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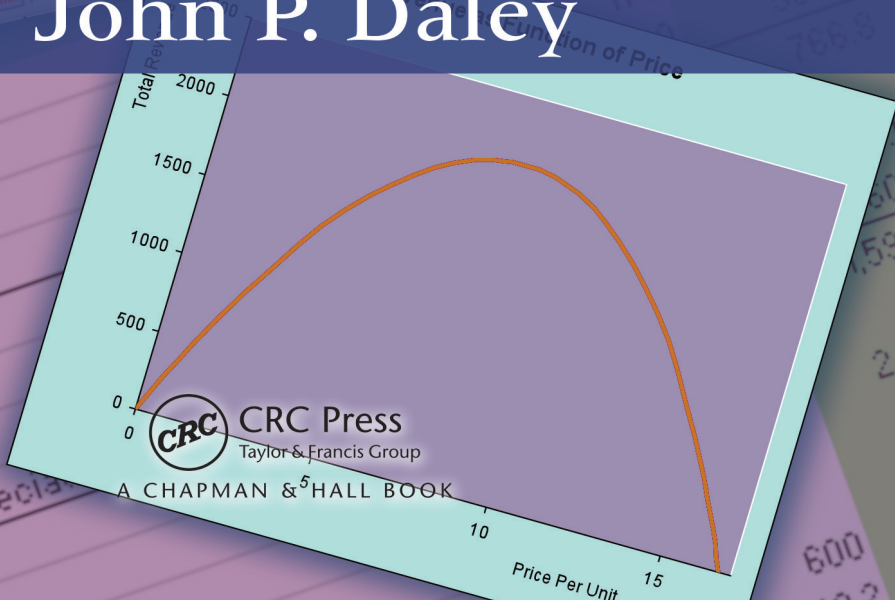
Introduction to Financial Models for Management and Planning

James R. Morris
John P. Daley



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James R. Morris and John P. Daley



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*To my family: JAM, RSM, and KDM.
Thanks for your love and support.*

—Jim

*To my wife and partner for life, Cynthia. Her untiring
support made my contribution possible.*

—John



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Preface to the Second Edition

This book is an introduction to the fascinating world of corporate financial modeling. It offers an overview of the power and flexibility of financial models and their role in financial planning. This book is intended for financial managers who wish to improve their financial decision making and for students who wish to become financial managers. In our increasingly complex world, the relationships between the choices faced by a manager and their subsequent outcomes have been obscured. A properly structured financial model can take into account much of this increased complexity and provide decision makers with a powerful planning tool, one that helps them identify the consequences of their decisions before they are put into practice.

The purpose of this book is to help the reader learn how to develop and use computer-based models for financial planning. By financial planning we mean planning the investment and financing decisions for a firm. We provide the reader with tools for his or her financial toolbox, and then show how to use those tools to build models. Throughout this book, we emphasize the structure of the models—models consistent with the theory of finance that are practical and usable. It is not the intent of this book to instruct the reader about the various software programs that we use. It is assumed that the reader either knows how to use most of the software or can find software instruction from other sources.

The main focus of this book is on modeling the problems of financial management, those associated with the planning and decisions faced by a firm's financial manager. Even though our primary emphasis is on models related to corporate financial management, we also introduce the reader to a variety of models related to security markets, stock and bond investments, portfolio management, and options.

This book introduces tools that relate to the financial management of the operating business. These tools include interactive cash budgets, dynamic pro forma financial statements that balance even under the most extreme assumptions, valuation techniques, forecasting techniques that range from simple averages to time series methods, Monte Carlo simulation, linear programming, and optimization. The toolbox is used to solve the problems of planning the firm's investment and financing decisions that lend themselves to financial modeling. These include evaluating capital projects, planning the financing mix for new investments, capital budgeting under capital constraints, optimal capital structure, cash budgeting, working capital management, mergers and acquisitions, and constructing efficient security portfolios.

One particularly valuable aspect of this book is that we continually focus the models on firm valuation. It makes little sense to build a financial model unless it provides a link between the decision being analyzed and the desired objective. Maximizing firm value is our objective; financial theory supplies the links. Therefore, firm value is the nexus of a model structure and the theory of finance.

An ancillary benefit of our focus on firm value is that by developing models that include valuation sectors, the reader learns about the theory of valuation and how to apply it to practical problems—how to build models that are consistent with the theory of valuation and simultaneously enhance one’s understanding about how numerous financial decisions influence firm value.

One of the challenges of writing a book such as this is deciding what to include and what to exclude. As we wrote each chapter, we would say to ourselves, “but we did not cover this topic, and we need to elaborate on that.” However, if we covered all the great topics in finance that were amenable to modeling, we would have to write several volumes. Unfortunately, we had to leave more out than we could include. The primary consideration about what to include was what volume of material could be covered in a one-semester university course. In addition, the topics chosen were primarily the corporate financial management topics that the student is likely to face in the environment of financial management. In each case, our coverage and models can only scratch the surface. For the material we have included in this text, the intent is to teach the student about building models, enough to get him or her started. Later he or she can provide the necessary details and elaborations as his or her professional tasks require.

ORGANIZATION

This book is organized into five major sections. [Section I \(Chapters 2 and 3\)](#) introduces the *tools* for financial planning. These include basic financial analysis ([Chapter 2](#)) and the different concepts of cash flow and growth ([Chapter 3](#)).

[Section II](#) explains the structure of a financial simulation model ([Chapter 4](#)), the equity and invested capital approaches to valuation ([Chapter 5](#)), and Monte Carlo simulation ([Chapter 6](#)). [Chapter 4](#), “Financial Statement Simulation,” is the core of this book. In it we explain the structure of a financial statement simulation model. The student learns how to develop a simple flexible model that balances under all scenarios with a structure that is consistent with the theory of finance. Valuation is a focus for most of our models, and it is important to link decisions to value. [Chapter 5](#) shows how to model the value of equity and deals with more specialized questions such as how to model value per share for a firm that is issuing new equity.

One problem with basic financial models is that they produce point estimates of their objective, be it intrinsic value, internal rate of return, or any number of other measures of financial performance. Such point estimates hide the uncertainty lurking beneath them. We introduce Monte Carlo simulation in [Chapter 6](#) to make this implicit uncertainty explicit and then go on to show how it can be used to explore the consequences of our decisions. Although we noted earlier that this text is not intended to give software instruction, this chapter is an exception. As most students have not been exposed to Monte Carlo

simulation or the software necessary to perform it, we introduce the Monte Carlo software @RISK. With this diverse set of tools in hand, the student is equipped to proceed with the remaining chapters of this book. Access to @RISK software for Monte Carlo simulation is provided with this book.

Most financial planning models are driven by a forecast of a basic input variable such as sales. So an important part of modeling is an ability to develop a forecast. [Section III \(Chapters 7 through 9\)](#) is a practical introduction to methods for forecasting a firm's sales and costs. As forecasting is a discipline unto itself, the most that we can hope to accomplish in a few chapters in a book devoted to modeling is to introduce the reader to a few basic forecasting methods. Forecasting is introduced in [Chapter 7](#) with simple time trend extrapolation methods. This chapter also discusses how to assess and compare the results of different forecasts. [Chapter 8](#) shows how to use linear regression to develop structural econometric forecasts. We show how to relate the firm's sales to measures of economic activity such as gross domestic product (GDP) and interest rates.

[Chapter 9](#) covers smoothing methods such as moving averages, exponential smoothing, and seasonal adjustment. At the end of each of these chapters, there are forecasting problems for student practice.

[Section IV \(Chapters 10 through 12\)](#) elaborates on the basic financial planning model of [Chapter 4](#), adding details and explaining how to expand the model to deal with more specialized questions. [Chapter 10](#) expands our coverage of valuation to the investment in long-term assets. It explains how fixed assets should be handled in the planning model, how to model the capital budgeting decision, and finally how to model the decision to acquire and merge with another firm. It presents extended examples and models for analyzing the capital budgeting decision and the merger decision.

Part of the problem of analyzing capital investment revolves around the financing of the project, so [Chapter 11](#) deals with the financing decision and the firm's capital structure. The first part of this chapter continues with the investment project modeled in [Chapter 10](#) and shows how to model its financial structure. Then, we use Monte Carlo simulation to show how to model a firm's optimal capital structure. The last section of this chapter provides models dealing with the more specialized topic of duration and debt swaps.

The last topic of [Section IV](#) is working capital in [Chapter 12](#). [Chapter 12](#) delves into the modeling of working capital accounts. We develop different models for managing cash, marketable securities, and receivables.

[Section V](#) is devoted to modeling investment securities and investment portfolios. [Chapter 13](#) shows how to model security prices as a binomial process and as a random walk Weiner process. Armed with the models of security prices, [Chapter 14](#) models the portfolio decision and shows how to construct a mean-variance efficient portfolio and model the efficient frontier. [Chapter 15](#) explains basic option models using both the binomial model and the continuous time Black–Scholes model.

The last section, [Section VI \(Chapters 16 and 17\)](#), is about optimization—how to use linear programming to find the best investment and financing decisions. We introduce linear programming in [Chapter 16](#) with a Weingartner type capital rationing model. We expand that model so that the student understands the transition from an investment model to a

complete planning model. [Chapter 17](#) is devoted to the application of linear programming to working capital planning. We use an Orgler type model to show how to plan short-term investment and financing decisions. These optimization chapters provide the student with detailed help in using the Solver optimization add-in to Excel. Shortly after this book was published, the user interfaces for Excel and @RISK changed dramatically. As a result, many of the exhibits in the first edition were quickly out of date. This edition uses the latest versions of Excel (Office 2016) and @RISK (v.7). In addition, we added cash control decision variables that control the internal and external flows of free cash flow. Finally, we extend the coverage of valuation techniques to include time-varying market-value based costs of capital and both the equity cash flow and value of invested capital approaches to firm valuation.

HOW TO USE THIS BOOK IN THE UNIVERSITY SETTING

At the university level, this book can be used for a graduate level or advanced undergraduate level course in finance. Our course that uses this material is part of our MS-Finance program, after the students have taken courses in financial management, investments, statistics, and operations management.

As the methods of financial modeling are learned by practice and experience, we view a course in financial modeling as a learning-by-doing course. We seldom give examinations. We structure our financial modeling course around a set of problems that require the student to construct models that help with planning and decision making. The imperative is that the models should be consistent with the theory of finance. To fulfill this imperative, it is necessary for the student to combine financial theory with modeling. To do this, the student needs to review the theory and figure out how to apply it at a practical level in a model. The result is that the student learns the theory and, more importantly, learns how that theory is applied in the real world.

The problems in this text provide the opportunity to apply the text material to a comprehensive set of fairly realistic situations. The problems posed to the students require them to set up the models and solve them. By the end of the course, the students will have enhanced their skills and knowledge of spreadsheet software, statistical software and methods, Monte Carlo simulation, and optimization. These are valuable skills that are in demand by the businesses that employ our students.

The solutions provide insights into business problems, of course, but it is the model structuring process and the linking of financial theory to real world scenarios that provide the important lessons. Students learn how to make the links between the business problem and the structure of a planning model. The ability to develop the structure of the planning model is the most valuable feature of our course. At the end of the course, we consistently get very positive feedback from the students, with comments such as “this has been the most valuable course in my finance program,” “I’ve learned more from this class than any other class in my MBA program,” and “this class really helped to bring the material together from my various other courses.” The students’ struggle to develop functioning models brings together the disparate ideas and theories they have learned in their business education programs. It is in the integration of the areas of study that we also get very

favorable feedback from our students. We have had comments such as “this course forced me to review my other courses such as statistics, operations management, and marketing, and really helped me bring them together.”

It would be very difficult to cover every chapter in this book in a semester. Indeed, we only cover the first 10 chapters in our course. It takes plenty of time to explain the concepts to the students, and the students spend a great deal of time outside of class building their models. In our course, we usually require the students to complete 6–10 of the larger end of chapter problems during a term. This is a heavy workload. Consequently, there is not really sufficient time in a typical semester to cover all the chapters and have the students do the extended problems that accompany each chapter.

The instructor is encouraged to pick and choose which topics will be covered during the term. It is not necessary to cover all of the chapters, nor is it necessary to cover them in sequence. Most of the chapters stand on their own. However, the key chapters are as follows: the last section of [Chapter 3](#) that covers sustainable growth; [Chapter 4](#), financial statement simulation; [Chapter 5](#), modeling value; and [Chapter 6](#), Monte Carlo simulation. These chapters constitute the heart of the topic of financial modeling. They feed into material in the chapters that follow. If you cover these chapters, you can generally pick and choose the other chapters you cover without much loss of background or continuity.

The financial analysis [Chapters 2](#) and [3](#) show the student how to set up models to analyze the firm’s condition. In addition, [Chapter 3](#) sets the stage for subsequent modeling chapters by explaining the details of cash flow and growth. Our students indicate that they think this material is valuable. However, if the students are already well versed in financial analysis and cash flow, these chapters can be skipped without losing much for understanding later chapters.

If students have already had substantial exposure to statistical forecasting methods, the forecasting [Chapters 7](#) through [9](#), can be skipped. On the other hand, although most students have had a statistics course, the typical statistics course spends very little time on forecasting. Consequently, the students gain a lot from working the very practical problems in [Chapters 7](#) through [9](#).

Most of chapter 10 is devoted to capital budgeting. This chapter and the problems give the student the opportunity to practice modeling the capital budgeting decision. However, most finance students have been exposed to these concepts in other courses. The last section of this chapter explains how to model a merger decision, and this is something students may not have covered elsewhere. Nevertheless, with scarce time, this chapter is not necessary for understanding subsequent material.

On the other hand, the debt financing chapter presents material many students have not been exposed to before. Of particular interest is the use of Monte Carlo simulation to analyze the debt financing decision. Although this chapter is not required for continuity, it is unique and is an important part of this book.

The security investment chapters in [Section V](#) are not necessary for continuity with other chapters, and this material can be covered independently of the earlier chapters. However, these topics are current, and students learn a lot by trying to model the concepts and methods they only see as theories in other courses. They may have learned about a

random walk, but it sinks in when the student builds a model. The same is true for the efficient frontier in [Chapter 14](#) or options in [Chapter 15](#).

[Chapters 16](#) and [17](#) introduce optimization. As with any of the topics, if you have time, students should be exposed to these methods and get the opportunity to build the optimization models.

In a nutshell, this book offers you a wide variety of topics that are amenable to modeling. All are practical. It is not necessary to cover this book from start to finish. You have great flexibility to choose from the smorgasbord of topics according to the interests of the instructor and the students. Whichever topics you choose to cover, the students will learn the rudiments of building financial models in Excel and the theory underpinning those models. The exposure to modeling different decision problems and to different methods gives the student a foundation to approach the wide variety of modeling tasks that would be impossible to cover in a single volume.

A WORD ABOUT SOFTWARE

The software that we use for our modeling includes spreadsheets, statistics programs, the Solver add-in to Excel, and the Monte Carlo add-in, @RISK. We find that most of our students are already proficient in the basics of using spreadsheets such as Excel, and they typically have been exposed to statistical packages such as MiniTab, or perhaps E-Views or SPSS in their statistics courses. In addition, they may have had some exposure to linear programming software such as the Solver add-in to Excel.

However, most students have not been exposed to @RISK, which is the software used in the Monte Carlo section of the course. It is a useful tool for a wide variety of modeling applications. The @RISK add-in for Excel is one of the most valuable parts of this book. As noted at the back of this book, the purchaser of this book has access to a one-year license for a fully functional version of @RISK, along with the range of other companion software by Palisade.

As most students have been exposed to much of the software we use, we do not focus this book on teaching the software. We try to give some guidance and instruction in the software as we go along, but we do not provide elementary software instruction. If the student needs an elementary introduction to the standard software such as Excel, it is assumed that they can get this from another source. On the other hand, for the more specialized software such as Solver and @RISK, we provide basic instruction in the chapters where these tools are used.

Our financial modeling course is one of the most popular courses in our graduate finance program. Students complain about the workload, but at the end, they consistently have high praise for the experience and the fact that they have learned to build a wide variety of very applicable models. We hope you will find this material equally valuable.

FOR THE INSTRUCTOR

At the end of each chapter, we have exercises for the students. Most of these are substantial problems that ask the student to build the models discussed in the chapter. All of the data

and the solutions to the problems are included in the instructor's disk. Although most of the data for the problems are shown with the problem at the back of the chapter, these data are included in the instructor's disk so that it can be given to the students without them having to copy all the data from the text. In addition, we have developed an extensive library of other problems plus solutions that are available on the instructor's disk. Many instructors will find ways to improve on our problems. We would invite you to share your problem sets with us so that they can be used widely. Finally, we have tried to eliminate mistakes in our text. However, inevitably there is much that we did not find and correct. We would appreciate corrections and suggestions for improvement.

Thank you for using our book. We hope that you find it instructive and useful.

—James R. Morris and John P. Daley



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An Overview of Financial Planning and Modeling

Planning:

Always plan ahead. It wasn't raining when Noah built the ark.

—RICHARD CUSHING

If you don't know where you are going, any road will get you there.

—LEWIS CARROLL

If you have accomplished all that you have planned for yourself, you have not planned enough.

—EDWARD EVERETT HALE

In preparing for battle, I have always found that plans are useless, but planning is indispensable.

—DWIGHT EISENHOWER

THIS CHAPTER PROVIDES AN OVERVIEW of the purpose of financial modeling and planning and explains the ingredients that go into the financial planning effort.

1.1 WHAT IS PLANNING?

Most managers engage in some aspect of planning. After all, the first step in business is the establishment of the enterprise—a step that is the result of the entrepreneur's dream about the wealth that could accrue from providing some good or service to others. That initial dream is part of the process of business planning. From its establishment throughout the firm's existence, the manager may be thinking about the future, wondering what will

happen tomorrow, asking what will be the results of current decisions, and perhaps even dreaming of new enterprises.

However, not every effort to know the future is planning. For example, forecasting is not active planning but a passive process. It involves the projection of future events, but it does not involve the consideration of the actions that can be taken to affect the future. Although forecasting is a necessary input to business planning, planning is much more than forecasting. Planning combines elements of forecasting with the specifications of decisions, actions, and intended results. A broad definition of planning is the specification of future decisions and actions to reach a stated goal. Very simply put, planning is deciding what you are going to do to get where you want to go.

It is useful to think of a traveler planning a trip. The traveler first sets the primary objective by deciding where to go. He or she may also have subgoals or constraints regarding when he or she wants to reach her goal and how much he or she wants to spend. The travel plan specifies the route, the signposts that will guide him on the route, where to turn, and where to stop.

If there is no uncertainty about future events on the trip, planning is relatively simple. Only one route needs to be chosen. The best route is chosen in advance with the assurance that it is the correct one: no detours will be necessary and the traveler will not get lost. No alternative routes or actions need to be considered, because under conditions of certainty, no random events can interfere with the completion of the trip.

However, the traveler invariably faces some uncertainty about the trip. Random events may interfere with the plan. A landslide may block the route, a storm may close the airport, or the plane may crash. The smart traveler takes uncertainty into account by planning alternative routes to the goal should obstructions be encountered. For example, in considering whether the plane may crash, the traveler assesses whether the benefits of flying are worth the risks. In the presence of uncertainty, it is especially important for the traveler to make plans that anticipate possible difficulties so as to reduce the resulting inconvenience and delay should the plan not be met.

Similarly, the greater the uncertainty in the business environment, the greater the necessity for and benefits from planning. One might think that with greater uncertainty, planning becomes less useful. After all, “the best laid plans of mice and men often go awry,” so why plan if even the best laid plans are destined to go awry? However, if the plans are truly best laid, they make allowance for things going awry. As Murphy’s law says, “Everything that can go wrong, will go wrong.” The best laid plans should take this unfortunate truth into account.

Whether complex and detailed or broad and simple, a plan should specify the objectives and the actions that lead to those objectives. The concept and process of planning are pointless unless one decides first what is to be achieved. Deciding on a destination seems so obvious that it hardly needs to be discussed. Nonetheless, the setting of goals is so fundamental to business planning that it needs extra stress. We plan in order to achieve a goal. Without first stating the goal, there is no way to decide along the way which actions should

be taken and no way to judge which of the actions were successful. Thus, the first step in planning a trip or a business venture is deciding on the goal or objective.

Once the primary objective is set, it is necessary to specify some criteria for judging progress toward that objective. The traveler uses signposts along the road to tell whether he or she is on the correct route and how far it is to her destination. The business planner also must develop some *signposts*, standards by which he or she can measure the firm's progress. For example, if the primary objective is the maximization of the market value of the firm, a set of measures of financial performance such as return on assets, profit margin, and asset turnover can help measure progress toward the objective. When properly specified, such measures of financial performance can be used as signposts (subgoals or intermediate objectives) toward which management's decisions can be directed. Thus, day-to-day management decisions need not necessarily be considered with the objective of maximizing the firm's value. Instead, operational decisions can be made based on more mundane standards, such as attaining a target profit margin. Of course, it is crucial that reaching these subgoals be consistent with attaining the primary objective.

Often, defining the link between the decision and the objective is the primary source of complexity and difficulty in business planning. If you know that a particular decision will lead unambiguously to the objective, there is very little problem in either planning or decision making. However, the effect a decision will have on progress toward the objective is usually unclear. There are often complex and poorly understood relationships between management's actions and the objective. For example, suppose the goal is to maximize the value of the firm's stock, and management is considering issuing bonds to finance a capital investment. To consider these actions, the planner needs to know how issuing bonds and adopting the investment will affect the value of the stock. Although the planner will not know all the effects, he or she can utilize the theory of finance to shed light on the links between the investment and financing decisions and the value of the stock, and can then trace through these complex relations to determine the best decision. A primary goal of this book is to show how to model the links between financial decisions and firm value, so that the planner can analyze the impact of decisions on the firm's objectives.

1.2 WHAT IS FINANCIAL PLANNING?

Finance is concerned with the problems and decisions relating to assets and liabilities, the two sides of a firm's balance sheet. Almost all management decisions that we regard as financial fall under one of these categories. Investment decisions identify which assets should be purchased. Financing decisions concern the sources of the funds (liabilities) necessary to purchase these assets. Financial planning is concerned with setting the objectives for future investment and financing decisions, judging the effect of various decisions on progress toward the objectives, and then deciding, based on these judgments, which investment or financing alternatives should be undertaken.

The range of problems addressed under the heading of financial planning is enormous. The problems range from very detailed specifications of working capital

decisions over a planning horizon as short as a day or a week to very broad plans about a business strategy over the next decade. For example, a commercial bank must decide how to satisfy the weekly reserve requirements set by the Federal Reserve Bank. The bank has a range of choices about which days of the week will show reserve surpluses and which will show deficits, about how it can invest any excess funds, and about how it can obtain funds to cover any deficits. All these decisions involve borrowing and lending on a daily or even hourly basis. In this case, the financial plan has a very short horizon and may involve very detailed analyses of a small number of specialized asset and liability accounts.

An example at the other end of the spectrum is the development of a long-range plan for strategic investment and expansion. A broad strategic analysis might consider the lines of business in which the firm should be involved. Such an analysis tends to have a very long planning horizon, so it does not develop fine detail in terms of specific assets and liabilities. Rather, it emphasizes business strategy in the broadest sense. This type of strategic planning fits the description by Brealey et al. (2008) of strategic investments that may not directly have attractive values. Rather, they constitute the purchase of options that will enable the firm to undertake attractive investments in the future because the firm positions itself strategically now.

In the quintessential financial planning problem, the firm's sales and production forecast is taken as given, and the investment and financing plan necessary to support the sales must be developed. An example of such a problem will be presented in [Chapter 4](#).

1.3 THE INPUT TO FINANCIAL PLANNING

Obviously, there are many different needs and applications for financial planning. Nonetheless, there are a number of ingredients that all financial plans should have. For more insights into these ingredients, we return to the travel plan analogy discussed earlier.

Some of the necessary ingredients for a thorough plan have already been discussed; still, it is informative to elaborate on them. First, the destination must be specified. Second, the various possible alternative routes and methods of transportation should be enumerated. Without a careful and complete listing of the possibilities, some of the most attractive routes might be overlooked. Third, each route should be evaluated as to whether it actually leads to the desired destination. In addition, the costs, advantages, and disadvantages of each route should be determined. Fourth, the resources available for making the trip must be considered. The amount of money and time available, suitable clothing and luggage, and the availability and condition of a car are all resources that can affect decisions about the trip. As each resource is limited, it imposes a constraint on what can be accomplished during the trip. There are other constraints, too. It may be desirable to see particular sights along the way, constraining the route selected. Visiting certain countries might be prohibited by legal restrictions. Finally, the traveler must decide what kind of detail is wanted in the plan, and how specific the time schedule should be. The plan could be made very broad, specifying only the countries to be visited and the approximate dates. Or, the plan could be made extremely detailed, scheduling the activity for each hour of each day. The advantages of more detailed planning must be weighed against the extra time and

effort in making the detailed plan. Furthermore, the traveler may not really want to be tied to the strict requirements of a tightly scheduled plan.

The following list of ingredients for a travel plan is not exhaustive, but it is a good start. The ingredients are

1. The destination or goal.
2. The alternative routes.
3. The relation between the route and destination.
4. The resources and constraints.
5. The level of detail.

Obviously, every travel plan would not explicitly consider each item on the list. The trip may be so routine and simple that an elaborate plan is unnecessary. Some travelers may not even require an explicit listing; they may develop the plan in their head without bothering to write it down. Still, most of the list's ingredients will be considered in any plan, even if only perfunctorily.

A financial plan is similar to a travel plan. Whatever the financial planning problem, whether it involves detailed analysis or a very broad consideration of strategy, the financial planning process is intended to aid in making future decisions to reach the firm's goal: a financial trip from the present into the future. The ingredients to the plan form the input to the planning process. Indeed, most financial plans should begin with an enumeration of the necessary inputs. The factors or ingredients that form the major input categories to a financial plan look much like the ingredients to the travel plan:

1. The goal
2. The decision alternatives
3. The link between the decisions and the goal
4. The resources for implementing the decisions and constraints inhibiting goal attainment
5. The planning horizon and the desired amount of planning detail

The planner may specify these elements explicitly or only have an intuitive idea of some or all of them. In either case, these elements are needed to develop a complete financial plan. There are considerable benefits to be derived from going through the exercise of carefully and explicitly specifying the elements in this list.

1.3.1 The Goals of Financial Decisions

With respect to investment and financing decisions, there is a general agreement among financial economists that the proper objective is to maximize the value of the firm's equity shares. In other words, management should choose the decision alternatives that make

the market price of the stock as high as possible.* A number of arguments support this position. First, management serves as the fiduciary agent of the stockholders and has the responsibility to act in the best interests of the stockholders. The market price of the stock serves as a barometer of stockholder preferences. If stockholders and investors like management's decisions, they will purchase the stock and thereby drive up its price. Conversely, if they do not like management decisions, investors will sell their stock and thereby drive down its price. Second, maximization of the share price makes all stockholders wealthier, which is one of the reasons they buy shares in the company. Third, from a broader point of view, decisions that maximize share value are decisions that use the firm's resources efficiently and result in a more efficient allocation of resources for society as a whole.

Of course, management and stockholders may have other objectives, ranging from offering stable employment to the workforce to protecting the environment from pollution. In some instances, the additional goal is consistent with share value maximization; in others, it is not. In the latter case, management chooses decisions that maximize share price subject to the constraint that the additional goal also be met.

1.3.2 The Decision Alternatives

The purpose of financial planning is to specify the future actions and decisions that will lead to the goal of making the value of the firm's equity shares as large as possible. So the second necessary ingredient is a complete list of the possible decisions and actions. Although such a list may seem rather obvious, this step is frequently overlooked. The mistake many managers make in planning is to ask *what will happen if I make this decision?* They do not fully consider the many different alternative decisions and ask which one is best. Consequently, they overlook many potentially superior decision possibilities.

The planner should develop a list of decision alternatives that is as comprehensive as possible. The list should include all possible actions, ranging from doing nothing to actions that, at first, seem infeasible or even, perhaps, counterproductive. Certainly, many possibilities will be quickly eliminated. Still, it is useful to engage in the brainstorming exercise of listing them prior to eliminating them.

1.3.3 The Links between Decisions and Goals

To decide which actions will maximize value, one must know how the action will affect share value. To do this, the link between the action and value must be identified. This is where the theory of finance makes its contribution—by providing us with a framework, or way of thinking about financial problems, that explains how various decisions affect the market value of the firm and its stock. It tells us what the hypothetical value of the firm is, given the prospective decision ([Exhibit 1.1](#)). This book will not present a complete survey of the theories of valuation.† However, we discuss the main ideas and develop valuation models that are always in the background of our discussions of

* We assume an efficient market populated by rational agents such that the market price will reflect the long-term intrinsic value of the firm.

† Valuation models are explained in greater detail in Brealey, Myers and Allen (2017); Brigham and Ehrhardt (2017), Damodaran (2012), or Jacob and Pettit (1988); a very practical reference is Pratt and Niculita (2008).

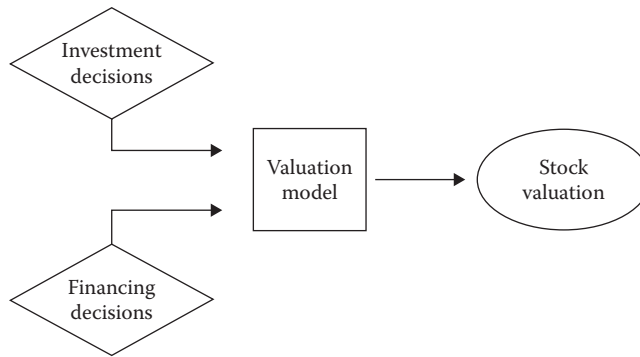


EXHIBIT 1.1 Valuation module as a link between decisions and objectives.

planning. Throughout this book, we relate the decision problem to the objective of value maximization by way of a valuation model.

In terms of the problems of financial planning, one must relate the decision problem at hand to the value of the firm and the stock. However, you do not necessarily have to determine the value of stock in all cases. Rather, the objective for the problem at hand can be stated in operational terms that are consistent with the objective of stock value maximization. For example, in capital budgeting, the investment project is chosen so as to maximize net present value; and in deciding on the best mix of debt and equity, the mix that minimizes the weighted average cost of capital is chosen. In each case, the objective is stated in narrower, more specific terms that are consistent with share price maximization.

1.3.4 Resources and Constraints

The purpose of financial planning is to decide on future actions to reach a goal. Therefore, a plan must take into account the factors that will be available to assist in attaining the goal. There also will be factors that limit the firm's actions or ability to attain the goal. We call the positive factors resources and the negative factors constraints. The dichotomy is not always clear, however. As nearly all resources are limited, they also act as constraints.

Resources and constraints occur in both the present and the future. Those in the present describe the company's current situation; those in the future are anticipated to affect the company in the future. Present and future resources and constraints can be classified as

1. Company specific.
2. Industry specific.
3. Related to the economy as a whole.
4. Influenced by the legal and political environment.

Resources and constraints are important inputs to the planning process and should be considered explicitly in evaluating the desirability and feasibility of a plan of action. The financial planning effort should begin with a careful assessment of present resources and

constraints and then should develop the plan based on the assumptions and forecasts for future resources and constraints.

1.3.4.1 *Company-Specific Resources and Constraints*

The firm's present condition is the starting point from which it will develop and the base on which it will build in the future. The firm's present condition is described by its balance sheet, its production technology, and the sources and nature of the supply of its productive inputs—raw materials, products, labor, and management.

1.3.4.1.1 Balance Sheet The balance sheet describes a firm's assets and liabilities. The firm's assets are simultaneously resources and constraints. They are part of the resources available to implement a plan, but because they are limited, they also are constraints on efforts to attain the goals. For example, liquid assets are potentially available to finance planned expenditures, but because they are limited, they limit or constrain expenditures financed from this source. A firm's fixed assets are a resource representing investment in capacity to produce output in the future. However, future potential production is limited by existing capacity unless more capital investment is undertaken. The firm's liabilities represent obligations to make promised payments in the future and are therefore constraints on future actions.

1.3.4.1.2 Production Technology The firm's ability to generate output and profits is determined by the capacity of its plant and equipment. Hence, production capacity is a resource for implementing its plans. Conversely, the firm's future opportunities are constrained by the nature of its technology and its current capacity. The firm's production technology determines what inputs must be used, their relative amounts, and the level of output attainable. The firm's production equipment, plant, and so on determine the costs and profit margins. Production technology also influences the variability of the firm's income and thus the firm's business risk. For example, the mix of fixed and variable costs of production determine the firm's operating leverage, its break-even point, and the variability of its operating income.

1.3.4.1.3 Sources of Supply Established sources of raw materials and good relationships with suppliers are resources for the future. Interruption of these supplies may be a major source of risk for future plans, and limitations on these supplies constrain the rate of production. The costs of materials influence the mix of inputs in the production process and are an important factor in the firm's current and future profitability.

1.3.4.1.4 Labor and Management People in an organization are an important resource. They can also be a source of problems and difficulties for the firm. As an economy becomes more oriented toward service and information, personnel increasingly become the major resource for reaching the firm's goals. This makes the availability of qualified personnel a primary constraint. In evaluating any financial plan, the reliability and productivity of the firm's labor supply must be considered. Similarly, the strength, ambition, and creativity of management are major factors in determining whether the firm can carry out its plans as well as whether the firm can respond effectively to a changing environment and the subsequent surprises that can undo their plans.

1.3.4.1.5 **Products** The nature of the firm's products, the uniqueness of its products, and any special product advantages, such as patents, are important factors in the plans for the future. The financial planning effort must evaluate the firm's product mix and the future prospects for its products and must relate both the product mix and the products themselves to the firm's industry and market.

1.3.4.2 *Industry-Specific Constraints*

The ability of the company to attain its goals is highly dependent on the environment of and the future prospects for the industry within which it operates. In most cases, the industry context presents constraints on the attainment of goals rather than resources that assist the firm, although the opportunities for growth in the industry may be considered a resource.

1.3.4.2.1 **Demand** Probably, the most important input to any planning effort is the forecast of future demand for the product. In fact, for most financial planning efforts, the sales forecast is the factor on which almost all other forecasts depend—the variable that drives the model. One of the first steps in generating a forecast of company sales is preparing a forecast of industry sales. Industry sales constrain a firm because they represent the maximum size of the financial pie available to all firms in the industry. In addition to a basic forecast of industry sales, there are numerous other more specific factors under the heading of demand that may be important inputs to the financial planning effort. These other factors include pricing, advertising by the company and its competitors, growth of market, and product life cycle.

Demand for the product depends on the price charged for the product. Therefore, a major input to analysis of the industry's and company's future sales is an estimate of the demand function for the product, how demand changes as the price is changed. Pricing policies of other firms in the industry also serve as important constraints on the company's decisions and affect the financial plans. The future demand for the product and the growth in the demand will depend on prices, the economy, and sociological and demographic trends. All of these factors may have to be considered in developing a sales forecast.

1.3.4.2.2 **Industry Structure** The structure of the industry will influence sales by the company and constrain many of the company's actions. Company sales may be expressed as the product of the total industry sales and the company's share of the market. In turn, the share of the market depends on the number of firms in the industry, their competitiveness, and the degree of concentration in the industry. In competitive industries with many firms, the sales of a particular company will constitute only a small share of the total market, and the firm will be constrained in its ability to change prices and to increase its share of the market. Just the opposite is true for more concentrated, less competitive industries. Competition and industry structure may be influenced by whether patents and licenses protect the various products and production methods. In addition, the nature of the production process, which may exhibit decreasing, constant, or increasing returns to scale, will affect industry concentration and competition. Finally, marketing and distribution practices in the industry will be important factors that constrain the ability of the company to attain its goals.

1.3.4.3 *Constraints Imposed by the Economy*

Every company is to some degree dependent on the vicissitudes of national and even international economic conditions. Indeed, it is well accepted that one of the best measures of a firm's risk is the extent to which the firm's fortunes are systematically related to the economy as a whole. Thus, we cannot ignore the dependence of the company's fortunes on what happens in the economy as a whole. This makes a forecast of general economic conditions (and perhaps more specific regional forecasts), one of the most important inputs to any financial planning effort. Of course, different companies and industries vary in terms of how much they are influenced by economic conditions. Some industries, such as automobiles, housing, and machine tools, are particularly vulnerable to the economic cycle. Others, such as utilities and grocery chains, are less sensitive. In still other sectors, such as banking, a single, specialized economic variable, such as interest rates, may be of prime importance in developing a company forecast.

1.3.4.4 *Constraints Imposed by the Legal and Political Environment*

The rules and regulations promulgated by government and the demands of political reality typically impose constraints and limitations on the actions the firm can undertake and may influence the results of the actions taken by the firm. In developing a financial plan, it is important to recognize and take explicit account of these constraints. As one purpose of financial planning is to plan for the unexpected and to be prepared to respond to surprises, it may also be important to attempt to forecast future legal and political conditions—one of the most difficult forecasting tasks. The company should be prepared to respond to society's demands for such things as pollution control, a low carbon footprint, equal opportunity, and product liability. In addition, it should consider the effects on the company of major political upheavals such as wars, foreign revolutions, expropriations, and devaluations. If a company depends on international trade, it should try to anticipate the effects of currency exchange rate fluctuations.

1.3.5 The Planning Horizon and the Amount of Detail

The last item on the list of ingredients in a financial plan is the length of the planning horizon (the length of time over which the plan will apply) and the amount of detail to be generated for the financial plan. Before developing the financial plan, the analyst must decide how many months or years should be considered in the plan. In addition, the analyst must decide how detailed to make the plan. The appropriate length of the planning horizon and the desired amount of detail are important decisions that are parts of the initial input to the planning process.

1.3.5.1 *The Length of the Planning Horizon*

A financial plan can have a planning horizon of almost any length—one day, one month, one year, five years, or longer. The choice of the planning horizon is important because it affects the results and usefulness of the plan. The planning horizon influences the amount of effort and cost necessary to develop the plan as well as the amount of detail incorporated in the plan. Therefore, it is important to view the planning horizon as a crucial ingredient

of the process. Factors that should be considered in deciding on the planning horizon are the nature of the planning problem, the amount of output detail desired from the plan, the horizon of reasonable forecasts, and the period over which the planning horizon makes a difference.

The nature of the planning problem obviously influences the planning horizon. As a general rule, you should make the planning horizon sufficiently long to show the full effects of the decision over a full cycle. If you are concerned with the long-run effects of a broad strategic decision, then a long planning horizon is appropriate. For example, analyzing the effects of a research and development program involving new product development over the next 10 years calls for a long planning horizon. It makes little sense to limit the analysis to, say, one year. On the other hand, if the problem is determining the financing requirements over the firm's seasonal production cycle, a one-year planning horizon would usually be appropriate because the cycle would tend to repeat itself each year, and seasonal financing would not normally be arranged for more than one year. In this case, a six-month planning horizon would be too short to show the effects of the cycle and the results of the decisions being considered.

The nature of the problem also influences the division of the planning horizon into subperiods, the time intervals that make up the planning horizon. For example, a one-year planning horizon could be divided into 12 monthly subperiods or four quarterly subperiods. The length of the subperiod determines how frequently prospective decisions and forecasted results can be changed in the model. With quarterly subperiods, prospective decisions and results can be changed each quarter, but not more frequently. If a broad, long-term strategic policy is under consideration, in which the basic decisions would not be varied more often than annually, then you probably would not be interested in developing a plan that generates quarterly or even more frequent results. Conversely, if the problem is one of managing the marketable security portfolio where daily decisions may be important, it would make sense to use daily, or perhaps weekly, subperiods, and it would be pointless to consider a five-year planning horizon composed of annual subperiods.

1.3.5.2 The Amount of Detail

The length of the planning horizon and the number of subperiods influence the amount of detail considered in the financial plan. In determining the length of the planning horizon and the subperiods, you need to consider how much detail is useful. It is too easy, particularly with computers, to generate more detail than can be evaluated. If the planning horizon is composed of many subperiods and the decisions and results are considered for each subperiod, an excessive amount of detail can be produced. One frequently finds that the additional detail is a disadvantage. Too much detail tends to overwhelm the user so that she loses track of how the results relate to the decisions and objectives. In addition, generating and evaluating detail are costly in terms of effort and time. Therefore, the information and detail must be balanced against the cost and effort involved.

It is useful to distinguish between cross-sectional detail and time series detail. Cross-sectional detail refers to the amount of data, variables, and decisions considered for a given period. Financial statements broken down into many account categories for a given period,

Time series detail

	Period 1	Period 2	Period 3	Period 4	Period 5
Sales					
Income					

Cross-sectional detail

	Period 1	Period 2
Sales		
Operating cost		
Operating income		
Interest expense		
Income		

EXHIBIT 1.2 Time series detail versus cross-sectional detail.

say June 2006, generate a large amount of cross-sectional detail. Time series detail refers to data, variables, and decisions considered at different points in time. A forecast of net income for each month of a 10-year planning horizon would include much time series detail, but because there is only one variable (net income) for each subperiod (one month), there would be little cross-sectional detail.

[Exhibit 1.2](#) demonstrates the idea. Both charts have 10 pieces of data. In the first chart, greater time series detail is shown at the expense of less cross-sectional detail. In the second chart, more cross-sectional detail is shown with less time series detail.

The amount of detail should be minimized, kept to a level sufficient to help the analyst understand the problem. If the problem involves a long planning horizon, you can control the size of the problem by decreasing the amount of cross-sectional detail (fewer accounts for each subperiod) and limiting the amount of time series detail (fewer [longer] subperiods across the planning horizon). If the planning problem involves a short planning horizon and requires a great deal of cross-sectional detail (e.g., many different investment and financing accounts), you can control the amount of detail by shortening the planning horizon and lengthening the subperiods.

The length of the planning horizon should be limited to the horizon of reasonable forecasts. If the financial plan depends on forecasts (of sales, interest rates, gross national product, or the like), it does not make much sense to extend the analysis beyond the time over which you can make reasonable forecasts. For example, sales forecasts for specific months make sense for one year into the future, but it is nonsense to try to distinguish between June and July sales five years hence. Consequently, developing a forecast of month-by-month financing needs is appropriate for the next year or so, but inappropriate over a five-year planning horizon. On the other hand, estimates of long-run growth trends may be quite reasonable and are useful for broad, general financial planning with a very long planning horizon. For example, an electric utility may need to plan its construction to meet growth in energy demand over the next 25 years. Although it cannot forecast month-by-month demand 25 years hence, it can make useful forecasts of long-run growth based on demographic trends. In this case, a 25-year planning horizon divided into annual subperiods may be appropriate.

1.4 INGREDIENTS OF A FINANCIAL MODEL

In the first part of this chapter, we introduced concepts of financial planning using the analogy of a travel plan. We then presented and explained the ingredients of a financial plan. This section extends the concepts of financial planning to financial modeling. Just as a road map connects the traveler's current location with her destination, a financial model connects the firm's current financial state with its future financial objective. In this section, we explain the concept of a financial model and the role played by a financial model in financial planning, and then show how the ingredients of a financial model correspond to the ingredients of a financial plan.

1.4.1 What Is a Model?

Webster defines a model as “a small copy or imitation of an existing object; or, a preliminary representation of something, serving as a plan from which the final object is to be constructed.” Everyone is familiar with model airplanes and recognizes that they are intended to imitate real airplanes. However, the manner in which they imitate the real thing depends on their intended use. For a child, the model only needs to have the shape of an airplane. A simply shaped block of wood is sufficient to excite the child's imagination. On the other hand, for an aircraft manufacturer, the model must be more detailed. It must resemble the full-sized prototype in enough important respects that the performance potential or other aspects of the real plane can be tested. Yet even in this case, the model need not include every detail of the real plane to be useful. Only the features necessary to test specific aspects of the plane are required. Features of the real plane irrelevant to the particular aspect being tested can be omitted. For example, to test air worthiness, only the shapes of fuselage and wings must be precise, whereas to test cockpit ergonomics, only the interior dimensions must be precise.

It is easy to understand what a model airplane is and how it imitates a real airplane. It is more difficult to understand what a financial model is and how it imitates the real thing. In business finance, the object to be modeled is the set of interrelationships and linkages between the firm's environment, prospective decisions and actions, and the firm's objectives. For example, the question for the financial model may be: What is the effect of an increase in interest rates on the earnings and value of the firm? Interest rates, earnings, and value are considerably more abstract than the shape of an airplane's fuselage, wings, or interior. Nonetheless, these relationships often can be expressed verbally or mathematically with enough essential detail to trace out their potential effects. This, then, is the essence of a financial model—a set of verbal or mathematical statements that expresses the system of relations and links between the firm's environment, its decisions and actions, and its objectives.

Consider the following example. Suppose we wish to analyze the effects of the firm's pricing decision on its sales. The following simple model shows the links between the price, units sold, and total revenue. This model consists of two equations and three variables: *Total Revenue_t*, *Units Sold_t*, and *Price per Unit_t*, where the subscript *t* denotes the time period. Management decides what price to charge for the product; the model returns the total revenue to be expected. *Price per Unit* is called the decision variable. *Units Sold* is a

variable that management cannot set. But management can influence *Units Sold* through the value they set for the decision variable, *Price per Unit*. The question to be answered is what price should be set so that the firm can best attain its objective, perhaps maximizing *Total Revenue*.

A Model of *Total Revenue*:

$$\text{Total Revenue}_t = \text{Units Sold}_t * \text{Price per Unit}_t \quad (1.1)$$

$$\text{Units Sold}_t = 500 - 30 * \text{Price per Unit}_t \quad (1.2)$$

Equation 1.2, what economists call a demand function, links the firm (its pricing decision) to its competitive environment (the number of units sold). It shows the quantity of units demanded at different prices. Over the range of prices to be considered, each \$1 increase in the price of a unit causes the quantity of the product demanded to decrease by 30 units. Thus, if *Price per Unit* is \$5, *Units Sold* will be 350 units, but if *Price per Unit* is increased to \$6, *Units Sold* decreases to 320 units. The competitive structure of the industry, the nature of consumer demand for the product, and the general condition of the economy all influence the way product demand responds to the price of the product. The demand function summarizes these effects. Through the demand function, these factors constrain management's ability to set an arbitrarily high price to attain its goal.

Equation 1.1 represents the link between the firm's decisions (price) and its objective (maximize revenue). It is essentially a definition that shows how price and demand combine to determine *Total Revenue*. If we assume that the goal of management is to maximize *Total Revenue*, then this expression is what is called the objective function—the expression or target that is to be optimized. In this case, the decision variable, *Price per Unit*, is supposed to be chosen so as to make the objective, *Total Revenue*, as large as possible.

Exhibit 1.3 shows how *Total Revenue* varies as *Price per Unit* changes. When *Price per Unit* is less than \$8.33, an increase in *Price per Unit* increases the *Total Revenue*. When the *Price per Unit* is greater than \$8.33, an increase in *Price per Unit* decreases *Total Revenue*. Therefore, *Total Revenue* attains its maximum value when *Price per Unit* equals \$8.33. Hence, the optimal decision—the one that maximizes *Total Revenue*—is to set the *Price per Unit* at \$8.33. At this price, 250 units would be sold for total revenue of \$2,083.

This model contains parameters in addition to equations and variables. Parameters are the constants (numbers) in the equations that operate (in this case, multiply or add) on the variables to yield the value of the equations. In Equation 1.2, the parameters are 500 and 30. These parameters represent the way the quantity demanded changes with price. Different parameters imply a different relationship between price and units sold. The parameters in any model are usually estimates of how one variable responds to another. They represent economic behavior or technological relationships.

The general form of an equation shows the relationships between variables without specifying specific values for the parameters. For example, the general form of the demand function (Equation 1.2) is



EXHIBIT 1.3 Revenue as a function of price.

$$\text{Units Sold} = a + b * \text{Price per Unit} \quad (1.3)$$

where a and b are generalized parameters. The general form is based on the modeler's estimate of the general relationships between variables. To make Equation 1.3 usable, the values of parameters a and b must be estimated.

This means that the builder of our simple model faces two tasks: (1) determining the general form of the relationships and (2) estimating the parameter values. Though in some cases the modeler may just guess structural forms and the parameter values, the preferred procedure is to identify the best structural form and estimate the parameters using historical data. Hence, forecasting and econometrics are an important part of the modeling process. These methods are explained in the forecasting chapters.

Time is another explicit element in Equations 1.1 and 1.2. The variable Units Sold_t refers to sales during period t . As Units Sold must be defined in terms of sales per period, the length of period t is an important part of the model. The parameters in Equation 1.2 would be very different for periods of one month versus one year.

Equations 1.1 and 1.2 do not specify the length of or the number of periods in the planning horizon. The model could apply over a number of periods, with an equation assigned to each period in the planning horizon. If the same equations with the same parameters apply identically to each of, say, 10 periods, the model could be written in the general form:

$$\text{Total Revenue}_t = \text{Units Sold}_t * \text{Price per Unit}_t \quad (1.4)$$

$$\text{Units Sold}_t = a - b * \text{Price per Unit}_t \quad \text{for } t = 1, \dots, 10 \quad (1.5)$$

where $t = 1, \dots, 10$ means that the expression is repeated for each period t from 1 through 10. Parameters can also vary across time. When this is the case, a subscript is added to the parameter, for example, a_t, b_t for $t = 1, \dots, 10$. This indicates that there are different parameters a and b in the demand function for each of the 10 periods in the planning horizon.

Though simple, the structure of this two-equation model contains most of the key elements of more complex financial models. It is made up of variables, equations, and parameters. Some of the variables represent prospective decisions, others represent the effects of those decisions. The equation for *Total Revenue* shows the relation between these variables and the goal or objective. We refer to this expression as the objective function. Finally, our simple model has parameters whose values must be estimated using historical data. More complex models will contain other expressions representing more complex relations between variables, and perhaps expressions that portray constraints and limitations on the firm's decisions. Whether simple or complex, the model links the decision to the objective. The equations in the model portray the influence of resources and constraints on the firm's ability to attain the goals set by the financial planning process. The relationships between variables, expressed as a set of equations, comprise the financial model.

1.4.2 What Does a Model Do?

As a small copy of the firm, the financial model serves as a testing platform. Using the model, the manager can assess the effect of various decisions before actually implementing them and thereby can avoid subjecting the firm itself to experimentation.

A financial model of the firm traces through the complex financial relationships in the firm. This is helpful and sometimes essential because of the complexity of the relationships between the firm's financial variables. For some problems, it may be possible to discern the general direction of the result of a decision simply by inspection, but it may be difficult to figure out the magnitude of the effect without a model. For other problems, it may not be possible even to discern the direction of an effect without a model. For either type of problem, the intent of the model is to aid and improve decision making by providing more information to the decision maker—the model's output.

1.5 HOW TO DEVELOP THE MODEL

There are as many different ways to build a financial model as there are potential model builders. As any given problem can be approached from many directions, there seldom is just one correct model suited to a given problem. Given such a variety of possible approaches, the modeler will find it helpful to begin by considering a set of questions that help to organize and structure the modeling effort. These questions relate to the problem, the output, and the input. The answers should be found in the financial plan.

1.5.1 The Decision Problem

1.5.1.1 What Is the Problem?

This is the most fundamental question in approaching the modeling effort. The model must be suited to the decision problem. This requires a thorough assessment of the nature of the financial planning problem. This assessment should be as broad as possible, exploring all

facets of the problem. More precisely, the model must be at least as broad as the problem and be suited to the analysis of all important facets of the problem and all reasonable decisions. A model that is too narrow and limited may not allow the planner to consider some important decision alternatives, and thereby may cause her to miss possibly useful solutions. Thus the initial definition of the problem should be broad enough, so the subsequently developed model is sufficiently general to allow a full analysis of the problem.

1.5.1.2 What Questions Must Be Answered?

For most problems, there are questions that, when answered, suggest the solution. You may say to yourself, “If I just knew the answer to *this question*, then the decision would be simple.” The model builder should enumerate these questions as completely as possible. Such a list of questions helps define the model’s structure and influences the kind of output data and answers one wants from the model. For example, suppose we are evaluating a new marketing program. We pose some basic questions: Will the program increase sales? Will earnings and cash flow increase? Will it benefit the stockholders with an increase in the value of the stock? These questions indicate which variables should be the focus for our model: sales, earnings, cash flow, and stock value.

1.5.1.3 What Is the Time Horizon of the Problem?

Financial planning problems are almost always concerned with time. There are various aspects of time in modeling, and these are important determinants of the structure of the model. The length of the planning horizon and the subperiods should be consistent with the timing of the decision. If the problem being considered is of a long-run nature, or has long-run consequences, then the planning horizon and subperiods should be long. If the problem involves frequently made decisions over short time intervals, then the planning horizon and subperiods should be short.

1.5.1.4 How Important Is the Problem?

The importance of the problem for the goals and survival of the firm should be considered when deciding how much time, effort, and expense to commit to model development. If a complex and expensive modeling effort is necessary to provide solutions to an unimportant problem, the effort is probably not justified. If a problem can be handled with a scaled-down model, then, by all means, scale down. In any case, these issues should be considered before embarking on a costly model development effort that promises little benefit.

1.5.2 The Output

1.5.2.1 What Kind of Information Is Needed?

After considering the planning/modeling problem facing the firm and broadly defining what questions need answering, the planner should decide what specific information must be generated by the model. Typically, this information will answer the specific questions identified in the first phase of the modeling effort (What is the problem?). Continuing with the previous example, this would mean a model that outputs revenue, cash flow, and so on.

All too often, a model is developed that generates mountains of information, little of which actually helps the planner to make a better decision. This is particularly true when models are designed before deciding what output is needed. For example, if the problem is to evaluate an investment project, a model that merely generates future pro forma financial statements likely will be insufficient for a valid analysis. If the investment analysis requires calculating the expected net present value and assessing the risk of the investment, the model should be developed to do that.

1.5.2.2 How Will the Information Be Evaluated?

You should decide ahead of time how the output is to be evaluated. For example, suppose the problem involves comparing alternative financing decisions. By deciding in advance how you are going to rank the alternatives, you can develop the model so that its output makes such an evaluation feasible. Many models generate standard financial statements with so much detail that the user is unable to evaluate the alternative decisions correctly. This mistake can be avoided by designing the model to produce output that is both relevant to the problem and useful to the decision process. This means relating the information (output) to the firm's objectives. As the correct decision is the one that will lead to the objective, you cannot evaluate the decision unless you have information that shows the effect of the decision on the objective.

1.5.2.3 What Kind of Detail Is Necessary?

In deciding what output is necessary to make a planning decision, it is important to specify the level of detail needed to evaluate progress toward the planning objective. The planner/modeler should ask two questions. First, what is the smallest amount of detail needed to make the decision? This is the lower limit on the amount of detail required. Second, at what point does more detail lead to the same decision as less detail? This is the upper limit on the amount of detail required. Too much detail is more often a problem than is too little. Structure the model so that it generates no less detail than the lower limit and no more detail than the upper limit.

1.5.2.4 Who Will Use the Information?

The type and format of the output should be tailored to the audience who will use it. It may be appropriate to generate a large volume of finely detailed data for use by the planning analyst. However, as one moves up through the levels of the organization, the users of the data may be less concerned with fine detail and more concerned with a broad picture of the firm. For the executive who takes a broad view of the firm's future, the output should be consolidated and summarized, so it can be quickly and easily grasped.

1.5.3 The Structural Input

After the planner has broadly defined the problem and decided what kind of output is necessary, the model itself is developed and solved with two kinds of input—structural input that determines the structure of the model and data input that determines the solution to

the model. The structural input is the set of equations and relationships that make up the model. The data input are the values for the unknown variables in the structural input. For example, the equations are part of the structure of the sales model.

$$\text{Total Revenue}_t = \text{Units Sold}_t * \text{Price per Unit}_t \quad (1.4 \text{ reprised})$$

$$\text{Units Sold}_t = a - b * \text{Price per Unit}_t \quad (1.5 \text{ reprised})$$

The data input (from forecasts or decisions) is fed into the model by specifying

$$\text{Price}_t = \$5.00$$

$$a = \$500$$

$$b = -30$$

In a spreadsheet model, the structure is the set of formulas in the cells of the spreadsheet, and the data input is the numbers that we feed into the model.

More generally, the structural input is based on the set of ideas and questions that motivate the modeling process. The structural input quantifies the plan. It determines the number and form of the equations, the number and type of variables in the equations, and the form of the objective function. The ideas and questions that underlie the structural input are the factors discussed previously as the ingredients of the financial planning process: the goal, decisions, linkages, constraints, and planning horizon.

1.5.3.1 What Is the Goal?

Most models should have an objective function that yields summary measures of the results of the decisions put into the model. In some models, the objective function will be a specific equation that generates a measure of the goal, such as the value of the stock. In others, there may not be a specific equation; instead, the model provides output that summarizes progress toward the goal of the decision process. Equation 1.4 is the objective function in our simple model because it provides as output the value of the thing we are concerned with, total revenue in this case.

1.5.3.2 What Are the Decision Alternatives?

One purpose of the financial model is to help identify the set of decisions out of all possible alternative sets that best fulfills our goal. Hence, a fundamental input to the model is a list of the decision variables. The decision variables should be clearly specified and the model should be clearly structured, so that they can be easily manipulated and changed to explore alternative decisions. For example, in structuring a spreadsheet model, the decision variables should be placed in an input section of the model where the user can change their values easily to see the effect on the objective. In our sales model, the decision variable to be manipulated is *Price per Unit*.

1.5.3.3 What Are the Linkages between the Decisions and the Goal?

One reason to build a model is to determine how various decisions affect the firm's goals. An objective function summarizes these linkages. The model should be structured so that the decision variables feed through the model and into the objective function that represents the firm's goals. In the sales model, we feed the pricing decision through the model, and we get sales as the output. The two equations of the model provide the linkage from *Price per Unit_t* to *Total Revenue_t*. Note that we need the whole model (both Equations 1.4 and 1.5) to provide the linkage.

1.5.3.4 What Are the Constraints?

The environment in which the decision maker operates constrains her choices over the decision variables. Constraints can arise from limitations on the amount of resources available to implement the financial plan; from technological relationships that limit choices; and from the industry, economy, or legal environment. There also may be constraints that are true by definition.

1.5.3.4.1 Definitional Constraints A major part of most financial models, particularly financial statement models, is a set of equations that are definitions. Typically, these are based on accounting rules. We treat these definitional equations as constraints because they constrain the model to conform to these definitions. Definitional equations maintain consistency within the model, aggregate groups of accounts into broader categories, and link account balances from one period to the next. An example of a definition that aggregates accounts is

$$\text{Total Assets}_t = \text{Current Assets}_t + \text{Fixed Assets}_t \quad (1.6)$$

An example of a definitional equation that links accounts from one period to the next is

$$\text{Earned Surplus}_t = \text{Earned Surplus}_{t-1} + \text{Retained Earnings}_t \quad (1.7)$$

Equation 1.4 in the sales model is a definitional constraint that simply holds the system together with the definition that *sales is equal to price times quantity*.

1.5.3.4.2 Constraints from the Industry and Economy In the sales model, the demand Equation 1.5 is a constraint imposed by the economy and the industry that sets limits on the pricing decision. There is also an implicit constraint in that it is understood that the price cannot be negative. However, in a computer model, it may be necessary to make the nonnegativity constraint explicit (*Price per Unit_t ≥ 0*) because the computer would not otherwise understand that price must be positive.

If we expanded our model into something more detailed, we might encounter various other constraints such as limitations imposed by technological relationships, resources, the economy, and regulations.

1.5.3.4.3 Technological Constraints Technology often places constraints on the actions available to management. For example, management might like to produce more product, but the technology of its manufacturing process limits how much can be produced. Suppose a production function describes the maximum number of units that can be produced with a specified amount of labor and capital equipment, according to Equation 1.8 as follows:

$$\text{Units Produced}_t = 5 * \text{Labor}_t^{0.33} \text{Capital}_t^{0.67} \quad (1.8)$$

This production function limits how much can be produced for a given endowment of resources. So we also need to consider the constraints imposed by our limited resources.

1.5.3.4.4 Resource Constraints Assume the maximum number of units of *Capital* and *Labor* available are 40 and 60, respectively. Our constraints are stated as

$$\begin{aligned} \text{Capital}_t &\leq 40 \\ \text{Labor}_t &\leq 60 \end{aligned} \quad (1.9)$$

which allows us to use less than the amounts available but prevents us from using more.

In both the production function and the resource constraints, the general form of the expressions is a part of the structure of the model, and the parameters (5, 0.33, and 0.67) in Equation 1.8 and the parameters (40 and 60) in Equation 1.9 are the data input. Throughout this book, as we build models you will be encouraged to separate the data input from the structural model. Doing so allows the structural model to remain general, whereas the input section is variable. This makes it easy to change the parameters in the model without having to change the model itself and, as an added benefit, greatly facilitates scenario simulation and sensitivity testing.

1.5.3.5 What Is the Planning Horizon?

The length of the planning horizon and the number of subperiods contained therein are important structural inputs to the model. Together they influence the number of equations and variables in the model and the volume of output from the model. Many of the considerations related to the planning horizon have been discussed previously, with respect to the length of the subperiods and the amount of detail in the plan. Suffice it to say that as the number of subperiods increases, the number of variables, the number of equations, and the complexity of the model also increase. Evaluate these factors before actually specifying the model.

Carefully considering the questions relating to the problem, the output, and the structural input provides the modeler with a rudimentary structure with which to begin developing the model in its general form. This consists of a set of equations (and/or inequalities), a set of decision variables that are the arguments in the equations, generalized parameters that enter the equations as constants or coefficients of the decision variables, and possibly an objective function. At this stage in the development, most of the parameters are shown only in their general form (such as *a*, *b*, and *c*). Data must be input to give specific values to the parameters before the model can be solved.

1.5.4 The Data Input

Up to this point, only the general form of the model has been developed. No specific numerical values for the parameters of the model have been supplied. To make the model usable, these parameters must be specified. This is the role of the input data—to fill in the blanks (e.g., the values for a and b)—in the general model so that all values except those for the decision variables are specified.

Data input can be classified into three broad categories: data that describe the current state of the system, data that describe the relationships between variables, and data that forecast future conditions.

1.5.4.1 *The Current State of the System*

Financial planning plots the trajectory of the firm through time—where it was, where it is, and where it hopes to be. It can be thought of as specifying decisions and anticipating their consequences over a number of periods of time in order to reach a stated objective. Obviously, where the firm goes in the future depends on where it starts. The current conditions of the firm and its environment are important determinants of where it goes in the future. Therefore, the financial plan and the financial model must take these current conditions into account. This is what is meant by the current state of the system—the condition or situation from which the firm begins its projected trip into the future. The firm's resources and constraints describe the current state of the system. These were discussed as input to the financial planning process. Internal data describe the firm's current state; external data describe its environment.

1.5.4.1.1 Internal Data: The Firm The firm's current condition consists of its endowment of resources, its productive capacity, and its obligations and commitments. The firm's financial statements describe most of these factors. They establish the initial condition from which the future will evolve, particularly for accounting models that generate future pro forma financial statements. In addition to the financial statements, the model may also require input data that describe the firm's production technology, sources of supply, labor and management, and products.

Whatever the input data required, there must be congruence between the model and the data. The level of input detail demanded by the model must be matched to the amount of detail available in the accounting statements used as input data. If the required detail exceeds what is available from the firm's accounting system, then the model should be revised so that the data required match the data available.

1.5.4.1.2 External Data: The Environment The current conditions in the firm's operating environment also influence the actions that the firm can take and where it can go in the future. Most of the factors that describe current conditions relate to economic conditions, such as interest rates, stock prices, and prices of goods and labor.

1.5.4.2 *Relations between Variables*

Variables in the financial model may be related to each other in a way that is not completely known to the modeler. Nonetheless, the modeler can estimate these relationships

by assuming a mathematical structure (an equation) for the relationship and using historical data to estimate its parameters. For example, suppose accounts receivable is assumed to be related to past sales according to the distributed lag relationship in Equation 1.10:

$$\text{Accounts Receivable}_t = a + b_1 * \text{Sales}_{t-1} + b_2 * \text{Sales}_{t-2} + b_3 * \text{Sales}_{t-3} \quad (1.10)$$

The parameters a , b_1 , b_2 , and b_3 in Equation 1.10 are unknown. Using past observations of accounts receivable and sales, these parameters can be estimated by least squares regression techniques. The estimates for a , b_1 , b_2 , and b_3 become data input for the model.

1.5.4.3 Forecasts of Future Conditions

If we knew the future, we would not need to do much modeling and planning. Unfortunately, we cannot know what the future will bring. Indeed, a major purpose of planning and modeling is to help the firm deal with the uncertainty of the future. This requires forecasting future conditions and relationships. Hence, forecasts are an important input to the modeling process. As forecasts are basic to solving the model and developing a financial plan, an important issue is the sensitivity of the plan and its results to forecast errors. The modeler should maintain a healthy skepticism of the accuracy of the forecasts and consider how forecast errors affect the financial plan and goal attainment. Forecasting techniques will be discussed in later chapters to introduce the modeler to the tools necessary to consider and evaluate alternative forecasting methods.

If one has asked and answered the questions relating to the nature of the planning problem, the output required, and the type and availability of input data, much of the modeling effort will have been completed.

1.6 TYPES OF MODELS

There are two general approaches to financial modeling: simulation and optimization. Simulation models show the results of a decision or action that is specified by the user; optimization models solve for the best set of decisions. Though simulation models are more widely used and understood, optimization models also can be very useful. To aid the prospective user in choosing the approach that best suits her needs, both approaches are discussed in this book.

1.6.1 Simulation

Simulation is best understood as a technique that allows the user to answer the question *what will happen if...?* Simulation is the process of imitating the firm or organization so that the consequences of alternative decisions, actions, and strategies can be analyzed. Simulation allows management to view the hypothetical results of undertaking a particular decision, making it possible for them to evaluate the decision's consequences before implementing it.

The simulation model links the prospective decision to the hypothetical results. The model consists of a set of mathematical expressions that reflects the relationships and links

between the firm’s environment and the organization, between units of the organization, and between decision and result. Thus, the simulation model provides the necessary bridge between the input (the firm’s current state) and the output (the firm’s projected future state) of the simulation process.

As shown in Exhibit 1.4, the input to the model normally consists of the current state of the system (the initial condition of the firm and its environment), forecasts of future conditions, and decisions or actions to be considered. A simulation model takes the set of decisions as input, and then combines this input with current and future economic conditions to generate the output—the consequences of those decisions. These consequences, in turn, partly describe the state of the system for a subsequent period.

The simulation approach does not tell the user which decision is best. Rather, the user specifies a decision, the model traces through the relationships, and then the user evaluates the results. The consequences of alternative decisions and strategies can be examined, but the assessment of which strategy is best is left to the user. For example, suppose the decision being considered is whether to finance a new investment project by issuing new common stock or by borrowing from a bank. The typical simulation would trace through the consequences of each of these alternatives and present the user with sets of financial statements and measurements of the objective such as value of the stock. The user then uses the simulated output to evaluate the results of the decisions.

Simulation models offer several advantages over optimization models: they are usually easier to develop and solve; managers usually find the simulation process easier to

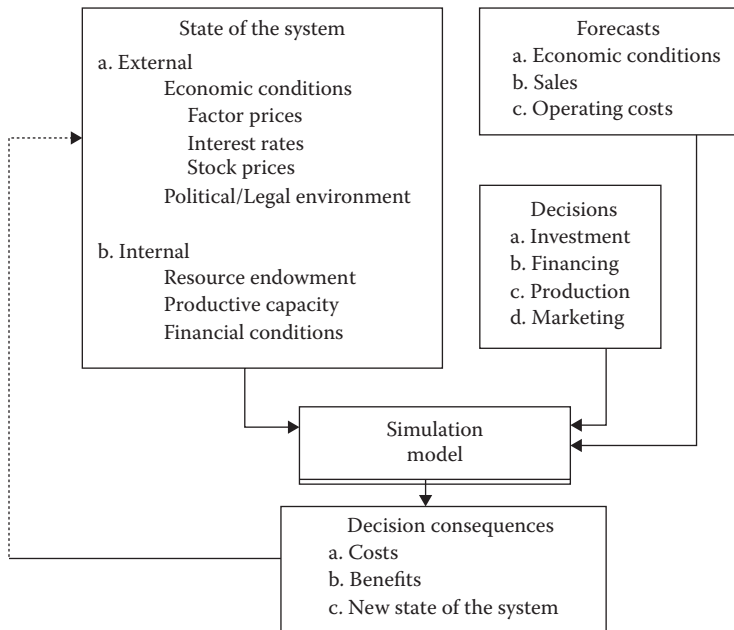


EXHIBIT 1.4 Information flow in a simulation model.

understand; and managers may find the output, in the form of standard financial statements, easier to understand and accept. In addition, managers may have more confidence in, and provide more support for, a decision that they make themselves based on the prospective consequences generated by the simulation model.

One problem with simulation models is that they can easily generate enough output to overwhelm the user, leaving her unable to evaluate the results and determine the best decision. For example, suppose that in the debt-versus-equity financing problem mentioned earlier, the output consists of an income statement and balance sheet for each of many periods in the planning horizon and for each decision alternative. In this case, the volume of total output could be so overwhelming and confusing that it is of no help to the decision maker.

Another problem is that users of simulation models develop models capable of generating detailed financial statements, but fail to specify the criteria necessary for determining the best decision. If many different decision variables are to be considered and many different financial factors are to be compared, it is difficult to compare financial statements and judge one as superior. For example, one decision alternative may result in a more liquid current position but greater long-term debt, whereas another results in the opposite scenario. This leaves the user to decide how to evaluate the trade-off between liquidity and leverage. In such a situation, it is useful to add an objective function that measures the relative desirability of the results of the various decisions under consideration.

An objective function facilitates the ranking of alternative decisions, making it possible to analyze different decision scenarios and select the best one. However, there still may be too many decision alternatives to inspect all of the results, even when they are summarized by an objective function. For example, suppose a planning problem involves just 10 different decision variables—the amount of cash balance to carry, level of inventory, investment in fixed plant, and so forth—and suppose each of these decision variables can take on any of the 15 different values. The number of different decision combinations would be $15^{10} = 5.77 * 10^{11}$. It would certainly take a long time for the planner to examine the result of each possible decision combination.* What is needed is an efficient way to search the myriad of decision combinations to find the best one. This is the task performed by optimization models. An optimization model can quickly determine which of the many possible combinations of decisions best meets the specified objective.

1.6.2 Optimization

An optimization model determines which decision alternative leads to a desired objective. Although a simulation model simply shows the consequences of a decision specified by the user, the optimization model solves for the best decision.

An optimization model is like a simulation model to which has been added an expression that represents the goal of the decision and a set of equations and inequalities that

* Assuming a tireless individual could evaluate one scenario per minute, work 24 hours per day, and 365 days per year; it would take 2,194,256 years to evaluate all possible scenarios.

constrain the choices. The constraints are usually closely related to the sets of equations that constitute the simulation model. The expression representing the goal of the decisions is the objective function.

The objective function is an equation that portrays the influence of the decision variables on the objective. The objective is the thing that the user wants to make as large as possible (maximize) or as small as possible (minimize). For example, if the user wants to identify the decisions that maximize the market value of the firm's common stock, the objective function would be a mathematical expression that shows the effect of the decision variables on the market value of the stock.

The decision variables represent the choices available to the decision maker. They might include the level of investment in inventory and how it is to be financed, with the financing choices being the amount to be borrowed and number of new shares to be issued. If so, the objective function would show the influences of the level of investment in inventory, the amount borrowed, and the number of new shares issued on the market value of the firm's stock. The purpose of the optimization model would be to find the values of these three variables that make the value of the stock as large as possible.

The constraints in an optimization model are sets of equations (or inequalities) that express the limitations that prevent the model from choosing the decision variables that make the objective arbitrarily large or small. Some of the constraints may express subgoals and other objectives that the user wants to attain in addition to the primary goal specified by the objective function. Other constraint equations might be identical to those in a simulation model.

The optimization approach has two advantages over the simulation approach. First, it forces the user to clarify and clearly state the objectives. Second, it relieves the user of the confusing task of searching through a mountain of simulated results to evaluate numerous decision alternatives. However, because they remove the decision from managers, optimization models may receive less managerial support. [Chapters 16 and 17](#) explain optimization models in greater detail.

1.7 WHAT DO WE GET OUT OF IT?

The financial planning and modeling process can be expensive in terms of time, effort, and money. What benefits do we derive from this process? Are they worth more than the costs? To evaluate the cost/benefit trade-off, we need to consider what we get out of the process and compare that to the costs. There are explicit and implicit payoffs from the modeling process. We need to consider both.

1.7.1 Explicit Benefits

The primary benefit of the modeling process is the opportunity for management to test their ideas and decisions before actually implementing them. Management can experiment with the model rather than the firm, exploring how their decisions relate to their overall objectives. With the ability to better anticipate the results of their decisions, managers should be able to make better decisions. In addition, a well-documented plan allows the planner to review and analyze the plan after it has been implemented in order to determine

where triumphs and mistakes occurred. Moreover, the ease of exploring decision alternatives gives managers more time to consider broader issues, such as setting, defining, and evaluating policy alternatives. Thus, models help managers use their talents more effectively.

1.7.2 Implicit Benefits

Many benefits from the planning process are intangible and implicit in the process itself. The process forces the planner to systematize her thoughts about objectives, decisions, assumptions, and the interactions among parts of the firm and between variables. As pointed out by Ackoff:

The value of planning to managers lies more in their participation in the process than in their consumption of its product.... Such participation stimulates the development of a deeper understanding of the business and its environment, and it forces the systematic formulation and evaluation of alternatives that would not otherwise be considered. (Ackoff 1970, p. 137)

The process of building a model forces the planner to think carefully about the complex relationships between her decisions and the financial variables that constitute the firm. Managers must wrestle with the most basic questions about the firm's objectives and how to measure progress toward those objectives. In addition, a financial plan helps to build consistency between different parts of the firm in terms of assumptions, forecasts, decisions, and results. Finally, involving management in the planning process increases the likelihood they will trust and support the implementation of the planning output.

The costs of modeling and planning are also difficult to measure. Some of the costs are direct and explicit—the costs of the software and time of the planning staff. However, even when the costs are fully counted, it will be difficult to objectively evaluate the net payoff because it is so hard to know what decisions would have been made if the process had never occurred. So although we encourage you in your modeling efforts, unfortunately we do not yet have a model that will allow you to objectively and accurately evaluate the net benefits from planning and modeling.

1.8 SUMMARY

This book seeks to help readers learn how to construct and use models for corporate financial planning. As these models should not be constructed in a vacuum, this chapter provides an overview of the planning process and helps set the stage on which models are developed and used. We provide a framework for thinking about how to create financial plans, how to relate models to those plans, and how to structure those models to assist in realizing those plans.

There are many ingredients or inputs to any plan. At the broadest level, the key ingredients are the goals we seek, the decisions we must make, and the links between the decisions and goals. With respect to financial planning, the primary goal is to maximize the firm's value, and one of the basic functions of the model is to show how the decisions relate to that goal.

In future chapters, we consider a variety of corporate financial planning models. [Section I](#) is the starting point. We show how the basic tools of financial analysis provide the foundation for planning and developing planning models. [Chapter 2](#) reviews basic financial ratio analysis and starts the model building process with a model that facilitates the analysis of a company. [Chapter 3](#) reviews the tools and concepts of financial analysis that will be used in the financial planning models that follow.

With this foundation of financial analysis in place, [Section II](#) shows how to build useful financial planning models. [Chapter 4](#) develops a simple but complete financial model of a firm. That model serves as the basis for many of the models and methods discussed in the remainder of this book. [Chapter 5](#) delves into the details of modeling firm value. [Chapter 6](#) introduces Monte Carlo simulation and shows how it is used to understand the uncertainty that firms face.

[Section III](#) takes up forecasting, one of the most basic inputs to financial planning models. [Chapters 7 through 9](#), are a primer on forecasting methods. They give the prospective model builder a set of basic tools for developing the forecasts that go into their models.

[Section IV](#) expands the simple model of the firm developed in [Chapters 4 and 5](#). [Chapter 10](#) shows how to model the firm's capital investment decisions; [Chapter 11](#) shows how to model the firm's financing decisions; and [Chapter 12](#) shows how to model working capital accounts.

[Section V](#) considers models of security prices and investment portfolios. [Chapter 13](#) shows how to model stock prices; [Chapter 14](#) shows how to construct efficient portfolios or risky assets; and [Chapter 15](#) shows how to model option prices.

The last section of this book, [Section VI](#), considers financial optimization models. It introduces several optimization methods and then develops models for optimizing the firm's capital investment and working capital decisions.

If you have the time and are so diligent as to work your way through these chapters, you will have been exposed to a wide variety of financial models. All are practical and useful. Along the way, you will have learned not just about models, but about finance. Indeed, our students consistently comment about how much they learned about the theory of finance, how building models helped them to understand how the ideas of finance fit together, how the parts of the firm fit together, and how the theory relates to the practical world. We certainly hope you will gain this same perspective from the following chapters.

I

Tools for Financial Planning and Modeling: Financial Analysis



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The Tools for Financial Planning

Financial Analysis

2.1 INTRODUCTION

Suppose you are planning your trip and laying out your route. If so, you need to know your starting point, your strengths and weaknesses, and the resources you have to support your trip. In addition, once you start the trip, you will need some criteria for judging your progress toward your destination. In other words, you need signposts along the way to tell whether you are on the right road and whether you are making acceptable progress. Financial planning is no different. Financial analysis provides methods and tools for assessing your current condition and strengths and weaknesses and for understanding how you got to that condition, and provides data that describe the starting point for the trip into the future. This chapter and the next chapter explain some of the basic tools of financial analysis that are useful for financial planning and modeling.

The financial planning tools that we will discuss in this chapter are financial analysis:

- Analysis of financial ratios
- Break-even analysis
- Analysis of operating and financial leverage

[Chapter 3](#) adds to our planning toolbox with explanations of budgeting and planning:

- Cash Flow
- Cash budgeting

- Sustainable growth
- Generation of pro forma financial statements

We assume you have some familiarity with the tools of financial analysis that we discuss, and so present the methods here as a refresher. More detailed discussions can be found in Brigham and Ehrhardt (2017), Foster (1986), Helfert (2001), and Higgins (2012).

2.2 FINANCIAL RATIO ANALYSIS

In this section, we engage in our first modeling effort. The modeling objective is to construct a set of template models into which we can insert data from any target company, so as to generate output spreadsheets that enable us to analyze the financial ratios of the target company and compare its performance to its industry. Financial ratios are important in financial modeling for two reasons. First, they provide guidelines for judging performance and identifying sources of strength and weakness in the firm's performance. Second, we use financial ratios as input parameters in the planning model of the firm that we will develop in [Chapter 4](#).

The template system consists of several sheets in an Excel workbook as shown in [Exhibit 2.1](#). This workbook is structured so that we enter the data for the target company and related industry data in the input spreadsheets. The output spreadsheets then process the data so that we can analyze the company, compare it to its industry, and obtain data for our planning model. In the following example, we enter the data from the target company's financial statements into sheets 1, 2, and 3. Sheets 4 and 5 process the data into a format suitable for our needs. To analyze another company, simply replace the first company's data with data for the second company in the input sheets and read the results for the second company in the output sheets.

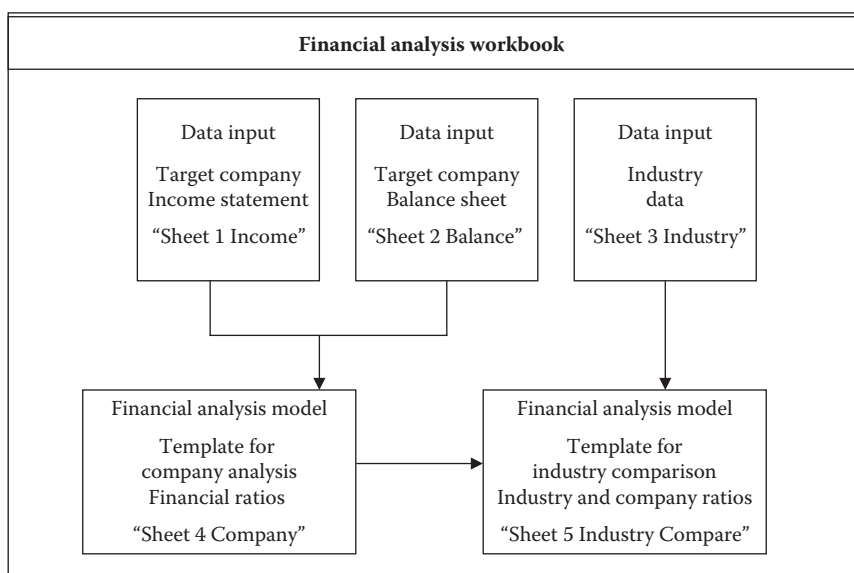


EXHIBIT 2.1 Spreadsheet workbook for financial analysis.

	A	B	C	D	E	F
1	DATA INPUT					
2	User enters Income Statement data for					
3	target company					
4						
5						
6	ODD & RICH CORP					
7	Income Statement					
8	(\$ Millions)					
9		2012	2013	2014	2015	2016
10		-4	-3	-2	-1	0
11						
12	Sales	748.5	792.6	800.2	944.0	1,000.0
13	Cost of Goods Sold	470.1	496.7	504.8	566.7	607.7
14	Gross Profit	278.4	295.9	295.5	377.3	392.3
15	Selling, General & Administrative	123.4	130.1	131.1	169.3	122.3
16	Operating Income Before Deprec.	155.1	165.8	164.4	208.0	270.0
17	Depreciation	57.8	53.6	54.3	49.9	53.8
18	Operating Profit	97.3	112.2	110.1	158.1	216.2
19	Interest Expense	12.8	3.6	18.7	3.1	3.4
20	Non-Operating Income/Expense & Special Items	(2.2)	2.7	6.8	0.0	0.0
21	Pretax Income	82.3	111.3	98.2	155.0	212.8
22	Income Tax	29.7	29.8	26.4	62.0	85.0
23	Net Income Before Extraordinary Items	52.7	81.5	71.8	93.0	127.8
24	Extraordinary Items	0.0	0.0	0.0	0.0	0.0
25	Net Income	52.7	81.5	71.8	93.0	127.8
26						
27	Dividends	21.1	32.6	43.1	55.8	76.7
28	Retained Earnings	31.6	48.9	28.7	37.2	51.1

EXHIBIT 2.2 Input data “Sheet 1 Income.” O&R Corp. income statement.

2.2.1 Example: The Odd & Rich Corporation

The target company for financial analysis and subsequent modeling will be the Odd & Rich Corporation (O&R). O&R is an industrial firm that manufactures a range of plastic components that are used as inputs by other manufacturers of consumer products, automobiles, aircraft, and computers. The manufacturing process involves plastic injection molding and automated assembly. The company uses its own sales force in addition to independent representatives and distributors who sell the product as independent contractors. The company is somewhat cyclical, but it has been growing at a respectable rate in recent years, and it is expected that its sales will continue to grow in the foreseeable future.

Assume the current date is year-end 2016, we have gathered income statement and balance sheet data for O&R for the past five years (2012–2016) and entered it into a spreadsheet as shown in Exhibits 2.2 and 2.3. The data ranges from the earliest year (2012) are designated as year –4 shown on the left to the year just ended (2016) designated as year 0 shown far right. These reports correspond to sheets 1 and 2 referenced in the diagram of Exhibit 2.1. We have also assembled average ratios for other companies in the same industry that we entered into the spreadsheet shown as Exhibit 2.4, which corresponds to the data input sheet 3 in Exhibit 2.1.*

* This industry ratio report is adapted from the sector ratios report of Standard & Poor’s Compustat Research Insight database.

	A	B	C	D	E	F
1	DATA INPUT					
2	User enters Balance Sheet data for					
3	target company					
4						
5						
6	ODD & RICH CORP					
7	Balance Sheet					
8	(\$ Millions)					
9		2012	2013	2014	2015	2016
10		-4	-3	-2	-1	0
11						
12	Assets					
13	Cash & Equivalents	129.1	149.8	143.1	160.4	170.0
14	Net Receivables	169.1	189.1	166.1	177.5	208.1
15	Inventories	127.3	132.0	128.0	142.8	148.2
16	Other Current Assets	119.0	117.6	119.1	189.5	183.7
17	Total Current Assets	544.5	588.6	556.2	670.2	710.0
18	Gross Plant, Property & Equipment	1,142.6	1,200.6	1,238.3	1,356.7	1,517.8
19	Accumulated Depreciation	288.4	342.0	395.6	449.2	502.8
20	Net Plant, Property & Equipment	854.2	858.6	842.7	907.5	1,015.0
21	Other Assets	237.7	280.0	342.9	435.9	409.0
22	Total Assets	1,636.4	1,727.3	1,741.8	2,013.7	2,134.0
23						
24	Liabilities					
25	Long Term Debt Due In One Year	8.7	9.3	11.8	6.9	7.0
26	Notes Payable	0.0	21.5	35.6	0.0	0.0
27	Accounts Payable	152.9	173.5	144.3	175.5	279.1
28	Taxes Payable	18.7	14.0	15.8	41.9	45.3
29	Accrued Expenses & Other Liabilities	117.0	135.0	128.2	342.0	268.6
30	Total Current Liabilities	297.3	353.2	335.7	566.4	600.0
31	Long Term Debt	31.4	26.7	21.8	37.7	73.3
32	Deferred Taxes & Other Liabilities	42.6	33.5	21.5	0.0	0.0
33	Common Stock	20.0	20.0	20.0	20.0	20.0
34	Capital Surplus	80.0	80.0	80.0	80.0	80.0
35	Retained Earnings	1,165.1	1,214.0	1,262.9	1,311.8	1,360.7
36	Less: Treasury Stock	0.0	0.2	0.0	2.1	0.0
37	Common Equity	1,265.1	1,313.8	1,362.9	1,409.6	1,460.7
38	Total Liabilities & Equity	1,636.4	1,727.3	1,741.8	2,013.7	2,134.0

EXHIBIT 2.3 Input data “Sheet 2 Balance.” O&R Corp. balance sheet.

We distinguish between data input and the financial analysis model itself. Sheets 1, 2, and 3 are the data that feed into the financial analysis model that is composed of Sheets 4 and 5 in [Exhibit 2.1](#). The *model* consists of the formulas for calculating financial ratios and the formatting of output, so we have the information in a usable form: Sheets 4 and 5 have no numbers in them—only formulas that operate on the input data from sheets 1, 2, and 3. For example, as shown in [Exhibit 2.5](#), Sales in cell B10 are picked up from the input sheet with the formula =“Sheet 1 Income”!B13, which means that the content of cell B10 is equal to the content of cell B13 on the sheet named “Sheet 1 Income.”

In this case, we want the model to generate financial ratios and other measures that enable us to evaluate the financial performance of the target company. [Exhibit 2.5](#) shows the template equations for the income statement section from “Sheet 4 Company” showing

	A	B	C	D	E	F
1	DATA INPUT					
2	User enters Industry Average Ratios					
3	for target company					
4						
5		Industry Average Ratios				
6		Plastic Products				
7		SIC 3089				
8						
9		2012	2013	2014	2015	2016
10		-4	-3	-2	-1	0
11						
12	Liquidity					
13	Current Ratio	2.3	2.1	2.1	2.2	2.1
14	Quick Ratio	1.4	1.2	1.2	1.2	1.2
15						
16	Activity: Operating Efficiency					
17	Receivables Turnover	6.6	6.7	6.8	8.8	7.9
18	Average Collection Period (Days)	57.9	58.4	54.2	48.4	49.4
19	Inventory Turnover	5.3	5.2	5.8	6.2	6.3
20	Days to Sell Inventory	85.0	90.8	81.5	76.2	75.6
21	Operating Cycle (Days)	142.9	149.1	135.7	124.6	125.1
22	Sales/Net Fixed Assets	3.9	3.7	4.0	4.5	5.3
23	Total Asset Turnover	1.1	1.1	1.1	1.3	1.3
24						
25	Profitability %					
26	EBITDA/Sales	13.8	13.6	12.6	13.8	13.5
27	EBIT/Sales	7.8	7.6	6.8	8.0	8.3
28	Pretax Profit Margin	5.0	0.9	3.4	4.1	5.8
29	Net Profit Margin (NI / Sales)	2.6	(0.7)	2.3	3.0	4.7
30						
31	Return on Investment %					
32	Return on Assets	3.3	(2.7)	0.7	2.9	5.4
33	Return on Equity	9.4	(0.1)	(16.4)	8.6	16.1
34	Return on Invested Capital	4.3	(5.2)	(0.9)	3.5	8.3
35						
36	Leverage					
37	Interest Coverage Before Tax	261.8	129.2	36.2	145.5	67.0
38	Interest Coverage After Tax	231.3	119.3	32.6	98.1	58.5
39	Long-Term Debt/Common Equity	91.8	175.6	456.1	346.1	168.0
40	Total Debt/Invested Capital	64.8	65.9	67.7	60.3	60.3
41	Total Debt/Total Assets	36.6	42.8	41.4	40.5	38.3
42	Total Assets/Common Equity	4.3	4.8	8.6	7.0	4.3
43						
44	Dividends					
45	Dividend Payout (%)	9.4	14.4	11.4	13.9	9.6
46	Dividend Yield (%)	1.0	1.2	1.0	1.2	0.9

EXHIBIT 2.4 Input data “Sheet 3 Industry.” Industry average financial ratios. (From S&P Research Insight Sector Ratio Report.)

	A	B
1		
2		
3		=Sheet 1 Income!A6
4		=Sheet 1 Income!A7
5		=Sheet 1 Income!A8
6		
7		=Sheet 1 Income!B9
8		=Sheet 1 Income!B10
9		
10	Sales	=Sheet 1 Income!B12
11	Cost of Goods Sold	=Sheet 1 Income!B13
12	Selling, General & Admin	=Sheet 1 Income!B15
13	Operating Income Before Deprec	=Sales-CGS-SGA
14	Other Inc/Exp	=Sheet 1 Income!B20
15	EBITDA	=B13+B14
16	Depreciation	=Sheet 1 Income!B17
17	EBIT	=EBITDA-Depr
18	Interest	=Sheet 1 Income!B19
19	EBT	=EBIT-Intr
20	Tax	=Sheet 1 Income!B22
21	Income Before Extraordinary Items	=EBT-Tax
22	Extraordinary Items	=Sheet 1 Income!B24
23	Net Income	=IBEI+ExI
24		
25	Dividends	=Sheet 1 Income!B27
26	Retained Earnings	=NI-Div

EXHIBIT 2.5 Income statement model template data drawn from “Sheet 1 Income.”

that each cell equation points to “Sheet 1 Income,” which is the sheet in which the user entered the basic data. With just the formulas entered in the analysis sheet, when a different company is the target, the user needs to change only the input sheet data. The balance sheet (in cells A29:F59 of “Sheet 4 Company”) is set up in the same manner with each cell formula pointing to the corresponding cell in “Sheet 2 Balance.”

The income statement and balance sheet are repeated on the model sheet to facilitate the computation of the financial ratios that are the main outputs of the company analysis model. There are three ratio sections in the model that are shown as Exhibits 2.6 through 2.9, with the “A” sheets showing the numerical output and the “B” sheets showing the formulas.

The output of sheet 4 in Exhibit 2.6 is shown in two parts. Part 1 summarizes some of the standard financial ratios that help to evaluate the company’s performance. An extra column on the right side computes the average over five years for the various ratios. Part 2 shows the financial statement in “common-size” format where the income data are stated as a percent of sales, and the balance sheet is stated as a percent of total assets. In other words, the income statement ratios are calculated for each line and each year as the income or cost amount divided by that year’s sales. Similarly, the common-size balance sheet section divides each balance sheet item by the total assets for that year.

	A	B	C	D	E	F	G
61	ODD & RICH CORP						
62	Financial Ratios						
63							
64		2012	2013	2014	2015	2016	
65		-4	-3	-2	-1	0	Average
66	Profitability Ratios						
67	Return on Investment						
68	EBITDA / TA	9.3%	9.8%	9.8%	10.3%	12.7%	10.4%
69	EBIT / TA (BEP)	5.8%	6.7%	6.7%	7.9%	10.1%	7.4%
70	NI / TA (ROA)	3.2%	4.7%	4.1%	4.6%	6.0%	4.5%
71	NI / Equity (ROE)	4.2%	6.2%	5.3%	6.6%	8.7%	6.2%
72							
73	Profit Margin						
74	Gross Profit Margin	37.2%	37.3%	36.9%	40.0%	39.2%	38.1%
75	CGS / Sales	62.8%	62.7%	63.1%	60.0%	60.8%	61.9%
76	SGA / Sales	16.5%	16.4%	16.4%	17.9%	12.2%	15.9%
77	Op Cost / Sales	79.3%	79.1%	79.5%	78.0%	73.0%	77.8%
78	EBITDA / Sales	20.4%	21.3%	21.4%	22.0%	27.0%	22.4%
79	Deprec / Sales	7.7%	6.8%	6.8%	5.3%	5.4%	6.4%
80	EBIT / Sales	12.7%	14.5%	14.6%	16.7%	21.6%	16.0%
81	Interest / Debt (-1)		9.0%	32.5%	4.5%	7.6%	13.4%
82	Tax Rate	36.0%	26.8%	26.9%	40.0%	39.9%	33.9%
83	NI bEI / Sales	7.04%	10.28%	8.97%	9.85%	12.78%	9.8%
84	NI / Sales	7.04%	10.28%	8.97%	9.85%	12.78%	9.8%
85							
86	Operating Efficiency						
87	Sales / TA	0.46	0.46	0.46	0.47	0.47	46.3%
88	Sales / NFA	0.88	0.92	0.95	1.04	0.99	95.5%
89	Sales / AR	4.43	4.19	4.82	5.32	4.81	471.2%
90	AR / Sales	22.6%	23.9%	20.8%	18.8%	20.8%	21.4%
91	Receivables Period (days)	81.3	85.9	74.7	67.7	74.9	76.9
92	Inventory Turnover	3.7	3.8	3.9	4.0	4.1	389.3%
93	Inventory Period (days)	97.5	95.7	91.3	90.7	87.8	92.6
94	Payable Period (days)	117.1	125.7	102.9	111.5	165.3	124.5
95	Payable /Sales	20.4%	21.9%	18.0%	18.6%	27.9%	21.4%
96	Operating Period (days)	178.8	181.6	166.0	158.4	162.7	169.5
97	Cash Cycle (days)	61.7	55.8	63.1	46.9	-2.6	45.0
98							
99	Leverage						
100	Total Debt / TA	22.7%	23.9%	21.8%	30.0%	31.6%	26.0%
101	IB Debt / TA	2.5%	3.3%	4.0%	2.2%	3.8%	3.1%
102	Debt / Equity	29.4%	31.5%	27.8%	42.8%	46.1%	35.5%
103	IB Debt / Equity	3.2%	4.4%	5.1%	3.2%	5.5%	4.3%
104	EBIT / Interest	7.44	31.88	6.25	51.00	63.59	3203.2%
105	Total Assets / Equity	1.29	1.31	1.28	1.43	1.46	135.5%
106							
107	Liquidity						
108	CA / CL	1.83	1.67	1.66	1.18	1.18	150.4%
109	(Cash + AR) / CL	1.00	0.96	0.92	0.60	0.63	82.2%
110	Cash / (Op Cst/360)	78.3	86.1	81.0	78.5	83.8	8153.4%
111	Cash / Sales	17.2%	18.9%	17.9%	17.0%	17.0%	17.6%
112							
113	Other Relationships						
114	Dividend Payout	40.0%	40.0%	60.0%	60.0%	60.0%	52.0%

(a)

EXHIBIT 2.6 (a) Financial section of financial analysis model template “Sheet 4 Company.”
(Continued)

	A	B	C	G
61	=D29			
62	Financial Ratios			
63				
64		=B33	=C33	
65		=B34	=C34	Average
66	Profitability Ratios			
67	Return on Investment			
68	EBITDA / TA	=EBITDA/TA	=C15/C46	=AVERAGE(B68:F68)
69	EBIT / TA (BEP)	=EBIT/TA	=C17/C46	=AVERAGE(B69:F69)
70	NI / TA (ROA)	=NI/TA	=C23/C46	=AVERAGE(B70:F70)
71	NI / Equity (ROE)	=NI/Equity	=C23/C58	=AVERAGE(B71:F71)
72				
73	Profit Margin			
74	Gross Profit Margin	=(Sales-CGS)/Sales	=(C10-C11)/C10	=AVERAGE(B74:F74)
75	CGS / Sales	=CGS/Sales	=C11/C10	=AVERAGE(B75:F75)
76	SGA / Sales	=SGA/Sales	=C12/C10	=AVERAGE(B76:F76)
77	Op Cost / Sales	=B75+B76	=C75+C76	=AVERAGE(B77:F77)
78	EBITDA / Sales	=EBITDA/Sales	=C15/C10	=AVERAGE(B78:F78)
79	Deprec / Sales	=Depr/Sales	=C16/C10	=AVERAGE(B79:F79)
80	EBIT / Sales	=EBIT/Sales	=C17/C10	=AVERAGE(B80:F80)
81	Interest / Debt (-1)		=C18/(STD+LTD)	=AVERAGE(B81:F81)
82	Tax Rate	=Tax/EBT	=C20/C19	=AVERAGE(B82:F82)
83	NI BEI / Sales	=IBEI/Sales	=C21/C10	=AVERAGE(B83:F83)
84	NI / Sales	=NI/Sales	=C23/C10	=AVERAGE(B84:F84)
85				
86	Operating Efficiency			
87	Sales / TA	=Sales/TA	=C10/C46	=AVERAGE(B87:F87)
88	Sales / NFA	=Sales/NFA	=C10/C44	=AVERAGE(B88:F88)
89	Sales / AR	=Sales/AR	=C10/C37	=AVERAGE(B89:F89)
90	AR / Sales	=AR/Sales	=C37/C10	=AVERAGE(B90:F90)
91	Receivables Period (days)	=360/B89	=360/C89	=AVERAGE(B91:F91)
92	Inventory Turnover	=CGS/Invent	=C11/C38	=AVERAGE(B92:F92)
93	Inventory Period (days)	=360/B92	=360/C92	=AVERAGE(B93:F93)
94	Payable Period (days)	=AP/(CGS/360)	=C49/(C11/360)	=AVERAGE(B94:F94)
95	Payable / Sales	=AP/Sales	=C49/C10	=AVERAGE(B95:F95)
96	Operating Period (days)	=B91+B93	=C91+C93	=AVERAGE(B96:F96)
97	Cash Cycle (days)	=B96-B94	=C96-C94	=AVERAGE(B97:F97)
98				
99	Leverage			
100	Total Debt / TA	=(CL+LTD+OLTLiab)/TA	=(C52+C54+C55)/C46	=AVERAGE(B100:F100)
101	IB Debt / TA	=(STD+LTD)/TA	=(C50+C54)/C46	=AVERAGE(B101:F101)
102	Debt / Equity	=(CL+LTD+OLTLiab)/Equity	=(C52+C54+C55)/C58	=AVERAGE(B102:F102)
103	IB Debt / Equity	=(STD+LTD)/Equity	=(C50+C54)/C58	=AVERAGE(B103:F103)
104	EBIT / Interest	=EBIT/Intr	=C17/C18	=AVERAGE(B104:F104)
105	Total Assets / Equity	=TA/Equity	=C46/C58	=AVERAGE(B105:F105)
106				
107	Liquidity			
108	CA / CL	=CA/CL	=C40/C52	=AVERAGE(B108:F108)
109	(Cash + AR) / CL	=(Csh+AR)/CL	=(C36+C37)/C52	=AVERAGE(B109:F109)
110	Cash / (Op Cst/360)	=Csh/((CGS+SGA)/360)	=C36/((C11+C12)/360)	=AVERAGE(B110:F110)
111	Cash / Sales	=Csh/Sales	=C36/C10	=AVERAGE(B111:F111)
112				
113	Other Relationships			
114	Dividend Payout	=Div/NI	=C25/C23	=AVERAGE(B114:F114)

(b)

EXHIBIT 2.6 (Continued) (b) Formulas for first two columns and averages of financial section "Sheet 4 Company."

	I	J	K	L	M	N	O
1							
2							
3		ODD & RICH CORP					
4		Income Statement					
5		Common Size: % of Sales					
6							
7		2012	2013	2014	2015	2016	
8		-4	-3	-2	-1	0	Average
9							
10	Sales	100.0%	100.0%	100.0%	100.0%	100.0%	
11	Cost of Goods Sold	62.8%	62.7%	63.1%	60.0%	60.8%	61.87%
12	Selling, General & Admin	16.5%	16.4%	16.4%	17.9%	12.2%	15.89%
13	Operating Income Before Deprec	20.7%	20.9%	20.5%	22.0%	27.0%	22.24%
14	Other Inc/Exp	-0.3%	0.3%	0.9%	0.0%	0.0%	0.18%
15	EBITDA	20.4%	21.3%	21.4%	22.0%	27.0%	22.42%
16	Depreciation	7.7%	6.8%	6.8%	5.3%	5.4%	6.39%
17	EBIT	12.7%	14.5%	14.6%	16.7%	21.6%	16.04%
18	Interest	1.7%	0.5%	2.3%	0.3%	0.3%	1.03%
19	EBT	11.0%	14.0%	12.3%	16.4%	21.3%	15.00%
20	Tax	4.0%	3.8%	3.3%	6.6%	8.5%	5.22%
21	Income Before Extraordinary Items	7.0%	10.3%	9.0%	9.9%	12.8%	9.78%
22	Extraordinary Items	0.0%	0.0%	0.0%	0.0%	0.0%	0.00%
23	Net Income	7.0%	10.3%	9.0%	9.9%	12.8%	9.78%
24							
25	Dividends	2.8%	4.1%	5.4%	5.9%	7.7%	5.18%
26	Retained Earnings	4.2%	6.2%	3.6%	3.9%	5.1%	4.61%

(a)

	H	I	J	K
1				
2				
3		=B3		
4		=B4		
5		Common Size: % of Sales		
6				
7		=B7	=C7	
8		=B8	=C8	
9				
10		=A10	=Sales/Sales	=C10/C10
11		=A11	=CGS/Sales	=C11/C\$10
12		=A12	=SGA/Sales	=C12/C\$10
13		=A13	=B13/Sales	=C13/C\$10
14		=A14	=B14/Sales	=C14/C\$10
15		=A15	=EBITDA/Sales	=C15/C\$10
16		=A16	=Depr/Sales	=C16/C\$10
17		=A17	=EBIT/Sales	=C17/C\$10
18		=A18	=Intr/Sales	=C18/C\$10
19		=A19	=EBT/Sales	=C19/C\$10
20		=A20	=Tax/Sales	=C20/C\$10
21		=A21	=BEI/Sales	=C21/C\$10
22		=A22	=ExI/Sales	=C22/C\$10
23		=A23	=NI/Sales	=C23/C\$10
24				
25		=A25	=Div/Sales	=C25/C\$10
26		=A26	=RE/Sales	=C26/C\$10

(b)

EXHIBIT 2.7 (a) Target company common-size percent of sales income statement “Sheet 4 Company.” (b) Formulas for the first two columns and averages of common-size income statement section “Sheet 4 Company.”

Two ways to gain perspective from the financial ratios are (1) to look at the trend of ratios over time, and (2) to compare them to other companies in the same industry and with the average ratios for the industry. The trend of the ratios over time is a good starting point for our analysis because significant changes or trends in ratios frequently signal potential problems or promising improvements. Although scanning across the line of ratios may quickly alert you to changes that may be important, graphing the trend of ratios is an even more effective means to highlight the trends. For example, [Exhibit 2.10](#) shows an Excel graph of O&R's profit margins clearly showing an upward trend. The picture of this trend helps us to quickly see one of the important aspects of O&R's operations. Similarly, pictures of the trends of other ratios can help us to visualize what is happening to the firm. The usefulness of your financial analysis model can be enhanced by including graphs of the more important ratios.

	I	J	K	L	M	N	O
29	ODD & RICH CORP						
30	Balance Sheet						
31	Common Size: % of Total Assets						
32							
33		2012	2013	2014	2015	2016	
34		-4	-3	-2	-1	0	Average
35	Assets						
36	Cash	7.9%	8.7%	8.2%	8.0%	8.0%	8.1%
37	Accounts Receivable	10.3%	10.9%	9.5%	8.8%	9.8%	9.9%
38	Inventory	7.8%	7.6%	7.3%	7.1%	6.9%	7.4%
39	Other CA	7.3%	6.8%	6.8%	9.4%	8.6%	7.8%
40	Current Assets	33.3%	34.1%	31.9%	33.3%	33.3%	33.2%
41							
42	Gross Fixed Assets	69.8%	69.5%	71.1%	67.4%	71.1%	69.8%
43	Accum Deprec	17.6%	19.8%	22.7%	22.3%	23.6%	21.2%
44	Net Fixed Assets	52.2%	49.7%	48.4%	45.1%	47.6%	48.6%
45	Other Long Term Assets	14.5%	16.2%	19.7%	21.6%	19.2%	18.2%
46	Total Assets	100.0%	100.0%	100.0%	100.0%	100.0%	
47							
48	Liabilities						
49	Accounts Payable	9.3%	10.0%	8.3%	8.7%	13.1%	9.9%
50	Short Term Debt	0.5%	1.8%	2.7%	0.3%	0.3%	1.1%
51	Other CL	8.3%	8.6%	8.3%	19.1%	14.7%	11.8%
52	Current Liabilities	18.2%	20.5%	19.3%	28.1%	28.1%	22.8%
53							
54	Long Term Debt	1.9%	1.5%	1.2%	1.9%	3.4%	2.0%
55	Other LT Liabilities	2.6%	1.9%	1.2%	0.0%	0.0%	1.2%
56	Common Stock	6.1%	5.8%	5.7%	4.9%	4.7%	5.4%
57	Earned Surplus	71.2%	70.3%	72.5%	65.1%	63.8%	68.6%
58	Common Equity	77.3%	76.1%	78.2%	70.0%	68.4%	74.0%
59	Total Liabilities & Equity	100.0%	100.0%	100.0%	100.0%	100.0%	

(a)

EXHIBIT 2.8 (a) Target company common-size balance sheet percent of total assets "Sheet 4 Company."

(Continued)

	I	J	K
29	=D29		
30	=B30		
31	Common Size: % of Total Assets		
32			
33		=J7	=K7
34		=B34	=C34
35	=A35		
36	=A36	=Csh/TA	=C36/C\$46
37	=A37	=AR/TA	=C37/C\$46
38	=A38	=Invent/TA	=C38/C\$46
39	=A39	=OCA/TA	=C39/C\$46
40	=A40	=CA/TA	=C40/C\$46
41			
42	=A42	=GFA/TA	=C42/C\$46
43	=A43	=AccumDep/TA	=C43/C\$46
44	=A44	=NFA/TA	=C44/C\$46
45	=A45	=OAssets/TA	=C45/C\$46
46	=A46	=TA/TA	=C46/C\$46
47			
48	=A48		
49	=A49	=AP/TA	=C49/C\$46
50	=A50	=STD/TA	=C50/C\$46
51	=A51	=OCL/TA	=C51/C\$46
52	=A52	=CL/TA	=C52/C\$46
53			
54	=A54	=LTD/TA	=C54/C\$46
55	=A55	=OLTLiab/TA	=C55/C\$46
56	=A56	=CS/TA	=C56/C\$46
57	=A57	=ErnSrpls/TA	=C57/C\$46
58	=A58	=Equity/TA	=C58/C\$46
59	=A59	=TLiab/TA	=C59/C\$46

(b)

EXHIBIT 2.8 (Continued) (b) Formulas for first two columns and averages of common-size balance sheet section “Sheet 4 Company.”

Ratio analysis is also used to compare a firm to its industry. [Exhibit 2.11](#) compares our target company with average ratios for the industry* taken from the input data sheet 3. When comparing a company with its industry, both the industry and company ratios must be computed in the same manner. The average ratio for the industry simply gives us something to which we can compare our company’s ratio. However, keep in mind that the industry average is only an indication of what other companies are doing. It does not mean that the other companies are doing it right. Whole industries can be unhealthy, making their averages nothing to aspire to. Nevertheless, using these averages provides a starting point to gain perspective about how our target company is doing relative to others in its industry.

* In this example, the company and industry ratios were based on data from Standard & Poor’s Research Insight database and put into the Research Insight format. Many of the ratios in [Exhibit 2.11](#) were computed with definitions described in the Research Insight documentation. For example, Return on Assets is often computed as $EBIT/TA$, but Research Insight computes it as NI/TA , so for comparison on sheet 5, ROA is computed with this latter definition.

Look for indications that the company is performing either better or worse than the industry average. Also look for trends and significant changes. Scanning for significant differences between the company and industry and watching for trends can alert us to potential problems or sources of strengths for our company. For example, the industry average net profit margin ranged from a low of -0.74% to a high of 4.68% , with a recent upward trend. O&R's margin has been well above the industry and is trending upward. This is clearly a very positive indicator for O&R. On the other hand, O&R's total asset turnover has averaged about 0.45 with little trend, whereas the industry has enjoyed a turnover almost three times as high at an average of about 1.2, with clear improvement over the last five years. This indicates a major weakness in O&R's performance. After this preliminary examination of the company and industry ratio and trends, we decompose our company's performance using DuPont analysis.

2.2.2 DuPont Analysis

The main purpose of financial analysis is to find and understand the strengths and weaknesses of the company. DuPont Analysis provides a remarkably useful structure for this analysis. It decomposes the return on equity into three components: the net profit margin,

	I	J	K	L	M	N	O
62	Assets & Liabilities as a Percent of Sales						
63							
64		2012	2013	2014	2015	2016	
65		-4	-3	-2	-1	0	Average
66	Assets						
67	Cash / Sales	17.2%	18.9%	17.9%	17.0%	17.0%	17.6%
68	Accounts Receivable / Sales	22.6%	23.9%	20.8%	18.8%	20.8%	21.4%
69	Inventory / Sales	17.0%	16.7%	16.0%	15.1%	14.8%	15.9%
70	Other CA / Sales	15.9%	14.8%	14.9%	20.1%	18.4%	16.8%
71	Current Assets / Sales	72.7%	74.3%	69.5%	71.0%	71.0%	71.7%
72							
73	Gross Fixed Assets / Sales	152.7%	151.5%	154.7%	143.7%	151.8%	150.9%
74	Accum Deprec / Sales	38.5%	43.2%	49.4%	47.6%	50.3%	45.8%
75	Net Fixed Assets / Sales	114.1%	108.3%	105.3%	96.1%	101.5%	105.1%
76	Other Long Term Assets / Sales	31.8%	35.3%	42.9%	46.2%	40.9%	39.4%
77	Total Assets / Sales	218.6%	217.9%	217.7%	213.3%	213.4%	216.2%
78							
79	Liabilities						
80	Accounts Payable / Sales	20.4%	21.9%	18.0%	18.6%	27.9%	21.4%
81	Short Term Debt / Sales	1.2%	3.9%	5.9%	0.7%	0.7%	2.5%
82	Other CL / Sales	18.1%	18.8%	18.0%	40.7%	31.4%	25.4%
83	Current Liabilities / Sales	39.7%	44.6%	41.9%	60.0%	60.0%	49.2%
84							
85	Long Term Debt / Sales	4.2%	3.4%	2.7%	4.0%	7.3%	4.3%
86	Other LT Liabilities / Sales	5.7%	4.2%	2.7%	0.0%	0.0%	2.5%
87	Common Stock / Sales	13.4%	12.6%	12.5%	10.4%	10.0%	11.8%
88	Earned Surplus / Sales	155.7%	153.2%	157.8%	139.0%	136.1%	148.3%
89	Common Equity / Sales	169.0%	165.8%	170.3%	149.3%	146.1%	160.1%
90	Total Liabilities & Equity / Sales	218.6%	217.9%	217.7%	213.3%	213.4%	216.2%
91							
92	Other Relationships						
93	Deprec / GFA ₋₁		4.7%	4.5%	4.0%	4.0%	4.3%
94	Deprec / NFA ₋₁		6.3%	6.3%	5.9%	5.9%	6.1%

(a)

EXHIBIT 2.9 (a) Target company percent of sales balance sheet and liabilities "Sheet 4 Company."
(Continued)

	I	J	K
62	Assets & Liabilities as a Percent of Sales		
63			
64		=B64	=C64
65		=B65	=C65
66	=A35		
67	Cash / Sales	=Csh/Sales	=C36/C\$10
68	Accounts Receivable / Sales	=AR/Sales	=C37/C\$10
69	Inventory / Sales	=Invent / Sales	=C38/C\$10
70	Other CA / Sales	=OCA/Sales	=C39/C\$10
71	Current Assets / Sales	=CA/Sales	=C40/C\$10
72			
73	Gross Fixed Assets / Sales	=GFA/Sales	=C42/C\$10
74	Accum Deprec / Sales	=AccumDep/Sales	=C43/C\$10
75	Net Fixed Assets / Sales	=NFA/Sales	=C44/C\$10
76	Other Long Term Assets / Sales	=OAssets/Sales	=C45/C\$10
77	Total Assets / Sales	=TA/Sales	=C46/C\$10
78			
79	Liabilities		
80	Accounts Payable / Sales	=AP/Sales	=C49/C\$10
81	Short Term Debt / Sales	=STD/Sales	=C50/C\$10
82	Other CL / Sales	=OCL/Sales	=C51/C\$10
83	Current Liabilities / Sales	=CL/Sales	=C52/C\$10
84			
85	Long Term Debt / Sales	=LTD/Sales	=C54/C\$10
86	Other LT Liabilities / Sales	=OLTLiab/Sales	=C55/C\$10
87	Common Stock / Sales	=CS/Sales	=C56/C\$10
88	Earned Surplus / Sales	=ErnSrpls/Sales	=C57/C\$10
89	Common Equity / Sales	=Equity/Sales	=C58/C\$10
90	Total Liabilities & Equity / Sales	=TLiab/Sales	=C59/C\$10
91			
92	Other Relationships		
93	Deprec / GFA ₋₁		=C16/GFA
94	Deprec / NFA ₋₁		=C16/NFA

(b)

EXHIBIT 2.9 (Continued) (b) Formulas for first two columns of percent of sales balance sheet “Sheet 4 Company.”

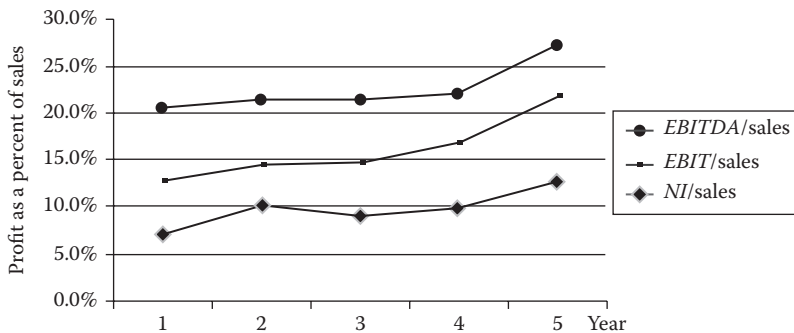


EXHIBIT 2.10 Excel chart of O&R profit margins.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q						
	Company		Industry		Year -3		Year -2		Year -1		Most Recent Year		Year -4		Year -3		Year -2		Year -1		Most Recent Year		
	Comparison of Company and Industry Performance Ratios											Comparison of Company and Industry Performance Ratios											
1	Company	ODD & RICH CORP																					
2	Industry	Plastic Products																					
3	SIC	3089																					
4																							
5																							
6	Profitability %																						
7	EBITDA/Sales	Company	20.72%	20.92%	20.54%	22.03%	13.80	13.51															
8		Industry	13.79	13.57	12.64	13.80	13.80	13.51															
9																							
10	EBIT/Sales	Company	13.00%	14.16%	13.75%	16.75%	8.01	8.29															
11		Industry	7.83	7.64	6.77	8.01	8.29																
12																							
13	Pretax Profit Margin	Company	11.00%	14.05%	12.27%	16.42%	21.28%	5.79															
14		Industry	4.97	0.90	3.39	4.08	5.79																
15																							
16	Net Profit Margin (NI / Sales)	Company	7.04%	10.28%	8.97%	9.85%	12.78%																
17		Industry	2.55	-0.74	2.29	3.00	4.68																
18																							
19	Leverage																						
20	Long-Term Debt/Common Equity	Company	0.02	0.02	0.02	0.03	0.05																
21		Industry	91.75	175.60	456.08	346.12	167.99																
22																							
23	Total Debt/Invested Capital	Company	3.1%	4.2%	4.8%	3.1%	5.2%																
24		Industry	64.84	65.93	67.70	60.27	60.35																
25																							
26	Total Debt/Total Assets	Company	2.5%	3.3%	4.0%	2.2%	3.8%																
27		Industry	36.59	42.79	41.36	40.47	38.35																
28																							
29	Activity: Operating Efficiency																						
30	Inventory Turnover	Company	3.69	3.76	3.94	3.97	4.10																
31		Industry	5.28	5.16	5.81	6.22	6.31																
32																							
33	Receivables Turnover	Company	4.43	4.19	4.82	5.32	4.81																
34		Industry	6.64	6.66	6.83	8.84	7.92																
35																							
36	Total Asset Turnover	Company	0.46	0.46	0.46	0.47	0.47																
37		Industry	1.13	1.12	1.14	1.30	1.30																
38																							
39	Dividends																						
40	Dividend Payout (%)	Company	40.0%	40.0%	60.0%	60.0%	60.0%																
41		Industry	9.35	14.41	11.40	13.86	9.55																

EXHIBIT 2.11 "Sheet 5 Output." Comparison of company and industry ratios.

total asset turnover, and the equity multiplier. This approach provides a systematic way to pinpoint the sources of strength and weakness in a firm's operations. The DuPont format is

$$\begin{aligned} \text{Return on Equity} &= (\text{Net Profit Margin}) \times (\text{Total Asset Turnover}) \\ &\quad \times (\text{Equity Multiplier}) \\ \left(\frac{\text{Net Income}}{\text{Equity}} \right) &= \left(\frac{\text{Net Income}}{\text{Sales}} \right) \times \left(\frac{\text{Sales}}{\text{Total Assets}} \right) \times \left(\frac{\text{Total Assets}}{\text{Equity}} \right) \end{aligned} \quad (2.1)$$

Note the relationship of the three components to financial statements. The net profit margin reflects the income statement, total asset turnover—the left (asset) side of the balance sheet, and the equity multiplier—the right (liabilities) side of the balance sheet. If the analyst needs to dig deeper into a particular ratio, these links indicate where to start the excavation process.

For Odd & Rich (O&R) in the most recent year (2016), this is calculated as

$$\begin{aligned} \left(\frac{127.8}{1,460.7} \right) &= \left(\frac{127.8}{1,000} \right) \times \left(\frac{1,000}{2,134} \right) \times \left(\frac{2,134}{1,460.7} \right) \\ 8.75\% &= 12.78\% \times 0.469 \times 1.46 \end{aligned} \quad (2.2)$$

These ratios are not particularly meaningful without some benchmark for comparison. Industry ratios provide such benchmarks. For the industry in which O&R competes, these ratios are*

$$\begin{aligned} \text{Return on Equity} &= \text{Profit Margin} \times \text{Total Asset Turnover} \times \text{Equity Multiplier} \\ 16.1\% &= 4.7\% \times 1.3 \times 4.3 \end{aligned} \quad (2.3)$$

O&R's return on equity of 8.7% is well below the industry average of about 16%, so we would like to know why O&R is not doing nearly as well as the average company in its industry.

The first component we consider is net profit margin. With a net profit margin of 12.8%, O&R is earning 12.8¢ for each \$1.00 of sales, almost three times higher than the industry average of 4.7¢. The second component is asset turnover. At 0.47 times per year, O&R's asset turnover is very low compared to the industry average of about 1.3. The third component is the equity multiplier. An equity multiplier of 1.46 is about a third of the industry average of 4.3. We see immediately that in spite of generating a much higher profit margin than the industry, O&R earned a lower ROE because both its asset turnover and its equity multiplier were considerably lower than the industry. O&R uses its assets less efficiently and less leverage than its industry.

Now it is time to dig a little deeper to understand the sources of O&R's low ROE. To do that, we need to consider what affects these three major performance factors.

* The product of the three industry average component ratios does not equal the industry average return on equity given Exhibit 2.4 because the industry average data may exclude some companies because of missing data or negative numbers.

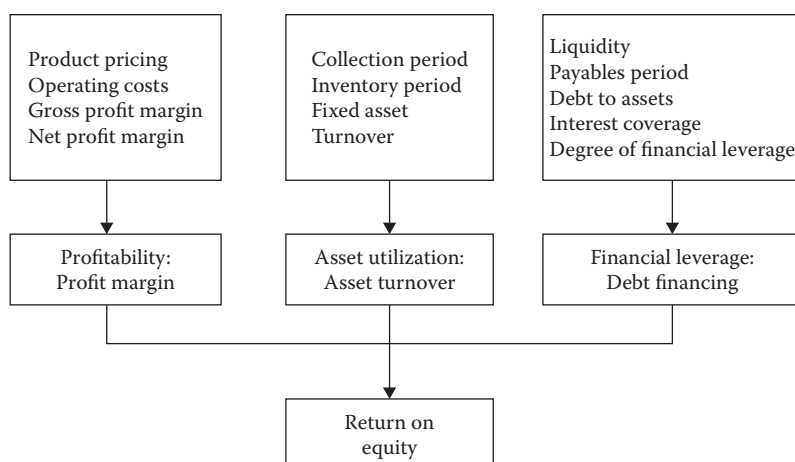


EXHIBIT 2.12 DuPont format: The three factors of performance.

Exhibit 2.12 shows selected performance measures that link to these three factors and to return on equity and suggests the ratios we should consider further to understand O&R's performance.

2.2.2.1 Pricing and Costs

As O&R's profit margin is very good, we do not need to be too concerned with it. However, had it been too low, we would have needed to dig into its components as suggested by the diagram. We would examine the firm's pricing structure to see if it is in line with the industry, and what factors such as product demand and the competitive structure of the industry would prevent the firm from increasing its prices. Next, we would look at the cost structure using the common-size income statement that shows cost as a percent of sales. Are the various cost categories such as costs of goods sold (CGS), selling, general and administrative expenses (SGA), depreciation, and interest in line with industry cost ratios? What has been their trend? If any one of these general cost categories seems to be out of line, we dig deeper to identify the component of that cost which is the culprit.

2.2.2.2 Asset Utilization

The second component in the DuPont format is asset turnover, which we now know is much lower than the industry average. Total asset turnover measures the efficiency with which all assets are being used. Is the firm generating too few sales given its investment in assets, or, to put it another way, has it invested too much in assets given its level of sales? Whichever way we state it, the company is not getting as much out of its investment in assets as other companies in the industry. However, total asset turnover only gives us a clue to the problem. We need to dig deeper to see which type of asset is being used inefficiently. This entails looking at the turnover and other measures of efficiency and activity for each major asset category. In O&R's case, these main categories are accounts receivable,

	Receivables	Inventory	Net fixed assets
Formula	$\frac{\text{Sales}}{\text{Accounts receivable}}$	$\frac{\text{Cost of goods sold}}{\text{Inventory}}$	$\frac{\text{Sales}}{\text{Net fixed assets}}$
O&R	$\frac{1000}{208.1} = 4.8$	$\frac{607.7}{148.2} = 4.1$	$\frac{1000}{1015} = 0.99$
Industry	7.9	6.3	2.3

EXHIBIT 2.13 O&R Corp. rates of asset turnover.

inventories, and fixed assets. Exhibit 2.13 shows the turnover rates for each category for both the company and its industry.

In each category, O&R' turnover rates are much lower than the industry, suggesting that O&R has too much money tied up in each of these categories. The lower turnover of inventories and receivables indicates that the company is carrying items on its shelves longer than the industry before they are sold and is then taking too long to collect the receivables from its customers after they are sold. The lower turnover of fixed assets shows that the company invests proportionally more in plant and equipment (fixtures, equipment, and real estate) than others in the industry. Thus, all three categories are problems for this company.

2.2.2.3 Operating Cycle

You can gain a better perspective of the company's management of its inventory and receivables by examining the operating cycle. The operating cycle is the average length of time between inventory being purchased from a supplier and cash being collected from a customer. This length of time consists of the inventory period and the receivables period. Both are directly related to asset turnover—the lower the turnover ratio, the longer the cycle period.

Inventory period is the average time that an item is held in inventory from the date of purchase to the date of sale. It is calculated as

$$\begin{aligned}
 \text{Inventory Period} &= \frac{\text{Inventory}}{\text{Cost per day}} \\
 &= \frac{\text{Inventory}}{\left(\frac{\text{Cost of Goods Sold}}{360 \text{ days}} \right)} \\
 &= \frac{148.2}{\left(\frac{607.7}{360 \text{ days}} \right)} = \frac{148.2}{1.69} = 87.7 \text{ days}
 \end{aligned} \tag{2.4}$$

or

$$= \frac{360 \text{ days}}{\text{Inventory Turnover}} = \frac{360 \text{ days}}{4.1} = 87.8 \text{ days}$$

On the average, O&R is carrying items on its shelves for 87.8 days, as compared to the industry average of about 76 days.

After selling an item, the company must collect the cash from the customer. If no credit is granted, the cash is collected immediately. But if the item is sold on credit, it takes time to collect from the customer. The average period of time it takes to collect from the customer is called the receivables period (or collection period). This is calculated as

$$\begin{aligned} \text{Receivables Period} &= \frac{\text{Receivables}}{\text{Sales per day}} = \frac{\text{Receivables}}{\left(\frac{\text{Sales}}{360 \text{ days}}\right)} \\ &= \frac{208.1}{\left(\frac{1000}{360 \text{ days}}\right)} = \frac{208.1}{2.78} = 74.9 \text{ days} \end{aligned} \quad (2.5)$$

or

$$= \frac{360 \text{ days}}{\text{Receivables Turnover}} = \frac{360 \text{ days}}{4.8} = 74.9 \text{ days}$$

It takes O&R an average of 74.9 days to collect from its customers as compared to about 49 days for the industry.

The length of the operating cycle is the time it takes from purchasing the inventory to selling it plus the time it takes from selling an item to collecting the cash. Hence, it is the time between buying the inventory and collecting the cash from selling it is the sum of the inventory period and the receivables period:

$$\begin{aligned} \text{Operating Cycle} &= \text{Inventory Period} + \text{Receivables Period} \\ \text{Operating Cycle}_{\text{O\&R}} &= 87.5 \text{ days} + 74.9 \text{ days} \\ &= 162.7 \text{ days} \end{aligned} \quad (2.6)$$

It takes O&R an average of 162.7 days from the time it purchases the raw materials until the date that it collects cash from the customer, as compared to about 125 days for the industry.

Looking at the turnover of various asset categories shows us that O&R has too much investment tied up in receivables, inventories, and fixed assets. If they could bring their turnover in line with the industry, they would reduce the capital tied up in asset investment and thereby would increase their return on assets and return on equity. There are two additional aspects of O&R's operating cycle and turnover that could be sources of inefficiency. These are cash and accounts payable. We leave it to the reader to dig into these to see if O&R is carrying too much or too little liquidity, or perhaps not managing their trade credit properly.

2.2.2.4 The Equity Multiplier and Financial Leverage

The last component in the DuPont decomposition of the return on equity is the equity multiplier, a measure of financial leverage. The equity multiplier is the ratio of total assets to stockholder's equity. It is the inverse of the equity-to-total assets ratio, and so is related to the use of debt in a firm's capital structure. If a lot of debt is used, total assets will greatly exceed equity and the multiplier and its effect on return on equity will be large. If very little debt is used, the multiplier will be close to one, and its effect on return on equity will be small. Thus, the equity multiplier measures the use of leverage from debt financing. In O&R's case, the equity-to-asset ratio is

$$\begin{aligned}\frac{\text{Stockholders' Equity}}{\text{Total Assets}} &= \frac{1,460.7}{2,134} \\ &= 0.684 \\ &= 68.4\%\end{aligned}\tag{2.7}$$

So 68.4% of the funds used to purchase the firm's assets were supplied by stockholders through purchases of shares of stock and through retained earnings. The remaining 32% of O&R's capital came from sources other than equity.

The equity multiplier tells us the total amount of assets supported by each \$1 of equity and is calculated as

$$\begin{aligned}\text{Equity Multiplier} &= \frac{\text{Total Assets}}{\text{Stockholders' Equity}} \\ &= \frac{2,134}{1,460.7} \\ &= 1.46\end{aligned}\tag{2.8}$$

Each dollar of equity supports the purchase of \$1.46 worth of assets. O&R's equity multiplier of 1.46 is considerably lower than the industry's equity multiplier of 4.3. If the firm used more debt, it could potentially increase its return on equity. However, the very high multiplier for the industry might suggest that other companies in the industry are too highly levered. If so, we would not want to follow their lead if it imposed too much risk.

The problem is that debt is a two-edged sword. Just as more leverage magnifies gains when the firm prospers, it magnifies losses when the firm does poorly. In addition, increasing the equity multiplier may decrease another component in the DuPont format, the profit margin. If the firm uses more debt, it will have to pay more interest. Its net profit margin may decrease as its financial risk increases. Whether the firm is better off or not will depend on the trade-off between the positive effects of leverage on return on equity and the negative effects of leverage on profit margin and financial risk. Analysis of leverage, liquidity ratios, and debt coverage ratios can help us to determine the firm's ability to service the debt.

We start our analysis of O&R's debt with various leverage ratios. These include the total debt ratios and the debt-to-equity ratio. The broadest measure of debt is the total debt ratio that takes into account all the firm's liabilities, and is measured as compared to a much higher debt ratio for the industry of about 77%.

$$\begin{aligned} \text{Debt Ratio} &= \frac{\text{Total Liabilities}}{\text{Total Assets}} \\ &= \frac{673.3}{2,134} \\ &= 31.6\% \end{aligned} \tag{2.9}$$

where:

$$\text{Total Liabilities} = \text{Current Liabilities} + \text{Long-term Debt}$$

O&R's debt ratio means that about 32% of the investment in the firm's assets are financed by debt sources, including liabilities such as trade credit and accruals, as well as interest-bearing debt such as notes payable and long-term debt.

As total liabilities includes many items that are not debt that must be repaid and are unlikely to lead to financial difficulty, we modify our measure of leverage to reflect the type of debt that is of greater concern for our analysis of leverage: interest-bearing debt. Interest-bearing debt includes short-term debt, such as notes payable, and long-term debt. Focusing on the interest-bearing debt (denoted by "ib"), O&R's debt ratio is

$$\begin{aligned} \text{Debt Ratio}_{ib} &= \frac{\text{Short-term Debt} + \text{Long-term Debt}}{\text{Total Assets}} \\ &= \frac{7.1 + 73.3}{2,134} \\ &= 3.8\% \end{aligned} \tag{2.10}$$

An alternative leverage measure emphasizes invested capital. Invested capital refers to permanent capital that was contributed by both lenders and equity investors, and consists of interest-bearing debt and equity:

$$\begin{aligned} \text{Invested Capital} &= \text{Debt}_{ib} + \text{Equity} \\ &= \frac{\text{Short-term Debt}}{7.0} + \frac{\text{Long-term Debt}}{73.3} + \frac{\text{Equity}}{1,460.7} \\ &= \$1,541.0 \end{aligned} \tag{2.11}$$

O&R's invested capital is making O&R's debt-to-invested capital ratio $80.3/1,541 = 5.2\%$, as compared to 60.3% for the industry.

Comparing a company's debt ratios with its industry averages may not tell us much more than that our company uses more or less debt than the average company in the industry. Except in extreme cases, it does not indicate whether the company's situation is good or bad. A couple of problems with evaluating a debt ratio and comparing it with the industry are that (1) the absolute level of the ratio, either for the company or the industry, does not really tell us how much risk is involved; and (2) if we do not know that the industry average is a sensible standard, knowing that the company has more or less debt than the industry average will not necessarily tell us whether the company's position is good or bad.

Interest coverage ratios are more useful for evaluating the risk of the debt position because they measure the cushion of earnings that is available to service the debt. The most common interest coverage ratio, commonly known as times interest earned, is calculated as

$$\begin{aligned} \text{Interest Coverage} &= \frac{\text{EBIT}}{\text{Interest}} \\ &= \frac{216.2}{3.4} \\ &= 63.6 \end{aligned} \tag{2.12}$$

This tells us that the income available to pay interest is almost 64 times greater than the interest obligation. With an industry average of about 67, both the company and the industry have very safe coverage levels in spite of the fact that the industry appears to be highly levered when measured by other ratios.

This brief analysis of O&R pointed out some of the O&R's strengths and weaknesses. We know that O&R has an attractive profit margin, but still earns a lower return on equity than the average of its industry. The reason for this lower return is that the company exhibits poor asset utilization and may be under leveraged. But we have only touched the surface with our analysis. There are many aspects of financial analysis such as the firm's liquidity that we did not discuss simply because our objective is to get you started on modeling rather than teaching all the details of financial analysis. We leave it to you, the reader, to dig deeper into the company with the methods that are discussed in greater depth in the other texts we have referenced. In the next two sections, we will look at additional tools for assessing the risk and performance of the firm.

2.3 BREAK-EVEN ANALYSIS

One significant issue in financial planning is the division of costs between fixed and variable components. This dichotomy has important implications for the firm's ability to operate profitably, as well as for the variability and riskiness of the firm's income. A useful set of tools for financial planning relate to the analysis of fixed and variable costs. These tools are break-even analysis and the analysis of operating and financial leverage. In this section, we discuss break-even analysis.

2.3.1 Fixed and Variable Costs

Before discussing break-even analysis and the measures of operating and financial leverage, we need to recast the income statement in terms of fixed and variable costs. Consider [Exhibit 2.14](#), which displays the income statement for the O&R Corporation in two formats. The first format is the one typically seen in an annual report. Costs are categorized as to the general purpose of the expense, but not as fixed or variable. CGS and selling, general, and administrative expenses include both fixed and variable components. For example, depreciation expense is frequently included in CGS. Also, all costs and expenses are shown with no intermediate calculation of income. In the revised format, expenses are classified as fixed or variable, and income is shown at intermediate levels, such as *EBITDA* and *EBIT*, on its way to the bottom line, net income.

Two questions that arise in financial analysis and financial planning are: What is the break-even point, and how likely is it that the firm will be able to operate above that point? Break-even analysis is the tool that helps to answer these questions. We define the break-even point as the level of output for which revenue just covers all costs. For production processes that involve variable costs only, the firm will be profitable at all output levels so long as the sale price exceeds the variable cost per unit. However, when some costs are fixed and do not vary with the level of output, some minimum level of output is necessary before revenue covers both fixed and variable costs.

In break-even analysis, fixed costs are costs that do not vary (in the short run) with the number of units of product produced or sold; variable costs are costs that change with the volume of production or sales. Examples of fixed costs are salary expenses for personnel

Standard Income statement	
Sales	\$ 1,000.00
Cost of Goods Sold	661.50
Sell, General & Administrative Expenses	122.30
Interest Expense	3.40
Income Tax	85.00
Net Income	<u>\$ 127.80</u>

Revised Format Fixed & Variable Cost	
Sales	\$ 1,000.00
Less:	
Variable Operating Expenses	563.00
Fixed Operating Expenses	167.00
Earnings Before Interest, Taxes, Depreciation and Amortization (EBITDA)	<u>\$ 270.00</u>
Fixed Cost: Depreciation	53.80
Earnings Before Interest & Taxes (EBIT)	<u>\$ 216.20</u>
Interest Expense	3.40
Earnings Before Tax (EBT)	<u>\$ 212.80</u>
Income Tax	85.00
Net Income (NI)	<u>\$ 127.80</u>

EXHIBIT 2.14 O&R Corp. income statements in two formats.

who are employed regardless of the level of production; rental and lease expenses that depend on time but not on sales volume; and, of course, noncash expenses such as depreciation and amortization. Examples of variable costs include sales commissions, labor costs related to the manufacturing process that depend on the volume of output, and the costs of materials used in production. The classification of costs as fixed or variable depends on the time horizon of the analysis. Although all costs are variable in the long run, many costs are independent of sales volume in the short run and are therefore fixed in the short run. The shorter the horizon, the more costs are fixed.

We use the example of the O&R Corporation in our discussion of break-even analysis, and consider *EBITDA* break-even, the level of sales where *EBITDA* will be zero. The first step is to separate fixed from variable costs. O&R Corporation has fixed-cost components in both the *CGS* and selling, general, and administrative costs. These include the costs of management salaries, rent, insurance, property taxes, and so on. Assume that these are about \$167 million per year, excluding depreciation.* The remaining costs are variable. They include items such as the cost of merchandize, direct labor, and sales commissions. The variable costs are estimated to be 56.3% of sales. In terms of units sold, the average item sells for \$10, with an average variable cost of \$5.63. The \$4.37 difference between sale price and variable cost is the contribution margin per unit.

[Exhibit 2.15](#) shows a graph of revenue and costs.† Sales are shown on the horizontal axis; revenue, costs, and profit are shown on the vertical axis. Sales revenue is shown by the dashed line that starts at the origin and slopes upward at a 45° angle. Fixed costs are shown by the flat, horizontal dashed line that intersects the vertical axis at \$167 million. The horizontal line indicates that these costs remain at \$167 million regardless of the number of units sold. Total costs are the sum of fixed and variable costs and are shown as the solid line that slopes upward from \$167 on the vertical axis. The slope of this line is 0.563, the rate at which variable and total costs increases as the level of sales increases.

The point of intersection between the total cost and revenue lines is the break-even point, which occurs at a sales level of \$382 million. At levels of sales below \$382 million, total operating costs exceed revenue and the firm suffers a loss; at sales levels higher than \$382 million, revenue exceeds operating costs and the firm has positive *EBITDA*. At sales of \$382 million, revenue equals total cost and the firm breaks even (*EBITDA* is zero). Hence, we call sales of \$382 million the break-even point.

* *Fixed* and *Variable Costs* were estimated from O&R's five-year income data with the regression

$$\text{Total operating cost} = F + v (\text{Sales}),$$

where the intercept, F , is the estimate of *Fixed Cost*, and the coefficient, v , is the estimate of the variable cost as a percent of sales. With five observations, the estimated equation was

$$\text{Total operating cost} = 167.3 + 0.58 (\text{Sales}) \quad R^2 = 0.93 (\text{adjusted}) \\ (t = 2.53) \quad (t = 7.56)$$

For ease of exposition of the O&R example, the variable cost rate was assumed to be 0.563 to fit with the fifth year's income statement figures in [Exhibit 2.14](#).

† The appendix to this chapter provides a brief instruction on how to use a data table to help make [Exhibit 2.15](#).

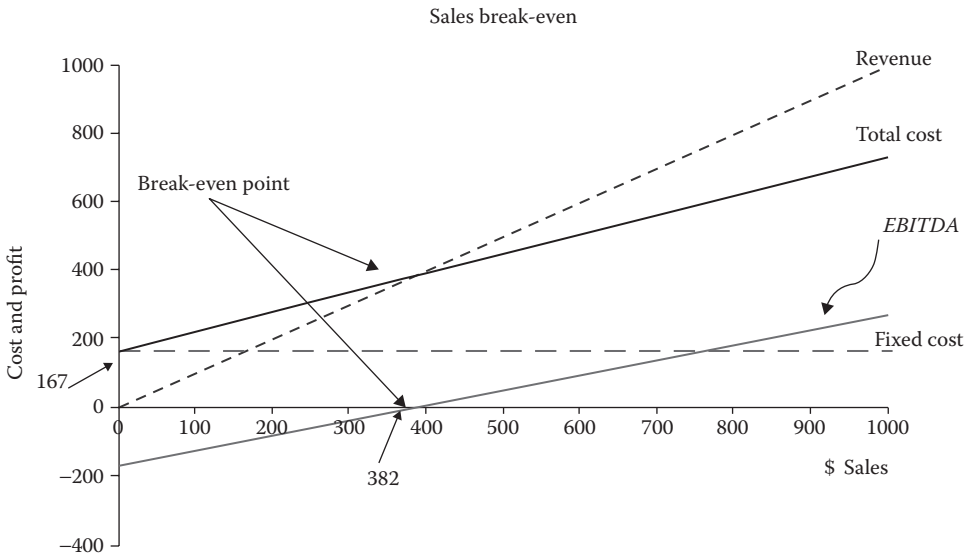


EXHIBIT 2.15 O&R Corp. break-even chart.

The break-even point is imposed by the presence of fixed costs. In the absence of fixed costs, the operation would be profitable at all sales levels so long as the sales price exceeds the variable cost per unit. In the presence of fixed costs, the sales level must be high enough so that the contribution margin (sales price minus variable cost) covers the fixed costs. With a percentage variable cost of 56.3%, O&R’s percentage contribution margin is $(1 - 0.563) = 0.437$, and the break-even sales level (denoted as Sales*) is calculated as*

$$Sales^* = \frac{Fixed\ Cost}{(1 - Variable\ Cost\ Rate)}$$

$$Sales^* = \frac{\$167\ million}{(1 - 0.563)} \tag{2.13}$$

$$= \$382.2\ million$$

This example defined break-even as the sales level where *EBITDA* would be zero. Break-even can also be defined in terms of cash flow, *EBIT*, or net income. In each case, the method is the same: to determine the sales level where the incremental profit covers

* For an analysis in terms of physical units, the break-even number of units, Q^* , is calculated as

$$Q^* = \frac{Fixed\ cost}{(Price - Variable\ cost\ per\ unit)} = \frac{\$167\ million}{(\$10 - \$5.63)}$$

$$Q^* = \frac{\$167\ million}{(\$4.37)} = \$38.21\ million\ units$$

where the sale price is \$10 per unit, and the variable cost per unit is \$5.63.

fixed costs, we divide fixed costs by the variable profit rate (percent or dollar contribution margin). For example, stepping down one level on the income statement to *EBIT*, O&R's fixed costs amount to \$220.8 million, including depreciation of \$53.8 million, in addition to the fixed operating costs of \$167 million that we have already discussed. The variable cost rate is still 56.3%, so the sales level for *EBIT* break-even is calculated as

$$\begin{aligned} \text{Sales}^* &= \frac{\text{Fixed Cost}}{(1 - \text{Variable Cost Rate})} \\ \text{Sales}^* &= \frac{\$220.8 \text{ million}}{(1 - 0.563)} \\ &= \$505.3 \text{ million} \end{aligned} \tag{2.14}$$

Thus at a sales level of \$382 million, *EBITDA* is zero, and because of the fixed expense of depreciation, *EBIT* will be negative. However, at the higher sales level of \$505.3 million, this additional fixed cost will be covered, and *EBIT* will be zero instead of negative. If sales exceed \$505.3 million, *EBIT* will be positive.

2.4 ANALYSIS OF OPERATING AND FINANCIAL LEVERAGE

Fixed costs not only impose break-even points, they also make the firm's earnings more volatile than they would be if all costs were variable. In fact, using fixed-cost methods of production and financing tend to cause earnings to fluctuate more than sales. This magnification effect is called leverage of which there are two types: operating leverage and financial leverage. As operating and financial leverages can increase the risk and uncertainty for the firm and its stakeholders, it is important to understand and measure it. In this section, we introduce three leverage measures—degree of operating leverage, degree of financial leverage and, putting the two together, the degree of combined leverage.

The magnification effect of O&R's leverage is shown in [Exhibit 2.16](#). It shows the effects of changes in sales on *EBITDA*, *EBIT*, *EBT*, and *NI*. The base level of sales of \$1,000 is shown in the middle, the columns on the left of base show the impact of sales declining 1% and 10%, respectively from the base, and the columns on the right show the effects of sales increases of 1% and 10%, respectively. The bottom five rows show the percentage change from the base level for each of the four earnings measures. We see that the percentage change in earnings is a multiple of the percentage change in sales. For example, a 1% change in sales up or down results in a 2.02% change in the same direction in *EBIT* and a 2.05% change in Net Income. Similarly, if sales changes by 10%, *EBIT* changes by 20.21%, and *NI* change by 20.54%. In this case, *EBIT* changes 2.02 times the percent change in sales, and *NI* changes by 2.05 times the change in sales.

The sensitivity of earnings to changes in sales which are like elasticity in economics. In economics, the price elasticity of demand is measured by the percentage change in demand resulting from a 1% change in the price: (% change demand)/(% change in price). In the case of leverage, we have an elasticity of earnings with respect to changes in sales. The elasticity of *EBIT* with respect to changes in sales is

Degrees of Leverage Sensitivity Analysis							
Variation From Base			-10%	-1%	Base Level	1%	10%
Sales			900.0	990.0	1,000.0	1,010.0	1,100.0
Variable Operating Cost			506.7	557.4	563.0	568.6	619.3
Fixed Operating Cost			167.0	167.0	167.0	167.0	167.0
EBITDA			226.3	265.6	270.0	274.4	313.7
Depreciation			53.8	53.8	53.8	53.8	53.8
EBIT			172.5	211.8	216.2	220.6	259.9
Interest			3.4	3.4	3.4	3.4	3.4
Earnings Before Tax			169.1	208.4	212.8	217.2	256.5
Income Tax @	40%		67.6	83.4	85.1	86.9	102.6
Net Income			101.5	125.1	127.7	130.3	153.9
% Change in Sales			-10%	-1%		1%	10%
% Change in EBITDA			-16.19%	-1.62%		1.62%	16.19%
% Change in EBIT			-20.21%	-2.02%		2.02%	20.21%
% Change in EBT			-20.54%	-2.05%		2.05%	20.54%
% Change in NI			-20.54%	-2.05%		2.05%	20.54%

EXHIBIT 2.16 O&R Corp. sensitivity of earnings to changes in sales.

$$\frac{(\% \text{ Change in EBIT})}{(\% \text{ Change in Sales})} = \frac{2.02\%}{1\%} = 2.02 \quad (2.15)$$

and the elasticity of *NI* with respect to change in sales is

$$\frac{(\% \text{ Change in NI})}{(\% \text{ Change in Sales})} = \frac{2.054\%}{1\%} = 2.054 \quad (2.16)$$

This elasticity or sensitivity is what is measured by the degrees of leverage: Degree of Operating Leverage (*DOL*), Degree of Financial Leverage (*DFL*), and Degree of Combined Leverage (*DCL*).

2.4.1 Degree of Operating Leverage

Earnings are elastic relative to sales (elasticity greater than 1) because of the presence of fixed costs. If all costs were directly proportional to sales, then earnings would vary directly with sales on a one-for-one basis. However, when some costs are fixed, independent of sales, increases in sales can occur without concomitant increases in all the costs, so that earnings can increase proportionally more than sales. The use of fixed-cost methods of operation (e.g., the use of machinery rather than labor) is referred to as using operating leverage. We measure the *DOL* by calculating the sales elasticity of *EBIT*. This elasticity or *DOL* is calculated (at a given level of sales) as

$$\begin{aligned}
 DOL &= \frac{\text{Sales} - \text{Variable Costs}}{\text{Sales} - \text{Variable Cost} - \text{Fixed Costs}} \\
 &= \frac{\text{Sales}(1 - v)}{\text{Sales}(1 - v) - F} = \frac{\text{Sales}(1 - v)}{EBIT}
 \end{aligned}
 \tag{2.17}$$

where:

v denotes variable cost as a proportion of sales

F denotes fixed costs

At a sales level of \$1,000 million, O&R Corp.'s DOL is

$$DOL = \frac{1,000(1 - 0.563)}{1,000(1 - 0.563) - 220.8} = \frac{437}{216.2} = 2.02
 \tag{2.18}$$

At this sales level, a 1% change in sales will result in a 2.02% change in $EBIT$ as shown in [Exhibit 2.16](#). However, note that the DOL depends on the sales level. At a different sales level, the DOL will be different. In fact, the closer the firm is operating to its break-even point, the greater the DOL .

2.4.2 Degree of Financial Leverage

Another source of fixed cost is the interest expense resulting from debt financing. Debt financing provides leverage to the equity owners. Thus, the ideas about break-even points, leverage, and elasticity that apply to operating leverage also apply to the use of the fixed-cost source of financing, debt. In the absence of debt financing, any variation in $EBIT$ is translated directly into an equally proportional variation in net income. However, with debt financing, because of the necessity to make a fixed interest payment, any variation in $EBIT$ will be leveraged (or magnified) into a more than proportional change in net income. This effect is referred to as financial leverage and is measured by the elasticity of net income with respect to $EBIT$. This elasticity is the DFL , and is calculated with the formula

$$DFL = \frac{EBIT}{EBIT - I}
 \tag{2.19}$$

where I denotes the interest payment. For example, at a sales level of \$1,000, the DFL is

$$DFL = \frac{216.2}{216.2 - 3.4} = 1.016
 \tag{2.20}$$

meaning that a 1% change in $EBIT$ would be translated into a 1.016% change in after-tax net income.

2.4.3 Degree of Combined Leverage

The equity owners of the firm are subject to both operating leverage and financial leverage. Variations in sales are translated into magnified variations in net income by the combined effects of fixed-cost production methods and fixed-cost financing methods. This effect is summarized and measured as the *DCL*, which is calculated as

$$\begin{aligned}
 DCL &= DOL \times DFL \\
 &= \frac{\text{Sales}(1-v)}{EBIT} \times \frac{EBIT}{EBIT - \text{Interest}} \\
 &= \frac{\text{Sales}(1-v)}{EBIT - \text{Interest}}
 \end{aligned} \tag{2.21}$$

at a given level of sales. For our example, at a sales level of \$1,000, the *DCL* is

$$\begin{aligned}
 DCL &= \frac{1,000(1-0.563)}{216.3-3.4} \\
 &= \frac{437}{212.8} \\
 &= 2.05
 \end{aligned} \tag{2.22}$$

or, expressed as the product of *DOL* and *DFL* as

$$\begin{aligned}
 DCL &= DOL \cdot DFL \\
 &= 2.021 \cdot 1.016 \\
 &= 2.05
 \end{aligned} \tag{2.23}$$

This means that at a sales level of \$1,000, a 1% change in sales would be translated into a 2.05% change in net income. With relatively little financial leverage, most of the combined leverage is due to O&R's fixed production costs.

The ideas of sales elasticity and *DOL* are of use to the planner when contemplating the effects of the structure of the firm on the firm's future performance. However, recall that both these measures depend on the level of sales. Generally, as the level of sales increases (above the break-even point), the degree of leverage decreases.

This elementary analysis assumed that costs and revenues are linearly related to the number of units sold. In other words, the graphs of revenues and costs as shown in [Exhibit 2.15](#) are straight lines, as opposed to curved lines. This assumption is a useful approximation for many situations and is well suited to introducing the concepts. In real life, the relationships

may be more complex, and indeed, the concept of break-even point itself may be more complicated than is apparent from this simple one period example.

2.5 CONCLUSION

This chapter introduced some basic tools for financial analysis. The tools of financial analysis help us to evaluate the financial condition of the firm. They tell us about the firm's present condition and provide an insight into how and why the firm reached its present condition. This is the starting point for our trip into the future. Knowing more about our starting point helps us to develop better plans for the future. In addition, the financial relationships within the firm revealed by financial analysis provide the input for developing financial models that will help to fine tune our financial plans. The next chapter adds to our stock of tools for financial analysis and planning by honing our understanding of cash flows, cash budgets, and sustainable growth.

PROBLEMS

1. O&R Corporation

Financial statements for O&R Corporation were shown in [Exhibits 2.2](#) and [2.3](#). Using that financial data as the basic input, develop a spreadsheet model that will show the various ratios discussed in this chapter. Your spreadsheet for the O&R Corporation should be set up in a way that it separates the data input from the calculations of the financial ratios. In other words, the data input should be in a separate section or separate sheet of the model, and the formulas that calculate the performance ratios should be in their own section or own sheet. This formula section should include only formulas and labels, but none of the input data. Thus, the ratios section will be a template for calculating ratios that can be used for evaluating performance for any company whose data you input.

2. K.D. Nickel Corporation

K.D. Nickel Corporation and Willards, Inc. are both department store retailers with stores located throughout the country. Six years of last financial statements for each of these companies, in addition to average ratios for their industry, are shown below. Your task is to set up a spreadsheet model so that you can perform a financial analysis of these two companies, comparing them with each other and with the industry averages. The objective of the financial analysis is to determine each company's strengths and weaknesses by digging as deeply into their performance as the data allow.

Modeling task: The spreadsheet you construct should be a template for doing financial analysis. By template, we mean the spreadsheet with formulas that will compute the

various ratios, and so on, that you use. The template should not contain any of the data input. The data input should come from a separate spreadsheet. In this way, you can use the template for any company, and simply input that company's data from a separate data sheet on to the analysis template.

Should you not want to enter the data yourself, the financial data are to be found on the class disk included with this book (Exhibits 2.17 through 2.21).

What the Data Table did was fill in the values for line 6 as if you had simply repeated the formula of I6 across the line. This provides you with an easy way to do a sensitivity analysis where you want to construct the sensitivity table showing the values of a dependent variable corresponding to a range of values of an independent variable. In this case, the Data Table includes cells I5:M6, and lines 8 to 12 were not really specified as part of the Data Table. However, these model lines take the Data Table output of line 6 and complete the remainder of the income statement, so you can see the sensitivity impact of various Units Sold on *EBITDA*.

	A	B	C	D	E	F	G
45	K.D. NICKEL						
46							
47	Ticker Symbol	KDNC					
48	SIC Code	5311					
49							
50	INCOME STATEMENT						
51		(\$ MILLIONS)					
52		2016	2015	2014	2013	2012	2011
53	Sales	17,786.0	32,347.0	32,004.0	31,846.0	32,510.0	31,380.0
54	Cost of Goods Sold	10,772.0	21,948.0	22,193.0	22,458.0	23,665.0	22,466.0
55	Gross Profit	7,014.0	10,399.0	9,811.0	9,388.0	8,845.0	8,914.0
56	Selling, General, & Admin	5,809.0	8,658.0	8,459.0	8,637.0	7,164.0	6,759.0
57	Operating Income Before Deprec.	1,205.0	1,741.0	1,352.0	751.0	1,681.0	2,155.0
58							
59	Depreciation & Amort	394.0	667.0	717.0	695.0	710.0	637.0
60							
61	Operating Profit	811.0	1,074.0	635.0	56.0	971.0	1,518.0
62	Interest Expense	261.0	407.0	426.0	477.0	673.0	663.0
63	Non-Operating Income & Spec Iter	-4.0	-83.0	-6.0	-465.0	233.0	100.0
64	Pretax Income	546.0	584.0	203.0	-886.0	531.0	955.0
65	Income Taxes	182.0	213.0	89.0	-318.0	195.0	361.0
66							
67	Income Before Extraord						
68	Items & Discontin Op	364.0	371.0	114.0	-568.0	336.0	594.0
69	Preferred Dividends	25.0	27.0	29.0	33.0	36.0	38.0
70	Available for Common	339.0	344.0	85.0	-601.0	300.0	556.0
71	Extraord Items & Discontin Op	-1,292.0	34.0	-16.0	-137.0	0.0	0.0
72	Net Income	-953.00	378.00	69.00	-738.00	300.00	556.00
73							
74	EPS -Basic Before Extra Items	1.25	1.28	0.32	-2.29	1.16	2.20
75	Dividends Per Share	0.259	0.743	0.754	0.806	0.780	0.848

EXHIBIT 2.17 K.D. Nickel income statement.

	A	B	C	D	E	F	G
2	K.D. NICKEL						
3							
4	Ticker Symbol	KDNC					
5	SIC Code	5311					
6							
7		(\$ MILLIONS)					
8	BALANCE SHEET						
9		2016	2015	2014	2013	2012	2011
10	ASSETS						
11	Cash & Equivalents	2,994.0	2,474.0	2,840.0	944.0	1,233.0	511.0
12	Net Receivables	233.0	705.0	698.0	893.0	1,138.0	4,415.0
13	Inventories	3,156.0	4,945.0	4,930.0	5,269.0	5,947.0	6,031.0
14	Other Current Assets	130.0	229.0	209.0	151.0	154.0	168.0
15	Total Current Assets	6,513.0	8,353.0	8,677.0	7,257.0	8,472.0	11,125.0
16							
17	Gross Plant, Property & Equipment	5,637.0	8,154.0	8,317.0	8,062.0	8,195.0	8,333.0
18	Accumulated Depreciation	2,122.0	3,253.0	3,328.0	2,948.0	2,883.0	2,875.0
19	Net Plant, Property & Equipment	3,515.0	4,901.0	4,989.0	5,114.0	5,312.0	5,458.0
20	Other Assets	8,272.0	4,613.0	4,382.0	7,371.0	7,104.0	7,055.0
21	TOTAL ASSETS	18,300.0	17,867.0	18,048.0	19,742.0	20,888.0	23,638.0
22							
23	LIABILITIES						
24	Notes Payable	260.0	288.0	935.0	250.0	955.0	2,362.0
25	Accounts Payable	1,167.0	1,792.0	1,551.0	1,948.0	1,480.0	1,496.0
26	Taxes Payable	119.0	123.0	158.0	195.0	179.0	232.0
27	Accrued Expenses	1,051.0	1,373.0	1,077.0	656.0	1,613.0	1,597.0
28	Other Current Liabilities	1,157.0	583.0	778.0	1,186.0	238.0	283.0
29							
30	Total Current Liabilities	3,754.0	4,159.0	4,499.0	4,235.0	4,465.0	5,970.0
31	Long Term Debt	5,114.0	4,940.0	5,179.0	5,448.0	5,844.0	7,143.0
32	Other Liabilities	4,007.0	2,398.0	2,241.0	3,800.0	3,351.0	3,356.0
33	EQUITY						
34	Preferred Stock	304.0	333.0	363.0	399.0	446.0	475.0
35	Common Stock	137.0	134.5	132.0	131.5	130.5	125.0
36	Capital Surplus	3,394.0	3,288.5	3,198.0	3,162.5	3,135.5	2,725.0
37	Retained Earnings	1,590.0	2,614.0	2,436.0	2,566.0	3,516.0	3,844.0
38	Common Equity	5,121.0	6,037.0	5,766.0	5,860.0	6,782.0	6,694.0
39	TOTAL EQUITY	5,425.0	6,370.0	6,129.0	6,259.0	7,228.0	7,169.0
40							
41	TOTAL LIABILITIES & EQUITY	18,300.0	17,867.0	18,048.0	19,742.0	20,888.0	23,638.0
42							
43	Shares Outstanding - Common	274.0	269.0	264.0	263.0	261.0	250.0

EXHIBIT 2.18 K.D. Nickel balance sheet.

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	I	J	K	L	M	N	O
45	WILLARDS INC						
46							
47	Ticker Symbol	WLLS					
48	SIC Code	5311					
49							
50	INCOME STATEMENT						
51		(\$ MILLIONS)					
52		2016	2015	2014	2013	2012	2011
53	Sales	7,848.1	8,169.6	8,388.3	8,817.8	8,921.2	8,011.7
54	Cost of Goods Sold	5,170.2	5,254.1	5,507.7	5,802.1	5,762.4	5,286.1
55	Gross Profit	2,677.9	2,915.5	2,880.6	3,015.6	3,158.8	2,725.6
56	Selling, General, & Admin	2,174.3	2,232.1	2,264.2	2,295.9	2,275.9	1,979.2
57	Operating Income Before Deprec.	503.5	683.4	616.5	719.8	882.9	746.4
58							
59	Depreciation & Amort	290.7	301.4	310.8	303.2	292.7	239.7
60							
61	Operating Profit	212.9	382.0	305.7	416.6	590.2	506.8
62	Interest Expense	187.8	185.4	195.8	229.0	241.8	199.8
63	Non-Operating Income & Spec Ite	-9.1	14.6	1.6	-46.7	-64.5	-87.9
64	Pretax Income	16.0	211.1	111.6	140.9	283.9	219.1
65	Income Taxes	6.7	74.8	45.8	44.0	120.2	83.8
66							
67	Income Before Extraord						
68	Items & Discontin Op	9.3	136.3	65.8	96.8	163.7	135.3
69	Preferred Dividends	0.0	0.0	0.0	0.0	0.0	0.0
70	Available for Common	9.3	136.3	65.8	96.8	163.7	135.2
71	Extraordinary Items	0.0	-534.7	6.0	-102.7	0.0	0.0
72	Net Income	9.344	-398.405	71.798	-5.85	163.721	135.237
73							
74	EPS -Basic Before Extra Items	0.11	1.61	0.78	1.06	1.55	1.26
75	Dividends Per Share	0.16	0.16	0.16	0.16	0.16	0.16

EXHIBIT 2.19 Income statement for Willards, Inc.

This Data Table was for the case where you have just one independent variable. Excel also has the capacity to build a Data Table with two independent variables. Two ways to gain perspective from the financial ratios are (1) to look at the trend of ratios over time, and (2) to compare them to other companies in the same industry and with the average ratios for the industry. The trend of the ratios over time is a good starting point for our analysis because significant changes or trends in ratios frequently signal potential problems or promising improvements. Scanning across the line of ratios may quickly alert you to changes that may be important, and graphing the trend of ratios is even more

	I	J	K	L	M	N	O
2	WILLARDS INC						
3							
4	Ticker Symbol	WLLS					
5	SIC Code	5311					
6							
7		(\$ MILLIONS)					
8	BALANCE SHEET						
9		2016	2015	2014	2013	2012	2011
10	ASSETS						
11	Cash & Equivalents	160.9	142.4	153.0	194.0	198.7	72.4
12	Net Receivables	1,191.5	1,338.1	1,074.9	979.2	1,104.9	1,192.6
13	Inventories	1,632.4	1,594.3	1,561.9	1,616.2	2,047.8	2,157.0
14	Other Current Assets	39.0	55.5	24.7	53.5	72.2	15.7
15	Total Current Assets	3,023.7	3,130.3	2,814.5	2,842.9	3,423.7	3,437.7
16							
17	Gross Plant, Property & Equipm	5,070.5	5,134.6	5,120.4	5,015.9	4,961.0	5,407.9
18	Accumulated Depreciation	1,873.0	1,764.1	1,664.7	1,507.6	1,341.8	1,723.2
19	Net Plant, Property & Equipment	3,197.5	3,370.5	3,455.7	3,508.3	3,619.2	3,684.6
20	Other Assets	189.9	175.2	804.3	848.0	875.3	1,055.2
21	TOTAL ASSETS	6,411.1	6,675.9	7,074.6	7,199.3	7,918.2	8,177.6
22							
23	LIABILITIES						
24		549.7	140.7	100.5	211.3	110.6	166.7
25	Accounts Payable	457.5	429.1	562.5	399.0	351.7	524.1
26	Taxes Payable	106.5	69.8	19.4	17.6	32.4	5.9
27	Accrued Expenses	222.4	246.8	245.7	248.9	315.9	397.1
28	Other Current Liabilities	0.0	0.0	0.0	0.0	0.0	0.0
29							
30	Total Current Liabilities	1,336.1	886.5	928.1	876.7	810.6	1,093.8
31	Long Term Debt	2,072.8	2,743.2	2,676.6	2,928.2	3,450.9	3,561.2
32	Other Liabilities	765.1	782.1	801.5	764.6	823.9	681.1
33	EQUITY						
34	Preferred Stock	0.0	0.0	0.0	0.0	0.0	0.4
35	Common Stock	1.2	1.2	1.2	1.2	1.2	1.2
36	Capital Surplus	45.6	61.9	49.6	69.7	252.1	407.1
37	Retained Earnings	2,190.3	2,201.2	2,617.6	2,558.9	2,579.6	2,432.8
38	Common Equity	2,237.1	2,264.2	2,668.4	2,629.8	2,832.8	2,841.1
39	TOTAL EQUITY	2,237.1	2,264.2	2,668.4	2,629.8	2,832.8	2,841.5
40							
41	TOTAL LIABILITIES & EQUITY	6,411.1	6,675.9	7,074.6	7,199.3	7,918.2	8,177.6
42							
43	Shares Outstanding - Common	83.5	84.8	83.9	85.0	98.8	106.9

EXHIBIT 2.20 Balance sheet for Willards, Inc.

	A	B	C	D	E	F	G
78	DEPARTMENT STORES						
79							
80	SIC	5311					
81							
82	AVERAGE ANNUAL RATIOS						
83							
84	Year	2016	2015	2014	2013	2012	2011
85							
86	LIQUIDITY						
87	Current Ratio	2	2.25	2.51	2.51	2.67	2.44
88	Quick Ratio	0.78	0.92	1.05	1	1	0.87
89	Cash Flow Per Share	3.83	3.69	3.35	3.01	3.63	2.98
90							
91	ACTIVITY						
92	Inventory Turnover	3.35	3.5	3.32	3.31	3.28	3.23
93	Days to Sell Inventory	112	107	113	114	115	118
94	Receivables Turnover	33.67	29.3	24.63	25.1	18.61	14
95	Average Collection Period (Days)	43	53	53	48	46	48
96	Operating Cycle (Days)	155	160	166.00	162.00	160.00	166.00
97	Total Asset Turnover	1.35	1.43	1.44	1.45	1.48	1.45
98							
99	PROFITABILITY						
100	Operating Margin Before Depr (%)	4.8	5.86	9.32	8.85	9.14	9.66
101	Operating Margin After Depr (%)	3.19	3.79	6.21	5.85	6.42	7.04
102	Pretax Profit Margin (%)	1.57	2.4	4.22	3.5	3.35	4.93
103	Net Profit Margin (%)	1.46	2.84	2.64	2.14	1.93	3.11
104	Return on Assets (%)	-28.48	7.49	3.36	2.95	2.74	4.24
105	Return on Equity (%)	-32.57	4.11	8.33	7.31	7.07	10.78
106	Return on Investment (%)	1.37	3	4.72	4.15	3.86	6.17
107							
108	LEVERAGE						
109	Interest Coverage Before Tax	2.8	3.17	4.42	3.80	3.31	4.16
110	Interest Coverage After Tax	60.29	5.95	3.13	2.71	2.36	2.99
111	Long-Term Debt/Equity(%)	92.97	53.06	82.87	88.57	88.28	82.85
112	Total Debt/Invested Capital (%)	28.67	35.09	46.46	48.07	51.03	50.17
113	Total Debt/Total Assets (%)	4.01	3.15	31.37	33.01	34.93	34.27
114	Total Assets/Common Equity	2.98	2.57	2.75	2.79	2.78	2.69
115							
116	DIVIDENDS						
117	Dividend Payout (%)	0.86	1.21	11.04	24.02	4.24	20.29

EXHIBIT 2.21 Average ratios for department stores.

effective in highlighting the trends. For example, [Exhibit 2.10](#) shows an Excel graph of O&R's profit margins clearly showing an upward trend. The picture of this positive trend might not be worth a thousand words, but it surely helps us to quickly see one of the important aspects of O&R's operations. Similar pictures of the trends can help us to visualize what is happening for the firm, and the usefulness of your financial analysis model can be enhanced by including graphs of the more important ratios.

The Tools for Financial Planning

Growth, Cash Flows, and Cash Budgeting

IN THE PREVIOUS CHAPTER, we considered several methods for financial analysis. In this chapter, we introduce additional useful tools and methods for financial analysis and planning. These include ways to project proforma financial statements, the concept of sustainable growth, various definitions of cash flow, and the development of cash budgets. Although these topics do not contribute directly to building financial models, they are important for understanding the structures of the models we build in the following chapters. Building correct models requires having a clear understanding proforma financial statements, growth, cash flows, and cash budgets. Reviewing the definitions and interrelationships between these factors will make it easier to understand subsequent modeling concepts, as well as help you develop your own models.

3.1 PROJECTING *PROFORMA* FINANCIAL STATEMENTS

This section introduces a simple method for forecasting the firm's future financial statements—the percent-of-sales format. This framework provides the basis for much of our discussion of financial planning models in subsequent chapters, as many of the examples in this book use a sales-driven model. Clearly, the primary input to our model is a forecast of sales, and that forecast drives the rest of the model. As shown in [Exhibit 3.1](#), with the sales-driven model, the sales forecast determines the required asset levels based on the idea that there is a minimum level of assets necessary to support sales. In turn, the asset requirements determine the level of financing necessary to purchase the assets.

To explain the percent-of-sales approach for developing financial projections, we build on the example of the Odd & Rich Corporation that we used in the previous chapter. We use our knowledge of O&R to determine what its financial condition will be in the future as it grows. The past financial statements and the ratios we computed provide the starting point

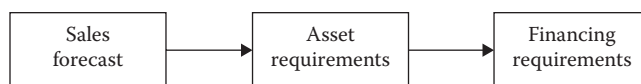


EXHIBIT 3.1 The flow of a sales-driven model.

for our analysis. [Exhibit 3.2](#) shows the financial statements for Odd & Rich Corporation for the last two years, 2015 and 2016, and the 2016 account balance as a percent of sales.

The percent-of-sales approach assumes that a link exists between the firm's sales level and its expenses, assets, and liabilities; hence, the sales forecast drives the forecasts for income, expenses, assets, and liabilities. In the simplest percent-of-sales model, this link is represented by the assumption that most of these categories maintain a constant ratio to sales. This is often a reasonable approximation, because the structure of the firm's operations determines its expenses, assets, and liabilities. For example, Odd & Rich manufactures plastic components. In order to be able to generate \$1.0 billion in sales, it needs to have a certain minimum number of machines, amount of warehouse space, delivery trucks, and administrative offices. To service its customers, it needs an inventory, and each plant must have an appropriate number of employees. To attract the number of customers necessary to buy \$1.0 billion worth of merchandize, it needs some minimum level of marketing. In addition, it will probably need to borrow some funds or obtain equity financing to finance its inventories and facilities. This supports our assumption of the link between the firm's sales and the expenses, assets, and liabilities that support the firm's sales. With respect to O&R, given its present mode of operation, it is unlikely that the firm can generate a much higher level of sales without increasing its expenses, assets, and liabilities in pace with sales.

Naturally, the assumption of a constant percent of sales will not be appropriate in all situations. Even for a firm where it fits in general, there will be exceptions. Some expenses will be fixed rather than varying with sales, or the firm may be able to accommodate moderate increases in sales without having to increase its asset base. For other firms, there may be only the weakest link between sales and assets. For example, a service firm such as an accounting firm may be able to serve many more clients without investing in more assets, or a software company can sell many more disks or file downloads of software without expanding its plant or hiring additional employees. So, like any method, one needs to be careful how they apply the percent-of-sales approach and make sure that their assumptions match the reality of the situation. Still, in those cases where it fits, the percent-of-sales approach is a very useful way to begin planning and modeling.

In the case of Odd & Rich, we assume that the constant percent-of-sales model is appropriate.* We have forecasted Odd & Rich' sales to increase by 15% from their level of \$1,000 in 2016 to \$1,150 in 2017. We want to see how this growth will affect the firm's needs for funds in 2017. To implement this, we assume that each of the expense and asset categories in [Exhibit 3.2](#) continues to maintain the proportional relationship with sales that it has had in the past. Most of these account percentages are summarized in [Exhibit 3.3](#). We will

* For the remainder of this chapter, we will assume that O&R's costs are directly proportional to sales, and that there are no fixed costs.

ODD & RICH CORP			
Income Statement			
(\$ Millions)			
	Year		Percent of Sales 2016
	2015	2016	
Sales	944.0	1,000.0	
Cost of Goods Sold	566.7	607.7	60.8%
Selling, General & Administrative	169.3	122.3	12.2%
EBDIT	208.0	270.0	27.0%
Depreciation	49.9	53.8	5.4%
EBIT	158.1	216.2	21.6%
Interest	3.1	3.4	0.3%
EBT	155.0	212.8	21.3%
Tax	62.0	85.0	8.5%
Net Income	93.0	127.8	12.8%
Dividends	55.8	76.7	7.67%
Retained Earnings	37.2	51.1	5.11%

ODD & RICH CORP			
Balance Sheet			
(\$ Millions)			
	Year		Percent of Sales 2016
	2015	2016	
Assets			
Cash	160.4	170.0	17.0%
Accounts Receivable	177.5	208.1	20.8%
Inventory	142.8	148.2	14.8%
Other CA	189.5	183.7	18.4%
Current Assets	670.2	710.0	71.0%
Gross Fixed Assets	1,792.7	1,927.0	192.7%
Accumulated Depreciation	449.2	503.0	50.3%
Net Fixed Assets	1,343.5	1,424.0	142.4%
Total Assets	2,013.7	2,134.0	213.4%
Liabilities			
Accounts Payable	175.5	279.1	27.9%
Short Term Debt	6.9	7.0	0.7%
Other CL	383.9	313.9	31.4%
Current Liabilities	566.3	600.0	60.0%
Long-term Debt	37.8	73.3	7.3%
Common Stock	100.0	100.0	10.0%
Earned Surplus	1,309.6	1,360.7	136.1%
Common Equity	1,409.6	1,460.7	146.1%
Total Liabilities & Equity	2,013.7	2,134.0	213.4%

EXHIBIT 3.2 Percent-of-sales approach.

Expenses and Earnings		
Cost of Goods Sold / Sales	CGS / Sales	60.8%
Selling, General & Administrative / Sales	SGA / Sales	12.2%
Earnings Before Interest, Taxes, Depreciation & Amortization / Sales	EBITDA/Sales	27.0%
Earnings Before Interest & Taxes / Sales	EBIT / Sales	21.6%
Net Income / Sales	NI / Sales	12.8%
Dividend Payout Ratio	Dividends / NI	60.0%
Assets and Liabilities		
Cash / Sales	Cash / Sales	17.0%
Accounts Receivable / Sales	AR / Sales	20.8%
Inventories / Sales	Inventory / Sales	14.8%
Other Current Assets / Sales	OCA / Sales	18.4%
Current Assets / Sales	CA / Sales	71.0%
Net Fixed Assets / Sales	NFA / Sales	142.4%
Total Assets / Sales	TA / Sales	213.4%
Account Payable / Sales	AP / Sales	27.9%
Other Current Liabilities / Sales	OCA / Sales	31.4%

EXHIBIT 3.3 O&R Corp. expenses, earnings, assets, and liabilities as a percent of sales.

use these percentages to estimate each of the income statement and balance sheet items by multiplying the forecast sales with the respective percentage shown in [Exhibit 3.3](#).

With costs proportional to sales, most of the income and expense items are straight forward. For example, for 2017, the cost of goods sold and $EBITDA_t$ are estimated as

$$\begin{aligned}
 \text{Cost of Goods Sold}_{2017} &= \left(\frac{\text{COGS}}{\text{Sales}} \right) \text{Sales}_{2017} \\
 &= (0.608) * 1,150 \\
 &= \$699.2 \text{ million}
 \end{aligned} \tag{3.1}$$

and

$$\begin{aligned}
 EBITDA_{2017} &= \left(\frac{EBITDA}{\text{Sales}} \right) \text{Sales}_{2017} \\
 &= (0.27) * 1,150 \\
 &= \$310.5 \text{ million}
 \end{aligned} \tag{3.2}$$

The calculation of selected items on the income statement and balance sheet is shown in [Exhibit 3.4](#).

For each type of asset, the ratio ($Asset/Sales$) is assumed to be the same each year; hence, level of a given asset required to support the sales is

$$\text{Asset}_t = \left(\frac{\text{Assets}}{\text{Sales}} \right) \text{Sales}_t \tag{3.3}$$

Pro-forma Income Statement

$$\begin{aligned}
\text{Sales}_1 &= \text{Sales}_0 (1 + g) = \$1,000 (1.15) = \$1,150 \\
\text{Cost of Goods Sold}_1 &= \text{Sales}_1 (\text{CGS}/\text{Sales}) = 1,150 (0.608) = 699.2 \\
\text{Selling, Gen \& Admin}_1 &= \text{Sales}_1 (\text{SGA}/\text{Sales}) = 1,150 (0.122) = 140.3 \\
\text{EBITDA}_1 &= \text{Sales}_1 (\text{EBITDA}/\text{Sales}) = 1,150 (0.27) = 310.5 \\
\text{EBIT}_1 &= \text{Sales}_1 (\text{EBIT}/\text{Sales}) = 1,150 (0.216) = 248.4 \\
\text{Net Income}_1 &= \text{Sales}_1 (\text{NI}/\text{Sales}) = 1,150 (0.128) = 147.2 \\
\text{Dividends}_1 &= \text{NI}_1 * \text{Payout Ratio} = 147.2 (0.60) = 88.3 \\
\text{Retained Earnings}_1 &= \text{NI}_1 - \text{Dividends}_1 = 147.2 - 88.3 = 58.9
\end{aligned}$$

Pro-forma Balance Sheet: Assets & Liabilities

$$\begin{aligned}
\text{Cash}_1 &= \text{Sales}_1 (\text{Cash}/\text{Sales}) = 1,150 (0.17) = \$195.5 \\
\text{Accounts Receivable}_1 &= \text{Sales}_1 (\text{AR}/\text{Sales}) = 1,150 (0.208) = 239.2 \\
\text{Inventories}_1 &= \text{Sales}_1 (\text{Invent}/\text{Sales}) = 1,150 (0.148) = 170.2 \\
\text{Other Current Assets}_1 &= \text{Sales}_1 (\text{OCA}/\text{Sales}) = 1,150 (0.184) = 211.6 \\
\text{Current Assets}_1 &= \text{Sales}_1 (\text{CA}/\text{Sales}) = 1,150 (0.71) = 816.5 \\
\text{Net Fixed Assets}_1 &= \text{Sales}_1 (\text{NFA}/\text{Sales}) = 1,150 (1.424) = 1,637.6 \\
\text{Total Assets}_1 &= \text{Sales}_1 (\text{TA}/\text{Sales}) = 1,150 (2.134) = 2,454.1 \\
\text{Accounts Payable}_1 &= \text{Sales}_1 (\text{AP}/\text{Sales}) = 1,150 (0.279) = 320.9 \\
\text{Other Current Liabilities}_1 &= \text{Sales}_1 (\text{OCL}/\text{Sales}) = 1,150 (0.314) = 361.1 \\
\text{Earned Surplus}_1 &= \text{Earned Surplus}_0 + \text{Retained Earnings}_1 = 1,360.7 + 58.9 = 1,419.6
\end{aligned}$$

EXHIBIT 3.4 Calculation of expense, earnings, assets, and liabilities using percent-of-sales method.

For example, we assume that the total asset-to-sales ratio will be maintained at 2.134. Therefore, with sales growing at 15% from \$1,000 million to \$1,150 million in 2017, the total assets required in 2017 will be

$$\begin{aligned}
\text{Total Assets}_{2017} &= (2.134) * 1,150 \\
&= \$2,454.1 \text{ million}
\end{aligned} \tag{3.4}$$

A similar calculation would apply for each of the individual asset accounts as shown in [Exhibit 3.4](#).

Having estimated the assets needed to support sales of \$1,150 million, we shift our focus to the liability accounts, or how the assets are to be financed. The liabilities identify the sources of funds for purchasing the assets needed to support sales. We split these sources into three categories—internal equity financing, spontaneous liabilities, and discretionary liabilities. Two of these are internally generated, internal equity financing, and spontaneous liabilities. The third source is external capital, what we call discretionary financing.

This section begins the development of an expression for discretionary financing as a function of sales. As per the chart in [Exhibit 3.1](#), we first derive the sales forecast. Based on that forecast, we estimate the assets required to support it. Finally, we parse the sources of the funds needed to purchase the assets needed to support the sales forecast.

We begin with internal equity financing, otherwise known as retained earnings. Retained earnings for the period are calculated as

$$\text{Retained Earnings}_t = \text{Net Income}_t - \text{Dividends}_t \quad (3.5)$$

To characterize *Retained Earnings* as a function of sales, assume that dividends are based on a dividend payout ratio that remains constant over time. If so,

$$\text{Retained Earnings}_t = (1 - \text{Payout Ratio}) \text{Net Income}_t \quad (3.6)$$

where $(1 - \text{Payout Ratio})$ is called the retention ratio. Assuming that net income is a constant percentage of sales based on the net profit margin yields

$$\text{Retained Earnings}_t = (1 - \text{Payout Ratio}) \left(\frac{\text{NI}}{\text{Sales}} \right) \text{Sales}_t \quad (3.7)$$

With a net profit margin of 12.8% and a 60% payout ratio, Odd & Rich's retained earnings for 2017 are expected to be

$$\begin{aligned} \text{Retained Earnings}_{2017} &= (1 - 0.60)(0.128) * 1,150 \\ &= \$58.9 \text{ million} \end{aligned} \quad (3.8)$$

The retained earnings for 2017 are added to the earned surplus (retained earnings accumulated from past periods), so Odd & Rich's earned surplus at the end of 2017 will be

$$\begin{aligned} \text{Earned Surplus}_{2017} &= \text{Earned Surplus}_{2016} + \text{Retained Earnings}_{2017} \\ &= 1,360.7 + 58.9 \\ &= \$1,419.6 \text{ million} \end{aligned} \quad (3.9)$$

The second source of internal funding stems from the increase in spontaneous liabilities. Spontaneous liabilities refer to liability accounts that are related directly to the production and sales cycle. These include accounts such as accounts payable, wages payable, and various liability accrual accounts that normally increase with production and sales. As these accounts are closely linked to production and sales, their increase in value is considered to be spontaneous with respect to sales. This spontaneous increase in liabilities helps the finance part of the required increase in assets. To the extent that the liabilities are maintained at a constant proportion of sales, the financing provided will be a constant proportion of the increase in sales. In our example, one spontaneous liability is *Accounts Payable*. *Accounts Payable* for 2017 is expected to be

$$\begin{aligned} \text{Accounts Payable}_{2017} &= \left(\frac{\text{AP}}{\text{Sales}} \right) \text{Sales}_{2017} \\ &= (0.279) * 1,150 \\ &= \$320.9 \text{ million} \end{aligned} \quad (3.10)$$

	2015	+	Change in 2016	=	Year End 2016
Total Assets	2,134.0	+	320.1	=	2,454.1
Accounts Payable	279.1	+	41.8	=	320.9
Other Current Liabilities	313.9	+	47.2	=	361.1
Notes Payable	7.0	+	(no change)	=	7.0
Long-term Debt	73.3	+	(no change)	=	73.3
Common Stock	100.0	+	(no change)	=	100.0
Earned Surplus	1,360.7	+	58.9	=	1,419.6
Total Liabilities & Equity	2,134.0	+	147.9	=	2,281.9
Required External Financing = Excess of Assets over Liabilities =					172.2

EXHIBIT 3.5 Provisional asset and liability balances prior to obtaining additional discretionary financing.

We also assume that *Other Current Liabilities* increase spontaneously with sales, so the level of *Other Current Liabilities* in 2017 will be

$$\begin{aligned}
 \text{Other Current Liabilities}_{2017} &= \left(\frac{\text{OCL}}{\text{Sales}} \right) \text{Sales}_{2017} \\
 &= (0.314) * 1,150 \\
 &= \$361.1 \text{ million}
 \end{aligned}
 \tag{3.11}$$

Discretionary liabilities are those sources of funds that do not accrue with the firm's production and sales. Instead, an explicit action must be taken to obtain these funds. The most common discretionary sources are borrowing and the issuance of new equity shares. These are sources of funds external to the firm, and the decision to use these sources is discretionary. In the case of the percent-of-sales approach, the discretionary sources of funds are the *plug* variables that make the balance sheet balance. These discretionary sources of financing were omitted from Exhibits 3.3 and 3.4 because they will be calculated as residuals.

Exhibit 3.5 shows the provisional balance sheet with the assets, internal equity financing, and spontaneous liabilities resulting from the projected growth in sales. If we take no additional action to obtain financing, total assets will exceed total liabilities by \$172.2 million. This is the *plug* required to bring the 2017 balance sheet into balance. We call this deficiency the Required External Financing (*REF*), because it is the amount of external, discretionary financing needed to make things balance.*

Assume that long-term debt is the discretionary source and we borrow enough to meet our needs. Long-term Debt will increase by \$172.2 million from \$73.3 million to \$245.5 million. This additional debt brings the proforma statements into balance—sources equal uses, and liabilities equal assets as shown in Exhibit 3.6.

Exhibit 3.6 shows the completed balance sheet based on our percent-of-sales projection, Exhibit 3.4 shows the income statement. These proforma financial statements were

* Another commonly used equation for *REF* is “Additional Funds Needed” or *AFN*.

