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Reference
Manual

Structural Engineering Reference Manual

Eighth Edition

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Alan Williams, PhD, SE, FICE, C Eng

Structural Engineering Reference Manual

Eighth Edition

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Table of Contents

Preface and Acknowledgments

I wrote the *Structural Engineering Reference Manual* to be a comprehensive resource that helps you prepare for the National Council of Examiners for Engineering and Surveying (NCEES) 16-hour Structural Engineering (SE) exam. As such, each of this book's eight chapters presents the most useful equations in the exam-adopted codes and standards, and each chapter also provides guidelines for selecting and applying these equations.

For this eighth edition, all nomenclature, equations, examples, and practice problems have been checked and updated so that they are consistent with NCEES-adopted codes and specifications. Additionally, significant changes have been made to the following chapters.

Chapter 1, Reinforced Concrete Design, includes significant new material on concrete anchoring. Existing content was revised to conform to the *Building Code Requirements for Structural Concrete and Commentary*, 2011 edition.

Chapter 4, Structural Steel Design, includes new material on nominal flexural strength, compact sections, noncompact sections, slender sections, lateral-torsional buckling, moment redistribution in continuous beams, buckling, bolt types and connections, and welds. Existing content was revised to conform to the *Steel Construction Manual*, fourteenth edition.

Chapter 5, Timber Design, includes new material on load combinations, reference design values, and adjustment factors. The chapter was also updated to include both exam-adopted ASD and LRFD design methods. Existing content was revised to conform to the *National Design Specification for Wood Construction ASD/LRFD*, 2012 edition.

Chapter 6, Reinforced Masonry Design, includes significant new material on required strength, allowable stress, masonry beams in flexure, reinforcement requirements, the design of reinforced masonry beams, minimum and maximum reinforcement area, shear beam design, masonry column design, and anchor bolt placement and design. The chapter was also updated in order to present both exam-adopted ASD and SD design methods. Existing content was revised to conform to the *Building Code Requirements and Specification for Masonry Structures*, 2011 edition.

Chapter 7, Lateral Forces, includes new material on shear wall-frame systems, steel systems, subdiaphragms, seismic parameters and building height, and wind loads. Existing content was revised to conform to the *Seismic Design Manual*, 2012 edition.

Finally, if you find an error in this book, please let me know by using the error reporting form on the PPI website at **ppi2pass.com/errata**. Valid submitted errors will be posted to the errata page and incorporated into future printings of this book.

Alan Williams, PhD, SE, FICE, C Eng

Introduction

PART 1: HOW TO USE THIS BOOK

This *Structural Engineering Reference Manual* is intended to help you prepare for the 16-hour Structural Engineering (SE) exam administered by the National Council of Examiners for Engineering and Surveying (NCEES). The NCEES SE exam will test your knowledge of structural principles by presenting problems that cover the design of an entire structure or portion of a structure. The exam is given in four modules—two concerning vertical forces and two concerning lateral forces. The eight chapters of this book are organized around the eight areas in which these forces are applied. These eight areas include

- reinforced concrete design
- foundations and retaining structures
- prestressed concrete design
- structural steel design
- timber design
- reinforced masonry design
- lateral forces (wind and seismic)
- bridge design

Each chapter presents structural design principles that build on the ones before, so you should read the chapters in the order in which they are presented. The examples in each chapter should also be read in sequence. Taken together in this way, they constitute the solution to a complete design problem similar to that on the exam.

Your solutions to the SE exam problems must be based on the NCEES-adopted codes and design standards. Therefore, you should carefully review the appropriate sections of the exam-adopted design standards and codes that are presented, analyzed, and explained in each chapter of this book. Each of the examples in this book focuses on one specific code principle and offer a clear interpretation of that principle.

Table 1 lists the SE design standards that code-based problems on the exam will reference. You will not receive credit for solutions based on other editions or standards. All problems are in customary U.S. (English) units, and you will not receive credit for solutions using SI units.

Abbreviations are used throughout this book to refer to the design standards and codes referenced by the SE exam. This book's "Codes and References" section lists these abbreviations in brackets after their appropriate design standard or code. This book also cites other publications that discuss pertinent structural design procedures, which may also be found in the "Codes and References" section. Text references to any other publications are numbered as endnotes in each chapter, and the publications are cited in the "References" section that precedes each chapter's practice problems. These references are provided for your additional review.

As you prepare for the SE exam, the following suggestions may also help.

- Become intimately familiar with this book. This means knowing the order of the chapters, the approximate locations of important figures and tables, and so on.
- Use the subject title tabs along the side of each page.
- Skim through a chapter to familiarize yourself with the subjects before starting the practice problems.
- To minimize time spent searching for often-used formulas and data, prepare a one-page summary of all the important formulas and information in each subject area. You can then refer to this summary during the exam instead of searching in this book.
- Use the index extensively. Every significant term, law, theorem, and concept has been indexed. If you don't recognize a term used, look for it in the index. Some subjects appear in more than one chapter. Use the index to learn all there is to know about a particular subject.
- Use the code index extensively. The most commonly used chapters, equations, and tables have been indexed for your quick reference.

PART 2: EVERYTHING YOU EVER WANTED TO KNOW ABOUT THE SE EXAM

ABOUT THE EXAM

The SE exam is offered in two components. The first component—vertical forces (gravity/other) and incidental lateral forces—takes place on a Friday. The second component—lateral forces (wind/earthquake)—takes place on a Saturday. Each component comprises a

Table 1 *NCEES SE Exam Design Standards*

abbreviation	design standard title
AASHTO	*AASHTO LRFD Bridge Design Specifications*, Sixth ed., 2012, American Association of State Highway and Transportation Officials, Washington, DC.
ACI 318	*Building Code Requirements for Structural Concrete and Commentary*, 2011 ed., American Concrete Institute, Farmington Hills, MI.
AISC	*Steel Construction Manual*, Fourteenth ed., 2011, American Institute of Steel Construction, Inc., Chicago, IL.
AISC	*Seismic Design Manual*, Second ed., 2012, American Institute of Steel Construction, Inc., Chicago, IL.
AISI	*North American Specification for the Design of Cold-Formed Steel Structural Members*, 2007 ed., with Supplement no. 2 (2010), American Iron and Steel Institute, Washington, DC.
ASCE/SEI7	*Minimum Design Loads for Buildings and Other Structures*, 2010 ed., American Society of Civil Engineers, Reston, VA.
IBC	*International Building Code*, 2012 ed. (without supplements), International Code Council, Country Club Hills, IL.
MSJC[a,b]	*Building Code Requirements and Specification for Masonry Structures* (and companion commentaries), 2011 ed., The Masonry Society, Boulder, CO; American Concrete Institute, Detroit, MI; and American Society of Civil Engineers, Reston, VA.
NDS	*National Design Specification for Wood Construction ASD/LRFD*, 2012 ed., and *National Design Specification Supplement, Design Values for Wood Construction*, 2012 ed., American Forest & Paper Association, Washington, DC.
PCI	*PCI Design Handbook: Precast and Prestressed Concrete*, Seventh ed., 2010, Precast/Prestressed Concrete Institute, Chicago, IL.
SDPWS	*Special Design Provisions for Wind and Seismic with Commentary*, 2008 ed., American Forest & Paper Association, Washington, DC.

[a]MSJC refers to TMS 402/ACI 530/ASCE 5.
[b]*MSJC Specification* refers to TMS 602/ACI 530.1/ASCE 6.

morning breadth and an afternoon depth module, as outlined in Table 2.

The morning breadth modules are each four hours and contain 40 multiple-choice problems that cover a range of structural engineering topics specific to vertical and lateral forces. The afternoon depth modules are also each four hours, but instead of multiple-choice problems, they contain constructed response (essay) problems. You may choose either the bridges or the buildings depth module, but you must work the same depth module across both exam components. That is, if you choose to work buildings for the lateral forces component, you must also work buildings for the vertical forces component.

According to NCEES, the vertical forces (gravity/other) and incidental lateral depth module in buildings covers loads, lateral earth pressures, analysis methods, general structural considerations (e.g., element design), structural systems integration (e.g., connections), and foundations and retaining structures. The depth module in bridges covers gravity loads, superstructures, substructures, and lateral loads other than wind and seismic. It may also require pedestrian bridge and/or vehicular bridge knowledge.

The lateral forces (wind/earthquake) depth module in buildings covers lateral forces, lateral force distribution, analysis methods, general structural considerations (e.g., element design), structural systems integration (e.g., connections), and foundations and retaining structures. The depth module in bridges covers gravity loads, superstructures, substructures, and lateral forces. It may also require pedestrian bridge and/or vehicular bridge knowledge.

WHAT DOES "MOST NEARLY" REALLY MEAN?

One of the more disquieting aspects of the exam's multiple-choice questions is that the available answer choices are seldom exact. Answer choices generally have only two or three significant digits. Exam questions ask, "Which answer choice is most nearly the correct value?" or they instruct you to complete the sentence, "The value is approximately ..." A lot of self-confidence is required to move on to the next question when you don't find an exact match for the answer you calculated,

Table 2 *NCEES SE Exam Component/Module Specifications*

Friday: vertical forces (gravity/other) and incidental lateral forces

morning breadth module
  4 hours
  40 multiple-choice problems
	analysis of structures (30%)
	  loads (10%)
	  methods (20%)
	design and details of structures (65%)
	  general structural considerations (7.5%)
	  structural systems integration (2.5%)
	  structural steel (12.5%)
	  light gage/cold-formed steel (2.5%)
	  concrete (12.5%)
	  wood (10%)
	  masonry (7.5%)
	  foundations and retaining structures (10%)
	construction administration (5%)
	  procedures for mitigating nonconforming work (2.5%)
	  inspection methods (2.5%)

afternoon depth module[a]
  4 hours
  essay problems
	buildings[b]
	  steel structure (1-hour problem)
	  concrete structure (1-hour problem)
	  wood structure (1-hour problem)
	  masonry structure (1-hour problem)
	bridges
	  concrete superstructure (1-hour problem)
	  other elements of bridges (e.g., culverts, abutments, and retaining walls) (1-hour problem)
	  steel superstructure (2-hour problem)

Saturday: lateral forces (wind/earthquake)

morning breadth module
  4 hours
  40 multiple-choice problems
	analysis of structures (37.5%)
	  lateral forces (10%)
	  lateral force distribution (22.5%)
	  methods (5%)
	design and detailing of structures (60%)
	  general structural considerations (7.5%)
	  structural systems integration (5%)
	  structural steel (10%)
	  light gage/cold-formed steel (2.5%)
	  concrete (12.5%)
	  wood (7.5%)
	  masonry (7.5%)
	  foundations and retaining structures (7.5%)
	construction administration (2.5%)
	  structural observation (2.5%)

afternoon depth module[a]
  4 hours
  essay problems
	buildings[c]
	  steel structure (1-hour problem)
	  concrete structure (1-hour problem)
	  wood and/or masonry structure (1-hour problem)
	  general analysis (e.g., existing structures, secondary structures, nonbuilding structures, and/or computer verification) (1-hour problem)
	bridges
	  columns (1-hour problem)
	  footings (1-hour problem)
	  general analysis (e.g., seismic and/or wind) (2-hour problem)

[a]Afternoon sessions focus on a single area of practice. You must choose *either* the buildings or bridges depth module, and you must work the same depth module across both exam components.
[b]At least one problem will contain a multistory building, and at least one problem will contain a foundation.
[c]At least two problems will include seismic content with a seismic design category of D or above. At least one problem will include wind content with a base wind speed of at least 110 mph. Problems may include a multistory building and/or a foundation.

or if you have had to split the difference because no available answer choice is close.

NCEES has described it like this.

> Many of the questions on NCEES exams require calculations to arrive at a numerical answer. Depending on the method of calculation used, it is very possible that examinees working correctly will arrive at a range of answers. The phrase "most nearly" is used to accommodate answers that have been derived correctly but that may be slightly different from the correct answer choice given on the exam. You should use good engineering judgment when selecting your choice of answer. For example, if the question asks you to determine the load on a beam, you should literally select the answer option that is most nearly what you calculated, regardless of whether it is more or less than your calculated value. However, if the question asks you to size a beam to carry a load, you should select an answer option that will safely carry the load. Typically, this requires selecting a value that is closest to but larger than the load.

The difference is significant. Suppose you were asked to calculate "most nearly" the volumetric pure water flow required to dilute a contaminated stream to an acceptable concentration. Suppose, also, that you calculated 823 gpm. If the answer choices were (A) 600 gpm, (B) 800 gpm, (C) 1000 gpm, and (D) 1200 gpm, you would go with answer choice (B), because it is most nearly what you calculated. If, however, you were asked to select a pump or pipe with the same rated capacities, you would have to go with choice (C). Got it?

HOW MUCH MATHEMATICS IS NEEDED FOR THE EXAM?

There are no pure mathematics questions (algebra, geometry, trigonometry, etc.) on the SE exam. However, you will need to apply your knowledge of these subjects to the exam questions.

Generally, only simple algebra, trigonometry, and geometry are needed on the SE exam. You will need to use trigonometric, logarithm, square root, exponentiation, and similar buttons on your calculator. There is no need to use any other method for these functions.

Except for simple quadratic equations, you will probably not need to find the roots of higher-order equations. Occasionally, it will be convenient to use the equation-solving capability of an advanced calculator. However, other solution methods will always exist. For second-order (quadratic) equations, it does not matter if you find roots by factoring, completing the square, using the quadratic equation, or using your calculator's root finder.

There is little or no use of calculus on the exam. Rarely, you may need to take a simple derivative to find a maximum or minimum of some function. Even rarer is the need to integrate to find an average, moment of inertia, statical moment, or shear flow.

Basic statistical analysis of observed data may be necessary. Statistical calculations are generally limited to finding means, medians, standard deviations, variances, percentiles, and confidence limits. Usually, the only population distribution you need to be familiar with is the normal curve. Probability, reliability, hypothesis testing, and statistical quality control are not explicit exam subjects, though their concepts may appear peripherally in some problems.

The SE exam is concerned with numerical answers, not with proofs or derivations. You will not be asked to prove or derive formulas.

Occasionally, a calculation may require an iterative solution method. Generally, there is no need to complete more than two iterations. You will not need to program your calculator to obtain an "exact" answer. Nor will you generally need to use complex numerical methods.

IS THE EXAM TRICKY?

Other than providing superfluous data, the SE exam is not a "tricky exam." The exam questions are difficult in their own right. NCEES does not provide misleading or conflicting statements to try to get you to fail. However, commonly made mistakes are represented in the available answer choices. Thus, the alternative answers (known as distractors) will be logical.

Questions are generally practical, dealing with common and plausible situations that you might experience in your job. You will not be asked to design a structure for reduced gravity on the moon, to design a mud-brick road, to analyze the effects of a nuclear bomb blast on a structure, or to use bamboo for tension reinforcement.

WHAT MAKES THE QUESTIONS DIFFICULT?

Some questions are difficult because the pertinent theory is not obvious. There may be only one acceptable procedure, and it may be heuristic (or defined by a code) such that nothing else will be acceptable.

Some questions are difficult because the data needed is hard to find. Some data just isn't available unless you happen to have brought the right reference book. Many of the structural questions are of this nature. There is no way to solve most structural steel questions without the *Steel Construction Manual.* Designing an eccentrically loaded concrete column without published interaction diagrams is nearly impossible in six minutes.

Some questions are difficult because they defy the imagination. Three-dimensional structural questions fit this description. If you cannot visualize the question, you probably cannot solve it.

Some questions are difficult because the computational burden is high, and they just take a long time.

Some questions are difficult because the terminology is obscure, and you may not know what the terms mean. This can happen in almost any subject.

WHAT REFERENCE MATERIAL IS PERMITTED IN THE EXAM?

The SE exam is an open-book exam. Check your state's exam requirements and restrictions, as some states restrict which books and materials can be used for the exam. (The PPI website has a listing of state boards at **ppi2pass.com/stateboards**.)

Personal notes in a three-ring binder and other semipermanent covers can usually be used. Some states use a "shake test" to eliminate loose papers from binders. Make sure that nothing escapes from your binders when they are inverted and shaken.

The references you bring into the exam room in the morning do not have to be the same as the references you use in the afternoon. However, you cannot share books with other examinees during the exam.

A few states do not permit collections of solved problems such as *Schaum's Outline Series*, sample exams, and solutions manuals. A few states maintain a formal list of banned books.

Strictly speaking, loose paper and scratch pads are not permitted in the exam. Certain types of preprinted graphs and logarithmically scaled graph papers (which are almost never needed) should be three-hole punched and brought in a three-ring binder. An exception to this restriction may be made for laminated and oversize charts, graphs, and tables that are commonly needed for particular types of questions. However, there probably aren't any such items for the SE exam.

MAY TABS BE PLACED ON PAGES?

It is common to tab pages in your books in an effort to reduce the time required to locate useful sections. Inasmuch as some states consider Post-it notes to be "loose paper," your tabs should be of the more permanent variety. Although you can purchase tabs with gummed attachment points, it is also possible to use transparent tape to attach the Post-its you have already placed in your books.

CAN YOU WRITE AND MARK IN YOUR BOOKS?

During your preparation, you may write anything you want, anywhere in your books, including this one. You can use pencil, pen, or highlighter in order to further your understanding of the content. However, during the exam, you must avoid the appearance of taking notes about the exam. This means that you should write only on the scratch paper that is provided. During the exam, other than drawing a line across a wide table of numbers, you should not write in your books.

WHAT ABOUT CALCULATORS?

The SE exam requires the use of a scientific calculator. It is a good idea to bring a spare calculator with you to the exam.

NCEES has banned communicating and text-editing calculators from the exam site. Only select types of calculators are permitted. Check the current list of permissible devices at the PPI website (**ppi2pass.com/calculators**). Contact your state board to determine if nomographs and specialty slide rules are permitted.

The exam has not been optimized for any particular brand or type of calculator. In fact, for most calculations, a \$15 scientific calculator will produce results as satisfactory as those from a \$200 calculator. There are definite benefits to having built-in statistical functions, graphing, unit-conversion, and equation-solving capabilities. However, these benefits are not so great as to give anyone an unfair advantage.

It is essential that a calculator used for the SE exam have the following functions.

- trigonometric and inverse trigonometric functions
- hyperbolic and inverse hyperbolic functions
- π
- $\sqrt{x}$ and x^2
- both common and natural logarithms
- y^x and e^x

You may not share calculators with other examinees.

Laptops, tablet computers, and electronic readers are not permitted in the exam. Their use has been considered, but no states actually permit them.

You may not use a walkie-talkie, cell phone, or other communications or text-messaging device during the exam.

Be sure to take your calculator with you whenever you leave the exam room for any length of time.

HOW ARE THE EXAM COMPONENTS GRADED AND SCORED?

For the morning multiple-choice problems, answers are recorded on an answer sheet that is machine graded. The minimum number of points for passing (referred to by NCEES as the "cut score") varies from administration to administration. The cut score is determined through a rational procedure, without the benefit of knowing examinees' performance on the exam. That is,

the exam is not graded on a curve. The cut score is selected based on what you are expected to know, not on allowing a certain percentage of engineers "through."

The grading of multiple-choice problems is straightforward, since a computer grades your score sheet. Either you get the problem right or you don't. There is no deduction for incorrect answers, so guessing is encouraged. However, if you mark two or more answers, no credit is given for the problem.

Solutions for the afternoon essay problems are evaluated for overall compliance with established scoring criteria and for general quality. The scores from each of the morning and afternoon modules are combined for a component's final score.

Exam results are given a pass/fail grade approximately 10–12 weeks after the exam date. You will receive the results of your exam from either your state board by mail or online through your MyNCEES account. You will receive a pass or fail notice only and will not receive a numerical score. Diagnostic reports that outline areas of strength and weakness are provided to those who do not pass.

HOW YOU SHOULD GUESS

NCEES produces defensible licensing exams. As a result, there is no pattern to the placement of correct responses. Therefore, it most likely will not help you to guess all "A," "B," "C," or "D."

The proper way to guess is as an engineer. You should use your knowledge of the subject to eliminate illogical answer choices. Illogical answer choices are those that violate good engineering principles, that are outside normal operating ranges, or that require extraordinary assumptions. Of course, this requires you to have some basic understanding of the subject in the first place. Otherwise, it's back to random guessing. That's the reason that the minimum passing score is higher than 25%.

You won't get any points using the "test-taking skills" that helped with tests prepared by amateurs. You won't be able to eliminate any [verb] answer choices from "Which [noun] . . ." questions. You won't find problems with options of the "more than 50" and "less than 50" variety. You won't find one answer choice among the four that has a different number of significant digits, or has a verb in a different tense, or has some singular/plural discrepancy with the stem. The distractors will always match the stem, and they will be logical.

CHEATING AND EXAM SUBVERSION

There aren't very many ways to cheat on an open-book exam. The proctors are well trained in spotting the few ways that do exist. It goes without saying that you should not talk to other examinees in the room, nor should you pass notes back and forth. You should not write anything into your books or take notes on the

contents of the exam. The number of people who are released to use the restroom may be limited to prevent discussions.

NCEES regularly reuses good problems that have appeared on previous exams. Therefore, exam integrity is a serious issue with NCEES, which goes to great lengths to make sure nobody copies the questions. You may not keep your exam booklet, enter text from questions into your calculator, or copy problems into your own material.

The proctors are concerned about exam subversion, which generally means activity that might invalidate the exam or the exam process. The most common form of exam subversion involves trying to copy exam problems for future use.

NCEES has become increasingly unforgiving about the loss of its intellectual property. NCEES routinely prosecutes violators and seeks financial redress for loss of its exam problems, as well as invalidating any engineering license you may have earned by taking one of its exams while engaging in prohibited activities. Your state board may impose additional restrictions on your right to retake any exam if you are convicted of such activities. In addition to tracking down the sources of any exam problem compilations that it becomes aware of, NCEES is also aggressive in pursuing and prosecuting examinees who disclose the contents of the exam in internet forum and "chat" environments. Your constitutional rights to free speech and expression will not protect you from civil prosecution for violating the nondisclosure agreement that NCEES requires you to sign before taking the exam. If you wish to participate in a dialogue about a particular exam subject, you must do so in such a manner that does not violate the essence of your nondisclosure agreement. This requires decoupling your discussion from the exam and reframing the question to avoid any exam particulars.

PART 3: HOW TO PREPARE FOR AND PASS THE SE EXAM

WHAT SHOULD YOU STUDY?

The exam covers many diverse subjects. Strictly speaking, you don't have to study every subject on the exam in order to pass. However, the more subjects you study, the more you'll improve your chances of passing. You should decide early in the preparation process which subjects you are going to study. The strategy you select will depend on your background. The four most common strategies are as follows.

- A broad approach is the key to success for examinees who have recently completed their academic studies. This strategy is to review the fundamentals of a broad range of undergraduate subjects (which means studying all or most of the chapters in this book). The exam

includes enough fundamental problems to make this strategy worthwhile. Overall, it's the best approach.

- Engineers who have little time for preparation tend to concentrate on the subject areas in which they hope to find the most problems. By studying the list of exam subjects, some have been able to focus on those subjects that will give them the highest probability of finding enough problems that they can answer. This strategy works as long as the exam has enough of the types of questions they need. Too often, though, examinees who pick and choose subjects to review can't find enough problems to complete the exam.

- Engineers who have been away from classroom work for a long time tend to concentrate on the subjects in which they have had extensive experience, in the hope that the exam will feature lots of problems in those subjects. This method is seldom successful.

- Some engineers plan on modeling their solutions from similar problems they have found in textbooks, collections of solutions, and old exams. These engineers often spend a lot of time compiling and indexing the example and sample problem types in all of their books. This is not a legitimate preparation method, and it is almost never successful.

HOW LONG SHOULD YOU STUDY?

We've all heard stories of the person who didn't crack a book until the week before the exam and still passed it with flying colors. Yes, these people really exist. However, I'm not one of them, and you probably aren't either. In fact, I'm convinced that these people are as rare as the ones who have taken the exam five times and still can't pass it.

A thorough review takes approximately 300 hours. Most of this time is spent solving problems. Some of it may be spent in class; some is spent at home. Some examinees spread this time over a year. Others cram it all into two months. Most classroom review courses last for three or four months. The best time to start studying will depend on how much time you can spend per week.

DO YOU NEED A REVIEW SCHEDULE?

It is important that you develop and adhere to a review outline and schedule. Once you have decided which subjects you are going to study, you can allocate the available time to those subjects in a manner that makes sense to you. If you are not taking a classroom review course (where the order of preparation is determined by the lectures), you should make an outline of subjects for self-study to use for scheduling your preparation. A fill-in-the-dates schedule is provided in Table 3 at the end of this introduction. If you purchased this book directly from PPI, you'll also have access to an interactive, adjustable, and personalized study schedule. Log on to your PPI account to access your custom study schedule.

A SIMPLE PLANNING SUGGESTION

Designate some location (a drawer, a corner, a cardboard box, or even a paper shopping bag left on the floor) as your "exam catch-all." Use your catch-all during the months before the exam when you have revelations about things you should bring with you. For example, you might realize that the plastic ruler marked off in tenths of an inch that is normally kept in the kitchen junk drawer can help you with some soil pressure questions. Or you might decide that a certain book is particularly valuable, or that it would be nice to have dental floss after lunch, or that large rubber bands and clips are useful for holding books open.

It isn't actually necessary to put these treasured items in the catch-all during your preparation. You can, of course, if it's convenient. But if these items will have other functions during the time before the exam, at least write yourself a note and put the note into the catch-all. When you go to pack your exam kit a few days before the exam, you can transfer some items immediately, and the notes will be your reminders for the other items that are back in the kitchen drawer.

HOW YOU CAN MAKE YOUR REVIEW REALISTIC

During the exam, you must be able to recall solution procedures, formulas, and important data quickly. You must remain sharp for eight hours or more. If you played a sport back in school, your coach tried to put you in game-related situations. Preparing for the SE exam isn't much different than preparing for a big game. Some part of your preparation should be realistic and representative of the exam environment.

There are several things you can do to make your review more representative. For example, if you gather most of your review resources (i.e., books) in advance and try to use them exclusively during your review, you will become more familiar with them. (Of course, you can also add to or change your references if you find inadequacies.)

Learning to use your time wisely is one of the most important lessons you can learn during your review. You will undoubtedly encounter questions that end up taking much longer than you expected. In some instances, you will cause your own delays by spending too much time looking through books for things you need (or just by looking for the books themselves!). Other times, the questions will entail too much work. It is important that a portion of your review involves solving problems so that you learn to recognize these situations and so that you can make intelligent decisions about skipping such questions during the exam. Two realistic review books that you can use during your exam preparation to practice solving problems include *Structural Engineering Solved Problems* (published by PPI), and *16-Hour Structural Engineering (SE) Practice Exam for Buildings* (published by PPI).

WHAT TO DO A FEW DAYS BEFORE THE EXAM

There are a few things you should do a week or so before the exam.

You should arrange for childcare and transportation. Since the exam does not always start or end at the designated time, make sure that your childcare and transportation arrangements are flexible.

Check PPI's website for last-minute updates and errata to any PPI books you might have and are bringing to the exam.

Obtain a separate copy of this book's index. You can photocopy the actual index; alternatively, the index is available as a download on PPI's website, **ppi2pass .com/strmindex**.

If it's convenient, visit the exam location in order to find the building, parking areas, exam room, and restrooms. If it's not convenient, you can find driving directions and/or site maps online.

Take the battery cover off your calculator and check to make sure you are bringing the correct size replacement batteries. Some calculators require a different kind of battery for their "permanent" memories. Put the cover back on and secure it with a piece of masking tape. Write your name on the tape to identify your calculator.

If your spare calculator is not the same as your primary calculator, spend a few minutes familiarizing yourself with how it works. In particular, you should verify that your spare calculator is functional.

PREPARE YOUR CAR

[ ] Gather snow chains, shovel, and tarp to kneel on while installing chains.
[ ] Check tire pressures.
[ ] Check your car's spare tire.
[ ] Check for tire installation tools.
[ ] Verify that you have the vehicle manual.
[ ] Check fluid levels (oil, gas, water, brake fluid, transmission fluid, window-washing solution).
[ ] Fill up car with gas.
[ ] Check battery and charge if necessary.
[ ] Know something about your fuse system (where they are, how to replace them, etc.).
[ ] Assemble all required maps.
[ ] Fix anything that might slow you down (missing wiper blades, etc.).
[ ] Check your car's taillights.
[ ] Affix the current DMV registration sticker.
[ ] Fix anything that might get you pulled over on the way to the exam (burned-out taillight or headlight, broken lenses, bald tires, missing license plate, noisy muffler, etc.).
[ ] Treat the inside windows with anti-fog solution.

[ ] Put a roll of paper towels in the back seat.
[ ] Gather exact change for any bridge tolls or toll roads.
[ ] Put $20 in your car's glove box.
[ ] Check for current registration and proof of insurance.
[ ] Locate a spare door and ignition key.
[ ] Find your roadside-assistance cards and phone numbers.
[ ] Plan alternate routes.

PREPARE YOUR EXAM KITS

Second in importance to your scholastic preparation is the preparation of your two exam kits. The first kit consists of a bag, box (plastic milk crates hold up better than cardboard in the rain), or wheeled travel suitcase containing items to be brought with you into the exam room.

[ ] your exam authorization notice
[ ] government-issued photo identification (e.g., driver's license)
[ ] this book
[ ] other textbooks and reference books
[ ] regular dictionary
[ ] scientific/engineering dictionary
[ ] review course notes in a three-ring binder
[ ] cardboard boxes or plastic milk crates to use as bookcases
[ ] primary calculator
[ ] spare calculator
[ ] instruction booklets for your calculators
[ ] extra calculator batteries
[ ] straightedge and rulers
[ ] compass
[ ] protractor
[ ] scissors
[ ] stapler
[ ] transparent tape
[ ] magnifying glass
[ ] small (jeweler's) screwdriver for fixing your glasses or for removing batteries from your calculator
[ ] unobtrusive (quiet) snacks or candies, already unwrapped
[ ] two small plastic bottles of water
[ ] travel pack of tissue (keep in your pocket)
[ ] handkerchief
[ ] headache remedy
[ ] personal medication
[ ] $5.00 in miscellaneous change
[ ] back-up reading glasses
[ ] light, comfortable sweater
[ ] loose shoes or slippers
[ ] cushion for your chair
[ ] earplugs

[ ] wristwatch
[ ] several large trash bags ("raincoats" for your boxes of books)
[ ] roll of paper towels
[ ] wire coat hanger (to hang up your jacket)
[ ] extra set of car keys

The second kit consists of the following items and should be left in a separate bag or box in your car in case they are needed.

[ ] copy of your application
[ ] proof of delivery
[ ] light lunch
[ ] beverage in thermos or cans
[ ] sunglasses
[ ] extra pair of prescription glasses
[ ] raincoat, boots, gloves, hat, and umbrella
[ ] street map of the exam area
[ ] parking permit
[ ] battery-powered desk lamp
[ ] your cell phone
[ ] piece of rope

PREPARE FOR THE WORST

All of the occurrences listed in this section have happened to examinees. Granted, you cannot prepare for every eventuality. But even though each occurrence is a low-probability event, taken together these occurrences are worth considering in advance.

- Imagine getting a flat tire, getting stuck in traffic, or running out of gas on the way to the exam.

- Imagine rain and snow as you are carrying your cardboard boxes of books into the exam room. Would plastic trash bags be helpful?

- Imagine arriving late. Can you get into the exam without having to make two trips from your car?

- Imagine having to park two blocks from the exam site. How are you going to get everything to the exam room? Can you actually carry everything that far? Could you use a furniture dolly, a supermarket basket, or perhaps a helpmate?

- Imagine a Star Trek convention, a square-dancing contest, construction, or an auction taking place in the next room.

- Imagine a site without any heat, with poor lighting, or with sunlight streaming directly into your eyes.

- Imagine a hard folding chair and a table with one short leg.

- Imagine a site next to an airport with frequent take-offs, or next to a construction site with a pile driver, or next to the NHRA State Championship.

- Imagine a seat where someone nearby chews gum with an open mouth; taps his pencil or drums her fingers; or wheezes, coughs, and sneezes for eight hours.

- Imagine the distraction of someone crying or of proctors evicting yelling and screaming examinees who have been found cheating. Imagine the tragedy of another examinee's serious medical emergency.

- Imagine a delay of an hour while they find someone to unlock the building, turn on the heat, or wait for the head proctor to bring instructions.

- Imagine a power outage occurring sometime during the exam.

- Imagine a proctor who (a) tells you that one of your favorite books can't be used during the exam, (b) accuses you of cheating, or (c) calls "time's up" without giving you any warning.

- Imagine not being able to get your lunch out of your car or find a restaurant.

- Imagine getting sick or nervous during the exam.

- Imagine someone stealing your calculator during lunch.

WHAT TO DO THE DAY BEFORE THE EXAM

Take the day before the exam off from work to relax. Do not cram. A good night's sleep is the best way to start the exam. If you live a considerable distance from the exam site, consider getting a hotel room in which to spend the night.

Practice setting up your exam work environment. Carry your boxes to the kitchen table. Arrange your "bookcases" and supplies. Decide what stays on the floor in boxes and what gets an "honored position" on the tabletop.

Use your checklist to make sure you have everything. Make sure your exam kits are packed and ready to go. Wrap your boxes in plastic bags in case it's raining when you carry them from the car to the exam room.

Calculate your wake-up time and set the alarms on two bedroom clocks. Select and lay out your clothing items. (Dress in layers.) Select and lay out your breakfast items.

If it's going to be hot on exam day, put your (plastic) bottles of water in the freezer.

Make sure you have gas in your car and money in your wallet.

WHAT TO DO THE DAY OF THE EXAM

Turn off the quarterly and hourly alerts on your wristwatch. Leave your cell phone in your car. If you must bring it, set it on silent mode. Bring a morning newspaper.

You should arrive at least 30 minutes before the exam starts. This will allow time for finding a convenient parking place, bringing your materials to the exam room, adjusting to room and seating changes, and calming down. Be prepared, though, to find that the exam room is not open or ready at the designated time.

Once you have arranged the materials around you on your table, take out your morning newspaper and look cool. (Only nervous people work crossword puzzles.)

WHAT TO DO DURING THE EXAM

All of the procedures typically associated with timed, proctored, computer-graded assessment tests will be in effect when you take the SE exam.

The proctors will distribute the exam booklets and answer sheets if they are not already on your tables. You should not open the booklets until instructed to do so. You may read the information on the front and back covers, and you should write your name in the appropriate blank spaces.

Listen carefully to everything the proctors say. Do not ask your proctors any engineering questions. Even if they are knowledgeable in engineering, they will not be permitted to answer your questions.

Answers to questions are recorded on an answer sheet contained in the test booklet. The proctors will guide you through the process of putting your name and other biographical information on this sheet when the time comes, which will take approximately 15 minutes. You will be given the full four hours to answer questions. Time to initialize the answer sheet is not part of your four hours.

The common suggestions to "completely fill the bubbles, and erase completely" apply here. NCEES provides each examinee with a mechanical pencil with HB lead. Use of ballpoint pens and felt-tip markers is prohibited.

If you finish the exam early and there are still more than 30 minutes remaining, you will be permitted to leave the room. If you finish less than 30 minutes before the end of the exam, you may be required to remain until the end. This is done to be considerate of the people who are still working.

When you leave, you must return your exam booklet. You may not keep the exam booklet for later review.

If there are any questions that you think were flawed, in error, or unsolvable, ask a proctor for a "reporting form" on which you can submit your comments. Follow your proctor's advice in preparing this document.

HOW TO SOLVE MULTIPLE-CHOICE QUESTIONS

When you begin each session of the exam, observe the following suggestions.

- Use only the pencil provided.

- Do not spend an inordinate amount of time on any single question. If you have not answered a question in a reasonable amount of time, make a note of it and move on.

- Set your wristwatch alarm for five minutes before the end of each four-hour session, and use that remaining time to guess at all of the remaining questions. Odds are that you will be successful with about 25% of your guesses, and these points will more than make up for the few points that you might earn by working during the last five minutes.

- Make mental notes about any questions for which you cannot find a correct response, that appear to have two correct responses, or that you believe have some technical flaw. Errors in the exam are rare, but they do occur. Such errors are usually discovered during the scoring process and discounted from the exam, so it is not necessary to tell your proctor, but be sure to mark the one best answer before moving on.

- Make sure all of your responses on the answer sheet are dark. Completely fill the bubbles.

SOLVE QUESTIONS CAREFULLY

Many points are lost to carelessness. Keep the following items in mind when you are solving the end-of-chapter questions. Hopefully, these suggestions will be automatic during the exam.

[ ] Did you recheck your mathematical equations?
[ ] Do the units cancel out in your calculations?
[ ] Did you convert between radius and diameter?
[ ] Did you convert between feet and inches?
[ ] Did you convert from gage to absolute pressures?
[ ] Did you convert between kPa and Pa?
[ ] Did you recheck all data obtained from other sources, tables, and figures?

SHOULD YOU TALK TO OTHER EXAMINEES AFTER THE EXAM?

The jury is out on this question. People react quite differently to the exam experience. Some people are energized. Most are exhausted. Some people need to unwind by talking with other examinees, describing every detail of their experience, and dissecting every exam question. Other people need lots of quiet space. Most engineers are in this latter category.

Since everyone who took the exam has seen it, you will not be violating your "oath of silence" if you talk about the details with other examinees immediately after the exam. It's difficult not to ask how someone else approached a question that had you completely stumped. However, keep in mind that it is very

disquieting to think you answered a question correctly, only to have someone tell you where you went wrong.

To ensure you do not violate the nondisclosure agreement you signed before taking the exam, make sure you do not discuss any exam particulars with people who have not also taken the exam.

AFTER THE EXAM

Yes, there is something to do after the exam. Most people go home, throw their exam "kits" into the corner, and collapse. A week later, when they can bear to think about the experience again, they start integrating their exam kits back into their normal lives. The calculators go back into the drawer, the books go back on the shelves, the $5.00 in change goes back into the piggy bank, and all of the miscellaneous stuff brought to the exam is put back wherever it came.

Here's what I suggest you do as soon as you get home.

[ ] Thank your spouse and children for helping you during your preparation.
[ ] Take any paperwork you received on exam day out of your pocket, purse, or wallet. Put this inside your *Structural Engineering Reference Manual.*
[ ] Reflect on any statements regarding exam secrecy to which you signed your agreement.
[ ] Call your employer and tell him/her that you need to take a mental health day on Monday.

A few days later, when you can face the world again, do the following.

[ ] Make notes about anything you would do differently if you had to take the exam over again.
[ ] Consolidate all of your application paperwork, correspondence to/from your state, and any paperwork that you received on exam day.
[ ] If you took a live review course, call or email the instructor (or write a note) to say, "Thanks."
[ ] Return any books you borrowed.
[ ] Write thank-you notes to all of the people who wrote letters of recommendation or reference for you.
[ ] Find and read the chapter in this book that covers ethics. There were no ethics questions on your SE exam, but it doesn't make any difference. Ethical behavior is expected of an SE in any case. Spend a few minutes reflecting on how your performance (obligations, attitude, presentation, behavior, appearance, etc.) might be about to change once you are licensed. Consider how you are going to be a role model for others around you.
[ ] Put all of your review books, binders, and notes someplace where they will be out of sight.

FINALLY

By the time you've "undone" all of your preparations, you might have thought of a few things that could help future examinees. If you have any sage comments about how to prepare, any suggestions about what to do during or bring to the exam, any comments on how to improve this book, or any funny anecdotes about your experience, I hope you will share these with me.

AND THEN, THERE'S THE WAIT . . .

Waiting for the exam results is its own form of mental torture.

Although the actual machine grading "only takes seconds," consider the following facts: (a) NCEES prepares multiple exams for each administration, in case one becomes unusable (i.e., is inappropriately released) before the exam date. (b) Since the actual version of the exam used is not known until after it is finally given, the cut score determination occurs after the exam date.

I wouldn't be surprised to hear that NCEES receives dozens, if not hundreds, of claims from well-meaning examinees who were 100% certain that the exams they took were fatally flawed to some degree—that there wasn't a correct answer for such-and-such question— that there were two answers for such-and-such question—or even, perhaps, that such-and-such question was missing from their exam booklet altogether. Each of these claims must be considered as a potential adjustment to the cut score.

Then the exams must actually be graded. Since grading nearly 50,000 exams (counting all the FE and PE exams) requires specialized equipment, software, and training not normally possessed by the average employee, as well as time to do the work (also not normally possessed by the average employee), grading is invariably outsourced.

Outsourced grading cannot begin until all of the states have returned their score sheets to NCEES and NCEES has sorted, separated, organized, and consolidated the score sheets into whatever sequence is best. During grading, some of the score sheets "pop out" with any number of abnormalities that demand manual scoring.

After the individual exams are scored, the results are analyzed in a variety of ways. Some of the analysis looks at passing rates by such delineators as degree, major, university, site, and state. Part of the analysis looks for similarities between physically adjacent examinees (to look for cheating). Part of the analysis looks for exam sites that have statistically abnormal group performance. And some of the analysis looks for exam questions that have a disproportionate fraction of successful or unsuccessful examinees. Anyway, you get the idea: Grading is not merely putting your exam sheet in an electronic reader. All of these steps have to be completed for 100% of the examinees before any results can go out.

Once NCEES has graded your test and notified your state, when you hear about it depends on when the work is done by your state. Some states have to approve the results at a board meeting; others prepare the certificates before sending out notifications. Some states are more computerized than others. Some states have 50 examinees, while others have 10,000. Some states are shut down by blizzards and hurricanes; others are administratively challenged—understaffed, inadequately trained, or over budget.

There is no pattern to the public release of results. None. The exam results are not released to all states simultaneously. (The states with the fewest examinees often receive their results soonest.) They are not released alphabetically by state or examinee name. The people who failed are not notified first (or last). Your coworker might receive his or her notification today, and you might be waiting another three weeks for yours.

Some states post the names of the successful examinees, unsuccessful examinees, or both on their official state websites before the results go out. Others update their websites after the results go out. Some states don't list much of anything on their websites.

Remember, too, that the size or thickness of the envelope you receive from your state does not mean anything. Some states send a big congratulations package and certificate. Others send a big package with a new application to repeat the exam. Some states send a postcard. Some send a one-page letter. Some states send you an invoice for your license fees. (Ahh, what a welcome bill!) You just have to open it to find out.

AND WHEN YOU PASS ...

- [ ] Celebrate.
- [ ] Notify the people who wrote letters of recommendation or reference for you.
- [ ] Ask your employer for a raise.
- [ ] Tell the folks at PPI (who have been rootin' for you all along) the good news.

Table 3 *Schedule for Self-Study*

chapter number	subject	date to start	date to finish
1	Reinforced Concrete Design		
2	Foundations and Retaining Structures		
3	Prestressed Concrete Design		
4	Structural Steel Design		
5	Timber Design		
6	Reinforced Masonry Design		
7	Lateral Forces		
8	Bridge Design		

Codes and References

EXAM-ADOPTED DESIGN STANDARDS

AASHTO: *AASHTO LRFD Bridge Design Specifications*, Sixth ed., 2012. American Association of State Highway and Transportation Officials, Washington, DC.

ACI: *Building Code Requirements for Structural Concrete*, 2011. American Concrete Institute, Farmington Hills, MI.

AISC: *Seismic Design Manual*, Second ed., 2012. American Institute of Steel Construction, Inc., Chicago, IL.

AISC: *Steel Construction Manual*, Fourteenth ed., 2011. American Institute of Steel Construction, Inc., Chicago, IL.

AISI: *North American Specification for the Design of Cold-Formed Steel Structural Members*, 2007 ed., with Supplement no. 2 (2010). American Iron and Steel Institute, Washington, DC.

ASCE/SEI7: *Minimum Design Loads for Buildings and Other Structures* (through Supplement no. 2), 2010. American Society of Civil Engineers, Reston, VA.

IBC: *International Building Code* (without supplements), 2012 ed. International Code Council, Falls Church, VA.

NDS: *National Design Specification for Wood Construction ASD/LRFD*, 2012 ed. and *National Design Specification Supplement: Design Values for Wood Construction*, 2012 ed. American Forest & Paper Association, Washington, DC.

PCI: *PCI Design Handbook: Precast and Prestressed Concrete*, Seventh ed., 2010. Precast/Prestressed Concrete Institute, Chicago, IL.

SDPWS: *Special Design Provisions for Wind and Seismic with Commentary*, 2008 ed. American Forest & Paper Association, Washington, DC.

MSJC: *Building Code Requirements and Specification for Masonry Structures* (and companion commentaries), 2011. The Masonry Society, Boulder, CO; American Concrete Institute, Detroit, MI; and Structural Engineering Institute of the American Society of Civil Engineers, Reston, VA.

ADDITIONAL RECOMMENDED REFERENCES

American Institute of Steel Construction. *AISC Basic Design Values Cards*. Chicago, IL: American Institute of Steel Construction. (This resource is available online as a downloadable PDF.)

Building Seismic Safety Council of the National Institute of Building Sciences. *NEHRP Recommended Seismic Provisions for New Buildings and Other Structures*. Washington, DC: Building Seismic Safety Council; the Earthquake Hazards Reduction Program; the Federal Emergency Management Agency; and the U.S. Department of Homeland Security.

Simpson Strong-Tie Company. *Wood Construction Connectors Catalog*. Pleasanton, CA: Simpson Strong-Tie Company. (This resource is available online as a downloadable PDF.)

United States Department of the Army. *Seismic Design for Buildings*. Washington, DC: United States Army COE.

1 Reinforced Concrete Design

1. GENERAL REQUIREMENTS

The *International Building Code*[1] (IBC) adopts by reference the American Concrete Institute's[2] *Building Code Requirements for Structural Concrete and Commentary* (ACI). Some sections of the ACI code are modified by the IBC, and these exceptions are given in IBC Sec. 1905.

The *alternate design method*, also known as the *working stress design method*, was the original method of designing concrete structures. Based on elastic theory, the alternate design method calculates the stresses produced in the member. Service loads are applied to the member, and the calculated service load stresses produced in the member must not exceed the specified allowable stresses. The method provides an unsatisfactory indication of conditions in the structure as loading increases and failure approaches. When loading increases and failure approaches, the assumption of a linear relationship between stress and strain is no longer valid. In general, this method is less satisfactory than the strength design method because it does not provide a uniform factor of safety against failure in different types of structures. Though ACI Sec. R1.1 does permit the use of the alternate design method, the strength design method is the code's preferred design method and it is the only design method presented in this chapter.

In the *strength design method*, factored loads are applied to the member to determine the required ultimate strength. This required strength must not exceed the design strength, which is calculated as the member's nominal strength multiplied by a resistance factor, ϕ.

2. STRENGTH DESIGN PRINCIPLES

Nomenclature

D	dead load	kips or lbf
E	earthquake load	kips or lbf
F	load due to weight and pressure of fluids	kips or lbf
H	load due to pressure of soil	kips or lbf
L	live load	kips or lbf
L_r	roof live load	kips or lbf
Q	service level force	kips or lbf
R	load due to rainwater	kips or lbf
S	snow load	kips or lbf
T	effect of temperature, shrinkage, creep, differential settlement, and shrinkage-compensating concrete	kips or lbf
U	required strength to resist factored load	kips or lbf
w	distributed load	kips/ft
W	wind load	kips or lbf

Symbols

γ	load factor	–
ϕ	strength-reduction factor	–

Required Strength

The required ultimate strength of a member consists of the most critical combination of factored loads applied to the member. Factored loads consist of working (i.e., service) loads, Q, multiplied by the appropriate load factor, γ. In accordance with ACI Sec. 9.2.1, the required strength, U, is defined by seven combinations as follows.

$$U = 1.4D \qquad \text{[ACI 9-1]}$$

$$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \qquad \text{[ACI 9-2]}$$

$$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) \qquad \text{[ACI 9-3]}$$
$$+ (0.5L \text{ or } 0.5W)$$

$$U = 1.2D + 1.0W + 0.5L + 0.5(L_r \text{ or } S \text{ or } R) \qquad \text{[ACI 9-4]}$$

$$U = 1.2D + 1.0E + 0.5L + 0.2S \qquad \text{[ACI 9-5]}$$

$$U = 0.9D + 1.0W \qquad \text{[ACI 9-6]}$$

$$U = 0.9D + 1.0E \qquad \text{[ACI 9-7]}$$

Replace $0.5L$ with $1.0L$ in ACI Eq. 9-3, Eq. 9-4, and Eq. 9-5 for garages, places of public assembly, and all

areas where L is greater than 100 lbf/ft^2. Replace $0.2S$ with $0.7S$ for roof configurations that do not shed snow.

The cumulative structural effects of temperature, creep, shrinkage, differential settlement, and shrinkage-compensating concrete, T, must be considered where appropriate. The load factor on T should be established by considering its likely magnitude and the probability that the maximum effect of T will occur simultaneously with other loads.

When a fluid load, F, is present, it must be included with the same load factor as the dead load, D, in ACI Eq. 9-1 through Eq. 9-5 and Eq. 9-7. When a lateral soil pressure load, H, is present, it must be included in the load combinations with load factors that depend on how H interacts with other loads. When H acts alone or adds to the effect of other loads, it has a load factor of 1.6. When the effect of H is permanent and counteracts the effects of other loads, it has a load factor of 0.9. When the effect of H is not permanent but counteracts the effects of other loads when present, it should not be included.

Example 1.1

The illustration shows a typical frame of a six-story office building. The loading on the frame is as follows.

roof dead load, including
  cladding and columns, $w_{Dr} = 1.2$ kips/ft
roof live load, w_{Lr} $= 0.4$ kip/ft
floor dead load, including
  cladding and columns, $w_D = 1.6$ kips/ft
floor live load, w_L $= 1.25$ kips/ft
horizontal wind pressure, $p_h = 1.0$ kips/ft
vertical wind pressure, p_v $= 0.5$ kip/ft

Determine the maximum and minimum required loads on the first-floor columns.

Illustration for Ex. 1.1
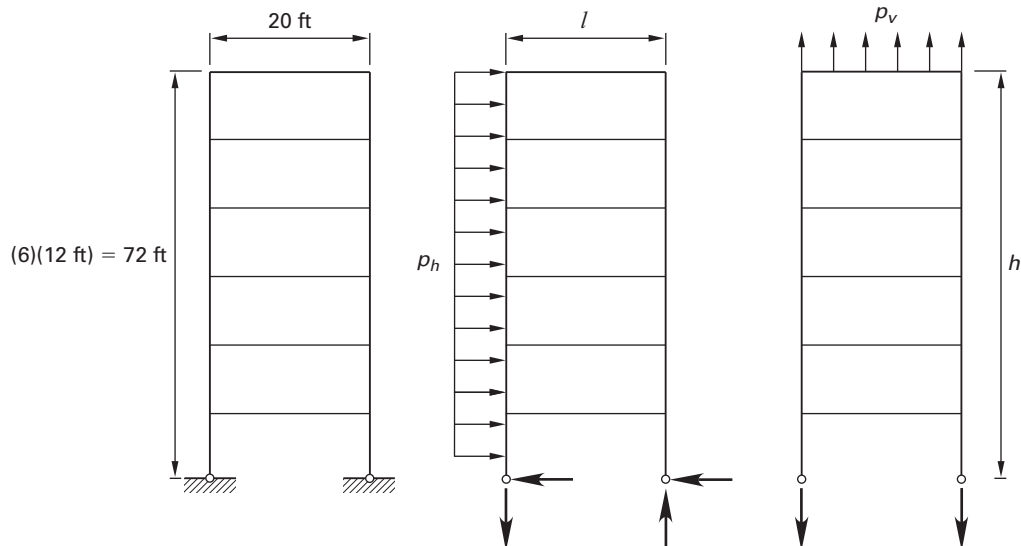
Solution

The axial load on one column due to the dead load is

$$D = \frac{l(w_{Dr} + 5w_D)}{2}$$

$$= \frac{(20 \text{ ft})\left(1.2 \dfrac{\text{kips}}{\text{ft}} + (5 \text{ stories})\left(1.6 \dfrac{\text{kips}}{\text{ft}}\right)\right)}{2}$$

$$= 92 \text{ kips}$$

The axial load on one column due to the floor live load is

$$L = \frac{l(5w_L)}{2}$$

$$= \frac{(20 \text{ ft})\left((5 \text{ stories})\left(1.25 \dfrac{\text{kips}}{\text{ft}}\right)\right)}{2}$$

$$= 62.5 \text{ kips}$$

The axial load on one column due to the roof live load is

$$L_r = \frac{lw_{Lr}}{2}$$

$$= \frac{(20 \text{ ft})\left(0.4 \dfrac{\text{kip}}{\text{ft}}\right)}{2}$$

$$= 4 \text{ kips}$$

The axial load on one column due to horizontal wind pressure is obtained by taking moments about the base of the other column and is given by

$$
\begin{aligned}
W_h &= \pm\frac{p_h h^2}{2l} = \pm\frac{\left(1\ \dfrac{\text{kip}}{\text{ft}}\right)(72\ \text{ft})^2}{(2)(20\ \text{ft})} \\
&= \pm 129.6\ \text{kips}
\end{aligned}
$$

The axial load on one column due to the vertical wind pressure is obtained by resolving forces at the column bases and is given by

$$
\begin{aligned}
W_v &= -\frac{p_v l}{2} = -\frac{\left(0.5\ \dfrac{\text{kip}}{\text{ft}}\right)(20\ \text{ft})}{2} \\
&= -5\ \text{kips}
\end{aligned}
$$

The maximum strength level required load on a column is

$$
\begin{aligned}
U &= 1.2D + 1.6L + 0.5L_r && [\textit{ACI 9-2}] \\
&= (1.2)(92\ \text{kips}) + (1.6)(62.5\ \text{kips}) \\
&\quad + (0.5)(4\ \text{kips}) \\
&= 212\ \text{kips} \quad [\text{compression}] \\
U &= 1.2D + 1.6L_r + 0.5L && [\textit{ACI 9-3}] \\
&= (1.2)(92\ \text{kips}) + (1.6)(4\ \text{kips}) \\
&\quad + (0.5)(62.5\ \text{kips}) \\
&= 148\ \text{kips} \quad [\text{compression}] \\
U &= 1.2D + 1.6L_r + 0.5W && [\textit{ACI 9-3}] \\
&= (1.2)(92\ \text{kips}) + (1.6)(4\ \text{kips}) \\
&\quad + (0.5)(129.6\ \text{kips} - 5\ \text{kips}) \\
&= 179\ \text{kips} \quad [\text{compression}] \\
U &= 1.2D + 1.0W + 0.5L + 0.5L_r && [\textit{ACI 9-4}] \\
&= (1.2)(92\ \text{kips}) \\
&\quad + (1.0)(129.6\ \text{kips} - 5\ \text{kips}) \\
&\quad + (0.5)(62.5\ \text{kips}) + (0.5)(4\ \text{kips}) \\
&= 268\ \text{kips} \quad [\text{compression; governs}]
\end{aligned}
$$

The minimum strength level design load on a column is

$$
\begin{aligned}
U &= 0.9D + 1.0W_h + 1.0W_v && [\textit{ACI 9-6}] \\
&= (0.9)(92\ \text{kips}) + (1.0)(-129.6\ \text{kips}) \\
&\quad + (1.0)(-5\ \text{kips}) \\
&= -52\ \text{kips} \quad [\text{tension}]
\end{aligned}
$$

Design Strength

The design strength of a member consists of the nominal, or theoretical ultimate, strength of the member multiplied by the appropriate strength reduction factor, ϕ. The reduction factor is defined in ACI Sec. 9.3 as

$\phi = 0.90$ [for flexure of tension-controlled sections]

$\phi = 0.75$ [for shear and torsion]

$\phi = 0.75$ [for columns with spiral reinforcement]

$\phi = 0.65$ [for columns with lateral ties]

$\phi = 0.65$ [for bearing on concrete surfaces]

$\phi = 0.75$ [for strut-and-tie models]

3. STRENGTH DESIGN OF REINFORCED CONCRETE BEAMS

Nomenclature

a	depth of equivalent rectangular stress block	in
$A_{\max}$	maximum area of tension reinforcement, $\rho_t bd$	in²
A_s	area of tension reinforcement	in²
A_s'	area of compression reinforcement	in²
A_{sf}	reinforcement area to develop the outstanding flanges	in²
A_{sw}	reinforcement area to balance the residual moment	in²
A_t	additional tension reinforcement	in²
b	width of compression face of member	in
b_w	web width	in
c_f	compressive force in flanges	kips
c	distance from extreme compression fiber to neutral axis	in
C_u	compressive force in the concrete	kips
d	distance from extreme compression fiber to centroid of tension reinforcement	in
d'	distance from extreme compression fiber to centroid of compression reinforcement	in
f_c'	compressive strength of concrete	lbf/in²
f_s'	stress in compression reinforcement	lbf/in²
f_y	yield strength of reinforcement	lbf/in²
h_f	flange depth	in
K_u	design moment factor, M_u/bd^2	lbf/in²
M_f	design moment strength of the outstanding flanges	in-lbf or ft-kips
$M_{\max}$	maximum design flexural strength for a tension-controlled section	in-lbf or ft-kips
M_n	nominal flexural strength of a member	in-lbf or ft-kips
M_r	residual moment ($M_u - M_{\max}$)	in-lbf or ft-kips
M_u	factored moment on the member	in-lbf or ft-kips
T_u	tensile force in the reinforcement	kips

Symbols

β_1	compression zone factor	–
ϵ_c	strain at external compression fiber	–
$\epsilon_{c(\max)}$	maximum strain at external compression fiber, 0.003	–
ϵ_t	strain in tension reinforcement	–
ϵ'_s	strain in compression reinforcement	–
ρ	ratio of tension reinforcement, A_s/bd	–
ρ_b	reinforcement ratio producing balanced strain conditions	–
$\rho_{\max}$	maximum tension reinforcement ratio in a rectangular beam with tension reinforcement only	–
$\rho_{\min}$	minimum allowable reinforcement ratio	–
ρ_t	reinforcement ratio producing a tension-controlled section	–
ρ_w	residual reinforcement ratio	–
ω	tension reinforcement index, $\rho f_y/f'_c$	–
$\omega_{\max}$	maximum tension reinforcement index in a rectangular beam with tension reinforcement only	–

Beams with Tension Reinforcement Only

In accordance with ACI Sec. 10.2, a rectangular stress block is assumed in the concrete, as shown in Fig. 1.1, and it is also assumed that the tension reinforcement has yielded.[2,3] The nominal flexural strength of a rectangular beam is derived[3] as

$$M_n = A_s f_y d\left(1 - \frac{0.59\rho f_y}{f'_c}\right)$$

Equating the tensile and compressive forces acting on the section gives the depth of the equivalent rectangular stress block as

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$M_n = A_s f_y\left(d - \frac{a}{2}\right)$$

The maximum permissible factored moment on the member, or required moment strength, must not exceed ϕM_n. For a tension-controlled section, where $\phi = 0.9$,

$$M_u = 0.9 M_n$$

Figure 1.1 *Rectangular Stress Block[3]*

The required reinforcement ratio for a given factored moment is then

$$\rho = \frac{(0.85 f'_c)\left(1 - \sqrt{1 - \dfrac{K_u}{0.383 f'_c}}\right)}{f_y}$$

$$K_u = \frac{M_u}{bd^2}$$

These expressions may be readily programmed using a handheld calculator.[4] Alternatively, design tables may be used,[5,6,7] and rearranging the expression[4] in terms of the tension reinforcement index, ω, gives

$$\frac{M_u}{f'_c bd^2} = \omega(0.9 - 0.5294\omega)$$

$$\omega = \frac{\rho f_y}{f'_c}$$

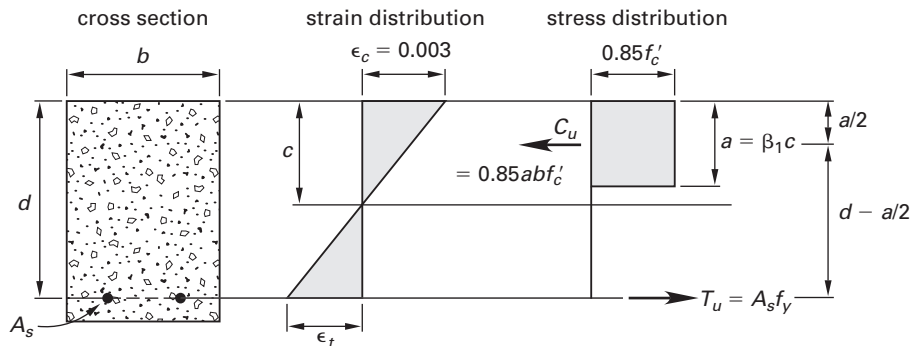Appendix A provides a design aid that tabulates the tension reinforcement index, ω, against $M_u/f'_c bd^2$.

Tension-Controlled and Compression-Controlled Sections

As specified in ACI Sec. 10.2.3 and shown in Fig. 1.1, the nominal flexural strength of a member is reached when the strain in the extreme compression fiber reaches a value of 0.003. Depending on the strain in the tension steel, the section is classified as either tension-controlled or compression-controlled, and the strength-reduction factor varies from a value of 0.90 to 0.65.

ACI Sec. 10.3.4 defines a tension-controlled section as one in which the strain in the extreme tension steel $\epsilon_t \geq 0.005$ when the concrete reaches its ultimate strain of $\epsilon_c = 0.003$. From Fig. 1.1, for a value of $\epsilon_t = 0.005$, the neutral axis depth ratio is given by

$$\frac{c}{d} = 0.375$$

The following relationships are obtained from Fig. 1.1.

$$\phi = 0.90$$
$$\epsilon_t = 0.005$$
$$a = 0.375\beta_1 d$$
$$C_u = 0.319\beta_1 f'_c bd$$
$$\rho_t = 0.319\beta_1 \frac{f'_c}{f_y}$$
$$K_u = \omega f'_c (0.9 - 0.529\omega)$$
$$\omega = 0.319\beta_1$$
$$\beta_1 = 0.85 \quad [f'_c \leq 4000 \ \text{lbf/in}^2]$$
$$= 0.85 - \frac{f'_c - 4000 \ \frac{\text{lbf}}{\text{in}^2}}{20,000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$[4000 \ \text{lbf/in}^2 < f'_c \leq 8000 \ \text{lbf/in}^2]$$
$$= 0.65 \ \text{minimum} \quad [f'_c > 8000 \ \text{lbf/in}^2]$$

The strength reduction factor for a tension-controlled section is given by ACI Sec. 9.3.2.1 as

$$\phi = 0.90$$

In a tension-controlled section at failure, the strength of the reinforcement is fully used and wide cracks and large deflections are produced, giving adequate warning of impending failure.

Table 1.1 lists various factors and their values for typical concrete strengths in tension-controlled beams where the tension strain, ϵ_t, is 0.005 and the yield strength, f_y, is 60,000 lbf/in^2.

Table 1.1 Typical Values for Singly Reinforced Concrete Beams with $\epsilon_t = 0.005$, $f_y = 60,000$ lbf/in^2, and $\phi = 0.90$

f'_c (lbf/in^2)	β_1	ρ_t	$\rho_{\min}$	K_u (lbf/in^2)	ω
3000	0.85	0.0136	0.0033	615	0.271
3500	0.85	0.0158	0.0033	718	0.271
4000	0.85	0.0181	0.0033	820	0.271
4500	0.83	0.0199	0.0034	906	0.265
5000	0.80	0.0213	0.0035	975	0.255

ACI Sec. 10.3.3 defines a compression-controlled section as that in which the strain in the extreme tension steel, $\epsilon_t \leq f_y/E_s$ when the concrete reaches its ultimate strain of $\epsilon_c = 0.003$. For grade 60 reinforcement bars, ACI Sec. 10.3.3 assumes a strain limit of

$$\epsilon_t = 0.002$$

The neutral axis depth ratio is given by

$$\frac{c}{d} = 0.600$$

The strength reduction factor for this condition, for members with rectangular stirrups, is given by ACI Sec. 9.3.2.2 as

$$\phi = 0.65$$

For sections that lie in the transition region between the tension-controlled and compression-controlled limits, the strength reduction factor is obtained from ACI Fig. R9.3.2 as

$$\phi = 0.48 + 83\epsilon_t$$
$$= 0.23 + \frac{0.25}{\frac{c}{d}}$$

Maximum Reinforcement Ratio

To ensure an under-reinforced section with ductile behavior and adequate warning of impending failure, ACI Sec. 10.3.5 limits the maximum reinforcement ratio in accordance with a prescribed strain distribution. The maximum reinforcement ratio is that which produces a tensile strain in the reinforcing bar closest to the edge of the beam, of

$$\epsilon_t = 0.004$$

From ACI Sec. R10.3.5, the maximum concrete compressive strain is

$$\epsilon_c \approx 0.003$$

The depth of the neutral axis is obtained from Fig. 1.1 as

$$c = \left(\frac{\epsilon_c}{\epsilon_c + \epsilon_t}\right)d$$
$$= \left(\frac{0.003}{0.003 + 0.004}\right)d$$
$$= 0.429d$$

The corresponding strength reduction factor is

$$\phi = 0.812$$

The maximum allowable reinforcement ratio is derived from Fig. 1.1 as

$$\rho_{\max} = 0.364\beta_1 \frac{f'_c}{f_y}$$

Minimum Reinforcement Ratio

A minimum reinforcement ratio is required to ensure that the flexural strength of the reinforced section is greater than that of the uncracked concrete section. If this is not the case, sudden failure will occur when the modulus of rupture of the concrete is exceeded and the first flexural crack forms.

In accordance with ACI Sec. 10.5, the minimum permissible reinforcement ratio is

$$\rho_{\min} = \frac{3\sqrt{f'_c}}{f_y} \geq \frac{200}{f_y}$$

The exception is that the minimum reinforcement need not exceed 33% more than that required by analysis. For slabs and footings, ACI Sec. 7.12 requires a minimum reinforcement area for grade 60 deformed bars of

$$A_{s,\min} = 0.0018bh$$

Analysis Procedure for a Singly Reinforced Beam

Given the section properties b, d, A_s, f_y, and f'_c, the analysis of the section consists of calculating the

- stress block depth using $a = A_s f_y / 0.85 f'_c b$
- nominal strength using $M_n = A_s f_y (d - a/2)$
- design strength using ϕM_n

Example 1.2

A reinforced concrete slab is simply supported over a span of 12 ft. The slab has a concrete compressive strength of 3000 lbf/in^2, and the reinforcement consists of no. 4 grade 60 bars at 11 in on center with an effective depth of 6 in. The total dead load, including the self-weight of the slab, is 120 lbf/ft^2.

(a) Consider a slab with a 12 in width. What is the tension reinforcement area provided?

(b) What is the depth of the rectangular stress block?

(c) What is the lever-arm of the internal resisting moment?

(d) What is the nominal flexural strength of a 12 in wide slab?

(e) What is the maximum permissible factored moment on a 12 in wide slab?

(f) What is the applied factored dead load moment in ACI Eq. 9-2 on a 12 in wide slab?

(g) What is the maximum permissible strength level live load moment on a 12 in wide slab?

(h) What is the maximum permissible service level live load moment on a 12 in wide slab?

(i) What is the permissible service level live load?

Solution

(a) Consider a 12 in wide slab.

The area of one no. 4 bar is 0.20 in^2. The reinforcement area provided in a 12 in width is

$$A_s = \frac{(0.20 \text{ in}^2)(12 \text{ in})}{11 \text{ in}}$$
$$= 0.22 \text{ in}^2$$

(b) Equating the tensile and compressive forces acting on the section gives the depth of the equivalent rectangular stress block as

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(0.22 \text{ in}^2)\left(60{,}000 \dfrac{\text{lbf}}{\text{in}^2}\right)}{(0.85)\left(3000 \dfrac{\text{lbf}}{\text{in}^2}\right)(12 \text{ in})}$$
$$= 0.43 \text{ in}$$

(c) The lever-arm of the internal resisting moment is obtained from Fig. 1.1 as

$$d - \frac{a}{2} = 6 \text{ in} - \frac{0.43 \text{ in}}{2}$$
$$= 5.78 \text{ in}$$

(d) The nominal moment of resistance is

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$
$$= \frac{(0.22 \text{ in}^2)\left(60 \dfrac{\text{kips}}{\text{in}^2}\right)(5.78 \text{ in})}{12 \dfrac{\text{in}}{\text{ft}}}$$
$$= 6.36 \text{ ft-kips}$$

(e) The limiting reinforcement ratio for a tension-controlled section is

$$\rho_t = 0.319\beta_1 \frac{f'_c}{f_y}$$
$$= (0.319)(0.85)\left(\frac{3000 \dfrac{\text{lbf}}{\text{in}^2}}{60{,}000 \dfrac{\text{lbf}}{\text{in}^2}}\right)$$
$$= 0.0136$$

The reinforcement ratio provided is

$$\rho = \frac{A_s}{bd}$$
$$= \frac{0.22 \text{ in}^2}{(12 \text{ in})(6 \text{ in})}$$
$$= 0.003$$
$$< \rho_t$$

The section is tension-controlled and the strength reduction factor is

$$\phi = 0.9$$

The maximum permissible factored moment is

$$M_u = \phi M_n = (0.9)(6.36 \text{ ft-kips})$$
$$= 5.73 \text{ ft-kips}$$

(f) The applied factored dead load moment is

$$M_{uD} = \frac{1.2 w_D l^2}{8}$$
$$= \frac{(1.2)\left(0.12 \dfrac{\text{kip}}{\text{ft}}\right)(12 \text{ ft})^2}{8}$$
$$= 2.59 \text{ ft-kips}$$

(g) From ACI Eq. 9-2, the maximum permissible strength level live load moment is

$$M_{uL} = M_u - M_{uD}$$
$$= 5.73 \text{ ft-kips} - 2.59 \text{ ft-kips}$$
$$= 3.14 \text{ ft-kips}$$

(h) The maximum permissible service level live load moment is

$$M_L = \frac{M_{uL}}{1.6} = \frac{3.14 \text{ ft-kips}}{1.6}$$
$$= 1.96 \text{ ft-kips}$$

(i) The permissible service level live load is

$$w_L = \frac{8 M_L}{l^2} = \frac{(8)(1.96 \text{ ft-kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{(12 \text{ ft})^2}$$
$$= 109 \text{ lbf/ft}$$

Design Procedure for a Singly Reinforced Beam

The procedure to select a suitable section to resist a given bending moment, M_u, consists of the following steps.

step 1: Assume beam dimensions and concrete strength.

step 2: Calculate the design moment factor from

$$K_u = \frac{M_u}{bd^2}$$

step 3: Calculate the ratio

$$\frac{K_u}{f'_c}$$

step 4: Assume a tension-controlled section, since generally this is the case, and determine the reinforcement index, ω, from App. A.

step 5: Determine the required reinforcement from

$$\rho = \frac{\omega f'_c}{f_y}$$

step 6: Check that the beam complies with the maximum reinforcement requirements of ACI Sec. 10.3.5.

$$\rho \le 0.364 \beta_1 \frac{f'_c}{f_y}$$

Increase the beam size or f'_c if necessary.

step 7: Check that the beam complies with tension-controlled reinforcement requirements of ACI Sec. 10.3.4.

$$\rho \le 0.319 \beta_1 \frac{f'_c}{f_y}$$

Increase the beam size or f'_c if necessary.

step 8: Check that the beam complies with minimum reinforcement requirements of ACI Sec. 10.5.

$$\rho_{\min} = \frac{3\sqrt{f'_c}}{f_y} > \frac{200}{f_y}$$

Increase the beam size or f'_c if necessary.

Example 1.3

A reinforced concrete beam with an effective depth of 16 in and a width of 12 in is reinforced with grade 60 bars and has a concrete compressive strength of 3000 lbf/in². Determine the area of tension reinforcement required if the beam supports a total factored moment of 150 ft-kips.

Solution

The design moment factor is

$$K_u = \frac{M_u}{bd^2}$$
$$= \frac{(150 \text{ ft-kips})\left(12 \dfrac{\text{in}}{\text{ft}}\right)\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{(12 \text{ in})(16 \text{ in})^2}$$
$$= 586 \text{ lbf/in}^2$$

$$\frac{K_u}{f'_c} = \frac{586 \dfrac{\text{lbf}}{\text{in}^2}}{3000 \dfrac{\text{lbf}}{\text{in}^2}} = 0.195$$

From App. A, assuming a tension-controlled section, the corresponding tension reinforcement index is

$$\omega = 0.255$$

The required reinforcement ratio is

$$\rho = \frac{\omega f'_c}{f_y}$$

$$= \frac{(0.255)\left(3000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}}$$

$$= 0.0128$$

The limiting reinforcement ratio for a tension-controlled section is

$$\rho_t = 0.319\beta_1 \frac{f'_c}{f_y}$$

$$= (0.319)(0.85)\left(\frac{3000\ \dfrac{\text{lbf}}{\text{in}^2}}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}}\right)$$

$$= 0.0136$$

$$> \rho$$

Therefore, the section is tension-controlled.

The minimum allowable reinforcement ratio is

$$\rho_{\min} = \frac{200\ \dfrac{\text{lbf}}{\text{in}^2}}{f_y}$$

$$= \frac{200\ \dfrac{\text{lbf}}{\text{in}^2}}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}}$$

$$= 0.0033$$

$$< \rho \quad [\text{satisfactory}]$$

The reinforcement area required is

$$A_s = \rho b d = (0.0128)(12\ \text{in})(16\ \text{in})$$

$$= 2.45\ \text{in}^2$$

Beams with Compression Reinforcement

A reinforced concrete beam with compression reinforcement is shown in Fig. 1.2. Compression reinforcement and additional tension reinforcement are required when the factored moment on the member exceeds the design flexural strength of a singly reinforced member with the strain in the tension steel, $\epsilon_t = 0.005$. The residual moment is given by

$$M_r = M_u - M_{\max}$$

The area of compression reinforcement is

$$A'_s = \frac{M_r}{\phi f'_s(d - d')}$$

The area of additional tension reinforcement is

$$A_t = \frac{A'_s f'_s}{f_y}$$

The stress in the compression reinforcement is

$$f'_s = \left(87{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)\left(1 - \frac{d'}{c}\right)$$

$$\leq f_y$$

The neutral axis depth is

$$c = 0.375d$$

Analysis Procedure for a Beam with Compression Reinforcement

To analyze a beam with compression reinforcement, an initial estimate of the neutral axis depth is required. Assuming that the compressive strain in the concrete is 0.003, the strain and the stress in the compression and tension reinforcement may be determined. The total compressive force in the concrete and the compression reinforcement is then compared with the tensile force in the tension reinforcement. The initial estimate of the neutral axis depth is adjusted until these two values are equal. The maximum nominal moment provided by

Figure 1.2 Beam with Compression Reinforcement[2]
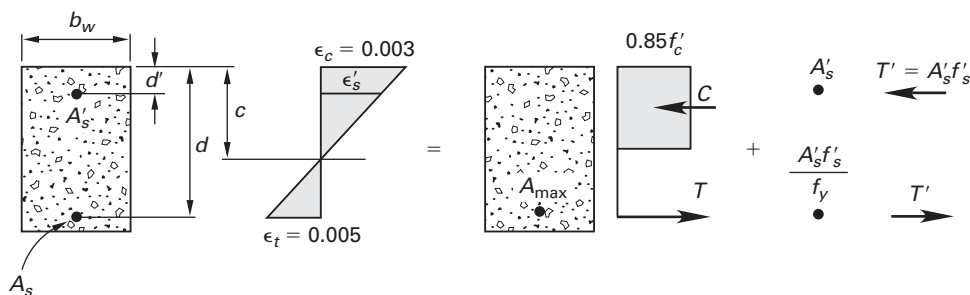
the section is obtained by taking moments of the forces in the concrete and in the compression reinforcement about the centroid of the tension reinforcement.

Design Procedure for a Beam with Compression Reinforcement

The procedure to select a suitable section to resist a given bending moment, M_u, consists of the following steps.

step 1: Assume beam dimensions and concrete strength.

step 2: Determine ρ_t for $\epsilon_t = 0.005$ from Table 1.1.

step 3: Calculate the maximum area of tension reinforcement from

$$A_{\max} = \rho_t bd$$

step 4: Determine the design moment factor, K_u, from Table 1.1.

step 5: Calculate the maximum design flexural strength from

$$M_{\max} = K_u bd^2$$

step 6: Calculate the residual moment from

$$M_r = M_u - M_{\max}$$

step 7: Determine the additional tension steel from

$$A_t = \frac{M_r}{\phi f_y(d - d')}$$

step 8: Calculate the required total area of tension reinforcement from

$$A_s = A_{\max} + A_t$$

step 9: Find the neutral axis depth for $\epsilon_t = 0.005$ from

$$c = 0.375d$$

step 10: Calculate the stress in compression reinforcement from

$$f'_s = \left(87{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)\left(1 - \frac{d'}{c}\right)$$

step 11: Calculate the required area of compression steel from

$$A'_s = \frac{A_t f_y}{f'_s}$$

Increase beam size or f'_c if necessary.

Example 1.4

A reinforced concrete beam with an effective depth of 16 in and a width of 12 in is reinforced with grade 60 bars and has a concrete compressive strength of 3000 lbf/in. The depth to the centroid of the compression reinforcement is 3 in. Determine the areas of tension and compression reinforcement required if the beam supports a total factored moment of 178 ft-kips.

Solution

From Table 1.1, the maximum allowable tension reinforcement ratio in a tension-controlled beam with a concrete strength of 3000 lbf/in^2 and grade 60 reinforcement bars is

$$\rho_t = 0.0136$$

The corresponding tension reinforcement area is

$$\begin{aligned}
A_{\max} &= \rho_t bd \\
&= (0.0136)(12 \text{ in})(16 \text{ in}) \\
&= 2.611 \text{ in}^2
\end{aligned}$$

The corresponding tension reinforcement index is

$$\begin{aligned}
\omega = \frac{\rho_t f_y}{f'_c} &= \frac{(0.0136)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{3 \ \dfrac{\text{kips}}{\text{in}^2}} \\
&= 0.271
\end{aligned}$$

From App. A, the corresponding maximum design flexural strength is

$$\begin{aligned}
M_{\max} &= 0.205 f'_c bd^2 \\
&= \frac{(0.205)\left(3000 \ \dfrac{\text{lbf}}{\text{in}^2}\right)(12 \text{ in})(16 \text{ in})^2}{\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)} \\
&= 157.4 \text{ ft-kips}
\end{aligned}$$

The residual moment is

$$\begin{aligned}
M_r &= M_u - M_{\max} \\
&= 178 \text{ ft-kips} - 157.4 \text{ ft-kips} \\
&= 20.6 \text{ ft-kips}
\end{aligned}$$

The additional area of tension reinforcement required is

$$A_t = \frac{M_r}{\phi f_y (d - d')}$$

$$= \frac{(20.6 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(0.9)\left(60 \frac{\text{kips}}{\text{in}^2}\right)(16 \text{ in} - 3 \text{ in})}$$

$$= 0.352 \text{ in}^2$$

The total required area of tension reinforcement is

$$A_s = A_{\max} + A_t = 2.611 \text{ in}^2 + 0.352 \text{ in}^2$$

$$= 2.963 \text{ in}^2$$

The neutral axis depth is

$$c = 0.375d$$

$$= (0.375)(16 \text{ in})$$

$$= 6.0 \text{ in}$$

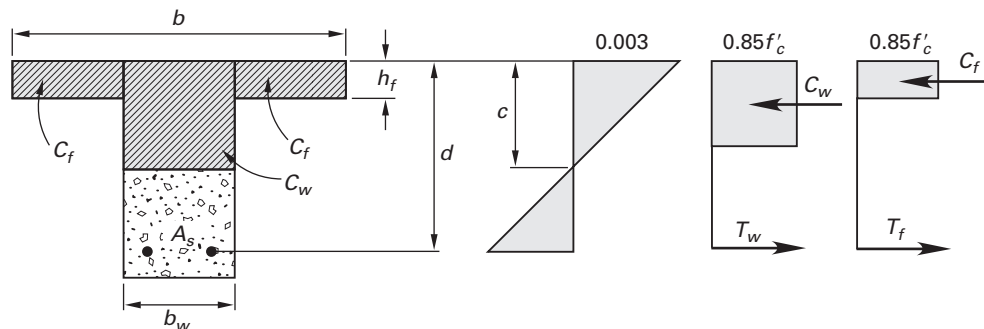The stress in the compression steel is

$$f'_s = \left(87{,}000 \frac{\text{lbf}}{\text{in}^2}\right)\left(1 - \frac{d'}{c}\right)$$

$$= \left(87{,}000 \frac{\text{lbf}}{\text{in}^2}\right)\left(1 - \frac{3 \text{ in}}{6.0 \text{ in}}\right)$$

$$= 43{,}500 \text{ lbf/in}^2$$

The required area of compression reinforcement is

$$A'_s = \frac{A_t f_y}{f'_s} = \frac{(0.352 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)}{43.50 \frac{\text{kips}}{\text{in}^2}}$$

$$= 0.486 \text{ in}^2$$

Figure 1.3 *Flanged Section with Tension Reinforcement*[2]

Flanged Section with Tension Reinforcement

When the rectangular stress block is wholly contained in the flange, a flanged section may be designed as a rectangular beam.

When the depth of the rectangular stress block exceeds the flange thickness, the flanged beam is designed as shown in Fig. 1.3. The area of reinforcement required to balance the compressive force in the outstanding flanges is

$$A_{sf} = \frac{0.85 f'_c h_f (b - b_w)}{f_y}$$

The corresponding design moment strength is

$$M_f = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right)$$

The beam web must develop the residual moment, which is given by

$$M_r = M_u - M_f$$

The value of $M_r / f'_c b_w d^2$ is determined. The corresponding value of ω is obtained from App. A, and the additional area of reinforcement required to balance the residual moment is

$$A_{sw} = \frac{\omega b_w d f'_c}{f_y}$$

The total area of reinforcement required is

$$A_s = A_{sf} + A_{sw}$$

Example 1.5

A reinforced concrete flanged beam with a flange width of 24 in, a web width of 12 in, a flange depth of 3 in, and an effective depth of 16 in is reinforced with grade 60 reinforcement. If the concrete compressive strength is 3000 lbf/in², determine the area of tension reinforcement required to support an applied factored moment of 250 ft-kips.

Concrete

Solution

Assume that the depth of the rectangular stress block exceeds the depth of the flange.

The area of tension reinforcement required to balance the compression force in the flange is

$$A_{sf} = \frac{0.85 f_c' h_f (b - b_w)}{f_y}$$

$$= \frac{(0.85)\left(3 \ \dfrac{\text{kips}}{\text{in}^2}\right)(3 \text{ in})(24 \text{ in} - 12 \text{ in})}{60 \ \dfrac{\text{kips}}{\text{in}^2}}$$

$$= 1.53 \text{ in}^2$$

Assuming the section is tension-controlled, the corresponding design moment strength is

$$M_f = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right)$$

$$= \frac{(0.9)(1.53 \text{ in}^2)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)\left(16 \text{ in} - \dfrac{3 \text{ in}}{2}\right)}{12 \ \dfrac{\text{in}}{\text{ft}}}$$

$$= 99.83 \text{ ft-kips}$$

The residual moment to be developed by the web is

$$M_r = M_u - M_f$$

$$= 250 \text{ ft-kips} - 99.83 \text{ ft-kips}$$

$$= 150.17 \text{ ft-kips}$$

$$\frac{M_r}{f_c' b_w d^2} = \frac{(150.17 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}{\left(3 \ \dfrac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})(16 \text{ in})^2}$$

$$= 0.196$$

From App. A, the corresponding tension reinforcement index is

$$\omega = 0.257$$

The reinforcement required to develop the residual moment is

$$A_{sw} = \frac{\omega b_w d f_c'}{f_y}$$

$$= \frac{(0.257)(12 \text{ in})(16 \text{ in})\left(3 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{60 \ \dfrac{\text{kips}}{\text{in}^2}}$$

$$= 2.47 \text{ in}^2$$

The total tension reinforcement area required is

$$A_s = A_{sf} + A_{sw}$$

$$= 1.53 \text{ in}^2 + 2.47 \text{ in}^2$$

$$= 4.00 \text{ in}^2$$

The depth of the equivalent rectangular stress block is given by

$$a = \frac{A_{sw} f_y}{0.85 f_c' b}$$

$$= \frac{(2.47 \text{ in}^2)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3 \ \dfrac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})}$$

$$= 4.84 \text{ in}$$

$$> h_f \quad \text{[as assumed]}$$

For a tension-controlled section, the maximum depth of the equivalent rectangular stress block is given by

$$a_t = 0.375 \beta_1 d = (0.375)(0.85)(16 \text{ in})$$

$$= 5.10 \text{ in}$$

$$> a \quad \text{[The section is tension-controlled as assumed.]}$$

Analysis of a Flanged Section

The following steps are used to analyze a flanged beam when the depth of the stress block exceeds the flange thickness.

step 1: Calculate the compressive force developed by the outstanding flanges from

$$C_f = 0.85 f_c' h_f (b - b_w)$$

step 2: Calculate the area of tension reinforcement needed to balance the compressive force from

$$A_{sf} = \frac{C_f}{f_y}$$

step 3: Calculate the corresponding design moment strength of the outstanding flanges from

$$M_f = \phi A_{sf} f_y \left(d - \frac{h_f}{2}\right)$$

step 4: Calculate the residual reinforcement area from

$$A_{sw} = A_s - A_{sf}$$

step 5: Calculate the residual reinforcement ratio referred to the web from

$$\rho_w = \frac{A_{sw}}{b_w d}$$

step 6: Calculate the design moment strength of the residual reinforcement from

$$M_w = \phi A_{sw} f_y d \left(\frac{1 - 0.59\rho_w f_y}{f'_c} \right)$$

step 7: Calculate the total design moment strength of the section from

$$M_u = M_f + M_w$$

Alternatively, the value of $M_w/(f'_c b_w d^2)$ may be determined from App. A using the calculated value of the reinforcement index, $\omega_w = \rho_w f_y/f'_c$. The value of M_w can then be determined from $M_w/(f'_c b_w d^2)$ using the known values f'_c, b_w, and d.

4. SERVICEABILITY REQUIREMENTS FOR BEAMS

Nomenclature

A_b	area of individual bar	in^2
A_s	area of tension reinforcement	in^2
A_{sk}	area of skin reinforcement per unit height in one side face	in^2
A_{ts}	area of nonprestressed reinforcement in a tie	in^2
c_c	clear cover to tension reinforcement	in
d_b	diameter of bar	in
E_c	modulus of elasticity of concrete, $33w_c^{1.5}\sqrt{f'_c}$	lbf/in^2
E_s	modulus of elasticity of reinforcement, 29,000	kips/in^2
f_r	modulus of rupture of concrete, $7.5\sqrt{f'_c}$	lbf/in^2
f_s	calculated stress in reinforcement at service loads	kips/in^2
h	overall dimension of member	in
I_{cr}	moment of inertia of cracked transformed section, $b_w(kd)^3/3 + nA_s(d-kd)^2$	in^4
I_e	effective moment of inertia	in^4
I_g	moment of inertia of gross concrete section, $b_w h^3/12$	in^4
k	neutral axis depth factor at service load, $\sqrt{2\rho n + (\rho n)^2} - \rho n$	–
l	span length of beam or one-way slab, projection of cantilever	ft
M_a	maximum moment in member at stage deflection is required	in-lbf or ft-kips

M_{cr}	cracking moment, $2f_r I_g/h$	in-lbf or ft-kips
n	modular ratio, E_s/E_c	–
s	center-to-center spacing of tension reinforcement	in
w_c	unit weight of concrete	lbf/ft^3

Symbols

|		
δ	deflection	in
ξ	time-dependent factor for sustained load	–
λ_Δ	multiplier for additional long-time deflection	–
ρ'	reinforcement ratio for compression reinforcement, A'_s/bd	–

Control of Crack Widths

Limitations on reinforcement distribution are specified to control cracking and to protect the reinforcement from corrosion. Crack width and reinforcement corrosion increase when reinforcement stress increases, concrete cover increases, or the area of concrete surrounding each bar increases.

In accordance with ACI Sec. 10.6, crack width is controlled by limiting the spacing of tension reinforcement to a value given by ACI Eq. 10-4, where f_s is in units of kips/in^2.

$$s = \frac{600}{f_s} - 2.5c_c$$
$$\leq \frac{480}{f_s}$$

As shown in Fig. 1.4, s is the center-to-center spacing, in inches, of the tension reinforcement nearest to the extreme tension face, and c_c is the clear concrete cover, in inches, from the nearest surface in tension to the surface of the tension reinforcement. Where there is only one bar nearest to the extreme tension face, s is taken as the width of the extreme tension face. Controlling the spacing of tension reinforcement limits the width of surface cracks to an acceptable level.

Figure 1.4 *Tension Reinforcement Details*
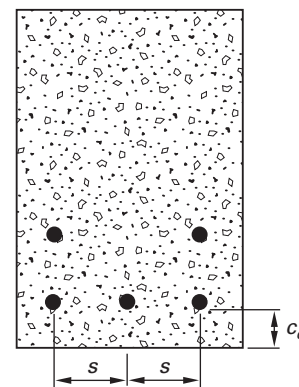
The stress in the reinforcement at service load may be either calculated or assumed to be equal to $\frac{2}{3}f_y$.

When the depth of the beam exceeds 36 in, ACI Sec. 10.6.7 requires that skin reinforcement be placed along both side faces of the web, in the lower half of the beam.

