable oscillating pattern is very similar to a natural pattern, found in a nonlinear chemical oscillator called Belousov-Zhabotinsky reaction (or BZ reaction).

The ability to simulate complex behavior of natural systems, by means of relatively simple and local rules is a well known feature of the cellular automata and one of the reasons why they are studied.

The main applications of cellular automata are in computer graphics (image cleanup, pattern generation, etc.), biological systems simulations (artificial life), physical phenomena simulations (heat-flow, turbulence), the design of massively parallel computers, cryptography and random number generation.

One-dimensional elementary cellular automata

The simplest *CA* is the *one-dimensional elementary cellular automaton*. It has just a single row of cells that can have only two states.

Each cell evolves depending only on its current state and on the state of its two neighboring cells. Three cells can have only eight configurations and, therefore, to specify completely the automaton rule (i.e. the next cell state for each possible configuration), we must provide eight bits of information.

There are only 256 different sequences of eight bits and therefore there are just 256 different rules, or only 88 fundamentally nonequivalent rules if we do not consider reflections or color swap.

Each rule can be identified by a number whose binary representation contains the eight next states for each possible cell configuration. The picture below shows rule #30 (00011110 in binary).

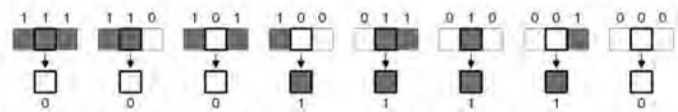

Fig. 4 The rule 30 (in binary: 00011110)

The evolution of a 2D automaton can be represented by putting all the generations one below the other. Next picture shows the evolution of rule #30, starting from a single black cell.

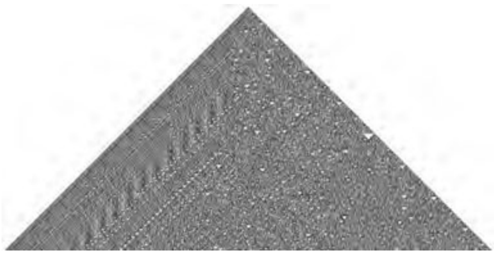

Fig. 5 The evolution of rule-30 cellular automaton

Even in this very simple example some interesting patterns emerge: white triangles, of different scales, pointing down. The pattern resembles the shell of the widespread cone snail species *Conus textile*.

Fig. 6 Conus textile shell, with a pattern similar to the automaton evolution

The cellular automata are actually used in the field of computer graphics to reproduce this kind of “natural” decoration.

The pattern contains order, but also chaos: the size and position of the triangles do not follow a simple scheme. Actually the automaton evolution is so chaotic that this rule is used in the program *Mathematica* to generate random numbers.

1 The universal constructor

The concept of cellular automaton has been invented in the early 1950s by the mathematicians Stanislaw Ulam and John von Neumann.

Von Neumann was studying the theoretical concept of *universal constructor*: that is a machine that can build anything, even a copy of itself.

Biological organisms are able to replicate themselves, and the offspring can be more complex than the ancestors by effects of mutations. On the contrary, mechanical artifacts are produced by factories that are much more complex of their products. It seems that a machine produced by another machine is doomed to be less complex than its originator.

Von Neumann investigated the possibility to create a machine complex enough to be able to reproduce (without assistance) a copy of itself, and also to introduce changes in the project in order to increase the complexity of the product.

He concluded that such a machine must be made of two different parts: the assembler and a description of the object to assemble. Of course the description cannot describe itself and therefore one of tasks performed by the assembler is to replicate the description as-is, without interpreting it.

The idea that the self-replicating system must be made of two parts, the machine and the description, and that the description must be both interpreted and copied to the target machine, resembles very much the biological world, where the role of the description is played by DNA. This is remarkable, because von Neumann described its work in 1948, while the double helix structure of DNA was discovered in 1953.

Ulam suggested von Neumann to use what he called *cellular space* to build an abstract prototype of his machine. Following his suggestion Von Neumann designed a universal constructor, in a cellular automaton with 29-state cells.

A mathematical puzzle

In 1970 Conway invented a game, trying to simplify the von Neumann automaton, but keeping its capacity to generate complex patterns. After a long period of experimentation he was able to find a set of rules, “simple, but not too simple”, in order to put the system in an unstable equilibrium between chaos and order.

He called his cellular automaton G*ame of Life*, because of its analogies with the rise, fall and alternations of a society of living organisms.

The rules of the game are:

- Cells are placed on an (ideally) infinite square grid. Each cell has eight close neighbors and only two states: alive or dead.
- If a live cell has more than 3 living neighbors it dies of overpopulation.
- If a live cell has less than 2 living neighbors, it dies of loneliness.
- If a dead (empty) cell has exactly 3 living neighbors it became alive.

As in any other cellular automaton, all the cells evolve at the same time, synchronously.

Martin Gardner described the game in his column “Mathematical Games” on Scientific American, and *Game of Life* became immediately very popular. Even today, after more than forty years, there is an active community that continues to explore the game, discover new patterns, name them (e.g. *Garden of Eden*, *Phonix*, etc.) and try to solve the many unsolved theoretical problems that the game presents.

A random distribution of cells evolves in a complex way. The cells arise and die, teeming of life and fully justifying the name of the game. Most random configurations eventually stabilize and in the final configuration many interesting and simple shapes appear spontaneously (they are called *ash*).

Interestingly, patterns that have no symmetry tend to become symmetrical. A symmetric pattern can become more complex and more symmetrical, but cannot lose its symmetry, until another pattern collides with it.

It is possible to understand the fate of simple configurations, just by using pen and pencil. A single isolated cell or a couple die of loneliness. Four cells, packed in a square, resist without change forming a stable pattern called *block*.

The pattern that do not evolve are called *still life*.

A number of configurations evolve continuously, but each few cycles they assume again the starting shape: these are named *oscillators*. The first example we explored is one of them. The most simple oscillator is called a *blinker* and is made of a row of three living cells.

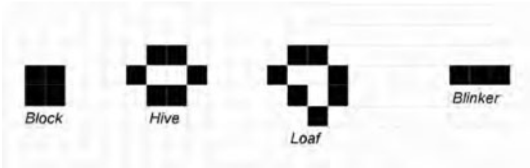

Fig. 7 Three common still-life form and the simplest oscillator

In general it is not so easy to predict the evolution of even simple shapes. Consider, for instance, the following pattern.

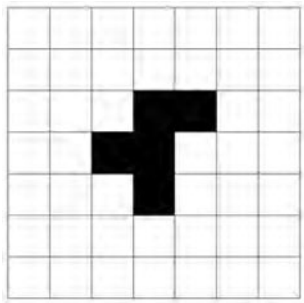

Fig. 8 The R-Pentomino

It is made of five adjacent cells and is called the *R-pentomino*. The letter 'R' has no connection with the pattern, that looks rather like a lowercase 'f', but Conway used the last consecutive letters of the alphabet to label all the 12 possible pentominoes. All the other pentominoes stabilize in maximum 10 generations, but this pattern continues to evolve steadily.

In 1970 the cost, speed and availability of computers was completely different from today. Conway suggested to use a *Go* board to experiment with his game. The procedure is slow and error prone. To follow the evolution of an even simple configuration can require many hours of work.

