The Pendulum

Scientific, Historical, Philosophical & Educational Perspectives

Edited by
Michael R. Matthews, Colin F. Gauld
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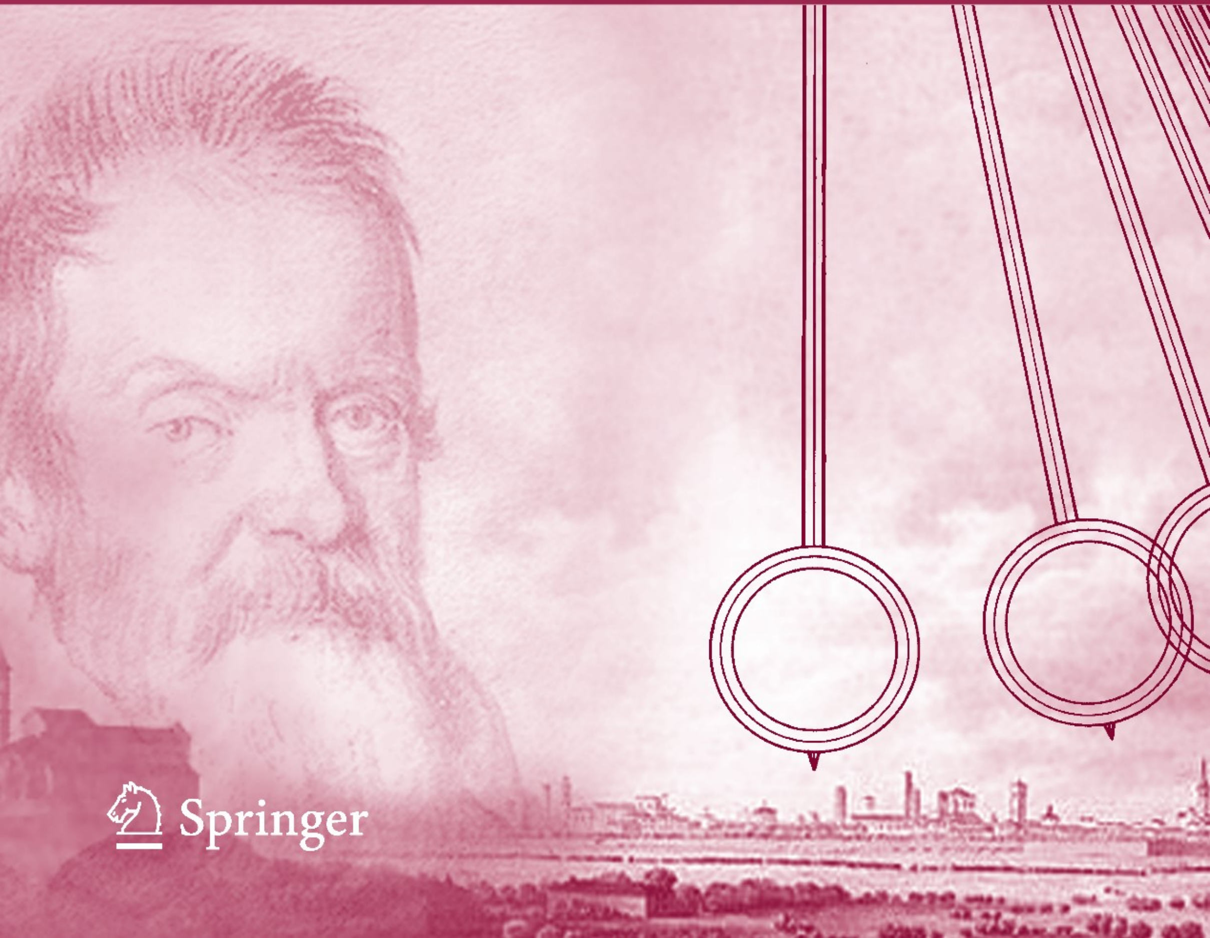

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The Pendulum: Its Place in Science, Culture and Pedagogy

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Abstract. The study and utilisation of pendulum motion has had immense scientific, cultural, horological, philosophical, and educational impact. The International Pendulum Project (IPP) is a collaborative research effort examining this impact, and demonstrating how historical studies of pendulum motion can assist teachers to improve science education by developing enriched curricular material, and by showing connections between pendulum studies and other parts of the school programme especially mathematics, social studies and music. The Project involves about forty researchers in sixteen countries plus a large number of participating school teachers.[1] The pendulum is a universal topic in university mechanics courses, high school science subjects, and elementary school programmes, thus an enriched approach to its study can result in deepened science literacy across the whole educational spectrum. Such literacy will be manifest in a better appreciation of the part played by science in the development of society and culture.

The Pendulum in Western Science

The pendulum has played a significant role in the development of Western science, culture and society. The pendulum was studied by Galileo, Huygens, Newton, Hooke and all the leading figures of seventeenth-century science. The pendulum was crucial for, among other things, establishing the collision laws, the conservation laws, the value of the acceleration due to gravity **g**, ascertaining the variation in **g** from equatorial to polar regions and hence discovering the oblate shape of the earth, and, perhaps most importantly, it provided the crucial evidence for Newton's synthesis of terrestrial and celestial mechanics.

The pendulum was important for the Galileo's new science, and it had a central place in Newton's physics, with the historian Richard Westfall remarking that 'without the pendulum, there would be no *Principia*' (Westfall 1990, p. 82). Subsequently the pendulum was at the core of classical mechanics as it developed through the eighteenth, nineteenth and early twentieth centuries, with the work of Stokes, Atwood and Eötvos being especially notable. Foucault's pendulum, as well as providing dynamical evidence for the rotation of the earth, also played a role in the popularisation of science in the late nineteenth and early twentieth centuries (Conlin 1999, Aczel 2003). Pendulum measurements enabled the shape of the

M.R. Matthews et al. (eds.), The Pendulum, 1–17.

earth to be determined, and were pivotal for the science of geodesy (Heiskanen and Vening Meinesz 1958).

The simple pendulum, when displaced through a small amplitude ($<10°$) oscillates with a natural frequency that depends solely upon its length. The pendulum manifests simple harmonic motion, whereby the restoring force on the bob (the tangential vector component of the pull of gravity) varies linearly with displacement. This is a marvellous physical system and is emblematic of a wide range of other such oscillating natural and perhaps social systems. The ideal, non-damped, simple pendulum is a conservative system in which the potential energy associated with the displacement is retained in the system when it swings. Galileo had an understanding of this, and demonstrated it so simply by showing how the pendulum, once released, retained its initial height, but did not exceed it. Low-level mathematical models can 'capture' the motion of simple pendulums. With more complicated pendulums – when the mass of the string, air disturbance, and fulcrum resistances are taken into account – more sophisticated mathematics and differential equations are required in order to 'capture' the behaviour. With double and triple pendulums chaotic motion can be induced which in turn requires still more sophisticated mathematics in order to be properly modelled. The whole pendulum system becomes more complex when the pendulum is driven by a varying torque at its point of suspension and the limits on its amplitude are removed. Then the pendulum's behaviour becomes more complex and consequently more resistant to mathematical capture. In recent decades mathematicians and physicists have jointly worked on this problem.[2]

The pendulum can support an extended and integrated pedagogical journey from elementary school to graduate programmes, in which the interplay of mathematics, technology, philosophy, culture, and experiment can be explored and appreciated. The dependence of science upon mathematics is beautifully illustrated at every stage of the pendulum story. The point can be made very early when students, through there own investigations, 'see' that period varies as length. With more sophisticated mathematical tools they can plot T against length (L) and, using simple curve fitting procedures, eventually see that if T is plotted against $\sqrt{(L)}$ a straight line is obtained. This leads to the mathematical relationship $T = k\sqrt{(L)}$. The square root of length is a mathematical construct rather than something commonly used in our everyday life and this exercise demonstrates the importance of mathematics in doing science.

The Pendulum and Timekeeping

The pendulum played more than a scientific role in the formation of the modern world. The pendulum was central to the horological revolution that was intimately tied to the scientific revolution. Huygens in 1673, following Galileo's epochal analysis of pendulum motion, utilised the pendulum in clockwork and so provided the world's first accurate measure of time (Yoder 1988). The accuracy of mechanical clocks went, in the space of a couple of decades, from plus or minus half-an-hour per day to a few seconds per day. This quantum increase in accuracy of timing enabled hitherto unimagined degrees of precision measurement in mechanics, navigation and astronomy. It ushered in the world of precision characteristic of the scientific revolution (Wise 1995). Time could then confidently be expressed as an independent variable in the investigation of nature.

Accurate time measurement was long seen as the solution to the problem of longitude determination which had vexed European maritime nations in their efforts to sail beyond Europe's shores. If an accurate and reliable clock was carried on voyages from London, Lisbon, Genoa, or any other port, then by comparing its time with local noon (as determined by noting the moment of an object's shortest shadow or, more precisely, by using optical instruments to determine when the sun passes the location's north-south meridian), the longitude of any place in the journey could be ascertained. As latitude could already be determined, this enabled the world to be mapped. In turn, this provided a firm base on which European trade and colonisation could proceed. The chances of being lost at sea were greatly decreased. This story has been enormously popularised by Dava Sobel (1995). By utilising her work, and that of others, students can realize that the *chronological method* rather than the *astronomical method* was the most practical way to solve the problem of locating the longitude of a point on earth. Using Galileo's approach of correlating the occultation of the moons of Jupiter, the timing of a planetary transit, or the timing of a solar or lunar eclipse, were all beset with difficulties of observation and were generally unreliable. John Harrison's marine chronometer, which followed on his extensive pendulum clock constructions, solved the longitude problem.[3]

The clock transformed social life and customs: patterns of daily life could be 'liberated' from natural chronology (the seasonally varying rising and setting of the sun) and subjected to artificial chronology; labour could be regulated by clockwork and, because time duration could be measured, there could be debate and struggle about the length of the working day and the wages that were due to agricultural and urban workers; timetables for stage and later train and ship transport could be enacted; the starting time for religious and cultural events could be specified; punctuality could become a virtue; and so on. The transition from 'natural' to 'artificial' hours was of great social and psychological consequence: technology, a human creation, begins to govern its creator.[4]

The clock did duty in philosophy. It was a metaphor for the new mechanical worldview that was challenging the entrenched Aristotelian, organic and teleological, view of the world that has sustained so much of European intellectual and religious life. In theology, the clock was appealed to in the influential argument from design for God's existence – if the world functions regularly like a clock, as Newton and the Newtonians maintained, then there must be a cosmic clockmaker.[5]

Horology

The link between the seventeenth century revolution in timekeeping, and developments in physics and methodology is oft-ignored. Despite there being scores of excellent books, and hundreds of research articles, on the technical, social and comparative history of timekeeping, there are few studies that connect the pendulum clock to Galileo and Huygens' discoveries of the physics of the pendulum, and even less studies that connect the pendulum clock to the Galilean revolution in scientific methodology. Galileo's law of isochronous motion, and hence his directions for using the pendulum in timekeeping, could not be accepted until he threw off the straight- jacket placed on science by the epistemological primacy given by Aristotelians to experience and the evidence of the senses. As long as scientific claims were judged by what could be seen, and as long as mathematics and physics were kept separate, then Galileo's pendulum claims could not be substantiated. Their substantiation required not just a new science, but a new way of judging scientific claims, a new methodology of science.

The Seconds Pendulum as a Universal Standard of Length

Huygens, in the process of elaborating his theory of pendulum motion and clockwork design argued in 1673 that the seconds pendulum could provide a new international standard of length (its length is effectively one modern metre). Undoubtedly this would have been a major contribution to simplifying the chaotic state of measurement existing in science and everyday life. He thought that this standard was dependent only upon the force of gravity, which he took to be constant all over the earth, and thus the length standard would not change with change of location. The standard was to be portable over space and time. Alas, Jean Richer's Cayenne voyage of 1672 suggested that the Paris seconds pendulum had to be very slightly shortened to beat seconds in tropical Cayenne (Matthews 2000, pp. 144–146). Still, if a specific latitude were agreed upon (Paris? London? Berlin? Madrid?) then Huygens' proposal would answer to the pressing need of a natural, invariant length unit. Once a subsidiary volume standard was created, by filling this volume with rain water, an international mass unit would also be created. How Huygens' 1673 proposal of the seconds pendulum as a universal length standard was related to the century later (1793) decree of the French Revolutionary Assembly establishing the metre length standard as one 40th million part of the circumference of the earth, is an intriguing story with rich methodological, social and political overtones.[6]

Philosophy of Science and Pendulum Studies

Philosophy of science should be informed by history of science. This is one of the important contributions of Thomas Kuhn's legacy to science studies. Historical study of the pendulum case shows how Galileo initiated the methodological transition which was to culminate in the Galilean-Newtonian Paradigm (GNP) which quickly came to characterise the Scientific Revolution, and the subsequent centuries of modern science. The Aristotelian epistemological taboo on manipulating nature, or experimenting, was lifted, as was the Aristotelian hesitancy to mix mathematics with science. The long entrenched conviction that only undisturbed or 'natural' states-of- affairs would reveal their essence was slowly replaced by the view that nature has to be simplified, that variables had to be controlled, that 'inputs' and 'outputs' needed to be measured and represented mathematically, and that scientific understanding was something other than grasping the essence or nature of things and ascertaining their final causes or teleological purposes.[7]

There are, admittedly, problems with 'historicised' philosophy of science. One is that history can be mined merely to find support for antecedently arrived at epistemological positions. History is then 'reconstructed' to suit whatever philosophical position is being advocated. Albert Schweitzer, in his monumental 1910 work on *The Quest of the Historical Jesus* that traced the history of Christian interpretation of Jesus, remarked that 'each successive epoch of theology found its own thoughts in Jesus .... But it was not only each epoch that found its reflection in Jesus; each individual created Him in accordance with his own character' (Schweitzer 1910, p. 4). Schweitzer could equally have been talking of Galileo. It is notorious that Galileo has been made out to be a shining example of the full range of epistemological positions: from rationalist, through empiricist and experimentalist, to positivist, and to methodological anarchist (Crombie 1981). The common thread is that the epistemology attributed to Galileo is usually the one favoured by the biographer or interpreter.

There is a chicken-and-egg problem with the Kuhnian stance. If philosophy of science emerges from history of science, how is the history first demarcated? Independently of a philosophical, normative, position what will count as the subject matter of history from which our methodological lesson is to be drawn? Do we draw lessons equally from Christian Science, National Socialist Science, Lysenkoism, Astrological Science, Islamic Science, Hindu Science, New-Age Science as well as classical mechanics, thermodynamics, and quantum mechanics?

The two standard ways around these problems are essentialist approaches on the one hand, and nominalist approaches on the other. For essentialists, history is ignored and science is characterised on *a priori* grounds – usually philosophical, political or sometimes religious. For nominalists, philosophy is ignored, and science is taken to be whatever people claiming to do science actually do. This option is popular among cultural historians of science and sociologists of science. It is better to steer a path between these two alternatives by focussing on an episode that all can agree upon as being good science, and then teasing out some methodological lessons from that. If the achievements of Galileo and Newton are not considered good, or at least, representative, science, then the very question of the epistemology of science loses its cogency. This is a version of the common 'paradigm case' argument in philosophy: to understand something, first find an exemplary instance of it, and examine its features and ramifications.[8]

Galileo's Methodological Revolution

The seventeenth century's analysis of pendulum motion is a particularly apt window through which to view the methodological heart of the scientific revolution. More particularly, the debate between the Aristotelian Guidobaldo del Monte and Galileo over the latter's pendular claims, represents, in microcosm, the larger methodological struggle between Aristotelianism and the new science. This struggle is about the legitimacy of idealisation in science, and the utilisation of mathematics in the construction and interpretation of experiments. Del Monte was a prominent mathematician, engineer and patron of Galileo (Renn et al. 2000, Matthews 2000, pp. 100–108). He kept indicating how the behaviour of pendulums contradicted Galileo's claims about them. Galileo kept maintaining that refined and ideal pendulums would behave according to his theory. Del Monte said that Galileo was a great mathematician, but a hopeless physicist. This is the methodological kernel of the scientific revolution. The development of pendular analyses by Huygens, and then Newton, beautifully illustrates the interplay between mathematics and experiment so characteristic of the emerging Galilean-Newtonian Paradigm. If students can be made familiar through their own investigations with some highlights of this nascent history of the pendulum, then they will have learnt something important about the origins and nature of modern science.

It is acknowledged that science has moved on, and that it can be claimed that understanding seventeenth century debates about the pendulum is irrelevant to understanding modern techno-industrial science and its methodology. This is a complex issue but, in brief, understanding origins, and development, is important for understanding and judging the present. This is true in just about all spheres – political, religious, social and personal – and no less so in conceptual matters.[9] Further modern science has not so outgrown its methodological roots as to make irrelevant an examination of central seventeenth century epistemological debates. Even if it could be shown that modern science is methodologically different from its origins, nevertheless understanding where modern science has come from and, consequently, what occasioned the change, is still important.

In education it is sensible to begin with simple or idealised cases. Presenting students with the full story – the truth, the whole truth, and nothing but the truth – is rarely a good idea. Concentrating on just some key aspects of a topic, be it in history, economics, biology, or what ever, makes pedagogical sense. Galileo's debate with del Monte debate does capture in comprehensible form some of the core issues of epistemology – the distinction between observation and experiment, the relationship of evidence to knowledge claims, the role of theory in guiding experiment, and so on – and this gives an educational justification for its presentation. Provided students are made aware that the complete picture, or the modern picture, might be more complex, and provided they are encouraged to examine how science may have changed, then dealing with the seventeenth century is educationally and philosophically justified. These claims conform to the 'Genetic Method' in pedagogy; a method that consciously endeavours to have students re-tread the intellectual and experimental path that science has moved along from its origins.

'Big Picture' History of Science

The pendulum story fits into the 'big picture' or 'grand narrative' genre of history of science: it deals with the interrelatedness of timekeeping, pendulum science, philosophy and social forces; and it endeavours to draw methodological lessons from all this. Big Pictures in the history of science need not be painted with broad brush strokes. The IPP endeavours to compose a big picture but does so with fairly fine brushes. The IPP deals with both *internal* matters concerning the development and refinement of scientific concepts; and *external* matters such as the social and cultural contexts in which science develops. This distinction needs detailed attention, and ultimately it is somewhat conventional. For instance, a change in epistemology was fundamental to Galileo's achievements in understanding pendulum motion. Is then epistemology internal or external to science? Huygens' recognition of the isochronous nature of cycloidal motion rested upon the new geometrical analysis of the cycloid curve. Is then mathematics internal or external to science? Neither Galileo's nor Huygens' proposals for utilising the pendulum in timekeeping could be experimentally tested until technological advances in metallurgy, gear-cutting and escapement design were made. Is then technology internal or external to science? Once science is recognised as part of the intellectual culture of a society then the separation of 'internal' and 'external' elements borders on being conventional.

That the distinction is blurred, does not mean that it cannot be made in some form. It is clear that the longitude problem played a major role in the development of clockwork. Solving longitude was one of the major preoccupations of European nations from the fifteenth to the eighteenth centuries. King's ransoms were offered for its solution. Despite all the external financial and political pressure, a solution had to wait on scientific, methodological and mathematical progress. The world was the judge of putative solutions, not political or ideological interests. This is an important point to be appreciated at a time when many maintain that science simply dances to the tune of the last patron who paid the fiddler. In science, paying the fiddler and getting a good dance, are two different things.

The Pendulum and Piagetian Research

The pendulum entered into educational research and cognitive psychology with the publication in 1958 of the English translation of Bärbel Inhelder and Jean Piaget's *The Growth of Logical Thinking from Childhood to Adolescence* (Inhelder and Piaget 1958). Chapter Four of the book describes the pendulum tasks that Piaget and Inhelder gave to children to ascertain the extent to which they could isolate and manipulate potential variables (length, amplitude, weight, impetus) that affected the periodicity of the pendulum. The chapter is titled 'Operations of Exclusion of Variables' because only one of the four potential variables impact upon the duration of swing. Performing the task of isolating and uncoupling (controlling) the variables was seen as a window onto the child's cognitive structures or capacities and their developmental sequencing. The tasks subsequently became a commonplace in diagnostic testing, being labelled 'Piagetian Reasoning Tasks' (PRT); as they involved extensive engagement with the child, the test procedure was called '*Méthode Clinique*' (or, the Clinical Method). Successful completion of the tasks was seen as indicative of the change from concrete to formal operational thinking. The subheadings of the chapter indicate the cognitive sequencing:

Stage I	Indifferentiation between the subject's own actions and the motion of the pendulum.
Stage II	Appearance of serial ordering and correspondence, but without separation of variables.
Stage IIIa	Possible but not spontaneous separation of variables.
Stage IIIb	The separation of variables and the exclusion of inoperant links.

The pendulum did for reasoning and formal thinking tests what it centuries earlier had done for timekeeping. Subsequently Piaget's cognitive theory, and his test protocols, have been extensively scrutinised.[10] Contributors to the IPP appraise this research tradition, commenting on its strengths, weaknesses and seeing how pendulum investigations might still be used to assess higher order mental capacities and children's ability to reason proportionally, to control variables, to make inferences, to draw conclusions about the truth of hypotheses given certain evidence – in brief, to think scientifically.

Enriched Scientific Literacy

Science literacy should be interpreted in a broad and generous sense, so that literacy is seen as involving an understanding and appreciation the nature of science, including its history, methodology and interrelations with culture. This is a demanding objective, but given the centrality of science to the development of society, culture and self-understanding, it is one that should be pursued by educationalists. In the USA, the *National Science Education Standards* (NRC 1996), and AAAS's reports *Project 2061* (Rutherford and Ahlgren 1990) and *The Liberal Art of Science* (AAAS 1990) all endorse this wider, liberal idea of scientific literacy. They recognise that:

> Science courses should place science in its historical perspective. Liberally educated students – the science major and the non-major alike – should complete their science courses with an appreciation

of science as part of an intellectual, social, and cultural tradition .... Science courses must convey these aspects of science by stressing its ethical, social, economic, and political dimensions. (AAAS 1990, p. 24)

This view is shared by the National Curriculum in the UK, a number of provincial science curricula in Canada, the Norwegian science curriculum, the Danish science curriculum, and the New South Wales state syllabus in Australia. Most science programmes aspire to having students know more than just a certain amount of science content, and having a certain level of competence in scientific method and scientific thinking. Most programmes want students to have some sense of the 'big picture' of science: its history, philosophy and relationship to social ideologies, institutions and practices (McComas and Olson 1998). In most countries, science education has dual goals: promoting learning *of* science, and also learning *about* science. Or, as it has been stated, science education has both *disciplinary* and *cultural* goals (Gauld 1977). Teaching the history and philosophy of pendulum motion is an ideal vehicle for realising some of these more ambitious aspirations for scientific literacy.

Teaching the Physics of the Pendulum and Its History

The pendulum is a remarkably simple device and has long been part of the physics curriculum, a fact well documented in the IPP bibliography of pendulum articles that have appeared over the past fifty years in major science education journals (Gauld 2004). In its basic form – a string supporting a heavy bob – the pendulum demonstrates clearly the interchange between gravitational potential energy and kinetic energy and, with appropriate measuring instruments, the constancy of the total energy throughout its motion. Teachers have used the simple pendulum, swinging through small angles, to teach the skills of measurement and graphical techniques for deriving the relationship between dependent (in this case, period) and independent variables (length of the string).

More complex types of pendulums (such as the physical, spring-mass, torsional and Wilberforce pendulums) can be used to demonstrate dramatically a wide range of physical phenomena and provide a context in which students can become acquainted with the process of mathematical modelling. In the classroom pendulum motion provides a model for many everyday oscillatory phenomena such as walking and the movement of a child's swing.

At the tertiary level there has been renewed interest in the pendulum to demonstrate chaotic behaviour. For these investigations the pendulum amplitude is unrestricted and the point of suspension is vibrated at varying amplitudes and frequencies. By removing the requirement that the amplitude be small the behaviour of the pendulum as a non-linear oscillator can clearly be seen.

The history of the uses of the pendulum in the study of kinematics and dynamics contains almost everything required to teach the fundamentals of kinematics and dynamics. The following is a brief history of the pendulum and a list of suggestions for the physics classroom.[11] Clearly, teachers who would use a historical approach like this one must have more than a cursory acquaintance with the history of science.

The inclined plane and the pendulum were crucial in the development of Galileo's kinematics and Newton's dynamics in the seventeenth century. In many of the key problems of Galileo these simple devices were connected and used in creative ways to study motion, first without considering the forces involved (kinematics), and later investigate the forces that caused this motion (dynamics). Galileo 'diluted gravity' and extrapolated to free fall in an attempt to understand what Aristotle called 'natural motion'. Studying the pendulum, Galileo thought that an arc of a circle represented the 'least time' path of an object in a vertical plane.

Huygens went beyond Galileo and used the pendulum to find the expression for 'centrifugal' force on a body moving in a circle, as well as the modern formula for the period of a pendulum for small angles. He was the first to find the modern formula, namely that $T = 2\pi\sqrt{(L/g)}$ for the simple pendulum and also the first to write the mathematical statement for 'centrifugal' acceleration as $a = v^2/R$. He used long and heavy pendula to determine the value of gravitational acceleration. He later correlated latitude and the local value of **g** to test his ideas. Huygens was also the first show (geometrically) that the path along which a pendulum would show isochronous motion was a cycloid and not the arc of a circle. From this background we can generate many experiments and problems that cover all those found in textbooks and beyond and in more interesting ways (Stinner and Metz 2003).

Huygens constructed the first pendulum clock that kept fairly accurate time. However, he failed to realize that the cycloid also represented the 'least time' path of descent of a particle in a vertical plane. It was left to Newton, Leibniz and Johannes Bernoulli to lay the foundation of a new branch of the calculus, in order to solve problems such as the brachistochrone, or 'least time' of descent between two points in a vertical plane. In the capable hands of Euler their approach then became a powerful method to solve minimum and maximum problems, called 'variational calculus'. Contemporary teachers can build a simple apparatus using two wires, one straight and the other roughly shaped as a cycloid, with two steel beads sliding down the wires. The bead travelling the longest path (the cycloid) takes the shortest time! This an example of a discrepant event that is sure to generate much discussion.

The work of Robert Hooke, a contemporary of Newton, should be included in this historical presentation. Textbooks mention Hooke only in connection with his law of springs. Hooke has been called 'the British Leonardo'. He was a polymath: scientist, inventor and arguably the greatest experimenter of the seventeenth century. He was the curator of the Royal Society and sometime friend of Newton.[12] He used his law ($F = -kx$) to show that simple harmonic motion (SHM), like that of the pendulum, or an oscillating mass attached to a spring, arises when this law holds. His scientific battles with Newton were legendary. When Newton became the president of the Royal Society in 1705, he expunged all vestiges of Hooke from the Society. We identify Robert Hooke by the famous drawing he made in his revolutionary *Micrographia* that he published at the age of 30 years. Discussing the confrontation between Newton and Hooke, students quickly come to realize that science is very much a human endeavor, and that scientists embody the full range of human foibles.

Students can be asked the question: 'What experiments did Newton perform that suggested and confirmed his three laws of motion?' Textbooks seldom discuss the experimental work of Newton beyond his optical experiments. It is not generally known that in his study of dynamics Newton used pendula to test his second and third laws of motion, as well as centripetal acceleration. Inertia, or his first law of motion, was seen as the consequence of a thought experiment that could not be tested directly. Newton went beyond Galileo's idea of inertia as 'the circumnavigation of an object on a perfectly smooth Earth' to the idea of 'straight line motion with a constant speed in deep space when there are no forces acting on the object'. His second law, $F = ma$, can be applied to a pendulum to demonstrate that if Hooke's law holds (restoring force is proportional to the displacement of the mass of the pendulum from the vertical) then we have simple harmonic motion. This part of the story is often told in textbooks, but Newton's experiments to test his third law is seldom mentioned.

The third law, 'action is equal to reaction', was demonstrated by Newton using two long (3–4m) pendula and having them collide. He used a result of Galileo (that the speed of a pendulum at its lowest point is proportional to the chord of its arc) and applied it to the collision by comparing the quantities mass times chord length, before and after collision. This is one of the few detailed accounts found in the *Principia* that high school students can read and understand. Students soon see that the third law is really equivalent to the principle of the conservation of linear momentum (Gauld 1993, 1998, 1999). Corollary III to his Laws of Motion states that 'The quantity of motion, which is obtained by taking the sum of the motions directed towards the same parts, and the difference of those directed to contrary parts, suffers no change from the action of bodies among themselves' (Newton 1729/1934, p. 17). For Newton this concept of 'quantity motion' represents what we call momentum and this corollary states what we call the law of conservation of momentum (Cohen 2002). Finally, Newton also used long bifilar pendula to test the equivalence of inertial and gravitational mass and came to the conclusion that to a 'thousandth part of the whole' they were equivalent. It is possible to replicate the experiments of Newton, using long pendula consisting of large wooden spheres, or bowling balls, suspended by wires.

The pendulum also played an important role in the next two centuries. Benjamin Robins in 1742 adapted the pendulum in his ballistic device to measure the muzzle velocity of bullets. Count Rumford, famous as the debunker of the caloric theory, in 1781 adapted Robins' method and patented it. This method of finding the muzzle velocity of bullets was used until the recent effective application of high speed photography. Here we have an experiment that can be replicated using a ‘Gauss gun’ that propels ball bearings at low speeds.

Later, in 1790, George Atwood used the pendulum incorporated in his famous machine, named after him, as a research apparatus. One of the experiments he performed was to test Newton’s second law of motion. Atwood’s machine is forever enshrined in physics textbooks problems, but it is seldom mentioned that Atwood’s approach was the first direct ‘test’ of Newton’s second law of motion. The pendulum in this experiment is part of the apparatus. A simple pulley can be used with two dissimilar weights and a pendulum to calculate the value of acceleration due to gravity.

In 1851 Jean Foucault designed a very long and heavy pendulum to demonstrate for the first time directly that the Earth revolves around its axis (Aczel 2003). Teachers can offer a good discussion of this dramatic and celebrated demonstration. Replication in the classroom is difficult but many science museums and centres have a Foucault pendulum demonstration.

Included in a rich history of the pendulum should be Hermann von Helmholtz’s studies of resonance. Although the original studies were made for sound, Helmholtz found an analogue for his colleagues Bunsen and Kirchhoff to explain the dark absorption lines of the solar spectrum. The important phenomenon of resonance can be dramatically demonstrated by using coupled pendula and, at the same time resonance demonstrations made using tuning forks imbedded in resonance boxes.

Teachers can discuss what may be the last of the great classical experiments to use a pendulum at the turn of the early twentieth century, namely the Eötvos experiment, to test the ratio of inertial and gravitational masses. This experiment is important even today and is connected with Einstein’s General Theory of Gravity and with a recent hypothesis of a ‘fifth force’ in nature.

Recently the pendulum has obtained a high profile in the demonstration of chaos theory. The study of the harmonic oscillator in all its manifestations in dynamics, electricity, and even atomic theory, can be traced back to the properties of the pendulum.

Curriculum Considerations

The educational usefulness of the IPP can be gauged from looking at the recently adopted US National Science Education Standards (NRC 1996). The *Standards* adopt a liberal or expansive view of scientific literacy saying that it ‘includes understanding the nature of science, the scientific enterprise, and the role of science in society and personal life’ (NRC 1996, p. 21). The *Standards* also devote two pages to the pendulum (pp. 146–147): however there is no mention of the history, philosophy, or cultural impact of pendulum motion studies; there is no mention of the pendulum’s connection with timekeeping; no mention of the longitude problem; and in the suggested assessment exercise, the obvious opportunity to connect standards of length with standards of time, is not taken, rather students are asked to construct a pendulum that makes six swings in 15 seconds (Matthews 1998). The *Standards* document was reviewed by tens of thousands of teachers and educators, and putatively represents current best practice in science education. It is clear that a little historical and philosophical knowledge about the pendulum could have transformed the treatment of the subject in the *Standards* and would have encouraged teachers to realise the expansive goals of the document through their treatment of the pendulum. This would have resulted in a much richer and more meaningful science education for US students. That this historical and philosophical knowledge is not manifest in the *Standards*, indicates the amount of work that needs to be done in having science educators become more familiar with the history and philosophy of the subject they teach.

The same point is recognised in the joint study undertaken by the Biological Sciences Curriculum Study and the Social Science Education Consortium when they say that the first barrier to school students understanding anything of the history and nature of science and technology is 'the preparation of teachers is inadequate' (Bybee et al. 1992, p. xiii). The problem is not confined to the US: it is an international problem. Hopefully the research publications and classroom materials generated by the IPP will do something to ameliorate this problem.

Liberal Education and Pendulum Teaching

The contextual, intellectualist, cross-disciplinary proposals advanced by the IPP find their natural home in the liberal education tradition, whose core commitment is that education is concerned with the development of a range of knowledge and a depth of understanding, and with the cultivation of intellectual and moral virtues.[13] The intellectual virtues certainly include developing capacities for clear, logical and critical thought. These liberal goals are contrasted with goals such as professional training, job preparation, promotion of self-esteem, social engineering, entertainment, or countless other putative purposes of schooling that are enunciated by politicians and administrators. The AAAS well states the matter when it says:

> Ideally, a liberal education produces persons who are open-minded and free of provincialism, dogma, preconception, and ideology; conscious of their opinions and judgments; reflective of their actions; and aware of their place in the social and natural worlds. (AAAS 1990, p. xi)

And then adds: 'The experience of learning science as a liberal art must be extended to all young people so that they can discover the sheer pleasure and intellectual satisfaction of understanding science' (ibid). On this liberal view, science education is seen as contributing to the overall education of students, and thus considerations about aims and purposes of education constrain decisions about science education. The development of an educated person is the *telos* of school science teaching; this is the 'prize' that teachers' eyes need to be kept on.

Participants in the IPP believe that the pendulum provides an accessible point of entry, or door, for students to learn important components of scientific knowledge, key features of scientific method, and important aspects of the interplay between science and its social and cultural context. A good pendulum-based, or pendulum-assisted, course allows students to learn:

(i) Basic scientific knowledge, such as the laws of fall, laws of motion, collision laws, and the laws of conservation of momentum and energy.

(ii) Essential features of scientific inquiry, such as observation, measurement, data collection, control of variables, experimentation, idealisation, and the use of various mathematical representations.

(iii) Important aspects of how science interrelates with society, culture and technology, as manifest in the use of the pendulum in timekeeping, navigation, length standards, and so on.

Further, the same door is available at all stages of a student's education, from elementary school to graduate studies in physics. What is 'behind the door' will change with teacher sophistication, student preparedness and curricular demands. It is hoped that the IPP research publications appearing in two special issues of *Science & Education* (Vol. 13, Nos. 4–5, 7–8), an associated anthology (Matthews, Gauld and Stinner 2005), and the pedagogical materials that will follow, will assist teachers to convey this richer view of science to students, and consequently deepen students appreciation of science and its impact on human thought and well being.

Notes

[1] The Project is coordinated by Michael Matthews at the University of New South Wales. The book *Time for Science Education: How Teaching the History and Philosophy of Pendulum Motion can Contribute to Science* Literacy (Matthews 2000) provides an overview of some of the scholarly and pedagogical matters with which the Project is concerned. IPP details can be seen at www.arts.unsw.edu.au/pendulum/ .

[2] Many books deal with the physics of the pendulum. Specifically: Tavel (2002, pp. 219–231) deals with the progressive elaboration of the pendulum from simple to chaotic; Barger and Olsson (1973, pp. 63–75) work through the mathematics of Lagrangian formulations of pendulum motion; Rogers (1960), a text written for the PSSC Physics Course, has an excellent chapter on the pendulum; Pólya (1977) deals with Galileo's analysis (pp. 82–105) and gives an illuminating derivation of the central period/length equation (pp. 210–224).

[3] Dava Sobel has given the Longitude Problem enormous exposure (Sobel 1995). Other more detailed and wide-ranging treatments are in Andrewes (1998), Gould (1923) and Howse (1980).

[4] Many books deal with the social and cultural history of timekeeping, among them are: Cipolla (1967), Landes (1983), Macey (1980) and Rossum (1996).

[5] Macey 1980, Pt.II is a nice introduction to the utilisation of the clock in eighteenth century philosophy and theology.

[6] Accounts of the development of the standard metre can be found in Alder (1995, 2002), Berriman (1953, chap. XI), Heilbron (1989), Kline (1988, chap. 9), and Kula (1986, chaps. 21–23). Some of the methodological and political story is told in Matthews (2000, pp.141–150).

[7] Some especially insightful discussions of Galileo's methodological revolution are McMullin (1978, 1990), Machamer (1998), and Mittelstrass (1972).

[8] For an exemplary discussion of this paradigm case alternative to essentialism and nominalism, see Suchting (1995).
[9] Ernst Mayr, in the opening pages of his *The Growth of Biological Thought*, commends historical study to scientists in these terms:

I feel that the study of the history of a field is the best way of acquiring an understanding of its concepts. Only by going over the hard way by which these concepts were worked out – by learning all the earlier wrong assumptions that had to be refuted one by one, in other words by learning all past mistakes – can one hope to acquire a really thorough and sound understanding. In science one learns not only by one's own mistakes but by the history of the mistakes of others. (Mayr 1982, p. 20)

[10] Some contributions are: Bond and Bunting (1995), Kuhn and Brannock (1977), Siegler, Liebert, and Liebert, (1973), Shayer and Adey (1981) and Sommerville (1974).
[11] For a more complete treatment of the use of the pendulum in physics programmes, see Stinner and Metz (2003).
[12] For the life and achievements of Hooke see Drake (1996), Jardine (2003) and contributions to Hunter and Schaffer (1989).
[13] Some of the more prominent advocates of liberal education have been: Mortimer Adler (Adler 1939/1988), G.H. Bantock (Bantock 1981), Paul Hirst (Hirst 1974), Richard McKeon (McKeon 1994), John Henry Newman (Tristram 1952), Richard Peters (Peters 1966) and Israel Scheffler (Scheffler 1973). See Kimball (1986), also contributions to Orrill (1995), and to Schneider and Shoenberg (1998). Elliot Eisner, in his review of curriculum ideologies, calls this educational tradition 'rational humanism' (Eisner 1992). There are connections with the Germanic educational idea of *Bildung* (Bauer 2003).

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The Pendulum in the 21st Century-Relic or Trendsetter

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Abstract. When identifying instruments that have had great influence on the history of physics, none comes to mind more quickly than the pendulum. Though first treated scientifically by Galileo in the 16th century, and in some respects nearly 'dead' by the middle of the 20th century; the pendulum experienced 'rebirth' by becoming an archetype of chaos. With the resulting acclaim for its surprising behavior at large amplitudes, one might expect that there would already be widespread interest in another of its significant nonlinearities. Such is not the case, however, and the complex motions of small amplitude physical pendula are barely known. The present paper shows that a simply-constructed metallic rod pendulum is capable of demonstrating rich physics in a largely unstudied area.

1. Introduction

Some students of physics have been described by the expression, "a formula looking for a problem". In fact, it appears that nearly all of us have difficulty rising above this tendency. In the present context, it is manifest through our association with a sacrosanct equation of motion – the simple harmonic oscillator (SHO) with viscous damping.

It took the science of chaos to recognize the futility, for some applications, of the approximation $\sin\theta \approx \theta$, as applied to the equation of motion for the pendulum. Here, θ is the angular displacement of the pendulum and, if the approximation were valid, SHO motion would result because the approximation is equivalent to the potential function being written as $U \propto \theta^2/2$. But in fact, for the pendulum, $U \propto 1 - \cos\theta$, at least as a first-approximation. To describe chaotic behavior, $\cos\theta \approx 1 - \theta^2/2$ is never acceptable.

Later in this paper we will provide evidence for the author's conviction that a real pendulum is not even properly described by $U \propto 1 - \cos\theta$. The difficulties arise from considerations of damping. It will be shown that the assumption of viscous damping is untenable, in any serious attempt to understand the physics; because internal friction is the most important source of energy loss. Moreover, internal friction implies that the potential well is complex, assuming that the concept of a potential function has any meaning to begin with. As with a non-Hookean

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spring, the atoms do not remain in fixed lattice positions. Consequently, to assume a potential well requires that the well be 'modulated by fine structure'. The problem is made difficult by the fact that the fine structure is not even constant in time. Because it is associated with hysteresis that ultimately involves temperature, through diffusion processes, pendulum motion is extremely complex.

It might be supposed that a very sophisticated instrument would be necessary to see such phenomena. This paper has been written to demonstrate that this is not the case at all. Rather, it will be seen that many of the complex features can be observed with a pendulum and sensor that are easily built. In fact, some features can be studied in the absence of an electronics sensor altogether – simply through visual observation.

Properties of the pendulum, as just described, would probably not be surprising if the physics community had taken note (as did the engineering community) of the work of two of our number in the 1920's. In their studies of the creep of alloys under stress, Portevin and Le Chatelier (1923) observed strains that were complex– ones that don't even conform to the fundamental theorem of calculus, because of 'jumps'.

2. Background

2.1. CREEP

When a wire is subjected to a constant tensile stress (force per unit area), the strain (fractional change in length) evolves through three phases of creep: (i) primary, (ii) secondary, and (iii) tertiary. An example of the first two phases of creep is shown in Figure 1.

In this case, the creep occurred in the coiled spring of the instrument, of LaCoste 'zero-length' type. Stress and strain of a coiled spring is more difficult to describe than the tensile elongation of a wire, but all cases of anelasticity demonstrate the creep phases indicated.

In the primary stage of creep, the sample is deformed by processes involving organization of defects in the crystalline structure. Influence of the disordering mechanisms is progressively reduced as the sample undergoes work hardening (such as pinning of dislocations). Work hardening would result in a purely exponential creep in the absence of thermal effects, which strive to undo the hardening via diffusion processes. At zero Kelvin the creep would eventually cease, if described by a single time constant. In the secondary stage a balance between work hardening and thermal softening is attained, in which the strain versus time has converted from exponential to linear.

In Figure 1 the creep resulted after rebalancing the instrument following a severe accidental disturbance. Whether a compound pendulum or, as in this case, a mass-spring oscillator in the form of a vertical seismometer; these long-period instruments always creep during initial operation.

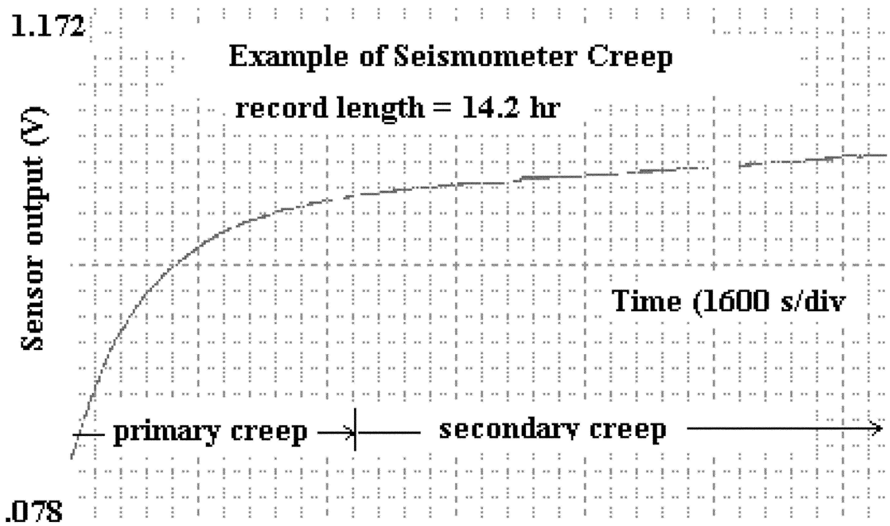

Figure 1. Illustration of creep in a vertical seismometer.

The amount of creep in Figure 1 deserves mention. In the indicated 14.2 h, the mass of the seismometer (11 kg of lead) moved a vertical distance of only 1/4 mm, as can be ascertained from the ordinate axis, using the sensor calibration constant of 2000 V/m.

2.2. DAMPING COMPLEXITY

A variety of studies by the author have suggested that physical pendulum motion is never simple. It is rare, for example, to observe a truly symmetric decay; i.e., one in which the decline of the amplitude of turning points on one side of equilibrium matches the decline of those on the opposite side of equilibrium. Furthermore, neither set of turning points is truly pure exponential. Creep can cause asymmetry in the decay pattern through a change in equilibrium position with time. Even if the equilibrium position is not drifting noticeably during decay, there can still be non-exponential behavior. The decrement from one cycle to the next can vary unpredictably because of anelasticity. [Note: the prefix ‘an’ means ‘other than’, and the word anelastic is not equal to inelastic = not elastic.]

The observations noted above result from measurements with the highly sensitive and linear sensors invented by Peters (1993). Departure from ideal SHO might not be as observable with sensors of lower quality. However, even most of them should be capable of demonstrating a property of long-period motions that is not widely known. Specifically, consider the free-decay of a viscous damped oscillator as the period of the motion is altered. For the damping of the motion to be as commonly taught, the decrement of the decay δ must be proportional to the period.

2.3. DECAY DECREMENT δ

The solution to the equation of motion for a viscous damped harmonic oscillator contains an exponential term that multiplies the harmonic term. It provides for the decay of amplitude with time, which can be expressed in terms of the *n*th full-cycle turning point as

$$A_n = A_0 e^{-n\beta T}, \qquad n = 0, 1, 2, \ldots \tag{1}$$

where A_0 is the starting amplitude ($n = 0$) and T is the period. Although the parameter β is frequently called the decay 'constant', it is not a true constant, instead being a function of frequency.

A convenient measure of the free-decay of any oscillator is the fractional change in amplitude that occurs in one cycle of the motion; i.e.,

$$\delta = \frac{A_n - A_{n+1}}{A_n} = \beta T, \qquad \beta T \ll 1. \tag{2}$$

In the case of exponential decay, this fractional change in amplitude is called the logarithmic decrement because $\delta = \beta T = \ln(A_n/A_{n+1})$. For exponential decay, δ is independent of amplitude. For nonlinear damping, δ can increase with time (Coulomb; i.e., sliding friction) or decrease with time (amplitude dependent dissipation, such as fluid friction with quadratic velocity dependence).

Studies by the author over the last fourteen years have shown that the functional dependence on period of the log decrement is not $\delta \propto T$ as commonly assumed, but usually closer to $\delta \propto T^2$ (see, for example, Peters & Pritchett 1997). In other words, $\beta \propto T$ rather than being constant. A previous paper by the author showed that this requires a nonlinear damping term in the equation of motion (Peters 2001). It was further shown that this term could be understood in terms of secondary creep. Whereas previous pendula used to demonstrate the indicated behavior were somewhat sophisticated, the rod pendulum described in the present paper is easy to fabricate. It also shows, at least in the case of a pendulum built from solder, the importance of physical properties, such as malleability. The effect is best illustrated by building two pendula, one of brass (low damping) and the other of solder (high damping) and comparing their free decays. It should be noted that one doesn't even need a sensor in order to see huge differences between the two pendula. As the center of mass of the solder pendulum begins to approach the axis of rotation, the damping increases in dramatic fashion. Also dramatic in this limit is the obvious importance of structural integrity, as one tries to realize long periods of the motion. In the case of solder it is difficult to configure the rod for sustained oscillations with a period in excess of 6 s.

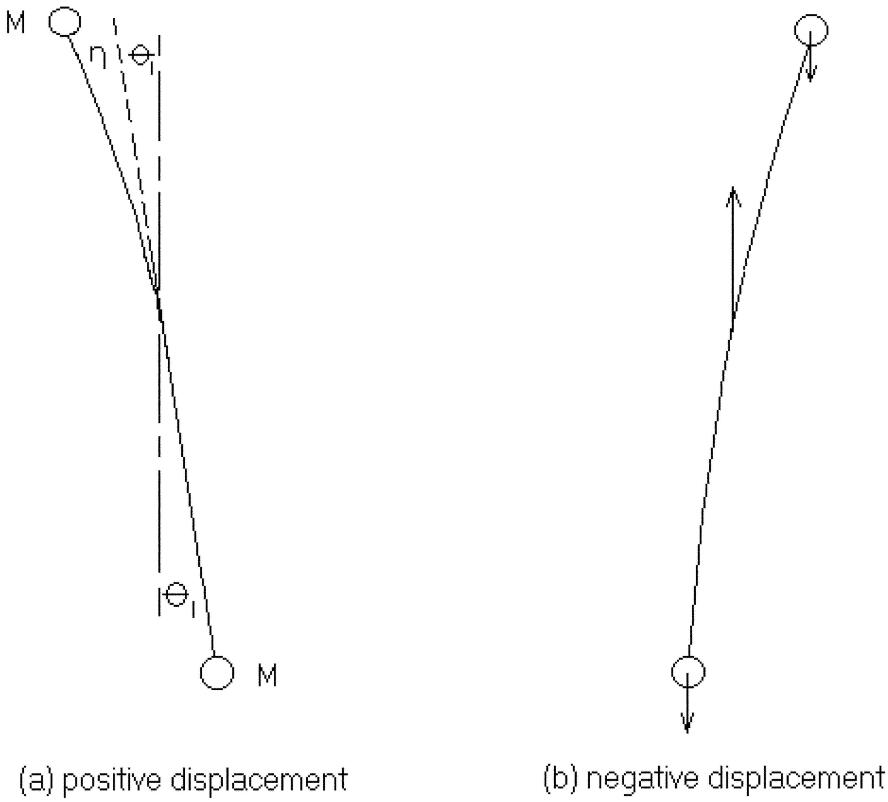

Figure 2. Point mass model of a low-frequency idealized compound pendulum.

3. Idealized Low Frequency Pendulum

Complex pendulum behavior is most readily demonstrated by lengthening the period of the motion. Means to achieve lower frequency can be understood from a consideration of Figure 2.

The mass located above the axis is responsible for a 'destoring' torque, whereas the larger torque from the lower mass provides a 'restoring' torque. As the two torques approach the same magnitude (achieved, for example, by moving the upper mass farther from the axis), the period lengthens dramatically. Period lengthening causes creep to become visible, since the sensitivity to both internal and external influence is proportional to the square of the period.

Assume that the masses are separated a distance of $2L$ with the center of mass located a small distance ΔL below the geometric center of the instrument. The equation of motion is readily obtained from Newton's 2nd law using the restoring and destoring torques just mentioned. The result is

$$\ddot{\theta}_1 + \frac{g}{2L}\left(1 + \frac{\Delta L}{L}\right)\sin\theta_1 - \frac{g}{2L}\left(1 - \frac{\Delta L}{L}\right)\sin\theta_2 = 0 \tag{3}$$

where

$$\theta_2 = \theta_1 + \eta, \quad \eta = c[\theta_1 \cos\delta_p - (\dot{\theta}_1/\omega)\sin\delta_p], \quad \omega = \left(\frac{g\Delta L}{L^2}\right)^{1/2}. \tag{4}$$

The constant c is related to the elastic constants of the material of the support structure, and the constant δ_p is cause for damping of the motion (dissipation of the energy). In terms of the log decrement δ

$$\delta_p = \frac{4L}{g}\frac{\omega^2}{c}\frac{\delta}{2\pi} \tag{5}$$

and the quality factor Q of the oscillator, which is defined as $2\pi|\Delta E|/E$ (with E being the energy of oscillation and ΔE the energy loss to friction per cycle) is given by

$$Q = \frac{2L}{gc\delta_p}\omega^2. \tag{6}$$

Because L, g, c and δ_p are all true constants, we see from Equation (6) that $Q \propto \omega^2$ as required to describe the 'universal' form of internal friction (and not $Q \propto \omega$ of the common theory).

The expressions, and particularly the angular frequency indicated in Equation (4), are valid in the limit of small amplitudes, where the sine of the angle is very nearly the same as the angle itself in radians.

It should be noted that the above equations describe the motion of a classical long-period pendulum; i.e., one in which the decay is exponential. Like the equations generated by all other models known to the author, they fail to describe the deviations from exponential that derive from complexities of mesoscale friction (Peters 2003). Later examples of rod pendulum decay, especially the case of the solder pendulum, illustrate such complexities.

4. The Pendulum

The pendulum is pictured in Figure 3. The rod is made from an ideally straight piece of malleable metal of uniform circular cross section, whose diameter is about 3 mm and whose length is about 50 cm.

For the picture, the solder pendulum has been mounted on the knife edge support which is a '5 g' aluminum mass of general education laboratory variety that has been secured to the shoulder of a short horizontal brass dowel using an 8/32 screw. The shoulder was formed with a hacksaw and then drilled and tapped for the screw. The dowel is held on the other end by a clamp whose position can be altered on the vertical rod which is supported by a sawed-off tripod base. The components rest on a rectangular wooden board. At the bottom of the rod is a thin copper plate,

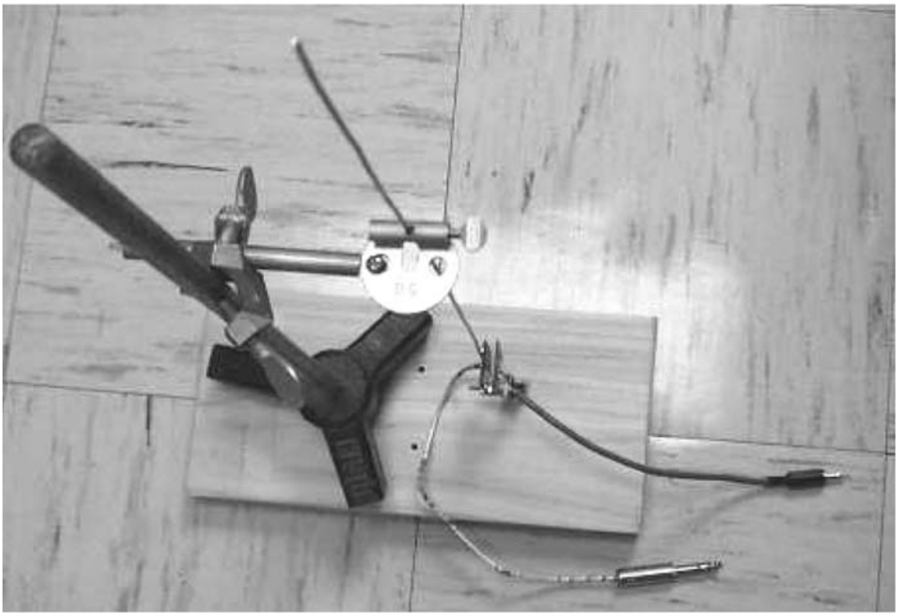

Figure 3. Top view of the pendulum.

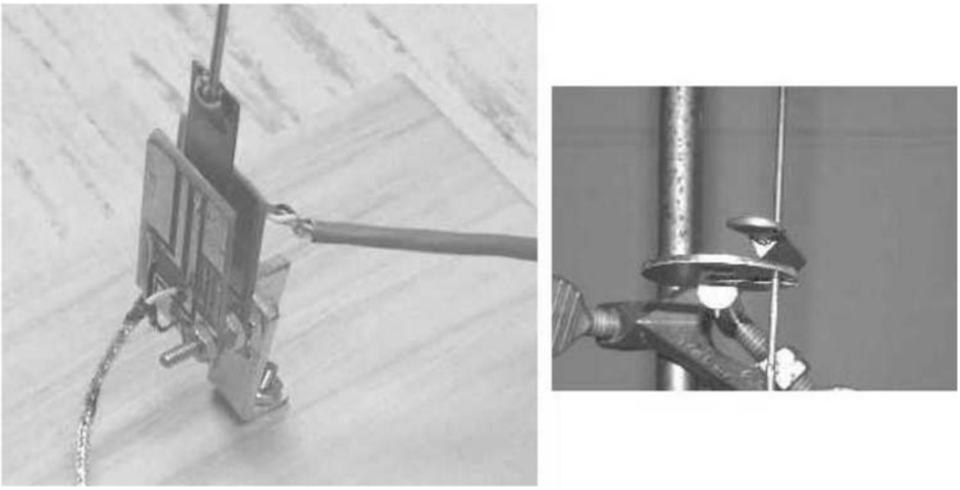

Figure 4. Close-up views of (i) the bottom of the pendulum inside the sensor (left), and (ii) the knife edge (right).

which has been attached to the rod by soldering, as shown in the left picture of Figure 4.

The plate is about 1.2 cm $\times$ 5 cm and it oscillates between the stationary electrodes of the sensor as the pendulum moves. The cables which connect to the stationary sensor electrodes on one end have been at the other end unplugged from their electronics support box. The small phono-plug carries the oscillator signal from the box, and the large phono-plug carries the 'sense' signals to the instrumentation amplifier in the box. (Note: Sensors of other type could be used to demonstrate the pendulum complexities that are the heart of this study. It is important that they be relatively non-perturbing of the instrument, which is best accomplished by using capacitive or optical types.)

The period is determined by the placement of the knife edge, which is clamped with a screw at different positions along the upper half of the rod. For the copper plate to remain in proper position in an altitude sense, it is necessary to reposition the clamp which holds the dowel, subsequent to any significant change in the period of the pendulum.

The 'knife edge', which is also pictured in a close-up photo of Figure 4, is made of brass, using the same stock as the brass dowel. A perpendicular hole was drilled through the center of this short cylinder to accommodate the rod. After drilling, the piece was filed to form two flats that intersect at an angle of about 70 degrees. Finally, an axial hole was drilled and tapped from an end to the center, to accept the set-screw that holds it to the rod. This set-screw is clearly visible in Figure 4.

The motion of the pendulum is recorded on a computer by feeding the output from the sensor electronics through an analog to digital converter. For this work, the a to d converter was the inexpensive Dataq DI 154RS. Though only a 12-bit converter, it is adequate for observing complex motions of the pendulum in free decay.

Before generating a temporal record, motion was initiated by displacing the pendulum by hand, roughly 20 mrad, and then letting it decay freely. Especially at longer periods, because the sensitivity of the instrument is proportional to T^2, it is important to shield against air currents. For the present work, this was done by placing a large trash can over the system after the pendulum motion was initiated.

In the present work, as well as in other studies by the author, it has been shown that internal friction (solid) damping is an important decay mechanism. In the case of the present rod pendulum, this is easily illustrated by comparing the motion of pendula constructed from two different alloys, one of brass and the other of solder. An earlier paper (Peters 2001) also postulated that creep of secondary type is the basis of the internal friction, at least for low-Q mechanical oscillators. That conclusion is supported by the present observations by noting that the soft solder pendulum decays dramatically faster than the brass pendulum. The fact that creep is a major factor with the rod pendulum is easily demonstrated in the solder case. The weight of rod material located above the knife edge is supported only by the structural 'integrity' of the material. Left to itself, the rod will quickly deform, because of creep. Most of the motion associated with creep deformation comes from the upper rod trying to relocate itself below the knife edge.

Of course the center of mass of the system changes during creep. Continual shape adjustments are therefore necessary to maintain the bottom sensor plate in an operational region of the sensor. With every repositioning of the knife edge, a satisfactory equilibrium relative to the sensor must be established by bending the rod, using finger forces that exceed the elastic limit of the material. This is very easy with the solder pendulum and a bit more difficult with the brass pendulum. By this means, the knife edge is translated fore or aft of the rod ends to effect a change in the position of the center of mass. Additionally, to insure that the end plate moves without touching the static electrodes of the sensor, it is sometimes necessary to bend the lower part of the rod in a direction perpendicular to that required for balancing.

4.1. RELATED INSTRUMENTS

As compared to other pendula described in the literature, the present instrument has some similarities but also major differences. It is most like the instrument used by the author in his earliest study of low-frequency mechanical oscillations; i.e., a compound pendulum built with a pair of 'point-like' masses that dominate the moment of inertia of the instrument (Peters 1990).

With the proper choice of an elastic element in the upper structure, so as to yield large flexure of the upper mass; the pendulum becomes a Duffing oscillator that can be chaotic. In this regard it is similar to the flexible component of the 'inverted pendulum' treated by Duchesne et al 1991. If properly configured, the flexure can even be the basis for improved isochronism (Peters 2003).

It should be noted that the present instrument differs greatly from a pendulum commonly labeled 'inverted', and which was first treated by Kapitza. In the Kapitza pendulum, the otherwise unstable equilibrium (upside down) is made stable by rapid vertical vibration of the axis of rotation (Kapitza 1930).

5. Some Results

Figure 5 is a typical decay for the brass rod. Although it looks reasonably ideal, a close inspection reveals fine structure in the turning points, as illustrated by the magnified upper turning points, shown in Figure 6.*

One cannot say with certainty that the cycle to cycle irregularities evident in Figure 6 are the result of anelastic structure changes in the metal of the pendulum. There can be little doubt, however, that such changes are present when one looks at the decay of the solder rod, as shown in Figure 7.

The influence of anelasticity is now dramatic. Curiously, as the solder oscillates at longer periods, such as this case; there is not an obvious bending of the pendulum. Evidently opposite sides of the rod, just above the knife edge, undergo phase reversed compression/extension cycles involving creep and creep-recovery.

It is true that the equilibrium position of the pendulum was shifting during the collection of the data for Figure 7. After oscillation died out, the mean position

* Note: All of the decay curves presented in this paper are in terms of sensor output voltage rather than pendulum amplitude. There is no loss of generality because of a calibration constant that relates the two.

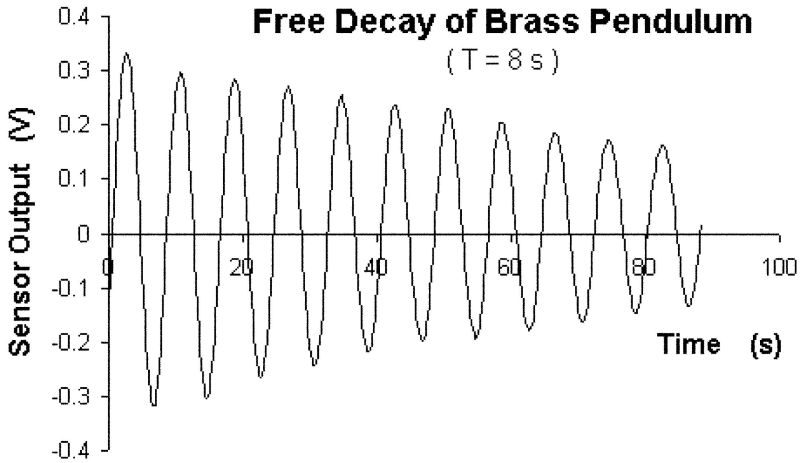

Figure 5. Brass rod pendulum free decay. The period of the motion was 8 s.

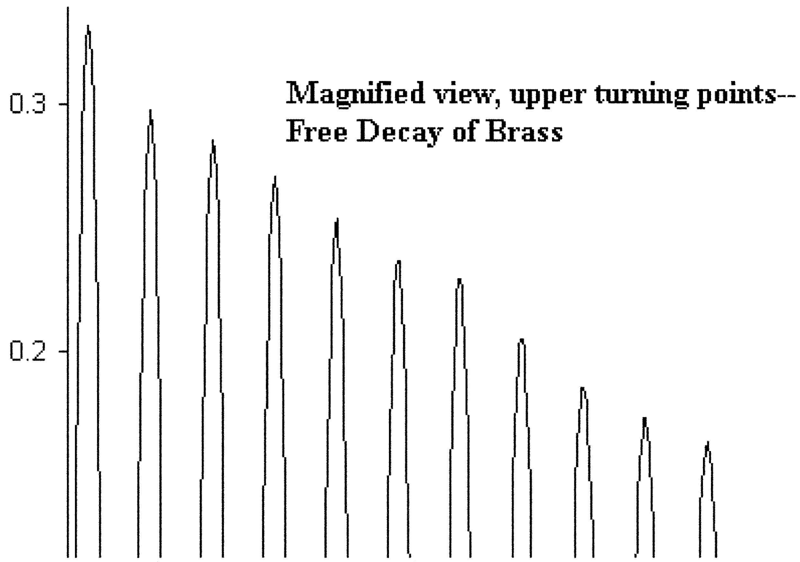

Figure 6. Magnified portion of Figure 5, showing some upper turning points.

continued to drift until the copper plate eventually exited the sensor electrodes. The evolution of the system during this time was reminiscent of avalanche phenomena, such as observed in sandpiles, although much slower.

6. Frequency Dependence of β

As noted previously, the decay 'constant' β should not depend on frequency (T^{-1}) if the motion is consistent with viscous damping. Such behavior of a physical pendulum has never been observed by the author and it was not expected with the

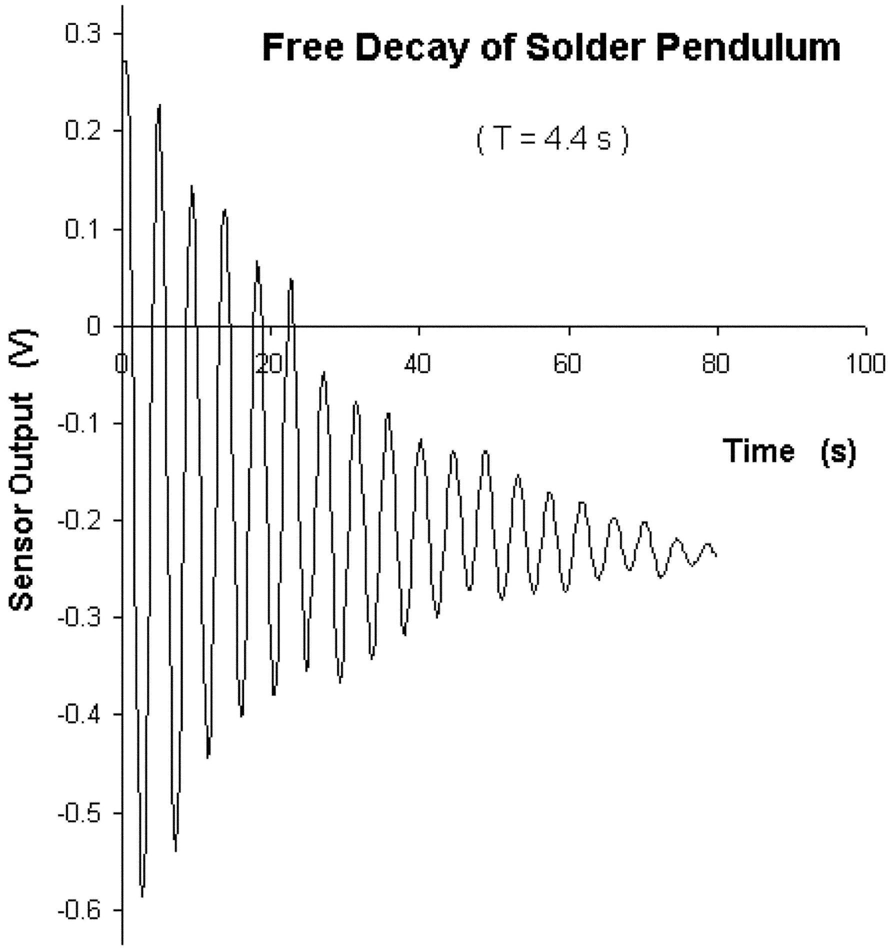

Figure 7. Solder rod pendulum free decay. The period of the motion was 4.4 s.

rod pendula of the present studies. To test the matter, damping was measured as a function of the period, which was altered by changing the position of the knife edge clamp, as previously indicated. The decay constant was estimated from the turning points, using the generalized method described in the Appendix. Instead of plotting β in the graphs which follow, the logarithmic decrement (βT) is plotted versus the square of the period. If the system were in agreement with viscous damping, a straight line should result when βT is plotted versus period instead of versus period squared.

Shown in Figure 8 are the results for the brass pendulum. The rather large error bars are thought to be associated with nonsteady effects of anelasticity.

These may relate to the Portevin LeChatelier effect mentioned previously. The fit to the data is closer to a linear regression using T^2 rather than T for the abscissa, in support of the claim that the damping is nonlinear and due to internal structure

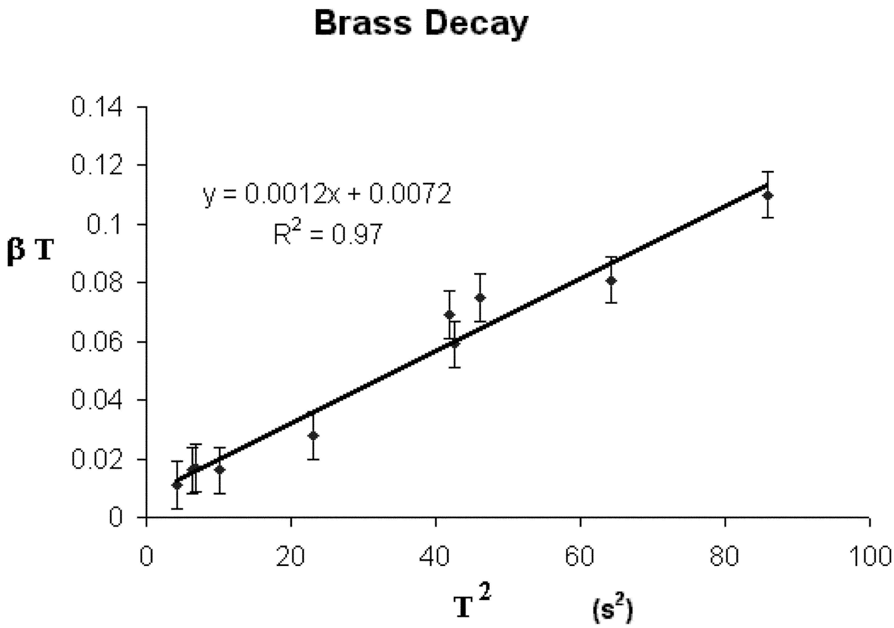

Figure 8. Logarithmic decrement versus period squared for the brass pendulum.

changes, rather than to some viscosity effect. Moreover, note that there is a remanent damping as the period goes toward zero. In a plot versus T the intercept is found to be negative and therefore unrealistic.

A similar behavior is found for the solder pendulum, as shown in Figure 9, although the slope of the line is significantly greater.

Consider the slopes of the indicated linear fit equations. For the same value of the period, the damping of the solder pendulum is seen to be 4.5 times greater than the damping of the brass pendulum. This is in keeping with the reduced structural integrity of the much softer solder. In turn, the upper limit on operational period of the solder pendulum is only about one-half that of the brass pendulum. As noted previously, this results from the difficulty of establishing a stable operating point in the presence of creep.

7. Conclusions

The pendula that have been studied show clearly that pendulum motion is complex. Especially for the solder pendulum, it is clear that damping of the motion is not from external forces such as air. The internal friction responsible for the decay is not simple. It may at times have quasi-periodic features, but it also is generally erratic. Moreover, cycle to cycle decrement changes can occur that are suggestive of ‘jerkiness’ reminiscent of the Portevin LeChatelier effect.

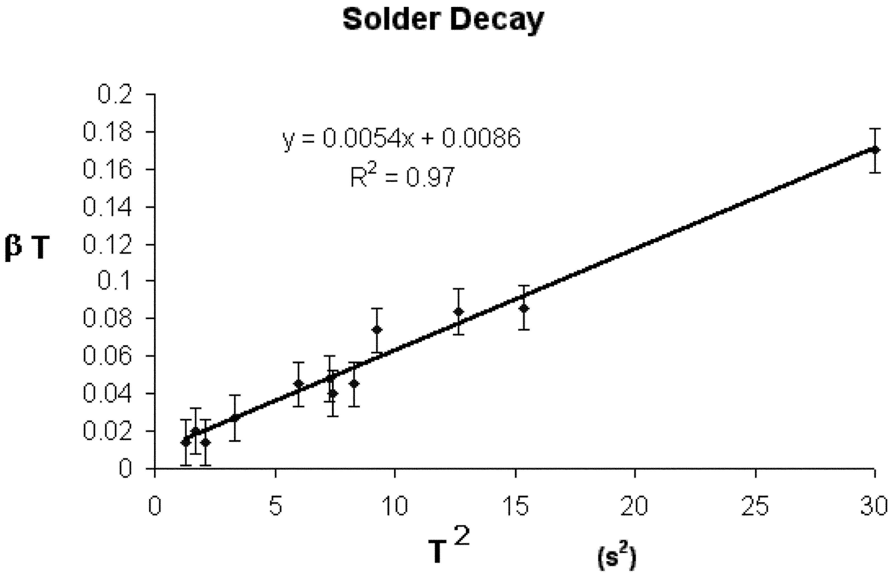

Figure 9. Logarithmic decrement versus square of the period for the solder pendulum.

The primary challenge to any meaningful attempt to model the pendulum's complex motion derives from the multiplicity of interactions that are obviously present. Responsible for the conversion of pendulum motion to heat, these interactions also preclude the possibility of a conventional treatment via the mechanics of Newton or Lagrange. Specifically, the technique of the Lagrangian is hampered by the inability for confidence in any potential function which might be assumed.

It is not the intent of this paper to follow the usual path of published results, in which one gives experimental results and then describes them with a theory. For the present work, it is felt that physics is woefully lacking in any of the tools thought necessary to explain the observations. It is a sad fact that science has not apparently progressed far beyond the earliest efforts to understand anelasticity, beginning with figures like Maxwell.

From studies such as this, it is clear that the pendulum is still important to engineering. The author also believes that it is important in a fundamental sense – that its consideration could increase our understanding of defect properties of materials. Thus it should not be considered a relic from an earlier era.

Appendix: Estimating the Decay Constant from Turning Points

A method for estimating the decay constant of oscillators was developed by Dr. Ken Hayes of Hillsdale College in Michigan. He expanded upon a method which uses three turning points, to improve precision in the presence of random errors– when the total cycles of the motion is acceptably large in a statistical sense. Hayes' technique for an arbitrary odd number of turning points is presented below. His method was first described in the manual for the computerized Cavendish balance designed by the author and sold by TEL-Atomic Inc. (described at Web URL http://www.telatomic.com/sdct1.html) As previously presented, the Hayes method contains a serious deficiency, for those cases where the equilibrium position is not constant. This will happen, for example, when the torsion fiber experiences creep. Fortunately, a simple fix for the deficiency exists and is described below. To illustrate the importance of using the generalized method, two examples of torsional oscillation involving creep will be provided.

ORIGINAL METHOD

For an ideal exponential decay, the position is given by

$$\theta(t) = \theta_e + A\mathrm{e}^{-b\,t}\cos(\omega t). \tag{7}$$

Inclusion of the equilibrium position, θ_e is necessary because it is virtually impossible to establish an operating point for which this term is zero, even though it might be constant.

Consider the turning points, given by

$$\theta_n = \theta_e + (\theta_1 - \theta_e)\mathrm{e}^{-(n-1)\beta T/2}(-1)^{n-1}, \qquad n = 1, 2, 3, \ldots \tag{8}$$

Using Equation (2), it can be shown that the decay parameter, $x = \mathrm{e}^{-\beta T/2}$ may be estimated from any three adjacent turning points by

$$x = -(\theta_{n+2} - \theta_{n+1})/(\theta_{n+1} - \theta_n). \tag{9}$$

Using three, rather than two points, eliminates the need to know θ_e, assumed constant.

The Hayes modification of this equation is as follows. Let N, odd only, be the total number of turning points in the set. For decay in which $\beta T \approx 0.25$, he showed by means of error analysis that the optimal size of the set is $N \approx 11$. By employing the 3-point expression recursively, he obtained

$$x = 1 - (\theta_1 - \theta_N)/(\theta_1 - \theta_2 + \theta_3 - \theta_4 \ldots - \theta_{N-1}). \tag{10}$$

In Equation (10) the set used for computation begins with 1 and ends with an odd integer, such as 11. In other words, the set comprises an integral number of total cycles of the motion. The larger the value of N the better, up to a maximum of about 11 for the Cavendish balance – to provide for averaging over random errors to reduce the uncertainty in the estimate of x, from which β is obtained.

It should be noted that Equation (10) assumes that the decay is a pure exponential; i.e., symmetric with θ_e constant. In practice, this may be the exception rather than the rule. Therefore, the following ‘fix’ is presented.

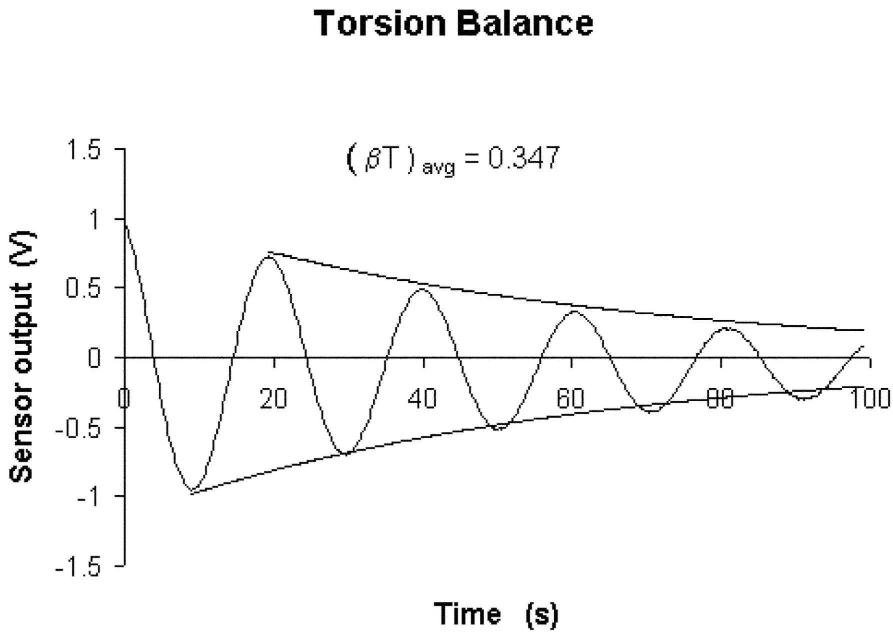

Figure 10. Torsion pendulum decay that is asymmetric.

GENERALIZED METHOD

Notice in Equation (10) the numerator term that involves a set which begins with 1 and ends with N. It is also possible to use the set that begins with 2 and ends with $N + 1$ (with the denominator adjusted to $(\theta_2 - \theta_3 + \theta_4 - \theta_5 \ldots - \theta_N)$. This half-cycle shift in the set gives essentially the same result as the former when the decay is ideal. For a non-ideal decay, however, the two cases yield different results. A better estimate for the decay is then obtained by using the mean value of the two calculations.

Examples

Among oscillators, a torsion balance typically comes closest to being a pure harmonic oscillator. The potential can be close to quadratic, and if the damping is primarily from a fluid such as air; then the conventional SHO with damping equation of motion is fairly good. It should be noted, however, that asymmetry of the decay can be present. Shown in Figure 10 is the decay of the boom in a Cavendish balance of the type mentioned above. Instead of the tungsten fiber used with this instrument to measure the Newtonian constant G, the data of Figure 10 were obtained with the tungsten replaced by a silk thread. The period of 20.2 s is much shorter than that of the instrument as normally operated. This was done so that some decay data could be easily and quickly generated.

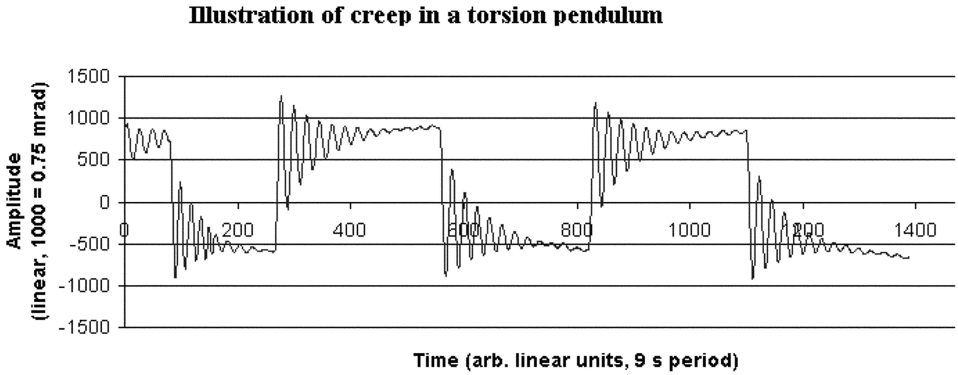

Figure 11. Example of creep from mean position shifts in a 'torsion-gravity' pendulum.

In addition to the free-decay oscillations of the boom in Figure 10, two curves have been added to 'guide the eye'. They correspond to an assumed symmetric decay with $\beta T = 0.347$, where the period of the oscillator is $T = 20.2$ s. Notice that the 'top' turning points are declining more rapidly at 0.362, whereas the 'bottom' ones are less rapid at 0.333. The value of 0.347 is the mean of these two values. These numbers were calculated by the generalized method described above, with $N = 7$; i.e., three cycles of the motion.

The need for two calculations and use of the mean probably results from creep. Creep is very often important, even in torsion pendula of other design types. For example, Figure 11 shows decays of a torsional oscillator which uses a tungsten wire. Even though tungsten is very hard, the metal is seen to creep significantly as the mean position of the boom is cycled back and forth through an angle of about 0.75 mrad.

The torsion wire of this pendulum is tied to both the top and bottom of the case and the instrument is therefore tilt sensitive. For that reason, the mean position could be easily adjusted using a piezo-translator, because the restoration depends on the Earth's gravitational field as well as the shear modulus of the tungsten wire. It is a pendulum similar to another that was used to study chaos (Peters 1995).

No additional discussion of this pendulum will be provided here. The Figure 11 was chosen for inclusion in the paper to simply illustrate the importance of creep, even in materials that might be erroneously thought of as being very stable.

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The Pendulum: A Paradigm for the Linear Oscillator

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Abstract. The simple pendulum is a model for the linear oscillator. The usual mathematical treatment of the problem begins with a differential equation that one solves with the techniques of the differential calculus, a formal process that tends to obscure the physics. In this paper we begin with a kinematic description of the motion obtained by experiment and a dynamic description obtained by the application of Newton's laws. We then impose the constraint of compatibility on the two descriptions. This method leads to a fuller understanding of the physics of the oscillator. The paper demonstrates the ubiquity of the linear oscillator as an idealisation of real physical phenomena. It treats the general case of damping with forced motion, including the phenomenon of resonance.

1. Introduction

Quite apart from its intrinsic utility as a timing device, the pendulum is a superb learning tool for science education. It can serve as a model for the study of the linear oscillator. In developing a complete picture of the phenomenon, it provides real understanding of the interaction between theory and experiment. It also illuminates the meaning of a 'solution' to a differential equation and therefore the role of mathematics in physics. As Matthews (2000) has shown so beautifully, the pendulum contributes greatly to science literacy.

In this paper I wish to show how the kinematic observations of pendulum motion combined with Newton's laws lead to a dynamic equation describing pendulum motion. The second part will show that an equation of this form governs a wide variety of disparate physical phenomena unified by their harmonic behaviour. These phenomena are all examples of the linear oscillator, modified to include the effects of damping.

2. Motion of the Simple Pendulum

Consider a pendulum consisting of a bob of mass m suspended from a pivot point by a string of length L, as shown in Figure 1. Let the mass of the string be negligible compared with that of the bob. Let the dimensions of the bob be so much smaller than the string length that we can treat it as a point mass. We shall also construct the

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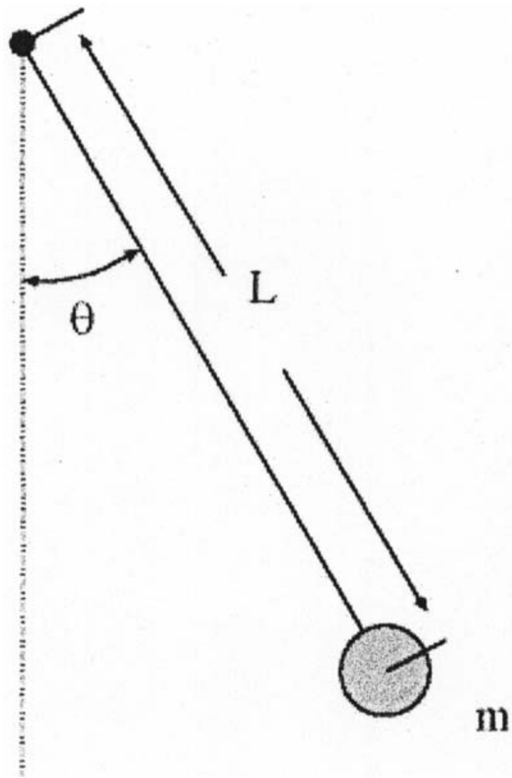

Figure 1. The simple pendulum of mass m and length L.

pendulum with exceedingly low friction so that we can neglect damping. Finally we displace the bob from equilibrium by a very small angle (less than 5°) and let it oscillate. What do we observe?

If we measure the angular displacement as a function of time, we note several facts. The motion is bounded. If our initial displacement is Θ, the bob position never exceeds this limit. We also note that the lower the friction, the closer to Θ does the bob return on each swing. If the friction is completely negligible, it returns to Θ exactly. The second fact is that the motion is periodic and isochronous as long we deal with very small angular displacements. Hence we can summarise our measurements of angle and time by the equation

$$\theta = \Theta \cos(Kt). \tag{1}$$

In the equation Θ is the maximum excursion of the bob (the amplitude), and K is an experimental constant related to the period T of the motion. It is also called the natural angular frequency ω_0 of the system,

$$K = 2\pi/T = \omega_0. \tag{2}$$

Equation (1) is a kinematic description of the motion containing two experimentally determined constants, the angular amplitude, Θ, and the natural frequency, K. As yet there is no physical explanation of the equation. It is purely experimental.

However, Newton's laws provide a dynamic description of the system. The rotational form of Newton's second law may be written as

$$\tau = I\alpha, \tag{3}$$

in which τ is the torque, I the moment of inertia, and α the angular acceleration. The torque is $mgL \sin \theta$ (where g is the acceleration of gravity), the moment of inertia for a point mass is mL^2, and the angular acceleration is $d^2\theta/dt^2$. Inserting these quantities into Equation (3) and simplifying, we obtain

$$d^2\theta/dt^2 = -(g/L)\sin\theta. \tag{4}$$

The negative sign for the torque reflects the fact that it is a restoring torque always acting to return the bob to its equilibrium position.

At this point we introduce the fact that the bob swings through a very small angle (less than 5°) so that we can use the small angle approximation

$$\sin\theta \sim \theta, \tag{5}$$

which is valid for $\theta \ll 1$, when θ is measured in radians. This gives us our final dynamic equation

$$d^2\theta/dt^2 = -(g/L)\theta. \tag{6}$$

Equations (1) and (6) look quite different, yet describe the same phenomenon. Therefore, in spite of their difference, they must be compatible. Differentiating Equation (1) twice yields

$$d^2\theta/dt^2 = -K^2\Theta\cos Kt = -K^2\theta = -\omega_0^2\theta. \tag{7}$$

Comparing Equations (6) and (7) enables us to evaluate the experimentally determined natural frequency, K (or ω_0), in terms of the dynamic parameters of the system, g and L,

$$K = 2\pi/T = (g/L)^{1/2} = \omega_0. \tag{8}$$

This is far more than an exercise in mathematical manipulation. Indeed, it serves as a model of the process of doing physics. One begins with an experimental determination of the motion. Equation (1), determined solely by experiment, describes the motion but offers no explanation. Only by invoking Newton's laws, laws that are actually the axioms of classical physics, can we relate the experimental constants to the dynamic parameters of the system (Newburgh 2001).

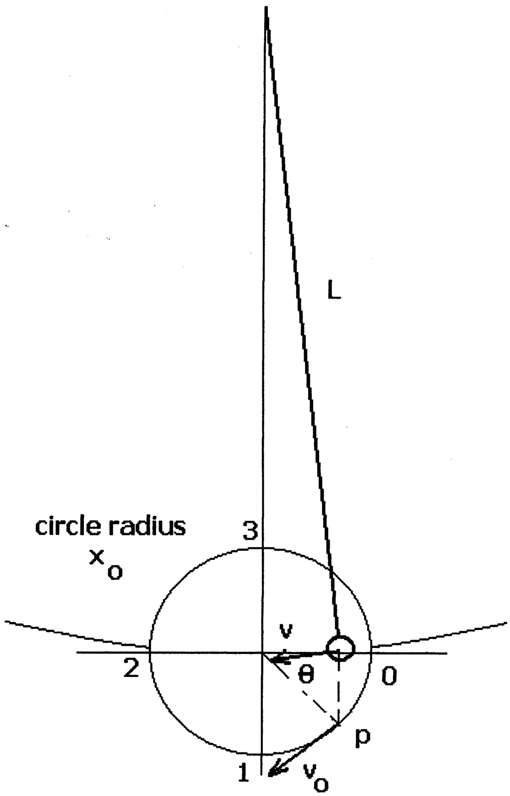

Figure 2. The reference circle for pendulum motion. The velocity of the bob is indicated by v. The velocity of the point p on the circle is indicated by v_0.

3. An Alternative Derivation of Pendulum Motion

Peters (2003) has also combined kinematic observations with a dynamic analysis to obtain a straightforward description of harmonic motion without using calculus. His description extends the unit reference circle to velocity and acceleration. I have not seen this approach developed elsewhere.

Peters begins, not with the unit circle, but one with radius equal to x_0, the linear amplitude of the motion (see Figure 2). The point p on the circle moves with constant speed. The condition of constant speed is an implicit statement of the fact that friction is negligible. The points shown – 0, 1, 2, and 3 – represent time differences of one-quarter period, $T/4$, with 0 corresponding to $t = 0$ or nT, where n is an integer ($n = 1, 2, 3, \ldots$). At any time t, p may be found by dropping a vertical from the center of the bob to the circle. As shown in Figure 2, p is about 1/8 cycle (45°) from the initial position.

The reference circle is commonly used to describe position of the oscillating body only. However, Peters applies it to determine velocity and acceleration as well. These are all one-dimensional vectors for which direction is determined by algebraic sign relative to the horizontal x-axis onto which all projections are made. This is possible because we are restricting the oscillations to small angles. In the figure velocity vectors only are shown, one for the circular motion of p (constant magnitude v_0) and one for the pendulum bob (variable magnitude v).

For the point p there are two constant quantities, its speed v_0 and its acceleration a_0. Since the point p moves along a circle at constant speed, we can apply the theorem for the rate of change of a rotating vector to obtain a_0 (Newburgh 1998).

$$a_0 = \omega v_0 = -\omega^2 x_0, \tag{9}$$

where $\omega^2 = 2\pi/T$. This theorem allows the calculation of the rate of change without resort to calculus. From the figure we conclude

$$x = x_0 \cos\theta, \tag{10}$$

$$v = -v_0 \sin\theta,$$

$$a = -a_0 \cos\theta = -\omega^2 x_0 \cos\theta = -\omega^2 x. \tag{11}$$

These expressions are valid for small oscillations, since $x_0 \ll L$.

Until now our analysis has been purely kinematic. To obtain the period T in terms of L and g we apply Newton's second law. The net force acting on the bob is a restoring force in the x-direction.

$$F_{\text{net}} = -mgx/L = ma = -m\omega^2 x. \tag{12}$$

From this we can determine the angular frequency and the period in terms of the acceleration due to gravity, g, and the pendulum length L.

$$\omega = (g/L)^{1/2} \tag{13}$$

and

$$T = 2\pi(L/g)^{1/2}. \tag{14}$$

Note that Equation (10) is equivalent to Equation (1) as long as we restrict ourselves to small oscillations. For small angles $\sin\theta \sim \theta$. Therefore

$$x_0/L = \sin\Theta \sim \Theta. \tag{15}$$

4. The Linear Oscillator

There are numerous physical phenomena that can be described by equations of the form of Equation (7), namely the temporal second derivative of a quantity that equals that quantity multiplied by a negative constant. Two examples are (a) the displacement x of a mass attached to a massless spring with a spring constant k and (b) the temporal variation of charge q in a circuit made up of a capacitor C and an inductance L. Note that an idealised circuit without resistance is the electrical analogue of a frictionless mechanical system such as our pendulum. The corresponding equations are

$$d^2x/dt^2 = -(k/m)x \tag{16}$$

and

$$d^2q/dt^2 = -(1/LC)q. \tag{17}$$

Referring to the previous section, we see immediately that for the spring

$$K = (k/m)^{1/2} = \omega_0 \tag{18}$$

and for the circuit

$$K = (1/LC)^{1/2} = \omega_0. \tag{19}$$

Moreover, we can immediately write the ‘solutions’ of Equations (9) and (10) as

$$x = A\cos[(k/m)^{1/2}t] \tag{20}$$

and

$$q = Q\cos[(1/LC)^{1/2}t]. \tag{21}$$

In these equations A and Q are the respective amplitudes and we have chosen the initial times appropriately.

The sinusoidal behaviour of the ‘displacement’ – be it angular displacement, linear displacement, or charge – is reflected in an energy oscillation. Since we are dealing with idealised, frictionless systems, there are no dissipative energy losses. Therefore the total energy of the system is conserved. However, it consists of two forms of energy, kinetic and potential for the mechanical systems and electric and magnetic for the LC-circuit. Figure 3 shows these energies as a function of position for the pendulum. Both the kinetic and potential energies exhibit quadratic behaviour with displacement. The potential energy is a maximum at the extremities and a minimum at the equilibrium position (zero with appropriate choice of our reference point). The kinetic energy is just the reverse, having its maximum value as the bob goes though equilibrium and zero at the extremities (turn-around points imply zero velocity).

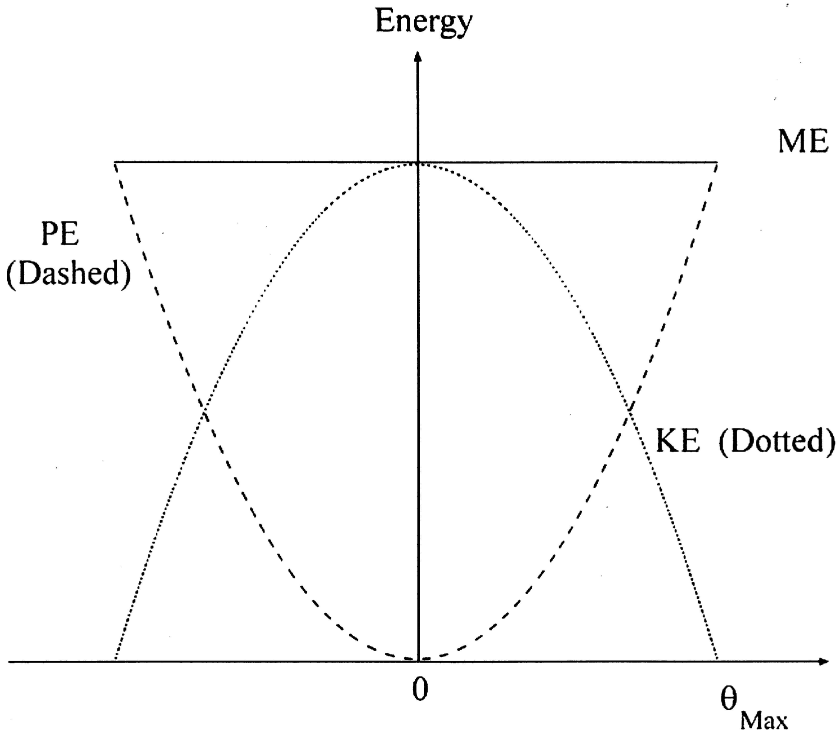

Figure 3. Energy of the simple pendulum as a function of angular displacement. The dashed line is the potential energy. The dotted line is the kinetic energy. The solid line is the mechanical energy. Note the parabolic behaviour of the potential and kinetic energies.

5. The Forced, Damped Oscillator

In reality it is impossible to construct frictionless systems. All mechanical systems with moving parts have friction. Air resistance is omnipresent. In electrical circuits the resistance of conducting elements is the source of dissipative heating. Therefore in setting up the differential equation governing real systems we must include dissipative forces. We shall treat those that are proportional to the first derivative of the 'displacement' velocity in mechanical systems and current in electrical circuits. Taking u to be the symbol of our generalised 'displacement', we can write our differential equation as

$$ad^2u/dt^2 + bdu/dt + cu = 0. \tag{22}$$

In Equation (22) a represents an inertia term such as mass, moment of inertia, or inductance. The symbol b is a resistance term. For small velocities friction is proportional to the velocity, du/dt, the proportionality constant being b. For an electrical circuit du/dt is the current I or dq/dt and b is the resistance R. The symbol c is the spring constant k for a mechanical system or $1/C$, the reciprocal capacitance for an electrical RLC-circuit.

Displacing our system from equilibrium, we observe free oscillations that are damped to zero, quickly or slowly depending on the strength of the frictional forces. Numerous books give the mathematical details of damped motion. A particularly lucid treatment is that of Melissinos and Lobkowicz (1975).

Given the reality of friction, we ask how can the pendulum serve as a time-keeper? True, all clocks run down with time. Therefore we must wind them or replace batteries. However, while they run, their ticks are periodic and isochronous. The act of winding puts energy into the system. The escapement releases this energy in discrete, periodic packets, energy that compensates for the friction losses. Thus the escapement provides a periodic driving force (and therefore torque) that we may write as $F_0 e^{i\omega t}$, where the frequency ω may differ from the natural frequency of the system. (The appendix has a brief discussion of the use of complex notation and phasor quantities.) The equation that governs the system becomes

$$ad^2u/dt^2 + bdu/dt + cu = F_0 e^{i\omega t}, \tag{23}$$

As before the natural frequency K is

$$K = \omega_0 = (c/a)^{1/2}. \tag{24}$$

We can rewrite Equation (23) in the form

$$d^2u/dt^2 + (b/a)du/dt + (\omega_0)^2 u = (F_0/a)e^{i\omega t}. \tag{25}$$

When the external force is applied, the resultant motion is a combination of the transient free oscillations and the forced motion. The transients die down with time, leaving the forced motion only. We find, following Melissinos and Lobkowicz (loc. cit.) and taking the real part of the solution, that the 'displacement' is a function of time given by:

$$u = (F_0/a)\{1/[(\omega_0^2 - \omega^2)^2 + (\omega b/a)^2]^{1/2}\} \cos(\omega t + \delta). \tag{26}$$

The angle δ is the lag of u with respect to F.

This result shows that the motion has the same frequency as the driving force but is not in phase with it. It also illustrates the phenomenon of resonance. As ω approaches ω_0, the amplitude increases reaching a maximum when the two frequencies are almost (but not quite) equal, as shown in Figure 4. Because of friction the amplitude can never become infinite. This means that the system can be driven at its natural frequency with constant mechanical energy.

6. Discussion

In this paper we have not 'solved' the differential equation for the frictionless pendulum. We must remember that finding the solution of a differential equation

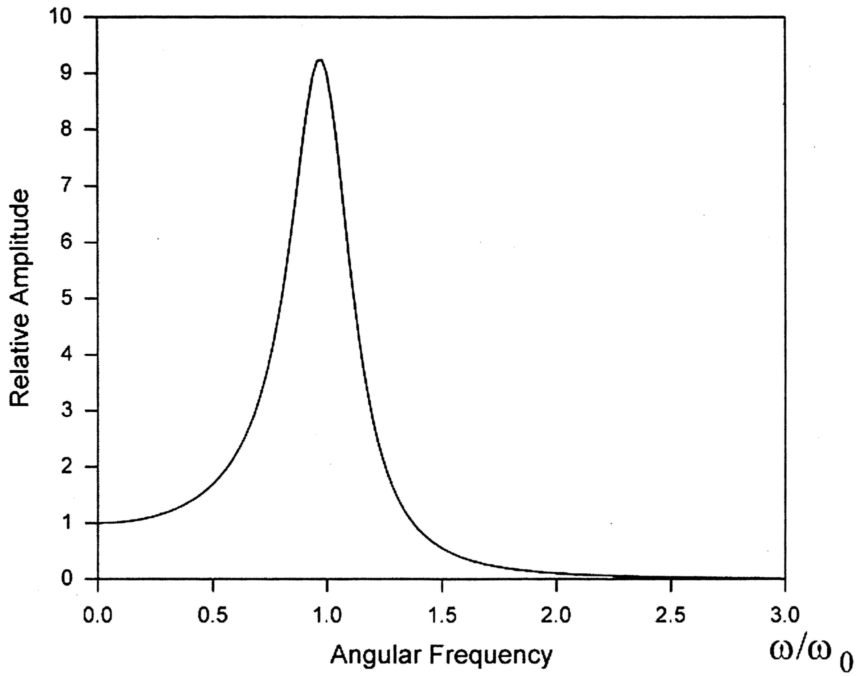

Figure 4. Amplitude as a function of driving frequency ω of the forced, damped oscillator.

is, of necessity, a trial and error process. Although we have a process whereby we can differentiate all functions, we can integrate only by knowing the answer. The only way of knowing if we have integrated correctly is by differentiating our solution and obtaining the original equation. The approach of this paper is somewhat different, beginning as it does with experimental observations. These data were reduced to a kinematic equation, Equation (1). Newton's laws provided a second, dynamic description of the phenomenon. We arrived at the 'solution' by demanding that the two descriptions be compatible.

We have only touched on the variety of physical phenomena that are examples of the linear oscillator. These include the pendulum, the physical pendulum, the oscillating mass on a spring and the LRC-electrical circuit. Musical instruments, both strings and horns, depend on vibration. Electromagnetic oscillations occur inside closed cavities with conducting walls. (Microwave ovens can set up standing waves. That is why they have rotating platforms so that all parts of the food will be warmed.) Atomic and molecular systems can be treated as a collection of linear oscillators. Therefore we can apply our theory of resonance to treat anomalous dispersion.

All these physical phenomena, dissimilar as they may be, do possess one common feature, namely inertia. In dealing with the pendulum, the moment of inertia affects the period. In the mass-spring system mass plays the same role, as does inductance in the LC-circuit. Thus, we may consider inductance to be an electrical inertia. In all three situations the 'inertia' tries to maintain a steady velocity (or current). Mathematically, the minus sign in Equations (7), (16), and (17) expresses this behavior. The inertia opposes the driving force. In other words the acceleration and displacement have opposite directions at all times.

Another reason for the ubiquity of the oscillator in the natural world is a consequence of stable equilibrium. A criterion for stability relates the effect of a small displacement from equilibrium. If it results in small, bounded motion about equilibrium, we have stable equilibrium. It is unstable, if the motion is unbounded. Goldstein (1951) in his chapter on small oscillations shows that a Taylor series expansion of the potential energy leads to a quadratic term for the potential energy for a small displacement. Since the force is the negative gradient of the potential energy, the result is a negative restoring force proportional to the displacement. In other words, small displacements from stable equilibrium give rise to linear oscillations.

A final point of this paper is its illustration of the role of mathematics in physics. Physics is an inductive science, beginning with and dependent on observation. It differs from geometry that starts with axioms to which one applies deductive reasoning. With the oscillator we first find experimental data that may be summarised in trigonometric form. Only when we have recognised this, do we begin to erect the mathematical scaffolding that has proven so powerful.

In the 18th century deists considered God to be a master clockmaker. Perhaps the term 'master oscillator' is even more appropriate.

Appendix: Phasor Quantities

We have written the applied force in Equation (23) using complex notation. This is an example of phasor notation that was first introduced by Steinmetz. At the end of the nineteenth century he popularised 'phasors' or 'vectors' for a-c quantities such as voltage, current, or electric field. (The term 'vector' is not to be confused with the normal use of the word for space vectors. Calling the voltage a complex vector does not imply that it is a space vector.) In 1893 at the International Congress in Chicago he introduced phasors, emphasizing the elegance and usefulness of the approach.

> The method of calculation is considerably simplified. Whereas before we had to deal with periodic functions of an independent variable 'time', now we obtain a solution through the simple addition, subtraction, etc, of constant numbers ... Neither are we restricted to sine waves, since we can construct a general periodic function out of its sine wave components ... With the aid of Ohm's Law in its complex for many circuit or network of circuits can be analysed in the same way, and just as easily, as for direct current, provided that all the variables are allowed to take on complex values. (Steinmetz, 1893)

The Chicago paper was not well understood, although the ideas became known by word of mouth. In 1897 Steinmetz described the approach in a book (Steinmetz & Berg 1897).

Acknowledgment

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Introduction to the Treatment of Non-Linear Effects Using a Gravitational Pendulum

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Abstract. We show that the treatment of pendulum movement, other than the linear approximation, may be an instructive experimentally based introduction to the physics of non-linear effects. Firstly the natural frequency of a gravitational pendulum is measured as function of its amplitude. Secondly forced oscillations of a gravitational pendulum are investigated experimentally without limiting amplitudes. By this arrangement new phenomena, the bistability and the jump-effect, can be observed. In the case of bistability the driven gravitational pendulum can oscillate in two different stable modes. Either it oscillates with a small amplitude and approximately in phase with the exciting torque or it oscillates with a larger amplitude and approximately anti-phase. The jump effect is the spontaneous transition from one mode of oscillation to the other. Both effects can be demonstrated and explained.

1. Introduction

The gravitational pendulum plays an important role in physics and in physics teaching. It is well known by students. We show that it may also facilitate the transition to the treatment of effects due to non linearity, which are characteristic of many areas of modern research.

Physics and physics teaching concentrate on the solutions of the linear approximation of the restoring force. Since a mathematical treatment of the gravitational pendulum for the domains of larger amplitudes involves elliptic integrals these domains are usually omitted in textbooks and only dealt with in specialized monographs (Pain 1984, Marion & Thornton 1995). We show that this omission is not justified because the qualitative effects due to non-linearity can be understood completely (Weltner et al. 1994, 1995). Far from the region of linear restoring force the natural frequency is no longer constant but decreases with increasing amplitudes. This can be demonstrated easily (Khosropur& Milland 1992, Weltner et al. 1994). Consequently the tuning curve of a driven gravitational pendulum is bent and, due to this non-linearity, two different modes of stable oscillations occur. Besides this, a transition between these modes happens under certain conditions a phenomenon which is known as the jump-effect. These new effects can be

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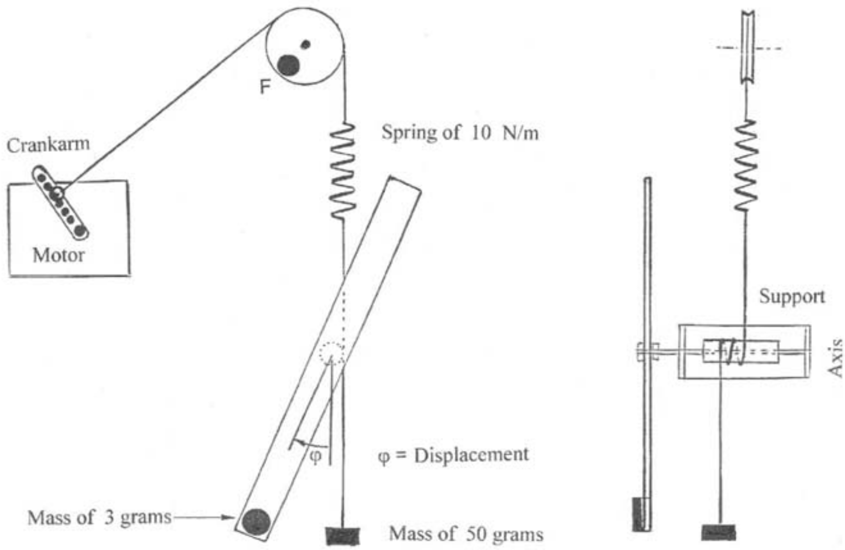

Figure 1. Experimental arrangement. Left: front view. Right: side view.

demonstrated, analyzed and understood and thus the role of non-linearity and its implications may be introduced. Thus the pendulum, which played a fundamental role in history of science and in education can be used as well as an introduction to modern parts of physics (Matthews 2000). The experimental setup is transparent, familiar to students and easy to arrange for lecturers. Consequently we start the discussion of non-linear effects by summarizing the properties of the linear gravitational pendulum and then proceeding to the new effects.

2. Properties of the Pendulum with linear restoring Force

For this and the following experiments, a rigid physical gravitational pendulum is used. The pendulum is constructed from a symmetrical bar, which is able to rotate about a horizontal axle through its center in a support as shown by Figure 1. This experimental arrangement allows unlimited amplitudes.

A small mass fixed to one end of the bar produces the restoring torque. The dimensions are not critical. Details are given in an appendix at the end of this paper. To investigate forced oscillations an electric motor is used which allows the frequency to be varied. To provide the exciting torque the exciting force acts from above by means of a spring (C= 10 N/m) and a thread twisted around the axle. To give the thread sufficient tension 50 grams mass is fastened to its lower end.

A dial plate behind the pendulum facilitates control and measurement of amplitudes. A marker F fixed to the pulley above the pendulum facilitates the observation of phases between the exciting torque and the oscillation of the pendulum.

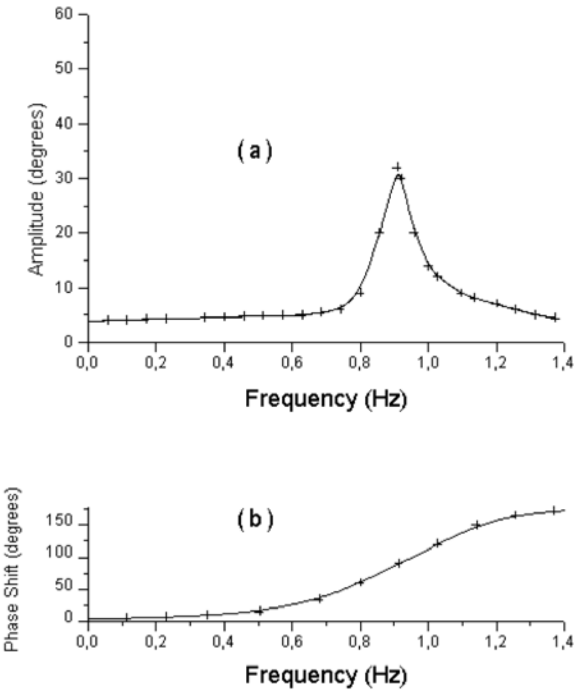

Figure 2. (a) Tuning curve of a forced gravitational pendulum for small amplitudes. (Natural frequency: 0.89 Hz). (b) Phase shift as function of frequency.

As a starting point, the properties of pendulum movement in the region of valid linear approximation can easily be revised and demonstrated. First, the independence between the pendulum's frequency (or period) and its amplitude can be demonstrated experimentally for the domain of small amplitudes (amplitudes less than 30°) by showing that the time of five or more periods is the same regardless of the amplitude, but differs if the amplitudes exceed substantially the domain of 30°.

Second, the forced oscillations of a gravitational pendulum in the domain of a linear restoring force can be demonstrated. The tuning curve with a sharp peak – the resonance maximum – and the phase shift between exciting force and the forced oscillation can easily be determined (see Figure 2).

The tuning curve represents the amplitudes of stationary oscillations excited by an external oscillating torque of constant maximal value. In the stationary case, there is an equilibrium between energy losses of the pendulum and energy input by the driving mechanism which compensates these losses. In the following we denote energy transmission the energy received by the pendulum.

The oscillating pendulum looses energy by friction. There are two important parts: air friction and friction between axle and support. The air friction may be regarded in first approximation proportional to velocity and, thus, amplitude. Since losses are the product of friction and length of displacement, the latter proportional to amplitude too, this results in losses proportional to the square of amplitude.

The friction between axle and support may be regarded in first approximation to be constant. Its product with length of displacement results in losses proportional to amplitude. The total losses are the sum of both and increase between the first and second power of amplitude.

The energy received by the pendulum from the driving torque is the product of the torque and the angular velocity of the axle. The amplitude of the torque being constant, the amplitude of the velocity being proportional to the oscillation's amplitude. Both factors can in first approximation be represented by sine-functions of equal frequency. But beyond this the product of two sine-functions depends on the phase between the two functions. The product of two sine functions is maximum if the phase shift between both is zero. In this case the torque acts always in direction of the pendulum's velocity and is always transmitting energy to it. Since it is the displacement we observe we must relate our considerations to the observed displacement φ. Velocity and displacement are represented by sine-functions with a phase shifted by 90°. Consequently energy transmission is maximum if driving torque and the pendulum's displacement have a difference of phase of exactly 90°. Energy transmission vanishes if the difference of phase between driving torque and the pendulum's displacement is either zero or 180°. In the steady oscillation the energy losses are compensated by energy input thus establishing an equilibrium. The amplitude represented by the tuning curve is, thus, an indicator for energy transmission too. Roughly speaking, energy transmission depends on the phase shift between driving torque and driven oscillation and, thus, on the difference between the pendulum's natural frequency and the exciting frequency. Energy transmission is maximum if the phase shift between exciting frequency and the pendulum's oscillation is 90°. This condition is given if the exciting frequency coincides with the natural frequency of the driven pendulum. Energy transmission decreases when the difference between natural frequency of the pendulum and exciting frequency increases.

3. Natural Frequency of a Gravitational Pendulum as a Function of Its Amplitude

If the restoring torque is proportional to displacement the frequency of a pendulum does not depend on its amplitude. This is a good approximation for a gravitational pendulum if its amplitude is restricted to values less than 30°. This is usually shown by observing and comparing the oscillations of two identical pendula oscillating with very different amplitudes. This independence may be derived theoretically by assuming a restoring torque linear to displacement, since this case may be dealt with by means of elementary mathematics. However, if the correct term describing the restoring torque is used, the solution involves elliptic integrals and mathemat-

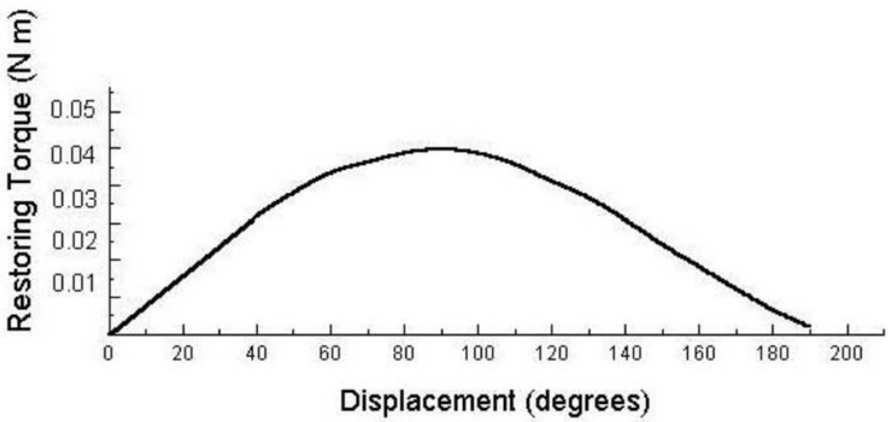

Figure 3. Restoring torque as a function of displacement.

ical difficulties rise for students. Thus these solutions are generally omitted in basic physics courses.

Nevertheless, the properties of oscillations with amplitudes beyond the domain of linear approximation can be treated qualitatively and investigated experimentally.

First, we measure the restoring torque of the gravitational pendulum within the given experimental arrangement using a dynamometer. The restoring torque is approximately linear to displacements up to 30°. Beyond displacements of 90° the restoring torque begins to decrease and vanishes for a displacement of 180°. The equation describing the restoring torque of a rigid gravitational pendulum is given by

$$\mathrm{T} = -m \cdot g \cdot l \cdot \sin \vartheta.$$

In this equation we neglect the additional restoring torque caused by the spring which connects the pendulum with the exciting device. Figure 3 is based on experimental data and shows the effect of the additional restoring torque due to the connecting spring for displacements of 180°.

Thus, an increase of period and a decrease of frequency are to be expected due to the decreasing restoring torque for larger amplitudes. This can be demonstrated easily by comparing the oscillations of two equal pendula oscillating with different amplitudes one within the domain of linearity and the other with amplitudes far outside this domain. If we start both pendula simultaneously, it can be observed that the oscillation of the larger amplitude pendulum takes noticeably more time to complete. Next we determine the period of oscillation as function of amplitude, starting from an amplitude A_1 and taking the time required to return to amplitude A_2 which is a bit less due to friction losses. By this procedure we assess the period of an intermediate amplitude $\mathrm{A} = (A_1 + A_2)/2$ for which the frequency is calculated.

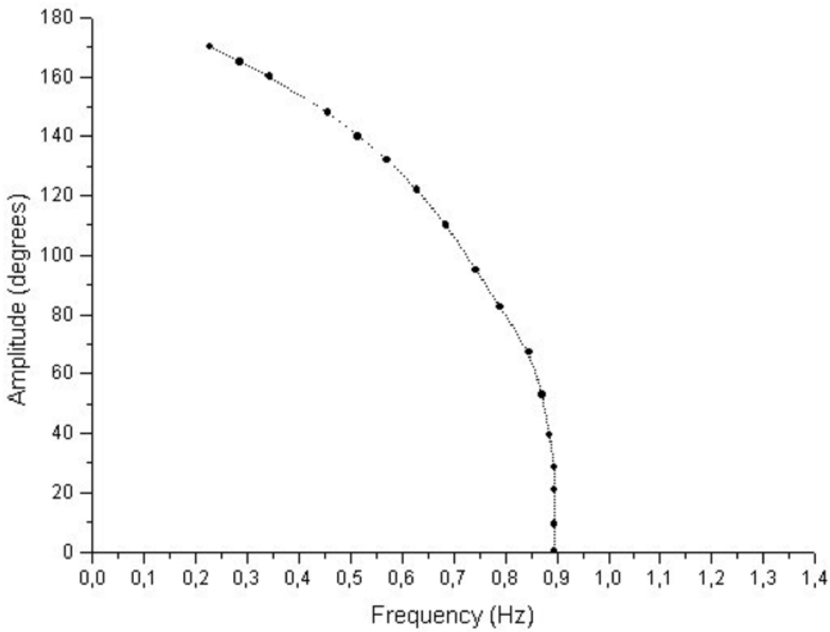

Figure 4. Amplitude versus natural frequency for a rigid gravitational pendulum.

Proceeding this way we establish the relationship between amplitude versus natural frequency for the full domain of amplitudes from 0° to 180°. This gives a curve, which starts vertically at the natural frequency f_0 for small amplitudes but then bends smoothly to the left. For amplitudes approaching 180° the correspondent natural frequencies vanish, because 180° represents a point of unstable equilibrium in which position the pendulum can remain forever. As one approaches this point the amplitude increases and the torque decreases so that the period time growths larger and larger.

4. Jump Effect and Bistability

New effects appear if we repeat the experimental investigation of forced oscillations of the gravitational pendulum discussed in Section 2 increasing the exciting torque. In Section 2 we restricted the exciting force to keep the amplitudes of the driven pendulum within the limits of linear restoring torque. Now we increase the exciting torque by enlarging the crank arm by a factor two or three (see Figure 1). We expect a tuning curve with amplitudes increased by the same factor.

In a first attempt to assess the tuning curve we start from frequencies well below the natural frequency f_0 for small amplitudes. Due to the increased value of the exciting torque we expect and find the amplitudes at the beginning increased by the same factor. In the following we increase the exciting frequency gradually step by step allowing the oscillation to approximate its stationary value after each step.

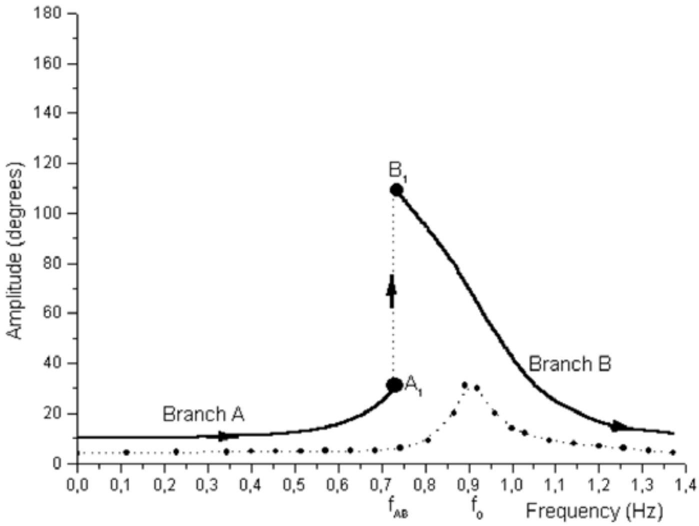

Figure 5. Tuning curve of forced oscillations of a gravitational pendulum showing the jump effect. (Dotted curve: tuning curve for linear restoring torque).

The amplitude increases with the frequency. The oscillations are approximately in phase with the exciting torque. When the amplitude reaches a certain value of about 30° (point A_1 in Figure 5) a new phenomenon is observed. The amplitude does not approximate a stationary value any more, but it increases steadily up to reach a new and different stable mode with an amplitude of approximately 115° (point B_1 in Figure 5). During this process the phase shift increases too and the oscillation is finally anti-phase. This transition from one stable mode of oscillation to an entirely different mode is called the jump effect. The jump frequency f_{AB} depends on experimental conditions. Roughly speaking it happens if the amplitudes exceed the linear domain. If we increase the exciting frequency even more the amplitudes decrease as is shown in Figure 5. We have labeled the branch of the tuning curve for exciting frequencies less than the corresponding natural frequency branch A. Branch B holds for exciting frequencies exceeding the corresponding natural frequency. At the end of branch B, the amplitudes follow qualitatively the tuning curve of the linear case, increased by the factor by which the exciting torque was increased.

We now determine the tuning curve starting with frequencies well above the natural frequency for the linear case f_0, represented by branch B. Initially the amplitudes coincide with those of the preceding experiment. Decreasing frequencies correspond to increasing amplitudes. When we reach the frequency f_{AB} of the jump effect observed in the preceding experiment represented by point B_1 the oscillations remain stable. If the exciting frequency is decreased even more the amplitude continues to increase steadily. The oscillations remain approximately antiphase with respect to the driving torque. We thus get an extended branch B

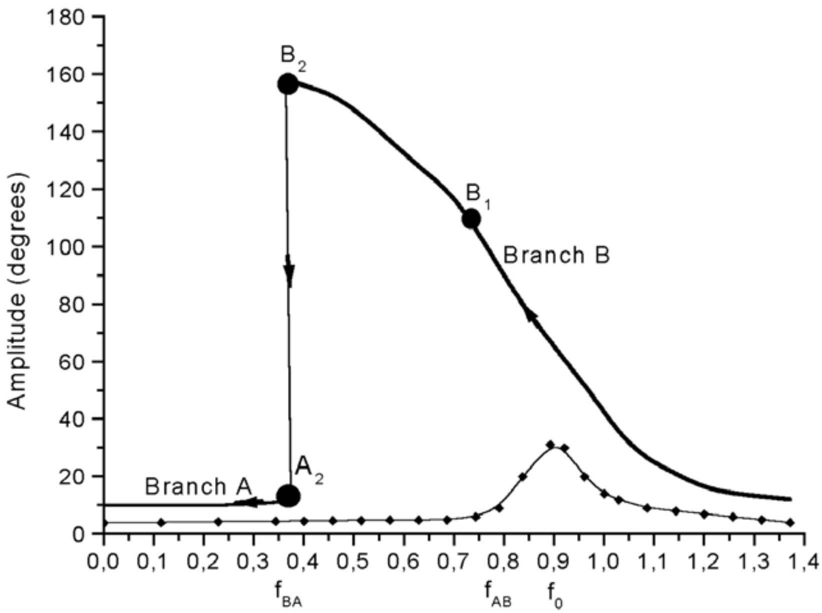

Figure 6. Tuning curve of a forced gravitational pendulum starting from high frequencies.

of the tuning curve (point B_2 in Figure 6). Then, at a frequency f_{BA} well below the former jump frequency, a reverse jump occurs. This time the amplitude of the oscillation decreases steadily, reaching a small value on branch A (point A_2 in Figure 6). During this process the phase difference decreases and almost vanishes. Diminishing the exciting frequency even more, we get the values of the former experiment represented by Figure 5. Thus we obtain different tuning curves depending on the initial conditions and the direction in which the exciting frequency varies.

We found that the tuning curve of the driven gravitational pendulum has two different branches depending how the experiment is carried out. The branches differ in the frequency domain from f_{AB} to f_{BA} and coincide outside this domain. Within this domain, there exist two different modes of stable oscillations for the same exciting torque. This phenomenon is called bistability. It is most fascinating to observe two identical pendula excited by the same torque oscillating in the same mode or in different modes simultaneously. Each of them can be brought to oscillate in both modes independently of the other. The jump effect and bistability are new effects never observed in a simple gravitational pendulum in a conventional physics course. These effects show that non-linearity in variables may cause a qualitatively new behavior.

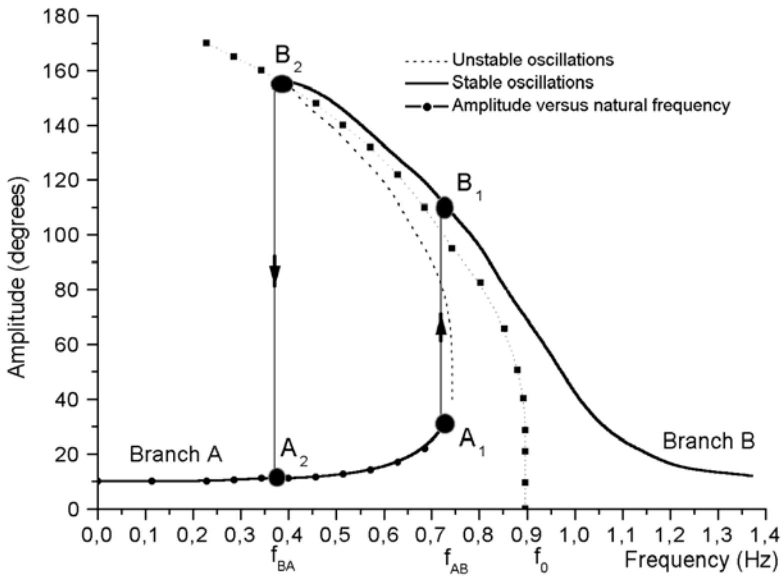

Figure 7. Combined and completed tuning curve for a gravitational pendulum.

5. Explanation of Bistability and Jump-effect as a Consequence of a Bent Tuning Curve

A clue to understanding bistability and the jump effect is obtained if we combine the tuning curves just established (Figures 5 and 6) to the amplitude versus natural frequency curve (Figure 4) shown in Figure 6.