Equality of Sources & Uses with Discretionary Financing			
Uses	Flow of Funds		Sources
Purchase of assets	320.1	Increase in Accounts Payable	41.8
		Increase in Other Current Liabilities	47.2
		Retained Earnings	58.9
		Additional Borrowing	172.2
		Total Financing	320.1
Assets & Liabilities			
Cash	195.5	Accounts Payable	320.9
Accounts Receivable	239.2	Other Current Liabilities	361.1
Inventories	170.2	Notes Payable	7.0
Other Current Assets	211.6	Long-term Debt	245.5
Total Current Assets	816.5	Common Stock	100.0
Net Fixed Assets	1,637.6	Earned Surplus	1,419.6
Total Assets	2,454.1	Liabilities & Equity	2,454.1

EXHIBIT 3.6 Equality of sources and uses with discretionary financing.

the immediate objective of our percent-of-sales approach. The percent-of-sales approach is a simple but powerful method that we use throughout this book. However, our example glossed over a lot of detail in order to demonstrate the concept of *REF*. For example, by assuming that the net profit margin was constant over time, we implicitly assumed that all expenses were strictly proportional to sales. This poses a difficulty when we issue debt to fund our external financing needs. The interest expense incurred to service the new debt will not be strictly proportional to sales; thus, the net profit margin will not be strictly proportional to sales as sales grow. Problems like this are why we will spend a considerable amount of time learning how to structure more detailed planning models. Future chapters delve deeper into these problems and show how to handle them.

In the next two sections, we extend the percent-of-sales technique to formalize two useful concepts for financial planning: *REF* and sustainable growth.

3.2 GROWTH AND THE NEED FOR FINANCING

One of the more difficult phases in the life cycle of a firm is the period of rapid growth. It seems that growth should be a blessing, yet there are countless stories of firms that failed because they were unable to plan and manage their growth. The difficulty is that as a firm grows rapidly, its need for funds to finance expansion outruns its ability to generate funds from its operations. Faced with the opportunity to expand, but lacking the internal financing, these firms often turn to debt as the source of financing. Frequently, they find themselves with debt servicing obligations that exceed their ability to pay. Failure is eventually brought about because the needs of expansion overwhelm the firm's resources for expansion. To paraphrase Robert Higgins, these firms literally grow broke. The difficulties and dangers of expanding too rapidly can be mitigated by anticipating the firm's financing requirements and developing a management policy to control growth at a pace consistent with the firm's ability to finance it.

Two concepts or tools are particularly helpful for managing growth—*REF* and sustainable growth. These two closely related concepts help us to understand the financing

implications of the firm's growth and to plan the appropriate financing strategies. The concepts of *REF* and sustainable growth help to answer questions about:

1. How much financing will be necessary to support a given growth rate in sales?
2. How rapidly can the firm's sales increase without having to obtain external financing?
3. How rapidly can sales grow if we constrain our borrowing?

3.2.1 Required External Financing

REF is the additional funds that must be obtained from sources external to the firm in order to finance the assets needed to support desired growth. *REF* is based on the assumption that the firm requires a certain level of assets and liabilities to support its sales. As sales grow, the firm needs to expand its asset base to be able to generate the sales. This required growth in assets needs to be financed. Part of the financing will be internal and/or spontaneous; the remainder likely will be external. This logic is general. It does not depend on a stable percent-of-sales situation. However, we discuss *REF* in the context of a percent-of-sales model, and continue to use Odd & Rich as our example.

In the last section, our objective was to project pro forma financial statements. In the process, we calculated the amount of external funds that Odd & Rich required to meet the needs imposed by its sales growth. This was referred to as *REF* in [Exhibit 3.5](#), and amounted to \$172.2 million. We now take a more granular look at *REF* by expanding the discussion from the previous section. The result will be a simple, back-of-the-envelope calculation of *REF* as a function of sales.

We begin with the forecast that Odd & Rich's sales will grow 15% during 2017. We seek to determine the amount of external financing necessary to support that growth. As in the previous section, we first determine the total amount of asset investment that will be required. We then estimate the amount of funds that will be generated from internal and spontaneous sources. The remainder is the amount of funds that we need to obtain from external sources, *REF*.

Assume that the amount of assets required to support sales is more or less a constant proportion of sales (i.e., the ratio [*Total Assets/Sales*] is the same each year). If so, the assets required to support forecasted future sales in period t are

$$Asset_t = \left(\frac{Assets}{Sales} \right) Sales_t \quad (3.1 \text{ reprised})$$

If sales increase from one year to the next, the investment in assets required to support the increase in sales from year $(t - 1)$ to year t is

$$Required\ Investment_t = \left(\frac{Total\ Assets}{Sales} \right) * (Sales_t - Sales_{t-1}) \quad (3.12)$$

With Odd & Rich's total asset-to-sales ratio at 2.134, and sales growing at 15% from \$1,000 million to \$1,150 million in 2017, the investment required during 2017 will be

$$\begin{aligned} \text{Required Investment}_{2017} &= 2.134 * (\$1,150 - 1,000) \\ &= (2.134) * 150 \\ &= \$320.1 \text{ million} \end{aligned} \quad (3.13)$$

This investment in new assets must be financed using a combination of the sources previously mentioned—internal equity, spontaneous financing, and external financing. Our question is how much external financing will be needed?

From the previous section, internal equity financing is the product of sales, net profit margin, and the retention ratio:

$$\text{Retained Earnings}_t = (1 - \text{Payout Ratio}) \left(\frac{NI}{Sales} \right) Sales_t \quad (3.14)$$

With a net profit margin of 12.8%, and a 60% payout ratio, Odd & Rich's retained earnings for 2017 are expected to be

$$\begin{aligned} \text{Retained Earnings}_{2017} &= (1 - 0.60)(0.128) * 1,150 \\ &= \$58.9 \text{ million} \end{aligned} \quad (3.15)$$

Spontaneous financing is the increase in spontaneous current liabilities, calculated as

$$\left[\begin{array}{l} \text{Spontaneous} \\ \text{Financing}_t \end{array} \right] = \frac{\text{Spontaneous Liabilities}}{\text{Sales}} * (Sales_t - Sales_{t-1}) \quad (3.16)$$

The spontaneous liabilities for O&R include both *Accounts Payable* and *Other Current Liabilities*. Combined, these are equal to about 59.3% of sales, making spontaneous financing in 2017.

$$\begin{aligned} \text{Spontaneous Financing}_{2017} &= (0.593) * (\$1,150 - 1,000) \\ &= \$89.0 \text{ million} \end{aligned} \quad (3.17)$$

Provided the relation between sales and spontaneous current liabilities remains unchanged, there will be an increase in these accounts of \$89 million due to the increase in sales.*

To recap, if O&R's sales grow 15% in 2017, it will need to invest \$320.1 million in new assets. A portion of this \$320.1 million will be financed using internal sources—\$58.9 million from the increase in internal equity and \$89 million from the

* Note that the ratio of accounts payable to sales is related to the length of the payables period. If the firm takes longer to pay its accounts payable, the ratio (*Spontaneous Current Liabilities/Sales*) will increase, so more financing would be available from this source.

increase in spontaneous liabilities. The remainder, \$172.2 million, needs to come from external sources, hence the name *REF*. This calculation is summarized as

$$\begin{array}{r}
 \text{Required Investment} \qquad \qquad \qquad \$320.1 \text{ million} \\
 - \text{Internal Funding} \qquad \qquad \qquad \qquad \qquad 58.9 \\
 - \text{Spontaneous Funding} \qquad \qquad \qquad \qquad \qquad 89.0 \\
 = \text{Required External Financing (REF)} \qquad \underline{\$172.2 \text{ million}}
 \end{array} \tag{3.18}$$

Given our percent-of-sales assumptions, combining Equations 3.12, 3.14, and 3.16 yields an expression for *REF* as a function of *Sales*, thus:

$$\begin{aligned}
 \left(\begin{array}{c} \text{Required External} \\ \text{Financing}_t \end{array} \right) &= \left[\left(\frac{\text{Total Assets}}{\text{Sales}} \right) - \left(\frac{\text{Spontaneous Liabilities}}{\text{Sales}} \right) \right] * (\text{Sales}_t - \text{Sales}_{t-1}) \\
 &\qquad - \text{Sales}_t * \left(\frac{\text{Net Income}}{\text{Sales}} \right) * (1 - \text{Payout Ratio})
 \end{aligned} \tag{3.19}$$

This makes *REF* in 2017:

$$\begin{aligned}
 \left(\begin{array}{c} \text{Required External} \\ \text{Financing}_{2017} \end{array} \right) &= (2.134 - 0.593) * (1,150 - 1,000) - 1,150 * 0.128 * (1 - 0.60) \\
 &= (1.541) * (150) - (.0512) * 1,150 \\
 &= 231.1 - 58.9 \\
 &= \$172.2 \text{ million}
 \end{aligned} \tag{3.20}$$

Note the kinds of trade-offs that can be considered with this analysis. An increase in the turnover of total assets (a decrease in the *Total Assets/Sales*) decreases the amount of external financing required. For example, if total asset turnover could be increased to the industry average of 1.3 from the company's present level of 0.469 (*Total Assets/Sales* decreasing from 2.134 to 0.769), external financing needs would be eliminated. Alternatively, holding the other ratios unchanged, decreasing the payout ratio to zero decreases *REF* to \$84 million. Another trade-off is the rate of growth itself because the need for external financing is imposed by growth. If sales grow more slowly, the firm's external financing requirements decrease. We consider the link between growth and the need for external financing next.

3.2.2 Sustainable Growth

In the previous section, we took the rate of growth in sales as given and asked how much external financing would be required to support that growth. We saw that if O&R's sales grow 15% next year, the firm will need to obtain about \$172 million of external financing. However, if O&R grows at a slower rate, it will not require as much new investment and

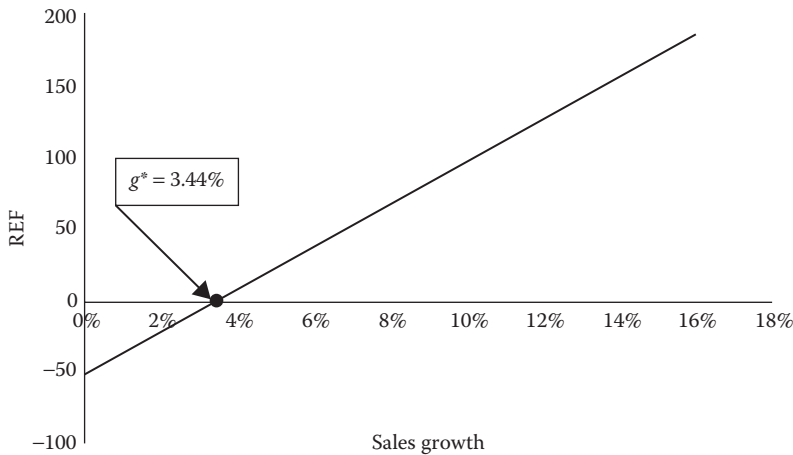


EXHIBIT 3.7 Required external financing.

external financing. This section asks how rapidly O&R could grow without having to seek external financing.

Exhibit 3.7 shows the relationship between sales growth and *REF* for O&R, with *REF* on the vertical axis and sales growth on the horizontal axis. At very low growth, *REF* is negative. Rather than requiring external funds, O&R generates funds that can be returned to its investors or used to retire debt. As the growth rate increases, the need for funds increases, so that at very high growth rates, a great deal of external financing is needed. *REF* goes from being negative for slow growth to positive for rapid growth. This implies that some growth rate exists at which *REF* is zero. At that point, all of the firm's funding needs are met from internal sources. This point is where the *REF* line crosses the horizontal axis, at which point the growth rate is about 3.44%. This tells us that, given its operating characteristics as represented by the various ratios, O&R's sales could grow 3.44% per year financed solely by trade credit and retained earnings. Although faster growth is certainly possible, it would have to be partially financed from external sources.

This leads us to the concept of sustainable growth. Sustainable growth is a rate of growth in sales that can be sustained without having to resort to external sources of financing. We consider two different versions of sustainable growth. The first, denoted by g^* , is the fastest rate at which sales can grow without forcing the firm to use any external financing. The second version, denoted by g^{**} , is the rate of sales growth that the firm can sustain while keeping its debt ratios constant and meeting its equity needs with internal equity financing. We now explain these two versions of sustainable growth.

In Exhibit 3.7, the point where the *REF* line crosses the horizontal axis is the growth rate where no external financing is required—no debt need be issued nor new shares sold. All financing needs are supplied from spontaneous liabilities and internal equity (retained earnings). In this example, the growth associated with zero *REF* is 3.44%. Setting Equation 3.21 to zero and solving for the growth rate yield the equation for sustainable growth, g^* , shown in Equation 3.21.

$$\begin{aligned}
 g^* &= \frac{\frac{\text{Net Income}}{\text{Sales}} * (1 - \text{Payout Ratio})}{\left(\frac{\text{Total Assets}}{\text{Sales}} - \frac{\text{Spontaneous CL}}{\text{Sales}}\right) - \left(\frac{\text{Net Income}}{\text{Sales}}\right) * (1 - \text{Payout Ratio})} \\
 &= \frac{(0.128) * (1 - 0.60)}{(2.134 - 0.593) - 0.128 * (1 - 0.60)} \\
 &= 0.0344 = 3.44\%
 \end{aligned}
 \tag{3.21}$$

If the firm's annual growth in sales exceeds 3.44%, outside financing (either debt or equity) will be required; the higher the growth rate, the more external funds will be required. If the firm's annual growth rate in sales is less than 3.44%, it will be able to meet its needs with spontaneous financing and internal equity financing and will be able to generate excess cash.

Confining the financing of growth to spontaneous current liabilities and internal equity may constrain the firm's growth unnecessarily. If management is willing to utilize debt financing in addition to these other sources, then greater growth may be possible without exceeding the limits of prudence. We could ask how much growth the firm could finance without increasing the proportional amount of debt in the firm's permanent capital structure. Higgins (1981) provides a formula similar to the one for g^* above that represents the highest sustainable rate of growth in sales, g^{**} , that the firm can generate and finance without changing the financing mix of debt and equity. Higgins' formula is

$$g^{**} = \frac{\left(\frac{\text{Net Income}}{\text{Sales}}\right) * (1 - \text{Payout Ratio}) * \left[1 + \left(\frac{\text{Debt}}{\text{Equity}}\right)\right]}{\left(\frac{\text{Total Assets}}{\text{Sales}}\right) - \left(\frac{\text{Net Income}}{\text{Sales}}\right) * (1 - \text{Payout Ratio}) * \left[1 + \left(\frac{\text{Debt}}{\text{Equity}}\right)\right]}
 \tag{3.22}$$

which for O&R, with a debt-to-equity ratio of 0.46, yields

$$\begin{aligned}
 g^{**} &= \frac{(0.128)(1 - 0.60)(1 + 0.46)}{2.134 - (0.128)(1 - 0.60)(1 + 0.46)} \\
 &= 0.0363 = 3.63\%
 \end{aligned}
 \tag{3.23}$$

Thus, O&R could sustain sales growth of about 3.63% per year with the assumed profit margin, payout ratio, and asset turnover by financing the expansion in such a way that the firm's total debt (both spontaneous current liabilities and discretionary debt) to total equity is maintained at its current level of 0.46 to 1.

In later years, Higgins moved to a simpler, more intuitive formula for g^{**} called *PRAT*. *PRAT* is the product of a firm's Profit margin, Retention rate, Asset turnover, and the ratio of Total assets to the previous year's equity, thus,

$$\begin{aligned}
 PRAT_t &= (\textit{Retention Ratio}) * (\textit{Return on Equity}) \\
 &= \textit{Retention Ratio} * \left(\frac{\textit{Net Income}}{\textit{Sales}} * \frac{\textit{Sales}}{\textit{Total Assets}} * \frac{\textit{Total Assets}}{\textit{Equity}_{t-1}} \right) \quad (3.24)
 \end{aligned}$$

where the latter three terms express the firm's return on equity based on the DuPont formula. There is one technical caveat to $PRAT$, however. This formula is based on the return earned on the incremental investment in retained earnings. It does not relate directly to the growth in sales, nor does it represent a limit on the growth rate. Still, it typically returns a number similar to g^{**} and is far more intuitive. For these reasons, we use $PRAT$ going forward.

It is important to keep in mind what these sustainable growth concepts mean, and what their limitations are. They are useful first approximations of the limits of the firm's long-run growth rate—its speed limit, if you will. This does not mean that the firm cannot grow more rapidly than the formulas indicate. More rapid growth is attainable. However, the firm can only grow faster by either obtaining outside financing or changing one of the limiting ratios. It can grow faster by increasing its profit margin, decreasing its dividend payout, increasing its asset turnover, increasing its spontaneous borrowing or its discretionary borrowing, or, finally, issuing new equity. To pursue more rapid growth, something has to give. These formulas help to remind the manager which items have to give.

The use of various ratios to determine required future financing and estimate growth limitations is useful. However, in more realistic applications, the formulas are just approximations that tend to gloss over the specifics of the situation. For example, when the various financial ratios are not constant from year to year, the formulas will not be exact. When costs include both fixed and variable components, profit margins will change as sales grow, and the formulas will be inaccurate. And, when the firm has excess capacity, it may be able to grow without making new investments, so it can grow more rapidly than the sustainable growth formulas indicate. To gain greater accuracy, we need more detailed models. In subsequent chapters, we develop models that allow us to analyze our plans in greater detail.

3.3 CASH FLOW

From the point of view of financial planning, financial management, and firm valuation, cash flow is usually of more importance than earnings as derived with standard accounting methods. This section defines various cash flows and explains how to calculate them. There are two lessons to be learned in this section. First, there are many different versions of cash flow; second, you need to know how to calculate the version of cash flow that is appropriate for your application.

Cash flow refers to the amount of cash that has been generated during a specified period of time. However, one must be careful defining and measuring *the amount of cash*. For example, one common method for calculating the cash flow is

$$\textit{Cash Flow} = \textit{Net Income} + \textit{Depreciation Expense} \quad (3.25)$$

We call this *Earnings Cash Flow*. This version of cash flow is based on the idea that *Net Income* understates the amount of cash flowing into the firm because Depreciation is a noncash expense and therefore should be added back to net income to determine how much cash was earned during the period.

There are, however, several difficulties with this definition of cash flow. First, this definition assumes that depreciation is the only noncash expense, when in fact there may be other expenses incurred during the period that did not involve cash payments. Second, it assumes that sales revenue actually generated cash, when in fact sales may not have brought in any cash—for example, they may have been credit sales. Third, there may be other transactions that are not reflected in the income statement that involves cash inflows and outflows. These can include items such as increases in working capital, purchases of fixed assets, repayment of debt, funds provided by borrowing, funds obtained by issuing new stock, and the payment of dividends. None of these items appear on the income statement, nor are they included in the calculation of net income. The point is, the “*Net income + Depreciation*” version of cash flow is not a complete answer to the question, *how much cash did the firm generate?* Cash flow is a lot more complicated than that.

When we ask *how much cash did the firm generate*, we should be more specific and clarify *cash for whom*, or *cash for what purpose*. For example, we could ask *how much cash was generated by sales for the purpose of paying our expenses?* The answer would involve the determination of cash payments by customers, but would be before any expenses. Or, we could ask *how much cash was generated to cover our interest expense?* This would be a very different number than the first one, and in neither case would the answer be “*Net Income + Depreciation*.” The point is that the question *how much cash did the firm generate?* must first answer *cash flow to whom?* or *cash flow for which purpose?*

In this book, we deal with several different definitions of cash flow. These include cash flow from operations, cash flow available to all the firm’s investors, and cash flow available to stockholders. There are other versions we could consider, but these are the most important ones. As noted above, the version used depends on the question being asked and the intent of the application.

In accounting, questions are usually related to what actually happened. But in financial planning, questions usually relate to what could happen, or perhaps, how much will something (usually an asset) be worth? In these cases, we frequently distinguish between the actual, the necessary, and the forecast. For example, free cash flow is a term often used in planning and valuation contexts. There are numerous definitions of free cash flow, but a commonly accepted definition is *the cash flow that is available to be distributed to investors after the firm has met all its needs for ongoing operations, including investments in working capital and fixed assets*. Central to this concept of free cash flow is the idea that some investments are necessary for the ongoing, normal operations and growth of the firm. Hence, when planning for or valuing an ongoing business, the free cash flow calculation must include these necessary investments. Failure to do so will result in significant errors in the model’s output.

We begin our discussion using the data from Odd & Rich’ financial statements in [Exhibit 3.2](#) to construct a Statement of Cash Flows in a standard accounting format.

<i>Cash flows from operating activities</i>	
Net income	127.8
Adjustments for non-cash items:	
Depreciation expense	53.8
Earnings cash flow	181.6
Subtract:	
Increase in accounts receivable	30.6
Increase in inventories	5.4
Increase in other current assets	-5.8
Funds used to increase current assets	-30.2
Add:	
Increase in accounts payable	103.6
increase in other current liabilities	-70.0
Funds provided by increase in current liabilities	33.6
Cash flow from operations	185.0
<i>Cash flows from investing activities</i>	
Purchase of fixed assets	134.4
Cash flow from investing	-134.4
<i>Cash flows from financing activities</i>	
Issuance of short-term debt	0.1
Issuance of long-term debt	35.6
Issuance of common stock	0.0
Payment of dividends	-76.7
Cash flow from financing	-41.0
Net cash flow	9.6
Plus:	
Cash balance 12/31/15	160.4
Equals	
Cash balance 12/31/16	170.0

EXHIBIT 3.8 O&R Corp. 2016 Statement of Cash Flows (\$ millions).

The Statement of Cash Flows for Odd & Rich is shown in [Exhibit 3.8](#). This *indirect format* is how cash flow usually is reported in an annual report. It starts with net income and makes adjustments to take account of other transactions that affect cash flow. Transactions are classified into operating, investing, and financing activities. Operating activities refer to transactions that are a normal part of the ongoing business. Investing activities refer to transactions that are not part of the regular operating activities, such as the purchase or sale of long-term assets. Financing activities refer to (nonoperating) transactions that raise new capital or make payments to investors (creditors, stockholders, etc.).

The purpose of the cash flow statement is to summarize all the firm's transactions in terms of their impact on the cash balance at the end of the year. Thus, the cash flow is the \$9.6 shown on the third line from the bottom. In answer to the question *cash flow for what purpose*, this statement defines cash flow as the cash flow that is available to increase the

cash balance. The answer is that during 2016, the firm generated \$9.6 million that was used to increase the cash balance. However, this definition of cash flow is not the one used in the financial planning and modeling mentioned earlier: cash flow from operations, cash flow available to the firm's investors, and cash flow available for the stockholders. In the field of finance, there is no consistent name for each of these definitions. Each version has a slightly different method of computation, depending on the author. We try to be consistent throughout this book with our terminology and computation. Still, along the way, we will learn a variety of terms used for each version of cash flow. We begin this discussion by considering the three components of the formal Statement of Cash Flows.

3.3.1 Cash Flow from Operations

Cash flow from operations refers to the cash flow generated by the normal operating activities of the firm. Consider a firm producing and selling a product. Its normal operations bring in cash from selling the product and pay out cash to produce the product, for example, to purchase raw materials, pay wages, and so on. In addition, as part of its regular operations, it may use cash to increase its cash balance, accounts receivable, and inventories; and it may use trade credit to purchase materials. The cash flow from operations segment of the Statement of Cash Flows considers nearly all these items in its calculation. These activities are shown in the first section of [Exhibit 3.8](#) except for the increase in the cash balance. [Exhibit 3.9](#) shows this calculation with the Increase in Cash explicitly omitted. The increase in cash is not shown in that section because it is shown at the bottom of the statement as the residual, the target of the definition of cash flows in the formal Statement of Cash Flows. This will not be the case when we consider the definitions for Cash Flow to Equity and Cash Flow to all Investors in the next section. Absent the change in cash, the cash flow from operations in the Statement of Cash Flows is calculated in [Exhibit 3.9](#) below.

Net Income	\$ 127.8
+ Depreciation & Other Non-cash Expenses	<u>53.8</u>
= Earnings Cash Flow	181.6
- Increase in Cash*	n.a.
- Increase in Accounts Receivable	30.6
- Increase in Inventories	5.4
- Increase in Other Current Assets	(5.8)
+ Increase in Accounts Payable	103.6
+ Increase in Current Liabilities	<u>(70.0)</u>
= <i>Cash Flow from Operations</i>	185.0
* Object or purpose of cash flow	

EXHIBIT 3.9 Cash flow from operations available to the firm (Statement of Cash Flows).

Another item generally not included in the definition of cash flow from operations is short-term debt. Although short-term debt is a current liability, it is a discretionary source of funds. This makes it a component of the cash flow from financing rather than cash flow from operations.

3.3.2 Cash Flow from Investing

The second section of [Exhibit 3.8](#) shows the Cash Flow from Investing Activities as a cash outflow of \$134.4. This refers to the investment in long-term assets such as plant and equipment or even the purchase of another company. In the format presented in that Exhibit, this investment also takes account of the replacement of depreciated assets. There are two ways to calculate this from the financial statements. These are shown in [Exhibit 3.10](#).

Both of these methods yield the total amount invested, which consists of the amount spent to replace the assets that were depreciated during the period plus the amount invested to purchase new assets beyond what was worn out. In this case, the firm spent \$134.4 million to purchase fixed assets, which consisted of \$53.8 to buy assets to replace those that wore out during the period (and thereby maintain productive capacity), and \$80.6 million to purchase assets in excess of those that wore out.

This calculation is correct assuming that depreciation expense has been included in the source of funds. In other words, if the source of funds is calculated as (*Net Income + Depreciation Expense*), then because depreciation is treated as a source, you must include an expenditure for replacement of depreciated assets as part of the fixed asset investment. On the other hand, if depreciation expense is not included in the calculation of the sources of funds, then you do not need to include the replacement as part of the expenditures. In that case, the expenditures for fixed assets would be just the increase in net fixed assets, \$80.6 million.

Method 1: Change in Gross Long-term Fixed Assets	
Gross Fixed Assets (GFA), year end 2016	1,518.0
Other Long-term assets	409.0
Gross Long-term assets, year end 2016	1,927.0
Less:	
Gross Fixed Assets (GFA), year end 2015	1,356.7
Other Long-term assets	435.9
Gross Long-term assets, year end 2015	1,792.6
Equals:	
Investment in Long-term Assets, 2016	134.4
Method 2: Change in Net Fixed Assets plus Depreciation Expense	
Net Long-term Assets (including Other Long-term Assets), year end 2016	1,424.0
Less:	
Net Long-term Assets, year end 2015	1,343.4
Expenditures for New Fixed Assets	80.6
Plus:	
Depreciation Expense, 2016	53.8
Equals:	
Investment in Fixed Assets, 2016	134.4

EXHIBIT 3.10 Two methods for calculating Investment in Fixed Assets.

The balance sheet in [Exhibit 3.2](#) is simplified. It does not show the whole range of accounts that might be shown on a more complex statement. In that case, the investment section of the cash flow statement must show the changes in all asset accounts just as was done here for fixed assets and other long-term assets.

3.3.3 Cash Flows from Financing

[Exhibit 3.8](#) shows the Cash Flow from Financing Activities as $-\$41.0$ million. This summarizes the flow of capital to and from the firm's investors, where *investors* include both the creditors and stockholders. The cash flows to the creditors from the firm include repayment of debt principal. Flows from the creditors to the firm include payments by the creditors in the form of new loans. In the current example, there was a net inflow from the creditors to the firm of $\$35.7$ million in the form of short- and long-term loans. If there had been repayments of debt principal to the lenders, it would have been shown as an outflow from the firm to the lenders. Alternatively, the payments from and to the creditors can be combined as the net flow to the creditors. Interest payments are not included here because they were already taken into account in the calculation of net income.

Flows to the stockholders include dividend payments and share repurchases. Flows from the stockholders include purchases of shares by the stockholders. The other investor flow shown in [Exhibit 3.8](#) is a dividend payment of $\$76.7$ million that is an outflow from the firm to the stockholders. If the firm repurchased shares from stockholders, the amount they paid to do so would also be shown as an outflow.

3.3.4 Other Definitions of Cash Flows

By now you are comfortable with the idea that the definition and calculation of a cash flow depend on the purpose or target of that cash flow. The purpose of the formal Statement of Cash Flows as shown in [Exhibit 3.8](#) is to summarize the cash flow *to the firm* from all sources, with the increase in the cash balance shown as a residual at the bottom because the cash account is the recipient of the cash flow into the firm. The treatment of component flows depends on the purpose of the cash flow statement, the answer to the question *cash flow to whom?* We refer to the group or entity *to whom* the cash is flowing as the *object* of the cash flow. In [Exhibit 3.8](#), the object of the cash flow is the firm, or the firm's cash balance; the cash flow available to increase the cash balance is the residual of the calculation. The firm gets whatever remains after everyone else has been paid. However, the definition or purpose for cash flow from operations changes in the definitions for Cash Flow to Equity and Cash Flow to all Investors. As the cash balance is an important part of ongoing business operations, it is not available to investors. Therefore, in the discussion of working capital that follows, cash is treated as a spontaneous current asset. The increases in current assets and current liabilities are included as uses and sources of funds, respectively, when calculating cash flow from operations.* We summarize the impact of these current

accounts on operating flow by treating the change in net working capital as a cash outflow. Net working capital is defined as

$$\text{Net Working Capital} = \text{Current Assets} - \text{Current Liabilities}$$

or

$$NWC_t = CA_t - CL_t \quad (3.26)$$

where:

NWC_t denotes net working capital

CA_t and CL_t denote current assets and liabilities at time t , respectively, and current liabilities exclude short-term debt.

The increase in NWC during period t is

$$\begin{aligned} \text{Increase in } NWC_t &= NWC_t - NWC_{t-1} \\ &= (CA_t - CL_t) - (CA_{t-1} - CL_{t-1}) \\ &= (CA_t - CA_{t-1}) - (CL_t - CL_{t-1}) \end{aligned} \quad (3.27)$$

For the year 2016, this was

$$\begin{aligned} \text{Increase in } NWC_{2016} &= (710.0 - 670.2) - (593.0 - 559.4) \\ &= 39.8 - 33.6 \\ &= \$6.2 \end{aligned} \quad (3.28)$$

where all current assets are included and current liabilities exclude short-term debt. Thus, all current account inflows and outflows can be summarized as the change in net working capital. The calculation of the cash flow from operations is summarized in [Exhibit 3.11](#).

* There is an important caveat that we need to make in this definition of cash flow: What we include in each category depends on our intended use for the cash flow calculation. For example, if we are measuring the actual cash flow, we would include all the items just as they are shown on the financial statement. However, if we want to measure the operating cash flow that potentially would be available to the investors, we would consider the *necessary*, or required, changes in these current accounts. Cash and Securities is a case in point. While cash may actually increase by \$9.6 million, the firm may need an increase of only \$3.6 million to meet its normal operating requirements. If so, \$6 million of the actual increase is excess. This excess potentially could be paid out to the firm's investors without impinging on normal operations. So, if the question is what was the actual operating cash flow, the \$9.6 million is included. If the question is "what is the cash flow from operations that are available for the investors," the answer takes account of only a necessary increase in cash balance of \$3.6 million. An extension of the same idea is that there may be other current accounts that do not relate to the normal operations of the firm. For example, if the firm has large investments in marketable securities that are not necessary for the regular operations, then these investments would not be considered in the calculation of operating cash flow. Similarly, the categories "Other Current Assets" and "Other Current Liabilities" are included above as catch-all categories. If these accounts do not relate to normal operations, they would be excluded.

Net Income	\$ 127.8
+ Depreciation & Other Non-cash Expenses	53.8
= Earnings Cash Flow	<u>181.6</u>
- Increase in Cash	9.6
- Increase in Accounts Receivable	30.6
- Increase in Inventories	5.4
- Increase in Other Current Assets	(5.8)
+ Increase in Accounts Payable	103.6
+ Increase in Current Liabilities	<u>(70.0)</u>
= <i>Cash Flow from Operations</i>	175.4

or

Net income	127.8
+ Depreciation Expense	<u>53.8</u>
= Earnings cash flow	181.6
- Increase in working capital	<u>-6.2</u>
= <i>Cash flow from operations</i>	175.4

EXHIBIT 3.11 Cash flow from operations available to all investors.

This cash flow from operations, \$175.4, differs from the Cash Flow from Operations shown in [Exhibit 3.8](#) (\$185.0) by the amount of \$9.6. This difference stems from the fact that the increase in Cash is included in the Increase in *Net Working Capital* when calculating Cash Flow from Operations to all investors, but it is not included in the Cash Flow from Operations in the formal Statement of Cash Flows ([Exhibit 3.8](#)) where it is the residual.*

During the course of the firm's normal operations, there is usually a need to increase its current assets to support the growth in sales. The increase in current assets is an investment that needs to be financed. The funds necessary to finance the increase have to come from somewhere. Part of these required funds will be supplied by increases in current liabilities. The part of the investment in current assets that is not financed by the increase in current liabilities must be financed from other sources. This is what the increase in net working capital represents—the part of the new investment in current assets that must be financed from sources other than current liabilities.

Another part of the cash flow statement also changes with the change in purpose or object to investors—the cash flow from financing. The investor group that is the object of the cash flow is left out of the financing section of the cash flow statement, and the flow available to that group is the residual of the calculation. For example, if our question is, *what is the cash flow available to be paid to the stockholders*, then the stockholders are the object of the cash flow statement. Once all the other flows are accounted for, what remains is the amount available to the stockholders. In this case, the financing section of the cash

* Reminder: The cash flow at the bottom of the Statement of Cash Flows answers the question “How much cash was generated by the firm to increase the cash balance?”

flow statement would show the flows from and to the other investors such as the creditors, and the flow available to the stockholders would be the remainder at the bottom of the statement.

3.3.4.1 Equity Cash Flow

Equity Cash Flow is the term generally used to refer to the cash flow that is available to the stockholders. Several interchangeable terms are commonly applied to the Equity Cash Flow, such as cash flow to equity, cash flow available for the stockholders, and dividend paying ability. We will probably use all of these terms at various times. All refer to the cash flow that is left for the owners after the firm's other needs for funds have been met. Think of a small business on Main Street. The cash flow to equity is the amount that the owner could take out of the business at the end of the operating period after paying all the bills, paying his creditors whatever was due them this period, and meeting all the needs for plowing money back into the business. What is left over at the end of the period belongs to the owner and can be withdrawn from the business without affecting its operations in the future. This is the equity cash flow.

We calculate equity cash flow in [Exhibit 3.12](#) using data from [Exhibit 3.8](#). The object of the cash flow calculation changes to the firm's current shareholders instead of the firm's cash balance. The blank lines represent categories that can be included, but are not present in Odd & Rich's case. Note that the equity cash flow equals the dividends that were actually paid to the stockholders. In other cases, it is possible that these would differ because some of the cash that could have been paid to the stockholder was retained as (say) marketable securities, and not paid as dividends.

Cash flow from operating activities	
Net income	127.8
+ Depreciation expense	53.8
= Earnings cash flow	181.6
- Increase in net working capital	-6.2
= Cash flow from operations	175.4
Less:	
Cash flow from investment activities	
Expenditures for net long-term assets	80.6
+ Depreciation expense	53.8
= Investment in fixed assets	-134.4
Plus:	
Cash flow from financing activities	
+ Proceeds from short-term borrowing	0.1
- Repayment of short-term principal	0
+ Proceeds from issuance of long-term debt	35.6
- Repayment of long-term principal	0
+ Proceeds from issuance of stock	0
= Cash flow from financing	35.7
Equity cash flow	76.7

EXHIBIT 3.12 Equity cash flow.

As with any of our cash flow definitions, this version of equity cash flow has variations. In this case, we calculated the equity cash flow from the point of view of the current stockholders, asking the question, *what is the cash flow available to be distributed to the current stockholders?* We included the proceeds from the issuance of stock as a cash inflow that enhanced the cash flow to equity, because the flow of funds into the firm from new stockholders adds to the cash available in this period to be paid as dividends to the current stockholders. Offsetting this positive effect of selling new shares is the fact that the ownership position of the existing stockholders would be diluted, so they would get a smaller share of the total dividends that are paid in future periods. An alternative definition of equity cash flow would not treat the proceeds from the issuance of new shares as a cash inflow. Rather, it would exclude all equity transactions and simply ask *how much cash flow was generated by the firm from all sources other than equity transactions?* For some applications, this latter definition is appropriate. However, for others, the former is best. In this book, we use the former definition where we treat the issuance of stock as a cash inflow that enhances the cash flow available for the current stockholders.

3.3.4.2 Cash Flow to Invested Capital

The cash flow to invested capital means the cash flow that is available to be paid to all the groups that have invested capital in the firm. Other terms are used to refer to this cash flow including *cash flow to investors*, *overall cash flow*, *cash flow from assets*, *enterprise cash flow*, *entity cash flow*, and *cash flow from the firm*. Regardless of the name used, this definition asks *after taking account of both the cash flow from operations and the need to invest funds in new assets, how much cash flow was generated that would be available for both the creditors and the equity investors?*

In this context, the term *creditors* refers to both long-term and short-term lenders, but does not include trade creditors or other liabilities that are not explicit debt. Generally, any debt that is interest bearing is included in this category, whether it is long term or short term. We distinguish between spontaneous credit and discretionary credit. Spontaneous credit refers to the liabilities that normally increase in step with the growth in sales, such as trade credit. Discretionary credit is credit that does not increase spontaneously and normally requires an explicit action to initiate, such as signing a promissory note. This is usually an interest-bearing debt. Short-term debt such as notes payable is considered discretionary, so we consider the short-term lenders as part of the group of permanent investors. The reason for this is that for most firms, short-term debt is constantly renewed and used on an ongoing basis, so there is always some short-term debt on the balance sheet. Hence, we treat it as a permanent source of capital.

Obviously, stockholders also are considered as investors. In addition, there may be other investors that provide permanent capital but who cannot be clearly defined as creditors or stockholders. Examples include claims that are hybrids of debt and equity such as convertible debt, preferred stock, and other more exotic securities. For our purposes, it does not really matter whether they are considered debt or equity.

Cash flow from operating activities	
Net income	127.8
+ Depreciation expense	53.8
+ Interest Expense	3.4
- Increase in net working capital	-6.2
= Cash flow from operations	178.8
Less:	
Cash flow from investment activities	
Expenditures for net long-term assets	80.6
+ Depreciation expense	53.8
= Investment in fixed assets	-134.4
Cash flow to all investors	44.4

EXHIBIT 3.13 Cash flow to investors.

We put all investors, debt and equity, in the same category, simply referring to them as *the investors*, or the providers of invested capital.

Having specified who is included in the category *investors*, we now determine the cash flow to investors, or the cash flow to invested capital (Exhibit 3.13). The cash flow to investors is the cash flow from operations plus the cash flow from investing activities. It does not yet take account of any of the payments to the creditors (interest-bearing debt) or to the stockholders.

Note that the cash flow from operations now includes a term that adds back interest expense. This is to restate net income without the effect of interest expense. Interest is part of the payment to the creditors, so in asking how much cash flow the firm generated for the creditors, we remove interest as an expense and show it as part of the funds available for the creditors. The cash flow to investors of \$44.4 is distributed in the manner shown in Exhibit 3.14.

This format is from the perspective of the investors, so the flow from the firm *to* the investors is shown as positive and *from* investors to the firm is shown as negative. From the stockholders' point of view, the dividend payment is a positive flow to them. From the lenders' point of view, interest is a flow to the lenders, and the issuance of new

Interest Paid to Lenders	3.4
- Proceeds from short-term borrowing	-0.1
+ Repayment of short-term principal	0.0
- Issuance of long-term debt	-35.6
+ Repayment of long-term principal	0.0
+ Dividends paid to stockholders	76.7
- Purchases of stock by stockholders	0.0
= Cash Flow Received by Invested Capital	\$ 44.40

EXHIBIT 3.14 Distribution of cash flows received by invested capital.

debt is a payment of \$35.7 from the lenders to the firm. On a net basis, the investors as a group (both lenders and stockholders) have received a total of \$44.4 from the firm.

There is yet another caveat to defining cash flow in terms of *cash flow for whom* or *cash flow for what purpose*? In the above calculation of cash flow to the invested capital, we added back interest expense to net income and showed it as part of the cash flow available to the investors. We did not make an adjustment for the tax deductibility of interest. Later, when we consider firm valuation and are discounting the cash flow to investors at the firm's weighted average cost of capital, we change the definition of cash flow to investors once again. At that point, we remove the effect of the tax deductibility of interest. In the calculation above, we added back interest of \$3.4 to restate cash flow from operation. However, when using the weighted average cost of capital as the discount rate, we add back $Interest(1 - T)$, where T denotes the tax rate, instead of *Interest*. We will not go into this issue any further now, but this is a warning to be prepared to encounter yet another way to calculate cash flow.

In this section, we examined the statement of cash flows and discussed various definitions of cash flow. In the next section, we consider another type of cash flow statement—the statement of cash receipts and disbursements. This will give us a tool to analyze the short-run flows of cash into and out of the firm and the potential need for short-term financing.

3.4 CASH RECEIPTS AND DISBURSEMENTS

Thus far, we have considered cash flows to a particular target—the firm, current shareholders, or all investors. We now consider a different type of cash flow, the periodic (monthly in the example that follows) cash flow that keeps the wheels of business turning.

A common problem faced by financial managers is the lag between disbursements of cash for expenses of production and receipts of cash from sales. This problem is particularly acute for firms that have a seasonal sales cycle. The lag between receipts and disbursements creates a need for seasonal financing. The financial planning problem is to anticipate these seasonal financing needs, to determine the amount of financing required, and to find the best way to meet this requirement. The following example is typical. It demonstrates seasonal financing needs by developing a statement of cash receipts and disbursements. This statement (cash budget) is applicable to a wide variety of problems, both short term and long term, but it is especially useful for considering short-term (quarterly, monthly, or even weekly or daily) cash flows.

3.4.1 Example: The Mogul Corporation

The Mogul Corporation manufactures skis that are sold on credit to retail outlets. The sales are seasonal, as retailers stock up on skis in late fall in anticipation of winter sales. The production cycle, the seasonal sales pattern, and the credit terms granted to the retailers influence the firm's financing requirements. The question is: Given the anticipated sales, what will be Mogul's financing requirements for the coming season?

The starting point to answering this question is developing a pro forma statement of cash receipts and disbursements. This requires a set of assumptions about the company's resources and constraints and forecasts for future conditions of the economy. These major assumptions are shown in [Exhibit 3.15](#); the monthly sales forecasts for the coming year are shown in line 1 of [Exhibit 3.16](#). The fiscal year starts in July and ends in June, with the sales for the previous year and the following year by following the same pattern month-to-month. Sales are at a seasonal low in June and July, start increasing rapidly in August, reach a peak in November, and then decline over the winter and spring months.

- a. Mogul Corporation manufactures skis to meet demand one month ahead of anticipated sales.
- b. Purchases of materials (line 7) are 50 percent of sales and occur two months prior to sales. Purchases are made on trade credit and are paid one month after the purchase. Thus, July purchase disbursements of \$4,000 are 50 percent of August sales.
- c. Wages and salary payments (line 8) are 20 percent of sales and are paid in the month the skis are manufactured. Thus, July wages of \$1,600 are 20 percent of August sales because the skis sold in August are manufactured in July.
- d. Rent (line 9) for the building is \$2,000 per month.
- e. Interest payments on an outstanding loan (line 10) are \$600 per month.
- f. Quarterly principal payments (line 11) of \$1,500 are due in March, June, September, and December.
- g. Quarterly payment of estimated income taxes (line 12) of \$1,000 must be made in April, June, September, and January.
- h. Miscellaneous other expenses (line 13) will amount to \$500 per month.
- i. All receipts are from sales. Ten percent of each month's sales are typically on a cash basis (line 2) and 90 percent are on credit.
- j. Forty percent of sales are collected the first month after the sale (line 3), 30 percent are collected the second month (line 4), 15 percent are collected three months after the sale (line 5), and 5 percent are bad debts and are never collected.

EXHIBIT 3.15 Assumptions used to develop the cash budget for Mogul Corp. with links to [Exhibit 3.16](#).

[Exhibit 3.16](#) shows the cash receipts and disbursements statement showing the cash flows and required financing over a one-year planning horizon that is divided into monthly subperiods. The sales forecasts are the model's primary input. They *drive* the calculation of the firm's financing needs, the output that is the focus of this exercise. This statement, however, is only a first approximation. It is not yet complete because it does not show interest expenses related to financing the deficit. This is because we first need to work through the cash budget to determine how much must be borrowed before calculating the financing costs. We take financing costs into account when we explicitly consider the form of financing. Similarly, any earnings on investment of excess cash are not yet considered in this preliminary statement. At this early stage, we only seek to develop a preliminary estimate of Mogul's short-term financing needs.

Notice that noncash expenses such as depreciation are not included in the cash budget. This is because they are noncash items. Similarly, no bad debt expense appears in the model because bad debts are cash not collected. Instead, bad debt is captured by sales never collected.* Also, in order to show the disbursements for June that depend on

* The sum of cash collected is $0.95 = 0.10 + 0.40 + 0.30 + 0.15$, leaving 0.05 unrecognized.

Mogul Corporation												
Cash budget												
Revised to include debt balance												
	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
1 Sales	3,000	8,000	18,000	40,000	60,000	35,000	20,000	15,000	5,000	3,000	2,000	1,000
Receipts												
2 Cash sales	300	800	1,800	4,000	6,000	3,500	2,000	1,500	500	300	200	100
Credit sales *												
3 First month	400	1,200	3,200	7,200	16,000	24,000	14,000	8,000	6,000	2,000	1,200	800
4 Second month	600	300	900	2,400	5,400	12,000	18,000	10,500	6,000	4,500	1,500	900
5 Third month	450	300	150	450	1,200	2,700	6,000	9,000	5,250	3,000	2,250	750
6 Total Receipts	1,750	2,600	6,050	14,050	28,600	42,200	40,000	29,000	17,750	9,800	5,150	2,550
Disbursements												
7 Purchases	4,000	9,000	20,000	30,000	17,500	10,000	7,500	2,500	1,500	1,000	500	1,500
8 Wages/salary	1,600	3,600	8,000	12,000	7,000	4,000	3,000	1,000	600	400	200	600
9 Rent	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000	2,000
10 Interest	600	600	600	600	600	600	600	600	600	600	600	600
11 Debt principal	0	0	1,500	0	0	1,500	0	0	1,500	0	0	1,500
12 Taxes	0	0	1,000	0	0	0	1,000	0	0	1,000	0	1,000
13 Misc.	500	500	500	500	500	500	500	500	500	500	500	500
14 Total Disbursements	8,700	15,700	33,600	45,100	27,600	18,600	14,600	6,600	6,700	5,500	3,800	7,700
15 Net Cash Flow	-6,950	-13,100	-27,550	-31,050	1,000	23,600	25,400	22,400	11,050	4,300	1,350	-5,150
16 Beginning Cash	5,000	-1,950	-15,050	-42,600	-73,650	-72,650	-49,050	-23,650	-1,250	9,800	14,100	15,450
17 Cumulative Cash	-1,950	-15,050	-42,600	-73,650	-72,650	-49,050	-23,650	-1,250	9,800	14,100	15,450	10,300
18 Desired cash	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000
19 Required financing	-6,950	-20,050	-47,600	-78,650	-77,650	-54,050	-28,650	-6,250	0	0	0	0

EXHIBIT 3.16 Mogul cash receipts and disbursements.

sales forecasted beyond June and the receipts on credit accounts that depend on sales prior to July, we assumed that the past months' and future months' (not shown) follow the same monthly sales pattern as the current cycle.

Net cash flow ([Exhibit 3.16](#), line 15) equals total receipts (line 6) less than total disbursements (line 14). Net cash flow plus the cash balance at the beginning of the month yields the month-end cash balance. Cumulative cash (line 17) is calculated as if no additional funds are obtained. Thus, cumulative cash shows what the cash balance would be at the end of the month if no funds were borrowed (or invested). The cumulative cash balance of (\$1,950) at the end of July indicates that a cash deficit of \$1,950 must be financed to restore the balance to zero. In addition, because the firm maintains an operating cash balance of \$5,000, the total required financing (line 19) for July is \$6,950.

The cumulative cash item carries the deficit (or surplus) from month-to-month. In other words, the deficit in July is \$1,950, in August it is \$13,100, and the total of these two amounts is \$15,050. This cumulative deficit plus the \$5,000 desired cash balance means that a total of \$20,050 must be financed in August—\$6,950 left over from July, plus an additional loan of \$13,100 to cover the August deficit.

This cumulative required financing is shown as required financing (line 19). Required financing increases from July through October, reaching a peak of \$78,650. This \$78,650 represents the total amount of financing that will be necessary over the one-year production cycle. Required financing decreases after October because the firm's net cash flow becomes positive. This allows some of the financing to be repaid, so required financing decreases. For example, using November's net cash flow of \$1,000 to repay the loan reduces required financing by \$1,000. By March, there will have been sufficient cash inflow to repay any loans and to build up the cash balance. Indeed, by the end of March, the required financing is zero and there is excess cash of \$4,800 (indicating that the cash balance would be \$4,800 in excess of the \$5,000 desired cash balance).

[Exhibits 3.17](#) and [3.18](#) show the funding needs and cash flows from [Exhibit 3.16](#) in graphical format. [Exhibit 3.17](#) shows the month-by-month funding needs and funds available. The additional required financing represents the marginal deficit for each month that must be funded either through borrowing or through drawing down the cash balance (as in June). Funds available represent the funds that can be used to repay debt, build up the cash balance, or invest in securities. [Exhibit 3.18](#) shows the cumulative required financing or excess cash. Required financing builds up to \$78,650 in October and then decreases over the following months, reaching zero in March (assuming net cash inflows are used to repay the debt).

The rapid increase in required financing results from expenses leading (occurring in advance of) sales and receipts lagging (occurring after) sales. The lead of expenses and lag of receipts magnify the deficits and extend their duration. This suggests that management should consider actions that can be taken to narrow the time gap between the disbursements and receipts. In terms of the ingredients of financial planning, these actions are the decision alternatives that must be specified.

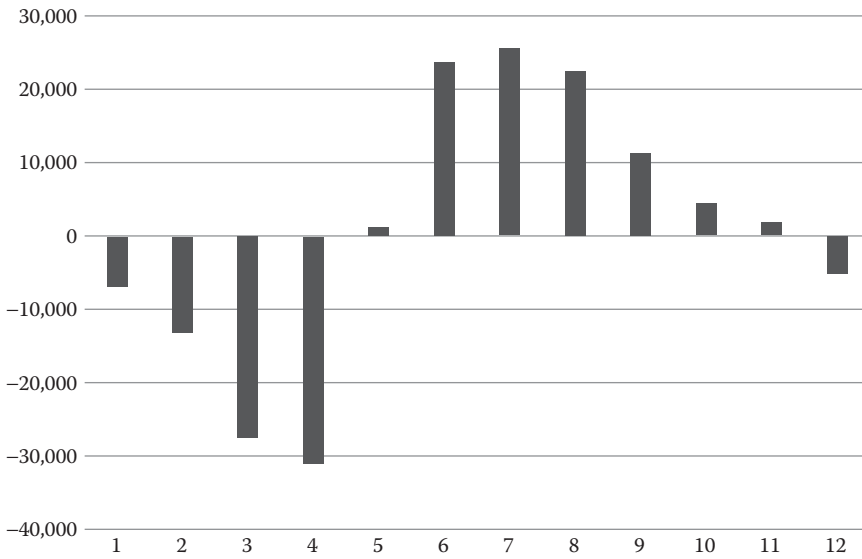


EXHIBIT 3.17 Required financing, Mogul Corp.

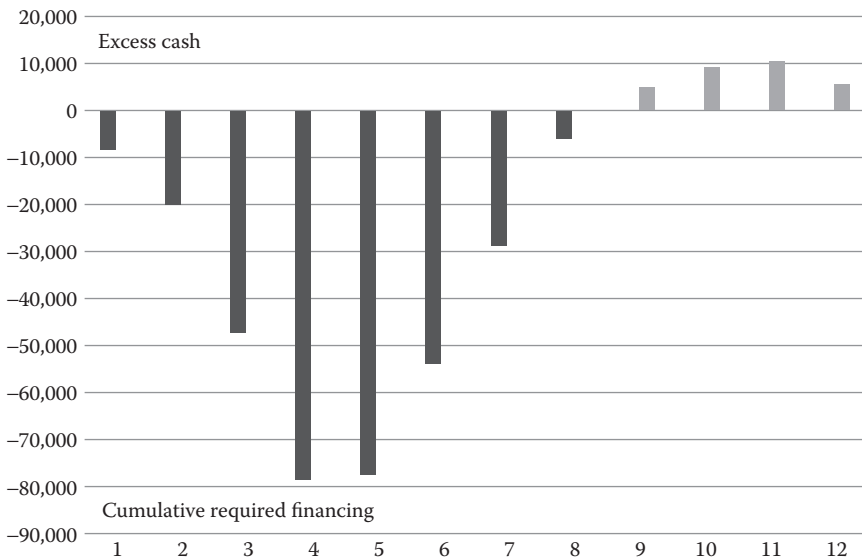


EXHIBIT 3.18 Cumulative required financing and excess cash, Mogul Corp.

Note the critical role played by the length of the planning horizon (one year in this example) and the length of the subperiods (months in this case). If the analyst had looked only at the annual figures, total receipts would have been \$199,500, total disbursements would have been \$194,200, and annual net cash flow would have been \$5,300. The annual figures hide the fact that O&R will require short-term financing of \$78,650 over

the cycle. By neglecting to consider the monthly cash flows, the analyst would have been completely unprepared to meet the firm's financing needs. An unprepared firm just starting out might open for business in July and might shut down in September or October due to lack of funds.

We previously noted that planning involves more than forecasting; planning is the specification of future decisions and actions to reach a stated goal. In terms of this definition, the cash-budgeting problem just analyzed was not really planning; rather, it was forecasting. It forecasts future required financing based on a forecast of sales and a set of assumptions about how customers pay their accounts and how the firm produces and pays for the goods it sold. Planning would extend this simple forecast of financing needs to explore alternative policies and decisions so as to determine the best way to reach the firm's goal. This requires that the goal and the various decision alternatives be specified and analyzed.

It is generally agreed that with respect to financial decisions, the proper objective is to maximize firm value (make the market value of the firm as large as possible). We make this broad objective operational by specifying a set of goals that lead to value maximization. For present purposes, assume that the goal is to make decisions that maximize the present value of the net cash flow available to equity over the planning horizon. In the context of this example, the net cash flow available to equity would be the amount of cash flow available that could potentially be paid to the firm's owners after all expenses and after servicing any debt associated with financing the seasonal cash deficits.* As we are calculating the present value of net cash flows to equity, the appropriate discount rate is the firm's cost of equity. It is estimated to be 20% and is assumed not to vary by any meaningful amount under the different decision alternatives.

The analysis begins by considering the effect of the present policy on the goal. This requires determining the present value of the net cash flows given the present policy, against which the present value of the net cash flows under the alternative policy will be compared. [Exhibit 3.19](#) is a revised cash budget based on [Exhibit 3.16](#). Total receipts from [Exhibit 3.16](#) are shown as the first line of [Exhibit 3.19](#), with the line number 6 on the left indicating that this matches line 6 of [Exhibit 3.16](#); similarly, the second line of [Exhibit 3.19](#) reproduces the total disbursements, line 14, of [Exhibit 3.16](#).

We now explicitly take account of the costs of financing. We assume the cost of borrowing is 10% per year with interest paid monthly in arrears, and that debt is repaid from any excess cash held above the desired level of \$5,000. Thus, line a shows the interest that would be paid for funds borrowed. The \$58 interest paid in August is one month's interest on the \$6,950 borrowed in July ($\$6,950 \times 0.10/12 = \58). Funds Borrowed is shown in line d, with negative values beginning in November indicating debt repayment. The net increase in cash, shown on line e, represents the cash available from operations after paying interest and principal on the funds borrowed, and is the amount that potentially

* The net cash flows to equity are shown on the Net Increase in cash line, line "e" in [Exhibits 3.19](#) and [3.20](#).

Mogul Corporation												
Cash budget												
Revised to include debt balance												
	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
6 Total receipts	1,750	2,600	6,050	14,050	28,600	42,200	40,000	29,000	17,750	9,800	5,150	2,550
14 Total disbursement	8,700	15,700	33,600	45,100	27,600	18,600	14,600	6,600	6,700	5,500	3,800	7,700
a. Incremental interest	0	58	168	399	661	658	467	259	74	0	0	0
b. Collection costs	0	0	0	0	0	0	0	0	0	0	0	0
c. Net cash flow	-6,950	-13,158	-27,718	-31,449	339	22,942	24,933	22,141	10,976	4,300	1,350	-5,150
d. Funds borrowed	6,950	13,158	27,718	31,449	-339	-22,942	-24,933	-22,141	-8,918	0	0	0
e. Net increase in cash	0	0	0	0	0	0	0	0	2,058	4,300	1,350	-5,150
f. Beginning cash	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	7,058	11,358	12,708
g. Ending cash	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	7,058	11,358	12,708	7,558
h. Loan balance	6,950	20,108	47,825	79,274	78,935	55,992	31,059	8,918	0	0	0	0
i. Present value of net increase in cash		\$2,320										

EXHIBIT 3.19 Mogul cash budget under current credit policy.

could be paid to the firm's owners after all expenses and debt servicing without decreasing the cash balance or requiring other financing. The present value of the net increase in cash, \$2,320 (line i), represents the contribution of this 12-month period's operations to the value of the equity of the firm. It serves as the benchmark for evaluating alternative policies.

There are numerous alternative policies and decisions that could be evaluated to determine the one that is most likely to help the firm reach its objective. These alternatives include changing credit terms to shorten the lag between sales and collections, changing the production schedule and purchasing and payment policies so as to decrease the lead of disbursements over sales, and considering various ways to fund the financing requirements. Not all decision alternatives will be analyzed here. Rather, just one policy alternative is considered as an example to demonstrate the approach.

The policy change to be evaluated in this case involves an effort to speed up collection of receivables, thereby narrowing the gap between sales and collections. Ten percent of sales would be cash, and 90% would be credit sales, with 70% of sales collected one month after sales, 15% collected in the second month, and the 5% remaining uncollected as bad debt. In addition, accelerating collections will cost 2% of sales per month. As with the base case, the required financing will be funded with a revolving line of credit at a cost of 10% per year with interest paid monthly in arrears and with debt repaid out of the positive net cash flows.

[Exhibit 3.20](#) shows the receipts and disbursements for this alternative. The line numbers correspond to those of [Exhibit 3.16](#), with line 6 showing receipts and line 14 showing the total disbursements. Line a shows the incremental interest that must be paid for the funds borrowed, analogous to line a in [Exhibit 3.19](#); and line b shows the additional 2% collection costs associated with the increased collection efforts. The net increase in cash (line e) corresponds to line e of [Exhibit 3.19](#) and represents the cash flow available to equity associated with this policy. The present value of the net increase in cash is \$207, far less than the current policy's \$2,320. Hence, despite more rigorous efforts to accelerate the collections, the present value of receiving these funds sooner is not sufficient to offset the cost of the collection effort. The old policy should be retained. Alternatively, other credit policies could be tested.

This example shows the usefulness of the cash receipts and disbursements statement. It helps to focus attention on the potential financing needs over the planning horizon, showing when financing will be necessary, how much will be necessary, and when it can be repaid. Cash budgets can be used to analyze many types of financial planning problems, regardless of the length of the planning horizon. They are particularly suited to short planning horizons that are broken up into shorter periods such as days, weeks, or months, as in the foregoing example. In cases where there is a seasonal pattern to the flow of funding, the cash budget is an ideal tool.

Mogul Corporation												
Cash budget												
Revised to include debt balance												
	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
6 Total receipts	1,300	3,050	7,850	17,800	36,700	51,500	35,500	20,750	14,000	6,050	3,050	1,950
14 Total disbursement	8,700	15,700	33,600	45,100	27,600	18,600	14,600	6,600	6,700	5,500	3,800	7,700
a. Incremental interest	0	62	169	388	626	565	302	133	19	0	0	0
b. Collection costs	60	160	360	800	1,200	700	400	300	100	60	40	20
c. Net cash flow	-7,460	-12,872	-26,279	-28,488	7,274	31,635	20,198	13,717	7,181	490	-790	-5,770
d. Funds borrowed	7,460	12,872	26,279	28,488	-7,274	-31,635	-20,198	-13,717	-2,276	0	0	1,165
e. Net increase in cash	0	0	0	0	0	0	0	0	4,905	490	-790	-4,605
f. Beginning cash	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	9,905	10,395	9,605
g. Ending cash	5,000	5,000	5,000	5,000	5,000	5,000	5,000	5,000	9,905	10,395	9,605	5,000
h. Loan balance	7,460	20,332	46,612	75,100	67,826	36,191	15,993	2,276	0	0	0	1,165
i. Present value of net increase in cash											\$207	

EXHIBIT 3.20 Mogul cash budget under tighter credit policy.

3.5 CONCLUSION

This chapter introduced several standard tools and analytical methods of financial planning. These included *pro forma* financial statements, estimating *REF* by relating investment and financing to the growth in sales, sustainable growth, calculating the numerous versions of cash flow, and developing a cash receipt and disbursement statement.

None of these techniques really requires a computer with a spreadsheet program such as Microsoft's Excel, but spreadsheet programs make building, using and sharing models far easier. The remainder of this book is concerned with the methods for constructing models for financial planning. The next chapter builds on the concepts of projecting future financial statements, the corner stone of properly constructed financial planning models.

PROBLEMS

1. Cash Flows: Odd & Rich Corporation

Problem 1 of [Chapter 2](#) asked you to develop a template model for financial ratios using the data from the Odd & Rich Corporation. Now you are asked to expand on that model, so it will generate cash flow statements for the company. Show 3 different cash flow statements for Odd & Rich: (a) cash flow in the accounting format of [Exhibit 3.8](#) with the cash balance as the residual; (b) cash flow to the equity investors as was shown in [Exhibit 3.12](#); and (c) cash flow to all investors as was shown in [Exhibit 3.13](#). As with the assignment in [Chapter 2](#), your model should be a template with only labels and formulas in the template, and the data in a separate section of the sheet, or on a different sheet.

2. Cash Budgeting: Taurus Corporation

The Taurus Corporation has a seasonal sales pattern with sales reaching their peak in late summer, and the low point occurs in the winter. The manager plans to arrange a line of credit from the bank to meet its seasonal cash needs. One question is: how big should the line of credit be? The manager has asked you to prepare a *pro forma* cash budget to determine the firm's financing needs. In addition, he wants you to evaluate the effects of a possible change in credit policy.

Facts that need to be considered in this analysis are as follows:

The current date is June 1, 2010.

Taurus manufactures two products, Product A and Product B. The sale price of A is \$22 per unit, and the sale price of B is \$15 per unit. Past and future expected future unit sales are as follows:

	April 2010	May	June	July	August
Product A	1500	1700	2500	3500	4000
Product B	2100	2790	3100	3850	3000
	September	October	November	December	January 2011
Product A	2500	2100	1700	1300	1000
Product B	3000	2700	2200	1500	1200
	February	March	April	May	June
Product A	800	1500	2000	2500	3000
Product B	1600	2100	2300	3100	3900
	July	August	September		
Product A	3800	4200	3000		
Product B	4000	3500	2200		

Each month's sales occur at a constant daily rate. In other words, each day's sales are 1/30 of the total monthly sales. Five percent of sales are on a cash basis, and 95% of the sales are to credit customers.

The firm grants a 2% discount for accounts paid within 20 days. Otherwise, the customers are supposed to pay the full invoice amount within 30 days. Of those who buy the firm's product on credit, 15% pay within the 20-day discount period, 50% pay on the 30th day following the sale, 30% ignore the 30-day terms and pay 45 days after the sale, and the remaining 5% never pay and are written off as bad debts 60 days after the sale.

Cost of raw materials purchased for production is 60% of sales, with the purchase occurring two months in advance of sales. SSC pays for its purchase of raw materials in the month following the date of the purchase and delivery.

Wages and salaries consist of a fixed component that amounts to \$20,000 per month, plus a variable component that is 5% of that month's sales. Wages and salaries are paid in the month that they occur. Property rental is \$4,000 per month. In addition, in June, a payment of \$50,000 must be made to buy back the stock of a minority stockholder.

The current cash balance is \$3,000, but management has determined that it wants to maintain a cash balance of at least \$20,000 at all times.

If Taurus uses a bank line of credit to fund its cash shortage, it will pay 8% (annual rate) interest each month that there is a positive loan balance. The interest paid in a given month is based on the loan balance outstanding at the end of the previous month. The lender will require that Taurus maintain a compensating balance in its cash account of at least 15% of the loan balance, and loan principal can be repaid only in the months of March, June, September, and December. Assume that if the company has excess cash, it will use it to repay debt in the months allowed. If it has excess cash in the other months, it invests it in

marketable securities that yield 6% annually. It can liquidate these securities to repay debt in the months allowed.

Assignment

- A. Use a spreadsheet program to prepare a cash budget for the next 12 months (June–May) that shows how much will have to be borrowed in each month to maintain a constant cash balance of at least \$20,000. Your model should reflect any interest paid on borrowing.

Your model should consist of two modules: A Formula Module and a Data Module. The Formula Module should show only the formulas, with none of the data (numbers) shown in the formulas. The Data Module will contain all the numbers that represent parameters, forecasts, and assumptions.

- B. The company's collection of accounts receivable is regarded as a little slow. In your spread sheet, evaluate the desirability of the following change in credit policy: The credit customers will be granted a 3% discount if they pay in 10 days, otherwise they are supposed to pay in 30 days. In addition, the company will increase its collection efforts to accelerate the payments of its other customers. The additional collection efforts will cost \$2,500 per month. The new 3/10 net 30 policy will result in 25% of the credit customers paying in 10 days and taking the discount. With the additional collection effort, 60% will pay in 30 days, 12% will pay in 45 days, and write-offs will drop to 3%. Is this change beneficial? What will be the effect on the firm's required borrowing?

3. Planning For Growth: Fly-by-Night Airlines, Inc.

The Fly-by-Night Airlines, Inc., is a mid-cap regional airline company that operates on the west coast. It is a point-to-point carrier that competes with Southwest Airlines and United Express, among others. The income statement and balance sheets for 2007 through 2010 appear in [Exhibit 3.21](#). Assume that the various average financial ratios indicated by these statements will prevail in future years.

Senior management of Fly-by-Night has asked you to analyze their capacity for growth. Over the next five years, they would like to expand sales at a rate of 20% per year. However, they are not sure about the implications of this growth. They have posed several questions that they would like you to answer.

- A. As a first step, management has requested that you create a common-size income statement and balance sheet (percent of sales for both) in the same format as the given accounting statements. In other words, your common-size statements will look exactly like the given statements except that they will have percentages of sales rather than dollar amounts in the cells. The exceptions (marked by an *) are that Depreciation expense is a percentage of the previous year's Plant, Property, and Equipment; Interest expense is a percentage of last year's total debt; and that

Fly-by-night Airlines - Income statement (000)					Percent of sales
	2013	2014	2015	2016	2013-2016
Sales	1,592,200	1,739,400	1,897,700	2,082,000	
COGS	958,700	951,600	961,700	1,103,900	54.7%
SG&A	477,000	580,500	649,900	693,400	32.7%
Gross income	156,500	207,300	286,100	284,700	
Depreciation	65,500	66,300	73,000	83,800	5.6% *
Amortization	2,000	2,000	2,100	1,000	
Operating Income	89,000	139,000	211,000	199,900	8.6%
Interest Expense	38,400	33,600	21,200	16,300	4.8% *
Interest Income	13,700	18,200	14,600	37,100	
Income before taxes	64,300	123,600	204,400	220,700	
Taxes	26,300	51,200	80,000	86,500	40.2% *
Net Income	38,000	72,400	124,400	134,200	
Shares outstanding	14,241	14,785	23,388	26,372	
Fly-by-night Airlines - Balance sheet (000)					Percent of sales
Assets	2013	2014	2015	2016	2013-2016
Cash & equivalents	101,800	212,900	306,600	329,000	12.6%
Accounts receivable	69,700	72,800	70,600	75,600	4.0%
Inventories	47,800	47,200	44,100	54,300	2.7%
Prepaid expenses	80,900	92,100	107,500	124,000	5.5%
Current assets	300,200	425,000	528,800	582,900	
Plant, property, and equip	1,189,700	1,334,400	1,478,300	1,954,100	80.8%
Accumulated depreciation	326,300	373,800	417,000	486,700	
Net fixed assets	863,400	960,600	1,061,300	1,467,400	
Intangibles	61,600	59,600	57,500	56,500	
Other assets	86,200	88,300	84,200	76,300	4.6%
Total assets	1,311,400	1,533,500	1,731,800	2,183,100	91.7%
Liabilities and shareholder equity					
Accounts payable	65,400	73,900	84,300	104,200	4.5%
Other current liabilities	349,300	370,700	414,400	448,000	21.7%
Short term debt	47,000				
Current portion l.t. debt	24,100	28,700	27,200	66,500	
Total current liabilities	485,800	473,300	525,900	618,700	
Long-term debt	503,600	512,200	317,200	486,700	
Deferred taxes	49,500	72,300	99,200	144,000	65.1% *
Total liabilities	1,038,900	1,057,800	942,300	1,249,400	
Common stock	17,200	22,000	30,400	31,300	
Paid in capital	166,800	292,800	473,800	482,900	
Retained earnings	88,500	160,900	285,300	419,500	
Total shareholder equity	272,500	475,700	789,500	933,700	
Total liabilities and equity	1,311,400	1,533,500	1,731,800	2,183,100	*Not percent of sales

EXHIBIT 3.21 Fly-by-Night Airlines Financial Statements.

Deferred Taxes are a percentage of last year's Taxes. No portion of long-term debt becomes current over the next five years. Verify that the percentages given are indeed the four-year averages (three years for the lagged ratios of depreciation expense, interest expense, and deferred taxes) of the values in your common-size income statement and balance sheet.

Based on the Fly-by-Nights' 2010 accounting statements, the questions that management would like answer are

- B. How rapidly could sales grow if they use no discretionary debt financing in future years and meet all their equity financing needs using internal sources. Assume that they do not change their dividend payout ratio (0%) from its last level. Use the g^* formula.
- C. Approximately how rapidly could sales grow if they utilize debt financing at a level such that they maintain their most recent total debt-total capital ratio ($[\text{current liabilities} + \text{debt}]/[\text{Total assets}]$), and all their equity needs are met from retained earnings. Use the *PRAT* formula.
- D. What would happen to g^* and *PRAT* if Fly-by-Night decided to start paying out 30% of their net income as dividends.
- E. Assume that they could change their debt-to-total capital and their dividend payout ratios to new levels that would be maintained at that new level in future years, and that the other ratios (net profit margin and asset turnover) do not change. Could they handle the desired 20% growth without changing their capital structure thereafter? (Hint: use *PRAT* for your analysis.)

What are the different combinations of dividend payout and total asset-to-equity ratios that could be used to support a 20% annual growth in sales? Graph this relationship. Let the horizontal axis be the payout ratio and the vertical axis be the asset/equity ratio. Show the combinations of these ratios that will support 20% growth according to the *PRAT* formula.

- F. Assume: (a) the firm's operating profit margin ($EBIT/Sales$) is maintained in future years; (b) the interest rate on debt new debt (both short- and long-term) is 4.8%; (c) no portion of the outstanding debt becomes current for the next five years; (d) the payout ratio is held at 0%; (e) no new external equity financing is used (any equity financing that is used comes entirely from retained earnings); (f) other ratios (turnover, etc.) are held constant, but net profit margin is allowed to change because of the effect of interest expense; (g) intangibles, other assets, and deferred taxes are held constant; and (h) assets necessary to support growth in year 2002 must be in place at the first of the year (same for 2003).

Approximately what amount of external financing will be required if the sales grow at 20% per year over the next two years? What will this do to their net profit margins in years 2011, 2012, and 2013?

Note: Do not use pro forma statements to answer these questions. Rather, use the formulas from this chapter along with the average ratios calculated in part 1.

- G. If Fly-by-Night were able to grow their sales more rapidly, would it be more valuable? Give a brief explanation and discussion.

What you turn in: A memo that answers all of the above questions, including the requested graph, in a well-organized, logical manner. Place the common-size statements with average ratios in an appendix.

4. Planning For Growth At Medtric, Inc.

Medtric, Inc. is a large-cap medical equipment company that manufactures devices to treat a variety of cardiac problems. Its income statements and balance sheets for 2012 through 2016 appear in [Exhibit 3.22](#); a variety of average financial ratios derived from them appear in [Exhibit 3.23](#). The reported ratios are three-year averages, taken over the years 2014–2016. You will find the data for this assignment in the Problem 3 data workbook on the Unit 2 Problem 3 page in our Canvas shell ([Exhibits 3.22](#) and [3.23](#)).

Senior management has asked you to analyze Medtric’s capacity for growth. Medtric’s sales have increased steadily over the past four years, in no small part due to an aggressive acquisition program. These recent acquisitions have set the stage for faster growth beginning in 2017. The question being considered is *how fast*? Two factions on Medtric’s board and in Medtric’s management are engaged in a vigorous debate about future revenue growth rates and dividend payout targets. The moderate growth faction is arguing for a revenue growth rate of 12% per year and a dividend payout of 35% of net income, both of which they hope can be funded internally. This position is based on the belief that the acquisition program over the last several years has filled Medtric’s new product pipeline sufficiently to support 12% revenue growth, and that Medtric’s cash flow is enough to fund both this growth rate and an increase in dividends. The other faction argues that Medtric should expand its acquisition program due to the many interesting potential targets that currently exist. If so, they expect revenues to grow 20% per year. They believe that Medtric has enough cash flow from operations and sufficient borrowing capacity to fund this program, especially if the dividend payout is decreased to 20% of net income. However, neither faction is certain about the overall financial implications of these goals. They have posed several questions for you to answer as you evaluate the two proposed scenarios.

PART I

How does Medtric’s current performance (measured by its 2016 financial statements) compare to the performance of the average firm in its industry sector as shown below?

Sector ROE Decomposed

$$ROE_t = \frac{\text{Net Income}_t}{\text{Sales}_t} * \frac{\text{Sales}_t}{\text{Total Assets}_t} * \frac{\text{Total Assets}_t}{\text{Equity}_t}$$

$$9.42\% = 8.57\% * 0.70 * 1.57$$

Medtric, Inc.					
Annual Income Statement					
In Millions of U.S. Dollars					
	2012	2013	2014	2015	2016
Total Revenue	10,054.6	11,292.0	12,299.0	13,515.0	14,599.0
Cost of Goods Sold	2,446.4	2,815.3	3,160.0	3,442.0	3,518.0
Gross Profit	7,608.2	8,476.7	9,139.0	10,073.0	11,081.0
Selling, General & Admin. Expenses	3,213.6	3,659.4	4,153.0	4,707.0	5,152.0
Research & Development	951.3	1,112.9	1,239.0	1,275.0	1,355.0
EBDIT	3,443.3	3,704.4	3,747.0	4,091.0	4,574.0
Depreciation Expense	127.0	143.0	334.0	275.0	86.0
EBIT	3,316.3	3,561.4	3,413.0	3,816.0	4,488.0
Interest Expense	101.0	122.0	395.0	307.0	347.0
Interest Income	100.2	107.0	179.0	96.0	60.0
Unusual Expense	654.4	463.8	174.0	879.0	1,555.0
EBT	2,661.1	3,082.6	3,023.0	2,726.0	2,646.0
Income Tax	739.6	614.6	713.0	654.0	425.0
Net Income After Taxes	1,921.5	2,468.0	2,310.0	2,072.0	2,221.0
Gross Dividends - Common Stock	404.9	464.8	504.0	565.0	843.0
Retained Earnings	1,516.6	2,003.2	1,806.0	1,507.0	1,378.0
Medtric, Inc.					
Annual Balance Sheet					
In Millions of U.S. Dollars					
	2012	2013	2014	2015	2016
Cash	2,232.2	2,994.3	1,256.0	1,060.0	1,271.0
Short Term Investments	1,159.4	3,107.1	1,822.0	553.0	405.0
Receivables	2,292.7	2,429.0	2,737.0	3,360.0	3,123.0
Inventory	981.4	1,176.9	1,215.0	1,280.0	1,426.0
Other Current Assets	755.8	669.3	888.0	1,069.0	1,235.0
Total Current Assets	7,421.5	10,376.6	7,918.0	7,322.0	7,460.0
Property/Plant/Equipment - Gross	3,628.6	3,794.1	4,309.0	4,743.0	4,887.0
Accumulated Depreciation	1,769.3	1,913.0	2,247.0	2,522.0	2,608.0
Property/Plant/Equipment - Net	1,859.3	1,881.1	2,062.0	2,221.0	2,279.0
Goodwill & Intangibles	5,299.2	5,937.6	5,760.0	9,712.0	10,672.0
Other Long Term Assets	2,037.4	1,469.5	3,772.0	2,943.0	3,250.0
Total Assets	16,617.4	19,664.8	19,512.0	22,198.0	23,661.0
Accounts Payable	371.8	318.6	282.0	383.0	382.0
Notes Payable	478.6	2,436.8	504.0	1,143.0	508.0
Other Current liabilities	2,529.6	1,650.4	1,772.0	1,998.0	2,243.0
Total Current Liabilities	3,380.0	4,405.8	2,558.0	3,524.0	3,133.0
Total Long Term Debt	1,973.2	5,486.3	5,578.0	5,802.0	6,772.0
Other Long Term Liabilities	964.7	545.0	337.0	1,050.0	703.0
Total Liabilities	6,317.9	10,437.1	8,473.0	10,376.0	10,608.0
Common Stock	121.0	115.5	114.0	112.0	112.0
Retained Earnings	10,178.5	9,112.2	10,925.0	11,710.0	12,941.0
Treasury Stock	-	-	-	-	-
Total Equity	10,299.5	9,227.7	11,039.0	11,822.0	13,053.0
Total Liabilities & Shareholders' Equity	16,617.4	19,664.8	19,512.0	22,198.0	23,661.0

EXHIBIT 3.22 Medtric, Inc. financial statements.

Income Statement (% Sales*)	
	3-year Average
Total Revenue*	0.089
Cost of Goods Sold	0.251
Gross Profit	0.749
Selling, General & Admin. Expenses	0.346
Research & Development	0.096
EBDIT	0.307
Depreciation Expense*	0.057
EBIT	0.289
Interest Expense*	0.050
Interest Income*	0.033
Unusual Expense	0.062
EBT	0.210
Income Tax*	0.212
Net Income After Taxes	0.164
Gross Dividends - Common Stock*	0.290
Retained Earnings	0.118
*Unless otherwise noted	
Balance Sheet (% Sales)	
	3-year Average
Cash	0.089
Marketable Securities	0.072
Receivables	0.228
Inventory	0.097
Other Current Assets	0.079
Total Current Assets	0.566
Property/Plant/Equipment, Gross	0.345
Accumulated Depreciation	0.183
Property/Plant/Equipment, Net	0.163
Goodwill and Intangibles	0.639
Other Long Term Assets	0.249
Total Assets	1.617
Accounts Payable	0.026
Notes Payable	0.053
Other Current liabilities	0.149
Total Current Liabilities	0.228
Long Term Debt	0.449
Other Liabilities, Total	0.051
Total Liabilities	0.728
Common Stock, Total	0.008
Retained Earnings (Accumulated Deficit)	0.880
Treasury Stock	0.000
Total Equity	0.889
Total Liabilities & Shareholders' Equity	1.617

(a)

Balance Sheet (% Assets)	
	3-year Average
Cash	0.055
Marketable Securities	0.045
Receivables	0.141
Inventory	0.060
Other Current Assets	0.049
Total Current Assets	0.350
Property/Plant/Equipment, Gross	0.214
Accumulated Depreciation	0.113
Property/Plant/Equipment, Net	0.101
Goodwill and Intangibles	0.395
Other Long Term Assets	0.154
Total Assets	1.000
Accounts Payable	0.016
Notes Payable	0.033
Other Current liabilities	0.092
Total Current Liabilities	0.141
Long Term Debt	0.278
Other Liabilities, Total	0.031
Total Liabilities	0.450
Common Stock, Total	0.005
Retained Earnings (Accumulated Deficit)	0.545
Treasury Stock	0.000
Total Equity	0.550
Total Liabilities & Shareholders' Equity	1.000

(b)

EXHIBIT 3.23 (a) Medtronic average percent-of-sales and (b) percent-of-total-assets ratios.

PART II

Management has requested that you create a five-year common-size income statement (mostly percent of sales) and two common-size balance sheets (one percent of assets, the other percent of sales) in the same format as the given five-year financial statements.* Your common-size statements will look exactly like Medtric's five-year income statement and balance sheet, except that they will report percentages rather than dollar amounts. The income statement ratios that are not percentages of sales follow.†

- a. Sales growth is the year-over-year growth in sales.
- b. Depreciation expense is a percentage of the previous year's Gross Plant, Property, and Equipment.
- c. Interest expense is calculated in arrears, based on the sum of the previous year's short- and long-term debt.
- d. Interest income is earned in arrears, based on the sum of the previous year's cash and short-term investments.
- e. Income Taxes are a percentage of current earnings before taxes.
- f. Dividends are a percentage of current net income.

Verify that the percentages shown on pages 6 and 7 are indeed the three-year averages of the values in your common-size income statement and balance sheets.

- A. What do the common-size financial statements tell you about the current state of Medtric Corporation?
- B. Are there any particularly favorable or unfavorable trends?
- C. Which of the average ratios do not reflect Medtric's current state?

PART III

Parts I and II are concerned with Medtric's past and present financial situation. Management would also like answers to the following forward-looking questions. Calculate all ratios and growth rates using the values in Medtric's 2016 financial statements (other than the 2015 level of equity used to calculate *PRAT*).

- A. How rapidly could sales grow if Medtric meets all of its financing needs using internal sources only? Assume a payout ratio of 20%. Use the g^* formula.

* While common-size balance sheets are typically done as percents of total assets, please calculate them as both percent of assets and percent of sales. We will use many of the percent-of-sales ratios to construct pro forma financial statements in Problem 4.

† A number of these items will not have observations for 2012.

- B. Approximately how rapidly could sales grow if Medtric utilizes debt financing at a level such that they maintain their current total-asset-to-lagged-equity ratio and all their equity needs are met from retained earnings? Assume a payout ratio of 20%. Use the *PRAT* formula.
- C. What would happen to g^* and *PRAT* if Medtric decided to pay out 35% of their net income as dividends?
- D. This question considers the effect of manipulating two of Robert Higgins' *ROE levers* on a firm's sustainable growth rate (*PRAT*). The two levers are the equity multiplier, T (the ratio of total assets to lagged equity); and the retention rate, R (1 minus the payout ratio).

Suppose that management could change their total-assets-to-lagged-equity ratio or their dividend payout rate to new levels that then would be maintained in future years. Suppose, too, that Medtric's net profit margin and asset turnover will remain at their 2016 levels. Could Medtric grow its revenues 12% per year and pay out 35% of its net earnings without changing their capital structure thereafter? Alternatively, could Medtric grow its revenues at 20% and pay out 20% of its net earnings without changing their capital structure thereafter? To answer these questions, perform the following calculations using the formula for *PRAT*:

- Calculate the total-asset-to-lagged-equity ratio needed to sustain growth at 12% and maintain a payout ratio of 35%.
- Now calculate the total-asset-to-lagged-equity ratio needed to sustain growth at 20% and maintain a payout ratio of 20%.
- Calculate the retention rate needed to sustain growth at 12% and maintain Medtric's 2016 total-asset-to-lagged-equity ratio.
- Now calculate the retention rate needed to sustain growth at 20% and maintain Medtric's 2016 total-asset-to-lagged-equity ratio.*
- Suppose Medtric were to omit its dividend altogether. What is the total-asset-to-lagged-equity ratio needed to sustain growth at 20%?

PART IV

This question asks you to estimate the external financing required over the next two years to support 12%/35% and 20%/20% sales growth/payout scenarios using the *back-of-the-envelope* method presented in Morris–Daley's [Chapter 3](#), "Tools for Financial Planning." Assume that the firm's profit margin ($NI/Sales$), total assets/sales, and spontaneous current liabilities/sales ratios remain at their 2016 levels. (Note: Do not use pro forma statements to answer these questions. Instead, use the formula for *REF* from Morris–Daley's [Chapter 3](#)).

* The retention rate is sometimes greater than one for questions c. and d. If so, what does this tell you?

- A. If sales grow at 12% over the next two years and the dividend payout ratio is 35%, approximately what amount of external financing will be required in 2017 and 2018?
- B. If sales grow at 20% over the next two years and the dividend payout ratio is 20%, approximately what amount of external financing will be required in 2017 and 2018?
- C. Is Medtric necessarily more valuable under the higher growth rate scenario? Briefly explain your answer.
- D. Why do your estimates in parts A and B understate the actual amount of external financing that would be required under either of the growth/dividend scenarios?

PART V

Given your analysis, which scenario should Medtric pursue? Briefly support your answer.

II

Tools for Financial Planning and Modeling: Simulation



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Financial Statement Simulation

4.1 INTRODUCTION

[Chapter 1](#) introduced financial planning and financial models in general terms. [Chapter 3](#) explained how to develop pro forma financial statements. This chapter discusses financial models in greater detail and presents a generic simulation model of a firm. Specific equations of the model are presented and discussed so that the reader will have a thorough understanding of the structure of such a model.

There are at least as many different types of financial simulation models as there are uses for them.* The most common financial simulation model is a financial statement model that consists of a set of mathematical expressions that are used to generate pro forma financial statements, so the effect of alternative decisions on the firm's financial condition can be evaluated. The mathematical equations specify the relationships, or links, between the economic variables, environmental variables, decision variables, and the financial state of the firm. After these relationships are properly specified, the input (environmental and economic forecasts, and decision variables) is fed through the model to generate output consisting of the state of the firm as described by its financial statements.

Our discussion focuses on a financial statement model driven by a sales forecast. The model allows us to explore the investment and financing necessary to meet the sales forecast. The model is structured in modules, or blocks of equations, with a module for each of the following tasks:

- Accounting
- Income generation
- Investment

* The References at the end of this book include many different discussions of simulation models. Some of the more comprehensive discussions of models and model building include Bryant (1982), Francis (1983), Gershefski (1968, 1969), Meyer (1977), Naylor (1976, 1979), Schrieber (1970), and Warren and Shelton (1971).

- Financing
- Cash flow
- Valuation

Exhibit 4.1 diagrams the flow between the modules. The income generation module utilizes the economic forecasts, decision specifications, and parameter estimates to generate estimates of revenues and costs. These are then input into the accounting module, that, in turn, generates trial values for the various accounts of the financial statements. Those trial values are used by the investment module to calculate the amount of new investment in assets necessary to support the forecast level of sales, and by the financing module to estimate the amount and form of new financing necessary to purchase these assets.

The new investments and financing are fed back to the accounting module to generate the pro forma financial statements consistent with the sales forecast. These financial statements are part of the output of the model, from which management can evaluate the forecast and their prospective decisions. Each of these modules will be explained, and an example model will be presented. The accounting module will be discussed first because it is of central importance in the model.

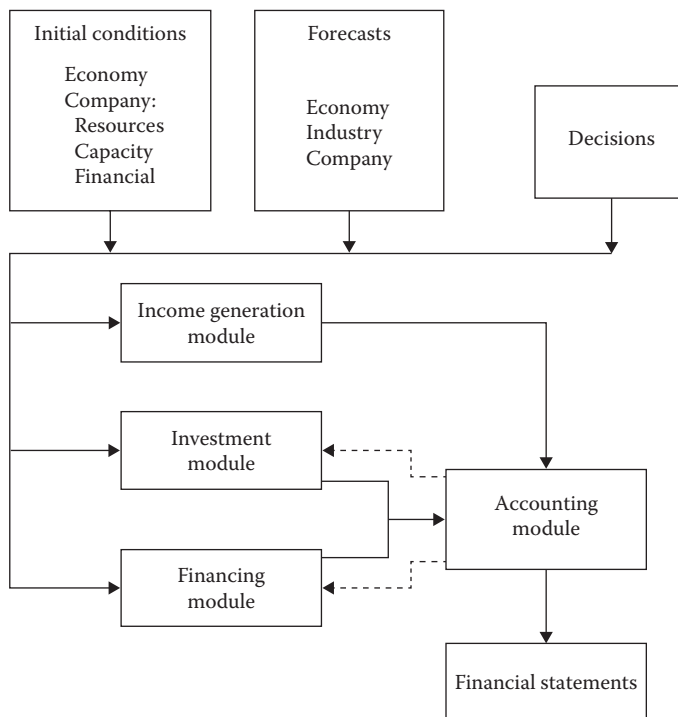


EXHIBIT 4.1 Structure and flow of a financial statement simulation model.

4.2 THE ACCOUNTING MODULE

4.2.1 Equations of the Module

The skeleton of most financial simulation models is a set of equations that are accounting definitions and relationships. These equations maintain accounting consistency, aggregate the detailed accounts into their broad groups, and link the account balances from one period to the next. These equations generate the financial statements for the firm: the income statement, the balance sheet, and the statement of cash flows.

To demonstrate our modules, we consider a simplified model for a firm. Exhibit 4.2a, b, and c show, in generalized form, examples of the types of equations necessary to generate the major account categories for the income statement (Exhibit 4.2a) and the balance sheet (Exhibit 4.2b) and allocate cash flow (Exhibit 4.2c). The subscript t denotes time period t , and variables that must be provided as data or generated elsewhere in the simulation system are enclosed in parentheses. For example, the *Growth Rate* of sales is an external forecast; it is data input in (A.1). Similarly, in this sales-driven model, costs are expressed as a percent of sales, and the ratio (*Operating Cost/Sales*) is a parameter forecast that

(a)		
A.1	$Sales_t$	= $Sales_{t-1} (1 + \text{Growth Rate})$
A.2	$Operating\ Costs_t$	= $Sales_t (\text{Operating Cost/Sales})$
A.3	$EBITDA_t$	= $Sales_t - Operating\ Costs_t$
A.4	$Depreciation\ Expense_t$	= $Gross\ Fixed\ Assets_{t-1} (\text{Depreciation Rate})$
A.5	$EBIT_t$	= $EBITDA_t - Depreciation\ Expense_t$
A.6	$Interest\ Expense_t$	= $Debt_{t-1} (\text{Interest Rate})$
A.7	$Earnings\ Before\ Tax_t$	= $EBIT_t - Interest\ Expense_t$
A.8	$Income\ Tax_t$	= $Earnings\ Before\ Tax_t (\text{Tax Rate})$
A.9	$Net\ Income_t$	= $Earnings\ Before\ Tax_t - Income\ Tax_t$
A.10	$Dividend_t$	= $Cash\ Available\ for\ Dividend_t$
A.11	$Retained\ Earnings_t$	= $Net\ Income_t - Dividend_t$
(b)		
Assets		
B.1	$Cash_t$	= $Cash_{t-1} + \text{Increase in Cash}_t$
B.2	$Other\ Current\ Assets_t$	= $Other\ Current\ Assets_{t-1} + \text{Other CA Investment}_t$
B.3	$Current\ Assets_t$	= $Cash_t + Other\ Current\ Assets_t$
B.4	$Gross\ Fixed\ Assets_t$	= $(Gross\ Fixed\ Assets/Sales) Sales_t$
B.5	$Accumulated\ Depreciation_t$	= $Accumulated\ Depreciation_{t-1} + Depreciation\ Expense_t$
B.6	$Net\ Fixed\ Assets_t$	= $Gross\ Fixed\ Assets_t - Accumulated\ Depreciation_t$
B.7	$Total\ Assets_t$	= $Current\ Assets_t + Net\ Fixed\ Assets_t$
Liabilities		
B.8	$Current\ Liabilities_t$	= $Current\ Liabilities_{t-1} + \text{Spontaneous Financing}_t$
B.9	$Debt_t$	= $Debt_{t-1} + Debt\ Issued_t - Debt\ Repaid_t$
B.10	$Common\ Stock_t$	= $Common\ Stock_{t-1} + Stock\ Issued_t$
B.11	$Earned\ Surplus_t$	= $Earned\ Surplus_{t-1} + Retained\ Earnings_t$
B.12	$Stockholders\ Equity_t$	= $Common\ Stock_t + Earned\ Surplus_t$
B.13	$Total\ Liabilities\ \&\ Equity_t$	= $Current\ Liabilities_t + Debt_t + Stockholders\ Equity_t$

EXHIBIT 4.2 (a) Accounting module: Income statement equations. (b) Accounting module: Balance sheet equations. (Continued)

C.1	Earnings Cash Flow _t	= Net Income _t + Depreciation Expense _t
C.2	NWC Investment _t	= Other CA Investment _t – Spontaneous Financing _t
C.3	Cash Flow from Operations _t	= Earnings Cash Flow _t – NWC Investment _t
C.4	Capital Expenditures _t	= Fixed Asset Investment _t
C.5	Capital Financing _t	= (Debt _t – Debt _{t-1}) + (Common Stock _t – Common Stock _{t-1})
C.6	Net Cash Inflow _t	= Cash Flow from Operations _t – Capital Expenditures _t + Capital Financing _t
C.7	Increase in Cash _t	= (Cash/Sales) Sales _t – Cash _{t-1}
C.8	Cash Available for Dividend _t	= Net Cash Inflow _t – Increase in Cash _t
(c)		
D.1	Increase in Cash _t	= (Cash/Sales) Sales _t – Cash _{t-1}
D.2	Other CA Investment _t	= (Current Assets/Sales) Sales _t – Other Current Assets _{t-1}
D.3	Current Asset Investment _t	= Increase in Cash _t + Other CA Investment _t
D.4	Investment in New Fixed Assets _t	= Net Fixed Assets _t – Net Fixed Assets _{t-1}
D.4	Fixed Asset Investment	= (Gross Fixed Assets/Sales) Sales _t – Gross Fixed Assets _{t-1}
D.5	Total Investment _t	= Current Asset Investment _t + Fixed Asset Investment _t
(d)		
E.1	Required Financing _t	= Total Investment _t
E.2	Earnings Cash Flow _t	= Net Income _t + Depreciation Expense _t
E.3	Minimum Dividend _t	= Net Income _t (Payout Ratio)
E.4	Internal Financing _t	= Earnings Cash Flow _t – Minimum Dividend _t
E.5	Spontaneous Financing _t	= (Current Liabilities/Sales) Sales _t – Current Liabilities _{t-1}
E.6	Required External Financing _t	= Required Financing _t – Internal Financing _t – Spontaneous Financing _t
E.7	Debt Issued _t	= Required External Financing _t (Debt Portion)
E.8	Stock Issued _t	= Required External Financing _t – Debt Issued _t
E.9	Total Financing _t	= Spontaneous Financing _t + Internal Financing _t + Debt Issued _t + Stock Issued _t
(e)		

EXHIBIT 4.2 (Continued) (c) Cash flow module equations. (d) Investment module: Asset investment equations. (e) Financing module equations.

is exogenous to this sector. In the income statement sector, [Exhibit 4.2a](#), each equation defines a revenue or expense account in terms of input variables or in terms of other preceding accounts. The balance sheet sector, [Exhibit 4.2b](#), consists of an equation for each account category in the balance sheet and equations that total the accounts. The cash flow module, [Exhibit 4.2c](#), consists of expressions that calculate the flow of cash based on the income statement and changes in the balance sheet accounts.

The system of equations in [Exhibit 4.2a](#), [b](#), and [c](#) is a simple model. The idea is to show the principles without being overwhelmed with detail and complexity. For example, we show just two current asset accounts, Cash and Other Current Assets, because this is all we need to introduce model construction. We could easily expand the model to show the accounts in as much detail as necessary. For example, we might want to add accounts for marketable securities, receivables, and inventories, but this is not necessary now. We expand our discussion of these other categories later.

Every model will differ because of the different type and detail of the input data available and the output data desired. Any given set of expressions usually can be collapsed

into a single, less detailed, more general expression, or expanded by adding more expressions and variables. The detail generated by the model and, thus, the extent to which sections of the model are expanded by adding more variables and equations or are shrunk by aggregating variables, and decreasing the number of equations is dependent on the requirements of the user. The user who is more concerned with the effects of sales or production decisions may elaborate and expand into minute detail the equations simulating revenue (A.1) or operating costs (A.2), and at the same time eliminate much of the unnecessary detail represented by the remainder of the model. On the other hand, the financial officer may prefer to take the revenue and costs as given and may emphasize the financial implications of security investments, capital investments, and financing decisions. This could involve expansion of the number of equations and variables relating to these decisions, such as those relating to interest expense (A.6); dividends (A.10); working capital (B.1), (B.2), (B.3), and (B.8); fixed assets (B.4), (B.5), and (B.6); and capital accounts (B.9), (B.10), (B.11), and (B.12).

The financial simulation module consists of a set of equations that must be fulfilled for each period t over the planning horizon. Thus, each equation in [Exhibit 4.2](#) is subscripted with t to denote period t . In most such models, the solution proceeds sequentially, with the first simulated period being solved based upon the initial conditions of the firm, usually the firm's most recent financial statements. The solution from the first period then is used as input for the solution for the second period, and so on, to the end of the planning horizon. Each sector (income, balance sheet, and cash flow) of this set of equations is related to the other sectors. Therefore, they must be solved as a coherent whole. This is not to say that they are necessarily simultaneous equations in a mathematical sense—they may be, but it is not necessary. Certainly, the equations are interrelated. However they can and should be structured so the total set can be solved sequentially rather than simultaneously. Sequential solution refers not just to the time sequence from the earliest period to the last, but also to the order in which the equations are solved for a given period, for example, from top to bottom. The first input variable, the driver of the model, is usually the sales forecast that typically is generated exogenously. The consequences of this sales forecast are generated by working through the list of equations in descending order. Simultaneity is avoided by letting a variable for period t to be a function of either a variable from period $t - 1$ (instead of the same period, t), or of a variable that has already been calculated for the same period, t . In Excel, this means referencing a value from the previous column ($t - 1$) or a higher row in the current column (t). For example, expression (A.6) calculates interest expense for period t based on the amount of debt outstanding at the end of period $t - 1$. The amount of debt outstanding at t is as yet unknown, and the equations would be simultaneous if interest expense for t (A.6) required the debt balance at t instead of $t - 1$.*

The first block or sector of equations shown as [Exhibit 4.2a](#) is the income sector. It takes the forecast of sales as input and generates a corresponding forecast of expenses and income,

* Spreadsheet software such as Excel will resolve the circularity, but in complex models this can cause difficulty if there are several different competing simultaneous systems in the model. At a minimum, simultaneously-solved models are quite opaque. It is almost always better to build a model without any simultaneity, making for a more transparent model.

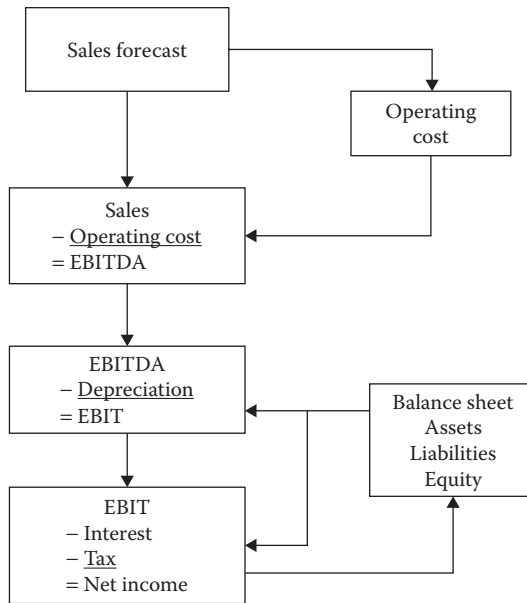


EXHIBIT 4.3 Income statement generation.

simulating the income statement in steps shown diagrammatically in [Exhibit 4.3](#). The income statement equations consist of both definitions and functions. The definitions are those expressions that simply calculate the value of a new variable based on values of previously determined variables; the expressions are true by definition. For example, Earnings Before Interest, Taxes, Depreciation, and Amortization (EBITDA) is defined as Total Revenue minus operating Costs. This expression, by itself, requires no direct parameter estimation or decision specification. Expressions A.3, A.5, A.7, A.9, and A.11 are all definitions. Functional equations are those expressions that require the specification of some exogenous parameter or decision variable; they are indicated in parentheses. For example, $Sales_t$ will be related to sales in the previous period, $Sales_{t-1}$, and the forecast of the rate of growth of sales, $Growth Rate_t$, where $Growth Rate_t$ must be forecast or specified as a parameter. Operating costs are dependent on the cost ratio, $(Operating Cost/Sales)$, forecast elsewhere and input as a parameter. We discuss how to obtain these parameter forecasts later.

Financial simulation tracks the financial status of the firm through time—where it has been, is now, and will be in the future. Therefore, the results in one period must be properly reflected as input for the next period. This function is served, in part, by the balance sheet equations that use balances at the end of the previous period ($t - 1$) as the beginning balances for current period (t). Thus, whatever happens in period $t - 1$ affects the results for the next period, t . For example, (B.1) shows $Cash_t$ as dependent on $Cash_{t-1}$, the cash balance at the end of the previous period. In (B.9), the debt balance at the end of the period, $Debt_t$, depends on the previous balance, $Debt_{t-1}$. In addition, expenses and income in period t are related to asset and liability balances at the end of the previous period. The most obvious example is Equation (A.4), which calculates *Depreciation Expense* during period t as a proportion of

*Gross Fixed Assets*_{*t-1*}, the balance at the end of the previous period. Similarly, Interest Expense (A.6) for period *t* is proportional to the amount of debt at the end of the previous period.

The cash flow module, [Exhibit 4.2c](#), provides the link between the income statement and the balance sheet, and is a crucial part of the mechanism for maintaining equilibrium in the model. The starting point is *Earnings Cash Flow (Net Income + Depreciation)* from the income statement. The subsequent expressions take account of changes in all asset and liability accounts. In (C.2), Other CA Investment is the change in current assets, excluding cash. It represents the cash outflow required to increase noncash current assets. Spontaneous Financing is the increase in current liabilities that occurs in response to increasing sales. NWC Investment (net working capital investment) is the difference between the increase in current assets other than cash and the spontaneous financing from current liabilities. It represents the investment in current assets other than cash that is not financed by the spontaneous increase in current liabilities. *Cash Flow from Operations* (C.3) is the cash flow from normal operating activities net of the investment in net working capital (excluding the cash balance). *Capital Expenditures* (C.4) is the amount spent to replace and expand plant and equipment and other long-term assets, and is based on the projected ratio of required Gross Fixed Assets-to-Sales, and the change in sales. Capital Financing (C.5) is the amount of funds flowing into the firm from the issuance of new securities (debt and equity). The total of these inflows and outflows is the *Net Cash Inflow* (C.6), the amount of cash from all sources that is available either to increase the cash account or to pay dividends. In this case, the cash balance is increased proportionally to the increase in sales (C.7), and the remaining cash flow is paid as a dividend to the stockholders (C.8).

Cash must flow to a specific spot in the model as opposed to flowing into a financial black hole. In our current model, part of the *Net Cash Inflow* is added to the cash account; the remainder is paid as dividends. In a more complex model, *Net Cash Inflow* can be invested in marketable securities or used to retire debt or repurchase equity. However the cash is allocated, one of the categories must receive the residual. For example, in the current format, we use the cash to maintain the cash balance at a target percent of sales and payout the remainder (the residual) as dividends. In this way, dividends become the residual, ensuring that all the cash flow goes somewhere. We could easily reverse the logic and have the dividend determined first (e.g., based on the dividend payout ratio), and allow the addition to cash to be the residual. In that case, because cash would go to the cash account as a residual, it would not be specified as a percent of sales.

An example of cash flowing into a financial black hole would be if the cash balance was determined as a percent of sales, and dividends were set by a dividend payout ratio, with no specification of where any leftover cash would go. For example, consider the following data ([Exhibit 4.4](#)):

		Portion allocated
Net Cash Inflow	\$ 1,000	
Addition to Cash Balance	300	30%
Dividends	500	50%
Unallocated Excess Cash Flow	\$ 200	

EXHIBIT 4.4 Cash flowing into a black hole.

If the model does not specifically put the \$200 excess cash someplace, it will simply disappear into a financial black hole, neither being added to an asset account (cash or marketable securities) nor being paid out as dividends. In other words, we should not have both the cash balance driven by sales and dividends driven by income, without otherwise specifying a target for any residual cash flow. An alternative target for the residual cash flow could be security investments. Continuing with the current example, the difference between *Net Cash Inflow* and the sum of the Addition to Cash Balance and Dividends, \$200 in this case, could be used to buy marketable securities. The point is our model must account for all the cash flow, else it will not balance.

4.2.2 The Income Generation Module

The income generation module generates variables related to revenues and costs that are, in turn, used in the accounting module. To generate these intermediate inputs to the accounting module, the income sector utilizes forecasts of future economic conditions and forecasts of firm relationships and decisions. These forecasts are input data to the system of equations comprising the income generation module. Of course, every model will approach the task of forecasting revenue and cost differently. The amount of detail in the model will depend on the purpose of the modeling effort. A more detailed income module could go into great detail calculating different categories of revenues and costs. For now, to better concentrate our attention on the basic structure of the model, we use the simple approach of embedding the income generation equations in the Accounting Module. We model revenue using a simple growth model in which revenue grows at the forecast rate, and we model costs proportional to sales. The inputs are the sales growth forecast and the cost parameter forecasts. In [Exhibit 4.2a](#), the driver of the model is the equation,

$$Sales_t = Sales_{t-1} * (1 + Growth Rate) \quad (4.1)$$

The forecast parameters are the items in parentheses: (*Growth Rate*), (*Operating Cost/Sales*), (*Depreciation Rate*), (*Interest Rate*), and (*Tax Rate*). We will consider different ways to model revenues and costs in greater detail in a later section.

One of the primary uses of a financial simulation model is to determine the level of future investment and/or financing that may be necessary to meet the firm's growth objectives. The role of the investment and financing modules is to specify these required levels of investment and financing. We consider how these modules fit into the simulation model next.

4.2.3 The Investment Module

The investment module simulates the firm's investment in assets and sets the investment schedule so that the cash inflows and outflows related to the investment can be used in the other sectors of the simulation model to generate the output necessary to evaluate the firm's financial plans. There are two approaches to modeling asset investment. One is a sales-driven model in which assets are assumed to be functions of sales. In this type of model,

investment occurs such that the level of assets is sufficient to support sales. In the second approach, investment is the driver of the model. Investment plans are input directly for the purpose of evaluating specific investments; sales and production are part of the secondary data describing the investment. Going forward, we will use the first approach, in which investment adjusts to the sales forecast because its simpler nature is a better fit with our continuing focus on understanding the structure. We take up the second structure later when modeling an acquisition.

The simplest investment model calculates the total stock of assets proportional to the sales. Investment is calculated as the change in required assets from one period to the next. For example, assume that the income generation module specifies $Sales_t$, and the required stock of assets is directly proportional to sales, so that

$$Assets_t = \left(\frac{Assets}{Sales} \right) * Sales_t \quad (4.2)$$

where $(Assets/Sales)$ is the parameter representing the required ratio of assets-to-sales. This parameter may be obtained by estimating it from historical data regarding sales and asset levels, or it may be based on anticipated technological relationships or policy decisions that are being analyzed by the simulation.

The investment required by the percent-of-sales model is

$$\begin{aligned} Asset\ investment_t &= Assets_t - Assets_{t-1} \\ &= \left(\frac{Assets}{Sales} \right) * (Sales_t - Sales_{t-1}) \end{aligned} \quad (4.3)$$

This model is unrealistically simple, since different asset categories will have different relations to sales. To develop a more detailed model, simply specify different functional links between each asset category and sales. For now, we will distinguish just three asset categories: Cash, Other Current Assets, and Fixed Assets. The expressions that calculate the investment in assets are shown in [Exhibit 4.2d](#).

4.2.4 The Financing Module

The financing module is the set of equations that specifies the sources of funds needed to acquire the assets determined in the Investment Module. The financing in the model is an important part of the equilibrating mechanism that balances assets and liabilities and ensures that sources are equal to uses of funds. [Exhibit 4.2e](#) shows expressions for a simple financing module.

We can separate the sources into spontaneous and discretionary financing, and distinguish between internal and external financing. Spontaneous financing refers to those sources of funds that are generated more or less automatically by the firm's normal operations and growth. Some liabilities, particularly current liabilities, grow with the firm's operations, and provide financing that grows spontaneously as sales increase. Trade credit and accruals are examples of sources that usually grow with sales, providing a spontaneous

source of financing.* In our model, we combine these spontaneous sources into one account, *Current Liabilities*, and call the increase in current liabilities *Spontaneous Financing*, as shown in (E.5). Discretionary financing refers to sources of funds that require explicit decisions to use them. Discretionary sources include borrowing and the issuance of new securities. These sources are discretionary in the sense that they need to be specified as a decision variable that is entered into the model.

We can also divide sources into internal or external financing. Internal financing consists of the sources internal to the firm's operations. External refers to the funds that come from outside the firm and need to be specified as an explicit decision. We usually think of retained earnings as internal equity financing. However, our model uses a slightly broader definition of internal financing. Internal Financing is shown in (E.4) as Earnings Cash Flow less Dividends, where *Earnings Cash Flow (Net Income + Depreciation)* is generated by the normal operation of the firm, and provides funds that can be used to purchase new assets, repay debt, or pay dividends. This source of financing is internal to the firm because no application needs to be made to outsiders to obtain the funds. It is also a spontaneous source of funds, but for our purposes, we will use the term spontaneous financing to refer to the increase in current liabilities, and the term internal financing to refer to internal equity financing.

The financing module ([Exhibit 4.2e](#)) takes *Total Investment_t* from the investment module as its initial input. *Required Financing_t* is set to equal to *Total Investment_t* (E.1). Subtracting the sum of *Internal Financing_t* and *Spontaneous Financing_t* from *Required Financing_t* yields *Required External Financing_t* (E.6). The user of the model decides how much of the *External Financing* will come from debt by specifying the decision variable (*Debt Portion*). Any remaining financing will come from the issuance of new stock (E.8). Assigning all remaining financing needs to stock ensures that all investment will be financed, and the model will balance. Investment and financing must be consistent with each other and with the overall financial structure of the firm. In other words, the investment and financing must be specified in a way that the firm is in equilibrium.

4.3 EQUILIBRIUM IN THE SIMULATION MODULE

Virtually all financial models must have a mechanism that balances the system. This section discusses the balancing process in the simulation model. A model is balanced or in equilibrium when assets equal liabilities and the sources of funds equal the uses of funds. We need to build a mechanism into our model that ensures this equilibrium.

There are two ways to equilibrate the model: flow balancing or stock balancing. A flow balancing model reaches its equilibrium by making sure that sources of funds equal uses. Balance of the inflows and outflows can be maintained by variations of expressions of the form

$$\textit{Total Uses of Funds} = \textit{Total Sources of Funds}$$

or

(4.4)

$$\textit{Net Change in Cash Balance} = \textit{Net Cash Inflow}$$

* Balance sheets frequently contain spontaneous long-term liabilities. When they do, the change in the long-term liabilities must be included in the financing module.

A stock balancing model equilibrates by making the stock (level) of liabilities equal assets. Stock balancing is accomplished by various expressions of the form

$$\textit{Total Assets} = \textit{Total Liabilities}$$

or

(4.5)

$$\textit{Total Assets}_t = \textit{Total Liabilities}_{t-1} + \textit{New Financing}_t$$

Although each of these approaches can be used by itself, it is often useful to include both stock and flow balancing in the same model to serve as a check that the model is performing properly. In the example to be discussed shortly, we will include elements of both approaches.

A simulation model cannot make decisions; it can only compute and indicate the results. Consequently, even though provision is made in the accounting equations for sources to equal uses, or for assets to equal liabilities, the model must be told specifically how to reach equilibrium. If new financing is required, the user must tell the model the composition of the new financing: whether it is debt or equity or something more exotic. In many financial simulation models, this is accomplished with a *plug* variable that forces equality between sources and uses or between assets and liabilities. The role of the investment and financing modules is, in part, to specify the amount and composition of these plug variables.

Suppose that for a given sales forecast, the investment module shows that \$100 million of additional investment in fixed assets is required. This means that we must obtain \$100 million of financing. Suppose, too, that \$25 million will be generated from spontaneous and internal sources. Without additional adjustment, the model will show assets exceeding liabilities and shareholder equity by \$75 million. The financing module makes the necessary adjustment. It calculates a *plug* variable equal to the required external financing that balances assets with liabilities, \$75 million in this case. The user must specify the form of this financing.

The logical steps to reach a stock balancing equilibrium are shown as a flowchart in [Exhibit 4.5](#). Based on a sales forecast, the investment module determines the composition and level of assets necessary to generate the sales. If these required assets differ from the existing level of assets, the amount of new investment is determined. The financing module takes this number and calculates the level of new financing necessary to acquire the investment. Total new financing is calculated as the difference between total assets (with the new investment) and the level of liabilities and shareholder equity that would exist without the new financing. The amount of the new financing is a plug variable that balances assets and liabilities.

[Exhibit 4.6](#) shows the steps for a flow balancing model. A cash receipts and disbursement model is an example of a flow balancing model where the difference from the stock balancing approach is most apparent. As shown in [Exhibit 4.6](#), this type of model determines the net cash inflow based on a sales forecast and the schedule of receipts and disbursements. The ending cash balance is compared to the minimum cash balance.

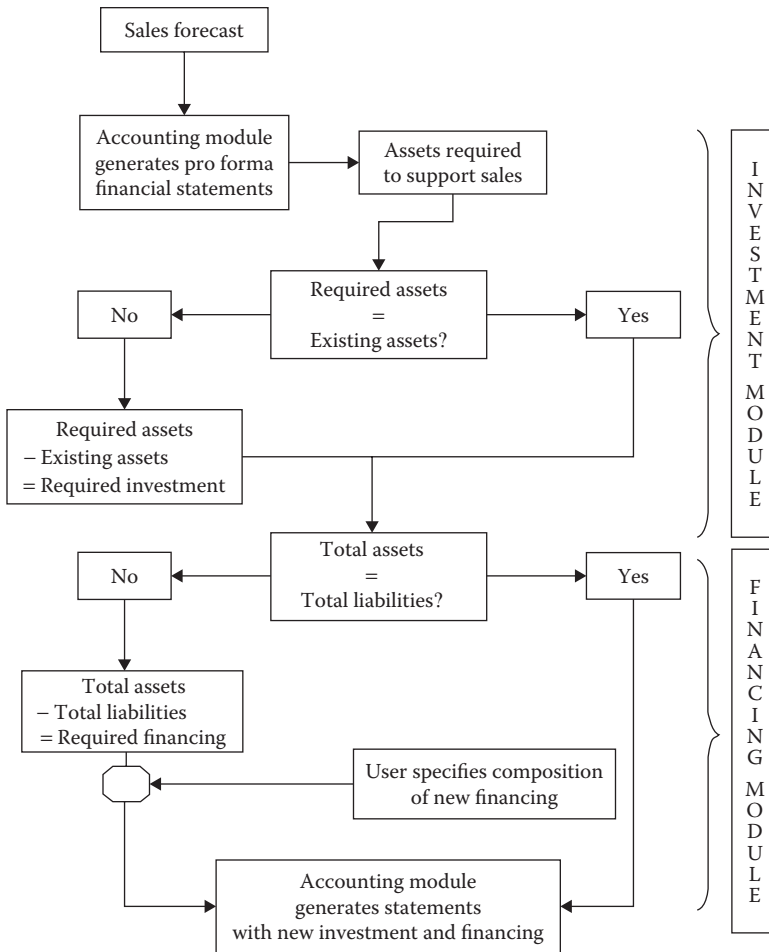


EXHIBIT 4.5 Flowchart for balancing a financial model: Stock balancing.

The model then checks for excess cash or a cash deficit. If ending cash exceeds the minimum balance, the investment module invests the excess in marketable securities. If ending cash is below the required minimum, the financing module borrows to finance the deficit. The plug variable in this case is the amount to be invested in marketable securities, or the amount of borrowing necessary to make the ending cash balance equal to the required cash balance.

A flow balancing model is ideally suited to the problems of simulating working capital decisions, but the same task can be accomplished with a stock balancing model. Moreover, the flow balancing model can be used for long-term investment and financing simulations just as easily as can the stock balancing approach. These two approaches are interchangeable because, in a properly specified model, actions that balance the cash flows also balance the stock of assets and liabilities. In a more aggregative model, especially one that is specified in terms of longer time periods, the stock adjustment approach is often

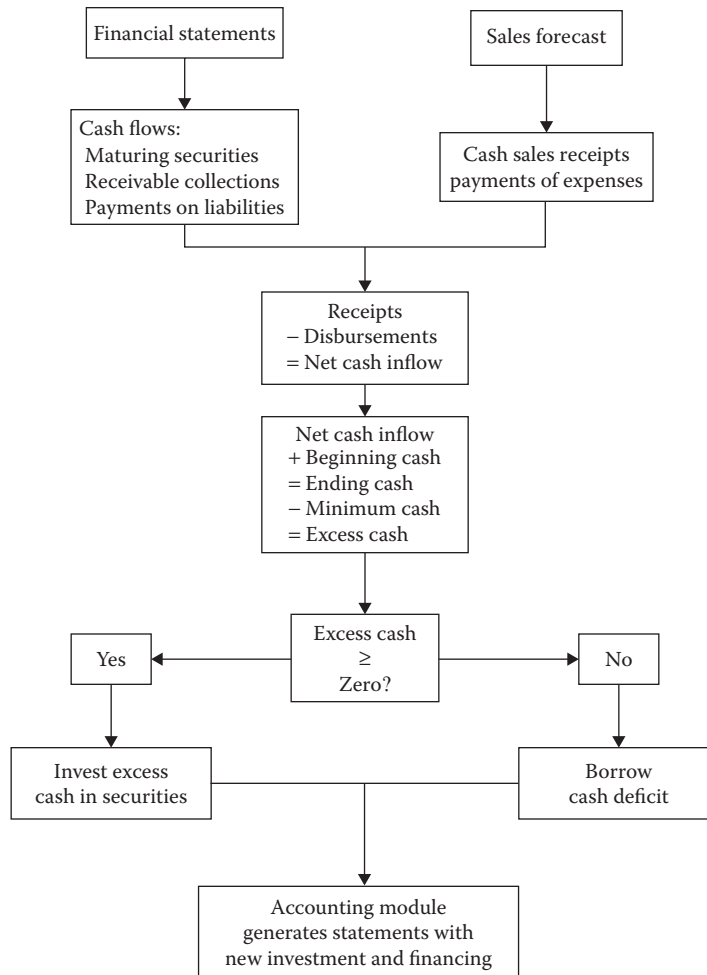


EXHIBIT 4.6 Flowchart for a financial model: Balancing sources with uses.

easier to develop. In a more detailed model, especially one with shorter time periods, the flow adjustment approach is frequently more useful. Although either approach can be used, we have found that fewer modeling mistakes occur when we combine elements of both approaches in our models. The example in the next section has elements of both stock balancing and flow balancing.

4.4 BUILDING A LONG-RANGE PLANNING MODEL

4.4.1 Model of O&R Corp.

In this section, we build a model of a firm's financial statements that is then used to project the firm's future financial condition. The ultimate purpose of the model is to evaluate the impact of different policies on the value of the firm's stock. To do this, we need to develop

a valuation model for the firm. However, at this stage, we just want to develop the first part of the model, the one that simulates the financial statements. We add the valuation module later. The starting point for our model is constructing the modules explained in general terms earlier in this chapter—the accounting, investment, and cash flow modules. We use the same basic structure of the model shown in [Exhibit 4.2](#), with some elaborations added as necessary. We continue using O&R Corp. as our representative company.

As you recall from [Chapter 2](#), Odd & Rich Corporation (O&R Corp) is an industrial firm that manufactures a range of plastic components used as input by other manufacturers of consumer products, automobiles, aircraft, and computers. The company's sales are somewhat cyclical, but have been growing at a respectable rate in recent years. It is expected that its sales will continue to grow in the foreseeable future. Our task is to develop a model to help us to analyze the firm's investment and financing needs over the next five years.

The financial statements for last two years serve as the starting point for a model that will simulate the firm's financial performance. Based on an examination of past performance and forecasts for the future, our assumptions about the firm's future growth, costs, and operating ratios are given in [Exhibit 4.7](#).

Our forecast of the economy and financial markets suggests that the rate on U.S. Treasury bills will be about 2%, and the firm will be able to borrow at an interest rate of 4%.

Having collected last financial statements and our assumptions and projections, we develop the data input section of our model, shown as part of our spreadsheet in [Exhibit 4.8a](#). The income statements and balance sheets for the last two years are shown in lines 17–51, columns C and D. Other data input, forecasts, and policy parameters are shown in rows 4–13, columns C and F.

The next step to name the input variables. This is done by first entering the desired names next to the cells to be named ([Exhibit 4.8b](#)). Choose short but meaningful names where possible. Next, highlight the arrays C5:D13, F5:G9, and F12:G13 (hold down the Ctrl key and use the mouse to highlight the arrays). Lastly, click Formulas > Create from Selection > OK.

We begin with a quick and dirty version of the model ([Exhibit 4.9a](#), formulas in [Exhibit 4.9b](#)). This model is balanced externally using a minifinancing module (lines 54–57) to calculate the plug, then feeding the plug into Debt on the balance sheet. This simplicity comes at a price. Besides giving the modeler very little control, this model takes depreciation expense after *Net Fixed Assets* have become zero, allows *Net Fixed Assets* to become negative, pays negative dividends, and pushes Long-term Debt negative when negatively

Annual growth in Sales:	6%
Operating Costs / Sales	70%
Depreciation / Gross Fixed Assets	3%
Tax Rate	40%
Dividend Payout Ratio	60%
Target Ratio of Cash / Sales	17%
Non-Cash Current Assets / Sales	54%
Gross Fixed Assets / Sales	193%
Current Liabilities / Sales	60%

EXHIBIT 4.7 Percent-of-sales inputs for O&R pro forma statements.

	B	C	D	E	F	G	H	I
54	INVESTMENT MODULE	2015	2016	2017	2018	2019	2020	2021
55	Current Asset Investment			42.6	45.2	47.9	50.7	53.8
56	Investment in New Fixed Assets (Δ NFA)			61.2	61.4	65.1	69.0	73.1
57	Replacement of Depreciated Assets			57.8	61.4	65.1	69.0	73.1
58	Fixed Asset Investment			119.0	122.7	130.1	137.9	146.2
59	Total Investment			161.6	167.9	178.0	188.7	200.0
60								
61	FINANCING MODULE			2017	2018	2019	2020	2021
62	Required Financing			161.6	167.9	178.0	188.7	200.0
63	Sources of Funds							
64	Net Income			154.4	163.5	173.4	183.8	194.9
65	Depreciation			57.8	61.4	65.1	69.0	73.1
66	Earnings Cash Flow			212.2	224.9	238.4	252.8	268.0
67	Minimum Dividend			92.6	98.1	104.0	110.3	116.9
68	Internal Equity Financing			119.5	126.8	134.4	142.5	151.1
69	Spontaneous Financing			36.0	38.2	40.4	42.9	45.4
70	Total Internal & Spontaneous Financing			155.5	164.9	174.9	185.4	196.5
71	Required External Financing			6.1	3.0	3.1	3.3	3.5
72	Debt Issued			6.1	3.0	3.1	3.3	3.5
73	Equity Issued			-	-	-	-	-
74	Total Discretionary Financing			6.1	3.0	3.1	3.3	3.5
75								
76	CASH FLOW MODULE			2017	2018	2019	2020	2021
77	Earnings Cash Flow			212.2	224.9	238.4	252.8	268.0
78	Current Asset Investment (except Cash)			32.4	34.3	36.4	38.6	40.9
79	Spontaneous Financing			36.0	38.2	40.4	42.9	45.4
80	NWC Investment			(3.6)	(3.8)	(4.0)	(4.3)	(4.5)
81	Cash Flow from Operations			215.8	228.7	242.5	257.1	272.5
82	Capital Expenditures			119.0	122.7	130.1	137.9	146.2
83	Capital Financing			6.1	3.0	3.1	3.3	3.5
84	Net Cash Flow			102.8	108.9	115.5	122.4	129.8
85	Increase in Cash			10.2	10.8	11.5	12.1	12.9
86	Cash Available for Dividends			92.6	98.1	104.0	110.3	116.9
87	Check: NCF- Δ Cash-Dividends			-	-	-	-	-
88								
89	INTEREST AND DEBT SERVICE	2015	2016	2017	2018	2019	2020	2021
90	Interest Expense	3.1	3.4	2.9	3.2	3.3	3.4	3.5
91	Total Interest-Bearing Debt	37.7	73.3	79.4	82.3	85.4	88.7	92.2
92	Total Debt (e.g., Total Liabilities)	604.1	673.3	715.4	756.5	800.0	846.2	895.1

(b)

EXHIBIT 4.10 (Continued) (b) O&R investment, financing, and cash flow modules. (Continued)

	B	E	F
54	INVESTMENT MODULE	2017	2018
55	Current Asset Investment	=E38-D38	=F38-E38
56	Investment in New Fixed Assets (Δ NFA)	=E42-D42	=F42-E42
57	Replacement of Depreciated Assets	=E23	=F23
58	Fixed Asset Investment	=E56+E57	=F56+F57
59	Total Investment	=E55+E58	=F55+F58
60			
61	FINANCING MODULE	2017	2018
62	Required Financing	=E59	=F59
63	Sources of Funds		
64	Net Income	=E28	=F28
65	Depreciation	=E23	=F23
66	Earnings Cash Flow	=SUM(E64:E65)	=SUM(F64:F65)
67	Minimum Dividend	=MAX(payout*E28,0)	=MAX(payout*F28,0)
68	Internal Equity Financing	=E66-E67	=F66-F67
69	Spontaneous Financing	=E46-D46	=F46-E46
70	Total Internal & Spontaneous Financing	=E68+E69	=F68+F69
71	Required External Financing	=E62-E70	=F62-F70
72	Debt Issued	=MAX(debt_portion*E71,0)	=MAX(debt_portion*F71,0)
73	Equity Issued	=MAX(E71-E72,0)	=MAX(F71-F72,0)
74	Total Discretionary Financing	=E72+E73	=F72+F73
75			
76	CASH FLOW MODULE	2017	2018
77	Earnings Cash Flow	=E28+E23	=F28+F23
78	Current Asset Investment (except Cash)	=E37-D37	=F37-E37
79	Spontaneous Financing	=E46-D46	=F46-E46
80	NWC Investment	=E78-E79	=F78-F79
81	Cash Flow from Operations	=E77-E80	=F77-F80
82	Capital Expenditures	=E42-D42+E23	=F42-E42+F23
83	Capital Financing	=E47-D47+E48-D48	=F47-E47+F48-E48
84	Net Cash Flow	=E81-E82+E83	=F81-F82+F83
85	Increase in Cash	=E36-D36	=F36-E36
86	Cash Available for Dividends	=E84-E85	=F84-F85
87	Check: NCF- Δ Cash-Dividends	=E84-(E36-D36)-E30	=F84-(F36-E36)-F30
88			
89	INTEREST AND DEBT SERVICE	2017	2018
90	Interest Expense	=E25	=F25
91	Total Interest-Bearing Debt	=E47	=F47
92	Total Debt (e.g., Total Liabilities)	=E46+E47	=F46+F47

(d)

EXHIBIT 4.10 (Continued) (d) O&R formulas for investment, financing, and cash flow modules.

The expressions for *Depreciation*, *Dividends*, and *Allowance for Depreciation* need some explanation. They are more complicated than in [Exhibit 4.2](#) or [Exhibit 4.9b](#) so as to take into account various exceptional cases. A computer model is very obedient, but also very stupid. It does exactly what you tell it to do. Unless you anticipate every eventuality, this obedience can lead to perverse results. For example, *Depreciation expense* was shown in [Exhibit 4.2](#) (A.4) as

$$\begin{aligned} \text{Depreciation Expense}_t &= \text{Gross Fixed Assets}_{t-1} * (\text{Depreciation Rate}) \\ &= \text{GFA}_{t-1} * \left(\frac{\text{Depreciation}}{\text{GFA}} \right) \end{aligned} \quad (4.6)$$

The problem is that if, as time passes, the firm's asset investment has not exceeded its depreciation, its *Net Fixed Assets* decrease. This can lead to a situation where *Accumulated Depreciation* exceeds *Gross Fixed Assets (GFA)*, so that *Net Fixed Assets (NFA)* become negative. To prevent such a perverse modeling result, we modify the *Depreciation Expense* and *Allowance for Depreciation* lines. For *Depreciation expense*, we use the following logic:

$$\begin{aligned} \text{IF } \text{GFA}_{t-1} \left(\frac{\text{Dep}}{\text{GFA}} \right) &\geq \text{NFA}_{t-1}, \text{ then } \text{Depreciation Expense} = \text{NFA}_{t-1} \\ \text{but} & \\ \text{IF } \text{GFA}_{t-1} \left(\frac{\text{Dep}}{\text{GFA}} \right) &< \text{NFA}_{t-1}, \text{ then } \text{Depreciation Expense} = \text{GFA}_{t-1} \left(\frac{\text{Dep}}{\text{GFA}} \right) \end{aligned} \quad (4.7)$$

Converting this to Excel code yields ([Exhibit 4.10c](#), line 23):

$$\text{IF} \left(\text{GFA}_{t-1} \left(\frac{\text{Dep}}{\text{GFA}} \right) \geq \text{NFA}_{t-1}, \text{NFA}_{t-1}, \text{GFA}_{t-1} \left(\frac{\text{Dep}}{\text{GFA}} \right) \right) \quad (4.8)$$

However, this alone will not prevent *NFA* from becoming negative. To do that, for *Allowance for Depreciation* we use the logic:

$$\begin{aligned} \text{IF} \left(\left[\begin{array}{l} \text{Allowance for} \\ \text{Depreciation}_{t-1} \end{array} \right] + \left[\begin{array}{l} \text{Depreciation} \\ \text{Expense}_t \end{array} \right] < \text{GFA}_t, \right. \\ \left. \text{then } \left[\begin{array}{l} \text{Allowance for} \\ \text{Depreciation}_t \end{array} \right] = \left[\begin{array}{l} \text{Allowance for} \\ \text{Depreciation}_{t-1} \end{array} \right] + \left[\begin{array}{l} \text{Depreciation} \\ \text{Expense}_t \end{array} \right] \right) \\ \text{but} \end{aligned} \quad (4.9)$$

$$\text{IF} \left(\left[\begin{array}{l} \text{Allowance for} \\ \text{Depreciation}_{t-1} \end{array} \right] + \left[\begin{array}{l} \text{Depreciation} \\ \text{Expense}_t \end{array} \right] < \text{GFA}_t, \text{ then } \left[\begin{array}{l} \text{Allowance for} \\ \text{Depreciation}_t \end{array} \right] = \text{GFA}_t \right)$$

Converting this to Excel code yields (Exhibit 4.10c, line 41):

$$\begin{bmatrix} Allowance\ for \\ Depreciation_t \end{bmatrix} = MIN \left(\begin{bmatrix} Allowance\ for \\ depreciation_{t-1} \end{bmatrix} + \begin{bmatrix} Depreciation \\ expense_t \end{bmatrix}, GFA_t \right) \quad (4.10)$$

In this way, if we reach the point where further depreciation would drive *NFA* negative, *Depreciation expense* will be zero, and *NFA* will go no lower than zero.

Another case where we may need to anticipate perverse modeling is the dividend. If we specify the dividend as a simple percent of *Net Income* in the Income Statement, thus

$$Dividend = Net\ Income * Payout\ Ratio \quad (4.11)$$

they could go negative. In addition, the model could not distribute additional cash to shareholders. We fix the second effect in the Cash Flow module (Exhibit 4.10b, line 86). We fix the first effect in the Financing module (Exhibit 4.10b, line 67) by calculating the minimum dividend to be paid as

$$Dividend = MAX(Net\ Income * Payout\ Ratio, 0) \quad (4.12)$$

The formulas of the Investment, Financing, and Cash Flow Modules are shown in Exhibit 4.10d. The Investment Module calculates the required investment based on the increase in assets—both current and fixed. *Total Investment* (line 59) carries over to the Financing Module where Required Financing (line 62) equals *Total Investment*. The Financing Module then determines how much of the Required Financing will be provided by Internal Financing (line 68) and Spontaneous Financing (line 69), allocating the remaining financing to external sources of debt and equity. Required External Financing is split between debt and equity by the user's decision input, Debt Portion, in cell F12 of the Policies and Decisions section (Exhibit 4.8a).

The expressions in these modules follow Exhibit 4.2 with some minor elaboration. For example, lines 72 and 73 use MAX(.) functions. The expression for Debt issued in 2017 is

$$Debt\ Issued = MAX(Debt_portion * E71, 0) \quad (4.13)$$

This prevents *Debt Issued* from being negative in case the Required External Financing is negative. The same provision is made for Equity Issued (line 73).

The equations in the O&R model are simplified, so we do not get too bogged down in detail. As you add features to the model, you will have to modify the equations. For example, this model does not provide for debt repayment, nor does it distinguish between short-term and long-term debt. If you add short-term debt to the model, you should provide for its repayment in the Financing Module above the *REF* line, so you have enough financing to repay the maturing short-term debt. Similarly, if long-term debt has to be repaid, the Financing Module would have to provide sufficient funds to cover the principal payment.

This model sets the target cash balance equal to a percent of sales and allows the dividend to be the residual that disposes of any cash flow that would otherwise cause the cash balance to exceed the required amount. Line 84 calculates the *Net Cash Inflow* from all sources. This is then split between the Increase in Cash (line 85) and Cash Available for Dividends (line 86). In this case, the Increase in Cash is proportional to the increase in sales, and whatever is left over is paid out as dividends. The dividend payment (Exhibit 4.10a, line 30) is linked to Cash Available for Dividends. The Minimum Dividend (line 67) in the Financing Module ensures that the dividend will be at least the amount required by the *Payout Ratio* specified by the user (Exhibit 4.8a, cell F13). However, the dividend will exceed the payout rate if excess cash flow is available. This ensures that all of the *Net Cash Inflow* goes somewhere—either to build up cash or to be paid out as a dividend. Alternatively, we could have directed the cash in excess of the minimum dividend back into the firm to increase *Marketable Securities* or to retire debt or repurchase shares. We add this capability to the model later.

Note how this model reached its equilibrium. Following the logic of Exhibit 4.5, the sales forecast drives all asset categories. The increase in assets determines *Total Investment* (line 59) and feeds into Required Financing in the Financing Module (line 62). The Financing Module then calculates the internal and spontaneous sources (line 70) that are available to fund the investment. The remainder, Required External Financing (line 71), is split between the discretionary sources of debt and external equity. Balance is maintained by making sure that the increase in liabilities and shareholder equity (retained earnings, increase in current liabilities, and increase in debt) is equal to the increase in assets. This model is a hybrid of stock and flow balancing. By balancing assets and liabilities, it is stock balancing, but since it does this by focusing on the changes in asset and liabilities, it resembles a flow balancing model. In addition, the Cash Flow module reconciles the equality of sources and uses, and ensures that the cash flow goes either to the cash account or is paid out as a dividend.

Many things can go wrong as you build these models. It is helpful to include features in your model that check whether it is balancing and doing what you want it to do. Lines 32, 52, and 87 are such checks. Line 52 checks to see if assets and liabilities are equal. Obviously, *Total Assets—Total Liabilities & Equity* should always be zero. However, it is easy to overlook something and have an imbalance. Line 52 provides a quick check on that. The line can be eliminated after you have tested the model under a variety of assumptions and find that it performs accurately under all scenarios.* A problem that frequently occurs for the novice modeler is that the model performs well when sales are growing, but goes out of balance if sales decrease or follow unusual patterns. So, when testing the model, try a variety of different growth assumptions to see that it balances under all conditions, including extreme negative growth.

Line 87 checks that all cash flow is being allocated either to cash or to dividends. This check is: *Net Cash Inflow – Increase in Cash – Dividends*. A nonzero value means that

* Alternatively, just decrease the size of the font and change its color. We like to include a sum of the check cells in lines 52 and 87 in the Input sector to provide an immediate check on the model whenever a parameter is changed. Given that approximately 60% of models created in a firm are used by others, this is a useful feature to add.

some of the cash is flowing into a financial black hole. This error would occur if you set cash equal to a percent of sales, dividends equal to a percent of income, and the cash flow exceeds the amount necessary to meet these two requirements. The check in line 32 is not required, nor must it equal zero. However, it allows the user a quick check to see if the firm is paying dividends in excess of the amount required by the payout ratio.

We have developed the basic model and it seems to balance. We now discuss two additional aspects in the following sections. First, we stress test the model to see that it works as intended. Second, after testing the model, we evaluate the financial results of our plan.

4.4.2 Testing the Model

The model developed thus far seems to balance. However, before concluding that it is a robust model, we need to stress it to see if it performs well under a variety of circumstances and to identify any weaknesses it might have. For example, as noted earlier, a model may seem to work fine with moderate assumptions about growth, but under extreme negative growth, it may not work. Alternatively, we may find that the model balances and otherwise performs as designed, but its results seem unrealistic. If so, this suggests changing some of the underlying assumptions. For example, if sales decrease, would we actually divest assets proportionally to the decrease in sales as this model does? If we think not, and if it is an important issue, we need to change the model so that it accounts for more realistic scenarios.

The first step is trying out different sales growth rates and see if the model remains in balance. Try extreme rates of growth for sales: $-50%$, $0%$, and $+50%$ per year. In each instance, check the asset and liability difference (line 52), the cash flow disposition (line 87), and the dividends and minimum dividends difference (line 32). In addition, look for other anomalies that might indicate a problem in our modeling.

[Exhibit 4.11](#) summarizes our main observations for each of the growth rates tried. In all cases, the model balances. However, some results are problematic at the extremes. At very high growth rates, the need to invest in new assets requires a very large amount of borrowing because the model says all external funds must come from debt. This is not really a problem for the model but it alerts us to a common problem associated with rapid growth—growth requires investment and a growing firm can easily outrun its ability to finance the growth. As far as the model goes, there is nothing wrong. However, we may decide to include a provision in the financing section that requires equity financing instead of debt if the debt ratios get too high.

Line	Issue	Growth Rate		
		-50%	0%	50%
52	Liabilities = Assets?	ok	ok	ok
87	Cash flow fully allocated?	ok	ok	ok
32	Dividend = Payout x Net Income?	Div > NI	Div > NI	ok
Comments		Assets decrease; Debt constant; Debt > TA	Equity decreases; Debt/Equity increases	Declining dividend; Heavy investment; Extreme debt

EXHIBIT 4.11 Stress test results.

At zero growth, the model balances, but the firm is generating enough cash flow each period that it can pay a dividend that exceeds net income. As a result, the contribution to retained earnings becomes negative. This, in turn, reduces the value of equity. As there is no provision in this simple model to repay debt, the debt balance remains constant and the debt-to-equity ratio increases.

When sales decrease rapidly, the model remains balanced. It pays dividends that greatly exceed net income because the firm, in essence, is being liquidated. *NFA* go to zero in 2018, followed by depreciation expense in 2019. Equity decreases while debt remains constant, so not only does the debt-to-equity ratio increase, eventually debt exceeds total assets. This suggests that we may want to put a provision in our model that repays debt from the cash flow generated by the liquidation. We add this option later.

Also on display is a weakness common to percent-of-sales models: The model performs as designed with assets increasing and decreasing proportionally to sales. However, it might be more realistic to assume that some asset categories, particularly fixed assets, increase in discrete increments as sales increase, but do not decrease (in the short run) when sales decrease. For example, if sales increase 10%, fixed assets may not increase at all because the firm has unused capacity that enables it to increase production without increasing the plant. On the other hand, if sales decrease 10% the firm probably will not immediately dispose of 10% of its productive capacity. It may keep the existing plant for several periods until it can dispose of a whole production unit, especially if the plant is not divisible. In [Chapter 10](#), we discuss how to modify the model so that it accounts for the capacity to expand without increasing assets and how to tolerate moderate sales declines without divesting assets.

None of these observations indicate that the model is working incorrectly. Rather, they point to problems that could be encountered by the actual business under conditions of extreme growth or extreme decline. If we believe that the extreme scenarios are unlikely, then we may feel that modifications to the model to account for such extreme occurrences are unnecessary. However, it would be worthwhile to try out additional growth scenarios to identify the limits of the model. In other words, at what points does the model start to generate unrealistic results?

4.4.3 Tracking Performance

Models should include a section that summarizes the results of our projections. After all, we build models to answer just such questions. Some of the results have already been shown—the pro forma financial statements for the planning horizon. However, it is difficult to digest all the data of these statements. We need to summarize the results in a way that we can evaluate the firm's prospective performance. We discuss two aspects of that evaluation: analysis of the financial ratios and estimation of the value of the firm.

[Exhibit 4.12a](#) shows a selection of financial ratios; [Exhibit 4.12b](#) shows the formulas used to calculate them. The ratios are shown in order of the DuPont format that emphasizes the interplay of profit margin, asset turnover, and financial leverage in determining the return on equity. Other standard ratios relating to liquidity (Current Ratio) and financial leverage (Debt Ratio, Debt-to-Equity, etc.) are also shown. In the exhibit, *TD* denotes total debt, including current liabilities such as accounts payable, whereas *TD_{ib}* denotes interest-bearing debt.

B		C		D		E		F		G		H		I	
PERFORMANCE MODULE		2015		2016		2017		2018		2019		2020		2021	
94	Return on Assets (EBIT/TA)	7.9%	10.1%	11.6%	11.8%	11.9%	12.0%	12.0%	12.1%						
95	Return on Equity (NI/Equity)	6.6%	8.7%	10.1%	10.3%	10.5%	10.6%	10.8%	10.8%						
96	Profit Margin (NI/Sales)	9.9%	12.8%	14.6%	14.6%	14.6%	14.6%	14.6%	14.6%						
97	Asset Turnover (Sales/TA)	0.47	0.47	0.47	0.48	0.48	0.49	0.49	0.49						
98	Asset Intensity (TA/Sales)	2.13	2.13	2.11	2.09	2.06	2.04	2.02	2.02						
99	Equity Multiplier (TA/Equity _{t-1})	1.43	1.46	1.47	1.48	1.48	1.49	1.49	1.49						
100	Current Ratio (CA/CL)	1.18	1.18	1.18	1.18	1.18	1.18	1.18	1.18						
101	Debt Ratio (TD/TA)	30.0%	31.6%	32.0%	32.3%	32.6%	32.8%	32.8%	33.1%						
102	Debt to Equity (TD/Equity)	42.9%	46.1%	47.0%	47.6%	48.3%	48.9%	49.5%	49.5%						
103	Debt to Equity (TD _{ib} /Equity)	2.7%	5.0%	5.2%	5.2%	5.2%	5.1%	5.1%	5.1%						
104	Debt to Total Capital (TD _{ib} /(Debt _{ib} + Equity))	2.6%	4.8%	5.0%	4.9%	4.9%	4.9%	4.9%	4.9%						

(a)

B		E		F	
PERFORMANCE MODULE		2017		2018	
94	Return on Assets (EBIT/TA)	=E24/E43	=F24/F43		
95	Return on Equity (NI/Equity)	=E28/E50	=F28/F50		
96	Profit Margin (NI/Sales)	=E28/E20	=F28/F20		
97	Asset Turnover (Sales/TA)	=E20/E43	=F20/F43		
98	Asset Intensity (TA/Sales)	=E43/E20	=F43/F20		
99	Equity Multiplier (TA/Equity _{t-1})	=E43/E50	=F43/F50		
100	Current Ratio (CA/CL)	=E38/E46	=F38/F46		
101	Debt Ratio (TD/TA)	=(E46+E47)/E51	=(F46+F47)/F51		
102	Debt to Equity (TD/Equity)	=(E46+E47)/E50	=(F46+F47)/F50		
103	Debt to Equity (TD _{ib} /Equity)	=E47/E50	=F47/F50		
104	Debt to Total Capital (TD _{ib} /(Debt _{ib} + Equity))	=(E47)/(E47+E50)	=(F47)/(F47+F50)		

(b)

EXHIBIT 4.12 (a) O&R performance module. (b) O&R formulas for performance module.

This model shows just a few of the wide variety ratios and performance measures that could have been included. In general, the performance measures reported will be determined by the purpose of the model. For example, this model did not break the assets into categories such as accounts receivable, inventories, and various other types of fixed assets. If it had been important to consider these more detailed accounts, they could be easily have been added.

Most of the ratios reported are quite stable over the planning horizon. This is not surprising given that the assets and most of the expenses are modeled as a constant percent of sales. Our conclusion for Odd & Rich is that the firm can handle moderate growth without being burdened by excessive debt.

This brief analysis points out one of the most common uses for such a model—the exploration of the consequences of a set of policies. The model enables us to see what might happen under a variety of scenarios if we follow the policies specified in the model. If we do not like the projected results, the model allows us to try out alternative policies, so we can find the mix that will best suit our objectives.

Although ratio analysis can be helpful for showing the strengths and weaknesses of a plan, it is not particularly helpful when trying to decide which of many possible plans is best. This raises the question, *how do you measure what is best?* Generally, the objective will be to maximize the value of the equity of the firm. Given this objective, estimating the value of equity for each policy provides a single measure related directly to the objective. This enables us to determine which policies have the best chance of reaching the objective. Consequently, models should include a valuation module. We will discuss this next.

4.4.4 Valuation

4.4.4.1 The Valuation Module

The valuation module estimates the value of the equity of the company being modeled. We use the variable growth dividend discount model shown in Equation 4.14 to do this, thus:

$$\text{Equity Value}_0 = E_0 = \sum_{t=1}^h \frac{D_t}{(1+k_e)^t} + \frac{E_h}{(1+k_e)^h} \quad (4.14)$$

where:

D_t denotes the dividend paid at time t

k_e is the discount rate

h is the number of periods in the forecast horizon

E_h is the terminal value of the equity at the end of the forecast horizon.

We use the capital asset pricing model (CAPM) to estimate the cost of equity, k_e , and the constant growth model to estimate the terminal value, E_h . The horizon (h) is five years. The valuation model generates an estimate of the aggregate value of the equity, total equity capitalization. At this stage, we are not concerned with the value per share. We consider the number of shares and the price per share in [Chapter 5](#) when we take up the dilutive effects of issuing additional shares.

[Exhibit 4.13a](#), shows the valuation module; [Exhibit 4.13b](#) shows the formulas. The data for calculating the discount rate are found in the Input Sector cells E4:F9 ([Exhibit 4.8a](#));

	B	C	D	E	F	G	H	I
	EQUITY VALUATION MODULE							
	2016	2015	2016	2017	2018	2019	2020	2021
107								
108								
109			0.02					
110			0.07					
111			0.98					
112			8.86%					
113								
114								
115				92.6	98.1	104.0	110.3	116.9
116			404					
117								
118								3.00%
119								120.4
120								2,055
121			1,344					
122								
123			1,748					

(a)

	B	C	D	E	F	G	H	I
	EQUITY VALUATION MODULE							
	2016	2015	2017	2018	2019	2020	2021	
107								
108								
109			=rf					
110			=mnp					
111			=beta					
112			=rf+beta*E110					
113								
114								
115			=E30	=F30	=G30	=H30	=I30	
116			=NPV(Discoun_ Rate,E115:I115)					
117			Terminal value					
118								=lt_growth
119								=(1+lt_growth)*I115
120								=I119/(Discount_Rate-I115)
121								
122								
123			=D116+D121					

(b)

EXHIBIT 4.13 (a) O&R valuation module. (b) O&R formulas for valuation module.

beta in C12; and the long-run growth rate for the constant growth model in F9. These input data are repeated in cells D109:D111. The beta is estimated to be 0.98.* The cost of equity is

$$\begin{aligned} k_e &= r_f + (r_M - r_f)\beta \\ &= 0.02 + 0.07 * 0.98 \\ &= .0886 = 8.86\% \end{aligned} \tag{4.15}$$

The dividends shown in line 115 come from the dividends shown in line 30 of the Income Statement. We calculate the present value of dividend stream (D116) using Excel's NPV function. The terminal value at the end of 2016 is shown in cell I120 based on the constant growth model,

$$\begin{aligned} E_5 &= \frac{D_5(1+g)}{(k_e - g)} \\ &= \frac{116.9*(1.03)}{(0.0886 - 0.03)} \\ &= \$2,055 \end{aligned} \tag{4.16}$$

Its present value is calculated in D121. The value of the equity in cell D123 is the sum of the present value of the dividends and the present value of the terminal value.

4.4.4.2 Evaluation and Sensitivity Analysis

Given the conditions and assumptions listed in the input section, we have estimated the aggregate value of the equity to be \$1,748. Suppose we want to know the impact on the value with different forecasts for variables such as growth rates. To determine what happens when the firm experiences different rates of growth of sales for the next five years (holding the posthorizon growth rate constant at 3%), four different growth rates were put into the model, other variables remained unchanged. [Exhibit 4.14a](#) reports the results.

The higher equity value and the higher debt ratio in 2021 for the -10% growth scenario stems from the fact that the firm is reducing its equity over time and pays it out as dividends. Once the growth rates turn positive, debt increases as it is used to fund O&R's growth. The fact that the equity is worth more when sales decrease at 10% per year warrants closer inspection. The firm is throwing off a lot cash as it liquidates assets when sales decrease. The additional

* For a more detailed model capable of analyzing the effects of alternative capital structures, the beta should adjust to the amount of financial leverage. In that case, the input beta could be the unleveraged beta, and the leveraged beta would adjust internally in the model using the Hamada (1969) formula,

$$\beta_L = \beta_U [1 + (\text{Debt}/\text{Equity}) (1 - T)]$$

where:

L is leveraged

U is unleveraged.

So as to not make the model too complex at this stage, we have not yet dealt with this issue. We discuss the effects of leverage on cost of equity in greater detail in [Chapter 5](#).

5 Year Growth Rate	-10%	0%	6%	10%
Debt Ratio 2017	33.8%	32.4%	32.0%	34.9%
Debt Ratio 2021	52.2%	36.4%	33.1%	45.6%
Value of Equity (a)	\$ 4,172	\$ 3,099	\$ 1,748	\$ 2,019

Debt Portion	0%	25%	50%	75%	100%
Debt Ratio 2017	31.3%	32.2%	33.1%	34.0%	34.9%
Debt Ratio 2021	30.6%	34.3%	38.0%	41.8%	45.6%
Value of Equity (b)	\$ 2,092	\$ 2,074	\$ 2,056	\$ 2,037	\$ 2,019

EXHIBIT 4.14 (a) Sensitivity analysis: Debt ratios and equity values with varying sales growth rates. (b) Sensitivity analysis: Debt ratios and equity values with varying debt portions.

cash over the minimum dividend in year 2021 is compounded in perpetuity even though it is only a one-time cash flow. In addition, the long-run growth (after year 5) was held constant at 3%, so the 10% sales decrease only applies to the first five years. Given these findings, we might conclude that stockholders would be better off shrinking the firm over the next five years. But this model is not yet complete, so such a conclusion would be unwarranted.

The sensitivity analysis above looked at the effect of varying the growth rate on equity value and the debt ratio. The financing policy was unchanged from the base model where it was assumed that 100% of external financing came from debt. We now explore the implication of different financing policies on firm value. We assume 10% growth in sales during the five-year horizon and vary the proportion of debt that is used to fund our external financing requirements. [Exhibit 4.14b](#) shows the effect of debt financing on equity value as the percentage of external financing that is debt (debt_portion, cell F12) ranges from 0 to 100%.

Equity value decreases as we use more debt to fund our external financing needs over the next five years. The reason for this is that debt financing and the resulting interest expense eats into the equity cash flow. Less is left for the stockholders, so equity value is smaller.* However, this table reports the aggregate value of equity value, not the value per share. Thus far, we have not included share values in our model, so we can say very little about the value per share. The only thing we know is that if we use 100% debt financing and issue no new equity, the number of shares will remain the same over the planning horizon. However, as the equity portion of new financing increases, the number of shares will increase. Thus, even though the aggregate value of equity is higher with 0% debt financing, it could be the case that the value per share is lower. To address that issue, we would have to model the number of shares. We leave that for later when we delve more deeply into models of valuation in [Chapter 5](#).

There are many other variables and policies that could be explored with sensitivity analysis. The variety of ways to manipulate the model is limited only by the needs and imagination of the model builder. In this section, we demonstrated the approach for just two: growth and debt financing.

* Had we linked the cost of equity to the D/E ratio, it would have increased with the amount of debt, further depressing the value of equity.

4.5 CONTROLLING THE FLOW OF EXCESS CASH

Our model as currently structured lacks two critical features—it does not account for the effects of issuing or repurchasing equity, nor does it offer any uses for excess cash other than paying it out as dividends. We address the problems created by issuing or repurchasing equity in the next chapter; we address other uses for excess cash now.

When a firm generates more cash than it needs to support its sales, service its debt, and pay scheduled dividends, it must do something with that cash. We consider three basic uses for excess cash—paying it out as a special dividend, purchasing marketable securities, or retiring debt and repurchasing equity. This sounds simple. It is not. It forces you to think through how cash flows through the various modules and onto the balance sheet or out to shareholders. Nearly every sector and module of the model require modifications. Doing so represents an interesting challenge and tests your understanding of the material in this chapter.

We build this ability into the model in two steps. First we add the option to allocate cash for internal uses by purchasing marketable securities. Once that works, we add the ability to retire debt and repurchase shares.

4.5.1 Purchasing Marketable Securities

As mentioned in the introduction to this section, adding the ability to allocate cash for both internal and external uses affects nearly every sector and module of our model. Consider the complexity of adding the option to purchase Marketable Securities: It is a new item on the balance sheet; it matures each year, generating cash that must be accounted for in the financing and cash flow modules; and it is an investment, affecting the valuation module and Statement of Cash Flows. We consider these effects one sector or module at a time.

A word on notation. When we refer to an item or concept in general, we use all lower case. For example, marketable securities are short-term investments firms purchase to park cash. When we refer to a titled line item our model, we used a formal title followed initially by a line reference.* For example, the line on the balance that reports the current balance of marketable securities is referred to as Marketable Securities (line 38). Finally, when we refer to a value for a specific year or an input variable to be used in a formula, the name will be written in italics and, if an income statement or balance sheet item, subscripted. For example, the balance of Marketable Securities in year t will be referred to as *Marketable Securities_t*.

4.5.1.1 Adding Transparency to Required External Financing

The Required External Financing line is the heart and soul of our model. It calculates the number that brings our model into balance—the plug. When REF is positive, the model is transparent. It is easy to see how it works. The discretionary input variable *debt_portion* allocates external financing between issuing new debt and equity, thereby bringing the balance sheet into balance. However, when REF is negative, the model is opaque. Code must be added to prevent negative debt and equity from being issued. Also, thinking of excess cash as a negative number feels somehow odd. To resolve these problems, we split required external financing into two lines. The name of the first line remains Required External

* For clarity, all line references in [Section 4.5.1.1](#), refer to [Exhibits 4.15](#) for values and [Exhibits 4.16](#) for code.

Financing (line 102); it reports REF_t as a positive number, else zero. We name the second line Excess Cash (line 103); it reports $Excess\ Cash_t$ as a positive number (when REF_t would otherwise be negative), else zero.

This has two significant benefits. First, it makes the model far more transparent. The user instantly sees whether external funds are needed or excess cash has been generated. Second, it simplifies coding for issuing debt and equity and for allocating excess cash. The code for the two lines, shown in Equation 4.17, is simple:

$$REF_t = \text{MAX} \left(\left[\begin{array}{c} \text{Required New} \\ \text{Investment}_t \end{array} \right] - \left[\begin{array}{c} \text{Spontaneous \& internal} \\ \text{sources of financing}_t \end{array} \right], 0 \right)$$

and (4.17)

$$Excess\ Cash_t = -\text{MIN} \left(\left[\begin{array}{c} \text{Required New} \\ \text{Investment}_t \end{array} \right] - \left[\begin{array}{c} \text{Spontaneous \& internal} \\ \text{sources of financing}_t \end{array} \right], 0 \right)$$

Having split REF, we are ready to modify the sectors and modules necessary to allocate cash between dividends and purchasing marketable securities. We work from the top down.

Input Sector: The decision being modelled is whether to direct excess cash internally to purchase Marketable Securities or externally to pay a special dividend. We use a decision input variable that the user varies between 0 and 1, called *cash_returned*. When *cash_returned* is set to 1, all excess cash will be paid out as dividends; when set to 0, all excess cash will be used to purchase marketable securities.

Balance Sheet: Marketable Securities is a current asset (it can readily be converted to cash). Add a line called Marketable Securities (line 38) to the current assets section of the balance sheet, immediately under Cash (line 37). If necessary, change the name of the item called Cash and Securities (or Cash and Equivalents) to Cash. Cash now represents the cash balance required to sustain day-to-day operations; Marketable Securities represents accumulated excess cash. Make sure that this new item is reflected in Total Current Assets (line 40).

Income Statement: No change is needed, unless you wish to have marketable securities earn interest. If so, you will need to add a line to the income statement.*

Investment Module: Marketable Securities (line 38) is a discretionary item. Funding it is not required, so long as company policy does not require a minimum balance. Therefore, it will not affect the Investment Module, with the following caveat. If you have calculated the the Current Asset Investment (line 85) as the change in Total Current Assets (line 40), it needs to be modified. Modify the code so that it calculates the change in each current asset other than Marketable Securities. Otherwise, a circular reference will be created a bit later and wreak havoc with solving your model.

Financing Module: Marketable Securities (line 38) mature each year, resulting in a spontaneous source of internally generated cash that can be used to finance new investment. Therefore,

* Typically, the balances in these accounts earn very little, hence, adding an interest earned line rarely materially affects firm value. This situation will change if you find yourself modeling a firm with a substantial balance in cash equivalents.

you need to add a line (Maturing Marketable Securities, line 98) to the Financing Module just above the Internal Equity Financing (line 99). The cells in this line refer to the previous year's Marketable Securities (*Marketable Securities_{t-1}*) in the Balance Sheet. Be sure to include the new cash inflow in your calculations for Total Internal and Spontaneous Financing (line 101).

Cash Flow Module: Adding marketable securities affects the Cash Flow module in two ways. First, it is a cash inflow that needs to be recognized above the Net Cash Flow line (line 117). As Marketable Securities (line 38) mature each year, the Cash Flow module must include the previous year's balance in the calculation of Net Cash Flow. Add the line Maturing Marketable Securities (line 111) above the Net Cash Flow line and include it when calculating Net Cash Flow.

Second, purchasing Marketable Securities is a decision that must be made below the Net Cash Flow line. Add a line called Marketable Securities Purchased (line 119) directly beneath the line that calculates the Increase in Cash (line 118). Calculate the amount of Marketable Securities purchased as $(Excess\ Cash_t) * (1 - cash_returned)$, where *Excess Cash_t* refers to the current year's value of the Excess Cash line in the Financing Module. Have *Marketable Securities_t* in the balance sheet reference this line, for example, reference *Marketable Securities Purchased_t*. (If this creates a circular reference, it is most likely because you inadvertently included Marketable Securities in the Increase in Current Assets in the Investment Module). Adjust Cash Available for Dividends (line 120) to reflect this new use of cash. While you are in the Cash Flow Module, adjust its check line by subtracting this year's Marketable Securities (from the balance sheet).

Statement of Cash Flows: The maturing and purchasing of marketable securities are cash flows from investing activities. Add an Increase in Marketable Securities line (line 71) to the Cash Flows from Investing Activities panel in the Statement of Cash Flows. Be sure to pick up this addition in Cash Flows from Investing Activities (line 72).

Valuation Module: Marketable Securities are a nonoperating asset (an asset that is not necessary to the ongoing operations of a business). Hence, any Marketable Securities in the terminal year need to be accounted for separately in the valuation module. Add a line to your valuation module that calculates the present value of Marketable Securities in year 2021, then add that present value to your aggregate equity calculation.*

[Exhibit 4.15a, b](#) and [c](#) show the model as modified above. [Exhibit 4.16a, b](#) and [c](#) show the code. The shaded items have been modified.

Test your model: It is time to test your model. Set the *cash_returned* variable to 0 and the growth rate to 0 so that O&R generates excess cash. You should see all excess cash accumulating in Marketable Securities on the balance sheet. Next, set *cash_returned* to 1. All excess cash should be flowing through to Cash Available for Dividends. Finally, set *cash_returned* to 0.5. Half of the excess cash generated should be accumulating in Marketable Securities, half should be flowing through to dividends. Throughout it all, your model should remain in balance.

* Accounting for marketable securities in this way tacitly assumes that you are building a planning model. Put differently, it assumes that you or the user will control how excess cash is handled.

	B	C	D	E	F	G	H	I
84	INVESTMENT MODULE	2015	2016	2017	2018	2019	2020	2021
85	Current Asset Investment			-	-	-	-	-
86	Investment in New Fixed Assets (ΔNFA)			(54.6)	(57.9)	(57.9)	(57.9)	(57.9)
87	Replacement of Depreciated Assets			57.8	57.9	57.9	57.9	57.9
88	Fixed Asset Investment			3.2	0.0	(0.0)	(0.0)	0.0
89	Total Investment			3.2	0.0	(0.0)	(0.0)	0.0
90								
91	FINANCING MODULE			2017	2018	2019	2020	2021
92	Required Financing			3.2	0.0	(0.0)	(0.0)	0.0
93	Sources of Funds							
94	Net Income			143.6	143.5	143.5	143.5	143.5
95	Depreciation			57.8	57.9	57.9	57.9	57.9
96	Earnings Cash Flow			201.4	201.4	201.4	201.4	201.4
97	Minimum Dividend			86.1	86.1	86.1	86.1	86.1
98	Maturing Marketable Securities			-	56.0	85.7	100.5	107.9
99	Internal Equity Financing			115.2	171.3	201.0	215.8	223.2
100	Spontaneous Financing			-	-	-	-	-
101	Total Internal & Spontaneous Financing			115.2	171.3	201.0	215.8	223.2
102	Required External Financing			-	-	-	-	-
103	Excess Cash			112.0	171.3	201.0	215.8	223.2
104	Debt Issued			-	-	-	-	-
105	Equity Issued			-	-	-	-	-
106	Total Discretionary Financing			-	-	-	-	-
107								
108	CASH FLOW MODULE			2017	2018	2019	2020	2021
109	Earnings Cash Flow			201.4	201.4	201.4	201.4	201.4
110	Current Asset Investment (except Cash)			-	-	-	-	-
111	Maturing Marketable Securities			-	56.0	85.7	100.5	107.9
112	Spontaneous Financing			-	-	-	-	-
113	NWC Investment			-	(56.0)	(85.7)	(100.5)	(107.9)
114	Cash Flow from Operations			201.4	257.4	287.1	301.9	309.3
115	Capital Expenditures			3.2	0.0	(0.0)	(0.0)	0.0
116	Capital Financing			-	-	-	-	-
117	Net Cash Flow			198.2	257.4	287.1	301.9	309.3
118	Increase in Cash			-	-	-	-	-
119	Marketable Securities Purchased			56.0	85.7	100.5	107.9	111.6
120	Cash Available for Dividends			142.1	171.8	186.6	194.0	197.7
121	Check: NCF-ΔCash-Dividends			-	-	-	-	-
122								
141	EQUITY VALUATION MODULE	2015	2016	2017	2018	2019	2020	2021
142	Cost of Equity							
143	Risk-free Rate		0.02					
144	Market Risk Premium		0.07					
145	Beta		0.98					
146	Discount Rate		8.86%					
147								
148	Dividend Valuation Model							
149	Dividends			142.1	171.8	186.6	194.0	197.7
150	PV(Dividends)		688					
151	Marketable Securities							111.6
152	PV(Marketable Securities)		73					
153	Terminal value							
154	Long-term Growth Rate							0.03
155	Forecast Dividend t = 6							203.63
156	Terminal Value at t=5							3,475
157	PV(Terminal Value)		2,273					
158								
159	Aggregate Value of Stock		3,034					

(c)

EXHIBIT 4.15 (Continued) (c) Investment, financing, cash flow, and valuation modules with the option to purchase marketable securities: Growth set to 0, *cash_returned* to 0.50.

previous section. We now add *cash_invested* and *debt_retired*. The three cash control variables are:

1. *Cash_returned*: allocates excess cash between internal uses and paying a special dividend.
2. *Cash_invested*: allocates the excess cash directed toward internal uses between purchasing marketable securities and retiring outstanding securities (debt and/or equity), and
3. *Debt_retired*: allocates the excess cash directed toward retiring outstanding securities between retiring debt (debit Long-term Debt in the balance sheet) or repurchasing stock (debit Treasury Stock in the balance sheet).

	B	D	E	F
84	INVESTMENT MODULE	2016	2017	2018
85	Current Asset Investment		=E37-D37+E39-D39	=F37-E37+F39-E39
86	Investment in New Fixed Assets (Δ NFA)		=E44-D44	=F44-E44
87	Replacement of Depreciated Assets		=E24	=F24
88	Fixed Asset Investment		=E86+E87	=F86+F87
89	Total Investment		=E85+E88	=F85+F88
90				
91	FINANCING MODULE		2017	2018
92	Required Financing		=E89	=F89
93	Sources of Funds			
94	Net Income		=E29	=F29
95	Depreciation		=E24	=F24
96	Earnings Cash Flow		=SUM(E94:E95)	=SUM(F94:F95)
97	Minimum Dividend		=MAX(payout*E29,0)	=MAX(payout*F29,0)
98	Maturing Marketable Securities		=D38	=E38
99	Internal Equity Financing		=E96-E97+E98	=F96-F97+F98
100	Spontaneous Financing		=E48-D48	=F48-E48
101	Total Internal & Spontaneous Financing		=E99+E100	=F99+F100
102	Required External Financing		=MAX(E92-E101,0)	=MAX(F92-F101,0)
103	Excess Cash		=-MIN(E92-E101,0)	=-MIN(F92-F101,0)
104	Debt Issued		=debt_portion*E102	=debt_portion*F102
105	Equity Issued		=E102-E104	=F102-F104
106	Total Discretionary Financing		=E104+E105	=F104+F105
107				
108	CASH FLOW MODULE		2017	2018
109	Earnings Cash Flow		=E29+E24	=F29+F24
110	Current Asset Investment (except Cash)		=E39-D39	=F39-E39
111	Maturing Marketable Securities		=D38	=E38
112	Spontaneous Financing		=E48-D48	=F48-E48
113	NWC Investment		=E110-E112-E111	=F110-F112-F111
114	Cash Flow from Operations		=E109-E113	=F109-F113
115	Capital Expenditures		=E44-D44+E24	=F44-E44+F24
116	Capital Financing		=E49-D49+E50-D50	=F49-E49+F50-E50
117	Net Cash Flow		=E114-E115+E116	=F114-F115+F116
118	Increase in Cash		=E37-D37	=F37-E37
119	Marketable Securities Purchased		=E103*(1-Cash_returned)	=F103*(1-Cash_returned)
120	Cash Available for Dividends		=E117-E118-E119	=F117-F118-F119
121	Check: NCF- Δ Cash-Dividends		=E117-(E37-D37)-E31-E38	=F117-(F37-E37)-F31-F38
122				
141	EQUITY VALUATION MODULE	2016	2017	2018
142	Cost of Equity			
143	Risk-free Rate	=rf		
144	Market Risk Premium	=mrp		
145	Beta	=beta		
146	Discount Rate	=rf+beta*D144		
147				
148	Dividend Valuation Model			
149	Dividends		=E31	=F31
150	PV(Dividends)	=NPV(discount_rate,E149:I149)		
151	Marketable Securities			
152	PV(Marketable Securities)	=I151/(1+discount_rate)^5		
153	Terminal value			
154	Long-term Growth Rate			
155	Forecast Dividend t = 6			
156	Terminal Value at t=5			
157	PV(Terminal Value)	=I156/(1+discount_rate)^5		
158				
159	Aggregate Value of Stock	=D150+D157+D152		

(c)

EXHIBIT 4.16 (Continued) (c) Code for investment, financing, cash flow, and valuation modules with the option to purchase marketable securities: Growth set to 0; *cash_returned* to 0.50.

To get started, we need to add two new items to the balance sheet—Short-term Debt (line 49), a current liability; and Treasury Stock (line 54), a shareholders equity item.

Short-term Debt: Short-term Debt behaves like Marketable Securities in several ways. It matures each year and therefore must be accounted for in the financing module above

Required External Financing, and it requires an active decision by management to issue. To introduce the coding for this problem, \$25 of Long-term Debt in year 2016 has been moved to Short-term Debt. This has thrown the model out of balance by \$150. The situation will right itself once the model is adjusted as instructed below.

Short-term Debt (line 49) matures each year creating a liability that must be financed. For example, Short-term Debt in year 2016 must be paid off in year 2017. To do this, add a Maturing Short-term Debt (line 103) to your Financing Module. Make sure this cash outflow is picked up by Spontaneous Financing (line 104). Because retiring short-term debt is a use of cash, it reduces spontaneous financing. In addition, Short-term Debt is another source of interest-bearing finance. Adjust Interest Expense (line 26) in the income statement to pick up this additional interest expense.

Treasury Stock: Add a line titled Treasury Stock (line 54) to the balance sheet, just under Earned Surplus (line 53). Treasury Stock is stock that has been temporarily retired. It can be reissued without having to register it with the SEC. Treasury Stock is a contra account. As equity is repurchased, this account is debited. Adjust the Total Shareholders Equity (line 55) to reflect this addition.

Cash Flow Module: Pick up the change in Short-term Debt and Treasury Stock in Capital Financing (line 123). Next, modify the Marketable Securities Purchased (line 126) using the following formula, where *Excess Cash_t* refers to the *Excess Cash* line in the Financing Module.

$$\left[\begin{array}{c} \text{Marketable} \\ \text{Securities Purchased}_t \end{array} \right] = [\text{Excess cash}_t](1 - \text{cash_returned})(\text{cash_invested}) \quad (4.18)$$

Note the logic. $[\text{Excess Cash}_t]$ is the amount of cash O&R generated in excess of its financing needs at time t . $[\text{Excess Cash}_t](1 - \text{cash_returned})$ is the amount of excess cash allocated for internal uses. Finally, $[\text{Excess Cash}_t](1 - \text{cash_returned})(\text{cash_invested})$ is the amount of internally directed excess cash used to purchase marketable securities. (Any remaining excess cash will be used to retire debt or repurchase equity.) Adjust the code in Cash Available for Dividends (line 127) to reflect the purchase of marketable securities.

Statement of Cash Flows: In the Statement of Cash Flows, add a line to the Cash flow from financing activities to reflect the Increase in Short-term Debt (line 78). Also, adjust the Increase in Equity (line 80) to reflect the change in the Treasury Stock.

4.5.1.2.1 Incorporate Decision Variables into the Financing Module We now incorporate the three cash control variables (*cash_returned*, *cash_invested*, and *debt_retired*) into the Financing Module.

Retiring debt: The first line to be modified is the Debt Retired (line 110). Several calculations are necessary to get to the amount of excess cash used to retire debt. First, the amount of excess cash for internal uses in year t must be calculated. This is $[\text{Excess Cash}]_t (1 - \text{cash_returned})$. Some of this internally directed cash may be directed toward purchasing marketable securities. This leaves $[\text{Excess cash}_t](1 - \text{cash_returned})(1 - \text{cash_invested})$ available for retiring outstanding

securities. Of this amount, the portion determined by *debt_retired* will be used to retire Long-term Debt. The calculation in its entirety is

$$\left[\begin{array}{c} \text{Debt} \\ \text{retired}_t \end{array} \right] = [\text{Excess cash}_t](1 - \text{cash_returned})(1 - \text{cash_invested})(\text{debt_retired}) \quad (4.19)$$

Incorporate this code into a MIN(.) function to prevent to Long-term Debt (line 51) from becoming negative (see extending the Financing Module, Unit 3 [section 3.2](#)). Finally, adjust the Long-term Debt to reflect Debt Retired (line 110).

Repurchasing stock: The logic for repurchasing equity is identical to that above, save for one term. Any cash directed toward retiring securities and not used to retire debt will be used to repurchase stock. To capture this, subtract the amount of debt retired from the amount of cash allocated to retiring securities. This serves as the residual and ensures that all cash allocated to retiring securities is used for that purpose. The amount of equity repurchased is equal to:

$$\left[\begin{array}{c} \text{Equity} \\ \text{repurchased}_t \end{array} \right] = [\text{Excess cash}_t](1 - \text{cash_returned})(1 - \text{cash_invested}) - \left[\begin{array}{c} \text{Debt} \\ \text{retired}_t \end{array} \right] \quad (4.20)$$

Debit Treasury Stock (line 54) on the balance sheet by this amount. Also, adjust Capital Financing (line 123) in the Cash Flow Module to reflect the change in Treasury Stock if you have not yet done so. Finally, make sure your Common Stock (line 52) line on the balance sheet does not capture Stock Repurchased (line 112).

Adjust Total Discretionary Financing (line 113) to account for cash used to retire securities. Once you have adjusted all of the aggregating calculations correctly, your model will balance. The finished model for this phase with growth set to zero 0 and each of the three cash control variables set to 0.50 is reported in [Exhibit 4.17a, b and c](#); the code in [Exhibit 4.18a, b and c](#).

4.5.1.2.2 Test Your Model Having integrated these three uses for excess cash into your model, it is time to see if it behaves as intended. You will probably need to tweak various codes to keep your model in balance calculations as you make the following changes.

1. Set growth to zero and *cash_returned* to 1. This should generate Excess Cash of \$115.8 in years 2018–2021. This excess cash should be flowing into dividends. The check line in the income statement called Dividends – Minimum Dividend (line 33) should equal the \$115.8 of Excess Cash in years 2018–2021.
2. Set *cash_returned* and *cash_invested* to 0 and *debt_retired* to 0.5. Now, all Excess Cash should flow into Debt Retired and Equity Repurchased. As debt was so small to begin with, all Excess Cash will be flowing into Equity Repurchased by 2019. The check line in the income statement, Dividends – Minimum Dividend, should equal 0.

	B	C	D	E	F	G	H	I
87	INVESTMENT MODULE	2015	2016	2017	2018	2019	2020	2021
88	Current Asset Investment			-	-	-	-	-
89	Investment in New Fixed Assets (ΔNFA)			(54.6)	(57.9)	(57.9)	(57.9)	(57.9)
90	Replacement of Depreciated Assets			57.8	57.9	57.9	57.9	57.9
91	Fixed Asset Investment			3.2	0.0	(0.0)	(0.0)	0.0
92	Total Investment			3.2	0.0	(0.0)	(0.0)	0.0
93								
94	FINANCING MODULE			2017	2018	2019	2020	2021
95	Required Financing			3.2	0.0	(0.0)	(0.0)	0.0
96	Sources of Funds							
97	Net Income			143.6	144.4	144.8	145.2	145.3
98	Depreciation			57.8	57.9	57.9	57.9	57.9
99	Earnings Cash Flow			201.4	202.3	202.7	203.1	203.2
100	Minimum Dividend			86.1	86.6	86.9	87.1	87.2
101	Maturing Marketable Securities			-	21.8	34.4	37.5	38.4
102	Internal Equity Financing			115.2	137.4	150.2	153.5	154.4
103	Maturing Short-term Debt			25.0	-	-	-	-
104	Spontaneous Financing			(25.0)	-	-	-	-
105	Total Internal & Spontaneous Financing			90.2	137.4	150.2	153.5	154.4
106	Required External Financing			0.0	0.0	0.0	0.0	0.0
107	Excess Cash			87.0	137.4	150.2	153.5	154.4
108	Short-term Debt Issued			0.0	0.0	0.0	0.0	0.0
109	Debt Issued			0.0	0.0	0.0	0.0	0.0
110	Debt retired			10.9	17.2	18.8	1.5	0.0
111	Equity Issued			0.0	0.0	0.0	0.0	0.0
112	Equity repurchased			10.9	17.2	18.8	36.9	38.6
113	Total Discretionary Financing			(21.8)	(34.4)	(37.5)	(38.4)	(38.6)
114								
115	CASH FLOW MODULE			2017	2018	2019	2020	2021
116	Earnings Cash Flow			201.4	202.3	202.7	203.1	203.2
117	Current Asset Investment (except Cash)			-	-	-	-	-
118	Maturing Marketable Securities			-	21.8	34.4	37.5	38.4
119	Spontaneous Financing			-	-	-	-	-
120	NWC Investment			-	(21.8)	(34.4)	(37.5)	(38.4)
121	Cash Flow from Operations			201.4	224.0	237.0	240.7	241.5
122	Capital Expenditures			3.2	0.0	(0.0)	(0.0)	0.0
123	Capital Financing			(46.8)	(34.4)	(37.5)	(38.4)	(38.6)
124	Net Cash Flow			151.4	189.7	199.5	202.3	202.9
125	Increase in Cash			-	-	-	-	-
126	Marketable Securities Purchased			21.8	34.4	37.5	38.4	38.6
127	Cash Available for Dividends			129.6	155.3	161.9	163.9	164.3
128	Check: NCF-ΔCash-Dividends			-	-	-	-	-
129								
148	EQUITY VALUATION MODULE	2015	2016	2017	2018	2019	2020	2021
149	Cost of Equity							
150	Risk-free Rate		0.02					
151	Market Risk Premium		0.07					
152	Beta		0.98					
153	Discount Rate		8.86%					
154								
155	Dividend Valuation Model							
156	Dividends			129.6	155.3	161.9	163.9	164.3
157	PV(Dividends)		600					
158	Marketable Securities							38.6
159	PV(Marketable Securities)		25					
160	Terminal value							
161	Long-term Growth Rate							0.03
162	Forecast Dividend t = 6							169.28
163	Terminal Value at t=5							2,889
164	PV(Terminal Value)		1,890					
165								
166	Aggregate Value of Stock		2,515					

(c)

EXHIBIT 4.17 (Continued) (c) Investment, financing, cash flow, and valuation modules with full cash control: Growth set to 0, all cash control variables set to 0.50.

	B	D	E	F
87	INVESTMENT MODULE	2016	2017	2018
88	Current Asset Investment		=E37-D37+E39-D39	=F37-E37+F39-E39
89	Investment in New Fixed Assets (ΔNFA)		=E44-D44	=F44-E44
90	Replacement of Depreciated Assets		=E24	=F24
91	Fixed Asset Investment		=E89+E90	=F89+F90
92	Total Investment		=E88+E91	=F88+F91
93				
94	FINANCING MODULE		2017	2018
95	Required Financing		=E92	=F92
96	Sources of Funds			
97	Net Income		=E29	=F29
98	Depreciation		=E24	=F24
99	Earnings Cash Flow		=SUM(E97:E98)	=SUM(F97:F98)
100	Minimum Dividend		=MAX(payout*E29,0)	=MAX(payout*F29,0)
101	Maturing Marketable Securities		=D38	=E38
102	Internal Equity Financing		=E99-E100+E101	=F99-F100+F101
103	Maturing Short-term Debt		=D49	=E49
104	Spontaneous Financing		=E48-D48-E103	=F48-E48-F103
105	Total Internal & Spontaneous Financing		=E102+E104	=F102+F104
106	Required External Financing		=MAX(E95-E105,0)	=MAX(F95-F105,0)
107	Excess Cash		=MIN(E95-E105,0)	=MIN(F95-F105,0)
108	Short-term Debt Issued		0	0
109	Debt Issued		=debt_portion*E106	=debt_portion*F106
110	Debt retired		=MIN(E107*(1-Cash_returned)*(1-Cash_invested)*Debt_retired,D51)	=MIN(F107*(1-Cash_returned)*(1-Cash_invested)*Debt_retired,E51)
111	Equity Issued		=E106-E109	=F106-F109
112	Equity repurchased		=E107*(1-Cash_returned)*(1-Cash_invested)-E110	=F107*(1-Cash_returned)*(1-Cash_invested)-F110
113	Total Discretionary Financing		=E109+E111-E110-E112	=F109+F111-F110-F112
114				
115	CASH FLOW MODULE		2017	2018
116	Earnings Cash Flow		=E29+E24	=F29+F24
117	Current Asset Investment (except Cash)		=E39-D39	=F39-E39
118	Maturing Marketable Securities		=D38	=E38
119	Spontaneous Financing		=E48-D48	=F48-E48
120	NWC Investment		=E117-E119-E118	=F117-F119-F118
121	Cash Flow from Operations		=E116-E120	=F116-F120
122	Capital Expenditures		=E48-D48+E24	=F48-E48+F24
123	Capital Financing		=E51-D51+E52-D52+E54-D54+E49-D49	=F51-E51+E52-E52+E54-E54+F49-E49
124	Net Cash Flow		=E121-E122+E123	=F121-F122+F123
125	Increase in Cash		=E37-D37	=F37-E37
126	Marketable Securities Purchased		=E107*(1-Cash_returned)*Cash_invested	=F107*(1-Cash_returned)*Cash_invested
127	Cash Available for Dividends		=E124-E125-E126	=F124-F125-F126
128	Check: NCF-ΔCash-Dividends		=E124-(E37-D37)-E31-E38	=F124-(F37-E37)-F31-F38
129				
130	INTEREST AND DEBT SERVICE	2016	2017	2018
131	Interest Expense	=D26	=E26	=F26
132	Total Interest-Bearing Debt	=D51	=E51	=F51
133	Total Debt (e.g., Total Liabilities)	=D48+D51	=E48+E51	=F48+F51
134				
135	PERFORMANCE MODULE	2016	2017	2018
136	Return on Assets (EBIT/TA)	=D25/D45	=E25/E45	=F25/F45
137	Return on Equity (NI/Equity)	=D29/D55	=E29/E55	=F29/F55
138	Profit Margin (NI/Sales)	=D29/D21	=E29/E21	=F29/F21
139	Asset Turnover (Sales/TA)	=D21/D45	=E21/E45	=F21/F45
140	Asset Intensity (TA/Sales)	=D45/D21	=E45/E21	=F45/F21
141	Equity Multiplier (TA/Equity _{t-1})	=D45/D55	=E45/E55	=F45/F55
142	Current Ratio (CA/CL)	=D40/D48	=E40/E48	=F40/F48
143	Debt Ratio (TD/TA)	=(D48+D51)/D56	=(E48+E51)/E56	=(F48+F51)/F56
144	Debt to Equity (TD/Equity)	=(D48+D51)/D55	=(E48+E51)/E55	=(F48+F51)/F55
145	Debt to Equity (TD _t /Equity)	=(D51+D49)/D55	=(E51+E49)/E55	=(F51+F49)/F55
146	Debt to Total Capital (TD _t /(Debt _t + Equity))	=(D51)/(D51+D55)	=(E51)/(E51+E55)	=(F51)/(F51+F55)
147				
148	EQUITY VALUATION MODULE	2016	2017	2018
149	Cost of Equity			
150	Risk-free Rate	=rf		
151	Market Risk Premium	=mnp		
152	Beta	=beta		
153	Discount Rate	=rf+beta*D151		
154				
155	Dividend Valuation Model			
156	Dividends		=E31	=F31
157	PV(Dividends)	=NPV(discount_rate,E156:1156)		
158	Marketable Securities			
159	PV(Marketable Securities)	=158/(1+discount_rate)^5		
160	Terminal value			
161	Long-term Growth Rate			
162	Forecast Dividend t = 6			
163	Terminal Value at t=5			
164	PV(Terminal Value)	=163/(1+discount_rate)^5		
165				
166	Aggregate Value of Stock	=D157+D164+D159		

(c)

EXHIBIT 4.18 (Continued) (c) Code for investment, financing, cash flow, and valuation modules with full cash control: Growth set to 0, all cash control variables set to 0.50.

You now have a model capable of answering a variety of questions concerning the use of excess cash. However, even though it can issue and repurchase equity, it does not yet have the ability to account for the effects of these transactions on value per share. In the next chapter, we address that problem and take a closer look at terminal value.

4.6 CONCLUSION

This chapter is the core of this book—structuring financial simulation models. There are many different ways to model a given problem and a huge variety of problems that can be addressed. Hence, the variety of models is virtually innumerable. Obviously, not

all types of models can be presented and explained. This chapter presented and discussed the basic form of a *generic* financial statement simulation model. The types of equations that constitute such a model were explained. This generic model consists of modules that serve different functions in the model—the accounting, income generation, investment module, financing, and cash flow modules. In addition, we introduced simple valuation concepts and applied them in the valuation module. Each module consists of equations that express the relationships within the firm, and between the firm and its environment.

