UNIT 3

FRACTIONS, DECIMALS AND THE FOUR OPERATIONS

Unit Outcomes: After completing this unit you should be able to:

- **know types of fractions.**
- **understand concept of percentage and principles of conversion of percentage to fraction and decimal.**
- **know the method of comparing fractions.**
- **perform the four basic operations on fractions and decimals.**

Introduction

In earlier grades, you have learnt about fractions. After a review of your knowledge about fractions, you will continue studying fractions, decimals and the four operations in the present unit. Here, you will learn about types of fractions, conversion of percentages to fractions and decimals, comparing fractions and performing the four basic operations on fractions and decimals.

3.1 Types of Fractions

Activity 3.1

1. Find a fraction which is represented by the diagram shown below.

a)

c)

b)

d)
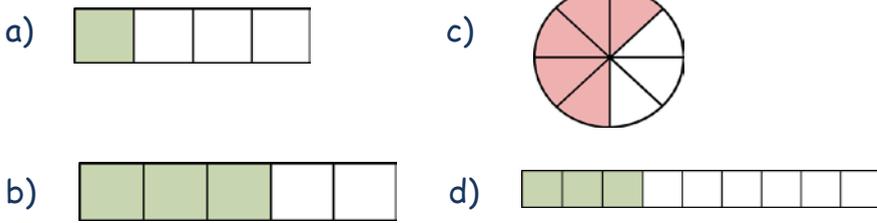
Figure 3.1

2. Write each fraction in simplest form.

a) $\dfrac{18}{20}$ b) $\dfrac{42}{60}$ c) $\dfrac{24}{40}$ d) $\dfrac{42}{56}$ e) $\dfrac{65}{75}$

3. Use <, >, or = to Compare each pair of fractions.

a) $\dfrac{7}{8} \square \dfrac{3}{8}$

d) $\dfrac{10}{15} \square \dfrac{8}{15}$

b) $\dfrac{4}{5} \square \dfrac{6}{10}$

e) $\dfrac{4}{9} \square \dfrac{7}{9}$

c) $\dfrac{6}{12} \square \dfrac{4}{8}$

f) $\dfrac{7}{16} \square 1$

Do you remember what you have studied about fractions in you grade 4 mathematics lessons? In this sub-unit you will study about types of fractions.

Remember that a **fraction** is a number (usually written as $\dfrac{a}{b}$, where a and b are whole numbers and b is not 0) equal to the quotient of a and b or

a divided by b. Fractions are used in everyday life. For example, you can find what fraction of a week 4 days is:

4 days = $\dfrac{4}{7}$ week or

you can find what fraction of a month 7 days (a week) is:

$$7 \text{ days} = \dfrac{7}{30} \text{ month.}$$

For the fraction $\dfrac{3}{4}$, the number 3 is called the **numerator** and the number 4 is called **denominator**

$$\dfrac{3}{4} \quad \begin{array}{l} \longleftarrow \textbf{numerator} \\ \longleftarrow \textbf{denominator} \end{array}$$

The denominator of a fraction tells us the number of equal parts into which a whole has been divided and the numerator tells us how many of these parts are being considered. Thus $\dfrac{3}{4}$ tells us that the whole (a cup) has been divided in to 4 equal parts and that 3 parts are being used.

Figure 3.2

You can represent fractions by using a diagram such as the following.

1 part shaded out of 2. Thus $\dfrac{1}{2}$ is shaded.

2 parts shaded out of 3. Thus $\dfrac{2}{3}$ is shaded.

3 parts shaded out of 4. Thus $\dfrac{3}{4}$ is shaded.
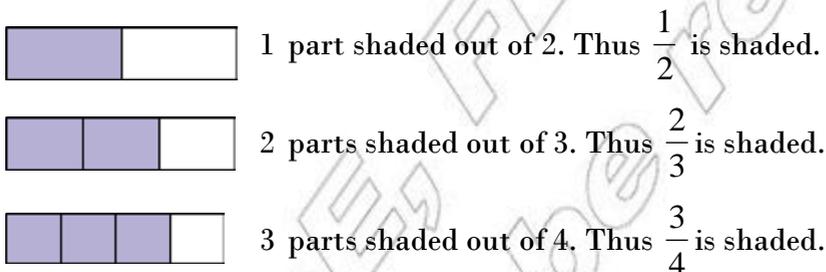
Figure 3.3

Observe, infractions such as $\dfrac{1}{2}$ or $\dfrac{2}{3}$ or $\dfrac{3}{4}$, that the value of the numerator is less than the value of the denominator. Such fractions are called **proper fractions.**

A **proper fraction** has a value less than one; its numerator is smaller than its denominator.

Example 1

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{10}, \frac{1}{12}, \frac{4}{7}, \frac{3}{8}$$ **are some examples of proper fractions.**

Can you give an example of a proper fraction of your own?

How much sleep do you get at night? Doctors recommend that we get 8 to $8\frac{1}{2}$

hours of sleep. What fraction is equivalent to $3\frac{1}{4}$?

Numbers such as $8\frac{1}{2}$ and $3\frac{1}{4}$ are called **mixed numbers**.
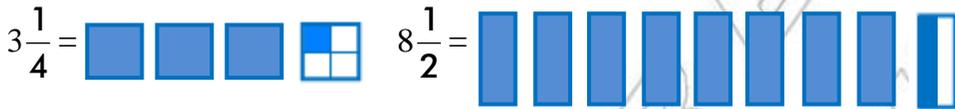
Figure 3.4

Mixed numbers show the sum of a whole number and a fraction. Mixed numbers can also be written as fractions.

Activity 3.2

Work with a partner.

Materials: paper, pencil, ruler

Draw a **model for** $1\frac{3}{4}$

- Draw a rectangle like the one shown below. Shade the rectangle to represent 1.

- Draw an identical rectangle beside the first one. Separate the rectangle on the right into four equal parts to show fourths. Shade three parts to present $\frac{3}{4}$
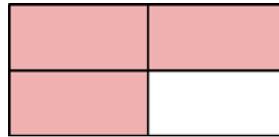
- Separate the whole number portion in to one-fourths.
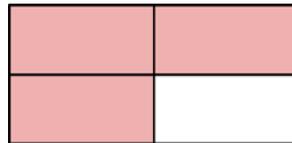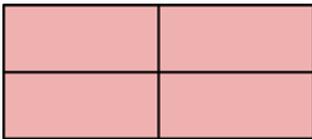
Figure 3.5

- How many shaded one-fourths are there?

- What fraction is equivalent to $1\frac{3}{4}$?

A fraction, like $\frac{8}{5}$ or $\frac{5}{4}$ with a numerator that is greater than or equal to the denominator is called an **improper fraction.**

From the Activity, you can conclude that it is possible to express a mixed number as an improper fraction. Here is one such example.

Example 2

Write $3\dfrac{1}{2}$ as an improper fraction.

Solution: **Find the number of parts in the whole numbers.**
Then add the fraction.

$\dfrac{2}{2}$ + $\dfrac{2}{2}$ + $\dfrac{2}{2}$ + $\dfrac{1}{2}$ = $\dfrac{7}{2}$

$3\dfrac{1}{2}$ $\dfrac{7}{2}$

$3\dfrac{1}{2}$ units   7 halves
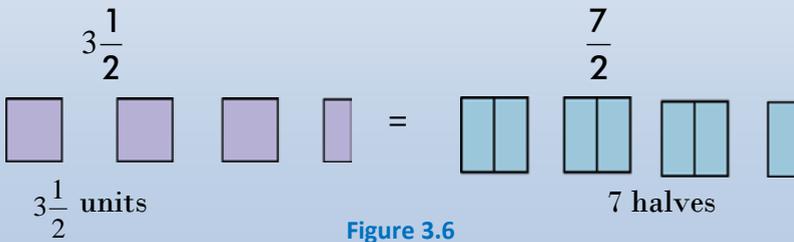
Figure 3.6

A short-cut is to multiply the whole number by the denominator and add the numerator. Then write this sum over the denominator.

$$3\frac{1}{2} = \frac{(3\times 2)+1}{2} = \frac{7}{2}$$

Study how you can write $3\dfrac{1}{4}$ as an improper fraction.

$$3\frac{1}{4} = 3 + \frac{1}{4} = \frac{13}{4}$$

Figure 3.7

$$3\frac{1}{4} = \frac{13}{4}$$
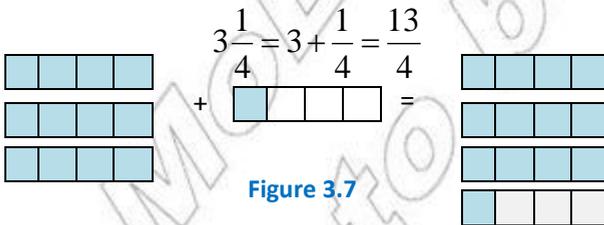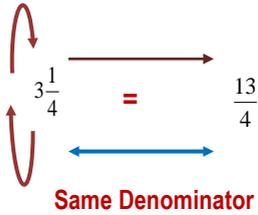
Same Denominator

1. Multiply the denominator (4) by the whole number part (3)
2. Add the numerator (1). This is the new numerator.
3. Use the same denominator.

Here is the procedure. To write a mixed number as an improper fraction:

Converting mixed numbers to improper fractions

Step 1. Multiply the denominator of the fraction by the whole number.

Step 2. Add the product from step 1 to the numerator of the old fraction.

Step 3. Place the total from step 2 over the denominator of the old fraction to get the improper fraction.