With paper and pencil and the *Go* table Conway did not manage to follow the complete evolution of the *R-pentomino*, and could track only 460 moves using a *PDP-7* computer, when the article by Gardner was published. The final fate of the pattern was still unknown.

We know now, that the *R-pentomino* requires 1103 generation to stabilize, reaching a population of 116 cells. This kind of pattern (i.e. a simple configuration that evolves for many generations before becoming stable) is called *methuselah*.

The glider

After 69 steps, the *R-pentomino* generates a very interesting pattern that was observed and described by Richard K. Guy in 1970.

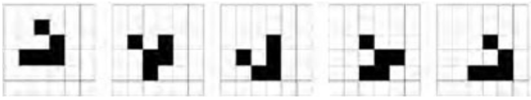

Fig. 9 The Glider

This pattern is made of 5 cells but is not a pentomino, because its cells are not connected along the edges. After 2 iterations the pattern regenerates itself, but translated and rotated. This kind of movement is called by mathematicians a *glide reflection* and therefore this pattern has been dubbed *Glider*. After 4 iterations the *Glider* assumes again the initial shape, but translated diagonally by a cell. In other words the pattern moves diagonally by one cell every 4 generations.

In *Life,* no effect can propagate at a speed greater than one cell per cycle; and this speed is called *speed of light* and is represented with the letter *c* (for analogy with the speed of light in the physical world). Thus the *glider* moves diagonally at c/4.

Patterns that move, i.e. that return to their initial state after a number of generations but in a different location, are called *spaceships*.

The existence of this new category of patterns that move (instead of die, stay still or oscillate) has great effects on the game, as we will see in the next paragraphs. Therefore the Glider, the first one discovered and the simplest of the category, has a strong symbolic role. For instance it has been proposed as an emblem for the *hacker* culture by Eric S. Raymond in 2003. In 2012 an engraved message containing a glider has been discovered in the Mac Book Pro *retina display* (inside the hardware, where no regular customer is supposed to see it).

The bet

Conway originally conjectured that no pattern can grow indefinitely. He guessed that for any initial configuration with a finite number of living cells, the population cannot grow beyond some finite upper limit. In the article published on Scientific American he offered a $50 prize to the first person who could prove or disprove the conjecture before the end of the year.

The conjecture could be disproved by the discovery of a *gun*, a cyclical pattern that creates other moving patterns (e.g. gliders) at each cycle, or a *puffer train*, that is a moving pattern that leaves behind it a trail of persistent "smoke".

An MIT team lead by Bill Gosper won the prize thanks to a configuration called *Gosper glider gun*. This configuration is an oscillator with a 30-cycle period. At cycle #15 it creates a glider that flies away. In an empty board, the *glider gun* generates a stream of gliders that move in a line, without interacting with each other. The total number of live cells keeps growing beyond any limit.

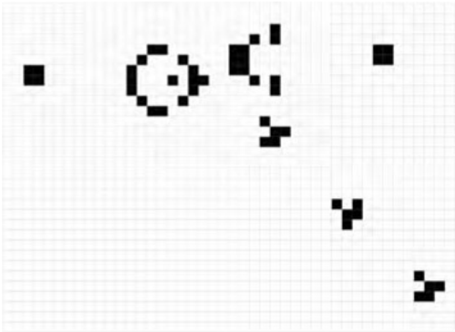

Fig. 10 The Gosper glider gun

The discovery of gliders and glider guns changed the scenario completely and made it possible to create an amazing variety of complex patterns.

Streams of gliders (or of spaceships) can be manipulated in many ways. The *guns* act as sources, but there are also patterns that can act as sinks and absorb the streams: they are called *eaters*. Moreover it is possible to bend the direction of a stream using a *reflector*.

Gliders can interact in many different ways: collisions can leave debris with different shapes or can completely destroy the colliding gliders.

The first process can be used to synthesize new patterns (e.g. to build more complex guns: an example is given later in the article) and the annihilation process proves very useful as shown in the next paragraph.

The logical gates

A stream of gliders can carry a bit of information (1 if the glider is present and 0 if it is not). The gliders annihilation reaction can be used to implement logical gates: i.e. devices that implement Boolean functions.

In the following configuration two glider streams, labeled A and B,represent the inputs, while the stream at the bottom is the output. It is active if and only if both inputs are active. Therefore this diagram implements the *AND* logical function.

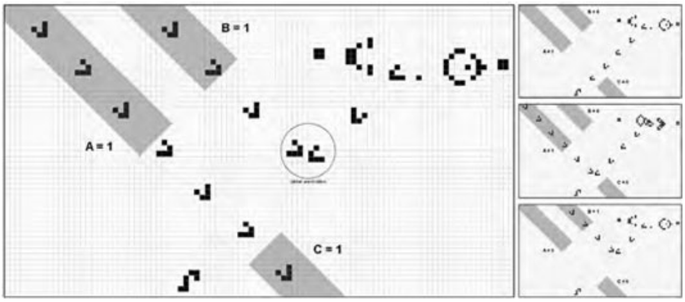

Fig. 11 A logical AND-gate simulated with glider streams

The implementation uses an orthogonal stream. The gliders of this stream can interact with gliders of both stream A and stream B. The interaction destroys the involved gliders. If stream A is not active the orthogonal stream destroys any glider coming from B. Only if both streams are active the glider from B can reach the output.

With similar configurations it is possible to implement other logical gates (NOT, OR, etc.). Moreover it is also possible to create patterns that store information, like a memory.

In summary it is possible to implement all the basic components of a computer. Actually it is possible to demonstrate that the game has the same power of a universal *Turing machine* (an abstract model of a computer) and therefore that anything that can be computed algorithmically can be computed through *Life* patterns. Of course this is a theoretical result and the diagrams capable to perform interesting computations require very many cells, but it is actually possible to make them. For instance Dean Hickerson in 1991 built a *Life* pattern that computes the sequence of prime numbers.

Faster computers and better programs

Since *Life* was invented forty years ago, the computing speed has dramatically increased. Today's commonly available computers (costing around 1,000 USD) are roughly 10.000.000 times faster than the *PDP-7* computer available to Conway (which costed around 70,000 USD).

Moreover, in the early 1980s Bill Gosper developed an algorithm called *hashlife* that exploits the time and space redundancy of many interesting patterns: in a typical configuration, most cells are empty and the living cells can be grouped in a number of identical sub patterns that evolve in the same way (until they interact with one another).

The algorithm memorizes these common sub patterns and computes their evolution only once. This consumes more memory and is slower at the beginning, but then the simulation speed can increase tremendously.

There are very many programs available online today that can simulate effectively *Life* and other cellular automata. For instance *Golly* is an open source application running on Windows, Max OS X and Linux, and implementing, among other features, the hashlife algorithm.

With the proper application and using an average laptop computer it is possible today to follow complex patterns made of millions of live cells for a large number of generations (billions of billions and even more) in a reasonable amount of cpu time.

Interesting patterns

There are many interesting patterns to explore and it is fascinating to observe their evolution on the computer screen.