We first consider the development of branch B. Starting the experiment with high exciting frequencies and reducing them step by step the tuning curve approximates the curve of amplitude versus natural frequency. We remember that highest amplitudes are obtained with exciting frequencies near the corresponding natural frequencies of the gravitational pendulum. The equilibrium between energy input by the driving mechanism and energy losses of the driven pendulum ends for the exciting frequency f_{BA}. At this point the losses prevail. The oscillation ceases to be stationary and a transitional jump effect occurs. The amplitude decreases to a value represented by branch A. During this process the conditions for energy transmission become less favorable, because the difference between exciting frequency and the natural frequency associated to the actual amplitude is still increasing during this transition. Thus energy losses exceed energy transmission resulting in a decreasing amplitude.

Starting the experiment with low exciting frequencies, which are increased step by step, results in increasing amplitudes, as shown by branch A of the tuning curve. Approaching the jump frequency f_{AB}, the amplitude exceeds 30° when the oscillation ceases to be stable and the jump effect starts. This can be explained again considering the equilibrium between energy input by the driving mechanism

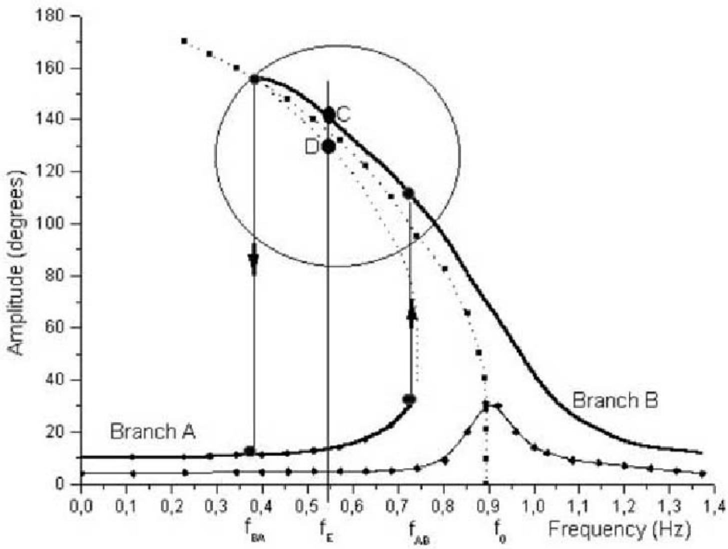

Figure 8. Complete tuning curve to analyze stability of oscillations. Exciting frequency f_E.

and energy losses of the driven pendulum. This time the conditions for energy transmission grow to be more favorable with increasing amplitude since the difference between exciting frequency and the respective natural frequency associated to the actual amplitude decreases. This process ends if a new equilibrium is reached represented by a value on branch B.

We complete the two branches by the dotted parts in Figure 7 to obtain a complete tuning curve, which may be understood as a bent tuning curve. The value of the amplitude increases if the difference between exciting frequency and natural frequency decrease. Consequently we find the highest values of the amplitudes in the vicinity of the amplitude versus natural frequency curve.

A question remains: why we do not observe the oscillations represented by the dotted part of the bent tuning curve?

In other words, how do we explain the stability of stationary oscillations represented by the full lined parts and the instability of those represented by the dotted parts.

Let us consider an oscillation represented by point C on branch B in Figure 8.

An amplified part (circled in Figure 8) is shown by Figure 9. The exciting frequency is f_E. Point C represents an oscillation which will be shown to be stable. The natural frequency associated to the given amplitude is $f_{C'}$. If by some reason the amplitude is increased to a value C' the natural frequency associated to that amplitude will be $f_{C'}$. The difference between the exciting frequency and the corresponding natural frequency is thus increased and consequently the conditions for energy input are less favorable causing the amplitude to fall back to the point of equilibrium again, which is given by point C.

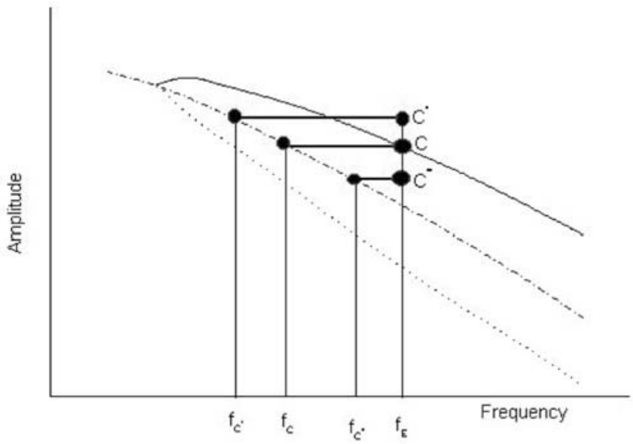

Figure 9. Amplified part of Figure 8.

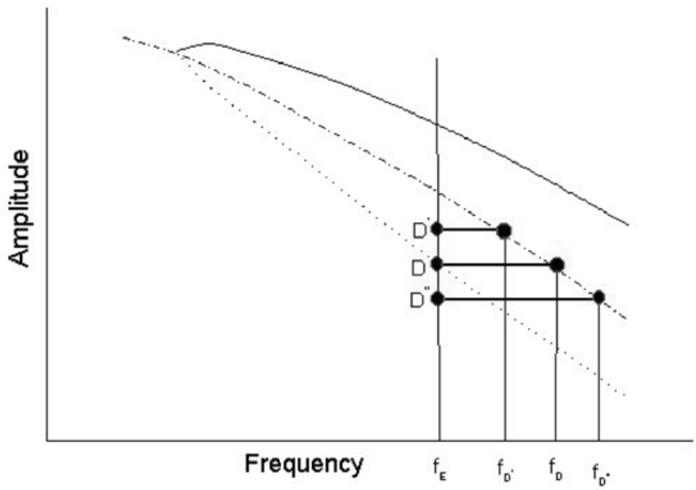

Figure 10. Amplified part of Figure 8 Point D represents an oscillation which will be shown to be unstable.

If by some reason the amplitude of the stationary oscillation is reduced to a value represented by C'' the natural frequency associated to this reduced amplitude is $f_{C''}$. This time the difference between $f_{C''}$ and the exciting frequency f_E is reduced causing an increased transmission of energy and consequently an increasing amplitude. Thus the oscillation approaches again the equilibrium represented by point C. This explains the stability of the oscillation represented by point C. The given consideration holds for any point of the full lined branch B of the tuning curve.

Now we consider an oscillation represented by the point D at the dotted part of the tuning curve; see Figure 10.

If for some reason the amplitude is increased to a value D' the natural frequency associated to that amplitude is $f_{D'}$. The difference between the exciting frequency and the natural frequency is thus reduced and consequently energy transmission increases causing the amplitude to rise even more. This process will not stop until a new equilibrium between energy transmission and energy losses is reached with an oscillation represented by point C on branch B.

If for some reason the amplitude represented by D is reduced to D'' the natural frequency associated to this oscillation will be $f_{D''}$. The difference between $f_{D''}$ and the exciting frequency increases and consequently the transmission of energy decreases causing a reduction of its amplitude. This process stops only if a new equilibrium between energy transmission and energy losses is reached. This equilibrium is represented by a point, which belongs to branch A in Figure 8. Branch A is characterized by small amplitudes where the natural frequency of the pendulum remains constant. In this case the conditions for energy input to the pendulum related to differences between driving frequency and the pendulum's natural frequency remain unchanged. Thus, the equilibrium at branch A is only determined by energy input and energy losses as explained in chapter 2.

Thus, oscillations represented by the dotted parts of the tuning curve are unstable. Any perturbation will start a process, which finally ends with oscillations represented, by the full-lined parts of branch A or branch B.

6. Experiments with Different Non-linear Restoring Forces

The gravitational pendulum is an example for a softening restoring torque. For a softening torque the graph torque against displacement is curved downwards while for a hardening torque this graph is curved upwards. Our introduction to non-linear effects can be extended if we investigate the consequences of different but well-defined non-linear restoring torques.

We modify our experimental arrangement. We replace the mass, which provided the restoring torque by an arrangement of springs to produce a restoring torque. A hardening restoring torque is obtained if two identical springs act nearly vertical from above and from below on the pendulum, as is shown by Figure 11. The springs are connected with the pendulum by fine threads fixed by pins or fine nails at a distance r to the axle.

Experimental values of the restoring torque as function of displacement are shown by Figure 12.

i) For small displacements, ($\varphi \leq 30°$), the restoring torque is approximately linear. Small displacements do not change the force of the two springs $F_0 = -x_0 \cdot C$ acting on an effective lever arm $r \sin \varphi$ which results in a torque T:

   $$\mathrm{T} = -r \cdot \sin \varphi \cdot 2 \cdot C \cdot x_0;$$

   r = distance between axle and pin. Thus the effective lever arm is: $r \sin \varphi$;
   x_0 = initial displacement of spring.

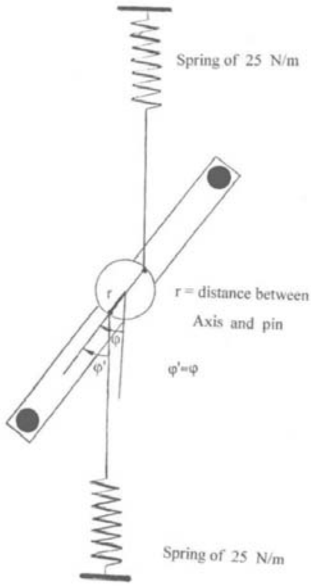

Figure 11. Arrangement to produce a hardening restoring torque.

ii) With increasing displacements the springs are stretched causing an increasing restoring force and this results in an additional restoring torque given by:
$T = -r \cdot \sin\varphi \cdot (C[x_{)} + r(1 - \cos\varphi)])$.
The restoring torque hardens up to displacements of about 80°.

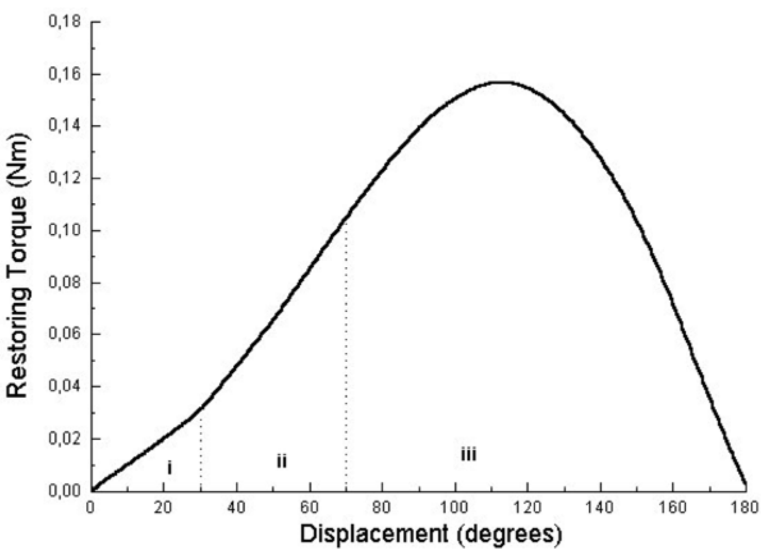

Figure 12. Restoring torque of the arrangement of springs according Figure 11.

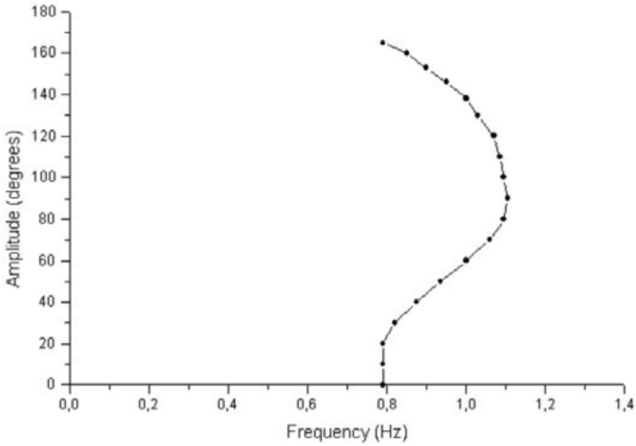

Figure 13. Amplitude versus natural frequency.

iii) With displacements exceeding this value the restoring torque decreases due to the reduction of the effective lever arm expressed by $r \cdot \sin\varphi$ and vanishes with a displacement of 180°.

The natural frequency of this pendulum depends in a non-trivial manner on the amplitude.

i) With small amplitudes below 30° we expect an approximately constant natural frequency.

ii) In the domain $30° < \varphi < 80°$ the restoring torque hardens and consequently the natural frequency increases, the curve amplitude versus natural frequency is bent to the right.

iii) In the domain $\varphi > 80°$ the restoring torque reaches a maximum and then it decreases. Consequently the natural frequency decreases resulting in a doubly bent curve amplitude versus natural frequency.

With this experimental arrangement we can demonstrate three different types of forced oscillations and associated tuning curves.

i) For small amplitudes resulting from small exciting torques we get the well known type of tuning curve with a distinct resonance maximum at the natural frequency f_0 for the linear case similar to Figure 2.

ii) If the exciting torques are increased two stable branches of the tuning curve can be identified repeating the procedures described for the gravitational pendulum. Starting from low frequencies we observe stable oscillations up to frequencies above f_0 with increasing amplitudes. At a certain point A_1, the branch A ends and we observe a jump effect and the amplitude decreases to a point B_1 on branch B (see Figure 14).

If we start the experiment from high frequencies well above f_0 we get stable oscillations represented by branch B. The amplitudes of which increase while

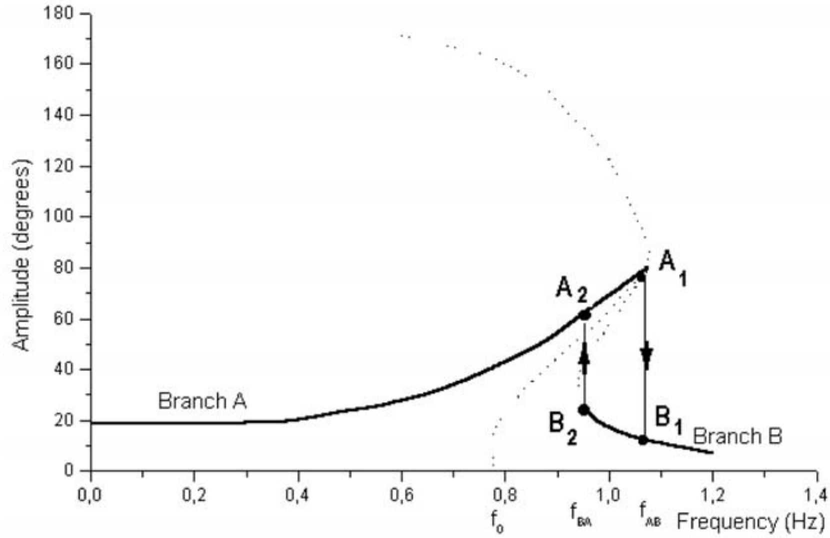

Figure 14. Two branches of a tuning curve due to a hardening restoring torque.

the exciting frequencies decrease. At point B_1 new jump effect happens and the amplitudes increase up to a point A_2 on branch B.

Both branches overlap and we notice a domain of bistability, where two different modes of forced oscillations are possible. As it has been shown in the previous section, we might complete the experimentally established branches to a complete tuning curve by a dotted part which represents unstable oscillations.

iii) If we increase the exciting torque even more we obtain three parts of the tuning curve representing stable oscillations. Two of these parts B_1 and B_2 (see Figure 15) represent oscillations with antiphase and may be regarded as belonging to the same branch B of the tuning curve since $f_E > f_{\text{natural}}$.

Within a small interval, three different modes of stable oscillations can be established caused by the same exciting torque. The dotted parts of the complete tuning curve again represent unstable oscillations.

To obtain oscillations represented by part B_2, we release the pendulum with appropriate initial conditions. Since this branch is the extension of the branch B starting from high frequencies its phase should be approximately anti-phase with respect to the exciting torque. Its amplitude should be about 150°. If thus released the pendulum oscillates at first with some perturbations, which damp out and after a few oscillations the stable mode is approximated. Varying the exciting frequency step-by-step the experimental values to establish branch B_2 can be obtained.

Within a certain interval (for our experimental design $0.98 \leq f \leq 1.06$ Hz) there exist three stable modes of oscillation for the same exciting frequency.

The oscillations of branch A are roughly in phase with the exciting torque while those of branch B_1 and B_2 are in antiphase.

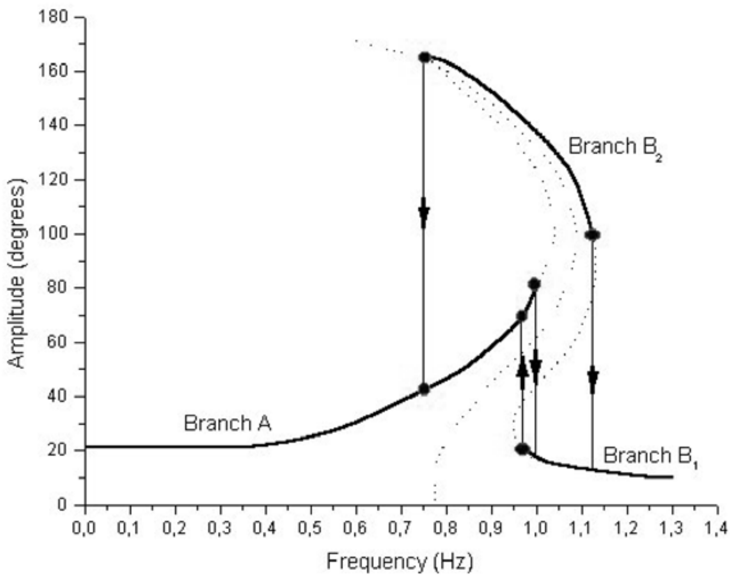

Figure 15. Doubly bent tuning curve. The three full-lined parts representing stable oscillations.

If two pendula are arranged it is a fascinating demonstration to have two identical pendula oscillate simultaneously both in antiphase but with two entirely different amplitudes.

7. Transition to Chaotic Behavior

Using the gravitational pendulum, the transition from regular oscillations to chaotic behavior can be demonstrated if the exciting torque is increased to get amplitudes approaching 180° (crank arm about 5 cm). Starting the experiment from frequencies well above f_0 the exciting frequency is gradually reduced obtaining amplitudes approaching values near 180°. Before this value is reached, however the oscillations become somewhat unstable. Small perturbations result in a turn over which initiates chaotic and irregular movements. If, as recommended before, two identical pendula are used it is most impressive to observe the transition from identical and regular movements of both pendula to irregular and uncorrelated movements. In all former experiments regular behavior prevailed- the pendula oscillated similarly or in a well-defined relation regarding phase and amplitude. In the chaotic mode there is no simple relation and no predictability.

8. Appendix

Details of the experimental arrangement are shown in Figure 1 in Section 2.

The pendulum is constructed by a symmetrical bar, which is to rotate about a horizontal axle in a support. The bar may be made of any material. We used a plastic bar 25 cm $\times$ 2 cm the mass of which is about 20 grams. The dimensions are not critical and may be changed.

A small mass fixed to one end produces the restoring torque. We used a coin the mass of which is about 3 grams. The given details of our arrangement may be changed, but for demonstration purposes the natural frequency should not exceed 1 Hz to facilitate observation. The axle is a spoke of a bicycle wheel, which can rotate freely in the support. The friction can be reduced using graphite powder as lubrication. A cardboard vane should be fixed to the rear end of the axle to generate air friction which can be regarded approximately proportional to velocity. Since energy loss is the product of friction and displacement, both proportional to amplitude, energy loss is proportional to the square of amplitude.

To investigate forced oscillations an electric motor is used which allows the frequency to be varied. To provide the exciting torque the exciting force acts from above by means of a spring (C= 10 N/m) and a thread twisted around the axle. To achieve sufficient torque the axle must be extended to a diameter of about 0.5 cm. The easiest way to achieve this is to roll adhesive tape around the axle. To avoid the thread slipping a piece of rubber tube or plastic tube is glued to the axle. To give the thread sufficient tension a 50 gram mass is fastened to its lower end.

It has been mentioned, with this driving device, an additional linear restoring torque is generated, which adds slightly to the restoring torque (see Figure 3). However with the given arrangement, these influences do not cause important errors.

The exciting torque is controlled by the motor's crank arm, which can vary from 0.5 cm to 5 cm.

A dial plate behind the pendulum facilitates control and measurement of amplitudes φ. A marker F fixed to the pulley above the pendulum facilitates the observation of phases between the exciting torque and the oscillations of the pendulum.

All demonstrations are more convincing if two identical pendula are arranged. This allows comparison of different oscillations and the easier observation of different amplitudes and phases. Both pendula have to be identical in natural frequency and damping. To control this, both should be released simultaneously from an initial dislocation of 90° observing simultaneity of its movements for about 10 oscillations. If there are differences in frequency these can be eliminated by readjusting the mass at the end of the bar. This changes mainly the restoring force and thus the natural frequency. Furthermore the damping should be similar.

To demonstrate qualitatively the dependency of period from amplitude two identical pendula can be released simultaneously with substantially different amplitudes. Within one period the differences show up.

To investigate quantitatively the dependency of natural frequency on amplitude, the period of one full oscillation should be assessed several times to reduce errors. From the periods the frequencies can be calculated.

To determine the tuning curve for the gravitational pendulum for the linear case, a small crank arm of about 1 cm or even less is to be used. In this case the friction should be as less as possible to get a distinct resonance peak.

To show the jump effect and bistability a crank arm of 2 cm to 3 cm is appropriate. Thus the exciting torque is increased proportional to the crank arm. The exciting frequency should be increased or decreased in small steps. Each variation causes disturbances and therefore it is necessary to wait a while to let the oscillation settle into its new stationary state. In this case air friction by a cardboard vane of about 6 cm $\times$ 6 cm fixed to the end of the axle helps to shorten the waiting time.

The basic arrangement for a hardening restoring torque is shown in Figure 11. Each spring is characterized by C= 25 N/m.

Two equal masses of 6 grams, are fastened symmetrically to the ends of the bar to increase the moment of inertia of the pendulum and to establish an appropriate initial natural frequency similar to the gravitational pendulum. The threads are connected with pins or fine nails fixed to the pendulum at a distance r of 1.0 cm to 1.5 cm from the axle.

9. Acknowledgements

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Experimental Control of Simple Pendulum Model

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Abstract. This paper conveys information about a Physics laboratory experiment for students with some theoretical knowledge about oscillatory motion. Students construct a simple pendulum that behaves as an ideal one, and analyze model assumption incidence on its period. The following aspects are quantitatively analyzed: vanishing friction, small amplitude, not extensible string, point mass of the body, and vanishing mass of the string.

It is concluded that model assumptions are easily accomplished in practice, within small experimental errors. Furthermore, this way of carrying out the usual pendulum experiments promotes a better understanding of the scientific modeling process. It allows a deeper comprehension of those physical concepts associated with model assumptions (small amplitude, point mass, etc.), whose physical and epistemological meanings appear clearly related to the model context. Students are introduced to a scientific way of controlling the validity of theoretical development, and they learn to value the power and applicability of scientific modeling.

1. Introduction

The direct references of the scientific theories are not the natural phenomena (because their are so complex) but the models, i.e., intellectual constructions based on generalizations, abstractions and idealizations (Bunge 1985).

In particular, simple models, in addition to their scientific relevance, are valuable didactic tools. By means of them, students can perform activities and make decisions consistent with those accepted by the scientific community, and control the adjustment between theory and reality.

2. The Problem

Our proposal is a Physics laboratory experiment for students with some theoretical knowledge about oscillatory motion. They have to construct a simple pendulum that behaves as an ideal one, and analyze model assumptions which affect its period. The following aspects are quantitatively analyzed: vanishing friction, small amplitude, not extensible string, point mass of the body, and vanishing mass of the string.

Among the various textbooks that treat the topic at an adequate level, for easy access to the lecturer, we chose a well known one: *Physics* (Vol. I) by Resnick et al. (1992).

M.R. Matthews et al. (eds.), The Pendulum, 67–76.

In particular, students must know the equation for the periods of (a) physical pendulum: any rigid body, suspended from some axis through it, that can oscillate on a vertical plane, and (b) ideal simple pendulum: a particle suspended from a light, not extensible string (Resnick et al. 1992):

(a) Physical pendulum period

$$T_p = 2\pi\sqrt{\frac{I}{mgd}} \tag{1}$$

where T_p represents the period of the physical pendulum, I the moment of inertia, m the mass of body, g the acceleration due to gravity and d the distance between the axis and the center of gravity of the system.

(b) Ideal simple pendulum period

$$T_s = 2\pi\sqrt{\frac{l}{g}} \tag{2}$$

where l is the length of the string.

Equation (1) was deduced assuming:

A_1: negligible friction (the resultant torque on the system about the horizontal axis is solely due to the weight of the body).

A_2: small oscillation amplitudes (in the equation of motion, the sine of the amplitude angle can be replaced by the angle in radians).

A_3: the pendulum is a rigid body (invariable mass distribution, constant moment of inertia).

In order to consider a simple pendulum as a particular case of a physical one, we must reformulate assumption A3. In fact, a string cannot be considered a rigid body, but the pendulum mass distribution can be considered invariable if the string keeps its length while the pendulum moves. Thus, we have:

A_3': the string must keep its length.

The system constructed by the students must satisfy these conditions, as well as two additional ones which are based on two requirements that allow us to pass from (1) to (2). These are associated with a simple expression for the moment of inertia and with the condition $d = l$. Thus,

A_4: the string mass must be negligible.

A_5: the body mass must be concentrated at a point.

In fact, under these conditions, Equation (1) can be written

$$\begin{aligned} T_p &= 2\pi\sqrt{\frac{I}{m \cdot g \cdot d}} \\ &= 2\pi\sqrt{\frac{m_p \cdot l^2}{m_p \cdot g \cdot l}} \end{aligned} \tag{3}$$

$$T_s = 2\pi\sqrt{\frac{l}{g}}$$

where m_p is the mass of the particle.

3. Analysis Of The Error Introduced By The Model Assumptions

Once the pendulum is properly built, students are asked to obtain g from (2) with a random error usually given in relative terms: $\varepsilon_g = \Delta g/g$ (suitable values for ε_g are about 5×10^{-2}). ε_g is associated with a random error in T:

$$\varepsilon_T = \varepsilon_g/2 \tag{4}$$

Equation (4) was deduced using (2) and assuming

$$\Delta l/l \lll \Delta g/g \tag{5}$$

which easily holds because l and ε_g are arbitrary.

Students must realize that, besides the random error, there is a systematic error in T including several independent terms, introduced by the fact that assumptions A_1 to A_5 are not fulfilled.

In the following analysis we will derive equations to value these terms, namely, the contributions due to friction (ε_f), initial amplitude (ε_α), variable length of the string due to a variable tension during oscillation ($\varepsilon_{\mathcal{T}}$), mass distribution of the body (ε_b), and mass of the string (ε_s). These equations can help students to design experimental system by determining the suitable values of certain variables so that real pendulum fits the model, i.e., the system can be designed so that the systematic error is negligible. Conversely, given a specific experimental system, students can use these equations to value the effect of model systematic errors, i.e., the systematic error in any given system may be not negligible, but can be subtracted from T. In any case, these equations allow students to control the agreement between a real pendulum and an ideal simple one.

Assuming the former case (negligible systematic error), the equation for the compound error, using (4), stands

$$\varepsilon_f + \varepsilon_\alpha + \varepsilon_{\mathcal{T}} + \varepsilon_b + \varepsilon_s \lll + \varepsilon_g/2 \tag{6}$$

3.1. VANISHING FRICTION

In a simple pendulum a friction effect may exist between the system and the medium in which it oscillates, and between the string and the oscillation axis. The latter is easier to avoid than to compute. Therefore, we suggest fastening the string to the axis firmly enough to avoid any movement of the knot. In particular, coiling must be avoided as well "loose ring" knots. If the string is coiled, during the oscillation it will coil and uncoil a fraction of turn, varying its effective length in each oscillation. On the other hand, if the knot is made as a loose ring, it will introduce an excessive friction the effect of which is difficult to calculate.

In order to evaluate the air friction, the torque, τ_R, that it produces on the system can be written:

$$\tau_R = F_R \cdot d \tag{7}$$

where d denotes the effective force arm, and F_R, the friction force. Both of them depend on the shape and size of the body, and F_R also depends on the velocity. Since the body is supposed to be small, we can assume that d is the distance between the oscillation axis and the center of gravity. Furthermore, F_R is

$$F_R = b \cdot v \tag{8}$$

which is valid for small velocities, b being the friction coefficient between the air and the body; and v, the tangential relative velocity. Including (7) and (8) in the differential equation of motion leads to

$$\alpha = \alpha_0 \cdot e^{\frac{-bt}{2m}} \cdot \cos(\omega' t + \varphi) \tag{9}$$

where α and α_0 are the displacement angle and the amplitude, respectively, the exponential is the damping factor, and ω' is equal to $2\pi f'$ where f' is the frequency of the damped pendulum.

Thus, for a damped simple pendulum, the period becomes

$$T' = 2\pi/\omega' = \frac{2\pi}{\frac{g}{l} - \left(\frac{b}{2m}\right)^2} \tag{10}$$

In Equation (10), it can be seen that ω' is different from the angular velocity of a not damped pendulum, namely, $\omega = (g/l)^{1/2}$. If the second term in the root of (10) is much smaller than he first one, the air friction will be negligible. Unless the body is made of an extremely low density material, this condition easily holds in most of the common cases. b can be computed from (9), by eliminating the cosine dependence, setting $t = nT$ in the exponential, and measuring the amplitude after an entire number, n, of oscillations.

The relative systematic error in the period, introduced by using (2) instead of (10) is

$$\frac{T' - T_s}{T_s} = \frac{\dfrac{2\pi}{\sqrt{\frac{g}{l} - \left(\frac{b}{2m}\right)^2}} - \dfrac{2\pi}{\sqrt{\frac{g}{l}}}}{\dfrac{2\pi}{\frac{g}{l}}} \tag{11}$$

After dividing numerator and denominator of (11) by $2\pi(g/l)^{-1/2}$, this equation becomes

$$\frac{T' - T_s}{T_s} = \left(1 - \frac{\left(\frac{b}{2m}\right)^2}{\frac{g}{l}}\right)^{-1/2} - 1 \tag{12}$$

Expanding the parenthesis on the right hand side as a binomial series, and neglecting third and higher order terms, we obtain

$$\frac{T' - T_s}{T_s} = \frac{1}{2}\frac{\left(\frac{b}{2m}\right)^2}{\frac{g}{l}} \tag{13}$$

Therefore, for the friction effect to be vanishing, it suffices to equate the right hand side of (13) to ε_f fitted by (6).

3.2. SMALL INITIAL AMPLITUDE

The general equation for the period, including amplitude dependency, is (Resnick et al. 1992):

$$T_\alpha = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1^2}{2^2}\sin^2\left(\frac{\alpha}{2}\right) + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2}\sin^4\left(\frac{\alpha}{2}\right) + \cdots\right) \tag{14}$$

Thus, truncation on the second term, in order to compute the error, leads to

$$T_\alpha = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1^2}{2^2}\sin^2\left(\frac{\alpha}{2}\right)\right) \tag{15}$$

Therefore, the relative systematic error introduced by using (2) instead of (15), which depends on amplitude, is

$$\frac{T_\alpha - T_s}{T_s} = \frac{1^2}{2^2}\sin^2(\alpha/2) \tag{16}$$

from which the maximum initial amplitude allowed can be valued, after the right hand side is set equal to ε_α fitted by (6).

3.3. INEXTENSIBLE STRING

To analyze this condition we must consider two features: (i) string deformation while applying the weight statically, and (ii) string deformation during the oscillation.

(i) While applying the weight statically, the string can undergo a considerable elongation without altering the model. It suffices that the deformation remains under the elastic limit and the lineal density remains constant, in order to compute the moment of inertia.

(ii) During the oscillation, one must keep in mind that a variable tension is applied to the string, due to the radial component of the weight, and the centripetal force associated with the circular motion:

$$\mathcal{T} = W \cdot \cos\alpha + m \cdot \frac{v^2}{l} \tag{17}$$

where W is the weight of the body, and v, its velocity with respect to the earth.

Equating the maximum kinetic energy of the body to its maximum potential energy, one can compute its maximum velocity, v_m, when it passes through its equilibrium position:

$$v_m = \sqrt{2.g.l(1 - \cos\alpha_0)} \tag{18}$$

A minimum tension is exerted when the pendulum is in its maximum amplitude, and a maximum tension when it passes through its equilibrium position. The difference of tension between these extreme values is

$$\Delta\mathcal{T} = 2W(1 - \cos\alpha_0). \tag{19}$$

Therefore, once the initial amplitude is fixed using (16), one must choose a string strong enough (whether due to its Young coefficient and/or to its size), or a body light enough, so that the difference of tension given by (19) stretches the string an amount not greater than that fixed for the error of its length (we have assumed $\Delta l/l \ll \Delta g/g$). Hence, it is suggested that vinyl strings and textile fibers be avoided.

3.4. POINT MASS OF THE BODY

The distribution of the oscillating body mass affects its moment of inertia. The moment of inertia about an arbitrary axis may be written as follows (Resnick et al. 1992):

$$I = I_G + m \cdot d^2 \tag{20}$$

where I_G is the moment of inertia about a parallel axis through the center of mass of the body, m is the mass of the body, and d is the distance between the two axis.

The second term on the right hand side in (20) is the moment of inertia of a point mass about the suspension point of the pendulum, and we can interpret the first term as being the error due to the non-point character of the mass of the body. So, the mass of the body may be considered to behave as a "point mass" if the ratio between the first and the second terms is very small.

To know how small this quotient must be, let us consider the relative error due to the use of the simple pendulum period expression (T_s) instead of the physical pendulum period expression (T_p). We will assume that a string of vanishing mass and a body of finite size form the physical pendulum. On the other hand, in the simple pendulum period expression we will use "d" instead of "l", given their identity in the case under consideration. We obtain:

$$\frac{T_p - T_s}{T_s} = \frac{2\pi\sqrt{\dfrac{I_G + m \cdot d^2}{m \cdot g \cdot d}} - 2\pi\sqrt{\dfrac{d}{g}}}{2\pi\sqrt{\dfrac{d}{g}}}$$

$$= \frac{2\pi\sqrt{\dfrac{d}{g}\left(\dfrac{I_G}{m \cdot d^2}\right)} - 2\pi\sqrt{\dfrac{d}{g}}}{2\pi\sqrt{\dfrac{d}{g}}}$$

$$= \left(\frac{I_G}{m \cdot d^2} + 1\right)^{1/2} - 1 \tag{21}$$

Expanding the statement between parenthesis in (21) as a binomial series and neglecting third and higher order terms, it can be seen that the mass of the body may be considered a "point mass" if the following relationship is verified:

$$\frac{1}{2}\frac{I_G}{m \cdot d^2} \leq \varepsilon_b \tag{22}$$

where ε_b must be fitted by (6).

3.5. VANISHING MASS OF THE STRING

The mass of the string affects the oscillation period because:

- it contributes to the moment of inertia of the system
- it changes the position of the center of gravity of the system
- it changes the mass of the system.

Let us first consider how the mass of the string influences the moment of inertia of the system. The moment of inertia of the system about the point of suspension is:

$$I = I_{\text{string}} + I_{\text{body}}. \tag{23}$$

In equation (1), this effect of I_{string} appears as an increase in the oscillation period.

Let us now consider how the mass of the string influences the position of the center of gravity of the system. The distance between the oscillation axis and the gravity center is (Resnick et al. 1992):

$$d = \frac{m_s \cdot l/2 + m_b \cdot l}{m_s + m_b} \tag{24}$$

In Equation (24), it can be seen that $d < l$. So, in Equation (1), this effect of m_{string} appears as an increase in the oscillation period.

Let us finally consider how the mass of the string influences the mass of the system.

The total mass of the system is:

$$m = m_{\text{string}} + m_{\text{body}}. \tag{25}$$

In Equation (1), this effect of m_{string} appears as a decrease in the oscillation period.

Now we will insert (23), (24) and (25) in Equation (1), to find an expression which shows the quantitative difference between the period values predicted by the physical and the simple pendulum models.

We will take into account that the moment of inertia of a homogeneous string is given by the following expression (Resnick et al. 1992):

$$I_{\text{string}} = \frac{1}{3} \cdot m_{\text{string}} \cdot l^2 \tag{26}$$

and we will define:

$$k = m_s/m_b. \tag{27}$$

We obtain:

$$T_p = 2\pi\sqrt{\frac{I}{m \cdot g \cdot d}}$$

$$= 2\pi\sqrt{\frac{I_s + I_b}{(m_s + m_b) \cdot g \cdot \dfrac{m_s \cdot 1/2 + m_b \cdot l}{m_s + m_b}}}$$

$$= 2\pi \sqrt{\frac{\left(\frac{1}{3}k + 1\right) \cdot m_b \cdot l^2}{g \cdot \left(\frac{1}{2}k + 1\right) \cdot m_b \cdot l}}$$

$$T_p = 2\pi \sqrt{\frac{l}{g}} \cdot \sqrt{\frac{\frac{1}{3}k + 1}{\frac{1}{2}k + 1}} \tag{28}$$

The second root in the right hand side of (28) is less than unity. Thus, $T_p < T_s$, i.e., the total influence of the mass of the string is to increase the period value predicted by the simple pendulum model.

We can write:

$$\frac{T_p - T_s}{T_s} = \sqrt{\frac{l}{g}} \cdot \sqrt{\frac{\frac{1}{3}k + 1}{\frac{1}{2}k + 1}} - 1 \tag{29}$$

According to the difference just mentioned between T_p and T_s, this expression gives a negative relative error due to a non-vanishing mass of the string. Thus, it suffices for the mass of the string effect to be vanishing that the right hand side of (29) is equal ε_s fitted by (6).

4. Conclusions

From quantitative analysis of systematic error we have shown that model assumptions are easily accomplished in practice, within small experimental errors. Considered separately, within an error of 1%:

– an initial amplitude of 23° is “small”.
– a sphere, whose diameter is 30% of the length of the string, is “a point mass”.
– a mass of the string equal to 10% of the mass of the body is “vanishing”.
– any elastic elongation suffered by the string during the static process of loading is negligible, providing the string length is measured after the loading.
– without loosing its property of ’not extensible’, the string may vary its length during oscillation (due to a variable tension), providing this variation is less than the measurement error of the string length.

This way of carrying out the usual pendulum experiments:

– promotes a better understanding of the scientific modeling process.
– allows a deeper comprehension of those physical concepts associated with model assumptions (small amplitude, point mass, etc.), whose physical and epistemological meanings appear clearly related to the model context.

– introduces students to a scientific way of controlling the validity of theoretical development, and helps them to value the power and applicability of scientific modeling.

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Soup-can Pendulum

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Abstract. In these studies, a vegetable can containing fluid was swung as a pendulum by supporting its end-lips with a pair of knife edges. The motion was measured with a capacitive sensor and the logarithmic decrement in free decay was estimated from computer-collected records. Measurements performed with nine different homogeneous liquids, distributed through six decades in the viscosity η, showed that the damping of the system is dominated by η rather than external forces from air or the knife edges. The log decrement was found to be maximum (0.28) in the vicinity of $\eta = 0.7$ Pa s and fell off more than 15 fold (below 2×10^{-2}) for both small viscosity ($\eta < 1 \times 10^{-3}$ Pa s) and also for large viscosity ($\eta > 1 \times 10^{3}$ Pa s). A simple model has been formulated, which yields reasonable agreement between theory and experiment by approximating the relative rotation of can and liquid.

1. Introduction

Pendulum damping is usually thought of as originating from forces external to the oscillating member – as for example, from air or knife edges. There are many mechanical oscillators, however, for which the primary damping mechanism is internal friction. A recently studied example is that of the long-period pendulum studied by Peters and Pritchett (1997). The present paper describes another pendulum, whose period is short (<0.5 s), and which is also influenced primarily by internal friction. The study was partly motivated by the mechanics of rolling vegetable cans. Although a proper interpretation of some results can be tricky as shown by Nickas (1989), it has become commonplace for physics teachers and their students to compare the rolling speed of two different vegetable cans on an inclined plane. The popularity of these demonstrations suggested that it might also be fascinating to study a "pendulating" vegetable can. Part of the fascination with the soup-can pendulum derives from early observations in which behavior differences of the type illustrated in Figure 1 were noted.

Shown in this figure is the decay of oscillatory amplitude for each of two different vegetables: (i) blackeyed peas, and (ii) sweet peas. Unlike the blackeyed peas case, for which there is little damping, a dramatic loss occurs when the pendulum is a can of sweet peas. In all studies presently reported, the period of oscillation is in the vicinity of 0.45 s, using common vegetable cans of size 7.4 cm dia. by 11.2 cm length, and 56 g empty can mass. Although some variations in period were noted from case to case, as expected; these changes were small compared to the primary

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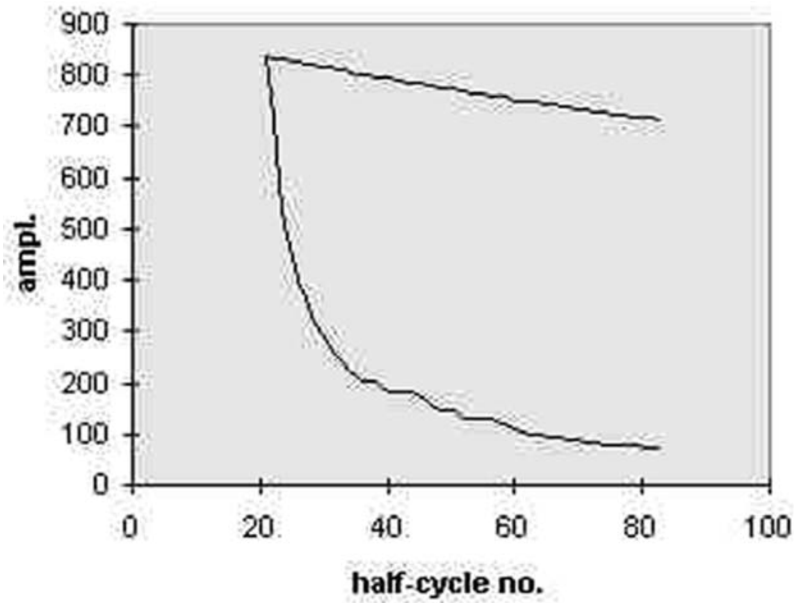

Figure 1. Comparison of the decay of two different vegetable cans.

variation, which is the damping. Later studies are planned in which the second order effects of period will be addressed. The remarkable difference between the sweet peas and blackeyed peas was not anticipated by means of other comparisons. For example, shaking the cans revealed a significant volume of water packed with each of the vegetables. In a rolling comparison it was found that the sweet peas were faster than the blackeyed peas down an incline, but not with as huge a difference as in Figure 1. Another interesting feature of Figure 1 are the "steps" in the decay of the sweet peas. Apparently the peas tend to organize in groups, the size of which depends on pendulum amplitude. Thus there is evidence for granularity giving rise to self-organized criticality (Bak et al. 1988). The understanding of these effects must also await future studies.

2. Pendulum Design

Shown in Figure 2 are (i) the support structure for the pendulating can and (ii) the placement of the symmetric differential capacitive (SDC) sensor to monitor position.

The static electrodes of the sensor were attached to the support frame with super glue, near the top end of the can, using two pieces of 8 mm thick lucite. The frame was constructed from a 11 cm long section of "c" channel aluminum of 8 mm wall thickness and 15.3 cm width. Two small aluminum pieces to hold the knife edges were welded to the 4 cm high sides, using a tungsten inert gas, or TIG welder. These optionally could have been attached with screws. The clearance between

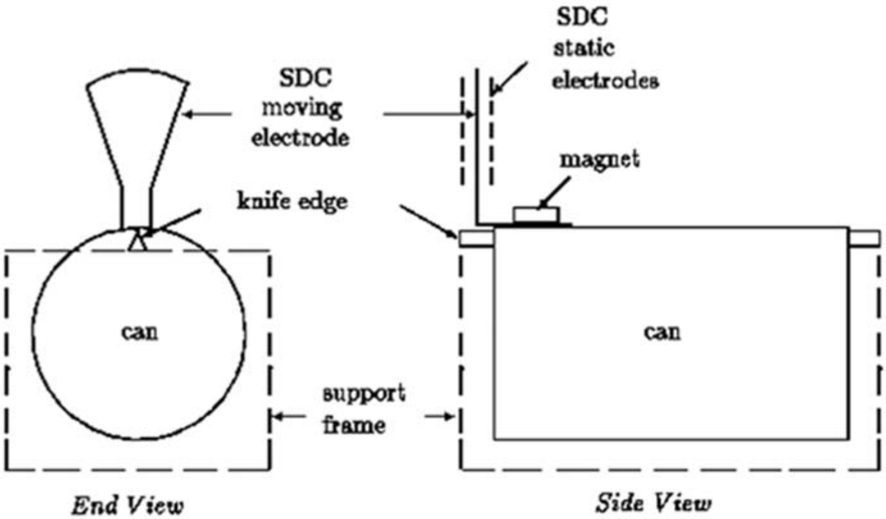

Figure 2. Illustration of the soup-can pendulum.

the bottom of a can and the frame is about 1 cm. The knife edges were made from a section of bandsaw blade 0.5 mm thickness, 1.2 cm width. The teeth were ground off the blade, and each knife edge was sharpened in the vicinity of the end which contacts the lip of the can. One of these was press fitted into a slot cut in its small aluminium holder, and the other was hinged with a small steel pin so that the can may be easily mounted and dismounted from the frame. The doubly differential capacitive sensor, which is described elsewhere (Peters 1993) comprises a pair of static electrodes held in parallel proximity, and a third planar electrode which moves between the static pair. For the present experiments, the moving electrode was cut with scissors from thin sheet aluminum. A near right angle bend in the lower section of this "fan-shaped" piece permits it to be fixed in position on top of the can by a small magnet.

3. Experimental Technique

When filled with inhomogeneous vegetables, the motion of the can pendulum is hopelessly complicated, relative to a first effort at theoretical modelling of the system. The difficulty of such a task may be appreciated by simply inspecting the sweet pea decay case in Figure 1. For this reason we chose to first look at decays (for serious study) in which the can is filled with a variety of different homogeneous liquids, whose primary difference is their viscosity, η. In the results which follow, it will be seen that a range in η of more than six orders of magnitude is readily achieved, using only liquids which are common to most physics departments. To insure a meaningful comparison among runs with different liquids, the vegetable can selected for use was in all cases the one whose dimensions were indicated in the discussion of Figure 1. Following the purchase of a can from the grocery store, the vegetable contents had to be emptied. To facilitate mechanical integrity after refilling, the lid was separated from the body of the can using a can-opener (Culinare) that cuts through the narrow outside crimp in the end-lip. Not only does this technique result in safe products of separation, since there are no sharp edges on either the can or its lid; but also their smooth separation permits the pair to be rejoined, after filling with a test liquid, by means of a thin layer of glue.

3.1. DATA COLLECTION

The analog data from the sensor electronics is input to the 33 MHz 486 PC computer by means of a Metrabyte 1401 analog to digital (A/D) converter. Software of both acquisition and processing types was written in QuickBasic (compiled), and the hybrid code had in some cases been written by Metrabyte and in other cases by the author. Two different modes of operation are employed. The setup mode is a real-time one in which the duration of the record graphed on the monitor, and the full-scale sensitivity of the electronics, are chosen after a prompt is displayed. This permits the operator to adjust the electronics offset for a mean output that is in the vicinity of zero. The pendulum displacement is mapped against time on the monitor using the 'pset' software command. In this mode, the computer emulates an old-fashioned strip-chart recorder. The second mode is one in which a record of 2048 points (2 K) is written to memory of the computer for later analysis. During collection of the record, no graphical information is available for viewing. For all cases presently reported, the 2 K records from which figures were produced, were collected using a full scale sensitivity of ± 0.1 V. For the sensor used in these experiments, the calibration constant corresponding to this A/D sensitivity, was $1,5 \times 10^5$ counts/rad for counts in the range -4095 to 4096.

It is possible to view the raw data directly, as illustrated in Figure 3, which shows the vast difference between a can filled with glycerin and a reference case for which there is insignificant internal friction.

For this comparison, the ordinate values were normalized to the initial peak amplitude, which is a straightforward operation with the Microsoft Excel software that was used to produce all graphs. For a given case, one simply imports from memory to Excel the 2 K record of interest, and then responds to prompts generated by the chart "wizard". The abscissa values (time) are integer $\cdot \Delta t$ where $\Delta t = 30$ s/2048. In addition to the obvious difference between the decay constants in the two cases of Figure 3, one can also see that the period of the motion is slightly greater when the can contains glycerin. As compared to the huge difference in damping coefficients, the variation with period is second order, as previously mentioned. To produce the reference decay, brass weights were fixed inside an empty can (56 g mass), the amount selected to approximate the mass of a water filled can (510 g). The log decrement in this reference ($5.0 \pm 0.4 \times 10^{-3}$, $R^2 = 0.993$) is evidently influenced largely by the knife edges. By contrast, the

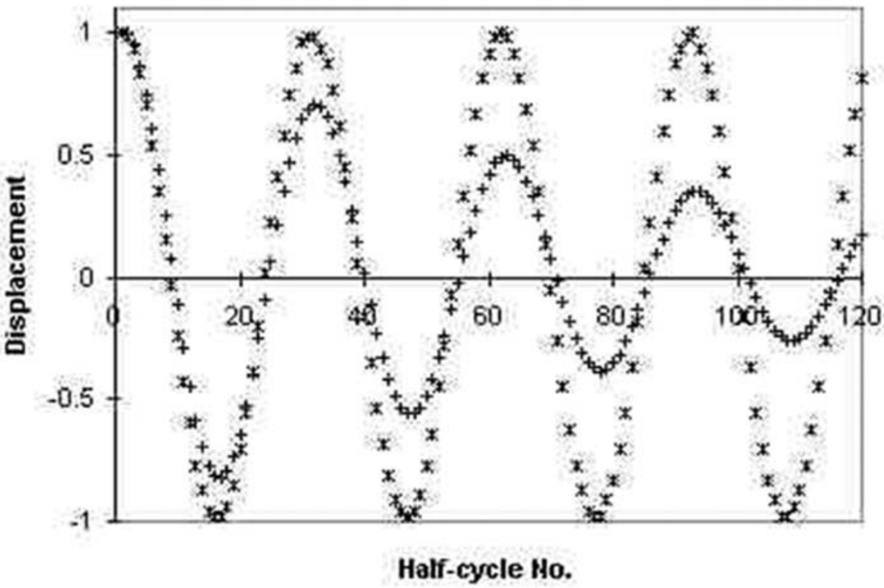

Figure 3. Decay of glycerin compared to a reference decay.

empty can ($8.1 \pm 0.9 \times 10^{-3}$, $R^2 = 0.998$) is evidently influenced primarily by the viscosity of surrounding air.

The decay constant which is referred to as the log decrement is defined as

$$\text{L.D.} = \ln(\theta_N/\theta_{N+1}) \tag{1}$$

where θ_N and θ_{N+1} are the displacements of a pair of turning points of like sign separated in time by one period of the oscillation.

In a graph which follows (Figure 6) of pendulum damping against viscosity, the reference damping was subtracted from the measured damping to yield the internal friction part. Only at low values of the damping was this correction significant.

4. Theoretical Model

The system was modelled by two coupled classical differential equations, based on Newton's 2nd law:

$$\ddot{\theta} + c \cdot \eta^{1/2}(\omega - \omega_L) + \theta = 0 \tag{2}$$

and

$$\ddot{\theta}_L = \eta^{1/2}(\omega - \omega_L) \tag{3}$$

where θ is the angular displacement of the can, θ_L (less well defined) describes the displacement of the liquid, $\omega = \mathrm{d}\theta/\mathrm{d}t$ and $\omega_L = \mathrm{d}\theta_L/\mathrm{d}t$. The constant c is the one adjustable parameter in the model, and η is the viscosity. Being concerned primarily with trends in the damping versus viscosity, the factor that would normally multiply the term in θ has been set to unity. Thus the period of oscillation of the model system is 6.28 when $\eta = 0$, rather than the actual experimental period in the neighborhood of 0.45 s.

The model equations were first tested in a limiting simple case; i.e., by removing the θ term in Equation (2) and noting that ω and ω_L approach a common value exponentially, with a time constant inversely proportional to $\eta^{1/2}$ To solve (2) and (3) numerically, they were first rewritten as an equivalent coupled set of four first order equations:

$$\dot{\omega} = -c\eta^{1/2}(\omega - \omega_L) - \theta \tag{4}$$

$$\dot{\omega}_L = \eta^{1/2}(\omega - \omega_L) \tag{5}$$

$$\dot{\theta} = \omega \tag{6}$$

$$\dot{\theta}_L = \omega_L \tag{7}$$

Although the liquid motion is undoubtedly complicated in most cases, a simplifying assumption has been made – that the effective angular momentum of the liquid in this "normalized" model, is proportional to $\eta^{1/2}$. This assumption is based on comparisons of theory and experiment with rotating liquids (Greenspan 1968).

5. Numerical Technique

The equations of motion (4)–(7) were integrated, using QuickBasic, on the basis of the last point approximation (LPA) (Cromer 1981). The author has used this algorithm instead of Runge Kutta or other techniques since the 1980's. Even in celestial mechanics calculations performed for orbital rendezvous and antisatellite intercepts, the LPA was found to be quite acceptable as far as errors are concerned, and much easier to both understand and implement than the algorithms traditionally known to the computational physics community. Careful comparative studies over the past decade by graduate students under the direction of Prof Tom Gibson at Texas Tech University, have shown that the LPA is also unsurpassed in terms of code size and CPU times for execution. The most common use of LPA has been for systems described by fewer equations than the present pendulum, even though the previous equations involved nonlinear terms necessary to produce chaos. An example is provided in Peters (1995). A testament to the prowess of LPA in the present modelling is the following observation: When Equations (4)–(7) were integrated (single precision) with approximately 20 widely distributed values of η, and the turning points fitted to an exponential; the R^2 of the resulting fit was in every case unity according to Excel – meaning a perfect fit to within at least 4 significant figures. This was true for a particular value of the time step, Δt, and the essential (uncommented) code which was used is supplied in Table I.

Table I. QuickBasic LPA code to integrate Equations (4)–(7). (The viscosity parameter is set to $\eta = 1.0$, corresponding to glycerin).

```
SCREEN12
VIEW (0, 0)–(600, 470)
WINDOW (0, −1)–(1, 1)
L = 1: LL = 0: th = 0: dt = 0.005
eta = 1.0
start:
t = t + dt
L = L − th * dt − 0.16 sqrt(eta) * (L − LL) * dt
LL = LL + sqrt(eta) * (L − LL) * dt
th = th + L * dt
PSET (0.01 * t, 0.5 * LL), 2
PSET (0.01 * t, 0.5 * L)
GOTO start
STOP
```

It should be noted that the integration of ω_L (LL in the code) to obtain θ_L is not performed since this variable was not used. To estimate the log decrement, whether of experimental data or of output from the code of Table I written to a file (the write statement is not indicated in the table), a QuickBasic software program was produced to identify the turning points of the damped sinusoid. The peak-to-peak amplitude of the motion, which is the sum of the absolute value of adjacent turning points of opposite sign, was then plotted vs time expressed in half-cycle integers.

6. Model Features

Before considering a detailed theory of the pendulum, it was clear that the log decrement would exhibit a maximum at some midrange value of the viscosity, since the damping mechanism must depend on relative rotation of can and liquid. At very high viscosity, the "liquid" is fully coupled to the can, and the absence of relative motion eliminates damping. At very low viscosity there is maximum relative rotation but the absence of coupling prevents exchange of energy and so damping is also low.

6.1. MODEL PARAMETER VARIATIONS WITH η

Using Equations (4)–(7), the phase and amplitude features of the liquid were determined as a function of η. As used here, phase is the angle with which the liquid's

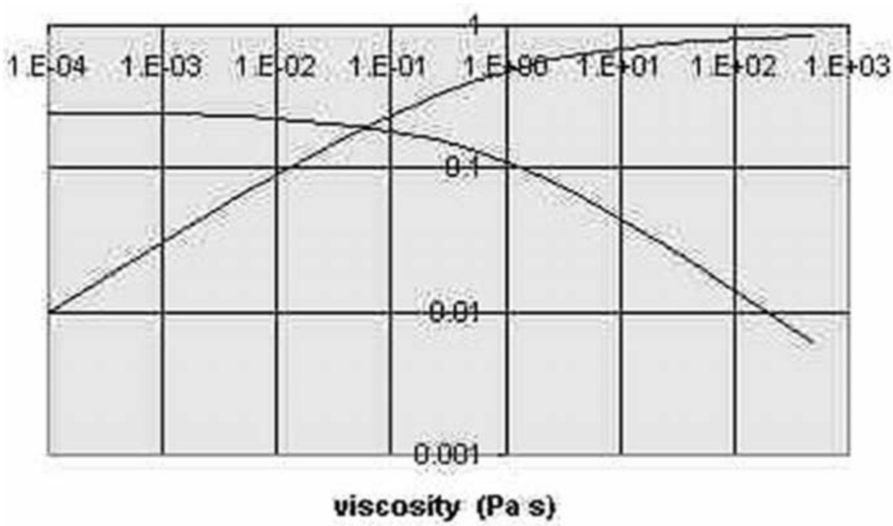

Figure 4. Phase and amplitude of liquid angular momentum vs viscosity.

angular momentum $\mathrm{d}\theta_L/\mathrm{d}t$ lags behind that of the can, $\mathrm{d}\theta/\mathrm{d}t$ "Amplitude" is the value of the first peak of $\mathrm{d}\theta_L/\mathrm{d}t$, obviously influenced by the initial conditions, which were in all cases $\theta = 0 = \omega_L$ and $\omega = 1$.

The results are shown in Figure 4, where the phase is seen to decrease with increasing η, from an initial value of 0.25 ($\times 2\pi$).

It should be noted that an increase in amplitude of θ_L (toward 1 as $\eta \to \infty$ in Figure 4) corresponds to a decrease in relative motion between can and liquid.

6.2. PERIOD

The period variation was not compared directly with experiment for this study; however, the model does predict that it should increase by about 8% as η increases from 10^{-3} to 10^2, as illustrated in Figure 5.

7. Comparison of Theory and Experiment

To compare theory with experiment, the value of η in the model code was set at the viscosity appropriate to the liquid considered (value shown in Table I being $\eta = 1$, corresponding to glycerin). A record was then written to memory (every 10th point, separated in time by 0.05), which could be compared to the corresponding experimental record for that liquid. In all cases, the log decrement was computed by first finding the turning points in a given record. Then the peak-to-peak values were computed as previously indicated. Finally, an exponential fit to these peak-to-peak values was generated using Excel, from which the log half-decrement was obtained as the coefficient in the exponential fit. For the model sets, $R^2 = 1$ in all

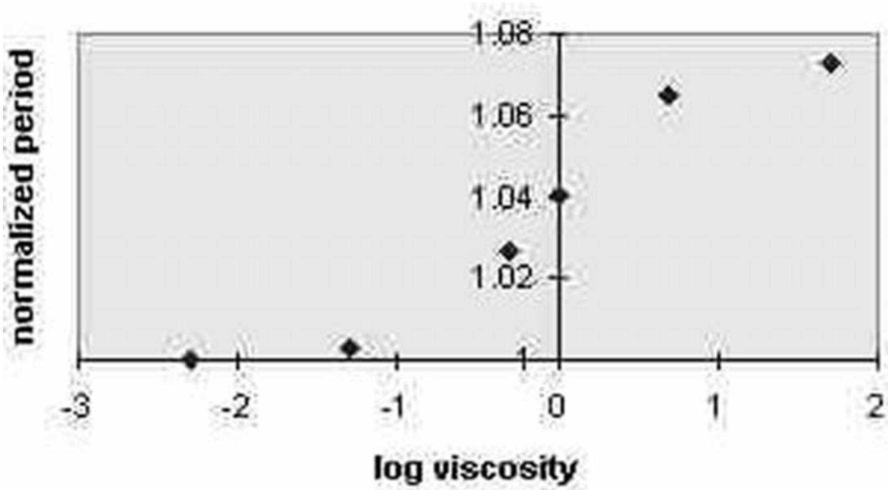

Figure 5. Variation of normalized model period with viscosity.

Table II. Liquids considered in the study.

Liquid	Viscosity (Pa s)	Mass density (g/cm^3)
Acetone	3×10^{-4}	0.79
Water	1×10^{-3}	1.00
Sugar water	7×10^{-3}	1.02
Vegetable oil	9×10^{-2}	0.86
Mineral oil	1×10^{-1}	0.82
Glycerin	1×10^{0}	1.26
Corn syrup	3×10^{0}	1.28
Honey	1×10^{1}	1.30
Corn starch	1×10^{3}	1.03

cases as noted previously. The experimental data typically showed some amplitude dependence to the decay constant, but $R^2 > 0.93$ in all cases. The liquids which were considered in this study are indicated in Table II.

The sugar water was made by dissolving 40% by weight of sucrose in water to produce a handbook listed viscosity of the indicated amount. All values of viscosity less than or equal to that of glycerin in Table II were obtained from handbooks. The value of η for liquids of higher viscosity was estimated relative to that of glycerin, using Stoke's Law. A small steel sphere was dropped in a test tube full of the liquid and the descent time was measured with a stopwatch. This time was then compared against the fall time using glycerin.

It should be noted that η is sensitive to temperature and therefore all experiments were performed at a laboratory temperature close to 23 °C. In addition to

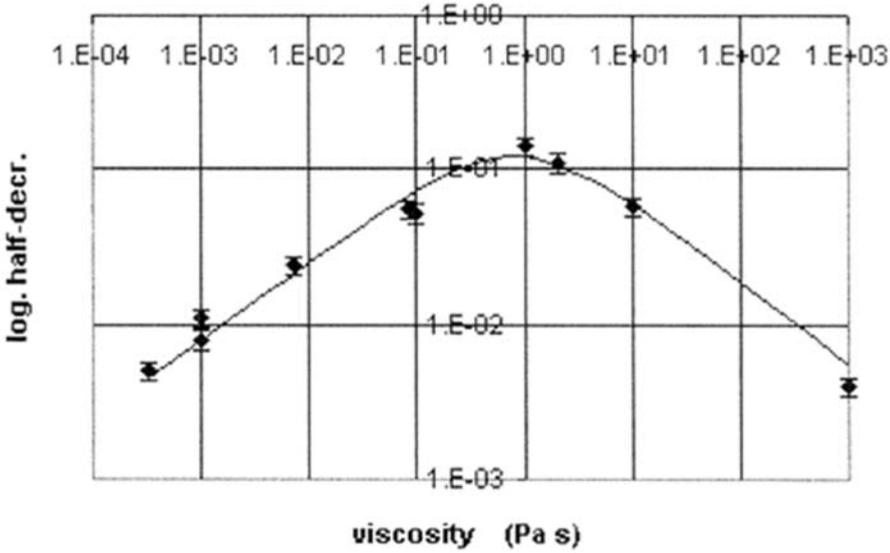

Figure 6. Damping vs viscosity-comparison of theory and experiment.

the viscosities, the densities of the liquids are also provided in Table II. These were estimated from mass and volume measurements and their uncertainty is about 3%. For an ideal comparison of experiment and present theory, all liquids would have the same density, which is not possible. This complication will be discussed later. Theoretical damping (solid curve) is compared against experiment (data points with error bars in the log half-decrement) in Figure 6.

For this graph the single adjustable parameter of the model, c of Equation (4) was set to 0.16 – the value which was found by trial and error to give the best agreement with experiment. The $\pm 16\%$ 1σ-uncertainty (error bars of Figure 6) is based on careful measurements done on glycerin, corn syrup, and the sugar water, using a set of 24 different records in each case. The range of the results for the three was from 14% to 17%, and smaller sample statistics with the other liquids suggested that an uncertainty of 16% is fairly representative of all cases. The uncertainty in η is much harder to quantify, particularly for the corn starch, which may be non-Newtonian. The estimate of its value at 1000 Pa s could be wrong by as much as 50%. For the other liquids, the uncertainty in viscosity is probably in the neighborhood of 20%. In Figure 6 two points are shown for water, to emphasize that there is amplitude dependence to the damping in this experiment. The larger value of the log half-decrement was obtained with the pendulum oscillating at a ten times larger amplitude. It was also noted that the R^2 declined from 0.99 for the least viscous liquids to 0.94 for the most viscous ones.

7.1. INFLUENCE OF MASS DENSITY DIFFERENCES

In the model, the mass of the pendulum has been assumed constant from one liquid to the next, which clearly is not true. The invariant quantity being the volume of the liquid, and since the can mass is only about one-tenth the liquid mass; we estimate from the densities of Table II that the mass of the acetone pendulum is about 80% that of water and that of the honey pendulum is about 130% that of water. These are the extremes of the variation for the liquids used in the present study. Future studies will consider whether the kinematic viscosity would be the better variable with which to make the comparison; i.e., division of η by the density. This seems reasonable since the damping constant, for a given viscosity, should decrease with increasing mass. Moreover, the kinematic viscosity is routinely used instead of absolute viscosity in engineering comparisons of gases. For the data of Figure 6, the difference between the graph given and one based on kinematic viscosity is not great enough to warrant redrawing the figure. A significant reduction in the viscosity uncertainties, however, would make this meaningful.

8. Use of the Can-Pendulum as a Viscosimeter

The reasonably good agreement between theory and experiment suggests that the system may be used as an instrument for measuring viscosity. Most liquids for which one would want to measure η are less viscous than glycerin. Therefore a power law fit to the low viscosity segment of Figure 6 was performed. The resulting $R^2 = 0.986$ is not outstanding, but still close enough to unity that ≈20% estimates in η should be possible from measured log decrements, by inverting the expression: log half-decr. $= 0.145\eta^{0.3936}$ for $\eta < 1$. Note that the log half-decrement has been used in these graphs, rather than the log decrement. To get the latter from the former, one need only multiply by a factor of 2.

9. Conclusions

It has been shown that some features of a liquid-filled can-pendulum can be readily understood, whereas others may be so complicated as to defy simple explanation. Pedagogically, it appears to be a system that is rich in new possibilities for improved teaching of the old physics of classical mechanics. Particularly when the fluid of the can is made inhomogeneous by mixing solid particles with a pure liquid, unexpected behaviour can result. For example, it appears that dynamic organization of particles can then occur for some-conditions, the nature of which are not yet understood. Planned future studies will attempt to understand these peculiar features. Future studies will also deal with the importance of pendulum mass, as well as size of the can. The theory which motivated the $\eta^{1/2}$ feature in Equations (4) and (5) also predicts that the dimensions of the can are important to the damping. Thus, an obvious follow-on experiment would be one to verify the functional dependence on can diameter.

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What Makes the Foucault Pendulum Move Among the Stars?*

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Abstract. Foucault's pendulum exhibition in 1851 occurred in an era now known by development of the theorems of Coriolis and the formulation of dynamical meteorology by Ferrel. Yet today the behavior of the pendulum is often misunderstood. The existence of a horizontal component of Newtonian gravitation is essential for understanding the behavior with respect to the stars. Two simple mechanical principles describe why the path of oscillation is fixed only at the poles; the principle of centripetal acceleration and the principle of conservation of angular momentum. A sky map is used to describe the elegant path among the stars produced by these principles.

1. History

On March 26 1851, 150 years ago, the Panthéon was the scene of a dramatic exhibition as Leon Foucault demonstrated his 67-m pendulum. The slow clockwise precession of the oscillation path amazed the onlookers, and the news spread rapidly around the world. Pendulums were immediately set up on all continents. For the first time it was possible to witness the turning of the earth in a closed room, with no reference to the skies. No longer was it reasonable to think that the sun and stars revolved around the earth; the evidence for the revolving earth was simply too strong (Deligeorges 1990). The demonstration took place at the time that meteorology, as we know it, was beginning to be formulated as a branch of mechanics (Ferrel 1859).

Foucault was an extraordinary experimental physicist, with major discoveries in optics as well as the pendulum and gyroscope.[1] However in his report to the Academy he avoided presenting a mathematical explanation of the pendulum precession (Foucault 1851). He did however suggest that the precession rate was proportional to the sine of the latitude.

$$\text{Period of precession} = 24 \text{ sidereal hours divided by the sine of the latitude.} \quad (1)$$

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Others immediately leaped to fill the mathematical gap, and the next several years saw many articles in the scientific literature that attempted to explain the phenomenon. The sine of latitude factor is now taught to beginning students of mechanics, using the theorems of Coriolis (Coriolis 1835). The development of vector algebra and calculus by Heaviside and Gibbs also made it easier to explain mathematically the effects of the earth's rotation. The standard classical explanation in rotating coordinates is that by Sommerfeld (1965).

Although the physics of the precession as seen from the earth are simple – at least after one has understood the Coriolis force! – the physics as seen from non-rotating space seems to have been confusing. Even Foucault seems to have believed that somehow the plane of precession was fixed with respect to the stars. This is true at the pole, but not elsewhere. The following essay is an attempt to describe the forces that bring about the precession, avoiding mathematics and using only the simplest mechanical principles. Hopefully it will be understandable to museum audiences as well as scientists.[2]

2. It Does Move Among the Stars

Consider for example the pendulum in the Panthéon. The sine of the latitude of Paris is close to 0.75, and according to Foucault's formula, 32 sidereal hours is required for a full circle. Thus, the plane of oscillation after 24 h has rotated only 3/4 of a full circle, and is oriented *perpendicular* to its initial location in the Panthéon. But since the Panthéon is oriented with respect to the stars precisely as it was 24 h earlier, the precession path must also be perpendicular to its initial orientation with respect to the stars. In spite of this direct explanation – it deserves to be called an *observation*, it is still common to find museums (and their web sites) where it is maintained that the pendulum path is fixed in space. This misconception seems to be based on the idea that there is no horizontal force that could exert a torque to make the path change in space; the pendulum restoring force, after all, is directed always toward the equilibrium point of the pendulum. But there is an additional force acting, a force that greatly simplifies our everyday life although not sensed by any of us.

3. The Resting Pendulum

To examine the forces acting on the bob it is essential to consider first the simple situation when the pendulum is not oscillating. It then acts as a simple 'plumb bob', defining the local vertical. But it is also travelling around with the earth from west to east in a circle as the earth rotates. What makes the non-oscillating bob move in this circle? It cannot be the tension in the wire because that tension acts only vertically upward along the wire – it is not dragging the bob around the earth; and if the wire were cut, the bob would continue to move around with the earth. When one whirls a stone on a string, it is necessary to pull on the string to make the stone move in a circle instead of a straight line. There must therefore be a similar force acting on the resting pendulum bob. This force is a component of Newtonian gravitation that acts poleward and parallel to the surface of the earth. Figure 1 shows how this force 'G' acts on the resting pendulum bob while Figure 2 illustrates motion in a circle with the force G accelerating the resting bob toward the pole.

It is not only the pendulum that experiences G; if G were not present, the water in the oceans would start to accelerate toward the equator instead of travelling around with the earth. The ellipsoidal shape of the earth, first imagined by Newton, ensures that Newtonian attraction is oriented with a component along the surface toward the pole just sufficient to keep the oceans and atmosphere, etc. in place. And of course it keeps us at the latitude where we choose to live.

How big is this force? It must be big enough to balance the familiar outward centrifugal force:

Necessary centripetal force = mass of bob (m) times square of eastward speed (V) divided by the distance to the axis(R) = m V^2/R. (2)

For a body resting on the earth, V equals the distance R times the rotation rate of the earth, or

$V = R$ times 2π radians per 24 h.

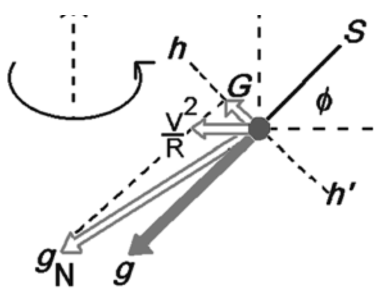

Figure 1. Forces acting on a resting pendulum bob hanging from the support S, as seen in a meridional plane. hh′ is a plane tangent to the earth and perpendicular to the solid arrow, which represents gravity, $\boldsymbol{g}$. $\boldsymbol{g}$ is composed of the Newtonian attraction g_N minus the centripetal acceleration V^2/R, where R is the distance to the rotation axis and V is the eastward speed of a point on the earth's surface. g_N has a component G acting poleward along the surface hh′ perpendicular to the support wire. The latitude is the angle Ø.

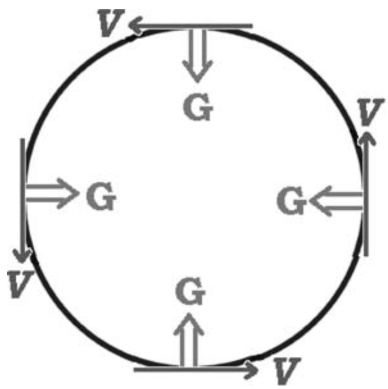

Figure 2. The horizontal component G of Newton's attraction acts to accelerate a mass in a circle around the earth with the rotational speed V of the earth at that latitude.

Since R in middle latitudes is about 4000 kms, V is about 1000 kms h^{-1}, or 278 m s^{-1}. And for a 1 kg bob, the force is:

Centripetal force $= (1)(278^2)/4{,}000{,}000 = 0.02$ Newtons.

(The kilogram weighs 10 Newtons.)

But in this way we have computed the total centripetal force, directed radially inward perpendicular to the rotation axis and parallel to the equatorial plane, as shown in the Figure 2. We are really interested only in how much is acting in the horizontal plane, since the vertical part of the centripetal force is contained in ordinary gravity. Therefore this estimate of 0.02 Newtons must be multiplied by the sine of the latitude to get the horizontal component. This will reduce the value to approximately $G \approx$ 0.015 Newton, say, in middle latitudes. (Note that the horizontal component will be zero at the equator.)

G is also present when the bob is oscillating. *We have already deduced one major fact* – there is a real horizontal force G acting on the bob that is not part of the tension in the wire or of ordinary gravity acting downward.[3] Furthermore, this force does not act only in the plane of oscillation; it is always directed toward the pole. This frees us from the restraint (superstition?) that there is no force available to make the pendulum path change its orientation with respect to the stars. But we will find that the effect of G on the precession is subtle and indirect.

How big is the force that makes the bob oscillate? If the bob, hanging on a wire of length L is displaced a distance x from the equilibrium point, there is a component of gravity equal to:

mass times g times (x/L)

acting perpendicular to the wire and accelerating the bob back toward the equilibrium position.

For $g = 9.8$ m s^{-2}, $L = 67$ m, $x = 3$ m, and a 1 kg bob, we get $RF \approx$ 0.5 Newton. Thus the restoring force is 33 times stronger than G. (The period of the oscillation will be about 16.4 s.)

4. The Changing Centrifugal Balance

We have now the problem of finding out how G, pointing always to the pole, acts to make the oscillation path precess clockwise. To resolve this we will examine how G operates in two different situations, one when the bob is oscillating from west to east, and secondly when the bob is oscillating from south to north.

To focus attention on G, we ignore the pendulum restoring force for a moment. Consider the bob as it moves from west to east with a speed v *as we observe it from the earth*. Its total eastward motion is then $V + v$, larger than V. If it were to continue moving in the same latitude circle as the equilibrium point, it would require a force directed towards the pole that is larger than G, according to Equation (2). Since that larger force is not present, the bob will not follow the west–east line of constant latitude, but will accelerate towards the equator, in a less curved line, somewhat as the whirling stone on our string would attempt to do if it were speeded up. When the bob is moving from east to west at a speed v in the return part of its oscillation, its total speed would be $V - v$, and the existing poleward force G will exceed the amount needed to keep the bob on the same latitude circle. The bob would accelerate poleward. These displacements are pictured in Figure 3.

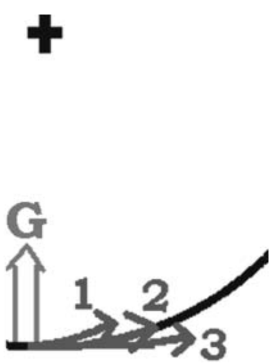

Figure 3. A mass moving eastward with the speed of the earth's surface will move as shown by displacement 2. A mass moving slower to the east (i.e. westward with respect to the earth) will accelerate northward as shown by displacement 1. A mass moving faster to the east will accelerate to the south of the latitude circle traversed by mass 2.

In both cases we see that the bob is deflected to the right of its motion with respect to the earth, just as observed in the precessing pendulum. (In Figure 3, note that the displacement 1 is for a bob moving westward with respect to the earth.) The stronger restoring force of the pendulum wire will not cancel out this effect completely, but it will resist sidewise deflection of the bob and reduce the clockwise precession. (Detailed mathematics shows that it cuts the deflection in half.)

5. Conservation of Angular Momentum

Now turn attention to the case of the bob moving from south to north. In this situation the gravitational force G and the pendulum restoring force are both directed either at the pole or away from the pole. At this instant *they cannot change the angular momentum of the bob about the rotation axis*. The angular momentum AM about the rotation axis is the product of the total eastward speed and the radius:

$$\text{AM} = VR$$

Conservation of AM is what increases the spin (i.e. 'V') of the iceskater when she pulls in her arms (decreases 'R'). When the bob moves poleward in its south–north oscillation, its distance R from the earth's axis decreases. V must then increase for AM to remain constant. This process is illustrated graphically as displacement 3 in Figure 4. This means that the bob will tend to acquire an eastward component of motion relative to the earth – it will no longer move in a strict south–north path but be deflected to the right. And when the bob moves southward on the return swing, the conservation of AM will make V decrease, and the bob will acquire a motion to the west with respect to the earth underneath. (Displacement 1 in Figure 4.) In both situ-

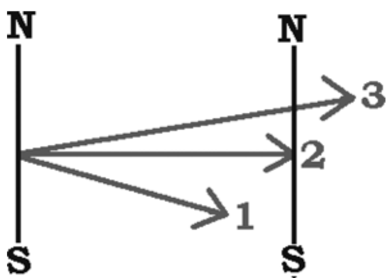

Figure 4. A mass moving eastward with the speed of the earth will move in displacement 2 (A parallel of latitude is shown here as a straight line.) A mass with the same initial eastward motion but given an added velocity poleward will acquire an additional eastward velocity from conservation of angular momentum as shown in displacement 3. A mass with the same initial eastward speed but given an added velocity toward the equator will lose eastward speed and appear to move westward with respect to the earth (displacement 1).

ations, the bob is deflected to the right of its motion with respect to the earth. And the mathematics for the complete solution again shows that when the stronger restoring force of the pendulum wire is allowed to act, the clockwise deflection is only half of what can be deduced from the above argument.

The angular momentum argument of Figure 4 illustrates a truly remarkable aspect of Foucault's pendulum. In its south–north oscillations of several meters, the pendulum recognizes that it has moved a little closer to, or a little further from, the earth's axis, by perhaps only 4 m in 4 million meters. Yet it adjusts its motion to the east or west in an attempt to conserve its angular momentum about the earth's axis!

We may conclude our discussion of the forces on the pendulum by noting that at the equator the horizontal component, G, of the gravitational attraction will vanish and the balance with the centripetal force takes place completely in the vertical axis. Additional west–east motion therefore does not result in a horizontal acceleration to the right. Furthermore, small poleward displacements from the equator when the pendulum is moving south–north do not change the distance R to the axis of the earth; conservation of angular momentum does not then require a change in the west–east motion. At the equator the precession is therefore zero, as foreseen intuitively by Foucault.

6. The Path Among the Stars

What does the path look like among the stars? As we observe it the bob reaches extreme points twice during each oscillation, when it comes to a momentary halt in its motion. Let us consider only alternate extremes, beginning with the point of release. We imagine a line drawn from the equilibrium point of the pendulum to these alternate extreme points, like a hand of a clock moving on the floor, as these points occur once in every oscillation period. This line as we observe it on earth will slowly rotate clockwise, as noticed in 1851 by the observers in the Panthéon. We will follow the path of this line when it is continued onto the sky, where it always selects a slowly moving point F on our horizon. If the release point is north of the equilibrium point, the point F in the heavens begins at the star located at that moment on the north horizon.

How does point F move on the sky? The motion of the pendulum path will consist of two superimposed rotations when observed from space:

A. a *clockwise* rotation with respect to the Panthéon about the local vertical axis, whose period is that given by Foucault's formula, and
B. a faster *counterclockwise* rotation as the Panthéon turns with the earth in a period of 24 h.

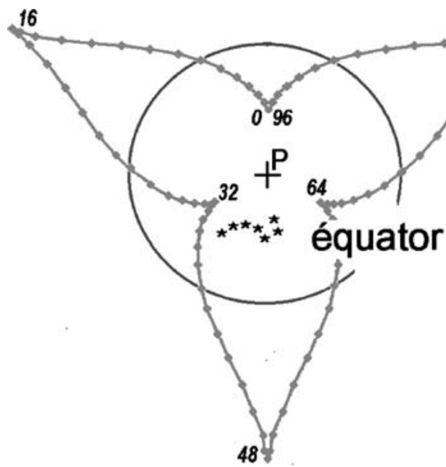

Figure 5. A sky map containing the North Pole (P) and the Equator. Stars in the Southern Hemisphere lie outside the circle. (The South Pole is at infinity on this map.) 96 h is shown for the path of the point *F* among the stars that is pointed to by the oscillation axis for a pendulum located at 48.6°. The release point is to the north of the equilibrium point and the precession period is 32 h. Cusps occur every 16 h when the path reaches an extremum at latitudes 41.4° north latitude and 41.4° south latitude on the sky map. *Ursa Major* is only a schematic indication.

Shortly after Foucault's demonstration in 1851 E. Silvestre designed a device to demonstrate mechanically how these two rotations combine (Silvestre 1851).[4] Nowadays it is possible to easily compute the resulting behavior on a sky map, using a home personal computer. The results of such a computation are shown in Figure 5 for a pendulum at 48.6° North latitude. For this example the pendulum was released from rest at the point north of its equilibrium point, so the initial location of *F* is at the star located at that moment on the northern horizon. (That star is located at latitude 41.4° on the sky map.) The sense of *F* 's rotation is counterclock wise, that of B, the more rapid of the two oscillation components. *F* starts to move then counterclockwise toward the equator on the sky map. At 16 h it will point to the southern horizon and it will halt its motion in the southern heavens at 41.4° south latitude. At that time it also halts temporarily its counterclockwise motion among the stars since the two rotations momentarily cancel one another. The result is a cusp. *F* then reverses its southward motion, and returns again to the northern skies. At 32 h it will have moved as far north in the skies as it was at the beginning. It will come again to a temporary halt and then repeat its path, although at a different longitude on the sky map.

Latitude 48.6 is such that three pendulum precessional periods (96 sidereal hours) is equal to four sidereal days. Therefore the path of *F* on the sky map is reentrant every 96 h for a pendulum at this latitude, since both the pendulum and the earth will have returned to their initial orientation with respect to the stars every 4 days.

Acknowledgements

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Notes

[1] See Amir Aczel (2003) and William Tobin (2003) [note added by editor].

[2] A less detailed explanation has been written recently by A. Marillier (1998).

[3] This fundamental fact is sometimes overlooked: Somerville 1972, bottom of p. 45; Hart et al. 1987, left column of p. 69.

[4] This device (inventory number 8044) was drawn to my attention by Professor William Tobin from his wide acquaintance with the work of Foucault. A photograph exists in Jacques Foiret, Bruno Jacomy & Jacques Payen, '*Le pendule de Foucault au Musée des Arts et Métiers*', Musée National des Techniques, Conservatoire National des Arts et Métiers, Paris, 1990 (ISBN2-908207-04-4) and in the Deligeorges reference listed above. It seems likely that such a device was useful at a time when the Coriolis theorems were not yet widely known, and when vector analysis had not yet been invented.

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Galileo and the Pendulum: Latching on to Time

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Abstract. Galileo changed the very concepts or categories by which natural philosophy could deal with matter and motion. Central to these changes was his introduction of time as a fundamental concept. He worked with the pendulum and with the inclined plane to discover his new concept of motion. Both of these showed him that acceleration and time were important for making motion intelligible.

1. Introduction

Many accounts of the work that Galileo did and why he became "the father of modern science" have been given. We will give yet another one that shows how the pendulum was crucial in Galileo's thought. First comes a very general historical narrative outlining a new overview of Galileo's work. Second we will present textual evidence from 1590 to 1609 that shows how Galileo used the pendulum and, by focusing on time, changed his way of thinking. Finally we will suggest what might be done with science students in ways that parallel the Galilean exemplar.

2. The Outline of the Galileo Story

Galileo wanted to reconstitute the whole of natural philosophy. What Galileo accomplished was a replacement of one set of analytical concepts with another. Some researchers might phrase this claim in terms of mental models. However phrased, Galileo's move was from the Aristotelian categories of the one celestial and four terrestrial elements and their directional natures of movement to only one element, matter. He then sought the important properties of matter and its motion trying first, relative heaviness, then specific gravity, then *momento* and force of percussion, and finally, acceleration and time. Galileo began his critique in the 1590 manuscript, *De Motu*, where he argued that the balance could be used as a model for treating all problems of motion, and heaviness (weight of the object minus weight of the medium) was the characteristic of all matter. What was not worked out was the positive characterization of the replacement categories, which probably contributed to his never publishing *De Motu*. Later in the 1600 *De Meccaniche* (Galileo 1600/1960) he introduces *momento* and begins to look at the properties of percussion of bodies of different specific gravities. Still, the details of how to properly treat weight and

M.R. Matthews et al. (eds.), The Pendulum, 99–113.

movement elude him. The problem is that the Archimedian simple machines that Galileo is using as his model of intelligibility are not dynamic enough and, except for the inclined plane, time is not an aspect that one would normally attend too. The details will not fall into place until 1603–1604, when Galileo works with pendula and inclined planes.

The pendulum showed Galileo that acceleration and, therefore, time is a crucial variable. The regularity of the period of a pendulum goes someway towards showing that equilibirium of times is the form of ratio that needs to be explicit in representing pendulum motion. Work on the force of percussion and inclined planes also emphasized acceleration and time. But he would not publish this until 1638 in *Discourses on Two New Sciences* (Galileo 1638/1954, hereafter *Discorsi*). In *The Starry Messenger*, (*Sidereus Nuncius*) published in 1610, he would begin his dismantling of the celestial/terrestrial distinction. But he had already laid the grounds for treating all matter as having the same nature back in 1590.

The rest of Galileo's story is well known, but we briefly retell it in this context. With *The Starry Messenger* (1610) and in *Letters on the Sunspots* (1612), Galileo enumerated many reasons for the breakdown of the celestial/terrestrial distinction. In the latter he even went so far as saying that the new evidence supported the Copernican theory.

Yet even with all these changes, two things were missing. First, there was no way to accurately describe the nature of matter in the new system. He had a start, but his developed matter theory would not come until Days One and Two of *Discorsi*. In the first part Galileo would attempt to show mathematically how bits of matter solidify and stick together, and do so by showing how they break into bits. The ultimate explanation of the "sticking" eluded him since he felt he would have to deal with infinitesimals to really solve this problem. The second science, Days Three and Four of *Discorsi*, dealt with proper principles of local motion, but this was now motion for all matter (not just sublunary stuff) and used the idea of time and acceleration as basic. The Fifth day dealt with the force of percussion, which had become an important aspect of Galileo's thinking.

The second missing part of the new natural philosophy was showing how a unified theory of matter could actually be applied to a moving earth. The change here was not just the shift from a Ptolemaic, Earth centered planetary system to a Copernican solar centered one. It was also a shift from a mathematical planetary model to a physically realizable cosmography. Galileo needed to show how, on a solar centered scheme, one could intelligibly use any laws of local motion. This he did by introducing two new principles, that all natural motion was circular, in Day One of his *Dialogues on the Two Chief World Systems* (Galileo 1632/1967, hereafter *Dialogo*) in 1632, and in Day Two the famous principle of the relativity of observed motion. The joint effect of these two principles was to say that all matter shares a common motion, circular, and so only motions different from the common could be directly observed.

This sketch provides the basis for understanding Galileo's changes. He has a new science of matter, a new physical cosmography, and a new science of local motion. In all these he is using a mathematical mode of description based upon, though somewhat changed from, the proportional geometry of Euclid, Book VI and Archimedes. (For details on the change, see Palmieri 2002).

3. In Search Of the Replacement Categories

Our historical theme is that Galileo's law of free fall arises out of his struggle to find the proper categories for his new science of motion. Galileo accepts, perhaps as early as the 1594 draft of *De Meccaniche*, that natural motions might be accelerated. But that accelerated motion is properly measured against time is an idea enabled only later, chiefly through his failure to find any satisfactory dependence on place. Galileo must have observed that the speeds of bodies increase as they move downwards and, perhaps, do so naturally. He would have seen speeds changing particularly in the cases of the pendulum or the inclined plane. Believing the same causes were at work in free fall and projectile motion, Galileo would have been more open to acceleration there too.

At this time he also begins thinking about percussive force that a body acquires during its motion and which shows upon impact. For many years he thinks the correct science of these changes should describe how speeds change according to where bodies are on their paths. Specifically, it seems that height is crucial. The percussive force of a dropped body is directly related to height and the motion of the pendulum seems to involve essentially equilibrium between the height of the bob at the start of the swing and its height at the end. This gives height the same role in understanding the pendulum that weight plays in understanding the balance. Of course times are also in some sort of equilibrium, each swing taking the same period (ignoring frictional losses), fixed by the length of the pendulum. This isochrony also seems to hold regardless of the initial displacement, making the radius of the path (i.e., the pendulum length) the sole determinant of the motion. However, these times do not analogize easily with weights on a balance.

The law of free fall, expressed as the distance in fall from rest being proportional to the time squared (the time-squared law), is discovered by Galileo through his inclined plane experiments. These were begun sometime before Galileo's letter of 1604 to Paolo Sarpi in which he gives a "proof" of the time-squared law by deriving it from the assumption that velocita (a new concept that amount to degrees of speed) are proportional to the distance fallen. In this proof the overall speed, which results from the accumulation of velocita, comes out proportional to distance squared and hence to time to the fourth (see below for details).

What is puzzling is that it is widely known at the time, thanks to the Mertonians and also to Oresme, that a distance-time-squared relation can easily be derived from an assumption of uniform acceleration. Galileo remains intent however, on finding an explanation of the time-squared law in the form of some relation between speed and height. The problem is that taking velocity proprotional to time might serve as a definition of free fall (i.e., defined as uniformly accelerated) but it provides no more explanation than does taking distance proportional to time squared. That this is Galileo's attitude is even in evidence in the *Discorsi*.

The key to his eventual confidence in the definition is the mean proportional. Any proportionality between a ratio and the square of a ratio can be expressed in terms of a mean proportional, and what quantity this mean proportional represents depends on the geometric and the physical context. Galileo's eventual definition of natural acceleration is an insight gained through the combination of pendulum and inclined plane "contexts" and recognition of the physical significance of the mean proportional relation.

What follows is a brief chronology intended to illustrate this conceptual shift. We begin with a description of how Galileo's first attempt fails in *De Motu* (1590), point out some key changes in *De Meccaniche* (1600) and finally we discuss the proofs of the working papers and the context that the pendulum and the inclined plane provide for the mean proportional relation.

4. The Galilean Texts

An early work in Galileo's search for the replacement categories is *De Motu* (1590) which ends with an incomplete attempt at describing projectile motion (Chapter 23, pp. 110–114). Addressed specifically is why a cannonball flies farther the more vertical the shot. According to the balance model Galileo just laid out for motion on inclined planes (Chapter 14, pp. 63–69), an object encounters no resistance to motion in the horizontal direction (which he calls neutral motion). But maximum range, well known from artillery, is achieved for shots of 45 degrees. The balance model must therefore be inadequate. The balance model had already been supplemented with an accidental, leaking impressed force (leaky impetus) to account for acceleration through a changing "effective" or net weight (i.e. intrinsic weight minus impressed upward force). The artillery example forces Galileo to make even greater accommodations.

Galileo's attempted reconciliation is twofold. More force will be imparted to a cannonball shot vertically, he believes, because it offers greater resistance to the motion than one shot horizontally or at any acute angle.. Secondly, an object attempting to turn downward will encounter more resistance the more directly opposed the downward and upward paths are. For any shot not directly upward, (see Figure 1).

> ... at the time when the ball begins to turn down [from the straight line], its motion is not contrary to the [original] motion in a straight line; and, therefore, the body can change over to the [new] motion without the complete disappearance of the impelling force (Galileo 1590, additions are Drabkin's.)

The resistance offered by the original projecting force must be overcome before the ball can turn downward, and so a greater decay of the projecting force is required. The result is that the more vertical the shot, the longer the trip before it turns

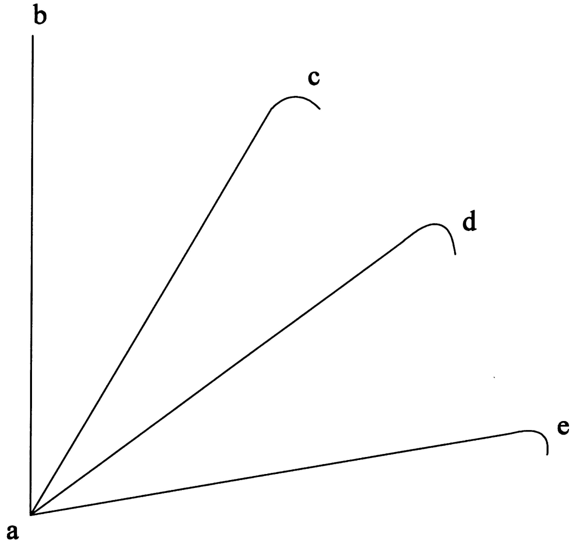

Figure 1.

downward. But "when the body moves along *ae*, which is almost parallel to the horizon, the body can begin to turn downward almost immediately" (Galileo 1590, p. 114).

This is a coherent picture. If the body falls obliquely to the projecting force, it is plausible that it would encounter less resistance. The picture is incomplete though. Galileo has no way of determining the relation between the remaining projecting force and points on the trajectory. Near the end of *De Motu* Galileo mentions the pendulum and how a lead bob will oscillate longer than a lighter bob, claiming that this is due to the greater retention of the impressed force by the heavier material (cf. p.108). It is probable that in order to complete the picture Galileo turned to investigating the rates of leakage of impressed force through observing pendulums. We will pick up this thread in a moment, but first we consider some other changes in Galileo's mechanical theory.

De Meccaniche (ca. 1600) is the product of a period of Galileo's extensive work on machines. It begins with three definitions that are, at this stage, his proposals for the new categories of motion. The definitions are of heaviness (*gravitas*), defined as the "tendency to move naturally downward" (Drake 1978, p.56); of *momento*, also a tendency to move downward caused not only by the weight, but also compounded by speed and the geometry of the Archimedian simple machines (something like mechanical advantage); and the *center of gravity*. Both the early (ca. 1593–1594) and later (ca. 1601–1602) versions also conclude with a section on percussion.

But the main focus is to rail against:

> those people who think they can raise very great weights with a small force, as if with their machines they could cheat nature, whose instinct – nay, whose most firm constitution – is that no resistance can be overcome by a force that is not more powerful than it. (Drake 1978, p.56)

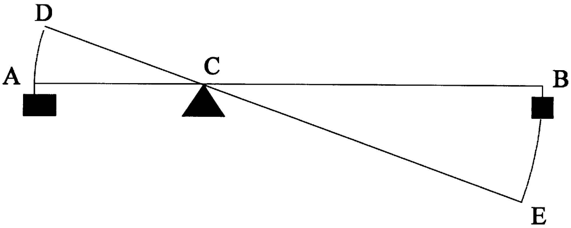

Figure 2.

Through machines like the lever, resistances do seem to be overcome by less powerful forces. Galileo therefore distinguishes *momento*, as a force, from a body's intrinsic weight. Galileo's new conception is that a trade-off can occur between speed and momento. A weight can overcome a greater resistance but must do so by travelling faster than the resistance. Because the lighter object is mechanically linked to the heavier one, their motions occur in the same time. The relation of the speeds can therefore be stated, as Galileo does, in terms of the distances each object move . . .

it is seen in all other instruments that any great resistance [or weight] may be moved by any given little force (*momento*), provided that the space through which this force is moved shall have to the space through which the resistant shall be moved that ratio which exists between this large resistant and the small. . . . (Drake 1978, p. 62)

Galileo provides an analysis of the lever in terms of the new category of *momento*. He has by now abandoned a leaking impressed force and believes instead that speed and force are internal to bodies themselves (See Figure 2).

The heavy body A being placed at the point D, and the other at the point E, it will not be unreasonable that the former, falling slowly to A, raises the latter swiftly to B, restoring with its heaviness that which comes to be lost by its slowness of motion. And from this reasoning we may arrive at the knowledge that speed of motion is capable of increasing *momento* in the moveable body in the same ratio as that in which this speed of motion is increased. (Ibid.)

Beside the ability to overcome resistance, Galileo is also interested in percussive force, both of which are measured as a body's *momento*. Speed allows a small weight to overcome a larger one due to the increased momento of the small weight. Speed also increases percussive effect. The greater the speed of the percussing object the greater its ability to overcome resistance. Speed acquired in fall then likewise contributes to *momento*.This commensurability, through the momento concept, between lifting force and falling force, suggests an interpretation of pendulum motion as an equilibrium between the *momento* or velocity gained by the weight in the downswing and the force required to overcome the weight's own resistance and carry it back to its original height.

Galileo's further pendulum discovery was the isochrony of the swings, of which he was convinced by at least November 1602. In a letter to Guidobaldo with that date Galileo writes:

You must excuse my importunity if I persist in trying to persuade you of the truth of the proposition that motions within the same quarter-circle are made in equal times. For this having always appeared to me remarkable, it now seems even more remarkable that you have come to regard it as false. (Drake 1978, p. 69)

Notice that Galileo is thinking of isochrony on the quarter circle. The symmetry of the pendulum path suggested equilibrium between the force gained in fall from an initial height and that required to climb back up to the same height, but this symmetry was ignored. What preoccupied Galileo was finding a relationship between speed and height that made the initial height not matter. Isochrony still did not suggest to Galileo that time ought to play any role in his causal account of the phenomena. Of course times mattered – speed was defined through distances and times. However, since for isochrony all times were the same, the features that seemed to make the difference were the path and distances (i.e. the heights of the initial displacement).

In *De Meccaniche* Galileo was initially misled with the lever because time seems to play no part, while with the pendulum the fundamental feature is equal times. What is even more deceiving in the case of isochrony is that the equality of times *is the thing to be explained*. It is only natural that the explanation would be in terms of something other than time. Moreover, the determinations or explanations that Galileo sought were through ratios but the ratio of equal times is one to one. Nonetheless, his consideration of these phenomena was fundamentally new. Time itself, even if only through the phenomena of equal times, had never before been subject to *mechanical* investigation.

Galileo's investigation of isochrony begins with his chord theorems. The key theorem makes it into the *Discorsi* as:

THEOREM VI, PROPOSITION VI: If from the highest or lowest point in a vertical circle there be drawn any inclined planes meeting the circumference the times of descent along these chords are each equal to the other. (Galileo 1638/1954, pp. 188–189)

The connection between these theorems and the pendulum is that the circle on which the chords are transcribed represents a pendulum attached at the center (see Figure 4). The ordering of theorems in the *Discorsi* does not, we contend, reflect the chronology of their discovery. The discovery of isochrony was almost certainly through observations of the pendulum. On the other hand, the chord law was probably first conjectured as a step towards achieving a physical explanation of isochrony and only later, if ever, was it verified by experiment. The letter to Guidobaldo suggests no distinction between the causes of the equality of times along the chords and along the arc:

Until now [that is, up to the chord theorems] I have demonstrated without transgressing the terms of mechanics; but I cannot manage to demonstrate how the arcs SIA and IA have been passed through in equal times and it is this that I am looking for.

Galileo's investigation of the chords is thus part of his seeking the causes behind the isochrony of the pendulum.

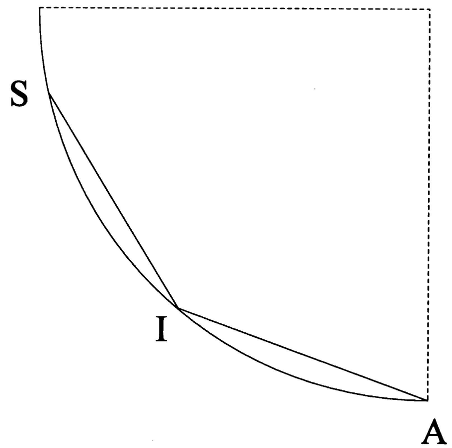

Figure 3.

One of the first things Galileo would have done with the chords was apply his balance model and so construct a perpendicular from the first point of each chord in order to find the ratio of their *moment* (See Figure 4). He would have immediately realized that those perpendiculars all intersect at the top of the pendulum circle (Euclid Book III, props 21, 31). Since Galileo knew the times of descent along each of these chords were the same, the vertical height would have seemed a natural feature of the diagram by which to represent their common time. All the chords share the same relationship to this height, which is itself also a chord. Galileo speculated that a ball rolling down one of the inclined planes represented by a chord would take the same time as a ball freely falling along the diameter (and he later carried out experiments to verify this, see (Drake 1978, pp. 217–218) and Drake, 1989). This would have justified Galileo's investigations of free fall through inclined planes since it linked the times and distances of both.

The time-squared law is discovered during Galileo's experiments with inclined planes (see Drake, 1975). The experiments involved rolling a brass ball in a highly polished groove down an inclined plane and marking the ball's location at successive equal time intervals. The results of the experiment were recorded on folio 107v. as a column of distances numbered 1 through 8, the numbers thus representing the times for each distance-the first distance was for one time increment, the second distance was for two time increments, and so on (See Galileo 1890-1909.) At some, possibly later, point Galileo added to this folio, to the left of the times, their squares, indicating he had recognized the correct relation of distances to the square of the times.

As mentioned above, a standard way for geometers of the time to deal with proportionalities to squares of ratios was through a mean proportionality. The physical significance of the distance that represented the mean proportional, or even which

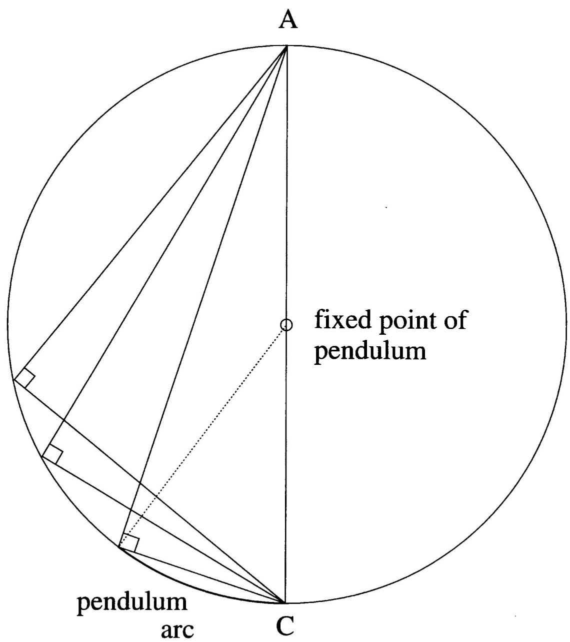

Figure 4.

mean proportional he should consider, was not clear. Nor did Galileo have any understanding of what caused the time-squared relation to hold.

That Galileo had not recognized the full importance of time and was not prepared to consider velocity as proportional to time, even after having worked with the mean proportionality, is indicated by his attempted explanation, given on f85v., of the time-squared relation (See Figure 5).

> I assume that acceleration of bodies falling along line AL to be such that *velocita* grow in the ratio of the spaces traversed, so that the *velocita* at C is to the *velocita* at B as space CA is to space BA, etc. ... But since *velocita* are increased successively at *all* points of line AE ... therefore all these *velocita* [taken together] are related, one [case] to another, as all the lines [together] drawn from all points of line AE parallel to the said BM, CN, and DO. But these [parallels] are infinitely many, and they constitute the triangle AEP; therefore the *velocita* at all points of line AB are, to the *velocita* at all points in line AC, as is triangle ABM to triangle ACN, and so on for the others; that is, these [overall speeds through AB and AC] are in the squared ratio of lines AB and AC.
>
> But since, in the ratio of increases of [speed in] acceleration, the times in which such motions are made must be diminished, therefore the time in which the moveable goes through AB will be to the time in which it goes through AC as line AB is to that [line] which is the mean proportional between AB and AC . (Drake 1978, pp. 98–99)

This is the proof of the time-squared law that Galileo sends to Sarpi in 1604. *Momento* has been replaced by *velocita* which are proportional to distance fallen. The overall speed the body has at any point in the fall (which contributes to the *momento*) is a result of the accumulation of all the degrees of *velocita* to that point. Speed is essentially an integral of the triangle and hence proportional to distance

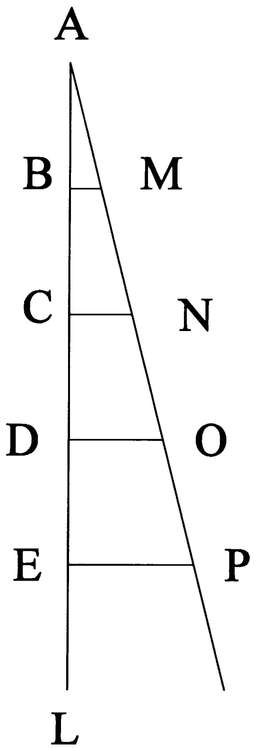

Figure 5.

squared. Then, rather than state the time-squared law explicitly, Galileo gives the conclusion as a mean proportionality relation: the ratio of the times through AB and AC is in proportion to the ratio of distance AB to the mean proportional between AB and AC. Eventually Galileo will arrive at the correct definition of natural acceleration but this attempt to find a cause for the time-squared law and the mean proportional relation is far from that. Here, since speed is proportional to distance squared and distance is proportional to time square, speed comes out proportional to time to the fourth.

He has accepted acceleration but thinks a correct physical explanation must be something like a velocity-distance relation. With hindsight we know that Galileo's search for a direct proportionality between velocity and distance would have to take him in the wrong direction. Moreover, percussion too is leading him in this way.

(f. 128) I suppose (and perhaps I shall be able to demonstrate this) that the naturally falling body goes continually increasing its *velocita* (speed) according as the distance increases from the point from which it parted. ... This principle appears to me very natural, and one that corresponds to all experiences we see in instruments and machines that work by striking, where the percussent works so much the greater effect, the greater the height from which it falls. (Drake 1978, pp. 102–103)

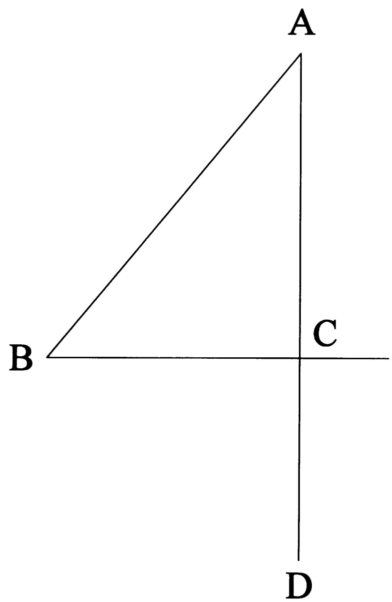

Figure 6.

Although Galileo may not be committed to a *direct* relation between speed and height, it is definitely height that in some way determines both speed and percussent force, the two concepts linked in *De Meccaniche*. Something puzzling arises in the speed-height relation when Galileo considers speeds along different inclinations (see Figure 6)

The motion along (*motus per*) the perpendicular AD is not perhaps quicker (*velocior*) than that along the inclined plane AB? It seems so; in fact, equal spaces are traversed more quickly (*citius conficiuntur*) along AD than along AB; still it seems not so; in fact, drawing the horizontal BC, the time along AB is to the time along AC as AB is to AC; then, the moments of velocity (*momenta velocitatis*) are equal along AB and along AC. ... (Wisan trans., p. 202)

Galileo expects the last to be true because percussion tells him the accumulated moments of velocity at B and C must be the same. Now imagine Galileo asked himself "under what conditions would both the chord law hold and percussive force be the same for all inclined planes of identical height?" In *Discorsi*, a proof is given that the speed of a body is the same when it traverses different inclines having the same height. We present that proof and then argue that this suggests how Galileo might have arrived at an answer to the above paradox that was physically intelligible, albeit in a new way (Figure 7).

Galileo's balance analysis of inclined planes says that the ratio of the force along the incline and along the vertical of an inclined plane are in proportion to the ratio of the length and height of the plane. Based on this he first argues that:

the speed at C is to the speed at D as the distance AC is to the distance AD. ... But, according to the definition of accelerated motion, the speed at B is to the speed of the same body at D as the

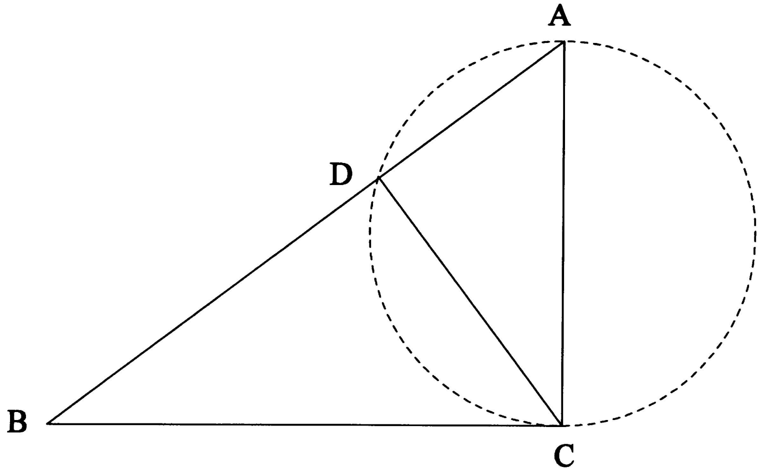

Figure 7.

time required to traverse AB is to the time required for AD; and, according to the last corollary of the second proposition, the time of passing through the distance AB bears to the time of passing through AD the same ratio as the distance AC (a mean proportional between AB and AD) to AD. Accordingly the two speeds at B and C each bear to the speed at D the same ratio, namely, that of the distances AC and AD; hence they are equal. … (*Discorsi*, p.184)

Let's restate the argument with $S(AB)$ representing the speed through distance AB, and $t(AB)$ the time for that distance. Therefore $S(AB) = AB/t(AB)$. Whether $S(AB)$ represents the instantaneous speed at the point B or average speed over the distance AB makes no difference to the ratios in this argument since we are dealing with uniform acceleration. The average speed $S(AB) = \frac{1}{2}S(B)$, where $S(B)$ is the speed at B. In all reasoning with *ratios* of speeds the $\frac{1}{2}$ drops out.

Notice also that, if the speeds at C and D are in inverse proportion to the distances AC and AD, then the times $t(AC)$ and $t(AD)$ are equal. Thus, the first claim, though stated in *Discorsi* as resulting from the balance analysis of the forces, is also equivalent to the chord theorem, even though this theorem does not appear until later in *Discorsi*. The argument reads then

1. $S(C)/S(D) = AC/AD$ (equivalent to the chord theorem)
2. $S(B)/S(D) = t(AB)/t(AD)$ (definition of accelerated motion, see below)
3. $t(AB)/t(AD) = AC/AD$ (mean proportionality)
4. $S(B)/S(D) = S(C)/S(D)$ (from 1,2 and 3)

The third premise is a corollary of the time-squared version of uniformly accelerated motion, i.e., distance is proportional to time squared. 2. also follows because free fall is uniformly accelerated and it holds for both average speed over a distance and instantaneous speed at the end of a distance. Again, any equivocation on average and instantaneous speeds does not invalidate the argument. Our conjecture is that this argument is the reverse of Galileo's actual reasoning. Rather, Galileo first wondered under what conditions the speeds – and so the percussive Force – could be the same at points C and D. This would immediately imply (as we show next) the mean proportional relation. The real breakthrough would then follow when Galileo realized that this implied that velocity must change uniformly with respect to time. Time has been lurking in the background all along and now leaps to the fore as the essential measure of, in effect, all natural motion.

Refer again to Figure 7, but now reverse (almost) the reasoning. By the chord law, $t(AC) = t(AD)$. With these times equal then the speeds must be proportional to the distances, i.e., $S(AC)/S(AD) = AC/AD$. Now, the speeds at $S(AB)$ and $S(AC)$ would be the same, as implied by percussion, if $S(AB)/S(AD) = S(AC)/S(AD)$ (two things are equal if their ratios with a third thing are proportional). Combining this with the first result for the speeds, we get $S(AB)/S(AD) = AC/AD$. This is where the geometrical context of the pendulum lends to Galileo's understanding of the mean proportional. The circle in Figure 7 represents a pendulum and AC, a mean proportional between AB and AD, is twice the pendulum's length. Galileo also took double the pendulum length as characteristic of the time when investigating isochrony. This connects the speed of free fall with his broader category of natural circular motion. Our argument is not that this is how Galileo discovers the mean proportional for times. As we pointed out he had conjectured this relation earlier. He is aware first of the time-squared law, probably on the basis of the inclined plane experiment, which would imply a mean proportionality. However, this gives him no physical explanation of the phenomena, and Galileo fails in his attempted explanation through accumulation of *velocita*. What we've outlined is the probable way in which t^2, the chord law, percussive force and the mean proportional are brought together in a way that, while it still may not provide a satisfactory *causal* explanation of the phenomena, unites them as consequences of the simple and correct definition of uniform acceleration as velocities proportional to time.

It is worth pointing out briefly how our reconstruction compares with others. In ours the pendulum has played a crucial role in many ways. It exhibits acceleration, is the source of the chord law and for Galileo is subject to the same explanation that percussion is. The symmetry of the pendulum suggests to Galileo the link between speeds obtained by a body in fall and its ability to overcome resistance such as its own weight. A recent article by Renn, et al. (Renn, et al. 2002) argues that the importance of symmetry chiefly arises in Galileo's consideration of projectile motion. In Drake's various reconstructions (Drake 1978; Drake 1989) acceleration is accepted by Galileo only after he has overcome the hurdle of accepting instantaneous velocities. In Wisan (Wisan 1974, p.175) it is suggested that the search for the brachistochrone (the path of least time between two points) is what led Galileo to ask the right questions and that the answer to those questions turned out to be the law of free fall. Our view does not exclude any of these histories but emphasizes the pendulum: a simple machine, closely related to the balance, which beautifully exhibits acceleration, symmetry and the importance of the relation between path and time.

5. Science Students Discovery

Briefly, we'll sketch how science students might "re-do" the Galilean discovery of time as crucial. This strategy accords with the idea of active learning that has dominated the science education literature in recent years. First, one might present students with a balance, an inclined plane and pendulum (and the necessary measuring instruments,) and ask them to discover and describe the basic properties of each with regard to motion. This might take the form of simply asking them to make a list, after viewing each experiment independently of one another, of nouns and verbs they could use to describe what they saw. After all three had been observed, the students would try to come up with a single, minimal set to describe the experiments.

Here the teacher ought to introduce the geometry of the circle and its chords, so that students may see the geometrical relation among the machines. If acceleration is not noted by the students as a feature of the inclined plane and the pendulum it should be brought to their attention. The problem is to find a way to represent time such that it can be seen as the important factor in the inclined plane and the pendulum, and why it is not necessary for balance problems. The final problem is to use what was observed and measured on the inclined plane and the pendulum to transfer to the concept of a freely falling body. Discussion at this point ought to encourage students to reflect on the basic nature of motions as observed in the world.

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The Treatment of Cycloidal Pendulum Motion in Newton's *Principia*

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Abstract. The discovery of the near isochrony of the simple pendulum offered the possibility of measuring time intervals more accurately than had been possible before. However, the fact that it was not strictly isochronous for all amplitudes remained a problem. The cycloidal pendulum provided this strict isochrony and, over a thirty year period from 1659 the analysis of the motion of this pendulum was developed. Newton's analysis in his *Principia* was both elegant and comprehensive and his argument is illustrated in this paper. It provides insights into the revolutionary nature of Newton's thinking especially compared to the Galilean approach to understanding the motion of the simple pendulum found in early 18th century textbooks.

Introduction

The near isochrony of simple pendulum motion for small amplitudes suggested early on that it might be used for measuring the time of events. However because the period of oscillation for different amplitudes was not strictly constant a search was carried on for ways to render pendulum motion strictly isochronous (Matthews 2000). Yoder (1988, pp. 48–64) has traced the path Huygens followed in 1659 (and published in 1673) to show for the first time that a pendulum in which the bob followed a cycloidal path rather than a circular one would perform oscillations in the same time regardless of its amplitude. Before this time various people had investigated the properties of the simple cycloid curve. For example, Roberval in 1634 (Kline 1972, p. 350) had shown how its area could be calculated while Wren in 1659 (Kline 1972, pp. 354–355; Yoder 1988, pp. 77–78) had devised a method for determining its length. Newton, in Book I of his *Principia* (Propositions XLVIII and XLXIX), showed how to determine the lengths of the hypo- and epi-cycloids and, in the propositions which followed, presented an analysis of the motion of a cycloidal pendulum under the action of a central force. The first edition of the *Principia* was published in 1687 and Newton acknowledged in it the work of Wren and Huygens on the common cycloid and the cycloidal pendulum (Newton 1729/1960, p. 158).

In the following sections the nature of Newton's reasoning is illustrated by adapting his discussion of the motion of a cycloidal pendulum under the action

M.R. Matthews et al. (eds.), The Pendulum, 115–125.

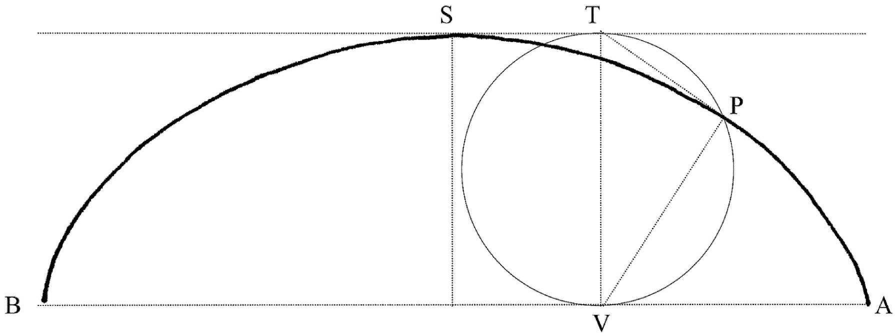

Figure 1. Some properties of the cycloid.

of central forces as he might have applied it to the motion of a pendulum moving along the path of a common cycloid.

Some Properties of the Common Cycloid

The cycloid is the curve traced out by a point P on the circumference of a circle when the circle rolls a distance equal to its circumference. The circle is called the generating circle (see Figure 1).

The generating circle begins with the point P at the bottom and coincident with A. As it rolls the centre of the circle moves forward. Points T and V are at the top and the bottom of the circle as it rolls along the line from A to B. The distance between A and B is equal to the length of the circumference of the circle.

It can be seen that as the circle rolls the cycloid is traced out by the point P at the end of the line VP where the point V is fixed for a short time. Thus the cycloid at P is perpendicular to the line VP[1]. In the circle angle VPT is a right angle so that PT is a tangent to the cycloid [2].

The following proof for the common cycloid is adapted from Newton's more general proof for finding the lengths of the hypo- and epi-cycloids.

As the wheel with radius r rolls along the line AB the point P on the perimeter traces out the cycloid APSB (see Figure 2). It is helpful to imagine that the small movement of this point from *m* to P takes place in two steps. In the first the wheel slides to the left so that the point moves from *m* to *n*. The tangent to the cycloid at the point *m* slides across to become T*n*. In the next step the circle rotates anti-clockwise about O so that the point moves from *n* to P. The distance *mn* is equal to the distance *n*P because the circle rolls from one position to the next.[1] Line *nq* is part of a circle with centre T and radius T*n* so that, if the rotation of the circle is small, the point *q* approximately bisects the line P*m* and T*n* equals T*q*. While the point moves from *m* to P, the length of the chord from the top of the circle to the point on the cycloid (i.e. the tangent to the cycloid) changes from T*n* to TP. Thus P*m* is the increase in the length of the cycloid as the circle rolls, P*q* is the decrease

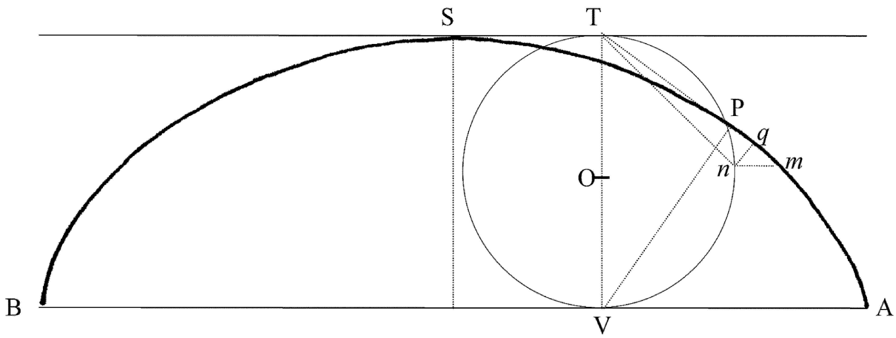

Figure 2. Finding the length of the cycloid.

in the length of the chord from the top of the circle to the point on the cycloid and Pm is equal to 2Pq.

Adding all such small increments from the time the circle begins to roll from A it can be seen that ΣPm = 2ΣPq or AP = 2(TV − TP). Thus the length of the half cycloid AS is equal to 2TV or 4r [3]. The length of the arc, SP, of the cycloid is equal to AS − AP or 2TV − 2(TV − TP) or 2TP [4].

The Cycloidal Pendulum

In Proposition L of Book I of the *Principia* Newton dealt with the problem of causing a pendulum to move in a cycloidal path and showed that the motion of the string must be constrained by plates shaped themselves like cycloids. Again his actual argument was a more general one referring to cycloidal motion under the action of central forces but it can be adapted to our situation by moving the centre of the force to infinity. His argument was as follows.

The three cycloids are generated by circles with the same radii (see Figure 3). As the bottom circle rolls to the right from A to O the line NT rotates through an angle of 180° and point T moves from A to S. The top circle rolls to the left and the point P moves from K to A. When their centres are on the same vertical line as shown with angle TNV equal to θ the angle PMQ will be equal to $180° - \theta$. Thus MP is parallel to TN and TV is parallel to PV so that PT is a straight line with PV equal to TV.

The length AP of the top left cycloid is equal to 2PV (from [4]) or PT so that, for all positions of the pendulum bob T, the total string length of the pendulum KPT is equal to the total length of the cycloid KPA and so the pendulum bob follows the path of the bottom cycloid [5].

In Proposition LI of Book I of the *Principia* Newton derived the force which acted on the bob of a cycloidal pendulum and which was responsible for its oscillatory motion. In Figure 4 the weight of the bob, W, is proportional to the length of the line TZ. This force is resolved into two components, F (proportional to TX) along the cycloidal path (from [2]), and a force (proportional to XZ) perpendicular to that

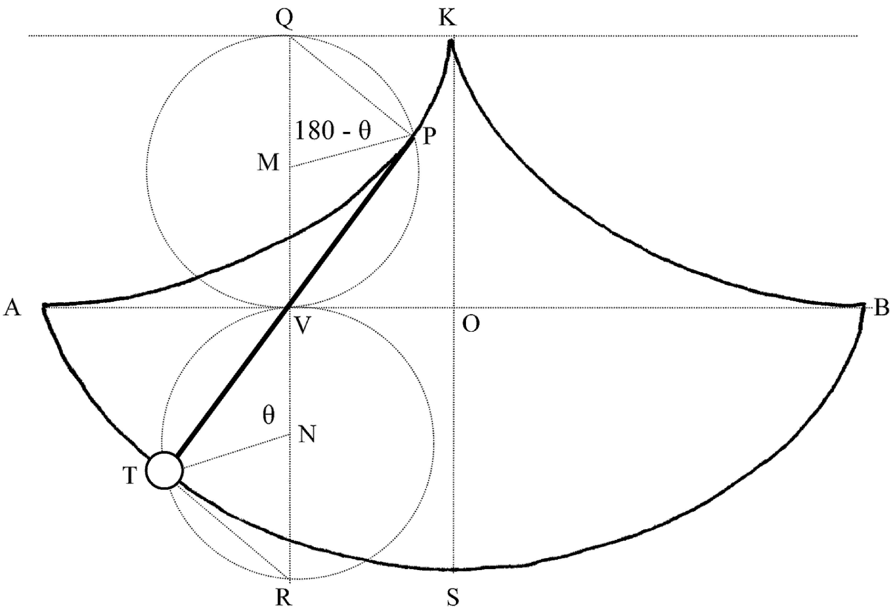

Figure 3. A cycloidal pendulum.

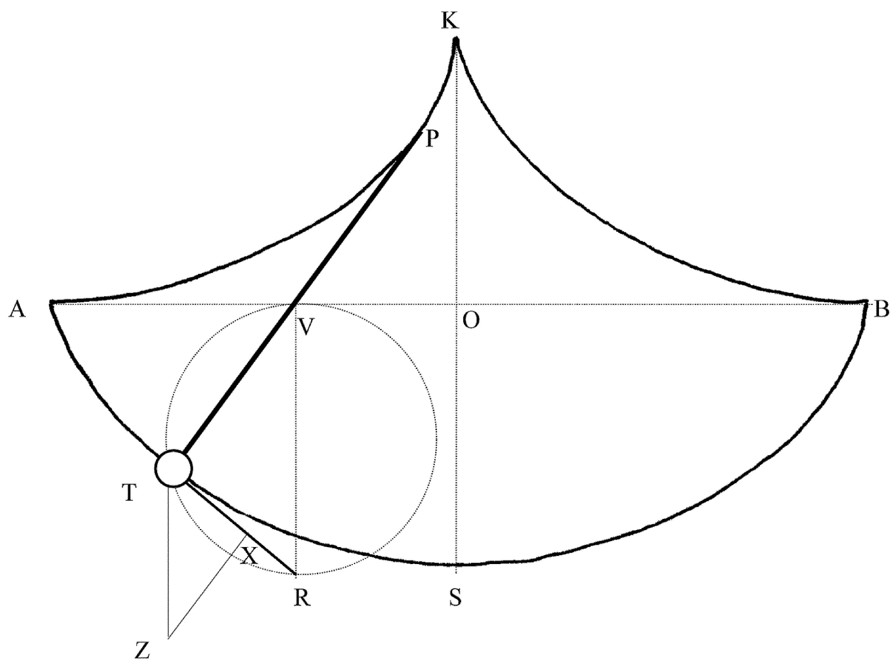

Figure 4. The force acting on a cycloidal pendulum.