Group work 3.1

Express as improper fraction.

a) $3\frac{5}{7}$ b) $6\frac{1}{4}$ c) $8\frac{1}{2}$

Example 3

Express each mixed number as improper fraction.

a) $4\frac{1}{2}$ b) $7\frac{2}{5}$

Solution

Find $4\frac{1}{2} = \frac{(4 \times 2) + 1}{2}$

$\qquad\qquad = \frac{9}{2}$

$7\frac{2}{5} = \frac{(7 \times 5) + 2}{5}$

$\qquad = \frac{37}{5}$

A whole number can be changed in to an improper fraction.

$$2 = \frac{2}{1} \quad = \quad \frac{4}{2} \quad = \quad \frac{6}{3} \quad = \quad \frac{8}{4}$$

2 units   **4 halves**   **6 thirds**   **8 fourths**
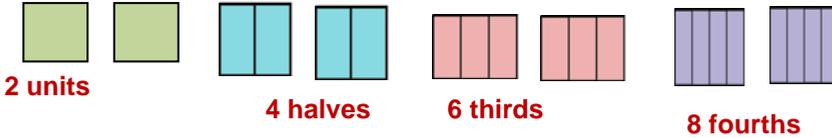
Figure 3.8

An improper fraction can also be changed in to either a whole number or a mixed number.

$$\frac{5}{2} \qquad\qquad 2\frac{1}{2}$$

$=$
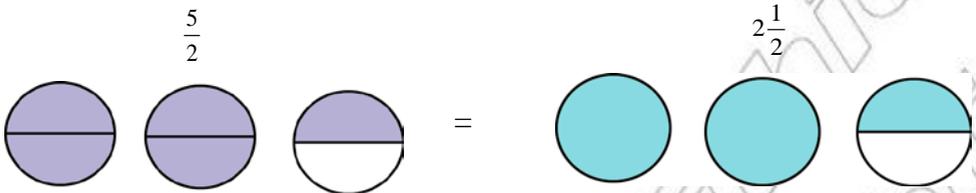
Figure 3.9

To convert an improper fraction to a whole number or a mixed number, divide the numerator by the denominator. Here is a diagram illustrating the procedure.

Divide the numerator (17) by the denominator (5)

The answer (3) is the whole number part of the mixed number

The remainder is the numerator of the fractional part

remainder

Improper fraction $\left\{\dfrac{17}{5} = 17 \div 5 = 3 \right.$ With remainder $\qquad 2 = 3\dfrac{2}{5} \left. \right\}$ Mixed number

Whole number

Same denominator
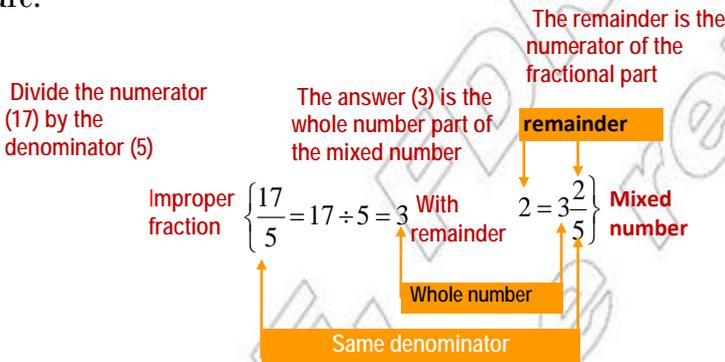
Note here the diagram for $\dfrac{17}{5}$
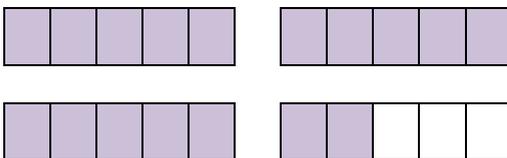
Figure 3.10

Note that $3\frac{2}{5} = 3 + \frac{2}{5}$, the sum of a whole number and a proper

fraction. Similarly, $\frac{5}{3} = 1\frac{2}{3}$ and $\frac{8}{5} = 1\frac{3}{5}$.

Converting improper fractions to whole or mixed numbers:
Step 1. Divide the numerator of the improper fraction by the denominator.
Step 2. a) If you have no remainder, the quotient is a whole number.
b) If you have a remainder, the whole number part of the mixed number is the quotient. The remainder is placed over the old denominator as the proper fraction of the mixed number.

Example 4

Convert each improper fraction to a mixed number in simplest form or a whole number.

a) $\dfrac{21}{4}$ b) $\dfrac{24}{3}$ c) $\dfrac{77}{8}$

Solution: a) $\dfrac{21}{4} = 5\dfrac{1}{4}$ since $4\overline{)21}$
$$\underline{20}$$
$$1$$

with quotient 5

b) $\dfrac{24}{3} = 8$ since $3\overline{)24}$
$$\underline{24}$$
$$0$$

with quotient 8

c) $\dfrac{77}{8} = 9\dfrac{5}{8}$ since $8\overline{)77}$
$$\underline{72}$$
$$5$$

with quotient 9

Exercise 3.A

1. Identify whether each of the following statements is true or false.

   a) $\dfrac{n}{n} = 1$ for any number n different from zero.

   b) $\dfrac{n}{1} = n$ for any number n.

   c) $\dfrac{0}{n} = 0$ for any number n different from zero.

   d) $\dfrac{15}{16}$ is an improper fraction.

   e) $\dfrac{n}{0}$ is not defined for any number n different from 0.

   f) $\dfrac{47}{5} = 9\dfrac{2}{5}$

   g) $\dfrac{23}{6} = 5\dfrac{1}{6}$

2. Classify the given fraction as proper or improper.

   a) $\dfrac{13}{15}$ b) $\dfrac{17}{5}$ c) $\dfrac{9}{9}$ d) $\dfrac{0}{5}$ e) $\dfrac{8}{1}$

3. Write the fraction as a mixed number.

   a) $\dfrac{21}{10}$ c) $\dfrac{18}{7}$ e) $\dfrac{29}{6}$ g) $\dfrac{69}{9}$ i) $\dfrac{101}{10}$

   b) $\dfrac{46}{5}$ d) $\dfrac{59}{8}$ f) $\dfrac{39}{2}$ h) $\dfrac{97}{3}$ j) $\dfrac{98}{9}$

4. Write the mixed number as an improper fraction.

   a) $8\dfrac{1}{7}$ c) $6\dfrac{1}{10}$ e) $1\dfrac{2}{11}$ g) $8\dfrac{3}{10}$ i) $2\dfrac{1}{16}$

   b) $7\dfrac{1}{9}$ d) $5\dfrac{3}{11}$ f) $4\dfrac{2}{13}$ h) $9\dfrac{4}{11}$ j) $9\dfrac{7}{8}$

5. A person slept for 7 hours. What fraction of the day (24 hours) is that?

6. A woman has worked for 5 hours. If her work day is 8 hours long, what fraction of the day has she worked?

7. What fraction of an hour (60 minutes) is fourty-five minutes?

8. A cake was cut in to 8 equal parts. Five pieces were eaten.

    a)   What fraction of the cake was eaten?

    b)   What fraction of the cake was left?

3.2. Percentage as Fractions

Activity 3.3

Work with a partner.
Materials: grid paper, markers

- Draw three 10 ×10 squares on your grid paper.
- For each percent below, shade three different 10 × 10 grids, each in a different way.

  a) 60%   b) 25%   c) $35\frac{1}{2}$ %

- How can you find the percent represented by the shaded area at the right if you don't count the squares:
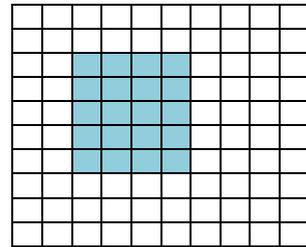
In this sub-unit you will deal with expressing a percentage as a fraction. The shaded area in the grid at the right shows that 43 out of 100 are shaded. Another name for the fraction $\frac{43}{100}$ is 43 **percent.**

A **percent** is a quotient that compares a number to 100. In symbols: $\frac{n}{100} = n\%$

The symbol % means **percent** or **per hundred** or **for every hundred.**
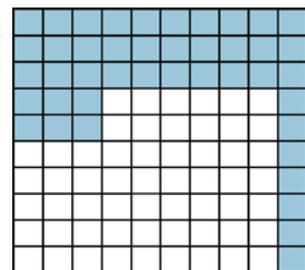
Figure 3.12

Example 5

Express each fraction as a percentage.

a) $\dfrac{37}{100} = 37\%$

b) a student answered 43 out of 100 = 43%

c) $9\dfrac{1}{2}$ per hundred = $9\dfrac{1}{2}\%$

Example 6

Write a percent to represent the number of shaded squares.

Solution: The grid has one hundred squares in all.
         Count the number that are shaded.

**There are 41 squares shaded.
So, 41% represents the
shaded area.**
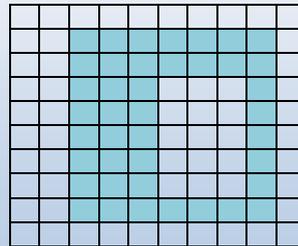
Figure 3.13

From the above discussion, perhaps you have got some idea about percentage. Now, we write 3% = 3 parts out of 100 equal parts $= \dfrac{3}{100}$

So, here we get a relation between **percentage** and **fraction.** Similarly, we can write 1% = 1 per hundred $= \dfrac{1}{100}$

Therefore $20\% = \dfrac{20}{100} = \dfrac{1}{5}$, $25\% = \dfrac{25}{100} = \dfrac{1}{4}$, $60\% = \dfrac{60}{100} = \dfrac{3}{5}$, etc

Conversion of fraction into percentage:

Step 1. Multiply both numerator and denominator by 100.

Step 2. Convert $\dfrac{1}{100}$ to '%' symbol.

Step 3. Simplify the fractional part if required

Group work 3.2

Express as percentage

a) 0.28　　　b) $\dfrac{3}{80}$　　　c) 0.7　　　d) 3.6

Example 7

Express each fraction as a percentage.

a) $\dfrac{4}{5}$　　　b) $\dfrac{3}{8}$　　　c) $\dfrac{6}{17}$　　　d) 0.4　　　e) 2.5

Solution: a) $\dfrac{4}{5} = \dfrac{4\times100}{5\times100}$ .... Step 1

$= \left(\dfrac{4\times100}{5}\right)\times\dfrac{1}{100}$ .... Step 2

$= \left(\dfrac{4\times100}{5}\right)\%$ .... Step 3

$= 80\%$

Therefore, $\dfrac{4}{5} = 80\%$

Another way to express a fraction as a percentage is to find an equivalent fraction with denominator of 100.

$$\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100} = 80\%$$

b) $\frac{3}{8} = \frac{3 \times 100}{8 \times 100} = \left(\frac{3 \times 100}{8}\right) \times \frac{1}{100} = 37.5\%$

c) $\frac{6}{17} = \frac{6 \times 100}{17 \times 100} = \left(\frac{6 \times 100}{17}\right) \times \frac{1}{100} = 35\frac{5}{17}\%$

d) 0.4 = $\frac{4}{10} = \frac{4 \times 100}{10 \times 100} = \left(\frac{4 \times 100}{10}\right) \times \frac{1}{100} = 40\%$

Activity 3.4

Work with a partner.