A critical component of any such model is the mechanism for balancing the model—for bringing it into equilibrium. Two major approaches to equilibrating the model were explained: a cash flow balancing system and an asset-liability balancing system. Each approach represents a way to balance investment and financing and to maintain consistency in the model.

The model that we developed was small, with detail kept to a minimum. Our emphasis was on the structure of the model. Additional details can be added once a model has been created that balances and is robust to widely varying assumptions. Later chapters discuss different ways this basic model can be adapted to address more specialized decision problem.

PROBLEMS

1. Spreadsheet Model for Growth: Fly-by-Night Airlines, Inc.

The Fly-by-Night Airlines, Inc. company was described in Problem 3, [Chapter 3](#). This is a continuation of that problem, so the income statement and the balance sheets for 2013 through 2016 that were shown in that problem are assumed to apply to this problem.

Your assignment is to develop a spreadsheet model of the Fly-by-Night Airlines, Inc. Your model should enable you to generate pro forma financial statements (income statement, balance sheet, and a sources and uses of cash statement) for each of the next five years (2017–2021).

In developing your model, make the following assumptions:

- Future ratios of COGS/Sales, SGA/Sales, Cash/Sales, Receivables/Sales, Inventories/Sales, Prepaid Expenses/Sales, Plant Property and Equipment/Sales, Other Assets/Sales, Accounts Payable/Sales, and Other current liabilities/Sales will be maintained at their average levels for the last four years.
- The ratios of *Depreciation Expense* to Plant Property and *Equipment*_{*t*-1} and the change in Deferred taxes to *Taxes*_{*t*-1} will be constant (three-year average).
- The ratio of Taxes to Taxable income will be constant (four-year average).
- Intangibles will be amortized at \$1,000 per year.
- No short-term financing will be used, and, after the current portion of long-term debt is paid off in 2011, no other portion of the long-term debt will become current.

- Investment and financing are in place at the beginning of each year. In other words, the assets required to support sales in 2017 have been purchased and paid for by 12/31/16.
- Interest is paid *in arrears*, meaning that interest paid on 12/31/17 is based on the debt balance outstanding at the end of the previous period, 12/31/16.

The goal of the previous two entries is that

$Depreciation\ expense_t$ is calculated as $Depreciation\ rate * GFA_{t-1}$

$Interest\ expense_t$ is calculated as $Interest\ rate * Debt_{t-1}$

- All external financing will be long-term debt, at an interest rate of 4.8%. Use any excess cash to retire debt. Payout all remaining excess cash as dividends (over and above the planned dividend). Hint: you will need to write a logical statement in the debt repayment cell of your financing module that prevents long-term debt from becoming negative. You will also need to calculate dividends in your financing module, then feed them into the income segment of the accounting module.
- Fly-by-Night expects to earn no interest income in the future.
- The dividend payout is 0%.

Assignment

- Show the results of your model assuming that sales grow at (1) g^* and (2) $PRAT$ (based on ratios as of 2016) for the next five years. What are your external financing requirements in each year for each scenario? What is the trend in the profit margin under each scenario?
- Show the results with sales growing at (1) 20% and (2) 30% per year. What are your external financing requirements in each year for these scenarios? What is the trend in the profit margin under each scenario? What happens to the profit margin as sales growth increases beyond $PRAT$?
- Show the results when sales grow 30% per year and half of all earnings are paid out as dividends. What are your external financing requirements in each year for this scenario? What is the relationship between dividends and external financing requirements?
- The sustainable growth rate $PRAT$ is significantly easier to apply and remember than g^{**} . However, it understates somewhat the actual sustainable growth rate. At what rate can Fly-by-Night grow its sales over the next five years and end up with the same $Total\ debt/Total\ assets$ ratio that it had at the end of 2016, assuming a zero dividend payout ratio? a 50% dividend payout ratio? (Hint: use Goal Seek)

Your model should be constructed so that it balances: Assets should equal liabilities, and sources should equal uses. Put checks into your model to verify that it

balances. In addition, your model should be constructed in such a way that the sections that contain equations do not have any numbers as parameters. The parameters should be in a separate section, with the two decision variables—sales growth and dividend payout—clearly delineated. Use Excel’s naming utility to name your parameters, for example, *cash_sales* for cash divided by sales. This will help you organize your model and make it easier to interpret your formulas.

What you hand in: a brief narrative in memo form explaining your answers to Parts A, B, C, and D. You will probably find it useful to include one or two tables in your memo. In an appendix, attach the pro forma statements for each scenario (formatted such that income statement and balance sheet are on one page, sources and uses of cash and the financing module are on another) and a printout of the formulas for your model. (There should be only one formula sheet. You answer the above questions simply by changing the values of your decision parameters and observing the effects.) Also include in your appendix a flowchart/decision tree diagram of the logic used to code the *Debt repaid* line of your financing module.

2. Modeling Growth & Financing: Fly-by-Night Airlines, Inc.

Fly-by-Night Airlines, Inc. was described in Problem 1. This is a continuation of that problem, so the income statement and the balance sheets for 2013–2016 that apply to Problem 1 are assumed to apply to this problem.

Your assignment is to continue to work on the same model, with the same assumptions as were described in Problem 1.

You have developed the basic model of the financial statements. Hopefully, it worked the first time. However, it is important to make sure that it balances under a variety of circumstances. For this assignment, assume a normal dividend payout of 50%. Check the balancing by varying the rate of growth of sales. Use high growth rates like 20% and 50% per year and see how it performs. Then use low growth scenarios like –10% and –20% per year. Assume that extra cash is used first to repay debt; if any is left over, pay it out as dividends. As before, assets should equal liabilities and all cash flow should be traceable and go to some use.

You will need to modify your model slightly for it to perform correctly. In particular, you will need to add a conditional statement (IF, THEN) to prevent taking a depreciation deduction once NFA reach zero. (Note: in extremely negative growth, you are selling assets so the balance of NFA can become negative. But you should no longer take depreciation expenses after this occurs.) As was the case in Problem 1, you will need a conditional statement to prevent negative values of debt in the balance sheet after total debt is retired, and a separate line to calculate total dividends (regular dividend plus return of free cash flow).

After testing the model and making any necessary changes so that it balances, make the following additions and changes. Your model should be set up in a way that allows you to specify the proportions of debt and equity as a parameter in its assumptions section.

Add several lines to your financing module that allows you to vary the way the firm finances its funds needs: It allows you to choose between debt and equity financing. Test it with three different assumptions:

- A. All new financing is done with long-term debt. The interest rate is 5%.
- B. You use a mix of debt and equity such that any new financing is 60% debt and 40% equity.
- C. All new financing is with new equity.

Test your model with varying sales growth rates.

3. Medtric, Inc., Spreadsheet Model for Growth

Medtric, Inc. was described in Problem 3. This problem is a continuation of that problem, so the income statement and the balance sheets for 2012 through 2016 that you used in Problem 3 also apply to this problem. You will find a new Excel workbook for this assignment next to these instructions on the Unit 3, Problem 4 assignment page in our Canvas shell.

This problem focuses on the current board discussion concerning Medtric's future growth and dividend policies: Should Medtric pursue a more modest 12% growth rate that requires no further acquisitions and increase its payout rate to 35% of net earnings? Or, should Medtric pursue a higher growth rate of 20% that requires an aggressive acquisition program and reduce its dividend payout rate to 20% of net earnings? Management has asked your team to develop a financial statement simulation model for Medtric, Inc. to assist them in answering this question. Your model should generate pro forma financial statements (income statement, balance sheet, and a formal statement of cash flows) that cover the next five years (2017–2021). Your model should include the following modules/sectors: input, accounting, investment, financing, cash flow, simple valuation (aggregate equity only), and performance tracking (ROA, ROE, D/E*, and the sustainable growth rates g^* and $PRAT$ and their components). Assume that any required external financing will be funded by issuing new debt, that the Notes Payable in 2016 will be retired in 2017, and that no equity will be repurchased. In addition, add a ComboBox to your model so that the user can toggle between the 12/35 and 20/20 growth/payout scenarios.

The two scenarios share many common input variable values. However, the values for five input variables are different under the two scenarios. Of course, the five-year growth rates and dividend payout ratios will differ as per the names of the scenarios. In addition, research and development expense will be higher—12.5% versus 9.5% of sales—for the 12/35 scenario (under which Medtric develops new products and technology in-house) than for the 20/20 scenario (under which Medtric plans to purchase new products and technology). Finally, under the 12/35 scenario, Medtric plans no new investment in Goodwill and Other Long-term Assets (as it

* Defined as the ratio of interest-bearing debt to equity in a given year throughout this assignment

will suspend its acquisition program), whereas under the 20/20 scenario, the new investment in Goodwill will be 50% of the increase in sales and the new investment in Other Long-Term Assets will be 22% of the increase in sales. [Exhibit 4.19](#) summarizes these differences (it is also the table I used when setting up my ComboBox and Choose functions).

Goodwill and Other Long-Term Assets

As two drastically different assumptions are made regarding acquisitions, Goodwill and Other Long-Term Assets cannot be modeled as simple percents of sales. Instead, they must be coded to increase incrementally, by calculating the new investment required for each pro forma year and then adding it to the previous year's balance. Consider the forecast for Goodwill in 2017. Under the 12/35 scenario, the increase will be zero (reflecting no new acquisitions). This is calculated by first calculating the increase in sales ($growth * Sales_{2016}$), multiplying it by the Goodwill percent of sales under the 12/35 scenario (0.000), then adding it to last year's Goodwill. Under the 20/20 scenario, you would use 0.500 for the Goodwill percent of the increase in sales and add the resulting product to last year's Goodwill. The math for Goodwill in 2017 under the 20/20 scenario is as follows.

$$\begin{aligned} Goodwill_{2017} &= Goodwill_{2016} + 0.50 * Sales_{2016} * growth \\ &= 10,672 + 0.50 * 14,599 * 0.20 \\ &= 12,131.9 \end{aligned}$$

Treat Other Long-Term Assets in the same way. The other expense that is not a simple percent of sales is Unusual Expense. Simply feed the values for this item shown in the base worksheet into the Income Statement under the interest paid and earned lines.

In developing your model, assume that the model parameters and economic variables shown in [Exhibit 4.20](#) will hold in the future. (You will recognize many of these parameters from your answers to Problem 3.) Please note that the Decision/Sensitivity settings shown in [Exhibit 4.20](#) are for a test scenario for the 12/35 scenario. In particular, *cash_returned* and *cash_invested* are set to 0.5 so that if you make a coding error in your handling of excess cash, you will not get the same numbers in your Summary Statistics as those shown in [Exhibit 4.20](#).

ComboBox controled assumptions						
	1	Growth	Goodwill	Other Lt_assets	R&D	Payout
Grow organically		0.12	0	0	0.125	0.35
Grow by acquisition		0.20	0.5	0.22	0.095	0.2

EXHIBIT 4.19 Medtrac scenario assumptions.

	B	C	D	E	F	G	H
4			Medtric				
5			Scenario: Grow organically (12/35)				
6	Input Sector						
7	Percent of sales inputs						
8	COGS	0.24					
9	SGA	0.35					0.12
10	Research & Development	0.125					4.0%
11	Depreciation Expense	0.06					0.35
12	Cash & Equivalents	0.1					
13	Total Receivables	0.23					
14	Total Inventory	0.1					
15	Other Current Assets, Total	0.08					1
16	Property/Plant/Equipment - Gross	0.34					0.5
17							0.5
18	Accounts Payable	0.025					1
19	Other Current liabilities	0.15					
20	Other Liabilities, Total	0.05					
21							
22	Economy						(0.0)
23	Interest Expense	0.070					\$ 46,683
24	Interest Income	0.030					0.48
25	beta - unleveraged	0.500					11.3%
26	riskfree rate	0.040					14.2%
27	market risk premium	0.065					8.02%
28	taxes	0.380					2,380.6
29	GDP growth	0.040					4.02%
30							
31	Scenario variables		2017	2018	2019	2020	2021
32	Unusual Expense (Income)	tbd	800.0	400.0	-	-	-
33	Goodwill & Intangibles	0	-	-	-	-	-
34	Other Long Term Assets	0	-	-	-	-	-

EXHIBIT 4.20 Medtric input sector.

The two scenarios present interesting modeling problems. Under the 12/35 scenario, Medtric generates significant excess cash. The question Medtric would like answered is should they return this excess cash to their shareholders as a special dividend, should they invest it in Short-Term Investments where it will earn interest until needed to fund an acquisition or future growth, or should they use it to retire outstanding debt? Under the 20/20 scenario, Medtric will require a moderately large amount of external debt financing. Medtric would like to know how this new debt will affect its financial position.

The 12/35 scenario (incorporating the lessons of Combex IV)

Answering the cash policy questions raised by the 12/35 scenario necessitates adding three cash control decision variables to the model: *cash_returned*, *cash_invested*, and *debt_retired*. This adds considerable complexity to the basic models we have considered thus far. These decisions can be modeled best by considering them sequentially. I found it convenient to first allocate excess cash between external (a special dividend) and internal (marketable securities and retiring debt or equity) uses (calling the decision variable *cash_returned*). I then allocate the internally directed cash between marketable securities and retiring debt and/or equity (calling this decision variable *cash_invested*). Finally, I allocate the cash directed toward repurchasing securities (calling the decision variable *debt_retired*) between retiring debt and repurchasing equity. In addition, Short-Term Investments and Short-Term Debt need to be incorporated into the model.*

Once you finish incorporating short-term investments and debt and the cash control decisions into your model, test your model to make sure it is doing what you think it is doing. Set *cash_returned* and *cash_invested* to 0.50 and *debt_retired* to 1. Make sure the excess cash for each year is allocated correctly. For example, suppose excess cash flow is 200. Then, assuming the *cash_returned* and *cash_invested* are each set to 0.50 and *debt_retired* is set to 1; you should see 100 paid out as a special dividend, 50 invested in marketable securities, and 50 used to retire debt.

When finished, this model will have five decision variables. The first, the *payout*, determines the portion of net income to be paid out as the minimum dividend.

* There are two ways to code the repurchase equity line in the finance module. One way assumes that a specific portion of excess cash be used to retire debt and equity. Should all debt be retired, the unused portion flows through to a special dividend. This code is

$$[\text{Excess Cash}] * (1 - \text{cash_returned}) * (1 - \text{cash_invested}) * (1 - \text{debt_retired})$$

The second way assumes that all cash allocated to retiring debt and equity only will be used for that purpose. Typically, this means that once all debt has been retired, all remaining unused cash flows will be used to repurchase equity. This code is

$$[\text{Excess Cash}] * (1 - \text{cash_returned}) * (1 - \text{cash_invested}) - [\text{Debt retired}]$$

Both methods are correct, depending on how you want your model to work, so long as you understand the different cash flows that result from your choice. However, for this problem, please use the first method.

The second, *debt_portion*, allocates required external financing between debt and equity. It should remain set to 1 throughout this problem. The third, *cash_returned*, allocates excess cash flow between paying a special dividend and internal uses. The fourth, *cash_invested*, allocates internally directed cash between purchasing Short-Term Investments and retiring outstanding debt or equity. The fifth, *debt_retired*, allocates cash directed to retiring debt and equity between retiring debt or repurchasing shares. It should remain set to 1 (100% used to retire debt) throughout Problem 4.

Once you are satisfied that your model works as you intend it, you will be ready to move on to the 20/20 scenario.

The 20/20 scenario

Under the 20/20 scenario, Medtric will require a fair amount of external financing (assumed to be 100% debt for this problem) to support the planned acquisition program. They want to know the effect of this additional financing on their financial position, particularly how it affects their D/E ratio and the intrinsic value of their aggregate equity.

Other miscellaneous instructions

The Valuation Module. The parameters needed by the valuation module are shown in the Economy section of the Input Sector in Exhibit Problem 4.20. Use beta to calculate Medtric's cost of equity according to the CAPM. Set long-term growth equal to the growth rate of the economy and use it to calculate terminal value. Base your forecast for the 2022 dividend on the Cash Available for Dividends in 2021. Finally, include the present value of Short-Term Investments in 2021 in your valuation calculations. The accumulation of cash in Short-Term Investments represents a nonoperating asset whose value is independent from the firm's operating cash flows. This simple valuation module returns the estimated value for all equity at the end of 2016.

General considerations and stress tests. Your model should be constructed so that it balances: assets should equal liabilities and sources should equal uses of cash. Put checks into your model to verify that it balances. (Be sure your Cash Flow Module check accounts for purchases of Short-Term Investments.) In addition, your model should be constructed in such a way that any equations that refer to input parameters do not contain numbers. The parameters should be in a separate input section, with the five decision/sensitivity variables—dividend payout, debt portion, cash returned, cash invested, and debt retired—clearly identified. Name all input parameters using Excel's naming utility. This will help you organize your model and make it easier for me to understand it.

When the above steps are completed, you will have developed a basic financial statement simulation model. Hopefully, it worked the first time. However it is important to make sure that it balances and remains logical under a variety of circumstances. Check the balancing by varying the sales growth rate. Use high growth rates such as 30% or 50% per year and see how your model performs. Then use low growth

rates such as -20% and -30% per year. Assets should equal liabilities, and all cash flows should go to some use.

You probably will need to modify your model slightly under these extreme scenarios for all of the accounts to remain logical. In particular, you will need to add conditional statements to prevent *NFA* from becoming negative and to prevent your model from taking a depreciation deduction once *NFA* reach zero. The Odd & Rich and Combex models show one way to do this. If your model balances and remains logical under the various extreme growth assumptions, return the five-year growth rate to the test scenario (0.12) and increase the SG&A/Sales ratio to 0.65. Check that you are not paying negative dividends. Be sure to return the ratio to its correct value of 0.35 once your model works.

Set your model to the 12/35 scenario and answer the following questions:

- Given the assumptions, including that the model is correct, how should Medtric use its excess cash? Should it pay all of it out as a special dividend, retain all of it for future uses, or use all of it to retire some of Medtric's outstanding debt?
- Why is there such a disparity between the intrinsic value of Medtric's equity when all cash is paid out versus when all cash is retained?
- Why does the D/E decrease under this scenario regardless of the cash control settings?

Now set the cash control variables to the setting that maximizes the intrinsic value of Medtric's equity under this scenario.

- Why does the intrinsic value decrease as the payout rate increases from 35%?
- What is the payout rate that maximizes Medtric's intrinsic value in this scenario? (Hint: It is less than 35%.)
- What factors cause Medtric's intrinsic value to change as the payout rate changes?

Now set your model to the 20/20 scenario and answer the following questions:

- Why does not changing the cash control variable settings affect Medtric's intrinsic value under this scenario?
- Will the acquisition program significantly increase Medtric's D/E ratio? Why or why not?

Suppose that Medtric's debt covenants restrict Medtric's D/E ratio to no higher than 1.0 in any year from 2017–2021.

- How fast can Medtric grow without violating its debt covenants?
- Is Medtric more valuable at this higher growth rate?
- If Medtric were to set their growth rate target to this higher rate, what might go wrong?



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Modeling Value

5.1 INTRODUCTION

Throughout the first four chapters of this book, we emphasized the need to link the decisions in your model to an objective. That objective was assumed to be maximizing the value of the equity of the firm. This chapter digs deeper into the details of modeling equity value. We start simply, and then add multiple layers of increasing complexity. The final result is a complete model of firm value.

We begin with the basic equity valuation model introduced at the end of [Chapter 4](#)—the variable growth discounted dividend model (reprinted from Equation 4.14):

$$\text{Equity value}_0 \equiv E_0 = \sum_{t=1}^h \frac{D_t}{(1+k_E)^t} + \frac{E_h}{(1+k_E)^h} \quad (5.1)$$

where:

D_t is the aggregate dividend paid at time t

k_E is the discount rate (cost of equity)

h is the number of periods in the forecast horizon

E_h is the terminal value of the equity at the end of the forecast horizon (when $t = h$).

The simplicity of Equation 5.1 masks a great deal of complexity. Each of its components (dividends, discount rate, and terminal value) has its own nuance or complication. This chapter first considers each component in its most basic form, then assembles the components into a simple but working valuation model. We begin by considering equity cash flow at its most basic. We then link the cost of equity to a firm's capital structure, simply at first, then correctly. Next, we take up how to account for the dilutive effects of issuing new shares of equity. The result is a complete model of equity value based on equity cash flows. The second half of this chapter introduces the value of invested capital approach to firm valuation, followed by a close look at terminal value, and finishes with more nuanced aspects of dividends, equity cash flow, and dividend-paying ability.

5.2 CASH FLOW TO EQUITY

5.2.1 Dividends versus Dividend-Paying Ability

Equation 5.1 shows equity value as the present value of a stream of dividends and a terminal value. If so, it would seem that computing equity value is relatively easy. All we need to do is forecast future dividends. Unfortunately, calculating value is more complicated. The complications arise from the manner in which dividends are defined. [Chapter 3](#) introduced definitions of cash flows other than dividends, including the two we use in this chapter, equity cash flow and cash flow available to invested capital. This section considers the differences between dividends and equity cash flow and how to handle them in valuation.

We begin by reprising the definition of Equity Cash Flow. Equity cash flow is the cash flow that can potentially be paid to the owners of the firm after meeting the need to invest in assets and fulfilling all other financial obligations. [Exhibit 5.1](#) shows a format for calculating equity cash flow. These calculations account for all cash flows into and out of the firm. In the plain vanilla case under consideration, equity cash flow is identical to the dividends paid to the firm's stockholders. It is the amount of money left over at the end of the year that the owner of the firm can take home.

Owners need to plow money back into their firms to keep them in business and growing. Investments in working capital and fixed assets are necessary to support production and sales, hence, the *Investment in Net Working Capital* and *Investment in Fixed Assets* lines in [Exhibit 5.1](#). So long as these investments are no more than what is required to keep the business growing, dividends will equal dividend-paying ability.

[Exhibit 5.2](#) shows the input sector, financing module, and cash flow module for the Odd & Rich as we left it at the end of [Chapter 4](#), with one exception. We lowered the dividend payout rate to 0.50 so that O&R generates excess cash beginning in 2018 (line 107).

Old Ex. 9.1, new Exhibit 5.1

Cash Flow from Operating Activities:	
Net Income	
+ Depreciation Expense	
= Earnings Cash Flow	
– Investment in Net Working Capital	
	= Cash Flow from Operations
less:	
Cash Flow from Investment Activities	
+ Expenditures for New Fixed Assets	
+ Depreciation Expense	
	– Investment in Fixed Assets
plus:	
Cash Flow from Financing Activities	
+ Proceeds from Borrowing	
– Repayment Debt Principal	
+ Proceeds from Issuance of Stock	
	+ Cash Flow from Financing
	= Equity Cash Flow

EXHIBIT 5.1 Equity cash flow.

	B	C	D	E	F	G	H	I
2	ODD & RICH Corporation							
3	INPUT SECTOR							
4	Parameter Estimates	External Conditions				Summary Statistics		
5	COGS/Sales	0.70		Risk-free rate	0.02		Master Check	0
6	Depreciation/GFA	0.03		Mrkt Rsk premiun	0.07		Aggregate Value	1,901
7	Tax Rate	0.40		Borrow rate	0.04		k	8.23%
8	Cash/Sales	0.17		Sales Growth	0.06		Debt ratio 2017	31.3%
9	CA (non-cash)/Sales	0.54		LT Growth	0.030		Debt ratio 2021	32.1%
10	GFA/Sales	1.93						
11	CL/Sales	0.60		Policies and Decisions			Cash Control	
12	unleveraged beta	0.87		Debt portion	1.00		Cash returned	1.00
13	Number of shares	100		Payout ratio	0.50		Cash invested	0.00
14							Debt retired	0.00
15								
94	FINANCING MODULE		2017	2018	2019	2020	2021	
95	Required Financing		161.6	167.9	178.0	188.7	200.0	
96	Sources of Funds							
97	Net Income		154.4	163.9	173.8	184.3	195.5	
98	Depreciation		57.8	61.4	65.1	69.0	73.1	
99	Earnings Cash Flow		212.2	225.3	238.9	253.3	268.6	
100	Minimum Dividend		77.2	81.9	86.9	92.2	97.7	
101	Maturing Marketable Securities		-	-	-	-	-	
102	Internal Equity Financing		135.0	143.3	152.0	161.1	170.8	
103	Maturing Short-term Debt		25.0	-	-	-	-	
104	Spontaneous Financing		11.0	38.2	40.4	42.9	45.4	
105	Total Internal & Spontaneous Financing		146.0	181.5	192.4	204.0	216.3	
106	Required External Financing		15.6	0.0	0.0	0.0	0.0	
107	Excess Cash		0.0	13.6	14.4	15.3	16.3	
108	Short-term Debt Issued		0.0	0.0	0.0	0.0	0.0	
109	Debt Issued		15.6	0.0	0.0	0.0	0.0	
110	Debt retired		0.0	0.0	0.0	0.0	0.0	
111	Equity Issued		0.0	0.0	0.0	0.0	0.0	
112	Equity repurchased		0.0	0.0	0.0	0.0	0.0	
113	Total Discretionary Financing		15.6	0.0	0.0	0.0	0.0	
114								
115	CASH FLOW MODULE		2017	2018	2019	2020	2021	
116	Earnings Cash Flow		212.2	225.3	238.9	253.3	268.6	
117	Current Asset Investment (except Cash)		32.4	34.3	36.4	38.6	40.9	
118	Maturing Marketable Securities		-	-	=cash*E21-D37	-	-	
119	Spontaneous Financing		36.0	38.2	=(cash/sales,)*sales-cash _{t-1}	42.9	45.4	
120	NWC Investment		(3.6)	(3.8)	=(E107*(1-Cash_returned)*Cash_invested	(4.5)	(4.5)	
121	Cash Flow from Operations		215.8	229.1	=[Excess Cash](1-Cash_returned)*Cash_invested	1.1	1.2	
122	Capital Expenditures		119.0	122.7	=E124-E125-E126			
123	Capital Financing		(9.4)		=Net Cash Inflow - Increase in Cash - MS Purchased			
124	Net Cash Flow		87.4	106.3				
125	Increase in Cash		10.2	10.8		11.5	12.1	12.9
126	Marketable Securities Purchased		-	-		-	-	-
127	Cash Available for Dividends		77.2	95.5	101.3	107.5	114.1	

EXHIBIT 5.2 Input sector, financing module, and cash flow module of Odd & Rich model, with a payout of 0.50, growth set to 6%, and all excess cash paid out as dividends.

The five-year sales growth rate is 6%. Cash control variables have been set to return to shareholders all cash in excess of what is needed to maintain O&R's business. We draw your attention to lines 100 (Minimum Dividend), 107 (Excess Cash), 125 (Increase in Cash), and 127 (Cash Available for Dividends). Now focus on the year 2018, column E. In line 107, Excess Cash is 13.6. This means that after all necessary investments in working capital and fixed assets, and paying the minimum dividend of 81.9 (line 100), 13.6 in cash still remained. Now consider the cash available for dividends, line 127. It is 95.5, exactly 13.6 larger than the minimum dividend. The 95.5 represents the plain vanilla cash flow in this scenario. It is the actual dividend paid, the equity cash flow, and the cash flow that feeds into the equity valuation model.

5.3 COST OF EQUITY

Section 5.2 defined basic equity cash flow. When discounted by the cost of equity, these cash flows produce an estimate of equity value. However, just as dividends required more explanation, so too does the cost of equity. We begin with beta, the input to the cost of equity that reflects the risk of that equity.

5.3.1 A Simplistic Approach to the Cost of Equity

When we presented the simple valuation model in [Chapter 4](#), the cost of equity, k_E , was based on the CAPM, but the beta coefficient was assumed to be fixed; it did not vary as the capital structure changed. However, the risk of a firm's equity is a function of both systematic risk (the risk of the firm if financed entirely by equity) and financial risk (leverage); hence, total risk increases with financial leverage. As the firm's financial leverage increases, the beta of the company's stock should also increase. In this section, we use Hamada's (1969) formula to model the cost of equity in the presence of a changing capital structure.*

Hamada's formula for leveraged beta captures systematic risk through an unleveraged beta and financial risk through the firm's D/E ratio. Specifically, his formula expresses leveraged beta as a function of the unleveraged beta and the debt-to-equity ratio:

$$\beta_L = \beta_U \left[1 + \frac{D}{E}(1-T) \right] \quad (5.2)$$

where:

β_U is what the beta of the firm would be if the firm is operated with no debt

β_L is the beta of the firm operating with debt

T is the firm's tax rate.

Ideally, beta would adjust each year to reflect any changes in the market value of the firm's equity. We show how to do this in the next section. For now, we use the average D/E ratio from the pro forma years. This works as a first approximation when the capital structure is not expected to change significantly over the planning horizon.

We continue with the O&R model from the previous section, but with one addition: a line that calculates the D/E ratio for each of the pro forma years ([Exhibit 5.3](#), line 153). The average of these D/E ratios is 0.038 (cell D154). As before, the risk-free rate is 0.02, the market risk premium is 0.07, and the tax rate is 0.40. Finally, assume that the unleveraged beta is 0.87. The leveraged beta is calculated in Equation 5.4:

$$\begin{aligned} \beta_L &= \beta_U \left[1 + \frac{D}{E}(1-T) \right] \\ &= 0.87 \left[1 + 0.038(1-0.40) \right] \\ &= 0.890 \end{aligned} \quad (5.3)$$

* Many academics and practitioners (including Pratt and Grabowski and Miles and Ezzell) use more complicated formulas for beta than does Hamada (1969). The Miles and Ezzell use the following formula.

$$\beta_L = \beta_U + \left(\frac{D}{E} \right) (\beta_U - \beta_d) \left[1 - \frac{T k_d}{1 + k_d} \right]$$

	B					D	E	F	G	H	I
	EQUITY VALUATION MODULE					2016	2017	2018	2019	2020	2021
148	<i>Cost of Equity (average approach)</i>										
149	Risk-free Rate					0.02					
150	Market Risk Premium					0.07					
151	Unleveraged beta					0.87					
152	D/E ratio						0.042	0.040	0.038	0.036	0.035
153	Average D/E					0.038					
154	Leveraged beta					0.890					
155	Discount Rate					8.23%					
156											
157											
158	<i>Dividend Valuation Model</i>										
159	Dividends						77.2	95.5	101.3	107.5	114.1
160	PV(Dividends)					388					
161	Marketable Securities										0.0
162	PV(Marketable Securities)					-					
163	<i>Terminal value</i>										
164	Long-term Growth Rate										0.03
165	Forecast Dividend t = 6										117.48
166	Terminal Value at t=5										2,247
167	PV(Terminal Value)					1,513					
168	Aggregate equity value					1,901					

EXHIBIT 5.3 Valuation module from O&R model when growth equals 6% and all excess cash is returned to shareholders.

Using the leverage beta, β_L , the cost of equity is*

$$\begin{aligned} k_E &= k_{rf} + (k_M - k_{rf})\beta_L \\ &= 0.02 + 0.07 * 0.890 \\ &= 8.23\% \end{aligned} \tag{5.4}$$

This is the appropriate discount rate to use to calculate the present value of dividends or equity cash flow. We do this in lines 158–168 of [Exhibit 5.3](#). The resulting aggregate equity value is \$1,901.

The use of the average D/E ratio in this instance produces a reasonable estimate of the cost of equity, as the annual D/E ratios are quite stable, ranging from 0.035 to 0.042. Suppose, however, that O&R will soon release a new blockbuster product and expects sales to increase to 15% per year over the next five years, financing this growth with debt. Holding all other model inputs the same, increasing growth to 15% results in D/E ratios ranging from 0.155 in 2017 to 0.594 in 2021. In this scenario, the use of the average D/E ratio is no longer justified. Further, these D/E ratios are based on the book values of debt and equity, whereas the correct application of Hamada's formula requires market values to be used. We address these problems next.

5.3.2 Time-Varying Market-Value-Based Beta

This section expands the simplistic solution for beta by allowing beta to adjust annually and reflect market values. We continue using Hamada's (1969) formula to adjust beta as the firm capital structure changes, first using book values for equity and capital ratios, then moving to market values. Now, however, we allow beta to change each year, and with it, the cost of equity. For example, at the end of 2016, O&R is leveraged with \$73.3 million of debt. Using the book value of equity as a first approximation, the leveraged beta would be

$$\begin{aligned} \beta_L &= \beta_U \left(1 + \frac{D}{E}(1-T) \right) \\ &= 0.87 \left(1 + \frac{73.3}{1460.7}(1-0.40) \right) \\ &= 0.87 * 1.03 \\ &= 0.896 \end{aligned} \tag{5.5}$$

* Had the unleveraged beta been used, the cost of equity would have been 8.09%.

In this case, O&R does not have much debt relative to the book value of equity, so the leveraged beta is not substantially greater than the unleveraged beta. It is enough, though, to change the cost of equity from

$$k_E = 0.02 + 0.87 * 0.07 = 0.0809$$

to (5.6)

$$k_E = 0.02 + 0.896 * 0.07 = 0.0827$$

Hamada's formula, however, requires the debt-to-equity ratio to be measured in terms of market values—the market value of debt and the market value of equity. This creates a circularity problem: beta is used to calculate cost of equity, that, in turn, is used to calculate the value of equity that, in turn, is used to calculate beta. We need to structure the model to take this circularity into account. To keep things from getting too complicated, we assume that the market value of debt equals its book value, and the beta of the debt is zero.

We continue with our model for Odd & Rich. The input variables remain the same; sales growth is set to 6%, the target payout ratio to 0.50, all excess is cash paid out as dividends, and external financing needs are met only with long-term debt. Without repeating all the financial statements, a revised version of the equity valuation section is shown in [Exhibit 5.4a](#). [Exhibit 5.4b](#) shows the formulas for selected columns.

This model of equity value differs from the version shown earlier as [Exhibit 5.3](#), in that it calculates beta and the cost of equity for each year. This means this model can be modified to allow for market conditions to change over time. Suppose our forecasts call for interest rates, market risk premia, costs of debt, or even unleveraged beta to change over time. By showing these rates in each column for each period, we can precisely modify the model to take changing market conditions or expectations into account. This model also accounts for changes in the capital structure over time rather than muddling those changes in an average. Most important for our current discussion, however, the beta can be set up to reflect the market value rather than the book value of the equity.

In the earlier model, the cost of equity was calculated with a beta based on the average book value of D/E. Now, beta is calculated annually using the Hamada formula based on the market value of equity. The inputs (risk-free rate, market risk premium, unleveraged beta, and book value of debt) for calculating the cost of equity are shown in lines 149 through 153. Line 154 is an *Assumed Equity Value* that will be explained shortly; line 155 is the debt-to-equity ratio based on the Assumed Equity Value; and line 156 calculates the leveraged beta based on that debt-to-equity ratio. The cost of equity is calculated in line 157.

The equity cash flows are in line 160. The present values of those cash flows are in line 161, where the numbers in each column are the present value of the equity cash

	B	D	E	F	G	H	I
148	EQUITY VALUATION MODULE	2016	2017	2018	2019	2020	2021
149	<i>Cost of Equity (average approach)</i>						
150	Risk-free Rate	0.02	0.02	0.02	0.02	0.02	0.02
151	Market Risk Premium	0.07	0.07	0.07	0.07	0.07	0.07
152	Unleveraged beta	0.87	0.87	0.87	0.87	0.87	0.87
153	Interest-bearing Debt (ST + LT)	73.3	63.9	63.9	63.9	63.9	63.9
154	Assumed value of equity*	1,913.1	1,993.3	2,061.4	2,129.2	2,196.2	2,262.2
155	D/E ratio	0.03832	0.032	0.031	0.030	0.029	0.028
156	Leveraged beta	0.890	0.887	0.886	0.886	0.885	0.885
157	Discount Rate	8.230%	8.21%	8.20%	8.20%	8.20%	8.19%
158							
159	<i>Dividend Valuation Model</i>						
160	Dividends		77.2	95.5	101.3	107.5	114.1
161	PV(Dividends)	388.1	342.9	275.5	196.8	105.4	-
162	Marketable Securities						0.0
163	PV(Marketable Securities)	-					
164	<i>Terminal value</i>						
165	Long-term Growth Rate						0.03
166	Forecast Dividend t = 6						117.5
167	Terminal Value at t=5						2,262
168	PV(Terminal Value)	1,525	1,650	1,786	1,932	2,091	2,262
169	Aggregate (derived) equity value	1,913.1	1,993.3	2,061.4	2,129.2	2,196.2	2,262.2
170	<i>*Initially assumed to be book value</i>						
171							
172	Total firm value						
173	Total debt	73	64	64	64	64	64
174	Equity	1,913	1,993	2,061	2,129	2,196	2,262
175	Total debt + Equity	1,986	2,057	2,125	2,193	2,260	2,326

(a)

	B	D	E	F
148	EQUITY VALUATION MODULE	2016	2017	2018
149	<i>Cost of Equity (average approach)</i>			
150	Risk-free Rate	=rf	=rf	=rf
151	Market Risk Premium	=mrp	=mrp	=mrp
152	Unleveraged beta	=un_beta	=un_beta	=un_beta
153	Interest-bearing Debt (ST + LT)	=D49+D51	=E49+E51	=F49+F51
154	Assumed value of equity*	=D169	=E169	=F169
155	D/E ratio	=D153/D154	=E153/E154	=F153/F154
156	Leveraged beta	=un_beta*(1+D155*(1-taxes))	=un_beta*(1+E155*(1-taxes))	=un_beta*(1+F155*(1-taxes))
157	Discount Rate	=rf+D156*D151	=rf+E156*E151	=rf+F156*F151
158				
159	<i>Dividend Valuation Model</i>			
160	Dividends		=E31	=F31
161	PV(Dividends)	=(E160+E161)/(1+D157)	=(F160+F161)/(1+E157)	=(G160+G161)/(1+F157)
162	Marketable Securities			
163	PV(Marketable Securities)	=I162/(1+discount_rate)^5		
164	<i>Terminal value</i>			
165	Long-term Growth Rate			
166	Forecast Dividend t = 6			
167	Terminal Value at t=5			
168	PV(Terminal Value)	=E168/(1+D157)	=F168/(1+E157)	=G168/(1+F157)
169	Aggregate (derived) equity value	=D161+D168+D163	=E161+E168+E163	=F161+F168+F163
170	<i>*Initially assumed to be book value</i>			
171				
172	Total firm value			
173	Total debt	=D153	=E153	=F153
174	Equity	=D169	=E169	=F169
175	Total debt + Equity	=D173+D174	=E173+E174	=F173+F174

(b)

EXHIBIT 5.4 (a) Equity valuation module with time-varying market-value-based cost of equity.
 (b) Formulas for equity valuation module with time-varying book value cost of equity.

flow from that date forward until the end of the planning horizon in 2021. For example, \$196.8 for 2019 (cell G161) is the present value at year-end 2019 of the equity flows in years 2020 and 2021. This works back to 2016, where \$388.1 is the present value at year-end 2016 of all the flows from 2017 through 2021. Lines 164–168 calculate the present value of the terminal value of equity based on the constant growth model. In cell I166, \$117.5 is the forecast cash flow for 2022, calculated as the cash flow in 2021 compounded at the long-term growth rate of 3% (114.1×1.03). The \$2,262 for 2021 in line 168 is the present value at the end of 2021 of the perpetuity of growing cash flows using the constant growth formula, $(114.1 \times 1.03) / (0.0819 - 0.03)$. The present value of this terminal value is shown in each earlier year back to the beginning of the planning horizon, year-end 2016.

The Aggregate Value of Equity in line 169 is the sum of the present value in each year of the cash flows until the end of the planning horizon and the present value of the terminal value. The aggregate value of equity at the end of 2016 is \$1,913. Total Firm Value is the sum of the values of debt and equity in lines 172–175, where the debt is the book value of short-term debt and long-term debt. In this example, the value of the firm at year-end 2016 is \$1,986, the sum of the debt of \$73.3 and equity of \$1,913.

5.3.2.1 Circularity

As noted above, the proper application of the Hamada leveraged beta formula requires that the debt-to-equity ratio be calculated based on market values.* But it is market value that we are trying to estimate, and that requires using beta. Excel's iterative calculation capability resolves this circularity problem.

To see how this works, we first show how to converge on a solution using manual iterations. Exhibit 5.5 shows the results of manual iterations to converge on our estimate of equity value. For our first iteration, suppose we assume that the value equity is equal to the book value of equity of \$1,460.7. We insert this value in line 154 of the valuation model

Iteration	Assumed Equity Value	Debt/Equity	Leveraged Beta	Cost of Equity	Value of Equity
1	\$1,460.7	5.018%	0.896	8.273%	\$1,903.3
2	\$1,903.3	3.851%	0.890	8.231%	\$1,912.9
3	\$1,912.9	3.832%	0.890	8.230%	\$1,913.0
4	\$1,913.0	3.832%	0.890	8.230%	\$1,913.1
5	\$1,913.1	3.832%	0.890	8.230%	\$1,913.1

EXHIBIT 5.5 Results of manual iterations to converge on the value of equity.

* Our definition of market value assumes an efficient market in which market values equal the present value of future cash flows.

(also inserting the book values for the subsequent years). Based on the book value of equity, the debt-to-equity ratio is 5.018% as shown in column 3 of [Exhibit 5.5](#), the resulting beta is 0.896, the cost of equity is 8.273%, and the estimated value of equity is \$1,903.3. This is substantially different from the book value of equity.

Iteration 2: Copy the values of equity from iteration 1 (cells D169:I169) and Paste Special those values into the assumed values of equity (D154:I154). This generate new values for debt-to-equity, leveraged beta, and cost of equity that result in value of equity of \$1,912.9, shown in the last column.

Iterations 3–5: Repeat the above step until the aggregate values of equity in line 169 equal the assumed values of equity in line 154.

The estimated value of equity for each from each iteration is used as the assumed value of equity, resulting in new values for the beta, cost of equity, and value of equity for every year. The model quickly converges on an estimate that is equal to the assumed value. In this case, it took only five iterations. This converged value is the proper estimate because the derived value is equal to the assumed value. In other words, this model uses the estimated market value of equity in the Hamada formula, as is required for the formula's proper application.

5.3.2.2 Automatic Iterations

These manual iterations demonstrate the process to converge on a solution where the market value of equity is used in the leveraged beta formula. Excel has the capability to automatically solve this circularity. Set Excel to iterate to a solution using the following steps. On the *File* tab, select *Options*, then *Formulas*. In the *Formula* dialog box, select *Enable Iterative Calculation*, then OK. This tells Excel to iterate to resolve circular references.

In the Equity Valuation Module of our model, line 154 is Assumed Equity Value. This is the value used for Equity in debt-to-equity in line 155, which, in turn, is used in the Hamada formula for the leveraged beta, line 156. The derived value of equity is line 169, the Aggregate Value of Equity. Assuming that you have enabled Excel's iterative calculation, set the Assumed Equity Values in line 154 equal to their respective derived values of equity (the present value of future cash flows to equity) in line 169, as shown in line 154 of [Exhibit 5.4b](#). This creates the circularity that Excel iteratively resolves. With this change, the assumed equity value equals the derived value for every period. The debt ratios, beta, and costs of capital now reflect the market value of equity on a period-by-period basis. The final result is that the model reflects the time-varying market-value-based cost of equity.

5.4 DILUTION: VALUE PER SHARE AND ISSUING NEW EQUITY

Thus far, we have been measuring aggregate equity value, the present value of cash flows to all shareholders. However, the objective of financial decisions should be to make the current owners of the firm as wealthy as possible—or to maximize the value of their investment in the firm. We operationalize this objective by maximizing the value per share of the stock owned by the current owners, otherwise known as diluted value per share, rather than maximizing the aggregate value of equity. Consequently, we

need to discuss how to calculate value per share in a model that can issue new shares or repurchase outstanding shares.

Modeling value per share seems like it should be easy: simply divide the aggregate value of equity by the number of shares. However, this is correct only when the number of shares is not going to change. When new equity is issued, the number of shares outstanding during the planning period changes. This creates a circularity problem: when shares are sold to raise new equity capital, the number of shares to be sold depends on the price per share; in turn, the price per share depends on the number of shares issued. Unless the model is carefully structured, this problem results in a seemingly endless cycle of calculations. This problem could be resolved using Excel's iterative capabilities, but doing so creates a remarkably unstable model. Instead, this section shows how to address this problem without the need of an iterative solution.

The problem is how to account for dilution when raising new equity capital by issuing new shares. The apparent simultaneity of the share price and the number of shares to be issued is demonstrated by the following expressions:

$$\begin{aligned} \text{Shares Issued}_t &= \frac{\text{Equity Issued}_t}{\text{Price Per Share}_t} \\ \text{Price Per Share}_t &= \frac{\text{Equity Value}_t}{(\text{Initial Shares}_t + \text{Shares Issued}_t)} \end{aligned} \quad (5.7)$$

where:

Equity Value_t is the aggregate value of equity

Equity Issued_t is the aggregate amount of new capital to be raised

Shares Issued_t is the number of shares sold

Initial Shares_t is the number of shares outstanding prior to selling new shares

Initial Shares + Shares Issued is the number of shares outstanding after the new issue of shares.

In the first Equation of 5.7, the number of shares issued depends on the price per share, whereas in the second equation, the price per share depends on the number of shares issued—clearly a circular problem.

One solution to this circularity problem derives from Miller and Modigliani (1961). It expresses the price per share as a function of the number of shares outstanding at time 0, thus:

Price Per Share₀ =

$$\frac{\sum_{t=1}^h \left[\frac{(\text{Net Income}_t) - (\text{Internal Equity Financing}_t) - (\text{Equity Issued}_t)}{(1+k)^t} \right] + \frac{E_h}{(1+k)^h}}{\text{Shares}_0} \quad (5.8)$$

This expression splits *Net Income* between dividends and reinvestment in the firm. The share price on a given date depends on the shares outstanding at that date, prior to the issuance of new shares. Alternatively, *Price Per Share* can be expressed as

$$Price\ Per\ Share_0 = \frac{(Equity\ Value_0) - \sum_{t=1}^h [Equity\ Issued_t / (1+k)^t]}{Shares_0} \quad (5.9)$$

where:

$$Equity\ Value_0 = \sum_{t=1}^h \frac{[(Net\ Income_t) - (Internal\ Equity\ Financing_t)]}{(1+k)^t} + \frac{E_h}{(1+k)^h} \quad (5.10)$$

The term $\sum_{t=1}^h [(Equity\ Issued_t) / (1+k)^t]$ in Equation 5.9 represents the present value of the new equity to be issued in future periods.

These expressions avoid the circularity problem that frequently occurs in calculating the price per share when we issuing new equity. As the price per share in Equations 5.8 and 5.9 is expressed in terms of the number of shares outstanding at time $t = 0$, $Shares_0$, we do not need to know the number of shares to be issued at each future date. We only need to know the aggregate value of new equity issued at each future date, $Equity\ Issued_t$. Note that $Equity\ Issued_t$ appears in Equations 5.8 and 5.9 with a negative sign. This captures the dilution that occurs for the original stockholders (the stockholders at time $t = 0$) when new shares are sold. Dilution is captured by reducing the total value of equity in a given period by the amount of new equity financing used in that period. Subtracting the aggregate value of the new shares issued implicitly accounts for the dilution that occurs when more shares are going to be outstanding in future periods.

5.4.1 Example: Mythic Corporation

The following example shows how the Miller and Modigliani relationship can be used to model the issuance of new shares. It has two scenarios: A, in which no new equity is issued; and B, in which new equity is issued.

Scenario A: The Mythic Corporation is financed entirely with equity, has 100 shares of common stock outstanding, and a cost of equity of $k_E = 12\%$. At the end of five years, it will cease operations and liquidate. The first scenario assumes that Mythic will make no capital investment over the next five years and therefore will not have to issue any new shares. The equity cash flow available to be paid as dividends is \$1,000 for each of the next five years, as shown in [Exhibit 5.6](#).

Given that all equity cash flow is paid as dividends, the aggregate value of the stock at the end of each period is simply the present value of future dividends, as shown on line 23. The aggregate value of the stock at $t = 0$ is \$3,605, the present value of the dividends paid in years 1–5. With 100 shares outstanding, the value per share is \$36.05 at $t = 0$ (line 26). The value shown for $t = 5$ is zero because there are no future cash flows after the last dividend has been paid.

	B	C	D	E	F	G	H
2	Mythic Corporation						
3	Equity Cash Flow						
4	Scenario A: No New Equity Issued						
5	Period						
6		0	1	2	3	4	5
7	Net Income		700	700	700	700	700
8	+ Depreciation		300	300	300	300	300
9	Earnings Cash Flow		1,000	1,000	1,000	1,000	1,000
10	- Net Working Capital Investment		0	0	0	0	0
11	- Capital Investment		0	0	0	0	0
12	+ Capital Financing: Equity Issued		0	0	0	0	0
13	Equity Cash Flow		1,000	1,000	1,000	1,000	1,000
14							
15	Equity Valuation Module						
16	Period						
17		0	1	2	3	4	5
18	Cost of Equity						
19	Discount Rate	12.0%					
20							
21	Dividend Valuation Model						
22	Dividends		1000	1000	1000	1000	1000
23	Aggregate value of Stock (ex-dividend)	\$3,605	\$3,037	\$2,402	\$1,690	\$893	\$0
24							
25	Number of Shares	100	100	100	100	100	100
26	Value per Share	\$ 36.05	\$ 30.37	\$ 24.02	\$ 16.90	\$ 8.93	\$ -

EXHIBIT 5.6 Equity cash flow and valuation. Scenario A: No new equity issued.

Scenario B: This scenario considers what happens when new shares are sold to raise additional equity capital. Assume that \$200 must be spent for capital equipment in period 2 to maintain Mythic's productive capacity. Assume that this \$200 will be financed by issuing new equity. If the firm did not sell new shares, the investment would have to be financed with retained earnings (internal equity), so dividends would be reduced by \$200 to a level of \$800.

However, because new shares are sold to finance the purchase of the equipment, the equity flow available for dividends remains at \$1,000, as shown in [Exhibit 5.7](#) (cells E11:E13).

	B	C	D	E	F	G	H
2	Mythic Corporation						
3	Equity Cash Flow						
4	Scenario B: New Equity Issued						
5	Period						
6		0	1	2	3	4	5
7	Net Income		700	700	700	700	700
8	+ Depreciation		300	300	300	300	300
9	Earnings Cash Flow		1,000	1,000	1,000	1,000	1,000
10	- Net Working Capital Investment		0	0	0	0	0
11	- Capital Investment		0	200	0	0	0
12	+ Capital Financing: Equity Issued		0	200	0	0	0
13	Equity Cash Flow		1,000	1,000	1,000	1,000	1,000

EXHIBIT 5.7 Equity cash flows. Scenario B: New equity issued.

As the aggregate equity cash flow remains the same, the aggregate value of the equity is still \$3,605; but because new shares were issued, the value *per share* will change.

Consider the position of the original stockholders. Under scenario A, the investors who owned shares at $t = 0$ expect to receive 100% of the equity cash flow. Under scenario B, new shares will be sold at $t = 2$ to finance the new investment. The sale of the new shares will increase the number of shares outstanding, thereby diluting the ownership of the original shareholders. Though the aggregate equity cash flow is unchanged, it will be distributed across more shares beginning in year 3, thereby decreasing the dividends per share in years 3, 4, and 5. As the dividend is smaller, the present value of the dividends per share is smaller, making the value per share smaller than it would have been otherwise.

The effect of dilution can be seen by digging deeper into scenario B. The calculations of the aggregate and per share value of equity based on the issuance of \$200 of new equity in period 2 are shown in Exhibit 5.8a. (Exhibit 5.8b shows the formulas for two columns of the model.) At $t = 0$, the aggregate value of equity is \$3,605, but the value per share has fallen to \$34.45. Why? This lower value per share reflects stockholders' knowledge that new shares will be issued at $t = 2$.

	B	C	D	E	F	G	H
15	Mythic Corporation						
16	Equity Cash Flow						
17	Scenario B: New Equity Issued						
18				Period			
19		0	1	2	3	4	5
20	Cost of Equity						
21	Discount Rate	12.0%					
22	Dividend Valuation Model						
23	Scenario A						
24	Dividends		1,000	1,000	1,000	1,000	1,000
25	Aggregate value of Stock (ex-dividend)	\$3,605	\$3,037	\$2,402	\$1,690	\$893	\$0
26							
27	Number of Shares	100	100	100	100	100	100
28	Value per Share	\$ 36.05	\$ 30.37	\$ 24.02	\$ 16.90	\$ 8.93	\$ -
29							
30	Scenario B						
31	Equity Financing						
32	Amount of Equity Issued	0	0	200	0	0	0
33	Aggregate Stock Value (ex-dividend)	\$3,605	\$3,037	\$2,402	\$1,690	\$893	\$0
34	PV Equity Issued	\$159	\$179	\$200	\$0	\$0	\$0
35	Equity Value - PV (Equity Issued)	\$3,445	\$2,859	\$2,202	\$1,690	\$893	\$0
36							
37	Shares Outstanding						
38	Number of Shares - beginning	100.00	100.00	100.00	109.08	109.08	109.08
39	Shares Issued	0.00	0.00	9.08	0.00	0.00	0.00
40	Shares Ending	100.00	100.00	109.08	109.08	109.08	109.08
41							
42	Issue Price per Share	\$ 34.45	\$ 28.59	\$ 22.02	\$ 15.49	\$ 8.19	
43	Dividends per Share		\$ 10.00	\$ 10.00	\$ 9.17	\$ 9.17	\$ 9.17
44	PV of Dividends per Share	\$ 34.45	\$ 28.59	\$ 22.02	\$ 15.49	\$ 8.19	
45	Value per Share - ex dividend	\$ 34.45	\$ 28.59	\$ 22.02	\$ 15.49	\$ 8.19	

(a)

EXHIBIT 5.8 (a) Equity value, aggregate and per share, when new shares are issued. (Continued)

	B	C	D	E
21	Discount Rate	0.12		
22	Dividend Valuation Model			
23	Scenario A			
24	Dividends		=D13	=D24
25	Aggregate value of Stock (ex-dividend)	=NPV(\$C\$21,D24:H24)	=NPV(\$C\$21,E24:I24)	=NPV(\$C\$21,F24:J24)
26				
27	Number of Shares	=C38	=C27	=D27
28	Value per Share	=C25/C27	=D25/D27	=E25/E27
29				
30	Scenario B			
31	Equity Financing			
32	Amount of Equity Issued	0	0	200
33	Aggregate Stock Value (ex-dividend)	=NPV(\$C\$21,D24:H24)	=NPV(\$C\$21,E24:I24)	=NPV(\$C\$21,F24:J24)
34	PV Equity Issued	=NPV(\$C\$21,D32:H32)+C32	=NPV(\$C\$21,E32:I32)+D32	=NPV(\$C\$21,F32:J32)+E32
35	Equity Value - PV (Equity Issued)	=C33-C34	=D33-D34	=E33-E34
36				
37	Shares Outstanding			
38	Number of Shares - beginning	=100	=C40	=D40
39	Shares Issued	0	=D32/D42	=E32/E42
40	Shares Ending	=C38+C39	=D38+D39	=E38+E39
41				
42	Issue Price per Share	=C35/C38	=D35/D38	=E35/E38
43	Dividends per Share		=D24/D38	=E24/E38
44	PV of Dividends per Share	=NPV(\$C\$21,D43:H43)	=NPV(\$C\$21,E43:I43)	=NPV(\$C\$21,F43:J43)
45	Value per Share - ex dividend	=C44	=D44	=E44

(b)

EXHIBIT 5.8 (Continued) (b) Selected formulas for equity financing section of Exhibit 5.8a.

At time $t = 2$, \$200 of external equity is raised by selling 9.083 shares (line 39) at a price of \$22.02 (line 42). Let us see where these numbers come from. First, we use expression 5.9 to determine the price per share.

$$\begin{aligned}
 \text{Price Per Share}_t &= \frac{\text{Equity Value}_t - PV_t(\text{Equity Issued})}{\text{Shares}_{t-1}} \\
 &= \frac{2402 - 200}{100} = \$22.02
 \end{aligned}
 \tag{5.11}$$

where:

Equity Value_t (line 33 repeating line 25) is the aggregate value of equity at time t

$PV_t(\text{Equity Issued})$ (line 34) is the present value at time t of all future issues of new equity

$\text{Equity Value}_t - PV_t(\text{Equity Issued})$ (line 35) is the aggregate value of the original 100 shares at time t .

Line 42 calculates value per share by dividing the net equity value of current shareholders (\$2,202 in line 35) by the number of shares at the beginning of the period (100 in line 38), Shares_{t-1} . As price per share at time t depends on Shares at time $t - 1$, simultaneity is avoided. Dividing the amount of new equity financing (\$200) by the price per share at time $t = 2$ yields the number of shares issued (line 39).

	B	C	D	E	F	G	H
50	Period						
51	0	1	2	3	4	5	
52	Dividend Per Share w/o Equity Issue		10.000	10.000	10.000	10.000	10.000
53	Dividend Per Share w/ Equity Issue		10.000	10.000	9.167	9.167	9.167
54	Difference in Dividend Per Share		0.000	0.000	0.833	0.833	0.833
55							
56	PV of Difference in Dividends per Share	\$1.59	\$1.79	\$2.00	\$1.41	\$0.74	\$0.00
57							
58	Price Per Share w/o Equity Issue	\$ 36.05	\$ 30.37	\$ 24.02	\$ 16.90	\$ 8.93	\$ -
59	Price Per Share w/ Equity Issue	\$ 34.45	\$ 28.59	\$ 22.02	\$ 15.49	\$ 8.19	\$ -
60	Difference in Price Per Share	\$ 1.594	\$ 1.79	\$ 2.00	\$ 1.41	\$ 0.74	\$ -
61							
62	Equity Issued	\$ -	\$ -	\$ 200.00			
63	PV Equity Issued	\$159.44	\$178.57	\$200.00			

EXHIBIT 5.9 Difference in dividends and equity values: Equity issued versus no equity issued.

Issuing new shares decreases the value of the previously existing shares because the aggregate dividends in years 3, 4, and 5 will be shared with nine additional shareholders. The reduction in value due to this dilution equals the present value of the reduction in dividends per share. This has several important implications. First, as Miller and Modigliani emphasize, if the firm increases its dividend, it will have to issue shares to finance the dividend (holding all other variables constant). The reduction in share value from this dilution will exactly equal the value of the additional dividend paid. Hence, there is no net benefit from paying the extra dividend.*

Even though the aggregate value of the equity is the same under both scenarios, \$3,605, the values per share in the two scenarios differ. The value per share when no new equity is issued is \$36.05; when equity is issued, it is \$34.45, a difference of \$1.59. The value per share decreases because dividends per share decrease after new equity is issued at time $t = 2$. Exhibit 5.9 shows the effects of this dilution. Issuing new shares in period 2 causes the dividends per share to drop by \$0.833 in periods 3, 4, and 5. The present value, at time $t = 0$, of this reduction is \$1.59. For the 100 original shares, this reduction in aggregate value amounts to \$159.44, which equals the present value at $t = 0$ of the equity issued at $t = 2$.

The second implication of this analysis, and the one most useful to financial planning, is how to handle the problem of issuing new equity while avoiding the problem of simultaneity. The Miller and Modigliani solution calculates the aggregate value of equity, independent of the number of shares, subtracts the value of new equity financing from that aggregate value, and divides the difference by the number of shares outstanding prior to the new issue.

5.4.2 Adding Dilution to the O&R Model

The Miller & Modigliani approach makes accounting for the effects of issuing or repurchasing equity in the valuation module quite simple. Exhibit 5.10a, b, and c demonstrate

* This can be demonstrated in the scenario B model by supposing that the \$200 of equity is issued to increase the dividend in year two rather than pay for capital expenditures. Simply change capital expenditures to zero. The resulting price per share is identical to the price per share absent issuing new equity.

how this is done. Exhibit 5.10a shows the input sector and financing module, whereas 5.10b shows the valuation module for our O&R model (Exhibit 5.10c shows the formulas for the valuation module). We change *debt_portion* (cell F12) from 1 to 0.50, thereby setting the percentage of new external financing from equity to 50%. All other inputs remain unchanged. Only two additional lines in the valuation module are required: one that calculates the net equity issued (line 179), the other that reports the present value of the net equity issued (line 180).^{*} Subtracting the present value of equity issued (D180) from aggregate equity value (D178 repeating D169) yields the equity value of the original shareholders (D181). These calculations parallel those in Exhibit 5.8a, lines 33–35 as well as Equation 5.10.

Dividing by the number of shares outstanding at time zero yields the fully diluted value per share. In this case, issuing equity to finance 50% of O&R's future growth increases value per share by \$0.01, from \$19.13 to \$19.14.

	B	D	E	F	G	H	I
2	ODD & RICH Corporation						
3	INPUT SECTOR						
4	Parameter Estimates	External Conditions				Summary Statistics	
5	COGS/Sales		Risk-free rate	0.02		Master Check	0
6	Depreciation/GFA		Mrkt Rsk premiur	0.07		Value (Equity)	1,921
7	Tax Rate		Borrow rate	0.04		Diluted VPS	\$ 19.14
8	Cash/Sales		Sales Growth	0.06		k	8.23%
9	CA (non-cash)/Sales		LT Growth	0.030		Debt ratio 2017	30.9%
10	GFA/Sales					Debt ratio 2021	31.8%
11	CL/Sales		Policies and Decisions			Cash Control	
12	unleveraged beta		Debt portion	0.50		Cash returned	1.00
13	Number of shares		Payout ratio	0.50		Cash invested	0.00
14						Debt retired	0.00
93							
94	FINANCING MODULE		2017	2018	2019	2020	2021
95	Required Financing		161.6	167.9	178.0	188.7	200.0
96	Sources of Funds						
97	Net Income		154.4	164.1	174.0	184.5	195.7
98	Depreciation		57.8	61.4	65.1	69.0	73.1
99	Earnings Cash Flow		212.2	225.5	239.1	253.5	268.8
100	Minimum Dividend		77.2	82.0	87.0	92.3	97.8
101	Maturing Marketable Securities		-	-	-	-	-
102	Internal Equity Financing		135.0	143.4	152.1	161.2	170.9
103	Maturing Short-term Debt		25.0	-	-	-	-
104	Spontaneous Financing		11.0	38.2	40.4	42.9	45.4
105	Total Internal & Spontaneous Financing		146.0	181.6	192.5	204.1	216.4
106	Required External Financing		15.6	0.0	0.0	0.0	0.0
107	Excess Cash		0.0	13.7	14.5	15.4	16.4
108	Short-term Debt Issued		0.0	0.0	0.0	0.0	0.0
109	Debt Issued		7.8	0.0	0.0	0.0	0.0
110	Debt retired		0.0	0.0	0.0	0.0	0.0
111	Equity Issued		7.8	0.0	0.0	0.0	0.0
112	Equity repurchased		0.0	0.0	0.0	0.0	0.0
113	Total Discretionary Financing		15.6	0.0	0.0	0.0	0.0

(a)

EXHIBIT 5.10 (a) Input sector and financing module for O&R with dilution.

(Continued)

^{*} Clearly, in any given year net equity issued will report either equity issued or equity repurchased. However, by subtracting the difference from equity cash flow, equity issued will reduce the cash flow to original shareholders while equity repurchased will increase the cash flow to original shareholders.

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	B	D	E	F	G	H	I
148	EQUITY VALUATION MODULE	2016	2017	2018	2019	2020	2021
149	<i>Cost of Equity (average approach)</i>						
150	Risk-free Rate	0.02	0.02	0.02	0.02	0.02	0.02
151	Market Risk Premium	0.07	0.07	0.07	0.07	0.07	0.07
152	Unleveraged beta	0.87	0.87	0.87	0.87	0.87	0.87
153	Interest-bearing Debt (ST + LT)	73.3	56.1	56.1	56.1	56.1	56.1
154	Assumed value of equity*	1,920.9	2,001.8	2,070.1	2,138.0	2,205.4	2,271.6
155	D/E ratio	0.03816	0.028	0.027	0.026	0.025	0.025
156	Leveraged beta	0.890	0.885	0.884	0.884	0.883	0.883
157	Discount Rate	8.229%	8.19%	8.19%	8.19%	8.18%	8.18%
158							
159	<i>Dividend Valuation Model</i>						
160	Dividends		77.2	95.7	101.5	107.7	114.2
161	PV(Dividends)	388.8	343.6	276.1	197.2	105.6	-
162	Marketable Securities						0.0
163	PV(Marketable Securities)	-					
164	<i>Terminal value</i>						
165	Long-term Growth Rate						0.03
166	Forecast Dividend t = 6						117.7
167	Terminal Value at t=5						2,272
168	PV(Terminal Value)	1,532	1,658	1,794	1,941	2,100	2,272
169	Aggregate (derived) equity value	1,920.9	2,001.8	2,070.1	2,138.0	2,205.4	2,271.6
170	*Initially assumed to be book value						
171							
172	Total firm value						
173	Total debt	73	56	56	56	56	56
174	Equity	1,921	2,002	2,070	2,138	2,205	2,272
175	Total debt + Equity	1,994	2,058	2,126	2,194	2,261	2,328
176							
177	<i>Diluted value per share</i>						
178	Aggregate equity value	1,920.9	2,001.8	2,070.1	2,138.0	2,205.4	2,271.6
179	Net equity issued		7.8	-	-	-	-
180	PV(Net equity issued)	7.2	-	-	-	-	-
181	Aggregate equity value or original shares	1,913.7					
182	Number of original shareholders	100					
183	Diluted value per share	19.14					

(b)

	B	D	E	F
148	EQUITY VALUATION MODULE	2016	2017	2018
149	<i>Cost of Equity (average approach)</i>			
150	Risk-free Rate	=rf	=rf	=rf
151	Market Risk Premium	=mrrp	=mrrp	=mrrp
152	Unleveraged beta	=un_beta	=un_beta	=un_beta
153	Interest-bearing Debt (ST + LT)	=D49+D51	=E49+E51	=F49+F51
154	Assumed value of equity*	=D169	=E169	=F169
155	D/E ratio	=D153/D154	=E153/E154	=F153/F154
156	Leveraged beta	=un_beta*(1+D155*(1-taxes))	=un_beta*(1+E155*(1-taxes))	=un_beta*(1+F155*(1-taxes))
157	Discount Rate	=rf+D156*D151	=rf+E156*E151	=rf+F156*F151
158				
159	<i>Dividend Valuation Model</i>			
160	Dividends		=E31	=F31
161	PV(Dividends)	=(E160+E161)/(1+D157)	=(F160+F161)/(1+E157)	=(G160+G161)/(1+F157)
162	Marketable Securities			
163	PV(Marketable Securities)	=I162/(1+discount_rate)^5		
164	<i>Terminal value</i>			
165	Long-term Growth Rate			
166	Forecast Dividend t = 6			
167	Terminal Value at t=5			
168	PV(Terminal Value)	=E168/(1+D157)	=F168/(1+E157)	=G168/(1+F157)
169	Aggregate (derived) equity value	=D161+D168+D163	=E161+E168+E163	=F161+F168+F163
170	*Initially assumed to be book value			
171				
172	Total firm value			
173	Total debt	=D153	=E153	=F153
174	Equity	=D169	=E169	=F169
175	Total debt + Equity	=D173+D174	=E173+E174	=F173+F174
176				
177	<i>Diluted value per share</i>			
178	Aggregate equity value	=D169	=E169	=F169
179	Net equity issued		=E111-E112	=F111-F112
180	PV(Net equity issued)	=(E179+E180)/(1+D157)	=(F179+F180)/(1+E157)	=(G179+G180)/(1+F157)
181	Aggregate equity value or original shares	=D178-D180		
182	Number of original shareholders	=shares		
183	Diluted value per share	=D181/D182		

(c)

EXHIBIT 5.10 (Continued) (b) Valuation module for O&R with dilution. (c) Formulas for valuation module for O&R with dilution.

5.5 VALUE OF INVESTED CAPITAL

Chapter 4 introduced the discounted dividend model of equity valuation. The previous sections of this chapter expanded that model by allowing the discount rate (cost of equity) to adjust to changes in a firm's capital structure on a period-by-period basis and address the problem of dilution when additional shares are issued. This section introduces a second valuation approach, the value of invested capital, that uses the same structure but with different cash flows and a different discount rate. This approach values the firm as a whole, from which the value of equity is extracted.

A simple example demonstrates the difference between firm value and equity value. Suppose you own a house that you bought several years ago. You made a down payment and then borrowed funds to finance the remainder of the purchase price. The current mortgage loan balance is \$250,000. A real estate appraiser tells you that you could sell the house now for \$1,000,000. These data are summarized as follows:

Prospective Sale Price of House	\$1,000,000
Less: Balance on Mortgage Loan	\$250,000
Equity	\$750,000

The question is: What is the value of the house? Is the value \$750,000 or \$1,000,000? We would all agree that the market value of the house is \$1,000,000, the amount for which it could be sold. It is the value to those who have invested capital in the house—you (your equity) and the bank (debt). The value of *equity* is \$750,000, the amount remaining after paying off the *debt* (mortgage balance). The value of the *house*, as opposed to the equity, is the value of the whole entity, \$1,000,000.

This same distinction applies to the value of a firm. Suppose we have a firm whose assets have a book value of \$600,000, and there is a debt of \$250,000. The total assets have been appraised at a current market value of \$1,000,000. The book value and market-value balance sheets would be:

	Book Value	Market Value
Total Assets	\$600,000	\$1,000,000
Debt	\$250,000	\$250,000
Equity	\$350,000	\$750,000
Total Liabilities and Equity	\$600,000	\$1,000,000

How much is this firm worth, and how much is the equity worth? The market value of the *firm* is \$1,000,000 and the market value of the *equity* is \$750,000 in the same sense that the market value of the house was \$1,000,000 and the equity value was \$750,000. The value of the firm refers to the whole entity, whereas the value of the equity is the value of the residual claims of the equity owners.

The value of the whole entity goes by various names, including *firm value*, *value of the business entity*, *total capital value*, *total company value*, *corporate value*, *value of operations*,

enterprise value, and the previously introduced *value of invested capital*. All of these terms refer to the value of the whole firm, which is the sum of the value of the equity and the value of the debt.

We use the term *invested capital* to refer to all the capital, both debt and equity, including hybrid claims that have characteristics of both debt and equity such as convertible stock. The value of invested capital is the sum of the values of all these different claims. *Debt* refers to interest-bearing debt that is a part of the firm's capital structure; it excludes current liabilities such as accounts payable and trade credit, but normally includes short-term interest-bearing debt such as notes payable that are an ongoing component of the firm's financial structure. This section explains the invested capital approach and highlights its differences with the equity approach.

Both the equity and invested capital methods of valuation calculate value as the discounted value of future cash flows. They differ in terms of the definition of cash flows to be discounted, the discount rate applied to those cash flows, and, of course, the output: the value that is estimated. Exhibit 5.11 summarizes these differences and introduces the notation that we use.

Calculating the cash flow to invested capital is less complicated than calculating the cash flow to equity. Pull EBIT directly from the Income Statement, then adjust it for taxes by

	Equity Method	Invested Capital Method
Discount Rate	Cost of Equity, k_E	Weighted Average Cost of Capital, k_A
Cash Flow	Dividends or Equity Cash Flow	Total Cash Flow to the Firm
Cash Flow Calculation	CFE: $(EBIT - I) * (1 - T)$ –Asset Investment –Debt Principal Payment = Cash Flow to Equity	CFIC: $EBIT * (1 - T)$ –Asset Investment = Cash Flow to Invested Capital
Output: Value	Value of Equity, E	Value of Invested Capital, $V = D + E$
Definitions & Notation: k_E = required return on equity, cost of equity capital k_A = weighted average cost of capital ($WACC$) = $\left(\frac{D}{V}\right)k_D(1 - T) + \left(\frac{E}{V}\right)k_E$ k_D = required return on the debt, cost of debt D = market value of the firm's interest bearing debt (excluding current operating liabilities) E = market value of the firm's equity $V = D + E$, value of invested capital, the market value of the firm as a whole T = tax rate EBIT = Earnings before interest and taxes I = Interest expense Asset Investment = amount invested in new fixed assets plus additions to net working capital Debt Principal Payment = amount paid to lenders to reduce outstanding debt principal balance		

EXHIBIT 5.11 Input and output for the equity and invested capital methods of valuation.

	B	D	E	F	G	H	I
208	Cash flows to invested capital						
209	EBIT*(1-T)		156.12	165.42	175.35	185.87	197.02
210	Investment in working capital		6.60	7.00	7.42	7.86	8.33
211	Investment in fixed assets		61.20	61.37	65.06	68.96	73.10
212	Total cash flow to invested capital		88.32	97.05	102.88	109.05	115.59

EXHIBIT 5.12 Total cash flow to invested capital for O&R Corp.

multiplying by one less than the tax rate. From this subtract the net investment in working capital and the investment in net fixed assets (the change in net fixed assets). Exhibit 5.12 shows these calculations using the model that created Exhibit 5.10.

Just as the value of equity was calculated as the present value of the cash flows to equity, the value of invested capital is calculated as the present value of the cash flows to invested capital:

$$V_0 = \sum_{t=1}^h \left[\frac{CFIC_t}{(1+k_A)^t} \right] + \left[\frac{V_h}{(1+k_A)^h} \right] \quad (5.12)$$

where:

$CFIC_t$ is the cash flow to invested capital (all investors, lenders, and stockholders) at time t (as defined in Exhibit 5.11)

V_h is the value of the whole firm at the end of the h -period investment horizon

k_A is the after tax weighted average cost of capital

With the invested capital method, the value of the equity is calculated as the residual after deducting the debt, D_0 , thus:

$$E_0 = V_0 - D_0 \quad (5.13)$$

5.5.1 The Weighted Average Cost of Capital, WACC

The discount rate used to calculate the value of invested capital using the invested capital approach is the weighted average cost of capital, or WACC. As its name implies, WACC is a weighted average of the return required by the equity investors (cost of equity, denoted by k_E) and the return required by the lenders (cost of debt, denoted by k_D). WACC is calculated as

$$k_A = \left(\frac{Debt}{Total\ Capital} \right) k_D (1-T) + \left(\frac{Equity}{Total\ Capital} \right) k_E \quad (5.14)$$

where:

The subscript A denotes average

Total Capital is the sum of *Debt* plus *Equity*

$(Debt/Total\ Capital)$ is the proportion of interest-bearing debt in the firm's capital structure

$(Equity/Total\ Capital)$ is the proportion of equity used to finance the firm.

We multiply the cost of debt by $(1 - T)$ to put our costs on an after corporate income tax basis.

Continuing with Odd and Rich, the book value of *Debt* in 2016 is 73.3, the book value of *Equity* is 1,460.7, so *Total Capital* equals 1,534.0. The cost of debt is $k_D = 4\%$, the cost of equity is $k_E = 8.27\%$, and the tax rate is $T = 0.40$. Then WACC for 2016 is

$$\begin{aligned} k_A &= \left(\frac{\text{Debt}}{\text{Total Capital}} \right) k_D (1 - T) + \left(\frac{\text{Equity}}{\text{Total Capital}} \right) k_E \\ &= \left(\frac{73.3}{1,534.0} \right) 4\% (1 - 0.40) + \left(\frac{1,460.7}{1,534.0} \right) 8.27\% \\ &= 0.048 * 4\% * (1 - 0.40) + 0.952 * 8.27\% \\ &= 7.99 \end{aligned} \quad (5.15)$$

Often, this is the discount rate applied to expected future cash flows. However, as was the case when calculating leveraged beta and the cost of equity, as time passes, the company's capital structure is subject to change. As it does, the cost of equity changes as previously discussed, but so, too, do the proportions of debt and equity to total capital. The solution, as it was for beta, is to use time-varying market-value-based ratios of debt to total capital, equity to total capital, and, of course D/E when calculating leveraged beta. We address this now in a manner similar to that used for leveraged beta and the cost of equity.

5.5.2 The Invested Capital Approach Using Time-Varying Market-Value-Based Capital Ratios

[Exhibit 5.13a](#) shows the Invested Capital Valuation Module for the O&R Corporation that corresponds to the Equity Valuation Module shown in [Exhibit 5.10](#). [Exhibit 5.13b](#) shows the formulas for selected columns of the model. The structure of the invested capital model parallels that of the equity valuation module ([Exhibit 5.10](#)).

The inputs for cost of capital are lines 200–202. The Assumed Value of Equity (line 204) is set equal to the derived Equity Value (line 235), which is calculated as the residual: Value of Invested Capital—Debt. Total Invested Capital (line 205) is the sum of debt (line 203) and assumed equity (line 204) at market values; the proportions of debt and equity in the capital structure are in lines 206 and 207. The debt-to-equity ratio (line 213) is based on the Assumed Value of Equity (line 204) and the book value of debt (line 203). This feeds into the Hamada formula for leveraged beta in line 214, that, in turn, is used to calculate the cost of equity capital (line 215). Armed with the proportions of debt and equity and the costs of debt (lines 210 and 211) and the cost of equity (line 215), the Weighted Average Cost of Capital is calculated in line 216.

Cash Flow to Invested Capital (*CFIC*) is calculated according to the formula:

$$CFIC = EBIT(1 - T) - \left[\frac{\text{Investment in Net}}{\text{Working Capital}} \right] - \left[\frac{\text{Investment in}}{\text{Fixed Assets}} \right] \quad (5.16)$$

in lines 218–224. *CFIC* is discounted at the weighted average cost of capital in line 225, and the present value of the Terminal Value is calculated in lines 227–231 based on the constant growth model for *CFIC* from the end of the horizon (2021) onward. Line 233 shows the value of invested capital at the end of each year of the planning horizon. Year-end 2016 is the current date (end of year 0), and the value of invested capital is \$2,012.5 million. This is our estimate of the value of the firm as represented by the values of the debt and equity securities.

In this simple O&R example, the only capital securities are short-term debt, long-term debt, and common equity. We have assumed that the value of debt is equal to its book value, so based on the invested capital approach, the value of equity is equal to the

	B	D	E	F	G	H	I
197	INVESTED CAPITAL VALUATION MODULE	2016	2017	2018	2019	2020	2021
198	<i>Cost of Capital</i>						
199	<i>Components</i>						
200	Risk-free Rate	0.02	0.02	0.02	0.02	0.02	0.02
201	Market Risk Premium	0.07	0.07	0.07	0.07	0.07	0.07
202	Unleveraged beta	0.87	0.87	0.87	0.87	0.87	0.87
203	Debt (ST & LT interest-bearing)	73.3	63.9	63.9	63.9	63.9	63.9
204	Assumed value of equity	1,939.2	2,021.6	2,091.9	2,162.2	2,231.9	2,300.7
205	Total Invested Capital	2,012.5	2,085.5	2,155.9	2,226.1	2,295.8	2,364.7
206	Debt / Total Capital	0.036	0.031	0.030	0.029	0.028	0.027
207	Equity / Total Capital	0.964	0.969	0.970	0.971	0.972	0.973
208	<i>Costs</i>						
209	<i>Debt</i>						
210	Cost of Debt (Borrowing Rate)	4.00%	4.00%	4.00%	4.00%	4.00%	4.00%
211	Cost of debt after tax	2.40%	2.40%	2.40%	2.40%	2.40%	2.40%
212	<i>Equity</i>						
213	Debt/Equity	3.78%	3.16%	3.06%	2.96%	2.86%	2.78%
214	Beta (leveraged)	0.89	0.89	0.89	0.89	0.88	0.88
215	Cost of Equity	8.23%	8.21%	8.20%	8.20%	8.19%	8.19%
216	Weighted Average Cost of Capital	8.02%	8.03%	8.03%	8.03%	8.03%	8.03%
217							
218	<i>Cash Flow to Invested Capital</i>						
219	EBIT(1-T)		156.1	165.4	175.3	185.9	197.0
220	EBIT(1 - T) + Dep		213.9	226.8	240.4	254.8	270.1
221	Investment in Net Working Capital		6.6	7.0	7.4	7.9	8.3
222	Investment in Fixed Assets		119.0	122.7	130.1	137.9	146.2
223	Total Asset Investment		125.6	129.7	137.5	145.8	154.5
224	Total Cash Flow to Invested Capital		88.3	97.1	102.9	109.0	115.6
225	Present value of cash flow to capital	405.2	349.4	280.4	200.0	107.0	
226							
227	<i>Terminal Value</i>						
228	Long Term Growth Rate						0.03
229	Cash Flow to Invested Capital t = 6						119.1
230	Terminal Value of Invested Capital						2,364.7
231	PV of Terminal Value	1,607.3	1,736.1	1,875.5	2,026.1	2,188.8	2,364.7
232							
233	<i>Value of Invested Capital</i>	2,012.5	2,085.5	2,155.9	2,226.1	2,295.8	2,364.7
234	Debt	73.3	63.9	63.9	63.9	63.9	63.9
235	Equity Value	1,939.2	2,021.6	2,091.9	2,162.2	2,231.9	2,300.7
236							
237	<i>Value per share</i>						
238	Aggregate value of equity	1,939.2					
239	Net equity issued (issued - repurchased)		0.0	0.0	0.0	0.0	0.0
240	PV(Net equity issued)	-	-	-	-	-	-
241	Aggregate value of original equity	1,939.2					
242	Shares outstanding	100					
243	Value per share	19.39					

(a)

EXHIBIT 5.13 (a) Invested capital valuation model with time-varying market-value-based costs of capital. (Continued)

	B	D	E	F
197	INVESTED CAPITAL VALUATION MODULE	2016	2017	2018
198	<i>Cost of Capital</i>			
199	<i>Components</i>			
200	Risk-free Rate	=rf	=rf	=rf
201	Market Risk Premium	=mrp	=mrp	=mrp
202	Unleveraged beta	=un_beta	=un_beta	=un_beta
203	Debt (ST & LT interest-bearing)	=D50+D52	=E50+E52	=F50+F52
204	Assumed value of equity	=D235	=E235	=F235
205	Total Invested Capital	=D203+D204	=E203+E204	=F203+F204
206	Debt / Total Capital	=D203/D205	=E203/E205	=F203/F205
207	Equity / Total Capital	=D204/D205	=E204/E205	=F204/F205
208	<i>Costs</i>			
209	<i>Debt</i>			
210	Cost of Debt (Borrowing Rate)	=int_expense	=int_expense	=int_expense
211	Cost of debt after tax	=D210*(1-taxes)	=E210*(1-taxes)	=F210*(1-taxes)
212	<i>Equity</i>			
213	Debt/Equity	=D203/D204	=E203/E204	=F203/F204
214	Beta (leveraged)	=un_beta*(1+D213*(1-taxes))	=un_beta*(1+E213*(1-taxes))	=un_beta*(1+F213*(1-taxes))
215	Cost of Equity	=rf+D214*mrp	=rf+E214*mrp	=rf+F214*mrp
216	Weighted Average Cost of Capital	=D206*D211+D207*D215	=E206*E211+E207*E215	=F206*F211+F207*F215
217				
218	<i>Cash Flow to Invested Capital</i>			
219	EBIT(1-T)		=E26*(1-taxes)	=F26*(1-taxes)
220	EBIT(1 - T) + Dep		=E219+E25	=F219+F25
221	Investment in Net Working Capital		=(E41-D41)-(E49-D49)	=(F41-E41)-(F49-E49)
222	Investment in Fixed Assets		=(E45-D45)+E25	=(F45-E45)+F25
223	Total Asset Investment		=E221+E222	=F221+F222
224	Total Cash Flow to Invested Capital		=E220-E223	=F220-F223
225	Present value of cash flow to capital	=(E224+E225)/(1+D216)	=(F224+F225)/(1+E216)	=(G224+G225)/(1+F216)
226				
227	<i>Terminal Value</i>			
228	Long Term Growth Rate			
229	Cash Flow to Invested Capital t = 6			
230	Terminal Value of Invested Capital			
231	PV of Terminal Value	=E231/(1+D216)	=F231/(1+E216)	=G231/(1+F216)
232				
233	Value of Invested Capital	=D225+D231	=E225+E231	=F225+F231
234	Debt	=D203	=E203	=F203
235	Equity Value	=D233-D234	=E233-E234	=F233-F234
236				
237	<i>Value per share</i>			
238	Aggregate value of equity	=D235		
239	Net equity issued (issued - repurchased)		=E112-E113	=F112-F113
240	PV(Net equity issued)	=(E239+E240)/(1+D216)	=(F239+F240)/(1+E216)	=(G239+G240)/(1+F216)
241	Aggregate value of original equity	=D238-D240		
242	Shares outstanding	=shares		
243	Value per share	=D241/D242		

(b)

EXHIBIT 5.13 (Continued) (b) Formulas for invested capital valuation model with time-varying market-value-based costs of capital.

value of invested capital minus the book value of debt at the beginning of the planning period (line 234: \$73.3 million), resulting in estimated equity value of \$1,939.2 million (line 235).

5.5.3 Value Difference between the Equity Approach and the Invested Capital Approach

The value of equity at time $t = 0$ (year-end 2016) using the equity approach was \$1,913; using the invested capital approach, it was \$1,939, a difference of \$26, and an error of 1.37%. In principle, the value of equity using either approach should be the same. In other words, if you get an equity value at a given date of, say, \$1,919 using the equity approach, you should get the same equity value of \$1,919 using the invested capital approach. However, it is rare that you will get exactly the same values with the two approaches. There are several reasons for this. For these two approaches to yield the same

values, everything must be consistent and in a precise balance. The growth rates must be consistent with the firm's financial ratios as shown by Higgins' sustainable growth (g^{**}) or *PRAT* by the time the constant growth model is applied. Without this balance, the assumption that the long-run growth rate will be the same for equity cash flow and the *CFIC* may be incorrect.

For example, we assumed an annual growth rate in sales of 6% for the examples we used in [Chapter 5](#). Given a 6% growth rate, O&R's *PRAT* is around 5.6%. Hence, equity values from the two approaches are very close. However, increasing the assumed growth rate to 10% markedly increases distance between the two estimates. *Equity value* using the equity method is \$1,691; using the invested capital method, it is \$499. The difference is -1,193, or -71%, as shown in [Exhibit 5.14](#), Panel A.

The underlying factor in these discrepancies is the growth rate. So long as this growth rate is less than or very near to g^{**} and *PRAT*, 6% in the O&R example, the minimum dividend (equity cash flow) is funded entirely or nearly so from internal sources, and equity values from the two methods are quite similar. However, when the growth rate for the planning horizon is considerably greater than g^{**} and *PRAT*, a significant portion of the minimum dividend is funded by external financing, whereas no cash flow to invested capital is funded by external sources. When these differences are compounded in perpetuity, the discrepancy between the equity values of the two approaches is magnified. For example, when the horizon growth rate is 6%, equity cash flow and the *CFIC* are 114.1 and 115.6, respectively; when the horizon growth rate is 10%, these case flows are 114.9 and 25.1. In this case, 99 of the 114.9 equity cash flow is funded by debt, something that cannot continue in perpetuity. Now suppose that in 2021, sales growth falls to 3%, the forecast long-term growth rate for the economy. This reduces the estimated equity value error from -71% to 5.1% (Panel B, [Exhibit 5.14](#)). The driving factor in all of this is the terminal value calculation. We take this up next.

2021		Approach	
Growth	PRAT	Diluted Equity	Invested Capital
Panel A			
<i>50% Equity financing</i>			
10.00%	5.85%	1,672	260
<i>Difference</i>			-84.5%
<i>0% Equity financing</i>			
10.00%	6.28%	1,691	499
<i>Difference</i>			-70.5%
Panel B			
<i>50% Equity financing</i>			
3.00%	5.38%	3,076	3,025
<i>Difference</i>			-1.6%
<i>0% Equity financing</i>			
3.00%	5.77%	3,058	3,215
<i>Difference</i>			5.1%

EXHIBIT 5.14 Equity value under different assumptions.

5.6 TERMINAL VALUE

Equation 5.1 shows equity value as the present value of the dividends over an h -period horizon plus the value of the equity at the end of the h periods. Equation 5.12 shows the value of invested capital as the present value of the *CFIC* over an h -period horizon plus the value of invested capital at the end of the h periods. The value of equity or invested capital at the end of h periods is called the terminal or horizon value. This presents a dilemma. We want to know the equity value today, but this value depends on the value h periods in the future. If it is difficult to estimate the value today, it is much more difficult to determine that value in the future. Yet, today's value depends on our forecast of the future value. There are several ways to handle this.

5.6.1 The Constant Growth Model

The most common way to estimate terminal value is to use the constant growth model of the form:

$$E_h = \frac{D_{h+1}}{(k_E - g)} \quad (5.17)$$

where:

E_h is the aggregate value of equity at time h

D_{h+1} is the aggregate dividend expected at time $h + 1$

g is the rate of growth in the dividends expected to prevail from time $h + 1$ into perpetuity

k_E is the firm's cost of equity

This formula can also be used for the invested capital cash flow valuation by substituting the *CFIC* for dividends and WACC for k_E .

The constant growth model is a remarkably useful way to estimate value. However, like nearly all financial models, it derives from several strong assumptions. As these assumptions create potential pitfalls when applying the model, they should be understood before the model is used. First, as implied by its name, the model assumes that dividends will grow at the same rate in every period, forever. As in the song made famous by Johnny Mathis, "that's a long, long time." As of this heroic assumption, the model only should be applied to stable-growth situations. So long as the growth rate is moderately stable and the growth rate used (g) is an unbiased estimate of future growth, in other words, as long as the firm has reached maturity and is growing no faster than the economy, the model will yield a reasonable estimate of value.*

The model is inappropriate when near-term growth rates depart systematically from the average perpetual growth rate. As the near future has a bigger impact on value than the more distant future, if near-term growth rates are applied in perpetuity they may yield excessively high valuations. The solution to this problem is to forecast dividends (equity cash flow) or *CFIC* period-by-period until the time in the future when the firm is expected to reach maturity. Practically, this is done by first forecasting how long the firm will remain in its rapid growth phase, the rapid growth rate, and subsequently how rapidly its growth

* Unbiased means that actual growth rates are symmetrically distributed around their mean, g .

will decline to the long-term growth rate of the economy. Only then should the constant growth model be used to estimate terminal value.

One benefit of this approach is that the further into the future you push the terminal value calculation, the smaller its impact will be on the current valuation. However, the difficulty is accurately forecasting the rapid growth rate, how long it will last, and how long the glide path to maturity will take. The difficulty of doing this should not be underestimated.*

5.6.1.1 Components of the Terminal Growth Rate

Given the critical role played by g in the constant growth model, a closer look is warranted. Two factors influence the long-run growth of the firm: (1) the rate of growth in the economy and the firm's industry; and (2) the limit imposed by the firm itself, measured by its sustainable growth rate.

5.6.1.1.1 Growth of the Industry and Economy Most industries follow a life cycle, with rapid growth early in their life followed by slowing growth as the industry matures. Firms tend to grow with their industry, with variations among companies within the industry as they strive for competitive advantage. Consequently, it is normal for a firm's earnings and cash flows to grow rapidly in the early stages of its industrial life cycle, followed by slowing growth as the firm and its industry mature and competition increases. Eventually, as the industry stabilizes and matures, its growth will be limited by the rate of growth of the economy as a whole. In the long run, neither the industry nor its companies can grow faster than the economy. The date in the future when we expect the industry and firm to mature and reach stable growth consistent with the economy is the point where we should consider using the constant growth model.

5.6.1.1.2 Sustainable Growth A company's growth is limited not just by the economy and the industry. It is also limited by internal constraints imposed by the company's own structure and production technology. In [Chapter 3](#), we introduced the concept of sustainable growth, which is an excellent indicator of the growth that the firm can handle within the limits of its capital structure and other operating characteristics.† As a firm matures, its

* Another limitation of the constant growth model is that the growth rate must be less than the discount rate, that is, $g < k_B$, otherwise the constant growth model is invalid. However, this is unlikely to be a problem so long as the constant growth model is applied only to firms that have reached maturity.

† The sustainable growth formulas in [Chapter 3](#) were

$$PRAT_t = (1 - \text{Payout Ratio}) \left[\left(\frac{\text{Net Income}}{\text{Sales}} \right) \left(\frac{\text{Sales}}{\text{Total Assets}} \right) \left(\frac{\text{Total Assets}}{\text{Equity}_{t-1}} \right) \right]$$

or

$$g^{**} = \frac{\left(\frac{\text{Net Income}}{\text{Sales}} \right) (1 - \text{Payout ratio}) \left(1 + \frac{\text{Debt}}{\text{Equity}} \right)}{\left(\frac{\text{Total Assets}}{\text{Sales}} \right) - \left(\frac{\text{Net Income}}{\text{Sales}} \right) (1 - \text{Payout ratio}) \left(1 + \frac{\text{Debt}}{\text{Equity}} \right)}$$

Consistent with the assumptions that underlie the constant growth model, these measures assume the operating ratios are stable, hence should not be applied until the firm reaches maturity.

operating ratios stabilize, so its profit margin, asset utilization, and dividend payout would not be expected to fluctuate widely. It also will tend to maintain its debt ratios within a narrow range. This suggests that a sustainable growth rate, for example, *PRAT*, serves as a good estimator of the long-run rate of growth that the firm can handle. This does not mean that the firm cannot grow more rapidly in some periods, or that its growth cannot vary from *PRAT* from time to time. Rather, it means that the firm's average growth rate over long periods cannot exceed *PRAT* without it having to change its operating methods and structure. This makes *PRAT* something of a recommended speed limit. Just as a car's speed is constrained by its mechanical makeup, the firm is limited by its structural and financial makeup. The structure and growth rate are expected to stabilize when the firm reaches maturity, making *PRAT* a sensible candidate to use in the constant growth model.

Together, the growth of the economy and the sustainable growth of the firm limit the long-run growth of the firm. The rate that prevails at maturity will be the limit that the firm reaches first. In other words, the long-run growth rate is the lower of either the economic growth limit or the sustainable growth limit imposed by the firm's structure. For example, the structural sustainable growth might be as high as 15%, but the economy is expected to grow at, say, 3% in the very long run. Even though the firm might have the structural capacity to grow at 15%, it would not make sense to forecast that it will grow forever at 15% when the economy will grow at just 3%. Besides, if the company's sustainable growth rate is 15%, the company likely has not yet reached maturity. Conversely, if the firm's sustainable growth rate is 1.5%, then assuming a long-term growth rate of 3% also is incorrect.

5.6.1.1.3 A Simple Solution There is one very simple, if flawed, solution to this dilemma—use the assumed long-term growth rate of the economy to forecast sales for the last year of your planning horizon. For the models we have built thus far, this would mean using the long-term growth rate in the fifth pro forma year. However, there is nothing special about a five-year planning horizon. The length of the planning horizon should be determined by the specifics of the firm being modeled.

5.6.2 Other Models of Terminal Value

There are many cases where using the constant growth model to estimate terminal value is inappropriate. For these cases, multipliers provide a simple alternative. Typical multipliers include the ratios of price-to-earnings, price-to-revenue, market-to-book, dividends-to-price (dividend multiple), and price-to-cash flow (cash flow multiple). This section considers the price-earnings multiplier. Other multipliers can be estimated and used in the same or similar manner.

The price-to-earnings ratio, or *PE* ratio, is one of the most widely used metrics of value. It provides an easy way to estimate terminal value that is consistent with market practices. Aggregate terminal value of equity is estimated as

$$E_t = PE_t * (\text{Net Income}_t) \quad (5.18)$$

and terminal per share value is estimated as

$$P_h = PE_h * (\text{Earnings per Share}_h) \quad (5.19)$$

Although the model provides the forecast of income or earnings, we need to provide the *PE* ratio. Obviously, we cannot know the *PE* ratio that will prevail several years into the future. Therefore, a forecast for the *PE* ratio either must be provided as an input to the model or calculated by a module as a function of other internal variables.

Two logical forecasting methods are available. One is to use the historical average *PE* ratio for the industry or company as an indicator of what we expect at the end of the horizon. This method is justifiable because markets exhibit a tendency to *regress to the mean*. If *PE* ratios are currently higher than the average, there is a tendency for them to work their way down toward the historical mean in future periods; the converse is true if the *PE* ratio is currently below its historical mean. The second forecasting method is to model the future *PE* ratio based on its historical relationship to other fundamental variables. For example, we would expect that the *PE* ratio will be higher for firms that exhibit more stable earnings, less debt, higher growth rates, longer potential duration of growth, or a combination of the above.

Various authors have examined the empirical relation between *PE* ratios and fundamental company variables. An early example of such a relationship is the simple model by Whitbeck and Kisor (1963). They estimate the empirical relationship between *PE* ratios and company fundamentals with a linear regression equation, thus,

$$\begin{aligned} PE &= a + b_1 \left(\frac{\text{Earnings}}{\text{Growth}} \right) + b_2 \left(\frac{\text{Dividend}}{\text{Payout}} \right) + b_3 \left(\frac{\text{Standard Deviation}}{\text{of Earnings Growth}} \right) \\ &= 8.2 + 1.5 \left(\frac{\text{Earnings}}{\text{Growth}} \right) + 6.7 \left(\frac{\text{Dividend}}{\text{Payout}} \right) - 0.2 \left(\frac{\text{Standard Deviation}}{\text{of Earnings Growth}} \right) \end{aligned} \quad (5.20)$$

Cragg and Malkiel (1968) updated the Whitbeck and Kisor study using the stock's beta coefficient as the measure of risk in place of the standard deviation of earnings growth. Their fitted equation using 1965 data was

$$PE = 0.96 + 2.74(\text{Earnings Growth}) + 5.01(\text{Dividend Payout}) - 0.35\beta \quad (5.21)$$

Damodaran (2012) updates this model using data from 2000. His estimated equation is

$$PE = -6.37 + 83.56(\text{Expected Growth Rate}) + 5.83(\text{Dividend Payout}) + 5.06\beta \quad (5.22)$$

Using these models to estimate terminal value presents several problems. The models fit the data better for some periods than for others and the relationships vary over time.

These problems can be addressed in several ways. First, we can re-estimate the regression model using recent data from companies in the same industry as the subject firm. Second, recognizing that the relationships change with market conditions and that we do not know what conditions will prevail at the end of the planning horizon, we can estimate the relationship over several periods so as to get a smoothed, normalized relationship, and then use the estimated coefficients as input to our model. By doing this, the terminal *PE* ratio and resulting equity value are adjusted for the characteristics of the firm at the horizon date. Finally, since the *PE* ratio is being used to forecast the value of a mature firm, the input variables—earnings growth and dividend payout—should reflect the expected growth and payout rates of the firm in maturity.

Other multipliers such as price-to-book, price-to-revenue, price-to-dividend, or price-to-enterprise-value can also be used to generate estimates for terminal value. As with the *PE* ratio, we do not know the ratio that will prevail at the terminal date, so it must be forecast. We can use methods similar to those suggested for estimating the *PE* ratio to model our forecast of these future multipliers.

5.7 CAVEATS AND EXTENSIONS

This completes our consideration of the basics of equity valuation. A few caveats remain with regard to the equity approach—valuing firms that do not pay dividends and valuing firms that do not return all free cash flow to their shareholders.

5.7.1 Value for a Nondividend Paying Firm

Our discussion of dividends raises the question of how to handle the situation where the firm pays no dividends. As we use a dividend valuation model, this suggests that firms that pay zero dividends have zero value. This is clearly wrong.

There are at least four situations in which a firm pays no dividends. First, there is the firm that has no earnings and is not expected to have earnings or cash flow to pay out. The value of this firm's dividends is zero, but the firm may have value in its assets. The value of such a firm would be based on the market value of those assets.

Second, there is the small business where the owner, as an employee, takes the firm's surplus in the form of salary. As a result, the firm earns little or no profits and pays no dividends. The value of this type of firm would be based on its ability to pay dividends assuming the owner/manager is paid the going market wage for managerial services, and the excess wages are treated as dividends.

Third, and closely related to the second situation, are firms, particularly small, closely held ones, that pay no dividends either because the stockholders do not need the funds or because the owners want to avoid paying taxes on the dividends. This firm retains the earnings and invests them in other assets, including perhaps marketable securities, that are not necessary for the normal operations of the firm. These are the nonoperating assets referred to earlier. The value of such a firm has two parts: the value of the operating business and the value of the nonoperating assets. The value of the operating business would be based on its ability to pay dividends assuming normal investment in assets necessary for its ongoing operations. The nonoperating assets would be valued separately.

Fourth, there is the growth firm that pays no dividends because it needs to plow its earnings back into the firm to finance current operations and growth. This investment enhances the firm's earnings capacity and dividend's paying ability in future years. In terms of our dividend valuation model with a terminal value at the horizon, all of this firm's value would be based on the terminal value; the value of the zero dividends over the planning horizon is zero. In this case, the valuation horizon should be extended to the point where the firm is expected to have the ability to pay dividends. However, if that point is at some indeterminate time in the future, the horizon will easily exceed the ability to accurately forecast those dividends.

5.7.2 Valuing Equity Cash Flow When Not All Free Cash Flow Is Paid Out

The estimates of equity value calculated in Sections 5.2 and 5.3 used plain vanilla equity cash flow or dividend-paying ability as the input. This flavor of equity cash flow assumes that the firm makes only the minimum investment necessary in working capital and fixed assets and pays out all remaining cash flow as dividends. This section considers calculating equity value when those investments do not equal the minimum amount necessary.

When a firm's investments in working capital and fixed assets differ from what is necessary, a firm's equity cash flow departs from the plain vanilla case of simple dividends. This complicates estimating the equity value. For example, the investment in net working capital includes the increase in cash and marketable securities. As a firm grows, these accounts should increase to support the larger operation. However, suppose more funds are invested in marketable securities than is necessary for normal business operations. The amount of funds by which the investment exceeds what is necessary is the amount that could have been paid to the stockholders as dividends but was not.

For example, when the excess investment in securities is included in the investment in net working capital, the resulting equity cash flow still will represent the funds available to be paid as dividends. However, the dividends paid will be less than those that could have been paid had the excess investment not occurred, so dividends paid will be less than equity cash flow (a.k.a., dividend-paying ability). In this case, equating dividends and equity cash flow is correct, but only from the perspective of a minority investor unable to affect the marketable securities decision.*

This difference creates a problem for modeling equity value: absent the ability to control the flow of excess cash through the firm, a model's calculations of investment and cash available for dividends (based on its *necessary investment* assumptions) differ from the firm's actual level of investment and dividends paid. As investment assumptions differ, the cash available to pay dividends and actual dividends paid differ. As a consequence, the valuations based on these cash flows differ.

This is one reason that we introduced the cash control variables at the end of Unit 4. Thus far, those variables have been set to return all equity cash flow in excess of the minimum dividend (that we refer to as Excess Cash) to shareholders by setting *cash_returned* set to 1. Changing *cash_returned* to 0 and *cash_invested* to 1 has the model invest all

* Still, under some conditions the value of the excess holdings will be reflected in the market value of the equity.

Excess Cash in Marketable Securities. Exhibit 5.15a shows the cash flow and equity valuation modules when *cash_returned* is set to 1; Exhibit 5.15b shows the cash flow and equity valuation modules with *cash_returned* set to 0 and *cash_invested* set to 1. Note the drop in equity value when the investment in marketable securities is greater than necessary.

In the valuation literature, the distinction between the two approaches (actual dividends vs. dividend-paying capacity) is regarded as the difference between the value to an investor who purchases a controlling interest in the firm and the value to an investor who purchases a minority position with no control.* Controlling value should be based on the dividend-paying ability of the firm, because the control investor can determine the dividend policy and does not have to settle for the present dividend policy. The minority value should be based on the forecast of actual dividends paid, because the minority investor cannot dictate the dividend or investment policies of the firm and must settle for whatever dividends the

	B	D	E	F	G	H	I
115	CASH FLOW MODULE		2017	2018	2019	2020	2021
116	Earnings Cash Flow		212.2	225.3	238.9	253.3	268.6
117	Current Asset Investment (except Cash)		32.4	34.3	36.4	38.6	40.9
118	Maturing Marketable Securities		-	-	-	-	-
119	Spontaneous Financing	=cash*E21-D37	36.0	38.2	40.4	42.9	45.4
120	NWC Investment	=(cash/sales _t)*sales _t -cash _{t-1}	(3.6)	(3.8)	(4.0)	(4.3)	(4.5)
121	Cash Flow from	=E107*(1-Cash_returned)*Cash_invested	215.8	229.1	242.9	257.6	273.1
122	Capital Expendit	=[Excess Cash _t](1-Cash_returned)*Cash_invested	119.0	122.7	130.1	137.9	146.2
123	Capital Financ	=E124-E125-E126	(9.4)	-	-	-	-
124	Net Cash Flow	=Net Cash Inflow - Increase in Cash - MS Purchased	87.4	106.3	112.8	119.7	126.9
125	Increase in Cash		10.2	10.8	11.5	12.1	12.9
126	Marketable Securities Purchased		-	-	-	-	-
127	Cash Available for Dividends		77.2	95.5	101.3	107.5	114.1
128	Check: NCF-ΔCash-Dividends		-	-	-	-	-
129							
148	EQUITY VALUATION MODULE		2016	2017	2018	2019	2020
149	<i>Cost of Equity (average approach)</i>						
150	Risk-free Rate	0.02	0.02	0.02	0.02	0.02	0.02
151	Market Risk Premium	0.07	0.07	0.07	0.07	0.07	0.07
152	Unleveraged beta	0.87	0.87	0.87	0.87	0.87	0.87
153	Interest-bearing Debt (ST + LT)	73.3	63.9	63.9	63.9	63.9	63.9
154	Assumed value of equity*	1,913.1	1,993.3	2,061.4	2,129.2	2,196.2	2,262.2
155	D/E ratio	0.03832	0.032	0.031	0.030	0.029	0.028
156	Leveraged beta	0.890	0.887	0.886	0.886	0.885	0.885
157	Discount Rate	8.230%	8.21%	8.20%	8.20%	8.20%	8.19%
158							
159	<i>Dividend Valuation Model</i>						
160	Dividends		77.2	95.5	101.3	107.5	114.1
161	PV(Dividends)	388.1	342.9	275.5	196.8	105.4	-
162	Marketable Securities						0.0
163	PV(Marketable Securities)	-					
164	<i>Terminal value</i>						
165	Long-term Growth Rate						0.03
166	Forecast Dividend t = 6						117.5
167	Terminal Value at t=5						2,262
168	PV(Terminal Value)	1,525	1,650	1,786	1,932	2,091	2,262
169	Aggregate (derived) equity value	1,913.1	1,993.3	2,061.4	2,129.2	2,196.2	2,262.2
170	<i>*Initially assumed to be book value</i>						

(a)

EXHIBIT 5.15 (a) Cash flow module set to return all free cash flow in excess of the minimum dividend to shareholders. (Continued)

* For a discussion of control versus. minority value, see Chapter 15, "Control and Acquisition Premiums," Pratt et al. (2000).

	B	D	E	F	G	H	I	
115	CASH FLOW MODULE		2017	2018	2019	2020	2021	
116	Earnings Cash Flow		212.2	225.3	238.9	253.3	268.6	
117	Current Asset Investment (except Cash)		32.4	34.3	36.4	38.6	40.9	
118	Maturing Marketable Securities		-	-	13.6	28.0	43.4	
119	Spontaneous Financing	=cash*E21-D37	36.0	38.2	40.4	42.9	45.4	
120	NWC Investment	=(cash/sales _t)*sales _t -cash _{t-1}	(3.6)	(3.8)	(17.6)	(32.3)	(47.9)	
121	Cash Flow from	=E107*(1-Cash_returned)*Cash_invested	215.8	229.1	256.5	285.6	316.5	
122	Capital Expendit	=[Excess Cash _t](1-Cash_returned)*Cash_invested	119.0	122.7	130.1	137.9	146.2	
123	Capital Financ	=E124-E125-E126	(9.4)	-	-	-	-	
124	Net Cash Flow	=Net Cash Inflow - Increase in Cash - MS Purchased	87.4	106.3	126.4	147.7	170.3	
125	Increase in Cash		10.2	10.8	11.5	12.1	12.9	
126	Marketable Securities Purchased			13.6	28.0	43.4	59.7	
127	Cash Available for Dividends		77.2	81.9	86.9	92.2	97.7	
128	Check: NCF-ΔCash-Dividends		-	-	-	-	-	
129								
148	EQUITY VALUATION MODULE		2016	2017	2018	2019	2020	2021
149	<i>Cost of Equity (average approach)</i>							
150	Risk-free Rate	0.02	0.02	0.02	0.02	0.02	0.02	0.02
151	Market Risk Premium	0.07	0.07	0.07	0.07	0.07	0.07	0.07
152	Unleveraged beta	0.87	0.87	0.87	0.87	0.87	0.87	0.87
153	Interest-bearing Debt (ST + LT)	73.3	63.9	63.9	63.9	63.9	63.9	63.9
154	Assumed value of equity*	1,684.2	1,702.4	1,760.6	1,818.4	1,875.7	1,932.0	1,932.0
155	D/E ratio	0.04352	0.038	0.036	0.035	0.034	0.033	0.033
156	Leveraged beta	0.893	0.890	0.889	0.888	0.888	0.887	0.887
157	Discount Rate	8.249%	8.23%	8.22%	8.22%	8.21%	8.21%	8.21%
158								
159	<i>Dividend Valuation Model</i>							
160	Dividends		77.2	81.9	86.9	92.2	97.7	97.7
161	PV(Dividends)	342.8	293.9	236.1	168.6	90.3	-	-
162	Marketable Securities							59.7
163	PV(Marketable Securities)	40						
164	<i>Terminal value</i>							
165	Long-term Growth Rate							0.03
166	Forecast Dividend t = 6							100.7
167	Terminal Value at t=5							1,932
168	PV(Terminal Value)	1,301	1,409	1,524	1,650	1,785	1,932	1,932
169	Aggregate (derived) equity value	1,684.2	1,702.4	1,760.6	1,818.4	1,875.7	1,932.0	1,932.0
170	*Initially assumed to be book value							

(b)

EXHIBIT 5.15 (Continued) (b) Cash flow module set to invest all free cash flow in excess of the minimum dividend in marketable securities.

directors choose to pay. The two approaches can yield different estimates of value. As the model's forecast of equity cash flow and the firm's dividends differ, valuations based on the assumption that equity cash flow equals dividends may be incorrect.

5.7.3 Additional Dividend Nuances

In addition to our cash control variables, there are several approaches to resolve this conflict. One is to separate the value of the firm's nonoperating assets from the value of its normal business operations and then sum the two values. For example, if a firm is making excess investments in cash and securities, estimate the value of the equity based on the dividends expected to be paid, given their low payout, and then add the value of the excess cash and securities to the value of the operating firm. The value obtained for the combined entity (operating firm value + value of nonoperating assets) is the value of the firm, given that it continues making excess security investments and paying dividends that are less than its dividend-paying capacity.

A second approach is to estimate value assuming the plain vanilla case: the firm pays dividends based on its full dividend-paying capacity and does not accumulate excess nonoperating assets. In this case, we assume that the asset investments are based on business need (i.e., they are less than what is actually made) and that equity cash flow and dividends are equal.

Our discussion of excess investment in securities also applies to other types of discretionary short- or long-term investments. If the firm pays out smaller dividends than it has capacity to pay, then the funds are used elsewhere—for example, to purchase unnecessary assets, to repay debt, or perhaps even to pay higher expenses than necessary (executive salaries, for example)—and not valued.

The relevance of this discussion to modeling is that we can change the way we model decisions such as dividends and investment based on the policy that we think will prevail. Going forward, we use equity cash flow, as our focus is on modeling for financial planning. For example, [Exhibit 5.15a](#) shows the cash flow sector of our Odd & Rich model from [Chapter 4](#). Line 124 is the Net Cash Inflow after taking into account of all the needs of the business except for increasing the cash balance and paying dividends. It is the flow available to be used either to add to cash and securities or to be paid out as dividends. Line 125 is the investment (increase) in the cash balance. It is set as a percent of sales based on the liquidity needs of the firm. Line 126 is Marketable Securities Purchased, currently set to zero. Cash Available for Dividends is the firm's residual cash flow—the cash flow left over after meeting all the business needs of the firm, including the cash balance. This model sets the dividend equal to the residual cash flow, thereby basing it on the firm's ability to pay dividends. The cash balance shown on line 125 is the *necessary* investment used in the model previously discussed.

5.8 SUMMARY

This chapter developed two complete models of firm valuation: one using cash flow to equity, the other using cash flows to invested capital. We defined both cash flows and showed a simple approach to allow the cost of equity to vary with financial risk, the correct but more complex way to allow the costs of capital to vary as time passed and/or the market values of debt and equity changed. We also considered the Miller and Modigliani (1961) approach to accounting for the dilutive effects of issuing additional shares of equity. We concluded with a much closer look at terminal value and nuanced views of dividends, equity cash flow, and dividend-paying ability.

PROBLEMS

1. Tellall Inc. Valuation

Tellall, Inc., is a midcap company engaged in designing and marketing equipment to telecommunications service providers in the United States and internationally. The company also provides deployment and professional services to support its products. Its income statements and balance sheets for 2012 through 2016 are shown in [Exhibit 5.16](#). [Exhibit 5.17](#) shows forecast parameters and policy variables that will apply for the next five years.

Annual Income Statement					
Tellall, Inc.					
In Millions of U.S. Dollars					
	2012	2013	2014	2015	2016
Total Revenue	1,317	980	1,232	1,883	2,041
Cost of Revenue, Total	831	626	575	1,028	1,108
Selling, General and Administrative Expenses	298	243	237	271	293
Research & Development	335	286	250	344	357
EBITDA	-147	-175	170	240	283
Depreciation/Amortization	9	13	18	36	29
Restructuring Charge	174	77	231	15	8
EBIT	-330	-265	-79	189	246
Interest Expense	0	0	0	10	11
Interest Income	33	31	34	53	69
Net Income Before Taxes	-297	-234	-45	232	304
Income Tax	-15	-3	20	37	91
Net Income After Taxes	-282	-231	-65	195	213
Dividends	0	0	0	0	0
Retained Earnings	-282	-231	-65	195	213
Annual Balance Sheet					
Tellall, Inc.					
In Millions of U.S. Dollars					
	2012	2013	2014	2015	2016
Cash & Equivalents	454	223	269	876	168
Short Term Investments	566	847	914	490	1,435
Receivables	217	197	309	319	411
Inventory	175	42	116	109	167
Other Current Assets	206	137	89	74	66
Total Current Assets	1,618	1,446	1,697	1,868	2,247
Property/Plant/Equipment - Gross	770	644	619	614	638
Accumulated Depreciation	349	362	380	416	445
Net Fixed Assets	421	282	239	198	193
Goodwill and Intangibles	526	660	1,248	1,229	1,197
Other Long Term Assets	143	133	133	113	185
Total Assets	2,708	2,521	3,317	3,408	3,822
Accounts Payable	188	208	216	250	301
Notes Payable/Short Term Debt	0	0	204	181	289
Other Current Liabilities, Total	0	0	16	38	153
Total Current Liabilities	188	208	436	469	743
Long Term Debt	0	0	0	0	0
Other Liabilities, Total	158	180	212	175	222
Total Liabilities	346	388	648	644	965
Common Stock	548	550	1,151	1,244	1,400
Retained Earnings (Accumulated Deficit)	1,944	1,713	1,648	1,843	2,056
Treasury Stock - Common	-130	-130	-130	-323	-599
Total Equity	2,362	2,133	2,669	2,764	2,857
Total Liabilities & Shareholders' Equity	2,708	2,521	3,317	3,408	3,822
Total Common Shares Outstanding	412	415	464	449	439

EXHIBIT 5.16 Tellall financial statements 2012–2016.

Tellall, Inc.	
Input Sector	
Parameters	
Cost of Revenue, Total	0.520
Selling, General and Administrative Expenses	0.144
Research & Development	0.180
Depreciation	0.053
Cash & Equivalents	0.100
Receivables	0.190
Inventory	0.080
Other Current Assets	0.030
Property, Plant & Equipment	0.320
Goodwill & Intangibles	0.620
Other Long Term Assets	0.100
Accounts Payable	0.140
Other Current Liabilities	0.080
Other long term liabilities	0.080
Economy	
Interest Expense	0.060
Interest Earned	0.050
riskfree rate	0.050
market risk premium	0.045
unleveraged beta	1.400
Tax rate	0.320
Long-term GDP growth	0.040

EXHIBIT 5.17 Input module for Tellall, base case.

Your assignment is to

- a. Develop a financial statement simulation model to generate pro forma financial statements (income statement, balance sheet, and a statement of cash flows) that cover the years 2017–2021. It should include the following modules/sectors: accounting (income statement and balance sheet), investment, financing, cash flow, and valuation. For the initial model, assume that the forecast parameters and policy variables are as listed in [Exhibit 5.17](#) apply. In addition, assume that any required external financing will be funded by issuing new debt and that the Notes Payable in 2016 will be retired in 2017. Assume also that the short-term investments in 2016 will be liquidated in 2017 and that no long-term debt will be repaid nor equity will be repurchased

Your model should be constructed so that it balances: assets should equal liabilities (plus shareholders equity) and sources should equal uses of cash. Put checks into your model to verify that it balances. In addition, your model should be constructed in such a way that any equations that refer to exogenous parameters do not contain numbers. The parameters should be in a separate input section, with the two decision/sensitivity variables—*payout* and *debt_portion*—clearly identified.

When the above steps are completed, you will have developed a basic financial statement simulation model. Hopefully, it worked the first time. However, it

is important to make sure that it balances and remains logical under a variety of circumstances. Check the balancing by varying the rate of growth of sales. Use high growth rates such as 30% or 50% per year and see how it performs. Then use low growth rates such as -20% and -30% per year. As before, assets should equal liabilities, and all cash flows should go to some use. You probably will need to modify your model slightly under these extreme scenarios for all of the accounts to remain logical. In particular, you will need to add conditional statements to prevent Net Fixed Assets from becoming negative and to prevent your model from taking a depreciation deduction once net fixed assets reach zero. If your model balances and remains logical under the various extreme growth assumptions, return the five-year growth rate to the base case (10%). Once your model balances, you will be ready to move on to part b.

- b. The focus of this portion of the assignment is upon the valuation module.

The parameters needed for the valuation module are shown in the Economy/Government section of the Input Sector. Use Hamada's (1969) formula to calculate Tellall's leveraged beta, in which the debt-to-equity ratio is the average of the forecast D/E^* ratios for 2017–2021. Calculate Tellall's cost of equity using the Capital Asset Pricing Model. Set the terminal growth rate equal to g^* based on ratios in 2021. Base your forecast for the 2022 dividend on the Cash Available for Dividends in 2021 compounded to 2022. This simple valuation module returns the estimated value for all equity at the end of 2016 (since we are only using debt financing, you need not worry about dilution).

When you are sure your valuation module works well, try a variety of growth rates to see the impact of growth on value. Try growth rates of 0%, 5%, 10%, 20%, and 30%. In each case, explain what happens to the capital structure, dividend payments, and the value of equity.

2. Continuing with Problem 5.1, modify your valuation module to calculate the value of Tellall's equity using the equity cash flow approach and the time-varying market-value-based cost of capital.
3. Continuing with Problem 5.1, modify your valuation module to calculate the value of Tellall's equity using the invested capital approach and the time-varying market-value-based cost of capital.
4. Return to the Medtrac, Inc., problem (#3) in [Chapter 4](#). Give that model the ability to issue and repurchase equity. Once that is done, estimate the value of Medtrac's equity using both the equity cash flow approach and the invested capital approach. Use time-varying market-value-based costs of capital. In the 12/35 scenario, what financing policy (the setting for *debt_portion* and the three cash control variables) maximizes Medtrac's intrinsic value per share? What is the resulting VPS?

* The ratio of interest-bearing debt to equity.

When you change to the 20/20 scenario, you will discover one of the hazards of working with iterative solutions. Equity goes to zero for some years, driving the D/E to infinity. There are several ways to interpret this, but the safest is to assume that the 20/20 model as specified is not feasible. To make it functional, you need to extend the model far enough into the future that the firm reaches maturity.

Planning for Uncertainty

Monte Carlo Simulation

Murphy's Laws:

- Anything that *can* go wrong, *will* go wrong.
- If anything simply cannot go wrong, it will anyway.
- If you perceive that there are four possible ways in which things can go wrong, and you circumvent these, then a fifth way, unprepared for, will promptly go wrong.
- Everything goes wrong all at once.

Murphy was an optimist.

6.1 INTRODUCTION

Uncertainty—knowing with near certainty that something will go wrong—motivates much of financial planning and the financial models on which those plans are based. Although we cannot know what will happen in the future, we can prepare for the most severe eventualities. Building models and developing plans helps minimize the pain and costs of encountering uncertainty in the future. Up to now, we have discussed only how to build simulation models. Those models implicitly suggest certainty. They estimate the value of a firm based on the expected values of the inputs used to drive them. As such, they obscure the uncertainty that lies beneath the expected surface. This chapter considers how to build models that address the uncertain world of business using Monte Carlo simulation.

Uncertainty surrounds nearly all future economic events: prices, sales levels, market shares, costs, lives of assets and projects, interest rates, and even availability of financing. These factors, and more, are important to the success of most business ventures, and most vary over time—we do not know what they will be in the future. Not knowing what will happen in the future, makes it difficult to be certain we are making the right decision now. We want to make the best decision possible given our limited knowledge. In this regard, models can help. Still, all the modeling and planning in the world cannot ensure that we will have made the best decision when viewed in hindsight. We can only hope to arrive at the best decision possible given the information that is available before the fact.

Information is the key. With the best information, properly evaluated, we can improve the likelihood of making the best decision. Modeling and planning provide a way to process and analyze the available information to gain the maximum amount of insight about our uncertain future. Models help organize what we know about our firm, the possible events that could influence it, and its relation to its environment. Planning involves evaluating that information and preparing multiple plans of action as the future rolls toward us.

No matter how thorough our models or how careful our plans, our decisions will have been wrong when we look back on them. Modeling and planning help minimize the cost of being wrong. If we have prepared for our uncertain future, if we have anticipated that the best laid plans—our plans—are bound to go awry, then we will be ready to respond to events that turn out differently than expected. Not only do we need to plan, we need to plan to be wrong.

One way to plan for being wrong is to consider the various events that can possibly occur, assess our situation given these different events, and then develop contingency plans. Modeling adds value to this process by providing an inexpensive mechanism to explore the effects of possible futures on our firm ahead of time.

One use for simulation models is scenario simulation—what is known as *what if analysis* or sensitivity analysis. Scenario simulation involves trying out different scenarios and asking *What if this or that happens?* It allows us to trace through the intricate relations that make up our firm by trying different scenarios one at a time. For example, you could ask what would be the value of Odd & Rich's (O&Rs) equity if sales grow at 3% versus 20%. Having built and tested the model, you need only to input these growth assumptions to view the forecast of the results for each scenario.

This sounds easy. However, suppose your colleague or boss says that many other growth rates are possible besides 3% and 20%, including zero and negative growth, and that we should also plan for those. To placate her, you run simulations for all the growth rates ranging between -5% and $+25\%$ in 0.5% increments; 61 in all. Now you have the output from 61 models to evaluate. The quantity of the output and the difficulty of evaluating each result is overwhelming. You suggest looking only at the more likely outcomes. Your boss agrees, but says that she is not really sure which rate is most likely to occur. In addition, it is not just sales growth that is uncertain—most of the other inputs, such as costs and interest rates, are also uncertain. To be

thorough, she suggests you should look at all the various combinations of growth and expense ratios. You answer that there are so many different scenario combinations that both of you would be overwhelmed and unable to evaluate the results. Besides, most of these combinations are very unlikely. Your boss suggests Monte Carlo simulation.

6.2 MONTE CARLO SIMULATION

Monte Carlo simulation extends what we have called scenario simulation by explicitly taking into account the probabilities of different scenarios, thereby determining the likelihood of different outcomes. In doing so, Monte Carlo simulation provides a richer information set on which to base decisions. Not only does it tell us the worst, most likely, and best case outcomes, tempered by a prudent consideration of the pessimistic and optimistic possibilities for each input, it also tells us the likelihood of these and every other possible outcome. Put differently, Monte Carlo simulation makes invisible uncertainty visible.

To see how this process works, consider a model that has three uncertain inputs—prices, sales, and interest rates—and that each input can assume one of many possible values. A simple scenario approach would assign a specific value to each of these inputs, what we call an event, and trace out the consequences. Monte Carlo is like running a series of scenario simulations, one for each different possible event, taking into account the probability of that event occurring. It does this by assuming that each uncertain input is a random variable with a known probability distribution. The Monte Carlo engine then randomly generates or draws many events, tracing out and tracking the consequences of each one. The end result is a frequency distribution of consequences that approximates the true relative likelihoods of the different outcomes.

The real value of making the random draws from probability distributions and then working the results through the model is that we can combine and explore the consequences of a number of different random variables in the system. If there are many different random variables that affect the outcomes of our decisions, it is extremely difficult to discern how these variables will interact *a priori*. However, by simulating their random behavior, we can begin to understand the combined effects of the uncertainty of the various inputs and estimate the relative likelihood of the consequences.

To show how Monte Carlo simulation works, we begin with a simple example that moves from scenario simulation to Monte Carlo simulation. Suppose a company currently has sales of \$1,000 and expects them to grow 6% in the coming year. Operating costs are expected to be 80% of sales. If so, next year's expected EBITDA will be $\$1,000 \times (1.06) \times (1 - 0.80) = \212 . Suppose, too, that the company recognizes that its forecasts are not entirely accurate. Instead of a single estimate for EBITDA, they want a range of possible values. From past experience, they believe that sales will not grow slower than 3%, nor faster than 9%. They also believe that operating expenses will never exceed 85% of sales, nor be lower than 75% of sales. Given the minimum, expected and maximum values for each variable, nine different scenarios are possible. These are shown in [Exhibit 6.1](#). EBITDA ranges from \$155 in the

Operating Expense	Sales Growth		
	3%	3%	9%
75%	\$258	\$265	\$273
80%	\$206	\$212	\$218
85%	\$155	\$159	\$164

EXHIBIT 6.1 EBITDA with various growth and cost scenarios.

low growth, high expense scenario, to \$273 in the high growth, low expense scenario, with the expected outcome in the middle of the range at \$212.

Scenario analysis gives us an idea of the different outcomes, but it is silent with respect to the likelihoods of those different outcomes. Monte Carlo simulation breaks that silence and much more. It simulates the distribution of next year's EBITDA. In Monte Carlo simulation, input variables are drawn randomly from their own probability distributions. The result of each set of draws is run through the spreadsheet model, producing a different result for each set. After many sets of random draws we have many different results, allowing us to compute the relative frequency of the results and determine relative probabilities of those outcomes. For example, suppose we make one random draw from the distributions for growth and expense. Suppose that, by chance, the growth rate draw is -2% and the expense ratio draw is 90% . Then the resulting EBITDA forecast is \$98. This is just one out of many possible outcomes that might occur. Certainly, it would be regarded as a disastrous outcome, but then we need to recognize that negative growth and very high expenses are seldom likely to occur together. If many draws are made, realizing an EBITDA, this low will not happen often. In this way, Monte Carlo simulation allows us to consider the frequency of occurrences.

The crucial inputs for Monte Carlo simulations are the probability distributions of the relevant input random variables. In this example, the rate of sales growth and the proportional operating costs are the random variables. Suppose that after examining past data we conclude that both sales growth and proportional operating costs follow the normal distribution, with growth showing a mean of 6% and standard deviation of 3% per year, and operating costs showing a mean of 80% and standard deviation of 5% of sales. Graphs of these frequency distributions are shown in [Exhibit 6.2a](#) and [b](#).

Armed with these distributions, Monte Carlo simulation produces a simulated distribution for EBITDA. It does so by making draws from each of the two input distributions, calculating an EBITDA, and then repeating these two steps many times. The result is a histogram ([Exhibit 6.3](#)) that shows the relative frequency of the EBITDA outcomes. In this simulation, EBITDA ranged from a low of about 40 to a high of about 410. The mean of the distribution was 212, the standard deviation was 53.4.

This is much richer information than what we got from scenario simulation. The histogram gives us a good idea of the likelihood of the different possible EBITDA outcomes because it reflects the uncertainty of the growth rate and operating costs. Although scenario simulation gave us an idea of the range of outcomes, it told us nothing about the likelihoods of those outcomes. Nor did scenario simulation capture the full range of outcomes, because it

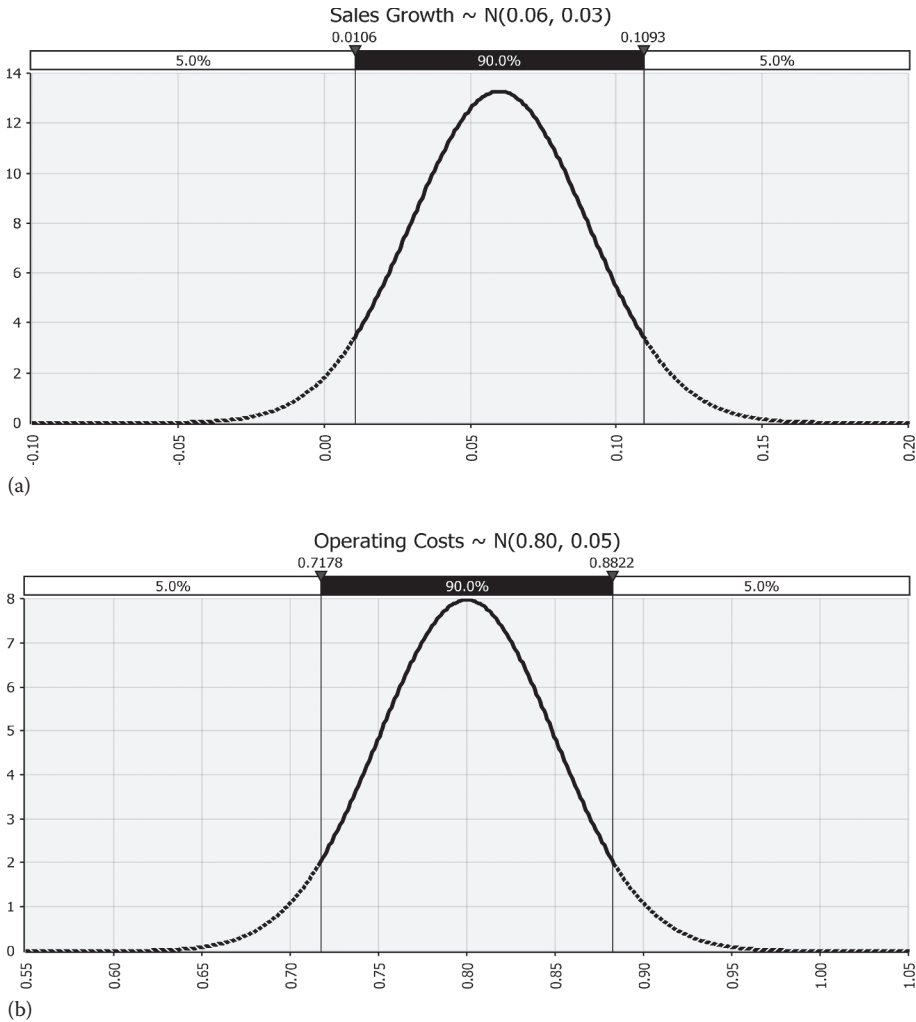


EXHIBIT 6.2 (a) Probability distribution of sales growth. (b) Probability distribution of percent operating costs.

considered only three possible values each for growth and cost, and these values were not at the extreme ends of their ranges. Monte Carlo simulation gives us more information about what might happen. [Exhibit 6.4](#) summarizes the differences between scenario and Monte Carlo simulation.

In the next section, we introduce @RISK, the software from Palisade Corp. that we use to add Monte Carlo simulation to our models.*

* @RISK is available from Palisade Corporation (www.palisade.com). There are two competing Monte Carlo software packages that are very similar, @RISK and Crystal Ball. Both are add-ins for Excel. Each one has some advantages and weaknesses, although we tend to have a slight preference for @RISK. That is why we use it with this text. Generally, both packages are useful and easy to use. Crystal Ball is available from the Oracle Corp. at (www.crystalball.com).

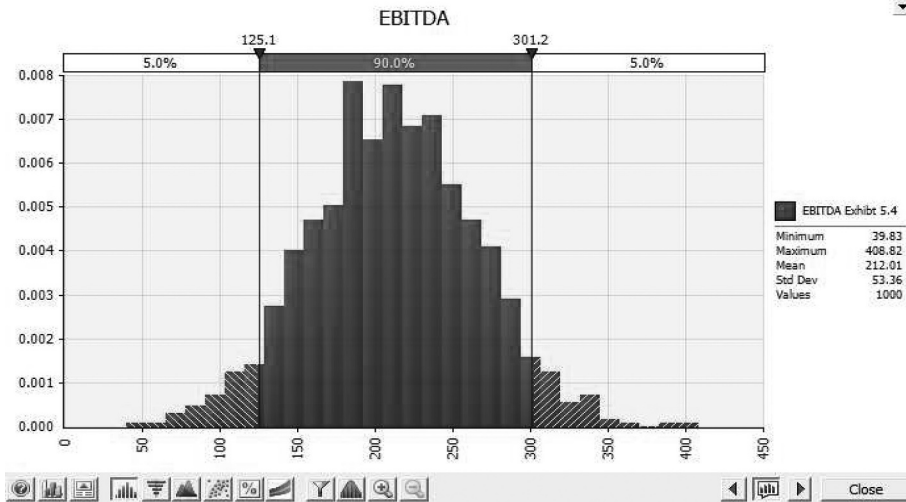


EXHIBIT 6.3 Simulated frequency distribution of EBITDA.

Scenario Simulation		Monte Carlo Simulation																			
3%, 6% or 9%	Possible growth rates																				
75%, 80% or 85%	Possible operating costs as a percent of sales																				
<table border="1"> <thead> <tr> <th rowspan="2">Operating Expense</th> <th colspan="3">Sales Growth</th> </tr> <tr> <th>3%</th> <th>3%</th> <th>9%</th> </tr> </thead> <tbody> <tr> <td>75%</td> <td>\$258</td> <td>\$265</td> <td>\$273</td> </tr> <tr> <td>80%</td> <td>\$206</td> <td>\$212</td> <td>\$218</td> </tr> <tr> <td>85%</td> <td>\$155</td> <td>\$159</td> <td>\$164</td> </tr> </tbody> </table>	Operating Expense	Sales Growth			3%	3%	9%	75%	\$258	\$265	\$273	80%	\$206	\$212	\$218	85%	\$155	\$159	\$164	Possible EBITDA outcomes	
Operating Expense		Sales Growth																			
	3%	3%	9%																		
75%	\$258	\$265	\$273																		
80%	\$206	\$212	\$218																		
85%	\$155	\$159	\$164																		
n.a.	$P[\text{EBITDA} \leq 150] = ?$	 $P[\text{EBITDA} \leq 150] = 12.3\%$																			

EXHIBIT 6.4 Comparing scenario simulation with Monte Carlo simulation.

6.3 MONTE CARLO SOFTWARE: @RISK

This text uses @RISK for Monte Carlo simulation. Access to a free textbook version is provided with this text.* We begin with a quick overview of the software. Readers desiring a more detailed explanation should consult the extensive @RISK online tutorials (@RISK ribbon > Help > Video) and literature. The books by Wayne Winston (1996, 2000, 2001) are particularly helpful.

@RISK integrates seamlessly with Excel, giving it Monte Carlo simulation capability through functions that are the probability distributions from which we make our random draws. @RISK even has its own ribbon ([Exhibit 6.5](#)). The ribbon provides easy access to the icons that call up @RISK functions. The crucial buttons for getting started are the Define Distributions, Add Output, Simulation Settings, and Start Simulation.

The essence of @RISK is a portfolio of probability distribution functions with which to describe and draw random variables. As this is also the area of @RISK with the greatest freedom, we take a bit of time to describe some of the multiple ways to enter these functions into Excel. These functions can be entered into a cell via an @RISK point-and-click dialog box, or they can be entered much like any other Excel function. Most of our discussion focuses on entering the distribution functions manually. A short list of the common functions includes the following:

- Normal distribution =RiskNormal(mean, standard deviation)
- PERT =RiskPert(minimum, most likely, maximum)
- Uniform =RiskUniform(minimum, maximum)
- Triangular =RiskTriang(minimum, most likely, maximum)
- Binomial =RiskBinomial(draws, probability of success)
- Bernoulli =RiskBernoulli(probability of success)
- Lognormal =RiskLognorm(mean, standard deviation)

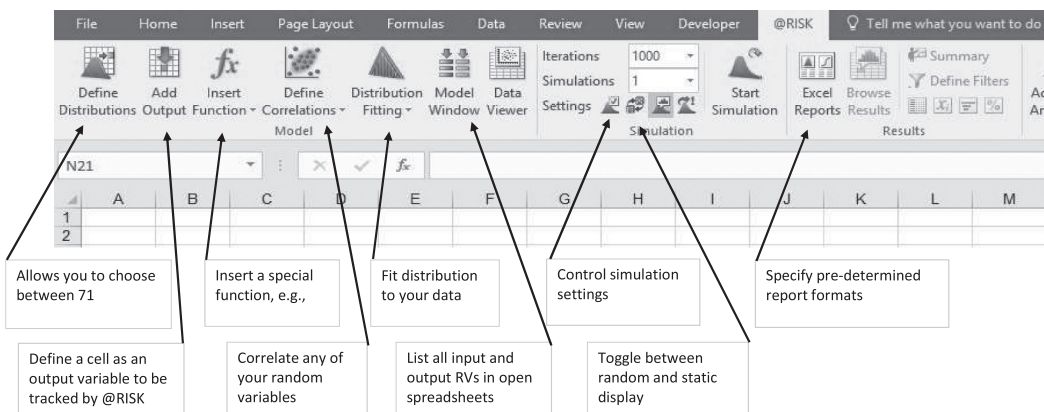


EXHIBIT 6.5 @RISK version 7.5 ribbon in Excel 2016.

* The link to the free textbook version and download instructions can be found at the beginning of this book.

	A	B	C	D	E	F	G	H	I	J
1										
2		Alternative Ways to Enter Parameters in @ Risk Functions								
3										
4	A	Parameters Entered Directly Into Distribution Function								
5							<code>=RiskNormal(212,53)</code>			
6			EBITDA		212.0					
7										
8	B	Parameters Specified in Other Cells								
9										
10			Mean		212.0					
11			Standard Deviation		53.0					
12							<code>=RiskNormal(E10,E11)</code>			
13			EBITDA		212.0					
14										
15	C	Parameters Computed Elsewhere & Random Parameters								
16										
17			Price		10.0		<code>=200 - 10*E17 + RiskUniform(-5,7)</code>			
18			Quantity Sold		101.0		<code>=RiskNormal(E17*E18,0.05*(E17*E18))</code>			
19			Sales		1,010.0		<code>=RiskTriang(0.75,0.8,0.9)</code>			
20			Operating Costs %		81.7%					
21										
22			EBITDA		185.2					
23										

EXHIBIT 6.6 Parameter entry in @RISK distribution functions.

Even though the @RISK library includes 71 distributions, we rarely use more than these seven functions.

Entering an @RISK distribution function in a cell in your spreadsheet model defines that cell as a random variable to be drawn from the specified distribution. There are various ways to insert parameters into the functions. Exhibit 6.6 shows several of them. Parameters can be entered directly as numbers (Example A), as cell references (Example B), or even as functions of cell references and numbers (Example C).

Cell E6 in Section A shows EBITDA being calculated with the mean and standard deviations specified directly in the =RiskNormal function. In Section B, the mean and standard deviation in the =RiskNormal function reference cells E10 and E11. Section C shows a more complicated entry for the parameters. The price is set as a constant in E17. The quantity in E18 is based on a demand function with a random error drawn from a uniform distribution ranging from -5 units to +7 units. Sales in E19 are a normal random variable with a mean that is the product of price (E17) and the random quantity (E18). The standard deviation is also the product of price and quantity. This makes both the mean and standard deviation of sales as random parameters. Operating Costs % (E20) is drawn from a triangular distribution. The result is that EBITDA is random variable based on draws from a uniform, a normal, and a triangle distribution, where some of the parameters of the distributions are themselves random variables. This shows the flexibility available for portraying the randomness in @RISK.

Another way to enter a distribution function in your model is to use @RISK’s graphic *Pop Up Window* accessed via the Define Distribution icon at the far left of the @RISK

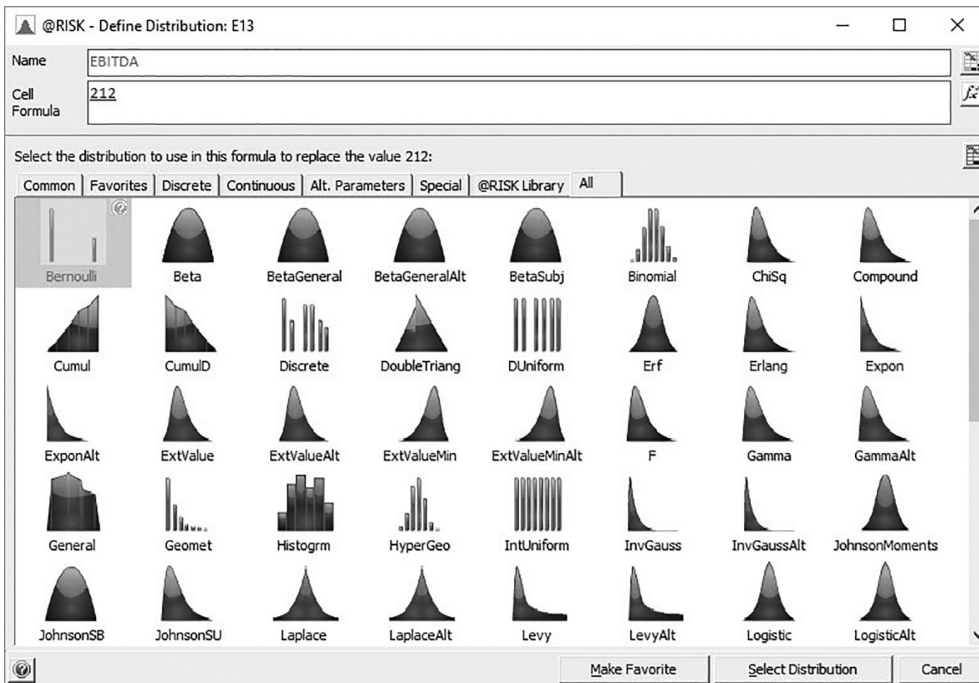


EXHIBIT 6.7 Define Distributions dialog box.

ribbon. This graphic approach allows you to see a graph of the distribution and set the parameters in a dialog box. [Exhibit 6.7](#) shows the Define Distribution dialog box that opens when you click the Define Distributions icon. This allows us to define a distribution for cell E13 because that was the highlighted cell when we clicked the Define Distributions icon. Click on the picture of the distribution you want to use, then click on the Select Distribution button at the bottom. This brings up the Define Distribution dialog box shown in [Exhibit 6.8](#).

The right panel in [Exhibit 6.8](#) shows a picture of the distribution; the left side shows the parameters entered to describe that distribution (indicated by the cursor inside the circle). The mean box (μ) references the model cell where the mean is located (E10), and the standard deviation box (σ) shows where its parameter is located (E11). You can change the entries in this section, including the type of distribution, the parameters (mean and standard deviation), and make other adjustments such as truncating the distribution. For example, you can enter parameters directly in the parameter box (i.e., as the number [212]), change the cell reference, or even enter a formula that would compute the parameter. If you want to change the type of distribution, say from normal to uniform, you click the Function arrow to see the list distributions from which you can choose. After entering the parameters, click the OK button and the distribution formula is entered into the cell.

In the next section, we introduce the remaining pieces of @RISK that you need to know to run a simulation by converting the EBITDA scenarios simulation model to a Monte Carlo simulation model, running a simulation, and analyzing the output.

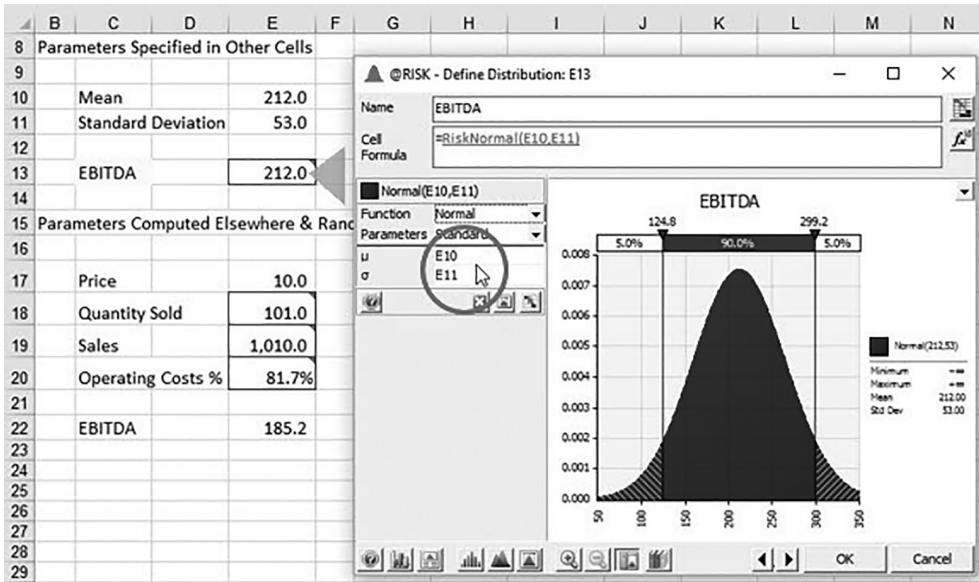


EXHIBIT 6.8 Defining a specific distribution in @RISK.

6.4 THE EBITDA MODEL WITH @RISK

Our simple EBITDA problem introduced you to the power of Monte Carlo simulation and hinted at the richness of its output. We now show how @RISK was used to convert the scenario simulation model to Monte Carlo simulation. Exhibit 6.9 shows the simple EBITDA model converted to an @RISK Monte Carlo simulation model. The input to the model consists of the initial (period 0) sales level of \$1,000 (cell E14), the initial operating costs (E15), the sales growth rate (G9), and percent operating costs (G10).

	A	B	C	D	E	F	G	H	I	J	K
1											
2		Simulation of EBITDA									
3		With Random Sales Growth and Operating Costs									
4											
5		Probability Assumptions									
6											
7											
8											
9											
10											
11											
12											
13											
14											
15											
17											
18											

The table shows the simulation of EBITDA with random sales growth and operating costs. The probability assumptions table is as follows:

	Distribution Type	Parameters	Actual or Expected Value
		Mean	Standard Deviation
Sales Growth %	Normal	6%	3%
Operating Costs %	Normal	80%	5%

The simulation results table is as follows:

Period	0	1
Sales	1,000	1060
Operating Costs	800	848
EBITDA	200	212

The formulas used in the simulation are:

- `=RiskNormal(E9,F9)` for Sales Growth %
- `=RiskNormal(E10,F10)` for Operating Costs %
- `=E14*(1+G9)` for Sales at Period 1
- `=F14*G10` for Operating Costs at Period 1
- `=RiskOutput("EBITDA")+ F14-F15` for EBITDA at Period 1

EXHIBIT 6.9 EBITDA model with Monte Carlo simulation.

To reprise, we assume that the values of the two input variables in this model—sales growth and operating costs in cells G9 and G10—are normally distributed random variables. These cells contain @RISK formulas that define the input variables as random variables, with values to be drawn from normal probability distributions. The code for the @RISK normal probability function is

$$=RiskNormal(\textit{mean}, \textit{standard deviation}) \tag{6.1}$$

The formula in G9 is =RiskNormal(E9,F9), where cells E9 and F9 contain our estimates for the mean and standard deviation of the growth rate. The formula for operating costs is G10 =RiskNormal(E10,F10), where E10 and F10 are the *mean and standard deviation* of the percentage operating costs.

6.4.1 Adding an Output Variable

The next step is to define F17 (the cell that calculates EBITDA) as an output variable so that @RISK knows which result to track during the simulation. To do this, make F17 the active cell, and then click on the Add Output icon on the @RISK ribbon. This brings up the dialog shown in Exhibit 6.10. @RISK opens with dialog box with a default name based on the first text entries to the right of and above of the cell being defined. You have the option to accept this name (click OK) or changing it by entering a name of your choice. We chose EBITDA. Click OK and @RISK inserts “=RiskOutput(“name”) +” in front of any other code in the cell (Exhibit 6.9, cell F17 code).

6.4.2 Simulation Settings

One step remains before running a simulation—checking the simulation settings. This is done via the simulation panel on the @RISK ribbon (Exhibit 6.11). The simulation panel provides quick access to four very useful functions. First, set the number of iterations in the *Iterations*

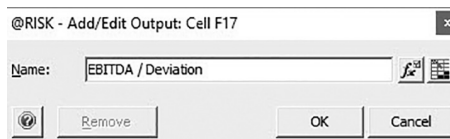


EXHIBIT 6.10 Adding and output variable.

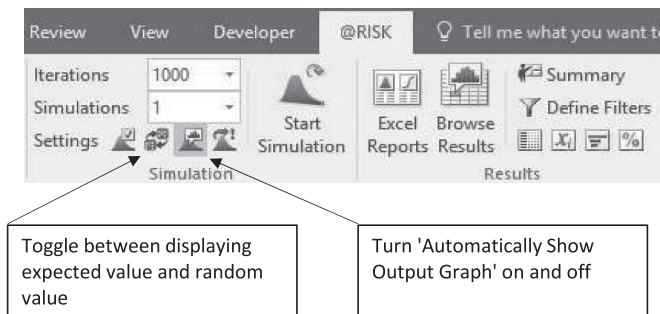


EXHIBIT 6.11 The simulation panel on the @RISK ribbon.

window, either by entering the number you want or using the dropdown menu. Next, the dice icon toggles your worksheet between showing the expected values of the random variables in your model or a number drawn from their distributions. This is a quick way to see if the randomness that you think you added to the model is in fact the randomness that you did add to the model. As a further check, hit F9 several times to force Excel to recalculate your model and watch the values of your random variables change. The icon directly to the right of the dice turns the *Automatically Show Output Graph* on and off. When this icon is on, the histogram of the active cell opens automatically upon the completion of the run. *Start Simulation* does just that.

6.4.3 Running a Simulation

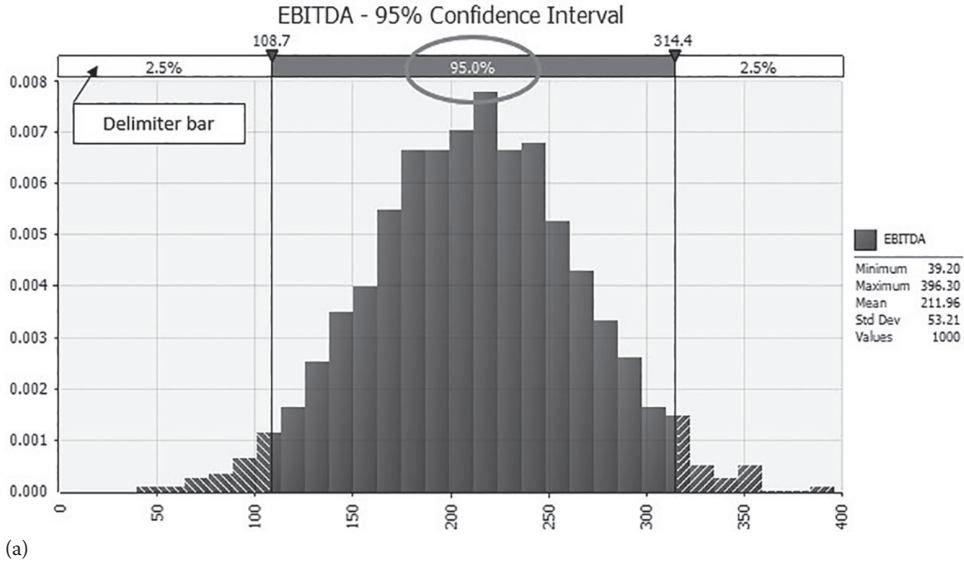
Set the number of iterations to 1,000, start the simulation, and watch @RISK go to work. Here is what @RISK is doing. On the first iteration of the simulation, a single random draw is made from each of the two specified distributions in the model. These random values are inserted into cells G9 and G10 and the results fed through the model to produce EBITDA (F17). The three values—growth, costs, and EBITDA—are saved, completing one random draw or iteration. Then another random draw is made producing another EBITDA, and those three values are saved. You get as many different EBITDAs as the number of iterations you specify. Since we set the number of iterations to 1,000, @RISK made 1,000 draws from the distributions and calculated 1,000 EBITDAs. Values that are unlikely are drawn less frequently; values that are more likely are drawn more frequently. The end result is shown as a frequency distribution. In this case, we have 1,000 different EBITDA values, with most occurrences in the middle, clustered around the mean or expected value, and relatively few occurrences at the extremes. [Exhibit 6.12a](#) reports the result of another simulation. As expected, it is slightly different from [Exhibit 6.3](#). If @RISK does not automatically open the frequency distribution, make the output cell as the active cell and click the *Browse Results* icon to bring it up.

6.4.4 Analyzing the Results

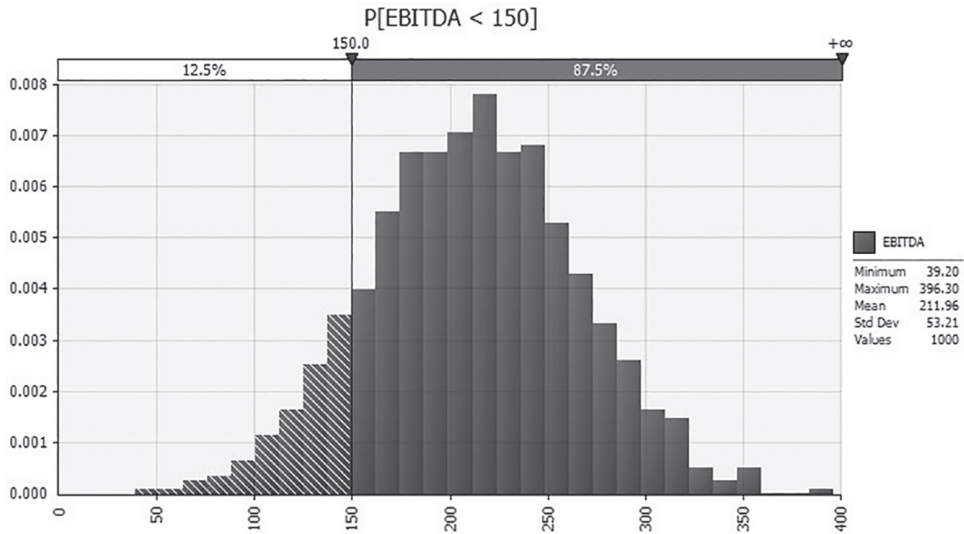
Monte Carlo simulation is not a magic bullet. Knowing the shape of the distribution of EBITDA or any other output variable does not magically solve a decision problem. However, it does provide insight into what might happen. With this additional information, we can make more informed decisions because we have a better idea of which outcomes are likely and which are unlikely. In this example, we learn more about the range of EBITDA and how likely it is to be at any given level. Such information gives us some confidence about what might happen.

One way to measure that confidence is with confidence intervals. The typical range for a confidence interval is 95%. A 95% confidence interval gives us a lower limit and an upper limit between which a value should fall 95% of the time. @RISK calculates this interval for us. All we need to do is enter 95 in middle portion of the delimiter bar and read the limits above each delimiter ([Exhibit 6.12a](#)). For this run, the 95% confidence interval is [108.7, 314.4].*

* If we know the details about the distribution of outcomes, we can determine the likelihood of the outcomes falling within a specified range with simple statistical theory. We know that 95% of the time, a normally distributed random variable will occur within the interval $mean \pm 1.96\sigma$. This makes the 95% confidence limits for EBITDA $212.0 \pm 1.96 \times 53.2 = 212.0 \pm 104.3$; hence, the lower limit equals $212.0 - 104.3 = 107.7$, and the upper limit equals $212.0 + 104.3 = 316.3$. These calculations tell us that there is a 95% chance that EBITDA will fall between 107.3 and 316.7. These are not the same as shown in [Exhibit 6.13](#) because that distribution is not perfectly normally distributed.



(a)



(b)

@RISK - Results Summary

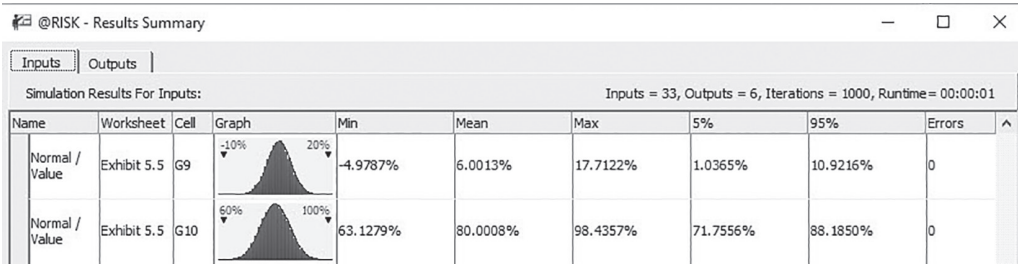
Inputs Outputs

Simulation Results For Outputs: Inputs = 33, Outputs = 6, Iterations = 1000, Runtime = 00:00:01

Name	Worksheet	Cell	Graph	Min	Mean	Max	5%	95%	Errors
EBITDA	Exhibit 5.5	F17		16.61753	212.0322	406.5274	125.4278	300.5811	0

(c)

EXHIBIT 6.12 (a) Find the 95% confidence interval with @RISK. (b) Measuring the probability of EBITDA being less than 150. (c) Summary output statistics from simulation run. (Continued)



(d)

EXHIBIT 6.12 (Continued) (d) Summary input statistics from simulation run.

We can also use this information to evaluate our ability to meet our obligations. Suppose that next year we will have to make a debt payment of \$150 using funds from earnings (EBITDA). What is the likelihood that we will not be able to make the payment? To answer this question, drag the right delimiter bar to the right-hand side of the histogram, enter 150 in the number box above the left delimiter, and then read the result in the left-hand panel of the delimiter bar (Exhibit 6.12b). In this case, there is a 12.5% likelihood that we will be unable to make our debt payment.* Knowing there is a 12.5% chance we will be unable to service our debt and helps us decide if we want to accept that risk.

6.4.5 Detailed Results

You can see more results by clicking the *Summary* icon on the @RISK ribbon. This brings up displays like those shown in Exhibit 6.12c and d. The Summary window reports the summary statistics for the input and output variables for the simulation: the minimum, mean, maximum, and the upper and lower limits of the 90% confidence interval.

For still more detail, click the small Simulated Detailed Statistics icon on the *Results* panel under Summary. This brings up the output shown in Exhibit 6.13. In addition to the material shown in the Summary Statistics window, the Detailed Statistics window shows standard deviation, skewness, kurtosis, and the variable values at percentile points in 5% intervals.

6.4.6 Putting the Output into a Spreadsheet

Although all this output is interesting in its own right, its sheer quantity can be overwhelming and not always germane to the problem under consideration. Drilling down through the various menus also can be confusing. Sometimes, too, a permanent record of the output for each simulation will be useful. @RISK addresses these issues with its Excel Reports option. Excel Reports allows the user to place @RISK output into a spreadsheet

* Assuming EBITDA is normally distributed, we can calculate the probability that EBITDA will be less than the required 150 debt payment. The debt payment of 150 is 1.16 standard deviations below the EBITDA mean, thus

$$\frac{(150 - \text{mean})}{\sigma} = \frac{(150 - 212)}{53.2} = -1.16$$

Using a standard normal probability table, we determine that the probability of a standard normal variable, x , occurring 1.16σ below the mean is

$$\Pr [x \leq (\text{mean} - 1.16\sigma)] = 12.3\%$$

@RISK - Detailed Statistics												
Name	EBITDA Exhibit 5.4		EBITDA		NPV		IRR		NPV		Normal / Value	
Description	Output	Exhibits 5.2 - 5.4...	Output	Exhibits 5.2 - 5.4...	Output	Exh 5.22IE71	Output	Exh 5.22IE72	Output	Exh 5.24IB23	Output	RiskNormal(F9,G9)
Cell	Exhibits 5.2 - 5.4...	Exhibits 5.2 - 5.4...	Exh 5.22IE71	Exh 5.22IE72	Exh 5.24IB23	Exh 5.24IB23	Exh 5.24IB23	Exh 5.24IB23	Exh 5.24IB23	Exh 5.24IB23	Exh 5.24IB23	Exhibits 5.2 - 5.4...
Minimum	45.50762	16.61753	2415.843	17.2455%	26445.97	-38492.95	26445.97	17.2455%	26445.97	-38492.95	0.6385526	-4.9787%
Maximum	374.8563	406.5274	6327.973	28.2246%	26445.97	56378.2	26445.97	28.2246%	26445.97	56378.2	0.9550779	17.7122%
Mean	2111.9779	212.0322	4544.451	23.2094%	26445.97	18854.34	26445.97	23.2094%	26445.97	18854.34	0.7999841	6.0013%
Std Deviation	53.22467	53.66613	629.614	1.7799%	0	18854.34	0	1.7799%	0	18854.34	0.0495603	3.0098%
Variance	2832.866	2880.054	396413.8	3.167902E-04	0	3.554862E+08	0	3.167902E-04	0	3.554862E+08	2.496031E-03	9.058741E-04
Skewness	4.206439E-02	2.530186E-02	-0.1138948	-9.517629E-02	n/a	0.8043426	n/a	-9.517629E-02	n/a	0.8043426	-8.211768E-03	1.259751E-02
Kurtosis	2.979867	3.035459	2.993472	2.895653	n/a	3.393517	n/a	2.895653	n/a	3.393517	2.954929	3.108011
Errors	0	0	0	0	0	0	0	0	0	0	0	0
Mode	180.459	186.3262	4594.946	23.1897%	26445.97	-8377.247	26445.97	23.1897%	26445.97	-8377.247	0.7866851	5.5842%
5% Perc	126.8878	125.4278	3487.599	20.2140%	26445.97	-32426.87	26445.97	20.2140%	26445.97	-32426.87	0.717512	1.0365%
10% Perc	143.3689	142.9552	3716.638	20.9157%	26445.97	-29962.2	26445.97	20.9157%	26445.97	-29962.2	0.7358376	2.1411%
15% Perc	155.8391	156.201	3880.937	21.3365%	26445.97	-27640.72	26445.97	21.3365%	26445.97	-27640.72	0.7480875	2.8892%
20% Perc	167.6834	165.9933	4028.637	21.7272%	26445.97	-9596.729	26445.97	21.7272%	26445.97	-9596.729	0.7578807	3.4743%
25% Perc	176.2647	176.1535	4152.554	22.0459%	26445.97	-9280.296	26445.97	22.0459%	26445.97	-9280.296	0.7661897	3.9726%
30% Perc	183.5138	183.9354	4221.647	22.2804%	26445.97	-9047.837	26445.97	22.2804%	26445.97	-9047.837	0.7736863	4.4198%
35% Perc	190.767	191.4632	4322.471	22.5485%	26445.97	-8866.632	26445.97	22.5485%	26445.97	-8866.632	0.7806838	4.8435%
40% Perc	198.5454	197.5496	4406.489	22.8380%	26445.97	-8716.845	26445.97	22.8380%	26445.97	-8716.845	0.7872299	5.2348%
45% Perc	204.8005	204.2437	4489.429	23.0423%	26445.97	-8598.241	26445.97	23.0423%	26445.97	-8598.241	0.7936813	5.6158%
50% Perc	211.3162	211.2457	4561.695	23.1960%	26445.97	-8488.671	26445.97	23.1960%	26445.97	-8488.671	0.7998961	5.9979%

EXHIBIT 6.13 Detailed statistics output.

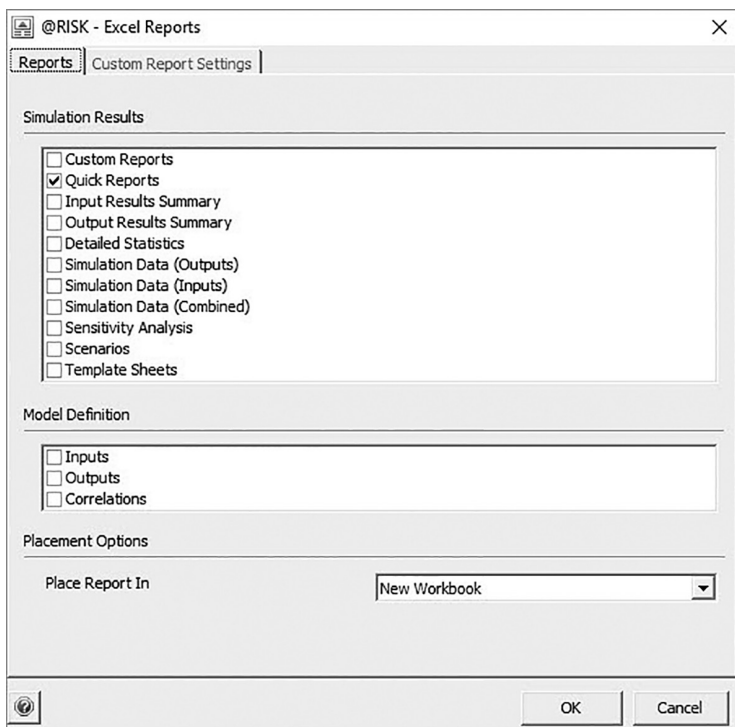


EXHIBIT 6.14 @RISK Reports dialog box.

so it can be printed or manipulated in the spreadsheet. You access this option through the Excel Reports icon that brings up the Excel Reports dialog box shown in [Exhibit 6.14](#). Checking an option such as *Quick Reports* and clicking OK will place the report in a new spreadsheet. The option that you select determines which output report is generated and inserted into Excel. Quick Reports was the option that generated the Excel output shown in [Exhibit 6.15](#). It is formatted to fit perfectly on one page in a Word document.

6.4.7 Correlating Variables

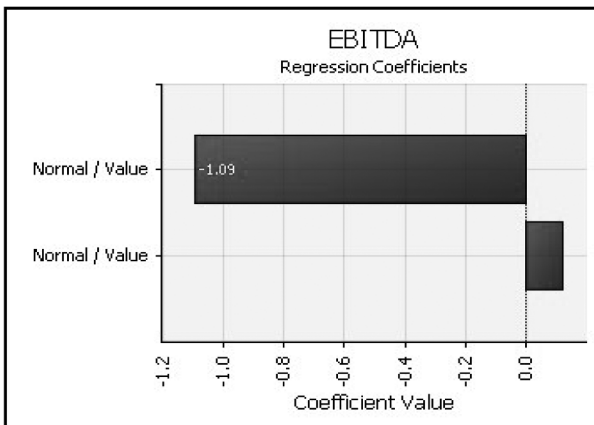
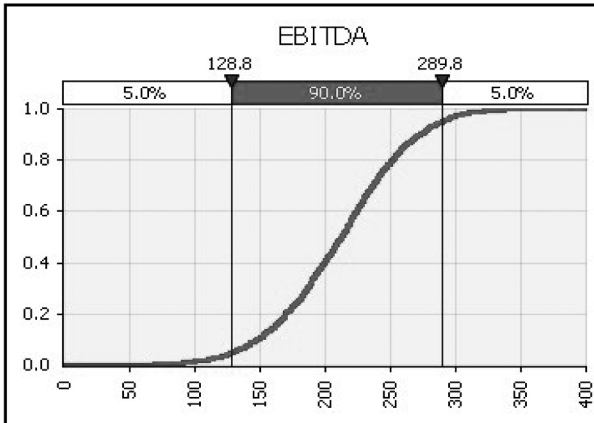
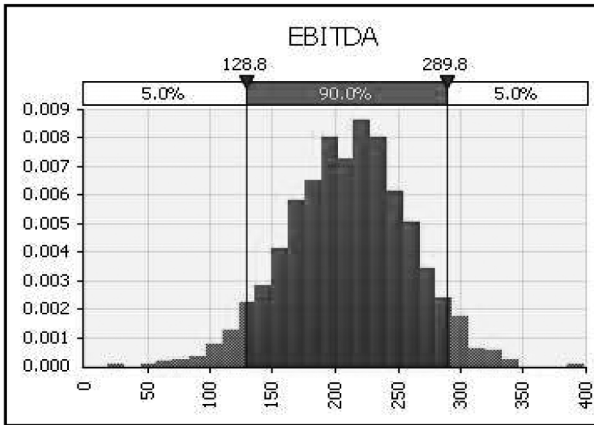
The two random variables in the EBITDA model, growth of sales and operating cost, were drawn independently—operating cost was drawn from its normal distribution without regard for the value of growth that was drawn. Sometimes it will be more realistic to assume that these variables are correlated. For example, suppose we expect that sales will grow more rapidly when the economy is doing well. We also expect that our costs will tend to be higher during periods of prosperity. This induces a positive relationship between the two input variables. We build this kind of linkage into our model by correlating the random variables, in this case, correlating the draws for sales growth and operating costs.

We measure correlation with the correlation coefficient. The correlation coefficient is an index number between -1 and $+1$: $+1$ means perfect positive correlation and -1 means perfect negative correlation. All the other values between -1 and $+1$ represent the different degrees of imperfect correlation. Positive correlation (between 0 and $+1$) means that two variables tend to move with each other, or, more accurately, when one variable is drawn randomly from the

@RISK Output Report for EBITDA

Performed By:

Date:



Simulation Summary Information

Workbook Name	EBITDA Monte Carlo.xls
Number of Simulations	1
Number of Iterations	1000
Number of Inputs	7
Number of Outputs	1
Sampling Type	Latin Hypercube
Simulation Start Time	7/13/08 10:40:07
Simulation Duration	00:00:04
Random # Generator	Mersenne Twister
Random Seed	719557342

Summary Statistics for EBITDA

Statistics	Percentile
Minimum	5% 128.780104
Maximum	10% 148.003994
Mean	15% 161.453483
Std Dev	20% 169.637535
Variance	25% 179.225809
Skewness	30% 186.184097
Kurtosis	35% 192.269853
Median	40% 199.712813
Mode	45% 205.958682
Left X	50% 213.000099
Left P	55% 218.514097
Right X	60% 223.950236
Right P	65% 229.751709
Diff X	70% 236.389106
Diff P	75% 243.499403
#Errors	80% 251.830619
Filter Min	85% 259.869926
Filter Max	90% 271.372363
#Filtered	95% 289.778763

Regression and Rank Information for EBITDA

Rank	Name	Regr	Corr
1	Normal / Valu	-1.095	-0.997
2	Normal / Valu	0.124	-0.741

EXHIBIT 6.15 Quick Report for EBITDA.

upper end of its probability distribution, the other variable will also tend to be drawn from the upper end of its distribution. Negative correlation (between -1 and 0) is when the variables tend to move in opposite directions, so that when one is high, the other tends to be low.

@RISK allows the user to specify the correlation coefficients between pairs of random variables using the *Define Correlations* dialog box. To see how this is done in the EBITDA example, assume that the estimated correlation between sales growth and the operating cost ratio is $+0.80$. Growth and costs are highly positively correlated, but not perfectly so. Most of the time when growth is high, the cost ratio will also be high. However, with imperfect correlation, occasionally growth and costs will move in the opposite directions.

To correlate two or more random variables, highlight the cells containing them (G9 and G10 in this example), then click the *Define Correlations* icon. The correlation window that opens is shown in [Exhibit 6.16](#). When the window first opens it shows a correlation matrix with 1s on the diagonal and 0s off the diagonal. Your dialog box will look like that shown in [Exhibit 6.16](#). Click in the lower left-hand cell and enter 0.80 (see cursor in the circle). Next, name the matrix and select where you want to place it in your worksheet ([Exhibit 6.17](#)).

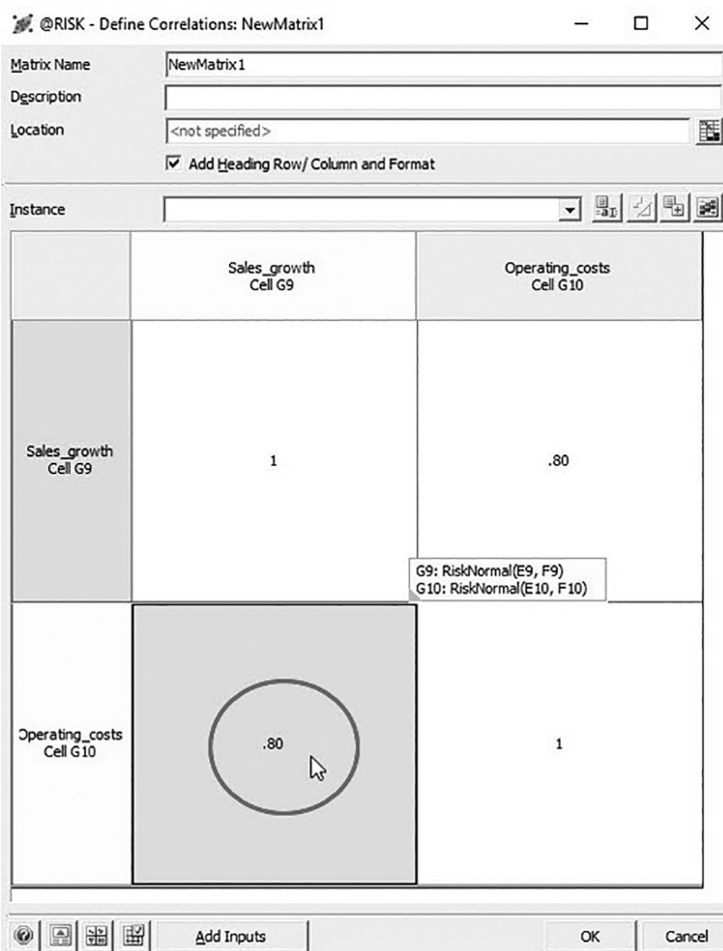


EXHIBIT 6.16 Define Correlations dialog box.

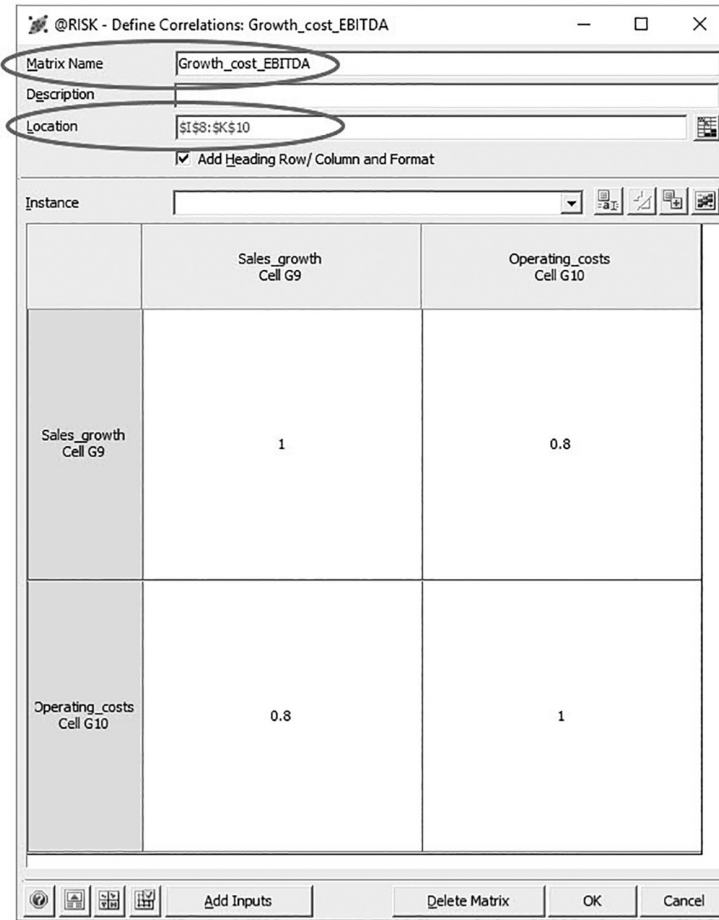


EXHIBIT 6.17 Completed Define Correlations dialog box.

Click OK. @RISK then inserts a matrix of correlation coefficients into the spreadsheet and modifies the probability distribution formulas for the correlated variables.

Exhibit 6.18 shows the end result. The correlation matrix shown in Exhibit 6.17 has been placed in cells I8:K10, as specified. The probability distribution functions have been modified by adding RiskCorrmat(.) functions. This can be seen for Sales growth in the *Formulas* bar. The RiskCorrmat function specifies the correlations between the variables by referencing the correlation matrix that is put into your model.

6.4.7.1 Model Results with Correlated Variables

We now consider the difference that adding correlation made. Exhibit 6.19 shows the EBITDA distributions without (bars) and with (solid line) correlation between sales growth and operating costs. When sales growth and operating costs are positively correlated, more observations lie closer to the mean. This is reflected by the standard deviation of 48.4 for the correlated model that is about 10% smaller than the 53.4 for the uncorrelated model.

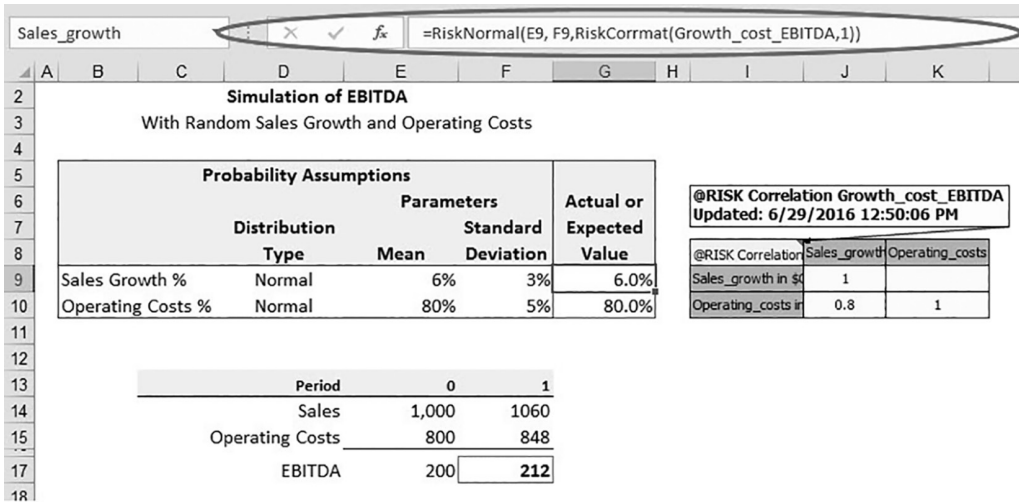


EXHIBIT 6.18 EBITDA model with correlation added.

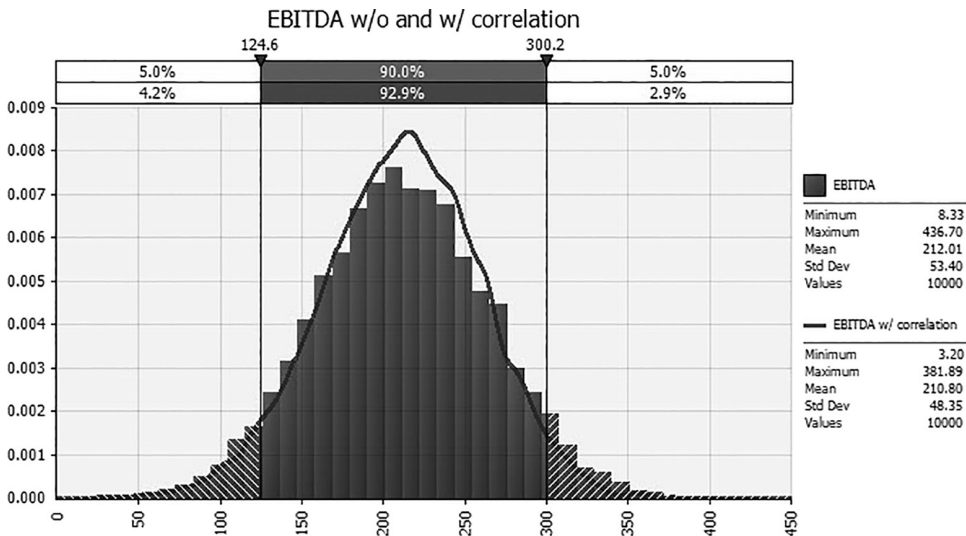


EXHIBIT 6.19 EBITDA histograms for correlated and uncorrelated models.

In addition, the mean of the correlated distribution is smaller, 210.8 versus 212.0. Finally, the range of the correlated distribution is narrower, 378.7 versus 428.4.

The decreased uncertainty is because when sales are low, costs also tend to be low, so earnings do not decline as much as they might otherwise. If the correlation was negative, we would get the opposite result—increased uncertainty—because when sales are low, costs tend to be high, so earnings will be even lower than otherwise, and vice versa.

This brief introduction to @RISK should be enough for you to begin running Monte Carlo simulations of your own creation. There are many additional options and features in @RISK that we have not discussed. Consult the @RISK documentation to learn more

and use it to its full capability.* The next section adds Monte Carlo simulation to the Odd & Rich model from [Chapter 4](#) and looks at several additional @RISK tools.

6.5 USING MONTE CARLO SIMULATION TO EVALUATE A COST-MANAGEMENT PROPOSAL

@RISK's power to visualize uncertainty is matched by its ease of use. To demonstrate both, we use it to evaluate a cost-management program recently submitted to Odd & Rich. Suppose that O&R has been approached by an entrepreneur who has developed a raw materials cost management software program that uses sophisticated financial derivatives to reduce raw material price volatility. It is called Risk Away. She claims her program reduces the average cost of raw materials by more than 7%. As most of O&R's production costs stem from the purchase of raw materials, management is interested. They have asked you to evaluate Risk Away's claims. In particular, management wants to know if Risk Away will add value to O & R's equity.

6.5.1 Establishing the Benchmark

The first step is determining O&R's value without and with Risk Away. This is easy, as you recently built a financial statement simulation model ([Chapter 4](#)). Given O&R's current operations, its equity is worth \$2,019 million (going forward we omit the *million* for simplicity). This model assumes that costs average 70% of sales. Risk Away claims it will reduce this to 65%. If so, equity value would increase to \$2,457, an increase of \$438. Based on this static analysis, O&R should purchase Risk Away. However, \$2,019 and \$2,457 are just point estimates. A great deal of uncertainty lies underneath them. Hence, management asked you to make this hidden uncertainty visible by adding Monte Carlo simulation to your base model.

6.5.2 Odd & Rich under Uncertainty

You know that a great deal of uncertainty is hidden beneath the surface of the two point estimates for equity value. If so, the \$438 increase in value promised by Risk Away may be illusory. Monte Carlo simulation is one way to reveal this type of uncertainty. However, you need to identify and quantify the uncertainty before you can build a realistic Monte Carlo simulation model. Therefore, your next step is to talk to the experts at O&R—the people in charge of O&R's working capital and capital investment decisions. After doing so, you develop a list of what you believe to be the key uncertainties and reasonable estimates of their attributes. These follow.

6.5.2.1 Operating Costs

Without Risk Away, operating costs will most likely remain at 70% of sales. However, there is a reasonable possibility of supply shortages. If they materialize, costs could rise as high as 75% of sales. On the other hand, a remote chance exists (i.e., possible but highly unlikely) that new suppliers will enter the market. If so, costs could fall to 60% of sales. The PERT distribution is ideal for capturing this type of uncertainty. You also believe that costs and

* Patrick Leach's book, *Why Can't You Just Give Me the Number* (2014), provides an excellent overview of the many ways Monte Carlo simulation can be used.

sales will rise or fall with the economy, inducing a strong positive correlation between sales growth and costs. You estimate the coefficient of correlation will be 0.75.*

6.5.2.2 *Current Assets*

After consulting with literally everyone in the working capital management department, the best estimate for the uncertainty of current asset investment is that it will fall somewhere between 50% and 58% of sales. Given your lack of information, you decide to use the uniform distribution, the distribution of maximum ignorance. In addition, if the economy grows faster than expected, O&R's inventories are likely to shrink as its inventory is depleted. This implies a negative correlation between current asset investment and sales growth. You decide on a coefficient of correlation of -0.50 .

6.5.2.3 *Current Liabilities*

The working capital management team has a much better understanding of the uncertainty underlying O&R's current liability accounts. They believe that current liabilities will mostly likely be around 60% of sales, though they could slip to 55% if O&R's profitability comes under pressure. However, there is a remote chance that O&R will be able to demand better terms from its trade creditors. If so, current liabilities could increase to 75% of sales. Once again, the PERT distribution best captures this uncertainty.