Example 1.6

The beam shown is reinforced with eight no. 9 grade 60 bars. Clear cover of $1\frac{1}{2}$ in is provided to the no. 4 stirrups. Determine the skin reinforcement required, and check that the spacing of the reinforcement conforms to ACI Sec. 10.6.
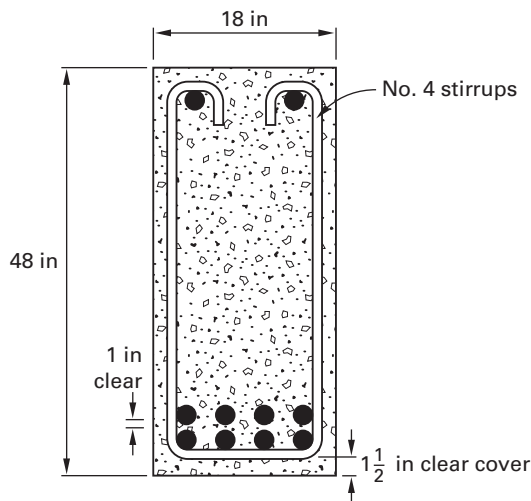
Solution

The clear cover provided to the tension reinforcement is given by

$$c_c = 1.5 \text{ in} + 0.5 \text{ in}$$
$$= 2 \text{ in}$$

The stress in the reinforcement at service load is assumed equal to

$$f_s = \tfrac{2}{3}f_y$$
$$= \left(\tfrac{2}{3}\right)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 40 \text{ kips/in}^2$$

The maximum allowable bar spacing is given by ACI Eq. 10-4 as

$$s = \frac{600 \ \dfrac{\text{kips}}{\text{in}^2}}{f_s} - 2.5c_c$$
$$= \frac{600 \ \dfrac{\text{kips}}{\text{in}^2}}{40 \ \dfrac{\text{kips}}{\text{in}^2}} - (2.5)(2 \text{ in})$$
$$= 10 \text{ in}$$

The actual bar spacing is given by

$$s' = \frac{18 \text{ in} - (2)(1.5 \text{ in}) - (2)(0.5 \text{ in}) - 1.128 \text{ in}}{3}$$
$$= 4.29 \text{ in}$$

Since 4.29 in is less than 10 in, this bar spacing is satisfactory.

The depth of the beam is
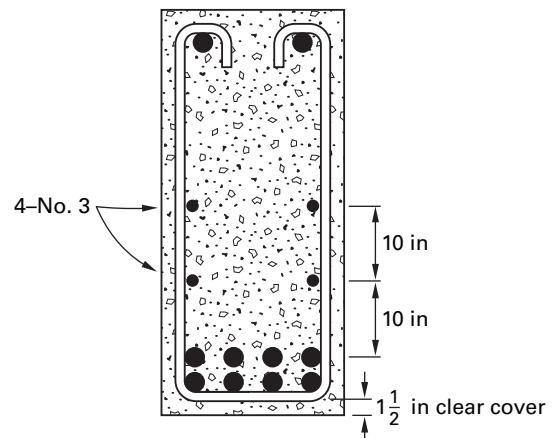
$$h = 48 \text{ in}$$
$$> 36 \text{ in}$$

Therefore, skin reinforcement is required.

Using no. 3 bars, the maximum allowable spacing is

$$s_{sk} = 10 \text{ in}$$

The bars shall extend for a distance, $h/2$, from the tension face.

The reinforcement layout is shown.

Deflection Limitations

Two methods are given in ACI 318 to control deflections. One method is to provide a minimum overall thickness for beams or slabs. The other method is to calculate the beam and slab deflections and compare them to permissible values.

Calculation of the deflections is not required if the limiting thickness requirements for beams and slabs are met.

The allowable, immediate deflection of flexural members supporting nonsensitive elements is specified in ACI Table 9.5(b) as $l/180$ for flat roofs and $l/360$ for floors due to the applied live load. The total deflection occurring after the attachment of nonsensitive elements is limited to $l/240$. The total deflection occurring after the attachment of deflection sensitive elements is limited to $l/480$.

For normal weight concrete and grade 60 reinforcement, ACI Table 9.5(a) provides span/depth ratios applicable

to members supporting nonsensitive elements. These ratios are shown in Table 1.2.

For grade 40 reinforcement, the tabulated values are multiplied by the factor 0.8. For lightweight concrete, the tabulated values are multiplied by the factor

$$R = 1.65 - 0.005w_c$$
$$\geq 1.09$$

Table 1.2 *Span/Depth Ratios*

end conditions	beam	slab
simply supported	$\dfrac{l}{16}$	$\dfrac{l}{20}$
one end continuous	$\dfrac{l}{18.5}$	$\dfrac{l}{24}$
both ends continuous	$\dfrac{l}{21}$	$\dfrac{l}{28}$
cantilever	$\dfrac{l}{8}$	$\dfrac{l}{10}$

Deflection Determination

Short-term deflections may be calculated by using the effective moment of inertia given by ACI Sec. 9.5.2.3 and illustrated in Fig. 1.5 as

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_a}\right)^3\right) I_{cr} \quad \text{[ACI 9-8]}$$

Additional long-term deflection is estimated from ACI Sec. 9.5.2.5 by multiplying the short-term deflection by the multiplier

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} \qquad \text{[ACI 9-11]}$$

ξ is the time-dependent factor for sustained load defined in ACI Sec. 9.5.2.5 and shown in Table 1.3.

Table 1.3 *Value of ξ*

time period (mo)	ξ
60	2.0
12	1.4
6	1.2
3	1.0

The deflection is calculated for each loading case using the appropriate value of the effective moment of inertia. Thus, the short-term deflection, δ_D, may be calculated for dead load only and the short-term deflection, $\delta_{(D+L)}$, may be calculated for the total applied load. The live load deflection is then given by

$$\delta_L = \delta_{(D+L)} - \delta_D$$

The final total deflection, including additional long-term deflection, is given by

$$\delta_T = \delta_D(1 + \lambda_\Delta) + \delta_L$$
$$= \delta_{(D+L)} + \lambda_\Delta \delta_D$$

Example 1.7

A reinforced concrete beam of normal weight concrete and spanning 12 ft has an effective depth of 16 in, an overall depth of 18 in, and a compressive strength of 3000 lbf/in², and it is reinforced with three no. 8 grade 60 bars. The beam is 12 in wide. The bending moment due to sustained dead load is 60 ft-kips. Neglect the weight of the nondeflection sensitive elements, which are attached immediately after removing the falsework.

The transient floor live load moment is 30 ft-kips. Compare the beam deflections with the allowable values and determine the final beam deflection due to long-term effects and transient loads.

Figure 1.5 *Service Load Conditions*
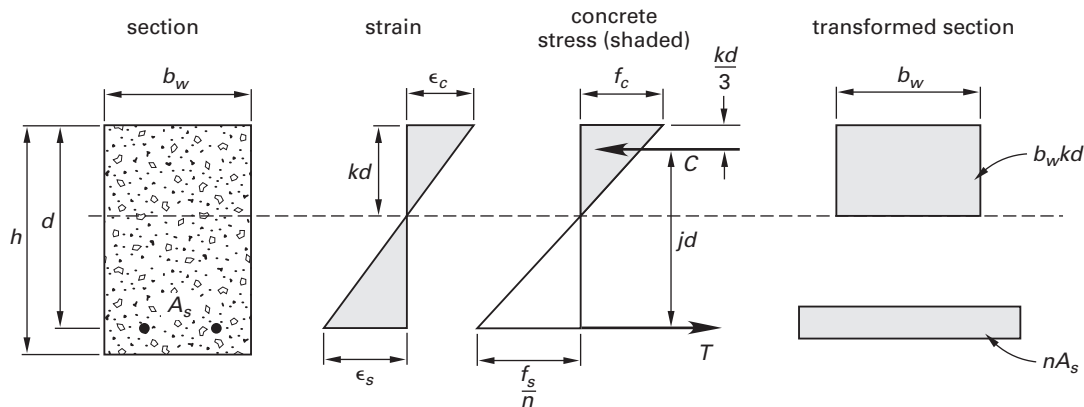
Solution

The allowable live load deflection for floors is given by ACI Table 9.5(b) as

$$\delta_L = \frac{l}{360} = \frac{(12 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{360} = 0.40 \text{ in}$$

The allowable deflection after attachment of nonsensitive elements is

$$\delta_{(\lambda D + L)} = \frac{l}{240} = \frac{(12 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{240} = 0.60 \text{ in}$$

From ACI Sec. 8.5,

$$E_c = 33\sqrt{\omega_c^3 f_c'} = \frac{33\sqrt{\left(150 \frac{\text{lbf}}{\text{ft}^3}\right)^3 \left(3000 \frac{\text{lbf}}{\text{in}^2}\right)}}{1000 \frac{\text{lbf}}{\text{kip}}}$$

$$= 3320 \text{ kips/in}^2$$

$$E_s = 29{,}000 \text{ kips/in}^2$$

$$n = \frac{E_s}{E_c} = \frac{29{,}000 \frac{\text{kips}}{\text{in}^2}}{3320 \frac{\text{kips}}{\text{in}^2}} = 8.73$$

$$\rho = \frac{A_s}{b_w d} = \frac{2.37 \text{ in}^2}{(12 \text{ in})(16 \text{ in})} = 0.0123$$

$$\rho n = (0.0123)(8.73) = 0.108$$

From App. B, the corresponding neutral axis depth factor is

$$k = 0.3691$$

The moment of inertia of the cracked transformed section is

$$I_{cr} = \frac{b_w (kd)^3}{3} + nA_s(d - kd)^2$$

$$= \frac{(12 \text{ in})\big((0.3691)(16 \text{ in})\big)^3}{3}$$

$$\quad + (8.73)(2.37 \text{ in}^2)\big(16 \text{ in} - (0.3691)(16 \text{ in})\big)^2$$

$$= 2932 \text{ in}^4$$

$$I_g = \frac{(12 \text{ in})(18 \text{ in})^3}{12} = 5832 \text{ in}^4$$

The modulus of rupture, where $\lambda = 1.0$ for normal weight concrete (ACI Sec. 8.6.1), is given by ACI Eq. 9-10 as

$$f_r = 7.5\lambda\sqrt{f_c'} = (7.5)(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 411 \text{ lbf/in}^2$$

The cracking moment is given by ACI Eq. 9-9 as

$$M_{cr} = \frac{2f_r I_g}{h} = \frac{(2)\left(411 \frac{\text{lbf}}{\text{in}^2}\right)(5832 \text{ in}^4)}{(18 \text{ in})\left(12 \frac{\text{in}}{\text{ft}}\right)(1000 \text{ lbf})}$$

$$= 22.18 \text{ ft-kips}$$

$$< 60 \text{ ft-kips} \quad \text{[Section is cracked.]}$$

The effective moment of inertia for the dead load bending moment is given by ACI Eq. 9-8 as

$$I_e = \left(\frac{M_{cr}}{M_D}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_D}\right)^3\right) I_{cr}$$

$$= \left(\frac{22.18 \text{ ft-kips}}{60 \text{ ft-kips}}\right)^3 (5832 \text{ in}^4)$$

$$\quad + \left(1 - \left(\frac{22.18 \text{ ft-kips}}{60 \text{ ft-kips}}\right)^3\right)(2932 \text{ in}^4)$$

$$= 3078 \text{ in}^4$$

The corresponding short-term deflection due to dead load is

$$\delta_D = \frac{180 M_D L^2}{E_c I_e} = \frac{(180)(60 \text{ ft-kips})(12 \text{ ft})^2}{\left(3320 \frac{\text{kips}}{\text{in}^2}\right)(3078 \text{ in}^4)} = 0.152 \text{ in}$$

The effective moment of inertia for the dead load plus live load is

$$I_e = \left(\frac{M_{cr}}{M_{(D+L)}}\right)^3 I_g + \left(1 - \left(\frac{M_{cr}}{M_{(D+L)}}\right)^3\right) I_{cr}$$

$$= \left(\frac{22.18 \text{ ft-kips}}{90 \text{ ft-kips}}\right)^3 (5832 \text{ in}^4)$$

$$\quad + \left(1 - \left(\frac{22.18 \text{ ft-kips}}{90 \text{ ft-kips}}\right)^3\right)(2932 \text{ in}^4)$$

$$= 2975 \text{ in}^4$$

The corresponding short-term deflection due to the dead load plus live load is

$$\delta_{(D+L)} = \frac{(180)(90 \text{ ft-kips})(12 \text{ ft})^2}{\left(3320 \frac{\text{kips}}{\text{in}^2}\right)(2975 \text{ in}^4)} = 0.236 \text{ in}$$

The short-term deflection due to transient live load is

$$\delta_L = \delta_{(D+L)} - \delta_D = 0.236 \text{ in} - 0.152 \text{ in}$$

$$= 0.084 \text{ in}$$

$$< 0.40 \text{ in} \quad \text{[satisfactory]}$$

The multiplier for additional long-term deflection is given by ACI Eq. 9-11 as

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} = \frac{2}{1 + 0} = 2$$

The deflection due to short-term live loads and long-term dead loads is

$$\delta_{(\lambda D + L)} = \lambda_\Delta \delta_D + \delta_L = (2)(0.152 \text{ in}) + 0.084 \text{ in}$$
$$= 0.388 \text{ in}$$
$$< 0.60 \text{ in} \quad [\text{satisfactory}]$$

The final deflection due to long-term and short-term effects is

$$\delta_T = \delta_D(1 + \lambda_\Delta) + \delta_L = (0.152 \text{ in})(1 + 2) + 0.084 \text{ in}$$
$$= 0.540 \text{ in}$$

5. ELASTIC DESIGN METHOD

Nomenclature

f_c	actual stress in concrete	lbf/in^2
f_s	actual tensile stress in reinforcement	lbf/in^2
j	lever-arm factor	–
j_{bal}	balanced lever-arm factor	–
k_{bal}	balanced neutral axis depth factor	–
M	service design moment	ft-kips
M_{bal}	balanced service design moment	ft-kips
p_{cb}	permissible concrete stress	lbf/in^2
p_{st}	permissible steel stress	lbf/in^2

Symbols

ρ_{bal}	balanced tension reinforcement ratio, $p_{cb}k_{\text{bal}}/2p_{st}$	–

Determination of Working Stress Values

The elastic design method is referred to in ACI Sec. R1.1 as the alternate design method. The straight-line theory, illustrated in Fig. 1.5, is used to calculate the stresses in a member under the action of the applied service loads and to ensure that these stresses do not exceed permissible values. The permissible stresses are

$$p_{cb} = \text{maximum permissible stress in the concrete}$$
$$= 0.45 f'_c$$
$$p_{st} = \text{maximum permissible stress}$$
$$\quad \text{in the reinforcement}$$
$$= 20 \text{ kips/in}^2 \quad [\text{grade 40 reinforcement}]$$
$$= 24 \text{ kips/in}^2 \quad [\text{grade 60 reinforcement}]$$

From Fig. 1.5, the neutral axis depth factor is derived as

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

Appendix B tabulates values of k against ρn. In addition, the lever-arm factor is derived as

$$j = 1 - \frac{k}{3}$$

The stress in the reinforcement due to an applied service moment, M, is

$$f_s = \frac{M}{A_s j d}$$

The stress in the concrete is

$$f_c = \frac{2M}{jkb_w d^2}$$

For a balanced design, the stress in the reinforcement and the maximum stress in the concrete should simultaneously reach their permissible values. Then, the corresponding design values will be

$$k_{\text{bal}} = \frac{np_{cb}}{p_{st} + np_{cb}}$$
$$j_{\text{bal}} = 1 - \frac{k_{\text{bal}}}{3}$$
$$\rho_{\text{bal}} = \frac{p_{cb}k_{\text{bal}}}{2p_{st}}$$
$$M_{\text{bal}} = A_{s(\text{bal})}p_{st}j_{\text{bal}}d$$

Example 1.8

A reinforced concrete beam with an effective depth of 16 in and a width of 12 in is reinforced with grade 60 bars and has a concrete cylinder strength of 3000 lbf/in^2. Using the elastic design method, determine the area of tension reinforcement required if the beam supports a total service moment of 50 ft-kips.

Solution

From Ex. 1.7, the modular ratio is given as

$$n = 8.73$$

The permissible concrete and reinforcement stresses are

$$p_{cb} = 1350 \text{ lbf/in}^2$$
$$p_{st} = 24,000 \text{ lbf/in}^2$$

Sufficient accuracy is obtained by assuming that the neutral axis depth factor equals the balanced value. Then,

$$k_{\text{bal}} = \frac{np_{cb}}{p_{st} + np_{cb}} = \frac{(8.73)\left(1350 \, \frac{\text{lbf}}{\text{in}^2}\right)}{24,000 \, \frac{\text{lbf}}{\text{in}^2} + (8.73)\left(1350 \, \frac{\text{lbf}}{\text{in}^2}\right)}$$
$$= 0.329$$

$$j_{\text{bal}} = 1 - \frac{k_{\text{bal}}}{3} = 1 - \frac{0.329}{3} = 0.89$$

$$A_{s(\text{bal})} = \frac{M_{\text{bal}}}{p_{st} j_{\text{bal}} d} = \frac{(50 \text{ ft-kips})\left(12 \, \dfrac{\text{in}}{\text{ft}}\right)}{\left(24 \, \dfrac{\text{kips}}{\text{in}^2}\right)(0.89)(16 \text{ in})}$$

$$= 1.76 \text{ in}^2$$

$$f_c = \frac{2 A_s p_{st}}{b_w k d}$$

$$= \frac{(2)(1.76 \text{ in}^2)\left(24{,}000 \, \dfrac{\text{lbf}}{\text{in}^2}\right)}{(12 \text{ in})(0.329)(16 \text{ in})}$$

$$= 1334 \text{ lbf/in}^2$$

$$< p_{cb} \quad [\text{satisfactory}]$$

6. BEAMS IN SHEAR

Nomenclature

a_v	shear span, distance between concentrated load and face of supports	ft
A_{cs}	effective cross-sectional area of a strut in a strut-and-tie model taken perpendicular to the axis of the strut	in^2
A_{cp}	area enclosed by outside perimeter of concrete cross section	in^2
A_f	area of reinforcement in bracket or corbel resisting factored moment	in^2
A_h	area of shear reinforcement parallel to flexural tension reinforcement	in^2
A_l	total area of longitudinal reinforcement to resist torsion	in^2
A_n	area of reinforcement in bracket or corbel resisting tensile force N_{uc}	in^2
A_{nz}	effective cross-sectional area of the face of a nodal zone	in^2
A_o	gross area enclosed by shear flow, $0.85 A_{oh}$	in^2
A_{oh}	area enclosed by centerline of the outermost closed transverse torsional reinforcement	in^2
A_s	area of nonprestressed tension reinforcement	in^2
A_t	area of one leg of a closed stirrup resisting torsion within a distance s	in^2
A_v	area of shear reinforcement perpendicular to flexural tension reinforcement	in^2
A_{vf}	area of shear-friction reinforcement	in^2
A_{vh}	area of shear reinforcement parallel to flexural tension	in^2
b	width of a deep beam	in
b	width of compression face of member	in
b_w	web width or diameter of circular section	in
C	compressive force acting on a nodal zone	kips
f_{ce}	effective compressive strength of concrete in a strut or node	lbf/in^2
f_y	yield strength of reinforcement	lbf/in^2
f_{yt}	yield strength of transverse reinforcement	lbf/in^2

h	overall thickness of member	in
l_a	anchorage length of a reinforcing bar	in
l_b	width of bearing plate	in
l_n	clear span measured face-to-face of supports	ft
M_u	factored moment at section	ft-kips
N_{uc}	factored tensile force applied at top of corbel	kips
p_{cp}	outside perimeter of the concrete cross section	in
p_h	perimeter of centerline of outermost closed transverse torsional reinforcement	in
R	support reaction acting on a nodal zone	kips
s	spacing of shear or torsion reinforcement in direction parallel to longitudinal reinforcement	in
s_2	spacing of horizontal reinforcement	in
T	tension force acting on a nodal zone	kips
T_n	nominal torsional moment strength	ft-kips
T_u	factored torsional moment at section	ft-kips
V_c	nominal shear strength provided by concrete	kips
V_s	nominal shear strength provided by shear reinforcement	kips
V_u	factored shear force at section	kips
w_s	effective width of strut perpendicular to the axis of the strut	in
w_t	effective width of concrete concentric with a tie	in

### Symbols

α	angle between inclined stirrups and longitudinal axis of member	degrees
β_n	factor to account for the effect of the anchorage of ties on the effective compressive strength of a nodal zone	–
β_s	factor to account for the effect of cracking and confining reinforcement on the effective compressive strength of the concrete in a strut	–
λ	correction factor related to unit weight of concrete, as given in ACI Sec. 8.6.1	–
μ	coefficient of friction	–
ρ	reinforcement ratio, $A_s/b_w d$	–

Design for Shear

When the support reaction produces a compressive stress in the member, as shown in Fig. 1.6, the critical section for shear is located at a distance, d, from the support equal to the effective depth. This location is applicable only when loads are applied near or at the top of the beam, and no concentrated load occurs within a distance from the support equal to the effective depth. When the applied factored shear force, V_u, exceeds the shear capacity of the concrete, ϕV_c, shear reinforcement, with a capacity ϕV_s, is added to the section to give a combined shear capacity of

$$\phi V_c + \phi V_s > V_u \quad [\phi = 0.75]$$

Figure 1.6 *Critical Section for Shear*
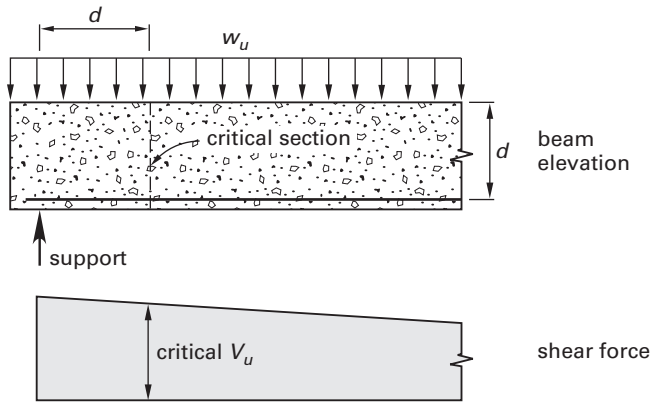
When V_u is less than $\phi V_c/2$, the concrete section is adequate to carry the shear without any shear reinforcement. Within the range $\phi V_c/2 \le V_u \le \phi V_c$, a minimum area of shear reinforcement is specified by ACI Sec. 11.4.6.3 as

$$A_{v(\min)} = \frac{0.75 b_w s \sqrt{f'_c}}{f_{yt}} \qquad \text{[ACI 11-13]}$$

$$\ge \frac{50 b_w s}{f_{yt}}$$

When $f'_c > 4.44$ kips/in^2, ACI Eq. 11-13 governs.

Shear Capacity of Concrete

The nominal shear capacity of the concrete section is given by ACI Sec. 11.2.1.1 as

$$V_c = 2 b_w d \lambda \sqrt{f'_c} \qquad \text{[ACI 11-3]}$$

This value is conservative and is usually sufficiently accurate. A more precise value is provided by ACI Sec. 11.2.2.1 as

$$V_c = \left(1.9\lambda\sqrt{f'_c} + \frac{2500\rho_w V_u d}{M_u}\right) b_w d \qquad \text{[ACI 11-5]}$$

$$\le 3.5\lambda\sqrt{f'_c} b_w d$$

$$\frac{V_u d}{M_u} \le 1.0$$

$$\sqrt{f'_c} \le 100 \ \text{lbf/in}^2$$

M_u is the factored moment occurring simultaneously with V_u at the section being analyzed.

$$\lambda = 1.0 \quad \text{[for normal weight concrete]}$$

$$= 0.85 \quad \text{[for sand lightweight concrete]}$$

$$= 0.75 \quad \text{[for all lightweight concrete]}$$

Shear Capacity of Stirrups

The nominal shear capacity of the inclined stirrups shown in Fig. 1.7 is given by ACI Sec. 11.4.7.4 as

$$V_s = \frac{A_v f_{yt}(\sin\alpha + \cos\alpha)d}{s} \qquad \text{[ACI 11-16]}$$

Figure 1.7 *Beam with Inclined Stirrups*
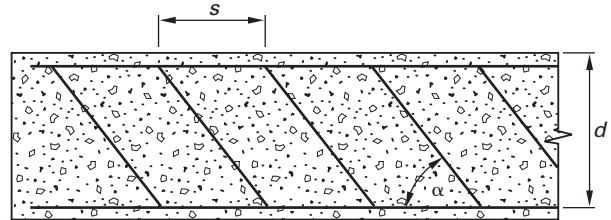
When the shear reinforcement is vertical, ACI Sec. 11.4.7.2 gives the nominal shear capacity as

$$V_s = \frac{A_v f_{yt} d}{s} \qquad \text{[ACI 11-15]}$$

The nominal shear strength of the shear reinforcement is limited by ACI Sec. 11.4.7.9 to a value of

$$V_s = 8\sqrt{f'_c} b_w d$$

If additional shear capacity is required, the size of the concrete section must be increased.

The spacing of the stirrups is limited to a maximum value of $d/2$ or 24 in when

$$V_s \le 4\sqrt{f'_c} b_w d$$

The spacing of the stirrups is limited to a maximum value of $d/4$ or 12 in when

$$V_s > 4\sqrt{f'_c} b_w d$$

Example 1.9

A reinforced concrete beam of normal weight concrete with an effective depth of 16 in and a width of 12 in is reinforced with grade 60 bars and has a concrete compressive strength of 3000 lbf/in^2. Determine the shear reinforcement required when

(a) the factored shear force $V_u = 9$ kips, the factored moment $M_u = 20$ ft-kips, and the reinforcement ratio $\rho_w = 0.015$

(b) the factored shear force $V_u = 14$ kips

(c) the factored shear force $V_u = 44$ kips

(d) the factored shear force $V_u = 71$ kips

(e) the factored shear force $V_u = 120$ kips

Solution

(a) The shear strength provided by the concrete is given by ACI Eq. 11-3 as

$$\phi V_c = 2\phi b_w d\lambda \sqrt{f_c'}$$

$$= \frac{(2)(0.75)(12\text{ in})(16\text{ in})(1.0)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}}{1000\ \frac{\text{lbf}}{\text{kip}}}$$

$$= 15.8\text{ kips}$$

$$< 2V_u$$

Using ACI Eq. 11-5 to verify the section gives

$$\frac{V_u d}{M_u} = \frac{(9\text{ kips})(16\text{ in})}{(20\text{ ft-kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)}$$

$$= 0.60$$

$$< 1.0 \quad \text{[satisfactory]}$$

$$\phi V_c = \phi\left(1.9\lambda\sqrt{f_c'} + \frac{2500\rho_w V_u d}{M_u}\right) b_w d$$

$$= \frac{(0.75)\left(\begin{array}{c}(1.9)(1.0)\sqrt{3000\ \frac{\text{lbf}}{\text{in}^2}}\\ + \dfrac{(2500)(0.015)(9\text{ kips})(16\text{ in})}{(20\text{ ft-kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)}\end{array}\right)}{1000\ \frac{\text{lbf}}{\text{kip}}}$$
$$\times (12\text{ in})(16\text{ in})$$

$$= 18.2\text{ kips}$$

$$> 2V_u$$

In accordance with ACI Sec. 11.4.6.1, shear reinforcement is not required.

(b) Because $\phi V_c/2 < V_u < \phi V_c$, the minimum shear reinforcement specified by ACI Sec. 11.4.6.3 is required, and this is given by

$$\frac{A_{v(\text{min})}}{s} = \frac{50 b_w}{f_y} \quad \text{[governs]}$$

$$= \frac{\left(50\ \frac{\text{lbf}}{\text{in}^2}\right)(12\text{ in})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{60{,}000\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.12\text{ in}^2/\text{ft}$$

Shear reinforcement consisting of two arms of no. 3 bars at 8 in spacing provides a reinforcement area of

$$\frac{A_v}{s} = 0.33\text{ in}^2/\text{ft}$$

The spacing of 8 in does not exceed $d/2$ and is satisfactory.

(c) The factored shear force exceeds the shear strength of the concrete, and the shear strength required from shear reinforcement is given by ACI Eq. 11-1 and Eq. 11-2 as

$$\phi V_s = V_u - \phi V_c = 44\text{ kips} - 15.8\text{ kips}$$

$$= 28.2\text{ kips}$$

$$< 2\phi V_c$$

In accordance with ACI Sec. 11.4.5.1, stirrups are required at a maximum spacing of $d/2 = 8$ in. The area of shear reinforcement required is given by ACI Eq. 11-15 as

$$\frac{A_v}{s} = \frac{\phi V_s}{\phi d f_{yt}} = \frac{(28.2\text{ kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{(0.75)(16\text{ in})\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}$$

$$= 0.47\text{ in}^2/\text{ft}$$

Shear reinforcement consisting of two arms of no. 4 bars at 8 in spacing provides a reinforcement area of

$$\frac{A_v}{s} = 0.60\text{ in}^2/\text{ft}$$

$$> 0.47\text{ in}^2/\text{ft} \quad \text{[satisfactory]}$$

(d) The shear strength required from the shear reinforcement is given by ACI Eq. 11-1 and Eq. 11-2 as

$$\phi V_s = V_u - \phi V_c$$

$$= 71\text{ kips} - 15.8\text{ kips}$$

$$= 55.2\text{ kips}$$

$$> 2\phi V_c$$

In accordance with ACI Sec. 11.4.5.3, stirrups are required at a maximum spacing of $d/4 = 4$ in. The area of shear reinforcement required is given by ACI Eq. 11-15 as

$$\frac{A_v}{s} = \frac{\phi V_s}{\phi d f_{yt}} = \frac{(55.2\text{ kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{(0.75)(16\text{ in})\left(60\ \frac{\text{kips}}{\text{in}^2}\right)}$$

$$= 0.92\text{ in}^2/\text{ft}$$

Shear reinforcement consisting of two arms of no. 4 bars at 4 in spacing provides a reinforcement area of

$$\frac{A_v}{s} = 1.2\text{ in}^2/\text{ft}$$

$$> 0.92 \quad \text{[satisfactory]}$$

(e) The shear strength required from the shear reinforcement is given by ACI Eq. 11-1 and Eq. 11-2 as

$$\phi V_s = V_u - \phi V_c$$
$$= 120 \text{ kips} - 15.8 \text{ kips}$$
$$= 104.2 \text{ kips}$$
$$> 4\phi V_c$$

In accordance with ACI Sec. 11.4.7.9, the section size is inadequate.

Shear Capacity of Inclined Bars

When a single, bent-up bar or group of bars equidistant from the support is used as shear reinforcement, the nominal shear capacity is given by ACI Sec. 11.4.7.5 as

$$V_s = A_v f_y \sin \alpha \qquad [\textit{ACI 11-17}]$$
$$\leq 3 b_w d \sqrt{f_c'}$$

When a series of equally spaced bent-up bars is used, as shown in Fig. 1.8, the nominal shear capacity is given by ACI Sec. 11.4.7.4 as

$$V_s = \frac{A_v f_y (\sin \alpha + \cos \alpha) d}{s} \qquad [\textit{ACI 11-16}]$$

Figure 1.8 *Beam with Inclined Bars*
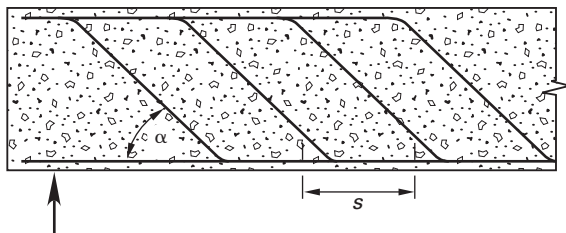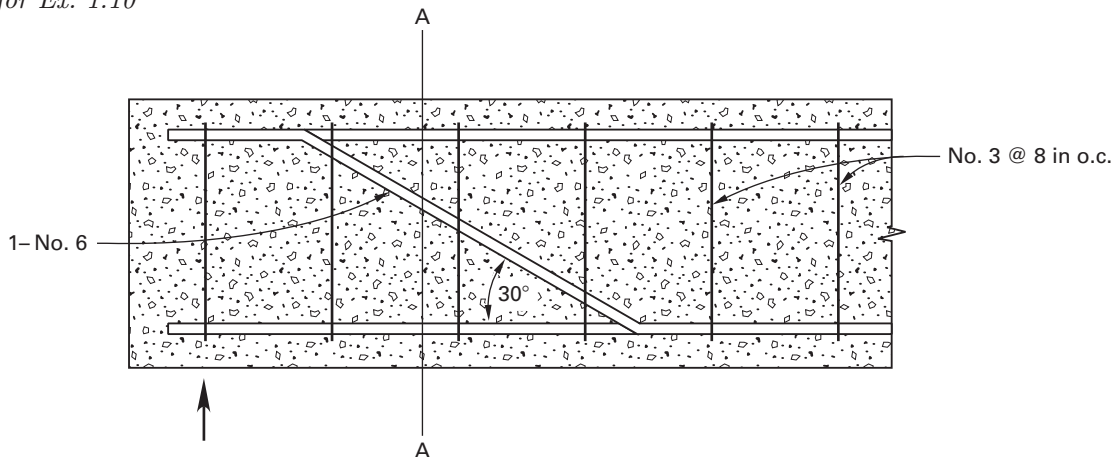
Only the center three-fourths of the inclined bar is considered effective; this limits the spacing, measured in a direction parallel to the longitudinal reinforcement, to a maximum value of

$$s_{max} = 0.375 d (1 + \cot \alpha)$$

This value is halved, in accordance with ACI Sec. 11.4.5.3, when V_s exceeds $4\sqrt{f_c'} b_w d$.

In accordance with ACI Sec. 11.4.1.2, the minimum permitted angle of inclination of the inclined bars is 30°. When shear reinforcement consists of both stirrups and inclined bars, the total combined shear resistance is given by the sum of the shear resistances of each type. The nominal combined shear resistance shall not exceed $8\sqrt{f_c'} b_w d$.

Example 1.10

The reinforced concrete beam of normal weight concrete shown has an effective depth of 16 in, a width of 12 in, a concrete compressive strength of 3000 lbf/in², and is reinforced with grade 60 bars. Determine the design shear capacity provided at section A-A.

Solution

The nominal shear strength of the concrete is given by ACI Eq. 11-3 as

$$V_c = 2 b_w d \lambda \sqrt{f_c'}$$
$$= \frac{(2)(12 \text{ in})(16 \text{ in})(1.0)\sqrt{3000 \dfrac{\text{lbf}}{\text{in}^2}}}{1000 \dfrac{\text{lbf}}{\text{kip}}}$$
$$= 21.0 \text{ kips}$$

Illustration for Ex. 1.10

The nominal shear strength of the vertical stirrups is given by ACI Eq. 11-15 as

$$V_{s(\text{str})} = \frac{A_v f_{yt} d}{s} = \frac{\left(0.33 \ \frac{\text{in}^2}{\text{ft}}\right)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)(16 \ \text{in})}{12 \ \frac{\text{in}}{\text{ft}}}$$

$$= 26.4 \ \text{kips}$$

The nominal shear strength of the inclined bar is given by ACI Eq. 11-17 as

$$V_{s(\text{bar})} = A_v f_y \sin\alpha = (0.44 \ \text{in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)(\sin 30°)$$

$$= 13.2 \ \text{kips}$$

The combined shear strength of the shear reinforcement is

$$V_s = V_{s(\text{str})} + V_{s(\text{bar})}$$

$$= 39.6 \ \text{kips}$$

$$< 2V_c$$

In accordance with ACI Sec. 11.4.5.1, stirrup spacing must not exceed $d/2 = 8$ in, and the spacing provided is satisfactory. The minimum shear reinforcement required is specified by ACI Eq. 11-13 as

$$\frac{A_v}{s} = \frac{50 b_w}{f_{yt}} \quad [\text{governs}]$$

$$= \frac{\left(50 \ \frac{\text{lbf}}{\text{in}^2}\right)(12 \ \text{in})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{60,000 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.12 \ \text{in}^2/\text{ft}$$

$$< 0.33 \quad [\text{satisfactory}]$$

The total design shear capacity at section A-A is

$$\phi V_n = \phi(V_c + V_s) = (0.75)(21 \ \text{kips} + 39.6 \ \text{kips})$$

$$= 45.5 \ \text{kips}$$

7. DEEP BEAMS

As shown in Fig. 1.9, a *deep beam*, as defined in ACI Sec. 10.7.1 and Sec. 11.7.1, is a beam in which the ratio of clear span to overall depth does not exceed 4. Deep beam conditions also apply to regions of beams loaded with concentrated loads within twice the beam depth from a support. The nominal shear strength of a deep beam is limited by ACI Sec. 11.7.3 to a maximum of

$$V_n = 10 b_w d \sqrt{f_c'}$$

As indicated in Fig. 1.9, minimum areas of vertical and horizontal reinforcement are specified in ACI Sec. 11.7.4 in order to restrain cracking. Alternatively, reinforcement may be provided as specified in ACI Sec. A.3.3.

Strut-and-Tie Model[8]

In accordance with ACI Sec. 11.7.2, deep beams must be designed using either nonlinear analysis or by the *strut-and-tie* method given in ACI App. A. In the strut-and-tie method, a member is divided, as shown in Fig. 1.10, into *discontinuity*, or *D-regions*, in which the beam theory of ACI Sec. 10.2 does not apply, and *B-regions* in which beam theory does apply. In addition, for the strut-and-tie method to apply, the deep beam must be loaded so that compression struts can develop between the loads and the supports.

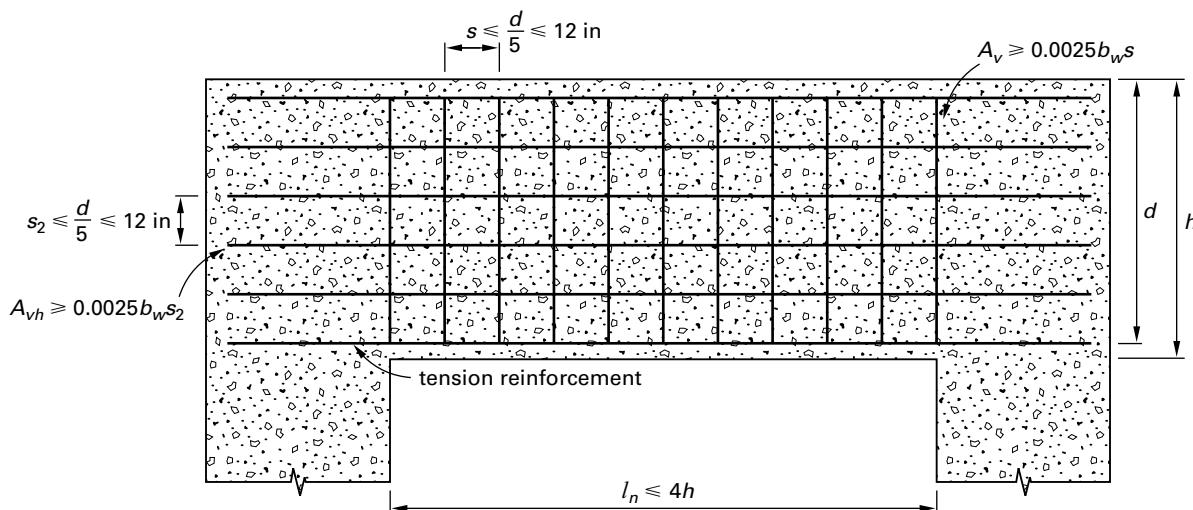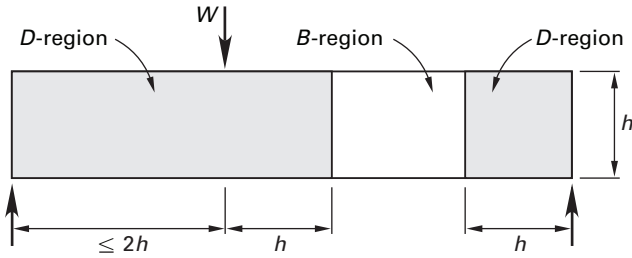**Figure 1.9** *Minimum Shear Reinforcement for a Deep Beam*

Figure 1.10 B- and D-Regions

As shown in Fig. 1.11, a strut-and-tie model may be constructed to represent the internal forces in a deep beam. Compression struts are formed in the concrete to resist compressive forces. The strength of these struts is governed by the transverse tension developed by the lateral spread of the applied compression force. Using crack control reinforcement, as specified in ACI Sec. A.3.3, to resist the transverse tension increases the strength of the strut.

As shown in Fig. 1.12, struts may be either prism shaped or bottle shaped.

Figure 1.11 Strut-and-Tie Model
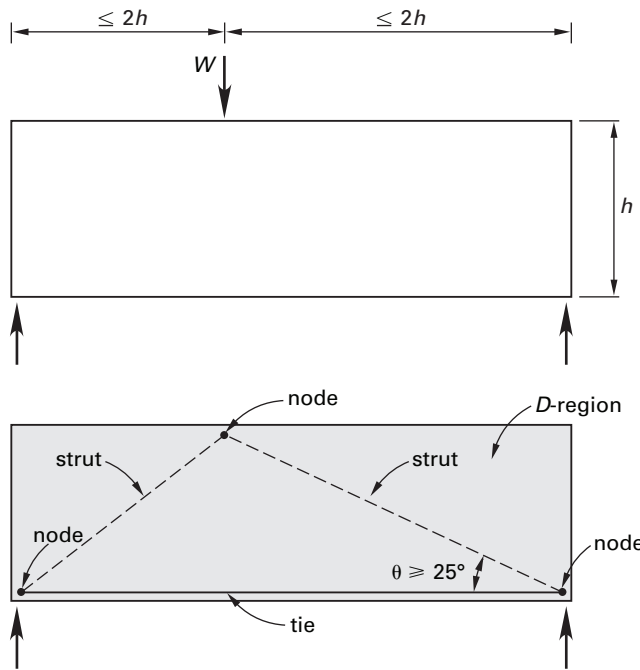
Figure 1.12 Prism and Bottle-Shaped Struts
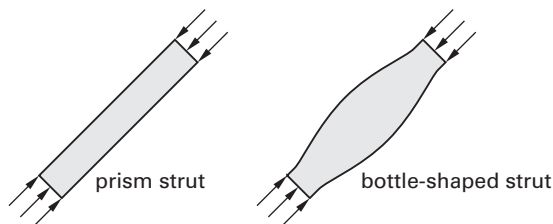
Ties consist of tension reinforcement and the surrounding concrete that is concentric with the axis of the tie. The concrete does not contribute to the strength of the tie.

Nodes occur where the axes of struts, ties, concentrated loads, and support reactions acting on the joint intersect. The angle between the axes of a strut and a tie at a node is limited by ACI Sec. A.2.5 to a minimum of $\theta = 25°$ in order to mitigate cracking.

Strut Nominal Strength

The effective compressive strength of the concrete in a strut is specified in ACI Sec. A.3.2 as

$$f_{ce} = 0.85\beta_s f_c' \qquad \text{[ACI A-3]}$$

$\beta_s = 1.0 \quad \left[\begin{array}{l}\text{for a strut of uniform cross section,} \\ \text{as in the compression zone of a beam}\end{array}\right]$

$\beta_s = 0.60\lambda \quad \left[\begin{array}{l}\text{for an unreinforced,} \\ \text{bottle-shaped strut}\end{array}\right]$

$\beta_s = 0.75 \quad \left[\begin{array}{l}\text{for a bottle-shaped strut with reinforce-} \\ \text{ment as specified in ACI Sec. A.3.3}\end{array}\right]$

$\beta_s = 0.40 \quad \left[\begin{array}{l}\text{for struts in a tension member} \\ \text{or the tension flange of a member}\end{array}\right]$

$\beta_s = 0.60\lambda \quad \text{[for all other cases]}$

$\lambda = 1.0 \quad \text{[for normal weight concrete]}$

$\lambda = 0.85 \quad \text{[for sand lightweight concrete]}$

$\lambda = 0.75 \quad \text{[for all lightweight concrete]}$

In accordance with ACI Sec. A.3.1, the nominal compressive strength of a strut is

$$F_{ns} = f_{ce}A_{cs} \qquad \text{[ACI A-2]}$$
$$= f_{ce}w_s b$$

The strength reduction factor for strut-and-tie models is given by ACI Sec. 9.3.2.6 as

$$\phi = 0.75$$

Tie Nominal Strength

In a reinforced concrete beam, the nominal strength of a reinforcing bar acting as a tie is given by ACI Sec. A.4.1 as

$$F_{nt} = A_{ts}f_y$$

If the bars in a tie are in one layer, as shown in Fig. 1.13, the width of the tie may be taken as the diameter of the bars in the tie plus twice the cover to the surface of the bars.

Nodal Zone Nominal Strength

As defined in ACI Sec. A.1 and shown in Fig. 1.13, a *nodal zone* is the volume of concrete surrounding a node that is assumed to transfer strut-and-tie forces through the node. The node illustrated in Fig. 1.13 is classified as C-C-T, with two of the members acting on the node in compression and the third member in tension. Similarly, when all three members acting on the node are in compression, the node is classified as C-C-C. The effective compressive strength of the concrete in a node is specified in ACI Sec. A.5.2 as

$$f_{ce} = 0.85 \beta_n f_c' \qquad [\text{ACI A-8}]$$

$$
\beta_n = 1.0 \quad \left[\begin{array}{c} \text{for a nodal zone bounded on all sides} \\ \text{by struts or bearing areas or both} \end{array} \right]
$$

$$
= 0.80 \quad \left[\begin{array}{c} \text{for a nodal zone anchoring} \\ \text{one tie} \end{array} \right]
$$

$$
= 0.60 \quad \left[\begin{array}{c} \text{for a nodal zone anchoring} \\ \text{two or more ties} \end{array} \right]
$$

In accordance with ACI Sec. A.5.1, the nominal compressive strength of a nodal zone is

$$F_{nn} = f_{ce} A_{nz} \qquad [\text{ACI A-7}]$$
$$= f_{ce} w_s b$$

Figure 1.13 Nodal Zone
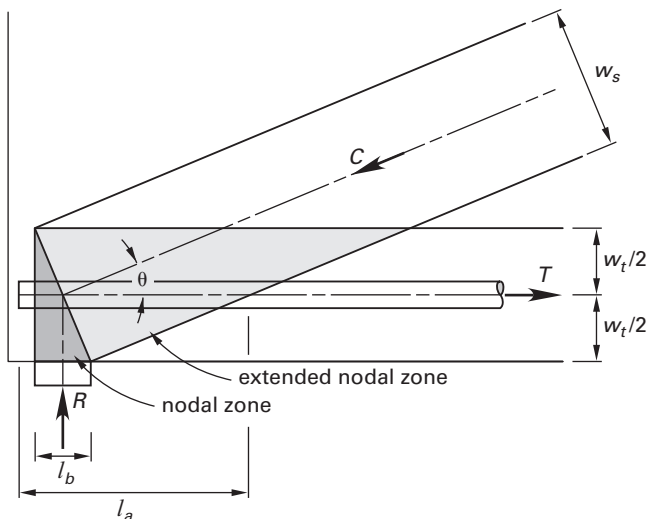
The faces of the nodal zone shown in Fig. 1.13 are perpendicular to the axes of the strut, tie, and bearing plate, and the lengths of the faces are in direct proportion to the forces acting. The node has equal stresses on all faces and is termed a *hydrostatic nodal zone*. The effective width of the strut shown in Fig. 1.13 is

$$w_s = w_t \cos\theta + l_b \sin\theta$$

The *extended nodal zone* shown in Fig. 1.13 is that portion of the member bounded by the intersection of the effective strut width and the effective tie width. As specified in ACI Sec. A.4.3.2, the anchorage length of the reinforcement is measured from the point of intersection of the bar and the extended nodal zone. The reinforcement may be anchored by a plate, by hooks, or by a straight development length.

Modeling Procedure

The modeling procedure for struts, ties, and nodes involves the following steps.

step 1: Determine the reactions on the model.

step 2: Select the location of the members by aligning the direction of struts in the direction of the anticipated cracking.

step 3: Determine the areas of struts, ties, and nodes necessary to provide the required strength.

step 4: Provide anchorage for the ties.

step 5: Provide crack control reinforcement.

Example 1.11

A reinforced concrete beam of normal weight concrete with a clear span of 6 ft, an effective depth of 26 in, and a width of 12 in, as shown, has a concrete compressive strength of 4500 lbf/in². The factored applied force of 80 kips includes an allowance for the self weight of the beam. Determine the number of grade 60, no. 8 bars required for tension reinforcement and check that the equivalent concrete strut and nodal zone at the left support comply with the requirements of ACI App. A.

Solution

The clear span-to-depth ratio is given by

$$\frac{l_n}{h} = \frac{72 \text{ in}}{28 \text{ in}}$$
$$= 2.6$$
$$< 4 \quad [\text{satisfies ACI Sec. 11.7.1}]$$

Illustration for Ex. 1.11

strut-and-tie model

nodal zone
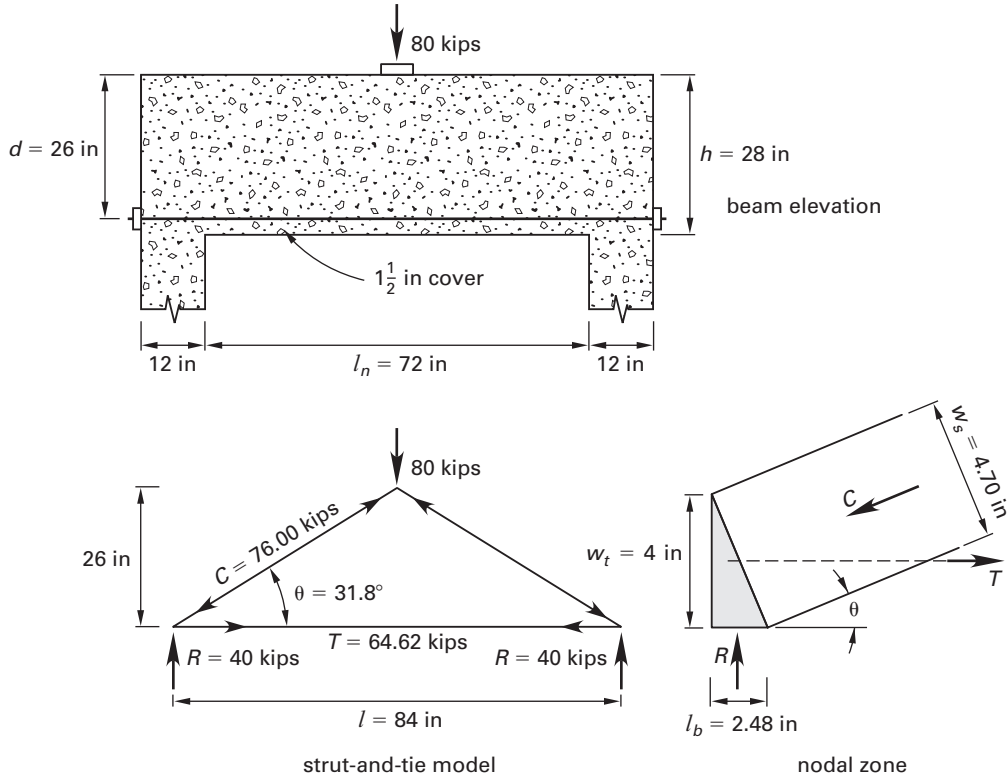
The idealized strut-and-tie model is shown in the illustration, and the angle between the struts and the tie is

$$\theta = \arctan \frac{26 \text{ in}}{42 \text{ in}}$$
$$= 31.8°$$
$$> 25° \quad \text{[satisfies ACI Sec. A.2.5]}$$

The equivalent tie force is determined from the strut-and-tie model as

$$T = \frac{(40 \text{ kips})(42 \text{ in})}{26 \text{ in}}$$
$$= 64.62 \text{ kips}$$

The strength reduction factor is given by ACI Sec. 9.3.2.6 as

$$\phi = 0.75$$

The necessary reinforcement area is given by

$$A_{ts} = \frac{T}{\phi f_y}$$
$$= \frac{64.62 \text{ kips}}{(0.75)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)}$$
$$= 1.44 \text{ in}^2$$

Use two no. 8 bars, which gives an area of

$$A = 1.58 \text{ in}^2$$
$$> A_{ts} \quad \text{[satisfactory]}$$

As shown in the illustration, the dimensions of the nodal zone are

$$w_t = \text{equivalent tie width}$$
$$= d_b + 2c$$
$$= 1 \text{ in} + (2)(1.5 \text{ in})$$
$$= 4 \text{ in}$$

$$l_b = \text{width of equivalent support strut}$$
$$= w_t \tan \theta$$
$$= (4 \text{ in})\tan 31.8°$$
$$= 2.48 \text{ in}$$

$$w_s = \text{width of equivalent concrete strut}$$
$$= \frac{w_t}{\cos \theta}$$
$$= \frac{4 \text{ in}}{\cos 31.8°}$$
$$= 4.70 \text{ in}$$

The stress in the equivalent tie is

$$f_T = \frac{T}{bw_t} = \frac{64.62 \text{ kips}}{(12 \text{ in})(4 \text{ in})} = 1.35 \text{ kips/in}^2$$

For a hydrostatic nodal zone,

f_C = stress in the equivalent concrete strut

$\quad = f_T$

$\quad = 1.35 \text{ kips/in}^2$

f_R = stress in the equivalent support strut

$\quad = f_T$

$\quad = 1.35 \text{ kips/in}^2$

For an unreinforced, bottle-shaped strut with normal weight concrete, the design compressive strength of the concrete in the strut is given by ACI Eq. A-3 as

$$\phi f_{ce} = 0.85\phi\beta_s f_c' = (0.85)(0.75)(0.6)(1.0)\left(4.5 \ \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 1.72 \text{ kips/in}^2$$

$$> f_C \quad \text{[satisfactory]}$$

The design compressive strength of a nodal zone anchoring one layer of reinforcing bars without confining reinforcement is given by ACI Eq. A-8 as

$$\phi f_{ce} = \phi 0.85\beta_n f_c' = (0.75)(0.85)(0.8)\left(4.5 \ \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 2.30 \text{ kips/in}^2$$

$$> f_C \quad \text{[satisfactory]}$$

The anchorage length available for the tie reinforcement, using 2 in end cover, is

$$l_a = \frac{h-d}{\tan\theta} + \frac{l_b}{2} + 6 \text{ in} - 2 \text{ in}$$

$$= \frac{28 \text{ in} - 26 \text{ in}}{\tan 31.8°} + \frac{2.48 \text{ in}}{2} + 6 \text{ in} - 2 \text{ in}$$

$$= 8.47 \text{ in}$$

The development length for a grade 60, no. 8 bar, with 2.5 in side cover and 2 in end cover and with a standard 90° hook, is given by ACI Sec. 12.5.2 as

$$l_{dh} = \frac{(0.7)\left(1200 \ \frac{\text{lbf}}{\text{in}^2}\right)d_b}{\sqrt{f_c'}} = \frac{(0.7)\left(1200 \ \frac{\text{lbf}}{\text{in}^2}\right)(1 \text{ in})}{\sqrt{4500 \ \frac{\text{lbf}}{\text{in}^2}}}$$

$$= 12.5 \text{ in}$$

$$> l_a \quad \text{[Anchorage length is inadequate.]}$$

Use an end plate to anchor the bars.

8. CORBELS

A corbel is a cantilever bracket supporting a load-bearing member. As shown in Fig. 1.14 and specified in ACI Sec. 11.8, the shear span-to-depth ratio, a/d, and the ratio of horizontal tensile force to vertical force, N_{uc}/V_u, are limited to a maximum value of unity. The depth of the corbel at the outside edge of bearing area must not be less than $d/2$.

Figure 1.14 *Corbel Details*
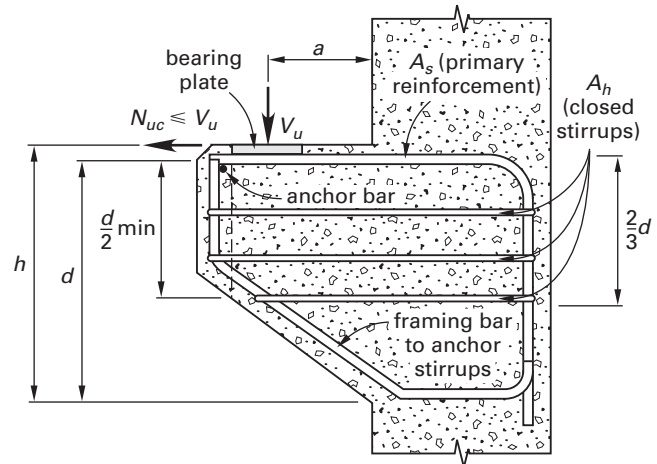
At the face of the support, the forces acting on the corbel are a shear force, V_u, a moment $(V_u a + N_{uc}(h-d))$, and a tensile force, N_{uc}. These require reinforcement areas of A_{vf}, A_f, and A_n, respectively. The shear friction reinforcement area is specified by ACI Sec. 11.8.3.2 and derived from ACI Eq. 11-25 as

$$A_{vf} = \frac{V_u}{\phi f_y \mu}$$

Also, from ACI Sec. 11.8.3.2.1, the factored shear force on the section is

$$V_u \leq 0.2\phi f_c' b_w d$$

$$\leq (480 + 0.08 f_c')\phi b_w d$$

$$\leq 1600\phi b_w d$$

μ = coefficient of friction at face of support, as given by ACI Sec. 11.6.4.3

$\quad = 1.4\lambda \quad \text{[for concrete placed monolithically]}$

The correction factor related to the unit weight of concrete is defined by ACI Sec. 11.6.4.3 as

$\lambda = 1.0 \quad \text{[for normal weight concrete]}$

$\quad = 0.75 \quad \text{[for all lightweight concrete]}$

The tensile force N_{uc} may not be less than $0.2V_u$, and the corresponding area of reinforcement required is given by ACI Sec. 11.8.3.4 as

$$A_n = \frac{N_{uc}}{\phi f_y}$$

The required area of primary tension reinforcement is given by ACI Sec. 11.8.3.5 and Sec. 11.8.5 as

$$A_{sc} = A_f + A_n$$
$$\geq \frac{2A_{vf}}{3} + A_n$$
$$\frac{A_{sc}}{bd} \geq \frac{0.04f_c'}{f_y}$$

The minimum required area of closed ties distributed over a depth of $2d/3$ is given by ACI Sec. 11.8.4 as

$$A_h = \frac{A_{sc} - A_n}{2}$$

ACI Sec. 11.8.3.1 gives the value of the strength reduction factor as $\phi = 0.75$.

Example 1.12

The reinforced concrete corbel shown has a width of 15 in, is reinforced with grade 60 bars, and has a concrete compressive strength of 3000 lbf/in². Determine whether the corbel is adequate for the applied factored loads indicated.
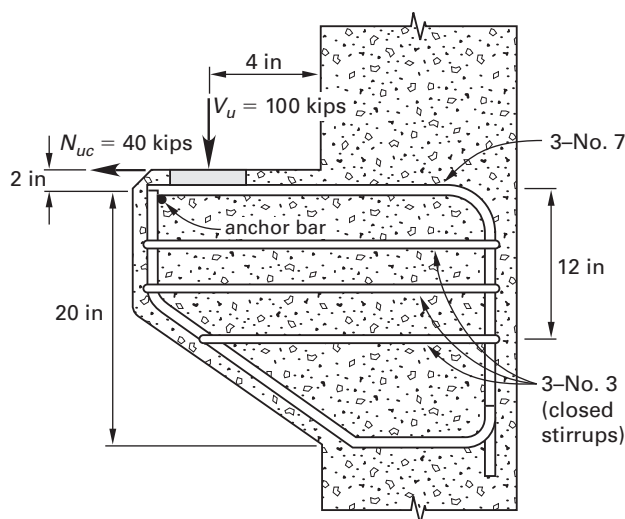
Solution

$$0.2\phi f_c' b_w d = (0.2)(0.75)\left(3\ \frac{\text{kips}}{\text{in}^2}\right)(15\text{ in})(20\text{ in})$$
$$= 135\text{ kips}$$
$$> V_u$$
$$(480 + 0.08f_c')\phi b_w d = (0.72)(0.75)(15\text{ in})(20\text{ in})$$
$$= 162\text{ kips}$$
$$> V_u$$
$$1.6\phi b_w d = (1.6)(0.75)(15\text{ in})(20\text{ in})$$
$$= 360\text{ kips}$$
$$> V_u$$

The corbel conforms to ACI Sec. 11.8.3.2.

The shear friction reinforcement area is given by ACI Sec. 11.6.4 as

$$A_{vf} = \frac{V_u}{\phi f_y \mu} = \frac{100\text{ kips}}{(0.75)\left(60\ \dfrac{\text{kips}}{\text{in}^2}\right)(1.4)}$$
$$= 1.59\text{ in}^2$$

The tension reinforcement area is given by ACI Sec. 11.8.3.4 as

$$A_n = \frac{N_{uc}}{\phi f_y} = \frac{40\text{ kips}}{(0.75)\left(60\ \dfrac{\text{kips}}{\text{in}^2}\right)}$$
$$= 0.889\text{ in}^2$$

The factored moment acting on the corbel is

$$M_u = V_u a + N_{uc}(h - d)$$
$$= (100\text{ kips})(4\text{ in}) + (40\text{ kips})(2\text{ in})$$
$$= 480\text{ in-kips}$$

The area of flexural reinforcement required for $\phi = 0.75$ as given by ACI Sec. R11.8.3.1 is

$$A_f = \frac{0.85bdf_c'\left(1 - \sqrt{1 - \dfrac{M_u}{0.319b_w d^2 f_c'}}\right)}{f_y}$$
$$= 0.545\text{ in}^2$$

The primary reinforcement area required is given by ACI Sec. 11.8.3.5 as

$$A_{sc} = A_f + A_n = 0.545\text{ in}^2 + 0.889\text{ in}^2$$
$$= 1.434\text{ in}^2$$

Three no. 7 bars are provided, giving an area of

$$A_s' = 1.80 \text{ in}^2$$
$$> 1.434 \text{ in}^2 \quad \text{[satisfactory]}$$

Also, from ACI Sec. 11.8.3.5, the area of primary reinforcement must not be less than

$$\frac{2A_{vf}}{3} + A_n = \frac{(2)(1.59 \text{ in}^2)}{3} + 0.889 \text{ in}^2$$
$$= 1.95 \text{ in}^2$$
$$> A_s' \quad \text{[unsatisfactory]}$$

The area of closed stirrups required is given by ACI Sec. 11.8.4 as

$$A_h = \frac{A_{sc} - A_n}{2} = \frac{1.95 \text{ in}^2 - 0.889 \text{ in}^2}{2}$$
$$= 0.53 \text{ in}^2$$

Three no. 3 closed stirrups are provided, giving an area of

$$A_h = 0.66 \text{ in}^2$$
$$> 0.53 \text{ in}^2 \quad \text{[satisfactory]}$$

9. BEAMS IN TORSION

The terminology used in torsion design is illustrated in Fig. 1.15 and Fig. 1.16.

Figure 1.15 Torsion in Rectangular Section
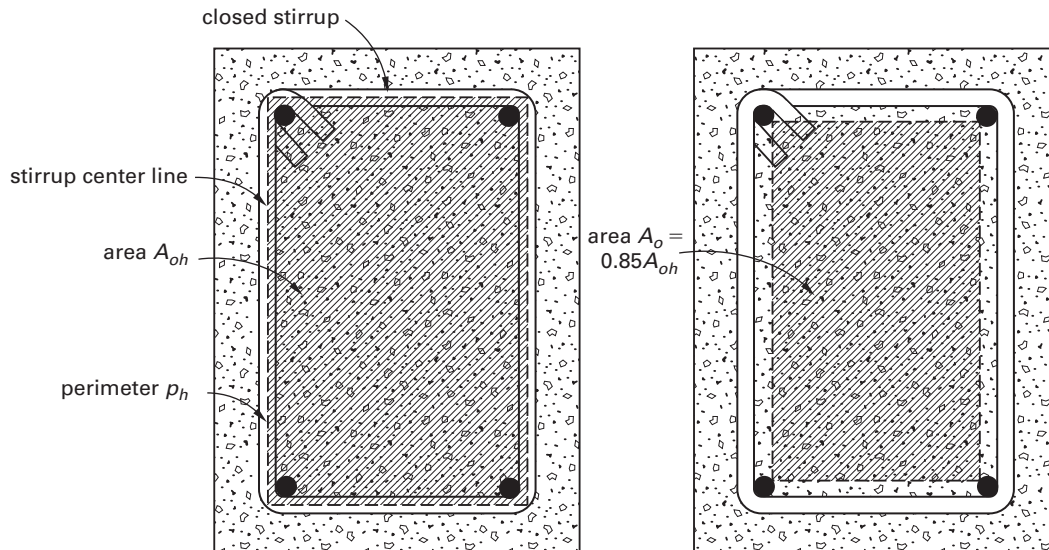
Figure 1.16 Torsion in Flanged Section
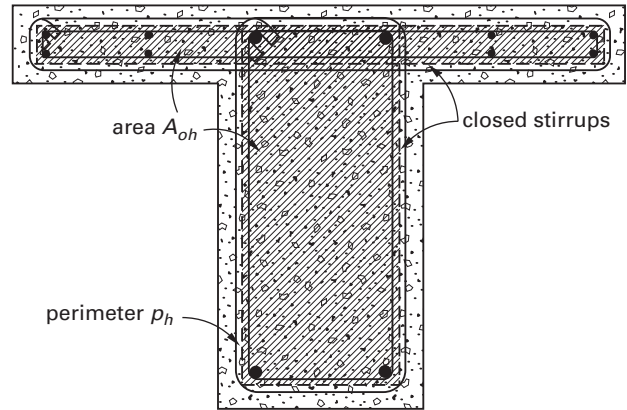
In accordance with ACI Sec. 11.5.1, for a statically determinate member the equilibrium torsional effects may be neglected, and closed stirrups and longitudinal torsional reinforcement are not required when the factored torque does not exceed

$$T_u = \phi\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

When this value is exceeded, reinforcement must be provided to resist the full torsion. When both shear and torsion reinforcements are required, the sum of the individual areas must be provided.

ACI Sec. 11.5.3.6 specifies the required area of one leg of a closed stirrup as

$$\frac{A_t}{s} = \frac{T_u}{2\phi A_o f_{yt}} \quad \begin{bmatrix} \text{for compression} \\ \text{diagonals at } 45° \end{bmatrix}$$
$$= \frac{T_u}{1.7\phi A_{oh} f_{yt}} \qquad [ACI\ 11\text{-}21]$$

The corresponding area of longitudinal reinforcement required is specified in ACI Sec. 11.5.3.7 as

$$A_l = \frac{A_t p_h f_{yt}}{f_y s} \qquad \text{[ACI 11-22]}$$

The minimum area of longitudinal reinforcement required is specified in ACI Sec. 11.5.5.3 as

$$A_l = \frac{5 A_{cp} \sqrt{f_c'}}{f_y} - \frac{A_t p_h f_{yt}}{f_y s} \qquad \text{[ACI 11-24]}$$

$$\frac{A_t}{s} \geq \frac{25 b_w}{f_{yt}}$$

The minimum diameter, specified in ACI Sec. 11.5.6.2, is

$$d_b = 0.042 s$$
$$\geq \text{no. 3 bar}$$

The minimum combined area of stirrups for combined shear and torsion is given by ACI Sec. 11.5.5.2 as

$$\frac{A_v + 2 A_t}{s} = \frac{0.75 \sqrt{f_c'} b_w}{f_{yt}} \qquad \text{[ACI 11-23]}$$

$$\geq \frac{50 b_w}{f_{yt}}$$

The maximum spacing of closed stirrups is given by ACI Sec. 11.5.6.1 as

$$s = \frac{p_h}{8}$$
$$\leq 12 \text{ in}$$

In accordance with ACI Sec. 11.5.2.2, when redistribution of internal forces occurs in an indeterminate structure upon cracking, a member may be designed for the factored torsion causing cracking, which is given by

$$T_u = 4\phi \sqrt{f_c'} \left(\frac{A_{cp}^2}{p_{cp}} \right)$$

Example 1.13

A simply supported reinforced concrete beam of normal weight concrete with an overall depth of 19 in, an effective depth of 16 in, and a width of 12 in is reinforced with grade 60 bars and has a concrete compressive strength of 3000 lbf/in² . Determine the combined shear and torsion reinforcement required when

(a) the factored shear force is 5 kips and the factored torsion is 2 ft-kips

(b) the factored shear force is 15 kips and the factored torsion is 4 ft-kips

Solution

(a) The area enclosed by the outside perimeter of the beam is

$$A_{cp} = (19 \text{ in})(12 \text{ in})$$
$$= 228 \text{ in}^2$$

The length of the outside perimeter of the beam is

$$p_{cp} = (2)(19 \text{ in} + 12 \text{ in})$$
$$= 62 \text{ in}$$

Torsional reinforcement is not required in accordance with ACI Sec. 11.5.1 when the factored torque does not exceed

$$T_u = \phi \lambda \sqrt{f_c'} \left(\frac{A_{cp}^2}{p_{cp}} \right)$$

$$= (0.75)(1.0) \sqrt{3000 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$\times \left(\frac{(228 \text{ in}^2)^2}{(62 \text{ in})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)} \right)$$

$$= 2.87 \text{ ft-kips}$$

$$> 2.0 \quad \text{[Closed stirrups are not required.]}$$

The shear strength provided by the concrete was determined in Ex. 1.9 as

$$\phi V_c = 15.8 \text{ kips}$$

$$> 2 V_u \quad \text{[Shear stirrups are not required.]}$$

(b) Because $\phi V_c / 2 < V_u < \phi V_c$, minimum shear reinforcement is required, and because $T_u > 2.87$ ft-kips, closed stirrups are necessary. Using no. 3 stirrups with 1.5 in cover, the area enclosed by the centerline of the stirrups is

$$A_{oh} = (19 \text{ in} - 3 \text{ in} - 0.375 \text{ in})$$
$$\times (12 \text{ in} - 3 \text{ in} - 0.375 \text{ in})$$
$$= 134.77 \text{ in}^2$$

From ACI Eq. 11-21, the required area of one arm of a closed stirrup is given by

$$\frac{A_t}{s} = \frac{T_u}{1.7 \phi A_{oh} f_{yt}}$$

$$= \frac{(4 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{(1.7)(0.75)\left(\dfrac{134.77 \text{ in}^2}{12 \ \frac{\text{in}}{\text{ft}}} \right)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)}$$

$$= 0.056 \text{ in}^2/\text{ft} \quad \text{[per arm]}$$

From ACI Eq. 11-23, the governing minimum combined shear and torsion reinforcement area is given by

$$\frac{A_v + 2A_t}{s} = \frac{50b_w}{f_{yt}}$$

$$= \frac{(50)(12 \text{ in})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.12 \text{ in}^2/\text{ft} \quad [\text{governs}]$$

The perimeter of the centerline of the closed stirrups is

$$p_h = (2)\big(19 \text{ in} + 12 \text{ in} - (2)(3.375 \text{ in})\big)$$

$$= 48.50 \text{ in}$$

The governing maximum permissible spacing of the closed stirrups is specified in ACI Sec. 11.5.6.1 as

$$s_{\max} = \frac{p_h}{8}$$

$$= \frac{48.50 \text{ in}}{8}$$

$$= 6 \text{ in}$$

Closed stirrups consisting of two arms of no. 3 bars at 6 in spacing provides an area of

$$\frac{A}{s} = 0.44 \text{ in}^2/\text{ft}$$

$$> 0.12 \quad [\text{satisfactory}]$$

The required area of the longitudinal reinforcement is given by ACI Eq. 11-22 as

$$A_l = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yt}}{f_y}\right)$$

$$= \frac{\left(0.056 \ \frac{\text{in}^2}{\text{ft}}\right)(48.50 \text{ in})\left(\dfrac{60 \ \frac{\text{kips}}{\text{in}^2}}{60 \ \frac{\text{kips}}{\text{in}^2}}\right)}{12 \ \frac{\text{in}}{\text{ft}}}$$

$$= 0.23 \text{ in}^2$$

Because the required value of $A_t/s = 0.056$ in^2/ft per arm is less than $25b_w/f_{yt} = 0.060$ in^2/ft, the minimum

permissible area of longitudinal reinforcement is given by ACI Eq. 11-24 as

$$A_{l(\min)} = \frac{5A_{cp}\sqrt{f_c'}}{f_y} - \left(\frac{25b_w}{f_{yt}}\right)p_h\left(\frac{f_{yt}}{f_y}\right)$$

$$= \frac{(5)(228 \text{ in}^2)\sqrt{3000 \ \frac{\text{lbf}}{\text{in}^2}}}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$\quad - \left(0.0050 \ \frac{\text{in}^2}{\text{in}}\right)(48.50 \text{ in})\left(\dfrac{60 \ \frac{\text{kips}}{\text{in}^2}}{60 \ \frac{\text{kips}}{\text{in}^2}}\right)$$

$$= 1.041 \text{ in}^2 - 0.242 \text{ in}^2$$

$$= 0.799 \text{ in}^2 \quad [\text{governs}]$$

Using eight no. 3 bars around the perimeter of the closed stirrups gives a longitudinal steel area of

$$A_l = 0.88 \text{ in}^2$$

$$> 0.799 \text{ in}^2 \quad [\text{satisfactory}]$$

10. CONCRETE COLUMNS

Nomenclature

A_{ch}	area of core of spirally reinforced compression member measured to outside diameter of spiral	in^2
A_g	gross area of concrete section	in^2
A_{st}	total area of longitudinal reinforcement	in^2
C_m	a factor relating actual moment diagram to an equivalent uniform moment diagram	–
E_c	modulus of elasticity of concrete	lbf/in^2
h	overall thickness of member	in
I_g	moment of inertia of the gross concrete section	–
k	effective length factor for compression members	–
l_c	length of a compression member in a frame, measured from center-to-center of the joints in the frame	ft or in
l_u	unsupported length of compression member	ft or in
M_c	factored moment to be used for design of compression member	ft-kips
M_1	smaller factored end moment on a compression member, positive if member is bent in single curvature, negative if bent in double curvature	ft-kips

M_{1ns}	factored end moment on a compression member at the end at which M_1 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis	ft-kips
M_{1s}	factored end moment on compression members at the end at which M_1 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis	ft-kips
M_2	larger factored end moment on compression member, always positive	ft-kips
M_{2ns}	factored end moment on compression member at the end at which M_2 acts, due to loads that cause no appreciable sidesway, calculated using a first-order elastic frame analysis	ft-kips
M_{2s}	factored end moment on compression member at the end at which M_2 acts, due to loads that cause appreciable sidesway, calculated using a first-order elastic frame analysis	ft-kips
P_c	critical load, $\pi^2 EI/(kl_u)^2$	kips
P_n	nominal axial load strength at given eccentricity	kips
P_o	nominal axial load strength at zero eccentricity	kips
P_u	factored axial load at given eccentricity $\leq \phi P_n$	kips
Q	stability index for a story, $\sum P_u \Delta_o / V_u l_c$	–
r	radius of gyration of cross section of a compression member	in
V_{us}	factored horizontal shear in a story	kips

Symbols

β_{dns}	ratio of sustained load to maximum load ≤ 1.0	–
γ	the ratio of the distance between centroids of the longitudinal reinforcement to the overall diameter of the column	–
δ	moment magnification factor for frames braced against sidesway to reflect effects of member curvature between ends of compression members	–
δ_s	moment magnification factor for frames not braced against sidesway to reflect lateral drift resulting from lateral and gravity loads	–
Δ_o	relative lateral deflection between the top and bottom of a story due to V_{us}, computed using a first-order elastic frame analysis	in
ρ	ratio of A_{st} to A_g	–
ρ_s	ratio of volume of spiral reinforcement to total volume of core (out-to-out of spirals) of a spirally reinforced compression member	–
Ψ	stiffness ratio at the end of a column	–

Reinforcement Requirements

ACI Sec. 10.9 limits the area of longitudinal reinforcement to not more than 8% and not less than 1% of the gross area of the section. For columns with rectangular or circular ties, a minimum of four bars is required. For columns with spirals, a minimum of six longitudinal bars is required. The minimum ratio of volume of spiral reinforcement to volume of core is given by ACI Sec. 10.9.3 as

$$\rho_s = 0.45 f_c' \left(\frac{\dfrac{A_g}{A_{ch}} - 1}{f_{yt}} \right) \qquad [\textit{ACI 10-5}]$$

In accordance with ACI Sec. 7.10, the clear spacing between spirals cannot exceed 3 in nor be less than 1 in, and the minimum diameter of the spiral is $^3/_8$ in.

For rectangular columns, the minimum tie size specified by ACI Sec. 7.10.5 is no. 3 for longitudinal bars of no. 10 or smaller and no. 4 for longitudinal bars larger than no. 10. The maximum vertical spacing of ties is given by

$$s_{\max} \leq 16 \times \text{longitudinal bar diameters}$$
$$\leq 48 \times \text{tie bar diameters}$$
$$\leq \text{least dimension of the column}$$

Ties shall be provided, as shown in Fig. 1.17, to support every corner and alternate bar, and no bar shall be more than 6 in clear from a supported bar.

Figure 1.17 Column Ties
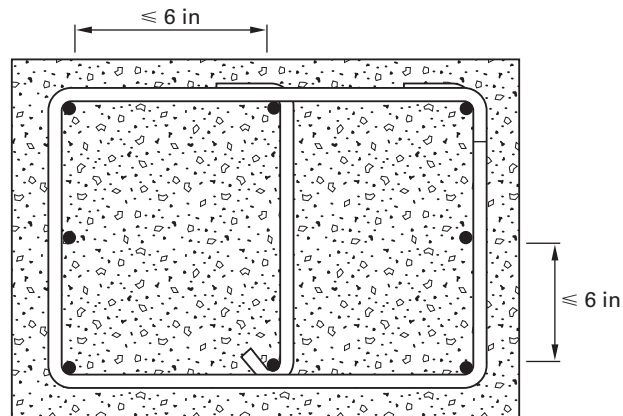
Example 1.14

A 24 in diameter spirally reinforced column with a 1.5 in cover to the spiral is reinforced with grade 60 bars and has a concrete compressive strength of 4500 lbf/in². Determine the required diameter and pitch of the spiral.

Solution

From ACI Sec. 7.10.4.2, the minimum permissible diameter of spiral reinforcement is

$$d_b = 0.375 \text{ in}$$

$$A_b = \text{area of spiral bar} = 0.11 \text{ in}^2$$

$$A_g = \text{gross area of column}$$

$$= \frac{\pi(24 \text{ in})^2}{4}$$

$$= 452 \text{ in}^2$$

$$A_{ch} = \text{area of core}$$

$$= \frac{\pi(21 \text{ in})^2}{4}$$

$$= 346 \text{ in}^2$$

$$d_s = \text{mean diameter of spiral}$$

$$= 21 \text{ in} - 0.375 \text{ in}$$

$$= 20.625 \text{ in}$$

From ACI Eq. 10-5, the minimum allowable spiral reinforcement ratio is

$$\rho_s = 0.45 f'_c \left(\frac{\dfrac{A_g}{A_{ch}} - 1}{f_{yt}} \right)$$

$$= (0.45) \left(4.5 \ \frac{\text{kips}}{\text{in}^2} \right) \left(\frac{\dfrac{452 \text{ in}^2}{346 \text{ in}^2} - 1}{60 \ \dfrac{\text{kips}}{\text{in}^2}} \right)$$

$$= 0.0103$$

$$= \frac{\text{volume of spiral bar per turn}}{\text{volume of concrete core per turn}}$$

$$= \frac{A_b \pi d_s}{A_{ch} s}$$

$$= \frac{(0.11 \text{ in}^2) \pi (20.625 \text{ in})}{(346 \text{ in}^2) s}$$

$$s = 2 \text{ in} \quad [\text{satisfactory}]$$

The calculated pitch lies between the maximum of 3 in and the minimum of 1 in specified in ACI Sec. 7.10.4.3.

Effective Length and Slenderness Ratio

The effective column length may be determined from the alignment charts given in ACI Sec. R10.10 and shown in Fig. 1.18.

Figure 1.18 *Alignment Charts for k^2*
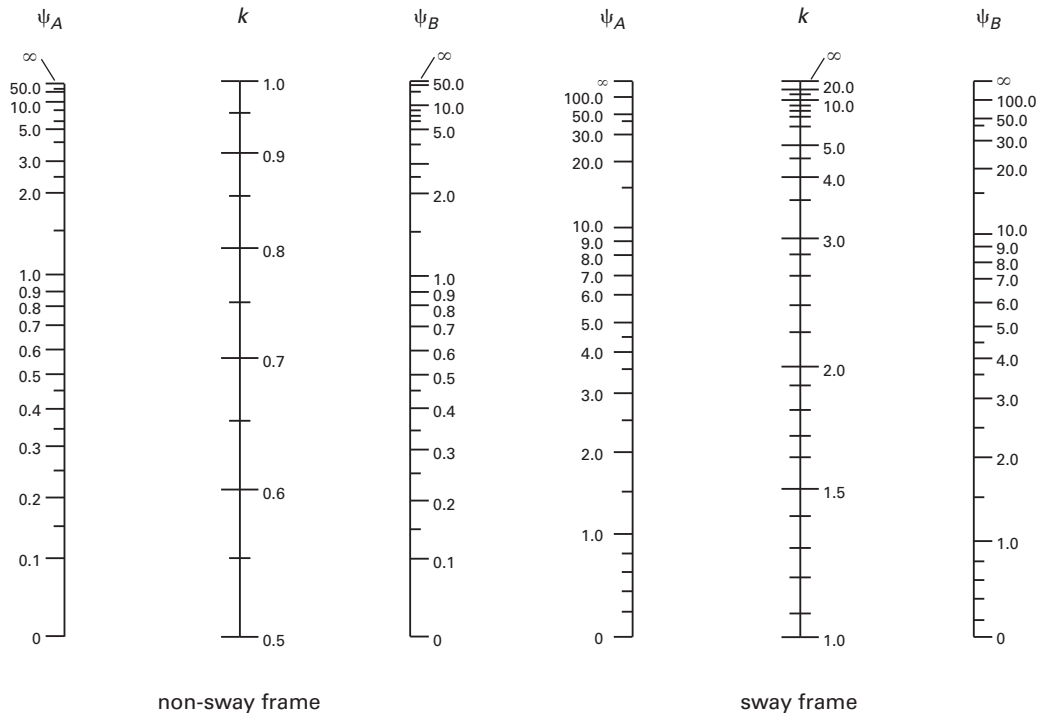
non-sway frame

sway frame

To use the alignment charts, the stiffness ratios at each end of the column must be calculated, and this is given by ACI Fig. R10.10.1.1 as

$$\Psi = \frac{\sum \dfrac{E_c I_c}{l_c}}{\sum \dfrac{E_b I_b}{l_b}}$$

The subscript c refers to the columns meeting at a joint, and the subscript b refers to the beams meeting at a joint.

For a non-sway frame, the effective length factor k may be conservatively taken as unity, as indicated in ACI Sec. 10.10.6.3. The slenderness ratio is defined as kl_u/r, and the radius of gyration is given by ACI Sec. 10.10.1.2 as

$$r = 0.25 \times \text{diameter of circular column}$$
$$= 0.30 \times \text{dimension of a rectangular column}$$
$$\text{in the direction stability is being considered}$$

A non-sway column is defined in ACI Sec. 10.10.5.1 as one in which the secondary moments due to P-delta effects do not exceed 5% of the primary moments due to lateral loads. ACI Sec. 10.10.5.2 specifies a story within a structure as non-sway, provided that the stability index, Q, does not exceed 0.05 where the stability index is given by

$$Q = \frac{\sum P_u \Delta_o}{V_{us} l_c} \qquad [\textit{ACI 10-10}]$$

V_{us} is the story shear, and $\sum P_u$ is the total vertical load on a story.

Example 1.15

Determine the slenderness ratio of columns 12 and 34 of the sway frame shown. The columns are 18 in square and have an unsupported height of 9 ft. All members of the frame have identical EI values.
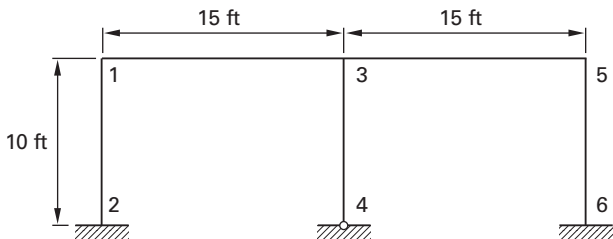
Solution

For column 12, the stiffness ratio of the fixed base is given by AISC 360[9] Comm. App. 7.2 as

$$\Psi_2 = 1.0$$

At joint 1, the relative stiffness value of the beam is

$$\sum \frac{E_b I_b}{l_b} = \frac{1}{15}$$

At joint 1, the relative stiffness value of the column is

$$\sum \frac{E_c I_c}{l_c} = \frac{1}{10}$$

The stiffness ratio at joint 1 is

$$\Psi_1 = \frac{\sum \dfrac{E_c I_c}{l_c}}{\sum \dfrac{E_b I_b}{l_b}} = \frac{15}{10}$$
$$= 1.5$$

From the alignment chart, for a sway frame, the effective length factor for column 12 is

$$k_{12} = 1.38$$

The radius of gyration of the column, in accordance with ACI Sec. 10.10.1.2, is

$$r = 0.30h$$
$$= (0.30)(18 \text{ in})$$
$$= 5.4 \text{ in}$$

The slenderness ratio of column 12 is

$$\frac{kl_u}{r} = \frac{(1.38)(9 \text{ ft})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}{5.4 \text{ in}}$$
$$= 27.6$$

For column 34, the stiffness ratio of the pinned base is given by AISC 360 Comm. App. 7.2 as

$$\Psi_4 = 10$$

At joint 3, the sum of the relative stiffness values of the two beams is

$$\sum \frac{E_b I_b}{l_b} = \frac{1}{15} + \frac{1}{15}$$
$$= \frac{2}{15}$$

At joint 3, the relative stiffness value of the column is

$$\sum \frac{E_c I_c}{l_c} = \frac{1}{10}$$

The stiffness ratio at joint 3 is

$$\Psi_3 = \frac{\sum \dfrac{E_c I_c}{l_c}}{\sum \dfrac{E_b I_b}{l_b}}$$
$$= \frac{15}{20}$$
$$= 0.75$$

From the alignment chart, for a sway frame, the effective length factor for column 34 is

$$k_{34} = 1.85$$

The slenderness ratio of column 34 is

$$\frac{kl_u}{r} = \frac{(1.85)(9 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{5.4 \text{ in}}$$
$$= 37.0$$

Short Column with Axial Load

In accordance with ACI Sec. 10.10.1, a column in a sway frame is classified as a short column, and slenderness effects may be ignored, when the slenderness ratio is

$$\frac{kl_u}{r} \leq 22 \qquad \text{[ACI 10-6]}$$

For a non-sway frame, a column is classified as a short column, in accordance with ACI Sec. 10.10.1, when the slenderness ratio is

$$\frac{kl_u}{r} \leq 34 - \frac{12M_1}{M_2} \leq 40 \qquad \text{[ACI 10-7]}$$

The term $(34 - 12M_1/M_2) \leq 40$ and M_1/M_2 is positive if the column is bent in single curvature. In addition, columns in a story may be considered braced when the bracing elements in the story have a total stiffness of at least 12 times the gross stiffness of the columns within the story.

For a short column with spiral reinforcement, ACI Sec. 10.3.6.1 gives the design axial load capacity as

$$\phi P_n = 0.85\phi \begin{pmatrix} 0.85f'_c(A_g - A_{st}) \\ + A_{st}f_y \end{pmatrix} \quad [\phi = 0.75]$$
$$\text{[ACI 10-1]}$$

For a short column with lateral tie reinforcement, ACI Sec. 10.3.6.2 gives the design axial load capacity as

$$\phi P_n = 0.80\phi \begin{pmatrix} 0.85f'_c(A_g - A_{st}) \\ + A_{st}f_y \end{pmatrix} \quad [\phi = 0.65]$$
$$\text{[ACI 10-2]}$$

Example 1.16

An 18 in square column is reinforced with 12 no. 9 grade 60 bars and has a concrete compressive strength of 4000 lbf/in^2. The column, which is braced against sidesway, has an unsupported height of 9 ft and supports axial load only without end moments. Determine the lateral ties required and the design axial load capacity.

Solution

The minimum tie size specified by ACI Sec. 7.10.5.1 is no. 3 for longitudinal bars of size no. 9. ACI Sec. 7.10.5.2 specifies a tie spacing not greater than

$$h = 18 \text{ in}$$
$$48d_t = (48)(0.375 \text{ in})$$
$$= 18 \text{ in}$$
$$16d_b = (16)(1.128 \text{ in})$$
$$= 18 \text{ in}$$

From ACI Sec. 10.10.6.3, the effective length factor, k, for a column braced against sidesway is

$$k = 1.0$$

The radius of gyration, in accordance with ACI Sec. 10.10.1.2, is

$$r = 0.3h$$
$$= (0.3)(18 \text{ in})$$
$$= 5.4 \text{ in}$$

The slenderness ratio is

$$\frac{kl_u}{r} = \frac{(1.0)(9 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{5.4 \text{ in}}$$
$$= 20.0$$

In accordance with ACI Eq. 10-7, the column, may be classified as a short column provided that

$$\frac{kl_u}{r} \leq 34 - \frac{12M_1}{M_2} \leq 40$$
$$20 < 34$$

Where,

$$M_1 = 0$$
$$M_2 = M_{\min}$$

The column is a short column, and the design axial load capacity is given by ACI Eq. 10-2 as

$$\phi P_n = 0.80\phi\bigl(0.85 f_c'(A_g - A_{st}) + A_{st}f_y\bigr)$$

$$= (0.80)(0.65)\left(\begin{array}{c} (0.85)\left(4\ \dfrac{\text{kips}}{\text{in}^2}\right) \\ \times\ (324\ \text{in}^2 - 12\ \text{in}^2) \\ + (12\ \text{in}^2)\left(60\ \dfrac{\text{kips}}{\text{in}^2}\right) \end{array} \right)$$

$$= 926\ \text{kips}$$

Short Column with End Moments

The axial load carrying capacity of a column decreases as end moments are applied to the column. Design of the column may then be obtained by means of a computer program, such as Structure Point's *spColumn*[10], based on ACI 318. Alternatively, approximate design values may be obtained from the interaction diagrams given in App. C through App. H.