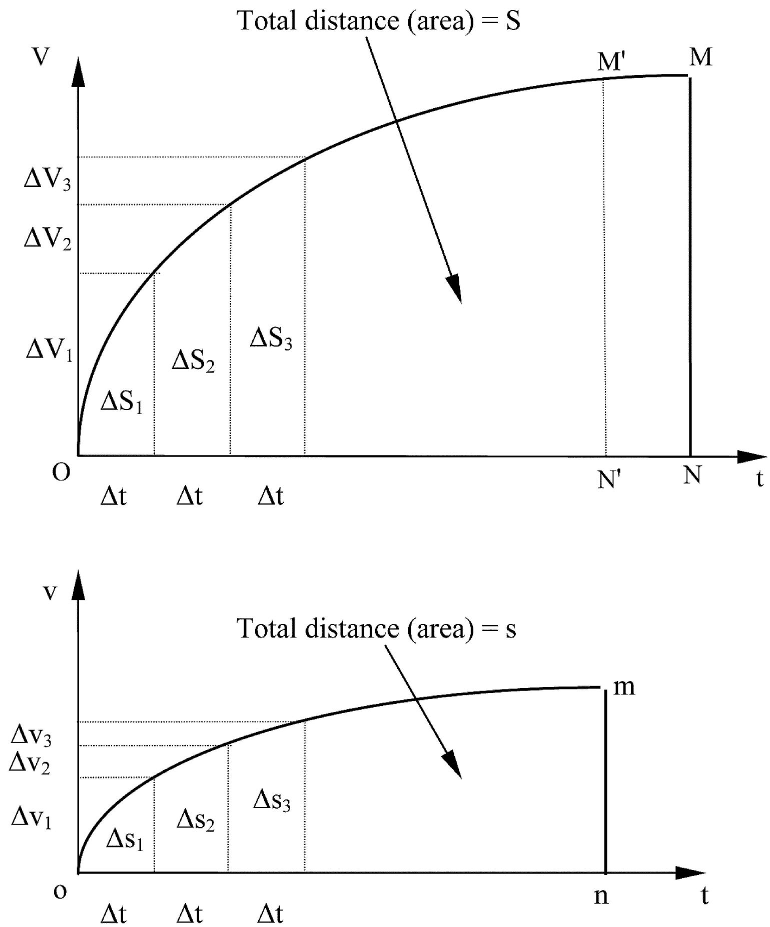

Figure 5. Graphs of the distances covered by two cycloidal pendulums.

path. Because it is at right angle to the motion the component of force along XZ has no effect on the motion of the bob and the acceleration of the bob is produced solely by the force, F, in the direction TX.

From Figure 4

$$\frac{F}{W} = \frac{TX}{TZ} = \frac{TR}{VR}$$

so that $F \propto TR$ since W and VR are constants. [6]

But the length of the arc of the cycloid TS is equal to 2TR (from [4]) so that $F \propto TS$ the length of the cycloid still to be passed over [7].

Consequently the force on the bob along its path is at all times proportional to the distance, along the path, from the lowest point.

Table I. Motion of the pendulums during the first small interval of time

Pendulum	Initial velocity	Final velocity	Change in distance	Distance remaining
b	0	Δv_1	$\Delta s_1 = \frac{1}{2}\Delta v_1 \Delta t$	$dr_1 = s - \Delta s_1$
B	0	ΔV_1	$\Delta S_1 = \frac{1}{2}\Delta V_1 \Delta t$	$DR_1 = S - \Delta S_1$

Isochrony of the Cycloidal Pendulum

Newton dealt with the periodic time of the cycloidal pendulum in Proposition LI of Book I of the *Principia*. Again he was interested in the more general context of pendulum motion under the action of a central force but his treatment applies just as well to the common cycloidal pendulum studied by Wren and Huygens. Newton's proof that, for two pendulums following the same cycloidal path but with different amplitudes, the periods are equal, reads as follows:

If therefore two pendulums APT [KPT in Figure 4], Apt, be unequally drawn aside from the perpendicular AR [KS in Figure 4], and let fall together, their accelerations will always be as the arcs to be described TR [TS in Figure 4], tR. But the parts described at the beginning of the motion are as the accelerations, that is, as the whole spaces that are to be described at the beginning, and therefore the parts which remain to be described, and the subsequent accelerations proportional to those parts, are also as the whole, and so on. Therefore the accelerations, and consequently the velocities generated, and the parts described with those velocities, and the parts to be described, are always as the whole; and therefore the parts to be described preserving a given ratio to each other will vanish together, that is, the two bodies oscillating will arrive together at the perpendicular AR [KS].

During the first small time interval, Δt, after starting the situation for the two pendulums is shown in Table I.

$$\frac{f_1}{F_1} = \frac{\Delta v_1/\Delta t}{\Delta V_1/\Delta t} = \frac{\Delta v_1}{\Delta V_1} = \frac{s}{S}. \qquad [8]$$

Inspection of the information in the Table I shows that

$$\frac{\Delta s_1}{\Delta S_1} = \frac{s}{S} \qquad \text{and so} \qquad \frac{dr_1}{DR_1} = \frac{s}{S}. \qquad [9]$$

Δs_1 and ΔS_1 are Newton's "parts described at the beginning of the motion" and are "as the whole spaces [s and S] that are to be described at the beginning". DR_1 and dr_1 are "the parts which remain to be described".

Thus during the second small time interval, Δt, the situation now is shown in Table II.

Again, since $f_2 \propto dr_1$ and $F_2 \propto DR_1$ it can be seen from [9] that

$$\frac{f_2}{F_2} = \frac{\Delta v_2/\Delta t}{\Delta V_2/\Delta t} = \frac{\Delta v_2}{\Delta V_2} = \frac{s}{S}.$$

Table II. Motion of the pendulums during the second small interval of time

Pendulum	Initial velocity	Final velocity	Change in distance	Distance remaining
b	Δv_1	$\Delta v_1 + \Delta v_2$	$\Delta s_2 = \frac{1}{2}(2\Delta v_1 + \Delta v_2)\Delta t$	$dr_2 = dr_1 - \Delta s_2$
B	ΔV_1	$\Delta V_1 + \Delta V_2$	$\Delta s_2 = \frac{1}{2}(2\Delta V_1 + \Delta V_2)\Delta t$	$DR_2 = DR_1 - \Delta S_2$

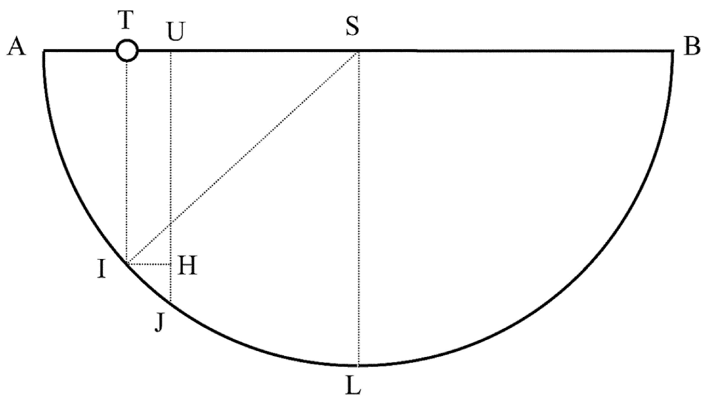

Figure 6. Calculation of the speed of the bob of a cycloidal pendulum.

Inspection of the information in Table II shows that

$$\frac{\Delta s_2}{\Delta S_2} = \frac{s}{S} \qquad \text{and so} \qquad \frac{dr_2}{DR_2} = \frac{s}{S}.$$

For each small interval of time "the accelerations, and consequently the velocities generated, and the parts described with those velocities, and the parts to be described, are always as the whole".

Thus when the pendulum, b, travelling the smaller distance has covered the distance s, the other pendulum, B, will have covered the distance S in the same time [10] (and so the line MN in Figure 5 should be in the position indicated by M′N′). It follows that the bob of the cycloidal pendulum will always reach the lowest point in the same time regardless of the position from which it is released.

The Velocity of the Pendulum along its Path and the Time for One Oscillation

When showing how to calculate the velocity of the pendulum bob at any point on its path Newton used a device which was also found in many of the textbooks of the early 18th century when dealing with a force which was proportional to the distance from a fixed point. Figure 6 illustrates the situation.

The line ASB is the same length as the cycloid ASB in Figure 4 and the semicircle is drawn with ASB as the diameter. The pendulum bob, shown in Figure 6 at the point T, moves along the line from A to B and back under the action of a force which is proportional to the distance TS. A line, TI, perpendicular to ASB meets the semicircle ALB at I. As the bob moves from A to B along the line ASB the point I moves from A to B along the semicircle ALB. In a small time, Δt, the bob moves from T to U and the point on the semicircle moves from I to J. The line IH is perpendicular to UJ.

Many writers, including Newton, used this device to analyse the motion of the cycloidal pendulum (see for example, Keill 1720, pp. 234–242; Maclaurin 1748, pp. 215–217). Newton used it in Propositions XXXVIII and LII of Book I of his *Principia*. The basic reasoning behind the device is given in the following section.

In Figure 6, if the time interval is small enough, the triangles TSI and HJI are similar so

$$\frac{TS}{HJ} = \frac{SI}{JI} = \frac{TI}{HI},$$

where

(i) SI, the radius of the semicircle is a constant,
(ii) TS, which is proportional to the force, is therefore also proportional to Δv, the increase in velocity between T and U, and
(iii) HI is equal to TU which is equal to $v\Delta t$ and is therefore proportional to v, the velocity at T.

If JI is constant, then HJ, like TS, is proportional to Δv, and TI, like HI, is proportional to v. These relationships are internally consistent since, if the velocity at T is proportional to TI, then the velocity at U is proportional to UJ and so the difference, HJ, is proportional to the increase in velocity.

The above leads to the conclusion that the point I, moving around the semicircle and mirroring the movement of the point moving along the line ASB, moves at a constant velocity along the semicircle ALB.[2] This velocity is proportional to the length of the line SL (half the length of the cycloid), and is equal to the velocity of the bob at its lowest point. Thus the time which the pendulum takes to complete one swing from A to B is equal to the time the point takes to travel around the semicircle ALB with a speed equal to the speed, V, of the bob at the bottom of its path, that is $t_{AB} = ALB/V$ [11].

A body falling freely through the axis of the cycloid (OS in Figure 4) would reach the same velocity, V, at S as the pendulum bob has at S (see Newton's proof in the Appendix) and the time taken to fall would be 2OS/V (see [4] in Gauld, 2002), that is, $\frac{1}{2}$ASB/V (see [3] this paper). Hence it follows that the ratio of the time, t_{AB}, for the pendulum bob to move from A to B to that, t_{OS}, for a body to fall freely through the axis of the cycloid is equal to ALB/$\frac{1}{2}$ASB or 2ALB/ASB, that is, the ratio of the circumference of a circle to its diameter [12] a conclusion well known in the early 18th century (Keill 1720, p. 242; Maclaurin 1748, p. 217; Pemberton 1728, pp. 73-75; 'sGravesande 1721, p. 63).

Proceeding from this point in more modern terms

$$OS = \frac{1}{2}gt_{OS}^2$$

so that

$$t_{OS} = \sqrt{(2OS/g)} = \sqrt{(l/g)},$$

where l is the length of the pendulum. Thus, from [12], the time taken for the pendulum to move from A to B and back again, $T = 2t_{AB}$, is given by

$$T = 2\pi\sqrt{(l/g)}.$$

It was soon realised by 18th century textbook writers (for example, Keill 1720, pp. 244–245; Maclaurin 1748, p. 218; Pemberton 1728, pp. 73–74) that the cycloidal pendulum oscillating through a small angle would have a period very close to that of a simple pendulum oscillating through a small angle and so the above result for the cycloidal pendulum provided a way of calculating the period of a simple pendulum given some knowledge of how bodies fell vertically under gravity.

Conclusion

Most 18th century textbooks aimed at presenting Newton's philosophy and way of thinking rather than slavishly following Newton's own presentation. Newton's *Principia* was simply not a textbook but a presentation of a radically new way of thinking about the world. Most post-Newtonian textbooks dealt with the motion of the cycloidal pendulum and were consistent with Newton's treatment presented above while not following it in every detail. It is interesting that in their treatment of the motion of the cycloidal pendulum they used Newton's new approach more consistently than they did when dealing with the motion of the simple pendulum where they tended to base their argument on a framework like that provided by Galileo (Gauld 2002).

The above case study provides a sample of thinking which Newton himself might have used and can provide a source of illustrations when teaching mechanics from a historical perspective.

Appendix, Newton's Proof of Proposition XL in book I of the *Principia*

Body 1, moves along the straight path DEC while Body 2, moves along the path ITKk (see Figure 7). The forces on the bodies acts towards the centre C. ID and KE are circles with centre C and the speeds of the bodies at I and D are equal at a particular time (i.e. $v_1 = v_2$). IC intersects KE at N. The points E and K are on the paths of the two bodies and, under the action of the forces at I and D, the bodies arrive at E and K after small intervals of time Δt_1 and Δt_2 respectively.

The accelerative force on body 1 at D causes it to accelerate to E and increases its speed by Δv_1 in the time Δt_1 while the force on the body 2 at I, acting along the line IN, accelerates the body to K and increases its speed by Δv_2 in the time Δt_2.

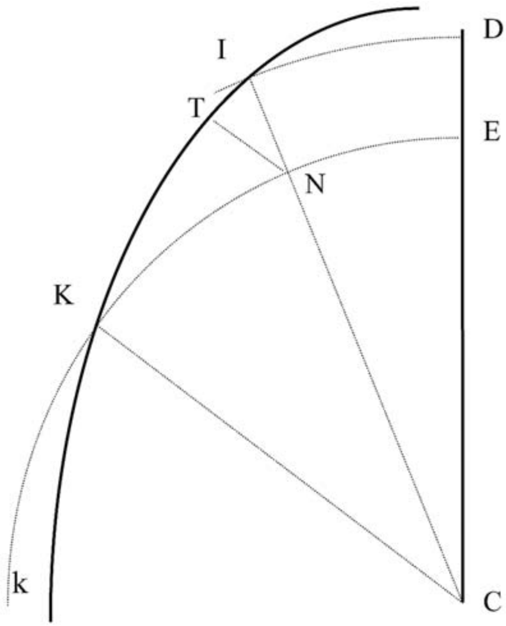

Figure 7. Newton's proof for the equality of velocities at equal altitudes.

NT is a line perpendicular to IK and so IT represents the component of the force from I to C which causes the increase in speed of body 2.

$$\frac{F_1}{F_2} = \frac{a_1}{a_2} \Rightarrow \frac{DE}{IT} = \frac{\Delta v_1/\Delta t_1}{\Delta v_2/\Delta t_2}$$

$$v_1 = v_2 \Rightarrow \frac{DE}{\Delta t_1} = \frac{KI}{\Delta t_2},$$

$$\frac{\Delta v_1}{\Delta v_2} = \frac{DE.\Delta t_1}{IT.\Delta t_2} = \frac{DE.DE}{IT.KI} = \frac{IN^2}{IT.KI}$$

But in ΔKNIT angle KNI = angle NTK = 90°

so

$$\frac{IN}{IK} = \frac{IT}{IN} \quad \text{or} \quad IN^2 = IT.IK \text{ (if IK is small enough)}$$

thus

$$\Delta v_1 = \Delta v_2$$

and so

$$v_1 + \Delta v_1 = v_2 + \Delta v_2$$

or the velocity of body 1 at E is the same as the velocity of body 2 at K.

If the point C is moved downwards an infinite distance the directions of the forces on the two bodies are parallel. Newton's conclusion implies that, if two bodies moving along different smooth surfaces under the action of gravity have the same velocities at one particular altitude, they will have the same velocities at all equal altitudes.

Notes

[1] I am indebted to Michael Fowler, Department of Physics, University of Virginia, for pointing this out to me.

[2] Newton's procedure here is akin to the more modern device in which the simple harmonic motion of a point along a line is represented as the projection of the uniform motion of a point around a circle of which the line is a diameter. The velocity and acceleration of the point executing simple harmonic motion are simply the components of the velocity, v, and the acceleration, v^2/r, onto the diameter.

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Pendulums, Pedagogy, and Matter: Lessons from the Editing of Newton's *Principia*★

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Abstract. Teaching Newtonian physics involves the replacement of students' ideas about physical situations with precise concepts appropriate for mathematical applications. This paper focuses on the concepts of 'matter' and 'mass'. We suggest that students, like some pre-Newtonian scientists we examine, use these terms in a way that conflicts with their Newtonian meaning. Specifically, 'matter' and 'mass' indicate to them the sorts of things that are tangible, bulky, and take up space. In Newtonian mechanics, however, the terms are defined by Newton's Second Law: 'mass' is simply a measure of the acceleration generated by an impressed force. We examine the relationship between these conceptions as it was discussed by Newton and his editor, Roger Cotes, when analyzing a series of pendulum experiments. We suggest that these experiments, as well as more sophisticated computer simulations, can be used in the classroom to sufficiently differentiate the colloquial and precise meaning of these terms.

1. Introduction

Teaching Newtonian physics involves, perhaps first and foremost, the replacement of students' uncritically held ideas about physical situations with a set of precise concepts appropriate for mathematical applications. The theoretical concept of 'force', for example, is known to be different than the one naturally invoked by students (Brown 1989) and novel methods for correcting this disparity have been offered (Gauld 1998; 1999). The idea of 'matter' and its theoretically precise kin 'mass', however, have been discussed less. Empirical studies regarding these either follow the work of Piaget and Inhelder by examining the relationship between 'density', 'volume', 'mass' and 'weight' in everyday reasoning (Smith et al. 1997; Baker & Susanne 2001), focus on teaching the distinction between 'weight' and 'mass' (Galili & Kaplan 1996), or test students' understanding of atomic theory and its implications regarding inter-atomic vacua (Novick & Nussbaum 1981). None examine the relationship between 'volume', 'matter', and 'mass' in a specifically Newtonian context. Although not an empirical study, this paper will suggest that

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the untutored use of these terms embodies assumptions that are foreign to their precise Newtonian meaning. Specifically, we suggest that the pre-theoretic use of the terms implies that ‘matter’ and ‘mass’ are the sorts of things that are tangible, bulky, and take up some definite volume. In Newtonian mechanics, however, the terms are given operational meaning only by their use in Newton’s Second Law: a body’s ‘mass’ or ‘quantity of matter’ is simply a measure of the body’s acceleration under an impressed force. Although there is no need to think of a massive body as spatially extended, we suggest that ‘massive’ is often taken to indicate that a body not only has a certain reaction to impressed forces, but also a determinate volume.

Grasping the distinction between ‘mass’ as a measure of the reaction to impressed forces and ‘mass’ as a representation of overall size is crucial to understanding Newtonian mechanics at both advanced and introductory levels. At an advanced level, a clear view of the distinction can be used to motivate the concepts of field theory and to teach about the implications of Newtonian mechanics for atomic physics. At an introductory level, the distinction can be used to counter students’ basic misconceptions about the relationship between the size of an object and its ability to exert force. Special care should be taken at this level, since the confusion between the two senses of ‘mass’ is embodied in our most basic physics teaching tools. In many collision diagrams, for example, the mass of an object is represented by the size of a ball. The collision of a massive body with a less massive body is represented as the meeting of a big ball with a small one (Resnik et al. 1992, pp. 210–217; Young & Freedman 1996, p. 242). This implicitly suggests that mass is related to spatial extension, and moreover, that bigger objects are necessarily more massive. Paradoxically, these diagrams are used most often in a student’s education exactly when it is most crucial that she divorce herself from the assumptions embodied in them.

In order to demonstrate how this feature of Newtonian mechanics can be better handled in the classroom, we will examine a short exchange between Isaac Newton and Roger Cotes, the editor of the *Principia*’s second edition, concerning pendulum motion. As several studies have suggested, there exists a correlation between contemporary students’ opinions about mechanics and those of pre-Newtonian scientists (Wandersee 1985; Squeira & Leite 1991), a fact which we will take for granted. Section 2 examines the historical basis for the distinction between the two senses of ‘mass’, particularly how it evolved from two different conceptions of the nature of ‘matter’. Section 3 turns to the pendulum experiments and the exchange between Newton and Cotes. The pendulum experiments demonstrate that at a fixed distance gravity depends only on the mass of an object, yet Newton attempts to draw from this further consequences regarding the composition of bodies and the nature of matter. What makes the exchange particularly interesting is that Newton himself plays the role of the pre-Newtonian scientist, and it is his editor who realizes the true implications of Newtonian mechanics. In Section 4, we draw morals from this exchange. Overall, this paper can be taken as yet another example of how the wrangling over the meaning of concepts that occurred during their discovery can be recast and utilized in contemporary pedagogy.

2. Two senses of 'Matter' and 'Mass'

Newton's *Principia* appeared in 1687, when the 'Mechanical Philosophy' dominated the scientific and philosophical scene in Europe. According to this philosophy, all physical phenomena were to be explained by the motion and configuration of bits of matter, where 'matter' was taken to mean 'a substance extended, divisible, and impenetrable' (Boyle 1666 [1979], p. 18). There was no well-formed notion of 'force' in this program, and a rejection of the idea that bits of matter could affect one another at a distance.[1] All physical interaction was thought to be reducible to direct contact of homogeneous, though differently sized, material particles. The Mechanical Philosophy was in large part a critical reaction to the ontological excesses of Scholastic philosophers, who postulated a multiplicity of 'substantial forms' to explain the observable characteristics of matter. For example, the inherence in a body of the substantial form 'red' was thought to explain why that body was red; the form 'heaviness' was thought to explain why it was heavy, etc. The Mechanists, on the other hand, sought to minimize explanatory principles and account for the observable characteristics of matter through only a few of its basic properties (e.g., size and impenetrability) which were thought to be truly essential to it – without them, matter would cease to exist *as* matter.[2] This explanatory strategy suggested that an explicit identification of matter *as such* with its essential properties was philosophically valuable. After all, if all the observable characteristics of matter could be explained by its essential properties, and if these properties were definitional of what matter is in itself, it would behove a philosopher to invoke them directly instead of invoking the less fundamental notion of 'matter'.

A radical version of this identification was endorsed by Rene Descartes, who claimed that matter was nothing other than spatial extension. He based this on the idea that spatial extension is the only property of matter which we cannot imagine eliminated from our experience and is thus matter's only truly essential property:

> *There is no real difference between space and corporeal substance.*
> It is easy for us to recognize that the extension constituting the nature of a body is exactly the same as that constituting the nature of a space ... Suppose we attend to the idea we have of some body, for example a stone, and leave out everything we know to be non-essential to the nature of body: we will first of all exclude hardness, since if the stone is melted or pulverized it will lose its hardness without thereby ceasing to be a body; next we will exclude colour, since we have often seen stones so transparent as to lack colour ... (Descartes 1644 [1985], p. 227)

After similarly discussing heaviness, heat, and cold, Descartes concludes:

> After all this, we will see that nothing remains in the idea of the stone except that it is something extended in length, breadth and depth. (Descartes 1644 [1985], p. 227, original emphasis)

The very idea of matter is thus reduced to the idea of spatial extension.

However absurd this identification may seem to the modern reader, it allowed Descartes to give a quantitative measure of matter: the quantity of matter in a body was simply the amount of space which it filled. The question of how to measure quantities of matter has a venerable history, dating back to various attempts by St. Thomas Aquinas and his followers in the 13th century to explain the preservation of the observable characteristics of the Eucharist through transubstantiation.[3] In the 17th century, however, the problem needed to be answered for the sake of getting any mathematical physics off the ground. Although his physics was an overall failure, Descartes understood the need to mathematize matter and stated unequivocally that what is important about matter is simply its quantity. He thus wrote shortly before the passage quoted above:

> *The distinction between quantity or number and the thing that has quantity or number is merely a conceptual distinction.* (Descartes, 1644 [1985], p. 226, original emphasis)

In other words, the quantity of matter in a body is not on equal footing with other properties of that body; it is the *only one* we need to understand in order to understand the body at all.

Without delving into further details, we note that analogous positions were held by a variety of mechanical philosophers. Galileo, for example, conceived of matter primarily in terms of its spatial limits. In this passage from *The Assayer*, he wrote that he cannot even entertain the notion of matter that is not contained in some volume:

> Now I say that whenever I conceive any material or corporeal substance, I immediately feel the need to think of it as bounded, and as having this or that shape; as being large or small in relation to other things, and in some specific place at any given time ... (Galilei 1623 [1957], p. 274).

Similarly, Robert Boyle, referring to the properties of the atomic constituents of matter, wrote:

> [We] must admit three essential properties of each entire or undivided, though insensible, part of matter: namely *magnitude* (... which ... we in English oftentimes call the *size* of a body), *shape*, and either *motion* or *rest*. (Boyle 1666 [1979], p. 20, original emphasis)

These examples should make clear that in the mid-17th century the commonplace scientific opinion was that one of matter's definitional properties was its spatial extension.[4] What we call a *geometrical* conception of matter was thus one of the core commitments of the mechanical philosophy. Although the scientists above did not use the term 'mass' to describe the geometrical measure of a quantity of matter before Newton popularized the term, what is important for our purposes is that insofar as they were concerned with quantifying matter, they invoked a geometrical measure. As will become clearer later, when the term 'mass' came to stand for 'quantity of matter' (after the *Principia*'s publication), the question still remained which quantification strategy was appropriate.

Newton himself held the geometrical view of matter throughout his long scientific career.[5] For example, in *De Gravitatione*, a treatise that may have been written around 1684 as part of the composition sequence of the *Principia*, Newton described his conception of matter in terms of matter's creation by God. Bodies, he wrote, were nothing other than:

determined quantities of extension which omnipresent God endows with certain [additional] conditions . . . :
(1) that they [the quantities of extension] be mobile; and therefore I did not say that they are numerical parts of space which are absolutely immobile, but only definite quantities which may be transferred from space to space;
(2) that two of this kind cannot coincide anywhere; that is, that they may be impenetrable . . . ;
(3) that they can excite various perceptions of the senses and the fancy in created minds. (Hall & Hall 1962, p. 140, original emphasis)

What is important about this passage is that it begins the discussion of matter with the assumption that bodies are "quantities of extension". Newton did not doubt that the physics of material bodies – for which he is setting the scene by discussing the impenetrability and motion of bodies – must be directly concerned with volumes. A later passage in the same treatise confirms the importance of the geometrical conception. Newton offered the following definition of momentum ("motion", as he called it):

[M]otion is either more intense or more remiss, as the space traversed in the same time is greater or less, for which reason a body is usually said to move more swiftly or more slowly . . . motion is more or less in extension as the body moved is greater or less, or as it is acting in a larger or smaller body. And the absolute quantity of motion is composed of both the velocity and the magnitude of the moving body. (Hall & Hall 1962, p. 149)

Newton argued that momentum is proportional to the product of the velocity and the size ("larger or smaller") of the body. For modern readers, this is a curious statement, since momentum should be equal to the product of velocity and mass, not velocity and size! It seems that Newton believed the two to be equivalent, at least as far as the atomic particles of matter were concerned.

During the same period, however, Newton was establishing a new conception of matter that was radically different from the one outlined above. This innovative *dynamical* conception of matter related the mass of a body not to its size, but to its ability to resist impressed forces; i.e., to its inertia.[6] Since this conception is familiar to all those who have some facility in Newtonian mechanics, we only need to mention that Newton's second law, put anachronistically as $F = ma$, is the ultimate embodiment of this view. According to this law, given a force a body's mass can be calculated based on the observable acceleration of the body in response to that force. Since the second law is one of the foundational principles of Newtonian mechanics, it is clear that the *dynamical* sense of matter underlies Newtonian mechanics. An appeal to the *geometrical* sense of matter, on the other hand, is completely out of place: geometric properties of the fundamental particles are irrelevant to Newton's mechanics.

It should be noted, however, that when coming to explicitly *define* mass, Newton did not state its relation to impressed forces. Rather, Newton defined mass as "a measure of the matter that arises from its density and volume jointly" (Newton, 1726 [1999], Defn. 1, p. 403). This more geometrical definition is seldom used in the *Principia* and is virtually forgotten in later treatments of Newtonian mechanics. Starting with Euler's *Mechanica* and ending in modern textbooks, the concept of mass has been traditionally introduced exclusively as a measure of a body's resistance to impressed forces.[7]

The history of physics in the 18th and 19th centuries shows that this *dynamical* conception was one of Newton's greatest contributions to physical theory. In the 18th century, Roger Boscovich suggested that matter should be conceived as a set of point-sized particles, each considered only as the locus of forces, while in the 19th century, the work of Faraday and Maxwell developed the idea of point sources into the field concept that still dominates physics today. Overall, the shape and size of the fundamental constituents of matter turned out to be of no importance for doing theoretical physics and the *geometrical* conception was left by the way-side. Nevertheless, the intuitive appeal of this conception continues to exert its power. The opinions of 17th century scientists demonstrate that even at a sophisticated level it is possible to believe that the concept of 'matter' appropriate for doing physics implies that matter is to be understood through its spatial properties. Yet as far as the theoretical structure of Newtonian mechanics is concerned, this is an unwarranted belief. We now turn to the exchange between Newton and Cotes that illustrates this theme.

3. Pendulum Motion and Conceptual Confusion

The conflation of the two conceptions of matter discussed above was first noted by Newton's young editor, Roger Cotes. Cotes' prodigious mathematical talent earned him an appointment as the first Cambridge Plumian Professor of Astronomy and Experimental Philosophy at the tender age of 26. In 1709 he was chosen as the editor of the *Principia*'s second edition, and his correspondence with Newton over the ensuing four years reveals that Cotes was one of the *Principia*'s most astute readers. Here we will focus only on an exchange between Newton and Cotes in the winter of 1712. Cotes noticed that Newton himself combined the two conceptions of 'matter' in drawing out the implications of ingenious pendulum experiments reported in the *Principia*, and attempted to dispel the confusion in exchanges with Newton.

Newton's predecessors thought that gravity (or 'heaviness') could depend upon a wide variety of a body's properties. Advocates of the geometrical conception usually explained gravity in terms of a fluid aether pressing upon bodies, and in this context it was natural to suppose that gravity might depend on a body's volume or shape (Descartes 1644 [1985], p. 268). Others thought that gravity resembles magnetism, which clearly affects some bodies differently than others. Newton devised a pendulum experiment to rule out the dependence of gravity on anything but mass (Newton 1726 [1999], Book III, Prop. 6). As Newton notes, this experiment is a more controlled version of Galileo's famous experiment comparing the free fall of bodies with different densities. Newton constructed two pendulums of equal length side by side, filled the empty pendulum bobs with equal weights of a variety of materials, and set them in motion. He had earlier derived the following relationship between the period of oscillation, the weight of a pendulum, and its mass, $m \propto w \cdot p^2$ (Book II, Prop. 24). In addition, the period of a simple pendulum depends only its length and the accelerative force due to gravity acting on the bob. Since the pendulums had the same length, any variation in the periods would reflect a difference in the accelerative force gravity imparts to different materials. Newton found that the pendulums exhibited *nearly identical periods* for all the materials he tested, and thereby confirmed that the mass and weight of an object are always directly proportional to one another, $m \propto w$, far more accurately than the Galilean experiment. In modern parlance, this proportionality shows that the 'inertial mass' m in $F = ma$ (which measures the response of the bob to the force acting on it) and the 'gravitational mass' m in $W = m\mathbf{g}$ (which measures the magnitude of the force acting on the bob) are one and the same.

This novel feature sets the gravitational force apart from other forces encountered in physics. In the corollaries to Prop. 6, Newton draws attention to this feature via a comparison to magnetism: unlike gravity, the force of magnetism has strikingly different effects on various materials. In anachronistic terms, gravitational 'charge' in the force law for gravitation is exactly equal to the inertial mass, unlike other force laws (such as Coulumb's law) which include a separate charge that determines the magnitude of the force. For forces with a separate charge one can imagine experimentally varying the ratio of the electric charge to the inertial mass to distinguish inertial effects from those due to the force. However, the exact equality of inertial and gravitational mass prevents this in the case of gravitation. Newton's notion of 'mass' is thus uniquely distinguished from other properties of matter. A little over two centuries later, Einstein would take this surprising fact to be one of the guiding principles – he called it the 'equivalence principle' – in his successful search for a new theory of gravitation.[8]

Newton thought that the pendulum experiments established more than just the proportionality between mass and weight. In the first edition, he goes on to argue in a corollary (Book III, Prop 6, Cor. 2) that matter must be filled with empty spaces![9] Newton's argument is based on two additional premises. The first is that pendulum motion would be impossible if the density of the pendulum bob was equal to the density of the medium in which it was moving. This is just a consequence of the fact that bodies cannot rise and descend unless their specific gravity is different from that of the ambient medium. Newton's second premise is that matter is composed of atoms of fixed density. If atoms have a fixed density yet there is a variation in the densities of gross bodies, the variation must be due to the presence of a different number of atoms in equal volumes of gross bodies. This can happen only if different amounts of empty spaces separate the atoms in each of these bodies.

Cotes challenged this argument. Although Cotes' objection may appear to be a minor quibble related to an unimportant corollary, it actually focuses clearly on the theme of this paper – the notion of 'matter' and its measure via 'mass' that is appropriate in Newtonian mechanics. Cotes asked Newton if it was not possible to imagine two identically sized atoms which nonetheless possess different masses. If this is possible, Cotes held, then Newton's argument fails to show that gross bodies must possess internal vacua. It would be possible for two identically sized pendulum bobs to have different masses by virtue of being made from two types of atomic matter (each with a different density), rather than from the same type of atomic matter differently distributed in space. One could even image a matter that is so dense that gross bodies made of it contain no vacua whatsoever. What is important about this observation, however, is not so much what it says about the existence of vacua, but what it says about the Newtonian concept of 'mass'.

Cotes claims that it is inappropriate, in the strict context of Newtonian mechanics, to treat bodies as if they were spatially extended. The *only* concept of 'mass' that is relevant for Newtonian mechanics is the one given by the second law. In Cotes' own words:

> Let us suppose two globes A & B of equal magnitudes to be perfectly fill'd with matter without any interstices of void Space; I would ask the question whether it be impossible that God should give different *vires inertia* [i.e., forces of inertia] to these Globes. I think it cannot be said that they must necessarily have the same or an equal *Vis Inertia*. Now You do all along ... estimate the quantity of matter [i.e., mass] by the *Vis Inertia* ... Tis possible then, that ye equal spaces possess'd by ye Globes A & B may be both perfectly fill'd with matter, so no void interstices remain, & yet that the quantity of matter in each space shall not be the same. Therefore when You define or assume the quantity of Matter to be proportionable to its *Vis Inertia*, You must not at the same time define or assume it to be proportionable to ye space which it may perfectly fill ... (Turnbull 1977, Volume V, Doc. 893, p. 228)

In other words, the *dynamical* conception of mass essential to the formulation of Newtonian mechanics is in no way tied to the geometrical conception advocated by Newton and the other mechanical philosophers. When Newton assumed that similarly sized atomic particles must necessarily possess the same mass – i.e., that their volume was a measure of their quantity of matter – , he was stepping beyond the limits of his own mechanics. As Cotes noted, it is possible for similarly sized atoms to possess different masses because 'mass' is simply a measure of a body's inertia – it need not be at the same time a measure of a body's volume. Although the experiments fix the relationship between inertial and gravitational mass, they have no bearing on the relationship between mass and volume of the fundamental particles.

The remainder of the exchange between Newton and Cotes is fascinating, yet beyond the scope of this paper (Biener & Smeenk 2001 for further discussion). It shows that although Newton ultimately acquiesed and formulated the corollaries as conditionals, he did not fully understand Cotes' complaint. Instead, Newton kept insisting that the proportionality between mass and volume, just as the proportionality between inertial and gravitational mass, was proved by the pendulum experiments. As we've discussed, however, only the latter was strictly proved by these experiments. Newton's belief in the former seems to have been due to a subtle confusion between the conception of matter he invented and the one generally accepted at his time. In other words, he failed to sufficiently differentiate in his proofs when 'mass' and 'quantity of matter' were used to indicate inertia and when they were used to indicate volume. The unfounded conclusions drawn from his pendulum experiments were simply artifacts of this confusion.

4. Morals for Teachers

If Newton's misconceptions about his own theory are any indication, physics teachers would do well to actively counter students' insufficient discrimination between the geometrical and dynamical senses of 'matter' and their quantification via the concept of 'mass'. This involves both an explicit confrontation with students' confused views and a more subtle use of existing teaching tools, so that they will not exacerbate existing confusions.

Newton's pendulum experiment can be easily modified for use in the classroom. Although some students might be familiar with the idea that the weight of bodies depends only on their mass, reproducing the experiment will provide a vivid demonstration of their knowledge. The easiest way to reproduce the experiment would be to construct two pendulums side by side, with small boxes for pendulum bobs, and ask students to fill the boxes with a variety of materials. Alternatively, one could use a single pendulum and have students measure the period for different materials. In either case, students should be asked to discuss the patterns they have observed and what they imply regarding the nature of the gravitational force. In the theoretical treatment of the experiments, it is crucial to distinguish inertial and gravitational 'mass' as mentioned above rather than simply assuming their equality. Students will discover that the two are very precisely equal, but it is important for them to recognize that m plays two distinct roles.

On a more introductory level, a series of experiments can be run with pendulum bobs of different sizes but equal lengths. A ball should be placed at the lowest point in the pendulum trajectory, so that a collision occurs when the pendulum is moving directly in the horizontal direction. As students can see, changing the size of the pendulum bob without changing the mass will not affect the motion of the struck ball. Changing the mass, however, will affect this motion. From this, the teacher can stress the point that almost all of a body's properties – save its *dynamical* mass – are irrelevant for Newtonian mechanics. In this experiment, the overall length of the pendulum should be large in relation to the size of the pendulum bobs, so that the difference in the length of the pendulum trajectory will be negligibly changed by a change in the bob size.

Several computer simulations can also be used to press home the idea that a body's geometrical properties are irrelevant to the dynamical notion of mass. For example, a computer simulation in which the volume of a pendulum bob may be varied by the student *while* the pendulum is oscillating illustrates that no change in the motion of the pendulum occurs. From this students should conclude that the volume of the bob is of no consequence. A similar simulation involving collisions can make an even more powerful teaching tool. In this case, both the volume and *mass* of colliding balls may be changed by the student during the course of collision. Students can see that when volumes are changed, no change in the motion results. However, a change in mass affects the motion. We believe that such a demonstration will be most useful if it is performed as a proper part of teaching Newton's Third Law and divorcing students' from the view that a larger body necessarily contains a larger 'internal force' (Gauld 1999). The idea of momentum and its relation to mass can then be developed more forcefully.

On a more subtle level, a better use of diagrams can also discourage the confusion in the two senses of 'mass'. As noted earlier, collision diagrams ordinarily use the size of an object as a representation of its mass. This enforces the notion that mass and volume are somehow related and even suggest that they are related linearly. As a corrective, collision diagrams should represent the mass of an object by internal shading, not size. Different masses should no longer be represented as differently sized balls, but as differently filled ones. It is important to use fillings which have a straightforward numerical interpretation (such as a set of dots with a certain number per unit area) and not color gradients, whose mathematical interpretation is less clear.

Before concluding, we must sound a single cautionary note: there are plenty of applications of Newtonian mechanics where size *does* matter. For example, in calculating angular momentum, fluid resistance, or coefficients of friction the size of the bodies involved determines the magnitude of the forces. The point of this paper is not that 'size doesn't matter', but rather that distinguishing between 'mass' as a dynamical measure and 'mass' as geometrical measure is crucial for understanding the foundations of Newtonian mechanics. The historical precedent suggests that students are likely to confuse these senses and thus teachers should combat this confusion at all pedagogical education.

5. Conclusion

The concept of 'mass' (i.e., a 'quantity of matter') in Newtonian mechanics is different than the one used colloquially. The difference lies in the fact that the Newtonian concept is derived from other precise concepts – force and acceleration – while the colloquial concept is derived from our everyday interaction with more or less massive material bodies. In learning Newtonian mechanics, students need to divorce themselves from the colloquial concept and understand that a Newtonian 'mass' (i.e., a 'quantity of matter') is a different notion than the one they are accustomed to. The difference between these two notions was historically noted in connection with pendulum experiments and we believe that such experiments can still be used today to clarify their conceptual interrelations.

Notes

1 See Westfall (1971) for the evolution of the force concept in the 17th century.

2 See Dijksterhius (1960), especially §III.G.

3 See Jammer 1961, chap. 4) on the medieval notion of 'quantitas materiae' and its theological origins.

4 See Hall (1963) for an extended analysis of similar matter theories.

5 For more on the impact of this view on Newton's physical theories, in connection with his views on the explanation of gravitation by aetherial mechanisms, see Biener and Smeenk (2001).

6 Newton was not the only one to recognize that the mechanical philosophy's conception of matter as bare, inert geometric extension was insufficient for physics. Newton's contemporary (and rival) Leibniz also prominently criticized the mechanical philosophy (see, for example, Leibniz 1989, pp. 245–256).

7 See Jammer (1961, Chap. 8) for an analysis of 'mass' in Newton's successors. An example of a modern definition of 'mass' via Law II can be found in Resnick and Halliday (1960, §5.4.

8 See Einstein (1916) for Einstein's brief statement of the principle, and Norton (1985) for a detailed discussion of the principle and its role in Einstein's discovery of general relativity.

9 Newton revised these corollaries in light of his discussion with Cotes, and the second corollary from the first edition was revised and expanded into corollaries three and four in later editions.

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The Treatment of the Motion of a Simple Pendulum in Some Early 18th Century Newtonian Textbooks

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Abstract. The treatment of pendulum motion in early 18th century Newtonian textbooks is quite different to what we find in today's physics textbooks and is based on presuppositions and mathematical techniques which are not widely used today. In spite of a desire to present Newton's new philosophy of nature as found in his *Principia* 18th century textbook analysis of pendulum motion appears to owe more to Galileo's insights than to those of Newton. The following case study outlines this analysis and identifies some of its distinctive features as a resource for teachers wishing to refer to this period in the history of science.

1. Introduction

Following the publication of Newton's *Principia* in 1687 many other writers began to promulgate the new mathematical and experimental philosophy which Newton had presented there. That book was structured on the Euclidean model of Axioms, Propositions and Theorems (see Newton 1729/1960) and many of those who adopted Newton's philosophy structured their books in the same way. Many parts of Newton's book were complex with much detail or sketchy with important detail omitted so that one main purpose of his disciples was to present the bones of his argument as clearly as possible. Although Newton, along with Leibniz, was credited with the invention of the calculus the *Principia* did not use that device. Instead its arguments were often based on the use of small and vanishing quantities which preceded the introduction of the calculus. He (and others before and after) rarely used equations but instead based his arguments on proportional relationships. Newton did not discuss the theory behind motion of the simple pendulum (except in a very condensed form when discussing Proposition XXIV of Book II of the *Principia*; see Densmore 1995, pp. 317–332) but instead focused his attention, as far as pendulum motion is concerned, in Section X of Book I of the Principia, on the motion of the cycloidal pendulum under the influence of a central force. In Section VI of Book II he deals with the motion of a cycloidal pendulum in a resisting medium (Newton 1729/1960).

Some of the books produced to present Newton's philosophy were for use in university lecture halls (for example, Desaguliers 1734, 1745; Keill 1720;

M.R. Matthews et al. (eds.), The Pendulum, 139–150.

'sGravesande 1720; Musschenbroek 1744) while others were simply for the edification of the public (Voltaire 1738) some of these being directed to young people (Tom Telescope 1779) or women (Algarotti 1739). Books in the former group contained a full treatment with as much mathematical rigour as necessary while those in the latter contained little mathematics.

While the treatments of pendulum motion in the university textbooks differ from each other in some details the basic sequence of the argument is the same in each. For most the mechanics of pendulum motion was related closely to an understanding of motion down an inclined plane and treatment of the mechanics of motion on inclined planes precedes the analysis of pendulum motion.

What follows is a case study to illustrate the types of arguments early 18th century textbook writers called on to help their readers understand, from a Newtonian perspective, the reason a simple pendulum moves as it does.

2. Motion on Inclined Planes

The background to 17th century studies of the mechanics of inclined planes (especially the movement of bodies down the plane compared to their motion in free vertical fall) lies in the 13th century with an interest in the conditions of equilibrium of two bodies connected by a string, one hanging vertically and the other resting on an inclined plane (Figure 1).

A writer belonging to the school of Jordanus (Clagett 1961, pp. 104–108; Dijksterhuis 1961, pp. 249–250) presented a proof that, when equilibrium existed, the ratio of these weights, W/w was equal to AB/AC. In the 16th century Stevin (Dijksterhuis 1961, pp. 326–327) provided an elegant argument for this conclusion based on the impossibility of perpetual motion. By the 17th century one implication of this fact, namely that the effective weight of a body on an inclined plane to its full weight was as AC/AB [1] (see Galileo 1590/1960, p. 65; 1600/1960, pp. 171, 174; Matthews 2000, p. 97), was taken for granted and used as the basis for comparing the motion of bodies falling freely or moving down an inclined plane.

The study of uniformly accelerated motion in which the speed of a moving body increased by the same amount in equal intervals of time [2] was assisted in the 14th century by the development of a graphical methods of portraying its properties. Nicholas Oresme (Clagett 1979, II: pp. 285–289; Dijksterhuis 1961, pp. 185–200) introduced his two dimensional graph on which the time was plotted vertically and the speed was plotted horizontally (Figure 2; see also 'sGravesande 1721, p. 53).

It could easily be shown from this presentation that the distance travelled (starting from rest) by the uniformly accelerated body (given by the area under the graph) was equal to the distance travelled by a body moving with a constant speed equal to half that acquired by the accelerated body at the end of its fall (see Figure 2). Others in the 17th century expressed the distance travelled by the accelerated body as half that acquired by a body moving with a constant speed equal to that

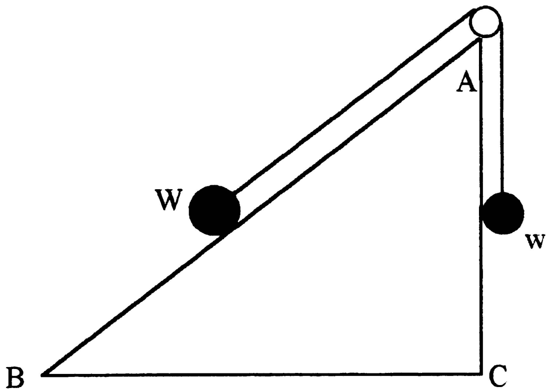

Figure 1. Weights in equilibrium on an inclined plane.

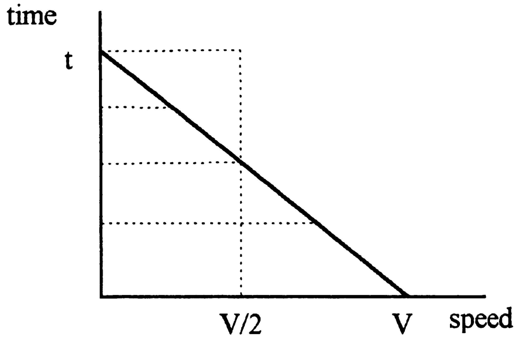

Figure 2. Graph of time against speed.

acquired by the accelerated body at the end of its fall. This fact applied equally to the uniformly accelerated motion of free fall and to that down an inclined plane.

For a particular uniformly accelerated motion beginning from rest it was well known in the 17th century that

total distance travelled = $\frac{1}{2}Vt$ (V = velocity at the end) [3]

$V \propto t$ [4]

the total distance travelled $\propto$ the square of the time taken (from [3] and [4]) [5]

the total distance travelled $\propto$ the square of the velocity at the end (from [3] and [4]) [6]

The comparison between free fall and motion down an inclined plane was another topic of interest and because constants of proportionality were not part of the mathematical vocabulary of the time, care had to be taken to make this comparison. Accelerations were often understood as proportional to the change in speed from

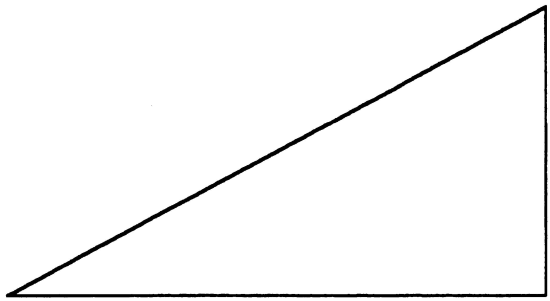

Figure 3. Motion down an inclined plane.

rest in the first small period of time, Δt, (or in the first second of the motion) rather than defined as $\Delta v/\Delta t$ at any point of time during the motion. From the publication of Newton's *Principia* acceleration was understood to be proportional to the "impressed" force. The accelerative force of gravity was the same for all bodies at a particular place but was reduced for bodies on inclined planes in the ratio of the height of the plane to its length. Therefore, if one knew the effective accelerative force of a body (Newton's Definition VII) on an inclined plane and the force of the same body in free fall one could then compare their motions.

Keill (1720, Theorem XXXV, pp. 207–208) showed that the ratio of the speed acquired, in a given time, by a body falling from rest along an inclined plane (Figure 3) to the speed acquired by the same body falling vertically in the same time is equal to the ratio of the height, AC, of the plane to its length, AB (both measured from A to the same horizontal line). He did this using the known fact, established by Galileo, that the ratio, w/W, of the effective weights of the body on the plane and freely falling is the same as the ratio AC/AB. From Newton's second law it follows that the increases in speed in equal intervals of time, $\Delta v/\Delta V$ are also in the same ratio [7]. Thus for any given interval of time the ratio of the total speeds, v/V, is equal to AC/AB [8].

Another important property of motion on an inclined plane was the distance a body moved down the plane AB in the same time it took the body to fall vertically through the height, AC, of the plane. Keill (1720, Problem V, 209–211) solved this problem as follows.

D is the point reached by the body moving down the inclined plane in the time it takes the body to fall from A to C. The previous Theorem XXXV shows that when the speed of the body at C is V the speed of the body at D will be V where $v/V = AC/AB$ [8]. However, because of relation [3] above v/V is equal to AD/AC so that

$$\frac{AC}{AB} = \frac{AD}{AC}. \qquad [9]$$

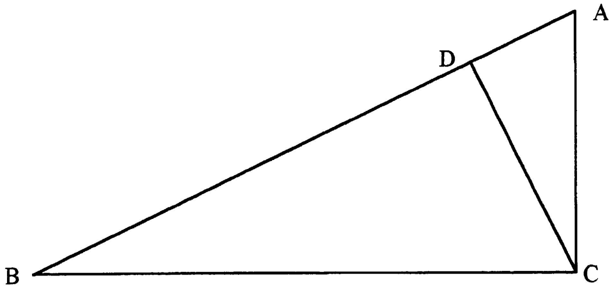

Figure 4. Keill's diagram for distances travelled in equal times.

If this relationship holds in the two triangles ABC and ACD then they must be similar and so angle ADC is a right angle. Thus for any inclined plane the foot of the perpendicular from C onto the plane indicates the distance moved by a body in the time it takes a freely falling body to move from A to C [10].

From this it can be shown (Keill 1720, Theorem XXXVI, p. 212) that the ratio of the times the bodies take to travel from A to B and C respectively is equal to AB/AC. For, from relationship [5],

$$\frac{AD}{AB} = \frac{(t_{AD})^2}{(t_{AB})^2} = \frac{(t_{AC})^2}{(t_{AB})^2}$$

but from [9]

$$\frac{AD}{AC} = \frac{AC}{AB}$$

so that

$$\frac{AC^2}{AB^2} = \frac{(t_{AC})^2}{(t_{AB})^2}$$

or

$$\frac{t_{AC}}{t_{AB}} = \frac{AC}{AB} \qquad [11]$$

A final important property of the inclined plane concerns the equality of the speeds at B and C after two bodies have fallen from rest, one moving from A to B, the other moving from A to C (Keill 1720, Theorem XXXVII, pp. 213–214).

From [3]

$$\frac{V_C}{V_D} = \frac{AC}{AD}.$$

Also from [6]

$$\frac{V_B}{V_D} = \frac{\sqrt{AB}}{\sqrt{AD}}$$

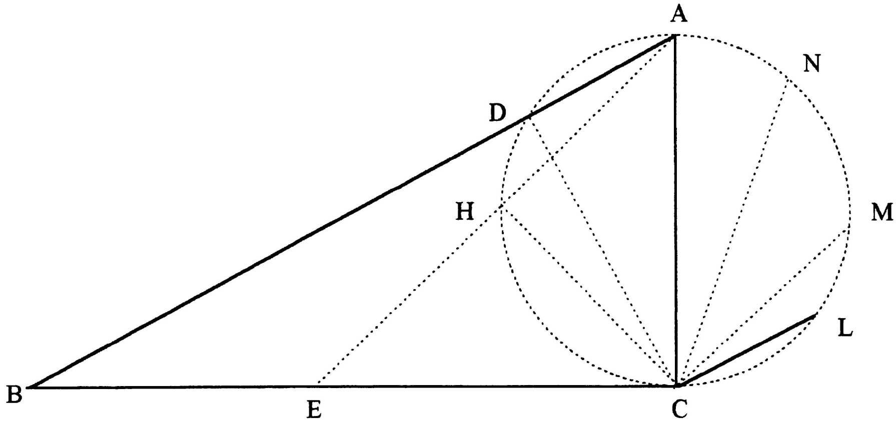

Figure 5. Motion along chords in a circle.

but from relation [9]

$$AC^2 = AB.AD$$

so that

$$\frac{V_B}{V_D} = \frac{AC}{AD}.$$

Thus

$$V_B = V_C. \qquad [12]$$

Newton in his *Principia* (Proposition XL, Theorem XIII) showed more generally that

> if a body, acted upon by any centripetal force [including one where the centre was infinitely far away], is moved in any manuer, and another body ascends or descends in a right [i.e., straight] line, and their velocities be equal in any one case of equal altitudes, their velocities will be also equal at all equal altitudes.

A version of Newton's proof of this proposition is provided in the Appendix.

3. Motion along Chords in a Circle

Relationship [10] presented in the previous section provided an important bridge to the mechanics of the motion of the pendulum.

Because the angle ADC (in Figures 4 and 5) is a right angle, one can draw a circle with AC as a diameter passing through A, D and C as shown in Figure 5 (Desaguliers 1734, pp. 369–370). For another inclined plane AE with the same height AC the point H where it intersects the circle represents the point a body

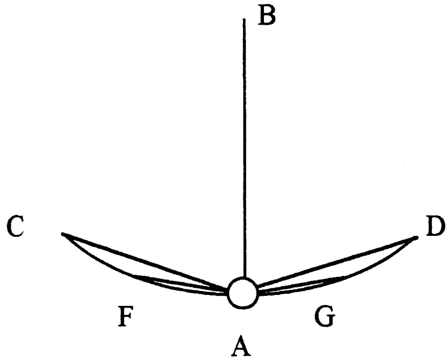

Figure 6. A simple pendulum.

moving from rest along AE would reach in the same time a body took to fall from rest vertically from A to C. Thus two bodies, one moving down AB and the other moving down AE from rest at A, would reach points D and H respectively in the same time. If chords CL and CM are drawn parallel to AD and AH these will also be equal in length to AD and AH. Thus bodies moving from rest down LC and MC (and NC) would all do so in the same time as one moving from rest along AC [13].

4. The Simple Pendulum

For Keill (1720, pp. 225–225; see also Musschenbroek 1744, p. 157) "if the Pendulum is swung about B, so that the heavy Body may describe the Arch CAD, the same Motion will happen to this heavy Body, as would to any Body descending by its Gravity along the spherical Superficies CAD, if that Superficies was perfectly hard and smooth" (see Figure 6). The link with the inclined plane was established by this assumption.

Keill (1720, Theorem XLI, pp. 225–226) added the chords CA, DA, FA and GA to the path of the pendulum (Figure 6) and argued that the times for a body to move from rest from C, D, F, or G to A were all equal (from [13]). If the pendulum bob moved between C and D or between F and G along the chords (see also 'sGravesande 1721, pp. 61–62) the times of oscillation would be equal. If the angle of swing was small the chords are almost the same lines as the equivalent arcs of the circle and so, Keill and 'sGravesande reasoned, the times of oscillation of the pendulum along the large or small arc of the circular path were equal [14] "as far as our Senses can distinguish" ('sGravesande 1721, p. 62; see also Desaguliers 1734, p. 370). It is interesting to note that this argument is very similar to that outlined by Galileo in his letter of November 29, 1602 to del Monte (see Matthews 2000, pp. 102–103).

If two pendulums with different string lengths (Figure 7) are set swinging so that they swing through the same angle the chords EB and GD are parallel as are

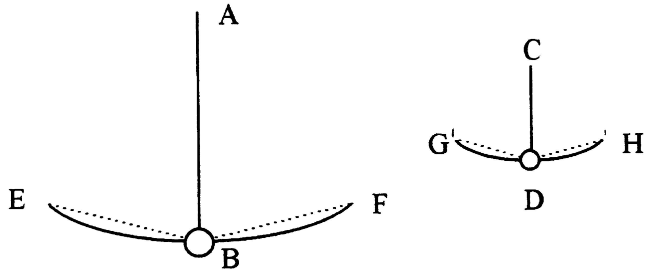

Figure 7. Two similar simple pendulums.

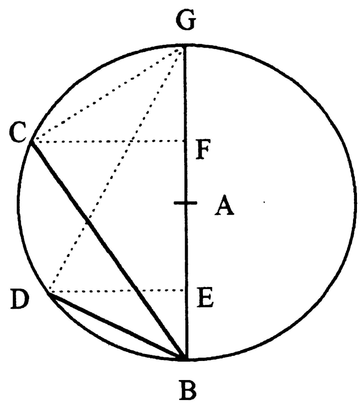

Figure 8. The velocity at the bottom of the path of a pendulum.

BF and DH. Thus the ratio of the time for a body to move from E to B along the chord to that for a body to move from G to D along the chord is equal to the ratio of the square roots of the lengths of the chords (from [5]). But, because of the similarity of the diagrams, this is equal to the ratio of the square roots of the lengths of the strings AB and CD. Thus the ratio of the periods of oscillation of two pendulums is equal to the ratio of the square roots of their lengths [15].

When a pendulum is released from a particular point it reaches the lowest point with a particular speed. In Theorem XLIII Keill (1720, pp. 228–230) demonstrated that this speed was proportional to the length of the chord drawn between the lowest point and the point from which it was released [16]. Keill argued as follows.

From [6], for bodies falling vertically from points E and G respectively (Figure 8)

$$\frac{V_{EB}}{V_{GB}} = \frac{\sqrt{EB}}{\sqrt{GB}}$$

where V_{EB} and V_{GB} are the velocities at B at the end of each fall.

But, because triangles GBD and DBE are similar

$$\frac{GB}{DB} = \frac{DB}{EB}$$

and so

$$\frac{V_{EB}}{V_{GB}} = \frac{DB}{GB}.$$

From [12] $V_{EB} = V_{DB}$ where V_{DB} is the velocity at B of a body moving down the chord or the arc DB starting from rest from B.

Thus

$$\frac{V_{DB}}{V_{GB}} = \frac{DB}{GB}.$$

For the same reasons

$$\frac{V_{CB}}{V_{GB}} = \frac{CB}{GB}$$

so that

$$\frac{V_{CB}}{V_{DB}} = \frac{CB}{DB}. \qquad [17]$$

This relationship was employed by many experimenters (including Newton) who used colliding pendulums in the 17th and 18th centuries to test the laws if impact and, in particular, the law of conservation of momentum which followed directly from Newton's third law of motion (Gauld 1998; Yokoyama 1972).

5. Conclusion

Successful historical treatments of scientific episodes should refer to modes of thought and things known at the time rather than being presented in the context of today's knowledge. Some of the foundations upon which the above treatment of pendulum motion was developed include:

(i) a dependence on proportional reasoning;

(ii) the widespread belief that pendulum motion was identical to that of a body sliding, under the action of gravity, along a smooth circular surface;

(iii) an understanding of the near coincidence of the arc of a circle and its chord for small angles.

The treatment of pendulum motion followed by many of the textbook writers of the early 18th century often owed more to the thinking of Galileo (see Matthews 2000, pp. 102–104) than to that of Newton in his *Principia*. As shown above the explanation of pendulum motion followed from known properties of motion down an inclined plane while in Newton's treatment it was based on the forces which acted on the pendulum bob (Gauld 2002). The above case study is offered to provide access to thinking about the simple pendulum after Newton's new philosophy was

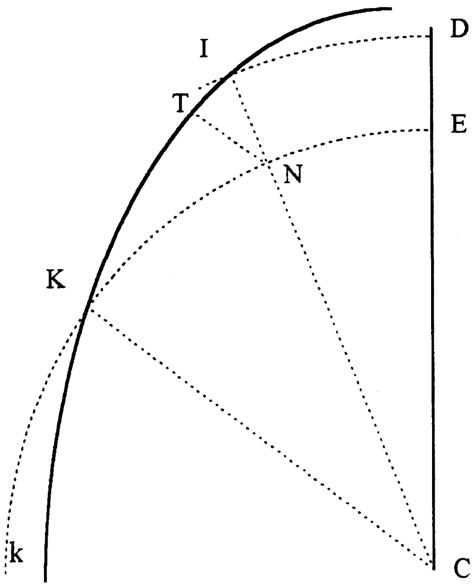

Figure 9. Newton's proof for the equality of velocities at the bottom of an inclined plane.

presented so it can be embedded in a more authentic early 18th century context by teachers today.

Appendix: Newton's proof of Proposition XL in Book I of the *Principia*

Body 1, moves along the straight path DEC while Body 2, moves along the path $ITKk$ (see Figure 9). The forces on the bodies acts towards the centre C. ID and KE are circles with centre C and the speeds of the bodies at I and D are equal at a particular time (i.e. $v_1 = v_2$). IC intersects KE at N. The points E and K are on the paths of the two bodies and, under the action of the forces at I and D, the bodies arrive at E and K after small intervals of time Δt_1 and Δt_2 respectively.

The accelerative force on body 1 at D causes it to accelerate to E and increases its speed by Δv_1 in the time Δt_1 while the force on the body 2 at I, acting along the line IN, accelerates the body to K and increases its speed by Δv_2 in the time Δt_2.

NT is a line perpendicular to IK and so IT represents the component of the force from I to C which causes the increase in speed of body 2.

$$\frac{F_1}{F_2} = \frac{a_1}{a_2} \Rightarrow \frac{DE}{IT} = \frac{\Delta v_1/\Delta t_1}{\Delta v_2/\Delta t_2}$$

$$v_1 = v_2 \Rightarrow \frac{DE}{\Delta t_1} = \frac{KI}{\Delta t_2}$$

$$\frac{\Delta v_1}{\Delta v_2} = \frac{DE.\Delta t_1}{IT.\Delta t_2} = \frac{DE.DE}{IT.KI} = \frac{IN^2}{IT.KI}.$$

But in $\Delta KNIT$ angle KNI = angle NTK

$$= 90^\circ$$

so

$$\frac{IN}{IK} = \frac{IT}{IN} \text{ or } IN^2 = IT.IK \quad \text{(if } IK \text{ is small enough)}$$

thus

$$\Delta v_1 = \Delta v_2$$

and so

$$v_1 + \Delta v_1 = v_2 + \Delta v_2$$

or the velocity of body 1 at E is the same as the velocity of body 2 at K.

If the point C is moved downwards an infinite distance the directions of the forces on the two bodies are parallel. Newton's conclusion implies that, if two bodies moving along different smooth surfaces under the action of gravity have the same velocities at one particular altitude, they will have the same velocities at all equal altitudes.

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Newton's Path to Universal Gravitation: The Role of the Pendulum

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Abstract. Much attention has been given to Newton's argument for Universal Gravitation in Book III of the *Principia*. Newton brings an impressive array of phenomena, along with the three laws of motion, and his rules for reasoning to deduce Universal Gravitation. At the centre of this argument is the famous 'moon test'. Here it is the empirical evidence supplied by the pendulum and Huygens' results which drive Newton's argument. This paper explores Newton's argument while paying close attention to the role the pendulum plays in the argument.

1. Introduction

The centrepiece of Newton's Principia is the argument for universal gravitation contained in Book III of the *Principia*. The book begins with a statement of four 'Rules of Reasoning in Philosophy' followed by six 'phenomena'.[1] Newton's basic argument consists of seven propositions that depend on the phenomena, the mathematical demonstrations of Book I, and on the Rules of Reasoning. For Newton phenomena are not simply data but, rather, are generalisations that fit not only the best data available to Newton but are expected to fit new data as they become known. Phenomena, for instance, include Kepler's harmonic and area laws. Newton moves on to deduce universal gravitation from these phenomena and we will explore this deduction. With this in mind, let us turn to Newton's argument for universal gravitation.

2. The Path to Universal Gravitation

Newton's deductions from phenomena are used ultimately to show universal gravitation. Newton's initial volley is to establish two propositions which are very similar: (1) That the moons of Jupiter and of Saturn are kept in their respective orbits by an inverse-square force directed toward Jupiter and Saturn, respectively. (2) Likewise, the primary planets are kept in their orbits

M.R. Matthews et al. (eds.), The Pendulum, 151–169.

by an inverse-square centripetal force directed toward the sun. Let use examine these in turn.

Newton reasons that the moons of Jupiter are deflected into their orbits by an inverse-square centripetal force because we know that these moons satisfy the area and harmonic laws (Phenomenon I). Because their orbits satisfy these laws Newton can apply Propositions II or III of Book I and Corollary VI, Proposition IV of the same book. The first part of the reasoning yields that there is a force directed toward the centre of Jupiter and, from the second part, we arrive at the fact that this force varies inversely as the square of the distance.

Newton's argument for the second proposition is similar to the preceding argument. In the second proposition Newton turns to the forces acting on the planets except, here, he turns to the harmonic law and area law for the planets, Phenomenas IV and V, respectively. In addition Newton notes that according to Corollary I, Proposition XLV, Book I the slightest deviation from inverse square centripetal force would manifest itself in a motion of the line of apsides. That is, orbital precession would indicate a deviation from an inverse-square relationship.

In his discussion of the Phenomena, Newton proceeded from the harmonic law and area law of Jupiter, Saturn, and their respective satellites to the planets, and finally to the moon. Likewise, the progression with his deductions proceeds accordingly. In Proposition III, Newton turns to the moon. Here, Newton argues that the moon is held in her orbit around the earth by an inverse-square centripetal force in the direction of the earth's centre.[2]

In this proposition Newton pointed out that given that the lunar apogee precesses 3°3′ per revolution *(in consequentia)*, the measure of the centripetal force would not be inverse-square but inversely as 2(4/243) power of the distance. Furthermore, Newton suggested that we might neglect this precession as being due to the action of the sun on the moon in her orbit around the earth. He further suggested that he will show this further on. It turns out that this lunar theory posed a serious empirical challenge to universal gravitation. This doesn't undermine the task at hand, what is of value here is that Newton is clearly remarking that we can use precession to measure an inverse square relationship. That the moon's orbit does not exemplify this relationship is due to a perturbative effect. In short, if precession is to measure inverse-square variation[3] one must show that the precession is due to perturbations. Newton suggests that this is the case with the moon but it was not successfully shown until Clairaut's address in 1749 to the Paris Academy.

Proposition IV further carries the discussion of the moon and confirms the previous proposition's assertion of inverse-square variation in the earth–moon system. Newton argues, here, that the moon is continually deflected from rectilinear motion by the force of gravity and by this force is retained in her orbit around the earth. Newton has thus inferred that the force holding the moon in her orbit is the same force, which we count as terrestrial heaviness, i.e., gravity. Prior to the publication of the *Principia,* 'gravitas' literally meant 'terrestrial heaviness'. To identify the centripetal force that pulls the moon off of tangential motion with gravity, terrestrial heaviness, was a radical departure. It was an admission that what causes heavy objects to fall to the earth also continuously deflected the moon toward the earth and away from (naturally) following her tangential motion. The planets had been considered to be of a different sort than terrestrial objects. The 'Newtonian' revolution was a revolution that resulted in seeing terrestrial bodies and celestial bodies as of the same kind. Newton infers this 'same cause' by appealing to Rules of Reasoning I and II. Newton computes how far the moon, at 60 earth radii from the earth's centre, would fall in 1 min if it were deprived of all forward motion.

> And now if we imagine the moon, deprived of all motion, to be let go, so as to descend towards the earth with the impulse of all that force by which it is retained in its orb, it will in the space of one minute of time, describe in its fall 15 1/12 *Paris* feet.[4]

Assuming, as in Proposition III, that the centripetal force holding the moon in orbit obeys an inverse-square law then we can calculate the centripetal accelerative force operating at the distance of the moon. Newton imagines what would happen if the moon were brought down to the surface of the earth. The moon is at a distance of 60 earth radii from the earth's centre. At just above the earth's surface it would be at one earth radius away from the centre. The ratio of the accelerative force at the current orbit to what it is at the surface is as 1 to 60×60 on the assumption of inverse-square variation. Therefore the force at the earth's surface would be 3600 times greater. On the assumption that gravity is an inverse-square force that extends to the moon, it follows that a heavy object on the earth's surface would freely fall, in 1 min, 3600 of these 15 1/12 Paris feet. In the increment of one second this heavy object, then, would freely fall 15 1/12 Paris feet, or 15 feet, 1 in., and 1 line 4/9, which Newton claims to be more accurate.[5] These calculations are made on the assumption that the lunar distance is 60 earth radii away. Newton cites other estimates of the lunar distance. To accommodate these estimates I will derive a general equation, so that the substitution of these various estimates into this equation will permit some analysis. Let,

c = Circumference of the earth = 123,249,600 Paris feet,
r = Radius of the earth $= \frac{12349600}{2\pi} = 19615783.07$ Paris feet,
R = Lunar distance = $n\,r$,
T = Lunar orbital period = 27d7h43m = 39343 min,
D = Diameter of lunar orbit = $2nr$,

C = Circumference of lunar orbit $= 2\pi R = \pi D = 2\pi nr = 2\pi rn$.
Since $2\pi r$ is the earth's circumference, c,

$$C = cn = 12349600n \text{ (in Paris feet).}$$

Since we know the distance travelled by the moon in one-orbital period. Therefore we know the distance she will travel in 1 min.

$$\frac{C}{T} = \frac{\text{Distance in 1 min}}{1\,\text{min}}.$$

Let $y =$ distance travelled in 1 min. Therefore

$$y = \frac{C * 1}{T} = \frac{12349600n}{39343} = 3132.6945n \text{ Paris feet (i.e., distance travelled in orbit in 1min).}$$

Now suppose the moon was deprived of this tangential motion. We could then calculate how far it would fall in 1 min under the influence of the same centripetal force, which held the moon in her orbit. Newton informs us that this calculation can be done using Proposition XXXVI or Proposition IV, Cor. IX of Book I. Corollary IX of Proposition IV, Book I states:

> From the same demonstration it likewise follows, that the arc which a body, uniformly revolving in a circle with a given centripetal force, describes in any time, is a mean proportional between the diameter of the circle, and the space which the same body falling by the same given force would describe in the same given time.[6]

According to this corollary, the ratio of the distance the moon would fall in 1 min to the distance travelled in orbit in 1 min, y, is equal to the ratio of the distance travelled in orbit in 1 min, y, to the diameter of the lunar orbit, D. So, if $\delta =$ distance the moon would fall in 1 min,

$$\frac{\delta}{y} = \frac{y}{2nr},$$

$$\delta = \frac{y^2}{2nr} = \frac{(3132.6945n)^2}{2(19615783.07)n} = 0.25015n. \text{ (Paris feet)}$$

Newton also informs us how to correct for the action of the sun on the moon. We are trying to isolate the action of the earth on the moon and so a correction (Corollary to Proposition III, Book III) is needed to offset the action of the sun on the moon. This correction amounts to

$$\frac{178(29/40)}{177(29/40)} = \frac{178.725}{177.725} = 1.00563.$$

So, more accurately, the distance the moon would fall in 1 min is

$$d = 0.251015(1.00563) = n = 0.25156n = \text{Paris feet},$$

where n refers to the number of diameters used in the calculation.

This is the general equation for the one-minute fall of the moon at the location of her orbit. But suppose the moon was brought down to an orbit of one earth-radius. We can now calculate how far she will drop in one second and compare this to the length of a seconds pendulum.

Let, F is the force on the moon in orbit and f is the force on the moon at the earth's surface.

$$F \alpha \frac{1}{R^2},$$

$$f \alpha \frac{1}{r^2},$$

$$\frac{F}{f} = \frac{r^2}{R^2} = \frac{t^2/\delta}{T^2/d}$$

where δ is the distance the moon would fall at the distance of one earth-radius. Thus,

$$\frac{r^2}{R^2} = \frac{t^2 d}{T^2 \delta},$$

$$\frac{d}{\delta} = \frac{r^2 T^2}{R^2 t^2}.$$

However $R = nr$ and $D = 0.25156n$. Furthermore, T is equal to 60 s and t is equal to 1 s. So $T = 60t$. Making these substitutions and solving for δ, which is the distance the moon would fall in 1 s at one earth-radius, we have the general equation

$$\delta = \frac{0.25156n^3}{3600}.$$

Note that this general equation incorporates Newton's correction for the action of the sun on the moon. Without this correction and depending on the estimate of the lunar distance in earth-radii, the distance the moon would fall in 1 s is

$$\frac{0.25015n^3}{3600}.$$

Newton sets 60 as the value for n. Table I lists the computed distances corresponding to a 1 s fall at the surface of the earth corresponding to the values listed in the first edition, the third edition, and the *System of the World*.

Huygens had shown that, at the latitude of Paris, a seconds pendulum will be 3 Paris feet, 8 lines 1/2 in. in length (i.e., 3.06 Paris feet).

Tabel I. Distance corresponding to a 1-second fall of the moon

First edition citation	Mean distance of the moon from the earth at syzygies (in earth-radii)	Distance corresponding to a 1 s fall at 1 earth-radius	Distance corresponding to a 1 s fall at 1 earth-radius (with two body correction)
Most astronomers	59	14.271	14.351
Vendelin	60	15.009	15.094
Copernicus	60.33	15.258	15.344
Kircher	60.5	15.387	15.474
Tycho (corrected for parallax)	61	15.772	15.861
Mean	60.166	15.139	15.225
Sample standard deviation		0.558	0.561
t-confidence (95%)		0.642	0.646
Third edition			
Ptolemy and most astronomers	59	14.271	14.351
Vendelin	60	15.009	15.094
Huygens	60	15.009	15.094
Copernicus	60.33	15.258	15.344
Street	60.4	15.311	15.397
Tycho (corrected for parallax)	60.5	15.387	15.474
Mean	60.038	15.041	15.126
Sample standard deviation		0.409	0.411
t-confidence (95%)		0.429	0.431
System of the world			
Ptolemy, Kepler, Bou11iau, Hewelcke, and Riccioli	59	14.271	14.351
Flamsteed	59.33	14.512	14.594
Tycho (corrected for parallax)	60 or 61 (choose 60.5)	15.387	15.474
Vendelin	60	15.009	15.094
Copernicus	60.33	15.258	15.344
Kircher	62.5	16.964	17.060
Mean	60.277	15.234	15.319
Sample standard deviation		0.951	0.956
t-confidence (95%)		0.998	1.003
Mean of all values	60.166	15.138	15.223
Sample standard deviation		0.648	0.651
t-confidence (95%)		0.404	0.406

> And the space which a heavy body describes by falling in one second of time is to half the length of this pendulum in the duplicate ratio of the circumference of a circle to its diameter (as Mr. *Huygens* has also shown), and is therefore 15 *Paris* feet, 1 inch, 1 line 7/9.[7]

Thus

$$\frac{d}{L/2} = \left(\frac{\text{circumference}}{\text{diameter}}\right)^2 = \left(\frac{2\pi r}{2r}\right)^2 = \pi^2.$$

Finally,

$$d = (L/2)\pi^2,$$

where

$$L = 3.06 \text{ Paris feet.}$$

$$d = 15.09 \text{ Paris feet.}$$

The thought experiment result (all values without the two body correction) of a free fall of the moon just above the earth's surface resulted in a drop of 15.138 ± 0.404 *Paris* feet. Huygens' value of 15.09 Paris feet falls well within the error bounds. We note, first, that Newton chooses to use the value of 60 earth-radii for the mean lunar distance. This value, as can be seen from Table I, is near the mean values of the citations in the *Principia* and in the *System of the World*. Second, the result of the moon-test was not dependent on Newton choosing 60 as the value corresponding to the number of earth-radii for the lunar distance.[8] Third, if we turn our attention to the results obtained when the correction for the sun's action on the moon, we notice again that Huygens' value falls well within the bounds. The positive result of the moon-test is not dependent on this correction factor.[9]

These results agree so well that, by Rules I and II, 'the force by which the moon is retained in its orbit is the very same force which we commonly call gravity'.[10] Here we have two phenomena that yield agreeing measurements of the same inverse-square force gravity – toward the earth's centre. The length of a seconds pendulum and the centripetal acceleration of the lunar orbit are two phenomena which measure the same force. The agreement in measured values is another phenomenon, which relates the two phenomena in question. By Rule I if we do not claim that the same force accounts for the centripetal acceleration of the moon and the length of the seconds pendulum at Paris, then we will have to claim that there are two separate causes for these phenomena. The centripetal acceleration of the moon and the length of a seconds pendulum each measure a force resulting in accelerations at a distance of one earth-radius (i.e., at the surface of the earth). The moon-test shows that these accelerations are not only equally directed toward the centre of the earth, but that they are equal in value. Now we have this higher order phenomenon of the agreement in measurements of the two phenomena. The parsimony invoked by the use of the first rule informs us not to infer another cause for this agreement.

Notice Newton's use of Rule II, namely, to the same effect assign the same cause. That is, we note that something attracts the moon toward the earth and something attracts heavy objects to the earth. By Rule II, that 'something' is the same. We note that it is not just a qualitative effect that is generally the same but that it is the same as close as experiments show that they are the same (in this case, Huygens' pendulum experiment). It is Newton's ideal of empirical success which drives his reasoning to show that what we call terrestrial gravity reaches to the moon and, therefore, does not discriminate between terrestrial objects and, thus far, the moon. That is, we have agreeing measurements of the same inverse-square acceleration field from the length of the seconds pendulum and the moon test.

Newton moves on to say that

> were gravity another force different from that, [from the centripetal accelerative force on the moon] then bodies descending to the earth with the joint impulse of both forces would fall with a double velocity, and in the space of one second of time would describe 30 1/6 *Paris* feet; altogether against experience.[11]

This indirect argument reinforces the appeal, via Rules I and II, to the unification of the measure of the centripetal acceleration of the moon with the measure of the length of a seconds pendulum. Notice that the indirect argument would not count against an alternative hypothesis, say, one which posited a force maintained the moon in her orbit (an inverse-square force to boot) but which did not act on terrestrial bodies. So Newton's appeal to Rules I and II helps rule out just this sort of alternative. This alternative hypothesis would demand a separate account of cause for each of the two (or whatever number) basic phenomena.[12] Not only would this alternative hypothesis require a separate cause for each of the phenomena, it would need to accommodate or explain the agreement of the measurements of these phenomena. Furthermore, according to Rule II we are to assign to the same effects the same cause. Until we have evidence to the contrary, there is nothing to indicate that the phenomena are of sufficiently different kinds to warrant us to claim that they have different causes.

Placed in the context of the proof for universal gravitation, Proposition IV carries a new constraint for Newton's theory and for any alternative to the theory. The unification of these phenomena in order to identify the lunar centripetal force with terrestrial gravity demanded Newton to constrain systematically the development of the theory. Newton transformed the notion of terrestrial gravity, heaviness, to count as varying inversely with the square of the distance from the earth's centre. Gravity applies not only to terrestrial objects but to the moon as well. After this proposition Newton is committed to counting any phenomena which measure gravity as also measuring the centripetal force on the moon, be it the length of a seconds pendulum in Peru or one in Lapland. Rules I and II impose systematic constraints on theory development such that the measures of the parameters of phenomena which are to be explained by a theory count as accurate measurements of the theory. A rival hypothesis to the claim that the moon is held in her orbit by the very same force which accounts for terrestrial heaviness would have to account for the equivalence between the centripetal force on the moon and the length of a seconds pendulum. Newton raised the stakes considerably for theory choice. Before moving on we should also note that Proposition IV emphasises the 'empirical' aspect of Newton's ideal of empirical success. That is, in answering the theoretical question regarding the force holding the moon in her orbit and drawing her away from tangential motion, Newton drew our attention to two phenomena which give us agreeing measurements of the same theoretical parameter.

In Proposition V, Rule II is now used to prove that the satellites of Jupiter and of Saturn have heaviness, or gravitate toward their respective centres, Jupiter and Saturn. Furthermore, this proposition also informs us that the circumsolar planets gravitate toward the sun. Newton has already shown in his list of phenomena that the satellites of Jupiter are drawn off of rectilinear motion in their orbits about Jupiter as their centre. The same holds for the satellites of Saturn. And finally, the planets orbit the sun. Newton showed this via the harmonic and area laws. The unification of phenomena shown in Proposition IV is now used again to claim that

> The circumjovial planets gravitate towards Jupiter; the circumsaturnal towards Saturn; the circumsolar towards the sun; and by the forces of their gravity are drawn off from rectilinear motions, and retained in curvilenear orbits[13]

Since Jupiter's satellites about Jupiter and the planets about the sun behave with respect to their centres, or primaries, as the moon behaves with respect to the earth then, by Rule II, we ought to assign them the same cause, gravity. Again, though, the application of this rule is quite specific:

> especially since it has been demonstrated, that the forces upon which revolutions depend tend to the centres of Jupiter, of Saturn, and of the Sun; and that these forces, in receding from Jupiter, from Saturn, and from the sun, decrease in the same proportion, and according to the same law, as the force of gravity does in receding from the earth.[14]

Newton cites two ways in which their behaviour is like that of our moon. First, Propositions I and II of Book III demonstrated that Jupiter is the centre of force for its satellites, Saturn is the centre of force for its satellites, and the sun is the centre of force for the planets. By Proposition III we know that the centre of force on which the moon depends is the earth. Second, from Propositions I and II of Book III Newton determined that the respective forces acting on the satellites of Jupiter and Saturn and on the planets about the sun vary as inversely to the square of the distance. This is the same as what was determined for the moon in Proposition III. By Proposition IV the force on the moon was shown to be the same as the force which accounts for heaviness on the earth, gravity. We now have an equivalence of the measure of the centripetal force on the moon with terrestrial heaviness. By Rule II, then, we are warranted in concluding that since gravity is the force which retains the moon in her orbit and draws the moon off of tangential motion and since the satellites of Jupiter and Saturn and all the planets are like the moon (i.e., compare Propositions I and II to Proposition III and then use Proposition IV) then this force must also be gravity for all these bodies.[15]

Newton gives three corollaries to Proposition V. The first corollary explicitly states that 'there is, therefore, a power of gravity tending to all planets' (p. 329) But notice that Mercury, Venus, and all 'moonless' planets do not have satellites affected by centripetal acceleration so that we can compare effects of the same kind. It is not clear how Newton is to apply Rule II here unless we grant him that 'doubtless, Venus, Mercury, and the rest, are bodies of the same sort with Jupiter and Saturn'. (p .329) Admittedly, we have not observed bodies in the vicinity of these 'moonless' planets but we may believe that such bodies would be drawn to these planets, by Rule. II. The counterfactual claim is in line with the sense of parsimony enshrined in Rule I such that we ought to be inclined to accept the claim that gravity tends to all these planets. There is more to Corollary 1:

> And since all attraction (by Law III) is mutual, Jupiter will therefore gravitate towards all his own satellites, Saturn towards his, the earth towards the moon, and the sun towards all the primary planets.[16]

That is, consider either Jupiter or Saturn and their respective moons. Saturn, for instance, orbits the sun by gravity. The satellites of Saturn orbit Saturn by gravity. It stands to reason, then, that an object near one of Saturn's satellites would be attracted to this satellite according to gravity. As it turns out there is such an object, Saturn itself. Notice, though, that Newton does not appeal to the Rules in his justification but to Law III:

> To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.[17]

Newton is treating gravity as a force of direct interaction. There is mutual attraction between a planet and its satellites. Aiton has pointed out that Huygens

> accepted the Newtonian system, though with important reservations ... Huygens declared that he had nothing against the *vis centripeta* or the gravity of the planets towards the sun, not only because it was established by experience but also because it could be explained by mechanical principles.[18]

What Huygens objected to was not the inverse-square centripetal force characterisation of gravity but to gravity being a force of direct action or mutual interaction between a planet and its satellites.

> I have nothing against *Vis Centripeta,* as Mr. Newton calls it, which causes the planet to weigh (or gravitate) toward the Sun, and the Moon toward the Earth, but here I remain in agreement without difficulty because not only do we know through experience that there is such a manner of attraction or impulse in nature, but also that is explained by the laws of motion, as we have seen in what I wrote above on gravity.[19]

Parting company with Newton, Huygens notes

> I say that I agree that the gravity of bodies corresponds to the quantity of their matter, and I have even demonstrated this in the present Discourse. But I have also shown that the gravity can well be imparted to these bodies that we call heavy, by the centrifugal force of a matter that does not itself weigh (or gravitate) toward the center of the Earth, because of its very rapid and circular motion, but that tends to move away from it.[20]

According to Huygens, without some such mechanical explanation to back up the account, Newton's appeal to universal gravitation would be occult. A mechanical explanation would show that the proper application of Law III would focus on the interaction between the planet's satellite and the surrounding vortical particles creating a pressure gradient that would deflect the satellite away from tangential motion.

In the second corollary Newton extends the force of gravity

> which tends to any one planet is reciprocally as the square of the distance of place from that planet's centre.[21]

The inverse-square variation is shown by the series of propositions of Book III which have led us to this point. That is, the inverse-square variation has been demonstrated for Jupiter and Saturn in Proposition I, for the primary planets in Proposition II, and for the moon in Proposition III. The identification of centripetal inverse-square variation for all these bodies with gravity takes place in Proposition IV (for the moon) and V (for the rest).

Picking up on the theme of mutual interaction in Corollary I, Newton concludes the third corollary to Proposition V with the following:

> All the planets do gravitate towards one another, by Cor. 1 and 2. And hence it is that Jupiter and Saturn, when near their conjunction, by their mutual attractions sensibly disturb each other's motions. So the sun disturbs the motions of the moon; and both sun and moon disturb our sea, as we shall hereafter explain.[22]

The first part is a generalisation resulting from Corollaries 1 and 2. The Newtonian ideal, as we have already stated, points to all these systematic dependencies. That is, the action of the sun on the moon, and the action of the sun and moon on our oceans are unified in the theory of universal gravitation.[23]

Newton claims in the Scholium to Proposition V, that gravity will be the term used for the centripetal force that produces and keeps celestial bodies in their orbits. It is worth repeating this scholium here:

2.1. Scholium

> The force which retains the celestial bodies in their orbits has been hitherto called centripetal force; but it being now made plain that it can be no other than a gravitating force, we shall hereafter call it gravity. For the cause of that centripetal force which retains the moon in its orbit will extend itself to all the planets, by Rule 1, 2, and 4.[24]

This scholium, added in the third edition, contains the only explicit citation of Rule IV in the argument for universal gravitation. Propositions I–V are propositions gathered from the phenomena by induction. These propositions, as we have seen, are backed up by the best data available at that time. We know from Rule IV that these propositions are to be considerded either exactly or very nearly true notwithstanding hypotheses to the contrary. We do this until such time as other phenomena make these propositions either more accurate or liable to exceptions. Newton adds that we do this so that hypotheses do not undercut our inferences. Rule IV endorses the inference to gravity between all the planets because it delivers on the explanations of the phenomena being supported by measurements of theoretical parameters.

What remains for us is to show how Newton's argument can withstand the challenge of the vortex hypothesis accompanied by the metaphysical commitment to explanations of phenomena by physical contact. To challenge Newton's theory, a rival would have to account for systematic dependencies to which we have alluded (for example the moon test) and to any new ones not yet mentioned. It turns out there were some serious challenges to Newton's theory. Those that Newton foreshadowed in the third corollary to Proposition V are listed here. Chief among these challenges is the effect of the sun on the moon in her orbit about the earth. Taking Newton seriously meant following the spirit of Rule IV. The natural philosophers following Newton knew perfectly well that Newton's theory accounted for a vast number of phenomena. Furthermore, these phenomena become unified by giving agreeing measurements. To undercut his theory they would need to emphasise that part of Rule IV until such time as other phenomena make these propositions either more accurate or liable to exceptions.

Turning to Proposition VI we find Newton reasoning for a direct proportional relationship between gravitation on a body and the inertial mass of that body. Here is Newton's statement of the proposition:

> That all bodies gravitate towards every planet; and that the weights of bodies towards any one planet, at equal distances from the centre of the planet, are proportional to the quantities of matter which they severally contain.[25]

Newton offers several arguments for this proposition and, of these, I wish to focus on the often-cited set of pendulum experiments. In these experiments, pendulums of equal weights of different materials were found to oscillate with equal periods and thus were measured to have equal inertial masses (all the while accounting for air resistance):

> I tried experiments with gold, silver, lead, glass, sand, common salt, wood, water, and wheat. I provided two wooden boxes, round and equal: I filled the one with wood, and suspended an equal weight of gold (as exactly as I could) in the centre of oscillation of the other. The boxes, hanging by equal threads of 11 feet, made a couple of pendulums perfectly equal in weight and figure, and equally receiving the resistance of air. And, placing the one by the other, I observed them to play together forwards and backwards, for a long time, with equal vibrations. And therefore the quantity of matter in the gold (by Cor. 1 and 6, Prop. XXIV, Book II) was to the quantity of matter in the wood as the action of the motive force (or *vis motrix)* upon all the gold to the action of the same upon all the wood; that is, as the weight of the one to the weight of the other: and the like happened in the other bodies.[26]

Newton's appeal to Corollary 1 of Proposition XXIV is an appeal to some of the major results of Huygens' work.[27] For two synchronised pendulums A and B, according to this proposition[28] we would have the following relation:

$$\frac{m_A}{m_B} = \frac{w_A}{w_B} \cdot \left(\frac{t_A}{t_B}\right)^2.$$

If $t_A = t_B$ and, *ex hypothesi,* $w_A = w_B$, then we know that $m_A = m_B$. This is the first corollary. The equality of periods, a phenomenon, is used to establish the equivalence of gravitational mass and inertial mass. In order to achieve this Newton points out that the experiment itself (in Book III) is accurate in measurement of inertial mass to one part in a thousand. Since $w_A = w_B$ then

$$\frac{m_A}{m_B} = \left(\frac{t_A}{t_B}\right)^2.$$

Suppose, now, that according to Newton's cited tolerance we have

$$\frac{m_A}{m_B} = \frac{1000}{1001}$$

and

$$\frac{t_A}{t_B} = \sqrt{\frac{m_A}{m_B}} = \sqrt{\frac{1000}{1001}} = 0.9995.$$

This difference of 0.0005 is a tolerance of one part in two thousand.[29] Thus, in order to determine the equality of inertial masses to one part in a thousand Newton needed to show that the difference in periods does not differ by more than one part in two thousand. For illustrative purposes only, let us examine the sort of observations Newton needed to make. We can get some idea of what is involved in the accuracy by assuming that the centre of oscillation is at the length of the pendulum which Newton cited to be 11 feet (= 3.3528 m)[30], let the acceleration due to gravity, g, be 9.82 m^2. Then

$$T = \pi\sqrt{\frac{L}{g}} = \pi\sqrt{\frac{3.3528\text{m}}{9.82\,\frac{\text{m}}{\text{s}^2}}} = \pi\sqrt{\frac{3.528}{9.82}\text{s}^2} = 1.88\,\text{s}.$$

Thus 1.88 s (roughly) is the time needed to make one swing. To make a back and forth swing we multiply by 2 to get 3.77 s. Notice that a tolerance of one part in two thousand would mean that Newton would have had to measure synchronicity for roughly 531 swings, or roughly just over a half-hour's worth of measurements. Recall that Newton claimed that he observed the pendulums' synchronisation 'for a long time'. Although the cited tolerance would be difficult to detect in just a few swings, over many such swings it would be detectable. Newton showed sensitivity to this and thus made observations over a long time.[31,32]

There are five corollaries to proposition VI. The first two corollaries extend the argument in proposition VI, the third corollary argues that all spaces are not equally full,[33] the fourth argues that provided atoms are identical and that any difference in large bodies can be explained by the different arrangements of their atoms then the existence of a vacuum is plausible, and the fifth corollary lists the differences between gravity and magnetism. Let us focus our attention to the first two corollaries. Here is the first:

> Cor. I. Hence the weights of bodies do not depend upon their forms and textures; for if the weights could be altered with the forms, they would be greater or less, according to the variety of forms, in equal matter; altogether against experience.[34]

Newton is claiming that an object's weight cannot be altered simply by changing its shape or texture. Thus melting a piece of wax does not change its weight. In support of this corollary is a body of evidence in accord with experience.

The second corollary explicitly appeals to Rule III and implies for all bodies universally gravitation toward the earth.

> Cor. II Universally, all bodies about the earth gravitate towards the earth; and the weights of all, at equal distances from the earth's centre, are as the quantities of matter which they severally contain. This is the quality of all bodies within the reach of our experiments; and therefore (by Rule III) to be affirmed of all bodies whatsoever.[35]

Newton uses Rule III to argue that for all bodies, universally, at equal distances from the centre of the earth the ratio of weight toward the earth to the quantity of matter (i.e., inertial mass) is equivalent. At any given distance from the centre of the earth the ratio of weight to inertial mass is a 'quality of all bodies' which does not lend itself to intensification or to remission and belongs to all bodies within the reach of experiment. If weight were to depend on form (Newton cites Aristotle, Descartes, and others as holding this point) and could be altered (increased or decreased) by transformations then Corollary 1 would be violated. Since the weights of bodies do not depend on their forms and textures, by Rule III, at equal distances from the earth's centre the weights of bodies are proportional to their inertial mass.

And now· we come upon Proposition VII with its corollaries:

> That there is a power of gravity tending to all bodies, proportional to the several quantities of matter which they Contain.
>
> Cor.1. Therefore the force of gravity towards any whole planet arises from, and is compounded of, the forces of gravity towards all its parts.
>
> Cor.2. The force of gravity towards the several equal particles of any body is reciprocally as the square of the distance of places from the particles.[36]

Proposition VII asserts both that all bodies have a power of gravity in proportion to their masses and that the force of gravity toward several particles of equal mass is proportional to the square of the distance from the particles. In his argument Newton invokes Law III (to every action corresponds an equal reaction) to identify that all the planets are attracted by and attract all other planets. With Proposition VII Newton inferred the equivalence of gravitational mass with inertial mass. Newton extends this result such that the gravitational power of bodies is in proportion to their masses.[37]

3. Conclusion

The argument for Universal Gravitation makes use of Newton's deductions from phenomena. Stein (1991) has shown that these do not end at Proposition VII but encompass the entire third book. Nonetheless the engine of the argument lies in the moontest (Propositions III and IV) and the proportionality of gravitational and inertial mass (Proposition VI). In both of these we see the crucial role of the pendulum. The length of the seconds pendulum is a phenomenon. The one second fall of the moon at the surface of the earth is another phenomenon. The unification of the two is a higher-order phenomenon upon which Newton's theory delivers success. It is precisely this sort of unification that raised the stakes for Newton's competitors and convinced Newton's audience of the importance of his work. Westfall, as can be seen from the preceding, was right: 'without the pendulum, there would be no *Principia*'.

Notes

1 The argument for Universal Gravitation is contained in Book III of the *Principia*. The book begins with a statement of four 'Rules of Reasoning in Philosophy' followed by six 'phenomena'. Newton's basic argument consists of seven propositions that depend on the phenomena, the mathematical demonstrations of Book I, and on the Rules of Reasoning.

2 In 'Newton' scholarship the first three propositions of Book III are often dealt with quickly. It is with Proposition IV, the famous moon test, which historians of science have noted that Newton's system stands apart from previous scientific thinking. Proposition IV is taken to be the crucial link in the argument to universal gravitation. As we will see later, it was Proposition III of Book III that offered a major empirical challenge to universal gravitation. It was due to the work of Clairaut, d'Alembert, and Euler that this particular challenge was resolved.

3 In a sense, we have non-Keplerian orbits. That is, Kepler's ellipse law seems to be violated (after all it is an idealized law). This may be why Newton does not include the ellipse law in his list of phenomena.

4 *Principia,* p. 408.

5 *Principia*, p. 408.

6 *Principia*, p. 46.

7 *Principia*, p. 408.

8 The value of 60 certainly made the computations easier. Our analysis above indicates that the value Huygens obtained clearly fell in between the error bounds.

9 Harper takes Westfall (1973) to task regarding the latter's accusation that Newton 'fudged' his results to get a close fit between his moontest and the length of Huygens' seconds pendulum. Our analysis here is consistent with Harper's point that carrying out the moontest, as per Newton's instructions, yields the values Newton claims and that Westfall's accusation is not credible.

10 *Principia*, p. 409.

11 *Principia*, p. 408.

12 Buffon used a similar line to argue against Clairaut's intial lunar theory which postulated a force which varies according to $(1/r^2) + (1/r^4)$

13 *Principia*, p. 410.

[14] *Principia*, p. 410.
[15] Harper (1989) discusses Newton's application of the Rules of Reasoning as natural kind reasoning. His interpretation is informed by Whewell's colligation of facts. Aiton (1995) in 'The Vortex Theory' has shown that the vortex theorists, notably Leibniz and Huygens, did not dispute inverse-square centripetal acceleration. That is, the explicit use of Rule II in the proof of Proposition V did not pose any problems for the vortex theorists. Their task, rather, was to give a vortex account of Newton's results.
[16] *Principia*, p. 410.
[17] *Principia*, p. 13.
[18] Aiton (1995, p. 7).
[19] Huygens (1986, p. 160).
[20] Huygens (1986, p. 163).
[21] *Principia*, p. 410.
[22] *Principia*, p. 410.
[23] See George Smith, 'Planetary Perturbations: The Interaction of Jupiter and Saturn', forthcoming in I.B. Cohen (1999) *Guide to the Principia*, The University of California Press. See also Wilson (1985).
[24] *Principia*, p. 410.
[25] *Principia*, p. 411.
[26] *Principia*, p. 411.
[27] According to corollary 1, if the times are equal, the quantities of matter in the individual bodies will be as the weights. It is an immediate consequence of proposition. 24. Huygens had made a major breakthrough when he used the slow motions of seconds pendulums to measure the acceleration of gravity far more accurately than it could be measured by attempting to directly determine how far bodies would fall in one second. The much slower motion, in addition to being easier to accurately measure, greatly reduced the relative effect of air resistance.
The slower motion, while minimizing the effect of air resistance, does not minimize the fact that balances in air compare relative buoyancies with respect to air rather than the weights themselves. Proposition 24 is proved for pendulums in vacuums.
[28] Proposition XXIV, Book II reads: 'The quantities of matter in pendulous bodies, whose centres of oscillation are equally distant from the centre of suspension, are in a ratio compounded of the ratio of the weights and the squared ratio of the times of the oscillation in a vacuum'. (p. 303)
[29]

$$0.0005 = \frac{5}{10,000} = \frac{1}{2000}$$

[30] Unlike the moontest, Newton does not here stipulate Paris feet, so we assume that Newton is using English feet. Densmore points out that one Paris foot is 1.066 English feet: Based on this, the length of the pendulum thread was 10.32 feet.
[31] See Harper (1991, 1993), and Wilson (1999).
[32] The modern reader would benefit from examining Wilson (1999), where Newton's pendulum experiments are replicated. There, Wilson highlights an affirmative answer to the question:
'Can the experiment described by Newton ... be carried out with the precision he claims for it, using only such means as were available to him (no stopwatches!)?'
[33] *Principia*, p. 414.
[34] *Principia*, p. 413.
[35] *Principia*, p. 413.
[36] *Principia*, p. 414–415.

[37] For a useful discussion of Newton's reasoning to universal gravitation see Stein (1991, pp. 209–222). Stein argues that it is open to doubt that Newton showed, in the progression to Proposition VII, that gravity is just the sort of force of interaction that warrants the use of Law III. Stein also points that Newton's deductions from the phenomena for universal gravitation do not end with Proposition VII but include the whole of Book III. The point of our brief discussion here is to show the argument. That Propositions III and IV figure prominently will form the substance of later discussion.

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Léon Foucault: His Life, Times and Achievements[1]

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Abstract. Léon Foucault's dramatic demonstration of the rotation of the Earth using a freely-rotating pendulum in 1850 shocked the world of science. Scientists were stunned that such a simple proof of our planet's rotation had to wait so long to be developed. Foucault's public demonstration, which was repeated at many locations around the world, put an end to centuries of doubt about the Earth's rotation – skepticism that had been bolstered since antiquity by Aristotelian philosophy and scripture. This paper puts Foucault's pendulum experiments in context, surveying the life and work of this extraordinary physicist, a man who achieved much – including work on measuring the speed of light, microscopy, astronomy, and photography – without formal training in the sciences.

1. Foucault's Childhood

Jean Bernard Léon Foucault was born in Paris on September 18, 1819, to a comfortable middle-class family. His father was a successful publisher who was known especially for his publication of a series of highly regarded volumes on the history of France. The father retired with his family to the city of Nantes, possibly because he suffered from poor health. He died there in 1829. After his father's death, the mother took her ten-year-old son back to Paris. They settled in a comfortable house at the corner of the Rue de Vaugirard and Rue d'Assas, in the heart of the then-as-now fashionable district of Saint Germain. Léon Foucault would live with his mother in this house his entire life; he would never marry.

As a boy, Foucault was weak and small, and he suffered from poor health. He was reserved, slow to respond, and reluctant to talk or act. Early biographers, who knew him personally, paint an unappealing portrait of a frail boy with a small head and asymmetrical eyes, which never looked in the same direction; one was myopic, and the other far-sighted.

Mme. Foucault had great ambitions for her son. She sent him to the prestigious Collège Stanislas in Paris. But young Léon Foucault was not a good student, and for long periods of time he had to be educated at home by a tutor, since his school performance was so poor. Through the help of a

M.R. Matthews et al. (eds.), The Pendulum, 171–183.

series of tutors, Foucault was able to finish his years of schooling and earn his high school diploma, allowing him to continue his studies at a university.

As a boy, Foucault was clearly not interested in what he was taught at school. But at the age of 13, he suddenly showed where his true talents, and his passions, lay. He began to work with his hands, using a variety of tools, working with great precision and care. He constructed various ingenious toys and machines. First, Foucault built himself a boat. Then he made a telegraph, which was a copy of the telegraph he observed in operation near his home, by the Church of Saint Sulpice. Later, the boy constructed a small steam engine, which really worked.

2. An Interrupted Education and Early Scientific Progress

His mother became convinced that the gift he had of working with his hands would make him a successful surgeon. And since she wanted her son to become a medical doctor, she promptly enrolled him in medical school. Foucault entered medical school in Paris in 1839. He planned to specialize in surgery, as his mother had suggested. At medical school, he studied and participated in the hospital work required of all who aspired to become doctors. One of Foucault's favorite classes was a course of microscopy taught by Professor Alfred Donné (1801–1878). The admiration Foucault developed for Professor Donné seems to have been mutual. The professor recognized the talents of his young student and cultivated them. He entrusted his young student with additional, special assignments and was impressed with his performance. The young student seemed destined to a great career in medicine.

But soon something unforeseen happened. Foucault saw blood in the course of his work in the hospital, and it made him sick. In addition, the sight of the suffering of patients was too much for him to handle. He became so ill that he could not perform his duties any longer. Foucault had no choice: he had to withdraw from the medical program, to his mother's great disappointment.

While still a student of medicine, just before he dropped out, Foucault discovered the work of Louis-Jacques Daguerre (1787–1851) on photography. Foucault shared his excitement about this form of photography with his friend Armand Hippolyte Louis Fizeau (1819–1896), and the two of them studied Daguerre's process carefully. Foucault and Fizeau noticed that Daguerre always kept the camera still in front of its subject for at least half an hour before removing the plate and beginning the development process. And his subject was always the view outside the window or a still object – never people. The two talked about it and decided that the technique could have much greater applicability if it could be used for making portraits. But people could not easily sit still for half an hour.

Foucault and Fizeau began to experiment with daguerrotype on their own. They experimented with various chemicals and – biographers say it was Fizeau who first had the idea – discovered that adding bromine to the plate sensitized it. Thus, in principle, using bromine to pre-treat the copper plate coated with silver should allow for a shorter period to record the image. But how would they implement this discovery? The two young students worked for many days, and finally Foucault was able to determine the exact way of submitting the plate to the action of the mercury vapor so that the necessary exposure time would be only 20 s. This would be their first joint scientific success. Unfortunately, neither of them was to share in the economic rewards of their discovery; for the most part, daguerrotype was soon replaced by other methods of photography.

Even though he was no longer a medical student, Foucault continued to work for Professor Donné. He earned tremendous respect from his professor, and Donné began to trust his young assistant more and more, eventually elevating him to the position of a co-researcher. In 1845, Donné's textbook, *A Course of Microscopy*, was published.[2] The second volume of the work was an atlas of 80 daguerrotypes of the subjects of microscopy discussed in the book. These photographs were all made by Foucault, and Donné described his co-author in the preface to the volume as: 'a young scholar and distinguished amateur of photography.'

Foucault continued to study light, photography, microscopes, and technical devices with enthusiasm. He made a few scientific advances but, since he was only a laboratory assistant to Donné and not a trained scientist, he did not earn much recognition.

3. The Science Reporter

Foucault did not have the mathematical background considered essential for understanding physics, and had not attended the right schools for science. He did not have a Ph.D. and was not even enrolled in a program that would enable him to earn one. Such a degree would constitute a minimal prerequisite for entrance to France's intellectual elite: a group so exclusive and aware of its social status that, within it, members addressed each other as: 'Cher Confrère Savant' (Dear Scholarly Colleague). Within this group, the mathematicians were the crème-de-la-crème. They were called 'geometers,' echoing the way mathematicians of ancient Greece referred to themselves since they considered geometry the purest form of mathematics.

In 1845, Alfred Donné retired from his duties as scientific editor for the newspaper *Journal des Debats* (the Journal of Debates), an important daily paper in France at that time. He passed the assignment on to his co-worker, Léon Foucault. The science editor had the responsibility of reporting in the newspaper any important news about science discussed at the weekly Monday meeting of the Academy of Sciences. Foucault performed this job admirably, and for years wrote these occasional articles explaining to the public about the trajectories of comets, total solar eclipses, and advances in chemistry. Through this job, which required attendance at the meetings of the Academy, Foucault got to know the Perpetual Secretary of the Academy of Sciences, as well as the many distinguished scientists who were members of this elite group of scholars.

Joseph Bertrand, who three decades later was the Perpetual Secretary of the Academy of Sciences, described Foucault as follows in his short biography of him, published in 1882.

> At the age of 25, having learned nothing in the schools, and even less from books, avid for science but loving study less, Léon Foucault accepted the mission of making known to the public the works of the savants and of judging their discoveries. From the beginning, he demonstrated much sense, much finesse, and a liberty of judgment tempered by more prudence than one would expect from a biting and severe spirit. His first articles were remarkable; they were spiritual. He took his task seriously. Introduced without apprenticeship and without a guide into this academic pell-mell, abundant and confused mixture of all the problems and all the sciences, he showed no awkwardness, and, in a role in which mediocrity would never be tolerated, obtained a complete success.[3]

To compound his problems with the established academic community, through his articles in the *Journal des Debats*, Foucault made some enemies and called unhelpful attention to himself who, as a journalist, expressed strong and controversial views about science without having proper training. Bertrand described Foucault's role as science journalist, revealing aspects of his personality, in the following words:

> Persons of considerable esteem in science solicited Foucault's attention, less fearful perhaps of his opinion than of his praise. Coolly polite, attentive only to the truth, Foucault judged with study and reflection, and without complacency. This unknown young man, who had no published scientific work to his name, no discovery justifying his quickly-acquired authority, made them impatient with his assured tranquility, irritated them by his audacious frankness, exasperated them at times with his thin irony.... He excited lively resentments and gave rise to deep rancors.[4]

4. Experiments to Measure the Speed of Light

Foucault and Fizeau worked for François Arago (1786–1853) on measuring the speed of light. Arago was one of France's greatest scientists and the director of the Paris Observatory; he had done work on measuring the speed of light, mainly trying to prove that light travels slower in water. However, diminishing eyesight made Arago give up on his experiments and search for younger researchers to continue the project. At first, Foucault and Fizeau worked together, but suddenly the two friends had a falling out. For four years they had studied science together, and had achieved much. According to Foucault's chronicler Lissajous, despite the fact that Foucault and Fizeau had built a common fund of knowledge through their joint work, each man had his ideas and wanted to pursue them alone.[5]

From collaborators, the two young men became competitors. Fizeau continued on Arago's track, using the old scientist's apparatus and technique. He also pursued his own research on determining the absolute speed of light. In a now-famous experiment in July, 1849, Fizeau used an experimental design that spanned 8633 m: from his parents' house at Suresnes to Montmartre, the high hill on the Right Bank in Paris, which during this time was not settled and was covered with vines. His result was 315,000 km per second, which was closer to the actual value than any estimate obtained until that time. This estimate was 5.1% higher than the value of the speed of light we know today. The next estimate of the speed of light would be obtained a few years later by Foucault, and his error would be one-tenth that of Fizeau's.

Foucault continued to work exclusively on the project of determining the relative speed of light in air and in water, and here chose his own route. He was eager to achieve a good result, now that his friend and new competitor had won recognition for his experiment to estimate the absolute speed of light.

Foucault built a small steam engine, and used it to drive a spinning polygonal mirror at a speed of 800 revolutions per second. His experimental setup was only 4 m long and consisted of the spinning mirror and a stationary one. Because light travels at a limited speed, and one mirror spins fast, the reflected light does not arrive at its starting point, but rather is deflected somewhat. This relative deflection can be measured when air separates the two mirrors and also when a transparent tube of water is inserted between them. In April 1850, Foucault successfully completed his experiment, proving that light traveled slower in water than in air, as predicted by the wave theory of light.[6]

5. Foucault's Discovery of a Proof of the Rotation of the Earth

But on January 6, 1851, Foucault made a discovery the world of science could not ignore. From his journal, we know that he made the discovery at exactly 2 o'clock in the morning. He was down in the cellar of the house he shared with his mother. He had been working feverishly in the cellar for weeks, but no one walking on the fashionable street above could suspect that down below, an experiment was being prepared – one that would forever change the way we view the world. During the last few months of 1850 and into 1851, Léon Foucault had been concentrating all his efforts on a different kind of problem. He was attempting to solve the most persistent scientific problem of all time: one that had plagued Copernicus, Kepler, Descartes, Galileo, and Newton in the sixteenth to the eighteenth centuries, and that – surprisingly – remained unresolved as late as Foucault's own time. Foucault was determined to provide the ultimate proof of a historically contentious scientific theory, the theory that the Earth rotates.

He had prepared his experiment carefully, perfecting it during long hours of concentrated work in his cellar over a period of months. Foucault's remaining problems with the experiment were technical ones, and he was an expert at doing precision work with his hands. He worked with wires, metal cutters, measuring devices, and weights. He finally secured one end of a 2-m long steel wire to the ceiling of the cellar in a special way that allowed it to rotate freely without resulting torque. At the other end of the wire, he attached a 5 kg bob made of brass. Foucault had thus created a free-swinging pendulum, suspended from the ceiling.

Once the pendulum was set in motion, the plane in which it oscillated back and forth could change in any direction. Designing a mechanism that could secure this property was the hardest part of his preparations. And the pendulum had to be perfectly symmetric: any imperfection in its shape or distribution of weight could skew the results of the experiment, denying Foucault the proof he desired. Finally, the pendulum's swing had to be initiated in such a way that it would not favor any particular direction because a hand pushed it slightly in one direction or another. The initial conditions of the pendulum's motion had to be perfectly controlled.

Since such a pendulum had never been made before, the process of building it also required much trial and error, and Foucault had been experimenting with the mechanism for a month. Finally, he got it right. His pendulum could swing in any direction without hindrance.

On January 3rd, 1851, Foucault's apparatus was ready, and he set the devise in motion. He held his breath as the pendulum began to swing. Suddenly the wire snapped, and the bob fell heavily to the ground. Three days later, he was ready to try again. He carefully set the pendulum in motion and waited. The bob swung slowly in front of his eyes, and Foucault attentively followed every oscillation.

Finally, he saw it. He detected the slight but clearly perceptible change he was looking for in the plane of the swing of the pendulum. The pendulum's plane of oscillation had moved away from its initial position, as if a magic hand had intervened and pushed it slowly but steadily away from him. Foucault knew he had just observed the impossible. The mathematicians, and among them France's greatest names: Laplace, Cauchy, and Poisson, had all said that such motion could not occur, or, if it did, could never be detected. Yet he, not a mathematician and not a trained physicist, somehow always knew that the mysterious force would be there. And now, he finally found it. He saw a clear shift in the plane of the swing of the pendulum. Léon Foucault had just seen the Earth turn. This was the first terrestrial, rather than astronomical, proof of the rotation of the Earth.

6. The Demonstration at the Observatory of Paris

As Foucault later described it, Arago kindly agreed to allow him to present his pendulum at the Observatory. Thus the largest, highest, and most famous room in the Observatory, Meridian Hall, was put at Foucault's disposal.

Foucault had much riding on this experiment, and he wanted it to proceed perfectly – no snapping wires – and with great precision. Accuracy would be of paramount importance, if he wanted to prove that the pendulum reflected the motion of the Earth underneath it. So, at his own expense, he hired the best craftsman he could find: Paul Gustave Froment (1815–1864), a man who was well known in France for his high-quality work with brass and other metals. It was important to have a perfect pendulum, hung just right, and started on its motion very carefully, so that its natural movement would not be disturbed by the human hand. For otherwise, the motion of the plane of the swing of the pendulum could be blamed on the initial conditions of motion. Froment produced such perfect pendulum bobs that people still marvel at how they look and perform today. His pendulums made for Foucault are now displayed at the museum of the Conservatoire National des Arts et Métiers (CNAM) on Rue Saint-Martin in Paris.[7] In the actual experiment, Froment would burn a woolen thread securing the pendulum to the wall, so that it would start to swing with no perturbation from human touch.

Foucault and Froment checked that the apparatus was in order. They performed a few trial runs in the high-vaulted hall with its arched windows, and the pendulum – its center aligned with the Paris meridian passing right underneath it – was ready to go.

It was now time to write the invitations to this great scientific demonstration, one that would – Foucault hoped – establish him as a scientist of repute. For, after all, Foucault had by now invented a revolutionary lighting technology used in science and the theater; he had measured the speed of light and proved it was lower in water than in air; and he had come up with the first piece of irrefutable evidence that the world turns. He deserved credit for all of these contributions, and this was his great opportunity to impress the *savants* of the *Académie des Sciences*, the mathematicians and scientists who carried the torch of French scholarship and research.

He made invitation cards, and wrote on each one:

> *You are invited to come to see the Earth turn, tomorrow,*
> *from three to five, at Meridian Hall of the Paris Observatory.*

On February 3rd, 1851, Foucault sent this invitation to all the known scientists in Paris. France's scientists and savants did come to the Observatory, and they did 'see the world turn'. Foucault's pendulum performed exceptionally well. There is an elegance to a large, heavy pendulum swinging slowly back and forth. This pendulum had the added advantage of not only swinging in a stately manner across the stark surroundings of Meridian Hall – it slowly shifted its orientation, rotating ever so slowly over the Paris meridian. And the scientists who gathered around this pendulum immediately understood what they were seeing.

But soon enough, the questions arose: How was it possible that no one had thought of this before? The experiment seemed so incredibly simple. Why had not any of the scientists and mathematicians who had spent lifetimes studying rotations and gravity and astronomy thought of this experiment? The mathematicians were angry that their equations had not predicted this phenomenon, and the physicists were equally upset that their physical intuition and analysis never led them to the 'beautiful experiment' demonstrating so clearly that the Earth rotates. More importantly, another question that begged an answer was: What does mathematics say about this experiment? Shouldn't the equations of motion, developed by generations of mathematicians and physicists (who were often the same people) from Galileo to Kepler, to Newton, to their own members have predicted this phenomenon? Some of the members had already begun to say: 'But I could have told you so. It's all in the equations.' And, in fact, many of them had over the years developed equations that dealt with rotational motion and moving bodies and the Earth.

However, not one of them had thought up such an experiment; not one of them had predicted that a pendulum would exhibit such change in its plane of oscillation in response to the turning of the Earth. *Au contraire*: Some of them had claimed that this would *not* be possible. Cauchy never thought it possible that a pendulum should change its plane of oscillation, and Poisson, as early as 1837, had said that a pendulum would *not* move in such a way. But there were now equations galore to explain the movement Foucault had just shown them. And not one of the mathematicians or physicists of the Academy had an equation or formula that would tell them how fast the pendulum's plane of swing must change at any given location on Earth.

7. Foucault's Sine Law

Foucault himself, the 'non-mathematician' as the members of the Academy thought of him, not only provided the proof of the rotation of the Earth: he had even derived an equation to describe it. Already on February 3, Foucault presented to the Academy a discussion of his experiment and its proof of the rotation of the Earth. In addition, Foucault presented his formula – now called the *sine law* – for determining the length of time it takes, at any given latitude, for the pendulum to sweep a full circle with its plane of oscillation and return to its starting point.

At the north (or south) pole, it takes 24 h to complete the cycle; on the equator, the plane of swing of the pendulum does not move at all. And at intermediate locations, the period is equal to 24 h divided by the sine of the latitude.

$$\textit{Foucault's Sine Law}: T = 24/\mathrm{Sin}(\theta)$$

where T is the time required to complete a circle; Sin is the trigonometric sine function; an θ is the latitude.

Hence in Paris, latitude 48°, 51 min, north it takes just under 32 h for the plane of oscillation of the pendulum to return to its starting point.[8] (Note that the latitude of the pole is 90°; Sin (90) = 1, and hence $T = 24$ h. At the equator the latitude is zero; Sin (0) = 0, and hence $T = 24/0 =$ 'infinity,' meaning that the pendulum's plane of oscillation does not change at all.)

This was an incredible finding, since it is not obvious why the sine function is the correct one to use in an expression to describe the time it should take the plane of the pendulum to complete a circle; and proofs of the sine law are not trivial.[9] Foucault had obtained this surprising result without mathematical training or experience in deriving mathematical equations, and he did it before the mathematicians had even begun to think about the problem.

But the mathematicians refused to be impressed by Foucault's formula. A week after Foucault's demonstration at the Observatory and his presentation of his paper describing the pendulum experiment and the sine law, members of the Academy scrambled to explain his experiment their way, as well as to protect themselves from criticism. They had all been put to shame by Foucault's achievement. Was it really possible that Foucault should discover something that the mathematicians' own equations did not predict? And how in the world would someone with no training in mathematics develop a law describing how fast the pendulum's plane must rotate for a pendulum placed *anywhere* on the planet? Foucault only had one observation point: Paris. They knew that this was a stunning achievement for Foucault, and an equally embarrassing situation for them, the 'experts'.

Joseph Bertrand summed up the circumstances of the proof of the rotation of the Earth as follows.

> Let us say very clearly, for that is true, that the geometers had signaled the direction; and add, for that is just, that they had not explored it; that deplorably quickly, Poisson had judged it unworthy of attention; and that it was Foucault, without any help and without a guide, who was the first to advance it.[10]

8. The Dramatic Public Demonstration at the Panthéon

Louis-Napoleon Bonaparte, Napoleon I's nephew and the President of France, heard about Foucault's experiments and *decreed* that Foucault's experiment be repeated in a grand public display under the highest dome in Paris: that of the Panthéon. Foucault and his engineer Froment worked hard to implement the experiment commanded by the President of the Republic. They and their workers climbed to the top of the high dome and installed the mechanism that would allow this huge pendulum to swing freely in any direction and change its plane of oscillation without any torque preventing the movement.

Toward the end of March, newspapers in Paris announced the upcoming public display of Foucault's pendulum in the Panthéon, saying that the President, demonstrating his 'support for science,' had decided that the experiment should be conducted in a grand public forum. On March 26, 1851, the science reporter Terrien wrote an article in *Le National*:

> Have you seen the Earth go round? Would you like to see it rotate? Go to the Panthéon on Thursday, and, until further notice, every following Thursday, from ten A.M. until noon. The remarkable experiment devised by M. Léon Foucault is carried out there, in the presence of the public, under the finest conditions in the world. And the pendulum suspended by M. Froment's expert hand from Sufflot's dome clearly reveals to all eyes the movement of rotation of the Earth.[11]

The Parisians flocked to the nation's shrine to see the great experiment in physics. This would be a historical moment, in which Galileo, Bruno, and of course Copernicus and Kepler, would be vindicated – inside a magnificent church now dedicated to greatness in science and letters and politics.

The Prince-President was there, along with dukes and duchesses and counts and countesses, leaders of industry and business, and average citizens. A smiling Foucault stood by the wooden circle, waiting. Froment was standing to one side under the great dome, from which the steel wire hung down holding the large pendulum bob. The bob itself was secured by a thread to a post on the side, near Froment. When Foucault issued the order, Froment touched a lit match (safety matches had just been invented) to the thread. As it caught fire, the thread released the pendulum, which swung down to the center of the circle and on to the opposite side. People were watching the swinging pendulum with great interest and curiosity. As Foucault later described it in an article in the newspaper *Journal des Débates*:

> After a double oscillation lasting sixteen seconds, we saw it return approximately 2.5 millimeters to the left of its starting point. As the same effect continued to take place with each new oscillation of the pendulum, this deviation increased continuously, in proportion to the passing of time.[12]

The public was fascinated – taken in by science in the making. According to an eyewitness, every Thursday there were many people inside the Panthéon, looking at the strange pendulum hanging from the high dome above – even during times the pendulum was motionless and the experiment not in action. Foucault himself was there for hours every day, explaining to those around him what the pendulum was doing and how it demonstrated that, in fact, it was the Earth itself that was rotating under the pendulum. The plane of the swing of the pendulum was actually 'fixed in absolute space,' as he put it, 'while we and the planet rotated right under it.'

Foucault was a celebrity. But the world of science continued to ignore him. According to his biographer Stéphane Deligeorges, the attention given to Foucault by Louis-Napoléon made it worse for him, because it made the scientists and members of the Academy jealous. Not only was he not one of them, but now he was getting all the media attention and was becoming the darling of Paris high society and the President of the Republic.

A short time later that year, Prince Louis-Napoléon Bonaparte bestowed on Foucault one of the greatest honors the French nation can accord its heroes: Foucault became a member of the Legion of Honor. But he was still not a member of the Academy of Sciences – and thus not recognized by the scientific community as a peer–and he would not be granted this status for many years to come.

9. Further Experiments

The very next experiment with Foucault's pendulum took place not much over a month after the first demonstration in the Panthéon, in early May, in the cathedral of the city of Reims, in France's fertile and prosperous region of Champagne, northeast of Paris. Others around the world quickly caught the Foucault pendulum fever. Within a few days, an experiment was carried out at another French cathedral, this time in the city of Rennes in Brittany, at almost the same latitude as Paris. There were experiments at the Radcliffe Library at Oxford, in Geneva, in Dublin, Ireland; three more pendulum tests were carried out in the U.K. – in Bristol, in the York Cathedral, and in London. Across the Atlantic, Foucault's pendulum was demonstrated in New York that very same year.[13]

A very important Foucault pendulum experiment was carried out in Rio de Janeiro in September and October of 1851. Later described as 'the marvelous demonstration in Rio,' the experiment was the first one, other than the Panthéon experiment, to be reported by experts in the *Proceedings of the Academy of Sciences*.[14] Another experiment in low latitudes was done in Colombo, Ceylon. This one was carried out by Lamprey and Schaw. The pair used a line made of silk, which was a novelty. The line was 22 m long. It held a 15 kg spherical bob made of lead, which was attached to the silk line with an iron ring. The silk held up well in the experiment. The results accorded with Foucault's sine law.

The final test of Foucault's pendulum in 1851 took place – very significantly–in Rome.The experiment was, in fact, performed in the Jesuit Church of Saint Ignacius in the Vatican. Father Angelo Secchi (1818–1878) carefully supported a twenty-eight-and-a-half kilogram pendulum with a wire of 31.89 m suspended from the high dome of the baroque church. This was an especially important experiment, since it was carried out in what was until then the bastion of anti-Copernican belief. Its successful completion would signal a major change in the attitude of the Church toward science, and particularly toward the Copernican theory that the Earth rotates.

Notes

1 Further details of the life,times and achievements of Foucault can be found in Aczel (2003).

2 Donné & Foucault 1845.

3 J. Bertrand, *Eloge Historique de Léon Foucault*. Paris: Institut de France, 1882, p. 3. Author's translation.

4 ibid. p. 4. Author's translation.

5 Lissajous 1875.

6 It should be noted that other experiments provide equally definite evidence for the particle theory of light – in particular, Albert Einstein's 1905 work on the photoelectric effect, for which he won the Nobel Prize.

7 The CNAM is housed in what used to be, until 1799, the priory of the medieval church of Saint-Martin des Champs, built in the eleventh century. The Foucault pendulum in this converted ancient church impressed Umberto Eco so much that he decided to use it as the setting for the opening, and title, of his novel, *Foucault's Pendulum*.

8 Latitude 48° and 51 min means that $\theta = 48.85°$ (since $51/60 = 0.85$). Using a calculator, we find: Sin (48.85) = 0.753. Now we compute: $T = 24/\text{Sin}\ (\theta) = 24/0.753 = 31.9$ h. Try this with the latitude of your own location. You can find your latitude by consulting any map of your area. For U.S. addresses, the exact latitude (even as accurate as the location of many public buildings and landmarks, rather than generally for an entire city or town) can be found at the U.S. Geological Survey Web site http://geonames.usgs.gov.