Materials: paper and pencil

What percentage of the students in your class do you think are in each category? Estimate by using one of the choices listed at the right.

a) left handed

b) right handed

c) male

d) female

e) less than 4 years old

f) greater than 10 years old

0%

Less than 10%

About 25%

About 50%

At least 75%

100%

To Write a percentage as a fraction, write a fraction with a denominator of 100. Then write the fraction in simplest form.

Example 8

Express each percentage as a fraction and decimal

a) 20%   b) 45%   c) $18\frac{2}{3}\%$

Solution: a) $20\% = \dfrac{20}{100} = \dfrac{1}{5}$

$\overset{\div 20}{\frown}$

$\underset{\div 20}{\smile}$

20% = 0.20 = 0.2

b) $45\% = \dfrac{45}{100} = \dfrac{9}{20}$

$\overset{\div 5}{\frown}$

$\underset{\div 5}{\smile}$

45% = 0.45

c) $18\dfrac{2}{3}\% = \dfrac{18\frac{2}{3}}{100} = 18\dfrac{2}{3} \div 100 = \dfrac{56}{3} \div 100$

$= \dfrac{56}{3} \times \dfrac{1}{100}$ **To divide by 100, multiply by** $\dfrac{1}{100}$.

$= \dfrac{56}{300} = \dfrac{14}{75}$

$\overset{\div 4}{\frown}$

$\underset{\div 4}{\smile}$

$18\dfrac{2}{3}\% = \dfrac{18.666\ldots}{100} = \dfrac{18.\dot{6}}{100} = 0.18\dot{6}$

You have observed that in order to write a percentage as a decimal, you need to divide by 100 and remove the % symbol.

Exercise 3.B

1. Express each fraction as a percentage

    a) $\dfrac{14}{15}$

    c) $\dfrac{18}{25}$

    e) $\dfrac{1}{8}$

    g) $\dfrac{7}{7}$

    i) $\dfrac{12}{25}$

    b) $\dfrac{23}{30}$

    d) $\dfrac{13}{20}$

    f) $\dfrac{5}{8}$

    h) $\dfrac{19}{20}$

    j) $\dfrac{3}{50}$

2. Express each percentage as a fraction in simplest form and decimal

    a) 55%

    c) 75%

    e) 90%

    g) $19\dfrac{1}{2}$%

    i) $9\dfrac{1}{4}$%

    b) 12%

    d) 10%

    f) $36\dfrac{2}{3}$%

    h) $14\dfrac{1}{3}$%

    j) $16\dfrac{1}{5}$%

3. Express each decimal as a percentage

    a) 0.18

    c) 0.7

    e) 0.375

    g) 0.086

    b) 0.01

    d) 0.025

    f) 0.681

    h) 0.0625

4. Write a percentage to represent the shaded area.

a.

c.

e.

b.

d.
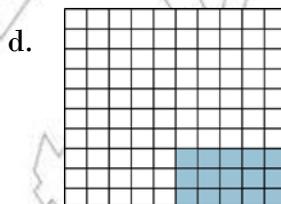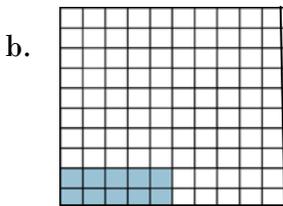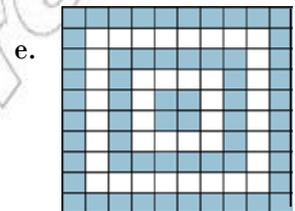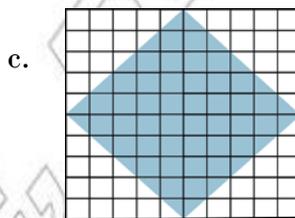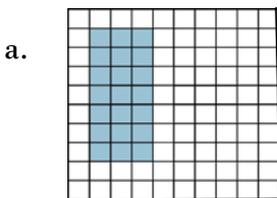
Figure 3.14

3.3. Comparison and Ordering of Fractions

Activity 3.5

Work with a partner.

The figure below is called a **Venn diagram.** The circle at the lower left contains all fractions greater than $\frac{1}{2}$. The circle at the lower right contains all fractions less than 1. Thus the region labeled E or G, where only these two circles overlap, contains all fractions that are greater than $\frac{1}{2}$ and less than 1.

Figure 3.15

- Identify the region where each of the following fractions would be located.

$$\frac{1}{8}, \frac{2}{3}, \frac{7}{4}, \frac{8}{9}, 1\frac{1}{2}, \frac{3}{5}, \frac{4}{4}, \frac{3}{7}$$
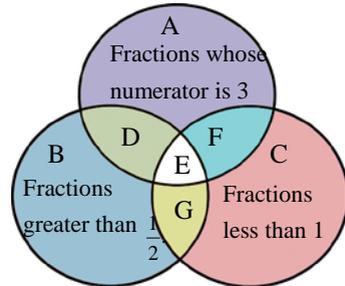
The fact that the numerator and denominator of a fraction can be multiplied by the same non zero number without changing its value is used to compare fractions. In this sub-unit you will study comparison and ordering of fractions in more detail.

Consider the fractions $\frac{3}{2}$ and $\frac{1}{2}$ (two fractions with the same denominator). Which one do you think is greater? Here is the rule to compare two fractions:

Grade 5 Student Text

Comparing fractions: Same denominator

To compare two fractions with the same denominator, compare the numerators. The one with the greater numerator is greater.

$\dfrac{1}{2}$ you can see that

$\dfrac{3}{2}$ $\dfrac{3}{2}$ is greater than $\dfrac{1}{2}$
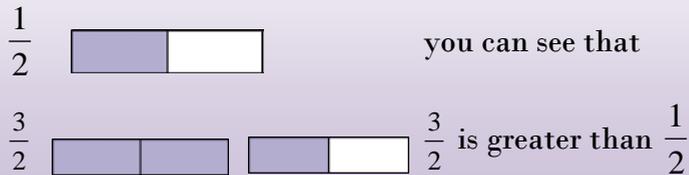
Figure 3.16

Since $\dfrac{3}{2}$ has a greater numerator, $\dfrac{3}{2}$ is greater than $\dfrac{1}{2}$. In this case, we write $\dfrac{3}{2} > \dfrac{1}{2}$.

Can you tell which fraction is greater, $\dfrac{1}{2}$ or $\dfrac{3}{4}$?

Since $\dfrac{1}{2}$ and $\dfrac{3}{4}$ do not have the same denominator, we first must write both fractions as fractions with the same denominator. Here is the way to do it:

Comparing fractions: Different denominators
To compare two fractions with different denominators, write both fractions with a denominator equal to the product of the original ones.

$\dfrac{1}{2} = \dfrac{1 \times 4}{2 \times 4} = \dfrac{4}{8}$ $\dfrac{1}{2} = \dfrac{4}{8}$

$\dfrac{3}{4} = \dfrac{3 \times 2}{4 \times 2} = \dfrac{6}{8}$ $\dfrac{3}{4} = \dfrac{6}{8}$
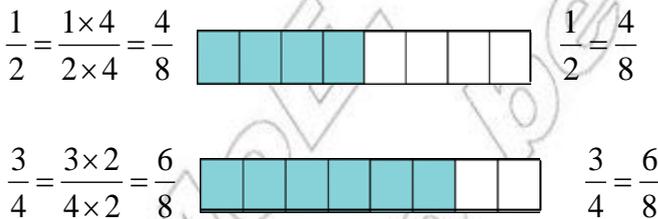
Figure 3.17

Since the numerator in $\dfrac{6}{8}$ (6) is greater than the one in $\dfrac{4}{8}$ (4), $\dfrac{3}{4} = \dfrac{6}{8}$ is greater than $\dfrac{4}{8} = \dfrac{1}{2}$.

In this case, we write $\dfrac{6}{8} > \dfrac{4}{8}$ or $\dfrac{3}{4} > \dfrac{1}{2}$.

 A generalization of comparing fractions is given as follows. Study the example given below.

Example 9

In basic science class, Lemlem has earned 30 points out of possible 35 points on tests. In English class she worked hard writing short story and presentation, earning 42 out of a possible 48 points. In which class has Lemlem earned a great portion of the possible points.

Solution: First write each fraction in simplest form.

$$\overset{\div 5}{\frac{30}{35}} = \frac{6}{7} \qquad \overset{\div 6}{\frac{42}{48}} = \frac{7}{8}$$

To compare $\dfrac{6}{7}$ and $\dfrac{7}{8}$, rewrite each fraction using the same denominator. Then you need only compare the numerators.

$$\frac{6}{7} = \frac{6 \times 8}{7 \times 8} = \frac{48}{56} \qquad \frac{7}{8} = \frac{7 \times 7}{8 \times 7} = \frac{49}{56}$$

Now, compare $\dfrac{49}{56}$ and $\dfrac{48}{56}$. Since 49>48, then

$\dfrac{49}{56} > \dfrac{48}{56}$., and Lemlem has earned a greater portion of the possible points in English than in basic science.

Note: 1. If two fractions have the same denominator, then fraction which has greater numerator is greater. Thus $\dfrac{4}{7} > \dfrac{2}{7}$ and $\dfrac{11}{20} > \dfrac{9}{20}$.