Example 1.17

A 24 in diameter tied column with a 1.5 in cover to the $^3/_8$ in diameter ties is reinforced with 14 no. 9 grade 60 bars and has a concrete compressive strength of 4000 lbf/in^2. The column, which is braced against side-sway, has an unsupported height of 9 ft and is bent in single curvature with factored end moments of $M_1 = M_2 = 400$ ft-kips. Determine the maximum axial load that the column can carry.

Solution

The column may be classified as a short column, and slenderness effects do not have to be considered. The ratio of the distance between centroids of the longitudinal reinforcement to the overall diameter of the column is

$$\gamma = \frac{24\ \text{in} - (2)(1.5\ \text{in}) - (2)(0.375\ \text{in}) - 1.125\ \text{in}}{24\ \text{in}}$$

$$= 0.80$$

The reinforcement ratio is

$$\rho = \frac{A_{st}}{A_g} = \frac{14\ \text{in}^2}{\pi(12\ \text{in})^2}$$

$$= 0.031$$

$$\frac{M_u}{A_g h} = \frac{(400\ \text{ft-kips})\left(12\ \dfrac{\text{in}}{\text{ft}}\right)}{\pi(12\ \text{in})^2(24\ \text{in})}$$

$$= 0.44\ \text{kip/in}^2$$

From App. D, with $\gamma = 0.75$, the design axial stress is

$$\frac{P_u}{A_g} = 1.2\ \text{kips/in}^2$$

From App. E, with $\gamma = 0.90$, the design axial stress is

$$\frac{P_u}{A_g} = 1.7\ \text{kips/in}^2$$

By interpolation, for $\gamma = 0.80$, the design axial stress is

$$\frac{P_u}{A_g} = 1.2\ \frac{\text{kips}}{\text{in}^2} + \left(0.5\ \frac{\text{kip}}{\text{in}^2}\right)\left(\frac{5}{15}\right)$$

$$= 1.37\ \text{kips/in}^2$$

The maximum factored axial load that the column can carry is

$$P_u = 1.37 A_g$$

$$= 1.37\pi(12\ \text{in})^2$$

$$= 620\ \text{kips}$$

Long Column Without Sway[11]

ACI 318 requires that the design of long columns be based on the factored loading from a second-order analysis, which must satisfy one of three potential analysis approaches: nonlinear second-order analysis, elastic second-order analysis, or moment magnification. The nonlinear second-order analysis is too complex to be demonstrated in this chapter, as is the elastic second-order analysis. Both are typically performed using frame analysis software. More commonly, the moments due to second-order effects are estimated by multiplying the first-order solution by appropriately defined moment magnification (or amplification) factors.

ACI 318 contains simplified criteria for determining when slenderness amplification factors do not have to be calculated.

The moment magnification analysis takes the secondary bending stresses caused by *P*-delta effects, which are estimated by amplifying the primary bending moments by a moment magnification factor. The column is then designed for the axial force and the magnified bending moment using the short column design procedure. Non-sway and sway frames are treated separately. For a non-sway column, the magnification factor is given by ACI Sec. 10.10.6 as

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75 P_c}} \qquad \text{[ACI 10-12]}$$

$$\geq 1.0$$

P_u is the factored axial load, and the Euler critical load is given by

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \qquad [ACI \ 10\text{-}13]$$

The flexural rigidity, in accordance with ACI Sec. R10.10.6.1, may be taken as

$$EI = \frac{0.4E_c I_g}{1 + \beta_{dns}} \qquad [ACI \ 10\text{-}15]$$

For simplification, it may be assumed that $\beta_{dns} = 0.6$. Then, as given by ACI Sec. R10.10.6.2,

$$EI = 0.25E_c I_g$$

The factor C_m corrects for a nonuniform bending moment on a column and is defined by ACI Sec. 10.10.6.4 as

$$C_m = 0.6 + \frac{0.4M_1}{M_2} \qquad [ACI \ 10\text{-}16]$$

$$= 1.0 \text{ for columns with transverse}$$
$$\text{loads between supports}$$

The column is now designed for the magnified moment given by ACI Sec. 10.10.6 as

$$M_c = \delta M_2 \qquad [ACI \ 10\text{-}11]$$

The design procedure for a slender non-sway column is as follows.

step 1: Calculate forces on the column using first-order analysis.

step 2: Calculate end restraints and effective length.

step 3: Calculate C_m, P_c, δ, and M_c.

step 4: Use interaction diagrams to check the adequacy of the column.

Example 1.18

A 24 in diameter tied column with a 1.5 in cover to the $^3/_8$ in diameter ties is reinforced with 14 no. 9 grade 60 bars and has a concrete compressive strength of 4000 lbf/in^2. The column, which is braced against sidesway, has an unsupported height of 12 ft and is bent in single curvature with factored end moments of $M_1 = M_2 = 400$ ft-kips. Determine whether the column can carry a factored axial load of 700 kips.

Solution

$k = 1.0$ and $r = 6.0$ in. The slenderness ratio is

$$\frac{kl_u}{r} = \frac{(1.0)(12 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{6.0 \text{ in}}$$
$$= 24$$

In accordance with ACI Eq. 10-7, slenderness effects may be neglected when

$$\frac{kl_u}{r} \leq 34 - \frac{12M_1}{M_2} \leq 40$$

$$34 - \frac{(12)(400 \text{ ft-kips})}{400 \text{ ft-kips}} = 22$$

$$< 24$$

The column is a long column, and secondary effects must be considered.

The modulus of elasticity is given by ACI Sec. 8.5.1 as

$$E_c = 57{,}000\sqrt{f_c'}$$
$$= 57{,}000\sqrt{4000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 3.61 \times 10^6 \text{ lbf/in}^2$$

The gross moment of inertia is given by

$$I_g = \frac{\pi(12 \text{ in})^4}{4}$$
$$= 16{,}288 \text{ in}^4$$

The effective flexural rigidity is given by ACI Sec. R10.10.6.2 as

$$EI = 0.25E_c I_g$$
$$= (0.25)\left(3.61 \times 10^3 \ \frac{\text{kips}}{\text{in}^2}\right)(16{,}288 \text{ in}^4)$$
$$= 14.68 \times 10^6 \text{ in}^2\text{-kips}$$

The critical load is given by ACI Eq. 10-13 as

$$P_c = \frac{\pi^2 EI}{(kl_u)^2}$$
$$= \frac{\pi^2(14.68 \times 10^6 \text{ in}^2\text{-kips})}{\left((1.0)(12 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\right)^2}$$
$$= 6987 \text{ kips}$$

The moment correction factor is defined by ACI Eq. 10-16 as

$$C_m = 0.6 + \frac{0.4M_1}{M_2}$$
$$= 0.6 + \frac{(0.4)(400 \text{ ft-kips})}{400 \text{ ft-kips}}$$
$$= 1.0$$

The moment magnification factor is given by ACI Eq. 10-12 as

$$\delta = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}}$$
$$= \frac{1.0}{1 - \dfrac{700 \text{ kips}}{(0.75)(6987 \text{ kips})}}$$
$$= 1.15$$

The magnified end moment is given by ACI Eq. 10-11 as

$$M_c = \delta M_2$$
$$= (1.15)(400 \text{ ft-kips})$$
$$= 461 \text{ ft-kips}$$

From Ex. 1.17, $\gamma = 0.80$, $\rho = 0.031$, and

$$\frac{M_u}{A_g h} = \frac{(461 \text{ ft-kips})\left(12 \dfrac{\text{in}}{\text{ft}}\right)}{\pi (12 \text{ in})^2 (24 \text{ in})}$$
$$= 0.51 \text{ kip/in}^2$$

From App. D, with $\gamma = 0.75$, the design axial stress is

$$\frac{P_u}{A_g} = 0.4 \text{ kip/in}^2$$

From App. E, with $\gamma = 0.90$, the design axial stress is

$$\frac{P_u}{A_g} = 1.2 \text{ kips/in}^2$$

By interpolation, for $\gamma = 0.80$, the allowable design axial stress is

$$\frac{P_u}{A_g} = 0.4 \text{ } \frac{\text{kip}}{\text{in}^2} + \left(0.8 \text{ } \frac{\text{kip}}{\text{in}^2}\right)\left(\frac{5}{15}\right)$$
$$= 0.67 \text{ kip/in}^2$$

The maximum factored axial load that the column can carry is

$$P_u = 0.67 A_g$$
$$= (0.67)\pi(12 \text{ in})^2$$
$$= 303 \text{ kips}$$
$$< 700 \text{ kips}$$

The column cannot carry the axial load of 700 kips.

Long Column with Sway

The magnification factor for end moments produced by the loads that cause sway is given by ACI Sec. 10.10.7.4 as

$$\delta_s = \frac{1}{1 - \dfrac{\sum P_u}{0.75 \sum P_c}} \qquad \text{[ACI 10-21]}$$
$$\geq 1.0$$

The summations extend over all the columns in a story. The sway moments are multiplied by the magnification factor, and the non-sway moments are added, in accordance with ACI Sec. 10.10.7, to give the final design end moments in the column of

$$M_1 = M_{1ns} + \delta_s M_{1s} \qquad \text{[ACI 10-18]}$$
$$M_2 = M_{2ns} + \delta_s M_{2s} \qquad \text{[ACI 10-19]}$$

Example 1.19

A 24 in diameter, tied column with a 1.5 in cover to the $^3/_8$ in diameter ties is reinforced with 14 no. 9 grade 60 bars and has a concrete compressive strength of 4000 lbf/in^2. The column is not braced and has an unsupported height of 12 ft, an effective length factor of 1.3, and factored end moments due to sway and non-sway moments of $M_{2s} = 300$ ft-kips and $M_{2ns} = 50$ ft-kips. In the story where the column is located, the sum of the column critical loads is $\Sigma P_c = 29{,}600$ kips, and the sum of the factored column loads is $\Sigma P_u = 2700$ kips. Determine whether the column can carry a factored axial load of 900 kips.

Solution

The slenderness ratio with $r = 6$ in is

$$\frac{kl_u}{r} = \frac{(1.3)(12 \text{ ft})\left(12 \dfrac{\text{in}}{\text{ft}}\right)}{6 \text{ in}}$$
$$= 31.2$$

In accordance with ACI Sec. 10.10.1, slenderness effects must be considered when

$$\frac{kl_u}{r} > 22$$

$$31.2 > 22 \quad \text{[consider slenderness]}$$

The moment magnification factor for the sway moments is given by ACI Eq. 10-21 as

$$\delta_s = \frac{1}{1 - \dfrac{\sum P_u}{0.75 \sum P_c}}$$

$$= \frac{1}{1 - \dfrac{2700 \text{ kips}}{(0.75)(29{,}600 \text{ kips})}}$$

$$= 1.14$$

The magnified end moment is given by ACI Eq. 10-19 as

$$M_2 = M_{2ns} + \delta_s M_{2s}$$

$$= 50 \text{ ft-kips} + (1.14)(300 \text{ ft-kips})$$

$$= 392 \text{ ft-kips}$$

From Ex. 1.17, $\gamma = 0.80$, $\rho_g = 0.031$, and

$$\frac{M_2}{A_g h} = \frac{(392 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}{(452 \text{ in}^2)(24 \text{ in})}$$

$$= 0.43 \text{ kip/in}^2$$

From App. D, with $\gamma = 0.75$, the design axial stress is

$$\frac{P_u}{A_g} = 1.3 \text{ kips/in}^2$$

From App. E, with $\gamma = 0.90$, the design axial stress is

$$\frac{P_u}{A_g} = 1.7 \text{ kips/in}^2$$

By interpolation, for $\gamma = 0.80$, the design axial stress is

$$\frac{P_u}{A_g} = 1.3 \ \frac{\text{kips}}{\text{in}^2} + \left(0.4 \ \frac{\text{kip}}{\text{in}^2}\right)\left(\frac{5}{15}\right)$$

$$= 1.43 \text{ kips/in}^2$$

The maximum factored axial load that the column can carry is

$$P_u = 1.43 A_g$$

$$= (1.43)(452 \text{ in}^2)$$

$$= 648 \text{ kips}$$

$$< 900 \text{ kips} \quad \text{[The column is unsatisfactory.]}$$

11. DEVELOPMENT AND SPLICE LENGTH OF REINFORCEMENT

Nomenclature

A_b	area of an individual bar	in^2
A_{tr}	total cross-sectional area of all transverse reinforcement that is within the spacing s and that crosses the potential plane of splitting through the reinforcement being developed	in^2
c	spacing or cover dimension	in
c_b	distance from center of bar to nearest concrete surface	in
d_b	nominal diameter of bar	in
f_{ct}	average splitting tensile compressive strength of lightweight aggregate concrete	kips/in^2
h	overall thickness of member	–
K_{tr}	transverse reinforcement index, $A_{tr} f_{yt}/1500sn$	in
l_a	additional embedment length at support or at point of inflection	in
l_d	development length, l_{db} applicable modification factors	in
l_{db}	basic development length	in
l_{dh}	development length of standard hook in tension, measured from critical section to outside end of hook (straight embedment length between critical section and start of hook (point of tangency) plus radius of bend and one bar diameter), $l_{hb} \times$ applicable modification factors	in
l_{hb}	basic development length of standard hook in tension	in
l_s	lap splice length	in
n	number of bars or wires being spliced or developed along the plane of splitting	–
s	maximum center-to-center spacing of transverse reinforcement within l_d	in

Symbols

β_b	ratio of area of reinforcement cutoff to total area of tension reinforcement at section	–
λ	lightweight aggregate concrete factor $= 0.75$ for all lightweight aggregate concrete $= f_{ct}/6.7\sqrt{f_c'} \leq 1.0$ when f_{ct} is specified $= 1.0$ for normal weight concrete	–
Ψ_e	coating factor $= 1.5$ for epoxy-coated bars with cover $< 3d_b$ or clear spacing $< 6d_b$ $= 1.2$ for all other epoxy-coated bars $= 1.0$ for uncoated bars	–
Ψ_s	reinforcement size factor $= 0.8$ for no. 6 and smaller bars $= 1.0$ for no. 7 and larger bars	–
Ψ_t	reinforcement location factor $= 1.3$ for horizontal bar with more than 12 in of concrete below $= 1.0$ for all other bars	–

Development Length of Straight Bars in Tension[12]

The development length for tension reinforcement is given by ACI Sec. 12.2.3 as

$$\frac{l_d}{d_b} = \frac{0.075 f_y \Psi_t \Psi_e \Psi_s}{\lambda \dfrac{\sqrt{f'_c}(c_b + K_{tr})}{d_b}} \qquad [ACI\ 12\text{-}1]$$

$$l_d \geq 12\ \text{in}$$

$$\frac{c_b + K_{tr}}{d_b} \leq 2.5$$

From ACI Sec. 12.2.4,

$$\Psi_t \Psi_e \leq 1.7$$

From ACI Sec. 12.1.2,

$$\sqrt{f'_c} \leq 100\ \text{lbf/in}^2$$

The derivation of the transverse reinforcement index is illustrated in Fig. 1.19.

ACI Sec. 12.2.2 also provides the following simplified, conservative values.

- Using a minimum clear spacing of $2d_b$, a minimum clear cover to flexural reinforcement of d_b, and in the absence of stirrups,

$$\frac{l_d}{d_b} = \frac{0.04 f_y \Psi_t \Psi_e}{\lambda \sqrt{f'_c}} \quad \text{[for } d_b \text{ no. 6 or smaller]}$$

$$\frac{l_d}{d_b} = \frac{0.05 f_y \Psi_t \Psi_e}{\lambda \sqrt{f'_c}} \quad \text{[for } d_b \text{ no. 7 or larger]}$$

- Using a minimum clear spacing of d_b, a minimum clear cover to flexural reinforcement of d_b, and minimum stirrups specified in ACI Eq. 11-13,

$$\frac{l_d}{d_b} = \frac{0.04 f_y \Psi_t \Psi_e}{\lambda \sqrt{f'_c}} \quad \text{[for } d_b \text{ no. 6 or smaller]}$$

$$\frac{l_d}{d_b} = \frac{0.05 f_y \Psi_t \Psi_e}{\lambda \sqrt{f'_c}} \quad \text{[for } d_b \text{ no. 7 or larger]}$$

- For all other cases,

$$\frac{l_d}{d_b} = \frac{0.06 f_y \Psi_t \Psi_e}{\lambda \sqrt{f'_c}} \quad \text{[for } d_b \text{ no. 6 or smaller]}$$

$$\frac{l_d}{d_b} = \frac{0.075 f_y \Psi_t \Psi_e}{\lambda \sqrt{f'_c}} \quad \text{[for } d_b \text{ no. 7 or larger]}$$

Using uncoated, grade 60 reinforcement, where no more than 12 in of normal weight concrete is below the bar gives

$$\Psi_e = \Psi_t = \lambda = 1.0$$

The resulting values are shown in Table 1.4.

For bundled bars, ACI Sec. 12.4.1 specifies that the development length shall be that for an individual bar increased by 20% for a three-bar bundle and 33% for a four-bar bundle. No increase is required for a two-bar bundle. The equivalent diameter d_b of a bundle is specified in ACI Sec. 12.4.2 as that of a bar with an area equal to that of the bundle.

Figure 1.19 Derivation of K_{tr}

Table 1.4 Values of l_d/d_b for Grade 60 Bars with $\Psi_e = \Psi_t = \lambda = 1.0$

f'_c (lbf/in^2)	bar size ≤ no. 6		bar size ≥ no. 7	
	case 1[a]	case 2[b]	case 1[a]	case 2[b]
3000	44	66	55	82
3500	41	61	51	76
4000	38	57	47	71
4500	36	54	45	67
5000	34	51	42	64

[a]For case 1, clear cover ≥ d_b and clear spacing ≥ $2d_b$; or, clear cover ≥ d_b, clear spacing ≥ d_b, and with minimum stirrups.
[b]For case 2, all other conditions apply.

ACI Sec. 12.2.5 specifies that when excess reinforcement is provided in a member, the development length may be reduced by multiplying by the factor $A_{s(\text{required})}/A_{s(\text{provided})}$. This reduction factor may not be applied when development is required for the yield strength of the reinforcement, as is the case for shrinkage and temperature reinforcement specified in ACI Sec. 7.12, integrity reinforcement specified in ACI Sec. 7.13, positive moment reinforcement specified in ACI Sec. 12.11, and tension lap splices specified in ACI Sec. 12.15.

Example 1.20

The simply supported reinforced concrete beam of normal weight concrete shown is reinforced with grade 60 bars and has a concrete compressive strength of 3000 lbf/in^2. The maximum moment in the beam occurs at a point 2 ft from the end of the bars, and 10% more flexural reinforcement is provided than is required. Determine whether the development length available is satisfactory.
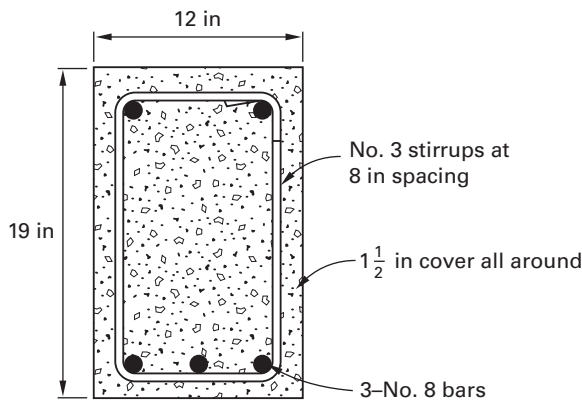
Solution

From ACI Sec. 12.2.4,

$$\Psi_t = \Psi_e = \Psi_s = \lambda = 1.0$$

The excess reinforcement factor is

$$E_{xr} = \frac{A_{s(\text{required})}}{A_{s(\text{provided})}} = \frac{100 \text{ in}^2}{110 \text{ in}^2}$$
$$= 0.91$$

The cover dimension to the flexural reinforcement is

$$c = 1.5 \text{ in} + 0.375 \text{ in} + \frac{1.0 \text{ in}}{2}$$
$$= 2.375 \text{ in}$$

The spacing dimension to the flexural reinforcement is

$$c_s = \frac{12.0 \text{ in} - (2)(1.5 \text{ in}) - (2)(0.375 \text{ in}) - 1.0 \text{ in}}{4}$$
$$= 1.81 \text{ in} \quad [\text{horizontal cracking governs}]$$
$$c_b = 1.81 \text{ in}$$

The area of transverse reinforcement crossing the horizontal crack is

$$A_{tr} = (2)(0.11 \text{ in}^2) = 0.22 \text{ in}^2$$

The number of bars being developed along the cracking plane is

$$n = 3$$

The transverse reinforcement index is

$$K_{tr} = \frac{A_{tr}f_{yt}}{\left(1500 \frac{\text{lbf}}{\text{in}^2}\right)sn} = \frac{(0.22 \text{ in}^2)\left(60{,}000 \frac{\text{lbf}}{\text{in}^2}\right)}{\left(1500 \frac{\text{lbf}}{\text{in}^2}\right)(8 \text{ in})(3)}$$
$$= 0.367 \text{ in}$$
$$\frac{c_b + K_{tr}}{d_b} = \frac{1.81 \text{ in} + 0.367 \text{ in}}{1.0 \text{ in}}$$
$$= 2.18$$
$$< 2.5 \quad [\text{satisfactory}]$$

From ACI Eq. 12-1,

$$\frac{l_d}{d_b} = \frac{0.075 f_y \Psi_t \Psi_e \Psi_s E_{xr}}{\dfrac{\lambda\sqrt{f'_c}(c_b + K_{tr})}{d_b}}$$
$$= \frac{(0.075)\left(60{,}000 \frac{\text{lbf}}{\text{in}^2}\right)(1)(1)(1)(0.91)}{(1)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}(2.18)}$$
$$= 34.3$$

The development length required is

$$l_d = 34.3d_b = (34.3)(1.0 \text{ in})$$
$$= 34.3 \text{ in}$$
$$> 24 \text{ in}$$

The development length available is unsatisfactory for straight bars.

Development Length of Straight Bars in Compression

The basic development length for bars in compression is specified in ACI Sec. 12.3.2 as

$$l_{dc} = \frac{0.02d_b f_y}{\lambda\sqrt{f'_c}}$$

$$\geq 0.0003 d_b f_y \quad \begin{bmatrix} \text{governs for normal weight concrete} \\ \text{when } f'_c \geq 4444 \text{ lbf/in}^2 \end{bmatrix}$$

The actual development length, l_d, is obtained by multiplying l_{dc} by the

- excess reinforcement factor specified in ACI Sec. 12.3.3(a)

$$E_{xr} = \frac{A_{s(\text{required})}}{A_{s(\text{provided})}}$$

- confinement factor specified in ACI Sec. 12.3.3(b)

$$C_f = 0.75$$

The confinement factor is applicable when spiral reinforcement is provided with a minimum diameter of 0.25 in and a maximum pitch of 4 in. Alternatively, no. 4 ties or larger may be provided at a maximum spacing of 4 in. The minimum allowable value for l_d is 8 in. Table 1.5 provides values of the basic development length for grade 60 bars.

The requirements for bundled bars in compression are identical to those applicable to bundled bars in tension.

Table 1.5 *Values of l_{db} for Grade 60 Bars in Compression*

bar size no.	concrete strength (lbf/in^2)			
	3000	3500	4000	$\geq$4444
3	8.2	7.6	7.1	6.8
4	11.0	10.1	9.5	9.0
5	13.7	12.7	11.9	11.3
6	16.4	15.2	14.2	13.5
7	19.2	17.8	16.6	15.8
8	21.9	20.3	19.0	18.0
9	24.7	22.9	21.4	20.3
10	27.8	25.8	24.1	22.9
11	30.9	28.6	26.8	25.4

Example 1.21

A spirally reinforced column of normal weight concrete with a 0.25 in diameter spiral at a pitch of 4 in is reinforced with no. 9 grade 60 bars in bundles of three and has a concrete compressive strength of 4500 lbf/in^2. The compression reinforcement provided is 15% more than is required. Determine the required development length of an individual bar.

Solution

From ACI Sec. 12.4, the development length of a bar in a three-bar bundle is increased 20%.

From ACI Sec. 12.3.3, the excess reinforcement factor is

$$E_{xr} = \frac{100}{115}$$
$$= 0.87$$

And the confinement factor is

$$C_f = 0.75$$

For a concrete strength in excess of 4444 lbf/in^2, ACI Sec. 12.3.2 and Sec. 12.3.3 give the required development length as

$$l_d = 1.2 E_{xr} C_f l_{dc}$$
$$= (1.2)(0.87)(0.75)(0.0003)d_b f_y$$
$$= (1.2)(0.87)(0.75)(0.0003)(1.13 \text{ in})\left(60{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)$$
$$= 15.9 \text{ in}$$

Development of Hooked Bars in Tension

The basic development length for hooked bars in tension is specified in ACI Sec. 12.5.2 as

$$l_{hb} = \frac{0.02\Psi_e d_b f_y}{\lambda\sqrt{f'_c}}$$

The lightweight aggregate concrete factor specified in ACI Sec. 12.5.2 is

$$\lambda = 0.75$$
$$= 1.0 \quad \text{[for all other cases]}$$

The epoxy-coated reinforcement factor specified in ACI Sec. 12.5.2 is

$$\Psi_e = 1.2$$
$$= 1.0 \quad \text{[for all other cases]}$$

The actual development length, l_{dh}, is obtained by multiplying l_{hb} by the factors given in ACI Sec. 12.5.3.

- the cover factor for bars not exceeding size no. 11, with side covers not less than 2.5 in and end covers not less than 2.0 in

$$C_b = 0.7 \quad \text{[for conditions specified above]}$$

$$= 1.0 \quad \text{[for all other conditions]}$$

- the tie factor for bars not exceeding size no. 11, with ties provided perpendicular to the bar being developed over the full development length at a spacing not exceeding $3d_b$; or for a 90° hook with ties provided, parallel to the bar being developed, along the tail extension of the hook plus bend at a spacing not exceeding $3d_b$

$$T_f = 0.8$$

- the excess reinforcement factor

$$E_{xr} = \frac{A_{s(\text{required})}}{A_{s(\text{provided})}}$$

For hooked bars at the discontinuous end of a member with cover of less than 2.5 in, ties must be provided, over the full development length, at a spacing not exceeding $3d_b$. The modification factor for this condition is 1.0.

In accordance with ACI Sec. 12.5.1, the minimum allowable value for the actual development length l_{dh} is

$$l_{dh} \geq 8d_b$$

$$\geq 6 \text{ in}$$

Example 1.22

Assume that the reinforced concrete beam for Ex. 1.20 is reinforced with hooked bars. Determine whether the development length provided for the hooked bars is satisfactory.

Solution

Because the bars terminate at the discontinuous end of the beam and the cover provided is less than 2.5 in, the modification factor is 1.0 and ties must be provided over the full development length at a maximum spacing of

$$s = 3d_b = (3)(1 \text{ in})$$

$$= 3 \text{ in}$$

The excess reinforcement factor derived in Ex. 1.20 is

$$E_{xr} = 0.91$$

All other modification factors are equal to unity.

The required development length is given by ACI Sec. 12.5.1 and Sec. 12.5.2 as

$$l_{dh} = E_{xr}l_{hb} = \frac{(0.91)\left(1200 \dfrac{\text{lbf}}{\text{in}^2}\right)d_b\Psi_e}{\lambda\sqrt{f_c'}}$$

$$= \frac{(0.91)\left(1200 \dfrac{\text{lbf}}{\text{in}^2}\right)(1 \text{ in})(1.0)}{(1.0)\sqrt{3000 \dfrac{\text{lbf}}{\text{in}^2}}}$$

$$= 20 \text{ in}$$

$$< 24 \text{ in}$$

The development length provided is satisfactory.

Curtailment of Reinforcement

ACI Sec. 12.10.3 and Sec. 12.10.4 specify that reinforcement may extend a distance beyond the theoretical cut-off point not less than the effective depth of the member or twelve times the bar diameter and may extend beyond the point at which it is fully stressed not less than the development length. This is illustrated in Fig. 1.20, where the four no. 9 bars are assumed to be fully stressed at the center of the simply supported beam. These requirements are not necessary at supports of simple spans or at the free ends of cantilevers.

Figure 1.20 *Curtailment of Reinforcement*
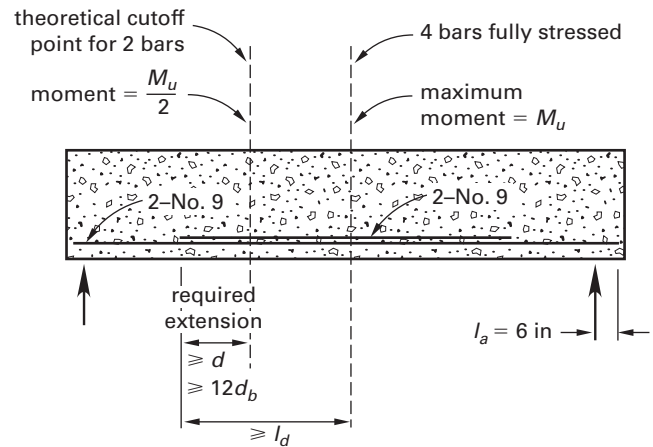
In addition to the previous requirements, ACI Sec. 12.10.5 requires that one of the following conditions be satisfied at the physical cutoff point.

- The factored shear force at the cutoff point does not exceed two-thirds of the shear capacity, ϕV_n.

- Additional stirrups with a minimum area of $60b_w s/f_{yt}$ are provided along the terminated bar for a distance of $0.75d$ at a maximum spacing of $d/8\beta_b$.

- For no. 11 bars and smaller, the continuing reinforcement provides twice the flexural capacity required, and the factored shear force at the cutoff point does not exceed three-fourths of the shear capacity, ϕV_n.

Example 1.23

The simply supported reinforced concrete beam, of normal weight concrete, shown in Fig. 1.20 is reinforced with four no. 9 grade 60 bars and has a concrete strength of 3000 lbf/in². The maximum moment in the beam occurs at midspan, and the flexural reinforcement is fully stressed. The bending moment reduces to 50% of its maximum value at a point 3 ft from midspan. The effective depth is 16 in, the beam width is 12 in, and the development length of the no. 9 bars may be taken as $55d_b$. The factored shear force is less than two-thirds the shear capacity of the section everywhere along the span. Determine the distance from midspan at which two of the no. 9 bars may be terminated.

Solution

From ACI Sec. 12.10.3 and Sec. 12.10.4, the physical cutoff point may be located a minimum distance from midspan given by the largest of

- $12d_b + 36 \text{ in} = (12)(1.128 \text{ in}) + 36 \text{ in} = 49.5 \text{ in}$

- $d + 36 \text{ in} = 16 \text{ in} + 36 \text{ in} = 52 \text{ in}$

- $l_d = 55d_b = (55)(1.128 \text{ in})$

  $= 62 \text{ in}$ [governs]

Development of Positive Moment Reinforcement

To allow for variations in the applied loads, ACI Sec. 12.11.1 requires a minimum of one-third of the positive reinforcement in a simply supported beam or one-fourth of the positive reinforcement in a continuous beam to extend not less than 6 in into the support. This is shown in Fig. 1.21.

Figure 1.21 *Positive Moment Reinforcement*
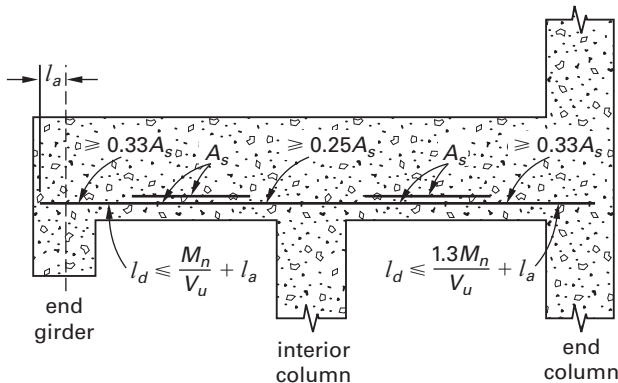
To ensure that allowable bond stresses are not exceeded, ACI Sec. 12.11.3 requires a bar diameter to be chosen such that its development length, in the case of a beam framing into a girder, is given by

$$l_d \leq \frac{M_n}{V_u} + l_a \qquad \text{[ACI 12-5]}$$

M_n is the nominal strength assuming all reinforcement is stressed to the specified yield stress, f_y, V_u is the factored shear force at the section, and l_a is the embedment length beyond the center of the support.

For the case of a beam framing into a column, ACI Sec. R12.11.3 requires a bar diameter to be chosen such that its development length is given by

$$l_d \leq 1.3 \frac{M_n}{V_u} + l_a$$

At a point of inflection, PI, ACI Sec. 12.11.3 requires a bar diameter to be chosen such that its development length is given by

$$l_d \leq \frac{M_n}{V_u} + \text{maximum of } d \text{ or } 12d_b$$

Alternatively, at a simple support, ACI Sec. 12.11.3 specifies that the reinforcement may terminate in a standard hook beyond the centerline of the support.

Example 1.24

Assume that the reinforced concrete beam for Ex. 1.23, which is shown in Fig. 1.20, has a factored end reaction of 30 kips and that the beam frames into concrete girders at each end. Determine whether the two no. 9 bars at the support satisfy the requirements for local bond.

Solution

The development length for the no. 9 bars was given in Ex. 1.23 as

$$l_d = 55d_b$$

$$= 62 \text{ in}$$

The nominal flexural strength at the support is given by ACI Sec. 10.2 as

$$M_n = A_s f_y d \left(1 - \frac{0.59 A_s f_y}{b_w d f_c'} \right)$$

$$= (2 \text{ in}^2) \left(60 \ \frac{\text{kips}}{\text{in}^2} \right) (16 \text{ in})$$

$$\times \left(1 - \frac{(0.59)(2 \text{ in}^2) \left(60 \ \frac{\text{kips}}{\text{in}^2} \right)}{(12 \text{ in})(16 \text{ in}) \left(3 \ \frac{\text{kips}}{\text{in}^2} \right)} \right)$$

$$= 1684 \text{ in-kips}$$

For a beam framing into a concrete girder, the appropriate factors given by ACI Sec. 12.11.1 and Sec. 12.11.3 are

$$\frac{M_n}{V_u} + l_a = \frac{1684 \text{ in-kips}}{30 \text{ kips}} + 6 \text{ in}$$

$$= 62.1 \text{ in}$$

$$> l_d$$

Local bond requirements are satisfied.

Development of Negative Moment Reinforcement

To allow for variation in the applied loads, ACI Sec. 12.12 requires a minimum of one-third of the negative reinforcement at a support to extend past the point of inflection not less than the effective depth of the beam, 12 times the bar diameter, or one-sixteenth of the clear span. This is shown in Fig. 1.22.

Figure 1.22 *Negative Moment Reinforcement*
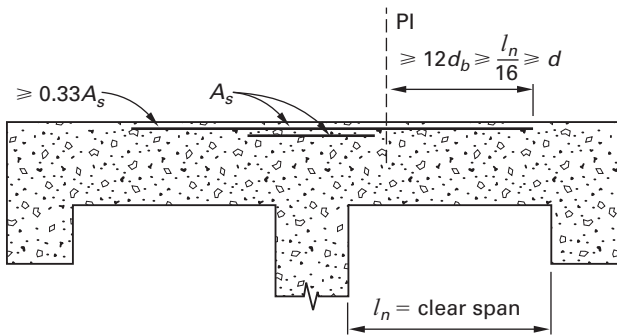
Example 1.25

The reinforced concrete continuous beam shown has an effective depth of 16 in, is reinforced with grade 60 bars, and has a concrete compressive strength of 3000 lbf/in². Determine the minimum length, x, at which the bars indicated may terminate.
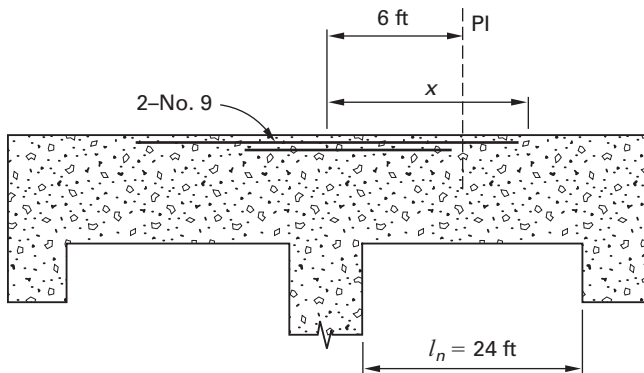
Solution

The cutoff point of the bars may be located a minimum distance beyond the point of inflection given by ACI Sec. 12.12 as the greater of

$$12d_b = (12)(1.128 \text{ in})$$

$$= 13.5 \text{ in}$$

$$d = 16 \text{ in}$$

$$\frac{l}{16} = \frac{(24 \text{ ft})(12 \text{ in})}{16}$$

$$= 18 \text{ in} \quad [\text{governs}]$$

The minimum allowable length for x is given by

$$x = 6 \text{ ft} + 1.5 \text{ ft}$$

$$= 7.5 \text{ ft}$$

Splices of Bars in Tension

Lap splices for bars in tension, in accordance with ACI Sec. 12.14, may not be used either for bars larger than no. 11 or for bundled bars. In flexural members, the transverse spacing between lap splices shall not exceed one-fifth of the lap length, or 6 in. Within a bundle, individual bar splices must not overlap, and the lap length for each bar must be increased by 20% for a three-bar bundle and 33% for a four-bar bundle.

The length of a lap splice may not be less than 12 in nor less than the values given by ACI Sec. 12.15, which are

$$\text{class A splice length} = 1.0l_d$$

$$\text{class B splice length} = 1.3l_d$$

A class A splice may be used only when the reinforcement area provided is at least twice that required and when, in addition, not more than one-half of the total reinforcement is spliced within the lap length. Otherwise, as shown in Table 1.6, a class B tension lap splice is required.

Table 1.6 *Tension Lap Splices*

$\dfrac{A_s \text{ provided}}{A_s \text{ required}}$	maximum percentage of A_s spliced within lap length	
	50	100
2 or more	class A	class B
less than 2	class B	class B

In determining the relevant development length, all applicable modifiers are used with the exception of that for excess reinforcement. The development length, l_d, is determined by using the values for clear spacing, c_s, indicated in Fig. 1.23 and Fig. 1.24.

Figure 1.23 *Value of c_s for Lap Splices*
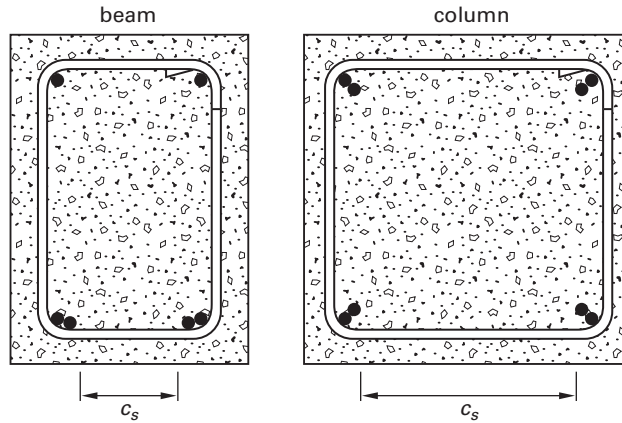
Figure 1.24 *Value of c_s in Slabs and Walls*
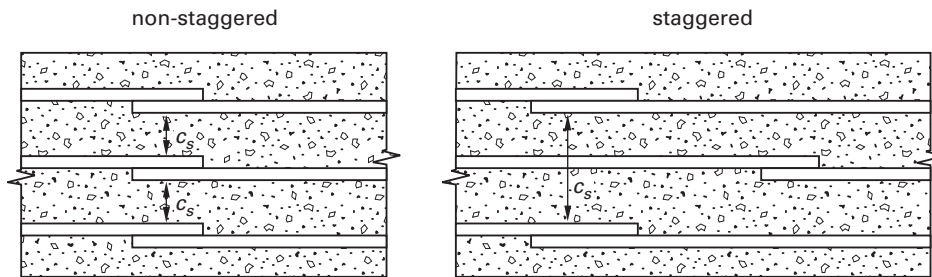
Example 1.26

A reinforced concrete beam is reinforced with two no. 8 grade 60 bars and has a concrete compressive strength of 3000 lbf/in^2. Both bars are lap spliced at the same location, and the development length of the no. 8 bars may be taken as $50d_b$. Determine the required splice length.

Solution

The development length is given as

$$l_d = 50d_b = (50)(1 \text{ in}) = 50 \text{ in}$$

Because both bars are spliced at the same location, a class B splice is required and the splice length is

$$l_s = 1.3l_d = (1.3)(50 \text{ in}) = 65 \text{ in}$$

Splices of Bars in Compression

In accordance with ACI Sec. 12.16.1, the length of a lap splice for bars in compression must not be less than 12 in and is given by

$$l_s = 0.0005f_y d_b \quad [f_y \leq 60{,}000 \text{ lbf/in}^2]$$
$$l_s = (0.0009f_y - 24)d_b \quad [f_y > 60{,}000 \text{ lbf/in}^2]$$

An increase in the lap length of 33% is required when the concrete strength is less than 3000 lbf/in^2. When bars of different sizes are lap spliced, ACI Sec. 12.16.2 specifies that the lap length must be the larger of the splice length of the smaller bar or the development length of the larger bar.

In accordance with ACI Sec. 12.17.2, lap lengths for columns may be reduced by 17% when ties are provided with an effective area of $0.0015hs$ and may be reduced by 25% in a spirally reinforced column. When tensile stress exceeding $0.5f_y$ occurs in a column, a class B tension lap splice can be used. A class A tension lap splice is adequate provided that the tensile stress does not exceed $0.5f_y$, not more than one-half of the bars are spliced at the same location, and alternate splices are staggered by l_d; otherwise, a class B tension lap splice is required.

Example 1.27

The reinforced concrete column shown is reinforced with grade 60 bars that are fully stressed and has a concrete compressive strength of 4000 lbf/in^2. The column is subjected to compressive stress only. Determine the required lap splice for the no. 8 and no. 9 bars.

12 in × 12 in

4–No. 8

No. 3 @ 12 in o.c.

4–No. 9
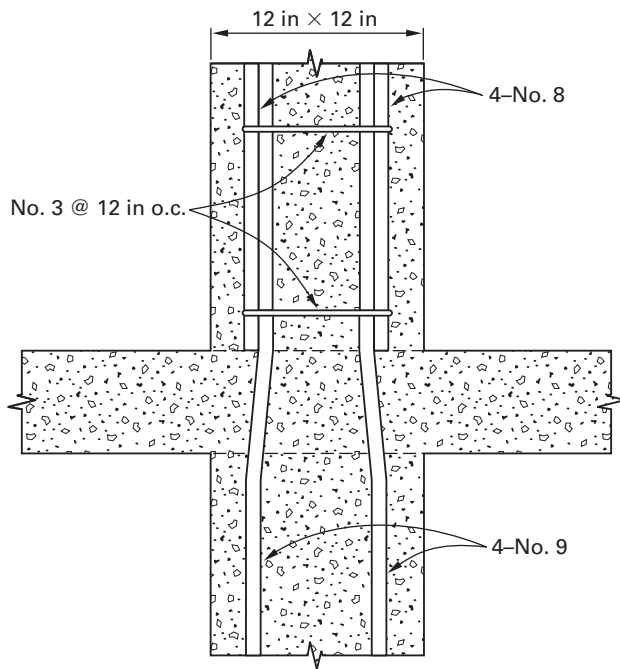
Solution

The development length of a no. 9 bar is given by ACI Sec. 12.3.2 and Sec. 12.3.3 as

$$l_d = \frac{0.02 d_b f_y}{\sqrt{f_c'}} = \frac{(0.02)(1.128 \text{ in})\left(60,000 \ \frac{\text{lbf}}{\text{in}^2}\right)}{\sqrt{4000 \ \frac{\text{lbf}}{\text{in}^2}}}$$

$$= 21.4 \text{ in}$$

To qualify for the 17% reduction in lap splice length of the no. 8 bars, ACI Sec. 12.17.2.4 requires the two arms of the no. 3 ties to have a minimum effective area of

$$\frac{A_t}{s} = 0.0015h = (0.0015)(12 \text{ in})\left(12 \ \frac{\text{in}}{\text{ft}}\right)$$

$$= 0.22 \text{ in}^2/\text{ft}$$

Two arms of the no. 3 ties provide an area of

$$\frac{A_t}{s} = 0.22 \text{ in}^2/\text{ft}$$

The 17% reduction applies.

The compression lap splice length of the no. 8 bars is given by ACI Sec. 12.16.1 and Sec. 12.17.2.4 as

$$l_s = 0.0005 f_y d_b (0.83)$$

$$= (0.0005)\left(60,000 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0 \text{ in})(0.83)$$

$$= 25 \text{ in} \quad \text{[governs]}$$

$$> 21.4 \text{ in}$$

The required lap splice length is 25 in.

12. TWO-WAY SLAB SYSTEMS

Nomenclature

b_o	perimeter of critical section	in
c_1	size of rectangular or equivalent rectangular column, capital, or bracket measured in the direction of the span for which moments are being determined	in
c_2	size of rectangular or equivalent rectangular column, capital, or bracket measured transverse to the direction of the span for which moments are being determined	in
C	cross-sectional constant to define torsional properties $\left(\Sigma(1 - 0.63x/y)x^3y\right)/3$; for T- or L-sections, it is permitted to be evaluated by dividing the section into separate rectangular parts and summing the values of C for each part.	–
E_{cb}	modulus of elasticity of beam concrete	lbf/in^2
E_{cs}	modulus of elasticity of slab concrete	lbf/in^2
I_b	moment of inertia about centroidal axis of gross section of beam	in^4
I_s	moment of inertia about centroidal axis of gross section of slab $= h^3/12$ times width of slab defined in symbols α and β_t	in^4
l_n	length of clear span in direction that moments are being determined, measured face-to-face of supports $= l_1 - c_1 > 0.65l_1$	ft or in
l_1	length of span in direction that moments are being determined, measured center-to-center of supports	ft or in
l_2	length of span transverse to l_1, measured center-to-center of supports	ft or in
M_o	total factored static moment	ft-kips
q_u	factored load per unit area	kips/ft^2
x	shorter overall dimension of rectangular part of cross section	in
y	longer overall dimension of rectangular part of cross section	in

Symbols

α_f	ratio of flexural stiffness of beam section to flexural stiffness of a slab width bounded laterally by center lines of adjacent panels (if any) on each side of the beam $= E_{cb}I_b/E_{cs}I_s$	–
α_s	constant used to compute V_c	–
α_{f1}	α_f in direction of l_1	–
α_{f2}	α_f in direction of l_2	–
β	ratio of long side to short side of column	–
β_t	ratio of torsional stiffness of edge beam section to flexural stiffness of a slab width equal to span length of beam, center-to-center of supports $= E_{cb}C/ 2E_{cs}I_s$	–

Figure 1.25 *Equivalent Beam and Slab Dimensions*
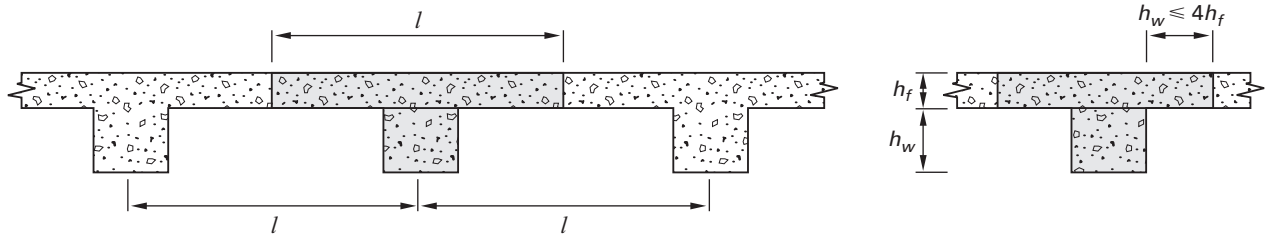
Design Techniques[13]

The different types of slab systems are as follows.

- *One-way slabs* are supported by beams on two opposite sides and span in one direction between the beams.

- *Two-way slabs* span in two orthogonal directions and are supported by beams on all four sides. When the ratio of long span to short span exceeds two, the slab acts as a one-way slab spanning in the direction of the long slab.

  Several different methods may be used to design two-way slabs. The *direct design method* divides the slab into a column strip and a middle strip. The *equivalent frame method* divides the slab into rows of columns supporting the tributary slab width. The *yield line method* is based on limit state theory applied to an appropriate collapse mechanism.

- *Flat plates* are supported at the corners by columns without any supporting beams.

- *Flat slabs*, which are similar to flat plates, have increased depth at the columns in order to produce a drop panel.

- *Waffle slabs* are flat slabs constructed with voids formed in the soffit. At the columns, the voids are omitted in order to produce a solid slab similar to a drop panel.

Direct Design Method

ACI Sec. 13.6.1 permits the direct design method to be used provided the following conditions exist.

- A minimum of three continuous spans exist in each direction.

- Panels are rectangular with an aspect ratio not exceeding 2.

- Successive span lengths do not differ by more than one-third of the longer span.

- Columns are not offset by more than 10% of the span.

- Loading consists of uniformly distributed gravity loads with the service live load not exceeding twice the service dead load.

- For beam supported slabs, the ratio of the beam stiffnesses in two perpendicular directions, $\alpha_{f1} l_2^2 / \alpha_{f2} l_1^2$, is between 0.2 and 5.0 where the moments of inertia of the equivalent beam and slab are based on the sections shown in Fig. 1.25.

The slab is divided into design strips, as shown in Fig. 1.26, with a column strip extending the lesser of $0.25 l_1$ or $0.25 l_2$ on each side of a column centerline. A middle strip consists of the remainder of the slab between column strips.

Figure 1.26 *Details of Design Strips*
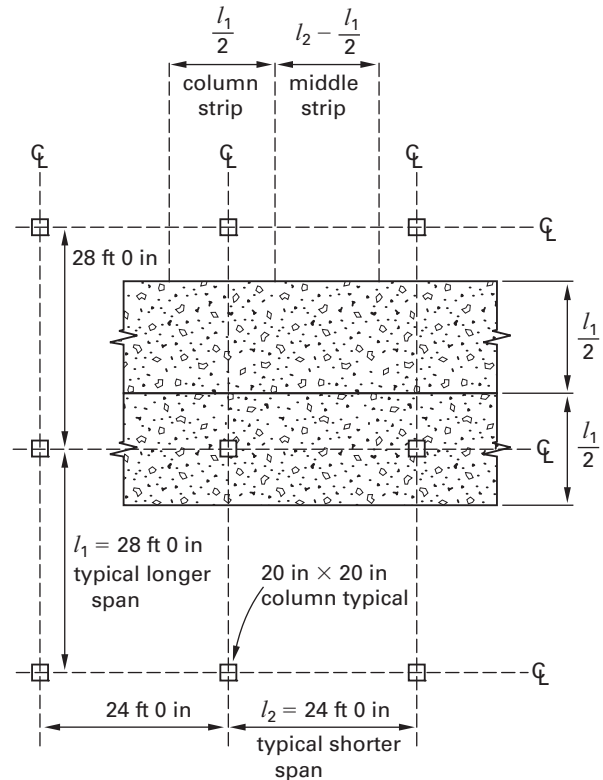
Example 1.28

A 9 in thick flat plate floor has plan dimensions between column centers of 24 ft and 28 ft as shown in Fig. 1.26. Calculate the widths of the column strip and the middle strip for the direction parallel to the 28 ft side of the panel.

Solution

From ACI Sec. 13.2, the width of the column strip is

$$w_c = 0.5l_{\min}$$
$$= (0.5)(24 \text{ ft})$$
$$= 12 \text{ ft}$$

The width of the middle strip in the direction of the longer span is

$$w_m = l_{\min} - 0.5l_{\min}$$
$$= 24 \text{ ft} - 12 \text{ ft}$$
$$= 12 \text{ ft}$$

Design for Flexure

The total factored static moment on a panel is calculated by ACI Sec. 13.6.2 as

$$M_o = \frac{q_u l_2 l_n^2}{8} \qquad [\textit{ACI 13-4}]$$

The total factored moment is now distributed in accordance with ACI Sec. 13.6.3 into positive and negative moments across the full width of the panel, depending on the support conditions, as shown in Fig. 1.27.

These distributed moments are now further subdivided between the column and middle strips as specified in ACI Sec. 13.6.4 through Sec. 13.6.7. That portion of the moment not attributed to a column strip is assigned to the corresponding half middle strips. The negative moment at an interior support is distributed to a column strip as shown in Table 1.7.

The negative moment at an exterior support is distributed to a column strip as shown in Table 1.8.

The positive moment at midspan is distributed to a column strip as shown in Table 1.9.

Beams at panel edges shall be assigned the percentage of column strip moment as shown in Table 1.10.

Example 1.29

Assume that the flat plate floor for Ex. 1.28, which is shown in Fig. 1.26, supports a total factored distributed load of 200 lbf/ft^2. The column size is 20 in by 20 in. Determine the factored moments in the column strip and middle strip in the direction of the longer span.

Figure 1.27 M_o Distribution Factors

(a) interior panel any support

(b) end panel simply supported

(c) flat plate, end panel with edge beam

(d) flat plate, end panel without edge beam

(e) end panel beam supported all round

(f) end panel fully restrained end

Table 1.7 *Percentage Distribution of Interior Negative Moment to Column Strip*

l_2/l_1	0.5	1	2
$\alpha_{f1}l_2/l_1 = 0$	75	75	75
$\alpha_{f1}l_2/l_1 \geq 1.0$	90	75	45

Table 1.8 *Percentage Distribution of Exterior Negative Moment to Column Strip*

l_2/l_1		0.5	1	2
$\alpha_{f1}l_2/l_1 = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$\alpha_{f1}l_2/l_1 \geq 1.0$	$\beta_t = 0$	100	100	100

Table 1.9 *Percentage Distribution of Positive Moment to Column Strip*

l_2/l_1	0.5	1	2
$\alpha_{f1}l_2/l_1 = 0$	60	60	60
$\alpha_{f1}l_2/l_1 \geq 1.0$	90	75	45

Table 1.10 *Percentage Distribution of Column Strip Moments to Edge Beam*

$\alpha_{f1}l_2/l_1$	0	≥ 1.0
% assigned to beam	0	85

Solution

From ACI Sec. 13.6.2.5, the clear span in the direction of the longer span is

$$l_n = l_1 - c_1 = 28 \text{ ft} - \frac{20 \text{ in}}{12 \frac{\text{in}}{\text{ft}}}$$

$$= 26.33 \text{ ft}$$

$$> 0.65l_1 \quad [\text{satisfactory}]$$

From ACI Sec. 13.6.2, the total factored static moment is

$$M_o = \frac{q_u l_2 l_n^2}{8} = \frac{\left(0.20 \, \frac{\text{kip}}{\text{ft}^2}\right)(24 \text{ ft})(26.33 \text{ ft})^2}{8}$$

$$= 416 \text{ ft-kips}$$

From ACI Sec. 13.6.3 and Fig. 1.27(a), the total positive moment across the panel is

$$M_m = 0.35M_o = (0.35)(416 \text{ ft-kips})$$

$$= 146 \text{ ft-kips}$$

From ACI Sec. 13.6.4 and Table 1.9, $\alpha_{f1}l_2/l_1 = 0$ and the column strip positive moment at midspan is

$$M_{cm} = 0.60M_m = (0.60)(146 \text{ ft-kips})$$

$$= 87 \text{ ft-kips}$$

From ACI Sec. 13.6.6, the middle strip positive moment at midspan is

$$M_{mm} = M_m - M_{cm} = 146 \text{ ft-kips} - 87 \text{ ft-kips}$$

$$= 59 \text{ ft-kips}$$

From ACI Sec. 13.6.3 and Fig. 1.27(a), the total negative moment across the panel is

$$M_c = 0.65M_o = (0.65)(416 \text{ ft-kips})$$

$$= 270 \text{ ft-kips}$$

From ACI Sec. 13.6.4 and Table 1.7, $\alpha_{f1}l_2/l_1 = 0$ and the column strip negative moment at an interior support is

$$M_{cc} = 0.75M_c = (0.75)(270 \text{ ft-kips})$$

$$= 203 \text{ ft-kips}$$

From ACI Sec. 13.6.6, the middle strip negative moment at an interior support is

$$M_{mc} = M_c - M_{cc} = 270 \text{ ft-kips} - 203 \text{ ft-kips}$$

$$= 67 \text{ ft-kips}$$

Design for Shear

The design for shear at column supports must consider both flexural or one-way shear and punching or two-way shear, as shown in Fig. 1.28. The flexural shear capacity of the panel, in a direction parallel to the side l_1, is given by ACI Sec. 11.2.1.1 as

$$\phi V_c = 2\phi d l_1 \lambda \sqrt{f_c'} \qquad [\textit{ACI 11-3}]$$

The critical perimeter for punching shear is specified in ACI Sec. 11.11.1.2 as being a distance from the face of the column equal to one-half of the effective depth. The length of the critical perimeter is given by

$$b_o = (2)(c_1 + c_2) + 4d$$

For a corner column located less than $d/2$ from the edge of a panel, the critical perimeter is two sided. For a similarly situated edge column, it is three sided, as shown in Fig. 1.29.

Openings in a panel within 10 times the thickness of the panel from the edge of a column reduce the critical perimeter, as shown in Fig. 1.30.

In accordance with ACI Sec. 13.4, openings of any size are permitted in the area common to two intersecting middle strips. In the area common to two intersecting column strips, the maximum width of opening is limited to one-eighth of the column strip width. In the area common to one column strip and one middle strip, not more than one-fourth of the reinforcement in either strip shall be interrupted by openings. In all cases, the area of reinforcement interrupted by openings shall be replaced by an equivalent amount added on the sides of the opening.

Figure 1.28 *Critical Sections for Shear*
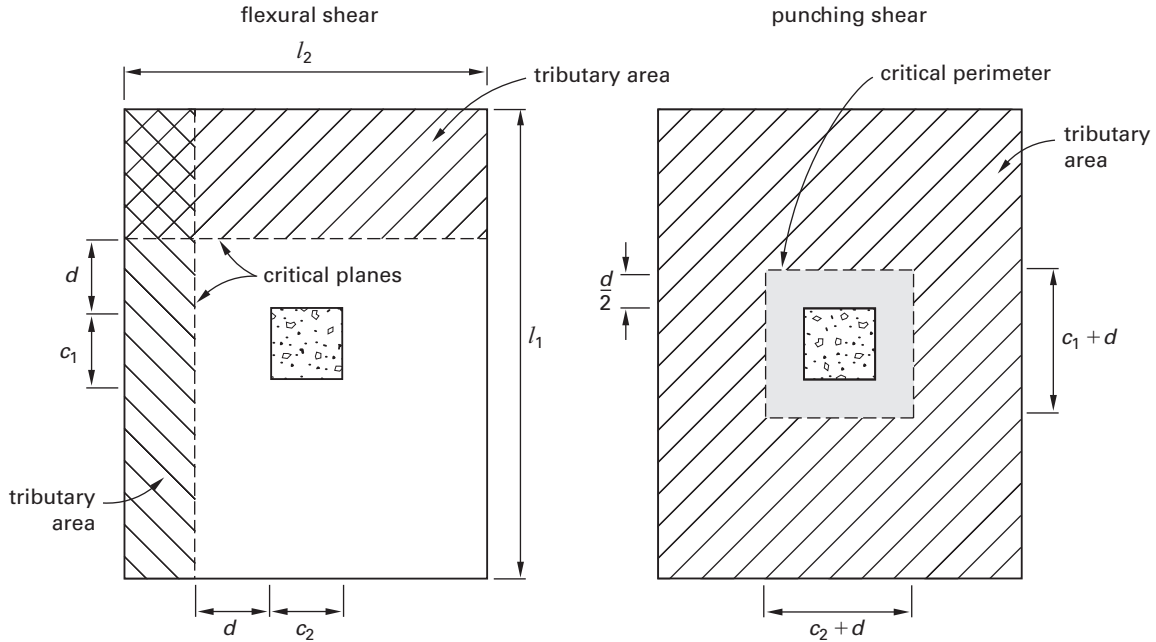
Figure 1.29 *Corner and Edge Columns*
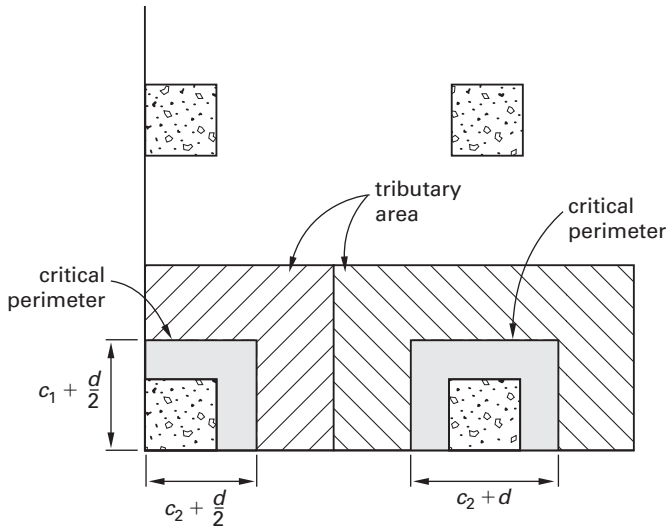
The punching shear capacity of the panel is the smallest of the three values given by ACI Sec. 11.11.2.1, which are

$$\phi V_c = 4\phi db_o \lambda \sqrt{f'_c} \qquad [\text{ACI 11-33}]$$

$$\le \phi db_o \left(2 + \frac{4}{\beta}\right) \lambda \sqrt{f'_c} \qquad [\text{ACI 11-31}]$$

$$\le \phi db_o \left(2 + \frac{\alpha_s d}{b_o}\right) \lambda \sqrt{f'_c} \qquad [\text{ACI 11-32}]$$

$$\alpha_s = 40 \text{ for an interior column}$$

$$= 30 \text{ for an edge column}$$

$$= 20 \text{ for a corner column}$$

Figure 1.30 *Reduction in Critical Perimeter*
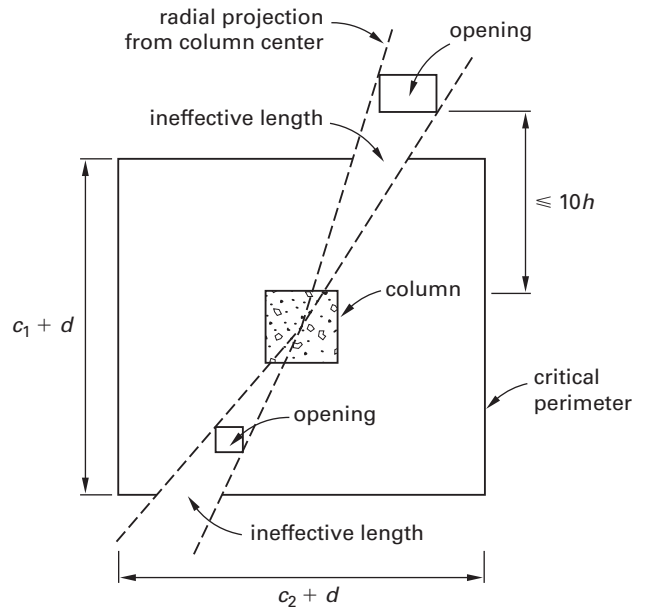
Example 1.30

The flat plate floor of normal weight concrete for Ex. 1.28 and Ex. 1.29 has an effective depth of 7.5 in and a concrete compressive strength of 4000 lbf/in². Determine whether the shear capacity of the plate is adequate for an interior column.

Solution

The critical section for flexural shear is located a distance from the center of the panel given by

$$x = \frac{l_1}{2} - \left(d + \frac{c_1}{2} \right)$$

$$= \frac{28 \text{ ft}}{2} - \left(\frac{7.5 \text{ in}}{12 \frac{\text{in}}{\text{ft}}} + \frac{20 \text{ in}}{(2)\left(12 \frac{\text{in}}{\text{ft}} \right)} \right)$$

$$= 12.54 \text{ ft}$$

The factored applied shear at the critical section for flexural shear is

$$V_u = q_u l_2 x$$

$$= \left(0.2 \frac{\text{kip}}{\text{ft}^2} \right) (24 \text{ ft})(12.54 \text{ ft})$$

$$= 60 \text{ kips}$$

The flexural shear capacity at the critical section is given by ACI Eq. 11-3 as

$$\phi V_c = 2\phi d l_2 \lambda \sqrt{f_c'}$$

$$= \frac{\begin{array}{c} (2)(0.75)(7.5 \text{ in})(24 \text{ ft}) \\ \times \left(12 \frac{\text{in}}{\text{ft}} \right)(1.0)\sqrt{4000 \frac{\text{lbf}}{\text{in}^2}} \end{array}}{1000 \frac{\text{lbf}}{\text{kip}}}$$

$$= 205 \text{ kips}$$

$$> V_u \quad [\text{satisfactory}]$$

The length of one side of the critical perimeter for punching shear is

$$b = c + d$$

$$= 20 \text{ in} + 7.5 \text{ in}$$

$$= 27.5 \text{ in}$$

The factored applied shear at the critical perimeter for punching shear is

$$V_u = q_u(l_1 l_2 - b^2)$$

$$= \left(0.2 \frac{\text{kip}}{\text{ft}^2} \right) \left((28 \text{ ft})(24 \text{ ft}) - \frac{(27.5 \text{ in})^2}{\left(12 \frac{\text{in}}{\text{ft}} \right)^2} \right)$$

$$= 133 \text{ kips}$$

The length of the critical perimeter for punching shear is

$$b_o = 4b$$

$$= (4)(27.5 \text{ in})$$

$$= 110 \text{ in}$$

The punching shear capacity of the plate is governed by ACI Eq. 11-33 as

$$\phi V_c = 4\phi d b_o \lambda \sqrt{f_c'}$$

$$= \frac{(4)(0.75)(7.5 \text{ in})(110 \text{ in})(1.0)\sqrt{4000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}}$$

$$= 157 \text{ kips}$$

$$> V_u \quad [\text{satisfactory}]$$

Equivalent Frame Method

No limitations are placed on the use of the equivalent frame method. As shown in Fig. 1.31, the structure is divided into a series of equivalent frames by cutting the building along lines midway between columns. The resulting frames are considered separately in the longitudinal and transverse directions of the building.

Figure 1.31 *Equivalent Frame Method*
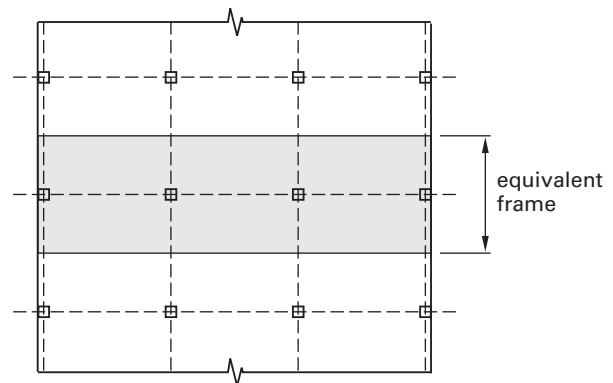
The equivalent frames may be analyzed by any frame analysis technique to determine the positive and negative moments in the slab. These moments are distributed across the slab using the same proportions as are used in the direct design method.

The equivalent frames may be analyzed for both vertical loads and lateral loads.

When the unfactored live load does not exceed three-quarters of the unfactored dead load, it may be assumed that maximum factored moments occur at all sections with the full factored live load on the entire slab system. For other conditions, patch loading must be used with only three-quarters of the full factored live load.

13. ANCHORING TO CONCRETE[14]

Nomenclature

A_{brg}	net bearing area of the head of stud, anchor bolt, or headed deformed bar	in^2
A_{Nc}	projected concrete failure surface of a single anchor or group of anchors	in^2
A_{Nco}	projected concrete failure surface of a single anchor	in^2
$A_{se,N}$	effective cross-sectional area of anchor in tension	in^2
$A_{se,V}$	effective cross-sectional area of anchor in shear	in^2
A_{Vco}	projected concrete failure area of a single anchor for calculation of shear strength in shear, if not limited by corner influences, spacing, or member thickness	in^2
c_{ac}	critical edge distance required to develop the basic concrete breakout strength of a post-installed anchor in uncracked concrete without supplementary reinforcement to control splitting	in
$c_{a,\max}$	maximum distance from center of an anchor shaft to the edge of concrete	in
$c_{a,\min}$	minimum distance from center of an anchor shaft to the edge of concrete	in
c_{a1}	distance from the center of an anchor shaft to the edge of concrete in one direction. If shear is applied to anchor, c_{a1} is taken in the direction of the applied shear. If tension is applied to the anchor, c_{a1} is the minimum edge distance.	in
c_{a2}	distance from center of an anchor shaft to the edge of concrete in the direction perpendicular to c_{a1}	in
d_a	outside diameter of anchor or shaft diameter of headed stud, headed bolt, or hooked bolt	in
d_a'	value substituted for d_a when an oversized anchor is used	in
e_h	distance from the inner surface of the shaft of a J- or L-bolt to the outer tip of the J- or L-bolt	in
e_N'	distance between resultant tension load on a group of anchors loaded in tension and the centroid of the group of anchors loaded in tension	in
f_{uta}	specified tensile strength of the anchor	lbf/in^2
f_{ya}	specified yield strength of anchor	lbf/in^2
h_a	thickness of member in which an anchor is located, measured parallel to anchor axis	in
h_{ef}	effective embedment depth of anchor	in
k_{cp}	pryout coefficient	–
l_e	load-bearing length of the anchor for shear	in
n	number of anchors	–
N_b	basic concrete breakout strength in tension of a single anchor in cracked concrete	lbf
N_{cb}	nominal concrete breakout strength in tension of a single anchor	lbf
N_{cbg}	nominal concrete breakout strength in tension of a group of anchors	lbf
N_n	nominal strength in tension	lbf
N_p	pullout strength in tension of a single anchor in cracked concrete	lbf
N_{pn}	nominal pullout strength in tension of a single anchor	lbf
N_{sa}	nominal strength of a single anchor or group of anchors in tension as governed by the steel strength	lbf
N_{sb}	side-face blowout strength of a single anchor	lbf
N_{sbg}	side-face blowout strength of a group of anchors	lbf
N_{ua}	factored tensile force applied to anchor or group of anchors	lbf
V_{cb}	nominal concrete breakout strength in shear of a single anchor	lbf
V_{cbg}	nominal concrete breakout strength in shear of a group of anchors	lbf
V_{cp}	nominal concrete pryout strength of a single anchor	lbf
V_{cpg}	nominal concrete pryout strength of a group of anchors	lbf
V_n	nominal shear strength	lbf
V_{sa}	nominal strength in shear of a single anchor or group of anchors as governed by the steel strength	lbf
V_{ua}	factored shear force applied to a single anchor or group of anchors	lbf

Symbols

ϕ	strength reduction factor	–
$\Psi_{c,N}$	factor used to modify tensile strength of anchors based on the presence or absence of cracks in concrete	–
$\Psi_{c,P}$	factor used to modify pullout strength of anchors based on presence or absence of cracks in concrete	–
$\Psi_{c,V}$	factor used to modify shear strength of anchors based on presence or absence of cracks in concrete and presence or absence of supplementary reinforcement	–
$\Psi_{cp,N}$	factor used to modify tensile strength of post-installed anchors intended for use in uncracked concrete without supplementary reinforcement	–
$\Psi_{ec,N}$	factor used to modify tensile strength of anchors based on eccentricity of applied loads	–
$\Psi_{ec,V}$	factor used to modify shear strength of anchors based on eccentricity of applied loads	–
$\Psi_{ed,N}$	factor used to modify tensile strength of anchors based on proximity to edges of concrete member	–
$\Psi_{ed,V}$	factor used to modify shear strength of anchors based on proximity to edges of concrete member	–
$\Psi_{h,V}$	factor used to modify shear strength of anchors located in concrete members with $h_a < 1.5c_{a1}$	–

Code Requirements

ACI App. D provides design requirements for cast-in and post-installed anchors in concrete that are used to transmit structural loads by means of tension, shear, or a combination of tension and shear. Tensile loading failure modes are shown in Fig. 1.32, and shear loading failure modes are shown in Fig. 1.33.

Figure 1.32 *Tensile Failure Modes*
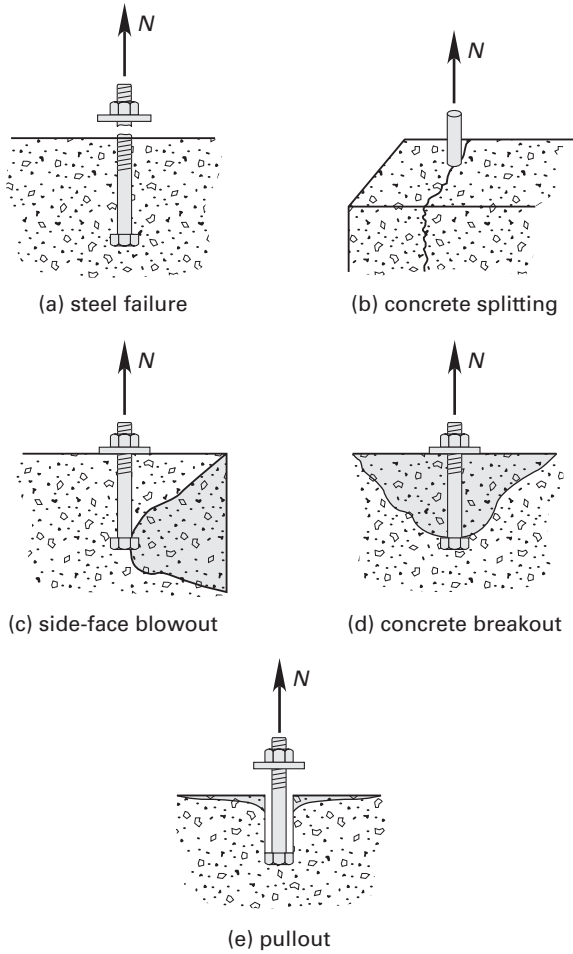
(a) steel failure

(b) concrete splitting

(c) side-face blowout

(d) concrete breakout

(e) pullout

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Cast-in anchors and post-installed anchors have different design requirements. This section covers the requirements of cast-in headed studs, headed bolts, and hooked bolts.

ACI requires reduced anchorage strengths when investigating anchorages for seismic loads. The reduction is required for structures in seismic design categories C, D, E, or F. IBC Sec. 1905.1.9 modifies ACI Sec. D.3.3.4 through Sec. D.3.3.7. Some industry confusion exists because IBC-12 references ACI 318-08 in Sec. 1905.1.9, yet ACI 318-11 is the referenced standard for IBC-12. For this chapter, the following seismic design requirements are based on the IBC Sec. 1905.1.9 modifications.

Figure 1.33 *Shear Failure Modes*
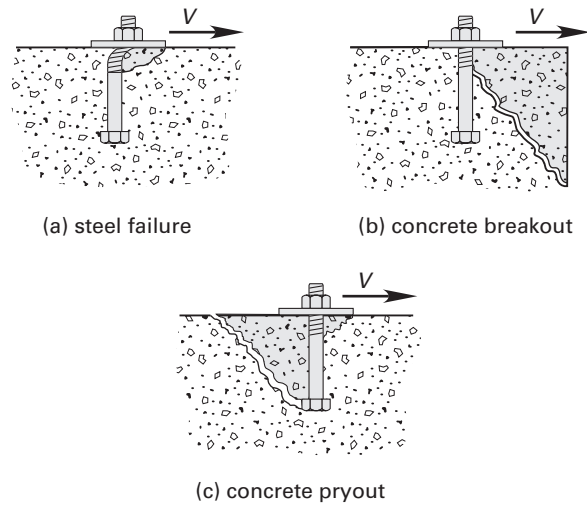
(a) steel failure

(b) concrete breakout

(c) concrete pryout

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- IBC-modified ACI Sec. D.3.3.4 requires the design strength for concrete failure be taken as $0.75\phi N_n$ and $0.75\phi V_n$. The values of N_n and V_n are determined assuming the concrete is cracked, unless it can be demonstrated that the concrete remains uncracked.

- To preclude brittle failure in the concrete, IBC-modified ACI Sec. D.3.3.5 and Sec. D.3.3.6 require the strength of the connection to be governed by the strength of ductile steel elements or ductile attachments.

- An exception to IBC-modified ACI Sec. D.3.3.5 and Sec. D.3.3.6 (for ductile steel elements or attachments) is that anchors in concrete designed to support nonstructural components are not governed by the strength of ductile steel elements. The design strength for concrete failure is taken as $0.75\phi N_n$ and $0.75\phi V_n$.

- An exception to IBC-modified ACI Sec. D.3.3.5 and Sec. D.3.3.6 (for ductile steel elements or attachments) is provided for anchors in concrete designed for the wall anchorage force specified in ASCE/SEI7 Sec. 12.11.2.1, which is

$$F_p = 0.4 S_{DS} k_a I_e W_p$$

The design strength for concrete failure is taken as $0.75\phi N_n$ and $0.75\phi V_n$.

- If a ductile failure cannot be achieved, IBC-modified ACI Sec. D.3.3.7 requires the design strength for concrete failure to be taken as

$$0.4(0.75\phi N_n) = 0.3\phi N_n$$
$$0.4(0.75\phi V_n) = 0.3\phi V_n$$

In light-frame construction of bearing or nonbearing walls, anchors of 1 in diameter or less that are attaching sill plate or steel track to foundations or foundation stem walls need not satisfy this requirement when the design strength of the anchors is determined by ACI Sec. D.6.2.1(c).

Anchors in Tension

The strength reduction factor for tension loads is given by ACI Sec. D.4.3 as

$$\phi = 0.75 \quad \text{[ductile steel anchor]}$$
$$= 0.65 \quad \text{[brittle steel anchor]}$$

Ductile bolts include ASTM A307 grade A bolts having a minimum specified tensile strength of 60 kips/in^2.

Steel Strength of Anchor

The nominal strength of an anchor in tension is given by ACI Eq. D-2 as

$$N_{sa} = A_{se,N} f_{uta}$$

The specified tensile strength of an anchor is given by ACI Sec. D.5.1.2 as

$$f_{uta} \leq 1.9 f_{ya}$$
$$\leq 125{,}000 \text{ lbf/in}^2$$

Concrete Breakout Strength

The strength reduction factor for an anchor or anchor group governed by concrete breakout, side-face blowout, pullout, or pryout strength is given by ACI Sec. D.4.3 as

$$\phi = 0.75 \quad \text{[supplementary reinforcement]}$$
$$= 0.70 \quad \text{[no supplementary reinforcement]}$$

As shown in Fig. 1.34, for a single anchor not near the edges of the concrete element, the failure surface in the concrete is shaped like a pyramid. The pyramid's apex is at the centerline of the anchor at the bearing contact surface of the head. The failure surface radiates outward to the surface at a slope of 1 to 1.5. The maximum projected failure area for a single anchor is given by ACI Eq. D-5 as

$$A_{Nco} = 9h_{ef}^2$$

h_{ef} is the effective embedment depth of an anchor. When the failure surface for a single anchor is limited by adjacent edges, the reduced projected failure area is denoted by A_{Nc}.

Figure 1.34 *Concrete Breakout of Anchor in Tension*
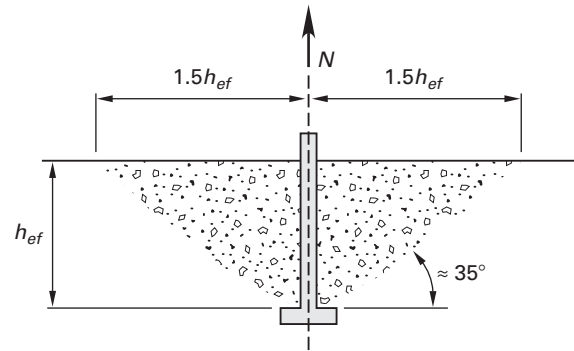
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Failure occurs and concrete breakout results when the tensile stress on the failure surface exceeds the tensile strength of the concrete. The nominal concrete breakout strength for a single cast-in anchor in tension is given by ACI Eq. D-3 as

$$N_{cb} = \frac{A_{Nc}}{A_{Nco}} \Psi_{ed,N} \Psi_{c,N} \Psi_{cp,N} N_b$$

ACI Sec. D.5.2.5 gives the modification factor for edge effects, $\Psi_{ed,N}$. When $c_{a,\min} \geq 1.5 h_{ef}$,

$$\Psi_{ed,N} = 1.0$$

$c_{a,\min}$ is the smallest of the influencing edge distances.

When $c_{a,\min} < 1.5 h_{ef}$,

$$\Psi_{ed,N} = 0.7 + 0.3 \left(\frac{c_{a,\min}}{1.5 h_{ef}} \right)$$

The modification factor for cracked concrete from ACI Sec. D.5.2.6 is

$$\Psi_{c,N} = 1.0 \quad \text{[concrete cracked at service load levels]}$$
$$= 1.25 \quad \text{[concrete uncracked at service load levels]}$$

From ACI Sec. D.5.2.7, the modification factor for post-installed anchors is

$$\Psi_{cp,N} = 1.0 \quad \text{[cast-in anchors]}$$

The basic concrete breakout strength of a single cast-in anchor in tension in cracked concrete as defined in ACI Sec. D.5.2.2 is

$$N_b = k_c \lambda \sqrt{f_c'} h_{ef}^{1.5}$$

The coefficient for concrete breakout strength is

$$k_c = 24 \quad \text{[cast-in anchors]}$$
$$= 17 \quad \text{[post-installed anchors]}$$

ACI Eq. D-7 applies when $11 \text{ in} < h_{ef} < 25 \text{ in}$, and the basic concrete breakout strength is

$$N_b = 16\lambda_a\sqrt{f'_c}h_{eb}^{5/3}$$

λ_a is the modification factor for lightweight concrete per ACI Sec. D.3.6. For normal weight concrete using cast-in anchors, $\lambda_a = 1.0$.

When anchors are spaced closer than three times their embedment depth, the failure surfaces of adjacent anchors intersect. The failure surface for such an anchor group is determined by projecting the failure surface outward from the outer bolts in the group, as shown in Fig. 1.35. From Fig. 1.35, the projected area of this failure surface when distanced from the element edges is

$$A_{Nc} = (a + 3h_{ef})(b + 3h_{ef})$$
$$\leq nA_{Nco}$$

n is the number of anchors in the group, a is the distance between outside anchors in the group, and b is the distance between outside anchors in the group perpendicular to a.

The nominal concrete breakout strength for a cast-in anchor group in tension is given by ACI Eq. D-4 as

$$N_{cbg} = \frac{A_{Nc}}{A_{Nco}}\Psi_{ec,N}\Psi_{ed,N}\Psi_{c,N}\Psi_{cp,N}N_b$$

Figure 1.35 *Concrete Breakout Surface for an Anchor Group*
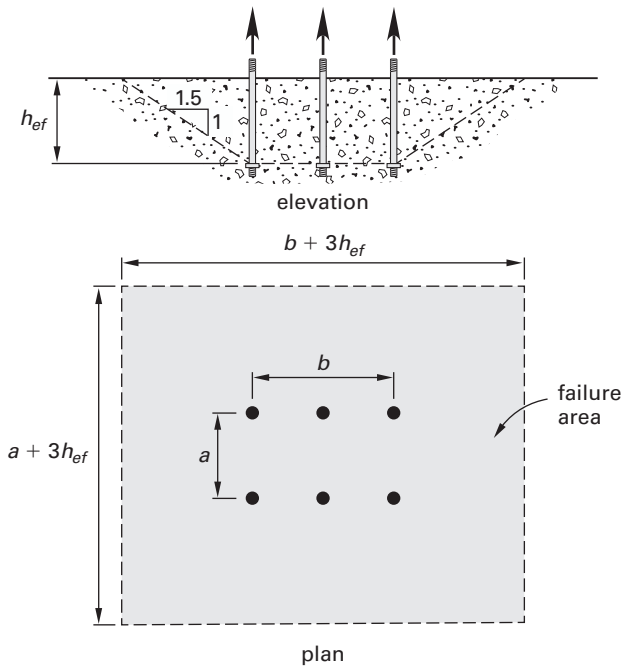
The modification factor for eccentrically loaded anchor groups is given by ACI Eq. D-8 as

$$\Psi_{ec,N} = \frac{1}{1 + \dfrac{2e'_N}{3h_{ef}}}$$

$$\leq 1.0 \quad \text{[concentrically loaded groups]}$$

Pullout Strength of Anchor

The nominal concrete pullout strength for a single anchor in tension is given by ACI Eq. D-13 as

$$N_{pn} = \Psi_{c,P}N_p$$

For a headed bolt or stud, where A_{brg} is the bearing area of the bolt or stud head, the nominal pullout strength in cracked concrete is given by ACI Eq. D-14 as

$$N_p = 8A_{brg}f'_c$$

Bearing areas may be obtained from Cook.[15] For a hooked bolt, the nominal pullout strength in cracked concrete is given by ACI Eq. D-15 as

$$N_p = 0.8e_h d_a f'_c$$
$$3d_a < e_h < 4.5d_a$$

e_h is the distance from the outer tip of a hooked bolt to the inner surface of the shaft, and d_a is the diameter of the bolt.

The modification factor for cracked concrete is given by ACI Sec. D.5.3.6 as

$$\Psi_{c,P} = 1.0 \quad \text{[concrete cracked at service load levels]}$$
$$= 1.4 \quad \text{[concrete uncracked at service load levels]}$$

Side-Face Blowout Strength of Anchor in Tension

Side-face blowout is caused by spalling of the concrete surface adjacent to the head of an anchor that is close to the face of the concrete. The nominal concrete blowout strength for a single anchor in tension, with $h_{ef} > 2.5c_{a1}$, is given by ACI Eq. D-16 as

$$N_{sb} = \left(160c_{a1}\sqrt{A_{brg}}\right)\lambda_a\sqrt{f'_c}$$

c_{a1} is the minimum distance from the center of the anchor shaft to the edge of concrete, and c_{a2} is the distance from the center of the anchor shaft to the edge of the concrete perpendicular to c_{a1}. If c_{a2} for the single headed anchor is less than $3c_{a1}$, the value of N_{sb} is multiplied by the factor $(1 + c_{a2}/c_{a1})/4$, where $1.0 \leq c_{a2}/c_{a1} \leq 3.0$.

Concrete Splitting

Unless supplementary reinforcement is provided to control splitting and minimum spacing, edge distances for

anchors must conform to the requirements of ACI Sec. D.8. The requirements for cast-in anchors are as follows.

- The minimum center-to-center spacing of anchors is $4d_a$ for untorqued cast-in anchors.

- The minimum center-to-center spacing is $6d_a$ for cast-in anchors that will be torqued.

- The minimum edge distances for cast-in headed anchors that will not be torqued are the same as for normal reinforcement cover requirements as specified in ACI Sec. 7.7.

- The minimum edge distance is $6d_a$ for cast-in headed anchors that will be torqued.

Example 1.31

A glulam crosstie between diaphragm chords is supported at one end in a steel beam shoe as shown. The shoe is attached to a tilt-up concrete wall of normal weight concrete with a compressive strength of $f'_c = 4000$ lbf/in². The four hex ASTM A307 grade A anchor bolts have a $1/2$ in diameter, an effective minimum specified tensile strength of $f_{uta} = 60$ kips/in², an effective cross-sectional area in tension of $A_{se,N} = 0.142$ in², an effective cross-sectional area in shear of $A_{se,V} = 0.196$ in², and an effective bearing area of the bolt head of $A_{brg} = 0.291$ in². The effective embedment length of the anchor bolts is $5^1/2$ in. The anchor bolts are not near any concrete edges and are not torqued. Supplementary reinforcement is not provided, and the concrete may be considered cracked. The floor diaphragm is flexible, and the wall anchor force is determined from ASCE/SEI7 Sec. 12.11.2.1. The structure is assigned seismic design category C, and the governing strength load combination gives a tension force on the beam shoe of $N_{ua} = 15$ kips and a shear force of $V_{ua} = 4$ kips. Determine if the anchorage is adequate for the tension force on the shoe.
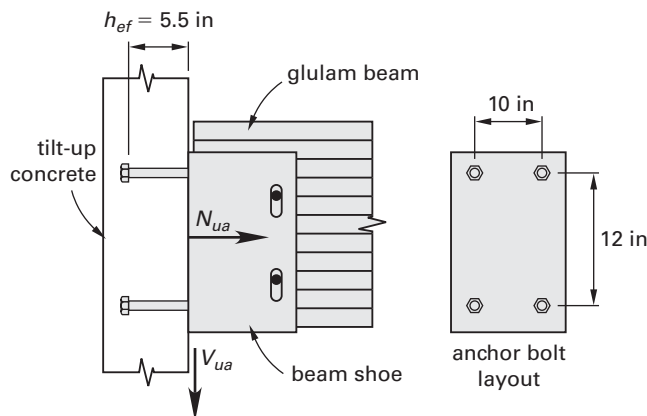
Solution

The effective embedment depth is

$$h_{ef} = 5.5 \text{ in}$$

The edge distance exceeds required cover requirements, and spacing exceeds $4d_a$. In accordance with ACI Sec. D.8, side-face blowout and splitting are not considered.

The anchor bolts are ductile, so from ACI Sec. D.4.3, the strength reduction factors are

$$\phi = 0.75 \quad \text{[tension on a ductile anchor bolt]}$$

$$= 0.70 \quad \left[\begin{array}{l}\text{concrete breakout or pullout} \\ \text{without supplemental reinforcement}\end{array}\right]$$

The wall anchor force is determined from ASCE/SEI7 Sec. 12.11.2.1. In accordance with IBC-modified ACI Sec. D.3.3.5, the strength of the anchorage need not be governed by the strength of the steel elements.