9 There are several different proofs of the sine law, which can be found in books on Newtonian mechanics.

10 J. Bertrand, *Eloge Historique de Léon Foucault*. Paris: Institut de France, 1882, p. 21. Author's translation.

11 Stéphane Deligeorges, *The Foucault Pendulum in the Panthéon*. Paris: Musée du Conservatoire national des arts et métiers, 1997, p. 9.

12 *Journal des Débates*, Monday, March 31, 1851,p.4

13 See discussion in Conlin (1999).

14 On October 24, 1887, a demonstration of Foucault's pendulum took place at the St. Jacques tower in Paris. W. De Fontveille published a commentary on it in a publication called *French Expeditions to Tonkin*. The event was attended by the Emperor of Brazil and the Chinese general Tcheng Ki-Tong. In his commentary, Fontveille described the experiment in Brazil in great detail.

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The Pendulum: From Constrained Fall to the Concept of Potential

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Abstract. Kuhn underlined the relevance of Galileo's gestalt switch in the interpretation of a swinging body: from constrained fall to time metre. But the new interpretation did not eliminate the older one. The constrained fall, both in the motion of pendulums and along inclined planes, led Galileo to the law of free fall. Experimenting with physical pendulums and assuming the impossibility of perpetual motion Huygens obtained a law of conservation of *vis viva* at specific positions, beautifully commented by Mach. Daniel Bernoulli generalised Huygens results introducing the concept of potential and the related independence of the 'work' done from the trajectories (paths) followed: *vis viva* conservation at specific positions is now linked with the potential. Feynman's modern way of teaching the subject shows striking similarities with Bernoulli's approach. A number of animations and simulations can help to visualise and teach some of the pendulum's interpretations related to what we now see as instances of energy conservation.

1. A Swinging Body and a Gestalt Switch: Constrained Fall and Isochronism

In Thomas Kuhn's *Structure of Scientific Revolutions* we read:

> Since remote antiquity most people have seen one or another heavy body swinging back and forth on a string or chain until it finally comes to rest. (Kuhn 1970, p. 118).

But did they 'see' (Figure 1) the same 'thing'?

> To the Aristotelians, who believed that a heavy body is moved by its own nature from a higher position to a state of natural rest at a lower one, the swinging body was simply falling with difficulty. Constrained by the chain, it could achieve rest at its low point only after a tortuous motion and a considerable time. Galileo, on the other hand, looking at the swinging body, saw a pendulum, a body that almost succeeded in repeating the same motion over and over again ad infinitum. (Kuhn 1970, pp. 118–119).

M.R. Matthews et al. (eds.), The Pendulum, 185–208.

Figure 1. A swinging body.

A Gestalt shift had occurred, and thus Galileo "observed other properties of the pendulum as well and constructed many of the most significant parts of his new dynamics around them" (Kuhn 1970, p. 119).

For Kuhn the switch was made possible by Galileo's knowledge of late medieval impetus and latitude of forms theories, which were modifications of the Aristotelian paradigm.

But "Galileo could still, when he chose, explain why Aristotle had seen what he did. Nevertheless, the immediate content of Galileo's experience with falling stones was not what Aristotle's had been" (Kuhn 1970, p. 125).

Two interpretations are available, constrained fall and isochronism of oscillations, and while at a first reading Kuhn attributes the first to the Aristotelians and the second to Galileo, it is actually well known that the 'swinging body' played a main role in Galileo's interpretation of the fall of bodies (Matthews 2000, pp. 2–3).

Here we tell a story dealing with the contributions of Galileo, Huygens and Daniel Bernoulli to the less well-known but more ancient interpretation of the swinging body: the one that still sees it as a constrained fall. It will deliver a number of unexpected goods and introduced us to the concept of 'potential' and eventually to the interplay between 'actual' and 'potential' 'energy' in the principle of energy conservation. Modern terminology itself reveals that this tradition started with Aristotle.[1]

2. Galileo: Equal Heights of Ascent and Descent

Studying the constrained fall on an inclined plane Galileo in his '*De motu' (On Motion),* written between 1589 and 1592, asserts that:

> [...] a heavy-body tends downward with as much force as it is necessary to lift it up" (Galilei 1890–1909a, p. 297; Galilei 1960, p. 63).[2]

Thus the basic assumption Galileo makes observing the swinging body appears at an early stage of his career: this is a shift of attention, from the actual movement and the actual trajectory of the body to the height of descent and of ascent. Galileo immediately establishes a first step (Matthews 2000, p. 97) towards an analogy between inclined planes and another form of constrained fall: the pendulum (Galilei 1890–1909a, pp. 297–298; Galilei 1960, pp. 64–65) The leap was made in his '*Mecaniche (On Mechanics)*', written between 1598 and 1600 (Galilei 1960, p. 137), through the analysis of "pendulum motion as motion in a circular rim and as motion in a suspended string. [...] [Galileo] was able to consider pendulum motions as a series of tangential motions down inclined planes" (Matthews 2000, p. 100):

> Consider the circle AIC and in this the diameter ABC with center B, and two weights at the extremities A and C, so that the line AC being a lever or balance, movable about the center B, the weight C will be sustained by the weight A. Now if we imagine the arm of the balance as bent downward along the line BF [...] then the moment of the weight C will no longer be equal to the moment of the weight A, since the distance of the point F from the line BI, which goes from the support B to the center of the earth, has been diminished.
>
> Now if we draw from the point a perpendicular to BC, which is FK, the moment of the weight at F will be as if it were hung from the line KB; and as the distance KB is made smaller with respect to the distance BA, the moment of the weight F is accordingly diminished from the moment of the weight A. Likewise, as the weight inclines more, as along the line BL, its moment will go on diminishing, and it will be as if it were hung from the distance BM along the line ML, in which point L a weight placed at A will sustain one as much less than itself as the distance BA is greater than the distance BM. You see, then, how the weight placed at the end of the line BC, inclining downward along the circumference CFLI, comes gradually to diminish its moment and its impetus to go downward, being sustained more and more by the lines BF and BL. But to consider this heavy body as descending and sustained now less and now more by the, radii BF and BL, and as constrained to travel along the circumference CFL, is not different from imagining the same circumference CFLI to be a surface of the same curvature placed under the same moveable body, so that this body, being supported upon it, would be constrained to descend along it. For in either case the moveable body traces out the same path, and it does not matter whether it is suspended from the center B and sustained by the radius of the circle, or whether this support is removed and it is supported by and travels upon the circumference, CFLI. [...] Now when the moveable

> body is at F, at the first point of its motion it is as if it were on an inclined plane according to the tangent line GFH, since the tilt of the circumference at the point F does not differ from the tilt of the tangent FG, part from the insensible angle of contact (Galilei 1890–1909c, p. 181–182; Galilei 1960, pp. 173–174).

It is possible to follow the transformation of this analogy between pendulum motions and motions along inclined planes into an equivalence through Galileo's entire scientific work,[3] but we confine ourselves here to its full and mature expression of 1638 in the *Discorsi* (First Day):

> As may be clearly seen in the case of a rather heavy pendulum which, when pulled aside fifty or sixty degrees from the vertical, will acquire precisely that speed and force (virtù) which are sufficient to carry it to an equal elevation save only that small portion which it loses through friction on the air. (Galilei 1890–1909b, p. 138; Galilei 1954, pp. 94).

Certainly this shift of attention to the same height of descent and of ascent in the constrained fall was not an easy step. Obviously it was an assumption and not an observation: Galileo correctly notes that the pendulum in standard conditions does not rise to the same height of descent.

Today it is easy to compare through a computer simulation the ideal and the real case and to show what Galileo had in mind: removing 'impediments' such as air resistance, the pendulum actually oscillates in agreement with Galileo's assumption (Figures 2 and 3).

We do not follow here the story of the interpretation of the isochronism of the oscillations, that in its relation with time measurement has already been brilliantly told (Matthews 2000), but instead, shifting our attention towards the constrained fall of the swinging body in a vacuum, we want to focus on the far-reaching role that the assumption that the height of descent and of ascent are the same plays in Galileo's physics, both in the case of inclined planes and of pendulums. This indeed is the main assumption of the third day of the *Discorsi:*

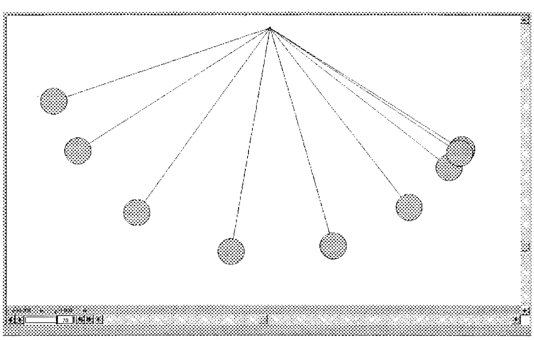


Figure 2. Computer simulation of pendulum motion with air.

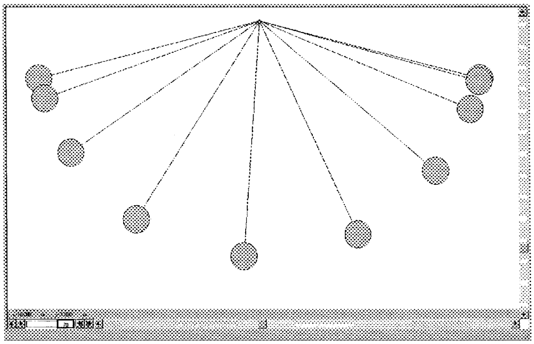


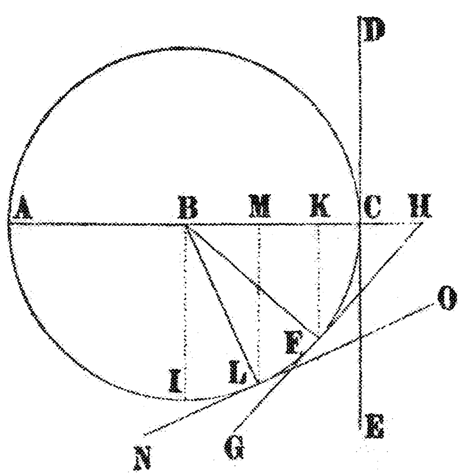
Figure 3. Computer simulation of pendulum motion without air.

> The speeds acquired by one and the same body moving down planes of different inclinations equal when the heights of these planes are equal. (Galilei 1890–1909b, p. 205; Galilei 1954, p. 169).

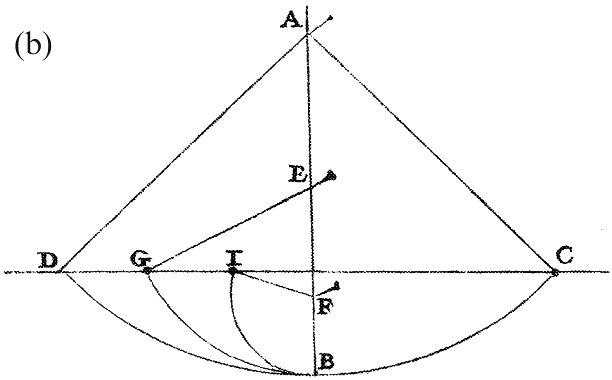
This means that the final velocity of fall depends on the vertical height of descent (elevation) and not on the trajectories actually followed (inclinations). Galileo deals here with inclined planes: the correspondence established between pendulum trajectories and planes with differing inclinations is a basic aspect of his work. In fact it is now a 'constrained' pendulum motion that 'established' the assumption:

> SALV. Your words are very plausible; but I hope by experiment to increase the probability to an extent which shall be little short of a rigid demonstration.

(a)



(b)



Imagine this page to represent a vertical wall, with a nail driven into it; and from the nail let there be suspended a lead bullet of one or two ounces by means of a fine vertical thread, AB, say from four to six feet long, on this wall draw a horizontal line DC, at right angles to the vertical thread AB, which hangs about two finger-breadths in front of the wall. Now bring the thread AB with the attached ball into the position AC and set it free; first it will be observed to descend along the arc CBD, to pass the point B, and to travel along the arc BD, till it almost reaches the horizontal CD, a slight shortage being caused by the resistance of the air and the string; from this we may rightly infer that the ball in its descent through the arc CB acquired a momentum *[impeto]* on reaching B, which was just sufficient to carry it through a similar arc BD to the same height. Having repeated this experiment many times, let us now drive a nail into the wall close to the perpendicular AB, say at E or F, so that it projects out some five or six finger-breadths in order that the thread, again carrying the bullet through the arc CB, may strike upon the nail E when the bullet reaches B, and thus compel it to traverse the arc BG, described about E as center. From this we can see what can be done by the same momentum *[impeto]* which previously starting at the same point B, carried the same body through the arc BD to the horizontal CD. Now, gentlemen, you will observe with pleasure that the ball swings to the point G in the horizontal, and you would see the same thing happen if the obstacle were placed at some lower point, say at F, about which the ball would describe the arc BI, the rise of the ball always terminating exactly, on the line CD. But when the nail is placed so low that the remainder of the thread below it will not reach to the height CD (which would happen if the nail were placed nearer B than to the intersection of AB with the horizontal CD) then the thread leaps over the nail and twists itself about it.

> This experiment leaves no room for doubt as to the truth of our supposition; for since the two arcs CB and DB are equal and similarly placed, the momentum *[momento]* acquired by the fall through the arc CB is the same as that gained by fall through the arc DB; but the momentum *[momento]* acquired at B, owing to fall through CB, is able to lift the same body *[mobile]* through the arc BD; therefore, the momentum acquired in the fall BD is equal to that which lifts the same body through the same arc from B to D; so, in general, every momentum acquired by fall through an arc is equal to that which can lift the same body through the same arc. But all these momenta *[momenti]* which cause a rise through the arcs BD, BG, and BI are equal, since they are produced by the same momentum, gained by fall through CB, as experiment shows. Therefore all the momenta gained by fall through the arcs DB, GB, IB are equal.
>
> SAGR. The argument seems to me so conclusive and the experiment so well adapted to establish the hypothesis that we may, indeed, consider it as demonstrated (Galilei 1890–1909b, pp. 205–207; Galilei 1954, pp. 170–172).[4]

Here again a reconstruction (Figure 4) and an animation (Figure 5) can help us 'see' what Galileo has established: when the fall of the pendulum is constrained by nails fixed on the vertical, whenever possible the weight rises to the same height, even if not in a symmetrical position, and when even that becomes impossible (the nail is in such a position that the length of the string left free is too short) it shows it still has a capacity of movement that makes it revolve around the 'impediment'.

Image now that the pendulum trajectories are substituted by a set of differently inclined planes (see Figure b) and the achievements will be extraordinary (Galilei 1890–1909b, pp. 207–208; Galilei 1954, p. 172):

> SALV. I do not wish, Sagredo, that we trouble ourselves too much about this matter, since we are going to apply this principle mainly in motions which occur on plane surfaces, and not upon curved, along which acceleration varies in a manner greatly different from that which we have assumed for planes. So that, although the above experiment shows us that the descent of the moving body through the arc CB confers upon it momentum *[momento]* just sufficient to carry it to the same height through any of the arcs BD, BG, BI, we are not able, by similar means, to show that the event would be identical in the case of a perfectly round ball descending along planes whose inclinations are respectively the same as the chords of these arcs. It seems likely, on the other hand, that, since these planes form angles at the point B, they will present an obstacle to the ball which has descended along the chord CB, and starts to rise along the chord BD, BG, BI. In striking these planes some of its

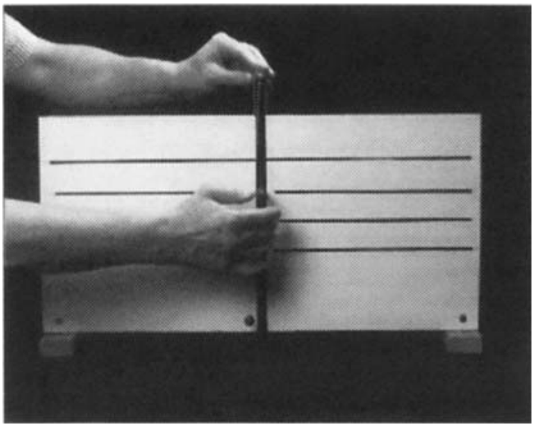


Figure 4. Reconstruction of Galileo's experiment with pendulum motion constrained by nails.

> momentum *[impeto]* will be lost and it will not be able to rise to the height of the line CD; but this obstacle, which interferes with the experiment, once removed, it is clear that the momentum [*impeto*] (which gains in strength with descent) will be able to carry the body to the same height. Let us then, for the present, take this as a postulate, the absolute truth of which will be established when we find that the inferences from it correspond to and agree perfectly with experiment (Galilei 1890–1909b, pp. 207–208; Galilei 1954, p. 172).

The emphasis is always on the 'impetus' the 'velocity', the 'virtue', the 'momentum' acquired during the fall. This is such to raise the weight (now a perfect sphere) to its initial height in the absence 'impediments'.

But what lies behind Galileo's assumption? The principle, already expressed long before by Leonardo, that bodies cannot be raised to a higher

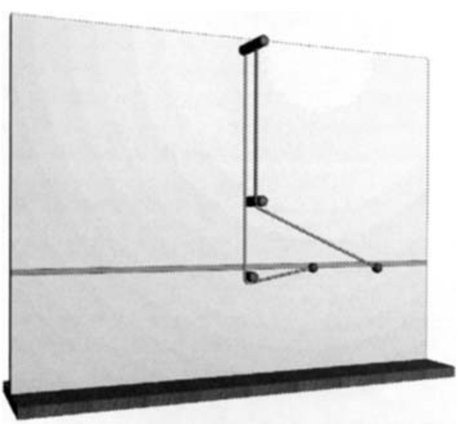


Figure 5. Animation of Galileo's experiment showing that the weights rise at the same height.

level by virtue of their own weight, an early statement of the principle of impossibility of perpetual motion. The bob of the pendulum in its periodical idealised motion cannot rise to a higher level than that of the first descent; otherwise 'work' would be produced out of nothing.

The important quantity connected with the initial and final height is thus the velocity acquired during the fall. To every height of fall corresponds a final velocity acquired during the fall. A link between a static, positional, quantity (height) and a kinetic one (velocity) is indicated. What is the mathematical relation that connects the two?

3. Galileo's Law of Free Fall and the Final Velocity of Fall

Much has been said about Galileo's formulation of the law of fall (Naylor 1974; Wisan 1974; Drake 1995). Its theoretical roots have been discussed in relation to the doctrine of the quantification of qualities and to the average speed theorem, and its experimental roots in relation to Galileo's manuscripts and to his famous passage of the *Discorsi* (see Figure 6):

> SALV. [. . .] A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball. Having placed this board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse-beat. Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the plane, i.e., of the channel, along which we rolled the ball. We also observed that the times of descent, for various inclinations of the plane, bore to one another precisely that ratio which, as we shall see later, the Author had predicted and demonstrated for them. For the measurement of time, we employed a large vessel of

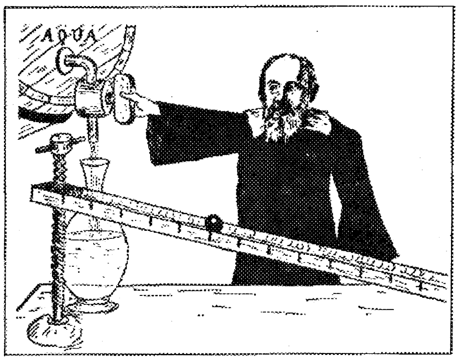


Figure 6. Illustration by Gamow of Galileo's determination of the law of fall (See Gamow 1961, chap. 2).

> water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times, and this with such accuracy that although the operation was repeated many, many times, there was no appreciable discrepancy in the results (Galilei 1890–1909b, pp. 212–213; Galliei 1954, pp. 1780–179).

A simulation (Figure 7) can help understand Galileo's procedures in this passage, where he measures time through a constant flow of water.

The results of Galileo's efforts can be summarised in modern terms as follows (g^* is the vertical acceleration, $a = g^* \sin \theta$. is the acceleration along the groove that varies with inclination, $s = h/\sin \theta$ is the length of Galileo's wooden groove, h its height)

- the instantaneous velocity is proportional to the time elapsed: $v = at$
- space is proportional to the square of the time: $s = at^2/2$
- from (a) and (b) we get: $s = v^2/2a$,
  that is:
- the final velocity is proportional[5] to the square root of the height $v_f = \sqrt{2g^*h}$.

This is a basic law because it connects, perhaps for the first time, position and velocity, statics and kinematics. One of the extraordinary features of this law, lost in modem textbooks, is that the two quantities, position and velocity, are not taken at the same instant. The velocity is the one that the body acquires falling from the height, that is, it is the 'virtual' or 'potential' velocity that it would acquire if it fell from that height. Reversing the two we can also say

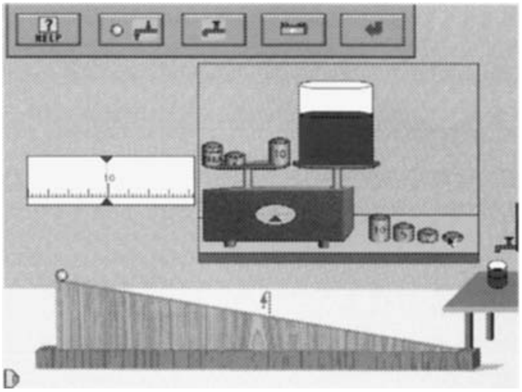


Figure 7. Simulation of Galileo's experiments.

that a body with such a velocity can raise itself to such a position. The pendulum thus acquires a new meaning: in the first quarter of period the weight falling acquires a velocity that, without friction and other impediments, will raise it on the symmetrical side in the second quarter to the same height of descent. The same happens in the third and fourth quarters till the weight reacquires its original height.

4. Huygens: From the Centre of Oscillation of a Compound Pendulum to *Vis Viva* Conservation at Specific Positions

In 1673 Christiaan Huygens in his *Horologium Oscillatorium* makes a significant contribution to our story by solving a difficult and important problem (Gabbey 1980; Yoder 1988; Erlichson 1996). Pendulums in nature are not ideal objects, but real ones with weights that are not concentrated in a point at the end of a weightless string. In the context of his time-measuring efforts, Huygens needed an answer to the question: what is the centre of oscillation of a compound pendulum? That is: what is the length of a simple pendulum that oscillates with the same period as the given compound pendulum? The search for the solution of this problem, perhaps the greatest among his many achievements, produced important results also for our story of the constrained fall. Huygens' early attempts date back to 1661 but 1664 but we refer here to the 1673 account.

The problem had originally been proposed by Mersenne and while "Descartes, Honoré Fabri, and other famous men who held the promise of success, also failed to hit the target except in a few of the easier cases" (Blackwell 1986, p. 105; Huygens 1888–1950, p. 243), Huygens asserts that:

> "In the light of what we will present here [...] I think, is demonstrated by more certain principles and will be found to be entirely in

agreement with experience" (Huygens 1888–1950, p. 243; Huygens 1986, p. 105).

In fact:

> [...] by starting from the first origin and by using a better approach, I overcame all the difficulties, and found not only the solution to Mersenne's problem but also other more difficult things. I also found a method by which one can determine with certainty the center of oscillation in lines, surfaces, and solid bodies (Huygens 1888–1950, pp. 243–245; Huygens 1986, p. 106).

Huygens gives a number of definitions (Huygens 1888–1950, pp. 245–247; Huygens 1986, pp. 106–107) and formulates two basic hypothesis. Here too it is not difficult to see a generalization of Galileo's approach: namely the generalization of the assumption, mentioned above, that Galileo proposed in the third day of the *Discorsi*. While Galileo was concerned with a single body, Huygens deals with a number of them, that is with a system of connected bodies, and thus his concern is with their center of gravity:

> [Hypothesis] I. If any number of weights begin to move by the force of their own gravity, their center of gravity cannot rise higher than the place at which it was located at the beginning of the motion (Huygens 1888–1950, p. 247; Huygens 1986, p. 108).

This statement in its apparent simplicity will have the most extraordinary consequences. It is no wonder then that Huygens makes an effort to explain its meaning. He actually states here, and a number of times after, that the real content of the hypothesis is simply that bodies cannot by virtue of their own weight rise to height higher than the one of fall, a statement that, he asserts, is largely shared:

> But lest our hypothesis create a doubt, we will show that it states only what no one has ever denied; namely, that heavy bodies do not move upwards. For first, if we consider only one heavy body, it is beyond doubt that it cannot ascend higher by the force of its own gravity, where 'ascend' is taken to mean that its center of gravity ascends. Next the same thing must be conceded in the case of any number of weights joined to each other by inflexible lines, for there is no reason not to consider this as only one body. And thus their common center of gravity cannot rise higher (Huygens 1888–1950, p. 249; Huygens 1986, p. 108).

But now Huygens introduces a comment that was implicit in Galileo's *Discorsi* and that is at the root of our interpretation:

> Indeed, if those builders of new machines who tried in vain to produce perpetual motion [motum perpetuum] had known how to use this

> hypothesis, they would have easily seen their errors and would have understood that this is in no way possible through mechanical means [mechanica ratione] (Huygens 1888–1950, p. 251; Huygens 1986, p. 110).

In other words, in modern terms, an endless quantity of work cannot be produced without compensation, perpetual motion is impossible.

A second hypothesis follows:

> II. Air and any other manifest impediment having been removed, as we wish to be understood in the following demonstrations, the center of gravity of a rotating pendulum crosses through equal arcs in descending and in ascending (Huyens 1885–1950, p. 251; Huygens 1986, p. 110).[6]

In fact one cannot imagine a pendulum that after each half period rises 'by virtue of its own weight' to a higher position! But Huygens' great achievement here is to apply this principle to the centre of gravity of the compound pendulum. From this extension of a Galilean line of thought (Mach 1974, p. 210) extraordinary consequences will follow.

Huygens' long analysis now presents a number of 'propositions' followed by a comment. Proposition III and IV are relevant for our analysis of the constrained fall interpretation of the pendulum; Proposition V, that deals with the determination of the centre of oscillation, is related to the interpretation of the pendulum as an isochronous device.

In Proposition III Huygens defines some important properties of the centre of gravity:

> If any number of bodies all fall or rise, but through unequal distances, the sum of the products of the height of the descent of each, multiplied by its corresponding magnitude, is equal to the product of the height of the descent or ascent of the center of gravity of all the bodies, multiplied by the sum of their magnitudes (Huygens 1888–1950, p. 255; Huygens 1986, p. 112).

In modern terms:

$$\Sigma m_i h_i = H \Sigma m_i.$$

Thus,

$$H = \Sigma m_i h_i / \Sigma m_i,$$

where H is the height of ascent–descent of the centre of gravity, m_i are the 'Weights' and h_i their heights of ascent–descent.[7] This is the definition for the ascent or descent of the centre of gravity of a physical body or of a system of bodies.

Now Huygens, in Proposition IV, states that the removal of the constraints between the bodies or parts of the bodies does not influence the equivalence between height of ascent and descent. In modern words, these constraints do not perform work:

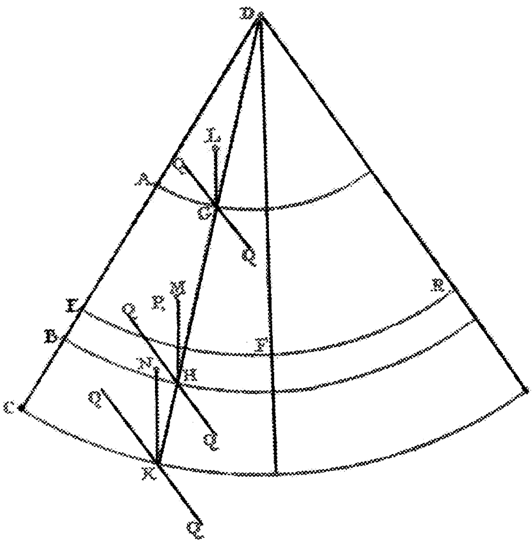
> Assume that a pendulum is composed of many weights, and beginning from rest, has completed any part of its whole oscillation. Imagine next that the common bond between the weights has been broken and that each weight converts its acquired velocity upwards and rises as high as it can. Granting all this, the common center of gravity will return to the same height which it had before the oscillation began (Huygens 1888–1950, p. 255; Huygens 1986, p. 113).

The consequences of the assumption that the constraints can be removed during motion while the centre of gravity regains its initial height will be far-reaching. Huygens in fact implicitly asserts here that the centre of gravity regains its initial height not only after a free fall and a free ascent, not only after a constrained fall and a constrained ascent, but also after a constrained fall and a free ascent.

Huygens' explanation is based on a generalised application of Galileo's relation, between height of free fall and final velocity acquired, to a compound pendulum made by a number of weights fixed to a line. But here we have a constrained fall and not a free fall. When we remove the constraints the weights' velocities do not correspond to those they would have in the same positions in the case of free fall. Before Huygens we did not have a relation to predict them. Huygens' brilliant solution is based on the application of Galileo's law not to the first but to the second part of the trajectory, the one after the removal of the constraints. Now we have a free movement and Galileo's relation can be applied in reverse order: the final velocity of constrained fall is now the initial velocity of the free trajectory of ascent for each single weight. Each weight, now free, will rise to a specific height, lower or higher than the corresponding initial one of constrained fall. From this height of ascent we can calculate the actual velocity acquired by each body at the end of the constrained fall, because after the removal of the constraints the ascent is free and Galileo's law can be applied. Huygens does not actually use the mathematical law but an imagined experimental device: a number of inclined planes able to show each height of ascent. The weights bounce off the inclined planes and rise vertically.[8] This perhaps is an indication of an experimental pattern towards the result: Huygens soon realised that the weights closer to the point of suspension rise to a smaller height, an indication that they were delayed during the constrained fall. In contrast the weights farther away rise to a higher height, indication that they were accelerated during the constrained fall. But the result in any case 'follows' from the principles stated. The centre of gravity we are dealing with here is the one of the system of weights each 'frozen' at the maximum height of ascent. In fact while each weight, depending on its distance from the centre of suspension, acquires a higher or lower height of ascent than in the case of free fall, the centre of gravity of the system of now free weights has to rise to the same height from which it fell because bodies do not rise by their own weight and do not violate the impossibility of perpetual motion.

Let us read Huygens' explanation of Proposition IV:

(c)



Let there be a pendulum which is composed of any number of weights attached to a rod or to a weightless surface. Let this pendulum be suspended from an axis which is drawn through the point D and which is understood to be perpendicular to the plane which is seen in the diagram. In this same plane let E be the center of gravity of the weights A, B, and C. Also let the center line DE be inclined to the perpendicular line DF by the angle EDF, since the pendulum has been pulled back this far. From this point it begins to move and completes any part of its oscillation such that the weights A, B, and C arrive at the points G, H, and K. Next, imagine that the common bond between the weights is broken and that each converts its acquired velocity upwards (which could occur if each encountered an inclined plane) and rises as high as it can; namely, to L, M, and N. When they have arrived there, the common center of gravity is at the point P. I say that P has the same height as the point E.

For first it is clear from the first of our assumed hypotheses that P is not higher than E. But that it is not lower we will show as follows. Assume, as if it were possible, that P is lower than E. And imagine that the weights fall again from the heights LG, MH, and NK, to which they had ascended. From this it is clear that they would acquire the same velocity which they

> had in order to ascend to those heights [Proposition 4, Part II], that is, the velocity they acquired from the motion of the pendulum from CBAD to KHGD. Consequently, if the weights are now returned with those velocities to the rod or surface to which they had been attached, and if they are, reattached to it and continue their motion along the original arc – which could occur if before they touch the rod they are imagined to rebound from the inclined planes QQ – then the pendulum restored in this way will complete the remaining part of its oscillation just as if it had continued its motion without any interruption. Thus the center of gravity, E, of the pendulum crosses equal arcs EF and FR in falling and ascending, and hence is found to have the same height at R and at E. But it was assumed that E was higher than P, which is the center of gravity of the weights located at L, M, and N. Therefore R is higher than P. Consequently, the center of gravity of the weights falling from L, M, and N rises higher than the place from which it descended, which is absurd [Hypothesis 1]. Therefore, the center of gravity P is not lower than E. Nor is it higher than E. Hence it must have he same height. Q.E.D. (Huygens 1888–1950, pp. 255–259; Huygens 1986, pp. 113–114).

Huygens interestingly asserts the possibility first to eliminate the constraints during the motion and second to detect the velocities of constrained fall acquired. He eventually asserts that the centre of gravity of the weights, 'frozen' each at its highest height of ascent after a constrained descent, rises at the same height of descent. The interplay between theory and experiment is interesting here. The assertion that the heights of ascent of each weight is higher or lower than the corresponding height of descent could not be easily detected experimentally, because obviously the now free ascents are not completed in the same time. To calculate the position of the centre of gravity one would need, after the constrained fall and the release of the constraints, to record each single height of ascent: without doubt a difficult experimental task. Then the centre of gravity of this new system had to be calculated. Certainly the assumption of the impossibility of perpetual motion helped Huygens in the formulation of the principle of same ascent–descent of the centre of gravity.

5. Huygens' Results Reinterpreted by Mach: Centre of Gravity and the Conservation of *Vis Viva*

A simpler version of this complex undertaking was suggested by Mach (Figure 8):[9] let us imagine that the pendulum is falling from left to right and that the constraints are removed at the end of the first quarter of period on the vertical line and that the pendulum's weights can ascend freely in the second quarter with the initial velocities of ascent equal to the final velocities of constrained fall acquired on the vertical line (no use here of inclined planes).[10] Some of the weights will rise to a lower level than in the case of free fall and some to a higher level. If we manage to ‘freeze’ the single weights at their maximum heights of ascent (due to different lengths of the pendulums they will be reached in different times) we could actually calculate the position of the centre of gravity of this new system and understand that Huygens’ hypotheses are perfectly reasonable and, of course, correct.

But how to remove the constraints without perturbing the motions? Through an experimental device inspired by Mach’s thought experiment and built by one of us as described in Figure 9.

We imagine that on the vertical line at the end of the first quarter of period a compound pendulum made by weights (iron balls or marbles) connected by a weightless constraint (balsa wood) hits an equal number of equal weights individually suspended and thus free to move. Assuming the conservation of momentum in the impact, we see that the weights rise to different heights, higher or lower than the corresponding initial ones, in agreement with Huygens’ statements. We also realise the difficulty of a precise experimental assessment of the hypotheses.

Today a much clearer visualisation can be achieved through a computer simulation (Figure 10): the automatic removal of the constraints, the tracking that shows the different maximum heights of ascent and the automatic calculation of the position of the centre of gravity are of great help not in proving the hypotheses (the software is built around the mechanical laws we are dealing with) but in visualising and understanding them.

We know now that the centre of gravity descends and ascends to the same height for whichever combination of constrained or free movements (of course if the unconstrained weights are ‘frozen’ at their maximum height of ascent). Trajectories are not important but only initial and final heights. Which results can be derived from Huygens’ third and fourth propositions?

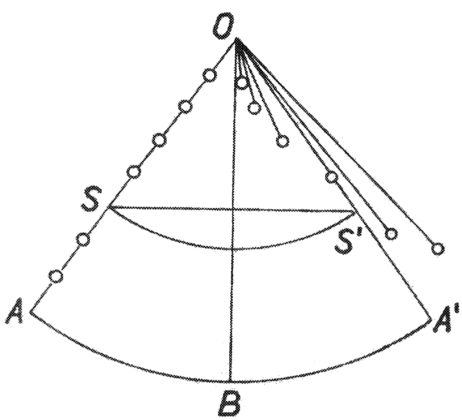


Figure 8. Mach’s interpretation of Huygens’ compound pendulum.

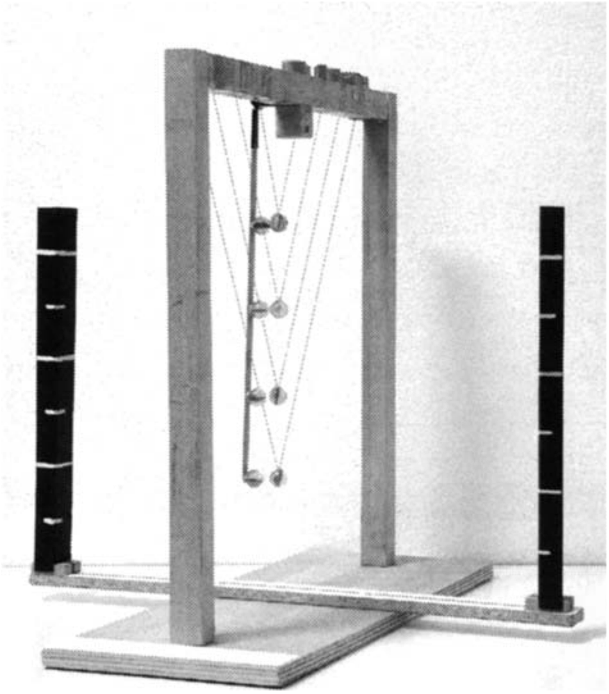


Figure 9. Reconstruction of Mach's compound pendulum by Paolo Mascheretti, Physics Department, Pavia University.

We want to develop the expression $H = \Sigma m_i h_i / \Sigma m_i$ resulting from Huygens' Proposition III, that equates the height of descent and of ascent of the centre of gravity of a compound pendulum, made by a number of weights constrained along a line.

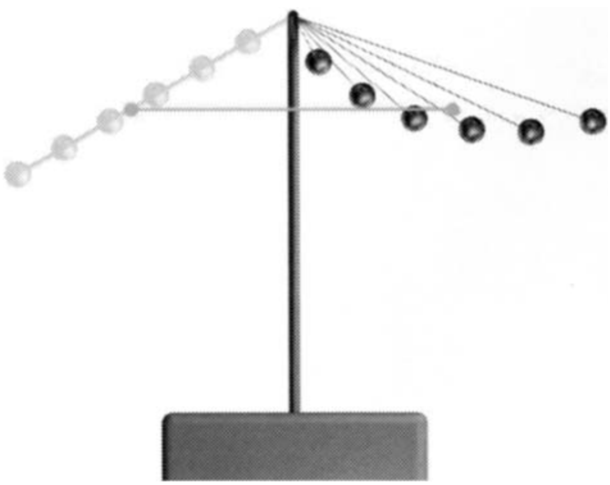


Figure 10. Simulation of the compound pendulum experiment.

Specifically we want to equate the height of descent of the centre of gravity of the pendulum without constraints (H_f), that is with each weight falling independently of the others, with the height of ascent of the centre of gravity of the same compound pendulum that falls with constraints (H_e) till the vertical line and then loses its constraints and rises without them, as in Huygens' Proposition IV.

As to free descent, according to Galileo's result, the vertical distance covered by a heavy body in free fall starting from rest is proportional to the square of the velocity acquired in the fall, with which velocity it could rise to the same height. Applying the relation $v_f = \sqrt{2gh}$ to each free falling weight:

$$H_f = \Sigma m_i h_i / \Sigma m_i = (\Sigma m_i v_i^2 / 2g) / \Sigma m_i.$$

In the case of the constrained descent and free ascent only experimentally, through the removal of the constraints, we can detect the individual heights of ascent (h_i'). But from these heights we can express the final velocities (on the vertical line) of the constrained fall with Galileo's same law (the letter u is used to indicate velocities acquired in the constrained fall):

$$H_c = \Sigma m_i h_i' / \Sigma m_i = (\Sigma m_i u_i^2 / 2g) / \Sigma m_i.$$

Thus Huygens' equivalence of the height of ascent and descent of the centre of gravity:

$$H_f = H_c$$

acquires the form:

$$(\Sigma m_i v_i^2 / 2g) / \Sigma m_i = (\Sigma m_i u_i^2 / 2g) / \Sigma m_i$$

and thus:

$$\Sigma m_i v_i^2 = \Sigma m_i u_i^2.$$

This equation was called the theorem of conservation of *vis viva*.

Huygens' result consists in this: for a system of bodies under the effect of gravity, the sum of the products of the masses multiplied by the squares of the final velocities is the same, whether the bodies move together constrained or whether they move freely from the same vertical height. It appears from this result that $\Sigma m_i v_i^2$ is an important quantity, which is characteristic of the position of the system (the vertical heights of its parts) and does not depend on the paths followed to get to that position. Again we have to remember that in this quantity, characteristic of a system in a given position, the velocities, whether constrained or free, are the final velocities of the 'virtual' or 'potential' fall.

Thus the (compound) pendulum has delivered a very good result: it helped identify one very important physical quantity, the so-called *vis viva,* the capacity of a body to perform 'work', its dependence on the position of the system of bodies and its independence from those constraints which do not perform 'work'. Returning back to the initial position through a closed path, independently of the trajectories, the value of the *vis viva* does not change: it is a constant of the system for a given position. This is here the meaning of conservation of *vis viva*. In fact the *vis viva* during motion varies at each instant due to the variation of the actual velocities.

6. Daniel Bernoulli: From *Vis Viva* Conservation to the Concept of Potential

Daniel Bernoulli, the first to introduce the potential function, contributes to the development of Galileo's and Huygens' results in an important way. In his *Hydrodynamica* of 1738 he discusses at length the relations between '*descensus actualis*' and '*ascensus potentialis*' (Bernoulli 1738, Sectio prima, passim).

In 1748 (Bernoulli 1748) his contribution was to refer back, through Galileo's relation, $\Sigma m_i v_i^2$ (now a positional conserved property) to displacements and to positions and then to the external forces (gravity).

Daniel Bernoulli's approach clearly starts from the conservation of *vis viva* derived from Huygens' results:

$$mv^2 + m'v'^2 + m''v''^2 + \cdots = mu^2 + m'u'^2 + m''u''^2 + \cdots$$

In Bernoulli the notation is inverted: v are the velocities acquired by the weights after the removal of the constraints, u are the velocities acquired when the weights fall freely (that is the mechanical constraints of the system are neglected).

How can this law be utilised for the connection of velocities with external forces? Through Galileo's theorem: in fact, in the case of uniform and parallel gravity, the square of the velocity gained is proportional to the displacement and since this is independent of the path of the body: "there is always conservation of *vis viva* with respect to the height from which the fall takes place" (Bernoulli 1748).

Assuming the acceleration due to gravity as equal to unity and the vertical fall distances equal to x, x' and so on:

$$u^2 = 2x, u'^2 = 2x', u''^2 = 2x'', \ldots$$

the expression of conservation of *vis viva* becomes

$$mv^2 + m'v'^2 + m''v''^2 + \cdots = 2mx + 2m'x' + 2m''x'' + \cdots$$

and thus "the total *vis viva* is equal to the product of the total mass of the system with twice the vertical distance the centre falls" (Bernoulli 1748).

From a modem point of view, the second member expresses double the 'work' done by the forces acting on the system (due to a uniform gravitational field with unit field strength). The *vis viva* of the system in a certain position (velocities are here still the final velocities of a potential fall) equals the 'work' done to get to that position or the capacity to do 'work' falling from that position.

In this sense, the equation is formally equivalent to d'Alembert's work-*vis viva* theorem (Lindsay 1975, p. 391), still used in textbooks as the work-kinetic energy theorem.

Bernoulli next considers the variation of *vis viva* for a body under a central force (Figure 11) and he shows that it only depends on the distance from the centre of attraction (*E*) and not on the trajectory:

Vis viva at D and C is the same, moving from C to D there is no change of *vis viva* (no work is performed along paths perpendicular to the force, the difference in *vis viva* between A and D thus does not depend on the trajectory followed (straight down from A to D or going through C). The advantage is that the principle in Daniel Bernoulli's version immediately furnishes an equation connecting the final velocities of the bodies of the system in question and the variables determining their position in space. *Vis viva* at D depends only on the distance from the centre of attraction E, it is now a positional quantity. In the closed path DACD there is no gain or loss of 'work'. The difference in the 'work' done depends only on the initial or final positions and not on the path. The 'positional' *vis viva* is thus an indication of potential 'work', later to be called 'potential energy'. The variation of the *vis viva* is equal to the variation of the potential 'work'.

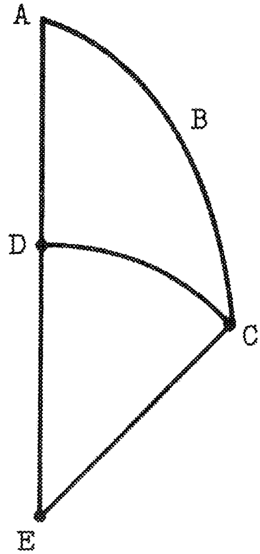


Figure 11. Diagram used by Bernoulli in his demonstration about central forces.

7. Modern textbooks Still Utilise Bernoulli's Approach

The Feynman's Lectures on Physics of 1963 (Feynman et al. 1975)[2] is revealing. In discussing work done by gravity, Feynman wants to show that the total work done in going around a complete cycle is zero (Figure 12), in agreement with the impossibility of perpetual motion (Feynman et al. 1975,[2] Vol. 1, p. 13.6). He thus analyses a closed path in a radial gravitational field (M is the centre of attraction) and shows that on the circular paths the work is zero because the force is at right angles to the curve, and on the radial paths the total work is again zero because it is the sum of the same amount of work done the first time in the direction of the centre of attraction and the second time in the opposite direction.

Is the situation different for a real curve? No, because we can refer back to the same analysis (see Figure 13): the work done in going from a to b and from b to c on a triangle is the same as the work done in going directly from a to c.

In the same chapter there is also an implicit reference to Huygens' discussion of the compound pendulum and to the resulting *vis viva* conservation at specific positions, independently of trajectories and constraints. Feynman, dealing with planetary motion, asserts that:

> So long as we come back to the same distance, the kinetic energy will be the same. So whether the motion is the real undisturbed one, or is changed in direction by channels, by frictionless constraints, the kinetic energy with which the planet arrives at a point will be the same (Feynman et al. 1972,[2] Vol. 1, p. 13.8).

A clear, even if implicit and perhaps unaware, reference to Daniel Bernoulli's results (through the mediation of the tradition of rational mechanics): work only depends on the initial and final positions (difference of potential) and not on the actual path (trajectory).

Thus the insight that pendulums without impediments can only rise back to their original heights has produced, through a number of achievements, a very important and lasting historical result: from *vis viva* conservation at specific positions we get the concept of potential, a remarkable gestalt switch from isochronism and a big step towards what is now energy conservation.

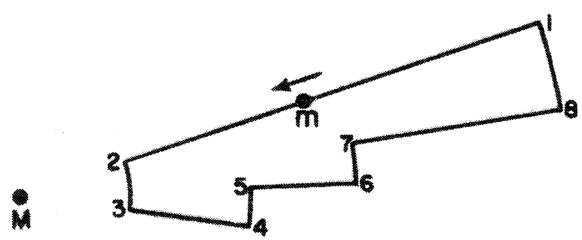


Figure 12. (See Feynman et al. I, 13, 3).

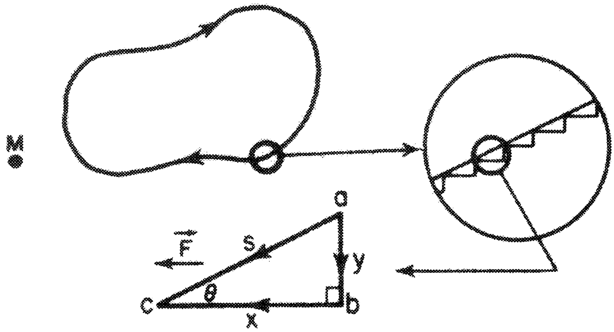


Figure 13. (See Feynman et al. I, 13, 4).

Acknowledgements

We would like to thank three referees (Colin Gauld, Art Stinner and Michael Fowler) for their useful and detailed suggestions and Carla Garbarino for her help in the editing of this paper.

Notes

[1] Leibniz's contributions, however Important, are not discussed here.
[2] This work is discussed in Drake (1976), Fredette (1976), see also Camerota (1992).
[3] Galileo's equivalence is correct as far as heights of descent and ascent are considered. It does not hold for the instantaneous velocities: the rotation of the ball on the inclined plane has also to be taken in account (the moment of inertia reduces the actual velocities).
[4] On the different meanings of 'momento' in Galileo, see Galluzzi (1979).
[5] In Galileo's case the vertical acceleration g^* differs from today' s acceleration of gravity g because of the moment of inertia of the ball rolling and not sliding down the groove: $g^* = 5/7\, g$.
[6] Here Blackwell's translation has been modified.
[7] The centre of gravity after the removal of the constraints is calculated with each body 'frozen' at its maximum height of ascent. In fact the bodies do not get to their final positions at the same time.
[8] In Huygens diagram below the positions L, M, N are not 'real' heights of ascent but part of his 'by absurd' demonstration.
[9] We will utilise Mach's 1883 analysis of Huygens 1673, see (Mach 1974, pp. 213–217).
[10] 'Free ascent' of course still implies a constraint (weightless string) as in the simple pendulum.

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Idealisation and Galileo's Pendulum Discoveries: Historical, Philosophical and Pedagogical Considerations

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Abstract. Galileo's discovery of the properties of pendulum motion depended on his adoption of the novel methodology of idealisation. Galileo's laws of pendulum motion could not be accepted until the empiricist methodological constraints placed on science by Aristotle, and by common sense, were overturned. As long as scientific claims were judged by how the world was immediately seen to behave, and as long as mathematics and physics were kept separate, then Galileo's pendulum claims could not be substantiated; the evidence was against them. Proof of the laws required not just a new science, but a new way of doing science, a new way of handling evidence, a new methodology of science. This was Galileo's method of idealisatioin. It was the foundation of the Galilean–Newtonian Paradigm which characterised the Scientific Revolution of the 17th century, and the subsequent centuries of modern science. As the pendulum was central to Galileo's and Newton's physics, appreciating the role of idealisation in their work is an instructive way to learn about the nature of science.

1. Introduction

In a letter of 1632, ten years before his death, Galileo surveyed his achievements in physics and recorded his debt to the pendulum for enabling him to measure the time of free-fall, which, he said, 'we shall obtain from the marvellous property of the pendulum, which is that it makes all its vibrations, large or small, in equal times' (Drake 1978, p. 399). To use pendulum motion as a measure of the passage of time was a momentous enough achievement, but the pendulum is also central to Galileo's treatment of free fall, the motion of bodies through a resisting medium, the conservation of 'energy', and the rate of fall of heavy and light bodies. The pendulum is even more central to Newton's elaboration of the scientific programme begun by Galileo. The historian Richard Westfall remarked that 'It is not too much to assert that without the pendulum there would have been no *Principia* (Westfall 1990, p. 82). Thus Galileo's and Newton's account of pendulum motion is central to the overthrow of Aristotelian physics and the development of the modern science, a development about which the historian Herbert Butterfield has said:

M.R. Matthews et al. (eds.), The Pendulum, 209–235.

> Of all the intellectual hurdles which the human mind has confronted and has overcome in the last fifteen hundred years, the one which seems to me to have been the most amazing in character and the most stupendous in the scope of its consequences is the one relating to the problem of motion. (Butterfield 1949, p. 3)

Consequently if students go through again the pendulum arguments, analyses and experiments of Galileo and Newton, one should expect them to learn a good deal of physics and a good deal of the methodology of physics; that is, to learn about the nature of science.

Galileo at different stages makes four novel claims about pendulum motion. Despite people seeing swinging pendulums for thousands of years, no one, not even the great Leonardo da Vinci who studied pendulum motion, saw what Galileo 'saw'. Galileo claimed that:[1]

(1) Period varies with length and later, more specifically, the square root of length; the Law of Length.
(2) Period is independent of amplitude; the Law of Amplitude Independence.
(3) Period is independent of weight; the Law of Weight Independence.
(4) For a given length, all periods are the same; the Law of Isochrony.

These laws are taught in all high school and university physics programmes, with the topic frequently being voted the 'most boring' in physics. But some knowledge of the rich history behind Galileo's pendulum discoveries allows much more than just the physics of pendulum motion to be taught; such knowledge allows one of the defining methodological features of the Scientific Revolution and of modern science to be appreciated, namely the importance of idealisation in the revolutionary scientific achievements of Galileo and Newton.

2. Teaching the Nature of Science

Most science programmes rightly aspire to having students appreciate something of the methodology of science; this is frequently expressed as the importance of teaching and learning about the 'nature of science' (NOS) . One reviewer of curricular developments has said:

> Although varying ideas exist for what components constitute a scientifically literate citizenry, two attributes that have been consistently identified in the literature are an understanding of the nature of science and an understanding of the nature of scientific knowledge. (Meichtry 1993, p. 429)

Another reviewer has stated: 'For more than half a century there has been an overwhelming consensus of science education literature and science organizations of instructing science teachers and/or their students in the nature of science' (Alters 1997, p. 39). NOS goals are found in numerous national, state and provincial curricula.[2]

The 17th century debates about the pendulum provide exemplary material for methodological, or epistemological, reflection. Understanding the Scientific Revolution is especially important for theories of knowledge, or for epistemology. An epistemology that pays no attention to the Scientific Revolution, or is at odds with its achievements and its methodological innovations, is at best ill-nourished and at worst irrelevant to science.

3. Epistemology and History of Science

The view that the history of science has a prime importance for epistemology, or theories of knowledge, has a long and distinguished heritage, going back at least to Bacon, Spinoza and Locke in the seventeenth century, and including Kant in the eighteenth century, Whewell in the nineteenth century, and Popper, Putnam and many others in the twentieth century. All these philosophers thought it incumbent to articulate their theories of knowledge in the light of their understanding of the new science of Galileo and Newton. Karl Popper is perhaps the best known twentieth-century advocate of the position, saying that:

> The central problem of epistemology has always been and still is the problem of the growth of knowledge. *And the growth of knowledge can be studied best by studying the growth of scientific knowledge*. (Popper 1934/1959, p. 15, italics in original)

However, linking epistemology to history of science is not without problems. There is a well recognised problem with philosophers studying the growth of scientific knowledge, namely they see their own philosophical predispositions as the ones that are responsible for the growth of science. This Pygmalion projection has been especially well documented in the case of Galileo, perhaps for no other reason than he is the scientist most written about by philosophers. The historian Alistair Crombie, in accepting the 1968 Galileo Prize given by the Domus Galileiana, Pisa, remarked that:

> Galileo has been described as a cultural symbol, transcending history. Rather it seems to me that his reputation illustrates the universal human habit of creating myths to justify attitudes taken to the present and future, myths intimately tied in Western culture to our conception of time and history. As a scientific thinker Galileo has been made by an astonishing variety of philosophical reformers whatever their hearts desired: an experimentalist contemptuous of speculation, a mathematical idealist indifferent to experiment; a positivist in fact hostile to ideas although he may not always have known this himself, an illustration of the role of ideas in scientific discovery; a Platonist, a Kantian, a Machian operationalist. … But our intellectual inheritance is also an essentially critical one, predisposing each generation to take to pieces the history written by its predecessors in their image, before re-writing it in its own. (Crombie 1970, p. 361)[3]

Figure 1.

So, mindful of this past ‘projectionist’ history, nevertheless epistemological lessons can, with caution, be drawn from Galileo’s pendulum discoveries; and these lessons are of a kind that students can easily enough appreciate.

4. Guidobaldo del Monte’s Criticism of Galileo’s Pendulum Claims

The seventeenth century’s analysis of pendulum motion is a particularly apt window through which to view the methodological heart of the Scientific Revolution. More particularly, the debate between the Aristotelian Guidobaldo del Monte and Galileo over the latter’s pendular claims, represents, in microcosm, the larger methodological struggle between Aristotelianism and the new science. This struggle is in large part about the legitimacy of idealisation in science, and the utilisation of mathematics in the construction and interpretation of experiments.

The most significant opponent of Galileo’s nascent views about the pendulum was his own academic patron, the distinguished Aristotelian Guidobaldo del Monte (1545–1607) (Figure 1). Del Monte was one of the great mathematicians and mechanicians of the late-16th century. He was a translator of the works of Archimedes, the author of a major book on mechanics (Monte 1581/1969), a book on geometry (*Planispheriorum universalium theorica*, 1579), a book on perspective techniques *Perspectiva* (1600), and an unpublished book on timekeeping *De horologiis* that discussed the theory and construction of sun dials. He was a highly competent mechanical engineer and Director of the Venice Arsenal. Additionally he was an accomplished artist, a minor noble, and the brother of a prominent cardinal.

And he was the patron of Galileo who secured for Galileo his first university position as a lecturer in mathematics at Pisa University (1588–1592), and his second academic position as a lecturer in mathematics at Padua University (1592–1610).[4]

Del Monte was not only a patron of Galileo, but from at least 1588 to his death in 1607, he actively engaged in Galileo's mechanical and technical investigations. They exchanged many letters and manuscripts on broadly Archimedean themes. Del Monte believed that theory should not be separated from application, that mind and hand should be connected. As he said in the Preface of his *Mechanics:* 'For mechanics, if it is abstracted and separated from the machines, cannot even be called mechanics' (Drake & Drabkin 1969, p. 245). Del Monte was concerned with the long-standing Aristotelian problem of how mathematics related to physics. In his *Mechaniche* he says:

> Thus, there are found some keen mathematicians of our time who assert that mechanics may be considered either mathematically, removed [from physical considerations], or else physically. As if, at any time, mechanics could be considered apart from either geometrical demonstrations or actual motion! Surely when that distinction is made, it seems to me (to deal gently with them) that all they accomplish by putting themselves forth alternately as physicists and as mathematicians is simply that they fall between stools, as the saying goes. For mechanics can no longer be called mechanics when it is abstracted and separated from machines. (Drake & Drabkin 1969, p. 245)

The methodological divide between del Monte and Galileo, between Aristotelian science and the embryonic New Science of the Scientific Revolution, was signalled in del Monte's criticism of contemporary work on the balance, including perhaps drafts of Galileo's first published work, *La Bilancetta* (Galileo 1586/1961). Del Monte cautioned that physicists are:

> … deceived when they undertake to investigate the balance in a purely mathematical way, its theory being actually mechanical; nor can they reason successfully without the true movement of the balance and without its weights, these being completely physical things, neglecting which they simply cannot arrive at the true cause of events that take place with regard to the balance. (Drake & Drabkin 1969, p. 278)

In a 1580 letter to Giacomo Contarini, del Monte says:

> Briefly speaking about these things you have to know that before I have written anything about mechanics I have never (in order to avoid errors) wanted to determine anything, be it as little as it may, if I have not first seen by an effect that the experience confronts itself precisely with the demonstration, and of any little thing I have made its experiment. (Renn et al. 1988, p. 39)

This then is the methodological basis for del Monte's criticism of Galileo's mathematical (or geometric) treatment of pendulum motion. Del Monte, a mathematician and a great technician, is committed to the core Aristotelian principle that physics, or science more generally, is about the world as experienced, and that sensory evidence is the bar at which putative physical principles are examined. Vision was Aristotle's primary sense; it provided the material for mind or *nous*. Aristotle reverses Plato's ordering of reason and observation. In his *On the Generation of Animals*, Aristotle remarks:

> Credit must be given rather to observation than to theories, and to theories only if what they affirm agrees with observed facts. (Barnes 1984, p. 1178)

This is also the commonsense understanding of science. Del Monte believed that Galileo was a great mathematician, but that he was a hopeless physicist. This is the methodological kernel of the Scientific Revolution. The subsequent development of pendular analyses by Huygens, and then Newton, beautifully illustrate the interplay between mathematics and experiment so characteristic of the emerging Galilean–Newtonian Paradigm.

Galileo and del Monte had had an early exchange of letters about motion in a semi-circle, and Galileo's belief that such motion was tautochronous;[5] unfortunately these letters were lost by the time Galileo's *Opere* was collected and edited by Antonio Favaro (20 volumes, Florence 1890–1909). Del Monte could not believe Galileo's pendular claims, and found them contradicted when he rolled balls inside an iron hoop. He was a scientist-engineer, and enough of an Aristotelian, to believe that tests against experience were the ultimate adjudicator of claims in physics. Galileo's claims failed the test. But Galileo replies that *accidents* interfered with del Monte's test: his wheel rim was not perfectly circular and it was not smooth enough. These are perfectly understandable qualifications, yet it needs to be appreciated that they are *modern* qualifications. Galileo introduced this, now well established, process of abstracting from real circumstances to ideal ones.

The crucial surviving document in the exchange between Galileo and his patron is a 29th November 1602 letter where Galileo writes of his discovery of the isochrony of the pendulum and conveys his mathematical proofs of the proposition.[6] The letter warrants discussion because it is a milestone in the science of mechanics, and as it illustrates important things about Galileo and his scientific style.

> You must excuse my importunity if I persist in trying to persuade you of the truth of the proposition that motions within the same quarter-circle are made in equal times. For this having always appeared to me remarkable, it now seems even more remarkable that you have come to regard it as false. Hence I should deem it a great error and fault in myself if I should permit this to be repudiated by your theory as something false; it does not deserve this censure, nor yet to be banished from your mind – which better than any other will be able to keep it more readily from exile by the minds of others. And since the experience by which the truth has been made clear to me is so certain, however confusedly it may have been explained in my other [letter], I shall repeat this more clearly so that you, too, by making this [experiment], may be assured of this truth.

Therefore take two slender threads of equal length, each being two or three braccia long [4–6 feet]; let these be AB and EF (Figure 2). Hang A and E from two nails, and at the other ends tie two equal balls (though it makes no difference if they are unequal). Then moving both threads from the vertical, one of them very much as through the arc CB, and the other very little as through the arc IF, set them free at the same moment of time. One will begin to describe large arcs like BCD while the other describes small ones like FIG. Yet in this way the moveable [that is, movable body] B will not consume more time passing the whole arc BCD than that used up by the other moveable F in passing the arc FIG. I am made quite certain of this as follows.

The moveable B passes through the large arc BCD and returns by the same DCB and then goes back toward D, and it goes 500 or 1000 times repeating its oscillations. The other goes likewise from F to G and then returns to F, and will similarly make many oscillations; and in the time that I count, say, the first 100 large oscillations BCD, DCB and so on, another observer counts 100 of the other oscillations through FIG, very small, and he does not count even one more – a most evident sign that one of these large arcs BDC consumes as much time as each of the small ones FIG. Now, if all BCD is passed in as much time [as that] in which FIG [is passed], though [FIG is] but one-half thereof, these being descents through unequal arcs of the same quadrant, they will be made in equal times. But even without troubling to count many, you will see that moveable F will not make its small oscillations more frequently than B makes its larger ones; they will always be together.

The experiment you tell me you made in the [rim of a vertical] sieve may be very inconclusive, perhaps by reason of the surface not being perfectly circular, and again because in a single passage one cannot well observe the precise beginning of motion. But if you will take the same concave surface (Figure 3) and let ball B go freely from a great distance, as at point B, it will go through a large distance at the beginning of its oscillations and a small one at the end of these, yet it will not on that account make the latter more frequently than the former.

Then as to its appearing unreasonable that given a quadrant 100 miles long, one of two equal moveables might traverse the whole and [in the same time] another but a single span, I say that it is true that this contains something of the wonderful, but our wonder will cease if we consider that there could be a plane as little tilted as that of the surface of a slowly running river, so that on this [plane] a moveable will not have moved naturally more than a span in the time that on another plane, steeply tilted (or given great impetus even on a gentle incline), it will have moved 100 miles. Perhaps the proposition has

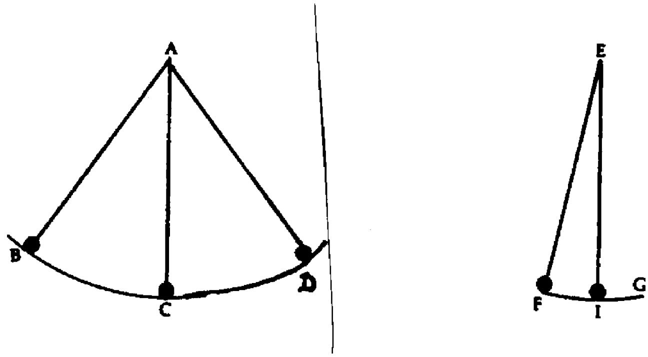

Figure 2.

Figure 3.

> inherently no greater improbability than triangles between the same parallels and on equal bases are always equal [in area], though one may be quite short and the other 1000 miles long. But keeping to our subject, I believe I have demonstrated that the one conclusion is no less thinkable than the other.
>
> Let BA be the diameter of circle *BDA* erect to the horizontal, and from point *A* out to the circumference draw any lines *AF*, *AE*, *AD*, and *AC* (Figure 4). I show that equal moveables fall in equal times, whether along the vertical *BA* or through the inclined planes along lines *CA*, *DA*, *EA* and *FA*. Thus leaving at the same moment from points *B, C, D, E*, and *F*, they arrive at the same moment at terminus A; and line FA may be as short as you wish.
>
> And perhaps even more surprising will this, also demonstrated by me, appear: That line SA being not greater than the chord of a quadrant, and lines SI and IA being any whatever, the same moveable leaving from S will make its journey SIA more swiftly than just the trip IA, starting from I. This much has been demonstrated by me without transgressing the bounds of mechanics. But I cannot manage to demonstrate that arcs SIA and IA are passed in equal times, which is what I am seeking. (Drake 1978, pp. 69–71)

Thus in 1602 Galileo is claiming two things about motion on chords within a circle:

1. That in a circle, the time of descent of a body free-falling along all chords terminating at the nadir, is the same regardless of the length of the chord.
2. In the same circle, the time of descent along two composite chords is shorter than along a single chord joining the beginning and end of the composites, even though the composite route is longer than the direct route.

This gets him tantalisingly close to a claim about motion along the *arcs* of the circle, the pendulum case, but not quite there. He is not prepared to make the leap, saying ‘But I cannot manage to demonstrate that arcs SIA and IA are passed in equal times, which is what I am seeking’. Although he has confident intuitions about the isochronism of circular motion, nevertheless these intuitions are mistaken. In just a few decades Christiaan Huygens will establish geometrically that it is motion in a cycloid, not in a circle, that is isochronic; but nevertheless, for small amplitudes, the circle and the cycloid coincide (Matthews 2000, Chapter 6).

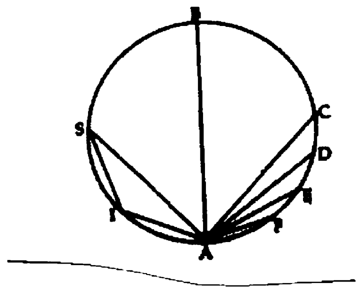

Figure 4.

The above diagram (Figure 4), of the circle and its chords, has great potential in Galileo's physics. He is asserting that the time of free fall along BA is the same as the motion along the inclines DA, EA, FA – given the contrary-to-fact idealised assumption that the chord-planes are perfectly smooth and without friction. In just a few years (1604) he will formulate his 'times-squared' law saying that distance fallen on an inclined plane, or in free fall, varies as the square of the time elapsed ($s \propto t^2$); or, as we can say, time elapsed varies as the square root of distance fallen ($t \propto \surd s$).

Consider a redrawing of the above figure, as in Figure 5. Galileo says that a sphere released from B will drop to A in the same time as a sphere released from E will travel down a perfectly smooth plane EA of length d.

This can be easily be shown. If $<$EAK $= \theta$, then $<$ABE $= \theta$, because $<$AEK $= <$EAB, and together $<$EAK $+ <$AEK $= 90$ °. The sphere at E has a vertical acceleration of g (acceleration due to gravity) along EK. If EA is inclined at an angle θ to the horizontal AK, then the sphere's acceleration component along EA is $g \sin \theta$. So the time t taken to travel the distance d from rest to A, will be derived from $d = ½a\ t^2$. Thus:

$$t = \sqrt{2d/a} = \sqrt{2d/g} \sin \theta$$

Consider now the sphere falling from B to A. It falls the distance 2l, with an acceleration of g. So the time (t_1) it takes to fall is given by

$$t_1 = \sqrt{2.2l/g} = 2\sqrt{l/g}$$

From the Δ ABE, $d = 2l \sin \theta$, and substituting in the first equation, we have:

$$t = \sqrt{2.2l \sin \theta/g} \sin \theta = 2\sqrt{l/g}$$

Thus $t = t_1$, so the time to fall vertically BA equals the time to travel on the incline EA. And the same holds for decent along all other chords in the circle: the time of decent is the same as the time of free fall from B to A.

The redrawn figure allows us to connect motion in free fall, motion on an inclined plane, and pendulum motion. Movement along the arc EA ($= s$) is precisely that of a pendulum suspended at O with length l and a bob or sphere at E. The amplitude of the pendulum swing is 2θ. The period of this pendulum (T) is four times the time taken to travel along the arc EA (period equals time for the bob to descend EA, then up the other side, and back again to E). If θ is small, then arc length s (l rad 2θ) is approximately equal to chord length d, and sin 2θ approximates radian 2θ. With approximations, we can derive the familiar formulae:[7]

$$T \approx 4t \approx 4.2\sqrt{l/g} \approx 2\pi\sqrt{l/g}$$

Thirty years later, in his *Dialogue Concerning the Two Chief World Systems* (1633), Galileo returns to this example of del Monte's, saying, in defence of his claims for the tautochronism of circular motion, that:

> Take an arc made of a very smooth and polished concave hoop bending along the curvature of the circumference *ADB* (Figure 6), so that a well-rounded and smooth ball can run freely in it (the rim of a sieve is well suited for this experiment). Now I say that wherever you place the ball, whether near to or far from the ultimate limit *B* … and let it go, it will arrive at the point *B* in equal times … a truly remarkable phenomenon. (Galileo 1633/1953 p. 451)

5. Testing Idealised Laws

Galileo did not develop a system of rational mechanics in the way that medieval scientists constructed mathematical models of physical systems,

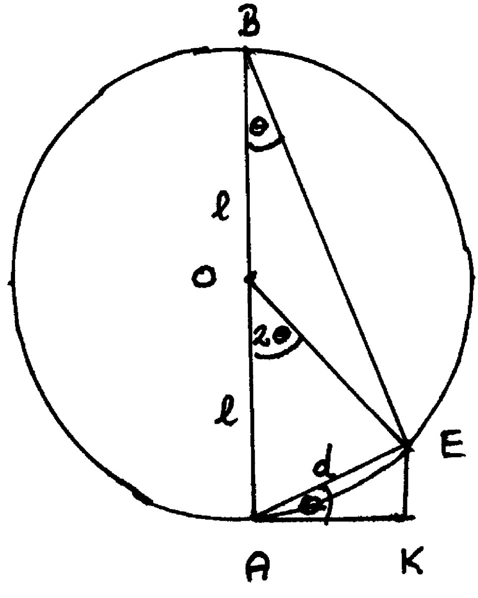

Figure 5.

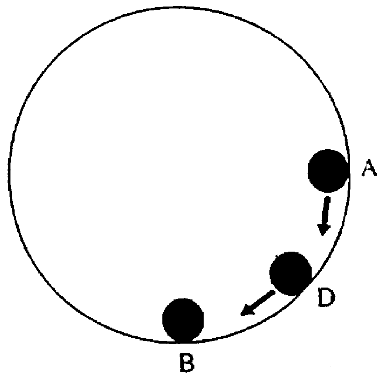

Figure 6.

and then proceeded no further. He was not content with merely making a 'world on paper'. In contrast to the medievals, Galileo's theoretical constructions are the means for engaging with, and working on, the natural world. For him the theoretical object provides a plan for interfering with the material world, and where need be, for making the real world in the image of the theoretical. When del Monte tells Galileo that he has done an experiment with balls in an iron hoop and the balls do not behave as Galileo asserts, Galileo replies that the hoop must not have been smooth enough, that the balls were not spherical enough and so on. These suggestions for improving the experiment are driven by the theoretical object that Galileo has already constructed. Without the theoretical object he would not know whether to correct for the colour of the ball, the material of the hoop, the diameter of the hoop, the mass of the ball, the time of day or any of a hundred other factors.

In his more candid moments, Galileo acknowledged that events do not always correspond to his theory; that the material world and his so-called 'world on paper', the theoretical world, do not correspond. Immediately after geometrically establishing his famous law of parabolic motion of projectiles, he remarks that:

> I grant that these conclusions proved in the abstract will be different when applied in the concrete and will be fallacious to this extent, that neither will the horizontal motion be uniform nor the natural acceleration be in the ratio assumed, nor the path of the projectile a parabola. (Galileo 1638/1954, p. 251)

Galileo is not deterred by the 'perturbations', 'accidents', and 'impediments' that interfere with the behaviour of the free falling, rolling, and projected bodies with which his New Science is dealing.[8] His procedure is explicitly stated immediately after the disclaimer about the behaviour of real projectiles in contrast to his ideal ones. Galileo says:

> Of these properties [*accidenti*] of weight, of velocity, and also of form [*figura*] infinite in number, it is not possible to give any exact description; hence, in order to handle this matter in a scientific way, it is necessary to cut loose from these difficulties; and having discovered and demonstrated the theorems, in the case of no resistance, to use them and apply them with such limitations as experience will teach. (Galileo 1638/1954, p. 252, 253)

In an historical understatement, Galileo adds: 'And the advantage of this method will not be small' (ibid.).

One can imagine the reaction of del Monte and other hardworking Aristotelian natural philosophers and mechanicians when presented with such a qualification. When baldly stated, it confounded the basic Aristotelian and empiricist objective of science, namely to tell us about the world in which we live. Consider, for instance, the surprise of Giovanni Renieri, a gunner who attempted to apply Galileo's theory of projectile motion to his craft, who when he complained in 1647 to Torricelli that his guns did not behave according to Galileo's predictions, was told by Torricelli that 'his teacher spoke the language of geometry and was not bound by any empirical result' (Segre, 1991, p. 43).

The law of parabolic motion was supposedly true, but not of the world we experience: this was indeed as difficult to understand for del Monte as it is for present-day students. Furthermore it confounded the Aristotelian methodological principle that the evidence of the senses is paramount in ascertaining how the world functions. That is, for a healthy observer what the eye sees is how the world is. Aristotle more than once asks that: 'If we cannot trust our eyes what can we trust?'

There is, of course, a problem of idealisation hiding fundamental mechanisms in the world. Keeping one's eye on the essential property is scientifically commendable, provided that it is the essential property, and that concentration on it does not blind one to other significant influences or properties. This is the case when Galileo maintains the isochrony of circular motion, dismissing experimental deviations as 'accidents' – due to air resistance, friction, compounding effect of the weight of the string, etc. Some of the deviation was accidental, but not all of it. The core deviation of experiment from theory was because the theory was wrong: it was the cycloid, not the circle that was isochronous. For theories that explicitly deal with idealised cases or situations, there is always the danger of maintaining that 'my theory is right, do not bother me with the facts'.[9]

Undoubtedly there was an element of metaphysics in Galileo's adherence to the circle as the tautochrone. The same conviction perhaps that lead him to discuss and defend Copernicus's theory of *circular* planetary orbits, despite Kepler's *elliptical* refinement of Copernicus's views being published in 1619, 14 years before Galileo's great *Dialogue*, and Galileo having a copy of the work in his library. The same conviction perhaps led Galileo to the doctrine of *circular* inertia.[10]

6. Galileo's Idealisations and the Beginning of Modern Science

What Galileo does with pendulum and projectile motions, he also does more generally with free fall. In his 1638 *Discourse on Two New Sciences* he explicitly turns away from Aristotelian attention to the kaleiodoscopic variety of behaviours exhibited by different shaped objects falling in different media – think of the innumerable ways that autumn leafs fall in air, to say nothing of how different bodies fall in water – and concentrates on how bodies would fall in a vacuum. As there was then no vacuum that could be observed or manipulated, and further as Aristotle's physics denied the very possibility of a vacuum, Galileo's idealisation was thought to be fundamentally insane. He was investigating another world, so to speak. But his method of idealisation marked the beginnings of modern science.[11]

Newton adopted and extended Galileo's method of idealisation. This is why the methodology of modern science can be referred to as the 'Galilean–Newtonian Paradigm'.[12] At the beginning of the *Principia* Newton announces that: 'in philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them' (Newton 1729/1934, p. 8). The whole structure of the *Principia* manifests this methodology. In Book One, Newton considers bodies as point masses moving in an infinite void. This allows him to formulate mathematically the physics of the simplest, or ideal, case. In Book Two, he introduces resistance and considers bodies moving in a medium. In the final section of the *Principia* he considers 'The System of the World' where the orbit of the earth is studied in full interaction with the moon and other planets.

In the beginning, as soon as Newton states his First Law of Motion [the Law of Inertia], he elaborates by saying: 'Projectiles continue in their motions, so far as they are not retarded by the resistance of air, or impelled downwards by the force of gravity' (Newton 1729/1934, p. 13). This is the statement of a massive contrary-to-fact idealisation. We are asked to consider first, the behaviour of a body in the absence of air, and no such situation had been observed; and second, the behaviour of a body in the absence of any other body, and no such situation is even possible, as if an observer is there to see what happens, then gravitational [attractive] forces have been introduced by the observer.

Again, early in the *Principia* Newton uses pendulum-collision experiments to prove his law of conservation of momentum: 'the sum of the motions [after collision] directed towards the same way, or from the difference of those that were directed towards contrary ways, was never changed' (Newton 1729/1934, p. 24). Having stated this, he immediately adds: 'the bodies were either absolutely hard, or at least perfectly elastic (whereas no such bodies are to be found in Nature)' (ibid). That is, baldly stated, the law is true for bodies that do not exist!

Bernard Cohen wrote of Newton's methodology that:

> The great power of the Newtonian style was that it made possible the study of forces of different sorts in relation to motions in general, and in relation to those motions observed in the external world, without any inhibiting considerations as to whether such forces can actually (or do actually) exist in nature. (Cohen 1980, p. xiii)

Aristotle had taken experience as the foundation of science, and so his 'law' of motion stated that 'speed of motion varied directly as the force being applied and inversely as the resistance of the medium through which the body was being moved'. This is commonsense, it is the articulation of everyday experience, but it was also an 'epistemological obstacle' to the birth of modern science.[13]

7. Idealisation and Experiment

Galileo abstracted from 'impediments' and 'accidents' – the shape and colour of bodies, wind, air resistance, friction, and so on – in order to get a mathematical formulation of the principal causal relationships. But he did not confine himself to 'a world on paper', he did not just make speculations and draw pictures – as many of the medieval physicists had done. Through experimental manipulation, elimination of impediments, and progressive approximations, he tried to have the real world mirror his ideal. As he said: 'having discovered and demonstrated the theorems, in the case of no resistance, to use them and apply them with such limitations as experience will teach' (Galileo 1638/1954, p. 252). Galileo defended this procedure in a 1637 letter to Pierre Carcavy:

> I argue *ex suppositione*, picturing to myself motion with respect to a point from which [a thing] leaving from rest goes accelerating, increasing its speed in the same proportion with which time increases, and in this way I conclusively demonstrate many events; then I add that if experience shows that such events are found verified in the motion of naturally descending heavy things, we can without error affirm this [natural motion] to be the same motion that I defined and assumed. (Drake 1978, p. 378)

This sounds fairly straightforward, but then Galileo immediately adds a qualification:

> If [they are] not, my demonstrations founded on my assumption lose nothing of their force and conclusiveness, just as it in no way prejudices the conclusions proved by Archimedes about the spiral that no naturally moving body moves spirally in that manner. (ibid.)

Newton followed the same procedure. He proposed three laws of motion for a thoroughly idealised world, but then bit by bit introduced complications. Nowak and colleagues call this the process of 'concretization'; progressively making concrete or real the idealised situation or model. This is how scientific experimentation is to be understood. The 200-year history of classical mechanics can be considered as a long attempt to make the world conform to Newtonian theory!

This fairly simple interpretation of experiment ('simple', once Aristotelian objections to the very idea of interference with nature are put aside) allows a number of perplexing matters about science to be understood. Not the least of which is the disjunction between scientific laws and everyday observation.

Michael Scriven, forty years ago, arrestingly remarked that 'The most interesting thing about laws of nature is that they are virtually all known to be in error' (Scriven 1961, p. 91). This position was subsequently taken up by Nancy Cartwright who in her *How the Laws of Physics Lie* says that if the laws of physics are interpreted as empirical, or phenomenal, generalisations, then the laws lie (Cartwright 1983). As Cartwright states the matter: 'My basic view is that fundamental equations do not govern objects in reality; they govern only objects in models' (Cartwright 1983, p. 129). The world does not behave as the fundamental equations dictate. This claim is not so scandalous: the gentle and random fall of an autumn leaf obeys the law of gravitational attraction, but its actual path is hardly as described by the equation $s = \frac{1}{2}gt^2$. This equation refers to idealised situations. A true description, a phenomenological statement, of the falling autumn leaf would be complex beyond measure. The law of fall states an *idealisation*, but one that can be experimentally approached. These laws are usually stated with a host of explicit *ceterus paribus*, or 'other things being equal', conditions.[14] For the laws of pendulum motion to hold at least the following *ceterus paribus* conditions need to be stated:

1. the string is weightless (so no dampening occurs);
2. the bob does not experience air resistance;
3. there is no friction at the fulcrum;
4. all the bob's mass is concentrated at a point;
5. the pendulum moves in a plane and does not experience any elliptical motion;
6. that gravity and tension are the only forces operating on the bob.

But these conditions can only be approached, never realised – something that Ronald Giere has been at pains to point out (Giere 1988, pp. 76–78; 1994; 1999, chapter 5). Giere believes that not only are scientific laws false, they are also neither universal or necessary (Giere 1999, p. 90). He says:

> On my alternative interpretation, the relationship between the equations and the world is *indirect*. … the equations can then be used to construct a vast array of abstract mechanical systems … I call such an abstract system a *model*. By stipulation, the equations of motion describe the behavior of the model with perfect accuracy. We can say that the equations are exemplified by the model or, if we wish, that the equations are *true*, even *necessarily* true, for the model. (Giere 1999, p. 92)

8. Philosophical Approaches to Idealisation

Despite the popularity, both professional and lay, of empiricist, and even inductivist, interpretations of Galileo's achievements, many philosophers have recognised the centrality of idealisation in his science. Immanuel Kant famously wrote in the Preface to his *Critique of Pure Reason* that:

> When Galileo caused balls, the weights of which he had himself previously determined, to roll down an inclined plane; when Torricelli made the air carry a weight which he had calculated beforehand to be equal to that of a definite volume of water … a light broke upon all students of nature. They learned that reason has insight only into that which it produces after a plan of its own, and that it must not allow itself to be kept, as it were, in nature's leading-strings, but must itself show the way with principles of judgment based upon fixed laws, constraining nature to give answer to questions of reason's own determining. Accidental observations, made in obedience to no previously thought-out plan, can never be made to yield a necessary law, which alone reason is concerned to discover. … It is thus that the study of nature has entered on the secure path of a science, after having for so many centuries been nothing but a process of merely random groping. (Kant 1787/1933, p. 20)

One hundred years later, Pierre Duhem wrote about the development of Galileo's and Newton's account of inertia, pointing out that:

> Now is it clear merely in the light of common sense that a body in the absence of any force acting on it moves perpetually in a straight line with constant speed? Or that a body subject to a constant weight constantly accelerates the velocity of its fall? On the contrary such opinions are remarkably far from common-sense knowledge; in order to give birth to them, it has taken the accumulated efforts of all the geniuses who for two thousand years have dealt with dynamics. (Duhem 1906/1954, p. 263)

Alexandre Koyré, in his influential 1943 essay on 'Galileo and the Scientific Revolution' supported Kant's account, when he wrote that:

> Aristotelian physics is based on sense-perception and is therefore decidedly non-mathematical. It refuses to substitute mathematical abstractions for the colourful,

> qualitatively determined facts of common experience, and it denies the very possibility of a mathematical physics. (Koyré 1943a/1968, p. 5)

Latter he would write:

> … observation and experience – in the meaning of brute, common-sense observation and experience – had a very small part in the edification of modern science; one could even say that they constituted the chief obstacles that it encountered on its way. … the empiricism of modern science is not *experiential*; it is *experimental*. (Koyré 1953/1968, p. 90)

Gaston Bachelard, stressed these matters in his influential 1930s work:

> Empirical notions derived from ordinary experience have to be revised and modified repeatedly before they can be of any use to microphysics, which defines reality by *interference* rather than *discovery*. (Bachelard 1934/1984, p. 160)

Laura Fermi and Gilberto Bernardini draw attention to the same methodological point regarding the centrality, for Galileo's science, of abstracting from everyday experience. They put the matter this way:

> In formulating the 'Law of Inertia' the abstraction consisted of imagining the motion of a body on which no force was acting and which, in particular, would be free of any sort of friction. This abstraction was not easy, because it was friction itself that for thousands of years had kept hidden the simplicity and validity of the laws of motion. In other words friction is an essential element in all human experience: our intuition is dominated by friction. (Fermi & Bernadini 1961, p. 116)

The historian Richard Westfall stated well the more general methodological principle that distinguished Galileo and early modern science from del Monte and the medieval and ancient traditions:

> Beyond the ranks of historians of science, in my opinion, the scientific revolution is frequently misunderstood. A vulgarized conception of the scientific method, which one finds in elementary textbooks, a conception which places overwhelming emphasis on the collection of empirical information from which theories presumably emerge spontaneously, has contributed to the misunderstanding, and so has a mistaken notion of the Middle Ages as a period so absorbed in the pursuit of salvation as to have been unable to observe nature. In fact medieval philosophy asserted that observation is the foundation of all knowledge, and medieval science (which certainly did exist) was a sophisticated systematization of common sense and of the basic observations of the senses. Modern science was born in the sixteenth and seventeenth centuries in the denial of both. (Westfall 1988, p. 5)

Leszek Nowak and his Polish colleagues have over the past forty years provided a very developed account of the logic of idealisation in science, and of the role of idealisation in the history of natural and social science.[15] And the topic has been addressed by other philosophers.[16] In an early paper (1975) Nowak says that 'In brief, the Galilean breakthrough consisted in the introduction of the method of idealization in physics' (Nowak 1980, p. 36). By this he meant:

> According to the proper scientific method, and investigator should separate the principal and secondary factors for a given phenomenon and abstract from them establishing the law connecting the phenomenon with the principal determinants of it. Thus idealization laws express scientists' convictions concerning the essential stratification of the world. (Nowak 1980, p. 37)

In a later paper he states the matter as follows:

> The Galilean revolution consisted in making evident the misleading nature of the world image which senses produce. We only see phenomena which are the joint effect of all the relevant influences. As a result, senses do not contribute in the slightest to the understanding of the facts. In order to understand phenomena the work of reason is necessary which selects some features of the objects through idealization and in their idealized models recognizes some other features of the empirical originals. These models differ a great deal from their sensory prototypes, what is more, they present images of hidden relationships which could not be grasped with the aid of experience at all. (Nowak 1994, p. 123).

9. Idealisation and Anti-Science Sentiment

A failure to appreciate what idealisation is and is not has been at the basis of a lot of anti-science criticism. It was, of course, Newtonian idealisation that the Romantic reaction was directed against. For Keats, Goethe and other Romantics and Naturalists, the rich world of lived experience was not captured by the colourless point masses of Newton. In the twentieth century, Sartre, Marcuse, Husserl, Tillich and others of an Existentialist or Personalist bent, have repeated this charge. Sartre at one point says that evil is 'the systematic substitution of the abstract for the concrete' (Passmore 1978, p. 70). This substitution is precisely the charge of del Monte against Galileo. This substitution is exactly what Gaston Bachelard identified as the *raison d'être* of the scientific revolution. Although it might be fashionable to agree with Sartre, the price is the rejection of the unequalled methodology of the seventeenth century's new science. There is obviously a problem here. Aldous Huxley, at the end of World War Two, intelligently commented on the matter saying:

> The scientific picture of the world is inadequate, for the simple reason that science does not even profess to deal with experience as a whole, but only with certain aspects of it in certain contexts. All of this is quite clearly understood by the more philosophically minded men of science. … [Unfortunately] our times contains a large element of what may be called 'nothing but' thinking. (Huxley 1947, p. 28)

A historically and philosophically literate science teacher can assist students to grasp just how science captures, and does not capture, the real, subjective, lived world. An HPS-illiterate teacher leaves students with the unhappy choice between disowning their own world as a fantasy, or rejecting the world of science as a fantasy.

10. Idealisation, Constructivism and Science Learning

Children have the same difficulty seeing the properties of pendulum motion that the sixteenth century Aristotelians had. School children, even with highly refined laboratory pendulums, struggle to see isochrony of large and small amplitude swings (the theoretical limit for 'largeness' is about 23°); their cork and brass pendulums are soon out of synchrony, with the cork one ceasing to swing well before the brass one. All of this experiential evidence is hard to reconcile with the 'laws' of pendulum motion. Children are in the position of the early pioneers of a science. Children can eventually, with lots of prompting, see that period increases as length increases, but they are unlikely to 'see' isochronic motion. Looking is important, but something else is required, namely a better appreciation of what science is and what it is aiming to do, an epistemology of science.

Aristotle, Oresme, Buridan, Bradwardine, da Vinci, del Monte, and countless hundreds of other natural philosophers had all 'seen' pendulum motion, but they did not see what Galileo saw. The historian E.J. Dijksterhuis had an appreciation of this when he observed:

> To this day every student of elementary physics has to struggle with the same errors and misconceptions which then [in the seventeenth century] had to be overcome … in the teaching of this branch of knowledge in schools, history repeats itself every year. (Dijksterhuis 1961/1986, p. 30)

Dijksterhuis goes on to make a fundamental point. Classical mechanics is not only not verified in experience, but its direct verification is fundamentally impossible: 'one cannot indeed introduce a material point all by itself into an infinite void and then cause a force that is constant in direction and magnitude to act on it; it is not even possible to attach any rational meaning to this formulation'.

Galileo's exchange with del Monte, and more generally the recognition of the centrality of idealisation to science, basically undermines all naïve experiential and inductivist approaches to learning; they also undermine most constructivist and 'discovery' learning approaches to teaching. The young Dewey shared with Ernst Mach and John Stuart Mill the wide-spread 19th century view that it was by looking carefully at nature, and then proceeding with cautious inductions, that Galileo and Newton launched modern science. He thought that if children likewise 'experienced' the world at first hand, they would come to know the world. In 1898 he confidently wrote that: 'After the conquest of the inductive method in all spheres of scientific inquiry, we are not called upon to defend its claims in pedagogy' (Prwawat 2003, p. 281). This confidence was instilled in the Progressive Education Association that Dewey founded. But sometime around 1915 Dewey changed his mind on inductivism; he no longer saw it as the method of modern science, saying rather that:

> It would be difficult to imagine a doctrine more absurd than the theory that general ideas or meanings arise by the comparison of a number of particulars, eventuating in the recognition of something common to them all. (Dewey 1929, p. 155)

Unfortunately many progressive educators either did not notice or did not attend to Dewey's change of mind; they persisted with various forms of experiential, student-centred, learning. A good deal of this tradition has been resurrected by contemporary constructivists, who have persisted in the mantra that 'knowledge cannot be transferred, it has to be constructed by the knower' (Matthews 2000a).

The weakness of simple inductivist views of learning is that intuitive beliefs and 'natural' interpretations are so strongly influenced by everyday, concrete experience. Lewis Wolpert, in his *The Unnatural Nature of Science* (1992), has correctly remarked that:

> Scientific ideas are, with rare exceptions, counter-intuitive: they cannot be acquired by simple inspection of phenomena and are often outside everyday experience … doing science requires a conscious awareness of the pitfalls of 'natural' thinking. (Wolpert 1992, p. xi)

Some decades ago Myron Atkin, a science educator pointed out this very problem with the community's enthusiasm for discovery learning:

> A basic flaw in the process is the apparent assumption that science is a sort of commonsensical activity, and that the appropriate 'skills' are the primary ingredients in doing productive work. There seems to be no explicit recognition of the powerful role of the conceptual frames of reference within which scientists and children operate and to which they are firmly bound. These general views of the physical world demand careful nurture … by a variety of means. (Glass 1970, p. 20)

And clearly language, as in telling, explaining, defining, clarifying etc., is integral to this 'careful nurture'.

Schecker (1992) has addressed some of these questions in an interesting way. He asked 254 high school students to comment upon the following statement:

> In physics lessons there are often assumptions or experiments of thought, which obviously cannot be realized in actual experiments, like completely excluding air resistance and other frictional effects or assuming an infinitely lasting linear motion.

The students were asked to comment on whether the method was useful or not useful. Eleven percent said it was useless, 'Why should I consider something that does not exist?' A large group, up to 50% said it was useful, but only for physics because physics did not deal with reality, 'I don't need to refer everything to reality. I am simply interested in physics … physics is not about the world' (Schecker 1992, p. 75). Only 25% had any comprehension of the method of idealisation in science.

César Medina and colleagues at the University of Tucumán have conducted pendulum-focussed practical programmes for university physics classes that are based explicitly on the programme of concretisation (Medina, Velazco & Salinas 2004). In class students:

> have to construct a simple pendulum that behaves as an ideal one, and analyze model assumptions which affect its period. The following aspects are quantitatively analyzed: vanishing friction, small amplitude, non extensible string, point mass of the body, and vanishing mass of the string.

From theory, the students have Galileo's 'world on paper', but the practical work imitates Galileo's efforts to 'embody' or make concrete this world. Students experimentally investigate each of the idealised assumptions, and ascertain that:

> Considered separately, within an error of 1%:

- an initial amplitude of 23° is "small".
- a sphere, whose diameter is 30 % of the length of the string, is "a point mass".
- a mass of the string equal to 10 % of the mass of the body is "vanishing".
- any elastic elongation suffered by the string during the static process of loading is negligible, providing the string length is measured after the loading.
- without losing its property of 'not extensible', the string may vary its length during oscillation (due to a variable tension), providing this variation is less than the measurement error of the string length.

Medina and colleagues say of this way of carrying out the usual pendulum experiments, that it:

> Promotes a better understanding of the scientific modeling process.
> Allows a deeper comprehension of those physical concepts associated with model assumptions (small amplitude, point mass, etc.), whose physical and epistemological meanings appear clearly related to the model context.
> Introduces students to a scientific way of controlling the validity of theoretical development, and helps them to value the power and applicability of scientific modeling.

11. Conclusion

The science curriculum and classroom can give students an appreciation of the scientific tradition and of the nature of science, but only if the scientific tradition is included in the programme. Teachers need to be selective, not everything can be dealt with historically, or by the 'genetic' method. However the pendulum story, and more specifically the del Monte and Galileo debate is a simple, yet scientifically and methodologically rich, part of the tradition that can easily be included in science programmes. The story is tailor-made for teaching the core scientific principles of mechanics; for teaching the chief aspects of scientific methodology, especially the importance of idealisation; and finally for displaying the human face of science.

The argument of this paper has been made many times over – with Ernst Mach, F.W. Westerway, Gerald Holton, Lloyd Taylor, Arnold Arons, James Rutherford and Leo Klopfer being familiar names to science educators. The argument was also made 40 years ago by Arthur Koestler, a person well outside the science classroom. He lamented the 'anti-humanism' and 'boredom' of school science, writing:

> To derive pleasure from the art of discovery … the student must be made to re-live, to some extent, the creative process. In other words he must be induced, with proper aid and guidance, to make some of the fundamental discoveries of science himself, to experience in his own mind some of those flashes of insight which have lightened its path. This means that the history of science ought be made an essential part of the curriculum. (Koestler, 1964, p. 268)

Galileo's discovery and utilisation of the properties of pendulum motion is a stunning example of the 'creative process'. If students are fortunate enough to have teachers capable of providing 'proper aid and guidance', then re-living the process can be a manageable and illuminating task.

Notes

[1] For a more complete account of Galileo's pendulum discoveries, see Matthews (2000, chap.5).

[2] A comprehensive survey of such 'big-picture' curriculum goals can be found in McComas and Olsen (1998).

[3] See also Crombie (1981) for a more extended discussion of this projection phenomena in philosophy.

[4] Del Monte's *Mechanics* is translated in Drake and Drabkin (1969) who describe del Monte as the greatest mechanician of the sixteenth century. Dugas (1988, pp. 100-101) discusses sympathetically del Monte's contribution to mechanics. See also the pen-picture of del Monte in Drake (1978, p. 459); a more extensive picture in Rose (1980, 1992); accounts of del Monte's patronage of Galileo in Biagioli (1993, pp. 30 and 31) and Sharratt (1994, p. 43 and 44); accounts of del Monte in Henninger-Voss (2000) and Bertoloni Meli (1992). An extensive discussion of del Monte as representative of the newly emerging engineer-scientist class is in Renn et al. (1998, pp. 36–40). These latter authors also translate a number of key letters between del Monte and Galileo, and provide documentation of archival sources.

[5] There are three different kinds of curve that need to be separated:

Tautochrone, meaning the curve where a body falling freely reaches the lowest point in the same time, regardless of where on the curve it was released. (from *tauto* = same)

Brachistochrone, meaning the curve where a body freely falling (but not in a straight line) will reach its lowest point fastest. (from *brachis* = short)

Isochronous, meaning the curve on which each successive oscillation takes the same period of time. (from *iso* = equal)

Tautochronous and Isochronous motion, though conceptually different, are in reality much the same thing.

[6] The letter was written in October 1602 (*Opere*, Edizione Nazionale, Florence 1934, vol. 10, pp. 97–100), and a translation has been provided by Stillman Drake (Drake 1978, pp. 69–71). Arguably a better translation is in Renn et al. (1998, pp. 104–106). Ronald Naylor (1980, pp. 367–371) and W.C. Humphreys (Humphreys 1967, pp. 232–234) discuss the letter in the context of Galileo's work on the law of fall.

[7] For derivations of the formulae that are cognisant of its historical origins, see French (1965, pp. 434–440), Kline (1959, pp. 288–293), Pólya (1977, pp. 211-224), and Taylor (1941, pp. 188–194). An excellent discussion is in Stinner and Metz (2002).

[8] On Galileo's recourse to 'accidental' factors, and their crucial role in his scientific methodology, see Koertge (1977) and McMullin (1985).

[9] There are everyday parallels of this 'blinkering', as when ideologues maintain that their 'party', 'church', 'economic system' etc. is the correct one and that contrary evidence (numerous gulags, systemic sexual abuse and massive unemployment) counts only against 'imperfect' realisations of the ideal.

[10] Alexandre Koyré (Koyré 1943b/1968) and Edwin Burtt (Burtt 1932, pp. 61–95) regarded this metaphysical conviction as evidence of Galileo's Platonism.

[11] The following are especially comprehensive accounts of Galileo's methodological innovation and its relation to other medieval and renaissance traditions: McMullin (1978, 1985, 1990), Wisan (1978, 1981) and Wallace (1981). In terms of the epistemological lessons to be learnt, few articles are better than Mittelstrass (1972) and Suchting (1995).

[12] The expression is used by Wallis Suchting (1995). Ernan McMullin remarked that: 'a philosophy of science shaped by the mechanics of the *Principia* was rapidly accepted as appropriate for natural science generally. Newtonian mechanics was to become for a time the paradigm of what any science of nature ought to look like.' (McMullin 2001, p. 289).

[13] The expression comes from Gaston Bachelard (1934/1984).

[14] For a historical and critical discussion of *Ceterus Paribus* laws see Earman et al. (2002).

[15] See numerous issues of the *Poznań Studies in the Philosophy of the Sciences and Humanities*. Also Nowak (1972, 1980, 1992, 1994, 2000).

[16] See for example Barr (1971, 1974), Dilworth (1989), Harré (1989), Hughes (1990), Laymon (1985), Niiniluoto (1990), Nola (2004) and Shaffer (2001).

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Pendula, Models, Constructivism and Reality

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Abstract. It is argued that Galileo made an important breakthrough in the methodology of science by considering idealized models of phenomena such as free fall, swinging pendula and the like, which can conflict with experience. The idealized models are constructs largely by our reasoning processes applied to the theoretical situation at hand. On this view, scientific knowledge is not a construction out of experience, as many constructivists claim about both the methods of science and about the learning of science. In fact Galileo's models can, depending on their degree of idealization or concretization, be at variance with experience. This paper considers what is meant by idealization and concretization of both the objects and properties that make up theoretical models, and the ideal laws that govern them. It also provides brief illustrations of ideal laws and how they may be made more concrete, and briefly considers how theories and models might be tested against what we observe. Finally some difficulties are raised for a radical constructivist approach to both science and learning in the light of Galileo's methodological approach. The upshot is that both the dialogue structure of Galileo's writings and his method of model building provide a rich resource for science education that rivals that of the standard varieties of constructivism, and at the same time gives a much better picture of the actual procedures of science itself.

Key words: Models in science, pendulum motion, realism in science, constructivism, Galileo, idealization.

> *Galileo systematically applied the method of idealization. And that was the real meaning of the revolution in the natural sciences which was named after him.* (Nowakowa & Nowak 2000, p. 21)

1. Galileo and the Subversion of Experience by Reason

It is commonly agreed that Galileo has an important place in the history of science, particularly concerning his astronomical observations, the development of concepts such as momentum and theories of motion on the Earth, such as free-fall, pendulum motion, projectile motion, and the like. It is also agreed, as the above quotation suggests, that Galileo was innovative in the methodological breakthrough that he made in science, and that, in respect of what has been commonly called the 'scientific revolution' of the 17th and 18th centuries, his methodological contribution was revolutionary. In this respect Galileo's methodology was taken up by Newton and fully exploited by him as his new theory of dynamics was applied to various models

M.R. Matthews et al. (eds.), The Pendulum, 237–265.

of the real world. However, there is less agreement about how the Galileo–Newton methodological breakthrough is to be characterized. Lesek Nowak has given one useful characterization:

> The Galilean revolution consisted in making evident the misleading nature of the world image which senses produce. We only see phenomena which are the joint effect of all the relevant influences. As a result, senses do not contribute in the slightest to the understanding of the facts. In order to understand phenomena the work of reason is necessary which selects some features of the objects through idealization and in their idealized models recognizes some other features of the empirical originals. These models differ a great deal from their sensory prototypes; what is more, they present images of hidden relationships which could not be grasped with the aid of experience at all. Science idealizing phenomena opposes commonsense .... (Nowak 1994, p. 123)

Two aspects of Nowak's remarks will be discussed here. The first is the two-part claim that the senses can be misleading, and that our senses may not be able to reveal the hidden joint causes which bring about happenings we can observe with our senses. This leads to the second point concerning how idealizations are to be made in science, even when the idealizations and/or their consequences run contrary to commonsense and what we in fact experience. This raises issues about the important role of reason in constructing idealizations which are models of phenomena, such as swinging pendula, where such models are not given directly in experience, and may not be fully in accordance with experience but only approximately so. If Nowak is right about Galileo's methodology, then it provides an important contrast with any account of scientific method that is too strongly oriented to empiricism, or claims that science is a 'construct' out of experience, one of the common central tenets of constructivism within science education. So a further aspect of this paper will be devoted to noting the contrast between a methodology based on Galilean idealizations and the tenets of constructivism in science education. This latter doctrine adopts the epistemological thesis that scientific knowledge is a construct out of experience for scientists, and then extends this to a theory of learning for students of science.

A quite different kind of construction goes on in Galileo's science (and the science of others), as characterized by Nowak. If we can expand the 'construction' metaphor, we can say that scientific knowledge is a construct out of reason. The role of reason is two-fold; in the first place to 'construct' idealizations or models, and then to make inferences from the models about possible observations that might only fit our experience to some degree of approximation.

Here we cannot discuss the various views that historians and philosophers of science have taken about the contrasting *a priori* or rationalist *versus* empiricist approaches that Galileo took in his science. Suffice to say that many note the often strong 'rationalism' to be found as opposed to the empiricism of commonsense or sense perception. As a single illustration from historians, consider Alexander Koyré, an eminent historian of Galilean science. He expresses a kindred contrast when he speaks of the difference between thought in Galileo and an appeal to experience that is often infused with commonsense views about the world that he wishes to challenge. This, in Koyré's view, is particularly the case in our understanding of motion. He says that for Galileo our natural ways of imagining lead us to talk of effort and impetus. But we need to overcome these natural tendencies through thought that leads us to the more appropriate notion of momentum, a notion which is in many respects 'unnatural' to us. The contrast he expresses as follows:

Thus we must choose: either to think or to imagine. To think with Galileo, or to imagine with common sense. For it is thought, pure unadulterated thought, and not experience or sense-perception, as until then, that gives the basis for the new science of Galileo Galilei. Galileo is perfectly clear about it. (Koyré 1968, p. 13)

More dramatically Koyré sees the new approach that Galileo took in building mathematical models of motion as a victory of a more Platonic and abstract idealizing approach to science over that of Aristotle and Galileo's contemporary Aristotelians who appealed to experience: 'for the contemporaries and pupils of Galileo, as well as Galileo himself, the Galilean philosophy of Nature, appeared as a return to Plato, a victory of Plato over Aristotle' (*ibid.*, p. 15). Though such contrast can mean many things, the emphasis on Plato will be understood here as one in which reason plays a major role in the construction of idealized models in science, even where such models, and what they give rise to as allegedly observable consequences, may go against commonsense experience.

Is Novak (amongst many others such as Koyré) right in his characterization of Galileo's methodology? Much evidence for it can be found in Galileo's own writings, a little of which will be indicated here. Nowhere does Galileo give an explicit account of his method. But as he develops his theory of motion he makes comments aside about his methodological procedure. We will look at just a few of his comments. First, there is the famous passage in the *Two Chief World Systems* in which Galileo says, through his mouthpiece Salviati, that we should make reason conquer the senses:

You wonder that there are so few followers of the Pythagorean opinion whereas I am astonished that there have been any .... Nor can I ever sufficiently admire the outstanding acumen of those who have taken hold of this opinion and accepted it is true; they have through sheer force of intellect done such violence to their own senses as to prefer what reason told them over that which sensible experience plainly showed them to the contrary. For the arguments against the whirling of the earth which we have already examined are very plausible, as we have seen; and the fact that the Ptolemaics and Aristotelians and all their disciples took them to be conclusive is indeed a strong argument of their effectiveness. But the experiences which overtly contradict the annual movement are indeed so much greater in their apparent force that, I repeat, there is no limit to my astonishment when I reflect that Aristarchus and Copernicus were able to make reason so conquer sense that, in defiance of the latter, the former became mistress of their belief. (Galileo 1967, p. 328)

Not only was Galileo aware that in some cases in science we would have to act and think contrary to our experiences, but he was also aware that the models we construct might not accurately fit what we experience. Thus in making the Earth move rather than the Sun, we appear to do violence to our very sensory observations as when we speak of the Sun rising or setting. However the moving Earth and stationary Sun are an essential part of the Copernican model of the solar system. This model is not given as some 'construct' out of experience. It is still a 'construct' but out of different materials, especially reason as employed in model building. As Nowak put it in the above: 'in order to understand phenomena the work of reason is necessary which selects some features of the objects through idealization and in their idealized models recognizes some other features of the empirical originals' (*loc. cit.*)

That experience can be an obstacle to model construction, especially where it is infused with allegedly 'commonsense' beliefs, is a point found in Galileo and commonly commented upon by historians from Koyré to Nowak. It is also a centerpiece of Feyerabend's account of Galileo, especially his critique of the 'natural interpretations' that must be exposed in reports of experience. In chapters 6 to 9 of *Against Method* (1975) Feyerabend makes much of the point that 'natural interpretations' infuse experience and our reports of experience, this being one aspect of Feyerabend's view that all observations are theory laden. In criticizing earlier theories we may also have to criticize the quite deeply hidden and embedded natural interpretations and replace them by what may appear to be quite 'unnatural' interpretations employing 'unnatural' concepts. Our commonsense reports of experience may, from a quite different point of view, be a quite unsuitable base from which any new theory can be constructed. On Feyerabend's understanding of Galileo's method, we need to overcome, and even contradict, the commonsense deliverances of experience, especially when new, profoundly deep theories are being developed.

Feyerabend makes much of the above quotation from Galileo, and many other remarks, that show that experience, and reports of it, need to be subverted. And he goes on to cite other remarks from Galileo such as 'they [the Copernicans] were confident of what reason told them' as opposed to the deliverances of experience upon which the Aristotelians relied. And again he cites approvingly Galileo saying 'with reason as his guide he [Copernicus] resolutely continued to affirm what sensible experience seemed to contradict' (Feyerabend 1975, p. 101). Particularly significant for Feyerabend's understanding of Galileo's procedure is the latter's discussion of the tower experiment in which a rock, dropped from the top of the tower, falls to the bottom. Does this show that the Earth is stationary, as many Aristotelians argued? Or does this result have to be understood anew if the Earth is taken to be rotating? On this Galileo says: 'for just as I ... have never seen nor ever expect to see the rock fall any way but perpendicularly, just so do I believe that it appears to the eyes of everyone else. It is therefore better to put aside the appearance, on which we all agree, and to use the power of reason either to confirm its reality or to reveal its fallacy' (Galileo 1967, p. 257; cited in Feyerabend 1975, p. 7; chapter 7 deals with Galileo's 'tower experiment'). Galileo uses strong words when he says that there may be a *fallacy* in experience to be overcome by reason. He gives an illustration of this when he continues, saying that 'one may learn how easily anyone may be deceived by simple appearances, or let us say by the impressions of one's senses. This event is the appearance to those who travel along a street at night of being followed by the moon, with steps equal to theirs, when they see it go gliding along the eaves of the roofs. ... an appearance which, if reason did not intervene, would only too obviously deceive the senses' (*loc. cit.*).

Galileo's point is well taken. For him, neither the deliverances of our senses, nor even our commonsense beliefs, are a sufficient basis for science; both may be called into question or even overturned if reason requires. And the converse can also be the case where what reason delivers fails to accord with experience. For Galileo, neither dominates the other; instead there is a complex dialectic between the two. Of interest here are the cases where our natural presuppositions built into our experience must be called into question and replaced by what seems 'unnatural' if science is to advance at all (the understanding of what we observe in the 'tower experiment' being a case in point).

We must be careful about the two aspects of the points being made. The first point concerns the understanding of the role of experience in science. The very building of models may, in some examples Galileo considers such as that proposed by Copernicus for the solar system, go against the commonsense view of the world. Reports of relevant experience may contain 'natural interpretations' of the world that are second nature to us, yet they must be overturned if science is to advance. For Galileo, the very same lessons that the Copernican modeling of the solar system taught us are to be extended to our understanding of motion itself (as Koyré emphasises), and to the construction of models for particular kinds of motion such as projectiles or swinging pendula. Here the second point emerges in that particular models of phenomena we can observe may be idealizations; the observational consequences of these models will involve approximations that may be at variance with what is experienced. In what follows we will focus on this second aspect of models as idealizations. But given the account of Galileo's methodology so far, the fact that a model may well be inconsistent with what we (report of) experience shows that an empiricist account of science in which theory is somehow a 'construct' out of experience is at variance with, and cannot capture, Galileo's methodological procedure. What is missing is the crucial role of 'reason', as we may put it, in providing models in science, and in persisting with these models even when they go against experience.

2. A Brief Characterization of Galileo's Methodology Using Idealization

Let us now turn to model building. In constructing models we may make a number of idealizations; but these can be made more concrete when we drop some of the idealizations and approach something like the real systems we are investigating. Terms (or their Italian equivalents) such as 'concrete' or 'material', which are applied to ordinary real objects, are contrasted by Galileo when he talks sometimes of 'ideal' but more commonly of 'abstract' or 'immaterial' objects. As an illustration, see the many passages (for example Galileo 1967, pp. 206–208) where

Galileo talks of material planes and spheres in contrast to immaterial planes and spheres and considers what might happen in concrete actual cases when they touch as compared to ideal or abstract cases. Part of the matter for debate is whether actual spheres and planes touch in one or many points in contrast with ideal planes and spheres which are said, by their very definition, to touch in only one point. For Galileo's opponents it is obvious that real, material spheres will, by their very weight, press down on a plane over many points. As a result they cannot see the rationale for adopting such an abstract and idealized model that is defined into existence and that is not true of real, material planes and spheres. But Galileo's response is to say 'I grant you all these things but they are beside the point' (*ibid.*, p. 206), the point being one about idealized spheres and planes and not their actual counterparts.

The above terminology of 'abstract', 'concrete', etc is, for us now, a quite natural mode of discourse to adopt, and such usage introduced by Galileo has become part and parcel of the discourse of contemporary theorists when they discuss the processes of model building in science. In comparing the manner in which idealized and perfect spheres and planes touch when compared with real spheres and planes, Galileo says:

> just as the computer who wants his calculations to deal of sugar, silk, and wool must discount the boxes, bales and other packings, so the mathematical scientist (*filosofo geometra*), when he wants to recognise in the concrete the effects which he has proved in the abstract, must deduct the material hindrances, and if he is able to do so, I assure you that things are in no less agreement than arithmetical computations. (Galileo 1967, p. 207)

As a further example of idealization and model building we find Galileo responding in the *Two New Sciences* to objections to his method of idealization through Salviati when he says of projectile motion:

> All these difficulties and objections which you urge are so well founded that it is impossible to remove them: and, as for me, I am ready to admit them all, which indeed I think our Author would also do. I grant that these conclusions proved in the abstract will be different when applied in the concrete and will be fallacious to this extent, that neither will the horizontal motion be uniform nor the natural acceleration be in the ratio assumed, nor the part of the projectile a parabola etc. But, on the other hand, I ask you not to begrudge our Author that which other eminent men have assumed even if not strictly true. The authority of Archimedes alone will satisfy everybody. (Galileo 1954, p. 251)

Galileo sees in the work of Archimedes the same methods of idealization as he proposes to use in his theory of motion. In the context above, Galileo is imagining a perfectly smooth ball rolling with uniform motion along a flat frictionless surface which, upon reaching its edge, acquires a downward motion which is to be added to its original horizontal motion. Galileo's task is to give an account of this new motion, but he recognises that the model he has given of it is ideal and does not fully specify what happens in real systems. Other participants in the *Dialogue* tell him that he has ignored the resistance due to the medium through which the object falls, he has ignored the fact that when the body moves along the horizontal plane it will have a variable distance from the centre of the earth, and so on. Galileo admits that all these are idealizing assumptions, and recognizes that any models which ignore them cannot be 'strictly true', as he says. However he does claim that in his models he has set out the *central*, *primary* or *essential* features of what is happening in such motion. It sets aside *peripheral*, *secondary* or all *non-essential* features of the motion; but these can be taken into account by dropping idealizations when the model is made more concrete.

What is important about Galileo's methodological revolution is the construction of models by reasoning about the theory of motion which, when applied to some situation, gives the essential features of the motion; but these features are not given in experience at all. Moreover, the model leaves out other features that one might envisage holding of real systems, but which are inessential to the motions being modeled. The distinction between *essential* and *inessential*, or *primary* versus *secondary*, features of models is an important aspect of Galileo's scientific method. Making this distinction is not one that can be based in experience but must be determined by reasoning, in the light of the theory, about the model being constructed. It is also important to note that the models might fit the observed facts only approximately; but they do capture the essential hidden features of the motion not given immediately in experience. In the next section a theory of idealization in models will be set out which reflects much of Galileo's methodological procedure.

Of course Galileo was aware of the important role that experience and experimentation play in science. But in his view these do not play the *only* role in determining what theories we are to accept. Importantly experience may play no role when it comes to constructing models which get to the essentials of what is happening in ways not evident in, or even controverted by, experience. An important role must be given to reason in constructing models of real systems that are then to be compared to reality itself, or with what we observe. Galileo was aware that many contemporary Aristotelians held the view that experience was the only determinant of what theories we should accept. Thus Galileo has the Aristotelian mouthpiece Simplicio say the following: 'Aristotle would not give assurance from his reasoning of more than was proper, despite his great genius. He held in his philosophizing that sensible experiments were to be preferred above any argument built by human ingenuity, and he said that those who would contradict the evidence of any sense deserve to be punished by the loss of that sense' (Galileo 1967, p. 32). For Galileo the theories of Aristotle and Aristotle's contemporaries were already, in their science, quite close to experience; there was no need to bring them any closer to experience. The problem was, however, to find an analysis through reason of what we experience where what the analysis reveals is not immediately evident in that experience, and may even go against it. In this respect Galileo was divided from his contemporaries, (such as Del Monte, as will be seen in the final section), over the role experience is to play in theory construction. For Galileo there is a paramount role of reason in constructing idealized models of real systems. Of course there is a role for experience in comparing models, or the consequences deduced from them, with observation or experimentation to determine the extent to which the models approximate real systems. But experience may have to be set aside when reason is applied to model construction and development.

3. Abstraction and Idealization

Though there is no agreed way of using the terms ‘abstraction’ and ‘idealization’ in the literature, they can be used to mark an important distinction. In this section an account is given of how they will be used. In this respect I follow, but not completely, the use of these terms as in Nowak (1994), and Nowakowa and Nowak (2000). Once these terms have been carefully defined it will be see that the distinction drawn has considerable consequences for our characterization of theoretical models.

All objects with which we are familiar, or postulate in the sciences, have (intrinsic) properties such as colour, weight, charge, etc; and they have relational (or extrinsic properties) such as position, spatial relationships to one another, ownership relations, etc. Now consider some real object such as a blob of metal stuck on the end of fine wire (i.e., a pendulum). From the point of view of mechanics we are not interested in some of its extrinsic properties, such as who owns it. But note that from the point of view of, say, theory in economics or sociology, whether or not it is owned, or who owns it, is of interest. We will say that the dynamicist abstracts away from the extrinsic or relational property of ownership in that it is irrelevant to the dynamicist whether it is owned or not. But the economist does not abstract away from such an extrinsic property since it is part of his science to consider ownership relations; but the economist does abstract away from the energy properties of the swinging pendulum. However both the dynamicist and the economist will abstract away from its relational property of being so far from, say, the Grand Canal in Venice. Neither have an interest in their theories with this relational property of the real object. Both will also *abstract* away from intrinsic properties such as colour; what colour the pendulum has is not a matter of interest in their theories. Any real, existing, actual, pendulum will have some colour (including black or white). But what colour it has is irrelevant to both theories of dynamics, or economics (but maybe not irrelevant to some other theory to do with the optical properties of objects, or the aesthetic properties prized by an art collector).

In general, we may say that we *make an abstraction* from a real object, such as a pendulum or the Moon, when the real object has a property P but it is of no concern to, or it is irrelevant to, some theory T whether the real object has that very property P. Note the emphasis here on *real* or *actual* objects and their real and actual properties. The actual object is of interest to some theory T (of dynamics, or of economics) and so are some of its actual properties (such as respectively, mass or production cost); but other actual properties are not considered in theory T (e.g., for both dynamics and economics, the colour of the object). Finally we need to note that some abstractions may be erroneous. Thus classical dynamicists erroneously thought that they could abstract away from the frame of reference in which a body was moving and talk about, say, its absolute mass. This is an erroneous abstraction, as pointed out within Einsteinian Special Theory of Relativity. Of course, scientists might make some such abstraction for various purposes and later discover they were wrong to make it. However dealing with such matters really takes us into the territory of idealization (as defined here.)

The term 'idealization' will be used differently. In the case of abstraction an object is still a real object with property P, but we ignore property P for certain purposes, such as whether it is a property with which our theory deals. But in the case of idealization we do not merely ignore a property; we regard P as a property that the object *definitely does not possess*. Thus we idealize humans when we consider that they are always just, honest, loving or act with good intentions. It is not that we merely ignore or set aside their unjust behaviour, their dishonesty, their propensity for hatred, or their bad intentions. Of course, we could merely abstract from these, in the sense of allowing that we have these negative features but we simply set them aside. But when we idealize humans and think of beings that lack these negative characteristics, then we are not talking of real humans at all. They have features in common with humans, but they are not actual human beings; they are more strictly akin to a God, or are angels or saints. What we are talking about is best described as an *idealized* human. Here the word 'idealized' carries with it the connotation that it is not a real item to be found in the actual, real world. But it is still an item of our scientific discourse that we can characterise as 'abstract' or 'ideal'.

Consider now real tables, chairs, rocks, plants, and animals. These all have some colour or other. Now if we abstract from the colour of these items then we are still considering these items as real items but ignore their colour properties. But what happens if we idealize in the sense above, and consider them as definitely lacking colour properties? Most would agree that no item can be real and yet not be coloured in some way. It is not that we have here a real object, but that it is colourless. Rather we do not have real object at all. But we still have an object of some sort since we continue to claim that it has other properties such as mass, or volume, or inertia or the power to gravitationally attract. In so far as the object *lacks* colour we can conclude that it cannot be a real item that we can bump into in our real spatio-temporal system. We do not find middle-size objects lacking colour in the real spatio-temporal system. But such 'objects' lacking colour are not nothing. They are, let us say, *ideal* objects or *idealizations* of real objects. Such items are, to use the terminology of philosophers, *abstract* objects and not *concrete*, *actual*, objects found in the space-time system. Note however that we do attribute temporal and spatial properties to ideal objects even though we do not expect to find such ideal objects in the actual space-time system we inhabit.

Another example might also help. Often in dynamical theory we consider objects to be point-like, that is, they lack any volume. Thus in simple models of the solar system the Sun, the Moon, and the planets are often considered to be point-like objects with forces either acting from, or acting upon, that point. But is there such a thing in the world as a point-Sun, point-Moon, or point-planet? No. But in dynamics they are not nothing. They are, let us say, idealized objects, or idealizations of real objects; they are not objects to be found in the real actual world. However we might be able to show, by means of a proof along the lines suggested by Newton, that treating an object as point-like with all its mass at its centre of gravity is equivalent to a body with volume in three-dimensional space and with its mass evenly distributed about the centre of gravity. Such an equivalence does not necessarily undermine the point that we are still idealizing an object when we treat it as if it were point-like. But what such an equivalence does show is that we are not making an idealization which is at a large distance from reality; in one respect the idealized and the real object share some common features, such as gravitational attraction, which are the object of investigation.

Hopefully the illustrations should make the difference between abstraction and idealization clear without setting out necessary and sufficient general characterisations of what these terms mean in this context. We abstract from real objects when we still consider them as real objects but ignore some of their properties. We do not say they *lack* these properties; rather we set them aside for reasons to do with our theories and what properties of real objects we wish to consider. But when we idealize, we are no longer considering real or actual objects, but non-real or non-actual ideal objects. This is so because we consider the object to lack some of the properties that would be necessary for it to have if it is to be a real object. The difference here is between an *epistemic* matter, as when we ignore properties while abstracting, and an *ontological* matter, as when we claim that an object lacks, does not have, certain properties when we idealize. In both cases it is humans that do the abstracting, and do the idealizing. To some extent the metaphor of humans constructing such objects can be helpful. Human activity is involved in both cases; in particular it is we who construct ideal objects since they do not actually exist outside our idealizing activities in science.

Just how far can the idealization process go? It can go quite far, but not so far as to denude the ideal object of all the properties of its real counterpart. But in treating a real object as, say, a point-like object one has gone a considerable distance in that the point-like object is considered only to have, say, mass, inertia and the power of gravitational attraction, along with velocity and acceleration, viz., just those properties of dynamics. Using some notation developed by Nowak let us assume that some real object, or class of real objects, O, have real properties A, B, C, $\ldots$, Q. We can represent the object as a whole by '$O\{A, B, C, \ldots, P, Q\}$'. In idealizing we consider O to lack properties, say, D to Q and indicate this by a negative sign '$\sim$'. Thus we can represent the idealized object as '$O\{A, B, C, \sim D, \sim E, \ldots \sim P, \sim Q\}$'. Nowak then suggests that the remaining properties may be linked in an idealized law L (see Section 4), where L is a function of the positive properties O has, such as A, B and C only. We can represent such a law as follows: $L(A, B, C)$, where in the law A, B and C are related in some way. For example, such an idealized law might be Newton's second law, which relates only mass, force and acceleration, $\mathbf{F} = \mathbf{m}$ (where bold **F** and **a** indicate vectors). The law clearly does not relate other properties if we abstract away from real situations. And nor does it relate other properties if we idealize the objects concerned since under such idealization the objects do not have any of these other properties.

Given the above we can now easily introduce Nowak's idea of concretization. This is the reverse of the process of idealization. It is the process of making ideal objects more like real actual objects, by attributing to the ideal object more of the actual properties possessed by its real counterpart. Thus consider ideal object $O\{A, B, C, {\sim}D, {\sim}E, \ldots, {\sim}P, {\sim}Q\}$. This might be made less ideal, or more concrete, by regarding O not as lacking properties D and E but as actually possessing them while still lacking F to Q. In so concretizing we move to an object which is still ideal, viz., $O\{A, B, C, D, E, {\sim}F, \ldots, {\sim}Q\}$; but we do move in the direction of greater concretization. In complete concretization we would consider an object with all its real properties.

On what grounds are some properties retained in an ideal object, and others abandoned? This is a question that can only be answered by considering what properties are postulated in a theory and what resources there are for handling them. Thus from the point of view of certain idealizations, the properties which are retained are said to be *essential* or *primary* for the understanding of the phenomena under consideration, while the abandoned properties are said to be *inessential* or *secondary* to that understanding. But this is not to say that these inessential or secondary properties are unimportant in all respects; they can be accommodated in models which are more concrete, and so do come to play some role in understanding the phenomena being considered. But they are not essential to that understanding, as Galileo says.

Note that so far we have treated only *objects* as idealizations, and have characterized this in terms of properties they are considered to either have or lack. But we could do exactly the same thing with the *properties* themselves and treat them as idealizations from real properties in much the same way; some of the features of properties are retained while others are abandoned. Thus we might idealize a property of being an ovoid, or oviform (egg-shaped). Such properties might be mathematically difficult to deal with; so initially we might treat the property as the idealized property of sphericity, and later the property of being ellipsoidal. In such a case there is an initial idealization of the property as one of sphericity, but this is made progressively more concrete, but still ideal, as when we move to considering the property of being an ellipsoidal. Such property idealization might be viewed as a case of object idealization, as in the case of the Earth. In dynamical theory initially the Earth was considered to be point-like; but with successive concretisation it was considered to be a perfect sphere, and then a sphere rotating about an axis, and then an oblate spheroid rotating about an axis. But if we view this as a case of property idealization we need to prescind from the objects in which the properties are instantiated and adopt a more Platonic view in which we consider the geometrical properties themselves, and not their instantiation in some object. In this way the sequence of properties, say, from being an ovoid, to being ellipsoidal, and then to being spherical, can be regarded as successive idealizations (in which, say, the different lengths of the axes and their different points of intersection in the case of an ellipsoid become identical and yield sphericity). Laws can also be idealized; this is the topic of the next section.