A bigger gun

The first pattern is depicted in Fig. 12. This is a sort of factory: it creates a sequence of complex spaceships that move to the right of the image.

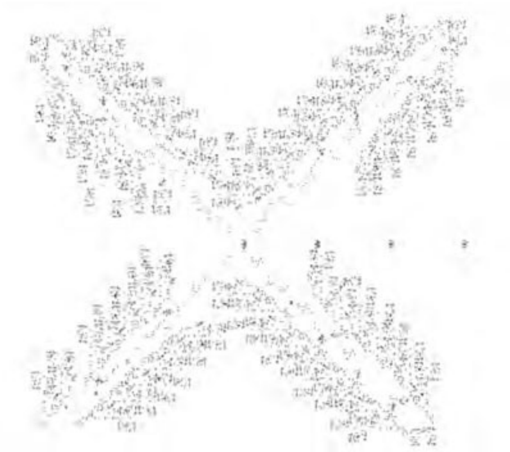

Fig. 12 The P416 60P5H2V0 gun

The spaceships have been invented by Tim Coe in 1996 and, not having a nice nickname, are called $60P5H2V0$ (the name refers to the period, the speed and the direction).

The whole pattern is called *P416 60P5H2V0 gun* and has been invented in 2003 by Dave Green.It creates many bunches of gliders that converge to the center. The collisions generate debris that eventually assemble the spaceship.

The first reactions create still life forms. Only the last reaction "activates" the group so that the complete spaceship starts moving toward the right, while other gliders are arriving to start the construction of the next spaceship.

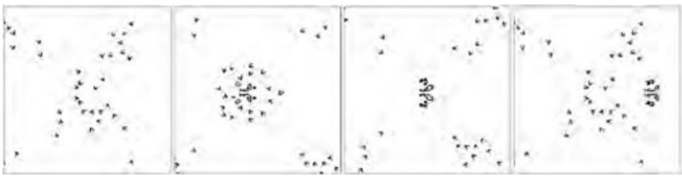

Fig. 13 A close up of the central section of the gun

Game of life playing at game of life

The second pattern is even more impressive and exploits all the power of this cellular automaton as a general computing device. The pattern demonstrates that it is actually possible to implement the *Game of life* inside of another *Game of life*. In other words, it is possible to create a pattern that simulate the automaton itself, at a larger scale.

In 2006, Brice Due, created the *Outer Totalistic Celullar Automata Meta-Pixel* (OTCA metapixel, for short). The pattern is 2048 x 2048 cells wide, and can have two states: *on* and *off*. In the *on* state, two orthogonal rows of "guns" fire continuous flows of spaceships that collide along the cell diagonal. In the *off* state, the guns are idle. Therefore when the metapixel is *on*, its area is almost completely covered with live cells. In the other state it is empty almost everywhere (except for the borders, that host the "logic").

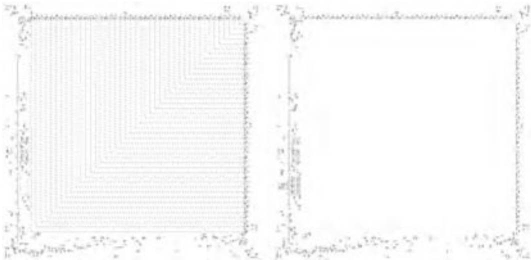

Fig. 14 The OTCA metapixel. (a) on-state (b) off-state

When many metapixels are packed together in a square grid, they communicate with each other, by means of streams of gliders. Each pattern interacts with its neighbors following the rules of *Life*: it switches to *on* or to *off* state according to the number of neighbors in the *on* state. The metapixels require 35328 step to update their state.

To summarize, the simulated Game of Life is 2048 times larger and 35328 slower of the original one.

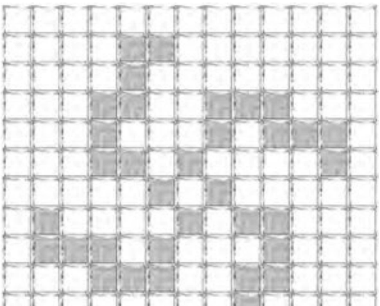

Fig. 15 The Kok's galaxy simulated with 169 OTCA metapixels. 5.3 millions cells

In principle it is possible to assemble a “meta-metapixel”, made of 2048 x 2048 metapixels i.e. it is possible to have a game of life that simulates a game of life that simulate a game of life.

A ticker

In the last pattern there are many streams of gliders on the right. The gliders run in loops: they are reflected at four corners and move perpetually forward and backward. One of the reflectors is more complex and generates a spaceship each time it reflects a glider. The stream of spaceships move to the left, organized in different rows.

In the gliders loops there are gaps that consequently cause gaps in the stream of spaceships. Eventually these gaps form a pattern that flows smoothly from right to left. By carefully designing the positions of the gaps one can shape the stream of spaceships as desired and compose a moving image or text like in a news ticker.

Conclusions

Our universe is made of elementary particles that are much simpler than the reality they form. Human bodies are made of cells and brains are made of neurons. The strong and potentially chaotic forces that drive our globalized world descend from the interaction of very many individuals, each one pursuing relatively simple goals.

The Game of Life is a nice toy that suggests, and in some sense demonstrates, how complex behaviors can be the result of many simple interactions.

I think that this intuition is very useful to understand what science is about and to interpret the world we live in.

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21. Konishi H., Sugita S.: Game of Space, the model was presented at SICF12 (Tokyo, 2011) and Hiroshima City Museum of Contemporary Art (Hiroshima, 2011) http://homesthetics.net/?p=3234
22. Game of Life, play by Rose Lewenstein, directed by Russel Bender. The Yard Theatre, 5-22 september 2012. http://theyardtheatre.co.uk/show/game-of-life/
23. Cook J.V.: Wolfrule, Art from Cellular Automata. http://www.wolfrule.com/
24. Myskja K.: Regel 30. Electrohype 2009. http://www.withoutatraceroute.com/2009/01/electrohype/

Homage to Alan Turing

Alan M. Turing (1912-1954)

Gabriele Lolli

The decision to remember Alan Mathison Turing in the anniversary of his birth in a conference dedicated to "Mathematics and Culture" is quite appropriate, since Turing's vision has determined some of the most important social and cultural changes of the these one hundred years, especially in the last part of the century we have lived through, and he was not going to see due to his untimely death.

We will briefly sketch his many varied work, then we will discuss why, notwithstanding its range, depth and practical impact it is not so widely known as it deserves; on the contrary his ideas are often misinterpreted and vulgarized.

Turing's thought is always eminently scientific, also when he discusses popular issues, and appears to be too difficult to accept: it is a delicate equilibrium between imagination, hypotheses and experiments, and mathematical development; hence it gets reduced to simplified recipes; when it borders on philosophy it is interpreted in terms of rather gross traditional philosophical questions, precisely those he wanted to substitute with a scientific outlook.

Turing has given original and decisive contributions in many fields: mathematical logic, computability theory, applied mathematics, criptography, computers design, computer science, artificial intelligence, morphogenesis.