Steel Strength of Anchor

The steel strength is based on the effective area of the threaded bolt. For a $1/2$ in diameter threaded anchor, the effective area is $A_{se,N} = 0.142$ in². The minimum specified tensile strength of the four ASTM A307 grade A anchor bolts is 60 kips/in². The design strength of the four $1/2$ in diameter ductile anchors is given by ACI Eq. D-2 as

$$4\phi N_{sa} = 4\phi A_{se,N} f_{uta}$$

$$= (4)(0.75)(0.142 \text{ in}^2)\left(60 \text{ } \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 25.56 \text{ kips} > N_{ua} \quad \text{[satisfactory]}$$

Concrete Breakout Strength in Tension

The horizontal spacing of the anchor bolts is

$$s_1 = 10 \text{ in} < 3h_{ef} = 16.5 \text{ in}$$

The vertical spacing of the anchor bolts is

$$s_2 = 12 \text{ in} < 3h_{ef} = 16.5 \text{ in}$$

The projected area of the failure surface is

$$A_{Nc} = (s_1 + 3h_{ef})(s_2 + 3h_{ef})$$

$$= (10 \text{ in} + 16.5 \text{ in})(12 \text{ in} + 16.5 \text{ in})$$

$$= 755 \text{ in}^2$$

$$< 4A_{Nco} \quad \text{[satisfies ACI Sec. D.5.2.1]}$$

The projection of the failure surface for a single anchor on the concrete outer surface has an area of

$$A_{Nco} = 9h_{ef}^2$$

$$= (9)(5.5 \text{ in})^2$$

$$= 272 \text{ in}^2$$

The basic concrete breakout strength in tension of a single anchor in cracked concrete as defined in ACI Sec. D.5.2.2 and ACI Eq. D-6 is

$$N_b = k_c \lambda_a \sqrt{f'_c} h_{ef}^{1.5}$$
$$= \frac{(24)(1.0)\sqrt{4000 \ \dfrac{\text{lbf}}{\text{in}^2}}(5.5 \ \text{in})^{1.5}}{1000 \ \dfrac{\text{lbf}}{\text{kip}}}$$
$$= 19.58 \ \text{kips}$$

The modification factors are 1.0 as specified by ACI Sec. D.5.

The nominal concrete breakout strength for the anchor group is given by ACI Eq. D-4 as

$$N_{cbg} = \frac{A_{Nc}}{A_{Nco}} \Psi_{ec,N} \Psi_{ed,N} \Psi_{c,N} \Psi_{cp,N} N_b$$
$$= \left(\frac{755 \ \text{in}^2}{272 \ \text{in}^2}\right)(1.0)(1.0)(1.0)(1.0)(19.58 \ \text{kips})$$
$$= 54.35 \ \text{kips}$$

The design concrete breakout strength for the anchor group for seismic loading is given by IBC-modified ACI Sec. D.3.3.5 and ACI Eq. D-4 as

$$0.75\phi N_{cbg} = (0.75)(0.7)(54.35 \ \text{kips})$$
$$= 28.53 \ \text{kips} > N_{ua} \quad [\text{satisfactory}]$$

Pullout Strength of Anchor in Tension

For a headed bolt, the nominal pullout strength in cracked concrete is

$$N_p = 8A_{brg}f'_c$$
$$= \left(\frac{(8)(0.291 \ \text{in}^2)}{1000 \ \dfrac{\text{lbf}}{\text{kip}}}\right)\left(4000 \ \frac{\text{lbf}}{\text{in}^2}\right)$$
$$= 9.31 \ \text{kips}$$

The modification factor for cracked concrete is 1.0 as specified by ACI Sec. D.5.

The nominal concrete pullout strength for a single anchor in cracked concrete is given by ACI Eq. D-13 as

$$N_{pn} = \Psi_{c,P} N_p$$
$$= (1.0)(9.31 \ \text{kips})$$
$$= 9.31 \ \text{kips}$$

The design concrete pullout strength for a single anchor in tension is given by IBC-modified ACI Sec. D.3.3.4 and ACI Eq. D-13 as

$$0.75\phi N_{pn} = (0.75)(0.7)(9.31 \ \text{kips})$$
$$= 4.89 \ \text{kips}$$

The design concrete pullout strength of the four bolts is

$$(4)(0.75)\phi N_{pn} = (4)(4.89 \ \text{kips})$$
$$= 19.56 \ \text{kips} > N_{ua} \quad [\text{satisfactory}]$$

Anchors in Shear

Steel Strength of Anchor

The strength reduction factor is given by ACI Sec. D.4.3 as

$$\phi = 0.65 \quad [\text{ductile steel anchor}]$$
$$= 0.60 \quad [\text{brittle steel anchor}]$$

The nominal strength of a headed stud anchor in shear is given by ACI Eq. D-28 as

$$V_{sa} = A_{se,V} f_{uta}$$

The nominal strength of a headed bolt and hooked bolt anchor in shear is given by ACI Eq. D-29 as

$$V_{sa} = 0.6A_{se,V} f_{uta}$$

The specified tensile strength of an anchor is given by ACI Sec. D.5.1.2 as

$$f_{uta} \le 1.9 f_{ya}$$
$$\le 125{,}000 \ \text{lbf/in}^2$$

f_{ya} is the specific yield strength of the anchor.

Ductile bolts include ASTM A307 grade A bolts with a minimum specified tensile strength of 60 kips/in^2.

Concrete Breakout Strength in Shear

ACI Sec. D.4.3 gives the strength reduction factor for a single anchor or an anchor group in shear, governed by concrete breakout or pry-out, as

$$\phi = 0.75 \quad [\text{supplementary reinforcement provided}]$$
$$= 0.70 \quad [\text{supplementary reinforcement not provided}]$$

As shown in Fig. 1.36, for a single anchor not near edges perpendicular to the shear force, the failure surface in the concrete is shaped like a half pyramid. The half pyramid has a side length of $3c_{a1}$ and a depth of $1.5c_{a1}$. The projected area of this failure surface on the concrete outer surface is given by ACI Eq. D-32 as

$$A_{Vco} = 4.5c_{a1}^2$$

Figure 1.36 *Concrete Breakout Surface in Shear*
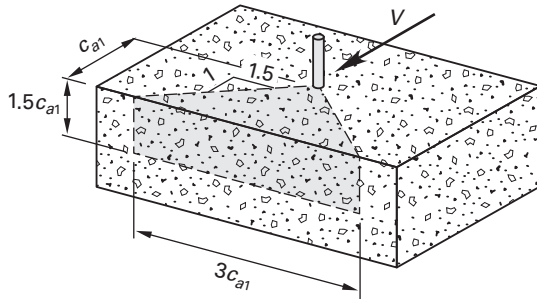
When the failure surface for a single anchor is limited by corner influences and element thickness, the reduced projected area is denoted by A_{Vc}.

The nominal concrete breakout strength for a single cast-in anchor in shear is given by ACI Eq. D-30 as

$$V_{cb} = \frac{A_{vc}}{A_{vco}} \Psi_{ed,V} \Psi_{c,V} \Psi_{h,V} V_b$$

The modification factor for edge effects is given by ACI Sec. D.6.2.6 as

$$\Psi_{ed,V} = 1.0 \quad [c_{a2} \geq 1.5c_{a1}]$$
$$= 0.7 + \frac{0.3c_{a2}}{1.5c_{a1}} \quad [c_{a2} < 1.5c_{a1}]$$

The modification factor for cracked concrete is given by ACI Sec. D.6.2.7 as

$$\Psi_{c,V} = 1.0 \quad [\text{no supplementary reinforcement}]$$
$$= 1.4 \quad [\text{with supplementary reinforcement}]$$

The modification factor for anchors located in a member with $h_a < 1.5c_{a1}$ is given by ACI Eq. D-39 as

$$\Psi_{h,V} = \sqrt{\frac{1.5c_{a1}}{h_a}}$$

ACI Eq. D-33 and ACI Eq. D-34 give the basic concrete breakout strength in shear of a single anchor in cracked concrete as the smaller of

$$V_b = \left(7\left(\frac{l_e}{d_a}\right)^{0.2} \sqrt{d_a}\right) \lambda_a \sqrt{f'_c} c_{a1}^{1.5}$$
$$V_b = 9\lambda_a \sqrt{f'_c} c_{a1}^{1.5}$$

l_e is the load-bearing length of the anchor and is equal to h_{ef} for anchors with a constant stiffness over the embedded section. In all cases, $l_e \leq 8d_a$.

When anchors are spaced closer than three times their edge distance in the direction of the shear, the failure surfaces of adjacent anchors intersect. The failure surface for such an anchor group is determined by projecting the failure surface outward from the outer bolts in the group, as shown in Fig. 1.37. From Fig. 1.37, and

Figure 1.37 *Concrete Breakout Surface for an Anchor Group in Shear*
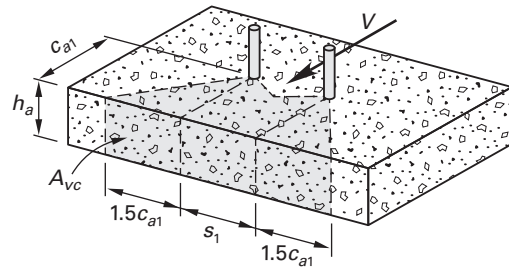
where $h_a < c_{a1}$, the projected area of the failure surface shown is

$$A_{Vc} = (3c_{a1} + s_1)h_a$$

When the projected area is limited by edge distances, the reduced projected area must be calculated; it is also designated by A_{Vc}.

The nominal concrete breakout strength for an anchor group in shear is given by ACI Eq. D-31 as

$$V_{cbg} = \frac{A_{vc}}{A_{vco}} \Psi_{ec,V} \Psi_{ed,V} \Psi_{c,V} \Psi_{h,V} V_b$$

The modification factor for eccentrically loaded anchor groups is given by ACI Eq. D-36 as

$$\Psi_{ec,V} = \frac{1}{1 + \dfrac{2e'_v}{3c_{a1}}}$$
$$= 1.0 \quad [\text{concentrically loaded groups}]$$

Concrete Pryout Strength of Anchor in Shear

The nominal concrete pryout strength for a single cast-in anchor is given by ACI Eq. D-40 as

$$V_{cp} = k_{cp}N_{cp}$$
$$N_{cp} = N_{cb}$$
$$= \frac{A_{Nc}}{A_{Nco}} \Psi_{ed,N} \Psi_{c,N} \Psi_{cp,N} N_b$$

The nominal concrete pryout strength for a group of cast-in anchors is given by ACI Eq. D-41 as

$$V_{cpg} = k_{cp}N_{cpg}$$
$$N_{cpg} = N_{cbg}$$
$$= \frac{A_{Nc}}{A_{Nco}} \Psi_{ec,N} \Psi_{ed,N} \Psi_{c,N} \Psi_{cp,N} N_b$$

The pryout coefficient is given by ACI Sec. D.6.3.1(b) as

$$k_{cp} = 1.0 \quad [h_{ef} < 2.5 \text{ in}]$$
$$= 2.0 \quad [h_{ef} \geq 2.5 \text{ in}]$$

Example 1.32

Using the information provided in Ex. 1.31, determine if the bolts are adequate for the shear force on the shoe.

Solution

The effective embedment depth is

$$h_{ef} = 5.5 \text{ in}$$

The anchor bolts are not near the concrete edges, and concrete breakout in shear is not applicable.

The anchor bolts are ductile. From ACI Sec. D.4.3, the strength reduction factors are

$$\phi = 0.65 \quad \text{[shear on a ductile anchor bolt]}$$

$$= 0.70 \quad \begin{bmatrix} \text{concrete pryout without} \\ \text{supplemental reinforcement} \end{bmatrix}$$

The wall anchor force is determined from ASCE/SEI7 Sec. 12.11.2.1. In accordance with IBC-modified ACI Sec. D.3.3.5, the strength of the anchorage is not governed by the strength of the steel elements.

Strength of Anchor Bolts in Shear

The steel strength in shear is based on the nominal area of the bolt. For a $1/2$ in diameter bolt, the nominal area is $A_{se,V} = 0.196 \text{ in}^2$. The minimum specified tensile strength of the four ASTM A307 grade A headed anchor bolts is 60 kips/in^2. The design strength of the four $1/2$ in diameter ductile anchor bolts is given by ACI Eq. D-29 as

$$\begin{aligned} 4\phi V_{sa} &= 4\phi 0.6 A_{se,V} f_{uta} \\ &= (4)(0.65)(0.6)(0.196 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right) \\ &= 18.3 \text{ kips} > V_{ua} \quad \text{[satisfactory]} \end{aligned}$$

Concrete Pryout Strength in Shear

The design concrete breakout strength for the cast-in anchor group in tension is

$$0.75\phi N_{cbg} = 28.53 \text{ kips} \quad \text{[from Ex. 1.31]}$$

$$k_{cp} = 2.0 \text{ for } h_{ef} > 2.5$$

The design concrete pryout strength for the cast-in anchor group in shear is given by ACI Eq. D-41 as

$$\begin{aligned} 0.75\phi V_{cpg} &= k_{cp} N_{cpg} = k_{cp}(0.75\phi N_{cbg}) \\ &= (2.0)(28.53 \text{ kips}) \\ &= 57.06 \text{ kips} \\ &> V_{ua} \quad \text{[satisfactory]} \end{aligned}$$

Interaction of Tensile and Shear Forces

When the factored shear force applied to a single anchor or group of anchors, V_{ua}, is greater than $0.2\phi V_n$, and when the factored tensile force applied to an anchor or a group of anchors, N_{ua}, is greater than $0.2\phi N_n$, the interaction expression of ACI Eq. D-42 applies.

$$\frac{N_{ua}}{\phi N_n} + \frac{V_{ua}}{\phi V_n} \leq 1.2$$

ϕN_n is the smallest of one of the following: steel strength of anchor in tension, concrete breakout strength in tension, pullout strength of anchor in tension, or side-face blowout strength. ϕV_n is the smallest one of the following: steel strength of anchor in shear, concrete breakout strength in shear, or the pryout strength.

When $V_{ua} < 0.2\phi V_n$, shear effects are neglected, the full design strength in tension is permitted, and

$$\phi N_n \geq N_{ua}$$

When $N_{ua} < 0.2\phi N_n$, tension effects are neglected, the full design strength in shear is permitted, and

$$\phi V_n \geq V_{ua}$$

Example 1.33

Using the information provided in Ex. 1.31, determine if the bolts are adequate for the combined tension and shear force on the shoe.

Solution

The applied loads are

$$N_{ua} = 15 \text{ kips} \quad \text{[tension]}$$
$$V_{ua} = 4 \text{ kips} \quad \text{[shear]}$$

The governing design strength in tension is the anchor bolt pullout strength, so

$$\phi N_{pn} = \phi N_n = 19.56 \text{ kips}$$

The governing design strength in shear is the anchor bolt shear strength, so

$$\phi V_{sa} = \phi V_n = 18.3 \text{ kips}$$

$$\begin{aligned} 0.2\phi N_n &= (0.2)(19.56 \text{ kips}) \\ &= 3.91 \text{ kips} < N_{ua} \end{aligned}$$

The full design strength in shear is not permitted.

$$\begin{aligned} 0.2\phi V_n &= (0.2)(18.3 \text{ kips}) \\ &= 3.66 \text{ kips} < V_{ua} \end{aligned}$$

The full design strength in tension is not permitted.

The interaction expression of ACI Eq. D-42 applies, so

$$\frac{N_{ua}}{\phi N_n} + \frac{V_{ua}}{\phi V_n} \le 1.2$$

$$\frac{15 \text{ kips}}{19.56 \text{ kips}} + \frac{4 \text{ kips}}{18.3 \text{ kips}} = 0.99 < 1.2$$

The anchorage is adequate.

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Concrete

PRACTICE PROBLEMS

1. A reinforced concrete beam continuous over four spans and integral with columns at the ends is shown. The clear distance between supports is 15 ft and the beam supports a factored load of 10 kips/ft.
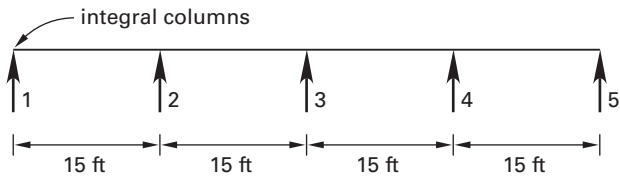
Design assumptions

- ACI Sec. 8.3.3 is applicable.

(a) The design shear force at the face of the first interior support is most nearly

  (A) 75 kips

  (B) 79 kips

  (C) 82 kips

  (D) 86 kips

(b) The design shear force at the face of the outer support is most nearly

  (A) 75 kips

  (B) 79 kips

  (C) 82 kips

  (D) 86 kips

(c) The design moment at the outer support is most nearly

  (A) 140 ft-kips

  (B) 160 ft-kips

  (C) 205 ft-kips

  (D) 230 ft-kips

(d) The design moment at the first interior support is most nearly

  (A) 140 ft-kips

  (B) 160 ft-kips

  (C) 205 ft-kips

  (D) 230 ft-kips

(e) The design moment at the center support is most nearly

  (A) 140 ft-kips

  (B) 160 ft-kips

  (C) 205 ft-kips

  (D) 230 ft-kips

(f) The design moment in the first span is most nearly

  (A) 140 ft-kips

  (B) 160 ft-kips

  (C) 205 ft-kips

  (D) 230 ft-kips

(g) The design moment in the second span is most nearly

  (A) 140 ft-kips

  (B) 160 ft-kips

  (C) 205 ft-kips

  (D) 230 ft-kips

2. A reinforced concrete beam of normal weight concrete with an effective depth of 20 in and a width of 12 in is reinforced with 3 in^2 of grade 60 reinforcement and has a concrete compressive strength of 3000 lbf/in^2. The maximum applied ultimate moment that the beam can support is most nearly

  (A) 210 ft-kips

  (B) 230 ft-kips

  (C) 260 ft-kips

  (D) 280 ft-kips

3. The reinforced concrete beam for Prob. 2 supports a factored shear force of 9 kips at the critical section. Is shear reinforcement required?

  (A) Yes, it is required (26.3 kips > 2V_u).

  (B) Yes, it is required (19.7 kips < 2V_u).

  (C) No, it is not required (19.7 kips > 2V_u).

  (D) No, it is not required (26.3 kips < 2V_u).

4. A short reinforced concrete column, 20 in square, is reinforced with ten no. 9 grade 60 bars and has a concrete strength of 4000 lbf/in^2. What is (a) the design axial load capacity and (b) the required spacing of the lateral ties?

  (A) 700 kips; 20 in

  (B) 1000 kips; 18 in

  (C) 1020 kips; 18 in

  (D) 1400 kips; 20 in

5. A simply supported reinforced concrete beam is reinforced with no. 9 grade 60 bars in bundles of three and has a concrete compressive strength of 4000 lbf/in^2. The reinforcement provided is 10% in excess of that required and has a clear cover equal to the equivalent diameter of the bundled bars and a clear spacing of twice the equivalent diameter. The required development length of an individual bar is most nearly

    (A) 40 in

    (B) 60 in

    (C) 65 in

    (D) 70 in

6. A reinforced concrete flat plate floor without beams has 18 in square columns at 20 ft centers in one direction and 24 ft centers in the other direction, and it supports a factored distributed load of 200 lbf/ft^2.

Design assumptions

- ACI Sec. 13.6 is applicable.
- $\alpha_{f1} = \alpha_{f2} = \beta_t = 0$
- $\dfrac{\alpha_{f1} l_2}{l_1} = 0$
- Consider moments in the direction of the shorter span only.

(a) The column strip moment at the end bay span is most nearly

    (A) 35 ft-kips

    (B) 40 ft-kips

    (C) 65 ft-kips

    (D) 110 ft-kips

(b) The column strip moment at the interior support of the end bay is most nearly

    (A) 35 ft-kips

    (B) 40 ft-kips

    (C) 65 ft-kips

    (D) 110 ft-kips

(c) The column strip moment in an interior bay span is most nearly

    (A) 35 ft-kips

    (B) 40 ft-kips

    (C) 65 ft-kips

    (D) 110 ft-kips

(d) The middle strip moment at the end bay span is most nearly

    (A) 30 ft-kips

    (B) 35 ft-kips

    (C) 40 ft-kips

    (D) 110 ft-kips

(e) The middle strip moment at the interior support of the end bay is most nearly

    (A) 30 ft-kips

    (B) 35 ft-kips

    (C) 40 ft-kips

    (D) 110 ft-kips

(f) The middle strip moment in an interior bay span is most nearly

    (A) 30 ft-kips

    (B) 35 ft-kips

    (C) 40 ft-kips

    (D) 110 ft-kips

SOLUTIONS

1. (a) The shear at the face of the first interior support is given by ACI Sec. 8.3.3 as

$$V_u = \frac{1.15 w_u l_n}{2}$$

$$= \frac{(1.15)\left(10 \ \frac{\text{kips}}{\text{ft}}\right)(15 \ \text{ft})}{2}$$

$$= 86.3 \ \text{kips} \quad (86 \ \text{kips})$$

The answer is (D).

(b) The shear at the face of all other supports is

$$V_u = \frac{w_u l_n}{2} = \frac{\left(10 \ \frac{\text{kips}}{\text{ft}}\right)(15 \ \text{ft})}{2}$$

$$= 75 \ \text{kips}$$

The answer is (A).

(c) The bending moment is given by

$$M_u = \tau w_u l_n^2$$

$$= \tau \left(10 \ \frac{\text{kips}}{\text{ft}}\right)(15 \ \text{ft})^2$$

$$= 2250\tau \ \text{ft-kips}$$

Values of the bending moment coefficients, τ, and the bending moments, M_u, are

$\dfrac{\tau}{M_u}$	support 1	span 12	support 2	span 23	support 3
τ	1/16	1/14	1/10	1/16	1/11
M_u (ft-kips)	141	161	225	141	205

From the table, the design moment at the outer support is 141 ft-kips (140 ft-kips).

The answer is (A).

(d) From the table, the design moment at the first interior support is 225 ft-kips (230 ft-kips).

The answer is (D).

(e) From the table, design moment at the center support is 205 ft-kips.

The answer is (C).

(f) From the table, the design moment in the first span is 161 ft-kips (160 ft-kips).

The answer is (B).

(g) From the table, the design moment in the second span is 141 ft-kips (140 ft-kips).

The answer is (A).

2. The nominal moment of resistance is

$$M_n = A_s f_y d \left(1 - \frac{0.59 A_s f_y}{b_w d f_c'}\right)$$

$$= \frac{(3 \ \text{in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)(20 \ \text{in})}{12 \ \frac{\text{in}}{\text{ft}}}$$

$$\times \left(1 - \frac{(0.59)(3 \ \text{in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)}{(12 \ \text{in})(20 \ \text{in})\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)}\right)$$

$$= 255.8 \ \text{ft-kips}$$

The reinforcement ratio of the beam is

$$\rho = \frac{A_s}{b_w d} = \frac{3 \ \text{in}^2}{(12 \ \text{in})(20 \ \text{in})}$$

$$= 0.0125$$

The limiting reinforcement ratio for a tension-controlled section is

$$\rho_t = \frac{0.319 \beta_1 f_c'}{f_y} = \frac{(0.319)(0.85)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)}{60 \ \frac{\text{kips}}{\text{in}^2}}$$

$$= 0.0136 > \rho$$

The section is tension controlled, and the strength reduction factor is

$$\phi = 0.9$$

The maximum allowable ultimate moment is, then,

$$M_u = 0.9 M_n = (0.9)(255.8 \ \text{ft-kips})$$

$$= 230.2 \ \text{ft-kips} \quad (230 \ \text{ft-kips})$$

The answer is (B).

3. The design shear capacity of the concrete section is, then,

$$\phi V_c = 2\phi b_w d\lambda\sqrt{f_c'}$$

$$= \frac{(2)(0.75)(12 \text{ in})(20 \text{ in})(1.0)\sqrt{3000 \ \frac{\text{lbf}}{\text{in}^2}}}{1000 \ \frac{\text{lbf}}{\text{kip}}}$$

$$= 19.7 \text{ kips} > 2V_u$$

Shear reinforcement is not required.

The answer is (C).

4. (a) The design axial load capacity is

$$\phi P_n = 0.80\phi\big(0.85f_c'(A_g - A_{st}) + A_{st}f_y\big)$$

$$= (0.80)(0.65)$$

$$\times \left(\begin{array}{c} (0.85)\left(4 \ \frac{\text{kips}}{\text{in}^2}\right)(400 \text{ in}^2 - 10 \text{ in}^2) \\ \\ + (10 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right) \end{array} \right)$$

$$= 1002 \text{ kips} \quad (1000 \text{ kips})$$

(b) The minimum allowable tie size is

$$d_t = \text{no. 3 bar}$$

The maximum tie spacing shall not be greater than

$$h = 20 \text{ in}$$

$$48d_t = (48)(0.375 \text{ in})$$

$$= 18 \text{ in}$$

$$16d_b = (16)(1.128 \text{ in})$$

$$= 18 \text{ in} \quad [\text{governs}]$$

The answer is (B).

5. The development length of a bar in a three-bar bundle is increased 20%.

The excess reinforcement factor is

$$E_{xr} = \frac{100}{110} = 0.91$$

The required development length is given by

$$l_d = 1.2E_{xr}d_b\left(\frac{0.05f_y\Psi_t\Psi_e}{\lambda\sqrt{f_c'}}\right)$$

$$= (1.2)(0.91)(1.13 \text{ in})\left(\frac{(0.05)\left(60{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)}{(1.0)\sqrt{4000 \ \frac{\text{lbf}}{\text{in}^2}}}\right)$$

$$= 59 \text{ in} \quad (60 \text{ in})$$

The answer is (B).

6. The clear span is

$$l_n = l_1 - c_1 = 20 \text{ ft} - 1.5 \text{ ft}$$

$$= 18.5 \text{ ft} > 0.65l_1 \quad [\text{satisfactory}]$$

The total factored static moment is

$$M_o = \frac{q_u l_2 l_n^2}{8}$$

$$= \frac{\left(0.2 \ \frac{\text{kip}}{\text{ft}^2}\right)(24 \text{ ft})(18.5 \text{ ft})^2}{8}$$

$$= 205 \text{ ft-kips}$$

The relevant coefficients, as given in the problem statement, are

$$\alpha_{f1} = \alpha_{f2} = \beta_t = 0$$

$$\frac{\alpha_{f1}l_2}{l_1} = 0$$

The total static moment is distributed as shown in the following table.

strip	coefficient/ moment	end span	interior support	interior span
full width	distribution coeff.	0.52	0.70	0.35
	moment (ft-kips)	107	144	72
column strip	distribution coeff.	0.60	0.75	0.60
	moment (ft-kips)	64	108	43
middle strip	distribution coeff.	0.40	0.25	0.40
	moment (ft-kips)	43	36	29

(a) From the table, the column strip moment in the end bay span is 64 ft-kips (65 ft-kips).

The answer is (C).

(b) From the table, the column strip moment at the interior support of the end bay is 108 ft-kips (110 ft-kips).

The answer is (D).

(c) From the table, the column strip moment in an interior bay span is 43 ft-kips (40 ft-kips).

The answer is (B).

(d) From the table, the middle strip moment at the end bay span is 43 ft-kips (40 ft-kips).

The answer is (C).

(e) From the table, the middle strip moment at the interior support of the end bay is 36 ft-kips (35 ft-kips).

The answer is (B).

(f) From the table, the middle strip moment in an interior bay span is 29 ft-kips (30 ft-kips).

The answer is (A).

2 Foundations and Retaining Structures

1. STRIP FOOTING

Nomenclature

A_1	loaded area at base of column	ft^2
A_2	area of the base of the pyramid, with side slopes of 1:2, formed within the footing by the loaded area	ft^2
b_o	perimeter of critical section for punching shear	in
B	length of strip footing parallel to wall, length of short side of a rectangular footing	ft
c	length of side of column	in
D	dead load	kips
e	eccentricity with respect to center of footing	in
e'	eccentricity with respect to edge of footing, $(L/2 - e)$	in
h	depth of footing	in
H	lateral force due to earth pressure	kips
l	distance between column centers	ft
L	length of strip footing perpendicular to wall, length of long side of a rectangular footing, length of side of a square footing	ft
L	live load	kips
M_u	factored moment at critical section	ft-kips
P	column axial service load	kips
P_{bn}	nominal bearing strength	kips
P_D	column axial service dead load	kips
P_L	column axial service live load	kips
P_u	column axial factored load	kips
q	soil bearing pressure due to service loads	lbf/ft^2
q_s	equivalent bearing pressure due to service loads	lbf/ft^2
q_u	net factored pressure acting on footing	lbf/ft^2
s	spacing of reinforcement	in
V_c	shear capacity of footing	kips
V_u	factored shear force	kips
w_c	specific weight of concrete	lbf/ft^3
x	distance from edge of footing or center of column to critical section	ft
x_o	distance from edge of property line to centroid of service loads	ft

Symbols

β	ratio of long side to short side of loaded area	–

Pressure Distribution

To determine soil pressure under a footing, the unfactored self-weight of the footing is added to the unfactored applied load from the wall. The footing dimensions are adjusted to ensure that the soil pressure as calculated from the unfactored loads does not exceed the allowable pressure. To determine the forces acting on a footing, the net pressure is required as determined from the applied wall load only. For a strip footing of length B parallel to the wall, the net pressure acting on the footing, as shown in Fig. 2.1, is given by

$$q = \frac{P}{BL} \quad [e = 0]$$

$$q_{max} = \frac{P\left(1 \pm \dfrac{6e}{L}\right)}{BL} \quad [e \leq \tfrac{L}{6}]$$

$$q_{max} = \frac{2P}{3Be'} \quad [e > \tfrac{L}{6}]$$

Design of a footing is based on the net factored applied loads, determined in accordance with ACI Sec. 9.2.

Example 2.1

The 18 in deep strip footing of normal weight concrete shown in the following illustration supports a 12 in concrete wall that is offset 1 ft from the center of the footing. The applied service loads are indicated in the figure, and the allowable soil pressure is 5000 lbf/ft^2. Determine the required footing dimensions, the soil pressure under the footing, and the net factored pressure acting on the footing.
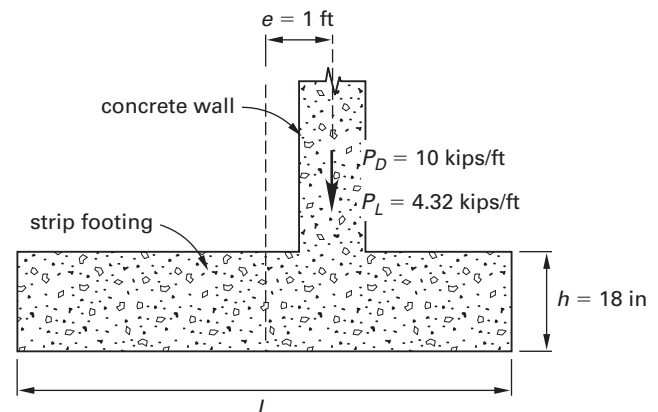
Figure 2.1 *Net Pressure Distribution on a Footing*
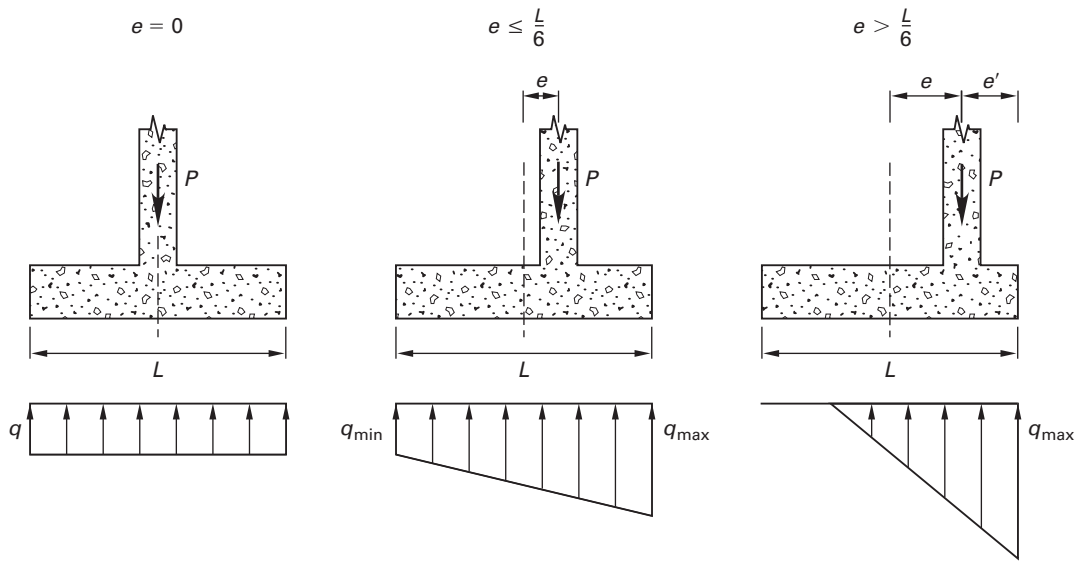
Solution

The total applied service load per foot run is

$$P_1 = P_D + P_L = 10{,}000 \ \frac{\text{lbf}}{\text{ft}} + 4320 \ \frac{\text{lbf}}{\text{ft}}$$
$$= 14{,}320 \ \text{lbf/ft}$$

The self-weight of the footing per foot run is

$$P_2 = w_c h L = \left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)(1.5 \ \text{ft})L$$
$$= 225L \ \text{lbf/ft}^2$$

Assuming that $e < L/6$, the maximum soil pressure is given by

$$q_{\max} = \frac{P_1\left(1 + \dfrac{6e}{L}\right)}{L} + \frac{P_2}{L}$$
$$= \frac{(14{,}320 \ \text{lbf})\left(1 + \dfrac{6}{L}\right)}{L} + \frac{225L \ \dfrac{\text{lbf}}{\text{ft}^2}}{L}$$
$$= 5000 \ \text{lbf/ft}^2 \quad \text{[as given]}$$

Solving for the footing length gives

$$L = 6 \ \text{ft}$$

The maximum soil pressure under the footing is

$$q_{\max} = \frac{P_1\left(1 + \dfrac{6e}{L}\right)}{BL} + \frac{P_2}{BL}$$
$$= \frac{(14{,}320 \ \text{lbf})\left(1 + \dfrac{(6)(1 \ \text{ft})}{6 \ \text{ft}}\right)}{(1 \ \text{ft})(6 \ \text{ft})} + \frac{1350 \ \text{lbf}}{(1 \ \text{ft})(6 \ \text{ft})}$$
$$= 5000 \ \text{lbf/ft}^2$$

The minimum soil pressure under the footing is

$$q_{\min} = \frac{P_1\left(1 - \dfrac{6e}{L}\right)}{BL} + \frac{P_2}{BL}$$
$$= \frac{(14{,}320 \ \text{lbf})\left(1 - \dfrac{(6)(1 \ \text{ft})}{6 \ \text{ft}}\right)}{(1 \ \text{ft})(6 \ \text{ft})} + \frac{1350 \ \text{lbf}}{(1 \ \text{ft})(6 \ \text{ft})}$$
$$= 225 \ \text{lbf/ft}^2$$

The net factored load on the footing, in accordance with ACI Sec. 9.2, is

$$P_u = 1.2P_D + 1.6P_L = (1.2)(10 \ \text{kips}) + (1.6)(4.32 \ \text{kips})$$
$$= 18.91 \ \text{kips}$$

The maximum net factored pressure acting on the footing is

$$q_{u(\max)} = \frac{P_u\left(1 + \dfrac{6e}{L}\right)}{BL} = \frac{(18.91 \ \text{kips})\left(1 + \dfrac{(6)(1 \ \text{ft})}{6 \ \text{ft}}\right)}{(1 \ \text{ft})(6 \ \text{ft})}$$
$$= 6.30 \ \text{kips/ft}^2$$

The minimum net factored pressure acting on the footing is

$$q_{u(\min)} = \frac{P_u\left(1 - \dfrac{6e}{L}\right)}{BL}$$
$$= \frac{(18.91 \ \text{kips})\left(1 - \dfrac{(6)(1 \ \text{ft})}{6 \ \text{ft}}\right)}{(1 \ \text{ft})(6 \ \text{ft})}$$
$$= 0$$

Factored Soil Pressure

A reinforced concrete strip footing may be designed for flexure or for flexural (i.e., one-way) shear. A reinforced concrete isolated footing may additionally be designed for punching (i.e., two-way) shear.

The critical section for flexure and shear is located at a different position in the footing, and each must be designed for the applied factored loads. The soil pressure distribution due to factored loads must be determined. Since the self-weight of the footing produces an equal and opposite pressure in the soil, the footing is designed for the net pressure due to the column load only, and the weight of the footing is not included.

Design for Flexural Shear

The critical section for flexural shear is defined in ACI Sec. 15.5.2 and Sec. 11.1.3.1 as being located a distance d from the face of the concrete or masonry wall, as shown in Fig. 2.2. The shear strength of the footing is determined in accordance with ACI Sec. 11.2.

Example 2.2

The strip footing for Ex. 2.1 has an effective depth of 14 in and a concrete strength of 3000 lbf/in^2. Using the following illustration, determine whether the shear capacity is adequate.

Illustration for Ex. 2.2
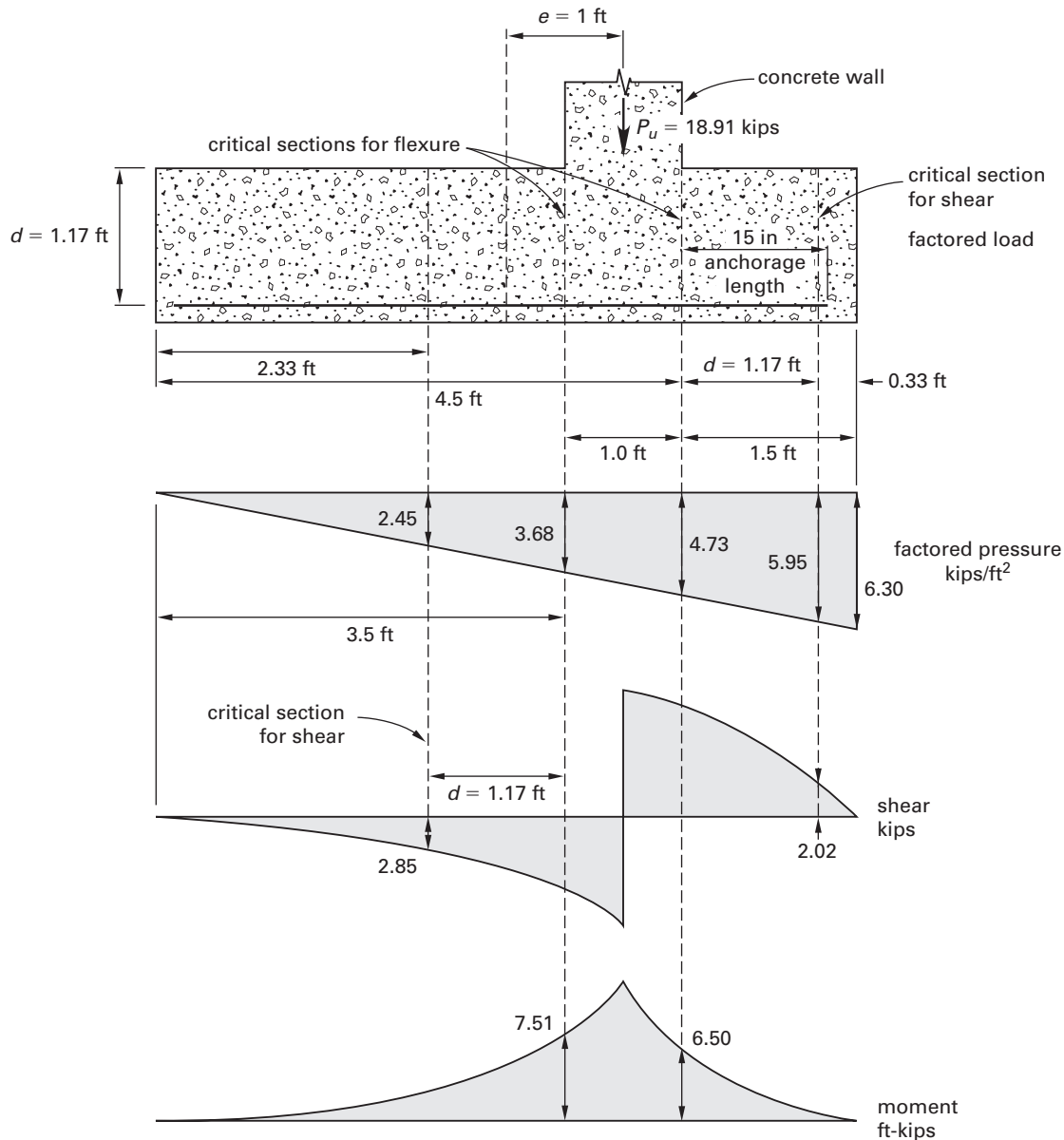
Figure 2.2 *Critical Sections for Flexure and Flexural Shear*
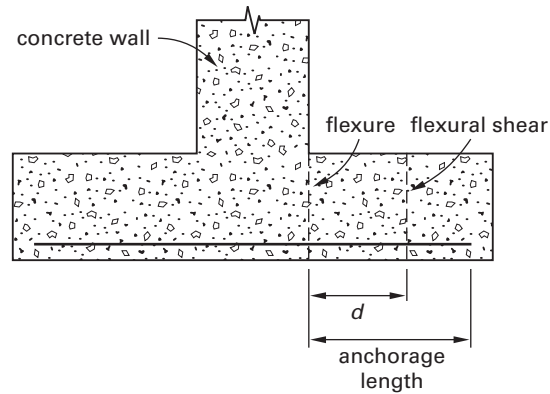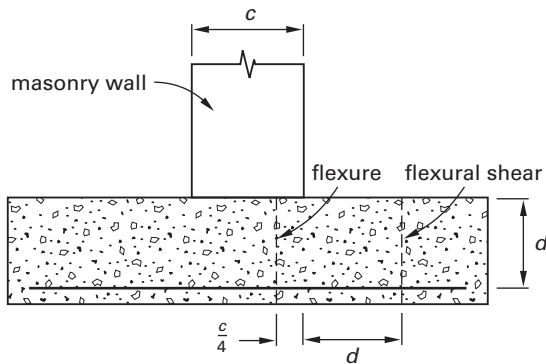
Solution

The net factored pressure acting on the footing is shown in the illustration, and the pressure at the critical section for shear on the right side of the footing is

$$q = \frac{q_u(5.67 \text{ ft})}{6 \text{ ft}}$$

$$= \frac{\left(6.30 \frac{\text{kips}}{\text{ft}^2}\right)(5.67 \text{ ft})}{6 \text{ ft}}$$

$$= 5.95 \text{ kips/ft}^2$$

For a 1 ft strip, the factored shear force at the critical section is

$$V_u = \frac{(0.33 \text{ ft})(1 \text{ ft})(q + q_u)}{2}$$

$$= \frac{(0.33 \text{ ft})(1 \text{ ft})\left(5.95 \frac{\text{kips}}{\text{ft}^2} + 6.30 \frac{\text{kips}}{\text{ft}^2}\right)}{2}$$

$$= 2.02 \text{ kips}$$

The pressure at the critical section for shear on the left side of the footing is

$$q = \frac{q_u(2.33 \text{ ft})}{6 \text{ ft}}$$

$$= \frac{\left(6.30 \frac{\text{kips}}{\text{ft}^2}\right)(2.33 \text{ ft})}{6 \text{ ft}}$$

$$= 2.45 \text{ kips/ft}^2$$

For a 1 ft strip, the factored shear force at the critical section is

$$V_u = \frac{(2.33 \text{ ft})(1 \text{ ft})q}{2}$$

$$= \frac{(2.33 \text{ ft})(1 \text{ ft})\left(2.45 \frac{\text{kips}}{\text{ft}^2}\right)}{2}$$

$$= 2.85 \text{ kips} \quad [\text{governs}]$$

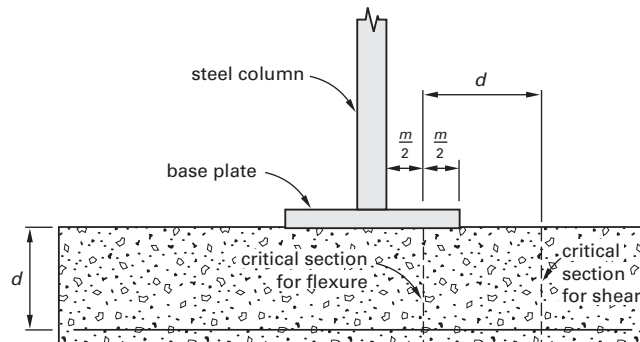The shear capacity of the footing is given by ACI Eq. 11-3 as

$$\phi V_c = 2\phi b d \lambda \sqrt{f'_c}$$

$$= \frac{(2)(0.75)(12 \text{ in})(14 \text{ in})(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}}$$

$$= 13.80 \text{ kips}$$

$$> V_u \quad [\text{satisfactory}]$$

Design for Flexure

The critical section for flexure is defined in ACI Sec. 15.4.2 as being located at the face of a concrete wall and halfway between the center and the face of a masonry wall, as shown in Fig. 2.2. The required reinforcement area is determined in accordance with ACI Sec. 10.2. For a footing, the minimum ratio, $\rho_{\min}$, of reinforcement area to gross concrete area is specified in ACI Sec. 10.5.4 and Sec. 7.12.2, for both main reinforcement and distribution reinforcement, as 0.0018 for grade 60 bars. The maximum spacing of the main reinforcement shall not exceed 18 in or three times the footing depth. The diameter of bar provided must be such that the development length does not exceed the available anchorage length. Distribution reinforcement may be spaced at a maximum of 18 in or five times the footing depth.

Example 2.3

Determine the reinforcement required in the strip footing for Ex. 2.1 and Ex. 2.2.

Solution

As shown in the illustration for Ex. 2.2, on the right side of the footing, the factored pressure at the critical section for flexure, which is at the right face of the wall, is given by

$$q = \frac{q_u(4.5 \text{ ft})}{6 \text{ ft}} = \frac{\left(6.30 \ \frac{\text{kips}}{\text{ft}^2}\right)(4.5 \text{ ft})}{6 \text{ ft}} = 4.73 \text{ kips/ft}^2$$

For a 1 ft strip, the factored moment at the critical section is

$$M_u = \frac{(1.5 \text{ ft})^2 (1 \text{ ft})(q + 2q_u)}{6}$$
$$= \frac{(1.5 \text{ ft})^2 (1 \text{ ft})\left(4.73 \ \frac{\text{kips}}{\text{ft}^2} + (2)\left(6.30 \ \frac{\text{kips}}{\text{ft}^2}\right)\right)}{6}$$
$$= 6.50 \text{ ft-kips}$$

As shown in the illustration for Ex. 2.2, on the left side of the footing the factored pressure at the critical section for flexure, which is at the left face of the wall, is given by

$$q = \frac{q_u(3.5 \text{ ft})}{6 \text{ ft}}$$
$$= \frac{\left(6.30 \ \frac{\text{kips}}{\text{ft}^2}\right)(3.5 \text{ ft})}{6 \text{ ft}}$$
$$= 3.68 \text{ kips/ft}^2$$

For a 1 ft strip, the factored moment at the critical section is

$$M_u = \frac{(3.5 \text{ ft})^2 (1 \text{ ft})q}{6}$$
$$= \frac{(3.5 \text{ ft})^2 (1 \text{ ft})\left(3.68 \ \frac{\text{kips}}{\text{ft}^2}\right)}{6}$$
$$= 7.51 \text{ ft-kips} \quad \text{[governs]}$$

Assuming a tension-controlled section, the required reinforcement ratio is given by

$$\rho = \frac{0.85 f'_c \left(1 - \sqrt{1 - \dfrac{M_u}{0.383 b d^2 f'_c}}\right)}{f_y}$$

$$= \frac{(0.85)\left(3 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{60 \ \dfrac{\text{kips}}{\text{in}^2}}$$
$$\times \left(1 - \sqrt{1 - \dfrac{\begin{array}{c}(7.51 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right) \\ \times \left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)\end{array}}{\begin{array}{c}(0.383)(12 \text{ in})(14 \text{ in})^2 \\ \times \left(3000 \ \frac{\text{lbf}}{\text{in}^2}\right)\end{array}}}\right)$$

$$= 0.0007$$

The minimum reinforcement area governs and, for a footing, is given by ACI Sec. 7.12.2 as

$$A_s = 0.0018 bh$$
$$= (0.0018)(12 \text{ in})(18 \text{ in})$$
$$= 0.39 \text{ in}^2$$

Providing no. 4 bars at 6 in on center for both main and distribution reinforcement gives

$$A_s = 0.40 \text{ in}^2/\text{ft} \quad \text{[satisfactory]}$$
$$\Psi_t = \Psi_e = \lambda = 1 \quad \left[\begin{array}{c}\text{uncoated bottom bars} \\ \text{in normal weight concrete}\end{array}\right]$$
$$\Psi_s = 0.8 \quad \text{[for no. 4 bars]}$$
$$\frac{c_b + K_{tr}}{d_b} = 2.5 \quad \text{[from ACI Sec. 12.2.3]}$$

ACI Eq. 12-1 for development length reduces to

$$l_d = \frac{(0.075)(0.8 d_b f_y)}{2.5 \lambda \sqrt{f'_c}} = \frac{0.06 d_b \left(60{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)}{(2.5)(1.0)\sqrt{3000 \ \frac{\text{lbf}}{\text{in}^2}}}$$

$$= 26.3 d_b$$
$$= 13.2 \text{ in} \quad \text{[for no. 4 bars]}$$
$$< 15 \text{ in anchorage length provided} \quad \text{[satisfactory]}$$

2. ISOLATED COLUMN WITH SQUARE FOOTING

Nomenclature

A_g	gross area of footing cross section	in^2
A_s	area of distribution reinforcement	in^2
b_1	width of the critical perimeter measured in the direction of M_u, $c_1 + d$	in
b_2	width of the critical perimeter measured perpendicular to b_1, $c_2 + d$	in
b_o	length of the critical perimeter, $2(b_1 + b_2)$	in
B	length of the footing measured perpendicular to L	ft
c_1	width of the column measured in the direction of M_u	in
c_2	width of the column measured perpendicular to c_1	in
d	average effective depth	in
d_b	bar diameter	in
J_c	polar moment of inertia of critical perimeter	in^4
J_c/y	$(b_1 d(b_1 + 3b_2) + d^3)/3$ [for a footing with central column as specified by ACI Sec. R11.11.7.2]	in^3
L	length of the footing measured in the direction of M_u	ft
M_u	moment applied to the column	ft-kips
P_u	factored axial force on column	kips
s	spacing of reinforcement	in
v_u	shear stress at critical perimeter	lbf/in^2
V_u	factored shear force acting on the critical perimeter, $P_u(1 - b_1 b_2 / BL)$	kips
y	distance from the centroid of the critical perimeter to edge of critical perimeter, $b_1/2$ [for footing with central column]	in

Symbols

α_s	constant used to compute shear in slabs and footings	–
β	ratio of long side to short side of column	–
γ_v	fraction of the applied moment transferred by shear as specified by ACI Sec. 11.11.7.1 and Sec. 13.5.3.2, $1 - 1/(1 + 0.67\sqrt{b_1/b_2})$	–

Reinforcement Details

The maximum spacing of the principal reinforcement in footings is limited by ACI Sec. 7.6.5 and Sec. 10.5.4 to

$$s = 3h$$

$$\leq 18 \text{ in}$$

The maximum spacing of the distribution reinforcement, which is required to resist shrinkage and temperature stresses, is limited by ACI Sec. 7.12.2.2 to

$$s = 5h$$

$$\leq 18 \text{ in}$$

The minimum spacing of reinforcement must be adequate to allow full consolidation of the concrete around the bars. The minimum clear spacing between parallel bars in a layer is specified by ACI Sec. 7.6.1 and Sec. 3.3.2 as

$$s_{\min,\text{clear}} = d_b$$

$$\geq 1 \text{ in}$$

$$\geq 1.33(\text{maximum aggregate size})$$

The minimum reinforcement ratio specified for distribution steel in ACI Sec. 7.12.2.1 is

$$\frac{A_s}{A_g} = 0.0018 \quad \text{[grade 60 reinforcement]}$$

A_s is the area of distribution reinforcement, and A_g is the gross area of footing cross section. When concrete is cast against and exposed to earth (as is the case with the soffit of footings), ACI Sec. 7.7.1 specifies the minimum concrete cover provided for reinforcement to be 3 in. When concrete is exposed to earth or weather, ACI Sec. 7.7.1 specifies the minimum cover to be $1\frac{1}{2}$ in for no. 5 bars and smaller, and 2 in for no. 6 through no. 18 bars.

Design for Punching Shear

The critical perimeter for punching shear is specified in ACI Sec. 15.5.2 and Sec. 11.11.1.2 and illustrated in Fig. 2.3. For a concrete or masonry column, the critical section is a distance from the face of the column equal to one-half the effective depth. For a steel column with a base plate, the critical section is one-half the effective depth from a plane halfway between the face of the column and the edge of the base plate. The punching shear strength of the footing is determined by ACI Sec. 11.11.2.1 as the smallest of ACI Eq. 11-31, Eq. 11-32, and Eq. 11-33.

$$\phi V_c = 4\phi d b_o \lambda \sqrt{f_c'} \qquad \text{[ACI 11-33]}$$

When $\beta > 2$,

$$\phi V_c = \phi d b_o \left(2 + \frac{4}{\beta}\right)\lambda\sqrt{f_c'} \qquad \text{[ACI 11-31]}$$

$$\phi V_c = \phi \left(\frac{\alpha_s d}{b_o} + 2\right)\lambda\sqrt{f_c'}\, b_o d \qquad \text{[ACI 11-32]}$$

Where,

$$\alpha_s = 40 \quad \text{[interior columns]}$$

$$= 30 \quad \text{[edge columns]}$$

$$= 20 \quad \text{[corner columns]}$$

$$\phi = 0.75$$

ACI Eq. 11-33 governs for a square column, ACI Eq. 11-31 governs for a column when $\beta > 2$, and ACI Eq. 11-32 governs for a column when $b_o > 20d$.

Figure 2.3 *Critical Perimeter for Punching Shear*
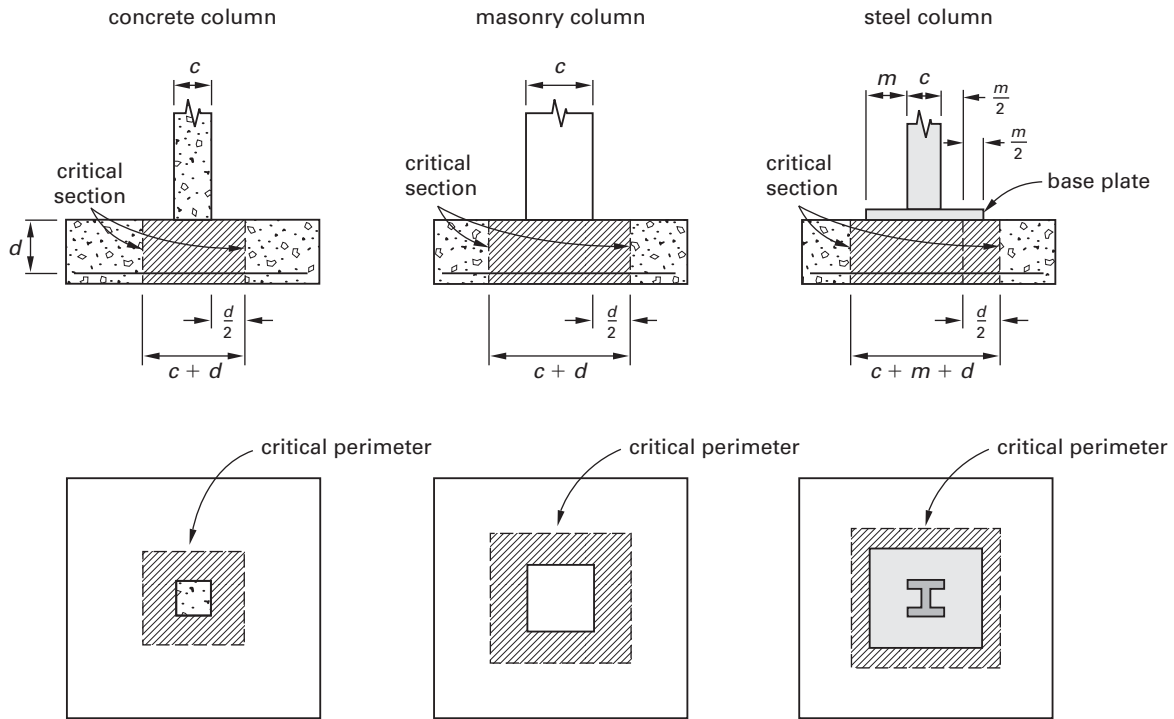
When the column supports only an axial load, shear stress at the critical perimeter is uniformly distributed around the critical perimeter, and is

$$v_u = \frac{V_u}{db_o}$$

The factored shear force acting on the critical perimeter is

$$V_u = P_u\left(1 - \frac{(c_1 + d)(c_2 + d)}{BL}\right)$$

The depth of the footing is typically governed by the punching shear capacity.

When, in addition to the axial load, a bending moment, M_u, is applied to the column, an eccentric shear stress is introduced into the critical section with the maximum value occurring on the face nearest the largest bearing pressure. When both axial load and bending moment occur, the shear stresses from both conditions are combined as specified in ACI Sec. R11.11.7.2 to give a maximum value of

$$v_u = \frac{V_u}{db_o} + \frac{\gamma_v M_u y}{J_c}$$

Example 2.4

A 6 ft square reinforced concrete footing of normal weight concrete with an effective depth of 12 in supports a 12 in square column with a factored axial load of 200 kips. The concrete strength is 3000 lbf/in^2. Determine whether the punching shear capacity of the footing is satisfactory.
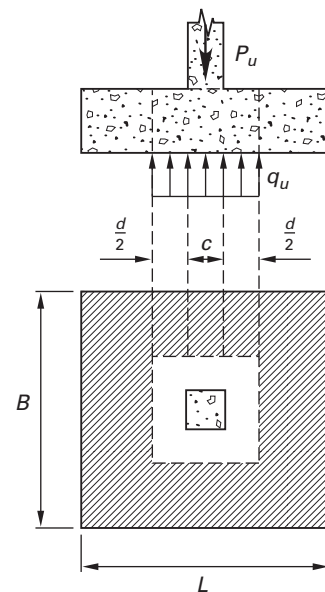
Solution

The net factored pressure on the footing is

$$q_u = \frac{P_u}{LB} = \frac{200 \text{ kips}}{(6 \text{ ft})(6 \text{ ft})} = 5.56 \text{ kips/ft}^2$$

The length of the critical perimeter is

$$\begin{aligned} b_o &= 4(c + d) \\ &= \frac{(4)(12 \text{ in} + 12 \text{ in})}{12 \frac{\text{in}}{\text{ft}}} \\ &= 8 \text{ ft} \end{aligned}$$

Shear at the critical perimeter is

$$\begin{aligned} V_u &= P_u - q_u(c + d)^2 \\ &= 200 \text{ kips} - \left(5.56 \frac{\text{kips}}{\text{ft}^2}\right)\left(\frac{12 \text{ in} + 12 \text{ in}}{12 \frac{\text{in}}{\text{ft}}}\right)^2 \\ &= 177.7 \text{ kips} \end{aligned}$$

Shear capacity of the footing is given by ACI Eq. 11-33 as

$$\begin{aligned} \phi V_c &= 4\phi d b_o \lambda \sqrt{f'_c} \\ &= \frac{(4)(0.75)(12 \text{ in})(96 \text{ in})(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 189.3 \text{ kips} \\ &> V_u \quad [\text{satisfactory}] \end{aligned}$$

Design for Flexural Shear

For concrete and masonry columns, the location of the critical section for flexural shear is identical with that for a strip footing and is shown in Fig. 2.2. As shown in Fig. 2.4, the critical section for a steel column with a base plate is located an effective depth from the plane halfway between the face of the column and edge of the base plate.

Figure 2.4 *Critical Sections for a Footing with Steel Base Plate*

Example 2.5

For the reinforced concrete footing of Ex. 2.4, determine whether the flexural-shear capacity is adequate.
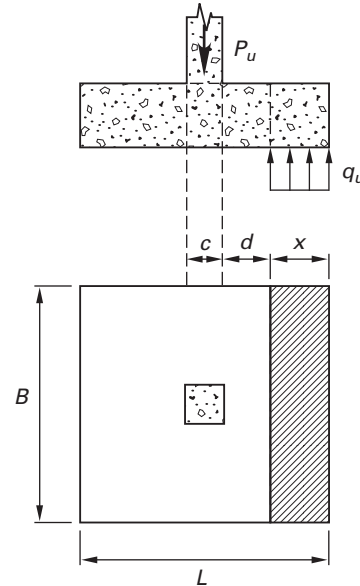
Solution

The distance from the edge of the footing of the critical section for flexural shear is

$$\begin{aligned} x &= \frac{L}{2} - \frac{c}{2} - d = \frac{6 \text{ ft}}{2} - \frac{1 \text{ ft}}{2} - 1 \text{ ft} \\ &= 1.5 \text{ ft} \end{aligned}$$

The factored shear force at this section is

$$\begin{aligned} V_u &= q_u B x = \left(5.56 \frac{\text{kips}}{\text{ft}^2}\right)(6 \text{ ft})(1.5 \text{ ft}) \\ &= 50.0 \text{ kips} \end{aligned}$$

The shear capacity of the footing is given by ACI Eq. 11-3 as

$$\begin{aligned} \phi V_c &= 2\phi b d \lambda \sqrt{f'_c} \\ &= \frac{(2)(0.75)(72 \text{ in})(12 \text{ in})(1.0)\sqrt{3000 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}} \\ &= 71.0 \text{ kips} \\ &> V_u \quad [\text{satisfactory}] \end{aligned}$$

Design for Flexure

For concrete and masonry columns, the location of the critical section for flexure is identical with that for a strip footing and is shown in Fig. 2.2. As shown in Fig. 2.4, the critical section for flexure for a steel column with a base plate is at a plane halfway between the face of the column and the edge of the base plate.

Example 2.6

Determine the reinforcement required in the square footing for Ex. 2.4. The depth of the footing is 15.5 in, and the reinforcement is grade 60.

Solution
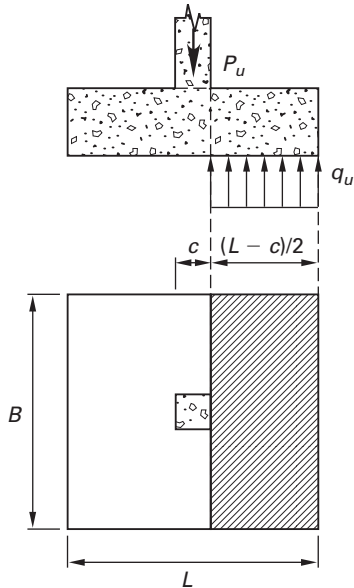
The factored moment at the critical section for flexure, which is at the face of the column, is given by

$$M_u = \frac{q_u B \left(\frac{L}{2} - \frac{c}{2}\right)^2}{2} = \frac{\left(5.56 \, \frac{\text{kips}}{\text{ft}^2}\right)(6 \, \text{ft})\left(\frac{6 \, \text{ft}}{2} - \frac{1 \, \text{ft}}{2}\right)^2}{2}$$
$$= 104.3 \, \text{ft-kips}$$

Assuming a tension-controlled section, the required reinforcement ratio is given by

$$\rho = \frac{0.85 f_c' \left(1 - \sqrt{1 - \frac{M_u}{0.383 b d^2 f_c'}}\right)}{f_y}$$

$$= \frac{(0.85)\left(3 \, \frac{\text{kips}}{\text{in}^2}\right)}{60 \, \frac{\text{kips}}{\text{in}^2}} \times \left(1 - \sqrt{1 - \frac{(104.3 \, \text{ft-kips})\left(12 \, \frac{\text{in}}{\text{ft}}\right) \times \left(1000 \, \frac{\text{lbf}}{\text{kip}}\right)}{(0.383)(72 \, \text{in})(12 \, \text{in})^2 \times \left(3000 \, \frac{\text{lbf}}{\text{in}^2}\right)}}\right)$$

$$= 0.0023$$

The maximum allowable reinforcement ratio for a tension-controlled section is given by

$$\rho_t = 0.319 \beta_1 \frac{f_c'}{f_y}$$
$$= 0.0136$$
$$> \rho \quad \begin{bmatrix} \text{satisfactory, the section} \\ \text{is tension-controlled} \end{bmatrix}$$

The required reinforcement area is

$$A_s = \rho b d$$
$$= (0.0023)(72 \, \text{in})(12 \, \text{in})$$
$$= 1.99 \, \text{in}^2$$

Providing 10 no. 4 bars gives a reinforcement area of

$$A_s = 2.0 \, \text{in}^2 \quad \text{[satisfactory]}$$

The minimum allowable reinforcement area is given by ACI Sec. 7.12.2 as

$$A_{s(\text{min})} = 0.0018 bh$$
$$= (0.0018)(72 \, \text{in})(15.5 \, \text{in})$$
$$= 2.0 \, \text{in}^2 \quad \text{[satisfactory]}$$

From Ex. 2.3, the development length of a no. 4 bar is

$$l_d = 13.2 \, \text{in}$$

The anchorage length provided (in inches) is

$$l_a = \frac{L}{2} - \frac{c}{2} - \text{end cover}$$
$$= \frac{72 \, \text{in}}{2} - \frac{12 \, \text{in}}{2} - 3 \, \text{in}$$
$$= 27 \, \text{in}$$
$$> l_d \quad \text{[satisfactory]}$$

Transfer of Force at Base of Column

Bearing Capacity of the Column Concrete

In accordance with ACI Sec. 15.8, load transfer between a reinforced concrete column and the footing may be affected both by bearing on concrete and by reinforcement.

The bearing capacity of the column concrete at the interface is given by ACI Sec. 10.14.1.

$$\phi P_{bn} = 0.85 \phi f_c' A_1$$
$$= 0.553 f_c' A_1 \quad \text{[for } \phi = 0.65\text{]}$$

A_1 is the area of the column.

Bearing Capacity of the Footing Concrete

The bearing capacity of the footing concrete at the interface is given by ACI Sec. 10.14.1. (Refer to Fig. 2.5.)

$$\phi P_{bn} = 0.85\phi f_c' A_1 \sqrt{\frac{A_2}{A_1}}$$

$$\leq (0.85\phi f_c' A_1)(2)$$

A_2 = area of the base of the pyramid, with side slopes of 1:2, formed within the footing by the column base

In accordance with ACI Sec. R15.8.1.2, when the bearing strength at the base of the column or at the top of the footing is exceeded, reinforcement must be provided to carry the excess load. This reinforcement may be provided by dowels or extended longitudinal bars, and the capacity of this reinforcement is

$$\phi P_s = \phi A_s f_y$$

A minimum area of reinforcement is required across the interface, and this is given by ACI Sec. 15.8.2.1 as

$$A_{s(\min)} = 0.005 A_1$$

Example 2.7

Assume that the column in Ex. 2.4 has a concrete compressive strength of 3000 lbf/in^2 and carries a factored axial load of 280 kips. Design the dowels required at the interface.

Solution

The bearing capacity of the column concrete is given by ACI Sec. 10.14 as

$$\phi P_{bn} = 0.553 f_c' A_1$$

$$= (0.553)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)(144 \ \text{in}^2)$$

$$= 239 \ \text{kips}$$

Figure 2.5 *Bearing on Footing Concrete*

Excess column load to be carried by dowels is

$$\phi P_s = P_u - \phi P_{bn}$$

$$= 280 \ \text{kips} - 239 \ \text{kips}$$

$$= 41 \ \text{kips}$$

The required area of dowels is given by

$$A_{s(\text{reqd})} = \frac{\phi P_s}{0.65 f_y} = \frac{41 \ \text{kips}}{(0.65)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)}$$

$$= 1.05 \ \text{in}^2$$

Providing four no. 5 dowels gives an area of

$$A_{s(\text{prov})} = 1.24 \ \text{in}^2 \quad \text{[satisfactory]}$$

The minimum dowel area allowed is given by ACI Sec. 15.8.2.1 as

$$A_{s(\min)} = 0.005 A_1 = (0.005)(144 \ \text{in}^2)$$

$$= 0.72 \ \text{in}^2$$

$$< A_{s(\text{prov})} \quad \text{[satisfactory]}$$

Allowing for the excess reinforcement provided, the development length of the dowels in the column and in the footing is given by ACI Sec. 12.3.2 as

$$l_{dc} = \frac{\left(\dfrac{A_{s(\text{reqd})}}{A_{s(\text{prov})}}\right)(0.02 d_b f_y)}{\lambda\sqrt{f_c'}}$$

$$= \frac{\left(\dfrac{1.05 \ \text{in}^2}{1.24 \ \text{in}^2}\right)\left((0.02)(0.63 \ \text{in})\left(60{,}000 \ \dfrac{\text{lbf}}{\text{in}^2}\right)\right)}{(1.0)\sqrt{3000 \ \dfrac{\text{lbf}}{\text{in}^2}}}$$

$$= 12 \ \text{in}$$

A_1 = loaded area = c^2

$c + 4d$

A_2 = base of pyramid = $(c + 4d)^2$

This length exceeds the minimum length of 8 in specified in ACI Sec. 12.3.1 and is satisfactory. In the footing, the length of the base of the pyramid, with side slopes of 1:2, formed within the footing by the loaded area is

$$L_p = c + 4d = 12 \text{ in} + (4)(12 \text{ in})$$

$$= 60 \text{ in}$$

$$< L \quad [\text{satisfactory}]$$

The area of the base of the pyramid is

$$A_2 = L_p^2 = (60 \text{ in})^2 = 3600 \text{ in}^2$$

$$\sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{3600 \text{ in}^2}{144 \text{ in}^2}} = 5$$

Use a maximum value of 2.

Then, bearing capacity of the footing concrete is given by ACI Sec. 10.14 as

$$\phi P_{bn} = 2(0.533 f'_c A_1)$$

$$= (2)(0.553)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)(144 \text{ in}^2)$$

$$= 478 \text{ kips}$$

$$> P_u \quad [\text{satisfactory}]$$

3. ISOLATED COLUMN WITH RECTANGULAR FOOTING

Nomenclature

A_b	area of reinforcement in central band	in^2
A_s	total required reinforcement area	in^2
B	length of short side of a rectangular footing	ft
c_1	length of short side of a rectangular column	in
c_2	length of long side of a rectangular column	in
L	length of long side of a rectangular footing	ft

Symbols

β	ratio of the long side to the short side of the footing L/B	–

Design for Flexure

Bending moments are calculated at the critical sections in both the longitudinal and transverse directions. The reinforcement required in the longitudinal direction is distributed uniformly across the width of the footing. Part of the reinforcement required in the transverse direction is concentrated in a central band width equal to the length of the short side of the footing, as shown in Fig. 2.6.

The area of reinforcement required in the central band is given by ACI Sec. 15.4.4.2 as

$$A_b = \frac{2A_s}{\beta + 1}$$

The remainder of the reinforcement required in the transverse direction is

$$A_r = \frac{A_s(\beta - 1)}{\beta + 1}$$

This is distributed uniformly on each side of the center band.

Figure 2.6 *Rectangular Footing: Reinforcement Areas*
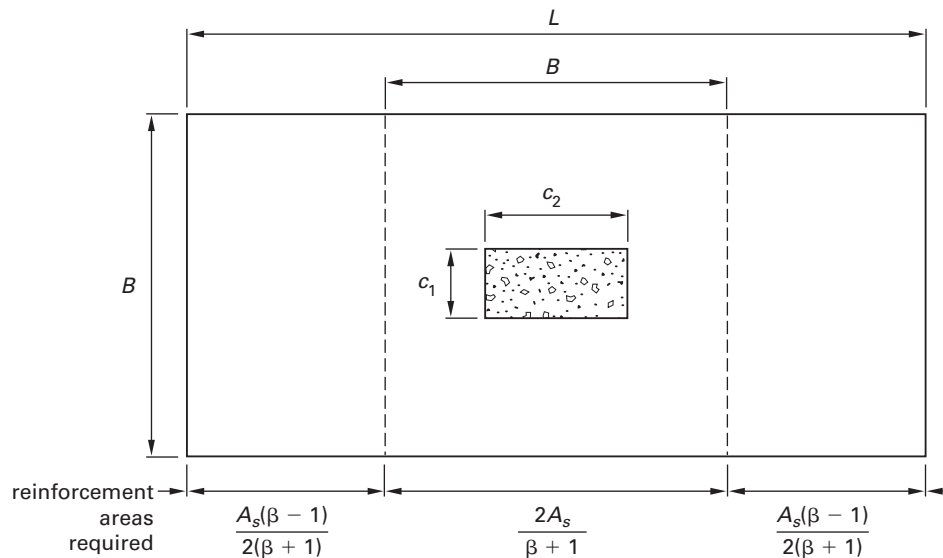
Example 2.8

The reinforced concrete footing shown in the following illustration is 10 ft long and 7 ft wide, has an effective depth of 12 in and an overall depth of 16 in, and has a concrete compressive strength of 5000 lbf/in². The footing supports a column with dimensions 12 in by 18 in and is reinforced with grade 60 bars. Determine the transverse reinforcement required when the net factored pressure acting on the footing is 4.8 kips/ft².
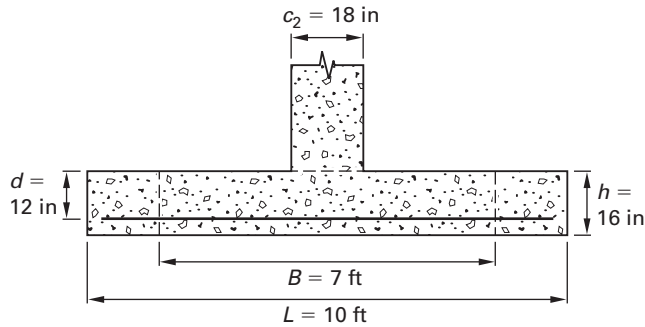
Solution

The factored moment in the transverse direction at the critical section, which is at the face of the column, is

$$M_u = \frac{q_u L \left(\frac{B}{2} - \frac{c_1}{2}\right)^2}{2}$$

$$= \frac{\left(4.8 \; \frac{\text{kips}}{\text{ft}^2}\right)(10 \text{ ft})\left(\frac{7 \text{ ft}}{2} - \frac{1 \text{ ft}}{2}\right)^2}{2}$$

$$= 216 \text{ ft-kips}$$

Assuming a tension-controlled section, the required reinforcement ratio is given by

$$\rho = \frac{0.85 f'_c \left(1 - \sqrt{1 - \frac{M_u}{0.383 b d^2 f'_c}}\right)}{f_y}$$

$$= \frac{(0.85)\left(5 \; \frac{\text{kips}}{\text{in}^2}\right)}{60 \; \frac{\text{kips}}{\text{in}^2}}$$

$$\times \left(1 - \sqrt{1 - \frac{(216 \text{ ft-kips})\left(12 \; \frac{\text{in}}{\text{ft}}\right) \times \left(1000 \; \frac{\text{lbf}}{\text{kip}}\right)}{(0.383)(120 \text{ in})(12 \text{ in}^2)^2 \times \left(5000 \; \frac{\text{lbf}}{\text{in}^2}\right)}}\right)$$

$$= 0.0028$$

The maximum allowable reinforcement ratio for a tension-controlled section is given by

$$\rho_t = 0.319 \beta_1 \frac{f'_c}{f_y}$$

$$= 0.0213$$

$$> \rho \quad \begin{bmatrix} \text{satisfactory, the section} \\ \text{is tension-controlled} \end{bmatrix}$$

The minimum allowable reinforcement area is given by ACI Sec. 7.12.2 as

$$A_{s(\min)} = 0.0018 bh = (0.0018)(120 \text{ in})(16 \text{ in})$$

$$= 3.46 \text{ in}^2$$

The required reinforcement area is

$$A_s = \rho b d = (0.0028)(120 \text{ in})(12 \text{ in})$$

$$= 4.032 \text{ in}^2$$

$$> A_{s(\min)} \quad \text{[satisfactory]}$$

The reinforcement required in the central 7 ft band width is

$$A_b = \frac{2 A_s}{\beta + 1} = \frac{(2)(4.032 \text{ in}^2)}{\frac{10 \text{ ft}}{7 \text{ ft}} + 1} = 3.32 \text{ in}^2$$

Providing 11 no. 5 bars gives a reinforcement area of

$$A_{b(\text{prov})} = 3.41 \text{ in}^2 \quad \text{[satisfactory]}$$

The remaining reinforcement is

$$A_r = A_s - A_{b(\text{prov})} = 4.032 \text{ in}^2 - 3.41 \text{ in}^2$$

$$= 0.62 \text{ in}^2$$

Providing two no. 4 bars on each side of the central band gives a reinforcement area of

$$A_{r(\text{prov})} = (4)(0.20 \text{ in}^2)$$

$$= 0.80 \text{ in}^2 \quad \text{[satisfactory]}$$

By inspection, the anchorage length provided is adequate.

4. COMBINED FOOTING

Nomenclature

A_{1b}	area of reinforcement required in the band width under column no. 1	in²
P_1	service load for column no. 1	kips
P_2	service load for column no. 2	kips
P_{1u}	factored load for column no. 1	kips
P_{2u}	factored load for column no. 2	kips
q_e	equivalent soil bearing pressure	lbf/ft²
V_c	shear capacity of concrete footing	kips
V_u	factored shear force	kips

Pressure Distribution

A combined footing is used when a column is located adjacent to a property line. A second column is placed on the combined footing, and the length of the footing is adjusted until its centroid coincides with the centroid of the service loads on the two columns. A uniformly distributed soil pressure is produced under the combined footing, as shown in Fig. 2.7. The footing width is adjusted to ensure that the soil bearing pressure does not exceed the allowable pressure.

Figure 2.7 *Combined Footing with Applied Service Loads*
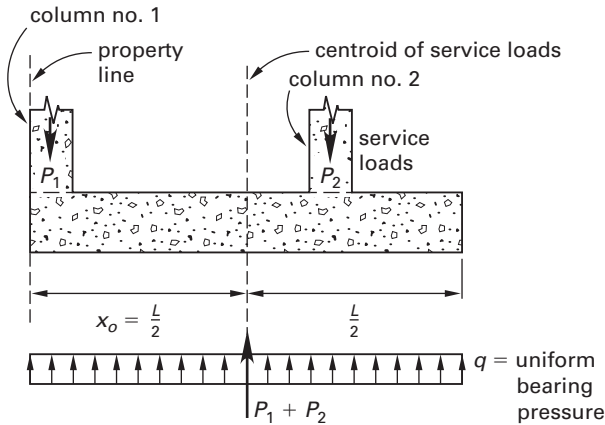
The footing must be designed for punching shear, flexural shear, and flexure. Each of these must be designed for the applied factored loads. The soil pressure distribution due to factored loads must be determined, as shown in Fig. 2.8. The soil pressure will not be uniformly distributed unless the ratios of the factored loads to service loads on both columns are identical. Since the self-weight of the footing produces an equal and opposite pressure in the soil, the footing is designed for the net pressure due to the column load only, and the weight of the footing is not included.

The footing is designed in the longitudinal direction as a beam continuous over two supports. As shown in

Fig. 2.8, the maximum negative moment occurs at the section where there is zero shear. The maximum positive moment occurs at the outside face of column no. 2. In the transverse direction, it is assumed that the footing cantilevers about the face of both columns. The reinforcement required is concentrated under each column in a band width equal to the length of the shorter side. The area of reinforcement required in the band width under column no. 1 is given by ACI Sec. 15.4.4.2 as

$$A_{1b} = \frac{2A_s P_{1u}}{(\beta + 1)(P_{1u} + P_{2u})}$$

Figure 2.8 *Combined Footing with Applied Factored Loads*
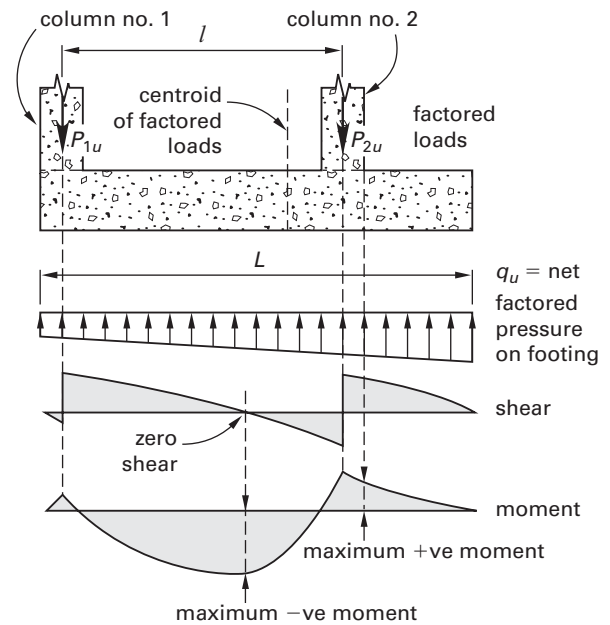
Example 2.9

Determine the plan dimensions required for the combined footing shown in the following illustration to provide a uniform soil bearing pressure of 4000 lbf/ft^2 under the service loads indicated.

Illustration for Ex. 2.9
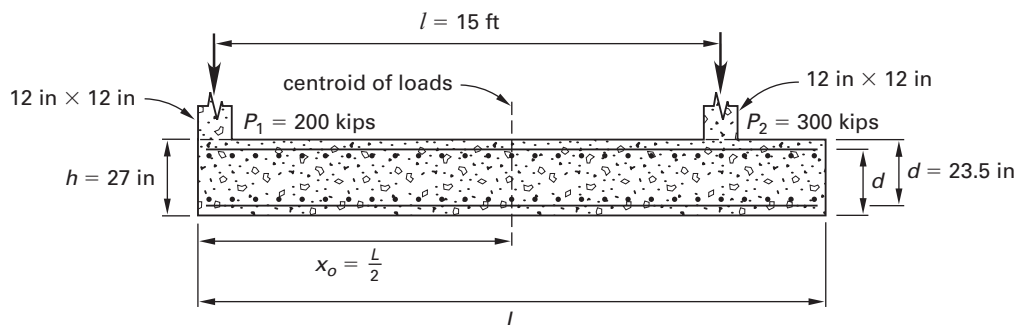
Solution

Allowing for the self-weight of the footing, the maximum allowable equivalent soil bearing pressure is

$$q_e = q - w_c h$$

$$= \frac{4000 \ \frac{\text{lbf}}{\text{ft}^2} - \dfrac{\left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)(27 \ \text{in})}{12 \ \frac{\text{in}}{\text{ft}}}}{1000 \ \frac{\text{lbf}}{\text{kip}}}$$

$$= 3.663 \ \text{kips/ft}^2$$

The centroid of the column service loads is located a distance x_o from the property line, which is obtained by taking moments about the property line and is given by

$$x_o = \frac{0.5 P_1 + 15.5 P_2}{P_1 + P_2}$$

$$= \frac{(0.5 \ \text{ft})(200 \ \text{kips}) + (15.5 \ \text{ft})(300 \ \text{kips})}{200 \ \text{kips} + 300 \ \text{kips}}$$

$$= 9.5 \ \text{ft}$$

The length of footing required to produce a uniform bearing pressure on the soil is

$$L = 2 x_o = (2)(9.5 \ \text{ft})$$

$$= (2)(9.5 \ \text{ft})$$

$$= 19 \ \text{ft}$$

The width of footing to produce a uniform pressure on the soil of 4000 lbf/ft^2 is

$$B = \frac{P_1 + P_2}{q_e L}$$

$$= \frac{500 \ \text{kips}}{\left(3.663 \ \frac{\text{kips}}{\text{ft}^2}\right)(19 \ \text{ft})}$$

$$= 7.2 \ \text{ft}$$

Design for Punching Shear

The critical perimeter for punching shear in a combined footing is identical with that in an isolated column footing and is located a distance from the face of the column equal to one-half the effective depth.

For the interior column, the length of the critical perimeter is

$$b_o = 4(c + d)$$

For the end column, the length of the critical perimeter is

$$b_o = (c + d) + 2\left(c + \frac{d}{2}\right)$$

The design punching shear strength of the footing is determined by ACI Sec. 11.11.2.1.

The net factored pressure on the footing must be determined from the factored applied column loads, as shown in Fig. 2.8. It will not necessarily be uniform unless the ratios of the factored loads to service loads on both columns are identical.

Example 2.10

The combined footing of normal weight concrete for Ex. 2.9 has a concrete strength of 5000 lbf/in^2 and a factored load on each column that is 1.5 times the service load. Determine whether the punching shear capacity is adequate.

Solution

Because the ratios of the factored loads to service loads on both columns are identical, the net factored pressure on the footing is uniform and has a value of

$$q_u = 1.5 q_e = (1.5)\left(3.663 \ \frac{\text{kips}}{\text{ft}^2}\right)$$

$$= 5.5 \ \text{kips/ft}^2$$

For column no. 1, the factored load is

$$P_{1u} = 1.5 P_1 = (1.5)(200 \ \text{kips})$$

$$= 300 \ \text{kips}$$

The length of the critical perimeter, as shown in the following illustration, is

$$b_o = (c + d) + 2\left(c + \frac{d}{2}\right)$$

$$= (12 \ \text{in} + 23.5 \ \text{in}) + (2)\left(12 \ \text{in} + \frac{23.5 \ \text{in}}{2}\right)$$

$$= 83 \ \text{in}$$
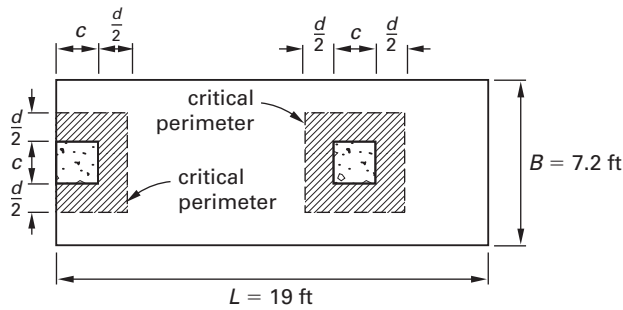
The punching shear force at the critical perimeter is

$$V_u = P_{1u} - q_u(c + d)\left(c + \frac{d}{2}\right)$$

$$= 300 \text{ kips} - \frac{\left(5.5 \dfrac{\text{kips}}{\text{ft}^2}\right)(35.5 \text{ in})(23.75 \text{ in})}{\left(12 \dfrac{\text{in}}{\text{ft}}\right)^2}$$

$$= 268 \text{ kips}$$

The punching shear capacity is given by ACI Eq. 11-33 as

$$\phi V_c = 4\phi d b_o \lambda \sqrt{f'_c}$$

$$= \frac{(4)(0.75)(23.5 \text{ in})(83 \text{ in})(1.0)\sqrt{5000 \dfrac{\text{lbf}}{\text{in}^2}}}{1000 \dfrac{\text{lbf}}{\text{kip}}}$$

$$= 414 \text{ kips}$$

$$> V_u \quad [\text{satisfactory}]$$

For column no. 2, the factored load is

$$P_{2u} = 1.5P_2$$

$$= (1.5)(300 \text{ kips})$$

$$= 450 \text{ kips}$$

The length of the critical perimeter is

$$b_o = 4(c + d)$$

$$= (4)(35.5 \text{ in})$$

$$= 142 \text{ in}$$

The punching shear force at the critical perimeter is

$$V_u = P_{2u} - q_u(c + d)^2$$

$$= 450 \text{ kips} - \frac{\left(5.5 \dfrac{\text{kips}}{\text{ft}^2}\right)(35.5 \text{ in})^2}{\left(12 \dfrac{\text{in}}{\text{ft}}\right)^2}$$

$$= 402 \text{ kips}$$

The punching shear capacity is given by ACI Eq. 11-33 as

$$\phi V_c = 4\phi d b_o \lambda \sqrt{f'_c} = \frac{(414 \text{ kips})(142 \text{ in})}{83 \text{ in}}$$

$$= 708 \text{ kips}$$

$$> V_u \quad [\text{satisfactory}]$$

Design for Flexural Shear

The critical section for flexural shear in a combined footing is identical with that in an isolated column footing and is located a distance d from the face of the column. The shear force at the critical section is determined from the shear force diagram, as shown in Fig. 2.8. The depth of the footing is usually governed by flexural shear.

Example 2.11

Determine whether the flexural shear capacity is adequate for the combined footing of Ex. 2.10.

Solution

At the center of column no. 1, the shear force is

$$V_1 = P_{1u} - \frac{q_u B c}{2}$$

$$= 300 \text{ kips} - \frac{\left(5.5 \dfrac{\text{kips}}{\text{ft}^2}\right)(7.2 \text{ ft})(1 \text{ ft})}{2}$$

$$= 280 \text{ kips}$$

At the center of column no. 2, the shear force is

$$V_2 = V_1 - q_u B l$$

$$= 280 \text{ kips} - \left(5.5 \dfrac{\text{kips}}{\text{ft}^2}\right)(7.2 \text{ ft})(15 \text{ ft})$$

$$= -314 \text{ kips}$$

The shear force diagram is shown in the following illustration, and the critical flexural shear is a distance $(d + c/2)$ from the center of column no. 2.