6.5.2.4 *Gross Fixed Assets*

Odd & Rich is a capital-intensive business. Its investment in gross fixed assets averages 193% of sales. The capital investment team believes that this relationship is likely to continue. However, they offer two caveats. First, there is some chance that investment needs could fall to 170% of sales if new efficiency measures currently being tested work out. Alternatively, O&R is considering adding several new product lines involving capital-intensive manufacturing processes. Should these go forward, fixed asset investment could reach 220% of sales. These estimates are highly speculative, but this has more information than you had about current assets, so you are not maximally ignorant—just one step removed. You opt to use the triangle distribution to capture this uncertainty, at least until you have more information about the possible extreme values.

6.5.2.5 *Growth, Long and Short*

Sales are expected to grow 10% per year for the next five years, with a 68% chance they will fall between 9% and 11%. However, both extreme positive and negative growth rates are remote possibilities. The normal distribution is ideal for capturing this type of uncertainty.

You know considerably less about O&R's long-term growth prospects. In fact, given that maturity is five years away, the best you can do is making an educated guess. You believe that long-term growth will never be higher than 4% or lower than 0.5%. You choose the uniform distribution.

* Patrick Leach believes that using ballpark estimates for correlations that cannot be measured is better than ignoring correlation altogether. He suggests correlations of 0.0, 0.25, 0.50, and 0.75 based on your beliefs of no, low, average, or high correlation.

6.5.2.6 Adding Monte Carlo Simulation

The hard work of quantifying the uncertainty is done. All that remains is to incorporate this uncertainty into your financial statement simulation model and compare O&R’s equity value with and without Risk Away.

First, organize the data. Build a table that lists the inputs to be defined as random variables, the distributions you expect each to follow, and the parameters for those distributions. Exhibit 6.20 shows one way to do this. Add this table and the correlation matrix to the Input sector of your model.

Next, incorporate this uncertainty into your input variables using @RISK’s distribution functions. Link the functions to the parameters in the table rather than coding hard numbers. Doing so creates a much more transparent model, as well as one that can be easily adapted to changing conditions. It will be particularly helpful for evaluating the benefits of Risk Away. Do not forget to add correlation.

This is a good time to add a *Summary Statistics* panel to the Input sector. No set structure for this panel exists. Rather, this panel allows you to track your output variables of interest—the reasons for building the model in first place. We like to include a *Master check* that sums the check lines for the balance sheet, the cash flow module, and the Statement of Cash Flows. Aggregate equity value is a must, as it is used to rank scenarios. We also like to track g^* and *PRAT* from the last year of the model, as they provide useful information regarding feasible long-term growth rates for the terminal value calculation. In addition, a cell that reports the maximum value of a debt ratio, either interest-bearing debt to equity or interest-bearing debt to total assets, is quite useful. Debt is the most common source of external financing, and it comes with covenants attached. Typically, these covenants restrict the maximum value of a debt ratio. You might well find that the scenario that maximizes equity value violates your debt covenants.

Lastly, define the aggregate equity cell in the *Summary Statistics* panel as an output variable. (We named it *Aggregate Equity—w/o Risk Away*.) The converted input sector is shown in Exhibit 6.21a; the formulas in Exhibit 6.21b.

Distribution table for MC simulation					
Variable	Distribution	Parameters			St. Dev.
		Low	Expected	High	
COGS	PERT	0.60	0.70	0.75	
CA	Uniform	0.50		0.58	
GFA	Triangle	1.70	1.93	2.20	
CL	PERT	0.55	0.60	0.75	
Growth rate assumptions					
Growth	Normal		0.10		0.01
Long-term growth	Uniform	0.005		0.04	

	Growth	CA	CL
Growth	1		
CA	0.75	1	
CL	-0.5	0	1

EXHIBIT 6.20 Table of distributions and parameters.

	B	C	D	E	F	G	H	I
2	ODD & RICH Corporation							
3	INPUT SECTOR							
4	Parameter Estimates							
5	COGS/Sales	0.69		Riskfree rate	0.02			Summary Statistics
6	Depreciation/GFA	0.03		Market Risk premium	0.07		Aggregate Equity	1,895
7	Tax Rate	0.40		Interest Rate on Borrowing	0.04		Max(D/E)	30.2%
8	Cash/Sales	0.17		Annual Sales Growth	0.10			
9	CA (non-cash)/Sales	0.54		Long-term Growth	0.02		Sustainable growth (2021)	
10	GFA/Sales	1.94					g*	4.1%
11	CL/Sales	0.62		Policies and Decisions			PRAT	5.3%
12	beta	0.98		Debt portion	1.00			
13	Number of shares	100		Payout ratio	0.60			
14								
15								
16								
17								
18								
Distribution table for MC simulation								
Parameters								
19	COGS			PERT	0.60	0.70	0.75	
20	CA			Uniform	0.50		0.58	
21	GFA			Triangle	1.70	1.93	2.20	
22	CL			PERT	0.55	0.60	0.75	
23	Growth rate assumptions							
24	Growth			Normal		0.10		0.01
25	Long-term growth			Uniform	0.005		0.04	
26								
27								
28	Growth			Growth	CA	CL		
29	CA			1				
30	CL			0.75	1			
				-0.5	0	1		

(a)

EXHIBIT 6.21 (a) O&R Input sector with Monte Carlo simulation.

(Continued)

	B	C	D	E	F	G	H	I
2					ODO & RICH Corporation			
3					INPUT SECTOR			
4	Parameter Estimates		External Conditions		Summary Statistics			
5	COGS/Sales	=RiskPert(F19,G19,H19,RiskCorrmat(base.2))	Riskfree rate	0.02	Master Check	=SUM(ES9:ES104,I04)		
6	Depreciation/GFA	0.03	Market Risk premium	0.07	Aggregate Equity	=RiskOutput("Aggregate Equity - w/o Risk Away")*HD149		
7	Tax Rate	0.4	Interest Rate on Borrow	0.04	Max(D/E)	=RiskOutput("Max(D/E) - w/o Risk Away")*MAX(E121:I121)		
8	Cash/Sales	0.17	Annual Sales Growth					
9	CA (non-cash)/Sales	=RiskUniform(F20:H20,RiskCorrmat(base.3))	Long-term Growth					
10	GFA/Sales	=RiskTriang(F21,G21,H21)						
11	CL/Sales	=RiskPert(F22,G22,H22)	Policies and Decisions					
12	beta	0.98	Debt portion	1				
13	Number of shares	100	Payout ratio	0.6				
14								
15								
16								
17								
18								
19			Variable	Distribution	Low	Expected	High	St. Dev.
20			COGS	PERT	0.6	0.7	0.75	
21			CA	Uniform	0.5		0.58	
22			GFA	Triang	1.7		1.93	2.2
23			CL	PERT	0.55		0.6	0.75
24			Growth rate assumptions					
25			Growth	Normal				0.01
26			Lt growth	Uniform	0.005		0.04	
27			Growth					
28			Growth 1					
29			CA	0.75	1			
30			CL	-0.5	0			

(b)

EXHIBIT 6.21 (Continued) (b) Formulas for O&R Input sector with Monte Carlo simulation.

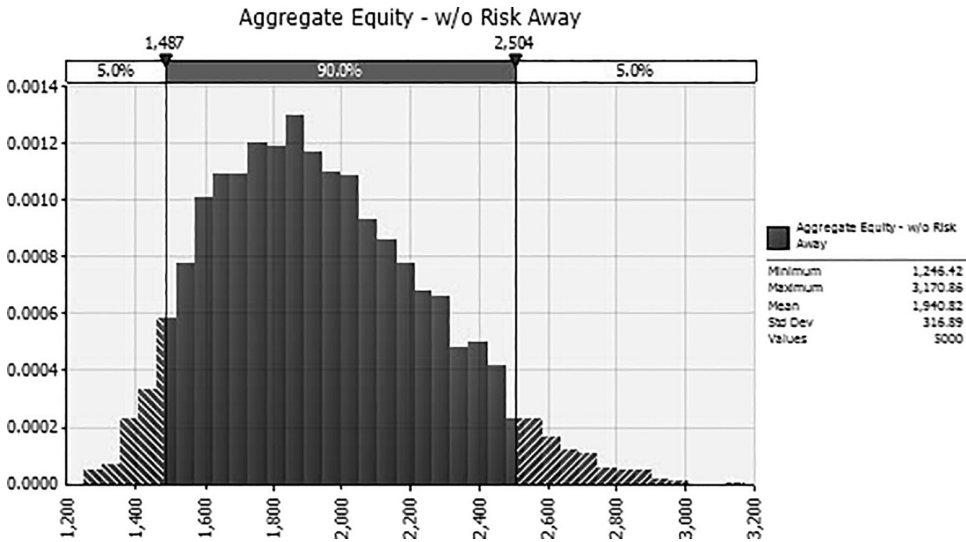


EXHIBIT 6.22 Simulated distribution of O&R's equity value before adding Risk Away.

It is time to run a simulation. Set the number of iterations to 5,000 and click the *Start Simulation* icon. The output window that opens ([Exhibit 6.22](#)) shows the histogram for the benchmark value of O&R's equity, with minimum, expected and maximum values of 1,250, 1,940, and 3,170, respectively (values rounded).

6.5.2.7 Measuring Risk Away's Promise

The results of this simulation are the benchmark for evaluating Risk Away. To measure the benefit of using Risk Away, change the distribution parameters for operating costs in your distribution table. The entrepreneur promises a 7% reduction of operating costs and reduced volatility. This would make the most likely value of the PERT distribution 65% of sales. The minimum and maximum values also decrease with Risk Away, the minimum value from 0.60 to 0.58, and the maximum value from 0.75 to 0.69. These changes decrease the standard deviation of operating costs from 2.8% to 2.0%. However, these statistics do not tell us much. The real questions are *How much value is Risk Away likely to add to our firm, and how likely is it that it adds any value at all?* To answer the first question, change the input parameters and run a simulation ([Exhibit 6.23](#)).

Risk Away appears to add value to Odd & Rich. Expected value increases by about 380, from 1,940 to 2,320, and both the minimum and maximum values are higher. However, the ranges of the two distributions display considerable amounts of overlap. This means that sometimes O&R's current program will outperform Risk Away. Your board will want to know how extensive the overlap is and how often their current cost management program will outperform Risk Away. Simply changing distribution parameters will not answer this question. You need to run the two models simultaneously. To do this, right-click on the tab of the worksheet on which your model is built, select *Move or Copy* and check the *Create a copy* box, then click OK. Change the parameters in your new worksheet to those assumed by Risk Away, then

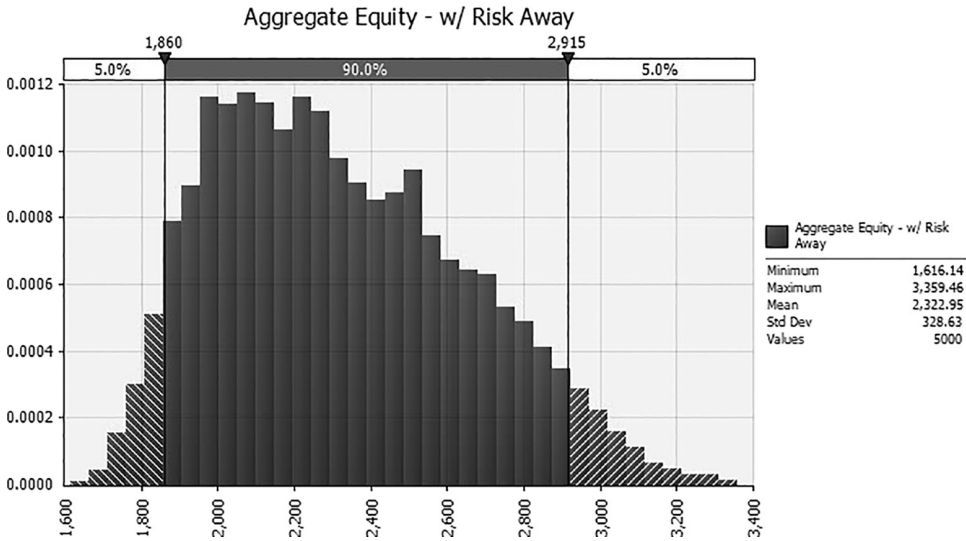


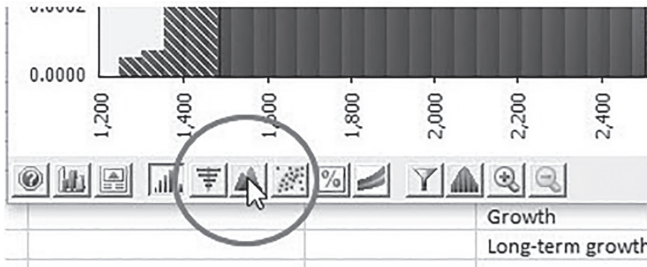
EXHIBIT 6.23 Simulated distribution of O&R’s equity value after adding Risk Away.

change the name of your output variable (just click on the cell and change the name inside the (“...”) being careful to stay inside the quotation marks). Now, running a simulation creates two separate outputs, one that reflects O&R’s current program, the other Risk Away’s.

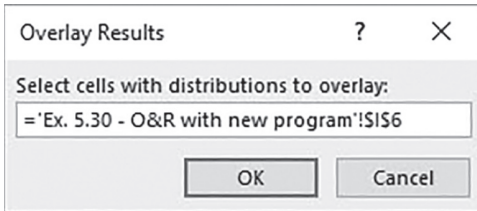
Run another simulation with the benchmark model open and equity value as the active cell. When the histogram window opens, click on the *Add Overlay to Graph* icon on the left side of bottom toolbar, navigate to the Risk Away model, and click on the equity value cell, then OK. This sequence can be seen in [Exhibit 6.24a–c](#).

As before, Risk Away adds about \$380 million to O&R’s equity value. However, viewing the superimposed distributions reveals the extent to which they overlap. So although Risk Away appears to add value on average, it will not always do so. Hence, the second question: *How likely is it that Risk Away adds to any value?*

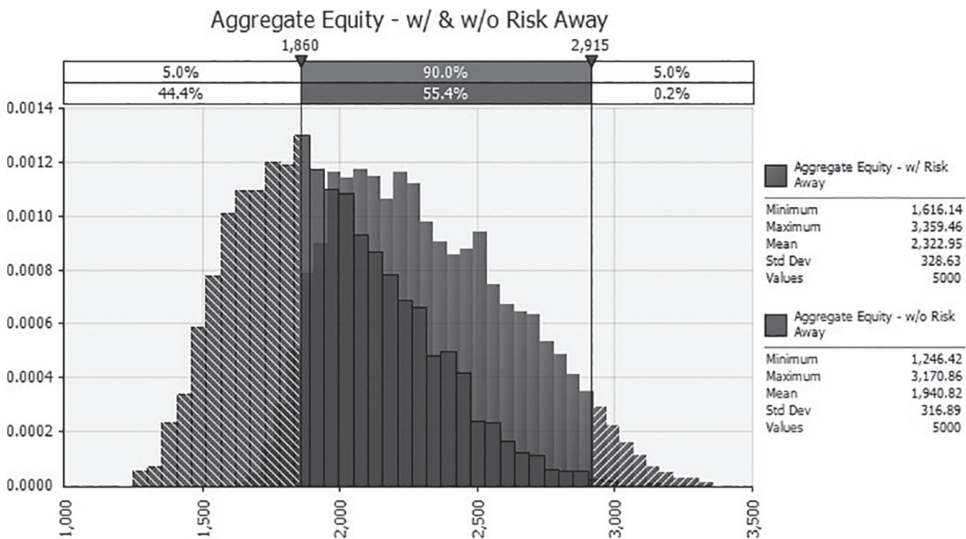
Answering this question is easy. It also demonstrates the combined power of Excel and @RISK. However, it requires thinking beyond the active worksheet. The variable of interest is the difference between equity values for O&R with and without Risk Away. To capture this, add another entry in the Summary Statistics panel. Label it *Difference*. Make the cell in which you will calculate the difference the active cell, then enter the “=” sign and navigate to the Risk Away worksheet. Click on the equity value on that sheet, type a “-” sign, navigate to the benchmark worksheet and click on the equity value, then hit *Enter*. Define this cell as an output variable, choose a name or accept @RISK’s, and run a simulation. Open the Difference histogram. It should look something like [Exhibit 6.25a](#). As you can see, the difference is negative a fair amount of the time, meaning that Risk Away will not always add value. To find the likelihood that Risk Away will not add value, click on the number above the left delimiter (circled—384 [Exhibit 6.25a](#)) and enter 0. Read the likelihood of a negative difference in the left panel of the delimiter bar—around 20% ([Exhibit 6.25b](#)). Hence, there is a 20% likelihood



(a)



(b)

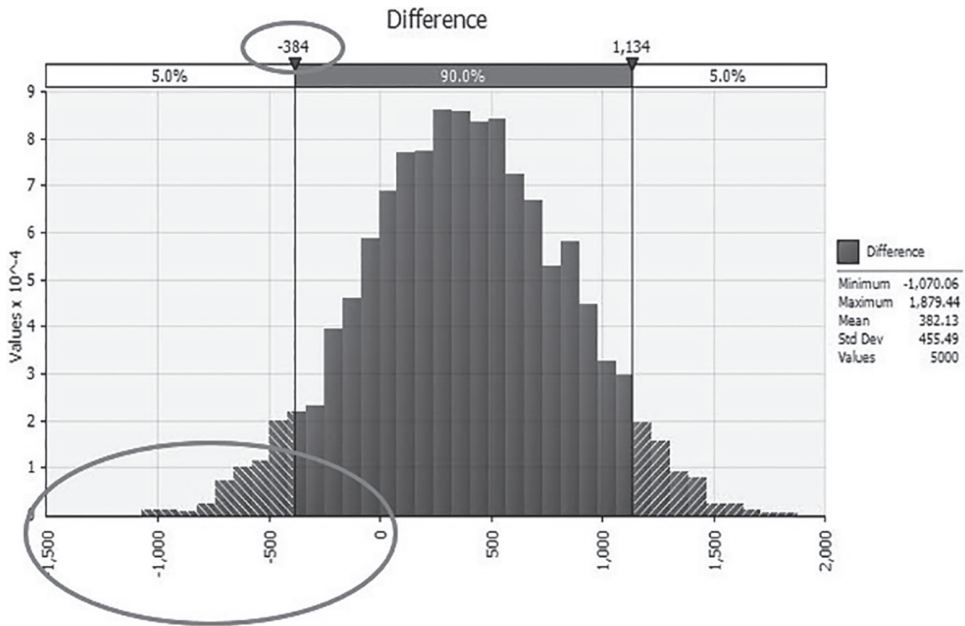


(c)

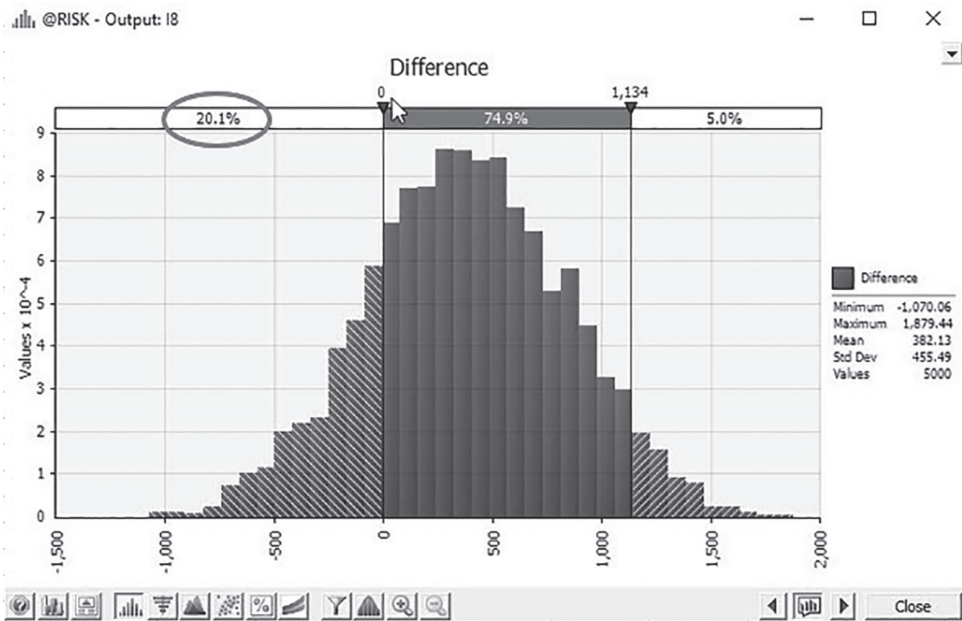
EXHIBIT 6.24 (a) Add Overlay to Graph icon. (b) Overlay selection dialog box. (c) Simulated histograms for O&R's equity with and without Risk Away.

that Risk Away will not add value, on average, or one year out of five. And this is all predicated on Risk Away being free and always effective. Of course, it will be neither.

Should you invest in Risk Away? That will depend on its price and you or your CFO's tolerance for risk. The simulation results provide additional information on which to base the decision.



(a)



(b)

EXHIBIT 6.25 (a) Chart of difference in O&R's equity value with and without Risk Away. (b) Measuring likelihood that Risk Away will decrease O&R's equity.

6.6 SUMMARY

This chapter introduced Monte Carlo simulation and the software to perform it in Excel, @RISK. Monte Carlo simulation helps us understand the effects of risk and uncertainty on our decisions. It does not provide answers or quick solutions to our questions. Rather, it gives us perspectives on what might happen, given the variety of risks that we face.

The output from the simulation is typically a frequency distribution that tells us how likely the different outcomes are. Armed with that information, we should be able to make more informed decisions.

Lastly, @RISK software was introduced. This seamless Add-In to Excel enables users to perform Monte Carlo simulation with little fuss. In future chapters we will see other ways that Monte Carlo simulation and @RISK can help us analyze difficult problems.

PROBLEM

1. Adding Monte Carlo simulation to Medtric's 12/25 scenario

This problem asks you to add Monte Carlo simulation to the solution to 12/35 scenario of the Medtric model that you built for Problem 4-3 (at the end of [Chapter 4](#)). As noted at that time, the 20/20 model becomes unstable once time-varying market-value-based discount rates are applied.

Jonsey, Medtric's CFO, is concerned that if Medtric's D/E ratio exceeds 1.0 in any of the five pro forma years (2017–2021), Medtric would be in technical default according to its loan covenants. Jonsey also observes that many of the inputs to this model are forecasts. The actual values realized will be either less than or greater than their expected values. Jonsey asks you to adapt your current model to use Monte Carlo simulation in order to better gauge these risks.

Input Parameters. Convert the simulation model from the Medtric model at the end of [Chapter 4](#) to a Monte Carlo simulation model. We make the heroic (and convenient) assumption that the distributions for the random variables described below are correctly identified.

- Cost of Goods Sold is expected to come in around 24% of sales and is unlikely to fall below 20% of sales. However, there is a remote likelihood that competition for skilled labor and raw materials could increase cost of goods sold to 35% of sales. Use the PERT distribution.
- Selling, General and Administrative Expenses are expected to run 35% of sales. They will not be less than 30% or greater than 40% of sales. Little else is known about their distribution. Use the triangle distribution.
- Research and Development expense is expected to be 12.5% of sales, and, in any event, will not be less than 9% or greater than 16% of sales. Use the triangle distribution.

- Accounts Receivable is expected to run between 19% and 23% or so of sales. However, if the economy falters they could balloon to 39% of sales. Use the PERT distribution.
- Inventory investment is expected to fall somewhere between 8% and 10% of sales. Similar to Accounts Receivable, should the economy falter Inventory investment could balloon to 18% of sales. Use the PERT distribution.
- Plant, Property, and Equipment investment is expected to fall anywhere between 30% and 38% of sales. Use the uniform distribution.
- Accounts Payable is expected to come in between 1.5% and 5% of sales. Most likely it will be 2.5% of sales. Little else is known about its distribution. Use the triangle distribution.
- Other Current Liabilities should fall somewhere between 13% and 17% of sales. Similarly, other long-term liabilities could fall anywhere between 3% and 7% of sales. Use the uniform distribution for both.
- Cost of Goods Sold are unlikely to remain stable for the duration of the planning horizon, as Medtric's rapid growth likely will increase the demand for the raw materials and labor required for its products, thereby increasing the Costs of Goods Sold in the later years of this model. Jonsey suggests that you model this effect by adding a line to your input sector that calculates the Costs of Goods Sold as separate random variables for 2020 and 2021. Jonsey has a fairly good idea of the minimum, expected, and maximum values for each of these three years, but has little other information. Use the triangle distribution.
- Finally, the target five-year growth rate is 12% per year, but there is a small probability that it could be considerably larger or smaller. Still, the actual growth rate should not display excessive variability. Use the normal distribution with a mean of 0.12 and standard deviation equal to 0.018.

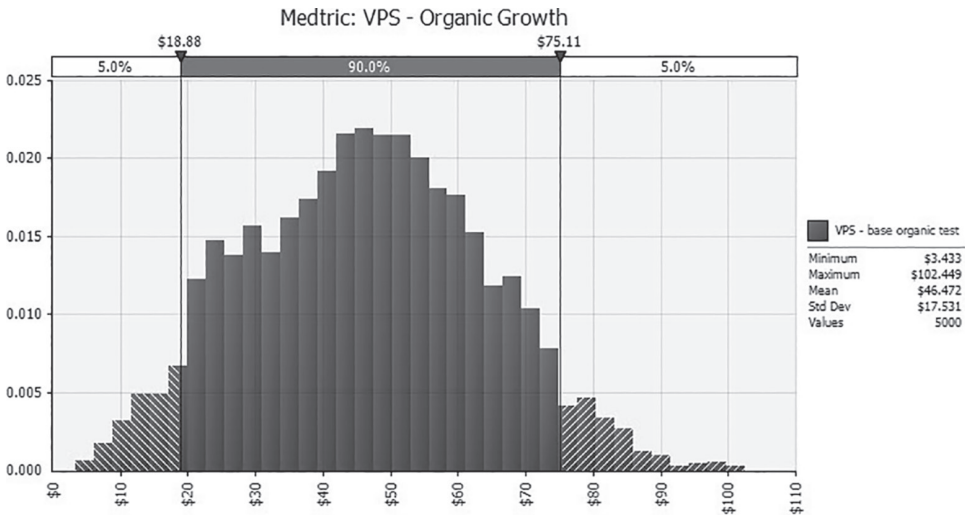
These values are summarized in [Exhibit 6.26a](#).

Once you have adapted your model to Monte Carlo simulation, define VPS and the Max D/E ratio as output variables. Set the *debt_portion* to the value that maximizes VPS and run a simulation of 5,000 iterations. Your graph should look quite similar to that shown in [Exhibit 6.26b](#). Use the results of your simulation to address the following questions:

- What is the expected VPS?
- What is the 95% confidence interval for the VPS?
- What are the five greatest influences on Medtric's VPS?
- What is the likelihood that Medtric will violate its debt covenants? Is this risk acceptable?

	K	L	M	N	O	P
5	Uncertainty assumptions, 12/35 scenario					
6	Parameter	Distribution	Min	Expected	Max	Sigma
7	Percent of sales					
8	COGS	PERT	0.200	0.240	0.350	
9	SGA	Triangle	0.300	0.350	0.400	
10	R&D	see ComboBox control cells				
11	Dep	Constant				
12	Cash	Policy				
13	Receivables	PERT	0.190	0.230	0.390	
14	Inventory	PERT	0.080	0.100	0.180	
15	Other CA	Constant				
16	PPE - Gross	Uniform	0.300		0.380	
17	AP	Triangle	0.015	0.025	0.050	
18	Other CL	Uniform	0.130		0.170	
19	Other LT Liab	Uniform	0.030		0.070	
20						
21	Distribution for COGS			Terminal		
22	Triangle	2020	2021	Year		
23	min	0.216	0.261	0.288		
24	mean	0.240	0.290	0.320		
25	max	0.312	0.377	0.416		
26						
27	Growth					
28	5-year	Normal		0.12		0.018

(a)



(b)

EXHIBIT 6.26 (a) Distribution table for Medtric, Inc., 12/35 scenario. (b) Value per share for Medtric under the 12/35 scenario.

III

Introduction to Forecasting Methods



Taylor & Francis

Taylor & Francis Group

<http://taylorandfrancis.com>

Forecasting: Time Trend Extrapolation

Forecasting is the substitution of error for chaos.

—ANONYMOUS

7.1 AN INTRODUCTION TO FORECASTING

Forecasts are a major input to financial plans and the models within those plans. They are basic to solving forward-looking models. Certainly, models can be built without forecasts. Still, we need forecasts to make the models work. This and the following two chapters are concerned with forecasting. The intent is to expose you to a variety of forecasting methods so you will be able to develop and use basic forecasts. We recognize that a few short chapters can barely introduce you to forecasting methods. We direct readers interested in going beyond this brief introduction to any of the many fine texts devoted to forecasting.*

There are many different methods and techniques for generating the forecasts necessary for financial models. The best technique to use depends on the problem being modeled. The choice will be determined by such factors as the time horizon over which the forecast will be made, the level of detail and accuracy required, the costs of being wrong, the type of data available, and the expertise of the forecaster.

The forecasting technique selected should suit the task, and its benefits should outweigh its costs. For example, consider the choice between highly sophisticated and simple methods. Highly sophisticated techniques may be too costly for the task, too complex to engender the confidence of the people in the organization, or too detailed in their output for the task at hand. Less sophisticated forecasting methods may produce equally reliable results at a lower cost and, because they are easier to understand, gain a higher level of acceptance.

* Useful forecasting texts include DeLurgio (1998), Hanke, Wichern, and Reitsch (2014), Levine, Berenson, and Stephan (2007), and Pindyck and Rubinfeld (1998).

This chapter and the two that follow are intended to give the reader a sufficient perspective to make this type of evaluation.

7.1.1 Qualitative versus Quantitative Forecasts

Forecasting methods can be either qualitative or quantitative. Before computers, qualitative forecasts were the primary means of forecasting.

Humans possess unique knowledge and inside information not available to quantitative methods. However, empirical studies and laboratory experiments have shown that their forecasts are not more accurate than those of quantitative methods.

SPYROS MAKRIDAKIS

The Art and Science of Forecasting, 1986

Quantitative forecasting methods have distinct advantages over qualitative methods—they are consistent, replicable, and testable. These attributes make them amenable to careful evaluation and academic study. Ultimately, perhaps, the best technique may be to combine both qualitative and quantitative methods.

This being a modeling text, we focus on replicable and testable quantitative methods—not because they are inherently superior, but because they are more easily taught and evaluated. This chapter introduces the use of linear regression for extrapolating a time trend. [Chapter 7](#) extends our treatment of regression-based models by introducing econometric models. [Chapter 8](#) discusses smoothing methods.

7.2 STEPS FOR DEVELOPING A FORECASTING MODEL

The development of your forecasting model begins with a hypothesis about what factors influence the quantity to be predicted. Even if you are not using a quantitative approach and your sales forecast is based on judgment and an intuitive understanding of your company and industry, your projections will be based on an idea of the factors and events that will affect your company. Similarly, with a quantitative model, you begin with an idea of the influential factors. These ideas form the basis of the hypothesis that supports the forecast.

With the hypothesis in mind, the next steps are to gather and examine data and then use that data to test and refine your hypothesis. In the example to follow, we have assembled past sales data for a company called The Speckled Band Corp., with the intent to forecast its sales for the next several quarters. We have some ideas about what affects our company's sales. At the top of our list is a time trend. We know that past sales generally have been increasing over time, although with considerable variation from period to period. This suggests that sales depend on time. Next, we take a more systematic look at the sales data to refine and test our initial hypothesis that sales are following a time trend.

The first step in data analysis is looking at the data—drawing a graph. A visual image of the data can help identify possible patterns in the data and perhaps suggest alternative hypotheses. After examining the data, we specify a model that allows us to evaluate our hypotheses and develop a forecast. Finally, we test our forecasting models to see if they produce usable forecasts.

Forecasting can be a discouraging task. The forecaster almost invariably will be wrong. Over the next several chapters, we seek to understand the limitations of both the data and the methods for building forecasting models. To this end, the forecasting model we develop in this chapter does not provide forecasts that are as accurate as we would like. We improve on those forecasts in the following chapter. The statistical methods we use require a lot of well-behaved data that conforms to the assumptions that underlie statistical analysis. Economic and business data seldom is that well behaved. Compounding our difficulties is the fact that we rarely have as much data as we would like. Nevertheless, our advice is to forge ahead, but always with an understanding of where the potential pitfalls lie and the various things that can go wrong.

7.3 TIME TREND EXTRAPOLATION

We begin by introducing simple methods to extrapolate a trend—average growth and linear regression. Our example forecasts sales, but the methods discussed can be used to forecast most business and economic variables. We use quarterly sales for Speckled Band shown in [Exhibit 7.1](#); the Roman numerals denote quarters of the year, the numbers 1 to 44 denote the quarters. Our goal is to forecast Speckled Band's quarterly sales for year 12 (quarters 12.I–12.IV, or, quarters 45 through 48).

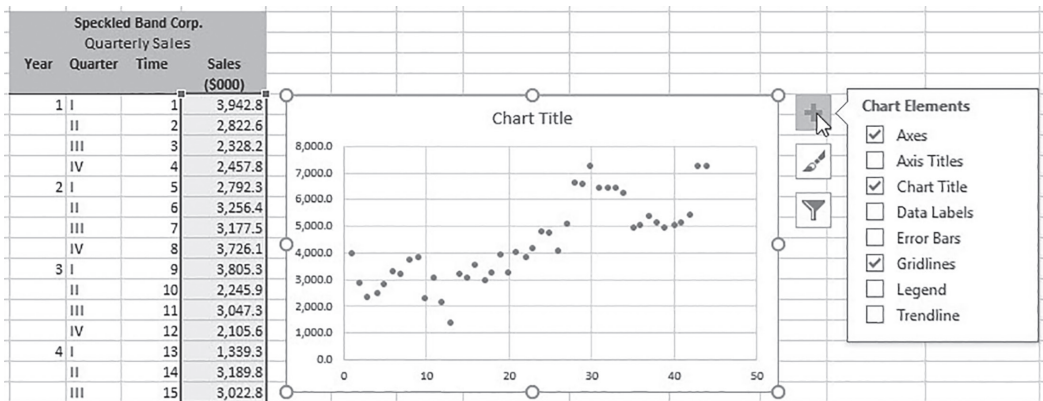
Speckled Band Corp. Sales							
Year	Quarter	Time	Sales (\$000)	Year	Quarter	Time	Sales (\$000)
1	I	1	3,942.8	7	I	25	4,698.2
	II	2	2,822.6		II	26	4,056.8
	III	3	2,328.2		III	27	5,074.0
	IV	4	2,457.8		IV	28	6,597.9
2	I	5	2,792.3	8	I	29	6,562.9
	II	6	3,256.4		II	30	7,226.5
	III	7	3,177.5		III	31	6,423.4
	IV	8	3,726.1		IV	32	6,409.5
3	I	9	3,805.3	9	I	33	6,400.5
	II	10	2,245.9		II	34	6,216.7
	III	11	3,047.3		III	35	4,930.6
	IV	12	2,105.6		IV	36	4,986.7
4	I	13	1,339.3	10	I	37	5,338.0
	II	14	3,189.8		II	38	5,087.5
	III	15	3,022.8		III	39	4,907.6
	IV	16	3,498.1		IV	40	5,021.5
5	I	17	2,947.6	11	I	41	5,122.1
	II	18	3,237.3		II	42	5,392.3
	III	19	3,917.1		III	43	7,238.7
	IV	20	3,230.1		IV	44	7,219.5
6	I	21	3,985.3				
	II	22	3,825.6				
	III	23	4,159.0				
	IV	24	4,746.9				

EXHIBIT 7.1 Speckled Band Corporation's quarterly sales.

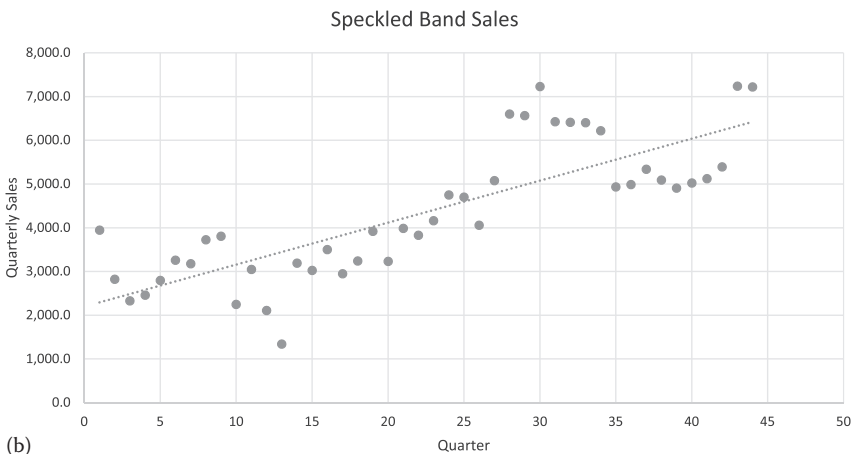
The first step is to examine the data and get an intuitive sense of any patterns in the data (e.g., trends and/or cycles). Excel makes drawing a chart of your data relatively easy. The first step is, of course, entering the data in a spreadsheet. Next, highlight the series and navigate to the *Insert* ribbon. On the *Charts* panel, click on *Scatter*. As very basic chart will appear on your spreadsheet with a large “+” on the right. Clicking on that “+” brings up the *Chart Element* menu, seen in Exhibit 7.2a below. Select the elements you wish to add to your chart (we added Axis Titles and Trendline), then click on the chart. Fill in the chart and axis titles and you are finished.

Exhibit 7.2b charts the sales against time. A visual check and the addition of a trend line tells us that sales have trended higher over the past 12 years. The chart also tells us that this growth has not been smooth. Instead, it tends to run in a cycle around the trend, alternating between longish trends: above-trend and below-trend of the 11 years of the sample.

The presence of a trend gives us our first simple time trend extrapolation method—average growth. Over the past 11 years, sales have increased at an average rate of 4.2% per



(a)



(b)

EXHIBIT 7.2 (a) Building a chart in Excel. (b) Speckled Band quarterly sales.

quarter and an average compound rate of 1.42% per quarter.* Though the average growth rate is frequently thought of as a descriptive statistic rather than a forecasting method, it can be used to generate a simple short-term forecast. For example, to forecast the next quarter's sales, take the average percentage quarterly change and apply it to the current quarter's sales. Speckled Band's sales were \$7,220 in the fourth quarter of year 11 (11.IV). The sales forecast for the first quarter of year 12 is

$$\begin{aligned} Sales_{12,I} &= Sales_{11,IV} * 1.0421 \\ &= 7,220 * 1.0421 \\ &= 7,524 \end{aligned} \quad (7.1)$$

The sales forecast for the second quarter of year 12 takes the forecast sales for the first quarter as its input, thus

$$\begin{aligned} Sales_{12,II} &= E[Sales_{12,I}] * 1.0421 \\ &= 7,524 * 1.0421 \\ &= 7,851 \end{aligned} \quad (7.2)$$

Forecasts using the geometric average growth rate are calculated in the same way, but use the geometric average quarterly growth rate instead of the simple average quarterly growth rate. Though this process can be continued indefinitely, it is not recommended. There is no reason to expect either of the average growth rates to continue into the future.

7.3.1 Estimation Period versus Hold-Out Period

Extrapolating a simple average growth rate is a very quick way to forecast the next period's value. However, it is not a viable longer-term forecast method. We now develop the first of

* The average percentage change is what is called an arithmetic average—the simple average of the % change for each of the 43 quarters. The compound growth, also called the geometric average growth is based on the compound growth expression $Sales_N = Sales_1 * (1 + g)^{N-1}$ where g is the compound rate of growth over $N - 1$ periods.

Over $N - 1$ periods, the average compound growth in sales is calculated as

$$g = \left(\frac{Sales_N}{Sales_1} \right)^{\frac{1}{N-1}} - 1$$

With 44 periods of data and 43 periods of growth, Speckled Band's geometric average growth is

$$\begin{aligned} g &= \left(\frac{Sales_{44}}{Sales_1} \right)^{\frac{1}{43}} - 1 = \left(\frac{7,219.5}{3,942.8} \right)^{\frac{1}{43}} - 1 = 0.0142 \\ &= 1.42\% \text{ average compound growth per quarter} \end{aligned}$$

our forecasting models. Once the forecasting models are developed, we will need to decide which ones seem to generate the best predictions. Obviously, we will not know which model yields the best prediction until we see what actually happens. However, since we do not want to wait to see how accurate our forecasts are, we will apply our model to past data and measure how well it would have worked in the past. We do this by partitioning the data into an *estimation period* and a *hold-out period*. We use data from the estimation period to estimate our model's parameters. Then, we use that model to predict the values for the hold-out period. This way we can check the forecasting accuracy of our model by comparing its forecasts to their respective actual values in the hold-out period. The model that appears to generate the best forecasts for the hold-out period is then reestimated using data from the entire sample—both the original estimation period and the hold-out period. Finally, the reestimated model is used to forecast the variables for the still unknown future. For Speckled Band, we use the first eight years as the initial estimation period and the last three years as the hold-out period. The next section uses linear regression to extrapolate the trend in Speckled Band's sales.

7.3.2 Time Trend Extrapolation Using Linear Regression

We consider two time trend regression models using one independent variable, *Time*. The time variable simply numbers the periods in the estimation period (1 to 44 in this example). To apply the model, the series to be forecast is regressed on this time sequence. The two models we consider differ in their assumptions about growth. The constant-change model assumes that the series being modeled increases by a constant dollar amount at each period. The compound growth model assumes that the series grows at a constant percentage rate. We begin with the constant-change model.

7.3.2.1 The Constant-Change Model

The general form of the constant-change model is shown in Equation 7.3.

$$\text{Sales}_t = a + b * \text{Time}_t + e_t \quad (7.3)$$

The task is to estimate the intercept parameter, a , and the slope parameter, b ; the random error term, e_t , denotes the remaining variation that this model cannot explain. We use the regression tool from Excel to estimate this model.* You access the regression tool in Excel through the Data Analysis icon on the *Data* ribbon.† In the Data Analysis list of Analysis Tools, click Regression. The Regression dialog box pops up and you fill it in as shown in [Exhibit 7.3](#). For the Input Y Range select the column (or row) of dependent variable data

* We are using the Excel regression routine because it is accessible in Excel. There are numerous other statistical software programs that are well-suited to developing a forecasting model. These include E-Views from Quantitative Micro Software (www.eviews.com), SPSS from SPSS Corp (www.spss.com), CB Predictor from Decisioneering (www.decisioneering.com), and StatTools from Palisade Corporation (www.Palisade.com).

† The Data Analysis ToolPak, an Excel Add-in, can be found on the right-hand side of the *Data* ribbon once added in. To add it in, click on File > Options > Add-ins. Next, select *Excel options* in the Manage box, then *Go*. Select Analysis ToolPak, then OK.

	B	C	D	E	F	G	H	I	J	K
3	Quarterly Sales									
4	Year	Quarter								
5			Time	Sales						
6	1	I	1	3,942.8						
7		II	2	2,822.6						
8		III	3	2,328.2						
9		IV	4	2,457.8						
10	2	I	5	2,792.3						
11		II	6	3,256.4						
12		III	7	3,177.5						
13		IV	8	3,726.1						
14	3	I	9	3,805.3						
15		II	10	2,245.9						
16		III	11	3,047.3						
17		IV	12	2,105.6						
18	4	I	13	1,339.3						
19		II	14	3,189.8						
20		III	15	3,022.8						
21		IV	16	3,498.1						
22	5	I	17	2,947.6						

Regression

Input

Input Y Range:

Input X Range:

Labels Constant is Zero

Confidence Level: %

Output options

Output Range:

New Worksheet Ply:

New Workbook

Residuals

Residuals Residual Plots

Standardized Residuals Line Fit Plots

Normal Probability

Normal Probability Plots

OK Cancel Help

EXHIBIT 7.3 Dialog box for Excel’s regression wizard.

(Sales in E5:E37). Note that E5 is the label Sales. The Input X Range is the independent variable data (D5:D37 that includes the label Time). With variable name labels Sales and Time included in the data range, check the Labels box, telling the program that the data range includes the label. G5 is entered in Output Range to indicate that you want the regression output to start at G5. Click ok and the output is inserted in the sheet as shown in Exhibit 7.4.

	F	G	H	I	J	K	L	M	N
4									
5		SUMMARY OUTPUT							
6		<hr/>							
7		<i>Regression Statistics</i>							
8		Multiple R	0.78						
9		R Square	0.60						
10		Adjusted R Square	0.59						
11		Standard Error	928.12						
12		Observations	32.00						
13		<hr/>							
14		ANOVA							
15			<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>		
16		Regression	1	39,453,941	39,453,941	45.80	0.00		
17		Residual	30	25,842,201	861,407				
18		Total	31	65,296,142					
19		<hr/>							
20			<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	
21		Intercept	1,886.20	335.99	5.61	0.00	1,200.03	2,572.37	
22		Time	120.26	17.77	6.77	0.00	83.97	156.55	
23		<hr/>							

EXHIBIT 7.4 Regression output.

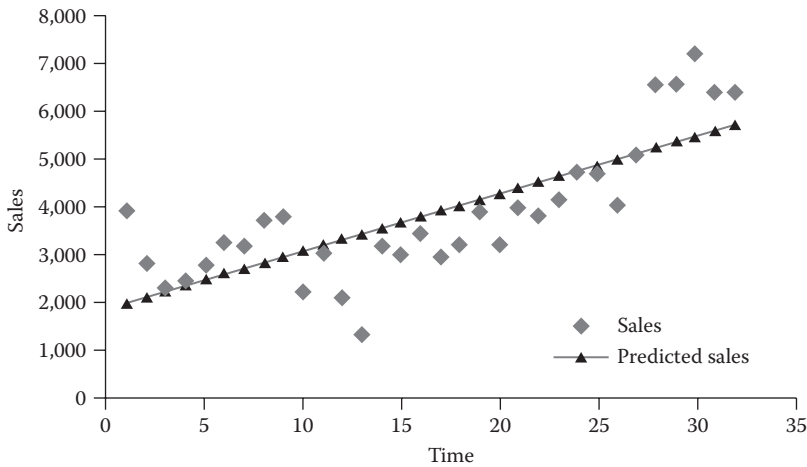


EXHIBIT 7.5 Regression output: Time line fit plot.

You will find the regression coefficients at the bottom of the regression output. The intercept, a , is 1,886.20; the slope coefficient, b , is 120.26. The rest of the output is the diagnostic statistics that we will discuss later. For now, we note that the estimated coefficients are statistically significant with t -statistics of 5.61 and 6.77 and p -values of 0.00. The adjusted R-square of 0.59 in cell G6 tells us that 59% of the variability of sales is accounted for by its relation with *Time*. Substituting the estimated parameters into the general form of the model gives us our following forecasting model:

$$Sales_t = 1,886.2 + 120.26 * Time_t \quad (7.4)$$

In the regression dialog box we checked the Line Fit Plots. This generates a graph of the regression relationship as shown in [Exhibit 7.5](#), where the dots are the actual sales and the line is the trend line that represents the *Sales* that would be predicted based on the estimated regression equation (we cleaned up this chart a bit). The deviations of the actual sales around the regression line represent the errors in the regression.

To forecast sales in future quarters, substitute the appropriate period's number for $Time_t$. Our estimation period ended with 32. To forecast the first two quarters of the hold-out period, we substitute 33 and 34 for $Time_t$:

$$\begin{aligned} Sales_{33} &= 1886.2 + 120.26 * 33 = 5,854.8 \\ Sales_{34} &= 1886.2 + 120.26 * 34 = 5,975.0 \end{aligned} \quad (7.5)$$

If you want to bypass the detail of the regression and are concerned only with the trend and the most basic statistics, the Excel functions INTERCEPT and SLOPE provide the regression coefficients. Excel's TREND function generates predicted future values based on the regression trend. These functions and their formulas are shown in [Exhibit 7.6](#).

	B	C	D	E	F	G	H
30	7	I	25	4,698.2			
31		II	26	4,056.8			
32		III	27	5,074.0			
33		IV	28	6,597.9	=INTERCEPT(E6:E37,D6:D37)		
34	8	I	29	6,562.9		Intercept	1,886.20
35		II	30	7,226.5		Slope	120.26
36		III	31	6,423.4	=SLOPE(E6:E37,D6:D37)		
37		IV	32	6,409.5			
38					=TREND(\$E\$6:\$E\$37,\$D\$6:\$D\$37,D40)		
39	Hold-out Portion				TREND forecast		
40	9	I	33	6,400.5		5,854.8	
41		II	34	6,216.7		5,975.1	
42		III	35	4,930.6		6,095.3	
43		IV	36	4,986.7		6,215.6	
44	10	I	37	5,338.0		6,335.8	
45		II	38	5,087.5		6,456.1	
46		III	39	4,907.6		6,576.4	
47		IV	40	5,021.5		6,696.6	
48	11	I	41	5,122.1		6,816.9	
49		II	42	5,392.3		6,937.1	
50		III	43	7,238.7		7,057.4	
51		IV	44	7,219.5		7,177.7	

EXHIBIT 7.6 INTERCEPT, SLOPE, and TREND functions.

One problem with the constant-change model is that the percentage rate of growth declines as time passes. This is because the level of sales increases by the same dollar amount in each period. A compound growth regression model may be better suited when growth is proportional to the level of sales.

7.3.2.2 The Compound-Growth Model

A firm's sales may grow by a given percent at each period rather than by a constant dollar amount. If so, forecasts made by the constant-change model will significantly underestimate future sales. The compound growth model addresses this problem.

If sales grow at a compound rate, g , sales in year t will be

$$Sales_t = Sales_0(1 + g)^t \quad (7.6)$$

where $Sales_0$ is the level in the base period $t = 0$. To use linear regression to estimate the growth rate, transform the expression into the logarithmic form

$$\ln(Sales_t) = a + b * t \quad (7.7)$$

where:

t is the number of the period

$\ln(\cdot)$ denotes the natural logarithm.

When we regress the log of sales against time as the independent variable, the slope coefficient, b , is the estimate of the compound growth rate, and the intercept, a , corresponds to $\ln(\text{Sales}_0)$.*

To run this regression in Excel, the dependent variable, *sales*, is transformed with the function =LN(.); time remains the independent variable. Using data for the 32-quarter estimation period, our estimated equation for Speckled Band is

$$\ln(\text{Sales}_t) = \underset{(307.9)}{7.715} + \underset{(7.868)}{0.029} * \text{Time}_t \quad (7.8)$$

$$R^2 = 51.7\%$$

where the numbers in parentheses are the t -statistics. The estimated compound growth rate is 2.9% per quarter.

To generate forecasts, substitute the appropriate period's number for *Time*. This yields the forecast for the natural log of sales for that period. Exponentiating this value yields the dollar forecast.† For the 33rd quarter ($\text{Time} = 33$), the forecast is

$$\begin{aligned} \ln(\text{Sales}_{33}) &= 7.715 + 0.029 * 33 \\ &= 8.672 \end{aligned} \quad (7.9)$$

so

$$\begin{aligned} \text{Forecast Sales}_{33} &= e^{8.672} \\ &= 5,837.2 \end{aligned}$$

The compound growth model addresses the decreasing growth-rate criticism of the constant-change assumption. Unfortunately, we cannot know a priori which model generates the better forecasts. To make this determination, we must measure and compare the accuracy of each model's forecasts. This is the purpose for the hold-out segment of the data. We take up this subject next.

7.3.3 Assessing Model Validity and Accuracy

Thus far we have considered two regression models to forecast quarterly sales. This section considers how to determine which model is most likely to produce the best forecasts. Our

* When $\text{Sales}_t = \text{Sales}_0(1 + g)^t$, the log transformation is

$$\begin{aligned} \ln(\text{Sales}_t) &= \ln(\text{Sales}_0 * (1 + g)^t) \\ &= \ln(\text{Sales}_0) + (\ln(1 + g)) * t \\ &= a + b * \text{Time}_t \end{aligned}$$

† The inverse function of $\ln(x)$ is the exponential function e^y , so $e^{\ln(x)} = x$ transforms the logarithmic forecast back into dollars. The respective Excel functions are =LN(.) and =EXP(.

assessment proceeds in two steps. First, we evaluate which forecasting model fits the data best for the initial estimation period. Then, we determine which model generates the best forecasts of sales for the hold-out period.

7.3.3.1 Evaluating Model Validity

This section reviews how to use the diagnostic statistics from Excel's regression output to evaluate how well the regression model fits the data.* Keep in mind, however, that evaluating statistical validity is not the same as forecasting. A model can fit past data very well but still not be a very good forecasting tool. Nevertheless, the first step is to evaluate the model against the past data in terms of its statistical validity and fit.

The first thing to understand is the *error* term, also called the *residual* or *deviation*, denoted by e_t . The error is the difference between the actual, observed value of the dependent variable, and the value predicted by the regression model. For example, the constant-change regression was estimated as

$$Sales_t = 1,886.2 + 120.26 * Time \quad (7.10)$$

For the first quarter, the estimated or predicted sales would be

$$\begin{aligned} Sales_t &= 1,886.2 + 120.26 * 1 \\ &= 2,006.5 \end{aligned} \quad (7.11)$$

and the actual value was 3,942.8. The error in the prediction is

$$Error_t = e_t = Actual Value_t - Predicted Value_t \quad (7.12)$$

For quarter 1 the error is

$$\begin{aligned} e_t &= 3,943 - 2,006 \\ &= 1,936 \end{aligned} \quad (7.13)$$

In other words, actual sales exceeded the prediction by 1,936.3. [Exhibit 7.7](#) shows the calculation of the predicted values and error terms for the constant-change model (Model 1) for the estimation period. Column D shows actual sales, column F is the value predicted by the regression model, and column G are the errors, or residuals. At the bottom are the totals and averages.

The mean error (*ME*) in cell G44 is the simple average of the errors over the estimation period. A mean error of zero ($ME = 0$) tells us that the model is unbiased; the forecasts

* An in-depth explanation of statistical methods and regression techniques is beyond the scope of this text. For greater detail and depth, consult texts such as DeLurgio (1998), Hanke, Wichern and Reitsch (2014), Levine, Berenson, and Stephan (2007), and Pindyck and Rubinfeld (1998).

	B	C	D	E	F	G	H	I
2	Speckled Band Corp.					Model 1: Constant Change		
3	Quarterly Sales					Intercept	1,886.2	
4	Year	Quarter	Time	Sales		Slope	120.3	
5				(000)		Predicted	Error	Squared Error
6	1	I	1	3,942.8		2,006	1,936	3,749,416
7		II	2	2,822.6		2,126.7	696	484,250
8		III	3	2,328.2		2,247.0	81	6,597
9		IV	4	2,457.8		2,367.2	91	8,201
10	2	I	5	2,792.3		2,487.5	305	92,902
11		II	6	3,256.4		2,607.8	649	420,732
12		III	7	3,177.5		2,728.0	449	202,030
13		IV	8	3,726.1		2,848.3	878	770,563
14	3	I	9	3,805.3		2,968.5	837	700,162
15		II	10	2,245.9		3,088.8	(843)	710,486
16		III	11	3,047.3		3,209.1	(162)	26,168
17		IV	12	2,105.6		3,329.3	(1,224)	1,497,502
18	4	I	13	1,339.3		3,449.6	(2,110)	4,453,303
19		II	14	3,189.8		3,569.8	(380)	144,435
20		III	15	3,022.8		3,690.1	(667)	445,297
21		IV	16	3,498.1		3,810.4	(312)	97,510
22	5	I	17	2,947.6		3,930.6	(983)	966,342
23		II	18	3,237.3		4,050.9	(814)	661,925
24		III	19	3,917.1		4,171.1	(254)	64,540
25		IV	20	3,230.1		4,291.4	(1,061)	1,126,376
26	6	I	21	3,985.3		4,411.7	(426)	181,791
27		II	22	3,825.6		4,531.9	(706)	498,902
28		III	23	4,159.0		4,652.2	(493)	243,237
29		IV	24	4,746.9		4,772.5	(26)	653
30	7	I	25	4,698.2		4,892.7	(195)	37,835
31		II	26	4,056.8		5,013.0	(956)	914,264
32		III	27	5,074.0		5,133.2	(59)	3,508
33		IV	28	6,597.9		5,253.5	1,344	1,807,431
34	8	I	29	6,562.9		5,373.8	1,189	1,414,070
35		II	30	7,226.5		5,494.0	1,732	3,001,509
36		III	31	6,423.4		5,614.3	809	654,685
37		IV	32	6,409.5		5,734.5	675	455,578
39					Total		0	25,842,201
40					Mean		0	807,569
41					Root Mean Squared Error (RMSE)			899
42					Residual Standard Error (RSE)			928

EXHIBIT 7.7 Errors from the constant-change model.

from the regression are not consistently in error one way or the other, and the overpredictions are balanced by the underpredictions. If the mean error is positive, the actual sales tend to be greater than the predicted values, and if the mean error is negative, the forecasts tend to be above the actual values. Even if some of the regression error terms are very large, they will still average zero. It is a function of the regression algorithm. So a mean error of zero does not mean that the model is accurate—only that it is unbiased. To get a better

	Model 1 Constant Change	Model 2 Compound Growth
Dependent Variable	Sales	ln(Sales)
Independent Variable	Time	Time
Intercept	1886.2	7.715
(t-statistic)	(5.61)	(82.1)
Slope Coefficient	120.26	0.029
(t-statistic)	(6.77)	(5.85)
Adjusted R-square	59.1%	51.7%
Standard Error of the Estimate	928	0.26
F-Statistic	45.8	34.2

EXHIBIT 7.8 Diagnostic statistics for time trend regressions.

idea of accuracy we need to look at other statistics like the mean absolute deviation, mean squared error, root mean square error, and standard error.

Least squares regression focuses on the squared errors that are shown in Column I, where, for example, the squared error for quarter 1 is $1,936.3^2 = 3,749,395$ in cell I6. When the squared errors are totaled, we get the *sum of the squared errors* in cell I39. This is the same as the Residual SS (sum of squares) in the regression output in [Exhibit 7.4](#), line I17. The mean (simple average) of the squared errors is $25,842,303/32 = 807,572$ in cell I40. This is what we call the mean square error (abbreviated *MSE*). The square root of the mean square error is called the root mean square error (*RMSE*), shown as 899 in I41.

The last item is the standard error (a.k.a. residual standard error), 928 in cell H42. This corresponds to cell H11 in the regression output of [Exhibit 7.4](#). The standard error is the standard deviation of the error terms around the regression line. This standard deviation is also called the standard error of the estimate or the standard error of the regression. The distinction between the standard error and the root mean square error is that the standard error is adjusted for degrees of freedom and the root mean square error is not.*

[Exhibit 7.8](#) summarizes the main diagnostic statistics for the two regression models. The coefficient of determination, R^2 , measures the proportion of variation of the dependent variable that is accounted for by the regression. An R^2 of 1 (100%) means that 100% of the variation of the dependent variable (around its mean) is attributable to its linear relationship with the independent variable, and the regression is a perfect fit. The R^2 for model 1 of 59.1% shows that 59.1% of the variability of sales is accounted for by the trend line and 40.9% is unrelated to the trend. This is a slightly better fit than model 2 with its R^2 of 51.7%. However, this small difference, by itself, is not enough to say that model 1 is a better model of the data.

Whether or not the R^2 is strong or weak depends on the form of the regression and the type of data. In some cases, such as a regression using cross-sectional data, an R^2 of 59% might

* The adjustment for degrees of freedom takes account of the number of observations and the number of variables in the regression. The divisor of the sum of squared residual errors (cell H13 in the regression output) is $(n - k)$, where n is the number of observations, and k is the number of variable coefficients estimated. For a regression with 1 independent variable, k is 2 to account for the slope coefficient and intercept term.

be considered a relatively good fit. In other cases, 59% might be considered low. Regressions using time series data frequently yield very high R^2 , often in the 90% range. In this case, with time series data, we would assess the R^2 of 59% as being relatively low. However, as the remaining diagnostic statistics will indicate, there is strong evidence that there is a time trend in the data. The problem is that there may be other forces at work in addition to just the time trend. We will deal with these other factors in the following chapter.

For model 1, the coefficient of the time variable is estimated to be 120.26, meaning that each quarter sales are expected to increase by 120.26. We assess the significance of this estimate with the t -statistic. The t -statistic tells us whether the estimated coefficient is significantly different from zero.* The t -statistic of 6.77 indicates that it is very unlikely that we could have gotten an estimated coefficient of 120.26 if, in fact, the true value was zero. Thus, we say the coefficient is significantly different from zero. It seems obvious that 120.26 is significantly different from zero. However, when we look at the time coefficient of 0.029 for model 2, we might be inclined to conclude that this is so small that it might as well be zero. The t -statistic of 5.85 tells us that, in spite of the coefficient's small magnitude, it is very unlikely that the real value is zero. In other words, the time coefficient of 0.029 in the logarithmic regression is almost as significant as the 120.26 in the constant-change model. Our conclusion for either regression model is that there is a statistically significant link between sales and time.

The standard error of the estimate (a.k.a., standard error of the regression or residual standard error) is the estimate of the standard deviation of the error terms around the regression line. This allows us to calculate a confidence interval for the dependent variable around the line. For example, for quarter 16, regression model 1 predicts sales to be

$$\begin{aligned} \text{Predicted Sales}_{16} &= 1,886.2 + 120.26 * 16 \\ &= 3,810.4 \end{aligned} \quad (7.14)$$

Due to random error, however, the actual sales will be drawn from a distribution of possible values around the predicted value. With the standard error of $\sigma_e = 928.12$, the 95% confidence interval would be

$$\begin{aligned} \text{Confidence Interval} &= \text{Predicted Sales} \pm t * \sigma_e \\ &= \text{Predicted Sales} \pm 2.04 * 928.12 \\ &= 3,810.4 \pm 1,893.4 \end{aligned} \quad (7.15)$$

where $t = 2.04$ is the *critical statistic* that represents the number of standard deviations around the mean wherein we expect to observe the actual dependent variable 95% of the

* The rough rule of thumb (depending on sample size) is that t should be greater than 2 for the coefficient to be significantly different from zero. The t -statistic tests the hypothesis of whether the true value of the regression coefficient is zero. For example, for a large sample, if $t \geq 1.98$, the probability of having obtained the estimated coefficient is less than 5% when in fact the true value is zero. The t tells us, in standard deviation units, how far the estimated coefficient is from zero. The $t = 6.77$ indicates that our estimated coefficient is 6.77 standard deviations from zero. If the true value was zero, it would be almost impossible to obtain an estimate that was 6.77 standard deviations from zero, so we say that the estimated coefficient is significantly different from zero. The output from most regression packages also reports of the likelihood that the estimated parameter is the result of chance alone. It is reported as the p -value.

time. In this case, with the large standard error, the 95% confidence interval is very wide, ranging from about 1,917 to 5,704.

The last line of [Exhibit 7.8](#) shows the F -statistics of 45.8 and 34.2 for regression models 1 and 2. The F -statistic is an overall significance test of the regression, and is more relevant when we have multiple independent variables. The F -statistic is analogous to the t -statistic in that it tests the hypothesis that there is no relation between the dependent variable and the independent variables. If F is large, it supports the hypothesis that there is in fact a significant relation between the dependent variable and the set of independent variables. The critical value of the F -statistic depends on the sample size, n , and the number of independent variables, k . In this example, with 32 observations and 2 independent variables (including the intercept), the critical value for F at the 5% significance level is $F_{k-1, n-k} = F_{1,14} = 4.6$. For either model, the F -statistic is far greater than the critical level, supporting the hypothesis that either regression model is valid.*

These diagnostic statistics show slightly greater support for model 1 than model 2, indicating that model 1 fits the data from the estimation period better than model 2. The next step is to see which model seems to give us better predictions.

7.4 EVALUATING FORECAST ACCURACY

7.4.1 Diagnostic Measures

In the previous section, we evaluated how well the two regression models fit data from the 32-quarter estimation period. We now measure how well the models forecast sales for the 12-quarter hold-out period.

[Exhibit 7.9a](#) shows actual sales, forecast sales, and error terms for the constant-change and compound-growth regression models for quarters 33 through 44. The error terms (*Actual Sales – Predicted Sales*) are used to calculate the statistics for evaluating forecast accuracy. The statistics we consider are the mean error (ME), mean absolute deviation (MAD), root mean squared error ($RMSE$), and the residual standard error (RSE). Each statistic is calculated in the same way as for the estimation period, except now we look only at the hold-out period. [Exhibit 7.9b](#) shows the code.

7.4.1.1 Mean Error

The most obvious measure of forecasting accuracy is an average of the error terms, known as the mean error (ME). It is calculated as a simple average of the errors. The mean errors over the hold-out period for both of the regression models are shown in line 19 of [Exhibit 7.9](#). Both models have negative ME 's, indicating that, on the average over the hold-out period, the forecast is biased with the actual sales less than the forecast. As the compound growth model's ME is larger (–1,226.5 versus –861), we are getting a more biased forecast from the compound growth model.

Even if the forecast is unbiased, it may not be accurate. We could have a model with very large errors, but they could average zero. To get a better idea of the accuracy of our forecasts, we look at the mean absolute deviations.

* As with the t -statistic, most regression packages report the p -value for the F -statistic.

	B	C	D	E	F	G	H	I	J	K
2	Speckled Band Corp.					Model 1: Constant Change		Model 2: Compound Growth		
3	Quarterly Sales					Intercept 1886.2		Intercept 7.715		
4	Hold-out Portion					Slope 120.26		Slope 0.029		
5	Year	Quarter	Time	Sales		Forecast	Error term	Forecast	Error term	
6	9	I	33	6,400.5		5,855	546	5,837	563	
7		II	34	6,216.7		5,975	242	6,009	208	
8		III	35	4,930.6		6,095	(1,165)	6,186	(1,255)	
9		IV	36	4,986.7		6,216	(1,229)	6,368	(1,381)	
10	10	I	37	5,338.0		6,336	(998)	6,555	(1,217)	
11		II	38	5,087.5		6,456	(1,369)	6,748	(1,660)	
12		III	39	4,907.6		6,576	(1,669)	6,947	(2,039)	
13		IV	40	5,021.5		6,697	(1,675)	7,151	(2,129)	
14	11	I	41	5,122.1		6,817	(1,695)	7,361	(2,239)	
15		II	42	5,392.3		6,937	(1,545)	7,578	(2,186)	
16		III	43	7,238.7		7,057	181	7,801	(562)	
17		IV	44	7,219.5		7,178	42	8,030	(811)	
18										
19						ME	(861)			(1,226)
20						MAD	1,029			1,354
21						RMSE	1,189			1,515
22						RSE	1,303			1,659

(a)

	B	C	D	E	F	G	H
2	Speckled Band Corp.					Model 1: Constant Change	
3	Quarterly Sales					Intercept =1886.2	
4	Hold-out Portion					Slope 120.26	
5	Year	Quarter	Time	Sales		Forecast	Error term
6	9	I	33	6400.5		=H\$3+\$H\$4*D6	=E6-G6
7		II	34	6216.7		=H\$3+\$H\$4*D7	=E7-G7
8		III	35	4930.6		=H\$3+\$H\$4*D8	=E8-G8
9		IV	36	4986.7		=H\$3+\$H\$4*D9	=E9-G9
10	10	I	37	5338		=H\$3+\$H\$4*D10	=E10-G10
11		II	38	5087.5		=H\$3+\$H\$4*D11	=E11-G11
12		III	39	4907.6		=H\$3+\$H\$4*D12	=E12-G12
13		IV	40	5021.5		=H\$3+\$H\$4*D13	=E13-G13
14	11	I	41	5122.1		=H\$3+\$H\$4*D14	=E14-G14
15		II	42	5392.3		=H\$3+\$H\$4*D15	=E15-G15
16		III	43	7238.7		=H\$3+\$H\$4*D16	=E16-G16
17		IV	44	7219.5		=H\$3+\$H\$4*D17	=E17-G17
18							
19						ME	=AVERAGE(H6:H17)
20						MAD	=AVERAGE(ABS(H6:H17))
21						RMSE	=SQRT(SUMSQ(H6:H17)/COUNT(H6:H17))
22						RSE	=SQRT(SUMSQ(H6:H17)/(COUNT(H6:H17)-COUNT(H3:H4)))

(b)

EXHIBIT 7.9 (a) Forecasts and errors for hold-out segment with goodness-of-forecast statistics. (b) Code used to create Exhibit 7.9a.

7.4.1.2 Mean Absolute Deviation

The mean absolute deviation (*MAD*) provides another perspective on the accuracy of the forecasting models. It gives us the magnitude of the average error, in either direction, without considering the sign. The absolute values of the error terms are summed and divided by the number of observations, as shown in Equation 7.16.

$$MAD = \frac{\sum_{i=1}^n |e_i|}{n} \quad (7.16)$$

The constant-change model's *MAD* of 1,029.5 versus 1,354 for the compound growth model, telling us the constant-change model makes smaller forecasting errors on average.*

7.4.1.3 Root Mean Square Error

The root mean squared error (*RMSE*) is the square root of the average squared error term. It is a better indicator of the accuracy of the forecast model than *ME* or *MAD* when avoiding the occasional large error is important. Squaring the error term overweights large errors. Hence, *RMSE* penalizes models with a few large errors versus models with multiple small to midsized errors. The formula for *RMSE* is shown in Equation 7.17.

$$RMSE = \sqrt{\frac{\sum_{i=1}^n e_i^2}{n}} \quad (7.17)$$

The fact that *RMSE* for the constant-change model is smaller than for the compound growth model (1,189 versus 1,515) is additional evidence that the constant-change model yields better forecasts for the hold-out period.

7.4.1.4 Residual Standard Error

The residual standard error, also known as the standard error of the estimate, is a more robust standard for assessing the forecasting accuracy of a model. Up to now, the goodness-of-forecast statistics have been simple averages. None has taken into account the number of parameters estimated in the model. This is important, because as the number of explanatory variables (hence, estimated parameters) in a model increases, the model's forecasting accuracy can appear to increase even if the additional variables have little or no ability to explain the data. The goal of forecasting is to find the most parsimonious model or to find the model that gives the best forecasts with the fewest parameters. The residual standard error tells us if adding more variables produces better forecasts.

The first two steps in calculating the residual standard error are identical to those in calculating the root mean squared error. The error terms are squared and then summed. However, instead of dividing by the number of observations (*n*), we divide by the number of observations less the number of estimated parameters (*k*), (*n*−*k*). As the number of parameters in a model increases, the denominator decreases, thereby increasing the residual standard error. This penalizes more complex models: Unless the additional variables produce better forecasts, the residual standard error increases. Equation 7.18 gives the formula; Equation 7.19 shows the calculations for the *RSE* of the constant-change model.

* Calculating the *MAD* is quite simple in Excel if an array function is used. Excel's array functions operate on arrays of numbers rather than single numbers. In [Exhibit 7.11](#), the code for the *MAD* of the constant change model is =AVERAGE(ABS(F6:F17)). Array functions are entered by pressing Ctrl+Shift+Enter.

$$RSE = \left[\sum_{i=1}^n \frac{e_i^2}{(n-k)} \right]^{\frac{1}{2}} \quad (7.18)$$

where k is the number of estimated parameters in the model.

$$RSE = \sqrt{\frac{546^2 + 242^2 + (-1165)^2 + (-1229)^2 + (-998)^2 + (-1369)^2 + (-1669)^2 + (-1675)^2 + (-1695)^2 + (-1545)^2 + 181^2 + 42^2}{(12-2)}} \quad (7.19)$$

$$= 1,303$$

Once again, the constant-change model yields better forecasts than the compound growth model. In this case, all our statistics for evaluating the forecasts agree: the constant-change regression model works better for Speckled Band in the hold-out period. Note however, that the goodness-of-forecast statistics will not necessarily always agree. We could find that one model is less biased, but does not generate as good a forecast (a smaller ME but larger MAD); or, a model might yield smaller $RMSE$ but if the results are due to spurious correlations associated with using many irrelevant independent variables.

7.4.2 Combining the Estimation Period and the Hold-Out Period

To complete the forecasting process, reestimate the parameters for best model using the entire data set. For example, assume that we have tested several models including both the constant change and the compound growth regression models, and we have concluded that the constant-change model is our best forecaster. We now reestimate the constant-change model using all 44 quarters. The output from this regression is shown as [Exhibit 7.10](#). Our reestimated regression for sales as a function of time is now estimated as

$$Sales_t = 2,196.9 + 96.0 * Time_t \quad (7.20)$$

(7.71) (8.71)

$$R_{adj}^2 = 63.5\%$$

The diagnostic statistics are slightly improved over the shorter estimation period, so the regression seems robust. The regression parameter estimates are different from those of the original estimation period shown in [Exhibit 7.4](#). It is expected that they will vary a little with the addition of the data from the hold-out period. If the estimated parameters are substantially different, it may indicate that the link between sales and time had changed between the two periods. Or, perhaps there is something else besides the time trend that is important for our forecast.

7.4.3 The Last Step: Making the Forecast

Having reestimated the relationship between sales and time using data from the entire sample period, we now use the estimated equation to forecast future sales. Unfortunately, we will not know how accurate the forecasts are until the future unfolds. We want to

	G	H	I	J	K	L	M
4	SUMMARY OUTPUT						
5							
6	<i>Regression Statistics</i>						
7	Multiple R	0.80					
8	R Square	0.64					
9	Adjusted R Square	0.64					
10	Standard Error	928.65					
11	Observations	44.00					
12							
13	ANOVA						
14		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
15	Regression	1	65,405,896	65,405,896	75.84	0.00	
16	Residual	42	36,220,126	862,384			
17	Total	43	101,626,022				
18							
19		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
20	Intercept	2,196.91	284.84	7.71	0.00	1,622.09	2,771.74
21	Time	96.01	11.02	8.71	0.00	73.76	118.26

EXHIBIT 7.10 Regression output for whole sample period, constant-change model.

forecast the next 4 quarters: periods 45 through 48. Applying the reestimated regression to the 45th quarter, our forecast is

$$\begin{aligned} Sales_{45} &= 2,196.9 + 96.0 * 45 \\ &= 6,516.9 \end{aligned} \quad (7.21)$$

Each of the following quarters is forecasted similarly as shown in [Exhibit 7.11a](#) and charted in [Exhibit 7.11b](#). The bottom lines of the table show the confidence intervals for our forecasts, explained next.

7.4.4 Assessing Forecast Accuracy: Confidence Intervals

One advantage of quantitative over qualitative methods is that they provide a way to estimate the accuracy of the forecasts—a margin of error. This is particularly useful in financial modeling, where the goal is to determine the joint consequences of forecast events and decisions. As forecasts almost will never be correct, it is important to know how far off the mark they might be. This is what the confidence intervals tell us.

Even when the model is correctly specified, there are 3 interrelated sources of error. To see why, consider the regression model with 1 independent variable X_t , and dependent variable Y_t :

$$Y_t = a + b * X_t + e_t \quad (7.22)$$

First, even when the estimates of a and b are correct, there is the error term e_t . We assume that it is normally distributed with zero mean and standard deviation σ_e . This error term means that the actual Y will be distributed around the forecast Y due to pure chance. However, this assumes that our estimates of a and b are correct, bringing us to the second and third sources of error. We only estimated the regression coefficients, a and b . The true

	Period	45	46	47	48
Sales Forecast		6,516.9	6,612.9	6,708.9	6,804.9
95% Confidence interval					
Upper Bound		8,477	8,580	8,682	8,784
Lower Bound		4,557	4,647	4,737	4,827

(a)

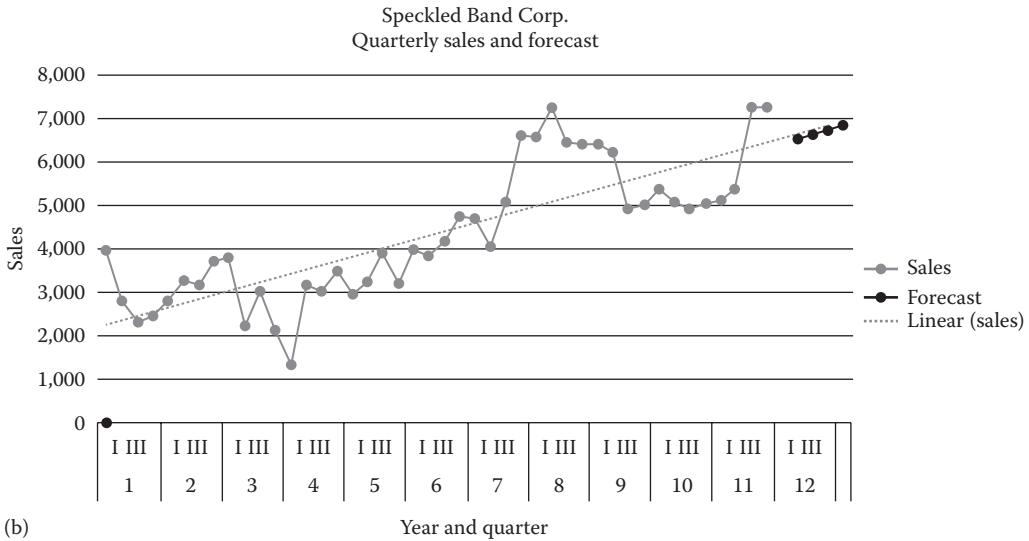


EXHIBIT 7.11 (a) Forecast of Speckled Band Corporation quarterly sales. (b) Speckled Band Corporation sales and forecasts.

coefficients fall within some confidence interval around the estimates. Given all of this opportunity for error, we know that our forecasts probably will be wrong. The question is, how wrong? Confidence intervals tell us the interval around our forecast in which we would expect the actual value to occur.

Confidence intervals are based on the standard error of the estimate, a statistic reported regression software. The standard error or *RSE* is converted to the standard error of the forecast. In turn, the standard error of the forecast is used to calculate the confidence intervals for the forecast.

When the estimated regression has considerable explanatory power (a high adjusted R^2 and small standard error), the confidence interval is a narrow band. We can be reasonably confident in our forecast. However, when the regression lacks explanatory power (a low adjusted R^2 and large standard error), the confidence interval is a wide band. We cannot have much confidence in our forecast.

The general expression for a confidence interval when the error terms are normally distributed is given by Equation 7.23, where Z is the standard unit-normal variable.*

* For smaller samples, we use the *t*-distribution instead of the standardized unit normal distribution, Z , to calculate the confidence interval.

$$\text{Confidence Interval}_t = \text{Forecast}_t \pm Z * SE_{\text{forecast}_t} \quad (7.23)$$

The confidence interval forms a band around the forecasts within which the true value of a predicted variable should fall a specified percentage of the time.

Three elements are needed to calculate confidence intervals: the forecast, the standard error of the forecast, and the critical statistic. The forecast is derived in the usual way. The other two arguments require some discussion. To explain the confidence interval, we will use the one factor (one independent variable) model for Speckled Band. When we have more than 1 independent variable it becomes more complicated, and the reader should consult a more specialized source such as Neter (1996).

7.4.4.1 The Standard Error of the Forecast

The standard error of the forecast SE_{forecast} measures the scatter of individual actual observations about the sample regression line. It is the estimated error when making a forecast for Y given X . The standard error of the forecast is the mechanism that adjusts the width of the confidence interval to reflect the distance from the center of the data. The formula is given in Equation 7.24.

$$SE_{\text{forecast}} = \sigma_e \left[1 + \frac{1}{n} + \frac{(X_f - \bar{X})^2}{\sum_{i=1}^n (X_i - \bar{X})^2} \right]^{\frac{1}{2}} \quad (7.24)$$

where:

σ_e is the standard error of the regression or the residual standard error from fitting the predicted values

X_f is the forecast for the independent variable

The confidence interval is narrowest at the center of the data, where the independent variable, X , and the dependent variable, Y , are both near their means. As the forecasts move away from the data's center (i.e., as X moves further from its mean, \bar{X}), the numerator of the third term inside the brackets (in Equation 7.24) increases at an increasing rate. This increases the confidence interval and induces the curvature of the confidence bands shown in [Exhibit 7.12](#). This curvature means that as we move away from our range of experience as represented by the center of the data, our forecasts become less and less accurate. In other words, if we extrapolate Y for values of X that are far from the values of X and Y that were used to estimate our model, we are less and less confident that our forecast will be accurate. If we use recent data to generate our forecast, we are more confident about our forecast for next year than we are forecasting 20 years into the future.

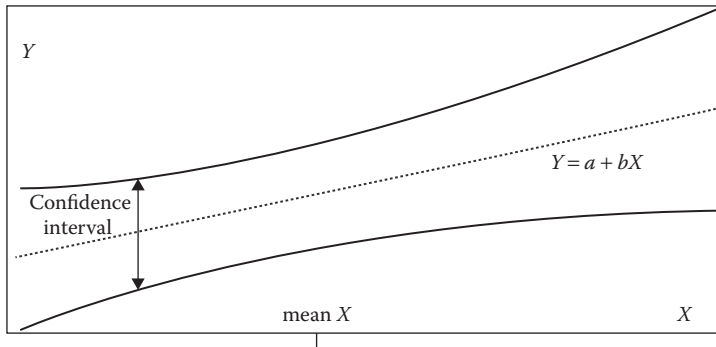


EXHIBIT 7.12 Forecast confidence intervals.

For our Speckled Band forecast, we want to calculate the standard error of the forecast for period 45 (quarter 12.I), the quarter following the last one in our sample period. The first step is to gather the following numbers from the regression output and a spreadsheet program.

$$\begin{aligned}
 n &= 4 \\
 \sigma_e &= 928.65 \\
 \bar{X} &= 22.5 \text{ (} X \text{ is Time)}
 \end{aligned}
 \tag{7.25}$$

$$\sum (X_i - \bar{X})^2 = 7,095$$

Next, use these numbers to calculate the standard error of the forecast from Equation 7.24. The standard error of the forecast for period 45 (where $Time = X = 45$) is calculated as

$$\begin{aligned}
 SE_{forecast} &= 928.65 * \left[1 + \frac{1}{44} + \frac{(45 - 22.5)^2}{7095} \right]^{\frac{1}{2}} \\
 &= 971.4
 \end{aligned}
 \tag{7.26}$$

7.4.4.2 The Critical Statistic

The last argument in the expression for the confidence interval is the critical statistic that specifies the width of the confidence interval. The confidence interval is the range, or band, around the value predicted by the regression model, within which we say the actual sales will be likely to occur. If we are willing to tolerate a higher likelihood of having the actual sales occurring outside this band, we would set a narrow confidence interval. For example, our model predicts that sales in period 45 will be \$6,516.9. However, we know that sales will not be exactly of this amount. Suppose we predict that sales will be within \$100 of this amount—between \$6,416.9 and \$6,616.9. Given the large standard error of the regression, there is a very low likelihood that sales will actually fall within this narrow interval. If we want greater

confidence that sales will actually turn out to be within the band we specify, then we need to set a wider band. For example, suppose we predict period 45 sales to be 6,516.9, plus or minus \$2,000—between \$4,516.9 and \$8,516.9. Clearly, we will be more confident that actual sales will be within this wide band than within the narrower \$100 band.

The critical statistic sets the width of the confidence band based on our specification of our desired probability of being right and the acceptable probability of being wrong. The link between the target probability and the width of the confidence band is the standard deviation (standard error). The critical statistic specifies the width of the confidence interval in standard deviation units. For example, suppose we can tolerate a 5% chance that we are wrong. This means we want a 95% chance that sales will actually be within the specified interval around the predicted value. If sales are normally distributed around the prediction, we can consult a normal probability table to determine the 95% bounds. For a normally distributed variable, the 95% bounds are mean -1.96σ and mean $+1.96\sigma$. The 1.96 is the critical statistic, specifying the number of standard deviations above and below the expected value such that there is a 95% chance that the actual value will be within this range. If the sample is small or the distribution is unknown, we use the t -statistic and the t -distribution table instead of the standard normal table to determine the critical statistic.

The current goal is to find the interval that is likely to contain the actual sales for period 45 ninety-five percent of the time. In this example, we use the t -statistic because we have a relatively small sample of 44 observations. The critical value of the t -statistic is determined by the degrees of freedom in our model and the level of confidence we desire. We used 44 observations to estimate our model's two parameters. This leaves 42 ($44 - 2$) degrees of freedom. Our desired level of confidence is 95% (2.5% of the observations should fall in either tail of the distribution). Therefore, we find the t -statistic for a two-tailed test and a 5% level of significance. From a t -distribution table or Excel's =TINV(.) function, the t -statistic is 2.018.* All of the pieces necessary to calculate the confidence interval are now in place. All that remains is to calculate the confidence interval:

$$\begin{aligned}
 E[Sales_{45}] &= 2,197 + 96.0 * 45 \\
 &= 6,517 \\
 \text{Confidence Interval} &= E[Sales_{45}] \pm t_{df=42, \alpha=0.05} * SE_{forecast} & (7.27) \\
 &= 6,517 \pm 2.018 * 971.4 \\
 &= 6,517 \pm 1,960 \\
 &= \text{lower limit of } 4,557 \text{ to upper limit of } 8,477
 \end{aligned}$$

This confidence interval is very wide, providing little confidence in our ability to forecast with the simple time trend (single factor) model. So what is wrong with our forecasting model? In the next section we discuss some of the problems with our model.

* For $n > 30$ or so, t - and Z -statistics are virtually identical.

7.5 PROBLEMS WITH THE FORECASTING MODEL

We have used a simple time trend regression model to show the method and steps for developing a forecasting model. However, we have seen that this model generates a forecast within a very wide range, so it is not as accurate as we might like. There are myriad reasons why we may not be able to generate a forecast that is as accurate as we would like. We discuss two of them. First, the data may be inconsistent with the assumptions of the regression model. Second, we may have the wrong model in that (a) sales may not really follow a trend, or (b) there may be other influences in addition to the trend.

1. Assumptions of the Regression Model

The reliability of linear regression depends on whether the data are consistent with the assumptions underlying the mathematics of regression. The major assumptions are

- a. The form of the relationship between the dependent variable, Y_t , and the independent variable, X_t , is a linear function of the form

$$Y_t = a + bX_t + e_t \quad (7.28)$$

- b. The error terms, e_t , are independent and normally distributed with mean zero and a standard deviation, σ_e , that is constant over time
- c. The error terms are uncorrelated between periods.

Assumption (a) is that we have the right model. If the real relationship is actually $Y_t = a + bX_t^2$, then Equation 7.28 is simply the wrong model. Similarly, if there are other variables, W and Z , that have an important influence on Y , and the actual relation is of the form

$$Y_t = a + bX_t + cW_t + dZ_t + e_t \quad (7.29)$$

then, Equation 7.28 may not yield reliable estimates of coefficients a and b , and may not provide a good forecast.

Assumption (b) is that the error terms are homoskedastic, or the dispersion of error terms around the regression line is the same at all values of the independent variable. If the spread of error terms around the regression line changes systematically with the independent variable, then the errors are said to be heteroskedastic. In that case the regression can still yield valid estimates of the regression coefficients, but the estimated standard errors will be biased.

Assumption (c) is that there is no serial correlation in the error terms. Serial correlation of error terms means that the error or deviation away from the regression line in one period is related to the size and direction of the error in a previous period. For example, our time series regression exhibits serial correlation. In [Exhibit 7.5](#), in the early period the data points tend to cluster above the regression line, in the middle periods they are all below the regression line, and in the last four periods they are above the line. This is a visual evidence of serial correlation of the error terms. The statistical evidence of

serial correlation is the Durbin–Watson (DW) statistic. A Durbin–Watson statistic near 2 indicates that serial correlation is not a problem. Positive serial correlation will result DW being below 2. DW will be between 2 and 4 if there is negative serial correlation. For Speckled Band sales for the 32-quarter estimation period, the DW statistic is 0.676.* This low value of DW is a strong indicator of positive serial correlation in the residuals.

Like heteroskedasticity, the presence of serial correlation does not necessarily invalidate estimates of the regression coefficients, but it does bias the estimates of their standard errors. This means, for example, that although our estimate of slope coefficient b is consistent and unbiased, the estimated standard error of b may be smaller than the actual standard error. In this case, we would be led to believe that the confidence interval for b is narrower than it really should be, and that the forecast is more accurate than it really is. We have already seen that the confidence interval for our forecast is very wide. With the presence of serial correlation, we conclude that this already wide confidence interval may be even wider than we think it is.

There are several reasons why we may have serial correlation in the regression. Frequently, we get serially correlated errors when we have left out of the model other factors that influence the dependent variable. In other words, we have omitted variables. If we add these omitted variables to the regression model, we will tend to get a better fitting model, and hopefully a better forecast. This is the subject of the next chapter.

7.6 SUMMARY

This chapter is an introduction to using linear regression to develop a simple time trend forecasting model. We used two simple models to demonstrate the basic methods of developing and testing a forecasting model. Though more complex models might be able to provide better forecasts, the methods for developing and testing the models would still follow the same basic steps.

The steps recommended for developing and testing the forecasting model includes the following:

- Develop hypotheses about what factors and variables affect the forecast variable.
- Gather data to test your hypotheses.
- Develop the models for testing.
- Separate the time series of data into an estimation period and a hold-out period.
- Test your alternative models on data from the estimation period.
- Evaluate the validity and fit of the models for the estimation period.
- Apply the superior models to forecasting the hold-out period.
- Assess which models generate the best forecast results for the hold-out period.

* Regression in Excel does not compute the Durbin–Watson statistic. Other regression software such as E-Views routinely shows this statistic.

- Reestimate the relationships for the whole sample period for the models that provide the best forecasts.
- Use the reestimated model to forecast period following the sample period.

Evaluating a forecasting model can differ from the standard methods for testing statistical hypotheses and evaluating regressions. The regression model that provides the best fit of the data may not necessarily provide the best forecast. For comparing different forecasting models, several summary measures were discussed as follows:

- Mean error.
- Mean absolute error.
- Root mean square error.
- Residual standard error.

In the next chapter we extend these methods to consider relationships more complex than a simple time trend.

PROBLEMS

1. Antsy Corporation

The following [Exhibit 7.13](#) reports the annual sales for Antsy Corp. over the past nine years. Use this information to answer questions a–d. Estimate your models using all nine years of data.

Antsy Corp.	
Year	Revenue
1	26.1
2	14.7
3	20.1
4	29.5
5	23.1
6	37.9
7	57.4
8	91.1
9	120.7

EXHIBIT 7.13 Antsy Corp. annual sales.

- Forecast Antsy's sales for years 10–13 using the simple average percentage increase method.
 - Forecast Antsy's sales for years 10–13 using the geometric average percentage increase method.
 - Forecast Antsy's sales, along with their 95% confidence intervals, for years 10–13 using the constant-change linear model.
 - Forecast Antsy's sales for years 10–13 using the compound-growth log-linear model.
- ### 2. Zephyr Corporation

The Zephyr Corporation markets wingbrats in the United States that it imports from Zongthia. Its wingbrats are of the highest quality, and the market is limited. However,

they are gaining in acceptance and popularity, and Zephyr's sales have been growing over the past eight years. The CFO of Zephyr has asked you to analyze their sales data and develop a simple forecasting model. Zephyr's quarterly sales for the past 7½ years are shown in [Exhibit 7.14](#) below.

Zephyr Corp.						
Year	Quarter	Sales		Year	Quarter	Sales
1	I	303.0		5	I	467.7
	II	278.9			II	560.6
	III	354.8			III	628.8
	IV	411.1			IV	556.5
2	I	422.9		6	I	587.4
	II	459.3			II	562.4
	III	317.7			III	571.1
	IV	410.6			IV	620.9
3	I	478.8		7	I	644.1
	II	406.5			II	605.3
	III	453.0			III	682.3
	IV	428.9			IV	703.8
4	I	504.8		8	I	740.4
	II	561.1			II	735.4
	III	572.9				
	IV	609.3				

EXHIBIT 7.14 Zephyr Corp. quarterly sales.

To develop your forecasting model, you will use the simple trend methods discussed in [Chapter 7](#). Begin by graphing the series. Next, determine which of the following methods yields the best fit with the historical data. The methods to consider are

- i. Average percentage increase per quarter
- ii. Geometric mean percentage increase
- iii. Constant-change linear model
- iv. Compound growth log-linear model

Evaluate the goodness-of-fit of each method using mean error, mean absolute deviation, root mean squared error, and residual standard error. Determine the method that yields the best fit, and then use that method to predict sales for the next four quarters (8.III through 9.II). Add your predictions to the initial graph of Zephyr's sales.

WHAT YOU TURN IN

Submit a report in memo format that describes the results of your work, including which goodness-of-fit statistic determined your choice of model and why you chose that statistic. Include a time-series graph of sales, actual and predicted, in your memo. Your graph should contain a title and axis labels. Also include a table that summarizes all of goodness-of-fit statistics similar to [Exhibit 7.9](#).

3. G&A Acquisitions

Imagine that you work for an econometric consulting firm. Imagine, too, that you have a client named G&A Acquisitions who is interested in acquiring Lindberg, Inc., a small consumer staples firm that has been growing steadily for the past 10 years. G&A has asked your firm to build a model to forecast Lindberg's quarterly sales over the next year and a half. They have supplied you with the data shown in [Exhibit 7.15](#) below.

Lindberg Corp.					
Year	Quarter	Sales	Year	Quarter	Sales
1	I	759.2	6	I	1,491.2
	II	761.4		II	1,674.4
	III	863.1		III	1,790.2
	IV	629.5		IV	1,704.6
2	I	838.8	7	I	1,825.2
	II	917.6		II	1,975.0
	III	962.0		III	1,795.8
3	I	978.8	8	I	1,667.4
	II	1,272.0		II	1,732.8
	III	1,051.4		III	1,956.0
	IV	1,325.0		IV	2,030.7
4	I	1,249.8	9	I	1,855.0
	II	1,378.2		II	2,088.7
	III	1,269.0		III	2,264.3
	IV	1,258.8		IV	2,057.7
5	I	1,301.5	10	I	2,180.6
	II	1,514.0		II	2,101.0
	III	1,602.0		III	2,417.4
	IV	1,397.5		IV	2,364.2
		1,556.3			2,116.3

EXHIBIT 7.15 Lindberg, Inc. quarterly sales.

Your task is to identify the model that best forecasts Lindberg's sales—the constant-change model or the compound-growth model. Begin by graphing Lindberg's sales. Estimate your preliminary models using the first seven years of data and test your models using the last three years of data. Measure their forecasting ability using the goodness-of-forecast statistics discussed in [Chapter 7](#) (*RMSE*, *ME*, *MAD*, and *RSE*).

After identifying the model that produces the best forecasts, reestimate that model using all 10 years of data. Label your graph appropriately. Finally, use this newly-estimated model to forecast sales for the next six quarters. Add the six forecasts to your original graph as a separate series so that your forecasts are clearly identified.

WHAT YOU TURN IN

Submit a report in memo format that describes the purpose and results of your work, including the criteria you used to select your forecasting model. Place the graphs of sales (including predicted sales) in the body of your memo. Also include a table in your memo that reports the model used (e.g., $Sales_t = 45 + 4 * Time_t$) and the sales predicted. In an appendix, place a well-organized print-out of the spreadsheet showing the goodness-of-forecast calculations you used to identify the best model. Use [Exhibit 7.9](#) as your template.

Forecasting with Econometric Models

CHAPTER 7 INTRODUCED FORECASTING using linear regression to extrapolate a time trend. For most companies, though, this model is too simple. Other forces can influence a company's sales. Most businesses are subject to the cycles of the economy, the vagaries of interest rates, and other economic and demographic factors. Knowledge of these links may improve our forecasts. In this chapter we explain how to account for these other economic influences and include them in a forecasting model. We call the resulting model a *structural econometric model* to reflect the fact that our company is affected by the structure and performance of its industry and the economy.

8.1 DEVELOPING A STRUCTURAL ECONOMETRIC MODEL

The first step in developing a structural econometric model is to state our hypotheses about the linkages between our company and economic variables that influence it. A good starting point is a description of our company and how we think it is related to other parts of the economy. Armed with this description, we can refine our hypotheses until we have captured the most important influences.

In this regard, keep the list short. The goal is to create a *parsimonious* model, one that uses only the most important variables, each of which captures a different aspect of the company's relationship with the economy. Indiscriminately including many different variables can distort our forecast with spurious correlation, meaning we have added variables that add little real explanatory power to our model. In addition, we do not want multiple variables that measure the same influence. Highly correlated independent variables in a regression model reduce the reliability of the estimated relationships in the model. Finally, keep in mind that we are building a forecasting model. Any variable that we use as an input to the forecast must, itself, be known or more easily forecasted than the variable being forecasted.

Once the hypotheses are formulated about the variables that influence our company, we gather the data, estimate the models, and then test the hypotheses. Based on a statistical analysis, some models will be eliminated immediately. The few remaining are then tested for their forecasting ability.

8.1.1 Example: Speckled Band, Inc.

Our task once again is to develop a forecasting model for Speckled Band, Inc., first introduced in [Chapter 7](#). Its quarterly sales data are reprised in [Exhibit 8.1](#). In [Chapter 7](#) we used simple linear regression to provide a time trend forecast of Speckled Band's sales. However, the time trend forecast is unlikely to be reliable because the confidence intervals are so wide. Also, we found that the residuals were serially correlated. This led us to suspect that we had omitted other important variables from our regression. We should be able to improve on our time trend forecasting model if we take account of the variables that we did not consider in the time trend model.

The next step is to decide what variables we might have overlooked. This is where our hypothesis building comes into play. We need to think about the industry and economic environment of the company and specify the factors that we think affect Speckled Band's sales.

Speckled Band, Inc.							
Year	Quarter	Time	Sales	Year	Quarter	Time	Sales
1	I	1	3,942.8	7	I	25	4,698.2
	II	2	2,822.6		II	26	4,056.8
	III	3	2,328.2		III	27	5,074.0
	IV	4	2,457.8		IV	28	6,597.9
2	I	5	2,792.3	8	I	29	6,562.9
	II	6	3,256.4		II	30	7,226.5
	III	7	3,177.5		III	31	6,423.4
	IV	8	3,726.1		IV	32	6,409.5
3	I	9	3,805.3	9	I	33	6,400.5
	II	10	2,245.9		II	34	6,216.7
	III	11	3,047.3		III	35	4,930.6
	IV	12	2,105.6		IV	36	4,986.7
4	I	13	1,339.3	10	I	37	5,338.0
	II	14	3,189.8		II	38	5,087.5
	III	15	3,022.8		III	39	4,907.6
	IV	16	3,498.1		IV	40	5,021.5
5	I	17	2,947.6	11	I	41	5,122.1
	II	18	3,237.3		II	42	5,392.3
	III	19	3,917.1		III	43	7,238.7
	IV	20	3,230.1		IV	44	7,219.5
6	I	21	3,985.3				
	II	22	3,825.6				
	III	23	4,159.0				
	IV	24	4,746.9				

EXHIBIT 8.1 Speckled Band, Inc. quarterly sales.

8.1.1.1 The Economic and Industry Context

Speckled Band manufactures industrial efficiency equipment used by other manufacturers. As the products it sells constitute capital investment for the buyers, we would expect that Speckled Band's sales are cyclical, tracking the overall level of industrial equipment purchases in the economy. When the economy turns up, manufacturers increase investment in additional equipment; when the economy turns down, they decrease those investments. Consequently, we expect Speckled Band's sales to be highly correlated with the general level of economic activity. In addition, if sales of their equipment follow the pattern of other capital equipment, we would expect that their sales would tend to decline if interest rates increase, because higher rates decrease the present value of their benefits for the purchasers.

However, discussions with the sales manager of Speckled Band suggest that this relationship might go the other way. In other words, he has observed that the company's sales cycle seems to follow the interest rate cycle. His observation is that rates increase when demand for industrial investment is high, so that even though high rates tend to decrease the NPV of customer purchases, this happens when demand is coincidentally high. So high rates are a measure of high demand for capital goods, and this effect tends to overshadow the NPV-depressing effect of interest rates. Given these observations, we should be able to use the links between economic activity, interest rates, and company sales to develop a forecasting model. Our hypothesis is that Speckled Band's sales will increase with the level of economic activity as measured by gross domestic product (GDP) and the level of interest rates as measured by the rate on *T-Note*.

The starting point of the analysis is to plot Speckled Band's sales over time ([Exhibit 8.2](#)) and look for possible influences in the data. In the last chapter we confirmed a time trend in sales, with sales growing from about \$4,000 to more than \$7,200 over 11 years. However, the plot in [Exhibit 8.2](#) does not show purely random variation about a trend. Rather, sales

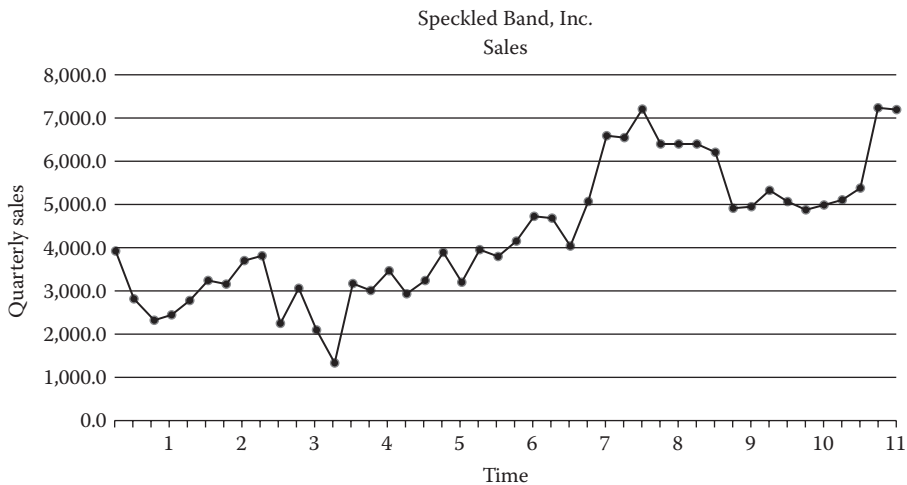


EXHIBIT 8.2 Speckled Band, Inc. sales.

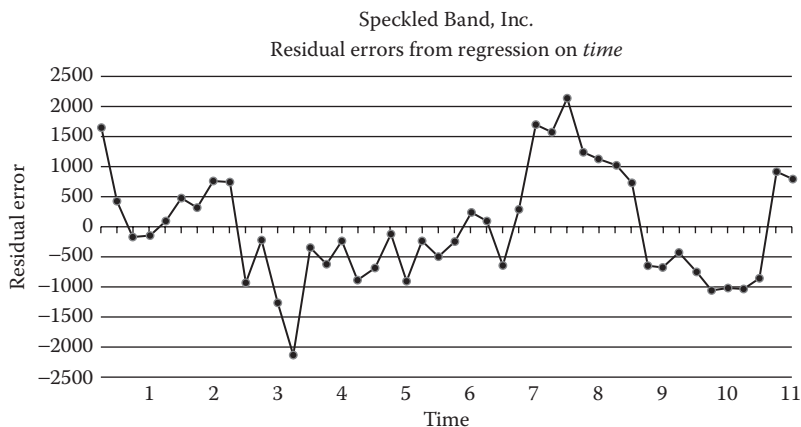


EXHIBIT 8.3 Speckled Band, Inc.: Residual plot from simple time trend regression.

tend to remain flat for the first six years, grow rapidly between years 6 and 9, only to fall back and then grow again in years 9 through 11. This pattern could indicate the presence of a cyclical influence.

A cyclical element, if present, can be made more visible by graphing the residual error terms that result from fitting a simple trend line. Most regression packages offer the option of reporting the residual errors from a regression. (In Excel, simply check the *Residuals* box in the regression wizard.) The residual plot for Speckled Band is shown in [Exhibit 8.3](#). The error terms do not appear to be random. They first decrease, then remain negative beginning in year 3, then increase beginning in year 7, then decrease again in year 9. Each error appears to be related to the errors that immediately precede or follow it, producing a smooth, nonperiodic wave-like pattern, a pattern similar to that identified in [Exhibit 8.2](#). Such a pattern may indicate the presence of a long-term cycle.

Given our previous conjecture about the relationship between Speckled Band's sales and the business cycle, [Exhibit 8.3](#) suggests that adding variables that capture the business cycle could improve our model's forecasts. We hypothesize the following model, using *GDP* and the 10-year Treasury Note (*T-Note*) rate to capture the influence of the business cycle.

$$\text{Sales}_t = a + b_1 * \text{GDP}_t + b_2 * \text{T-Note rate}_t + b_3 * \text{Time}_t \quad (8.1)$$

The data needed to estimate and test this model are given in [Exhibit 8.4](#). As we did in [Chapter 7](#), we partition the data into a 32-observation estimation segment and a 12-quarter hold-out period. Regression results for model 8.1 using the 32-observation estimation period are shown in [Exhibit 8.5](#).

The model appears to explain much of the variation in Speckled Band's sales. The adjusted R^2 is 93%, as compared to 59% for the time trend regression, and the t -statistics confirm the link between sales and *GDP* and interest rates. However, there are other statistics that we should check. First, as with the time trend model, we should check for serial

Speckled Band, Inc. Quarterly Sales & Economic Data											
Year	Quarter	Sales	Time	GDP	10-year TN	Year	Quarter	Sales	Time	GDP	10-year TN
1	I	3,942.8	1	6,542.7	6.6	7	I	4,698.2	25	9,066.6	4.72
	II	2,822.6	2	6,612.1	5.97		II	4,056.8	26	9,174.1	5.18
	III	2,328.2	3	6,674.6	5.81		III	5,074.0	27	9,313.5	5.79
	IV	2,457.8	4	6,800.2	5.33		IV	6,597.9	28	9,519.5	6.11
2	I	2,792.3	5	6,911.0	5.75	8	I	6,562.9	29	9,629.4	6.66
	II	3,256.4	6	7,030.6	6.97		II	7,226.5	30	9,822.8	5.99
	III	3,177.5	7	7,115.1	7.3		III	6,423.4	31	9,862.1	6.05
	IV	3,726.1	8	7,232.2	7.74		IV	6,409.5	32	9,953.6	5.74
3	I	3,805.3	9	7,298.3	7.78	Hold Out Period					
	II	2,245.9	10	7,337.7	7.06						
	III	3,047.3	11	7,432.1	6.28	9	I	6,400.5	33	10,024.8	5.16
	IV	2,105.6	12	7,522.5	6.04		II	6,216.7	34	10,088.2	5.14
4	I	1,339.3	13	7,624.1	5.65	III	4,930.6	35	10,096.2	5.24	
	II	3,189.8	14	7,776.6	6.51	IV	4,986.7	36	10,193.9	4.57	
	III	3,022.8	15	7,866.2	6.87	10	I	5,338.0	37	10,329.3	5.04
	IV	3,498.1	16	8,000.4	6.53		II	5,087.5	38	10,428.3	5.21
5	I	2,947.6	17	8,113.8	6.58	III	4,907.6	39	10,542.0	4.65	
	II	3,237.3	18	8,250.4	6.89	IV	5,021.5	40	10,623.7	3.94	
	III	3,917.1	19	8,381.9	6.22	11	I	5,122.1	41	10,735.8	4.05
	IV	3,230.1	20	8,471.2	6.03		II	5,392.3	42	10,846.7	3.96
6	I	3,985.3	21	8,586.7	5.54	III	7,238.7	43	11,107.0	3.98	
	II	3,825.6	22	8,657.9	5.64	IV	7,219.5	44	11,262.0	4.29	
	III	4,159.0	23	8,789.5	5.46						
	IV	4,746.9	24	8,953.8	4.53						

EXHIBIT 8.4 Speckled Band, Inc.: Quarterly sales and economic data.

SUMMARY OUTPUT							
<i>Regression Statistics</i>							
Multiple R		0.97					
R Square		0.94					
Adjusted R Square		0.93					
Standard Error		376.72					
Observations		32					
ANOVA							
		<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
Regression		3	61,322,342	20,440,781	144.03	0.00	
Residual		28	3,973,792	141,921			
Total		31	65,296,134				
		<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept		(61,514.41)	5,178.47	(11.88)	0.00	(72,122.02)	(50,906.80)
Time		(970.30)	92.85	(10.45)	0.00	(1,160.49)	(780.11)
GDP		9.81	0.82	11.90	0.00	8.12	11.50
10-year TN		255.36	95.80	2.67	0.01	59.11	451.61

EXHIBIT 8.5 Regression output from model (8.1).

	<i>Sales</i>	<i>Time</i>	<i>GDP</i>	<i>10-year TN</i>
<i>Sales</i>	1.000			
<i>Time</i>	0.777	1.000		
<i>GDP</i>	0.819	0.997	1.000	
<i>10-year TN</i>	(0.152)	(0.389)	(0.383)	1.000

EXHIBIT 8.6 Correlation matrix.

correlation of the error terms. Second, we want to make sure that the independent variables are not too closely related to each other.

We check for serial correlation of the residuals with the Durbin–Watson (DW) statistic. If the DW statistic is near 2.0, it indicates that serial correlation among the error terms is not a problem. For this regression, the DW statistic is 1.96.* The inclusion of the two economic variables has eliminated the serial correlation that was present in the time trend model.

The second issue is whether there is multicollinearity in the model—or whether the independent variables are highly correlated. If they are, estimates of their regression coefficients are unreliable. Multicollinearity is a common problem in regressions with time series data where some or all of the variables are following a similar time trend. This model has two variables that could be highly related: *Time* and *GDP*. If *GDP* tends to follow a trend and increase with time, *GDP* and *Time* will be highly correlated. In Excel we quickly check for correlation with the correlation tool in Data Analysis on the Data ribbon. Click on *Data Analysis* and select *Correlation*. This produces the correlation matrix shown in Exhibit 8.6. We see that all the independent variables are correlated. But the relation that draws our attention is the 99.7% correlation between *GDP* and *Time*, which is considerably higher than the correlation between *Sales* and *Time*. This very high correlation means that the estimated coefficients may not be reliable.

To obtain more reliable estimates, we can eliminate one of the highly correlated independent variables, or perhaps change the form of the model. For example, we could eliminate *Time* and use *GDP* to capture both the trend and cyclical aspects of *Sales*. However, we would like to separate these two components in our model so we can see the effects of the business cycle separate from the trend. One way to do this is to keep the trend variable, *Time*, and measure the cyclical component of *GDP* that is independent of the time trend. One way to do this is to estimate the trend of *GDP* with the regression

$$GDP_t = a + b * Time_t + u_t \quad (8.2)$$

and then use the residuals from that regression, u_t , to measure the *GDP* cycle around its trend. To do this in Excel, click *Residuals* in the Regression dialog box when setting up this regression. The regression output will then report the residuals, u_t , for each of the 32 periods in our sample. These residuals then are used as a new independent variable that measures the cyclical component of *GDP*, denoted by $GDPu_t$. Our revised sales regression becomes

* Regression in Excel does not provide the Durbin–Watson statistic, whereas, most statistical software provides it. The E-Views program was used to check these other statistics such as Durbin–Watson.

SUMMARY OUTPUT						
<i>Regression Statistics</i>						
Multiple R	0.97					
R Square	0.94					
Adjusted R Square	0.93					
Standard Error	376.72					
Observations	32					
<i>ANOVA</i>						
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>	
Regression	3	61,322,342	20,440,781	144.03	0.00	
Residual	28	3,973,792	141,921			
Total	31	65,296,134				
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	430.47	657.37	0.65	0.52	(916.09)	1,777.02
Time	108.18	8.06	13.42	0.00	91.66	124.69
GDPu	9.81	0.82	11.90	0.00	8.12	11.50
10-year TN	255.36	95.80	2.67	0.01	59.11	451.61

EXHIBIT 8.7 Regression output for model (8.3).

$$Sales_t = a + b_1 * GDPu_t + b_2 * T\text{-Note rate}_t + b_3 * Time_t + e_t \quad (8.3)$$

The output for the three-factor least squares regression is shown in Exhibit 8.7 for the 32-quarter estimation period.

The fitted model is

$$Sales_t = 430.5 + 9.8 * GDPu_t + 255.4 * T\text{-Note rate}_t + 108.2 * Time_t \quad (8.4)$$

Its adjusted- R^2 of 0.93 indicates a very good fit: the three-factor model explains 93% of the variation in Speckled Band's quarterly sales. In addition, the t -statistics for each of the explanatory variables are significant at the 1% level of confidence, telling us that each variable contributes important information to our forecasts. In spite of the apparent good fit, the standard error of the regression, 376.7, is about 10% of the mean sales level. This is a relatively large standard error for a forecasting model, and it will lead to wide confidence intervals around the forecasts. Statistics not shown are the DW, which at 1.96 shows no evidence of serial correlation, and the correlations between the independent variables that show no evidence of multicollinearity. We conclude that adding $GDPu$ and the $T\text{-Note rate}$ to the time trend model appears to capture much of the cyclical component in Speckled Band's sales.

These statistics, however, do not tell us which model (the time trend model or the three-factor model) will produce the most accurate forecasts. To answer this question, we use the models to forecast sales in the hold-out segment of our data (observations 33–44), then calculate goodness-of-forecast measures for both sets of forecasts. To calculate forecasts, plug historical values for $Time$, the $T\text{-Note rate}$, and $GDPu$, into each model, as appropriate. For example, during the first quarter of the year 9, $Time$ was 33, the $T\text{-Note rate}$ was 5.16%, and the $GDPu$ was 84.3. Plugging these values into the three-factor model (Equation 8.4)

produces the predicted sales for the first quarter of year 9, \$6,145. Plugging the value for *Time* into the one-factor model (Equation 8.5) produces the predicted sales for the first quarter of year 9 of \$5,855.

$$\begin{aligned}
 Sales_{33} &= 430.5 + 9.8 * GNPu_{33} + 255.4 * T\text{-}Note\ rate_{33} + 108.2 * Time_{33} \\
 &= 430.5 + 9.8 * 84.3 + 255.4 * 5.16 + 108.2 * 33 \\
 &= 6,145
 \end{aligned}
 \tag{8.4}$$

$$\begin{aligned}
 Sales_t &= 1886.2 + 120.3 * Time_t \\
 &= 1886.2 + 120.3 * 33 \\
 &= 5,856
 \end{aligned}
 \tag{8.5}$$

Repeating the calculations for each quarter through quarter 44, then calculating the error terms and goodness-of-forecast statistics, produces the table shown in [Exhibit 8.8](#). Comparing the goodness-of-forecast statistics shows that the three-factor model produces the more accurate forecasts. Its root mean squared error and mean absolute deviation are smaller, suggesting that the three-factor model produces fewer large forecast errors; its mean error is smaller (on an absolute basis), suggesting that it is less biased; and its residual

Speckled Band, Inc. Quarterly Sales & Economic Data									
Year	Quarter	Sales	Time	GDPu	10-year TN	1-Factor Model		3-Factor Model	
						Forecast	Error	Forecast	Error
9	I	6,400.5	33	84.3	5.16	5,855	546	6,145	255
	II	6,216.7	34	37.8	5.14	5,975	242	5,792	425
	III	4,930.6	35	-64.1	5.24	6,095	(1,165)	4,926	5
	IV	4,986.7	36	-76.4	4.57	6,216	(1,229)	4,743	244
10	I	5,338.0	37	-50.9	5.04	6,336	(998)	5,221	117
	II	5,087.5	38	-61.8	5.21	6,456	(1,369)	5,266	(178)
	III	4,907.6	39	-58.0	4.65	6,576	(1,669)	5,268	(360)
	IV	5,021.5	40	-86.2	3.94	6,697	(1,675)	4,918	103
11	I	5,122.1	41	-84.0	4.05	6,817	(1,695)	5,076	46
	II	5,392.3	42	-83.0	3.96	6,937	(1,545)	5,171	222
	III	7,238.7	43	67.3	3.98	7,057	181	6,759	480
	IV	7,219.5	44	112.4	4.29	7,178	42	7,389	(169)
							ME	(861)	99
							MAD	1,030	217
							RMSE	1,189	258
							RSE	1,303	316
Model Parameters									
Intercept						1,886.20	430.47		
GDPu						---	9.81		
T-Note						---	255.36		
Time						120.26	108.18		

EXHIBIT 8.8 Forecast sales and goodness-of-forecast statistics.

Speckled Band, Inc.						
Forecast Sales & Cyclical Variables						
Year	Quarter	Sales	Time	GDP		
				Residual	10-year TN	
12	I	7,594	45	115.1	4.15	
	II	7,726	46	119.0	4.08	
	III	7,855	47	124.0	3.95	
	IV	7,979	48	130.0	3.75	

EXHIBIT 8.9 Speckled Band, Inc.: Forecast sales and cyclical variables.

standard error is smaller, suggesting that the two additional variables (GDP_u and $T\text{-Note rate}$) significantly increase the model's explanatory ability.*

The three-factor model is clearly the superior forecasting model. However, before using it to forecast the sales for year 12, we reestimate it using all 11 years of data. Doing so yields our final forecasting model (Equation 8.6).

$$E[\text{Sales}_t] = 624.7 + 9.95 * E[GDP_u_t] + 223.8 * E[T\text{-Note}_t] + 108.8 * \text{Time}_t \quad (8.6)$$

Note the terms $E[GDP_u_t]$ and $E[T\text{-Note}_t]$. In order to forecast future sales, we must first forecast future values of GDP_u and $T\text{-Note rates}$, adding still more uncertainty to our forecasts.† In this case, suppose we consult various services that provide forecasts of national economic data. We use these forecasts as input to our sales forecasting model. The resulting forecasts are shown in Exhibit 8.9 for the next four quarters.

8.1.2 Confidence Interval of the Forecast

The next step in our forecasting process is to calculate the confidence intervals for our forecast so we have a better idea how far from the forecast the actual sales might turn out to be. In this multivariate case, we use the standard error of the regression to calculate the confidence interval.‡ The standard error of the regression associated with estimating Equation 8.6 for

* The residual standard errors were calculated using $(n - 2)$ for the one-factor and $(n - 4)$ for the three-factor model.

† The forecast of the GDP residual, GDP_u , warrants some explanation. The GDP in quarter 44 was \$11,262 billion. Economists' consensus forecast for each of the next 4 quarters is shown in the table below as the *Actual GDP Forecast*. The time trend regression of GDP (Equation 8.2) was estimated to be $GDP_t = 6313.3 + 109.9 * \text{Time}_t$. This time trend model yields the Time Trend GDP Forecast shown in the table. The difference between the amounts actually forecast, and the time trend regression forecast provides the residuals, GDP_u , for the next four quarters that are the input to our forecasting model:

	Quarter 45	Quarter 46	Quarter 47	Quarter 48
Actual GDP Forecast	11,374.6	11,488.4	11,603.3	11,719.3
Time Trend GDP Forecast	11,259.5	11,369.4	11,479.3	11,589.3
Difference (Residual)	115.1	119.0	124.0	130.0

‡ In Chapter 7 we calculated the confidence interval for the forecast when we had one independent variable. Based on the standard error of the forecast calculated according to Equation 8.15, we saw that the confidence interval widened as we forecasted further into the future. However, when we have several independent variables, the calculation and application of the equivalent standard error of the forecast is beyond the scope of this book. Consequently, in this example we use the standard error of the regression as given by the Excel regression routine. The resulting confidence intervals are slightly narrower than their true values. However, the differences are quite small.

the whole estimation period of 44 quarters was 342.06, with 40 degrees of freedom $[(n - k) = (44 - 4)]$. The 95% confidence interval for each forecast period, t , will be

$$\begin{aligned} E[\text{Sales}_t] \pm t_{\alpha, n-k} * \text{Standard Error} &= E[\text{Sales}_t] \pm 2.021 * 342.06 \\ &= E[\text{Sales}_t] \pm 691.3 \end{aligned} \quad (8.7)$$

where $t_{\alpha, n-k} = t_{0.05, 40} = 2.021$, obtained with the Excel function =TINV (0.05, 40). The forecasts and the upper and lower 95% confidence bounds are shown in [Exhibit 8.10](#).

Quarter	Lower Bound	Sales Forecast	Upper Bound
45	\$6,902	\$7,593	\$8,587
46	7,035	7,726	8,417
47	4,164	7,855	8,546
48	7,288	7,979	8,670

EXHIBIT 8.10 Speckled Band, Inc.: Forecast sales and confidence intervals.

These confidence intervals are considerably narrower than those for the time trend model. Nevertheless, at about 9% on each side of the forecast value they still are quite wide. And, of course, these confidence intervals are slightly understated due to the fact that they do not take account of possible errors in the regression coefficients.

If we are not satisfied with the accuracy of this forecasting model, we could try to improve the model, perhaps by looking for independent variables with greater explanatory ability. However, the costs in time and effort of finding such variables are likely to exceed the benefit of the additional accuracy. As with nearly everything else in business, the costs of finding a better solution (in this case, a model) must be balanced against the benefits of that better solution (in this case, improved accuracy).

8.2 SUMMARY

This chapter presented the basic steps for developing an econometric forecasting model. Our goal was to use economic data to capture the effect of the economy on a variable of interest, the target variable. We began by hypothesizing the relationships between the target variable and several econometric variables in very general terms. We then moved to the specific example of forecasting Speckled Band's quarterly sales. We hypothesized a linear relationship between Speckled Band's sales and several economic variables to be used as a forecasting model. We then estimated the parameters of that model using multiple linear regression. Next, we measured the forecasting ability of our model. Finally, we used the model to forecast future sales.

This and the previous chapter considered how to build forecasting models for time series that exhibit increasing or decreasing trends through time. Sometimes, however, a time series exhibits no trend whatsoever, but instead varies randomly about a central tendency. Sometimes, too, more recent data contains more information about the future than data further in the past. In these situations, smoothing techniques are useful short-term forecasting tools. We consider several of these next.

PROBLEMS

1. Speckled Band

In [Chapters 7](#) and [8](#) we presented quarterly sales data for the Speckled Band Corporation and developed 2 regression models. The one-factor model used time as the single independent variable, and the three-factor model used time, interest rates, and *GDP*. However, we did not assess the effectiveness of basing our forecasts on other combinations of these 3 variables. For this exercise, you are asked to develop, test, and compare regression forecast models that use (a) *GDP* only, (b) interest rates (only), (c) *GDP* and time, and (d) interest rates and time.

2. Corrigan, Inc.

The forecasting firm for which you work has been hired by Corrigan, Inc. to forecast Corrigan's sales for the next four quarters. Corrigan supplies industrial equipment to small and mid-sized businesses. Corrigan's management suspects its sales are influenced by the macroeconomy and general level of interest rates. To this end, they have supplied you with quarterly time series for the percentage change in hours worked (to capture the overall economy) and the annualized yield on the *T-Note* (to capture interest rates).

One aspect of the data complicates the model selection process. The economy experienced robust growth over the early years the data was collected. This could produce spurious results in your regression models. It means that you will need to rely more on goodness-of-forecast measurements and less on regression statistics to identify the best model for forecasting.

Your task is the following. Estimate the hypothesized models for Corrigan's sales shown below using the first seven years of data and test their forecasting accuracy using the last four years of data.

Hypothesized forecasting models for Corrigan, Inc.

$$Sales_t = a + b_1 * Trend_t$$

$$Sales_t = a + b_1 * Trend_t + b_2 * Hours_worked_t$$

$$Sales_t = a + b_1 * Trend_t + b_3 * T_bond_yield_t$$

$$Sales_t = a + b_1 * Trend_t + b_2 * Hours_worked_t + b_3 * T_bond_yield_t$$

After identifying the model that produces the best forecasts, reestimate that model using all 11 years of data. Have Excel report of the residuals so that you can graph them and look for evidence of a missing factor. Label your graph appropriately. Finally, forecast sales for the next four quarters. In Excel, draw a graph that shows Corrigan's actual and predicted quarterly sales ([Exhibit 8.11](#)). Draw actual and predicted sales as separate series, so that your forecasts are clearly identified. Be sure the company name appears in the graph title.

Corrigan, Inc.											
Economic Data						Economic Data					
Change in						Change in					
Quarterly			Worked			Quarterly			Worked		
Year	Quarter	Trend	Sales	Hours	T-bond	Year	Quarter	Trend	Sales	Hours	T-bond
1	I	1	680.2	-6.02	6.60	7	I	25	778.2	2.97	4.72
	II	2	683.9	-6.18	5.97		II	26	837.4	2.52	5.18
	III	3	761.8	-6.33	5.81		III	27	788.4	2.46	5.79
	IV	4	664.9	-5.09	5.33		IV	28	806.8	1.91	6.11
2	I	5	632.8	-4.34	5.75	8	I	29	776.3	1.05	6.66
	II	6	648.1	-3.89	6.97		II	30	922.2	0.30	5.99
	III	7	670.7	-3.35	7.30		III	31	1004.5	-0.16	6.05
	IV	8	689.9	-3.00	7.74		IV	32	1013.4	-0.71	5.74
3	I	9	746.1	-1.86	7.78	9	I	33	1032.4	-0.37	5.16
	II	10	536.9	-0.01	7.06		II	34	1082.8	-0.22	5.14
	III	11	600.5	1.53	6.28		III	35	1060.1	-0.77	5.24
	IV	12	523.6	3.08	6.04		IV	36	1131.6	-0.43	4.57
4	I	13	551.7	3.62	5.65	10	I	37	1035.1	-0.78	5.04
	II	14	434.4	4.57	6.51		II	38	1202.3	-1.44	5.21
	III	15	435.9	4.92	6.87		III	39	1238.2	-1.89	4.65
	IV	16	424.1	5.46	6.53		IV	40	1279.9	-3.45	3.94
5	I	17	579.4	5.31	6.58	11	I	41	1224	-3.60	4.05
	II	18	619.1	4.95	6.89		II	42	1327.7	-4.26	3.96
	III	19	477.1	4.60	6.22		III	43	1434.7	-4.41	3.98
	IV	20	623.6	4.04	6.03		IV	44	1491.8	-5.16	4.29
6	I	21	542.4	4.29	5.54	12	I	45		-5.44	4.16
	II	22	655.6	3.63	5.64		II	46		-5.92	4.01
	III	23	732.0	3.38	5.46		III	47		-6.15	3.89
	IV	24	744.1	3.13	4.53		IV	48		-5.91	4.05

EXHIBIT 8.11 Corrigan, Inc.: Quarterly sales and economic data.

WHAT YOU TURN IN

Submit a report in memo format that describes the purpose and results of your work, including the criteria you used to select your model. Place the graph of actual and predicted sales in the body of your memo. Include a table in your memo that reports the model you used to predict sales (e.g., $Sales_t = 45 + 4 * Time_t$) and the sales predicted. In your appendix, place clearly labeled graphs of the error terms, before and after model identification, where the *before* error terms are those from a simple time trend regression. In addition, include a well-organized print-out of the spreadsheet showing the goodness-of-forecast calculations you used to identify the best model.

A suggestion:

To help organize your work, we suggest you use three worksheets: (1) one to run the initial time trend regression and plot the resulting error terms; (2) another to determine the best-forecasting model; (3) to reestimate the final model, forecast sales, and draw the graphs for sales and the residuals from your final model.

3. Extravagant Goodies, Inc.

You have been assigned to forecast the next four quarters of sales for Extravagant Goodies, Inc (hereafter EGI). EGI is a retailer of high-end products—fancy toys for individuals with more disposable income than common sense. Its sales have been growing fairly steadily over the past seven years. EGI's CFO believes their sales are closely related to overall economic activity and/or inflation. Sales tend to decrease during recessions, increase during expansions, and increase during periods of inflation. As a result, their sales are highly cyclical. Management believes that sales respond to expectations for economic growth (measured by the Treasury spread).^{*} In addition, casual observation suggests that EGI's sales increase with the Consumer Price Index (CPI).

In the below [Exhibit 8.12](#), find 7.5 years of quarterly sales data and concurrent time series for the Treasury spread and the CPI. The data also includes forecasts for both the Treasury spread and CPI over the next four quarters.

Extravagant Goodies, Inc. (EGI)											
Year	Quarter	Time	T_sprd	Change	Quarterly	Year	Quarter	Time	T_sprd	Change	Quarterly
				CPI	Sales					CPI	Sales
1	I	1	1.54	0.71	274.7	5	I	17	4.13	0.54	321.4
	II	2	0.52	0.95	320.7		II	18	4.22	0.86	341.8
	III	3	0.24	1.05	303.6		III	19	3.91	0.96	346.6
	IV	4	-0.07	0.69	291.4		IV	20	3.09	0.79	373.3
2	I	5	0.50	0.98	328.6	6	I	21	2.44	0.26	351.1
	II	6	1.91	0.46	319.7		II	22	1.97	1.36	423.3
	III	7	2.24	0.57	270.5		III	23	1.26	0.62	398.7
	IV	8	3.18	0.11	256.5		IV	24	1.03	2.10	472.3
3	I	9	4.04	0.06	262.3	7	I	25	0.41	-0.10	413.9
	II	10	4.13	0.90	247.7		II	26	0.62	0.90	429.7
	III	11	3.83	0.39	255.5		III	27	0.30	1.20	
	IV	12	3.42	0.67	267.5		IV	28	0.02	-0.69	
4	I	13	3.85	0.61	296.2	8	I	29	-0.03	0.67	
	II	14	3.78	0.55	298.7		II	30	0.08	1.40	
	III	15	4.02	0.27	308.4						
	IV	16	4.29	0.60	326.9						

EXHIBIT 8.12 Extravagant Goodies, Inc.: Quarterly sales and economic data.

Your job is to determine which of the four hypothesized linear models shown below produces the best forecasts and then use that model to forecast EGI's sales. Hypothesized models for EGI quarterly sales are

^{*} The Treasury spread for this problem is the difference between the yield on 20-year Treasury bonds less the yield on six-month Treasury bills.

Model 1:

$$\text{Quarterly Sales}_t = a + b * \text{Time}_t$$

Model 2:

$$\text{Quarterly Sales}_t = a + b * \text{Time}_t + d * \text{CPI}_t$$

Model 3:

$$\text{Quarterly Sales}_t = a + b * \text{Time}_t + c * T_Spread_t$$

Model 4:

$$\text{Quarterly Sales}_t = a + b * \text{Time}_t + c * T_Spread_t + d * \text{CPI}_t$$

After identifying the model that produces the best forecasts, reestimate it using all 7.5 years of data and use it to forecast EGI's sales for the next four quarters. Have Excel report of the residuals of your final regression so that you can graph them and look for evidence of a missing factor. In Excel, draw a graph that shows EGI's actual and predicted quarterly sales. Draw actual and predicted sales as separate series, so that your forecasts are clearly identified. Label your sales and residuals graphs appropriately.

WHAT YOU TURN IN

Submit a report in memo format that describes the purpose and results of your work, including the criteria you used to select your model. Identify the model that is the least biased, the model that produces the smallest average error regardless of direction, and the model that is the most efficient with respect to the number of independent variables used. Place your graph of EGI's sales (including predicted sales) in the body of your memo. Include a table in your memo that reports the model used to predict EGI's sales (e.g., $\text{Sales}_t = 45 + 4 * \text{Time}_t$) as well as the predicted sales. Place the following in an appendix: A clearly labeled graph of the error terms from the time series regression used to estimate your forecasting model and a well-organized print-out of the spreadsheet showing the goodness-of-forecast calculations you used to identify the best model.

Forecasting with Smoothed Data

9.1 INTRODUCTION

Economic and business data is not always well behaved. It frequently exhibits extreme variations, regular patterns, or both that can mask underlying trends or cycles. There are a variety of techniques to smooth extreme variation or remove regular patterns. These techniques result in better behaved data series that in turn produce more precise models and improved forecasts. Several of these techniques are the focus of this chapter—moving averages, Exponential Smoothing, and seasonal decomposition. All of these techniques share one common element—past observations on a variable are used to forecast future values of that same variable.

9.2 MOVING AVERAGE

The simplest smoothing technique is a moving average forecast. In the simplest approach, the forecaster decides on the length of the horizon over which the average is to be taken (the number of periods, n , in the average). The average value for the observations within this horizon becomes the forecast for the next period. As the series moves forward from period to period, the last observation is dropped, the most recent observation added, and a new average computed. This new average becomes the forecast for the next period. The average moves through time, hence its name. In general, the n -period moving average forecast for period t is given by Equation 9.1.

$$\text{Forecast Sales}_t = \frac{\sum_{i=t-n}^{t-1} \text{Sales}_i}{n} \quad (9.1)$$

The forecast can also be thought of as a weighted average of past values, where the weight, $1/n$, is the same for each period. The moving average forecast can be an effective method

for very short-term (one-period) forecasts of simple series that do not exhibit cyclical or seasonal patterns or a trend.

To see how this works, consider the quarterly sales data for Tudor Corporation, shown below in Exhibit 9.1. The chart of Tudor’s sales is shown in Exhibit 9.2. From the chart we see that Tudor’s sales do not exhibit any obvious cyclical pattern or growth trend.

Tudor Corp. Quarterly Sales			
Year	Quarter	Time	Sales (\$000)
1	I	1	822.1
	II	2	899.9
	III	3	859.8
	IV	4	904.7
2	I	5	882.3
	II	6	880.8
	III	7	883.5
	IV	8	884.7
3	I	9	872.3
	II	10	822.4
	III	11	817.0
	IV	12	811.7
4	I	13	855.6
	II	14	862.2
	III	15	840.1
	IV	16	904.2
5	I	17	861.4
	II	18	903.7
	III	19	872.2
	IV	20	824.0
6	I	21	894.6
	II	22	871.1
	III	23	865.9
	IV	24	821.6

EXHIBIT 9.1 Tudor Corp. quarterly sales.

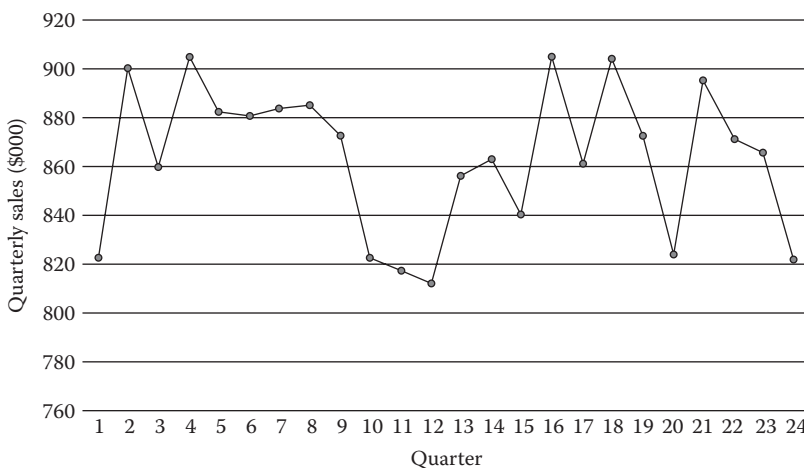


EXHIBIT 9.2 Tudor Corp. sales.

	B	C	D	E	F	G	H	I
2	Tudor Corp. Quarterly Sales							
3								
4	4-period Moving average							
5	Year	Quarter	Time	Sales (\$000)	Forecast			
6		1 I	1	822				
7		II	2	900				
8		III	3	860				
9		IV	4	905				
10	2	I	5	882	872			
11		II	6	881	887			
12		III	7	883	882			
13		IV	8	885	888			
14	3	I	9	872	883			
15		II	10	822	880			
16		III	11	817	866			
17		IV	12	812	849			
18	4	I	13	856	831			
19		II	14	862	827			
20		III	15	840	837			
21		IV	16	904	842			
22	5	I	17	861	866			
23		II	18	904	867			
24		III	19	872	877			
25		IV	20	824	885			
26	6	I	21	895	865			
27		II	22	871	874			
28		III	23	866	865			
29		IV	24	822	864			

EXHIBIT 9.3 Tudor Corp.: Four-period moving average forecasting model.

Exhibit 9.3 illustrates the calculations for a four-period moving average in Excel. The sales forecast for quarter 5 in cell F12, 872, is the average of sales in quarters 1–4 as shown in Equation 9.2.

$$E[Sales_5] = \left(\frac{822 + 900 + 860 + 905}{4} \right) = 872 \quad (9.2)$$

The sales forecast for quarter 6, 887, is the average of sales in quarters 2–5, as shown in Equation 9.3.

$$E[Sales_6] = \left(\frac{900 + 860 + 905 + 882}{4} \right) = 887 \quad (9.3)$$

The average moves forward with each new period.

Moving averages smooth a series. The idea is to remove the period-to-period variation to better discern the long-term tendency in a time series. The greater the number of periods in the moving average, the greater the smoothing effect. Consider Exhibit 9.4a–d.

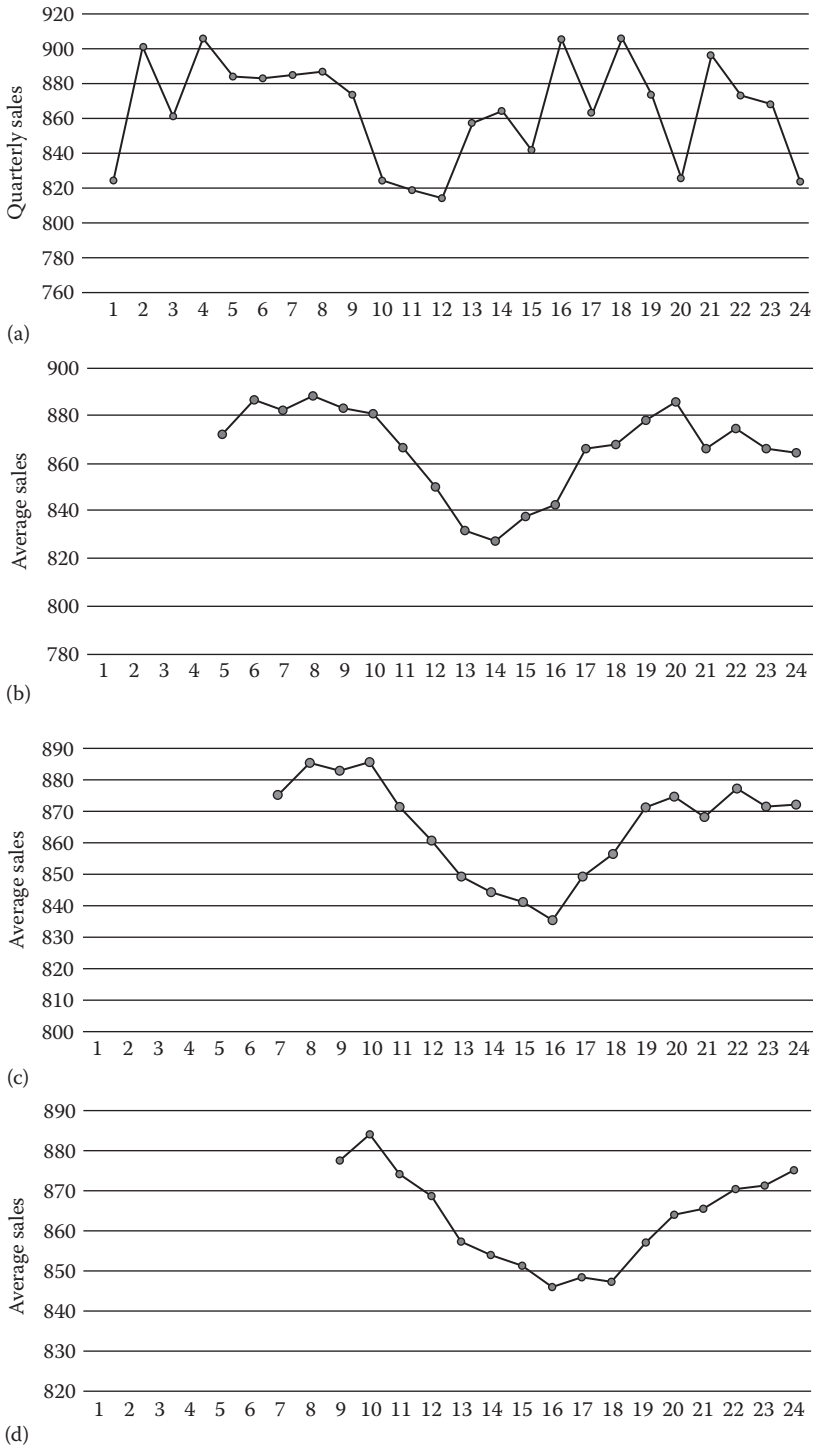


EXHIBIT 9.4 (a) Quarterly, Tudor Corp. sales. (b) Four-period moving average, Tudor Corp. sales. (c) Six-period moving average, Tudor Corp. sales. (d) Eight-period moving average, Tudor Corp. sales.

The number of quarters used to smooth sales increases from zero to four to six to eight periods. Note that the longer the moving average period, the smoother the series.

Moving average forecasting suffers from several problems. It can be difficult to determine the appropriate number of periods. Often, though, a graph of the series helps the forecaster to determine the appropriate number.* In addition, moving averages give equal weight to all observations in the average. Sometimes, however, more recent observations contain more information, whereas old observations contain less, if any, information. In this case, the solution is to give more weight to recent observations. This leads us into a discussion of Exponential Smoothing.

9.3 EXPONENTIAL SMOOTHING

Exponential Smoothing is a moving average in which some observations are more important than others. It is based on the notion that the most recent observations of the variable being forecast convey more information about the future values of that variable than more distant observations. To capture this information, Exponential Smoothing weights each observation in the moving average, typically assigning the greatest weight to the most recent observation and the smallest weight to the most distant observation. It is calculated according to Equation 9.4.

$$\begin{aligned} \text{Forecast sales}_{t+1} = & a * \text{Sales}_t + a(1-a) * \text{Sales}_{t-1} \\ & + a(1-a)^2 * \text{Sales}_{t-2} + \cdots + a(1-a)^j * \text{Sales}_{t-j} \end{aligned} \quad (9.4)$$

where $0 < a < 1$. As $0 < a < 1$, the terms $a(1-a)^j$ decrease monotonically as j increases. Although this expression looks daunting, it reduces to the simple expressions as shown below.

$$\text{Forecast sales}_{t+1} = a * \text{Sales}_t + (1-a) * \text{Forecast sales}_t \quad (9.5)$$

or, alternatively,

$$\text{Forecast sales}_{t+1} = \text{Forecast sales}_t + a * (\text{Sales}_t - \text{Forecast sales}_t) \quad (9.6)$$

In Equation 9.5, the forecast for period $t + 1$ is a weighted average of the sales observed in period t , Sales_t , and the forecast for sales in period t that was made in period $t - 1$, Forecast Sales_t . Equivalently, Equation 9.6 shows the forecast for period $t + 1$ as a function of the forecast for period t sales, Forecast Sales_t , and the error in that forecast, $\text{Sales}_t - \text{Forecast sales}_t$.

There are two issues to consider when building an Exponential Smoothing model—the initial forecast and the value of the smoothing constant. The forecast for period t depends

* You can also calculate goodness-of-fit statistics for these models. Goodness-of-fit statistics are identical to goodness-of-forecast statistics. The difference is that goodness-of-fit statistics are used when the model is fit to the entire data set instead of just the hold-out segment.

on the forecast for period $t - 1$, raising the question of where the initial forecast comes from. Forecasters frequently use either of two methods for the initial forecast. One method assumes a random walk and uses the observation for period 1, $Sales_1$, as the forecast for period 2, $Forecast\ Sales_2$. The other method uses a moving average to get the first forecast. For example, the average of the first 4 observations can be used to forecast the 5th observation. In the example that follows, we use the first approach. As for the smoothing parameter, a , an arbitrarily chosen value is used to build the model. The final value used is the one that minimizes some measure of the model's goodness-of-fit, such as the root mean square error (RMSE).

Return to Tudor Corp.'s quarterly sales. To start the Exponential Smoothing algorithm, we need a value for the smoothing constant and the actual and forecast sales for a given period. Let the smoothing constant, a , be 0.10. Observed sales in quarter 2 were 899.9. Finally, the first or *seed* forecast will be for quarter 2. To start the algorithm we set the forecast for period 2 equal to the actual sales in period 1, 822.1. With these numbers in place, we now proceed to forecast quarter 3 using Equation 9.5.

$$\begin{aligned} Forecast\ Sales_3 &= a * Sales_2 + (1 - a) * Forecast\ Sales_2 \\ &= 0.10 * 899.9 + 0.90 * 822.1 \\ &= 829.8 \end{aligned} \tag{9.7}$$

Alternatively, Equation 9.6 can also be used to calculate the forecast for quarter 3.

$$\begin{aligned} Forecast\ Sales_3 &= Forecast\ Sales_2 + a * (Sales_2 - Forecast\ Sales_2) \\ &= 822.1 + 0.10 * (899.9 - 822.1) \\ &= 829.8 \end{aligned} \tag{9.8}$$

As successive forecasts combine a with the sales and forecast sales from the previous period, they use the same code. Therefore, all that is needed to complete this model is to make the cell address of the smoothing constant an absolute reference (*dollar sign* it) and copy your code into the remaining rows of your model. The final model, assuming $a = 0.10$, is shown in [Exhibit 9.5](#).

The value of the smoothing constant, a , determines that how much influence the most current observation has on the forecast value. To see this, we repeat Equation 9.5.

$$Forecast\ sales_{t+1} = a * Sales_t + (1 - a) * Forecast\ sales_t \tag{9.5\ reprised}$$

When a is close to one, $(1 - a)$ is close to zero, so the most recent observation has a significant influence on the next period's forecast. Very little smoothing of the series takes place and the series approaches a random walk. When a is close to zero, the most recent observation

	B	C	D	E	F	G	H
2	Tudor Corp. Quarterly Sales						
3					Exponential Smoothing		
4				constant = 0.100			
5						Error Term	=E7
6	Year	Quarter	Time	Sales (\$000)	Forecast		
7		1 I	1	822.1			
8		II	2	899.9	822.1	77.9	=constant*E8+(1-constant)*F8
9		III	3	859.8	829.8	30.0	
10		IV	4	904.7	832.8	71.8	
11		2 I	5	882.3	840.0	42.2	
12		II	6	880.8	844.3	36.6	
13		III	7	883.5	847.9	35.6	
14		IV	8	884.7	851.5	33.2	
15		3 I	9	872.3	854.8	17.5	
16		II	10	822.4	856.5	-34.1	
17		III	11	817.0	853.1	-36.1	
18		IV	12	811.7	849.5	-37.8	
19		4 I	13	855.6	845.7	9.8	
20		II	14	862.2	846.7	15.5	
21		III	15	840.1	848.3	-8.2	
22		IV	16	904.2	847.5	56.8	
23		5 I	17	861.4	853.1	8.3	
24		II	18	903.7	854.0	49.8	
25		III	19	872.2	858.9	13.2	
26		IV	20	824.0	860.3	-36.3	
27		6 I	21	894.6	856.6	38.0	
28		II	22	871.1	860.4	10.7	
29		III	23	865.9	861.5	4.4	
30		IV	24	821.6	861.9	-40.4	
31							
32					ME	15.6	
33					MAD	32.3	
34					RMSE	37.7	

EXHIBIT 9.5 Tudor Corp. exponentially smoothed sales.

has little influence on the new forecast. There is a substantial degree of smoothing. The new forecast will be very similar to the old forecast.

From another perspective, the smoothing constant determines the speed at which past observations lose their importance to the forecast. When a is small, random variations are smoothed and the influence of distant observations on the forecast is reasonably large. Forecasts will be stable from period to period. When a is large, the influence of distant observations will be small. The forecasts will respond rapidly to recent observations and will exhibit more variation from period to period.

We now consider how to determine the best smoothing constant, a . The standard approach is to find the smoothing constant, a , that minimizes the root mean square error, RMSE using a nonlinear optimization program such as Excel's Solver. Solver is found on the *Data* ribbon.* Clicking on Solver opens the dialog box shown in [Exhibit 9.6](#).

* Solver is an Excel Add-In. To add Solver in, click on *File*, select *Options*, then *Add-ins* and select Solver.

Year	Quarter	Time	Sales (\$000)	Forecast	Error Term	
1	I	1	822.1			
2	II	2	899.9	822.1	77.9	
3	III	3	859.8	829.8	30.0	
4	IV	4	904.7	832.8	71.8	
5	I	5	882.3	840.0	42.2	
6	II	6	880.8	844.3	36.6	
7	III	7	883.5	847.9	35.6	
8	IV	8	884.7	851.5	33.2	
9	I	9	872.3	854.8	17.5	
10	II	10	822.4	856.5	-34.1	
11	III	11	817.0	853.1	-36.1	
12	IV	12	811.7	849.5	-37.8	
13	I	13	855.6	845.7	9.8	
14	II	14	862.2	846.7	15.5	
15	III	15	840.1	848.3	-8.2	
16	IV	16	904.2	847.5	56.8	
17	I	17	861.4	853.1	8.3	
18	II	18	903.7	854.0	49.8	
19	III	19	872.2	858.9	13.2	
20	IV	20	824.0	860.3	-36.3	
21	I	21	894.6	856.6	38.0	
22	II	22	871.1	860.4	10.7	
23	III	23	865.9	861.5	4.4	
24	IV	24	821.6	861.9	-40.4	
					ME	15.6
					MAD	32.3
					RMSE	37.7

Solver Parameters

Set Objective:

To: Max Min Value Of:

By Changing Variable Cells:

Subject to the Constraints:

constant <= 1

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method
 Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

EXHIBIT 9.6 Solver dialog box for minimizing RMSE.

The steps to fill out the Solver dialog box are

1. *Set Objective:* This is the cell that we want to optimize. In this case we want to make the RMSE as small as possible. RMSE is in cell G34, so select and enter G34 in the Set Target Cell box.
2. *To:* Click Min because we want to minimize the RMSE that is the Target Cell.
3. *By Changing Variable Cells:* The decision variable that we want to adjust is the smoothing constant, a , that is in E4, the cell named *constant*. Click in the By Changing Variable Cells box, and select and enter either E4 or type in *constant*.
4. *Subject to the Constraints:* The constraints limit the range of values of the decision variable. In this case, we consider values of a between 0 and 1. To enter the maximum constraint of 1, click the Add button to bring up the constraint dialog box shown in [Exhibit 9.7](#). The dialog box is shown filled in with the constraint E4 (or constant) ≤ 1 . To add the lower constraint, change the inequality to ≥ 0 (click the down arrow indicated by the cursor). Alternatively, select *Make unconstrained variables nonnegative*.
5. *Solve:* Click the Solve button and Solver follows your directions to find the value of the smoothing constant (the Changing Cell) that minimizes RMSE (Target Cell). In this case, the optimal solution is $a = 0.505$, yielding the minimum RMSE of 34.2, and the forecasts as shown in column D of [Exhibit 9.6](#).

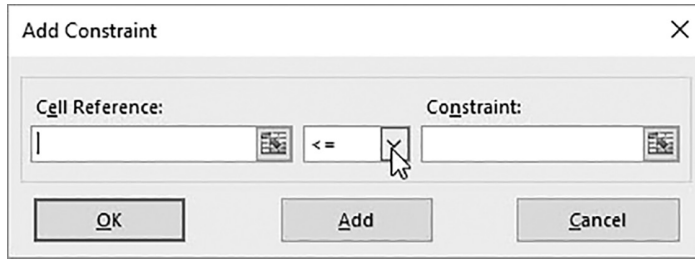


EXHIBIT 9.7 Constraint dialog box.

	B	C	D	E	F	G	H
1							
2		Tudor Corp. Quarterly Sales					
3						Exponential	
4			constant =	0.505		Smoothing	
5						Error	
6		Year	Quarter	Time	Sales (\$000)	Forecast	Term
7		1	I	1	822.1	#N/A	
8			II	2	899.9	822.1	
9			III	3	859.8	860.6	
10			IV	4	904.7	860.2	
11		2	I	5	882.3	882.2	
12			II	6	880.8	882.3	
13			III	7	883.5	881.6	
14			IV	8	884.7	882.5	
15		3	I	9	872.3	883.6	
16			II	10	822.4	878.0	
17			III	11	817.0	850.5	
18			IV	12	811.7	833.9	
19		4	I	13	855.6	822.9	
20			II	14	862.2	839.1	
21			III	15	840.1	850.5	
22			IV	16	904.2	845.4	
23		5	I	17	861.4	874.5	58.9
24			II	18	903.7	868.0	-13.1
25			III	19	872.2	885.7	35.7
26			IV	20	824.0	879.0	-13.5
27		6	I	21	894.6	851.8	-55.0
28			II	22	871.1	873.0	42.8
29			III	23	865.9	872.0	-1.9
30			IV	24	821.6	869.0	-6.2
31							-47.4
32					ME		2.1
33					MAD		25.7
34					RMSE		34.2

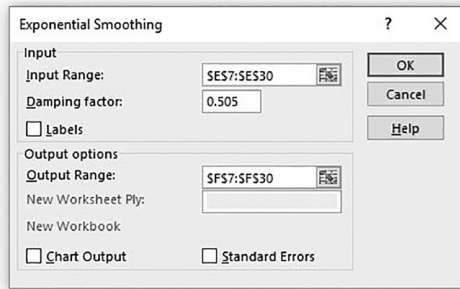


EXHIBIT 9.8 Excel's Exponential Smoothing tool dialog box.

In the above model, we entered the Exponential Smoothing formula (9.6) in the spreadsheet to develop our forecast. Excel's Exponential Smoothing tool in Data Analysis also does this. Click on Data Analysis and select Exponential Smoothing. This brings up the dialog box that you fill in as shown in Exhibit 9.8. To use the Exponential Smoothing tool, you specify the damping factor, the input data (the column of actual sales, E7:E30 in this example) and the output range (F7:F30). The Exponential Smoothing routine inserts the forecasts in the output range column using the formula

$$\begin{aligned} \text{Forecast Sales}_t = & (1 - \text{Damping Factor}) \text{Sales}_{t-1} \\ & + (\text{Damping Factor}) \text{Forecast Sales}_{t-1} \end{aligned} \quad (9.9)$$

where the *Damping Factor* that is entered in the dialog box is $(1 - a)$.

There are several quirks with this tool. First, Excel's *Damping Factor* is not the same as our smoothing constant; rather, it is the complement of the smoothing constant. Our optimal smoothing constant is $a = 0.505$, so the *Damping Factor* we entered was $(1 - a) = (1 - 0.505) = 0.495$, as shown in the dialog box. Second, the Exponential Smoothing tool enters #N/A for the first period's forecast. It sets the second period forecast equal to the first period's actual sales. The result is the same as our earlier manual approach. Lastly, the tool only accepts a hard number in the damping factor box. It will not accept a cell reference for the damping factor, not even when preceded by an "=" sign. Hence, the resulting model is static. Given how easy it is to build this model from scratch, we recommend doing precisely that.

9.4 EVALUATING THE MODEL

We now have two models for the quarterly sales of Tudor Corp., raising the question of which one is likely to produce more accurate forecasts. To answer this question in the previous chapters, we measured forecasting accuracy. A portion of the data set was used to estimate the models and then the models were used to generate forecasts for the values of the remaining hold-out period. Error terms were calculated and then used to calculate four goodness-of-forecast statistics—mean error, mean absolute deviation, root mean squared error, and residual standard error. Unfortunately, we cannot use this same procedure to rank the two smoothing models for Tudor Corp. because smoothing models can only forecast a limited number of periods, typically just one. This means that there is insufficient data to measure forecasting accuracy. Instead, we measure how well each model *fits* the data.

Return to the four-period moving average model of Tudor Corp.'s quarterly sales. It forecasts sales for all but the first four periods in the data sample. The Exponential Smoothing model forecasts sales for all but the first period that starts the algorithm. To compare forecasts, we exclude the first four periods, thereby considering only the periods for which both methods make forecasts. Subtracting these forecasts from their respective actual values gives us error terms that can be used to calculate each model's mean error, mean absolute deviation, and root mean squared error. These statistics are calculated in precisely the same way as the goodness-of-forecasting statistics in the previous section. They also convey the same information that are as follows: (1) Models with lower root mean squared errors are said to fit the data better, (2) models with lower mean errors are less biased, and (3) models with lower mean absolute deviations have smaller absolute errors, on average.*

Exhibit 9.9 reports the forecasts for Tudor made by the four-period moving average model and an Exponential Smoothing model with a smoothing constant of 0.505. The error terms and the diagnostic statistics are also shown. The Exponential Smoothing model produces a lower root mean squared error (it fits the data better) and a lower mean absolute deviation (its absolute errors are smaller, on average) than the four-period moving average

* Goodness-of-fit statistics can also be used in lieu of goodness-of-forecast statistics if the data set is not large enough to permit partitioning into estimating and testing segments.

	B	C	D	E	F	G	H	I
2	Tudor Corp. Quarterly Sales							
3								
4					4-period		Exponential	
5					Moving average		Smoothing	
6							constant = 0.505	
7	Year	Quarter	Time	Sales (\$000)	Forecast	Error	Forecast	Error
8		1 I	1	822.1				
9		II	2	899.9			822.1	
10		III	3	859.8			861.4	
11		IV	4	904.7			860.6	
12		2 I	5	882.3	871.6	10.6	882.9	-0.6
13		II	6	880.8	886.7	-5.8	882.6	-1.7
14		III	7	883.5	881.9	1.6	881.7	1.8
15		IV	8	884.7	887.8	-3.1	882.6	2.1
16		3 I	9	872.3	882.8	-10.5	883.7	-11.4
17		II	10	822.4	880.3	-57.9	877.9	-55.5
18		III	11	817.0	865.7	-48.7	849.9	-32.9
19		IV	12	811.7	849.1	-37.4	833.3	-21.5
20		4 I	13	855.6	830.9	24.7	822.4	33.2
21		II	14	862.2	826.7	35.5	839.2	23.1
22		III	15	840.1	836.6	3.5	850.8	-10.7
23		IV	16	904.2	842.4	61.8	845.4	58.8
24		5 I	17	861.4	865.5	-4.1	875.1	-13.7
25		II	18	903.7	867.0	36.7	868.2	35.5
26		III	19	872.2	877.4	-5.2	886.1	-14.0
27		IV	20	824.0	885.4	-61.4	879.1	-55.1
28		6 I	21	894.6	865.3	29.3	851.3	43.3
29		II	22	871.1	873.6	-2.5	873.1	-2.1
30		III	23	865.9	865.5	0.4	872.1	-6.3
31		IV	24	821.6	863.9	-42.3	869.0	-47.4
32								
33					ME	-3.7		-3.7
34					MAD	24.2		23.5
35					RMSE	32.3		30.7

EXHIBIT 9.9 Comparing the goodness-of-fit for two smoothing models.

model. Both models show the same bias with a mean error of -3.7 . These results indicate that Exponential Smoothing generates forecasts that fit Tudor's sales better than the four-period moving average.

9.4.1 Making the Forecast

We have evaluated each of these methods relative to past data, and found that the Exponential Smoothing method provides the best fit of past data. The last step is to use the model to forecast the next period's sales. Our best smoothed model is

$$Sales_{t+1} = 0.505 * Sales_t + (1 - 0.505) Forecast Sales_t \quad (9.10)$$

For quarter 25 the forecast is

$$\begin{aligned} Sales_{25} &= 0.505 * 821.6_{24} + (1 - 0.505) * 869.0_{24} \\ &= 845.1 \end{aligned} \quad (9.11)$$

We will not know if this is an accurate forecast until the events of quarter 25 unfold.

As previously mentioned, smoothing methods are intended only to provide short-term (one period) forecasts; they are not well suited for longer-term forecasts. Furthermore, both moving average and Exponential Smoothing are most effective with stable time series with minimal trend, seasonal, or cyclical components.* If the forecaster wishes to use these models for series that exhibit trends or seasonal components, these components must first be removed. We consider the process of removing seasonal variation next.

9.5 SEASONALITY AND SEASONAL DECOMPOSITION

In the previous sections we considered two ways to smooth a data series. Our purpose was to reduce the influence of extreme observations on short-term forecasts. Frequently, however, data will also exhibit a periodic variation about its mean or trend. The most common type of periodic variation in economic data is seasonal, in which patterns repeat weekly, monthly, or quarterly. These seasonal swings often mask the other influences, and though predictable, decrease the precision of our forecasts. Therefore, most economic forecasting utilizes data that has had the seasonal pattern removed. That is what you see when you look at a series that is labeled *seasonally adjusted*. This section introduces another smoothing method, seasonal adjustment, also known as seasonal decomposition.

We are all familiar with the fact that retail stores tend to receive an overwhelming proportion of their sales in the Christmas season, followed by very low sales after the first of the year. Suppose we are at the end of the year, and we are using monthly data as input to our forecast for the coming year. We observe that monthly sales have increased at a rate of 20% per month over the past three months. Should we extrapolate that trend and forecast sales will continue increasing at 20% per month for the coming year? Being familiar with the company, you say “of course not. It is normal for monthly sales to go up 20% per month during the holiday season, and that based on this data, there is no reason to think that next year’s total sales will be any greater than this year’s.” What you have just done intuitively is deseasonalize the data to help you make a forecast.

9.5.1 Sources of Variation in Data

Many economic and business data series reflect several sources of variation: seasonal, trend, business cycle, firm or industry specific variation, and random fluctuations. [Exhibit 9.10](#) displays the sales for a company that exhibit four different influences: a simple trend with cyclical, seasonal, and random variation. The data we actually observe are the dots on the graph. The underlying trend is the straight line; the long dashed wave indicates the presence of a long-term economic cycle; the short solid wave

* Both moving average and Exponential Smoothing have been extended to handle trends and seasonal effects. These extensions are beyond the scope of this book. The interested reader can consult a more specialized forecasting text to learn more about methods such as the double moving average, Holt’s method of Exponential Smoothing, and the Holt–Winter method, that handle trends and seasonality.

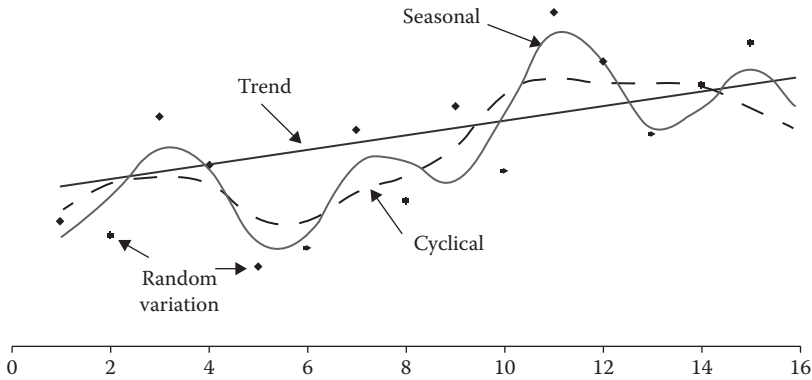


EXHIBIT 9.10 Sources of variation in data.

indicates the presence of seasonal variation; and the displacements of the observations from the other 3 influences represent a random component.

When such influences are present, they must be removed or captured by the modeling process if reliable forecasts are to be made. As we saw in the previous chapter, econometric models can be used to capture cyclical components. Seasonal decomposition uses the data to identify and remove the systematic components of season. Any remaining trend or economic cycle effects are then used to develop a forecast with a method such as smoothing or linear regression. The process is sequential. First, remove seasonality. Next, look for a trend. What is left should be either random or the result of a long-term cycle. After generating the forecast that excludes the seasonal component, we add back the seasonal factor to obtain a complete forecast. Through using this method, our forecast takes into account all of the influences: trend, cycle, and seasonal.

9.5.2 Seasonal Adjustment Factors

In its most general form, seasonal decomposition characterizes a given period's sales as a function of the four influences discussed in the previous section. This is shown formally in Equation 9.12.

$$Y = f(T, C, S, e) \quad (9.12)$$

where:

Y is the actual value of the time series

T is the trend

C is the cyclical influences

S is the season influences

e is the random error term

The multiplicative form of this model is

$$Y_t = TC_t * S_t * e_t \quad (9.13)$$

where:

T represents the level we would expect without taking account of the cyclical, seasonal, or random effects

C and S are proportional indexes centered on 1

Ignoring the error term, when the product $CS > 1$, sales will be above the trend line; when $CS < 1$, sales will be below the trend line.

Consider the following example. Suppose the trend component of sales is shown by the trend regression equation,

$$T_t = \text{Sales Trend}_t = 4,761 + 178.4 * \text{Time}_t \quad (9.14)$$

Before accounting for the stage of the economic cycle or the fact that it is the holiday season, the expected sales for quarter 16 are

$$\begin{aligned} T_{16} &= 4,761 + 178.4 * 16 \\ &= 7,645.4 \end{aligned} \quad (9.15)$$

Now suppose the economy is in a recession so sales are expected to be 3% below normal; then C would be 0.97. Suppose, too, that even though we are in a recession, this is the holiday season and sales are expected to be 20% higher than the *average* for the year; then S would be 1.20. The combined effect of cycle and seasonal effects is that sales are $C \times S = (0.97)(1.20) = 1.164$ times higher than that would be indicated by the trend by itself. Taking account of trend, cycle, and season, sales are expected to be

$$\begin{aligned} \text{Forecast Sales}_{16} &= T_{16} C_{16} S_{16} \\ &= 7615.4 * C_{16} S_{16} \\ &= 7615.4 * 0.97 * 1.20 \\ &= 8,864.3 \end{aligned} \quad (9.16)$$

In this example, sales are 20% higher in the holiday season. But how did we arrive at the 20% figure? We first determined the normal level of sales without the seasonal effects, and then calculated how much sales exceed the normal level during the holiday season. To estimate the normal level, we use a special type of moving average.

Consider the quarterly sales of Warwick Corp. in [Exhibit 9.11](#), shown graphically in [Exhibit 9.12](#). The seven years of quarterly data suggest a seasonal component: sales tend to rise in the first quarter, then fall back over the next three quarters.

Our goal is to develop a model to forecast quarterly sales for Warwick Corp. Following the procedure used in the previous chapter, we partition the data into two segments—one to estimate the model, the other to test it. First we use Warwick's quarterly sales from the years 1 through 4 to estimate seasonal adjustment factors for each quarter (S_t). We then

Warwick Corp. Quarterly Sales		
Period	Quarter	Reported Sales
1	I	6,284
	II	5,796
	III	5,084
	IV	3,734
2	I	6,763
	II	6,048
	III	5,359
	IV	5,496
3	I	6,321
	II	6,775
	III	6,222
	IV	6,383
4	I	7,563
	II	8,081
	III	7,199
	IV	6,571
5	I	8,066
	II	8,586
	III	6,758
	IV	6,085
6	I	8,854
	II	9,084
	III	7,693
	IV	7,312
7	I	9,793
	II	9,054
	III	8,870
	IV	8,984

EXHIBIT 9.11 Warwick Corp. quarterly sales.

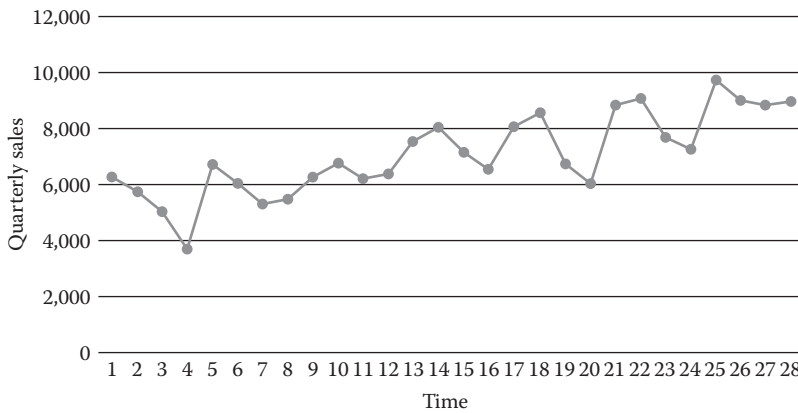


EXHIBIT 9.12 Warwick Corp. sales.

use these factors to remove the seasonality from the quarterly sales figures. Next, we graph the deseasonalized data and look for any trend or cycle that might be present. If a trend or cycle is present, we fit a model to capture those effects and then use it to estimate the seasonally-adjusted (or deseasonalized) quarterly sales for years 5–7. Finally, we use the seasonal adjustment factors to restore seasonality to our forecasts. This last step produces forecasts for the actual sales in each quarter of years 5–7.

9.5.2.1 Estimating the Seasonal Adjustment Factors

The first step is to determine the seasonal adjustment factors for each period.* For Warwick, each period is one quarter. Although there are various approaches, we use seasonal variation relative to an annual moving average to estimate each quarter's seasonal adjustment factor, S_t . The model is shown in Equation 9.17 as

$$S_t = \frac{Y_t}{TC_t} \quad (9.17)$$

where:

S_t is the seasonal adjustment factor for period t

Y_t is the actual sales for period t

TC_t is the centered moving average annual sales centered around period t

We begin by calculating the centered four-quarter average sales for each quarter t , TC_t . Why a centered average? The center of a year that runs from quarter 1 through quarter 4 lies midway between quarters 2 and 3. Similarly, the center of a year that runs from quarter 2 through quarter 1 of the next year lies between quarters 3 and 4. We want our seasonal adjustment factor to be centered precisely on a given quarter, quarter 3 in the example just introduced. Doing so aligns the numerator, Y_t , with the denominator, TC_t , thereby reducing any estimation error caused by asynchronous estimates.† By averaging quarterly sales for the years that run from quarters 1 through 4 and 2 through 1, we achieve an average quarterly sales figure that is precisely centered on quarter 3. This centered average becomes our estimate for TC_t for quarter 3. Dividing each quarter's actual sales (Y_t) by its respective centered average (TC_t) yields estimates of the seasonal adjustment factors, four in this example because the data is quarterly. This process is shown in Excel in Exhibit 9.13. Note how the centered moving average in cell E8 falls precisely in between the values in cells D6:D10.††

To calculate the centered four-quarter average sales for quarter 1.III, calculate the average quarterly sales for the year that runs from 1.I through 1.IV and the average quarterly sales for the year that runs from 1.II through 2.I. These are 5,225 and 5,344, respectively. Taking the average of these two averages gives us the average quarterly sales centered on quarter 1.III, thus

$$\begin{aligned} TC_3 &= \frac{5225 + 5344}{2} \\ &= 5284 \end{aligned} \quad (9.18)$$

* Our explanation of seasonal adjustment is relatively simplified. More complex algorithms and programs are available for more complex data sets. A very widely used program is the X-12 ARIMA program from the U.S. Census Bureau, available via the internet at www.census.gov.

† Certainly, we could accomplish this using a five-quarter average (e.g., quarter 1 through the next year's quarter 1). But doing so would overweight one quarter, quarter 1 in this case.

†† You can calculate a centered moving average directly in Excel by using arrays as arguments in Excel's AVERAGE function. For example, in the above example the centered moving average for quarter 1.III is =AVERAGE(R84:R87,R85:R88).

Warwick Corp. Quarterly Sales		Centered Moving Average	Seasonal Adjustment Factor	Smoothed Sales
Period	Quarter	Reported Sales		
1	I	6,284		
	II	5,796		
	III	5,084	0.96	=D8/E8
	IV	3,734		
2	I	6,763		
	II	6,048		
	III	5,359		
	IV	5,496		
3	I	6,321		

EXHIBIT 9.13 Calculating a third quarter seasonal adjustment factor.

The seasonal adjustment factor for quarter 3 (S_3) can now be calculated as shown in Equation 9.19.

$$\begin{aligned}
 \text{Seasonal adjustment factor}_3 &= \frac{\text{Sales for 1.III}}{\text{Centered 4-quarter average}_{1,\text{III}}} \\
 S_3 &= \frac{Y_3}{TC_3} \\
 &= \frac{5084}{5284} \\
 &= 0.96
 \end{aligned}
 \tag{9.19}$$

This process is repeated for each subsequent period (quarters in the current example). For example, for the fourth quarter of year 1, calculate the average quarterly sales for the year 1.II through 2.I and the year 1.III through 2.II, average the two averages, then divide quarter 1.IV's sales by this centered average. Continuing the process for the subsequent 10 quarters yields three estimates for each seasonal adjustment factor. Finally, the estimated adjustment factors for each quarter are averaged to give us the four seasonal adjustment factors that we will use to deseasonalize the data. The final results are shown in [Exhibit 9.14](#). The final four adjustment factors should sum to four (there are four quarters), providing a check on your calculations.

The seasonal adjustment factors tell us the portions above or below average seasonal sales that the actual seasonal observations should fall. For example, Warwick's first quarter seasonal adjustment factor is 1.11, telling us that Warwick's first quarter sales tend to be 111% of its average quarterly sales. Similarly, Warwick's fourth quarter sales tend to be 85% of its average quarterly sales.

9.5.3 Removing Seasonality

Once estimated, these seasonal adjustment factors are used to deseasonalize the series' observed sales. Divide each quarter's actual sales by its respective average seasonal

	B	C	D	E	F	G
2	Warwick Corp.					
3	Quarterly Sales			Centered	Seasonal	
4			Reported	Moving	Adjustment	Smoothed
5	Period	Quarter	Sales	Average	Factor	Sales
6	1	I	6,284			
7		II	5,796			
8		III	5,084	5,284	0.96	
9		IV	3,734	5,376	0.69	
10	2	I	6,763	5,442	1.24	
11		II	6,048	5,696	1.06	
12		III	5,359	5,861	0.91	
13		IV	5,496	5,897	0.93	
14	3	I	6,321	6,096	1.04	
15		II	6,775	6,314	1.07	
16		III	6,222	6,581	0.95	
17		IV	6,383	6,899	0.93	
18	4	I	7,563	7,184	1.05	
19		II	8,081	7,330	1.10	
20		III	7,199			
21		IV	6,571			
34				=AVERAGE(F10,F14,F18)		
35					SAFs	
36	(Hold out segment hidden)			SAF_I	1.11	
37				SAF_II	1.08	
38				SAF_III	0.94	
39				SAF_IV	0.85	

EXHIBIT 9.14 Estimating average seasonal adjustment factors for the estimation segment.

adjustment factor. This grosses up the sales for quarters with traditionally low sales and grosses down the sales for quarters with traditionally high sales. For example, actual quarter 1.I sales were 6284 and the first quarter seasonal adjustment factor is 1.11, so seasonally-adjusted 1.I sales are $6284/1.11 = 5657$; similarly, actual 1.IV sales were 3,734 and the fourth quarter seasonal adjustment factor is 0.85, so seasonally-adjusted 1.IV sales are $3734/0.85 = 4390$. Exhibit 9.15a shows the reported and seasonally-adjusted (smoothed) quarterly sales for Warwick Corp.

Once the data has been deseasonalized, look for other influences that might be present, for example, trends or business cycles. The reported and deseasonalized quarterly sales for Warwick Corp. are graphed in Exhibit 9.15b. With seasonality removed, a clear upward trend in Warwick Corp.’s quarterly sales can be seen.

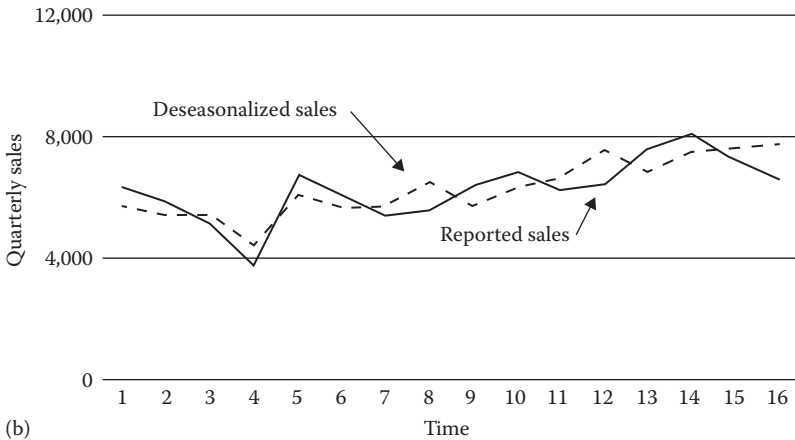
The next step is to fit a trend model to the data. We use the constant-change model* of the form

$$\text{Forecast Sales}_t = a + b\text{Time}_t \tag{9.20}$$

* In actual application, both the constant-change and constant-growth models should be estimated, goodness-of-forecast statistics calculated and compared, and the model that yields the best forecasts selected.

	B	C	D	E	F	G	H	
2	Warwick Corp. Quarterly Sales							
3				Centered	Seasonal	Smoothed		
4			Reported	Moving	Adjustment	Sales		
5	Period	Quarter	Sales	Average	Factor			
6		1 I	6,284			5,657	=D6/SAF_I	
7		II	5,796			5,371	=D7/SAF_II	
8		III	5,084	5,284	0.96	5,405	=D8/SAF_III	
9		IV	3,734	5,376	0.69	4,390	=D9/SAF_IV	
10		2 I	6,763	5,442	1.24	6,088		
11		II	6,048	5,696	1.06	5,605		
12		III	5,359	5,861	0.91	5,697		
13		IV	5,496	5,897	0.93	6,461		
14		3 I	6,321	6,096	1.04	5,690		
15		II	6,775	6,314	1.07	6,279		
16		III	6,222	6,581	0.95	6,615		
17		IV	6,383	6,899	0.93	7,504		
18		4 I	7,563	7,184	1.05	6,808		
19		II	8,081	7,330	1.10	7,489		
20		III	7,199			7,653		
21		IV	6,571			7,725		
34								
35				Name	SAF			
36				SAF_I	1.11			
37				SAF_II	1.08			
38				SAF_III	0.94			
39				SAF_IV	0.85			

(a)



(b)

EXHIBIT 9.15 (a) Warwick Corp. reported and seasonally-adjusted sales. (b) Chart of Warwick Corp. reported and seasonally-adjusted sales.

The resulting forecasting model is

$$\text{Forecast Sales}_t = 4761 + 178.4 * \text{Time}_t \tag{9.21}$$

Warwick’s deseasonalized quarterly sales data with a fitted trend line are shown in Exhibit 9.16.

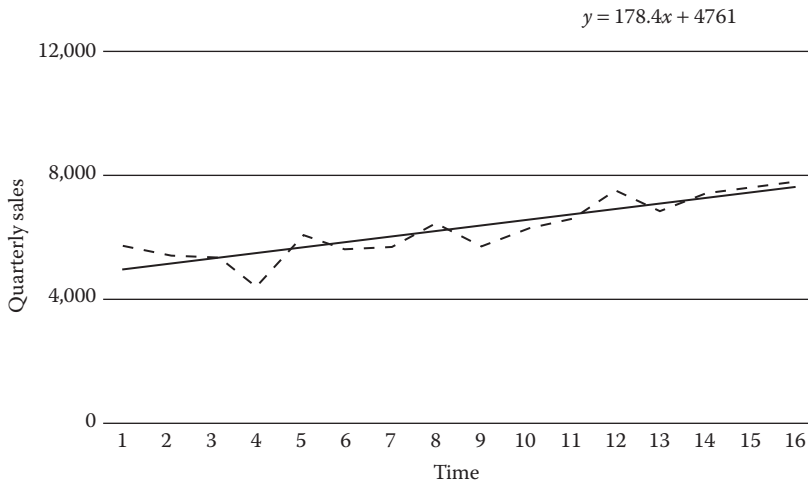


EXHIBIT 9.16 Deseasonalized sales with fitted trend line.

9.5.4 Forecasting Sales

Everything is now in place to forecast Warwick Corp.’s quarterly sales for years 5–7, the goal stated at the beginning of this extended process. We do this by combining the seasonal adjustment factors in [Exhibit 9.14](#) and our model of the growth trend (Equation 9.21). We assume that both the seasonality and trend will continue through years 5–7. The forecast for seasonally adjusted sales in quarter 5.I is shown in Equation 9.22.

$$\begin{aligned}
 E[\textit{Seasonally-Adjusted Sales}_{5,I}] &= 4761 + 178.4 * \textit{Time}_{5,I} \\
 &= 4761 + 178.4 * 17 \\
 &= 7,794
 \end{aligned}
 \tag{9.22}$$

Repeating the calculations for the next 11 quarters yields the seasonally-adjusted quarterly sales forecasts for years 5 through 7. Multiplying these forecasts by their respective average seasonal adjustment factors yields the actual quarterly sales forecasts. For example, the seasonally-adjusted forecast for 5.I is \$7,794 and the first quarter seasonal adjustment factor is 1.11. Multiplying the two yields the actual 5.I sales forecast of \$8,657, as shown in Equation 9.23.

$$\begin{aligned}
 E[\textit{Actual Sales}_{5,I}] &= E[\textit{Seasonally-Adjusted Sales}_{5,I}] * \textit{SAF}_I \\
 &= 7794 * 1.11 \\
 &= 8,657
 \end{aligned}
 \tag{9.23}$$

The seasonally-adjusted and actual quarterly sales forecasts for years 5–7 are reported in [Exhibit 9.17](#). [Exhibit 9.18](#) shows the forecast added to the actual sales for quarters 1–16 from [Exhibit 9.11](#).

	B	C	D	E	F	G
2	Warwick Corp. Sales					
3	Model:			Intercept	4,761.0	
4				Time	178.4	
5				Forecast		
6				Deseasonalized	Forecast	
7	Period	Quarter	Time	Sales	Seasonal	
24	5	I	17	7,794	8,658	=E24*SAF_I
25		II	18	7,972	8,602	
26		III	19	8,151	7,667	
27		IV	20	8,329	7,085	
28	6	I	21	8,507	9,450	
29		II	22	8,686	9,372	
30		III	23	8,864	8,338	
31		IV	24	9,043	7,692	
32	7	I	25	9,221	10,243	
33		II	26	9,399	10,142	
34		III	27	9,578	9,009	
35		IV	28	9,756	8,299	
36						
37			Name	SAF		
38	Estimation segment hidden		SAF_I	1.11		
39			SAF_II	1.08		
40			SAF_III	0.94		
41			SAF_IV	0.85		

EXHIBIT 9.17 Forecast seasonal sales for Warwick Corp.

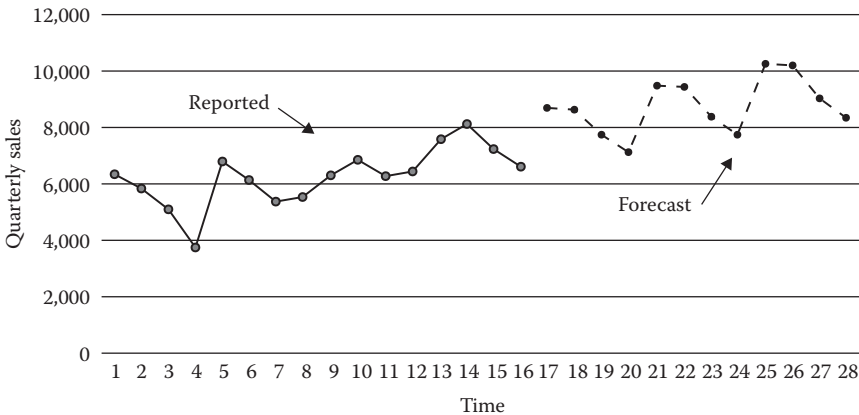


EXHIBIT 9.18 Reported and forecast sales for Warwick Corp.

The penultimate step is to calculate the goodness-of-forecast statistics and compare them with those for other models being considered, such as the compound-growth model. The statistics for the constant-change model are shown in Exhibit 9.19. Recall that the residual standard error adjusts for the number of parameters estimated. In the current situation, the number of estimated parameters is six—the 2 regression coefficients and the 4 seasonal adjustment factors.

Finally, look at the forecasting errors. If they do not display any obvious cyclicity, your model has probably done an acceptable job of capturing the structure of the data.

Warwick Corp. Sales						
Period	Quarter	Forecast		Forecast Seasonal Sales	Errors	
		Reported Sales	Deseasonalized Sales			
5	I	8,066	7,794	8,658	-592	
	II	8,586	7,972	8,602	-16	
	III	6,758	8,151	7,667	-909	
	IV	6,085	8,329	7,085	-1,000	
6	I	8,854	8,507	9,450	-596	
	II	9,084	8,686	9,372	-288	
	III	7,693	8,864	8,338	-645	
	IV	7,312	9,043	7,692	-380	
7	I	9,793	9,221	10,243	-450	
	II	9,054	9,399	10,142	-1,088	
	III	8,870	9,578	9,009	-139	
	IV	8,984	9,756	8,299	685	

Goodness-of-Forecast Statistics	
ME	-452
MAD	566
RMSE	649
RSE (denominator: 12-6)	917

EXHIBIT 9.19 Goodness-of-forecast analysis of simple trend model.