2. If the numerator of two fractions are equal, the fraction which has smaller denominator is greater. Thus $\dfrac{5}{9} > \dfrac{5}{11}$.

Group work 3.3

Which one is the least?

$\dfrac{3}{5}, \quad \dfrac{4}{7}$ or $\dfrac{5}{8}$?

Example 10

Compare the fractions $\dfrac{9}{25}$ and $\dfrac{13}{40}$.

Solution: $\dfrac{9}{25} = \dfrac{9}{25} \times \dfrac{40}{40} = \dfrac{360}{1000}$

And $\dfrac{13}{40} = \dfrac{13 \times 25}{40 \times 25} = \dfrac{325}{1000}$

Since 360 > 325

Then $\dfrac{360}{1000} > \dfrac{325}{1000}$

Thus, $\dfrac{9}{25} > \dfrac{13}{40}$

Example 11

Arrange

a) $\dfrac{3}{4}$, $\dfrac{4}{3}$ and $\dfrac{6}{7}$ in an ascending order.

b) $\dfrac{3}{8}$, $\dfrac{1}{2}$ and $\dfrac{7}{5}$ in a descending order.

Solution

a) $\dfrac{3}{4} = \dfrac{3 \times 3 \times 7}{4 \times 3 \times 7} = \dfrac{63}{84}$

$\dfrac{4}{3} = \dfrac{4 \times 4 \times 7}{3 \times 4 \times 7} = \dfrac{112}{84}$

And $\dfrac{6}{7} = \dfrac{6 \times 4 \times 3}{7 \times 4 \times 3} = \dfrac{72}{84}$

Since 112 > 72 > 63, then

$\dfrac{112}{84} > \dfrac{72}{84} > \dfrac{63}{84}$

Therefore, $\dfrac{4}{3} > \dfrac{6}{7} > \dfrac{3}{4}$

That is, $\dfrac{3}{4}, \dfrac{6}{7}, \dfrac{4}{3}$ are in ascending order.

b) $\dfrac{3}{8} = \dfrac{3 \times 2 \times 5}{8 \times 2 \times 5} = \dfrac{30}{80}$

$\dfrac{1}{2} = \dfrac{1 \times 8 \times 5}{2 \times 8 \times 5} = \dfrac{40}{80}$

And $\dfrac{7}{5} = \dfrac{7 \times 2 \times 8}{5 \times 2 \times 8} = \dfrac{112}{80}$

Since 112 > 40 > 30, then

$\dfrac{112}{80} > \dfrac{40}{80} > \dfrac{30}{80}$

Thus, $\dfrac{7}{5} > \dfrac{1}{2} > \dfrac{3}{8}$

That is $\dfrac{7}{5}, \dfrac{1}{2}, \dfrac{3}{8}$ are in descending order.

Group work 3.4

Which one is greater?

$\dfrac{5}{6}$ or $\dfrac{7}{8}$

Example 12

Robel walks $\dfrac{3}{5}$ part of a certain distance and Molla

walks $\dfrac{5}{8}$ part of the same distance in the same time.

Who walks faster?

Solution.

$$\frac{3}{5} = \frac{3 \times 8}{5 \times 8} = \frac{24}{40} \qquad \text{and} \qquad \frac{5}{8} = \frac{5 \times 5}{8 \times 5} = \frac{25}{40}$$

Since 25>24, we see that $\dfrac{25}{40} > \dfrac{24}{40}$. That is, $\dfrac{5}{8} > \dfrac{3}{5}$

Therefore, Molla walks faster.

Exercise 3.C

1. Find the greater of the two numbers.

a) $\dfrac{5}{18}, \dfrac{7}{18}$

b) $\dfrac{4}{11}, \dfrac{5}{11}$

c) $\dfrac{3}{20}, \dfrac{1}{20}$

d) $\dfrac{7}{12}, \dfrac{9}{10}$

e) $\dfrac{7}{16}, \dfrac{6}{15}$

f) $\dfrac{4}{7}, \dfrac{14}{15}$

g) $\dfrac{7}{6}, \dfrac{9}{10}$

h) $\dfrac{5}{14}, \dfrac{3}{28}$

i) $\dfrac{5}{8}, \dfrac{11}{10}$

j) $1\dfrac{4}{7}, 1\dfrac{5}{7}$

k) $6\dfrac{1}{3}, 6\dfrac{2}{5}$

l) $11\dfrac{2}{7}, 11\dfrac{3}{8}$

2. Arrange the fractions in ascending order.

   a) $\dfrac{5}{2}, \dfrac{4}{3}, \dfrac{7}{4}$

   b) $\dfrac{8}{15}, \dfrac{14}{35}, \dfrac{11}{21}$

   c) $\dfrac{5}{6}, \dfrac{1}{18}, \dfrac{23}{36}$

   d) $\dfrac{3}{7}, \dfrac{4}{9}, \dfrac{15}{21}$

   e) $\dfrac{2}{30}, \dfrac{1}{10}, \dfrac{3}{5}$

3. Arrange the fractions in descending order.

   a) $\dfrac{2}{3}, \dfrac{5}{6}, \dfrac{3}{8}$

   b) $\dfrac{7}{2}, \dfrac{3}{4}, \dfrac{5}{16}$

   c) $\dfrac{9}{10}, \dfrac{7}{6}, \dfrac{11}{15}$

   d) $\dfrac{4}{5}, \dfrac{5}{6}, \dfrac{7}{12}$

   e) $\dfrac{3}{7}, \dfrac{5}{14}, \dfrac{8}{21}$

4. Senait reads 24 out of 84 pages of a book within a day. But Hanan reads 21 out of 63 pages of another book within a day, who reads faster?

3.4 Operations on Fractions

3.4.1 Addition and Subtraction of Fractions

Activity 3.6

Add or subtract. Write each answer in simplest form.

a) $\dfrac{2}{7} + \dfrac{3}{7}$

b) $\dfrac{6}{13} - \dfrac{2}{13}$

c) $\dfrac{5}{9} - \dfrac{5}{9}$

d) $\dfrac{8}{12} + \dfrac{7}{12}$

e) $\dfrac{8}{9} + \dfrac{5}{9}$

f) $\dfrac{6}{15} - \dfrac{4}{15}$

g) $\dfrac{1}{10} + \dfrac{2}{10} + \dfrac{3}{10} + \ldots + \dfrac{9}{10}$

Do you remember what you have leant about addition and subtraction of fractions with the same denominators in your previous mathematics lessons? To add (subtract) fractions with the same denominators, add (subtract) the numerators.

Suppose a man spends about $\frac{1}{3}$ of his weekly income on food, $\frac{1}{6}$ on clothes and $\frac{1}{9}$ on entertainment. What is the fraction of money spent per week on food and entertainment? To find the fraction, you must add $\frac{1}{3}$ and $\frac{1}{9}$.

Activity 3.7

Work with a partner

Materials: fraction models

- To add $\frac{1}{2}$ and $\frac{1}{4}$, the common unit of measure is fourths.
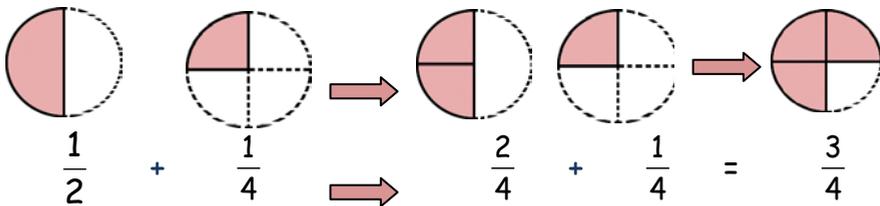
Figure 3.18

What conclusion can you draw about units of measures for fractions that are to be added or subtracted?

Grade 5 Student Text

From the Activity you may conclude the following: To find the sum or difference of two fractions with different denominators, rename the fractions as fractions with the same denominator. Then add or subtract and simplify.

That is, if $\dfrac{a}{b}$ and $\dfrac{c}{d}$ are two fractions (where b, d≠0), then

$$(i) \quad \frac{a}{b} + \frac{c}{d} = \frac{a \times d}{b \times d} + \frac{c \times b}{d \times b} = \frac{ad+cb}{bd}$$

$$(ii) \quad \frac{a}{b} - \frac{c}{d} = \frac{a \times d}{b \times d} - \frac{c \times b}{d \times b} = \frac{ad-cb}{bd} \quad (ad - cb > 0)$$

Now solve the problem at the beginning of this section .

$$\frac{1}{3} + \frac{1}{9} = \frac{3}{9} + \frac{1}{9} = \frac{4}{9}$$

The man spends about $\dfrac{4}{9}$ of his weekly income on food and entertainment.

Example 13

Add a) $\dfrac{1}{5}$ and $\dfrac{1}{2}$ b) $\dfrac{2}{5}$ and $\dfrac{1}{3}$ c) $2\dfrac{3}{10} + \dfrac{7}{20} + 1\dfrac{3}{5}$

Solution. a) Here in $\dfrac{1}{5}$ and $\dfrac{1}{2}$, the denominators are 5 and 2.

You can write

$$\frac{1}{5} = \frac{1 \times 2}{5 \times 2} = \frac{2}{10} \text{ and } \frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

Therefore $\dfrac{1}{5} + \dfrac{1}{2} = \dfrac{2}{10} + \dfrac{5}{10} = \dfrac{2+5}{10} = \dfrac{7}{10}$

b) Here, the denominators are 5 and 3.