Illustration for Ex. 2.11
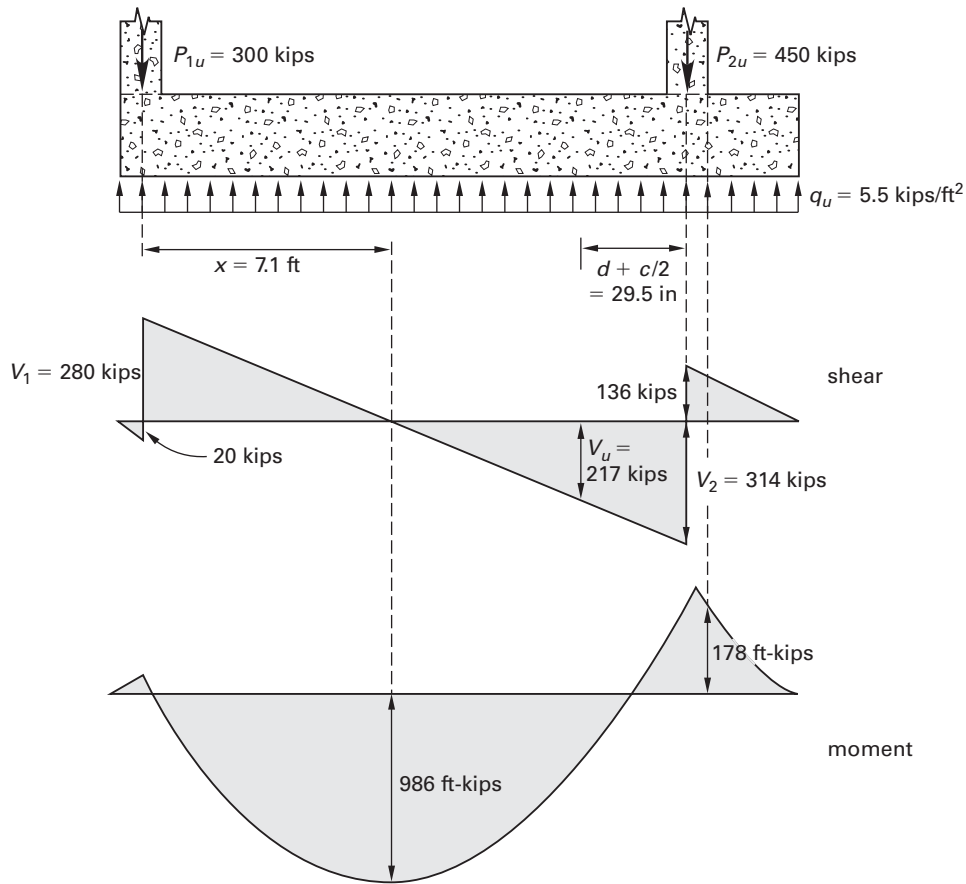
The critical flexural shear at this section is

$$V_u = V_2 - q_u B\left(d + \frac{c}{2}\right)$$

$$= 314 \text{ kips} - \frac{\left(5.5 \ \dfrac{\text{kips}}{\text{ft}^2}\right)(7.2 \text{ ft})(29.5 \text{ in})}{12 \ \dfrac{\text{in}}{\text{ft}}}$$

$$= 217 \text{ kips}$$

The flexural-shear capacity of the footing is given by ACI Eq. 11-3 as

$$\phi V_c = 2\phi b d \lambda \sqrt{f_c'}$$

$$= \frac{\begin{array}{c}(2)(0.75)(7.2 \text{ ft})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right) \\ \times (23.5 \text{ in})(1.0)\sqrt{5000 \ \dfrac{\text{lbf}}{\text{in}^2}}\end{array}}{1000 \ \dfrac{\text{lbf}}{\text{kip}}}$$

$$= 215 \text{ kips}$$

$$\approx V_u \quad [\text{satisfactory}]$$

Example 2.12

Determine the required longitudinal and transverse grade 60 reinforcement for the combined footing of Ex. 2.11.

Solution

From Ex. 2.11, the point of zero shear is a distance from the center of column no. 1 given by

$$x = \frac{V_1}{q_u B} = \frac{280 \text{ kips}}{\left(5.5 \ \dfrac{\text{kips}}{\text{ft}^2}\right)(7.2 \text{ ft})} = 7.1 \text{ ft}$$

The maximum negative moment at this point is

$$M_u = P_{1u} x - \frac{q_u B\left(x + \dfrac{c}{2}\right)^2}{2}$$

$$= (300 \text{ kips})(7.1 \text{ ft})$$

$$- \frac{\left(5.5 \ \dfrac{\text{kips}}{\text{ft}^2}\right)(7.2 \text{ ft})\left(7.1 \text{ ft} + \dfrac{1 \text{ ft}}{2}\right)^2}{2}$$

$$= 986 \text{ ft-kips}$$

For the reinforcement in the top of the footing, assuming a tension-controlled section, the reinforcement ratio required is

$$\rho = \frac{0.85f_c'\left(1 - \sqrt{1 - \dfrac{M_u}{0.383bd^2f_c'}}\right)}{f_y}$$

$$= \frac{(0.85)\left(5\ \dfrac{\text{kips}}{\text{in}^2}\right)}{60\ \dfrac{\text{kips}}{\text{in}^2}}$$

$$\times \left(1 - \sqrt{1 - \frac{(986\ \text{ft-kips})\left(12\ \dfrac{\text{in}}{\text{ft}}\right)\times\left(1000\ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.383)(86.4\ \text{in})(23.5\ \text{in})^2\times\left(5000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}}\right)$$

$$= 0.0048$$

The maximum allowable reinforcement ratio for a tension-controlled section is given by

$$\rho_t = 0.319\beta_1\frac{f_c'}{f_y}$$

$$= 0.0213$$

$$> \rho \quad \left[\begin{array}{c}\text{satisfactory, the section}\\ \text{is tension-controlled}\end{array}\right]$$

The required reinforcement area in the top of the footing is

$$A_s = \rho bd$$

$$= (0.0048)(86.4\ \text{in})(23.5\ \text{in})$$

$$= 9.75\ \text{in}^2$$

Providing 10 no. 9 bars gives an area of 10 in^2 (satisfactory).

The minimum permissible reinforcement area is given by ACI Sec. 7.12.2 as

$$A_{s(\text{min})} = 0.0018bh$$

$$= (0.0018)(86.4\ \text{in})(27\ \text{in})$$

$$= 4.20\ \text{in}^2$$

$$< A_s \quad [\text{satisfactory}]$$

The maximum positive moment at the outside face of column no. 2 is

$$M_u = \frac{q_uB(L - l - c)^2}{2}$$

$$= \frac{\left(5.5\ \dfrac{\text{kips}}{\text{ft}^2}\right)(7.2\ \text{ft})(19\ \text{ft} - 15\ \text{ft} - 1\ \text{ft})^2}{2}$$

$$= 178\ \text{ft-kips}$$

For the reinforcement in the bottom of the footing, assuming a tension-controlled section, the reinforcement ratio required is

$$\rho = \frac{0.85f_c'\left(1 - \sqrt{1 - \dfrac{M_u}{0.383bd^2f_c'}}\right)}{f_y}$$

$$= \frac{(0.85)\left(5\ \dfrac{\text{kips}}{\text{in}^2}\right)}{60\ \dfrac{\text{kips}}{\text{in}^2}}$$

$$\times \left(1 - \sqrt{1 - \frac{(178\ \text{ft-kips})\left(12\ \dfrac{\text{in}}{\text{ft}}\right)\times\left(1000\ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.383)(86.4\ \text{in})(23.5\ \text{in})^2\times\left(5000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}}\right)$$

$$= 0.00083$$

The combined area of reinforcement in the top and bottom of the footing exceeds the minimum required value of $0.0018bh$, and in accordance with ACI Sec. 10.5.3, the required reinforcement area in the bottom of the footing is

$$A_s = 1.33\rho bd = (1.33)(0.00083)(86.4\ \text{in})(23.5\ \text{in})$$

$$= 2.24\ \text{in}^2$$

Providing 10 no. 5 bars gives an area of 3.10 in^2 (satisfactory).

The factored moment in the transverse direction at the face of the columns is

$$M_u = \frac{q_uL\left(\dfrac{B}{2} - \dfrac{c}{2}\right)^2}{2}$$

$$= \frac{\left(5.5\ \dfrac{\text{kips}}{\text{ft}^2}\right)(19\ \text{ft})\left(\dfrac{7.2}{2}\ \text{ft} - \dfrac{1}{2}\ \text{ft}\right)^2}{2}$$

$$= 502\ \text{ft-kips}$$

Foundations

For the transverse reinforcement in the bottom of the footing, assuming a tension-controlled section, the reinforcement ratio required is

$$\rho = \frac{0.85 f_c' \left(1 - \sqrt{1 - \dfrac{M_u}{0.383 b d^2 f_c'}} \right)}{f_y}$$

$$= \frac{(0.85)\left(5 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{60 \ \dfrac{\text{kips}}{\text{in}^2}} \times \left(1 - \sqrt{1 - \dfrac{(502 \ \text{ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right) \times \left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.383)(228 \ \text{in})(23.5 \ \text{in})^2 \times \left(5000 \ \dfrac{\text{lbf}}{\text{in}^2}\right)}} \right)$$

$$= 0.00089$$

The minimum permissible reinforcement area governs, and the reinforcement area required in both the top and bottom of the footing transversely is

$$A_s = \frac{0.0018 bh}{2} = \frac{(0.0018)\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)(27 \ \text{in})}{2}$$

$$= 0.29 \ \text{in}^2/\text{ft}$$

Providing no. 4 bars at 8 in centers gives an area of $0.30 \ \text{in}^2/\text{ft}$ (satisfactory).

5. STRAP FOOTING

Nomenclature

A_1	base area of pad footing no. 1	in^2
A_2	base area of pad footing no. 2	in^2
B_S	length of short side of strap	ft
B_1	length of short side of pad footing no. 1	ft
B_2	length of short side of pad footing no. 2	ft
h_S	depth of strap	in
h_1	depth of pad footing no. 1	in
h_2	depth of pad footing no. 2	in
l	distance between column centers	ft
l_R	distance between soil reactions	ft
L_S	length of long side of strap	ft
L_1	length of long side of pad footing no. 1	ft
L_2	length of long side of pad footing no. 2	ft
R_1	soil reaction under pad footing no. 1	kips
R_2	soil reaction under pad footing no. 2	kips
w_c	unit weight of concrete	lbf/ft^3
W_S	weight of strap beam	kips
W_1	weight of pad footing no. 1	kips
W_2	weight of pad footing no. 2	kips

Pressure Distribution

The strap footing shown in Fig. 2.9 has the strap beam, which connects the two pad footings, underlaid by a layer of Styrofoam™ so that the soil pressure under the strap may be considered negligible. Because of the stiffness of the strap beam, the strap and pad footings act as a rigid body producing uniform soil pressure under the pad footings. The base areas of the two pad footings may be adjusted to produce equal soil pressure q under both footings.

The total service load acting is

$$\sum P = P_1 + P_2 + W_1 + W_2 + W_S$$

$$q = \frac{\sum P}{A_1 + A_2}$$

The soil reactions act at the center of the pad footings and are given by

$$R_1 = q A_1$$

$$R_2 = q A_2$$

Pad footing no. 2 is located symmetrically with respect to column no. 2 so that the lines of action of P_2 and R_2 are coincident.

$$l_R = l + \frac{c_1}{2} - \frac{B_1}{2}$$

$$L_S = l_R - \frac{B_1 + B_2}{2}$$

Equating vertical forces gives

$$R_2 = \sum P - R_1 \quad \text{[equilibrium equation no. 1]}$$

Taking moments about the center of pad footing no. 2 gives

$$R_1 = \frac{P_1 l + W_1 l_R + \dfrac{W_S(L_S + B_2)}{2}}{l_R}$$

[equilibrium equation no. 2]

To determine suitable dimensions that will give a soil bearing pressure equal to the allowable pressure q, suitable values are selected for h_1, h_2, h_S, B_1, B_2, and B_S. l_R and L_S are determined, and

$$W_S = w_c L_S B_S h_S$$

An initial estimate is made of R_1, and

$$A_1 = \frac{R_1}{q}$$

$$W_1 = w_c A_1 h_1$$

Figure 2.9 *Strap Footing with Applied Service Loads*
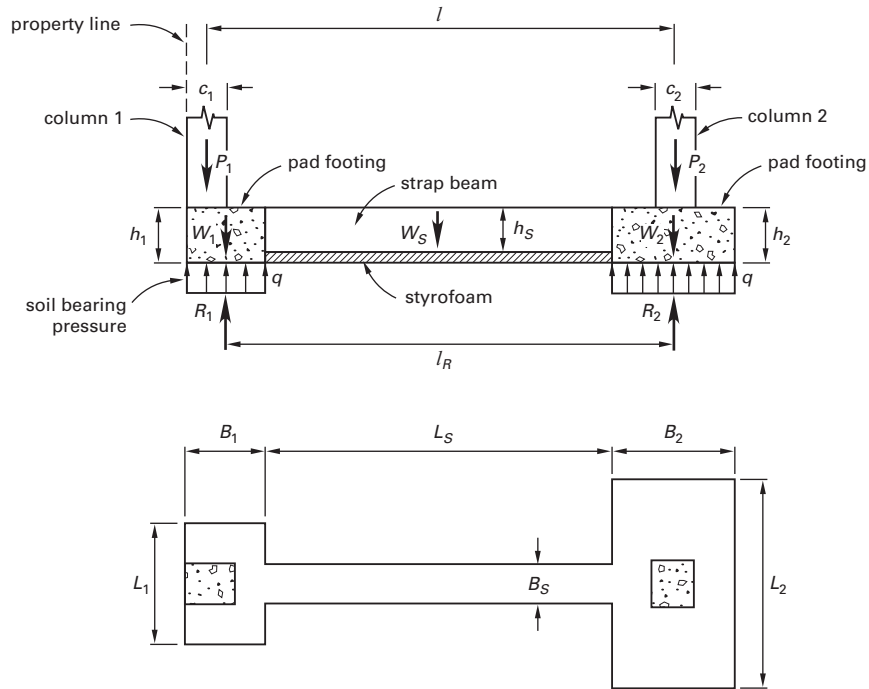
An initial estimate is made of R_2, and

$$A_2 = \frac{R_2}{q}$$

$$W_2 = w_c A_2 h_2$$

$$\sum P = P_1 + P_2 + W_1 + W_2 + W_S$$

Substituting in the two equilibrium equations provides revised estimates of R_1 and R_2, and the process is repeated until convergence is reached.

Example 2.13

Determine the plan dimensions required for the strap footing shown in the following illustration to provide a uniform bearing pressure of 3000 lbf/ft^2 under both pad footings for the service loads indicated.
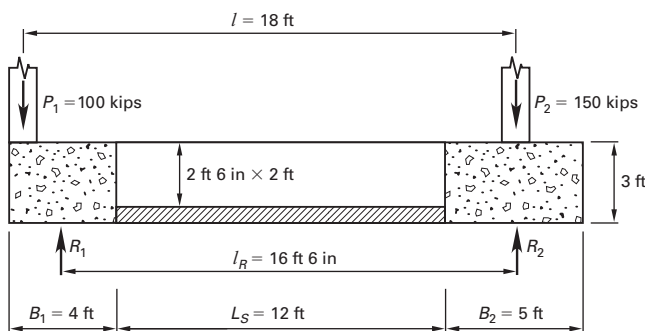
Solution

From the dimensions indicated in the illustration,

$$W_S = w_c L_S B_S h_S$$

$$= \left(0.15 \ \frac{\text{kip}}{\text{ft}^3} \right) (12 \text{ ft})(2 \text{ ft})(2.5 \text{ ft})$$

$$= 9 \text{ kips}$$

Assuming that $R_1 = 134$ kips, then

$$A_1 = \frac{R_1}{q} = \frac{134 \text{ kips}}{3 \ \frac{\text{kips}}{\text{ft}^2}} = 44.67 \text{ ft}^2$$

$$W_1 = w_c A_1 h_1 = \left(0.15 \ \frac{\text{kip}}{\text{ft}^3} \right) (44.67 \text{ ft}^2)(3 \text{ ft})$$

$$= 20.1 \text{ kips}$$

Assuming that $R_2 = 171$ kips, then

$$A_2 = \frac{R_2}{q} = \frac{171 \text{ kips}}{3 \ \frac{\text{kips}}{\text{ft}^2}} = 57 \text{ ft}^2$$

$$W_2 = w_c A_2 h_2 = \left(0.15 \ \frac{\text{kip}}{\text{ft}^3} \right) (57 \text{ ft}^2)(3 \text{ ft})$$

$$= 25.7 \text{ kips}$$

$$\sum P = P_1 + P_2 + W_1 + W_2 + W_S$$

$$= 100 \text{ kips} + 150 \text{ kips} + 20.1 \text{ kips}$$

$$\qquad + 25.7 \text{ kips} + 9 \text{ kips}$$

$$= 304.8 \text{ kips}$$

Equating vertical forces gives

$$R_2 = \sum P - R_1$$
$$= 304.8 \text{ kips} - 134 \text{ kips}$$
$$= 170.8 \text{ kips}$$
$$\approx 171 \text{ kips} \quad [\text{satisfactory}]$$

Taking moments about the center of pad footing no. 2 gives

$$R_1 = \frac{P_1 l + W_1 l_R + \dfrac{W_S(L_S + B_2)}{2}}{l_R}$$

$$= \frac{\begin{array}{c}(100 \text{ kips})(18 \text{ ft}) + (20.1 \text{ kips})(16.5 \text{ ft}) \\[4pt] + \dfrac{(9 \text{ kips})(12 \text{ ft} + 5 \text{ ft})}{2}\end{array}}{16.5 \text{ ft}}$$

$$= 133.8 \text{ kips}$$
$$\approx 134 \text{ kips} \quad [\text{satisfactory}]$$

The initial estimates were sufficiently accurate, and the required pad footing areas are

$$A_1 = 44.67 \text{ ft}^2$$
$$A_2 = 57 \text{ ft}^2$$

Figure 2.10 *Factored Forces on Strap Footing*

Design of Strap Beam for Shear

The factored forces acting on the footing are shown in Fig. 2.10. The total factored load on the footing is

$$\sum P_u = P_{1u} + P_{2u} + W_{1u} + W_{2u} + W_{Su}$$

Taking moments about the center of pad footing no. 2 gives

$$R_{1u} = \frac{P_{1u} l + W_{1u} l_R + \dfrac{W_{Su}(L_S + B_2)}{2}}{l_R}$$

Equating vertical forces gives

$$R_{2u} = \sum P_u - R_{1u}$$

The shear at the left end of the strap is

$$V_{Su} = R_{1u} - P_{1u} - W_{1u}$$

The shear at the right end of the strap is

$$V'_{Su} = P_{2u} + W_{2u} - R_{2u}$$

Example 2.14

The strap footing of normal weight concrete for Ex. 2.13 has a concrete strength of 3000 lbf/in^2 and a factored load on each column that is 1.5 times the service load. The strap beam has an effective depth of 27.5 in. Determine whether the shear capacity is adequate.

Solution

The factored forces are

$$P_{1u} = 1.5P_1 = 150 \text{ kips}$$

$$P_{2u} = 1.5P_2 = 225 \text{ kips}$$

$$W_{1u} = 1.2W_1 = 24 \text{ kips}$$

$$W_{2u} = 1.2W_2 = 31 \text{ kips}$$

$$W_{Su} = 1.2W_S = 11 \text{ kips}$$

$$\sum P_u = 441 \text{ kips}$$

$$R_{1u} = \frac{P_{1u}l + W_{1u}l_R + \dfrac{W_{Su}(L_S + B_2)}{2}}{l_R}$$

$$(150 \text{ kips})(18 \text{ ft}) + (24 \text{ kips})(16.5 \text{ ft})$$

$$= \frac{+\dfrac{(11 \text{ kips})(12 \text{ ft} + 5 \text{ ft})}{2}}{16.5 \text{ ft}}$$

$$= 193 \text{ kips}$$

$$R_{2u} = \sum P_u - R_{1u}$$

$$= 248 \text{ kips}$$

The shear at the right end of the strap is

$$V'_{Su} = P_{2u} + W_{2u} - R_{2u}$$

$$= 225 \text{ kips} + 31 \text{ kips} - 248 \text{ kips}$$

$$= 8 \text{ kips}$$

The shear at the left end of the strap is

$$V_{Su} = R_{1u} - P_{1u} - W_{1u}$$

$$= 193 \text{ kips} - 150 \text{ kips} - 24 \text{ kips}$$

$$= 19 \text{ kips} \quad \text{[governs]}$$

The design shear capacity of the strap beam is given by ACI Eq. 11-3 as

$$\phi V_c = 2\phi bd\lambda\sqrt{f'_c}$$

$$= \frac{(2)(0.75)(24 \text{ in})(27.5 \text{ in})(1.0)\sqrt{3000 \ \dfrac{\text{lbf}}{\text{in}^2}}}{1000 \ \dfrac{\text{lbf}}{\text{kip}}}$$

$$= 54 \text{ kips}$$

$$> 2V_{Su} \quad \text{[No shear reinforcement is required.]}$$

Design of Strap Beam for Flexure

From Fig. 2.10, the factored moment at the left end of the strap is

$$M_{Su} = P_{1u}\left(B_1 - \frac{c_1}{2}\right) - \frac{(R_{1u} - W_{1u})B_1}{2}$$

The factored moment at the right end of the strap is

$$M'_{Su} = \frac{(R_{2u} - W_{2u} - P_{2u})B_2}{2}$$

Example 2.15

Determine the required grade 60 flexural reinforcement for the strap beam of Ex. 2.14.

Solution

The factored moment at the right end of the strap is

$$M'_{Su} = \frac{(W_{2u} + P_{2u} - R_{2u})B_2}{2}$$

$$= \frac{(31 \text{ kips} + 225 \text{ kips} - 248 \text{ kips})(5 \text{ ft})}{2}$$

$$= 20 \text{ ft-kips}$$

The factored moment at the left end of the strap is

$$M_{Su} = P_{1u}\left(B_1 - \frac{c_1}{2}\right) - \frac{(R_{1u} - W_{1u})B_1}{2}$$

$$= (150 \text{ kips})\left(4 \text{ ft} - \frac{1 \text{ ft}}{2}\right)$$

$$- \left(\frac{(193 \text{ kips} - 24 \text{ kips})(4 \text{ ft})}{2}\right)$$

$$= 187 \text{ ft-kips} \quad \text{[governs]}$$

Assuming a tension-controlled section, the required reinforcement ratio is

$$\rho = \frac{0.85f'_c\left(1 - \sqrt{1 - \dfrac{M_{Su}}{0.383bd^2f'_c}}\right)}{f_y}$$

$$= \frac{(0.85)\left(3 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{60 \ \dfrac{\text{kips}}{\text{in}^2}}$$
$$\times \left(1 - \sqrt{1 - \dfrac{(187 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right) \times \left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.383)(24 \text{ in})(27.5 \text{ in})^2 \times \left(3000 \ \dfrac{\text{lbf}}{\text{in}^2}\right)}}\right)$$

$$= 0.0023$$

The controlling minimum reinforcement ratio is given by ACI Sec. 10.5.1 and Sec. 10.5.3 as the lesser of the following results.

$$\rho_{\min} = \frac{200}{f_y} = \frac{200 \; \dfrac{\text{lbf}}{\text{in}^2}}{60{,}000 \; \dfrac{\text{lbf}}{\text{in}^2}}$$

$$= 0.0033$$

$$\rho_{\min} = \left(\frac{4}{3}\right)(0.0023) = 0.0031 \quad \text{[governs]}$$

The reinforcement required in the top of the strap beam is

$$A_s = bd\rho_{\min}$$

$$= (24 \text{ in})(27.5 \text{ in})(0.0031)$$

$$= 2.05 \text{ in}^2$$

Providing four no. 7 bars gives an area of 2.4 in² (satisfactory).

6. CANTILEVER RETAINING WALL

Nomenclature

F	frictional force at underside of base	kips
$\overline{h}$	equivalent additional height of fill, w/γ_S	ft
h_B	depth of base	ft
h_K	height of shear key	ft
h_T	total height of retaining wall, $h_B + L_W$	ft
h_W	stem thickness	ft
H_A	total active earth pressure behind wall	kips
H_L	total pressure behind wall due to live load surcharge	kips
H_P	total passive earth pressure in front of wall	kips
K_A	Rankine coefficient of active earth pressure $(1-\sin\phi)/(1+\sin\phi)$	–
K_P	Rankine coefficient of passive earth pressure $(1+\sin\phi)/(1-\sin\phi)$	–
L_B	length of base	ft
L_H	length of heel	ft
L_T	length of toe	ft
L_W	height of stem	ft
p_A	active lateral pressure due to a fluid of specific weight γ_A, $K_A\gamma_S$	lbf/ft²
p_L	lateral pressure due to live load surcharge, wK_A	lbf/ft²
p_P	passive lateral pressure due to a fluid of specific weight γ_P, $K_P\gamma_S$	lbf/ft²
q	earth pressure under the base	lbf/ft²
w	live load surcharge	lbf/ft²
W_B	weight of base	kips
W_K	weight of key	kips
W_L	weight of surcharge	kips
W_S	weight of backfill	kips
W_W	weight of stem	kips

Symbols

γ_S	specific weight of backfill	lbf/ft³
μ	coefficient of friction	–
ϕ	angle of internal friction	degree

Pressure Distribution

Figure 2.11 shows the forces acting on a cantilever retaining wall. The total active earth pressure behind the wall is given by Rankine's theory as

$$H_A = \frac{p_A h_T^2}{2} = \frac{K_A \gamma_S h_T^2}{2} = \frac{\left(\dfrac{1-\sin\phi}{1+\sin\phi}\right)\gamma_S h_T^2}{2}$$

$$= \frac{30h_T^2}{2} \quad \text{[for } \gamma_S = 110 \text{ lbf/ft}^3 \text{ and } \phi = 35°\text{]}$$

$$= \begin{array}{l}\text{pressure exerted by a fluid}\\ \text{of density 30 lbf/ft}^3\end{array}$$

The total active earth pressure acts at a height of $h_T/3$ above the base.

The total surcharge pressure behind the wall due to a live load surcharge of w is

$$H_L = p_L h_T = wK_A h_T = \frac{wp_A h_T}{\gamma_S}$$

The surcharge may be represented by an equivalent height of fill given by

$$\overline{h} = \frac{w}{\gamma_S}$$

$$H_L = p_A \overline{h} h_T$$

The total surcharge pressure acts at a height of $h_T/2$ above the base.

The total passive earth pressure in front of the wall is

$$H_P = \frac{p_P h_K^2}{2} = \frac{K_P \gamma_S h_K^2}{2} = \frac{\left(\dfrac{1+\sin\phi}{1-\sin\phi}\right)\gamma_S h_K^2}{2}$$

$$= \frac{400h_K^2}{2} \quad \text{[for } \gamma_S = 110 \text{ lbf/ft}^3 \text{ and } \phi = 35°\text{]}$$

$$= \begin{array}{l}\text{pressure exerted by a fluid}\\ \text{of density 400 lbf/ft}^3\end{array}$$

The total passive earth pressure acts at a height of $h_K/3$ above the bottom of the key. The frictional force acting on the underside of the base is given by

$$F = \mu\sum W$$

$\sum W$ is the total weight of the retaining wall plus backfill plus live load surcharge.

Figure 2.11 *Cantilever Retaining Wall with Applied Service Loads*
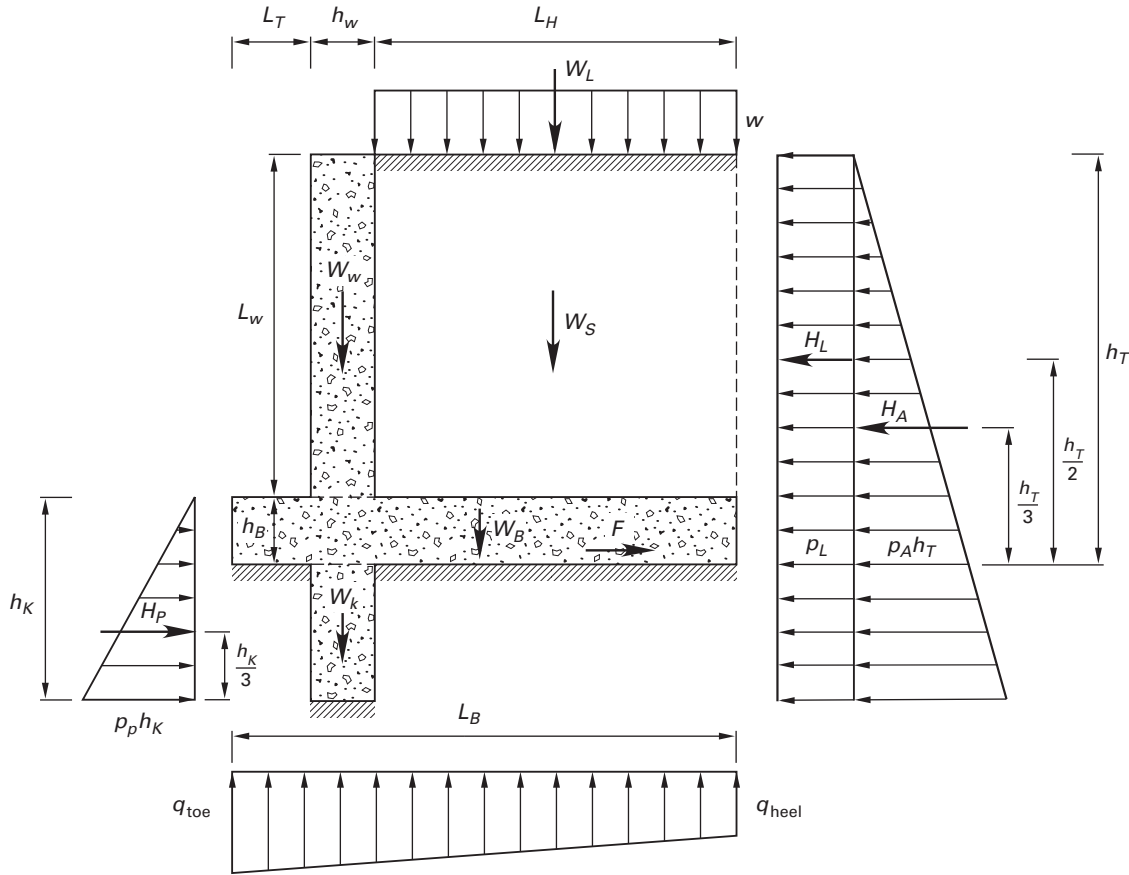
A factor of safety of 1.5 against sliding is required, which gives

$$F + H_P \geq \begin{array}{l} \text{1.5 times the active pressure acting} \\ \text{from top of wall to bottom of key} \end{array}$$

A factor of safety of 1.5 is required for overturning about the toe.

Reinforcement Details

The minimum reinforcement required in the stem wall is specified by ACI Sec. 14.3. For grade 60 bars larger than no. 5, the reinforcement ratios are based on the gross concrete area for vertical and horizontal reinforcement, and are

$$\rho_{\text{vert}} = 0.15\%$$

$$\rho_{\text{hor}} = 0.25\%$$

For grade 60 bars no. 5 and smaller, the corresponding ratios are

$$\rho_{\text{vert}} = 0.12\%$$

$$\rho_{\text{hor}} = 0.20\%$$

A single layer of horizontal reinforcement is permitted in walls not exceeding 10 in thickness. For walls exceeding 10 in thickness, two layers of horizontal reinforcement are required, and the limiting reinforcement ratio is based on the total reinforcement ratio for both layers combined. The reinforcement layer for the air face should be between one-half and two-thirds of the total reinforcement. It should be placed between 2 in and one-third the thickness of the wall from the air face. The layer for the earth face, consisting of the balance of the required horizontal reinforcement, should be placed a distance from the earth face not more than one-third the thickness of the wall, but not less than 2 in for bars larger than no. 5, or $1^{1}/_{2}$ in for bars no. 5 or smaller.

The area of vertical reinforcement in the earth face is governed by flexural requirements. Nominal vertical reinforcement is required in the air face to give a total reinforcement ratio of not less than 0.0018% for grade 60 reinforcement.

Vertical and horizontal reinforcement is spaced apart no more than three times the wall thickness and no further apart than 18 in.

Example 2.16

The retaining wall shown in the following illustration retains soil with a unit weight of 110 lbf/ft³ and an equivalent fluid pressure of 30 lbf/ft² per foot. The live load surcharge behind the wall is equivalent to an additional height of 2 ft of fill. Passive earth pressure may be

assumed equivalent to a fluid pressure of 450 lbf/ft^2, and the coefficient of friction at the underside of the base is 0.4. Determine the factors of safety against sliding and overturning and the bearing pressure distribution under the base.
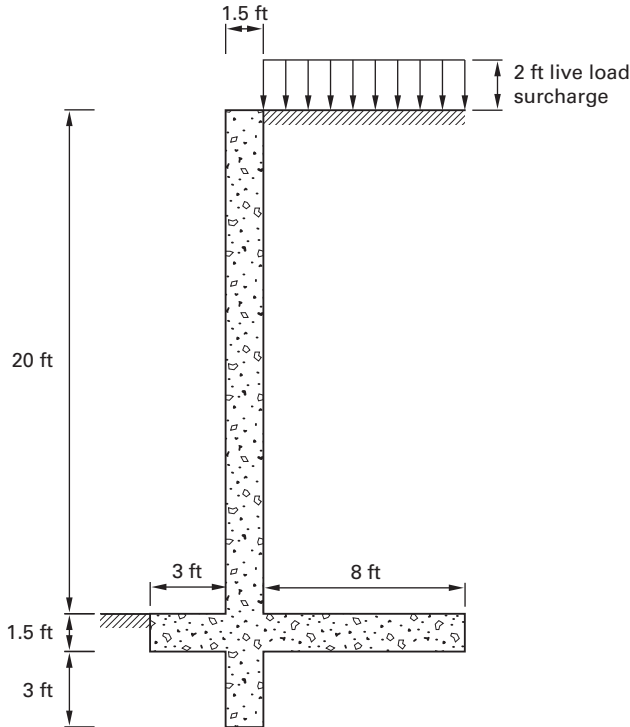
Solution

The lateral pressures from the backfill and the surcharge are

$$H_A = \frac{p_A h_T^2}{2} = \frac{\left(30 \ \frac{\text{lbf}}{\text{ft}^2}\right)(21.5 \ \text{ft})^2}{2}$$

$$= 6934 \ \text{lbf}$$

$$H_L = p_A h_T \overline{h} = \left(30 \ \frac{\text{lbf}}{\text{ft}^2}\right)(21.5 \ \text{ft})(2 \ \text{ft})$$

$$= 1290 \ \text{lbf}$$

Taking moments about the toe gives

$$M_o = \frac{H_A h_T}{3} + \frac{H_L h_T}{2}$$

$$= \frac{(6934 \ \text{lbf})(21.5 \ \text{ft})}{3} + \frac{(1290 \ \text{lbf})(21.5 \ \text{ft})}{2}$$

$$= 63{,}561 \ \text{ft-lbf}$$

The gravity loads acting are

$$W_W + W_K = w_c h_W L_W + w_c(h_K - h_B)h_W$$

$$= \left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)(1.5 \ \text{ft})(20 \ \text{ft})$$

$$+ \left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)(3 \ \text{ft})(1.5 \ \text{ft})$$

$$= 5175 \ \text{lbf}$$

$$W_B = w_c h_B L_B$$

$$= \left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)(1.5 \ \text{ft})(12.5 \ \text{ft})$$

$$= 2813 \ \text{lbf}$$

$$W_L + W_S = \gamma_S(h_T - h_B + \overline{h})L_H$$

$$= \left(110 \ \frac{\text{lbf}}{\text{ft}^3}\right)(20 \ \text{ft} + 2 \ \text{ft})(8 \ \text{ft})$$

$$= 19{,}360 \ \text{lbf}$$

$$\sum W = 5175 \ \text{lbf} + 2813 \ \text{lbf} + 19{,}360 \ \text{lbf}$$

$$= 27{,}348 \ \text{lbf}$$

The distance of the resultant vertical load from the toe is

$$x_o = \frac{\begin{array}{c}(W_W + W_K)\left(L_T + \dfrac{h_W}{2}\right) + \dfrac{W_B L_B}{2} \\[2mm] + (W_L + W_S)\left(L_T + h_W + \dfrac{L_H}{2}\right)\end{array}}{\sum W}$$

$$= \frac{\begin{array}{c}(5175 \ \text{lbf})(3.75 \ \text{ft}) + (2813 \ \text{lbf})(6.25 \ \text{ft}) \\[2mm] + (19{,}360 \ \text{lbf})(8.5 \ \text{ft})\end{array}}{27{,}348 \ \text{lbf}}$$

$$= 7.37 \ \text{ft}$$

The factor of safety against overturning is

$$\frac{x_o \sum W}{M_o} = \frac{(7.37 \ \text{ft})(27{,}348 \ \text{lbf})}{63{,}561 \ \text{ft-lbf}} = 3.2$$

$$> 1.5 \quad [\text{satisfactory}]$$

The eccentricity of all applied loads about the toe is

$$e' = \frac{x_o \sum W - M_o}{\sum W} = \frac{(7.37 \ \text{ft})(27{,}348 \ \text{lbf}) - 63{,}561 \ \text{lbf}}{27{,}348 \ \text{lbf}}$$

$$= 5.05 \ \text{ft}$$

The eccentricity of all applied loads about the midpoint of the base is

$$e = \frac{L_B}{2} - e' = \frac{12.5 \ \text{ft}}{2} - 5.05 \ \text{ft}$$

$$= 1.20 \ \text{ft} \quad [\text{within middle third}]$$

The pressure under the base is given by

$$q = \frac{\sum W \left(1 \pm \frac{6e}{L_B}\right)}{BL_B}$$

$$= \frac{(27{,}348 \text{ lbf})\left(1 \pm \frac{(6)(1.2 \text{ ft})}{12.5 \text{ ft}}\right)}{(1 \text{ ft})(12.5 \text{ ft})}$$

$$q_{\text{toe}} = 3448 \text{ lbf/ft}^2$$

$$q_{\text{heel}} = 928 \text{ lbf/ft}^2$$

The frictional resistance under the base is

$$F = \mu \sum W = (0.4)(27{,}348 \text{ lbf})$$

$$= 10{,}939 \text{ lbf}$$

The passive pressure in front of the wall is

$$H_P = \frac{p_P h_K^2}{2} = \frac{\left(450 \frac{\text{lbf}}{\text{ft}^2}\right)(4.5 \text{ ft})^2}{2}$$

$$= 4556 \text{ lbf}$$

The lateral pressures behind the wall from the backfill and surcharge are

$$H_A = \frac{p_A h_T^2}{2} = \frac{\left(30 \frac{\text{lbf}}{\text{ft}^2}\right)(24.5 \text{ ft})^2}{2} = 9004 \text{ lbf}$$

$$H_L = p_A h_T h = \left(30 \frac{\text{lbf}}{\text{ft}^2}\right)(2 \text{ ft})(24.5 \text{ ft}) = 1470 \text{ lbf}$$

The factor of safety against sliding is

$$\frac{F + H_P}{H_A + H_L} = \frac{10{,}939 \text{ lbf} + 4556 \text{ lbf}}{9004 \text{ lbf} + 1470 \text{ lbf}} = 1.48$$

$$\approx 1.5 \quad [\text{satisfactory}]$$

Design for Shear and Flexure

To determine the shear and flexure at the critical sections in the wall, the soil pressure under the footing is recalculated by using the factored forces given by ACI Sec. 9.2 as

$$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \qquad [\textit{ACI 9-2}]$$

Shear is generally not critical. The location of the critical section for flexure in the stem is at the base of the stem; for flexure in the toe, at the front face of the stem; and for flexure in the heel, at the rear face of the stem.

Example 2.17

Determine the reinforcement areas required in the toe, heel, and stem of the retaining wall for Ex. 2.16. The concrete strength is 3000 lbf/in^2, and grade 60 reinforcement is provided.

Solution

The factored overturning moment about the toe is

$$M_{ou} = 1.6 M_o$$

$$= (1.6)(63{,}561 \text{ ft-lbf})$$

$$= 101{,}698 \text{ ft-lbf}$$

The factored total vertical load is

$$\sum W_u = 1.2(W_W + W_B + W_S + W_K) + 1.6 W_L$$

$$= (1.2)(5175 \text{ lbf} + 2813 \text{ lbf} + 17{,}600 \text{ lbf})$$

$$\quad + (1.6)(1760 \text{ lbf})$$

$$= 6210 \text{ lbf} + 3376 \text{ lbf} + 21{,}120 \text{ lbf} + 2816 \text{ lbf}$$

$$= 33{,}522 \text{ lbf}$$

The factored restoring moment is

$$M_{Ru} = (6210 \text{ lbf})(3.75 \text{ ft}) + (3376 \text{ lbf})(6.25 \text{ ft})$$

$$\quad + (21{,}120 \text{ lbf})(8.5 \text{ ft}) + (2816 \text{ lbf})(8.5 \text{ ft})$$

$$= 247{,}844 \text{ ft-lbf}$$

The eccentricity of the factored loads about the toe is

$$e'_u = \frac{M_{Ru} - M_{ou}}{\sum W_u}$$

$$= \frac{247{,}844 \text{ ft-lbf} - 101{,}698 \text{ ft-lbf}}{33{,}522 \text{ lbf}}$$

$$= 4.36 \text{ ft}$$

The eccentricity of the factored loads about the midpoint of the base is

$$e_u = \frac{L_B}{2} - e'$$

$$= \frac{12.5 \text{ ft}}{2} - 4.36 \text{ ft}$$

$$= 1.89 \text{ ft} \quad [\text{within middle third}]$$

The factored pressure under the base is given by

$$q_u = \frac{\sum W_u \left(1 \pm \frac{6e_u}{L_B}\right)}{BL_B}$$

$$= \frac{(33{,}522 \text{ lbf})\left(1 \pm \frac{(6)(1.89 \text{ ft})}{12.5 \text{ ft}}\right)}{(1 \text{ ft})(12.5 \text{ ft})}$$

$$q_{u(\text{toe})} = 5115 \text{ lbf/ft}^2$$

$$q_{u(\text{heel})} = 249 \text{ lbf/ft}^2$$

Design of Toe

The factored pressure distribution under the base is shown in the following illustration, and the maximum factored bending moment in the toe is

$$
M_u = \frac{L_T^2 B(q_{uF} + 2q_{u(\text{toe})})}{6} - \frac{1.2\,W_B L_T^2}{2L_B}
$$

$$
= \frac{(3\text{ ft})^2(1\text{ ft})\left(3947\ \frac{\text{lbf}}{\text{ft}^2} + (2)\left(5115\ \frac{\text{lbf}}{\text{ft}^2}\right)\right)}{6}
$$

$$
- \frac{(3376\text{ lbf})(3\text{ ft})^2}{(2)(12.5\text{ ft})}
$$

$$
= 20{,}050\text{ ft-lbf}
$$
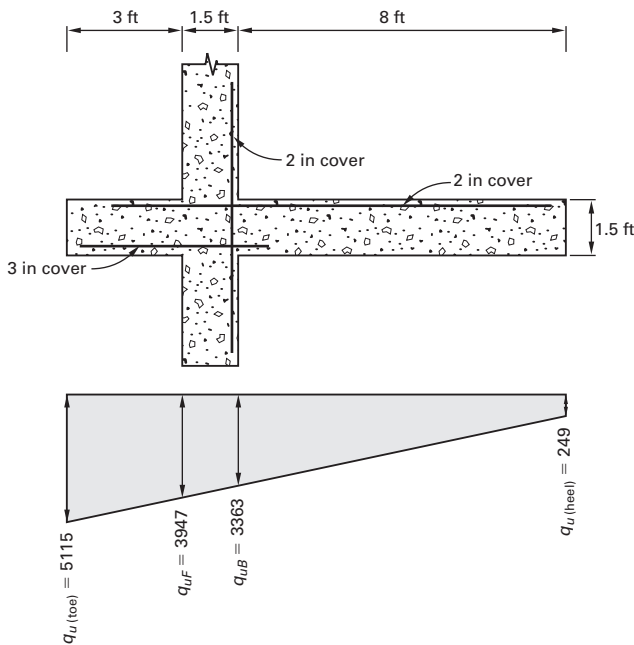
Assuming a tension-controlled section, the required reinforcement ratio is

$$
\rho = \frac{0.85f_c'\left(1 - \sqrt{1 - \dfrac{M_u}{0.383bd^2f_c'}}\right)}{f_y}
$$

$$
= \frac{(0.85)\left(3\ \dfrac{\text{kips}}{\text{in}^2}\right)}{60\ \dfrac{\text{kips}}{\text{in}^2}}
$$

$$
\times\left(1 - \sqrt{1 - \dfrac{(20{,}050\text{ ft-lbf})\left(12\ \dfrac{\text{in}}{\text{ft}}\right)}{(0.383)(12\text{ in})(14.5\text{ in})^2 \times \left(3000\ \dfrac{\text{lbf}}{\text{ft}^2}\right)}}\right)
$$

$$
= 0.0018
$$

The maximum allowable reinforcement ratio for a tension-controlled section is given by

$$
\rho_t = 0.319\beta_1\frac{f_c'}{f_y}
$$

$$
= 0.0136
$$

$$
> \rho \quad [\text{satisfactory, the section is tension-controlled}]
$$

$$
A_s = \rho bd = (0.0018)(12\text{ in})(14.5\text{ in}) = 0.314\text{ in}^2
$$

$$
A_{s(\text{min})} = 0.0018bh = (0.0018)(12\text{ in})(18\text{ in})
$$

$$
= 0.389\text{ in}^2 \quad [\text{governs}]
$$

Design of Heel

The maximum factored bending moment in the heel is

$$
M_u = \frac{L_H\left(1.2\,W_S + 1.6\,W_L + \dfrac{1.2\,W_B L_H}{L_B}\right)}{2}
$$

$$
- \frac{L_H^2 B\left(q_{uB} + 2q_{u(\text{heel})}\right)}{6}
$$

$$
= \frac{(8\text{ ft})\left(\begin{array}{c}21{,}120\text{ lbf} + 2816\text{ lbf} \\ + \dfrac{(3376\text{ lbf})(8\text{ ft})}{12.5\text{ ft}}\end{array}\right)}{2}
$$

$$
- \frac{(8\text{ ft})^2(1\text{ ft})\left(3363\ \dfrac{\text{lbf}}{\text{ft}^2} + (2)\left(249\ \dfrac{\text{lbf}}{\text{ft}^2}\right)\right)}{6}
$$

$$
= 63{,}203\text{ ft-lbf}
$$

Assuming a tension-controlled section, the required reinforcement ratio is

$$
\rho = \frac{0.85f_c'\left(1 - \sqrt{1 - \dfrac{M_u}{0.383bd^2f_c'}}\right)}{f_y}
$$

$$
= \frac{(0.85)\left(3\ \dfrac{\text{kips}}{\text{in}^2}\right)}{60\ \dfrac{\text{kips}}{\text{in}^2}}
$$

$$
\times\left(1 - \sqrt{1 - \dfrac{(63{,}203\text{ ft-lbf})\left(12\ \dfrac{\text{in}}{\text{ft}}\right)}{(0.383)(12\text{ in})(15.5\text{ in})^2 \times \left(3000\ \dfrac{\text{lbf}}{\text{in}^2}\right)}}\right)
$$

$$
= 0.0052
$$

$$
< \rho_t \quad \left[\begin{array}{c}\text{satisfactory, the section} \\ \text{is tension-controlled}\end{array}\right]
$$

$$
A_s = \rho bd = (0.0052)(12\text{ in})(15.5\text{ in})
$$

$$
= 0.97\text{ in}^2
$$

Design of Stem

The maximum factored bending moment in the stem is

$$M_u = 1.6\left(\frac{p_A L_W^3}{6} + \frac{p_A \overline{h} L_W^2}{2}\right)$$

$$= (1.6)\left(\frac{\left(30 \ \frac{\text{lbf}}{\text{ft}^2}\right)(20 \ \text{ft})^3}{6} \right.$$

$$\left. + \frac{\left(30 \ \frac{\text{lbf}}{\text{ft}^2}\right)(2 \ \text{ft})(20 \ \text{ft})^2}{2}\right)$$

$$= 83{,}200 \ \text{ft-lbf}$$

The required reinforcement ratio is

$$\rho = 0.0070 < \rho_t \quad [\text{satisfactory}]$$

$$A_s = \rho bd = (0.0075)(12 \ \text{in})(15.5 \ \text{in}) = 1.30 \ \text{in}^2$$

7. COUNTERFORT RETAINING WALL

Nomenclature

a	lever-arm of resisting couple in counterfort	ft
b_c	width of counterfort	in
C	compression force in resisting couple	kips
l_c	clear height of counterfort	ft
l_n	clear span between counterforts	ft
q	earth pressure at a depth of $0.6l_c = 0.6p_A l_c$	lbf/ft
s_c	spacing, center to center, of counterforts, $l_n + b_c$	ft
T	tension force in resisting couple	kips

Design of Stem and Base

The stem spans horizontally between counterforts and cantilevers from the base. For ratios of l_n/l_c between 0.5 and 1.0, the stem may be designed for a value of the earth pressure at a depth of $0.6l_c$. The horizontal span moments are given by ACI Sec. 8.3.3 as $ql_n^2/11$ at counterfort supports and $ql_n^2/16$ between counterforts. The cantilever moment (tension on earth face) at the base is $0.035p_A l_c^3$. The bending moment producing tension on the air face is $0.009p_A l_c^3$. The distribution of bending moment in the stem is shown in Fig. 2.12, and more precise values of moment may be obtained from tabulated coefficients.[1,2]

The base slab is similarly designed for the net factored pressure as a slab spanning longitudinally between counterforts.

Example 2.18

For the counterfort retaining wall shown in the following illustration, determine the design moments in the stem. The fill behind the wall has an equivalent fluid pressure of 40 lbf/ft^2 per foot.
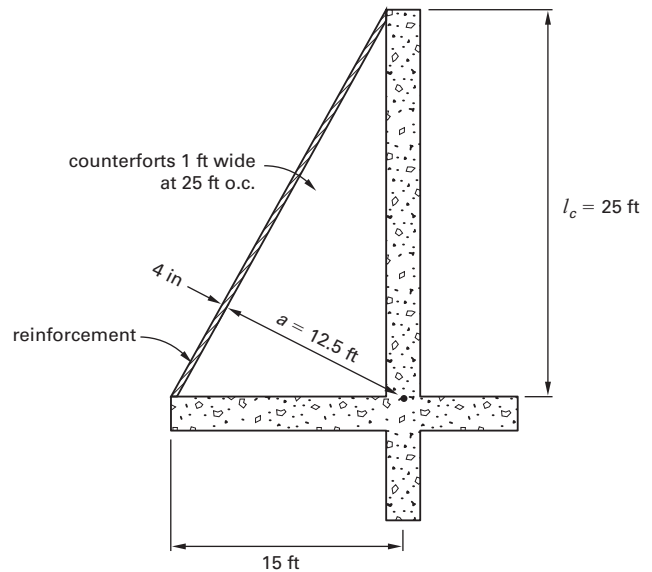
Solution

The lateral earth pressure acting on a 1 ft horizontal strip at a depth of 0.6 times the stem height is

$$q = 0.6p_A l_c = (0.6)\left(40 \ \frac{\text{lbf}}{\text{ft}^2}\right)(25 \ \text{ft})$$

$$= 600 \ \text{lbf/ft}$$

At the counterfort supports, the factored design moment is given by ACI Sec. 8.3.3 and Sec. 9.2.1 as

$$M_u = \frac{1.6ql_n^2}{11} = \frac{(1.6)\left(0.6 \ \frac{\text{kip}}{\text{ft}}\right)(24 \ \text{ft})^2}{11}$$

$$= 50.27 \ \text{ft-kips}$$

Between counterforts, the factored design moment is

$$M_u = \frac{1.6ql_n^2}{16} = \frac{(1.6)\left(0.6 \ \frac{\text{kip}}{\text{ft}}\right)(24 \ \text{ft})^2}{16}$$

$$= 34.56 \ \text{ft-kips}$$

The factored design cantilever moment at the base is

$$M_u = 1.6(0.035p_A l_c^3)$$

$$= (1.6)(0.035)\left(0.04 \ \frac{\text{kip}}{\text{ft}^2}\right)(25 \ \text{ft})^3$$

$$= 35.00 \ \text{ft-kips}$$

Design of Counterforts

The bending moment produced by the earth pressure at the base of the stem is resisted by the couple produced by the tension in the reinforcement at the rear of the counterfort and the compression in the stem concrete. As shown in Fig. 2.12, the lever-arm of the couple acts at right angles to the reinforcement. The thrust produced by the earth pressure acting on the rear face of the stem is resisted by the horizontal ties in the counterfort.

Figure 2.12 *Details of Counterfort Retaining Wall*

side elevation for B–B

rear elevation

moment on B–B

moment on A–A
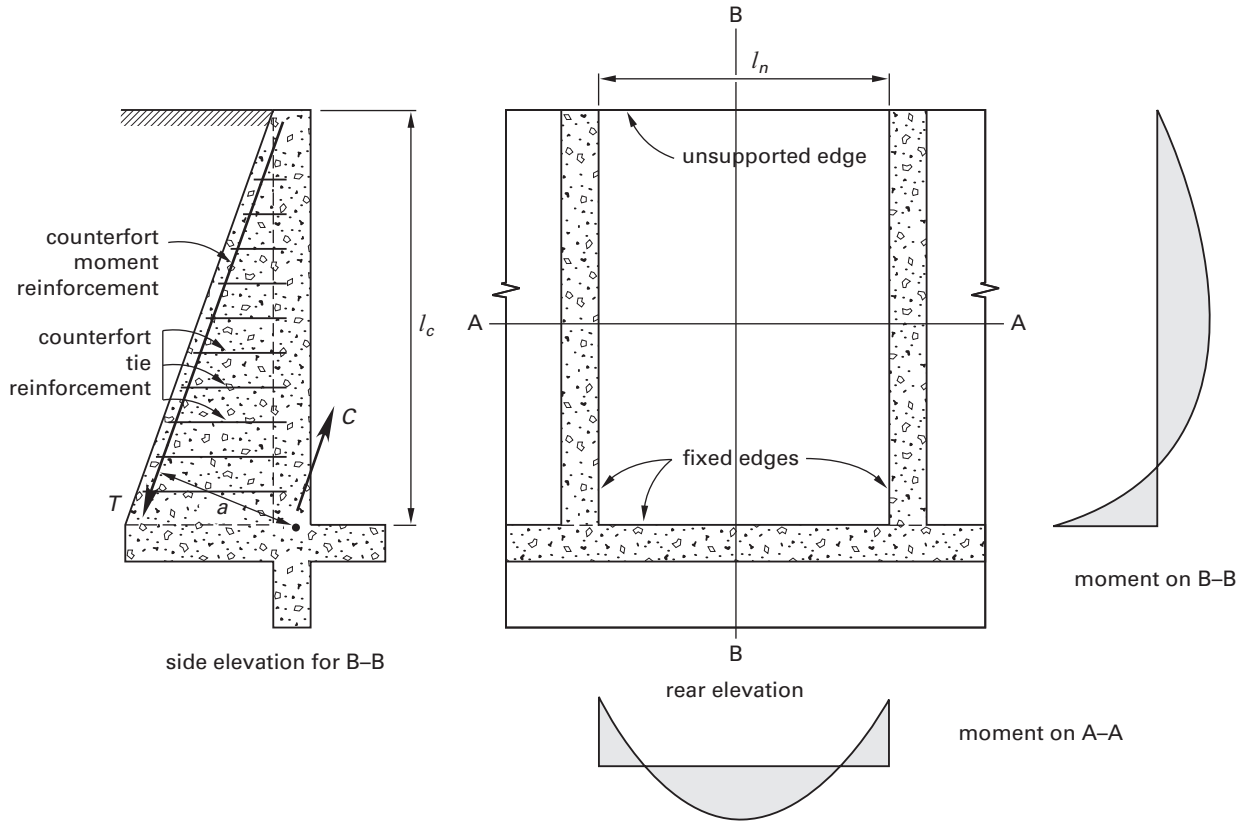
Example 2.19

For the counterfort retaining wall of Ex. 2.18, determine the reinforcement area required in the rear of the counterfort and the tie area required at the base of the counterfort. Grade 60 reinforcement is provided.

Solution

The factored moment produced by the earth pressure at the base of the stem over one bay is

$$M_u = \frac{1.6 p_A s_c l_c^3}{6} = \frac{(1.6)\left(40 \ \dfrac{\frac{\text{lbf}}{\text{ft}^2}}{\text{ft}}\right)(25 \ \text{ft})(25 \ \text{ft})^3}{(6)\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}$$

$$= 4167 \ \text{ft-kips}$$

The reinforcement area required in the rear of the counterfort to resist this moment is

$$A_s = \frac{M_u}{\phi a f_y}$$

$$= \frac{4167 \ \text{ft-kips}}{(0.9)(12.5 \ \text{ft})\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)}$$

$$= 6.17 \ \text{in}^2$$

The factored lateral pressure from the backfill on a 1 ft horizontal strip at the base of the stem over one bay is

$$Q_u = 1.6 p_A l_c l_n$$

$$= \frac{(1.6)\left(40 \ \dfrac{\text{lbf}}{\text{ft}^2}\right)(25 \ \text{ft})(24 \ \text{ft})}{1000 \ \dfrac{\text{lbf}}{\text{kip}}}$$

$$= 38.40 \ \text{kips}$$

The tie reinforcement area required at the base of the counterfort to resist this lateral pressure is

$$A_s = \frac{Q_u}{\phi f_y} = \frac{38.40 \ \text{kips}}{(0.9)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)} = 0.72 \ \text{in}^2$$

REFERENCES

1. Portland Cement Association. *Rectangular Concrete Tanks*. Skokie, IL: Portland Cement Association, 1998.

2. Reynolds, Charles E., James C. Steedman, and Anthony J. Threlfall. *Reinforced Concrete Designer's Handbook*. Wexham Springs, UK: Cement and Concrete Association, 1981.

PRACTICE PROBLEMS

1. What is the factored net pressure on the footing shown in the following illustration?
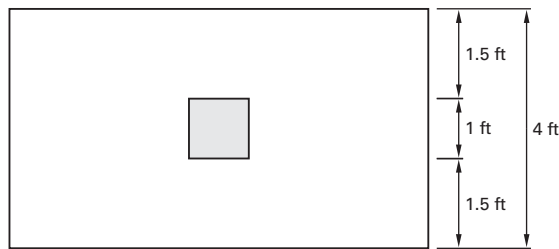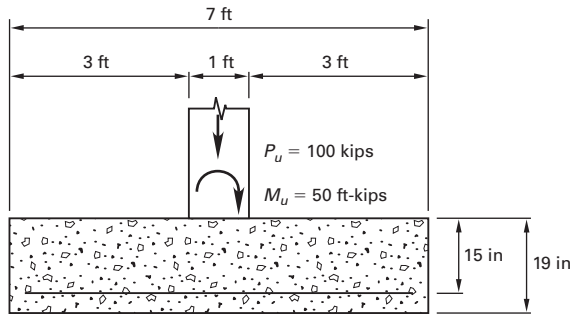
(A) 2.55 kips/ft^2 max., 1.02 kips/ft^2 min.

(B) 5.10 kips/ft^2 max., 2.04 kips/ft^2 min.

(C) 6.25 kips/ft^2 max., 0.89 kips/ft^2 min.

(D) 9.67 kips/ft^2 max., -2.55 kips/ft^2 min.

2. The rectangular footing of normal weight concrete of Prob. 1 has a concrete strength of 4000 lbf/in^2. Is the punching shear capacity adequate?

(A) Yes, 228 lbf/in$^2 > v_u$.

(B) Yes, 190 lbf/in$^2 > v_u$.

(C) No, 53 lbf/in$^2 < v_u$.

(D) No, 60 lbf/in$^2 > v_u$.

3. The flexural-shear capacity for the rectangular footing of Prob. 1 is most nearly

(A) 68 kips

(B) 73 kips

(C) 82 kips

(D) 91 kips

4. The area of grade 60 reinforcement required in the direction of the applied moment for the rectangular footing of Prob. 1 is most nearly

(A) 1.50 in^2

(B) 1.55 in^2

(C) 1.65 in^2

(D) 1.70 in^2

5. For the retaining wall of Ex. 2.17, what is most nearly the minimum area of horizontal reinforcement required in the stem? Use no. 3 grade 60 bars.

(A) 0.3 in^2/ft

(B) 0.4 in^2/ft

(C) 0.7 in^2/ft

(D) 1.0 in^2/ft

Foundations

<div style="float:left">**Foundations**</div>

SOLUTIONS

1. The equivalent eccentricity is

$$e = \frac{M_u}{P_u} = \frac{50 \text{ ft-kips}}{100 \text{ kips}} = 0.5 \text{ ft}$$

$$< \frac{L}{6} \quad \text{[within middle third]}$$

The net factored pressure on the footing is

$$q_u = \frac{P_u\left(1 \pm \dfrac{6e}{L}\right)}{BL}$$

$$= \frac{(100 \text{ kips})\left(1 \pm \dfrac{(6)(0.5 \text{ ft})}{7 \text{ ft}}\right)}{(4 \text{ ft})(7 \text{ ft})}$$

$$q_{u(\text{max})} = 5.10 \text{ kips/ft}^2$$

$$q_{u(\text{min})} = 2.04 \text{ kips/ft}^2$$

The answer is (B).

2. The length of the critical perimeter is

$$b_o = 4(c + d) = \frac{(4)(12 \text{ in} + 15 \text{ in})}{12 \dfrac{\text{in}}{\text{ft}}}$$

$$= 9 \text{ ft}$$

Shear caused by the axial load at the critical perimeter is

$$V_u = P_u - 0.5\left(q_{u(\text{max})} + q_{u(\text{min})}\right)\left(\frac{b_o}{4}\right)^2$$

$$= 100 \text{ kips} - (0.5)\left(5.10 \frac{\text{kips}}{\text{ft}^2} + 2.04 \frac{\text{kips}}{\text{ft}^2}\right)$$

$$\times (2.25 \text{ ft})^2$$

$$= 82 \text{ kips}$$

The polar moment of inertia of the critical perimeter is

$$\frac{J_c}{y} = \frac{b_1 d(b_1 + 3b_2) + d^3}{3} \quad \begin{bmatrix} \text{for a footing with a} \\ \text{central column} \end{bmatrix}$$

$$= \frac{(27 \text{ in})(15 \text{ in})\big(27 \text{ in} + (3)(27 \text{ in})\big) + (15 \text{ in})^3}{3}$$

$$= 15{,}705 \text{ in}^3$$

The fraction of the column moment transferred by shear is

$$\gamma_v = 1 - \frac{1}{1 + 0.67\sqrt{\dfrac{b_1}{b_2}}}$$

$$= 1 - \frac{1}{1.67}$$

$$= 0.40$$

The combined shear stress due to the applied axial load and the column moment is

$$v_u = \frac{V_u}{db_o} + \frac{\gamma_v M_u y}{J_c}$$

$$= \frac{(82 \text{ kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{(15 \text{ in})(108 \text{ in})}$$

$$+ \frac{(0.4)(600 \text{ in-kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{15{,}705 \text{ in}^3}$$

$$= 66 \text{ lbf/in}^2$$

The ratio of the long side to the short side of the column is

$$\beta_c = \frac{c_2}{c_1}$$

$$= \frac{12 \text{ in}}{12 \text{ in}}$$

$$= 1.00$$

$$< 2$$

The allowable shear stress for two-way action is given by ACI Eq. 11-33 as

$$\phi v_c = 4\phi\lambda\sqrt{f'_c}$$

$$\phi = \text{strength reduction factor}$$

$$= 0.75 \text{ from ACI Sec. 9.3}$$

$$\phi v_c = (4)(0.75)(1.0)\sqrt{4000 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 190 \text{ lbf/in}^2$$

$$> v_u \quad \text{[satisfactory]}$$

The answer is (B).

3. The distance of the critical section for flexural shear from the edge of the footing is

$$x = \frac{L}{2} - \frac{c}{2} - d = \frac{7 \text{ ft}}{2} - \frac{1 \text{ ft}}{2} - 1.25 \text{ ft}$$

$$= 1.75 \text{ ft}$$

The net factored pressure on the footing at this section is

$$q_{ux} = q_{u(\max)} - \frac{x\left(q_{u(\max)} - q_{u(\min)}\right)}{L}$$

$$= 5.10 \ \frac{\text{kips}}{\text{ft}^2} - \frac{(1.75 \ \text{ft})\left(5.10 \ \frac{\text{kips}}{\text{ft}^2} - 2.04 \ \frac{\text{kips}}{\text{ft}^2}\right)}{7 \ \text{ft}}$$

$$= 4.34 \ \text{kips/ft}^2$$

The factored shear force at the critical section is

$$V_u = \frac{Bx\left(q_{u(\max)} + q_{ux}\right)}{2}$$

$$= \frac{(4 \ \text{ft})(1.75 \ \text{ft})\left(5.10 \ \frac{\text{kips}}{\text{ft}^2} + 4.34 \ \frac{\text{kips}}{\text{ft}^2}\right)}{2}$$

$$= 33.04 \ \text{kips}$$

The flexural-shear capacity of the footing is given by ACI Eq. 11-3 as

$$\phi V_c = 2\phi B d\lambda\sqrt{f_c'}$$

$$= \frac{(2)(0.75)(48 \ \text{in})(15 \ \text{in})(1.0)\sqrt{4000 \ \frac{\text{lbf}}{\text{in}^2}}}{1000 \ \frac{\text{lbf}}{\text{kip}}}$$

$$= 68 \ \text{kips}$$

$$> V_u \quad [\text{satisfactory}]$$

The answer is (A).

4. The net factored pressure on the footing at the face of the column is

$$q_{uc} = q_{u(\max)} - \left(\frac{L}{2} - \frac{c}{2}\right)\left(\frac{q_{u(\max)} - q_{u(\min)}}{L}\right)$$

$$= 5.10 \ \frac{\text{kips}}{\text{ft}^2} - (3.5 \ \text{ft} - 0.5 \ \text{ft})$$

$$\times \left(\frac{5.10 \ \frac{\text{kips}}{\text{ft}^2} - 2.04 \ \frac{\text{kips}}{\text{ft}^2}}{7 \ \text{ft}}\right)$$

$$= 3.79 \ \text{kips/ft}^2$$

The factored moment at the face of the column is

$$M_u = \frac{B\left(\dfrac{L}{2} - \dfrac{c}{2}\right)^2\left(2q_{u(\max)} + q_{uc}\right)}{6}$$

$$= \frac{(4 \ \text{ft})(3 \ \text{ft})^2\left((2)\left(5.10 \ \frac{\text{kips}}{\text{ft}^2}\right) + 3.79 \ \frac{\text{kips}}{\text{ft}^2}\right)}{6}$$

$$= 83.94 \ \text{ft-kips}$$

Assuming a tension-controlled section, the required reinforcement ratio is

$$\rho = \frac{0.85 f_c'\left(1 - \sqrt{1 - \dfrac{M_u}{0.383 b d^2 f_c'}}\right)}{f_y}$$

$$= \frac{(0.85)\left(4 \ \frac{\text{kips}}{\text{in}^2}\right)}{60 \ \frac{\text{kips}}{\text{in}^2}}$$

$$\times \left(1 - \sqrt{1 - \frac{(83.94 \ \text{ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right) \times \left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(0.383)(48 \ \text{in})(15 \ \text{in})^2 \times \left(4000 \ \frac{\text{lbf}}{\text{in}^2}\right)}}\right)$$

$$= 0.0018$$

$$A_s = \rho B d$$

$$= (0.0018)(48 \ \text{in})(15 \ \text{in})$$

$$= 1.30 \ \text{in}^2$$

The minimum reinforcement area is given by ACI Sec. 7.12.2 as

$$A_{s(\min)} = 0.0018 B h$$

$$= (0.0018)(48 \ \text{in})(19 \ \text{in})$$

$$= 1.64 \ \text{in}^2 \quad (1.65 \ \text{in}^2) \quad [\text{governs}]$$

The answer is (C).

5. From ACI Sec. 14.3.3, the required ratio of horizontal reinforcement in the stem is

$$\rho_{\text{hor}} = 0.0020$$

$$A_{sh} = \rho_{\text{hor}} b h$$

$$= (0.0020)(12 \ \text{in})(18 \ \text{in})$$

$$= 0.432 \ \text{in}^2/\text{ft} \quad (0.4 \ \text{in}^2/\text{ft})$$

The answer is (B).

Foundations

3 Prestressed Concrete Design

1. DESIGN STAGES

Nomenclature

a	depth of equivalent rectangular stress block	in
A_{ct}	area of concrete section between the centroid and extreme tension fiber	in^2
A_g	area of concrete section	in^2
A_{ps}	area of prestressed reinforcement in tension zone	in^2
A_s	area of nonprestressed tension reinforcement	in^2
A'_s	area of compression reinforcement	in^2
b	width of compression face of member	in
c	distance from extreme compression fiber to neutral axis	in
C_u	total compression force in equivalent rectangular stress block	lbf
d	distance from extreme compression fiber to centroid of nonprestressed reinforcement	in
d'	distance from extreme compression fiber to centroid of compression reinforcement	in
d_p	distance from extreme compression fiber to centroid of prestressed reinforcement as defined in Fig. 3.8	in
e	eccentricity of prestressing force	in
E_c	modulus of elasticity of concrete	$kips/in^2$
E_p	modulus of elasticity of prestressing tendon	$kips/in^2$
f_{be}	bottom fiber stress at service load after allowance for all prestress losses	lbf/in^2
f_{bi}	bottom fiber stress immediately after prestress transfer and before time-dependent prestress losses	lbf/in^2
f'_c	specified compressive strength of concrete	lbf/in^2
f'_{ci}	compressive strength of concrete at time of prestress transfer	lbf/in^2
f_{ps}	stress in prestressed reinforcement at nominal strength	$kips/in^2$
f_{pu}	specified tensile strength of prestressing tendons	$kips/in^2$
f_{py}	specified yield strength of prestressing tendons	$kips/in^2$
f_r	modulus of rupture of concrete	lbf/in^2
f_s	permissible stress in prestressed reinforcement at the jacking end	$kips/in^2$
f_{se}	effective stress in prestressed reinforcement after allowance for all prestress losses	$kips/in^2$
f_{si}	stress in prestressed reinforcement immediately after prestress transfer	$kips/in^2$
f_{te}	top fiber stress at service loads after allowance for all prestress losses	lbf/in^2
f_{ti}	top fiber stress immediately after prestress transfer and before time-dependent prestress losses	lbf/in^2
f_y	specified yield strength of nonprestressed reinforcement	$kips/in^2$
h	height of section	in
I_g	moment of inertia of gross concrete section	in^4
l	span length	ft
M_{cr}	cracking moment strength	ft-kips
M_D	bending moment due to superimposed dead load	ft-kips
M_G	bending moment due to self-weight of member	ft-kips
M_L	bending moment due to superimposed live load	ft-kips
M_n	nominal flexural strength	ft-kips
M_S	bending moment due to sustained load	ft-kips
M_T	bending moment due to total load	ft-kips
M_u	factored moment	ft-kips
P_e	prestressing force after all losses	kips
P_i	initial prestressing force	kips
r	distance of tendon from the neutral axis	in
S_b	section modulus of the concrete section referred to the bottom fiber	in^3
S_t	section modulus of the concrete section referred to the top fiber	in^3
w_c	unit weight of concrete	lbf/ft^3
$\overline{y}$	height of centroid of the concrete section	in

Symbols

β_1	compression zone factor	—
γ_p	factor for type of prestressing tendon	—
ϵ_c	strain at extreme compression fiber at nominal strength, 0.003	—

ϵ_p prestrain in prestressed reinforcement   –
       due to the final prestress

ϵ_s strain produced in prestressed   –
       reinforcement by the ultimate loading

ϵ_t strain at extreme tension steel at   –
       nominal strength

λ correction factor related to unit weight   –
       of concrete as given by ACI Sec. 8.6.1

ϕ strength reduction factor   –

General Requirements

According to ACI Sec. R18.2, there are three major design stages that must be investigated when designing prestressed concrete members.

- *Transfer design stage*

    At the transfer design stage, a prestressing force is transferred from a hydraulic jack to the concrete member. Immediate prestress losses occur due to elastic deformation of the concrete and, in the case of post-tensioned concrete, due to anchor set and friction losses. At this stage, the prestressing force is at its maximum, the concrete compressive strength and the dead load are at their minimums, and the live load is zero. The member must be designed so that stresses produced by the applied forces do not exceed the allowable values. Since the transfer stresses are temporary, the allowable values are higher than they are for the serviceability stage. The applied forces consist of the prestressing force and the self-weight of the member.

- *Serviceability design stage*

    At the serviceability design stage, all time-dependent prestress losses have occurred due to the concrete's creep and shrinkage and the relaxation of the tendon stress. At this stage, the prestressing force is at its minimum, and the concrete compressive strength, dead load, and live load are at their maximums. The member must be designed so that stresses produced by the applied forces do not exceed the allowable values.

- *Strength design stage*

    In the strength design stage, the design strength of the member is calculated. It must not be less than the required strength of the member, which is calculated using the most critical combination of the factored loads. At this stage, a rectangular stress block is assumed with a maximum strain in the concrete of 0.003.

Transfer Design Stage

The permissible stresses at transfer are specified in ACI Sec. 18.4.1. As shown in Fig. 3.1, the initial prestressing force mobilizes the self-weight of the member producing the stresses. The permissible stresses are shown in Fig. 3.2.

$$f_{ti} = P_i\left(\frac{1}{A_g} - \frac{e}{S_t}\right) + \frac{M_G}{S_t} = P_i R_t + \frac{M_G}{S_t}$$

$$\geq -6\sqrt{f'_{ci}} \quad \begin{bmatrix} \text{at ends of simply supported beams} \\ \text{without auxiliary reinforcement} \end{bmatrix}$$

$$\geq -3\sqrt{f'_{ci}} \quad \begin{bmatrix} \text{at all other locations without} \\ \text{auxiliary reinforcement} \end{bmatrix}$$

P_i = force in prestressing tendon immediately after prestress transfer

$$= A_{ps}f_{si}$$

$$R_t = \frac{1}{A_g} - \frac{e}{S_t}$$

$$f_{bi} = P_i\left(\frac{1}{A_g} + \frac{e}{S_b}\right) - \frac{M_G}{S_b} = P_i R_b - \frac{M_G}{S_b}$$

$$\leq 0.70f'_{ci} \quad \text{[at ends of simply supported beams]}$$

$$\leq 0.60f'_{ci} \quad \text{[at all other locations]}$$

$$R_b = \frac{1}{A_g} + \frac{e}{S_b}$$

Figure 3.1 *Transfer Design Stage*
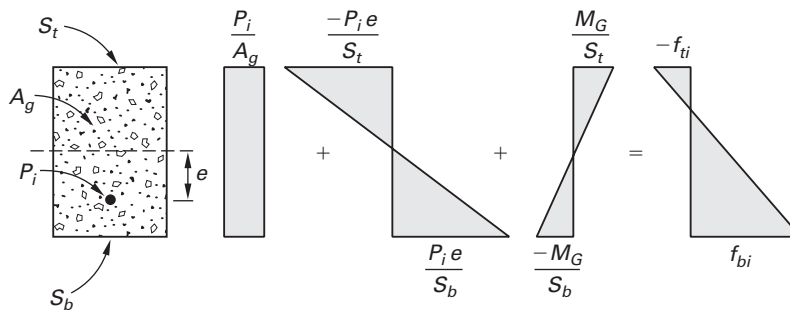
Figure 3.2 *Specified Concrete Stress at Transfer*

$-6=\overline{f_{ci}'}$...at member ends without auxiliary reinforcement
$-3=\overline{f_{ci}'}$...at other locations without auxiliary reinforcement

tension   compression

0.70f_{ci}'...at member ends
0.60f_{ci}'...at other locations
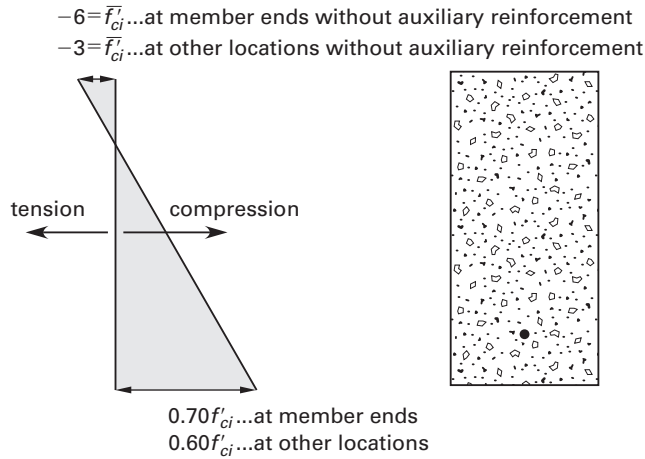
In accordance with ACI Sec. 18.5.1, the permissible stress should not exceed the following. At post-tensioning anchorages and couplers, the permissible stress is

$$f_{si} = 0.70\,f_{pu}$$

The maximum permissible stress due to the tendon jacking force is

$$f_s = 0.94\,f_{py}$$
$$\leq 0.80\,f_{pu}$$

The permissible tendon stresses are shown in Fig. 3.3.

Example 3.1

The pretensioned beam of normal weight concrete shown is simply supported over a span of 20 ft and has a concrete strength at transfer of 4500 lbf/in². Determine the magnitude and location of the initial prestressing force required to produce satisfactory stresses at midspan, immediately after transfer, without using auxiliary reinforcement.

Figure 3.3 *Specified Stress in Prestressing Tendons*

pretensioned member

stress

0.94f_{py}
< 0.80f_{pu}

distance along tendon

jacking end

post-tensioned member

stress

before seating loss

0.94f_{py}
< 0.80f_{pu}

0.70f_{pu}

after seating loss

distance along tendon

jacking end
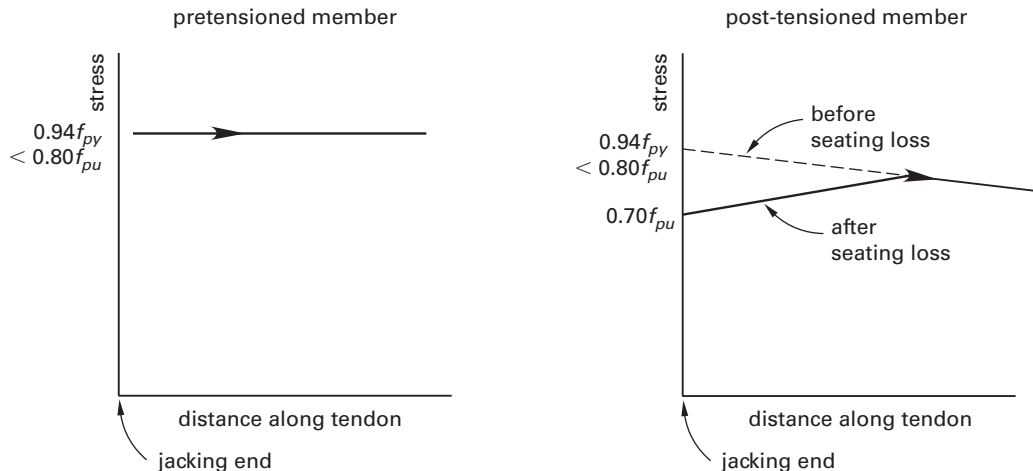
Solution

The properties of the concrete section are

$$A_g = 72 \text{ in}^2$$
$$I_g = 863 \text{ in}^4$$
$$\overline{y} = 4.67 \text{ in}$$
$$S_t = 118 \text{ in}^3$$
$$S_b = 185 \text{ in}^3$$

At midspan, the self-weight moment is

$$M_G = \frac{w_c A_g l^2}{8}$$
$$= \frac{\left(150\ \frac{\text{lbf}}{\text{ft}^3}\right)(72\text{ in}^2)(20\text{ ft})^2}{(8)\left(12\ \frac{\text{in}}{\text{ft}}\right)}$$
$$= 45{,}000 \text{ in-lbf}$$

At midspan, the permissible tensile stress in the top fiber without auxiliary reinforcement is given by ACI Sec. 18.4.1 as

$$f_{ti} = -3\sqrt{f_{ci}'} = -3\sqrt{4500\ \frac{\text{lbf}}{\text{in}^2}}$$
$$= -201 \text{ lbf/in}^2$$
$$= \frac{P_i}{A_g} - \frac{P_i e}{S_t} + \frac{M_G}{S_t}$$
$$= \frac{P_i}{72\text{ in}^2} - \frac{P_i e}{118\text{ in}^3} + \frac{45{,}000\text{ in-lbf}}{118\text{ in}^3}$$
$$-582\ \frac{\text{lbf}}{\text{in}^2} = \frac{P_i}{72\text{ in}^2} - \frac{P_i e}{118\text{ in}^3} \quad [\text{Eq. 1}]$$

Prestressed Concrete

At midspan, the permissible compressive stress in the bottom fiber is given by ACI Sec. 18.4.1 as

$$f_{bi} = 0.6f'_{ci} = 2700 \text{ lbf/in}^2$$

$$= \frac{P_i}{A_g} + \frac{P_i e}{S_b} - \frac{M_G}{S_b}$$

$$= \frac{P_i}{72 \text{ in}^2} + \frac{P_i e}{185 \text{ in}^3} - \frac{45{,}000 \text{ in-lbf}}{185 \text{ in}^3}$$

$$2943 \, \frac{\text{lbf}}{\text{in}^2} = \frac{P_i}{72 \text{ in}^2} + \frac{P_i e}{185 \text{ in}^3} \quad [\text{Eq. 2}]$$

Solving Eq. [1] and Eq. [2] gives

$$P_i = 113{,}056 \text{ lbf}$$

$$e = 2.25 \text{ in}$$

Auxiliary Reinforcement

ACI Sec. 18.4.1 specifies that when the computed tensile stress exceeds the permissible stress, bonded auxiliary reinforcement shall be provided to resist the total tensile force in the concrete. The tensile force is computed by using the properties of the uncracked concrete section, and in accordance with ACI Sec. R18.4.1, the permissible stress in the auxiliary reinforcement is $0.6f_y$ or 30 kips/in^2 maximum.

From Fig. 3.4, the depth to the location of zero stress is given by

$$c = \frac{h f_t}{f_t + f_b}$$

The tensile force in the concrete is

$$T = \frac{c f_t b}{2}$$

Illustration for Ex. 3.1
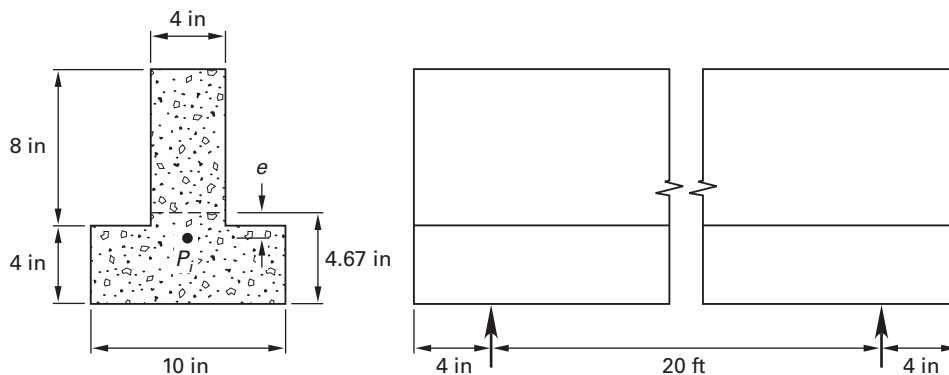
The area of auxiliary reinforcement required is given by ACI Sec. R18.4.1 as

$$A_s = \frac{T}{0.6f_y} \geq \frac{T}{30 \, \dfrac{\text{kips}}{\text{in}^2}}$$

Example 3.2

The pretensioned beam of Ex. 3.1 is prestressed with tendons providing an initial prestressing force of 110,100 lbf at an eccentricity of 2.37 in at midspan. Determine the area of grade 60 auxiliary reinforcement required.

Solution

$$R_b = \frac{1}{A_g} + \frac{e}{S_b}$$

$$= \frac{1}{72 \text{ in}^2} + \frac{2.37 \text{ in}}{185 \text{ in}^3}$$

$$= 0.0267 \, 1/\text{in}^2$$

$$R_t = \frac{1}{A_g} - \frac{e}{S_t}$$

$$= \frac{1}{72 \text{ in}^2} - \frac{2.37 \text{ in}}{118 \text{ in}^3}$$

$$= -0.0062 \, 1/\text{in}^2$$

The top and bottom fiber stresses are

$$f_t = P_i R_t + \frac{M_G}{S_t}$$

$$= (110{,}100 \text{ lbf})\left(-0.0062 \, \frac{1}{\text{in}^2}\right)$$

$$+ \frac{45{,}000 \text{ in-lbf}}{118 \text{ in}^3}$$

$$= -301 \text{ lbf/in}^2$$

This is less than the minimum permissible value of $-3\sqrt{f_c'} = -201$ lbf/in² determined in Ex. 3.1, and auxiliary reinforcement is required.

$$f_b = P_i R_b - \frac{M_G}{S_b}$$

$$= (110{,}100 \text{ lbf})\left(0.0267 \ \frac{1}{\text{in}^2}\right) - \frac{45{,}000 \text{ in-lbf}}{185 \text{ in}^3}$$

$$= 2696 \text{ lbf/in}^2$$

This is less than the maximum permissible value of $0.6f_{ci}' = 2700$ lbf/in² determined in Ex. 3.1 and is satisfactory.

Depth to the neutral axis is obtained from Fig. 3.4 as

$$c = \frac{h f_t}{f_t + f_b}$$

$$= \frac{(12 \text{ in})\left(301 \ \frac{\text{lbf}}{\text{in}^2}\right)}{301 \ \frac{\text{lbf}}{\text{in}^2} + 2696 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 1.21 \text{ in}$$

Figure 3.4 *Determination of Tensile Force*

The tensile force in the concrete is

$$T = \frac{c f_t b}{2}$$

$$= \frac{(1.21 \text{ in})\left(301 \ \frac{\text{lbf}}{\text{in}^2}\right)(4 \text{ in})}{2}$$

$$= 728 \text{ lbf}$$

The area of auxiliary reinforcement required is obtained from ACI Sec. R18.4.1 as

$$A_s = \frac{T}{30{,}000 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.024 \text{ in}^2$$

Serviceability Design Stage

The permissible stresses under service loads after all prestressing losses have occurred are specified in ACI Sec. 18.3.3 and Sec. 18.4.2. The stress conditions are shown in Fig. 3.5, and the stresses are given by

$$f_{te} = P_e R_t + \frac{M_G + M_D + M_L}{S_t}$$

$$\leq 0.45 f_c' \quad \text{[for sustained loads]}$$

$$\leq 0.60 f_c' \quad \text{[for total loads]}$$

$$f_{be} = P_e R_b - \frac{M_G + M_D + M_L}{S_b}$$

$$= -7.5\sqrt{f_c'} \quad \text{[for class U member]}$$

$$= -12\sqrt{f_c'} \quad \text{[for class T member]}$$

$$< -12\sqrt{f_c'} \quad \text{[for class C member]}$$

Class U members are assumed to behave as uncracked members. Class C members are assumed to behave as cracked members. Class T members are assumed to be in transition between uncracked and cracked.

Figure 3.5 *Serviceability Design Stage After all Losses*

In accordance with ACI Sec. R18.3.4, stresses at the serviceability design stage in class U and class T members may be computed using uncracked section properties, and no crack control measures are necessary. Stresses in class C members are computed using cracked section properties, and crack control measures are necessary as specified in ACI Sec. 10.6.4, Sec. 10.6.7, and Sec. 18.4.4.1. Deflections for class U members are based on uncracked section properties, and for class T and class C members are based on the cracked transformed section properties, as specified in ACI Sec. 9.5.4.2.

$$P_e = \text{force in prestressing tendon at service}$$
$$\text{loads after allowance for all losses}$$
$$= A_{ps} f_{se}$$

The permissible stresses are shown in Fig. 3.6.

Figure 3.6 *Permissible Concrete Stress at Service Load*

0.45f_c'...for sustained loads

0.60f_c'...for total loads

tension　　　compression

$-7.5\sqrt{f_c'}$...for class U

$-12\sqrt{f_c'}$...for class T

$<-12\sqrt{f_c'}$...for class C
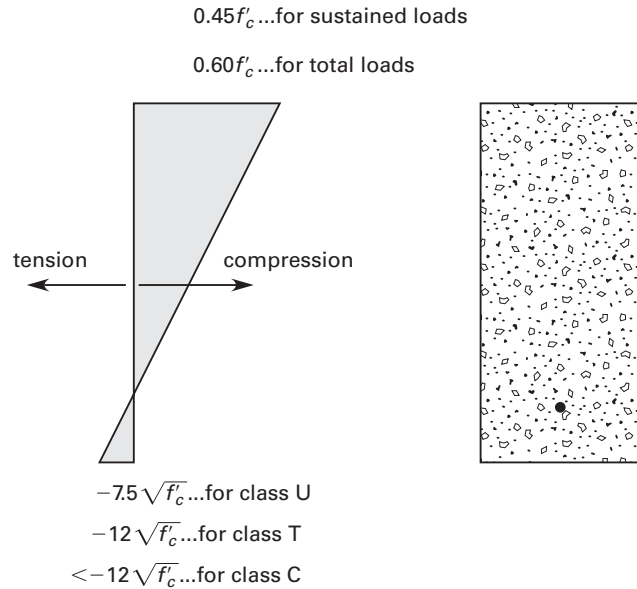
Example 3.3

The class U pretensioned beam of Ex. 3.1 has a long-term loss in prestress of 25% and a 28-day compressive strength of 6000 lbf/in². The initial prestressing force is $P_i = 112{,}850$ lbf with an eccentricity of $e = 2.25$ in. Determine the maximum bending moment the beam can carry if the sustained load is 75% of the total superimposed load.

Solution

The relevant parameters are $e = 2.25$ in, $R_b = 0.0261$ in^{-2}, $P_i = 112{,}850$ lbf, $P_i R_b = 2940$ lbf/in², $P_i R_t = -584$ lbf/in², and $R_t = -0.00518$ in^{-2}.