Now we can introduce the notion of a *theoretical model*. This is a set of idealized objects having idealized properties and being in idealized relations to one another and obeying idealized laws. An excellent example of this is provided by Lakatos in his discussion of scientific research programmes in which he illustrates how they might progress by considering a sequence of models of the solar system that begin in a highly idealized manner but become successively more concrete (Lakatos 1978, p. 50). Thus he says that Newton began by considering the solar system as a collection of separate planetary systems, one for each point-like planet orbiting a point-like Sun as centre of gravity. But it was known, even by Newton as he worked on his models, that the centre of gravity of a single Sun-planet model cannot be where the point-like Sun is, but outside it. When Newton dropped this idealizing assumption, he worked on an even more concrete, but still ideal, model by considering all the planets in one unified model, but only under the action of the Sun's gravitational force. Clearly the theory on which Newton was working suggests even further concretizations, such as: allowing the planets to gravitationally affect one another (this gives rise to the difficult many-body problem in dynamics); removing the assumption that the Sun and planets are point-like objects; allowing the planets to rotate about an axis, or even wobble; and so on.

Importantly Lakatos considers how such progressive concretization can occur even when it is known by those developing the models that they are inconsistent with what can be observed, and even always remain so. Lakatos' main point is that model building is an entirely theoretical process, or to use the vocabulary of Galileo, is a rational process. It is not a 'construction out of experience', though empirical claims are an input into the process of model building. In fact Lakatos' discussion of this issue is in a section with a title that includes the phrase 'the relative autonomy of theoretical science'. Here the autonomy is of what we can construct from theory independently of what we might be able to construct from experience, a point about model construction on which Lakatos and Galileo are at one.

Such models are common in dynamics, but they also have a role in the rest of physics and chemistry. The role of models also looms large in the human sciences from psychology to economics. In economics one can find a range of models about the economic behaviour of rational humans being made more concrete by adding empirically discovered hypotheses about actual human behaviour in economic situations. (It should be noted that there are many different senses of the term 'model' at work in the sciences, and different kinds of model as well. Here we are considering only one kind of model, viz., that defined above.[1])

4. Idealized Laws

Some philosophers of science would argue that all scientific laws are really idealizations of what happens in reality and that no law is strictly obeyed; however as the laws are made more concrete they come to apply more accurately to real observed happenings. Since this will be the position adopted here, what needs to be done is to give an account of what an idealized law is like. We will find that this characterises quite closely Galileo's approach to the laws he discovered. In what follows we will adopt the view of idealized laws set out by Nowak and his school.

Let, $I_1, I_2, \ldots, I_n, I_c$ be $n + 1$ idealizing assumptions made about the objects and properties and their magnitudes in some theoretical model. For example, these might be the assumptions made in models of the Solar System that the Earth has zero volume and is a point-like object, or that it does not rotate; or that it is a perfect sphere, or that its centre of gravity is the geometrical centre of a perfect sphere, and so on. In the case of free fall the idealizing assumptions often include the claims that the body falls through a constant gravitational field, or that it experiences no drag of the air as it falls, and so on.

The final idealizing assumption has been labelled as 'I_c', where the letter 'c' stands for a 'catch-all' idealization. That is, the last idealizing assumption says that $I_2, I_2, \ldots, I_n$ are *all* the idealizing assumptions to be made. Of course we could be wrong about this and there are a number of matters we have not taken into account in making our idealizations about them; we might not even be aware of them. For example, early work on the solar system assumed that the only forces at work were gravitational and inertial; but we now know that this is false. If the idealized models of the solar system did not include a catch-all idealization, then the model would have failed to idealize to only the dynamical forces under consideration. So there is a need for a final catch-all idealization assumption.

Now we can set out a general schema for an idealized law, and then illustrate it in some simple cases. The general schema is:

If $I_1, I_2, \ldots, I_n, I_c$, idealizing assumptions were to hold then law L would hold.

Let us fill in this schema with some particular cases, the first being Newton's First Law of motion (stated in the *Principia*, Book I, under 'Axioms, or Laws of Motion'): 'Every body continues in its state of rest, or of uniform motion in a right line, unless compelled to change that state by the forces impressed upon it'. Now as many have noted, there may be no example of a body in the entire cosmos that is under the action of no forces. It is not that the sum of the resultant forces acting on it is zero; rather there is always some resultant non-zero force due to gravitational attraction, or whatever. So there is no actual instantiation of Newton's First Law in the entire cosmos. But this does not mean that the Law is false; rather it is an idealized law that has an idealizing condition about zero resultant force in an antecedent clause. So understood the Law is unproblematic, even in an idealized universe in which there are only gravitational forces and they are ubiquitous throughout space and time.[2]

Consider now Galileo's free fall law. He discovered that the distance a body fell in free fall when close to the Earth was proportional to the square of the time of the fall. We now know that this law can be expressed as follows:

$$s(x) = \frac{1}{2}gt^2(x),$$

where 'x' stands for some body, 's' is the distance fallen by x, 't' is the time of the fall from initial movement to impact on the surface of the Earth, 'g' is the acceleration on Earth due to gravity.

Now we can consider the idealizing assumptions.

I_1 is the assumption that the body x falls in a perfect vacuum and suffers no air resistance; that is, if F are the forces due to air friction then I_1 is the assumption that $F = 0$. I_2 is the assumption that the Earth's gravitational field exerts, contrary-to-fact, a constant force at all times in the fall of x; that is, $g =$ constant, and is not, say, a function of distance.

There is also the 'catch-all' idealizing assumption that the model specifies all and only the items to be considered in the model and that none have been left out. Of course the idealizations that have been adopted do not hold in real systems, including the idealizing 'catch-all' that only the dynamical forces mentioned are those at work.

There is a further assumption that is not strictly an idealizing assumption of the sort indicated in the previous section. Rather it is more like an abstraction. We have not considered whether x had an initial motion before falling; it could have had some component of motion in the direction of the Earth, say a velocity of $u(x)$; but we have ignored this rather than take it into account. That is, we make the assumption that at the initial time of to the initial velocity of $xu(x, t_0) = 0$. If we remove this abstraction then the law under consideration becomes:

$$s(x) = u(x, t_0)t(x) + \frac{1}{2}gt^2(x).$$

But if we set the first initial velocity factor $u(x, t_0) = 0$, we abstract from (i.e., ignore) any initial velocity rather than idealize it.

It is fairly evident in a case like this what are the essential or primary features of free fall motion and what are the inessential or secondary features of that motion. And it is fairly evident that such a law will not give a correct account of free fall; in particular it ignores the fact that there is a terminal velocity of free fall in the atmosphere (but one which can vary according to height). However the inessential or secondary features can be incorporated into the theoretical model through a process of concretisation that will give a better accord with the free fall motions that we can observe. What is important to note is that Galileo was fully aware of what his procedures were in considering free fall, and other motion; he was constructing a series of idealized models that could be concretized once one dropped idealizing assumptions. But the construction is not one out of experience; it is a construction based on reasoning about the hidden essential features of the motion under consideration.

A further example, that of the pendulum, will help illustrate assumptions about idealized objects, properties and laws. In most physics textbooks there is a proof, based on what is called the 'simple' pendulum, of the period law $T = 2\pi\sqrt{(l/g)}$ where 'T' is the period of swing, 'l' the length of the pendulum, and 'g' the acceleration due to the Earth's gravity. And the same textbooks usually make clear the idealizing assumptions made in the deduction of the law, all of which Galileo was one of the first to be aware. Compare the idealized model of a simple pendulum with a real swinging device such as wire attached at one end and to the other end of which is attached a lump of lead, and the whole allowed to freely swing back and forth. In the idealized model of such a real system the top of the simple pendulum is suspended from a frictionless point; the body of the pendulum is a weightless, dimensionless, frictionless, rigid (and so idealized) line-like rod. The bob is attached to the free end; but it is treated as a point-like mass that swings in a plane. The gravitational center of the whole system is situated at the same position as the center of gravity of the point-like mass.

Such an idealized model fits the real system of the pendulum to only to some extent. In the real system the pendulum is attached to a joint where there is a frictional force affecting the motion of the pendulum. After a short while the real pendulum will have slowed considerably while the idealized one swing indefinitely. The pendulum rod also swings in the air, which causes friction, as does the massive blob that is attached to the free end of the pendulum. The air friction is not always a linear function of the object's velocity; usually it is a non-linear function. This pertains not so much to the idealized objects in the model as to the idealized laws in which, in most cases, all frictional forces are disregarded. The bob itself is not point-like and occupies volume. And the center of gravity of the whole system is not situated at the center of gravity of the bob, but some distance away from it.

The model also assumes that the Earth's gravitational field is uniform. This would be the case if the Earth were a perfect homogeneous sphere; but it is not. Moreover the field is affected by the buildings, hills and mountains, not to mention the changes in the field due to the presence of the Sun, Moon and the planets. There is also a central assumption about the restoring force function at work. It is usually assumed to be linear, and is generally known as 'Hooke's Law'. The force varies with the distance x along the arc of the swing, $F = -kx$ (where k is a constant and the minus sign indicates the direction of the force is back towards the vertical). Using Hooke's Law, and the force due to gravitational attraction, one can then deduce a formula close to the period formula given above, except that there is a trigonometrical function of the angle of swing, θ, to take into account, such as $\cos\theta$. But where the angle of swing is small, this factor can be neglected.

Granted all these idealizing assumptions the period law, first noted by Galileo, can be deduced from the model. Of course with various kinds of concretization a better swing law can be deduced. (This is discussed more fully in other places such as Morrison (1999), section 3.4.1 entitled 'Theoretical models meet the world').

If the swinging pendulum were to be of a different sort, such as that in a Grandfather clock, then different idealizing assumptions would have to come into play. One important assumption of the ideal pendulum model is that nothing is said of the periodic force acting on the Grandfather clock pendulum by the escapement mechanism. This abruptly starts and stops the motion. Further, through a system of falling weights or a spring, it applies a force that prevents the swing from gradually decreasing its amplitude, thereby ensuring a more even way of indicating time. The real system of the pendulum of a Grandfather clock is at a considerable distance from the idealized model of the simple pendulum; less ideal models which move in the direction of greater concretization, and more mathematics, are needed to even approximate to such a real system.

As a final example to illustrate how a series of concretizations can lead to a succession of less ideal laws, consider the case of the Boyle–Charles Law

$$PV = RT, \tag{1}$$

where P is the pressure of a gas, V its volume, T its temperature and R is a constant. This law can be deduced from the (idealized) laws of Newtonian mechanics as applied to a simple model of a gas in which the gas molecules are perfectly elastic corpuscles bouncing off one another and the perfectly elastic walls of the container; it is also assumed that they are point-like and take up no room in the container. If the assumption that the gas molecules take up no room is dropped through one concretizing move, then van der Waals recognized that the above equation becomes

$$P(V - b) = RT, \tag{2}$$

where b is a factor related directly to the volume occupied by the molecules. He also recognised that if one takes into account the fact that the gas molecules attract one another thereby changing their force of interaction with the container then a further equation can be developed to take into account this concretization:

$$(P + an^2/V^2)(V - b) = RT, \tag{3}$$

where 'a' is a constant and 'n' is the number of molecules.

This last equation is really a cubic equation in V. Yet other equations based on models of the molecules that are quite different from those just mentioned, lead to further equations (some named after physicists) such as:

$$\textit{Dieterci};\ P(V - b) = RT(\exp -a/VRT) \tag{4}$$

$$Bertholet:\ (P + a/TV^2)(V - b) + RT \quad (5)$$

$$Virial\ Equation:\ PV/RT = 1 + B/V + C/V^2 + D/V^3 \cdots \quad (6)$$

(where B, C, etc are functions of T for which further mathematical equations are to be given).[3]

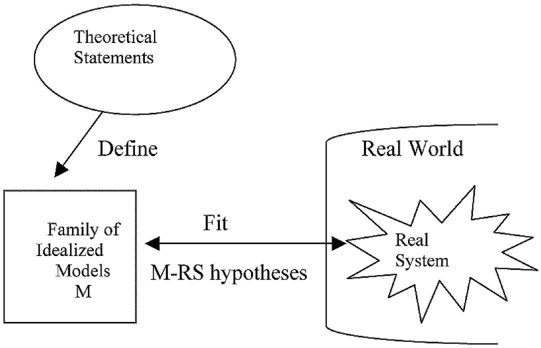
Further concretizations are possible when one takes into account electrical attraction forces, quantum effects, and so on.

What the above sequence of concretizations of the first ideal gas law equation (1) show is that from each equation lower down one can deduce the equation immediately above by setting certain values in the lower equation equal to zero. Thus there is an important relationship between the various idealized equations as concretisation takes place. More generally, this relationship can be expressed by what has been known since Bohr as the *Correspondence Principle*. Roughly this says that a preceding theory should be obtained from a subsequent theory as a special case of latter, especially when some factor is set as zero. This principle applies not only in Quantum Mechanics but also in, for example, the Special Theory of Relativity in which classical laws of motion can be obtained by letting the velocity of light go to infinity, in which case the 1/c factor tends to zero. As can be seen the Correspondence Principle applies to the above gas law equations and is simply another expression of increasing idealization, or in reverse, and expression of increasing concretization. As such the Correspondence Principle is an important notion in science given a rationale in terms of the theory of idealization developed here.[4]

5. Theories, Models, Reality and Test

In this section a number of the above points about ideal models will be brought together and linked into the question of how we might compare our theories and models with reality. The underlying ideas of this section have been advocated by Ronald Giere in what he calls constructive realism (Giere 1988, chapters 3 and 4). What will be presented here are some of Giere's basic ideas, but with some modifications. Only a sketch can be given; and it will be assisted by some diagrams that hopefully will convey the basic points quite graphically.

A *theoretical model*, M, as we have defined it, is a system of idealized objects, with idealized properties and obeying idealized laws. We can now ask just how well such a model M (of pendulum motion or gases) *fits* the real system RS (or a real swinging pendulum or a real gas) The notion of *fit* is to be understood in terms of the degree of similarity that holds between the real items in RS and the idealized items of M. Clearly many features of real systems will be absent from models; the more that are absent the lesser will be the degrees of resemblance that hold between RS and M, and so the smaller the measure of fit. But with successive concretizations of M so that it approaches RS more closely, the higher the various degrees of resemblance, and so the greater the measure of overall fit.



A *theory*, we will say, has two elements; it is a set of theoretical statements, and a family of models that are 'defined' as we might say, by the statements. The first diagram sets this out. It also specifies a relationship of *fit* between the model M (its idealized features indicated by a regular square) and a Real System, RS (indicated by an irregular figure – real systems are always 'messy'). The $M - RS$ hypotheses describe these relationships of fit and the degree to which they do, or do not, fit. Using the M-RS hypotheses we can derivatively speak of the degree of verisimilitude of theory T (that defines the models); that is, T has some degree of truth-likeness with respect to reality.

The scientific realism implicit in the account of the relationship between T, M and RS departs radically from a constructive empiricist view of theories of the sort advocated by van Fraassen (see van Fraassen 1980, chapters 2 and 3). For constructive empiricists, there can be a fit between the observable aspects of a theoretical model and what we observe. What is denied is that we can meaningfully speak of a fit between the unobservable, or theoretical, features of the theory plus model and what is out there in unobservable reality.

In contrast the *constructive realist* view maintains not only that there can be a fit between the observational aspects of a model and observable reality, but also that there are relations of fit to be considered between the unobservable items of the model and unobservable reality. Thus while the constructive empiricist and the constructive realist can agree about what may be the relations of fit between the observable aspects of a model and observable reality of, say, the temperature, volume and pressure of a gas, they disagree about whether or not we can have a relation of fit between the idealized items of the model, the idealized molecules, and molecular reality.

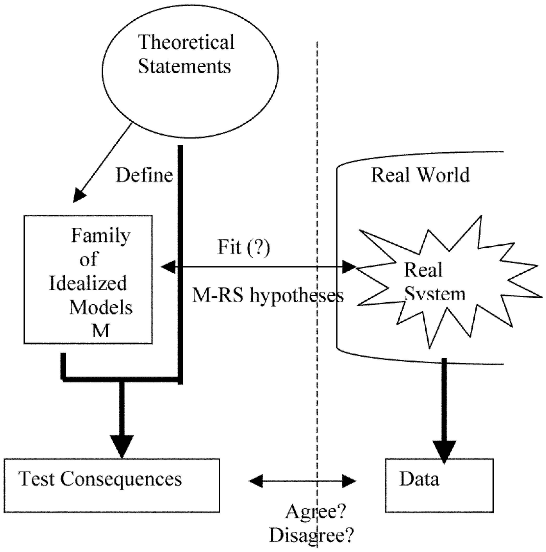
The claims of the realist clearly go beyond what we can observe. Because of this the constructive empiricist denies what the constructive realist asserts, viz., that there are such relations of fit between idealized unobservables and real unobservables; or if there are such relations of fit, then we can know nothing of them. Realists presuppose that such comparisons are an integral part of the relationship between theory and models on the one hand and reality on the other and that a measure of them can be given; these claims are denied by those of an empirical constructivist persuasion. Arguing the case for constructive realism is one of the major controversies of recent philosophy of science that cannot be entered into here.[5]

Constructive empiricism of the sort advocated by van Fraassen does share one of the features of the kind of radical constructivism found in science education. This is the claim that relations of fit cannot hold between the unobservable idealized reality postulated in M and unobservable reality itself. But where they part company is over whether there can be such relations of fit between observable idealized reality postulated in M and observable reality itself. Constructive empiricists say that such relations of fit are possible, and we can know of them; radical constructivists deny this. It is a feature of many varieties of constructivism about scientific theories that such relations of fit cannot be established at all; if they do exist we can know nothing of them.[6] We will return to this and other matters in the next section.

Let us now turn to the matter of testing. Again we can adapt some useful diagrams in Giere (1991, pp. 28–37) for our purposes.

We can observe real systems such as the solar system and obtain much observational data about planetary positions, their juxtapositions and opposition, their paths against the background of the stars, and the like. Or we can perform controlled experiments on some real system RS, such as a swinging pendulum, and record the values of certain observable or detectable features of RS. This is indicated in the second diagram in which, on the right hand side of the dotted line a real system in the world is either observed, or experimented upon, and data collected. Also indicated in the second diagram is the relationship of fit, or lack of fit, between the idealized model M and what it models, viz., a real system RS.

Now we are in a position to consider the important matter of the agreement or disagreement (within some degree of error) between the test consequences and the data. There are two alternatives here. First, there might be a satisfactory agreement (up to some degree of error) between the test consequences and the data. All that one can legitimately infer from this is that the conjunction of the set of statements T, and the model M (and any other auxiliary statements A not indicated in the diagram that assist in the deduction of the test consequences), have jointly passed the test. With some theory of confirmation, one might even be able to ascertain what degree of confirmation the conjunction of these items has by the data, and how it can be distributed amongst T, M and A.



Second, there might be disagreement between the test consequences and the data that exceeds any compatibility within the allowable degree of error. So, if the test consequences are to be rejected as false, then so must the items from which they were derived. That is, either all of T, M and A are false, or any two of these, or just one. This leads to the well-known Quine–Duhem thesis in which no hypotheses can be refuted in isolation from other hypotheses. But the situation may not always be as grim as the Quine–Duhem thesis suggests. Perhaps one has good independent grounds to say that the auxiliary assumptions A are true, and so A is not to be blamed; the blame must fall on T, or on M. Now one can readily see a way to sheet the blame home to M with all its idealizations. All models are idealized to some extent; and as such, when combined with theory may not lead to test consequences that fit the data within an acceptable degree of error. It is then possible to concretize aspects of the model to improve the values of the test consequences that flow from it so that the discrepancy between test consequences and data might be lessened, or removed altogether. That is, when confronted with a conflict between test consequences and data, there is a procedure of concretization to be carried out to bring the test consequences and the data into accord. But it might also be the case that successive concretizations, leading to the complete concretization of the model in which all idealizing assumptions have been removed, still do not remove the discrepancies. In such a case one would have grounds for thinking that the fault lies not with the model proposed but the theory that is used in combination with the model. (On testing idealized laws, see Nowakowa & Nowak (2000, pp. 130–134).)

In summary, there is a test procedure for determining which of our theory, or the models to which theory is applied, are to be accepted, modified or rejected. The process of concretization plays an important role here in enabling us to improve on our models by dropping idealized assumptions. Such a procedure is an important part of the methodology of idealization that Galileo introduced and applied to a number of dynamical phenomena, such as free-fall or pendulum motion.

6. Consequences for a Radical Constructivist Account of Science

Galileo was aware that his contemporaries would not fully grasp his novel methodology based on models and idealization. In his dialogues he put a number of considerations against it in the speeches of those who opposed the views of his mouthpiece, Salviati. His opponents were Aristotelians who adopted the common-sense and empirically based conception of science that Galileo was at pains to re-evaluate. There were also live, contemporary opponents who were expert mechanics and engineers in their own right and who challenged Galileo's procedures. One such was Del Monte who complains of the considerable distance between the ideal models proposed by pure mathematicians (such as Galileo) and the actual physical systems and machines they purport to describe.

> Thus, there are found some keen mathematicians of our time who assert that mechanics may be considered either mathematically, removed from physical considerations, or else physically. As if, any time, mechanics could be considered apart from either geometrical demonstrations or actual motion! Surely when that distinction is made, it seems to me (to deal gently with them) that all they accomplish by putting themselves forth alternatively as physicists and as mathematicians is simply that they fall between stools, as the saying goes. For mechanics can no longer be called mechanics when it is abstracted and separated from machines. (cited in Matthews 2000, p. 101)

This remark, and others like it by Del Monte, can be found in Matthews book on the pendulum along with a longer discussion of the issues involved (Matthews 2000, pp. 100–107). What these remarks of Del Monte show is that mechanics is to be strongly connected to what we can experience of the workings of machines and the like. Any other alternative view of mechanics, such as Galileo's treatment of motion by means of models that might not even fit experience, is to be regarded with suspicion.

Moving to our own time, how does Galileo's methodological approach compare with constructivism as it arises in science education? In some respects it provides a serious challenge – depending on how constructivism is understood. There are at least three aspects to constructivism depending on whether it is (a) a philosophical doctrine that provides an epistemology and a view of the world and science that flows from it, or (b) an account of how science has grown throughout its history, or (c) an account of how students in fact do, or ought to, learn about the world or science.

Let us consider the first aspect (a), using the remarks of a leading advocate of radical constructivism, von Glasersfeld. Radical constructivism is not radical in an immediately political sense (though it is this too); it is radical in that it aims to overthrow the domination of traditional epistemology of the last two and half millennia. What is to be overthrown in that tradition is the idea that any of our theories are true, or are a copy of reality: 'Although Piaget said dozens of times that, in his theory, "to know" does not mean to construct a picture of the real world, most of his interpreters still cling to the notion that our knowledge must correspond to a world thought to be independent of the knower' (von Glasersfeld 2000, p. 4). Now it is not necessary for a realist theory of truth to be committed to a 'copy' view, whatever that means. All that is required is, as Aristotle put it: 'to say of what is that it is not, or of what is not that it is, is false; while to say of what is that it is, and of what is not that is not, is true' (*Metaphysics* 1011b, 25–28). But we will not labour this point. The point that von Glasersfeld wishes to make is against the realist's epistemological claim that we can have knowledge of an external reality. Instead it is alleged that we can never *know* that anything we say is true or false (even if it be true, or false).

According to von Glasersfeld skeptics have, from ancient times onwards, shown that the realist claim of knowability is unrealizable; but they failed to shake people's convictions about realism because they did not propose a plausible alternative. However an alternative explanation might be that people were not convinced by the skeptics' argument; some reasons are given for rejecting the skeptics' conclusion in Nola (2003) and will not be repeated here. Von Glasersfeld's rejection of (a) is radical in two ways. First, he appears to be rejecting, because it is otiose, one of the central conditions that distinguish knowledge from belief or opinion, viz., that if one knows some proposition, that *p*, then it follows that *p* is true. In this respect the epistemology of von Glasersfeld and others is seriously at odds with the philosophical tradition. Second, it should be noted that the radical constructivist's rejection of the realists' claim of knowability is quite broad; it has the consequence that we should give up on ever knowing the truth of even observational claims such as 'there is an open page of a book in front of me now'. That is, not even for the observational realm can we say that we know that our observational reports are true of an external reality in which there is a book in front of one now and it is open at a page. Radical constructivists take skepticism about the external world seriously as part of their position.

This distinguishes radical constructivism from the constructive empiricism of the previous section. On this issue constructive empiricists join hands with realists; both acknowledge the possibility of the truth of reports about what we can observe, and our *knowing* that they are true. However constructive empiricists join hands with radical constructivists against realists in denying that we can know the truth about reports of unobservable, or theoretical, objects and happenings. Their common objection is that we cannot check our claims about the unobservable by directly comparing the claim with a bit of reality – which in the nature of the case is unobservable. To illustrate, see the second diagram of the previous section where realists wish to talk not only of some degree of fit holding between observable aspects of models and observable reality, but also of some degree of fit holding between unobservable aspects of models and unobservable reality. Realists would insist that even if the comparison cannot be directly made in the latter case, it does not follow that there are not other indirect means for testing truth of claims about the unobservable, or the unobserved.[7]

Von Glasersfeld is also aware of moves that attempt to replace talk of truth by that of verisimilitude; his objections are meant to apply also to claims about the truth-likeness of our theoretical beliefs to unobservable reality (see von Glasersfeld 2000, p. 5). It follows from this that there is no room for talk of how well the idealized models advocated by Galileo or Newton fit reality, that is, how truth-like they are. For radical constructivists this applies to talk not only of the unobservable but also to talk of the observable. In this respect the doctrine is radical. At no point, either at the observable or unobservable level, is there any contact between, or correspondence with, our theories and/or models and reality, or if there is some contact or correspondence we can never know that there is (this being the point made by skeptics about our knowledge of the external world). To underline this point, in his discussion of Popper's distinction between realism and instrumentalism, von Glasersfeld tells us: 'Radical constructivism is uninhibitedly instrumentalist' (von Glasersfeld 1995, p. 22). But it is an instrumentalism that must reach right down to the very reports of observations themselves and cannot stop at them as the pivotal points at which we make contact with the world. Such contact with the world is not to be countenanced even at the level of our reports of what we see, or hear.

So what positive epistemological position does radical constructivism advocate against traditional epistemology? 'Radical constructivism ... holds that knowledge is under all circumstances constructed by individual thinkers as an adaptation to their subjective experience. ... The task is to show *that* and *how* what is called knowledge can be built up by individual knowers within the sensory and conceptual domain of individual experience and without reference to ontology' (von Glasersfeld 2000, p. 4). The last clause 'without reference to ontology' we can take to reiterate the point that we should make no assumptions about how external reality is, or whether our non-traditional constructivist conception of 'knowledge' is true of it, or even a copy of it. Reality is to be set aside as a 'something-we-do-not-know-what' that plays no role in the construction process. The only thing that plays a role in the construction is individual sensory experience – and, though their role is not clear, concepts about one's individual experience also get into the constructing act.

In the sense of radical constructivism, can everything we claim to 'know' be constructed in this way? Von Glasersfeld acknowledges that there is at least a problem about how the constructors themselves get constructed (see *ibid.*, pp. 5–6). Each constructor can, on this doctrine, only be constructed out of their own 'sensory and conceptual domain of individual experience'. We will not explore here the difficulties for the self-construction of the constructors. There are also difficulties for the construction of 'things' other than the individual constructor. Such other 'things' must include other constructors; at best they can only be the construction of each constructor. (Is there not an incoherence in the idea that each constructor constructs all the other constructors that is akin to the problems of solipsism?)

Given the importance radical constructivists attach to social processes in education and elsewhere, it should be noted that the social Other is entirely an individual construction. Not to notice this is to help oneself to items in ontology of the Other, which is independent of each individual constructor and their constructions. But this has been explicitly ruled out given the above account of 'knowledge' within radical constructivism with its ban on any reference to ontology.

Let us set these problems aside and consider items in the external world that physical science might examine, such as rock, trees, tables, pages of books, and mass, gravitational attraction, electrons, and the like. All items in the external world, the 'Other', are to be constructed out of individual experience. They are, as von Glasersfeld says, to be 'constructed by individual thinkers as an adaptation to their subjective experience'. But to most this seems nothing other than a version of extreme empiricism, or of phenomenalism. The very external objects such as rocks or books are really epistemological constructs of an individual and are not really out there at all.

But let us set these points of contention aside and consider the doctrine in relation to Galileo's methodology. It is prone to all the difficulties raised by Galileo and mentioned in Section 1 about how we might have to correct our experiences. It is also prone to Feyerabend's problem of 'natural interpretations' that infuse experience, or our reports of experience (that is, how concepts get applied to experience in making the reports). Radical empiricism also has a restricted view of what can count as the materials out of which the constructions emerge. Only individual experience is mentioned. However, as pointed out in Sections 1 and 2, in model building experience may play only a minimal, or even no, role; what plays the central role is, as Galileo put it, reason or thought. This is the missing dimension in the radical constructivist's store of materials out of which constructions, such as ideal models, are to be made.

So far radical constructivism seems not to be able to give an account of Galileo's methodological procedure; in fact it runs counter to it. Elsewhere von Glasersfeld tells us that he follows Piaget who 'saw cognition as an instrument of adaptation, as a tool for fitting ourselves into the world of our experience' (von Glasersfeld 1995, p. 14). If we think of cognition here as including the kinds of theoretical models that Galileo constructed, then they may never fit the world of experience and may run counter to it. Think of the idealizations of Section 4, such as Newton's Law of Motion which may fit no actual motion, whether we experience it or not.

If realist truth is under a heavy cloud of suspicion, with what do radical constructivists replace it? With the notion of *viability*: 'Actions, concepts, and conceptual operations are viable if they fit the purposive or descriptive contexts in which we use them. Thus, in the constructivist way of thinking, the concept of viability in the domain of experience, takes the place of the traditional philosopher's concept of Truth, that was to indicate a 'correct' representation of reality' (von Glasersfeld 1995, p. 14). Here we need to focus on the idea of viability of which von Glasersfeld says in another context, invoking Piaget's idea of equilibration:

'"viability" is tied to the concept of equilibrium. ... In the sphere of cognition, though indirectly linked to survival, equilibrium refers to a state in which an epistemic agent's cognitive structures have yielded and continue to yield expected results, without bringing to the surface conceptual conflicts or contradictions' (von Glasersfeld 1998, p. 16). An important issue here is whether the notion of viability must depend ultimately on the notion of truth and that it cannot be avoided altogether. First, the idea of a contradiction can not be understood without an appeal to the notion of truth: that two claims are contradictory just means that if one is true, then, as a matter of logical necessity, the other must be false. Here the notions of truth (and of falsity, and of logical necessity) enter into the definition and cannot be avoided.

Second, how do we tell of our 'cognitive structures' (these will include theories and models we entertain, let us suppose) that they are *viable*; that is, they '*yield expected results*' (in a contradiction-free manner). Surely we must be able to *tell*, even be able to know, that some result we expect, *E* (on the basis of our 'cognitive structures') has actually been yielded, that is, that it has turned up as a bit of reality *R*. In other words, we need to be able to tell that expectation *E* is made true by *R* (or, if one wants to use 'copy' or 'correspondence' talk, that *E* copies, or corresponds to, bit of reality *R*). Thus it appears that if the notion of viability is to be understood at all, the notion of truth that reaches out to the world, matching expectations with reality, has to invoked. We need to lift the ban that tells us to proceed 'without reference to ontology' if the notion of viability is to have any content at all. If some such 'cognitive structure'/world relationship is available then our 'cognitive structures' do not become merely castles that we build in the air; that would produce a miracle, the miracle that the very castles we build in the air manage to help us cope with a world that bears no connection at all to the airy castles of cognition in which we must, perforce, live. But that it is no miracle is something realists can explain and radical constructivists cannot.[8]

Given the large role that Piaget's views are to be given as a part of radical constructivism, perhaps there is another way in which the doctrine is to be understood other than merely a construction out of individual experience. We should also invoke Piaget's model of schema, perturbation, followed by accommodation that leads to equilibration and the rest of his theory of cognitive development. That is, we need to add these Piagetian elements to our understanding of von Glasersfeld's account of how we construct out of individual experience. Some see in this a version of the commonly invoked theory of 'trial and error' that is not too distant from Popper's theory of conjectures and refutations. Let us go along with this understanding of radical constructivism, and then make the analogy with 'trial and error'.

But does the analogy hold? It is quite unclear that anything like the kind of conscious model building, and more generally scientific method, that Galileo prescribes is actually to be taken on board as part of what happens generally in such cognitive development. Piaget's account of cognitive development is one thing and scientific method is another; there is no reason why they should be the same (despite what followers of Piaget say about the psychogenesis of science throughout its history). For Galileo the application of method is a deliberate conscious activity using a quite specific scientific theory, and understanding the model as capturing the essential, and dropping the non-essential, aspects of what is going on in real systems. (Galileo's idea that we are capturing the essence of what is going on in some real system undercuts any attempt to understand this along the lines of the instrumentalism that von Glasersfeld 'uninhibitedly' adopts).

The analogy fails in a deeper way pointed out by Popper (see Popper 1999, chapter 1, the paper originally appearing in 1972). He claims quite broadly that 'all life is problem solving', including the life of an amoeba, the life of a learning child, or the life of a Galileo or a Newton or an Einstein and the scientific problems they faced and solved. He suggests a three-part model that broadly captures all of these problem-solving activities:

(1) problem → (2) attempted solutions → (3) elimination [→ new problem, etc.]

The problems to be solved may be those to do with brute survival, including problems in which evolutionary change is required; or they might be intellectual scientific problems. Those confronted with the problems attempt various solutions. Then these various attempts are put to the test; some are eliminated because they fail as solutions while one attempted solution is taken up since it does not fail. (If an agent fails to come up with any non-failing solution then they may well cease to be, such is the magnitude of the problem facing them.)

However for Popper there is an important difference at stage (3) in the case of science. It is we who have to conduct the elimination process through the applications of methodology that lead to elimination or acceptance. Popper, of course, is an advocate for his own methodological theory of falsificationism. However we need not adopt his view; we can instead consider a range of extant theories of method. This leads to a four-part model:

(1) old problem → (2) formation of tentative theories → (3) attempts at elimination through critical discussion and testing → (4) new problems → etc.

The big difference for Popper is step (3), the element of critical, rationally based discussion of the tentatively formed theories which members of the scientific community carry out amongst themselves. If a comparison is to be made between Piaget's equilibration model for cognitive development and Popper's two models, then Piaget's is akin to the first three-stage model and more distant from the second four-stage model. But what is crucially missing is the kind of appeal to consciously used theories of method as part of the critical discussion of theories.

Various kinds of critical discussion are clearly indicated in Galileo's dialogues in which he actively proposes new concepts and tentative theories, including his theoretical models, and then subjects them to critical evaluation by comparing them with rival views, or developing less idealized models and so on. More generally this critical discussion has been vastly enhanced in the 20th century through the development of theories of statistical method and analysis that goes well beyond anything that is envisaged in Piaget's model of cognitive development. The important point that Popper makes for our purposes is that the critical discussion we carry out is consciously carried out using such principles of method; in large part it is constitutive of our idea of rationality. It is this that also takes us away from Piaget's model of cognitive development despite superficial similarities to Popper's two quite broad models indicated above.

In sum, von Glasersfeld's characterization of radical constructivism as individual construction out of experience ('without reference to ontology') is inadequate in that it must appeal at least to a notion of viability that involves truth (and so to the banned appeal to ontology). It is also inadequate on other grounds; its appeal to experience leaves out all the richness of Galilean methodology indicated in Sections 1 and 2 that puts great store on the role of reason in model building. If we add to this account of radical constructivism all the elements of Piaget's theory of cognitive development, then we still fall short of Popper's notion, expressed in the four-part schema, of subjecting our 'cognitive structures', theories, models, or whatever, to an explicit rational, critical evaluation through the application of our theories of scientific method.

As indicated at the beginning of this section, radical constructivism is not only (a) an epistemological doctrine, but it is also (b) an account of how science has grown throughout its history, and (c) an account of how students do, or ought to, learn about the world or science. We have said enough of (a). And (b) must be passed over with the bare comment that the actual history of science has proved to be recalcitrant when it comes to squeezing it into any simple account of the growth of science suggested by philosophers (such as Popper, Lakatos, Kuhn, the Bayesians, and so on), historians, or sociologists, as well as those who have picked up theories of cognitive development from psychology. This includes Piaget's psychogenetic account of the growth of the history of science. While each of these attempts to explain the growth of science does, in varying degrees, have something useful to say, they often fall far short of an adequate account of the growth of science.

But what of (c)? Enough has been said to show that learning on the basis of construction out of individual experience can be very misleading. This is evident in Section 1 where Galileo shows that our commonsense beliefs and experience are often inadequate to the task of obtaining a proper understanding of even quite simple dynamical matters such as what is really going on in the tower experiment, or when a pendulum swings. We need to expose misleading 'natural interpretations' that infuse (reports of) experience; and we may have to replace them by 'unnatural' (to the pupil) concepts which are more correct. And as was shown in Section 1 and 2, it is not enough to merely appeal to experience. Galileo made the case for an appeal to reason, since reason is the crucial ingredient in the construction of models. It is the very idea of constructing models that is at the core of Galileo's account of the scientific revolution. What is suggested above is that radical constructivism, while it employs the metaphor of construction that can have application (but not in a way that it spells out), is neither an adequate epistemology for science, nor an adequate account of its growth, nor an adequate account of how science ought to be learned. We need other theories to perform these three tasks. Galileo's dialogues suggest an alternative theory (that needs to be given more serious consideration than it has) as a way of approaching the first and the third tasks.

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Notes

[1] For a range of different kinds of model to be found in science, see Giere (1991, chapter 2.3) who distinguishes between scale models, models as analogies, theoretical models and models as maps. Achinstein (1968, chapters 7 and 8) considers these and other models. In the above the emphasis is on theoretical models only.

[2] There are several accounts of the idealized version of Newton's Law of Motion; for one account in line with the approach to idealization taken here see Nowakowa & Nowak (2000, Chapter 1, section III, especially pp. 52–53).

[3] The term 'virial' is Clausius' term, derived from *vis* for force, which has to do with the stresses due to inter-molecular attraction, repulsion and impact. These increasingly concretized equations are commonly given in physics textbooks; the source used here is Bromberg (1980, chapter 2). For a further fuller discussion of the different models and equations and their simplifying assumptions, see Morrison (2000, pp. 47–52).

[4] Much more could be said of the role of idealisation in science than has been said here. Important is the notion that our idealised fundamental laws of science are false of the actual world, a matter clearly indicated in Cartwright's (1983) book that has the provocative title *How the Laws of Physics Lie*.

[5] A fuller account can be found in Niiniluoto (1999, especially chapter 5) where diagrams akin to those in the text can be found. The treatment in this book of the rivalry between realists and their opponents is at an advanced level.

[6] That radical versions of constructivism, especially those found in science education, are committed to the view that we cannot even compare our observation and experiential reports with reality is discussed more fully in Nola (2003).

[7] Realists often use inference to the best explanation to argue for their realism about scientific theories. This cannot be discussed here; for an account, see Niiniluoto (1999 chapter 6).

[8] See the reference in the previous footnote for why it is no miracle.

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The Poet and the Pendulum

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Abstract. We begin with the pendulum and the curious authority of the expression for the period of its swing, $T = 2\pi\sqrt{l/g}$. That this is not an empirical result $-\pi$ is an irrational number -leads to an examination of the nature of physics. In the course of things, we come to Plato's critique of poetry in *The Republic* and the fundamental differences he points to between the authority of the particular and that of reason. Extending this distinction to physics, we show how the study of the pendulum illustrates Plato's project. The study of the pendulum not only prompts the question, "What is the nature of physics?" it also proves to be an excellent way for students to come to appreciate the kind of reasoning that is at the heart of physics.

1. Introduction

The period of a pendulum is given by the expression: $T = 2\pi\sqrt{l/g}$. What is the authority for this expression? It is clearly not empirical. No one has ever measured the time it takes for a pendulum bob to swing back and forth to be $T = 2\pi\sqrt{l/g}$. Nor is it the average of carefully determined data. It cannot be, because π is an irrational number. You can't get there from the data. You can't get there from experience.

This expression is an invitation to look at the nature of physics with an eye on just what kind of enterprise it is. Moreover, by following its lead, we shall find ourselves considering not only the nature of physics but also that of poetry, and we shall find that there is a special value to the pendulum in leading students to appreciate the kind of reasoning that is at the heart of physics. In the end, the pendulum proves to be a delightful pedagogical tool for getting students to engage the distinctive character of physics.

2. What is Physics About?

Curiously, it is common for physics textbooks not to discuss this question. Instead of defining the discipline, authors tend to offer a few observations about the scientific method and the importance of measurement, or perhaps they sketch the over-all parameters of the universe. (Halliday et al. 1998; Holton and Brush 1973; Caspar and Noer 1997) Then they get down to business. This is unfortunate, in that unlike other science disciplines such as astronomy, biology, or geology, there is no reason to assume that students know what physics is about.

M.R. Matthews et al. (eds.), The Pendulum, 267–276.

Physics seems to have become essentially a set of topics. For example, *Conceptual Physics*, a highly successful text, describes physics as "the most basic of all the sciences", and proceeds to explain: "It's about the nature of basic things such as motion, forces, energy, matter, heat, sound, light, and the inside of atoms" (Hewitt 1998). Hewitt offers no clear way to characterize this set, other than these are the topics traditionally dealt with in physics courses. That is, physics is as physics does.

There is nothing wrong with this. Tradition is an adequate framework for practice. And indeed, it may be that the real character of physics lies in some ineffable quality or sensibility that only emerges in the interaction with the discipline, such as an apprentice might gain at the feet of a master. Nevertheless, there is a value to raising the question – why are these topics in the same set? What binds them together? That is, what is the proper character of an introduction to physics?

The curiously non-empirical authority of the expression for the period of a pendulum suggests that unlike other sciences, the domain of physics might be defined by its analytical character rather than by a set of objects. In this vein, consider this passage from *Physics* by Halliday and Resnick (1964): "Indeed, if idealizations or approximations are not made, the vast majority of significant problems of all kinds in physics and engineering cannot be solved at all". This is an intriguing observation because, as the pendulum expression makes clear, we are not simply talking about a casual attitude toward the data. Something is going on.

So if idealizations and approximations are of a distinctive significance to physics, how should we understand them? To get a handle on this question, we may turn to the history of physics where we find an interesting episode involving Galileo. Galileo is associated with a host of remarkable discoveries from the thermometer to the telescope, as well as for his work on falling bodies and his critique of Aristotelean physics. In this context he came to the study of the trajectory of projectiles, such as cannon balls. Galileo argued that a projectile would move uniformly in the forward direction as it rises and falls subject to the force of gravity. The result would be a parabola.

It is not, and Galileo knew it was not.

The actual trajectory of a cannon ball lacks the symmetry of a parabola. It rises in an arc, but then drops rather precipitously. This is not far removed from the Aristotelean view, and scientists before Galileo had carefully analyzed this motion and given it sophisticated form. (Whewell 1837)
Galileo knew this, too.

The problem was, Galileo explained, that this work was too close to the phenomena. The actual behavior of a cannon ball was supernatural! Its natural motion, its real motion was parabolic –despite the fact that this is not what we see. This is an extraordinary claim, especially when we so often extol the virtues of careful observation in the sciences. Galileo has denied a rival theory because it fitted the data! From his perspective, the data carried more than it should, and of course, he was right – air resistance so distorts the *real* motion as to make it un-natural.

Galileo's work and his language suggest how idealization works in physics. It carries you passed the distortions of experience down to the genuine contours of underlying reality. Idealizations undo the supernatural. Michael Matthews (2000) describes a delightful example of this in an exchange between Galileo and his patron and mentor, Guidobaldo del Monte. Del Monte is stunned by Galileo's suggestion that a pendulum's swing keeps uniform time. How could it, when it slows down to a standstill? Here again, it is clear that the ideal is curiously detached from experience. And so we return to our initial problem. Galileo's notion of the real motion beneath actual experience reveals physics to be a curiously un-empirical discipline where reasoning is somehow independent of the data.

3. What Physics is about – First Go

To try to make sense of things, I propose to follow the advice of George Grote. Over a century ago, Grote asked why it was that the writings of ancient Greece had such a firm hold on the Western imagination, so that after more than 2,000 years not only were they read, but they were at the heart of a good education. The answer, he offered, was not the direct relevance of the writings themselves. Modern science had far outstripped its Greek proportions, as had modern philosophy and historical criticism. No, the real claim on the modern mind was that in the ancients every generation could find anew the discovery of reason itself, and so uncover the deepest questions and stirrings within the various disciplines (Grote, 1844). Perhaps, then, we can find the distinctive character of physics by looking at its emergence in classical antiquity.

In a fine study of the origins of science, Giorgio de Santillana (1961) makes an intriguing observation. He links the Greek word *physis* and the Latin *natura*. Both were agricultural notions referring to the push of a plant as it breaks through the soil. From this perspective then, Anaximander – by tradition the author of the first prose text, an essay on 'the physis of things' – had characterized what would become the sciences with an organic metaphor that tapped into the quality of agency, of push. We still use 'nature' in this sense when we speak of an individual's nature or character, something within that lies at the root of their behavior and makes them what they are.

De Santillana's notion is that the origins of science rest in the effort to understand the 'nature' of things, that is their internal push. We may go a long way toward capturing what physics is about by seeing it as the systematic examination of agency. Where biology is about life and geology about the earth, so physics is about the causes of things, what makes them what they are -the roots of their behavior.

Such a notion explains many of the most striking features of physics. In the first place, it explains the need for an introduction to physics. We can see why ambiguity surrounds just what physics is about. Instead of studying a particular patch of natural phenomena, it studies the 'nature' of nature itself. It also explains the distinctive status of physics as the most basic of the sciences, in that other disciplines draw upon the conceptualization of agency within physics.

It does not, however, explain the pendulum. It does not explain that distinctive quality we have seen in idealizations and approximations. For that we need to extend our discussion to Plato and his critique of poetry.

4. Second Go – Physics and Poetry

It was clear to the ancients that Thales, Anaximander, Anaximenes, Heraclitus and Parmenides had marked a profound change in the character of story telling. With the work of Plato, however, we find that reflective moment which most clearly signals the birth of a new approach to things. So let us begin by considering the *Republic*, perhaps the most widely read of Plato's dialogues. The *Republic* sets forth a utopian society founded upon the ideal form of the good and the just. In the course of things, there is a curious problem with this text. The *Republic* is divided into ten books. The tenth book features a critique of poetry. Not a simple sorting out of the good and the bad, but a principled condemnation of the whole. There would be no poetry in the republic. How is this possible? How can the culminating chapter of a lengthy treatise on the just society come down to a matter of rhyming couplets?

Remarkably, the answer to this question will prove to be the key to our problem regarding the pendulum and the nature of physics.

Plato's critique of poetry has been somewhat of a puzzle for a long time. Some commentators, presumably quite fond of poetry, have been hard-pressed to see how it could threaten a just state. But Eric Havelock in his fine study, *Preface to Plato* (1963), explains why Plato's critique is exactly right. The problem, Havelock urges, is with us. We have taken poetry to mean . . . well, poetry.

Havelock carefully establishes his case, but we can, perhaps, sketch it with a few bold lines. First we must recognize that for Plato poetry would have meant first and foremost the epics of Homer, and further more that the *Iliad* and the *Odyssey* were oral 'documents'. Though they had been written down by Plato's day, they had long been preserved from story teller to story teller by the extraordinary effort of memorizing what we now see as two and three hundred page texts.

Such a massive amount of memorizing carries with it a certain set of the mind that makes criticism very difficult. Plato lays this out in terms of *mimesis* where both audience and story teller relive the tale. In this way, a false image of events is taken for the real thing. At the same time, the act of supporting this false image cuts out that distance which is needed for examination and criticism. We cannot pause to consider when we are re-enacting.

Reflecting on this, we may observe that the systematic preservation of such lengthy 'texts' would have required extraordinary efforts. What would warrant such commitment? More than entertainment. We shade here onto the grounds of the sacred. Poetry, and Homer's in particular, was the leading institution in classical antiquity. It was the central authoritative voice within the culture, setting values and guiding judgements. The Homeric encyclopaedia, as Havelock terms it, was more than tales of epic proportion. It was the way through the thicket.

The literature of ancient Greece has continued to capture the imagination of successive generations, but we need to be careful. The colorful tales we read to our children – the exploits of Hercules or Achilles, the cruelty of Prometheus' punishment, and the cleverness of the young Theseus – these meant much more to the ancients.

The bold print of Odysseus' story is an excellent example. It is the plight of a man caught between his obligations as a Greek and those of a father, a husband, and the head of an extended household. To have read of his doubts about joining the Greek expedition in the 1960's would have underlined the continued relevance of such issues and sensibilities, visible in the torn sense of obligation that divided citizens in the United States and elsewhere over the war in Vietnam.

This is the hallmark of the Homeric encyclopaedia. Again and again throughout these epics, Homer has portrayed people caught between great forces and great principles, and everywhere they must resolve tensions and draw conclusions. What Homer had drawn would prove to be the central body of lines and proprieties that would shape ancient Greek life.

Homer informed the character of Greek life more deeply than we can readily imagine. These were not just good stories. They had a formal authority, as well as a general influence. One way to appreciate this authority is to consider an ancient court. Plaintiff and defendant would stand before their ruler/judge and make their case. But judgement was not a matter of the law. It was not a matter of ferreting out who was telling the truth and who was lying, nor was it a question of which statute applied to what degree. Rather, it was a matter of linking the issues in the case to an episode within Homer. Judgement would then unfold in a natural, that is, an Homeric fashion.

These epics were not simply romantic tales about great adventures. They provided a set of conceptual devices whereby the ancient Greek could come to an understanding of his world: What it was; How it had come to be the way it is, and What claims it made upon you. But these devices were not laid out as principles, bare and abstract like theorems of geometry or the code of a constitution; rather, they were embedded in stories, textured, rich and suggestive. To return to Odysseus, that was how loyalties to the state, to one's family, and so on were configured -not in legal code, but in the unfolding of Odysseus' decisions and the events that befell him and his family.

The brilliance of Plato lay in his ability to push to the most fundamental matters. The problem was *not* the Homeric encyclopaedia itself. It was the way decisions were being made. In this sort of decision-making, there was a misplaced authority in particular episodes. The wise ruler could 'see' the right analogy to draw between the present conflict and the Homeric corpus, but this rested justice upon particular just acts. Plato sought a new, radical alternative where just decisions would be drawn from principles of justice.

5. Physics and Reason – from Poetry to Orbits

To appreciate Plato's radical shift and with it to appreciate the pendulum and the distinctive character of physics, we may pause to consider the paradigm for Plato of non-poetic reasoning, geometry. Years ago, I was teaching geometry and the theorem before the class was – the angle bisector of the vertex angle of an isosceles triangle is also a median. When asked to prove this, a student took a meter stick to the board and found the theorem to be false. One section of the base of the triangle I had drawn was longer than the other. The angle bisector had not divided the base evenly. The student had made a categorical error. He was, of course, right; but more profoundly, he was dead wrong. There were two triangles, the one I had drawn and the one in our minds. Geometry is a discipline, a schooling that teaches us how to play upon a mental field, tracing our reasoning not in the sand but in our minds.

When Plato sought a foundation for the practice of justice in his ideal state, it was to have it approach the authority of geometry, a move reflected in the language he used. The Greek word for the form or shape of an object was 'eidos', from which we derive 'idea'. Plato's theory of forms, his theory of ideas, his view that we needed to understand the underlying idea of such notions as the good, the just, or the beautiful, was a mathematical vision. And of course, it was a politically charged vision, perfectly appropriate for the culminating chapter of the *Republic*. Plato sought to completely re-define the authority of the state, anchoring it in a new kind of reasoning.

Plato had distinguished between the muse of poetry and that of mathematics. There is a problem with poetry. It instructs through the authentic particular. It gets us to look at the world through a lens set by these particulars, and in this way, Plato argued, we are led to mistakes. It is like seeking the truths of geometry with a meter stick.

The rejection of the particular, the heart of Plato's critique of poetry, also figured prominently in one of Plato's most extraordinary contributions to natural philosophy, his radical reform of astronomy. Astronomy was as old to Plato as he is to us. In a stroke, he so deeply transformed this science that what was done before would not be done again after, and what has been done since, has been done in the spirit of his work. Plato invented the orbit.

What is an orbit? It is not a thing, at least not in an ordinary sense. We need not worry about banging into it or tripping over it. It is a path, a path traced by the mind's eye. We do not see orbits; yet they are real. They are the idea that informs the behavior of the heavenly bodies. Astronomy had erred by fixing its eye upon the patterns of the night sky. Instead, Plato argued, it should seek the underlying structures that inform these appearances. The first models of the heavens are a response to his call, explaining the behavior of the planets via combinations of circular motions.

Plato's critique of poetry was not about rhyming couplets, but was a call for a new way to see the truth, a truth that lay behind or beneath experience. It ranged from the proportions of the just state and the character of virtue to the forms that guided the heavenly bodies, and in each case the alternative to poetry was the art of geometry.

There is more here than a framework of axioms, postulates, and theorems. The art of geometry is the discipline of a mental field ruled by reason. A key issue within this field was how can one push, poke, and prod and so come to new understandings when it is not by examining particulars more carefully. This is precisely the problem of the pendulum. When we look at the formative models of Platonic instruction, that is the dialogues, we find Socratic teaching. Here notions are offered, questions asked, challenges raised, and alternative views explored. These texts do not present answers so much as a process for coming to answers. They are manuals on how to push.

6. The Pendulum, Pedagogy, and Reason

Our discussion thus far has led us to see physics as a study of agency characterized by the use of idealizations. Inspired by Grote, we sought the origins of this approach to making sense of things, and we came to Plato and, in particular, to his critique of poetry. Plato has given us a context for meaningful statements that do not directly connect with experience. That is, the approximations and idealizations that Halliday and Resnick talked about are somehow in the manner of geometry and those two triangles that are always there – the one we draw and the one traced in the mind.

We come now to a new problem. Physics is a body of knowledge. It is a set of expressions like the expression for the period of a pendulum. Indeed, there is a whole alphabet soup of expressions. But physics is also an approach to making sense of things -a process that makes critical use of idealizations and approximations. It would certainly be possible to teach physics as a body of knowledge. Indeed, I suspect it most often is taught that way. The question is how would we go about teaching physics so that it highlights the distinctive character we have been considering? The answer involves the processes of the art of geometry, and most particularly the derivation.

Take the pendulum.

Play with a pendulum pretty quickly suggests that the period of a pendulum is largely independent of the mass of the pendulum bob and the vigor of its swing, and further that it is very sensitive to differences in the length of the string. If we take a set of data on the time of swing and the length of string and ask students to find an expression, the result is intriguing. Most students I have taught have no idea what they should do and the rest treat it as a mathematical problem, seeking a function that will satisfy the data.

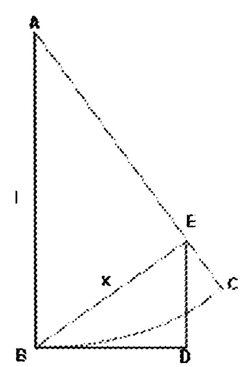
What follows when we come together to consider the task is a really important conversation. Students have been led to see the connection between mathematics and the sciences in essentially algebraic terms. That is, they map the data. In this case, every year several students will come up with delightfully ornate third degree equations relating time and length that they have drawn from their graphing calculators.

When I suggest that we might want to try a more geometric approach, they give me a strange look. But I push, saying that we should try to derive an expression in much the same way we would derive a theorem in geometry. What do you know, I ask, that might provide a framework for understanding the swing of a pendulum?

As I typically do this exercise very early in the year when their background in physics is really only kinematics, their resources here are pretty limited. Yet, delightfully, it is enough to get the ball rolling, as it were. Start, I suggest, by thinking about the back and forth motion of the pendulum bob as a ball rolling in a smooth bowl, or along an arc of track. We had already established that as a ball rolls down a ramp the acceleration is a function of the angle. In this case, however, the angle is constantly changing.

We need to simplify things.

Since my goal is to have students come to see how to derive relations from what they know, I suggest we reduce the curvature of the track to a straight line. Even here we have several options. Referring to the diagram below, the pendulum swings from C to B. We could use the chord from C to B, for example, but I prefer another hypothetical slope. The line from B to E is perpendicular to AC and is the hypotenuse of the right triangle, BDE. Note, the angle ABD is also a right angle, and thus, the angle EBD is of the same measure as the angle at the top, angle A – both of these angles are compliments of angle ABE.



A number of things now fall into place. To begin with, we can see that the acceleration down the slope E to B would be: $a = g \sin A$. Furthermore, we can express the length of BE in terms of the sine of A in triangle ABE; that is, letting l represent the length of the pendulum, AB, and x represent the line BE, we have: $x = l \sin A$. We can now use the kinematic expression: $x = v_1 t + 1/2at^2$. Since $v_1 = 0$, substituting the above values leaves the following expression: $l \sin A = 1/2g \sin At^2$. If we re-arrange this to solve for time, we get: $t^2 = 2l/g$ or $t = \sqrt{}2l/g$. As this value for t is for one trip down the slope, the period of this simplified, approximate pendulum would be $4t$ or $T = 4\sqrt{}2(\sqrt{}l/g)$.

There are one or two things worth noting about this expression. In the first place, we have addressed our initial question. We have seen how you can have an irrational number in the equation for the period of a regular motion. Plato's geometric approach, by allowing us to derive an expression rather than map the data, has yielded a curiously un-empirical result. As one student would observe late in the year in an essay on relativity: "The first thing we learned is that you don't do physics with a ruler". Furthermore, comparing this expression to the standard one, we find that the numerical constants, $4\sqrt{}2$ and 2π are not significantly different values. One is slightly under 6, the other slightly over. More importantly, the functional dependencies here are identical.

It is clear to everyone that this simplification is only an approximation of the real pendulum. The fact that the numerical constant we derive differs from the real constant is not at all surprising. Indeed, the real surprise is that our expression is so close to the right answer. This thereby suggests the value of further study. Kinematics could take us just so far. Perhaps with further study, we might find other frameworks where we could make less distorting approximations that would bring us closer to the standard result. Across the year, we do return to the pendulum, re-deriving expressions for the period of its swing. In each visit, we try to gauge the amount of distortion introduced by our simplification, and the power we find in the new approach. By the way, we leave it to the reader to find other approaches to the pendulum, including approaches that yield the standard result.

Derivations are central to physics, and we do a real disservice to our students when we do not help them to earn this more formal aspect to the discipline. The sort of reasoning used in this exercise with the pendulum helps students to see a new aspect of the mathematical face of physics. In addition to the basic enterprise of solving problems and practicing various algorithms where the relevant expressions are given, here students were to determine the expression. As they engaged this problem, they came to see the fundamental differences between finding an expression that maps the data, basically an algebraic process, and starting from established principles to derive an expression. This more geometric sort of mathematical reasoning is, we believe, the key to seeing what physics is really about. Derivations are the opposite of poetry. They are played out in a mental field quite removed from the particular. As for the pendulum, it is a lovely challenge that enables students to see a host of nuances in this enterprise and so gain a real feel for what physics is about.

And there is more. The Socratic method, as Plato would have us extend it to physics, shifts the politics of discourse in the classroom. Rather than seeing the flow through the material as a matter of preparing students so that they may receive the right answer – defining terms, practicing algorithms, and verifying these relations in various activities and lab exercises -Plato points us in a different direction. Progress is earned through a series of problems and puzzles. Students are everywhere encouraged to engage these puzzles and draw upon what they know in working out how to solve them. And the solutions, by involving derivations, take them back to what they know. The flow becomes more essentially a matter of making sense of things, and that seems to me to be right where it ought to be.

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Methodology and Politics: A Proposal to Teach the Structuring Ideas of the Philosophy of Science through the Pendulum

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Abstract. This article refers to a framework to teach the philosophy of science to prospective and in-service science teachers. This framework includes two components: a list of the main schools of twentieth-century philosophy of science (called *stages*) and a list of their main theoretical ideas (called *strands*). In this paper, I show that two of these strands, labelled intervention/method and context/values, can be taught to science teachers using some of the instructional activities sketched in Michael Matthews's *Time for Science Education*. I first explain the meaning of the two selected strands. Then I show how the pendulum can be used as a powerful organiser to address specific issues within the nature of science, as suggested by Matthews.

Introduction

The importance of the philosophy, history and sociology of science (collectively referred to as the *nature of science*, from now on) in the science curriculum and in science teacher education has been acknowledged all over the world by educational researchers and policy makers (AAAS 1989; Matthews 1994; Driver et al. 1996; NRC 1996; Millar and Osborne 1998). Accordingly, a number of important rationales and practical proposals have been put forward in the last twenty years or so with the aim of teaching elements of the nature of science to different populations (Duschl 1990; Matthews 1991, 2000; Jiménez Aleixandre 1996; McComas 1998).

The available theoretical developments in this research line (known by its acronym NOS) generally point to the need for an identification of the epistemological foundations of science teaching. Authors usually focus on establishing connections between well-known philosophical views (e.g., rationalism, hypothetico-deductivism, revolutionism, constructivism) and models of teaching (Nussbaum 1983; Cleminson 1990; Mellado and Carracedo 1993; Izquierdo and Adúriz-Bravo 2003). As an instance of this procedure, following a critical review during the 1990s of the extensive use of constructivism in science education, some authors have highlighted the urgent need to recover *temperate* versions of realism and

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rationalism,[1] which are more compatible than relativist philosophies with the aims of a liberal science education for all (Matthews 1994, 1997; Giere 1999; Good and Shymansky 2001; Cobern and Loving 2003). I think with these authors that currently available perspectives on realism and rationalism are modern and accurate views of NOS that have enormous value to achieve a science education of quality.

Instructional proposals typically select topics from the nature of science on whose relevance for science education there is reasonable consensus amongst researchers, for instance: scientific method, theory change, realism, scientific explanation. The proposals infuse such topics into classroom activities using various strategies (for a broad range of strategies, see McComas 1998). For instance, a number of authors have turned to Giere's (1988) decisional model of scientific judgement in order to design nature-of-science activities for prospective and in-service science teachers (Duschl 1990; Jiménez Aleixandre 1996; Izquierdo 2000). These activities select and discuss famous episodes from the history of science.

In spite of the impressive base of proposals that has been thus generated, a weakness in the connection between theory and practice can be detected in many cases. Some instructional units employ content from the nature of science that can be considered outdated, or they combine contents from incompatible schools of thought. Another problem is a lack of reflection about the specific role of the nature of science in science teachers' professional induction.⋆ I have been developing, in previous publications (Adúriz-Bravo 2001b, c, 2002b; Adúriz-Bravo and Izquierdo, 2001; Adúriz-Bravo et al. 2001, 2002), some ideas that seek to provide criteria for a more theoretically founded *selection* of nature-of-science content. Such criteria should permit the adaptation of existing instructional procedures and also the development of new ones. Therefore, they could prove to be a powerful instructional tool both for science teachers and teacher educators.

This paper exemplifies the teaching of a few key elements from the nature of science selected through the use of this theoretical framework. The elements, bearing on scientific *method* and *values*, are infused into instructional activities by means of two historical episodes around the use of the pendulum, examined by Michael Matthews in his book *Time for Science Education* (Matthews 2000). Our selection of these two specific topics is supported by their appearance in many materials designed for NOS education of science teachers, but will be further justified below.

The first section of this paper identifies some available ideas to account for the need for a science teacher education that includes the nature of science as a central component. The second section makes a brief reference to my theoretical framework that could guide the inclusion of that component. I focus on one particular construct, the *strands* (i.e., structuring ideas) of the philosophy of science. The third section provides two examples of how episodes extracted from

⋆ Readers can refer to the proceedings of the very successful IHPST international conferences (e.g., Pavia 1999; Denver 2001; Winnipeg 2003) to spot instructional activities where the NOS topics to be taught are taken for granted without discussion, and where no specific theoretical basis for the *pedagogical* design is provided.

Matthews's work on the pendulum may be used for teaching the strands. I mainly profit from the 'Huygens section' of his book (Matthews 2000), later published independently (Matthews 2001). Finally, there is a short section that states some conclusive comments and future perspectives.

This paper is written with the intention of being a plausible example of the claim that theoretical reflection on the use of the nature of science in science teacher education permits both the *adaptation* of existing procedures (*evaluative* function) and the *creation* of new ones (*heuristic* function). In the instructional activities that I present, I take some available suggestions and further develop them along new directions. It is my hope that the ideas presented here can be of use and inspiration to other science teacher educators.

The Nature of Science in Science Teacher Education

A rapid review of the theoretical scenario related to science teacher education in the nature of science suggests that the available positions cover a broad spectrum. There is a small group of researchers who strongly object to an abusive use of the nature of science in compulsory education and therefore assume that this component has a restricted value when preparing science teachers. These researchers usually denounce the difficulties associated with the history of science in particular, stating that a heavily distorted 'pseudo-history' is often present in the secondary science curriculum (Brush 1974; Lombardi 1997; Fried 2001). Other authors contend that elements of the nature of science should be *implicit* in science education; there would be no need to contemplate specific instruction in the philosophy of science when designing the curriculum.

Among those frankly in favour of teaching the nature of science to science teachers, two main positions can be identified. One group concentrates on the intrinsic value that the nature of science has for the education of citizens; these authors resort to what Rosalind Driver and her colleagues label *democratic* and *cultural* arguments:

> An understanding of the nature of science is necessary if people are to make sense of socioscientific issues and participate in the decision-making process. (...) [It] is necessary in order to appreciate science as a major element of contemporary culture. (Driver et al. 1996, pp. 18–19)

Therefore, these authors argue that the nature of science needs to be introduced in science teacher education mainly because teachers are going to teach it in the classroom. I call this a *curriculum* perspective; it is well represented in the works of Hodson (1988) and Matthews (1994).

The other group looks rather at the participation of the nature of science in science teachers' professional development (Duschl 1990; Izquierdo 2000; Seroglou and Koumaras 2001), to a certain extent independently of curriculum considerations. The nature of science is assumed to represent a second-order reflection on the content and methods of science that positively contributes to teachers' autonomy in the task of *didactical transposition* (i.e., the decision-making when transforming scientists' science into school science). I call this a *meta-theoretical* perspective; my own proposals are in tune with this second perspective.

Along this latter line, one of the main points on which there is consensus is the idea of *functionality*. By this I mean a strong requirement that science teacher education in the nature of science must act as a *tangible* contribution to their own professional practice. That is, theoretical reflection on science is valuable in that it provides criteria and tools for science teachers to act in their classrooms (McComas 1998). Following this requirement, several constructs and activities have been diffused for public discussion.

I would like to argue that, although the enormous value of some of these available activities cannot be denied, some general directions are still lacking. According to many authors (Abimbola 1983; Gil-Pérez 1993; Izquierdo 2000; Leach 2001), science teacher education would require an explicit selection of some particular families of nature-of-science models. Choosing some models and rejecting others would ensure a *convergent* participation of this meta-theoretical component in teachers' thinking and practice. In my case, preference goes mainly towards the *cognitive model of science* (Giere 1988, 1999) and its counterpart in science education research (Izquierdo and Adúriz-Bravo 2003). I strongly adhere to Matthews's call for a rationalist and realist science curriculum; this conviction 'restricts' the diversity of epistemological and philosophical models at which I am looking when working with science teachers. Giere's account of the nature of science, and the ideas of the rest of authors that I claim it is worth examining with science teachers, fulfil this initial requirement.

In an attempt to achieve some degree of usefulness (or functionality, as defined above), I suggest that science teacher educators need an encompassing comprehension of the ideas on the nature of science that have been produced in academia, at least during last century. The constructs that I have developed are for the purpose of providing a *chart* of the available content and some criteria to prioritise and sequence it. The next section is devoted to one particular construct, which acts as a content *organiser* and inspires my pragmatic selection of two contributions by Matthews.

The 'Structuring Theoretical Fields' of the Philosophy of Science

My framework for teaching the nature of science to science teachers contains a number of elements that have been exposed in the previous publications mentioned above; in this sense, it is not my intention to repeat here material that readers have access to elsewhere. The core of the framework is related to a carefully guided *selection* of the content from the nature of science that can be taught to teachers once the role of this component in science teacher education is established and clarified. Selection is done by reviewing and articulating two key elements of twentieth-century philosophy of science – its major schools of thought and its main theoretical concepts.

I have proposed a coarse division of academic philosophy of science in three overlapping periods, which I call *stages*:

1. *Logical positivism and received view* (roughly covering from 1920 to 1965). This first stage sustains a strict, almost naïve, rationalist and realist reconstruction of science both as product and process. An initial division between the contexts of discovery and justification is respected. Formal logic and linguistics are extensively used. This stage is paradigmatically represented by the classical works of Carl Hempel (1966).
2. *Critical rationalism and the new philosophy of science* (approximately going from 1935 to 1980). This second stage represents a serious questioning of philosophical orthodoxy. Thomas Kuhn, Imre Lakatos and Stephen Toulmin are representatives of the 'irruption' of the history of science in the philosophy of science (Estany 1993); they defend the idea that a narrow internalism is theoretically insufficient.
3. *Postmodernism and contemporary accounts* (starting around 1970). Relevant authors representing postmodernism would be Larry Laudan and Paul Feyerabend, while Fred Suppe, Ulises Moulines and Ronald Giere, among a host of others, have produced what I rather broadly label 'contemporary' philosophical accounts. As I portray it, current philosophy of science comprises derivations and syntheses of both previous stages.

In parallel with this view of stages, I have put forward an abstract organisation of the stock of ideas on the nature of science, which I call the *structuring theoretical fields of the philosophy of science*, or more briefly *strands*. My design of this construct stems from a review of the literature in science education research that proposes a science curriculum development based on a few powerful pillar concepts, called 'structuring concepts' (Sanmartí and Izquierdo 1997). Accordingly, a 'structuring field' would be a set of interrelated concepts that give identity to a discipline. Some structuring concepts of physics would be 'energy' and 'interaction', while examples of structuring fields in the same discipline would be 'motion' or 'waves'.

I have been able to identify seven strands in twentieth-century philosophy of science, which roughly cover all the major theoretical concerns of this discipline produced by different schools of thought. Strands are labelled as follows:

1. *Correspondence and rationality.* This strand comprises two complementary aspects of the nature of scientific knowledge: the way in which it is believed that knowledge fits reality, and the criteria that scientists use in order to assess this fit.[2]
2. *Representation and languages.* This strand concerns the different structural units that philosophers of science have produced in order to account for the process of representation of the natural world (i.e., theories, models, laws, paradigms, ...). Abstract scientific entities are characterised by means of specialised language that is object of philosophical study.

3. *Intervention and method.* Approaches to the nature of science usually examine methodological matters pre-supposing various degrees of relationship between science and reality. 'Scientific method' is a construct that has generated strong debate among philosophers of science.
4. *Contexts and values*. This strand focuses on the relationships between science and the technological, socio-cultural and educational contexts, which are all characterised by their own aims and values.
5. *Evolution and judgement.* All models on the nature of science have included a diachronic component that provides assumptions on how science advances (Estany 1990).
6. *Demarcation and structure.* A philosophical issue as old as meta-theoretical reflection is that of distinguishing, or *demarcating*, between science and non-scientific intellectual enterprises.
7. *Normativity and recursion.* This last strand refers to the unique nature of the philosophy of science as a meta-scientific discipline, i.e., an academic discipline reflecting on science as a discourse and as an activity. Philosophers usually range from normative positions, in which *a priori* or absolutist parameters are sought, to a strong relativism.

Stages and strands permit us to *map* different theoretical models on the nature of science and to a certain extent assess their pertinence in science teacher education. In this sense, these constructs permit clearer options when selecting the nature-of-science elements to be taught. Both constructs work together in what I have called the *matrix of stages and strands* (Adúriz-Bravo and Izquierdo 2001). As they are 'orthogonal', they can be combined in a diachronic representation of the philosophy of science. The matrix thus obtained maps ideas, schools and authors placing them in a particular stage in time and in a particular thematic space (i.e., one or more strands).

My use of the matrix with science teachers can be exemplified with the topic of *scientific explanation*, which many NOS instructional proposals seek to teach. Figure 1 shows how we can trace three 'models of explanation', each one corresponding to a stage. Scientific explanation itself combines at least four strands, since it coalesces logical, linguistic, representational and methodological considerations and, at the same time, has been a nodal point in the task of demarcation.

This apparatus has allowed me to identify *scientific method* and *scientific values* as two interesting ideas for this paper (other key nature-of-science ideas are inspected in Adúriz-Bravo 2001c). I will show in the next section how I have designed specific instructional activities to teach these two ideas to prospective science teachers.

I follow earlier suggestions that contents of the nature of science can be successfully taught using central historical episodes as *case studies* (Irwin 2000; Matthews 2001). My source of materials from the history of science is Michael Matthews's extensive work on the role of the pendulum in Western culture.

Strands ↓	Stages →	Logical positivism & received view	Critical rationalism & new philosophy of science	Postmodernism & contemporary accounts
Correspondence & rationality		Explanation as a reasoning pattern (deductive logic)	Explanation as a text (pragmatics, rhetorics)	Explanation as modelling (analogies, abductive logic)
Representation & languages				
Intervention & method				
Demarcation & structure		Hempel's covering-law model	Achinstein's illocutionary model	Giere's model-based view

Figure 1. The 'matrix of stages and strands' permits tracing the evolution of meta-theoretical ideas on scientific explanation.

Practical Proposals Using Matthews's Work on the Pendulum

This section exemplifies two of the numerous and diverse instructional activities that can be constructed using material extracted from Matthews's (2000) book. The first activity is concerned with the use of formal logic in order to characterise algorithmic aspects of the scientific method, generally referred to as *scientific judgement*. A hypothetico-deductive model that regards theory testing as a *falsification* process is examined. The second activity examines the influence of contextual matters in science. Scientific progress is seen as a series of informed choices that take place within a scientific community holding a worldview and a set of values.

Rationales for including these two particular nature-of-science topics – method and context – in science teacher education have been extensively provided in NOS literature. It is easy to see how the methodological aspects of science matter in the international setting of new curricula that require students to answer, besides the usual scientific question 'what do we know?', the *epistemological* question, 'how have we come to know it?' (Duschl 1990; Osborne 1996). Scientific context and values have also been spotted as an important issue in science education; in this sense, the influence of Kuhn's inspiring ideas is enormous within our research community.

The activities that I present resort to well-known episodes spanning from seventeenth- to nineteenth-century history of science. These episodes are related to the European voyages of discovery and to the search of international standards for weights and measures. Both episodes can be connected to the life and works of the Dutch scientist Christiaan Huygens (1629–1695), who was one of the most active defenders of the use of the 'seconds pendulum' (i.e., a simple pendulum that swings at second intervals) as a universal standard of length.

I think that the Huygens section of *Time* (Matthews 2000, chapter 6, pp. 141–150) provides advantageous opportunities to address two of my strands: intervention/method and context/values. For the first strand, I select the key concept of scientific method and choose to teach it by analysing the advantages and limitations of a strictly logical account. To treat the second strand, the overall validity of *externalism* as a theoretical perspective on the nature of science is examined.

In relation to the scientific method, philosophers of science in the first half of last century favoured heavily rational reconstructions. Formal logic was extensively used within the so-called context of justification. Several versions of the method were put forward: an Aristotle-inspired inductive-deductive scheme, classical rationalist approaches following Newton and others, Popper's hypothetico-deductive falsificationism, and so on. The second half of last century saw the emergence of a more flexible view, acknowledging the existence of methodological diversity and identifying *modelling via abduction* as a key element. I suggest that Richer's voyage to Cayenne, as reported by Matthews, is an appropriate 'staging' to learn the distinction between the successive views on theory testing.

Regarding contextual factors, I adhere to a moderate externalism that is available in what I have called the third stage of the philosophy of science. The first stage completely disregarded the interference of social, cultural, economic and religious factors in scientific change. Authors from the second stage introduced as a great novelty a radical denial of this position (Estany 1990, 1993). But it can be argued that a *synthesis* of these two positions is more adequate for science teachers. The establishment of a metre that was accepted world-wide, as it is reconstructed by Matthews, is an excellent opportunity to reflect on these nature-of-science issues.

TEACHING THE STRAND OF INTERVENTION AND METHOD

Matthews briefly describes Jean Richer's trip to the French Guyana (South America), commissioned by the *Académie Royale des Sciences* in 1672–1673. One of the purposes of this trip was to confirm the invariability of the pendulum period with latitude (although the presence of a centrifugal effect due to Earth rotation had been predicted following Newton's framework, it was thought that this effect would prove too small to be detected). This assumed invariability came from the acceptance of the postulates of classical mechanics and the additional requirement of a perfectly spherical Earth.

Richer's negative results – he found that the seconds pendulum was shorter near the Equator – made it apparent that a revision of the apparatus underlying the study of pendulum motion and its use in time-keeping was needed. But several ways of doing this revision were suggested, attacking more or less deeply the theoretical core.

Under the label 'methodological matters' (p. 146 in the book), Matthews briefly exposes a standard process of falsification via a *modus tollens*, that is, a way of rejecting theories by means of a logical inference of a strict *deductive* nature. This scheme is best known through the work of Sir Karl Popper (1959). A theory T gives place through deduction to some predictions O that are 'observable'. According to classical formal logic, if these predictions are falsified (i.e., proved to be incorrect:

$\sim O$) when contrasted against experimental evidence, T needs to be rejected also ($\sim T$). The scheme would then be:

$$\begin{array}{l} T \rightarrow O \\ \sim O \\ \hline \sim T \end{array}$$

But this too direct account proves to be inapplicable in the Richer controversy. If the pendulum effectively changes its period, the source of the prediction O about invariability (that is, Newtonian mechanics T) is undermined. This overtly contradicts what happened in history.

A more elaborate version of the falsification process is obtained by means of the inclusion of a *ceteris paribus* clause C. This clause is implicitly attached to the deductive pattern and represents the hypothesis that 'other things are equal' when moving from theoretical predictions to experimental results. In the example with which we are dealing, C represents, among other things, the assumption of a spherical Earth. Within this new scheme, the premises of deduction include a *conjunction* between theory T and the clause C:

$$\begin{array}{l} (T.C) \rightarrow O \\ \sim O \\ \hline \sim T \text{v} \neq \sim C \end{array}$$

The conclusion of this reasoning represents the choice between rejecting theory *or* C. In the Richer example, this latter option is of course more sensible and the oblate form of the Earth is eventually accepted.

Up till here, I have more or less described the activity suggested by Matthews, with my additional proviso that it needs to be adjusted and specified for the population of secondary science teachers (Matthews does not mention the target of his proposal in this section of the book). But there are still more elements that can be added to deepen this proposal inside the corresponding strand and introduce more recent accounts on the nature of the scientific method.

My own contribution to this proposal consists in going further into the use of patterns of logical inference. I suggest using compact representations of three different forms of inference: *deduction*, *induction* and *abduction*. I follow Charles Sanders Peirce's canonical presentation of deductive, inductive and abductive argumentation patterns as permutations of the same three statements, alternatively functioning as premises and conclusions (Samaja 1994). It can be argued that a more complex account of the scientific method rises from the use of a refined version of what was traditionally named the *fallacy of the affirmation of the consequent*:

$T \rightarrow O$
O
. . . .
T

From a strictly classical viewpoint, this form of method is flawed (and thus the dotted line represents a fallacious inference). Observations that confirm the prediction O do not add to the *truth* of T. But an abductive framework focussing on the ample analogical relationships between evidence and theoretical models avoids this difficulty and seems to provide a plausible reconstruction of scientists' cognitive and social functioning (Giere 1988; Samaja 1994).

One of the instructional activities that I have designed to discuss these ideas with science teachers uses Agatha Christie's *Death on the Nile* in book and film format (Adúriz-Bravo 2001a, 2002a). The detective story works as an analogue for scientific research, respecting its three key elements: problem, solution, and inferential connection between them. There is explicit comparison of two approaches to explanation (return to Figure 1), namely the one purported by philosophers of the received view and the one favoured by contemporary philosophers of science. The deductive-nomological model is mapped to Agatha Christie's construction of the plot: she *deduces* the clues knowing the murderer beforehand. The abductive-analogical model would correspond to detective Hercule Poirot's reconstruction of the crime: he *abduces* the identity of the murderer only knowing the clues.

Historical episodes – in connection with the evolution of atomic models – are provided to stage the rather abstract reflections generated during the activity. More concretely, the transition between Thomson's 'pudding model' and Rutherford's 'planetary model' for the atom is reconstructed as an abductive process. Working on Geiger's and Mardsen's well-known experiments of the gold-foil lamina, student teachers reconstruct Rutherford's inference. A mechanical analogue ('If small balls are thrown against a grid of knots and empty spaces, the balls can go through or bounce at variable angles') inspires the major premise:

If alpha particles are projected against a gold lamina constituted of small atomic nuclei and big empty spaces, the particles can go through or bounce at variable angles.
Some particles go through and some others (very few) bounce at variable angles.

The lamina is constituted of small atomic nuclei and big empty spaces.

TEACHING THE STRAND OF CONTEXT AND VALUES

I use Matthews's proposal once again in order to illustrate what is an *externalist* approach to the study of the nature of science, that is, one taking into account variables other than the internal logic of scientific knowledge. A strictly classical view on the processes of scientific change would minimise the relevance of socio-cultural forces in scientists' choices. On the other hand, a relativist – i.e., a 'second stage' – approach is excessively externally-driven and blurs epistemic considerations. I suggest turning to a synthetic view that attends to contexts and values and provides a more accurate picture of how scientific change takes place (Estany 1990).

Under the label 'political matters' (p. 147 in the book), Matthews narrates the debate concerning the French post-revolutionary decision for the establishment of an international standard of length. Huygens had proposed one hundred years before that the seconds pendulum could be used as a cheap and portable universal measure; a recovery of such proposal was considered and rejected by the *Commission des Poids et Mesures*. Instead, the committee approved and financed a determination of the length standard by means of a *geodetic* procedure – measuring the span of a ten-millionth part of the quadrant of arc of an Earth meridian. The length of the selected meridian, notably that going through Paris, was measured by Delambre and Méchain between 1792 and 1799, and after these measures the (in)famous brass 'metre' was cast.

This apparently irrational choice – the more expensive and time-consuming method is favoured because of somewhat obscure political matters – suggests that externalism is a sound theoretical idea for understanding the nature of science. But the same example can show that some amount of internalism needs to be conserved for a more accurate reconstruction of the episode, since

> Huygens' seconds pendulum length standard did survive its rejection by the academy. After years of patient measurement of the meridian sector, and the expenditure of a great deal of state money, the academy chose a fraction of the meridian distance that coincided with Huygens "three horological feet", and accepted the seconds pendulum as a secondary reference for its new length standard. (Matthews 2000, pp. 149–150)

The epistemic values of economy and simplicity are present in this survival, as well as the connections of Huygens's proposal to established physical theories. And Matthews suggests another internal element that can be usefully analysed by science teachers. The committee's rejection of Huygens's idea was partly founded on the argument that time and length considerations should not be mixed when establishing consensual space standards. Some two hundred years later, the shift to a length standard dependent on the *speed* of light would therefore represent an important epistemic breakpoint.

This proposal as stated by its original author can be expanded with the addition of more theoretical elements (theory load, incommensurability, conceptual change) and the translation to new historical contexts. Along the first line, I suggest the use of Giere's (1988) decisional model, which tries to strike an adequate balance between cognitive and social components in the scientific enterprise. On the other hand, in order to historically re-contextualise the discussion, the dispute between phlogiston- and oxygen-defenders in eighteenth-century chemistry provides the opportunity for a rich case study (Izquierdo 2000; Adúriz-Bravo 2001c).

Final Remarks

In the last twenty years or so, a vast number of practical proposals have become available world-wide to teach elements of the nature of science to prospective and in-service science teachers. Although these proposals are very valuable, most of them suffer from an absence of theoretical support. Marilar Jiménez Aleixandre (1996), for instance, has pointed to the fact that many NOS activities for teachers make use *exclusively* of the 'new philosophy of science' (i.e., philosophical developments from the 1960s), referring to this school as 'recent' or 'contemporary' NOS.

The strands are my proposal – still being refined and tested – to oppose this tendency and construct plausible examples for teacher education that give to science teachers professional autonomy in the field of the nature of science. The strands permit identifying very basic, *structuring*, ideas that should not be omitted in science teacher education.

One crucial question that remains unanswered in this paper is whether pre-service science teachers can benefit from my approach to NOS instruction.[3] I have not conducted so far any specific investigations to support an affirmative answer. As anecdotal data, I can comment that the application of these two activities on ten separate instances in three different countries suggests that the matrix of stages and strands is a powerful device to help teachers 'navigate' the philosophy of science, which to them represents a very complex and unknown discipline.

In relation to Michael Matthews's specific contribution centred around the pendulum, I am convinced that *Time for Science Education* contains an enormous number of ideas, suggestions and materials that deserve being further explored. One possible exploration can be done by means of the theoretical framework to which I have briefly referred in this paper. The two practical examples developed here are intended to constitute a proposal together with an opportunity for further developments.

Some slight imprecisions (cf. de Castro Moreira 2001) in the historical treatment of the Richer and metre episodes do not hinder their use in instructional sequences. I regard these two specific incidents suggested by Matthews as clear examples of his claim of the profound contribution which a study of pendulum motion can make to the science curriculum (Matthews 2000, p. 14).

Notes

[1] By 'temperate' realism and rationalism I refer to recent critical re-formulations of these long-standing philosophical positions; for instance, *perspectival realism* as depicted by Ronald Giere (1988, 1999) and *moderate rationalism* as proposed by Norwood Hanson (1958).

[2] Giere (1988) considers the ideas of *representation* and *judgement*, which closely correspond to this first strand, a major tool for discriminating between different philosophies of science.
[3] I am grateful to an anonymous reviewer of the paper who suggested the inclusion of this remark in the final version.

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Degree of Influence on Perception of Belief and Social Setting: Its Relevance to Understanding Pendulum Motion

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Abstract. Modern visualization techniques in science education present a challenge of sorting out the contributions of perception to understanding science. These contributions range over degrees to which perception is influenced by belief (including systematic sets of beliefs which comprise scientific theories) and social setting. This paper proposes a (first-approximation) categorization of these perceptions. A perception is categorized according to the degree of influence on it from belief and social setting. The contributions of perception to understanding scientific phenomena are drawn from the history of the discovery of the secrets of pendulum motion.

1. Introduction

Even though visualization is important to science education and new visualization techniques continue to be introduced, few philosophers who address issues in school education attempt to sort out the contributions of perception to understanding scientific phenomena. Instead, philosophical discussion of perception's role in understanding scientific phenomena tends to focus on the claim that social settings and beliefs, including adherence to scientific theories, and other psychological attitudes (for example, desires and expectations) decisively influence what one perceives. One such claim is that of theory-ladenness of observation. Discussion also focusses on a range of related claims, including some claims with an anti-empiricism thrust.[1] Since the issues raised in these discussions have a direct bearing on how we acquire scientific knowledge, these discussions directly pertain to science education. However, from the perspective of how we come to understand scientific phenomena, something vital seems to have gone missing from the philosophical discussion. There is little appreciation of the various degrees in which belief and social setting influence perception. All perception tends to get painted with the same brush.

With a view to helping to redress the lack of attention paid to the degree of influence on perception of belief and social setting, I develop in this paper a

M.R. Matthews et al. (eds.), The Pendulum, 293–302.

first-approximation categorization (scheme of classification) based on the degree of influence. (This is a categorization of *visual* perception. Except for a brief mention of auditory perception, to which the categorization also seems to apply, no other mode of perception is considered.) For this purpose, from the outset I assume that perception displays degrees of influence from belief and social setting and that some perceptions are relatively free from this influence, while others are not.[2]

The discussion and development of the categorization runs as follows. I first propose a categorization with respect to the influence of belief and social setting. I then provide examples of the categories in the overall categorization, taken from how we come to understand pendulum motion. Matthews's historical study of the discovery of the secrets of pendulum motion acts as the main reference.

2. A Categorization of Perceptions

A natural place to start a categorization of perception according to the degree of influence of belief and social setting is at an extreme in which this influence is minimal, even in a perceptual learning stage. This extreme point I will call *primary* perception. A primary perception occurs when I see a big chunk of something or other, I do not know quite what, straight ahead as I am skiing. In the hospital, later, I am told it was a tree. Perceiving colour is another example of primary perception (naming colour is another matter). Belief and social setting can influence primary perception, for example, in the context of degraded stimuli or of a malfunction in the visual system.[3] However, in general, belief and social setting have little influence on primary perceptions, largely because primary perceptions are either innate or do not involve perceptual learning which takes place at the conscious level.

While beliefs and social setting have little influence on primary perception, their influence is much more pronounced on what I call *secondary* perception. Their influence occurs during the learning stage of a secondary perception.[4] Inference is involved in this learning stage. For example, I have a socially conditioned belief that a flittering pattern of flight, yellow colouration, and small size are properties sufficient to identify a goldfinch and, hence, I consciously infer the presence of a goldfinch when I perceive a conjunction of these properties. After sufficient time for perceptual learning, I perceive a goldfinch without conscious inference when I perceive this conjunction of properties. Secondary perceptions always encompass primary perceptions. In having a secondary perception of a goldfinch based on its flight pattern, colouration, and size, I have primary perceptions of these things. Although secondary perceptions have a derivative character in contrast to primary perceptions, secondary perceptions do not have a different 'feel' from primary perceptions.

That is, in both secondary and primary perceptions, we get the sense of immediate cognitive contact with an object and its properties and relations.[5]

Another fundamental distinction between primary and secondary perception concerns the extent to which perceptions can be intentionally revised. Generally, we cannot intentionally revise primary perceptions, even if we know that what we perceive is an illusion. In the Müeller–Lyer illusion, we perceive unequal lengths of lines, even though we know that this perception is mistaken. Secondary perceptions are intentionally revisable. Someone might tell me that goldfinches are not the only bird in my neighbourhood with a flittering flight pattern, yellow colouration, and small size, after which I intentionally unlearn the secondary perception of a goldfinch based on the conjunction of the three properties.

While secondary perception contrasts with primary perception with respect to intentional revisability, the two compare in another respect. Both occur without conscious inference (although conscious inference can occur in the learning stage of secondary perception). In this respect, they both contrast with *quasi-perception* (my term) which consists of a primary or secondary perception plus a quick, conscious inference. (Quasi-perception could be a later stage in learning a secondary perception.)

So we have a two-part (first-approximation) categorization based on degree of influence from belief and social setting. Belief and social setting influence primary perception very little. They influence secondary perception much more. There are plenty of ways in which one might consider refining the categorization. One might consider including, for example, quasi-perception (perhaps renamed) in the categorization. Additionally, one might consider differentiating among beliefs and social settings which influence perception. There is an evident difference, for example, between a scientific theory (a refined set of beliefs) which influences perception and tacit, non-verbal social practice which influences perception. Probably many refinements are possible. However, the two-part division given by primary and secondary perception seems to be fundamental and fundamental in the right way, for it marks a difference in the domain of immediate perceptual awareness. While refinements are possible, the two-part categorization concerning the influence of belief and social setting seems to be a good, maybe the best, starting point.[6]

Conditions for non-veridical primary perception contrast with those for non-veridical secondary perception. The ways primary perception can go wrong do not include unsound inference, but include degraded stimuli, hallucinations, and many others. (A long list could be composed.) In contrast, unsound inference in a learning stage can cause incorrect secondary perception. The unsound inference could result from untrue premises, or invalid application of principles of inference. The untrue premises for inference stem from false belief, which can be socially induced. I might, for example, based on the information provided by my friends, incorrectly infer the presence of a chickadee when I perceive a conjunction of a flittering flight pattern, yellow colouration, and small size. This inference could lead to incorrect secondary perception. Additionally, secondary perceptions can go wrong due to incorrectness of component primary perceptions.

Furthermore, conditions for veridical primary perception contrast with those for veridical secondary perception. A normally functioning visual system and clear stimuli generally make for veridical primary perception. True belief and a grasp of valid inference generally are also needed to make for veridical secondary perception.

We have seen that the two categories based on the influence of belief and social setting correspond to division of perceptions based on other factors. In terms of inclusion of one type of perception within others, secondary perceptions include primary perceptions; and primary perceptions include no other perceptions. In terms of intentional revisability of perception, it is difficult to shake primary perceptions, unlike secondary perceptions. In terms of learning, primary perceptions are generally innate or not learned at the conscious level, while a significant component of the learning of secondary perceptions involves conscious inference. In terms of error, inference is not involved in error in primary perception, while problems both in incorrect application of the principles of inference and in holding false beliefs which enter inference can arise in the learning process leading to secondary perception. (Patterns pertaining to lack of error in perception reflect those pertaining to its presence.) Thus, although the categorization is based on the degree of influence of belief and social setting, it corresponds to a division of perceptions according to other factors – inclusion, intentional revisability, and so on. The correspondences suggest that the categorization cuts the subject matter at its main joint.

Examples of the two categories are found in how science came to understand pendulum motion. These examples pertain to perception of ordinary pendulum motion, perception of experimental measurement in pendulum research, and diagrams used to investigation pendulum motion.

3. Primary Perception of Pendulum Motion

We begin with an example of primary perception of pendulum motion. A pendulum, once it is set in motion, sways back and forth before stopping (unless propelled by a driving force). A great insight due to Galileo and others holds that the motion is due (in Newtonian terms) to a composite of forces, including the natural force of oscillation, drag or friction and other perturbing factors, and a driving force. Primary perception does not reveal these refined features of this swaying, but it does reveal the brute fact of swaying. (Thus, a key function of primary perception is to bring potentially scientific phenomena to our attention.)

As long as the amplitude of the swaying remains large, belief and social setting will have little influence. However, once the amplitude becomes quite small, too small for the calibration of perceptual motion detection, belief and social setting might, for example, prompt perception of swaying when none obtains.

4. Secondary Perception of Pendulum Motion

While belief and social setting exercises only minimal influence on primary perceptions, belief and social setting can have significant influence on secondary perceptions of aspects of pendulum motion, at least a plausible case can be made in this regard. For example, acceptance of a claim that the period of an ideal pendulum depends, not on amplitude of the swing, but on the length of the pendulum (implying that an ideal pendulum is isochronic, i.e., all periods are equal) could engender a secondary perception in which discrepancies are perceived *as* due solely to friction and other perturbing factors. Such secondary perception could confirm a mistaken theory of isochronic motion. Secondary perception like this may lie behind Galileo's following remark about motion of two pendulums with equal length.

Referring to the representation of pendulum motions in Figure 1, he wrote:

> The moveable B passes through the large arc *BCD* and returns by the same *DCB* and then goes back toward D, and it goes 500 or 1,000 times repeating its oscillations. The other goes likewise from F to G and then returns to F, and will similarly make many oscillations; and in the time that I count, say the first 100 large oscillations *BCD*, *DCB* and so on, another observer counts 100 of the other oscillations through *FIG*, very small, and he does not count even one more – a most evident sign that one of these large arcs *BDC* consumes as much time as each of the small ones *FIG*. Now, if all *BCD* is passed in as much time [as that] in which *FIG* [is passed], though [*FIG* is] but one-half thereof, these being descents through unequal arcs of the same quadrant, they will be made in equal times. But even without troubling to count many, you will see that moveable F will not make its small oscillations more frequently than B makes its larger ones; they will always be together. (1969; quoted from Matthews, p. 103[7])

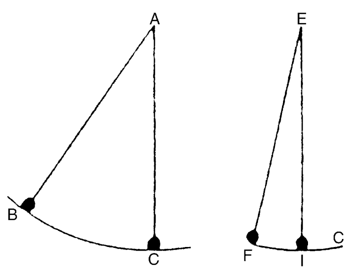

Figure 1. (taken from Matthews, p. 103).

Galileo is mistaken. The pendulums would not stay in motion anywhere close to 500 oscillations, nor would they stay in synchronization anywhere close to 100 oscillations.[8] At best, before stopping, they would stay in close synchronization for a little while, then to an unprejudiced eye the oscillations would diverge, although still approximately in synchronization, and then the oscillations would widely diverge. The reasons for Galileo's mistaken observational claims may never be known. Nonetheless, it seems plausible that these claims arise, in part, from secondary perception in which discrepancies are perceived as being due solely to friction and other perturbing factors. His theory of synchronization in ideal pendulum motion could be confirmed by this perception.

Because secondary perception can be so heavily influenced by belief and social setting, it can lead to error, as in the case of Galileo. His theory, which plausibly secondary perception helped confirm, turned out to be wrong. Ideal circular pendulums with equal length, but unequal amplitudes of oscillation, do not stay in synchronization. (Ideal cycloidal pendulums are required for that.) On the positive side, theory-informed secondary perception promotes seeing phenomena in light of theory. This can lead to greater insight. Additionally, theory-informed secondary perception makes for efficient reasoning.

5. Primary Perception Pertaining to Measurement

Because belief and social setting influence primary perception only very little, primary perception is a natural candidate for perception pertaining to measurement because primary perception typically does not depend on the theory being tested (although, of course, data interpretation often depends on the theory). In this regard, consider a well-known experiment by Mersenne in 1647, which used pendulum motion in an attempt to find a gravitational constant (the distance that a body falls in the first second after release). This experiment led to doubt about Galileo's theory of pendulum motion. About this experiment Matthews writes:

> Mersenne's earlier investigations pointed to three Parisian feet being the length of the seconds pendulum, and so in a famed experiment (1647) he held a 3 foot pendulum out from a wall and released it along with a freefalling mass [Figure 2]. He adjusted a platform under the mass until he heard both the pendulum strike the wall and the mass strike the platform at the same time. He reasoned that this should given him the length of freefall in half a second (a complete-one-way swing of the pendulum taking one second), and so, by the times-squared rule, he calculated the length of freefall in one second, the gravitational constant. Mersenne did not get consistent results. He was frustrated by his experiments and became convinced that the circular pendulum was not isochronic. He was not however able to proceed beyond this point of frustration. Huygens thought the ear could not separate the pendulum and freefall sounds to better than six inches of free fall. (Matthews, pp. 116–117)

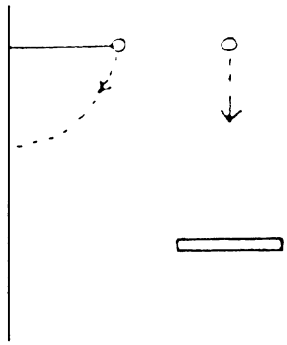

Figure 2. (taken from Matthews, p. 117).

The measurement crucially involves auditory perception of simultaneous striking, a primary perception, which does not depend on knowing the theory and is generally little influenced by it. Instead, the perception depends on auditory mechanisms. Huygen's criticism ('the ear could not separate the pendulum and freefall sounds to better than six inches of free fall') directly pertains to such considerations (and is illustrative of the concern within science for such matters).

As already mentioned, primary perception does not function totally free from the influence of belief and social setting. This influence could become a factor in Mersenne's experiment if the stimulus of simultaneous striking is not clear.

6. Secondary Perception Pertaining to Measurement

In Mersenne's experiment, the simultaneous striking requires interpretation. We need to know, for example, the cause of the striking – a pendulum hitting a wall, a ball hitting a platform. Some of this interpretation, which depends on belief and social setting, can become incorporated into secondary perceptions.

Perception of time as indicated by a clock is another example of secondary perception. (Horology was central to the development of the theory of pendulum motion, as Matthews points out. In particular, the accuracy of a clock provided evidence for the theory implemented in the workings of the clock.) The physical motion of the hands of a clock are merely that, physical motions. They, in themselves, do not tell time. Considerable interpretation, engendered by belief and social setting, must come into play in order for a perception of a clock face to count toward telling the time. Much of this interpretive information gets incorporated into secondary perception as in, for example, perceiving the time *as noon* when we perceive the hour and minute hands pointing directly upward during daylight hours.

7. Primary Perception of Scientific Diagrams

Scientific diagrams, including geometric diagrams, typically incorporate optimal conditions for primary perception (with the exception of sloppily drawn diagrams sometimes drawn by scientists in the course of working out problems).[9] These primary perceptions function to supply crucial information needed to understand the diagram. For example, basic perception can provide information concerning 'betweenness' relations. An example of such a relation occurs in a diagram in Figure 3 of a cycloid (of much importance in the theory of pendulum motion). Point K is between points A and D. Perception of this relation is primary perception because it is likely not consciously learned and may even be innate. Thus, it is little influenced by belief and social setting. In addition to this primary perception we have primary perception of other betweenness relations and in general primary perception of many elementary topological relations (of which betweenness is one). Without these primary perceptions, the diagram could not be perceived as a shape.

8. Secondary Perception of Scientific Diagrams

Diagrams also require interpretation. A diagram, for example, needs to be interpreted as a representation. This interpretation, evidently, is heavily influenced by belief and social setting. It is highly likely that some interpretation becomes incorporated into secondary perception in a learning stage. An example pertains to the diagram in Figure 4 which depicts acceleration throughout an oscillation of a pendulum. We do not have to pause to consider that the arrows indicate direction. This is incorporated in secondary perception.

To summarize briefly: regarding scientific phenomena, measurements, and scientific diagrams, primary perception provides basic information which is little influenced by belief and social setting, while secondary perception provides information with interpretation derived from belief and social setting. It is hoped that the categorization finds its way into theoretical dis-

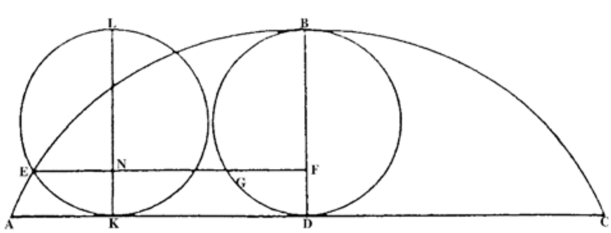

Figure 3. (taken from Matthews, p. 125).

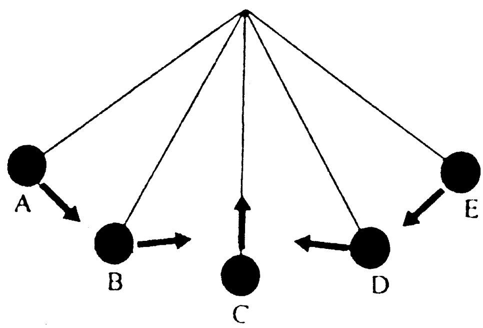

Figure 4. (taken from Matthews, p. 287).

cussion about science education where it seems relevant to current concerns such as that with visualization in teaching and learning science.[10]

Notes

I wish to thank anonymous referees for helpful comments.

1 See, for example, Mathews (2000, p. 257). All subsequent references to Matthews's work in this paper refer to Matthews (2000).

2 Although a defence of this assumption would take us too far afield in this paper, a good fit of the categorization with the practice of science would offer partial motivation for the assumption, which underlines the categorization. The examples drawn from understanding pendulum motion in this paper suggest that there is a fairly good fit.

3 See Goldman (1986, pp. 188–189).

4 Perceptual learning at the conscious level can also serve to unlearn previously learned secondary perceptions. Additionally, secondary perceptions can participate in the learning of other secondary perceptions, in this way allowing for the emergence of many secondary perceptions.

5 The primary–secondary division of perception follows a fairly common view in philosophy. See, for example, Price (1953, p. 45 ff.) who proposes the categories of primary and secondary recognition.

6 A typical objection to the categorization would deny the existence of primary perception, or anything like it, by maintaining that belief and social setting heavily influence all perception. (Using my terminology one might, along this line, hold that primary perception is really secondary perception.) However, as mentioned earlier, for the purposes of this paper, I assume some perceptions remain quite free from the influence of belief and social setting. The category of primary perception reflects this assumption.

Another objection might question where I place the boundary between primary and secondary perception. It might be contended that perceptions of a flittering pattern of flight, yellow colouration, and small size are not primary perceptions, but secondary perceptions; consequently, only something more basic can count as primary perception. Although I think I have provided a reasonable (albeit rough) boundary between primary and secondary perceptions, I have no fundamental objection to attempts to move this boundary, provided any such attempt does not have the category of primary perception come to refer only to 'pure' sensation (if there is such a thing), upon which judgements cannot be based because it is bereft of concepts.

[7] The square brackets in this passage are Matthews's.
[8] See Matthews (2000, p. 106).
[9] Although clear diagrams do not guarantee veridical primary perception (as illustrated by the Müller–Lyer illusion), clear diagrams are generally conducive to veridical perception.
[10] The categorization seems compatible with a view in science education which calls attention to ways in which science is fallible. In fact, secondary perception and, to a small degree, primary perception, can be sources of error due to the influence exerted on them from belief and social setting.

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Piaget and the Pendulum

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Abstract. Piaget's investigations into children's understanding of the laws governing the movement of a simple pendulum were first reported in 1955 as part of a report into how children's knowledge of the physical world changes during development. Chapter 4 of Inhelder & Piaget (1955/1958) entitled 'The Oscillation of a Pendulum and the Operations of Exclusion' demonstrated how adolescents could construct the experimental strategies necessary to isolate each of the variables, exclude the irrelevant factors and conclude concerning the causal role of length. This became one of the most easily replicable tasks from the Genevan school and was used in a number of important investigations to detect the onset of formal operational thinking. While it seems that the pendulum investigation fits nicely into Piaget's sequence of studies of concepts such as time, distance and speed suggested to him by Einstein, more recent research (Bond 2001) shows Inhelder to be directly responsible for the investigations into children's induction of physical laws. The inter-relationship between the pendulum problem, developing thought and scientific method is revealed in a number of Genevan and post-Piagetian investigations.

Key words: Cognitive development, Piaget, Inhelder, genetic epistemology, ceteris paribus, experimental method, induction, formal operations.

Introduction

1958 saw the Genevan treatise on the development of mature forms of thinking appear in its English language translation. What had appeared three years earlier as *De la logique de l'enfant à la logique de l'adolescent* ('From the logic of the child to the logic of the adolescent') was translated as *The growth of logical thinking from childhood to adolescence: an essay on the construction of formal operational structures* (*GLT*, Inhelder & Piaget 1955/1958). The translated title subtly moved the emphasis away from the development of adolescent thought out of its less sophisticated predecessor, to the development of logical thought *per se*. Half a century later *GLT* remains the singular detailed examination of the structure and functioning of adolescent thinking, describing and explaining the transition from the thinking that typifies childhood to that of which the adolescent becomes capable.

GLT contains chapters each reporting children's efforts to discover and then to explain the functioning of 15 experimental devices drawn from physics, mechanics, optics, chemistry etc. Each chapter provides exemplary excerpts from the protocols transcribed from the children's problem solving efforts, divided into the sequence

M.R. Matthews et al. (eds.), The Pendulum, 303–313.

of increasing cognitive sophistication posited by the authors, along with a detailed logico-mathematical modelling of those performances due largely to Jean Piaget. A final three integrative chapters summarize the operational structures and relate the discoveries of the previous chapters to a more general conceptualisation of the possibilities and realities of 'Adolescent Thinking'.

Although what is now commonly known as the 'pendulum task' is reported in detail in but one of these chapters (Chapter 4), and reprised in Chapters 16 and (esp.) 17 for logicomathematical modelling, it has reached a status in Piagetian research which reflects a number of features of the problem which have salience for science and experimental method in general, and for school science educators in particular. Central to the exalted status that the pendulum problem enjoys is the role that the *ceteris paribus* principle plays in determining that the period of the pendulum bears an inverse relationship to its length *and* the role that same key problem-solving strategy has as a defining feature of mature thought. Moreover, the basic law of pendulum motion (periodicity) is ubiquitous in school science texts and the materials required for a satisfactory display of that pendulum principle are so basic that a demonstration could be made in all but the most straightened circumstances.

Fifty years of research and commentary into the investigations and theory central to *GLT* leave the following issues for explanation and/or consideration:

- While *GLT* does describe a particular form of mature thought, is Piaget's logico-mathematical modelling of it inappropriate, irrelevant or just plain wrong-headed?
- Why would a text ostensibly about the development of adolescent thinking be so restricted to thinking in 'scientific' problem-solving situations?
- Why does the incidence of formal operational thinking detected by independent researchers seem considerably lower than that in the Genevan sample inferred from *GLT*?

There is the hint of a clue to the central role given to scientific reasoning in *GLT* in the preface of that book which reported a new style of synthesis of the research agendas of the two authors, Bärbel Inhelder and Jean Piaget:

> In other words while one of us was engaged in an empirical study of the transition in thinking from childhood to adolescence, the other worked out the analytical tools needed to interpret the results. It was only after we had compared notes and were making final interpretations that we saw the striking convergence between the empirical and analytic results. This prompted us to collaborate again, but on a new basis. The result is the present work. (Inhelder & Piaget 1958, p. xxiii)

The Induction of Physical Laws

Ostensibly, it seems that the pendulum investigation fits nicely into Piaget's sequence of studies of core scientific concepts such as time, distance and speed suggested to him by Einstein in Davos in 1928. Their discussion on the relationship between distance, speed and time (encapsulated in every high school child's notes as $s = d/t$) raised the question as to which of these interrelated notions was 'primitive' – in the epistemological sense. Piaget and his team launched themselves into a series of studies into the construction of key physical notions in children: quantity (1941a/1974), number (1941b/1965), time (1946a/1969), movement and speed (1946b/1970), space (1948/1967), and chance (1951/75) (Ducret 1990, p.61). The movement and speed book reported that the notion 'overtaking' and none of the earlier suspects, distance, speed or time, was the primitive notion from which the $s = d/t$ interrelationship was constructed during the early school years (Piaget 1946b/1970).

More recent research (Bond 2001) based on original source materials held at the Archives Jean Piaget in Geneva, shows Bärbel Inhelder to be directly responsible for the investigations into children's induction of scientific laws reported in *GLT*. About a dozen chapters of the original hand-written manuscript for *LELA/GLT* have been housed in the Archives Jean Piaget in Geneva for over a decade, and another three chapters were added to the manuscript in 1998 when Inhelder's academic papers were collected together. A note, added apparently later to the preface in Piaget's hand, indicated that a book on Induction, would 'be the subject of a special work by the first author' (i.e., Inhelder, n.d.). While I had previously found some 80 pages or so of draft materials for Inhelder's book on the induction of physical laws in various folders at the AJP, a mostly complete draft of some 150 type-written pages was collated from her papers in 1998.

Although the investigation into the pendulum and other devices reported in *GLT*, is well-grounded in Piaget's epistemological interests (1918, 1950 etc.) in general, and the work of Lalande on induction and experimentation (Lalande 1929; Piaget 1950, p. 161), all the existing evidence confirms that Inhelder conceptualised, conducted and interpreted the investigations into how children's conduct of scientific experiments changed over the school years. 'This experiment forms part of a complete research project on the development of induction currently underway at our institute under the direction of B. Inhelder.' (Piaget 1950, p. 199 footnote)

Two chapters of *GLT* are explicitly focussed on what might be more generally termed, the control of variables schema. Chapter 3, 'Flexibility and the Operations Mediating the Separation of Variables' is based on an experimental device, Flexible Rods. Chapter 4 'The Oscillation of a Pendulum and the Operations of Exclusion' reports the investigation into the induction of the qualitative length/period relationship of a simple pendulum. The complementary roles of these two situations can be gleaned from two of Inhelder's own reports. Her autobiographical chapter reports:

> One of the questions we asked was how the method called *ceteris paribus* (all other things being equal) was discovered. J. Rutschman, A. Weil-Sandler, and I designed an experiment in which the subjects were asked to determine the various factors, that make metal rods more or less flexible (length, thickness, shape of section, kind of metal). The results of a series of experiments of this kind were highly promising, and in the corridors of the Institute one could hear excited discussions about how we had discovered a new stage: formal thought is not achieved before the age of fifteen or so. (Inhelder 1989, p. 223)

This complements delightfully a section in a 1954 conference report to teachers at Saint-Cloud, recently translated into English as part of a *festschrift* for Inhelder (Tryphon and Vonèche 2001):

THE PENDULUM
An analogous, but much simpler, experiment can be arranged by means of a pendulum. It is used for children or adolescents to discover that the frequency of a pendulum is a function of its length, to the exclusion of all other invoked factors, such as, for example, the suspended weight, the momentum imparted or even the height of its drop.
Towards 14–15 years, but not earlier, adolescents correctly test all the possible hypotheses by combining them methodically. In varying the length, they take care to maintain the weight, the amplitude and momentum constant. In varying the weight, they hold constant the length of the string, as well as all the other factors, etc. The famous method, familiarly called "method of all other things equal", then always has recourse to combinatorial operations on the one hand and neutralisation or compensation of factors on the other. (Inhelder 1954/2001, pp. 200–201)

1. Pendulum Research in Geneva

Much of the original investigation reported in *GLT* was conducted by psychology students, directly under the supervision of a research *assistant* (usually one for each experimental device) and Inhelder. Each of the psychology students submitted an assessable report which addressed the ten or so cases personally conducted, while each of Inhelder's *assistants* produced annual reports which both summarised and analysed all the year's investigations on one problem. For example, graduate student F. Maire's report 'Recherche sur l'induction de la loi du pendule' (July 1950) on that academic year's pendulum investigations referred to matters left unresolved in his previous (June 1949) summary report, and along with the usual summary of the performances of school-aged subjects, he included the results of an investigation of the abilities of one superior adult university student. (Bond 1994)

The school children reported on by Maire numbered 33 children aged from 6 years 2 months to 16 years 3 months from six Genevan schools. Maire's report reveals two important features of the pendulum investigation not detailed in the *GLT* chapter. Firstly, a watch or stopwatch was provided to any subject who requested one (after all, at that time Geneva was the centre of the watchmaking world). Secondly, to help those subjects who appeared to confuse the speed of the bob with the frequency, the investigator would lightly tap the weight each time it reached one extremity of its swing. Remarkably, absence of a timing device and confusion of speed and frequency were specifically remarked upon by Bunting (1993) and Stafford (2002) in her detailed replication of the pendulum problem, which adopted the Genevan investigative technique.

The interrelated foci of the suite of scientific experimental situations reported in *GLT* underline several of the key reasons for the pendulum task appearing almost as archetypical for the adolescent phase of the Genevan research into the development of knowledge construction. The law of the periodicity of the simple pendulum is scientifically significant both in its own right and for its consequent contributions to scientific knowledge as well as to discovery, commerce etc. (see Mathews (2000)). While that law often might be taught formally in an expository mode in schools, the induction of the law from the use of a pendulum requires the interrelated use of a number of strategies central to scientific experimentation. These include: the identification of possible variables as well as the ordered values of those variables (i.e., increasing: lengths of string, weights of bobs, angles of amplitude and force of release); and the realization of an exhaustive multifaceted matrix of all possible combinations of these variables and their values. The subheadings of the pendulum chapter indicate the cognitive sequencing by stage and substage:

Stage I. Indifferentiation between the subject's own actions and the motion of the pendulum.
Stage II . Appearance of serial ordering and correspondence, but without separation of variables.
Stage IIIa. Possible but not spontaneous separation of variables.
Stage IIIb. The separation of variables and the exclusion of inoperant links. (Inhelder & Piaget 1955/1958, pp. 69–75)

Central to such an investigation is the strategy of systematic experimentation which varies but one variable at a time while holding all others constant – the *ceteris paribus* approach. Moreover, the experimenter must adopt hypothetico-deductive reasoning techniques in order that valid conclusions may be derived from the sequence of experimental manipulations: a suite of alternative hypotheses concerning possible operant variables are each disconfirmed or otherwise in turn as the sequence of manipulations are executed. In this manner the inoperant variables of impetus, amplitude and weight are then excluded from involvement in the final explanation of pendulum frequency. (In fact, variations in amplitude can affect periodicity, but not to an extent that is detectable in qualitative investigations such as those described here.) While this experimental strategy does not and can not overcome the ubiquitous philosophical problem of inducing general laws from restricted sets of observations (e.g., Russell 1993, pp. 209, 534), it is consistent with the views of French philosopher of science, Lalande (1929), endorsed separately by both Piaget (1950) and Inhelder (1954/2001), that induction makes an important contribution to scientific method in its own right: it is not merely the impoverished inferior of deduction.

2. The Contribution to Piagetian Theory

But what did Inhelder's 1949–1950 work on induction with the pendulum task contribute to Piaget's conception of developing intellectual competence during childhood? Recent research into original sources at the Archives Jean Piaget in Geneva (Bond 2001) clearly concludes that Piaget's understanding of the nature of formal thought persisted almost unchanged for three decades since his first foray into the topic in one of his very first psychological investigations (1922). Indeed, several of his accounts in that period contained no reference to cognitive development after childhood; Inhelder's claim for the discovery of a new stage at about age fifteen seems easily sustainable in hindsight (Bond 2001, p.80). The distinctive features of the Genevan account of formal operational thinking might now be summarized as follows: formal operational thinking is hypothetico-deductive, propositional and combinatorial. This is a succinct, almost cryptic summary of the complexities of schemata for formal operational thought laid out in chapter 17 of *GLT*, ‘Concrete and Formal Structures’.

It is clear that the Piagetian account of what was termed for three decades as ‘formal thought’ consisted of the logical deduction component – where the conclusions deduced from stated premises follow the well-known logical rules of *modus ponens*, *modus tollens* etc. After Inhelder’s investigation of the pendulum and other science problems this account was clearly insufficient. Her research team discovered that it was during adolescence that thinking became, in a word, scientific; the pendulum problem was not solved without the induction of experimental hypotheses, the propositional thinking instantiated in ‘if *p* then *q*’, ‘*p* whether or not *q*’ strategies, and the ability the develop an exhaustive list of all the possible combinations of variables. It is from the consideration of all *possible* combinations that the adolescent induces competing hypotheses in turn. The logically necessary conclusions deduced from each hypothesis are compared to the *actual* results observed, as a sequence of ‘fair tests’ based on the *ceteris paribus* principle is undertaken. Indeed, it seems quite a reasonable conclusion that Piaget’s early account of formal thought based on logical deduction became the 1955 Genevan account of formal operational thinking as hypothetico-deductive, propositional and combinatorial as a direct consequence of the scientific problem solving strategies that Inhelder and her team uncovered as school students confronted the pendulum and other problems at the end of the 1940s.

Scientific Thinking and Education

In the context of education generally and science education in particular, it is a shame that we had to wait half a century for Inhelder’s 1954 conference report to teachers on ‘The experimental approach of children and adolescents’ to be translated into English.

> Indeed it reflects so well the particular skills of Inhelder and the immediate relevance of her work to education and psychology that one could easily imagine quite a different scenario if this paper had been published in a key English language psychological journal at the same time as the French original had been. It appeared as “Les attitudes expérimentales de l’enfant et de l’adolescent” in the *Bulletin de Psychologie* in France, the year before *LELA* appeared in print. (Bond 2001, p. 70)

Inhelder’s interest in children and schooling was quite explicit; she concluded the paper with a direct challenge for education:

> Towards 14–15 years, gifted adolescents seem then to possess the psychological aptitude necessary for the acquisition of the experimental and inductive method – it’s up to the school to create the climate that is favourable to its implementation. (Inhelder 1954/2001, pp. 202, 282)

In the past, ideas generally attributed to Piaget have permeated the rationales for science education programs in the UK, the US and Australia to such an extent that science educators and reviewers for science education journals now often refer to 'having gone past that Piaget stuff', 'no longer interested in Piaget', or just plain, 'Piaget's been disproved'. Such was the impact of the Genevan work that for the decade that straddled 1970, two journals in the US, *Science Education* and *Journal of Research in Science Teaching* might have easily accounted for the bulk of all published research into formal operational thinking in that period. The impact was quite considerable in the UK where *Nuffield Science* and *Concepts in Secondary Mathematics and Science* (CSMS) paved the way for three decades of work by Shayer and Adey into the relationship between cognitive development and science education (Shayer & Adey 1981) and the impact of classroom interventions on cognitive development and achievement in secondary school science (Adey & Shayer 1994; Shayer & Adey 2002).

In Australia, the *Australian Science Education Project* (ASEP) of that era explicitly claimed a Piagetian formulation: 'ASEP materials have been designed for use at three levels of student cognitive development.

> Stage 1 materials are suitable for students at Piaget's concrete stage of thinking when thinking is dependent on the presence of concrete objects and examples.
> Stage 2 materials are for students in transition from the concrete to the formal stage.
> Stage 3 materials are for students at Piaget's formal stage when there is freedom from dependence on concrete examples, and hypothetical situations can be considered. (Tisher & Dale 1975, p. 3)

En passant, this style of description of formal operational thinking, 'when there is freedom from dependence on concrete examples, and hypothetical situations can be considered' was almost ubiquitous in science education research of the time; in direct contra-distinction, every single demonstration of mature scientific reasoning in *GLT* involves an adolescent actively physically manipulating actual concrete experimental devices such as the simple pendulum.

A Testing Time for the Pendulum Task

It appears that educational applications of the Genevan work – especially in science education settings – drove many of the replications and developments of Inhelder's investigatory procedures, including the pendulum task. While early research adopted techniques based more or less directly on the one-on-one 'Piagetian interview' style apparently evident in *GLT*, moves to standardize procedures and to administer the tasks to whole classes (rather than individuals) at a time resulted in versions of the pendulum problem being developed where class demonstrations, or even printed diagrams, prompted children to make written responses which were later scored.

Even if one were to try to maintain the view that the Genevan method of critical exploration (méthode critique or méthode clinique) was the gold standard for administering and interpreting the pendulum problem (following *GLT*), there would remain a plethora of problems to be overcome before that standard could be applied to evaluating the quality and outcomes of the secondary research in the area. First of all, it takes more than a superficial reading to be aware of the philosophical underpinnings of the Genevan work on formal thinking (Bond & Jackson 1991). Even those who are persistent about being sensitive, well-informed researchers would find great difficulty in replicating the Genevan interview technique, because there is much more to it that is ever revealed in the literature, whether in English translation or the original French (Bond 1994). It is only in the original French-language protocols or the summary reports such as that of Maire on the pendulum task where the provision of a timing device and the tapping of the bob are mentioned at all.

The Genevan method produced obviously qualitative data; Piaget was not interested in quantitative indices of his ideas (Piaget 1941/1965) and elsewhere he espoused sound philosophical reasons for rejecting the social sciences' techniques for quantification as being inadequate in scientific measurement terms (Piaget 1970). In the secondary research, the méthode critique thereby suffers on two rather contradictory grounds: it was dismissed as qualitative when the dominant paradigms in science education required quantification, and in the current post-modernist times, it is dismissed as passé when qualitative approaches are now in vogue.

The Pendulum Task Measures Up

Indeed, it has taken modern developments in measurement theory for the social sciences (Bond 1995a, b; Bond & Fox 2001) to reveal the fine psychometric qualities of the Genevan method, properly applied. Bunting's ground-breaking research used the Rasch partial credit model thoroughly to quantify the nuances of children's performances on the pendulum task (Bunting 1993; Bond & Bunting 1995; Bond & Fox 2001; Stafford 2002) and to demonstrate that Inhelder's pendulum task and its closest standardised analogue – the Piagetian Reasoning Task III – Pendulum (Shayer 1976) from the CSMS suite of Piagetian tasks – measure the same underlying formal operational ability. But there is a sting in the tail: the class task is considerably more difficult, on average, than is the Genevan original. Piaget insisted that the Genevan method aimed at the best estimate of a child's intellectual competence, not merely the child's quotidian performance. This gives some clue as to why the results of the CSMS survey of cognitive development were less optimistic than *GLT* seems to suggest was the case for Genevan school-children.

The Logic of Scientific Experimentation

In conclusion, it is still worth asking if Piaget's rather idiosyncratic logical modelling of formal operational thinking still has any role in Piaget's conception of adolescent or scientific reasoning. In the latest edition of the authoritative *Handbook of Child Psychology*, Moshman finds surprisingly strong support for Piaget's proposal that hypothetico-deductive reasoning plays an important role in mature thought 'but is rarely seen much before the age of 11 or 12'. He makes the routinely accepted observation that '[T]he theory of formal operations – strictly construed as the logical model proposed by Inhelder and Piaget (1958) – no longer plays much role in the literature.' (Moshman 1998, p. 972). The apparent truth of this conclusion begs two important questions. The first revolves around whether those who have dismissed the logico-mathematical modelling of formal operational thinking had read it in the context of Piaget's self-declared philosophical and epistemological perspective (Bond & Jackson 1991). The second asks how could the *Thinking Science* classroom interventions (Adey & Shayer 1994) continue to be so successful in increasing cognitive development and school achievement of adolescents (Endler & Bond 2001) if its explicit base in Piaget's formal operational schemata is not worthy of further attention? (Tryphon & Bond, 2002).

It is of more than mere passing interest, that Piaget recurred to the Inhelder work on induction, with specific reference to the pendulum (and flexible rods) problem, in *Psychogenesis and the History of Science* (Piaget and Garcia 1983/1989, p. 83) – the book whose final draft was completed the day before Piaget suffered the trauma which was to end his life. This final chapter in the Piagetian *oeuvre* was an integration of reflections on the psychogenetic evidence collected over more than half a decade with a reading of the history of science, especially geometry, mechanics and algebra. 'But it turns out that, as we studied, with B. Inhelder, the induction of the simple laws of physics in 11-year old pre-adolescents, who had received absolutely no academic instruction in the subject, we have been able to observe the formation of a methodology derived only from their logical reasoning, rather than experimental expertise or pre-existing theoretical knowledge' (Piaget & Garcia 1983/1989, p. 83). He then rehearses his old favourites from the 1950s:

> The only answer possible appears to be that, on the basis of the propositional operations constructed, such as conjunction, implication, and exclusive or nonexclusive disjunction, which enable the subjects to reason about simple hypotheses and to evaluate these by deriving from them logically necessary consequences, the subject then apply this logic to the problems we present them with. (Piaget & Garcia 1983/1989, p. 83)

In the context of the history of science, this poses a double conundrum for Piaget: 'And why, above all, did the great logicians and methodologists of the thirteenth century not elaborate one [a theory of physics] more scientific than that of Aristotle?' (p. 85) and what of the particular case of Aristotle, who, 'although the inventor of logic failed to apply it to experimentation' (p. 86). Clearly, Piaget had not resiled from his earlier explanation of mature thought, even though he regarded himself as the chief revisionist of his own theory.

What price, then, a doctoral dissertation rereading the history of the roles of various pendula in the progress of science from the perspective of Piaget's psychogenesis?