Now $\dfrac{2}{5} = \dfrac{2 \times 3}{5 \times 3} = \dfrac{6}{15}$ and $\dfrac{1}{3} = \dfrac{1 \times 5}{3 \times 5} = \dfrac{5}{15}$

Therefore $\dfrac{2}{5} + \dfrac{1}{3} = \dfrac{6}{15} + \dfrac{5}{15} = \dfrac{6+5}{15} = \dfrac{11}{15}$

c) Observe that $2\dfrac{3}{10} = 2 + \dfrac{3}{10}$ **and** $1\dfrac{3}{5} = 1 + \dfrac{3}{5}$

Thus $2\dfrac{3}{10} + \dfrac{7}{20} + 1\dfrac{3}{5} = 2 + \dfrac{3}{10} + \dfrac{7}{20} + 1 + \dfrac{3}{5} = 2 + 1 + \dfrac{3}{10} + \dfrac{7}{20} + \dfrac{3}{5}$

$$= 3 + \dfrac{3\times 2}{10\times 2} + \dfrac{7}{20} + \dfrac{3\times 4}{5\times 4} \quad \text{(why)?}$$

$$= 3 + \dfrac{6}{20} + \dfrac{7}{20} + \dfrac{12}{20}$$

$$= 3 + \dfrac{6+7+12}{20} = 3 + \dfrac{25}{20}$$

$$= 3 + \dfrac{20+5}{20}$$

$$= 3 + \dfrac{20}{20} + \dfrac{5}{20}$$

$$= 3 + 1 + \dfrac{5}{20}$$

$$= 4 + \dfrac{5}{20}$$

$$= 4\dfrac{1}{4}$$

Therefore $\quad 2\dfrac{3}{10} + \dfrac{7}{20} + 1\dfrac{3}{5} = 4\dfrac{1}{4}$

Group work 3.5

Add

a) $\quad 3\dfrac{1}{2} + 2\dfrac{3}{4}$ 　　　　 b) $11\dfrac{2}{3} + 3\dfrac{1}{2}$ 　　 c) $\quad 24\dfrac{1}{16} + 21\dfrac{3}{4}$

Example 14

Subtract (a) $\dfrac{3}{7}$ **from** $\dfrac{11}{21}$ **(b)** $\dfrac{5}{12}$ **from** $\dfrac{17}{24}$

Solution. a) $\dfrac{11}{21} - \dfrac{3}{7} = \dfrac{11}{21} - \dfrac{9}{21}$ $\left(\text{because } \dfrac{3}{7} = \dfrac{3 \times 3}{7 \times 3} = \dfrac{9}{21} \right)$

$$= \dfrac{11-9}{21} = \dfrac{2}{21}$$

Therefore, $\dfrac{11}{21} - \dfrac{3}{7} = \dfrac{2}{21}$

b) $\dfrac{17}{24} - \dfrac{5}{12} = \dfrac{17}{24} - \dfrac{10}{24} = \dfrac{17-10}{24} = \dfrac{7}{24}$ $\left(\text{because } \dfrac{5}{12} = \dfrac{5 \times 2}{12 \times 2} = \dfrac{10}{24} \right)$

Therefore, $\dfrac{7}{24} - \dfrac{5}{12} = \dfrac{7}{24}$

Group work 3.6

Evaluate

$$18\dfrac{11}{12} - \left(9\dfrac{1}{4} + 6\dfrac{2}{3} \right)$$

Example 15

Find the simplified value of $\dfrac{3}{2} + \dfrac{2}{3} - \dfrac{1}{5}$

Solution: $\dfrac{3}{2} + \dfrac{2}{3} - \dfrac{1}{5} = \dfrac{45}{30} + \dfrac{20}{30} - \dfrac{6}{30} = \dfrac{45+20-6}{30} = \dfrac{59}{30} = 1\dfrac{29}{30}$

Exercise 3.D

1. Add. Then write each sum in simplest form.

    a) $\dfrac{2}{5}+\dfrac{3}{4}$

    b) $\dfrac{5}{6}+\dfrac{3}{8}$

    c) $\dfrac{4}{15}+\dfrac{2}{25}$

    d) $\dfrac{5}{14}+\dfrac{8}{7}$

    e) $2\dfrac{3}{4}+1\dfrac{7}{8}$

    f) $1\dfrac{5}{16}+2\dfrac{3}{8}$

    g) $4\dfrac{3}{10}+\dfrac{9}{20}$

    h) $3\dfrac{2}{7}+2\dfrac{5}{14}$

    i) $\dfrac{3}{5}+\dfrac{5}{7}+2\dfrac{1}{35}$

    j) $4+3\dfrac{5}{18}+2\dfrac{1}{9}$

    k) $1\dfrac{7}{10}+2\dfrac{1}{20}+3\dfrac{4}{40}$

2. Subtract. Then write each difference in simplest form.

    a) $\dfrac{3}{4}-\dfrac{1}{8}$

    b) $\dfrac{7}{5}-\dfrac{3}{10}$

    c) $\dfrac{5}{12}-\dfrac{7}{36}$

    d) $7\dfrac{1}{2}-5$

    e) $\dfrac{11}{4}-2\dfrac{1}{3}$

    f) $3\dfrac{1}{5}-\dfrac{3}{8}$

    g) $4\dfrac{1}{6}-2\dfrac{1}{5}$

3. Find the simplified value.

    a) $\dfrac{7}{4}+\dfrac{5}{6}-\dfrac{1}{12}$

    b) $\dfrac{3}{4}+\dfrac{7}{2}-\dfrac{1}{8}$

    c) $\dfrac{5}{3}+\dfrac{3}{4}-\dfrac{1}{2}$

    d) $\dfrac{4}{15}+\dfrac{7}{9}-\dfrac{1}{3}$

    e) $\dfrac{7}{12}+\dfrac{5}{6}-\dfrac{3}{4}$

    f) $\dfrac{1}{4}+\dfrac{1}{7}-\dfrac{1}{28}$

4. Does $\dfrac{3}{4}+\dfrac{5}{8}-\dfrac{5}{6}=\dfrac{5}{8}+\left(\dfrac{5}{6}-\dfrac{3}{4}\right)$? Explain.

5. A bottle contains $1\dfrac{1}{2}$ litres of water. If $\dfrac{1}{4}$ litre of water is used up from the bottle, how much water is left in it?

6. What must be added to $\dfrac{3}{10}$ to get $\dfrac{1}{2}$?

7. What must be subtracted from $\dfrac{7}{12}$ to get $\dfrac{1}{4}$?

8. Mesfin cuts a rope of length 9 meters in to two pieces. If one piece is $4\dfrac{1}{6}$ metres long, what is the length of the other piece?

9. A father left $\dfrac{1}{4}$ of his money to his daughter, $\dfrac{1}{2}$ to his wife, and $\dfrac{1}{8}$ to his son. What fraction of the money remained?

3.4.2 Multiplication and Division of Fractions

a) Multiplication of Fractions

As in the multiplication of whole numbers, multiplication of fractions and mixed numbers represents repeated addition.

The picture below shows 3 cups, each containing $\dfrac{1}{4}$ cup of sugar. How much sugar do they contain altogether? To find the answer we must multiply 3 by $\dfrac{1}{4}$, that is, we must find $3 \times \dfrac{1}{4}$.

Figure 3.19

We have 3 one-quarter cups of sugar, which make $\dfrac{3}{4}$ cup. Thus to find the answer, we multiply 3 by $\dfrac{1}{4}$, obtaining

$$3 . \dfrac{1}{4} = \dfrac{3}{4}$$

We can show the idea pictorially like this:
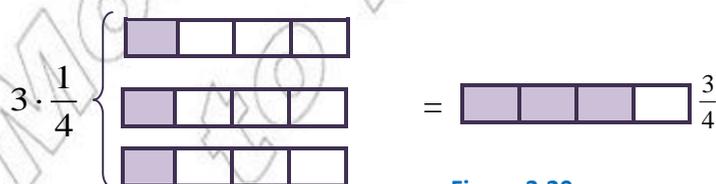
Figure 3.20

The diagram also suggests that multiplication is repeated addition; that is, $3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. Similarly, if a recipe calls for $\frac{1}{3}$ cup of flour and we wish to make only $\frac{1}{2}$ of the recipe, we have to find $\frac{1}{2}$ *of* $\frac{1}{3}$ (which means $\frac{1}{2} \times \frac{1}{3}$ because "of" is translated as "times"), that is, $\frac{1}{2} \cdot \frac{1}{3}$. Here is a diagram to help you do it.

1

| $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

Each rectangle on the diagram represents $\frac{1}{3}$.

$\frac{1}{2}$ of $\frac{1}{3}$ or $\frac{1}{6}$

Figure 3.21
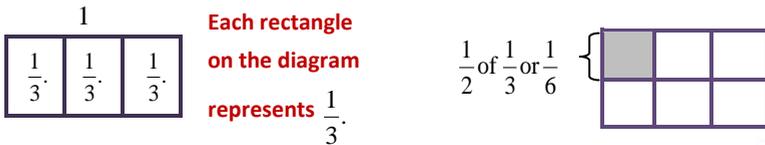
$$\frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

Activity 3.8

Fill in the blanks

a) $\dfrac{2}{5} + \dfrac{2}{5} + \dfrac{2}{5} = 3 \times \boxed{} = \boxed{}$

b) $\dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{3} + \dfrac{1}{3} = 4 \times \boxed{} = \boxed{}$

c) $\dfrac{3}{7} + \dfrac{3}{7} + \dfrac{3}{7} + \dfrac{3}{7} + \dfrac{3}{7} = \boxed{} \times \dfrac{3}{7} = \boxed{}$

Notice that you can also find the product of $\frac{1}{2}$ and $\frac{1}{3}$ as follows:

$$\frac{1}{2} \times \frac{1}{3} = \frac{1 \times 1}{2 \times 3} = \frac{1}{6}$$

Similarly $\dfrac{2}{9} \times \dfrac{4}{7} = \dfrac{2 \times 4}{9 \times 7} = \dfrac{8}{63}$

and $\dfrac{5}{3} \times \dfrac{9}{16} = \dfrac{5 \times 9}{3 \times 16} = \dfrac{45}{48} = \dfrac{15}{16}$ $\qquad$ $(45 \div 3 = 15$ and $48 \div 3 = 16)$

Rule for Multiplying Fractions

The **product** of two fractions is a fraction whose numerator is the product of numerators of the given fractions and whose denominator is the product of their denominators.