The permissible tensile stress at midspan, in the bottom fiber, due to the total load is given by ACI Sec. 18.3.3 as

$$f_{be} = -7.5\sqrt{f_c'}$$
$$= -7.5\sqrt{6000\ \frac{\text{lbf}}{\text{in}^2}}$$
$$= -581\ \text{lbf/in}^2$$
$$= P_e R_b - \frac{M_G}{S_b} - \frac{M_T}{S_b}$$
$$= (0.75)\left(2940\ \frac{\text{lbf}}{\text{in}^2}\right) - 243\ \frac{\text{lbf}}{\text{in}^2} - \frac{M_T}{185\ \text{in}^3}$$
$$M_T = 470{,}455\ \text{in-lbf}$$

The permissible compressive stress at midspan, in the top fiber, due to the sustained load is given by ACI Sec. 18.4.2 as

$$f_{te} = 0.45 f_c' = (0.45)\left(6000\ \frac{\text{lbf}}{\text{in}^2}\right)$$
$$= 2700\ \text{lbf/in}^2$$
$$= P_e R_t + \frac{M_G}{S_t} + \frac{M_S}{S_t}$$
$$= (0.75)\left(-584\ \frac{\text{lbf}}{\text{in}^2}\right) + 381\ \frac{\text{lbf}}{\text{in}^2} + \frac{0.75 M_T}{118\ \text{in}^3}$$
$$M_T = 433{,}770\ \text{in-lbf}$$

The permissible compressive stress at midspan, in the top fiber, due to the total load is given by ACI Sec. 18.4.2 as

$$f_{te} = 0.60 f_c' = (0.60)\left(6000\ \frac{\text{lbf}}{\text{in}^2}\right)$$
$$= 3600\ \text{lbf/in}$$
$$= P_e R_t + \frac{M_G}{S_t} + \frac{M_T}{S_t}$$
$$= (0.75)\left(-584\ \frac{\text{lbf}}{\text{in}^2}\right) + 381\ \frac{\text{lbf}}{\text{in}^2} + \frac{M_T}{118\ \text{in}^3}$$
$$M_T = 431{,}530\ \text{in-lbf} \quad \text{[governs]}$$

Cracking Moment

The cracking moment is the moment that, when applied to the member after all losses have occurred, will cause cracking in the bottom fiber. From Fig. 3.7, equating the bottom fiber stresses gives a value for the modulus of rupture of

$$f_r = \frac{M_{cr}}{S_b} - P_e R_b = 7.5\lambda\sqrt{f_c'} \qquad [\textit{ACI 9-10}]$$
$$M_{cr} = S_b(P_e R_b + f_r)$$

As specified in ACI Sec. 18.8.2, a prestressed beam with bonded tendons must have adequate reinforcement to support an applied factored moment $(M_u = M_n)$ at least 1.2 times the cracking moment. This requirement is

waived for members with both shear and flexural design strengths at least twice the required strength.

$$\phi M_n \geq 1.2 M_{cr}$$

Example 3.4

For the pretensioned beam of Ex. 3.3, determine the cracking moment strength.

Solution

The modulus of rupture is given by ACI Sec. 9.5.2.3 as

$$f_r = 7.5\lambda\sqrt{f_c'}$$
$$= (7.5)(1.0)\sqrt{6000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 581 \ \text{lbf/in}^2$$

The cracking moment strength is

$$M_{cr} = S_b(P_e R_b + f_r)$$
$$= (185 \ \text{in}^3)\left(\begin{array}{c}(0.75)\left(2940 \ \dfrac{\text{lbf}}{\text{in}^2}\right) \\ + \ 581 \ \dfrac{\text{lbf}}{\text{in}^2}\end{array}\right)$$
$$= 515{,}410 \ \text{in-lbf}$$

Figure 3.7 *Cracking Moment*

$$\frac{P_e}{A_g} \qquad \frac{P_e e}{S_b} \qquad \frac{-M_{cr}}{S_b} \qquad -f_r$$

Strength Design Stage

In accordance with ACI Sec. 18.7.3, nonprestressed reinforcement is assumed to contribute to the ultimate moment of resistance of the section at its yield strength. Equating the longitudinal forces shown in Fig. 3.8 gives

$$0.85 f_c' ab = A_{ps} f_{ps} + A_s f_y - A_s' f_y$$

The nominal flexural strength of the member is

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - \frac{a}{2}\right) + A_s' f_y\left(\frac{a}{2} - d'\right)$$

Using the notation in Fig. 3.8,

$$a = \frac{A_{ps} f_{ps} + A_s f_y - A_s' f_y}{0.85 f_c' b}$$

When the section does not contain compression reinforcement

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - \frac{a}{2}\right)$$
$$a = \frac{A_{ps} f_{ps} + A_s f_y}{0.85 f_c' b}$$

Figure 3.8 *Strain Distribution and Internal Forces at Flexural Failure*

When the section contains neither compression nor auxiliary reinforcement the depth of the stress block is

$$a = \frac{A_{ps}f_{ps}}{0.85f'_c b}$$

The nominal flexural strength is

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right)$$
$$= A_{ps}f_{ps}\left(d_p - \frac{0.59A_{ps}f_{ps}}{f'_c b}\right)$$

Tension- and Compression-Controlled Sections

In ACI Sec. 10.3.4, a section is defined as tension-controlled if the net tensile strain in the extreme tension steel, ϵ_t, is not less than 0.005 when the concrete reaches its maximum useable compressive strain of 0.003. ACI Sec. 9.3.2.1 gives the strength reduction factor as

$$\phi = 0.9$$

The following relationships may be derived from Fig. 3.9 using simple geometry.

$$\epsilon_t = 0.005$$
$$\frac{c}{d_p} = 0.375$$
$$a = 0.375\beta_1 d_p$$

In ACI Sec. 10.3.3, a section is defined as compression-controlled if the net tensile strain in the extreme tension steel is not more than 0.002 when the concrete reaches a strain of 0.003. The strength reduction factor is then given by ACI Sec. 9.3.2.2 as

$$\phi = 0.65$$

Figure 3.9 *Strain Distribution at Nominal Strength*
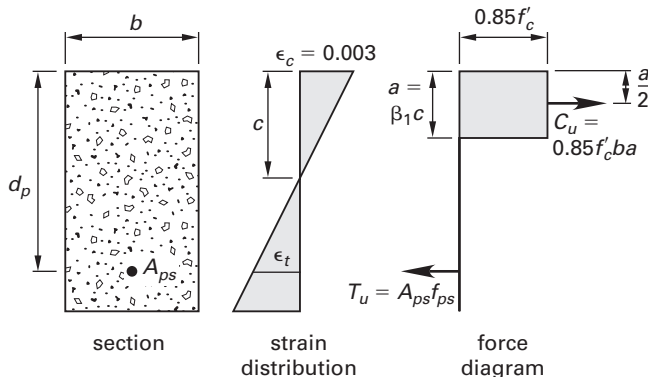
The following relationships may be derived from Fig. 3.9 using simple geometry.

$$\epsilon_t = 0.002$$
$$\frac{c}{d_p} = 0.600$$
$$a = 0.600\beta_1 d_p$$

Members having a tensile strain between 0.002 and 0.005 are in the transition zone. (See Fig. 3.10.) The strength reduction factor in the transition zone may be interpolated from Fig. 3.10.

Figure 3.10 *Variation of ϕ with ϵ_t*
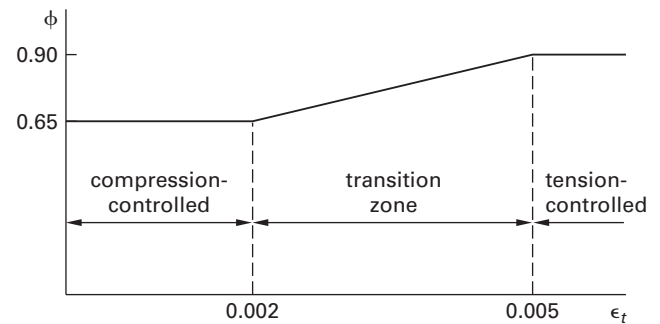
Example 3.5

For the pretensioned beam of Ex. 3.1, which has a 28-day compressive strength of 6000 lbf/in², determine the maximum possible value of the nominal flexural strength for a tension-controlled section.

Solution

The height of the centroid of the section is given in Ex. 3.1 as

$$\overline{y} = 4.67 \text{ in}$$

The eccentricity of the prestressing force is given in Ex. 3.1 as

$$e = 2.25 \text{ in}$$

The height of the section is given in Ex. 3.1 as

$$h = 12 \text{ in}$$

Then, the distance from the extreme compression fiber to the centroid of prestressed reinforcement is given by

$$d_p = h - \overline{y} + e$$
$$= 12 \text{ in} - 4.67 \text{ in} + 2.25 \text{ in}$$
$$= 9.58 \text{ in}$$

The maximum depth of the rectangular stress block for a tension-controlled section is given by ACI Sec. 10.3.4 as

$$a = 0.375\beta_1 d_p$$
$$= (0.375)(0.75)(9.58 \text{ in})$$
$$= 2.69 \text{ in}$$

The maximum nominal flexural strength is

$$M_n = 0.85f'_c ab\left(d_p - \frac{a}{2}\right)$$
$$= (0.85)\left(6000 \ \frac{\text{lbf}}{\text{in}^2}\right)(2.69 \text{ in})(4 \text{ in})$$
$$\times \left(9.58 \text{ in} - \frac{2.69 \text{ in}}{2}\right)$$
$$= 451{,}903 \text{ in-lbf}$$

$$\phi M_n < 1.2M_{cr} \quad \text{[unsatisfactory]}$$

Flexural Strength of Members with Bonded Tendons

Approximate values of f_{ps} in terms of the reinforcement index may be determined in accordance with ACI Sec. 18.7.2, provided that $f_{se} \geq 0.5f_{pu}$ and that all the prestressing tendons are located in the tensile zone.

The reinforcement indices are given by ACI Sec. 2.1 and Sec. 18.7.2 as

$$\omega = \text{reinforcement index of nonprestressed}$$
$$\text{tension reinforcement}$$
$$= \frac{\rho f_y}{f'_c}$$

$$\omega' = \text{reinforcement index of}$$
$$\text{compression reinforcement}$$
$$= \frac{\rho' f_y}{f'_c}$$

The reinforcement ratios are given by ACI Sec. 2.1 as

$$\rho = \text{ratio of nonprestressed}$$
$$\text{tension reinforcement}$$
$$= \frac{A_s}{bd}$$

$$\rho' = \text{ratio of compression reinforcement}$$
$$= \frac{A'_s}{bd}$$

$$\rho_p = \text{ratio of prestressed reinforcement}$$
$$= \frac{A_{ps}}{bd_p}$$

For bonded tendons, ACI Sec. 18.7.2 gives the value of the stress in the prestressed reinforcement at nominal strength as

$$f_{ps} = f_{pu}\left(1 - \left(\frac{\gamma_p}{\beta_1}\right)\left(\rho_p\left(\frac{f_{pu}}{f'_c}\right) + \frac{d(\omega - \omega')}{d_p}\right)\right)$$

[*ACI 18-1*]

The factor for type of prestressing tendon is given by ACI Sec. 18.7.2 as

$$\gamma_p = 0.55 \text{ for deformed bars with } f_{py}/f_{pu} \geq 0.80$$
$$= 0.40 \text{ for stress-relieved wire and strands,}$$
$$\text{and plain bars with } f_{py}/f_{pu} \geq 0.85$$
$$= 0.28 \text{ for low-relaxation wire and strands}$$
$$\text{with } f_{py}/f_{pu} \geq 0.90$$

When compression reinforcement is taken into account while calculating f_{ps} by ACI Eq. 18-1,

$$0.17 \leq \left(\rho_p\left(\frac{f_{pu}}{f'_c}\right) + \frac{d(\omega - \omega')}{d_p}\right)$$
$$d' \leq 0.15d_p$$

When the section contains no auxiliary reinforcement, the value for f_{ps} reduces to

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p \rho_p f_{pu}}{\beta_1 f'_c}\right)$$

Example 3.6

The pretensioned beam shown is simply supported over a span of 30 ft and has a 28-day concrete strength of 6000 lbf/in². The area of the low-relaxation prestressing tendons provided is 0.765 in² with a specified tensile strength of 270 kips/in², a yield strength of 243 kips/in², and an effective stress of 150 kips/in² after all losses. Determine the nominal flexural strength of the beam.
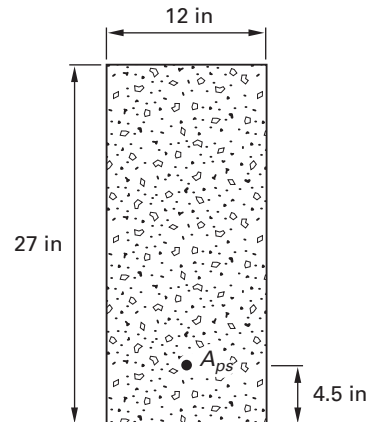
Solution

The relevant properties of the beam are

$$A_g = 324 \text{ in}^2$$

$$S_b = 1458 \text{ in}^3$$

$$e = \frac{h}{2} - 4.5 \text{ in}$$

$$= \frac{27 \text{ in}}{2} - 4.5 \text{ in}$$

$$= 9 \text{ in}$$

$$R_b = \frac{1}{A_g} + \frac{e}{S_b}$$

$$= \frac{1}{324 \text{ in}^2} + \frac{9 \text{ in}}{1458 \text{ in}^3}$$

$$= 0.00926 \ 1/\text{in}^2$$

The factor for this type of prestressing tendon is given by ACI Sec. 18.7.2 as

$$\gamma_p = 0.28 \quad [\text{for } f_{py}/f_{pu} \geq 0.9]$$

$$\rho_p = \frac{A_{ps}}{b d_p}$$

$$= \frac{0.765 \text{ in}^2}{(12 \text{ in})(22.5 \text{ in})}$$

$$= 0.00283$$

From ACI Sec. 10.2.7.3, the compression zone factor is given by

$$\beta_1 = 0.75$$

$$M_{cr} = S_b(P_e R_b + f_r)$$

$$= (1458 \text{ in}^3) \left(\begin{array}{c} (0.765 \text{ in}^2)\left(150 \ \dfrac{\text{kips}}{\text{in}^2}\right) \\ \times (0.00926 \text{ in}^{-2}) + 0.581 \ \dfrac{\text{kips}}{\text{in}^2} \end{array} \right)$$

$$= 2400 \text{ in-kips}$$

From ACI Eq. 18-1,

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p \rho_p f_{pu}}{\beta_1 f_c'}\right)$$

$$= \left(270 \ \frac{\text{kips}}{\text{in}^2}\right)\left(1 - \frac{\begin{array}{c}(0.28)(0.00283) \\ \times \left(270 \ \frac{\text{kips}}{\text{in}^2}\right)\end{array}}{(0.75)\left(6 \ \frac{\text{kips}}{\text{in}^2}\right)}\right)$$

$$= 257 \text{ kips/in}^2$$

The depth of the stress block is given by

$$a = \frac{A_{ps} f_{ps}}{0.85 f_c' b}$$

$$= \frac{(0.765 \text{ in}^2)\left(257 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(6 \ \dfrac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})}$$

$$= 3.21 \text{ in}$$

The maximum depth of the stress block for a tension-controlled section is given by ACI Sec. 10.3.4 as

$$a_t = 0.375 \beta_1 d_p$$

$$= (0.375)(0.75)(22.5 \text{ in})$$

$$= 6.33 \text{ in}$$

$$> a$$

Therefore, the section is tension-controlled and $\phi = 0.9$.

The nominal moment of resistance of the section is given by

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{0.59 A_{ps} f_{ps}}{b f_c'}\right)$$

$$= (0.765 \text{ in}^2)\left(257 \ \frac{\text{kips}}{\text{in}^2}\right)$$

$$\times \left(22.5 \text{ in} - \frac{\begin{array}{c}(0.59)(0.765 \text{ in}^2) \\ \times \left(257 \ \frac{\text{kips}}{\text{in}^2}\right)\end{array}}{(12 \text{ in})\left(6 \ \frac{\text{kips}}{\text{in}^2}\right)}\right)$$

$$= 4106 \text{ in-kips}$$

$$\phi M_n > 1.2 M_{cr} \quad [\text{satisfactory}]$$

Flexural Strength of Members with Unbonded Tendons

When the effective stress after all losses have occurred is not less than half the tensile stress of the tendon, ACI Sec. 18.7.2 permits the stress in the unbonded tendons at nominal strength to be calculated by one of the following methods.

- For unbonded tendons and a span-to-depth ratio ≤ 35, ACI Sec. 18.7.2 gives the stress in the tendons at nominal strength as

$$f_{ps} = f_{se} + 10{,}000 \ \frac{\text{lbf}}{\text{in}^2} + \frac{f_c'}{100 \rho_p} \quad [\textit{ACI 18-2}]$$

$$\leq f_{py}$$

$$\leq f_{se} + 60{,}000 \ \frac{\text{lbf}}{\text{in}^2}$$

f_{se}, f_{py}, and f_c' are in lbf/in^2.

- For unbonded tendons and a span-to-depth ratio > 35, ACI Sec. 18.7.2 gives

$$f_{ps} = f_{se} + 10,000 \ \frac{\text{lbf}}{\text{in}^2} + \frac{f_c'}{300\rho_p} \quad \text{[ACI 18-3]}$$

$$\le f_{py}$$

$$\le f_{se} + 30,000 \ \frac{\text{lbf}}{\text{in}^2}$$

f_{se}, f_{py}, and f_c' are in lbf/in^2.

In accordance with ACI Sec. 18.9.2, auxiliary bonded reinforcement is required near the extreme tension fiber in all beams with unbonded tendons. The minimum area required is independent of the grade of steel and is given by

$$A_s = 0.004 A_{ct} \quad \text{[ACI 18-4]}$$

A_{ct} is the area of the concrete section between the centroid of the section and the extreme tension fiber, as shown in Fig. 3.11.

Figure 3.11 Bonded Reinforcement Area
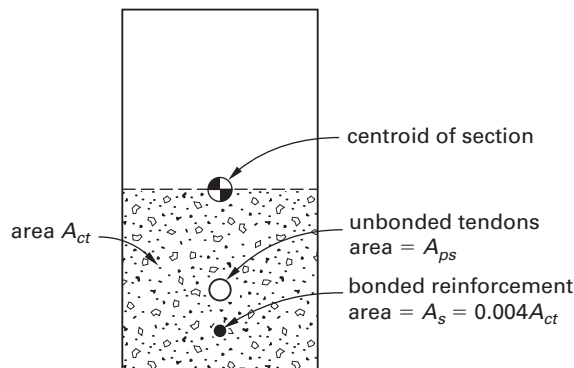
In flat slabs, when the tensile stress due to dead load plus live load is less than $-2\sqrt{f_c'}$, auxiliary reinforcement at a stress of $0.5f_y$ must be provided to resist the total tensile force in the concrete.

Example 3.7

The post-tensioned beam shown is simply supported over a span of 30 ft and has a 28-day concrete strength of 6000 lbf/in^2. The area of the low-relaxation unbonded tendons provided is 0.765 in^2 with a specified tensile strength of 270 kips/in^2, a yield strength of 243 kips/in^2, and an effective stress of 150 kips/in^2 after all losses. The area of the grade 60 auxiliary reinforcement provided is 0.8 in^2. Determine the nominal flexural strength of the beam.
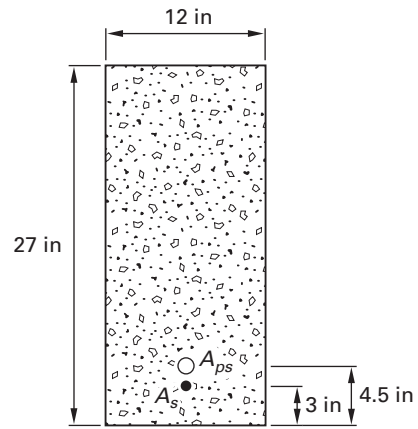
Solution

Because $f_{se}/f_{pu} > 0.5$, the method of ACI Sec. 18.7.2 may be used. The ratio of prestressed reinforcement is

$$\rho_p = \frac{A_{ps}}{bd_p}$$

$$= \frac{0.765 \ \text{in}^2}{(12 \ \text{in})(22.5 \ \text{in})}$$

$$= 0.00283$$

From ACI Sec. 18.7.2, the stress in the unbonded tendons at nominal strength is

$$f_{ps} = f_{se} + 10 \ \frac{\text{kips}}{\text{in}^2} + \frac{f_c'}{100\rho_p}$$

$$= 150 \ \frac{\text{kips}}{\text{in}^2} + 10 \ \frac{\text{kips}}{\text{in}^2} + \frac{6 \ \dfrac{\text{kips}}{\text{in}^2}}{(100)(0.00283)}$$

$$= 181 \ \text{kips/in}^2$$

$$< f_{py} \quad \text{[satisfactory]}$$

$$< f_{se} + 60 \ \text{kips/in}^2 \quad \text{[satisfactory]}$$

The minimum area of auxiliary reinforcement required is specified by ACI Sec. 18.9.2 as

$$A_s = 0.004 A_{ct}$$

$$= (0.004)(12 \ \text{in})(13.5 \ \text{in})$$

$$= 0.648 \ \text{in}^2$$

$$< 0.80 \ \text{in}^2 \quad \text{[satisfactory]}$$

Assuming full use of the auxiliary reinforcement, the depth of the stress block is

$$a = \frac{A_{ps}f_{ps} + A_s f_y}{0.85 f'_c b}$$

$$= \frac{\begin{aligned}(0.765 \text{ in}^2)\left(181 \ \frac{\text{kips}}{\text{in}^2}\right) \\ + (0.8 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)\end{aligned}}{(0.85)\left(6 \ \frac{\text{kips}}{\text{in}^2}\right)(12 \text{ in})}$$

$$= 3.04 \text{ in}$$

The maximum depth of the stress block for a tension-controlled section is given by ACI Sec. 10.3.4 as

$$a_t = 0.375\beta_1 d_p$$

$$= (0.375)(0.75)(22.5 \text{ in})$$

$$= 6.33 \text{ in}$$

$$> a \quad [\text{section is tension-controlled}]$$

The nominal flexural strength is

$$M_n = A_{ps}f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - \frac{a}{2}\right)$$

$$= (0.765 \text{ in}^2)\left(181 \ \frac{\text{kips}}{\text{in}^2}\right)\left(22.5 \text{ in} - \frac{3.04 \text{ in}}{2}\right)$$

$$+ (0.80 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)\left(24 \text{ in} - \frac{3.04 \text{ in}}{2}\right)$$

$$= 3980 \text{ in-kips}$$

$$\phi M_n > 1.2 M_{cr} \quad [\text{satisfactory}]$$

Flexural Strength of Members Using Strain Compatibility

When the approximate methods of determining flexural strength cannot be used, a member's flexural strength can be determined from the strain compatibility method using Fig. 3.12 and the following steps.

step 1: Per ACI 318, use 0.003 as the concrete's maximum strain, ϵ.

step 2: Make an initial estimate of the depth, c, to the neutral axis.

step 3: Calculate the strain in the tendons and auxiliary reinforcement from

$$\epsilon_s = 0.003\frac{r}{c}$$

step 4: To the strain in the tendons, add the pre-existing strain due to the final prestress which is given by

$$\epsilon_p = \frac{f_{se}}{E_p}$$

step 5: Determine the stresses in the tendons and auxiliary reinforcement from the appropriate stress-strain curve, and calculate the forces in the tendons and auxiliary reinforcement.

step 6: Compare the total tensile force, T, with the compressive force, C_u, in the concrete, and adjust the location of the neutral axis until

$$\sum T = C_u$$

step 7: Sum the moments of forces about the neutral axis to determine the required flexural strength.

Figure 3.12 *Flexural Strength by Strain Compatibility*

Example 3.8

The pretensioned beam shown has a 28-day concrete strength of 6000 lbf/in^2 and is pretensioned with five $^1/_2$ in diameter strands. The area of each strand is 0.153 in^2 with a specified tensile strength of 270 $kips/in^2$ and an effective stress of 150 $kips/in^2$ after all losses. Using the idealized stress-strain curve shown, determine the nominal flexural strength of the beam.

Solution

From the illustration, the depth to the neutral axis is $c = 4$ in. Then, the depth of the equivalent stress block is

$$a = \beta_1 c = (0.75)(4 \text{ in}) = 3 \text{ in}$$

The total compressive force in the concrete stress block is obtained from Fig. 3.12 as

$$C_u = 0.85 f'_c (ab - A_{s1})$$
$$= (0.85)\left(6 \ \frac{\text{kips}}{\text{in}^2}\right)\left((3 \text{ in})(12 \text{ in}) - 0.153 \text{ in}^2\right)$$
$$= 183 \text{ kips}$$

For an effective final prestress in each tendon of 150 $kips/in^2$, the prestrain in each tendon is

$$\epsilon_p = \frac{f_{se}}{E_p} = \frac{150 \ \dfrac{\text{kips}}{\text{in}^2}}{28{,}000 \ \dfrac{\text{kips}}{\text{in}^2}} = 5.36 \times 10^{-3}$$

The total strain in each tendon is given by

$$\epsilon_{\text{tot}} = \epsilon_s + \epsilon_p = 0.003\frac{r}{c} + \epsilon_p$$

Illustration for Ex. 3.8
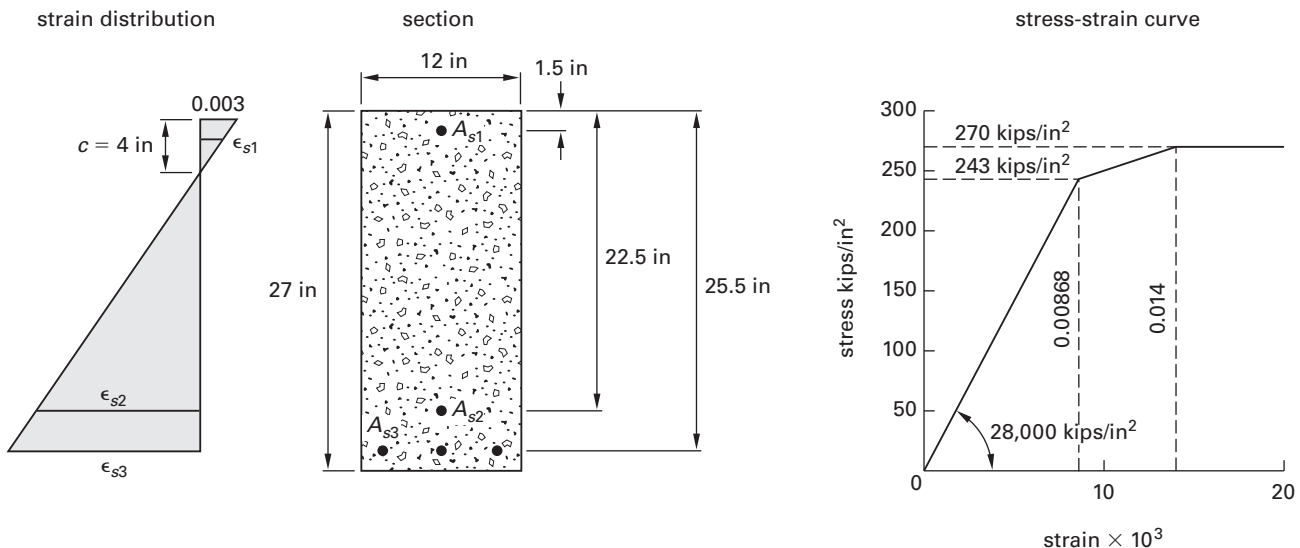
The tendons reach their specified tensile strength at a strain of

$$\epsilon_{pu} = 14 \times 10^{-3} \quad \text{[from stress-strain curve]}$$
$$\epsilon_{s1} = (0.003)\left(\frac{-2.5 \text{ in}}{4 \text{ in}}\right) + 5.36 \times 10^{-3}$$
$$= 3.49 \times 10^{-3}$$
$$\epsilon_{s2} = (0.003)\left(\frac{18.5 \text{ in}}{4 \text{ in}}\right) + 5.36 \times 10^{-3}$$
$$= 19.24 \times 10^{-3} \quad \text{[exceeds } \epsilon_{pu}\text{]}$$
$$\epsilon_{s3} = (0.003)\left(\frac{21.5 \text{ in}}{4 \text{ in}}\right) + 5.36 \times 10^{-3}$$
$$= 21.49 \times 10^{-3} \quad \text{[exceeds } \epsilon_{pu}\text{]}$$

The force in each tendon is given by

$$T = A_s f_s$$
$$T_1 = A_{s1} \epsilon_{s1} E_p$$
$$= (0.153 \text{ in}^2)(3.49 \times 10^{-3})\left(28{,}000 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 15 \text{ kips}$$
$$T_2 = (0.153 \text{ in}^2)\left(270 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 41 \text{ kips}$$
$$T_3 = (3)(0.153 \text{ in}^2)\left(270 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 124 \text{ kips}$$
$$\sum T = 15 \text{ kips} + 41 \text{ kips} + 124 \text{ kips}$$
$$= 180 \text{ kips}$$
$$\approx C_u \quad \text{[satisfactory]}$$

Taking moments about the neutral axis gives

$$M_n = T_{ps1}(-r_1) + T_{s2}r_2 + T_{s3}r_3 + C_u\left(c - \frac{a}{2}\right)$$

$$= (15 \text{ kips})(-2.5 \text{ in}) + (41 \text{ kips})(18.5 \text{ in})$$

$$+ (124 \text{ kips})(21.5 \text{ in})$$

$$+ (183 \text{ kips})\left(4 \text{ in} - \frac{3 \text{ in}}{2}\right)$$

$$= 3845 \text{ in-kips}$$

2. DESIGN FOR SHEAR

Nomenclature

A_{ps}	area of prestressed reinforcement in tension zone	in^2
A_v	area of shear reinforcement within a spacing s	in^2
b_w	web width	in
d	distance from extreme compression fiber to centroid of nonprestressed reinforcement	in
d	distance from extreme compression fiber to centroid of prestressed and nonprestressed reinforcement $\geq 0.8h$, as given in ACI Sec. 11.3.1	in
d_p	actual distance from extreme compression fiber to centroid of prestressing tendons as defined in Fig. 3.8 $\geq 0.8h$	in
f_d	tensile stress at bottom fiber of section due to unfactored dead load	kips/in^2
f_{pc}	compressive stress in concrete at the centroid of the section due to final prestressing force	kips/in^2
f_{pe}	compressive stress in concrete at the bottom fiber of the section due to final prestressing force	kips/in^2
f_{pu}	specified strength of prestressing tendons	kips/in^2
f_{se}	effective stress in prestressing reinforcement after allowance for all prestressing losses	kips/in^2
f_{yt}	yield strength of transverse reinforcement	kips/in^2
g	sag of the tendon	in
h	depth of member	in
h	overall thickness of member	in
l	span length	in
M_{cre}	moment causing flexural cracking at section	in-kips
M_{ct}	cracking moment	in-kips
$M_{\max}$	maximum factored moment at section due to externally applied loads	in-kips
M_u	factored moment at section	in-kips
R_u	end reaction	lbf
s	spacing of shear or torsion reinforcement in direction parallel to longitudinal reinforcement	in
S_b	section modulus of the section referred to the bottom fiber	in^3
V_c	nominal shear strength provided by concrete	kips

V_{ci}	nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment	kips
V_{cw}	nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in the web	kips
V_d	shear force at section due to unfactored dead load	kips
V_i	factored shear force at section due to externally applied loads applied simultaneously with $M_{\max}$	kips
V_p	vertical component of effective prestress force at section	kips
V_s	nominal shear strength provided by shear reinforcement	kips
V_u	factored shear force at section	kips
w_u	distributed load	lbf/ft
x	distance from support to section considered	in

Symbols

λ	correction factor related to unit weight of concrete	–
ϕ	strength reduction factor, 0.75 for shear and torsion	–

Critical Section

As shown in Fig. 3.13, the critical section for the calculation of shear in a prestressed beam is located at a distance from the support that is equal to half the overall thickness of the section. The maximum design factored shear force is given by

$$V_u = R_u - w_u\left(\frac{h}{2}\right)$$

As specified in ACI Sec. 11.1.3, sections located less than a distance $h/2$ from the face of the support may be designed for the shear force, V_u. This is permitted provided that the support reaction produces a compressive stress in the end of the beam, loads are applied at or near the top of the beam, and concentrated loads are not located closer to the support than half the overall depth.

Design for Shear

The nominal shear capacity of shear reinforcement perpendicular to the member is given by ACI Sec. 11.4.7.2 as

$$V_s = \frac{A_v f_{yt} d}{s} \qquad [\textit{ACI 11-15}]$$

The nominal shear strength of the shear reinforcement is limited by ACI Sec. 11.4.7.9 to a value of

$$V_s = 8 b_w d \sqrt{f_c'}$$

Figure 3.13 *Critical Section for Shear*
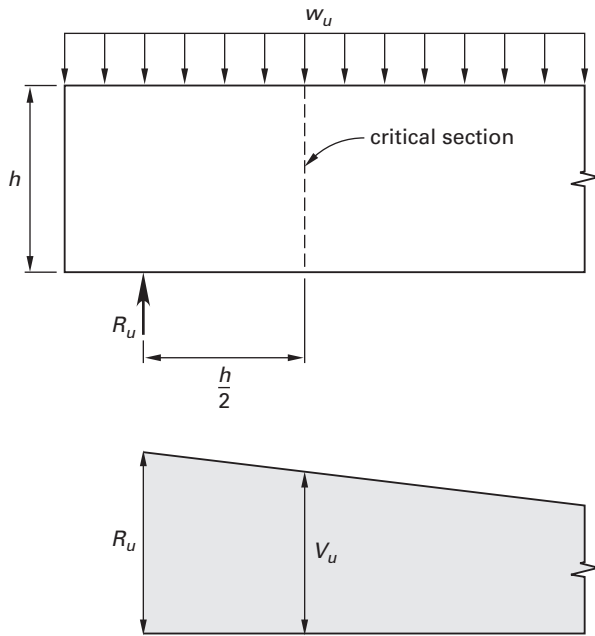
If additional shear capacity is required, the size of the concrete section must be increased. ACI Sec. 11.4.5 limits the spacing of the stirrups to a maximum value of $0.75h$ or 24 in, and when the value of V_s exceeds $4b_w d\sqrt{f_c'}$, the spacing is reduced to a maximum value of $0.375h$ or 12 in.

In accordance with ACI Sec. 11.1.1, the combined shear capacity of the concrete section and the shear reinforcement is

$$\phi V_n = \phi V_c + \phi V_s$$

When the applied factored shear force, V_u, is less than $\phi V_c/2$, the concrete section is adequate to carry the shear without any shear reinforcement. Within the range $\phi V_c/2 \le V_u \le \phi V_c$, a minimum area of shear reinforcement is specified by ACI Sec. 11.4.6; this area is given by the smaller of the results of the following equations.

$$A_{v(\min)} = 0.75\sqrt{f_c'}\left(\frac{b_w s}{f_{yt}}\right) \qquad \text{[ACI 11-13]}$$

$$A_{v(\min)} = \frac{A_{ps}f_{pu}s\sqrt{\dfrac{d}{b_w}}}{80f_{yt}d} \quad \text{[for } f_{se} > 0.4f_{pu}] \qquad \text{[ACI 11-14]}$$

Both ACI Eq. 11-13 and Eq. 11-14 must not be less than $(50b_w s)/f_{yt}$.

f_{yt}, f_{pu}, and f_c' are in lbf/in². As stated in ACI Sec. 11.4.3, d need not be taken less than $0.8h$.

When $f_{se} \ge 0.4f_{pu}$, the nominal shear strength of the concrete section is conservatively given by ACI Sec. 11.3.2 as

$$V_c = \left(0.6\lambda\sqrt{f_c'} + 700\frac{V_u d_p}{M_u}\right)b_w d \qquad \text{[ACI 11-9]}$$

$$\le 5\lambda\sqrt{f_c'}b_w d$$

$$\ge 2\lambda\sqrt{f_c'}b_w d$$

$$\frac{V_u d_p}{M_u} \le 1.0$$

$$\sqrt{f_c'} \le 100 \text{ lbf/in}^2 \quad \text{[from ACI Sec. 11.1.2]}$$

For simply supported beams with uniformly distributed loads, ACI Eq. 11-9 may be expressed as

$$\frac{V_u d_p}{M_u} = \frac{d_p(l - 2x)}{x(l - x)}$$

Example 3.9

The post-tensioned beam shown has a 28-day concrete strength of 6000 lbf/in² and is tensioned with five $^1/_2$ in diameter strands. The area of each strand is 0.153 in² with a specified tensile strength of 270 kips/in² and an effective stress of 150 kips/in² after all losses. The cable centroid, as shown, is parabolic in shape, and the value of $V_u d_p/M_u = 1.0$ at section A-A. Determine the nominal shear capacity at section A-A.

Solution

The equation of the parabolic cable profile is

$$y = \frac{gx^2}{a^2} = \frac{(10.5 \text{ in})x^2}{\left((15 \text{ ft})\left(12\,\frac{\text{in}}{\text{ft}}\right)\right)^2}$$

At section A-A, the rise of the cable is given by

$$y_A = \frac{(10.5 \text{ in})\left((15 \text{ ft})\left(12\,\frac{\text{in}}{\text{ft}}\right) - 23 \text{ in}\right)^2}{\left((15 \text{ ft})\left(12\,\frac{\text{in}}{\text{ft}}\right)\right)^2}$$

$$= 8 \text{ in}$$

The actual depth of the cable is

$$d_p = h - y_A - y_o$$

$$= 27 \text{ in} - 8 \text{ in} - 4.5 \text{ in}$$

$$= 14.5 \text{ in}$$

Illustration for Ex. 3.9
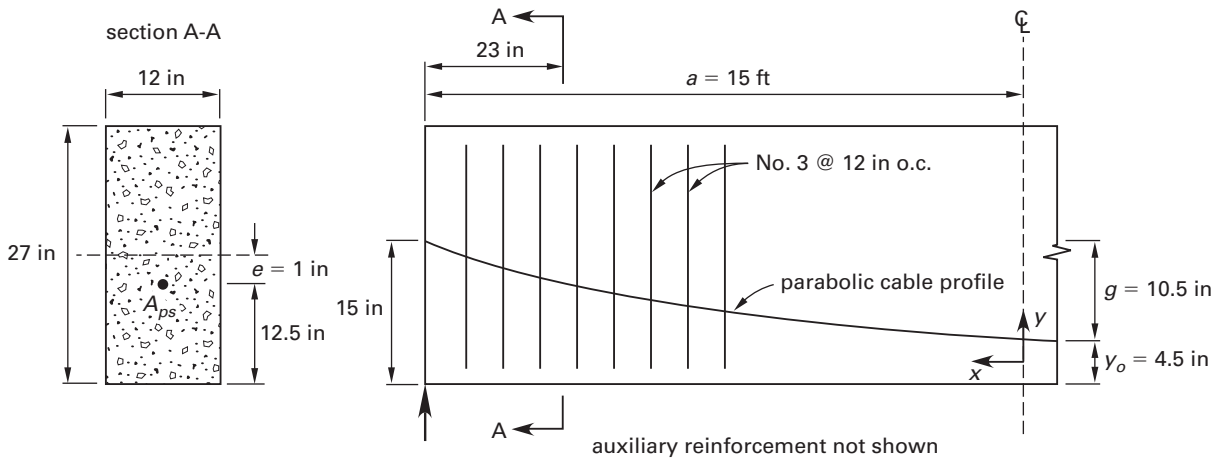
The effective depth of the section is

$$d = 0.8h$$
$$= (0.8)(27 \text{ in})$$
$$= 21.6 \text{ in} \quad [\text{governs}]$$

The nominal shear capacity is given by ACI Eq. 11-9 as

$$V_c = \left(0.6\lambda\sqrt{f_c'} + 700\frac{V_u d_p}{M_u}\right)b_w d$$

$$= \left((0.6)(1.0)\sqrt{6000 \frac{\text{lbf}}{\text{in}^2}} + (700)(1.0)\right)$$

$$\times \left(\frac{(12 \text{ in})(21.6 \text{ in})}{1000 \frac{\text{lbf}}{\text{kip}}}\right)$$

$$= 193 \text{ kips}$$

$$\leq 5\lambda\sqrt{f_c'}b_w d$$

$$= \frac{(5)(1.0)\sqrt{6000 \frac{\text{lbf}}{\text{in}^2}}(12 \text{ in})(21.6 \text{ in})}{1000 \frac{\text{lbf}}{\text{kip}}}$$

$$= 100 \text{ kips} \quad [\text{governs}]$$

The nominal shear capacity of the stirrups provided is given by ACI Eq. 11-15 as

$$V_s = \frac{A_v f_{yt} d}{s}$$

$$= \frac{(0.22 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)(21.6 \text{ in})}{12 \text{ in}}$$

$$= 24 \text{ kips}$$

The total nominal shear capacity is given by ACI Eq. 11-2 as

$$V_n = V_c + V_s$$
$$= 100 \text{ kips} + 24 \text{ kips}$$
$$= 124 \text{ kips}$$

Flexure-Shear and Web-Shear Cracking

A more precise value of the shear capacity is obtained by distinguishing between flexure-shear cracking and web-shear cracking. The two types of cracking are shown in Fig. 3.14.

Figure 3.14 *Cracking in Prestressed Concrete Beams*
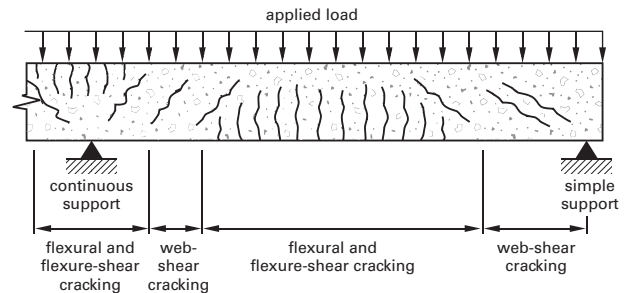
Adapted with permission from *Building Code Requirements for Structural Concrete and Commentary* (ACI 318-11) copyright © 2011, by the American Concrete Institute.

The nominal shear capacity is provided by the lesser value of V_{ci} or V_{cw} given by ACI Sec. 11.3.3.1 and Sec. 11.3.3.2.

Flexure-Shear Cracking

For flexure-shear cracking, the nominal shear capacity is given by

$$V_{ci} = 0.6b_w d_p \lambda\sqrt{f_c'} + V_d + \frac{V_i M_{cre}}{M_{\max}} \quad [\textit{ACI 11-10}]$$

$$\geq 1.7b_w d\lambda\sqrt{f_c'}$$

V_d is the shear force at the section due to the unfactored dead load, $M_{\max}$ is the maximum factored moment at the section due to the externally applied loads, V_i is the factored shear force at the section associated with $M_{\max}$, and d_p is the distance from the top fiber of the section to the centroid of the prestressing tendons, but not less than $0.8h$. The cracking moment due to the unfactored external applied loads is given by

$$M_{cre} = S_b \left(6\lambda\sqrt{f'_c} + f_{pe} - f_d \right) \qquad [\textit{ACI 11-11}]$$

f_d is the tensile stress at the bottom fiber of the section, due to the unfactored dead load, and f_{pe} is the compressive stress in the concrete, due to the final prestressing force, at the bottom fiber of the section.

For a uniformly loaded member, ACI Sec. R11.3.3 gives the variant of ACI Eq. 11-10.

$$V_{ci} = 0.6b_w d\lambda\sqrt{f'_c} + \frac{V_u M_{ct}}{M_u}$$

M_u is the total factored moment at the section, V_u is the factored shear force associated with M_u, and the total unfactored moment, including dead load, required to cause cracking is given by

$$M_{ct} = S_b \left(6\lambda\sqrt{f'_c} + f_{pe} \right)$$

For composite members, in accordance with ACI Sec. R11.3.3, the applicable equations are ACI Eq. 11-10 and Eq. 11-11, with the shear force, V_d, and the stress, f_d, determined from the unfactored dead load resisted by the precast unit and the unfactored superimposed dead load resisted by the composite member. Similarly, M_d is the bending moment at the section due to the unfactored dead load acting on the precast unit plus the moment due to the unfactored superimposed dead load acting on the composite member; V_d is the unfactored shear force associated with M_d. Then,

$$V_i = V_u - V_d$$
$$M_{\max} = M_u - M_d$$

Web-Shear Cracking

For web-shear cracking, the nominal shear capacity is given by ACI Sec. 11.3.3.2 as

$$V_{cw} = b_w d_p \left(3.5\lambda\sqrt{f'_c} + 0.3f_{pc} \right) + V_p$$

$$[\textit{ACI 11-12}]$$

f_{pc} is the compressive stress in the concrete due to the final prestressing force at the centroid of the section, and V_p is the vertical component of the effective prestress force at the section, in kips.

Example 3.10

For the post-tensioned beam of Ex. 3.9, determine the nominal shear capacity of the concrete at section A-A by using ACI Eq. 11-10 and Eq. 11-12. The unfactored

bending moment at section A-A due to dead load and live load is 500 in-kips.

Solution

At section A-A, the slope of the cable is given by

$$\frac{dy}{dx} = 2\left(\frac{gx}{a^2}\right)$$

$$= (2)\left(\frac{(10.5 \text{ in})\left((15 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right) - 23 \text{ in}\right)}{(180 \text{ in})^2}\right)$$

$$= 0.102$$

The vertical component of the final effective prestressing force at section A-A is

$$V_p = A_{ps}f_{se}\frac{dy}{dx}$$

$$= (0.765 \text{ in}^2)\left(150 \frac{\text{kips}}{\text{in}^2}\right)(0.102)$$

$$= 11.7 \text{ kips}$$

The compressive stress in the concrete, due to the final prestressing force, at the centroid of the section is

$$f_{pc} = \frac{P_e}{A_g}$$

$$= \frac{(0.765 \text{ in}^2)\left(150 \frac{\text{kips}}{\text{in}^2}\right)}{324 \text{ in}^2}$$

$$= 0.354 \text{ kips/in}^2$$

The nominal web-shear capacity of the concrete at section A-A is given by ACI Eq. 11-12 as

$$V_{cw} = b_w d_p \left(3.5\lambda\sqrt{f'_c} + 0.3f_{pc} \right) + V_p$$

$$= \frac{(12 \text{ in})(21.6 \text{ in})\left(\begin{array}{c}(3.5)(1.0)\sqrt{6000 \frac{\text{lbf}}{\text{in}^2}} \\ + (0.3)\left(354 \frac{\text{lbf}}{\text{in}^2}\right)\end{array}\right)}{1000 \frac{\text{lbf}}{\text{kip}}}$$

$$+ 11{,}700 \text{ lbf}$$

$$= 110 \text{ kips}$$

At section A-A, the cable eccentricity is

$$e = \frac{h}{2} - y_A - y_o$$

$$= 13.5 \text{ in} - 8 \text{ in} - 4.5 \text{ in}$$

$$= 1.0 \text{ in}$$

$$R_b = \frac{1}{A_g} + \frac{e}{S_b} = \frac{1}{324 \text{ in}^2} + \frac{1 \text{ in}}{1458 \text{ in}^3}$$

$$= 0.00377 \text{ 1/in}^2$$

The compressive stress in the bottom fiber, at section A-A, due to the final prestressing force is

$$f_{pe} = P_e R_b = (0.765 \text{ in}^2)\left(150 \ \frac{\text{kips}}{\text{in}^2}\right)\left(0.00377 \ \frac{1}{\text{in}^2}\right)$$

$$= 0.433 \text{ kips/in}^2$$

The applied moment required to produce cracking at section A-A is given by modified ACI Eq. 11-11 as

$$M_{ct} = S_b\left(6\lambda\sqrt{f_c'} + f_{pe}\right)$$

$$= (1458 \text{ in}^3)\left(\frac{(6)(1.0)\sqrt{6000 \ \dfrac{\text{lbf}}{\text{in}^2}}}{1000 \ \dfrac{\text{lbf}}{\text{kip}}} + 0.433 \ \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 1309 \text{ in-kips}$$

$$> 500 \text{ in-kips}$$

As this moment exceeds the given unfactored applied moment at section A-A, flexural cracking does not occur at section A-A; ACI Eq. 11-10 is not applicable, and ACI Eq. 11-12 governs. The nominal shear capacity of the concrete section is

$$V_c = V_{cw} = 110 \text{ kips}$$

3. DESIGN FOR TORSION

Nomenclature

A_{cp}	area enclosed by outside perimeter of concrete cross section	in^2
A_l	total area of longitudinal reinforcement to resist torsion	in^2
A_o	gross area enclosed by shear flow	in^2
A_{oh}	gross area enclosed by centerline of the outermost closed transverse torsional reinforcement	in^2
A_t	area of one leg of a closed stirrup resisting torsion within a spacing s	in^2
d_{bl}	diameter of longitudinal torsional reinforcement	in
f_y	yield strength of longitudinal torsional reinforcement	kips/in^2
p_{cp}	outside perimeter of concrete cross section	in
p_h	perimeter of centerline of outermost closed transverse torsional reinforcement	in
T_n	nominal torsional moment strength	in-kips
T_u	factored torsional moment at section	in-kips

Symbols

τ	shear stress	kips/in^2

General Principles

After torsional cracking occurs, the central core of a prestressed concrete member is largely ineffective in resisting applied torsion, and so it can be neglected.

When a member is subjected to torsion, ACI Sec. R11.5 assumes that it behaves as a thin-walled tube. In order to maintain a consistent approach, a member is also analyzed as a thin-walled tube prior to cracking. As shown in Fig. 3.15, the shear stress in the tube walls produces a uniform shear flow, q, that acts at the midpoint of the walls. The shear flow's magnitude is

$$q = \tau t$$

Figure 3.15 *Thin-Walled Tube Analogy*

thin-walled tube   area enclosed by shear flow
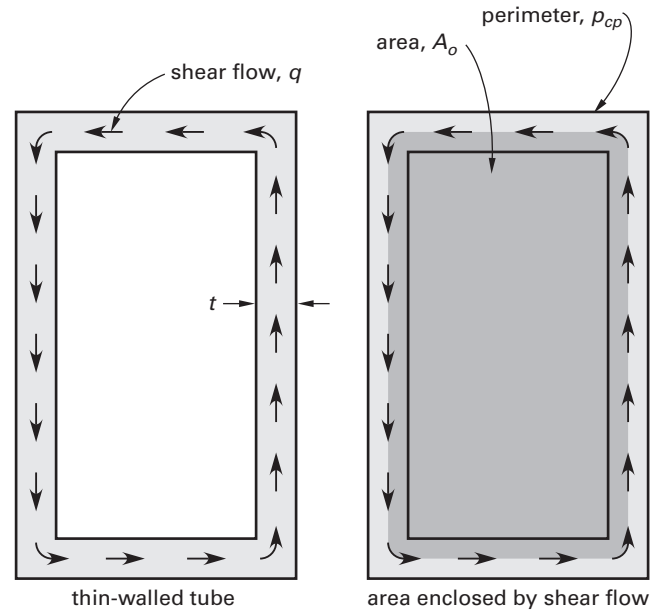
From ACI Sec. R11.5.1, the applied torsion is resisted by the moment of the shear flow in the walls about the centroid of the section and is given by

$$T = 2A_o q$$

The gross area enclosed by shear flow, A_o, is the area enclosed by the center line of the walls, A_{cp}, and is given by

$$A_o = \frac{2A_{cp}}{3}$$

The stress in the shear walls is

$$\tau = \frac{T}{2A_o t}$$

For flanged sections, the overhanging flange width used to calculate the values of A_{cp} and p_{cp}, the outside perimeter, is determined from ACI Sec. 11.5.1.1 and Sec. 13.2.4, and shown in Fig. 3.16.

Figure 3.16 *Overhanging Flange Width*
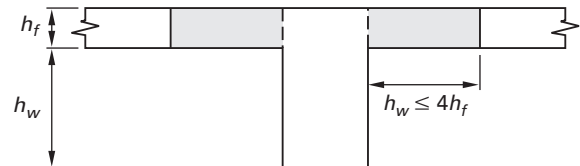
In accordance with ACI Sec. 11.5.2.5, the critical section for the calculation of torsion in a prestressed beam is located at a distance from the support equal to half the overall depth. When a concentrated torsion occurs within this distance, the critical section for design must be at the face of the support.

Cracking is assumed to occur in a member when the principal tensile stress reaches a value of

$$p_t = 4\sqrt{f'_c} = \tau$$

The cracking torsion is

$$T_{cr} = 4\lambda\sqrt{f'_c}\left(\frac{A^2_{cp}}{p_{cp}}\right)\sqrt{1 + \frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$$

Design Provisions

The design provisions for torsion in prestressed concrete are similar to those for reinforced concrete. In accordance with ACI Sec. 11.5.1, torsional effects may be neglected, and closed stirrups and longitudinal torsional reinforcement are not required when the factored torque does not exceed

$$T_u = \phi\lambda\sqrt{f'_c}\left(\frac{A^2_{cp}}{p_{cp}}\right)\sqrt{1 + \frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$$

When this value is exceeded, reinforcement must be provided to resist the full torsion. When both shear and torsion reinforcements are required, the sum of the individual areas must be provided.

ACI Sec. 11.5.3.6 specifies the required area of one leg of a closed stirrup as follows, with $\theta = 37.5°$ as permitted by ACI Sec. 11.5.3.6.

$$\frac{A_t}{s} = \frac{T_u}{2\phi A_o f_{yt}\cot\theta} = \frac{T_u}{1.7\phi A_{oh} f_{yt}\cot\theta}$$

The corresponding area of longitudinal reinforcement required is specified in ACI Sec. 11.5.3.7 and Sec. R11.5.3.10 as follows, with $\theta = 37.5°$ as permitted by ACI Sec. 11.5.3.7.

$$A_l = \left(\frac{A_t p_h f_{yt}}{f_y s}\right)\cot^2\theta \qquad [\textit{ACI 11-22}]$$

$$\geq \frac{5A_{cp}\sqrt{f'_c}}{f_y} - \frac{A_t p_h f_{yt}}{f_y s} \qquad [\textit{ACI 11-24}]$$

$$\frac{A_t}{s} \geq \frac{25b_w}{f_{yt}}$$

The minimum diameter is given by ACI Sec. 11.5.6.2 as

$$d_{bl} = 0.042s \text{ in}$$

$$\geq \text{no. 3 bar}$$

The minimum combined area of transverse closed stirrups for combined shear and torsion is given by ACI Sec. 11.5.5.2. Rearranging ACI Eq. 11-23,

$$\frac{A_v + 2A_t}{s} = \frac{0.75\sqrt{f'_c}b_w}{f_{yt}}$$

$$\geq \frac{50b_w}{f_{yt}}$$

The maximum spacing of closed stirrups is given by ACI Sec. 11.5.6.1 as

$$s = \frac{p_h}{8} \text{ in}$$

$$\leq 12 \text{ in}$$

When redistribution is possible in an indeterminate structure, the nominal torsional capacity of the member, in accordance with ACI Sec. 11.5.2.2, need not exceed

$$T_n = 4\lambda\sqrt{f'_c}\left(\frac{A^2_{cp}}{p_{cp}}\right)\sqrt{1 + \frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$$

Example 3.11

The post-tensioned beam for Ex. 3.9 is subjected to a factored shear force of 88 kips and a factored torsion of 100 in-kips at section A-A. Determine the combined shear and torsion reinforcement required.

Solution

The area enclosed by the outside perimeter of the beam is

$$A_{cp} = (27 \text{ in})(12 \text{ in}) = 324 \text{ in}^2$$

The length of the outside perimeter of the beam is

$$p_{cp} = (2)(27 \text{ in} + 12 \text{ in}) = 78 \text{ in}$$

The compressive stress at the centroid, due to the final prestressing force, was determined in Ex. 3.10 as

$$f_{pc} = 354 \text{ lbf/in}^2$$

Torsional reinforcement is not required in accordance with ACI Sec. 11.5.1 when the factored torque does not exceed

$$T_u = \phi\lambda\sqrt{f'_c}\left(\frac{A^2_{cp}}{p_{cp}}\right)\sqrt{1 + \frac{f_{pc}}{4\lambda\sqrt{f'_c}}}$$

$$= \left((0.75)(1.0)\sqrt{6000 \text{ }\frac{\text{lbf}}{\text{in}^2}}\left(\frac{(324 \text{ in}^2)^2}{(78 \text{ in})\left(1000 \text{ }\frac{\text{lbf}}{\text{kip}}\right)}\right)\right)$$

$$\times \sqrt{1 + \frac{354 \text{ }\frac{\text{lbf}}{\text{in}^2}}{(4)(1.0)\sqrt{6000 \text{ }\frac{\text{lbf}}{\text{in}^2}}}}$$

$$= 115 \text{ in-kips}$$

$$> 100 \text{ in-kips} \qquad \begin{bmatrix}\text{Closed stirrups are} \\ \text{not required.}\end{bmatrix}$$

The shear strength provided by the concrete was determined in Ex. 3.9 as

$$V_c = 100 \text{ kips}$$

From ACI Eq. 11-1 and Eq. 11-2, the required nominal capacity of the shear reinforcement is

$$V_s = \frac{V_u}{\phi} - V_c = \frac{88 \text{ kips}}{0.75} - 100 \text{ kips}$$

$$= 17.33 \text{ kips}$$

The minimum permissible area of shear reinforcement is the smaller value given by ACI Eq. 11-13 and Eq. 11-14.

$$\frac{A_{v(\min)}}{s} = \frac{A_{ps}f_{pu}\sqrt{\dfrac{d}{b_w}}}{80 f_{yt} d} \qquad [ACI\ 11\text{-}14]$$

$$= \frac{(0.765 \text{ in}^2)\left(270\ \dfrac{\text{kips}}{\text{in}^2}\right)\sqrt{\dfrac{21.6 \text{ in}}{12 \text{ in}}}}{(80)\left(60\ \dfrac{\text{kips}}{\text{in}^2}\right)(21.6 \text{ in})}$$

$$= 0.0027 \text{ in}^2/\text{in} \quad [\text{governs}]$$

Or,

$$\frac{A_{v(\min)}}{s} = \frac{50 b_w}{f_{yt}} = \frac{(50)(12 \text{ in})}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}}$$

$$= 0.010 \text{ in}^2/\text{in}$$

Or,

$$\frac{A_{v(\min)}}{s} = \frac{0.75 b_w \sqrt{f_c'}}{f_{yt}} \qquad [ACI\ 11\text{-}13]$$

$$= \frac{(0.75)(12 \text{ in})\sqrt{6000\ \dfrac{\text{lbf}}{\text{in}^2}}}{60{,}000\ \dfrac{\text{lbf}}{\text{in}^2}}$$

$$= 0.012 \text{ in}^2/\text{in}$$

From ACI Eq. 11-15, the shear reinforcement required is

$$\frac{A_v}{s} = \frac{V_s}{f_{yt}d} = \frac{17.33 \text{ kips}}{\left(60\ \dfrac{\text{kips}}{\text{in}^2}\right)(21.6 \text{ in})}$$

$$= 0.014 \text{ in}^2/\text{in}$$

$$> \frac{A_{v(\min)}}{s} \quad [\text{satisfactory}]$$

Provide no. 3 stirrups at 15 in spacing, which gives

$$\frac{A_v}{s} = \frac{0.22 \text{ in}^2}{15 \text{ in}} = 0.015 \text{ in}^2/\text{in}$$

$$> 0.014 \quad [\text{satisfactory}]$$

4. PRESTRESS LOSSES

Nomenclature

A_{ps}	area of prestressing tendon	in^2
c	anchor set	in
C	factor for relaxation losses	–
E_{ci}	modulus of elasticity of concrete at time of initial prestress	kips/in^2
E_p	modulus of elasticity of prestressing tendon	kips/in^2
f_{pd}	compressive stress at level of tendon centroid after elastic losses and including sustained dead load	lbf/in^2
f_{pi}	compressive stress at level of tendon centroid after elastic losses	lbf/in^2
f_{pp}	compressive stress at level of tendon centroid before elastic losses	lbf/in^2
g	sag of prestressing tendon	in
H	ambient relative humidity	%
J	factor for relaxation losses	–
K	wobble friction coefficient per foot of prestressing tendon	–
K_{re}	factor for relaxation losses	kips/in^2
K_{sh}	factor for shrinkage losses accounting for elapsed time between completion of casting and transfer of prestressing force	–
l_c	length of prestressing tendon affected by anchor seating loss	ft
l_{px}	length of prestressing tendon from jacking end to any point x measured along the curve	ft
m	loss of force per foot of cable due to friction	kips/ft
n_i	E_p/E_{ci}	–
p_{cp}	outside perimeter of the concrete cross section	in
P_c	prestressing tendon force at a distance of l_c from the jacking end	kips
P_i	prestressing tendon force after elastic losses	kips
P_p	prestressing tendon force before elastic losses	kips
P_{pj}	prestressing tendon force at jacking end	kips
P_{px}	prestressing tendon force at a distance of l_{px} from the jacking end	kips
$P_{\Delta c}$	loss of tendon force due to anchor set	kips
$P_{\Delta cr}$	loss of tendon force due to creep	kips
$P_{\Delta el}$	loss of tendon force due to elastic shortening	kips
$P_{\Delta f}$	friction loss due to curvature and unintentional wobble of the duct profile	kips
$P_{\Delta re}$	loss of tendon force due to relaxation	kips
$P_{\Delta sh}$	loss of tendon force due to shrinkage	kips
R	radius of curvature of tendon profile	ft

Symbols

α	angular change in radians of tendon profile from jacking end to any point x	radians
ϵ_{sh}	basic shrinkage strain	–
μ	curvature friction coefficient	–

Prestressed Concrete

Types of Prestress Losses

In a pretensioned concrete member, loss of tendon force is produced by

- elastic shortening of concrete, $P_{\Delta el}$
- long-term creep of concrete, $P_{\Delta cr}$
- long-term shrinkage of concrete, $P_{\Delta sh}$
- long-term relaxation of stress in the prestressed tendons, $P_{\Delta re}$
- anchor seating loss at transfer, $P_{\Delta c}$

The total loss of tendon force in a pretensioned concrete member is

$$P_{\Delta} = P_{\Delta el} + P_{\Delta cr} + P_{\Delta sh} + P_{\Delta re} + P_{\Delta c}$$

Additional friction losses, $P_{\Delta f}$, are produced in a post-tensioned concrete member, such as losses due to curvature of the duct profile and unintended wobble (i.e., out-of-straightness) of the duct.

The total loss of tendon force in a post-tensioned concrete member is

$$P_{\Delta} = P_{\Delta el} + P_{\Delta cr} + P_{\Delta sh} + P_{\Delta re} + P_{\Delta c} + P_{\Delta f}$$

To determine the effective stress in the prestressing tendons, f_{se}, all prestress losses must be calculated. ACI Sec. R18.6.1 does not provide methods to calculate these losses, rather, it refers to the journal article, *Estimating Prestress Losses.*[1]

Friction Losses

Friction losses occur in post-tensioned members due to curvature of the duct profile and wobble of the ducts.

The relationship between tendon force at a distance, l_{px}, from the jack and the tendon force, P_{pj}, at the jack is

$$P_{pj} = P_{px} \exp\left(Kl_{px} + \mu\alpha\right)$$

For a value of $Kl_{px} + \mu\alpha$ not greater than 0.3, this expression reduces to

$$P_{pj} = P_{px}(1 + Kl_{px} + \mu\alpha)$$

Values of friction coefficients may be obtained from the manufacturers of the prestressing system.

Example 3.12

The post-tensioned beam shown has a prestressing cable consisting of five $^1/_2$ in diameter low-relaxation strands. Each strand has an area of 0.153 in^2, a yield strength of 243 kips/in^2, and a tensile strength of 270 kips/in^2. The cable centroid, as shown, is parabolic in shape and is stressed simultaneously from both ends with a jacking force of 159 kips. The value of the wobble friction coefficient is 0.0015/ft, and the curvature friction coefficient is 0.25. Determine the cable force at midspan of the member before elastic losses.

Solution

The nominal radius of the cable profile is

$$R = \frac{a^2}{2g}$$

$$= \frac{(15 \text{ ft})^2 \left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}{(2)(10.5 \text{ in})}$$

$$= 129 \text{ ft}$$

The cable length along the curve, from the jacking end to midspan, is

$$l_{px} = a + \frac{g^2}{3a} = 15 \text{ ft} + \frac{\left(\dfrac{10.5 \text{ in}}{12 \ \dfrac{\text{in}}{\text{ft}}}\right)^2}{(3)(15 \text{ ft})}$$

$$= 15.02 \text{ ft}$$

Illustration for Ex. 3.12

The angular change of the cable profile over this length is

$$\alpha = \frac{l_{px}}{R} = \frac{15.02 \text{ ft}}{129 \text{ ft}}$$
$$= 0.117 \text{ radians}$$

$$Kl_{px} + \mu\alpha = (0.0015)(15.02 \text{ ft})$$
$$+ (0.25)(0.117 \text{ radians})$$
$$= 0.052$$
$$< 0.3 \quad [\text{ACI Eq. 18-2 is applicable.}]$$

The cable force at midspan is given by

$$P_{px} = \frac{P_{pj}}{1 + Kl_{px} + \mu\alpha} = \frac{159 \text{ kips}}{1 + 0.052}$$
$$= 151 \text{ kips}$$

Anchor Seating Loss

Anchor seating loss results from the slip or set that occurs in the anchorage when the prestressing force is transferred to the anchor device. In a pretensioned tendon with an anchor set of c, the loss in prestressing

tendon force is constant along the cable, as shown in Fig. 3.17. From Fig. 3.17, the anchor set is obtained as

$$c = \frac{\text{shaded area}}{A_{ps}E_p} = \frac{P_{\Delta c}l}{A_{ps}E_p}$$

The loss in prestressing tendon force is then found as

$$P_{\Delta c} = \frac{cA_{ps}E_p}{l}$$

In a post-tensioned tendon, friction in the duct resists the inward movement of the tendon[2] and limits the affected zone to the length l_c shown in Fig. 3.18. The anchor set is obtained from Fig. 3.18 as

$$c = \frac{\text{shaded area}}{A_{ps}E_p} = \frac{ml_c^2}{A_{ps}E_p}$$

$$l_c^2 = \frac{cA_{ps}E_p}{m}$$

$$P_c = P_{pj} - ml_c$$

Figure 3.17 *Seating Loss in a Pretensioned Tendon*
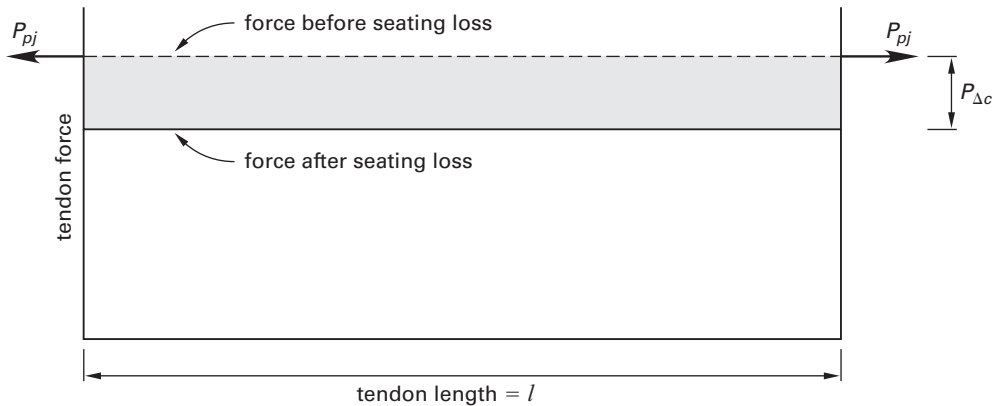
Figure 3.18 *Seating Loss in a Post-Tensioned Tendon*
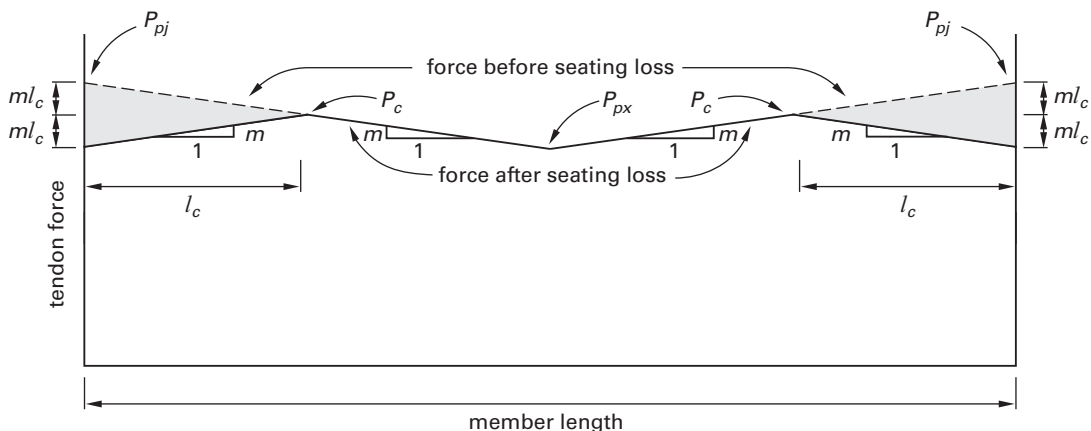
Example 3.13

The post-tensioned beam of Ex. 3.12 has tendon anchorages with a cable set of 0.05 in. Determine the residual tendon force, after anchorage and before elastic and long-term losses occur, at the jacking end and at a distance of l_c from the jacking end.

Solution

From Ex. 3.12, the stress loss per foot due to friction is

$$m = \frac{P_{pj} - P_{px}}{l_{px}} = \frac{159 \text{ kips} - 151 \text{ kips}}{15.02 \text{ ft}}$$

$$= 0.532 \text{ kips/ft}$$

$$l_c^2 = \frac{cA_{ps}E_p}{m}$$

$$= \frac{(0.05 \text{ in})(0.765 \text{ in}^2)\left(28 \times 10^3 \ \frac{\text{kips}}{\text{in}^2}\right)}{\left(0.532 \ \frac{\text{kips}}{\text{ft}}\right)\left(12 \ \frac{\text{in}}{\text{ft}}\right)}$$

$$l_c = 13 \text{ ft}$$

The cable force at a distance of l_c from the jacking end is

$$P_c = P_{pj} - ml_c$$

$$= 159 \text{ kips} - \left(0.532 \ \frac{\text{kip}}{\text{ft}}\right)(13 \text{ ft})$$

$$= 152 \text{ kips}$$

The cable force at the jacking end after anchoring is

$$P_{pj(\text{anc})} = P_{pj} - 2ml_c = 145 \text{ kips}$$

Elastic Shortening Losses

Losses occur in a prestressed concrete beam at transfer due to the elastic shortening of the concrete at the level of the centroid of the prestressing tendons. The concrete stress at the level of the centroid of the prestressing tendons after elastic shortening is

$$f_{pi} = P_i\left(\frac{1}{A_g} + \frac{e^2}{I_g}\right) - \frac{eM_G}{I_g}$$

Conservatively, the concrete stress at the level of the centroid of the prestressing tendons after elastic shortening is

$$f_{pi} = f_{pp} = P_p\left(\frac{1}{A_g} + \frac{e^2}{I_g}\right) - \frac{eM_G}{I_g}$$

In a pretensioned member with transfer occuring simultaneously in all tendons, the loss of prestressing force is

$$P_{\Delta el} = n_i A_{ps} f_{pi}$$

In a post-tensioned member with only one tendon, no loss from elastic shortening occurs.

In a post-tensioned member with several tendons stressed sequentially, the maximum loss occurs in the first tendon stressed, and no loss occurs in the last tendon stressed. The total loss is then one-half the value for a pretensioned member or

$$P_{\Delta el} = \frac{n_i A_{ps} f_{pi}}{2}$$

Example 3.14

The pretensioned beam shown is simply supported over a span of 30 ft and has a concrete strength at transfer of 4500 lbf/in². Five $1/2$ in diameter low-relaxation strands are provided, each with an area of 0.153 in². The initial force in each tendon after anchor seating loss is 32 kips. Determine the loss of prestressing force due to elastic shortening.
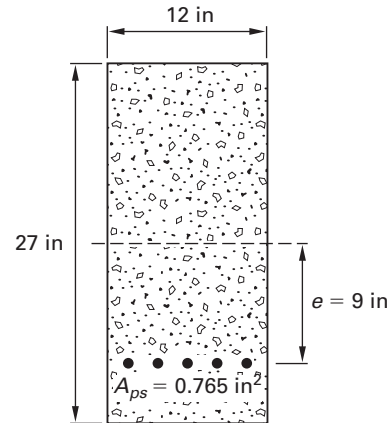
Solution

From ACI Sec. 8.5, the modulus of elasticity of the concrete at transfer is

$$E_{ci} = 57\sqrt{f'_{ci}} = 57\sqrt{4500 \ \frac{\text{lbf}}{\text{in}^2}} = 3824 \text{ kips/in}^2$$

$$n_i = \frac{E_p}{E_{ci}} = \frac{28 \times 10^3 \ \frac{\text{kips}}{\text{in}^2}}{3824 \ \frac{\text{kips}}{\text{in}^2}} = 7.32$$

$$f_{pp} = P_p\left(\frac{1}{A_g} + \frac{e^2}{I_g}\right) - \frac{eM_G}{I_g}$$

$$= (5)(32 \text{ kips})\left(\frac{1}{324 \text{ in}^2} + \frac{(9 \text{ in})^2}{19{,}683 \text{ in}^4}\right)$$

$$\quad - \frac{(9 \text{ in})(455 \text{ in-kip})}{19{,}683 \text{ in}^4}$$

$$= 1.152 \ \frac{\text{kips}}{\text{in}^2} - 0.208 \ \frac{\text{kip}}{\text{in}^2}$$

$$= 0.944 \text{ kip/in}^2$$

The total loss of prestressing force is

$$P_{\Delta el} \approx n_i A_{ps} f_{pp}$$

$$= (7.32)(5)(0.153 \text{ in}^2)\left(0.944 \text{ } \frac{\text{kip}}{\text{in}^2}\right)$$

$$= 5.3 \text{ kips}$$

Creep Losses

Creep occurs in a prestressed concrete member as a result of the sustained compressive stress. It causes a prestress loss that is proportional to the concrete's initial stress and inversely proportional to its modulus of elasticity. Creep increases the earlier the stress is applied to the concrete. Half the total creep will take place in the first month after transfer, and three-quarters of the total creep will take place in the first six months after transfer.

The concrete stress at the level of the centroid of the prestressing tendons after elastic shortening is

$$f_{pd} = P_i\left(\frac{1}{A_g} + \frac{e^2}{I_g}\right) - \frac{eM_G}{I_g} - \frac{eM_D}{I_g}$$

For post-tensioned members with transfer at 28 days, the creep loss is given by[1,3]

$$P_{\Delta cr} = 1.6nA_{ps}f_{pd}$$

For pretensioned members with transfer at three days, the creep loss is given by

$$P_{\Delta cr} = 2.0nA_{ps}f_{pd}$$

Example 3.15

For the post-tensioned beam of Ex. 3.12, the cable force at midspan after elastic losses is 151 kips and the 28-day concrete strength is 6000 lbf/in^2. The superimposed dead load moment is 800 in-kips. Determine the loss of prestressing force due to creep.

Solution

From ACI Sec. 8.5, the modulus of elasticity of the concrete at 28-days is

$$E_c = 57\sqrt{f_c'} = 57\sqrt{6000 \text{ } \frac{\text{lbf}}{\text{in}^2}}$$

$$= 4415 \text{ kips/in}^2$$

$$n = \frac{E_p}{E_c} = \frac{28 \times 10^3 \text{ } \frac{\text{kips}}{\text{in}^2}}{4415 \text{ } \frac{\text{kips}}{\text{in}^2}}$$

$$= 6.34$$

$$f_{pd} = P_i\left(\frac{1}{A_g} + \frac{e^2}{I_g}\right) - \frac{eM_G}{I_g} - \frac{eM_D}{I_g}$$

$$= (151 \text{ kips})\left(\frac{1}{324 \text{ in}^2} + \frac{(9 \text{ in})^2}{19,683 \text{ in}^4}\right)$$

$$- \frac{(9 \text{ in})(455 \text{ in-kips} + 800 \text{ in-kips})}{19,683 \text{ in}^4}$$

$$= 1.087 \text{ } \frac{\text{kips}}{\text{in}^2} - 0.574 \text{ } \frac{\text{kip}}{\text{in}^2}$$

$$= 0.514 \text{ kip/in}^2$$

The loss of prestressing force due to creep is

$$P_{\Delta cr} = 1.6nA_{ps}f_{pd}$$

$$= (1.6)(6.34)(0.765 \text{ in}^2)\left(0.514 \text{ } \frac{\text{kip}}{\text{in}^2}\right)$$

$$= 4.0 \text{ kips}$$

Shrinkage Loss

The shrinkage of a concrete member with time produces a corresponding loss of prestress. The basic shrinkage strain is given by[1,3]

$$\epsilon_{sh} = 8.2 \times 10^{-6} \text{ in/in}$$

For a pretensioned member, with allowance for the ambient relative humidity, H, and the ratio of the member's volume to surface area, A_g/p_{cp}, the shrinkage loss is

$$P_{\Delta sh} = A_{ps}\epsilon_{sh}E_p\left(1 - \frac{0.06A_g}{p_{cp}}\right)(100 - H)$$

For a post-tensioned member with transfer after some shrinkage has already occurred, the shrinkage loss is

$$P_{\Delta sh} = K_{sh}A_{ps}\epsilon_{sh}E_p\left(1 - \frac{0.06A_g}{p_{cp}}\right)(100 - H)$$

K_{sh} is the factor for shrinkage losses in a post-tensioned member, accounting for elapsed time between completion of casting and transfer of tendon force. These values[4] are given in Table 3.1.

Table 3.1 *Shrinkage Factor Values*

days	K_{sh}
1	0.92
3	0.85
5	0.80
7	0.77
10	0.73
20	0.64
30	0.58
60	0.45

Example 3.16

The post-tensioned beam of Ex. 3.15 is located in an area with an ambient relative humidity of 55% and transfer is effected 7 days after the completion of curing, giving a value[3] of 0.77 for K_{sh}. Determine the loss of prestressing force due to shrinkage.

Solution

The shrinkage loss is given by

$$P_{\Delta sh} = K_{sh}A_{ps}\epsilon_{sh}E_p\left(1 - \frac{0.06A_g}{p_{cp}}\right)(100 - H)$$

$$= (0.77)(0.765 \text{ in}^2)\left(8.2 \times 10^{-6} \text{ } \frac{\text{in}}{\text{in}}\right)$$

$$\times \left(28 \times 10^3 \text{ } \frac{\text{kips}}{\text{in}^2}\right)$$

$$\times \left(1 - \frac{(0.06 \text{ in}^{-1})(324 \text{ in}^2)}{78 \text{ in}}\right)(100 - 55)$$

$$= 4.6 \text{ kips}$$

Relaxation Losses

A prestressing tendon is subjected to relaxation over time. The loss in prestress depends on the tendon properties and the initial force in the tendon and on the losses due to creep, shrinkage, and elastic shortening. The relaxation loss is given by[1,3]

$$P_{\Delta re} = \left(A_{ps}K_{re} - J(P_{\Delta cr} + P_{\Delta sh} + P_{\Delta el})\right)C$$

Values of the relaxation parameters may be obtained from Zia[1] and Kelley.[4]

Example 3.17

For the post-tensioned beam in Ex. 3.15, the values of the relevant parameters are $K_{re} = 5$ kips/in^2, $J = 0.04$, and $C = 0.90$.[3] $P_{\Delta cr} = 4.0$ kips, $P_{\Delta sh} = 4.6$ kips, and $P_{\Delta el} = 2.7$ kips. Determine the loss of prestressing force due to relaxation.

Solution

The loss due to relaxation is

$$P_{\Delta re} = \left(A_{ps}K_{re} - J(P_{\Delta cr} + P_{\Delta sh} + P_{\Delta el})\right)C$$

$$= \left(\begin{array}{l}(0.765 \text{ in}^2)\left(5 \text{ } \frac{\text{kips}}{\text{in}^2}\right) \\ - (0.04)(4.0 \text{ kips} + 4.6 \text{ kips} + 2.7 \text{ kips})\end{array}\right)$$

$$\times 0.90$$

$$= 3.0 \text{ kips}$$

5. COMPOSITE CONSTRUCTION

Nomenclature

A_c	area of precast surface or area of contact surface for horizontal shear	in^2
A_v	area of ties within a distance s	in^2
A_{vf}	area of friction reinforcement	in^2
b_f	actual flange width	in
$b_{f(\text{eff})}$	effective flange width	in
$b_{f(\text{tran})}$	transformed flange width	in
b_v	width of girder at contact surface	in
b_w	width of girder web	in
E_f	modulus of elasticity of flange concrete	kips/in^2
E_w	modulus of elasticity of precast girder concrete	kips/in^2
f_{cb}	stress in the bottom fiber of composite section	kips/in^2
$f_{ci(\text{flan})}$	stress in the flange at interface of composite section	kips/in^2
$f_{ci(\text{web})}$	stress in the girder at interface of composite section	kips/in^2
f_{ct}	stress in the top fiber of composite section	kips/in^2
h	depth of composite section	in
h_f	depth of flange of composite section	in
h_w	depth of precast girder	in
I_{cc}	moment of inertia of composite section	in^4
l	span length	ft
M_F	moment due to flange concrete	ft-kips
M_G	moment due to precast girder self-weight	ft-kips
M_{prop}	moment due to removal of props	ft-kips
M_{Sht}	moment due to formwork	ft-kips
M_W	moment due to superimposed dead plus live load	ft-kips
n	modular ratio E_w/E_f	–
P_Δ	total loss of prestress	kips
s	spacing of ties	in
S	spacing of precast girder	ft
S_{cb}	section modulus at bottom of composite section	in^3
S_{ci}	section modulus at interface of composite section	in^3
S_{ct}	section modulus at top of composite section	in^3
V_{nh}	nominal horizontal shear strength	lbf
V_u	factored shear force at section	lbf

Symbols

ρ_v	ratio of tie reinforcement area to area of contact surface $A_v/b_v s$	–

Section Properties

The effective width of the flange of a composite section, as shown in Fig. 3.19, is limited by ACI Sec. 8.12 to the least of

- $\dfrac{l}{4}$

- $b_w + 16h_f$

- S

Figure 3.19 *Effective Flange Width*
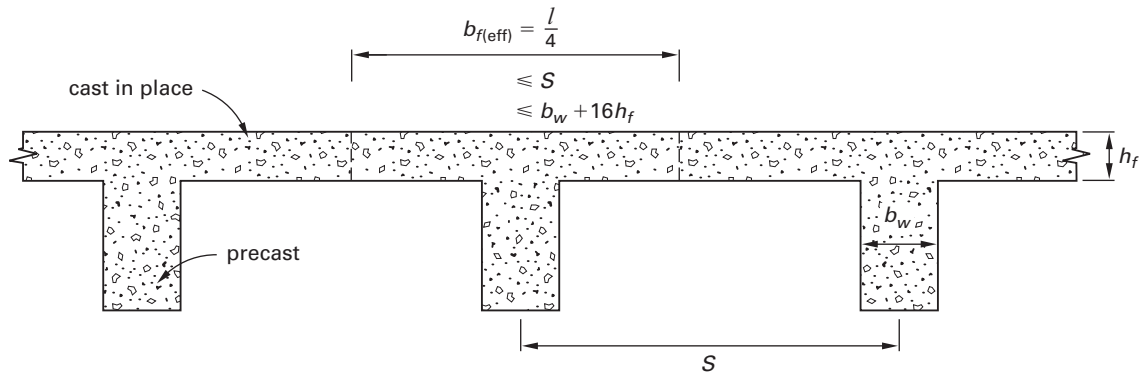
When the 28-day compressive strengths of the precast section and the flange are different, the transformed section properties are obtained, as shown in Fig. 3.20, by dividing by the modular ratio $n = E_w/E_f$. The stresses calculated in the flange by using the transformed section properties are converted to actual stresses by dividing by n.

Figure 3.20 *Transformed Flange Width*
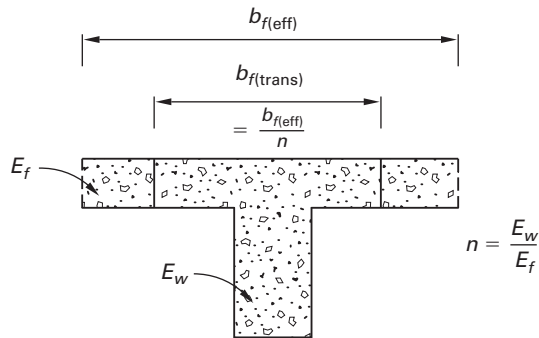
Example 3.18

A composite beam of normal weight concrete with an effective span of 25 ft is shown. The beam is an interior beam in the floor of a commercial building. The precast, pretensioned girder has a 28-day concrete strength of 600 lbf/in^2, and the flange has a 28-day concrete strength of 3000 lbf/in^2. Determine the transformed section properties of the composite section.

Illustration for Ex. 3.18
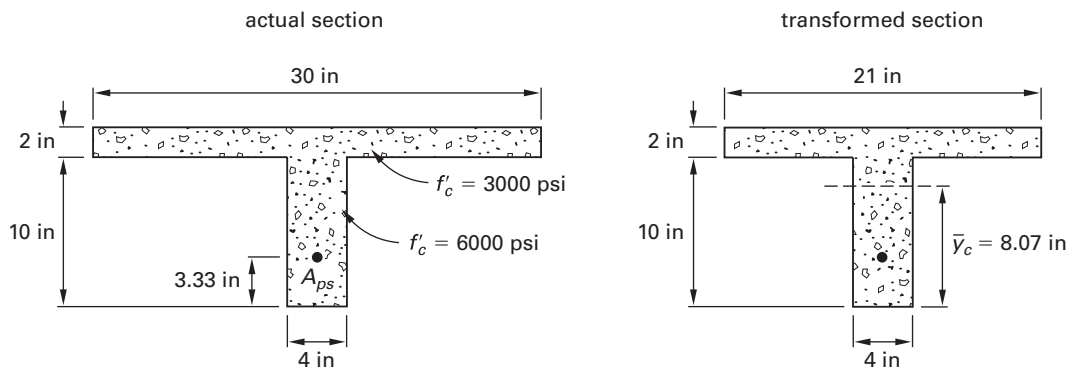
Solution

From ACI Sec. 8.5,

$$E_w = 57\sqrt{f_c'} = 57\sqrt{6000 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 4415 \ \text{kips/in}^2$$

$$E_f = 57\sqrt{f_c'} = 57\sqrt{3000 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 3122 \ \text{kips/in}^2$$

$$n = \frac{E_w}{E_f} = \frac{4415 \ \dfrac{\text{kips}}{\text{in}^2}}{3122 \ \dfrac{\text{kips}}{\text{in}^2}} = 1.41$$

The effective flange width is limited to the least of

- $b_{f(\text{eff})} = \dfrac{l}{4} = \dfrac{(25 \ \text{ft})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}{4}$

  $= 75 \ \text{in}$

- $b_{f(\text{eff})} = b_w + 16h_f = 4 \ \text{in} + (16)(2 \ \text{in})$

  $= 36 \ \text{in}$

- $b_{f(\text{eff})} = b_f = 30 \ \text{in}$ [governs]

The transformed flange width is

$$b_{f(\text{tran})} = \frac{b_{f(\text{eff})}}{n}$$
$$= \frac{30 \text{ in}}{1.41}$$
$$= 21 \text{ in}$$

The relevant properties of the precast girder are

$$A_g = 40 \text{ in}^2$$
$$I_g = 333 \text{ in}^4$$
$$S_b = S_t = 66.67 \text{ in}^3$$

The properties of the transformed section are obtained as shown in the following table.

part	A (in^2)	y (in)	I (in^4)	Ay (in^3)	Ay^2 (in^4)
girder	40	5	333	200	1000
flange	42	11	14	462	5082
total	82	–	347	662	6082

$$\overline{y}_c = \frac{\sum Ay}{\sum A} = \frac{662 \text{ in}^3}{82 \text{ in}^2} = 8.07 \text{ in}$$

$$I_{cc} = \sum I + \sum Ay^2 + \overline{y}_c^2 \sum A - 2\overline{y}\sum Ay$$
$$= 347 \text{ in}^4 + 6082 \text{ in}^4 + 5344 \text{ in}^4$$
$$\quad - (16.14 \text{ in})(662 \text{ in}^3)$$
$$= 1088 \text{ in}^4$$

$$S_{ct} = \frac{I_{cc}}{h - \overline{y}_c} = \frac{1088 \text{ in}^4}{3.93 \text{ in}}$$
$$= 277 \text{ in}^3$$

$$S_{ci} = \frac{I_{cc}}{h_w - \overline{y}_c} = \frac{1088 \text{ in}^4}{1.93 \text{ in}}$$
$$= 564 \text{ in}^3$$

$$S_{cb} = \frac{I_{cc}}{\overline{y}_c} = \frac{1088 \text{ in}^4}{8.07 \text{ in}}$$
$$= 135 \text{ in}^3$$

Horizontal Shear Requirements

To ensure composite action, full transfer of horizontal shear at the interface is necessary, and ACI Sec. 17.5.3 specifies that the factored shear force at a section must not exceed

$$V_u = \phi V_{nh} \qquad [\textit{ACI 17-1}]$$

ACI Sec. 17.5.3.1 specifies that when the interface is intentionally roughened, the nominal horizontal shear strength is given by

$$V_{nh} = 80b_v d$$

ACI Sec. 17.5.3.2 specifies that when the interface is smooth with minimum ties provided across the interface to give $A_v/s = 50b_w/f_y$,

$$V_{nh} = 80b_v d$$

ACI Sec. 17.5.3.3 specifies that when the interface is roughened to $^1/_4$ in amplitude with minimum ties provided across the interface to give $A_v/s = 50b_w/f_y$,

$$V_{nh} = (260 + 0.6\rho_v f_y)\lambda b_v d$$
$$\leq 500b_v d$$

The correction factor related to the unit weight of concrete is given by ACI Sec. 11.6.4.3 as

$$\lambda = 1.0 \quad \text{[for normal weight concrete]}$$
$$= 0.75 \quad \text{[for all lightweight concrete]}$$

ACI Sec. 17.5.3.4 requires that when the factored shear force exceeds $\phi(500b_v d)$, the design must be based on the shear-friction method given in ACI Sec. 11.6.4, with the nominal horizontal shear strength given by

$$V_{nh} = A_{vf}f_y\mu \qquad [\textit{ACI 11-25}]$$
$$\leq 0.2f'_c A_c$$
$$\leq 800A_c$$

ACI Sec. 11.6.4.3 gives the coefficient of friction as

$$\mu = 1.0\lambda \quad \begin{bmatrix} \text{interface roughened to} \\ \text{an amplitude of 1/4 in} \end{bmatrix}$$
$$= 0.6\lambda \quad \text{[interface not roughened]}$$

In accordance with ACI Sec. 17.6.1, the tie spacing must not exceed four times the least dimension of the supported element, nor exceed 24 in.