He has laid the basis of modern computability theory by proposing in 1936 his model of an abstract machine ("Turing's machine", as Alonzo Church first called it) and proving the existence of a universal machine, a machine which presented with a coded description of any machine would do the same computation as that of the machine given in input. Kurt Gödel would admit that Turing's was the definitive analysis of the concept of mechanical procedure. Using this tool Turing gave a negative answer to Hilbert's *Entscheidungsproblem* of finding a decision procedure for the logical calculus (contemporarily and independently of Church) and showed the existence of other undecidable problems.[1]

During World War II he gave an important contribution to the decryption of the German machine *Enigma* used to encrypt naval messages to submarines. After the

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[1] Not the *halt* problem, the original Turing's machines never halt; the problem has been formulated and proved undecidable by Martin Davis (1958) in relation to the modern presentation of Turing machine given by S. C. Kleene (1952).

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experience with the electronic machine *Colossus*, he became interested in the building of electronic computers, in 1945-48, and wrote the project for the first english computer ACE (Automatic Computing Engine) for the National Physical Laboratory (NPL), then collaborated to the Mark I built in Manchester. At the same time in USA J. P. Eckert e J. W. Mauchly, who had already built the last non electronic computer ENIAC, built the EDVAC with the supervision of von Neumann, and von Neumann himself with J. Bigelow built the Princeton computer MANIAC. In the *ACE Report* we see Turing interested in all aspects of the construction of the machine, from the flip-flop valves to the economic budget to the temperature conditions of the laboratories. A specific discussion is devoted to the different forms of memory, from mercury delay lines to cathodic tubes.

The machines designed by Turing are of course the concrete realization of his universal machine. Stored programs had been independently used by Eckert, but sporadically and as practical solutions, without understanding their decisive import, which was instead clear to von Neumann, and the basis of his Princeton computer; von Neumann used to tell his collaborators that everything was in the 1936 Turing's paper.

Captured by the practice, Turing laid the basis of computer science by using and conceiving the hierarchy of programming languages, inventing original solutions in the machine language, such as the conditional branching, or the stack automata for subroutines; he prefigured the assembler with the instructions he called in "popular form"; he wrote also the first programming manual for the Mark I; he gave the first example of a mathematical proof of the correctnes of a program; he deviced instructions to generate really random numbers through an external link.[2]

But Turing has also conceived machine architectures different from the sequential one; he has described and studied machines constituted by nets of nodes (possibly of other machines), a model for a nervous system with random distribution of neurons anticipating that of McCulloch e Pitts. These "non organized machines" Turing has shown that could be transformed in a universal machine by proper training, opening the way to machine learning. By considering architectures changing and modifing themselves through experience, Turing anticipates modern connectionism; he was inspired by the commmon predicament of people who, as is known, learn and follow rules witout being able to state them.

As for Artificial Intelligence (a term introduced in 1956 by J. MacCarthy)[3] Turing has immediately anticipated the fields of research and application where for sixty years practitioners have been engaged: games (chess), automatic translation, automated proofs, symbolic calculus, machine learning, conjectures formation by induction (later realized by Herbert Simon with the BACON programs)[4], numerical explorations; he has introduced the concept of nondeterminism in computability with the idea of *guess and check*. Turing has also determined the discussion on the possibility of machine intelligence still debated today.

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[2] To von Neumann are due the specification of the measure of *byte*, the *buffer*, the *flow charts*.

[3] Some of Turing's papers were classified and became known only in the nineties and later.

[4] Langley P., Simon H., Bradshaw G., Zytkow J.: Scientific Discovery: An Account of the Creative Process. MIT Press, Cambridge MA (1986).

Surely we are forgetting something. To say that for sixty years computer science and Artificial Intelligence have followed, or tried to follow the roads indicated by Turing is not to give a reductive judgement. It would be risible, if only we look around us to the changed world. On the contrary this appraisal is a proof of the fecundity of his vision and first realizations. Turing's however was not some alien knowledge but it was based on the clear understanding of the meaning of his universal machine.

Consider that still in the fifties could be advanced such sceptical valuations as the following, given by researchers who had given strong thrust to the development of digital computers:

> [I]f it should turn out that the basic logics of a machine designed for the numerical solution of differential equations coincide with the logics of a machine intended to make bills for a department store, I would regard this as the most amazing coincidence I have ever encountered.[5]

Turing on the contrary from the start had no doubts:

> This special property of digital computers, that they can mimic any discrete state machine, is described by saying that they are *universal* machines. The existence of machines with this property has the important consequence that, considerations of speed apart, it is unnecessary to design various new machines to do various computing processes. They can all be done with one digital computer, suitably programmed for each case.[6]

One can safely conclude that the only advances really independent of Turing have been those due to later tecnological innovations, such as microprocessors, printed circuits and miniaturization.

Let us not forget that Turing has done important research in pure and applied mathematic, from group theory in the study of almost periodic functions to probability,[7] to numerical analysis,[8] the zeros of the Riemann function, word problems.[9]

Finally Turing has built the first mathematical models of the growth of living organisms, in particular his reaction-diffusion model to explain the typical stripe colouring of the mantle of some animal species.[10] In chemistry, he has predicted the Bolousov-Zhabitinsky reaction, that is the existence of a non linear chemical oscillator.

Turing has been probably the last of universal scientists, before the new era of big science; if he could have continued, he could have become second only to Newton in Cambridge's Pantheon.[11]

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[5] Aiken H.: The Future of Automatic Computing Machinery. Elektronische Rechenanlage und Informationsverarbeitung (1956), n. 33. Aiken had built Harvard Mark I, a joint enterprise of Harvard University and IBM.

[6] Turing, 1950.

[7] He rediscovered independently the central limit theorem.

[8] He introduced for example the so called LU decomposition of a matrix in two triangular matrices.

[9] Group theory and almost periodic functions were the areas mentioned in his application for a Princeton scholarship in 1936.

[10] In the march 2012 issue of *Nature Genetics* it has been confirmed the discovery of two excitation and inhibition morphogenes, included in his model.

[11] So it has been surmised: Toulmin S.: The New York Review of Books, January 19), p. 3 (1984).

Instead of detailing Turing's scientific contributions we want to dwell on his method. He always tackles the problems with a totally candid curiosity, unencumbered by traditions of thought which have addressed the same questions, or by current opinions, by philosophical bias, by most commonly accepted tools, which he masters but is ready to discard and invent new ones if those available do not work.

The regular habits of thought are shattered, and laziness of mind reinterprets his answers as philosophical positions. For reasons of space we will consider only the problem of the intelligence of machines, but the same could be said on other issues, for example so called Turing's functionalism as a position in philosophy of mind.