In symbols, $\dfrac{a}{b} \times \dfrac{c}{d} = \dfrac{a \times c}{b \times d}$

Example 16

Multiply. Write each product in simplest form.

a) $\dfrac{2}{3} \times \dfrac{5}{7}$ b) $\dfrac{2}{9} \times \dfrac{7}{2}$ c) $4\dfrac{2}{3} \times 9$

Solution:

a) $\dfrac{2}{3} \times \dfrac{5}{7} = \dfrac{2 \times 5}{3 \times 7} = \dfrac{10}{21}$

b) $\dfrac{2}{9} \times \dfrac{7}{2} = \dfrac{\cancel{2}^{1} \times 7}{9 \times \cancel{2}} = \dfrac{7}{9}$

c) $4\dfrac{2}{3} = \dfrac{(4 \times 3) + 2}{3} = \dfrac{12 + 2}{3} = \dfrac{14}{3}$ or $4\dfrac{2}{3} = \dfrac{4}{1} + \dfrac{2}{3} = \dfrac{12 + 2}{3} = \dfrac{14}{3}$

Therefore, $4\dfrac{2}{3} \times 9 = \dfrac{14}{\cancel{3}} \times \dfrac{\cancel{9}^{3}}{1} = 42$

Note: **Multiplying Mixed Numbers: To Multiply Mixed numbers, rename each mixed number as an improper fraction. Then multiply the fractions.**

Example 17

Find $4\dfrac{1}{2} \times 1\dfrac{1}{3}$

Solution. $4\dfrac{1}{2} \times 1\dfrac{1}{3} = \dfrac{\cancel{9}^{3}}{\cancel{2}_{1}} \times \dfrac{\cancel{4}^{2}}{\cancel{3}_{1}} = 6$

b) Division of fractions

When you studied whole numbers in Unit 1, you saw how multiplication can be checked by division. The multiplication of fractions can also be checked by division, as you will see in this section on dividing proper fractions and mixed numbers.

Dividing proper fractions

The division of proper fractions introduces a new term the **reciprocal.** To use reciprocals, we must first recognize which fraction in the problem is the divisor. Let's assume the problem we are to solve $\frac{1}{4} \div \frac{2}{3}$. We read this problem as "$\frac{1}{4}$ divided by $\frac{2}{3}$." The divisor is the fraction after the division sign (or the second fraction). The steps that follow show how the divisor becomes a reciprocal.

> **Dividing proper fractions:**
> **Step 1: Invert (turn up side down) the divisor. The inverted number is the reciprocal.**
> **Step 2: Multiply the fractions**.
> **Step 3: Reduce the answer to lowest terms.**

Do you know why the inverted fraction number is a reciprocal? Reciprocals are two numbers that when multiplied give a product of 1. For example, 3 (which is the same as $\frac{3}{1}$) and $\frac{1}{3}$ are reciprocals because multiplying them gives 1.

> **Reciprocal: The product of a number and its reciprocal is 1. That is, for all fractions $\frac{a}{b}$, where $a, b \neq 0$, $\frac{a}{b} \times \frac{b}{a} = 1$.**

Example 18

Suppose a girl figures that a person will drink $1\frac{1}{3}$ cups of orange juice for breakfast. And she buys 4 cups of orange juice for seven people. Will there be enough juice?

To solve this problem, we need to find how many $1\frac{1}{3}$ cups are in 4 cups. Divide 4 by $1\frac{1}{3}$.

Figure 3.22

$1\frac{1}{3}$ $1\frac{1}{3}$ $1\frac{1}{3}$

Thus, $4 \div 1\frac{1}{3} = 3$
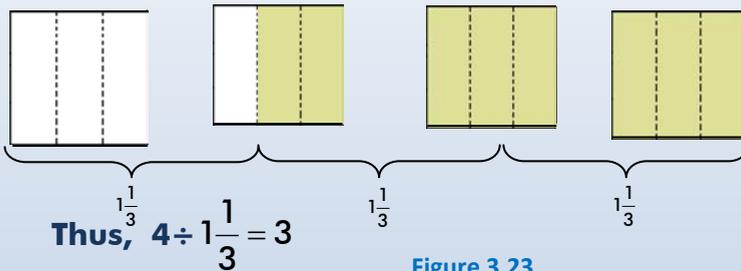
Figure 3.23

You can also divide by a fraction or mixed number. To do this multiply by its reciprocal.

$4 \div 1\frac{1}{3} = \frac{4}{1} \div \frac{4}{3}$ (Rename 4 as $\frac{4}{1}$ and $1\frac{1}{3}$ as $\frac{4}{3}$)

$= \frac{4}{1} \times \frac{3}{4}$ (Dividing by $\frac{4}{3}$ is the same as multiplying by $\frac{3}{4}$).

$= \frac{3}{1}$ or 3

4 Cups of orange juice will be enough for 3 people, not for 7 people.

Division of fractions and Mixed numbers: To divide by a fraction multiply by its reciprocal.

That is, $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \times \dfrac{d}{c}$ **where b, c and d ≠ 0.**

Thus $\dfrac{1}{4} \div \dfrac{2}{3} = \dfrac{1}{4} \times \dfrac{3}{2} = \dfrac{3}{8}$

Can you change order in division as you do in multiplication?

Example 19

Is $\dfrac{1}{4} \div \dfrac{1}{2} = \dfrac{1}{2} \div \dfrac{1}{4}$ **?**

Now, $\dfrac{1}{4} \div \dfrac{1}{2} = \dfrac{1}{4} \times \dfrac{2}{1} = \dfrac{2}{4} = \dfrac{1}{2}$ **and** $\dfrac{1}{2} \div \dfrac{1}{4} = \dfrac{1}{2} \times \dfrac{4}{1} = \dfrac{4}{2} = 2$ **but** $\dfrac{1}{2} \neq 2$

So the statement is false. Therefore, we cannot change order in division. That is $\dfrac{a}{b} \div \dfrac{c}{d} \neq \dfrac{c}{d} \div \dfrac{a}{b}$

Group work 3.7

Divide

    a) $6 \div 4\dfrac{1}{2}$ b) $42 \div 4\dfrac{2}{3}$ c) $\dfrac{15}{16} \div 1\dfrac{1}{3}$

Now you are ready to divide mixed numbers by using improper fractions.

Dividing Mixed numbers
Step 1. Convert all mixed numbers to improper fractions.
Step 2. Invert the divisor (take its reciprocal) and multiply
      If your answer is an improper fraction, reduce it to lowest terms.

Example 20

Divide

$6\dfrac{3}{4} \div 3\dfrac{5}{6}$

$= \dfrac{27}{4} \div \dfrac{23}{6}$ **step 1**

$= \dfrac{27}{4} \times \dfrac{6}{23}$ **step 2**

$= \dfrac{81}{46} = 1\dfrac{35}{46}$

Exercise 3.E

1. Multiply. Write each product in simplest form.

   a) $\dfrac{3}{5} \times \dfrac{10}{21}$ d) $\dfrac{2}{7} \times \dfrac{21}{6}$ g) $\dfrac{4}{5} \times \dfrac{2}{4} \times \dfrac{4}{6}$

   b) $\dfrac{5}{9} \times \dfrac{27}{35}$ e) $\dfrac{9}{5} \times \dfrac{35}{36}$ h) $6\dfrac{1}{8} \times \dfrac{8}{9}$

   c) $\dfrac{3}{4} \times \dfrac{8}{15}$ f) $\dfrac{20}{3} \times \dfrac{9}{40}$ i) $3\dfrac{1}{8} \times 3\dfrac{4}{5}$

2. Evaluate ab if a$= 1\dfrac{5}{7}$ and b$= 2\dfrac{5}{8}$.

3. Find the product $\dfrac{1}{2} \times \dfrac{2}{3} \times \dfrac{3}{4} \times \dfrac{4}{5} \times ...... \times \dfrac{99}{100}$

4. Find the value of

   a) $\dfrac{1}{4}$ of 100   c) $\dfrac{1}{2}$ of 64   e) $\dfrac{7}{6}$ of 120

   b) $\dfrac{1}{7}$ of 98   d) $\dfrac{3}{5}$ of 80

5. A book has 100 pages. Chala read $\dfrac{3}{10}$ of the book. How many pages are left to read?

6. Name the reciprocal of each number.

   a) $\dfrac{3}{7}$ b) 4   c) $2\dfrac{4}{5}$ d) $4\dfrac{5}{6}$

7. Divide. Write each quotient in simplest form

   a) $\dfrac{2}{5} \div \dfrac{1}{4}$ d) $8 \div 2\dfrac{1}{2}$ g) $1\dfrac{1}{9} \div 1\dfrac{2}{3}$

   b) $\dfrac{3}{14} \div \dfrac{2}{7}$ e) $2\dfrac{1}{4} \div \dfrac{2}{3}$ h) $4\dfrac{1}{2} \div 6\dfrac{3}{4}$

   c) $5 \div \dfrac{1}{6}$ f) $2\dfrac{2}{3} \div 5\dfrac{1}{3}$ i) $5\dfrac{1}{4} \div 3$

8. Will the quotient $5\dfrac{3}{8} \div 6\dfrac{3}{4}$ be a proper or a mixed number?

9. In a school with a total number of 2000 students, $\dfrac{3}{5}$ are girls. Find the number of boys.

3.5 Operations on Decimals

In this sub-unit you will deal with addition, subtraction, multiplication and division of decimals in more detail.

3.5.1 Addition and Subtraction of Decimals

The following Activity will help you revise what you have learnt about decimals in your previous mathematics lessons.

Activity 3.9

1. Fill in the blanks. The first one is done for you.

   a) 3.42 = __3__ ones __4__ tenths __2__ hundredths.

   b) 4.51 = ____ ones ___ tenths ___ hundredths.

   c) 0.345 =____ tenths ___hundredths ___ thousandths.

   d) 15.27 = ___ tens ___ ones ___ tenths ___hundredths.