Example 3.19

The composite section of Ex. 3.18 has a factored shear force of $V_u = 14$ kips at the critical section. Determine the area of grade 60 ties at a spacing of 12 in required at the interface that is intentionally roughened to an amplitude of $^1/_4$ in.

Solution

From ACI Sec. 17.5.2 and Sec. 17.5.3.3,

$$d = 0.8h = (0.8)(12 \text{ in}) = 9.6 \text{ in}$$

$$500\phi b_v d = \left(500 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.75)(4 \text{ in})(9.6 \text{ in})$$
$$= 14{,}400 \text{ lbf}$$
$$> V_u \ [\text{ACI Sec. 17.5.3.3 applies.}]$$

$$V_u = 14{,}000 \text{ lbf}$$
$$= \phi(260 + 0.6\rho_v f_y)\lambda b_v d$$
$$= (0.75)\left(260 \ \frac{\text{lbf}}{\text{in}^2} + 0.6\rho_v\left(60{,}000 \ \frac{\text{lbf}}{\text{in}^2}\right)\right)$$
$$\times (1.0)(4 \text{ in})(9.6 \text{ in})$$

$$\rho_v = 0.0063$$

The required area of vertical ties is

$$A_v = \rho_v b_v s = (0.0063)(4 \text{ in})(12 \text{ in}) = 0.30 \text{ in}^2$$

Nonpropped Construction

In nonpropped construction, the precast section supports its own self-weight, the formwork required to support the cast-in-place flange, and the weight of the flange. It may be conservatively assumed that all prestress losses occur before the flange is cast. As shown in Fig. 3.21, the composite section is subjected to the forces produced by removal of the formwork and by the superimposed applied load.

In accordance with ACI Sec. 17.2.4, the design of the composite section for the strength limit state is identical for both shored and unshored construction.

Figure 3.21 *Nonpropped Construction*

applicable section	loading case
precast section	P_e
precast section	$M_G + M_{Sht} + M_F$
composite section	$M_W - M_{Sht}$
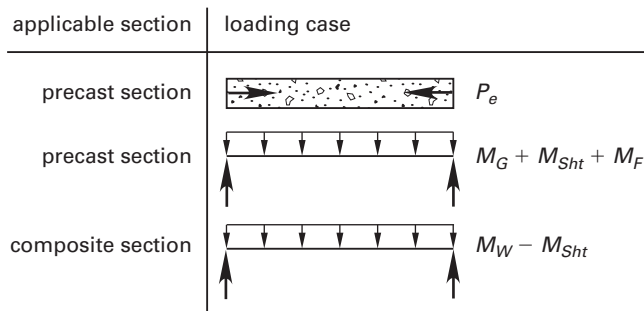
Example 3.20

The precast, pretensioned girder of the composite section of Ex. 3.18 is prestressed with an initial prestressing force of 65 kips. The total loss of prestress is 20% and may be assumed to occur before the flange is cast. The weight of the formwork to support the flange is 25 lbf/ft, the superimposed applied load is 250 lbf/ft, and the precast section is not propped. Determine the stresses at midspan in the composite section.

Solution

$$M_G = \frac{wl^2}{8} = \frac{\left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)(40 \text{ in}^2)(25 \text{ ft})^2}{(8)\left(12 \ \frac{\text{in}}{\text{ft}}\right)}$$
$$= 39{,}060 \text{ in-lbf}$$

$$M_{Sht} = \frac{wl^2}{8} = \frac{\left(25 \ \frac{\text{lbf}}{\text{ft}}\right)(25 \text{ ft})^2\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{8}$$
$$= 23{,}440 \text{ in-lbf}$$

$$M_F = \frac{wl^2}{8} = \frac{\left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)(60 \text{ in}^2)(25 \text{ ft})^2}{(8)\left(12 \ \frac{\text{in}}{\text{ft}}\right)}$$
$$= 58{,}600 \text{ in-lbf}$$

$$M_W = \frac{wl^2}{8} = \frac{\left(250 \ \frac{\text{lbf}}{\text{ft}}\right)(25 \text{ ft})^2\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{8}$$
$$= 234{,}400 \text{ in-lbf}$$

$$P_e = 0.8P_i = (0.8)(65 \text{ kips})$$
$$= 52 \text{ kips}$$

The prestressing force is applied at a height of $h_w/3$, and the stresses in the precast section after casting the flange are

$$f_t = \frac{M_G + M_{Sht} + M_F}{S_t}$$
$$= \frac{121{,}100 \text{ in-lbf}}{66.67 \text{ in}^3}$$
$$= 1816 \text{ lbf/in}^2$$

$$f_b = \frac{2P_e}{A_g} - \frac{M_G + M_{Sht} + M_F}{S_b}$$
$$= \frac{(2)(52{,}000 \text{ lbf})}{40 \text{ in}^2} - 1816 \ \frac{\text{lbf}}{\text{in}^2}$$
$$= 784 \text{ lbf/in}^2$$

The stresses in the composite section due to all loads are

$$f_{ct} = \frac{M_W - M_{Sht}}{nS_{ct}} = \frac{210{,}960 \text{ in-lbf}}{(1.41)(277 \text{ in}^3)}$$
$$= 540 \text{ lbf/in}^2$$

$$f_{cb} = f_b - \frac{M_W - M_{Sht}}{S_{cb}} = 784 - \frac{210{,}960 \text{ in-lbf}}{135 \text{ in}^3}$$
$$= -779 \text{ lbf/in}^2$$

$$f_{ci(\text{flan})} = \frac{M_W - M_{Sht}}{nS_{ci}} = \frac{210{,}960 \text{ in-lbf}}{(1.41)(564 \text{ in}^3)}$$
$$= 265 \text{ lbf/in}^2$$

$$f_{ci(\text{web})} = f_t + \frac{M_W - M_{Sht}}{S_{ci}} = 1816 + \frac{210{,}960 \text{ in-lbf}}{564 \text{ in}^3}$$
$$= 2190 \text{ lbf/in}^2$$

Propped Construction

In propped construction, the weight of the formwork and the flange act on the propped precast girder, producing moments in the girder and reactions in the props. As shown in Fig. 3.22, removal of the props is equivalent to applying forces, equal and opposite to the reactions in the props, to the composite section. The superimposed load is carried by the composite section.

When four or more props are used, the precast section may be considered continuously supported, and no stresses are produced in the precast girder by the weight of the formwork and the flange. Similarly, no stresses are produced by the removal of the formwork. On the removal of the props, the weight of the flange is carried by the composite section, as shown in Fig. 3.23.

Example 3.21

Before placing the formwork to support the flange and casting the flange, the precast, pretensioned girder of the composite section of Ex. 3.20 is propped at midspan. Determine the stresses at midspan in the composite section.

Figure 3.22 *Propped Construction*
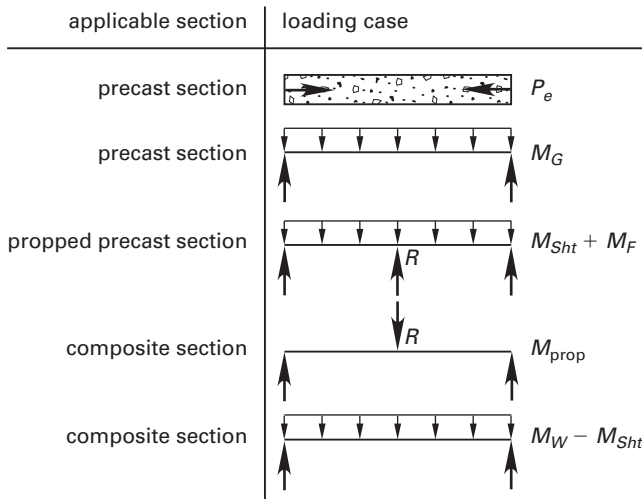
Figure 3.23 *Continuously Supported Section*
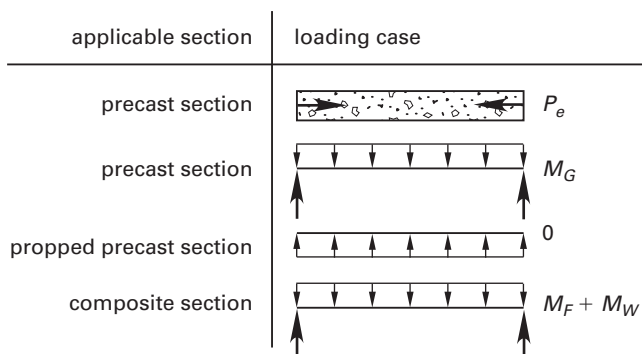
Solution

The prestressing force is applied at a height of $h/3$, and the stresses in the precast girder before propping are

$$f_t = \frac{M_G}{S_t}$$
$$= \frac{39{,}060 \text{ in-lbf}}{66.67 \text{ in}^3}$$
$$= 586 \text{ lbf/in}^2$$
$$f_b = \frac{2P_e}{A_c} - \frac{M_G}{S_b}$$
$$= \frac{(2)(52{,}000 \text{ lbf})}{40 \text{ in}^2} - 586 \frac{\text{lbf}}{\text{in}^2}$$
$$= 2014 \text{ lbf/in}^2$$

The central prop creates a continuous beam with two spans of 12.5 ft each. The reaction on the prop due to the formwork and the flange concrete is

$$R = 1.25wl$$
$$= (1.25)\left(25 \frac{\text{lbf}}{\text{ft}} + 62.5 \frac{\text{lbf}}{\text{ft}}\right)(12.5 \text{ ft})$$
$$= 1367 \text{ lbf}$$

The moment in the precast girder at midspan, due to the formwork and flange concrete, is

$$M_{Sht} + M_F = \frac{wl^2}{8}$$
$$= \frac{\left(87.5 \frac{\text{lbf}}{\text{ft}}\right)(12.5 \text{ ft})^2\left(12 \frac{\text{in}}{\text{ft}}\right)}{8}$$
$$= 20{,}510 \text{ in-lbf}$$

The stresses in the precast girder after casting the flange are

$$f'_t = 586 \frac{\text{lbf}}{\text{in}^2} - \frac{20{,}510 \text{ in-lbf}}{66.67 \text{ in}^3}$$
$$= 278 \text{ lbf/in}^2$$
$$f'_b = 2014 \frac{\text{lbf}}{\text{in}^2} + \frac{20{,}510 \text{ in-lbf}}{66.67 \text{ in}^3}$$
$$= 2321 \text{ lbf/in}^2$$

Removing the prop produces a moment at midspan of

$$M_{\text{prop}} = \frac{Rl}{4}$$
$$= \frac{(1367 \text{ lbf})(25 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{4}$$
$$= 102{,}530 \text{ in-lbf}$$

The stresses in the composite section due to all loads are

$$f_{ct} = \frac{M_{\text{prop}} + M_W - M_{Sht}}{nS_{ct}}$$

$$= \frac{313{,}485 \text{ in-lbf}}{(1.41)(277 \text{ in}^3)}$$

$$= 802 \text{ lbf/in}^2$$

$$f_{cb} = f_b' - \frac{M_{\text{prop}} + M_W - M_{Sht}}{S_{cb}}$$

$$= 2321 \frac{\text{lbf}}{\text{in}^2} - \frac{313{,}485 \text{ in-lbf}}{135 \text{ in}^3}$$

$$= -1 \text{ lbf/in}^2$$

$$f_{ci(\text{flan})} = \frac{M_{\text{prop}} + M_W - M_{Sht}}{nS_{ci}}$$

$$= \frac{313{,}485 \text{ in-lbf}}{(1.41)(564 \text{ in}^3)}$$

$$= 394 \text{ lbf/in}^2$$

$$f_{ci(\text{web})} = f_t' + \frac{M_{\text{prop}} + M_W - M_{Sht}}{S_{ci}}$$

$$= 278 \frac{\text{lbf}}{\text{in}^2} + \frac{313{,}485 \text{ in-lbf}}{564 \text{ in}^3}$$

$$= 834 \text{ lbf/in}^2$$

6. LOAD BALANCING PROCEDURE

Nomenclature

f_c	concrete stress	lbf/in^2
g	sag of prestressing tendon	in
M_B	balancing load moment due to w_B	in-lbf
M_O	out-of-balance moment due to w_O, $M_W - M_B$	in-lbf
M_W	applied load moment due to w_W	in-lbf
P	prestressing force	kips
w_B	balancing load produced by prestressing tendon	kips/ft
w_O	out-of-balance load, $w_W - w_B$	kips/ft
w_W	superimposed applied load	kips/ft

Figure 3.24 *Load Balancing Method*

section A-A
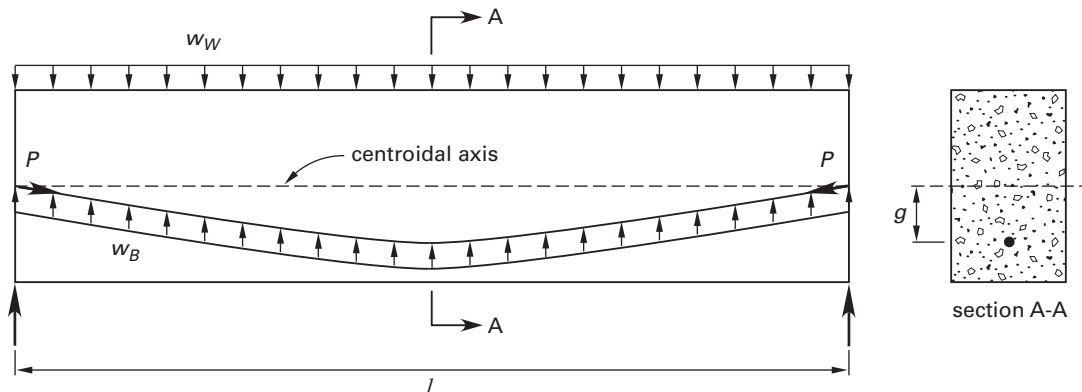
Design Technique

The prestressing tendon of the beam shown in Fig. 3.24 has a parabolic profile and produces a uniform upward pressure of

$$w_B = \frac{8Pg}{l^2}$$

If the total downward load on the beam is equal to w_B, the net load is zero and a uniform compressive stress of $f_c = P/A_g$ is produced in the beam. If the downward load is not fully balanced by the upward force, the out-of-balance moment is

$$M_O = M_W - M_B$$

The stress in the concrete is then given by

$$f_c = \frac{P}{A_g} \pm \frac{M_O}{S}$$

Balancing loads produced by alternative tendon profiles are available[5,6,7] and are shown in Fig. 3.25. This technique also facilitates the calculation of deflections.

Example 3.22

The post-tensioned beam shown supports a uniformly distributed load, including the weight of the beam, of 0.75 kip/ft. The tendon has a parabolic profile.

(a) Determine the prestressing force required in the tendon to balance the applied load exactly, and determine the resulting stress in the beam.

(b) Determine the stresses in the beam at midspan when an additional distributed load of 0.75 kip/ft is applied to the beam.

Solution

(a) The sag of the tendon is

$$g = 13.5 \text{ in} - 4.5 \text{ in} = 9 \text{ in}$$

Figure 3.25 *Alternative Tendon Profiles*
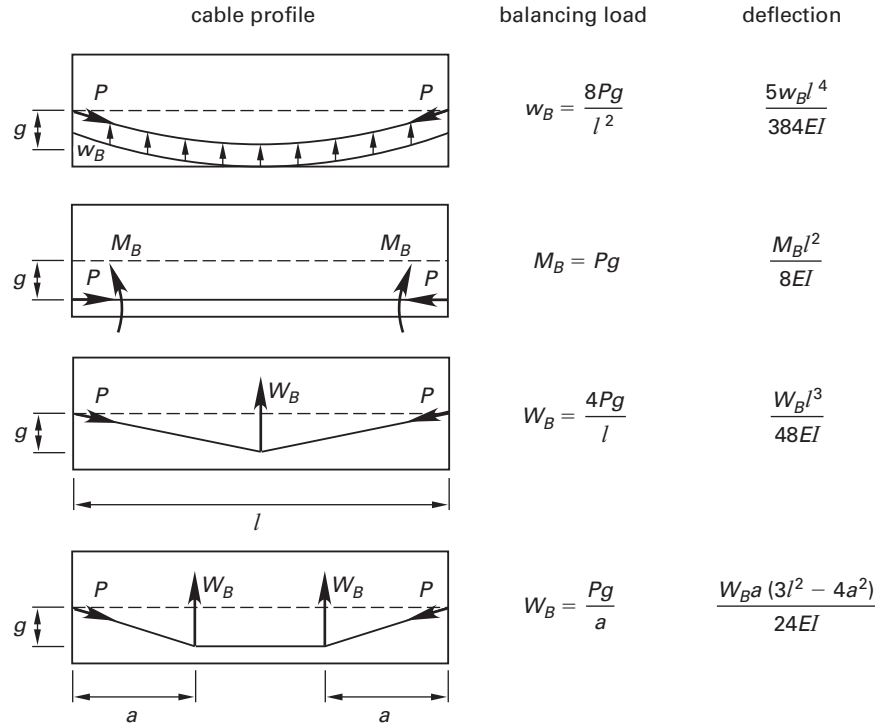
Illustration for Ex. 3.22
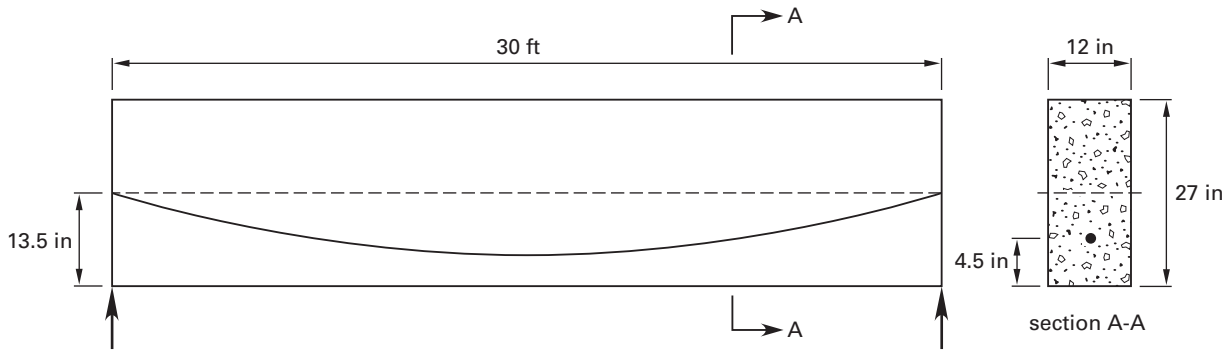
The prestressing force required to balance the applied load exactly is

$$P = \frac{w_W l^2}{8g} = \frac{\left(0.75 \ \frac{\text{kip}}{\text{ft}}\right)(30 \ \text{ft})^2 \left(12 \ \frac{\text{in}}{\text{ft}}\right)}{(8)(9 \ \text{in})}$$

$$= 112.5 \ \text{kips}$$

The uniform compressive stress throughout the beam is

$$f_c = \frac{P}{A_g} = \frac{112{,}500 \ \text{lbf}}{324 \ \text{in}^2}$$

$$= 347 \ \text{lbf/in}^2$$

(b) The out-of-balance moment produced at midspan by an additional load of 0.75 kip/ft is

$$M_O = \frac{w_O l^2}{8}$$

$$= \frac{\left(0.75 \ \frac{\text{kip}}{\text{ft}}\right)(30 \ \text{ft})^2 \left(12 \ \frac{\text{in}}{\text{ft}}\right)}{8}$$

$$= 1013 \ \text{in-kips} \quad (1{,}013{,}000 \ \text{in-lbf})$$

The resultant stresses at midspan are

$$f_{be} = f_c - \frac{M_O}{S}$$

$$= 347 \ \frac{\text{lbf}}{\text{in}^2} - \frac{1{,}013{,}000 \ \text{in-lbf}}{1458 \ \text{in}^3}$$

$$= 347 \ \frac{\text{lbf}}{\text{in}^2} - 694 \ \frac{\text{lbf}}{\text{in}^2}$$

$$= -347 \ \text{lbf/in}^2$$

$$f_{te} = f_c + \frac{M_O}{S} = 347 \ \frac{\text{lbf}}{\text{in}^2} + 694 \ \frac{\text{lbf}}{\text{in}^2}$$

$$= 1041 \ \text{lbf/in}^2$$

7. STATICALLY INDETERMINATE STRUCTURES

Nomenclature

e'	resultant cable eccentricity	in
m	moment produced by unit value of the redundant	in-kips
M_R	resultant moment due to prestressing force and secondary effects, $Pe + M_S$	in-kips
M_S	moment produced by secondary effects	in-kips
R	reaction, support restraint	kips

Design Principles

Prestressing an indeterminate structure may result in secondary moments, due to the support restraints, that produce the resultant moment

$$M_R = Pe + M_S$$

Figure 3.26 *Continuous Beam*
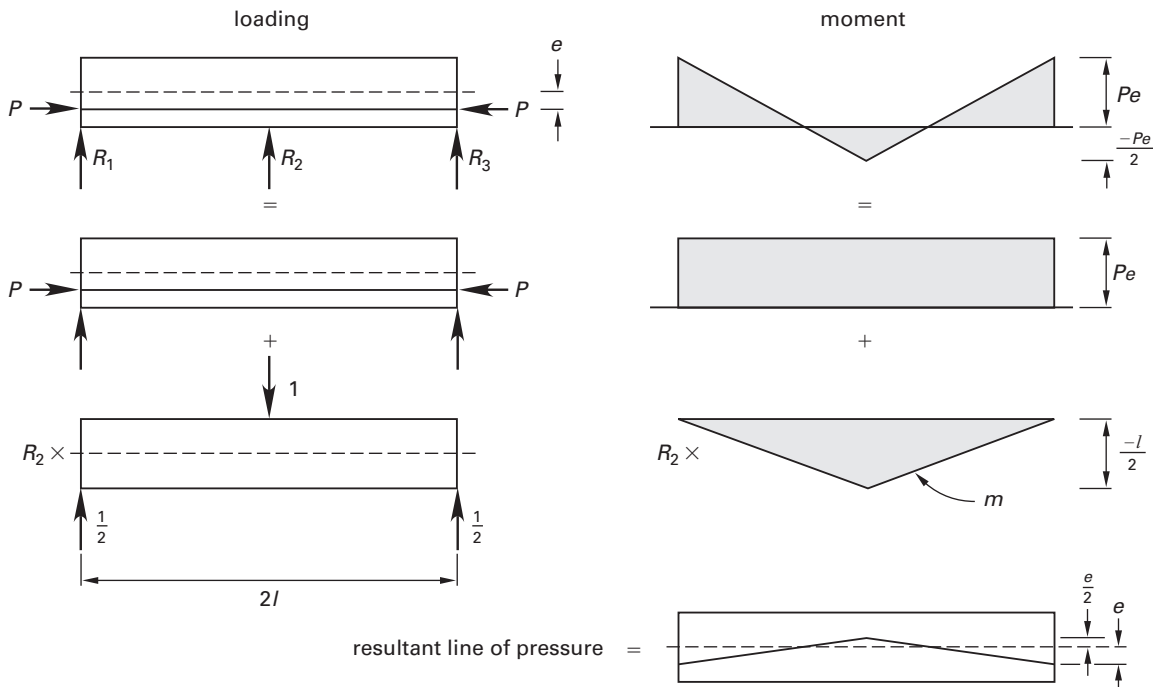
In the two-span beam shown in Fig. 3.26, the support restraint R_2 is taken as the redundant and a release introduced at 2 to produce the cut-back structure. Applying the prestressing force to the cut-back structure produces the primary moment $M_P = Pe$. Applying the unit value of R_2 to the cut-back structure produces the moment diagram, m, and the secondary moment is $M_S = R_2 m$. From the compatibility of displacements,

$$\frac{Pe}{EI} \int m \, dx = -\frac{R_2}{EI} \int m^2 \, dx$$

$$m = \frac{x}{2}$$

$$\frac{Pel^2}{2} = -\frac{R_2 l^3}{6}$$

$$R_2 = -\frac{3Pe}{l}$$

The secondary moment is given by

$$M_S = R_2 m = -\frac{3Pex}{2l}$$

The primary moment is given by

$$M_P = Pe$$

The resultant moment is given by

$$M_R = M_P + M_S = Pe - \frac{3Pex}{2l}$$

The resultant line of pressure, as shown in Fig. 3.26, is given by

$$e' = \frac{M_R}{P} = e - \frac{3ex}{2l}$$

At midspan,

$$M_{S(\text{midspan})} = -\frac{3Pe}{2}$$

$$M_{R(\text{midspan})} = Pe - \frac{3Pe}{2} = -\frac{Pe}{2}$$

$$e'_{(\text{midspan})} = e - \frac{3e}{2} = -\frac{e}{2}$$

A tendon with an initial eccentricity of e' produces no secondary effects in the member and is termed the concordant cable. Similarly, as shown in Fig. 3.27, the bending moment diagram for the external loads on a continuous beam is also a concordant profile because no support restraints will be produced. In addition, a concordant profile may be modified by means of a linear transformation by varying the location of the tendon at interior supports, as shown in Fig. 3.27, without changing the resultant moment.

Example 3.23

The post-tensioned two-span beam supports a uniformly distributed load, including the weight of the beam, of 0.75 kip/ft. The tendon profile is parabolic in each span and is located in span 12 as indicated.

(a) Determine the prestressing force required in the tendon and the required sag of the tendon in span 23 to balance the applied load exactly. Then, determine the resulting stress in the beam.

(b) Determine the stresses in the beam at the central support when an additional distributed load of 0.75 kip/ft is applied to the beam. Then, determine the location of the resultant line of pressure at the central support.

Figure 3.27 *Concordant Tendon Profile*
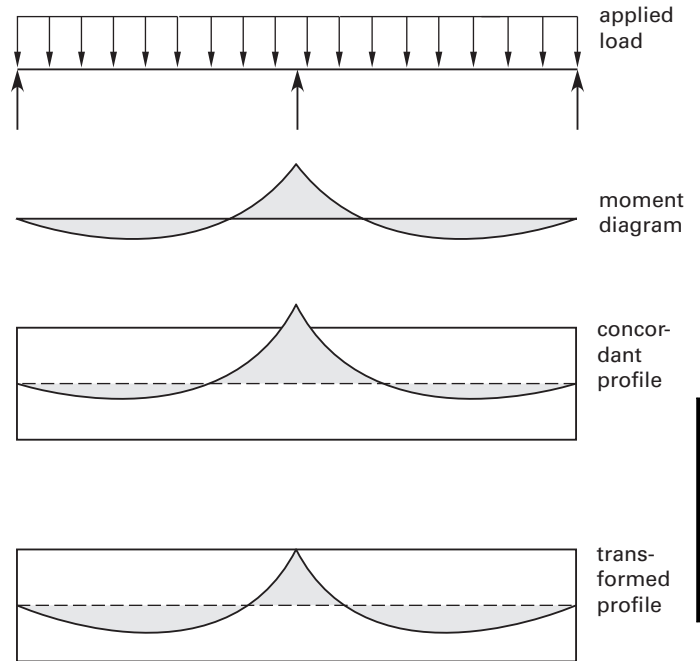
Solution

(a) The sag of the tendon in span 12 is given by

$$g_{12} = e_4 + \frac{e_2}{2} = 3.65 \text{ in} + \frac{10.7 \text{ in}}{2}$$

$$= 9 \text{ in}$$

The prestressing force required to balance exactly the applied load in span 12 is

$$P = \frac{w_w l_{12}^2}{8 g_{12}}$$

$$= \frac{\left(0.75 \dfrac{\text{kip}}{\text{ft}}\right)(30 \text{ ft})^2 \left(12 \dfrac{\text{in}}{\text{ft}}\right)}{(8)(9 \text{ in})}$$

$$= 112.5 \text{ kips}$$

Illustration for Ex. 3.23
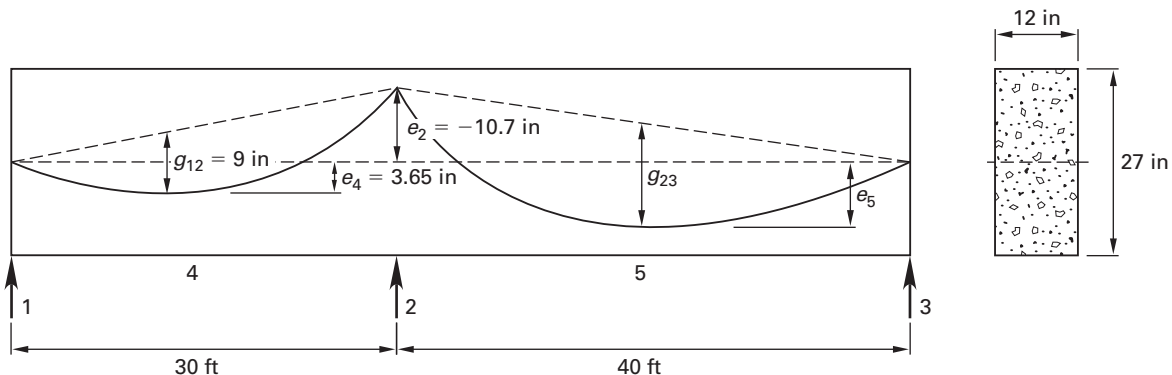
The required sag of the tendon in span 23 is given by

$$g_{23} = \frac{w_w l_{23}^2}{8P}$$

$$= \frac{\left(0.75\ \dfrac{\text{kip}}{\text{ft}}\right)(40\text{ ft})^2 \left(12\ \dfrac{\text{in}}{\text{ft}}\right)}{(8)(112.5\text{ kips})}$$

$$= 16\text{ in}$$

$$e_5 = g_{23} - \frac{e_2}{2} = 16\text{ in} - \frac{10.7\text{ in}}{2}$$

$$= 10.65\text{ in}$$

The uniform compressive stress throughout the beam is

$$f_c = \frac{P}{A_g} = \frac{112{,}500\text{ lbf}}{324\text{ in}^2}$$

$$= 347\text{ lbf/in}^2$$

(b) Allowing for the hinges at supports 1 and 3, the fixed-end moments produced by the additional load of 0.75 kip/ft are

$$M_{F21} = \frac{w_w l_{12}^2}{8} \qquad \text{[clockwise]}$$

$$= Pg_{12}$$

$$= (112.5\text{ kips})(9\text{ in})$$

$$= 1013\text{ in-kips}$$

$$M_{F23} = -Pg_{23} \qquad \text{[counterclockwise]}$$

$$= -(112.5\text{ kips})(16\text{ in})$$

$$= -1800\text{ in-kips}$$

The fixed-end moments are distributed as shown in the table, allowing for the hinges at the supports to eliminate carryover to ends 1 and 3.

joint	2	
member	21	23
relative $\frac{EI}{l}$	$\frac{3}{30}$	$\frac{3}{40}$
distribution factors	$\frac{4}{7}$	$\frac{3}{7}$
FEM	1013	−1800
distribution	449	+337
final moments	1463	−1463

The final moment at support 2 due to the distributed load is

$$M_{O2} = 1463\text{ in-kips} \quad (1{,}463{,}000\text{ in-lbf})$$

The resultant stresses at the central support are

$$f_{te} = f_c - \frac{M_{O2}}{S}$$

$$= 347\ \frac{\text{lbf}}{\text{in}^2} - \frac{1{,}463{,}000\text{ in-lbf}}{1458\text{ in}^3}$$

$$= -656\text{ lbf/in}^2$$

$$f_{be} = f_c + \frac{M_{O2}}{S}$$

$$= 347\ \frac{\text{lbf}}{\text{in}^2} + \frac{1{,}463{,}000\text{ in-lbf}}{1458\text{ in}^3}$$

$$= 1350\text{ lbf/in}^2$$

The location of the resultant line of pressure at the central support is

$$e_2' = -\frac{M_{O2}}{P} = -\frac{1463\ \dfrac{\text{in}}{\text{kips}}}{112.5\text{ kips}} = -13\text{ in}$$

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Prestressed Concrete

PRACTICE PROBLEMS

1. The pretensioned beam of normal weight concrete shown is simply supported over a span of 20 ft and has a concrete strength at transfer of 4500 lbf/in^2. What are the magnitude and the location of the initial prestressing force required to produce satisfactory stresses at midspan, immediately after transfer, without using auxiliary reinforcement?

(A) $P_i = 90$ kips, $e = 2.82$ in

(B) $P_i = 95$ kips, $e = 2.75$ in

(C) $P_i = 100$ kips, $e = 2.90$ in

(D) $P_i = 105$ kips, $e = 2.99$ in

2. The class U pretensioned beam in Prob. 1 has a long-term loss in prestress of 25% and a 28-day compressive strength of 6000 lbf/in^2; normal cover is provided to the tendons. The maximum bending moment the beam can carry if all the superimposed load is sustained is most nearly

(A) 350,000 in-lbf

(B) 360,000 in-lbf

(C) 390,000 in-lbf

(D) 400,000 in-lbf

3. For the pretensioned beam in Prob. 1, the cracking moment strength is most nearly

(A) 380 in-kips

(B) 410 in-kips

(C) 430 in-kips

(D) 470 in-kips

4. The pretensioned beam in Prob. 1 is prestressed with low-relaxation tendons. The area of the low-relaxation prestressing tendons provided is 0.306 in^2 with a specified tensile strength of 270 kips/in^2 and a yield strength of 243 kips/in^2. The nominal flexural strength of the beam is most nearly

(A) 470 in-kips

(B) 530 in-kips

(C) 570 in-kips

(D) 600 in-kips

5. The pretensioned beam of Prob. 1 supports two concentrated loads each of 2.5 kips.

(a) The prestressing force required in the tendons to balance the applied loads is most nearly

(A) 40 kips

(B) 50 kips

(C) 60 kips

(D) 70 kips

(b) The resulting stress in the beam is most nearly

(A) 885 lbf/in^2

(B) 890 lbf/in^2

(C) 900 lbf/in^2

(D) 930 lbf/in^2

6. For the beam of Prob. 5, what are the stresses in the beam at midspan due to the loads, W, and self-weight?

(A) $f_{be} = 570$ lbf/in^2; $f_{te} = 1200$ lbf/in^2

(B) $f_{be} = 585$ lbf/in^2; $f_{te} = 1210$ lbf/in^2

(C) $f_{be} = 590$ lbf/in^2; $f_{te} = 1220$ lbf/in^2

(D) $f_{be} = 610$ lbf/in^2; $f_{te} = 1160$ lbf/in^2

Illustration for Prob. 1
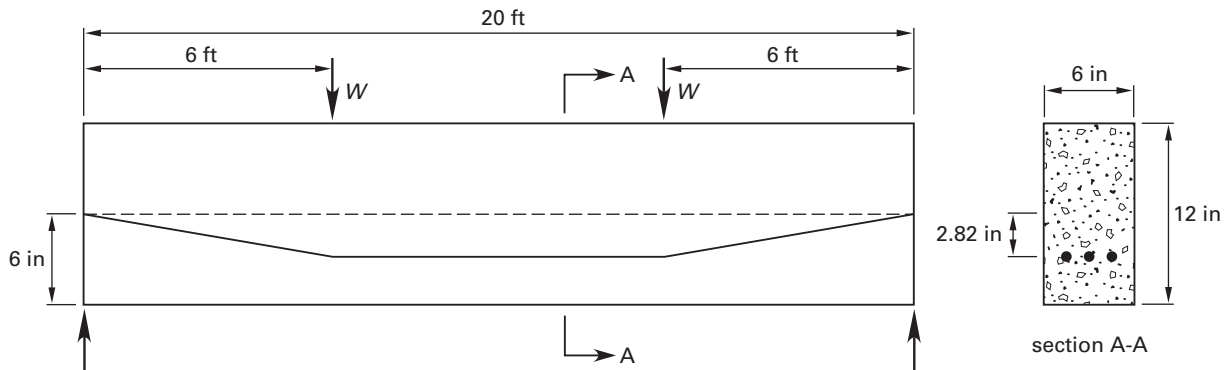
SOLUTIONS

1. The properties of the concrete section are

$$A_g = 72 \text{ in}^2$$
$$S_t = S_b = 144 \text{ in}^3$$

At midspan, the self-weight moment is

$$\begin{aligned}
M_G &= \frac{w_c A_g l^2}{8} \\
&= \frac{\left(150 \; \frac{\text{lbf}}{\text{ft}^3}\right)\left(\frac{72}{144 \text{ ft}^2}\right)(20 \text{ ft})^2\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{8} \\
&= 45,000 \text{ in-lbf}
\end{aligned}$$

At midspan, the permissible tensile stress in the top fiber without auxiliary reinforcement is given by ACI Sec. 18.4.1 as

$$\begin{aligned}
f_{ti} &= -3\sqrt{f'_{ci}} = -3\sqrt{4500 \; \frac{\text{lbf}}{\text{in}^2}} \\
&= -201 \text{ lbf/in}^2 \\
&= \frac{P_i}{A_g} - \frac{P_i e}{S_t} + \frac{M_G}{S_t} \\
&= \frac{P_i}{72 \text{ in}^2} - \frac{P_i e}{144 \text{ in}^3} + \frac{45,000 \text{ in-lbf}}{144 \text{ in}^3} \\
-514 \; \frac{\text{lbf}}{\text{in}^2} &= \frac{P_i}{72 \text{ in}^2} - \frac{P_i e}{144 \text{ in}^3} \qquad \text{[Eq. 1]}
\end{aligned}$$

At midspan, the permissible compressive stress in the bottom fiber is given by ACI Sec. 18.4.1 as

$$\begin{aligned}
f_{bi} &= 0.6 f'_{ci} = 2700 \text{ lbf/in}^2 \\
&= \frac{P_i}{A_g} + \frac{P_i e}{S_b} - \frac{M_G}{S_b} \\
&= \frac{P_i}{72 \text{ in}^2} + \frac{P_i e}{144 \text{ in}^3} - \frac{45,000 \text{ in-lbf}}{144 \text{ in}^3} \\
3013 \; \frac{\text{lbf}}{\text{in}^2} &= \frac{P_i}{72 \text{ in}^2} + \frac{P_i e}{144 \text{ in}^3} \qquad \text{[Eq. 2]}
\end{aligned}$$

Solving Eq. [1] and Eq. [2] gives

$$P_i = 90 \text{ kips}$$
$$e = 2.82 \text{ in}$$

The answer is (A).

2. The permissible compressive stress at midspan, in the top fiber, due to the sustained load is

$$\begin{aligned}
f_{te} &= 0.45 f'_c \\
&= (0.45)\left(6000 \; \frac{\text{lbf}}{\text{in}^2}\right) \\
&= 2700 \text{ lbf/in}^2 \\
&= 0.75 P_i \left(\frac{1}{A_g} - \frac{e}{S_t}\right) + \frac{M_G}{S_t} + \frac{M_T}{S_t} \\
&= (0.75)\left(-514 \; \frac{\text{lbf}}{\text{in}^2}\right) + 313 \; \frac{\text{lbf}}{\text{in}^2} + \frac{M_T}{144 \text{ in}^3} \\
M_T &= 399,240 \text{ in-lbf}
\end{aligned}$$

The permissible tensile stress at midspan, in the bottom fiber, due to the total load is

$$\begin{aligned}
f_{be} &= -7.5\sqrt{f'_c} \\
&= -7.5\sqrt{6000 \; \frac{\text{lbf}}{\text{in}^2}} \\
&= -581 \text{ lbf/in}^2 \\
&= 0.75 P_i\left(\frac{1}{A_g} + \frac{e}{S_b}\right) - \frac{M_G}{S_b} - \frac{M_T}{S_b} \\
&= (0.75)\left(3013 \; \frac{\text{lbf}}{\text{in}^2}\right) - 313 \; \frac{\text{lbf}}{\text{in}^2} - \frac{M_T}{144 \text{ in}^3} \\
M_T &= 363,996 \text{ in-lbf} \quad (360,000 \text{ in-lbf}) \quad \text{[governs]}
\end{aligned}$$

The answer is (B).

3. The modulus of rupture is

$$\begin{aligned}
f_r &= 7.5\lambda\sqrt{f'_c} \\
&= (7.5)(1.0)\sqrt{6000 \; \frac{\text{lbf}}{\text{in}^2}} \\
&= 581 \text{ lbf/in}^2
\end{aligned}$$

The cracking moment strength is

$$\begin{aligned}
M_{cr} &= S_b(P_e R_b + f_r) \\
&= \frac{(144 \text{ in}^3)\left((0.75)\left(3013 \; \frac{\text{lbf}}{\text{in}^2}\right) + 581 \; \frac{\text{lbf}}{\text{in}^2}\right)}{1000 \; \frac{\text{lbf}}{\text{kip}}} \\
&= 409 \text{ in-kips} \quad (410 \text{ in-kips})
\end{aligned}$$

The answer is (B).

4. The relevant properties of the beam are

$$\gamma_p = 0.28 \quad [\text{for } f_{py}/f_{pu} \geq 0.9]$$

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{0.306 \text{ in}^2}{(6 \text{ in})(8.82 \text{ in})}$$

$$= 0.00578$$

$$\beta_1 = 0.75 \quad [\text{from ACI Sec. 10.2.7.3}]$$

From ACI Eq. 18-1,

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p \rho_p f_{pu}}{\beta_1 f'_c}\right)$$

$$= \left(270 \ \frac{\text{kips}}{\text{in}^2}\right)\left(1 - \frac{\begin{array}{c}(0.28)(0.00578)\\ \times \left(270 \ \frac{\text{kips}}{\text{in}^2}\right)\end{array}}{(0.75)\left(6 \ \frac{\text{kips}}{\text{in}^2}\right)}\right)$$

$$= 244 \text{ kips/in}^2$$

The depth of the stress block is given by

$$a = \frac{A_{ps}f_{ps}}{0.85f'_c b} = \frac{(0.306 \text{ in}^2)\left(244 \ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(6 \ \frac{\text{kips}}{\text{in}^2}\right)(6 \text{ in})}$$

$$= 2.44 \text{ in}$$

The maximum depth of the stress block for a tension-controlled section is given by ACI Sec. 10.3.4 as

$$a_t = 0.375\beta_1 d_p = (0.375)(0.75)(8.82 \text{ in})$$

$$= 2.48 \text{ in}$$

$$> a$$

Therefore, the section is tension-controlled and $\phi = 0.9$.

The maximum nominal flexural strength is

$$M_n = (0.85f'_c ab)\left(d_p - \frac{a}{2}\right)$$

$$= (0.85)\left(6 \ \frac{\text{kips}}{\text{in}^2}\right)(2.44 \text{ in})(6 \text{ in})$$

$$\times \left(8.82 \text{ in} - \frac{2.44 \text{ in}}{2}\right)$$

$$= 567 \text{ in-kips} \quad (570 \text{ in-kips})$$

The answer is (C).

5. The sag of the tendon is

$$g = 2.82 \text{ in}$$

(a) The prestressing force required in the tendons to balance the applied load exactly is

$$P = \frac{Wa}{g}$$

$$= \frac{(2.5 \text{ kips})(72 \text{ in})}{2.82 \text{ in}}$$

$$= 63.8 \text{ kips} \quad (60 \text{ kips})$$

The answer is (C).

(b) The uniform compressive stress throughout the beam is

$$f_c = \frac{P}{A_g} = \frac{63{,}800 \text{ lbf}}{72 \text{ in}^2}$$

$$= 886 \text{ lbf/in}^2 \quad (885 \text{ lbf/in}^2)$$

The answer is (A).

6. The moment produced at midspan by the beam self-weight is

$$M_G = \frac{w_G l^2}{8}$$

$$= \frac{\left(75 \ \frac{\text{lbf}}{\text{ft}}\right)(20 \text{ ft})^2\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{8}$$

$$= 45{,}000 \text{ in-lbf}$$

The resultant stresses in the beam at midspan are

$$f_{be} = f_c - \frac{M_G}{S}$$

$$= 886 \ \frac{\text{lbf}}{\text{in}^2} - \frac{45{,}000 \text{ in-lbf}}{144 \text{ in}^3}$$

$$= 573 \text{ lbf/in}^2 \quad (570 \text{ lbf/in}^2)$$

$$f_{te} = f_c + \frac{M_G}{S}$$

$$= 886 \ \frac{\text{lbf}}{\text{in}^2} + 313 \ \frac{\text{lbf}}{\text{in}^2}$$

$$= 1199 \text{ lbf/in}^2 \quad (1200 \text{ lbf/in}^2)$$

The answer is (A).

4 Structural Steel Design

1. INTRODUCTION

The *Specification for Structural Steel Buildings*[1] (AISC 360) permits the design of steel buildings by either the load and resistance factor design (LRFD) method or the allowable strength design (ASD) method.

In the LRFD method, factored loads using LRFD load combinations are applied to a member to determine the required ultimate strength. This required ultimate strength must not exceed the design strength, which is calculated as the member nominal strength multiplied by a resistance factor.

The *allowable stress design* method was traditionally used to design steel structures. However, this method has been superseded by the *allowable strength design* method. In the allowable stress design method, nominal loads are applied to a member, and the calculated stress in the member cannot exceed the member's specified allowable stress. In general, the allowable stress is determined as the yield stress of the member divided by a safety factor. The effects of transient loads, such as wind and seismic loads, are allowed for by increasing allowable stresses by $1/3$ above the normal value.

In the *allowable strength design* (ASD) method, factored loads, using ASD load combinations, are applied to a member to determine the required strength. This required strength must not exceed the allowable strength, which is calculated as the member nominal strength divided by a safety factor. The LRFD *resistance factor* is designated by the symbol ϕ, and the ASD *safety factor* is designated by the symbol Ω. The load factors in the two methods are calibrated to

provide similar results at a live-to-dead load ratio of 3. The relationship between the resistance factor and safety factor is

$$\Omega = \frac{1.5}{\phi}$$

2. LOAD COMBINATIONS

Nomenclature

D	dead loads	kips or kips/ft
E	earthquake load	kips or kips/ft
H	load due to lateral pressure	kips/ft^2
L	live loads due to occupancy	kips or kips/ft
L_r	roof live load	kips or kips/ft
Q	load effect produced by service load	kips
R	load due to rainwater or ice	kips or kips/ft
R_n	nominal strength	–
S	snow load	kips or kips/ft
U	required strength to resist factored loads	–
W	wind load	kips or kips/ft

Symbols

γ	load factor	–
ϕ	resistance factor	–
Ω	safety factor	–

LRFD Required Strength

The required ultimate strength of a member consists of the most critical combination of factored loads applied to the member. Factored loads consist of working, or service, loads multiplied by the appropriate load factors.

Seismic and wind loads specified in the IBC are at the strength design level, in contrast to other loads that are at the service level. In the ASD load combinations, the load factor for seismic loads is 0.7 and the load factor for wind loads is 0.6, which reduces them to service level values. In the LRFD combinations, the load factor for both seismic and wind loads is 1.0.

In accordance with AISC 360[1] Sec. B2, load combinations shall be as stipulated by the applicable building code. The required strength, $\Sigma\gamma Q$, is defined by seven combinations in IBC[2] Sec. 1605.2.1. The combinations, with uncommon loads (self-straining loads and fluid pressure) omitted, are as follows.

$$\sum \gamma Q = 1.4D \qquad \text{[IBC 16-1]}$$

$$\sum \gamma Q = 1.2D + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$$
$$\text{[IBC 16-2]}$$

$$\sum \gamma Q = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + 1.6H$$
$$+ (f_1 L \text{ or } 0.5W) \qquad \text{[IBC 16-3]}$$

$$\sum \gamma Q = 1.2D + 1.0W + f_1 L + 1.6H$$
$$+ 0.5(L_r \text{ or } S \text{ or } R) \qquad \text{[IBC 16-4]}$$

$$\sum \gamma Q = 1.2D + 1.0E + f_1 L + f_2 S + 1.6H$$
$$\text{[IBC 16-5]}$$

$$\sum \gamma Q = 0.9D + 1.0W + 1.6H \qquad \text{[IBC 16-6]}$$

$$\sum \gamma Q = 0.9D + 1.0E + 1.6H \qquad \text{[IBC 16-7]}$$

For IBC Eq. 16-3, Eq. 16-4, and Eq. 16-5, use $f_1 = 1.0$ for garages, places of public assembly, and areas where $L > 100$ lbf/ft^2. Use $f_1 = 0.5$ for all other live loads. For IBC Eq. 16-5, use $f_2 = 0.7$ for roof configurations that do not shed snow, and use 0.3 for other roof configurations.

ASD Required Strength

The required strength of a member consists of the most critical combination of factored loads applied to the member. Factored loads consist of working, or service, loads multiplied by the appropriate load factors. In accordance with AISC 360 Sec. B2, load combinations must be as stipulated by the applicable building code. The required strength, $\sum \gamma Q$, is defined by nine combinations in IBC Sec. 1605.3.1. The combinations, with uncommon load conditions omitted, are as follows.

$$\sum \gamma Q = D \qquad \text{[IBC 16-8]}$$

$$\sum \gamma Q = D + L + H \qquad \text{[IBC 16-9]}$$

$$\sum \gamma Q = D + (L_r \text{ or } S \text{ or } R) + H \qquad \text{[IBC 16-10]}$$

$$\sum \gamma Q = D + 0.75L + H$$
$$+ 0.75(L_r \text{ or } S \text{ or } R) \qquad \text{[IBC 16-11]}$$

$$\sum \gamma Q = D + H + (0.6W \text{ or } 0.7E) \qquad \text{[IBC 16-12]}$$

$$\sum \gamma Q = D + H + 0.75(0.6W) + 0.75L$$
$$+ 0.75(L_r \text{ or } S \text{ or } R) \qquad \text{[IBC 16-13]}$$

$$\sum \gamma Q = D + H + 0.75(0.7E)$$
$$+ 0.75L + 0.75S \qquad \text{[IBC 16-14]}$$

$$\sum \gamma Q = 0.6D + 0.6W + H \qquad \text{[IBC 16-15]}$$

$$\sum \gamma Q = 0.6D + 0.7E + H \qquad \text{[IBC 16-16]}$$

Example 4.1

A typical frame of a six-story office building is shown. The loading on the frame is as follows.

roof dead load, including
    cladding and columns, w_{Dr} $= 1.2$ kips/ft
roof live load, w_{Lr} $= 0.4$ kip/ft
floor dead load, including
    cladding and columns, w_D $= 1.6$ kips/ft
floor live load, w_L $= 1.25$ kips/ft
horizontal wind pressure, p_h $= 1.0$ kip/ft
vertical wind pressure, p_v $= 0.5$ kip/ft

Determine the maximum and minimum required loads on the columns.
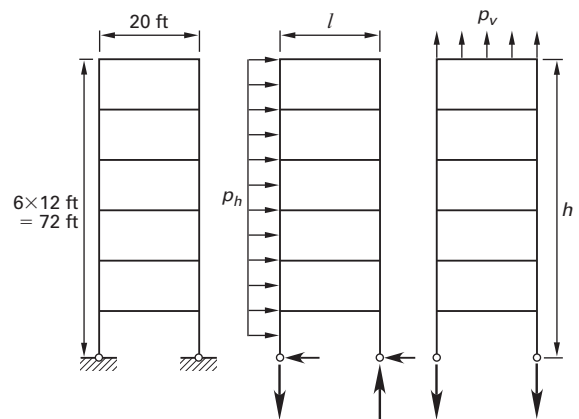
Solution

The axial load on one column due to the dead load is

$$D = \frac{l(w_{Dr} + 5w_D)}{2}$$
$$= \frac{(20 \text{ ft})\left(1.2 \dfrac{\text{kips}}{\text{ft}} + (5 \text{ stories})\left(1.6 \dfrac{\text{kips}}{\text{ft}}\right)\right)}{2}$$
$$= 92 \text{ kips}$$

The axial load on one column due to the roof live load is

$$L_r = \frac{l w_{Lr}}{2} = \frac{(20 \text{ ft})\left(0.4 \dfrac{\text{kip}}{\text{ft}}\right)}{2}$$
$$= 4 \text{ kips}$$

The axial load on one column due to the floor live load is

$$L = \frac{5 l w_L}{2} = \frac{(5)(20 \text{ ft})\left(1.25 \dfrac{\text{kips}}{\text{ft}}\right)}{2}$$
$$= 62.5 \text{ kips}$$

The axial load on one column due to the horizontal wind pressure is

$$W_h = \pm \frac{p_h h^2}{2l} = \pm \frac{\left(1.0 \ \dfrac{\text{kip}}{\text{ft}}\right)(72 \text{ ft})^2}{(2)(20 \text{ ft})}$$

$$= \pm 130 \text{ kips}$$

The axial load on one column due to the vertical wind pressure is

$$W_v = \frac{-p_v l}{2} = \frac{-\left(0.5 \ \dfrac{\text{kip}}{\text{ft}}\right)(20 \text{ ft})}{2}$$

$$= -5 \text{ kips}$$

LRFD Method

The terms H, S, and R are not applicable. From IBC Eq. 16-2, the maximum design load on a column is

$$\sum \gamma Q = 1.2D + 1.6L + 0.5L_r$$

$$= (1.2)(92 \text{ kips}) + (1.6)(62.5 \text{ kips})$$

$$+ (0.5)(4 \text{ kips})$$

$$= 212 \text{ kips} \quad [\text{compression}]$$

Alternatively, from IBC Eq. 16-4,

$$\sum \gamma Q = 1.2D + 1.0W + 0.5L + 0.5L_r$$

$$= (1.2)(92 \text{ kips}) + (1.0)(130 \text{ kips} - 5 \text{ kips})$$

$$+ (0.5)(62.5 \text{ kips}) + (0.5)(4 \text{ kips})$$

$$= 269 \text{ kips} \quad [\text{governs}]$$

From IBC Eq. 16-6, the minimum design load on a column is

$$\sum \gamma Q = 0.9D + 1.0W_h + 1.0W_v$$

$$= (0.9)(92 \text{ kips}) + (1.0)(-130 \text{ kips})$$

$$+ (1.0)(-5 \text{ kips})$$

$$= -52 \text{ kips} \quad [\text{tension}]$$

ASD Method

The terms H, S, and R are not applicable. From IBC Eq. 16-9, the maximum design load on a column is

$$\sum \gamma Q = D + L$$

$$= 92 \text{ kips} + 62.5 \text{ kips}$$

$$= 155 \text{ kips} \quad [\text{compression}]$$

Alternatively, from IBC Eq. 16-13,

$$\sum \gamma Q = D + 0.75(0.6W) + 0.75L + 0.75L_r$$

$$= 92 \text{ kips} + (0.75)(0.6)(130 \text{ kips} - 5 \text{ kips})$$

$$+ (0.75)(62.5 \text{ kips}) + (0.75)(4 \text{ kips})$$

$$= 198 \text{ kips} \quad [\text{governs}]$$

From IBC Eq. 16-15, the minimum design load on a column is

$$\sum \gamma Q = 0.6D + 0.6W_h + 0.6W_v$$

$$= (0.6)(92 \text{ kips}) + (0.6)(-130 \text{ kips})$$

$$+ (0.6)(-5 \text{ kips})$$

$$= -26 \text{ kips} \quad [\text{tension}]$$

LRFD Design Strength

The design strength of a member consists of the nominal, or theoretical ultimate, strength of the member, R_n, multiplied by the appropriate resistance factor, ϕ. The resistance factor is defined in AISC 360 as

$\phi_b = 0.90$ [for flexure]

$\phi_v = 1.0$ [for shear in webs of rolled I-shaped members]

$\phi_v = 0.90$ [for shear in all other flexural conditions]

$\phi_c = 0.90$ [for compression]

$\phi_t = 0.90$ [for tensile yielding]

$\phi_t = 0.75$ [for tensile fracture]

$\phi = 0.75$ [for shear rupture of bolts]

To ensure structural safety, AISC 360 Sec. B3 specifies that

$$\phi R_n \geq \sum \gamma Q$$

The *AISC Manual*[3] load tables incorporate the appropriate values of ϕ and provide a direct value of the design strength.

ASD Allowable Strength

The allowable strength of a member consists of the nominal strength of the member, R_n, divided by the appropriate safety factor, Ω. The safety factor is defined in AISC 360 as

$\Omega_b = 1.67$ [for flexure]

$\Omega_v = 1.5$ [for shear in webs of rolled I-shaped members]

$\Omega_v = 1.67$ [for shear in all other conditions]

$\Omega_c = 1.67$ [for compression]

$\Omega_t = 1.67$ [for tensile yielding]

$\Omega_t = 2.00$ [for tensile fracture]

$\Omega = 2.00$ [for shear rupture of bolts]

Steel

To ensure structural safety, AISC 360 Sec. B3 specifies that

$$\frac{R_n}{\Omega} \geq \sum \gamma Q$$

The *AISC Manual*[3] load tables incorporate the appropriate values of Ω and provide a direct value of the allowable strength.

Example 4.2

A pin-ended column of grade A50 steel 14 ft long is subjected to a factored axial load of $\sum \gamma Q = 450$ kips (LRFD) or 300 kips (ASD). Determine the lightest adequate W10 shape.

Solution

LRFD Method

From *AISC Manual* Table 4-1, for an effective height of 14 ft, a W10 × 49 column provides the design axial strength.

$$\phi R_n = \phi_c P_n$$
$$= 471 \text{ kips}$$
$$> \sum \gamma Q \quad [\text{satisfactory}]$$

ASD Method

From *AISC Manual* Table 4-1, for an effective height of 14 ft, a W10 × 49 column provides the allowable axial strength.

$$\frac{R_n}{\Omega} = \frac{P_n}{\Omega_c}$$
$$= 313 \text{ kips}$$
$$> \sum \gamma Q \quad [\text{satisfactory}]$$

3. DESIGN FOR FLEXURE

Nomenclature

BF	tabulated factor used to calculate the design flexural strength for unbraced lengths between L_p and L_r	–
C_b	lateral torsional buckling modification factor	–
F_r	compressive residual stress in the flange (10 kips/in^2 for rolled sections; 16.5 kips/in^2 for welded sections)	–
F_y	specified minimum yield stress	kips/in^2
I_y	moment of inertia about the y-axis	in^4
L_b	length between braces	ft or in
L_m	limiting laterally unbraced length for full plastic bending capacity ($C_b > 1.0$)	ft or in
L_p	limiting laterally unbraced length for full plastic bending capacity ($C_b = 1.0$)	ft or in
L_r	limiting laterally unbraced length for inelastic lateral torsional buckling	ft or in
M_a	required flexural strength	ft-kips
M_A	absolute value of moment at quarter point of the unbraced beam segment	ft-kips
M_B	absolute value of moment at centerline of the unbraced beam segment	ft-kips
M_C	absolute value of moment at three-quarter point of the unbraced beam segment	ft-kips
$M_{\max}$	absolute value of maximum moment in the unbraced beam segment	ft-kips
M_n	nominal flexural strength	ft-kips
M_p	plastic bending moment	ft-kips
M_r	$0.7 F_y S_x$	ft-kips
M_r	required bending moment	ft-kips
M_u	required flexural strength	ft-kips
M_y	yield moment	ft-kips
R_m	cross section monosymmetry parameter	–
S	elastic section modulus	in^3
Z	plastic section modulus	in^3

Symbols

λ_{pf}	limiting slenderness parameter for compact element	–
λ_{rf}	limiting slenderness parameter for noncompact element	–

Plastic Moment of Resistance

When a compact, laterally braced steel beam is loaded to the stage when the extreme fibers reach yield, as shown in Fig. 4.1(a), the applied moment, ignoring residual stresses, is given by

$$M_y = S F_y$$

Figure 4.1 *Stress Distribution in W Shape*

(a) initial yield   (b) plastic hinge
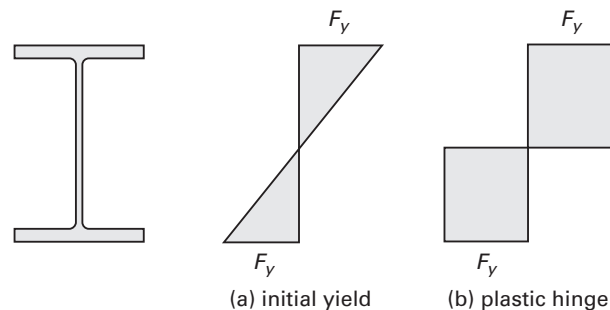
Taking into account the residual stress in the beam, the applied moment at first yielding is given by

$$M_r = 0.7 F_y S$$

Continued loading eventually results in the stress distribution shown in Fig. 4.1(b). A plastic hinge is formed,

and the beam cannot sustain any further increase in loading. The nominal strength of the member is given by

$$M_n = M_p = ZF_y$$

The shape factor is defined as

$$\frac{M_p}{M_y} = \frac{Z}{S} \approx 1.12 \quad \text{[for a W shape]}$$

A shape that is compact ensures that full plasticity will be achieved prior to flange or web local buckling. Compactness criteria are given in AISC 360 Table B4.1. Most W shapes are compact, and tabulated values of $\phi_b M_p$, in Part 3 of the *AISC Manual*, allow for any reduction due to noncompactness. Adequate lateral bracing of a member ensures that full plasticity will be achieved prior to lateral torsional buckling occurring.

Example 4.3

Determine the plastic section modulus and the shape factor for the steel section shown. Assume that the section is compact and adequately braced.

Solution

The properties of the elastic section are obtained as given in the following table.

part	A (in^2)	y (in)	I (in^4)	Ay (in^3)	Ay^2 (in^4)
flange	45.0	18.5	34	832	15,401
web	25.5	8.5	614	217	1842
total	70.5	–	648	1049	17,243

Illustration for Ex. 4.3
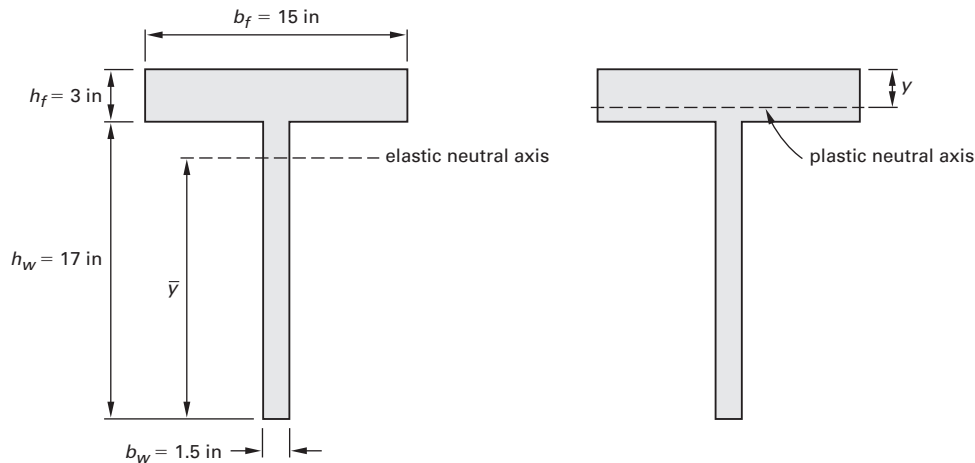
$$\overline{y} = \frac{\sum Ay}{\sum A} = \frac{1049 \text{ in}^3}{70.5 \text{ in}^2}$$

$$= 14.9 \text{ in}$$

$$I = \sum I + \sum Ay^2 + \overline{y}^2 \sum A - 2\overline{y} \sum Ay$$

$$= 648 \text{ in}^4 + 17,243 \text{ in}^4 + 15,610 \text{ in}^4$$

$$\quad - 31,260 \text{ in}^4$$

$$= 2241 \text{ in}^4$$

$$S_b = \frac{I}{\overline{y}} = \frac{2241 \text{ in}^4}{14.9 \text{ in}}$$

$$= 150 \text{ in}^3$$

The location of the plastic neutral axis is obtained by equating areas above and below the axis. The depth of the plastic neutral axis is given by

$$yb_f = (h_f - y)b_f + h_w b_w$$

$$y = \frac{\sum A}{2b_f} = \frac{70.5 \text{ in}^2}{(2)(15 \text{ in})}$$

$$= 2.35 \text{ in}$$

The plastic section modulus is obtained by taking moments of areas about the plastic neutral axis.

$$Z = \frac{y^2 b_f}{2} + \frac{(h_f - y)^2 b_f}{2} + A_w \left(\frac{h_w}{2} + h_f - y\right)$$

$$= \frac{(2.35 \text{ in})^2 (15 \text{ in})}{2} + \frac{(0.65 \text{ in})^2 (15 \text{ in})}{2}$$

$$\quad + (25.5 \text{ in}^2)(9.15 \text{ in})$$

$$= 278 \text{ in}^3$$

The shape factor is

$$\frac{Z}{S} = \frac{278}{150} = 1.85$$

Nominal Flexural Strength

Nominal flexural strength is influenced by several factors.

- *Plastic moment strength:* $M_n \leq M_p$

- *Flange local buckling:* a slender flange is prone to local buckling and most rolled I-shapes are compact for a yield stress of $F_y \leq 50$ kips/in^2; the exceptions are W21 $\times$ 48, W14 $\times$ 99, W14 $\times$ 90, W12 $\times$ 65, W10 $\times$ 12, W8 $\times$ 31, W8 $\times$ 10, W6 $\times$ 15, W6 $\times$ 9, W6 $\times$ 8.5, and M4 $\times$ 6

- *Web local buckling:* a slender web is prone to local buckling and all rolled I-shapes are compact for a yield stress of $F_y \leq 50$ kips/in^2; more common with thin web plate girders

- *Lateral-torsional buckling:* lateral bracing applied to the compression flange restrains the flange and prevents local buckling; governs when L_b exceeds L_p and $C_b = 0$

- *Lateral-torsional buckling modification factor, C_b:* a beam with a uniform bending moment has $C_b = 1.0$; for other moment gradients, $C_b > 1.0$ and lateral-buckling strength is obtained by multiplying basic strength by C_b, with a maximum permitted value of M_p for the lateral-buckling strength

Tabulated values of $\phi_b M_p$ and M_p/Ω_b in *AISC Manual* Table 3-2 are based on a value of $C_b = 1.0$ and allow for reduction in M_n due to slenderness effects.

Compact, Noncompact, and Slender Sections

Steel beams are classified as compact, noncompact, and slender, in accordance with the slenderness criteria in AISC 360 Table B4.1b. The flexural capacity of an adequately braced beam depends on the slenderness ratio of the compression flange and the web. When the slenderness ratios are sufficiently small, the beam can attain its full plastic moment, and the cross section is classified as compact. When the slenderness ratios are larger, the compression flange or the web may buckle locally before a full plastic moment is attained, and the cross section is classified as noncompact. When the slenderness ratios are sufficiently large, local buckling will occur before the yield stress of the material is reached, and the cross section is classified as slender. The flexural response of the three classifications is shown in Fig. 4.2.

Figure 4.2 *Variation of M_n with λ*
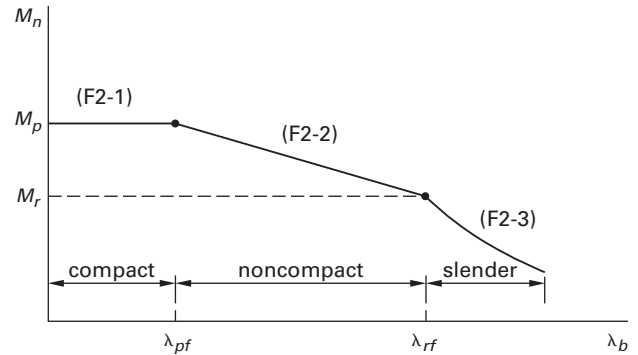
The limiting slenderness parameters for three flange buckling modes are as follows. (See AISC 360 Table B4.1b, Case 10.)

- *Compact flange:* a beam flange with a slenderness parameter of $\lambda = b_f/2t_f < \lambda_{pf}$. The limiting slenderness parameter for a compact flange is

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}}$$

An adequately braced beam will develop its plastic moment of resistance before the onset of lateral-torsional and local buckling. The nominal flexural strength of the compact section is given by AISC 360 Eq. F2-1 as

$$M_n = M_p = F_y Z_x$$

- *Noncompact flange:* a beam flange with a slenderness parameter of $\lambda_{pf} \leq \lambda \leq \lambda_{rf}$. The limiting slenderness parameter for a noncompact flange is

$$\lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}}$$

A beam with an unbraced length, L_r, will develop its yield moment, M_r, coincident with yielding of the beam's extreme fibers and before the onset of lateral-torsional and local buckling. Allowing for residual stresses in the beam, the yield moment is

$$M_r = 0.7F_y S_x$$

The limits of M_n are given by

$$M_r \leq M_n < M_p$$

The nominal flexural strength of the noncompact section is given by AISC 360 Eq. F3-1 as

$$M_n = M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)$$

- *Slender flange:* a beam flange with the slenderness parameter defined by $\lambda < \lambda_{rf}$. A slender section is one that cannot develop the yield stress prior to flange local buckling. The nominal flexural strength

of a section with slender flanges, with adequate lateral bracing, is given by AISC 360 Eq. F3-2 as

$$M_n = \frac{0.9Ek_cS_x}{\lambda^2}$$

The value of k_c is limited to

$$k_c = \frac{4}{\sqrt{\dfrac{h}{t_w}}}$$

$$\geq 0.35$$

$$\leq 0.76$$

Example 4.4

Determine the design flexural strength and allowable flexural strength of a W14 × 90 with a yield stress of 50 kips/in² and bent about its major axis. $L_b < L_p$, so lateral-torsional buckling will not govern.

Solution

The limiting slenderness parameter for the flange of a rolled I-shape is given by AISC 360 Table B4.1b as

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}}$$

$$= 0.38\sqrt{\frac{29{,}000\ \dfrac{\text{kips}}{\text{in}^2}}{50\ \dfrac{\text{kips}}{\text{in}^2}}}$$

$$= 9.15$$

From *AISC Manual* Table 1-1, a W14 × 90 has a value of

$$\lambda = \frac{b_f}{2t_f} = 10.2$$

$$> 9.15 \quad [\text{flange is not compact}]$$

A W14 × 90 with a yield stress of 50 kips/in² is not a compact section. From *AISC Manual* Table 1-1, the elastic section modulus about the *x*-axis is

$$S_x = 143\ \text{in}^3$$

The yield moment considering residual stresses is

$$M_r = 0.7F_yS_x$$

$$= \frac{(0.7)\left(50\ \dfrac{\text{kips}}{\text{in}^2}\right)(143\ \text{in}^3)}{12\ \dfrac{\text{in}}{\text{ft}}}$$

$$= 417\ \text{ft-kips}$$

The plastic moment of resistance is given by

$$M_p = Z_xF_y$$

$$= \frac{(157\ \text{in}^3)\left(50\ \dfrac{\text{kips}}{\text{in}^2}\right)}{12\ \dfrac{\text{in}}{\text{ft}}}$$

$$= 654\ \text{ft-kips}$$

From AISC 360 Table B4.1b, the limiting slenderness parameter for a noncompact flange is

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}}$$

$$= 1.0\sqrt{\frac{29{,}000\ \dfrac{\text{kips}}{\text{in}^2}}{50\ \dfrac{\text{kips}}{\text{in}^2}}}$$

$$= 24.1$$

From AISC 360 Eq. F3-1, the nominal flexural strength is given by

$$M_n = M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)$$

$$= 654\ \text{ft-kips} - (654\ \text{ft-kips} - 417\ \text{ft-kips})$$

$$\times \left(\frac{10.2 - 9.15}{24.1 - 9.15}\right)$$

$$= 637\ \text{ft-kips}$$

LRFD Method

The design flexural strength is

$$\phi_bM_n = (0.9)(637\ \text{ft-kips})$$

$$= 573\ \text{ft-kips}$$

ASD Method

The allowable flexural strength is

$$\frac{M_n}{\Omega_b} = \frac{637\ \text{ft-kips}}{1.67}$$

$$= 381\ \text{ft-kips}$$

These values are given in *AISC Manual* Table 3-2 as ϕ_bM_{px} and M_{px}/Ω_b.

Lateral-Torsional Buckling with C_b = 1.0

Lateral-torsional buckling of an I-shape occurs along the length of the beam between lateral supports. The compression flange tends to buckle similarly to a long column while the tension flange provides restraint. This results in lateral displacement of the compression flange and a torsional twisting of the cross section. An I-shape bent about its minor axis does not buckle. An HSS section bent about any axis does not buckle.

Steel

The maximum nominal moment capacity of a compact rolled I-shape is $M_n = M_p$. As shown in Fig. 4.3, when the unbraced length, L_b, between points of lateral support on a compact beam with $C_b = 1.0$ increases beyond the length, L_p, the nominal flexural strength of the beam decreases. The beam passes through the three phases: *plastic*, *inelastic*, and *elastic*. L_p is the limiting laterally unbraced length for full plastic flexural capacity with $C_b = 1.0$. L_r is the limiting laterally unbraced length for inelastic lateral torsional buckling. M_r is the limiting moment reached when $L_b = L_r$ and $M_r = 0.7 F_y S_x$.

Values of L_p, L_r, $\phi_b M_{px}$, M_{px}/Ω_b, $\phi_b M_{rx}$, and M_{rx}/Ω_b for beams with a yield stress of 50 kips/in^2 are given in *AISC Manual* Table 3-2.

Figure 4.3 *Variation of M_n with L_b for $C_b = 1.0$*
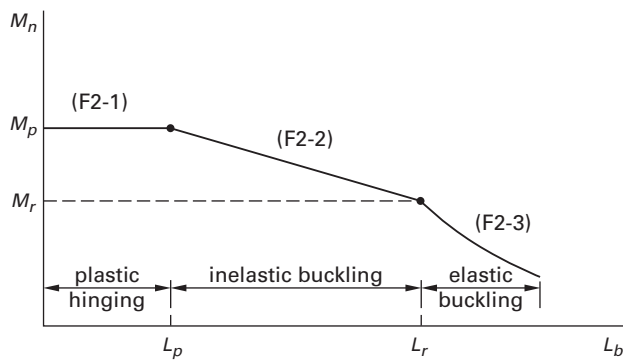
Plastic Phase: $L_b \leq L_p$

A compact section subjected to uniform bending moment will develop its full plastic moment capacity, provided that the laterally unsupported segment length is

$$L_b \leq L_p$$
$$M_n = M_p$$

The value of L_p is defined in AISC 360 Sec. F2.2 and is tabulated in *AISC Manual* Part 3. The values of $\phi_b M_{px}$ and M_{px}/Ω_b are also tabulated in Part 3.

Example 4.5

A simply supported beam of grade 50 steel is laterally braced at 4 ft intervals. If the beam is subjected to a uniform factored bending moment of 270 ft-kips (LRFD) or 180 ft-kips (ASD), with $C_b = 1.0$, determine (a) the lightest adequate W shape, and (b) the W shape with the minimum allowable depth.

Solution

LRFD Method

(a) From *AISC Manual* Table 3-2, the lightest satisfactory section is a W16 × 40, which has

$$\phi M_{px} = 274 \text{ ft-kips}$$
$$> 270 \text{ ft-kips} \quad [\text{satisfactory}]$$

$$L_p = 5.55 \text{ ft}$$
$$> 4 \text{ ft} \quad [\text{satisfactory}]$$

(b) From *AISC Manual* Table 3-2, the W shape with the minimum depth is a W10 × 60, which has

$$\phi_b M_{px} = 280 \text{ ft-kips}$$
$$> 270 \text{ ft-kips} \quad [\text{satisfactory}]$$
$$L_p = 9.08 \text{ ft}$$
$$> 4 \text{ ft} \quad [\text{satisfactory}]$$

ASD Method

(a) From *AISC Manual* Table 3-2, the lightest satisfactory section is a W16 × 40, which has

$$\frac{M_{px}}{\Omega_b} = 182 \text{ ft-kips}$$
$$> 180 \text{ ft-kips} \quad [\text{satisfactory}]$$
$$L_p = 5.55 \text{ ft}$$
$$> 4 \text{ ft} \quad [\text{satisfactory}]$$

(b) From *AISC Manual* Table 3-2, the W shape with the minimum depth is a W10 × 60, which has

$$\frac{M_{px}}{\Omega_b} = 186 \text{ ft-kips}$$
$$> 180 \text{ ft-kips} \quad [\text{satisfactory}]$$
$$L_p = 9.08 \text{ ft}$$
$$> 4 \text{ ft} \quad [\text{satisfactory}]$$

Inelastic Phase: $L_p < L_b \leq L_r$

When the laterally unsupported segment length equals L_r, the nominal flexural strength is given by

$$M_n = M_r = 0.7 F_y S_x$$

The value of L_r is defined in AISC 360 Eq. F2-6 and is tabulated in *AISC Manual* Part 3. The values of $\phi_b M_{rx}$ and M_{rx}/Ω_b are also tabulated in *AISC Manual* Part 3.

The nominal flexural strength for an unbraced length between L_p and L_r is obtained by linear interpolation between M_p and M_r and is given by AISC 360 Eq. F2-2 as

$$M_n = C_b \left(M_p - \frac{(M_p - M_r)(L_b - L_p)}{L_r - L_p} \right)$$

For LRFD,

$$\phi_b M_n = C_b\big(\phi_b M_{px} - (BF)(L_b - L_p)\big)$$

$$\leq \phi_b M_{px}$$

$$BF = \frac{\phi_b(M_p - M_r)}{L_r - L_p}$$

For ASD,

$$\frac{M_n}{\Omega_b} = C_b\left(\frac{M_{px}}{\Omega_b} - (BF)(L_b - L_p)\right)$$

$$\leq \frac{M_{px}}{\Omega_b}$$

$$BF = \frac{M_p - M_r}{\Omega_b(L_r - L_p)}$$

Values of BF are tabulated in *AISC Manual* Part 3. The variation of nominal flexural strength with unbraced length is shown in Fig. 4.3.

Example 4.6

A simply supported W16 × 40 beam of grade 50 steel is laterally braced at 6 ft intervals and is subjected to a uniform bending moment with $C_b = 1.0$. Determine the available flexural strength of the beam.

Solution

LRFD Method

From *AISC Manual* Table 3-2, a W16 × 40 has

$$\phi_b M_{px} = 274 \text{ ft-kips}$$

$$\phi_b M_{rx} = 170 \text{ ft-kips}$$

$$L_p = 5.55 \text{ ft}$$

$$< 6 \text{ ft}$$

$$L_r = 15.9 \text{ ft}$$

$$> 6 \text{ ft}$$

$$BF = 10.0 \text{ kips}$$

$$\phi_b M_n = C_b\big(\phi_b M_{px} - (BF)(L_b - L_p)\big)$$

$$= (1.0)\big(274 \text{ ft-kips} - (10.0 \text{ kips})(6 \text{ ft} - 5.55 \text{ ft})\big)$$

$$= 269 \text{ ft-kips} \quad \text{[design flexural strength]}$$

ASD Method

From *AISC Manual* Table 3-2, a W16 × 40 has

$$\frac{M_{px}}{\Omega_b} = 182 \text{ ft-kips}$$

$$\frac{M_{rx}}{\Omega_b} = 113 \text{ ft-kips}$$

$$L_p = 5.55 \text{ ft}$$

$$< 6 \text{ ft}$$

$$L_r = 15.9 \text{ ft}$$

$$> 6 \text{ ft}$$

$$BF = 6.67 \text{ kips}$$

$$\frac{M_n}{\Omega_b} = C_b\left(\frac{M_{px}}{\Omega_b} - (BF)(L_b - L_p)\right)$$

$$= (1.0)\big(182 \text{ ft-kips} - (6.67 \text{ kips})(6 \text{ ft} - 5.55 \text{ ft})\big)$$

$$= 179 \text{ ft-kips} \quad \text{[allowable flexural strength]}$$

Elastic Phase: $L_b > L_r$

When the laterally unsupported segment length exceeds L_r, the nominal flexural strength is governed by elastic lateral torsional buckling. The nominal flexural strength is equal to the critical elastic moment M_n and is defined in AISC 360 Eq. F2-3 as

$$M_n = F_{cr} S_x$$

Values of ϕM_n and M_n/Ω are graphed in *AISC Manual* Table 3-10.

Example 4.7

A simply supported beam of grade 50 steel has an unbraced length of 31 ft. Determine the lightest adequate W shape if the beam is subjected to a uniform factored bending moment of 190 ft-kips (LRFD) or 127 ft-kips (ASD) with $C_b = 1.0$.

Solution

LRFD Method

From *AISC Manual* Table 3-10, a W12 × 58 braced at 31 ft intervals has

$$\phi_b M_n = 196 \text{ ft-kips}$$

$$> 190 \text{ ft-kips} \quad \text{[satisfactory]}$$

ASD Method

From *AISC Manual* Table 3-10, a W12 × 58 braced at 31 ft intervals has

$$\frac{M_n}{\Omega_b} = 131 \text{ ft-kips}$$

$$> 127 \text{ ft-kips} \quad \text{[satisfactory]}$$

Steel

Lateral-Torsional Buckling Modification Factor

The lateral-torsional buckling modification factor, C_b, accounts for the influence of moment gradient on lateral torsional buckling. The value of C_b is defined in AISC 360 Sec. F1 as

$$C_b = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3 M_A + 4 M_B + 3 M_C} \quad \text{[AISC F1-1]}$$

A beam segment bent in single curvature and subjected to a uniform bending moment has a C_b value of 1.0. Other moment gradients increase the C_b value and increase the resistance of the beam to lateral torsional buckling. For any loading condition, the C_b value may conservatively be taken as 1.0; this is the value adopted in *AISC Manual* tables. When the C_b value exceeds 1.0, tabulated values of the design moment may be multiplied by C_b, with a maximum permitted value of the design moment of $\phi_b M_p$. For a beam with a compression flange continuously braced along its entire length, such as a beam supporting a composite deck slab, $C_b = 1.0$.

The terms used in determining the C_b value are illustrated in Fig. 4.4, and typical values are shown in Fig. 4.5.

Figure 4.4 Determination of C_b
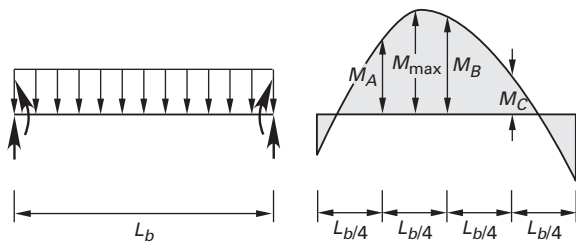
Figure 4.5 Typical Values of C_b
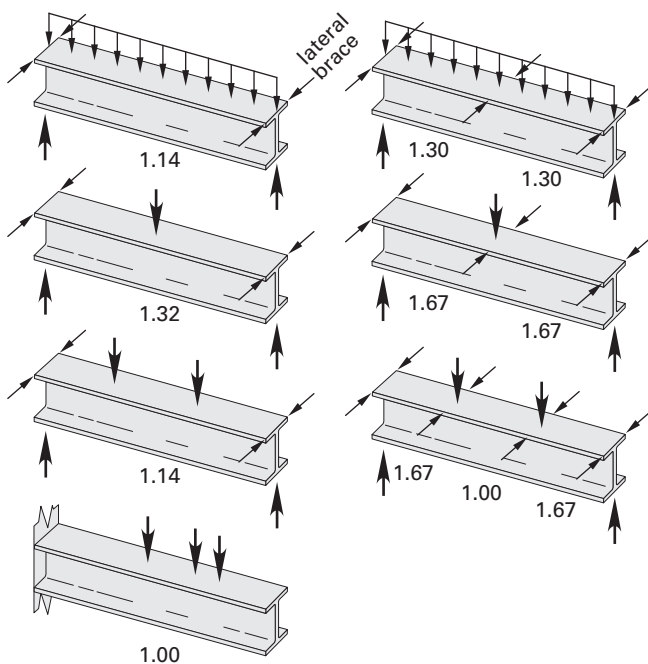
Example 4.8

The factored loads acting on a simply supported beam with a cantilever overhang, including the beam self-weight, are shown. The beam is laterally braced at the supports and at the end of the cantilever. Determine the relevant C_b values.
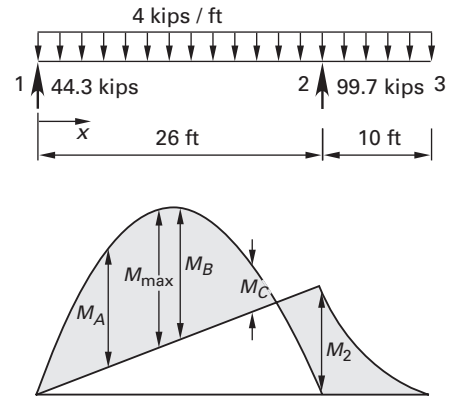
Solution

The bending moment at support 2 is

$$M_2 = \frac{wL^2}{2} = \frac{\left(4 \ \dfrac{\text{kips}}{\text{ft}}\right)(10 \ \text{ft})^2}{2}$$
$$= 200 \ \text{ft-kips}$$

The free bending moment in span 12 is

$$M_{\text{span}} = \frac{wL^2}{8} = \frac{\left(4 \ \dfrac{\text{kips}}{\text{ft}}\right)(26 \ \text{ft})^2}{8}$$
$$= 338 \ \text{ft-kips}$$

For the cantilever overhang, AISC 360 Sec. F1 specifies that

$$C_b = 1.0$$

For span 12, the relevant terms are

$$M_{12} = V_1 x - \frac{wx^2}{2}$$
$$= (44.3 \ \text{kips})x - \frac{\left(4 \ \dfrac{\text{kips}}{\text{ft}}\right)x^2}{2}$$
$$\frac{dM_{12}}{dx} = 44.3 \ \text{kips} - \left(4 \ \frac{\text{kips}}{\text{ft}}\right)x$$

M_{12} is a maximum at $x = 11.1$ ft.

$$M_{\max} = (44.3 \text{ kips})(11.1 \text{ ft}) - \frac{\left(4 \frac{\text{kips}}{\text{ft}}\right)(11.1 \text{ ft})^2}{2}$$

$$= 245 \text{ ft-kips}$$

$$M_A = 0.75M_{\text{span}} - 0.25M_2$$

$$= (0.75)(338 \text{ ft-kips}) - (0.25)(200 \text{ ft-kips})$$

$$= 204 \text{ ft-kips}$$

$$M_B = M_{\text{span}} - 0.5M_2$$

$$= 338 \text{ ft-kips} - (0.5)(200 \text{ ft-kips})$$

$$= 238 \text{ ft-kips}$$

$$M_C = 0.75M_{\text{span}} - 0.75M_2$$

$$= (0.75)(338 \text{ ft-kips}) - (0.75)(200 \text{ ft-kips})$$

$$= 104 \text{ ft-kips}$$

The lateral-torsional buckling modification factor is given by AISC 360 Eq. F1-1 as

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{(12.5)(245 \text{ ft-kips})}{(2.5)(245 \text{ ft-kips}) + (3)(204 \text{ ft-kips})}{+ (4)(238 \text{ ft-kips}) + (3)(104 \text{ ft-kips})}$$

$$= 1.23$$

Lateral-Torsional Buckling with $C_b > 1.0$

Figure 4.6 shows the effect of $C_b > 1.0$ on the relationship between L_b and M_n. The nominal moment in the elastic and inelastic regions is obtained by multiplying the tabulated nominal strength values in *AISC Manual* tables by C_b. The maximum permitted value of the nominal flexural capacity is limited to M_p.

Figure 4.6 *Variation of M_n with L_b for $C_b > 1.0$*
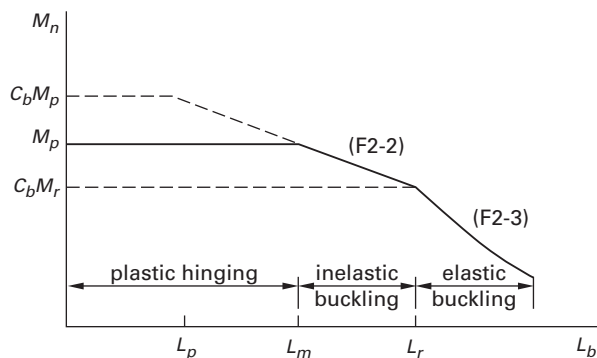
Plastic Phase: $L_b \le L_m$

For a beam with a value of C_b greater than 1.0, the unbraced length for a full plastic moment of resistance is extended beyond L_p to L_m, which is given by

$$L_m = L_p + \frac{(C_bM_p - M_p)(L_r - L_p)}{C_b(M_p - M_r)}$$

$$= L_p + \frac{\phi_b M_{px}(C_b - 1.0)}{C_b(BF)}$$

Inelastic Phase: $L_m < L_b \le L_r$

When L_b exceeds L_m and does not exceed L_r, collapse of the beam occurs prior to the development of the full plastic moment. When $L_b = L_r$, the nominal flexural strength of the beam is

$$M_n = C_bM_r = 0.7C_bF_yS_x$$

For a value of L_b between L_m and L_r, the nominal flexural strength is obtained by linear interpolation between M_r and M_p, and the design flexural strength is given by

$$\phi_bM_n = C_b\big(\phi_bM_{px} - (BF)(L_b - L_p)\big)$$

$$\frac{M_n}{\Omega_b} = C_b\left(\frac{M_{px}}{\Omega_b} - (BF)(L_b - L_p)\right)$$

Elastic Phase: $L_b > L_r$

Collapse of the beam occurs by elastic lateral-torsional buckling and when $M_n = F_{cr}S_x$.