In the literature and in popular discussions there is frequent talk of so called Turing's test. Arthur C. Clark e.g. says that the question whether (the computer) Hal could actually think had been solved by the english mathematician Alan Turing since the forties. Turing had pointed out that if one could maintain a prolonged conversation with a machine without being able to distinguish its answers from those a person could give, then the machine thinks, under any reasonable meaning of the word. Hal could easily pass the Turing test.[12]

The "prolonged conversation with a machine" is rather reminiscent of Weizenbaum's program ELIZA for psychiatric help.[13]

Statements similar to Clark's are being repeated in this period of Turing's celebrations, as have been in all the intervening years of the last half century. A particularly discussed issue is whether to pass the test is a necessary or sufficient, or both, condition to attribute intelligence to a machine.[14]

The philosopher John Searle has built his fortune on twenty years of discussion of Turing's test; he argues that if a machine built of beer cans could sustain a conversation in Chinese, we wouldn't for that accept that it knows Chinese.[15]

Turing has never used the term "test" in reference to machine intelligence, but once during a radiophonic debate at the BBC in 1952, in the heat of the discussion.[16] With this word, he referred to his imitation game, he began playing with in 1947, when he began to talk about intelligence of the machines.[17]

He starts reflecting on the surprise one feels in face of unexpected behaviours of machines, due to the complexity of instructions, possibly modified by the machine itself. Although it would be nonsensical to say that the machine can do other "processes that they are instructed to do", the results can appear unikely to the programmer. As usual he makes a comparison with "a pupil who has learnt much from

[12] Clark A.C.: 2001: A Space Odyssey. New American Library (1968).

[13] Weizenbaum J.: ELIZA - A computer program for the study of natural language communication between man and machine. Communications ACM, **9**, pp. 36-45 (1966). This was actually anticipated in 1948 by Turing, who observed that in order to ascertain the presence of consciousness in an interlocutor one should conduct a roundabout dialogue on subjects indirectly related to an argument, making connections and associations by means of literary images, analogies, metaphors.

[14] We prefer to mention only the sin, not the sinners.

[15] See for example Searle J.R.: Minds, Brains and Programs. Behavioral and Brain Sciences, **3**, pp. 417-58 (1980).

[16] The transcript has been published in Italian with the permit of Turing's estate curators in: Sistemi Intelligenti **X**(1), pp. 27-40 (1998).

[17] In the Lecture at the London Mathematical Society.

his master, but had added much more by his own work. When this happen I feel that one is obliged to regard the machine as showing intelligence".

In discussing memory, and the necessity of a rapid acces to it, Turing observes as an aside that:

> Certainly if they [digital computers] are to be persuaded to show any sort of genuine intelligence much larger capabilities [of quick access memory] than are yet available must be provided.

In this first paper he notices only the apparent contradiction in terms in talking of a machine with intelligence, considering "common catch phrases such as *acting like a machine*". However these phrases ignore possible more sophisticated uses of the terms intelligence and machine, on which he has began to reflect: if a machine must show some form of intelligence, we must "arrange that it gives occasional wrong answers" (for example in relation to undecidable questions).

In 1948, in a report for the NPL published only in 1956, Turing set to himself the task of "investigat[ing] the question as to whether it is possible for machinery to show intelligent behaviour".

Now the built in capacities are not enough: "the potentialities of the human intelligence can be only realized if suitable education is provided"; Turing describes here methods of training non organized machines based on the terminology of the educational process.

The last section is titled "Intelligence as an emotional concept".

> The extent to which we regard something as behaving in an intelligent manner is determined as much by our state of the mind and training as by the properties of the object under consideration. If we are able to predict its behaviour or if there seems to be little underlying plan, we have little temptation to imagine intelligence. With the same object therefore it is possible that one man would consider it as intelligent and another would not; the second man would have found out the rules of its behaviour.

In the last paragraph he describes an experiment he has done; he wrote a program (a "paper machine") to play chess, not badly. A medium human player had difficulties to understand whether he/she was playing against a human or this program. Now imagine three persons, A, B and C, A and C poor chess players, B a computer operator. In separate rooms, with obvious communication facilities, are located in one C and in the other either A or B who plays following the program: "C may find it quite difficult to tell with which he is playing".

The 1950 paper "Computing machinery and intelligence" begins with the imitation game. Turing proposes to discuss the question whether machine can think; one should preliminarily give a definition of "machine" and of "thought", a definition possibly respecting the common use; but to find such a definition would require an indefinite statistical and in the end inconclusive enquiry. So "I shall replace the question by another, which is closely related to it and is expressed in relatively unambiguous words".

The imitation game is briefly described in the following terms: the game involves three players, a man, a woman and an interrogator who by the (written) answers of the others to his questions has to guess who is who. The questions may refer to any subject. Turing adds that the woman tries to help the interrogator, the man to deceive, but this part is not clearly explained. The woman need not say e.g. “I am the woman” since the same can be declared by the man. Anyway Turing asks now what happens if the man is substituted by a machine: “Will the interrogator decide wrongly as often when the game is played like this as he does when the game is played between a man and a woman? These questions replace our original ‘Can machine think?’”.

Turing hypothesis is that there can be machines (his universal machine) able to play well in the game; he uses this hypothesis in the 1950 paper to counter preconceived beliefs against the possibility of thinking machines, starting with that based on Gödel’s incompleteness theorem, an argument which will be repeatedly resumed by many people.

Notice however that the game is not offered as a discriminant or as a criterion for definition, but only as a description of the circumstances which could justify our idiosyncratic modes of expression (including those used by Turing himself). The focus is on the interrogator. A game, or a series of games, is not a necessary criterion for the attribution of intelligence to a machine, nor a sufficient one: there is no test to pass but only better or worse performances, in the sense of greater or lesser difficulty for the interrogator to decide, compared to the games where the player is human.

To play well for a machine means that it makes the game difficult for the interrogator:

> I believe that in about fifty years’ time it will be possible to programme computers, with a storage capacity of about 10^9, to make them play the imitation game so well that an average interrogator will not have more than 70% chance of making the right identification after five minutes of questioning.

The prediction is not strongly optimistic. If the average interrogator has a 70% probability to give the right identification in five minutes, that means that the probability of the machine to deceive the interrogator is less than 30%.

Turing however does not evade the popular question:

> The original question “Can machine think?” I believe to be too meaningless to deserve discussion. Nevertheless I believe that at the end of the century the use of words and general educated opinion will have altered so much that one will be able to speak of machine thinking without expecting to be contradicted.[18]

In the seventies it was common practice to make mockery of the huge gap between the promises and the actual progresses in areas such as that of speech understanding and meaning recognition. Nowadays we read of programs (e.g. IBM’s Watson) that win against human players, in quiz contests where the questions, relating to literature, history, science require the understanding of nuances of meaning, of rhetorical figures; we read of cellular phones that understand irony and sarcasm.[19] The producers say that present systems still have a high margin of error (20 %), but people

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[18] He goes on observing that the scientist does not proceed from facts to facts, but is often guided by unproved conjectures: “Conjectures are of great importance since they suggest useful lines of research”.

[19] Tremolada L.: “Se il telefono ha il senso dell’umorismo”. *Il sole24ore*, January 2012, p. 49.

using the systems treat them as human beings, since they have the impression that they are human beings.

Turing's arguments, in this case as in others, for example the possibility of "building a brain", have a peculiar character, that they seem to admit of no confutation. It has been remarked that the style of the arguments, in the restatement of the problems, is that of passing to the adversary the burden of the proof.[20]

In the case of thinking machines, the restatement of the problem is not a rhetorical move, a kind of *ad hominem* argument, but follows from Turing's interests. A positive reason to believe in the possibility of building machine (called) intelligent consists in the fact that "it is possibile to make machinery to imitate any small part of a man". The universal Turing machine, although proved to be such on the basis of its explicitely written instructions, is conceived from the start and inspired by the observation of the human mechanical behaviour. The very word *computer* in the english lexicon meant up to then the human calculator.