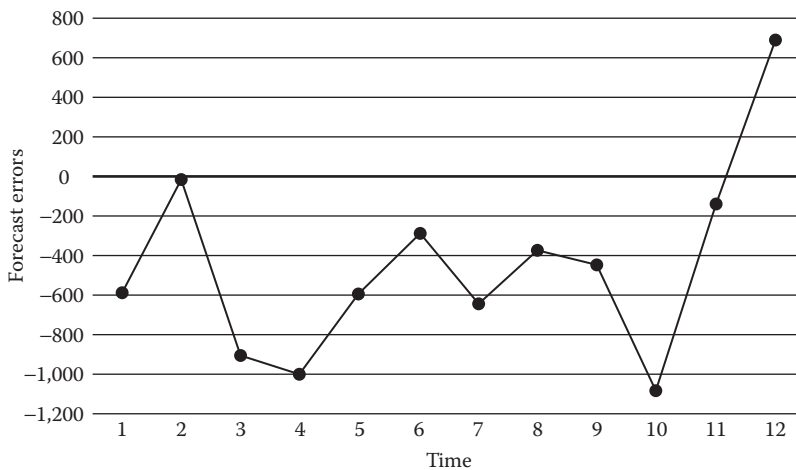


EXHIBIT 9.20 Forecast errors, Warwick Sales.

The forecasting errors are shown in Exhibit 9.20. In this case, a substantial number is consecutively below the 0 line, indicating that there may be serial correlation in the series. This suggests we look for another variable that, when added to the model, will improve its predictive power. But for our present purposes, we have accomplished our seasonal adjustment. It remains as further work for the forecaster to look at this remaining issue.

9.5.5 Review of Seasonal Adjustment

The steps to seasonal adjustment and forecasts are summarized below:

1. Calculate a centered moving average equal to the length of the seasons (12 months if the pattern is monthly, four quarters if quarterly, etc.).

2. Calculate the actual observation as a proportion of the centered moving average to obtain the seasonal index for each period. If you have enough data, consider calculating factors over several cycles, then averaging them.*
3. Deseasonalize the time series by dividing each observation by its respective average seasonal adjustment factor.
4. Estimate the trend and/or cyclical components using the deseasonalized data.
5. Check the model's goodness-of-forecast.
 - a. Use the model to forecast seasonally-adjusted values.
 - b. Multiply the seasonally-adjusted values by their appropriate average seasonal factors to compute the seasonal values.
 - c. Calculate errors and goodness-of-forecast statistics using known actuals.
6. Check to see if cyclical factors are important by plotting the error terms of your model. They should exhibit a random pattern. If they do not, add explanatory variables and repeat steps 5 and 6.

9.5.5.1 Cyclical^y

Return to [Exhibit 9.17](#). Having removed the seasonality from Warwick Corp.'s quarterly sales, the series exhibits no obvious cyclical^y. However, we do observe a possible cyclical effect in the forecast errors in [Exhibit 9.21](#). If cyclical^y is present, consider using an econometric model as discussed in [Chapter 8](#).

9.5.6 The Final Forecast

One step remains after completing the model selection process—estimating the final forecasting model, making the forecasts, then charting the results. This involves no new processes. We just repeat most of the steps that led to the test forecasting model, but with all of the data. The steps follow.

1. Estimate seasonal factors for every quarter possible, and then average them. In this example, this means seasonal adjustment factors for quarters 1.III–7.II.
2. Use the new seasonal adjustment factors to deseasonalize the data.
3. Estimate the model using all of the data—all seven years of deseasonalized sales in this example.
4. Generate the smoothed forecasts, and then add seasonality to them.
5. Look at your results.

The resulting model and sales chart are shown in [Exhibits 9.21](#) and [9.22](#).

* DeLurgio recommends adjusting outlying observations and seasonal adjustment factors, both of which are beyond the scope of this text.

Warwick Corp. Quarterly Sales							Constant Change 4,837.0 Intercept 179.5 Slope	
Period	Quarter	Reported	Centered	Deseasonalized		Time	Forecasts	
		Sales	MA	SAF	Sales		Smoothed	Seasonal
	I	6,284			5,652	1		
	II	5,796			5,329	2		
	III	5,084	5,284	0.96	5,430	3		
1	IV	3,734	5,376	0.69	4,407	4		
	I	6,763	5,442	1.24	6,082	5		
	II	6,048	5,696	1.06	5,561	6		
	III	5,359	5,861	0.91	5,723	7		
2	IV	5,496	5,897	0.93	6,486	8		
	I	6,321	6,096	1.04	5,685	9		
	II	6,775	6,314	1.07	6,229	10		
	III	6,222	6,580	0.95	6,646	11		
3	IV	6,383	6,899	0.93	7,533	12		
	I	7,563	7,184	1.05	6,801	13		
	II	8,081	7,330	1.10	7,430	14		
	III	7,199	7,416	0.97	7,688	15		
4	IV	6,571	7,542	0.87	7,755	16		
	I	8,066	7,550	1.07	7,254	17		
	II	8,586	7,434	1.15	7,894	18		
	III	6,758	7,472	0.90	7,217	19		
5	IV	6,085	7,633	0.80	7,181	20		
	I	8,854	7,812	1.13	7,963	21		
	II	9,084	8,082	1.12	8,352	22		
	III	7,693	8,353	0.92	8,216	23		
6	IV	7,312	8,467	0.86	8,630	24		
	I	9,793	8,610	1.14	8,807	25		
	II	9,054	8,966	1.01	8,324	26		
	III	8,870			9,473	27		
7	IV	8,984			10,603	28		
						29	10,043	11,167
						30	10,222	11,118
						31	10,402	9,739
						32	10,581	8,965
8			SAFs					
			SAF_I	1.11				
			SAF_II	1.09				
			SAF_III	0.94				
			SAF_IV	0.85				

EXHIBIT 9.21 Warwick Corp. final sales forecasting model.

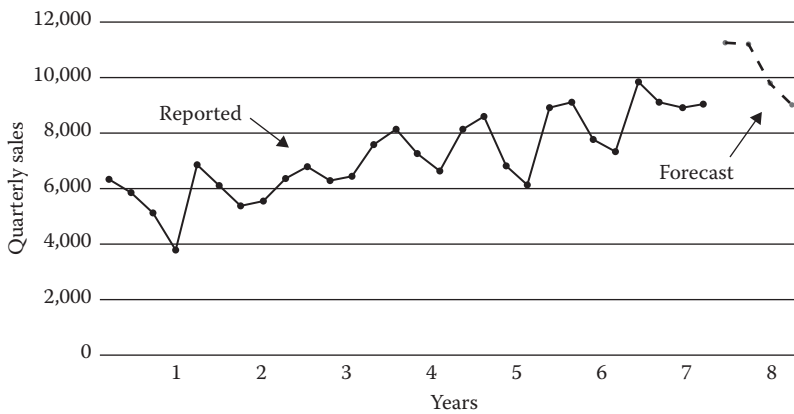


EXHIBIT 9.22 Warwick Corp. reported and final sales forecasts.

9.6 SUMMARY

Chapters 7 through 9 presented only the most basic methods of forecasting. The goal was to introduce future model builders to a few of the wide variety of forecasting techniques available. If you are going to do a lot of forecasting, and the forecasts are crucial input to your modeling, you will need to develop considerably more expertise as a forecaster than we have space to discuss here. Readers interested in pursuing the subject further should consult a text devoted exclusively to forecasting, such as DeLurgio or Pindyck and Rubinfeld listed in the References.

EXERCISES

1. Zeplin Corp.

Use the following data from Exhibit 9.23 for Zeplin Corp. to answer the following 2 questions.

Zeplin, Inc.							
Year	Quarter	Sales	Time	Year	Quarter	Sales	Time
1	I	853.0	1	5	I	1017.7	17
	II	828.9	2		II	1110.6	18
	III	904.8	3		III	1178.8	19
	IV	961.1	4		IV	1106.5	20
2	I	972.9	5	6	I	1137.4	21
	II	1009.3	6		II	1112.4	22
	III	867.7	7		III	1121.1	23
	IV	960.6	8		IV	1170.9	24
3	I	1028.8	9	7	I	1194.1	25
	II	956.5	10		II	1155.3	26
	III	1003.0	11		III	1232.3	27
	IV	978.9	12		IV	1253.8	28
4	I	1054.8	13	8	I	1290.4	29
	II	1111.1	14		II	1285.4	30
	III	1122.9	15				
	IV	1159.3	16				

EXHIBIT 9.23 Zeplin Inc. quarterly sales.

- Fit four- and six-period moving average models to Zeplin's sales. Calculate the mean error, mean absolute deviation, and mean squared error for each model. Based on these statistics, which model best fits the data? Use the best-fitting model to forecast Zeplin's sales for the next two quarters.
 - Fit an Exponential Smoothing model to Zeplin's sales. Calculate the mean error, mean absolute deviation, and mean squared error for each model. Use the average of the first four periods for your initial forecast and the smoothing constant that minimizes the mean squared error. Forecast Zeplin's sales for the next quarter using your model.
2. Boulder Brass Works, Inc.

Boulder Brass Works, Inc. (BBW) is a struggling young company that manufactures toy replicas of Civil War brass instruments. The owners realize that their sales move

with the seasons, but lack the necessary skills to analyze these seasonal fluctuations. In an effort to better plan their production levels, they have asked you to build a model that tells them, on average, how much each quarter's sales depart from BBW's average quarterly sales. They have given you BBW's sales data for the last 7.5 years. It is reported below in [Exhibit 9.24](#).

Boulder Brass Works, Inc						
Year	Quarter	Sales		Year	Quarter	Sales
1	I	923.0		5	I	1,456.8
	II	1,015.3			II	1,783.8
	III	289.2			III	404.9
	IV	434.6			IV	607.6
2	I	1,071.1		6	I	1,300.3
	II	1,420.3			II	2,009.9
	III	318.2			III	458.4
	IV	471.8			IV	721.4
3	I	1,020.7		7	I	1,621.4
	II	1,294.0			II	1,866.5
	III	375.0			III	533.2
	IV	545.0			IV	753.9
4	I	1,118.1		8	I	1,772.9
	II	1,449.7			II	2,913.2
	III	336.6				
	IV	592.4				

EXHIBIT 9.24 Boulder Brass Works, Inc.: quarterly sales.

Using the simple seasonal decomposition method discussed in [Chapter 8](#), estimate the seasonal adjustment factors for BBW's sales. Next, use those factors to remove the seasonality from BBW's sales. Finally, on the same chart, plot BBW's sales with and without seasonality.

3. Comfort Food Corp.

Comfort Food Corp. (hereafter CFC) is a wholesaler of high-end food products—food for individuals who miss their mothers' cooking. As part of their production planning, their management has asked you to forecast quarterly sales for the next six quarters.

CFC sales are highly seasonal. Each year, they grow steadily from the 1st quarter through the 4th quarter, then fall back at the beginning of the 1st quarter of the next year and repeat the process. In addition to a year-over-year increasing trend, casual observation suggests that their sales increase with consumer sentiment and, perhaps, disposable income. Some in management also believe that sales may increase with positive expectations for economic growth (measured by the Treasury spread) and decrease with the overall level of risk aversion (measured by the risk spread).*

* The Treasury spread in this case is the difference between the yield on 20-year Treasury bonds less the yield on six-month Treasury bills. The risk spread is the difference between the yields on Baa and Aaa 20-year corporate bonds.

CFC has supplied 11 years of quarterly data consisting of the change in consumer sentiment, the percentage change in disposable income, the Treasury and risk spreads, and CFC's quarterly sales. The data set also contains CFC's forecasts for the next six quarters of macroeconomic data. It is shown below in [Exhibit 9.25](#).

Comfort Foods							
Year	Quarter	Time	Change Sentiment	% change Disp. Inc.	Treasury Spread	Risk Spread	Quarterly Sales
1	I	1	-1.7	1.83	3.31	0.64	2,248
	II	2	-3.6	0.66	2.92	0.69	2,668
	III	3	3.7	1.39	2.69	0.63	3,111
	IV	4	4.9	0.61	1.76	0.62	3,298
2	I	5	-5.1	-0.08	1.68	0.57	2,182
	II	6	1.9	0.52	1.37	0.63	2,594
	III	7	-4.2	0.53	1.13	0.63	2,893
	IV	8	-0.9	1.12	1.19	0.66	3,411
3	I	9	3.4	0.83	1.92	0.69	2,394
	II	10	2	0.85	1.84	0.7	2,527
	III	11	1.8	0.51	1.79	0.68	3,247
	IV	12	0.9	0.96	1.81	0.67	3,838
4	I	13	4	0.82	1.83	0.61	3,013
	II	14	5.7	1.14	1.44	0.61	2,687
	III	15	-1.5	1.32	1.29	0.57	3,290
	IV	16	1	2.14	0.85	0.58	3,461
5	I	17	2.1	1.45	0.94	0.64	2,567
	II	18	-3.5	1.09	0.75	0.6	2,681
	III	19	-7.8	0.78	1.25	0.81	3,396
	IV	20	6.5	0.74	1.12	1.05	4,050
6	I	21	0.7	0.14	1.45	0.84	2,636
	II	22	1.4	0.50	1.7	0.76	3,076
	III	23	-2.8	1.45	1.68	0.83	3,325
	IV	24	8.8	2.22	1.36	0.55	3,825
7	I	25	-2.8	0.67	0.37	0.76	2,792
	II	26	-0.9	1.28	0.2	0.7	2,878
	III	27	-2.5	0.14	0	0.79	3,322
	IV	28	-11.1	0.32	0.7	0.78	3,714
8	I	29	-6.3	-0.42	1.93	0.87	2,879
	II	30	4	2.76	2.3	0.84	3,088
	III	31	-9.7	-1.43	3.22	0.88	3,196
	IV	32	10.3	2.78	3.96	1.32	4,641
9	I	33	0	0.47	3.92	1.27	3,411
	II	34	-4.9	-0.39	3.81	1.37	3,496
	III	35	-7.5	0.05	3.44	1.41	3,364
	IV	36	1.8	0.43	3.82	1.18	4,247
10	I	37	3.6	1.24	3.77	1.11	3,112
	II	38	4.9	1.55	3.97	1.13	3,848
	III	39	-1.3	0.42	4.21	1.03	4,261
	IV	40	14.2	0.96	4.04	0.9	4,616
11	I	41	-9.6	0.59	4.07	0.73	3,099
	II	42	2.5	0.68	3.58	0.8	3,564
	III	43	-5	1.82	2.85	0.74	3,878
	IV	44	3.8	-1.01	2.16	0.66	4,177
12	I	45	-7.8	0.11	1.7	0.72	
	II	46	8.8	-0.14	1.06	0.89	
	III	47	-22.3	1.35	0.75	0.95	
	IV	48	17	1.14	0.34	0.95	
13	I	49	-3.8	-0.38	0.5	0.84	
	II	50	-2.7	1.02	0.19	0.91	

EXHIBIT 9.25 Comfort Food Corp.: Quarterly sales and economic data.

Your job is to determine which of four hypothesized linear models produces the best forecasts and then use that model to predict CFC's sales for the next six quarters. The four models are shown below.

Hypothesized models for CFC's sales

Model 1:

$$\text{Quarterly Sales}_t = a + b * \text{Consumer sentiment}_t + c * \text{Disposable Income}_t \\ + d * \text{Treasury Spread}_t + e * \text{Risk Spread}_t + g * \text{Time}_t$$

Model 2:

$$\text{Quarterly Sales}_t = a + b * \text{Consumer sentiment}_t + c * \text{Treasury Spread}_t + d * \text{Time}_t$$

Model 3:

$$\text{Quarterly Sales}_t = a + b * \text{Consumer sentiment}_t + c * \text{Risk Spread}_t + d * \text{Time}_t$$

Model 4:

$$\text{Quarterly Sales}_t = a + b * \text{Consumer sentiment}_t + c * \text{Time}_t$$

Begin with a simple time-trend regression (seasonal sales on Time). Have Excel's regression package report the residual errors. Plot the errors to confirm the presence of seasonal and cyclical variation.

Next, identify which of the 4 hypothesized models best forecasts future quarterly sales. Estimate your preliminary models using the first seven years of data and test their forecasting accuracy using the past four years of data. Base your goodness-of-forecast statistics (RMSE, ME, MAD, and RSE) on seasonal sales forecasts. (As you will estimate your models using deseasonalized data, you will need to restore seasonality to your forecasts before calculating the error terms.)

After identifying the model that produces the best forecasts, reestimate that model using all 11 years of data. Have Excel report of the residuals from this regression so that you can graph them and look for evidence of a missing factor. Label your graph appropriately. Finally, forecast sales for the next six quarters. Add the 6 forecasts to your original graph of CFC's quarterly sales. Add them to your graph as a separate series so that your forecasts are clearly identified. Be sure the company name appears in the graph title.

WHAT YOU TURN IN

Submit a report in memo format that describes the purpose and results of your work. Include the criteria you used to select your model. Identify the model that is the least biased, the model that produces the smallest average error regardless of direction, and the model that is the most efficient with respect to the number of independent variables. Place the graph of sales for CFC (including predicted sales) in the body of your memo. Also include a table in your memo that reports the model used (e.g., $\text{Sales}_t = 45 + 4 * \text{Time}_t$) and the sales forecasts. Place the following in your appendix: Clearly labeled graphs of the error terms, before and after model identification; and a well-organized print-out of the spreadsheet showing the goodness-of-forecast calculations you used to identify the best model.

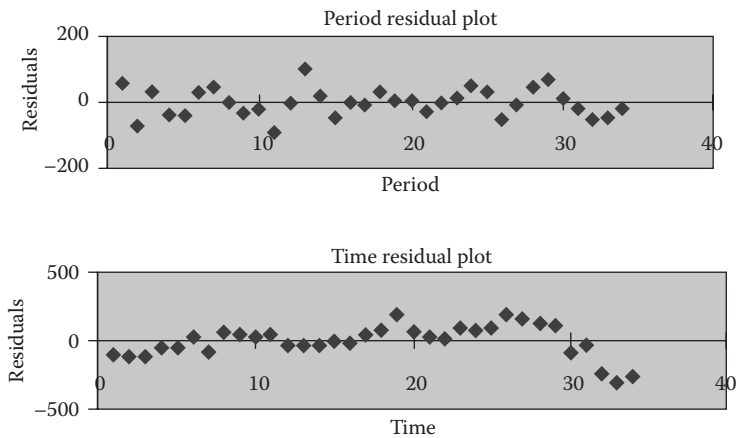
4. Brass Wind and Wood Wind

This problem asks you to apply the techniques of seasonal decomposition, trend analysis, and econometric time-series models to forecast quarterly sales for two companies: Brass Wind and Wood Wind. Both companies' sales exhibit seasonality. In addition, one exhibits a cyclical component related to the treasury spread (long-term treasury rate less T-bill rate) and possibly to changes in the GDP. The data you need to perform for this analysis is reported below in [Exhibit 9.26](#).

		Reported Quarterly Sales			
Year	Quarter	Brass Wind	Wood Wind	Change in GDP (%)	Spread: Treasury
1	I	209.4	105.8	1.5%	3.04
	II	260.8	140.5	1.8%	3.02
	III	247.4	238.7	1.1%	2.63
	IV	121.5	275.0	1.7%	2.32
2	I	161.5	159.8	1.1%	1.34
	II	503.2	277.6	0.6%	1.31
	III	295.1	320.0	1.2%	1.10
	IV	171.0	435.7	1.3%	0.81
3	I	197.2	232.4	1.3%	0.92
	II	473.5	301.0	2.0%	1.52
	III	203.0	528.6	1.0%	1.51
	IV	195.8	363.3	1.5%	1.49
4	I	332.4	198.5	1.8%	1.52
	II	606.1	270.1	1.9%	1.49
	III	285.1	487.7	1.3%	1.18
	IV	218.8	406.3	1.0%	1.03
5	I	279.5	270.3	1.7%	0.58
	II	696.5	383.9	0.8%	0.66
	III	364.9	782.1	1.4%	0.45
	IV	243.8	530.3	1.9%	0.81
6	I	294.4	268.4	1.2%	0.67
	II	716.1	354.4	0.7%	1.01
	III	409.6	685.9	1.5%	1.23
	IV	294.9	580.2	2.4%	0.87
7	I	366.3	329.7	1.5%	-0.22
	II	688.6	510.7	1.9%	1.06
	III	426.9	819.1	0.8%	-0.42
	IV	314.9	670.9	0.9%	-0.52
8	I	429.9	355.9	1.1%	0.39
	II	863.4	314.0	0.6%	1.66
	III	450.7	594.5	0.4%	2.05
	IV	279.9	268.2	0.0%	3.15
9	I	370.6	101.4	-1.0%	3.68
	II	878.8	179.6	0.0%	3.85
Forecasts	III			0.2%	4.00
	IV			0.5%	3.75
10	I			0.5%	3.35
	II			1.0%	2.75

EXHIBIT 9.26 Brass Wind and Wood Wind: Quarterly sales and economic data.

Begin by identifying which series displays only seasonality and which displays seasonality and cyclical. To do this, deseasonalize both series, fit a simple trend line, and plot the deseasonalized errors. One error term series will display a somewhat random pattern (below left), the other a clear long-term cycle (below right). The former is the series that displays only seasonality.



A. For the company that displays seasonality but not cyclicity:

1. Determine the model—the constant-change or compound-growth model—that generates the most accurate forecasts. Use the first 20 observations to estimate each model, the last 14 to test each model's forecasting ability. Measure forecasting accuracy using the residual standard error.
2. Reestimate the model with the lowest RSE using all 34 observations. Use this new model to predict the sales for the next four quarters.
3. Create a graph that displays actual and predicted sales (as separate series).

B. For the company whose sales display both seasonality and cyclicity:

1. Determine the linear model (trend + Treasury spread or trend + Treasury spread + GDP) that generates the best forecasts as measured by the residual standard error. As before, estimate each model using the first 20 observations and test them using the last 14.
2. Reestimate the model with the lowest RSE and use it to predict sales over the next four quarters.
3. As in part A., create a graph that displays actual and predicted sales (as separate series).

WHAT YOU TURN IN

Submit a report in memo format that describes the results of your work. Include the graphs for each firm's actual and forecast sales, with the company name appearing in the title of the graph. Also include a table in your memo that lists, by company, the model used (e.g., $Sales_t = 45 + 4 * Time_t$) and the forecast sales. Include the following in your appendix: Graphs of the error terms, clearly labeled; and well-organized print-outs of the spreadsheets showing your RSE calculations.

IV

A Closer Look at the Details of a Financial Model



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Modeling Long-Term Assets, Capital Budgeting, and Merger Decisions

IN CHAPTER 4, WE developed a model that simulated the financial statements in very general form. In order to introduce the subject and understand the structure of the model, the number and detail of the different accounts was kept to a minimum. Chapter 5 added the invested capital value method and valuation with dilution. Chapter 6 introduced Monte Carlo simulation. It was noted that once we developed a functioning model, we could always expand it to meet our specific needs. This chapter begins that expansion process by developing more detailed treatments of long-term or fixed assets. First, we consider ways to include fixed assets in the long run planning model where the assets are driven by the sales forecast; second, in the capital budgeting section, we will develop a model where the investment in assets is the driver for the purpose of evaluating the investment in fixed assets; and third, we look at the decision to acquire and/or merge with another company as a capital budgeting decision.

10.1 FIXED ASSETS IN A LONG RUN PLANNING MODEL

With a long planning horizon and long periods comprising the horizon, a percent-of-sales approach to fixed assets is usually adequate. In the long run, the firm's ability to increase product output is tied closely to its productive capacity, so that an increase in output and sales will require an increase in the fixed assets that constitute its productive capacity. Thus, the percent-of-sales relation is a good first approximation for total fixed assets, where

$$\text{Fixed Assets}_t = a * \text{Sales}_t \quad (10.1)$$

and the investment in fixed assets is

$$\text{Fixed Asset Investment}_t = a * (\text{Sales}_t - \text{Sales}_{t-1}) \quad (10.2)$$

This simplified approach leads to immediate concurrent responses of investment to changes in sales. However, although this approach may be adequate for very aggregative models where the time periods are relatively long, it can be misleading in a more detailed model that focuses on shorter time periods. One difficulty is that if sales decline, this model always shows disinvestment in response, whereas in the short run a firm can cut production in response to a sales decline without selling off assets. Conversely, most firms have some capacity to increase sales without an immediate increase in assets.

The Odd & Rich model introduced in [Chapter 4](#) linked fixed assets directly to sales and would disinvest with declines in sales. Fixed assets were modeled as

$$\text{Gross Fixed Assets}_t = \left(\frac{\text{GFA}}{\text{Sales}} \right) \text{Sales}_t \quad (10.3)$$

depreciation expense as

$$\text{Depreciation Expense}_t = [\text{Depreciation Rate}] * \text{Gross Fixed Assets}_{t-1} \quad (10.4)$$

and net fixed assets as

$$\text{Net Fixed Assets}_t = \text{Gross Fixed Assets}_t - \text{Allowance for Depreciation}_t \quad (10.5)$$

where the *Allowance for Depreciation* is the accumulated depreciation. In addition to disinvesting when sales decline, this model may even disinvest when sales are growing. If the depreciation rate exceeds the growth rate of sales, depreciation expense will exceed the net addition to assets at each period and net fixed assets will decline. So, even though sales are growing, depreciation is so large that the model shows net fixed assets declining in spite of the sales growth. This may or may not be appropriate for the situation being modeled, so this is just a caution for the modeler to be aware of the implications of the interplay between growth and the depreciation rate. Even if it is appropriate for net fixed assets to decline, you need to put checks in your model so that it does not produce nonsense results such as negative net fixed assets. For example, the Odd & Rich model in [Chapter 4](#) included an IF statement so that depreciation would be zero if it would otherwise cause net fixed assets to be negative:

$$\begin{aligned} \text{Depreciation Expense}_t &= (\text{Depreciation Rate})\text{GFA}_{t-1}, \\ &\text{IF}[(\text{Depreciation Rate})\text{GFA}_{t-1} \leq \text{NFA}_{t-1}], \\ &\text{ELSE} = \text{NFA}_{t-1} \end{aligned} \quad (10.6)$$

This statement is coded in Excel as

$$\text{Depreciation Expense}_t = \text{IF}(\text{Dep_Rate} * \text{GFA}_{t-1} \leq \text{NFA}_{t-1}, \text{Dep_Rate} * \text{GFA}_{t-1}, \text{NFA}_{t-1}) \quad (10.7)$$

There are various ways to modify the link between sales and investment so as to avoid this disinvestment problem. Francis and Rowell (1978) presented a model where investment occurs only if the sales forecast exceeds the level of sales that can be produced with existing assets. For example, assume the minimum stock of assets that must be available to produce a given level of sales is

$$\text{Required Fixed Assets}_t = a * \text{Sales}_t \quad (10.8)$$

If so, asset investment can be specified as

$$\begin{aligned} \text{Fixed Asset Investment}_t &= \text{MAX}(0, \text{Required Fixed Assets}_t - \text{Fixed Assets}_{t-1}) \\ &= \text{MAX}(0, a * \text{Sales}_t - \text{Fixed Assets}_{t-1}) \end{aligned} \quad (10.9)$$

With this code, if $\text{Fixed Assets}_{t-1}$ exceeds $a * \text{Sales}_t$, no new investment occurs in the model, nor do fixed assets decrease in periods where a decline in sales is fed into the model.

The technology of the production process may help us to model the link between sales and investment in fixed assets. The firm's production function, expressing its maximum output as a function of the factor inputs, can be utilized to derive the level of capital assets required to support a given level of sales. For example, in the planning model of AT&T (Davis, Caccappolo, & Chaudry [1973]), the firm's production function is estimated. Based on the production function, the required investment in capital assets can be expressed as a function of sales and the relative prices of the factors of production, including the cost of physical capital services.

One difficulty frequently encountered in structuring the simulation model, particularly when relating investment to sales, is whether to define output in physical units or in dollars. It can be useful to focus on the physical units of output because it is the capacity to produce physical units that most directly affects the need for new investment, and physical capacity is less likely to be confounded by the effects of inflation. In addition, the separation of units from product prices makes the model better suited to exploring pricing policies and the effects of competition and inflation.

For example, in the Francis and Rowell model the dollar amount of investment is linked to the forecast of unit industry sales with the following four simplified expressions:

$$\text{Unit Company Sales}_t = \text{Market Share} * \text{Industry Unit Sales}_t$$

$$\text{Capital Units Required}_t = A * \text{Unit Company Sales}_t$$

$$\text{Required Fixed Assets}_t = P * \text{Capital Units Required}_t$$

$$\text{Fixed Asset Investment}_t = \text{MAX}(0, \text{Required Fixed Assets}_t - \text{Fixed Assets}_{t-1})$$

where A is the ratio of physical capital to sales, and P is the price per unit of physical capital. Another example of focusing on physical units was Gershefski's (1968, 1969) oil company model that defined market share in terms of gallons of gasoline sold, and the model determined the investment in new service stations required to sell a target number of gallons for the forecast period.

The problem with defining variables in physical units is that it can lead to a model that is excessively detailed, perhaps requiring more data input than is available, and generating more output than is necessary to answer the questions that the model is intended to answer. This is especially true for a firm with many different product lines. The excessive detail demanded by unit definitions can be avoided either by sticking with a model defined in dollar amounts, or by using an index of unit output that summarizes the unit output of all the different product lines.

The modeling of fixed assets demonstrates a major difference between a planning model and an accounting model. An accounting model typically is oriented toward keeping track of the costs of the assets and tracking the accounts for specific classes of assets over their life. This is usually too much detail to be useful in a planning context. For planning purposes, it is better to focus attention on the broad aggregate levels of the asset accounts rather than develop excessive detail about the different account categories. An exception occurs when the purpose of the simulation is to analyze the acceptability and feasibility of specific investment proposals, the subject of the next section.

10.2 DIRECT INVESTMENT EVALUATION

When the purpose of the simulation is to evaluate specific investment projects, the modeling approach is different from the case where investment reacts passively to the sales forecast. In this case, the focus is on the specifics of the proposed investment, and the modeling problem is how to relate the details of the investment to the simulation model of the firm.

In the context of financial modeling and planning, there are two perspectives to the consideration of investment proposals. The first is evaluating the investment itself; the second is relating the investment to the firm as a whole so that the user can evaluate how the investment affects the firm's overall performance. The independent evaluation will be considered first, after which we briefly discuss how to integrate the investment into the larger firm model.

10.2.1 Investment Evaluation

There are various techniques that are available for evaluating capital investments by the firm. The two most generally accepted techniques are net present value (NPV) and internal rate of return (IRR).^{*} The NPV will be emphasized here. NPV is calculated according to the expression

^{*} More detailed discussions of the techniques for investment evaluation can be found in any standard financial management text such as Brigham and Ehrhardt (2017) or Ross, Westerfield, Jaffee and Jordan (2016).

$$NPV = \sum_{t=1}^h \frac{Inflow_t}{(1+k)^t} - \sum_{t=1}^h \frac{Outflow_t}{(1+k)^t} \quad (10.10)$$

where $Inflow_t$ is the increment in the cash inflow that is generated by the investment project in period t , $Outflow_t$ is the outflow, typically the cost of the project, h is the number of periods in the life of the project, and k denotes the rate of return that is required on the investment.

The decision criterion used to evaluate the acceptability of an investment project is that the NPV should *not be less than zero*, or $NPV \geq 0$. If the NPV is exactly equal to zero it means that the cash flows are just sufficient to return to the investor the original cost of the investment (return of investment), plus yield a rate of return on investment equal to the discount rate k . If $NPV > 0$, the investment yields a rate of return greater than k in addition to returning the original principal of the investment. Applying the NPV criterion is relatively straight forward so long as one is careful to use the correct definitions of cash flow, cost, and discount rate.

We consider two alternative applications of NPV : first, the weighted-average-cost-of-capital (WACC) or invested capital method; and second, the equity method. The invested capital method evaluates the project as a whole, whereas the equity method evaluates just the equity share of the project. The invested capital approach seeks to answer the question: Are the total (or overall) cash flows generated by the project sufficient to return the original total cost (return of investment principal) and provide the required returns on investment to all classes of security holders who provided capital to finance the cost of the project? The invested capital method is concerned with all the contributors of capital, hence its name: the invested capital method. The equity method asks whether the cash flows to the equity owners of the project will be sufficient to return the initial investment by the equity investors and give them their required return on investment. We will explain the invested capital approach first, then the equity method.

10.2.2 The Invested Capital Method

10.2.2.1 Cost of Investment

For the purpose of evaluating the acceptability of a project, the cost of the project should be the present value of all the incremental outlays necessary to complete the project and make it operational. The outlays typically include expenditures for the purchase of plant and equipment and any investment necessary to increase the balances of working capital, net of any spontaneous increase in current liabilities. As was the case for valuing a firm, how we calculate these costs depends on whether we are using the invested capital (WACC) method or the equity method of evaluation. The total cost of an investment project is financed either entirely or partly with equity capital. The part that is not financed with equity is financed with debt (or some other hybrid source). The total cost includes not just the investment by equity security holders, but also whatever funds were borrowed to finance the project. For example, suppose we invest \$2 million of our own money and borrow \$8 million to purchase assets costing \$10 million. The total project investment is \$10 million, but the equity investment is \$2 million. If we use the invested capital method, we consider the cost of the project as \$10 million; if we use the equity method, the investment cost is the \$2 million.

10.2.2.2 Discount Rate

The discount rate used to calculate NPV with the invested capital approach is a weighted average of the return required by the equity investors (cost of equity, denoted by k_E) and the return required by the lenders (cost of debt, denoted by k_D). WACC is calculated as

$$k_A = \left(\frac{\text{Debt}}{\text{Total capital}} \right) k_D (1 - T) + \left(\frac{\text{Equity}}{\text{Total capital}} \right) k_E \quad (10.11)$$

where the subscript A denotes average;

$(\text{Debt}/\text{Total capital})$ is the proportion of interestbearing debt in the firm's capital structure if the investment is undertaken.

$(\text{Equity}/\text{Total capital})$ is the proportion of equity used to finance the firm if the project is undertaken.

We multiply the cost of debt by $(1 - T)$ to put our costs on an after corporate income tax basis. If our project can earn a return of at least k_A , the project is expected to satisfy all investors by being able to provide each group of investors with their required return.

For example, suppose a project is financed with 80% debt and 20% equity. Assume that the cost of debt is $k_D = 7\%$, the cost of equity is $k_E = 18\%$, and the tax rate is $T = 0.40$. Then WACC will be

$$\begin{aligned} k_A &= \left(\frac{\text{Debt}}{\text{Total capital}} \right) k_D (1 - T) + \left(\frac{\text{Equity}}{\text{Total capital}} \right) k_E \\ &= 0.80 * 7\% * (1 - 0.40) + 0.20 * 18\% \\ &= 6.96\% \end{aligned} \quad (10.12)$$

The correct application of WACC requires that the proportions of debt and equity be the proportions of these capital sources for the firm as a whole if the project is adopted. It would be incorrect to use only the proportions that apply to the project financing in isolation. In this example it is assumed that the project is the whole firm so the 80%/20% mix is appropriate. On the other hand, if the project was part of a larger firm, it would be the firm's capital proportions that are relevant, not just the project's. Another important point about applying WACC is that the proportions of capital should be based on market values, not book values. In other words, the amounts of debt and equity used in calculating the proportions should be market values at the time the project is undertaken.

10.2.2.3 Cash Flow to Invested Capital

The cash flow that should be matched to WACC is the cash flow available to be distributed to those who contributed capital to finance the project. We refer to this as the cash flow to

Income Statement	
Sales	15,000
Operating costs	11,250
EBITDA	3,750
Depreciation	1,560
EBIT	2,190
Interest	433
EBT	1,757
Tax	703
Net Income	1,054

(a)

Cash flow to invested capital	
EBIT(1-T)	1,314
+ Depreciation	1,560
= Earnings flow to invested capital	2,874
- Investment in Net Working Capital	500
- Gross investment in Fixed Assets	1,000
= Cash flow to invested capital	1,374

(b)

EXHIBIT 10.1 (a) Expected incremental income. (b) Cash flow to invested capital.

invested capital. Since both lenders and equity investors contributed capital, this should be the after-tax cash flow available to be distributed to both lenders and equity investors, where, for purposes of cash flow calculation, interest expense, and the tax effects of interest are ignored because they will be taken into account in the calculation of k_A , or WACC. For example, suppose our project is expected to earn incremental income as reported in [Exhibit 10.1a](#).

In that same period, suppose that net working capital must be increased by \$500, and an additional \$1,000 must be invested in fixed assets.

For the WACC approach, we calculate the cash flow to invested capital on an after-tax basis as shown in [Exhibit 10.1b](#).

The cash flow calculations (10.1.b) show neither interest nor principal payments on the debt as deductions from the basic cash flow. Nor does it consider the tax deductibility of interest payments. The reason for this apparent omission is that WACC uses the interest rate on the debt on an after-tax basis, $k_D * (1 - T)$, so the question being asked is whether the cash flows will be large enough to cover these costs. We exclude interest from the cash flow to avoid double counting.

Care must be taken to match project costs and cash flows with the discount rate used. In the above example, the focus was on invested capital and the weighted average cost of capital, so the cost, cash flows, and discount rate were all defined to be consistent with this approach. To clarify these issues, we compare the invested capital method to the equity method next.

10.2.3 The Equity Method

The equity approach to investment evaluation asks whether the cash flows generated by the investment project are sufficient to return the original principal amount invested by the equity investors and provide the return on investment required by the equity investors. In the context of *NPV*, the cost of the investment is the amount of capital contributed by the equity investors; the cash flow is that which is available to be distributed to the equity investors; and the discount rate is the minimum expected return required by the investors as compensation for the risk of the investment. Thus, equity *NPV* is the present value of the cash flows available to stockholders (equity), discounted at the stockholders' required return.

Continuing our numerical example, but now focusing on equity, the initial cost of the project from the stockholders point of view is \$2,000. The discount rate is the required return to equity, $k_E = 18\%$. The operating cash flow to be discounted at 18% is the flow available to the stockholders. Assume in the period being considered, that an additional \$1,200 will be borrowed, and because of earlier borrowing attributable to the project, debt principal payment of \$1,075 will also have to be made. Starting with the net income from the statement shown in [Exhibit 10.1a](#), the equity cash flow would be calculated as in [Exhibit 10.2](#).

This equity cash flow represents the amount that could be taken out of the project by the equity investors after paying all expenses, making the necessary investments in new assets, and meeting the debt obligations. Note that the equity flow takes account of the debt payments. Interest expense and its tax effects have already been taken into account in the calculation of net income. Then, we subtract the principal payments on debt and add proceeds from borrowing to determine the equity flow. With the invested capital approach, neither the interest expense nor the principal flows from borrowing were considered. The major differences between these two approaches are summarized in [Exhibit 10.3](#).

To evaluate a project, we can use either the invested capital approach or the equity approach. Each is valid, provided the cost, cash flows, and discount rate are defined

Cash flow to equity	
Net Income	1,054
+ Depreciation	1,560
= Earnings flow to Equity	2,614
- Investment in Net Working Capital	500
- Gross investment in Fixed Assets	1,000
- Repayment of Debt Principal	1,075
+ Proceeds from new borrowing	1,200
= Cash flow to invested capital	1,239

EXHIBIT 10.2 Calculation of cash flow to equity.

	Invested Capital Method	Equity Method
Discount Rate	Weighted Average Cost of Capital (k_A)	Cost of Equity (k_E)
Cash Flow	Total project cash flow	Equity cash flow
Investment cost	Total Capital Invested (D+E) = Total Fixed Asset Investment + Net Working Capital Investment	Equity invested (E) = Total Fixed Asset Investment + Net Working Capital Investment - Amount Borrowed
Earnings Cash Flow Calculations	EBIT(1-T) + Depreciation Expense = Earnings flow to invested Capital (CF_{IC})	(EBIT - I) *(1-T)+Depreciation Expense = Earnings cash flow to equity (CF_E)
Output: Value	NPV of the project	NPV of the equity of the project

EXHIBIT 10.3 Input and output for Invested Capital and Equity methods of investment evaluation.

consistently. Both are discussed here to help the reader understand the differences in approach. The invested capital approach to *NPV* is the method that is most frequently used. Generally speaking, the nonexpert would tend to make fewer obvious mistakes in using the invested capital approach for investment evaluation.

10.2.4 Working Capital

Regardless of which approach to investment evaluation is used, it is especially important to use the correct method for calculating the various factors that enter the *NPV* calculation. One aspect that is frequently handled improperly is the determination of the cost of the project. A major source of confusion is the treatment of the investment in working capital.

The cost of undertaking an investment project should include not just the direct costs of purchasing and installing the long-term assets, but also any incremental investment in working capital necessary to make the investment operational. If the purchase and operation of a new plant requires that the plant be provided with an initial inventory of raw materials or an initial balance of cash or other current assets, the funds committed to these current assets must be considered part of the cost of the investment. Any funds provided by increases in spontaneous current liabilities, such as accounts payable, offset the increase in the current asset accounts, so we consider only the increase in the net working capital account as part of the cost that must be financed by the contributors of permanent capital.

We typically assume that the investment in net working capital will be recovered at the end of the project. This is analogous to the salvage value of long-term assets at the end of the project.

The foregoing discussion of methods for evaluating investments was intended to provide the general framework. Now, an example will be presented that will demonstrate some of the detail of modeling a capital budgeting problem.

10.3 EXAMPLE: EVALUATING AN INVESTMENT FOR STILIKO PLASTICS

10.3.1 The Details of the Investment

10.3.1.1 *The Opportunity*

Flavio Stiliko has developed a new process for manufacturing a wide variety of plastic parts and components that can be used in products ranging from toys to components for automobiles and aircraft. Flavio is considering a project that would supply the plastic body for a new sports car to be marketed by Rocky Mountain Motor Company. Supplying the bodies in the quantity and quality required by Rocky Mountain Motor would require that Flavio's nascent company, Stiliko Plastics, construct a new manufacturing plant from scratch. Land would have to be purchased, a new building constructed, and new machinery, moulds, and equipment purchased and installed. In addition, the project would require an investment in working capital. The project would be quite expensive for a small start-up company, requiring an investment of about \$10 million over two years. Although there is little doubt that this opportunity to contract with Rocky Mountain is the break that Flavio has been waiting for, the profitability and value of the project need to be rigorously evaluated.

10.3.1.2 *The Objective*

Flavio and his other investors agree that they will adopt the project only if it increases the value of their investment. Laric Amal, one of Flavio Stiliko's coinvestors, agrees, although the link between their decision to adopt the project and their main objective is intangible, the firm can use as its decision criterion the *NPV* of the project: It should accept this project only if its *NPV* is positive. Positive *NPV* serves as the operational goal that, if attained, would help Stiliko Plastic's investors reach their primary objective of making the value of their equity as large as possible.

10.3.1.3 *Financing*

Flavio Stiliko believes they can finance the \$10 million investment by borrowing \$8 million at an interest rate of 7%. The remaining \$2 million will come from Flavio and the other equity investors. They have estimated that with this capital mix, they would require an equity return of about 18%.

10.3.1.4 *The Planning Horizon*

The contract with Rocky Mountain Motors would last for six years. Hopefully, the life of the process and product will extend well beyond that horizon, but Flavio wants to plan conservatively so that if the project ends in six years, it still would be profitable. In addition, the equipment for the new process is expected to deteriorate sufficiently rapidly so that by the end of the six years any continuation would require substantial new capital investment. Consequently, the project will be evaluated over a six-year planning horizon. As annual data provide sufficient detail for the evaluation, the planning horizon is broken up into six subperiods of one year each. The capital investment in the project is

assumed to begin immediately, with the first capital investment occurring at time $t = 0$. Subsequent capital expenditures and cash inflows are assumed to occur at the end of each year.

10.3.1.5 The Constraints: Sales, Production, and Costs

The main constraints relevant to this project are the economic conditions and the technological relationships that limit the firm's ability to attain its objectives. The relevant economic conditions are those in Stiliko Plastic's product market, the requirements in their contract with Rocky Mountain Motor Company, and the costs of the factors of production. As the Rocky Mountain contract and the success of their car will determine the price of the product and the number of units that can be sold, these factors can be considered as constraints that limit the firm. On the basis of the features of the contract and the forecast of the future of the market for this model of car, Stiliko estimates sales of the plastic car bodies will be \$15 million per year, beginning in year 2 and ending in year 6. The firm's production technology and conditions in the markets for labor and materials serve as constraints on production and costs. Estimates of the level of production, the costs of labor and materials, and the technical specification of the new plant lead to a forecast for operating and administrative costs (excluding depreciation) equal to 75% of sales.

10.3.1.6 Project Investment

This project requires that Stiliko build a manufacturing facility on land that must be purchased. The building will cost \$3 million and the land \$1 million. The necessary machinery and equipment will cost an additional \$4.5 million. In addition, the project will require an investment in working capital of about \$1.5 million at the end of period 2. These costs will be incurred according to the schedule shown as [Exhibit 10.4](#) (numbers in thousands of dollars). A total of about \$10 million will have to be committed to the project, with \$5.5 million required immediately, another \$3.0 million at the end of the year 1, and \$1.5 million at the end of the year 2.

	Period		
	0	1	2
Depreciable Assets			
Equipment	3,000	1,500	-
Building	1,500	1,500	-
Non-Depreciable Assets			
Land	1,000		
Total Fixed Asset Investment	5,500	3,000	-
Net Working Capital Investment	-	-	1,500
Incremental Project Investment	5,500	3,000	1,500
Total Cumulated Capital Investment	5,500	8,500	10,000

EXHIBIT 10.4 Stiliko Plastics: Schedule of investment expenditures (\$000s).

10.3.1.7 Depreciation

The equipment will be depreciated using the three year modified accelerated cost recovery system (MACRS) life. Assume that the MACRS depreciation percentages for three-year assets are 33%, 45%, 15%, and 7% for years 1, 2, 3, and 4, respectively. The building will be depreciated over 40 years using straight line depreciation. The depreciation will begin in year 2, when the building and equipment are placed in service, and end in year 6. The land will not be depreciated.

10.3.1.8 Terminal (Salvage) Value

Stiliko expects that the building and machinery and equipment will decrease in market value over the life of the project: the building by 4% per year and the machinery and equipment by 15% per year. Hence, the prices at which these assets can be disposed of at the end of the project are expected to be less than their purchase prices. The land is expected to increase in market value by 3% per year.

10.3.2 Constructing the Spreadsheet Model

The purpose for our model is to determine the *NPV* and *IRR* for this project from the perspectives of all investors (the invested capital method) and of equity alone (the equity method). To this end, we will build a model consisting of three worksheets, named Data Input, Data Processing, and Evaluation. The first sheet (Data Input) lists the basic input data; the second sheet (Data Processing) processes the data in the Data Input sheet for input into the third sheet; and the third sheet (Evaluation) calculates and evaluates the cash flows to all investors and to equity. The Data Input sheet contains the assumptions, data, and information necessary to construct the evaluation model. The other two sheets only contain the formulas necessary to calculate cash flows, discount rates, *NPV*, *IRR*, and other ancillary details.*

The analysis begins with the current state of Stiliko Plastics, represented by the data and assumptions in the Data Input sheet. However, there is a gap between this raw data and the input needed by the Evaluation sheet. The components of future cash flows and the costs of capital used to evaluate them must be estimated. This is the job of the Data Processing worksheet. Fortunately, both evaluation methods share common inputs, so the raw data only need to be processed once. Using the processed data as input, the Evaluation sheet estimates future cash flows to all investors and to equity, and then calculates their *NPVs* and *IRRs*. All that is left is for the investors to interpret the results.

10.3.2.1 Data Input

The Data Input sheet, shown in [Exhibit 10.5](#), contains four sections: (1) The Global Assumptions and Data section lists the number of periods of initial investment and the horizon (cells F7 and F8), market data for calculating discount rates (cells F11:F14), and

* For simple models, it usually is more convenient to place all modules on the same page, similar to the structure used in our financial statement simulation model.

	A	B	C	D	E	F	G	H	I	J	K	
1	Stiliko Corporation											
2	Investment Evaluation											
3	Data Input											
4												
5	Global Assumptions and Data											
6												
7		Number of periods in Horizon				6	Dollars in \$000					
8		Number of periods of Initial Investment				3						
9												
10	Economic and Market Data						Working Capital Assumptions					
11		Riskfree Interest Rate				2.0%	Cash/Sales			1.0%		
12		Market Risk Premium				7.0%	Receivables/Sales			5.0%		
13		Borrowing Rate				7.0%	Inventory/Sales			6.0%		
14		Beta - Unleveraged				1.0	Payables/Sales			2.0%		
15												
16	Fixed Asset Assumptions											
17				Period	0	1	2	3				
18	Investment in Assets											
19	Depreciable Assets											
20		Equipment Investment				3000	1500					
21		Total Deprec. Cost - Equipment					4500					
22		Building Investment				1500	1500					
23		Total Deprec. Cost - Building					3000					
24	Non-Depreciable Assets											
25		Land Investment				1000						
26												
27	Depreciation			Period	0	1	2	3	4	5	6	
28		Depreciation Rate 3-year MACRS					33.0%	45.0%	15.0%	7.0%	0.0%	
29		Depreciation Rate Building					2.5%	2.5%	2.5%	2.5%	2.5%	
30	Appreciation in Value Per Year											
31		Equipment					-15.0%					
32		Building					-4.0%					
33		Land					3.0%					
34												
35	Income Assumptions & Forecast											
36				Period	0	1	2	3	4	5	6	
37		Initial Incremental Sales				0	0	15000				
38		Sales Growth						0	0	0	0	
39		Operating Costs/Sales					75.0%					
40		Tax Rate				40.0%						
41												
42	Financing Assumptions											
43				Period	0	1	2	3	4	5	6	
44		Debt Financing (%)				80.0%	80.0%	80.0%				
45		Debt Term to Maturity				6	5	4				
46		Interest Rate				7.0%	7.0%	7.0%				
47		Equity Financing (%)				20.0%	20.0%	20.0%				

EXHIBIT 10.5 Investment evaluation sheet: Data Input.

current assets and liabilities expressed as a percent of sales (cells K11:K14). The latter ratios allow the model to calculate the investment in net working capital associated with any given sales forecast; (2) The Fixed Asset Assumptions section contains the amounts to be invested in equipment, building and land in each period (lines 18–25), depreciation rates (lines 27–29), and asset appreciation rates (lines 30–33). The latter rates are used to calculate the terminal asset values; (3) The Income Assumptions and Forecast section contains sales and cost forecasts (lines 35–40); and (4) finally, the Financing Assumptions section contains assumptions about financing (lines 42–47). The data from the Data Input sheet feeds into the Data Processing and Evaluation sheets, leading to the ultimate goal of determining the project's *NPV* and *IRR* from the perspectives of invested capital and equity.

10.3.2.2 Data Processing Sheet

The Data Processing sheet contains the formulas and links needed to process the raw data from the Data Input sheet for input into the Evaluation sheet. It is organized in six sections and structured, so the calculations flow from the top to the bottom. The sections are shown in Exhibits 12.6a–c: Part (a) shows the Initial Investment and Asset Accounting sections (lines 1–51); part (b) shows the Debt Accounting and Income Forecast sections (lines 54–87); and part (c) shows the Financing and Costs of Capital

Stiliko Corporation									
Investment Evaluation									
Data Processing									
Initial Investment									
	Period	0	1	2	3	4	5	6	
Depreciable Assets									
Equipment		3,000	1,500						
Building		1,500	1,500						
Non-Depreciable Assets									
Land		1,000							
Total Fixed Asset Investment		5,500	3,000						
Net Working Capital Investment		-	-	1,500					
Incremental Project Investment		5,500	3,000	1,500					
Total Cumulated Capital Investment		5,500	8,500	10,000					
Working Capital Accounts									
	Period	0	1	2	3	4	5	6	
Cash				150	150	150	150	150	
Accounts Receivables				750	750	750	750	750	
Inventory				900	900	900	900	900	
Current Assets				1,800	1,800	1,800	1,800	1,800	
Accounts Payable				300	300	300	300	300	
Net Working Capital				1,500	1,500	1,500	1,500	1,500	
Net Working Capital Investment				1,500	-	-	-	-	
Asset Accounting									
Depreciation Schedule	Period	0	1	2	3	4	5	6	
Equipment				1,485	2,025	675	315	-	
Building				75	75	75	75	75	
Total				1,560	2,100	750	390	75	
Asset Book Value	Period	0	1	2	3	4	5	6	
Equipment		3,000	4,500	3,015	990	315	-	-	
Building		1,500	3,000	2,925	2,850	2,775	2,700	2,625	
Land		1,000	1,000	1,000	1,000	1,000	1,000	1,000	
Total Book Value of Fixed Assets		5,500	8,500	6,940	4,840	4,090	3,700	3,625	
Asset Market Value	Period	0	1	2	3	4	5	6	
Equipment		3,000	4,500	3,825	3,251	2,764	2,349	1,997	
Building		1,500	3,000	2,880	2,765	2,654	2,548	2,446	
Land		1,000	1,030	1,061	1,093	1,126	1,159	1,194	
Total Market Value of Fixed Assets		5,500	8,530	7,766	7,109	6,543	6,056	5,637	
Terminal Value	Period	0	1	2	3	4	5	6	
Net Working Capital Recovery								1,500	
Sale of Fixed Assets								5,637	
Fixed Asset Book Value								3,625	
Gain of Sale of Fixed Assets								2,012	
Tax on Gain								805	
Terminal Flow net of Taxes								6,332	

(a)

Debt Accounting									
	Period	0	1	2	3	4	5	6	
Amount Borrowed		4,400	2,400	1,200					
Rate		0.07	0.07	0.07					
Term		6	5	4					
Debt 0 Payment			923	923	923	923	923	923	
Interest			308	265	219	170	117	60	
Principal			615	658	704	754	806	863	
Balance			3,785	3,127	2,423	1,669	863	0	
Debt 1 Payment				585	585	585	585	585	
Interest				168	139	108	74	38	
Principal				417	447	478	511	547	
Balance				1,983	1,536	1,058	547	0	
Debt 2 Payment					354	354	354	354	
Interest					84	65	45	23	
Principal					270	289	309	331	
Balance					930	641	331	0	
Totals									
Debt Service			923	1,508	1,863	1,863	1,863	1,863	
Interest			308	433	442	342	236	122	
Principal			615	1,075	1,421	1,521	1,627	1,741	
Balance			3,785	5,109	4,888	3,368	1,741	0	
Income Forecast									
	Period	0	1	2	3	4	5	6	
Sales			-	15,000	15,000	15,000	15,000	15,000	
Operating Costs			-	11,250	11,250	11,250	11,250	11,250	
EBITDA			-	3,750	3,750	3,750	3,750	3,750	
Depreciation			-	1,560	2,100	750	390	75	
EBIT			-	2,190	1,650	3,000	3,360	3,675	
Interest			308	433	442	342	236	122	
EBT			(308)	1,757	1,208	2,658	3,124	3,553	
Tax			(123)	703	483	1,063	1,250	1,421	
Net Income			(185)	1,054	725	1,595	1,875	2,132	

(b)

Financing									
	Period	0	1	2	3	4	5	6	
Amount Borrowed		4,400	2,400	1,200					
Equity Capital Investment		1,100	600	300					
Additional Equity Investment			800	-	-	-	-	-	
Project Capital Investment		5,500	3,800	1,500					
Costs of Capital									
Total Invested Capital		10,800							
Total contributed by equity		2,800							
Total contributed by lenders		8,000							
Proportion of Debt		74.07%							
Proportion of Equity		25.93%							
Cost of Debt		7.0%							
Cost of Equity									
Kf		2.0%							
Km - Kf		7.0%							
Bu		1.0							
BL		2.71							
Ke		21.00%							
WACC		8.56%							

(c)

EXHIBIT 10.6 (Continued) Invested Capital (WACC) evaluation: (b) Data processing sheet II, and (c) Data processing sheet III.

sections (lines 89–109). Once the Data Processing sheet is complete, the processed data is fed into the appropriate cash flow section in the Evaluation sheet.

10.3.2.3 Initial Investment

The Initial Investment section (lines 5–25) of the Data Processing sheet I, determines the total outlays for fixed assets and the increase in net working capital for each period. The outlays for fixed assets (cells E8:F12) are taken from the Data Input sheet. The net working capital investment (line 14) links to the Working Capital Accounts sector (lines 18–25). Net working capital requirements (in \$000s) consist of the following items:

$$\begin{aligned} \text{Cash}_2 &= \left(\frac{\text{Cash}}{\text{Sales}} \right) * \text{Sales}_2 = 1\% * 15 \text{ million} \\ &= 150 \text{ (line 19)} \end{aligned}$$

$$\begin{aligned} \text{Accounts Receivable (AR)}_2 &= \left(\frac{\text{AR}}{\text{Sales}} \right) * \text{Sales}_2 = 5\% * 15 \text{ million} \\ &= 750 \text{ (line 20)} \end{aligned}$$

$$\begin{aligned} \text{Inventory}_2 &= \left(\frac{\text{Inventory}}{\text{Sales}} \right) * \text{Sales}_2 = 6\% * 15 \text{ million} \\ &= 900 \text{ (line 21)} \end{aligned} \tag{10.13}$$

less

$$\begin{aligned} \text{Accounts Payable (AP)}_2 &= \left(\frac{\text{AP}}{\text{Sales}} \right) * \text{Sales}_2 = 2\% * 15 \text{ million} \\ &= 300 \text{ (line 23)} \end{aligned}$$

yields

$$\text{Net Working Capital (NWC)}_2 = 1,500 \text{ (line 25)}$$

Stiliko's project will require an investment in net working capital of \$1.5 million at time $t = 2$. The investment in net working capital is the change in net working capital, calculated as

$$\text{NWC Investment}_t = (\text{CA}_t - \text{AP}_t) - (\text{CA}_{t-1} - \text{AP}_{t-1}) \tag{10.14}$$

where:

NWC denotes net working capital

CA is current assets

AP is accounts payable

As we have assumed that Stiliko has no working capital prior to the start of the project, the total net working capital represents the incremental amount that must be invested in

year 2. If we were dealing with an ongoing firm with previous working capital balances, the investment in net working capital would be the incremental increase required should the project be undertaken.

10.3.2.4 Accounting Sections

The next sections of the Data Processing sheet are Asset Accounting ([Exhibit 10.6a](#), lines 27–51) and Debt Accounting ([Exhibit 10.6b](#), lines 54–75). These sections track the asset and debt values needed by the Terminal Value sector (lines 45–51), the Income Forecast section (lines 77–87), and the operating cash flow sections of the Evaluation sheet.

10.3.2.4.1 Asset Accounting The Asset Accounting section tracks the depreciation, book values, and market values of the assets used in the project. Depreciation expense (lines 28–31) is based on the MACRS and 40-year straight-line percentages specified on the Data Input sheet. The book value of each type of asset, such as equipment, is tracked as

$$Equipment_t = Equipment_{t-1} + \left[\begin{array}{c} Equipment \\ Investment_t \end{array} \right] - \left[\begin{array}{c} Equipment \\ Depreciation_t \end{array} \right] \quad (10.15)$$

that, in period 2 is

$$\begin{aligned} G34 &= F34 - G29 \\ &= 4,500 - 1,485 \\ &= 3,015 \end{aligned}$$

The book value of the building is tracked in the same way. As land is not depreciated, its book value remains equal to the original purchase price.

The market values of the assets are based on their forecast rates of appreciation (a negative appreciation rate means the asset is depreciating). These rates, provided in the Data Input sheet, are –15% for equipment (F31), –4% for the building (F32), and +3% for the land (F33). For example, the market value of the equipment at the end of period t is

$$Equipment\ Value_t = Equipment\ Value_{t-1} * (1 + Equipment\ Appreciation\ Rate)$$

that, for period 2 is

$$\begin{aligned} G40 &= F40 * (1 + 'Data\ Input'\ !F31) \\ &= 4,500 * (1 + (-0.15)) \\ &= 3,825 \end{aligned} \quad (10.16)$$

Expressions for the building and land are similar.

The last sector in the Asset Accounting section calculates the project's terminal value and the cash flows at the end of the project. The figures in this sector come from the previous sectors of the Asset Accounting section. Net working capital recovery is the net working capital

balance at the end of the project. The proceeds from the sale of fixed assets at the terminal date (cell K47) are the sum of the market values of equipment, building, and land (cells K40:K42). It is assumed that each asset category is sold at its appreciated market value at the end of the project. The difference between the sale prices of the assets and their book values (cells K34:K36) is the net gain or loss. If the assets are sold at a gain, the gain is taxed; if they are sold at a loss, the loss reduces taxes. The terminal cash flows and net of taxes are summarized in cell K51.

10.3.2.4.2 Debt Accounting Stiliko will borrow a total of \$8 million over three years: \$4.4 million at time 0, \$2.4 million at time 1, and \$1.2 million at time 2. Each issue will bear an interest rate of 4%, mature at the end of year 6, and be serviced by equal annual payments. Borrowing terms are specified on lines (42–48) of the Data Input sheet. The Debt Accounting section (lines 54–75, [Exhibit 10.6b](#)) calculates the total payments, interest expense, principal payments, and balance for each of the debt issues, based on the borrowing terms given.

The annual payment for each issue is calculated with the function

$$\text{Payment} = \text{PMT}(\text{Interest Rate}, \text{Term to Maturity}, \text{Amount of Loan}) \quad (10.17)$$

The interest portion of the payment can be calculated either as

$$\text{Interest}_t = \text{Balance}_{t-1} * (\text{Interest Rate}) \quad (10.18)$$

or, using Excel's interest function, as

$$\text{Interest}_t = \text{IPMT}(\text{Interest Rate}, \text{Period in Life}, \text{Term to Maturity}, \text{Amount of Loan}) \quad (10.19)$$

If you use the *IPMT* function, “Period in the Life of the Loan” is the number of the period when they are numbered in ascending order from beginning to maturity. As the period changes in each column of the model, this is with the *COLUMNS* function. For example, for the loan taken out at $t = 0$, the interest in period 2 is

$$\text{G60} = -\text{IPMT}(\text{E57}, \text{E58} - \text{COLUMNS}(\text{H49:K49}), \text{E58}, \text{E56}) \quad (10.20)$$

where *COLUMNS* (H49:K49) counts the number of columns remaining from H to K (the number of periods remaining after $t = 2$ until the loan matures) and where the other cell references are

$$\text{E57} = \text{Interest Rate}$$

$$\text{E58} = \text{Term to Maturity} \quad (10.21)$$

$$\text{E56} = \text{Amount of Loan}$$

The principal portion of the annual payment is calculated in each case with the relation

$$\text{Principal Payment}_t = \text{Payment}_t - \text{Interest}_t \quad (10.22)$$

The principal balance for each debt issue is

$$Balance_t = Balance_{t-1} - Principal\ Payment_t \quad (10.23)$$

The last part of the debt accounting (lines 72–75) totals the total payments, interest expenses, principal payments, and balances for the three debt issues. Total interest (line 73) feeds into the Income Forecast section as interest expense (line 84); Total principal (line 74) feeds into the equity cash flow calculations in the Evaluation worksheet (line 33). The Total balance (line 75) feeds into the terminal flows to equity in the case when, at the end of the project, the remaining debt balance must be repaid before stockholders can receive their terminal cash flow. In this particular project, however, the debt will have been completely repaid by the end of the project.

10.3.2.4.3 Income Forecast The Income Forecast section (lines 77–87) calculates expected sales based on the specified initial sales (line 37, Data Input) and sales growth rates (line 38, Data Input). Operating costs are a percent of sales (line 39, Data Input); interest expense is drawn from the Debt Accounting section above (line 73); and depreciation expense is taken from the depreciation schedule in the Asset Accounting section (line 31, [Exhibit 10.6a](#)). Taxes are calculated based on the tax rate specified in cell F40 in the Data Input sheet. The Income Forecast section provides nearly all the basic information required to calculate the operating cash flows to invested capital and to equity in the Evaluation worksheet.

10.3.2.4.4 Financing Section The project's capital investment to be provided by the lenders and the equity investors in each period is shown in the Initial Investment section (line 15 of [Exhibit 10.6a](#)). Absent of any complications, the capital outlays for Stiliko would be \$5.5 million, \$3 million, and \$1.5 million at times 0, 1, and 2, respectively, for a total of \$10 million. With the 80% debt target, the amount of debt would be \$8 million and equity would be \$2 million. However, there is a complication: a cash shortfall of \$0.8 million in period 1. The shortfall results from a negative net income of \$185,000 and a debt principal payment of \$615,000. The funds to cover this shortfall must be provided by someone. We have assumed that operating losses will be covered by the equity investors, making equity's proportion of financing the sum of their direct project investment and the cash shortfall. The total funds that will have to be provided by the equity investors at $t = 1$ is calculated (in thousands) as

$$\begin{aligned} \left[\begin{array}{c} Total\ Equity\ Capital \\ Investment \end{array} \right] &= \left[\begin{array}{c} Equity\ Capital \\ Investment \end{array} \right] + \left[\begin{array}{c} Excess\ Equity \\ Investment \end{array} \right] \\ &= \quad F92 \quad + \quad F93 \quad (10.24) \\ &= 600 + 800 \\ &= \$1,400 \end{aligned}$$

Including the additional \$0.8 million needed by the project, the project's total capital investment is \$10.8 million (cell E97). Owing to the additional equity investment, the

proportional amounts of debt and equity will differ slightly from the target 80%/20% mix. The proportion of debt is calculated in E100 as

$$\text{Proportion of Debt} = \frac{\sum_{t=0}^2 \text{Amount Borrowed}_t}{\sum_{t=0}^2 \text{Amount Borrowed}_t + \sum_{t=0}^2 \text{Equity Capital Investment}_t} \tag{10.25}$$

$$E100 = \frac{\text{SUM}(E91:G91)}{\text{SUM}(E91:G91) + \text{SUM}(E92:G92) + \text{SUM}(E93:G93)}$$

$$74.07\% = \frac{8,000}{8,000 + 2,800}$$

The proportion of equity is calculated similarly in E101 based on the total equity investment at times 0, 1, and 2, and comes to 25.9%.

10.3.2.5 Costs of Capital Section

10.3.2.5.1 Costs of Capital There are two components to Stiliko’s costs of capital: the cost of debt and the cost of equity. Two cash flows will be evaluated in the Evaluation sheet—the cash flow to invested capital and cash flow to equity. The former uses the WACC and the latter the cost of equity (k_E).

The WACC uses both costs of capital as inputs. The cost of debt is relatively easy. Stiliko expects to borrow its funds at $k_D = 7\%$. The cost of equity, however, is more complex. It is based on the CAPM formula,

$$k_E = k_f + (k_m - k_f)\beta \tag{10.26}$$

The risk free interest rate is 2% and the market risk premium ($k_m - k_f$) is estimated to be 7%. However, β is more difficult. It should reflect the amount of debt used to finance the project. We assumed the systematic risk of the project equals that of the market, and given the debt-equity mix we have calculated, the beta coefficient for this project is estimated to be 2.714. Therefore, the cost of equity capital is

$$k_E = 2\% + 7\% * 2.714 = 21\% \tag{10.27}$$

as shown in E108.

With these estimates of capital proportions and costs of debt and equity, the WACC is calculated in E109 as

$$k_A = \left(\frac{\text{Debt}}{\text{Total Capital}} \right) * k_D * (1 - T) + \left(\frac{\text{Equity}}{\text{Total Capital}} \right) * k_E$$

$$E105 = (E100) * E102 * (1 - \text{'Data Input'!F40}) + (E101) * E108 \tag{10.28}$$

$$8.56\% = (0.7407) * 7\% * (1 - 0.40) + (0.2593) * 21\%$$

All of the input required to estimate and evaluate the cash flows to invested capital and to equity are finished. We estimate and evaluate the cash flows next, in the Evaluation sheet.

10.3.2.6 Evaluation Sheet

Exhibit 10.7 shows the Evaluation sheet that contains the Invested Capital and Equity evaluation methods. We consider the Invested Capital method first.

10.3.2.7 The Invested Capital Method

Evaluation of the project using the WACC as the discount rate is also referred to as the Invested Capital method. This method is shown in lines (5–21). It begins by calculating the costs and cash flows to all investors, then calculating the *NPV* and *IRR* for these flows. To calculate the appropriate cash flows, this section pulls the following data from the Data Processing sheet: capital invested and the increase in net working capital (line 15), depreciation expense and EBIT (lines 82 and 83), terminal cash flow (cell K51), and the WACC (cell E109). The tax rate is pulled from the Data Input sheet (cell F40). Using this data, the net cash flows to all investors is calculated in line 15.

Stiliko Corporation									
Investment Evaluation									
Invested Capital and Equity Evaluation Modules									
Invested Capital Perspective									
		Period	0	1	2	3	4	5	6
Initial Costs									
Capital Invested			5,500	3,000		-	-	-	-
Net increase in working capital					1,500				
Cash Flows from operations									
EBIT(1 - Tax)				-	1,314	990	1,800	2,016	2,205
Depreciation				-	1,560	2,100	750	390	75
Net cash flows from operations				-	2,874	3,090	2,550	2,406	2,280
Terminal Value					-	-	-	-	6,332
Net cash flows to invested capital			(5,500)	(3,000)	1,374	3,090	2,550	2,406	8,612
WACC	8.56%								
NPV	\$ 4,013					\$ 9,680			
IRR	19.3%					\$ 3,869			
						\$ 13,549			
						\$ 9,536			
						\$ 4,013			
Equity Perspective									
		Period	0	1	2	3	4	5	6
Initial Costs									
Capital Invested			1,100	600					
Net increase in working capital					300				
Cash Flows from operations									
Net income				(185)	1,054	725	1,595	1,875	2,132
+ Depreciation				-	1,560	2,100	750	390	75
- Payment to debt principal				615	1,075	1,421	1,521	1,627	1,741
Net Cash Flows from operations				(800)	1,539	1,404	824	638	466
Terminal Value				-	-	-	-	-	6,332
Net cash flows to invested capital			(1,100)	(1,400)	1,239	1,404	824	638	6,798
Ke	21.00%								
NPV	\$ 2,178					\$ 1,961			
IRR	45.0%					\$ 2,018			
						\$ 3,979			
						\$ 1,801			
						\$ 2,178			

EXHIBIT 10.7 Investment evaluation: Evaluation sheet.

All that remains is to calculate the *NPV* and *IRR* for these cash flows. The *NPV* for the project is calculated in cell C18, as follows:

$$NPV_{Project} = Net\ Cash\ Flows_0 + \sum_{t=1}^6 \frac{Net\ Cash\ Flows_t}{(1+k_A)^t} \quad (10.29)$$

$$C18 = E15 + NPV(C17, F15:K15)$$

$$= \$4,013$$

The *NPV* function is of the form $=NPV(\text{rate}, \text{CF1:CFh})$, where rate in this case refers to the discount rate, k_A (cell C17) and F15:K15 is the cell range that contains the cash flows.

Equation 10.29 shows an important quirk in Excel's $=NPV(.)$ function: It is inconsistent with the way we normally define *NPV* in the finance field. The proper definition of *NPV* includes the cash flows at time $t = 0$ (usually outflows), and $t = 0$ cash flows are not discounted. The Excel *NPV* function, however, assumes that the first cash flow in its series is to be discounted by 1 period. With this assumption, we must treat the capital expenditure at $t = 0$ separately so that it is not discounted. That is why we show E15, the investment at $t = 0$, outside the *NPV* function. The first capital expenditure to be discounted occurs at $t = 1$, so it is the first in the series F15:K15. The project has a positive *NPV*, meaning that it generates enough cash to return the investors' initial capital and exceed their required rates of return.

There are three components to the project's *NPV*: (1) the present values of the operating cash flows, (2) terminal value, and (3) initial costs. These are summarized in cells E17:H21. The present value of the operating flows from the project is calculated in cell H17 as

$$PV_{Operating\ flow} = \sum_{t=1}^6 \frac{Net\ Cash\ Flow\ from\ Operations_t}{(1+k_A)^t} \quad (10.30)$$

$$H17 = NPV(C17, F13:K13)$$

The present value of the terminal flow is calculated cell H18 as

$$PV_{Terminal\ Flow} = \frac{Terminal\ Flow\ Net\ of\ Taxes_6}{(1+k_A)^6} \quad (10.31)$$

$$H18 = K14 / [(1+C17)^{K6}]$$

where K6 is the number of periods to the end of the project, and K14 is the *Terminal Flow*.

Cell H19 is the present value of the project's cash flows excluding its initial costs. It represents the value of the project before subtracting the present value of the costs. In other words, the value of the project is

$$\begin{aligned}
\text{Project Value} &= \text{PV}(\text{Op Flow} + \text{Terminal Flow}) \\
&= \text{PV}(\text{Op Flow}) + \text{PV}(\text{Terminal Flow}) \\
\text{H19} &= \text{H17} + \text{H18} \\
&= 9,680 + 3,869 \\
&= \$13,549
\end{aligned}
\tag{10.32}$$

Cell H20 calculates the present value of the initial outlays for the project as

$$\begin{aligned}
\text{PV}_{\text{Initial Cost}} &= \text{Capital Expenditure}_0 + \sum_{t=1}^6 \frac{\text{Capital Expenditure}_t}{(1 + k_A)^t} \\
\text{H20} &= \text{E8} + \text{NPV}(\text{C17}, \text{F8:K9})
\end{aligned}
\tag{10.33}$$

The present value of the project costs is \$9,536. Subtracting these from the project's cash flows yields the *NPV* of the project, as follows:

$$\begin{aligned}
\text{Net Present Value} &= \text{PV}(\text{Op Flow} + \text{Terminal Flow}) - \text{PV}(\text{Initial Cost}) \\
\text{H21} &= \text{H19} - \text{H20} \\
&= 13,549 - 9,536 \\
&= \$4,013
\end{aligned}
\tag{10.34}$$

The project value exceeds its cost by \$4,013, so it is worthwhile. The *NPV* represents the addition to the value of the firm that should result from the adoption of the project. Thus, the *NPV* of \$4,013 is how much higher the value of Stiliko Plastics should be with the project as compared to not adopting the project.

The other measure of the acceptability of the project is the *IRR* of the project. The *IRR* is solved as the discount rate that will equate the present value of the project's cash inflows with the present value of the project's costs. In other words, *IRR* is the discount rate (r) that makes the present value of all the project cash flows (inflows and outflows) equal to zero, thus,

$$\sum_{t=0}^6 \frac{\text{Cash Flow to Invested Capital}_t}{(1 + r)^t} = 0
\tag{10.35}$$

The Total Net Flow to Invested Capital in row 15 (E15:K15) is the summary flow that is relevant for calculating *IRR*, so the *IRR* is calculated in C19 as

$$\text{C19} = \text{IRR}(\text{E15:K15})
\tag{10.36}$$

where $IRR(.)$ is the function that solves Equation 10.35. We see that the IRR for the project is 19.3%. This figure is to be compared with the firm's cost of capital, $k_A = 8.56\%$ in cell C17. The IRR exceeds the cost of capital, so that the project is acceptable.

10.3.2.8 Equity Method

The structure of the Equity Evaluation model is the same as that for the invested capital method. However, we calculate the cash flows differently and discount them using a different discount rate. The equity method evaluates the cash flows to equity as shown in [Exhibit 10.2](#). These cash flows are discounted by the cost of equity capital, the equity investors' required rate of return. The resulting NPV represents the value of the project to the equity investor.

In the case of Stiliko Plastics, the equity method evaluates the project from the point of view of Flavio Stiliko and the other equity investors. The equity investors will contribute \$2 million to help finance the purchase of project assets. In addition, equity investors will have to cover the negative equity flow of about \$0.80 million in year 1. For committing their capital and bearing much of the project's risk, the equity investors will want a return on their investment of about 21%. This is the rate that will be used to discount the equity cash flows to determine the NPV of the project.

The Equity Evaluation section is shown by lines (25–42) in [Exhibit 10.7](#). It is identical to the Invested Capital section in nearly all respects, differing only in that it calculates equity cash flow and discounts at the cost of equity. Otherwise, it uses the same equations as the other section and it draws its basic data from the Data Processing sheet. Lines (30–34) show the calculation of equity cash flow, following the format of [Exhibit 10.2](#). As before, terminal value is pulled from cell K51 on the Data Processing sheet. It matches the Invested Capital section because the lenders will have been repaid when the project has ended, so the stockholders will receive the entire terminal payoff. Lines (38–42) calculate the NPV as the present value of the cash flows to equity, using the cost of equity as the discount rate. The NPV for the equity investors is \$2,178 (thousand); their IRR is 45.0%.

The equity NPV of \$2.178 million means that the value of the project exceeds its cost by \$2.178 million. The value of the project, \$3.979 million, is the present value of the cash flows that will be earned by the stockholders. However, to earn these cash flows, the stockholders will have to invest their capital. They will invest a total of about \$2 million consisting of \$1.1 million at $t = 0$, \$0.6 million at $t = 1$, \$0.3 million at $t = 2$, and cover the \$0.8 million outflows at $t = 1$. The NPV of their capital investment, \$1,801, is the equity cost of the project. Thus, the amount that the value exceeds the cost, in present value terms is

Equity value of project	\$3.979 million
Equity cost of project	\$1.801 million
Equity net present value	\$2.178 million

This analysis indicates that the project will increase equity value by \$2.178 million and will be attractive to the investors who require a rate of return of 21.00%.

10.3.3 Evaluating the Investment's Impact on the Firm

In addition to evaluating the *NPV* of the investment by itself, it may be important to assess the effect of the investment on the financial position of the firm as a whole, and to see how it fits into the broader set of policies and strategies of the firm. In some cases, an investment project that is attractive when evaluated in isolation with the *NPV* criterion will have undesirable effects on other aspects of the firm. Management will want to know how the investment will affect the whole firm's sales, operating costs, earnings, ability to service debt, liquidity, and equity position. In addition, the financing for the project will affect the whole firm's financial position, and, in turn, the firm's financial mix will affect the cost of capital for the project. Thus, the effect of the investment on the firm's financial position should be explored by integrating the investment decision model with the broader model of the firm.

We can explore the impact of the proposed investment project on the firm as whole by linking the project investment model with the various modules in the firm planning model. Most of the links are straight forward—we add the project's sales and expenses to those the firm would have without the project, and we add the acquired assets and issued liabilities to those of the whole firm. In evaluating a capital investment project when integrated with the firm, both the cost of equity and the weighted average cost of capital should reflect the capital structure mix for the whole firm that results from adding the project financing to the firm's existing capital structure.

For example, suppose that Odd & Rich Corporation is an on-going firm that is considering investing in the same plastic project that Stiliko has been evaluating. Odd & Rich has an existing capital structure consisting of the components shown in the abbreviated balance sheet of [Exhibit 10.8](#) that shows both book values and market values of the capital structure prior to adopting the proposed project. For purposes of calculating costs of capital, we focus on the firm's invested capital—its interest-bearing debt and the equity. The bottom part of the table shows the invested capital components. The interest-bearing debt (both short term and long term), which has a book value of \$40.2 million, has a market value of \$45.9 million

Book and Market Values (\$000,000)		
	Book Value	Market Value
Current Assets	710.0	
Net Fixed Assets	1,424.1	
Total Assets	2,134.1	1,634.4
Non-Interest Bearing Liabilities	600.0	600.0
Short Term Debt	-	-
Long Term Debt	40.2	45.9
Equity	1,493.9	988.5
Total Liabilities & Equity	2,134.1	1,634.4
Invested Capital		
Interest Bearing Debt (ST + LT)	40.2	45.9
Equity	1,493.9	988.5
Invested Capital	1,534.1	1,034.4

EXHIBIT 10.8 O&R assets and capital structure.

because the long-term bonds are trading at premium. The book value of equity is \$1,493.9 million, but with Odd & Rich's common stock trading at a price of \$9.885 for each of its 100 million outstanding shares, the aggregate value of the stock is \$988.5 million.

Given the present capital structure, Odd & Rich's cost of debt is $k_D = 7\%$. The cost of equity, estimated using the CAPM based on a risk-free rate of 4%, a market risk premium of 5%, and Odd & Rich's stock beta of 0.98 is

$$\begin{aligned} k_E &= k_f + (k_m - k_f)\beta \\ &= 4\% + 5\% * 0.98 \\ &= 8.9\% \end{aligned} \tag{10.37}$$

The capital proportions for the WACC calculation are based on the market values of the invested capital components. Even though, in book value terms, the debt to total invested capital ratio is

$$\begin{aligned} \frac{Debt}{Debt + Equity} &= \frac{40.2}{40.2 + 1493.9} \\ &= \frac{40.2}{1534.1} \\ &= 2.6\% \end{aligned} \tag{10.38}$$

in market value terms, we have

$$\begin{aligned} \frac{Debt}{Debt + Equity} &= \frac{45.9}{45.9 + 988.5} \\ &= \frac{45.9}{1034.4} \\ &= 4.4\% \end{aligned} \tag{10.39}$$

Using the capital proportions based on market values, Odd & Rich's WACC is

$$\begin{aligned} k_A &= \left(\frac{Debt}{Total\ Invested\ Capital} \right) k_D (1 - T) + \left(\frac{Equity}{Total\ Invested\ Capital} \right) k_E \\ &= \left(\frac{45.9}{1034.4} \right) * 7\% * (1 - 0.40) + \left(\frac{988.5}{1034.4} \right) * 8.9\% \\ &= 8.7\% \end{aligned} \tag{10.40}$$

If Odd & Rich adopts the proposed project that costs \$10 million, assume that it will finance it with the same 80% debt and 20% equity mix that we discussed for Stiliko. This would require that Odd & Rich borrow \$8 million and provide \$2 million equity to get the project started. As Odd & Rich's resources are fully committed to their existing operations, the

equity component will require that Odd & Rich should issue new common stock to raise the equity portion of project financing. Raising this new capital will change the capital structure mix for the company and possibly change its cost of capital.

Assume that Odd & Rich can borrow the required \$8 million at 8%. The \$2 million of new equity capital will be raised by selling new common shares. The current stock price is \$9.885 per share, but the flotation costs of selling new shares will reduce the proceeds from the issue to \$9.00 per share. This will require that 222,222 new shares be issued ($\$2,000,000/9.00 = 222,222$ shares).

After raising the new capital, the capital structure will be as shown in [Exhibit 10.9](#). Note that when we add the new financing to the existing financial structure, the debt-to-capital ratio changes from 2.6% to 3.1% in book value terms, and from 4.4% to 5.2% in market value terms.

When we calculate the WACC for evaluating the proposed investment project, we need to use the new market value capital proportions for the firm as a whole. In other words, we use the 5.2%/94.8% mix shown in the table in [Exhibit 10.9](#). In this way, we are treating the project's financing as if we obtain some new capital, mix it with the old capital, and use this combined mix to purchase the new project.

Even though we include both the existing and new capital in the calculation of the proportions, our costs of capital should be the marginal costs for the project by itself. The marginal cost of borrowing is the current rate, $k_D = 8\%$ for the additional debt, regardless of what the historical rate that the old debt carries. For example, Odd & Rich's existing long-term debt has a coupon rate of 9%, and prior to this project, was trading at a yield to maturity of 7%. However, both the 9% embedded coupon cost and the prior market yield of 7% are irrelevant for calculating the cost of capital. To issue new debt, the cost will be 8%, and this marginal cost is the relevant cost for calculating the cost of capital.

We also use the current marginal cost of equity in WACC. This marginal cost should not only reflect current market conditions, but it also should be based on the equity risk of the project being evaluated. Prior to adopting the new project, the Odd & Rich's stock had an unleveraged beta of 0.958, a leveraged beta of 0.98, and a cost of equity of 8.9%. However, this cost of capital is not used to evaluate the new project. The cost of equity for the project

	Book Value		Market Value	
	Proportion		Proportion	
<i>Debt</i>				
Short Term	-		-	
Long Term - Existing	40.2		45.9	
Long Term - New	8.0		8.0	
Total Debt	<u>48.2</u>	3.1%	<u>53.9</u>	5.2%
<i>Equity</i>				
Equity - Existing	1,493.9		988.5	
Equity - New	2.0		2.0	
Total Equity	<u>1,495.9</u>	96.9%	<u>990.5</u>	94.8%
Total Invested Capital	<u>1,544.1</u>	100.0%	<u>1,044.4</u>	100.0%

EXHIBIT 10.9 O&R capital structure with new project financing (\$000).

should be based on the risk of the project itself. The project, by itself, with no debt financing is expected to generate equity returns that have volatility that is equal to that of the market in general. In other words, the project, with no debt financing, has a beta of 1.0. On the other hand, Odd & Rich will use some debt to finance the project, so that debt must be considered in calculation of the marginal cost of equity. The way we do this is to calculate a leveraged beta that reflects Odd & Rich's capital structure mix for the firm as a whole. When the new financing is combined with Odd & Rich's existing financing, the market value debt-to-equity ratio will be

$$\frac{Debt}{Equity} = \frac{53.9}{990.5} = 0.054 (5.4\%) \quad (10.41)$$

We use this new, total firm debt-to-equity to calculate the leveraged beta for the project:

$$\begin{aligned} \beta_L &= \beta_U \left(1 + \left(\frac{Debt}{Equity} \right) (1 - T) \right) \\ &= 1.0 * (1 + (0.054)(1 - 0.40)) \\ &= 1.032 \end{aligned} \quad (10.42)$$

and the leveraged beta is used to calculate Odd & Rich's marginal cost of equity capital for the project:

$$\begin{aligned} k_E &= k_f + (k_m - k_f) 1.032 \\ &= 4\% + 5\% * 1.032 \\ &= 9.16\% \end{aligned} \quad (10.43)$$

Using the marginal costs of debt and equity and the whole firm capital structure proportions from the table above, Odd & Rich's WACC for evaluating the project is

$$\begin{aligned} k_A &= \left(\frac{Debt}{Total\ Invested\ Capital} \right) k_D (1 - T) + \left(\frac{Equity}{Total\ Invested\ Capital} \right) k_E \\ &= \left(\frac{53.9}{1044.4} \right) * 8\% * (1 - 0.40) + \left(\frac{990.5}{1044.4} \right) * 9.16\% \\ &= 8.93\% \end{aligned} \quad (10.44)$$

Note that the weighted average cost of capital is really a weighted average of marginal costs.

For evaluating the project from Odd & Rich's viewpoint, the cash flows are calculated the same way as for Stiliko, but we use Odd & Rich's WACC of 8.93%. At this cost of capital, the NPV of the project is \$3.83 million, as compared to the NPV for Stiliko of \$4.013 million, which was based on Stiliko's WACC of 8.56%. The project is attractive for

both firms, but is slightly more valuable to Stiliko because of their lower cost of capital. Stiliko's WACC is lower because it is using proportionally more debt to finance the project. Although both firms are borrowing the same amount for the project, and the direct equity is the same in both cases, Odd & Rich is implicitly committing more equity to the project because its existing equity is backing up the project.

It is ironic that by committing more equity to the project and seeming to have less risk, Odd & Rich has a higher cost of capital. The reason for this is that Stiliko is using proportionally more of the lower cost source of funds (debt). Owing to the greater financial leverage, Stiliko's marginal cost of equity is higher than Odd & Rich's (21% vs. 9.16%). Nevertheless, Stiliko's cost of equity is not so high that it offsets the advantage of using a greater proportion of debt.

The estimation of *NPV* of an investment project is the most widely advocated method for evaluating the acceptability of investment projects. Although we have used it to evaluate a proposal to invest in new plant and equipment, the method applies to evaluating any type of long-term investment. The investment would not have to involve fixed assets. It could be a softer type investment such as a new marketing program or personnel training. These are investments in less tangible assets. The marketing program generates sales demand and brand identity, which is a type of asset; and the personnel training program increases the productivity of the employees. Both would have long-term benefits and potentially increase the firm's cash flow. Thus, both would have costs and benefits that could be evaluated in terms of *NPV*.

10.4 EXAMPLE: GENETIC SYSTEMS CORPORATION

In [Chapter 6](#), we used Monte Carlo simulation to capture the uncertainty in O&R's value. All of the risks used in that simulation were continuous. However, sometimes risk is binomial—an event will either happen or not for some specified probability. For example, when you flip a coin there is a 50% likelihood heads will come up. Or, in the pharmaceutical world, a new drug will receive FDA approval or it will not. This example uses Monte Carlo simulation to model this type of risk.

10.4.1 Success and Failure in Pharmaceutical Research and Development

This example introduces a model in which the ultimate success is contingent on the successful completion of one or more intermediate phases. Put slightly different, it models a series of dependent sequential binary risks.

Genetic Systems Corporation is engaged in the research and development of a new vaccine to treat tumors. This type of biotechnology research has to go through 3 phases of development and testing before reaching its market. In the first two phases, the prospective new vaccine undergoes internal development and testing. If the development and testing is successful at the end of one phase, the product goes on to the next phase where it faces further development, refinement, and testing. If it fails the earlier phase, the product is considered a failure and is abandoned. The third phase is where the vaccine has to receive approval by the FDA. After that, the company can market its product where it is assumed the demand for life saving products is price inelastic.

At each phase there is uncertainty regarding the success of the phase and the costs and revenues. However, before describing these uncertainties, we address the costs and revenues.

Phase 1 is the initial research. It will last for one year, and is expected to cost about \$10 million. If phase 1 is successful, phase 2 will proceed. Phase 2 tests the product in the lab, on animals, and on humans. It is expected to take two years, and will cost about \$15 million per year. If phase 2 is successful, Genetic Systems will apply to for FDA approval to market the drug (phase 3). Phase 3 will cost about \$8 million. It will take a year to get a decision. On receiving approval, the company will go to market with the vaccine.

The market is expected to contain about 1 million people who will purchase the product each year. The treatment will require 12 monthly injections for the year, with each injection costing the patient \$5,000. The company will receive about \$4,000 for each monthly dose, with the balance going to the doctors and hospitals that administer the medicine. Once the product reaches the market, the marginal cost of producing the vaccine is expected to be small relative to the sales price, costing only about \$1,000 per dose. Converting the production costs and revenues to annual figures, the company expects to earn about \$36 million in gross profit each year. They expect the market for this vaccine to last six years, after which it will be displaced by a competing product.

10.4.1.1 Conventional Analysis of the Project Net Present Value

Exhibit 10.10 shows a spreadsheet with the forecasts of the most likely costs and revenues. Should the vaccine make it to market, the NPV would be \$26.4 million. This is based on a discount rate of 20%, regarded as appropriate for risky drug R&D programs. The costs of the R&D phase are shown in D16:D19, and the gross profits from the marketing program for years 5–10 are shown in E20:E25. The net cash flows for the project are summarized in column F, with the present values of costs and cash inflows in C30 and C32, respectively. If the analysis stopped here, the project would be accepted. However, the NPV of \$26.4 million assumes that the product succeeds in all 3 phases of development—something that is by no means certain. Therefore, we need to take a closer look at the risks as reflected in the costs and probabilities of successfully completing each phase of the process.

10.4.1.2 Risk in the R&D Program

R&D programs in the pharmaceutical and biotechnology industries are notoriously risky. At each phase of the R&D process there is a substantial risk of failure. These risks of failure accumulate so that there is high probability that any given drug development project will fail. Some estimates suggest that only about 1 in 10 drug development projects ever get to market. With that in mind, we consider the probabilities for the Genetic Systems project. As a result of our closer examination, we make the following estimates of the various risks:

Phase 1: Costs in phase 1 are uncertain. We portray them using a triangular probability distribution with a minimum of \$9 million, most likely at \$10 million, and a maximum of \$12 million. The probability of phase 1 research being successful is considered to be only 40%.*

* The uniform distribution has been called the distribution of maximum ignorance. If so, the triangle distribution can be considered the distribution of near maximum ignorance. Both are used when data is insufficient to assume any other type of distribution.

	A	B	C	D	E	F
1						
2		Genetic Systems Corporation				
3		Investment Evaluation With Monte Carlo Simulation				
4		(thousands)				
5						
6		INPUT DATA				
7		Market				
8		Patients Per Year (thousar	1,000			
9		Revenue Per Patient	48			
10		Cost Per Patient	12			
11		Gross Profit Per Patient	36			
12		Expected Gross Profit	36,000			
13						
14		DATA PROCESSING				
15			Year	Cost	Cash In Flow	Net Cash Flow
16		Phase 1: Initial Developmen	1	10,000	-	(10,000)
17		Phase 2: Testing	2	15,000	-	(15,000)
18			3	15,000	-	(15,000)
19		Phase 3: FDA Approval	4	8,000	-	(8,000)
20		Phase 4: Market	5		36,000	36,000
21			6		36,000	36,000
22			7		36,000	36,000
23			8		36,000	36,000
24			9		36,000	36,000
25			10		36,000	36,000
26						
27		INVESTMENT EVALUATION				
28		Discount Rate	20.0%			
29		Present Value				
30		Cost	(31,289)			
31		Cash Flow	57,735			
32		Net Present Value	26,446			

EXHIBIT 10.10 NPV model for Genetic Systems' cancer vaccine.

Phase 2: Costs during the 1st year of phase 2 are similarly uncertain. We assume the lowest cost would be \$12 million, most likely at \$15 million, and a maximum at \$20 million. We portray this uncertain cost with a triangular distribution. Costs are expected to increase in the 2nd year of phase 2 by about 8%, but this increase is also uncertain. We think the rate of increase will be normally distributed with mean of 8% and standard deviation of 1%. If phase 1 is successful, the chance of phase 2 succeeding is estimated to be 70%. Success or failure of the 2nd phase will be determined when the phase ends in year 3 of the program.

Phase 3: The costs of applying to the FDA are uncertain, with a triangular distribution with minimum of \$7 million, most likely of \$8 million, and a maximum of \$10 million.

If Genetic Systems reaches this point, there is an 80% chance of success with the FDA application.

Phase 4: Success with the FDA will allow us to go to market. Even then, nothing is certain. The number of patients who will use the vaccine is uncertain. We think that the number of patients per year will be normally distributed with a mean of one million and a standard deviation of 100,000. Revenue and cost per patient are also uncertain. We estimate that the revenue per patient will be normally distributed with mean of \$48,000 annually with a standard deviation of \$5,000, and the cost per patient per year will be normally distributed with mean of \$12,000 and a standard deviation of \$2,000.

10.4.2 Monte Carlo Simulation of Project

Having identified the risky variables and defined their distributions, we put the probability distributions into our model and run a Monte Carlo simulation. [Exhibit 10.11a](#) shows the Excel model with the @RISK probability distribution functions and [Exhibit 10.11b](#) shows the formulas.* We model the discrete risks with a RiskBernoulli function with the form =RiskBernoulli(probability), which returns a 1 with the probability entered, otherwise 0, where 0 represents failure and 1 represents success. In G29 the formula is =RiskBernoulli(F9), where F9 = 0.40. When we run the Monte Carlo simulation, this function will work like flipping a coin where tails is failure, heads is success, and there is a 60% chance of tails. On any one draw in the simulation, this cell will either be 0 or 1 depending on the chance of the draw. But over many draws, 60% of them will be 0 and 40% will be 1.

Success or failure is carried through the model with the 0's and 1's. By multiplying any subsequent cell formula with the 0 or 1, we can make that cell a 0 or the value determined by the formula in the cell. For example, the cost of testing in phase 2 (cell D30) is drawn randomly from the distribution =RiskTriang(12000,15000,20000). However, if phase 1 fails, the research program ends, so there would be no costs (or revenues) in any period after the failure. Therefore, the code in D30 is

$$=RiskTriang(12000,15000,20000) \times G29 \quad (10.45)$$

If the project fails at the end of phase 1, G29 will be 0 and the contents of D30 will be 0. If phase 1 succeeds, G29 will be 1 and D30 will be the result of a draw from the triangular distribution. Thus, at each phase, multiplying by the success or failure of the previous phase, we can carry success or failure through to the end. If we fail at any phase, all subsequent phases will show 0, representing failure at that phase. If we succeed in phases 1 and 2, and then fail in phase 3, we will incur the costs of phases 1, 2, and 3, but we will not be able to go to the market, and we will not earn any of the income.

We set NPV as the output variable to be tracked and then run a simulation. [Exhibit 10.12](#) shows the frequency distribution of the NPV's from a simulation with 5,000 iterations.

* In [Exhibit 10.11a](#) there is only a 40% likelihood of success after phase 2, the expected value of phase 1 success (G29) is zero, zeroing out all subsequent cash flows.

	B	C	D	E	F	G
2	Genetic Systems Corporation					
3	Investment Evaluation With Monte Carlo Simulation					
4	(thousands)					
5						
6	INPUT DATA					
7	Market			Likelihood of Success		
8	Patients Per Year (\$000)	1,000		(RiskBernoulli)		
9	Revenue Per Patient	48		Phase 1	0.40	
10	Cost Per Patient	12		Phase 2	0.70	
11	Gross Profit Per Patient	36		Phase 3	0.80	
12	Expected Gross Profit	36,000				
13						
14	Uncertainty Table					
15	Random Variable					
16	Market	Distribution	Mean	St. Dev.		
17	Patients per year	Normal	1,000.0	100.0		
18	Revenue per patient	Normal	48.0	5.0		
19	Cost per patient	Normal	12.0	2.0		
20						
21	Costs	Distribution	Min	Mean	Max	St. Dev.
22	Phase 1	Triangle	9,000	10,000	12,000	
23	Phase 2	Triangle	12,000	15,000	20,000	
24	Phase 2 2 nd yr increase	Normal		0.08		0.01
25	Phase 3 - FDA	Triangle	7,000	8,000	10,000	
26						
27	DATA PROCESSING					
28		Year	Cost	Cash Inflow	Net Cash Flow	Success?
29	Phase 1: Initial Development	1	10,333	-	(10,333)	0
30	Phase 2: Testing	2	-	-	-	
31		3	-	-	-	0
32	Phase 3: FDA Approval	4	-	-	-	0
33	Phase 4: Market	5		-	-	
34		6		-	-	
35		7		-	-	
36		8		-	-	
37		9		-	-	
38		10		-	-	
39						
40	INVESTMENT EVALUATION					
41	Discount Rate	20.0%				
42	Present Value					
43	Cost	(8,611)				
44	Cash Flow	-				
45	Net Present Value	(8,611)				

(a)

EXHIBIT 10.11 (a) Monte Carlo simulation model for Genetic Systems' cancer vaccine.

(Continued)

	B	C	D	E	F	G
2	Genetic Systems Corporation					
3	Investment Evaluation With Monte Carlo Simulation					
4	(thousands)					
5						
6	INPUT DATA					
7	Market				<i>Likelihood of Success (RiskBernoulli)</i>	
8	Patients Per Year (\$000)	=RiskNormal(D17,E17)				
9	Revenue Per Patient	=RiskNormal(D18,E18)		Phase 1	0.4	
10	Cost Per Patient	=RiskNormal(D19,E19)		Phase 2	0.7	
11	Gross Profit Per Patient	=C9-C10		Phase 3	0.8	
12	Expected Gross Profit	=C8*C11				
13						
14	Uncertainty Table					
15	Random Variable					
16	Market	Distribution	Mean	St. Dev.		
17	Patients per year	Normal	1000	100		
18	Revenue per patient	Normal	48	5		
19	Cost per patient	Normal	12	2		
20						
21	Costs	Distribution	Min	Mean	Max	St. Dev.
22	Phase 1	Triangle	9000	10000	12000	
23	Phase 2	Triangle	12000	15000	20000	
24	Phase 2 2 nd yr increase	Normal		0.08		0.01
25	Phase 3 - FDA	Triangle	7000	8000	10000	
26						
27	DATA PROCESSING					
28		Year	Cost	Cash Inflow	Net Cash Flow	Success?
29	Phase 1: Initial Development	1	=RiskTriang(D22,E22,F22)	0	=E29-D29	=RiskBernoulli(F9)
30	Phase 2: Testing	2	=RiskTriang(D23,E23,F23)*G29	0	=E30-D30	
31		3	=D30*(1+RiskNormal(E24,G24))	0	=E31-D31	=RiskBernoulli(F10)*G29
32	Phase 3: FDA Approval	4	=RiskTriang(D25,E25,F25)*G31	0	=E32-D32	=RiskBernoulli(F11)*G31
33	Phase 4: Market	5		=C12*G32	=E33-D33	
34		6		=E33	=E34-D34	
35		7		=+E34	=E35-D35	
36		8		=+E35	=E36-D36	
37		9		=+E36	=E37-D37	
38		10		=+E37	=E38-D38	
39						
40	INVESTMENT EVALUATION					
41	Discount Rate	0.2				
42	Present Value		=NPV(C41,F29:F32)			
43	Cost	=NPV(C41,F29:F32)		=NPV(C41,E29:E38)		
44	Cash Flow	=NPV(C41,E29:E38)		=RiskOutput("NPV") + C43+C44		
45	Net Present Value	=RiskOutput("NPV") +				

(b)

EXHIBIT 10.11 (Continued) (b) Formulas for Monte Carlo simulation model for Genetic Systems' cancer vaccine.

This simulation [Exhibit 10.12](#) shows a mean NPV of −4.804 million, with outcomes ranging from −39.5 million to +61.9 million. In 77.8% of the iterations, the NPV was negative. After taking the risk of failure into account, this no longer appears to be a viable project.

In this simulation, we see multiple peaks or modes—hence, we call this a multimodal frequency distribution. Each of the peaks corresponds to the value of the program if we fail at a given phase. For example, if we fail at the end of phase 1, we will have incurred the cost of the research at that phase. The phase 1 cost is a random variable from a triangular distribution with a most likely value of \$10 million, but a mean of \$10.33 million. The present value of this cost is shown as the tallest spike in the frequency distribution because there is a 60% chance that we will incur these costs and never proceed to the next phase. The outcomes furthest

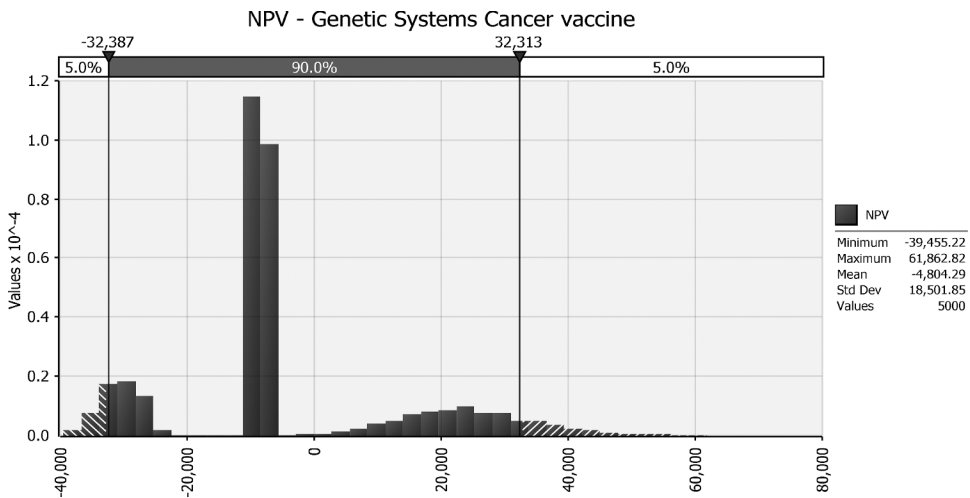


EXHIBIT 10.12 Frequency distribution of NPV for Genetic Systems' cancer vaccine.

to the left on the graph represent the present values of the greatest losses. These are the outcomes when we incur the costs at phases 1 and 2 and succeed at phases 1 and 2, then incur the costs of going to the FDA only to have them reject the application. On the other hand the outcomes that are positive—on the right side of the graph—correspond to the cases the cancer vaccine survives testing and the FDA and reaches the market. However, there is only a 23% chance that this success will happen.

The results of this Monte Carlo analysis are a far cry from the attractive *NPV* that was suggested by our preliminary analysis shown in [Exhibit 10.10](#) where the *NPV* was an attractive \$26.45 million. This helps to point out how Monte Carlo simulation can enhance our planning and decision making. Monte Carlo simulation is an important addition to our collection of tools for financial planning and modeling.*

10.5 MODELING MERGERS AND ACQUISITIONS

10.5.1 Overview

Mergers and acquisitions have become an important part of managing a business in recent years. Long a staple of the American business scene, they also have become increasingly common in Europe, both within and across borders. Mergers and acquisitions present interesting, complex, and challenging modeling problems. This section provides a brief overview of the M&A process and then suggests several ways to model the acquisition decision-taking uncertainty into account.

A primary attraction of mergers and acquisitions is that they offer many potential ways to increase firm value. Commonly cited sources of enhanced value that are amenable to modeling include the operating synergies that result from combining the buyer's business

* We leave it to the reader to analyze this project in the presence of possible price controls.

with the target's business in a way that improves the performance of both operations, the transfer of proprietary technology or unique skills, the use of complementary resources, and the makeover of an acquired operation. However, mergers and acquisitions can also destroy value. Sources of value destruction that are amenable to modeling include paying too much and failing to identify and maintain the value drivers of the transaction.

The track record of mergers and acquisitions is not encouraging. Though, on average M&A activity creates value, most acquisitions do not create value for the acquiring company's shareholders. Indeed, many acquisitions lose money and become significant drains on management resources. Financial modeling can help, especially when it includes Monte Carlo simulation. Many assumptions and projections are made when analyzing a potential acquisition. A financial model can identify which of the assumptions and projections must go right if the acquisition is to succeed.

The M&A process is remarkably complex. Much time and expense goes into just identifying potential targets worthy of further analysis.* Our focus in this section is limited to building a financial statement simulation model of the acquisition process once a target has been selected. We assume the stand-alone projections for the future sales and costs of both target and acquirer have been done and the nonoperational and nonrecurring cash flows have been identified and quantified. Given this information, we show how to combine the financial statements and adjust them to reflect the incremental changes stemming from the merger. This leads to complete pro forma financial statement projections for the target, acquirer, and prospective postacquisition firm and estimates for the stand-alone value of the target, the acquirer, the combined entity, and, consequently the value created by the merger.

The discounted cash flow valuation method we use provides many useful insights into the M&A decision. It clearly identifies the value drivers and value destroyers of the deal. Expected synergies are the motivating factor underlying many, if not most, acquisitions. They are the increase in cash flows over and above those the two companies can generate independently. Synergies derive from increases in operational efficiency, financial efficiency, or both.† The estimated values for these cash flows are highly dependent on the opinions of experienced operating personnel. As they are projections of future value, synergies introduce considerable uncertainty into the valuation for an acquisition or merger. Buying a company also introduces potentially large unanticipated risks. Transition plans have a hard time gauging operating risk—the risk that melding product lines, rearranging assets, and real-locating acquired personnel goes badly. More general risks include the effects of a potential recession on cash flows, the reaction of competitors (e.g., price wars), and the possibility that the expected synergies will be less beneficial than expected. This uncertainty is well suited to Monte Carlo simulation.

* Hooke (1997) and Bruner (2004) offer excellent practical overviews of the merger and acquisition process.

† A short list of synergies includes economies of scale that stem from increased operating efficiency in both the acquiring and target companies, elimination of duplicate costs and unnecessary overhead, increased pricing power, and the exchange of complementary skills or resources.

10.5.2 Modeling the Merger and Acquisition Problem

Given its inherent uncertainty and considerable sources for error, the merger and acquisition problem is an excellent candidate for financial modeling. Although modeling cannot address every uncertainty inherent in the M&A process, it can illuminate many issues related to valuation of the combined firm. In particular, Monte Carlo simulation can identify the primary value drivers and destroyers in a deal.

The remainder of this chapter proceeds as follows: We use a simple example to illustrate one approach to modeling the M&A process based on the dynamic financial statement simulation models introduced in [Chapter 4](#). We then convert this model to a Monte Carlo simulation model. Both models are driven by the Investment Module and take financing into account.

10.5.2.1 The Merger

Marks Pharma is considering approaching another pharmaceutical company, BAM Biotech, with a friendly takeover proposal. Marks' management believes the acquisition makes sense. Marks generates substantial free cash flow, but its drug pipeline is nearly empty, and its research and development department is losing its creative edge. BAM Biotech, on the other hand, has a nearly full drug pipeline and an innovative and successful approach to R&D, but it lacks the money needed to pursue its many promising drugs. The two companies complement each other in other ways as well. Marks has an efficient management structure, a low cost approach to research and development (R&D), and is much better at managing its working capital. BAM Biotech, in contrast, has developed a proprietary and remarkably efficient system of buying and managing its raw materials.

Marks' managers believe that the two firms could share their expertise and make the combined entity considerably more efficient and profitable than either company on its own. They have identified the following areas for potential synergies (value drivers):

- Cost savings, including reduced COGS and SG&A.
- More efficient use of working capital, including lower balances in cash, receivables and inventory, and increased balances in payables.
- Faster growth.

These relative advantages can be seen in the current cost structures and estimated growth rates for the two companies shown in [Exhibit 10.13](#). For the purpose of this model, the expected synergies resulting from the merger are captured by changes in the relationships between the affected income statement and balance sheet items and forecast sales. We discuss each potential source and the ratio by which it is realized next.*

All company parameters are percent of sales. Under Decision/Sensitivity, five-year growth rate is the forecast sales growth rate. Payout ratio is the portion of the firm's net

* Coming up with these estimates is the most difficult part of the M&A process. It is, however, not within the scope of this text. Interested readers are referred to the texts by Bruner (2004) and Hooke (1997).

Parameters (% of sales)	Marks	BAM
COGS	0.640	0.310
SG&A	0.160	0.300
R&D	0.070	0.120
Depreciation	0.020	0.030
Cash and Equivalents	0.090	0.210
Receivables	0.105	0.200
Inventory	0.086	0.100
Gross PPE	0.405	0.460
Goodwill & Intangibles	0.140	0.150
Accounts Payable	0.230	0.200
Economy		
Riskfree rate	0.045	
Market risk premium	0.070	
Unleveraged beta	0.800	0.600
Tax rate	0.400	0.400
Interest expense	0.060	
Decision/Sensitivity		
5-year growth rate	0.10	0.10
Payout ratio	0.30	0.55
Debt portion	0.25	0.25
Debt retired	1.00	0.50
Excess cash returned	0.00	0.00
Price paid for BAM Biotech	28.00	
Terminal rates		
Terminal growth rate	0.050	0.045
Terminal payout ratio	0.70	0.60
Terminal cash returned	1	1

EXHIBIT 10.13 Cost structure for Marks Pharma and BAM Biotech.

income paid out as dividends, Debt portion is the portion of new financing raised by issuing debt, Debt retired is the portion of excess cash used to retire outstanding debt, and Excess cash returned is the portion of excess cash returned to shareholders. Under Terminal rates, Terminal growth rate is the growth rate used to calculate terminal value, Terminal payout ratio is the portion of net income to be paid out as dividends in perpetuity, and Terminal cash returned is a failsafe variable to make sure all excess cash flow is returned to shareholders in perpetuity.

10.5.2.2 Synergies

10.5.2.2.1 Costs The synergies from cost savings are expected to come from reduced operating costs (COGS) and selling, general and administrative expenses (SGA). BAM Biotech has a much more effective cost-control system than Marks. BAM's COGS averages only 31% of sales versus 64% for Marks. Marks managers expect to employ BAM Biotech's methods to increase the efficiency of the combined firm's purchasing system. They also expect to negotiate better prices from their suppliers due to their increased size and financial clout. As a result, they expect the combined firm's COGS-to-sales ratio will be 55%.

The merger team also expects to be able to reduce overall SGA expenses. Sales forces will be integrated and redundancies eliminated. The merger team anticipates lower per unit overhead and advertising costs from increased economies of scale. In addition, Marks' acquisition team expects to institute their own cost-control methods to further reduce BAM's costs. As a result of these changes, they expect the postacquisition SG&A-to-sales ratio to be 15%.

Finally, the merger team believes it is cheaper to acquire BAM's drugs than to develop competing drugs internally. They are interested especially in BAM's full and innovative pipeline and the R&D methods used to develop it. And though they plan to impose some of their cost discipline on BAM's R&D department, the overall postmerger R&D budget will increase. Marks expects the postmerger ratio of R&D-expense-to-sales to be 9%.

10.5.2.2.2 Working Capital Synergies Marks' merger team believes they can free up much of the cash currently locked in BAM's working capital via Marks' greater managerial resources. Gains are expected to come from shorter product cycles, lower inventories and receivables, larger payables, and more efficient use of cash. Marks is more efficient at managing its inventories and receivables than is BAM. On average, Marks' inventory spends 48 days in COGS versus BAM's 88 days (8.6% of sales versus BAM's 10%). The merger team expects to quickly gain control of BAM's inventory, and for this reason they expect postmerger inventory as a percent of sales to remain at 8.6%. On average, credit sales spend 38 days in Marks receivables versus nearly 60 days in BAM's receivables (10.5% of sales versus BAM's 20% of sales). The merger team anticipates postmerger receivable turnover to slow slightly as they plan to aggressively market BAM's pipeline, so receivables are expected to be 11.5% of sales. Marks is marginally better at extracting favorable payment terms from its suppliers. Its payables-to-sales ratio is 23% versus BAM's 20%. Given the increased size of the combined entity, the merger team expects even more favorable terms are possible. Their forecast for postmerger payables is 24% of sales. Finally, BAM currently carries excess cash on its books. Its ratio of cash to sales is 21% versus Marks' 9%. After the merger, Marks expects this ratio to be 9%, in line with its current cash management policies. This will free up some cash to help with acquisition costs.

10.5.2.2.3 Growth Synergies The merger team expects a significant boost to the combined entity's sales. By cross-selling each other's products and integrating the sales force, they plan to move more products through the existing distribution system. By increasing R&D, they plan to develop an innovative new line of drugs. They anticipate that sales growth will be 15% over the next five years, settling into a sustainable growth rate of 5.5% thereafter. The increase in sustainable growth from 5% to 5.5% reflects the long-term value of BAM's drug pipeline. These scenarios are summarized in [Exhibit 10.14](#).

10.5.2.2.4 Costs of Integration Finally, the merger team is planning for substantial post-acquisition costs. The workforce will be trimmed or relocated, plants and warehouses shuttered, product lines merged, and computer and information systems integrated. All of this takes time and costs a great deal. They expect high costs in the first two years, decreasing to zero by the fifth year. [Exhibit 10.15](#) reports the merger team's estimates for these costs.

Variable	Post-merger Expected value
COGS	0.550
SG&A	0.150
R&D	0.090
Receivables	0.115
Inventory	0.086
PPE	0.410
Accounts Payable	0.240
5-year growth	0.150
Terminal growth	0.055
Price paid	30.00

EXHIBIT 10.14 Parameter scenarios after the Marks—BAM merger.

Integration Costs by Year				
1	2	3	4	5
8,000	10,000	4,000	2,000	0

EXHIBIT 10.15 Expected costs of integration.

We now have the inputs necessary to build our base model of Marks acquisition of BAM. We assume that the beta of the combined firm will be a book value-based weighted average of the two firm's betas. This yields an unleveraged beta of 0.74 for the postmerger entity. We also assume book value-based discount rates based on the CAPM, with a risk-free rate of 4.5% and a market risk premium of 7%.

The first step in the valuation process is to estimate the stand-alone intrinsic values for the acquiring and target firms. These values are estimated by percent-of-sales models that use the ratios reported in [Exhibit 10.13](#) and the current financial statements for each firm shown in [Exhibit 10.16](#). Marks' stand-alone intrinsic value is \$51,956 million; BAM Biotech's is \$45,117 million.* We assume that the acquisition occurs by the end of year 1, but that there will have been insufficient time for any expected synergies to be realized. Therefore, year 1 revenues will be the sum of Marks' and BAM's expected year 1 revenues, year 1 COGS will be the sum of the expected year 1 COGS for each firm, and so forth.

To integrate BAM's financial statements with Marks', account balances from BAM's year 1 income statement and balance sheet are entered into the Investment Module for Marks. The expanded Investment Module is shown in [Exhibit 10.17](#). The Investment Module then feeds these values into the Accounting Module. In this way, year 1 account values in Marks' *pro forma* statements are linked to BAM's stand-alone *pro forma* financial statements. The most obvious example of how this accounting adjustment is done can be seen by looking at

* These stand-alone models are not shown because they are simple percent-of-sales models similar to the ones previously described in this text. The assumed five-year growth rate for both firms is 10%. Little or no external financing is required by either firm to support this rate.

Income statement (\$millions)	Marks	BAM Biotech
Revenue	51,790	18,106
COGS	33,054	6,532
Gross profit	18,737	11,574
SG&A	6,187	5,267
R&D	2,677	2,206
Depreciation	205	239
Interest Expense	391	374
Merger Costs		
Net Income Before Taxes	10,214	3,488
Income Taxes	3,064	1,395
Net Income After Taxes	7,150	2,093
Dividends paid	3,211	2,164
Retained earnings	3,939	(71)
Balance Sheet (\$millions)		
Cash and Equiv	4,971	4,572
Receivables	5,423	2,968
Inventory	4,440	1,608
Total Current Assets	14,835	9,148
Gross PPE	20,984	8,706
Accumulated Depreciation	6,788	3,931
Net PPE	14,196	4,775
Goodwill & Intangibles	7,241	2,677
Goodwill from merger		
Total LT Assets	21,437	7,452
Total Assets	36,271	16,600
Accounts payable	11,567	3,561
Total Current Liabilities	11,567	3,561
Long-Term Debt	5,695	7,261
Total Liabilities	17,262	10,822
Common Stock	6,974	2,711
Retained Earnings	36,145	15,795
Treasury Stock	(24,109)	(12,728)
Total Shareholders Equity	19,009	5,778
Total Liabilities & Equity	36,271	16,600

EXHIBIT 10.16 Current income statements and balance sheets for Marks and BAM.

Revenues in cell D42 of [Exhibit 10.18b](#). Marks' revenues in year 1 are \$56,969. BAM's year 1 revenues of \$19,917 are added to Marks revenues for total revenues in year 1 of \$76,886. Less obvious is how the combined cost of goods sold is calculated. BAM's cost of goods sold for year 1 is \$6,174; Marks's is \$36,460. The merged firm's cost of goods sold for year 1 is \$42,635.

	B	C	D	E	F	G	H
87	INVESTMENT MODULE	0	1	2	3	4	5
88	Current Asset Investment		11,331	-436	3,859	4,438	5,104
89	Investment in Net Fixed Assets		6,378	3,212	4,531	5,211	5,993
90	Replacement of Dep. Assets		681	806	906	1,042	1,199
91	Capital Expenditures		7,058	4,018	5,438	6,253	7,191
92	Increase in Goodwill		57,374	1,415	1,857	2,135	2,456
93	Total Investment		75,764	4,997	11,154	12,827	14,751
94							
95	BM Biotech contribution	0	1				
96	<i>Income statement</i>						
97	Revenue	18,106	19,917				
98	COGS	6,532	6,174				
99	SG&A	5,267	5,975				
100	Research & Development	2,206	2,390				
101	Depreciation/Amortization	239	261				
102	Interest Expense	374	436				
103	<i>Balance sheet</i>						
104	Cash & Equivalents	4,572	4,182				
105	Accounts Receivable	2,968	3,983				
106	Inventory	1,608	1,992				
107	Gross PP&E	8,706	9,162				
108	Accumulated Depreciation	3,931	4,192				
109	Goodwill & Intangibles	2,677	2,987				
110	Accounts Payable	3,561	3,983				
111	Debt (assumed)	7,261	7,175				
112	<i>Merger-associated costs</i>		8,000	10,000	4,000	2,000	0
113	<i>Contribution to Goodwill</i>						
114	Book value of liabilities		11,158				
115	Book value of equity	5,778					
116	Number of shares	1,981					
117	Price per share paid	\$ 30.00					
118	Total paid		59,430				
119	Excess over book value to Goodwill		53,652				

EXHIBIT 10.17 Expanded Investment Module for Marks acquisition of BAM showing account balances from BAM's *pro forma* financial statements.

The Investment Module also includes estimates for the postacquisition integration costs associated with the merger as well as the calculation of goodwill that will go onto Marks' balance sheet.* Marks' postacquisition input sector, income statement, and balance sheet are shown in [Exhibit 10.18a](#) and [b](#). The formulas for the income statement and balance sheet are shown in [Exhibit 10.19](#).

Beginning in year 2 we assume that the expected synergies have been realized. This is captured by estimating all the usual income statement and balance sheet items as percentages of the combined revenues. The percentages used are those reported under the Merged Firm column of the Input Sector. Once the modifications to the Investment and Accounting modules are finished, your model is finished. Nothing else needs to be modified. The Financing, Free Cash Flow, and Investment modules automatically adjust to the changes made in the Accounting module.

The intrinsic values for Marks and BAM as stand-alone companies and the intrinsic value after the merger are reported in [Exhibit 10.20](#). On their own, Marks and BAM have

* To maintain simplicity, we assume that the goodwill associated with the acquisition will not be impaired in future and we ignore any associated tax consequences. A word about goodwill: An asset's value reflects the cash flows it throws off, not the cost of putting the asset into service. When a company is acquired, the price paid reflects the operating value of that company's assets, not their cost or book value. For these reasons, the price paid for a firm is typically greater than the book value of that firm. Something must make the total assets brought onto the books of the acquiring firm equal the price paid. That something is called goodwill.

a combined intrinsic value of \$97,073; when merged, their intrinsic value is \$172,420, an increase of \$75,346. On a fully diluted basis, Marks' value per share was \$22.82 before the merger and \$55.23 afterward.

This model, although encouraging, gives us an incomplete assessment of the proposed merger. It assumes that all expected synergies will be realized. This is unlikely to be the case. Some synergies may never be realized, and some costs may be grossly underestimated. Nor does this model tell us which of the potential synergies are responsible for Marks' dramatic increase in value. Although some expectations will likely be met, others will exceed or fall short of their expected values. Hence, there will be some canceling of good and bad draws. It is more realistic to assume that the input parameters can assume one of many different values within a given range. Monte Carlo simulation addresses all of these issues. In the next section we adapt the current model to Monte Carlo simulation.

	B	C	D	E
1	Merger of Marks and BAM Biotech			
2				Merged Firm
3	Parameters	Marks	BAM	
4	COGS	0.640	0.310	0.550
5	SG&A	0.160	0.300	0.150
6	R&D	0.070	0.120	0.090
7	Depreciation	0.020	0.030	0.025
8	Cash and Equivalents	0.090	0.210	0.090
9	Receivables	0.105	0.200	0.115
10	Inventory	0.086	0.100	0.086
11	Gross PPE	0.405	0.460	0.410
12	Goodwill & Intangibles	0.140	0.150	0.140
13	Accounts Payable	0.230	0.200	0.240
14	Economy			
15	Riskfree rate	0.045		
16	Market risk premium	0.070		
17	Unleveraged beta	0.800	0.600	0.740
18	Tax rate	0.400	0.400	
19	Interest expense	0.060		
20	Decision/Sensitivity			
21	5-year growth	0.10	0.100	0.150
22	Payout	0.30	0.550	
23	Debt portion	0.25	0.250	
24	Debt retired	1	0.500	
25	Excess cash	0	0.000	
26	Price paid for BM Biotech	30.00		
27	Terminal rates			
28	Terminal growth	0.050	0.045	0.055
29	Terminal payout	0.7	0.600	
30	Terminal cash	1	1	

(a)

EXHIBIT 10.18 (a) Input sector for Marks acquisition of BAM Biotech.

(Continued)

	B	C	D	E	F	G	H
41	Income statement (millions)	0	1	2	3	4	5
42	Total revenue		76,886	88,419	101,682	116,934	134,474
43	Marks	51,790	56,969				
44	BAM Biotech		19,917				
45	COGS	33,054	42,635	48,630	55,925	64,314	73,961
46	Gross profit	18,737	34,251	39,788	45,757	52,620	60,513
47	SG&A	6,187	15,090	13,263	15,252	17,540	20,171
48	R&D	2,677	6,378	7,958	9,151	10,524	12,103
49	Depreciation	205	681	806	906	1,042	1,199
50	Interest Expense	391	777	1,364	1,206	1,216	1,192
51	Merger Costs		8,000	10,000	4,000	2,000	0
52	Net Income Before Taxes	10,214	3,325	6,398	15,240	20,298	25,849
53	Income Taxes	3,064	1,330	2,559	6,096	8,119	10,339
54	Net Income After Taxes	7,150	1,995	3,839	9,144	12,179	15,509
55							
56	Dividends paid	3,211	599	1,152	2,743	3,654	4,653
57	Retained earnings	3,939	1,397	2,687	6,401	8,525	10,856
58							
59	Balance sheet (\$millions)	0	1	2	3	4	5
60	Cash and Equiv	4,971	9,310	7,958	9,151	10,524	12,103
61	Receivables	5,423	9,965	10,168	11,693	13,447	15,465
62	Inventory	4,440	6,891	7,604	8,745	10,056	11,565
63	Total Current Assets	14,835	26,166	25,730	29,589	34,028	39,132
64							
65	Gross PPE	20,984	32,234	36,252	41,689	47,943	55,134
66	Accumulated Depreciation	6,788	11,661	12,467	13,373	14,415	15,614
67	Net PPE	14,196	20,573	23,785	28,316	33,527	39,520
68	Goodwill & Intangibles	7,241	10,963	12,379	14,235	16,371	18,826
69	Goodwill from merger		53,652	53,652	53,652	53,652	53,652
70	Total LT Assets	21,437	85,188	89,815	96,204	103,550	111,999
71	Total Assets	36,271	111,354	115,545	125,793	137,578	151,131
72							
73	Accounts payable	11,567	17,086	21,221	24,404	28,064	32,274
74	Total Current Liabilities	11,567	17,086	21,221	24,404	28,064	32,274
75							
76	Long-Term Debt	5,695	22,737	20,107	20,272	19,872	18,358
77	Total Liabilities	17,262	39,823	41,327	44,676	47,936	50,632
78							
79	Common Stock	6,974	58,099	58,099	58,597	58,597	58,597
80	Retained Earnings	36,145	37,541	40,228	46,629	55,154	66,011
81	Treasury Stock	-24,109	-24,109	-24,109	-24,109	-24,109	-24,109
82	Total Shareholders Equity	19,009	71,531	74,218	81,117	89,642	100,499
83							
84	Total Liabilities & Equity	36,271	111,354	115,545	125,793	137,578	151,131
85	Check	0	0	0	0	0	0

(b)

EXHIBIT 10.18 (Continued) (b) Income statement and balance sheet after Marks' acquisition of BAM Biotech.

10.5.3 Monte Carlo Simulation

Converting the base model to a Monte Carlo simulation model is simple. All that needs to be done is to define the appropriate input parameters as random variables and define the output variable that we wish to track. The model's structure remains the same. The conversion involves replacing the merged firm parameters with @RISK random variable functions. The inputs to be defined as random variables, their distributions, and their distributions' parameters are shown in [Exhibit 10.21](#). We now briefly discuss the rationale behind the distributions for each of these random variables.

10.5.3.1 COGS

Marks anticipates being able to reduce their operating costs between 15 and 20% by adapting BAM Biotech's cost-control system. They most likely will be able to reduce operating costs to 55% of sales, and have a reasonable likelihood of being able to reduce them even

	B	C	D	E
41	Income statement (millions)	0	1	2
42	Total revenue	=D44+D43		=D42*(1+mgr_growth)
43	Marks	51790.3	=C43*(1+MRK_5_year_growth)	
44	BAM Biotech		=D97	
45	COGS	33053.6	=MRK_COGS*D43+D98	=mgr_cogs*E42
46	Gross profit	18736.7	=D42-D45	=E42-E45
47	SG&A	6186.8	=MRK_SG_A*D43+D99	=mgr_sga*E42
48	R&D	2677.2	=MRK_R_D*D43+D100	=mgr_rd*E42
49	Depreciation	304.9	=((MRK_Depreciation*C65+C67,MAX(C67,0)),MRK_Depreciation*C65)+D101	=((mgr_dep*D65>D67,MAX(D67,0)),mgr_dep*D65)
50	Interest Expense	390.8	=MRK_interest_expense*C76+D102	=MRK_interest_expense*D76
51	Merger Costs		=D112	
52	Net Income Before Taxes	10213.6	=D46-D47-D48-D49-D50-D51	=E46-E47-E48-E49-E50-E51
53	Income Taxes	3064.1	=MRK_Tax_rate*D52	=MRK_Tax_rate*E52
54	Net Income After Taxes	7149.500000000001	=D52-D53	=E52-E53
55				
56	Dividends paid	3210.57	=D150	=E150
57	Retained earnings	3938.930000000001	=D54-D56	=E54-E56
58				

(a)

	B	C	D	E
59	Balance sheet (\$millions)	0	1	2
60	Cash and Equiv	4971.2	=MRK_Cash_and_Equiv*D43+D104	=mgr_cash*E42
61	Receivables	5423.4	=MRK_Receivables*D43+D105	=mgr_rec*E42
62	Inventory	4440	=MRK_Inventory*D43+D106	=mgr_inv*E42
63	Total Current Assets	=SUM(C60:C62)	=SUM(D60:D62)	=SUM(E60:E62)
64				
65	Gross PPE	20983.6	=MRK_Gross_PPE*D43+D107	=mgr_pppe*E42
66	Accumulated Depreciation	6788	=MIN(C66+D49,D65)+D108	=MIN(D66+E49,E65)
67	Net PPE	14195.6	=D65-D66	=E65-E66
68	Goodwill & Intangibles	7241	=MRK_Goodwill___Intangibles*D43+D109	=E42*mgr_gdwl
69	Goodwill from merger		=D119	=D69
70	Total LT Assets	=C67+C68	=D67+D68+D69	=E67+E68+E69
71	Total Assets	=C63+C70	=D70+D63	=E70+E63
72				
73	Accounts payable	11566.9	=MRK_Accounts_Payable*D43+D110	=mgr_ap*E42
74	Total Current Liabilities	=C73	=D73	=E73
75				
76	Long-Term Debt	=4879+816	=C76+D133-D134	=D76+E133-E134
77	Total Liabilities	=C74+C76	=D74+D76	=E74+E76
78				
79	Common Stock	6974	=C79+D135	=D79+E135
80	Retained Earnings	36144.5	=C80+D57	=D80+E57
81	Treasury Stock	-24109.1	=C81-D136	=D81-E136
82	Total Shareholders Equity	19008.9	=SUM(D79:D81)	=SUM(E79:E81)
83				
84	Total Liabilities & Equity	=C77+C82	=D82+D77	=E82+E77
85		Check =C84-C71	=D84-D71	=E84-E71

(b)

EXHIBIT 10.19 Formulas for the postmerger (a) Income statement and (b) Balance sheet.

Tracking Intrinsic Value	
Panel a. Aggregate Equity	
Pre-Acquisition	Intrinsic Value
Marks	51,956
BAM Biotech	45,177
Total	97,133
Post Acquisition	Expected Value
Marks + BAM Biotech	172,420
Change	75,287
Panel b. Per Share (diluted)	
Pre-Acquisition	Intrinsic Value
Pre-Acquisition	22.82
Post-Acquisition	55.23
Change	32.41

EXHIBIT 10.20 Summary of value added by Marks' acquisition of BAM Biotech.

Random Variables and Their Distributions				
Variable	Distribution	Minimum	Expected	Maximum
COGS	PERT	0.52	0.55	0.61
SG&A	Triangle	0.14	0.15	0.22
R&D	Uniform	0.07		0.11
Receivables	PERT	0.105	0.115	0.25
Inventory	PERT	0.08	0.086	0.19
PPE	Uniform	0.405	0.41	0.45
Accounts Pay.	Triangle	0.19	0.24	0.25
5-year growth	PERT	0.07	0.15	0.18
Terminal growth	Uniform	0.035		0.055

EXHIBIT 10.21 Random input variables with their distributions and parameters.

more, to 52%. There is, however, a remote possibility that the integration will not go well. If so, the operating costs as a percent of sales would be 61%.

The PERT distribution is ideally suited to model this type of scenario.* Frame (2003) calls it “the Murphy’s Law distribution.” The reason can be seen in [Exhibit 10.22](#). Ninety-five percent of the draws from this distribution will cluster around 55%, whereas only a thin tail extends out to the worst case draw of 61%. This says that Marks believes it has talented managers who are doing their best and that the outcome of these efforts should be significantly lower operating costs. However, if something goes wrong when adapting BAM’s cost-control methods, it could go very wrong.

10.5.3.2 SG&A

Similar logic applies to the postmerger ratio of selling, general, and administrative costs to revenues. In this case, however, Marks’ merger team is less certain of the likelihood of

* PERT stands for Program Evaluation Review Technique. It was developed by the Navy in the 1950s to plan complex projects. However, it is well-suited for modeling asymmetric distributions with long thin tails.

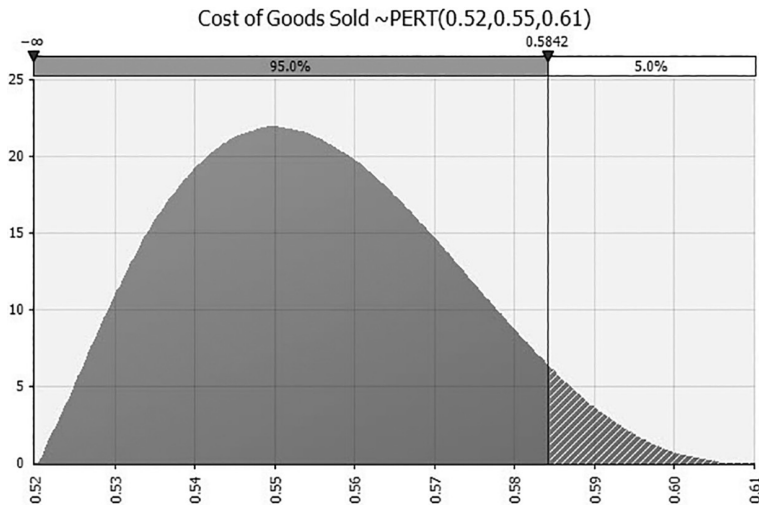


EXHIBIT 10.22 PERT distribution for COGS.

the worst case scenario. As of the difficulty of integrating different cultures, they believe an adverse outcome is more likely for SG&A than for COGS. Therefore, we use the triangular distribution to model this parameter.

10.5.3.3 R&D

Marks' only expectations for postmerger R&D expenses is that they will fall somewhere between 7 and 11% of sales, with any value considered equally likely. Use the uniform distribution to model this expense.

10.5.3.4 Receivables and Inventory

Marks management believes that these ratios will come very close to their expected values. However, should the economy suffer a severe downturn, both would balloon. We use the PERT distribution to model these items.

10.5.3.5 Gross PPE

Investment in fixed assets is expected to fall somewhere between 40.5% and 45% of sales, with all possible values viewed as equally likely. We use the uniform distribution to model this investment.

10.5.3.6 Accounts Payable

Marks is less secure about the value Accounts Payable will assume. Similar to SG&A expense, they have a fairly clear picture for the best, most likely, and worst case values this ratio may assume, but little more. We therefore model this ratio using the triangular distribution.

10.5.3.7 Growth Rates

Marks is fairly certain their growth over the next five years will fall somewhere between 15% and 18% as the synergies from the acquisition of BAM Biotech are realized. However,

there is a remote possibility of an economic downturn, in which case Marks’ short-term growth rate could fall to as little as 7%. We model this variable using the PERT distribution. Terminal growth is expected to fall somewhere between 3.5% and 5.5%, so we model it using the uniform distribution.

The converted input section, after making the above changes, is shown in Exhibit 10.23a: the formula view is shown in Exhibit 10.23b. Random variables are highlighted.

	B	C	D	E	F	G	H	I	J	K
1	Merger of Marks and BAM Biotech									
2	Merged Firm			Random Variables and Their Distributions						
3	Parameters	Marks	BAM	Merged Firm	Variable	Distribution	Minimum	Expected	Maximum	
4	COGS	0.640	0.31	0.555	COGS	PERT	0.52	0.55	0.61	
5	SG&A	0.160	0.30	0.173	SG&A	Triangle	0.15	0.15	0.22	
6	R&D	0.070	0.12	0.090	R&D	Uniform	0.07		0.11	
7	Depreciation	0.020	0.03	0.025	Receivables	PERT	0.105	0.115	0.25	
8	Cash and Equivalents	0.090	0.21	0.100	Inventory	PERT	0.08	0.086	0.19	
9	Receivables	0.105	0.20	0.136	PPE	Uniform	0.405	0.41	0.45	
10	Inventory	0.086	0.10	0.102	Accounts Pay.	Triangle	0.19	0.24	0.25	
11	Gross PPE	0.405	0.46	0.422	5-year growth	PERT	0.07	0.15	0.18	
12	Goodwill & Intangibles	0.140	0.00	0.140	Terminal growth	Uniform	0.035		0.055	
13	Accounts Payable	0.230	0.20	0.227						
14	Economy									
15	Riskfree rate	0.045								
16	Market risk premium	0.070								
17	Unleveraged beta	0.800	0.600	0.74						
18	Tax rate	0.400	0.40							
19	Interest expense	0.060								
20										
21	Decision/Sensitivity									
22	5-year growth	0.25	0.10	0.142						
23	Payout	0.30	0.55							
24	Debt portion	0.25	0.250							
25	Debt retired	1	0.500							
26	Excess cash	0	0.000							
27	Price paid for BM Biotech	\$ 30.00								
28	Terminal rates									
29	Terminal growth	0.050	0.045	0.045						
30	Terminal payout	0.7	0.600							
31	Terminal cash	1	1							

(a)

	B	C	D	E	F	G	H	I	J	K
1	Merger of Marks and BAM Biotech									
2	Merged Firm			Random Variables and Their Distributions						
3	Parameters	Marks	BAM	Merged Firm	Variable	Distribution	Minimum	Expected	Maximum	
4	COGS	0.64	0.31	=RiskPert(I4,J4,K4)	COGS	PERT	0.52	0.55	0.61	
5	SG&A	0.16	0.3	=RiskTriang(I5,J5,K5)	SG&A	Triangle	0.14	0.15	0.22	
6	R&D	0.07	0.12	=RiskUniform(I6,K6)	R&D	Uniform	0.07		0.11	
7	Depreciation	0.02	0.03	0.025	Receivables	PERT	0.105	0.115	0.25	
8	Cash and Equivalents	0.09	0.21	0.1	Inventory	PERT	0.08	0.086	0.19	
9	Receivables	0.105	0.2	=RiskPert(I7, J7, K7)	PPE	Uniform	0.405	0.41	0.45	
10	Inventory	0.086	0.1	=RiskPert(I8, J8, K8)	Accounts Pay.	Triangle	0.19	0.24	0.25	
11	Gross PPE	0.405	0.46	=RiskTriang(I9,J9,K9)	5-year growth	PERT	0.07	0.15	0.18	
12	Goodwill & Intangibles	0.14	0	0.14	Terminal growth	Uniform	0.035		0.055	
13	Accounts Payable	0.23	0.2	=RiskTriang(I10,J10,K10)						
14	Economy									
15	Riskfree rate	0.045								
16	Market risk premium	0.07								
17	Unleveraged beta	0.8	=0.6	0.74						
18	Tax rate	0.4	0.4							
19	Interest expense	0.06								
20										
21	Decision/Sensitivity									
22	5-year growth	0.25	0.1	=RiskPert(I11, J11, K11)						
23	Payout	0.3	0.55							
24	Debt portion	0.25	0.25							
25	Debt retired	1	0.5							
26	Excess cash	0	0							
27	Price paid for BM Biotech	30								
28	Terminal rates									
29	Terminal growth	0.05	0.045	=RiskUniform(I12,K12)						
30	Terminal payout	0.7	0.6							
31	Terminal cash	1	1							

(b)

EXHIBIT 10.23 (a) Input Sector after conversion to Monte Carlo simulation (outlined boxes indicate random input variables). (b) Formulas for Input Sector after conversion to Monte Carlo simulation.

10.5.3.8 Integration Costs

Marks has no strong beliefs about the distributions for the postmerger acquisition costs, other than their worst, most likely, and best cases. We therefore model these costs using the triangle distribution based on the parameters given for the scenario simulation model.

10.5.3.9 Output

Marks plans to use this simulation model to track the increase in the fully-diluted intrinsic value per share. The model allows the user to determine the relative portions of debt and equity when issuing new financing. At an expected price paid of \$30.00 per share, Marks will need to raise a total of \$71,475 million by the end of year 1. Of this, \$59,430 million will be paid to BAM Biotech's current shareholders. We assume the new financing with 25% debt and 75% equity. Although Marks would prefer to finance the acquisition with 100% debt, doing so would put its debt into the junk bond category and raise Marks cost of equity to 20%. Financing entirely with new equity would be too dilutive.*

All that remains is to make the above changes, run a simulation, and analyze the results. We first consider the increase in the fully-diluted intrinsic value per share resulting from the merger. As Marks is currently selling near its intrinsic value per share, \$22.82, Marks' managers believe that any increase in intrinsic value per share will be reflected in the price of their stock. The simulated frequency distribution is shown in Exhibit 10.24. The mean increase is \$8.03 per share or 35%. However, the range of the distribution (−\$37.47 to \$61.45) suggests that there is considerable risk in this acquisition, a conclusion supported by nearly 30% likelihood of the acquisition destroying the current shareholder value.

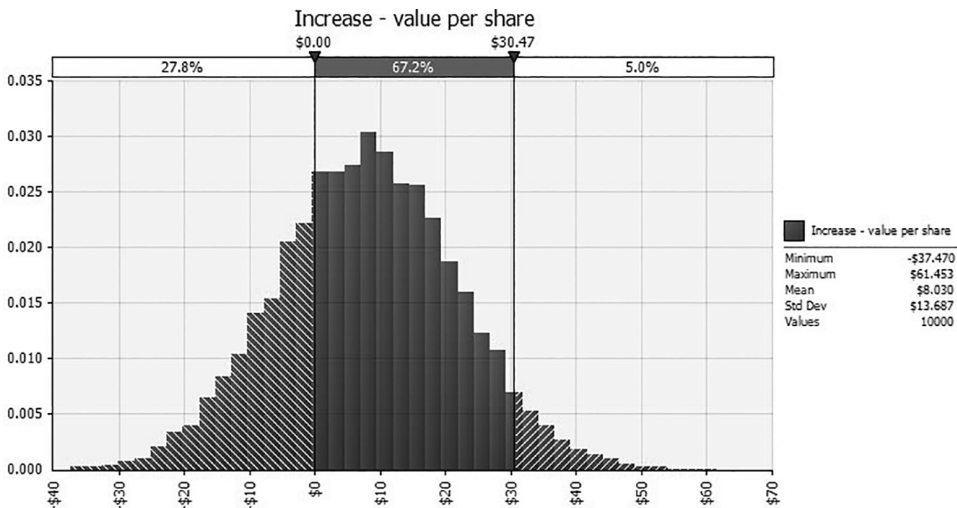


EXHIBIT 10.24 Simulated distribution of the increase in the fully diluted intrinsic value per share resulting from Marks' acquisition of BAM Biotech.

* These values were read directly from the model. The \$71,475 is the required external financing in year 1; the 20% cost of equity is found in the valuation module.

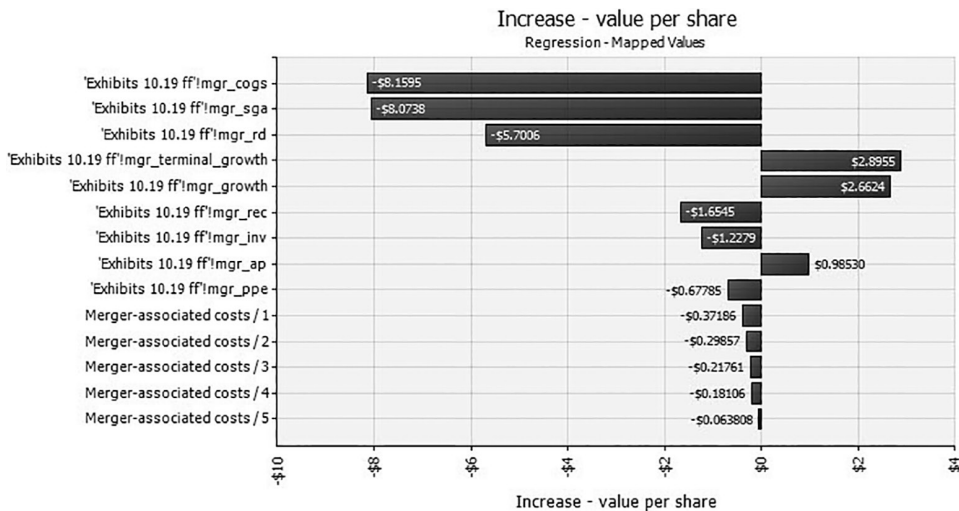


EXHIBIT 10.25 Regression sensitivity tornado graph for simulated increase in intrinsic value per share.

Regression sensitivity analysis as shown in [Exhibit 10.25](#) reveals the key value drivers of this deal. Not surprisingly, they are cost synergies—cost of goods sold, selling, general and administrative expenses, and R&D expenses. Marks will need to closely monitor the synergies related to these costs if it is to benefit from the deal. The terminal growth rate also has a significant effect on intrinsic value added. Marks should therefore try to position itself for stable, long-term growth at the high end of their projections.

Numbers in the bars reflect the relative influence of the variables listed. For example, the reported number for COGS is \$8.16. This means that a 1 standard deviation increase in COGS will decrease the intrinsic value per share added by \$8.16.

Should Marks move forward with the acquisition? Clearly, it has the potential to richly reward their current shareholders, but it is not without significant downside risk. And though this analysis does not reveal a clear answer, it does provide a much richer information set on which management can base their decision.

This example of applying Monte Carlo simulation to an acquisition is highly simplistic. Other than integration costs, it assumes that values for all parameters apply across all subperiods in each iteration. A more realistic model would make separate but correlated draws for each parameter for each subperiod. Additional complexity could be added by assuming that different scenarios apply for many of the random parameters, each with its own distribution. Although these enhancements add significant complexity, they are not particularly difficult to build. The interested reader is referred to Winston (2000).

10.6 SUMMARY

This chapter has discussed three main topics relating to modeling long-term assets. The first modeling approach discussed how to include long-term assets in a sales-driven planning model. The second focused on evaluating an investment in long-term assets. The third showed how to model the merger and acquisition decision.

Modeling long-term assets in the sales-driven model is fairly straight forward. We simply relate the asset level with the sales level. The modeling problems that arise are how to adjust the asset level to both positive and negative changes in assets.

The capital budgeting decision is one of the most important decisions for a firm. We showed how to model the capital budgeting decision. It was noted that the crucial modeling issue is to correctly calculate the cash flows and costs of capital for each approach—equity value or invested capital value. We then introduced the use of Monte Carlo simulation to model-dependent sequential binary risks.

In the last section of this chapter we showed how to model a merger decision. Due to the uncertainties of the benefits of the merger, it is important to explore the drivers of value and analyze the sensitivity of the decision to changes in the assumptions. Monte Carlo simulation was used to explore these aspects.

EXERCISES

1. Rocky Mountain Motors

Rocky Mountain Motors (RMM) manufactures and markets a line of sports cars. At the end of the current year, selected revenue, expense and asset accounts are as follows (expressed in \$ thousands):

Sales revenue	\$50,000
Depreciation expense	\$17,860
Gross fixed assets	\$125,000
Accumulated depreciation	\$63,500
Net fixed assets	\$61,500

Construct a model of Rocky Mountain's fixed assets. Assume that sales drives gross fixed assets, so that for each \$100 of car sales, it has to have gross fixed assets of \$250. Assume that annual depreciation expense is equal to 15% of the previous year's gross fixed assets. Your model should show 10 years of results.

Based on your model, answer the following questions:

- If sales grow at 5% per year, how much will the gross capital investment and net capital investment have to be next year (year 1)?
 - At 5% sales growth, what will be the annual rate of growth in net fixed assets over the next five years?
 - At 5% sales growth, what will be the gross fixed assets and net fixed assets in year 6?
 - At what rate would sales have to grow so that net fixed assets remain unchanged?
- ### 2. Rocky Mountain Motors (2)

Assume the same initial data and ratio relationships for Rocky Mountain Motors as in problem 1. Rocky Mountain Motors' sales tend to be cyclical. In other words,

sales tend to go up when the economy does, and sales decline during recessions. Although RMM cannot operate with less than \$250 worth of gross fixed assets for each \$100 of sales, there is no reason it cannot produce less than \$100 of output with \$250 of gross fixed assets. In other words, if sales decline in a given period it does not have to sell assets. It can maintain its asset level at excess capacity until sales turn up again.

Construct a 10-year model of Rocky Mountain's gross fixed assets so that assets are added when necessary, but gross fixed assets do not decline with decreases in sales.

- Test your model by varying the rate at which sales grow. Include sales that decline at high rates (e.g., -20% annually), low rates of sales growth (2%), and high rates of sales growth (+30% annually).
- Now show the results when the company faces a business cycle with sales growth rates as follows:

Year	1	2	3	4	5	6	7	8	9	10
Sales growth from prior year	+20%	+18%	-35%	-10%	+30%	+25%	+60%	-10%	-15%	+20%

What are the gross and fixed net assets at the end of the 10th year?

3. Xatos Corporation

The Xatos Corporation is considering the purchase of a new Xat machine that will enable it to expand its annual rate of production and increase its expected sales. The machine will have a direct cost of \$14,000 that will be paid right now ($t = 0$). In addition, the machine must be installed and tested. The installation and testing will occur over the next year, and the costs of installation and testing, which are assumed to be paid one year from now ($t = 1$) will amount to \$6,000. The depreciable cost of the machine includes the acquisition cost, installation cost, and testing costs. It will be depreciated according to the MACRS three-year schedule (Rates are 33%, 45%, 15%, and 7%). The machine will be placed in service at the end of the first year ($t = 1$), with depreciation taken for the first time at the end of its first year of service ($t = 2$).

The equipment will be operated for 10 years, with the first year of operation beginning at time 1, with sales for the first year of operations assumed to be received at time $t = 2$. The sales in this first year of operation are expected to be \$350,000. Then, sales will grow by 8% per year until $t = 11$ (the 10th year in operation). The annual operating costs (before depreciation) will consist of fixed cash operating costs of \$25,000 and variable operating costs equal to 75% of sales.

To support the increased level of production, the inventory of raw materials will have to be increased. The inventory will have to be available at the beginning of the year in which the sales are made. With the first sales occurring at $t = 2$, the initial investment in inventory occurs at $t = 1$. The value of the inventory at each date will be

a fixed amount equal to \$5,000, plus a variable amount that is equal to 5% of the sales expected at the end of the coming year. The additional inventory will be carried until the machine is scrapped, which is assumed to be at time 11. At that time the investment in inventory will be recovered. Assume there is no taxable gain or loss on the recovery of investment in inventory.

At the end of the 10-year operating life of the project (time $t = 11$) it is assumed that the equipment will be sold for \$6,000. Any gain over book value will be taxed at the firm's ordinary income tax rate of 40%.

If the new X at machine is purchased, it will be financed by borrowing the required funds at an interest rate of 8%. The loan will be repaid in 8 equal annual installments, with the first principal installment occurring at $t = 4$. Interest will be paid annually as it accrues. The funds will be borrowed at the dates that they will be needed to purchase the project assets.

If this project were considered by itself, with no debt financing, its beta would be 1.3.

Prior to undertaking this new investment, the company's balance sheet is as follows:

Current assets	\$25,000
Net fixed assets	\$50,000
Total assets	\$75,000
<hr/>	
Debtenture bond, 10% annual coupon, Due in 15 years	\$20,000
Common stock, \$1 par value, 1,000 shares	\$1,000
Paid in capital in excess of par value	\$16,000
Earned surplus	\$38,000
Total capital	\$75,000

The debentures are currently trading at a price of 117.29 (% of par value), to yield 8%.

The common stock is trading at a price of \$46.92 per share. The beta coefficient for the stock (before the new project) is 1.56, and recent estimates of the risk premium on the market portfolio are $(k_m - k_f) = 0.05$, and the current risk free rate is $k_f = 0.04$.

Assignment

Set this decision problem up as a spreadsheet to evaluate the acceptability of the X at project. Your model should calculate NPV and IRR for the project. The cost of capital should take account of both the risk of the project itself, and the capital structure mix of the firm as it would be when the project is undertaken.

4. Investment Evaluation with Monte Carlo Simulation: JN Pharmaceutical Corp. acquisition of Bri-Mark Pharmaceutical Company

Part I: Drug development under certainty

JN Pharmaceuticals is considering expanding its drug pipeline by purchasing Bri-Mark Pharmaceutical Company. Bri-Mark is a smaller biotech firm with one drug under development, an antidepressant they call Woebego. Woebego just finished

TABLE 10.1 Remaining Phases of Woebegon Development with their Durations and Expected Costs

Stage	Number of Years in Phase	Average Annual Cost (\$ million)	Minimum Cost (\$ million)	Maximum Cost (\$ million)
a. Phase III testing	3	8	6	10
b. Regulatory filing	2	5	4	8
c. Marketing	1	18		

phase II testing and begins phase III testing immediately. If phase III testing is successful, Bri-Mark will seek FDA approval and, subsequent to approval, begin a marketing campaign. Table 10.1 above lists the remaining phases in the drug's development, the average number of years for completion of each phase, and the average annual cost of completing that phase. Also shown are the minimum and maximum annual costs of completing each remaining phase.

If Woebegon reaches the marketing stage and is successful, the annual cash flow generated by the product's sales would be about \$60 million, beginning the year after the marketing campaign. These cash flows would be earned for eight years, at which point Woebegon would lose its patent protection and all meaningful cash flows from it would cease.

To evaluate this type of investment, the cash flows should be discounted at the cost of capital adjusted for the riskiness of the project. This type of investment is considered to be very risky: its cash flows are about twice as volatile as the average stock market investment. Consequently, in the context of the capital asset pricing model (CAPM), Bri-Mark's beta is about 2. Twenty-year Treasury bonds are currently yielding about 4.5%. Use this as the risk-free rate.* In the current moderately risk-tolerant environment, the risk premium (returns in excess of the risk free rate) on the market portfolio (or the average stock market investment) is about 4%.

Phase III will begin in year 1 and its first costs will be incurred at the end of year 1. Develop a spreadsheet model based on the most likely costs and cash flows with which you can evaluate the fair value of Bri-Mark Pharmaceuticals.

- What is Bri-Mark's *NPV* and the *IRR*?
- What is the maximum price JN should pay for Bri-Mark?

Part II: Modeling Uncertainty

There is considerable uncertainty about the ultimate success of Woebegon. This uncertainty should be considered in the evaluation. We can only guess about the possible success for each of the remaining phases of the research and marketing, and there is considerable variation in the possible cash flows.

In addition, there is a remote possibility that the U.S. Department of Justice (DoJ) will disallow the acquisition due to antitrust concerns. JN plans to file a petition with

* Ibbotson & Associates use the 20-year Treasury-bond yield as their risk-free rate of return, as they believe this is the investment horizon for most long-term investors.

TABLE 10.2 Probabilities of Success for Phase III, FDA Approval, and Marketing Campaign

	Stage		
	Phase III	Regulatory approval	Marketing
Probability of success	70%	85%	80% if aggressive 50% if less aggressive

the DoJ for a letter of approval as soon a purchase agreement is reached. The Justice Department then will seek comment from JN's competitors before making a decision. This alternative action will take one year, after which JN estimates that there is a 10% chance that the Justice Department will nullify the merger. In this case, JN would be out of the first full year of phase III testing expenses.

There are also sources of uncertainty related to each of the three remaining phases of development. Assume that each remaining phase has a probability of failure or success according to the following table (Table 10.2). Model these probabilities using the binomial distribution with n equal to 1 and the probability set appropriately.

If Woebegon is unsuccessful in the phase III testing or in seeking FDA approval, the project will be terminated and no additional costs will be incurred. If it is successful at the end of a given stage, the next stage will be carried to completion. Note also that failure is recognized only at the end of each stage. For example, if a stage lasts for three years, costs for that stage are incurred for the full three years, and then, if Woebegon fails, no future costs or revenues will be realized.

The costs of development at each stage are random variables drawn from a probability distribution. Assume that the development costs are from a triangular distribution with minimum, most likely, and maximum value as shown in Table 10.1. For example, the annual costs for phase 3 are drawn from a triangular distribution with a minimum of \$6 million, a most likely value of \$8 million, and a maximum value of \$10 million. You need to make a separate draw for each year of phase III testing and seeking regulatory approval. Marketing costs are a decision variable, not a random variable.

Marketing and Sales

If Woebegon makes it through the phase III testing and receives FDA approval, then it can be marketed to the medical industry and the public. Stage c. is a marketing program that will begin subsequent to FDA approval and last one year. After that year, Woebegon will be sold to the public, with the first cash flow being received one year after the end of the marketing effort. These flows will continue for eight years.

Bri-Mark is considering two marketing programs, a less aggressive one that costs \$4 million and a very aggressive one that costs \$18 million. The success of the marketing program depends on the amount spent. The \$4 million program has a probability of success of 50%; the \$18 million program has a probability of success of 80%.

If the marketing effort is successful, Woebegon will have a substantially different market than if it is less successful. Projections and data relating to the markets for successful and less successful marketing programs are reported in Table 10.3.

TABLE 10.3 Expected Revenues from Marketing Campaign

Population who use the Woebegon	Successful	Less Successful
Mean	1.0%	0.5%
Standard deviation	0.15%	0.10%
Days per year taken	365	365
Wholesale price per dose	\$0.125	\$0.110
Profit margin	40%	35%

The percentage of the total U.S. population who will use the treatment will be a normally distributed random variable with the parameters reported in Table 10.3. The profit margin represents the rate of profit earned from the wholesale price, and represents the incremental cash flow to the company that will be generated from the sales.

The population in the U.S. prior to beginning phase III testing was approximately 300 million people. It is expected that the population will grow at an annual rate of 0.8% for the next 30 years.

Expand your model to take account of the uncertainty of the Justice Department actions, phase III testing, FDA approval, and the marketing processes. Your model should permit you to conveniently switch between marketing plans. Once you have done so, run a test simulation of 1,000 iterations under the aggressive marketing campaign. Your distribution should look something like that shown in Exhibit 10.26 below. If it does, copy the worksheet and change the marketing scenario in the copy to the less aggressive program. Name the NPV cells in each worksheet according to the

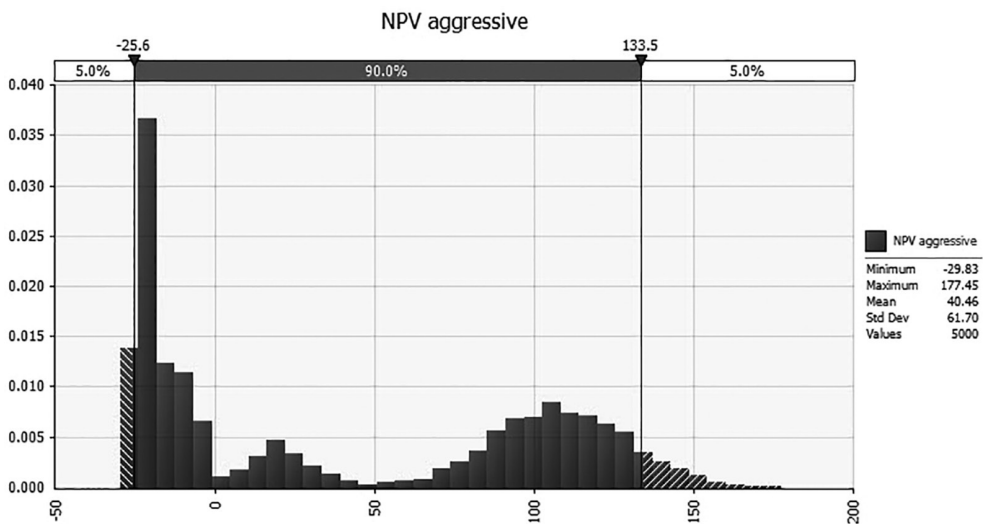


EXHIBIT 10.26 Distribution for NPV aggressive/C91.

marketing plan in effect. Now run a simulation of 10,000 iterations and address the following managerial concerns:

- Why is the simulated expected *NPV* so much smaller than the *NPV* reported by either the model under certainty or by the *NPV* cell in your models under uncertainty?
- The simulated distribution has three modes. Explain them. (You might find it useful to look at the probabilities of an outcome falling into one of the three modes for both proposed marketing campaigns.)
- Now, overlay the simulated *NPV* distribution for the less aggressive marketing campaign on the simulated distribution for the aggressive campaign. Does the less aggressive marketing campaign offer any advantages over its more expensive competitor? If Woebegon is approved by the FDA, should JN market it aggressively (i.e., spend \$18 million) or less aggressively (i.e., spend only \$4 million)?
- Assume that JN will market Woebegon aggressively should it acquire Bri-Mark, and that JN believes it can acquire Bri-Mark for \$33.5 million. At that price, what is the likelihood that doing so will decrease the value of JN's current shareholders. (Put another way, what is the likelihood that Bri-Mark's *NPV* is less than \$33.5 million?)
- JN has a company policy of not investing in projects or acquisitions that have a likelihood of destroying shareholder value greater than 40%. Given this policy, what is the maximum price JN should pay for Bri-Mark?
- Given your analysis, would you recommend that JN acquire Bri-Mark? Defend your position.

Finally, suppose that JN could increase the likelihood of successfully bringing Woebegon to market by 5 percentage points if it invests an additional \$1 million into managing either the DOJ review, phase III testing, or the FDA application. Assume that the \$1 million would be paid in the last year of the stage involved. (Hint: You do not need to test each of the three phases to answer this question.)

- Into which of these stages should JN invest?
- Which marketing campaign would realize the greatest increase in *NPV*?
- By how much would it increase Woebegon's *NPV*?

In a memo, briefly introduce the decisions faced by JN, describe your findings (the answers to the questions in Parts I and II), and give your recommendations (acquire Bri-Mark or not, market aggressively or not, and acquire a portfolio of companies or not). Your memo should contain the @RISK graphs of the probability distributions that support your findings.

5. Sanbull Trombones

Sanbull Corp. is evaluating a proposed project to purchase old student trombones, melt them down to create new brass sheet, then create new professional caliber instruments. The data relating to this project is as follows:

New Machine	
Initial cost at $t = 0$	\$35,000
Depreciation (eight-year straight line)	\$4,375 per year
Salvage value (after taxes)	\$5,000
Investment in working capital	
Inventory increases by	\$7,500 at $t = 0$
Investment in working capital is recovered at the end	\$7,500 for $t = 8$
Expected Income	
Incremental increase in sales	\$85,000 for $t = 1 \dots 8$
Growth rate of sales	0%
Incremental costs	
Costs of goods sold	65% of sales
Selling, general and administrative	20% of sales
Other operating expenses	\$2,500 per year
Income tax rate	34%
Cost of capital	
Required return to equity	12%

Assignment

- A. Develop a spreadsheet with which you can evaluate the acceptability of this proposal. Calculate the net present value (*NPV*) and the internal rate of return (*IRR*) on the project. Note that this part of the assignment does not treat uncertainty. You are asked to develop a deterministic spreadsheet model. Briefly summarize your results.
- B. Now assume that Sanbull's sales and costs are random variables with distributions characterized as follows: *Sales* are distributed normally with mean (average) of \$85,000 per year and standard deviation of \$17,000. (Try =RiskNormal[85000,17000]).

You are uncertain about costs of goods sold as a proportion of sales. You think the lowest this proportion could go is 60%, the most likely is 65%, and it could go as high as 70%. (Try =RiskTriang(0.60,0.65,0.70)).

Selling, general and administrative (SG&A) expenses are uncertain, and past data indicates that the SG&A as a percent of sales is normally distributed with a mean of 20%, and a standard deviation of 4%.

Other operating costs could be as low as \$2,000 per year, or as high as \$3,000 per year. You are very uncertain about these costs, and you think all values between these limits are equally likely. (Try =RiskUniform[2000,3000]).

Salvage value for the machinery at time 8 is uncertain, and is distributed as a normal distribution with a mean of \$5,000 and standard deviation of \$1,000. Treat the recovery of inventory at $t = 8$ as nonrandom: \$7,500.

Include the effects of these uncertainties in your analysis by the following methods:

- a. First, using scenario analysis to analyze the possible outcomes for the *NPV* and *IRR* of the project. With scenario analysis you explore the results of different outcomes without explicitly running a Monte Carlo simulation. In other words, simply make your evaluation based on low, most likely, and high outcomes. For most likely values, use the expected values for each distribution. For the low and high values, use the end points of the triangular and uniform distributions, and use the mean $\pm 2X$ standard deviations for the normal distributions. Report your results in a table for the worst, expected, and best case scenarios.
 - b. Use Monte Carlo simulation to analyze the distribution of *NPV* and *IRR* under the following three scenarios. Briefly summarize your results, including answers to the following two questions: (1) What is the likelihood that *NPV* will turn out to be negative? and (2) What is the range of outcomes for *IRR*? Be sure to include a table that displays your numerical results. Place the area histograms for *NPV* and *IRR* under each scenario in an appendix.
1. In specifying the random variables, set the first period's sales or cost as a distribution (e.g., Normal), and make the subsequent periods' sales or costs equal to the first period. This makes the variables perfectly correlated. Simulate the problem and answer the questions. For example,

Period 1	Period 2
Sales = RiskNormal(85000,17000)	=Sales in period 1

2. Next, make each period's sales or cost an independent random variable by specifying the distribution in each column (year). Simulate and answer the questions. For example,

Period 1	Period 2
Sales = RiskNormal(85000,17000)	=RiskNormal(85000,17000)

3. Give the sales a correlation between periods. *Sales* in period 2 have a correlation of 0.70 with sales in period 1, and sales in period 4 have a correlation of 0.20 with sales in period.
4. Simulate and give answers.
 - c. Sanbull intends to locate its smelters in a foreign country and there is some likelihood that their plant will be expropriated in the future. You have estimated that there is a 15% probability that it will be expropriated in year 5, and a 20% chance it will be expropriated in year 6. It is unlikely that the plant would be expropriated in any other year. Obviously, if it is expropriated, all subsequent cash flows will cease, starting with the year of the expropriation. However, because inventories will be warehoused in the United States, they will be recovered in the year of expropriation. Further, if the plant is expropriated in year 5, it cannot be expropriated in year 6. Use the assumptions of (b)-3 above for other parameters. Briefly summarize your results.

Suggestion:

First build the spreadsheet model entirely in Excel, without considering the uncertainty. Make sure it works well. I calculated the *NPV* of the project as \$3,544. Having completed the basic Excel model, start @RISK and open your Excel model. Change the relevant cells to have the @RISK distribution functions in them. Then, run the @RISK simulation model. In running @RISK, if it bombs because of the *IRR* function, remove the *IRR* and run it to examine *NPV*.

6. VitalRitha Pharmaceuticals

VitalRitha Pharmaceutical Company is a biotechnology company engaged in research, development, manufacture, and marketing of pharmaceuticals developed from genetic research. The company was founded in 1976 by Vitali Kopych, a venture capitalist and Saowarin Sritha, a scientist, each of whom invested \$50,000 to start the company. Today the company employs 1,300 and has annual revenues of about \$400 million.

The company has many new applications of its technology that could lead to significant breakthroughs in medicine, and to very profitable medical products. However, there is great risk in pursuing any one of its research projects because of the high costs of development and the likelihood that the product will not be successful, or will not get the necessary approvals. One of the decision problems faced by VitalRitha is to correctly choose which new research project to support, and how long to continue with this support.

Each potential new drug goes through a four-phase research development R&D process before it can be marketed, as illustrated by the following table that lists each phase, the average number of years for completion of the phase, and the average annual cost of completing that phase, along with the minimum and maximum annual costs of completing the phase.

VitalRitha Pharmaceuticals
Development Cost Estimates

Phase	Years	Average Annual Cost(\$)	Minimum Cost(\$)	Maximum Cost(\$)
1. Preclinical development	2	5 million	4 million	9 million
2. Phase II testing	1	15 million	12 million	18 million
3. Phase III testing	3	8 million	6 million	10 million
4. Regulatory filing	2	5 million	4 million	8 million
5. Marketing	1	14 million	10 million	18 million

(Marketing expenditure is a decision variable in Part 2 of the problem.)

Each different product would have a different cost structure, but these figures are representative. In addition, each product would have a different market potential. For the typical product, if it reaches the marketing stage and is successful, the annual net cash flow generated by the product's sales would be about \$60 million, and these cash flows would be earned for about eight years, at which point they would terminate.

To evaluate this type of investment, the cash flows should be discounted at the cost of capital adjusted for the riskiness of the project. This type of investment is considered to be very risky, with its cash flows about twice as volatile as the average stock market investment. Consequently, in the context of the capital asset pricing model (CAPM), the beta is regarded as about 2.

Assume that the risk free interest rate is 4% and the risk premium (in excess of the risk free rate) on the market portfolio (or the average stock market investment) is about 7%.

It is now the end of year 2010. If a new program is started, it will begin in 2011, and its first costs will be incurred at the end of 2011. The NPV should be evaluated as of this date, the end of 2010.

Assignment 1

Develop a spreadsheet model based on the most likely costs (average annual costs) and cash flows with which you can evaluate the prospect of investing in a new research project.

Your model should show the NPV and the IRR for the typical new research project.

Modeling Uncertainty

There is considerable uncertainty about new research projects that should be considered in the evaluation. We can only guess about the possible success of each phase of the research, and there is considerable variation in the possible cash flows.

To take account of the uncertainty of a project, assume the following:
Each Phase has a probability of failure or success:

Phase	1	2	3	4	5
Probability of success	40%	60%	70%	80%	80% if high marketing expenditures. 50% if low expenditures

If the project is unsuccessful at any of the first 4 stage, the project will be terminated, and no additional costs will be incurred. If it is successful at the end of a given stage, the next stage will be carried to completion. Note also that failure is recognized only at the end of the stage, so if, for example, a stage lasts three years, costs for that stage are incurred each year for the full three years, and then, with failure, the project ends and no future costs or revenues will occur.

The costs of development at each stage are random variables drawn from a probability distribution. Assume that the development costs are from a triangular distribution with minimum, most likely, and maximum value as shown in the table above. For example, the annual costs for Phase 3 are drawn from a triangular distribution with a minimum of \$6 million, a most likely value of \$8 million, and a maximum value of \$10 million. However, keep in mind that these costs would only be incurred if Phase 2 is successful; in which case the Phase 3 costs will be incurred for three years.

Marketing and Sales

If a drug makes it through the R&D process and receives FDA approval, then it can be marketed to the medical industry and the public. Stage 5 is a marketing program that will begin after stage 4 is successful and which lasts just one year. After the end of that year, the product will be sold to the public, with the first cash flow being received one year after the end of the marketing effort. These flows will continue for eight years. In other words, the earnings from the product will be received from year 10 through year 17.

The success of the marketing program depends on the amount spent on the program. If the company spends at least \$15 million in stage 5, the probability of success will be 80%. If it spends less than that, its probability of success will be 50%. There are two alternative marketing programs available, the less expensive program involves marketing expenditures of \$10 million, and the more expensive program requires that \$18 million be spent at stage 5.

If the marketing effort is successful, it will have a substantially different market than if it is less successful. Projections and data relating to the markets for successful and less successful marketing programs are as follows.

The percentage of the total U.S. population who will use the treatment will be a normally distributed random variable with the following parameters:	Successful	Less Successful
Mean percent of population who use the medication	1%	0.5%
Standard deviation of percentage usage	0.15%	0.10%
Days per year taken by each patient	365	365
Wholesale price per dose	\$0.148	\$0.12
Profit margin	40%	35%

The profit margin represents the rate of profit earned from the wholesale price, and represents the incremental cash flow to the company that will be generated from the sales.

Assume the population in the United States in 2010 is 300,000,000 people. It is expected that the population will grow at an annual rate of 0.8% (8/10 of 1%) for the next 30 years.

Assignment 2

Expand your model to take account of the uncertainty of the R&D and marketing processes. Evaluate the attractiveness of the investment in a typical pharmaceutical product in terms of *NPV* and *IRR*. If you have difficulty with *IRR*, you may eliminate it from your random model.



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Debt Financing

THIS CHAPTER EXPLAINS HOW to model the details of the firm's debt financing. In the first section we expand the basic planning model to handle greater detail in the debt financing sector of the model. In the second section, we use Monte Carlo simulation to determine an optimal capital structure. In the last section, we show how to model some of the more esoteric issues of debt financing, including duration and swaps.

11.1 DEBT IN THE LONG-TERM PLANNING MODEL

In earlier chapters we distinguished between spontaneous and discretionary sources of funds, and between internal and external sources of funds. Here we are concerned mainly with the discretionary or external sources of debt financing. Borrowing is considered a discretionary as opposed to a spontaneous source of funds, because an explicit decision and action must occur to tap this source, and most borrowing does not necessarily increase spontaneously with revenues.

One of the first tasks is to determine the amount of discretionary financing that will be required. In our sales-driven model of the Odd & Rich Corporation (O&R) in [Chapter 4](#), discretionary financing was based on required external financing, as shown in lines E6–E9 in [Exhibit 4.2e](#) (repeated here as [Exhibit 11.1](#)).

Required external financing (REF) (E.6) is the amount necessary to fund the firm's planned investment in new assets and pay the minimum dividend. The task then, is to decide how to split this financing requirement into debt and equity. In our O&R model, this decision is specified by the user in the decision variable, *Debt Portion* that splits the REF into debt and equity as shown in Equations 11.7 and 11.8.

In the model in [Chapter 4](#), we did not delve very deeply into the question of how we decide on the split between debt and equity financing. The purpose of this chapter is to understand how to model the firm's borrowing decisions in a way that is consistent with the theory of capital structure, so our models can help us to make the best financing decisions.

Naturally, there are many different approaches to specifying the discretionary debt financing in the simulation model. The approach to be discussed first is aggregative and

E.1	Required Financing _t	=	Total Investment _t
E.2	Earnings Cash Flow _t	=	Net Income _t + Depreciation Expense _t
E.3	Minimum Dividend _t	=	Net Income _t (Payout Ratio)
E.4	Internal Financing _t	=	Earnings Cash Flow _t – Minimum Dividend _t
E.5	Spontaneous Financing _t	=	(Current Liabilities/Sales) Sales _t – Current Liabilities _{t-1}
E.6	Required External Financing _t	=	Required Financing _t – Internal Financing _t – Spontaneous Financing _t
E.7	Debt Issued _t	=	Required External Financing _t (Debt Portion)
E.8	Stock Issued _t	=	Required External Financing _t – Debt Issued _t
E.9	Total Financing _t	=	Internal Financing _t + Spontaneous Financing _t + Debt Issued _t + Stock Issued _t

EXHIBIT 11.1 Financing module equations.

would be appropriate for long-term planning. The O&R model had only one kind of debt. We may want to provide for the issue of different varieties of debt. The most basic division would be short-term and long-term debt. Exhibit 11.2 extends the financing module of Exhibit 11.1 by splitting the debt issue into short term and long term, and then tracking the debt service and balances from period to period. Short-term (ST) Portion is the decision variable input by the user to specify the proportion of the total *Debt Issued* that will be short-term debt, with the remainder assumed to be long-term (LT) debt.

This model assumes that the short-term debt matures in one period, as reflected in Equation 11.4. Long-term debt is repaid according to a schedule specified by the user with the parameter *Long-term Repayment Rate*, which is the proportion of the outstanding long-term debt that is repaid each period. The interest expense would be calculated using *Short-Term Rate* and *Long-Term Rate* parameters that are the rates on *short-term* and *long-term debts*, respectively.

Debt Transactions

Debt Issued _t	=	Required External Financing _t (Debt Portion)	(11.1)
Short Term Debt Issued _t	=	Debt Issued _t (Short Term Portion _t)	(11.2)
Long Term Debt Issued _t	=	Debt Issued _t – Short Term Debt Issued _t	(11.3)
Short Term Debt Repaid _t	=	Short Term Debt _{t-1}	(11.4)
Long Term Debt Repaid _t	=	Long Term Debt _{t-1} (Long Term Repayment Rate _t)	(11.5)

Interest and Debt Service

Short Term Interest _t	=	Short Term Debt _{t-1} (Short Term Rate _t)	(11.6)
Long Term Interest _t	=	Long Term Debt _{t-1} (Long Term Rate)	(11.7)
Interest Expense _t	=	Short Term Interest _t + Long Term Interest _t	(11.8)
Debt Principal Payment _t	=	Short Term Debt Repaid _t + Long Term Debt Repaid _t	(11.9)
Total Debt Service _t	=	Interest Expense _t + Debt Principal Payment _t	(11.10)

Debt Balance

Short Term Debt _t	=	Short Term Debt _{t-1} – Short Term Debt Repaid _t + Short Term Debt Issued _t	(11.11)
Long Term Debt _t	=	Long Term Debt _{t-1} – Long Term Debt Repaid _t + Long Term Debt Issued _t	(11.12)

EXHIBIT 11.2 Debt sector equations.

11.1.1 Constraining Debt Ratios

The financing model in [Exhibit 11.2](#) determines the borrowing based on the *Debt Portion* parameter specified by the user, and then splits the debt into short term and long term based on the user input. Instead of having the user specify these parameters, we may want to allow the model to determine the financing mix based on targets or limits on the debt ratios. For example, suppose our policy is to maintain our ratios within the following limits:

$$\text{Current Ratio} \geq 2.0$$

$$\text{Total Debt-to-Total Assets} \leq 45\%$$

$$\text{Short Term Issued}_t = 30\% \text{ of Debt Issued}_t$$

$$\text{Long-Term Borrowing}_t = 70\% \text{ of Debt Issued}_t$$

An example of modifications to the Equations of [Exhibit 11.2](#) to handle these constraints is shown in [Exhibit 11.3](#), where we assume that the only current liabilities are accounts payable and *short-term debt*, and *long-term debt* is the only long-term liability.

In [Exhibit 11.3](#), Equation 11.13 uses the total debt-to-total asset ratio to set the limit on total debt, consisting of all liabilities. We will subtract the noninterest bearing debt from this to find our capacity for short-and long-term interest bearing debt. The first ST debt limit (Equations 11.14 and 11.15) is set by the requirement that the current ratio must be at least 2.0. Note that when we have a subtraction such as Equation 11.15, we usually put it inside a MAX function to prevent the difference from being negative. A second ST debt limit is set by a constraint on total debt, so Equation 11.16 says that *short-term debt* cannot

Input: Policy Limits

Debt/Asset Limit	=	45%
Current Ratio Limit	=	2.0
Short Term Portion	=	30%

Debt Financing

$$\text{Maximum Total Debt}_t = (\text{Debt/Asset Limit}) \text{ Total Assets}_t \quad (11.13)$$

$$\text{Maximum Current Liabilities}_t = \text{Current Assets}_t / (\text{Current Ratio Limit}) \quad (11.14)$$

$$\text{ST Debt Capacity}_t = \text{MAX} [\text{Maximum Current Liabilities}_t - \text{Accounts Payable}_t, 0] \quad (11.15)$$

$$\text{Maximum ST Debt}_t = \text{MIN} [\text{ST Debt Capacity}_t, \text{Maximum Total Debt}_t - (\text{LT Debt}_{t-1} - \text{LT Debt Repaid}_t) - \text{Accounts Payable}_t] \quad (11.16)$$

$$\text{ST Financing Capacity}_t = \text{MAX} [\text{Maximum ST Debt}_t - (\text{ST Debt}_{t-1} - \text{ST Debt Repaid}_t), 0] \quad (11.17)$$

$$\text{LT Debt Capacity}_t = \text{MAX} [\text{Maximum Total Debt}_t - \text{Maximum ST Debt}_t - \text{Accounts Payable}_t, 0] \quad (11.18)$$

$$\text{LT Financing Capacity}_t = \text{MAX} [\text{LT Debt Capacity}_t - (\text{LT Debt}_{t-1} - \text{LT Debt Repaid}_t), 0] \quad (11.19)$$

$$\text{Short Term Debt Issued}_t = \text{MIN} [(\text{ST Portion}) \text{ Required External Financing}_t, \text{ST Financing Capacity}_t] \quad (11.20)$$

$$\text{LT Debt Issued}_t = \text{MIN} [\text{Required External Financing}_t - \text{ST Debt Issued}_t, \text{LT Financing Capacity}_t] \quad (11.21)$$

EXHIBIT 11.3 Ratio constraints on debt financing.

exceed the smaller of (a) the limit set by the current ratio or (b) a limit set by the maximum total debt. Then, in Equation 11.17, the allowable increase in ST debt is the difference between the maximum total ST debt, and amount that is currently outstanding. The *LT debt capacity* in Equation 11.18 is set as the residual of total debt capacity less maximum *short-term debt*, and *LT financing capacity* is the difference between the capacity for LT debt and the current level of debt. Equations 11.20 and 11.21 issue ST and LT debt based on these capacity limits coupled with the policy of meeting the debt needs with 30% short term and 70% long term. So long as none of the limits are binding, the 30%–70% mix will prevail. But, if the firm has hit its debt limits, it will issue debt within these limits instead of using the 30%–70% mix. Naturally, there are other ways to model borrowing subject to constraints; in addition, other constraints such as interest coverage may apply. But, this simple model gives you a start.

The treatment of debt financing can be expanded or contracted according to the modeling needs of the user. For longer planning horizons, it may not be productive to distinguish between LT and ST debt, or between different issues or tranches of debt. For shorter term planning or more detailed models, this distinction may be much more important. As the need for greater detail increases, the detail can be expanded to show various types of debt financing having different features.

We have already looked at a more detailed accounting of different debt issues in the Stiliko example in [Chapter 10](#). In the Stiliko capital investment example, LT debt was issued at times 0, 1, and 2, with all issues amortized to mature at time 6. For each of the three different issues, we assumed constant annual debt service (principal and interest), and we noted that we could track the details of the debt with Excel functions PMT(.) functions:

$$\text{Debt Service Payment}_t = \text{PMT}(\text{Interest Rate}, \text{Term to Maturity}, \text{Amount of Loan})$$

$$\text{Interest Payment}_t = \text{IPMT}(\text{Interest Rate}, \text{Period in Life of Loan}, \text{Term to Maturity}, \text{Amount of Loan})$$

$$\text{Principal Payment}_t = \text{PPMT}(\text{Interest Rate}, \text{Period in Life of Loan}, \text{Term to Maturity}, \text{Amount of Loan})$$

In these functions, *Amount of Loan* is the original principal, *Term to Maturity* is the length of the loan at the date it is funded, *Period in Life of Loan* refers to the age of the loan, or period 1 means the first period in the life of a loan that might be a five year loan. The PMT(.) functions assume equal periodic payments including principal and interest, so we could not use these functions for loans with other payment terms.

The size and complexity of our model will increase as the number of types or issues of debt increase. As [Exhibit 11.2](#) showed, we need to calculate the amount issued, principal repaid, amount outstanding, and interest payment for each different type of debt and each period. Each issue requires at least 4 lines in the model. Adding to the modeling task, we usually need to keep track of which period it is in the life of the loan. If we track specific issues of debt it may be difficult to maintain a generalized model. In other words, as we add different types of debt, we need to make the model specific to the types of debt. This means that the model

is less flexible and applicable to other modeling problems and tasks. This is one reason it is helpful to build separate modules so that you can increase or decrease the complexity of one module or sector without having to change all the other sectors of the model. The following example shows how we can develop a more detailed sector of our model to handle various types of debt.

11.1.2 Example: A Borrowing Sector for Odd and Rich

O&R needs to finance the new production facility. O&R will need to borrow \$4,400 immediately (all amounts are expressed in thousands), \$2,400 in one year, and \$1,200 at the end of two years. There are several different types of debt that O&R can issue, and O&R is considering combinations of a fixed rate mortgage loan, a floating rate loan, and a fixed coupon bond. The basic terms for the different loans are as follows.

The mortgage loan would be for five years at an interest rate of 8%. It will not be fully amortizing, so it will have a balloon payment equal to 20% of the face amount of the loan at the end of the five years. The periodic payments will amortize the other 80% of the loan, with equal annual payments (principal + interest). The fees and closing costs will be 0.5% of the face amount of the loan plus \$10 based on the assumption that the amount of the loan will be between \$2,000 and \$5,000.

The floating rate loan must be repaid in 3 equal annual principal installments, and the interest rate on the loan will be equal to the 90-day LIBOR rate plus 3.5%, adjusted annually. There will be a 1% loan fee paid up front.

O&R can also issue fixed coupon bonds that will be repaid in four years when the bonds mature. Bonds can be issued at 8.5%, and will involve flotation and transaction costs of 2% plus a fixed cost of \$50.