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What the Pendulum Can Tell Educators about Children's Scientific Reasoning

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Abstract. Inhelder and Piaget (1958) studied schoolchildren's understanding of a simple pendulum as a means of investigating the development of the control of variables scheme and the *ceteris paribus* principle central to scientific experimentation. The time-consuming nature of the individual interview technique used by Inhelder has led to the development of a whole range of group test techniques aimed at testing the empirical validity and increasing the practical utility of Piaget's work. The Rasch measurement techniques utilized in this study reveal that the Piagetian Reasoning Task III – Pendulum and the *méthode clinique* interview reveal the same underlying ability. Of particular interest to classroom teachers is the evidence that some individuals produced rather disparate performances across the two testing situations. The implications of the commonalities and individual differences in performance for interpreting children's scientific understanding are discussed.

Introduction

Science educators have long held an interest in Piaget's philosophy and methods of understanding the development of scientific reasoning in children and adolescents. The 15 tasks outlined in Chapter 4 of the Inhelder and Piaget's *The Growth of Logical Thinking* (*GLT*) have been utilised, not merely as a means of teaching scientific principles but, more importantly, as a tool in measuring students' level of scientific reasoning through their approach to these tasks. Piaget developed the *méthode clinique* technique to investigate intellectual development. This clinical approach, as used in *GLT* consisted of physical tasks (developed by Inhelder) and an unstructured interview method. The interview situation is highly flexible, allowing the experimenter to prompt and question the child, searching for the strengths and limitations of the strategies which govern the child's actions in attempting to solve the task.

Amongst the tasks outlined in *GLT*, the pendulum experiment, in particular, has been widely used in science classrooms as a means of studying students' ability to use the principle of *ceteris paribus* in scientific reasoning. The pendulum problem utilised a simple apparatus consisting of a string, which could be shortened or lengthened, and a set of weights of varying masses. Other variables, which might at first be considered relevant, are the height of the release of the weight, and the

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force of push given by the subject. The task is used for children or adolescents to discover that the frequency of a pendulum is a function of its length, to the exclusion of all other factors.

The literal translation of *ceteris paribus*, 'other things being equal', alludes to a methodology which may be described as 'controlling variables', In order to investigate the effect (or lack of it) for any single factor, all other variables in the situation must be held constant while the variable of interest is manipulated and corresponding effects are noted. This concept is especially operative in the 'control' mechanism of the experimental method. It follows, therefore, that an awareness of the principle of *ceteris paribus* is critical to the current scientific understanding of the world around us.

MEASURING FORMAL OPERATIONS

While replications of Piaget's work (Lovell 1961; Pauli et al. 1974; Somerville 1974) have shown the accuracy of the *méthode clinique* in eliciting cognitive competence and differentiating between operational levels, they also describe its limitations in that the method is time-consuming and requires considerable expertise for administration and interpretation. Researchers interested in empirical studies have also argued "that results obtained by such a flexible procedure as the *méthode clinique* do not lend themselves to statistical treatment" (Wallace 1965, p. 58). These factors have restricted the utility of the m,thode clinique for teachers seeking to establish students' operational levels (Shayer & Wharry 1974).

In the literature, two lines of argument have derived from an awareness of the restrictions of the *méthode clinique*. Those concerned with empirical investigations have provided impetus for the proliferation of formal reasoning measures. These researchers maintained that:

> [i]t should not be presumed that the *méthode clinique* provides the only technique by which competence with the sixteen binary operations can be inferred. Indeed, there exists a variety of tests of formal operational ability which require a whole range of different responses such that the size and nature of the inferential leap from observed performance to inferred operational competence vary considerably. (Bond & Jackson 1991, p. 47)

Others interested in the structural aspects of the cognitive model suggest that only the flexibility of the *méthode clinique* interview and Piaget's method of structural analysis provide adequate insight into adolescent cognitive schemata (Easley 1974).

At a practical level, Inhelder and Piaget's pendulum experiment has been translated into a range of alternative test formats aimed at examining students' operational abilities with greater ease and efficiency than was provided by the original experiments. Possibly the most widely used of these testing formats is the Piagetian Reasoning Task (PRT) III – Pendulum developed by Shayer and his associates (Shayer & Adey 1981) working in the Concepts in Secondary Mathematics and Science (CSMS) Program, based at the Chelsea College in London. The

PRT III is advantageous in that its group test format allows the administration of the task to a class of students in a single session. Further, the authors claim that the provision of a timing device in the PRT III facilitates more accurate experiments for the students and a more valid assessment of intellectual competence.

However, the question of prime importance for science educators is – Do the PRT III and the *méthode clinique* administration of the pendulum task measure the same underlying ability?

Establishing students' operational levels is obviously significant for teachers attempting to develop Piagetian-based curriculum programs which begin at the level of students' abilities. The use of invalid classroom measures of operational ability is misleading at best and an impediment at worst. Given the apparent relevance of the PRT III in schools, the imperative to ensure its validity should be a necessary prerequisite to its implementation.

THE THREE TIERS

In view of the distinction Piaget made between performance and structural competence, his theory may be regarded as existing in three distinct tiers (Shayer & Adey 1981). At the first tier are the descriptions of behavioural responses by children of various ages on Inhelder's tasks. The second tier is the classification of these observations into stages which constitute a developmental pattern characterising preoperational, concrete operational and formal operational behaviours. These behaviours are held as being the overt products of the set of available covert cognitive structures. At the third tier is Piaget's metatheory. Here, tier two's behavioural and classificatory descriptions are couched in terms of symbolic logic. For Piaget, these three tiers constitute *a structure d'ensemble* in that the description at any one level is consonant with, and derivative of, descriptions at each of the other tiers.

For educators and developmental psychologists, the most salient applications of Piaget's work are tiers at one and two of the model, whereby observed behaviours are classified in terms of the various stages of cognitive development. This paper provides an analysis of two methods which have been used to examine aspects of tier one and tier two of Piaget's model of cognitive ability.

ANALYSING PERFORMANCES ON PIAGETIAN TASKS

Concurrent validity must be of foremost consideration in the selection of appropriate tests of formal reasoning for classroom use. The importance of concurrent validity of test instruments derives from the recognition that different research strategies display varying degrees of accuracy in their ability to display formal operational performance. This difference gives rise to a critical interaction between performance and assessment strategy which has vital implications for the degree and nature of the inferential leap between observed performance and cognitive competence. It follows, therefore, that teachers interested in establishing students' competence in the use of the control of variables scheme must adopt methods which provide a valid account of this scheme.

Most frequently, researchers have employed relational statistics to test the unifactorial hypothesis for group tests and, hence, the unidimensionality of formal reasoning (Bart 1971; Gray 1976; Lawson & Renner 1974; Lawson et al. 1978; Shayer 1979). As relational statistics assume equivalence of difficulty of test items, it follows that individual performance across test items should be consistent. Since it has been shown that this assumption is inapplicable to Piagetian data, the appropriateness of these statistics for such a measure have been questioned (Bond 1991, 1992; Hacker et al. 1985; Hautamäki 1989).

Relational statistics such as factor analysis and correlations are limited in that test item difficulty and the subject's ability can only be calculated in relation to the parameters of the sample being tested (Hautamäki 1989). It follows, therefore, that relatively low correlations between tests can be calculated simply due to a limited ability range in the selected sample. While Lawson (1979, 1985) and Shayer (1979) suggested that this problem can be overcome by the selection of an appropriately wide age-stage sample, the fact remains that the hierarchial structure of Piagetian tasks could produce any number of factors based on comparative differences between groups of test items (Ferguson 1941).

The inadequacy of psychometrically defined measures, such as factor analysis, in quantitative Piagetian research, is now widely accepted (Bond 1989, 1991, 1992; Gray 1990; Hacker et al. 1985; Hautamäki 1989). Attention has, instead, been directed toward latent trait theories or item-response theories. In particular, the Simple Logistic Model, developed by Rasch (1960), is held to be one analytic model appropriate for the measurement of performances on Piagetian tasks (Bond 1989; Bond & Fox 2001; Hacker et al. 1985; Hautamäki 1989).

At the most basic level, the Rasch analytic model differs from factor-analytic methods in that each item on a test is assumed to have equal discriminating power as part of that test. Hence, an underlying assumption of the Rasch model is that performance on a given test is determined by only two factors; the ability of the subject and the difficulty of the test item (Hautamäki 1989). The unique characteristic of the Rasch model is that it is the only fully unidimensional analytic model (Elliot 1983). Since formal reasoning is a unitary mental construct, this type of statistic has obvious applications to the Piagetian model (Hacker et al. 1985; Hautamäki 1989).

Rasch analysis considers "errors" in relation to the difficulty of the test item. Hence, it could be expected that there is a greater possibility of getting an easier test item correct than a more difficult one. The "necessary precondition" model treats success on easier items as a precursor to success on a more difficult item (Bond 1991). This treatment of item difficulty corresponds to the hierarchial structure of formal reasoning.

Wright and Masters' (1981, 1982) work has more been concerned with developing a variation on the rating-scale analysis model to deal with situations in which the relative difficulties or “steps” between achieved performance “levels” on an item is of greater significance. This type of Partial Credit Model is most appropriate to the form of data examined in the present research.

PIAGETIAN REASONING TASKS

The hybrid class tasks developed by Shayer and his colleagues have become widely used in secondary school science classrooms, particularly in England. The advantage of these Piagetian Reasoning Tasks (PRTs) is in the ease of administration for the teacher, the clear guidelines provided for marking and the requirement for only a single set of readily available apparatus. An important criterion for the development of the PRTs was that the child should be given at least two separate opportunities to display each of the criterion behaviours described in *GLT*. Further, since the tasks are closely based on Piagetian theory, which incorporates the *structure d’ensemble* of formal operations, a child’s performance on any one (or a maximum of two tasks) is considered sufficient to accurately determine the level of cognitive development (Wylam & Shayer 1978).

The validity and reliability of the series of PRTs was tested prior to their publication. Probably the most widely used of the tasks is the PRT III – Pendulum, based on Chapter 4 of *GLT* “The Oscillation of the Pendulum and the Operation of Exclusion”. Shayer and Adey (1981) report the following statistics for the PRT III: KR20 internal consistency 0.83, test-retest correlation 0.79 (n = 24), and task-interview correlation 0.79 (n = 24). While these statistics reflect favourably on the reliability and validity of the pendulum task, the small sample size involved in the testing restricts the inferential power of the results.

RASCH ANALYSIS AND THE PRT III

While it has been argued that Rasch analysis is appropriate for this type of data, investigations of the fit of the PRT III to the Rasch model have not been entirely conclusive. Bond (1989, 1991) and Hautamäki (1989) have both found that the PRT III data fit the Rasch model remarkably well. It could be inferred, then, that since the Rasch model can be held to represent the unidimensional structure of formal operations, the PRT IIII is a valid measure of those abilities.

On the other hand, Hacker et al.’s (1985) analysis of the PRTs suggest that six of the fourteen items on the PRT III do not fit the unidimensionality aspect of the model. They found a joint test of fit of $p < 0.001$ and, consequently, rejected the test unidimensionality. However, with the subsequent removal of these six items, the researchers obtained a joint test of fit of $p = 0.09$. They conclude that “task III is not recommended because of the larger number of misfitting items” (p. 30). Given the critical necessity of employing valid classroom measures (Ahlawat & Billeh 1987; Blake 1980; Bond 1992; Gray 1990; Lawson 1985), these criticisms merit further investigation of the PRT III.

Methodology

The purpose of the present study was to test the concurrent validity of the Piagetian Reasoning Task III – Pendulum with the traditional *méthode clinique* interview technique. Accordingly, it was fundamental to the design that each subject was exposed to both the PRT III and the *méthode clinique* measures, so that a comparative analysis could be performed between the two scores.

THE SAMPLE

The total tested sample for this study consisted of 72 students from a very large public secondary school in Townsville of approximately 2,000 students. The subjects were drawn from three science classes at grade 8 (n = 28), grade 9 (n = 23) and grade 10 (n = 21). There was an even mix of gender in the sample.

The PRT III was completed by 68 students, while the *méthode clinique* administration of the pendulum task was undertaken by 58 students. Complete data sets for students tested under both measurement conditions were obtained for a total of 57 students. This sample comprised 20 grade 8 students, 17 grade 9 students and 20 grade 10 students. In accordance with the cautions of Shayer (1976) and Lawson (1985) regarding the need for a distribution of abilities across the sample, the classes used in the study were selected by the science teachers at the school, on the basis of classroom performance and grades, to ensure that a wide range of ability levels would be obtained. Selection for participation in the *méthode clinique* situation was based on alphabetised placement on the class roll. Starting from the beginning of the roll, interviews were held with each nominated student present on the given day of testing.

DESIGN

A counterbalanced design was most suited to the methodological and analytic requirements of this study so that the order of presentation of the tasks for students in Group A would be reversed for Group B students. The experimental design is shown in Figure 1.

ADMINISTRATION

Administration of the test instruments took place over a period of approximately two months, with a two-week mid-semester break between the testing of group A and group B. The sample of grade 8–10 students was divided such that group A consisted of the grade 8 class and half of the grade 10 class, and group B was made up of the remainder of the grade 10 class and the grade 9 class.

Group A undertook the PRT III in the initial testing phase followed by a one week break before the second phase of individual interviews began. Group B began testing at a period two weeks after the completion of Group A's testing phase. For

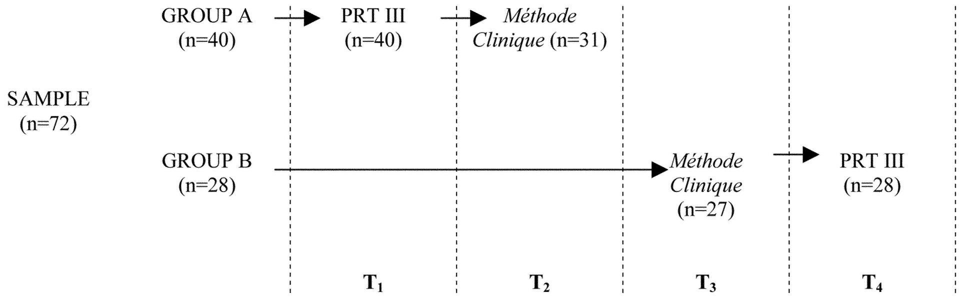

Figure 1. Counterbalanced design for the concurrent validation of the PRT III with the *méthode clinique*.

this group, a three week *méthode clinique* phase was completed before the group testing with the PRT III in the final phase.

APPARATUS

The apparatus for *the méthode clinique* administration was the same as that used in *GLT* (pp. 67–79), consisting of an adjustable string and a series of three weights (40 grams, 80 grams and 100 grams), which were experimented with in order to deduce which of the variables (weight, length, angle and push) affects the period of the pendulum. A general purpose laboratory stand, measuring 60 centimetres in height was used as the base for suspension.

The apparatus Shayer and Wharry (1974) describe for the PRT III is similar to that used in *GLT*, except that there are only two strings (long and short), from which either of two different weights may be hung. The two strings measured 69cm and 35 cm respectively, and the two weights were 40 grams and 100 grams.

Shayer and Wharry suggest the use of a stopclock to time 30 second intervals while the students count the swings. In the present study, a stopwatch was used as the timing device. The stopwatch was given to a chosen student who indicated to the class when each 30 second interval had elapsed. For the PRT III, each subject recorded individual responses to questions on the task paper.

TESTING PROCEDURES

In the investigation of the pendulum problem, each situation involved separate testing procedures and organisational requirements, in accordance with the guidelines provided by the respective test developers.

Piagetian Reasoning Task III – Pendulum (PRT III)

The Piagetian Reasoning Task III – Pendulum was conducted during three separate group test situations, the group size varying from nine to forty students. Each testing session lasted approximately one hour.

The PRT III was administered by a trained and experienced administrator of the test and was overseen by the researcher each time. Following the guidelines set down by Küchemann (1979) and in the Wylam and Shayer (1978) guidebook, testing consisted of a series of teacher-demonstrated manipulations of variables potentially related to the swing of the pendulum. At particular intervals, the students made written responses on the PRT III task paper.

In accordance with the stipulated organisational requirements (Wylam & Shayer 1978), the students were seated so as to minimise the risk of cheating, but still allowing all of them to see the apparatus from their seating positions. Wylam and Shayer (1978) attest the importance of the students having a clear understanding of the questions being asked. Hence, testing frequently involved repeating, rephrasing and clarifying questions to help ensure that each child understood the meaning of the question.

Méthode Clinique

The conditions of the *méthode clinique* replicated, as far as it was possible, those of the original researchers in *GLT*. The method is unique in that it allows the child to demonstrate mental operations through actions on concrete materials as well as through language, and incorporates an element of flexibility as the experimenter is able to pursue the direction of the child's thought (Ginsburg & Opper 1988). The experimenter modifies questions and experimental conditions, seeking clarification of genuineness and consistency in the child's responses.

Piaget recommended that experimenters using the *méthode clinique* should undergo extensive training in its effective usage. Such training was not possible in the present case. However, the study was piloted by a series of rehearsal interviews (n = 10) with the pendulum apparatus, according to the models set down by Piaget. The interviewer was not a complete novice in this regard, having completed *méthode clinique* interviews with the pendulum as part of earlier undergraduate work.

The individual interview with each subject lasted 10-20 minutes and was videotaped for later transcription and analysis. The interviews took place in an allocated quiet room in the school library. Students were individually presented with the apparatus, shown how to manipulate appropriate variables and encouraged to explain what they were thinking and doing. On all occasions, the students were aware that the interview was being recorded and would be reviewed later by the researcher.

SCORING PROCEDURES

In order to examine the concurrent validity of the *méthode clinique* with the PRT III, two levels of analysis were conducted. Firstly, the items comprising the PRT III and the classificatory guidelines for the *méthode clinique* administration of the pendulum were scored and analysed using Quest procedures for Rasch analysis. The aim of this step was to examine the unidimensionality of each of the measures, and the extent to which each of the measures could be found to be conforming to the stage placements suggested by their respective authors. Secondly, the items on the PRT III and the *GLT* criteria were together subjected to Rasch analysis to examine the degree to which items on the two scales fit with each other, fall into the indicated stage allocations, and can be said to be measuring the same underlying ability.

The different administration procedures used for the PRT III and *méthode clinique* situation necessitate similarly unique scoring procedures for the assessment of the performances on the tasks.

Piagetian Reasoning Task III – Pendulum (PRT III)

The PRT III answer sheets were collected at the end of the class testing time. Papers were marked by the present author using the clear guidelines set down by Wylam and Shayer (1978) and each was allocated a stage classification according to Küchemann's (1979) criteria.

Performances on each PRT III item are scored dichotomously, resulting in a total score out of 14. The Quest procedures produce item difficulty estimates, and associated error estimates, infit and outfit mean square values as well as transformed infit and outfit *t* statistics for each test item.

Méthode Clinique

Classificatory guidelines derived directly from Chapter 4 of the original *GLT* text (p. 67–79) were used as the sole basis for stage classification of the protocols obtained from the *méthode clinique* interviews. The initial set of behavioural criteria was extracted from *GLT* prior to the commencement of data collection. Subsequent elaboration and refinement of these descriptions produced 18 items (clusters of related criteria) representing the different pendulum problem-solving behaviours from the preoperational (I) to late formal operational (IIIB) levels of ability.

In the scoring of performances, the wealth of descriptions provided in *GLT* suggested that, while a simple dichotomous yes/no procedure would be appropriate for some of the identified behaviours, that would not provide sufficient detail for other areas of performance. Rather, an ordinal scale was used to allow for the inclusion of (polychotomous) items with three or more graded values across performances, reflecting lesser or greater operational ability. The resultant set of 34 substantive performance criteria (and 14 criteria representing null categories) is shown in Appendix A. Details of the scoring procedures used with the *GLT*-derived criteria are outlined in Bond and Bunting (1995).

Protocols from the *méthode clinique* interviews were assessed according to the 18 items (constituting 34 classificatory criteria) extracted from Chapter 4 of *GLT*. The data obtained from analysis according to these criteria were subjected to statistical analysis under Rasch principles using Quest software (Adams & Khoo 1992). The Quest program provides access to the validation of variables from both dichotomous and polychotomous information, in accordance with the Partial Credit option of the Rasch model (Wright & Masters 1982).

Results

Shayer and Adey's (1981) description of the three-tiered Piagetian model has been outlined as incorporating behavioural, classificatory and logico-mathematical levels. While this study incorporated analysis at each of these three tiers, behavioural and classificatory levels (tiers one and two) are of particular interest to educators and will be addressed here. Data analysis consisted of a statistical analysis of the empirical results from the PRT III and the *méthode clinique* interview. Empirical data collected in this study was analysed according to Rasch principles using Quest software (Adams & Khoo 1992). Quest is an Australian software package designed to perform the functions of Rasch analysis according to the partial credit models.

> Quest offers a comprehensive test and questionnaire analysis environment by providing a data analyst with access to the most recent developments in Rasch measurement theory.[It] can be used to construct and validate variables based on both dichotomous and polychotomous observations. It scores and analyses instruments such as multiple choice test scores, short answer items, and partial credit items. (Khoo & Adams 1992, p.1).

Elaboration of the use of the Partial Credit model in this study can be found in Bond and Bunting (1995).

TIER 1 – THE PROBLEM-SOLVING BEHAVIOURS

Piaget's description of the behaviours which constitute tier one of his theory incorporates two assumptions about the nature of these behaviours. Firstly, the behaviours displayed on the pendulum task must demonstrate some spread of difficulty consonant with the developmental acquisition of the abilities. Secondly, the behaviours should demonstrate homogeneity indicative of the inter-relatedness of the abilities used in solving the pendulum problem (Shayer 1979). Data from both of the testing procedures were analysed using Rasch procedures to determine their conformity to these premises.

Rasch Analysis of the PRT III Behaviors

Rasch analysis of the students' raw scores on the 14 assessable PRT III items produced the results shown below in Table I.

The conventional interpretation of item-fit data is that the transformed t statistic should range from -2 to $+2$ at the $p < 0.05$ level to demonstrate fit to the model. Results in Table I show that item 8 on the PRT III has infit and outfit t balues of $+2.1$ and $+2.5$ respectively, falling just outside the acceptable limits conventionally imposed under Rasch procedures. This result suggests that item 8

Table I. PRT III item statistics

Item	Difficulty estimate	Error estimate	Infit *t*	Outfit *t*
1	−2.65	0.32	+1.4	+3.1
2	+2.16	0.54	−0.4	−0.4
3	+0.09	0.33	+0.8	+0.1
4	−1.23	0.29	−0.7	−0.8
5	−0.82	0.30	−1.1	−0.8
6	+0.89	0.38	0.0	+0.1
7	−0.01	0.32	−1.3	−1.5
8	−2.74	0.33	+2.1	+2.5
9	+1.03	0.39	+1.2	+1.9
10	−1.06	0.30	−1.7	−0.9
11	+0.63	0.36	−0.6	−1.0
12	−0.90	0.30	−0.6	0.0
13	+2.45	0.59	−0.5	−0.1
14	+2.16	0.54	+0.1	−0.2

is not measuring precisely the same set of abilities as the other 13 items on the test. Item 8 (question A.5 on the test paper) is concerned with the ability to establish a relationship of inverse correspondence between the length of the string and the period of the pendulum. In Chapter 4 of *GLT*, Inhelder and Piaget note: "At sub-stage II-A.the subject discovers the inverse relationship between the length of the string and the frequency of the oscillations" (p. 70). Item 8 is the only PRT III item which measures abilities at the lower end of concrete operations; the remaining items relate to the late concrete and formal operational stages. It is possible, then, that item 8 appears misfitting relative to the other PRT III items merely because it is located at one extreme end of the measurement range for the PRT III.

Further, the test of joint fit of the PRT III items with the *GLT* items (results shown in Table III) shows that, when all items on both tests are analysed together, item 8 on the PRT III no longer appears to be misfitting (infit $t = +1.3$). Since the *GLT* items measure a range of abilities from II-A to III-B, the relatively greater amount of information provided about these easier items in the joint test might be regarded as providing a more accurate fit statistic for PRT III item 8.

Item 1 demonstrated an outfit t statistic of +3.1. However, the low infit t statistic relative to the outfit t indicates that the fit of this item is uncertain. At the $p < 0.05$ level, up to one item in a set of fourteen could fall outside acceptable boundaries, due to random fluctuations alone.

Rasch analysis of the test data produced the error estimates for each of the PRT III items, shown in Table I. For the PRT III error estimates ranged from 0.29 logits (for item 4) to 0.59 for logits (for item 13).

Item estimates of the PRT III items produced a spread of items along a logit scale from +2.45 logits for the most difficult item (item 13) to −2.74 logits for the easiest item (item 8), as shown in Figure 2. The spread of persons along the same logit scale located the least successful most successful person (person 9) at +3.48 logits, and the least successful (persons 1, 13, 20, 22, 33, 53) at −3.54 logits. Each X on the graph respresents one person.

Rasch Analysis of the Méthode Clinique Behaviors

Table II shows the results of the statistical analysis of the *GLT* raw scores. According to Rasch principles, any items on which all persons score correctly or incorrectly provide inadequate information about the difficulty level of that item. Hence, item 1 (on which all students scored correctly) was excluded from the analysis. This item concerns subjects' ability to accurately serially order the lengths of the string.

From Table II, it can be seen that the majority of *GLT* items fit the Rasch model as all test items, excepting items 8.1 and 8.2, produced infit t statistics within the acceptable ($-2.0 < t < +2.0$) boundaries. Items 8.1 and 8.2 appear as misfitting items in the set of 34 items. However, the low infit t statistic relative to the outfit t statistic, suggests that the fit of these items is uncertain. Estimates of the *GLT* item difficulties are spread along a logit scale from −2.41 logits for the easiest item (item 6.1) to +3.12 for the most difficult item (item 18). Error estimates for the *GLT* criteria are consistently larger than those for the PRT III, ranging from 0.29 (criterion 16) to 1.25 (criterion 6.1).

The spread of persons along this same scale showed the most successful person (65) at +4.38 logits and the least successful (person 21) at −1.14 logits. This information is shown in Figure 3.

Joint Test of Fit

A joint test of the fit of the two test measures to the Rasch model was undertaken in order to establish whether both tests could be considered as assessing the same ability or latent trait. Item 1 on the *GLT* criteria was again omitted from the calculations because all subjects scored correctly on this item. Table III shows the substantial fit of 30 of the 32 items to the Rasch model. Item 10 on the PRT III (infit $t = -2.1$; outfit $t = -2.0$) and item 15 on the *GLT* (infit $t = +2.7$; outfit $t = +1.7$) appear as two misfitting items from a total of 32 test items. However, for *GLT* item 15, the low outfit t statistic, relative to the infit t statistic again suggests that the determination of fit for this item is uncertain. At the conventional $p < 0.05$ level, it would be expected that one or two items in a set of 32 could show some variation from the model, based on random fluctuations alone.

MOST SUCCESSFUL		*MOST DIFFICULT*
X		
	3.0-	
		13
		2 14
XXX	2.0-	
X		
		9
XXX	1.0-	6
		11
XXX		
XXXXXXXX	.0-	3
		7
XXXXX		
		5
XXXXX	-1.0-	12
		10
XXXXXXXXX		4
XXXXXXXXXX	-2.0-	
XXXXXXXXXXXXXX		
		1
		8
	-3.0-	
XXXXXX		
LEAST SUCCESSFUL	-4.0-	*LEAST DIFFICULT*

Figure 2. PRT III person ability and item difficulty estimates.

Table II. *GLT* criteria statistics

Item	Difficulty estimate	Error estimate	Infit *t*	Outfit *t*
1	Item has perfect score			
2	−2.37	0.73	+0.3	+0.5
3	−2.37	0.73	+0.3	+0.5
4	−2.37	0.73	+0.1	−0.4
5	−2.37	0.73	+0.1	−0.4
6.1	−2.41	1.25	0.0	−0.2
6.2	+0.36	0.63	0.0	−0.2
7.1	−0.97	0.69	+1.0	+0.6
7.2	+0.89	0.58	+1.0	+0.6
8.1	−0.65	0.30	+0.6	+2.3
8.2	−0.65	0.30	+0.6	+2.3
9.1	−0.28	0.59	−0.5	−0.3
9.2	+0.72	0.55	−0.5	−0.3
10.1	−1.50	1.13	−0.3	−0.2
10.2	−1.22	1.08	−0.3	−0.2
10.3	+1.12	0.52	−0.3	−0.2
10.4	+3.11	0.64	−0.3	−0.2
11.1	−1.19	0.66	0.0	−0.2
11.2	+1.63	0.52	0.0	−0.2
11.3	+2.15	0.52	0.0	−0.2
12.1	−0.06	0.53	−0.7	−0.7
12.2	+1.73	0.53	−0.7	−0.7
12.3	+2.88	0.63	−0.7	−0.7
13.1	−2.06	0.97	−0.8	−0.9
13.2	+1.40	0.56	−0.8	−0.9
14.1	−1.59	0.78	+0.7	+0.5
14.2	+1.07	0.48	+0.7	+0.5
14.3	+1.12	0.49	+0.7	+0.5
15.1	+0.97	0.47	+1.8	+1.7
15.2	+1.22	0.48	+1.8	+1.7
15.3	+2.14	0.51	+1.8	+1.7
16	+1.42	0.29	−1.2	−0.9
17	+2.37	0.33	−1.2	−1.3
18	+3.12	0.41	−0.5	−0.9

MOST SUCCESSFUL		*MOST DIFFICULT*
X		
	4.0-	
X		10.4 18
	3.0-	
		12.3
X		
		17
XXX		
		11.3 15.3
XXXXX	2.0-	
XXX		12.2
XXXXX		11.2
XXXX		
		13.2 16
XXXX		15.2
XX	1.0-	10.3 14.2
XXXXXXXXXX		7.2 15.1 14.3
XXXXX		9.2
XXXX		
		6.2
XXXX		
	.0-	
XXX		12.1
		9.1
X		
X		8.1 8.2
	-1.0-	7.1
X		
		10.2 11.1
		10.1 14.1
	-2.0-	
		13.1
		2 3 4 5 6.1
	-3.0-	
LEAST SUCCESSFUL		*LEAST DIFFICULT*

Figure 3. GLT person ability and item difficulty estimates.

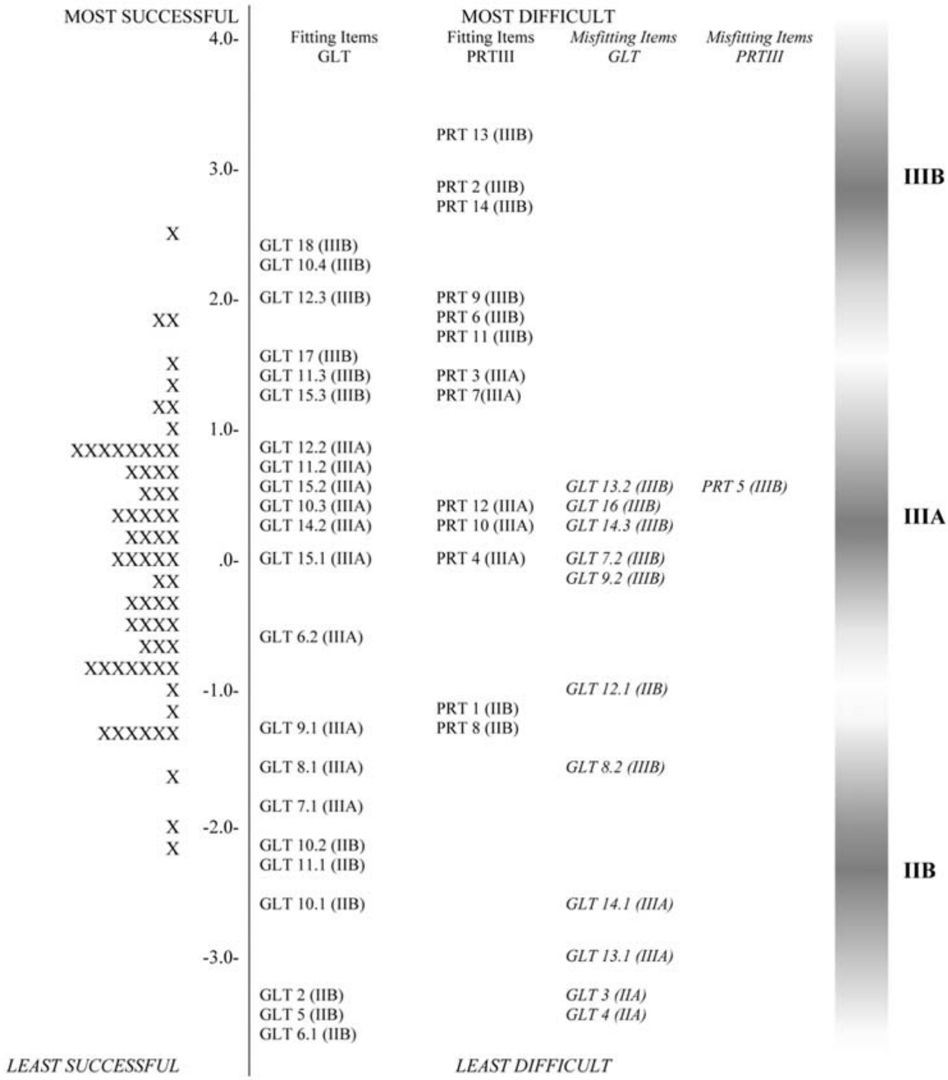

Figure 4. Person ability estimates, item difficulty estimates and stage placements for PRT III and GLT items together.

Estimates of the item difficulties of the 32 test items together produced a spread along a logit scale from +3.12 logits for the most difficult item (item 13 of PRT III) to – 3.38 logits for the easiest item (*GLT* criterion 6.1). These estimates are shown in Figure 4.

Tier 2 – Stage Placements of Problem-Solving Behaviours

At the second tier of Piaget's model, the behavioural descriptions of tier one are classified into developmental stages of preoperational, concrete operational and

Table III. Statistics for joint test of PRT III and revised *GLT* items

Item	Difficulty estimate	Error estimate	Infit *t*	Outfit *t*
PRT 1	−1.05	0.29	+1.4	+1.9
PRT 2	+2.88	0.48	+0.1	−0.5
PRT 3	+1.18	0.29	−0.6	−0.7
PRT 4	+0.13	0.26	−0.9	−0.8
PRT 5	+0.46	0.26	−1.9	−1.9
PRT 6	+1.83	0.34	−0.2	−0.5
PRT 7	+1.10	0.28	−0.9	−1.3
PRT 8	−1.14	0.29	+1.3	+0.8
PRT 9	+1.94	0.35	+0.4	+0.5
PRT 10	+0.26	0.26	−2.1	−2.0
PRT 11	+1.62	0.35	−0.8	−0.9
PRT 12	+0.39	0.26	−0.5	−0.9
PRT 13	+3.12	0.53	+0.2	+0.2
PRT 14	+2.88	0.48	0.0	−0.5
GLT 1	Item has perfect score			
GLT 2	−3.34	0.73	+0.3	+0.5
GLT 3	−3.34	0.73	+0.3	+0.5
GLT 4	−3.34	0.73	+0.2	−0.3
GLT 5	−3.34	0.73	0.0	−0.7
GLT 6.1	−3.38	1.29	−0.3	−0.6
GLT 6.2	−0.58	0.65	−0.3	−0.6
GLT 7.1	−1.88	0.69	+1.0	+1.0
GLT 7.2	0.0	0.54	+1.0	+1.0
GLT 8.1	−1.57	0.29	+1.0	+1.0
GLT 8.2	−1.57	0.29	+1.0	+1.0
GLT 9.1	−1.19	0.59	+0.1	+0.3
GLT 9.2	−0.18	0.53	+0.1	+0.3
GLT 10.1	−2.41	1.13	−0.1	0.0
GLT 10.2	−2.14	1.06	−0.1	0.0
GLT 10.3	+0.24	0.53	−0.1	0.0
GLT 10.4	+2.18	0.65	−0.1	0.0
GLT 11.1	−2.13	0.66	+0.4	+0.8
GLT 11.2	+0.74	0.53	+0.4	+0.8
GLT 11.3	+1.25	0.52	+0.4	+0.8
GLT 12.1	−0.94	0.50	+0.5	+0.6
GLT 12.2	+0.84	0.52	+0.5	+0.6
GLT 12.3	+1.96	0.59	+0.5	+0.6
GLT 13.1	−3.00	0.97	−0.6	−0.7

Table III. Continued

Item	Difficulty estimate	Error estimate	Infit *t*	Outfit *t*
GLT 13.2	+0.52	0.54	−0.6	−0.7
GLT 14.1	−2.50	0.75	+1.8	+1.3
GLT 14.2	+0.17	0.48	+1.8	+1.3
GLT 14.3	+0.23	0.48	+1.8	+1.3
GLT 15.1	+0.09	0.47	+2.7	+1.7
GLT 15.2	+0.33	0.47	+2.7	+1.7
GLT 15.3	+1.22	0.50	+2.7	+1.7
GLT 16	+0.53	0.28	−1.2	−1.3
GLT 17	+1.49	0.33	−0.6	−0.8
GLT 18	+2.23	0.40	−0.3	−0.6

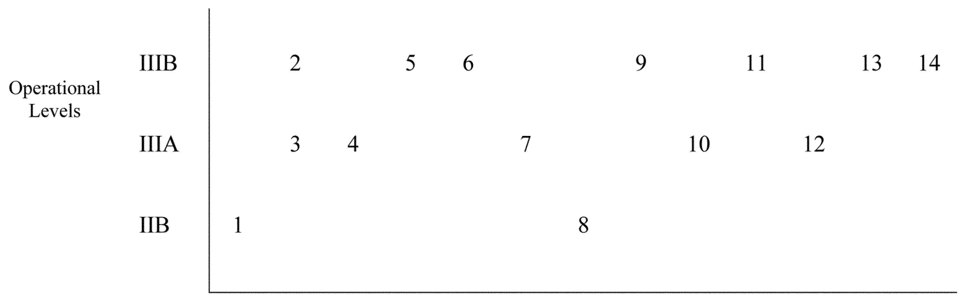

Figure 5. PRT III Items at each Piagetian level.

formal operational ability. In particular, the *GLT* problems are related to discrimination between concrete and formal levels of operational ability (Parsons 1958; Smith 1987).

A further consequence of Rasch analytic procedures is that, since item difficulty estimates are obtained for test items on the two measurement procedures, it is possible to make a comparison of those estimates relative to stage estimates of item difficulties, as stipulated by the test developers. Accordingly, difficulty estimates of PRT III items, and of the criteria derived from *GLT* were compared with the qualitative descriptions of behaviours suggested by the respective authors at each level of development.

PRT III Difficulties and Stage Allocations

In the development of the PRT III, Wylam and Shayer (1978) included assessable items at each of the Piagetian levels of cognitive development. Küchemann (1979) indicated levels of individual items, as shown in Figures 4 and 5.

When these allocations are interpreted in terms of the item difficulty estimates (see Table I), it can be seen that the results provide general support for Küchemann's stage classifications of the PRT III. These results are represented on Figure 4. The areas showing more intense shading indicate those items for which stage allocation is relatively certain. Where the shaded colour is less intense, the stage barriers and, consequently, the stage placement of the items is less certain.

The single exception is item 5, which Küchemann identified as a IIIB item. From the analysis of the data reported here, it appears that item 5 is easier than the other IIIB items, with a level of item difficulty within the band of the other IIIA items on the PRT III.

GLT Item Difficulties and Stage Allocations

The 34 performance criteria extracted from the descriptions in the Inhelder and Piaget text were ascribed a stage classification to each ordinal level, according to the *GLT* specifications. These stage allocations are shown in Figure 6.

Comparison of the difficulty estimates for the criteria derived from the sample data (see Table II) in relation to Piaget's specifications indicated that not all the items fall at the statistical levels of difficulty which might be seen as congruent with stage allocations based on Piaget's purely logical analysis. These results are also shown in Figure 4. The clustering of items where the colour is more intense indicates greater certainty of stage placement with those items.

Under these estimations, items 3, 4 and 12.1 appear to be empirically more difficult than predicted under the theory, and items 7.2, 8.2, 9.2, 13.1, 13.2, 14.1, 14.3 and 16 easier than the *GLT* allocations suggest. Each of these items, therefore, is considered somewhat problematic.

RELATIVE DIFFICULTY OF THE TWO TEST PROCEDURES

When independent person ability estimates and a joint-test of fit had been obtained for each test measure, it was possible to use these results to estimate the relative difficulty levels of the two tests. While the spread of item difficulties for each of the three analyses is centred around an arbitrary zero point for convenience, the comparison of difficulties for the closest to centre items (item 12.1 from the *méthode clinique* and item 7 for PRT III) on the joint test indicates that the written demonstrated class task version of the pendulum task is about two logits more difficult for this sample than was the *méthode clinique* administration of the pendulum task.

This estimate for Rasch analysis results was supported further by analysis which showed that 33 percent of the subjects performed at a higher level of ability in the individual *méthode clinique* situation than on the PRT III. When the general consistency of performances across test situations was analysed, it was found that 45% of subjects scored at exactly the same stage level across both tasks, and 89% of all subjects scored consistently within one substage on the tasks.

IIIB					7.2	8.2	9.2		10.4	11.3	12.3	13.2	14.3	15.3	16	17	18
IIIA				6.2	7.1	8.1	9.1		10.3	11.2	12.2	13.1	14.1	14.2	15.1	15.2	
IIB	2	5		6.1				10.1	10.2	11.1	12.1						
IIA	1	3	4														

Figure 6. Stage allocations for GLT criteria. (After *GLT*, pp. 67–79).

Discussion

The central purpose of the present study was to investigate the concurrent validity of the Piagetian Reasoning Task III – Pendulum (Shayer & Adey 1981) with the traditional Genevan *méthode clinique* technique for 'the oscillation of the pendulum and the operation of exclusion'. The results presented here generally provide support for the concurrent validity question. The results of the Rasch analysis of performance on the PRT III and the *méthode clinique* administration of the pendulum problem indicate that both scales are substantially unidimensional. Further, the combined scale consisting of all items form both tests was also substantially unidimensional, providing support for the claim that both scales are measuring the same underlying ability. However, the combined analysis also indicated that the *GLT* criteria is a somewhat more difficult test of formal operational abilities than the PRT III.

Analysis at tiers one and two of Piaget's model is a useful way in which to examine the results.

TIER 1 – RASCH ANALYSIS OF PROBLEM-SOLVING BEHAVIOURS

Rasch Analysis of PRT III Performances

The study provides support for the results reported in the work of Shayer and Adey (1981) suggesting that the test provides a useful classroom measure of the operational abilities encompassed in the *méthode clinique* administration of the pendulum task. However, the larger sample used in this study provides a more reliable basis for this conclusion. The present results also confirm the conclusions of Bond (1989, 1991) and of Hautamäki (1989), who both claimed that the PRT III may be considered to be substantially unidimensional in the terms encountered under Rasch analysis.

Rasch Analysis of Méthode Clinique Performance

The statistical analysis of the *GLT* criteria undertaken in this study provides the first published attempt to test the empirical validity of Piaget's theory using a detailed quantitative assessment of the original *méthode clinique* administration of the pendulum task. The results outlined above provide remarkable corroboration of Piagetian theory. The analysis of the *GLT* criteria (shown in Table II) provides support for the underlying unidimensionality ($p < 0.05$) of the detailed set of behaviours described by Inhelder and Piaget in Chapter 4 of *GLT*. The strength of the results obtained in this study attests to the validity of Piaget's theory of operational ability – statistical analysis of essentially qualitative data was used to indicate that the performance criteria may be regarded as essentially unidimensional.

Relative Test Difficulties

While Rasch analysis confirmed the joint fit of the PRT III items and the *GLT* criteria, analysis of the difficulty estimates of the PRT III items together with the *GLT* items (in Figure 4) showed that the three most difficult items across the scale are all PRT III items, and the fourteen easiest items on the scale are derived from the *GLT* items. Further, when the relative difficulties of these two test situations were compared, it was revealed that the difficulty level of the PRT III is two logits higher than that of the *méthode clinique* administration of the same task. Together, these results suggest that the PRT III is a more difficult test measure than the *méthode clinique* based on *GLT* performance criteria.

The results of the joint-fit analysis of the PRT III with the *GLT* criteria also relate to previous research using the PRT III. In Shayer's (1978) review of Somerville's (1974) replication of the pendulum task, he takes issue with the range of the sample used in that study. Somerville's results indicated that up to 97 percent of her 236 subjects (aged 10–14 years) had reached a late formal level of operational ability, and that none of her subjects were at the early concrete (IIA) stage. These results differed considerably from Shayer's (1976) findings of about 24 percent of his sample demonstrating formal operational thinking by 14 years of age. His results were obtained using the PRT III as the measure of problem-solving ability on the pendulum task. In his 1978 paper, Shayer claimed that the discrepancies between the results were due to the inadequate representativeness of Somerville's sample, and held that representative sampling would provide a more accurate reflection of the incidence of formal operational ability.

In this regard, the results obtained in this study provide general support for Shayer's arguments. In the present research, the same sample was tested in the traditional *méthode clinique* interview technique (used by Somerville) and with the PRT III (used by Shayer). However, analysis of student performance data indicates that the majority of students scored noticeably higher in the individual *méthode clinique* situation than on the PRT III. While 45% of students scored at exactly the same level across the two tasks, 33% were allocated a higher level of operational ability in the *méthode clinique* situation than on the PRT III. This finding of greater difficulty of the PRT III over the *méthode clinique* provides a competing explanation to Shayer's sampling argument for the discrepancy between Somerville's results, and those of Shayer's original investigation.

TIER 2 – ITEM ESTIMATES AND STAGE PLACEMENTS

At the second tier of Piagetian theory, the behaviours on the pendulum task are classified into stages of operational ability. The results of a comparison between item difficulty estimates (as obtained under Rasch analysis procedures) and suggested item stage placements for each test are shown in Figure 4.

Item Estimates and Stage Placements for PRT III Items

Comparison of the relative difficulty levels of the PRT III items with the stage classifications made by Küchemann (1979) showed general consistency between the estimates, with only item 5 appearing to be problematic. Küchemann suggests that this item tests abilities at the IIIB level. However, statistical estimates of item difficulty placed item 5 at −0.82 logits, closer to other IIIA items. This item (question A.6) on the test paper calls for the subject to identify possible tests to determine the effect of weight, and then exclude any tests which are not needed, such that a necessary and sufficient set of tests is obtained.

GLT Criteria Estimates and Stage Placements

Comparison of item difficulties and the stage estimates described in Chapter 4 indicated that some clusters of *GLT* criteria were placed at levels of difficulty along the scale which did not conform to stage estimates derived from *GLT*. From Figure 4 it was noted that *GLT* criteria 8.2, 12.1, 13.1 and 14.1 are particularly discrepant, while the positioning of criteria 3, 4, 7.2, 9.2, 14.3 and 16 requires further comment. Detailed discussion of these misfitting items is undertaken in Bond and Bunting (1995). The results of the Rasch analysis of the 34 *GLT* criteria suggest that some refinement may be required in terms of the relative difficulties involved in producing combinations of each of the factors of the problem.

Performance Discrepancies

While the results provide support for the concurrent validity of the PRT III, it was quite apparent that some individuals produced rather disparate performances across the two testing situations. In particular, four students (persons 013, 035, 050 and 068) were ascribed Piagetian levels which differed by more than two substages, across the two assessment conditions. This finding is of particular issue for teachers as it may indicate variability in the reliability of testing instruments for different students.

The nature of this study allowed for some additional information to be obtained about each of these subjects. While this information cannot provide conclusive reasons for performance discrepancies, it indicates the potential influence of the interaction between the particular measurement situations and individual factors that subjects bring to the experimental context (Hales 1986). These performance discrepancies are considered noteworthy and may have implications for the deployment of diagnostic tests and selection of assessment strategies in the classroom. They suggest that, at least for some students, a single test of operational ability may not provide an accurate reflection of that individual's level of intellectual development, either in performance or competence terms.

Subject 013

A male students from year 10 who is confined to a wheelchair because he has muscular dystrophy. He performed at the IIB level of concrete operations in the PRT III situation, but was able to produce a level of mature formal performance (IIIB) in the *méthode clinique* situation. During the individual interview, the experimenter manipulated the experimental variables according to the subject's instructions, as he was unable to perform the experiments himself. It may be deduced, then, that the interactional mode of the *méthode clinique*, and the lack of demands for writing, provide a more reliable assessment of this student's abilities.

Subject 035

A male student in year 8in Group A of the sample performed at the IIIB level on the PRT III test, but at the IIB level of concrete operations in the individual situation. According to his teacher, the student's family has a military background and has moved around the country a great deal. His teacher reported that, although the student worked well in class, he seemed to experience some difficulty relating to people on a one-to-one basis, particularly people with whom he is not familiar. His relatively poorer performance in the *méthode clinique* interview may have been affected by a difficulty in relating to the interviewer.

Subject 050

A male student in year 9, this student responded to every question on the PRT III paper, yet failed to answer a single item correctly, suggesting a complete lack of understanding of the problem. He was, consequently, allocated a IIB-, indicating that the test could not provide an accurate assessment of operational ability since he had not yet reached the level of concrete operations. Yet the student demonstrated performance at the early formal (IIIA) stage when interviewed in the individual situation. In the interview situation, the student was enthusiastic and appeared to enjoy the interactive nature of the testing situation.

Subject 068

A female student in year 10, this subject produced performance at the fully formal operational (IIIB) level during the individual interview, but in the group PRT III situation, she demonstrated performance at a low level of concrete operations (IIB-). The seating arrangements in this final PRT III test were, perhaps, more conducive to social distractions than Wylam and Shayer (1978) would have recommended; she was almost entirely distracted by her interest in a male neighbour. Infit t statistics for this subject were $+0.67$ for the PRT III and -0.78 for the *GLT*, indicating some systematic constraint on the student's performance in the *GLT* situation.

These cases are informative since they suggest that, while both the PRT III and the *méthode clinique* interview may provide substantially similar indications of operational abilities, neither can be unconditionally regarded as the accurate measure of individual operational ability on the pendulum task. While the PRT III was found to be a somewhat more difficult test of formal operational abilities, there are a number of items from the *GLT* criteria which did not conform to the expected stage placement, based on the descriptions given in *GLT*. Some refinement of these criteria may be required.

The results support the claims of Bond and Jackson (1991), Nagy and Griffiths (1982) and Flavell and Wohlwill (1969) that operational performances may be influenced by both the type of assessment task administered and the relationship between the individual and the task, and is consistent with Hales' (1986) description of the role of the human subject in psychological investigation. Given the differences for the four subjects discussed in this sample, it is clear that the possibilities for intervening variables remain indeterminable.

EVALUATION OF THE METHODS

This study was chiefly concerned with the collection of data through PRT III procedures and *méthode clinique* interviews. However, the study format also allowed the collection of additional data through direct observation of each method used and informal discussions with the students involved. This information is useful with regard to the identification of benefits and limitations associated with each method.

Evaluation of the PRT III

A recognised deficiency of paper and pencil testing is the demand placed on the reader's interpretation of written test questions and the marker's interpretation of the written responses. While such difficulties are a feature of all written tests, the degree of interpretation involved could potentially produce highly discrepant results. The administration and marking of the PRT III in the present study revealed that this was a limitation in the test and, consequently, provided another obstacle to interpretation of the nexus between individual's performance and intellectual competence. Indeed, when the two test formats were discussed with the year 10 students in Group B, several students noted the difficulties they had in clearly expressing their ideas on the test paper.

A second concern derives from the more structured format of the PRT III, as compared with the *méthode clinique* technique. Küchemann's (1979) guidelines for administration clearly set out a series of six experiments on the pendulum, performed in a specific sequence, with questions posed at key intervals. However, several students noted that the experiments provided nsufficient basis for their deductions and more experiments (such as available in the *méthode clinique* format) would have been advantageous.

The second limitation is well reflected in an incident which occurred at the completion of the second PRT III with the year 9 class. Two female students approached the administrator at the end of the session, asking if they could use the pendulum apparatus to do some more experiments. After these students had performed two more experiments themselves, one of the students exclaimed, “See, I was right! I told you that’s what it would do!” The restricted number and types of experiments demonstrated in the class-test procedure often resulted in a repetitive test format and extended periods of thinking and writing which might have constrained performances for some students, and indicating a potential limitation of the PRT III procedure.

Thirdly, the written answers on the PRT III format necessarily introduce a greater level of interpretation on the part of the teacher than is characteristic of the more interactive Genevan interview situation, where any responses can be followed up for immediate elaboration. While PRT III test questions were clearly explained during administration, and the students were encouraged to ask questions about the testing situations if they needed to, the frequent ambiguity of some individual answers remains an issue of concern. These problems arose despite the fact that students were asked to express their answers as clearly as possible. In the marking of the PRT III task papers, difficulties in the interpretation of students’ answers indicated that clearer wording and more specific instructions might be necessary in order to obtain amore accurate assessment of students’ abilities.

A fourth point of some note is that the group test situation inevitably introduces a social dimension to the assessment procedure, which might distract student’s performances in a number of ways, for example, the intentional omission of test questions, or making inappropriate responses. Wylam and Shayer (1978) stipulate that students should be seated so as to avoid copying answers. However, even with these seating positions, the social aspects which often characterise adolescent classrooms appear to be an unavoidable feature of the group test situation.

A related factor is that, even in the open and flexible structure of the PRT III, some students lagged behind the pace of the rest of the class. To this end, in the testing undertaken here, the experimenter was able to survey the classroom and assist individual as the administrator oversaw the class experiments. While it is likely that such assistance facilitated a more accurate assessment of competencies for some students in the study, it is notable that this type of extra intervention would not be possible under the conditions of a regular (one-teacher) classroom administration of the test. Rather, the effective implementation of the testing procedure would rely on the diligence and appropriate repertoire of management strategies used by individual teachers experienced with their own class.

In a more positive regard, however, the specifics of the PRT III testing procedures encompass a number of characteristics which might be considered beneficial to assessment in formal operations. In particular, the provision of a stopclock in the class administration of the PRT III facilitated accuracy in observations and the deduction of effects. The Grade 10 students in group B who discussed the administration formats stated that the use of a timing device was preferable since it allowed observations to be more accurate. Indeed, during the individual interviews with group A students (who had previously completed the PRT III), a number of students expressed an interest in timing, or made some crude attempt to time the pendulum swings, in order to make their observations more exact.

Evaluation of Méthode Clinique

As the original Genevan method, the *méthode clinique* remains the conventional means of investigating operational ability (Dale 1970; Lovell 1961; Somerville 1974). However, during the course of testing in the present study, three procedural aspects in the administration of the pendulum problem were identified which provide potential for misinterpretation of performances in the individual *méthode clinique* situation.

Firstly, the provision of a timing device in the PRT III apparatus facilitated a greater degree of objectivity and accuracy in observation, which the students themselves identified. The absence of a stopclock in the interview situation was frequently noted by subjects in group A (who had previously completed the PRT III) and some made attempts at timing the oscillations. In one case, a Year 10 student used his own digital watch to time the swing of the pendulum. Of course, the effective inclusion of a timing device would also require there be some initial demonstration of the use of the instrument so that subjects could the equipment to provide accurate feedback. However, it is not likely that this would limit the possibilities for the subject to experiment with the variables of the problem.

A second issue regards the influence of expectations on observations, as was noted in the early replication studies of the pendulum problem by Lovell (1961) and Somerville (1974). For many subjects, even those who otherwise demonstrated mature use of the control of variables scheme, observations of effects on the pendulum seemed to be heavily influenced by their expectations of the results. Piaget (1972, 1974) discussed this type of unconscious repression of observations as characterising the performances at the earlier preoperational level.

However, it should be considered that the discrepancy may be related to a deeper problem in the interpretation of the requirements of the pendulum problem. While the subjects manipulate the independent variables of length, weight, angle and impetus on the apparatus, their observations of effects may have been confused between the dependent variables of *frequency* of oscillations and *speed* or *velocity* of oscillation. This distinction is problematic, because the independent variables of length, angle and impetus affect velocity, but only the length of the string affects the frequency of oscillation. Consequently, observations which would otherwise be labelled as ‘faulty’ may be derived from an inability to distinguish between the two dependent variables, or a misdirection of attention to the variables in which relevant changes are noticed, even though this is not the intended focus of the experimental situation.

Even when the difference between the variables of velocity and frequency was clearly identified for the subjects, they were often unable to discriminate between the two in practice. This failure to attend to relevant variables is exemplified in the protocol of the individual interview with subject 048:

048 (14;2): "If you push it hard when it's up here, it keeps swinging out further" – "Does that make a difference to how quickly it goes?" – "It makes it go more slowly. Just giving it a little push, it doesn't have as far to go, so it goes quicker".

It is noteworthy that this confusion of independent variables was most apparent for group B subjects, for whom the first exposure to the pendulum apparatus was in the *méthode clinique* situation. Group A subjects, undertaking the PRT III prior to the individual interview, did not appear to experience the problem to the same extent. It is possible that the direction of attention to the frequency variable and the provision of the timing device used in the PRT III provided a means of avoiding the interpretational confusion.

COMPETENCE AND PERFORMANCE

The purpose of the recommended modifications to both the *méthode clinique* procedure and the PRT III test format is to use the information obtained from this study as a means of narrowing the inferential gap between children's performances on the tasks and their level of operational competence.

At the same time, the differences in the difficulty levels of the PRT III and the *méthode clinique* technique shown in this study illustrate an important educational issue in the investigation of student operational competence using performances on tests of operational ability. The point has been made by Bond and Jackson:

different research strategies exhibit varying degrees of tractability in their capacity to display formal operational performance. This gives rise to a critical interaction between the investigatory methods used in any given setting and the degree to which operational performance could be expected to show up in a cogent way. (1991, p. 48)

The outcomes of this research are significant in terms of the validity and utility of the PRT III as a classroom measure of problem-solving abilities on the pendulum task. Beyond these results, the wealth of information obtained through individual *méthode clinique* interviews, and the statistical analysis of those results enabled a quite substantial investigation and analysis of the Piagetian model itself.

EDUCATIONAL IMPLICATIONS

The results of the analysis reported in this paper have a number of significant implications for the nature of science teaching and education in general. In the first place, analysis of the performances of this group of students on the PRT III supported claims for the unidimensionality of the PRT III and for its validity and utility as a measure of concrete and formal operational abilities. The principle of *ceteris paribus* in the control of variables scheme constitutes an important ability in the areas of science and mathematics, with application apparent in school subjects of English and history, among others. The PRT III represents a useful classroom test of the ability to apply this principle in the use of the control of variables scheme.

However, since the analysis of the difficulty of this test relative to the *méthode clinique* administration of the same pendulum task indicated that the PRT III is two logits more difficult than the original technique, it seems that measurements of performance on the PRT III tend to underestimate subjects' ability to isolate and systematically test variables in a controlled experiment.

Secondly, if investigators using the *méthode clinique* format for the pendulum problem intend that it should provide maximal insight into student competence, then it seems that two variations are worth further consideration and investigation. The unavoidable recommendation is that the provision of a suitable timing device would allow students to make more objective observations of their experimental manipulations so that their deductions and conclusions may be exercised on interpretations of correct data. Likewise it seems worthwhile to trial further investigations of the pendulum problem, removing the inclusion of the independent variable push or impetus (Items 12 and 115, shown in Appendix A). Its less concrete nature makes push difficult to accurately quantify in the experimental procedures under investigation. Given that impetus directly affects the amplitude of swing, experimental procedures which varied release position (angle, amplitude) rather than release force (impetus, push) could reduce an unnecessary (and misleading) complication in the *méthode clinique* administration of the pendulum problem.

Finally, the degree to which the performance-competence nexus may be influenced by a range of interacting factors including test instruments, individual differences and environmental factors should be a primary consideration in assessing students' abilities. The qualitative and quantitative results reported here indicate that variability in each of these factors can greatly influence the degree to which an individual's formal reasoning abilities is revealed in any given situation. It follows, then, that while different test instruments vary in their capacity to elicit formal reasoning abilities, no single instrument is likely to provide a reliable indication of operational ability for *all* individuals under *all* situations.

Appendix A. Performance criteria derived from Chapter 4 of *GLT*

	IIA	IIB	IIIA	IIIB
1	Able to accurately serially order lengths			
2	Unable to accurately serially order weights	Able to accurately serially order weights		
3	Able to accurately serially order push			
4	Establishes inverse relationship between length and frequency of oscillation			
5	Unable to manipulate some variables	Able to vary all factors		
6	Does not make inferences. Limited to observations	**6.1** Makes inferences based only on observed concrete correspondence	**6.2** Makes inferences going beyond observations, without needing to test all possibilities.	
7		To test for length, manipulates incorrect variable and in an unsystematic manner	**7.1** Manipulates incorrect variable, but makes logical deductions by inference to results on earlier experiments	**7.2** Systematically manipulates lengths to test for their effects
8		Manipulates incorrect variable and is unsystematic in testing for weight	**8.1** Manipulates incorrect variable, but makes logical deductions by inference to results on earlier experiments	**8.2** Systematically manipulates weights to test for their effects
9		Manipulates incorrect variable and is unsystematic in testing for push	**9.1** Manipulates incorrect variable, but makes logical deductions by inference to results on earlier experiments	**9.2** Systematically manipulates impetus to test for the effect of push
10	**10.1** Makes illogical deductions about the role of length (including illogical exclusion of length in favour of weight or impetus)	**10.2** Excludes the effect of length (because of inaccurate observations)	**10.3** Logically deduces positive relationship of *affirmation* or *implication* for the role of length	**10.4** Deduces *equivalence* of length and frequency of oscillation
11		**11.1** Makes illogical deductions about the role of weight (either illogical exclusion or positive implications)	**11.2** Logically deduces a positive relationship of *affirmation* or *implication* for weight, based on inaccurate observations	**11.3** Excludes the role of weight

	IIA	IIB	IIIA	IIIB
12	Preoccupied with the role of impetus as the cause of variations in frequency of oscillation. Illogical deduction of positive implication	**12.1** Testing results in the illogical *exclusion* of the role of push	**12.2** Logically deduces a positive relationship of *affirmation* or *implication* for push, based on inaccurate observations	**12.3** Excludes the role of push
13		Does not produce combinations of length with other variables	**13.1** Produces combinations of different lengths with different weights or pushes to test for effects	**13.2** Produces sets of combinations of lengths with various weights and pushes to test for their effects
14		Does not produce combinations of weights with other variables	**14.1** Produces combinations of different weights with different lengths to test for effects **14.2** Produces combinations of different weights with different pushes to test for effects	**14.3** Produces sets of combinations of weights with various lengths and pushes to test for their effects
15		Does not produce combinations of push with other variables	**15.1** Produces combinations of different pushes with different lengths, to test for effects **15.2** Produces combinations of different pushes with different weights, to test for effects	**15.3** Produces sets of combinations of various pushes with lengths and weights, to test for their effects
16			Unsystematic method	Systematically produces all combinations, using the method of varying a single factor, while holding all else constant
17			Unable to exclude the effect of weights	Logically excludes the effect of weight
18			Unable to exclude the effect of push	Logically excludes the effect of push

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Pendulum Phenomena and the Assessment of Scientific Inquiry Capabilities

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Abstract. Phenomena associated with the *pendulum* present numerous opportunities for assessing higher order human capabilities related to *scientific inquiry* and the *discovery* of natural law. This paper illustrates how systematic *assessment of scientific inquiry capabilities*, using *pendulum* phenomena, can provide a useful tool for classroom teachers and program planners. *Structured inquiry*, a technique of teacher-facilitated student *inquiry* involving direct interaction between students and natural phenomena, is presented as a way to establish student competence in applying *scientific inquiry capabilities* (e.g., conceptualizing variation due to error). This approach to *assessment* can heighten student curiosity and provide a concrete referent for complementary cultural, historical, and scientific instruction. The role of *assessment* in constructively shaping science education programs is considered.

1. Introduction

Those who advocate *inquiry-based learning* and the development of process skills in science education must look back upon a century of failed attempts to realize these goals in mainstream science classrooms. While there are a number of social and particularly institutional forces that share responsibility for this state of affairs, there is also a technical impediment, one that, if not properly addressed, will continue to frustrate attempts to move the agenda forward. Namely, that in spite of extensive evidence that *assessment* methods mold instructional programs, we still have not developed methods of *assessment* that foster *inquiry-based learning* and *discovery learning* (Zachos, Hick, Doane, and Sargent 2000). In the United States, the science teacher, increasingly, must navigate a wary course between the Scylla of ‘high-stakes, norm-referenced tests’ and the Charybdis of the teacher-made classroom test. High-stakes, norm-referenced tests (Glaser 1963; Kohn 2000) exert enormous leverage over *curriculum* and instruction (Fredricksen 1984; Heubert & Hauser 1999), but typically do not produce information which is either useful or timely for classroom application (Madaus & Tan 1993). Moreover, such tests have contributed to trivializing the science *curriculum* (Resnick & Resnick 1990). On the other hand, studies of classroom tests (McMorris & Boothroyd 1993; Stiggins

M.R. Matthews et al. (eds.), The Pendulum, 349–362.

2001) indicate that teacher-made tests tend to be unreliable and inadequate to the task of providing the information needed to support instructional effectiveness.

Here, I present an approach to *assessment* that constitutes an alternative path, both for the science teacher and for stakeholders in the educational enterprise who are in search of ways to promote high quality *inquiry-based learning* programs. The approach is illustrated through explication of a *Classroom Event* that centers on the phenomenon of *pendulum* periodicity. The *pendulum*, in addition to its rich cultural context (Matthews 2000), has properties which make it particularly useful for developing *assessment* instruments that support instruction in scientific thinking, *scientific inquiry*, and the attainment of concepts through a process of *discovery*. Implied in all of the educational goals to be presented here is the overarching goal of developing the human capability to *discover* natural law.

2. A Classroom Event

A secondary school science teacher begins a session by telling students that they will be taking a journey into the past. She unfolds the vision of a young man, of their own age, sitting in a church in Renaissance Italy, staring upwards as if at the edge of a revelation. He is watching incense burners, suspended from the ceiling above, swinging to and fro on long chains. Periodically, when their contents have burned down, a long stick emerges from an upper balcony. Someone pulls the incense burners in, replenishes their charcoal and incense and sets them swinging again. The young man, watching, wonders if the complete swings are indeed taking different amounts of time to go back and forth as they appear to be. The teacher explains that this man does not have a wristwatch with a second hand to check his perceptions. In fact, she points out that the clock, as we know it today, had not yet been invented. Undeterred, this young man considers what might provide him with a stable indicator of the passage of time so that he could answer his question. He decides to use his pulse, for all its irregularities, probably the most dependable method available in his time for this purpose.

Yes, he found, the incense burners were taking different amounts of time to swing back and forth. But why? Was it how long they had been swinging? The weight of their contents? How hard they had been pushed?

The teacher brings the class back to today and presents the students with a mass suspended from a one meter long string. She coaxes these materials into swinging motion and asks if anyone knows what the resulting phenomenon is called. Typically, someone has heard of the *pendulum*.

She explains that this equipment will allow them to conduct a series of investigations into the properties of the *pendulum*, not unlike those that were conducted by Galileo Galilei and many others around the year 1600; investigations which became a foundation of what we think of as modern scientific methods.

She calls the students' attention to the period of time it takes for the suspended mass to swing back and forth and explains that this is called the *period of the* *pendulum*. She prods members of the class to explain in their own words what the *period of the pendulum* is until she feels secure that they have grasped the concept.

"Our goal today will be to consider how to construct a *pendulum* that has a period of one second. We may not be able to accomplish our goal in one day, but we'll take some important steps to getting there". Again she queries the class until they satisfy her that they have understood the goal.

"What do you think we need to do first"? A wide range of suggestions emerge. ... *We should try different masses. We need to drop it from a different point, etc.* The teacher guides the conversation so that the class realizes (hopefully based on a student's suggestion) that it is too early to start changing the period when the period associated with the existing configuration has not yet been established. This gives the teacher the support to steer the class to recognition of the virtues of starting with a measurement of the period.

"How shall we measure the period"? The teacher suggests to the class that they use a stopwatch. One student is called up to set the *pendulum* in motion and another to operate the stopwatch. Typically, students let it swing once and catch it. Almost inevitably the student with the stopwatch operator will call out something like "1.81".

The teacher asks, "1.81 what"?

"Seconds" responds another student.

"Write that down on your response sheet", the teacher instructs, "where it says, *Period of the Pendulum*. We're going to keep an accurate record of what we do here today".

The teacher then elicits from the students, factors that might affect the *period of the pendulum*. After compiling a list from their suggestions she says, "Today, we're going to consider just three of the factors that you have proposed: the mass of the bob, the length of the string, and the height from which the *pendulum* was dropped. Please take these into consideration in all of your reports today".

"Let's try one of them out. Let's change the height of the drop". The teacher calls on the student who first set the *pendulum* in motion to prepare to drop the bob from a different position.

"Before we set it swinging again, I would like each of you to predict what the new *period of the pendulum* will be. Write down your answer in the space provided for your prediction".

The *pendulum* is swung again and a student calls out the time.

"Now, as I mentioned before, we want to keep an accurate record of what we did and what happened. So please do the following: Give a complete description of the *pendulum* that we worked with, describe what we did, and what resulted".

"Was there a difference between what you predicted and the actual result? Describe what that difference was. Give as many reasons as you can think of to explain that difference. I want to know what *you* think; so, for now, don't share your answers with anyone else".

Next, the teacher suggests that she would like the last trial to be repeated. She asks the same student to release the *pendulum* again from the same height and members of the class, individually, to predict what the new period will be. Again, they are asked to record their description of the experimental setting and the results, and to elaborate on discrepancies between their predictions and the actual results.

The class is left with the value obtained from the stopwatch and asked to present reasons why that value might have been obtained. Then she moves forward, "Now when Galileo did such an experiment he might well have let it swing 50 times. Why do you think he would do that? Write down as many reasons as you can for why he may have wanted the *pendulum* to swing so many times".

3. The Assessment of Scientific Inquiry Capabilities

What is this teacher doing? Teaching history? ... science? ... critical thinking? Preparing students for an upcoming lesson in the *pendulum*? Indeed, all of these are intended. However, although significant, and perhaps even critical for the *learning* that is intended to follow, all these aims are put aside in deference to the teacher's immediate aim. The immediate aim is to obtain systematic information about how competent her students are in scientifically approaching tasks and problems in the natural world. In particular, this *Classroom Event* provides the teacher with information about the students' levels of competence regarding:

- describing the experimental setting,
- attending to the properties of experimental factors,
- distinguishing inference from observation,
- control of variation due to error, and
- consistent use of units of measure.

For each of the above capabilities, the teacher has available to her scales of levels of student competence that can be inferred from samples of student performance. For example, Figure 1 displays the levels of competence for 'Control of Variation Due to Error', and samples of student responses that exemplify the levels.

Instructions for rating student performance are supported by definitions, explanations, and examples that help the teacher to make reliable judgments of student performance. Once the judgment is made, it is entered into an information system that allows the teacher to keep track of students' progress on each of these *inquiry capabilities* over time. A sample report from the information system is presented in Figure 2 for a student and a class over time.

Figure 2 shows ratings of student competence in 'Control of Variation Due to Error' based on information derived from several administrations of tasks such as the *Classroom Event* (previous section). The bar graph shows that *assessments* were administered on two occasions – 8/27/02 and 9/26/02. In the 8/27 administration, 19 out of 20 students (95%) were performing at the lowest level of competence on this capability (0 – Does not attempt to control for error). One month later, evidence was available that 25% of the class expressed the need to control for

2 - Proposes a valid method to control for variation due to error
"If you take an average it will cancel out the little differences that you get each time you measure the period".
"50 trials can be more accurate than one trial".
"We should make sure that we release it from the exact same spot each time".

1 - Indicates the need to control for variation due to error, but proposes no valid method
"The period will never be the same".
"Yes, because it might not be in the same spot as before". (explaining variations by referring to differing positions from which the bob was released)

0 - Does not indicate the need to control for variation due to error

Figure 1. Levels of competence and student response samples (in italics) for a scientific inquiry capability: Control of variation due to error.

error (Level 1) and 10% were giving evidence of having available to them some means for exercising that control (Level 2 – e.g., repeated measures, averaging). This particular report, however, is intended to reveal the progress of one particular student and the classroom display is, in this case, the context against which to interpret the student's performance. Jane Jones' performance is presented in the growth chart at the top of Figure 2. Here, it becomes evident that she was the only student that had indicated the necessity to control for error on 8/27 (Level 1), and that by 9/26 she had shown evidence of a valid method of controlling for error (Level 2). Looking back into her performance history we find earlier entries indicating that in the previous school year she too had been performing at the lowest level of competence on this capability (Level 0).

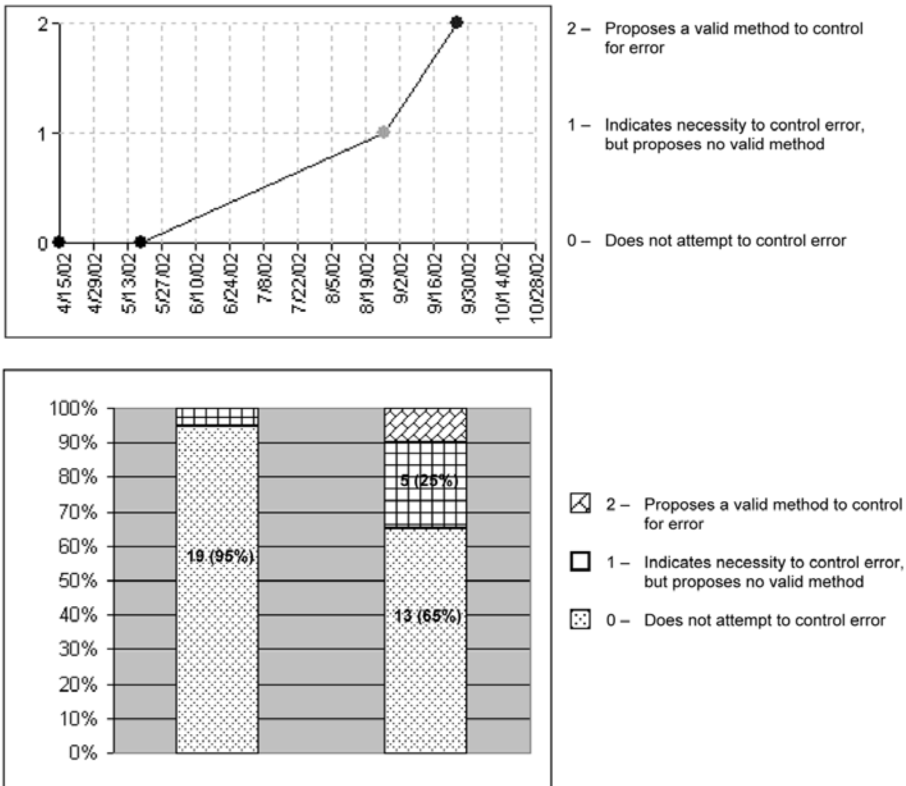
Looking back over the *Classroom Event* we can begin to see that the teacher was following a rigorous protocol, the purpose of which was to elicit information that would allow her to make valid judgments about students' level of competence regarding specific *scientific inquiry capabilities* that are *learning objectives* in her course. Explaining what is meant here by the terms *learning objectives* and *scientific inquiry capabilities* will permit a rigorous discussion of the role that *assessment* using *pendulum* phenomena can play in supporting *inquiry-based learning*.

4. Learning, Learning Objectives, and Assessment

At the heart of the educational enterprise there is a great intangible which we call *learning*. It is the goal of all educational processes. It has both process and product aspects but, in both cases, it refers to experiences or inherent qualities of students, which are ultimately imperceptible. This limitation makes it necessary to rely on careful study of observable student performance in order to make valid inferences about the state of the underlying hypothesized *learning*. To effectively make such inferences about *learning* it is necessary to have goals or objectives for *learning*

Student Summary
Jones, Jane – 20 (Spring 03-04)

Control of Variation Due to Error



	8/27/02 Pend-Obs	9/26/02 Pend-Obs
2	0	2 (10%)
1	1 (5%)	5 (25%)
0	19 (95%)	13 (65%)
Total	20 (100%)	20 (100%)

Figure 2. Sample assessment results for a student and a class over time.

and to collect evidence on the extent to which these goals and objectives have been achieved (Mager 1962). *Assessment* is the process of obtaining evidence to support inferences concerning the attainment of *learning objectives* (Zachos et al. 2000). *Assessment* is thus a necessary basis for determining whether an educational program has been successful and comparing the relative contributions to success of different methods of instruction (Johnson 1977; Tyler 1949). The present discussion will concentrate on *learning objectives* that refer to observable student performance. These are properly called *performance objectives. Performance objectives* provide practical goals of instruction and objects of *assessment* for teachers and stakeholders. In this paper the term *curriculum* refers to *learning objectives* and ordered sets of *learning objectives* (Johnson 1967).

5. A Central Problem for Student Assessment in Science Education

In the United States, high-stakes, norm-referenced tests increasingly drive *curriculum* and instruction in educational programs. Typically, these examinations do not provide reports that indicate how well students performed on specific *learning objectives* representing valued 'learning outcomes'. When such results are made available, it is often at the end of the year when it is too late to use the information to plan instruction for current students. Rather than planning instruction around valued intended *learning*, teachers find themselves gearing instruction to a 'shadow' *curriculum*, an extrapolation of what they believe students will have to know and do to perform well on the next year's high stakes test. Anyone who has worked in such settings has found teachers deliberating on how much attention is to be given to a *learning objective*, not on the basis of its contribution to further *learning* or value in solving life problems, but rather based on the likelihood that it will form a component of a high-stakes test. Often, a substantial portion of a school year is devoted simply to preparing students to do well on the exam. Ironically, this decision-making and instruction often occurs in the absence of reliable information about the level of competence of these students on the targeted *learning objectives*. The consequences of this system can be counter-productive for student, school, and state agency alike. This is a problem for the educational researcher as well because the absence of valid and reliable measures of valued *learning objectives* amounts to nothing less than the absence of dependent variables for the educational enterprise. The question that must arise from such a situation is, "How can we build a program for developing higher order cognitive capabilities in secondary school science in which *assessment* plays a supportive role to teaching and *learning*, rather than serves as a distraction from or inhibitor to these processes"?

6. Theoretical and Empirical Foundations for Assessing Scientific Inquiry and Discovery

6.1. THE EMPIRICAL DEFINITION OF SCIENTIFIC INQUIRY CAPABILITIES

In 2000, Zachos, Hick, Doane, and Sargent approached the question of how to identify the capabilities that should be part of a secondary school science *curriculum* in *scientific inquiry*. The study documented wide agreement on the notion that the *discovery* of concepts which bring to light relationships in the natural world was the chief aim of the scientific enterprise. (The word 'concepts' is used here as a surrogate for laws, rules, principles etc.). Moreover, that study reported, again, wide agreement among scientists, historians of science, philosophers of science, cognitive scientists, and science educators (in spite of their diverse terminologies) that *discovery* has two facets – 'concept building' and 'concept testing'. Operationally, for use in educational programs, *scientific discovery* was defined as growth in level of conceptualization of a phenomenon based on building and testing concepts in a direct phenomenal inquiry. Therefore, a *scientific inquiry capability* could be defined as any human attribute that contributes to success in making *scientific discoveries* as defined. A distinction was made in this study between the 'historical' *discoveries* made by scientists and the 'personal' *discoveries* made by students.

On this foundation, methods were developed for empirically validating human attributes as *scientific inquiry capabilities* (i.e., demonstrating that they contributed to success in making *discoveries*). The same body of research literature used to define *discovery* (above) was culled to identify human attributes (skills, knowledge, and dispositions) that were perceived as contributing to success in making *discoveries*. Some 30 or so *scientific inquiry capabilities* were identified. The method employed was a study of the correlation between the *inquiry capabilities* and success in *discovery* (i.e., growth in 'level of conceptualization' through building and testing concepts in the context of a phenomenon-based inquiry). Because this was an 'observational study' (Rosenbaum 1995), causal links between *scientific inquiry capabilities* and *discovery* could not be established. However, instruments were developed for measuring these variables and associative links were established that set the stage for future studies study of how these capabilities might contribute to success in *discovery. Curriculum* development can now go beyond the previous limitation of identifying *learning objectives* through judgments by experts and negotiation among stakeholders (Ravitch 1995). A *curriculum* for *scientific inquiry* could now be developed by setting *learning objectives* for *inquiry capabilities* that have been validated against the criterion of their contribution to success in making *discoveries*.

6.2. STRUCTURED INQUIRY

Tasks were developed which provided opportunities to elicit growth in conceptualization and competence in conducting *inquiry* (Zachos et al. 2000). It is a special characteristic of these tasks that success is based on building and testing conceptualizations of the phenomenon and that students receive no assistance related to concepts or methods. Three tasks were administered to 33 upper level secondary school students by a facilitator/teacher. Students were presented with a phenomenon and a problem. The tasks are structured in such a way that brings students into a direct conversation with Nature, in which each step of *inquiry* results in an immediate response, telling the students whether their concepts and methods of investigation are working. The facilitator elicits the students' concepts, reasoning, and dispositions without mediating the quality of student performance. This special way of presenting phenomenal tasks is referred to as *structured inquiry*. Three phenomena were adopted from the research of Inhelder and Piaget (1958) in order to take advantage of their thoughtful analyses of young people's conceptualizations and *inquiry* strategies. These phenomena were – Floating and Sinking, Equilibrium on the Balance Beam, and the *Period of the Pendulum*. The task associated with the *pendulum* was to come up with a method for building a *pendulum* for any given period that the facilitator might propose.

6.3. OVERALL FINDINGS - CRITICAL SCIENTIFIC INQUIRY CAPABILITIES

The results of that study (Zachos et al. 2000) revealed correlations between success in *discovering* the concepts underlying these phenomena on the one hand, and the following *scientific inquiry capabilities*:

- coordinating theories with evidence,
- searching for an underlying principle,
- concern for precision,
- identifying sources of error in measurement, and
- proportional reasoning.

7. Special Findings and Considerations Related To The Investigation of Pendulum Periodicity and the Role of the Pendulum in Assessing Scientific Inquiry Capabilities

While each of these three phenomena had special characteristics and points of interest for *assessment* and implications for subsequent instruction, the *pendulum* task was especially notable in that it posed striking problems for students as they attempted to build and test concepts that could explain periodicity. Inhelder and Piaget (1958) had already pointed out that the ability to control variables was crucial to success on this task. However, a number of other capabilities proved necessary even before control of variables could be properly applied. Most dramatically, students were unable to make progress identifying the factors which affect the *period of the pendulum* until they confronted problems associated with error.

Real progress, then, in establishing the factor underlying periodicity requires:

- systematic organization of measurements taken from observations,
- distinguishing variation attributable to undetermined factors, or factors not under the control of the experimenter, from variation attributable to manipulations of the independent variables,
- having means for controlling for the effects of indeterminate factors (e.g., repeated measures, averaging), and
- exercising systematic control over independent variables in testing for effects.

These elementary capabilities are needed in order to disentangle the factors affecting the *period of the pendulum*. *Learning objectives* associated with these capabilities should be parts of the foundation of a *curriculum* for scientific thinking and *scientific inquiry*. The *Classroom Event* depicted earlier in this paper is designed to elicit evidence of student competence related to measurement, error, and the control of variables based on the findings of this study.

Just as Galileo's investigation of the *pendulum* became a foundation for the establishment of what is now considered the scientific approach to investigating phenomena, so the investigation of *pendulum* periodicity by students today can be a foundation for assessing and developing the fundamental capabilities that are needed for a scientific approach to the investigation of phenomena. The search for the causes underlying *pendulum* periodicity has the marvelous property of serving as a guardian at the gate, refusing access to the properties of periodicity for those investigators who are not competent in measuring, working with error, and controlling variables.

This phenomenon and task do not, however, exhaust the value of the *pendulum* as a tool for eliciting capabilities related to *scientific inquiry*. The investigation of *pendulum* periodicity can be an opportunity to assess and develop many other *scientific inquiry capabilities* including:

- techniques for organizing and presenting data,
- using evidence to support hypotheses concerning correlation,
- causal reasoning,
- reasoning with proportions, and
- modeling relationships between variables through graphing and mathematical functions.

Indeed the phenomenon continues to be a rich source of opportunities for empirical and theoretical study beyond the issue of causes of *pendulum* periodicity. Investigations into cycloids, isochronicity, harmonic motion, and issues of friction and drag associated with different shaped bobs etc., can provide links to mathematics, physics, engineering, and technology, as well as opportunities to build and test concepts. Capabilities associated with more sophisticated techniques for dealing with error can be developed through a link to elementary and advanced statistical methods. A host of important *learning objectives* can be assessed and developed around *pendulum* phenomena.

8. Moving From Educational Research to Educational Practice: The Extension of the Assessment of Scientific Inquiry to the Classroom

From its inception, the research just described was intended to set foundations for teaching *scientific inquiry capabilities*. For this reason, the *scientific inquiry capabilities* were designed so that their top levels of competence constituted valued *learning objectives* appropriate for secondary school instruction and *assessment*. The integration of these *learning objectives* with *inquiry* tasks that are imbedded in natural phenomena assures that they are *performance objectives*, and therefore, that judgments of level of competence can be based on the evidence of observable student performance. The primary problem with the method of *structured inquiry* described above is that it requires one facilitator for each student assessed. While this is a highly desirable ratio for *assessment* or instruction, it cannot be practically realized in existing educational institutions.

8.1. EXPLICATING THE CLASSROOM EVENT

In view of this problem, versions of *structured inquiry* appropriate for classroom use have been developed. The *Classroom Event* presented above is part of just such an instrument. Classroom versions of *structured inquiry* can be administered to a full class of students who provide their responses on a special form. The teacher's presentation is intended to engage student interest in the phenomenon and the investigation. After administering such an *assessment task* the teacher reviews student responses and enters judgments of student competence on the targeted *learning objectives* into an information system in about the time it takes to score a test or writing assignment.

In administering a task of this sort to a class, there must be great flexibility in wording so as to allow the teacher to speak meaningfully to the group of students present, to their levels of knowledge and skills, their dispositions, and the contexts of their lives. Ideas and suggestions elicited from students as part of this conversation are not those that are being assessed. What is important is that the teacher's presentation elicits the targeted capabilities without providing leading information that would confound the aim of identifying the students' spontaneous conceptualizations and true levels of competence. In the *Classroom Event*, the repeated requests for comparisons between prediction and result require reflection and judgments concerning sources of variation, and particularly on the possible effects of error. The use of a stopwatch with readings to hundredths of a second ensures that students will run head on into problems of error related to their measurements and manipulations.

The story, which serves as prelude to the *assessment*, is left to the imagination of the teacher but must serve the purpose of motivating students, both to achieve their active participation in the *assessment* and to set the stage for subsequent instruction. The story, if successful, will activate the imaginative capacities of students, stimulate interest in divergent areas of study, and prepare the ground for subsequent empirical investigations. A virtue of the phenomenon of the *pendulum*, at the heart of the story, is that it can pose a real problem, one that "awakens students' interest and encourages them to ask questions and seek answers" (National Research Council 1996, p. 146).

The *assessment activity* described here is at the most elementary level of introduction to the scientific reasoning and the phenomenon of periodicity. It is also presented in a manner that is highly teacher-directed. Once the capabilities associated with this activity are attained, the class can move to more independent *assessment* and instructional activities. For instance, a logical next step is student-directed design and conduct of experiments to test for causal relationships between various factors and the *pendulum's* period.

8.2. CONTRIBUTIONS OF ASSESSMENT BASED ON STRUCTURED INQUIRY

The *Classroom Event* thus presents the actual protocol for administering a *performance assessment* of *scientific inquiry capabilities*. It is designed to combine the rigor of administration of a standardized test with the flexibility needed to interact with students at their level of comprehension and performance in the presence of a natural phenomenon of interest. The *Classroom Event* elicits many of the same capabilities that were associated with success in *discovery* in the research cited above, and that were identified as critical to success in investigating the factors underlying *pendulum* periodicity.

For the science teacher, the primary obstacles to using *performance assessments* have been inadequate awareness of the principles for constructing valid and reliable *assessments*, the time it takes to construct tests, and the time it takes to administer and score *assessments* (Stecher & Klein 1997). The *assessment* methods described here solve these problems by providing teachers with ready-made *assessment* instruments that yield reliable information concerning the extent of attainment of higher order *learning objectives*, but take no longer to administer and score than classroom tests. Because the students must apply their knowledge and skills in the context of a problem posed by a natural phenomenon, the associated *assessment* and instruction address the higher order capabilities associated with the upper levels of Bloom's taxonomy of educational objectives in the cognitive domain (Bloom et al. 1956).

Yeh suggests that:

> If state-mandated tests focused on critical thinking rather than more recall of factual knowledge, teachers who feel pressured to teach to the test, could focus on teaching critical thinking rather than on the universe of items that student might otherwise be asked to recall (2001, p. 12).

Assessments based on *structured inquiry* require students to demonstrate competence by applying their *inquiry capabilities* to a task or problem lodged in a phenomenon. To solve the problem, the student must build and test concepts about the phenomenon. Consequently, the use of such *assessments* to evaluate programs that aim at developing *inquiry capabilities* should support the type of instruction that will most effectively achieve these capabilities, this will more than likely be *inquiry-based learning*. Having students working with living phenomena can also serve as a remedy against excesses of 'intellectualism and cognitivism' in science education pointed out by Dahlin (2001). These excesses are thought to alienate academically successful science students from the natural world, and to alienate more experientially and aesthetically inclined students from the subject matter and process of science itself.

The great argument for high-stakes, norm-referenced tests have been the 'reliability' of the scores that they generate and this has been achieved through classical standardization procedures (Thorndike 1971). Black has pointed out the dangers of decisions based on unreliable *assessment* results (1998). However, 'reliability' can also be achieved through professional development (Brennan, 2001). Judgments emerging from rating student responses to *structured inquiry* sessions can be highly reliable when sufficient attention has been taken to prepare the teacher to understand the distinguishing characteristics of the levels of competence for each *learning objective*.

9. Conclusion

I hope that I have been able to show that *assessment*, far from necessarily trivializing educational programs, can contribute to the achievement of worthy educational goals, such as the development of the human capabilities leading to the potential to infer natural law. The *pendulum* has a special contribution to make towards achieving such goals. The *pendulum's* historical role in building the foundations of scientific approaches to the physical world can be re-experienced at the level of the individual. Its continued importance in the evolution of critical scientific concepts (Peters 2002) makes it a prime candidate for a core around which to build a *curriculum* of scientific thinking, *scientific inquiry capabilities*, and fundamental scientific concepts. Matthews (2000 & 2002) has made the case that learning about the *pendulum* can improve science education. It is also true that learning about how our students conceptualize and investigate the *pendulum* can improve science education.

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Roles of Abductive Reasoning and Prior Belief in Children's Generation of Hypotheses about Pendulum Motion

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Abstract. The purpose of the present study was to test the hypothesis that student's abductive reasoning skills play an important role in the generation of hypotheses on pendulum motion tasks. To test the hypothesis, a hypothesis-generating test on pendulum motion, and a prior-belief test about pendulum motion were developed and administered to a sample of 5th grade children. A significant number of subjects who have prior belief about the length to alter pendulum motion failed to apply their prior belief to generate a hypothesis on a swing task. These results suggest that students' failure in hypothesis generation was related to abductive reasoning ability, rather than simple lack of prior belief. This study, then, supports the notion that abductive reasoning ability beyond prior belief plays an important role in the process of hypothesis generation. This study suggests that science education should provide teaching about abductive reasoning as well as scientific declarative knowledge for developing children's hypothesis-generation skills.

1. Introduction

A hypothesis is defined as a single proposition proposed as a possible explanation for the occurrence of some observed phenomena (Barnhart 1953). Science educators know well that scientific reasoning involves generating as well as testing hypotheses. Specifically, hypothesis generation has been regarded as one of core reasoning processes in creative thinking and scientific discovery (Klahr & Dunbar 1988; Kuhn et al. 1988; Ohlsson 1992; Lawson 1995; Kwon et al. 2000). However, regardless of its importance, teaching of science and science textbooks have heavily concentrated on procedures for testing hypotheses (e.g., designing experiment, manipulating and controlling variables, collecting data, measurement, analyzing data) and largely ignore procedures for generating the hypotheses (Kwon et al. 2000).

A particular hypothesis might be generated quickly. On the other hand, a complex, revolutionary hypothesis might take some time to form. Sometimes a reasoner leaps to a hypothesis almost as soon as she/he sees the problem while another reasoner needs to puzzle a long time to generate such a hypothesis about the same problem. What, then, is the principal difference

M.R. Matthews et al. (eds.), The Pendulum, 363–376.

between good and poor reasoners in generating hypotheses? Generally speaking, there are two factors that influence hypothesis generation. One factor is reasoner's prior belief and the other is the reasoner's ability to retrieve that stored belief.

An important alternative factor exists that could explain the performance differences in terms of the presence or absence of declarative knowledge, as opposed to procedural knowledge (Anderson 1995), specific to the solving of each task (c.f., Korthagen & Lagerwerf 1995). Science instructors might assume that their students' prior declarative belief plays a crucial role in their ability to acquire new concepts. This view regarding the importance of prior declarative knowledge has been largely stated in terms of Ausubel's theory of meaningful learning (Ausubel et al. 1978; Novak & Musonda 1991). In other words, according to this domain-specific prior knowledge hypothesis, if students have acquired the necessary declarative knowledge, they will successfully generate alternative hypotheses. Lacking that knowledge, however, they will fail. The acquisition of declarative knowledge is not only a necessary condition for hypothesis generating, it is also sufficient. In this sight, it means that our students should have only specific declarative knowledge about the period of pendulum motion for solving successfully a pendulum task.

Traditionally, two types of reasoning, induction and deduction have been recognized in the logic of science. By means of induction one ascertains how often in the ordinary course of experience one phenomenon will be accompanied by another. No definite probability attaches to the inductive conclusion. By means of deduction one predicts the special results of things and calculates how often they will occur in the long run. A definite probability should be attached to make deductive conclusion. According to Hempel (1966) and Popper (1968), science proceeds by free formulation of a hypothesis from which empirical tests can be deduced or induced. However, a long line of discussion has shown that another type of scientific reasoning, called retroduction or abduction, in addition to induction and deduction exists in scientific endeavor (Peirce 1903; Burk 1946; Lawson 1995; Giere 1997; Kwon et al. 2000). For example, the American philosopher Charles S. Peirce recognized a distinctive type of scientific reasoning, called "abduction" in his terminology, which is a necessary condition for successful performance of scientific task. Abduction is the mental process of generating hypotheses in which an explanation that is successful in one situation is borrowed and applied as a tentative explanation in a new situation (Hanson 1958; Lawson 1995; Kwon et al. 2000; Fisher 2001).

Hypothesis generation involves presumably a reasoning procedure involving exploring, combining, comparing, and selecting possible alternatives (Kwon et al. 2000). First, the process of hypothesis generating starts with identifying qualitative constructs of the current causal question and the previously experienced world which may have a strong qualitative likeness to the current situation. Second, the scientific reasoner explores and combines several explanations of the previously experienced world. Third, the reasoner compares the combined several explanations and selects the most likely one based on the qualitative likeness between the current situation with its question and the previously experienced world. Finally, the reasoner uses the selected explanation as the hypothesis for the current question. This process of hypothesis generating is shown diagrammatically in Figure 1.

Figure 1 has four boxes, labeled "Questioning Situation", "Experienced Situation", "Hypothetical Explicans", and "Causal Explicans". These boxes mean prior beliefs which are represented during the process of scientific hypothesis generation. The questioning situation is a fact, event, type of behavior, etc., of an experimental situation. The experienced situation is an experience or prior belief that is similar to the questioning situation. The causal explicans are causal factors that explain the experienced situation. The hypothetical explicans are causal factors that explain the questioning situation.

Figure 1 has three kinds of arrows indicating the boxes of prior beliefs. The first one (↔) is "comparing" which is the process of comparing between the questioning situation and the experienced situation on the basis of similarity among these situations. The second one (↑) is "explaining" by which the situation are explained causally with the explicans. The last one (←) is "borrowing" by which the causal explicans are borrowed as the hypothetical exlicans for the questioning situation on the basis of the similarity between the questioning situation and the experienced situation.

Ohlsson (1992) also has argued that the structure of explanation, which is one of the types of theory articulation, is required to produce explanation patterns the person already knows. Furthermore, Ohlsson argued the most important type of theory articulation is explanation, which is a narrative of the cause/effect events that produced the observed phenomenon. In his arguments, to generate a hypothesis, in part, is to explain an observed phenomenon that is associated with a theory. For example, the Darwinian explanation for the long neck of the giraffe, the hypothesis of evolution consists of two basic causes, the random variation and the natural selection.

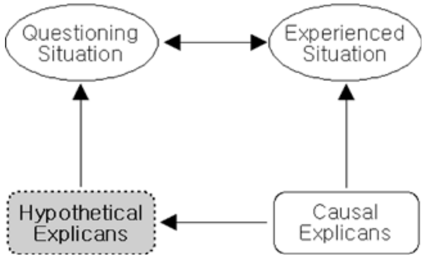


Figure 1. An abduction model of hypothesis-generating process.

These causes which are used to explain the giraffe's long neck can be used to explain other animals' anatomy. Ohlssen's notion might be an argument to support the abduction model of hypothesis-generating process in this study.

In other ways, recent studies have argued that in addition to prior belief, the skills to represent the prior belief from his/her cognitive structure are pivotal factors in hypothesis generation (Kwon et al. 2000; Kwon et al. 2003a; Kwon et al. 2003b). Therefore, the primary purpose of the present study is to test the notion that student's abductive reasoning skills exist in addition to prior belief as defined above. Furthermore, we test the hypothesis that one's abductive reasoning skill is required for successful generation of hypotheses in pendulum motion tasks. To test the hypothesis, we conducted two experiments.

Pendulum motion has been regarded as a significant role not only in educational and psychological research but also in teaching children's logical thinking since Jean Piaget's initial work. Piaget and Inhelder, in their *The Growth of Logical Thinking from Childhood to Adolescence* (Inhelder & Piaget 1958), describe the pendulum motion tasks that Piaget and Inhelder gave to children to ascertain the extent to which they could isolate and manipulate potential variables (length, amplitude, weight) that affected the period of pendulum motion. Performing the task of isolating and controlling the variables was considered as a window into the child's logical ability and their developmental process. The present study adapted the pendulum task as an experimental tool to investigate children's hypothesis generation.

2. Methods

2.1. SUBJECTS

For the experiment I, a sample of 290 five graders (164 female, 126 male) were selected from four elementary schools located in a Korean metropolitan area. The subjects' age ranged from 9.9 to 12.1 years (mean age = 11.1 years, SD = 0.4). In addition, a sample of 34 five graders (18 female, 16 male) was selected from one elementary schools for the experiment II. The subjects' age ranged from 10.0 to 12.2 years (mean age = 11.2 years, SD = 0.4).

2.2. INSTRUMENTS

In experiment I, we developed and administered the hypothesis-generation test and the Prior-Belief Test I to the subjects.

Hypothesis-generation test. A hypothesis-generation test was designed to test hypothesis generation about a swing situation which relates presumably to a simple pendulum motion. There were two swings moving at different speeds in the test. Children were asked to think about what cause makes a difference in the speed of the swings. A face validity of this test was investigated. To obtain the measure of face validity of the test, nine experts (a professor and eight graduate students majoring science education) were asked a question, "Which item in the test can assess 5th grade elementary school students' hypothesis-generation in a swing situation which presumably relates to the simple pendulum motion?" The result of responses indicated that the face validity of this test was 0.87. The hypothesis-generation test is as follows:

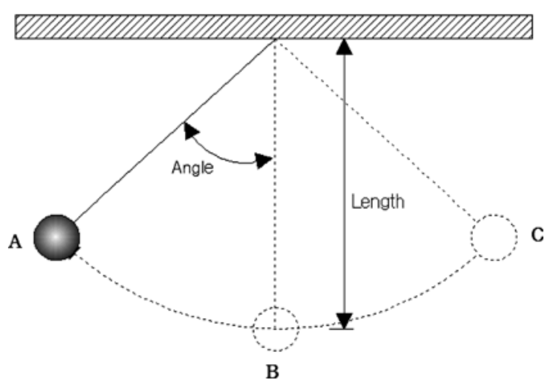
> While Mary and Joe got on each swing side by side, they detected that Joe's swing is faster than Mary's. So they decided to measure the speed of two swings. They didn't take any actions such as swinging their legs during the measuring. The measuring results are shown in following a table.

The periods of Mary's swing and Joe's swinging

Swing	Period (time for 10 swings)
Mary's	32
Joe's	25

> Why did Joe's swing go back-and-forth faster than Mary's? What caused the difference in the speeds between Joe's swinging and the Mary's swinging?

Prior-belief test I. Children's prior belief about pendulum motion was assessed by a multiple-choice test. In the test, subjects were asked to respond with one of three variables which affect the period of the pendulum motion. A face validity of this test was investigated. To obtain the measure of face validity of the test, nine experts (a professor and eight graduate students majoring science education) were asked a question, "Which item in the test can assess prior belief about pendulum motion of 5th grade elementary school students?" The result of responses indicated that the face validity of this test was 0.90. The prior belief test is as follows:



> In this experiment, the bob swings repetitively back-and-forth along the line from A through B to C and back again A. The time of each swing is the period of the pendulum motion. What factor causes to increase or decrease the pendulum period?

Following picture shows an experimental setting of a simple pendulum.

In experiment II, we also administered the hypothesis-generation test that was used in experiment I to identify children's hypothesis about swing. To assess the children's prior-belief about pendulum, we used the prior-belief test II which was open response type paper test. This test was slightly modified from the prior-belief test I which was a multiple-choice test.

2.3. PROCEDURES AND PREDICTION

In this study, there were two study hypotheses for test, prior-belief and belief-abduction hypotheses. The prior-belief hypothesis is that prior belief is the only one factor that influences children's hypothesis-generation. The belief-abduction hypothesis is that there are two factors that influence hypothesis generation. One factor is children's prior belief and the other is children's ability to use the stored belief.

In the experiment I, test procedures were divided into two sessions. In the first session, the hypothesis-generation test was administered to identify hypotheses of the children. In the second session, we assessed the children's prior belief about pendulum motion with the prior-belief test I. By the types of the children's prior belief, they were divided into one of three groups, namely angle, length, and weight prior-belief groups. According to the prior-belief hypothesis, if children have prior belief about potential variables that affect the period of pendulum motion, they should generate hypotheses on the swing task which is related to the variable already thought to vary the pendulum motion. The predicted patterns of results for the prior-belief hypothesis are shown as Figure 2.

An alternative explanation is the belief-abduction hypothesis. In this view, children's successful performance on the swing task is affected by their not

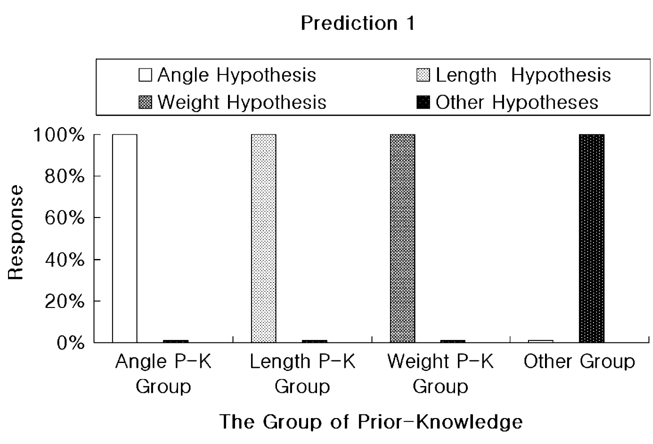


Figure 2. The predicted results of the prior-belief hypothesis.

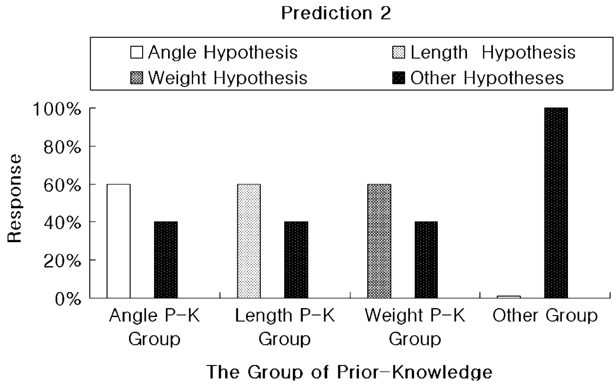


Figure 3. The predicted result of the belief-abduction hypothesis.

only prior belief but also abductive reasoning skills. The predicted result of this hypothesis is shown as Figure 3.

In the experiment II, test procedures were divided into two sessions. In the first session, we taught all children pendulum motion knowledge, and then assessed the pendulum motion belief with the prior-belief test II. In the second session, the hypothesis-generation test was administered to identify hypotheses of the children. And then the prior-belief test II was again administered to the children. Finally, we interviewed the children how they generated their hypotheses. Table 1 shows teaching and test procedures.

In experiment II, there were also two study hypotheses for test, prior-belief and belief-abduction hypotheses. According to the prior-belief hypothesis, all of the subjects who have prior belief that length of a pendulum affects period of pendulum motion should generate hypotheses using the belief on the swing task which is related to the pendulum motion. The predicted results of the prior-belief hypothesis are shown as the Prediction 3 of Figure 4.

On the other hand, the belief-abduction hypothesis leads to a prediction that some of children who have the pendulum belief cannot generate hypotheses on the swing task. These children are insufficient in the abduction

Table 1. Test procedures of experiment II.

Session	Step	Procedure
I	1	Teaching
	2	Prior-belief test II
II	1	Hypothesis-generation test
	2	Prior-belief test II
	3	Interview

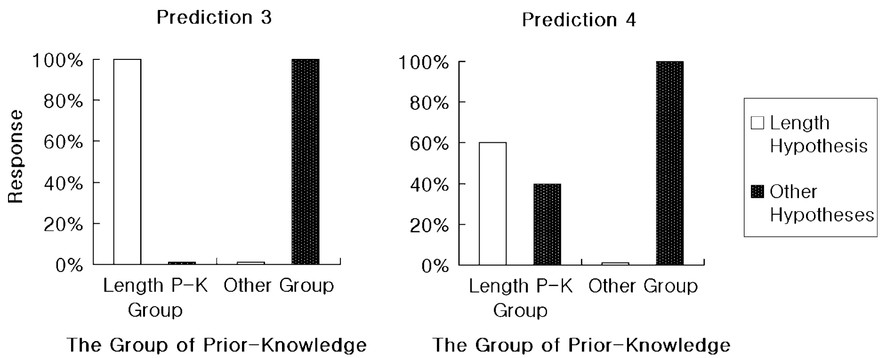


Figure 4. The predicted results of the prior-belief and the belief-abduction hypotheses.

ability of using the belief. The predicted result of this hypothesis is shown in the Prediction 4 of Figure 4.

3. Results and Discussions

3.1. TYPES OF CHILDREN'S HYPOTHESES

The types of children's hypotheses which were generated on the hypothesis-generation test in the first experiment are reported in Table 2.

As shown in the Table 2, there were three types of children's hypotheses. The first type (1–11 in Table 2) involves internal factors of a simple pendulum. In this type, 1–3 hypotheses are related to the angle of a pendulum, 4–6 are related to the length of a pendulum, and 7–11 are related to the weight of a bob. The second type (12–22) involves to external factors, such as skill, exercise, and wind. Hypotheses 23–25 are classified as a non-scientific hypothesis. The 23rd is just the repetition of the question while the 24th and 25th are far from an answer to the hypothesis-generation question.

3.2. FREQUENCY OF HYPOTHESES BY PRIOR-BELIEF GROUPS

Children's responses to the prior-belief test I are summarized in Table 3.

As shown in Table 3, 86 (29.7%) children from 290 respondents claimed that the angle of the swing determines the period of the pendulum motion. Sixty (20.7%) children responded to the length of the string, 135 (46.6%) children responded to the weight of the bob, and 9 (3.1%) children responded to that they didn't know.

To test the roles of subject's prior belief and abduction, we analyzed the first type hypotheses by prior-belief groups. Table 4 shows frequencies of the first type hypotheses by each prior-belief group.

Table 2. Children's hypotheses on the hypothesis-generation test in experiment I.

Types of hypotheses	Frequency
Internal factors	168
1. Joe's swing went up lower than Mary's.	18
2. The moving distance of Joe's swing was shorter than Mary's.	8
3. Joe's swing went up higher than Mary's.	10
4. The ropes of the swings were different in length.	4
5. The length of the ropes of Joe's swing was shorter than Mary's.	18
6. The length of the ropes of Joe's swing was longer than Mary's.	8
7. Mary's weight and Joe's weight were not the same.	6
8. Joe's weight was lighter than Mary's.	51
9. The weight of Joe's swing was lighter than Mary's.	6
10. Joe's weight was heavier than Mary's.	37
11. The weight of Joe's swing was heavier than Mary's.	2
External factors	90
12. Joe was able to get on a swing with specific skill.	17
13. Joe begun to get on a swing earlier than Mary did.	15
14. Joe swung better than Mary did.	12
15. Joe was stronger than Mary.	10
16. Joe's swing was new.	9
17. Wind stud against the movement of Mary's swing.	7
18. There was something wrong with Mary's swing.	6
19. Joe took exercise better than Mary did.	4
20. Joe had a fine physique.	4
21. Joe's swing was older than Mary's.	3
22. Joe's swing had little frictional resistance.	3
The others	32
23. Joe's swing was faster than Mary's.	15
24. Joe measured exactly the swing time.	3
25. The speeds of the swings of are same.	2
26. I don't know.	12
Total	290

As shown, the percentages for angle and the other (length, weight, and other) hypotheses were 19.8 and 80.2% in the angle prior-belief group. For the length prior-belief group, the percentage of the length and the other (angle, weight, and other) hypotheses were 30.0 and 70.0%, respectively. In addition, the percentage of the weight and the other (angle, length, and other)

Table 3. Summary of children's responses to the prior-belief test I.

Prior-belief	Number (%)
Angle	86 (29.7)
Length	60 (20.7)
Weight	135 (46.6)
I don't know	9 (3.1)
Total	290 (100)

Table 4. The frequency of hypotheses by prior-belief groups in experiment I.

Hypothesis	Prior-belief group				
	Angle (%)	Length (%)	Weight (%)	I don't know	Total
Angle	17 (19.8)	7 (11.7)	12 (8.9)		36 (12.4)
Length	7 (8.1)	18 (30.0)	5 (3.7)		30 (10.3)
Weight	22 (25.6)	14 (23.3)	66 (48.9)		102 (35.2)
Other	40 (46.5)	21 (35.0)	52 (38.5)	9 (100)	122 (40.1)
Total	86 (100)	60 (100)	135 (100)	9 (100)	290 (100)

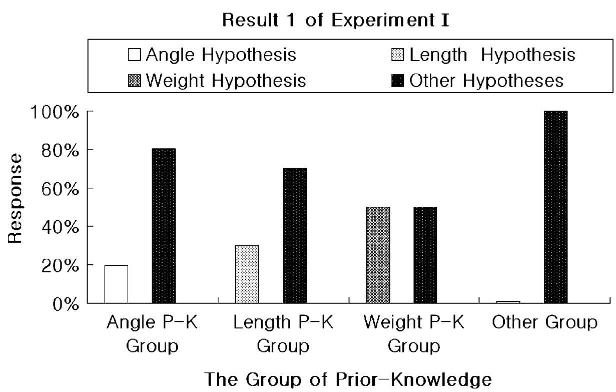


Figure 5. The percentages of hypotheses by prior-belief groups in experiment I.

hypotheses were 48.9 and 51.1%, respectively, for the weight prior-belief group. A bar chart showing the percentage is plotted as Figure 5.

To test the roles of subject's prior belief and abduction, we also analyzed the type hypotheses by prior-belief groups in experiment II. Table 5 shows frequencies of the first type hypotheses by each prior-belief group.

Table 5. The frequency of hypotheses by prior-belief groups in experiment II.

Hypothesis	Prior-belief group		
	Length (%)	Other (%)	Total
Length	19 (61.3)		19 (55.9)
Other	12 (38.7)	3 (100)	15 (44.1)
Total	31 (100)	3 (100)	34 (100)

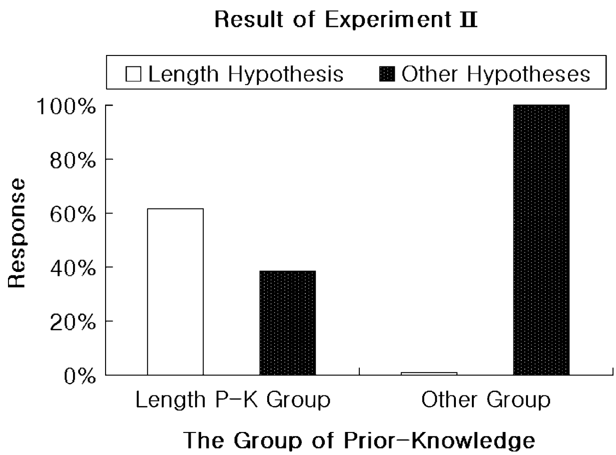


Figure 6. The percentages of hypotheses by prior-belief groups in experiment II.

As shown in Table 5, 31 subjects from 34 respondents claimed that the length of the swing determines the period of the pendulum motion. The percentages for length and other hypotheses were 61.3 and 38.7% in the length prior-belief group. For the other prior-belief group, the percentage of the length and other were 0 and 100.0%. A bar chart showing the percentage is plotted as Figure 6.

These results can be compared with the predicted results shown in Figures 2–4. According to the prior-belief explanation, all children who have acquired the necessary prior belief should successfully generate hypotheses related to their prior belief. However, Figures 5 and 6 reveal that many children did not employ their prior-belief about pendulum motion to generate hypotheses on the swing task.

In experiment I, 17 of the angle group, 18 of the length group, and 66 of the weight group students generated hypotheses related to their prior-belief. The result of the Chi-square test showed a statistically significant difference between the expected and the observed frequencies in the angle group ($\chi^2 = 5536018.50$, $p < 0.001$), the length group ($\chi^2 = 2939974.75$, $p < 0.001$) and the weight group ($\chi^2 = 3526599.00$, $p < 0.001$), respectively. In addition, only 19 of the length group students generated hypotheses related to the length prior-belief in experiment II. The result of the Chi-square test of experiment II indicated that the difference between the expected and the observed frequencies was statistically significant ($\chi^2 = 464501.41$, $p < 0.001$). Thus, these results do not conform to the expectation of the prior-belief hypothesis.

However, the Predictions 2 and 4 related to the belief-abduction hypothesis in Figures 2 and 4 fit more closely to the results of Figures 5 and 6. According to the belief-abduction hypothesis, some of the children who have the pendulum belief could not generate hypotheses on the swing task. Therefore, the results of the Figures 5 and 6 provide positive evidence to support the belief-abduction hypothesis in children's hypothesis generation.

Cognitive psychologists believe that the process of knowledge generation is an interaction between declarative knowledge and procedural knowledge (Anderson 1995; Gagne et al. 1997; Solso 2001). For example, a scientist might generate a hypothesis that is a proposition intended as a possible explanation for an observed phenomenon. The input information ("Why do male Dawson's bees exist in two distinctly different size groups? I need to generate a hypothesis for the phenomenon.") would be transformed to produce an output (the hypothesis: minor males are the incidental byproduct of external environmental factors) that looks quite different from the input. The previous example shows that procedural knowledge is used to operate to generate a hypothesis and knowledge (such as a hypothesis) generation is the result of interaction between declarative knowledge for prior knowledge and procedural knowledge for abductive reasoning.

In the experiment II, we interviewed the subjects to confirm the cognitive process of hypothesis-generation. We requested that the subjects say everything that went through their mind while they generated a hypothesis. The results of this additional interview support the belief-abduction hypothesis. Actually 17 (89.5%) children from 19 respondents who generated length hypotheses on the hypothesis-generation test responded that they represented the simple pendulum motion that was taught by the researchers during the teaching session. In addition, 9 (75.0%) children from 12 respondents instantly generated length hypotheses when we presented the simple pendulum motion task. These results indicate that nine subjects' failures were caused by insufficiency of representing a experienced situation that was similar to the questioning situation.

4. Conclusion and Implication

The results of this study provide support for the hypothesis that students' abductive reasoning skills beyond typical sorts of prior or declarative knowledge exist and might be required for generating scientific hypotheses in the hypothesis-generation test. This study shows that these skills appear to have been used by students to generate hypotheses for the swing task. Evidence suggests that the presence of prior belief of pendulum motion alone is not sufficient to produce successful hypothesis-generation performance. Hypothesis generation seems to require, we call, *abductive reasoning skills* of declarative knowledge, which is beyond possessing prior knowledge. This is not to say that prior belief is unimportant in generating hypotheses. Prior belief, such as that relating to angle, length, and weight about pendulum motion may have been used as resources for generating hypotheses. Results of this study, then, support the notion that knowledge generating in science is completed by a compensatory interaction between prior belief and scientific reasoning, such as abductive reasoning described in this study.

Results of experiment II showed that 12 from 31 children who claimed that length of pendulum affected the period of pendulum motion did not generate a length hypothesis for the swinging, and 9 (75.0%) children from 12 respondents generated length hypotheses for the hypothesis-generation test. These results support the notion that hypothesis generation requires reasoning skills to explore, select, and represent possible alternatives in explanations of the previously experienced situation. Clearly, this is an example of what we call abductive reasoning skills. Interestingly, the conclusion that a new cause to produce hypothesis generation in pendulum motion task exists seems to have been foretold by Charles S. Peirce (Peirce Edition Project 1998). Although Peirce's notion of retroductive reasoning was not widely recognized by philosophers of science and educators, several studies found that his notion of retroductive or abductive reasoning beyond inductive and deductive reasoning exists in scientific reasoning.

Hypothesis-generation skills could be applied to a broad range of students' reasoning development. School science teachers should not only concern themselves with introducing declarative knowledge in the classroom. They also should concern themselves with developing students' reasoning abilities, which are skills to generate and test hypotheses. To do this, a careful analysis and selection of various scientific issues and the development of those selected issues as educational materials should become a matter of concern in science teaching. Pendulum motion tasks possess systematic cognitive-sequencing and plenty of scientifically declarative knowledge. Therefore, pendulum motion can be used as an educational material for teaching and developing students' hypothesis-generating skills.

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Types of Two-Dimensional Pendulums and Their Uses in Education

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Abstract. Pendulums which swing in two dimensions simultaneously and are designed to leave a record of their motion are termed 'harmonographs'. The curves which they draw are known, alternatively, as 'Bowditch curves' or 'Lissajous curves'. A variety of designs of harmonographs have been invented over the years. These may be a 'Y-suspended' 'simple' pendulum, or they may be a complex 'physical' pendulum system. Harmonographs have been built as demonstration apparatus in physics (or mathematics) or as 'art' machines for enjoying the aesthetics of the curves produced.

1. Introduction

The pendulum and its motion are frequently topics in a beginning physics course. A 'simple' pendulum may be considered as a point mass fastened to the end of a massless cord. It is then allowed to swing in a plane, its motion being constrained to one dimension. The history and the educational importance of such a device have recently been discussed by Matthews (2000, 2001).

If a pendulum swings with a small amplitude, its motion is often referred to as 'simple harmonic motion' (SHM). This term was first introduced by Sir William Thomson (later, Lord Kelvin) and P.G. Tait in their classic texts on Natural Philosophy (Thomson & Tait 1873, p. 19). In addition they demonstrated that the projection, on the diameter of the circle, of a point that is moving with uniform circular motion also executes simple harmonic motion. This can easily be observed if one sights, edge on, a rod mounted near the circumference of a large turntable (such as a merry-go-round); as the turntable rotates at a constant angular speed, the rod's horizontal motion is SHM (French 1971, pp. 7–9).

In 1815 James Dean, Professor of Mathematics and Natural Philosophy at the University of Vermont, published an analysis of the motion of the moon about the Earth (Dean 1815; Crowell 1981). Instead of viewing the moon from the Earth, Dean placed an observer on the equator of the moon and observed the Earth. Following his analysis he noted that this motion 'may be easily imitated by a pendulum' that is hung with a Y-suspension (Figure 1) (Dean 1815, p. 245). The pendulum's mass is free to swing with two independent lengths – first, the length from the top of the 'Y' suspension; second, the length from the junction of the three

M.R. Matthews et al. (eds.), The Pendulum, 377–391.

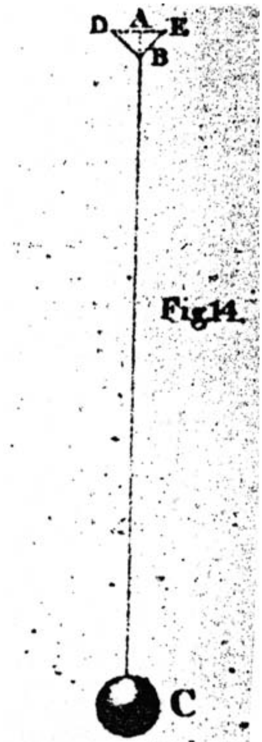

Figure 1. James Dean's 'Y-suspended' pendulum (Dean 1815, Plate I).

cords forming the 'Y'. Each of these, then, was free to swing in two independent, vertical planes.

Nathaniel Bowditch, in the same year, extended Dean's idea for a Y-suspended pendulum and provided a detailed mathematical analysis of its motion (Bowditch 1815). The curves, described by the equations of the motion of a (dimensionless) pendulum mass swinging with a small amplitude in two dimensions, are sometimes referred to as 'Bowditch curves'. While Bowditch was the first to derive and demonstrate these, one may more often find these same curves referred to as 'Lissajous curves', after Jules Antoine Lissajous who produced these curves with a pair of vibrating tuning forks and described his work in a series of papers beginning in 1855 (Lovering 1881, p. 298; Crowell 1981, p. 454; Whitaker 2001a, pp. 169–170).

The Y-suspended pendulum was again introduced around 1844 by Hugh Blackburn while a student at Cambridge. Blackburn was a classmate and friend of William Thomson (who would later be better known as Lord Kelvin). Curiously, Blackburn does not seem to have left any description of this pendulum himself. We learn of it from the secondary literature, where it is referred to as the 'Blackburn pendulum'. The difference in this design is that a permanent (or semi-permanent) record of its motion was made. Several alternative methods are mentioned in the literature. A heavy, hollow funnel is filled with fine sand (or ink) which falls through a small hole leaving a trace as the pendulum swings. Other pendulums have a sharp point which trace the motion on a plate of smoked glass or in a layer of sand. Later versions used electrical sparks to mark a sheet of paper placed on a conducting plate (Whitaker 1991, 2001a, pp. 163–164). Worland and Moelter (2000) have recently described the use of a spreadsheet to analyze the data produced by a modern spark generator of a Y-suspended pendulum in a student laboratory.

An alternative version, built by John Dobson around 1877, was described by J.G. Hagen (Hagen 1879, pp. 287, 297–299; Rigge 1926, pp. 68–71). Dobson's version had the advantage of having bifilar suspensions so that twisting was eliminated as the mass swung. A pen recorded its motion on paper.

2. The 'Physical' Pendulum

The Y-suspended pendulum was basically a simple pendulum which was free to swing in two independent directions simultaneously. These usually had large masses and long supporting cords. This limited the location where this pendulum could be demonstrated. If one removes the restrictions of a 'small' mass and a 'massless' support, one has a 'physical pendulum'. A common example of this in introductory physics is a rigid rod (such as a meter stick), fitted with a knife edge clamp, which may swing on an appropriate support. The knife edge is adjusted along different positions of the stick, and the period of this pendulum is found to be dependent on the position of the knife edge. Because of the uniformity of mass distribution along a meter stick, the period of the pendulum, as a function of mass distribution, may be readily obtained (Stephenson 1969, pp. 210–218; Halliday et al. 1993, pp. 390–393).

An early example of a physical pendulum for precision time measurements was introduced in 1817 by Captain Henry Kater (Kater 1818, pp. 33–102). Known now as 'Kater's reversible (or convertible) pendulum', Kater's pendulum

> consisted of a brass rod to which were attached a flat circular bob of brass and two adjustable weights, the smaller of which was adjusted by a screw. The convertibility of the pendulum was constituted by the provision of two knife edges turned inwards on opposite sides of the center of gravity. The pendulum was swung on each knife edge, and the adjustable weights were moved until the times of swing were the same about each knife edge. When the times were judged to be the same, the distance between the knife edges was inferred to be the length of the equivalent simple pendulum, ... (Lenzen & Multhauf 1965, p. 314)

Appropriate corrections for error (such as buoyancy of the air) were made, and this equivalent seconds pendulum made possible improved accuracy for measurements of gravitational acceleration at different locations to aid in determining the 'figure of the Earth'. While studies of this kind were a part of 'physics' in the Nineteenth Century, one must look for them today in references on 'geodesy' or 'geophysics' as these have evolved as separate disciplines. Kater's pendulum has served as a model for subsequent modifications and refinements in the measurement of gravitational acceleration or differences in acceleration (Garland 1965, pp. 6–27).

3. ‘Harmonographs’

The preceding discussion is indicative of the use made of the physical pendulum as part of the ‘mainstream’ physics research in the 19th century. The pendulums used also oscillated in one dimension. We now return to the use of those physical pendulums which, like the Y-suspended pendulum, were designed to oscillate in two dimensions. In 1873 S.C. Tisley reported on an apparatus made of two vertical rods, fitted with knife edges, which could swing at right angles to each other. This is illustrated in Figure 2 (*Engineering* 1874, p. 101). A ball-and-socket joint was attached to the top of each rod; a wire arm was attached to the joint and perpendicular to the rods. A pen, mounted at the intersection of the two wires, traces the curves resulting from the motion of the pendulums. The period of each pendulum could be adjusted by means of weights attached to each rod (Tisley 1873, p. 48). Tisley’s pendulum apparatus was soon offered for sale by the firm, ‘Tisley and Spiller, Opticians, etc.’. By May 1877 the apparatus was advertised as ‘Tisley’s Harmonograph. For drawing Lissajous’ and Melde’s figures ... ’. This seems to be the first use of the term, ‘harmonograph’, in the literature (Whitaker 2001a, p. 171). The term is subsequently applied to a variety of different designs of curve drawing apparatus.

A simplified modification of Tisley’s design was offered by Newton & Co. and described by Herbert Newton in 1909 (Goold et al. 1909, pp. 4–8). This is shown in Figure 3. The drawing table is mounted on the top of one of the pendulums, and the pen is attached, by means of a long rod, to the top of the second pendulum. Each pendulum is mounted on a knife edge and can be set in motion in any direction.