The computable numbers, in the 1936 paper, are defined as "those whose decimal are calculable by finite means. [...] No real attempt will be made to justify the definition given until we reach §9. For the present I shall only say that the justification lies in the fact that the human memory is necessarily limited". In the following, the description of the machine is full of mentalistic terms: "the 'scanned symbol' is the only one of which the machine is, so to speak, 'directly aware'. [...] However, by altering its *m*-configuration the machine can effectively remember some of the symbols which it has 'seen' (scanned) previously". The terminology is justified by the fact that the machine is described by observing the human behaviour.[21]

> Computing is normally done by writing certain symbols on paper. We may suppose this paper is divided into squares like a child's arithmetic book. [...] I shall also suppose that the number of symbols which may be printed is finite [...]. The behaviour of the computer at any moment is determined by the symbols which he is observing, and his 'state of mind' at that moment. We may suppose that there is a bound *B* to the number of symbols or squares which the computer can observe at any moment. If he whishes to observe more, he must use successive observations. We will also suppose that the number of state of mind which need to be taken into account is finite. [...] Again, the restriction is not one which seriously affects computation, since the use of more complicated states of mind can be avoided by writing more symbols on the tape.
>
> Let us imagine the operations performed by the computer to be split up into "simple operations" which are so elementary that it is not easy to imagine them further divided. Every such operation consists of some change of the physical system consisting of the computer and his tape. [...] Besides these changes of symbols, the simple operations must include changes in the distribution of observed squares. The new observed squares must be immediately recognisable by the computer. I think it is reasonable to suppose that they can only be squares whose distance from the closest of the immediately previously observed squares does not exceed a certain fixed amount.

Even for real computing machines, the imitation game confirms that the readiness to label a behaviour as intelligent depends not on the behaviour itself but on our culture and our mental disposition.

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[20] See among others, Meo A.R.: Guida alla Metrologia. Supplement to *Qualità*, n. 4, 1995.

[21] Though some statements seem to express the opposite direction: "We may compare a man in the process of computing a real number to a machine which is only capable of a finite number of conditions".

Further readings

Turing's biography by A. Hodges, *Alan Turing: the Enigma*, Hutchinson, London, 1983 (transl. it. *Alan Turing: una biografia*, Bollati Boringhieri, Torino, 1991) will long remain unsurpassed. There is also a biography written by Alan's mother: Sara Stoney Turing, *Alan M. Turing*, Heffers, Cambridge, 1959.

Turing's scientific works are published in four volumes of *Collected Works*, North Holland, Elsevier Science B.V., Amsterdam:

Pure Mathematics, J.L. Britton ed., 1992
Mechanical Intelligence D.C. Ince ed., 1992
Morphogenesis, P.T. Saunders ed., 1992
Mathematical Logic, R.O. Gandy, C. E. M. Yates eds., 2001.

The last one was delayed by the death of the editor, and Alan's friend, R.O. Gandy, but the very delay has made possible to insert in it, besides the logical papers, others not logical essays (e.g. one in criptography previously classified).

In Italian, a selection of papers from *Mechanical Intelligence* has been translated in A.M. Turing, *Intelligenza Meccanica* (G. Lolli ed.), Bollati Boringhieri, Torino, 1994. The volume comprises the translation of:

ACE Report (first part), 1945
Lecture at the London Mathematical Society, 1947
Intelligent Machinery, 1948
Computing machines and intelligence, 1950.

Translated from the Italian by Kim Williams

Alan Turing and the Poisoned Apple

Massimo Vincenzi

ALAN: (sings in the dark)
I'm wishing (I'm wishing)
for the one I love
to find me (to find me)
today (today)
I'm hoping (I'm hoping)
and I'm dreaming
of the nice things (the nice things)
he'll say (he'll say).

THE JUDGE The Queen versus Alan Turing, we may proceed to question the accused. Please bring him in.

ALAN Mother?! Mother, where are you? It's cold here. It's already fall and cold weather suddenly set in. Yesterday I was running along the river in a T-shirt and shorts and today the cold wind forced me to cover myself. And I'm also a bit sad. I know it may seem strange to you that I'm writing, but as you always said, even pumpkins mature sooner or later.

THE MOTHER Yes, even pumpkins mature… So, now you write. You, who never even read my letters, who threw them away…

ALAN It's true. But it was not out of indifference or because I didn't love you, mother. It was because I had confidence in your ability and in your strength. I threw them away saying: she'll be all right. I was always certain of that. I know it may seem horrible. But you always understood my not quite normal side. You must be smiling thinking about it. I mean, about my not quite normal side. Like when I tie the teacup to the radiator with a padlock and a heavy chain because I'm afraid that somebody will steal it when I'm not there. But there's nothing strange about that, believe me. You have no idea how many things get stolen in universities and colleges. Especially in America. And they steal them just like that, with their usual way of doing things. Somebody drops by and says: hey, you, lend me your cup, let

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me take a sip. Lend me your cup? A sip? I get the shivers even now thinking about it. A padlock? I would have put my teacup in a safe, if I could. I know, I know. Smile if you like, mother, smile…

THE MOTHER Like the time you asked me to mail the letter to Santa Claus? Remember? You were 22. You had just returned from Cambridge and your aunt was here. Remember the look on her face? Her white powdered grimace? Red with shame?

ALAN And her voice? A letter to Santa Claus? What's our young teacher doing, playing games? Play games? I never play, or maybe I always do. And plus, I am not a young teacher. That's why I can't tell whether Santa Claus really exists or not. And you, dear aunt, can you prove, on the basis of perfect algebraic expressions, that God is watching over you and is made in your image and likeness? Your idea of God is so clear that you are sure that now, right now, your God would bring you to safety in Paradise instead of me? But I never said anything. At that time, I was learning to be silent.

THE MOTHER You kept on talking about Santa Claus to me. That year, you told me you wanted a nice teddy bear for Christmas.

ALAN But not the hard type, with a hard nose, coarse hair, stiff joints and eyes staring into space like a mummy. No, that year for Christmas I wanted a soft teddy bear, one of those that usually wear a white bib with a yellow writing: I'm the king of spots. And their face is such that you can't avoid talking to them. They just sit there, listening to you, playing with you, giving you advice. In short, how can you not understand how important it was for me and how bad I wanted a teddy bear.

THE MOTHER As a matter of fact, that year Santa Claus brought you a beautiful teddy bear: you named it John.

ALAN John… He has been with me most of my life. I lost him in these last few weeks. I don't know how it could possibly happen. I don't know many things about the last few weeks. Mother, I don't know how many, I really don't… I'm sorry, but I must go to bed… just for a few minutes. See you soon.

THE JUDGE Alan Turing, please rise. Put your left hand on this Bible and promise to tell the truth, nothing but the truth. Say: I promise.