O&R would like to analyze different combinations of loans to meet its needs. As a first pass, O&R is considering funding the first \$4,400 with a mortgage loan, then using the floating rate loan for the \$2,400 needed at time 1, and the bond for the funds needed at time 2. The debt sector model to help analyze these borrowing alternatives is shown as [Exhibit 11.4](#), with formulas shown for selected cells shown in [Exhibit 11.4b](#). The model also allows for a short term loan, but for our present discussion, we will focus on the longer term loans. This module is intended to link with a broader capital budgeting model by way of the aggregated flows.

The loan sector model shown as [Exhibit 11.4a](#) has a section for each of the 3 types of long-term loans plus a short term loan in lines 36–50 that is not shown. For each loan it calculates the interest and principal payments, the balance, and the net interest cost of the loan taking account of the loan fees and transaction costs. Then, the payments and balances are aggregated to feed back to the broader firm planning model. This way, the loan model can be expanded or contracted as necessary without having to change the main planning model which only needs to know the total payments, but not the details for each loan. Of course, the number and types of loans could be expanded without changing the broader model.

This 3-loan model is structured to give the user flexibility to explore different loan features, amounts, and even borrowing dates. The loan details are entered by the user in lines

	A	B	C	D	E	F	G	H	I	J	K
1											
2						Odd & Rich Corporation					
3						Loan Sector					
4						Loan Tracking for Different Loan Types					
5											
6											
7	Loan Details										
8	Enter the Loan Features in this Section:										
9											
10	Loan	Amount	Interest	Borrowing	Years to	Repayment	Balloon	Fees %	Fees \$		
11	Type	Borrowed	Rate	Date	Maturity	Rate	Payment	%			
12						% per year	% of loan				
13											
14	Short Term	0	6.0%	0	1	100.0%	0.0%	0.0%	0		
15											
16	Mortgage	4,400	8.0%	0	5.0	Annuity	20%	0.5%	10		
17											
18	Floating	2,400	Libor +	1	3.0	33%		1.0%			
19	Rate		3.5%								
20											
21	Bond	1,200	8.5%	2	4.0	0%	100%	2.0%	50		
22											
23											
24	Economic Data										
25	Enter Interest Rate forecasts in this Section:										
26											
27											
28											
29	90 Day T-Bill Rate				3.00%						
30	90 Day Libor Rate				3.50%						
31	Libor Change from Initial Value (basis Points)			0		20	25	-15	20	30	30
32	Short Term Loan Rate				6.00%						
33	10 Year T-Note Rate				6.50%						
34	Corp Bond, AA, 10 yr				7.00%						
35											
52	Loan Tracking										
53											
54	Mortgage										
55											
56											
57	Amount Borrowed				4,432						
58	Loan Fees				32						
59	Funds Available				4,400						
60	Total Debt Service Payment					952	952	952	952	952	
61	Interest Payment					352	304	252	196	136	
62	Principal Payment					600	648	700	756	816	
63	Balloon Payment									880	
64	Ending Balance				4,432	3,832	3,184	2,484	1,728	32	32
65											
66	Period in Life of Loan				0	1	2	3	4	5	
67	Remaining Term to Maturity				5	4	3	2	1	0	
68											
69	Net Loan Cash Flow				4,400	-952	-952	-952	-952	-1,832	0
70	Net Cost of Debt				8.00%						
71											

(a)

	A	D	G	H
52	Loan Tracking			
53				
54	Mortgage			
55	Time		=G27	=H27
56	Amount Borrowed			
57	Loan Fees			
58	Funds Available			
59	Total Debt Service Payment			
60	Interest Payment		=IF(AND(G\$66>0,G\$66<=\$E\$16),-PMT(\$C\$16,\$E\$16,\$B\$16,\$G\$16,\$B\$16,-\$G\$16*\$B\$16),	=IF(AND(H\$66>0,H\$66<=\$E\$16),-PMT(\$C\$16,\$E\$16,\$B\$16,\$B\$16,-\$G\$16*\$B\$16),)
61	Principal Payment		=IF(AND(G\$66>0,G\$66<=\$E\$16),-PPMT(\$C\$16,\$G\$66,\$E\$16,\$B\$16,-\$G\$16*\$B\$16),	=IF(AND(H\$66>0,H\$66<=\$E\$16),-PPMT(\$C\$16,\$B\$16,\$B\$16,-\$G\$16*\$B\$16),)
62	Balloon Payment			
63	Ending Balance		=IF(G57>0,G57,MAX(F64-G62-G63,0))	=IF(H57>0,H57,MAX(G64-H62-H63,0))
64	Period in Life of Loan		=IF(AND(G55>=\$D\$16,G55<=\$D\$16+\$E\$16),MAX(G55-\$D\$16,0),"")	=IF(AND(H55>=\$D\$16,H55<=\$D\$16+\$E\$16),MAX(H55-\$D\$16,0),"")
65	Remaining Term to Maturity		=IF(ISNUMBER(G66),MAX(\$E\$16-G66,0),"")	=IF(ISNUMBER(H66),MAX(\$E\$16-H66,0),"")
66	Net Loan Cash Flow		=G59-G60-G63	=H59-H60-H63
67	Net Cost of Debt			
71				
72	Floating Rate Loan			
73	Time		=G27	=H27
74				
75	Libor		=\$E\$76+(G77/10000)	=\$E\$76+(H77/10000)
76	Rate Change (basis points)		=G31	=H31
77	Loan Rate		=G76+\$C\$19	=H76+\$C\$19
78				
79	Amount Borrowed			
80	Loan Fees			
81	Funds Available			
82	Total Debt Service Payment		=G84+G85	=H84+H85
83	Interest Payment		=IF(AND(G\$88>0,G\$88<=\$E\$18),F78*F86,)	=IF(AND(H\$88>0,H\$88<=\$E\$18),G78*G86,)
84	Principal Payment		=IF(AND(G\$88>0,G\$88<=\$E\$18),MIN(\$F\$18*\$B\$18,F86),)	=IF(AND(H\$88>0,H\$88<=\$E\$18),MIN(\$F\$18*\$B\$18,G86),)
85	Principal Payment		=IF(G80>0,G80,F86-G85)	=IF(H80>0,H80,G86-H85)
86	Ending Balance			
87				
88	Period in Life of Loan		=IF(AND(G74>=\$D\$18,G74<=\$D\$18+\$E\$18),MAX(G74-\$D\$18,0),"")	=IF(AND(H74>=\$D\$18,H74<=\$D\$18+\$E\$18),MAX(H74-\$D\$18,0),"")
89	Remaining Term to Maturity		=IF(ISNUMBER(G88),MAX(\$E\$18-G88,0),"")	=IF(ISNUMBER(H88),MAX(\$E\$18-H88,0),"")
90	Net Loan Cash Flow		=G82-G83	=H82-H83
91	Net Cost of Debt			
92				

(b)

EXHIBIT 11.4 (Continued) (b) Formulas for loan sector module.

(Continued)

	A	D	G	H
94	Bond			
95	Coupon Rate	Time	=G27	=H27
96	Amount Borrowed		=G98+G98	
97	Loan Fees		=IF(G97>0,\$H\$21*G97+\$I\$21)	
98	Funds Available		=B21	
100	Total Debt Service Payment			=H101+H102+H103
101	Interest Payment			=IF(AND(H\$106>0,H\$106<=\$E\$21),\$C\$21*G104)
102	Interim Principal Payment			
103	Balloon Payment			
104	Ending Balance		=F104+G97-G102-G103	=G104+H97-H102-H103
105	Period in Life of Loan		=IF(AND(G95>=\$D\$21,G95<=\$D\$21),MAX(G95-\$D\$21,0),"")	=IF(AND(H95>=\$D\$21,H95<=\$D\$21),MAX(H95-\$D\$21,0),"")
106	Remaining Term to Maturity		=IF((NUMBER(G106),MAX(\$E\$21-G106,0),"")	=IF((NUMBER(H106),MAX(\$E\$21-H106,0),"")
108	Net Loan Cash Flow		=G99-G100	=H99-H100
110	Net Cost of Debt			
111				
112	Summary of Loans Flows			
113	Time:		=G27	=H27
114				
115	Amount Borrowed		=G57+G80+G97	
116	Loan Fees		=G58+G81+G98	
117	Funds Available		=G59+G82+G99	
118	Total Debt Service Payment		=G60+G83+G100	=H60+H63+H100
119	Interest Payment		=G61+G84+G101	=H61+H64+H101
120	Principal Payment		=G62+G63+G85+G102+G103	=H62+H63+H85+H102+H103
121	Ending Balance		=G64+G86+G104	=H64+H86+H104
122				
123	Net Loan Cash Flow		=G68+G91+G109	=H69+H91+H109
124	Net Cost of Debt			
125				

(b)

EXHIBIT 11.4 (Continued) (b) Formulas for loan sector module.

14–21, and general interest rate data is entered in lines 29–34. For each loan, the user enters the details in the loan detail section as follows:

<i>Amount Borrowed</i>	Column B
<i>Interest Rate</i>	Column C
<i>Borrowing Date</i>	Column D (time 0, 1, 2, etc.)
<i>Years to Maturity</i>	Column E
<i>Repayment Rate per Year</i>	Column F (% of loan repaid)
<i>Balloon Payment</i>	Column G (% of loan repaid in addition to the normal repayment schedule)
Fees (%)	Column H (% fees based on size of loan)
Fees (\$)	Column I (fixed \$ fees independent of size)

Now, we look at the mortgage loan section to understand the model structure. For any loan, and particularly for a mortgage amortized loan, it is important to track the age of the loan and the remaining time to maturity because this information is used to calculate the interest and principal payments. Lines 66 and 67 of the mortgage section account for the age of the loan using the borrowing date and the original years to maturity as input. Ignoring for the moment the flexibility that we will discuss shortly, the period in the life of the loan would be

$$\text{Period in Life of Loan} = \text{Current Period} - \text{Borrowing Date} \quad (11.1)$$

For example, if the *current period* is $t = 3$ and the *borrowing date* was $t = 1$, then time $t = 3$ is the 2nd period in the loan life. However, we need to make sure that the period is not negative, so we do that with the function

$$= \text{MAX}(\text{Current Period} - \text{Borrowing Date}, 0) \quad (11.2)$$

Our model allows the user to specify different possible borrowing dates for each type of loan (for the mortgage loan, cell D16 of the loan detail input section). But this flexibility complicates the calculation of the period in the life of the loan. We handle this in cell G66 (time $t = 2$) with the formula,

$$\begin{aligned} \text{Period in Life of Loan} = & \text{IF}(\text{AND}(\text{G55} \geq \text{D16}, \text{G55} \leq \text{D16} + \text{E16}), \\ & \text{MAX}(\text{G55} - \text{D16}, 0), "") \end{aligned} \quad (11.3)$$

where:

G55 is the number of the current period

D16 is borrowing period

E16 is the years to maturity

This formula says that if the current date is between the borrowing date and the maturity date, the period in the life of the loan is *Current Period*—*Borrowing Date*. Otherwise, a blank is entered in the cell.

Cell G67 calculates the time remaining in the life of the loan in period 2 as

$$\text{Remaining Time to Maturity} = \text{IF}(\text{ISNUMBER}(\text{G66}), \text{MAX}(\text{E16} - \text{G66}, 0), "") \quad (11.4)$$

where:

E16 is the original years to maturity

G66 is the period in the life of the loan from the previous calculation

The `=ISNUMBER(G66)` function returns a *True* value if the entry in the cell G66 is a number as opposed to a non-number, so this formula calculates time to maturity as (*Original Maturity—Period in Life of Loan*) if there is a number in cell G66. There will be a number in G66 only if the current period is between the borrowing date and the maturity date of the loan.

The effect of these loan life formulas is to enable the user to specify any starting date for the loan and the formulas will track the loan life. For example, the user could input the borrowing date as any time within the model planning horizon, and the model will track the life of the loan until maturity. The loan life figures are particularly important for tracking the mortgage amortized loans when we use the Excel PMT functions. Turning now to those functions, the periodic loan payment for the mortgage amortized loan is calculated in line 60 with the payment function of the form:

$$= \text{PMT}(\text{Interest Rate}, \text{Years to Maturity}, \text{Amount Borrowed}, \text{Balloon Payment}) \quad (11.5)$$

However, we want the payment to be calculated only for periods between the borrowing date and the maturity date, both of which we want to be flexible within the model. This is done with the formula (for example for period 2, column G)

$$\begin{aligned} \text{Total Debt Service Payment} = & \text{IF}(\text{AND}(\text{G66} > 0, \text{G66} \leq \text{E16}), \\ & - \text{PMT}(\text{C16}, \text{E16}, \text{B16}, -\text{G16} * \text{B16}),) \end{aligned} \quad (11.6)$$

which calculates the debt service if the period in the life of the loan (G66) is between 0 and the time to maturity (E16), and where C16 is the interest rate, E16 is the time to maturity, B16 is the amount of the loan, and G16*B16 is the balloon payment at the end, with G16 being the balloon payment as a % of the amount of the loan (B16). The effect of the blank space at the end of the IF function is to return zero if the AND function is not TRUE. In other words, if the current period is not between the borrowing date and the maturity date, the cell content will be zero.

The total debt service payment of the mortgage amortized loan is split into interest (line 61) and principal (line 62) with the IPMT and PPMT functions. Both of functions follow the same format for the parameters to be entered in the function:

= *IPMT*(Interest Rate, Period in Life of Loan, Years to Maturity,
Amount Borrowed, Balloon Payment),

and

= *PPMT*(Interest Rate, Period in Life of Loan, Years to Maturity,
Amount Borrowed, Balloon Payment).

These functions differ from the total payment function because splitting the payment into interest and principal depends on the period in the life of the loan, which is the second parameter, G66, in each of the following statements where the loan payment functions are included in an IF statement of the same form as the earlier total payment calculation:

$$\begin{aligned} \text{Interest Payment} &= \text{IF}(\text{AND}(G66 > 0, G66 \leq E16, \\ &\quad - \text{IPMT}(C16, G66, E16, -G16 * B16),) \\ \text{Principal Payment} &= \text{IF}(\text{AND}(G66 > 0, G66 \leq E16, \\ &\quad - \text{PPMT}(C16, G66, E16, -G16 * B16),) \end{aligned} \quad (11.7)$$

The current O&R example calls for a balloon payment at the end of the five-year loan life, amounting to 20% of the amount of the loan. Line 63 of the mortgage loan section accounts for this balloon payment. The payment formula for period 1, for example, is

$$\text{Balloon Payment} = \text{IF}(G67 = 0, G16 * B16, 0) \quad (11.8)$$

where:

G67 is the remaining term to maturity

G16 is the balloon payment as a % of the face amount of the loan

B16 is the face amount of the loan

Line 64 is the balance on the loan at each date, which is calculated as

$$\text{Ending Balance} = \text{IF}(G57 > 0, E57, \text{MAX}(G64 - G62 - G63, 0)) \quad (11.9)$$

which says the ending balance will be equal to the amount borrowed (G57), if the funds were borrowed that period, and otherwise the balance will be:

Previous Balance	(G64)
–Principal Payment	(H62)
–Balloon Payment	(H63)
<u>= Ending Balance</u>	(H64)

However, the MAX function is used to assure that the balance is not negative.

The last part of the mortgage section is a calculation of the net interest cost of the loan using an internal rate of return function (*IRR*). The cash flows for the loan are calculated in line 69 for each period as

Amount Borrowed	(G57)
– Loan Fees	(G58)
<u>= Funds Available</u>	(G59)
– Total Debt Service Payment	(G60)
– Balloon Payment	(G63)
= Net Loan Cash Flow	(G69)

In the period the funds are borrowed, there will be no payments, so the cash flow is positive for the borrower. In subsequent periods, payments will be made, but the amount borrowed will be zero that period, and the cash flow will be negative.

The interest cost in cell E70 is

$$\text{Net Cost of Debt} = \text{IRR}(\text{E69:O69}) \quad (11.10)$$

so, the interest cost is net of loan fees and takes account of all payments made by the borrower. In this example, the stated rate on the five-year mortgage loan is 8%, and the loan fees are sufficiently small that the effective rate is still at 8.00%.

Each of the other types of loans has a section that follows the same general format. However, they are not mortgage amortized loans, so they do not use the PMT functions. In the case of the floating rate loan, the interest rate is calculated at each period based on the LIBOR + Spread for that period, and the principal payment is simply a percent of the face amount of the loan. In the case of the bond, the interest is based on the coupon rate and all the principal is paid at the end of the loan. In each case, we track the life of the loan to show when the loan starts and ends just as with the mortgage loan. Of course, the net cost of each loan is calculated with the *IRR* function.

The loan flows and costs are combined and summarized in the summary section (lines 115–128). Each of the lines in the summary section corresponds to the individual loan sections, with each summing the amounts borrowed, payments, and balances for the individual loans. The net cost in the summed section combines all the flows, so the cost is the consolidated cost for the package of loan. For the O&R example, with 3 loans of differing borrowing dates, amounts, interest rates, and repayment rates, the net cost is about 8.27% as shown in cell E128.

In the context of a larger firm planning model, these combined figures are the only ones that need to be passed to the broader model. In other words, the broader corporate planning model only needs to know the total debt issued, total debt payments, total interest expenses, and so on. It does not need to know the specific details for each of the firm's loans. This way, the loan section can be expanded to include any number of different loans with different features without having to change the firm-level planning model.

The original O&R model was set up to determine the firm's required financing based on a sales forecast. If we have a separate loan sector as we have been discussing, we are inserting our own specific financing decisions in the model, and we need to link these user specified decisions with the basic model. This is done in the financing module of the basic model. In the financing module of the O&R model, the debt financing was originally calculated as

$$\text{Debt Financing}_t = (\text{Debt Portion}) \text{ Required External Financing}_t \quad (11.11)$$

If part of the firm's financing is set by the user in a loan sector, we can feed this specific financing into the model and still allow the model to solve for any additional financing with the following modification:

$$\begin{aligned} \text{Debt Financing}_t = \text{MAX} \left[\begin{array}{l} (\text{Debt Portion}) \text{ Required External Financing}_t \\ - \text{Total Borrowing from Loan Sector}_t, 0 \end{array} \right] \\ + \text{Total Borrowing from Loan Sector}_t \end{aligned} \quad (11.12)$$

This allows the model to determine the financing necessary for the firm's growth, and also include the loan sector borrowing in the total.

11.1.3 Analyzing the Financing Decision

Our discussion has focused on ways to include debt in our planning model. The next step is to use the results of the model to evaluate the financing decision. Questions we might want our model to answer include the following:

1. Will the firm's ratios be within the bounds acceptable to management, lenders, and stockholders?
2. Will the company be able to make the required debt payments?
3. What happens if sales decline?
4. How will the financial structure affect the value of the stock?
5. Is there a better or best financial structure?

A planning model similar to our O&R is well suited to help answer most of these questions. The financial ratios are part of the output so we can look at the liquidity, leverage, and coverage ratios that are part of the usual analysis of debt financing. Sensitivity analysis that explores different scenarios will easily help us to see whether we will be able to service the debt under a range of likely scenarios. In addition, with cost of capital and valuation included as part of the model we can explore the interaction between financing, capital costs, and equity valuation. These tools give us some guidance regarding the effects of our financing plan. However, it is still difficult to find the best financial structure with this model. But, with the theory of capital structure to guide us, in the next section we will see how we can develop a model to find an optimal financial structure.

11.2 USING MONTE CARLO SIMULATION TO FIND THE OPTIMAL CAPITAL STRUCTURE

One of the most important and enduring questions in finance is what is the best financing mix for the firm? The issue has been debated for more than 60 years and is still not fully resolved. There is still wide disagreement as to whether an optimal capital structure even exists, and we will not restate the debate here.* In spite of a diversity of opinions, many experts now agree that the best capital structure is the one where the tax benefits of debt and the costs of financial distress are in balance. In this section, we show how to use Monte Carlo simulation to model this balance.

11.2.1 Default Risk, Tax Savings, and Bankruptcy Costs

Stewart Myers (1984) characterized the widely accepted view of optimal capital structure as the static trade-off hypothesis. He states, “A firm’s optimal debt ratio is usually viewed as determined by a trade off of the costs and benefits of borrowing... The firm is portrayed as balancing the value of interest tax shields against various costs of bankruptcy or financial embarrassment” (p. 577). According to this view, the firm’s optimal capital structure occurs at the mix of debt and equity where the *net benefit of debt* is maximized. The net benefit of debt can be expressed as

$$\begin{aligned} & \text{Expected Present Value of Tax Savings from Interest} \\ & - \text{Expected Present Value of Bankruptcy Costs} \\ & \hline & = \text{Net Benefit of Debt} \end{aligned}$$

or, abbreviated as

$$NB(D) = E[PV(TS(D))] - E[PV(BC(D))] \quad (11.13)$$

where:

$NB(D)$ denotes the net benefits from debt level D

$E[PV(TS(D))]$ and $E[PV(BC(D))]$ are the expectation of the present values of tax savings and bankruptcy costs, respectively.

The optimal capital structure would be the amount of debt where $NB(D)$ is maximized. We need to be more specific to be able to calculate $E[PV(TS(D))]$ and $E[PV(BC(D))]$.

11.2.1.1 Default Risk

In order to introduce the notation, we will not be specific at this stage about the events that bring about default, but we will say simply that if the firm cannot meet the terms required by its debt covenants, it will default, and we will assume that it enters the bankruptcy

* The capital structure literature is so extensive that we cannot begin to list all the relevant articles here. A selection of sources is listed in References at the end of this book. For a basic summary of the debates about capital structure, see [Chapters 14 and 17](#) in Brealey et al. (2017), and [Chapters 15 and 16](#) in Ross et al. (2016).

process. The probability of defaulting in period t , having survived for $t - 1$ periods will be denoted by q_t . The probability of not defaulting in period t is $p_t = (1 - q_t)$. The probability of surviving for t periods and being able to service the debt in period t is

$$P_t = \prod_{j=1}^t p_j = \prod_{j=1}^t (1 - q_j) \quad (11.14)$$

and the probability of surviving $t - 1$ periods and then failing in period t is

$$Q_t = P_{t-1} q_t \quad (11.15)$$

11.2.1.2 Tax Savings

In period t the taxes saved from paying interest I_t is $T * I_t$, where T is the tax rate. The present value of this tax saving is $T * I_t (1 + k_d)^{-t}$, and if the tax savings were certain in each period t , the present value of tax savings over the n period life of the debt would be

$$\sum_{t=1}^n T * I_t \left(\frac{1}{(1 + k_d)^t} \right) \quad (11.16)$$

However, the tax savings would not be certain. Assume that if bankruptcy occurs, the firm will not pay interest or taxes, and no tax savings will be realized subsequent to the bankruptcy event. As P_t is the probability of surviving through t periods, it represents the probability of paying interest for t periods, and realizing tax savings in each period through time t . The expected value of tax savings over the n period life of the debt is

$$E[PV(TS)] = \sum_{t=1}^n P_t * T * I_t \left(\frac{1}{(1 + k_d)^t} \right) \quad (11.17)$$

11.2.1.3 Bankruptcy Costs

Assume that bankruptcy cost of BC_t would be paid if bankruptcy occurs in period t . The expected present value of bankruptcy cost for period t would be

$$P_{t-1} q_t BC_t \left(\frac{1}{(1 + k_d)^t} \right) \quad (11.18)$$

and the expected value of bankruptcy costs over the n period life of the debt would be

$$E[PV(BC)] = \sum_{t=1}^n P_{t-1} q_t BC_t \left(\frac{1}{1 + k_d} \right)^t \quad (11.19)$$

11.2.1.4 Net Benefits from Debt

Combining these terms, the net benefit of using amount of debt D would be

$$NB(D) = \sum_{t=1}^n P_t * T * I_t \left(\frac{1}{(1+k_d)} \right)^t - \sum_{t=1}^n P_{t-1} q_t BC_t \left(\frac{1}{(1+k_d)} \right)^t \quad (11.20)$$

The Net Benefit would be maximized at the point where the 1st derivative of NB with respect to the amount of debt, D , is zero, or where the marginal $E[PV(TS)]$ is equal to the marginal $E[PV(BC)]$. Conceptually, this optimization is a simple calculus problem.* However, to take derivatives and solve is difficult in a realistic situation because of the complexity of the $E[PV(TS)]$ and $E[PV(BC)]$ terms. Even though the problem is difficult to solve analytically, it is relatively easy to solve with simulation. We will show how the problem can be solved with Monte Carlo simulation.

11.2.2 The Monte Carlo Simulation Model

With Monte Carlo simulation, for a given level of debt, D , we can simulate the frequency distribution of the present value of tax savings. The simulated mean of that frequency distribution is our estimate of the expected value of the present value of tax savings, $E[PV(TS(D))]$. Similarly, we can simulate the frequency distribution of the present value of bankruptcy costs. The mean of that simulated frequency distribution is our estimate of the expectation of the present value of bankruptcy costs. The difference of the simulated means is our estimate of the net benefit of the given level of debt. Then, by varying the amount of debt, we can find the debt level, D^* , that maximizes the net benefit of debt. This is the optimal amount of debt based on the trade-off model of capital structure.

We will explain this approach in stages. First, we will use Monte Carlo simulation to explore the risk of bankruptcy. Second, armed with our ability to determine the risk of bankruptcy, we can estimate the expected present value of tax savings and bankruptcy costs. Third, we vary the debt level until we find the D^* that maximizes the net benefit.

11.2.2.1 Simulating the Risk of Default and Bankruptcy

Default and bankruptcy are complex occurrences, and the line between corporate survival and corporate death is sometimes subtle and vague. Corporate death is usually the result of a complicated sequence of events of which default and bankruptcy are just the unfortunate denouement. Firms seldom go bankrupt suddenly from a single event. Typically, the sequence of events leading to bankruptcy is a story of decline: declining profits, declining liquidity, declining asset and equity values, and declining ability to meet debt payments and covenants. Owing to this complexity, two different firms could face substantially the same situation, and one would go bankrupt while the other would not. Consequently, it is often difficult to say when bankruptcy will occur. It is even more difficult to accurately model default and bankruptcy. Part of the difficulty is due to the fact that there are

* Morris (1982) has shown that under reasonable conditions, $NB(D)$ is concave, and reaches a maximum at less than 100% debt.

numerous terms dealing with financial failure, and some may not be very precise. For example, consider the following definitions of financial distress, failure, default, insolvency, and bankruptcy.

Financial distress is an imprecise term referring to the difficulty a firm faces when it struggles to meet its financial obligations, and when creditors and the financial markets become concerned that the firm may not be able to survive.

A firm is a failure when its costs exceed its revenue, or when it cannot earn a rate of return on its invested capital at least as high as its cost of capital. The firm has failed when it ceases to operate due to its inability to earn a profit.

Default is the failure to make payments required by creditors, and/or failing to meet conditions required by creditors.

Insolvency is generally regarded as having two definitions. A firm is insolvent in the equity sense when it is unable to meet its obligations as they fall due. A firm is insolvent in the bankrupt sense when the value of its assets is less than its debt obligations.

Bankruptcy also has two meanings: A firm is bankrupt when the value of its assets is less than its debt obligations (same as insolvency) and the second use of bankruptcy refers to the legal process. A firm is bankrupt when it applies for court protection for bankruptcy reorganization or for court supervised liquidation according to the bankruptcy laws.

In spite of the numerous concepts of financial distress, we will develop a model that will simulate financial distress, default, and bankruptcy. Our models will necessarily simplify the process, and will not reflect all of its complexity. With this caveat, we will proceed with our first simple model of financial failure.

To develop our initial model of financial failure, we will focus on default as the crucial event, and make the following assumptions. When the firm's operating earnings (*EBITDA*) are insufficient to cover the required debt service, it will default. In other words, default is assumed to occur when

$$EBITDA_t < Debt\ Service_t \quad (11.21)$$

where *Debt Service* includes required interest and principal payments. Note that this definition of default is simplified in that it does not make a distinction between principal and tax deductible interest payments in causing default, and it does not take account of other resources the firm might have to help service the debt and stave off default. On default, it immediately fails and is bankrupt. From the standpoint of current security holders, all payments cease; interest and principal payments to creditors end, and stockholders receive no further payments. With the termination of interest payments, the tax savings from interest payments end. Bankruptcy costs are incurred at the time of default and bankruptcy.

Let us apply these concepts with an example. Assume that the firm's *EBITDA* is normally distributed with mean, μ_{EBITDA} , of \$1,000; and standard deviation, σ_{EBITDA} , of \$100, with a probability density as shown in [Exhibit 11.5](#). The distribution is the same for each period *t*.

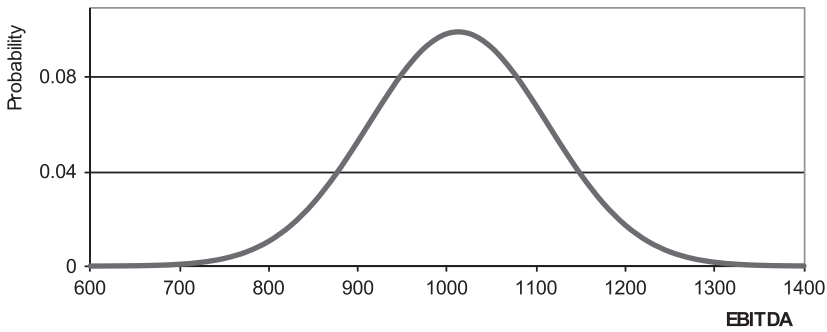


EXHIBIT 11.5 Probability distribution of EBITDA.

Assume the firm has \$3,673 of debt at an interest rate of 6% that requires equal annual payments of \$872 (principal + interest) for five years. The probability of default in period t is

$$q_t = Pr(EBITDA_t < 872) \tag{11.22}$$

The standardized value of the debt service of 872 is calculated as

$$Z = \frac{(872 - \mu_{EBITDA})}{\sigma_{EBITDA}} = \frac{(872 - 1,000)}{100} = 1.28 \tag{11.23}$$

The standardized value, Z , represents the distance, measured in standard deviation units, that 872 is from the mean, $\mu_{EBITDA} = \$1,000$. In other words, Debt Service of \$872 is \$128 less than the mean EBITDA of \$1000. With a standard deviation of \$100, this is 1.28 standard deviations units below the mean. If we look at a standard normal probability table, the probability of a standard normal variable being less than 1.28 σ below the mean is 10%, as is shown in Exhibit 11.6. In other words, the probability of default has been found to be $q_t = 10\%$.

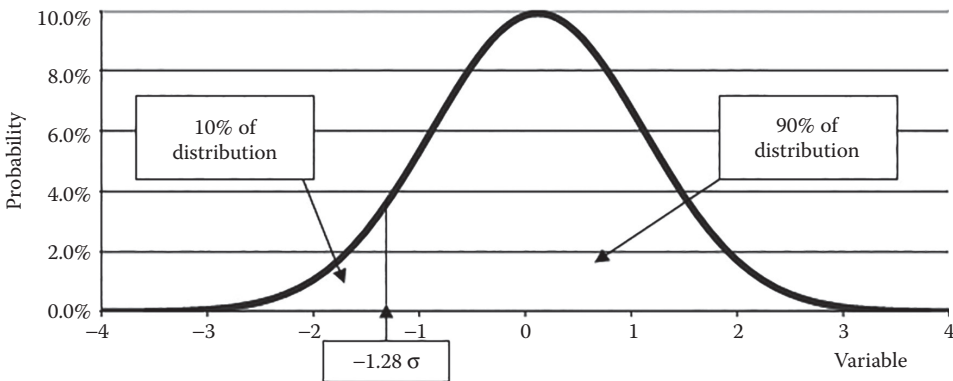


EXHIBIT 11.6 Standardized normal probability distribution.

With $q_t = 10\%$ for all periods t , we can calculate other probabilities. The probability of not failing in period t is $p_t = (1 - q_t) = (1 - 0.10) = 0.90$, and the probabilities of surviving

through t periods, P_t , and the probability surviving for $t - 1$ periods and then failing in period t , Q_t , are calculated as shown below.

$P_1 = (1 - q_1) = 0.90$	$Q_1 = q_1 = 0.10$
$P_2 = P_1(1 - q_2) = 0.90 (0.90) = 0.81$	$Q_2 = P_1q_2 = 0.90 (0.10) = 0.09$
$P_3 = P_2(1 - q_3) = 0.81 (0.90) = 0.729$	$Q_3 = P_2q_3 = 0.81 (0.10) = 0.081$
$P_4 = P_3(1 - q_4) = 0.729 (0.90) = 0.656$	$Q_4 = P_3q_4 = 0.729 (0.10) = 0.073$
$P_5 = P_4(1 - q_5) = 0.656 (0.90) = 0.59$	$Q_5 = P_4q_5 = 0.656 (0.10) = 0.066$

For example, the probability of surviving for three periods and then failing in period 4 is $P_3q_4 = 7.3\%$. The probability of surviving through the full five-period life of the debt is $P_5 = 59\%$, and the probability of failing sometime during the five periods is

$$\sum_{t=1}^5 Q_t = 0.10 + 0.09 + 0.081 + 0.073 + 0.066 = 41\% \tag{11.24}$$

$$= (1 - P_5) = 1 - 0.59 = 0.41$$

With the probability distribution for *EBITDA* given, what are the probabilities of failure for different amounts of debt? Using the same five year, 6% mortgage amortized loan, and varying the amount of debt, we get the probabilities as shown in [Exhibit 11.7](#) and are graphed in [Exhibit 11.8](#).

Calculation of the Probability of Default as a Function of the Amount of Debt			
Loan Terms			
Rate		6%	
Years to Maturity		5	
EBITDA Parameters			
Mean		\$1,000	
Std Dev		\$100	
Amount of Debt	Debt Service	Z	Probability q
\$1,000	\$237	-7.63	0.00%
2,000	475	-5.25	0.00%
3,000	712	-2.88	0.20%
3,200	760	-2.40	0.81%
3,400	807	-1.93	2.69%
3,600	855	-1.45	7.30%
3,673	872	-1.28	10.02%
3,800	902	-0.98	16.38%
4,000	950	-0.50	30.71%
4,200	997	-0.03	48.83%
4,400	1,045	0.45	67.20%
4,600	1,092	0.92	82.13%
4,800	1,140	1.40	91.85%
5,000	1,187	1.87	96.92%
6,000	1,424	4.24	100.00%

EXHIBIT 11.7 Calculating probability of default as a function of debt.

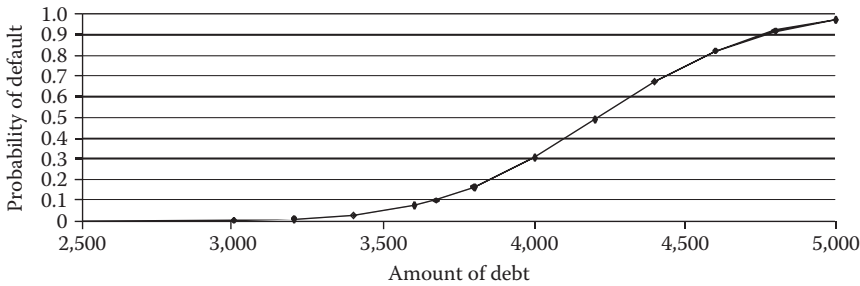


EXHIBIT 11.8 Probability of default as a function of the amount of debt.

The analysis thus far provides us with sufficient information to apply the concept of debt capacity popularized by Donaldson (1961, 1962). Donaldson's chance-constrained approach to debt capacity is the idea that the firm's management will determine a level of default risk that it is willing to tolerate, and issue debt up to the point where it reaches that risk level, but will not exceed it. In the above example, if the firm did not want to exceed a 10% probability of risk, with five-year, 6% debt, it could issue up to \$3,673 of debt. If they could tolerate a 30% chance of bankruptcy, they could issue more, and push the debt to about \$4,000.

This chance-constrained approach to debt capacity can also help us to compare different maturities in terms of default Risk. In our example, if we consider a 10-year, mortgage amortized loan at 7% (a higher interest rate because of the longer maturity), we can borrow \$6,125 (instead of \$3,673 for five years at 6%) and still be within our 10% Risk limit.

11.2.2.2 Simulating Default Risk

The equations and notation introduced above aid our discussion of default risk but they may look intimidating for the less mathematically inclined. The good news is that with the aid of Monte Carlo simulation we can simplify these calculations. This section will explain how to analyze default risk and debt capacity with simulation run with @Risk.

Exhibit 11.9a shows the output of an Excel model with @Risk to determine the probability of default. Line 9 is *EBITDA*, representing the amount available to service the debt. It is a normally distributed random variable with mean of \$1,000 and standard deviation of \$100.

The Excel model equations for the first few columns are shown in Exhibit 11.9b, which we will now explain. *EBITDA* is entered in line 9 as the @Risk probability function =*RiskNormal(mean, standard deviation)*, so that *EBITDA* is generated randomly from this normal distribution. Debt service (line 10) is from the *PMT* function in G23. Line 11 is

$$\text{Funds In Excess of Debt Service} = \text{EBITDA} - \text{Debt Service} \quad (11.25)$$

and line 12, Funds Surplus shows a positive amount when the Funds In Excess is positive and zero otherwise.

Our model is programmed so that once the firm fails, it remains failed, and it will receive no income subsequent to failure. The way this is done is to use [0, 1] variables to denote failure and success. The formula in line 13,

	A	C	D	E	F	G	H	I	J
1									
2									
3									
4									
5	Debtor Corporation								
6					Period				
7					1	2	3	4	5
8									
9	Funds Available to Service Debt:		EBITDA		\$1,006	\$1,059	\$843	\$0	\$0
10	Debt Service				\$872	\$872	\$872	\$872	\$872
11	Funds In Excess of Debt Service				\$134	\$187	(\$29)	(\$872)	(\$872)
12	Funds Surplus				134	187	0	0	0
13	Funds Deficit				0	0	29	872	872
14									
15	Failure Event				1	1	0	0	0
16	Failure Product				1	1	0	0	0
17									
18									
19	EBITDA Parameters			Loan Terms					
20	Mean	\$1,000		Amount of Debt	\$3,673		Input		
21	Std Dev	100		Rate	6%				
22				Years to Maturity	5				
23				Debt Service	\$872				
24									

(a)

	A	C	D	E	F	G
1						
2						
3						
4						
5	Debtor Corporation					
6					Period	
7					1	2
8						
9	Funds Available to Service Debt:		EBITDA		=RiskNormal(\$C\$20,\$C\$21)	=RiskNormal(\$C\$20,\$C\$21)*F16
10	Debt Service				=\$G\$23	=\$G\$23
11	Funds In Excess of Debt Service				=F9-F10	=G9-G10
12	Funds Surplus				=IF(F9>F10,F9-F10,0)	=IF(G9>G10,G9-G10,0)
13	Funds Deficit				=IF(F10>F9,F10-F9,0)	=IF(G10>G9,G10-G9,0)
14						
15	Failure Event				=IF(F13=0,1,0)	=IF(G13=0,1,0)
16	Failure Product				=F15	=F16*G15
17						
18						
19	EBITDA Parameters			Loan Terms		
20	Mean	1000		Amount of Debt		3673
21	Std Dev	100		Rate		0.06
22				Years to Maturity		5
23				Debt Service		=PMT(G21,G22,G20,0,0)
24						

(b)

EXHIBIT 11.9 (a) Simulating default with @RISK. Default occurs when EBITDA < Debt service. (b) Formulas for simulating default with @RISK.

$$\begin{aligned}
 \text{Funds Deficit} &= IF(\text{Debt Service} > \text{Funds Available}, \text{Debt Service} \\
 &\quad - \text{Funds Available}, 0) \\
 &= IF(F10 > F9, F10 - F9, 0)
 \end{aligned}
 \tag{11.26}$$

tests whether there is a deficit. If there is a deficit, the *Funds Deficit* is F10–F9, the positive amount by which the *Debt Service* exceeds *EBITDA*. Line 15, *Failure Event*, is a [0, 1] variable that tests whether there is a funds deficit in line 13. If there is a deficit, the *Failure Event* variable is 0. Line 16 tracks success or failure through time by multiplying the

values in line 15 in subsequent columns. Thus, if failure occurs in any period (column), the values in line 16 will be zero for all subsequent periods. Note in cell G9 the formula

$$= \text{Risk Normal}(\$C\$20, \$C\$21) * F16 \quad (11.27)$$

By multiplying the *RiskNormal* function by F16, we are setting the contents of G9 to zero if failure occurred in any previous period. Thus, if failure occurs in period 1, there can be no EBITDA in period 2. To check the probability of failure in each period, independently of previous periods, we would remove the [0, 1] variable, F16 from the formula in G9, and subsequent columns.

Now return to the results of the model shown in [Exhibit 11.9a](#). *Debt Service* is \$872 (G23), and we see in line 9 that EBITDA exceeds the debt service in periods 1 and 2, but in period 3 (column H), the random draw of EBITDA is only \$843, so it is insufficient to pay the debt service of \$872. Line 12 shows a zero surplus, and line 13 shows a deficit of \$29. The firm is assumed to fail, so the failure event cell for period 3 (H15) is zero. The zero in H15 is carried forward so we show zero EBITDA in periods 4 and 5. In other words, once the company is dead, it remains dead in all subsequent periods.

If EBITDA is insufficient to service the debt, the firm is assumed to fail, and all subsequent cash flows cease. In the Monte Carlo simulation, the proportional frequency of having insufficient cash flow to service the debt is our estimate of the probability of bankruptcy. To obtain this estimate, we designate the Funds in Excess of *Debt Service* as the @Risk Output variables. To do this, select cell F11:J11 and click the Add Output icon on the @Risk ribbon. When you run the simulation, the frequency distribution of Funds in Excess of *Debt Service* is the frequency distribution of survival/bankruptcy. The estimate of the probability of default is the relative frequency of Funds in Excess of Debt Service falling below zero.

[Exhibit 11.10](#) shows the frequency distribution from a simulation run of our default model in @Risk. The assumed amount of debt was \$3,673. With that amount of debt, and 5,000 trials of simulator, our estimated Risk of default is 10%. Note the slider (delimiter) at \$0 along the top. This is placed approximately zero, and represents the cut off between failure and survival. The proportion of observations to the right of the arrow represents the probability (90% shown in the top band) that Funds in Excess of *Debt Service* will be positive—the firm will survive that period. The probability of default is the proportion of observations where Funds in Excess of *Debt Service* are negative—to the left of the arrow, or $10\% = [1 - Pr(\text{survival})]$, shown on the left in the top band. In the probability notation introduced above, our probability estimates for the first period are $p_1 = 90\%$ and $q_1 = 10\%$.

To estimate p_t and q_t for each period, we would designate Funds in Excess of Debt Service in each period t as a forecast variable and check the frequency distribution for each period. The model shown as [Exhibit 11.9](#) would be modified by removing the 0, 1 variables that multiply EBITDA in line 9 to obtain these estimates for each period.

11.2.2.3 Balancing Taxes and Bankruptcy Costs

We have built a simple model to analyze the probability of default. Now we want to extend that model to determine the expected values of tax savings and bankruptcy costs, and to determine the financing mix that maximizes the net benefit of debt.

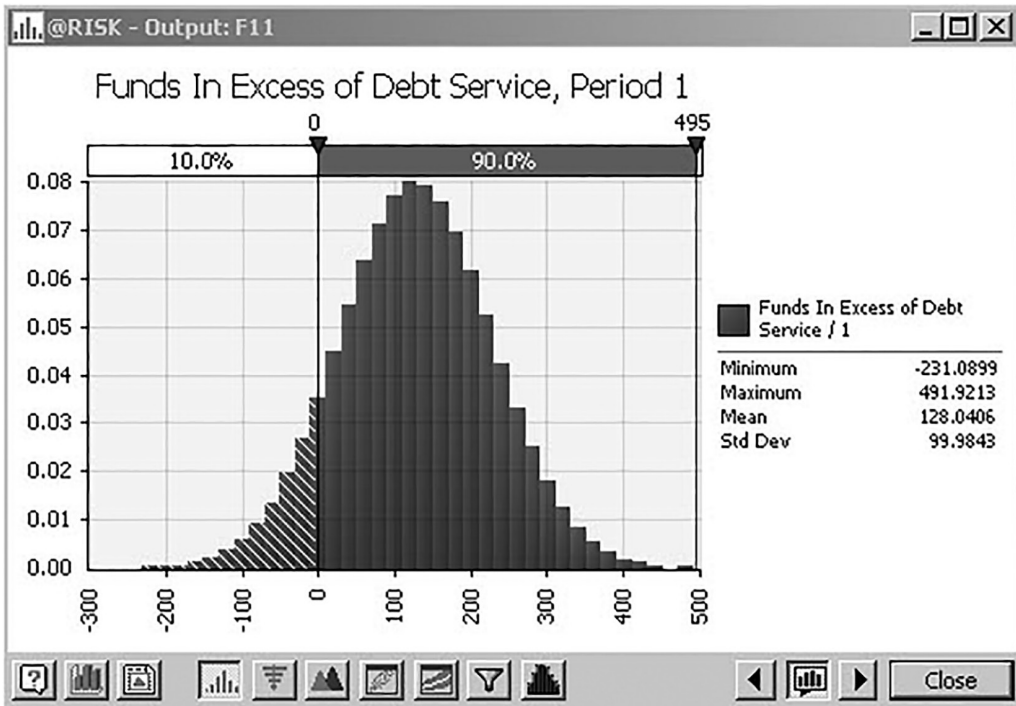


EXHIBIT 11.10 Simulated frequency distribution for probability of default.

As stated earlier, the net benefit of debt would be calculated as

$$NB(D) = \sum_{t=1}^n P_t * T * I_t \left(\frac{1}{(1+k_d)} \right)^t - \sum_{t=1}^n P_{t-1} * q_t * BC_t \left(\frac{1}{(1+k_d)} \right)^t \quad (11.28)$$

If we know the probabilities and the tax and bankruptcy parameters, we can easily apply this expression to determine the net benefit for an amount of debt, D . Then, we can increment D to find the debt level, D^* , that maximizes the net benefit.

To see how to calculate the net benefit of debt, we will continue the earlier example with EBITDA normally distributed with mean \$1,000, and standard deviation of \$100. Assume the debt amount is \$3,673 for five years at 6% interest, so the probability of bankruptcy each year is $q_t = 10\%$. Assume the income tax rate is $T = 40\%$, and bankruptcy costs are equal to 20% of the outstanding debt principal at the date of bankruptcy.

Exhibit 11.11 shows the calculation of the Net Benefit. This is the *analytical calculation* rather than the simulation. The first section shows the probabilities of default and survival. The probabilities of default in period t , $Q_t = P_{t-1}q_t$ sum to 40.95%, which is the probability of defaulting in any one of the five periods, and is equal to $1 - P_5 = 1 - 0.5905$, the probability of surviving for five periods. The next section of the exhibit shows the calculation of the present value of tax savings from interest. The last line of the section, the expected value of tax savings in each period t is calculated by multiplying the

Tax Savings & Bankruptcy Costs									
Loan Terms		Cost Parameters							
Amount of Debt	\$3,673	Tax Rate	40%						
Rate	6%	Bankruptcy Cost	20%						
Years to Maturity	5								
Debt Service	872.0								
Probabilities of Default & Survival				Year					
				1	2	3	4	5	
Probability of Default	q_t	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%	
Probability of Survival	p_t	90.00%	90.00%	90.00%	90.00%	90.00%	90.00%	90.00%	
Probability of Surviving t periods	P_t	90.00%	81.00%	72.90%	65.61%	59.05%			
Probability of Default in period t	Q_t	10.00%	9.00%	8.10%	7.29%	6.56%			
Value of Tax Savings from Interest									
Interest		\$220.38	\$181.29	\$139.85	\$95.92	\$49.36			
Tax Savings T^*I		88.15	72.51	55.94	38.37	19.74			
PVIF @ $k d$		0.9434	0.8900	0.8396	0.7921	0.7473			
PV(TS)		83.16	64.54	46.97	30.39	14.75			
$E[PV(TS)]$	$P_t PV(TS)$	74.85	52.28	34.24	19.94	8.71			
				Expected PV Tax Savings (Sum)					190.01
Value of Bankruptcy Costs									
Debt Balance at Start of Period		3673.00	3021.42	2330.75	1598.64	822.60			
Bankruptcy Cost		734.60	604.28	466.15	319.73	164.52			
PVIF @ $k d$		0.9434	0.8900	0.8396	0.7921	0.7473			
PV(BC t)		693.02	537.81	391.39	253.25	122.94			
$E[PV(BC t)]$	$Q_t PV(BC)$	69.30	48.40	31.70	18.46	8.07			
				Expected PV Bankruptcy Costs (Sum)					175.94
Net Benefit of Debt =	Expected Value of Tax Savings - Bankruptcy Costs								14.07

EXHIBIT 11.11 Calculating the net benefit of debt.

discounted tax savings by the probability of surviving through period t , P_t . The sum of \$190.01 is the expectation of the value of tax savings over the five-period life of the debt.

The last section of [Exhibit 11.11](#) shows the calculation of the expected present value of bankruptcy costs. Each period's bankruptcy cost is calculated as the amount of the debt at the beginning of the period multiplied by the 20% bankruptcy cost. That amount is discounted, and then multiplied by the probability of defaulting that period, Q_t , to obtain the period t expected bankruptcy cost. The summation over all periods, amounting to \$175.94, is the expected present value of bankruptcy costs for the five-year debt. The difference ($190.01 - 175.94$) of \$14.07 is the net benefit of borrowing \$3,673 over five years at 6%.

11.2.2.4 Simulating the Net Benefit of Debt

[Exhibit 11.11](#) showed the analytical calculation of the net benefit. A simulation can make the calculation easier. [Exhibit 11.12a](#) shows the tax savings–bankruptcy cost model in Excel with @Risk, and [Exhibit 11.12b](#) shows the first two columns of Excel code for the model. The tax savings–bankruptcy cost simulation model extends the default model, [Exhibit 11.11](#), repeating the top 16 lines of the default model. The tax savings and bankruptcy costs calculations are on lines 18–28, debt service is in lines 31–40, and the input data are in the bottom line 43–47. The amount of the contractual interest and principal payments are in lines 32–35. However, if *EBITDA* is insufficient to make the contractual

payment, lines 37–40 calculate the amount actually paid assuming that the entire *EBITDA* will be paid to cover the debt service, with interest being paid first. In the example shown, in period 3 *EBITDA* of \$861 is insufficient to cover the required debt service of \$872. The whole \$861 (line 37) is paid to the lenders, consisting an interest of \$139.85 (line 38), and the remaining amount of \$721 (line 39) going toward the principal. As this is insufficient, the firm is in default and is assumed to fail. Failure in period 3 results in zero *EBITDA* in periods 4 and 5. Zero *EBITDA* in periods 4 and 5 (I9 and J9) is modeled by the multiplication of *EBITDA* by the zero Failure Product variable in line 16. The tax savings (line 20) are \$88.20, \$72.50, and \$55.90 in periods 1, 2, and 3, and, because of failure in period 3, zero in periods 4 and 5. The present value of the three periods of tax savings amounts to \$194.677 (E21), where the interest rate on the debt is used as the discount rate.

	A	C	D	E	F	G	H	I	J
1									
2									
3					Model to Simulate Default				
4									
5	Debtor Corporation								
6				Period					
7					1	2	3	4	5
8									
9	Funds Available to Service Debt:		EBITDA		\$1,044	\$1,045	\$861	\$0	\$0
10	Debt Service				\$872	\$872	\$872	\$872	\$872
11	Funds In Excess of Debt Service				\$172	\$173	(\$11)	(\$872)	(\$872)
12	Funds Surplus				172	173	0	0	0
13	Funds Deficit				0	0	11	872	872
14									
15	Success or Failure				1	1	0	0	0
16	Failure Product				1	1	0	0	0
17									
18	Interest				\$220.38	\$181.29	\$139.85	\$0.00	\$0.00
19	Tx Sv				88.2	72.5	55.9	0.0	0.0
20	Tax Savings from Interest				88.2	72.5	55.9	0.0	0.0
21	PV(Tax Savings)			\$194.67					
22									
23									
24	Bnrpt Cst				734.6	604.3	466.2	322.0	322.0
25	Bankruptcy Cost				0.0	0.0	466.2	0.0	0.0
26	PV(Bankruptcy Cost)			\$391.39					
27									
28	Net Benefit			(\$196.72)					
29									
30									
31	Debt Service Accounting								
32	Debt Service				\$871.96	\$871.96	\$871.96	\$871.96	\$871.96
33	Interest (contract)				\$220.38	\$181.29	\$139.85	\$95.92	\$49.36
34	Principal Payment (contract)				\$651.58	\$690.67	\$732.11	\$776.04	\$822.60
35	Principal Balance (contract)			\$3,673	\$3,021.42	\$2,330.75	\$1,598.64	\$822.60	\$0.00
36									
37	Debt Service (actual)				\$871.96	\$871.96	\$860.84	\$0.00	\$0.00
38	Interest (actual)				\$220.38	\$181.29	\$139.85	\$0.00	\$0.00
39	Principal (actual)				\$651.58	\$690.67	\$721.00	\$0.00	\$0.00
40	Principal Balance (actual)			\$3,673	\$3,021.42	\$2,330.75	\$1,609.76	\$1,609.76	\$1,609.76
41									
42									
43	EBITDA Parameters		Loan Terms			Cost Parameters			
44	Mean	\$1,000		Amount of Debt	\$3,673		Tax Rate		40%
45	Std Dev	100		Rate	6%		Bankruptcy Cost		20%
46				Years to Maturity	5				
47				Debt Service	\$872				

(a)

EXHIBIT 11.12 (a) Monte Carlo simulation model for simulating tax savings and bankruptcy costs.
(Continued)

	A	C	E	F	G
3			Model to Simulate Default		
4					
5	Debtor Corporation				
6				Period	
7				1	2
8					
9	Funds Available to Service Debt:			=RiskNormal(\$C\$44,\$C\$45)	=RiskNormal(\$C\$44,\$C\$45)*F16
10	Debt Service			=\$G\$47	=\$G\$47
11	Funds In Excess of Debt Service			=RiskOutput(\$A\$11,1)+F9-F10	=RiskOutput(\$A\$11,2)+G9-G10
12	Funds Surplus			=IF(F9>F10,F9-F10,0)	=IF(G9>G10,G9-G10,0)
13	Funds Deficit			=IF(F10>F9,F10-F9,0)	=IF(G10>G9,G10-G9,0)
14					
15	Success or Failure			=IF(F13=0,1,0)	=IF(G13=0,1,0)
16	Failure Product			=F15	=F16*G15
17					
18	Interest			=MIN(F33,F9)	=MIN(G33,G9)
19	Tx Sv			=\$J\$44*F38	=\$J\$44*G38
20	Tax Savings from Interest			=F19	=G19*F16
21	PV(Tax Savings)		=NPV(G45,F20:J20)		
22					
23					
24	Bnrkpt Cst			=\$J\$45*E40	=\$J\$45*F40
25	Bankruptcy Cost			=IF(F16=0,F24,0)	=IF(G16=0,G24,0)*F16
26	PV(Bankruptcy Cost)		=NPV(G45,F25:J25)		
27					
28	Net Benefit		=E21-E26		
29					
30					
31	Debt Service Accounting				
32	Debt Service			=-PMT(\$G\$45,\$G\$46,\$G\$44,0,0)	=-PMT(\$G\$45,\$G\$46,\$G\$44,0,0)
33	Interest (contract)			=-IPMT(\$G\$45,F7,\$G\$46,\$G\$44,0)	=-IPMT(\$G\$45,G7,\$G\$46,\$G\$44,0)
34	Principal Payment (contract)			=-PPMT(\$G\$45,F7,\$G\$46,\$G\$44,0)	=-PPMT(\$G\$45,G7,\$G\$46,\$G\$44,0)
35	Principal Balance (contract)		=G44	=E35-F34	=F35-G34
36					
37	Debt Service (actual)			=MIN(F32,F9)	=MIN(G32,G9)
38	Interest (actual)			=MIN(F33,F37)	=MIN(G33,G37)
39	Principal (actual)			=F37-F38	=G37-G38
40	Principal Balance (actual)		=E35	=E40-F39	=F40-G39
41					
42					
43	EBITDA Parameters		Loan Terms		
44	Mean	1000	Amount of Debt		=IF(Default Risk!G20
45	Std Dev	100	Rate		=IF(Default Risk!G21
46			Years to Maturity		=IF(Default Risk!G22
47			Debt Service		=-PMT(G45,G46,G44,0,0)
48					

(b)

EXHIBIT 11.12 (Continued) (b) Formulas for simulating tax savings and bankruptcy costs.

The bankruptcy cost (line 24) is assumed to be equal to 20% of the debt principal balance at the end of the previous period. However, this cost is incurred only at the time default occurs, so the bankruptcy cost in line 25 is calculated as

$$\text{Bankruptcy Cost}_t = (\% \text{ Bankruptcy Cost})(\text{Principal Balance}_{t-1}) \text{ if default in } t \tag{11.29}$$

$$= 0 \text{ otherwise}$$

For period 2 (column G) this is

$$G25 = IF(G16 = 0, G24, 0) * F16 \tag{11.30}$$

where:

- G24 is 20% of the previous principal balance
- G16 is the failure product (0 if failure occurs)
- F16 is the previous period's failure product

In this way, if default occurs in period 2, *Bankruptcy Cost* is a positive amount, but if default occurred in a previous period, it is zero. In the example, default occurs in period 3, so *Bankruptcy Cost* is \$466 ($20\% \times 2,330$) at time 3, and zero in all other periods. The present value of bankruptcy cost is \$391.39 (E26). The Net Benefit (E28) of the debt is the difference between the present value of the tax savings (\$194.67) and the present value of bankruptcy costs (\$391.39), which amounts to $-\$196.72$. So, as it turned out, the benefits of debt were outweighed by the costs of financial distress. However, before the fact, we do not know which sequence of events will occur and we do not know if the firm will fail. That is why we run the simulation.

The results shown in [Exhibit 11.12a](#) are for one random draw in the simulation. Monte Carlo simulation makes many draws from the specified probability distributions so we can see the results of numerous scenarios, instead of just one of the possible outcomes. To simulate the expected Net Benefit, we use @Risk to run a Monte Carlo simulation of the model with Net Benefit, cell E28, designated as the @Risk output variable. The mean of the simulated distribution of Net Benefit will be our estimate of the expected Net Benefit.

[Exhibit 11.13](#) shows the frequency distribution from the simulation using 1,000 random draws. The mean of \$35.25 is our estimate of the expected net benefit associated with using \$3,673 of debt. The frequency distribution in [Exhibit 11.13](#) consists of 6 spikes, with each spike corresponding to each of 6 discrete outcomes. The 6 outcomes are survival, and failure at times 1, 2, 3, 4, and 5 respectively. If the firm survives the full five periods, the present value of tax savings is \$239.81, which, with no bankruptcy

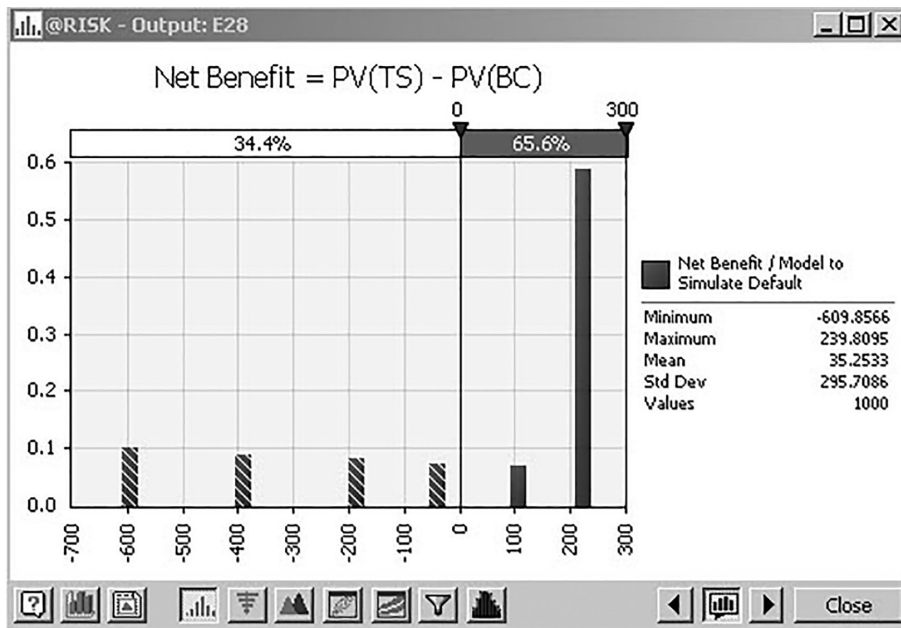


EXHIBIT 11.13 Frequency distribution of net benefit of debt.

costs, is the net benefit. If it survives for four periods and fails in the 5th period, the net benefit is \$116.87, and failure in periods 4, 3, 2, and 1 generate net benefits of $-\$28.20$, $-\$196.72$, $-\$390.11$, and $-\$609.86$, respectively. The spikes in the frequency diagram are at these points.

Although \$35.25 is a valid estimate of the expected benefit of the debt, a look at the frequency chart of net benefits will suggest that we need to be careful about interpreting this number. Note that the frequency distribution in [Exhibit 11.13](#) does not resemble a symmetric, continuous distribution such as the normal that we are familiar with. We will get 1 of the 6 possible outcomes, and not one of them is \$35.25. Yet, we evaluate the benefit of the debt as being \$35.25 because it is the expected value of the 6 possible outcomes, and we will use this number in comparison to other debt amounts and strategies.

We have calculated the net benefit from using \$3,673 of debt financing. Now we have to determine the optimal amount of debt. The question is whether we can obtain a higher net benefit with a different amount of debt. For example, would we obtain higher net benefits if we borrowed \$4,000 instead of \$3,673? To answer this question, we increment the debt to \$4,000, and run the simulation to estimate the expected net benefit. With \$4,000 of debt, the simulated mean of the Net Benefit turns out to be negative $-\$280.86$, so borrowing \$4,000 is worse than \$3,673.

To find the amount of debt that maximizes the net benefit, we vary the amount of debt in increments so we can trace out the values of tax savings and bankruptcy costs for each level of debt. Different levels of debt were put into the model and the simulation was run to obtain the distribution means of tax savings, bankruptcy costs, and net benefits, respectively. The results for various levels of debt are shown in tabular form in [Exhibit 11.14](#), and as a graph in [Exhibit 11.15](#). As can be seen on the graph, the net

Debt Amount	Default Probability	Expected Tax Savings	Expected Bankruptcy Cost	Expected Net Benefit
500	0.00%	32.64	0.00	32.64
1,000	0.00%	65.3	0.0	65.3
1,500	0.00%	97.9	0.0	97.9
2,000	0.00%	130.6	0.0	130.6
2,500	0.00%	163.2	0.0	163.2
2,750	0.55%	179.4	0.7	178.8
2,900	0.35%	189.2	1.2	188.0
3,000	0.45%	195.2	4.1	191.1
3,100	0.25%	201.6	5.9	195.7
3,250	1.25%	209.1	20.6	188.5
3,500	4.20%	216.4	75.9	140.6
3,673	10.85%	209.9	174.7	35.3
3,800	14.15%	204.1	267.9	-63.8
4,000	30.20%	180.2	449.8	-269.7
4,500	76.75%	124.3	790.3	-666.0
5000	96.70%	116.2	936.1	-819.9

EXHIBIT 11.14 Expected net benefit, tax savings, and bankruptcy costs at various levels of debt.

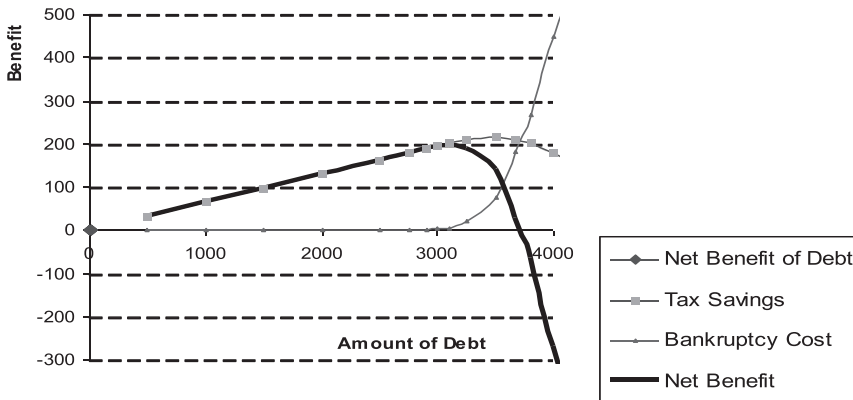


EXHIBIT 11.15 Net benefit, tax savings, bankruptcy costs, and probability of default.

benefit reaches its maximum at about \$3,100 of debt, where the probability of bankruptcy is still quite small at 0.25%. Beyond that level of debt, the probability of default starts to rise very rapidly, as does the expected present value of bankruptcy costs. This risk quickly overwhelms the tax savings. This simulation shows us that for this firm, with fairly large proportional bankruptcy costs, there is not much capacity to increase debt before it becomes too risky.

11.3 ADDITIONAL DEBT MODELING CONCEPTS

In this section we introduce two additional topics relating to modeling debt. First we briefly explain the concept of duration, and then show how to model duration in the spreadsheet. Second, we introduce swaps and show how to construct a model to analyze simple swap opportunities.

11.3.1 Duration

The duration of a security, measures two related aspects of the security: the length of its life and its sensitivity to changes in interest rates. These aspects are potentially important for both the issuer of the security and the investor who buys the security.

11.3.1.1 Duration as Average Life of Cash Flows

Duration is usually applied to debt securities, particularly bonds. For bonds we think of the life of the bond as the time until it matures. For example, a bond that pays the outstanding principal at the end of five years is considered to be a bond with five years to maturity. But because we receive several coupon interest payments in the intervening periods, the time to maturity may not tell us much about the timing of the cash flows we get from the bond. Duration is a measure of the life of the security that takes account of the timing of all the payments, not just the last one at the maturity date.

To understand duration, consider the following example. Suppose you loan me \$1,000. The interest rate on the loan is 10%, and I promise to make the following annual payments over four years to repay the loan and pay you 10% interest:

Period	1	2	3	4
Payment	100	900	150	78

At a 10% discount rate, these payments have a present value of \$1,000, calculated as

$$1,000 = 100 \left(\frac{1}{1.10} \right)^1 + 900 \left(\frac{1}{1.10} \right)^2 + 150 \left(\frac{1}{1.10} \right)^3 + 78 \left(\frac{1}{1.10} \right)^4 \quad (11.31)$$

confirming that the rate on the loan is 10%.

Ask yourself, *what is the time to maturity of the loan?* Surely, you will answer *four years*. However, when do you actually get your payments? Do you have to wait four years to get your payments? No! By the end of the second period, you have received \$1,000, the amount you originally loaned me. In terms of the length of time until you receive your money, the time is something less than four years because you receive most of your money sooner than four years. So, you are not really waiting a full four years to receive your money back.

Duration is a measure of the average time it takes to collect all the payments. Duration is a weighted average of the dates that you receive the payments, where the weights are the *proportion of the value* of the investment that you receive at each date.

In this example, the present value of all the payments is \$1,000. Of that \$1,000, the first payment of \$100 has a present value of \$100 (0.9091) = \$90.91, which is \$90.91/\$1,000 = 0.0991 = 9.91% of the value that you will receive. The second payment of \$900 has a present value of 900 (0.8264) = \$743.80, which constitutes 74.38% of the value of all the payments. Similar calculations show that the third payment of \$150 has a present value that is 11.27% of the total present value [\$150 (0.7513) = \$112.70], and the last payment of \$78 is 5.328% of the total value [\$78 (0.687) = \$53.28]. Of the total present value of \$1,000 that you receive, you will receive 9.91% in period 1, 74.38% in period 2, 11.27% in period 3, and 5.33% in period 4.

Although it takes four periods to receive all the funds, most of the money is received earlier than four periods. Duration takes account of this pattern. Duration is calculated as the weighted average:

$$\begin{aligned} Dur &= \sum_{t=1}^m w_t t \\ &= 0.0909(1) + 0.7438(2) + 0.1127(3) + 0.0533(4) \\ &= 2.13 \end{aligned} \quad (11.32)$$

where:

$$w_t = \frac{\frac{\text{Cash flow}_t}{(1+r)^t}}{\sum_{t=1}^n \frac{\text{Cash flow}_t}{(1+r)^t}} = \frac{\text{PV}(\text{Cash flow}_t)}{\text{Price of bond}} \tag{11.33}$$

The weights can be seen as the proportions of the present value received at each date relative to the bond’s price, and t is the date at which you receive the payment. In this case, you receive payments at times $t = 1, 2, 3,$ and $4,$ but the duration is 2.13 years. This is indicating that the average time to receive the value of the payments is 2.13 years, instead of four years.

Exhibit 11.16 shows the calculation of duration for 3 different bonds. Bond A is the one we just explained, bond B is an 8% annual coupon bond that matures in 10 years, and bond C is a 10% annual coupon bond that matures in 16 years. The cash flows from each bond are shown in columns C, H, and L, respectively. The numbers in column P are the present values of \$1, $(1/(1+r)^t)$, abbreviated PVIF (present value interest factor). The amounts in columns D, I, and M are the present values of each of the annual payments, with the sum of these present values being the value of the bonds in D6, I6, and M6. For bond A, cells E8:E11 are the weights calculated according to Equation 11.33, and F8:F11 are the product tw_t . The sum $\sum tw_t$ is the duration in F2, which as shown earlier is 2.13 years for bond A. For bonds B and C, J8:J17 and N8:N23 are the calculations of tw_t , and these figures are summed to obtain the durations in J2 and N2. The duration of the 10-year, 8% bond

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1			Bond A					Bond B				Bond C			
2	Duration					2.12				6.94				9.38	
3						=SUM(F8:F11)									
4															
5	Period		Cash Flow	Value	% of Value	t*(%_Value)		Cash Flow				Cash Flow			
6	0			981				823.32				926.208			
7															
8	1		100	90.09	0.092	0.0918		80	72.07	0.088		100	90.09	0.109	
9	2		900	730.38	0.745	1.4891		80	64.93	0.158		100	81.16	0.197	
10	3		150	109.39	0.112	0.3345		80	58.50	0.213		100	73.12	0.266	
11	4		78	51.08	0.052	0.2083		80	52.70	0.256		100	65.87	0.320	
12	5							80	47.48	0.288		100	59.35	0.360	
13	6							80	42.77	0.312		100	53.46	0.390	
14	7							80	38.53	0.328		100	48.17	0.410	
15	8							80	34.71	0.337		100	43.39	0.422	
16	9							80	31.27	0.342		100	39.09	0.427	
17	10							1080	380.36	4.620		100	35.22	0.428	
18	11											100	31.73	0.424	
19	12											100	28.58	0.417	
20	13											100	25.75	0.407	
21	14											100	23.20	0.394	
22	15											100	20.90	0.381	
23	16											1100	207.12	4.025	
24															
25	Discount Rate		11%												

EXHIBIT 11.16 Duration calculations for 3 bonds.

is 7.04 years; for the 16-year, 10% bond, 9.81 years. Note that with a duration of 9.81 years for the 16-year bond, the duration is just a little greater than half of the time to maturity.

Excel has a duration formula that is in cell J19 of [Exhibit 11.16](#), and shows the duration of the 10-year bond. The format for the duration function is

$$=Duration(settlement\ date, maturity\ date, coupon, yield, frequency, basis) \quad (11.34)$$

where:

settlement date is date you pay for the bond

maturity date is the last payment (principal plus interest)

yield is the yield to maturity based on the current market price of the bond

frequency is the number of coupon payments per year

basis is the number of days per year

As you can see, the duration formula returns the same answer as our model for this standard bond. However, it only applies to standard bonds, is not suited to more unusual cash flow patterns such as bond A. For securities other than standard fixed coupon bonds, the Excel duration formula does not work well, and you are better off making your own model similar to [Exhibit 11.16](#).

11.3.1.2 Duration as Interest Rate Sensitivity

Many discussions of duration emphasize that interpreting duration as time variable is only half the story. They say that the more relevant interpretation is that duration is a measure of the sensitivity of the value of a security to changes in the discount rate. This is due to the fact that in response to small changes in the discount rate, the (approximate) percentage change in the value of the security can be written as

$$\frac{\Delta Value}{Value} = -Dur \left(\frac{\Delta r}{1+r} \right) \quad (11.35)$$

where:

$\Delta Value$ denotes the change in the value

Dur is the duration

Δr is the change in the interest rate, r

This says that the greater the duration, the more the value will change in response to changes in the interest rate. Longer term bonds that have greater duration will be more sensitive to changes in interest rates than bonds with short duration.

To see the link between interest rates, values, and duration, return to the example of the 3 bonds of [Exhibit 11.16](#). Keep in mind that at an interest rate of 10%, the values of the bonds were \$1,000, \$877.11, and \$1,000 for A, B, and C, and their durations were 2.13 years, 7.04 years, and 9.81 years, respectively. [Exhibit 11.17](#) is the same spreadsheet, except we changed the interest rate from 10% to 11%. With the discount rate at 11%, the bond values have declined to \$981, \$823, and \$926, (in row 6) which are percentage

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	
1			Bond A					Bond B				Bond C					
2	Duration						2.12			6.94					9.38		
3							=SUM(F8:F11)										
4			Value	% of													
5	Period	Cash	Value	t-(%of Value)				Cash				Cash				PVIF	
6	0	Flow	981					Flow	823.32			Flow	926.2084			1/(1+r)	
7																	
8	1	100	90.09	0.092	0.0918			80	72.07	0.088		100	90.09	0.109		0.9009	
9	2	900	730.38	0.745	1.4891			80	64.93	0.158		100	81.16	0.197		0.8116	
10	3	150	109.39	0.112	0.3345			80	58.50	0.213		100	73.12	0.266		0.7312	
11	4	78	51.08	0.052	0.2083			80	52.70	0.256		100	65.87	0.320		0.6587	
12	5							80	47.48	0.288		100	59.35	0.360		0.5935	
13	6							80	42.77	0.312		100	53.46	0.390		0.5346	
14	7							80	38.53	0.328		100	48.17	0.410		0.4817	
15	8							80	34.71	0.337		100	43.39	0.422		0.4339	
16	9							80	31.27	0.342		100	39.09	0.427		0.3909	
17	10							1080	380.36	4.620		100	35.22	0.428		0.3522	
18	11											100	31.73	0.424		0.3173	
19	12											100	28.58	0.417		0.2858	
20	13											100	25.75	0.407		0.2575	
21	14											100	23.20	0.394		0.2320	
22	15											100	20.90	0.381		0.2090	
23	16											1100	207.12	4.025		0.1883	
24																	
25	Discount Rate		11%														

EXHIBIT 11.17 Bond values and duration at 11% interest.

changes of -1.9% , -6.13% , and -7.38% . Using Equation 11.35, the calculation of the percentage change in value for each of the 3 bonds would be

$$\begin{aligned} \frac{\Delta \text{Value}}{\text{Value}} &= -\text{Dur} \left(\frac{\Delta r}{1+r} \right) \\ &= -2.13 \left(\frac{+0.01}{1.10} \right) = -0.0194 = -1.94\% \text{ for bond A} \\ &= -7.04 \left(\frac{+0.01}{1.10} \right) = -0.064 = -6.4\% \text{ for bond B} \end{aligned}$$

and

$$= -9.81 \left(\frac{+0.01}{1.10} \right) = -0.0892 = -8.9\% \text{ for bond C}$$

As you see, these figures only approximate the change in values that we computed. However, they are just approximations because the formula is accurate only for very small changes in the interest rate. A rate change from 10% to 11% is relatively large in this context, and there is some error. The error is due to the fact that the relationships are nonlinear, and the duration formula does not pick that up. The nonlinear relation is referred to as convexity, and for a more accurate calculation we need to take the convexity into account. However, like so many topics, that is beyond the scope of this chapter, and you are referred to more detailed discussions such as Hull (2015), Ho and Lee (2004), and Reilly and Brown (2012).

11.3.1.3 Duration and Immunization

Duration is useful for understanding the interest rate risk of a security. But in the context of financial planning for corporate financial management, one of the most interesting applications of the concept is immunization. Immunization has two different meanings in this context. The first refers to balancing the risk of changes in *value* due to interest rate changes against *reinvestment risk*.^{*} The second meaning refers to insulating the value of the equity of the firm from changes in interest rates. We will confine the remainder of this section to the second interpretation.

From our discussion of duration we see that the longer the duration of an investment, the more sensitive the value of the security is to changes in interest rates. If rates go up, longer duration investments will decline more in value than those with shorter durations; if interest rates decline, longer duration investments will increase in value more than shorter duration investments.

Note that we used the term investment in reference to the sensitivity of the value. The reason is that the sensitivity to interest rates does not just apply to debt securities such as bonds. It applies to all investments whose values are based on the present value of future cash flows. With that in mind, we can think of a firm consisting of different claims on the cash flows. The assets generate a stream of future cash flows that come into the firm and are subsequently paid out to the creditors and the equity owners. With the equity receiving the residual flow after payments to the creditors, the value of equity is the value of the assets minus the value of the liabilities:

$$\text{Equity Value} = \text{Asset Value} - \text{Liability Value} \quad (11.36)$$

Each of the components has a duration of its cash flows, and each has a sensitivity to interest rate changes.

With Equity being the residual claimant, consider the effects of interest rate changes on the value of the equity. Suppose the asset cash flow is of very long duration, the liabilities are of short duration, and the interest rates increase. The increase in rates will make the longer duration assets decline in value more than the shorter duration liabilities. The net effect is that the value of the equity will decline simply because the asset value declines more than the liability value. On the other hand, if rates decline, the long duration assets will increase more in value than the shorter duration liabilities, and the value of equity will increase.

The point is that the duration of the assets and liabilities impacts the interest rate risk of the equity. The interest rate risk of equity can be managed by adjusting the maturities and duration of the assets and liabilities. If management wants to decrease equity's exposure to interest rate risk, they could restructure the debt to increase the duration of their liabilities. With appropriate balancing of the duration of assets and liabilities, the value of equity

^{*} Reinvestment risk is an important concept for managing investment portfolios—particularly those such as retirement funds that have a target payout in the future. Fabozzi (2012) and Granito (1984) provide greater detail about immunization in the presence of reinvestment risk.

should be immunized against changes in interest rates. We do not go more deeply into this interesting issue. Rather, we simply alert you to one of the applications of the duration concept for financial management.*

11.3.2 Swaps

In the previous section we discussed the effects of interest rate changes on the value of securities. In this section we explain a technique for dealing with interest rate variability: interest rate swaps. Interest rate swaps are a mechanism for borrowers and lenders to shift from fixed to variable rate, or variable rate to fixed rate securities.† For example, suppose a borrower has a variable rate loan, but a fixed rate loan would fit its needs better. The variable rate borrower could arrange a swap with another borrower who has a fixed rate loan and wants a variable rate. These two borrowers make the swap in a way that results in the formerly variable rate borrower now making fixed rate interest payments, and the formerly fixed rate borrower now making variable rate payments. The effect is as if the two borrowers each agree to make the payments for the other one.

The reason for interest rate swaps is that some firms are better able to handle the uncertainty of interest rate variability than others, but, the best deal for a loan that they can get carries a floating interest rate loan. With a swap, they can accept the floating rate loan and then swap it with another borrower whose best borrowing opportunity might have been a fixed rate loan and who may prefer to have a floating rate loan. An example will help to show how this works and show how we can model it.

11.3.2.1 Example: Swapping Plain Vanilla

The Varbic Company has a loan for \$100 million. The loan matures in five years, with the entire principal paid at the end of the five years. The interest rate on the loan is variable, being equal to the six-month LIBOR rate plus 1.2% (120 basis points). Interest is paid at the end of each six-month interval based on the rate at the beginning of the period.

Varbic's income available to pay interest (EBIT) tends to be relatively stable over the economic cycle. However, for several years they have been meeting their financing needs with variable rate borrowing. The result is that their income after paying interest (EBIT—Interest) is quite variable. [Exhibit 11.18](#) diagrams the pattern of the company's EBIT, interest cost, and income after interest. EBIT has been slowly trending upward with minor variations. But the variability of their interest costs translates directly into their income stream. The managers have concluded that if they could stabilize their interest costs, their income would be as smooth as their EBIT.

* The following sources provide detailed explanations of the effect of duration and immunization on the value of equity: Fabozzi (2012), Bierwag (1987), Haugen and Wichern (1974), (1975), Boquist et al. (1975), and Grove (1974), (1966).

† The fixed for variable rate swaps are the most common and the easiest to understand, and are termed *plain vanilla* swaps. The name suggests that there are other flavors. There are many kinds of swaps including currency swaps, and various kinds of swaps involving risk shifting. See Hull (2015) and Dufresne and Miller (2003) for more detailed discussions.

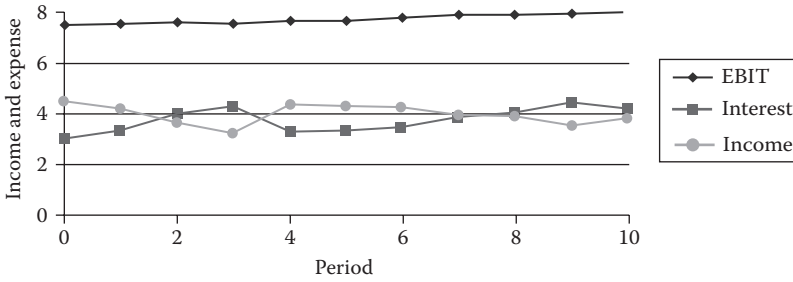


EXHIBIT 11.18 Varbic income with variable rate debt.

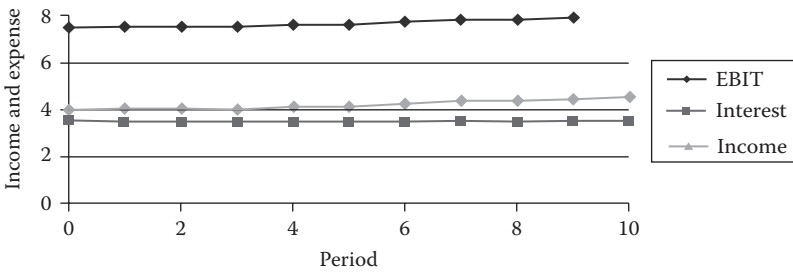


EXHIBIT 11.19 Varbic income with fixed rate debt.

Varbic management examined their income after interest costs assuming stable interest costs. Exhibit 11.19 shows the company’s past income assuming that their debt had been a fixed rate loan over the past five years, with interest at an annual rate of 7.5%, which they estimate would have been the rate they could have gotten at the start of the five-year period. They currently have a five-year variable rate loan that at the moment is carrying a rate of 7.2%. At current rates, they could obtain a new fixed rate loan at 8.2%, but they think they can make a better deal by engaging in swap with another company that has a fixed rate loan.

The Fixion Company has a \$100 million fixed rate loan that matures in five years. The rate on the loan is 7%, with interest paid every six months. Principal is paid at the end of the five years. The income stream generated by Fixion’s assets is variable and tends to follow the cycle of the economy and interest rates. Fixion’s management feels that they could stabilize their income by swapping to a variable rate loan. As shown in Exhibit 11.20, their income over that past five years shows considerable cyclical variability. The dotted line in the middle represents interest

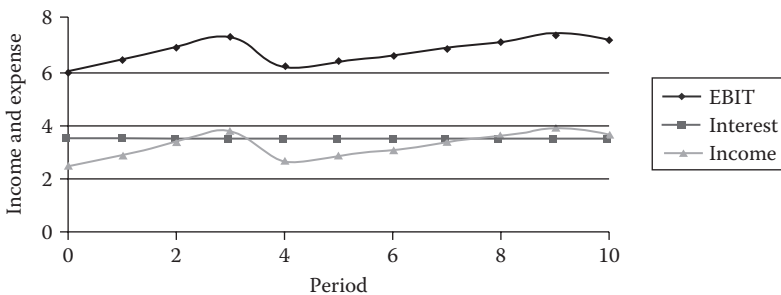


EXHIBIT 11.20 Fixion income and expense with fixed rate debt.

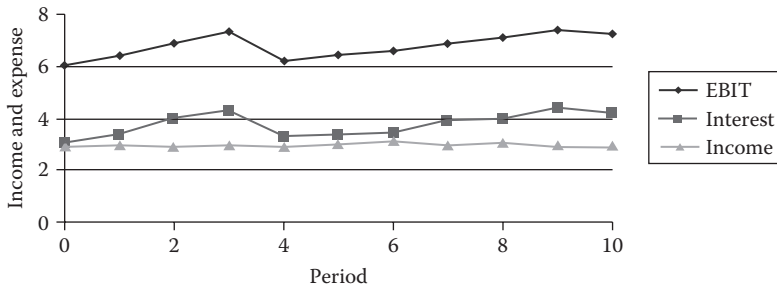


EXHIBIT 11.21 Fixion income and expense with variable rate debt.

expense, and the bottom line is the income after interest. The variability of income has been increased by the fact that their interest costs remain constant when their EBIT is fluctuating.

Exhibit 11.21 shows Fixion's income based on the assumption that they used floating rate debt with the same pattern as Varbic. The interest cost follows the same pattern as their EBIT, so the difference, EBIT–Interest, is quite stable as shown by the income line at the bottom.

Using a bank as the broker to make the arrangements, Varbic and Fixion agree to a swap. The swap has the effect of the two companies trading payments. It is as if Varbic makes Fixion's payments, and Fixion makes Varbic's payments. The way this is actually implemented is that the payments are netted against each other and one counterparty pays the other the difference.

Exhibit 11.22 models how this would work. The upper left section (A5:F14) shows the input. Cells D9:F9 indicate the rate each of the companies would pay for a fixed rate loan,

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P		
1	Swap																	
2																		
3	CounterParties		Fixion Co			Varbic Co												
4																		
5	Principal (\$ million)			100			100											
6	Term			5			5											
7	Number of Payments			10			10											
8																		
9	Fixed Interest Rate			7.0%			8.2%											
10	Interest Rate - Floating																	
11	Base LIB OR			6.00%			6.00%											
12	Margin above Base			0.30%			1.20%											
13																		
14	Loan - Fixed or Float?			Fixed			Floating											
15																		
16	Period						Difference		Fixion Co			Varbic Co						
17							Fx - Fl		Payment	Net		Payment	Net					
18									Received	Made	Payment	Received	Made	Payment				
19	0	6.00%							0.00	0.10	3.60	0.10	0.00	3.50				
20	1	7.00%	3.50		3.60		-0.10		0.00	0.60	4.10	0.60	0.00	3.50				
21	2	8.00%	3.50		4.60		-1.10		0.00	1.10	4.60	1.10	0.00	3.50				
22	3	8.50%	3.50		4.85		-1.35		0.00	1.35	4.85	1.35	0.00	3.50				
23	4	7.00%	3.50		4.10		-0.60		0.00	0.60	4.10	0.60	0.00	3.50				
24	5	6.00%	3.50		3.60		-0.10		0.00	0.10	3.60	0.10	0.00	3.50				
25	6	4.00%	3.50		2.60		0.90		0.90	0.00	2.60	0.00	0.90	3.50				
26	7	5.00%	3.50		3.10		0.40		0.40	0.00	3.10	0.00	0.40	3.50				
27	8	4.00%	3.50		2.60		0.90		0.90	0.00	2.60	0.00	0.90	3.50				
28	9	3.50%	3.50		2.35		1.15		1.15	0.00	2.35	0.00	1.15	3.50				

EXHIBIT 11.22 Swap payment by Varbic and Fixion.

and D11:F12 show the rates each would have for a variable rate loan. Fixion's loan is a fixed rate at 7%, and Varbic's is a floating rate at LIBOR plus 1.20%. The current LIBOR rate is 6%, and hypothetical future LIBOR rates are shown in B19:B27 to demonstrate what could happen over the five-year remaining life of the loans.

With the fixed rate loan at 7% on \$100 million principal, Fixion's interest payments would be \$3.5 million every six months for the next five years as shown in D19:F28. Given the hypothetical LIBOR rates shown in B18:B27, Varbic's interest payments would be as shown in F19:F28, varying every six months based on the variation of the LIBOR over the period. The figures in column H (H19:H28) show the difference between the fixed payments by Fixion and the floating rate payments by Varbic. The negative numbers indicate that Varbic's payments exceed the fixed rate payment by Fixion. Positive numbers indicate that Fixion's payments are greater.

In a swap deal, the counterparties do not take over each other's payments. Rather, they simply pay each other the difference between the payments. Look at the loan payments made by the two companies in the 1st month. Fixion pays \$3.5 million (D19), and Varbic pays \$3.6 million (F19), for a difference of $-\$0.10$ million (H10) against Varbic. To make up this difference, Fixion will pay Varbic \$0.10 million (K19). Varbic's receipt of the payment is shown in N19. The effect of Fixion paying this difference to Varbic is to stabilize Varbic's net interest payments and to turn Fixion's payments into floating rate payments. Without this payment from Fixion, Varbic's cost would be \$3.6 million (F19), but receiving a payment of \$0.10 million reduces Varbic's net cost to \$3.5 million as shown in P19. Without having to make the payment, Fixion's cost would be \$3.5 million (D19), but net of the payment, Fixion's cost is \$3.6 million (L19). Look at each payment period. The net payment by Fixion (column L) varies so it is equal to the payment that Varbic would have made without the swap (F19:F28). Now, with the swap, Varbic's net payments in column P are fixed at \$3.5 million every six months. The effect of the companies agreeing to pay the differences in the payments is as if they had traded loans—Fixion getting the variable rate loan, and Varbic getting the fixed rate loan.

Why would these companies do this? The first reason we have already discussed. The variable rate loan fits better with Fixion's pattern of income, and the fixed rate loan is better suited to Varbic's pattern of income. So why does not each company simply arrange the kind of loan they need? Without going into all the detail of the comparative advantage argument,^{*} it may be that Varbic can make a better deal with the lender by taking the variable rate loan, and then swapping, and Fixion can get its best deal by taking the fixed rate loan and then swapping subsequently.

The variable rates on LIBOR in [Exhibit 11.22](#) are hypothetical to demonstrate the impact of the variation in rates. Before the fact, we do not know what rates will do, and the borrower who ends up paying the floating rate will face the uncertainty of future rates. The model shown in [Exhibit 11.22](#) can easily be adapted to use Monte Carlo simulation to explore what might happen from the random variation of future rates. We show a Monte Carlo model next. First, we need to construct a model of randomly varying interest rates.

^{*} Hull (2015), [Chapter 7](#) elaborates on the comparative advantage argument.

11.3.2.2 Stochastic Interest Rates

Obviously, interest rates vary considerably based on conditions in the economy and financial markets. To show the variation of interest rates, we need to model the process that interest rates follow over time. Vasicek (1977)* developed a model of the random variation of interest rates where rates are assumed to follow a mean-reverting Weiner process, what he calls an elastic random walk, of the form

$$dr = a(b - r)dt + \sigma z\sqrt{dt} \quad (11.37)$$

where:

dr denotes the change in the short-term interest rate, r , over a short period of time, dt

b is a long-run average (mean) rate of interest

σ is the instantaneous standard deviation of the interest rate, expressed as an annual rate

z is the standard normal variable with mean 0 and standard deviation of 1

a is the response rate governing the speed of adjustment

The mean rate b is a rate toward which interest rates tend to drift over time. The term $(b - r)$ is the difference between the long-run mean and the current rate, r . When the current rate is below the average, $(b - r) > 0$, the rate will have a tendency to increase proportionally to the gap between the current rate and the long-run rate. Conversely, if the current rate is very high, $(b - r) < 0$, rates will have a tendency to drift down toward the long-run mean, b . The rate at which the adjustment occurs is governed by the response rate, a . If a is large, the adjustment will be rapid, and vice versa.

When interest rates follow the elastic random walk portrayed by Equation 11.37, they tend to vary randomly over short periods of time, but the further away the rate is from the average, the stronger the tendency to move back to the average, with random variation along the way. The random variation comes from the error term $\sigma z\sqrt{dt}$. In this expression, z represents a random draw from a standard normal distribution (a normal distribution with mean of 0 and standard deviation of 1).

We use the Vasicek elastic random-walk interest rate model to portray the movements of the LIBOR rate in our swap example. When we put some numbers into the interest rate equation it will be a little more understandable. Assume the long-run average interest rate is $b = 0.03$ (3%), the standard deviation of the interest rate change is $\sigma = 0.006$ (0.6%) per year, the response parameter is $a = 0.10$, and the time interval is $dt = 0.50$ (one half year).† Assume the current LIBOR rate is 6%, well above the long-run average of $b = 3\%$. The change in the rate in the next six-month interval will be

* Vasicek's (1977) interest rate model is one of many that portray the random variation of rates over time. Hull (2015) Chapter 30 provides an overview of many of these interest rate models.

† In the interest rate model, the time interval, dt , is intended to be just an instant. That is, the limit as $dt \rightarrow 0$. However, for our example, we approximate the process by assuming that dt is a discrete period of time such as six months.

$$\begin{aligned}
 dr &= a(b-r)dt + \sigma z\sqrt{dt} \\
 &= 0.10(0.03 - 0.06)0.50 + 0.006z\sqrt{0.50} \\
 &= 0.10(-0.03)0.50 + 0.00424z \\
 &= -0.0015 + 0.00424z
 \end{aligned}
 \tag{11.38}$$

where z is the random draw from the standard normal distribution. Suppose the random draw is $z = 1.40$. The change in the interest rate over the six-month interval would be

$$dr = -0.0015 + 0.00424(1.4) = 0.0044
 \tag{11.39}$$

and the interest rate will move from the initial rate of 6% to 6.44% ($0.06 + 0.0044$). In the next interval, a new random value of z will occur, and the rate will change from the base of 6.44%. Note that even though this process has a tendency to move back toward the long-run average, depending on the random draw, z , the rate can move up, as it did from 6% to 6.44%. But the further the rate goes above the average, the greater the tendency to move back toward the mean.

This elastic random walk of interest rates is easy to model. Exhibit 11.23 shows a Monte Carlo model to generate the interest rate series. The parameters from the example in the previous paragraph are input in cells F41:F44. Only the first 4 lines of the model are shown, with the formulas for periods 1 and 2 in lines 49 and 50, and the output for periods 3 and 4 in lines 51 and 52. The formulas in the drift column B are the $a(b-r)dt$ components of the model. The random part is in column D, which calculates the $\sigma z\sqrt{dt}$ part of process. The random variable z is drawn by @RISK with the =RiskNormal(0, 1) function. The drift and the random components are added together in column F, and the resulting interest rate change is added to the previous period's rate in column H. The formulas are copied down the column for as many periods as needed.

	A	B	C	D	E	F	G	H
34		Model of Stochastic Interest Rates						
35		$dr = a(b-r)dt + \sigma z\sqrt{dt}$						
36								
37								
38								
39								
40		Interest Rate Input Parameters						
41		Response Rate, a				0.1		
42		Long Run Average, b				0.03		
43		Rate Volatility, σ				0.006		
44		Time Interval, dt				0.5		
45								
46	Period	Drift	Random			Rate Change		LIBOR Rate
47								
48	0							=F12
49	1	=F\$41*(F\$42-H48)*F\$44		=RiskNormal(0,1)*F\$43*SQRT(F\$44)		=B49+D49		=H48+F49
50	2	=F\$41*(F\$42-H49)*F\$44		=RiskNormal(0,1)*F\$43*SQRT(F\$44)		=B50+D50		=H49+F50
51	3	-0.001604		0.0016157		0.0000114		0.062098
52	4	-0.001604		-0.001974		-0.003579		0.058519
53	5	=F\$41*(F\$42-H52)*F\$44		=RiskNormal(0,1)*F\$43*SQRT(F\$44)		=B53+D53		=H52+F53

EXHIBIT 11.23 Model to generate random interest rate changes.

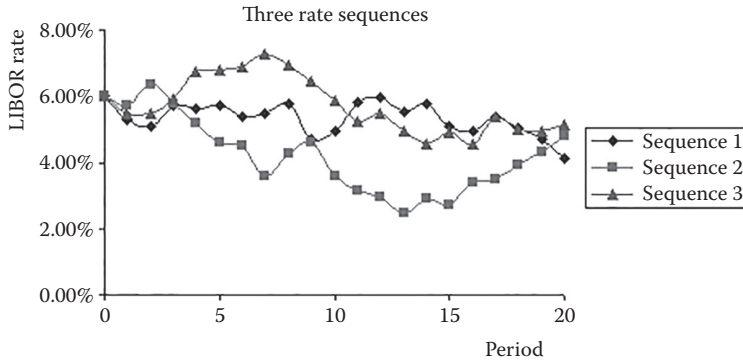


EXHIBIT 11.24 Different rate sequences from mean-reverting random walk.

When we run the model we get a sequence of rates starting from the initial value and varying randomly over time. Exhibit 11.24 shows the sequence of rates over 20 periods (10 years) generated with three different Monte Carlo simulation runs. Each different run produces as different sequence of rates. With the mean-reverting model, the sequences will tend to fluctuate randomly, but generally drift downward toward the long-run average, which in this example is assumed to be 3%. This example shows the Monte Carlo model of interest rates. The next step is to use the interest rate model in the swap model.

Exhibit 11.25 is the swap model shown earlier with the LIBOR rates generated as a mean-reverting random walk. One particular realization of the randomly generated LIBOR rates

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P		
1	Swap																	
2																		
3																		
4	CounterParties			Fixion Co			Varbic Co											
5																		
6	Principal (\$ million)			100			100			<div style="border: 1px solid black; padding: 5px; width: fit-content;"> Model of Stochastic Interest Rates $dr = a(b-r)dt + \sigma z(dt)^{1/2}$ </div>								
7	Term			5			5											
8	Number of Payments			10			10											
9																		
10	Fixed Interest Rate			7.0%			8.2%			Response Rate, a			0.10					
11	Interest Rate - Floating									Long Run Average, b			3.00%					
12	Base LIBOR			6.00%			6.00%			Rate Volatility, σ			0.60%					
13	Margin above Base			0.30%			1.20%			Time Interval, dt			0.50					
14																		
15	Loan - Fixed or Float?			Fixed			Floating											
16																		
17	Period						Difference			Fixion Co			Varbic Co					
18							Fx - Fl			Payment			Payment					
19										Received			Received					
20										Made			Made					
20	0	6.00%								3.60			3.50					
21	1	5.85%		3.50			3.60			-0.10			0.10 0.00 3.60			0.00 0.10 3.50		
22	2	5.71%		3.50			3.53			-0.02			0.02 0.00 3.53			0.00 0.02 3.50		
23	3	5.57%		3.50			3.45			0.05			0.00 0.05 3.45			0.05 0.00 3.50		
24	4	5.44%		3.50			3.39			0.11			0.00 0.11 3.39			0.11 0.00 3.50		
25	5	5.32%		3.50			3.32			0.18			0.00 0.18 3.32			0.18 0.00 3.50		
26	6	5.21%		3.50			3.26			0.24			0.00 0.24 3.26			0.24 0.00 3.50		
27	7	5.10%		3.50			3.20			0.30			0.00 0.30 3.20			0.30 0.00 3.50		
28	8	4.99%		3.50			3.15			0.35			0.00 0.35 3.15			0.35 0.00 3.50		
29	9	4.89%		3.50			3.10			0.40			0.00 0.40 3.10			0.40 0.00 3.50		
30	10			3.50			3.05			0.45			0.00 0.45 3.05			0.45 0.00 3.50		
31	Annual Rate over 5 Years			7.12%			6.75%						6.75%			7.12%		

EXHIBIT 11.25 Swap model with Monte Carlo simulation of stochastic interest rates.

are shown in cells B19:B28. The payments by each counterparty to the swap are in columns J through P, just as in [Exhibit 11.22](#). Note that Varbic ends up with fixed payments of \$3.5 million every six months no matter what the sequence of LIBOR rates that might be generated. On the other hand, Fixion ends up paying the floating rate.

As the LIBOR rate can vary over time, it is difficult ahead of time to know what the floating rate loan will cost. The Monte Carlo simulation allows us to see what the possibilities are. There are different ways to measure the cost. The way we will gauge the cost of the loan is to calculate the *IRR* on the loan over the five-year loan period. Treating the loan principal as a cash inflow at time $t = 0$, and the semiannual interest payment and then the principal payment at the end of five years as outflows, the cost of the loan over the five years can be calculated with the `=IRR()` function in Excel.

A single random sequence of rates will generate a single *IRR*. What we want to know is what does the probability distribution of all possible *IRR*'s look like. When we run the Monte Carlo simulation with many iterations, we generate a frequency distribution of *IRRs*.

[Exhibit 11.26](#) shows the frequency distribution of Fixion's annual interest cost (*IRR*) over the five years with the swap. The mean cost is 6.75% with a standard deviation of 0.623%. The maximum rate was 8.72% and the minimum was 5.04%. With the swap, even though the rate is floating randomly, Fixion is able to stabilize their income stream by swapping to a variable rate loan, and in this case, the company is able to decrease the expected interest cost. Without the swap, their cost would have been fixed at 7.12%, but with the swap, the expected (mean) cost has been reduced to 6.75%. The reason for the

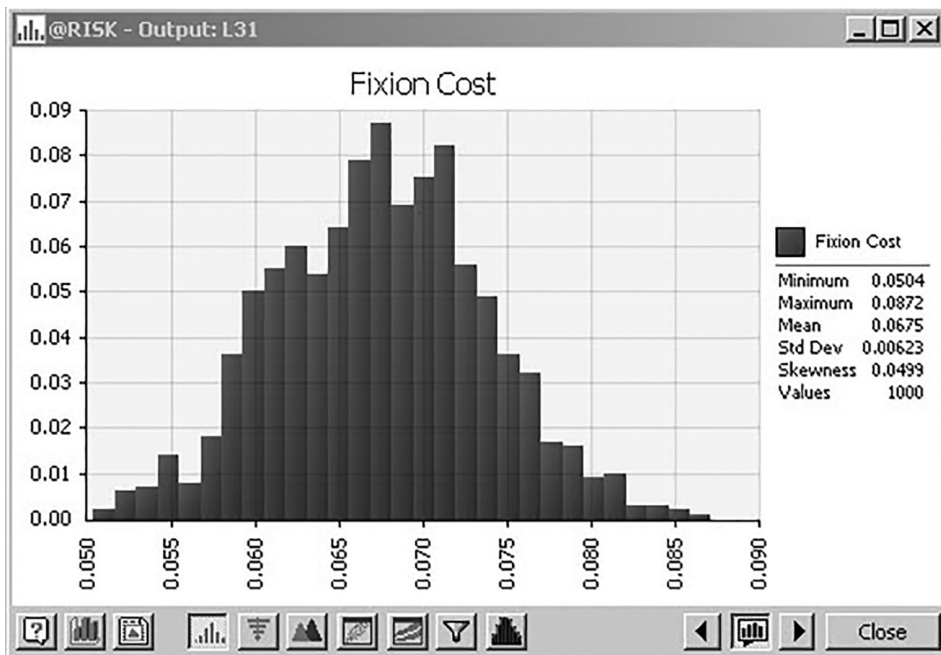


EXHIBIT 11.26 Frequency distribution of annual rate.

reduction in cost is that with the mean-reverting process, rates tend to drift downward if they are currently on the high end of their range. Because rates were high at the time of the swap, the counterparty with the floating rate is able to ride the rates downward. On the other hand, if rates had been low at the time of the swap, it is more likely that the floating rate borrower would end up with higher rates over the five years.

This section introduced the plain vanilla swaps, in which a fixed rate loan is swapped for a variable rate loan. The model we presented enables us to analyze the effects of the swap and see the distribution of possible interest costs for a borrower who undertakes the variable rate debt. This Monte Carlo model of borrowing rates and costs is helpful for understanding the costs and benefits of the swap. There is a wide variety of swaps and similar arrangements. This model merely scratches the surface. Still, it shows you how to begin developing a model that will analyze more complex situations.

11.4 SUMMARY

The purpose of this chapter was to show different ways to model the firm's debt. The first section developed a detailed model for tracking the debt in the context of a long-term planning model such as our ubiquitous O&R model. With this elaboration, the planning model can track the details of different debt issues so the user can explore the implications of alternative borrowing strategies.

The second section presented a Monte Carlo model for analyzing default risk and for determining the optimal capital structure. The Monte Carlo model allowed us to balance the tax savings and bankruptcy costs associated with the debt so as to determine the optimal amount of debt.

The last section of this chapter discussed two aspects of debt: duration and swaps. We introduced these topics to give you a starting point you should have to delve more deeply into the topics and build models to deal with these issues. Like so many topics in this text, we are only able to get you started. We leave it up to you to extend the models as required for your applications.

EXERCISES

1. Evaluating the Risk of Default for Laric Corporation

You are the financial manager of the Alaric Corporation. The CEO, Al Laric has stated that he would like you to determine the maximum amount of debt that the company could issue and not exceed certain limits on the risk of default on the debt. Specifically, he wants you to determine the maximum amount of long-term debt that could be issued and not exceed respectively, (a) 5% chance of bankruptcy; (b) 10% chance of bankruptcy; and (c) 20% chance of bankruptcy in 2021.

The current year end date is, 2020. Laric's financial statements through the end of 2020 are shown in [Exhibit 11.27](#). Dollar amounts are stated in thousands (\$000). For the purpose of making forecasts of Laric's future financial condition, the following assumptions are made.

Laric Corp					
Income Statement					
(\$ millions)					
Year Ended December 31:					
	2016	2017	2018	2019	2020
Sales	\$31,380.0	\$32,510.0	\$31,846.0	\$32,004.0	\$33,000.0
Cost of Goods Sold	22,466.0	23,665.0	22,458.0	22,193.0	22,800.0
Selling, General & Administrative	6,759.0	7,164.0	8,637.0	8,459.0	8,300.0
Other Inc/Exp	100.0	233.0	-465.0	-6.0	0.0
EBITDA	2,255.0	1,914.0	286.0	1,346.0	1,900.0
Dep	637.0	710.0	695.0	717.0	667.0
EBIT	1,618.0	1,204.0	-409.0	629.0	1,233.0
Interest	663.0	673.0	477.0	426.0	0.0
EBT	955.0	531.0	-886.0	203.0	1,233.0
Tax	361.0	195.0	-318.0	89.0	493.0
Net Income	594.0	336.0	-568.0	114.0	371.0
Extraordinary Items	0.0	0.0	-137.0	-16.0	0.0
Adjusted Net Income	594.0	336.0	-705.0	98.0	371.0
Dividend (Pfd + Cmn)	589.5	535.9	247.8	160.5	440.0
Pfd Dividend	38.0	36.0	33.0	29.0	0.0
Common Dividend	551.5	499.9	214.8	131.5	440.0
Retained Earnings	4.5	-199.9	-952.8	-62.5	300.0
Laric Corp					
Balance Sheet					
(\$ millions)					
Year Ended December 31:					
	2016	2017	2018	2019	2020
Assets					
Cash	\$511.0	\$1,233.0	\$944.0	\$2,840.0	\$2,474.0
Accounts Receivable	4,415.0	1,138.0	893.0	698.0	705.0
Inventory	6,031.0	5,947.0	5,269.0	4,930.0	4,945.0
Other CA	168.0	154.0	151.0	209.0	229.0
Current Assets	11,125.0	8,472.0	7,257.0	8,677.0	8,353.0
Gross Fixed Assets	8,333.0	8,195.0	8,062.0	8,317.0	8,154.0
Accum Depreciation	2,875.0	2,883.0	2,948.0	3,328.0	3,253.0
Net Fixed Assets	5,458.0	5,312.0	5,114.0	4,989.0	4,901.0
Other Assets	7,055.0	7,104.0	7,371.0	4,382.0	4,613.0
Total Assets	23,638.0	20,888.0	19,742.0	18,048.0	17,867.0
Liabilities					
Accounts Payable	1,496.0	1,480.0	1,948.0	1,551.0	1,792.0
Notes Payable	2,362.0	955.0	250.0	935.0	0.0
Other CL	2,112.0	2,030.0	2,037.0	2,013.0	4,477.0
Current Liabilities	5,970.0	4,465.0	4,235.0	4,499.0	6,269.0
Long Term Debt	7,143.0	5,844.0	5,448.0	5,179.0	0.0
Other LT Liabilities	3,356.0	3,351.0	3,800.0	2,241.0	0.0
Preferred Stock	475.0	446.0	399.0	363.0	0.0
Common Stock	2,850.0	3,266.0	3,294.0	3,330.0	3,423.0
Earned Surplus	3,844.0	3,516.0	2,566.0	2,436.0	8,175.0
Total Equity	6,694.0	6,782.0	5,860.0	5,766.0	6,037.0
Total Liabilities & Equity	23,638.0	20,888.0	19,742.0	18,048.0	17,867.0

EXHIBIT 11.27 Laric Corp. financial statements.

Laric's annual sales revenue is a normally distributed random variable that will be drawn *independently* from normal distributions with expected values and standard deviations as follows:

Year	2021	2022	2023	2024	2025
Expected sales	33,000	33,000	33,000	33,000	33,000
Standard deviation	3,000	3,000	3,000	3,000	3,000

Costs of Goods Sold (CGS), will consist of fixed and variable components. The variable cost is a random variable from a triangular distribution with a minimum value of 55% of Sales, a most likely value of 60%, and a maximum value of 65%. The fixed portion of CGS will be \$3,000 per year.

Selling, general and administrative expense (SGA) consists of a fixed component equal to \$5,000 per year plus a variable component that is a random variable from a uniform distribution that ranges from 8% of sales to 12% of Sales.

The Depreciation expense (a noncash expense) is equal to 8.5% of Gross Fixed Assets at the end of the previous year.

Interest expense is equal to 9% of the previous year's debt balance for both new and old debt.

The income tax rate is 40%.

The target Cash balance is 8% of expected Sales.

Receivables will be 2.2% of expected Sales.

Inventories will be 15.6% of expected Sales.

Gross Fixed Assets will be 25.5% of expected Sales.

Other Long-Term Assets will be equal to a fixed amount of \$1,400 plus a variable amount equal to 10% of expected sales.

Accounts Payable will be equal to 5.3% of expected Sales.

Other Current Liabilities will be equal to 14% of expected Sales.

All earnings and cash flow in excess of the amount necessary to maintain assets at their required levels and meet the current period's financial obligations are paid out as dividends to the stockholders.

At the end of the planning horizon (end of 2025), the firm will be liquidated. Liabilities will be paid from the proceeds from the sale of the assets, which will be sold for their book values. Any value in excess of the liabilities will be paid to the stockholders as a liquidating dividend. If the firm defaults and fails prior to the horizon, the stockholders receive nothing from that point forward, and the creditors receive the subsequent cash flow and proceeds from the asset sale at the end.

For purposes of testing different financial structures, assume a specified amount of debt is issued at the end of 2020. (i.e., you simply insert this amount of debt into the 2020 capital structure). The cash generated by this borrowing will be used, first to meet the firm's needs for external financing, and second, what is left will be paid to stockholders as a dividend. In other words, if issuing new debt at the end of 2020 results in excess cash, the excess cash will be used to pay an immediate dividend to the common stockholders at the end of 2020, or any other year when debt financing generates excess cash.

At the end of 2020, prior to issuing any new debt, there is no outstanding short-term or long-term debt. Any new long-term debt that is issued will be issued at the end of 2020. The debt issued at the end of 2010 will have a term to maturity of five years (i.e., it will mature at the end of 2025), and it will require 5 equal annual payments (principal + interest) at an interest rate of 9%. If any additional external financing is necessary after the end of 2020, it will be long-term debt at 9%, with equal annual payments (principal + interest) so as to be completely repaid by the end of 2025.

For purposes of this analysis, it is assumed that financial distress (bankruptcy) would occur if the *EBITDA* falls below the required debt service. *Debt service* is the sum of interest and principal (sinking fund) payments. Default is the event that $EBITDA < Debt\ Service$. The probability of default in a given year, t , having survived for $t - 1$ years, is

$$q_t = Pr(EBITDA_t < Debt\ Service_t)$$

The probability of surviving in year t is

$$p_t = (1 - q_t) = Pr(EBITDA_t \geq Debt\ Service_t)$$

The probability of surviving for t periods is

$$P_t = \prod_{j=1,t} p_j = \prod_{j=1,t} (1 - q_j)$$

and the probability of surviving for $t - 1$ periods and then failing in period t is

$$Q_t = P_{t-1} q_t$$

With no debt outstanding at the end of 2020, the first date that bankruptcy can occur is when the first debt service is due at the end of 2021.

The current five-year Risk-free interest rate is 3.5%, and this rate is expected to continue to prevail for the remainder of the planning horizon.

The debt issued for financial restructuring will be issued at the end of 2020. If there is new investment projected in any year, assume it occurs at the end of the year; and new financing necessary to meet growth needs in subsequent years will occur at the end of the year.

Assignment

Develop a spreadsheet model for Laric Corp. that will enable you to determine the probability of default associated with different levels of long-term debt. Find that the largest amount of long-term debt could be issued at the end of 2020 so as to yield probabilities of default of (a) 5%; (b) 10%; and (c) 20%, respectively, in year 2021. For each of these debt amounts, find the probability of defaulting in each subsequent year, 2022–2025. For each case, what is the probability that you will default at any time over the planning horizon? Note: Your estimate of probability of default should be based on running the Monte Carlo simulation and evaluating $\Pr(EBITDA_t < Debt\ Service_t)$ in each period.

2. Laric Corporation—Optimal Capital Structure

In Problem 1 you were asked to determine how much debt the company could incur and not exceed specified levels of default Risk. Now you are asked to use that same model to determine the company's optimal capital structure.

According to the widely accepted *trade-off model* of capital structure, the firm's optimal capital structure occurs at the mix of debt and equity where the *net benefits of debt* are maximized. In this context, the net benefits of debt are defined as

$$Net\ Benefits = E[PV(Tax\ Savings)] - E[PV(Bankruptcy\ Costs)]$$

$$NB = E[PV(TS)] - E[PV(BC)]$$

You can determine the optimal debt level by calculating the total *Net Benefits* (*NB*) for different levels of debt, and find the level of debt where the maximum is reached. In other words, you assume a level of debt and calculate *NB*. Then increase the assumed debt and recalculate *NB*. Keep doing this until you can graph the functional link between the amount of debt and *NB*.

To do this, make distribution assumptions just as you did in Problem 1, and assume

- Dividends are paid only if there are funds remaining after servicing the debt.
- Financial distress (bankruptcy) occurs if *EBITDA* falls below the required debt service. *Debt service* is defined as the sum of the interest payment and the principal payment on the debt.
- If bankruptcy occurs, bankruptcy costs will be incurred that will equal 15% of the face value of the outstanding debt.
- If bankruptcy occurs, the firm stops operating. However, in the period of bankruptcy, if the firm has any cash flow available, it pays what it has to the lenders. Interest is paid before principal. Assume that there are tax savings in that period equal to the tax rate x interest actually paid.
- Subsequent to bankruptcy, no further tax savings or bankruptcy costs will occur.

In the simulation, the value of tax savings is calculated as

$$PV(TS) = \sum_{t=1,n} TI_t (1 + k_d)^{-t}$$

And the value of bankruptcy costs is calculated as

$$PV(BC) = \sum_{t=1,n} BC_t (1 + k_d)^{-t}$$

for each single draw in the simulation. These are designated as the Output variables in @Risk.

The *means* of these value in the simulation are estimates of $E[PV(TS)]$ and $E[PV(BC)]$, respectively. If your model has a variable defined as

$$NB = PV(TS) - PV(BC)$$

as these items are calculated above, when you run the simulation, you get a frequency distribution of NB. The *mean* of this distribution is the estimate of the expected net benefit.

3. Swapping by Ying and Shuang

Ying Corporation and the Shuang Company each have loans that they are interested in swapping. Ying has a loan for \$200,000 that pays interest only for five years, with the principal due at the end of five years. The interest rate on this loan is fixed at 8% over the five years. Shuang has a variable rate loan that pays interest only for five years, with the principal due at the end of the five years. Shuang's variable rate is reset every six months, equal to the LIBOR plus 2% per year. The current LIBOR rate is 6.5%. If the swap is made, the interest paid at the end of each six-month period is based on the LIBOR at the start of the six-month period. The future LIBOR rate is unknown, but assume that the changes in the LIBOR follow an *elastic random walk* function of the form: $dr = 0.15(0.04 - r)dt + 0.005z(dt)^{1/2}$.

Assignment

Construct a spreadsheet model similar to [Exhibit 11.22](#) that will show the results of a potential swap. Your model should calculate the *IRR*'s for each loan. Using the random LIBOR function simulate with 5,000 draws, the interest rates over the next five years. Compute and show the frequency distribution of the *IRR*'s for the borrower who pays the variable rate in the swap.

Modeling Working Capital Accounts

THIS CHAPTER EXPLAINS SEVERAL different approaches to modeling the working capital accounts—the categories of accounts in current assets and current liabilities. The accounts considered include cash, marketable securities, receivables, inventories, and accounts payable.

If the purpose of the simulation is to analyze the effects of broad strategic decisions over a long planning horizon, then the details of working capital are unimportant. Only the general details of working capital would have to be modeled to pursue the implications of long-run policies. With a long planning horizon comprised of long subperiods, any seasonal variation in current assets and liabilities disappears. Generally, the longer the horizon, the less the need to distinguish between the component accounts of working capital. Thus, for longer run planning purposes, it is usually satisfactory to let the balances in the current asset and current liability accounts be directly proportional to sales, and even to lump all these accounts together as we did in the Odd & Rich model.

However, as the planning horizon and the planning periods are shortened, greater detail may be needed in modeling current asset and liability accounts. Indeed, one of the most useful applications of financial simulation models is to explore short-run decisions that affect seasonal current balances, so it is necessary to build much more detailed accounting for current accounts. This is done by increasing the number of accounts and variables, and more carefully linking current assets and liabilities to the sales forecast.

A sales-driven model such as the percent-of-sales approach is particularly appropriate for working capital because the primary motive for most of the current accounts is to meet the needs of sales. Inventory is a good example because it is normally held solely to meet the requirements of future sales. However, this statement itself points out one of the difficulties of modeling working capital investment: Should working capital balances in the model be related to current sales, past sales, or future sales? Other questions include whether working capital investment should be strictly proportional to sales, and what degree of detail

should be built into the working capital sector. We will consider these questions as we discuss the various working capital categories.

12.1 CASH

Our choice of the way to model the cash account depends on the type of model, the length of the subperiods and planning horizon, and, of course, the purpose of the modeling effort. If the subperiods and the planning horizon are long and the purpose of the planning model is to address very broad planning issues, then one would not be overly concerned with specific details of the balances of liquid assets. In such a case, it is usually appropriate to combine cash and marketable securities in a single account and use the percent-of-sales approach for modeling the desired balance. *Cash* normally is held to meet the transaction requirements that are generated by the firm's production and sales, so that the percent-of-sales approach fits the situation, and the cash balance held at the end of period t could be modeled as

$$\text{Cash}_t = C * \text{Sales}_t \quad (12.1)$$

where parameter C is obtained from historical data or is based on the anticipated policy for the planning period.

An alternative approach under the same circumstances is to relate the balance of liquid assets to the stock of *Total Assets*. *Total Assets* would depend on sales, and cash would be a constant proportion of *Total Assets*:

$$\text{Cash}_t = C * \text{Total Assets}_t \quad (12.2)$$

However, one must be aware of the way that *Total Assets* are computed in the model to avoid the problem of simultaneity. In other words, if the balance in *Cash* must be known to calculate *Total Assets*, then the model has simultaneous equations if the amount of *Total Assets* must also be known to calculate the *Cash* balance.

The approach to modeling the cash balance also depends on whether the planning model is a *stock balancing* or a *flow balancing* model. In the standard stock balancing model that is driven by the sales forecast, the sales forecast leads to the required asset balances, and the liability balances are adjusted to equal total assets. In this case, it is permissible to have the cash balance adjust to the forecast of sales or total assets independently from the balancing mechanism for the system that forces liabilities to equal assets.

However, in flow balancing models, it cannot be assumed that the liquid assets are a constant proportion of either sales or *Total Assets*. This is because in the typical flow balancing model, the mechanism for maintaining equilibrium involves adjusting the cash and liquid asset account. The equation that balances the system is of the general form:

$$\text{Net Change in Cash Balances}_t = \text{Net Cash Inflow}_t \quad (12.3)$$

One advantage of the flow balancing approach is that it helps avoid the problem of cash disappearing into a financial black hole. When we discussed balancing in reference to the Odd & Rich example, we noted the possible problem of the model generating cash

that does not seem to flow either to the cash balance or to stockholders in the form of dividends. When this happens, we have cash flowing into the firm but not showing up in any asset account. This is what we call a financial black hole. If our model clearly accounts for all the cash flow, we avoid the black hole. The flow balancing approach is better for accounting for all the cash flow. Moreover, a flow balancing model with cash as the balancing account, well suited to a planning problem where the planning horizon and the subperiods are short and greater detail about cash and marketable security accounts is needed.

For the cash flow balancing model to function properly, the model must be equipped with some decision rules that tell it how to balance the system in a way that realistically reflects management's policies. In the absence of such instructions, there is nothing to prevent the cash balance getting excessively large or small, or even negative. The kinds of decision rules necessary are instructions that state the desired balance of cash and what should be done if the actual balance departs from the desired balance. [Exhibit 12.1](#) shows an example of a cash balance sector for short-term planning model that is based on the cash flow equations of [Exhibit 4.2c](#). In developing the expressions in [Exhibit 12.1](#), the assumptions and policies are as follows:

- a. The firm will maintain its cash balance at a level assigned to the policy variable Desired Cash.
- b. Any cash in excess of Desired Cash that builds up from the firm's operations is invested in marketable securities.
- c. If operations generate a cash outflow so that the cash balance would be less than Desired Cash, the cash balance is restored to the desired level by first, selling marketable securities, and second by borrowing.
- d. If security sales are necessary, the amount of Securities Sold is limited to the amount of securities in the portfolio at the beginning of the period, *Marketable Securities*_{*t*-1}.
- e. If Cash Deficit exceeds the amount that can be obtained by selling securities, then the remainder of Cash deficit is financed by short-term borrowing.
- f. In reference to Net Working Capital Investment, Other Current Asset Investment excludes cash and marketable securities, and Spontaneous Financing refers to current liabilities other than short-term borrowing.
- g. Cash and security investments are the residual in the model, and dividends are based on a target payout ratio.

The equations in [Exhibit 12.1](#) provide the model with a mechanism for managing the cash account with the target Desired Cash. This target level of cash is not generated in the model; rather, it is specified by the user as a policy parameter. The target cash balance is influenced by needs such as compensating balances required by the bank, anticipated cash payments, and emergency requirements. There are several ways that modeling can help us

Cash Flow from Operations _t	=	Earnings Cash Flow _t – NWC Investment _t
Source of Cash _t	=	Cash Flow from Operations _t – Capital Expenditures _t + Capital Financing _t – Dividend _t + Securities Mated _t
Trial Cash Balance _t	=	Cash _{t-1} + Source of Cash _t
Trial Excess Cash _t	=	Trial Cash Balance _t – Desired Cash _t
Excess Cash _t	=	Maximum(0, Trial Excess Cash _t)
Cash Deficit _t	=	0 – Minimum(0, Trial Excess Cash _t)
Securities Purchased _t	=	Excess Cash _t
Securities Sold _t	=	Minimum (Cash Deficit _t , Marketable Securities _{t-1} – Securities Mated _t)
Short Term Borrowing _t	=	Maximum(0, Cash Deficit _t – Securities Sold _t)
Net Cash Flow _t	=	Trial Source of Cash _t – Securities Purchases _t + Securities Sold _t + Short Term Borrowing _t
Cash _t	=	Cash _{t-1} + Net Cash Flow _t

EXHIBIT 12.1 Cash balance adjustment sector.

to determine the minimum or target level for cash, including simulation to find a comfortable cash target or build optimization algorithm that solves for the best cash target.

A priori, we may not know the appropriate level of the Desired Cash Balance. We can get a better idea by specifying a Desired Cash balance and then exploring different scenarios for the inflows and outflows and tracing out the implications of different cash target levels. If too little cash is maintained, the firm will have to liquidate securities or borrow too frequently, and its transaction costs will be excessive. If too much cash is maintained, the transactions are avoided, but the firm foregoes the income it could have earned if the funds were invested.*

There are two ways to simulate: scenario simulation and Monte Carlo simulation. With scenario simulation, for a given policy, we simply try out different cash flow scenarios to trace out the results for each scenario. Then, we change the policy and iterate over the different cash flow scenarios again. We compare the results from the different policies and determine which one yields the best results. With Monte Carlo simulation, we draw the cash flow inputs as random variables from probability distributions, and see the results of a given policy as a probability, or frequency, distribution of outcomes. We evaluate the resulting probability distributions to see which policy yields the most desirable result.

Another approach to determining the appropriate cash balance has been to borrow from the techniques of inventory theory to derive decision rules regarding the inventory of cash. For example, Miller and Orr (1966) present a model for managing cash balances that allows the cash balance to wander randomly between an upper limit, H , and a lower limit, L , and when the cash balance hits either the upper or lower limit, actions are taken to restore the balance to a restoration level, Z . The limits and the restoration points are determined by a formula that considers the parameters of the probability distribution that describes the random daily cash flows, the transaction cost of restoring the cash balance, and the interest rate that represents the opportunity cost of holding funds as cash.† With the Miller and Orr

* Cash management is explained in greater detail in Kallberg and Parkinson (1996), Masson and Wikoff (1995), Smith (1979), and Orgler (1970).

† The Miller and Orr formula is $Z = (3b\sigma^2/4r)^{1/3}$, and, $H = 3Z$, $L = 0$, where b is the fixed dollar cost of a transaction, r is the interest rate per day, and σ^2 is the variance of the random, normally distributed daily cash flows.

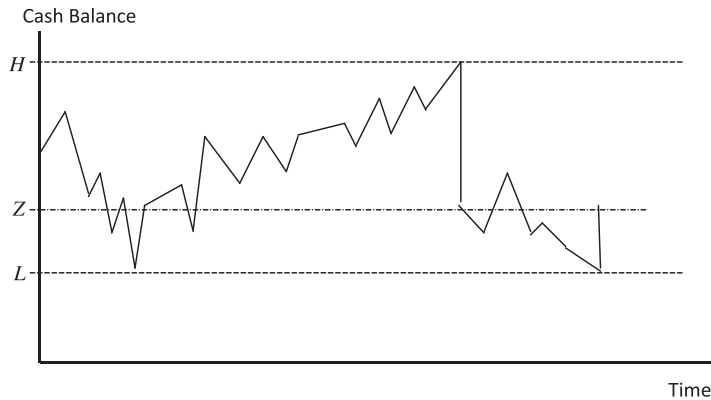


EXHIBIT 12.2 Cash balances using the Miller and Orr model.

Cash Flow from Operations _t	= Earnings Cash Flow _t – NWC Investment _t
Source of Cash _t	= Cash Flow from Operations _t – Capital Expenditures _t + Capital Financing _t – Dividend _t + Securities Matured _t
Trial Cash Balance _t	= Cash _{t-1} + Source of Cash _t
Z	= $3 * B * \text{Cash Flow Variance} / 4 * \text{Interest Rate}$
H	= $3 * Z$
L	= 0
Excess Cash _t	= Maximum(0, Trial Cash Balance _t – H)
Cash Deficit _t	= Maximum(0, L – Trial Cash Balance _t)
Securities Purchased _t	= Trial Cash Balance _t – Z, IF(Excess Cash _t ≥ 0) = 0 otherwise
Securities Sold _t	= Minimum(Z – Trial Cash Balance _t , Marketable Securities _{t-1}), IF(Cash Deficit _t > 0) = 0 otherwise
Borrowing _t	= Minimum(Z – Trial Cash Balance _t – Securities Sold _t , 0), IF(Cash Deficit _t > 0)
Net Cash Flow _t	= Source of Cash _t – Securities Purchased _t + Securities Sold _t + Borrowing _t
Cash _t	= Cash _{t-1} + Net Cash Flow _t

EXHIBIT 12.3 Equations for a cash management model using the Miller and Orr policy.

(H, Z, L) policy, the cash balance would follow the pattern shown in [Exhibit 12.2](#), where securities are purchased when the cash balance hits the upper limit H to restore the balance to Z , and securities are sold (or funds borrowed) when the lower limit L is breached. The Miller and Orr cash management decision rules could be built into a short-run planning model such as [Exhibit 12.1](#) with the expressions shown in [Exhibit 12.3](#). The parameters for the limit formulas would be estimated from a study of the firm's operating cash flows. In the model described by [Exhibit 12.3](#), it is assumed that the costs of borrowing are identical to the rate earned on marketable securities.

12.2 MARKETABLE SECURITIES

If the planning horizon and the subperiods making up the horizon are long, there is a little need to distinguish between cash and marketable securities; they can be combined in a liquid asset category for purposes of analyzing a long-term plan, and a straightforward percent-of-sales approach can be used. On the other hand, if the planning horizon is short and the purpose of the model is to consider the effects of short-run investment strategies, then more detailed modeling of marketable securities would be appropriate.

For the longer run model, the percent-of-sales approach might specify marketable securities as

$$\text{Marketable Securities}_t = M * \text{Sales}_t \quad (12.4)$$

where M is a parameter based either on historical relationships or on anticipated policy. Alternatively, securities could be specified as a percent of assets as in

$$\text{Marketable Securities}_t = M * \text{Total Assets}_t \quad (12.5)$$

but care must be taken to avoid simultaneity.

For short run models focusing on working capital management, the details of the securities accounts are of concern and more detailed modeling is required. A good starting point for modeling securities is simply to track the balance with an expression such as

$$\begin{aligned} \text{Marketable Securities}_t = & \text{Marketable Securities}_{t-1} - \text{Securities Matured}_t \\ & - \text{Securities Sold}_t + \text{Securities Purchased}_t \end{aligned} \quad (12.6)$$

where the balances are stated in terms of face value of the securities. We only need to expand on the components of this expression to have a complete model of marketable securities.*

Exhibit 12.4 shows expressions that would be developed to model security transactions over a short planning horizon. So that we can show the relationships concisely, we will introduce some notation—with apologies to the less mathematically inclined readers. Two subscripts will be used: the first subscript will denote the period in which a transaction occurs, and the second subscript will denote the period in which a security matures. For example, SP_{ij} is the amount of securities purchased in period t that will mature in period j . This is stated in terms of the face, or maturity, value of the security. SS_{ij} denotes the face value of securities sold in period t that mature in period j . The index 0 refers to securities held at the beginning of the planning horizon, and h denotes securities that mature at or beyond the end of the horizon. The expression at the bottom of **Exhibit 12.4**, for $t = 1, \dots, h - 1$, means that for each period in the planning horizon, there is an expression corresponding to (1) through (7).

* A model of short-term security transactions in the context of a linear programming model for working capital management is presented in Orgler (1970). A simplified example of Orgler's model is presented in **Chapter 17**.

$$\begin{aligned}
 (1) \text{ Securities Purchased}_t &= \sum_{j=t+1,h} SP_{tj} \\
 (2) \text{ Securities Sold}_t &= \sum_{j=t+1,h} SS_{tj} \\
 (3) \text{ Securities Matured}_t &= \sum_{i=0,t-1} SP_{it} - \sum_{i=1,t-1} SS_{it} \\
 (4) \text{ Security Balance}_t &= \text{Security Balance}_{t-1} + \text{Securities Purchased}_t \\
 &\quad - \text{Securities Sold}_t - \text{Securities Matured}_t \\
 (5) \text{ Marketable Securities}_t &= \sum_{i=0,t-1} \sum_{j=t+1,h} a_{j-t} SP_{ij} \\
 &\quad - \sum_{i=1,t-1} \sum_{j=t+1,h} a_{j-t} SS_{ij} \\
 (6) \text{ Cash Flow from Securities}_t &= \text{Securities Matured}_t - \sum_{j=t+1,h} a_{j-t} SP_{tj} \\
 &\quad + \sum_{j=t+1,h} a_{j-t} SS_{tj} \\
 (7) \text{ Security Income}_t &= \text{Securities Matured}_t - \sum_{i=0,t-1} a_{t-i} (SP_{it} - SS_{it}) \\
 &\quad + \sum_{j=t-1,h} (1 - a_{j-t}) SS_{tj} \\
 &\quad \text{for } t = 1, \dots, h-1.
 \end{aligned}$$

Note: Decision variables SP_{tj} , SS_{tj} , and the a summary balances are stated in terms of face value for discount securities. The coefficients, a_{t-j} , represent the discounted value per dollar of face value for a security maturing in $t-j$ periods.

EXHIBIT 12.4 A model for short-term marketable securities.

For this model it is assumed that the securities are short-term discount securities, such as Treasury bills, that are bought at a discount from face value, and the increase in value from purchase until sale or maturity is interest income, and is shown as *Security Income_t*. The total values of *Securities Purchased_t*, *Securities Sold_t*, *Securities Matured_t*, and the portfolio balance, *Security Balance_t*, are all stated in terms of face, or maturity, value. To convert from face value to market value, we define the coefficient $a_{t-j} = 1/(1 + r_{t-j})$ as the present value factor that converts the face (maturity) value of securities to the discounted value at time t , with r_{t-j} representing the interest rate for $t-j$ periods. For example, $a_{5-3} = a_2$ is the present value for the two periods from period 3 to period 5. Suppose securities are purchased at time 3 to mature at time 5 with a maturity value of \$1,000. With an interest rate of 1% per period, the amount spent for the purchase at time 3 is

$$a_{3,5} * SP_{35} = \left(\frac{1}{1.01} \right)^2 SP_{35} = (0.9803)1000 = 980.30 \quad (12.7)$$

Expressions (1) through (4) of [Exhibit 12.4](#) total the face values of securities purchased, sold, matured, and held. Expression (1) says the face value of securities purchased in period t is the sum of all securities purchased that period that mature in subsequent periods. Expression (2) totals the sales of securities that mature in future periods that are sold in period t . Expression (3) says that the face value of securities matured in period t is the sum of all past purchases that would mature in t , less the amount of securities sold in past periods that mature in period t . Expression (5) gives the market value of the security portfolio at the end of each period t based on all past purchases and sales; and (6) shows the cash flowing in and out of the portfolio each period. The income expression (7) calculates

income in terms of the gain from the discounted purchase price to the amount realized on the sale or maturity of the security.

Now we will see how to implement the equations shown in [Exhibit 12.4](#).

12.2.1 Example: Managing Securities for the Mogul Corporation

The Mogul Corporation has a portfolio of cash and short-term securities. In managing the cash and securities, the objective is to maximize value of cash & securities held at the end of the 3-month planning horizon. The planning horizon is divided into 1-month time intervals. At the start of the planning period, the firm has an initial balance of cash and an endowment of securities as follows:

Cash Balance	\$5,000		
Marketable Securities			
Maturity Date	Month 1	Month 2	Month 3
Face Value	\$1,000.00	\$2,000.00	\$500.00
Market Value	\$990.10	\$1960.59	\$485.30

In addition to holding the initial balance of cash & securities, the following cash flows are expected in each month:

	Month 1	Month 2	Month 3
Expected Cash Flow	\$1,000	-\$3,000	\$500

The cash inflow will be available to be invested, and the cash outflow of \$3,000 must be funded from the cash balance or from the security portfolio.

Securities are traded at prices that are discounted from face, or maturity value. The current and expected future short-term interest rates that we use to discount the face value to the market value of the securities are shown in the table below. The number in the first column indicates the month in which the security is purchased, and the numbers in the top row indicate the month the security matures. So, a security purchased in month 1 that matures in month 3 is expected to have a monthly interest rate of 1.2%. Securities purchased in period 0 refers to securities already held at the start of the planning horizon. Period 4 represents securities maturing after the end of the 3-month planning horizon. We will assume period 4 is 1 month beyond the end of the planning horizon.

Monthly Interest Rates for Marketable Securities

Period Security is Purchased	Period Security Matures			
	1	2	3	4
0	1.0%	1.0%	1.0%	1.0%
1		1.1%	1.2%	1.3%
2			1.3%	1.4%
3				1.4%

Our task is to construct a spreadsheet model to track our security portfolio for 3 months so we can see the results of alternative strategies for buying and selling securities. The decisions we can make are how much to invest in securities each month, which month the securities will mature, and, when, how many securities, and of which maturity should we sell? We can purchase securities at times 1, 2, and 3 to mature at any subsequent date up to time 4. The purchase price of the security is the present value of the face value of the security. Thus, if, in month 1 we purchase a security that matures in month 3 at face value of \$1,000, the purchase price (market value) will be

$$\begin{aligned}
 \text{Purchase Price} &= \text{Market Value} \\
 &= 1,000 \left(\frac{1}{1.012} \right)^2 \\
 &= 1,000(0.97643) \\
 &= \$976.43
 \end{aligned}
 \tag{12.8}$$

If we hold the security for the 2 months until it matures, we will receive the face value of \$1,000 at the end of month 3. The income will be the appreciation in price amounting to

$$\begin{aligned}
 \text{Income} &= 1,000 - 976.43 \\
 &= \$23.57
 \end{aligned}
 \tag{12.9}$$

The security can be sold prior to maturity. If this security is sold at time 2, the sale price will be

$$\begin{aligned}
 \text{Sale Price} &= 1,000 \left(\frac{1}{1.013} \right) \\
 &= 1,000(0.98717) \\
 &= \$987.17
 \end{aligned}
 \tag{12.10}$$

The income earned over the 1 month holding period would be

$$\begin{aligned}
 \text{Income} &= 987.17 - 976.43 \\
 &= \$10.74
 \end{aligned}
 \tag{12.11}$$

The discount rate used above is the 1.3% that is shown in the table as the rate for securities purchased at $t = 2$ to mature at $t = 3$.

In this example, the portfolio is given at $t = 0$, and we can make purchase decisions at times 1, 2 and 3, with time 4 being beyond the horizon. The six purchase decision variables are enumerated as follows:

Security Purchase Decision Variables			
Period of Purchase	Period That Security Matures		
	2	3	4
1	SP_{12}	SP_{13}	SP_{14}
2		SP_{23}	SP_{24}
3			SP_{34}

The six security sales decision variables are:

Security Sale Decision Variables			
Period of Sale	Period That Security Matures		
	2	3	4
1	SS_{12}	SS_{13}	SS_{14}
2		SS_{23}	SS_{24}
3			SS_{34}

To analyze the implications of different security transactions, we have constructed a spreadsheet model that tracks the security transactions, keeping track of purchases, sales, security balances, and earnings from the portfolio. [Exhibit 12.5a](#) shows the model with the decisions inserted that will be explained shortly. [Exhibit 12.5b](#) shows selected key formulas for the model. Now, we take a brief tour of the model.

In [Exhibit 12.5](#) the input data describing the endowment of cash and securities, the expected exogenous cash flow, and expected future interest rates is in block A5:G25. The data input by the model user includes:

Input Data	Cells
Initial cash balance	C10
Expected cash inflow	D14:G14
Beginning Security Balances (face value)	D16:G16
Expected Interest Rates	D21:G24

The user's security purchase and sale decisions are input in block A27:G45. Specifically, the decision variables are entered in:

Input Decision Variables	Cells
Security Purchases, SP_{ij}	D34:G37
Security Sales, SS_{ij}	E43:G45

where the amounts inserted are the face values of the securities to be purchased or sold at each date. The market values of the security purchase and sales amounts are calculated in the block J30:P45, to the right of the input fields. The market values are calculated using the present value factors shown in J18:O25 that are based on the monthly interest rates input by the user in A18:G24. Generally, the values on the left side of the model (columns A–H) are in face value (FV) terms, and the right side of the model (columns J–P) shows the market values (MV).

The FVs of the securities that mature at the end of each period are shown in D50:G50. The balances (FV) of securities held at the end of each period are shown in D55:H58. For example, cell E56 is the amount of securities held at the end of period 1 that mature in period 2, and F56 is the amount held at $t = 1$ that will mature at $t = 3$. H55:H58 shows the total amount of securities held at the end of each period. The corresponding MV is shown on the right side in J51:P58.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
2	Mogul Corporation															
3	Marketable Security Portfolio															
4																
5																
6	Input Data															
7	Enter initial data in this section															
8	Peri	0	1	2	3	4										
9																
10	Cash	\$5,000.00														
11	Marketable Securities (3,435.99	Do not enter data here													
12	Cash & Securities	8,435.99														
13																
14	Cash Inflow		1,000.00	-3,000.00	500.00	0.00										
15																
16	Securities Maturing	(face value)	1,000.00	2,000.00	500.00	0.00										
17																
18	Interest Rate															
19	Maturity Period		1	2	3	4										
20	Purchase Period															
21		0	0.010	0.010	0.010	0.010										
22		1		0.011	0.012	0.013										
23		2			0.013	0.014										
24		3				0.014										
25																
26																
27	User Input: Decisions															
28																
29	Enter purchases & sales in this section															
30	except the initial portfolio (time 0)															
31	Security Purchases	Face Value														
32	Maturity Period		1	2	3	4										
33	Purchase Period															
34		0	1,000.00	2,000.00	500.00	0.00	0	990.10	1,960.59	485.30	0.00	3,435.99				
35		1		1,000.00	4,000.00	500.00	1	989.12	3,905.70	481.00	5,375.82					
36		2			1,500.00	0.00	2		1,480.75	0.00	1,480.75					
37		3				500.00	3			493.10	493.10					
38																
39	Security Sales	Face Value														
40	Maturity Period		1	2	3	4										
41	Sale Period															
42		1		0	0	0	1		0.00	0.00	0.00	0.00				
43		2			500	0	2			493.58	0.00	493.58				
44		3				0	3				0.00	0.00				
45																

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
47	Securities Maturing															
48																
49	Maturity Period		1	2	3	4										
50			1,000.00	3,000.00	5,500.00	1,000.00										
51	Security Balance															
52																
53	Maturity Period		1	2	3	4	Total FV	Maturity Period		1	2	3	4	Total MV		
54	Period Held							Period Held								
55		0	1,000.00	2,000.00	500.00	0.00	3,500.00	0	990.10	1,960.59	485.30	0.00	3,435.99			
56		1	0.00	3,000.00	4,500.00	500.00	8,000.00	1	2,967.36	4,393.91	481.00	7,842.27				
57		2		0.00	5,500.00	500.00	6,000.00	2		5,429.42	486.29	5,915.71				
58		3			0.00	1,000.00	1,000.00	3			986.19	986.19				
59																
60	Cash Flow															
61			1	2	3											
62			-3,375.82	-987.17	5,506.90											
63	Security Income															
64			1	2	3											
65			9.90	58.58	123.40											
66	Security Accounting															
67																
68	Sales of Securities Purchased in Period 0															
69	Maturity Period		1	2	3	4										
70																
71	Sale Period			0.00	0.00	0.00	1	##	2,000.00	500.00	0.00					
72						0.00	2		2,000.00	500.00	0.00					
73					500.00		3			0.00	0.00					
74						0.00		3			0.00					
75																
76	Sales of Securities Purchased in Period 1															
77	Maturity Period		1	2	3	4										
78																
79	Sale Period					0.00	1		1,000.00	4,000.00	500.00					
80						0.00	2			4,000.00	500.00					
81						0.00	3				500.00					
82																
83	Sales of Securities Purchased in Period 2															
84	Maturity Period		1	2	3	4										
85																
86	Sale Period					0.00	1			1,500.00	0.00					
87							2				0.00					
88							3				0.00					

EXHIBIT 12.5 (a) Spreadsheet model for marketable securities.

(Continued)

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	J	K	L	M	N	O	P
13	Objective						
14	Cash & Security Value at Horizon:				=N63		
15							
16							
17							
18	Present Value Factors						
19	Maturity Period						
20	Purchase Period	1	2	3	4		
21	0	=1/(1+D21)	=1/(1+E21)^2	=1/(1+F21)^3	=1/(1+G21)^4		
22	1		=1/(1+E22)	=1/(1+F22)^2	=1/(1+G22)^3		
23	2			=1/(1+F23)	=1/(1+G23)^2		
24	3				=1/(1+G24)		
25							
26							
27							
28							
29							
30	Security Purchases						
31	Market Value						
32	Maturity Period	1	2	3	4		Total MV
33	Purchase Period						0
34	0	=(1/(1+D21))*D34	=(1/(1+E21)^2)*E34	=(1/(1+F21)^3)*F34	=(1/(1+G21)^4)*G34		=SUM(L34:O34)
35	1		=E35*M22	=F35*N22	=G35*O22		=SUM(M35:O35)
36	2			=F36*N23	=G36*O23		=SUM(N36:O36)
37	3				=G37*O24		=SUM(O37)
38							
39	Security Sales						
40	Market Value						
41	Maturity Period	1	2	3	4		Total MV
42	Sale Period						
43	1		=E43*M22	=F43*N22	=G43*O22		=SUM(M43:O43)
44	2			=F44*N23	=G44*O23		=SUM(M44:O44)
45	3				=G45*O24		=SUM(M45:O45)
46							
47							
48							
49							
50							
51	Security Balance						
52	Market Value						
53	Maturity Period	1	2	3	4		Total MV
54	Period Held						
55	0	=L21*D55	=M21*E55	=N21*F55	=O21*G55		=SUM(L55:O55)
56	1		=E56*M22	=F56*N22	=G56*O22		=SUM(L56:O56)
57	2			=F57*N23	=G57*O23		=SUM(L57:O57)
58	3				=G58*O24		=SUM(L58:O58)

	A	B	C	D	E	F	G	H
47	Securities Maturing							
48			Maturity Period	1	2	3	4	
49				=D16	=E16+E35-E43	=F16+F35+F36-F43-F44	=G16+G35+G36+G37-G43-G44-G45	
50								
51	Security Balance							
52			Maturity Period	1	2	3	4	Total FV
53								
54		Period Held						
55		0		=D16	=E16	=F16	=G16	=SUM(D55:G55)
56		1		=D55-D50	=E55+E35-E43	=F55+F35-F43	=G55+G35-G43	=SUM(D56:G56)
57		2			=E56-E50	=F56+F36-F44	=G56+G36-G44	=SUM(D57:G57)
58		3				=F57-F50	=G57+G37-G45	=SUM(D58:G58)
59								
60	Cash Flow							
61				1	2	3		
62				=D14+D50-P35+P43	=E14+E50-P36+P44	=F14+F50-P37+P45		
63	Security Income							
64				=(D34-L34)+E72*(M22-M21)+F72*(N22-N21)+G72*(O22-O21)				
65				=D34-L34)+E72*(M22	=E34-M34)+(E35-M33	=F34-N34)+(F35-N35)+(F		
66	Security Accounting							
67				=(E34-M34)+(E35-M35)-E72*(M22-M21)+F73*(N23-N21)+F80*(N23-N22)+G73*(O23-O21)+G80*(O23-O22)				
68	Sales of Securities Purchased in Period 0							
69			Maturity Period	1	2	3	4	
70								
71		Sale Period						
72		1			=MIN(E43,E55)	=MIN(F43,F55)	=MIN(G43,G55)	
73		2				=MIN(F44,M72)	=MIN(G44,N72)	
74		3					=MIN(G45,N73)	

EXHIBIT 12.5 (Continued) (b) Formulas for selected parts of the Mogul model.

(Continued)

	I	J	K	L	M	N
60	Cash & Security Balance		1		2	3
61		Cash		=C10+D14+D50-P35+P43	=L61+E14+E50-P36+P44	=M61+F14+F50-P37+P45
62		Securities (MV)		=P56	=P57	=P58
63		Cash & Securities		=L61+L62	=M61+M62	=N61+N62
64						
65						
66						
67						
68	Security Balance After Sale: Purchased @ 0					
69		Maturity Period	1	2	3	4
70	End of Period					
71	0		=D16	=E16	=F16	=G16
72	1			=L71-E72	=M71-F72	=N71-G72
73	2				=M72-F73	=N72-G73
74	3					=N73-G74
75						
76	Security Balance After Sale: Purchased @ 1					
77		Maturity Period	1	2	3	4
78	End of Period					
79	1		=E35		=F35	=G35
80	2				=M79-F80	=N79-G80
81	3					=N80-G81
82						
83	Security Balance After Sale: Purchased @ 2					
84		Maturity Period	1	2	3	4
85	End of Period					
86	2				=F36	=G36
87	3					=N86-G87

EXHIBIT 12.5 (Continued) (b) Formulas for selected parts of the Mogul model.

The cash flow generated each period is calculated in line 61. The cash flow is calculated according to expression (6) in Exhibit 12.4, taking account of the exogenous cash flow, maturing securities, purchases and sales.

The total balance of cash and securities (MV) is tracked in J61:N63. The objective is to maximize the value of the cash and security balance at the end of the horizon, time 3. Cell N63 is the value to be maximized. It is reflected at the top right of the model in N14 as the objective.

The income earned on the security portfolio in each period is shown in D64:F64. *Income* is calculated according to (7) in Exhibit 12.4 as to total increase in security values from purchase price to maturity or sale price. For example, the income earned in period 2 is calculated as

$$\begin{aligned}
 Income_2 = & (SP_{02} - a_{02}SP_{02}) + (SP_{12} - a_{12}SP_{12}) - (a_{12} - a_{02})SS_{12}^0 \\
 & + (a_{23} - a_{03})SS_{23}^0 + (a_{23} - a_{13})SS_{23}^1 + (a_{24} - a_{04})SS_{24}^0 + (a_{24} - a_{14})SS_{24}^1
 \end{aligned} \tag{12.12}$$

where SS_{2j}^n denotes sales in period 2 of securities maturing in period j that were purchased in period n . In spreadsheet formula in cell E64 is

$$\begin{aligned}
 E64 = & (E33 - M34) + (E35 - M35) - (M22 - M21) * E72 + (N23 - N21) * F73 \\
 & + (N23 - N22) * F80 + (O23 - O21) * G73 + (O23 - O22) * G80
 \end{aligned} \tag{12.13}$$

This calculates income period 2, taking account of (a) the gain on securities purchase in periods 0 and 1 that mature in 2:

$$\begin{aligned}
 Income_2 = & (SP_{02} - a_{02}SP_{02}) + (SP_{12} - a_{12}SP_{12}) \\
 = & (E34 - M34) + (E35 - M35)
 \end{aligned} \tag{12.14}$$

offset by (b) securities that should have matured in 2, but were sold in 1:

$$-(a_{12} - a_{02})SS_{12}^0 = -(M22 - M23)E*72 \tag{12.15}$$

plus (c) the gain on securities sold in 2 that would mature in later periods:

$$\begin{aligned} &(a_{23} - a_{03})SS_{23}^0 + (a_{23} - a_{13})SS_{23}^1 + (a_{24} - a_{04})SS_{24}^0 + (a_{24} - a_{14})SS_{24}^1 \\ &= (N23 - N21) * F73 + (N23 - N22) * F80 + (O23 - O21) * G73 \\ &\quad + (O23 - O22) * G80 \end{aligned} \tag{12.16}$$

Similar expressions apply to periods 1 and 3.

The security accounting block, A66:N87 tracks securities purchased in each period until they are sold or mature. It is assumed they are sold on a first in—first out basis. The reason this tracking is necessary is that we need to know when and at what price a security was purchased so we can calculate the gain on its sales. To see how this works, consider cell F73, the sales in period 2 of securities maturing in 3, which were originally purchased in period 0, SS_{23}^0 . The formula in F73 is

$$SS_{23}^0 = \text{MIN} \left(\text{Total } SS_{23}, \left[\begin{array}{l} \text{Balance at } t = 1 \text{ of securities maturing} \\ \text{at } t = 3 \text{ that were purchased at } t = 0 \end{array} \right] \right) \tag{12.17}$$

coded as

$$F73 = \text{MIN}(F44, M72)$$

where F44 = total purchases in at $t = 2$ to mature in 3, SS_{23} , and M72 = Balance at the end of the last period, $t = 1$, of securities maturing at $t = 3$, that were purchased at $t = 0$. This balance is tracked for each period's originating portfolio until it either matures or is completely sold.

Clearly, we do not want to buy securities if we do not have the money, nor do we want to sell securities that we do not own. Consequently, we need to include constraints on our decisions or put some warnings in the model so the user will know that an infeasible decision is being made. This model has a warning to the user.

Expressions in C35:C37 and C43:C45 test for feasibility and provide a warning when the security purchase and security sale decisions are infeasible. For the security purchase decisions, the budget constraint in each period t is

$$\left[\begin{array}{l} \text{Total Security} \\ \text{Purchase}_t \end{array} \right] \leq \text{Cash}_{t-1} + \text{Cash Inflow}_t + \left[\begin{array}{l} \text{Securities} \\ \text{Matured}_t \end{array} \right] + \left[\begin{array}{l} \text{Securities} \\ \text{Sold}_t \end{array} \right] \tag{12.18}$$

The cells in D35:D37 have a test of this constraint that gives the user a message if the purchase decision will violate the budget. For example, the formula in C35 is

$$= \text{IF}(P35 > C10 + D50 + P43, \text{"infeasible"}, "") \tag{12.19}$$

This way, if the user inputs a purchase decision that violates the budget, a message appears that it is an infeasible decision, and the model does not compute. Nothing shows in the cell if the constraint is not violated. That is why the cells are empty in [Exhibit 12.5a](#).

There are similar constraints in the security sales section, E43:G45, where the budget test is

$$\text{Security Sales}_t \leq \text{Securities Held}_{t-1} \quad (12.20)$$

For example, in the security sale decision input section (cells E43:G45), there are constraint tests that inform the user when a decision would violate the constraint. For period 2, the sale is infeasible if

$$\begin{aligned} \text{Security Sales}_{23} &> \text{Securities Held}_{13} \\ \text{or} \\ \text{Security Sales}_{24} &> \text{Securities Held}_{14} \end{aligned} \quad (12.21)$$

so the formula for second period sales in cell C44 is

$$= \text{IF}(\text{OR}(\text{F44} > \text{F56}, \text{G44} > \text{G56}), \text{"infeasible"}, \text{""}) \quad (12.22)$$

which informs the user when she tries to sell more securities that are left in the portfolio from last period.

At this stage we do not know what the best decisions are. But we will explore the consequences of some alternative choices with our spreadsheet model. The first decisions we test are to purchase securities in period 1 totaling \$5,500, consisting of \$1,000 to mature in period 2, \$4,000 to mature in period 3, and \$500 to mature in period 4. In period 2 we will purchase \$1,500 to mature in period 3, and in period 3, purchase \$500 to mature in period 4:

$$\begin{aligned} SP_{12} &= \$1,000 \\ SP_{13} &= \$4,000 \\ SP_{14} &= \$500 \\ SP_{23} &= \$1,500 \\ SP_{34} &= \$500 \end{aligned} \quad (12.23)$$

In addition, in period 2 we will sell \$500 of securities to mature in period 3:

$$SS_{23} = \$500 \quad (12.24)$$

With these decision variables input to our spreadsheet model, the results are shown in [Exhibit 12.5a](#). We see in cell N14 that the value of the objective function has a cash and security balance at time 3 of \$7,130.11. We can change the investment decisions to find if there is a better set of decisions. Note that selling securities at time 2 does not seem like a good decision because we give up interest income that would be earned on these securities. We change the security sale decision to $SS_{23} = 0$, and the objective function

increases to \$7,136.53. With a little more adjustment of the decision variables to make sure our money is fully invested, we increase SP_{13} to \$4,600, increase SP_{23} to \$1,530, set SP_{34} at 0, to make the ending cash & security balance reach \$7,138.96. At this stage we do not know if this is really the best we can do. To find the best, we need to turn to an optimization routine to optimize our investment decisions. We will leave that to [Chapter 17](#).

This section has presented a simple model for analyzing decisions to invest in short-term securities. This model allowed us to track the security portfolio to see the results of purchase and sale decisions over a three period horizon. This is the type of model where we would have to carefully consider the costs and benefits of developing it. With just three investment periods, we see that the model is relatively large and unwieldy. Even though the model is not complex, if we were to extend the horizon, the model would increase exponentially in size. The effort to develop such a model can be worthwhile if it enables us to analyze a wider range of decision alternatives than we could without a model. As the number of periods and securities expands we could find it almost impossible to evaluate the exponentially expanding number of decision combinations, when we may come to appreciate the ease with which we can easily explore the consequences of different decision combinations—once the model is built.

Our model just opens the door to many different types of security investment decisions. There are many types of short-term security problems we have not considered in this section, but this model gives you some ideas on how to get started. In the next section we will discuss ways to model receivables.

12.3 MODELING RECEIVABLES AND CREDIT

This section presents different methods for modeling the firm's accounts receivable or credit portfolio. The importance of credit to the firm's planning will depend on the role of credit in the business. For many firms, receivables arise in conjunction with selling their products on credit. For this type of firm, receivables would be secondary to the main line of business—selling the product, and receivables would be a spontaneous asset that would respond to sales. In this case, credit policy probably would be less crucial to the future of the business, and there would be fewer policy decisions to be explored in the model. At the other end of the spectrum would be a business where credit is the focus. Banks are the best example, where their *raison d'être* is to make loans. In addition, there are other firms whose main line may nominally be production or sales, but credit is major part of their business. Examples include an auto company whose auto credit division is a major contributor to its profitability.

Clearly, there is a wide range of problems that could be the subject of a model concerned with receivables, and a wide range of models that could be used. For long-range planning for a firm where credit is not central, we may not have to delve in the fine detail of the accounts receivable, and it would be sufficient to model receivables in broad, non-detailed terms. For short-term planning, and for firms where credit is central to their

business, we may want a model that allows us to see the detailed implications of our actions and policies. In this section, we will present examples for long range planning, and we will only scratch the surface of modeling possibilities for the firm where credit is central and a finely tuned, detailed credit model is necessary.

Accounts receivable are normally generated by credit sales, so we can link receivables directly to sales. For long-range planning when it is not important to follow the short-run details of the accounts receivable, it may be sufficient to use a percent-of-sales approach for receivables. For a model with a long horizon and planning periods of a year or more, receivables at the end of the year would depend on sales that year, and only in unusual cases would they depend at all on sales prior to that year. Consequently, receivables can be expressed as a simple proportion of sales:

$$\text{Receivables}_t = a * \text{Sales}_t \quad (12.25)$$

where parameter a is the anticipated ratio of receivables to sales. The receivables to sales ratio ($AR/Sales$) can be estimated directly from past observations of receivables and sales, or can be based on the anticipated credit policy of the firm. One way we can link receivables to sales is via the anticipated collection period, or days sales outstanding (DSO). The DSO is calculated as

$$\text{DSO} = \frac{\text{Accounts Receivables}}{\left(\frac{\text{Sales}}{360}\right)} \quad (12.26)$$

So the ratio of receivables to sales is

$$\frac{AR}{Sales} = \frac{DSO}{360} \quad (12.27)$$

For example, if annual sales are \$1,000,000 (sales per day of \$2,778) and accounts receivable are outstanding for an average of 30 days, the receivables balance is $30 * (2,778) = \$83,340$, and the ratio of receivables to sales is

$$\begin{aligned} \frac{AR}{Sales} &= \frac{\$83,340}{\$1,000,000} = 8.33\% \\ &\text{or} \\ \frac{AR}{Sales} &= \frac{DSO}{360} = \frac{30}{360} = 8.33\% \end{aligned} \quad (12.28)$$

In this case, the receivables in the planning model would be expressed as

$$\text{Accounts Receivable}_t = (0.0833)\text{Sales}_t \quad (12.29)$$

By linking DSO to *AR/Sales*, we have a way to link receivables in our percent-of-sales model to a widely understood measure of collection efficiency.

As the length of the periods is shortened, the amount of detail can be increased with the accounts receivable shown as dependent on the sequence of past observed sales. A distributed lag relationship of a form such as

$$\text{Receivables}_t = a_0 * \text{Sales}_t + a_1 * \text{Sales}_{t-1} + a_2 * \text{Sales}_{t-2} \quad (12.30)$$

may be appropriate, where parameters a_0 , a_1 , and a_2 are estimated from past data or, perhaps, based on credit policy and anticipated payment practices. For example, assume that the model has a planning horizon of one year and subperiods of one month, that credit sales are 80% of total sales, and 60% of the credit sales are collected in the first month after the sales occur, 30% are collected in the second month, and 10% are collected in the third month. Assuming that there is no uncertainty in the process, the receivables collected would be

$$\begin{aligned} \text{Receivables collected}_t &= 0.60(0.80)\text{Sales}_{t-1} + 0.30(0.80)\text{Sales}_{t-2} + 0.10(0.80)\text{Sales}_{t-3} \\ &= 0.48\text{Sales}_{t-1} + 0.24\text{Sales}_{t-2} + 0.08\text{Sales}_{t-3} \end{aligned} \quad (12.31)$$

and the balance at the end of period t would be

$$\begin{aligned} \text{Receivables}_t &= (0.80)\text{Sales}_t + 0.40(0.80)\text{Sales}_{t-1} + 0.10(0.80)\text{Sales}_{t-2} \\ &= 0.80 * \text{Sales}_t + 0.32\text{Sales}_{t-1} + 0.08 * \text{Sales}_{t-2} \end{aligned} \quad (12.32)$$

where the parameter estimates are $a_0 = 0.80$, $a_1 = 0.32$, and $a_2 = 0.08$. With this percent-of-sales model, the primary input is the sales forecast. For the distributed lag example, the *Receivables* forecast at any future date t would be generated by the forecast of *Sales* at dates $t - 1$ and t . Suppose the sales forecasts for the next 12 months are as shown in [Exhibit 12.6](#); with the parameters shown in Equation 12.32, the *Receivables* balance in each month will be as shown in the second line. For example, *Receivables* in the eighth month would be

$$\begin{aligned} \text{Receivables}_8 &= 0.80 * \text{Sales}_8 + 0.32 * \text{Sales}_7 + 0.08 * \text{Sales}_6 \\ &= 0.80 * 350 + 0.32 * 400 + 0.08 * 350 \\ &= \$436 \end{aligned} \quad (12.33)$$

To the extent that receivables and/or collections are affected by economic conditions, estimates may be improved by adding appropriate economic variables to the estimated

Month (<i>t</i>)	1	2	3	4	5	6	7	8	9	10	11	12
Sales (\$000)	100	150	200	250	300	350	400	350	300	250	200	150
Receivables (\$000)	144	164	216	276	336	396	456	436	384	324	264	204

EXHIBIT 12.6 Sales and Receivables forecast with the distributed lag equation: $Receivables_t = 0.80 Sales_t + 0.32 Sales_{t-1} + 0.08 Sales_{t-2}$.

predictive equation. Just remember that while such elaborations may generate better estimates using past data, to use them in the simulation of future periods, the future values of the economic variables must also be forecasted. The economic forecast may be more elusive than forecasting receivables.

Of course, there are numerous other ways in which receivables can be modeled. For example, the accounting relationship

$$Receivables_t = Receivables_{t-1} - Receivables\ Collected_t + Credit\ Extended_t \quad (12.34)$$

can serve as the basis for our prediction where we estimate the variables *Receivables Collected_t* and *Credit Extended_t* with regression equations that might be based on a mix of firm data and economic data. Credit collections in month *t* will be influenced by the past receivables balances and past and current economic conditions. Credit extended might be driven by firm sales and interest rates. This would suggest that we could estimate credit and collections with regression equations such as

$$\begin{aligned} [Receivables\ Collected_t] &= A_1 Receivables_{t-1} + A_2 Interest\ Rate_{t-1} + \\ &A_3 Unemployment_{t-1} \quad Credit\ Extended_t = B_1 Sales_t + B_2 Interest\ Rate_{t-1} \end{aligned} \quad (12.35)$$

With this econometric approach, we would estimate the regression coefficients based on past economic and firm data. Of course, we would have to investigate several alternative predictive models to find the one that best described and predicted our firm's variables.

12.4 INVENTORIES

As with other current asset categories, there are numerous ways in which to model inventories, ranging from the simplest with inventories as a proportion of sales, to more complex models that link inventories to a production sector of the planning model.

For the longer horizon model the simple proportional relation

$$Inventories_t = b * Sales_t \quad (12.36)$$

may be sufficiently accurate because inventories at the end of each year may bear a stable relation to sales over a period of years, and the relation could be estimated easily from past data. For long-run strategic planning purposes where rough approximations prove

sufficient, and where it is unnecessary to track the seasonal details of inventories, there is little reason to burden the model with greater complexity.

The relation between Inventories and *Sales*, such as Equation 12.36, can be based on past relationships and estimated from past data using a linear regression approach, or it may be included in the model so as to reflect an inventory policy decision. As an example of the latter approach, the policy might be to maintain an inventory balance equivalent to 50 days' sales (evaluated at cost of goods sold). Assume that cost of goods sold is 70% of sales, with sales per day of

$$\text{Sales per Day} = \frac{\text{Annual Sales}}{365} \quad (12.37)$$

Evaluated in terms of cost, the goods sold per day would be

$$\begin{aligned} &0.70 * \text{Sales per Day} \\ &\text{or} \\ &0.70 * \frac{\text{Annual Sales}}{365} \end{aligned} \quad (12.38)$$

and to have inventory to meet 50 days' sales, the firm would have

$$\begin{aligned} \text{Inventory} &= 50 * 0.70 * \frac{\text{Annual Sales}}{365} \\ &= 0.096 * \text{Annual Sales} \end{aligned} \quad (12.39)$$

so this inventory policy would call for inventories being maintained at 9.6% of *Sales* to meet the 50-day requirement.

Greater detail and accuracy may be attainable by showing inventories in terms of the variables that most directly influence them—specifically, both production and sales. For example, the expression

$$\text{Inventories}_t = \text{Inventories}_{t-1} + (\text{Units produced}_t - \text{Units sold}_t) * \text{Cost per Unit}_t \quad (12.40)$$

is true by definition and could accurately track inventories if production, sales, and costs are accurately forecasted. This would be especially suited to a short horizon where seasonal variations in production and sales are important, and where there is a production sector that deals in physical units. However, the relation need not be expressed in terms of physical units. It can be expressed directly in terms of dollar value so as to bypass the difficulties of working with physical units and developing data from which to evaluate the cost per unit.

As production is usually based on current and future expected sales, the inventory Equation 12.40 may be rederived in a form so inventories are driven solely by sales. For example, a simple adaptive model of production could be of the form:

$$\text{Units Produced}_t = \text{Units Sold}_t + a(\text{Units Sold}_t - \text{Units Sold}_{t-1}) \quad (12.41)$$

where current production is equal to current sales plus some proportion, a , of the current increase in sales so as to increase inventory to meet an anticipated sales increase next period. Substituting Equation 12.41 for $Units Produced_t$ in Equation 12.40 yields

$$Inventories = Inventories_{t-1} + a(Units Sold_t - Units Sold_{t-1}) * Cost per Unit \quad (12.42)$$

There are numerous decision models that are intended to specify an optimal inventory policy based on various factors of demand, interest rates, and ordering costs for goods held in inventory.* These optimization models can provide the basis for forecasting inventories in a simulation model when it is assumed that the firm follows the optimal policy. For example, the simplest inventory model is the economic order quantity (EOQ) model that prescribes the optimal amount of inventory to order and implies that the average inventory will be

$$Inventory = Sales^{\frac{1}{2}} * \frac{1}{2} \left(\frac{Ordering Cost}{Cost per Unit * Interest Rate} \right)^{\frac{1}{2}} \quad (12.43)$$

where Ordering Cost represents the fixed cost of placing one new order for the goods carried in inventory, and $(Cost per Unit) * (Interest Rate)$ is the opportunity cost of carrying one unit in inventory for a year.

For the firm following the EOQ policy, the average inventory would increase with sales at a decreasing rate. Thus, inventories would not be strictly proportional to sales, and the inventory as a proportion of sales would decrease as forecasted sales increase. Note that this model relates the average inventory to the Ordering cost and the Carrying cost, so that to utilize it in the financial planning context would necessitate forecasting not just sales, but these costs (including interest rates) as well.

There are numerous other more sophisticated inventory models that potentially could be useful in modeling inventories in a simulation model. However, the sophistication may be self-defeating. The more sophisticated models may impose an impractical burden of forecasting on the planner if it requires that too many variables be forecasted in excessive detail. Generally speaking, the simpler the model, the more useful it is.

12.5 SPONTANEOUS FINANCING

This section will deal with the sources of short-term financing that are spontaneous. Spontaneous financing refers to financing sources that grow more or less automatically with sales and the firm's assets, whereas discretionary means that a decision must be made to utilize a particular source of external financing. The spontaneous source we consider here is financing in the form of accounts payable. The other source that we would include in this spontaneous short-term category would be accruals, but our focus will be on payables.

Accounts payable is a good example of spontaneous financing from current liabilities. First, note that accounts payable for one firm are the accounts receivable for another firm,

* Inventory policy models are explained in Fogarty (2009).

so that the various methods for modeling receivables that we discussed earlier apply to modeling the payables. Accounts payable represents credit purchases of goods and materials used in producing the output. Thus, sales drives purchases, which in turn, determines the balance in accounts payable. In a very aggregative model with long periods comprising the planning horizon, these relationships would be summarized in a percent-of-sales model for payables of the form:

$$\text{Accounts Payable}_t = a * \text{Sales}_t \tag{12.44}$$

where a is the parameter that represents the payable as a percentage of sales. The parameter a would be determined from observations of past data, modified by judgments regarding the future relationship.

With respect to financing growth of sales and assets, it is the increase in current liabilities that is the relevant source of spontaneous financing. For accounts payable, the spontaneous increase, denoted by ΔAP , corresponding to Equation 12.44, is expressed as

$$\Delta AP_t = a(\text{Sales}_t - \text{Sales}_{t-1}) \tag{12.45}$$

As with other account balances, as the planning horizon and the periods comprising the horizon are shortened, it may be necessary to model payables in greater detail. There are many different approaches, and a good starting point is the definitional expression:

$$\left[\begin{array}{c} \text{Accounts} \\ \text{Payable}_t \end{array} \right] = \left[\begin{array}{c} \text{Accounts} \\ \text{Payable}_{t-1} \end{array} \right] - \left[\begin{array}{c} \text{Payments} \\ \text{on Account}_t \end{array} \right] + \left[\begin{array}{c} \text{Credit} \\ \text{Purchases}_t \end{array} \right] \tag{12.46}$$

Each of the components, *Payments on Account* and *Credit Purchases*, would be modeled separately. For example, if the length of the interval is one week and accounts are always paid in four weeks we would have

$$\text{Payments on Account}_t = \text{Accounts Payable}_{t-4} \tag{12.47}$$

And if purchases are equal to 60% of sales forecasted for two weeks in the future, we have

$$\text{Credit Purchases}_t = 0.60 * \text{Sales}_{t+2} \tag{12.48}$$

This example is based on a specific policy that is assumed to apply over the planning horizon. In other cases, it may be more appropriate to use observed past data to make econometric estimates of the relationships between payments or purchases and sales. An example of a linear regression relation that would be estimated using lagged variables is

$$\text{Payments on Account}_t = A + B * \text{Accounts Payable}_{t-1} + C * \text{Accounts Payable}_{t-2} \tag{12.49}$$

where the data are used to estimate parameters A , B , and C ; and purchases are estimated from a relation such as

$$\text{Credit Purchases}_t = A + B * \text{Sales}_{t-1} + C * \text{Sales}_t + D * \text{Sales}_{t+1} \tag{12.50}$$

Other types of current liabilities, such as accrued expenses, are usually spontaneous and would be modeled with a percent-of-sales approach. There is seldom much benefit from modeling such accounts in great detail in a planning context. The other major source of short-term financing would be short-term borrowing. We will consider that when we consider debt financing in [Chapter 17](#).

12.6 SUMMARY

This chapter has discussed ways in which we can model selected working capital accounts. We continued to assume that our task is to develop a sales-driven planning model for the firm, and presented suggestions for modeling the working capital accounts. In each case, you need to keep in mind that there are as many different ways to model particular problems or accounts as there are model builders. Each planning problem will be different and requires a different approach, and our models and methods will only provide a starting point. The challenge and the fun of financial modeling are in devising new approaches and solutions to planning problems.

PROBLEMS

1. M&O Corporation Cash Balance

The M&O Corporation has asked you to examine their cash management processes and recommend a policy for managing their cash balances. As part of this assignment, you have decided to test the Miller and Orr cash balance model with Monte Carlo simulation.

You have examined the company's cash processes and estimated the following parameters and data:

Current cash balance:	\$10,000
Current balance of marketable securities:	\$30,000
Interest rate earned annually on marketable securities:	10%
Interest rate paid annually on borrowing:	12%

The daily cash flow is a normally distributed random variable with a mean of \$1,000 and standard deviation of \$450.

If cash is transferred into or out of marketable securities, or funds are borrowed, there is a fixed cost per transaction of \$50.

Modeling assignment: Develop a model of the company's cash flow and cash balance where the cash account is managed according to the Miller and Orr model. After calculating the Miller and Orr transaction policy boundaries (H , Z , L), use @Risk to test the model. Make random draws from the daily cash flows to apply the model and follow the cash balances, security transactions, and borrowing. In other words, for one run

of the model over 30 days, each day's cash flow would be drawn from the @Risk function, RiskNormal(1000, 450). The transactions for each single simulation draw would be calculated and recorded. With 5,000 draws in a single 30-day simulation run, you will have distributions for each day's cash balance, the total transaction costs over the 30 days, the total number of securities purchased and sold, and the total amount borrowed, and the total interest paid. Provide a summary of the distributions for each of these variables.

2. Managing Marketable Securities: Placidia Corporation

The Placidia Corporation has a portfolio of cash and short-term securities. In managing the cash and securities, the objective is to maximize value of cash & securities held at the end of the 3-month planning horizon. The planning horizon is divided into intervals of 1 month. The first date that decisions can be implemented is time 1, the end of the first month which is when the first cash flow will occur, and securities will mature. The cash from maturing securities and securities sold will be available immediately to be used for other investments or other use.

At the start of the planning period, the firm has an initial balance of cash and an endowment of securities as follows:

Cash Balance	\$2,500		
Marketable Securities			
Maturity Date	Month 1	Month 2	Month 3
Face Value	\$400	\$1,000	\$500

In addition to holding the initial balance cash & securities, the firm expects a cash flow in each month. The cash flows expected in each of the three months of the planning horizon are as follows:

	Month 1	Month 2	Month 3
Expected Cash Flow	-\$3,000	\$700	\$800

The cash inflow will be available to be invested, and the cash outflow of \$3,000 must be funded from the cash balance or from the security portfolio.

Securities are traded at prices that are discounted from face, or maturity value. The current and expected future short-term interest rates that we use to discount the face value (FV) to the market value of the securities are shown in the table below. The number in the first column indicates the month in which the security is purchased, and the numbers in the top row indicate the month the security matures. So, a security purchased in month 1 that matures in month 3 is expected to have a monthly interest rate of 0.9%. Securities purchased in period 0 refers to securities already held at the start of the planning horizon. Period 4 represents securities maturing after the end of the 3 month planning horizon. We will assume that period 4 is 1 month beyond the end of the planning horizon.

Monthly Interest Rates for Marketable Securities

Period Security Is Purchased	Period Security Matures			
	1	2	3	4
0	0.5%	0.6%	0.7%	1.0%
1		0.8%	0.9%	1.2%
2			1.2%	0.8%
3				0.5%

Our task is to construct a spreadsheet model to track our security portfolio for three months, so we can see the results of alternative strategies for buying and selling securities.

We can purchase securities at times 1, 2, and 3 to mature at any subsequent date up to time 4. The purchase price of the security is the present value of the FV of the security. Thus, if, in month 1, we purchase a security that matures in month 3 at FV of \$1,000, the purchase price (market value) will be

$$\begin{aligned}
 \text{Purchase Price} &= \text{Market Value} = 1,000 \left[1 / (1 + .009)^2 \right] \\
 &= 1,000 [.98224] \\
 &= \$982.24
 \end{aligned}$$

If we hold the security for 2 months until it matures, we will receive the FV of \$1,000 at the end of month 3. The income will be the appreciation in price amounting to

$$\text{Income} = \$1,000 - 982.24 = \$17.76$$

A security can be sold prior to maturity. If this security is sold prior to maturity, it will be sold at a discounted price based on the discount rate prevailing at the date it is sold, with the rate reflecting the remaining time to maturity at the date it is sold.

The decisions you have to make are as follows:

- When to buy securities
- Which securities to buy
- When to sell securities
- Which securities to sell

You want to make these decisions with the objective of maximizing the value at time 3 of the cash balance and the market value of securities held at time 3. Of course, you may not allow your cash balance to be negative at any date. You cannot buy securities unless you have the cash to pay for them. You cannot sell securities that you do not have, and you cannot sell securities short.



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V

Modeling Security Prices and Investment Portfolios



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Modeling Security Prices

THIS AND THE FOLLOWING 2 chapters are concerned with modeling investment opportunities. This chapter starts by developing models of security prices that will be used in subsequent chapters. The objective of the price model is to portray mathematically and graphically the movement of security prices as time passes. This is useful for understanding the distribution of possible future prices. It is also useful for devising investment strategies, constructing security portfolios, and understanding the pricing of derivative securities. We look at two different ways to model the time path of security prices: the discrete time binomial model and the continuous time model.

13.1 THE BINOMIAL MODEL OF STOCK PRICE MOVEMENT

The binomial model assumes that the price of a security will move to either of two prices by the next point in time. Although this model may seem unrealistic, it can be made sufficiently accurate to portray most price patterns. It is particularly helpful for understanding and modeling the pricing of derivative securities such as call and put options.

We begin by assuming the length of the time period is one year, the current stock price is S_0 , and the price at the end of the year will be S_1 . Letting u denote the wealth relative return (1+ rate of return) when the stock moves up, and d denote the wealth relative return when the stock moves down, the year-end stock price will be either $S_1 = S_0u$ or $S_1 = S_0d$. For example, suppose the initial stock price is \$100 and the wealth relative returns over the year are either $u = 1.25$ or $d = 0.80$ (rates of return of 25% or -20%). Then the year-end price will be either

$$S_1 = S_0u = 100(1.25) = 125$$

or

$$S_1 = S_0d = 100(0.80) = 80 \tag{13.1}$$

as shown in [Exhibit 13.1](#).

We now extend the model to four periods and consider the prices that could result from this binomial process. [Exhibit 13.2a](#) shows the prices over four periods, starting with

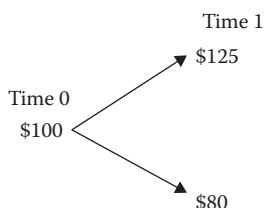


EXHIBIT 13.1 Binomial stock price movement.

$S_0 = 100$ at $t = 0$, assuming that at each node (date) the price can go up by proportion $u = 1.25$ or down by $d = 0.80$. Even though there are 16 different possible price paths to the 4th node (labeled a through p on the right), only 5 different prices can occur at $t = 4$: \$41, \$64, \$100, \$156.3, and \$244.1.* The reason there are relatively few possible prices is that there are numerous paths to the same price. For example, out of 16 price paths, \$100 occurs 6 times (nodes d, f, g, j, k, and m). This results from the fact that u and d are the same amounts in each period. This produces 6 sequences of u 's (ups) and d 's (downs) that lead from \$100 to \$100 in 4 steps. For $u = 1.25$ and $d = 0.80$ at every step, we have

$$S_4 = S_0 uudd = S_0 udud = S_0 duud = S_0 dduu \quad (13.2)$$

The order of sequence of up's and down's does not matter—they all lead to the same terminal price of \$100. This can be seen more clearly in [Exhibit 13.2b](#), in which the paths share common points.

One of our concerns is the probability distribution of prices at a given date. Monte Carlo simulation is well suited to this purpose. Given the probability of each of the moves, u and d , it is easy to simulate the probability of each stock price when there are only a few steps. However, determining the distribution of stock prices can be more difficult when there are many steps in the process. We can simulate the process to get a better estimate of the distribution of possible outcomes.

Suppose that we estimate the probability of an up move to be 78% and the probability of a down move is 22%, and the moves are independent in different periods. What is the probability of each possible price at time 4 given the initial price of \$100 at time 0?

[Exhibit 13.3](#) is a four-period model of stock prices with the up and down movement at each stage chosen randomly with an @RISK Monte Carlo simulation. The input data is described in cells A4:C16, and the output consisting of stock moves and prices at each of the 4 steps are shown in cells E8:H12. The initial stock price of \$100 is in cell H8. The price then moves up or down at times 1 through 4 based on a random draw of u and d , until it reaches time 4 in cell H12. The up move is 1.25 each period with a probability of 78% (specified in cell B16), and the down move is 0.80 with a probability of 22%

* With 2 outcomes at each node, the number of price paths over 4 periods is $2^4 = 16$, and in general, over n periods, there would be 2^n different paths. However, with u and d the same each period, the number of different prices is $n + 1$, or with 4 periods, 5 different prices.

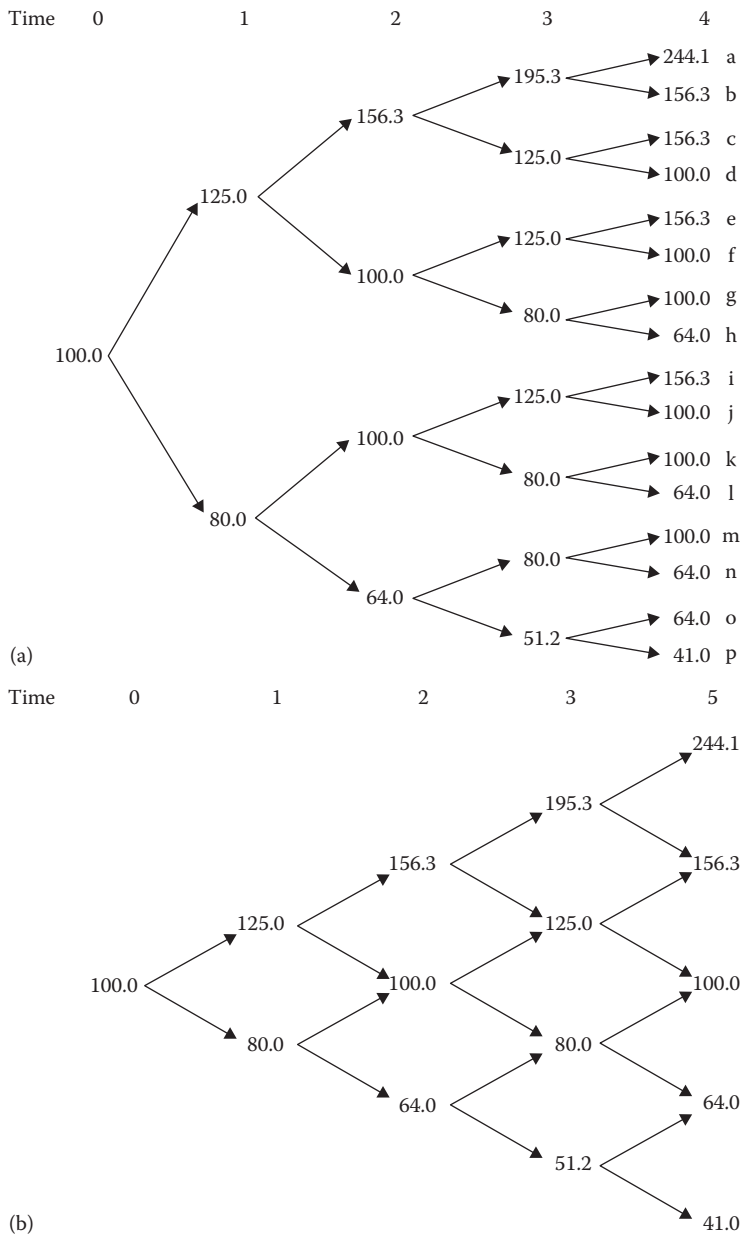


EXHIBIT 13.2 (a) Price paths over four periods with the binomial model. (b) Price paths over four periods that share common nodes with the binomial model.

(cell C16). The random draws are modeled in column F. For example, cell F9 has the @RISK uniform distribution function =RiskUniform(0,1) that generates a random variable between 0 and 1. G9 then translates this draw into the up (u) or down (d) by choosing $d = .80$ if the fraction drawn with the uniform distribution is less than .22, and choosing $u = 1.25$ if the fraction is greater than .22. By using the uniform distribution with all

	A	B	C	D	E	F	G	H	I
1	Binomial Stock Price Movement								
2	Four Period Model								
3									
4	Data Input				Model of Prices				
5									
6	Initial Stock Price	100			Step	Random	Price	Stock	
7						Draw	Move	Price	
8		Stock			0			100.00	
9		Returns & Movement			1	0.57	1.2500	125.00	
10					2	0.44	1.2500	156.25	
11		Outcome			3	0.12	0.8000	125.00	
12		#1	#2		4	0.90	1.2500	156.25	
13		Up	Down	=RiskUniform(0,1)				=H8*G9	
14	Rate of Return per Year	25%	-20%						
15	Wealth Relative Return (u & d)	1.25	0.80			=IF(F9<=\$C\$16,\$C\$15,\$B\$15)			
16	Probability	78%	22%						
17									

EXHIBIT 13.3 Excel model of binomial price movements over four periods.

fractions from 0 to 1 having an equal likelihood of occurring, $d = 0.80$ will occur 22% of the time, and $u = 1.25$ will appear 78% of the time.