4. Wheatstone’s Kaleidophone

While not a pendulum, *per se*, the motion of the ‘kaleidophone’ discussed by Charles Wheatstone (Wheatstone 1827) is closely related. The kaleidophone consists of a thin rod, clamped at one end. This could be set in vibration with a small hammer or violin bow. A small silvered glass bead, mounted on the end, reflects a beam of light incident upon it. Persistence of vision permits one to observe the path of the bead. William Sang, in 1832, produced a mathematical analysis of the kaleidophone in which there is asymetry in the vibration of the rod in two directions. The motion of the bead is described by equations identical to those of Bowditch (Greenslade 1992; Whitaker 1993). A common automobile radio antenna will exhibit the same effects under appropriate conditions (Annet 1979; Merivuori & Sands 1984; Newburgh & Newburgh 2000).

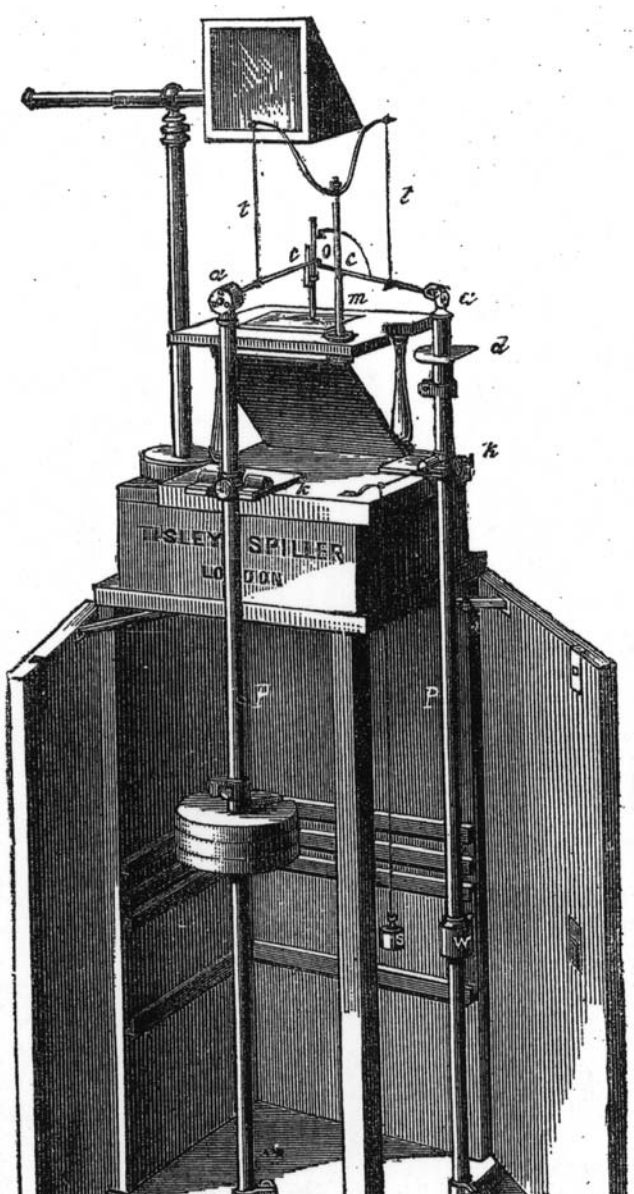

Figure 2. Tisley's compound pendulum (*Engineering* 1874, p. 101).

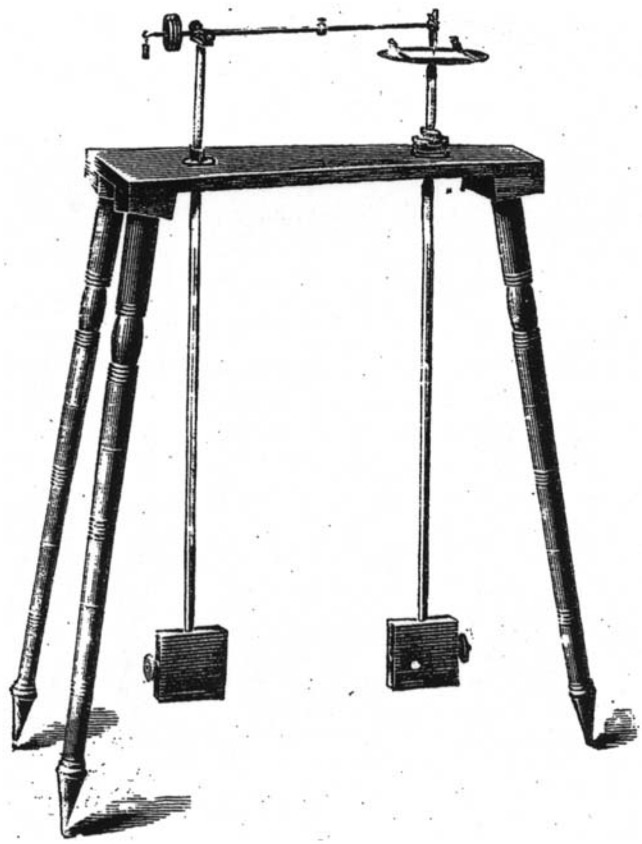

Figure 3. Newton's harmonograph (Goold et al. 1909, p. 5).

5. 'Bowditch/Lissajous' Curves

When the pendulums are set to vibrate perpendicular to one another, their motion traces the same system of curves described by Bowditch. Since the periods of each pendulum can be adjusted independently, a nearly infinite number of different curves of varying complexity may be drawn. Figure 4 shows two simple examples (Poynting & Thomson 1900, p. 74). Of particular interest are those curves produced when the periods of the two pendulums are in integer ratios of one another. A range of ideal curves (without damping) are shown in the plate reproduced as Figure 5.

The plate itself is of interest; a number of authors have illustrated their works with this identical plate. Zahm (1892, p. 416) and Tyndall (1894, p. 418), for example, used it to demonstrate the curves produced by properly adjusted tuning forks based on Lissajous' work. Poynting and Thomson (1900, p. 76) reproduced the same plate as an example of kaleidophone figures. Interestingly, this practice was not that uncommon. In his study of scientific illustration Knight has noted: 'Plates were expensive to make, and wherever possible were reused; otherwise,

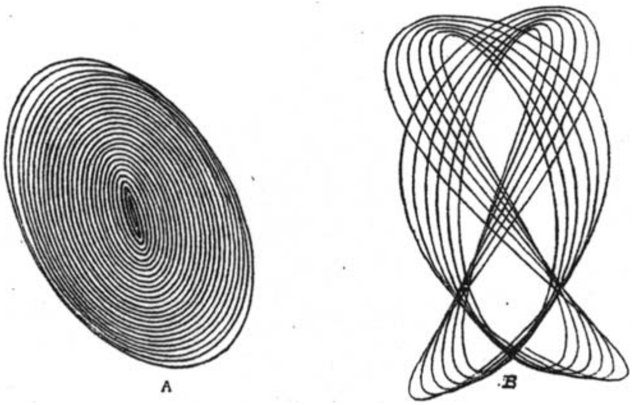

Figure 4. Damped harmonic curves (Poynting & Thomson 1900, p. 74).

they were often copied (sometimes reversed in the process) – we keep meeting recycled illustrations' (Knight, 1998, p. 249). This writer has made no effort to determine the first use of this plate, nor to list its (probably) many other locations.

6. Harmonographs in Popular Works

In addition to the 'ideal' curve the gradual reduction of amplitude of each pendulum adds to the complexity (and interest) of the resulting product. Thus, while the harmonograph was used to demonstrate the drawing of these curves, the demonstration was, as often as not, for the purposes of entertainment. As a result, authors of popular works soon began to include descriptions of commercial harmonographs in their books or to describe simplified versions that could be constructed by the home craftsman. J.H. Pepper, in the fourth edition of his *Cyclopaedic Science Simplified*, described Tisley's apparatus and reproduced a picture of it (Pepper 1877, pp. 562–565). Tissandier, in 1883, discussed Tisley's apparatus, but he also provided details for constructing one from simple materials (Tissandier 1883, p. 175; Whitaker 2001a, p. 167).

Cundy and Rollet wrote in 1961 that: 'The harmonograph was a popular diversion in Victorian drawing-rooms, since when it has suffered a decline and is rarely seen today' (Cundy & Rollett, 1961, p. 244). While one might not find the elaborate commercial versions that once were sold, harmonographs are still popular as science projects for students and for hobbyists. Directions for their construction are readily available. Cundy and Rollet, for example, provide detailed directions for the construction of a harmonograph similar to Newton's earlier commercial design. Bulman has provided directions for a similar device, as well as a simple Wheatstone kaleidophone (Bulman 1968a, pp. 86–94, 82—85). In a second book he describes a device for drawing curves that result from the rotation of gears (Bulman 1968b, pp. 12–31). Most recently Ashton has written a little book that describes the construction of a harmonograph similar to Newton's design, as well

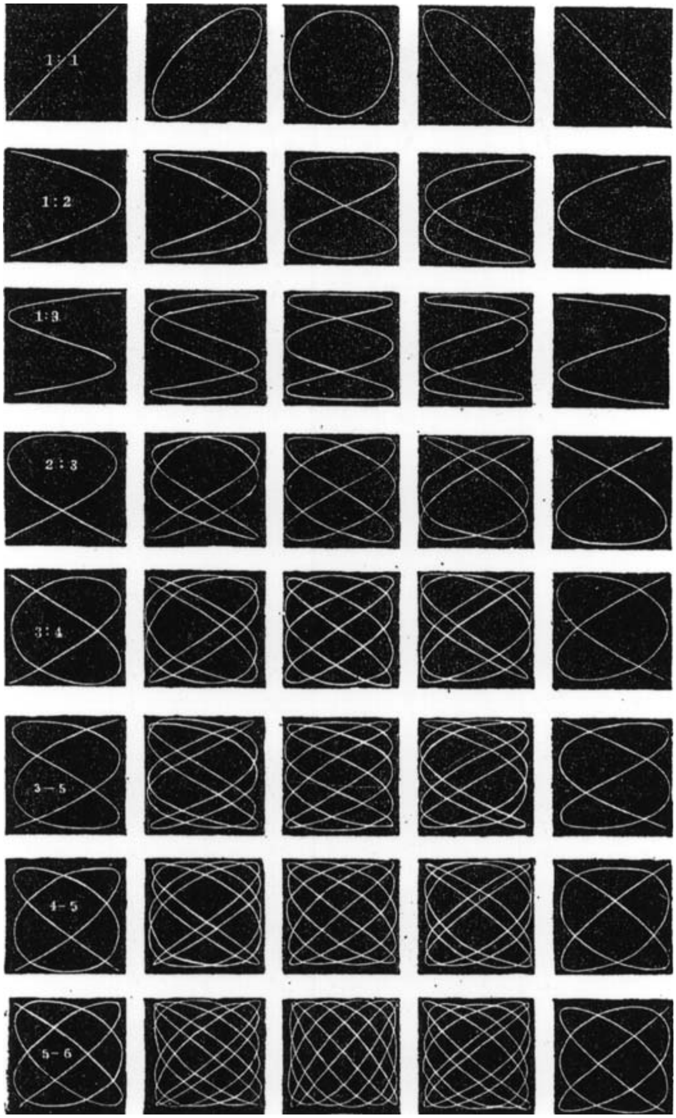

Figure 5. Ideal harmonic curves (Zahm 1892, p. 416; Tyndall 1894, p. 418).

as a three pendulum harmonograph, similar to Tisley's, which has the drawing table mounted on a third pendulum (Ashton 1999, pp. 3, 29). His account is illustrated with drawings produced by his apparatus as well as illustrations from older references. A variety of harmonograph designs, under different names, have been discussed by this author (Whitaker 2001a). These are summarized in Table I.

Table I. Various pendulum apparatus

Device	Year	Designer	Reference
'Y-suspended Pendulum'	1815	Dean; Bowditch	Crowell 1981, pp. 452-454
'Blackburn Pendulum'	1844	Blackburn	Whitaker 1991, pp. 330–333
'Tisley's Compound Pendulum'	1873	Tisley	Whitaker 2001a, pp. 164–165
'Tisley's Harmonograph'	1877	Tisley & Spiller	Whitaker 2001a, p. 165
'Sympalmograph'	1877	Browning; Benham	Goold et al. 1909, pp. 39–50; Whitaker 2001a, p. 166
'Dobson Duplex Pendulum'	1877	Dobson	Hagen 1879, pp. 287, 297–299; Rigge, 1926, pp. 68–71
'Double Pendulum'	1894	Bryan	Whitaker 2001a, p. 166
'Pendulograph'	1895	Andrew	Whitaker, 2001a, pp. 167–168
'Quadruple Harmonic-Motion Pendulum'	1899	Hoferer	Whitaker, 2001a, p. 168
'Twin-elliptic Pendulum'	1906	Benham	Goold et al. 1909, pp. 61–79
'Benham's Triple Pendulum'	1909	Benham	Goold et al. 1909, pp. 51–61

7. Harmonic Motion in College Textbooks

As with the simple pendulum, the compound pendulum (and Kater's pendulum in particular) has seen reduced discussion in introductory college physics textbooks. However some older textbooks present detailed accounts of the topic. Among several of these are Watson (1902, pp. 132–134) and Poynting & Thomson (1909, pp. 12–27). Pointing and Thomson, particularly, provide a detailed discussion of various pendulum methods and sources of error as well as extensive historical account of the problems involved in determining a standard unit of length and of the figure of the Earth. The textbook by Millikan, Roller, and Watson was noted for the introduction of historical material into the text. The authors' discussion of simple harmonic motion and pendulum motion (including the Kater pendulum) in its historical context is noteworthy (Millikan et al. 1937, pp. 330–345). Similar historical emphasis was provided by Lloyd William Taylor in his book published in 1941 (Taylor 1941, 1959, pp. 182–197). Feather's account is insightful (Feather 1959, pp. 182–194). And, in an intermediate level text, Stephenson includes the Kater pendulum as an example of a physical pendulum (1969, pp. 213–215).

One example of the Kater's pendulum, as an experiment in the laboratory, may be found in Searle (1934, pp. 7–15). Searle provides a thorough discussion of the theory, as well as sample data gathered from an experiment. Apparatus for the study of Kater's pendulum and for the study of the compound pendulum were sold for many years by Central Scientific Company (Cenco), and detailed instructions in their 'Selective Experiments in Physics' series were provided with these to assist in their use (Eaton 1940, 1941). A simplified version may be found in Ingersoll, Martin, and Rouse, who include it as part of a series of experiments on moment of inertia (1953, pp. 54–57). Modifications in the Kater pendulum have been discussed more recently as laboratory experiments in physics (Jesse 1980, pp. 785–786; Peters 1999, pp. 390–393; Candela et al. 2001, pp. 714–720). It is less clear, however, how many institutions may be using these as part of their instruction.

Arnold Sommerfeld, in his classic work on theoretical physics, wrote in the introduction to his chapter on 'oscillation problems':

> 'The investigations that are to follow will teach us nothing new about the principles of mechanics. So great, however, is the significance of oscillation processes for physics and engineering that their separate systematic treatment is deemed essential' (Sommerfeld 1952, p. 87).

Sommerfeld's mathematical analysis of a 'simple' pendulum and of a 'physical' pendulum is based on assumptions regarding the mass distribution of the pendulum as well as the location of its support. Similar restrictions are placed on 'coupled' pendulums, including the 'double pendulum', in which a mass is connected by a cord to a second mass, which is in turn connected to a support. (This is the basis of the 'twin-elliptic' pendulum harmonograph.) Assumptions again arise involving 'point' masses and 'massless' cords. His mathematical analysis of such a problem, which is usually restricted to advanced courses in mechanics, is a model of clarity (Sommerfeld 1952, pp. 87–117).

It should not be surprising, then, that none of the discussions of harmonographs includes a mathematical analysis of the motion of the pendulums. We find that the period of each is adjusted by trial and error. Small differences in adjustment may produce large variations in the curves produced. Under 'proper' adjustment the recording point of the pendulum traces the same curve but with a continually decreasing amplitude. These differences seem to be one of the fascinations with harmonographs. Rigge, for example, devoted a full chapter in his book on the beauty of curves (Rigge 1926, pp. 122–132).

A second class of harmonographs should be mentioned in passing. These make use of gears or pulleys in their operation. While they do not operate under pen-

Table II. Various ‘gear’ apparatus

Device	Year	Designer	Reference
‘Geometric Chuck’	1833	Ibbetson	Whitaker 2001b, p. 174
‘Lissajous’ Apparatus	1869	Pickering	Whitaker 2001b, p. 174
‘Harmonic Curve Apparatus	1873	Donkin	Whitaker 2001b, pp. 174–175
‘Cycloidotrope’	1883	Pumphery	Rigge 1926, pp. 74–75
‘Campylograph’	1900	Dechevrens	Rigge 1926, pp. 78–81
‘Wondergraph’	1913	Tuck	Tuck 1913, pp. 436–439;
	1931	Collins	Collins 1931, pp. 71–74
‘Cyclo-harmonograph’	1916	Moritz	Whitaker 2001b, p. 176
‘Creighton Compound Harmonic Motion Machine’	1924	Rigge	Rigge 1926, pp. 81–91; Whitaker 2001b, p. 178
‘Kukulograph’	1933	Hoferer	Whitaker 2001b, p. 177
‘Spirograph®’	1967	Kenner Products, Co.	Whitaker 1988; Whitaker 2001b, pp. 177, 179–180
‘Schemagraph’	1968	Bulman	Bulman 1968b, pp. 12–31
‘Turntable Oscillators’	1971	Project Physics	Whitaker 2001b, p. 177

dulum motion, they are related to an important problem in astronomy – the effort to describe the motion of the heavenly bodies with a system of circular motions. An appropriate example of one of these is the popular toy, Spirograph®, which came on the market in 1967. It is designed to produce that class of curves known as ‘trochoids’ (Whitaker 1988b, 2001b, pp. 179–180). A number of elaborate devices, however, have also been invented and described over the years; these are summarized in Table II (Whitaker 2001b).

8. Previous Surveys of Curve Drawing Apparatus

Several surveys of curve drawing apparatus have been published. Among the earlier was Hagen (1879). Lovering, in surveying the role of Bowditch, also summarized those devices of which he was aware. An extensive survey of a variety of devices was provided in Goold et al. (1909), many of which were sold by Newton and Co. Rigge (1926) provided an extensive mathematical description of the curves possible with various machines, including the highly complex 'Creighton Compound Harmonic Motion Machine' which he had begun in 1915 and completed in 1924. Greenslade, for a number of years, has encouraged continual interest in harmonographs in his series of articles (Greenslade 1979, 1992, 1993, 1998). This writer has recently provided a survey of the history of various harmonographs (Whitaker 1988a, b, 1991, 1993, 2001a, b).

9. Conclusions

This article has attempted to summarize those pendulums, constructed to swing in two dimensions, which produce records of those curves known as 'Bowditch curves' or 'Lissajous curves'. These 'harmonographs' were designed as demonstration devices to illustrate vibrations in mechanics as well as in the physics of sound and of harmonies in music. They were also designed to entertain through the ingenuity of the machine or the variety of the curves it produced. While the equations for these various curves may be programed into a computer, and the curves changed almost instantly, there is still a fascination in watching them being drawn by a mechanical device. Tolansky, in introducing his two pendulum harmonographs, has noted: '...it so happens that sophisticated electronics systems cannot create patterns which even remotely compare either in interest or in aesthetic appeal with those that can be formed by quite crude mechanical pendulum devices ...' (Tolansky 1969, p. 267). Romer, similarly, has described a 'corridor apparatus' for student interaction and has observed that '...this apparatus can be used to produce a wide variety of designs which seem to have considerable aesthetic appeal to many people. We have found it very valuable, both in stimulating an interest in the simple physics on which it is based and perhaps, more honestly, simply as an "art machine" ' (Romer 1970, p. 1116).

Today one may find large scale harmonographs as interactive displays in science museums. While we may no longer find elaborate harmonographs listed in the catalogs of apparatus companies, the continual discussion of these in the literature is indicative of their inherent interest to students, teachers, and the general public.

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The Pendulum as a Vehicle for Transitioning from Classical to Quantum Physics: History, Quantum Concepts, and Educational Challenges

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Abstract. In this article we use the pendulum as the vehicle for discussing the transition from classical to quantum physics. Since student knowledge of the classical pendulum can be generalized to all harmonic oscillators, we propose that a quantum analysis of the pendulum can lead students into the unanticipated consequences of quantum phenomena at the atomic level. We intend to illustrate how classical deterministic physical ideas are replaced by a point of view that contains both deterministic and probabilistic aspects. For example, the wave function contains probabilistic information but it evolves in time according to a fixed law, the Schrodinger equation. Discussion of the transition from classical to quantum thinking is historically grounded in the work of twentieth-century physicists who developed quantum ideas. We see application to current science in areas such as semiconductors, optics, GPS systems, and superconductivity. Our notion is that a scientifically-literate public should have a sense of the broad, conceptual schemes in modern physics, as well as those associated with classical physics. We discuss educational challenges and strategies connected to including quantum theory in a general education physics course. Our work would have other applications in college and secondary school settings.

1. Introduction

The quantum pendulum is an excellent model for introducing some of the basic principles of quantum mechanics to university-level 'non-science' physics students.[1] The authors – a science educator, a physicist, and a science historian – in this paper report their initial efforts on the construction of a team-taught introductory conceptual physics course that takes as one of its central models the quantum pendulum.

Our overarching motivation for this endeavor is the belief that a scientifically literate public should have a sense of the broad, conceptual schemes in modern physics, as well as those associated with classical physics. All too often even the best introductory physics courses leave the students with an understanding that stops with the nineteenth century.

Current, influential science education documents argue the importance of science literacy for all (American Association for the Advancement of Science (AAAS), 1993; National Research Council (NRC), 1996). Should some under-

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standing of modern physics be a component of science literacy? Many introductory, general education physics courses exclude quantum theory because it is abstract, highly mathematical, and often counter-intuitive. However, several arguments support its inclusion in general education settings (Bohm & Peat 1987; Johnston et al. 1998; Petri & Neidderer 1998) in order to:

- demystify it;
- dispell misconceptions about the nature of science and scientists;
- introduce a non-Newtonian world view in which probability is a foundational idea;
- connect to major advancements in modern life, such as lasers, transistors, semiconductors, modern optics, nanostructures, satellite technology;
- inquire into the quest for interrelated ideas, as science is evolving and constantly testing models and theories; and/or
- understand current views of reality and the contribution that science has made to them.

We need to be wary of the impact of specialization and fragmentation of ideas on student learning, especially in liberal arts settings. Transmissionist teaching strategies do not support the goal of scientific literacy if they postpone meaning making until students reach research-level experiences not accessible to non-science majors (Johnston et al. 1998). These same students are excluded from opportunities for critical thinking when their science courses focus entirely on mathematical procedures rather than conceptual tools (Bailin 2002).

We begin with a review of some of the pertinent history of the development of quantum ideas by examining the transition from classical to quantum thinking. Under the assumption that readers are familiar with classical mechanical ideas, some of the fundamental principles of quantum mechanics are then presented along axiomatic lines. The quantum oscillator (an approximation to the quantum pendulum) features prominently as an accessible illustration. The allure of quantum mechanics to general students is highlighted by stressing some philosophical controversies within quantum theory. The final section explores the challenges of teaching the highly abstract and mathematical discipline of quantum mechanics to conceptual physics students in light of the latest findings in science education research.

2. Historical Foundations: Transitioning from Classical to Quantum Ideas

2.1. THE CLASSICAL WORLD VIEW: THE PENDULUM AND PROBABILITY

The pendulum (or oscillator) has played such a vital role in the development of scientific thought, that it continues to be a useful tool for structuring the curriculum in classical mechanics (Matthews 2000, 2001). Can we also use the pendulum to effectively introduce students already familiar with classical concepts to quantum theory? By ‘quantizing’ the pendulum, we believe we can offer a means for introducing students to quantum thinking and the intellectual process by which quantum theory was created.

We begin with the classical picture. Consider a pendulum bob of mass, m, that is in oscillation. In both the classical and quantum treatment (below) we assume the bob oscillates with small amplitude oscillations, thereby treating the pendulum as a simple harmonic oscillator. To simplify matters we focus on the shadow of the bob cast on the floor below and orient the x-axis along the floor. Many of the basic mechanics principles, both classical and quantum mechanical, may be illustrated by this system.

In classical mechanics the initial *state* of the bob is given by the initial conditions, that is the position and velocity of the bob at time zero, $x(t = 0)$ and $v(t = 0)$. The state of the bob at any future time t is,

$$x(t) \quad \text{and} \quad v(t). \tag{1}$$

The fundamental problem in classical mechanics is to find the state at time t given the initial state and the forces, F, acting on the bob. The way this time dependence is determined is by solving the fundamental equation of classical mechanics, Newton's Second Law of Motion,

$$F = ma, \tag{2}$$

where a is the acceleration of the bob under the influence of the forces F acting on it. Equation (2) is called the *classical equation of motion*. In principle, classical mechanics is a deterministic description (Mach 1989), although – as a practical matter – it falls short of determinism due to the lack of precision with which the state of the system can be measured (Feynman et al. 1965, Vol. III, pp. 2–9).

With an eye toward directing students' thoughts to quantum thinking, probability ideas may be brought to bear on the classical description of the pendulum (Anderson 1971. See pp. 198–199 for the classical probability distribution for the simple harmonic oscillator). Imagine the following experiment: For the pendulum bob, use a sphere that contains sand and that has a tiny hole in its bottom. As the pendulum swings back and forth, the sand steadily falls out of the hole onto the floor below. After a period of time a pattern will emerge on the floor. Near the extremes of the swing, where the pendulum has a low speed, the sand will be piled high but near the lowest part of the swing; where the pendulum has its highest speed, there will not be much sand. The crucial question for the student is this: Without specific information of the location of the pendulum bob at a given time t, where is the bob most likely to be? The sand pattern directly reveals that the classical pendulum has its highest probability of being present near the extremes of its motion and its lowest probability at its lowest point, where $x = 0$. We will later return to this example after introducing the quantum viewpoint of the pendulum, showing the striking changes that appear in the sand pile in the quantum world.

2.2. INTO THE 1920S AND THE PRESENT VERSION OF QUANTUM MECHANICS

Physicists formulated the first quantum theory between 1900 and 1920,[2] the theory reaching its zenith with the Bohr model of the structure of the hydrogen atom (Eiseberg 1961). In constructing his model, Bohr's main theoretical goal was to explain the stability of the atom and hence account for its chemical properties. Building on the experimental results of Rutherford, Bohr needed to construct an atomic model that placed a dense, positively charged nucleus at the center of the atom with electrons in orbit around the nucleus. The immediate problem, however, was that accelerated charged particles emit energy. Assuming that the electrons within an atom are in continuous circular motion, electromagnetic theory predicts that atoms would quickly collapse, making molecules and higher order composites impossible. Bohr's initial solution was to limit the electron orbits to stable stationary states, each with a specific and unchanging quantum of energy. Unable to emit energy on a continuous basis, electrons would neither spiral into the nucleus nor continuously absorb energy and move to larger, more energetic orbits. Dubbed the 'quantum postulate', this assumption was nevertheless ad hoc; it saved the stability of the atom, but what did it actually mean? And would it ultimately rest on more fundamental principles?

A partial answer came soon after when Bohr hypothesized that electrons jumped between energy states, emitting radiation when jumping from a higher energy level to a lower energy level, absorbing radiation when jumping from a lower energy state to a higher energy state. The energy of the electromagnetic radiation emitted or absorbed could then be calculated by the difference between the two energy levels occupied by the electron. Close agreement with the established energies given by the Balmer formula for hydrogen and the prediction of as yet unobserved spectral lines helped to propel the theory into the limelight, despite its obvious limitations. Bohr set about trying to understand the meaning of the discontinuous electron jumps, but troubling questions remained. How did the electron 'know' to stay in particular energy states? Why does the electron, still considered to be a particle, not radiate energy on a continuous basis? What could it mean that electrons instantaneously go from one energy state to another (Folse 1985; Kragh 1999)?

The Bohr model was displaced in the early 1920s by a more wide-ranging theory comprising our current understanding of quantum mechanics (van der Waerden 1959). Beginning with the thoughts of de Broglie, it was argued that if radiation – normally thought of as consisting of electromagnetic waves – could in certain instances behave like a collection of particles (photons), then in some instances why could particles (e.g., electrons) not behave like waves? Einstein, taken with some of de Broglie's ideas, introduced them to Schrödinger, who soon developed a three-dimensional electron model (Kragh 1999). According to de Broglie and Schrödinger, electrons were to be represented as standing waves enshrouding the nucleus. Because standing waves can only exist in states with integer and half-integer wavelengths, an electron in a stable orbit would have a specific energy associated with the wavelength of that state. Thus, with the apparatus of Schrödinger's mathematics, particles could now be represented by wave functions that, presumably, captured all the information contained in the state of the particle itself. Furthermore, Schrödinger believed that this model saved the atom from Bohr's discontinuous quantum jumps by creating the possibility that the wave function Ψ could transform continuously in time (Whitaker 1996). In 1925, Schrödinger developed the fundamental equation from which the wave function and energy of the particle can be found.

But while Schrödinger (and Einstein) found solace in this physical interpretation of the wave function, Bohr and Heisenberg did not. Even before Schrödinger had created his 'wave mechanics', Heisenberg had created a 'matrix mechanics' that sought to account for all the experimental results related to measurements of the atom. Unlike wave mechanics, however, the power of matrix mechanics was that it absolved quantum theory of requiring physical models (Whitaker 1996); like the statistical mechanics of the late nineteenth century, matrix mechanics transcended the need for picturable physical events. From this point of view, Schrödinger's theory was not only mistaken; it was seen as entirely wrongheaded. While Bohr may not have been as ready to dismiss wave mechanics as so much nonsense, the question for him was, what does the wave function represent? Certainly it cannot represent a physical particle, since such particles were no longer the subject of direct observation.

Following the striking example of symmetrical reasoning (i.e., wave-particle duality), Max Born offered his postulate as to what in fact was waving in the Schrödinger waves. In a 1926 paper, Born suggested that the wave function in a region of space associated with a particle gives the probability of finding the particle in that region. In effect, the recognized power of Schrödinger's mathematical formalism combined with Born's probabilistic interpretation put the wave theory securely in the conceptual camp of Bohr and his colleagues (Kragh 1999).

The tidy quantum mechanical picture that emerged by the end of the 1920s was soon found to imply a number of features that troubled physicists (Bohm 1951). With the development of the wave picture of subatomic particles, harmonic variables like wavelength and frequency became fundamental to the understanding of how the atom 'worked'. But the application of wave properties to what had previously been thought to be particles raised a number of conceptual difficulties. The advantage of 'wave mechanics' was that it built on familiar concepts related to macroscopic wave phenomena. By applying a well known conceptual scheme to the poorly understood structure of the atom, physicists developed a theory that not only accounted for a great deal of observable phenomena; it also provided a means for making future predictions that could then be experimentally tested. At the same time, however, wave mechanics begged the question, what does it mean for particles to have wave properties?

The apparent contradiction between particle and wave phenomena implied that the subatomic world had to be understood on very different terms than the macroscopic world. The development of Heisenberg's 'matrix mechanics', on the other hand, offered an approach to quantum systems that, like the thermodynamic systems of the nineteenth century, focused on measurable quantities and their mathematical relationships. The seemingly contradictory nature of the quantum world and the increasingly abstract mathematical theories used to describe it fostered the notion that subatomic phenomena could not be conceived in the same visual terms as the macroscopic world. While the mathematics of Heisenberg and Schrödinger provided powerful tools for analyzing and predicting the energy states of subatomic particles, the mathematical picture became increasingly divorced from visual analogues.

Quantum mechanics as it is now understood and taught (Shankar 1994) thus consists of highly abstract rules and procedures. Understandably, this poses unique and interesting challenges to teachers of modern physics. Should students develop an understanding of the mathematics without worrying about the philosophical implications of the theories themselves? Should the historical development of quantum theory be included in a program of study? Or does the messiness of history prove to be too distracting, taking time away from internalizing the procedures of axiomatic knowledge?

2.3. EINSTEIN–BOHR DEBATES ON FOUNDATIONS OF QUANTUM MECHANICS

One of the assumptions of the International Pendulum Project (IPP) is that the history and philosophy of science can contribute greatly to an appreciation of both the content of science and the nature of science as a professional activity. The project thus recognizes that science is a process subject to the social and cultural conditions of the historical periods in which it is pursued as well as a set of received doctrines and procedures that students should know and understand.

Through a focus on process, a course on the conceptual features of quantum mechanics can look at how prominent scientists such as Bohr, Einstein, Schrödinger, and Heisenberg defined and dealt with the challenges that the new theory created for physicists. Through an instructional approach emphasizing the different scientific and philosophical styles these historical figures brought to their understanding of nature, and the marked disagreements that resulted from their encounters with one another, we believe students can come to see contemporary science as a dynamic, ongoing process of debate rather than a static body of predefined notions about the natural world. The abstract and relatively remote nature of quantum theory makes a historical consideration of the development of the theory even more relevant for younger students. The passions and philosophical commitments of the early framers of the theory can engage students on a variety of levels. It helps that Einstein, for instance, is already popularly known, even if rarely understood. The Einstein–Bohr debates provide a perfect case study of how different philosophical commitments and scientific methodologies can lead to different styles of doing science. Students will thus be encouraged to understand more deeply the contingencies of doing pioneering work in science as they develop an understanding of the scientific ideas and processes themselves (Matthews, 2004).

The Einstein–Bohr debates have drawn a considerable amount of historical attention over the years, so it remains the task of this section to consider how the themes and issues present in the debates can be presented in the classroom. It should be cautioned at the outset that the historical importance of the debate should not be exaggerated. As Kragh has pointed out (1999), the exchanges between Einstein and Bohr have been both romanticized and exaggerated in the literature. Yet the impact of the debates did not initially extend far beyond those physicists who were interested in or troubled by the philosophical implications of the quantum worldview. For most working physicists, the debates did not lead to any fundamental new equations for understanding the structure of matter, so they could safely be disregarded. But for the conceptual physics audience the series of exchanges between Einstein and Bohr should help generate awareness of the important philosophical problems the quantum theory raised and thus help to demonstrate the broader impact of modern physics on the way we think about nature. At the college level, where the mathematics of quantum mechanics may play a larger role in the classroom experience, the discussion can be expanded to include the work of Heisenberg and Schrödinger to demonstrate how quantum theory developed its high degree of abstractness and how Bohr's early visual planetary model of the atom gave way to what were, at first, competing mathematical models. Discussion of these models will also give the students an opportunity to consider the relationship between experimental results and the descriptive/predictive power of mathematical models. Heisenberg in particular was most eager to generate preexisting experimental results in such a way that visual models of the atom would no longer be necessary in order for the new quantum theory to be useful.

The series of encounters between Bohr and Einstein were not so much debates as they were challenges, with Einstein and others (including Schrödinger) posing thought experiments that were intended to undermine Bohr's notion that quantum theory, particularly as it rested on Heisenberg's Uncertainty Principle and Bohr's Complementarity Principle (see discussion below), gave a complete description of nature. At its core, the debate rested on fundamental disagreements about the nature of scientific explanations. Historians and philosophers have argued, however, that deep misunderstandings characterized the positions of both figures, with Bohr often frustrated that Einstein did not understand the revolutionary nature of quantum theory, and Einstein firmly convinced that the quantum theory led to logical and experimental absurdities and hence could not be a complete theory (Folse 1985).

Einstein's position rested on a strong philosophical commitment to the determinism of classical mechanics. According to the assumptions of classical mechanics, all material bodies existing in space and time have properties that can be theoretically determined without changing those properties in the process of measurement.

In the simplest case, the momentum and spatial position of a material object should be knowable without affecting those very same properties. The classical analysis of the simple pendulum exemplifies what is meant here. Once the position and momentum of a physical system has been determined, its future course of motion, both alone and in contact with other physical systems that are equally well known, will be fully determined by the known laws of nature. This point of view underscored the sciences throughout the eighteenth and nineteenth centuries. Einstein's Theory of Relativity expanded the notion of determinism by supplying the equations that would allow one to translate measurements of length, time, and mass from one inertial reference frame to another. As far as Einstein was concerned, a complete theory of nature would give a complete knowledge of any physical system. If a theory was unable to give such complete knowledge, then this implied that the theory itself was incomplete.

The necessary precondition for this view of nature is that the process of determining the physical properties of an object will in no way alter the properties themselves. But this was the very premise that quantum mechanics called into doubt. Instead, quantum theory claimed that at atomic levels the process of measuring a physical system imparted enough energy to change the state of the system being measured. Furthermore, theoretical consistency seemed to demand that energy states within an atom had to be discreet and discontinuous. The process of measurement, then, caused fundamental discontinuous changes in the energy states of atomic particles and hence made it impossible to make a certain determination of a particle's energy state *before* the measurement. The implication, therefore, was that the state of a physical system before measurement could not in principle be known. One could predict the probable states of the system, but this was not the deterministic system of Newtonian mechanics. Making the situation even more problematic was the notion that physical systems at the atomic level showed signs of having the properties of both waves and particles. Which set of properties showed up in experiments depended on the type of properties being measured for. What was one to make of such counter-intuitive ideas?

Bohr's attempts to make sense of these apparent contradictions led him to formulate a theoretical perspective that even he believed entirely recast our understanding of nature (Folse 1985). To Bohr, the uncertainty principle, quantum jumps, and quantized energy levels were not symptoms of an incomplete theory, but rather the inherent characteristics of nature at the atomic level. They thus had to be embraced rather than overcome. Of course, Einstein could not have disagreed more.

3. Formal Principles of Quantum Mechanics Illustrated by the Quantum Pendulum

Today, the practicing physicist or aspiring physics student rarely retraces the colorful and rich history of the development of quantum mechanics. As in most areas of physics, textbooks present a polished and consistent mathematical scheme with few traces of history remaining. To complete the picture of quantum mechanics we wish to paint for the 'non-science major', we now turn to a brief modern presentation of the basic principles of quantum mechanics (Shankar 1994) with the aid of using the pendulum to demonstrate the counter-intuitive nature of the theory. Although the mathematics used is elementary and accessible to general students, the concepts are far from trivial.

3.1. STATE OF THE PENDULUM

The quantum mechanical description of a pendulum is markedly different from the classical model reviewed in 2.1; however, this discussion parallels that of the classical pendulum. The quantum state of the bob is fully determined by the wave function or state, $|\Psi\rangle$, which we can visualize as an arrow. The initial state is assumed known $|\Psi(t = 0)\rangle$. The quantum state of the bob at any future time, t, we write as,

$$|\Psi(t)\rangle. \tag{3}$$

The primary problem in quantum mechanics is to find the wave function at time t given the initial state and the forces that act on the particle. The way this is determined is by solving the fundamental equation of quantum mechanics (i.e., the Schrödinger equation), which is called the *quantum equation of motion*,

$$|\Psi(t)\rangle = U_{op}(t)|\Psi(t = 0)\rangle, \tag{4}$$

where $U_{op}(t)$ is called the propagator operator. (We will not consider how this operator is found other than to say it depends on the forces acting on the particle.) The mathematical procedure by which $|\Psi(t)\rangle$ is found is not crucial to the explanation of quantum mechanics to general students.

3.2. WHAT DOES $|\Psi(t)\rangle$ TELL ABOUT THE PENDULUM?

One of the first hurdles students must jump is to gain an understanding of just what $|\Psi(t)\rangle$ is telling them. Whatever information it contains, it is dogma in quantum mechanics that it contains *all* the information we are able to know about the bob.

Suppose it is desired to know some property of the pendulum, for example, its energy, E. To this particular property we associate an *operator*, H_{op} that is called the Hamiltonian operator, which is simply a mathematical rule stipulating what H_{op} does when it is multiplied times the wave function, $|\Psi(t)\rangle$. Another fundamental principle of quantum mechanics states that multiplying H_{op} times $|\Psi(t)\rangle$ is the same as, for certain special choices of $|\Psi(t)\rangle$ (denoted $|\Psi_i(t)\rangle$), a number E_i, times $|\Psi_i(t)\rangle$,

$$H_{op}|\Psi_i(t)\rangle = E_i|\Psi_i(t)\rangle. \tag{5}$$

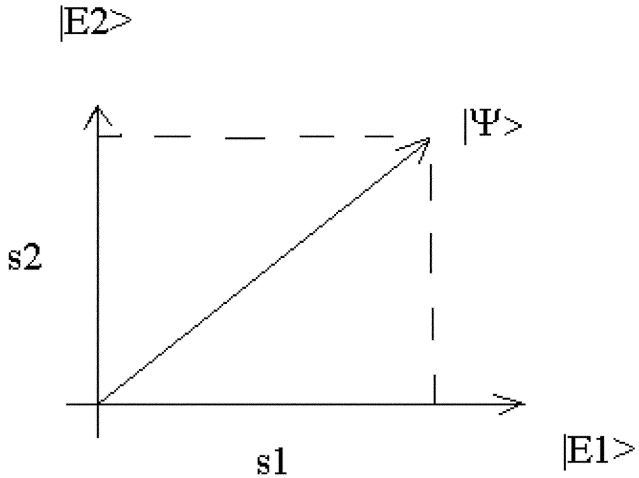

Figure 1. The two-state system in the mixed state. The eigenstates form the axes.

The above equation is called the *eigenequation*. Any wave function that obeys this equation is called an *eigenfunction* of H_{op} and the E_i is called an *eigenvalue* of H_{op} corresponding to eigenfunction $|\Psi_i(t)\rangle$. Henceforth, when $|\Psi(t)\rangle$ is an eigenfunction $|\Psi_i(t)\rangle$, we will use the common notation, $|E_i\rangle$, to refer to this *eigenstate*.

For conceptual physics students, it is not important to know how to solve the eigenequation; rather, it is essential to understand what the solutions tell about the system. Quantum mechanics assumes that, if the pendulum is described by a wave function $|\Psi(t)\rangle$ and an energy measurement is made on the pendulum, then the measurement will only yield one of the eigenvalues E_i of H_{op}. An important question is, given the wave function of the particle $|\Psi(t)\rangle$, how does this give the probability of measuring a given energy eigenvalue E_i? To answer this question, it is advantageous to discuss by example a simpler case, the so-called two-state system. We will then return to the pendulum.

As the name implies, a two-state system has only two energy eigenvalues. We denote these numbers as E1 and E2. The corresponding eigenstates are written, $|E_1\rangle$ and $|E_2\rangle$. Imagine a Cartesian coordinate system with $|E_1\rangle$ and $|E_2\rangle$ directed along the x- and y-axes, respectively. The state of the system $|\Psi(t)\rangle$ may be viewed as a vector in this coordinate system (Figure 1).

In Figure 1, the vectors, $s_1|E_1\rangle$ and $s_2|E_2\rangle$, represent the projection of $|\Psi\rangle$ onto the coordinate axes, i.e., they are the shadow of $|\Psi\rangle$ projected along the horizontal and vertical axes in the figure. In general, these projection vectors add vectorially to give the total vector,

$$|\psi\rangle = s_1|E_1\rangle + s_2|E_2\rangle. \tag{6}$$

The state given by Equation (6) is not a single eigenstate, but is called a ‘mixture’ of eigenstates.

The assumption is made that if the particle is in the mixed state then the probability, $P(E_i)$, of measuring the energy and obtaining the energy eigenvalue E_i is given by,

$$P(E_i) = s_i^2, \quad \text{where } i = 1, 2. \tag{7}$$

Accordingly, $s_1^2 + s_2^2 = 1$, since probabilities must add up to one.

The two-state system showcases the property of quantum mechanics that gives the field its name: quantization. In classical physics, most variables that describe a system, such as energy, can take on any value; i.e., energy is a continuous variable. In quantum mechanics, variables are often restricted or may not be continuous. In the two-state system, it is possible to measure only one of two values for the energy, E_1 or E_2, and so the energy of the system is quantized and not continuous. The best known example of quantization is the allowed energies of an atom.

During the measurement of the energy, the mixed state wave function suddenly changes to the eigenstate $|E_i\rangle$ whose eigenvalue E_i is found in the measurement. In Figure 1, the state vector shown, $|\Psi\rangle$, rotates suddenly to $|E_i\rangle$ in a transition aptly phrased 'the collapse of the wave function'. Notice *after* the measurement one is *certain* of the state of the system, $|E_i\rangle$, while *before* the measurement knowledge is *probabilistic* in the mixed condition. Before the measurement is made, the *expected* value of energy, written $\langle E\rangle$ and also called the mean or *average* value of the energy, is given by,

$$\langle E\rangle = P(E_1)E_1 + P(E_2)E_2 = s_1^2 E_1 + s_2^2 E_2. \tag{8}$$

For students who are comfortable with the concept of standard deviation, it is meaningful to introduce uncertainty at this juncture, particularly given the importance of the term in quantum mechanics. To accomplish this, imagine there are N identical and independent two-state systems, each one described by the same state vector $|\Psi\rangle$. Suppose measurement of the energy finds n_1 of the systems to have energy eigenvalue E_1 and finds n_2 of the systems with eigenvalue E_2. Evidently, $N = n_1 + n_2$ and

$$P(E_1) = s_1^2 = n_1/N \quad \text{and} \quad P(E_2) = s_2^2 = n_2/N. \tag{9}$$

The *uncertainty* in energy is the standard deviation of these measurements,

$$\Delta E = [\langle E^2\rangle - \langle E\rangle^2]^{1/2}. \tag{10}$$

Quantitative Example: A Biased Coin

To further illustrate the two-state system in the context of an everyday phenomenon familiar to students, suppose there is a collection of N identical coins. A coin can exist in a state of being a head, |Head⟩, or a tail |Tail⟩; these are the eigenstates of the 'flip the coin' operator, F_{op}. Notice the only possible measurable states are the eigenstates. Let the eigenvalue of |Head⟩ be +1 and that for |Tail⟩ be −1. Therefore,

$$F_{op}|\text{Head}\rangle = +1|\text{Head}\rangle \quad \text{and} \quad F_{op}|\text{Tail}\rangle = -1|\text{Tail}\rangle.$$

For the sake of the argument, assume the coins are a bit biased – there is a greater chance of the coin landing on heads than landing on tails. To be precise, let the probability of heads be 2/3 and tails 1/3. Each of the N coins in the collection of coins would then be described by the state,

$$|\Psi\rangle = (2/3)^{1/2}|\text{Head}\rangle + (1/3)^{1/2}|\text{Tail}\rangle.$$

The wave function $|\Psi\rangle$ would cast a longer shadow on |Head⟩ than on |Tail⟩ indicating the greater likelihood of landing on heads. The average value and uncertainty of F_{op} are then found to be,

$$\begin{aligned}\langle F\rangle &= (2/3)\cdot(1) + (1/3)\cdot(-1) = 1/3, \quad \textit{and}\\ \Delta F &= [(2/3)\cdot 1\cdot 1 + (1/3)\cdot(-1)\cdot(-1) - (1/3)^2]^{1/2} = [8/9]^{1/2}\end{aligned}$$

3.3. COMPLEMENTARITY AND THE HEISENBERG UNCERTAINTY PRINCIPLE

In quantum mechanics, certain properties or ideas are said to complement each other. For example, the concept of a particle complements that of a wave. Early experiments indicated that in some situations what was normally thought of as a wave behaved more like a particle. At a very fundamental level, nature exhibits this dualism in wave-particle duality. It should not surprise us that classical models which emerge from our experience with the macroscopic world might fail when extrapolated to the microscopic world, where the language and models of classical physics may prove inadequate.

The best known example of complementarity is the position and momentum of a particle. It is found that the more precisely the position of a particle is known, the less precisely the momentum is known. The Heisenberg Uncertainty Principle relates this imprecision between complementary variables in the mathematical statement,

$$\Delta x \Delta p > h/2 \tag{11}$$

where Δx is the uncertainty in position and similarly for the momentum. In Equation (11), h is a universal constant, Planck's constant. The uncertainty is an estimate of the error in measurement of that quantity; it is the standard deviation of the measurements.

Using the pendulum as an example, the roots of the uncertainty principle are found. When a measurement of the bob's position is made, the bob's momentum is changed by the measurement. Consequently, measurement of x and then measurement of p gives different results than if these measurements were carried out in the reverse order. Equation (11) ultimately arises from this 'non-commutative' aspect of quantum measurements. It turns out that for certain other pairs of variables, measuring one variable in the pair – such as, what time it is – disturbs the value one subsequently measures for the other member of the pair, the energy of the bob. Not all variables interfere with each other in this fashion; instead, some are commutative.

3.4. THE BORN POSTULATE

We return now to the discussion of the pendulum begun in Section 2.1. Recall that the classical pendulum had a greater chance of being found near the extremes of its motion, the sand piled higher there. In quantum mechanics the probability of finding the bob at a given location is an important issue. Suppose a student asked Born the question, "What is the chance of finding the bob between position x and position $x + \Delta x$"? Born would have argued that the probability, ΔP, is based on the value of the pendulum's wave function $\Psi(x)$ at position x and is given by,

$$\Delta P = \Psi^2(x)\Delta x. \tag{12}$$

In words, the probability of finding the bob near x is proportional to the square of the wave function at x. The wave function tells us where we would likely find the bob.

3.5. CORRESPONDENCE PRINCIPLE REQUIREMENT APPLIED TO THE PENDULUM

The Correspondence Principle introduced by Bohr begins with the observation that classical physics works very well in the everyday world. Quantum mechanics is needed primarily to describe the microscopic world. Bohr's principle states that any new theory (quantum mechanics) must agree with an older theory (classical mechanics) in those circumstances for which the older theory gives the correct predictions. Applied to quantum mechanics, this requires quantum mechanics to agree with classical predictions for macroscopic systems.

The pendulum affords a direct way of illustrating the Correspondence Principle. Moreover, an understanding of the pendulum has wide ramifications since any mechanical system that has an equilibrium will behave as an oscillator. Figure 2 is a graphic illustration typically used to visualize this relationship. In the figure, the vertical axis graphs (roughly) the probability of finding the pendulum bob in a given region around position x versus the position of the pendulum bob, x, on the horizontal axis. The dotted curve in the figure is the classical prediction of Section 2.1. Notice the higher chance of finding the bob at the extremes of the motion. The oscillatory curve is the quantum probability corresponding to the tenth energy

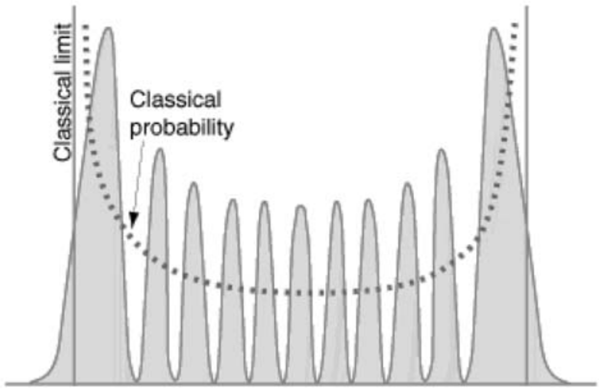

Figure 2. The classical (dotted curve) and quantum (solid curve) probability versus position for an oscillator. The oscillator is in the tenth energy level. The vertical lines show the limits of the classical motion, the classical turning points. The classical probability distribution is based on the fraction of time the bob is in a given interval on the x-axis. (For a detailed derivation of this distribution, see Anderson (1971, pp. 198–199.)) [Note: This figure is provided courtesy of Rod Nave, Georgia State University, Atlanta, Georgia, as retrieved from his educational website, http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/hosc6.html.]

level (an excited state) and therefore a nearly macroscopic pendulum, close to the everyday world. Because the Correspondence Principle states that, as the quantum numbers become large, the predictions of quantum mechanics become the same as those of classical mechanics, the diagram helps one visualize how the quantum pendulum is approaching the classical one. Not shown in the figure is the quantum probability for the ground state – the microscopic world of atoms. For this state, the quantum probability curve would look like a bell curve centered on the origin and much different than the classical curve.

In summary, quantum mechanics creates a shift from classical mechanistic determinism to a new quasi-deterministic viewpoint. In the nineteenth century, Poincare proved that any finite mechanical system obeying Newtonian mechanics will, given enough time, return to any initial mechanical state (Huang 1963). Taking this point of view to its extremes, some concluded that all mechanical properties of the universe are determined and, if thought is simply a result of mechanics, then all thought is determined as well. Since the time of Newton through Einstein, some have argued in favor of this 'clockwork' model of the universe.

In the sense that it introduces probability into the description of the state of a system, quantum mechanics is not deterministic. However, it is deterministic in the sense that this probability 'fluid' moves in space according to a fixed law of nature, the Schrödinger equation. Given the wave function at all points of space, then the wave function at all future times and places is strictly determined. The knowledge the wave function gives is of a probabilistic nature, but that knowledge evolves in time according to a fixed law.

4. The Challenges of Teaching Quantum Mechanics to Non-Scientists

In introducing quantum theory to non-science students, we have pondered several questions: Is there any purpose to having students learn mathematical procedures that they will most likely never use once they have left the science classroom? On the other hand, might not banishing mathematics from physics give a superficial, distorted understanding and does not this go against the grain of how physics progressed historically? How does one encourage students to take seriously a subject that at first seems so removed from everyday experience? How can we motivate students to take an active interest in the theory and its applications? Certainly quantum mechanics is fundamental to our understanding of nature. Furthermore, it is often as important as classical mechanics in the development of modern technologies. Consequently, we believe that any basic understanding of modern science and technology requires some understanding of quantum mechanics. But the question remains, how can we make the teaching and learning of the subject as accessible as possible without becoming overly vague and incoherent?

Since the onset of quantum thinking, considerable knowledge has accrued about how people learn (Arons 1990; NRC 2000). [For an overview of how people learn in relation to physics education, see Redish & Rigden 1996.] Interdisciplinary learning and memory research provide us with insights into how the mind processes stored and new information. Successful students are able to find relationships among ideas and not just remember isolated entities devoid of meaning and intellectual utility. In a review of literature, Brekke (1994) identifies myriad and complex variables that affect success in high school and college physics, including problem-solving ability, visual-spatial ability, gender-related factors, motivation, teaching style and curricula. Even decisions to take physics courses are affected by institutional, cultural, and social factors.

What, then, should be major goals of conceptual physics courses? In 1912, Mann wrote of the essence of physics in a book about physics teaching:

> The spirit of physics is not composed of Newton's laws of motion, Boyle's law, et al.; and this spirit cannot be imparted to pupils by imposing on them ideas, arranged in a logical system, to be learned by fair means or foul. The spirit of physics is the intuition of universal relatedness, which the pupils already have; and the function of physics teaching is to assist them in making that intuition concrete and in proving its validity. (p. 216)

Further, he offers the following suggestions that are relevant to the current context of physics learning:

> Our present problem is, (1) to find out how the pupils actually do observe and think, and (2) to discover by experiment how the material of physics may be used most effectively to develop ideals of scientific method while acquiring a mastery of the most useful physical principles. (p. 287)

How do students learn 'relatedness' in physics and how do we discern how they think as they attempt to learn physics? Research indicates that students learn through cognitive conflict, much like the emergence of new theories in a scientific revolution (Kwon et al. 2000; Villani 1992). Disequilibrium occurs when data contradict current theory and new conceptualizations result when equilibrium is restored, sometimes supporting a higher intellectual developmental level (Piaget 1985). Posner, Strike et al. (1982) assert that learning is inquiry by means of a conceptual change model (CCM) in which a current conception is no longer adequate and a new conception is produced by accommodation in the cognitive structure of the individual. Further, they hold that, because accommodation is tied to a student's fundamental assumptions and knowledge of the world, we as educators should aim to develop in students an awareness of their assumptions and those implied in scientific theories; consistency in their beliefs about the world; an appreciation of the historical and philosophical foundations of modern science; and, an appreciation of the worth of new conceptions (Posner et al. 1982, pp. 224–225). Students often pass through transitions in model making (Villani 1992). Tao and Gunstone (1999) concur with CCM, but found that conceptual change may occur progressively and differ by context, thus affecting transfer to new situations. Students need to be reminded of analogous contexts and commonalities when faced with a new situation. For example, computer simulations can be useful when used in groups in which students can explore, manipulate, and negotiate meanings in various contexts.

Context and personal cognitive elements must be addressed in physics learning. More insights are needed into intricacies of student learning pathways, especially those of non-science majors (Tobias 1990). Petri and Niedderer (1998) have identified examples of 'cognitive attractors' that are likely to appear on the road to understanding. Often scientifically incomplete or erroneous, these attractors are affected by meta-cognitive beliefs and may vary in strength as instructional experiences lead to more accurate conceptual understanding. Current knowledge is a powerful gatekeeper to developing new knowledge.

4.1. TEACHING STRATEGIES

Mental models are critical to science learning as they are means by which learners make predictions and formulate explanations. However, some students are mired in mathematical abstractions that have limited meaning for them. How, then, are the complex, abstract ideas constituting quantum theory to be communicated to students who may not pursue further formal science courses?

Using research reviewed and acknowledging the challenge in communicating quantum ideas to a general audience, we have identified a series of strategies that will support the general education conceptual physics course that we are planning. The course will be our initial teaching manifestation of the IPP, and we expect others (such as linked courses and faculty discussion groups) to follow.

1. Rutherford (2001) argues for the inclusion of history and philosophy of science as an ingredient of science literacy. Science is a complex human endeavor and the history can be depicted in a schematic to show the major players, their views, and emergence of new ideas (Leary & Kippany 1999). We intend to tell

the story of the dialog, controversy, and passion surrounding the emergence of quantum theory by the scientists in the early part of the twentieth century. We will explore the Bohr and Einstein debates as an example of the impact of world view on data interpretation (Bohm & Peat 1987). Quantum theory accepts duality, an understanding so important that Bohr actually used the yin yang symbol when knighted (Hobson 1996). The need for addressing conflict in views is reinforced by Kalman (2002), who advocates having students use a designated science philosopher's world view when studying their discipline. An interplay of learning models occurs as students study scientific process and how theories emerge, thereby refining their own views and their critical thinking skills.

2. We will introduce semantics (conceptual understanding) prior to syntax (mathematical formulas), in line with research reported by Greca and Moreira (2002), reflecting the work of the Physics Education Group. Their findings encourage tapping into students' tacit abilities to employ analogies, idealization, and abstraction as they develop their mental models, idiosyncratically and recursively. We will use their findings to explicitly treat the modeling process by using the pendulum as the transition vehicle from the locality and determinism of classical thinking (classical pendulum) to the probability, nonlocality, uncertainty, and dualism of quantum thinking (quantum pendulum). In so doing we will study the nature and impact of scientists' world views as we explore the shift from the classical to the quantum pendulum. For instance, we will end discussion of the classical pendulum by equating it to Einstein's world view.
3. We will acknowledge that learning occurs in social settings by using peer collaboration and reflection, rather than simply reproducing information (Johnston et al. 1998). Success in collaborative physics learning is related to resolving cognitive conflict by co-construction in which students consider and confront one another's varied ideas before constructing problem solutions. This kind of shared, reflective exercise involves implicit and tacit awareness and precedes individual meaning making, which seems to improve with exposure to varied contexts (Tao 2000). In addition, it mirrors the process of dialog and debate that led to the evolution of quantum theory.
4. As assessments often limit inquiry in a course (Lawrence & Pallrand 2000), we will assure that our assessment strategies are matched to our learning goals in order to provide appropriate feedback to students and to ourselves as instructors. Alternative assessments, such as student-generated concept maps, will assist us and the students in analyzing their sense of relevant concepts and how they are related. As we attempt to build bridges between students' conceptions and the fundamental tenets of quantum theory, we will map students' conceptual change (Dykstra et al. 1992), realizing the tenuous nature of many students' mental models, and identify 'cognitive attractors' that accompany the transition from classical to quantum thinking. Indeed, we look forward to

a research agenda that focuses on students' thinking about quantum theory as we explore the quantum pendulum and related conceptual schemes.

5. Acknowledging the need for assistance with spatial representations, we will use computer simulations to assist with conceptual transitions as we employ multiple modes of learning (Peña & Alessi 1999), supported with classroom dialog. Because the notion of uncertainty is fundamental to quantum theory and yet so often misunderstood, we will explore ways to make it more tangible (e.g., in exploring the meaning of standard deviation).
6. We will allow time for students to reorganize their thoughts and accommodate new information, realizing that a time investment early on will yield a return later in the course (Nussbaum 1998). In the spirit of science education reform, we will free ourselves to choose depth over breadth of content coverage and to link quantum ideas to current scientific and technological advances.

5. Conclusion

As a team with diverse individual backgrounds, we feel that we have benefited from the ongoing dialog and research that have informed our constructing a rationale and plan of action for teaching quantum theory to non-science students. In that conceptual change is often accompanied by cognitive and even emotional struggle, we hypothesize that learning key aspects of quantum theory, itself fraught with cognitive dissonance, will meet with less student resistance than would theories based on their own experiences in a classical world. Perhaps 'letting go' is easier in a quantum milieu. We intend to develop a research agenda around that possibility.

In addition to our plans to teach quantum ideas in a conceptually-based undergraduate course, we intend to work with high school teachers on their understanding of quantum theory and its importance, as we believe that quantum thinking should begin in secondary schools. The quantum world appeals to the imagination and students should not be deprived of opportunities to appreciate the stories and world views connected to the emergence of modern science. We believe that the pendulum will serve our students well as they journey toward understanding the duality of their worlds.

Acknowledgement

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Notes

[1] In this article we are primarily concerned with students majoring in the humanities and the social sciences. We presume that conceptual physics courses are designed to introduce such students to the ideas of modern physics with minimum use of the mathematics commonly associated with courses for prospective majors in the physical and biological sciences. We thus use the labels 'non-science students', 'conceptual physics students', and 'general students' synonymously to refer to these more traditional students. Also see Hobson (1996).

[2] The history of quantum mechanics is a rich field within the history of modern science. The best recent treatment can be found in Kragh (1999) (see references), which includes an extensive bibliography of the secondary literature. See also contributors list to *Science & Education* **12**:(5–6).

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Analyzing Dynamic Pendulum Motion in an Interactive Online Environment Using Flash *

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Abstract. A pendulum 'engine' with dynamic parameters can be created and pendulum functions manipulated and analyzed using interactive elements in Flash. The effects of changing the damping (convergence) properties, initial release angle and initial velocity conditions can be explored. The motions then can be digitized using the Flash Digitizer 1.1, exported and graphed. The powerful properties of actionscripting, coupled with the flexible interactivity of the Flash environment, allows for attractive and mathematically driven Flash movies. Along with the accessibility and interactive nature of the Web, and by contextualizing the history and mathematical applications of pendulum motions, this lesson becomes teacher and student-friendly for physics and mathematics classrooms.

1. Introduction and Rationale

If we want to understand why our instruction works or doesn't, we have to understand something about how our students' minds function.

– Robert Redish - Millikan Award Lecture, AAPT, Lincoln, NB, Aug 1998.

Based in current thinking about learning theories, educators are making predictions that effective learning environments in the 21st century will function quite differently from the structure of classroom settings in the past century (Pellegrino 1999). 'Extremely powerful information technologies will become ubiquitous in educational settings, fundamentally changing the nature of learning environments at all educational levels' (Schoenfeld 1992). In the last 10 years, new facilitating technologies have been developed and implemented with an 'exponential' growth in data, information and knowledge. Research that attempts to gain insight into and design a pattern for change of the shifting and developing roles of both the teacher and the student has begun in earnest. Definitive studies about 'naïve conceptions'

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that students may have about physics have been undertaken by various individuals and research groups for the last twenty five years and more, (Riche 2000).[1]

In addition, there are many others researching in the areas of cognition and learning, especially in the areas of educational psychology, innovative computer technologies and the study of computer-human interactions.[2]

Cognitive research has shown that learning is most effective when four fundamental characteristics are present: (1) active engagement of students in the lesson; (2) participation in a group setting; (3) frequent interaction with feedback; and (4) connections to real-world contexts are present (Roschelle et al. 2000). Brown (1992) proposes that richer and more lasting knowledge acquisition ensues as a more in-depth probing of students occurs. The effective use of Diagnoser is an example of this active engagement, when run in parallel with other instructional activities, such as problem-solving, laboratory activities, after the individual student 'facets' have been discovered and targeted (Minstrell 2001).[3]

Integrally, teachers need a fundamental understanding of the history and philosophy of science and its nature. 'Knowledge *of* science (science content and method), and knowledge *about* science (its history, philosophy and sociology) are both important components of scientific literacy' (Matthews 2000). In addition, educator-teachers should not only have a rich understanding of their content, but also its teaching context. Science is an explanatory and exploratory system used to account for natural phenomena (Cobern & Loving 1991).

The study of the pendulum offers a 'unifying concept' through which the study of the physical world can be taught in a powerful context. In the study of the history of mathematics and science, invention and progress have been accelerated by unexpected and novel applications of new discoveries.

2. Some Science Educational Research Perspectives

> The most serious criticism which can be urged against modern laboratory work in physics is that it often degenerates into a servile following of directions, and thus loses all save a purely manipulative value. Important as is dexterity in the handling and adjustment of apparatus, it can not be too strongly emphasized that it is a *grasp of principles, not skill in manipulation*, which should be the primary object of General Physics courses. (Robert A. Millikan, in Redish 1990).

An important area of cognitive research in physics is the study of misconceptions. Many researchers have shown that misconceptions:

(1) Are extremely common;
(2) Are not easily displaced;
(3) Can be found (even) among experts; and
(4) Hinder understanding.

It appears that people continually and unconsciously build models of how the world operates. The human brain seeks patterns and quickly establishes categories. Patterns of experience are put into models, but often, these models are based on insufficient experience. Furthermore, these models, misconceptions included, affect how later experiences are interpreted.

From the results of their 'balance scale task', Siegler and Klahr conclude that acquisition of new knowledge depends on the interaction between existing knowledge, 'encoding processes,' and the instructional environment (Siegler and Klahr 1999, p. 197).

Learning in interactive environments allows students not only to make decisions about the physical situation that is represented, but also to interact with it, proceeding from familiar schemas toward constructing new ones. There is wide-spread agreement by many educational researchers that prior experience and preconception of physical situations can 'be at odds' with newly presented concepts. Consequently, learners will distort this presented material, learning something opposed to the educator's intentions, no matter the quality of the lesson (Roschelle 2000). Prior knowledge appears to be simultaneously necessary and problematic. Thus, in order to invoke a move toward conceptual change, making the most of each novel experience, an 'anchor' in prior experience must be constructed and discussed. Further, instructional designs must include strategies that endeavor to illuminate, discuss and take steps to resolve these conflicts. This requires very careful consideration of the assumptions that are made about knowledge, experience and learning.

Many conceptual change explanations have been attempted, although they can at most claim some limited lasting success. Learners can succeed in conceptual change as long as appropriate care is taken in acknowledging students' ideas, embedding them in an appropriate social discourse, and providing ample support for the cognitive struggles that will occur, combining the theoretical frameworks of Piaget et al. (Roschelle 2000). There have been some exemplary endeavors, however, most notably:

- Clement, Brown & Zeitsman (1989) have developed a science curriculum based on 'anchoring analogies' – everyday concepts from which scientific concepts can grow.
- Minstrell (2001) has developed classroom techniques for gradually restructuring students' conceptions by identifying students' facets of learning.
- White (1993) has developed a computer-based curriculum called 'Thinker Tools' which develops a scientific concept of motion gradually over several months and includes explicit attention to differences between scientific discourse and ordinary discourse.
- Roschelle (1991) studied students' learning from similar computer software and concluded that students learn the scientific concept of acceleration through a series of gradual transformations of their prior knowledge.
- Van Heuvelen & O'Kuma developed Active Learning modules, using 'multiple representation problem-solving, and along with Dave Maloney, developed Ranking Tasks for physics problems, providing authentic and interactive learning environments for students.

Table I. Comparison of the differences in the learning styles of novices and experts

Experts	Novices
Knowledge characteristics	
Large store of domain-specific knowledg	Sparse knowledge set
Knowledge richly interconnected and hierarchically structured	Disconnected and amorphous structure
Integrated multiple representations	Poorly formed and unrelated representations
Problem-solving behavior	
Conceptual knowledge impacts problem-solving	Problem-solving largely independent of concepts
Performs qualitative analysis	Manipulates equations
Uses forward-looking concept-based strategies	Uses backward-looking means-ends techniques

Retrieved from, http://umperg.physics.umass.edu/perspective/researchFindings.

The above table summarizes some of the differences between experts and novices that cognitive research has studied and revealed. One of the tasks of excellent curriculum materials is to encourage beginners to think more like experienced problem-solvers.

3. Some Math Educational Research Perspectives

[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word.

– Galilei, Galileo, Opere Il Saggiatore p. 171 (Woodard 2000).

'It is widely accepted in the psychological literature that people organize their experiences mentally via mental representations of familiar *classes* of experience [schema]. ... Attached to a schema are its typical features, some knowledge related to it, and typical ways of behaving when that schema has been called to mind (Schoenfeld 1998).

A generally accepted perspective on the nature of learning is that it is the 'process of conceptual change', integrating new concepts with previous knowledge and experience, and integrating old conceptions with new perspectives (Riche 2000). Although much of what is written on conceptual change is built upon the Piagetian concepts of *assimilation*, *accommodation* and *cognitive disequilibrium*, some current research on the brain is critical even outright hostile to the methods of Piaget, Inhelder and others. Stanislas Dehaene, mathematician and cognitive neuropsychologist, has carried out careful and definitive research on how humans acquire and use number sense. 'According to the theory first set forth some fifty years ago by Jean Piaget, logical and mathematical abilities are progressively constructed in a baby's mind by observing, internalizing, and abstracting regularities about the external world. ... [He] believed that number.must be constructed in the course of sensorimotor interactions with the environment' (Dehaene 1997, p. 42). Dehaene targets what he calls 'Piaget's errors'. He counters some of Piaget's findings and research methods by asserting:

- Children not only have mental representation of numbers, but have it soon after birth.
- Piaget relied on an 'open dialog' between experimenters and children, other researches, examining similar tasks demonstrated that test results varied widely, depending on the context in which questions were framed and the motivation of the subjects tested (Dehaene 1997, p. 44).
- Children were led by Piagetian researchers to misinterpret instructions and chose the 'longest row' rather than the one that had more items.

The results of more recent 'brain research' has yielded evidence that children not only do not have an 'optimum developmental stage' at which numbers make sense. 'From an evolutionary viewpoint, it is rather remarkable that nature founded the bases of arithmetic on the most fundamental laws of physics. At least three laws are exploited by the human "number sense".'

- First, an object cannot simultaneously occupy several separate locations.
- Second, two objects cannot occupy the same location.
- Finally, a physical object cannot disappear abruptly, nor can it suddenly surface at a previously empty location; its trajectory has to be continuous' (Dehaene 1997, p. 60).

This link between discrete physical objects and numerical information (often called one-to-one correspondence) endures in many children up to a much older age and can even inhibit mathematical development.

As a child develops mathematical representations and conceptions about the physical world, conclusions they may draw are based on observation and experience with objects in the real world. When students learn physics, they bring these naive conceptions to the classroom. Careful questioning, discrepant situations and demonstrations may illuminate these 'native ideas' which must be recognized, vocalized and analyzed, before they can be incorporated into a more sophisticated view of the physical world. Bruer defines representations as the link between a real world and our internal processing systems. 'Our initial problem representations are important because they shape the course of our problem solving' (Bruer 1999, p. 32–22).

WHY THE PENDULUM? AN 'INTEGRATED' MODEL

U.S. physicists have solved a 350 year old riddle of why the pendulums of two clocks become synchronized. The clocks were the first example of spontaneous synchronization, a phenomenon found throughout nature from cells to the Solar System ('Ancient Pendulum Conundrum Solved') (Whitfield 2002).

Table II. Comparison of NCTM and science education standards, pendulum motion

NCTM, Mathematics Standards, 9–12 (2000).
The power of functions to simplify complex situations and to predict outcomes can be demonstrated by observing a phenomenon involving an underlying functional relationship between two variables, gathering and plotting observational data, fitting a graph to the plotted points, using the graph to formulate the relationship between the variables, and then predicting outcomes for unobserved values of one of the variables. For example, students could record the number of swings during a given time period for *pendulums of differing lengths*, graph the relationship between the number of swings and the length of the pendulum, formulate this relationship, use it to predict *the number of swings for pendulums* of other lengths, and validate their predictions of the *motion of a pendulum* through written lab reports.
National Science Education Standards, 9
Teaching Standard A. (Community of Learners.) Teachers of science plan an inquiry-based science program for their students. In doing this, 'Students will be using an inquiry-based approach to study *pendulum motion*. In addition, the students will develop computational models that add to their understanding of the *periodic motion of a pendulum*'. Content Standard A. (Understanding Scientific Inquiry). Science as Inquiry – As a result of activities in grades 9-12, all students should develop: Students will be using an inquiry-based approach in their initial study *of pendulum motion*. Content Standard E. (Understanding abilities of technological design). Science and Technology – As a result of activities in grades 9–12, all students should develop: 'Students will develop computational models of ipendulum motion using spreadsheets. Students will use computational modeling techniques using (STELLA).[4] And, they will also study the *pendulum problem* using modeling software.

Both the National Center for Teaching of Mathematics and the National Science Standards include pendulum motions specifically. A portion of each standard, dealing with pendulums and their motions is given in Table II.

Pendulum motions figure prominently in several accepted sources for student misconceptions about motion.[5] The study of pendulum motion has pervaded the development of Western Thought. Scientific investigations began in the fourteen century experiments of Galileo and the development of the pendulum clock by Huygens. This figured into the solution of the measurement of longitude. Next was the establishment of the behavior of objects under gravitational force, the conservation laws and Newton's synthesis of the laws of classical mechanics – supplanting an Aristotelian view of physics with the 'new' science of observation and measurement? Mathematics, physics, philosophy, politics, religion, sociology and historical perspectives are all intricately involved in the study of the pendulum and its motions.

Students are familiar from infancy with experiences involving such motions, from the swaying of a lamp to the tetherball on the playground. The pendulum

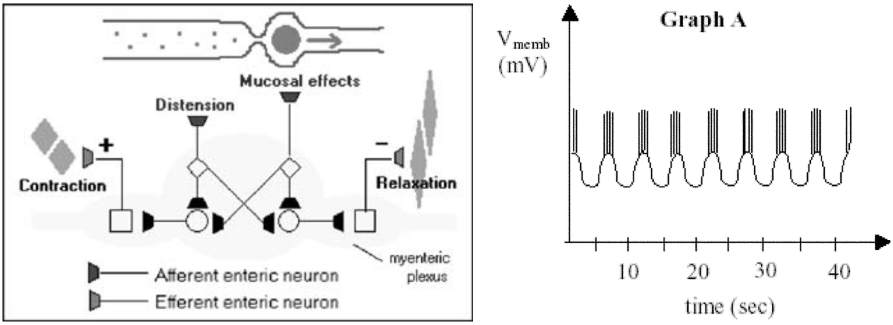

Figure 1. Picture represented by permission.[6]

represents a primordial motion not unlike those of the fetus in a mother's womb, in the simple harmonic motions of a rocking chair and in peristalsis, the progressive, wavelike contracting of the longitudinal and circular muscles, primarily in the digestive tract and in some other hollow tubes of the body. Peristaltic waves occur in the esophagus, stomach and intestines, as is shown in Figure 1a and b.

Personal and recurrent experience, however, does not make the mathematical representations of such motion intuitive. Careful analysis of concept and function must be undertaken in order for the student to understand the wide implications of these motions.

In the last half of the 20th century, information processing models became the dominant view of how knowledge was transferred. 'Information processing psychology builds on the metaphor of mind as a computer of symbolic data, … the major contribution of IP is the production of innovative representational systems and sound scientific methodology for analyzing learning processes' (Roschelle 2000).

In the last ten years, 'situated cognition theories,' have emerged as a counterpoint to IP theories, maintaining that learning occurs within experiential transactions, 'co-ordinations' between a student and the environment. Situated learning is a general theory of knowledge acquisition. It has since been applied in the context of technology-based activities for learning that focus on problem-solving skills (Kuhn 1992). Converting 'recalcitrant reasoning', into well-understood forms can succeed only so long as the older forms are of less prominence than those that fit with the constructed knowledge. Until the computer revolution, reasoning using non-sequential representation was soundly discounted as 'illogical', or 'anti-logical' (Barwise & Etchemendy 1998).

Tapping research in the areas delineated above, interactive stand-alone models can be developed to positively impact the process of student concept- building in physics. Beginning with the motions of the pendulum, which have been studied for centuries, if not millennia, as a 'vehicle for conceptual change', students can interact, derive and describe myriad situations in which these actions apply.

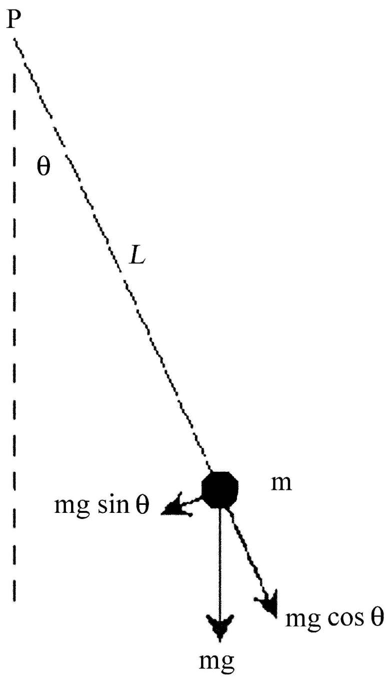

Figure 2. Force distributions for a simple pendulum.

4. Modeling the Motion of a Simple Pendulum

Modeling the mathematics in a modern way (for Galileo had no equations and solved his problems by geometric proof), Figure 2, shows a distribution of force for an idealized pendulum of mass (m), suspended from a 'massless' string or rod, of length (L). As the pendulum is moved to its maximum positive displacement from rest (as pictured), and released, it will swing to the left some distance (maximum negative displacement). The time it takes to swing one complete cycle (from positive maximum and back again, for example), is one *period* of the pendulum.

The forces acting on the pendulum bob are:

- The gravitational force on its mass (weight) compels the motion.
- The force exerted by the string (tension), constrains the circular path.
- A damping force (if any).

Pendulum Motion: Modeling $d\theta/dt$:

We make the following definitions and set notations. Let $\theta(t)$ be the corresponding angle with the vertical (see Figure 2). Then $-mg\sin\theta$ be the tangential force (the only force producing motion) and $mg\cos\theta$ be the radial force with respect to the path (exactly offset by the tension in the string). The basic law of motion, $F = ma$, will be applied where (a) is the acceleration of the bob and where $a = g$, the acceleration of gravity. Assume for now that the damping force = 0. The direction of the gravitational force g is downward (taken to be the negative direction). This is a constant 9.807 m/sec/sec; at sea level on the earth. The length of the string or radius of a complete circle and be measured in meters is L. It then follows that when the pendulum is moving to the left, $d\theta/dt < 0$. Conversely, as it swings to the right, $d\theta/dt > 0$. The restoring force acts in the direction opposite the swing.

From Newton's Second Law

$$ma = -mg \sin\theta$$

We have that $a = d^2s/dt^2$ and therefore in angular measure $a = Ld^2\theta/dt^2$. So, $m\ L\ d^2\theta/dt^2 = -mg \sin\theta$. We rewrite this as

$$d^2\theta/dt^2 = -g/L \sin\theta$$

Using the approximation for small angles $\theta \approx \sin\theta$, we arrive at the standard form for the undamped pendulum

$$d^2\theta/dt^2 = -g/L\theta.$$

This second order differential equation has a solution given by

$$\theta(t) = a \cos\sqrt{\frac{L}{g}}(t) + b \sin\sqrt{\frac{L}{g}}(t).$$

If we assume that the pendulum is held and then released; that is, it has zero initial velocity, the solution simplifies to

$$\theta(t) = a \cos\sqrt{\frac{L}{g}}(t).$$

Since the restoring force is proportional to the displacement, the pendulum is a model of a simple harmonic oscillator with a spring constant g/L. The period of the pendulum is $T = 2\pi\sqrt{L/g}$. The angular frequency (reciprocal of the period) is

$$\omega = -\sqrt{\frac{L}{g}}.$$

Now we model the damping by assuming that the damping force is proportional to the angular velocity (ω), the simplest case, with proportionality factor (b). This gives the damping force term to have the form

$$-b/md\theta/dt$$

where the constant of proportionality $b > 0$. The damped motion, with the $\theta \approx \sin\theta$, angular approximation is therefore

$$d^2\theta/dt^2 = -b/md\theta/dt - g/\theta.$$

The solution is more complex in this circumstance having the form

$$\theta(t) = a\,\mathrm{e}^{\left(-1/2\frac{(Lb-\sqrt{L^2b^2-4m^2gL})t}{mL}\right)} + b\,\mathrm{e}^{\left(-1/2\frac{(Lb-\sqrt{L^2b^2-4m^2gL})t}{mL}\right)} \times \mathrm{e}^{\left(-1/2\frac{(Lb-\sqrt{L^2b^2-4m^2gL})t}{mL}\right)}$$

For the small angle approximation, the amplitude of the pendulum has no effect on the period. As a pendulum is damped, it loses energy, however the period is constant – great for timekeeping.

Huygens, in his *Horologium Oscillatorium sive de motu pendulorum*, in 1673, (and later Johann Bernoulli), solved the correct path of quickest descent (brachistochrone) for a pendulum to be the cycloid curve. Huygens established that the cycloid curve was the tautochrone (path independent of time) and work on the compound pendulum (O'Connor & Robertson 2002).

'Torricelli was the first to publish the solution of the [cycloid] problem (in Opera *Geometrica, "De dimensione Parabolae, solidique Hyperbolici problemata duo* ... " pp. 85–90). Three demonstrations are found as an appendix to the chapter indicated, "by means of which we demonstrate", wrote Torricelli, "with the help of God that [the cycloidal space] is the triple [of the generating circle]". The first and the third demonstration are carried out using the method of indivisibles, the second is made in the manner of the ancients, by double reductio ad absurdum'.[7]

Properties of a cycloid curve:

$$x(t) = at - b\sin(t),$$

and

$$y(t) = a - b\cos(t)$$

In the following section, the modeling in Flash will include a comparison of the motions of the pendulum and the cycloid curves.

5. Flash Animations and Digitizing

Doing physics is not just creating a new map; it's about creating new understandings of the physical world, (Redish 1998).

Why Flash? There are many delightful and well-executed Java programs and software available commercially.[8] However, as the landscape of online learning (along with e-standards) is changing, educators are compelled to tap new authoring resources available, that they, themselves may use as tools, without the steep learning curve that programming in Java requires.

The plethora of computer resources available today yields a much richer and complete array of representations, and new forms of valid inference and application. Computers with their increasingly user-friendly graphical capabilities can

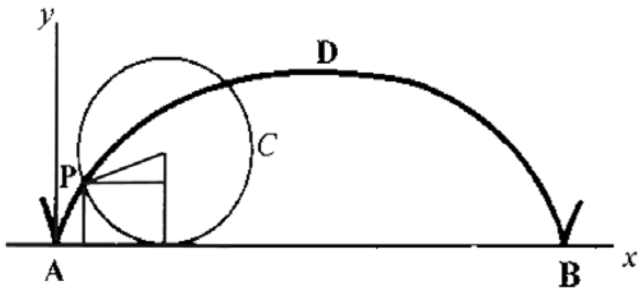

Figure 3. The cycloid curve I described by a point P, attached to a circle *C* that rolls, without sliding, on a fixed line AB. The full arc ABD has a length equal to 8r (r = the radius of the generating circle), and the surface included between one complete arc and the fixed line is 3pr2 . The cycloid curve is "brachistochrone", i.e., a curve of least time: given two points A, B in a vertical plane, a heavy point will take the least time to travel from A to B if it is displaced along an arc of a cycloid. It is also an "isochronic" curve, i.e., a curve of equal time. A heavy point which travels along an arc of cycloid placed in a vertical position with the concavity pointing upwards will always take the same amount of time to reach the slowest point, independent of the point from which it was released.

provide powerful tools for understanding a wide variety of representation patterns. So, a dynamic inference process can be captured and reproduced. Deductive systems can be constructed using complex and sophisticated representations (Barwise & Etchemendy 1998).

With Flash, almost everything that Java can be asked to do is also available and more easily accessible. A teacher can easily use many of the considerable resources available in the Flash environment. Every teacher can construct meaningful problem scenarios after only a few hours of training. Flash also offers a considerable built-in tutorial with online support. Many flash web resources are also freely accessible, including instructions on how to enhance Flash lessons for the novice through expert user. With Flash, teachers can create classroom activities, demos and lessons that are

- Interchangeable (easily editable)
- Durable (of lasting value).
- Assessable (Flash has many test and quiz templates built in).
- Accessible (Flash is web-friendly but also can function as a computer-based lesson).

Flash MX is a powerful development tool, with the ability to integrate a wide variety of media and external data sources (Reinhardt & Lentz 2001).

Using Flash allows:

- Interactivity (active learning between student and learning environment).
- Emulation of difficult-to-produce-in-the-real world elements in physics.
- Ability to isolate variables and watch their effect on motion.
- Powerful front-end graphics, integrated animations and presentations.

Table III. Student misconceptions of pendulum motion

Students naïve conceptions:
• The period of oscillation of the pendulum depends on the amplitude
• The restoring force is constant at all points in the oscillation
• The heavier a pendulum bob, the shorter the period
• All pendulum motion is perfect simple harmonic motion, for any initial angle
• Harmonic oscillators go on forever.
• A pendulum accelerates through the lowest point of its swing
• Amplitude of oscillations is measured peak to peak on a graph of pendulum motion
• The acceleration is zero at the end points of the swing of a pendulum

Table compiled from C3P data retrieved
from: http:www.shodor.org/cserd/applets_desk/Pendulum/educator.html.

If presented with interactive Flash components embedded into the context of a lesson on pendulum motions, along with the ability to digitize and compare related types of motions, understanding of the nuances and differences between motion types can help students recognize their own preconceptions and naive thinking and encourage them to explore other conceptions.

To this end, and based on the research findings delineated above, two simple pendulum Flash animations were designed in Flash and digitized – one pendulum transcribes a circle, the other a cycloid path. Each was digitized using Digitizer 1.1 (Allen 2002). Also, a Flash animation for the pendulum has been programmed so that the user can manipulate the variables of angle, length, damping factor, time step and tracking the period. The key ingredient is user interactivity. Teachers can modify each scenario fairly easily and can assemble them into a powerful lesson.

These Flash elements can be orchestrated into a coherent, dynamic Flash generated module, based on the elements of 'best practices' delineated above. Students' misconceptions regarding the pendulum are considered; these are organized in Table III.

Ultimately, the use of Flash to create 'learning objects, and analyze their motions, imbed them in a coherent lesson and plan for measurement of student learning, builds on accepted educational practice and reinforces the need to design instruction that is responsive not only to content but also to student needs.

'Minstrell conceives of physics as a sense-making activity, and he believes that students' classroom experience with physics should support the development of students' abilities as sense-makers' (Schoenfeld 1998). Making sense is the first step all students must take in the learning process. Awareness of their own difficulties, and what they may already know about a subject, (prior learning) along with recognition of what steps they may need to take in order to complete the 'whole picture', are all key elements in helping students toward active participation in their own learning.

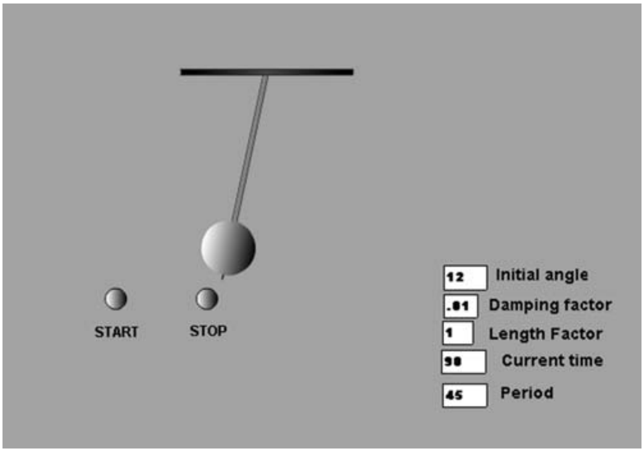

Figure 4. Dynamic interactive pendulum motion.

Although the original pendulum animation was begun in Flash 5, it was converted quickly to Flash MX, as MX is a considerably more powerful tool. In Flash, construction of a Flash animation (Fla) is done on several levels with a 'theatre' model. Action is directed:

- On the Stage
- On the Timeline
- In the Actions menu
- On the objects

An overview of the process of the production of the Dynamic Interactive Pendulum animation is described in Figure 4.

This was constructed partially on the timeline, where comments, actions and motions can be attached to individual frames, corresponding to time steps – the key ingredient to designing effective Flash movies, keeping the parts separate and editable. On the stage is where the action takes place. The programming is done in a code called ActionScript, an object oriented programming language.

The Dynamic Pendulum was programmed using ActionScript, attached to the pendulum, to frames in the movie, to buttons and other objects. Buttons were created for playing, stopping, stepping forward and stepping backward. Input boxes hold the variables, defined in the program.

The brachistochrone model was programmed, next.

After constructing the pendulum, and the cycloid Flash animations, each was imported into Digitizer 1.1 and 'coordinized.' Just about anything that can be measured can be analyzed from a variety of viewpoints: arithmetic, algebra, geometry, calculus and physics. Students can learn, first hand, the importance of coordinate systems, the value of system scales and the importance of multiple sampling. Students can learn many mathematical concepts and skills using digitization, such as:

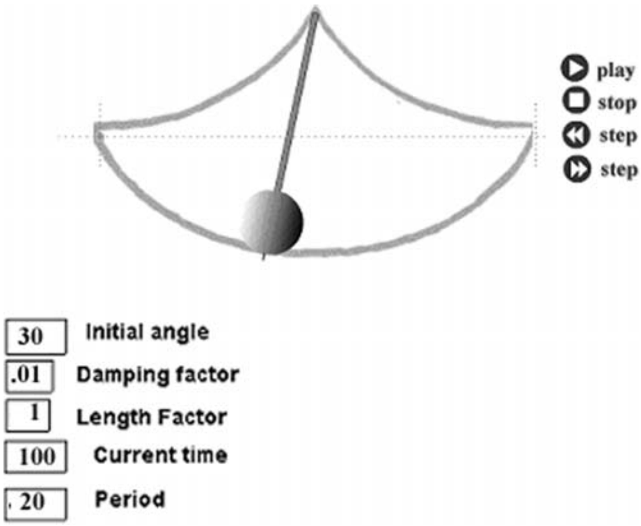

Figure 5. Cycloid motion.

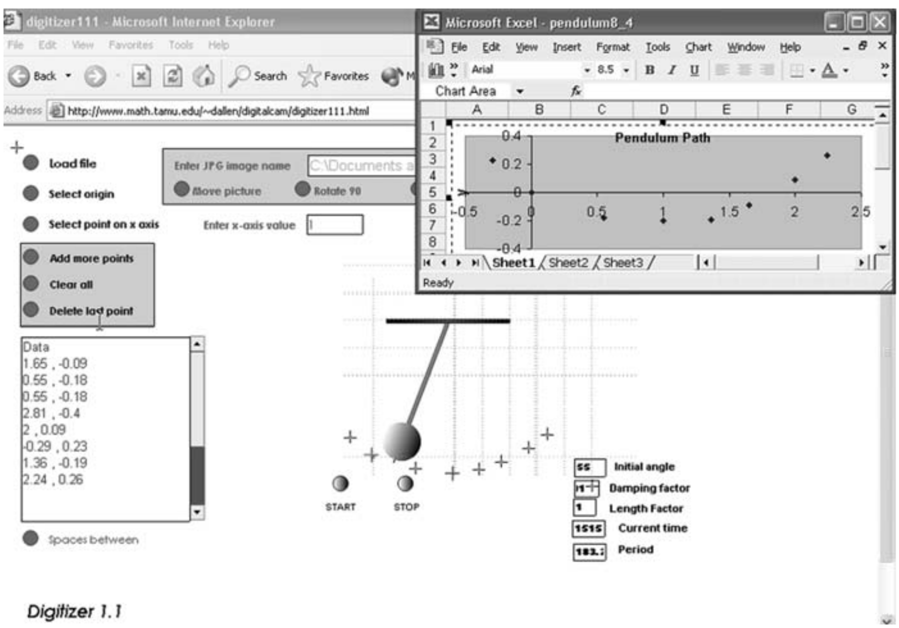

Figure 6. Digitized pendulum motion.

- Error of approximation
- Relationships between coordinates and other quantities
- Identification of shapes from data
- Use of spreadsheet software (Allen 2002).

The digitized images appear in Figures 5–11.

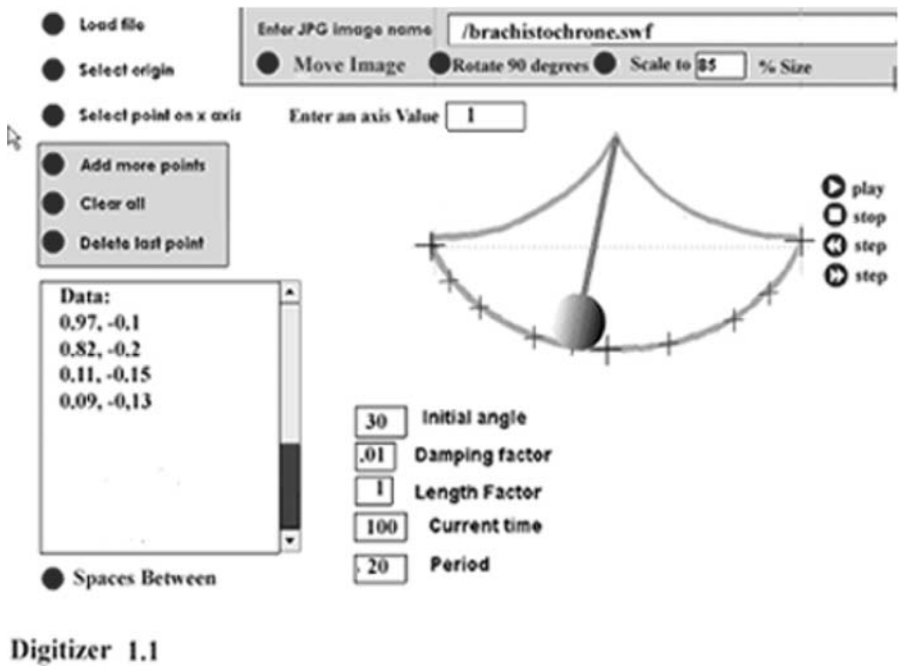

Figure 7. Digitized cycloid motion.

Digitization allows for the exporting of data sets to Excel and for the analysis of generated data. An example catenary curve digitization appears in Figure 8.

An example follows for the side by side pendulum and cycloid objects (Figure 9). The second Flash object created illustrates the difference between the pendulum motion and that of the cycloid (see previous discussion).

These were also each digitized, using Flash Digitizer 1.1, in Figure 10.

These data were then imported into Excel and graphed, in Figure 11.

The differences in motions as well as in their functions can then be visually compared. And, students are encouraged to repeat these data, fitting the formulae to the curves and comparing data.

These graphs were then inserted into a coherent 'active lesson' on motion, programmed in Flash – the presentation.[9] Thornton and Sokoloff, (1990) showed that traditional lectures, homework and recitations do not do well in helping students build good mental models. By changing the learning environment to an active-engagement one, most students gain success in grasping difficult physics concepts. Flash is an attractive medium in which to design and implement physics lessons, based on the best thinking in educational research. The use of Flash in interactive lesson design, allows the student the ability to not only change the parameters and observe the effect on motions, but also to apply the digitization of the motions and graphically analyze them. Combined with the accessibility and interactive nature of the Web and Flash, mathematical applications of pendulum motions can be

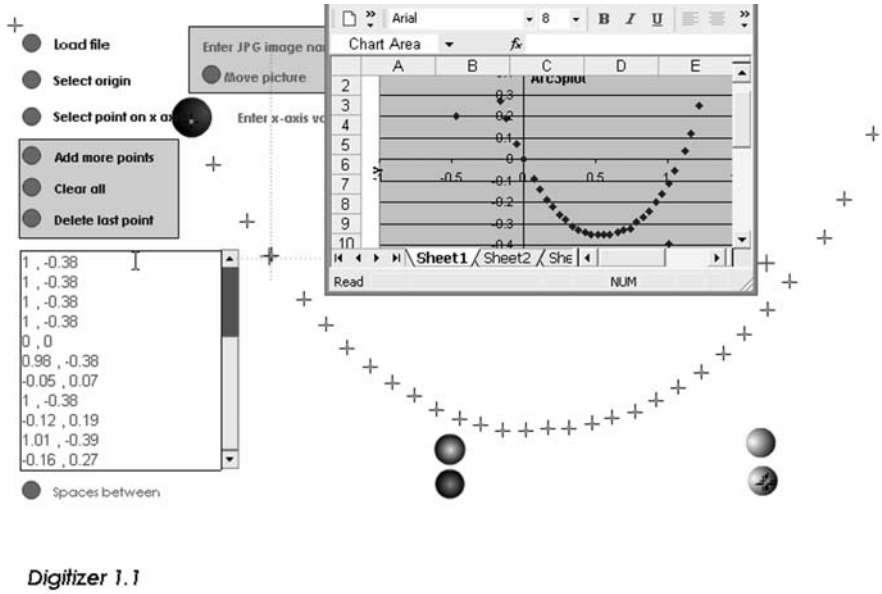

Figure 8. Comparisons with a catenary curve – digitized.

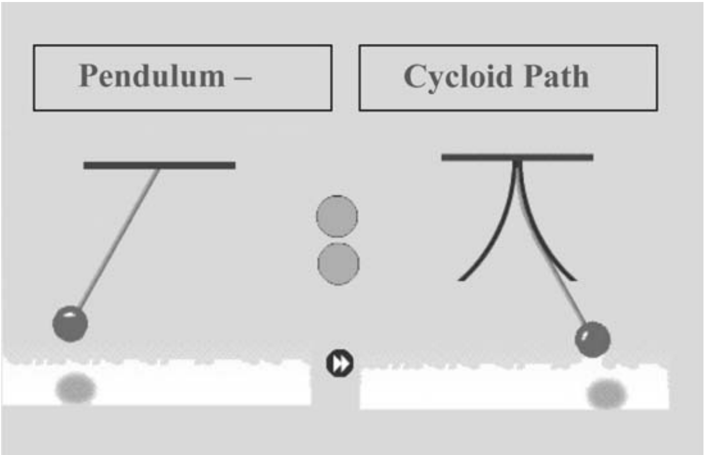

Figure 9. Side by side flash motions of the pendulum and cycloid.

taught and learned effectively and more holistically in physics and mathematics classrooms. 'To function in an intuitive mode, our brain needs images (Dehaene 1997, p. 88).

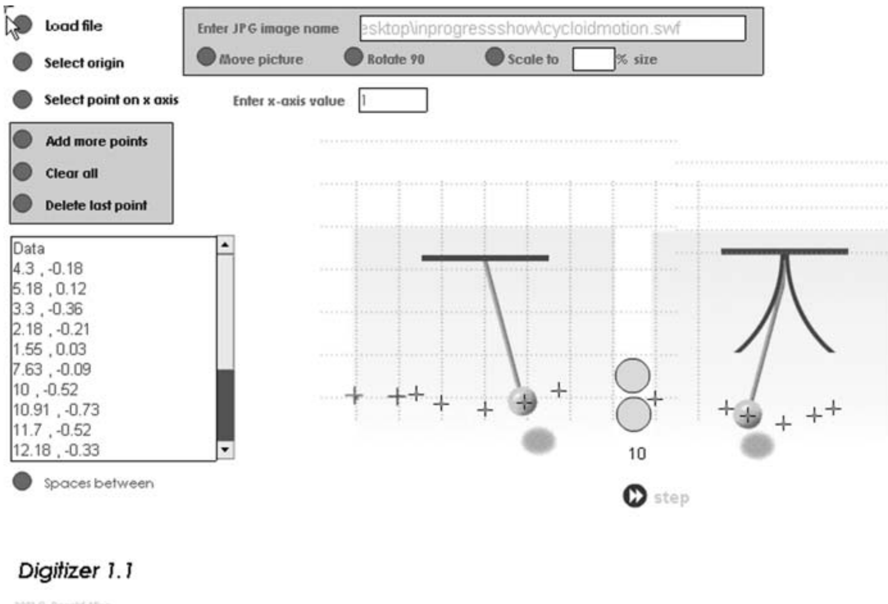

Figure 10. Side-by-side digitization of pendulum and cycloid.

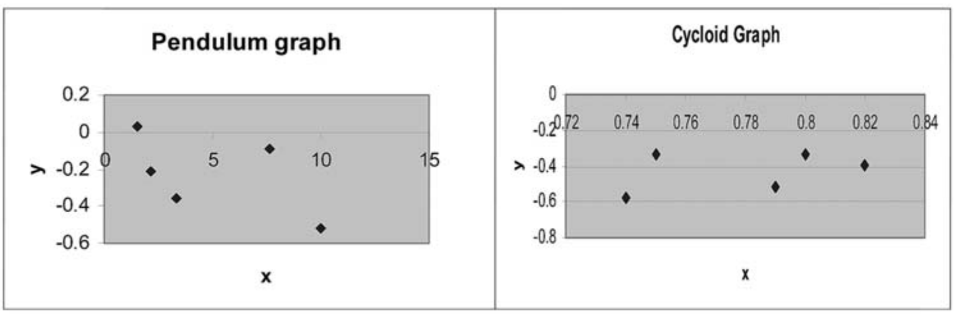

Figure 11. Comparison graphs of the pendulum and cycloid motions.

6. Conclusions and Implications

The union of the mathematician with the poet, fervor with measure, passion with correctness, this surely is the ideal

– (William James, in MacKay, 1977).

The motion of the pendulum, especially as Galileo perceived it, with its isosynchronicity (independence of path), was a 'paradigm shattering' event. Once set in motion, it culminated in a flurry of new discoveries and invention of many new mechanical devices. More importantly, Galileo effectively challenged the Aristotelian scientific world view and defined scientific methods and perspective for future generations. Although Galileo's use of the pendulum was as a simple timing device, it may be one of the rare examples where a machine was born from pure theory (Lienhard 1998).

Initially, the pendulum was chosen as the subject of interest for Flash animated lessons because it had an important impact on the authors as it arose in several contexts in the spring of 2002. Its mathematics and physics applications were explored at the same time that its scientific, historical and pedagogical contexts were examined. The opportunity to employ Flash in the programming of the pendulum and related motions, had special significance in that it would tie together the myriad aspects of the pendulum study, already underway. A Flash slideshow with branching options highlights the history and philosophy of the pendulum while analyzing its motions and comparing them with the cycloid and others will be online, shortly. Student interactions will be observed and the Flash modules modified as the data are collected and analyzed in a pilot project completed during the 2002–2003 academic year.

Implications for improved and interactive learning environments are many. Student and teacher access to immediate feedback on student pre- conceptions and learning progress will be enhanced. Interactivity of the Flash lessons in a 'complete' learning environment, will weave together the many threads making up the breadth of scientific concepts. The teacher has added control over modifications of the Flash environment, editable for each student's needs. The interactive Flash and physics and mathematics lesson responds to the best in educational research and, along with digitization as an enhanced analytical tool, has the potential to become a powerful new learning environment design for the future. Adding to this the Web accessibility, Flash lessons can be available for widespread use among teachers in the mathematics and scientific communities.

Erwin Schrödinger codified a simple but profound viewpoint about our scientific thinking,

> Thus, the task is, not so much to see what no one has yet seen; but to think what nobody has yet thought, about that which everybody sees. (Woodard 2000).

Notes

[1] Exhaustive research has been done by: Caramazza et al. (1981), Dykstra et al. (1982), Minstrell (1982), Posner et al. (1982), McCloskey (1983), Dobson (1985), Terry et al. (1985), Feldsine (1987), Ivowi & Ouvulton (1987), Lawson & McDermott (1987), Schultz et al. (1987), Tobias (1987), Brown (1989), Hammer (1989), Kyle et al. (1989), Marioni (1989), Maloney (1990), Renner et al. (1990), Scott et al. (1991), Hewson & Hewson (1991), Van Heuvelen (1991), Andre (1992), Mazur (1992), Roach (1992), Eckstein & Shamesh (1993), Linder (1993), Committee on Undergraduate Education (1994), Wandersee et al. (1994), Gordon (1996), Colman (1997), Seifert (1997), Tao & Gunstone (1999), Minstrell & Kraus (19??), Bransford, et al., at Vanderbuilt University, and others. The most prevalent physics topic researched in the area of student conceptions is that of force and motion.

[2] Research in the area of human-computer interactions and user-friendly 'physics-ware' is being done by educational research groups at: the University of Nebraska, led by Scott Henniger, http://pooh.unl.edu/ scotth/Welcome.html; York University, led by Michael Har-

rison, and others, http://www.cs.york.ac.uk/hci/formal/; University of Oldenburg, http://www-cg-hci.informatik.uni-oldenburg.de/; the University of Maryland, http://www.cs.umd.edu/hcil/; University of California, Irvine, http://www.isr.uci.edu/research-HCI.html; Carnegie-Mellon University, http://www.hcii.cs.cmu.edu/; Web Physics, at Davison College and IUPUI by Wolfgang Christian, Mario Belloni and Gregor Novak, http://webphysics.davidson.edu/Applets/Applets.html; JitT, Andy Gavrin, Evelyn Patterson, Gregor Novak; Web Assign – Larry Martin, Aaron Titus, et al. A synopsis of these and other groups' research can be found at http://futureofchildren.org/usr_doc/10-2d_tab1.pdf.

[3] Facets in Thinking refers to a description using 'middle language' to get at the right level of student thinking as it is seen or heard in a learning situation. Facets have been derived from research on student thinking during learning and derived from classroom situations during learning.

[4] Some additional sources are: CSERD, http://www.shodor.org/cserd/applets_desk/Pendulum/educator.html; Saskatchewan Education (1992), http://www.sasked.gov.sk.ca/docs/physics/u5d3phy.html; Robert Riche, http://www.bishops.ntc.nf.ca/rriche/ed6390/paper.html.

[5] Alternative modeling software can be used.

[6] Permission: http://arbl.cvmbs.colostate.edu/hbooks/pathphys/digestion/basics/peristalsis.html

[7] See 4, above.

[8] Illustration from the Online Enciclopedia Treccani, Institute and Museum of History of Science, Florence, ITALY, http://galileo.imss.firenze.it/multi/torricel/etorat32.html.

[9] The presentation will be active online so that the user can view the interactive elements, see the slideshow and get the flavor of the Flash Active-lesson environment, soon.

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Pendulum Activities in the Israeli Physics Curriculum: Used and Missed Opportunities

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Abstract. In light of the established validity of pendulum as a topic in physics curricula (Matthews 2000), the study looked at the place of pendulum motion in the physics curriculum of the high school in Israel. The data is available through presenting results of the nationwide matriculation examination in its units of Mechanics and Research Laboratory for the Advanced Placement program (several thousands students). Although the assessment questions and problems mainly tested students' performance, and less their understanding of the subject, the study, by discussing these problems and questions, allows a perception of the extent to which the pendulum topic is addressed in High School physics instruction. The results can support a discussion on the nature of the requirements and values encouraged by the particular educational system and the strengths and weaknesses of the adopted educational policies.

Background

To describe the educational activities related to pendulum motion, we first, very briefly, present the structure of the physics education system in Israeli High Schools. In grade 10, the Ministry of Education suggests that physics be taught three hours per week throughout the whole academic year. The extent of this instruction constitutes "one unit" (1-u) in the general score of the requirements for matriculation certificate – the formal goal of the secondary school studies. This 1-u program is compulsory. Further physics education is elective and is presented at two levels: two and four additional units, comprising regular (3-u) and expanded (5-u) curricula.[1] The 3-u program approximately corresponds to GCSE[2] level, and the 5-u program corresponds to A-level in the UK ("Advanced Placement" in the USA).

The curriculum at the elementary level (1-u) includes presentation of the simple pendulum, mainly for the purpose of mastering fundamental skills of measurement of physical quantities (time period of a periodic process) and their dependence on variables (length, mass, amplitude), which can be isolated. Although the formula for the time-period is not taught, students are required to graphically represent the functional dependence of time period on the length of the pendulum.

M.R. Matthews et al. (eds.), The Pendulum, 435–448.

Only within the extended 5-u curriculum do students have a chance to explore pendulum motion at a more comprehensive level. This can occur in three places within the curriculum: mechanics (within the topic on harmonic oscillation), rigid body (physical pendulum) and laboratory (different types of pendulums).[3]

Unlike many other countries, in Israel 3-u and 5-u streams include matriculation examination tests, which are administered simultaneously to all students across the country. This makes students' responses in the matriculation examination available for analysis and interpretation. Due to the large numbers of participants, the data can reliably indicate student's difficulties in learning pendulum motion. Secondly, it is easy to understand that the format of nationwide (or, as is said in Israel, "nationlong") examinations has a major influence on the content of the studies during years 11–12 at school, which has both positive and negative aspects. In any case, both perspectives invite and justify a review of the matriculation examinations with regard to pendulum motion. Investigation of the problems and tasks used in the exams may reveal the nature of student knowledge of the topic, accepted by the system as a standard. Therefore, we will demonstrate examples of problems and tasks, which were related to pendulum motion and used in the 5-u matriculation examinations in recent years. We will reproduce some statistics of students' successes and difficulties, analyze the requirements and discuss the rationale of the examination and the values adopted in physics education, at least in our country.

Representative Problems and Tasks

"MECHANICS" EXAMINATION

The mechanics exam lasts 90 minutes. The questionnaire includes five problems, of which three are required to be solved.

Problem M1 (1998)

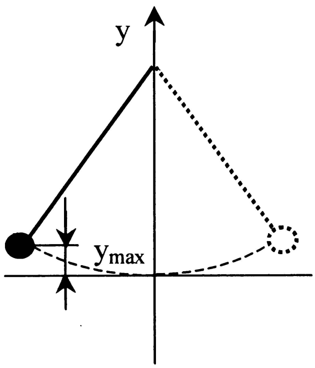
The following are the results of measurement of period (T) of several simple pendulums of different length (l) swinging at small amplitudes as performed by a student:

Period T (sec)	0.90	1.25	1.55	1.80	2.00
Length l (m)	0.2	0.4	0.6	0.8	1.0

a. Provide the graph that will facilitate your calculation of the free fall acceleration g (45%).*

b. Calculate from the graph the free fall acceleration (40%).

c. For the pendulum length 1.0 m and period 2.0 sec, the student measured the height y of the pendulum bob, as a function of time (see the figure). The student discovered that the function is periodic. What is the period of this function?
Explain your answer (15%).



* Percentage in brackets shows the relative weight of the particular sub-question.

Relevant results:

58% chose the problem. The average score was 83% (the average score for the whole test was 74%). If one summarizes the expected responses based on the knowledge required, we observe the following:

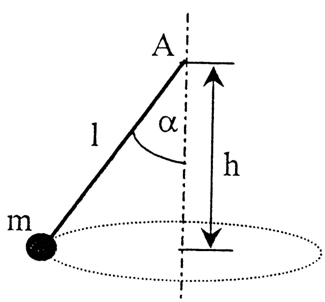
(a) The question required the graph T^2 versus l. Some provided the graph T versus l, which cannot support simple calculation of the free fall acceleration.

(b) The question required equating the slope in the graph T^2 versus l to $4\pi^2/g$. The typical mistake here was the calculation of the slope according to a single point, not the whole graph.

(c) The typical mistake here was identifying the period of y with the period of the pendulum. The failure was the greatest in comparison with the other sub-questions.

Problem M2 (1991)

A small body of mass m is suspended from a stationary point A on the rope of the length l. The body rotates with frequency f along a circular horizontal trajectory (see the figure). The angle between the rope and vertical is α.



a. Show the forces acting on the body during its movement (specify each force, its direction and its origin) (12%).

b. Using the equation of motion, obtain the expression for $\cos\alpha$ as a function of the given length l, and frequency f (45%).

c. After doubling the length of the rope the body continued to rotate with the same frequency. Did the distance h between the point of suspension and the center of the circular trajectory (see the figure) increase, decrease or not change? Explain your answer (27%).

d. Is it possible that the rope takes a horizontal position while rotating the body? (16%).

Relevant results:

64% (4700 students) chose the problem. The average score was 75% (with standard deviation 25) while the average score for the whole test was 73%. 45% of the students provided a full answer and 21% failed. If one groups here the expected performance with the prevailing flaws, the following summary is obtained:

(a) Two forces only act on the body: the gravitational attraction to the Earth and the tension of the rope. The typical mistake (50% of the students) was to mention the centripetal force as additional force acting on the body. Other students mentioned centrifugal force (unnecessary within the Newtonian framework of forces and required only in the description within the rotating frame of reference not studied at high school).

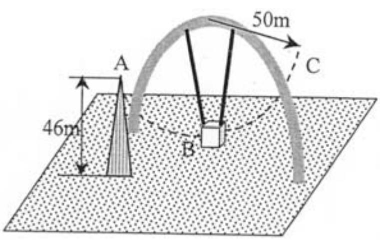
(b) Newton's second law written in vertical and horizontal axis for the considered motion yields: $\cos\alpha = g/(4\pi^2 l f^2)$.

(c) The obtained expression for $\cos\alpha$ implies its reduction by the factor of two, following the doubling of the length l preserves the distance h, which is equal to $l\cos\alpha$.

(d) The negative answer could be supported by the fact that there is always a vertical component of the tension (equal to mg).

<u>Problem M3 (2001)</u>

The figure schematically represents a swing in the amusement park. Long metallic ropes from the semicircular frame suspend the carriage (see the figure). Three passengers, with a total mass of 200 kg, are in the carriage. At the vertical position of the ropes the carriage is at the height of 1 m. To initiate swings the carriage is raised to the height of 46 m (the top of the tower nearby) and released.



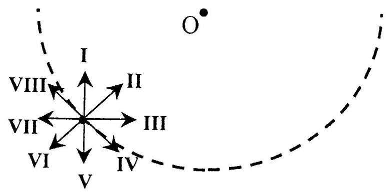
The trajectory of the oscillating carriage is along the arc ABC of the radius of 50 m. Assume that during the swings the ropes do not lengthen and their own mass is negligible. Neglect the friction with the air as well.

a. Calculate the speed of the carriage when it passes point B (24%).

b. Calculate the centripetal acceleration of the carriage when it passes point B (24%).

c. Calculate the total force of the ropes on the carriage when it passes point B (30%).

d. The drawing on the right presents the trajectory of the carriage movement ABC. A point is marked on the trajectory and eight directions labeled as I, II, ... VIII. What direction from the eight better represents the direction of the acceleration of the carriage when passing the point? (22%)



One may mention that the problem is presented without using the term pendulum, although it is equivalent to the simple pendulum motion, implying that all the required knowledge has been taught to the students.

Relevant results:
59% (5400 students) chose the problem. The average score was 83% (with standard deviation 18.2) while the average score for the whole test was 80%. 48% of the students provided a full answer and 9% failed. Grouping here the expected performance with the prevailing flaws, the following summary is obtained:

(a) Usage of energy conservation was required. A few students slipped up here by ignoring the finite height of 1 m in energy conservation balance.

(b) Calculation of the centripetal acceleration required the correct answer of the previously calculated speed and the knowledge of the corresponding formula.

(c) The calculation of the rope tension T required the previously obtained results and using Newton's Second Law. Some students mistakenly equated it to the weight of the carriage. Others mistakenly equated T with the centripetal force. Some students had confused the presence of two ropes mentioning the tension of each as equal to the calculated total result for T.

(d) The correct direction III represents the net force in this case and thus the total acceleration of the carriage. Some students mistakenly rendered only partial acceleration, either radial, or tangential.

In 2001 the twenty teachers employed to check the examination results were invited to comment on the questions used. Their evaluation is indicative. We present below several questions and the distribution of responses received regarding Problem M3.

Question 1: Did the question adequately reflect the material taught in class?

Very much so	Yes	Reasonably	Does not	No relation
10	9	1	0	0

Question 2: Was the presentation of the question clear?

Very clear	Clear	Reasonably	Not clear	Very unclear
6	7	3	1	1

Question 3: Were the provided drawings clear?

Very clear	Clear	Reasonably	Not clear	Very unclear
4	7	1	7	1

Question 4: Evaluate the loading of the question.

Too loaded	Very loaded	Reasonably loaded	Normally loaded	Underloaded
0	1	11	8	0

Question 5: Was the question frightening to the students?

Very much so	Yes	Reasonably	Only in the beginning	No
0	3	5	7	2

Question 6: Evaluate the overall level of difficulty of the question.

Too difficult	Difficult	Normal	Easy	Very easy
0	1	14	5	0

Such evaluation made by the teachers importantly represents the expectations and, to a certain extent, the views of practicing teachers regarding the assessment and curriculum.

"RIGID BODY" EXAMINATION

Unlike the Mechanics section, the Rigid Body section presented one of ten elective sections, each containing two questions from which students had to choose one. Students are given 90 minutes to solve two questions out of four in the two elective sections. The topic of pendulum appears with regard to the physical pendulum in the Rigid Body section. We represent this examination using one example.

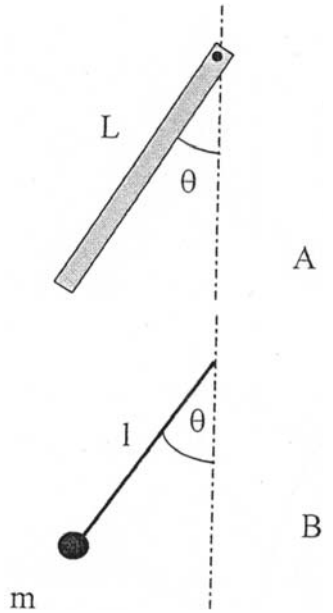
Problem RB (1997)

a. One can consider the simple pendulum as a limiting case of the physical pendulum. Develop expression for the period of oscillation of the simple pendulum from the corresponding formula for the physical pendulum. Explain your reasoning (40%).

b. A uniform rod of length L and mass M represents a physical pendulum. It swings round the axis through one of its edges (see figure A). Show that the length of the simple pendulum (see figure B) that has the same period is (40%):

$l = \frac{2}{3}L.$

c. Two pendulums, physical (length L) and mathematical of the length $l = \frac{2}{3}L$ are released from the same angle of initial deviation θ with the vertical (see figures A and B). Will the angular velocities of both pendulums be equal at the moment they are vertical? (20%)



Relevant results:
1% (35 students) chose this problem.[4] The average score was 85.6% (standard deviation 20), while the average score for the whole test was 75.4%. 71% of the students provided a full answer and only 6% failed. The answers expected were to include the following:

(a) Starting from the formula for physical pendulum, one requires the knowledge of the expression for the moment of inertia of the point body relative to a certain axis and of the torque relative to the pivot point to attain the answer. (b) Equating of the two appropriate formulas and the knowledge of the moment of inertia of the uniform rod relative to its edge could provide the answer.

(c) The answer of equal angular velocities follows directly from the equality of periods of oscillation of the two pendulums: being released together, the two pendulums swing synchronously. This implies the equality of angular velocities. Very few failed in this question.

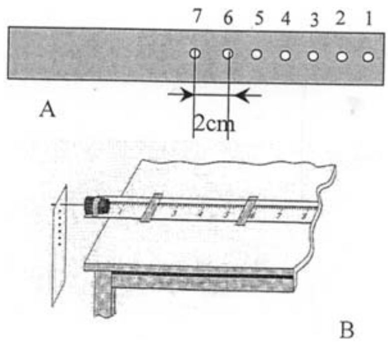
"RESEARCH LABORATORY" EXAMINATION

Currently in schools, the physics laboratory is included only at the highest level of instruction – 5-u (advanced placement curriculum). A special examination is provided for this section. We demonstrate it here with two tasks that touch on the topic of pendulum motion. The tasks appeared in the test mode of "unseen experiment" (one of the two forms adopted for laboratory examination[5]). Within this format students are invited, to follow explicit instructions, perform an experiment they have never seen before, demonstrate the skills of manipulation with simple and common physical apparatus, perform the requested measurements, elaborate the data and interpret it basing on the knowledge they have acquired in the subject. The examination lasts 120 minutes.

Experiment L1 (1992) "Oscillations of metallic rod"

The question regarding the experiment was: "What is the relationship between the distance of the center of a metallic plate to the pivot point and the period of oscillations of the plate?"

Basic apparatus:
A 25 cm aluminum ruler with seven equidistant holes at 2 cm (figure A), a thin rigid rod to hang the ruler through one of the holes allowing its free oscillation (figure B).[6]



Although the experiment dealt with a physical pendulum, it was designed to address all the students, including those who did not learn the elective section "Rigid Body". Thus, knowledge of the concepts of torque, momentum of inertia, etc. was not assumed. The subject for testing was the above mentioned skills required for carrying out a physical experiment. Explicit instructions guided the performance of the experiment. It was followed by a series of questions based on (1) the accumulated data regarding the period of oscillations with various pivot points and (2) the provided theoretical expression for the period of oscillations: $T = 2\pi\sqrt{\frac{k^2+l^2}{lg}}$, in which $k = \sqrt{\frac{a^2+b^2}{12}}$ (a, b are length and width of the plate), l – the distance between the center of mass of the plate and the point of suspension, g – the free fall acceleration.

The questions asked for: (1) determination of the length l for which the period T is a minimum, (2) explanation for the choice of axis for which the graph become linear, (3) determination of g from the graph and interpretation of this result, (4) elicitation of k from the same graph, its comparison with the calculated value and the interpretation of the difference between them; (5) the sensitivity of the results to the material of the plate.

One thousand students took the exam. The average success in the test was 87% (standard deviation 12) with 2% failure. The typical difficulties were (Rosen 1993; Vardin & Sela 1993):

1. Inappropriate choice of unit scale for graph representation (approaching zero at the time axis causes the graph to become inconvenient for later use).

2. Some students drew linear graphs regardless of the nature of the corresponding function.
3. Some students extended graphs to pass the zero point (crossing of the axes) regardless of the data and the nature of the function.
4. Instead of using the slope and point of crossing the vertical axis (in the graphic T^2l versus l) to determine g and k, some students used the formula and particular data points to calculate the requested unknowns.
5. In calculating the slope, some students took close points of the graph, thus increasing the inaccuracy of the evaluation.

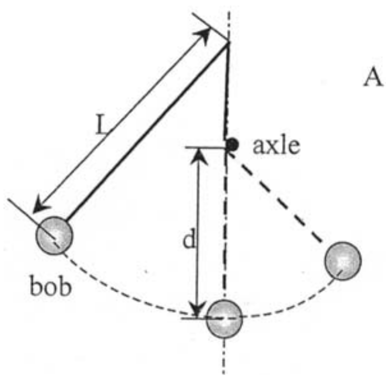
Experiment L2 (1999) "Euler's pendulum"

The question of the experiment was: "What is the relationship between the distance d (of the axle stopping the pendulum rope to the bob at its lowest location – see the figure) and the period of oscillations of the pendulum?"

Basic apparatus:
Simple pendulum of length L. A stand which provided variation of the distance d.
The students were instructed to collect data for the periods of oscillation for several distances d. They were also provided with the theoretical formula describing the period of oscillations at small amplitudes:

$$T = C + \pi\sqrt{\frac{d}{g}}$$

with constant C, depending on the geometry of the apparatus, and g – the free fall acceleration.



The questions asked for: (1) determination of the appropriate axis for the data presentation, which can produce a linear graph of the data, (2) explanation of the graphical meaning of the slope and crossing point of the graph with the vertical axis, (3) calculation of g (free fall acceleration) from the graph and explanation of the meaning of this calculation, (4) addressing the sensitivity of the error in the measurement of the period of oscillations to the change of distance d.

3250 students took the exam. The average score in the test was 87%, (standard deviation 9.2), 68% high scores and 1% failure. The typical difficulties they encountered were:

1. The length of the pendulum was mistakenly taken (by 33%) as the length of the rope (neglecting the size of the bob).
2. Incorrect choice for axes to obtain the linear graph (10%).
3. Incorrect interpretation of the points of crossing the axes by the graph (40%).
4. Failure to recognize the decrease of the relative error of the measured quantity when the absolute error remains the same, but the quantity itself increases (48%).

Discussion

The matriculation examination reliably reflects the curriculum. Its content communicate the accepted standards of the particular educational system, as well as the goals, values and standards of the subject matter knowledge to be acquired in high school. At the same time, the performance of students is expected to represent the extent to which these goals have been fulfilled.

The first feature of the accumulated results on which we reflect, is that the examination, in its "theoretical" as well as "practical" (laboratory) applications, does not require students to show any knowledge of physical theory and is solely aimed at: (1) formal problem-solving ability and (2) manipulating the empirical data.

The examples presented here (especially Problem M1), demonstrate that knowledge of a simple formula of pendulum motion for small amplitudes is sufficient to achieve full marks. The richness of the related physical knowledge (the appreciation of pendulum motion, the simplest among the complex motions, the appreciation of the origin of isochronism obtained as an approximation) is unnecessary for successful performance. The provision of the pendulum formula, as a given piece of exact knowledge ready to use, the request for simple algebraic manipulations with it do not encourage construction of a conceptual account of the topic by the student, but requires a sort of "instrumental" skill. Instead, one could use the vision of Galileo, who considered the simple pendulum as a modified free falling and thus conceptually bridged between the isochronism of a pendulum and the fact that all bodies fall with the same acceleration, regardless of their weight (Drake 1978, p. 73). This aspect is extremely important in the physics curriculum, and requires other kinds of testing questions.

Problem M2 was not presented as a pendulum problem and its original name "a conical pendulum" was not even mentioned. The relationship between conical and simple pendulums is often not addressed in instruction, although the two are simply related and can be accounted for by similar formulas of oscillation periods for small angles, showing isochronism. The difference between the two pendulums is the additional horizontal velocity of the bob, which significantly changes the appearance. In this regard, other conceptual questions are important for understanding elementary physics of pendulum motion, such as:

Why the conical pendulum does not "fall down" (a clear analogy with a gravitational satellite), while the regular pendulum does?
Given that the case of $h = 0$ ($\alpha = 90°$) is impossible, what happens to the velocity of the bob, period of rotation ("swinging") and tension of the rope while approaching this case?