ALAN (sings in the dark) "Put the fruit in the poison until it overflows – make yourself beautiful to tempt her and put her to sleep forever" – "We are evil beings wandering in despair and hungry on this hopeless Earth. The best we can aspire to is to navigate just below the line of nastiness". No, mother, this is not a sentence from a Greek tragedy. It's what me and others told ourselves in our barracks during the war. It was wonderful, even if it may seem crazy to say it now. But it was really wonderful: it was like a movie. And it was for us. We were soldiers, but we didn't take any chances. Instead, every time we made a mistake, a bomb fell on our friends' heads, on our families in London and our soldiers ended up under enemy fire. And this should have killed our conscience worse than one of Hitler's V2s, but we were young and crazy. We felt like the knights of King Arthur engaged in the eternal battle against evil, in the endless search for the Holy Grail that would give us peace and happiness forever. And we were all such frightful snobs. Even though we lived in rundown wooden barracks and not in medieval castles. And then the dragons used to spit fire, a fire that burned everything.

THE MOTHER I remember it well, Alan, I've seen the bombs...

ALAN When I thought of you was the only time I felt the weight of what I was doing. Then I would stop playing against the Nazi enemy and I earnestly put myself to work and suffer. But they, the Germans, were really good. That's when I started to think about God. If good and evil were so clear-cut, what the hell was going on? As soon as we discovered a code to spy on them, they changed it. And sometimes we didn't discover anything for weeks. Or months. It was the most dramatic moment. The bombs were falling everywhere. Every night we would see London in flames. And the desire to play had left us. Giving way to anxiety. That taste of rotten aluminium that you feel between your tongue and your palate the day after a night of partying. And you hold your head in your hands and say it will never happen again: I promise, mother, I won't go out tomorrow, tomorrow I'll study till late. Tomorrow...

THE MOTHER In those days, nobody thought tomorrow would come. But, luckily, we were wrong.

ALAN Yes. So we stopped following the enemy's moves. It's like in rowing. It's useless to pull when you realize that their pace is too high. Concentrate on your pace, think about your heart, listen to the beats, forget your breathing, can you hear? Can you hear that you're not panting now? Only the heart has a slow and regular beat, then it gets faster and faster, you can't imagine how fast. And the other boat is no longer there. Only the water and your heart. It's your head that makes it work. That's how we built our Colossus: my first intelligent car. The weapon that revealed the secrets of the evil dragons. Their mouths spit fire and we began to feel again like the knights of King Arthur.

THE MOTHER You told me this story a thousand times. But to those who say that you must process your grief and relive the traumas in order to free yourself of them, I say that only those who never experienced bereavement or suffered traumas speak so.

ALAIN I think the only things we should relive are nice things, mother. The ones that warm us from the cold that's out there. Where they are. Goodnight, mother.

THE MOTHER Dream, Alan. Always dream. Dreams are the children of imagination, just like words are the children of thoughts. You take after me, Alan. I've always dreamed too. Just think, a little while ago, your teddy bear came to see me. He was smiling. He told me you were fine. That you lost weight. That I should not worry. Dear Alan, you'll find your teddy bear. We will find it, you'll see.

THE PROSECUTION Tell us, professor Turing, do you recognize the boy sitting in the first row? Wait, don't answer right away. Let me finish the question. Professor

Turing, if you do know him, as it seems clear to me, can you tell us how you met him?

ALAN Mother, remember that from an early age I had in mind what I was going to be when I grew up? Or rather, I had numbers in my head. Numbers and love. It must have been the second semester of the first year of high school or maybe the summer after that. I can feel, as if it were now, the smell of our garden, it came through my bedroom window. In those days, I knew I had found my way. It was neither right nor wrong. I loved numbers and studied maths. I loved men and looked for men. I can't find other words. Simple, like breathing. It's nature. This is how the world goes. And now that they have extinguished all my desires, I have no regrets. If I were not trapped inside this body that is no longer mine, I would put back on my old running stuff and would jump in the lawn beyond the railway to turn off all the noise in the world and to hear only that of my heart. But races are over for me. At least for now. Do you remember how much I liked racing?

THE MOTHER Yes, I remember how good you were. I came to see you a few times. You arrived exhausted, all skin and bones. And yet you had strong muscles. Strong shoulders from rowing, short and constant speed acquired playing tennis. An athlete.

ALAN Yes, an athlete. I you'd see me now, mother, you would start crying. I'm old and fat. I even have a sort of wet-nurse's pendulous breasts. I can't look at myself in the mirror. I am not that thing it reflects. I can no longer ask the mirror on the wall who's the fairest of them all? It would answer me: are you blind? And it would be right. I'm even losing my hair. I gain weight and lose hair. It's the head in between that doesn't understand anything. When you have trouble understanding yourself, it's impossible to look outside.

THE JUDGE An acquaintance? Pardon me, professor, but I'm afraid that, in this courtroom, not everybody understands what you mean by "acquaintance". Can you explain the meaning of acquaintance?

ALAN The things I thought when I began to discover death came back to my mind. For a boy, for a child, the transition to adulthood is almost a ritual, snatched from the world of fairy tales. Body and soul. I thought a lot about this at that time and long after. Body and soul. The body as a strong physical presence, active and full of energy, is able to draw its own soul and to hold it tight for life. Or nearly. Because there comes a time, sooner or later, when the body no longer has the energy and the strength it had before. Like a planet that is unable to hold its own satellites. Therefore, the soul begins to move away. Makes longer turns. It flies, as if to see if it can find another body. A future. The body shuts down and the soul frees itself. It's a very slow process. And the numbers stop here. But that's how things go. As you shut your eyes taking your last breath, your soul goes far away taking along, locked in its immense experience, a vague knowledge of the body that once was. And with this trace, it will seek the body that will be.

THE MOTHER I know what you think about God. Once you talked about Him with that boy on the tennis court. Remember?

ALAN Our neighbours' son was sitting on the tennis court in the fog and didn't want to leave. His mother was screaming. I went over, sat on the red earth, dark with water, and asked him: but do you think that God would catch a cold if he sat on a wet tennis court in the winter? He looked at me very serious and said: but God is in the church, he doesn't play tennis. I answered him: but he would be sorry if you caught a cold. That's the last lie I told in the name of their God. Because, in fact, I didn't need a God who wouldn't play tennis with me just because I liked men…

THE MOTHER You know, Alan, that child has grown up. He's a handsome boy. He also studies in Cambridge. A few days ago, his mother told me that somebody spoke to him about you. Other students or some professor, I didn't understand. She lowered her eyes and mumbled some words. I didn't understand anything.

ALAN I would do the same. I'd be ashamed too, if I were like them. And your God, is he ashamed of them?

THE MOTHER I know what you think about God, but you'll see that I'm right. God plays tennis and he would be happy to play with you.

THE PROSECUTION So, professor, an acquaintance, as you have defined it, comes home with you and spends the night. Then comes back the following evening. And in all this, you were not curious to know how old your young acquaintance was? Because I believe you must have guessed that he was young. Am I right, professor?

ALAN Machines think. Mother, my machines think. But they don't have a soul that stays with them until they die. Machines are not men. They have the same logical framework we have in our heads, but they don't have a heart. And my enemies never understood this. I'm not a pervert who's in love with machines. But someone said that my passion for machines was "insane, morbid". I wanted to scream that machines are not my type at all, mother. But they would not have understood. I know that machines will become so small that, when they'll see how tiny, how fast and how useful they are, they'll understand that I'm not crazy. I only wanted to do. They wanted to stand still. At the end, this is the heart of the matter. Simple. Human matters are always too simple. That's why we need machines to move forward.