The output of interest is the frequency distribution of the price at time 4. Exhibit 13.4 shows an @RISK graph of the frequency distribution, with the relative frequencies of each of the 5 prices turning out to be as follows:

Price	\$41	\$64	\$100	\$156.3	\$244.1
Frequency	0.2%	3.2%	17.8%	42%	36.8%

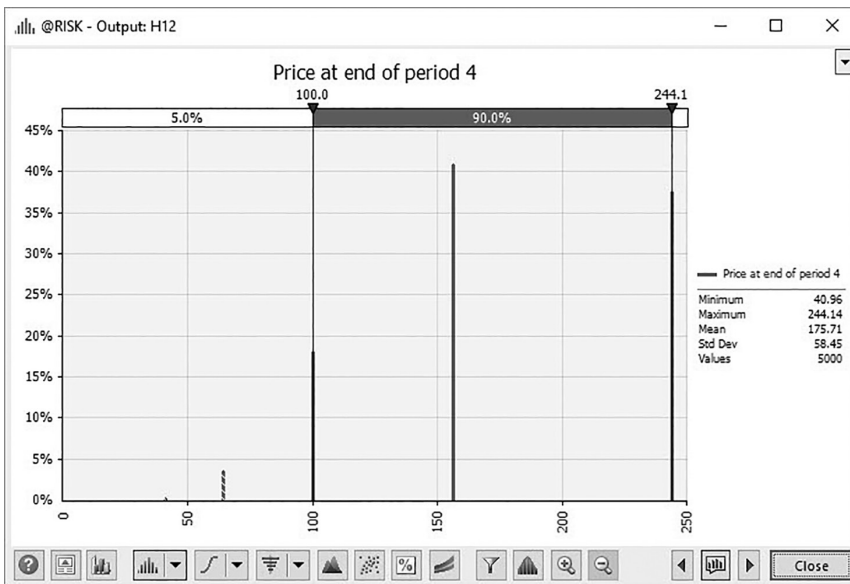


EXHIBIT 13.4 Distribution of price at time 4 with @RISK Monte Carlo simulation.

This section introduced the binomial model and showed how to construct a simple Excel model of binomial price movements. As noted earlier, you may be inclined to think such a model is unrealistic because stock prices seem to move in far more complex patterns than just up or down. Although it is true that a simple up or down pattern is an oversimplification, we can adjust the model to provide more realistic movements by dividing time into smaller units with prices going up or down in very small increments. In fact, we can make the model mimic very closely the stock movements that would result from a continuous process. We show how to use the binomial model to approximate a continuous process in a subsequent section, after explaining and modeling the continuous stock price process.

13.2 STOCK PRICES AS A RANDOM WALK IN CONTINUOUS TIME

The continuous time model of stock returns and prices was developed by Merton (1973) and underlies the Black & Scholes (1973) option pricing model. This model hypothesizes that stock prices follow a process where, over short periods of time of length dt ,* the return is generated according to the expression

$$\text{Return} = \frac{dP}{P} = \mu dt + \sigma z \sqrt{dt} \quad (13.3)$$

where:

P is the price

dP is the change in the price

dP/P is the rate of return

μ is the expected return per period

σ is the standard deviation of return per period

z is the standard normal variable with mean 0 and standard deviation of 1

Equation 13.1 says that the return on a given stock during a period of length dt will be the expected return plus an error that is a random variable from a normal distribution with mean 0 and standard deviation of σ . For example, if the time is defined as years, with μ and σ expressed as annual rates, the return over a quarter of a year ($dt = 0.25$) would be

$$\text{Return} = \frac{dP}{P} = \mu(0.25) + \sigma z \sqrt{0.25} \quad (13.4)$$

If the period was one year ($dt = 1$), the return over the year would be $\mu + \sigma z$.

When the return is generated by the process described in Equation 13.3, we say the return follows a Weiner process. This is similar to Brownian motion in chemistry that refers to the random motion of a particle suspended in a liquid, except in this case it refers to the random movement of stock prices. In this case, the stock price follows what is called a geometric Brownian motion, where the random movement of the stock price is proportional to

* In the continuous time model, dt denotes the limit as the time increment approaches zero, and in most discussions of the model, a distinction is made between dt and a discrete time change denoted by, for example, δt . Our price model will be a discrete time approximation of the continuous process, but we will not make the distinction between the limit dt and the discrete time version, and we use dt to denote the change in time over a small discrete increment.

the level of the price; it is the return that follows the Weiner process, or Brownian motion. This process has come to be known as the random walk of stock prices.

Using Equation 13.3 and Monte Carlo software, we simulate the distribution of returns on a stock by making random draws from a standardized normal [0, 1] distribution that is represented by z in the formula. Suppose the expected return per year on a stock is $\mu = 12\%$, with a standard deviation of return per year of $\sigma = 20\%$, and we wish to simulate the distribution of returns over 0.25 year. Exhibit 13.5 shows a model of stock returns that follow the random walk in a discrete time version of the Merton model. Cells B10, B11, and B15 are the inputs (μ , σ , and dt). B22 is the random variable, z that is drawn from a standard normal probability distribution, and C22 is the simulated return. The return in C22 is generated by the formula $= B10 * B15 + B11 * C22 * (B15^0.5)$. The display shows one particular random draw of the standard normal variable $z = 1.384$. This produces the random error of

$$\sigma z \sqrt{dt} = 0.20(1.384)\sqrt{0.25} = 0.1369 \tag{13.5}$$

and the quarter year return for this particular draw of the simulation is

$$Return = r = 0.12(0.25) + 0.1384 = 0.1684 \tag{13.6}$$

as shown in cell C22. Given the initial stock price of \$100, the stock price at the end of the first time interval is \$116.84 in cell D22.

	A	B	C	D
1	Stock Returns With Continuous Process			
2	Return = $\frac{dP}{P} = \mu dt + \sigma z \sqrt{dt}$			
3	Return = $\frac{dP}{P} = \mu dt + \sigma z \sqrt{dt}$			
4	Return = $\frac{dP}{P} = \mu dt + \sigma z \sqrt{dt}$			
5	Return = $\frac{dP}{P} = \mu dt + \sigma z \sqrt{dt}$			
6	Data Input			
7				
8	Initial Stock Price	100		
9				
10	Mean Return	12.0%	per year	
11	Standard Deviation	20.0%	per year	
12				
13	Length of Sub-Period	3 months		
14	Sub-Periods Per Year	4		
15	Length of Period	0.25	fraction of year	
16				
17	= RiskNormal(0,1)	Standard	Return	
18	Period	Normal	For	Price
19		Variate, z	Period	
20		= B10*B15+B11*B22*(B15^0.5)		
21		0		100
22	1	1.384	16.84%	116.84

EXHIBIT 13.5 Model of stock returns that follow a Weiner process (random walk).

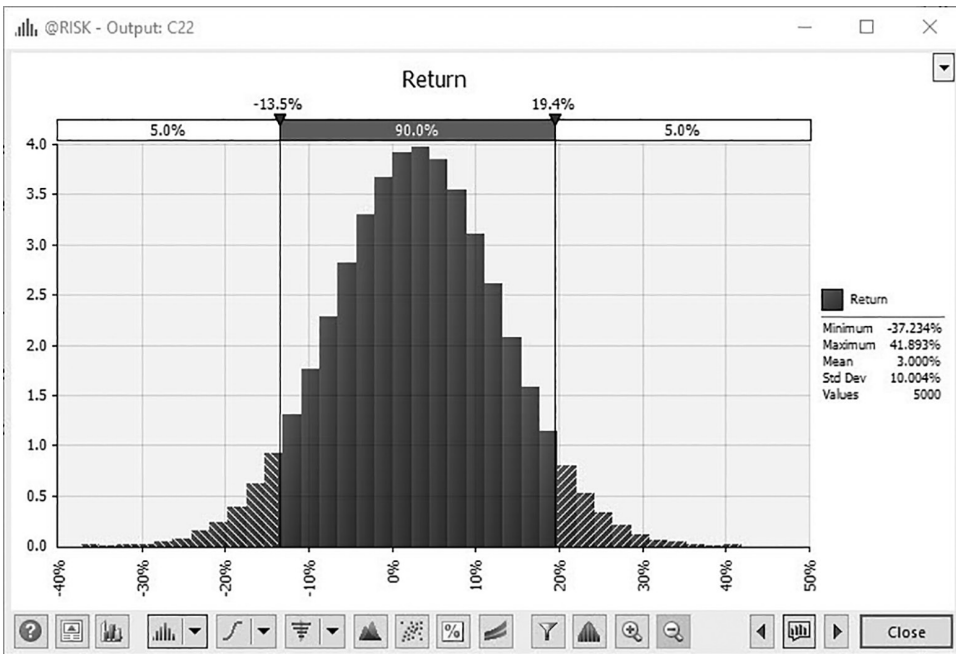


EXHIBIT 13.6 Frequency distribution of stock returns from @RISK simulation.

By making repeated draws for the random variable, z , and tracking the resulting return, we can estimate the probability distribution of the return at a single point in time. Suppose we wish to simulate the distribution of quarter year returns. We set the period at 0.25 year (cell B15), and make 5,000 draws. Exhibit 13.6 shows the results for simulated distribution of stock returns over a three-month period. The simulated three-month return is normally distributed with a mean of 3% and a standard deviation of 10%, with minimum and maximum returns of -37.23% and 41.89% . These figures describe the probability distribution for a three-month return generated by the Weiner process as specified by Equation 13.3.

Now we change the perspective of the simulation to see what might happen to the price of a share of stock over time, six months in this example. Assume the stock return follows the process described by Equation 13.3, and we use one week as the length of each subinterval. The return would be

$$\text{Return Per Week} = 0.12(0.0192) + 0.20z\sqrt{0.0192} \quad (13.7)$$

where the 0.12 and 0.20 are the expected return and standard deviation of return per year, and the length of the period is $dt = 1/52 = 0.0192$ of a year. With this process, the price of the stock at date t will be

$$\text{Price}_t = \text{Price}_{t-1}(1 + \text{Return}_t) \quad (13.8)$$

The @RISK model of the return and stock price is shown in Exhibit 13.7. The initial price of the stock is \$100.

	A	B	C	D	E	F	G	H
1	Stock Returns With Continuous Process							
2	$\text{Return} = \frac{dP}{P} = \mu dt + \sigma z \sqrt{dt}$							
3								
4								
5								
6	Data Input							
7								
8	Intitial Stock Price	100						
9								
10	Mean Return	12.0%	per year					
11	Standard Deviation	20.0%	per year					
12								
13	Length of Sub-Period	1 Week						
14	Sub-Periods Per Year	52						
15	Length of Period	0.019231	fraction of year					
16								
17		Standard	Return	Price				
18		Period	Per					
19		Normal	Period					
20		Variate, z						
21	=RiskNormal(0,1)			=B\$10*B\$15+B\$11*B22*(B\$15^0.5)				
22	0			=D21*(1+C22)				
23	1	-0.746	-1.84%	98.16				
24	2	-0.194	-0.31%	97.86				
25	3	-0.365	-0.78%	97.10				
26	4	-1.225	-3.17%	94.02				
27	5	0.720	2.23%	96.12				
28	6	0.816	2.50%	98.51				
29	7	-0.248	-0.46%	98.06				
30	8	1.777	5.16%	103.12				
31	9	0.674	2.10%	105.29				
32	10	-1.348	-3.51%	101.59				
33	11	-0.464	-1.06%	100.52				
34	12	0.025	0.30%	100.82				
35	13	-2.449	-6.56%	94.21				
36	14	-0.825	-2.06%	92.27				
37	15	-0.433	-0.97%	91.38				
38	16	-0.168	-0.24%	91.16				
39	17	0.616	1.94%	92.93				
40	18	0.703	2.18%	94.95				
41	19	-0.111	-0.08%	94.88				
42	20	-0.947	-2.40%	92.61				
43	21	-0.940	-2.38%	90.41				
44	22	0.417	1.39%	91.66				
45	23	-0.102	-0.05%	91.61				
46	24	-0.966	-2.45%	89.37				
47	25	-0.491	-1.13%	88.36				
48	26	2.366	6.79%	94.36				

EXHIBIT 13.7 Model of random stock prices.

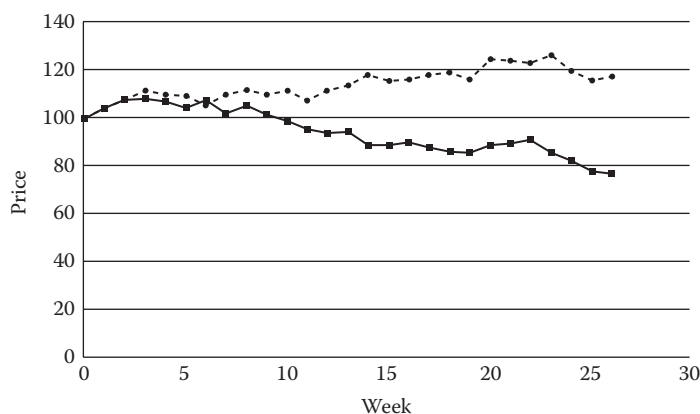


EXHIBIT 13.8 Two paths of stock prices from the same probability distribution of returns.

This is a model of weekly prices with a beginning price of \$100. The returns based on Equation 13.7 are drawn randomly from their distributions that are independent from week to week. The end-of-week price is calculated according to Equation 13.8. The Excel code for the end of week one (D22), along with the 26-week price series, is shown in Exhibit 13.7. Each iteration results in a different series of prices. Exhibit 13.8 shows two different price series resulting from two iterations of the simulation. The two series were drawn from the same probability distribution of returns, but by the chance of the draw, the prices follow different paths.

13.3 BINOMIAL APPROXIMATION OF THE CONTINUOUS PRICE PROCESS

In the last section, we modeled stock prices as a continuous process, although we used discrete time increments of one week as an approximation. For some purposes such as option pricing, it is easier to use a binomial model to approximate the continuous Weiner process. Instead of assuming that the return will be generated according to the Equation 13.3, we assume that the price follows a binomial process over very short increments in time. If we use sufficiently short time increments, the process can be made to closely approximate a continuous process.

For the binomial model to approximate the continuous time model, we use formulas developed by Cox, Ross, and Rubinstein (1979) to relate the binomial up and down moves to the length of the step periods and the continuous time standard deviation of return:

$$u = e^{\sigma\sqrt{dt}} \quad (13.9)$$

$$d = e^{-\sigma\sqrt{dt}} = \frac{1}{u} \quad (13.10)$$

and

$$p = \frac{e^{\mu\sqrt{dt}} - d}{u - d} = \text{probability of an up move, } u \quad (13.11)$$

For our example, as with the continuous time example, assume the mean return per year is $\mu = 0.12$, and the standard deviation of annual return is $\sigma = 0.20$. Based on Equations 13.9

and 13.10, this gives us $u = 1.0281$ and $d = 0.9726$, with the probability of prices moving either up by u with probability p , or down by d with probability $(1 - p)$, where

$$p = \frac{e^{u\sqrt{dt}} - d}{u - d} = \frac{e^{0.12\sqrt{0.0192}} - 0.9726}{1.0281 - 0.9726} = 0.535 \tag{13.12}$$

Exhibit 13.9 shows the binomial model with the parameters calculated as shown above, with return and price calculations for weekly subperiods over a horizon of 26 weeks. Cell B21 contains formula of Equation 13.9, B22 the formula of Equation 13.10, B24 the formula of Equation 13.11, and B19 the square root of the step length,

$$\sqrt{dt} = \sqrt{0.0192} \tag{13.13}$$

	A	B	C	D	E	F	G	H	I	J
1	Binomial Stock Price Movement									
2	Partial Year Model									
3										
4										
5										
6	Data Input				Step	Random Draw	Price Move	Stock Price		
7										
8	Initial Stock Price	100			0			100.00		
9					1	0.81	1.0281	102.81		
10	Annual Data				2	0.78	1.0281	105.70		
11	Risk Free Interest Rate	5%			3	0.46	0.9726	102.81		
12	Mean Return	12%			4	0.26	0.9726	100.00		
13	Standard Deviation Per Year	20%			5	0.93	1.0281	102.81		
14					6	0.10	0.9726	100.00		
15	Length of Sub-Period	1 Week			7	0.88	1.0281	102.81		
16	Sub-Periods Per Year	52			8	0.27	0.9726	100.00		
17					9	0.61	1.0281	102.81		
18	Step Length	0.0192			10	0.93	1.0281	105.70		
19	(Step Length) ^{1/2}	0.1387			11	0.11	0.9726	102.81		
20					12	0.27	0.9726	100.00		
21	Up Wealth Relative Return	1.0281			13	0.66	1.0281	102.81		
22	Down Wealth Relative Return	0.9726			14	1.00	1.0281	105.70		
23					15	0.86	1.0281	108.68		
24	Up Probability	0.535			16	0.30	0.9726	105.70		
25	Down Probability	0.465			17	0.71	1.0281	108.68		
26					18	0.17	0.9726	105.70		
27					19	0.83	1.0281	108.68		
28					20	0.40	0.9726	105.70		
29					21	0.44	0.9726	102.81		
30					22	0.70	1.0281	105.70		
31					23	0.37	0.9726	102.81		
32					24	0.55	1.0281	105.70		
33					25	0.25	0.9726	102.81		
34					26	0.49	1.0281	105.70		

EXHIBIT 13.9 Model of binomial stock price movement with weekly subperiods.

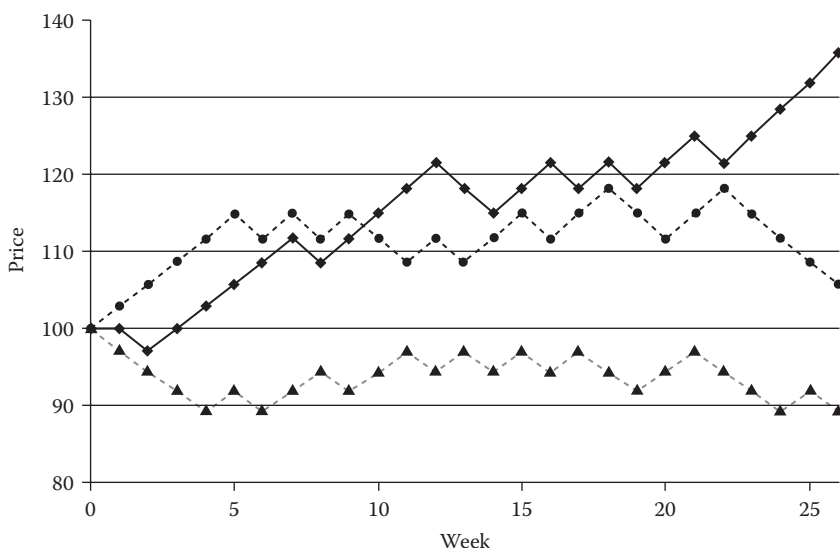


EXHIBIT 13.10 Price paths for three different simulations with weekly binomial price moves.

The returns and prices shown in [Exhibit 13.9](#) are for one particular iteration of the Monte Carlo simulation. Each iteration of the simulation results in a different sequence of prices. If you run 5,000 iterations, you will potentially generate 5,000 different price paths.* [Exhibit 13.10](#) shows three of those possible price paths that lead to ending prices of \$89.50, \$105.70, and \$131.96.

[Exhibit 13.11](#) shows the frequency distribution of prices at the end of 26 weeks when the simulation model was run with 5,000 iterations. The mean price at the end of week 26 was \$106.16, with standard deviation of \$14.76. The distribution is skewed to the right, which is confirmed by the positive skewness parameter of 0.3326. The positive skew of the distribution of stock prices means that the distribution is not normal. In fact, when the returns are normally distributed, the price distribution will be log-normally distributed with the positive skew we see here.

In the binomial model, at each step the price moves either up or down by a discrete percentage, so in a weekly model with a 26-week horizon the binomial model will end at any one of 27 different prices. In contrast, in the continuous model the price can move to any value within the domain of the probability distribution so there are an infinite number of different prices possible at the end of 26 weeks. Nevertheless, the binomial model can be made to approximate the distribution of continuous outcomes by using smaller time intervals. For example, we can easily adjust the binomial model in [Exhibit 13.9](#) by using daily price moves over the same half year. To do this, change the length of the subperiod in B15 in the spreadsheet to read “1 day” and the subperiods per year in B16 to read 250

* For the binomial process with 26 steps, the number of different price paths would be 2^{26} (about 67 million paths), leading to 27 different ending prices.

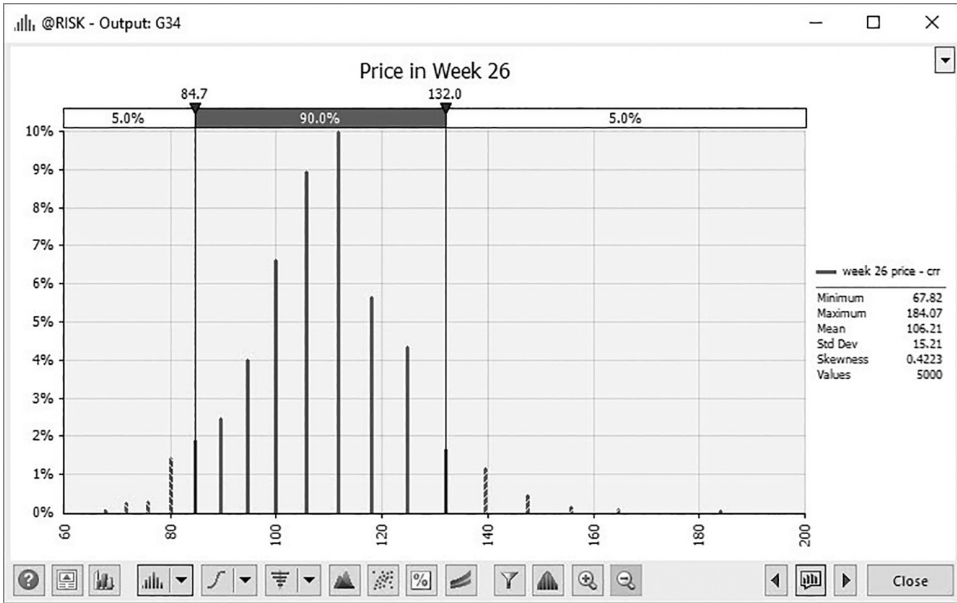


EXHIBIT 13.11 Simulated frequency distribution of stock price at the end of 26 weeks.

(number of trading days in a year). Increase the number of price outputs starting in D9 from 26 steps to 125 steps (i.e., Copy the Random Draw [columns E & J], Price Move (columns F & K), and Stock Price (columns G & L) to a total of 125 steps). Specify the cell that has the last price as the output and run the simulation. Exhibit 13.12 shows the frequency distribution of prices at the end of the half year. The mean and standard deviation

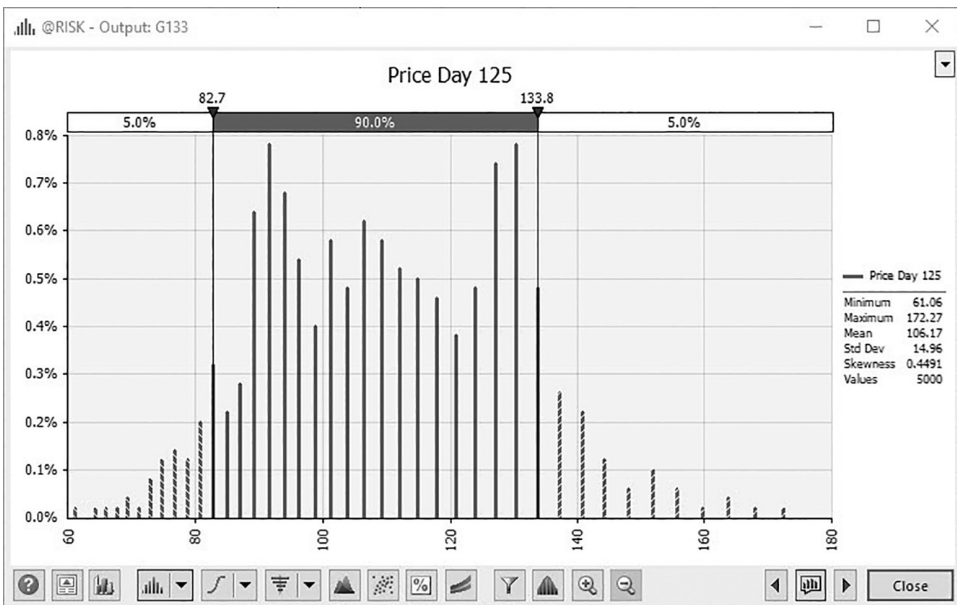


EXHIBIT 13.12 Frequency distribution of stock price at end of one and a half years with daily steps.

of the price are \$106.17 and \$14.96, respectively, as compared to \$106.21 and \$15.21 that we got with weekly steps (Exhibit 13.11), so the results are about the same. However, as you can see, the frequency distribution is almost completely filled in because there are many more possible terminal prices ($n + 1 = 126$ in this case), approximating what would occur with a continuous process.

13.4 RETURNS FOR A PORTFOLIO OF SECURITIES

We have seen that it is easy to model the process that a stock price might follow over time. In this section we will see how to simulate the results when we combine 2 or more securities in a portfolio. This will set the stage for the next chapter that delves more deeply into modeling portfolios.

In our portfolio model, we evaluate an investment in two securities, each of which follows the continuous time Weiner process that we introduced earlier. Exhibit 13.13 is similar to the model of stock returns for 1 stock shown in Exhibit 13.7. However, in this example there are two different investments, Stock A and Stock B, each of whose returns follow a Weiner process described by the expression

	A	B	C	D	E	F	G	H	I	J	K	L	M	
1	Stock Returns With Continuous Process													
2	$\text{Return} = \frac{dP}{P} = \mu dt + \sigma z \sqrt{dt}$													
3														
4														
5														
6	Data Input													
7		Portfolio	Stock A	Stock B										
8														
9	Total Amount Invested	100,000	70,000	30,000										
10	Proportion Invested	70%	70%	30%										
11														
12	Initial Stock Price		100.00	60.00										
13	Shares Purchased		700	500										
14														
15	Mean Return		12.0%	18.0%										
16	Standard Deviation		20.0%	35.0%										
17														
18	Correlation		0.30											
19														
20	Length of Sub-Period		1 Week											
21	Sub-Periods Per Year		52											
22	Length of Period		0.0192	fraction of year										
23														
24														
25														
26														
27														
28														
29														
30														
31	0			100	70,000				0		60	30,000		
32	1	0.735	2.27%	102.27	71,589	2.27%			1	1.417	7.22%	64.33	32,167	7.22%
33	2	0.620	1.95%	104.26	72,985	4.26%			2	0.667	3.59%	66.64	33,320	11.07%
34	3	1.101	3.28%	107.69	75,382	7.69%			3	1.643	8.32%	72.18	36,092	20.31%
35	4	0.186	0.75%	108.49	75,945	8.49%			4	0.383	2.21%	73.78	36,888	22.96%
36	5	-0.318	-0.65%	107.79	75,451	7.79%			5	-1.841	-8.59%	67.44	33,719	12.40%
37	6	0.786	2.41%	110.39	77,270	10.39%			6	1.224	6.29%	71.68	35,839	19.46%

EXHIBIT 13.13 Model portfolio of two stocks with returns that follow a Weiner process.

$$\text{Return} = \frac{dP}{P} = \mu dt + \sigma z \sqrt{dt} \quad (13.3 \text{ reprised})$$

Each security has its own mean return, μ , and standard deviation of return, σ . The mean returns are in cells C15 and D15 as 12% and 18%, respectively, and the standard deviation of annual returns are in cells C16 and D16 as 20% and 35%. The current market prices of the two stocks are \$100 and \$60 per share, as shown in cells C12 and D12. The other data input is the correlation coefficient between the returns on the two stocks, which has been estimated to be 0.30 as shown in cell C18. We explain correlation shortly.

For the investment portfolio, the decision variables are the total amount to and the proportions of our total investment allocated to that we will put in each stock. In this example, the total investment is \$100,000 (cell B9), and the proportions invested are 70% in stock A and 30% in stock B (cells C10 and D10). The returns are simulated for weekly subperiods over 26 weeks. The number of subperiods per year is input as 52 in C21, and C22 translates that into the fraction of the year as 1/52.

The Monte Carlo simulation of the returns on stock A start in line 31, and proceed by weeks in the same way as shown earlier in [Exhibit 13.7](#). Cell B32 is the probability distribution function =RiskNormal(0,1), and this and other return formulas are copied down column B for 26 periods to row 57. The return (cell C32), is calculated according to the Weiner process formula, Equation 3.3. The stock price (cell D32) is calculated according to Equation 13.14.

$$\begin{aligned} \text{Price}_t &= \text{Price}_{t-1} (1 + \text{Return}_t) \\ &= \text{D31} * (1 + \text{C32}) \end{aligned} \quad (13.14)$$

The stock values in column E are simply the price times the number of shares (C13) in the initial portfolio. Return to Date in column F calculates the cumulative return at each date as

$$\text{Cumulative Return}_t = \left(\frac{\text{Price}_t}{\text{Price}_0} \right) - 1 \quad (13.15)$$

The returns and prices for stock B are calculated in the same way in columns I through M.

Armed with total value of each of the two stocks at each date, we combine them in the portfolio. [Exhibit 13.14](#) shows the part of the model that does the return and value calculations for stock B and for the portfolio as a whole. The portfolio calculations are in columns O, P, and Q. Portfolio Value in column O simply adds together the total values of stock A and B to get the total of the two taking account of the price of each stock and the number of shares. Column P calculates the return for each subperiod as

$$\text{Portfolio Return for Period}_t = \left(\frac{\text{Portfolio Value}_t}{\text{Portfolio Value}_{t-1}} \right) - 1 \quad (13.16)$$

	N	O	P	Q	R	
21						
22	=Stock A + Stock B					
23			=(O32/O31) - 1			
24						
25						
26					=(O32/\$O\$31) - 1	
27		Portfolio	Portfolio Return			
28		Portfolio	for	to		
29		Value	Period	Date		
30						
31		100,000				
32		101,173	1.17%	1.17%		
33		101,017	-0.15%	1.02%		
34		106,439	5.37%	6.44%		
35		104,832	-1.51%	4.83%		
36		105,696	0.82%	5.70%		
37		110,983	5.00%	10.98%		
38		110,333	-0.59%	10.33%		
39		103,400	-6.28%	3.40%		
40		100,305	-2.99%	0.31%		
41		104,605	4.29%	4.61%		
42		102,057	-2.44%	2.06%		
43		99,153	-2.85%	-0.85%		
44		100,367	1.23%	0.37%		
45		98,768	-1.59%	-1.23%		
46		102,019	3.29%	2.02%		
47		106,028	3.93%	6.03%		
48		105,199	-0.78%	5.20%		
49		103,822	-1.31%	3.82%		
50		106,557	2.63%	6.56%		
51		107,487	0.87%	7.49%		
52		111,104	3.37%	11.10%		
53		110,487	-0.56%	10.49%		
54		110,293	-0.18%	10.29%		
55		110,772	0.43%	10.77%		
56		104,759	-5.43%	4.76%		
57		106,886	2.03%	6.89%		

EXHIBIT 13.14 Model of portfolio of two stocks.

and column Q calculates the return to date based on the original investment as

$$Portfolio\ Return\ to\ Date_t = \left(\frac{Portfolio\ Value_t}{Portfolio\ Value_0} \right) - 1 \tag{13.17}$$

All the formulas are copied down to row 57, so we have returns and values through week 26. Our primary concern is the value of the portfolio and the cumulative portfolio return at the end of the 26-week horizon. But before discussing the end results, we need to explain one of the most important aspects of the portfolio analysis—the correlation of returns.

13.4.1 Correlating Returns

When we combine two or more securities in a portfolio, we need to take account of the correlations in their returns. Correlation refers to the way in which the security prices and returns move with or against each other. If stocks tend to go up together, they are positively correlated; if they tend to move in opposite directions, they are negatively correlated. As we observe from daily news reports, share prices of individual companies tend to move in the same direction as other companies in the same industries and as the market as a whole. The market is, after all, the sum total of all the individual stocks, and movements in stock prices tend to be positively correlated.

Correlation is important in constructing a portfolio because it affects the overall risk of the portfolio as measured by the standard deviation of returns. Holding stocks that have low or negative correlation can reduce portfolio risk, significantly in some cases. However, holding stocks that are highly positively correlated does little to reduce portfolio risk. Hence, correlation must be considered when modeling portfolio value and returns.

In @RISK, variables are correlated by specifying the correlation between the probability distributions. In this model we do this by specifying a correlation between the draws of the standard normal distributions for stocks A and B that are in columns B and I. In other words, cells B32:B57 each contain the @RISK distribution function =RiskNormal(0,1) that generates the random draw of the standard normal variate, z , for stock A for each of the 26 periods; and cells I32:I57 contain the same distribution formulas for stock B. We want to make the random draws for stock B in column I correlated with the draws for stock A in column B. We do this with the correlation dialog box in @RISK. The steps for our model are as follows:

1. Select the cells containing probability distributions that you wish to correlate: Highlight cells B32:B57 (for stock A) and then, holding down the CTRL key, highlight cells I32:I57 (for stock B).
2. Click the Define Correlations icon on the @RISK ribbon (on the top left of [Exhibit 13.15](#)). This brings up the Define Correlations dialog box shown on the right that shows a few cells of the matrix of correlation coefficients for the selected random variable cells.
 - a. Matrix Name: In the Matrix Name box, type in a name for our correlation matrix. We name the matrix ReturnCorrelations1 and type it into the box as shown.
 - b. Location Box: Indicate the location on the spreadsheet where the correlation matrix will be placed. In this case, the correlation matrix will be placed in cells S30:BS82, which is indicated by entering the cell address for the upper left corner of the matrix. Click OK. @RISK places the correlation matrix in this location. [Exhibit 13.16](#) shows a portion of the initial matrix.

EXHIBIT 13.15 Defining correlations in @RISK.

	S	T	U	V	W	X
26						
27	@RISK Correlation ReturnCorrelations1 Updated: 8/8/2016 11:36:16 AM					
28						
29						
30	@RISK Correlations	1 / Variate, z	2 / Variate, z	3 / Variate, z	4 / Variate, z	5 / Variate, z
31	1 / Variate, z in \$B\$32	1				
32	2 / Variate, z in \$B\$33	0	1			
33	3 / Variate, z in \$B\$34	0	0	1		
34	4 / Variate, z in \$B\$35	0	0	0	1	
35	5 / Variate, z in \$B\$36	0	0	0	0	1
36	6 / Variate, z in \$B\$37	0	0	0	0	0

EXHIBIT 13.16 Upper left portion of the @RISK correlation matrix.

3. Enter Correlations: The correlation coefficients could have been entered in the matrix shown as part of the dialog box in step 2. However, in this case we want the correlations to be set equal to the contents of the correlation cell C18 that is part of the model input. One way to do this is to enter $=\$C\18 in all of the appropriate cells in the correlation matrix that was inserted in cells S30:BS82. The appropriate cells are those representing the intersection of each period's stock A and stock B =RiskNormal(0,1) functions. Note in [Exhibit 13.16](#) that the rows and columns of the correlation matrix are labeled with the cell addresses. For example, the first correlation cell in T32 shows the row heading as "1/Variate, z in $\$B\32 ," and the column heading is the same. This is a diagonal cell in the matrix so the correlation is shown as 1, indicating that the B32 is perfectly correlated with itself. Cell T33 is off diagonal, with row heading "2/Variate, z in $\$B\33 " and column heading "1/Variate, z in $\$B\32 ." This cell refers to the correlation between the random variable in B32 and the random variable in B33—the random draw for stock A in period 1 and the draw for stock A in period 2. A zero correlation in this case means that the returns in period 1 are uncorrelated with the returns in period 2 for stock A, which is what we want because we do not want the returns to be serially correlated over time.

The cells where we want to enter the correlations are cells linking stock A and stock B returns in the same period. [Exhibit 13.17](#) shows a few of these cells of the correlation matrix. Cell S57 has row heading "1/Variate, z in $\$I\32 " and column heading "1/Variate, z in $\$B\32 ." This intersection refers to cells B32 and I32 in the model. These are the random draws for stocks A and B in period 1. First, note that the number in the cell is 0.30, and second, note in the formula line at the top that the formula

	S	T	U	V	W
30	@RISK Correlations	1 / Variate, z in $\$B\32	2 / Variate, z in $\$B\33	3 / Variate, z in $\$B\34	4 / Variate, z in $\$B\35
31	1 / Variate, z in $\$B\32	1			
32	2 / Variate, z in $\$B\33	0	1		
33	3 / Variate, z in $\$B\34	0	0	1	
34	4 / Variate, z in $\$B\35	0	0	0	1
35	5 / Variate, z in $\$B\36	0	0	0	0
54	24 / Variate, z in $\$B\55	0	0	0	0
55	25 / Variate, z in $\$B\56	0	0	0	0
56	26 / Variate, z in $\$B\57	0	0	0	0
57	1 / Variate, z in $\$I\32	0.30	0	0	0
58	2 / Variate, z in $\$I\33	0	0.30	0	0
59	3 / Variate, z in $\$I\34	0	0	0.30	0
60	4 / Variate, z in $\$I\35	0	0	0	0.30

EXHIBIT 13.17 Correlating stock A and stock B returns.

in the cell is `=C$18`. This specifies that the correlation between the draws is 0.30, but that this number is drawn from the input cell C18. That way, if we change the input correlation in C18, it will be changed in the correlation matrix.

Now look at the other cells along a diagonal from T57, that each show 0.30 as the correlation. Each one refers to the correlation between stock A and stock B in periods 1, 2, 3, and so on. The formula `=C$18` has been copied into each cell along this diagonal down to cell AS82, which relates returns in the period 26, so the correlation between returns is 0.30 in every period. If we want to change the input correlation, we only have to change the input cell C18, and all the relevant cells in the correlation matrix will be changed.

13.4.2 Running the Simulation

Now that we have a model that simulates returns for stocks A and B, with the correlations specified, we run a simulation and examine the results. We look at two types of output: the simulated pattern of stock returns and the values of the stocks and the portfolio over the 26 weeks. There are an infinite number of different returns and values that could be generated. One particular random draw of the simulation produced the pattern of returns that is shown in [Exhibit 13.18](#). The dashed line shows the return for stock A that is slightly less volatile than stock B. The more volatile solid line represents the returns for stock B that has a higher standard deviation of returns. Observe that the returns on stocks A and B tend to move together some times and in opposite directions other times. For example, in weeks 3–11 the returns on the stocks move together, first increasing, then decreasing, then increasing again. This reflects the positive correlation between the two stocks. However, they do not perform exactly in tandem. Some periods B is doing well when A is doing poorly as in week 2. This demonstrates the fact that the returns are imperfectly correlated.

Contrast that pattern with one we might get for securities that are negatively correlated. We change the correlation by changing input cell C18 from 0.30 to -0.90 and run the simulation. [Exhibit 13.19](#) shows the results from one random draw of the simulation. Note that when one stock has high returns, the other tends to have low returns. For example, in week 7 stock B has a negative return and stock A has a positive return. On the other hand, even

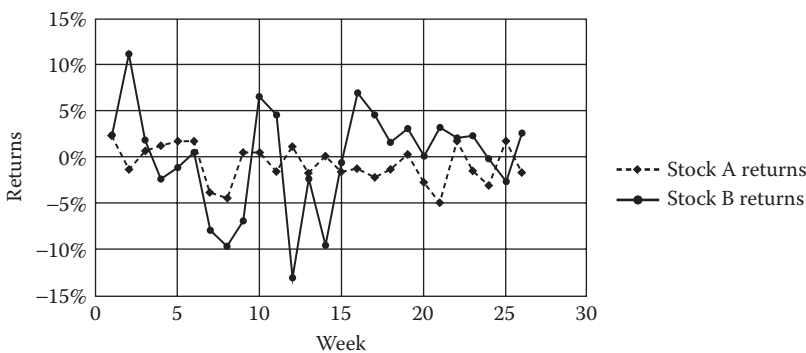


EXHIBIT 13.18 Simulated returns for stocks A and B with correlation of 0.30.

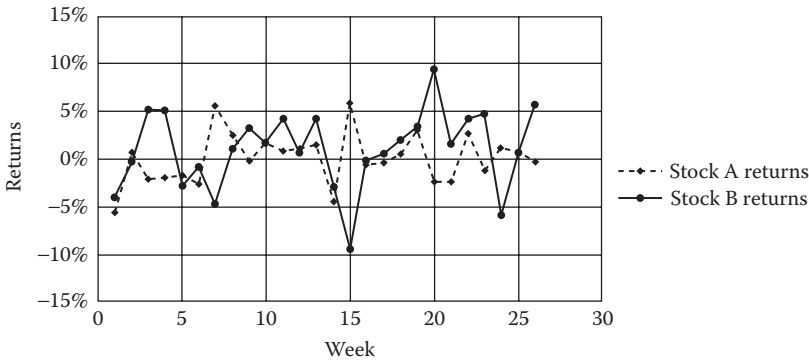


EXHIBIT 13.19 Simulated returns for stocks A and B with correlation of -0.90 .

with highly negative correlation, the stocks do not always move in the opposite direction. So long as the returns are not perfectly negatively correlated (correlation = -1.0) there is randomness in the relationship. For example, in week 7 both A and B are generating positive returns.

The simulation of returns on the two stocks did not really tell us what happened to the portfolio as whole. We now look at the distribution of returns for the portfolio that is a mix of the two securities. To see the distribution of portfolio returns over the whole investment period, we designate the cumulative return at the end of the 26-week period as the output for the simulation, and then run the simulation. Exhibit 13.20 shows the distributions of portfolio returns with 5,000 iterations for both the 0.30

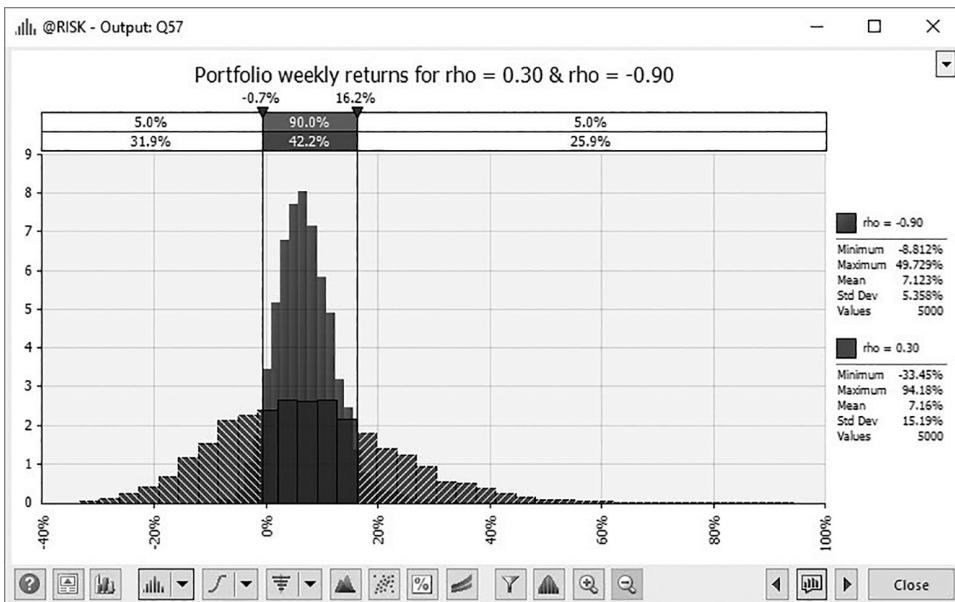


EXHIBIT 13.20 Frequency distribution of two-security portfolio returns for correlations of 0.30 and -0.90 .

and -0.90 correlation. The mean and standard deviation of the distribution when the correlation was 0.30 were 7.16% and 15.19% . For the correlation of -0.90 , they were 7.12% and 5.36% , a stunning reduction in portfolio risk. These are the 26 week, or semiannual rates. Converted to annual rates, the annual means and standard deviations are 13.83% and 21.48% when the correlation is 0.30 and 13.76% and 7.58% when the correlation is -0.90 .*

When you consider these results, the beneficial effects of diversification may not be immediately obvious. But consider that if we just took a weighted average of the risks (standard deviations) of the two stocks, where the weights are the portfolio proportions, we have $0.70(20\%) + 0.30(35\%) = 24.5\%$, which would be the portfolio standard deviation if the two stocks were perfectly correlated. In our example with a correlation of 0.30 , the portfolio standard deviation is about 21.5% . Although the difference is not great, it certainly helps that the range of variation of returns on the portfolio is smaller than would result with no benefit from diversification.

This distribution of returns is a function of the proportion of the total investment put in each of the two securities. In addition, it is a function of the correlation between the returns. It is left to the reader to explore these aspects by developing the model and varying these and other inputs. In the next chapter we will delve more deeply into modeling portfolios of securities.

13.5 SUMMARY

This chapter showed how we to simulate the pattern of stock prices and returns over time. We used two models of stock prices: the binomial model and the continuous time Weiner process. With the binomial model, stock prices are assumed to move up or down over discrete intervals of time. With the continuous time model, the return over very short intervals of time the return is generated as a Weiner process. The Weiner process results in stock prices following a random walk, meaning that the movement at the next moment in time is random and cannot be predicted from the past pattern of stock prices.

We showed how to construct a model of stock prices for each of these processes. Next, we showed how the binomial process can be used to approximate the continuous process by using small increments of time with small up and down movements. When the very small movements are repeated over many short periods, the resulting pattern of stock prices closely approximates the random walk pattern generated by the continuous process. Using these models and Monte Carlo simulation, we simulated price paths for a stock.

Having developed stock price models for a single stock, we then combined two stocks in a portfolio to show the importance of correlation between returns. First we showed how to add correlation to the @RISK model to simulate the returns on a portfolio. This portfolio model led us to consider how we can mix securities in portfolios and find the best risk-return trade-offs. This is the topic of the next chapter.

* We are using continuous compounding in the return model, where $(1+)$ the compound return over period dt is e^{rt} , where r is the annual return, and the standard deviation is $\sigma(dt)^{0.5}$. With $dt = 0.5$ year, we solve for the annual return as $r = \ln(1.0715)/0.5 = 0.13831$, and the annual standard deviation is $\sigma = 0.1519/(0.5)^{0.5} = 0.2148$ (for $\rho = 0.30$).

PROBLEMS

1. Construct a spreadsheet model of binomial stock prices.

Use monthly intervals and assume that each month the price of a stock can make an up move of $u = 1.106$ with a probability of 55%, or a down move of $d = 0.904$ each month with a probability of 45%. Assume the initial stock price is \$100 and the risk-free interest rate is 5%. Use @RISK to simulate the movements of the stock over the course of two years.

- a. Show the graph of the frequency distributions of stock values at the end of one year, and at the end of two years. What are the parameters of the distribution of stock values at the end of each of these periods?
 - b. Show the simulated distribution of stock returns at the end of one year, and at the end of two years. What are the parameters of the distribution of stock returns at each period?
 - c. What is the probability that you will lose money (negative cumulative return) over the intervals of one year and two years?
2. Construct a spreadsheet model for continuous returns similar to the model shown as [Exhibit 13.7](#). Assume the mean annual return of the stock is 18% and standard deviation of the annual return is 34.9%. With weekly intervals, use @RISK to simulate the stock price and the return on the stock over a period of 52 weeks.
 - a. Show the frequency distributions of stock prices and stock returns at the end of 52 weeks. What are the parameters of the frequency distribution?
 - b. Add a second stock whose mean annual return is 8%, with a standard deviation of annual return of 40%. Assume the correlation between the returns on the 1st and 2nd stocks is 0.50.

Assume you invest 40% in the 1st stock and 60% in the 2nd stock.
Simulate the returns on the portfolio over one year using weekly intervals.
Show the frequency distribution of returns for the portfolio at the end of the year. What are the parameters of the distribution of returns?

Modeling Investment Portfolios

14.1 INTRODUCTION

When Harry Markowitz (1952) published his paper on portfolio selection and mean-variance efficiency, he ushered in the new discipline of financial economics. His work earned him the 1990 Nobel Prize in Economics (along with William Sharpe and Merton Miller) and led to the development of the capital asset pricing model (CAPM) by William Sharpe (1964), Lintner (1965), and Mossin (1966). In the decades since, this seminal work continues to be used to determine asset allocation recommendations as well as to structure and create benchmarks for professionally managed portfolios.

Markowitz's contribution was to show how to measure the probabilistic effects of diversification on security returns. He showed how to calculate the mean and standard deviation of a portfolio of securities based on the parameters of the probability distributions of the component securities and the proportion of the total investment committed to each security, and he originated the concept of an efficient frontier and showed how to construct the efficient portfolios comprising it.

In this chapter, we use Markowitz's theory to model the risk and return of portfolios of securities and show how to use Solver to find efficient portfolios. We assume that the reader has been exposed to the basic ideas of portfolio analysis in previous financial management courses, and so will spend little time reviewing that material.* We show how to build models that combine a few securities into efficient portfolios.

* For a review of portfolio analysis, see any standard investments text such as Elton et al. (2014) or Reilly and Brown (2012), or a financial management text such as Brigham and Ehrhardt (2017), Brealy et al. (2017), Ross et al. (2016), or Berk and DeMarzo (2017).

However, these simple models are easily expanded to include many more.* The models we build require the use of some rudimentary linear algebra, Excel's array functions, and Excel's linear and quadratic program called Solver.

This chapter begins by reviewing the mean-variance problem in a two-asset world, the one typically introduced in introductory financial management and investments courses. We then review the elementary linear algebra needed to set up the problem when the opportunity set includes more than two assets. Next, we use Excel's array functions and Solver add-in to find the efficient frontier. We conclude this chapter with a brief discussion of the potential pitfalls of naïve mean-variance optimization and an overview of portfolio value-at-risk.

14.2 THE MEAN-VARIANCE PROBLEM FOR TWO ASSETS

Modern portfolio theory shows that holding two or more assets in a portfolio is often better than holding a single asset. This is because two securities do not always move in the same direction at the same time. When one security moves up and the other security moves down, the value of a portfolio containing both may change very little. Indeed, it is often possible to create portfolios of assets with risk and return attributes superior to any single asset. For example, consider [Exhibit 14.1](#). This graph plots the returns for portfolios comprised only of a bond fund and an *S&P* 500 index fund against their respective risks as measured by the standard deviation of return. Though the *S&P* index is considerably riskier than the bond fund, adding a little of the *S&P* index to the bond fund creates a portfolio with higher returns and less risk than the bond fund alone. For example, when 93% of a portfolio is allocated to the bond fund and when the remaining 7% is allocated to the *S&P* index fund, the expected return on that fund is 6.06% (versus the bond fund's 5.93%) and its standard deviation is 3.30% (versus the bond fund's 3.48%).

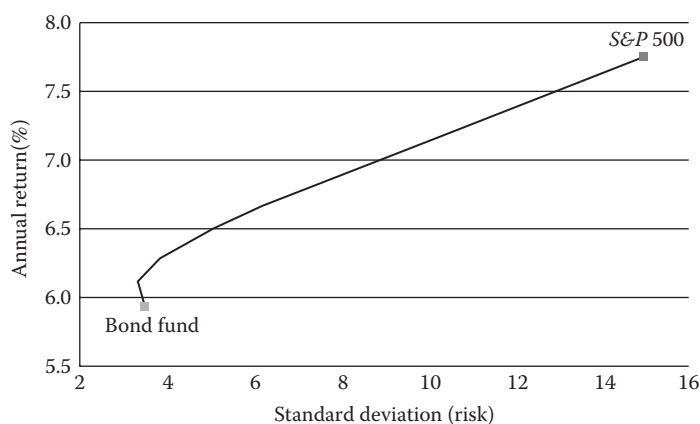


EXHIBIT 14.1 Portfolios comprised of a bond fund and the *S&P* 500 index.

* When a large number of securities are involved (e.g., considerably more than 100), the computational challenges are beyond the scope of this text and the capacity of less powerful personal computers. For greater detail and references relating to efficient portfolio computation, see "Techniques for Calculating the Efficient Frontier," [Chapter 6](#), in Elton et al. (2014).

The mathematics of two-asset portfolio return and risk are necessary to proceed. Expected portfolio return is a simple weighted average of the expected returns of the assets that comprise the portfolio. The formula is shown in Equation 14.1.

$$\begin{aligned} E[r_p] &= x_1 E[r_1] + x_2 E[r_2] \\ &= \sum_{i=1}^2 x_i E[r_i] \end{aligned} \quad (14.1)$$

where

$E[r_i]$ is the expected return on asset i

x_i is the proportion of asset i in the portfolio relative to total portfolio value

$x_1 + x_2 = 1$

The formula for portfolio risk, defined as the standard deviation of its returns, is shown in Equation 14.2.

$$\begin{aligned} \sigma_p &= [x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2x_1 x_2 \rho_{12} \sigma_1 \sigma_2]^{\frac{1}{2}} \\ &= \left[\sum_{i=1}^2 \sum_{j=1}^2 x_i x_j \rho_{ij} \sigma_i \sigma_j \right]^{\frac{1}{2}} \end{aligned} \quad (14.2)$$

where

x_1 and x_2 are the proportions of assets 1 and 2 relative to total portfolio value

σ_1 and σ_2 are the standard deviations of assets 1 and 2

$\rho_{1,2}$ is the correlation between the returns on assets 1 and 2

The source of the benefits from diversification can be seen in last term of the first line of Equation 14.2, $2x_1 x_2 \rho_{12} \sigma_1 \sigma_2$. The smaller the correlation, $\rho_{1,2}$, between two assets, the greater the potential benefits gained by creating a portfolio containing both of them. Indeed, under the extreme—and extremely unlikely—condition of a perfect negative correlation, portfolio risk can be eliminated.

Consider the following numerical example. Suppose an investor decides to allocate her total portfolio between two funds—a bond fund and an S&P 500 index fund (hereafter S&P index fund). The performance of these two funds, and the relationship between their returns, are summarized in [Exhibit 14.2](#).*

Suppose, too, that our investor allocates 40% of her total investments to the bond fund and allocates 60% to the S&P index. Then the expected return and standard deviation for this portfolio are shown in Equation 14.3.

* We use data for six Vanguard mutual funds from 1996 through 2007 throughout this chapter. End-of-month closing values, adjusted to reflect dividend and interest payments, were downloaded from finance.yahoo.com. Continuously compounded monthly returns and variances were calculated and then aggregated to create the annual statistics.

Fund statistics		
	Bond fund	S&P index
Average annual return	5.93	7.75
Standard deviation	3.48	14.92

Correlation matrix		
	Bond fund	S&P index
Total Bond	1.00	
S&P500	(0.09)	1.00

EXHIBIT 14.2 Correlation matrix and annual returns for bond and S&P index funds.

$$\begin{aligned}
 E[r_p] &= x_{Bond} E[r_{Bond}] + x_{S\&P} E[r_{S\&P}] \\
 &= 0.4 * 5.93\% + 0.6 * 7.75\% \\
 &= 7.02\% \\
 \sigma_p &= [x_{Bond}^2 \sigma_{Bond}^2 + x_{S\&P}^2 \sigma_{S\&P}^2 + 2x_{Bond} x_{S\&P} \rho_{Bond, S\&P} \sigma_{Bond} \sigma_{S\&P}]^{\frac{1}{2}} \\
 &= [0.4^2 * 3.48^2 + 0.6^2 * 14.92^2 + 2 * 0.4 * 0.6 * (-0.09) * 3.48 * 14.92]^{\frac{1}{2}} \\
 &= 8.9\%
 \end{aligned} \tag{14.3}$$

Before moving forward, we need to introduce the covariance matrix. Although the benefits of diversification derive from correlation, the mathematics of portfolio analysis takes variances and covariances as its inputs.* For example, using variances and covariances, Equation 14.3 may be rewritten as Equation 14.4.

$$\begin{aligned}
 \sigma_p &= [x_{Bond}^2 \sigma_{Bond}^2 + x_{S\&P}^2 \sigma_{S\&P}^2 + 2x_{Bond} x_{S\&P} \sigma_{Bond, S\&P}]^{\frac{1}{2}} \\
 &= [0.4^2 * 12.1 + 0.6^2 * 222.5 + 2 * 0.4 * 0.6 * (-4.9)]^{\frac{1}{2}} \\
 &= 8.9\%
 \end{aligned} \tag{14.4}$$

where:

$$\begin{aligned}
 \sigma_{Bond, S\&P} &= \text{the covariance between the bond and S\&P index funds} \\
 &= \rho_{Bond, S\&P} \sigma_{Bond} \sigma_{S\&P} = (-0.09) * 3.48 * 14.92 \\
 &= -4.9
 \end{aligned}$$

There is a compact way to organize variances and covariances—a covariance matrix. A covariance matrix uses the same structure as the correlation matrix, which was discussed in [Chapter 12](#), except that it reports covariances instead of correlations for the off-diagonal

* The covariance between two securities, denoted by $\sigma_{1,2}$, is the product of the correlation between the two securities and the standard deviations of each of the securities, expressed as $\sigma_{12} = \rho_{12} \sigma_1 \sigma_2$.

Covariance matrix		
	Bond fund	S&P index
Bond fund	12.1	(4.9)
S&P index fund	(4.9)	222.5

EXHIBIT 14.3 Covariance matrix for the bond and S&P index funds. Variances (12.1 and 222.5) are reported on the diagonal; the covariance (−4.9) on the off-diagonal.

terms (the terms not equal to 1 in the correlation matrix), and variances on the diagonal.* Equation 14.4 shows the variances of the bond and S&P index as 12.1 and 222.5, respectively, and the covariance between their returns as −4.9. Exhibit 14.3 shows how this data is organized in a covariance matrix.†

It is relatively straight forward to set up this two-asset problem in Excel as a dynamic model. The user enters him or her desired percentage allocation to the bond fund in cell A14, and the model returns the weight for the S&P index fund and the resulting portfolio's expected return and risk (standard deviation). An example with cell formulas is shown in Exhibit 14.4.

	A	B	C	D
2	Expected returns			
3		Bond fund	S&P index	
4	Annual return	5.93	7.75	
5				
6	Covariance matrix			
7		Bond fund	S&P index	
8	Bond fund	12.1	(4.9)	
9	S&P index fund	(4.9)	222.5	
10		=1-A14	=A14*B4+B14*C4	
11	Portfolio return and risk			
12	Weights		Return	Sigma
13	Bond fund	S&P index		
14	0.4	0.6	7.02	8.93
15	=SQRT(A14^2*B8+B14^2*C9+2*A14*B14*B9)			
16				

EXHIBIT 14.4 Asset returns, covariance matrix and the expected return and standard deviation for a two-asset portfolio.

* The covariance matrix is also referred to as the variance-covariance matrix because the entries on the diagonal are variances. However, variances are merely the covariance of an asset with itself, just as the correlation of an asset with itself is 1. To see this, consider $\sigma_{11} = \rho_{11}\sigma_1\sigma_1 = 1*\sigma_1\sigma_1 = \sigma_1^2$.

† Excel's DataAnalysis ToolPak has functions that will quickly calculate correlation and covariance matrices. The DataAnalysis ToolPak can be found on the "Data" ribbon in Excel. The DataAnalysis ToolPak is an Excel Add-In. To add it in, click on "File," select "Options," then "Add-ins," then "Manage: Excel Add-ins" and "Go." Select DataAnalysis ToolPak.

Now suppose that our investor is particularly risk averse. He or she wants to bear the minimum possible risk given a portfolio comprised of the bond and *S&P* index funds. What portion should he or she allocate to each fund? Our investor wishes to invest in the portfolio known as the minimum variance portfolio, or *MVP*. As its name implies, the *MVP* is the portfolio that has the least risk among all of the portfolios that can be formed from a given group of assets, in this case, from the bond fund and *S&P* index.

Mathematically, her problem is shown in Equation 14.5.

$$\text{MIN}_{\{x_{\text{Bond}}, x_{\text{Index}}\}} \sigma_p = [x_{\text{Bond}}^2 \sigma_{\text{Bond}}^2 + x_{\text{Index}}^2 \sigma_{\text{Index}}^2 + 2x_{\text{Bond}}x_{\text{Index}}\sigma_{\text{Bond,Index}}]^{\frac{1}{2}} \quad (14.5)$$

subject to:

$$x_{\text{Bond}} + x_{\text{Index}} = 1$$

Equation 14.5 says that the problem is to find the weights (x_{Bond} and x_{Index}) for the bond and *S&P* funds that minimize the standard deviation of a portfolio with the constraint (subject to:) that the weights must sum to 1. Certainly, the solution to this simple problem can be found quickly by trial and error, the calculus, or the formulas found in many investment textbooks. However, for larger portfolios, some type of quadratic optimization program is needed. Excel's Solver is one such program. We use Solver for this problem, even though the example and mathematics are transparent.

Before having Solver find the weights to the *MVP*, we need to add a cell to the model shown in [Exhibit 14.4](#) that calculates the sum of the portfolio weights (the condition under *subject to* in Equation 14.5). The portfolio model with this additional cell (B15) and the Solver dialog window are shown in [Exhibit 14.5](#).

Invoke Solver* and fill in its dialog window as follows:

- Set Objective: D14 (the cell that calculates the portfolio's standard deviation)
- To: Min (we seek to minimize the portfolio's standard deviation)
- By Changing Variable Cells: A14:B14 (the cells that contain the portfolio weights)
- Subject To the Constraints: B15 = 1 (the weights must sum to 1)

We assume no short selling. This means all weights must be positive. Select the "Make Unconstrained Variables Non-Negative" option. Finally, click on "Solve." Solver quickly returns the solution shown in [Exhibit 14.6](#).

Note the benefit from diversification. Combining a fund that is nearly four times riskier than the bond fund with the bond fund (a standard deviation of 14.9% vs. the bond fund's 3.5%) creates a portfolio with less risk and a higher expected return than the bond fund alone.

* Solver is invoked by clicking on Solver on the "Data" ribbon in Excel. It, too, is an Excel Add-in. Please see the previous footnote if you need to add it in.

	A	B	C	D	E	F	G
1							
2	Expected returns						
3		Bond fund	S&P index				
4	Annual return	5.93	7.75				
5							
6	Covariance matrix						
7		Bond fund	S&P index				
8	Bond fund	12.1	(4.9)				
9	S&P index fund	(4.9)	222.5				
10							
11	Portfolio return and risk						
12		Weights	Return	Sigma			
13		Bond fund	S&P index				
14		0.4	0.6	7.02	8.93		
15	Sum of weights	1					

Solver Parameters ✕

Set Objective: [fx]

To: Max Min Value Of:

By Changing Variable Cells: [fx]

Subject to the Constraints:

\$B\$15 = 1

Make Unconstrained Variables Non-Negative

Select a Solving Method: Options

Solving Method

Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

EXHIBIT 14.5 Two-asset portfolio with Solver dialog box set to find the MVP. Solving method is set to GRG Nonlinear.

Portfolio return and risk			
Weights		Return	Sigma
Bond fund	S&P index		
0.93	0.07	6.06	3.30

EXHIBIT 14.6 Two-asset minimum variance portfolio.

14.3 A LITTLE LINEAR ALGEBRA

The two-asset model is easy to set up using the standard Excel functions. However, the simple algebraic formulas quickly become intractable as more assets are considered. For example, for our two-asset portfolio, the code for the expected return cell contained two terms and code for the standard deviation cell contained three terms. This code is easy to manage. However, the number of terms required to calculate a portfolio's standard deviation expands quickly with the number of securities in the portfolio. For example, the formula for the standard deviation of a 20-asset portfolio requires 210 terms. It would be difficult to enter a formula with 210 terms into a cell in Excel without making an error. A little linear algebra and Excel's array functions quickly resolve this problem. Though instruction in linear algebra is beyond the scope of this text, we do show how to use Excel's array functions to make the necessary portfolio calculations for portfolios of many securities.*

Excel array functions operate on vectors and matrices, both of which are termed arrays in Excel. An array is a rectangular block of numbers. For example, consider [Exhibit 14.7](#). The numbers in the block B4:C4 form an array that has one row and two columns. We say it is a 1×2 array, where 1×2 refers to the number of rows by the number of columns.

An array can have any number of rows and columns. When it has more than one row and more than one column, we call it a matrix. In [Exhibit 14.8](#), the block of cells B8:C9 is a 2×2

	A	B	C
2	Expected returns		
3		BF	SP
4	Annual	5.93	7.75

EXHIBIT 14.7 A 1×2 array (darker shaded area).

	A	B	C
6	Covariance matrix		
7		BF	SP
8	Bond fund	12.1	(4.9)
9	S&P index fund	(4.9)	222.5

EXHIBIT 14.8 A 2×2 array (darker shaded area).

* We refer the interested to Strang (2003) or any other linear algebra textbook.

matrix—it has two rows and two columns. Covariance matrices are square and symmetric. Square means that the number of columns is the same as the number of rows. Symmetric means that the numbers in the upper right portion of the matrix (−4.9) are the same as those in the lower left portion of the matrix (−4.9). In [Exhibit 14.8](#), this is because the covariance of the returns on asset BF with those on SP is the same as the covariance of the returns on asset SP with those on BF. In mathematical terms, $\text{Cov}(r_{\text{BF}}, r_{\text{SP}}) = \text{Cov}(r_{\text{SP}}, r_{\text{BF}}) = -4.9$.

We use two array functions to calculate portfolio returns and standard deviations: =MMULT(array1, array2) to multiply arrays, and =TRANSPOSE(array) to change the shape of arrays. The steps for matrix multiplication are as follows:

1. Select (highlight) the block of cells into which the product of the matrix multiplication will be placed in the spreadsheet.
2. In a cell in the selected block, type the array function.
3. When you have finished typing the array function, enter it by simultaneously pressing the Ctrl+Shift+Enter keys.* This action puts {} brackets around the formula you have entered, telling Excel that it is an array formula, and completes the formula entry. Excel enters the formula into each of the cells in the selected block.

Before proceeding with the multiplication in the model, let us briefly look at matrix multiplication in general. Suppose we want to multiply the 1×2 array of portfolio weights for two securities, (w_1, w_2) times the 2×2 covariance matrix:

$$\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}$$

We write the matrix multiplication as follows:

$$[w_1, w_2] \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = [(w_1\sigma_{11} + w_2\sigma_{12}), (w_1\sigma_{12} + w_2\sigma_{22})]$$

The product is the 1×2 array $(w_1\sigma_{11} + w_2\sigma_{12}), (w_1\sigma_{12} + w_2\sigma_{22})$, where the terms in parentheses each represent 1 number.

To use the array functions in Excel, we need to have enough knowledge of matrix algebra to recognize that the product of the above operation will be a 1×2 array. Assume for the moment that we know these dimensions. Here is how to do the multiplication in Excel.

1. Select (highlight) the two-cell block B16:C16.
2. With the two cells selected, type in =MMULT(B13:C13,B8:C9). Do not hit the “Enter” key. The result of this sequence is shown in [Exhibit 14.9a](#).
3. Hit the key combination “Ctrl+Shift+Enter”

* If you inadvertently press “Enter” you will get Excel’s #VALUE! error message. If so, press F2 and then Ctrl+Shift+Enter.

	A	B	C
2	Expected returns		
3		BF	SP
4	Annual	5.93	7.75
5			
6	Covariance matrix		
7		BF	SP
8	Bond fund	12.1	(4.9)
9	S&P index fund	(4.9)	222.5
10			
11	Portfolio weights		
12		BF	SP
13		0.40	0.60
14			
15	Array Product		
16		=MMULT(B13:C13,B8:C9)	

(a)

		A	B	C	D	E	F
2	Expected returns						
3			BF	SP			
4	Annual		5.93	7.75			
5							
6	Covariance matrix						
7			BF	SP			
8	Bond fund		12.1	(4.9)			
9	S&P index fund		(4.9)	222.5			
10							
11	Portfolio weights						
12			BF	SP			
13			0.40	0.60			
14							
15	Array Product						
16			1.90	131.53			

(b)

EXHIBIT 14.9 (a) Entering an array function. (b) Entered array function (note the {.} in the formula bar).

The “Ctrl+Shift+Enter” enters the formula as an array formula into both the highlighted cells (B16:C16) and reports the products of the matrix multiplication as shown in Exhibit 14.9b. Note that the product of the 1 × 2 array multiplied into the 2 × 2 array is the 1 × 2 array of numbers, [1.90, 131.53]. These numbers are not meaningful for us at the moment. The example was used simply to show the method for array multiplication in Excel.

Armed with this brief introduction to matrix operations in Excel, we now show how to calculate the return and risk for a portfolio comprised of assets BF and SP.* Exhibit 14.10 shows the portfolio model for our two securities, BF and SP. The expected returns are

* The reader will recognize that these are the same calculations coded in simple algebra in section II.

	A	B	C	D	E	F	G	H
2	Expected returns							
3		BF	SP					
4	Annual	5.93	7.75					
5								
6	Covariance matrix							
7		BF	SP					
8	Bond fund	12.1	(4.9)					
9	S&P index fund	(4.9)	222.5					
10								
11	Portfolio weights							
12		BF	SP					
13		0.40	0.60					
14								
15	Array Product							
16		1.90	131.53					
17								
18	Portfolio statistics							
19	Expected Return	7.02	=MMULT(B13:C13,TRANSPOSE(B4:C4))					
20	Variance	79.68	=MMULT(MMULT(B13:C13,B8:C9),TRANSPOSE(B13:C13))					
21	Standard deviation	8.93	=SQRT(B20)					

EXHIBIT 14.10 Expected two-asset portfolio return and standard deviation calculated using array functions.

reported in cells B4:C4, the covariance matrix in B8:C9, and the decision variables, the portfolio weights, in B13:C13. Each of these blocks of numbers is an array.

We want to calculate the expected return and standard deviation of return for the portfolio consisting of securities BF and SP held in the weights indicated in B13:C13. The results of these array multiplications and the code used for each calculation are shown in [Exhibit 14.10](#). The portfolio expected return and standard deviation were computed to be 7.02% and 8.93% as shown in B19 and B21.

We now address a few remaining characteristics of array functions—dimension, multiplication order, the TRANSPOSE function, and nested MMULT statements. We have already encountered dimension—the number of rows and columns in an array. Multiplication order refers to the order the arrays are listed in the MMULT function. Dimension and multiplication order are inextricably linked. When multiplying one array by another, the column dimension (the second number) of the first array must match the row dimension (the first number) of the second array. For example, in [Exhibit 14.9b](#), we multiplied B13:C13 (a 1×2 array) into B8:C9 (a 2×2 array) and got the product B16:C16 (a 1×2 array). The multiplication worked because the dimensions “fit”: $(1 \times 2) \times (2 \times 2) = (1 \times 2)$.

Now consider the calculation for portfolio return. We multiply the weight array, B13:C13 (a 1×2 array) and the return array, B4:C4 (also a 1×2 array). The dimensions for these two arrays do not fit. The column dimension of the first array is 2 and the row dimension of the second array is 1, giving you $(1 \times 2) \times (1 \times 2)$. Here is where the TRANSPOSE function comes into play. If we change the dimensions of the second array (returns) to 2×1 , then the dimensions do fit— $(1 \times 2) \times (2 \times 1)$ —and the operation will succeed. The TRANSPOSE function does just that—it turns a row array into a column or a column array into a row. [Exhibit 14.11](#) shows how this works. Line 24 shows the return

	A	B	C	D	E	F	G	H
2	Expected returns							
3		BF	SP					
4	Annual	5.93	7.75					
5								
6	Covariance matrix							
7		BF	SP					
8	Bond fund	12.1	(4.9)					
9	S&P index fund	(4.9)	222.5					
10								
11	Portfolio weights							
12		BF	SP					
13		0.40	0.60					
14								
15	Array Product							
16		1.90	131.53					
17								
18	Portfolio statistics							
19	Expected Return	7.02	=MMULT(B13:C13,TRANSPOSE(B4:C4))					
20	Variance	79.68	=MMULT(MMULT(B13:C13,B8:C9),TRANSPOSE(B13:C13))					
21	Standard deviation	8.93	=SQRT(B20)					
22								
23	Array or function	Values	Dimension					
24	B4:C4	5.93	7.75	1x2				
25								
26	TRANSPOSE(B4:C4)	5.93		2x1				
27		7.75						

EXHIBIT 14.11 Transposing the return array.

array B4:C4 with dimension 1×2 . Lines 26 and 27 transpose this array from a row to a column, giving it the dimensions 2×1 . Using the transposed B4:C4 array, the operation =MMULT(B13:C13,TRANSPOSE (B4:C4)) succeeds.

This brings us to the code for portfolio variance, =MMULT(MMULT(B13:C13,B8:C9) TRANSPOSE(B13:C13)). This is a nested MMULT statement—one MMULT function is nested inside of another. The operation proceeds as follows. Excel calculates the nested MMULT command first, multiplying the arrays B13:C13 and B8:C9. As we saw earlier, this produces the array [1.90, 131.53]. Then, the outer MMULT multiplies this new 1×2 array into the 2×1 transposed array. This second product is the portfolio's variance, cell B20 in [Exhibit 14.11](#).

14.3.1 Naming the Arrays

When working with small arrays, it is a relatively simple and error-free process to type in the array function and enter the arrays by highlighting them or by typing in their cell addresses. However, when arrays become large, this process becomes quite cumbersome. In addition, models coded using cell addresses are not easy to troubleshoot. Excel's Name function resolves both of these problems. To see how it works, we recalculate portfolio returns and variance using named arrays.

The first step in using names is to name the arrays. To do this, highlight the array and type the name you wish to use for it into the name box, then press enter. This operation is shown in [Exhibit 14.12](#). Choose a name that will make sense to you should you revisit your model long after building it. I chose the name "omega_2" for the covariance matrix,

returns_2			
	A	B	C
2	Expected returns		
3		BF	SP
4	Annual	5.93	7.75

omega_2			
	A	B	C
6	Covariance matrix		
7		BF	SP
8	Bond fund	12.1	(4.9)
9	S&P index fund	(4.9)	222.5

weights_2			
	A	B	C
11	Portfolio weights		
12		BF	SP
13		0.40	0.60

EXHIBIT 14.12 Naming the return, covariance, and weight arrays.

because covariance matrices are frequently denoted by the Greek letter omega. The name “returns_2” applies to the array of expected returns, and “weights_2” names the array of portfolio weights. The extension “_2” is appended to the names to indicate the number of securities in the portfolio to distinguish these calculations from those for the six-security examples that come later in this chapter.

Having named the arrays, you need only type in their names as you write your code. [Exhibit 14.13](#) repeats [Exhibit 14.10](#), this time using names in place of cell references. We

	A	B	C	D	E	F	G	H
2	Expected returns							
3		BF	SP					
4	Annual	5.93	7.75	returns_2				
5								
6	Covariance matrix							
7		BF	SP					
8	Bond fund	12.1	(4.9)	omega_2				
9	S&P index fund	(4.9)	222.5					
10								
11	Portfolio weights							
12		BF	SP					
13		0.40	0.60	weights_2				
14								
15	Array Product							
16		1.90	131.53					
17								
18	Portfolio statistics							
19	Expected Return	7.02	=MMULT(weights_2,TRANSPOSE(returns_2))					
20	Variance	79.68	=MMULT(MMULT(weights_2,omega_2),TRANSPOSE(weights_2))					
21	Standard deviation	8.93	=SQRT(B20)					

EXHIBIT 14.13 Calculating portfolio return and variance using named arrays.

realize that using names for the first time can be daunting. However, the reward for perseverance is great. One last comment: The code shown in [Exhibit 14.13](#) will not change regardless of the number of securities in a portfolio. Only the names of the arrays will change.

14.4 THE EFFICIENT FRONTIER WITH MULTIPLE ASSETS

We now find the efficient frontier with no short selling* using six assets—the bond and S&P index funds from the previous example plus an international fund, a small capitalization fund (small cap), a real estate fund (REIT), and an emerging markets fund; all are Vanguard funds. Adjusted closing values were downloaded from finance.yahoo.com. The fund ticker symbols for the bond, index, international, small cap, REIT, and emerging market funds are VBMFX, VFINX, VWIGX, NAESX, VGSIX, and VEIEX, respectively. Monthly continuously compounded returns for the years 1996 through 2007 were then calculated and used to calculate average monthly returns and as input to Excel's Correlation and Covariance functions in the Data Analysis ToolPak. The annualized covariance matrix and the average annual returns were found by multiplying monthly covariances and average monthly returns by 12. The covariance and correlation matrices for these funds, along with their average annual returns and standard deviations, are reported in [Exhibit 14.14](#).

Annual statistics	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
Average return	5.93	7.75	8.00	8.62	10.19	12.29
Standard deviation	3.48	14.92	15.23	19.12	24.33	14.72

Correlation matrix	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
<i>Bond fund</i>	1.00					
<i>S&P index</i>	(0.09)	1.00				
<i>International</i>	(0.15)	0.79	1.00			
<i>Small cap</i>	(0.14)	0.75	0.76	1.00		
<i>Emerging markets</i>	(0.18)	0.72	0.84	0.74	1.00	
<i>REIT</i>	0.03	0.32	0.33	0.46	0.34	1.00

Covariance matrix	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
<i>Bond fund</i>	12.1	(4.9)	(8.0)	(9.4)	(15.2)	1.6
<i>S&P index</i>	(4.9)	222.5	179.8	212.9	260.8	71.0
<i>International</i>	(8.0)	179.8	231.9	222.7	309.8	74.0
<i>Small cap</i>	(9.4)	212.9	222.7	365.5	345.2	130.0
<i>Emerging markets</i>	(15.2)	260.8	309.8	345.2	592.1	122.5
<i>REIT</i>	1.6	71.0	74.0	130.0	122.5	216.6

EXHIBIT 14.14 Fund statistics used to calculate and graph the six-asset efficient frontier.

* Throughout this section, we assume that there is no short selling of securities. In the model, short selling would be indicated by portfolio weights less than zero. When solving the optimization problem with Solver, we require that the portfolio weights be nonnegative, thereby prohibiting short sales in our solution. We believe that no short selling is a realistic approach to finding the efficient frontier because the mutual funds held by most individual investors cannot be sold short. In addition, the unlimited short sales assumption presumes that the short seller will have use of the funds generated by the short sale. Again, this is rarely if ever the case for individual investors. Finally, the costs involved in short selling are typically large. Granted, ETFs are rapidly replacing mutual funds and they can be sold short. Even so, the latter two constraints on short selling would still obtain.

	A	B	C	D	E	F	G	H	I	J
3		Covariance matrix	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>	omega_6	
4		Total Bond	12.10	(4.90)	(8.05)	(9.43)	(15.16)	1.58		
5		S&P500	(4.90)	222.49	179.78	212.87	260.76	70.98		
6		Intl growth	(8.05)	179.78	231.92	222.70	309.79	73.96		
7		Small cap	(9.43)	212.87	222.70	365.52	345.22	129.99		
8		Emerging markets	(15.16)	260.76	309.79	345.22	592.12	122.50		
9		REIT	1.58	70.98	73.96	129.99	122.50	216.64		
10										
11							return_6			
12		Annual statistics	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>	<i>10-year treasuries</i>	
13		Return	5.93	7.75	8.00	8.62	10.19	12.29		4.98
14		Standard deviation	3.48	14.92	15.23	19.12	24.33	14.72		
15										
16		=SQRT(MMULT(MMULT(weight_6,omega_6),TRANSPOSE(weight_6)))								weight_6
17		=MMULT(weight_6,TRANSPOSE(return_6))								
18		Portfolio weights								
19		Solver Portfolio	Sigma	Return	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
20		Output row	12.28	8.80	0.17	0.17	0.17	0.17	0.17	0.17
21		Sum of weights			1.00	=SUM(weight_6)				
22		Target sigma	3.50							
23					return_6 = 1x6 vector of average annual returns					
24					omega_6 = 6x6 covariance matrix					

EXHIBIT 14.15 Solver portfolio model for six-asset efficient frontier.

The only changes to the code for portfolio returns and standard deviation that need to be made are to the names of the arrays. Once again we append numbers to the names to distinguish these arrays from those used in the two-asset example. The weight and return arrays (weight_6 and return_6) are 1 × 6 row arrays; the covariance matrix (omega_6) is a 6 × 6 array.

The only way to find the efficient frontier with no short selling is by finding one efficient portfolio at a time. Solver must be used repeatedly to maximize the expected returns for a given portfolio standard deviation and its output saved to find enough efficient portfolios to clearly trace out the efficient frontier. Therefore, the first step in finding the efficient frontier is to set up the portfolio calculations with an output structure that facilitates easy copying and pasting to a table. We call these portfolio calculations the “Solver portfolio” and show our structure in Exhibit 14.15.

We draw your attention to two aspects of this structure. First, the “Output row” (C20:J20) is set up to report the portfolio’s standard deviation first, followed by the return, and then the portfolio weights. This will facilitate drawing the efficient frontier in Excel once the necessary efficient portfolios have been identified by Solver. Second, all the output data is on one line, so that the entire line can easily be copied and pasted (as values) into a table. Third, note the “Target sigma” cell, C22. As mentioned earlier, Solver must be used repeatedly to map the efficient frontier. The modeler asks Solver to find the portfolio weights that maximize the return on a portfolio for a given standard deviation. The standard deviation is then increased, and Solver is asked to find the portfolio weights that maximize the return on a portfolio given the new standard deviation. Then the process is repeated. This means constantly having to change one of the constraints in Solver’s dialog window. One way to simplify the process is to link the standard deviation constraint in Solver to a specific cell (C22 in our model). Then one only needs to change the value in this cell before running Solver to find another efficient portfolio.

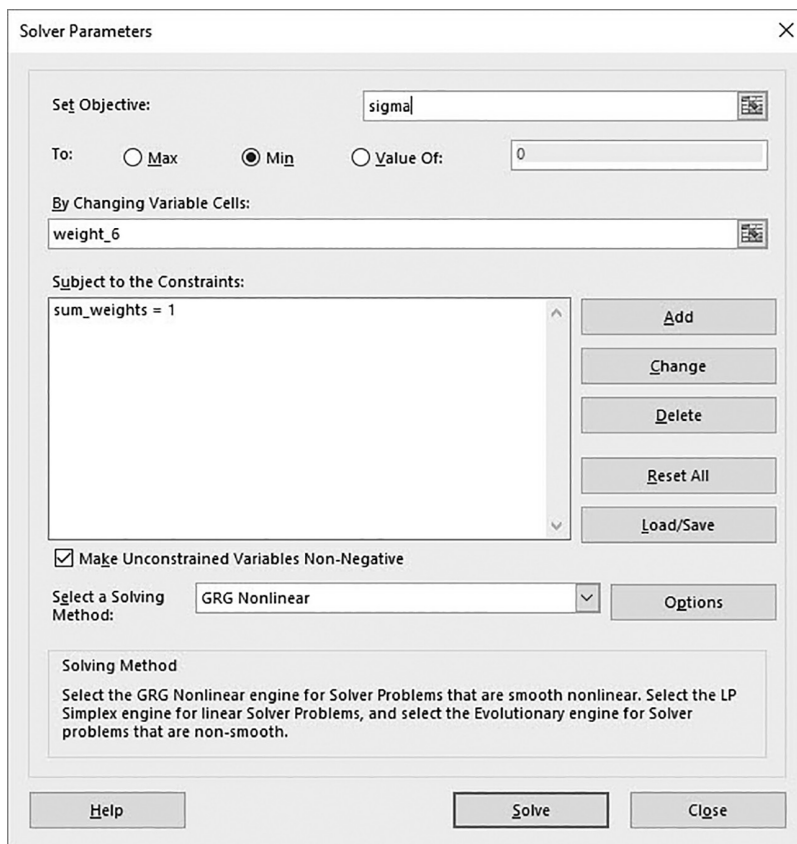


EXHIBIT 14.16 Solver dialog window set to find the minimum variance portfolio.

Next, set up the table into which Solver’s output will be pasted. [Exhibit 14.18](#) shows one way to structure this table. At a minimum, you need to copy and paste each efficient portfolio’s standard deviation and return. We recommend tracking the portfolio weights as well, as simple or naïve mean-variance optimization can produce counterintuitive recommendations (more on this later).

The efficient frontier begins with the *MVP*. As we did for the two-asset example, we use Solver to minimize the Solver portfolio’s standard deviation. The only constraints are that the weights sum to 1, and that the solution weights are nonnegative. This dialog window is shown in [Exhibit 14.16](#).^{*} Run Solver and copy and paste the output to the first row of your table.

The remaining efficient portfolios are found by using Solver to maximize the portfolio return for a given standard deviation, where the standard deviation constraint is entered in the “Target sigma” cell. Then the standard deviation is increased slightly, and Solver is run again. You need to “Add” this constraint to the Solver dialog box as shown

^{*} We named the portfolio standard deviation cell *sigma*, the portfolio returns cell *return*, and the sum of the weights cell *sum_weights*. Cells were named in the same way that the arrays were—by clicking on a given cell and typing the desired name in the “Name box.” While naming cells is certainly not necessary, it makes for a more transparent dialog window.

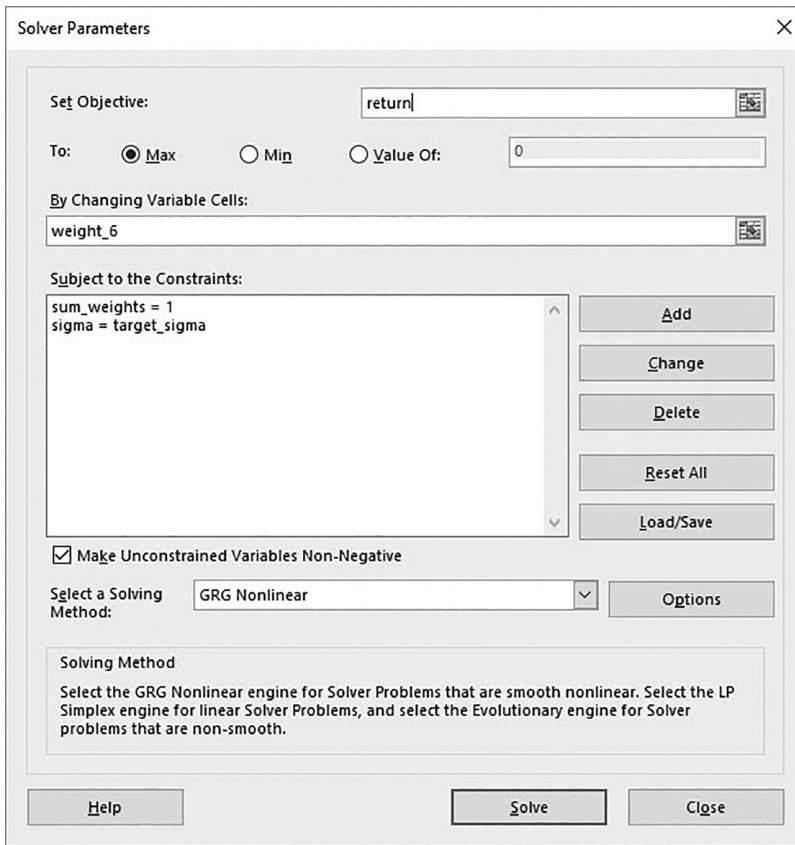


EXHIBIT 14.17 Solver dialog window set to maximize portfolio return for a target standard deviation.

in Exhibit 14.17. The incremental increases in the target sigma should be small at first, as small changes in sigma near the MVP result in relatively large increases in portfolio return. As the target sigma increases, the incremental changes can also increase.*

The table for the efficient frontier is shown in Exhibit 14.18. The table was constructed using Solver set as shown in Exhibit 14.17 and running it repeatedly. After each run, the output was copied and pasted as values into the table as another row. Then the value in the target sigma cell was increased so that the next run of Solver would find an efficient portfolio with a bit more risk. This process was repeated until the sigma constraint equaled the fund with the highest return, the end of the efficient frontier with no short selling.†

Once your table is complete, drawing the chart is easy. Highlight the sigma and return columns in the table, go to the “Insert” ribbon, and click on “Scatter” in the “Charts” panel and select “Scatter with smooth lines and markers” (we will remove the markers shortly). The chart will now float over your current worksheet. To add the underlying

* Alternatively, one can maximize the slope of the line connecting some arbitrarily chosen y -intercept to the efficient frontier, then increase the y -intercept.

† A program called SolverTable can automate this process. It can be downloaded from www.kelley.iu.edu/albright/freedownloads.htm.

	B	C	D	E	F	G	H	I	J
26		Portfolio weights							
27		Sigma	Return	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
28	MVP	3.24	6.18	0.91	0.01	0.06	0.00	0.00	0.02
29		3.24	6.25	0.90	0.01	0.06	0.00	0.00	0.03
30		3.25	6.31	0.89	0.01	0.06	0.00	0.00	0.04
31		3.26	6.35	0.89	0.01	0.06	0.00	0.00	0.04
32		3.28	6.41	0.88	0.01	0.06	0.00	0.01	0.05
33		3.30	6.46	0.87	0.01	0.06	0.00	0.01	0.06
34		3.35	6.55	0.86	0.00	0.05	0.00	0.01	0.07
35		3.40	6.63	0.85	0.00	0.05	0.00	0.01	0.09
36		3.50	6.76	0.83	0.00	0.05	0.00	0.01	0.11
37		3.60	6.86	0.82	0.00	0.04	0.00	0.02	0.12
38		3.80	7.04	0.80	0.00	0.03	0.00	0.02	0.15
39		4.00	7.19	0.78	0.00	0.03	0.00	0.02	0.17
40		4.50	7.53	0.73	0.00	0.01	0.00	0.03	0.23
41		5.00	7.82	0.69	0.00	0.00	0.00	0.04	0.27
42		6.00	8.35	0.61	0.00	0.00	0.00	0.04	0.35
43		8.00	9.32	0.45	0.00	0.00	0.00	0.04	0.51
44		10.00	10.23	0.31	0.00	0.00	0.00	0.04	0.65
45		12.00	11.12	0.17	0.00	0.00	0.00	0.04	0.79
46		14.72	12.29	0.00	0.00	0.00	0.00	0.00	1.00
47		14.75	11.58	0.00	0.00	0.00	0.00	0.34	0.66
48		14.80	11.56	0.00	0.00	0.00	0.00	0.35	0.65
49		14.90	11.53	0.00	0.00	0.00	0.00	0.36	0.64
50		15.00	11.51	0.00	0.00	0.00	0.00	0.37	0.63
51		16.00	11.28	0.00	0.00	0.00	0.00	0.48	0.52
52		17.00	11.11	0.00	0.00	0.00	0.00	0.56	0.44
53		20.00	10.69	0.00	0.00	0.00	0.00	0.76	0.24
54		22.00	10.45	0.00	0.00	0.00	0.00	0.88	0.12
55		24.30	10.19	0.00	0.00	0.00	0.00	1.00	0.00

EXHIBIT 14.18 Six-asset efficient portfolios with no short selling.

funds, right-click on the chart and choose “Select data.” Click on “Add” in the new dialog window, then add the assets, either individually (if you wish to identify them) or as a series (if you just wish to show the assets as a group). At this point, your chart will look something like that in [Exhibit 14.19](#).

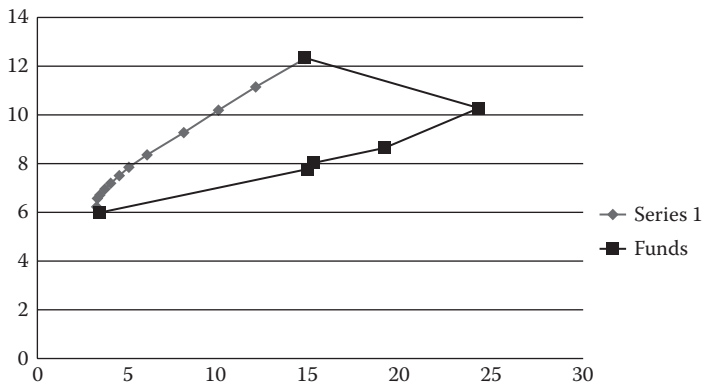


EXHIBIT 14.19 Chart of the six-fund efficient frontier and underlying funds as it is likely to appear before formatting and adding titles and legends.

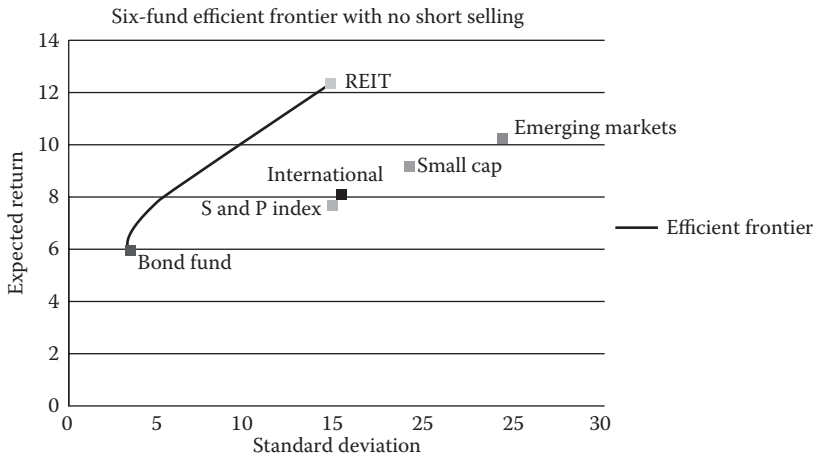


EXHIBIT 14.20 Efficient frontier.

To remove the markers from the efficient frontier curve, right-click on the data series, choose “Format Data Series,” then choose “Marker options,” and then choose “None.” To remove the lines connecting your assets series, right-click on the Assets series, select “Format Data Series,” select “Line Color,” and then select “No line.” To change the name of Series 1, right-click on the blue series, choose “Select Data,” highlight Series 1, and choose “Edit.” Type in the name you desire (e.g., Efficient frontier) or point the “Name” box to the cell containing the name you wish to use. To add chart and axis titles, click on your graph, then on the “Chart Tools” tab that floats above the ribbon names. Select the Chart Layout that you want, and then enter the desired titles. You are finished. Our finished chart is shown in [Exhibit 14.20](#).*

14.5 EXTENSIONS OF MODERN PORTFOLIO THEORY

The preceding pages explain how to find the efficient frontier with no short selling. We now consider some extensions to this basic model: finding the tangency portfolio (the market portfolio for the CAPM), and the problem of counterintuitive portfolio allocation recommendations and constrained optimization.

14.5.1 The Tangency Portfolio

It is common in most discussions of modern portfolio theory to assume that investors can borrow and lend at a risk-free rate of return. This assumption results in a linear efficient frontier that begins at the risk-free asset (the y -intercept) and lies tangent to the efficient frontier of risky assets. However, to draw this line, one needs to identify the tangency portfolio. In the discussion that follows, we assume a 10-year investment horizon and use as our risk-free rate of return the recent annual return on the 10-year U.S. Treasury note in 2007, 4.05%. It should be noted that the 10-year horizon is arbitrary. Investors should

* In the graph shown in [Exhibit 14.20](#), each asset was entered as a separate series. Once on the graph, each asset’s point was “right-clicked” and “Add Data Labels” was selected. By default, Excel reports the y -coordinate. Each series was right-clicked again, “Format Data Labels” was selected and the “Series name” option chosen.

match the risk-free rate of return to their investment horizon. For example, Lummer et al. (1994) assume an investment horizon of 20 years use the rate of return on Treasury bonds with maturities of 20 years.

Finding the tangency portfolio requires the addition of two cells to our Solver portfolio model: one containing the risk-free rate of return and the other containing the slope of the line connecting the risk-free asset to the efficient frontier. The numerator of the slope is the return on the Solver portfolio minus the risk-free rate; the denominator is the standard deviation of the Solver portfolio. The weights of the portfolio that Solver returns after maximizing this slope identify the composition of the tangency portfolio. The formula, using the names given in the arrays in our workbook is shown in Equation 14.6.

$$\begin{aligned} \text{Max}_{\{\text{weight}_6\}} [\text{slope}] &= \left[\frac{(\text{weight}_6 \times \text{TRANSPOSE}(\text{return}_6) - \text{riskfree_rate})}{(\text{weight}_6 \times \text{omega}_6 \times \text{TRANSPOSE}(\text{weight}_6))^{\frac{1}{2}}} \right] \\ &= \frac{(\text{return} - \text{riskfree_rate})}{\text{sigma}} \end{aligned} \tag{14.6}$$

These changes to our workbook, along with the tangency portfolio weights, are shown in Exhibit 14.21. The tangency portfolio weights were found by asking Solver to maximize Equation 14.6, the slope cell (C37) in Exhibit 14.21. The Solver dialog window used to find this solution, assuming no short sales, is shown in Exhibit 14.22.

Once the tangency portfolio is found, the line can be added to the chart by constructing a table with three observations. The first point on the line is the risk-free asset. The second point is the tangency portfolio just identified. The third point is a portfolio with an arbitrarily high standard deviation that extends the linear efficient beyond the tangency portfolio. We chose a standard deviation of 20% for this portfolio, large enough to extend the line beyond the REIT fund. The return on the third portfolio is found using the formula: $r_p = r_f + [(r_{\text{tangency}} - r_f) / \sigma_{\text{tangency}}] \sigma_p$, where σ_p is the standard deviation of this third high risk portfolio. (The reader likely will recognize this formula as the formula for the CAPM’s Capital Market Line.) To add this line to your portfolio, build the table shown in Exhibit 14.23, where “Extension” refers to this third portfolio. Right-click on your chart, select “Source Data,” click on “Add” and fill in the appropriate dialog boxes.

The final result will look like the chart shown in Exhibit 14.24.

	B	C	D	E	F	G	H	I	J
31		sigma	return	Portfolio weights					
32	Solver Portfolio	Sigma	Return	Total Bond	S&P500	Intl growth	Small cap	Emerging markets	REIT
33		3.86	7.09	0.79	-	0.03	-	0.02	0.16
34	Sum of weights			1.00					
35	Target sigma	3.50							
36	Risk free rate	4.05	riskfree_rate						
37	Slope	0.79	=(return-riskfree_rate)/sigma						
38									

EXHIBIT 14.21 The tangency portfolio found by having Solver maximize the slope of the linear efficient set.

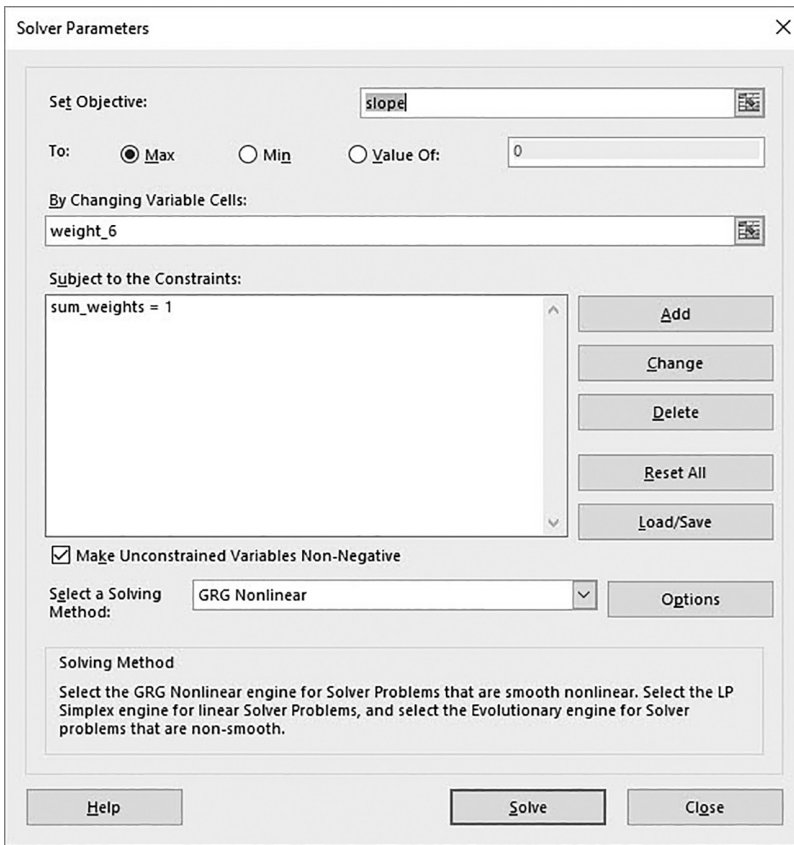


EXHIBIT 14.22 Solver dialog window to find the tangency portfolio.

	L	M	N
31	Linear efficient set		
32		<i>Sigma</i>	<i>Return</i>
33	Riskfree	0.00	4.05
34	Tangency	4.95	7.79
35	Extension	20.00	19.16
36	=N33+(N34-N33)/M34*M35		
37			

EXHIBIT 14.23 Table used to graph the linear efficient set.

14.5.1.1 Extension of the Model to the CAPM

The only differences between the linear efficient set just identified and the CAPM’s *ex post* Capital Market Line are the data used, and of course, the assumptions upon which the CAPM is based. If one were to accept the simple CAPM’s assumptions and use investors’ expectations for the future returns and covariances for all assets in the economy to construct the above graph, then the tangency portfolio identified would be the market portfolio, and the linear efficient set would be the Capital Market Line.

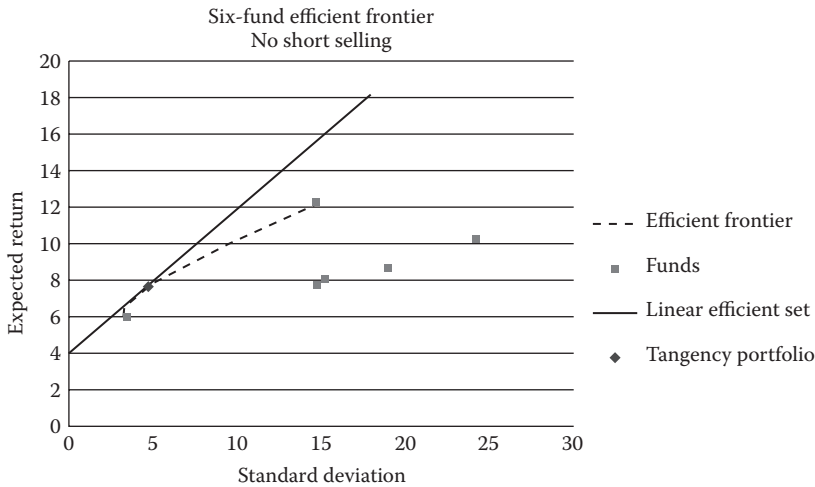


EXHIBIT 14.24 Linear efficient frontier with no short selling.

14.5.2 Counterintuitive Recommendations

Simple or naïve mean-variance optimization suffers from several criticisms. By simple or naïve, we mean that the optimizer is run without restricting its choice of weights other than no short selling. First, the recommended portfolios are often unstable, in that a subtle shift in the estimated asset returns, shifts that are well within estimation error, can produce a huge shift in recommended weights. Closely related to this criticism is the problem that the recommended portfolios can be counterintuitive.

Both of these problems are present in the recommended portfolios for the efficient frontier developed earlier in this chapter. Exhibit 14.25 reports the standard deviations, returns, and asset weights for this efficient frontier. Two assets dominate the portfolio recommendations—the bond fund and the REIT fund. Two portfolios that are typically assumed to be core assets, the *S&P* index and the international fund, receive almost

	M	N	O	P	Q	R	S	T
4	Portfolio weights							
5	Sigma	Return	Total Bond	S&P500	Intl growth	Small cap	Emerging markets	REIT
6	3.24	6.18	0.91	0.01	0.06	0.00	0.00	0.02
7	3.50	6.76	0.83	0.00	0.05	0.00	0.01	0.11
8	4.00	7.19	0.78	0.00	0.03	0.00	0.02	0.17
9	4.50	7.53	0.73	0.00	0.01	0.00	0.03	0.23
10	5.00	7.82	0.69	0.00	0.00	0.00	0.04	0.27
11	6.00	8.35	0.61	0.00	0.00	0.00	0.04	0.35
12	8.00	9.32	0.45	0.00	0.00	0.00	0.04	0.51
13	10.00	10.23	0.31	0.00	0.00	0.00	0.04	0.65
14	12.00	11.12	0.17	0.00	0.00	0.00	0.04	0.79
15	14.72	12.29	0.00	0.00	0.00	0.00	0.00	1.00

EXHIBIT 14.25 Efficient portfolios under naïve mean-variance optimization.

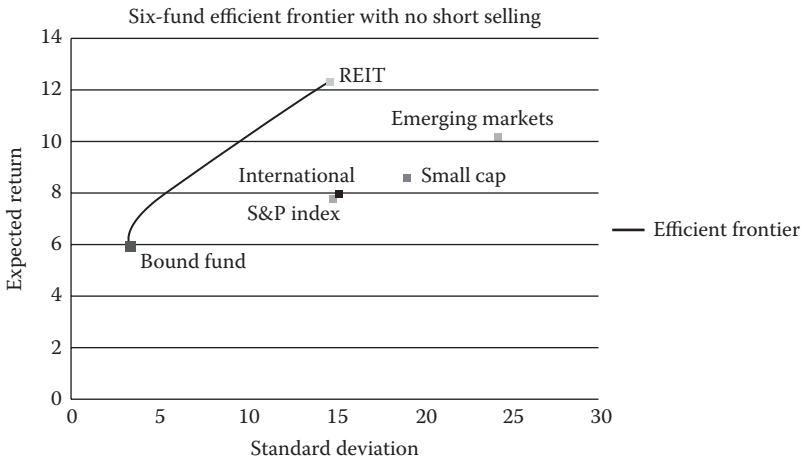


EXHIBIT 14.26 Naïve efficient frontier with underlying assets.

no weight. Further, the small cap index, arguably an important supporting part of any portfolio, is never included in the naïve optimizer's recommendations.

These problems stem from two problems in the data. First, the return on the REIT fund over our observation period is large relative to the other funds and its total risk (standard deviation) is small relative to its returns. Second, the returns on the emerging market fund are negatively correlated with the bond fund and larger than the returns on the *S&P* index, international and small cap funds. Hence, as the desired expected return increases, naïve mean-variance optimization selects the REIT and the emerging market funds to combine with the bond fund while excluding all others in the opportunity set.

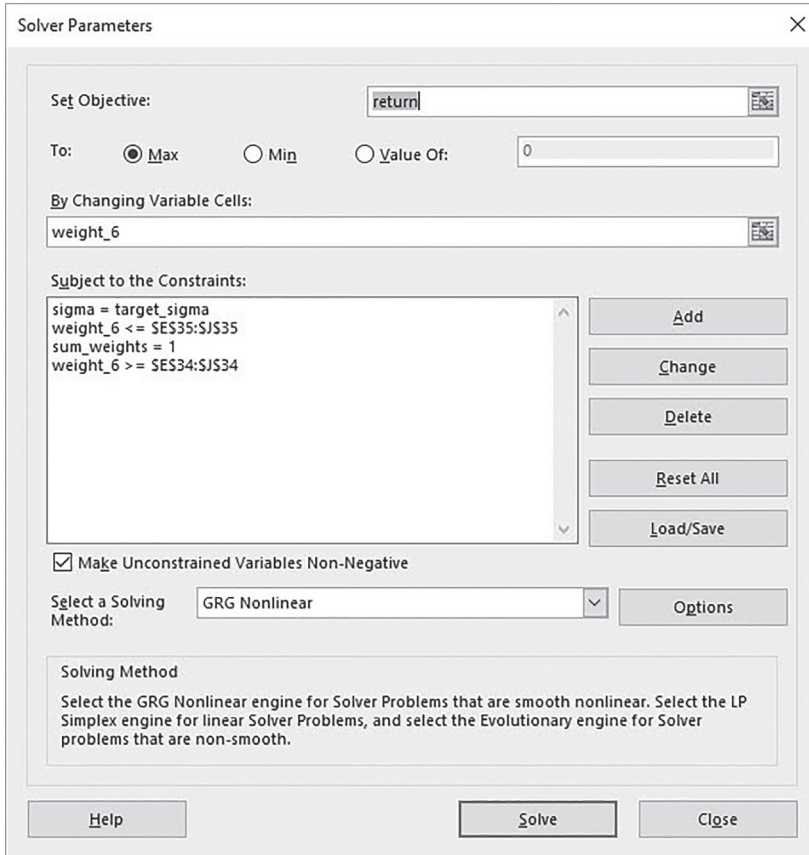
The second problem can be seen by looking at the *S&P* index-international-small-cap-fund cluster in [Exhibit 14.26](#). The international fund has a slightly higher return than the *S&P* index, lower risk than the small cap fund, and the lowest correlation with the bond fund. These attributes lead mean-variance optimization to select the international fund over the *S&P* index and small cap funds. Had either the *S&P* index or small cap fund had higher returns or lower correlations, or both, with the bond fund than the international fund, they would have dominated the other two funds in this cluster.

The solution to these problems is quite straight forward. The first step is to categorize the assets to be included in the optimal portfolio as either core or supporting assets. Suppose you have determined that you wish the core of your portfolio to be formed from the bond, *S&P* index, and international funds with minimum weights of 10%, 25%, and 15%, respectively. Suppose, too, that you wish the weights for each of the supporting funds to fall between 3% and 8%.*

* The values for these weight constraints are the subject of much debate among financial planning practitioners. The actual weights selected will be a function of an investor's age and level of risk aversion. The weights shown here are somewhat aggressive, given the low minimum for bonds and the relatively high maximums for the supporting funds. Still, the minimum variance portfolio of the constrained efficient frontier allocates 51% of the assets to the bond fund.

	B	C	D	E	F	G	H	I	J
28	Solver Portfolio			Portfolio weights					
29		Sigma	Return	Bond fund	S&P index	International	Small cap	Emerging markets	REIT
30		12.95	8.25	0.10	0.25	0.42	0.08	0.08	0.07
31	Sum of weights			1					
32	Target sigma	12.95							
33	Weight constraints								
34	Minimum			0.1	0.25	0.15	0.03	0.03	0.03
35	Maximum			1	1	1	0.08	0.08	0.08

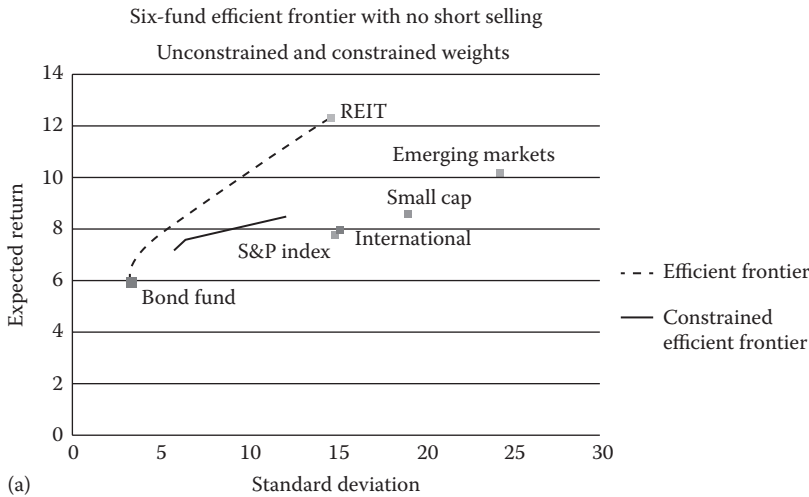
(a)



(b)

EXHIBIT 14.27 (a) Solver portfolio model with weight constraints. (b) Solver dialog window for constrained optimization.

These constraints are easy to integrate into our current Solver portfolio and Solver. One method is shown in [Exhibit 14.27](#). Once the weight constraints have been added to the Solver portfolio model, invoke Solver and constrain the weight vector to be greater than the minimum weights and less than the maximum weights by adding two more constraints in the Solver dialog window. The resulting dialog window is shown in [Exhibit 14.27](#). Calculate the constrained efficient frontier in the same manner as the unconstrained frontier. Identify the MVP; then maximize portfolio returns subject to the additional constraints. The results are shown in [Exhibit 14.28a](#) and [b](#).



	C	D	E	F	G	H	I	J
71	Portfolio weights							
72	Sigma	Return	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
73	6.95	7.09	0.51	0.25	0.15	0.03	0.03	0.03
74	7.00	7.14	0.50	0.25	0.15	0.03	0.03	0.04
75	7.50	7.46	0.45	0.25	0.15	0.03	0.04	0.08
76	8.00	7.57	0.42	0.25	0.15	0.03	0.07	0.08
77	9.00	7.74	0.37	0.25	0.15	0.07	0.08	0.08
78	10.00	7.89	0.29	0.25	0.22	0.08	0.08	0.08
79	12.00	8.18	0.16	0.25	0.35	0.08	0.08	0.08
80	12.86	8.30	0.10	0.25	0.41	0.08	0.08	0.08
81	12.95	8.25	0.10	0.25	0.42	0.08	0.08	0.07
82	13.00	8.12	0.10	0.54	0.15	0.08	0.08	0.05
83	13.20	8.01	0.10	0.56	0.15	0.08	0.08	0.03

(b)

EXHIBIT 14.28 (a) Unconstrained and constrained efficient frontiers. (b) Portfolio weights for constrained efficient frontier.

The constrained frontier differs significantly from the unconstrained frontier, beginning with the *MVP*. Although the bond fund comprised more than 90% of the unconstrained *MVP*, it comprises just 51% of the constrained *MVP*. This is the most that can be allocated to the bond fund, given the minimum values for the other assets in the opportunity set for this example. As a result, the constrained *MVP* has more risk than its unconstrained counterpart, and the constrained efficient frontier starts further to the right in Exhibit 14.28a. In addition, the expected returns on the constrained efficient frontier are considerably smaller than those for the unconstrained frontier. This is a direct outcome of constraining the maximum weight on the REIT fund to 8%. Finally, the constrained frontier is considerably shorter than the unconstrained efficient frontier. The maximum return of 8.3% (the end of the constrained efficient frontier) is achieved by assuming a portfolio standard deviation of 12.86%. In this portfolio, the two funds with the lowest returns (the bond and *S&P* 500 funds) are allocated to their minimum weights, the three supporting funds are allocated to their maximum weights (all have higher returns than the core assets), and the remaining weight (41%) is allocated to the highest yielding core fund, the international fund.

The counterintuitive recommendations in this chapter's example result, at least in part, from the use of historical returns instead of forecast returns. In particular, real estate and emerging market funds experienced historically high returns over our observation period, especially relative to the staid returns on the *S&P* index, international, and small cap funds. Using forecast returns partly resolved this problem. Lummer et al. (1994) suggest using CAPM betas and the security market line to forecast future returns. Haugen and Baker (1996) use as many as 40 independent variables to forecast future returns with impressive results.

Ultimately, the source of the returns used is a judgment call. Forecasting, whether it involves projecting historical returns forward, using an asset pricing model such as the CAPM or Arbitrage Pricing Theory (APT), or using an ad hoc multiple-factor model, must be viewed as the process of substituting error for chaos. In the end, mean-variance optimization recommendations are simply the starting point for one's investment decisions, not the ending point.

14.6 VALUE-AT-RISK

Up to this point, this chapter has defined portfolio risk as the standard deviation of portfolio returns. For those comfortable with statistics, and assuming normally distributed returns, the mean and standard deviation of a portfolio's returns provide the information necessary to compare the risks and rewards of different risky portfolios. However, many people desire a simple and intuitive measure of risk. Value-at-risk (hereafter *VaR*) is such a measure. *VaR* is a compelling measure of portfolio risk because it conveys the market risk of a portfolio in one number, and that number, either in terms of dollars or returns, is the major motivation for assessing risk—the potential loss in value of a portfolio (Hendricks, 1996).

VaR tells the investor the loss on a portfolio he or she can expect to experience some specified percent of the time over a given investment horizon. Typically, the critical probability (the small percent) is either 1% or 5%, and the given investment horizon is either one day or two weeks. For example, if the two-week 5% *VaR* for a given portfolio of \$100 million is a loss of \$1.5 million, this tells the investor that 5% of the time, or once in any given string of 20 two-week periods, he or she can expect to lose more than \$1.5 million. Put another way, *VaR* expresses the price risk of a portfolio of assets in terms of the frequency with which a specific loss will be exceeded some given percentage of the time (Boudoukh et al., 1995).

There are many ways to calculate *VaR*.^{*} We present three: computation, historical simulation, and Monte Carlo simulation. In the discussion that follows, we calculate the *VaR* for two portfolios—the constrained *MVP* and the constrained maximized return portfolio calculated earlier in this chapter. We use all three methods to estimate the *VaR* of both portfolios over one-day and two-week horizons for a critical value of 5%, leaving values-at-risk with 1% critical value as an exercise for the reader.

14.6.1 Computational Method

The computational method is perhaps the simplest and fastest way to estimate *VaR*. We begin by making the assumption that returns are normally distributed and serially

^{*} The interested reader is referred to Hendricks (1996) for 12 examples.

[†] Serially uncorrelated returns means that the returns for one period are not related to returns in previous periods. See [Chapter 7](#) for a more complete explanation.

uncorrelated, then combine the Z -statistic from the standard normal distribution for the critical p -value with the return and standard deviation of returns for a given portfolio to calculate that portfolio's VaR .[†] The application is simple. For a critical value of 5%, the return VaR is the return for the 5th percentile of the portfolio's probability distribution (the expected median return being the 50th percentile). Suppose the expected return for a risky portfolio is μ and the standard deviation of its returns is σ . The Z -statistic for the 5th percentile of a standard normal distribution (the left 5% tail) is -1.645 .^{*} The expression for the 5th percentile return is shown in Equation 14.7.

$$VaR_{\alpha=0.05} = \mu - 1.645\sigma \quad (14.7)$$

Applying Equation 14.7 to real data is straightforward. We begin with the constrained MVP . The expected annual return and standard deviation of returns for this portfolio are 0.0709 and 0.0695, respectively. Assuming that there are 240 trading days per year, the daily return for the MPV portfolio is calculated as

$$\begin{aligned} E_{\text{daily}}[r_{MVP}] &= 0.0709 / 240 \\ &= 0.0002954 \end{aligned} \quad (14.8)$$

The daily standard deviation of returns is calculated as

$$\begin{aligned} \sigma_{MVP,\text{daily}} &= \frac{\sigma_{MVP,\text{annual}}}{\sqrt{240}} \\ &= \frac{0.0695}{\sqrt{240}} \\ &= 0.00449 \end{aligned} \quad (14.9)$$

Substituting these values into Equation 14.7 gives us the 5th percentile return for the MVP portfolio, thus.

$$\begin{aligned} VaR_{MVP,\alpha=0.05} &= \mu - 1.645\sigma \\ &= 0.000295 - 1.645 * 0.00449 \\ &= -0.0071 \end{aligned} \quad (14.10)$$

This tells us that the MPV portfolio can expect one-day losses greater (more negative) than -0.71% , 1 out of every 20 trading days. If the portfolio is currently worth \$100 million, then the investor can expect losses to exceed \$710,000 once every 20 trading days, or once a month.

VaR is a measure of relative risk. When viewed in isolation, it tells us very little about the risk of a given portfolio. It is best used to compare the risks of different portfolios. For this reason, we calculate the VaR for the constrained maximized return portfolio ($MxRP$) from earlier in this chapter. This portfolio has an expected annual return of 0.0830 and a standard deviation of returns of 0.1286. Equation 14.11 converts these to daily values and calculates the 5% critical value VaR returns.

^{*} The Z statistic corresponding to the critical probability is returned by the Excel function `=NormSInv(0.05)`, where the argument of the function, 0.05, is the critical probability, Z .

$$\begin{aligned}
E_{\text{daily}}[r_{MxRP}] &= 0.0830/240 \\
&= 0.000346 \\
\sigma_{MxRP,\text{daily}} &= 0.1286/\sqrt{240} \\
&= 0.0083 \\
VaR_{MxRP,\alpha=0.05} &= 0.000346 - 1.645 * 0.0083 \\
&= -0.0133
\end{aligned}
\tag{14.11}$$

This tells us that an investor in the *MxRP* portfolio can expect losses in excess of -1.33% , 1 out of every 20 trading days. If this portfolio, too, is currently worth \$100 million, then the investor can expect losses to exceed \$1.33 million once every 20 trading days, or once a month. Now the relative nature of *VaR* is clear. The extra return earned by *MxRP* portfolio comes at a price. The losses expected to be exceeded 5% of the time on any given trading day are nearly 90% greater than the similarly defined losses for the *MVP* portfolio.

The 5% *VaR* over a 2-week or 10-trading-day horizon is found by increasing the daily returns by a factor of 10 and the daily standard deviations by the square root of 10. These calculations are shown in Equations 14.12 and 14.13.

$$\begin{aligned}
E_{2\text{-week}}[r_{MVP}] &= 0.000295 * 10 \\
&= 0.00295 \\
\sigma_{MVP,2\text{-week}} &= \sigma_{MxRP,\text{daily}} * \sqrt{10} \\
&= 0.00449 * \sqrt{10} \\
&= 0.0142 \\
VaR_{MVP,2\text{-week}} &= 0.00295 - 1.645 * 0.0142 \\
&= -0.0204
\end{aligned}
\tag{14.12}$$

$$\begin{aligned}
E_{2\text{-week}}[r_{MxRP}] &= 0.000346 * 10 \\
&= 0.00346 \\
\sigma_{MxRP,\text{daily}} &= \sigma_{MxRP,\text{annual}} * \sqrt{10} \\
&= 0.0083 * \sqrt{10} \\
&= 0.0262 \\
VaR_{MVP,\alpha=0.05} &= 0.00346 - 1.645 * 0.0262 \\
&= -0.0396
\end{aligned}
\tag{14.13}$$

As would be expected, the size of the loss exceeded 5% of the time over any given two-week horizon is considerably greater than the loss of a one-day horizon for both portfolios.

14.6.2 Historical Simulation

The historical simulation method is more data intensive but less mathematically intensive than the computational method. To implement it, collect a daily series of portfolio values over some given horizon, calculate the one-day and two-week dollar and percentage changes for that series, then sort the changes series by size, and lastly find the observation that lays one observation above the bottom 5% of observations. Alternatively, once the changes and returns have been calculated, you can use Excel's =PERCENTILE(array, percentile) function to find the values-at-risk. Be aware, however, that this function reports an extrapolated percentile rather than an actual observation.

To apply the first method to the *MVP* and *MxRP* portfolios, we collected the daily adjusted closing prices for the six funds in our observation set from May 1, 2006 through May 1, 2008. Next, we determine the number of shares of each fund in the opportunity set held by the *MVP* and *MxRP* portfolios. This is done using the size of the portfolio, the weights of the underlying funds in each portfolio, and the prices of the underlying funds. We assume that both portfolios were worth \$100 million on May 1, 2006. The weights are those determined by Solver for the constrained minimum variance and maximum return portfolios. The prices we take from the adjusted closing values on May 1, 2006 as reported by finance.yahoo.com. For example, suppose we wish to calculate the number of shares of the bond fund held by the *MVP* portfolio. The weight allocated to the bond fund in the *MVP* portfolio is 0.51. The bond fund's closing price was \$8.78. Hence, the number of shares of the bond fund in the *MVP* portfolio is

$$\begin{aligned} \text{(Number of shares)} &= \frac{\text{(Portfolio size)} * \text{weight}}{\text{Price}} \\ &= \frac{(\$100 \text{ million}) * 0.51}{\$8.78} && (14.14) \\ &= 5,808,656 \end{aligned}$$

[Exhibit 14.29](#) reports prices, weights, number of shares held in each portfolio, and the code used to calculate the number of shares.

Having determined the shareholdings for each fund, we constructed portfolio value series for our two target portfolios over the May 2006–May 2008 horizon. We then calculated one-day and overlapping two-week returns for both portfolios.* The first 14 observations and calculations are shown in [Exhibit 14.30](#).

Once the changes in values are calculated, copy the changes and paste them as values in another table. Next, sort each series from largest to smallest using Excel's "Sort" function (found on the "Data" ribbon). After sorting, determine the number of observations

* We wish to capture the greatest two-week loss. We acknowledge that this likely creates a downward bias for the two-week VaR, but nonoverlapping periods would likely introduce a positive bias. Since downside risk is the focus of VaR, we chose the former bias.

	I	J	K	L	M	N	O
21	Portfolio weights and shares						
22	Assumed portfolio value	100,000,000					
23	May 1, 2006 prices	8.78	115.48	19.69	31.13	22.02	19.39
24							
25	Weights	VBMFX	VFINX	VWIGX	NAESX	VEIEX	VGSIX
26	MVP	=J\$22*J26/J23	0.51	0.25	0.15	0.03	0.03
27	MxRP		0.10	0.25	0.41	0.08	0.08
28	Shares if purchased now	VBMFX	VFINX	VWIGX	NAESX	VEIEX	VGSIX
29	MVP	=J\$22*J27/J23	5,808,656	216,488	761,808	96,370	136,240
30	MxRP		1,138,952	216,488	2,082,275	256,987	363,306

EXHIBIT 14.29 Number of shares in each fund held by the MVP and MxRP portfolios on May 1, 2006.

	Q	R	S	T	U	V
6	MPV Portfolio					
7	Value	Daily Change	Daily return	2-week change	2-week return	Date
8	100,000,000	=P9-P8	=Q9/P8			5/1/2006
9	100,484,883	484,883	0.0048			5/2/2006
10	100,161,096	(323,787)	(0.0032)			5/3/2006
11	100,458,185	297,089	0.0030			5/4/2006
12	101,165,119	706,934	0.0070			5/5/2006
13	101,151,334	(13,785)	(0.0001)			5/8/2006
14	101,239,758	88,424	0.0009			5/9/2006
15	101,115,879	(123,880)	(0.0012)			5/10/2006
16	100,393,875	(722,003)	(0.0071)			5/11/2006
17	99,664,243	(729,632)	(0.0073)	=P18-P8	=S18/P8	5/12/2006
18	99,517,743	(146,500)	(0.0015)	(482,257)	(0.0048)	5/15/2006
19	99,436,596	(81,148)	(0.0008)	(1,048,287)	(0.0104)	5/16/2006
20	98,428,978	(1,007,617)	(0.0101)	(1,732,117)	(0.0173)	5/17/2006
21	98,189,006	(239,973)	(0.0024)	(2,269,179)	(0.0226)	5/18/2006
22	98,329,075	140,069	0.0014	(2,836,044)	(0.0280)	5/19/2006

EXHIBIT 14.30 Calculating one-day and overlapping two-week dollar and percentage changes in MVP portfolio value.

below the 5th percentile. For example, we have 504 observations in our one-day series. Five percent of 504 equals 25.2, making the 26th observation from the bottom the 5% VaR. The result is shown in Exhibit 14.31. For the MVP portfolio, the dollar loss that will be exceeded 5% of the time is \$998,331; the percentage loss that will be exceeded 5% of the time is -0.91%. For the MxRP portfolio, these values are \$2,050,494 and -1.87%, respectively.

Excel's PERCENTILE function offers an alternative to copying, pasting, and sorting the change data. After creating the dollar and percentage changes, invoke the percentile function (in the Statistics library of functions), highlight the data series for which you wish to find the VaR, then enter the percentile desired. Exhibit 14.32 reports the values-at-risk found using this function. Note the extrapolation that has occurred for the changes. For the MVP portfolio, the historical value VaR for the dollar change is -\$998,331; the PERCENTILE function reports it as -\$993,491, a slight understatement.

	AL	AM	AN	AO	AP	AQ	AR	AS
484		MPV Portfolio			MxR Portfolio		Observation #	
485		Change	% change		Change	% change	Descending	Ascending
486	5% VaR	(998,331)	(0.0091)		(2,050,494)	(0.0187)	26	479
487		(1,007,617)	(0.0093)		(2,065,505)	(0.0188)	25	480
488		(1,022,844)	(0.0094)		(2,066,267)	(0.0188)	24	481
489		(1,045,139)	(0.0095)		(2,079,663)	(0.0188)	23	482
490		(1,053,069)	(0.0096)		(2,109,657)	(0.0190)	22	483
491		(1,060,735)	(0.0097)		(2,143,471)	(0.0191)	21	484
492		(1,091,534)	(0.0097)		(2,193,755)	(0.0192)	20	485
493		(1,098,690)	(0.0101)		(2,207,737)	(0.0195)	19	486
494		(1,185,807)	(0.0103)		(2,222,962)	(0.0199)	18	487
495		(1,187,529)	(0.0103)		(2,372,672)	(0.0204)	17	488
496		(1,213,971)	(0.0104)		(2,429,668)	(0.0216)	16	489
497		(1,236,107)	(0.0110)		(2,527,226)	(0.0217)	15	490
498		(1,262,837)	(0.0111)		(2,649,717)	(0.0224)	14	491
499		(1,268,920)	(0.0113)		(2,667,849)	(0.0229)	13	492
500		(1,301,993)	(0.0116)		(2,675,749)	(0.0235)	12	493
501		(1,311,473)	(0.0116)		(2,682,048)	(0.0235)	11	494
502		(1,345,635)	(0.0116)		(2,872,417)	(0.0237)	10	495
503		(1,358,034)	(0.0117)		(2,913,205)	(0.0240)	9	496
504		(1,429,013)	(0.0127)		(2,937,319)	(0.0256)	8	497
505		(1,444,986)	(0.0127)		(2,963,026)	(0.0259)	7	498
506		(1,472,531)	(0.0128)		(3,159,096)	(0.0271)	6	499
507		(1,529,086)	(0.0132)		(3,170,356)	(0.0272)	5	500
508		(1,570,814)	(0.0135)		(3,239,132)	(0.0279)	4	501
509		(1,605,385)	(0.0137)		(3,477,286)	(0.0286)	3	502
510		(1,751,725)	(0.0154)		(3,699,570)	(0.0336)	2	503
511		(1,834,793)	(0.0166)		(3,887,848)	(0.0353)	1	504

EXHIBIT 14.31 One-day 5% VaR for the MPV and MxRP portfolios found using the historical simulation method.

5% VaR from Historical simulation				
Values found using =PERCENTILE(array, percentile)				
	MPV Portfolio		MxR Portfolio	
Horizon	Change	% change	Change	% change
1 day	(993,491)	(0.0091)	(2,050,011)	(0.0187)
2 week	(2,708,757)	(0.0238)	(5,837,581)	(0.0501)

EXHIBIT 14.32 Five percent VaR values as determined by Excel's PERCENTILE(.) function (that extrapolates).

14.6.3 Monte Carlo Simulation

The Monte Carlo simulation approach begins by assuming that the returns on the six funds in our opportunity set follow a simple Weiner process as described in [Chapter 13](#). We then model the daily price movements over two weeks (10 trading days) for each fund and track the dollar and percentage changes over the first day, and the cumulative dollar and percentage changes over the two-week horizon. Finally, we run a Monte Carlo simulation of 5,000 iterations and read the 5% VaRs for each portfolio from the simulated probability distributions.

Inputs						
Historical annual performance						
	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
Return	5.93%	7.75%	8.00%	8.62%	10.19%	12.29%
Standard deviation	3.48%	14.92%	15.23%	19.12%	24.33%	14.72%
Length of sub-period (days)	1					
Sub-periods per year	240					
Length of period	0.0042					

EXHIBIT 14.33 Input sector for Monte Carlo simulation of *VaR*.

The first step is to build the Input section, as shown in [Exhibit 14.33](#). It contains the annual returns and their standard deviations for each of the six funds. As we model one-day changes, the length of the subperiod in days is 1. Assuming 240 trading days per year, the subperiod length in years is $1/240 = 0.0042$.

Next, we model the standard normal random variables, return processes, and price series for each of the six funds. We assume that the standard normal variables are correlated for each day according to their historical correlations (the correlation matrix calculated earlier in this chapter), but are serially uncorrelated. Entering all of the correlations in one correlation matrix would require a 60×60 matrix. To simplify the process, we chose instead to create separate correlation matrices for each trading day in our model. This resulted in ten 6×6 matrices that were filled in by copying and pasting the correlation matrix, facilitating data entry and greatly reducing the likelihood of error. After creating the 10×6 array of standard normal random variables and correlating each day's draws, return and price series were built following the procedures described in [Chapter 13](#). Initial prices were based on the adjusted closing fund prices on May 1, 2008, though any arbitrary price could have been used. In practice, the model builder will know the current prices and share holdings for the assets their portfolios. The resulting return process model is shown in [Exhibit 14.34](#); three of the correlation matrices are shown in [Exhibit 14.35](#).

Once the models of the returns processes and individual fund price series have been built, all that remains is to use the simulated price series data to construct two two-week series that simulate the values for the *MVP* and *MxRP* portfolios, calculate and track the one-day and two-week dollar and percentage changes, run a Monte Carlo simulation, and read the desired values-at-risk from the simulated probability distributions.

Once again, the first step in this final model-building phase is determining the shares of each fund in the opportunity set held by the *MVP* and *MxRP* portfolios. The process is the same as used in the previous section on historical simulation, except in this case we used prices from May 1, 2008 as our opening prices. The results are shown in [Exhibit 14.36](#).

After determining the number of shares held in each portfolio, we construct the two-week portfolio value series for each fund. We use Excel's SUMPRODUCT function to multiply the number of shares of each fund held by a given portfolio times their respective simulated prices for each trading day in the series. Judicious use of the dollar sign (\$) to make the weight array an absolute reference (meaning that it will not change when the

	B	C	D	E	F	G	H
2	Inputs						
3	Historical annual performance						
4		<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
5	Return	5.93%	7.75%	8.00%	8.62%	10.19%	12.29%
6	Standard deviation	3.48%	14.92%	15.23%	19.12%	24.33%	14.72%
7							
8	Length of sub-period (days)	1					
9	Sub-periods per year	240					
10	Length of period	0.0042					
11		=RiskNormal(0,1,RiskCorrmat(day_1,1))					
12	Return processes						
13	Normal processes	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
14	1	0	0	0	0	0	0
15	2	0	0	0	0	0	0
16	3	0	0	0	0	0	0
17	4	0	0	0	0	0	0
18	5	0	0	0	0	0	0
19	6	0	0	0	0	0	0
20	7	0	0	0	0	0	0
21	8	0	0	0	0	0	0
22	9	0	0	0	0	0	0
23	10	0	0	0	0	0	0
24							
25	Simulated daily returns	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
26		=C5*\$C\$10+C6*C14*SQRT(\$C\$10)					
27	1	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
28	2	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
29	3	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
30	4	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
31	5	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
32	6	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
33	7	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
34	8	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
35	9	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
36	10	0.000247	0.000323	0.000333	0.000359	0.000425	0.000512
37		=(1+C27)*C39					
38	Simulated daily prices	<i>Bond fund</i>	<i>S&P index</i>	<i>International</i>	<i>Small cap</i>	<i>Emerging markets</i>	<i>REIT</i>
39	0	10.12	129.27	24.11	31.77	32.31	22.21
40	1	10.12	129.31	24.12	31.78	32.32	22.22
41	2	10.12	129.35	24.13	31.79	32.34	22.23
42	3	10.13	129.40	24.13	31.80	32.35	22.24
43	4	10.13	129.44	24.14	31.82	32.36	22.26
44	5	10.13	129.48	24.15	31.83	32.38	22.27
45	6	10.14	129.52	24.16	31.84	32.39	22.28
46	7	10.14	129.56	24.17	31.85	32.41	22.29
47	8	10.14	129.60	24.17	31.86	32.42	22.30
48	9	10.14	129.65	24.18	31.87	32.43	22.31
49	10	10.15	129.69	24.19	31.88	32.45	22.32

EXHIBIT 14.34 Return process model to simulate VaR.

cell is dragged to a new position) facilitates the series construction. Enter the initial portfolio value of \$100 million for day 0. In the next row, use SUMPRODUCT to multiply the weights by their respective prices and sum the products, dollar-signing both sides of the array address for the weights. Then drag the cell down nine more rows. The value series is finished. See the code in cell C65 in Exhibit 14.36 for an example of how to do this.

Next, calculate the cumulative dollar and percentage changes for the 1st and 10th trading day of each of the portfolio series. Again, see the code in cells D74 and D75 of Exhibit 14.36 for examples. These are the values we will track to find the one-day and two-week

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	J	K	L	M	N	O	P	
12	Day 1							
13	@RISK Correlations	1 / Bond fund in \$	@RISK Correlation day_1 Updated: 7/19/200	index in \$D\$14	1 / International in \$E\$14	1 / Small cap in \$F\$14	1 / Emerging markets in \$G\$14	1 / REIT in \$H\$14
14	1 / Bond fund in \$C\$14	1						
15	1 / S&P index in \$D\$14	-0.09441509	1					
16	1 / International in \$E\$14	-0.151943743	0.791424902	1				
17	1 / Small cap in \$F\$14	-0.141800106	0.746462021	0.764883749	1			
18	1 / Emerging markets in \$G\$14	-0.179064044	0.718441344	0.835974861	0.742054667	1		
19	1 / REIT in \$H\$14	0.030923262	0.323316812	0.329956348	0.461954351	0.342043079	1	
20	Day 2							
21	@RISK Correlations	2 / Bond fund in \$	@RISK Correlation day_2 Updated: 7/19/200	index in \$D\$15	2 / International in \$E\$15	2 / Small cap in \$F\$15	2 / Emerging markets in \$G\$15	2 / REIT in \$H\$15
22	2 / Bond fund in \$C\$15	1						
23	2 / S&P index in \$D\$15	-0.09441509	1					
24	2 / International in \$E\$15	-0.151943743	0.791424902	1				
25	2 / Small cap in \$F\$15	-0.141800106	0.746462021	0.764883749	1			
26	2 / Emerging markets in \$G\$15	-0.179064044	0.718441344	0.835974861	0.742054667	1		
27	2 / REIT in \$H\$15	0.030923262	0.323316812	0.329956348	0.461954351	0.342043079	1	
28	Day 3							
29	@RISK Correlations	3 / Bond fund in \$	@RISK Correlation NewMatrix1	index in \$D\$16	3 / International in \$E\$16	3 / Small cap in \$F\$16	3 / Emerging markets in \$G\$16	3 / REIT in \$H\$16
30	3 / Bond fund in \$C\$16	1						
31	3 / S&P index in \$D\$16	-0.09441509	1					
32	3 / International in \$E\$16	-0.151943743	0.791424902	1				
33	3 / Small cap in \$F\$16	-0.141800106	0.746462021	0.764883749	1			
34	3 / Emerging markets in \$G\$16	-0.179064044	0.718441344	0.835974861	0.742054667	1		
35	3 / REIT in \$H\$16	0.030923262	0.323316812	0.329956348	0.461954351	0.342043079	1	

EXHIBIT 14.35 Correlation matrices for the first three trading days.

	B	C	D	E	F	G	H
37		$=(1+C27)*C39$					
38	Simulated daily prices	Bond fund	S&P index	International	Small cap	Emerging markets	REIT
39	0	10.12	129.27	24.11	31.77	32.31	22.21
40	1	10.12	129.31	24.12	31.78	32.32	22.22
41	2	10.12	129.35	24.13	31.79	32.34	22.23
42	3	10.13	129.40	24.13	31.80	32.35	22.24
43	4	10.13	129.44	24.14	31.82	32.36	22.26
44	5	10.13	129.48	24.15	31.83	32.38	22.27
45	6	10.14	129.52	24.16	31.84	32.39	22.28
46	7	10.14	129.56	24.17	31.85	32.41	22.29
47	8	10.14	129.60	24.17	31.86	32.42	22.30
48	9	10.14	129.65	24.18	31.87	32.43	22.31
49	10	10.15	129.69	24.19	31.88	32.45	22.32
50							
51	Portfolio weights and shares						
52	Assumed portfolio value	100,000,000					
53							
54	Weights	VBMFX	VFINX	VWIGX	NAESX	VEIEX	VGSIX
55	MVP	0.51	0.25	0.15	0.03	0.03	0.03
56	MxRP	$=$C$52*C55/C39$	0.10	0.25	0.41	0.08	0.08
57	Shares if purchased now	VBMFX	VFINX	VWIGX	NAESX	VEIEX	VGSIX
58	MVP	$=$C$52*C56/C39$	193,394	622,148	94,429	92,851	135,074
59	MxRP	988,142	193,394	1,700,539	251,810	247,601	360,198
60		$=SUMPRODUCT($C$58:$H$58,C40:H40)$			$=SUMPRODUCT($C$59:$H$59,C40:H40)$		
61	10-day Simulated Portfolio Performance						
62		MVP Value	Cumulative Change	Return	MxRp Value	Cumulative Change	Return
63	0	100,000,000			100,000,000		
64	1	100,029,554	29,554	0.000296	100,034,573	34,573	0.000346
65	2	100,059,117			100,069,158		
66	3	100,088,689			100,103,756		
67	4	100,118,270			100,138,366		
68	5	100,147,860			100,172,988		
69	6	100,177,459			100,207,623		
70	7	100,207,068			100,242,270		
71	8	100,236,685			100,276,929		
72	9	100,266,312			100,311,601		
73	10	100,295,948	295,948	0.002959	100,346,285	346,285	0.003463
74							
75		$=RiskOutput("MVP: 10-day change w/ corr")+C74-C64$			$=RiskOutput("MPV: 10-day return w/ corr")+D74/C64$		
76							

EXHIBIT 14.36 Fund prices, share holdings, portfolio prices, and dollar and percentage changes in value for the MVP and MxRP portfolios.

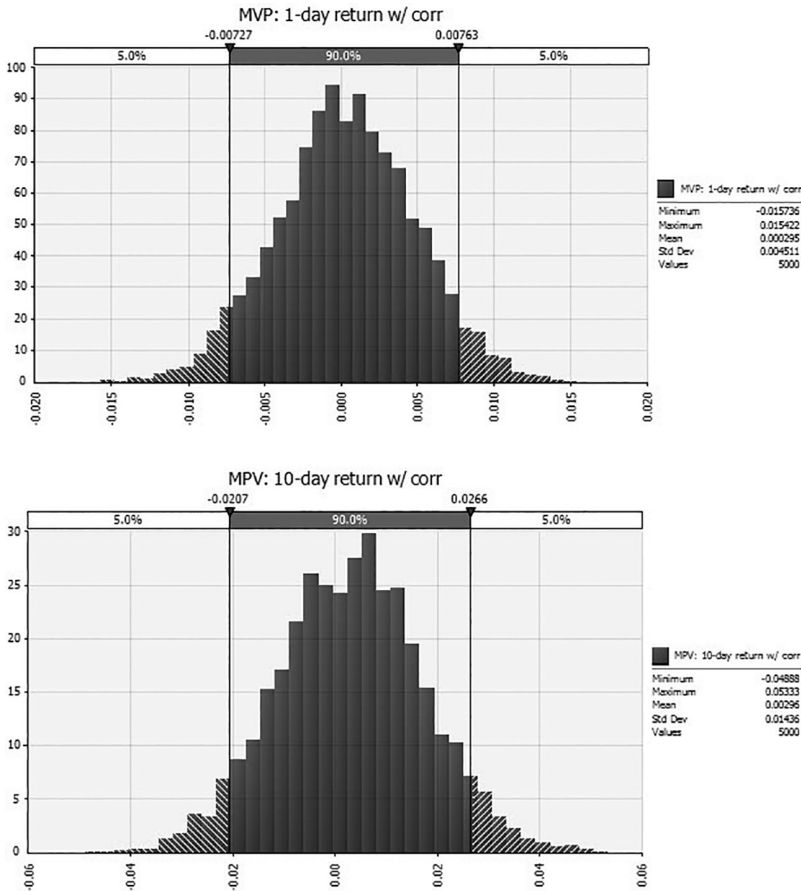


EXHIBIT 14.37 Simulated probability distributions of the 1-day and 2-week percentage changes for the *MVP* portfolio.

values-at-risk. Define each of these cells to be tracked (cells 065, E65, 074, E74, G65, H65, G74 and H74 in [Exhibit 14.36](#)) as @RISK output variables and run a 5,000 iteration simulation. Bring up the histograms for each of the output variables and take from them the values shown at the 5th percentile (the 90% confidence interval is @RISK's default setting). You are finished. These are the simulated 5% values-at-risk. [Exhibit 14.37](#) shows the histograms for the 1-day and 10-day (2-week) *MVP* portfolios. The 5th percentile one-day return is reported as -7.27 thousandths or -0.73% , and the two-week 5% return *VaR* is reported as -20.7 thousandths or -2.07% . [Exhibit 14.38](#) shows the histograms and values for the *MxRP* portfolio, -1.32% and 4.05% , respectively.

14.6.4 Discussion of Results

[Exhibit 14.39](#) summarizes the one-day and two-week return values-at-risk for the *MVP* and *MxRP* portfolios determined by the three methods considered in this chapter. Two important findings stand out. First, the computed values-at-risk are statistically insignificantly

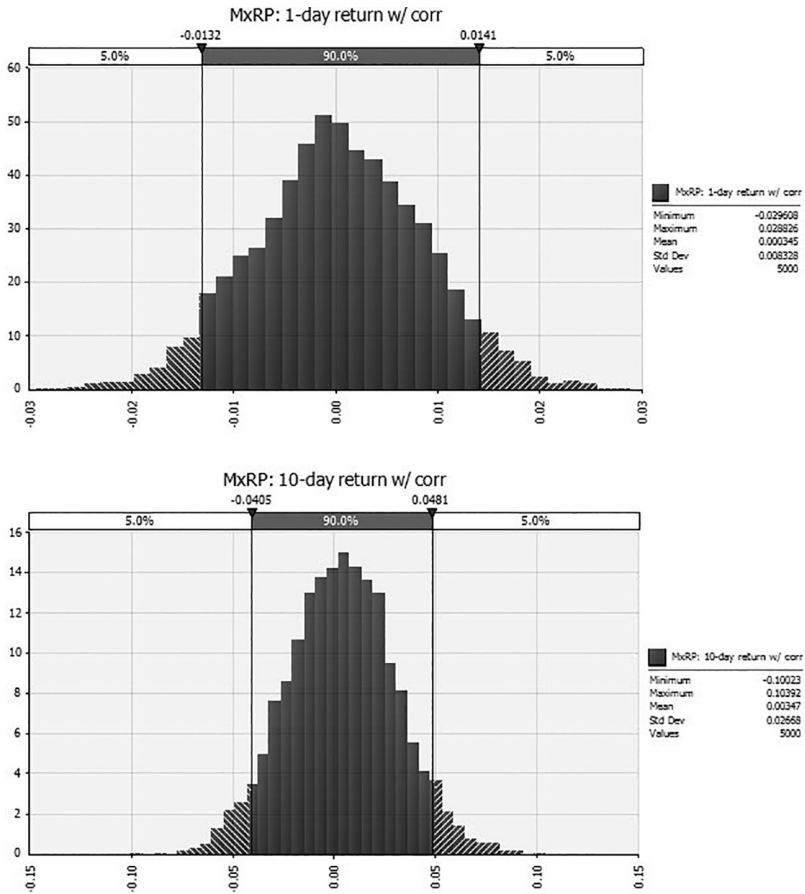


EXHIBIT 14.38 Simulated probability distributions of the 1-day and 2-week percentage changes for the *MxRP* portfolio.

5% VaR from computations		
Horizon	MVP % change	MxRP % change
1 day	-0.71%	-1.33%
2 week	-2.04%	-3.96%
5% VaR from Historical simulation		
1 day	-0.91%	-1.87%
2 week	-2.38%	-5.01%
5% VaR from Monte Carlo simulation		
1 day	-0.71%	-1.34%
2 week	-2.03%	-3.94%

EXHIBIT 14.39 Summary of findings for the three approaches to percentage return *VaR*.

different from those found using Monte Carlo simulation. This makes sense. All are based on the same underlying return process assumptions. Second, both the computational method and the Monte Carlo simulation method understate the actual *VaR* found using historical simulation by nearly 20%.

The cause of this difference likely stems from the assumption of normally distributed returns. Many returns distributions have fatter tails than predicted by the normal distribution (Duffie and Pan, 1997). *VaR* measures the tail risk for the returns on a given asset. If the tails of the actual return distribution are fatter than those predicted by the normal distribution, then values-at-risk based on the normal distribution will understate the tail risk. Although this problem more typically shows up when the critical value is 1%, it is nonetheless present for the 5% values-at-risk in our limited data set.

There are several solutions. The first is to use historical simulation to estimate *VaR*. This is the method recommended by Hendricks (1996). Alternatively, when the normal distribution is used, the analyst should recognize that *VaR* likely understates the true downside risk. Finally, the return process can be adjusted to produce fatter tailed return distributions. Interested readers are referred to Duffie and Pan (1997) and Zangari (1996).

14.7 SUMMARY

This chapter considered the risk and return for portfolios of risky assets. We began by showing how to use Excel's array functions to calculate the returns and the standard deviation of returns for portfolios with many securities. We first calculated the expected return and standard deviation of returns for a portfolio of six risky assets, each of which comprised 1/6th of the total portfolio's value. We then considered the benefits of efficient portfolios. We used Solver to find the portfolio that offered the least of amount of risk (the *MVP*) and portfolios that promised the highest returns for a given level of risk. The resulting portfolios formed the efficient frontier of risky assets.

Next, we showed how and why these portfolios are not necessarily the ones investors would want to hold. We then offered one solution to this problem—constraining Solver to find more intuitive portfolio allocation recommendations.

Lastly, we looked at an alternative measure of portfolio risk called *VaR*. *VaR* tells the investor the loss on a portfolio he or she can expect to experience some small percent of the time over a given investment horizon. It is a more intuitive way to measure portfolio risk than the standard deviation of returns, because it conveys the market risk of a portfolio in one number that is likely most important to an investor—the potential for significant losses.

PROBLEMS

1. Security Returns and Portfolio Composition

Use the following data to answer questions a–j. ([Exhibit 14.40](#)).

- a. Calculate the average quarterly returns and the standard deviations of those returns for each asset in the above table.
- b. Use Excel's Correlation function (in DataAnalysis) to calculate the correlation matrix for these assets. Which assets promise the greatest benefits from diversification? Why?

Quarterly Index Returns									
All returns are expressed as percentages									
Year	Quarter	Total Market	Commodities	Total Bond	Internationl	Small cap	Emerging Market Stocks	Emerging Market Bonds	High-yield Bonds
1	I	0.447	2.486	-0.775	2.306	-5.108	7.037	2.067	0.316
	II	15.024	-4.959	3.63	13.08	15.981	8.549	10.967	4.924
	III	8.439	2.907	3.321	-1.489	14.526	-2.07	7.265	3.83
	IV	1.919	-9.032	2.862	-9.92	-3.422	-12.219	-4.652	2.286
2	I	11.914	-5.001	1.401	12.103	9.642	4.31	5.595	3.06
	II	1.438	-9.822	2.406	0.774	-5.105	-21.878	-8.004	1.361
	III	-12.311	-6.529	4.157	-13.66	-22.499	-29.067	-36.075	-1.361
	IV	19.108	-15.073	0.325	16.365	15.283	12.687	12.327	2.436
3	I	3.041	4.058	-0.488	-1.006	-5.794	8.94	2.796	1.854
	II	7.212	2.029	-0.984	4.771	15.745	19.715	4.891	-0.791
	III	-7.209	11.152	0.82	-0.08	-6.182	-2.407	0	-1.064
	IV	14.709	-0.173	-0.164	19.774	17.131	36.603	13.174	2.639
4	I	4.042	6.547	2.425	5.829	6.652	8.48	8.229	-1.575
	II	-3.648	6.144	1.427	-3.289	-3.115	-10.84	-0.58	2.094
	III	0.406	2.042	3.101	-10.429	0.775	-13.001	4.916	1.287
	IV	-9.701	6.949	3.893	-1.221	-7.589	-15.274	1.647	-2.591
5	I	-13.253	-7.211	3.179	-12.695	-6.658	-9.03	3.215	4.114
	II	6.304	-4.873	0.85	-2.492	13.834	6.484	3.963	-2.293
	III	-17.229	-6.575	4.144	-17.411	-23.093	-28.172	-3.963	-2.611
	IV	10.749	-6.598	-0.135	11.7	18.978	24.867	5.804	3.637
6	I	0.627	11.063	0.135	0.444	3.908	10.548	6.267	1.267
	II	-14.385	0.07	2.802	-4.534	-8.199	-6.572	-7.267	-1.523
	III	-19.353	6.584	3.745	-23.293	-23.958	-17.081	-0.504	-3.117
	IV	7.217	3.687	1.385	8.094	5.894	10.046	8.547	4.892
7	I	-3.556	2.588	1.366	-7.86	-4.617	-7.053	5.561	2.726
	II	14.617	2.281	2.556	16.444	20.061	20.555	11.973	5.933
	III	2.941	4.321	-0.12	7.446	8.248	13	0.775	1.373
	IV	11.254	11.236	0.24	13.571	13.916	16.755	3.788	4.445
8	I	1.816	10.893	2.607	6.001	6.358	9.178	3.65	1.938
	II	0.909	-4.622	-2.487	-2.218	0.993	-9.411	-4.521	-0.857
	III	-2.345	6.162	3.07	0.155	-2.259	6.614	8.503	4.211
	IV	9.221	-5.077	0.926	13.421	13.001	17.502	6.231	3.046
9	I	-2.656	10.728	-0.462	-0.542	-3.814	1.695	1.18	-1.816
	II	1.784	-5.846	2.965	-1.092	4.736	5.886	7.098	2.613
	III	3.515	15.349	-0.676	10.974	5.068	18.731	4.751	0.593
	IV	1.547	-4.061	0.564	4.627	1.156	6.451	2.985	1.371
10	I	4.744	-3.546	-0.677	9.462	11.491	10.909	2.451	1.544
	II	-2.444	4.76	-0.227	-0.107	-4.821	-8.047	-2.635	-0.576
	III	4.094	-7.623	3.787	3.726	-0.269	6.896	6.246	3.409
	IV	6.412	3.707	1.411	10.04	8.118	17.959	4.895	3.477

EXHIBIT 14.40 Return data.

- Use Excel's Covariance function (in DataAnalysis) to calculate the covariance matrix for these assets. Fill in the upper right-hand portion to create a square, symmetric covariance matrix.
- Use Solver to find the allocation weights, expected quarterly return, and standard deviation of returns for the *MVP*. Assume no short selling.
- Identify 10 efficient portfolios by having Solver maximize the quarterly returns for 10 portfolios, where each portfolio has a different standard deviation. Begin with three standard deviations only incrementally larger than the standard deviation of the *MVP* you identified in question 4. Assume no short selling. Plot the efficient frontier and the underlying assets on a chart with expected quarterly returns on the *y*-axis and the standard deviation or returns on the *x*-axis. The efficient frontier begins at the *MVP* and ends at the asset that promises the highest return.

- f. Now assume that you can borrow and lend at the risk-free rate of 1% per quarter. What are the quarterly returns and the standard deviation of returns for the tangency portfolio?
- g. Compute the 5% *VaR* for the *MVP* over one quarter, expressed as a percentage. Assume that the parameters you estimated in question 1 will hold in the future.
- h. Use historical simulation to identify the 5% *VaR* for the *MVP* over one quarter, expressed as a percentage. Combine the weights for the *MVP* with the assets returns for each quarter to generate the time series of portfolio returns. (Hint: Excel's `SUMPRODUCT` function is useful for this type of calculation.)
- i. What are the 1% values-at-risk (dollars and percentages) over a one-day horizon for the *MVP* and the maximized return portfolio (*MxRP*) discussed in [Section 14.6](#) of this chapter? Use the computational and historical simulation methods. (Hint: You will find the *VaRs* for the latter method in [Exhibit 14.31](#).)
- j. How might you apply the methods in this chapter to manage your retirement savings?



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Options and Option Pricing

15.1 INTRODUCTION

This chapter reviews the basics of options and shows how to build some basic models of options. A thorough treatment of options is beyond the scope of this book, and it is assumed that the reader has already had some exposure to options and option valuation models.* The emphasis here is on how to build basic models of options so you have a foundation for undertaking more complex modeling tasks.

The first order of business is to introduce options and the terminology of options. Most of the discussion is confined to the standard options: calls and puts. If you learn about these basic derivatives and learn how to model them, it should not be difficult to extend your modeling to more complex derivatives at a later date. Here are some definitions and details.

15.1.1 Introducing Calls and Puts

The two most common options are calls and puts. A call is an instrument that gives the owner of the call the option to *buy* a share of stock at a specified price by a specified date. The price at which you can buy the stock is called the *exercise price*, and will be denoted by X . The last date by which you must exercise the option to purchase is called the *expiration date*, denoted by T . We refer to this instrument as an *option* because it gives the holder of the option the right, but not the obligation to buy the stock. In other words, the option holder is free not to exercise the option, and if he/she does not exercise the option, it expires.

Most options traded in the U.S. markets are referred to as American options. An American option can be exercised anytime until the expiration date. In contrast to the American option is the European option that can be exercised only on the date of expiration. Much of the development of the theory of option pricing was based on the European option simply because it is easier to analyze. But most of the analysis can be extended to American options.

* See the sources listed in the References at the end of this book for more detailed explanations of options. The more understandable introductions include Hull (2015), Dufresne and Miller (2003), Ross et al. (2016), and Berk and DeMarzo (2017).

Whereas a call is the option to buy the stock, a put is just the opposite; it is an option to *sell* the stock at a specified price by a specified date.

If you buy an option, there must be someone on the other side of the transaction who is the seller. In other words, if you buy a call giving you the right to buy the stock by a future date, there is someone else who is selling you this right. The person who is selling the call is termed the *writer* of the call. This person is undertaking an obligation to sell the stock should you choose to exercise your option. In other words, the call writer must sell you the shares, and does not have a choice. So the call *writer* does not have an option; he/she has the *obligation* and must perform if the option is exercised.

As with the call, for each buyer of a put option, there is seller on the other side of the transaction—the put writer. The writer of the put has the obligation to buy the stock should the buyer of the put option choose to exercise the right to sell the stock.

Acquiring an option involves a cost. The option writer who undertakes the obligation to sell the stock in the case of the call, or to buy the stock in the case of a put, would not normally be willing to undertake the obligation for free. The price the option writer earns for undertaking the obligation is referred to as the option *premium*, or simply the option price. We denote the premium paid for the call as C , and the premium paid for a put a P .

How much an investor should be willing to pay to acquire an option, and the factors that determine that price, or premium, has been one of the more interesting questions in finance. Black and Scholes (1973) and Merton (1973) developed models of option prices that have revolutionized the field of finance and earned Nobel Prizes for both Scholes* and Merton. One of the purposes of this chapter is to explain the famous Black–Scholes model. We also explain the more easily understood binomial model of option prices developed by Cox, Ross, and Rubinstein (1979).

15.2 PAYOFFS FROM OPTIONS

15.2.1 Buying Options

The starting point for modeling options is to understand the payoffs from the options. Most of the discussion focuses on the call. If we understand the call and can model its payoffs, the put easily follows.

The reason the call buyer wants to buy the call is to profit from an increase in the price of the underlying stock. At the expiration date, T , if the market price of the stock, S_T , is higher than the exercise price of the call, the call buyer can exercise the call and buy at the exercise price, X . The stock can be sold immediately for S_T , and the payoff is the difference, $S_T - X$. If the stock price is below the exercise price at expiration, the call buyer can refuse to exercise the call and get nothing. Thus, the call buyer will either make a profit on exercise, or nothing. In notation, before considering the premium originally paid for the call, the payoff to the call buyer at expiration, which is termed the *intrinsic value*, is

$$\text{Max}(S_T - X, 0) \quad (15.1)$$

* Black did not share the Nobel Prize simply because he died prior to its being awarded, and the prize is not given posthumously.

So long as $S_T > X$, the payoff at expiration is positive, and the call buyer will profit (prior to taking account of the call premium paid earlier). The higher the stock price at expiration, the greater the profit, so the call buyer is betting that the stock price will increase. In fact, as we will see later, buying a call is much like a leveraged, or margined, position in the stock.

The put is just the opposite of the call in that the put buyer will profit from a decline in the price of the stock. At expiration, the put buyer has the option to sell the stock at exercise price, X , when the stock price is S_T . If the market price is below the exercise price, the payoff is positive, and the lower the stock price the greater the payoff from exercising the put option. If the stock price is higher than the exercise price, the payoff from exercising the put is negative, so the put buyer will refuse to exercise the option and get nothing. The payoff to the put buyer at expiration (the intrinsic value of the put) can be written as

$$\text{Max}(X - S_T, 0) = \text{Max}(-(S_T - X), 0) \quad (15.2)$$

The payoffs for the options are easy to model. [Exhibit 15.1](#) shows a model of the payoffs from the call and the put. The input data are in B6:D11, indicating that the initial (current) stock price is \$100, and the exercise price for both the call and the put is \$100 (cells C9:D9); there is one year to expiration, and the cost premium to buy either option is \$10. Stock prices at expiration ranging from 0 to \$140 are shown in column A. The intrinsic value of the call at expiration [$\text{Max}(S_0 - X, 0)$] is shown in C17:C31, and the payoff net of the cost of buying the call for \$10 is shown in D17:D31. The same calculations for the put are in F17:G31.

At expiration, if the stock price is below the call exercise price of \$100, the call buyer will not exercise the option, and the intrinsic value is zero as shown in C17:C27. However, the call buyer paid \$10 to buy the option, so the payoff net of the \$10 premium is $-\$10$. At stock prices above \$100, the call has a positive intrinsic value (cells C28:C31). Of course, when the initial premium of \$10 is deducted, the profit is decreased as shown in D28:D31.

[Exhibit 15.2](#) shows a graph of the call payoffs from the source data in A17:D31. The horizontal axis is the stock price (A17:A31) as the independent variable in the XY scatter diagram, with the vertical axis showing the payoff associated with each different stock price. This is the typical pattern for the call: the intrinsic value (the upper line) is flat up to the exercise price, and then the payoff increases linearly at a slope of 1.0, indicating that for each \$1 increase in the stock price, the intrinsic call value increases by \$1. The lower line is the payoff net of the call premium, so it is at $-\$10$ at all stock prices up to \$100. Then it increases at the same slope as the intrinsic value line.

[Exhibit 15.3](#) shows a graph of the put payoffs from cells F17:G31 with a pattern just the opposite of the call. At stock prices above the exercise price of \$100, the intrinsic value is zero, and the payoff net of the put premium is $-\$10$. But the lower the price of the stock, the greater the profit from exercising, with the intrinsic value increasing by \$1 for each \$1 decrease in the price of the stock.

	A	B	C	D	E	F	G	
1	Option Payoff							
2	at Expiration							
3								
4	Data Input							
5								
6	Initial Stock Price	100						
7	Option Features							
8	Type of Option		Call	Put				
9	Exercise Price	X	100	100				
10	Time to Expiration	T	1	1				
11	Premium to Buy Option		10.00	10.00				
12								
13			=MAX(A17-\$C\$9,0)			=MAX(\$D\$9-A17,0)		
14	Stock Price		Call Payoff			Put Payoff		
15	at Expiration		Intrinsic	Net of		Intrinsic	Net of	
16			Value	Premium		Value	Premium	
17	0		0	-10.00		100	90.00	
18	10		0	-10.00		90	80.00	
19	20		0	-10.00		80	70.00	
20	30		0	-10.00		70	60.00	
21	40		0	-10.00		60	50.00	
22	50		0	-10.00		50	40.00	
23	60		0	-10.00		40	30.00	
24	70		0	-10.00		30	20.00	
25	80		0	-10.00		20	10.00	
26	90		0	-10.00		10	0.00	
27	100		0	-10.00		0	-10.00	
28	110		10	0.00		0	-10.00	
29	120		20	10.00		0	-10.00	
30	130		30	20.00		0	-10.00	
31	140		40	30.00		0	-10.00	
32	150		50	40.00		0	-10.00	
33	160		60	50.00		0	-10.00	
34	170		70	60.00		0	-10.00	
35	180		80	70.00		0	-10.00	
36	190		90	80.00		0	-10.00	
37	200		100	90.00		0	-10.00	

EXHIBIT 15.1 Model of payoffs for call and put.

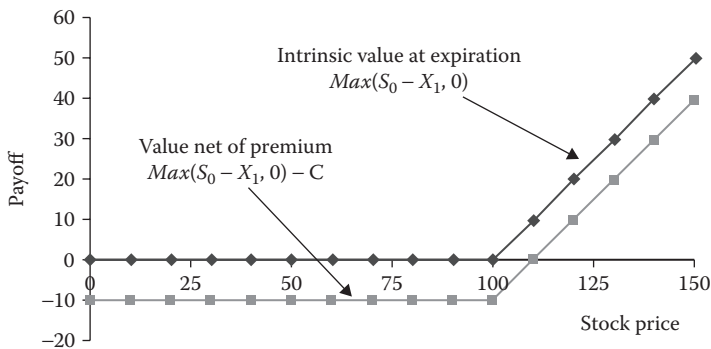


EXHIBIT 15.2 Payoffs at expiration from the call option.

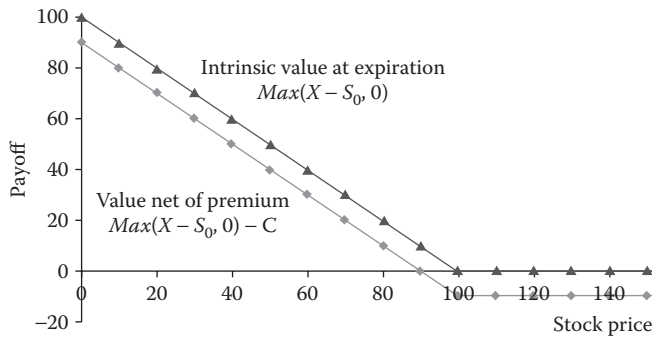


EXHIBIT 15.3 Payoffs at expiration for the put option.

15.2.2 Writing Options

The option writer is the person who sells the option to the buyer and undertakes the obligation to sell the stock in the case of the call, or to buy the stock in the case of the put. We refer to buying an option as taking a long position in the option; we refer to selling (writing) an option as taking a short position in the option. The payoffs for the short position are just the negative of the payoffs from the long position. The payoffs from the different positions are written as follows:

	Buyer (Long)	Seller (Short)
Call	$Max(S_T - X, 0)$	$-Max(S_T - X, 0)$
Put	$Max(X - S_T, 0)$ or, $Max(-(S_T - X), 0)$	$-Max(X - S_T, 0)$ $-Max(-(S_T - X), 0)$

When you graph the payoff to the writer, it is an upside-down image of the payoff to the option buyer. [Exhibit 15.4](#) shows the payoff to the call writer. Whereas the call buyer loses the call premium if the stock price does not increase, the call writer earns the premium. But if the stock price goes above the exercise price, the call writer loses $S_T - X$ because he or she has to sell the stock for X when it is worth $S_T > X$. So, at prices below X , the call writer earns the payoff of the call premium, but at prices above X , the call writer loses an amount equal to what the call buyer earns.

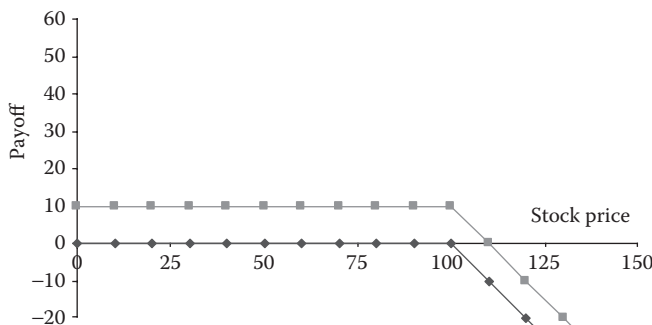


EXHIBIT 15.4 Payoff to the call writer.

15.2.3 Options as a Leveraged Investment in the Stock

We see from these payoff graphs the profit possibilities from buying options. For a very small investment, just \$10 per option per share in this example, you can acquire an opportunity to profit from price moves by a stock. In the case of the call, if the stock price increases, you can earn the same dollar profit as the stock without having to invest nearly as much. So this is like a leveraged, or margined, transaction in the stock. In fact, as we will see in a subsequent section, the payoff pattern from the option is exactly like a leveraged stock position. It is from this fact that we will be able to determine the value of the option. However, prior to discussing the value of the options, we will take a quick look at how we can model combinations of options.

15.2.4 Option Combinations

You need not confine your investment to *plain vanilla* options with just a long position in a call or a put. You can combine them in any number of different mixtures. Many of the combinations have exotic sounding names like strip, spread, and straddle, butterfly spread, bear spread, and even a strangle. Explaining and modeling each of these are beyond the scope of this chapter, so the interested reader is directed to texts devoted to derivatives such as Hull (2015) or Dubofsky and Miller (2003). Nevertheless, we can get you started by showing how to adopt a model as simple as [Exhibit 15.1](#) to analyze combinations of options.

[Exhibit 15.5](#) shows a model for analyzing different option combinations. In this case, the position is a *straddle*, a combination of buying a call and a put (with the same exercise prices and same expiration dates). This combination profits from stock prices moving in either direction away from the exercise price, with payoffs shown in [Exhibit 15.6](#).

In the spreadsheet, the input is the same as the example in [Exhibit 15.1](#), and the intrinsic values (payoff at expiration) of the call and put are in columns C and D. Columns F, G, and H model combinations of options. The user enters the transactions in G8:H9 by typing “Buy” or “Sell” in G8 and H8 for the call and the put, respectively. The number of calls or puts bought or sold are entered in G9 and G10. In this example, the transaction is to buy 1 call and buy 1 put. The formula in G12 is $=IF(G8 = \text{“Buy,” } 1, IF(G8 = \text{“Sell,” } -1, 0)) * G9$. If the user has entered “Buy” in G8, this formula returns +1, but if the user entered “Sell,” it returns -1; otherwise it returns 0. This gives the correct sign for the transaction, with +1 for buy and -1 for sell, and it is multiplied times the number of shares entered in G9 by the user. The formula in H12 is similar. The results are the payoff amounts for the call and put in F18:G38. Cells H18:H38 add the payoffs from the call and put as the payoff for the combination.

In [Exhibit 15.6](#), the gross payoff (before considering the premium) of the call is zero (following the horizontal axis) up to the exercise price of \$100, after which it increases dollar-for-dollar with the stock price. The put payoff is the mirror image going in the other direction, with positive payoffs for stock prices below \$100. The straddle yields a gross payoff shown by the top V-shaped line. When we net out the cost of each of the options at \$10 each, the net payoff is shown by the bottom V-shaped line. As you see, net of costs, the straddle profits for any stock price less than \$80 or greater than \$120. So the straddle is a good bet if you think that the stock price will move substantially, but you do not know which way.

	A	B	C	D	E	F	G	H
1		Option Payoff at Expiration				Combinations of Options		
2								
3								
4	Data Input					Transaction: User Input Indicate 'Buy' or 'Sell' and Number of Shares		
5	Interest Rate	5.0%						
6	Initial Stock Price	100						
7	Option Features							
8	Type of Option		Call	Put		Buy or Sell?	Call	Put
9	Exercise Price	X	100	100		Shares	Buy	Buy
10	Time to Expiration	T	1	1			1	1
11	Premium to Buy Option		10.00	10.00				
12						Long or Short:	1	1
13			=IF(G8="Buy",1,IF(G8="Sell",-1,0))*G9					
14			=IF(H8="Buy",1,IF(H8="Sell",-1,0))*H9					
15						Payoff at Expiration		
16	Stock Price at Expiration		Call Intrinsic Value	Put Intrinsic Value		Call	Put	Option Combination
17								
18	0		0	100		0	100	100
19	10		0	90		0	90	90
20	20		0	80		0	80	80
21	30		0	70		0	70	70
22	40		0	60		0	60	60
23	50		0	50		0	50	50
24	60		0	40		0	40	40
25	70		0	30		0	30	30
26	80		0	20		0	20	20
27	90		0	10		0	10	10
28	100		0	0		0	0	0
29	110		10	0		10	0	10
30	120		20	0		20	0	20
31	130		30	0		30	0	30
32	140		40	0		40	0	40
33	150		50	0		50	0	50
34	160		60	0		60	0	60
35	170		70	0		70	0	70
36	180		80	0		80	0	80
37	190		90	0		90	0	90
38	200		100	0		100	0	100

EXHIBIT 15.5 Payoffs from combinations of calls and puts: Straddle.

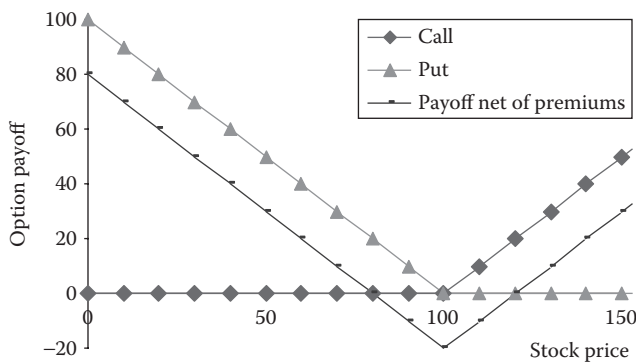


EXHIBIT 15.6 Payoffs from a straddle: Buy a call and buy a put.

If you are not quite sure which way the stock will move, but you are more bullish, thinking a price increase is more likely, you can buy a *strap*, which consists of 2 calls and 1 put. This places a bigger bet on the stock price increasing, but still profits if the stock price declines. In the model in [Exhibit 15.5](#), this transaction is input by typing 2 in the call cell G9 and 1 in the put cell H9.

On the other hand, if you are more bearish, but still not sure which way the stock price will move, you can create a *strip*, which is buying 1 call and 2 puts. This way if the stock moves down, you have greater profit, but you still have a profit if it goes up. This is input in the model by inserting 1 in G9 and 2 in H9.

As you can see, you can devise nearly any combination of options by changing the number of each option, and by going long or short by typing in “Buy” or “Sell” in G8 or H8 in the model. There are many different mixes to serve myriad purposes. We leave it to you to explore on your own a range of option combinations and exercise prices with a model such as [Exhibit 15.5](#). For more detail regarding derivative trading strategies, consult one of the texts referenced at the end of this chapter. Although we will not explain the wide range of option trading strategies, we now build on the model shown in [Exhibit 15.5](#) to demonstrate some of the results of mixing the options with the stock and with bonds.

15.2.5 Mixing Options with the Stock and a Bond

The model shown in [Exhibit 15.7](#) is an extension of the one in [Exhibit 15.5](#) with a section added to allow for buying or selling the stock and a bond. A stock purchase occurs at the initial price originally shown in B6 at \$100, with the stock being sold at the option expiration date at the prices shown in E18:E32. A sale of the stock at $t = 0$ is assumed to be a short sale, with the proceeds available. The model user indicates the stock transaction by typing in “Buy” or “Sell” in J8, and the number of shares in J9.

A bond transaction refers to either lending or borrowing money at the interest rate indicated in B5 (in [Exhibit 15.5](#)). Lending is indicated by typing in “Buy” in K8, and the amount to be lent in K9. The user indicates borrowing by typing “Sell” in K8, with the amount borrowed in K9. Funds lent or borrowed are repaid at the expiration date of the options.

The option payoffs at expiration are in F18:H32, as before. The payoffs from the stock and bond are in J18:K32. The payoffs from the total mix are in L18:L32. The total cost of the combination of call, put, stock, and bond is in N18:R32.

In the example shown, the transaction consists of buying

- One call (G8:G9).
- Selling one put (H8:H9).
- Selling one share of stock (J8:J9).
- Lending \$95.125 at 5% (buying the bond in K8:K9).

The payoffs at expiration shown in L18:L32 sum the payoffs from each of the individual instruments. For example, at expiration, if the stock price has dropped to \$30 (row 21), the call will not be exercised, and the call payoff to the call buyer will be zero (F21); the put will

	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1														
2		Combinations of Options			Other Securities in Portfolio									
3		Transaction: User Input			Transaction: User Input									
4		Indicate 'Buy' or 'Sell'			Stock									
5		and Number of Shares			(Shares)									
6														
7			Call	Put										
8		Buy or Sell?	Buy	Sell										
9		Shares	1	1		1	Buy							
10							95.125							
11														
12		Long or Short:	1	-1		-1								
13														
14			Payoff at Expiration			Payoff at Expiration				Initial Cost				
15	Stock Price							Total						
16	at Expiration	Call	Put	Combination	Option	Stock	Bond	Position		Call	Put	Stock	Bond	Total
17														
18	0	0	-100	-100	-100	0.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
19	10	0	-90	-90	-90	-10.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
20	20	0	-80	-80	-80	-20.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
21	30	0	-70	-70	-70	-30.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
22	40	0	-60	-60	-60	-40.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
23	50	0	-50	-50	-50	-50.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
24	60	0	-40	-40	-40	-60.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
25	70	0	-30	-30	-30	-70.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
26	80	0	-20	-20	-20	-80.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
27	90	0	-10	-10	-10	-90.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
28	100	0	0	0	0	-100.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
29	110	10	0	10	10	-110.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
30	120	20	0	20	20	-120.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
31	130	30	0	30	30	-130.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88
32	140	40	0	40	40	-140.00	100.00	0.00		10.00	-10.00	-100	95.125	-4.88

EXHIBIT 15.7 Model for portfolio of options, stock, and bond.

be exercised, so the put writer will have to buy the stock for \$30 and deliver to the put buyer at the exercise price of \$100, for a loss to the put writer of \$70 (G21). The total payoff from the two options is $\$0 - \$70 = -\$70$ (H21).

The stock was sold short at $t = 0$, for \$100. At expiration, the short position must be covered, so the stock will be purchased at the market price of \$30. The $-\$30$ in J21 shows the cost of the stock purchase. A bond was purchased for \$95.125 at the start, and at expiration, it matures for \$100 (principal + interest), which is shown in K21. The payoffs at expiration from the stock and the bond are $-\$30 + \$100 = \$70$, and the total payoff from the combination of call, put, stock, and bond is zero summarized as

Call (long)	\$0
Put (short)	-\$70
Stock (short)	-\$30
Bond (long)	+\$100
Total payoff at expiration	\$0

Note that the payoff from the total position (L18:L32) is zero in every case. In other words, no matter what happens, the payoff at expiration nets out to zero. Normally, no one wants an investment that pays zero. But now let us look at what the investment costs at the start of the transaction.

At the start of the transaction ($t = 0$), the cash flows are as follows:

Call (long: buy 1 call)	-\$10.00
Put (short: sell 1 put)	+\$10.00
Stock (short: sell 1 share)	+\$100.00
Bond (long: buy a bond)	-\$95.125
Total cash flow	+\$4.875

The positive cash flow of \$4.875 means that the investor collects \$4.875 at the start of the transaction from the mix of purchases and sales. This corresponds to -4.88 shown as the Total cost in R18:R32 of the model. In other words, the cost is negative. Instead of paying out something to make the investment, this investor collects some cash at the start.

This may seem strange and even a little confusing. We have an investment that gives us money at the start, and then gives us nothing at the end. This is an arbitrage opportunity, or what we might call a *money machine*. Imagine, if this opportunity persisted, how you could undertake transactions like this for thousands of shares instead of just one. Such an opportunity to make money up front at no cost would seem too good to be true. As they say, *If it seems too good to be true, it probably is*. So, what is the catch? The catch is that the prices are in disequilibrium. In the example, the option prices of \$10 for both the call and the put are such that the investor has this opportunity to make instant riskless profits. When prices offer such opportunities, investors will rush in to take advantage. The pressures of investors trying to take advantage of the opportunity will result in prices changing to the point where the excess profit opportunity disappears.

The question is, what must the prices of the options be so that there is no longer the opportunity for riskless arbitrage profits? The next section discusses pricing of options and presents option pricing models based on the idea that investors should not be able to engage in transactions that yield arbitrage profits.

15.3 OPTION PRICING MODELS

15.3.1 Binomial Option Pricing

The starting point for modeling option prices is a single period binomial model. This option pricing model derives prices of calls and puts based on the idea just discussed: In equilibrium, the options will be priced so there are no opportunities to make riskless arbitrage profits.

In [Chapter 12](#), we examined a stock price model where the stock price followed a binomial process. The basis of the option model in this section is also a binomial process. Assume that at time $t = 0$, a share of stock has a price of $S_0 = \$100$. By the end of the period ($t = 1$), the stock can move up by a multiple ($1 +$ rate of return) of $u = 1.25$ to a price of

$$S_1 = uS_0 = 1.25 * 100 = 125 \quad (15.3)$$

or, it can move down by a multiple of $d = 0.80$ to a price of

$$S_1 = dS_0 = 0.80 * 100 = 80 \quad (15.4)$$

The call option on the stock has an exercise price of $X = \$100$, and the call will expire at time $T = 1$. This is a European style call, so it can be exercised only at the expiration date.

[Exhibit 15.8](#) shows a spreadsheet model for this simple option. The input data are shown in lines 1–15, and the payoffs for the stock and the option are shown in lines 19–21. Cell B20 is the beginning price of the stock, and cells E19 and E21 are the stock values at time 1, the date that the call option expires. The stock prices are simply the beginning price multiplied by the up or down multiples, u and d . The payoffs for the call are shown in cell F19 and F21 using the formulas

$$\begin{aligned} \text{Call Value}_1 &= \text{Max}(S_1 - X, 0) \\ &= \text{Max}(E19 - E9, 0) = \text{Max}(125 - 100, 0) \\ &= \$125 \end{aligned} \quad (15.5)$$

or

$$\begin{aligned} &= \text{Max}(E21 - E9, 0) = \text{Max}(80 - 100, 0) \\ &= 0 \end{aligned}$$

in F19 and F21, respectively.

We see how easy it is to develop a model for the call payoffs for this simple one-period binomial case. Modeling the payoffs in more extended cases is not much harder. However,

	A	B	C	D	E	F	G
1	Binomial Option Model						
2	Single Period						
3							
4	Data Input						
5	Riskless Interest Rate	5.0%					
6	Initial Stock Price	100					
7	Option Features						
8	Type of Option			Call			
9	Exercise Price			X	100		
10	Time to Expiration			T	1		
11							
12	Stock Returns over 1 period						
13		Rate of Return		Wealth Relative Return			
14	Outcome 1	25%		u	1.25		
15	Outcome 2	-20%		d	0.80		
16							
17	Time	0		*****	1	*****	
18					Stock Price ₁	Call Payoff	
19				S u	125	25	
20	Stock Price ₀	100		S d	80	0	
21							
22							
23							

EXHIBIT 15.8 Excel model for call option on stock with binomial price moves.

we not only want to model payoffs, we also want to use the model to explore the value of the call. That is what we do next.

15.3.1.1 Option Pricing by Replicating the Payoffs

To understand option pricing models, it is important to understand the law of one price and arbitrage.* The law of one price is simply the idea that if two investments have the same cash flows, risks, and other features, then they must have the same price. The reason for this is that if they trade at different prices, investors will be able to engage in arbitrage by buying and selling these equivalent securities so that their market prices will adjust until the securities trade at the same price. This principle also applies to option pricing: if the market price of an option differs from the prices of other securities that have equivalent cash flows, the prices of the securities should all adjust until they trade at the same price. We use this principle to figure out what the price of the option should be when arbitrage does its job of equilibrating prices.

If we do not know what an option’s market price should be, we find other investments whose prices are known and construct a mix of those investments that will replicate the payoffs from the option. If this other investment has the same payoff as the option, then we can conclude that the market value of the option should be the same as this other replicating investment. For example, if we know that a call option has payoffs of either \$0 or \$25, we try to find other investments, whose prices we know, and mix those known investments in a way

* See Berk and DeMarzo (2017), Chapter 3 for a clear discussion of arbitrage and the law of one price.

so the mix has payoffs that are also \$0 or \$25. Since we know the prices of these alternative investments, and we can mix them in a way to duplicate the payoffs of the call, the market value of the call adjusts so as to be equal to the cost of constructing this replicating mix.

It turns out that we can duplicate the returns on the call with an appropriate mix of the stock and borrowing. In other words, as a call is much like a leveraged investment in the stock, we should be able to engage in a leveraged purchase of the stock that will yield payoffs identical to the call.