---

2. Use >, < or = to compare the decimals

a) 0.3 ___ 0.5

b) 0.04 __ 0.01

c) 1.31 ___ 1.13

d) 5.08 ____ 5.8

e) 0.9____ 0.09

f) 0.7 ____ 0.71

3. Write 3.8, 3.79, 3.67 and 3.81 in order from least to greatest.

Do you remember how to add decimals? Adding decimals is like adding whole numbers. Make sure that you line up the decimal points before you add or subtract.

Adding Decimals
1. Line up the decimal points.
2. Write zeros so that both numbers have the same number of decimal places.
3. Add.

Example 21

Add 12.5 and 27.21

Solution. First line up the decimal points

```
  12.50      You can write a zero so that (12.5=12.50) each
+ 27.21      addend has the same number of decimal
  39.71      places.
```

Therefore, 12.5+27.21=39.71

Example 22

A school paid Birr 234.50 for new jackets and Birr 175.35 for new shirts. What is the total cost?

Solution.
```
  234.50
+ 175.35        Therefore, total cost= Birr 409.85
  409.85
```

Group work 3.8

Add

a)
```
   3 8 2 . 4 1
 + 4 7 1 . 2 6
 ─────────────

 ─────────────
```

b)
```
   7 6 6 . 6 2
 + 8 6 5 . 3 3
 ─────────────

 ─────────────
```

Example 23

A bird model has a head 11.3 cm long and a neck 23.15cm. The rest of the body is 64.52cm long. How long is the entire bird?

Solution: To answer, add.

First line up the decimal points.

```
  11.30
  23.15         (11.3 = 11.30)
+ 64.52
  98.97      The bird is 98.97cm long.
```

Figure 3.23

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Example 24

Find the sum of 12.041, 26.706 and 321.24

Solution.

$$
\begin{array}{r}
12.041 \\
+\ 26.706 \\
\underline{321.240} \\
\underline{359.987}
\end{array}
\qquad (321.24 = 321.240)
$$

Group work 3.9

An elephant's speed is 40.001 kilometers per hour. A pig's speed is 17.601 kilometers per hour. What is the sum of the speeds of the elephant and pig?

Subtraction of decimal fractions can also be done in the same way as you did in case of whole numbers, only keep in mind the following steps:

Subtracting Decimals
1. Line up the decimal points.
2. Write zeros so that both numbers have the same number of decimal places.
3. Subtract as with whole numbers.

Example 25

Subtract 0.3 from 1.53

Solution.

$$
\begin{array}{r}
1.53 \\
-\ \underline{0.30} \\
\underline{1.23}
\end{array}
\qquad
\begin{array}{l}
\text{make sure to place the decimal point} \\
\text{correctly} \\
(0.3 = 0.30)
\end{array}
$$

Example 26

Subtract 41.32 from 543.431

Solution: 543.431   (41.32=41.320)
              - **41.320**
              502.111

Group work 3.10

Subtract

a) 3 . 8 4   b) 27.51 from 347.82
  - 1 . 7 2
  _____

Example 27

The weights of one bag of rice and one bag of wheat are 52.05kg and 63.375kg respectively. Which bag is heavier and by how much?

Solution

      63.375>52.05

This implies that the bag containing wheat is heavier. And the difference is given as

        63.375
       -**52.050**
        11.325

Thus, the bag containing wheat is heavier than the bag containing rice by 11.325kg.

1. Add
   a) 3.21 and 4.015
   b) 0.04, 2.132 and 4.013
   c) 25.002, 40.115 and 13.101
   d) 10.134,9.021 and 120.412

2. Subtract
   a) 3.21 from 5.623
   b) 7.341 from 18.451
   c) 4.3 from 17.591
   d) 12.53 from 20.639

3. Last year 2.15 million people visited a park. This year 3.26 million visited. How many more people visited the park this year?

4. Abetu drove 215.355km from his house to his sister's house. His friend's house was 14.1 km shorter. How far did Abetu travel on his way to his friend's house?

5. An office building is 125.3m high. The building next to it is 40.45m higher than that. How high is the second building?

6. A rope is 80m long. Three pieces of length 13.25m, 21.4m, 18.3m are cut off. How much rope is left?

3.5.2 Multiplication of Decimals

Activity 3.10

Multiply
   a) 35 × 21
   b) 47 × 82
   c) 124 × 35
   d) 853 × 46
   e) 236 × 103
   f) 343 × 59

The multiplication of decimals is similar to the multiplication of whole numbers except for the additional step of placing the decimal in the answer (product). The product will have the same number of decimal places as the sum of the number of decimals in the factors.

Example 28

Multiply

a)   0.13   ←——   **Two decimal places**
      × 2
      0.26   ←——   **Two decimal places**

b)   1.4   ←——   **One decimal place**
    × 0.3   ←——   **One decimal place**
      0.42   ←——   **Two decimal places**

c)   2.37   ←——   **Two decimal places**
    × 0.8   ←——   **One decimal place**
    1.896   ←——   **Three decimal places**

What do you understand? The steps that follow simplify the procedure of multiplication of decimals.

Multiplying decimals
Step 1. Multiply the numbers as whole numbers ignoring the decimal points.
Step 2. Count and total the number of decimal places in the multiplier and multiplicand.
Step 3. Starting at the right in the product, count to the left the number of decimal places totaled in step 2. Place the decimal point so that the product has the same number of decimal places as totaled in step 2. If the total number of places is greater than the places in the product, insert zeros in front of the product.

Example 29

a) 6.3 ⟵ one decimal place
   ×1.2 ⟵ one decimal place
   126
   63
   ─────
   7.56 ⟵ Two decimal places

b) 2.13 ⟵ two decimal places
   ×3.5 ⟵ one decimal place
   1065
   639
   ─────
   7.455 ⟵ Three decimal places

Activity 3.11

Find the product in each case

a) 1.2 × 10
   1.2 × 100
   1.2 × 1000

b) 0.37 × 10
   0.37 × 100
   0.37 × 1000

c) 0.048 × 10
   0.048 × 100
   0.048 × 1000

d) 3.65 × 10
   3.65 × 100
   3.65 × 1000

The following example illustrates short cut steps to solve multiplication **problems involving multiples of 10 (10, 100, 1000, 10,000, etc).** Study the shift in decimal point.

Example 30

2.43×10 = 24.3   (1 decimal place to the right)
2.43×100= 243   (2 decimal places to the right)
2.43×1000=2430   (3 decimal places to the right)

What do you understand? You may follow the following steps to solve multiplication problems involving multiple of 10.

> **Step 1.** Count the zeros in the multiplier.
>
> **Step 2.** Move the decimal point in the multiplicand the same number of places to the right as you have zeros in the multiplier.

Exercise 3.G

1. Multiply

     a) 0.12×3   d) 8.3×1.4   g) 0.47×0.32

     b) 0.17×4   e)7.6×5.6   h) 1.23×4.8

     c) 3.4×8   f) 4.25 ×2.3   i) 5.31×0.48

2. A piece of string is 0.32cm long. What is the total length of 12 such pieces of string?

3. The cost per hour to rent a medium-size car is Birr 36.75. What is the charge to rent this car for 9 hours?

4. Use >, < or = to compare the following

     a) 1.5×1.2 ☐ 3.6×0.5   d) 7.75×1.5 ☐ 77.5×2.5

     b) 3.2×2.4 ☐ 5.1×1.2   e) 0.86×0.8 ☐ 8.6×0.1

     c) 0.34×1.3 ☐ 0.4×1.2

5. Alemu says that he runs about 1.35 km in each football game. How many kilometers does he run in 3.5 games (or in three and half games)?

3.5.3 Division of Decimals

> **Activity 3.12**
>
> What is the quotient when
>
>   a) 4 is divided by 0.5?   b) 2 is divided by 0.1?

If the divisor in your decimal division problem is a whole number, first place the decimal point in the dividend. Then divide as usual. If the divisor has a decimal point, complete the steps that follow.

3 FRACTIONS, DECIMALS AND THE FOUR OPERATIONS

Dividing Decimals

Step 1. Make the divisor a whole number by moving the decimal point to the right.

Step 2. Move the decimal point in the dividend to the right the same number of places that you moved the decimal point in the divisor (step 1). If there are not enough places, add zeros to the right of the dividend.

Step 3. Divide as usual.

Example 31

a) $8 \div 0.5 = \dfrac{8}{0.5} \times \dfrac{10}{10} = \dfrac{80}{5} = 16$

b) $27 \div 0.9 = \dfrac{27}{0.9} \times \dfrac{10}{10} = \dfrac{270}{9} = 30$

c) $0.36 \div 0.04 = \dfrac{0.36}{0.04} \times \dfrac{100}{100} = \dfrac{36}{4} = 9$

d) $\dfrac{15.6}{0.13} = \dfrac{15.6}{0.13} \times \dfrac{100}{100} = \dfrac{1560}{13} = 120$

The following example discusses **dividing decimals by powers of ten.** Study the shift in decimal point.

Example 32

a) $3.87 \div 10 = .387$ (1 decimal place to the left)

b) $3.87 \div 100 = .0387$ (2 decimal places to the left)

c) $3.87 \div 1000 = .00387$ (3 decimal places to the left)

d) $3.87 \div 10000 = .000387$ (4 decimal places to the left)

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Activity 3.13

What is the quotient when 2.13 is divided by 10? by 100? by 1,000? by 10,000?

You may use the rule that follow:

Dividing decimals by powers of ten: To divide a decimal by 10, 100, 1000, etc. Shift the decimal point in the dividend to the left as many places as the number of zeros in the divisor.

Example 33

a) 0.4 ÷ 10 = 0.04

b) 12.6 ÷ 100 = 0.126

c) 34.5 ÷ 1,000 = 0.0345

Exercise 3.H

1. Divide

   a) $5 \div 0.1$

   b) $80 \div 0.02$

   c) $12 \div 0.06$

   d) $12.8 \div 0.64$

   e) $2.25 \div 1.5$

   f) $3 \div 0.04$

   g) $19.6 \div 0.14$

   h) $25.6 \div 0.16$

   i) $10 \div 0.001$

2. Fill in the blank

   a) $4.27 \div 10 = \square$

   b) $4.27 \div \square = 0.427$

   c) $4.27 \div 100 = \square$

   d) $4.27 \div 1000 = \square$

   e) $0.56 \div \square = 0.056$

   f) $5.6 \div \square = 0.56$

   g) $14.28 \div \square = 0.1428$

UNIT SUMMARY

Important facts you should know:

- **Types of fractions**

  (i) **Proper:** value less than 1; numerator smaller than denominator.