THE MOTHER Dear Alan, they can beat us, they can take everything away from us. Even our memories. They even change our faces, like they did with you, but not our hearts. Your heart will stay the same.

THE PROSECUTION Pardon me, professor, but I really don't understand. You insist that the boy, whose age you're not aware of, whose name you barely know, had your house keys and he used them to open the door to his accomplices. I'm sorry, but I don't understand. In my family, only my wife and my three children have the house keys. My children only since they've grown up. I find it difficult to think of this young man as one of your children. Well, the age fits, but you don't even know his name. And, though he has fine features, I find it difficult to think of him as your wife…

ALAN Mother, remember what I wrote to you a life ago? I was little more than a boy. Remember what I wrote to you when my friend Christopher died? I wrote these words. I've always kept them inside me and they will remain there forever.

I cry, but I'm not in despair. I feel, dear mother, that I'll meet Christopher again somewhere and that we'll have work to do together, as we did here. I feel I never had any friends besides him. He had the natural ability to make all the others appear uninteresting. This is what made him special. That's why I'm certain he's not dead. He simply went to do something important someplace else…

Well, dear mother. That's where I feel I want to go now. I must go there now, right away. There's no more time, I have no more time left here. It's like when I opened a calculus book and went to the end of the long sequence of numbers to see the result of the operation. I didn't have time. As if it were a novel to burn up in one night. I looked at that number on the last page, on the last line, marked with a small bold square, smiling happy. At that moment I knew there was something right at the end of everything. There was that black square with the result in the middle which gave me confidence. A hidden, mysterious, secret confidence.

Forgive me, mother. For not being what you wished me to be. Forgive me. You are the only one to whom I owe an apology. The only one who never asked me. You saw everything. You understood everything. There are no social masks that protect from true love. That's why I've been away from you for so long. You've always known who I really was and you never said a word. Now I'm going to look for Christopher and you're the only one I'll miss. The only one I must ask forgiveness. Maybe you deserved a better son. Different… Which is probably not the right word to use when speaking of the son you deserved. What does it matter if your son is a genius: someone who helped England win the war. It's not me who says so, they wrote it on the medal they gave me. A son who paved the way for artificial intelligence. What will it mean to you, if nothing will remain of this, dear mother? You know how I feel about all this. I never told you, but you know.

Alan says that machines think, Alan sleeps with men. Therefore, machines can't think.

It's a perfect syllogism. For them, I am and will always be a pervert, mother.

THE MOTHER I've always loved you and never judged you.

ALAN But that's what they call it: perversion. I ended up like this because of them. My degree, my prizes, my books counted nothing. I had so many words to defend myself, but now I've exhausted them together with my strength. I've spent all of them. I had won the war. I had gone to the United States to teach. I could have said: that's how I am, take it or leave it. People have always liked oddballs. I could even have gone to class wearing my pyjama coat, played tennis naked, with only my raincoat on, and society would have stood there, lined up on the sidelines, nudging: look at him, the professor is having a good time, genius and debauchery. And instead…

The brilliant professor is not just a bit extravagant. He's not just a stubborn man who persists in dreaming an imaginary future for some metal boxes. Yes, your gifted and loyal son, mother, is not just shy and bashful, he's not attracted to women. Yes, mother, your son not only likes men, your son is a pervert.

THE PROSECUTION Very well, professor. I thank you very much. The audience in this courtroom also thanks you, you'll see. Your confession, pardon me, your story has been of great help to us. I only regret that – I say this from professor to professor – it came after so many long days of trial. I have no more questions, your Honour. I retire and give the floor to the Court.

ALAN The arrest, the trial, the drugs. Me saying yes. Mother, the gross error I made was saying yes. If the truth is so trivial that you cannot tell that truth, then maybe that truth doesn't exist. Perhaps they're right, mother. That's when my body began losing energy. And my soul began to leave this body which, mutilated, horribly mutilated by my consent, is no longer mine.

I only ask one thing of you, mother, don't let them erase my life.

Machines think even if Alan sleeps with men.

THE JUDGE Today, March 31*st* 1952, in the name of God and of the Queen, this Court sentences Alan Turing, self-confessed offender of crimes against public morality and sexual assault contrary to nature, to undergo an antiandrogen drug treatment capable of inhibiting libido. The treatment shall be administered in the Gloucester clinic as from tomorrow and shall last until the doctors shall consider the accused cured. The Court retires.

ALAN Dear Snow White, you and me always liked apples. I never understood, or rather I really don't remember, if I liked apples or your story first. There's everything. There's pain, beauty, the struggle between good and evil, the joy of childhood, the forces of nature, love that surrounds all things, darkness, the sudden lights of the evil forest that turns into a clearing, the warmth of the house, white horses and Prince Charming, the not too evil hunters, small adorable dwarfs and even the witch is not as evil as she looks. The red apples have the quality of being amazingly simple. That elementary conquering force that only very few things and individuals are capable of achieving. Many years ago, but not too many, I had a friend. In short, this friend, if you looked at him, he was nothing special. One like many. Then you'd see him in a large room full of other boys like him. And suddenly, you'd understand that behind his apparent normality, there was a wonderful construction, as complex as the craziest and most talented architect wouldn't have been able to imagine. And all this mixture, all the effort you perceived was just a sensation, a feeling that you had to have the strength to understand. Because what he returned to the world was the strength which only true simplicity is able to give.

I already loved numbers when I met him, but looking at him, I understood why I loved them. As I loved him. Numbers, just like apples in their own way, were and are like him. So simple that they seem elementary. And you are stunned by their enormous romantic beauty.

You were right to eat the apple, Snow White. I'm eating mine and it's so good, just like yours, I presume. I can't even taste the poison. Poison is good. No, no, don't worry. I'm not hoping that soon the door of my room will open and Prince Charming will appear. Since Christopher died without leaving me even the memory of what true love could have been, I never even hoped for it. I have since removed that word from my thoughts, I cancelled my desires. And maybe this is the only mistake I made in my whole life. Who knows, Snow White, perhaps one day we will really meet. I content myself with the apple. It's cold outside.

I'm going to sleep now. I'll sleep a little. I haven't done so for too long. I turn off the light and seek that old dream of mine: there's me wearing the shiny white running shoes, black shorts and white T shirt; the grass is wet, the lawn slightly uphill. I run and keep on running. The lawn is wider and wider, I see only green around me. Even the sky is mixed with green. And I see my white shoes. Then the green disappears. And the shoes are still, or rather, they move so well that they seem motionless to me. Finally, I no longer see them either. I'm suspended, I run in empty space. And below me, I see a body which isn't mine, exerting itself in a desperate race, so I turn my head with a smile and leave. Weightless.

One more thing: say hello to Santa Claus, if you hear from him.

Film Credits

Alan Turing e la mela avvelenata 2012
by Massimo Vincenzi
Directed by Carlo Emilio Lerici.
Music by Francesco Verdinelli.
Voice off by Stefano Molinari.
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