Such questions could significantly contribute to students' understanding of why a satellite "does not fall" the question which has similar conceptual idea. This is a common problem and students of all ages can cope with it (Gardner 1984). All this is missed in the curriculum and the exam. Instead, the student is asked to develop the mathematical expression for $\cos\alpha$ through given parameters and use it in reasoning its variation, expecting a pure algebraic (if not arithmetical) consideration of compensation. Only question (d) in this problem, contributing 16% to the score, required qualitative understanding of conical pendulum.

Problem M3, presented in an everyday context of an amusement park, addressed vertical circular movement.[7] The relation to the pendulum was not mentioned and the term pendulum was not used. In fact, the term "swing" often replaces "pendulum" in physics classes. Within this terminology, many students may comprehend the pendulum to be solely an isochronic device. In a more general presentation, the isochronic feature of pendulum would represent only a special case. Generally, problem M3 requires formal and qualitative knowledge, presenting a good example of such a combination. As was shown, the teachers who were selected to check the solutions confirmed this evaluation with strong agreement.

Problem RB, in the Rigid Body unit, deserves even greater evaluation due to its strong conceptual emphasis addressing the important connection between physical and mathematical pendulums. It is significant to notice the small fraction of students, who chose it, compared to the second, alternative, question of this section (in the examination form), which was less related to the theory of the subject.[8]

Summarizing our inspection of the theoretical examination, one may infer that pendulum is widely used in high schools. However, in the assessment, comprehension of the pendulum topic was examined using problems which required mainly "instrumental" skills, significantly impoverishing this conceptually rich topic. Even the development of related skills, such as reading the numerical data, constructing a corresponding graph and its interpretation, cannot change the fact that the assessment did not invite students to explain phenomena, define concepts, present laws of Nature, illustrate their theoretical statements by choosing relevant experiments, etc. Instead, seeking success in the matriculation exam, the success which will determine the future career of students, forces them to focus on the development of problem solving ability of standard problems and manipulating with formulas which no-one expects them to explain.[9] Whether or not desired, this must cause a limitation of knowledge gained at school, restricting the scientific and cultural literacy of the individual. In the existing orientation on problem solving, students abandon regular physics textbooks, since they can manage without mastering the theoretical content of physics courses. *Teaching*, thus becomes similar to military *instruction*, and *learning*, to *training*. Physics ideology: ontology, epistemology, ethics, worldview, history of physics, become a useless burden (although interesting for many), a waste of time for a pragmatically oriented and ambitious youth, seeking a successful career and ascribing the highest value to the focused instruction on problem solving. Consequently, problem-solving manuals seemingly replace physics textbooks – a highly regretting phenomenon.

Using pendulum motion in the laboratory exam deserves special comment. Since the physics content of the practical activities used and their theoretical background were beyond the curriculum and thus unknown to students, the activities *solely* address the experimental skills, that is, the ability of the students to measure physical quantities and provide simple elaboration of the numerical data collected (Vardin 1997). The students are provided with a ready-to-use formula of an oscillation period of a particular device. Since the examination is individual, no discussion or questioning is allowed. Therefore, all the steps of "inquiry" have to be exactly set down in a rigid sequence and provided to the student who is not expected to deviate from them. Although the meaning of such a mode of examination was not investigated in depth, some questions can be raised regarding the appropriateness, efficacy and worthiness of this activity. Apparently, not much was learned by the students regarding pendulum motion, even though the experiments designed were elegant and interesting (for an expert). Among the questions physics educators should consider in this regard are:

1. Is any kind of inquiry really skill tested by the examination which involves step-by-step guidance through an experiment, when the student's theoretical background is unknown? What kind of knowledge about physics is encouraged by this kind of testing?

2. Does the fact that physics presents an experimental science (requiring elementary skills of accumulation and elaboration of numerical data), justify an isolated focus on these skills in a conceptually unknown to the student context?

3. What kind of advantage does the "unseen" experiment present, relative to the second mode of laboratory examination? In that, the student has to perform one of fifteen experiments, known in advance, chosen from various areas of physics. He or she has to orally explain its meaning and answer the questions of two physics teachers (one of them an external examiner). This examination is the only oral matriculation examination still existing in the school education system.

4. The test format of an "unseen" experiment is performed simultaneously by all students taking the exam in the country. This fact inevitably restricts the equipment to being the simplest possible, available in adequate numbers. This constraint excludes the use of advanced apparatus (V-scope, MBL, etc.) available in many schools but in small numbers. Is this restriction justified?

Thus, with regard to pendulum motion, advanced apparatus allows the measuring of variables such as rope tension in different bob locations and the investigation of pendulum deviation from isochronism, when the period of oscillation varies with amplitude. The latter case provides an elegant opportunity to "enter the historical discourse" and explore Galileo's claims in The First Day of his *Dialogues* (Galilei 1638/1914, pp. 95–97), leading him to the great insight of falling speed independent of weight (Drake 1978, p. 73). Another opportunity for such an activity with the pendulum is to reveal the meaning of the centripetal force (Krakover 1995). Is the neglect of these opportunities of knowledge checking justifiable?

5. At first glance, the "unseen" experiment mode of laboratory exam might look as an imitation of real research in physics. Is this a true resemblance? Does a physics researcher move in accordance with the script (prepared by somebody else) he/she never designed and discussed, without an idea, model or framework of what he/she is going to do in the experiment?

6. The mode of a written examination checked as an anonymous product by a teacher is usually claimed to be a preferable assessment, to be a more standardized and objective assessment. Do these advantages compensate for the apparent disadvantages: a restriction of the required knowledge content, an inability of an examiner to clarify an ambiguous answer, an inability of the student to argue for his/her answer, even an apparently incorrect answer? For instance, students may provide stipulating conditions and new meanings, which could testify to his/her creativeness and power of imagination.

Concluding Remarks

Although the pendulum topic is well entrenched in physics curriculum of Israeli High Schools, its presentation is usually restricted to a mere illustration of basic laws and concepts of Newtonian dynamics, and is used as a convenient setting for standard problem solving and simple laboratory measurements. At the same time, the focus of the assessment solely on the performance of simple standard problems naturally causes simplification of the acquired disciplinary knowledge of pendulum. It ignores such important issues as the approximate nature of its isochronism, different kinds of pendulums and their relationship, the rich history of the progress in human understanding of pendulum motion and the role of pendulum in establishing of the new paradigm of modern science (Matthews 2000). All these represent the missed opportunities of the pendulum topic in the Israeli physics curriculum. The single type of assessment regarding theoretical knowledge – the matriculation examination, being focussed primarily on problem solving, contributes to this situation, as well as to the depletion of literacy and a lack of cultural knowledge of physics. At the same time, one can use the advantages of the centralized nationwide assessment and by its virtue produce a fast improvement in the situation by changing the contents of the matriculation examination and/or by establishing alternative streams in the physics education in High Schools.[10] Of course, this change to a more liberal education presumes a reconsideration of the goals, values and theoretical framework of physics instruction in High Schools. Open discourse involving science educators, philosophers and historians of science can be useful in making physics curriculum culturally rich. The manner of presentation of the pendulum topic in the physics curriculum may serve as an exemplar of such a new approach. It can be a representative and reliable indicator of the new cultural scope, standards and norms adopted by particular education system.

Notes

[1] The two elective physics education levels are curricula of 3 and 5 units. Altogether both these curricula are taught to about 14% of the students. Others elect chemistry, biology or technology (electronics, mechanics, computer science), if they choose the science-technology stream. In total, the science-technology stream is taken by about 50% of the students.

[2] GCSE – General Certificate of Secondary Education.

[3] Similar situation is in the USA (American Association for the Advancement of Science, 2000) and in the UK (Grounds & Kirby 1995).

[4] Even within the elective section of "Rigid Body" (one of ten elective sections), this number was extremely low, only a small fraction compared to those who preferred the second question in this section (500).

[5] The other mode is a format (in the past the only one), in which the student is asked to perform and explain one of 15 known in advance laboratory units.

[6] We do not mention standard laboratory equipment such as, stopper, metric ruler, etc.

[7] Two ropes of suspension, instead of the usually used one rope with regard to a pendulum was another variation of the setting, reflecting the practically common case of a swing.

[8] It is a trivial fact that an individual chooses the simplest task in a test setting. The more standard and technical the question is, the more probable it is that students will choose it. A very high average score in this question hints at the fact that those who choose it were good students. It is however up to the designers of the test to establish standards that can encourage the student to achieve more desirable outcomes.

[9] This statement does not exclude the existence of enthusiastic teachers, who despite of the shortage of time deviate from the pragmatic needs of the curriculum and enrich the classroom teaching with cultural and conceptual contents, so abundant in physics. Their number, however, as well as the extent to which it is done, cannot be reliably checked within with the adopted form of assessment. Moreover, one may think that each such deviation is made at the expense of training in such skills as problem solving, the main skill required by the exam.

[10] We elaborated more on this in the paper "Physics Teaching in the Search for Itself: From Physics as a Discipline to Physics as a Discipline-Culture" to be published in 2004 in *Science & Education*.

[11] *Tehuda* [Resonance] – the Journal of the Israeli Association of Physics Teachers.

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The Pendulum as Presented in School Science Textbooks of Greece and Cyprus

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Abstract. When we refer to scientific knowledge, we, implicitly or explicitly, refer to its three components, namely its conceptual framework, its methodological principles and its cultural aspects. The pendulum is a topic of science teaching and learning where all three of these aspects can be examined with the aim of gaining a holistic appreciation of the transformation of a natural phenomenon into a phenomenon of the physical sciences and how this can then be recontextualized into a topic of school science learning. The main objective of this study is to examine whether this richness of the pendulum as a topic of teaching is revealed in the school science textbooks in Greece and Cyprus, for both primary and secondary education. We will use an analytical mapping instrument in order to determine, whether the pendulum is introduced at some grade level and, if so, in what context. We will then use an interpretive instrument, which relies on taxonomy of science curricula into traditional, innovative and constructivist programs, in order to attach meaning to the analysis. Finally, we will formulate a series of proposals in relation to the educational value of the simple pendulum at the Greek and Cypriot gymnasium level.

Key words: pendulum, science curriculum, science textbook

1. Introduction

The term *scientific knowledge* refers to wider appreciation of the three components of science, namely its concepts, methodology and cultural attributes (Bybee & DeBoer 1994). The pendulum constitutes an object of teaching and learning that makes it possible for someone to dwell equally well on all three dimensions of the scientific knowledge. On the other hand, there has been noted ‘a striking imbalance between the importance of the pendulum in the history of science and the meager attention it commands in science curricula’ (Matthews 2000, p. 3, ch.11).

The main objective of this study is to investigate if and how the above comment holds true in the case of the school science textbooks used in primary and secondary education in Greece and Cyprus. We use a model of analysis of school curricula and science textbooks, which is designed to investigate the way a thematic or conceptual topic is handled in the context of

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formal education. Based on this instrument, we shall prove that the study of the pendulum at all levels of the school system in both countries is restricted to the simple pendulum and reflects the main features of a traditional perspective to science curricula. The main attributes of the traditional approach are a mathematicized or 'pseudo-qualitative' conceptualization, an 'empiricist' methodological treatment and a minimal cultural emphasis. These features appear to contradict the opinion which regards the pendulum as an important object of teaching and learning by virtue of its conceptual, methodological and cultural richness.

Finally, based on the same model of analysis, we undertake to formulate proposals for improvement aimed at enhancing the role of pendulum study in Greek and Cypriot schools while at the same time providing practically feasible solutions in the context of established school curricula.

2. A Model of Analysis of School Curricula and Textbooks

Research in the field of science curricula and textbook analysis is multifaceted. A great number of research projects, conducted with the aim to analyze school science textbooks, concentrate on the content, while a substantially lower interest is in linguistic and sociological analysis (Koulaides & Tsatsaroni 1996). The model presented here is a model of analysis as to the content, revealing the epistemological and cultural features belonging to the school curricula and the textbooks. The originality of this model lies in that, as well as serving as a research tool, it can also operate as a teacher preparation tool for educators at all levels, since it has been designed as a medium for closing the gap between research and pedagogic practice. It is a generalized version of a model that was used, initially, for the analysis of school curricula and textbooks relevant to the concept of 'energy' (Koliopoulos & Ravanis 2000). This model proved to be especially useful in encouraging educators to appreciate the characteristics and constraints of their teaching, to be able to explain their practice and, potentially, to proceed in exploring alternative options for reorganizing their teaching (Koliopoulos & Ravanis 1998).[1]

The proposed model is based on the distinction between two frameworks[2] served by the school curricula and science textbooks that present a conceptual character (Driver & Millar 1986).[3] Each of these frameworks refers to the way a science school curriculum or a science school textbook manipulates the concepts, methodology or cultural characteristics of one or more thematic or conceptual units. At this point, we have to note that the proposed classification of frameworks, even though it derives from an empirical analysis of the existing school curricula, does not correspond to one or more specific curricula. Rather, it reflects an abstract entity integrating general epistemological characteristics which could appear partly or wholly in a concrete science curriculum or science textbook. The two frameworks are as follows:

(i) *The 'traditional' framework*. This framework is characterized by:

(a) the juxtaposition of small thematic units leading to *juxtaposition* or *dispersion* of various conceptual frameworks. A typical example is found in the study of the concepts of energy and light in Greek and international curricula of lower grades of education (elementary and secondary education). These concepts are dispersed in various thematic and/or conceptual unities. The dispersion is such that concepts acquire a different systemic meaning (i.e. a meaning that emanates from the relations of the concept with the other concepts of a conceptual system). They also acquire an empirical meaning (i.e. a meaning that emanates from assigning the concept to some real phenomenon in the course of the transformation of this phenomenon to a scientific object) in each of the thematic units (Baltas 1990).

(b) the *mathematical*, in higher grades of education, or the *'pseudo-qualitative'*, in lower grades, dealing of science concepts. In the mathematical approach the systemic meaning of concepts is favored. In contrast, in the 'pseudo-qualitative' approach, the systemic meaning of the concepts is totally cancelled, because of the lack of mathematical expressions, thus resulting to the domination of the empirical meaning of these concepts.[4] A typical example of this approach is the study of the concept of friction in Greek and international elementary curricula. In Greece, a research of analysis regarding the content of textbooks has shown that, at the elementary school, the concept of friction has been treated in the same way as at the university level. This means that mathematical language has been replaced by everyday language destroying the systemic meaning of the concept of friction.

(c) the *empirical – experimental* approach in which, carrying out one (usually) experiment is sufficient to introduce or confirm a conceptual framework (Joshua & Dupin 1993). In the school textbooks, this approach appears in the form of setting a sequence of instructions the students have to follow accurately in order 'for the experiment to work'. This framework is rooted in the empirical tradition according to which the methodological observation of a natural phenomenon leads to the formation of scientific models. It is found at all grades of education but, mostly, at the lower ones even for concepts that cannot be formed in that manner (Zogza et al. 2001).

(d) the *limited* use of cultural features which does not favor the possibility of developing scientific literature. A clear example of the limited use of cultural features in school textbooks is that, where everyday applications of science are mentioned, these are often not integrated into the main text but placed in a separate box.

(ii) *The 'innovative' framework*. This framework is rooted in the innovative changes on the science curriculum in the 1970s and 1980s and is characterized by:

(a) the formation of broad thematic/conceptual units in which the emphasis is placed on the structure of the unit or/and the so-called *directed theme*. A typical example of this framework is the unit of 'Optics' in the French school project *Sciences Physiques*, *Libres Parcours* (Chanut et al. 1979), which provides three alternative structures of teaching the unit. In the first, emphasis is placed on the construction of the conceptual model 'source – transmission – recipient', the second presents a more methodological character and is structured around the observation of astronomical phenomena while in the third, the approach is more technological with the study of various optical instruments. However, all three structures serve the same conceptual, methodological and cultural goals set by the curriculum. Another example is the American program *Physical Science II* (Physical Science Group 1972) where the concept of energy recommends an organized principle of the whole teaching program.

(b) The *'in-depth'* dealing with a conceptual framework which, at many times, is characterized by a 'qualitative/semi-quantitative' approach to science concepts trying to form a selected relation of the qualitative – quantitative. The approach of the energy concept through the conceptual framework of energy chains is a typical example of this approach.

(c) the effect of the '*hypothetico-deductive*' methodological approach showing the prime role of 'didactical activity – problem'. In this, the hypothetical substance of the science knowledge is shown, which arises out of the study of an open problem the students have to familiarize themselves with while participating, partly or wholly, in designing the experiments (Robardet 2001). Including this approach in the school textbooks is not easy. The example of the French school textbooks *Sciences Physiques, Libres Parcours* (Chanut et al. 1979), where the teaching activities play a prime role and which do not allow a lineal reading of the text, is indicative of this approach.

(d) the *organic enclosure* of the cultural dimension of science in the various thematic units. This means that daily/technological problems (e.g. energy saving, cooking, constructing a measuring instrument) or science historical texts support on their own starting points and frameworks within which the conceptual and methodological approach is formed (e.g. the German teaching project *Neue Physik, Das Energiebuch* (Falk & Herrmann 1981) and the American *Harvard Project Physics* – Holton et al. 1970).

3. The Approach of the Pendulum in Greek and Cypriot Teaching Programs

3.1. METHOD

A first step to analyzing the school textbooks was using an *inventory instrument* of the position, content and form of the pendulum study in every educational level.[5] The structure of the study defines the *unit of analysis* and supports information relevant to the thematic units or sub-units dealing with the pendulum and the coherence or dispersion level of the pendulum study in one or more educational levels. The content provides information on the conceptual, methodological and cultural approach of the pendulum. The *key – phrases* used in this part of analysis code the conceptual framework, the methodological approach and the cultural features of the unit. Finally, the form of the study refers to the *means of expression* (mostly texts, issues, exercises/problems, pictures/figures, experimental activities etc.) and the *way of studying* (simple reference, detailed study) the unit and offers information concerning the importance given by the school textbook to the specific unit. In Table I we give an extract of the instrument used.

The information collected for each analysis unit and coded underneath the column 'Comments/Remarks' could lead to conclusions which can be interpreted through the model of analysis already described in the previous unit.

3.2. RESULTS

3.2.1 *The pendulum study in different educational levels*

Both in Greece and Cyprus, the pendulum study is confined mainly to the study of a simple pendulum and never introduces a comprehensive unit with *its own character*. The simple pendulum is introduced in various units in each of which the conceptual framework of its study is different. The following conceptual frames of study have been identified:[6]

- [a] application of Newton's 2nd law or/and measurement of the pendulum period within the study of the phenomenon of oscillation (circular or/and cycloid curve of a simple pendulum or/and torsional pendulum, measure of frequencies of a system of matched pendula, qualitative approach of wave transmission through the analogy of matched pendula),
- [b] tracing interactions or/and measuring forces during the motion or/and the equilibrium of a pendulum,

Table I. Extract out of the inventory tool used during the analysis of a Greek school textbook

GRADE	STRUCTURE		CONTENT			FORM		COMMENTS REMARKS
	Unit	*Sub-Unit*	*Conceptual frame*	*Methodological approach*	*Cultural features*	*Expression means*	*Ways of study*	
3rd grade of gymnasium	Electromagnetism		–	–	–	–	–	
3rd grade of gymnasium	Electronic particles		–	–	–	–	–	
3rd grade of gymnasium	Motion – Force – Energy	...	...	...	...	...	...	...
3rd grade of gymnasium	Motion – Force – Energy	Force measure	'In which of the two pictures the glass stick and the plastic sphere have same electric charges?'			Question – figures	Simple reference	Conceptual frame (b)
	Motion – Force – Energy	Force features	'Name the forces acting on the iron ball in pictures 1 and 2'			Question – figures	Simple reference	Conceptual frame (b)
	Motion – Force – Energy	...	...	...	...	...	...	...
3rd grade of gymnasium	Swings	Periodic moves			What is the resemblance of a back – forth	Photographs (2)	Simple reference	

				swing move in a playground and a heart's rhythmical move?				
3rd grade of gymnasium	Swings	Swing study – Period	'Timing the move of the simple pendulum, estimating the time needed for a complete swing (or circle). This time is the pendulum period'	Activity 4.1: Swing timing (instructions of performing the experimental activity)		Experimental activity (issue)	Whole paragraph	Conceptual frame (a) Totally guiding experimental activity defined by the definition of the concept 'pendulum period'
3rd grade of gymnasium	Swings	…	…	…	…	…	…	…
3rd grade of gymnasium	Particles of matter structure		–	–	–	–		

- [c] qualitative representation of energy transfer or/and application of the laws of momentum and energy conservation (simple and/or torsional and/or ballistic pendulum),
- [d] tracing a noninertial frame of reference system or/and measuring the rotation period of the earth (Foucault's pendulum),
- [e] measurement of the gravitational constant or/and gravitational acceleration (Cavendish' torsion balance or/and acceleration meters) and
- [f] tracing or/and measuring the electric/magnetic force or/and of the electric charge (use of pendulum for detecting electric or magnetic field/ Coulomb's balance).

Table II presents a general picture of pendulum study in Greek and Cypriot curricula at all three educational levels (elementary, gymnasium, lyceum), based on the conceptual frameworks of this study. In this Table we can see that, in both countries, the pendulum is introduced quite early in science teaching as an instrument, which is appropriate while dealing with various conceptual frameworks. This does not signify that there is always a detailed study of the pendulum. In most conceptual frameworks, the study of the pendulum is incidental and limited. For instance, even though the simple pendulum forms one of the usual phenomena of the Newtonian study concerning the equilibrium of a material point, in Greek and Cypriot school textbooks, is not mean that it is also essential. The same applies to the study of the energy conservation of a mechanical system. It is quite a different thing to chose the simple pendulum chosen as the main phenomenon to introduce this concept (Holton 1985) than for it to simply be one of a number of examples of its application, as occurs in Greek and Cypriot textbooks. In these cases there is a *simple reference* for the simple pendulum also relating to

Table II. A general picture of the conceptual frameworks (see Section 3.2.1) within which the pendulum study is conducted in Greek and Cypriot school textbooks in different educational levels

	Greece		Cyprus	
5th Grade of elementary	[c]		–	
6th Grade of elementary	–		–	
2nd Grade of gymnasium	[f]		[a], [f]	
3rd Grade of gymnasium	[a], [c], [e]		[b]	
	General education	Direction	General education	Direction
1st Grade of lyceum	[b], [c]	–	[c], [b]	–
2nd Grade of lyceum	[a]	–	–	[b], [c], [f]
3rd Grade of lyceum	–	[a]	–	[a], [e], [c]

the conceptual dimension of scientific knowledge. In opposition, the simple pendulum on its own forms an object of detailed study only in the units of 'Oscillations' and 'Waves'. This appears in the 3rd grade of the gymnasium and the 2nd grade of the lyceum in the Greek curriculum and the 3rd grade of the lyceum in the Cypriot curriculum.

3.2.2 *The content of a detailed study of the pendulum*

In the Greek curriculum the detailed study of the pendulum occurs within the core curriculum. In contrast, to the Cypriot curriculum this study takes place within an optional curriculum for students of practical studies. Table III gives a comparative picture of detailed study of the pendulum.

Two approaches are apparent: the first in Greek textbook in the 3rd grade of the gymnasium and the second in the 2nd grade of lyceum in Greece and 3rd grade of lyceum in Cyprus. The first approach gives special meaning to the pendulum study since the pendulum is chosen as the favored phenomenological field for the study of the phenomenon of oscillation and especially the definition of the period of oscillation. This does not appear in the second approach in which the introduction to the phenomenon of oscillation and the study of its modeling (simple harmonic oscillation) is done based on the retrogressive motion of a sphere aided by a spring (spring-mass pendulum).[7] The second point on which the two approaches differ is the absence of the cultural dimension in the second approach. That is, we do not have these experiential features, which endow the concepts with what Baltas (1990) call 'excess meaning'. In contrast, in the first approach the cultural dimension appears in the form of historical or contemporary references to the sundial and Foucault's pendulum, however, without a thorough study. Finally, a

Table III. Comparison of the textbooks in those levels where the pendulum study is done in a detailed manner (see & Section 3.2.1)

	3rd Grade of gymnasium (Greece)	2nd Grade of lyceum (Greece)	3rd grade of lyceum (Cyprus)
A. Introduction and study of the oscillation phenomenon and the simple harmonic oscillation	X [a]	–	–
B. Study of the pendulum movement			
B1. Conceptual approach	X [a], [c], [e]	X [a]	X [a], [e], [c]
B2. Methodological approach	X	–	X
B3. Cultural approach	X	–	–
C. Study of matched oscillations	–	X [a]	X [a]
D. Introduction to waves	X	X [a]	X [a]

third difference is relative to the quantitative or qualitative character of the conceptual framework. (In the first approach, in both circumstances, there is an introduction of texts simply using common language to describe mathematical equations. In the second approach the treatment of force and energy analysis is carried out exclusively in mathematical terms.

Similarities are also apparent between the two approaches. One similarity concerns the methodological study of measuring the period of the simple pendulum as well as the study of the relation between the period and the measures ‘length of string’ and ‘gravitational acceleration’. More specifically, the derivation of this relation seems to occur in a ‘natural way’ through the experimental activity whereas the students are directed to check the nature of the relationship since the relation has been announced by naming the factors on which the pendulum period depends. Another similarity is related to the use of simple matching pendula for the qualitative study of the energy and elastic wave transmission.

4. Conclusions – Proposals

Both in Greece and Cyprus, the study of the pendulum occurs within the traditional framework over the school curricula and science textbooks.

(a) Whether in the case of a simple reference, or the case of a detailed study of the pendulum, the options of the educational grade and its position in the school curriculum and the relevant school textbook, do not seem to be derived from a specific pedagogical plan but rather follow the material separation into traditional thematic units as this appears in classic introductory university textbooks. Similar to these textbooks, in the traditional school curriculum all subjects, and mainly the concepts, have the same pedagogical value considering there is not some kind of ‘external’ criterion to the scientific knowledge (e.g. the social importance of a theme) that will give greater or lesser importance to a conceptual framework. At the same time, the feature of setting different conceptual frameworks within which the pendulum study occurs is apparent, even though a favored field seems to be the one of Newtonian analysis, at least in the lyceum grades. Nevertheless, the effort to elevate the simple pendulum into favored field of studying the phenomenon of oscillation, in the gymnasium grades, shows that innovative framework for the school curricula and science textbooks has influenced the authors of the corresponding school textbook.

(b) In the gymnasium grades, the ‘pseudo – qualitative’, conceptual approach is followed, considering this approach does not differ in anything from the one used in the lyceum but the exchange of the mathematical language with everyday language. So, the phrase ‘in extreme positions of oscillation the sphere acquires its maximum gravitational dynamic energy as to the equilibrium position and zero kinetic energy' can not acquire any meaning if not included in the quantitative/mathematical approach of the conservation of the mechanical energy. On the other hand, in the lyceum, we find a totally mathematicized approach. A series of mathematical equations replace the problem that can give meaning to the concepts corresponding to mathematical symbols. Issues such as 'which problem leads to the Newtonian analysis of the simple pendulum motion' or 'how did the connection of the motion of the simple pendulum with the law of energy conservation occur' (Matthews 2000, ch.8) are not included in any of the three educational levels.

(c) In the gymnasium, one more clearly sees the empirical methodological approach to the simple pendulum. For example, the dependence of the period on the length of the string of the simple pendulum and the acceleration of gravity does not appear as a conceptual problem, but as a technical problem solved through a series of instructions for the 'successful' performance of an experimental activity. Thus, the impression is given that the relation of period with the other physical entities emerges from a simple (however, systematic) observation of the pendulum motion. This impression leads to the empirical logic that scientific knowledge, especially mathematical relations expressing it, is 'hidden' within the natural phenomenon. In the two grades of the lyceum we find, apart from the specific experimental activity, the derivation of the formula $T = 2\pi\sqrt{\frac{l}{g}}$ from a series of mathematical relations.

(d) Lastly, the absence of any cultural reference relating to the pendulum in the lyceum as well as the loose connection between the scientific knowledge of the pendulum and the everyday/technological or/and historical applications in the gymnasium, sets all three approaches within the traditional framework for school curriculum and science textbooks. Still, the reference to time measuring and the Foucault's pendulum supports the possibility of curriculum change so as to attain the features of an innovative framework. This change from the traditional to the innovative framework will be occupying us afterwards.

As already pointed out, the instrument of analysis and interpretation of school textbooks we presented has another value since it can be used as a learning tool for educators of science of different grades. In a previous study relevant to how educators can modify the traditional tradition of the school curriculum in science and move towards the innovative framework (Koliopoulos & Ravanis 1998), the following ways of intervention were identified:

(*A*) Part or whole modification of the curriculum which relates to the union of thematic units or the formation of a broad unit over a directed theme and

(*B*) Modification of one or some features of the traditional framework within a certain thematic unit like the modification of the methodological framework for the introduction of experimental activities.

In the following, we are about to present an example of transformation of the traditional framework to an innovative one for teaching about the pendulum. Firstly, this example cannot be generalized because it refers to the existing Greek curriculum. This approach can replace or complete, if there is enough time, the traditional approach of the 3rd grade of the gymnasium (case *A*). In Table IV is presented a sequence of didactical units, based on didactical activities – problems while for each unit we propose the basic conceptual, methodological and cultural elements of the concerned study. This approach differs from the traditional approach as to the following points:

(a) A broad unit is formed in which time measurement constitutes the main theme, that is the (cultural) framework within which the desired conceptual and methodological features of the pendulum study acquire meaning.

Table IV. A sequence of units relevant to the teaching of the simple pendulum, which is based on the 'innovative' framework for the school curriculum

Activity – Problem	Conceptual Frame	Methodological Frame	Cultural Frame
Why is it needed for time measuring to be accurate?	Periodicity	Measuring accuracy	Sundial, mechanisms in Ancient Greece and Cyprus, pendulum clock, modern clocks
How can we measure time in the pendulum clock?	Period/frequency	Measuring accuracy Measuring the pendulum period, measuring faults	
From a true pendulum clock to the simple mathematical pendulum	Period/frequency Period – length of string relation	Measuring accuracy Measuring the pendulum period, measuring faults Showing factors on which the pendulum period depends	
Once again the issue of accuracy in time measuring	Period/frequency Period – gravity relation	Showing factors on which the pendulum period depends	Examples derived from science history (The differences in time measurement)

(b) There is an in-depth analysis of a conceptual framework that, in the specific occasion, relates to showing a qualitative/semi-quantitative relation between the period of the simple pendulum, the string length of the pendulum and gravitational acceleration. The mathematical approach to this relation is not necessary in this grade. At the same time, the paragraphs relating to other conceptual frameworks, like the Newtonian analysis, energy analysis and measure of gravitational acceleration, are omitted.

(c) A hypothetico-deductive approach to the relation between the period and the string length of the pendulum and gravitational acceleration is attempted. Concerning the length of the string, a practical problem is raised concerning the explanation of how a clock 'ticks the seconds'. This problem, with the educator's guidance, can lead the students to plan on their own the same experimental activity imposed by the school textbooks in the traditional approach.[8] In gravitational acceleration, a problem is presented through science history (Matthews 2000, ch.6) that can lead students to realize primarily, the qualitative relation between the pendulum period and the force of gravity. If there is also an analogy between the gravitational field and the magnetic field or one can be established, teaching practice can include an experimental activity of measuring the period while placing a magnet under the pendulum.

(d) The cultural dimension is specified as an essential element of the educational procedure. The cultural dimension not only (a) acts as a means for approaching the everyday/technological reality and of getting familiarized with the scientific/technological tradition (e.g. in Greece and Cyprus, familiarization with the sundial and the mechanism of Antikythera) and (b) constitutes a guiding principle of the broad unit but also (c) acquires an organic relation with the conceptual and methodological dimension thus attributing meaning to the study of these two dimensions. Hence, the function of the clock is not viewed as a simple application of the pendulum study. Instead, the study of the technological and natural phenomenon of the clock's operation leads to the procedure of its conversion to a physical phenomenon (study of the modeled simple pendulum) (Baltas 1990).

As we have already noted the limits of the above approach relate to the school curriculum tradition (see Table III) and its pedagogical frameworks (e.g. teaching methods and means) within which it is called to function. Another factor influencing this approach can be the mental representations the students acquire for the concepts of time measurement, pendulum period or/and for representing patterns/models in selected data. If this factor is also considered, then it is possible to have radical changes in the sequence and context of the approach proposed.

Epilogue

We consider this article as a small contribution to the area of research in science education by showing how a conceptually, methodologically and, most importantly, culturally rich object of teaching, such as the pendulum, is limited through the traditional framework for science school curricula. At the same time, the same instruments of analysis used in this case study seem to be suitable for identifying specific ways of reorganizing this framework. The transformation of this new framework into educational practice by suitably prepared teachers is the next phase of our research.

Notes

[1] The generalized version of the model was presented in a series of lectures given to the students of the Department of Educational Sciences of the University of Cyprus during the academic year 1999–2000. A preliminary evaluation of the results of these courses indicates the potential of this model as an effective teacher preparation tool for primary education.
[2] The complete version of the proposed model includes a third framework, the ‘constructivist’ framework, which is conceptualized not so much as a teaching methodology but more as a framework that determines the sequencing of many innovative teaching programs (Tiberghien et al. 1995, Koliopoulos & Ravanis 2000). Such programs are mostly experimental in nature and have not in general been transformed to established school curricula. It is for this reason that we have chosen to omit this third perspective in the context of the present paper.
[3] We do not include here curricula that emphasize other attributes such as STS or those that emphasize methodological process skills
[4] In the ‘pseudo-qualitative’ approach the replacement of mathematical language by everyday language leads to the breakdown of any logical connection between the two concepts. This occurs because this connection results exclusively from a mathematical model. That is, someone who is trying to comprehend a text belonging to the ‘pseudo-qualitative’ approach should have priorily comprehended the corresponding mathematical model. In contrast, in a ‘qualitative’ approach the natural language or other symbolic representation is used in such a way as to favor the construction of different forms of physical causality (e.g., the establishment of series of intermediaries and series ordered in time – Antoine 1982)
[5] In both Greece and Cyprus teaching is entirely implemented through one officially approved textbook. The fact that the last two years there are two officially approved textbooks for the primary school doesn’t change anything in our research since only one of these textbooks refers to pendulum.
[6] This is a synthetic description of many conceptual frameworks, which include both quantitative and qualitative approaches. Usually only some of the attributes of each framework appear in the school textbooks under examination. For instance, the reference in frame (c) to the level of primary education concerns the implementation of an experimental activity where a simple pendulum is used to familiarize children with the transfer of the sound that is generated when a glass is hit with a spoon.
[7] In the French bibliography the system of spring-sphere with vertical motion is called the «elastic pendulum».
[8] In Greece, a corresponding activity has been implemented successfully in the case of Hooke’s law of elasticity. In this case, 8th grade students propose by themselves the well

known experimental procedure with a coiled spring which results in an analogical relation between weight and extension, in the context of project work in response to the problem «How can we make a force meter?»

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The Public Understanding of Pendulum Motion: From 5 to 88 Years Old

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Abstract. This paper describes life-span development of understanding about pendulum motion and effects of school science. The subjects were 2,766 people ranging from kindergartners up to 88 years senior citizens. The conflict and consensus between children and their parent's understanding of pendulum motion were also analyzed. The kindergartner's understanding, mostly non-scientific, made a marked developmental change to another type of non-scientific understanding by the time they reach G 4. Parents with scientific understanding do not presumably nurture scientifically minded children, even though about half of them can apply scientific conceptions that shorter pendulums swing faster, and the amplitude and speed of pendulum motion do not depend on its weight. There seems to be another type of developmental change from scientific understanding to non-scientific understanding around their fifties. It is suggested that the scientific understanding in the public about pendulum motion become predominant due to the educational intervention through school science.

1. Introduction

Sometimes we find an expert in one area of specialization almost ignorant in another area. This characteristic of understanding can be explained by the cognitive framework which constrains cognition of an event in the external world; in other words the view of domain-specific understanding. Since the second half of the 1970s, there has emerged in science education a cognitive paradigm known as alternative conceptions movement (ACM) (Gilbert & Swift 1985), by which, based on a domain-specific view of understanding, children's conceptions on specific topics in school science curriculum are recorded and analyzed. Contributing to this approach was the emergence of post-behaviorism psychology referred to as the cognitive revolution (Gardner, 1985), and the influence on science education of relativist viewpoints from philosophy of science.

However, studies on knowledge acquisition and conceptual change in science based on a domain-specific view of understanding have the problem that few studies have been made on the understanding in science covering learners from a wide spectrum of ages. Cognitive research used in science education centers on how to enable children at specified school grades to acquire scientific knowledge set forth in the curriculum, and mainly concerns evaluation of the achievement of learning.

M.R. Matthews et al. (eds.), The Pendulum, 465–484.

A typical task of research is based on a rule common among, and understandable by nobody else, than learners of dynamics, such as the question asking to express the direction of force by an arrow, used by Watts and Zylbersztajn (1981) and Clement (1982). According to a survey of undergraduate students by Viennot (1979), scientific conceptions of dynamics presented in school science are forgotten over time, but it is yet to be studied and found out over how long a period and at what rate the frequency of occurrence of a scientific response decreases and continues to decrease. The relativist view of science after Kuhn (1970) is characterized by its focus on the social context in scientific discoveries and revolutions, and the cognitive theory of science learning, by its focus on cognition in an informal context. Nevertheless, few studies have been made on the scientific context of society surrounding children. Even in research on parents and their substitutes (guardians) who are likely to share much of their time with children outside school (Wang & Wildman 1995), no work has been done studying their understanding of science topics.

Furthermore, many of the cognitive studies made on specific science topics tend to assume the understanding by a learner at a given point of time as being something unitary and fixed. Against this tendency, Hewson (1985) proposed a dimension of commitment, and attempted to express the learning process by using a theory of conceptual ecology. Brown (1987) and Clement et al. (1989) used the level of commitment as a means to find an anchor conception out of the various conceptions a learner may have. While this task of commitment level is characterized by a more dynamic grasp of the process of conceptual change by checking the level of confidence in the subject's reply (Morifuji 1994), few cross-age studies have ever been made to find out how this level of confidence may vary with the difference in school grade or in the experience in science learning.

The "domain" of this study was the topic of dynamics. Out of this domain, pendulum motion, which Matthews (1994, 2000) claims to be by far the suitable topic for discussing science studies, cognitive science and science education was selected. The subjects would be selected from a wide spectrum of age brackets ranging kindergarteners to senior citizens, and surveyed in a cross-age manner using the same question to find what differences in the learner's understanding of pendulum motion are made by the age and the experience or inexperience in learning. Then, the understanding of pendulum motion by guardians of children of all school grades, from the kindergarten to the third grade of junior high school, would be examined. The purpose of study is to consider whether or not there is any difference in the trend of response between the children whose guardians choose scientific answers and the children whose guardians do not and assess the role of science learning at school in children's understanding in science from the life-span perspective.

2. Methodology

This study is intended to consider in a life-span perspective the understanding by a learner of the motion of an object and the effect of science learning at school by developing original tasks which concern pendulum motion and can be commonly used for subjects of all age brackets from kindergarteners to senior citizens and surveying and analyzing their responses in a cross-age manner. This study covered, as shown in Table 1, a total of 2,766 subjects living in Naruto City, Tokushima Prefecture, ranging from five-year-old kindergarteners to 88 year old senior citizens. The guardians selected had children ranging from the kindergarten age to the third grade of junior high school. Previous studies have pointed out that a variety of diseases and an extreme decline in physical strength may affect the intelligence of a senior citizen (Nakazato 1990). For this study, elderly subjects were selected from healthy senior citizens visiting a service facility for the aged only to receive day care, as they were considered suitable for the purpose of the study.

The subjects of this survey, those from the kindergarten age to the fifth grade of elementary school have not yet studied pendulum motion at school. On the other hand, those from the sixth grade of elementary school to the highest age have presumably learnt about pendulum motion in one way or another. Out of the guardians, two age groups, one of 40 to 47 and the other of 30 to 37, are extracted and classified as same curriculum experienced groups with the period of transition of curriculum being taken into account.

The tasks to be studied are basically two. One is the “length task” to ask about the speed of pendulum motion, the “length” of the pendulum being an independent variable (Figure 1a). The other is the “weight task” to ask about the amplitude and speed of the pendulum motion, the “weight” of the pendulum being an independent variable (Figure 1b, c).

For the “length task”, two similar bobs (wooden balls each weighing about 10 g) were hung by strings of different lengths and swung in equal amplitudes. The subjects were asked in a multiple choice question which of the two bobs would hit the wall first.

The “weight task” contains two further tasks: the “amplitude task" to examine the subject’s cognition of the relationship between the weight of the bob and the amplitude of the pendulum, and the “speed task” to check his or her cognition of the relationship between the weight of the bob and the swinging speed. For the “amplitude task”, two bobs differing only in weight (a wooden ball weighing about 10 g and an iron ball weighing about 200 g) were let swing freely from the left side at the same height. The subjects were asked in a multiple choice question how far the two bobs would reach in their first swing (going route). For the “speed task”, two bobs differing only in weight (a wooden ball weighing about 10 g and an iron ball weighing about 200 g) were let swing freely from two sides, right and left, at the same height. The subjects were asked in a multiple choice question where the

Table I. Subjects

Grade	K	Elementary school						Junior high school			High school					Guardians	Old	Total
Sex		G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	G11NS	G11S	G12NS	G12S			
Male	49	57	46	60	61	59	65	69	66	71	72	27	54	26	46	355	13	1196
Female	40	58	51	48	55	73	70	77	72	72	79	54	17	64	9	644	38	1521
Total	89	115	97	108	116	132	135	146	138	143	151	81	71	90	55	999(49)	51	2717(49)

S: Science major course students; NS: Non-science major course students.

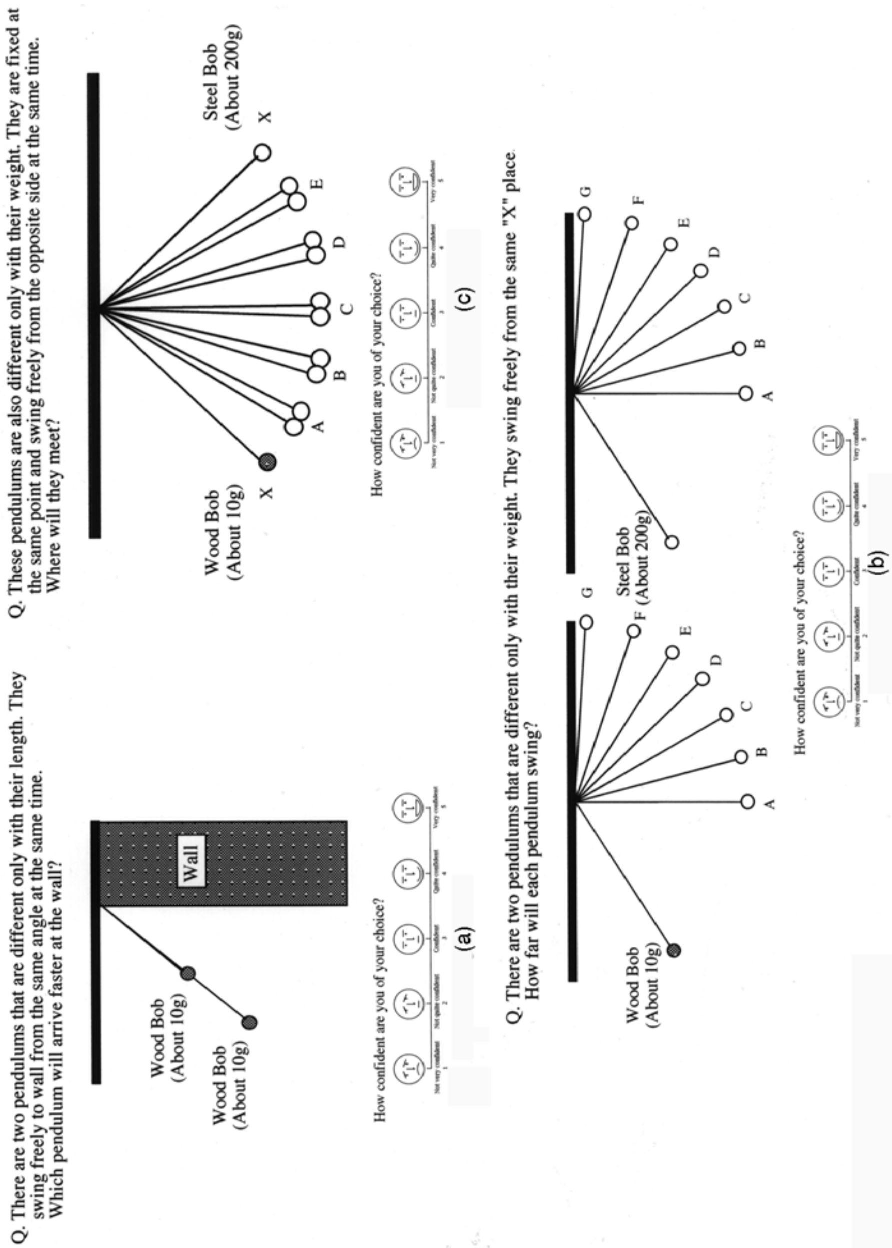

Figure 1. Tasks in this study.

two bobs would hit each other. Each of them was further asked to grade his or her confidence to the answer he or she gave on each task (Figure 1).

The surveying method for kindergarteners was interviewing, adapted from the questionnaire and having the same contents, asked by the same person for every child. That for elementary schoolchildren and junior and senior high school students used a printed questionnaire, and each subject was supposed to answer multiple choice questions in the sequence of question numbers. For guardians, the same questionnaire as that for junior high students was used. For each of these adults, an answering manual and a return envelope were enclosed with the questionnaire, and the sealed envelope containing them was entrusted to the child for delivery to his or her guardian. The reply was put into the return envelope, which was sealed and collected by the school through the child. For every school grade, guardians were surveyed after the children. The collection rate of requested data from the guardians was about 86%. For senior citizens, the same questions and method as for kindergarteners were used through one-to-one interviews by the same person.

3. Life-span Development of Understanding about Pendulum Motion and Effects of Science Learning at School

The responses of the subject to the length task can be classified into the following three types by the answer on the expected position of collision between two pendulums differing only in the length of string. The rates of occurrence of and the average levels of confidence to different types of response to the length task are classified by school grade, and shown in Figure 2 and Figure 3, respectively.

[Long < Short] type: The shorter pendulum swings faster.

[Long = Short] type: The swing is as fast irrespective of the pendulum length.

[Long > Short] type: The longer pendulum swings faster.

The results regarding the length task diagrammed in Figure 2 and Figure 3 can be described as follows. No significant difference in response type among children from the kindergarten level to the fifth grade of elementary school, who have been subject to no educational intervention in this regard, and many of the subjects in this age range chose the [Long < Short] type, which is scientifically correct. Science lessons at school on pendulum motion, which are given in the fifth grade of elementary school, seemingly contribute to an increase in the frequency of [Long < Short] type responses, but science lessons at the junior high and higher levels rather appear to be contributing to an increase in the frequency of [Long = Short] type responses, which is unscientific. Among the two groups of guardians, the frequency of [Long = Short] type responses is about the same as among senior high students, and if this means that science lessons at the junior high and higher levels contribute

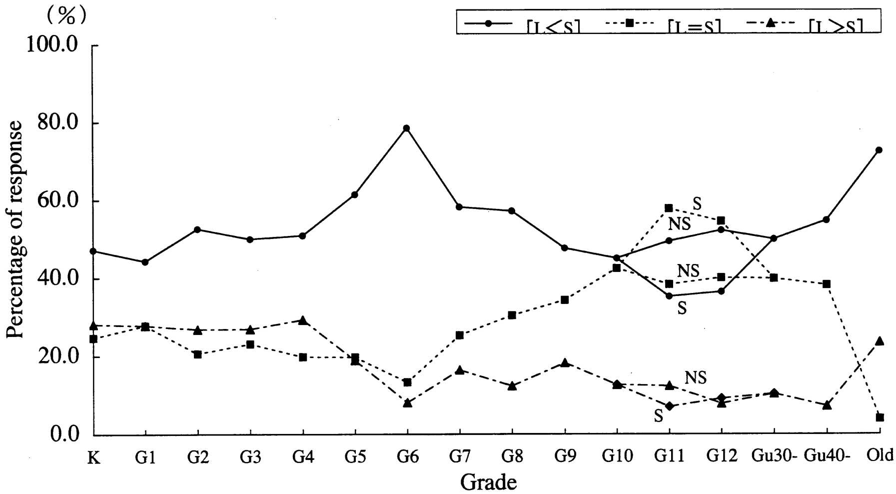

Figure 2. Prevalence of subjects' response to "Length Task".

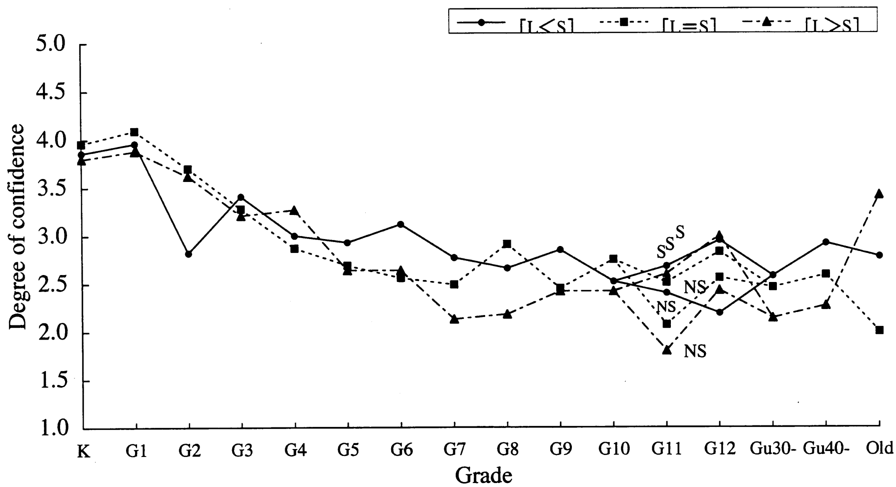

Figure 3. Prevalence of subjects' confidence to "Length Task".

to an increase in the frequency of [Long = Short] type responses, it may well be suspected that the effect of science learning at school is sustained on the subjects in their forties. Furthermore, since the response pattern of senior citizens significantly differs from those of subjects of all other age brackets including the guardian group whose average age is the closest to theirs, there is a possibility that in some age range after 48 years many learners substantially reconstruct what they learned at school.

Regarding the level of confidence, no significant difference in this respect was witnessed among the different response types in all the groups from kindergarteners to the fifth grade of elementary school but the elementary second grade. By

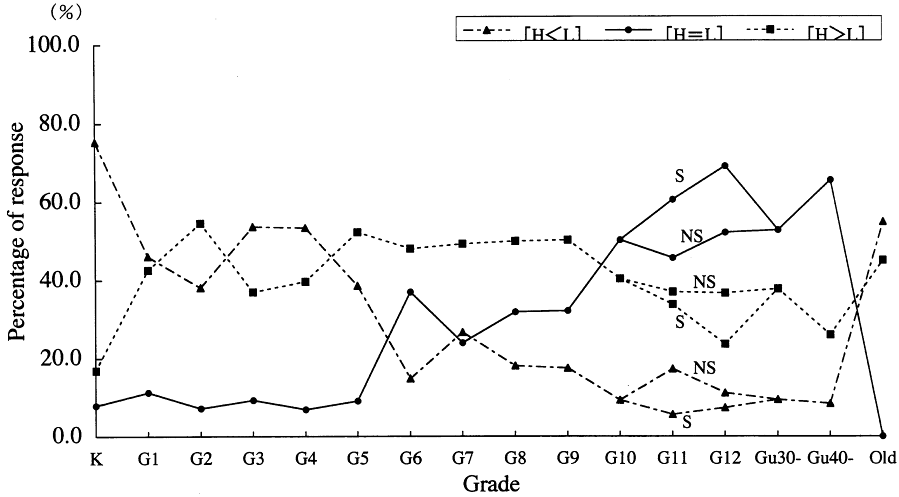

Figure 4. Prevalence of subjects' response to "Amplitude Task".

contrast, in almost all the age brackets of and above the sixth grade of elementary school, the level of confidence to the scientifically correct [Long < Short] type was higher than that to any other response type. To compare science course students and non-science course students at high school, the former were found more firmly committed irrespective of the response type.

Then, as regards the amplitude task, the responses of subjects can be classified into the following three types by comparing the replies of the same subject concerning the reach of the wooden ball of about 10 g referred to in the first question and concerning that of the iron ball weighing about 200 g in the second question. The rates of occurrence of and the average levels of confidence to different types are classified by school grade and response type, and shown in Figure 4 and Figure 5, respectively.

[Heavy > Light] type: The heavier pendulum swings more widely.

[Heavy = Light] type: The swing is as wide irrespective of the pendulum weight.

[Heavy < Light] type: The lighter pendulum swings more widely.

Figure 4 and Figure 5 reveal the following results regarding the amplitude task. While the [Heavy < Light] type is dominant among kindergarteners, the [Heavy < Light] type and the [Heavy > Light] type become about equal at and above the elementary first grade, but almost none of the kindergarten through the elementary fifth grade ages chooses the scientifically correct [Heavy = Light] type. Learning about dynamics including pendulum motion at the elementary fifth and junior high third grades and in later science classes seems to be contributing to the rise in the frequency of [Heavy = Light] type responses. Since the frequency of [Heavy =

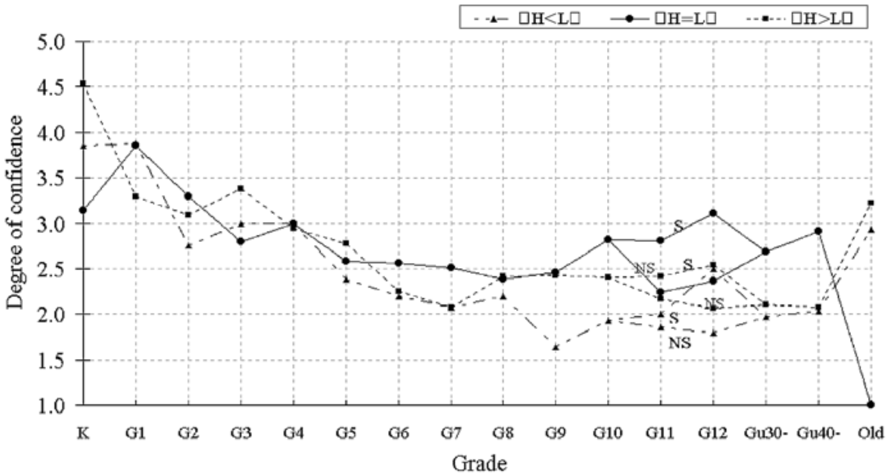

Figure 5. Prevalence of subjects' confidence to "Amplitude Task".

Light] type responses is high among both groups of guardians, who presumably are distant from the educational intervention by science lessons at school, and this seems to indicate that the earlier supposed effect of science learning at school as the understanding of the [Heavy = Light] type still sustains its functioning even on subjects in their forties. Since the response pattern of senior citizens significantly differs from subjects of all other age brackets (including the guardian group whose average age is the closest to theirs) with the exception of the elementary third grade, there is a possibility that in some age range after 48 years many learners substantially reconstruct what they learned at school about the amplitude task, too.

Regarding the level of confidence, as Figure 5 shows, no significant difference in this respect was witnessed among the different response types in all the groups from kindergarteners to the fifth grade of elementary school but the kindergarten age, and younger children were found more firmly committed. In the age brackets of and above the sixth grade of elementary school, the level of confidence to the scientifically correct [Heavy = Light] type was higher than that to the [Heavy < Light] type and to the [Heavy > Light] type. To compare science course students and non-science course students at high school, the former were found more firmly committed irrespective of the response type.

As regards the speed task, the responses of subjects can be classified into the following three types by comparing the replies of the same subject concerning the colliding position of bobs differing in weight. The rates of occurrence of and the average levels of confidence to different types are classified by school grade and response type, and shown in Figure 6 and Figure 7, respectively.

[Heavy > Light] type: The heavier pendulum swings faster.

[Heavy = Light] type: The swing is as fast irrespective of the pendulum weight.

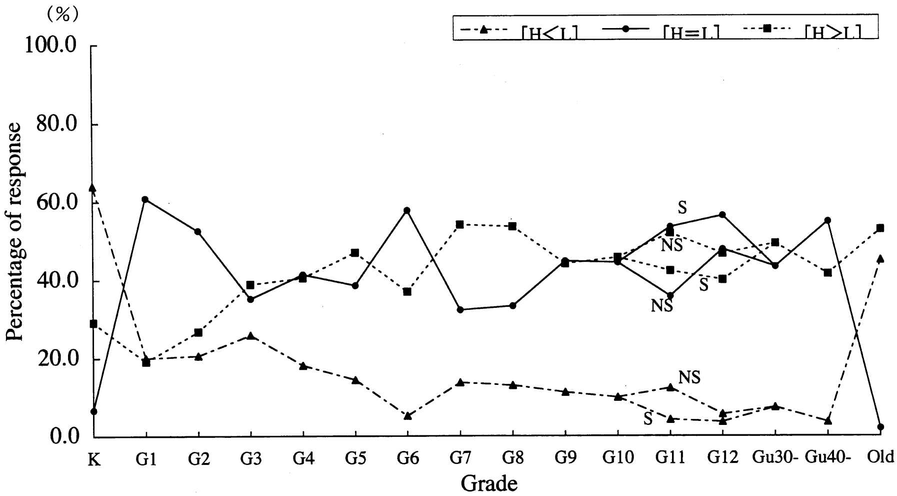

Figure 6. Prevalence of subjects' response about "Speed Task".

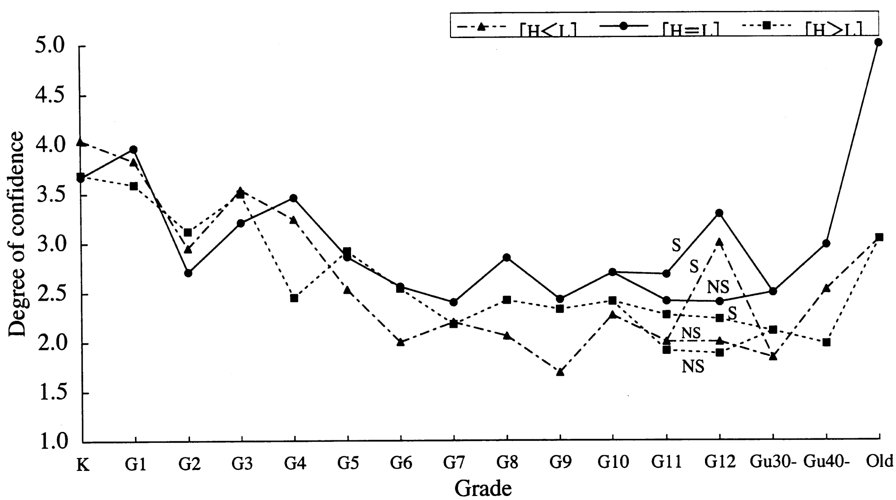

Figure 7. Prevalence of subjects' confidence to "Speed Task".

[Heavy < Light] type: The lighter pendulum swings faster.

As regards the rates of occurrence of different response types concerning the speed task, learners' cognition varies from the kindergarten age until the elementary fifth grade, the age range in which children have as yet experienced no formal learning on pendulum motion and the [Heavy = Light] type, which is scientifically correct, and the [Heavy > Light] type are more frequently found. Although learning about pendulum motion in school science lessons at the fifth grade of elementary school seems to be contributing to the increase in the frequency of choice of the [Heavy = Light] type, this learning effect is short-lived. Nor does science learning at and above the junior high school level contribute much to an increase in the frequency of choice of the [Heavy = Light] type. However, since about half of guardians in both age groups choose the scientifically correct [Heavy = Light] type, being away from school science cannot be regarded as a negative factor to the occurrence of the scientifically correct response type. However, the response pattern of senior citizens significantly differs from subjects of all other age brackets. This seems to suggest the possibility that in some age range after 48 years many learners substantially reconstruct what they learned at school about the speed task as they do about the length task and the amplitude task.

Regarding the level of confidence, as with the length task and the amplitude task, the difference in confidence level with the difference in response type is absent in most of the groups from kindergarteners to the fifth grade of elementary school. In the age brackets of and above the sixth grade of elementary school, the level of confidence to the scientifically correct [Heavy = Light] type was higher than that to the [Heavy < Light] type and to the [Heavy > Light] type. Science course students were found more firmly committed than non-science course irrespective of the response type.

4. Interrelatedness between Guardians and Children in Understanding about Pendulum Motion

In the foregoing section, we classified guardians into age groups by reason of curricular experience, and analyzed their responses. The groups of guardians taken up in this survey have children ranging from the kindergarten age to the junior high third grade. The findings described in the foregoing section suggest that the understanding of pendulum motion by children of this age range may vary from one grade to another. If children's understanding varies and guardians are a major factor in children's intellectual environment (Costa 1995; Phelan et al. 1991), the guardians' understanding may differ with the school grade of children. In view of this possibility, guardians' understanding is analyzed in this section as classified by the school grade of children. Further, any direct correspondence is identified between children and their guardians, children are classified by the response type of guardian, and any interrelatedness between guardians and children would be discussed.

Guardians' responses to the length task, amplitude task and speed task are sequenced according to the above-stated criteria of classification, and further classified by the rates of occurrence of and the average levels of confidence to different types according to the child's school grade. The results are shown in Figure 8 through Figure 13, respectively.

As is evident from Figure 8 through Figure 13, guardians' response patterns and levels of confidence do not significantly differ with the difference in the child's school grade. It is seen that, the age difference between a kindergartener's guardian (averaging 34.7 years in age) and a junior high third grade student's guardian

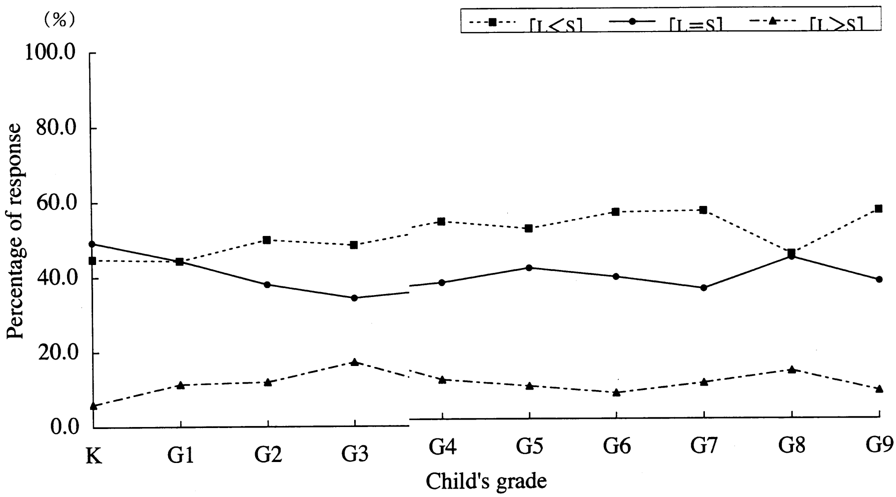

Figure 8. Prevalence of guardian's response to "Length Task".

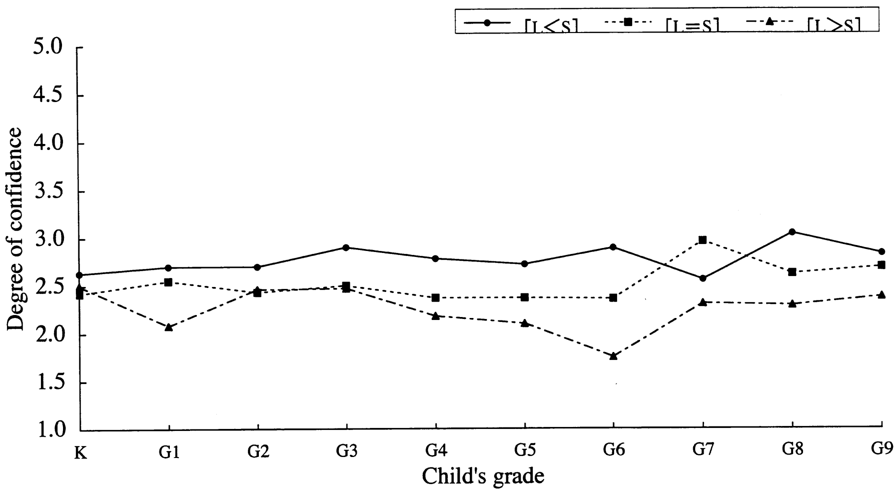

Figure 9. Prevalence of guardian's confidence to "Length Task".

(averaging 42.1) being assumed to be about the same as the children's difference in school grade, scientific conceptions are held with considerable stability by guardians farther away from formal science learning at school than children. Regarding the level of confidence, too, only the main effect of the response type was witnessed with respect to every task, and the confident to scientifically correct response types was found higher than that to other response types.

Thus, the children from the kindergarten age to the junior high third grade who were subjects of this survey are, with respect to pendulum motion, presumably in a relatively constant intellectual environment out of school. Broadly classified, this is true, with respect to the length task, of the guardian group of the scientifically

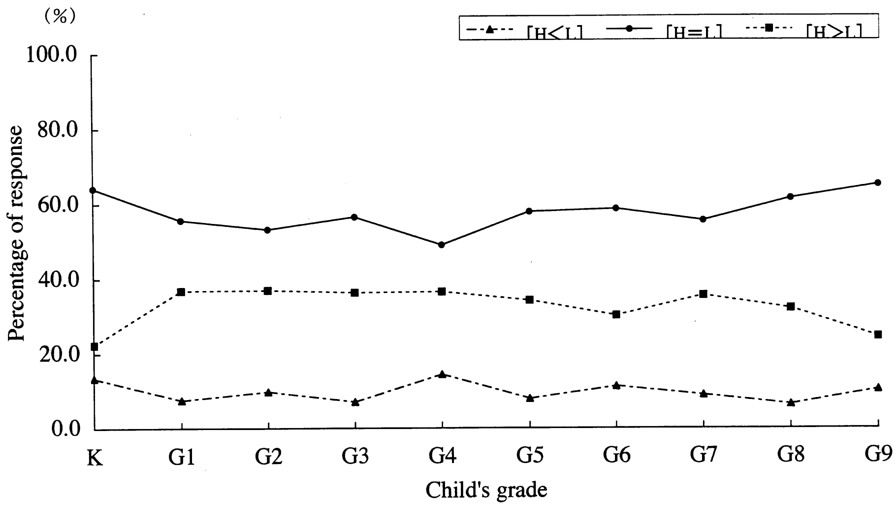

Figure 10. Prevalence of guardian's response to "Amplitude Task".

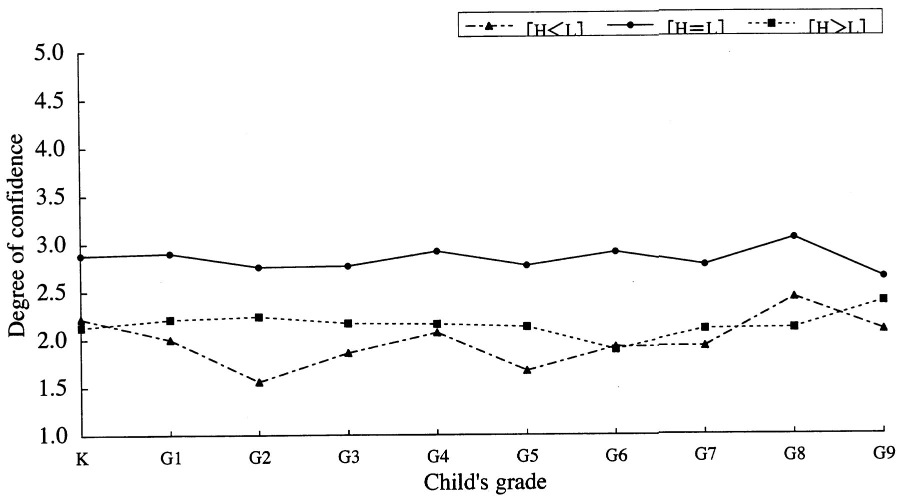

Figure 11. Prevalence of guardian's confidence to "Amplitude Task".

correct [Long < Short] type with a high level of confidence and that of the [Long = Short] type with a low level of confidence and, with respect to the amplitude and speed tasks, the scientifically correct [Heavy = Light] type with a high level of confidence and that of the [Heavy > Light] type with a low level of confidence.

Then, for each task, the difference in understanding between the children of the two main guardian groups was examined. With respect to neither the length task nor the speed task, there was any finding to demonstrate that the child's response type differed with a difference in the guardian's response type. Only with respect to the amplitude task, the interaction between the guardian's response type and the

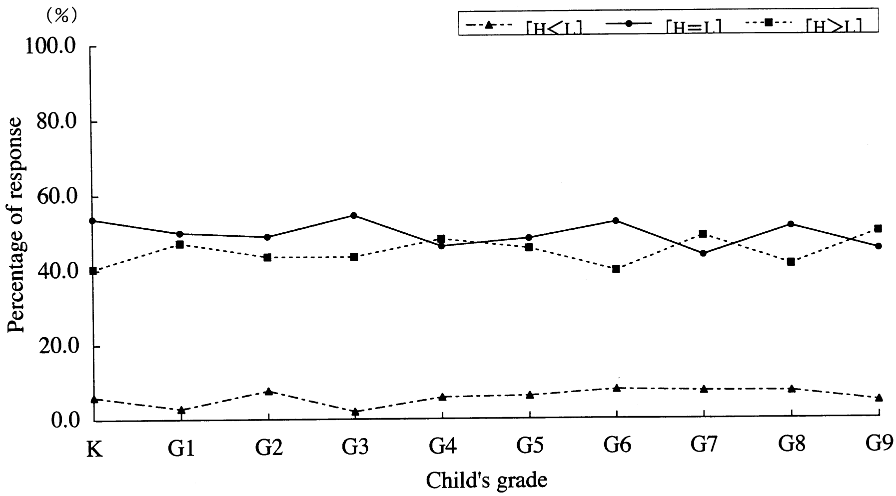

Figure 12. Prevalence of guardian's response to "Speed Task".

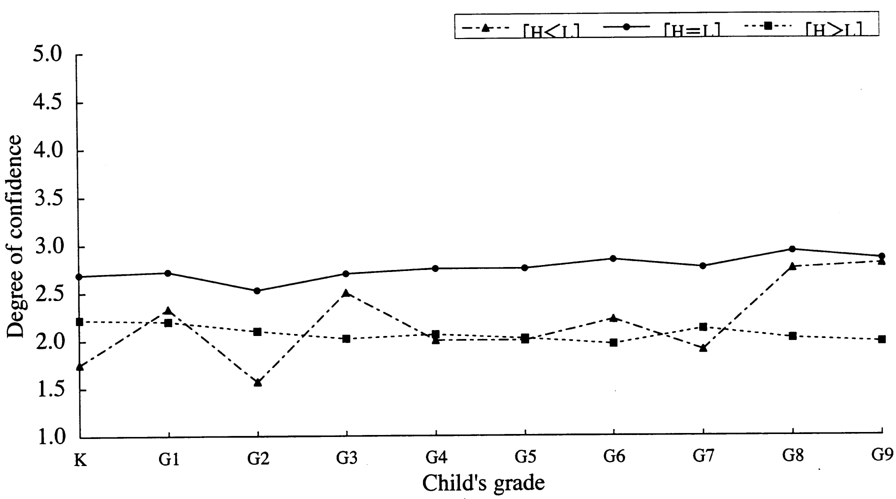

Figure 13. Prevalence of guardian's confidence to "Speed Task".

child's was found significant. In this aspect, when the guardian's response was of the [Heavy = Light] type, the child's was more likely to be of the [Heavy = Light] type, too, and when the guardian's was of the [Heavy > Light] type, the child's also was more likely to be of the [Heavy > Light] type. Figure 14 and Figure 15 suggest that this tendency is more conspicuous at and above the elementary sixth grade having undergone educational intervention. Then, the children were divided into two groups, one of the kindergarten age to the elementary fifth grade and the other of the elementary sixth to the junior high third grade. The result indicated significant interactions among the guardian's response type, the child's response type

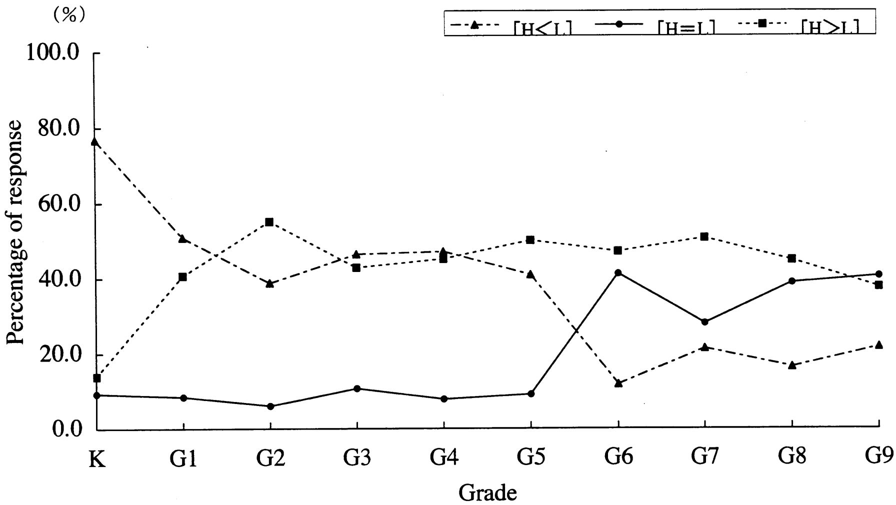

Figure 14. Children's responses to "Amplitude Task" (whose guardian responds scientific type).

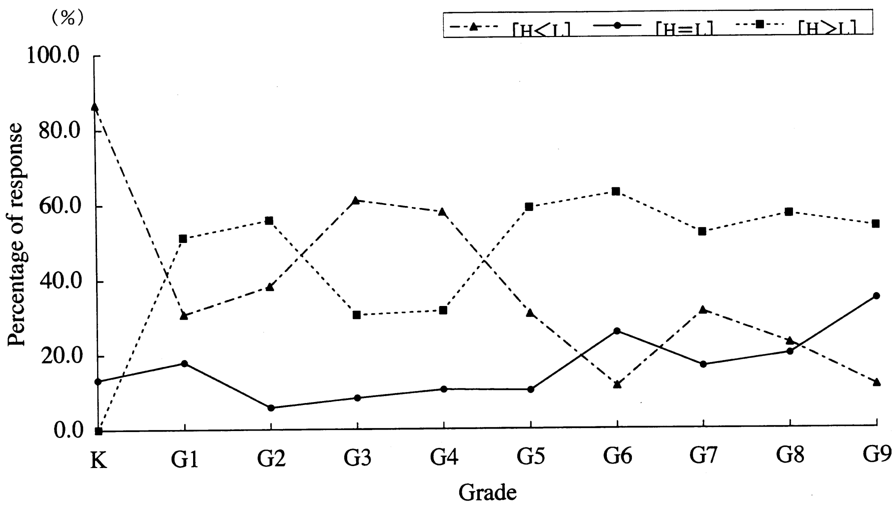

Figure 15. Children's responses to "Amplitude Task" (whose guardian responds [H > L] type).

and the child's experience of science learning. Table 2 shows children's responses classified by the guardian's response type for each of the two groups.

Thus, when viewed against the criterion of whether or not educational intervention has been experienced, in the comparison of children from the kindergarten age until the elementary fifth grade having experienced no formal science learning and children from the elementary sixth to the junior high third grade having experienced learning about pendulum motion at school, the frequency of the scientifically correct [Heavy = Light] type was higher, and that of the [Heavy < Light] type was

Table II. The cross-effect of guardian's understanding and educational intervention in school science on children's understanding

Children's group	Guardian's understanding	Frequency of response [H < L] type	[H = L] type	[H > L] type
Before educational intervention (K-G5)	[H = L] type	159	28	137
($N = 525$)	($N = 324$)	(49.1)	(8.9)	(42.3)
	[H > L] type	94	22	85
	($N = 201$)	(46.8)	(10.9)	(42.3)
After educational intervention (G6–G9)	[H = L] type	50	103	126
($N = 423$)	($N = 279$)	(17.9)	(36.9)	(45.2)
	[H > L] type	30	33	81
	($N = 144$)	(20.8)	(22.9)	(56.3)

lower, in the latter group than in the former. Further, while no difference in the child's response was observed with the guardian's response type in the subject group from the kindergarten age until the elementary fifth grade, the child whose guardian replied in the [Heavy = Light] type was found more likely to give the [Heavy = Light] type response, and the child whose guardian gave a [Heavy > Light] type response was more likely to reply in the [Heavy > Light] type in the subject group from the elementary six to the junior high third grade.

5. Discussion

The first thing to consider, on the basis of the findings of this study, is the realities of children's understanding about "pendulum motion", which is first taught in the elementary fifth grade in the formal curriculum of school science, and the character of their understanding. The survey results suggest that the kindergarteners did not respond at random to the tasks of the survey, but they had already formed understanding about pendulum motion. Since their understanding differs from guardians' in most cases, the origin of children's understanding about pendulum motion could perhaps be traced back to something innate. As a matter of fact, it is generally agreed among cognitive psychologists that naïve conceptions in physics are acquired considerably early, already in infancy (Elman et al. 1996; Karmiloff-Smith 1992; Spelke 1988, 1990). Since understanding varies, as the results of this study indicate, at a low age where children are not so familiar with the "pendulum" and have experienced no overt learning about it, it is well conceivable that not only infants but also many of the children learning science at elementary school already have a repertoire of many conceptions regarding the motions of objects. The repertoire of conceptions further expands in learners having undergone educational intervention by school science. This observation seems to well fit the argument of Yates et al. (1988) who labels the cognition of motion by learners as prototypes, rather than a theory, and that there are many prototypes which are like domain-specific images, and the context-dependent research findings about cognition reported by Clough and Driver (1986) and Wakimoto (1992).

Then, what seriously matters in assessing the "scientific correctness" of children's answer, children who learn about pendulum motion for the first time in a higher elementary grade, for instance learners who have arrived at the [Heavy > Light] type or the [Heavy = Light] type via the [Heavy < Light] type, may well have cognitions of various types already, including the scientifically correct cognition. Thus, a learner who has seemingly given a wrong answer may perhaps have simply chosen a wrong context of application. Conversely, it is well conceivable that senior high students and guardians who have experienced educational intervention and show a high proportion of scientifically correct responses give their scientifically correct answers in accordance with an unscientific understanding. This is particularly true of those subjects who, for instance, gave the [Heavy > Light] type response to the speed task in spite of their scientifically correct [Heavy = Light] response to the amplitude task. Learners having experienced no educational intervention, including infants, showed no significant response type-dependent difference in the level of confidence to their respective response types though the frequency of occurrence tended to concentrate on one specific response type or another. This finding suggests that, as one of conceivable possibilities, children's informal learning takes place from a very different ontological point of view from the criteria of classification we suppose, and their responses regarding pendulum motion vary as an effect or a byproduct of that learning (Chi et al. 1994; Bliss & Ogborn 1994).

In this study, subjects who consistently gave scientifically correct responses were scarcely found among those from the kindergarten age to the elementary fifth grade having experienced no formal educational intervention. Thus it is quite unlikely for children to acquire scientific knowledge spontaneously from their own experience in life. Since subjects of and above the elementary sixth grade, including guardians having finished learning school science decades ago, give scientifically correct type responses at a higher frequency than younger subjects, and it is not very likely for guardians, even if they have held scientific conceptions, to have communicated those conceptions to the children, science learning at school concerned with such contents necessarily play an important role in the communication and reproduction of scientific understanding in the public. We also identified another type of developmental change from scientific understanding to non-scientific understanding around their fifties. Baltes (1987) attempted to theorize two kinds of intelligence in life-span cognitive development. One is the crystallized intelligence that maintains even in senior citizen and another is the fluid intelligence that declines with aging. There may be the second/third phase of developmental change of understanding in science in the public.

No parallel is found with the level of confidence. Science learning at school does not contribute to any rise in the level of confidence to one's response, even if it is a scientifically correct type. Comparison of the level of confidence between children from the kindergarten age to the elementary fifth grade who have experienced no formal educational intervention and subjects of higher school grades reveals an evidently higher level of the former. However, the group of subjects having experienced school science and giving scientifically correct response is characterized by a higher level of confidence than others. The comparison of science course students and non-science course students at high school reveals a higher level of confidence of the former irrespective of the response type.

These findings can be interpreted in the following way. As a positive effect of science learning at school, not only learners who can give scientifically correct responses but also ones who do not, are enabled to compare their own responses and the scientifically correct ones. Thus the learners who give scientifically correct responses become even more scrupulous and therefore more likely to give scientifically correct responses, and even those who do not give scientifically correct responses gain learning experience that enables them to become more scrupulous and expect to some extent that theirs are not scientifically correct responses. An implication of the findings of this study is that science learning at school not only functions to reproduce scientific conceptions but also effectively and consistently performs an important role in the scientific understandings at the meta level.

Furthermore, the achievement of this study which, as revealed with respect to the amplitude task, suggests the effect of group affiliation in the scientific understanding seems to imply the need for research in a broader context and in a longer term perspective by expanding the choice of subjects to include not just children learning the given topic but also adults if the *raison d'etre* of science education is to be more clearly defined. From now on, it will become a basic issue in science education how coordination can be achieved between such scientific understandings and understandings acquired outside school while taking note of the contradictions and conflicts between the scientific cultures and the cultures outside school in the public.

Acknowledgments

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Using Excel to Simulate Pendulum Motion and Maybe Understand Calculus a Little Better

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Abstract. As part of a first-year college Introductory Physics course, I have students construct an Excel® spreadsheet based on the differential equation for pendulum motion (we take a pendulum having a light bar rather than a string, so it can go 'over the top'). In extensive discussions with the students, I find that forcing them to make the spreadsheet *themselves*, entering velocities as position differences divided by time, etc., leads to a firmer grasp of basic calculus concepts. And, the instant graphical response of the finished product gives a sense of accomplishment as well as a lot of fun while building intuition about pendulum motion.

Introduction

Anyone who has taught calculus-based introductory physics knows that many students have a hard time understanding acceleration. They soon learn to compute how high a ball thrown vertically upwards will go, but just ask them if the ball is accelerating at the topmost point, and many will say no. An analogous exercise with the simple pendulum is to have students draw diagrams showing the acceleration vector for the pendulum bob at various stages in the swing. The results can be disappointing if this assignment is given without a few hints!

Of course, acceleration is a difficult concept – the Greeks, for all their geometric intuition and clarity of thought, never analyzed falling motion carefully enough to come to grips with it. Galileo was the first to understand one-dimensional constant acceleration, and for half his life he understood acceleration as the rate of change of speed with respect to *distance* traversed, not *time* elapsed. It was a little over half a century later that Newton and others developed the full basis of classical dynamics, encoded in Newton's Laws. An essential part of this development was the invention of calculus.

It is no surprise, then, that student difficulties with the concept of acceleration are closely related to their difficulties with the basic concepts of calculus: the definition of velocity as a limit of distance moved divided by time taken for very short times, and the parallel expression for acceleration. (An added problem is that once these ideas are fully digested, they seem very straightforward. Consequently, some teaching assistants in college, and even beginning teachers, forget how much

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time they personally spent mastering the material, and become frustrated with the students' apparent inability to grasp these 'simple' concepts readily.)

I have found that an effective way of getting the students to seriously think about velocity, acceleration and differentiation is to have them construct Excel spreadsheets. I emphatically do *not* mean having them use an already constructed spreadsheet to find how, say, the path of a projectile depends on various parameters – that can be useful later, but will not help them with the fundamentals. One big reason the spreadsheet approach is so effective is that almost all students now come with Excel already on their laptops, and most of them are eager to develop spreadsheet skills.

Constructing a Very Simple Spreadsheet

The first exercise I give is something really simple: the most naïve possible numerical integration of the vertical motion of a falling ball without air resistance. I have the students construct a spreadsheet with four columns (perhaps 100 rows, from, say, the tenth spreadsheet row down) labeled: time, position, velocity and acceleration. The 100 rows will give the values of position, etc., at 100 successive times, as calculated by the spreadsheet. The rows left blank at the top of the spreadsheet are for name, date, a brief description of the exercise, and a list of the basic parameters: g, initial velocity and the time interval delta_t between successive rows.

The instructions for creating the spreadsheet are as follows: first, the basic parameters must be entered. This is straightforward: write in cell A4: g =. Then select cell B4, and from the toolbar at the top click Insert, Name, Define. Excel will suggest the name g for B4, click OK. Then enter a suitable value, say –10, in B4. (Units can be exhibited by writing m/sec∧2 in C4.) In exactly similar fashion, put the variables initial velocity v_init and time interval (between successive calculated rows) delta_t in cells B5 and B6, with initial numerical values 0, and 0.1 respectively.

Turning now to the four columns where the computation will take place, they can be labeled by writing 'time', 'position', 'velocity', 'accel' in A9, D9, C9, D9 respectively. In A10 enter the initial time, take it 0. Immediately underneath, in A11, enter =A10 + delta_t. A11 should now read 0.1. Now select A11 and drag down 100 rows using the small black square at the bottom right of the selecting frame around the cell. This should fill in all the times. In the acceleration column, enter =g in the first cell, D10, then drag that down. (This may seem a bit pointless, but sets up a structure in anticipation of variable acceleration problems, such as the pendulum.) In the velocity column, enter =v_init in C10, then in C11 enter = C10 + D10*delta_t. In the position column, enter 0 in B10, then in B11 enter = B10 + C10*delta_t.

These entries in B11, C11 – in effect; the first steps in numerically integrating the equation of motion – are the most important step in constructing the spreadsheet. The students should figure out what to enter in B11, C11 by themselves, or with the minimum guidance necessary. The instructor can later review how this connects with basic concepts of calculus, with simple diagrams of the discretized function and using the diagrams to make the point – which will be important in the next example – that greater accuracy would be gained by using the velocity in the middle of an interval rather than that at the beginning to find the change in position over the interval. But first, it is easy to complete the spreadsheet by drag-copying B11 and C11 down 100 rows, to find out if it's going to work.

Now comes the fun part. Many of the students will already be familiar with Excel's ChartWizard (the little colored bar graph up on the toolbar). ChartWizard can be used to graph the position of the falling object as a function of time as follows: select (highlight) all the numerical entries in the first two columns (time and position), click on ChartWizard, then XY Scatter, then Finish (refinements such as labeling, titles, and decorative touches can be added in a moment). The familiar half-parabola appears. Different values for v_init can be entered, and, if necessary, delta_t adjusted. An enlightening new graph can be constructed by selecting *all four* columns and clicking ChartWizard, XY Scatter, etc. Now three curves for position, velocity and acceleration appear. Putting in v_init = 40, the chart makes very clear how the acceleration remains constant at the topmost point! It is instructive for the students to dwell on the relationships between position, velocity and acceleration as exhibited by this graph.

The Simple Harmonic Oscillator: Wild Oscillations and the Leapfrog Solution

This same spreadsheet can be readily adapted to a simple harmonic oscillator: delete (or ignore) g, introduce a new named cell k, put k = 10, this is our spring constant. Replace =g in D10 (the first cell in the acceleration column) by = - k*B10 (B10 being the position x) and drag-copy this down column D. That's all – we've now replaces the constant gravitational force with a linear spring restoring force. We should see some oscillations. Putting v_init = 20, the chart displays oscillations, all right, but of a rather alarming kind – the amplitude is clearly diverging! We know this is not what happens with a real simple harmonic oscillator, and in fact we know the equation we are trying to solve numerically has a simple sine wave solution, so where did we go wrong? The answer is that our approximation of finding the new position (and velocity) by the Euler method, taking the old position and adding the old velocity multiplied by the time interval is just too crude. (It worked better for gravity because there the acceleration doesn't change.)

Fortunately, it's easy to improve our approximation dramatically. Instead of using the velocity at the *beginning* of a time interval to find the change in position during that interval, we need to use the velocity in the *middle* of the time interval. Similarly, to find the change in velocity over a small time interval we get a much more accurate result if we use the acceleration in the *middle* of the time interval multiplied by the time elapsed. To build this into our spreadsheet, we use the ‘leapfrog’ concept: we measure position and acceleration at times 0. 0.1, 0.2, 0.3, say, *but velocities at times 0.05, 0.15, 0.25, ….* To get the change in position from 0.1 to 0.2 we use the velocity at time 0.15: and to get the change in velocity from 0.15 to 0.25 we use the acceleration at time 0.2, so the position and velocity leapfrog over each other as time goes on, and for each numerical step forward in time we are using the derivative in the middle of the time interval.

Building the leapfrog into the spreadsheet is surprisingly simple: we replace =v_init in C10 with =v_init + 0.5*delta_t*D10. In other words, the velocity in C10 is now at time just halfway between the time in A10 and the time in A11. (Of course, this entry is *not* to be copied down the column – once the first entry in the column has been budged down half a time interval, the others automatically follow.) The position column can be left intact, as the existing formula B11 = B10 + C10*delta_t (and subsequent copies) will now be automatically using the velocity C10 at the *midpoint* of the *position* time intervals to find the successive changes in position. The acceleration column also stays unaltered. however, we are not quite through with the velocity column. Going from C10 to C11 is going in time from 0.05 to 0.15, so we need to use the acceleration at the midpoint time 0.1. In other words, in C11 we must replace =C10 + D10*delta_t with =C10 + D11*delta_t *and drag-copy that down the column.* The spreadsheet is now a fully-fledged leapfrog spreadsheet, and the oscillations no longer diverge. Our improved numerical method gives the right answer! It is not difficult to check this further in Excel, because the function sin(x) is built in. The student can plot Asin(Bx + C) and adjust the parameters appropriately to see how well the numerical solution matches the exact one. It is instructive to try different values of delta_t, etc.

A Spreadsheet Analysis of the Simple Pendulum

First, let me make clear what I mean by a simple pendulum: not a bob on a string but a bob at the end of a light rod, the rod being constrained to move in a vertical plane, so it can go ‘over the top’ but not ‘sideways’ – it always stays within the vertical plane. This is the same thing as a bob attached to the rim of a light wheel, which rotates about a fixed horizontal axis.

To adapt the simple harmonic oscillator leapfrog spreadsheet constructed above to the simple pendulum, all one need do is replace the acceleration –k*B10 in the fourth column by (g/L)*sin(B10) and drag-copy it down the column. (Of course, we also need to add cells for g and L to our ‘named cells’ list, and we might as well put one in for initial position x_init, then enter =x_init in A10. Strictly speaking, we should at this point replace x, v, accn., by theta, omega and alpha, and adjust the units appropriately, but first let’s look at the curves the spreadsheet generates!)

Many students will be surprised at first by the curves for the position of the simple pendulum as a function of time. Entering first g = –10, L = 1, x_init = 0, v_init = 0.001 gives the expected simple harmonic oscillator type curves. But things really change for high v_init! Try v_init = 6, then 7 – the curve goes from periodic to a sloping line with slight wiggles.

This is the time to introduce a physical model – perhaps a light wheel with a weight on the rim. It immediately becomes clear that the 'wiggly' curves correspond to a sufficiently high v_init for the bob to go over the top and the wheel continues to rotate in the same direction. It is amusing to try tuning the initial velocity in the spreadsheet to get the pendulum to stop at the top. One can then check the accuracy of the spreadsheet's computation by using conservation of energy. (In fact, one can add an extra column giving total energy as a function of time as an accuracy check, remembering that the velocity and position listed on the same row are at slightly different times, and correcting accordingly.)

In contrast to the simple harmonic oscillator, the simple pendulum's motion cannot be integrated using functions the students know, so the spreadsheet enables them to find new quantitative information. For example, they can find the period as a function of amplitude, by putting v_init = 0, and varying x_init, so they can figure the error if a pendulum clock swings with an amplitude other than that assumed by the manufacturer.

Are Spreadsheets Pedagogically Effective?

I have not tried to assess how well students learn this material compared with a course not using spreadsheets. That would certainly be worth doing. However, many students have told me that they understood calculus and acceleration much better after being forced to construct their own spreadsheets to predict position as a function of time for the examples given above. (But they will only learn if they are *not* handed the finished spreadsheet!) Once the spreadsheet is running, they enjoy playing with it. They can easily extend it from 100 to 2000 rows or more, and see how that improves accuracy. The falling ball spreadsheet can be extended to include air resistance, to go to two dimensions, even to planetary orbits. (Details can be found on my website.) The simple harmonic oscillator can be generalized to a driven damped oscillator (or pendulum), generating beautiful graphs of resonant behavior, critical damping, etc. The instant feedback from the graph means the students can explore parameter space fairly thoroughly in a reasonable time. My experience over several classes has been that students really enjoy constructing and using spreadsheets for physics problems, they are more positive about the course, and they learn a great deal.

Web Resources

A fuller discussion of the leapfrog method can be found on my website at <http://www.phys.virginia.edu/classes/152.mfli.spring02 /ExcelPendulum.html>.

A polished version of the finished spreadsheet can be downloaded from: <http://www.phys.virginia.edu/classes/581/SimpPend.xls> . That is part of my

Physics 581 website, which also contains spreadsheets for many other problems: <http://www.phys.virginia.edu/classes/581>.

Acknowledgment

My first physics spreadsheet efforts years ago were based on the excellent book *Spreadsheet Physics*, by Misner and Cooney which teaches the leapfrog method, and many other tricks. Unfortunately the book is now more than 10 years old, has not been revised, and is based on Lotus 1-2-3 for DOS, so students do not find it appealing, severely limiting its usefulness.

Teaching Cultural History from Primary Events

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Abstract. This article explores the relationship between specific cultural events such as Galileo's work with the pendulum and a curriculum design that seeks to establish in skeletal form a comprehensive epic narrative about the co-evolution of cultural systems and human consciousness. The article explores some of the challenges and some of the strategies needed to represent complex primary events in the concise, viscerally immediate form necessary to make this curriculum offering practical.

1. Foundational Perspectives on Curriculum

I have elsewhere argued that foundational perspectives should be used to enrich the teaching of individual subject areas as well as to frame the whole curriculum (Carson 1997a, b, 1998, 2002a, b). Foundational perspectives are those derived from bringing to bear upon the study of any subject area the disciplines of history, philosophy, sociology, anthropology, and psychology, such as 'history of science', 'philosophy of mathematics', and so forth. Thus, in the teaching of science, we may choose to teach science content as a canon of knowledge and skills to be acquired by the learner, or we can teach something of the history of the subject, showing how it evolved over time, introduce some insights from philosophy on the nature of knowledge, of perception, and of the act of knowing, and so forth (see Matthews 1994). We could also study how different societies construct their understandings (Shweder 1991) including those of the physical world, and how in the disciplines of science we strive to see things differently.

The argument for foundational perspective is that we end up with a richer, more authentic, and probably more intellectually honest account of the subject (Scheffler 1970). The teaching of some disciplines, such as music, art, and literature, would be severely impoverished if those works were taught with no regard to the social, historical, philosophical, and cultural contexts in which they have arisen. Thus, it is customary to pursue an education in art partially through art history and courses in the philosophy of art. But in

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science there is hesitation to do this because it seems to imply that the findings of science are somehow provisional (which they are) and context bound (which, in a qualified sense, they also are). And that would threaten the lofty public impression that scientific knowledge is timeless, objective, absolute, and that it is dynamic only in the sense of its showing incremental movement toward a fixed and perfect state. To be sure, scientific findings are not a matter of whim or fashion, subject to complete revision should the political winds shift or the social climate undergo a change, the Edinburgh strong program notwithstanding (Matthews 1994, p. 40, 93, 142). Science as a human undertaking produces an account of the natural world that is more clear, more reliable, more useful, and more accurate than any other cultural system, precisely because it has been designed with that end in mind. But how and why that has been possible, and making the case convincingly to neophytes, requires giving them some exposure to foundational perspectives.

In previous articles (Carson 2002a, b) I have argued for a comprehensive middle school curriculum framed by historical epochs over the whole three years, beginning with human prehistory and early societies and proceeding through the conventional historical epochs to the 21st century. In this approach, a kind of skeletal history of culture would be deployed, made up of the 'primary events' of cultural evolution. There is no definitive canon of primary events, of course, but sample listings, narrative histories, and anthologies are available (cf. Van Doren 1991; Thompson 2001; Carson, 2002b). In particular, our goal would be to 'tell the story' of the development of art, music and architecture; science and technology; literature and language; mathematics and logic; and that story would be told against the backdrop of an account of the world's huge spectrum of traditional cultures. The latter are intriguing evidence of humankind's tremendous adaptability and of the manner in which culture functions as one of the primary adaptive mechanisms between human beings and their various environments.

For each discipline examined, the account – viewed as a kind of narrative history, a story (here, consider Egan 1986) – would begin with a careful grounding in humankind's pre-historical conditions. In telling the story of mathematics, for example, we could acknowledge that the proverbial caveman did not need a numbering system to wake up each morning and know that he still had all of his fingers and toes. There is a relatively simple, intuitive basis for mathematics that even ducks and geese have. If humankind dates back three million years to our earliest evolutionary ancestors (Wenke 1990), it is fair to say that this intuitive-empirical grasp of number was sufficient for most of that time. But following the close of the last ice age some twelve thousand years ago, humans began to invent tools not only to modify their physical abilities but tools to modify their cognitive abilities (Eccles 1991). It is this discovery of our ability to create tools for the mind that results in such dramatic cultural change over the past four to five thousand years (Vygotsky 1986).

The early progress in cultural innovation left a pretty serviceable record, once historical records were kept. The Greeks, for example, were very conscious of the significance of their discoveries in almost every domain of learning (Snell 1982/1953), and they celebrated as heroes those who made substantial contributions. While the telling of their story has a somewhat mythological flavor about it, the events are at least developmentally pretty accurate. Heath's (1981/1921) account of the history of mathematics, for example, offers numerous corrections to the historical record as given by the Greeks themselves, but the developmental sequence of geometry given there seems generally quite accurate.

In any event, it is that story, of the primary discoveries, innovations, and conventions that make up the skeletal history of this curriculum project, that I have proposed. The project, named '*Ourstory*,' is composed of six subject matter areas: Social Studies, History and Geography; Music, Art and Architecture; Mathematics and Logic; Science and Technology; Literature and Language; and Traditional Cultures. Over a three year period, beginning with Early Cultures and Societies, a narrative history is deployed in which developmental events in each of these subject areas are reconstructed for the learner. Coordinating these subject area narratives by historical epoch permits the learner to see how cultural trends transgress disciplinary boundaries and affect whole cultures. The really big themes come to define entire cultural epochs, and they account for the seeming stylistic unity of a given age.

What this hints at is a comprehensive narrative history of intellectual culture in all of its major domains. Clearly, the time needed to deliver such a massive history would be prohibitive. Yet the goal is not to put forth a comprehensive history, but rather to supply the learner with a concise, parsimonious framework that establishes an initial index in space and time (geography and history) for the location of culturally significant historical events. The establishment of this framework then provides a basis for the operation of *locale* memory in nearly all subsequent learning.

1.1. PSYCHOLOGICAL AND CULTURAL CONSTRAINTS

There are psychological considerations that I have tried to heed in proposing this for a middle school, such as respecting the developmental limits of emergent teenagers and the limits typical of that age group in terms of language sophistication and cultural background. The use of narrative and simulations rather than a more didactic approach is intended to key into the one comprehensive framework all learners have, namely a sophisticated grasp of what it is like to be a human being, and to experience events from that highly visceral perspective (Damasio 1999). Simulations, enactments, and narratives are powerful precisely because learners know how and where to locate massive amounts of contextual information in 'setting the scene' both physically and psychologically, and can therefore construct and process fairly complex scenarios when the problem space itself has this human context as its frame.

There are also philosophical considerations I have tried to honor, since any statement offered as instructional fodder can be refuted ad infinitum for its epistemic and ontological ambiguities and deficiencies. The goal has been to teach the conflicts honestly without trying to resolve them, thus recognizing that unresolved and interesting philosophical issues are a stimulus to higher order thinking.

There are sociological and cultural issues too, since every culture has *its* story, and every human his or her unique variant of that story, so that attempting to tell 'our story' is presumptuous, bordering on folly.

Rather than deny the folly, I have preferred to let it stand out as a cautionary beacon, clear enough in its exposure that anyone can spot it, take it into consideration, and move beyond it as the value of this curricular offering gets absorbed and then subsumed by a richer understanding which hopefully follows. It is, after all, a heuristic, a kind of temporary scaffolding designed to establish a basis for learning that takes advantage of the brain's vast locale memory system (for an undergraduate level discussion of this, see Caine and Caine 1991). Heuristics have limitations if taken too literally, but they are so powerful in their capacity to tap higher order thinking that we tolerate those limitations. The goal is to be able to incorporate foundational perspectives in a way that is natural and coherent, that comes to the student in ways the mind is prepared to receive them. That means telling a story and embedding the important information into that historical narrative. It would be quite wonderful if somehow we could transport the learner back in time and take her to the exact time and place where each culturally significant event was unfolding, to experience for herself the problem and its context, as well as the humans who worked through the problem. She could gain vicarious connection to the whole situation and the epiphany that comes from seeing through the problem to the solution, whether it be a discovery, an innovation, establishment of a new cultural convention, or a new way of looking at something in the world or in our own mental landscape.

In the absence of foundational perspectives teaching risks being unassailably dogmatic, and that really disadvantages students who are trying to answer the 'why' questions, as well as those whose cultural backgrounds stand opposed to the things being taught. Providing foundational perspective means providing the means to recognize alternative views and, if well done, to assess their relative merits. The value of this project is that it provides an organizational framework (see Table I) for all that subject matter students are expected to learn, collates it in ways that promote richer connections, and

Table I. A history of formal intellectual culture as seen against the backdrop of the world's cultural traditions

Grades	Grade 6		Grade 7			Grade 8	
Historical epochs	Early Cultures and Societies (12thm.–8thc.BC)	Classical Civilizations Greece (600–320BC)	The European Middle Ages (410AD–1300)	The Renaissance (1300'S–1600)	The Enlightenment (1660–1800)	Modernity (1800–1920)	20th and 21st Centuries
School Subjects	The Agricultural Revolution	Literacy, Reason and Democracy	Christianity and Islam	Rebirth of Classical Learning	Science and Romanticism	Technology; Global Expansion	Post-Modern-ism. Cultural Pluralism
Social Studies, History and Geography	A1	A2	A3	A4	A5	A6	A7
Music, Art and Architecture	B1	B2	B3	B4	B5	B6	B7
Mathematics and Logic	C1	C2	C3	C4	C5	C6	C7
Science and Technology	D1	D2	D3	D4	D5	D6	D7
Literature and Language	E1	E2	E3	E4	E5	E6	E7
Traditional Cultures	F1 (Egypt, Africa Mesopotamia)	F2 (China, India, Israel, Rome)	F3 (Europe, Arabic civilization)	F4 (American Indians, Inca, Aztec, Mayan)	F5 (Africa Revis-ited: Tribal structures versus Colonialism)	F6 (Clash of cultures: Traditional versus Scientific. Asia revisited)	(Special topic: Philosophy of Language, Culture and Mind)

therefore aids the learner in acquiring, understanding, and remembering knowledge that must otherwise be encountered as disconnected, less meaningful chunks of information. Central to our ability to learn is our ability to detect and construct patterns and connections, and thus to link the pieces into coherent, meaningful wholes. One thing that makes this approach especially attractive is that entire cultural epochs tend to derive their character from the wholesale generalization of a relatively small number of very powerful ideas, the discovery of rationality, abstraction, and a meta-narrative on the nature of mind and culture among the first philosophers, for example. Seeing how these find expression across disciplinary boundaries provides the basis for a conceptually rich historical treatment of human cultures and human consciousness.

2. Large Scale History

We cannot tell the whole cultural story in all of its complex detail. But we can identify those cultural shifts that have to be understood in order to follow the evolving nature of each formal discipline. The key to restricting the number of episodes to a manageable number is to focus on dynamics of cultural development, and to highlight those events that precipitate the most dramatic and significant changes in the ways we think and act.

In mathematics, for example, a fundamental change takes place when Thales begins to ponder the possibility of a formal explanation for mathematical propositions. Seeing that something is true intuitively is different from being able to demonstrate that truth by means of a formal argument. Later, the Pythagoreans codify the method of proof. Still later, Plato extrapolates from the evident 'necessity' of mathematical truths to propose the immortal Forms upon which all things material and otherwise were patterned. These events are significant because they redefine general cultural trends far beyond their significance for geometry *per se*. It is events like these that we want to identify and use to create a kind of conceptual lattice-work, which we may then think of as a kind of epic narrative of the evolution of the world's formalized cultural systems.

In trying to flesh out what this whole project would look like I have identified approximately five hundred of these *primary events* (Carson 2002b). An event may be something as discrete as 'the advent of the phonetic alphabet,' or as broad and complex as 'the mathematization of physics in the 17th century'.

Matthews' work on the contributions of Galileo to the development of modern science (Matthews 2000) explores several of those primary cultural events in detail. Galileo is significant because both the discipline of science and the world view of the society he lived in were different as a result of his work. In general, this is what identifies primary developmental events in this approach to curriculum. They occur in response to various cultural pressures, and they are dramatic and far-reaching in scope and effect. Typically – and this is important – they affect more than the discipline in which they originate. They spill out by metaphorical transference into the whole of the culture and effect changes throughout. Removing humankind from the center of the universe, as Galileo did, in effect, has its correlates in every other cultural domain from art and literature to theology and philosophy. Typically, such shifts in human consciousness unfold gradually, such that the initial event can seem far more pedantic and distant from everyday life than it actually proves to be.

Galileo's work on the pendulum is a wonderful example of an (at first) seemingly esoteric and minor bit of tinkering by an eccentric old fellow. Looked at from the perspective of an immature student indifferent to such matters, interest in the topic is not immediate, but surface appearances are deceptive. The pendulum provided Galileo a means to study the effects of gravity in a controlled fall. It provided an encapsulated instance of complex motion, that is, motion in which the velocity was not constant, but increasing and decreasing in a regular manner. Basic concepts that would eventually enter mathematical thinking as foundational to the calculus began with the study of motion, acceleration, and other dynamic features of the natural world. It requires the kind of refined, patient, astute observation of a Galileo to realize the nature of the pendulum's motion, and then to figure out ways to portray that motion mathematically. As he did so, he contributed a set of conceptual templates that were destined to reshape human consciousness. Students can replicate the very development of those conceptual templates, and a helpful strategy for doing so is to relive the historical sequence of their unfolding.

On a more practical level, the work on the pendulum led to development of accurate clocks, which in turn had profound implications for a host of applications. It was crucial to the whole cultural shift of the 17th century, the beginnings of modern science (Matthews 2000, pp. 2–3), and, ultimately, the evolution of sophisticated industrial societies. The clock, based upon the action of the pendulum, is a machine that eventually came to represent metaphorically the precise workings of nature. These, too, are cultural developments the student can learn to appreciate, if provided access to the historical context.

In each historical epoch, we want to ask, how is *this* particular domain of learning (science, mathematics, literature, art, etc.) different at the end of the epoch from what it was at the beginning, and we want to account for that change by looking at the events around which those changes appear to have precipitated. A new way of looking at the world can change the way science (or art, or poetry) is done. An innovation, such as the microscope or the telescope, can take us into whole new ranges of experience. The advent of a new nomenclature, such as Lavoisier developed in chemistry, can help the mind see familiar phenomena in new ways. When these events have unfolded, they not only affect the original problem (perhaps resolving it) but they also result in a host of collateral effects that could not have been anticipated. The advent of writing systems, especially the highly compact phonetic alphabet, occurred because of the need to keep records, but once it happened it became a technology for crystalizing ideas as well. The invention of writing altered fundamentally our relationship to our own thoughts, making them tangible objects we could look at in our leisure rather than mysterious voices emanating from the silence of the mind.

3. Planning Instruction in *Ourstory*

The challenge is to develop *Ourstory* as an austere lattice-work that can be slipped in and amongst the interstices of the existing curriculum in middle schools, and then to engineer a gradual migration of the whole curriculum around that temporal framework. We are attempting to create this framework in the local middle schools in the community where I live, establishing it to begin with in the social studies curriculum with the hope that each subject area will then come to claim those portions that rightfully belong to them.

The standard US social studies curriculum could use some improvement anyway. Its purposes are confused, and somewhat anachronistic. It tends to be a compromised history, made up primarily of economic, political, and military topics, the purposes for which are questionable. Presumably, schools have as their primary obligation the moving along of a cultural heritage to the next generation, equipping that next generation to think for itself, and that requires an emphasis on cultural history rather than imperial history.

Understanding how we have made the transitions from traditional hunter-gatherer societies to modern scientific-technological societies, and encountering that history in a way that engenders neither arrogance nor bigotry, but understanding and humility, would be a goal worthy of the schools in any civilized society.

To tell this story, we have to separate it into modules which can form the units and lessons. These are the ‘primary events’ mentioned above. Then, we have to do something with those primary events in order to connect the learner with them. Simply telling about them, in lecture form, would quickly become tedious, and it is not the most vivid way to achieve a strong vicarious connection to those events. So the goal has been to get the learner to encounter and engage those primary events as if she were there when they first happened. We want to take the learner back in time, locate her in the problem space that gave rise to that event, and then guide the learner through the cultural transformation that makes the event so significant in the first place. Egan (1997) describes this pedagogical moment as a special case of recapitulation, one in which the historical dynamic of cultural change is recreated for the learner and is experienced as a shift of consciousness.

3.1. THE GALILEO EPISODE

What this takes, ironically, is thorough scholarship into each event, and then an almost catastrophic compression and simplification to make it both digestible at the middle school level and concise enough to make room for the other 499 or so events. The key to this is figuring out what the transformation of consciousness is that occurred during the denouement of these primary, threshold events. In the case of Galileo, at least two extraordinary changes in consciousness took place.

One had to do with the mathematization of physics. Galileo demonstrated that the motion of the pendulum was a modified example of free fall, and that it portrayed characteristics of acceleration similar to those of a free-falling body. He used mathematics as a descriptive language, and he crafted that description in the language of classical geometry. That in itself was a substantial achievement. In doing this he set the stage for the eventual use of analytical geometry and algebra as the new descriptive languages of science. His work with the pendulum, significant in itself because it reveals the dynamics of gravity, is significant in *Ourstory* because it is the subject matter around which students can understand the restructuring of consciousness that takes place when science begins to use mathematics for descriptive and predictive purposes.

A second threshold event attributable in part to Galileo was the establishment of the Copernican universe. In some respects, this is a far more difficult event to portray in the form of a compressed historical narrative, for most attempts end up as caricature, with the Church authorities looking foolish. The richness of this event has to do with fundamental questions of what is real and how we know, for the locus of authority shifts as a consequence of the revolution of thought Galileo contributes to. We are reminded of Plato's breath-taking pronouncement that 'The Good is not good because the gods approve; the gods approve because it is good.' In other words, Plato put the locus of authority in the natural order of the universe, rather than in the gods.

By a similar token, Galileo challenged the authority of holy writ and insisted that evidence and reason would reveal the way things really were. The refusal of cardinal Bellarmino to look through the telescope constitutes a moral stand in which he is refusing to acknowledge the authority of empirical evidence to arbitrate questions pertaining to the world. Like traditional cultures worldwide, Christian doctrine could not stand up to the dissembling influence of science. It is not that the doctrine is wrong necessarily, but that it is designed with different purposes in mind and that it resolves human problematics using an entirely different strategy from that of scientific culture. Thus, even though the immediate event has to do with whether the earth revolves around the sun or vice versa, the essential cultural dynamic we would be interested in with respect to *Ourstory* is the clash of two fundamentally different and incommensurable cultural systems, representing two very different ways of knowing, two different world views.

Telling that story in a way that respects both the traditions of a religious culture and the findings of scientific culture is no easy task. But in a world comprised of multiple, competing cultural systems we are faced with only two alternatives, one in which cultures go head to head in a process of elimination, or one in which the incommensurability of cultural systems is acknowledged as part of the cultural landscape. In our work we have taken the position that schools must not falsify the distinction between science and cultural traditions (which occurs when creationism is touted as 'science'), that those distinctions do exist and probably cannot be reconciled, that it is not within the competence of public schools to reconcile those differences, and that the role of the school is to teach *about the world as it is*, which includes teaching the conflicts in a manner that is intellectually honest. This constitutes an ethical stand, of course, but, more to the point, it represents a pedagogical strategy based on the assumption that the astringent of competing viewpoints has greater pedagogical value than does the resolution of all conflicts by selective omission or dogmatic certainty.

3.2. LANGUAGE AND CULTURE

Different cultural systems, formal and informal, constitute human consciousness in particular ways (Vygotsky 1978, 1986; Shweder 1991; Carroll 1995). The most powerful and ubiquitous cultural structure is natural language. As language has evolved for humankind consciousness has gained extension, clarity, and versatility (Lieberman 1984; Innis 1994; Deacon 1997). By a similar token, as language is acquired by the individual the capacity to think in a clear, extensive and versatile manner increases (Barrow 1993). Different languages, or different symbolic and representational systems, frame consciousness in different ways (Jacobi 1959; Carroll 1995; Jackendoff 1996). The 'language' of mathematics, which is also symbol-based in all of its advanced forms, structures human consciousness in ways that are uniquely different from natural language. It takes us into whole new ranges of thought that would otherwise be inaccessible to us. So too the 'languages' of artistic convention. They cause us not only to see things that are different by putting novel images before us, but to see in ways that are different even those things that are familiar to us.

Vygotsky's observation, that humans have learned to invent 'tools' for the mind, is an especially potent insight as we contemplate what events should be considered as the pivotal, threshold events in the history of formal intellectual culture. The advent of symbols and their accretions as languages are among the most powerful examples of human cultural innovation. So too are the unique ways we have learned to capture insights and concepts by various strategies for modeling, idealizing, representing, and structuring thought in science.

Clearly, the advent of a new system of thought, such as the calculus, the development of new tools to aid cognition, such as computers, the introduction of a new convention for the organization of perception, such as the metric system, or various conventions of twentieth century art that focus our attention on the mind's contributions to perception – all of these may be viewed as primary events that effect substantial change in the cultural landscape and that help to explain how and why the world we experience today is so different from the world that existed when humans first expressed themselves by painting on the walls of limestone caves. This focus on the evolution of cultural systems reflects Hirst's emphasis on the varieties of thought and experience made possible for humankind because of the various formal disciplines. His defense of liberal education hinged on the argument that each domain of learning opens up a territory in our mental landscape that would otherwise not exist for us (Hirst 1973). In this sense, education is about acquiring these different ways of knowing.

4. Conclusions

The domestication of human consciousness is the central story of any curriculum concerned with cultural education. How we arrange those episodes to tell our whole epic narrative, and how those stories are crafted so the learner will participate in the historical transformation each represents, are the central tasks demanded by this approach to schooling. As we contemplate these challenges from within the confines of any single discipline, we must also make the attempt to see how historical patterns of development in other disciplines run parallel to the patterns of development in our own. By aligning those disciplinary narratives we gain a synergistic advantage, revealing the larger themes that have defined entire cultural epochs and revealing the ingenuity of human imagination as it transposes those conceptual themes into the various media of art, music, architecture, mathematics, logic, science, technology, literature, historiography, philosophy, theology, poetry, theater... as well as social and political patterns, economic activity, jurisprudence, and so on. The style of an age is determined by the generalizing of fundamental aesthetic/philosophical/conceptual thematics that find expression in many or most of these domains. Taking into account the relationship between these larger thematics and the specific dynamics of primary cultural events is the challenge we are attempting to meet in the design of the *Ourstory* curriculum project.

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Pendulums in The Physics Education Literature: A Bibliography

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Abstract. Articles about the pendulum in four journals devoted to the teaching of physics and one general science teaching journal (along with other miscellaneous articles from other journals) are listed in three broad categories – types of pendulums, the contexts in which these pendulums are used in physics teaching at secondary or tertiary levels and a miscellaneous category. A brief description of the sub-categories used is provided.

1. Introduction

The pendulum has been dealt with in science textbooks for almost four centuries. The following bibliography consists of articles dealing mainly with pendulums – their nature and behaviour – found in four journals devoted specifically to the teaching of physics – *The American Journal of Physics* (Vols. 1–69), *The Physics Teacher* (Vols. 1–39), *Physics Education* (Vols. 1–36) and *European Journal of Physics* (Vols. 1–23) – and one general science teaching journal – *The Australian Science Teachers Journal* (Vols. 1–45). A number of miscellaneous articles from other journals have also been included. The articles have been classified into three broad groups: in the first are those articles concerned with the nature of different types of pendulums, in the second are those concerned with the pendulum in particular contexts and in the third are those articles in which the pendulum is not the main point of interest. Within the first two groups there are further subdivisions the natures of which are explained further below.

2. Types of Pendulums

The name 'pendulum' is associated with an oscillating system in which kinetic energy is converted into potential energy and back again. Usually, but not always, the potential energy is gravitational potential energy.

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M.R. Matthews et al. (eds.), The Pendulum, 505–526.

The simplest type of pendulum (the *simple pendulum*) consists of a spherical ball suspended from a thin string so that the ball can move backwards and forwards in one plane along a path which is a portion of a circle. As Matthews (2000) has pointed out this device has been central to the early beginnings of modern science. The *physical pendulum* (also known as the compound pendulum) consists of a solid object pivoted about a fixed point and the motions of both the simple and the physical pendulum are described by the same relationship, namely,

$$\tau = I\ddot{\theta}$$

or

$$I\ddot{\theta} = -mgd \sin \theta$$

where τ is the torque acting on the system, I is the moment of inertia of the object about the pivot point, m is the mass of the object, d is the length of the line between the centre of mass of the object and the pivot point and θ is the angle between this line and the vertical. In an ideal pendulum, for which the amplitude, θ_0, is constant, the periodic time is constant and depends the length, d, the value of g and the size of θ_0. In a real pendulum the amplitude decreases with time as energy is expended in the system so that the motion is *damped.* For small values of θ_0, $\sin \theta \approx \theta$, and the pendulum executes simple harmonic motion with a period, $T = 2\pi\surd(I/mgd)$, that is independent of θ_0. For a simple pendulum, $d = l$ and $I = ml^2$, so that $T = 2\pi\surd(l/g)$.

Kater's pendulum is a physical pendulum which has two points of support allowing it to be suspended upside down. When the position of a mass is changed on the pendulum rod or the position of one of the points is adjusted so that the period in both positions is the same, the length of the equivalent simple pendulum is equal to the distance between the supports.

If a pendulum is constructed so that the line between the point of support and the centre of mass when at rest is not vertical but at an angle φ to the vertical the effective value of g is reduced and the period of oscillation is $T = 2\pi\surd(l/\text{g} \cos \varphi)$. Escriche's pendulum is one example of a variable gravity pendulum.

Huygens (Matthews 2000) and Newton (1729/1960, Propositions 48–52) showed that if a pendulum moved along a cycloidal rather than a circular path its period was independent of its amplitude for all values of θ_0. The *cycloidal pendulum* was thus shown to be an ideally suited device to regulate the mechanism of a time-keeping instrument.

If the supporting string of a simple pendulum can stretch elastically the pendulum is called in the literature an *elastic pendulum.*

The *bifilar pendulum* is a simple or physical pendulum which is suspended from two points so that it is constrained to oscillate about a fixed horizontal axis through the line joining the two points of suspension rather than having the freedom to move in any direction. In 'Newton's cradle' a number of bifilar pendulums hang in a row so that their steel bobs just touch one another. An extended bob, supported in the above manner by two strings attached to different points on the bob, as well as oscillating about a fixed horizontal axis, can also oscillate about a vertical axis.

In the most common version of the *ballistic pendulum* a large block of wood is suspended by four strings so that it remains horizontal when it swings. It is used to determine the speed of a projectile fired into the block by measuring the height the block rises and using the law of conservation of momentum during the collision and the law of conservation of mechanical energy after the block has begun to move (Taylor 1941, pp. 205–206; Resnick and Halliday 1966, pp. 219–220). A more recent version, the *Blackwood pendulum*, consists of a solid rod which is suspended at its top (Blackwood 1973, p.104). The bob, at the bottom end of the rod, is a cage into which a spherical metal ball is fired and captured causing the pendulum to swing upwards. When it stops, a ratchet holds the bob in its highest position so that its height can be measured. This information provides the data from which the initial speed of the ball can be calculated.

The *ring pendulum* is a physical pendulum in which the object is a ring which is suspended from a point on the inside surface.

Because a simple pendulum is free to move in any direction the *two-dimensional pendulum* can be used to study the nature of the motion of the bob in two dimensions. In the *conical pendulum* the pendulum bob moves in a horizontal circle so that the string is at a fixed angle to the vertical. The period of the conical pendulum is $T = 2\pi\sqrt{(h/g)}$ where h is the distance of the point of suspension above the plane in which the bob moves.

The *Blackburn pendulum* is a bifilar pendulum in which the lower part is another pendulum supported by a single string attached to the V formed by the upper pair of strings. The pendulum as a whole is constrained to move in one plane while the lower portion can move at right angles to this plane. The motion of the bob thus consists of two perpendicular oscillatory motions.

If the material which is used to connect the pendulum bob to the point of suspension is solid and elastic the pendulum can be turned upside down so that the fixed point is at the bottom and the bob moves from side to side above it. This is called an *inverted pendulum*. If the connecting rod is rigid the inverted pendulum is unstable but stability can be achieved by driving it either vertically or horizontally from the bottom.

A *double pendulum* often consists of two physical pendulums with one usually but not always being suspended from the bottom of the other. It can also consist of two simple pendulums with one suspended from the bob of the other.

Pendulums can be arranged so that they are joined near the tops of the strings by a horizontal string. The motion of one pendulum is then communicated by this string to the pendulum to which it is linked. Such pendulums are called *coupled pendulums*.

The name 'pendulum' is also used for other systems in which the potential energy is not always (or not totally) gravitational. In the *spring-mass pendulum* the bob is suspended by a spring so that it can move up and down. In this case the kinetic energy which the system has in its equilibrium position is converted into gravitational and elastic potential energy as the spring stretches or compresses. Ignoring the mass of the spring, the period of such a pendulum is $T = 2\pi\sqrt{(m/\kappa)}$ where κ is the spring constant.

The *torsion pendulum* also consists of a mass suspended by a spring but in this case the potential energy is stored in the spring as it twists. Changes in gravitational potential energy are of minor consideration. The period of this pendulum is given by $T = 2\pi\sqrt{(I/k)}$ where k is the torsion constant. However, as the spring twists so that the coils close up the length of the spring decreases a little and when the spring unwinds its length becomes a little greater. If the spring constant and the moment of inertia of the object are carefully chosen the period of the torsional motion and the period of the natural vertical motion (as a spring-mass pendulum) can be made equal. When this is the case the torsional motion slowly decreases and the vertical oscillations gradually increase until all the original rotational kinetic energy appears as kinetic energy of vertical motion and torsional motion ceases. In this mode, as the spring stretches it also unwinds a little and when it compresses it winds up a little. Gradually, the vertical motion decreases and the torsional motion increases again. This type of pendulum is called a *Wilberforce pendulum* and demonstrates the conservation of energy as the mode of vibration changes.

When it is pulled aside and released a pendulum will fall to its lowest position along a plane which contains the initial line of the string and the line of the string when it is vertical. If no other external influences than gravity act the pendulum will continue to swing in this plane until it stops. However, if the frame to which the point of support is attached is rotated a torque can be exerted on the string and the plane of oscillation may rotate about a vertical axis with the rotation of the frame. On the other hand, if the point of support is carefully designed to prevent this torque from acting the initial plane of oscillation can be maintained even as the frame rotates. This property was used by *Foucault* to show that the Earth itself was rotating.

3. Pendulum Contexts

In teaching physics/science the pendulum has been used both as a device to be studied and as a tool for finding out other things. For example, in the

Principia Newton (1729/1960) presented the theory which lies behind the motion of the pendulum and also used it as a means of measuring the velocities of balls before and after they collided.

The simple pendulum is a physical system which is easy to make and to study and it is often used to teach *investigative skills and skills of measurement*. Its role in *timekeeping* is also something which students can explore.

Two everyday systems which can be modelled by pendulum motion are *walking* and *swinging* and both have been extensively discussed in the physics education literature. If one considers the leg as a simple or double pendulum (with a second pivot at the knee) then the most comfortable leg movement is related to the natural period of this pendulum. One of the implications of the law of conservation of momentum is that forces within a system are unable to change the total momentum of the system. This raises the question of how the kinetic energy of a child's swing can be increased by a person sitting on the swing and the subsequent discussion of this issue is most illuminating (see also Walker 1977, pp. 37–38).

In the 17th and 18th centuries 'laws of motion' referred to the laws which governed elastic and inelastic collisions between two bodies and the laws enabled predictions to be made about the outcomes of different types of collisions (Gauld 1998). *Colliding pendulums* were widely used to measure the velocities before and after the collisions to check the predictions. Today they can be very effective in demonstrating the law of conservation of momentum in a dynamic rather than a static context. 'Newton's cradle' consists of a series of colliding bifilar pendulums.

Coupled pendulums in which the motion of one pendulum influences the motion of a nearby pendulum can be used to demonstrate *resonance*. If one pendulum in a pair of equal-length, coupled pendulums is set in motion the second will begin to move and the first will begin to slow down. This continues until the second is moving with the same amplitude with which the first began and the first one has stopped. The total energy is transferred from one pendulum to the other and back again. Driven pendulums also demonstrate resonance at particular frequencies.

Chaotic motion can be demonstrated using a multiply-connected pendulum or a pendulum in which the point of suspension is driven backwards and forwards at different frequencies.

The importance of the pendulum in *Galileo's thinking* has also been discussed in the physics education literature.

Since Piaget's famous studies of adolescent thinking published in 1958 *student conceptions* of the pendulum and their explanations for its motion have been of interest to physics teachers and others. More recently error analysis of student responses to questions about the pendulum provide some idea of the pre-conceptions which students have when they first begin to learn physics.

4. Miscellaneous

In the miscellaneous category are articles in which other phenomena than pendulum motion is the focus of attention. There are also articles which cover a wide variety of pendulum types.

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Name Index

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