  E.g. $\dfrac{3}{7}, \dfrac{7}{9}, \dfrac{8}{19}$

  (ii) **Improper:** value equal to or greater than 1; numerator equal to or greater than denominator.

  E.g. $\dfrac{5}{5}, \dfrac{20}{13}$

  (iii) **Mixed:** Sum of whole number greater than zero and a proper fraction.

  Eg. $6\dfrac{3}{4}, 7\dfrac{8}{9}$

- **Fractions conversions**

  (i) Improper to whole or mixed: Divide numerator by denominator; place remainder over old denominator.

  Eg. $\dfrac{17}{4} = 4\dfrac{1}{4}$

  (ii) Mixed to improper:

  $$\dfrac{\text{whole number} \times \text{Denominator} + \text{Numerator}}{\text{old denominator}}$$

  Eg. $4\dfrac{1}{8} = \dfrac{32+1}{8} = \dfrac{33}{8}$

▪ **Adding and Subtracting fractions**

i) When denominators are the same, add numerators, place total over original denominator, and reduce to lowest terms.

$$\frac{5}{9}+\frac{1}{9}=\frac{6}{9}=\frac{2}{3}$$

ii) When denominators are different, change them to like fractions. Then add (or subtract) the numerators, place total over their common denominator, and reduce to lowest terms.

Eg. $\dfrac{4}{5}+\dfrac{2}{7}=\dfrac{28}{35}+\dfrac{10}{35}=\dfrac{38}{35}=1\dfrac{3}{35}$

▪ **Adding and Subtracting Mixed numbers**

Convert the mixed numbers to improper fractions, then add (or subtract) by writing both fractions as equivalent ones with the same denominators, reduce to lowest terms.

Eg. $4\dfrac{2}{5}+2\dfrac{3}{4}=\dfrac{22}{5}+\dfrac{11}{4}=\dfrac{88}{20}+\dfrac{55}{20}=\dfrac{143}{20}=7\dfrac{3}{20}$

$3\dfrac{1}{4}-1\dfrac{1}{8}=\dfrac{13}{4}-\dfrac{9}{8}=\dfrac{26}{8}-\dfrac{9}{8}=\dfrac{17}{8}=2\dfrac{1}{8}$

▪ **Multiplying proper fractions**

i) Multiply numerators and denominators

ii) Reduce answer to lowest terms

Eg. $\dfrac{3}{5}\times\dfrac{10}{18}=\dfrac{30}{90}=\dfrac{1}{3}$

▪ **Multiplying Mixed numbers**

i) Convert mixed numbers to improper fractions.

ii) Multiply numerators and denominators.

iii) Reduce answer to lowest terms.

Eg. $1\frac{1}{8} \times 2\frac{4}{5} = \frac{9}{8} \times \frac{14}{5} = \frac{126}{40} = \frac{63}{20}$

- **Dividing proper fractions**

    i) Invert divisor

    ii) Multiply

    iii) Reduce answer to lowest terms

    Eg. $\frac{2}{3} \div \frac{4}{9} = \frac{2}{3} \times \frac{9}{4} = \frac{18}{12} = \frac{3}{2} = 1\frac{1}{2}$

- **Dividing Mixed numbers**

    i) Convert mixed numbers to improper fractions

    ii) Invert divisor and multiply, If final answer is an improper fraction reduce to lowest terms.

    Eg. $1\frac{1}{2} \div 1\frac{5}{8} = \frac{3}{2} \div \frac{13}{8} = \frac{3}{2} \times \frac{8}{13} = \frac{24}{26} = \frac{12}{13}$

- **Addition and Subtraction of decimals**

    i) Line up the decimal points

    ii) Write zeros so that both numbers have the same number of decimal places

    iii) Add or subtract as with whole numbers

    Eg.

    $$\begin{array}{r} 13.40 \\ +5.12 \\ \hline 18.52 \end{array} \qquad \begin{array}{r} 24.963 \\ -\ 3.500 \\ \hline 21.463 \end{array}$$

- **Multiplication of decimals**

    i) Multiply the numbers as whole numbers ignoring the decimal point.

    ii) Count and total the number of decimal places in the multiplier and multiplicand.

    iii) Starting at the right in the product, count to the left the number of decimal places totaled in step 2. Place the decimal point so that the product has the same number of decimal places as totaled in step 2. If the total number of places is greater than the places in the product, insert zeros in front of the product.

        Eg. 2.3 ⟵ one decimal place

          × 0.6 ⟵ one decimal place

          1.38 ⟵ two decimal places

▪ **Division of decimals**

  i) Make the divisor a whole number by moving the decimal point to the right.

  ii) Move the decimal point in the dividend to the right that you moved the decimal point in the divisor (step1).
    If there are not enough places, add zeros to the right of the dividend.

  iii) Divide as usual.

$$\text{Eg. } 12 \div 0.25 = \frac{12}{0.25} \times \frac{100}{100}$$

$$= \frac{1200}{25} = 48$$

REVIEW EXERCISE

1. Match the fraction with its percentage.

A	B
a) $\dfrac{1}{8}$	i) 37.5%
b) $\dfrac{1}{6}$	ii) $33\dfrac{1}{3}\%$
c) $\dfrac{1}{3}$	iii) 50%
d) $\dfrac{3}{8}$	iv) 12.5%
e) $\dfrac{1}{2}$	v) $16\dfrac{2}{3}\%$
f) $\dfrac{3}{4}$	vi) $66\dfrac{2}{3}\%$
g) $\dfrac{2}{3}$	vii) 75%
	viii) 7.5%
	ix) 0.5%
	x) 5%

2. Find the value of the following.

a. 50% of 80

b. 35% of 60

c. $\dfrac{1}{4}$ of 100

d) $\dfrac{2}{5}$ of 120

e) $\dfrac{4}{3}$ of 450

3 Write each improper fraction as a mixed number or as a whole number.

a) $\dfrac{11}{2}$ = _____

f) $\dfrac{42}{14}$ = _____

b) $\dfrac{15}{8}$ = _____

g) $\dfrac{13}{12}$ = _____

c) $\dfrac{24}{7}$ = _____

h) $\dfrac{27}{8}$ = _____

d) $\dfrac{31}{6}$ = _____

i) $\dfrac{47}{5}$ = _____

e) $\dfrac{14}{3}$ = _____

4. Write each numbers as an improper fraction.

a) $2\dfrac{3}{4}$ = _____

d) $5\dfrac{2}{5}$ = _____

b) $3\dfrac{5}{8}$ = _____

e) $6\dfrac{1}{3}$ = _____

c) $7\dfrac{2}{5}$ = _____

f) $12\dfrac{3}{5}$ = _____

5. Add or subtract.

a) $\dfrac{1}{8}+\dfrac{2}{3}$

e) $5\dfrac{1}{2}-2\dfrac{4}{5}$

i) $3\dfrac{7}{8}-2\dfrac{3}{4}$

b) $\dfrac{4}{9}+\dfrac{3}{4}$

f) $6\dfrac{1}{3}+1\dfrac{5}{8}$

j) $18\dfrac{2}{5}-9\dfrac{1}{2}$

c) $2\dfrac{1}{5}+1\dfrac{5}{6}$

g) $8-1\dfrac{2}{3}$

k) $8\dfrac{5}{8}-4\dfrac{3}{5}$

d) $5\dfrac{3}{4}+2\dfrac{7}{8}$

h) $4\dfrac{1}{6}-2\dfrac{1}{3}$

l) $17\dfrac{1}{3}+9\dfrac{4}{9}+2\dfrac{6}{7}$

6. Which fraction is equivalent to $957\frac{3}{5}$?

   a) $\dfrac{4781}{5}$ c) $\dfrac{4783}{5}$

   b) $\dfrac{4788}{5}$ d) $\dfrac{9573}{5}$

7. Multiply. Simplify to lowest terms

   a) $2\frac{1}{3} \times 6\frac{2}{5}$ c) $1\frac{7}{8} \times \frac{5}{6}$

   b) $9 \times 2\frac{1}{2}$

8. Divide. Simplify to lowest terms

   a) $\dfrac{5}{9} \div \dfrac{1}{2}$ c) $2\frac{1}{4} \div 1\frac{2}{3}$

   b) $\dfrac{6}{11} \div \dfrac{5}{6}$ d) $5\frac{5}{6} \div 2\frac{2}{5}$

9. A bucket contains $20\frac{1}{2}$ litres of water. If $8\frac{1}{4}$ litres of water is used up,

   how much water remains in the bucket?

10. Nunu and her two friends ate lunch at a hotel. They decided to split the
    bill evenly. The total bill was Birr 82.50. How much was each person's
    share?

11. Melkamu measured the amount of rainfall at his house for 3 days. On
    Sunday, it rained 0.4 in. On Monday, it rained $\frac{5}{8}$ in. On Wednesday it
    rained 0.57 in. List the days in order from the least to the greatest
    amount of rainfall.

12. Find each product

   a)   3.42
      × 7.2

   b)   2.3
      × 4.1

   c)   5.12
      × 0.3

   d) 4.68
      × 5.8

   e) 2.8× 0.05

   f) 1.45× 0.7

13. Find each quotient

   a) 4÷0.01   b) 0.3÷ 0. 03   c) 3.5÷0.7   d) 3÷0.003   e) 11÷0.001

14. A father gave away half of his property to his wife and the remaining was equally divided among his three children. If his total property was worth Birr 120,000, then find the share of each member of the family.

15. A pair of foot ball shoes weighs 1.213 kilograms. How much do 10 such pairs weigh? 100 pairs? 1000 pairs?