Exhibit 15.9 repeats the option payoff model from above, and shows the replicating investment at the bottom in lines 26 through 34. Line 29 shows the cost of buying 1 share of stock for \$100 at time $t = 0$ (cell B29), and the payoffs from selling the stock at time $t = 1$ for either \$125 or \$80 (E29 and F29). Line 30 shows the cash flows from borrowing. Assume that at time $t = 0$, we borrow an amount equal to the present value of the “down” payoff from the stock. The “down” payoff when we sell the stock at $t = 1$ is \$80. Using continuous discounting at the riskless rate of 5% over one period, this present value is $80e^{-0.05(1)} = 80 * 0.9512 = \76.10 in cell B30. In other words, we borrow \$76.10 at the rate of 5% at $t = 0$, and repay \$80 (principal plus interest) at $t = 1$. At time $t = 0$, the cost of the

	A	B	C	D	E	F	G
1	Binomial Option Model						
2	Single Period						
3							
4	Data Input						
5	Riskless Interest Rate	5.0%					
6	Initial Stock Price	100					
7	Option Features						
8	Type of Option			Call			
9	Exercise Price			X	100		
10	Time to Expiration			T	1		
11							
12	Returns over 1 period						
13		Rate of Return		Wealth	Relative Return		
14	Outcome 1	25%		u	1.25		
15	Outcome 2	-20%		d	0.80		
16							
17	Time	0		*****	1	*****	
18					Stock Price ₁	Call Payoff	
19					125	25.0	
20	Stock Price ₀	100					
21					80	0.0	
22							
23	Replication of Call Payoffs						
24	Time	0		*****	1	*****	
25							
26	Stock Shares Purchased	1					
27					S d	S u	
28							
29	Stock Purchase	-100.00			80	125	
30	Borrow	76.10			-80	-80	
31	Total Cost	-23.90			Total Payoff	0	45
32							
33	Equivalent Number of Calls to Replicate Payoff						
34	Cost / Number of Calls						
35						1.80	
36						-13.28	
37							

EXHIBIT 15.9 Binomial option model: Replicating portfolio of stock plus borrowing.

stock was \$100, but we borrow \$76.10 to help finance the stock purchase, so the equity we need to invest is the remainder of $\$100 - \$76.10 = 23.90$ (cell B31).

At $t = 1$, when the option expires, the payoff from this leveraged position will be the proceeds from selling the stock less the amount we have to repay on the debt:

<i>Stock Sale</i> ₁	= \$80	or	\$125	(E29 & F29)
<u>- Debt Repayment</u> ₁	= \$80		\$80	(E30 & F30)
= <i>Net Payoff</i> ₁	= 0		\$45	(E31 & F31)

Note that regardless of whether the stock went up or down, we have to repay the \$80 debt. The debt repayment uses up the proceeds of the stock sale in the down state, and leaves \$45 in the up state.

Note that the \$0 or \$45 payoffs do not exactly match the payoffs from the call that are \$0 or \$25. So we need to adjust the number of calls to duplicate these payoffs. For example, since the \$45 payoff in the “up” state is 1.8 times the \$25 payoff from the call, we can see that if we bought 1.8 calls, we would have the same payoff as the leveraged stock. So, if the leveraged stock transaction costs \$23.90 (cell B31), then the value of one call must be $\$23.90/1.8 = \13.28 (cell F34). Or, to put it another way, we can scale the leveraged stock transaction down by a factor of $1/1.8$ and duplicate the payoffs of the call. In other words, buy $1/1.8 = 0.5556$ shares of stock, and borrow \$42.28 to leverage the investment, and get the same \$0 and \$25 payoff as the call. The cost of buying 0.5556 shares net of borrowing would be $\$55.56 - \$42.28 = \$13.28$. Since we can earn the same payoffs of \$0 or \$25 from this investment as from the call, we certainly should not pay more than \$13.28 for the call. If the price of the call was less than this, investors can engage in an arbitrage transaction that will drive up the price of the call and/or drive down the price of the stock until they are in equilibrium.

15.3.1.2 An Arbitrage Transaction

An alternative way to model the option payoffs and value is to model the arbitrage transaction. The basic model is the same, except we construct a transaction so that the payoffs are the same in each state. That way, no matter what happens, the investment pays off the same amount, so the investment is riskless.

The transaction we construct is called a covered call. This involves buying the stock and writing a call. Writing a call means that instead of buying a call option, we sell the call option (also called taking a short position in the call). When we sell the call option we are undertaking the obligation to sell the stock to the call buyer should the buyer choose to exercise the option to buy the stock. If the call option is in the money at the expiration date ($S_T > X$), the call buyer will exercise, and we will have to sell the stock at the exercise price, X . When we have to sell the stock at the exercise price of X when it is worth S_T , we lose the difference $S_T - X$. If the call expires out of the money ($S_T < X$), the call buyer will not exercise, and we do not have to sell the stock. In either case, as the call writer, we earn the call premium, the price that the call buyer paid at the start to buy the call option. As the call writer, the time pattern of cash flows is as follows:

Time:	0	Expiration Date, T
Cash Flow:	Call Premium, $+C$	$-Max(S_T - X, 0)$

	A	B	C	D	E	F
1	Binomial Option Model					
2	Single Period					
3						
4	Data Input					
5	Riskless Interest Rate	5.0%				
6	Initial Stock Price	100				
7	Option Features					
8	Type of Option			Call		
9	Exercise Price			X	100	
10	Time to Expiration			T	1	
11						
12	Returns over 1 period					
13		Rate of Return		Wealth Relative Return		
14	Outcome 1	25%		u	1.25	
15	Outcome 2	-20%		d	0.80	
16						
17	Time	0		***** 1 *****		
18					Stock Price ₁	Call Value
19						
20	Stock Price ₀	100		→ S u	125	25.0
21				→ S d	80	0.0
22						
23	Riskless Covered Call					
24	Time	0		***** 1 *****		
25						
26	Stock Shares Purchased	0.556				
27	Call Sold	1				
28	Call Premium	13.28			S d	S u
29	Stock Purchase	-55.56		Sell Stock	44.44	69.44
30	Call Proceeds	13.28		Call Payoff	0	-25.0
31	Total Cost	-42.28		Total Payoff	44.44	44.44
32						
33	PV of Payoff	42.28				
34						
35				Payoff Difference		0
36						

EXHIBIT 15.10 Binomial option model: Riskless covered call.

For the binomial case, it is possible to structure the covered call so that no matter what happens, we get the same amount at the expiration date. Exhibit 15.10 shows how this works.

The model in Exhibit 15.10 shows the same binomial case as the previous model. In this case, stock shares are purchased, and a call is sold. The results of one particular transaction are shown in the exhibit. The number of shares purchased is in B26 and is input by the model builder. The cost of buying the shares is in B29 and is simply the price per share times the number of shares purchased. B27 is the number of calls written, and B30 is the premium amount the writer receives, which is the number of calls written times the call premium price per call. Assume that we do not yet know the number of shares we will buy, nor the price of the call. The number of shares actually shown in B26 and the call price in B28 are the end products of our computations, but for now let us just use these numbers to understand the model.

The payoffs' calculations are shown in columns E and F. The \$44.44 and \$69.44 in E29 and F29 are the number of shares (0.556 shares in B26) times the price per share of either

\$80 or \$125. The call payoffs are in E30:F30. If the stock price is \$80, the call is not exercised, and we get nothing (E30) from the call. If the price is \$125, we exercise the call and buy the stock for the market price of \$125 and sell it to the call buyer for \$100, losing the difference of \$25 (F30).

The proceeds from the up and down state are the same. In the case where the stock went up, we were able to sell our 0.556 shares of stock for \$69.44, and we incurred a loss of \$25 from the call. The difference is our net proceeds of \$44.44 (F31). In the case where the stock went down, we sold our 0.556 shares for \$44.44, and got nothing from the call (E31). Note that in either case, we get \$44.44 at $t = 1$. Thus, we have structured our portfolio so that no matter what happens to the level of stock prices, we get \$44.44. This is a riskless transaction.

Had we chosen a different number of shares to purchase, the payoffs would have been different, and it would not necessarily have been a riskless transaction. Before the fact, we do not know precisely the number of shares to buy to construct this riskless transaction. There is formula for this, but for now, we will let the spreadsheet model solve this problem.

In the context of the spreadsheet model, we can use Goal Seek to solve for the number of shares to make the payoffs the same in both states. Note in the model, cell F34 is the difference between the payoffs in the two states, F31–E31. We would like to find the number of shares of stock to purchase, so this difference is zero. In other words, we want to make the payoffs the same in both states. To do this, go to the Data ribbon and click on What-if Analysis > Goal Seek. In the Goal Seek dialog box, Set Cell F34 > To Value 0 > By Changing Cell B26, and click OK. The number of shares will be varied to reach the goal. In this case, the number of shares will converge to 0.556 as shown on [Exhibit 15.10](#), and the Payoff Difference in F33 will be zero. Your model has now solved for the risk-free mix of stock and calls. You get \$44.44 no matter what happens.

The next step is to solve for the value of the call which, at this stage, you presumably do not yet know. If you have an investment that has payoff that is certain regardless of the performance of the stock price, then that investment is riskless and should earn the risk-free interest rate. So now we want to find the amount of the call premium that will make the return on the total investment equal to the risk-free interest rate. Once again, we use Goal Seek to find the call premium. Cell B33 is the present value of the riskless payoff. The riskless payoff is \$44.44 in F31, and B33 is the present value at the riskless rate:

$$\begin{aligned}
 PV \text{ of Payoff} &= B33 \\
 &= E31 * EXP(-B5 * E10) \\
 &= 44.44 * e^{-0.05(1)} \\
 &= 42.28
 \end{aligned}
 \tag{15.6}$$

We now know that the total cost of the investment mix has to be \$42.28 in order for it to earn the riskless rate of 5%. Since we know how much we have to invest in the stock (\$55.56 in B29), we need to find the amount we would need to earn as a call premium,

so the net cost is \$42.28. In this case, it is obvious that the premium would have to be $\$55.56 - \$42.28 = \$13.28$. In more complex cases, we could use Goal Seek that would find the Call Premium in B28 that would make the Total Cost in B31 equal to the \$42.28 in B33.

Note that this method of working out a riskless mix of stock and the call option results in the same value of the call and same number of shares as we obtained with the previous model that used the mix of stock and borrowing in [Exhibit 15.9](#). In this covered call example, it is easier to see how arbitrage would work. If the call premium was more than \$13.28, you could sell the call and buy the stock for a total cost lower than \$42.28 and thereby could earn riskless return greater than the 5% risk-free rate. If the call premium was lower than \$13.28, we could devise a mix of buying the call and selling the stock so as to earn a sure rate that exceeded the risk-free rate. Given the pattern of binomial stock prices and the current price of the stock, the only price for the call that will prevent investors from earning a disequilibrium return is the \$13.28 that we have solved for. With many investors trading in these instruments, the prices will move to this equilibrium value.

15.3.1.3 The Cox, Ross, and Rubinstein Binomial Option Formula

In order to demonstrate the basic binomial model of option pricing, we have used a roundabout method. In fact, in this binomial case, the value of the call that we have solved for can be calculated more directly with a formula that was developed by Cox, Ross, and Rubinstein (1979). The development of their formula follows the same logic that we have presented as a spreadsheet model. In the single-step binomial case where the length of the step is T years, and r is the risk-free rate per year, the value of the call should be

$$C = [pC_u + (1-p)C_d]e^{rT}$$

where

$$p = \frac{e^{rT} - d}{u - d}, \text{ and} \quad (15.7)$$

$$1 - p = \frac{u - e^{rT}}{u - d}$$

Applying this for our example where $u = 1.25$, $d = 0.80$, $C_u = \$25$, $C_d = 0$, and $T = 1$, and at the riskless rate of 5%, the present value of \$1 at time $T = 1$ is $e^{-rT} = e^{-0.05(1)} = 0.9512$, we have:

$$C = [pC_u + (1-p)C_d]e^{rT} = [0.5584 * 25 + 0.4416 * 0] * 0.9512 = \$13.28$$

where

$$p = \frac{e^{rT} - d}{u - d} = \frac{1.0513 - 0.80}{1.25 - 0.80} = \frac{0.2513}{0.45} = 0.5584, \text{ and} \quad (15.8)$$

$$1 - p = \frac{u - e^{rT}}{u - d} = \frac{1.25 - 1.0513}{1.25 - 0.80} = \frac{0.1987}{0.45} = 0.4416$$

Note that this is identical to the value generated in our models in Exhibits 15.9 and 15.10. Part of the derivation of the call value is the hedge ratio, H , that is the number of shares of stock purchased for each call sold in the covered call model, which is calculated as

$$H = \frac{C_u - C_d}{uS_0 - dS_0} = \frac{25 - 0}{125 - 80} = 0.556 \quad (15.9)$$

which matches the calculation in cell B26 of Exhibit 15.10.

15.3.1.4 Put Value

Having developed a simple one-step binomial model for a call, we show how to adapt the model to a put. The put option is just the reverse of the call in that it gives the buyer of the put the option to sell the stock for a specified exercise price by the expiration date. As the payoff pattern of the put is like a reverse image of the pattern for the call, the model of the put is almost identical to the call model, except that we reverse many of the signs. To see how this works, we will use the same single-step binomial data as before, but we modify the model to examine a put option to sell the stock for an exercise price of $X = \$100$ at time $T = 1$. As in the previous case, we assume the current stock price is $S_0 = \$100$, and it will move to either $dS_0 = 0.80 * 100 = \$80$ or $uS_0 = 1.25 * 100 = \$125$ by the option expiration date at $T = 1$. In this binomial case, the value of the put at the expiration date will be

$$\begin{aligned} P_1 &= \text{Max}(X - S_1, 0) = \text{Max}(-(S_1 - X), 0) \\ &= \text{Max}(100 - 125, 0) = \text{Max}(-25, 0) = 0 \Big|_{S_1=125} \\ &= \text{Max}(100 - 80, 0) = \text{Max}(20, 0) = 20 \Big|_{S_1=80} \end{aligned} \quad (15.10)$$

With these payoffs at expiration, we want to know how much the put option is worth at $t = 0$. We can solve for the put value just as we did for the call. We can determine the put value that results when we model either a replicating investment or a riskless portfolio, or we can also go directly to the solution with the Cox, Ross, and Rubinstein formula.

In the case of the call, we found we could replicate the payoffs for the call with a portfolio consisting of a long position in the stock plus borrowing. For the put, we simply reverse the signs of the transaction and duplicate the put payoff with a short position in the stock combined with lending.

15.3.2 Put–Call Parity

The values of the put and call are related, a relationship that flows from the fact that the payoffs from both types of options can be replicated using some combination of the underlying stock and a riskless bond. Hence, the prices for a put and call must be linked in equilibrium. The relationship is called put–call parity. Assuming the put and call

have the same exercise price and time to expiration, the link between the prices of the put and call can be expressed as

$$C + Xe^{-rT} = S + P \quad (15.11)$$

The left side of the equation is the call price plus the present value of exercise price; the right side is the stock price plus the put price. The equation can be rearranged to express the value of one variable in terms of all the other variables—the call expressed in terms of bond, stock, and put, or the put expressed in terms of the bond, stock, and call. If prices do not conform to this relationship, an arbitrage opportunity will enable investors to make abnormal profits.

For example, suppose prices are such that Equation 15.11 did not hold. To be specific, suppose

$$C + Xe^{-rT} > S + P \quad (15.12)$$

Or

$$C + Xe^{-rT} - S - P > 0 \quad (15.13)$$

To arbitrage this situation, assume that at time $t = 0$, we sell a call for price C , borrow an amount (Xe^{-rT}) equal to the present value of the exercise price on the call, buy a put for price P , and buy the stock for price S_0 . When we add up the cash inflows and outflows from these transactions, we have total flow as shown algebraically in line 6 of [Exhibit 15.11](#), where the inflows are shown with a positive sign, and outflows are shown with a negative sign. The algebraic sum of the cash flows in line 6 is identical to Equation 15.13, so it is positive by assumption. This means we have a positive amount of money flowing in at $t = 0$ because the cash inflow from selling the call and borrowing exceeds the outflow from buying a put and buying the stock.

Investment Transaction and Cash Flows at $t = 0$			Cash Flows at Expiration of Options	
1	Transaction	Flow	Stock Down $S_1 < X$	Stock Up $S_1 > X$
2	Sell Call	$+C$	0	$-(S_1 - X)$
3	Borrow	$+Xe^{-rT}$	$-X$	$-X$
4	Buy Put	$-P$	$+(X - S_1)$	0
5	Buy Stock	$-S_0$	$+S_1$	$+S_1$
6	Sum of Cash Flows	$C + Xe^{-rT} - P - S_0 > 0$ by assumption	$0 - X + (X - S_1) + S_1 = 0$	$-(S_1 - X) - X + 0 + S_1 = 0$

EXHIBIT 15.11 Cash inflow and outflow from transaction with stock, bond, call, and put.

Now look at the cash flows at the expiration date. Each line shows the flow from the purchase or sale, and the algebraic sum of the flows from each of the transactions is shown in line 6. No matter what happens, whether the stock goes up or down, the total payoff at expiration is zero, so there is no uncertainty regarding the payoff. Like the example in [Exhibit 15.7](#), the total transaction is one where we have a positive inflow at time $t = 0$

and zero flow at time $t = 1$. In other words, we structure the buying and selling, so money comes in at $t = 0$, but on a net basis, we do not have to pay anything out at time $t = 1$.

This is the nirvana that every greedy investor dreams about—a situation where money is produced at no cost and no risk. Unfortunately, financial nirvanas seldom exist and if they do exist, it will only be for a fleeting moment. If prices are as assumed in Equation 15.13, investors will quickly notice the disequilibrium, will rush to take advantage of it, and prices will change so that such a profitable arbitrage opportunity quickly disappears. This means that in equilibrium, Equation 15.11 must apply, and Equation 15.13 must hold as an equality. This relationship between prices is what is meant by put–call parity. Put and call prices have to conform to this relationship, or there will be arbitrage opportunities that cannot last.

We can build an Excel model that we can use to check whether prices are conforming to put–call parity and to devise trading strategies if price depart from parity. Exhibit 15.12 is a model for the one-period binomial case allowing the user to input the various buy and sell transactions and see the results in terms of cash flows at times $t = 0$ and 1. Formulas for rows 29–34 are shown in Exhibit 15.12b. The input section, A1:E15 is similar to the previous binomial models except that current market prices of the call and the put are in D11 and E11 as input data. Lines 19–21 show the binomial stock prices and the call and put payoffs at expiration. The transaction section is in A27:H34, showing the transactions and the cash flows at time $t = 0$ and 1. As we saw earlier, the put–call parity equation takes account of the prices of

	A	B	C	D	E	F	G	H
1	Binomial Option Model							
2	Put - Call Parity for Binomial Model							
3								
4	Data Input							
5	Riskless Interest Rate	5.0%						
6	Initial Stock Price	100						
7	Option Features							
8	Type of Option			Call	Put			
9	Exercise Price		X	100	100			
10	Time to Expiration		T	1	1			
11	Price of Option			15.00	8.40			
12	Stock Returns over 1 period							
13		Rate of Return		Wealth	Relative Return			
14	Outcome 1	25%		u	1.25			
15	Outcome 2	-20%		d	0.80			
16								
17	Time	0				***** 1 *****		
18					Stock Price t_1	Call Value	Put Value	
19					→ S u	125	25.0	0.0
20	Stock Price t_0	100			→ S d	80	0.0	20.0
21								
22								
23								
24	Time		***** 0 *****			***** 1 *****		
25						S d	S u	
26								
27		Price	Transaction					
28			Units	Cash Flow				
29	Stock	100	Buy	1	-100.00			
30	PV(Exercise)	0.9512	Borrow	100	95.12	-100	-100	
31	Call	15	Sell	1	15.00	0.0	-25.0	
32	Put	8.40	Buy	1	-8.40	20.0	0.0	
33								
34	Portfolio Cost				1.72	0.00	0.00	

(a)

EXHIBIT 15.12 (a) Put and call transaction for put–call parity.

(Continued)

	A	B	C	D	E	F	G	H	I	J
24	Time		++++++	0	++++++		*****	1	*****	
25							S d	S u		
26										
27		Price	Transaction							
28				Units	Cash Flow					
29	Stock	100	Buy	1	-100.00		80	125		
30	PV(Exercise)	0.9512	Borrow	100	95.12		-100	-100		
31	Call	15	Sell	1	15.00		0.0	-25.0		
32	Put	8.40	Buy	1	-8.40		20.0	0.0		
33										
34	Portfolio Cost				1.72		0.00	0.00		
35										
36	Cash flow									
37	=IF(C29="Buy",-1,1)*B29*D29				=IF(C29="Buy",1,-1)*F21*D29					
38	=D30*EXP(-B5*E10)*IF(C30="Borrow",1,-1)				=E30*EXP(B5*E10)					
39	=IF(C31="Buy",-1,1)*B31*D31				=MAX((F21-\$D\$9),0)*IF(C31="Buy",1,-1)*D31					
40	=IF(C32="Buy",-1,1)*B32*D32				=MAX(-(F21-\$D\$9),0)*IF(C32="Buy",1,-1)*D32					
41										
42										
43										

(b)

EXHIBIT 15.12 (Continued) (b) Put-call parity model with selected formulas.

the stock, call, and put, and the opportunity to borrow and lend. Stock is in line 29, borrowing or lending is in line 30, the call is in line 31, and the put is in line 32, with the totals for the portfolio in line 34. For each of these instruments, the current price is carried down from the input section. The user enters the desired transaction in C29:C32 by entering the words Buy, Borrow, or Sell in the appropriate lines. The number of units of each is in column D, where units refers to shares, and to the dollars borrowed or lent, expressed in terms of maturity value. The time $t = 0$ cash flows from each transaction are in E29:E34. The cash flows at the expiration date from each transaction are shown in G29:H34 for each of the binomial states.

In this example, all the input data are the same as in the previous binomial examples except that the call is priced at a disequilibrium price of \$15.00, as compared to the equilibrium price that should be \$13.28 as was shown in the previous examples. With the price of the call too high, it should be obvious that we should not buy the call. As the model will show, if we sell the call, combined with the other instruments, we have the opportunity to earn a positive return with no investment and no risk.

Cells C29:D32 show the transactions we have entered. In C29 we enter "Buy" for the stock, and indicate 1 share in D29; E29 shows the cash outflow at $t = 0$ of buying 1 share at \$100. In C30, we enter "Borrow" to indicate the transaction in the riskless bond. The amount borrowed is \$95.12, which is equal to the present value of \$100, the exercise price of the options. Line 31 shows the call transaction, where we enter "Sell" in C31, indicating 1 share in D31. Selling 1 call bring in a positive cash flow of \$15 in E31. Line 32 relates to the put, where we enter "Buy" in C32, and 1 share in D32, with the amount paid of \$8.40 in E32 (negative indicating a cash outflow). In column E, we show that we payout \$108.40 for the stock and put, we have an inflow of \$110.12 from the borrowing as call sale, for a net flow of \$1.72 in E34.

The payoffs at the expiration date ($t = 1$) are in columns G and H, based on the binomial stock prices of \$80 and \$125 for the down and up states, respectively. We buy 1 share of stock, and then sell it for either \$80 or \$125 (cell G20 and H29). We borrow \$95.12 at 5% at $t = 0$, and repay \$100 in both states. The call we sold will pay \$0 in the down state, but will cost us

\$25 in the up state (G31 and H31), and the put we bought will pay \$20 in the down state, or 0 in the up state, as shown in G32 and H32. Combining all these $t = 1$ payoffs, we see in G34 and H34 that whatever happens, we have a zero payoff. In other words, the payoff is riskless.

If an investment pays zero at $t = 1$ with certainty, it should be worth zero today ($t = 0$). What we see in this case is that this investment that pays zero at $t = 1$ will actually provide us with a positive cash flow at $t = 0$. So it is worth zero, but instead of costing zero, it is as if someone is paying us for the investment. This looks like a great deal. And, in contradiction to the saying that “If it seems too good to be true, it probably is,” so long as prices are as indicated in the example, it is a virtual money machine. But money machines seldom last long. Investors who are aware of the put–call parity relations, and with the ability to analyze the data with a model such as this, will quickly see and act on the arbitrage opportunity. Prices will change until the arbitrage opportunity disappears. Meanwhile, you, the model builder, will be able to analyze different transaction possibilities by trying them out in your model, and perhaps spot a disequilibrium that can make you rich.

15.3.3 Multistep Binomial Option Models

15.3.3.1 Two Periods

In the last section, we learned how to model puts and calls when it was assumed that stocks followed a binomial process over one period. In [Chapter 13](#), we noted that even though the binomial model of stock prices can be unrealistic if the length of the single binomial step is too long, we can make the model more realistic if we cut time into shorter and shorter intervals and assume that stock prices follow a binomial process over these very short time intervals. In this section, we will continue to use the binomial model for options, but we will use short time intervals to more closely approximate the actual behavior of stock prices.

The first step to understanding the multiperiod option models is to understand how we work backward through time, from the future back to the present, in order to derive today’s value of the option. Ultimately, we will use many periods, but to get started, assume that we have a 2-period model as shown in [Exhibit 15.13](#). Using the same binomial numbers,

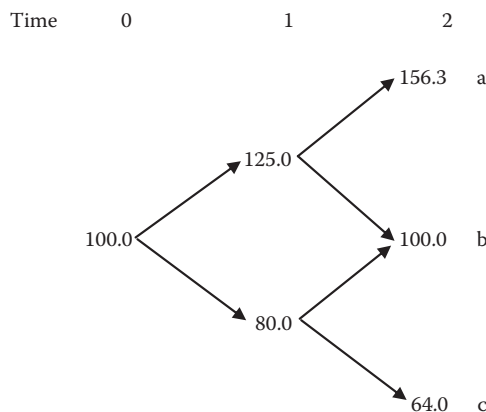


EXHIBIT 15.13 Stock price paths over two periods with binomial model.

we have used previously, assume that the stock starts out at $t = 0$ at a price of \$100 and can go up by $u = 1.25$ or down by $d = 0.80$ by $t = 1$. This yields prices of either \$125 or \$80 at $t = 1$. Then, from $t = 1$ to $t = 2$, the price can change again by 0.80 or 1.25 to any of the three different prices, \$156.3, \$100, or \$64. These three ending prices are the starting points for our analysis of the options.

As before, assume the exercise price for the call is $X = \$100$, but assume the expiration date is $t = 2$ instead of $t = 1$. At $t = 2$, the expiration date, the call will be worth $\text{Max}(S_2 - X, 0)$, which will be either $\$56.3 = \text{Max}(156.3 - 100, 0)$, or $0 = \text{Max}(100 - 100, 0) = \text{Max}(64 - 100, 0)$.

Next, we step back in time to $t = 1$, and calculate the value of the call at that date. It is easy to see that if the current stock price is \$80, the call will be worthless because there is no way for the stock to rise above the \$100 exercise price by $t = 2$. On the other hand, if the price at $t = 1$ is \$125, it could move to either \$100 or \$156.3 by $t = 2$, so the option will have value. We use the familiar binomial model at $t = 1$ based on the price of \$125 and the possible moves to \$100 or \$156.3 in the next step. To determine the value of the call at $t = 1$, we can either modify the one-step models shown in Exhibits 15.9 or 15.10 by changing the initial stock price to \$125, or we can go directly to the solution using the Cox, Ross, and Rubinstein (CRR) formula (15.3). Using that formula for the time one value we have

$$\begin{aligned} C &= [pC_u + (1-p)C_d]e^{-rT} \\ &= [0.5584 * 56.25 + 0.4416 * 0] * 0.9512 \\ &= \$29.88 \end{aligned} \tag{15.14}$$

where the risk neutral probabilities $p = 0.5584$ and $1 - p = 0.4416$ are calculated the same as they were earlier for the single-step problem. Now we know that if the price of the stock at $t = 1$ is \$125, the value of the call will be \$29.88, given that it expires at $t = 2$ with an exercise price of $X = \$100$.^{*} Next, we need to take another step back in time from $t = 1$ to $t = 0$.

Now, step back to $t = 0$ when the stock is priced at \$100, and there are two periods to go until the call can be exercised. If the stock moves from \$100 to \$125, the call will be worth \$29.88 at $t = 1$ as just shown. If the stock moves to \$80, the call will be worthless. Once again, we use the CRR formula with the risk neutral probabilities of $p = 0.5584$ and $(1 - p) = 0.4416$, to calculate the value of the call at $t = 0$ as

$$\begin{aligned} C &= [pC_u + (1-p)C_d]e^{-rT} \\ &= [0.5584 * 29.88 + 0.4416 * 0] * 0.9512 \\ &= \$15.87 \end{aligned} \tag{15.15}$$

* The call value at $t = 1$ of \$29.88 assumes that the call cannot be exercised at $t = 1$. If it could be exercised early, the value at $t = 1$ would be $\text{Max}(\text{Value if Exercised}, \text{Value if Wait}) = \text{Max}(156.3 - 100, 29.88) = \56.3 , because if it is exercised at $t = 1$ when the stock is priced at \$156.3, the call value will be the intrinsic value, the difference between the stock price and the exercise price.

For the two-period model, we now have the values of the call at each node. The call starts out at $t = 0$ with a value of \$15.87, and then either goes to \$29.88 or 0 at $t = 1$, depending on what happens to the stock. If $S_1 = \$125$, the call is worth \$29.88, but otherwise it remains worthless because there is no chance the stock will rise above \$100 by the expiration date at $t = 2$.

Reviewing the solution method for the multiperiod problem: We start at the end, when the option will expire, and work backward to the present. Next, we want to apply this method to the case when we use small-time increments, so we get a better approximation to the continuous movement of stock prices over time.

15.3.3.2 Modeling Option Prices with the Binomial Approximation of Continuous Prices
In [Chapter 12](#), we discussed the model of stock prices where the returns on the stock follow a continuous Weiner process. In that model, over a short interval of time, dt , the return on the stock is generated by the process expressed as

$$\text{Return} = \frac{dP}{P} = \mu dt + \sigma z \sqrt{dt} \quad (15.16)$$

where P is the price, dP is the change in the price, dP/P is the rate of return, μ is the expected return per period, σ is the standard deviation of return per period, and z is the standard normal random variable with mean 0 and standard deviation of 1. Having introduced the continuous time model, we showed how to use the binomial model to approximate the continuous movement of prices. This approximation chops time into smaller and smaller increments and allows prices to follow a binomial process over these very short intervals. The result is that when the stock price makes small binomial jumps over short time intervals, the pattern of prices over longer periods, made up of many small jumps, closely resembles the pattern that would result from the continuous Weiner process. Now, we use this approximation to model option prices. We use the binomial option model that was introduced in the previous section where we start at the end of the process and work backward, with the time interval for each step being short relative to the time until the option expires.

To approximate the continuous Weiner process of stock returns using the up and down binomial process, the up and down movements are specified as a function of the standard deviation of the continuous return on the stock. These relationships, developed by Cox, Ross, and Rubinstein (1979), are expressed as

$$u = e^{\sigma\sqrt{dt}} \quad (15.17)$$

$$d = e^{-\sigma\sqrt{dt}} = \frac{1}{u} \quad (15.18)$$

and

$$p = \frac{e^{\mu\sqrt{dt}} - d}{u - d} \quad (15.19)$$

= probability of an up move, u

where σ is the standard deviation (volatility) of the annual rate of return on the stock, dt is the length of the time interval, which in this context is a discrete time interval such as one day or one week, p is the risk neutral probability of the stock making the up move, and $(1 - p)$ is the risk neutral probability of the stock making the down move in the short, discrete time interval, dt .

To model the options, we will use the same numerical example of stock prices that we used in [Chapter 13](#). The stock is currently priced at \$100 and its returns are assumed to follow the process described by Equation 15.16 with the mean return per year on the stock of $\mu = 0.12$, and the standard deviation of annual return is $\sigma = 0.20$. Based on Equations 15.17 and 15.18, using intervals of one week so that $dt = 1/52 = 0.0192$, the stock movement each week will be either $u = 1.0281$ or $d = 0.9726$, with the risk neutral probability the price moving up calculated as

$$\begin{aligned} p &= \frac{e^{\mu\sqrt{dt}} - d}{u - d} \\ &= \frac{e^{0.12\sqrt{0.0192}} - 0.9726}{1.0281 - 0.9726} \\ &= 0.535 \end{aligned} \tag{15.20}$$

and the risk neutral probability of the price moving down by d is $(1 - p) = 0.465$. The call option on the stock expires in 13 weeks, and has an exercise price of $X = \$100$.

[Exhibit 15.14a](#) shows the Excel model of the stock price and the option, and [Exhibit 15.14b](#) shows the formulas in selected cells of the model. The data input is in A6:B22, and A24:B31 calculates the step length, the weekly stock price step sizes $u = 1.0281$ (B27) and $d = 0.9726$ (B28). The risk neutral probabilities of the up and down moves are in B30 and B31.

Based on this input, the stock will move up by $u = 1.0281$ or down by $d = 0.9726$ each week. Starting from the current price of \$100 per share, the price will move to the values shown by the tree of binomial prices in F11:S24. In the first week, the price can move to either \$102.81 (G11) or \$97.26 (G12). If the price at $t = 1$ is \$102.81, it can either move up to \$105.70 (H11), or down to \$100 (H12). If the stock goes down to \$97.26 at $t = 1$, it can either move up to \$100 (H12), or down to \$94.60 (H13). This process continues each week with the price moving up or down to the prices shown in this triangular matrix. The top row (row 11) is the sequence of stock prices moving up each week ($S_t = S_0 u_1 u_2 \dots u_t$). The bottom diagonal represents the sequence of prices dropping each period ($S_t = S_0 d_1 d_2 \dots d_t$). The cells in between are the other binomial branches. At each date t , there are $t + 1$ possible prices. So, by week 13, there are 14 possible prices ranging from \$143.41 (S11) to \$69.73 (S24).

Corresponding to each possible stock price at each date is a call value. These are shown in the F29:S42. For example, the call value of \$32.237 in R30 corresponds to the stock value in the same relative location in the stock price matrix; that is \$131.96 in R12. The call value calculations start at the expiration date in week 13 and work backward. Column S corresponds to the expiration date, and the call values in S29:S42 are calculated as the

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	
1	Option Value With Binomial Approximation																			
2																				
3																				
4																				
5																				
6	Data Input																			
7																				
8																				
9																				
10																				
11																				
12																				
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41																				
42																				

EXHIBIT 15.14 (a) Call valuation model based on binomial approximation of continuous price changes. (Continued)

	A	B	C	E	F	G	R	S
1	Option Value With Binomial Approximation							
2	$\frac{dP}{P} = \mu dt + \sigma z \sqrt{dt}$							
3								
4								
5								
6	Data Input	Tree of Binomial Stock Prices						
7	Stock Data		Step	0				
8	Initial Stock Price	100	Time	0	=F9+ $\$B\24			
9								
10								
11	Annual Data		Stock	=B9	=F11* $\$B\27			
12	Risk Free Interest Rate	0.05	Prices		=F11* $\$B\28			
13	Mean Return	0.12			=O1* $\$B\28			
14	Standard Deviation Per Year	0.2			=O1* $\$B\28			
15					=O14* $\$B\28			
16	Option Data				=O15* $\$B\28			
17	Type	Call			=O16* $\$B\28			
18	Exercise Price	100			=O17* $\$B\28			
19	Time to Expiration	13			=O18* $\$B\28			
20					=O19* $\$B\28			
21	Length of Sub-Period	1 Week			=O20* $\$B\28			
22	Sub-Periods Per Year	52			=O21* $\$B\28			
23					=O22* $\$B\28			
24	Step Length	=1/B22	Tree of Binomial Call Values					
25	(Step Length) ^{1/2}	=SQRT(B24)						
26	Up Wealth Relative Return	=EXP(B14*(B25))						
27	Down Wealth Relative Return	=1/B27						
28								
29	Up Probability	=(EXP(B13*(B24)-B28)/(B27-B28))	Call					
30	Down Probability	=1-B30	Values					
31								
32								
33								
34								
35								
36								
37								
38								
39								
40								
41								
42								

(b)

EXHIBIT 15.14 (Continued) (b) Formulas for Exhibit 15.14a.

intrinsic value, $Max(S_{13} - X, 0)$. The call values in each earlier period are calculated according to the Cox, Ross, and Rubinstein binomial formula

$$C_t = [pC_{u,t+1} + (1-p)C_{d,t+1}]e^{-rT} \quad (15.21)$$

where C_t is the call value at time t , and $C_{u,t+1}$, and $C_{d,t+1}$ are the call value at the next date given that the stock went up or down. For example, suppose the stock price in week 12 is \$131.96 (R12), and can move either up to \$135.67 (S12) or down to \$128.35 (S13). At the expiration date, the value of the call will be either $C_{u,13} = Max(135.67 - 100, 0) = \35.67 (S30) or $C_{d,13} = Max(128.35 - 100, 0) = \28.35 (S31). Stepping back to week 12 (column R), the call value will be

$$\begin{aligned} C_t &= [pC_{u,13} + (1-p)C_{d,13}]e^{-rT} \\ &= [0.535 * 35.67 + 0.465 * 28.35] * 0.999 \\ &= \$32.237 \end{aligned} \quad (15.22)$$

as shown in cell R30. All of the call values for week 12 are calculated similarly in column R. Then the process steps back to week 11 in column Q and calculates the call value according to Equation 15.11, where the $C_{u,t+1}$ and $C_{d,t+1}$ refer to the call values already calculated for week 12 in column R. We continue to step back week by week until we reach the current call value of $C = \$5.748$ in F29.

In this example, we used step lengths of one week so as to limit the size of the problem in the discussion and exhibits. A more accurate value would result if we cut time into smaller increments. For example, for the call expiring in 13 weeks, we could easily build our model with daily price moves. There would be 65 trading days, and the step length would be $1/260 = 0.0038$ years. The model in [Exhibit 15.14](#) would be expanded to 65 columns instead of 13. The structure would be exactly the same, with the same formulas, we just add columns and rows. By the 65th week, there would be 66 possible stock prices, so we would need 66 rows for stock prices and 66 rows for call prices. We will leave it as an exercise for you to calculate the value of the call with the same features as our example, but with daily price changes.

15.4 OPTION PRICING WITH CONTINUOUS STOCK RETURNS: THE BLACK-SCHOLES MODEL

In the previous section, we developed a binomial model of option prices where we approximated the continuous process with binomial stock price movements. In this section, we take the next step and work with the famous Black-Scholes (1973) option model that is based on the assumption that stock returns follow the continuous Weiner process described as

$$Return = \frac{dS}{S} = \mu dt + \sigma z \sqrt{dt} \quad (15.23)$$

that we introduced in [Chapter 13](#).

The Black–Scholes model of option prices starts with the assumption that the stock returns follow the continuous Weiner process Equation 15.12 and derives the formula expressing the value of a European call based on the current value of the stock, S_0 ; the annual rate of volatility of the stock return, σ ; the risk-free interest rate, r ; the exercise price for the call, X ; and the time until expiration, T :

$$C = S_0 N(d_1) - X e^{-rT} N(d_2) \quad (15.24)$$

where

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (15.25)$$

$$\begin{aligned} d_2 &= \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad (15.26) \\ &= d_1 - \sigma\sqrt{T} \end{aligned}$$

The subscripts on d_1 and d_2 do not denote time, they simply distinguish between the two formulas; and the terms $N(d_1)$ and $N(d_2)$ denote the cumulative standard normal probability functions. In other words, $N(d_1) = \text{Prob}(x \leq d_1)$, where x is a normally distributed random variable with mean 0 and standard deviation of 1. In this formula for the cumulative probability, you are asking: “What is the probability that the random variable x will be less than d_1 ?” The Excel function for the standard normal variable is =NormSDist(d_1), where d_1 is a number or cell reference, or even another formula.

If you have not worked with the Black–Scholes formula before, it may look rather intimidating. However, it is fairly straight forward, and it is very easy to model it in Excel. [Exhibit 15.15](#) shows the Black–Scholes option value calculation. The input is the same example we have been using with a stock priced at \$100 with mean annual return of 12% and standard deviation of return of 20%. The call has an exercise price of \$100, with $\frac{1}{4}$ year to expiration (it is a European call, so it is exercisable only at the expiration date).

In [Exhibit 15.15](#), the input data are in A3:B16, and the Black–Scholes option formulas are shown in the block on the upper right. The computed results are shown in D22:D39, with the value of the option shown as \$4.615 in D39. The Excel formulas used at each step of the calculation are shown to the right of the answers. The formulas could be combined for a more concise model, but the step-by-step calculations are shown, so you can follow it more easily.

For example, “ $\ln(S/X)$ ” is used to calculate d_1 and d_2 and is calculated in D22 as =LN(B6/B15).

So now you say to yourself, “Wow! I programmed the famous Black–Scholes option formula in Excel. So what? What good does that do me? How can I use it?”

	A	B	C	D	E	F	G				
1	Black-Scholes Option Value										
2											
3	Data Input			$C = S_0 N(d_1) - X e^{-rT} N(d_2)$ <p>where</p> $d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ $d_2 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$ $\frac{dS}{S} = \mu dt + \sigma z \sqrt{dt}$							
4											
5	Stock Data										
6	Initial Stock Price	100									
7											
8	Annual Data										
9	Risk Free Interest Rate	5%									
10	Mean Stock Return	12%									
11	Standard Deviation Per Year	20%									
12											
13	Option Data										
14	Type	Call									
15	Exercise Price	100									
16	Time to Expiration (Years), T	0.25									
17											
18											
19											
20											
21											
22		ln(S/X)		0.0000		=LN(B6/B15)					
23		r +(σ ² /2)		0.0700		=B9+(B11^2)/2					
24				0.0175		=D23*B16					
25		σ(T ^{1/2})		0.1000		=B11*SQRT(B16)					
26											
27		d 1		0.1750		=(D22+D23*B16)/D25					
28		d 2		0.0750		=D27-D25					
29											
30		N(d 1)		0.5695		=NORMSDIST(D27)					
31		N(d 2)		0.5299		=NORMSDIST(D28)					
32											
33		S N(d 1)		56.946		=B6*D30					
34											
35		PV of \$1 @ r		0.9876		=EXP(-B9*B16)					
36		PV(X)		98.758		=D35*B15					
37		PV(X)N(d2)		52.331		=B15*D35*D31					
38											
39		Call Value		4.615		=D33-D37					

EXHIBIT 15.15 Excel model for Black–Scholes option formula.

First answer: congratulations. Second answer: If you have the required formula inputs, you are able to easily calculate an approximate value for the option.* Even if the formula is not exact in real-world applications, it is close enough to give us a good indication of what the option should be worth. If the market price differs greatly from the computed value, it may indicate that there is profitable trading opportunity. On the other hand, it may also indicate that the numbers you plugged into the formula are wrong.

Let us look more closely at the figures you need to plug into the formula. One of the significant things about this and other option formulas is that most of the inputs are readily

* The Black-Scholes formula has been shown to have various systematic biases so that the computed value departs systematically from the observed market prices. For more detailed discussions see Cox and Rubinstein (1985), pp. 338–342; Dubofsky and Miller (2003), pp. 532–547, and Hull (2015), Chapter 18.

observable and not subject to guesses or forecasts. There is only one item in the formula that you cannot find out by simply consulting the newspaper: the volatility of the stock. The list of observable inputs is the stock price, S_0 ; the exercise price of the call, X ; the time to expiration, T ; and the risk-free interest rate. The item you cannot directly observe is the volatility of the stock return, σ . The items that you do not need to know or estimate include the expected return on the stock, or factors such as beta.

To use the formula, the one item we cannot directly observe and must be estimated is the stock's volatility, σ , the standard deviation of the return. What we would like to have is the volatility for the future period until the option expires. This we can never really know, so past observations of the returns are used to estimate the volatility. As we know, estimates are usually imperfect, and past volatility may not be a good predictor of future volatility. Nevertheless, volatility estimates from past data provide a good starting point and lead to useful valuation estimates.

One of the more interesting applications of the model is its use to estimate *implied volatility*. Even though we cannot observe volatility, we can observe the actual option price in the market and the other inputs to the model. By plugging these variables into the model, we can work backward to solve for the volatility that is consistent with, or *implied* by, the observed option price and other variables. This gives us an estimate of the volatility that other investors must be attributing to the stock in the future. If the implied volatility differs from our own prediction, it may signal a trading opportunity—if we think we can make better predictions than the rest of the investors in the market. Implied volatility is also used to estimate volatility for comparable options. For example, we may not have much information about a particular option that is traded infrequently. We turn to other options of similar companies that have similar option features, and that are traded more actively. Using the implied volatility of these comparable companies, we can estimate the volatility for the less actively traded option, and calculate its value.

How do we extract the implied volatility from option prices? Some of the more statistically accurate methods are beyond the scope of this text, but for a good first approximation, we can use our Excel model to solve for the implied volatility. For example, suppose we have the same data as in our previous example, with the stock trading at a price of \$100 per share, the exercise price of the call is \$100, the call expires in 90 days ($\frac{1}{4}$ year), and the risk-free interest rate is 5%. Unlike the previous example, the call is currently trading at a price of \$6.00. What volatility must investors be attributing to the stock in order for the call to trade at a price of \$6.00? We solve this easily with our model using Excel's Goal Seek with the following steps: Data > What-if Analysis > Goal Seek brings up the Goal Seek dialog box. Fill it in with these entries: Set Cell: F19 (the call value cell), To Value: 6.00, By Changing Cell: B12 (the standard deviation input cell), click OK. The solution value for volatility will be in B12, which is now 27.04%. Assuming the Black-Scholes model is reasonably accurate, the call price of \$6.00 implies that investors must be assuming the stock return volatility to be about 27%.

This demonstrates an important aspect of option valuation: the more volatile the stock, the more valuable the option. The reason for this is simply that if the stock price and return are more volatile, there is a higher probability that the call will be in the money when it expires.

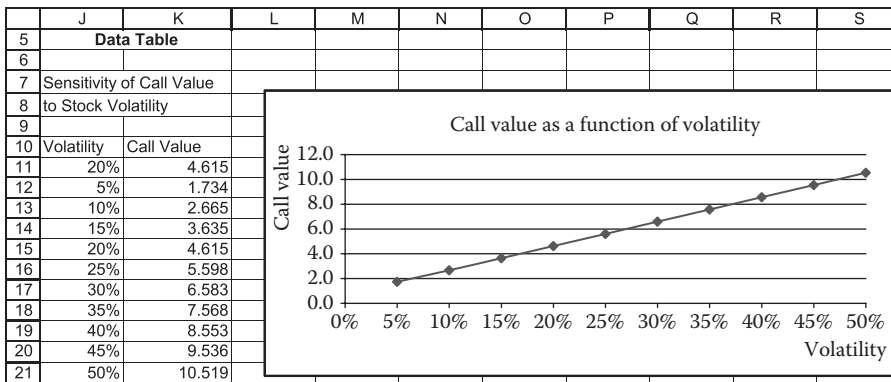


EXHIBIT 15.16 Volatility of stock returns and call value.

Exhibit 15.16 shows this link between volatility and call value,^{*} where we see the value for the call in our example increasing from \$1.73 when $\sigma = 5\%$ to \$10.52 when $\sigma = 50\%$.

With this model, you can explore many aspects of option valuation. In the exercises at the end of this chapter, you will have the opportunity to see the impacts of changing the variables in the pricing model and gaining greater understanding of options.

15.5 SUMMARY

This chapter introduced you to the basics of options and option pricing. We presented several different models that allow you to understand options and to analyze their prices. The first section of this chapter explained what options are and showed how to model their payoffs. Once we understand the payoffs at expiration, we can then analyze different combinations of options, stock, and bonds. One of the most useful aspects of our models is to see and understand how we can mix options with stock and bonds in a way to create a riskless investment. It is from this idea that we get option pricing models.

We looked at two kinds of option models. The first was the binomial model based on the assumption that stock prices will move either up or down. Although this is a very simple model, it not only helps us to understand option payoffs and values, but it can be fine tuned to approximate the more realistic continuous time model.

The second type of option model is based on the assumption that stock prices move according to a Weiner process in continuous time. The Black–Scholes model is a continuous time model. Although the formula looks foreboding at first, we found that it is quite easy to use it in a spreadsheet, and we encourage you to construct your own model, so you can explore the nuances of this useful formula.

Options, like so many other topics in this text, is a complex topic covered in many specialized articles and books. We cannot hope to cover all the interesting aspects of options in this chapter. But with this introduction to the topic, you should be well prepared to develop and extend your own models of options and option trading.

^{*} The sensitivity of the option value to changes in volatility is referred to in the option literature as the “vega.”

PROBLEMS

1. Binomial Model of Option Prices.

Develop a model to analyze the values of both a call and a put when it is assumed that the stock price follows a binomial process over monthly intervals. Assume the initial stock price is \$100 and you can either borrow or lend at the risk-free interest rate of 5% per year. Each month, the price of a stock can make an up move of $u = 1.106$, or a down move of $d = 0.904$. The call has an expiration date at the end of 12 months, and the exercise price is \$110. The put also expires in one year, and it has an exercise price of \$110.

- a. Use your model to structure a mix of the stock and bond (borrowing or lending) so as to replicate the payoffs from the call at expiration. Based on this, how much should the call be worth one month prior to expiration? Is there just one call value at that date?
- b. Use the Cox, Ross, and Rubinstein formula to determine the value of the call at each date at each stock price over the 12-month horizon.

2. Black–Scholes Model

Develop a spreadsheet model of the Black–Scholes option model. Assume that stock prices follow a continuous Weiner process with a mean annual return of 18%, and standard deviation of the annual return is 35%. Assume the stock price is currently \$100 per share. The call will expire in six months. The exercise price is \$110. The risk-free interest rate is 5% annually.

- a. Show the value of the call.

Now explore the impact on call value of varying the assumptions:

- b. What happens to the value of the call as time passes? Or, keeping all other factors the same, what happens when you change the time to expiration? Graph the relationship between time to expiration and call value.
- c. What happens when you change the interest rate? Graph the relationship between the interest rate and call value. Briefly explain why you get this relationship.



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VI

Optimization Models



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Optimization Models for Financial Planning

16.1 INTRODUCTION TO OPTIMIZATION

The modern theory of finance seeks to define the optimal investment and financing decisions for a firm, where optimal decisions are taken to mean those that maximize the wealth of the firm's current stockholders. The body of financial theory that has been developed seeks to explain the link between decisions and the value of the firm. If the effect of decisions on firm value can be determined, then we should be able to determine which particular policy or set of decisions will maximize the firm's value. The theory of finance, however, is not so perfected that we have a completely accepted and verified theory of valuation. Consequently, we are not yet in a position to make perfect financial decisions that are the simple result of some calculation. Nevertheless, we do have much of the necessary valuation framework to help us describe the links between decisions and the objective of value maximization. Armed with that framework, we should be able to improve financial decision making by constructing decision models that calculate optimal decisions. That is the topic of this chapter—the construction and use of optimization models to aid us in financial planning and decision making by providing the mechanism for calculating the optimal decisions in the context of our financial model.

Chapters 4 and 5 explained the construction and use of financial statement simulation models of the firm. Simulation models are useful for exploring the consequences of decisions. As such, they are used more widely all the time. However, simulation models have drawbacks: there may be too many decisions to evaluate, too much output for each decision to compare, or simply too much output to manage. In addition, when a problem presents many different decision alternatives, the number of different decision combinations that can be considered may be too large for a person to analyze and evaluate. For example, suppose a firm has identified 11 investment projects and wishes to choose the best combination of projects to be included in the firm's capital budget. The portfolio of projects could contain one project, all 11 projects, or any number in between, with the number

of different combinations of projects that could be considered over 2,000. Most decision makers would rather not be faced with the task of sifting through 2,000 alternatives to find the best combination. The optimization approach forces the model user to carefully define the objective of the decision and create a summary statistic. Optimization models then systematically analyze all possible combinations and determine the combination that yields the optimal solution.

A dictionary defines an optimum as “the best or most favorable point,” or “the best result obtainable under specific conditions.” Optimization is the process of finding the optimum. In other words, optimization is the process of searching or solving for a decision variable that will yield the best result obtainable under the conditions faced by the decision maker. For example, Equations 1.1 and 1.2 from [Chapter 1](#) constitute a simple model of the firm’s total revenue. They can be combined as one equation of the form:

$$\text{Total revenue} = 500 * \text{Price per unit} - 30 * \text{Price per unit}^2 \quad (16.1)$$

In this simple model, the decision variable is the *Price per unit*. If the objective of the decision is to make *Total revenue* as large as possible, the goal is finding the *Price per unit* that generates the largest *Total revenue*. Optimization is the process or technique of finding this optimal value.

Optimization techniques typically take one of the two forms. Some search systematically and efficiently through all the alternatives; others use information about the conditions characterizing an optimum to determine the optimal solution directly. One type of search technique enumerates the various possible values of the decision variable and the value of the objective function that results from that decision, then searches through the list to determine which was best. For example, suppose the prices to be considered by model (16.1) range from \$5 to \$10 in increments of \$1. The *Total revenues* associated with each *Price per unit* are:

<i>Price per unit</i>	\$5	\$6	\$7	\$8	\$9	\$10
<i>Total revenue</i>	\$1,750	\$1,920	\$2,030	\$2,080	\$2,070	\$2,000

A search for the highest *Total revenue* yields a value of \$2,080 at a *Price per unit* of \$8. There is no assurance, however, that there is not another *Price per unit* between \$7 and \$9 that yields a higher *Total revenue*. The search could be continued to consider prices in this second interval that might yield even better results.

One drawback of searching for the best alternative is that there may be too many alternatives, making the search process too time consuming and expensive. In this case, we turn to another method for finding the optimum. One familiar method is to use calculus to determine an extreme point in a mathematical function such as Equation 16.1. By taking the derivative of (16.1) with respect to the variable *Price per unit*, equating it to zero, and solving for *Price per unit*, we obtain the optimal solution of \$8.33. At this price, *Total revenue* is \$2,083.33, the largest revenue possible. Any other price produces less revenue.

[Exhibit 16.1](#) is a chart that shows *Total revenue* as a function of the *Price per unit*. The process of searching for a best price simply calculates values at different points along the

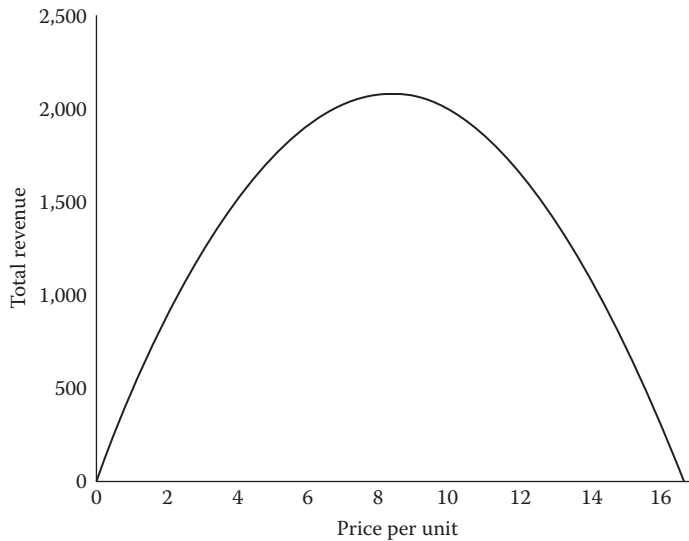


EXHIBIT 16.1 Revenue as a function of price.

curve. Each point represents the *Total revenue* earned at a given *Price per unit*. Using our knowledge that the maximum is reached when the slope of the curve is zero, calculus solves for that point.

Although the techniques of calculus are useful for many problems, the problems of financial modeling are usually too complex to be formulated in a way that can be solved directly with calculus. Fortunately, there are other techniques that are well suited to determining an optimal solution to a financial planning problem. These techniques come under the heading of mathematical programming—linear programming (LP), nonlinear programming, and dynamic programming. This chapter focuses on the use of LP to find optimal solutions to financial planning problems.

16.2 CONSTRAINED OPTIMIZATION

Mathematical programming is a technique for solving *constrained optimization* problems. Constrained optimization is the process of finding the optimal solution to a problem in the presence of constraints that limit the choices of the decision variables. For example, consider the problem of determining the quickest (optimal) way to get from New York to San Francisco. Clearly, the quickest way is to travel in a straight line at the speed of light—perhaps being beamed there by the Starship Enterprise. However, technology limits our ability to do that. In other words, constraints limit our choices and, ultimately, our ability to reach our goal.

The problem should be restated, saying that we want to find the quickest way to get from New York to San Francisco subject to several constraints. The first constraint is existing technology. Though traveling at the speed of light is preferred, traveling by airplane, train, and automobile are the only modes available. Second, we may want to impose a constraint on our expenditures, limiting our travel expenses to \$1,000. Finally, we want to make a stop in Bishop, California, to visit relatives. Given the transportation constraints, the airplane is

clearly the fastest mode of transportation. However, the commercial airlines do not stop in Bishop, so we must consider chartering a plane. Chartering a plane, however, would cause us to exceed our expenditure constraint of \$1,000. So we reject the airplane as a choice. Nor do trains go through Bishop. Therefore, we are constrained to traveling by car, if at all. The car would be a feasible choice if we can make the trip for no more than \$1,000; if not, then the trip is not feasible given our constraints. Given these constraints, the problem becomes finding the quickest route by car that is within our budget.

The pricing problem described by Equation 16.1 can also be formulated as a constrained optimization problem. Assume the objective remains maximizing *Total revenue*, but now the *Price per unit* must be at least \$9 in order to cover our costs. This problem is stated mathematically as

$$\underset{\{\text{Price per unit}\}}{\text{Maximize Total revenue}} = 500 * \text{Price per unit} - 30 * \text{Price per unit}^2 \quad (16.2)$$

subject to

$$\text{Price per unit} \geq \$9.00 \quad (16.3)$$

The expression to the right of the equal sign in Equation 16.2 is the objective function. It expresses the relationship between the decision variable, *Price per unit*, and the objective, *Total revenue*. Equation 16.3 is the constraint. It limits our choice of the decision variable to values that are \$9 or greater. In [Exhibit 16.1](#), it is obvious that a price of \$9 will yield the largest revenue, because *Total revenue* declines as the *Price per unit* increases for all prices greater than \$8.33, and \$9 is the lowest price that can be charged without violating the constraint.

16.2.1 Linear Programming

Linear programming is a method for formulating and solving certain kinds of constrained optimization problems. It is similar to the constrained maximization pricing example given above, in which the problem has an objective function that is to be either maximized or minimized, has one or more decision variables, and has one or more constraints that limit our choice among decision variables. The difference is that the objective function is linear. There are a number of different algorithms for solving LP problems that are quick and efficient. Quite large and involved problems can be solved quickly on a computer. Consequently, if a problem can be formulated as a LP problem, it can usually be solved quite easily.

To understand what problems can be solved with LP, it helps to understand the general structure of a LP problem. An LP problem consists of a linear objective function and linear constraints. What do we mean by linear objective function and constraints? Linear means that the relationships can be drawn as straight lines or planes, rather than curves. Equation 16.4 is an example of a linear objective function:

$$\underset{\{X_i\}}{\text{Max}} = 1.17 X_1 + 1.22 X_2 + 0.47 X_3 - 0.02 X_4 \quad (16.4)$$

Each X_i represents a decision variable whose value is to be determined so as to maximize the value of the maximand (the value to be maximized), Z ; the coefficients 1.17, 1.22, 0.47, and -0.02 are the contribution to the maximand per unit of their corresponding decision variables X_1 , X_2 , X_3 , and X_4 . What makes Equation 16.4 linear is that the decision variables are multiplied only by constant coefficients; none are multiplied by other decision variables, including themselves. For example, the expression $1.17X_1 + 1.22X_2$ is linear, but the expressions $1.17X_1X_2$ and $5X_1^2$ are not. The latter two expressions are drawn as curved relationships, whereas the former equations are drawn as straight lines.

Many problems in economics and finance are nonlinear. This was the case in the pricing problem presented earlier, in which the decision variable *Price per unit* appeared in the objective function as a squared term, *Price per unit*². This resulted in the objective function appearing on a graph as a curve [Exhibit 16.1](#). Another example would be if the objective is to maximize the value of equity, with the decision variables being the amounts of debt and equity financing. This will be a nonlinear problem if the discount rate used to determine the value of equity depends on the amount of debt financing, making the coefficients of the debt and equity financing variables depend on each other in a complicated way. As the problem would be nonlinear, it could not be solved with the techniques of LP. It potentially could be solved as a nonlinear programming problem, but the solution process would be more complicated. Fortunately, the increased power of the iterative solution algorithms packaged with today's spreadsheet software, such as Solver in Excel, renders this distinction relatively unimportant when dealing with relatively small-scale problems. Nevertheless, we focus on linear problems in this chapter.

We now look at the general structure of a LP problem. The standard form for a LP problem is as follows:

$$\underset{\{x_i\}}{\text{Max}} Z = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 \quad (16.5)$$

subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 &\leq b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 &\leq b_3 \end{aligned} \quad (16.6)$$

where Equation 16.5 is the objective function, and Equations 16.6 are the constraints; the x_i 's are the decision variables whose optimal values are to be found by solving the problem; the c_i 's and a_{ij} 's represent the numbers that are the coefficients (the subscripts distinguish among the coefficients so that, for example, a_{32} denotes the coefficient in the third row and second column); and the b_i 's are the constant numbers that are the constraint limitations.

If the problem can be stated in this general form, then it can be solved as a LP problem. Solution algorithms such as the simplex method are usually used to solve the problem, and these algorithms are readily available as computer software. Although understanding the solution process can enhance our understanding of the solution, we do not necessarily have to be knowledgeable in the technical details of the solution process to formulate and solve the problem. The reader who is interested in the mathematics of solving the linear program is referred to the operations research books cited at the end of this book, such as Hillier and Hillier (2003) and Ragsdale (in press). We do not emphasize the solution methods here. Rather, we focus on structuring the model with the view toward allowing Excel's Solver to handle the computations.

A short example will help demonstrate how a LP problem is formulated. Assume that your task is to choose how to allocate your investment budget among several alternative investment proposals and how to finance the investment program. Your decisions are constrained (limited) by a ceiling on your budget, borrowing capacity, and the size of certain investments.

In this case, your objective is to maximize the net present value of your investment portfolio. There are four different investment projects that can be undertaken (projects 1, 2, 3, and 4). Project 1 generates future cash flows with a net present value of \$1.17 for each \$1 invested; project 2 generates a net present value of \$1.22 for each \$1 invested; project 3 generates \$0.99 for each \$1 invested; and project 4 generates \$0.47 per dollar invested.

All four projects are attractive, as each has a positive net present value. So you would like to invest as much money as possible. However, there are a number of constraints that limit your choice of investments: you can invest no more than \$500 of your own money, you can borrow no more than an additional \$200, and you can invest no more than \$100 in project 1 and \$80 in project 2. These constraints must be taken into account in deciding the best mix of investments and financing.

This decision problem can be set up and solved as a linear program consisting of an objective function (containing five decision variables) and nine constraints. We now develop these expressions. First, we define the decision variables: let *Invest1*, *Invest2*, *Invest3*, and *Invest4* be the amounts to be invested in projects 1, 2, 3 and 4 respectively; let *Borrow* represent the amount to be borrowed. The problem is to choose the values of the decision variables *Invest1*, *Invest2*, *Invest3*, *Invest4*, and *Borrow* that will maximize our objective, the net present value of the cash flows from our investment portfolio.

The objective function is the mathematical expression that shows how each decision variable affects the objective. In this case, the objective function is written as

$$Z = 1.17 * Invest1 + 1.22 * Invest2 + 0.99 * Invest3 + 0.47 * Invest4 - 0.02 * Borrow \quad (16.7)$$

where the coefficient of each decision variable represents the net present value per dollar invested, or in the case of borrowing, the net present value per dollar

borrowed.* For example, each dollar invested in project 1 yields \$1.17 in net present value, and the decision variable *Invest 1* is the number of dollars invested in project 1. Therefore, the product $1.17 * \textit{Invest 1}$ represents the net present value of the cash flows generated by the total investment in project 1. If \$300 is invested in project 1, the net present value derived from the investment is

$$\begin{aligned} 1.17 * \textit{Invest 1} &= 1.17 * 300 \\ &= \$351 \end{aligned} \tag{16.8}$$

Next, we write the constraint equations, Equations 16.9.†

$$\begin{aligned} \textit{Invest 1} + \textit{Invest 2} + \textit{Invest 3} + \textit{Invest 4} - \textit{Borrow} &\leq 500 \\ \textit{Invest 1} &\leq 100 \\ \textit{Invest 2} &\leq 80 \\ \textit{Borrow} &\leq 200 \\ \textit{Invest 1} &\geq 0 \\ \textit{Invest 2} &\geq 0 \\ \textit{Invest 3} &\geq 0 \\ \textit{Invest 4} &\geq 0 \\ \textit{Borrow} &\geq 0 \end{aligned} \tag{16.9}$$

The first expression in Equations 16.9 constrains expenditures, net of borrowing, to be no greater than \$500. (When the variable *Borrow* is transposed to the right side, the gross expenditures are \$500 plus the amount borrowed.) The next two equations limit the expenditures for projects 1 and 2 to no more than \$100 and \$80, respectively. The fourth equation limits the amount borrowed to no more than \$200. The last five equations force the decision variables to be nonnegative.

* The borrowing coefficient is the net present value of principal and interest repaid on the debt. It is assumed that the interest rate on the debt is 12% and the discount rate is 10%, which for one period borrowing yields the net present value per dollar borrowed of $+\$1.00 - \$1.00 * 1.12 / 1.10 = -0.02$, where the $+\$1.00$ indicates the inflow of \$1.00 when the money is borrowed, and $-\$1.00 * 1.12$ represents the outflow of principal and interest when the debt is repaid one period later.

† We include the nonnegative constraints for clarity. However, the Solver dialogue box include the option to make unconstrained variables nonnegative, eliminating the need for these constraints to be specified individually.

Combining the objective function and constraints yields Equation system 16.10:

$$\begin{aligned}
 \text{Max}_{\{Invest\ i\}} Z &= 1.17 * Invest\ 1 + 1.22 * Invest\ 2 + 0.99 * Invest\ 3 + 0.47 * Invest\ 4 \\
 &\quad - 0.02 * Borrow \\
 \text{subject to} \\
 Invest\ 1 + Invest\ 2 + Invest\ 3 + Invest\ 4 - Borrow &\leq 500 \\
 Invest\ 1 &\leq 100 \\
 Invest\ 2 &\leq 80 \\
 Borrow &\leq 200 \\
 Invest\ 1 &\geq 0 \\
 Invest\ 2 &\geq 0 \\
 Invest\ 3 &\geq 0 \\
 Invest\ 4 &\geq 0 \\
 Borrow &\geq 0
 \end{aligned}
 \tag{16.10}$$

Using Solver in Excel, the solution is

$$Invest\ 1 = \$100$$

$$Invest\ 2 = \$80$$

$$Invest\ 3 = \$520$$

$$Invest\ 4 = \$0$$

$$Borrow = \$200$$

With these decision variables, the optimized objective (the net present value of the investment program) is $Z = \$725.40$. Any other feasible solution to the problem would yield a lower net present value.

This problem is so simple that one does not need a linear program to determine that projects 1 and 2 should be undertaken to their limits because they yield the greatest *net present value* (NPV) per dollar invested, that project 3 should be expanded as far as borrowing allows, and that borrowing should be undertaken to its limit. However, the problem demonstrates LP and highlights some of the features of the technique. It is helpful to understand the general structure of a LP problem such as the one shown in Equations 16.5 and 16.6. However, in actual practice, we may not have to put the problem in that format.

Some computer software oriented to LP can handle messier problems and still solve them as linear programs. For example, the Excel add-in, Solver, has the capability to treat a spreadsheet problem as a linear program, and solve it.

16.2.2 Using Solver to Find a Solution

This investment problem can be set up as an Excel spreadsheet and solved for the optimal solution using the Excel add-in “Solver.” Exhibit 16.2 is an Excel spreadsheet that shows the data for this problem. Cells E9 through I9 represent the decision variables whose value will be determined by Solver; cells E11 through I11 represent the coefficients of the decision variables in the objective function, whose values represent the NPVs of \$1 invested or borrowed. Cell C13 is the objective function. The formula in C13,

$$= \text{SUMPRODUCT}(E9:I9, E11:I11) \quad (16.11)$$

multiplies each cell in row 9 by its respective cell in row 11, then adds the products, an operation known as vector multiplication. In this case, the result is

$$= E9 * E11 + F9 * F11 + G9 * G11 + H9 * H11 + I9 * I11 \quad (16.12)$$

Solver provides an optimal solution to the problem by finding the values of the decision variables (E9 through I9) that maximize the objective function, C13, subject to the constraints specified in column L, lines 17 through 25.

	B	C	D	E	F	G	H	I	J	K	L
2	Capital Investment Problem										
3											
4		Investment Period		1	2	3	4	Borrowing			
5											
6											
7	Decision Variables										
8		Name of Variable		invest 1	invest 2	invest 3	invest 4	Borrowing			
9		Amount of Variable		100	80	520	0	200			
10											
11		NPV per \$1 Invested		1.17	1.22	0.99	0.47	-0.02			
12											
13		Objective	725.40								
14											
15											
16	Constraints								Constraint Usage	Constraint Limits	
17	Expenditure Constraints			100	80	520	0	-200	500	500	
18	Project 1 Constraint			100					100	100	
19	Project 2 Constraint				80				80	80	
20	Borrowing Constraint							200	200	200	
21	Non-Negativity Constraints			100					100	0	
22					80				80	0	
23						520			520	0	
24							0		0	0	
25								200	200	0	

EXHIBIT 16.2 Spreadsheet for Capital Investment Optimization.

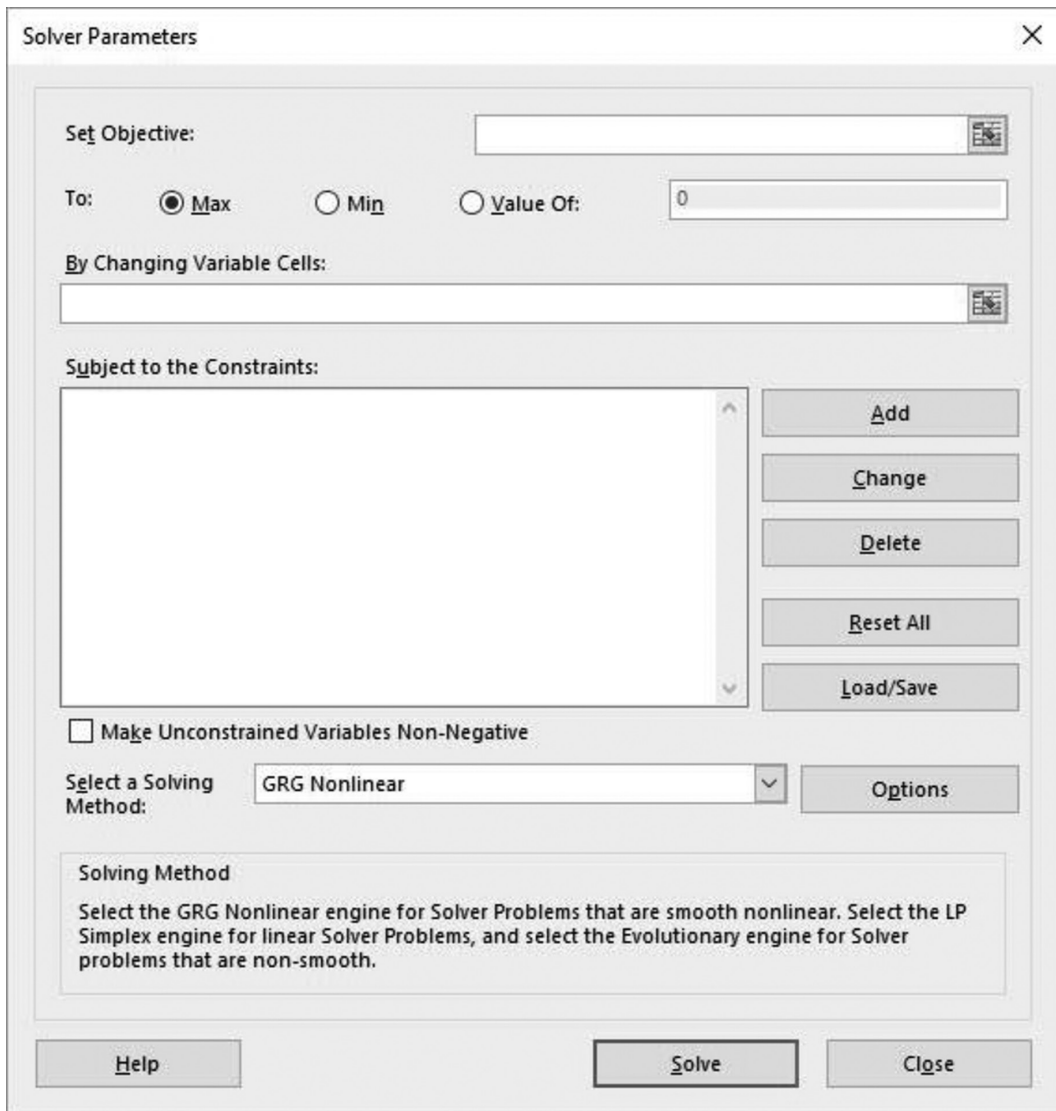


EXHIBIT 16.3 Solver dialogue box.

Having set up the spreadsheet, we need to tell Solver which cells represent the decision variables, the objective function, and the constraints. We access Solver by choosing Solver on the Data ribbon.* The Solver dialog box will be displayed as shown in [Exhibit 16.3](#).

We want Solver to work on the objective function, cell C13 in this example. So C13 is the “Target Cell.” To select it as the target cell, click on the square to the right of the “Set Objective” box, then click on cell C13, then click on the square again. C13 should now appear in the box to the right of “Set Objective.” We select “Max” to indicate that the

* Solver is not included in the default installation of Excel. You must add it in. To do so, click File>Options>Add-ins>“Manage” Excel Add-ins>Go.

objective function is to be maximized, and we select the range of cells that are the decision variables for insertion in the “By Changing Variable Cells” box (cell range E9:I9). To specify the constraints, we click “Add” to obtain the constraint dialog box shown in [Exhibit 16.4](#).

“Cell Reference” is the cell, or range of cells, that contains the formula for the left side of the constraint (cell range J17:J25), and the “Constraint” refers to right side of the constraint equation that is the fixed amount of resource or budget (cell range L17:L25). The middle box is used to specify the direction of the inequality of the constraint. It can also be used to specify an equality or to define the decision variable to be an integer or a binary number (0 or 1). The completed Solver parameter dialog box is shown in [Exhibit 16.5](#).

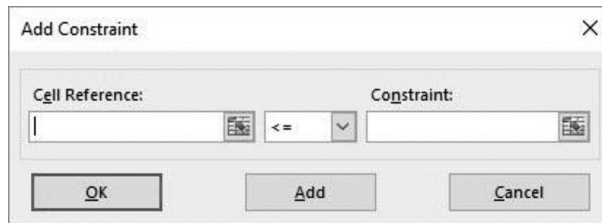


EXHIBIT 16.4 Add Constraint dialog box.

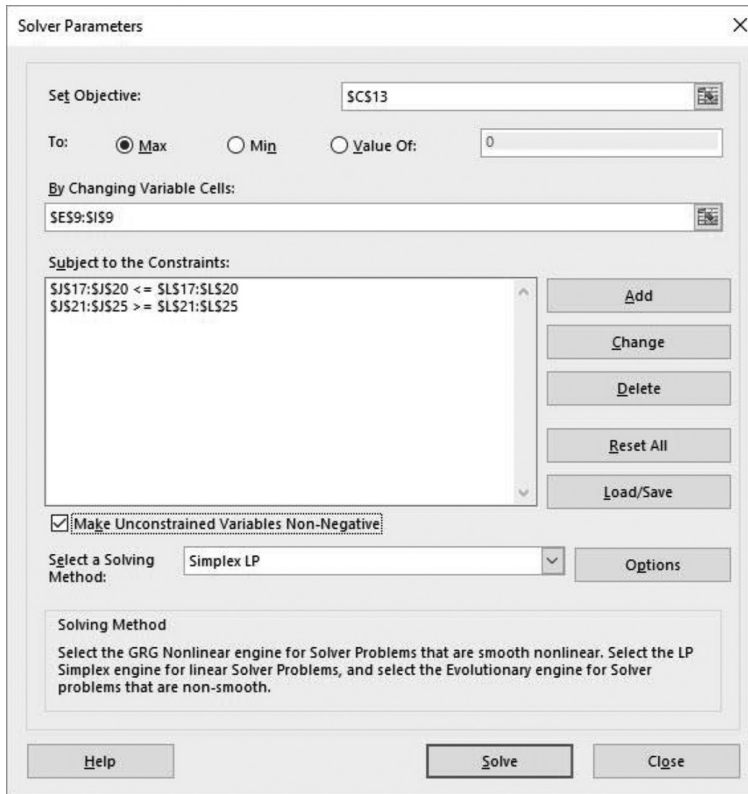


EXHIBIT 16.5 Completed Solve dialog box.

	B	C	D	E	F	G	H	I	J	K	L
2	Capital Investment Problem										
3											
4		Investment Period		1	2	3	4	Borrowing			
5											
6											
7	Decision Variables										
8		Name of Variable		invest 1	invest 2	invest 3	invest 4	Borrow			
9		Amount of Variable		100	80	520	0	200			
10											
11		NPV per \$1 Invested		1.17	1.22	0.99	0.47	-0.02			
12											
13	Objective	725.40									
14											
15									Constraint Usage	Constraint Limit	
16	Constraints										
17	Expenditure Constraints			100	80	520	0	200	500	500	
18	Project 1 Constraint			100					100	100	
19	Project 2 Constraint				80				80	80	
20	Borrowing Constraint							200	200	200	
21	Non-Negativity Constraints			100					100	0	
22					80				80	0	
23						520			520	0	
24							0		0	0	
25								200	200	0	

EXHIBIT 16.6 Optimal Solution for Capital Investment Problem.

Click “Solve,” and the Solver quickly computes the LP solution, with the optimal values of the decision variables inserted in their cells in the spreadsheet as shown in [Exhibit 16.6](#).

The decision variables cells E9 through I9 show the amounts to be invested in the four projects as 100, 80, 520, and 0, and the amount to be borrowed as 200. This yields the objective function value of 725.40 (cell C13). All the constrained budget amounts and the borrowing are used up to their limits, as shown in the Constraint Usage column, cells J17 through J25.

Although this LP model of the capital budgeting problem can be useful, in its simplest form, it is limited because it does not take into account the possibility of obtaining additional financing. However, it takes very little elaboration of the model to add various forms of financing, transforming it into a more general model for long-term financial planning. We will learn more about using Solver and about interpreting the LP output as we look at a more detailed example in the next section.

16.2.3 Example: Investment Decisions on a Limited Budget

The capital investment problem solved earlier was a variation of Weingartner’s (1967) LP approach to capital budgeting under conditions of capital rationing. In this section, we look at a more detailed problem. Weingartner pioneered the application of LP to capital budgeting problems, and his formulation is one of the best known and easiest to understand applications of LP to financial decision problems. Its objective is to choose the best set of investment projects given a limited capital budget.

It is a common mantra that a firm should adopt all the positive *NPV* projects that it can identify. But this assumes a nearly unlimited capital budget. This is not always the case. A firm's capital budget may be limited for several reasons: the officers of the firm may set a maximum limit on capital expenditures in each period; the capital expenditures may be limited to the amount of funds generated internally by the operations of the firm; management may assume that there is a limit on the amount of funds that can be raised externally in a given period; or, the officers of the firm may set a limit on the amount of funds that each division can spend for capital projects. Management also may feel that there is a limit on the rate of expansion of the firm; if the firm tries to expand too rapidly, it will overload the firm's managerial resources. Whatever the reason for constraining the capital budget, once the limit is specified, the task is choosing the best mix of investment projects while simultaneously ensuring that the budget allocation is not exceeded. An example demonstrates the problems of capital rationing and the LP approach to solving those problems.

Suppose the CFO of the Gartner Corporation asked the managers of its different divisions to submit proposals for major capital expenditures, so he can decide how to allocate the firm's capital budget. The divisional managers have screened the projects to see that they generally meet the firm's standards for acceptance: each project must have a positive *NPV* when evaluated at a discount rate that is appropriate for its level of risk. The firm's cost of capital is 10%, and that is the rate used to evaluate the projects. The preliminary screening process narrowed the field to just four major projects. Now the decision must be made as to which of the various projects should be adopted.

The major project proposals are all quite detailed and involved, but the staff at headquarters has summarized the relevant data relating to the costs and expected cash flows of the projects. The four proposals are designated as project 1 through project 4. [Exhibit 16.7](#) shows the estimated cash flows for each period for each project (expressed in thousands of dollars). Negative numbers indicate cash outlays; positive numbers indicate cash inflows. Cash flows are assumed to occur at the beginning of each period, so that period 1 cash flows are assumed to occur at time $t = 0$ (immediately), and period 2 cash flows are assumed to occur at the beginning of period 2, or one year from now (end of period 1). Project 1 requires an immediate outlay of \$120, and then generates positive flows until period 6. Projects 2, 3, and 4 require outlays in periods 1 and 2, then generate positive cash flows until period 6. All four projects incur costs associated with closing down, hence the negative cash flows in period 6. The *NPV* and the internal rate of return (IRR) for each project are shown at the bottom of the exhibit, with *NPV* evaluated at the 10% cost of capital.

As all four projects are acceptable with positive *NPVs* and IRRs greater than the cost of capital, the firm would accept all four if it did not face limits on capital expenditures set by the board of directors. However, the board set a maximum of \$500 on the first period's capital expenditures and \$130 on the second period's capital expenditures. The problem is to decide which portfolio of projects will generate the highest total *NPV* for the firm without exceeding the capital budget.

Gartner Corporation				
Project Cash Flows				
Discount rate = 0.10				
Period	Projects (\$000s)			
	1	2	3	4
1	-120	-50	-90	-450
2	30	-80	-90	-70
3	60	200	80	480
4	80	100	130	401
5	181	50	259	100
6	-1	-3	1.8	-20.4
7	0	0	0	0
NPV	140	150	170	240
IRR	43%	68%	41%	29%

EXHIBIT 16.7 Cash flows and NPV of Gartner investment projects.

The choice of the best portfolio of capital investment projects can be formulated as a LP problem. The objective is to choose the mix of projects that generates the highest NPV for the portfolio as a whole. The main constraints are that the limits on capital expenditures should not be exceeded. The decision variables are which projects to adopt and which to reject.

16.2.3.1 Decision Variables

To set this model up in LP format, the variables must be defined. We number the projects 1 through 4, letting the subscript i designate a project, so that $i = 3$ refers to project 3. A decision variable is designated as X_i , where $X_i = 1$ if project i is adopted, 0 if it is not. For example, $X_3 = 1$ in the solution means project 3 should be adopted; $X_3 = 0$ means project 3 should be rejected. With four projects ($i = 1, 2, 3, 4$), there will be four decision variables, X_1, X_2, X_3 , and X_4 , each one of which will be either 0 or 1 in the solution.

16.2.3.2 Objective Function

Let NPV_i denote the NPV of project i . For example, for project 3, $NPV_3 = \$170$; for project 4, $NPV_4 = \$240$. Using this notation, the NPV of the portfolio of projects can be expressed in general form as

$$Z = \sum_{i=1}^4 NPV_i * X_i \quad (16.13)$$

or explicitly as

$$Z = 140 X_1 + 150 X_2 + 170 X_3 + 240 X_4 \quad (16.14)$$

The numbers shown as the coefficients of the X_i 's are the *NPVs* of the projects, NPV_i . The product $NPV_i * X_i$ represents the contribution of project i to the *NPV* of the portfolio of projects. For example, if in the solution $X_3 = 1$, we have

$$\begin{aligned} NPV_3 * X_3 &= 170 * 1 \\ &= \$170 \end{aligned} \tag{16.15}$$

if $X_4 = 0$, we have

$$\begin{aligned} NPV_4 * X_4 &= 240 * 0 \\ &= 0 \end{aligned} \tag{16.16}$$

The sum, $\sum_{i=1}^4 NPV_i * X_i$, is the *NPV* for the whole portfolio. Suppose, for example, the solution values of the decision variables are $X_1 = X_3 = 1$ and $X_2 = X_4 = 0$. Then the *NPV* for the portfolio would be

$$\begin{aligned} \sum_{i=1}^4 NPV_i * X_i &= 140X_1 + 150X_2 + 170X_3 + 240X_4 \\ &= 140 * 1 + 150 * 0 + 170 * 1 + 240 * 0 \\ &= 310 \end{aligned} \tag{16.17}$$

As all four projects have positive net present value, the portfolio *NPV* could be made larger by adopting all four projects ($X_i = 1$ for $i = 1, 2, 3, 4$), in which case the portfolio *NPV* would be \$700. However, the essence of the problem is that not all projects can be adopted due to budget limitations.

16.2.2.3 Constraints

Some type of budget constraint is present in virtually all financial LP models. In the mathematical programming context, the limitations imposed by the budget are expressed as constraint equations or inequalities. The budget constraint for a given period t is the mathematical statement that the amount spent for capital investment cannot exceed the budget limit. For period t , the budget constraint is the equation:

$$\text{Total capital expenditure}_t \leq \text{Budget ceiling}_t \tag{16.18}$$

To express the budget constraint, define $Expend_{ti}$ as the net cash outflow in period t for project i . If there is a net cash *outflow* for the project, $Expend_{ti}$ is *positive*; if there is a net cash *inflow*, $Expend_{ti}$ is *negative*. For example, $Expend_{11} = 120$, meaning that in period 1, project 1 involves a net cash outlay of \$120, and $Expend_{21} = -30$ meaning that in period 2, project 1 produces a net cash inflow (negative outflow) of \$30. For project 2, $Expend_{12} = 50$ and $Expend_{22} = 80$, meaning that there are outflows of \$50 and \$80 in periods 1 and 2, respectively.

In the budget constraint, $Expend_{ti}$ shows the net incremental cash outflow, regardless of whether or not it is a capital expenditure. For example, project 1's outlay of \$120 in period 1 may consist of \$100 for plant and equipment and \$20 for incremental investment in working capital. Similarly, the net cash inflow in period 2 for project 1 could represent a capital expenditure of \$90 offset by a cash inflow from operations for the project of \$120.

Using this notation, the cash outflow in period t from project i will be

$$Expend_{ti} * X_i \tag{16.19}$$

where X_i is the decision variable. If $X_i = 0$, we have

$$Expend_{ti} * X_i = Expend_{ti} * 0 = 0 \tag{16.20}$$

and if the project is accepted, so that $X_i = 1$, we have

$$Expend_{ti} * X_i = Expend_{ti} * 1 = Expend_{ti} \tag{16.21}$$

In period t , the total expenditures for all four projects will be

$$\sum_{i=1}^4 Expend_{ti} * X_i \tag{16.22}$$

The constraint that total project expenditures do not exceed the budget ceiling is written for period t as

$$\sum_{i=1}^4 Expend_{ti} * X_i \leq Budget\ ceiling_t \tag{16.23}$$

There will be a constraint for each period that has a budget ceiling. For example, the immediate (period 1) constraint for Gartner Corporation is that expenditures do not exceed \$500, so the constraint is written as

$$120X_1 + 50X_2 + 90X_3 + 450X_4 \leq 500 \tag{16.24}$$

where the numerical coefficients are the values of $Expend_{ij}$ for $t = 1$ and $j = 1, 2, 3, 4$. As the firm has set budget limits for periods 1 and 2, the linear program will have two budget constraints, one for each period. The second period budget constraint is

$$-30X_1 + 80X_2 + 90X_3 + 70X_4 \leq 130 \tag{16.25}$$

In the second period, project 1 generates an inflow of 30. As outflows are shown as positive numbers in the budget constraint, this inflow of 30 is shown with a negative sign, indicating a negative expenditure. The impact of this is easier to understand if you transpose the $-30X_1$ to the right side of the constraint, so the budget limit becomes $130 + 30X_1$. Now we can see

that if project 1 is adopted ($X_1 = 1$), the funds available for the other projects is \$160 because \$30 generated by project 1 is available to help cover expenditures for the other projects.

Combining the ingredients developed thus far, our LP can be stated as

$$\text{Max } Z = 140X_1 + 150X_2 + 170X_3 + 240X_4 \quad (16.26)$$

$\{X_i\}$

subject to:

$$120X_1 + 50X_2 + 90X_3 + 450X_4 \leq 500 \quad (16.27)$$

$$-30X_1 + 80X_2 + 90X_3 + 70X_4 \leq 130$$

The solution to this problem, shown in Exhibit 16.8, is that $X_1 = 3.02$, $X_2 = 2.76$, and $X_3 = X_4 = 0$. This means that we should adopt 3.02 units of project 1, 2.76 units of project 2, and none of projects 3 or 4. Now we need to consider whether it makes sense to adopt 3.02 units of a project, and if it does not make sense, what to do about it.

In our discussion to this point, we assumed that the decision variables, X_i , would be either 0 or 1, indicating rejection or acceptance of the project. However, the solution above

	B	C	D	E	F	G	H	I
2	Gartner Corporation							
3	Project Cash Flows							
4	Discount rate = 0.10							
5	Projects (\$000s)							
6	Period	1	2	3	4			
7	1	-120	-50	-90	-450			
8	2	30	-80	-90	-70			
9	3	60	200	80	480			
10	4	80	100	130	401			
11	5	181	50	259	100			
12	6	-1	-3	1.8	-20.4			
13	7	0	0	0	0			
14								
15	NPV	140	150	170	240			
16	IRR	43%	68%	41%	29%			
17								
18	Decision Variables, X_i	3.02	2.76	0.00	0.00			
19								
20	Portfolio NPV	836						
21								
22								
23	Capital Expenditure						Actual	Limit
24	Period 1	120	50	90	450		500	500
25	Period 2	-30	80	90	70		130	130
26								
27	Non-negative solutions						Solver option selected	

EXHIBIT 16.8 Gartner Corp.: LP model for capital budgeting.

indicates that we should adopt 3.02 units of project 1 and 2.76 units of project 2. In other words, we should not only adopt more than 1 unit of project 2, but we should adopt it in fractional units. In some limited instances, this may make sense. However, in most other instances, it is nonsense. For example, if project 2 was a bridge across a river, it may or may not be nonsense to consider building two bridges; it is definitely nonsense to consider building an additional 76% of a bridge. On the other hand, the solution of 2.76 bridges may indicate that we should consider a bridge with greater capacity. This may require that we reformulate the problem and do a new analysis. Nevertheless, for most capital budgeting analyses, it makes sense to consider either accepting or rejecting the project, in which case it does not make sense to recommend a solution of 3 units of the project, much less 3.02 units. Consequently, we need to limit our analysis to accept–reject decisions.

When mathematical computations are done by computers, we have to tell the system what we want it to do. In this case, we need a way to tell it that the decision variables should be either 0 or 1. There are two ways to do this. We can use integer programming so that we only consider decision variables that are integers; or, we can use binary programming so that we only consider decision variables equal to 0 or 1. We discuss the integer approach first.

When only whole projects are to be considered, we confine the decision variables to integer values (0, 1, 2, 3, and so on.) using an integer programming method of solving the problem. Integer programming is a variation of LP that searches for the positive integer values of the decision variables that yield the optimal solution. To move from integer to binary programming, we only need to constrain the solution to be less than or equal to 1, thus:

$$X_i \leq 1 \quad (16.28)$$

As integer programming considers only positive integer values, this constraint ensures that the only values of the decision variables that will be considered are $X_i = 0$ or $X_i = 1$, where $X_i = 0$ indicates that we reject project i , and $X_i = 1$ indicates that we accept it.

Using integer programming, and adding the new constraints, the linear program becomes

$$\text{Max}_{\{X_i\}} Z = 140X_1 + 150X_2 + 170X_3 + 240X_4 \quad (16.29)$$

subject to:

$$\begin{aligned} 120X_1 + 50X_2 + 90X_3 + 450X_4 &\leq 500 \\ -30X_1 + 80X_2 + 90X_3 + 70X_4 &\leq 130 \\ X_1 &\leq 1 \\ X_2 &\leq 1 \\ X_3 &\leq 1 \\ X_4 &\leq 1 \end{aligned} \quad (16.30)$$

Using a solution algorithm that looks only at positive, integer solution values, the solution is $X_1 = 1$, $X_2 = 0$, $X_3 = 1$, and $X_4 = 0$. This means that we adopt projects 1 and 3 and reject projects 2 and 4, resulting in a portfolio NPV of \$310. We only use \$210 of our \$500 budget in period 1, and \$60 of our \$130 budget in period 2.

A second way to derive optimal 0, 1 solutions is to use binary programming. This is similar to the integer approach except that it considers only 0 or 1 as feasible solutions to the problem. In this case, we do not need to specify that $X_i \leq 1$. We need only to tell the system to use a binary solution method. When we do this, we should get the same optimal solution as with the integer approach. For our purposes, we do not need to go into the details of the computation of the solutions for either approach. We rely on the computer software to reach the solution. The use of Solver for this problem is explained below.

16.2.4 Using Excel and Solver for the Gartner Problem

Exhibit 16.9 shows the Gartner Corporation problem as an Excel spreadsheet. This spreadsheet is quite simple. All but three of the cells simply contain the numbers shown, as opposed to formulas. The other three cells contain key formulas that are used to complete the LP spreadsheet. These are shown below, with the number in parenthesis on the right indicating the number of the expression in the LP model discussed above.

Key Cell Formulas			
Cell	Excel Formula	Purpose	
C20	=SUMPRODUCT(C15:F15,C18:F18)	Objective function	(16.31)
H24	=SUMPRODUCT(C18:F18,C24:F24)	Constraint	(16.32)
H25	=SUMPRODUCT(C18:F18,C25:F25)	Constraint	

The decision variables (the cells changed by Solver) are in cells C18:F18. The coefficients for the objective function are the numbers in cells C15:F15. The coefficients for the integer constraints are in cells C27:F30. The budget ceilings for periods 1 and 2 are the constants in cells I24 and I25. Cells H24 and H25 perform the multiplication and addition that are shown as the left side of Equations 16.32. Cells H27:H30 simply point back to the decision variables; cells I27:I30 are constraints that these values have to be less than or equal to 1.

To solve this problem with Solver, open the Solver dialog box shown in **Exhibit 16.10**. This one is already filled in to indicate the objective function (Set Objective: C20) and the decision variables (by Changing Variable Cells: C18:F18), and that it is a maximization problem. The constraints are indicated as shown (Subject to the Constraints:). To impose the requirement that decision variables are integers, click “Add” to get the constraint dialog box (**Exhibit 16.11a**), then click the middle box to show the “int” choice. This constrains the variables indicated to be integers.

	B	C	D	E	F	G	H	I
2	Gartner Corporation							
3	Project Cash Flows							
4	Discount rate = 0.10							
5	Projects (\$000s)							
6	Period	1	2	3	4			
7	1	-120	-50	-90	-450			
8	2	30	-80	-90	-70			
9	3	60	200	80	480			
10	4	80	100	130	401			
11	5	181	50	259	100			
12	6	-1	-3	1.8	-20.4			
13	7	0	0	0	0			
14								
15	NPV	140	150	170	240			
16	IRR	43%	68%	41%	29%			
17								
18	Decision Variables, X_j	0.00	0.00	0.00	0.00			
19								
20	Portfolio NPV	0						
21								
22							Capital	Capital
23	Capital Expenditure						Expenditures	Budget
24	Period 1	120	50	90	450		0	500
25	Period 2	-30	80	90	70		0	130
26								
27		X1					0	1
28			X2				0	1
29				X3			0	1
30					X4		0	1

EXHIBIT 16.9 Gartner Corp.: Integer LP model for capital budgeting.

WARNING: You might think this is enough for Solver to use integer programming. However, it is not. In Excel 2016, there is one more step that is neither intuitive nor easy to find. From the Solver dialog box, click “Options.” In the dialog box that opens (Exhibit 16.11b), make the two changes (circled): deselect “Ignore integer constraints;” and set “Integer optimality” (%) to 0. Click “OK.”

Having found and completed all the necessary choices for Solver, the next step is to tell it to solve the problem. Click the “Solve” button, and the spreadsheet will show the results, including the optimal values for the decision variables, as shown in Exhibit 16.12. The solution spreadsheet shows the effect of the optimally chosen decision variables. With projects 1 and 3 chosen (line 18), the NPV of the investment portfolio is \$310. Any other feasible combination of projects would yield a smaller portfolio NPV.

A feasible program is one that does not violate the constraints. The capital budget allows for \$500 to be invested in period 1 and \$130 in period 2. However, the optimal

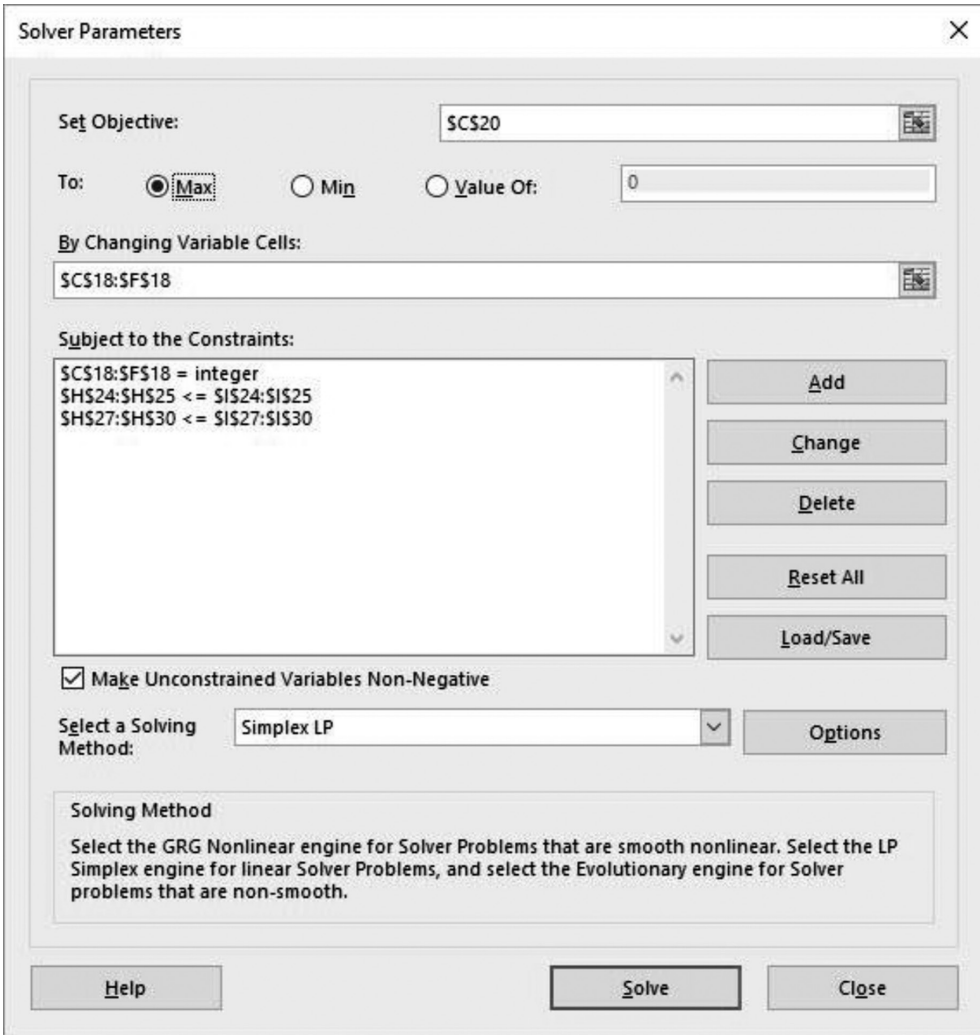
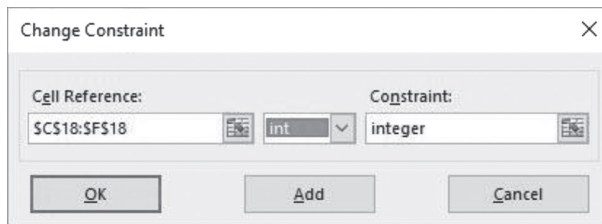


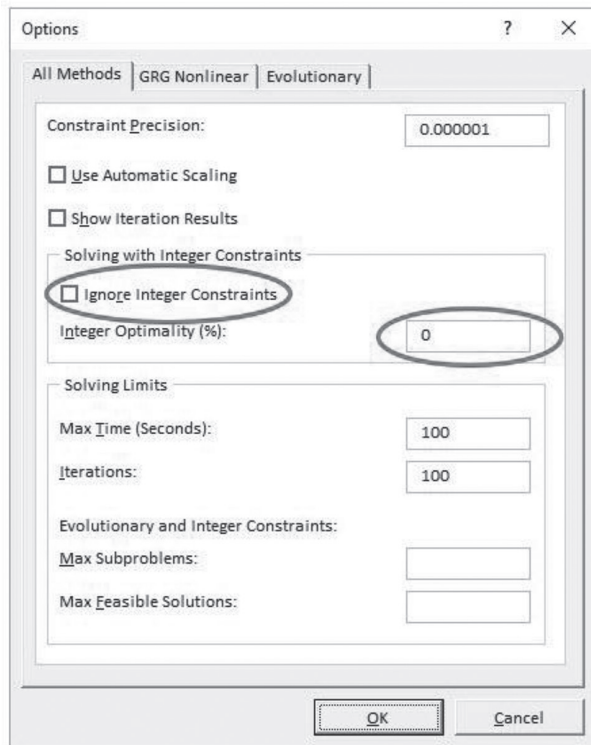
EXHIBIT 16.10 Solver dialogue box set to integer programming.



(a)

EXHIBIT 16.11 (a) First step: Define solution variables as integers.

(Continued)



(b)

EXHIBIT 16.11 (Continued) (b) Second step: Setting Solver so that it actually uses integer programming.

solution only uses \$210 of the period 1 budget (cell H24) and \$60 (cell H25) of the period 2 budget, so it is not using all the budget. With the excess funds in period 1, why not also adopt project 2? It would use only \$50 of the unused \$290 in period 1. But project 2 requires an additional expenditure of \$80 in period 2, making the period 2 expenditures \$140, thereby violating the period 2 budget constraint of \$130. Adding project 2 is therefore an infeasible solution.

This analysis shows one of the advantages of the LP approach. The LP takes into account all the possible decisions and all the constraints, and then picks the best decision in view of the constraints. In this simple problem, the optimal solution is relatively easy to determine, even without the LP. However, in more complicated problems, it can be very difficult to check how each possible decision affects the budget without the LP. The LP can handle many different complex constraints and decision combinations, taking into account the costs and benefits of each project relative to all other projects and simultaneously observing all of the constraints.

	B	C	D	E	F	G	H	I
2	Gartner Corporation							
3	Project Cash Flows							
4	Discount rate = 0.10							
5	Projects (\$000s)							
6	Period	1	2	3	4			
7	1	-120	-50	-90	-450			
8	2	30	-80	-90	-70			
9	3	60	200	80	480			
10	4	80	100	130	401			
11	5	181	50	259	100			
12	6	-1	-3	1.8	-20.4			
13	7	0	0	0	0			
14								
15	NPV	140	150	170	240			
16	IRR	43%	68%	41%	29%			
17								
18	Decision Variables, X_j	1.00	0.00	1.00	0.00			
19								
20	Portfolio NPV	310						
21								
22							Capital	Capital
23	Capital Expenditure						Expenditures	Budget
24	Period 1	120	50	90	450		210	500
25	Period 2	-30	80	90	70		60	130
26								
27		X1					1	1
28			X2				0	1
29				X3			1	1
30					X4		0	1

EXHIBIT 16.12 Gartner Corp.: Integer LP capital budgeting program solved.

16.3 ELABORATIONS ON THE BASIC MODEL

This example demonstrates the basic features of the Weingartner LP model for capital budgeting under conditions of capital rationing. But it is too simple to be useful as a general planning model. Fortunately, this base model is easily extended and elaborated on. The elaborations can include:

- Mutually exclusive projects
- Contingent projects
- Lending and borrowing
- A general schedule of investment opportunities
- Policy constraints on minimum rates of growth of earnings and dividends
- The issuance of new common stock

With elaborations such as these, the simple base model can be transformed into a more general planning model. We explain the first three elaborations in the next sections.

16.3.1 Mutually Exclusive Projects

Mutually exclusive projects occur when you can adopt one of two (or more) projects, but not both (or all). In other words, you must choose between projects. For example, one choice might be between building a new plant in California or building the same plant in Colorado. As you can choose one or the other, but not both, the projects are mutually exclusive. Another example would be starting a project immediately versus delaying the project for a year. The LP format allows us to compare these mutually exclusive choices.

The binary (0,1) decision variables make it easy to handle mutually exclusive projects in the LP. Suppose that eight projects are being considered, and that projects 3 and 8 are mutually exclusive. A constraint can be added to insure that one (either 3 or 8) but not both projects are adopted. The constraint would be of the form

$$X_3 + X_8 \leq 1 \quad (16.33)$$

As the decision variables must be either 0 or 1, Equation 16.33 ensures that we cannot accept both projects. Both projects can be rejected ($X_3 = 0$ and $X_8 = 0$), or one of the projects can be accepted (either $X_3 = 1$ and $X_8 = 0$ or $X_3 = 0$ and $X_8 = 1$). However, both projects cannot be accepted ($X_3 = 1$ and $X_8 = 1$) because in that case $X_3 + X_8 = 2$, violating Equation 16.22. The constraint is easily expanded to include more than two projects. Suppose, for example, there is a set of four projects among which we can choose only one, say projects 2, 3, 7, and 8. Then the constraint would take the form

$$X_2 + X_3 + X_7 + X_8 \leq 1 \quad (16.34)$$

16.3.2 Contingent Projects

An investment is contingent when it cannot be accepted unless another project is also accepted. For example, suppose project 1 is the purchase of a new computer, and project 2 is the purchase of a software system to be used on the new computer. The computer could be purchased without that particular software, but it would make no sense at all to purchase the software without also purchasing the computer. In this case, project 2 (software) is contingent on the acceptance of project 1 (computer). This type of contingency can be included in the LP with the addition of the constraint

$$X_2 \leq X_1 \quad (16.35)$$

We usually express the LP constraints with all the decision variables on the left side of the inequality sign and the constant number on the right, thus:

$$-X_1 + X_2 \leq 0 \quad (16.36)$$

If the solution is $X_1 = 0$, Equation 16.36 requires that X_2 also is equal to 0. When $X_1 = 1$, however, X_2 can be either 0 or 1, depending on its merits. Suppose another project, 6, is

an alternative software system competing with project 2. Then projects 2 and 6 are mutually exclusive and are contingent on the acceptance of project 1. This would be reflected in the constraint

$$-X_1 + X_2 + X_6 \leq 0 \quad (16.37)$$

If $X_1 = 1$, either project 2 or 6 could be accepted, but not both.

16.3.3 Lending and Borrowing

As the model is currently structured, nothing can be done to relax a budget constraint that binds, even when extra funds are present in other periods. The ability to transfer excess funds between periods might make it possible to relax such a binding constraint, thereby allowing us to accept additional projects and increase the *NPV* of the investment portfolio. For example, in the solution to the Gartner Corporation's capital budgeting problem, we had excess funds in period 1 but could not undertake an additional project because it required more funds in period 2 than were available. If we could transfer some of the excess funds from period 1 to period 2, we could undertake additional projects and realize a higher portfolio *NPV*. We need ways to transfer funds from one period to another. Lending and borrowing are just such mechanisms. Lending transfers funds from earlier periods to later ones; borrowing transfers funds from later periods to earlier ones. When added to the problem, lending and borrowing give us the beginnings of a complete financial planning model. To see how lending and borrowing opportunities can be added to the model, return to the Gartner example.

16.3.3.1 Lending

Lending (e.g., investing in marketable securities such as U.S. Treasury securities) enables the transfer of funds forward from one period to the next. For example, surplus funds in period 1 can be carried forward to period 2. Let $Lend_1$ represent the decision variable that is the amount to be invested in marketable securities in period 1. Assume that there is no limit on the amount that can be lent except for limitations imposed by the funds available for lending, so the decision variable is not required to be an integer variable. Assume the rate of return earned on funds lent for one period is

$$r_L = 0.10 \quad (16.38)$$

Assuming a tax rate of $T = 0.40$, the after-tax return is

$$\begin{aligned} r_L(1 - T) &= 0.10 * (1 - 0.40) \\ &= 0.06 \end{aligned} \quad (16.39)$$

Next, the effect of lending must be incorporated into the objective function. As the objective function is stated in terms of *NPV*, the coefficients of the lending decision variables in the objective function also must be defined in terms of *NPV*. The coefficients of the X_i 's for the investment projects are the net present values (*NPV_i*) of the after-tax cash flows generated by these projects. Consequently, the coefficients of the $Lend_i$ decision variables

must also be in terms of the *NPVs* of the after-tax cash flows. Lending \$1 in period 1 is a cash outflow of \$1 in period 1 and generates a cash inflow after taxes of \$1 * [1 + r_L (1 - T)] in period 2. Therefore, the *NPV* of the cash flows from lending \$1 in period 1 is

$$-1 + \frac{[1 + r_L(1 - T)]}{[1 + k]} \quad (16.40)$$

where k denotes the discount rate appropriate for discounting the cash inflow from lending. Suppose the appropriate discount rate is 12%; then the *NPV per dollar lent* in period 1 is

$$-1 + \frac{[1 + 0.10 * (1 - 0.40)]}{[1.12]} = -0.0536 \quad (16.41)$$

Given this discount rate,* investing in short-term securities does not add net value to the investment portfolio. However, it allows the transfer of slack or idle funds from one period to a future period in which they may be used in productive capital investments.

Lending also affects two budget constraints. In the objective function, lending \$1 reduces the *NPV* of the portfolio by -\$0.0536. Constraints, however, are expressed in terms of cash outflows. In the constraint from which funds are lent (the period during which T-bills are purchased), cash flows out of the budget and into T-bills. Therefore, $Lend_i$ has a coefficient of (+1). In the constraint for the period that receives the funds (the period during which T-bills are sold), cash flows in, relaxing the budget constraint by \$1.06 for each dollar lent. Therefore, the coefficient of the lending variable is a negative \$1.06 (-1.06).

16.3.3.2 Borrowing

Assume that even though a budget limit has been imposed on capital expenditures, management will allow a limited amount of borrowing to take place. This transfers cash from future periods to the present period. Suppose that the borrowing limit is \$200 and that we wish to borrow in period 1 and repay the loan in period 2. Further, assume that the current interest rate on borrowed funds is 12%, denoted by

$$r_d = 0.12 \quad (16.42)$$

As was the case with lending, borrowing enters the objective function and two budget constraints. Let $Borrow_1$ represent the decision variable that is the amount to be borrowed in period 1 and repaid in period 2. $Borrow_1$ is the cash inflow to the firm in period 1 that is available to help finance other investment projects. Retiring the debt in period 2 (principal and interest net of taxes) results in a cash outflow of

* The discount rate is a crucial variable in determining the value of lending or borrowing. Whether the *NPV* of lending or borrowing is positive or negative depends on whether the discount rate is higher or lower than the lending or borrowing rate. The borrowing rate would be appropriate as the discount rate because it represents the highest return that can be earned on an extra \$1 at this level of risk. In other words, an extra \$1 can be used to repay debt and save (i.e., earn) the borrowing rate.

$$\begin{aligned}
 [1 + r_d(1 - T)] * Borrow_1 &= [1 + 0.12 * (1 - 0.40)] * Borrow_1 \\
 &= 1.072 * Borrow_1
 \end{aligned}
 \tag{16.43}$$

Similar to lending's coefficient in the objective function, the coefficient of the borrowing decision variable in the objective function is the *NPV* of one dollar borrowed. Borrowing \$1 in period 1 generates a cash inflow of \$1 at the date of the loan and a cash outflow after taxes of $[1 + r_d(1 - T)]$ in period 2. Therefore, the *NPV* of the cash flows from period 1 borrowing is

$$\left\{ 1 - \frac{[1 + r_d(1 - T)]}{(1 + k)} \right\}
 \tag{16.44}$$

where k is the discount rate. We assume that the borrowing rate, $r_d = 0.12$, equals the discount rate, $k = 0.12$, because the borrowing rate is the opportunity cost of funds. Then the coefficient (the *NPV* per dollar borrowed) of $Borrow_1$ in the objective function is

$$\left\{ 1 - \frac{[1 + 0.12(1 - 0.40)]}{(1 + 0.12)} \right\} = 0.0429
 \tag{16.45}$$

and the borrowing variable will appear in the objective function as

$$+0.0429 * Borrow_1
 \tag{16.46}$$

To add the lending and borrowing opportunities to the Gartner LP (shown earlier as Equations 16.29 and 16.30), the objective function and constraints are expanded with the new terms shown in [Exhibit 16.13](#). In the exhibit, the negative coefficient of the borrowed amount, $-1 * Borrow_1$ in Equation 16.48a indicates a cash inflow from borrowing in period 1. To understand this, transpose $-1 * Borrow_1$ to the right side of Equation 16.48a. With the change of sign, we see that $Borrow_1$ adds to the funds available for investment. The positive coefficient $1.072 * Borrow_1$ in Equation 16.48b represents the after-tax cash outflow resulting from repayment of the debt. The last Equation, 16.48g limits the total amount that can be borrowed in period 1 to no more than \$200. The fact that all borrowing is repaid in period 2, coupled with the fact that there is no further borrowing in period 2, ensures that all borrowing is repaid from the cash flows generated by the investment projects.

Equation in Gartner Model	Lending Terms Added	Borrowing Terms Added
16.47	$-0.0536 Lend_1$	$+0.0429 Borrow_1$
16.48a	$+1 Lend_1$	$-1 Borrow_1$
16.48b	$-1 Lend_1$	$+1.072 Borrow_1$
16.48g		$Borrow_1 \leq 200$

EXHIBIT 16.13 Gartner Corp.: Terms added to LP for lending and borrowing.

Max Z	$140X_1$	$+150X_2$	$+170X_3$	$+240X_4$	$-0.0536Lend_1$	$+0.0429 Borrow_1$		16.47
Subject to:								
	$120X_1$	$+50X_2$	$+90X_3$	$+450X_4$	$+1 * Lend_1$	$-1 * Borrow_1$	≤ 500	16.48a
	$-30X_1$	$+80X_2$	$+90X_3$	$+70X_4$	$-1.06Lend_1$	$+1.072Borrow_1$	≤ 130	16.48b
	X_1						≤ 1	16.48c
		X_2					≤ 1	16.48d
			X_3				≤ 1	16.48e
				X_4			≤ 1	16.48f
						$Borrow_1$	≤ 200	16.48g

EXHIBIT 16.14 Gartner Corp.: LP objective with borrowing and lending constraints.

	B	C	D	E	F	G	H	I	J
2	Gartner Corporation								
3	Project Cash Flows								
4	Discount rate = 0.10								
5	Projects (\$000s)								
6	Period	1	2	3	4	Lending	Borrow		
7		1	-120	-50	-90	-450			
8		2	30	-80	-90	-70			
9		3	60	200	80	480			
10		4	80	100	130	401			
11		5	181	50	259	100			
12		6	-1	-3	1.8	-20.4			
13		7	0	0	0	0			
14									
15		NPV	140	150	170	240	-0.0536	0.0429	
16		IRR	43%	68%	41%	29%			
17									
18	Decision Variables, X_i	1.00	1.00	1.00	0.00	9.43	0.00		
19									
20	Portfolio NPV	459.43							
21									
22								Capital	Capital
23	Capital Expenditure							Expenditures	Budget
24	Period 1		120	50	90	450	1	-1	269
25	Period 2		-30	80	90	70	-1.06	1.072	130
26									
27			X1						1
28				X2					1
29					X3				1
30						X4			0
31								0.00	0.00
									200

EXHIBIT 16.15 Gartner Corp.: LP model with lending and borrowing.

The new LP is shown in Exhibit 16.14.

After adding borrowing and lending, the new solution is $X_1 = 1$, $X_2 = 1$, $X_3 = 1$, and $Lend_1 = 9.43$, as shown in line 18 of Exhibit 16.15. The addition of the opportunity to lend and borrow increases the portfolio NPV (the objective function) from \$310 in the original solution to a value of \$459.43. This is because Gartner can now accept project 2, which previously was rejected due to insufficient funds in period 2. Transferring unused funds from period 1 to period 2 (lending) relaxed the constraint.

Max Z	140 X_1	+150 X_2	+170 X_3	+240 X_4	-0.0536 $Lend_1$	+0.0429 $Borrow_1$		16.49
subject to:								
	120 X_1	+50 X_2	+90 X_3	+450 X_4	+1 * $Lend_1$	-1 * $Borrow_1$	≤500	16.50a
	-30 X_1	+80 X_2	+90 X_3	+70 X_4	-1.06 $Lend_1$	+1.072 $Borrow_1$	≤250	16.50b
	X_1						≤1	16.50c
		X_2					≤1	16.50d
			X_3				≤1	16.50e
				X_4			≤1	16.50f
						$Borrow_1$	≤200	16.50g

EXHIBIT 16.16 Gartner Corp: LP model with period 2 capital expenditure constraint of 250.

The program still does not accept project 4 even though it has the largest *NPV*. Why not? Project 4 requires a net cash outflow of \$450 in period 1. If we were to drop project 2 and borrow \$190, we could meet project 4's cash needs in period 1. The problem is that there will not be enough cash in period 2 to repay the debt. If you borrow \$190 in period 1 to finance project 4, you will not have enough funds in period 2 to cover the \$70 outflow required by project 4 and repay the \$190 plus interest. Had sufficient funds been available in period 2 to repay the borrowing in period 1, the LP would have recommended borrowing in period 1 to undertake project 4. To see this, let us revise the problem by increasing our period 2 budget constraint to \$250 (from \$130). The problem is now stated as [Exhibit 16.16](#).

The only difference from [Exhibit 16.14](#) is the change in the right side of constraint (16.50b) from 130 to 250. Solving this problem yields the optimal solution of $X_1 = 1$, $X_2 = 1$, $X_3 = 0$, $X_4 = 1$, $Lend_1 = 0$, and $Borrow_1 = 121.27$. The new solution is shown in [Exhibit 16.17](#). The value of the objective function is $Z = 535.12$. Increasing the period 2 budget limit increased the firm's borrowing capacity in period 1 by making more funds available for debt service in period 2. This permitted the adoption of project 4, which is expensive but has a very high *NPV*. Without the additional \$120 in period 2's budget, there was no way to borrow the funds needed to finance project 4's large initial investment in period 1 and repay the loan in period 2.

The period 1 cost of projects 1, 2, and 4 is $\$120 + \$50 + \$450 = \620 . Given a budget constraint of \$500, we need to borrow \$120 to be able to finance them. Yet, the LP solution calls for borrowing \$121.27 instead of \$120, so it is borrowing \$1.27 more than we seem to need. The reason for this is that the LP will try to increase the value of any decision variable that increases the value of the objective function, and there is a positive coefficient on the borrowing variable in the objective function. This points out one of the idiosyncrasies of including borrowing in an LP financial planning model: Unless we formulate the problem carefully, the model will typically push borrowing to its limit. In many cases, this may seem irrational. We will discuss borrowing in greater detail in the next section.

With this simple problem, we have seen how the LP takes account of the interrelationships between investment, financing, and budgeting. Problems larger than this that would send an analyst to the mental ward are easily solved with LP.

	B	C	D	E	F	G	H	I	J
2	Gartner Corporation								
3	Project Cash Flows								
4	Discount rate = 0.10								
5	Projects (\$000s)								
6	Period	1	2	3	4	Lending	Borrow		
7	1	-120	-50	-90	-450				
8	2	30	-80	-90	-70				
9	3	60	200	80	480				
10	4	80	100	130	401				
11	5	181	50	259	100				
12	6	-1	-3	1.8	-20.4				
13	7	0	0	0	0				
14									
15	NPV	140	150	170	240	-0.0536	0.0429		
16	IRR	43%	68%	41%	29%				
17									
18	Decision Variables, X_i	1.00	1.00	0.00	1.00	0.00	121.27		
19									
20	Portfolio NPV	535.12							
21									
22								Capital	Capital
23	Capital Expenditure							Expenditures	Budget
24	Period 1	120	50	90	450	1	-1	499	500
25	Period 2	-30	80	90	70	-1.06	1.072	250	250
26									
27		X1						1	1
28			X2					1	1
29				X3				0	1
30					X4			1	1
31								121.27	121.27
									200

EXHIBIT 16.17 Gartner Corp.: LP model with increased period 2 capital constraint.

16.4 BORROWING IN THE LP-PLANNING MODEL

One of the more difficult problems in developing an LP model for financial planning is the treatment of debt financing. We need a way to capture the advantages of debt financing and at the same time properly take account of the risks imposed by debt financing. The theory of finance says that a firm financed entirely with equity can increase its value by adding moderate and prudent amounts of debt, primarily due to the tax advantages of debt financing. There is a point, however, where the risks and potential costs of financial distress due to borrowing begin to outweigh the advantages of debt. Debt financing should not be expanded beyond that point. The ideal planning model would assess these trade-offs and arrive at an optimal mix of equity and debt financing. Although it is difficult to construct this ideal model as a linear program because of the nonlinearities inherent in this capital structure problem, we can handle some of this complexity in an LP model.

It is not difficult to show the tax advantage of debt in the LP model.* Following the adjusted present value approach by Myers and Pogue (1974), we calculate the value of an

* The standard method for evaluating investments and financing alternatives is to discount expected future cash flows at the firm's weighted average cost of capital, denoted by k_a . To the extent that k_a depends on the proportional mix of debt and equity financing, the present value of a cash flow, discounted at k_a , is a nonlinear function of the financing mix. In other words, if the proportion of debt is changed, the present value of a future cash flow changes in a nonlinear way, violating the requirement that the model be linear with respect to the decision variables, one of which is the proportion of debt financing.

investment project as if it is financed entirely with equity, and then add the value of the tax savings from interest. The pure equity value is obtained by discounting the project cash flows at the cost of unleveraged equity; the tax savings from interest are calculated separately at the cost of debt.

Consider the following example. Suppose an investment is financed entirely with equity and that the investment has a level of risk that would require its cash flows to be discounted at a rate k_u (where subscript u denotes that it is “unleveraged,” or has no debt financing). The unleveraged value of the cash flows would be

$$\sum_{t=1}^n \frac{\text{Cash flow}_t}{(1+k_u)^t} \quad (16.51)$$

To account for the effects of debt financing, we add a term that is the present value of the tax savings resulting from the tax deductibility of interest rather than adjusting the discount rate to show the effects of financial leverage. Therefore, the value of the leveraged cash flows would be

$$\sum_{t=1}^n \frac{\text{Cash flow}_t}{(1+k_u)^t} + T \sum_{t=1}^n \frac{\text{Interest}_t}{(1+k_d)^t} \quad (16.52)$$

The term on the right is the present value of these tax savings, because the tax shield from interest in a given year is $T * \text{Interest}_t$, where T is the tax rate and k_d is the interest rate on the debt. This value is linearly related to the amount of debt financing because $T * \text{Interest}_t$ is a linear function of the amount of debt financing. This approach allows us to avoid the complications of nonlinearity that would occur if we used the weighted average cost of capital as the discount rate.*

To see how these ideas can be applied in an LP model, return to the four-project investment decision problem represented by Exhibit 16.14. As before, $X_1, X_2, X_3,$ and X_4 are the investment decision variables, and $Borrow_1$ is the decision variable representing the amount borrowed in the first period. Assume that the NPVs for the investment projects were based on the cost of equity capital as though the firm was financed entirely with equity. Even with the addition of borrowing to the problem, we still use the same project NPVs in the objective function of the LP. We account for the effect of borrowing through the coefficient of the decision variable $Borrow_1$ in the objective function, rather than using

* This approach has the same effect as if the cash flows were discounted at the firm’s weighted average cost of capital, where the cost of equity changes with the debt-to-equity ratio according to the equation:

$$k_s = k_u + \frac{D}{E}(k_u - r_f)(1 - T)$$

where k_u is the discount rate for a pure equity (unleveraged) cash flow, and r_f is the risk-free rate of interest. This equation, developed by Modigliani and Miller (1958), leads to the result that the weighted average cost of capital declines as the proportional amount of debt financing increases.

the weighted average cost of capital as the discount rate. This coefficient is the NPV of \$1 borrowed. In general terms, for debt with an n -period life, this coefficient is

$$T \sum_{t=1}^n \frac{k_d}{(1 + k_d)^t} \quad (16.53)$$

where k_d is the borrowing rate on the debt. With the borrowing rate at 12% per year, and the tax rate is 40%, the borrowing coefficient is

$$\begin{aligned} T \frac{k_d}{(1 + k_d)} &= 0.40 * \frac{0.12}{1.12} \\ &= 0.0429 \end{aligned} \quad (16.54)$$

per dollar borrowed at the beginning of the first period and repaid in period 2. The borrowing decision variable appears in the objective function as

$$0.0429 * Borrow_1 \quad (16.55)$$

The shortcoming of this approach is that the positive coefficient on $Borrow_1$ in the objective function encourages the use of debt beyond the point most practitioners would consider prudent. This means that without constraints, an LP-planning model tends to generate solutions that recommend excessive expansion of debt financing. Debt appears as an especially attractive source of funds in planning models for two reasons: first, borrowing relaxes the budget constraint and thereby provides funds for expanding investments; second, borrowing generates tax savings because the interest payments are tax deductible. However, as everyone but the linear program knows, excessive borrowing can be hazardous to a firm's financial health. So, like cigarettes, debt should carry warnings of impending doom that limit its use by the linear program.

There are numerous ways to limit the use of debt financing in the LP model, but none is entirely satisfactory. One way that has already been introduced is to limit the amount to be borrowed by adding a constraint to the LP in the form

$$Borrow_t \leq Borrowing\ limit_t \quad (16.56)$$

This approach is appropriate when either the lender or the borrower sets a limit (e.g., a revolving credit agreement with a maximum cap). The problem with this approach is that, because of the apparent advantage of borrowing, the LP will usually push borrowing right to this limit without telling the user when it is best to borrow less than the maximum.

Financial ratios also can be used to set borrowing limits. For example, it is common to set a target for the ratio of total debt to total assets. This is easily done as an LP constraint.

Suppose you regard the prudent limit for debt to be 25% of total assets. The overall constraint would be

$$Total\ debt_t \leq 0.25 * Total\ assets_t \quad (16.57)$$

Here, $Total\ debt_t$ consists of debt that is already outstanding, denoted by $Debt_0$, plus any new borrowed funds as specified by the decision variable $Borrow_t$. Similarly, $Total\ assets_t$ consists of initial assets, $Assets_0$, plus additions specified by the investment decisions recommended by the model. For example, in period 1, we have

$$Total\ assets_1 = Assets_0 + \sum_{i=1}^4 Asset_i X_i \quad (16.58)$$

where $Asset_i$ is the amount of assets purchased in period 1 for investment project i . With these definitions of assets and debt, the constraint for the first period would be written as

$$Debt_0 + Borrow_1 \leq 0.25 * \left[Assets_0 + \sum_{i=1}^4 Asset_i X_i \right] \quad (16.59)$$

Following the LP convention of putting all decision variables on the left side of the inequality and all constants (nondecision variables) on the right, the constraint for the first period becomes

$$Borrow_1 - 0.25 \sum_{i=1}^4 Asset_i X_i \leq 0.25 * Assets_0 - Debt_0 \quad (16.60)$$

Similar constraints would be used for each period in the planning horizon.

16.4.1 Risk Constraint

Another way to constrain debt financing is to use a risk constraint. With this approach, management subjectively determines the maximum acceptable probability that the firm will be unable to service its debt. A constraint is imposed that prohibits debt from expanding past the point at which the probability of insolvency is unacceptable.

Define the risk of insolvency as the probability that the cash flow will be insufficient to service the debt. Suppose we are willing to live with the risk that there is a 10% probability that our firm will be unable to service its debt from its operating cash flow. Using information about the probability distribution of the cash flows, we can impose a constraint on the amount of debt so that we do not exceed the acceptable level of risk of insolvency. For example, suppose as a first approximation that the firm's free cash flow is normally distributed with a mean (expected value) of

$$E[Cash\ flow] = \$100,000 \quad (16.61)$$

and a standard deviation of

$$\sigma_{Cash\ flow} = \$60,000 \tag{16.62}$$

Using a standard normal probability table, we find the number of standard deviations below the expected value that we expect a draw less than 10% of the time (1.28 standard deviations). Translating this into the expected value and standard deviation of our firm’s cash flow yields \$23,200, as shown in Equation 16.63:

$$\begin{aligned} E[Cash\ flow] - 1.28\sigma_{Cash\ flow} &= 100,000 - 1.28 * 60,000 \\ &= 100,000 - 76,800 \\ &= \$23,200 \end{aligned} \tag{16.63}$$

Thus, there is a 10% probability that the cash flow will be less than \$23,200. Constraining the debt level such that debt service is no more than \$23,200 limits the chance that we will be unable to service the debt to 10%. If the annual debt service is equal to 20% of the outstanding debt, the period 1 constraint on debt would be

$$0.20(Debt_0 + Borrow_1) \leq 23,200 \tag{16.64}$$

or, putting the constant terms on the right-hand side,

$$0.20(Borrow_1) \leq 23,200 - 0.20(Debt_0) \tag{16.65}$$

The difficulty with this approach is that the probability distribution of cash flows may be dependent on the investment projects accepted, thereby making it necessary to incorporate the cash flow distribution assumptions in our constraints. Myers and Pogue (1974, p. 593) explain how to treat this type of complication.

When we develop our LP on a spreadsheet, we can use the spreadsheet’s statistical function NORMINV(.) to find the critical level of income as shown in Exhibit 16.18.

In this model, the user specifies the risk of default in cell C4 and the parameters of the probability distribution in cells C6 and C7. The \$23,107 in cell C9 is the output

	B	C	D
4	Maximum Acceptable Risk of Default	10%	
5	Probability Distribution for Cash Flows		
6	Mean	100,000	
7	Standard Deviation	60,000	
8			
9	Critical Point	23,107	
10		=NORMINV(C4,C6,C7)	
11			

EXHIBIT 16.18 Calculating debt limit based on the probability distribution of cash flows.

of the Excel function for the inverse of the normal probability function shown in Equation 16.66:

$$\begin{aligned}
 &= \text{NORMINV}(\text{probability}, \text{mean}, \text{standard deviation}) \\
 &= \text{NORMINV}(C4, C6, C7) && (16.66) \\
 &= \text{NORMINV}(0.10, 100000, 60000)
 \end{aligned}$$

This function takes the level of acceptable risk specified in C4 (10%) and the mean and standard deviation parameters specified in C6 and C7 and returns the critical point of the distribution where, in this example, 10% of the distribution is below and 90% is above. In the debt constraint (Equation 16.59) of the linear program, the =NORMINV(.) function would be inserted in place of the \$23,200 now shown. In this way, the user has the option of easily varying the acceptable risk to explore different policies or scenarios.

16.5 SUMMARY

This chapter introduced the ideas and techniques of using LP for solving financial planning problems. The advantages of using a constrained optimization technique like LP are that it forces the planner to specify the objective, the constraints, and the linkages between the decision variables and the objective; and most important, it provides a systematic way to identify the best set of decisions.

The essential features of an LP model for financial planning include the decision variables that represent the amounts or levels of the different decisions that the planner must make, the objective function that expresses mathematically the links between the decisions and the objective, and the constraints that express the various limitations on the planner's decisions.

The constraints in a financial planning model usually include expressions that balance the system, that link one period to the next, that limit access to financing, that represent limitations set by the firm's policies, and that express accounting definitions or technical relations between variables. The crucial constraints are those that balance the system by requiring assets to equal liabilities or sources of funds to equal uses of funds. The limits on access to financing usually refer to limits on borrowing when it is one of the decision variables and limits that prevent the model from pushing borrowing beyond what would be considered prudent. Policy constraints take many forms, including dividend payout ratios, target growth rates in earnings and dividends, and cash balance requirements.

If too many constraints are imposed on the model, the problem may not have a feasible solution. In other words, there may be no solution that simultaneously satisfies all constraints. Consequently, the user should not add constraints unless they are absolutely necessary to the problem being modeled. In fact, the modeler should make every effort to keep the model as simple as possible, using only the minimum number of constraints and decision variables necessary. The more complex the model, the greater the likelihood that it will confuse the user and contain undetected errors.

Models are merely guides to decision making; their results should not be enshrined. The user must constantly be alert for errors and illogical results. Finally, the results should be

recalculated as new information is received, in case the new information leads to a revision of the results, or even a restructuring of the model itself.

Naturally, not all problems of financial planning require linear programming—nor any other kind of optimization technique, for that matter—for their solution. Still, there are many planning problems that are handled best with an optimization technique, and they often can easily be fitted into the LP format. There are many examples of financial planning problems that have been solved with LP. Some of these have been referenced throughout this chapter and can be found in the References. These include problems of working capital management, capital budgeting under conditions of capital rationing, and more general long-range financial planning. This chapter has focused on problems of planning the firm's capital budgeting program. The next chapter will discuss models that deal with managing working capital.

EXERCISES

1. Optimal Capital Investment Portfolio: Prager Corporation

The CEO of the Prager Corporation has asked the managers of its various divisions to submit proposals for major capital expenditure programs. The proposals have been submitted and now the decision must be made as to which of the proposed projects should be adopted.

Summary data regarding the costs and expected cash flows of the proposed projects are shown in the table on the next page. There are 12 proposed projects, designated as projects A through L. The numbers in the body of the table are the estimated cash flows (in thousands of dollars), with negative numbers indicating cash outflows, and positive numbers indicating inflows. All project cash flows (with the exception of project A) should be discounted at the firm's cost of capital of 15%. Project A has a risk adjusted cost of capital of 18%.

All 12 projects meet Prager's acceptance criterion: they have positive *NPV* at the cost of capital of 15%, and their *IRR*'s equal or exceed 15%. If there were no limitations on the firm's ability to accept projects, all 12 would be adopted. However, the board of directors has set limits on capital expenditures. The maximum that can be spent right now (time 0) is set at \$950; for the end of the coming year (time 1), the limit is \$400; and for year 2 (time 2), the limit is \$200. Given the limits set by the directors, it must now be decided which of these 12 projects should be undertaken. The portfolio of projects that is chosen should be the one that maximizes the *NPV* of the portfolio as a whole while not exceeding the limited capital budget.

Assignment

- For each of the projects, calculate in a spreadsheet the present value of the costs and the cash inflows, calculate the *NPV*, the *IRR*, and the present value index (profitability index).
- Formulate this investment problem to be solved as a linear program (with continuous decision variables), and solve it.

- c. Now, set the problem up with integer (0,1) variables and solve.
- d. Assume that the company can borrow up to \$100 at time 0. These funds can be used to finance investment beyond the expenditure limits set by the directors. Assume that for each \$1 borrowed at time 0, \$1.20 must be repaid (including interest) at time 2, and the NPV of \$1 borrowed is -\$0.02. Add borrowing as a decision variable to your problem and determine the new solution.
- e. Assume you can lend money at times 0 and 1. You can lend any positive amount– there is no constraint on the amount you can lend. For each \$1 lent, there is an outflow of \$1 at the time of the lending and an inflow of \$1.10 one period later. The NPV of lending is - 0.015 for each \$1 lent. Add lending as a decision variable to the model used in part (c) and solve it to see if the lending opportunity changes any of your choices.

For each of the different cases (b)–(e), explain your solution and explain how & why the solution changes as you proceed down the list (Exhibit 16.19).

Optimal Capital Investment Portfolio													
CASH FLOW ESTIMATES FOR PROPOSED INVESTMENT PROJECTS													
PROJECT:	A	B	C	D	E	F	G	H	I	J	K	L	
TIME													
0	-23	-200	-600	-100	-150	-420	-200	-10	0	-400	-80	-50	
1	4.78	-150	0	-475	30	80	-600	-4	-100	-200	0	400	
2	4.78	20	200	-250	35	90	280	5	-475	-300	150	50	
3	4.78	80	350	290	40	180	280	8	-250	400	0	-510	
4	4.78	120	350	290	40	180	280	8	290	350		0	
5	4.78	150	100	290	40	130	280	8	290	300			
6	4.78	180	0	290	40	0	280	0	290	300			
7	4.78	90		290	40		280		290	250			
8	4.78	70		0	40		100		290	0			
9	4.78	50			35		100		0				
10	12.78	0			10		-950						
11	0				0		0						
CAPITAL INVESTMENT	0	23	200	600	100	150	420	200	10	0	400	80	50
	1	-4.78	150	0	475	-30	-80	600	4	100	200	0	-400
	2	-4.78	-20	-200	250	-35	-90	-280	-5	475	300	-150	-50
DISCOUNT RATE	0.18	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
PRESENT VALUE of Initial Costs													
Cash Inflows													
NET PRESENT VALUE													
INTERNAL RATE OF RETURN													
PRESENT VALUE INDEX													

EXHIBIT 16.19 Data Cash Flows for Capital Investment Proposals.

2. AOC Corporation

The CEO of AOC Corporation has asked the managers of its various divisions to submit proposals for major capital expenditure programs. The proposals have been submitted and now the decision must be made as to which of the proposed projects should be adopted.

Summary data regarding the costs and expected cash flows of the proposed projects are shown in Exhibit 16.20 (all values are in thousands). Eleven projects have been proposed, designated as projects A through K. The numbers in the body of the table are the estimated cash flows (in thousands of dollars), with negative numbers indicating cash outflows and positive numbers indicating inflows. Project K is contingent on the adoption of Project H. Also, Project E is simply Project D delayed by one year, Project I is project H delayed by one year, and Project J is Project C delayed by one year (treat each pair as two mutually exclusive projects). Discount all project cash flows at the firm's cost of capital of 15%.

CASH FLOW ESTIMATES FOR PROPOSED INVESTMENT PROJECTS												
PROJECT:												
TIME	A	B	C	D	E	F	G	H	I	J	K	
0	-150	0	-135	-275	0	-285	-100	-420	0	0	-150	
1	100	-250	-200	120	-275	165	-250	250	-420	-135	100	
2	50	-185	-100	95	120	155	135	250	250	-200	75	
3	50	150	200	75	95	75	140	100	250	-100	75	
4	50	150	200	75	75	40	245	100	100	200	75	
5	50	150	200	75	75	40	245	0	100	200	0	
6	50	150	200	75	75	40	245	0	0	200	0	
7	50	150	200	75	75	40	245	0	0	200	0	
8	50	150	200	75	75	40	50	0	0	200	0	
9	50	150	0	75	75	0	50	0	0	200	0	
10	0	150	0	75	75	0	-900	0	0	0	0	
11	0	0	0	0	75	0	0	0	0	0	0	

EXHIBIT 16.20 Data Cash Flows for Capital Investment Proposals.

All 11 projects meet AOC's acceptance criterion: they have positive *NPVs* when evaluated using the firm's cost of capital, and their *IRRs* equal or exceed 15%. If there were no limitations on the firm's ability to accept projects, all 11 would be adopted. However, the board of directors has set limits on capital expenditures. They wish to preserve cash should more attractive opportunities present themselves in the future. The maximum amounts that can be invested are \$500 for the current year (year 0), \$300 for year 1, and \$150 in year 2. Given the limits set by the directors, it must now be decided which of these 11 projects to undertake.

The following instructions walk you through how to set up of these tasks. Begin by finding the portfolio that maximizes firm value.

Your assignment

- In a spreadsheet, calculate each project's *NPV* and *IRR*.
- Model 1.** Formulate this investment problem to be solved as a linear program, and then solve it. Be sure to constrain the portfolio decision variables to be binary and select Solver's "Non-negative solutions" option.
- Model 2.** Copy the Model 1 worksheet and make the following changes. Assume that the company can borrow up to \$200 in years 0 and 1 (from years 1 and 2, respectively). These funds can be used to finance investment beyond the expenditure limits

set by the directors. Assume that for each \$1 borrowed, \$1.046 must be repaid (after taxes) the following year. The net present value of \$1 borrowed in year 0 and repaid in year 1 is \$0.022; the NPV of \$1 borrowed in year 1 and repaid in year 2 is \$0.021.* Add these borrowing decision variables and parameters to your problem and have Solver find the new solution.

- d. **Model 3.** Copy the Model 2 worksheet and make the following changes. You may observe slack cash in the capital constraints for years 0 and 1. Assume that you can lend this slack cash to years 1 and 2, respectively. Each \$1 lent constrains the capital budget in the year lent by \$1 and relaxes the capital budget in the year received by \$1.02. The NPV of lending from year 0 to year 1 is $-\$0.047$ for each \$1 lent; from year 1 to year 2, it is -0.044 for each \$1 lent. Add these lending decision variables and parameters to your model and solve it.
- Does the addition lending change your portfolio choices for this model?
- e. Tabulate the NPVs for these three scenarios and report them ([Exhibit 16.20](#)).
- Why did the addition of borrowing and/or lending increase the NPV of your optimal portfolio of projects? (Hint: Track the budget slack in each year.)

3. MGB Corp.

The CEO of MGB Corporation has asked the managers of its various divisions to submit proposals for major capital expenditure programs. The proposals have been submitted and now the decision must be made as to which of the proposed projects should be adopted.

Summary data regarding the costs and expected cash flows of the proposed projects are shown in [Exhibit 16.21](#) (all values are in thousands). Ten projects have been proposed, designated as Projects A through J. The numbers in the body of the table are the estimated cash flows (in thousands of dollars), with negative numbers

CASH FLOW ESTIMATES FOR PROPOSED INVESTMENT PROJECTS										
PROJECT:	A	B	C	D	E	F	G	H	I	J
0	-175	0	0	-135	0	-300	-100	-600	0	0
1	100	-250	0	-375	0	200	-250	600	-135	-200
2	85	-185	-250	-300	-400	100	125	300	-375	225
3	85	100	-185	275	100	75	150	0	-300	25
4	85	175	100	275	100	40	250	0	275	25
5	85	175	175	275	100	40	250	0	275	0
6	85	175	175	275	100	40	250	0	275	0
7	85	175	175	275	100	40	250	0	275	0
8	85	175	175	275	100	40	50	0	275	0
9	85	175	175	0	100	0	50	0	275	0
10	0	175	175	0	100	0	-950	0	0	0
11	0	0	175	0	100	0	0	0	0	0

EXHIBIT 16.21 Data Cash Flows for Capital Investment Proposals.

* The assumption underlying these values, and those for lending are a tax rate of 34%, a borrowing (and discount rate of 7%), and a T-bill rate (earned on money lent) of 3%.

indicating cash outflows and positive numbers indicating inflows. Project J is contingent on the adoption of Project H. Also, Project C is simply Project B delayed by one year, and Project I is Project D delayed by one year (i.e., treat Projects C & B and I & D as mutually exclusive pairs). Discount all project cash flows at the firm's cost of capital of 15%.

All 10 projects meet MGB's acceptance criterion: they have positive *NPVs* when evaluated using the firm's cost of capital and their *IRRs* equal or exceed 15%. If there were no limitations on the firm's ability to accept projects, all 10 would be adopted. However, the board of directors has set limits on capital expenditures. They wish to preserve cash should more attractive opportunities present themselves in later years. The maximum amounts that can be invested are \$600 for the current year (year 0), \$250 for year 1, and \$200 in year 2. Given the limits set by the directors, it must now be decided which of these 10 projects to undertake. The portfolio of projects that is chosen should be the one that maximizes the *NPV* of the portfolio as a whole while not exceeding the limited capital budget.

Assignment

- a. In a spreadsheet, calculate the *NPV* and the *IRR* for each project.
- b. Formulate this investment problem to be solved as a linear program, and solve it. (Hint: One of your constraints should define the portfolio decision variables as binary, and be sure to check the "Non-negative solutions" option in Solver.)
- c. Assume that the company can borrow up to \$200 in years 0 and 1 (from years 1 and 2). These funds can be used to finance investment beyond the expenditure limits set by the directors. Assume that for each \$1 borrowed, \$1.046 must be repaid (after taxes) the following year. The net present value of \$1 borrowed in year 0 and repaid in year 1 is \$0.022; the *NPV* of \$1 borrowed in year 1 and repaid in year 2 is \$0.021. Add these borrowing decision variables to your problem and determine the new solution.
- d. You should observe substantial amounts of slack cash in year 1's constraint in part (c) (and some in year 0, too). Assume that you can lend this slack cash to years 1 and 2. Each \$1 lent constrains the capital budget in the year lent by \$1 and relaxes the capital budget in the year received 3 by \$1.02. The *NPV* of lending from year 0 to year 1 is $-\$0.047$ for each \$1 lent; from year 1 to year 2, it is -0.044 for each \$1 lent. Add these lending decision variables to the model used in part (c) and observe how the lending opportunity changes your portfolio choices ([Exhibit 16.21](#)).
- e. Tabulate the results for these three scenarios and report them in a table. Why did the addition of borrowing and lending increase the *NPV* of your optimal portfolio of projects? (Hint: Track the budget slack in each year.)

Planning and Managing Working Capital with Linear Programming

17.1 OPTIMIZING WORKING CAPITAL DECISIONS

One of the tasks of a financial manager is to develop and execute plans for managing working capital. The management of working capital involves issues such as deciding what minimum amount of cash should be maintained, how to invest excess cash, which marketable securities should be purchased, which should be sold, how receivables and payables should be managed, how much funds should be borrowed and in what maturity, and when outstanding debt should be repaid. The planning and management of the current accounts involves a plethora of problems with myriad of potential solutions. Various approaches have been offered for the handling of individual problems such as cash management. The current accounts are interrelated, and the separate management of each account as an independent problem may be suboptimal. What is necessary is a method for considering the numerous interrelationships and making decisions that would be consistent with the objective of maximizing the value of the firm.

Chapter 16 introduced linear programming as a tool for optimization. In addition, linear programming is well suited to handling some of the problems and interdependencies of current assets and liabilities. Numerous linear programming models of the current accounts have been presented; two of the earliest and best known are those by Robichek et al. (1965) and Orgler (1970).

Robichek et al. (1965) developed a linear programming model to analyze short-term financing decisions. The problem they addressed was how to fund the seasonal demands for funds with different short-term and intermediate-term funding sources over a short-term planning horizon. The sources of funds they considered included (1) a line of credit,

(2) pledging of accounts receivable (AR), (3) delays in payments on trade accounts payable (AP), and (4) a term loan. In addition, excess cash could be invested in short-term marketable securities. The decision variables were the amounts of each of these funding sources to be used in each of the periods of the planning horizon, and the objective was the minimization of the interest cost over the planning horizon. The planning horizon for their example was 12 months. Their model would generate recommended financing decisions for each of the 12 months in the planning horizon. The general structure of their model can be summarized as follows:

Objective:	Minimize: Total Net Interest Cost over the Horizon
Constraints:	
<i>Line of Credit:</i>	
Total borrowing on Line of Credit	\leq Credit Limit
Cash Balance	\geq Required Compensating Balance
Payments on Line	\leq Balance on Line of Credit
<i>Pledging of AR:</i>	
Receivables Pledged	\leq Lender Limit on Receivables
Repayment on Receivable Loans	\leq Amount Borrowed
<i>Stretching (Delayed Payment) of AP:</i>	
Payables Stretched	\leq (X%) Total Payables
<i>Term Loan:</i>	
Amount Borrowed	\leq Lender Limit on Loan
Total Borrowing from All Sources	\leq Policy Limit on Borrowing
<i>Cash Balance:</i>	
Cash Balance	\geq Desired Cash Balance
<i>Cash Flow:</i>	
Cash Balance	$=$ Beginning Cash + Cash Inflow – Cash Outflow

Robichek et al. demonstrated their model with a 12-month example. It had 73 decision variables and 134 constraints. It is interesting to note that with the technology of the time (1964), using a Fortran program on an IBM 7090 mainframe computer, it took about 15 minutes to compile the program and 5 minutes to solve it. Now, it would seem almost instantaneous when solved on a PC with Excel's Solver routine.

Yair Orgler (1970) developed a linear programming model for working capital decisions that considered cash balance targets, marketable securities, accounts payable, and a line of revolving credit. His model of working capital decisions had the objective of maximizing the net earnings over a finite horizon. The maximization of net earnings (revenue minus cost) over the planning horizon has the effect of maximizing the future value of the net working capital accounts, evaluated at the end of the planning horizon. This has the same decision result as the maximization of present value, but it is an easier formulation in the working capital context. The decision variables include the amounts to be paid on trade

accounts payable, the amounts to be borrowed and repaid on a line of credit, purchases and sales of marketable securities, and the amount of cash to keep on hand. The constraints are similar to Robicheck et al.:

Payments on AP	\leq Payable Balances
Borrowing on Line of Credit	\leq Credit Limit
Securities Sold	\leq Securities Held
Cash Balance	\geq Desired Cash Balance
Cash Balance	$=$ Beginning Cash + Cash Inflow – Cash Outflow

One of the unique features of Orgler's approach is the use of unequal time periods. In the short-term planning context, the size of the problem can quickly get out of hand because of the number of periods and the number of different decision variables. For example, we may need to plan for daily or weekly transactions in securities and other short-term accounts, and if we have a six-month planning horizon, we can end up with an overwhelming number of decision variables. To make the problem more manageable, Orgler divides the planning horizon into periods of unequal length. For example, a six-month horizon is divided into six periods, with the first two periods of one day each, the third period lasting 10 days, followed by periods of 20 days, 60 days, and 90 days. The advantage of this unequal-period planning horizon is that it is unnecessary to make decisions regarding daily actions, say, four months hence, but it is necessary to worry about tomorrow and the next day. The decision problem is solved to generate decision recommendations for the next two days, and tentative decisions for the longer periods after that. Then the problem is solved again as time passes and market conditions change. In this way, it is generating recommendations every couple of days for current decisions. The recommendations for periods further in the future are tentative and are revised as those periods approach the present. This unequal-period approach decreases the detail of the plan for periods further in the future while allowing more detailed consideration of the near term.

Even with the efficiency of unequal periods, the problem can easily be quite large. In the small six-month example presented by Orgler, there were 69 decision variables and 25 constraints. As he notes, "a cash management model for a large corporation will include, approximately, 300 to 500 constraints and 2,000 to 4,000 variables" (Orgler 1970, p. 101). Although Orgler's model is considered smaller than more realistic applications, it is still sufficiently large that the reader quickly gets lost in the detail. So that we can understand the principles and structure without being overwhelmed by detail, we look at a considerably smaller example.

17.2 WORKING CAPITAL DECISIONS FOR THE STILKO COMPANY

The manager of the working capital accounts of the Stilko Corporation faces the task of planning the firm's short-term security investments, short-term borrowing, and managing the cash account. Over the next four weeks, the firm's operating cash flows will

alternate between inflows and outflows. The cash flows expected in each of the next four weeks are as follows:

Stilko Corporation				
Schedule of Cash Flows for the Next Four Weeks				
Week	1	2	3	4
Cash Flow	−3,000	+6,000	−4,000	+10,000

The decisions available to help manage the working capital accounts include purchasing and selling marketable securities and borrowing on a line of credit.

17.2.1 Marketable Securities

At the start of the planning period, the firm has a portfolio of marketable securities that will mature according to the following schedule:

Schedule of Maturing Marketable Securities				
Week	1	2	3	4
Maturing Securities	1,000	0	500	2,000

These securities were purchased at a discount, and the maturing amounts are the face values of the securities, representing the amount of cash that will be received. New securities can be purchased at a discount from face value, with the discount based on an annual yield to maturity of 10%, or weekly rate of 0.192%. For purchasing any new securities, the policy will be to purchase only securities that will mature in two weeks. None of the securities will be sold prior to maturity.

17.2.1.1 Borrowing

The company has a bank line of credit that will allow it to increase its short-term borrowing up to a balance of \$5,000 at an annual interest rate of 12%, or 0.231% per week. Interest will accrue on the debt at 0.231% per week, with principal and accrued interest repaid after the end of the planning horizon.

17.2.1.2 Cash Balance

The current cash balance is \$2,000, and the firm would like to carry a minimum cash balance of \$3,000.

17.2.1.3 Time

Assume that all cash flows, security transactions, and borrowing occur at the end of each week. Thus, the first exogenous cash flow (an outflow of \$3,000) will occur on Friday of the first week, and securities scheduled to mature will provide funds the same day.

17.2.1.4 Objective

The objective of the decisions is to maximize the value of net working capital at the end of the four-week planning horizon. Value of net working capital is defined as assets minus liabilities. In this model, the assets consist of cash and marketable securities, and debt balance (line of credit) is the liability. Thus, the objective function is

$$\text{Max } W = \text{Cash}_4 + \text{Marketable Securities}_4 - \text{Debt balance}_4$$

At the horizon date, $\text{Marketable Securities}_4$ is the value of securities held at the end of the fourth week that mature in a subsequent period. Debt balance_4 is the balance, including accrued interest at the end of the fourth week. The terminal value of net working capital will include all earnings on investments and all costs of borrowing, so it takes account of the time value of money. Consequently, the maximization of terminal value is identical with the maximization of present value.

17.2.1.5 Decision Variables

The model will have 12 decision variables: 4 security purchase variables— SP_{13} , SP_{24} , SP_{35} , and SP_{46} ; 4 borrowing variables— B_1 , B_2 , B_3 , and B_4 ; and 4 cash balance variables— C_1 , C_2 , C_3 , and C_4 . These decision variables are defined as follows:

17.2.1.5.1 Security Purchases SP_{tj} denotes purchases at the end of week t that mature in week j , expressed as the face value of the securities. With the restriction that we will consider purchasing only securities that mature in two weeks, the variables are

$$SP_{13}, SP_{24}, SP_{35}, \text{ and } SP_{46}$$

Let SP_{0j} denote the securities held at the beginning of the planning period. With the initial endowment of securities, we have

$$SP_{01} = 1,000, SP_{02} = 0, SP_{03} = 500, \text{ and } SP_{04} = 2,000$$

17.2.1.5.2 Borrowing B_t denotes the principal amount borrowed at the end of week t . The borrowing variables are

$$B_1, B_2, B_3, \text{ and } B_4$$

17.2.1.5.3 Cash Balance The cash balance is the result of our other decisions, and we want to maintain a minimum cash balance of at least \$3,000. As the balance itself is a result, it is not really a decision variable. Nevertheless, for expressing our model let C_t represent the cash balance at the end of week t , with $C_0 = \$2,000$ being our initial cash balance.

17.2.1.6 Constraints

We will have the following constraints that limit our decisions:

Borrowing:	$Debt\ Balance_t \leq Borrowing\ Limit_t$
Security Purchases:	$Total\ Security\ Purchases_t \leq Funds\ available_t$
Cash Balance:	$Cash\ Balance_t \geq Minimum\ Cash$
Cash Balance & Flow:	$Cash\ Balance_t = Cash\ Balance_{t-1} + Cash\ Inflow_t - Cash\ Outflow_t$

Now let us fill in the complete model so we can find a solution. We will explain the various coefficients as we proceed.

Exhibit 17.1 shows the completed model setup as a linear programming tableau. The decision variables are listed across the top. The objective function is shown as row 0, and the constraints are shown as rows 1 through 12.

17.2.1.7 Objective Function

The objective is to maximize the value of net working capital at the end of the fourth week. Net working capital consists of cash and securities, less the debt balance. Cash at the end of the fourth week is C_4 . At the end of the fourth week, the security portfolio consists of the securities that mature in later periods, which would be the securities purchased in week 3 to mature in week 5, SP_{35} , and the securities purchased in week 4 to mature in week 6, SP_{46} . There are no other securities at that time because we are only considering securities that mature in two weeks. As of the end of week 4, the value of these securities is the present value of the maturity payment (face value) to be received in one and two weeks respectively:

$$0.99808 SP_{35} + 0.99617 SP_{46}$$

The coefficients are simply the present value factors $1/(1+r)^t$ for $t = 1$ and 2 , where the discount rate is the weekly rate $r = 0.192\%$, based on the annual rate of 10% . At the end of week 4, securities purchased in week 3 to mature in week 5, SP_{35} , now have just one week to maturity, so their value is $1/1.00192 = 0.99808$ per dollar of face value. Similarly, the securities purchased in week 4 to mature in week 6 have two weeks to mature and their value is $1/(1.00192)^2 = 0.99617$ per dollar of face value.

The debt balance at the end of the fourth week includes principal and accrued interest. With no debt repaid during the planning horizon, the principal and interest are

$$1.00694 B_1 + 1.00463 B_2 + 1.00231 B_3 + B_4$$

At a nominal annual rate of interest of 12% , the weekly rate is $r_d = 0.231\%$. For each dollar borrowed in week 1, the amount owed at the end of week 4 is $(1+r_d)^3 = 1.00231^3 = 1.00694$. The coefficients for borrowing in weeks 2 and 3 are calculated similarly with 2 and 1 weeks of compounding, respectively. The coefficient for B_4 is 1 because no interest has yet accrued.

Combining terms, the value of the net working capital at the end of the fourth week is

$$C_4 + 0.99808 SP_{35} + 0.99617 SP_{46} - 1.00694 B_1 - 1.00463 B_2 - 1.00231 B_3 - B_4$$

	C_1	C_2	C_3	C_4	SP_{13}	SP_{24}	SP_{35}	SP_{46}	B_1	B_2	B_3	B_4	
0				C_4			.99808 SP_{35}	.99617 SP_{46}	-1.00694 B_1	-1.00463 B_2	-1.00231 B_3	B_4	≥ 3000
1	C_1												≥ 3000
2		C_2											≥ 3000
3			C_3										≥ 3000
4				C_4									≥ 3000
5									B_1				≤ 5000
6									B_1	$+B_2$			≤ 5000
7									B_1	$+B_2$	$+B_3$		≤ 5000
8									B_1	$+B_2$	$+B_3$	$+B_4$	≤ 5000
9	C_1				$+99617 SP_{13}$				$-B_1$				$= 0$
10	$-C_1$	C_2				$+99617 SP_{24}$				$-B_2$			$= 6000$
11		$-C_2$	C_3				$+99617 SP_{35}$				$-B_3$		$= -3500$
12			$-C_3$	C_4				$+99617 SP_{46}$				$-B_4$	$= 12000$

EXHIBIT 17.1 Stilko Corp. linear programming model: Table of objective function, decision variables and constraints.

This is the objective function that is to be maximized. Note that the other decision variables (cash and security purchases) for weeks 1, 2, and 3 do not appear in the objective function because their effects are reflected in the week 4 values of cash, securities, and borrowing. For example, the cash balance in week 4 derives from the cash in week 3 plus the cash flows resulting from security transactions and borrowing during the week. Similarly, the security portfolio held at the end of week 4 is the result of purchases in earlier weeks.

17.2.1.8 Constraints

Lines 1–4 are the cash balance constraints, $C_t \geq 3000$, that require that the cash balance at the end of each week be at least \$3,000. Lines 5–8 are the constraints on borrowing that limit the total principal amount borrowed to be less than \$5,000 in each week. In week 1, the total borrowed is B_1 , but with none of the debt repaid, each week's new borrowing adds to the principal balance so that the balance at the end of week 4 will be $B_1 + B_2 + B_3 + B_4$. Lines 9–12 are the cash flow equations that balance the model. At the end of week t , the cash balance is calculated as

$$\begin{aligned} \text{Cash}_t = & \text{Cash}_{t-1} + \text{Securities Matured}_t - \text{Securities Purchased}_t + \text{Borrowing}_t \\ & + \text{Exogenous Cash Flow}_t \end{aligned}$$

Expressed in terms of our notation, at the end of the first week, the cash balance is

$$C_1 = C_0 + SP_{01} - 0.99617 SP_{13} + B_1 + (-3000)$$

where the -3000 is the exogenous cash outflow of week 1. With the beginning cash balance, $C_0 = 2000$, and the initial endowment of securities that mature in week 1, $SP_{01} = 1000$, this equation becomes

$$C_1 = 2000 + 1000 - 0.99617 SP_{13} + B_1 - 3000$$

Putting the decision variables to the left and the constant terms on the right side, we have

$$C_1 + 0.99617 SP_{13} - B_1 = 2000 + 1000 - 3000$$

or

$$C_1 + 0.99617 SP_{13} - B_1 = 0$$

which is the way it is shown on line 9 of the exhibit. In week 2, the cash balance is

$$C_2 = C_1 + SP_{02} - 0.99617 SP_{24} + B_2 + 6000$$

With the initial security endowment of $SP_{02} = 0$, this is shown on line 10 as

$$-C_1 + C_2 + 0.99617 SP_{24} - B_2 = 6000$$

The remaining cash flow equations are similarly constructed.

Exhibit 17.2 displays an Excel spreadsheet setup in LP format so that it can be solved with Solver. The lines are numbered in column B to correspond to the equation numbers in **Exhibit 17.1**. The decision variables are labeled in spreadsheet row 21, with the actual decision variables in cells D22:O22. The numbers in the LP tableau are the coefficients of the decision variables. The numbers in cells D26:O26 are the coefficients of the decision variables that appear in the objective function, and the value of the objective function is calculated in cell C26 as `=SUMPRODUCT(D22:O22,D26:O26)`. The constraint section shows just the coefficients. For example, the “1” in cell D31 is the coefficient of decision variable C_1 in constraint #1, which is $1 \cdot C_1 \geq 3000$, with the minimum cash balance of \$3,000 shown in cell Q31. The product of the coefficient (1 in cell D31) and the decision variable C_1 (cell D22) is calculated in cell P31 as `=D31*D22`, in the column labeled “Resource Used.”

To solve the problem as a linear program, we use Tools/Solver to call up the Solver dialog box as is shown in **Exhibit 17.3a**. In the Solver dialog box, we have designated cell C26 as the objective function (Target Cell) to be maximized, and cells D22:O22 as the decision variables (Changing Cells). The constraints are shown in the “Subject to the Constraints” section. Having specified the objective function, decision variables, and constraints in the dialog box, we click “Options” and specify that we want to “Assume NonNegative” in order to require the decision variables to be nonnegative. When we click the Solve button, we obtain the solution shown in the body of the spreadsheet of **Exhibit 17.3b**.

The solution calls for borrowing \$3,000 in period 1 (cell L22), so we can fund the exogenous outflow and bring our cash balance up to the target. In period 3, we purchase \$2,509 of securities that will mature in period 5 (J22). This optimal plan will generate a net asset value at the end of period 4 of \$14,484 (C26), which is the largest balance that is feasible given our resources, obligations, and policy constraints. This solution results in a cash balance that meets or exceeds the target of \$3,000 in each period, and borrowing does not exceed the limit.

It is important to make sure that the solution makes sense. As the size of a problem expands, it is increasingly difficult to sort through the results to see that everything balances, and we are not getting nonsense results. Financial statements that summarize the results in a familiar format will help us to check the results. **Exhibit 17.4** is a part of the spreadsheet showing cash flow, working capital balances, and security transactions, so we can see the results of the recommended program. The top section is the cash flow showing each of the transactions that contributes to the cash balance at the end of the week. The net working capital balance is shown in the second section, representing the working capital assets net of borrowing. The balance at the end of week 4 was the value that was maximized. Security transactions are shown as security purchases and the portfolio balance at the end of each week. The last line is the balance in the security portfolio, stated in terms of market value. These statements help us to see how the various transactions feed into the cash flow and the account balances, so we can be sure that the model is doing what we want it to do. As we will see in the larger model in

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1																		
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EXHIBIT 17.2 Excel spreadsheet in linear programming format.

Solver Parameters

Set Objective:

To: Max Min Value Of:

By Changing Variable Cells:

Subject to the Constraints:

Make Unconstrained Variables Non-Negative

Select a Solving Method:

Solving Method
 Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

(a)

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
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44																		
45																		

(b)

EXHIBIT 17.3 (a) Solver dialog box for solving Stilko LP model. (b) Stilko LP model as solved using Solver.

	A	B	C	D	E	F	G	H	I	J	K	L	M
47					Stilko Corporation								
48					Financial Statements								
49		Cash Flow											
50					Week	1	2	3	4				
51													
52		Exogenous Cash Flow				-3,000	6,000	-4,000	10,000				
53		Maturing Securities				1,000	0	500	2,000				
54		Security Purchases				0	0	2,500	0				
55		Amount Borrowed				3,000	0	0	0				
56		Net Cash Inflow				1,000	6,000	-6,000	12,000				
57		Beginning Cash				2,000	3,000	9,000	3,000				
58		Ending Cash				3,000	9,000	3,000	15,000				
59													
60		Working Capital Balance											
61		Net Asset Balance											
62		Cash				3,000	9,000	3,000	15,000				
63		Marketable Securities	@ market value			2,487	2,491	4,496	2,505				
64		Debt Balance				3,000	3,007	3,014	3,021				
65		Net Assets				2,487	8,484	4,482	14,484				
66													
67													
68		Security Purchases										Securities Purchased	
69												@ Face	@ Market
70					Week Purchased	1	2	3	4	5	6		
71		@ Face Value			0	1,000	0	500	2,000			3,500	3,480
72					1			0				0	0
73					2				0			0	0
74					3					2,510		2,510	2,500
75					4						0	0	0
76		Total Maturing Securities				1,000	0	500	2,000	2,510	0		
77													
78		Security Portfolio										@market value	
79		Maturing in 1 Period				0	499	1,996	2,505				
80		Maturing in 2 Periods				498	1,992	2,500	0				
81		Maturing in 3 Periods				1,989	0						
82													
83		Market Value of Securities				2,487	2,491	4,496	2,505				

EXHIBIT 17.4 Stilko financial statements.

Section 17.3, without this kind of check, we could easily develop a model that gives us nonsense results but not even know they are nonsense.

17.3 ELABORATIONS AND EXTENSIONS

The Stilko example was simple so that we could see how to construct a working capital model without getting too bogged down with the detail of too many decision variables and too many planning periods. A more realistic model would usually be far more complex with more decision variables and planning periods. Now we look at a few elaborations that we might make that could help us handle some more complicated problems.

17.3.1 Debt Repayment

Our Stilko example allowed us to borrow, but it did not provide for the possibility of repaying any of the debt during the planning period. We can easily add decision variables for repaying debt. If we add the option to repay debt prior to maturity, we have to modify the model by adding decision variables, modifying the objective function, and adding constraints. Assume that Stilko can repay debt in weeks 2, 3, or 4 (we assume that there is no debt outstanding at the beginning, so no repayment is possible in week 1). Let R_t be the amount of debt repaid in period t .

17.3.1.1 Objective

Given that our objective is to maximize the value of net working capital (working capital assets—debt) at the end of the horizon, the repayment variables affect the objective directly by way of the debt balance at the end, and indirectly by affecting the asset balances. If we repay debt, we do not have as much debt outstanding at the end of the horizon, and by reducing principal during the planning period, we save interest expense that we would otherwise have to pay. The interest saved will show up as increased asset balances (cash or marketable securities) at the end of the horizon. The general form of our objective function is the same as before:

$$\text{Max } W = \text{Cash}_4 + \text{Marketable Securities}_4 - \text{Debt balance}_4$$

The debt balance at the end of week 4 will now show debt repayments:

$$\text{Debt balance}_4 = 1.00694 B_1 + 1.00463 B_2 + 1.00231 B_3 + B_4 - 1.00463 R_2 - 1.00231 R_3 - R_4$$

where the negative coefficients of the repayment variables, R_2 , R_3 , and R_4 , represent the principal and interest saved from the repayment date to the horizon. For example, debt repaid in week 2 saves two weeks of interest by the end of the fourth week. At a weekly interest rate of 0.231%, the principal and interest per dollar repaid is 1.00463. Adding these terms to the objective function, the expanded objective function is

$$\begin{aligned} \text{Max } W = & C_4 + 0.99808 SP_{35} + 0.99617 SP_{46} - 1.00694 B_1 - 1.00463 B_2 \\ & - 1.00231 B_3 - B_4 + 1.00463 R_2 + 1.00231 R_3 + R_4 \end{aligned}$$

17.3.1.2 Constraints

Debt repayment will require new constraints, so the amount repaid does not exceed the outstanding debt, and the repayment variables will appear in the borrowing limit constraints as well as the cash flow constraints.

17.3.1.3 Repayment

The amount repaid in any period cannot exceed the outstanding debt:

$$\begin{aligned} R_2 & \leq B_1 \\ R_2 + R_3 & \leq B_1 + B_2 \\ R_2 + R_3 + R_4 & \leq B_1 + B_2 + B_3 \end{aligned}$$

Putting the decision variables on the left side of the inequality yields

$$\begin{aligned} -B_1 + R_2 & \leq 0 \\ -B_1 - B_2 + R_2 + R_3 & \leq 0 \\ -B_1 - B_2 - B_3 + R_2 + R_3 + R_4 & \leq 0 \end{aligned}$$

17.3.1.4 *Borrowing Limit*

The total debt outstanding cannot exceed \$5,000:

$$B_1 \leq 5000$$

$$B_1 + B_2 - R_2 \leq 5000$$

$$B_1 + B_2 + B_3 - R_2 - R_3 \leq 5000$$

$$B_1 + B_2 + B_3 - R_2 - R_3 - R_4 \leq 5000$$

17.3.1.5 *Cash Flow Constraint*

The cash flow constraint requires that the ending cash equals the beginning cash plus net cash inflows. The repayment of debt needs to be added to these constraints by showing the repayment as an outflow. The general form of the cash flow constraint is

$$\begin{aligned} \text{Cash}_t = & \text{Cash}_{t-1} + \text{Securities Matured}_t - \text{Securities Purchased}_t + \text{Borrowing}_t \\ & - \text{Repayment}_t + \text{Exogenous Cash Flow}_t \end{aligned}$$

For example, the period 2 constraint would now be

$$-C_1 + C_2 + 0.99617 SP_{24} - B_2 + R_2 = 6000$$

where we have transposed the decision variables to the left side, so R_2 appears with a positive sign. The other cash flow expressions would be similarly modified by adding the repayment variable. Once these additional repayment variables are added to the model, we can solve it to find the optimal pattern of borrowing and repayment over the planning horizon.

17.3.2 *Another Source of Debt Financing: Accounts Payable*

In the simple Stilko problem, we considered only a single type of bank debt. We could easily expand the model to consider several types of loans with different terms and constraints. Most borrowing sources would be handled similarly to the borrowing in this model. However, one source of funds that is treated somewhat differently is trade credit. Trade credit is included in both the working capital models by Robichek et al. and Orgler. In their models, they assume that trade credit is a spontaneous source of financing, so that accounts payable (AP) is related to the firm's production operation and is taken as exogenously given in the context of the working capital LP. Thus, in their models, AP financing is not a decision variable. However, the pattern of payments is considered a decision variable to be included in the LP model. Let us see how we might handle AP in our model.

Assume that Stilko purchases materials from its suppliers on credit terms that grant a 2% discount for payment in the next week after the purchase, and require that the account be paid in three weeks. (Our terms are more stringent than normal practice so we can demonstrate the idea and still fit within the framework of our simple four-week model.)

The schedule of purchases on credit is given below, where the weeks with negative numbers indicate weeks prior to the start of our planning period. The amount purchased is the credit purchase in that week, and the last two columns are the weeks when the bill must be paid to receive the discount, and the week when the total balance is due, respectively. For example, the 900 purchased in week 0 (the week prior to the start of the planning period) must be paid in full by the end of week 3, but if it is paid by the end of week 1, the 2% discount is taken.

Week of Purchase	Amount Purchased	Week to Receive Discount	Week to Pay Total
-2	200	-1	1
-1	800	0	2
0	900	1	3
1	1,000	2	4
2	700	3	5
3	600	4	6
4	400	5	7

17.3.2.1 Decision Variables

Let P_{jt} represent the payment made in week t on a credit purchase made in week j . For example, $P_{-1,2}$ is the payment made in week 2 on a purchase made in week -1, the week prior to the start of our planning period. Assume that none of the purchases prior to week 1 have been paid, nor have the discounts been taken. Our decisions start in week 1, so we cannot take discounts on purchases in weeks -2 or -1, but we if we pay this week on purchases made in week 0, we can take the discount. There are 12 decision variables in our model, designated as follows:

$$\begin{aligned}
 &P_{-2,1}, \\
 &P_{-1,1}, \quad P_{-1,2}, \\
 &P_{0,1}, \quad P_{0,2}, \quad P_{0,3}, \\
 &P_{1,2}, \quad P_{1,3}, \quad P_{1,4}, \\
 &P_{2,3}, \quad P_{2,4}, \\
 &P_{3,4}
 \end{aligned}$$

Within our four-week model, we are not dealing with payment decisions after period 4.

17.3.2.2 Objective

Our objective function is the net working capital balance at the end of period 4, with the AP balance appearing with a negative sign:

$$\text{Max } W = \text{Cash}_4 + \text{Marketable Securities}_4 - \text{Debt balance}_4 - \text{Accounts Payable}_4$$

By the end of week 4, we will have paid for all purchases made up through week 1. We may still owe on purchases made in weeks 2, 3, and 4. The amount still owed on purchases made in week 2 is

$$Purchase_2 - 1.0204 P_{23} - P_{24}$$

The amount owed on purchases in week 3 is

$$Purchase_3 - 1.0204 P_{34}$$

Of course, we have not yet made any payments on week 4 purchases, so we owe $Purchase_4$. The AP balance at the end of period 4 would be

$$AP_4 = Purchase_2 - 1.0204 P_{23} - P_{24} + Purchase_3 - 1.0204 P_{34} + Purchase_4$$

With the purchases specified in the aforementioned table, this expression is

$$\begin{aligned} AP_4 &= 700 - 1.0204 P_{23} - P_{24} + 600 - 1.0204 P_{34} + 400 \\ &= 1,700 - 1.0204 P_{23} - P_{24} - 1.0204 P_{34} \end{aligned}$$

This expression is subtracted from the rest of the objective function to reflect the AP balance owed at the end of the fourth week. In the objective function, it is unnecessary to show the constant, \$1,700, so the terms subtracted from the rest of the objective function are just the decision variables with their coefficients:

$$-1.0204 P_{23} - P_{24} - 1.0204 P_{34}$$

The coefficient 1.0204 accounts for the 2% discount for early payment to convert the amount paid to the face amount owed on the account. In other words, suppose we owe \$100 face amount. With the 2% discount if we pay within 1 week, we can pay off the \$100 liability by paying \$98. The face amount that we have paid off is

$$(1 / (1 - .02)) * 98 = 1.0204 * 98 = 100$$

As the decision variable, P_{jp} , is the dollar amount paid, for the early payments, we multiply it by the 1.0204 coefficient to specify the face amount of the account that is paid.

17.3.2.3 Constraints

We have two constraints specific to A/P. First, we do not want to pay more than we owe; and second, we must pay the total amount due on each purchase by the end of the third week. In addition to these constraints, we need to include the payment variables in the cash flow constraints, with the payments shown as outflows. The A/P specific constraints would be as follows.

$$P_{-21} = 200 \tag{17.1}$$

$$P_{-11} \leq 800 \quad (17.2)$$

$$P_{-11} + P_{-12} = 800 \quad (17.3)$$

$$1.0204 P_{01} \leq 900 \quad (17.4)$$

$$1.0204 P_{01} + P_{02} \leq 900 \quad (17.5)$$

$$1.0204 P_{01} + P_{02} + P_{03} = 900 \quad (17.6)$$

$$1.0204 P_{12} \leq 1000 \quad (17.7)$$

$$1.0204 P_{12} + P_{13} \leq 1000 \quad (17.8)$$

$$1.0204 P_{12} + P_{13} + P_{14} = 1000 \quad (17.9)$$

$$1.0204 P_{23} \leq 700 \quad (17.10)$$

$$1.0204 P_{23} + P_{24} \leq 700 \quad (17.11)$$

$$1.0204 P_{34} \leq 600 \quad (17.12)$$

Expression (17.1) requires that the payables balance of purchases made in week -2 be paid in full in week 1. Expression (17.4) says that payments in week 1 on purchases in week 0 cannot exceed the \$900 owed. The coefficient 1.0204 converts the payment with the discount taken to the face amount owed. Expression (17.6) says that the total of payments in weeks 1, 2, and 3 on week 0 purchases must completely repay the amount owed. Similar interpretations apply to the other expressions. This formulation requires that the accounts be paid in full by the end of the normal payment period. A further extension suggested by Robichek et al. would allow stretching of AP beyond the normal period.

17.3.3 Selling Securities

Our four-week example allowed us to purchase securities, and we held the securities until they matured. We did not consider the possibility of selling securities from our portfolio prior to maturity. When we add the opportunity to sell securities prior to maturity, we expand the range of methods for obtaining funds when we need them. To consider selling securities, we add decision variables representing the sale of securities, and we add constraints. The primary constraint that we have to add is a restriction that says that we cannot sell more securities than we own. Of course, we have to include security sales in the cash flow constraints, with the proceeds from the sale shown as inflows at the date of the sale. The security sales affect the objective function by decreasing the value of securities held at the horizon, and by either adding cash or decreasing the debt balances at the end if we borrow less or use the proceeds to repay debt.

17.3.3.1 Decision Variables

Let SS_{tj} denote the sale of securities in week t that mature in week j , with the values expressed in terms of face value securities sold. In our original Stilko problem, we had an initial endowment of securities: $SP_{01} = 1,000$, $SP_{03} = 500$, and $SP_{04} = 2,000$. We allowed the purchase of securities that matured in two weeks, so we had four purchase variables: SP_{13} , SP_{24} , SP_{35} , and SP_{46} . Given the beginning portfolio and allowed purchases, we could sell securities prior to maturity that mature in periods 1, 3, 4, 5, and 6. Presumably, we could sell securities in any week, which would require 13 additional decision variables. So we do not get too complicated, we will decrease the number of new variables by considering only the possibility of selling securities in weeks 1 and 2. With this simplification, we have the following four decision variables: SS_{13} , SS_{23} , SS_{14} , and SS_{24} . In weeks 1 or 2, we cannot sell securities maturing in weeks 5 or 6, because we will not have acquired them yet.

17.3.3.2 Constraints

Assuming that we are not considering short sales, we cannot sell securities that we do not own, so we need constraints that say

$$\text{Security Sales} \leq \text{Securities Held}$$

These constraints would be formulated as follows:

Sales in week 1:

$$SS_{13} \leq 500$$

$$SS_{14} \leq 2000$$

where the constant amounts on the right side are the initial endowments, $SP_{03} = 500$ and $SP_{04} = 2,000$, respectively.

Sales in week 2:

$$SS_{23} \leq SP_{03} + SP_{13} - SS_{13}$$

$$SS_{24} \leq SP_{04} + SP_{14} - SS_{14}$$

With the initial endowments shown as constants and putting the decision variables on the left, the selling constraints for both periods would be written as

$$SS_{13} \leq 500$$

$$SS_{14} \leq 2000$$

$$SS_{13} + SS_{23} - SS_{13} \leq 500$$

$$SS_{14} + SS_{24} - SS_{14} \leq 2000$$

We would show similar constraints for each period as we expanded the problem to consider more decision variables.

In the cash flow constraints, we would show the inflow from security sales. As the variables are defined in terms of face value of the securities, the cash derived from sales would be the discounted value at the date of the sale. The weekly rate earned on securities is 0.192%, so the proceeds per dollar of face value of securities sold would be \$0.99808, \$0.99617, and \$0.99426 for securities sold in weeks 1, 2, and 3 prior to maturity. (These figures would be adjusted downward to account for transactions costs of selling.) The cash flow constraints for weeks 1 and 2 would be modified to show the cash inflow from security sales as

$$\text{Week 1: } C_1 + 0.99617 SP_{13} - 0.99617 SS_{13} - 0.99426 SS_{14} - B_1 = 0$$

$$\text{Week 2: } -C_1 + C_2 + 0.99617 SP_{24} - 0.99808 SS_{23} - 0.00617 SS_{24} - B_2 = 6000$$

Each subsequent cash flow constraint would also have to be modified to take account of the fact that if securities are sold in weeks 1 or 2, the securities that would otherwise have matured in weeks 3 or 4 would not be available to generate an inflow. For example, in week 3, the modified constraint would be

$$\text{Week 3: } -C_2 + C_3 + 0.99617 SP_{35} + SS_{13} + SS_{23} - B_3 = -3500$$

where the terms SS_{13} and SS_{23} reflect the fact that the sales in periods 1 and 2 of securities that mature in week 3 will reduce the cash flow in period 3. Their coefficients are 1 because their value and cash flow are at face value in the week the securities mature.

17.3.3.3 Objective Function

The objective is

$$\text{Max } W = \text{Cash}_4 + \text{Marketable Securities}_4 - \text{Debt balance}_4$$

If securities are sold during the planning period, the direct effect is to reduce the securities held at the end of the horizon. Taking account of possible sales in weeks 1 or 2, the ending security balance is expressed in general terms as

*Marketable Securities*₄ =

$$\sum_{t=0}^4 0.9981 SP_{t,5} + \sum_{t=0}^4 0.9962 SP_{t,6} - 0.9981 SS_{1,5} - 0.9981 SS_{2,5} - 0.9962 SS_{1,6} - 0.9962 SS_{2,6}$$

The summation terms reflect all purchases of securities maturing after period 4, including the initial endowments. The last four terms represent the sales in periods 1 and 2 of securities maturing beyond the horizon. However, in our particular problem, the only securities potentially held at the horizon would be those purchased in weeks 3 or 4, and the only sales we are considering are those in weeks 1 or 2. Consequently, the security sales would

not appear directly in the objective function. The indirect effects could be to increase the cash balance or reduce the debt balance when sales proceeds are used to repay debt, or to decrease the need for borrowing. Furthermore, if the funds are reinvested, sales in weeks 1 or 2 could increase the ending security balance by way of purchases in weeks 3 or 4.

17.4 SUMMARY

This chapter is an introduction to the use of linear programming for solving problems of working capital management. We have looked at a simple working capital problem to learn the principles. The problem we examined included decisions about cash balances, security purchases, and borrowing. We discussed how the model could be extended to handle debt repayment, management of AP, and security sales. The model would have to be expanded greatly to handle more realistic problems. But the problem we examined helps us to understand how to structure the larger problems.

PROBLEMS

1. Orgler Corporation: Optimization of Working Capital Decisions

At the end of December 2018, the CEO of the Orgler Corporation has asked you to develop the best plan for meeting the firm's working capital needs for the next two months. You are to develop an optimization model that will enable you to manage the cash account and manage the firm's short-term investments and financing over the next four weeks.

The objective of this set of decisions is to maximize the net value of the working capital accounts at the end of the four-week planning horizon. Net value is: Short-Term Assets–Short-Term Liabilities, where accrued interest is included in the values of assets and liabilities.

You have gathered the following information that relates to this problem:

Over the next four weeks, the firm's operating cash flows will alternate between inflows and outflows as the firm carries on its business. The following table shows these flows for the next four weeks. Assume that the flows for each week occur all on one day at the end of the week. Positive numbers mean inflows, and negative numbers mean outflows.

Week	1	2	3	4
Cash Flow	+12,000	–15,000	–4,000	+2,000

The firm has a portfolio of short-term securities that will mature during the planning horizon. The face value of the securities that will mature at the end (Friday) of each of the four weeks is shown as follows:

Week	1	2	3	4	5
Securities Maturing	1,000	500	2,000	3,500	0

Period 5 means “after the end of the planning horizon.” At the end of each of these weeks, these securities will mature for their face value. These are discount securities that have an annual yield of 8%. Their market values are calculated as the present value of the face value, discounted at the 8% annual rate. Assume each week has seven days.

If the firm has excess cash, it will be used either to repay debt or to invest in marketable securities. If marketable securities are purchased, they are purchased at the discounted price based on the 8% annual yield. You can purchase securities that will mature either one or two weeks after the purchase date. But no securities will be purchased to mature later than one week after the end of the four-week planning horizon. No securities can be sold prior to maturity.

The resources that are available to help the firm meet its objectives include its cash balance, its marketable securities, and its ability to borrow. The current cash balance is \$5,000. The marketable securities have already been explained. Its borrowing is based on a line of credit. The company can borrow up to \$10,000 at the end of any week. It can repay part or all of the debt at the end of any subsequent week. When debt is repaid, the repayment includes interest accrued on the debt. Over the four-week horizon, the firm does not have to repay any debt nor does it have to pay interest. Of course, if it does not pay interest, the interest accrues. The interest rate on the debt is 9% per year. Ignore the effects of income taxes.

The firm has a target desired cash balance of \$6,000.

Assignment

Set this problem up as a linear programming problem that can be solved with Solver. Solve the problem and explain the solution.



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