Values of ϕM_n and M_n/Ω are graphed in *AISC Manual* Table 3-10.

Example 4.9

Determine the lightest adequate W shape for the beam of Ex. 4.8 using grade 50 steel. The ASD load is 2.67 kips/ft.

Solution

LRFD Method

For section 23, from *AISC Manual* Table 3-2, a W18 × 40 has

$$\phi_bM_{px} = 294 \text{ ft-kips}$$
$$\phi_bM_{rx} = 180 \text{ ft-kips}$$
$$L_p = 4.49 \text{ ft}$$
$$< 10 \text{ ft}$$
$$L_r = 13.1 \text{ ft}$$
$$> 10 \text{ ft}$$

$BF = 13.2$ kips

$$\phi_b M_n = C_b\big(\phi_b M_{px} - (BF)(L_b - L_p)\big)$$
$$= (1.0)\big(294 \text{ ft-kips} - (13.2 \text{ kips})(10 \text{ ft} - 4.49 \text{ ft})\big)$$
$$= 221 \text{ ft-kips}$$
$$> 200 \text{ ft-kips} \quad \text{[satisfactory]}$$

For section 12, the equivalent design flexural strength required is

$$\phi_b M_n = \frac{M_{\max}}{C_b} = \frac{245 \text{ ft-kips}}{1.23}$$
$$= 199 \text{ ft-kips}$$

From *AISC Manual* Table 3-10, a W12 × 58 with an unbraced length of 26 ft has

$$\phi_b M_n = 227 \text{ ft-kips}$$
$$> 199 \text{ ft-kips} \quad \text{[satisfactory]}$$

Therefore, the W12 × 58 governs the design.

ASD Method

$$M_2 = 133 \text{ ft-kips}$$
$$M_{\max} = 163 \text{ ft-kips}$$

For section 23, from *AISC Manual* Table 3-2, a W18 × 40 has

$$\frac{M_{px}}{\Omega_b} = 196 \text{ ft-kips}$$

$$\frac{M_{rx}}{\Omega_b} = 119 \text{ ft-kips}$$

$$L_p = 4.49 \text{ ft}$$
$$< 10 \text{ ft}$$

$$L_r = 13.1 \text{ ft}$$
$$> 10 \text{ ft}$$

$$BF = 8.94 \text{ kips}$$

$$\frac{M_n}{\Omega_b} = C_b\left(\frac{M_{px}}{\Omega_b} - (BF)(L_b - L_p)\right)$$
$$= (1.0)\big(196 \text{ ft-kips} - (8.94 \text{ kips})(10 \text{ ft} - 4.49 \text{ ft})\big)$$
$$= 147 \text{ ft-kips}$$
$$> 133 \text{ ft-kips} \quad \text{[satisfactory]}$$

For section 12, the equivalent design flexural strength required is

$$\frac{M_n}{\Omega_b} = \frac{M_{\max}}{C_b} = \frac{163 \text{ ft-kips}}{1.23}$$
$$= 133 \text{ ft-kips}$$

From *AISC Manual* Table 3-10, a W12 × 58 with an unbraced length of 26 ft has

$$\frac{M_n}{\Omega_b} = 151 \text{ ft-kips}$$
$$> 133 \text{ ft-kips} \quad \text{[satisfactory]}$$

Therefore, the W12 × 58 governs the design.

Moment Redistribution in Continuous Beams

AISC 360 Sec. B3.7 allows for the additional capacity that occurs in continuous beams after plastic hinges have formed because of redistribution of the bending moment. In an indeterminate structure, the formation of a single plastic hinge in the structure does not cause collapse of the structure. The structure can continue to support increasing load, while the moment at the hinge remains constant at a value of M_p and the moments at other locations in the structure continue to increase.

The method is applicable to beams with fixed ends and beams continuous over supports but not to rigid or pin-jointed frames. It is not applicable to simply supported beams or to cantilevers, as redistribution cannot occur in these members. Similarly, the method may be applied only to beams with compact sections, as noncompact sections have inadequate plastic hinge rotation capacity to permit redistribution of moments. The method may be applied to a beam-column, provided that the axial force in the member does not exceed $0.15\phi_c F_y A_g$ for LRFD or $0.15 F_y A_g / \Omega_c$ for ASD. Redistribution applies only to moments computed from an elastic analysis. An inelastic analysis automatically accounts for redistribution; for this reason, additional redistribution of moments is not applicable.

The ductility of beams in the inelastic buckling phase may be significantly reduced and require a closer spacing of lateral bracing. The unbraced length of a beam must not exceed the value L_{pd}, given by AISC 360 Eq. A-1-5.

$$L_{pd} = \left(0.12 - 0.076\frac{M_1'}{M_2}\right)\left(\frac{E r_y}{F_y}\right)$$

M_2 is the larger moment at the end of the unbraced length. M_2 is taken as positive in all cases.

When the magnitude of the bending moment at any location within the unbraced length exceeds M_2,

$$\frac{M_1'}{M_2} = +1 \qquad \textit{[AISC A-1-6a]}$$

Otherwise, when $M_{\mathrm{mid}} \le (M_1 + M_2)/2$,

$$M_1' = M_1 \qquad \textit{[AISC A-1-6b]}$$

When $M_{\mathrm{mid}} > (M_1 + M_2)/2$,

$$M_1' = 2M_{\mathrm{mid}} - M_2 < M_2 \qquad \textit{[AISC A-1-6c]}$$

M_1 is the smaller moment at the end of the unbraced length, M_{mid} is the moment at the middle of the unbraced length, and M_1' is the effective moment at the end of the unbraced length opposite from M_2. The moments M_1 and M_{mid} are individually taken as positive when they cause compression in the same flange as the moment M_2 and negative otherwise.

The method is applied in accordance with AISC 360 Sec. B3.7, which states that negative moments at supports that are produced by gravity loads computed by an elastic analysis may be reduced by 10%, provided span moments are increased by 10% of the average adjacent support moments.

Example 4.10

The factored loading, including the beam self-weight, acting on a three-span continuous beam is shown. Continuous lateral support is provided to the beam. Determine the lightest adequate W shape using grade 50 steel.
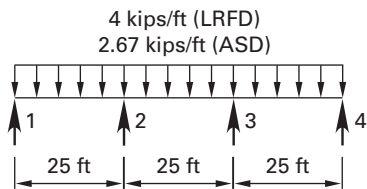
Solution

LRFD Method

From *AISC Manual* Table 3-23,

$$M_2 = \text{moment at interior support}$$
$$= -0.10wL^2$$
$$= (-0.10)\left(4 \ \frac{\text{kips}}{\text{ft}}\right)(25 \text{ ft})^2$$
$$= -250 \text{ ft-kips}$$
$$M_{12} = 0.08wL^2$$
$$= (0.08)\left(4 \ \frac{\text{kips}}{\text{ft}}\right)(25 \text{ ft})^2$$
$$= 200 \text{ ft-kips}$$

Allowing for redistribution in accordance with AISC 360 Sec. B3.7, the required flexural strengths are

$$M_{u12} = 200 \text{ ft-kips} + \frac{(0.1)(0 + 250 \text{ ft-kips})}{2}$$
$$= 212.5 \text{ ft-kips}$$
$$M_{u2} = (0.9)(-250 \text{ ft-kips})$$
$$= -225 \text{ ft-kips} \quad \text{[governs]}$$

From *AISC Manual* Table 3-2, a W18 × 35 has

$$\phi_b M_p = 249 \text{ ft-kips}$$
$$> 225 \text{ ft-kips} \quad \text{[satisfactory]}$$

ASD Method

From *AISC Manual* Table 3-23,

$$M_2 = \text{moment at interior support}$$
$$= -0.10wL^2$$
$$= (-0.10)\left(2.67 \ \frac{\text{kips}}{\text{ft}}\right)(25 \text{ ft})^2$$
$$= -167 \text{ ft-kips}$$
$$M_{12} = 0.08wL^2$$
$$= (0.08)\left(2.67 \ \frac{\text{kips}}{\text{ft}}\right)(25 \text{ ft})^2$$
$$= 134 \text{ ft-kips}$$

Allowing for redistribution in accordance with AISC 360 Sec. B3.7, the required flexural strengths are

$$M_{a12} = 134 \text{ ft-kips} + \frac{(0.1)(0 \text{ ft-kips} + 167 \text{ ft-kips})}{2}$$
$$= 142.4 \text{ ft-kips}$$
$$M_{a2} = (0.9)(-167 \text{ ft-kips})$$
$$= -150 \text{ ft-kips} \quad \text{[governs]}$$

From *AISC Manual* Table 3-2, a W18 × 35 has

$$\frac{M_p}{\Omega_b} = 166 \text{ ft-kips}$$
$$> 150 \text{ ft-kips} \quad \text{[satisfactory]}$$

Biaxial Bending

A beam subjected to bending moment about both the x- and y-axes may be designed in accordance with AISC 360 Sec. H1.1 by using the following interaction expressions.

$$\frac{M_{rx}}{\phi_b M_{nx}} + \frac{M_{ry}}{\phi_b M_{ny}} \leq 1.00 \quad \text{[LRFD]}$$

$$\frac{M_{rx}\Omega_b}{M_{nx}} + \frac{M_{ry}\Omega_b}{M_{ny}} \leq 1.00 \quad \text{[ASD]}$$

Example 4.11

A simply supported W16 × 36 beam with a span of 15 ft is subjected to a uniformly distributed factored vertical load of 4 kips/ft and a horizontal lateral factored concentrated load of 3 kips applied at midspan for LRFD combinations. The corresponding ASD values are 2.67 kips/ft vertical and 2 kips horizontal. Determine whether the beam is adequate if the beam of grade 50 steel is laterally braced at the supports.

Solution

LRFD Method

The maximum bending moments due to the factored loads are

$$M_{rx} = \frac{wL^2}{8} = \frac{\left(4\ \frac{\text{kips}}{\text{ft}}\right)(15\ \text{ft})^2}{8}$$
$$= 112.5\ \text{ft-kips}$$

$$M_{ry} = \frac{WL}{4} = \frac{(3\ \text{kips})(15\ \text{ft})}{4}$$
$$= 11.3\ \text{ft-kips}$$

For bending about the x-axis with an unbraced length of 15 ft, from *AISC Manual* Table 3-10, a W16 × 36 has

$$\phi_b M_{nx} = C_b(150\ \text{ft-kips})$$
$$= (1.14)(150\ \text{ft-kips})$$
$$= 171\ \text{ft-kips}$$

For bending about the y-axis, from *AISC Manual* Table 3-4, a W16 × 36 has

$$\phi_b M_{ny} = 40.5\ \text{ft-kips}$$

The left side of the interaction equation is

$$\frac{M_{rx}}{\phi_b M_{nx}} + \frac{M_{ry}}{\phi_b M_{ny}} = \frac{112.5\ \text{ft-kips}}{171\ \text{ft-kips}} + \frac{11.3\ \text{ft-kips}}{40.5\ \text{ft-kips}}$$
$$= 0.94$$
$$< 1.0 \quad [\text{satisfactory}]$$

ASD Method

The maximum bending moments due to the factored loads are

$$M_{rx} = \frac{wL^2}{8} = \frac{\left(2.67\ \frac{\text{kips}}{\text{ft}}\right)(15\ \text{ft})^2}{8}$$
$$= 75\ \text{ft-kips}$$

$$M_{ry} = \frac{WL}{4} = \frac{(2\ \text{kips})(15\ \text{ft})}{4}$$
$$= 7.5\ \text{ft-kips}$$

For bending about the x-axis with an unbraced length of 15 ft, from *AISC Manual* Table 3-10, a W16 × 36 has

$$\frac{M_{nx}}{\Omega_b} = C_b(100\ \text{ft-kips})$$
$$= (1.14)(100\ \text{ft-kips})$$
$$= 114\ \text{ft-kips}$$

For bending about the y-axis, from *AISC Manual* Table 3-4, a W16 × 36 has

$$\frac{M_{ny}}{\Omega_b} = 26.9\ \text{ft-kips}$$

The left side of the interaction equation is

$$\frac{M_{rx}\Omega_b}{M_{nx}} + \frac{M_{ry}\Omega_b}{M_{ny}} = \frac{75\ \text{ft-kips}}{114\ \text{ft-kips}} + \frac{7.5\ \text{ft-kips}}{26.9\ \text{ft-kips}}$$
$$= 0.94$$
$$< 1.0 \quad [\text{satisfactory}]$$

4. DESIGN FOR SHEAR

Nomenclature

A_{gt}	gross area subject to tension	in^2
A_{gv}	gross area subject to shear	in^2
A_{nt}	net area subject to tension	in^2
A_{nv}	net area subject to shear	in^2
A_w	web area	in^2
C_v	web shear coefficient	–
d	overall depth of member	in
d_b	nominal bolt diameter	in
d_h	diameter of bolt hole	in
F_u	specified minimum tensile strength	kips/in^2
F_{yw}	specified minimum yield stress of the web material	kips/in^2
h	for rolled shapes, the distance between flanges less the corner radius; for built-up sections, the clear distance between flanges	in
k	distance from outer face of flange to web toe of fillet	in
l_b	length of bearing	in
P_n	rupture strength in tension	kips
R	nominal reaction	kips
s	bolt spacing	in
t_f	flange thickness	in
t_w	web thickness	in
U_{bs}	reduction coefficient	–
V_a	required shear strength	kips
V_n	nominal shear strength	kips
V_u	required shear strength	kips

Shear in Beam Webs

The shear in rolled W shape beams is resisted by the area of the web that is defined as

$$A_w = dt_w$$

It is assumed that the shear stress is uniformly distributed over this area, and for a slenderness ratio $h/t_w \le 2.24\sqrt{E/F_y}$, the nominal shear strength is governed by yielding of the web. This is the case for most W shapes, and the nominal shear strength is given by AISC 360 Sec. G2.1 as

$$V_n = 0.6 F_y A_w C_v \qquad [\textit{AISC G2-1}]$$
$$C_v = 1.0$$

Steel

The design shear strength is

$$\phi_v V_n = 1.0 V_n$$

The allowable shear strength is

$$\frac{V_n}{\Omega_v} = \frac{V_n}{1.5}$$

Example 4.12

Check the adequacy in shear of the W12 × 53 grade 50 beam in Ex. 4.8 at support 2.

Solution

LRFD Method
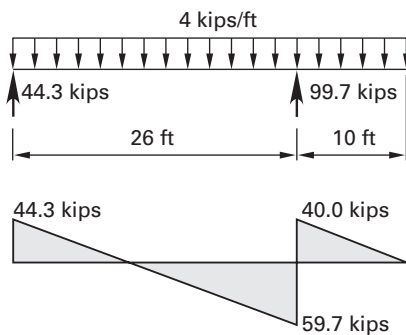
The shear force diagram is shown in the illustration, and the required shear strength is

$$V_u = 59.7 \text{ kips}$$

The design shear strength is obtained from *AISC Manual* Table 3-2 as

$$\phi_v V_n = 125 \text{ kips}$$
$$> V_u \quad [\text{satisfactory}]$$

ASD Method
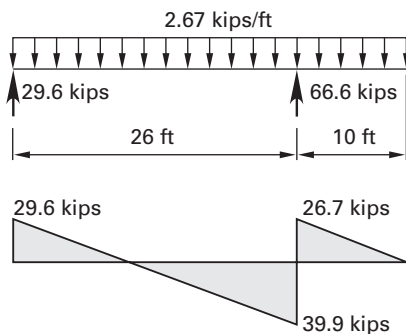
The shear force diagram is shown in the illustration, and the required shear strength is

$$V_a = 39.9 \text{ kips}$$

The allowable shear strength is obtained from *AISC Manual* Table 3-2 as

$$\frac{V_n}{\Omega_v} = 83.5 \text{ kips}$$
$$> V_a \quad [\text{satisfactory}]$$

Block Shear

As shown in Fig. 4.7, failure may occur by block shear in a coped beam. Block shear is a combination of shear along the vertical plane and tension along the horizontal plane. Block shear strength is the sum of the strengths of the shear area and tension area. Tension failure occurs by rupture in the net tension area. Shear failure occurs either by rupture in the net shear area or by shear yielding in the gross shear area, with the minimum value governing.

Figure 4.7 *Block Shear in a Coped Beam*
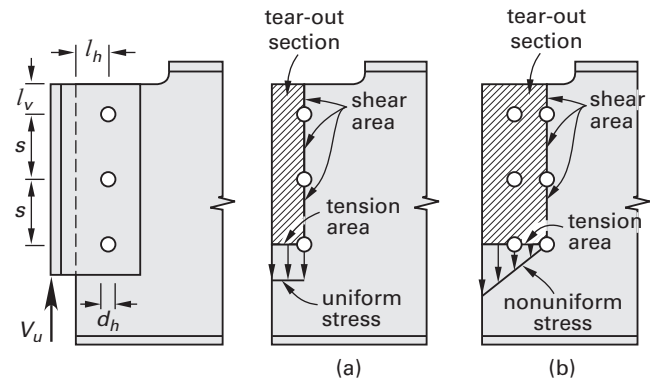
The nominal strengths are given in AISC 360 Sec. J4 and AISC 360 Sec. D2 as follows.

- Rupture strength in tension

$$P_n = U_{bs} F_u A_{nt} \qquad [\textit{AISC D2-2}]$$

- Yield strength in shear

$$V_n = 0.6 F_y A_{gv} \qquad [\textit{AISC J4-3}]$$

- Rupture strength in shear

$$V_n = 0.6 F_u A_{nv} \qquad [\textit{AISC J4-4}]$$

The nominal resistance to block shear is given by AISC 360 Eq. J4-5 as

$$R_n = 0.6 F_u A_{nv} + U_{bs} F_u A_{nt}$$
$$\leq 0.6 F_y A_{gv} + U_{bs} F_u A_{nt}$$

The reduction coefficient is

$$U_{bs} = 1.0 \text{ for uniform tension stress}$$

$$= 0.5 \text{ for nonuniform tension stress}$$

The resistance factor is

$$\phi = 0.75$$

The safety factor is

$$\Omega = 2.00$$

Example 4.13

Determine the resistance to block shear of the coped W16 × 40 grade A36 beam shown in Fig. 4.7(a). The relevant dimensions are $l_h = l_v = 1.5$ in and $s = 3$ in. The bolt diameter is $^3/_4$ in.

Solution

The hole diameter for a $^3/_4$ in diameter bolt is defined in AISC 360 Sec. B4.3b and AISC 360 Table J3.3 as

$$d_h = d_b + \frac{1}{8} \text{ in} = 0.75 \text{ in} + 0.125 \text{ in}$$

$$= 0.875 \text{ in}$$

$$t_w = 0.305 \text{ in}$$

$$A_{nv} = t_w(l_v + 2s - 2.5d_h)$$

$$= t_w\Big(1.5 \text{ in} + (2)(3.0 \text{ in}) - (2.5)(0.875 \text{ in})\Big)$$

$$= 5.31t_w \text{ in}^2$$

$$A_{nt} = t_w(l_h - 0.5d_h)$$

$$= t_w\Big(1.5 \text{ in} - (0.5)(0.875 \text{ in})\Big)$$

$$= 1.06t_w \text{ in}^2$$

The tensile stress is uniform, and the reduction coefficient is

$$U_{bs} = 1.0$$

$$U_{bs}F_uA_{nt} = \left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(1.06t_w \text{ in}^2)$$

$$= 61.48t_w \text{ kips}$$

$$0.6F_uA_{nv} = (0.6)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(5.31t_w \text{ in}^2)$$

$$= 184.79t_w \text{ kips}$$

$$A_{gv} = t_w(l_v + 2s)$$

$$= t_w\Big(1.5 \text{ in} + (2)(3 \text{ in})\Big)$$

$$= 7.5t_w \text{ in}^2$$

$$0.6F_yA_{gv} = (0.6)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(7.5t_w \text{ in}^2)$$

$$= 162t_w \text{ kips} \quad [\text{governs}]$$

$$< 0.6F_uA_{nv}$$

LRFD Method

Shear yielding governs and the design strength for block shear is given by AISC 360 Eq. J4-5 as

$$\phi R_n = \phi(0.6F_yA_{gv} + U_{bs}F_uA_{nt})$$

$$= (0.75)(0.305 \text{ in})\left(162 \ \frac{\text{kips}}{\text{in}} + 61.48 \ \frac{\text{kips}}{\text{in}}\right)$$

$$= 51.12 \text{ kips}$$

ASD Method

Shear yielding governs, and the allowable strength for block shear is given by AISC 360 Eq. J4-5 as

$$\frac{R_n}{\Omega} = \frac{0.6F_yA_{gv} + U_{bs}F_uA_{nt}}{\Omega}$$

$$= \frac{(0.305 \text{ in})\left(162 \ \frac{\text{kips}}{\text{in}} + 61.48 \ \frac{\text{kips}}{\text{in}}\right)}{2}$$

$$= 34.08 \text{ kips}$$

Web Local Yielding

As shown in Fig. 4.8, a bearing plate may be used to distribute concentrated loads applied to the flange to prevent web local yielding. The load is assumed to be dispersed, at a gradient of 2.5 to 1.0, to the web toe of fillet. For loads applied at a distance of not more than d from the end of the beam, the nominal strength is given by AISC 360 Sec. J10.2 as

$$R_n = F_{yw}t_w(2.5k + l_b) \qquad [\textit{AISC J10-3}]$$

Figure 4.8 *Web Local Yielding*
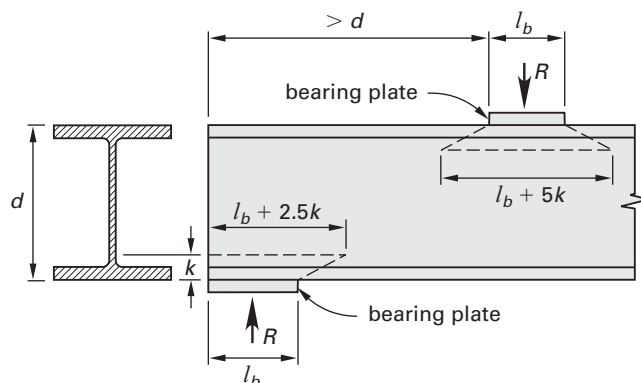
For loads applied at a distance of more than d from the end of the beam, the nominal strength is given as

$$R_n = F_{yw}t_w(5k + l_b) \qquad [\textit{AISC J10-2}]$$

The design strength is given by ϕR_n with $\phi = 1.0$. *AISC Manual* Table 9-4 tabulates values of

$$\phi R_1 = \phi(2.5 k F_y t_w)$$
$$\phi R_2 = \phi(F_y t_w)$$

The allowable strength is given by R_n/Ω with $\Omega = 1.5$. *AISC Manual* Table 9-4 tabulates values of

$$\frac{R_1}{\Omega} = \frac{2.5 k F_y t_w}{\Omega}$$
$$\frac{R_2}{\Omega} = \frac{F_y t_w}{\Omega}$$

Example 4.14

Determine the resistance to local web yielding of a W40 × 331 grade 50 beam with a 4.0 in long bearing plate at the end of the beam.

Solution

LRFD Method

AISC 360 Eq. J10-3 is applicable. Using values from *AISC Manual* Table 9-4, the design strength is

$$\phi R_n = \phi F_y t_w (2.5k + l_b)$$
$$= \phi(R_1 + l_b R_2)$$
$$= 505 \text{ kips} + (4.0 \text{ in})\left(61.0 \ \frac{\text{kips}}{\text{in}}\right)$$
$$= 749 \text{ kips}$$

ASD Method

AISC 360 Eq. J10-3 is applicable. Using values from *AISC Manual* Table 9-4, the allowable strength is

$$\frac{R_n}{\Omega} = \frac{F_y t_w (2.5k + l_b)}{\Omega}$$
$$= \frac{R_1 + l_b R_2}{\Omega}$$
$$= 337 \text{ kips} + (4.0 \text{ in})\left(40.7 \ \frac{\text{kips}}{\text{in}}\right)$$
$$= 500 \text{ kips}$$

Web Crippling

For a concentrated load applied at a distance of not less than $d/2$ from the end of the beam, the nominal strength against web crippling is given by AISC 360 Sec. J10.3 as

$$R_n = 0.80 t_w^2 \left(1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5} \right) \sqrt{\frac{E F_{yw} t_f}{t_w}}$$

[*AISC J10-4*]

For loads applied at a distance of less than $d/2$ from the end of the beam and for $l_d \leq 0.2$, the nominal strength is given by

$$R_n = 0.40 t_w^2 \left(1 + 3\left(\frac{l_b}{d}\right)\left(\frac{t_w}{t_f}\right)^{1.5} \right) \sqrt{\frac{E F_{yw} t_f}{t_w}}$$

[*AISC J10-5a*]

For loads applied at a distance of less than $d/2$ from the end of the beam and for $l_b/d > 0.2$, the nominal strength is given by

$$R_n = 0.40 t_w^2 \left(1 + \left(4\left(\frac{l_b}{d}\right) - 0.2\right)\left(\frac{t_w}{t_f}\right)^{1.5} \right) \sqrt{\frac{E F_{yw} t_f}{t_w}}$$

[*AISC J10-5b*]

The design strength is given by $\phi_r R_n$ with $\phi_r = 0.75$. Using values of $\phi_r R_3$, $\phi_r R_4$, $\phi_r R_5$, and $\phi_r R_6$ tabulated in *AISC Manual* Table 9-4 reduces AISC 360 Eq. J10-5a to

$$\phi_r R_n = \phi_r R_3 + l_b(\phi_r R_4)$$

AISC 360 Eq. J10-5b becomes

$$\phi_r R_n = \phi_r R_5 + l_b(\phi_r R_6)$$

The allowable strength is given by R_n/Ω_r with $\Omega_r = 2.00$. Using values of R_3/Ω_r, R_4/Ω_r, R_5/Ω_r, and R_6/Ω_r tabulated in *AISC Manual* Table 9-4 reduces AISC 360 Eq. J10-5a to

$$\frac{R_n}{\Omega_r} = \frac{R_3}{\Omega_r} + l_b\left(\frac{R_4}{\Omega_r}\right)$$

AISC 360 Eq. J10-5b becomes

$$\frac{R_n}{\Omega_r} = \frac{R_5}{\Omega_r} + l_b\left(\frac{R_6}{\Omega_r}\right)$$

Example 4.15

Determine the design web crippling strength of a W40 × 331 grade 50 beam with a 3.25 in long bearing plate at the end of the beam.

Solution

LRFD Method

For $l_b/d \leq 0.2$, using values from *AISC Manual* Table 9-4, the applicable expression is

$$\phi_r R_n = \phi_r R_3 + l_b(\phi_r R_4)$$
$$= 710 \text{ kips} + (3.25 \text{ in})\left(22.6 \ \frac{\text{kips}}{\text{in}}\right)$$
$$= 783 \text{ kips}$$

ASD Method

For $l_b/d \leq 0.2$, using values from *AISC Manual* Table 9-4, the applicable expression is

$$\frac{R_n}{\Omega_r} = \frac{R_3}{\Omega_r} + l_b\left(\frac{R_4}{\Omega_r}\right)$$

$$= 474 \text{ kips} + (3.25 \text{ in})\left(15.1 \ \frac{\text{kips}}{\text{in}}\right)$$

$$= 523 \text{ kips}$$

5. DESIGN OF COMPRESSION MEMBERS

Nomenclature

A	area of member	in^2
A_g	gross area of member	in^2
b_f	flange width of rolled beam	in
b_x	$8/9 M_{cx} \times 10^3$	(ft-kips)$^{-1}$
b_y	$8/9 M_{cy} \times 10^3$	(ft-kips)$^{-1}$
B	base plate width	in
B_1	moment magnification factor applied to the primary moments to account for the curvature of the members, with lateral translation inhibited, as defined in AISC 360 Eq. C2-2	–
B_2	moment magnification factor applied to the primary moments to account for the translation of the members, with lateral translation permitted as defined in AISC 360 Eq. A-8-3	–
C_b	bending coefficient	–
C_m	reduction factor given by AISC 360 Eq. A-8-4	–
d	depth of rolled beam	in
E	modulus of elasticity	kips/in^2
EA^*	reduced value of EA	kips
EI^*	reduced value of EI	kips-in^2
F_{cr}	critical stress	kips/in^2
F_e	elastic critical buckling stress	kips/in^2
F_y	yield stress	kips/in^2
G	ratio of total column stiffness framing into a joint to total girder stiffness framing into the joint	–
H	horizontal force	kips
H	story shear produced by the lateral forces used to compute Δ_H	kips
H_p	lateral load required to produce the design story drift	kips
I	moment of inertia	in^4
K	effective length factor	–
K_1	effective length factor in the plane of bending	–
K_2	effective length factor in the plane of bending for an unbraced frame	–
L	actual unbraced length in the plane of bending	ft
L	story height	ft
m	cantilever dimension for base plate along the length of the plate	in
M_a	augmented moment	ft-kips

M_c	available flexural strength	ft-kips
M_{lt}	calculated first-order factored moment in a member, due to lateral translation of the frame only	ft-kips
M_{nt}	calculated first-order factored moment assuming no lateral translation of the frame	ft-kips
M_{nx}	nominal flexural strength about strong axis in the absence of axial load	ft-kips
M_{ny}	nominal flexural strength about weak axis in the absence of axial load	ft-kips
M_r	required second-order factored flexural strength for LRFD or ASD load combinations	ft-kips
M_{rx}	required second-order factored bending moment about strong axis	ft-kips
M_{ry}	required second-order factored bending moment about weak axis	ft-kips
M_1	smaller moment at end of unbraced length of member, calculated from a first-order analysis	ft-kips
M_2	larger moment at end of unbraced length of member, calculated from a first-order analysis	ft-kips
n	cantilever dimension for base plate along the width of the plate	in
N	base plate length	in
N_i	notional lateral load applied at level i	kips
p	$1/P_c \times 10^3$	kips^{-1}
P_a	augmented axial force	kips
P_c	available axial compression strength	kips
$P_{e,\text{story}}$	sum, for all columns in a story of a moment frame, of the Euler buckling strength	kips
P_e	Euler buckling strength	kips
P_{e1}	Euler buckling strength of the member in the plane of bending as defined in AISC 360 Eq. A-8-5	kips
P_{lt}	calculated first-order factored axial force in a member, due to lateral translation of the frame only	kips
P_{mf}	total vertical load in columns in the story	kips
P_n	nominal axial strength	kips
P_{nt}	first-order factored axial force assuming no lateral translation of the frame	kips
P_r	required second-order axial strength for LRFD or ASD load combinations	kips
P_{story}	total factored vertical load supported by the story, including gravity columns loads	kips
P_y	axial yield strength	kips
r	radius of gyration	in
R	restraint	kips
R_M	system coefficient	–
t_{req}	required base plate thickness	in
Y_i	gravity load applied at level i independent of loads from above	kips

Steel

Symbols

α	gravity load multiplier	–
Δ_a	interstory drift due to applied loads	in
Δ_H	first-order interstory drift due to lateral forces	in
Δ_{oh}	translational deflection of the story under consideration	in
Δ_p	permissible interstory drift	in
Δ_{1st}	first-order drift	in
Δ_{2nd}	second-order drift	in
λ	F_y/F_e	–
$\sum(I_c/L_c)$	the sum of the I/L values for all columns meeting at a joint	in^3
$\sum(I_g/L_g)$	the sum of the I/L values for all girders meeting at a joint	in^3
τ_b	stiffness reduction coefficient	–
ϕ_b	resistance factor for flexure	–
ϕ_c	resistance factor for compression	–
Ω_b	safety factor for flexure	–
Ω_c	safety factor for compression	–

Effective Length

In the design of compression members, the effective length factor K is used to account for the influence of restraint conditions at each end of a column. The K factor is used to equate the nominal strength of a compression member of length L to that of an equivalent pin-ended member of length KL. The nominal strength of a compression member is dependent on the slenderness ratio, KL/r, which is limited to a maximum recommended value of 200.

AISC 360 Comm. Table C-A-7.1 specifies effective length factors for well-defined conditions of restraint; these are illustrated in Fig. 4.9. These values may only be used in simple cases when the tabulated end conditions are closely approximated in practice.

For compression members in a pin-jointed truss, an effective length factor of 1.0 is used. For load bearing web stiffeners on a girder, AISC 360 Sec. J10.8 specifies an effective length factor of 0.75. For columns in a rigid frame that is adequately braced, AISC 360 Comm. App. 7.2 specifies a conservative value for the effective length factor of 1.0. For compression members forming part of a frame with rigid joints, AISC 360 Comm. App. 7.2 presents alignment charts for determining the effective length for the two conditions of sidesway prevented and sidesway permitted; these charts are illustrated in Fig. 4.10.

To use the alignment charts, the stiffness ratio at the two ends of the column under consideration must be determined. This ratio is defined as

$$G = \frac{\sum\left(\dfrac{I_c}{L_c}\right)}{\sum\left(\dfrac{I_g}{L_g}\right)}$$

Figure 4.9 *Effective Length Factors*

illus.	end conditions	theoretical	design
(a)	both ends pinned	1	1.00
(b)	both ends built in	0.5	0.65
(c)	one end pinned, one end built in	0.7	0.8
(d)	one end built in, one end free	2	2.10
(e)	one end built in, one end fixed against rotation but free to translate	1	1.20
(f)	one end pinned, one end fixed against rotation but free to translate	2	2.0

For a braced frame with rigid joints, the girders are bent in single curvature and the alignment charts are based on a stiffness value of $2EI/L$. If one end of a girder is pinned, its stiffness is $3EI/L$; if one end is fixed, its stiffness is $4EI/L$. Therefore, for these two cases, the I_g/L_g values are multiplied by 1.5 and 2.0, respectively. For a sway frame with rigid joints, the girders are bent in double curvature and the alignment charts are based on a stiffness value of $6EI/L$. If one end of a girder is pinned, its stiffness is $3EI/L$ and the I_g/L_g values are multiplied by 0.5. For a column with a pinned base, AISC 360 Comm. App. 7.2 specifies a stiffness ratio of $G=10$. For a column with a fixed base, AISC 360 Comm. App. 7.2 specifies a stiffness ratio of $G=1$.

Figure 4.10 *Alignment Charts for Effective Length Factors*

braced frame
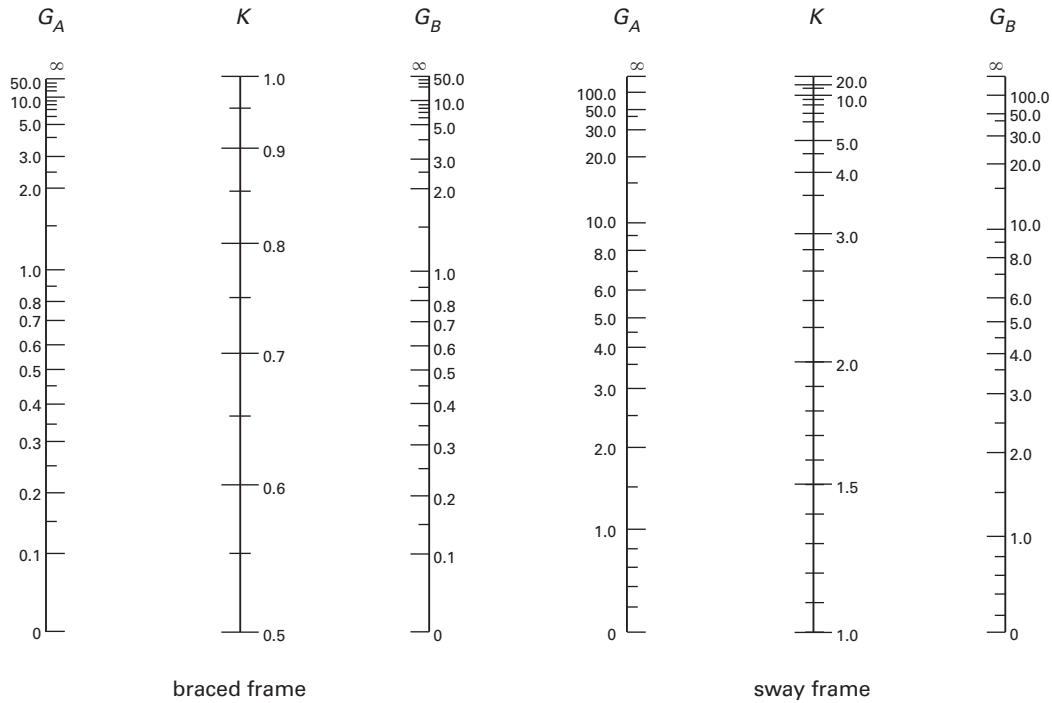
sway frame

Adapted from American Institute of Steel Construction, *Specifications for Structural Steel Buildings*, Commentary Fig. C-A-7.1 and Fig. C-A-7.2.

Example 4.16

The sway frame shown consists of members with identical I/L values. Determine the effective length factors of columns 12 and 34.
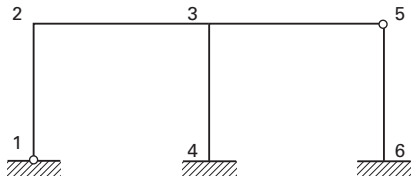
Solution

For the pinned connection at joint 1, AISC 360 Comm. App. 7.2 specifies a stiffness ratio of $G_1 = 10$.

At joint 2,

$$G_2 = \frac{\sum\left(\dfrac{I_c}{L_c}\right)}{\sum\left(\dfrac{I_g}{L_g}\right)} = \frac{1.0 \text{ in}^3}{1.0 \text{ in}^3} = 1$$

From the alignment chart for sway frames, the effective length factor is

$$K_{12} = 1.9$$

Allowing for the pinned end at joint 5, the sum of the adjusted relative stiffness values for the two girders connected to joint 3 is

$$\sum\left(\frac{I_g}{L_g}\right) = 1.0 \text{ in}^3 + 0.5 \text{ in}^3$$

$$= 1.5 \text{ in}^3$$

The stiffness ratio at joint 3 is given by

$$G_3 = \frac{\sum\left(\dfrac{I_c}{L_c}\right)}{\sum\left(\dfrac{I_g}{L_g}\right)} = \frac{1.0 \text{ in}^3}{1.5 \text{ in}^3}$$

$$= 0.67$$

For the fixed connection at joint 4, AISC 360 Comm. App. 7.2 specifies a stiffness ratio of $G_4 = 1.0$. From the alignment chart for a sway frame, the effective length factor for column 34 is

$$K_{34} = 1.27$$

Axially Loaded Members

The design strength in compression is given by

$$\phi_c P_n = 0.90 A_g F_{cr}$$

The allowable strength in compression is given by

$$\frac{P_n}{\Omega_c} = \frac{A_g F_{cr}}{1.67}$$

Short Column

For a short column with $KL/r \leq 4.71(E/F_y)^{0.5}$ and $F_e \geq 0.44F_y$, inelastic buckling governs, and AISC 360 Eq. E3-2 defines the critical stress as

$$F_{cr} = (0.658^\lambda)F_y$$

The parameter λ is defined by

$$\lambda = \frac{F_y}{F_e}$$

The elastic critical buckling stress is given by AISC 360 Eq. E3-4 as

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2}$$

Long Column

For a long column with $KL/r > 4.71(E/F_y)^{0.5}$ and $F_e < 0.44F_y$, elastic buckling governs, and the critical stress is given by AISC 360 Eq. E3-3 as

$$F_{cr} = 0.877F_e$$

Once the governing slenderness ratio of a column is established, the design stress, $\phi_c F_{cr}$, and the allowable stress, F_{cr}/Ω_c, may be obtained directly from *AISC Manual* Table 4-22 for steel members with a yield stress of 35, 36, 42, 46, or 50 kips/in^2.

Values of the design axial strength and the allowable axial strength are tabulated in *AISC Manual* Table 4-1 for rolled sections W14 and smaller with respect to r_y for varying effective lengths. These tabulated values may be used directly when $(KL/r)_y$ exceeds $(KL/r)_x$.

Example 4.17

Determine the lightest W12 grade 50 column that will support a factored load of 850 kips (LRFD) or 570 kips (ASD). The column is 12 ft high, is pinned at each end, and has no intermediate bracing about either axis.

Solution

LRFD Method

From *AISC Manual* Table 4-1, a W12 × 79 column with an effective length of 12 ft has a design axial strength of

$$\phi_c P_n = 887 \text{ kips} \quad \text{[satisfactory]}$$

ASD Method

From *AISC Manual* Table 4-1, a W12 × 79 column with an effective length of 12 ft has an allowable axial strength of

$$\frac{P_n}{\Omega_c} = 590 \text{ kips} \quad \text{[satisfactory]}$$

Buckling About Major Axis

When the minor axis of a W-shape is braced at closer intervals than the major axis, the slenderness ratio about both axes must be investigated to determine which governs. The larger of the two values will control the design. When the slenderness ratio about the minor axis, $(KL/r)_x$, governs, the available axial strength values tabulated in *AISC Manual* Table 4-1 may be utilized directly. When the slenderness ratio about the major axis, $(KL/r)_y$, governs, the effective length about the major axis is divided by r_x/r_y to give an equivalent effective length about the minor axis, which has the same load carrying capacity as the actual effective length about the major axis. *AISC Manual* Table 4-1 may then be used to obtain the available design strength of the member in compression by the equivalent length $(KL)_x/(r_x/r_y)$.

Equivalent Effective Length

When the effective lengths of a column about the x- and y-axes are different, the strength of the column must be investigated with respect to both axes. Dividing the effective length about the x-axis by the ratio r_x/r_y provides an equivalent effective length about the y-axis.

Example 4.18

Determine the available axial strength of a W12 × 106 grade 50 column that is 12 ft high, pinned at each end, and braced at midheight about the y-axis.

Solution

The effective length about the y-axis is

$$(KL)_y = 6 \text{ ft}$$

The effective length about the x-axis is

$$(KL)_x = 12 \text{ ft}$$

From *AISC Manual* Table 4-1, a W12 × 106 column has a value of

$$\frac{r_x}{r_y} = 1.76$$

The equivalent effective length about the major axis with respect to the y-axis is

$$(KL_y)_{\text{equiv}} = \frac{(KL)_x}{\dfrac{r_x}{r_y}} = \frac{12 \text{ ft}}{1.76}$$

$$= 6.8 \text{ ft} \quad [\text{governs}]$$

$$> (KL)_y$$

LRFD Method

From *AISC Manual* Table 4-1, a $\text{W12} \times 106$ column with an effective length, $(KL_y)_{\text{equiv}}$, of 6.8 ft has a design axial strength of

$$\phi_c P_n = 1334 \text{ kips}$$

ASD Method

From *AISC Manual* Table 4-1, a $\text{W12} \times 106$ column with an effective length, $(KL_y)_{\text{equiv}}$, of 6.8 ft has an allowable axial strength of

$$\frac{P_n}{\Omega_c} = 887 \text{ kips}$$

Built-Up Sections

For built-up sections and laced compression members, *AISC Manual* Table 4-22 tabulates $\phi_c F_{cr}$ and F_{cr}/Ω_c against KL/r for steel with yield stresses of 35 kips/in^2, 36 kips/in^2, 42 kips/in^2, 46 kips/in^2, and 50 kips/in^2, respectively.

Example 4.19

A laced column consisting of four $5 \times 5 \times {}^1\!/_2$ angles of grade A36 steel is shown. The column may be considered a single integral member and is 20 ft high with pinned ends. Determine the maximum design axial load.
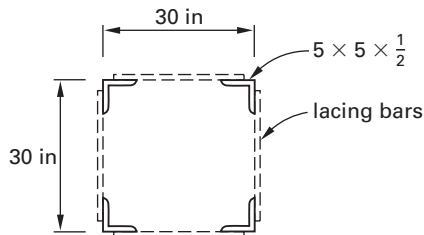
Solution

The relevant properties of a $5 \times 5 \times {}^1\!/_2$ angle are

$$A = 4.75 \text{ in}^2$$

$$I = 11.3 \text{ in}^4$$

$$y = 1.42 \text{ in}$$

The relevant properties of the laced column are

$$\sum A = 4A$$

$$= (4)(4.75 \text{ in}^2)$$

$$= 19 \text{ in}^2$$

$$\sum I = 4I + \sum A \left(\frac{d}{2} - y\right)^2$$

$$= (4)(11.3 \text{ in}^4) + (19 \text{ in}^2)(15 \text{ in} - 1.42 \text{ in})^2$$

$$= 3549 \text{ in}^4$$

The radius of gyration of the laced column is

$$r = \sqrt{\frac{\sum I}{\sum A}} = \sqrt{\frac{3549 \text{ in}^4}{19 \text{ in}^2}}$$

$$= 13.67 \text{ in}$$

The slenderness ratio of the laced column is

$$\frac{KL}{r} = \frac{(1.0)(20 \text{ ft})\left(12 \, \frac{\text{in}}{\text{ft}}\right)}{13.67 \text{ in}}$$

$$= 17.56$$

$$< 200 \quad [\text{satisfactory}]$$

From *AISC Manual* Table 4-22, the design stress is

$$\phi_c F_{cr} = 31.9 \text{ kips/in}^2$$

The allowable stress is

$$\frac{F_{cr}}{\Omega_c} = 21.2 \text{ kips/in}^2$$

The design axial strength is

$$\phi_c P_n = \phi_c F_{cr} \sum A$$

$$= \left(31.9 \, \frac{\text{kips}}{\text{in}^2}\right)(19 \text{ in}^2)$$

$$= 606 \text{ kips}$$

The allowable axial strength is

$$\frac{P_n}{\Omega_c} = \frac{F_{cr} \sum A}{\Omega_c}$$

$$= \left(21.2 \, \frac{\text{kips}}{\text{in}^2}\right)(19 \text{ in}^2)$$

$$= 402 \text{ kips}$$

Composite Columns

Concrete filled hollow structural sections and concrete encased rolled steel sections reinforced with longitudinal and lateral reinforcing bars are designed by using

AISC 360 Sec. I2. Values of the design axial strength for typical sizes of column are tabulated in *AISC Manual* Part 4.

Example 4.20

Determine the least weight rectangular composite column using an HSS section filled with 5000 lbf/in^2 concrete that can support a factored load of 730 kips (LRFD) or 485 kips (ASD). The column is 15 ft high and is pinned at each end.

Solution

LRFD Method

From *AISC Manual* Table 4-14, an HSS $14 \times 10 \times {}^5/_{16}$ in with an effective length of 15 ft has a design axial strength of

$$\phi_c P_n = 732 \text{ kips} \quad \text{[satisfactory]}$$

ASD Method

From *AISC Manual* Table 4-14, an HSS $14 \times 10 \times {}^5/_{16}$ in with an effective length of 15 ft has an allowable axial strength of

$$\frac{P_n}{\Omega_c} = 488 \text{ kips} \quad \text{[satisfactory]}$$

Second-Order Effects

In accordance with AISC 360 Sec. C2, the design of compression members must take into account secondary effects. The secondary moments and axial forces caused by the P-delta effects must be added to the primary moments and axial forces in a member, which were obtained by a first-order analysis. The P-delta effects are the result of the two separate effects P-δ and P-Δ, as shown in Fig. 4.11. The final forces in a frame (including secondary effects) may be obtained as the summation of the two analyses, sway and non-sway.

Figure 4.11 *P-delta Effects*

(a) braced frame   (b) sway frame
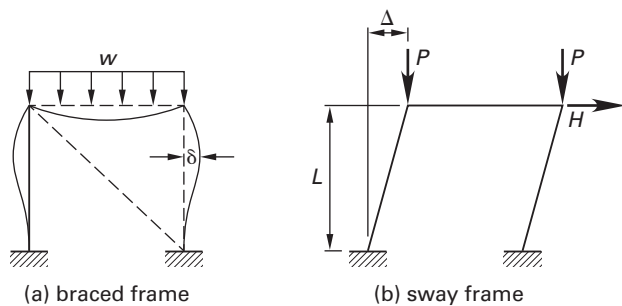
The *P-δ effect* produces an amplified moment due to the eccentricity of the axial force with respect to the displaced center line of the member. This is termed the *member effect*. The moment magnification factor which, when applied to the primary moments, accounts for the P-δ effect, is termed B_1.

The *P-Δ effect* produces an amplified moment due to the drift in a sway frame. This is termed the *sidesway effect*. The moment magnification factor which, when applied to the primary moments, accounts for the P-Δ effect, is termed B_2.

When sidesway is inhibited,

$$B_2 = 0$$

For a member not subject to axial compression,

$$B_1 = 1.0$$

Approximate Second-Order Analysis

As shown in Fig. 4.12, two first-order analyses are required in order to determine M_{nt} and M_{lt}. In the first analysis, imaginary horizontal restraints are introduced at each floor level to prevent lateral translation. The factored loads are then applied, the primary moments, M_{nt}, are calculated, and the magnitudes of the imaginary restraints, R, are determined. In the second analysis, the reverse of the imaginary restraints for the frame is analyzed in order to determine the primary moments, M_{lt}, and axial forces, P_{lt}.

Figure 4.12 *Determination of Secondary Effects*
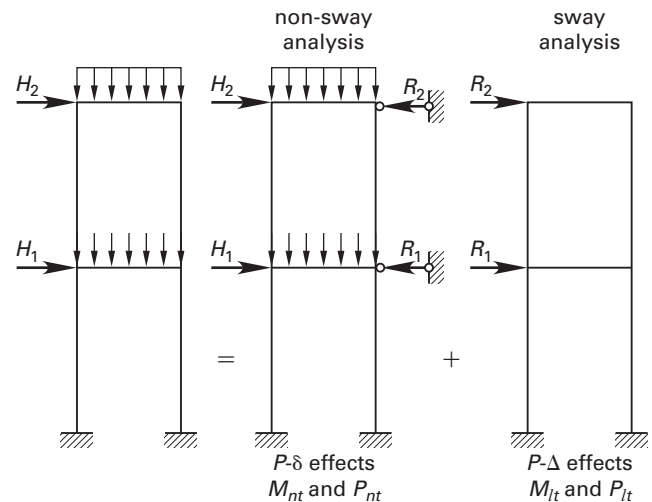
The required final second-order forces are then given by

$$M_r = B_1 M_{nt} + B_2 M_{lt} \qquad \text{[AISC A-8-1]}$$

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{[AISC A-8-2]}$$

The determination of the B_1 and B_2 multipliers is detailed in AISC 360 App. 8, and as follows.

Multiplier B_1 for P-δ Effects

The moment magnification factor to account for the P-δ effect, assuming no lateral translation of the frame, is defined in AISC 360 Eq. A-8-3 as

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \geq 1$$

$\alpha = 1$ for LRFD load combinations, and $\alpha = 1.6$ for ASD load combinations.

The Euler buckling strength of the member in the plane of bending is defined in AISC 360 Eq. A-8-5 as

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2}$$

$EI^* =$ flexural rigidity used in the analysis

$\qquad = 0.8\tau_b EI$ [for direct analysis method]

$\qquad = EI$ [for effective length and first-order methods]

To assist in the determination of the Euler buckling strength of a member, *AISC Manual* Table 4-1 tabulates values of $P_e(KL)^2/10^4$ for W-shapes with a yield stress of 50 kips/in^2.

The effective-length factor in the plane of bending, for a member in a frame with lateral translation inhibited, is given by

$$K_1 = 1.0$$

The reduction factor is given by AISC 360 Eq. A-8-4 as

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \quad \begin{bmatrix} \text{for a member not subjected} \\ \text{to transverse loading} \\ \text{between supports} \end{bmatrix}$$

$$= 1.0 \quad \begin{bmatrix} \text{for a member transversely loaded} \\ \text{between supports} \end{bmatrix}$$

$$= 1.0 \quad \begin{bmatrix} \text{for a member bent in single curvature} \\ \text{under uniform bending moment} \end{bmatrix}$$

M_1/M_2 is positive for a member bent in reverse curvature and negative for a member bent in single curvature.

In AISC 360 Eq. A-8-3, it is permitted to use the first-order estimate of

$$P_r = P_{nt} + P_{lt}$$

In accordance with AISC 360 Sec. C2.1(2), P-δ effects may be neglected when all of the following three conditions apply.

- The structure supports gravity loads primarily through nominally vertical columns, walls, or frames.

- The ratio $\Delta_{2nd}/\Delta_{1st}$, calculated using LRFD load combinations or 1.6 times ASD load combinations

with stiffness adjusted as specified in AISC 360 Sec. C2.3, is

$$\frac{\Delta_{2nd}}{\Delta_{1st}} \leq 1.7$$

- No more than one-third of the total gravity load on the structure is supported by columns that are part of moment-resisting frames.

AISC 360 Eq. A-8-1 is then

$$M_r = M_{nt} + B_2 M_{lt}$$

Multiplier B_2 for P-Δ Effects

The moment magnification factor to account for the P-Δ effect, with lateral translation of the frame allowed, is defined in AISC 360 Eq. A-8-6 as

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}}$$

$$= \frac{\Delta_{2nd}}{\Delta_{1st}}$$

$\alpha = 1.00$ for LRFD combinations and $\alpha = 1.60$ for ASD combinations.

P_{story} is the total vertical load supported by the story using LRFD or ASD load combinations, as applicable, including loads in columns that are not part of the lateral force resisting system. $P_{e\,story}$ is the elastic critical buckling strength for the story in the direction of translation being considered. $P_{e\,story}$ is determined by either sidesway buckling analysis or by using

$$P_{e\,story} = R_M \frac{HL}{\Delta_H} \qquad \text{[AISC A-8-7]}$$

$$R_M = 1 - 0.15\left(\frac{P_{mf}}{P_{story}}\right) \qquad \text{[AISC A-8-8]}$$

$$= 1.00 \quad \text{[braced frame]}$$

$$= 0.85 \quad \begin{bmatrix} \text{moment frames} \\ \text{and combined systems} \end{bmatrix}$$

P_{mf} is the total vertical load in columns in the story that are part of moment frames, in the direction of translation being considered, if any. H is the story shear in the direction of translation being considered that is produced by the lateral forces used to compute Δ_H.

When calculating AISC 360 Eq. A-8-6, use inches for the height of the story, L. Δ_H is computed using the stiffness required to be used in the analysis (stiffness is reduced as provided in AISC 360 Sec. C2.3 when the direct analysis method is used). When Δ_H varies over the plan area of the structure, it is the average drift weighted in proportion to the vertical load or the maximum drift. When a limit is placed on the drift index, Δ_H/L, the amplification factor, B_2, is determined by using this limit in AISC 360 Eq. A-8-7.

Alternatively, *P*-delta effects may be directly determined in a rigorous second-order frame analysis, and the members may then be designed directly for the calculated axial force and bending moment. The principle of superposition is not valid in a second-order analysis, and separate analyses are necessary for each combination of factored loads.

Analysis Methods[4,5,6]

AISC 360 Sec. C2 requires the design of compression members to take into account secondary effects. The factors that must be considered include

- flexural, shear, and axial deformations of members
- P-Δ second-order effects caused by structure displacements
- P-δ second-order effects caused by member deformations
- geometric imperfections caused by initial out-of-plumbness of the columns
- reduction in member stiffness due to inelasticity and residual stresses
- uncertainty in stiffness and strength

The first three of these issues are covered in the analysis of the structure.

Geometric imperfections are caused by the permitted tolerances in the plumbness of columns. The out-of-plumbness effects are duplicated by applying notional fictitious lateral loads to the structure that produce an equivalent effect. As specified by the American Institute of Steel Construction's *Code of Standard Practice for Steel Buildings and Bridges* (AISC 303) Sec. 7.13.1.1, the maximum tolerance on out-of-plumbness of a column is 1/500 of the height of the column. This produces a moment in a column of height L of

$$M = P_\Delta = \frac{PL}{500}$$

The same effect may be produced by applying a notional load of $P/500$ at the top of the column. This produces an identical moment at the bottom of the column of

$$M = \frac{PL}{500} = 0.002PL$$

As shown in Fig. 4.13, to account for initial imperfections in the members, notional lateral loads are applied at each story, in accordance with AISC 360 Sec. C2.2b(1), and are given by

$$N_i = 0.002\alpha Y_i$$
$$\alpha = 1.0 \quad \text{[LRFD]}$$
$$= 1.6 \quad \text{[ASD]}$$

Figure 4.13 *Notional Loads*
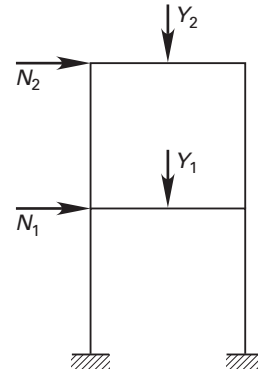
N_i is the notional lateral load applied at level i and Y_i is the gravity load applied at level i.

Residual stresses and *plastic yielding* cause increased deformations in the structure. These effects are compensated for by reducing the stiffness of members prior to analysis and adopting a value of $K = 1.0$ in design of the columns. Alternatively, the nominal stiffness of members is used in the analysis, and traditional K values are used in design of the columns.

In accordance with AISC 360 Sec. C2.3 the flexural and axial stiffness of members that contribute to the lateral stability of the structure are reduced to give

$$EI^* = 0.8\tau_b EI$$

$$EA^* = 0.8EA$$

The stiffness reduction parameter is

$$\tau_b = 1.0 \quad [\text{for } \alpha P_r \le 0.5 P_y]$$
$$= 4\left(\frac{\alpha P_r}{P_y}\right)\left(1 - \left(\frac{\alpha P_r}{P_y}\right)\right) \quad [\alpha P_r > 0.5 P_y]$$

The force level adjustment factor is

$$\alpha = 1.0 \quad \text{[LRFD]}$$
$$= 1.6 \quad \text{[ASD]}$$

P_r is the required second-order axial strength. P_y is the member yield strength and is equal to $F_y A$.

The stiffness reduction may be applied to all members in the structure, not only to those members contributing to the stability of the structure. This prevents unintended distortion of the structure and redistribution of forces.

Uncertainties in stiffness and strength of members are accounted for by the resistance factors and safety factors adopted.

Four methods are presented in AISC 360 and the *AISC Manual* for determining secondary effects.

- effective length method, detailed in AISC 360 App. 7.2

- direct analysis method, detailed in AISC 360 Sec. C2

- first-order analysis method, detailed in AISC 360 App. 7.3

- simplified method, detailed in *AISC Manual* Part 2

The effective length method and the direct analysis method both require a second-order frame analysis of the structure. This may be accomplished using a rigorous, second-order computer analysis. Alternatively, the approximate B_1-B_2 procedure specified in AISC 360 App. 8.2 may be used.

Effective Length Method

This method is restricted by AISC 360 App. 7.2 to structures with a sidesway amplification factor of

$$B_2 = \frac{\Delta_{2nd}}{\Delta_{1st}} \le 1.5$$

Drift is determined for LRFD load combinations or is 1.6 times the ASD load combinations.

In accordance with AISC 360 App. 7.2, the design forces may be determined either by a rigorous second-order computer analysis or by amplifying the results of a first-order analysis.

In applying the effective length method, the nominal stiffness of all members is used in the analysis with no reduction for inelasticity and residual stress. These effects are accounted for in the design of the columns by using an appropriate value for the effective length factor, K. The available strength of columns is determined using an

effective length factor as defined in AISC 360 Comm. Table C-A-7.1, or is calculated in accordance with AISC 360 Comm. App. 7.2. The empirical column curve then accounts for inelasticity and residual stress. A value of 1.0 may be used for the effective length factor of members in a braced frame. For columns that do not contribute to the lateral resistance of the structure, a value of $K = 1.0$ may be used for the effective length factor. As specified in AISC 360 App. 7.2.3(b), the value of $K = 1.0$ may also be used for all columns when the structure is sufficiently stiff that

$$B_2 = \frac{\Delta_{2nd}}{\Delta_{1st}} \le 1.1$$

Drift is determined for LRFD load combinations or 1.6 times the ASD load combinations.

To account for initial imperfections in the members, minimum lateral loads are applied at each story in accordance with AISC 360 Sec. C2.2b(1). These loads are given by

$$N_i = 0.002\alpha Y_i$$

The notional loads are applied solely in gravity-only load combinations in the effective length method.

Example 4.21

The first-order member forces produced in the outer column of a moment frame by the governing factored load combination are shown. Determine the second-order member forces in the column using the effective length method. The column consists of a W12 × 79 section with a yield stress of 50 kips/in². The bay length is 25 ft and the beams consist of W21 × 62 sections with a yield stress of 50 kips/in². No intermediate bracing is provided to the column about either axis.

Illustration for Ex. 4.21
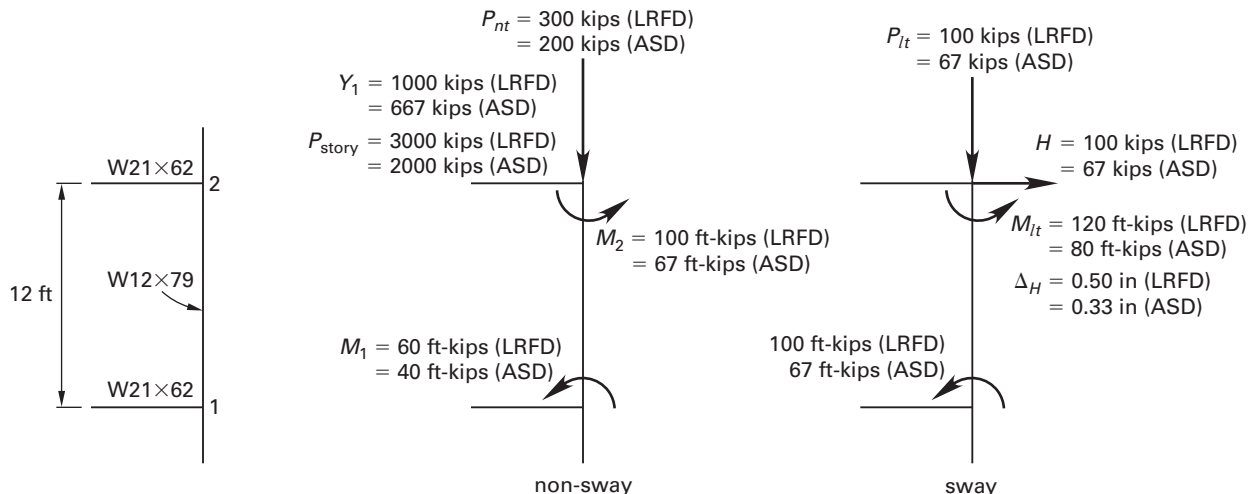
Solution

LRFD Method

The loading condition is not a gravity-only load combination, and the notional lateral load is not applicable.

Determine B_1.

The results of the first-order, non-sway analysis are shown in the illustration, and the reduction factor is given by AISC 360 Eq. A-8-4 as

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right)$$
$$= 0.6 - (0.4)\left(\frac{60 \text{ ft-kips}}{100 \text{ ft-kips}}\right)$$
$$= 0.36$$

The effective length factor in the plane of bending for a non-sway frame is given by

$$K_1 = 1.0$$

From *AISC Manual* Table 4-1, the Euler buckling strength of a $W12 \times 79$ column in the plane of bending is given by

$$P_{e1} = \frac{\pi^2 E I^*}{(K_1 L)^2}$$
$$= \frac{(18,900 \text{ kip-in}^2)(10^4)}{\left((1.00)(144 \text{ in})\right)^2}$$
$$= 9115 \text{ kips}$$

EI^* is the flexural rigidity used in the analysis. For the effective length method, $EI^* = EI$.

Assume the required second-order axial force in the column is

$$P_r = 415 \text{ kips}$$

The moment magnification factor to account for the P-δ effect, assuming no lateral translation of the frame, is defined in AISC 360 Eq. A-8-3 as

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}}$$
$$= \frac{0.36}{1 - \dfrac{(1.0)(415 \text{ kips})}{9115 \text{ kips}}}$$
$$= 1.0 \quad [\text{minimum}]$$

Determine B_2.

For a moment frame, $R_M = 0.85$.

The results of the first-order sway analysis are shown in the illustration, and the Euler buckling strength for the story is given by AISC 360 Eq. A-8-7 as

$$P_{e\,\text{story}} = \left(\frac{R_M L}{\Delta_H}\right) H$$
$$= \left(\frac{(0.85)(144 \text{ in})}{0.50 \text{ in}}\right)(100 \text{ kips})$$
$$= 24,480 \text{ kips}$$

The moment magnification factor to account for the sidesway effect, with lateral translation of the frame allowed, is given by AISC 360 Eq. A-8-6 as

$$\frac{\Delta_{2\text{nd}}}{\Delta_{1\text{st}}} = B_2 = \frac{1}{1 - \dfrac{\alpha P_{\text{story}}}{P_{e\,\text{story}}}}$$
$$= \frac{1}{1 - \dfrac{(1.0)(3000 \text{ kips})}{24,480 \text{ kips}}}$$
$$= 1.14$$

Calculate the second-order member forces.

The second-order member forces for the frame are given by AISC 360 Eq. A-8-1 and Eq. A-8-2 as

$$M_r = B_1 M_{nt} + B_2 M_{lt}$$
$$= (1.0)(100 \text{ ft-kips}) + (1.14)(120 \text{ ft-kips})$$
$$= 237 \text{ ft-kips}$$
$$P_r = P_{nt} + B_2 P_{lt}$$
$$= 300 \text{ kips} + (1.14)(100 \text{ kips})$$
$$= 414 \text{ kips}$$

ASD Method

The notional lateral load is not applicable.

Determine B_1.

The results of the first-order, non-sway analysis are shown in the illustration and the reduction factor is given by AISC 360 Eq. A-8-4 as

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right)$$
$$= 0.6 - (0.4)\left(\frac{40 \text{ ft-kips}}{67 \text{ ft-kips}}\right)$$
$$= 0.36$$

The effective length factor in the plane of bending for a non-sway frame is given by

$$K_1 = 1.0$$

From *AISC Manual* Table 4-1, the Euler buckling strength of a $W12 \times 79$ column in the plane of bending is given by

$$
\begin{aligned}
P_{e1} &= \frac{\pi^2 EI}{(K_1 L)^2} \\
&= \frac{(18{,}900 \text{ kip-in}^2)(10^4)}{\left((1.0)(12 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) \right)^2} \\
&= 9115 \text{ kips}
\end{aligned}
$$

Assume the required second-order axial force in the column is

$$
P_r = 277 \text{ kips}
$$

The moment magnification factor to account for the member effect, assuming no lateral translation of the frame, is defined in AISC 360 Eq. A-8-3 as

$$
\begin{aligned}
B_1 &= \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \\
&= \frac{0.36}{1 - \dfrac{(1.6)(277 \text{ kips})}{9115 \text{ kips}}} \\
&= 1.0 \quad [\text{minimum}]
\end{aligned}
$$

Determine B_2.

For a moment frame, $R_M = 0.85$.

The results of the first-order sway analysis are shown in the illustration, and the Euler buckling strength for the story is given by AISC 360 Eq. A-8-7 as

$$
\begin{aligned}
P_{e\,\text{story}} &= \left(\frac{R_M L}{\Delta_H} \right) H \\
&= \left(\frac{(0.85)(144 \text{ in})}{0.33 \text{ in}} \right)(67 \text{ kips}) \\
&= 24{,}850 \text{ kips}
\end{aligned}
$$

The moment magnification factor to account for the frame effect, with lateral translation of the frame allowed, is given by AISC 360 Eq. A-8-6 as

$$
\begin{aligned}
\frac{\Delta_{\text{2nd}}}{\Delta_{\text{1st}}} = B_2 &= \frac{1}{1 - \dfrac{\alpha P_{\text{story}}}{P_{e\,\text{story}}}} \\
&= \frac{1}{1 - \dfrac{(1.6)(2000 \text{ kips})}{24{,}850 \text{ kips}}} \\
&= 1.15
\end{aligned}
$$

Calculate the second-order member forces.

The second-order member forces for the frame are given by AISC 360 Eq. A-8-1 and Eq. A-8-2 as

$$
\begin{aligned}
M_r &= B_1 M_{nt} + B_2 M_{lt} \\
&= (1.0)(67 \text{ ft-kips}) + (1.15)(80 \text{ ft-kips}) \\
&= 159 \text{ ft-kips} \\
P_r &= P_{nt} + B_2 P_{lt} \\
&= 200 \text{ kips} + (1.15)(67 \text{ kips}) \\
&= 277 \text{ kips}
\end{aligned}
$$

Direct Analysis Method

The direct analysis method is applicable to all types of structures and must be used when

$$
B_2 = \frac{\Delta_{\text{2nd}}}{\Delta_{\text{1st}}} > 1.5
$$

The design forces may be determined either by a rigorous second-order computer analysis, or by amplifying the results of a first-order analysis.

In applying the method, the factored loads are applied to the structure using reduced flexural and axial stiffness of members that contribute to the lateral stability of the structure. The reduced stiffnesses account for elastic instability and inelastic softening effects, and are given by AISC 360 Sec. C2.3 as

$$
\begin{aligned}
EI^* &= 0.8\tau_b EI \\
EA^* &= 0.8 EA
\end{aligned}
$$

The stiffness reduction coefficient is given by

$$
\begin{aligned}
\tau_b &= 1.0 \quad [\alpha P_r \leq 0.5 P_y] \\
&= 4\left(\frac{\alpha P_r}{P_y} \right)\left(1 - \frac{\alpha P_r}{P_y} \right) \quad [\alpha P_r > 0.5 P_y]
\end{aligned}
$$

Alternatively, when $\alpha P_r > 0.5 P_y$, the stiffness reduction factor, τ_b, may be taken as 1.0, provided that the actual lateral loads are increased by a notional lateral load of

$$
N_i = 0.001 \alpha Y_i
$$

To account for out-of-plumbness in the columns, minimum lateral loads are applied at each story, in accordance with AISC 360 Sec. C2.2(b). These loads are given by

$$
N_i = 0.002 \alpha Y_i
$$

As specified in AISC 360 Sec. C2.2b(4), the notional loads are additive to the applied lateral loads when the sidesway amplification ratio is

$$
\frac{\Delta_{\text{2nd}}}{\Delta_{\text{1st}}} > 1.5
$$

Drift is calculated using the reduced elastic stiffness for LRFD load combinations, or 1.6 times the ASD load combinations.

Otherwise, it is permissible to apply the notional loads solely in gravity-only load combinations.

In designing the members of the frame for the calculated second-order forces, the appropriate effective length factor is specified in AISC 360 Sec. C3, as $K = 1.0$ for all members.

Example 4.22

The first-order member forces produced in the outer column of a moment frame by the governing factored load combination are shown. Reduced stiffness values of $EI^* = 0.8EI$ were used in the analyses. Determine the second-order member forces in the column using the direct analysis method. The column consists of a $W12 \times 79$ section with a yield stress of 50 kips/in^2. The bay length is 25 ft and the beams consist of $W21 \times 62$ sections with a yield stress of 50 kips/in^2. No intermediate bracing is provided to the column about either axis.

Solution

LRFD Method

The load combination is not a gravity-only combination and, assuming $B_2 < 1.7$, notional lateral loads are not applicable.

Determine B_1.

In accordance with AISC 360 Sec. C2.3, for the direct analysis method, the flexural and axial stiffness of members in the structure are reduced to

$$EI^* = 0.8\tau_b EI$$
$$EA^* = 0.8EA$$

Assuming $\alpha P_r \leq 0.5P_y$, the stiffness reduction parameter is $\tau_b = 1.0$, and the flexural stiffness becomes

$$EI^* = 0.8EI$$

From Ex. 4.21, the Euler buckling strength is

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2}$$

$$= \frac{(0.8)(18{,}900 \text{ kip-in}^2)(10^4)}{\left((1.0)(12 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)\right)^2}$$

$$= 7292 \text{ kips}$$

$$C_m = 0.36$$

Assuming the second-order axial force is

$$P_r = 415 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}}$$

$$= \frac{0.36}{1 - \dfrac{(1.0)(415 \text{ kips})}{7292 \text{ kips}}}$$

$$= 1.0 \quad [\text{minimum}]$$

Determine B_2.

For a moment frame, $R_M = 0.85$.

Illustration for Ex. 4.22

The results of the first-order sway analysis are shown in the illustration, and the Euler buckling strength for the story is given by AISC 360 Eq. A-8-7 as

$$P_{e\,\text{story}} = \left(\frac{R_M L}{\Delta_H}\right) H$$

$$= \left(\frac{(0.85)(12 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{0.60 \text{ in}}\right)(100 \text{ kips})$$

$$= 20{,}400 \text{ kips}$$

The moment magnification factor to account for the sidesway effect, with lateral translation of the frame allowed, is given by AISC 360 Eq. A-8-6 as

$$\frac{\Delta_{2\text{nd}}}{\Delta_{1\text{st}}} = B_2 = \frac{1}{1 - \dfrac{\alpha P_{\text{story}}}{P_{e\,\text{story}}}}$$

$$= \frac{1}{1 - \dfrac{(1.0)(3000 \text{ kips})}{20{,}400 \text{ kips}}}$$

$$= 1.17$$

Calculate the second-order member forces.

The second-order member forces for the frame are given by AISC 360 Eq. A-8-1 and AISC 360 Eq. A-8-2 as

$$M_r = B_1 M_{nt} + B_2 M_{lt}$$
$$= (1.0)(100 \text{ ft-kips}) + (1.17)(120 \text{ ft-kips})$$
$$= 240 \text{ ft-kips}$$
$$P_r = P_{nt} + B_2 P_{lt}$$
$$= 300 \text{ kips} + (1.17)(100 \text{ kips})$$
$$= 417 \text{ kips}$$

ASD Method

The load combination is not a gravity-only combination and, assuming $B_2 < 1.7$, notional lateral loads are not applicable.

Determine B_1.

From Ex. 4.21, the Euler buckling strength is

$$P_{e1} = \frac{\pi^2 E I^*}{(K_1 L)^2}$$

$$= \frac{(0.8)(18{,}900 \text{ kip-in}^2)(10^4)}{\left((1.0)(12 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)\right)^2}$$

$$= 7292 \text{ kips}$$

Assuming the second-order axial force is

$$P_r = 280 \text{ kips}$$

$$B_1 = \frac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}}$$

$$= \frac{0.36}{1 - \dfrac{(1.0)(280 \text{ kips})}{7292 \text{ kips}}}$$

$$= 1.0 \quad [\text{minimum}]$$

Determine B_2.

For a moment frame, $R_M = 0.85$.

The results of the first-order sway analysis are shown in the illustration, and the sum of the Euler buckling strength, for all columns in the story, is given by AISC 360 Eq. A-8-7 as

$$P_{e\,\text{story}} = \left(\frac{R_M L}{\Delta_H}\right) H$$

$$= \left(\frac{(0.85)(12 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{0.40 \text{ in}}\right)(67 \text{ kips})$$

$$= 20{,}502 \text{ kips}$$

The moment magnification factor to account for the sidesway effect, with lateral translation of the frame allowed, is given by AISC 360 Eq. A-8-6 as

$$\frac{\Delta_{2\text{nd}}}{\Delta_{1\text{st}}} = B_2 = \frac{1}{1 - \dfrac{\alpha P_{\text{story}}}{P_{e\,\text{story}}}}$$

$$= \frac{1}{1 - \dfrac{(1.6)(2000 \text{ kips})}{20{,}502 \text{ kips}}}$$

$$= 1.19$$

Calculate the second-order member forces.

The second-order member forces for the frame are given by AISC 360 Eq. A-8-1 and AISC 360 Eq. A-8-2 as

$$M_r = B_1 M_{nt} + B_2 M_{lt}$$
$$= (1.0)(67 \text{ ft-kips}) + (1.19)(80 \text{ ft-kips})$$
$$= 162 \text{ ft-kips}$$
$$P_r = P_{nt} + B_2 P_{lt}$$
$$= 200 \text{ kips} + (1.19)(67 \text{ kips})$$
$$= 280 \text{ kips}$$

First-Order Elastic Analysis

The first-order elastic analysis method is specified in AISC 360 App. 7.3. This method is restricted by AISC 360 App. 7.3.1(2) to structures with a sidesway amplification factor of

$$B_2 = \frac{\Delta_{2nd}}{\Delta_{1st}} \leq 1.5$$

Drift is determined for LRFD load combinations, or 1.6 times the ASD load combinations.

The design forces are determined by a first-order analysis with only notional loads applied to the structure using the nominal (unreduced) member stiffnesses.

In addition, a limit is placed on the required axial compressive strength such that

$$\alpha P_r \leq 0.5 P_y$$
$$P_y = \text{member yield strength}$$
$$= A F_y$$

To account for initial imperfections in the members, notional lateral loads are applied at each story and are given by

$$N_i = \frac{2.1 \Delta_{1st} \alpha Y_i}{L}$$
$$\geq 0.0042 Y_i$$

The notional loads are additive to the applied lateral loads and eliminate the need for a second-order analysis.

The non-sway amplification of column moments is considered by applying the B_1 amplifier to the total member moments.

In designing the members of the frame for the calculated second-order forces, the appropriate effective length
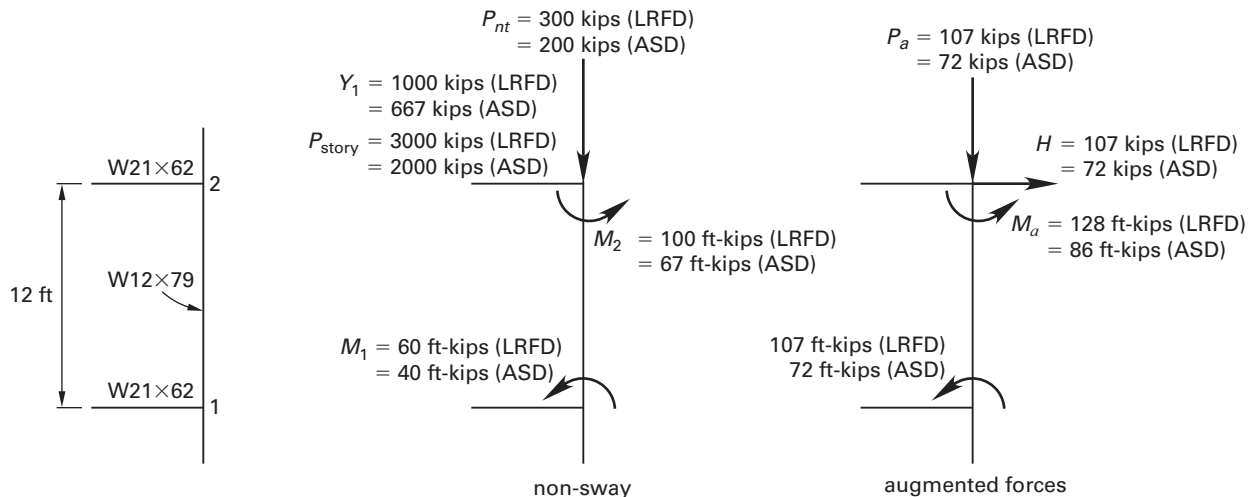
factor is specified in AISC 360 App. 7.3.3 as $K = 1.0$ for all members.

Example 4.23

The factored loads acting on the outer column of a moment frame are shown. Determine the required strength of the column using the first-order elastic analysis method. The column consists of a W12 × 79 section with a yield stress of 50 kips/in².

The bay length is 25 ft and the beams consist of W21 × 62 sections with a yield stress of 50 kips/in². No intermediate bracing is provided to the column about either axis.

The interstory drift is $\Delta_{1st} = 0.50$ in (LRFD) or $\Delta_{1st} = 0.33$ in (ASD).

Solution

LRFD Method

The column yield strength is

$$P_y = A F_y$$
$$= (23.2 \text{ in}^2)\left(50 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 1160 \text{ kips}$$

Assuming that the required axial load is

$$P_r = 417 \text{ kips}$$
$$\frac{\alpha P_r}{P_y} = \frac{(1.0)(417 \text{ kips})}{1160 \text{ kips}}$$
$$= 0.36$$
$$< 0.5$$

In accordance with AISC 360 App. 7.3.1(3), a first-order elastic analysis is permissible.

Illustration for Ex. 4.23

Determine the augmented loads.

The first-order interstory drift due to the design loads is

$$\Delta_{1st} = 0.5 \text{ in}$$

The sum of the gravity loads, applied at the story, independent of loads from the upper stories is

$$Y_1 = 1000 \text{ kips}$$

The notional lateral load on the story is given by AISC 360 App. 7.3.2(1) as

$$N_i = 0.0042\, Y_i$$
$$= (0.0042)(1000 \text{ kips})$$
$$= 4.2 \text{ kips}$$

However, this value may not be less than

$$N_i = \frac{2.1\Delta_{1st}\alpha\, Y_i}{L}$$
$$= \frac{(2.1)(0.5 \text{ in})(1.0)(1000 \text{ kips})}{144 \text{ in}}$$
$$= 7.3 \text{ kips} \quad [\text{governs}]$$

This is additive to the applied lateral loads, and the resulting augmented forces on the column are indicated in the illustration.

Determine B_1.

The moment magnification factor to account for the P-δ effect, assuming no lateral translation of the frame, is determined in Ex. 4.21 as

$$B_1 = 1.0$$

Calculate the required forces. The required forces are given by

$$M_r = B_1(M_{nt} + M_a)$$
$$= (1.0)(100 \text{ ft-kips} + 128 \text{ ft-kips})$$
$$= 228 \text{ ft-kips}$$
$$P_r = P_{nt} + P_a$$
$$= 300 \text{ kips} + 107 \text{ kips}$$
$$= 407 \text{ kips}$$

ASD Method

The column yield strength is

$$P_y = AF_y$$
$$= (23.2 \text{ in}^2)\left(50 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 1160 \text{ kips}$$

Assuming that the required axial load is

$$P_r = 278 \text{ kips}$$
$$\frac{\alpha P_r}{P_y} = \frac{(1.6)(278 \text{ kips})}{1160 \text{ kips}}$$
$$= 0.38$$
$$< 0.5$$

In accordance with AISC 360 App. 7.3.1(3), a first-order elastic analysis is permissible.

Determine the augmented loads.

The first-order interstory drift due to the lateral loads is

$$\Delta_{1st} = 0.33 \text{ in}$$

The sum of the gravity loads, applied at the story, independent of loads from the upper stories is

$$Y_1 = 667 \text{ kips}$$

The notional lateral load on the story is given by AISC 360 App. 7.3.2(1) as

$$N_i = 0.0042\, Y_i$$
$$= (0.0042)(667 \text{ kips})$$
$$= 2.8 \text{ kips}$$

However, this value may not be less than

$$N_i = \frac{2.1\Delta_{1st}\alpha\, Y_i}{L}$$
$$= \frac{(2.1)(0.33 \text{ in})(1.6)(667 \text{ kips})}{144 \text{ in}}$$
$$= 5.1 \text{ kips} \quad [\text{governs}]$$

This is additive to the applied lateral loads, and the augmented loads and the resulting forces on the column are indicated in the illustration.

The moment magnification factor to account for the member effect, assuming no lateral translation of the frame, is determined in Ex. 4.21 as

$$B_1 = 1.0$$

Calculate the required forces. The required forces are given by

$$M_r = B_1(M_{nt} + M_a)$$
$$= (1.0)(67 \text{ ft-kips} + 86 \text{ ft-kips})$$
$$= 153 \text{ ft-kips}$$
$$P_r = P_{nt} + P_a$$
$$= 200 \text{ kips} + 72 \text{ kips}$$
$$= 272 \text{ kips}$$

Simplified Method

The simplified method is specified in *AISC Manual* Part 2 and requires a first-order analysis only. The method is restricted to structures with a sidesway amplification factor of

$$\frac{\Delta_{2nd}}{\Delta_{1st}} \le 1.5$$

In addition, the ratio of the sway and non-sway amplification factors is restricted to

$$\frac{B_1}{B_2} \le 1.0$$

For members not subjected to transverse loading, it is unlikely that B_1 will be greater than B_2.

In applying the method, the factored loads are applied to the structure using the nominal stiffness of the members, and a first-order analysis is performed. To account for initial imperfections in the members, minimum lateral loads are applied at each story in accordance with AISC 360 Sec. C2.2(b)(1). These loads are given by

$$N_i = 0.002\alpha Y_i$$

This is only applied in gravity load cases.

Required strengths are determined by multiplying the forces obtained from the first-order analysis by tabulated values of B_2. These tabulated values are a function of the required story drift limit and the ratio of the total story gravity load to the lateral load that produces the drift limit.

From the first-order elastic analysis, the lateral load required to produce the required story drift is determined, and the ratio of the total story gravity load to the lateral load that produces this drift limit is calculated. Using α

times this value and the value of the required story drift, the appropriate amplification factor, B_2, may be obtained from the table in *AISC Manual* Part 2 or from *AISC Basic Design Values Cards.*[7] (See Table 4.1.)

In designing the members of the frame for the calculated forces, the appropriate effective length factor is specified as $K = 1.0$ for all members, provided that the amplification factor does not exceed 1.1.

For cases where the amplification factor exceeds 1.1, the effective length factors for the members are determined by analysis.

Example 4.24

The factored loads acting on the outer column of a moment frame are shown. Determine the required strength of the column using the simplified analysis method. The column consists of a W12 × 79 section with a yield stress of 50 kips/in². The bay length is 25 ft and the beams consist of W21 × 62 sections with a yield stress of 50 kips/in². No intermediate bracing is provided to the column about either axis. The required story drift is limited to the product of the story height and 1/240 for LRFD (1/360 for ASD).

Solution

LRFD Method

This is not a gravity-only loading case and the notional lateral load is not applicable.

The factored loads produce the first-order elastic forces, which are derived in Ex. 4.21 and are shown in the illustration. The interstory drift produced is

$$\Delta_a = 0.50 \text{ in}$$

Illustration for Ex. 4.24
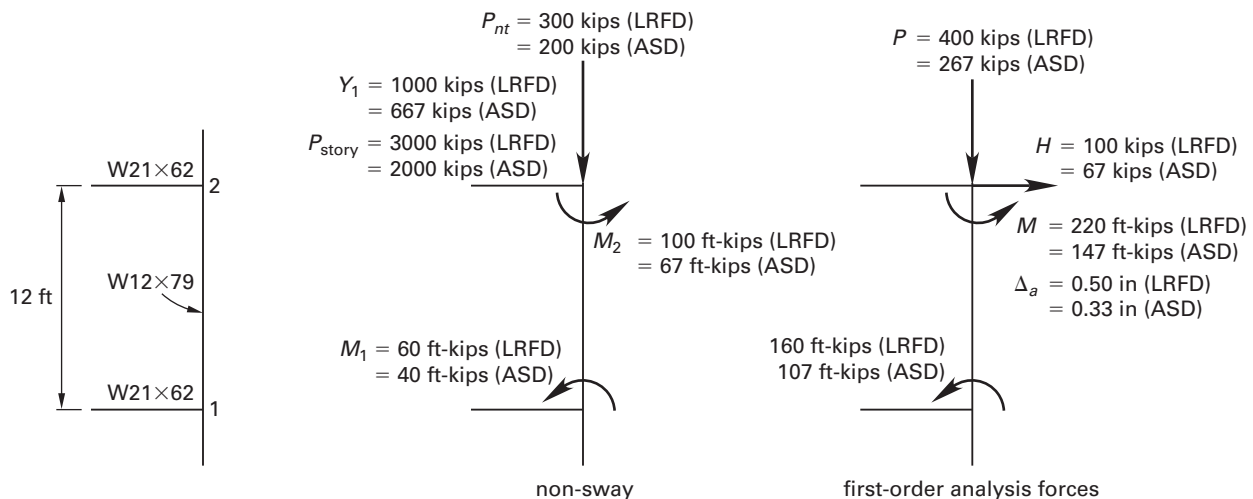
For a design story drift of 1/240, the permissible deflection is

$$\Delta_p = \frac{144 \text{ in}}{240}$$
$$= 0.60 \text{ in}$$

The lateral load required to produce the required story drift is

$$H_p = \frac{(100 \text{ kips})\Delta_p}{\Delta_a} = \frac{(100 \text{ kips})(0.60 \text{ in})}{0.50 \text{ in}}$$
$$= 120 \text{ kips}$$

The ratio of α times the total story gravity load to the lateral load that produces the drift limit is

$$\frac{\alpha P_{\text{story}}}{H_p} = \frac{(1.0)(3000 \text{ kips})}{120 \text{ kips}}$$
$$= 25$$

Interpolate from Table 4.1. For a drift limit of 1/240 and a load ratio of 25, the amplification factor is

$$B_2 = 1.13$$

The required forces are given by

$$M_r = B_2 M = (1.13)(220 \text{ ft-kips})$$
$$= 249 \text{ ft-kips}$$
$$P_r = B_2 P = (1.13)(400 \text{ kips})$$
$$= 452 \text{ kips}$$

ASD Method

This is not a gravity-only loading case and the notional lateral load is not applicable.

The factored loads produce the first-order elastic forces, which are derived in Ex. 4.21 and are shown in the illustration. The interstory drift produced is

$$\Delta_a = 0.33 \text{ in}$$

For a design story drift of 1/360, the permissible deflection is

$$\Delta_p = \frac{144 \text{ in}}{360}$$
$$= 0.40 \text{ in}$$

The lateral load required to produce the required story drift is

$$H_p = \frac{(67 \text{ kips})\Delta_p}{\Delta_a} = \frac{(67 \text{ kips})(0.40 \text{ in})}{0.33 \text{ in}}$$
$$= 81 \text{ kips}$$

The ratio of α times the total story gravity load to the lateral load that produces the drift limit is

$$\frac{\alpha P_{\text{story}}}{H_p} = \frac{(1.6)(2000 \text{ kips})}{81 \text{ kips}}$$
$$= 39.5$$

Interpolate from Table 4.1. For a drift limit of 1/360 and a load ratio of 39.5, the amplification factor is

$$B_2 = 1.14$$

Table 4.1 *Amplification Factor B_2 for Use with the Simplified Method*

design story drift limit	load ratio (times 1.6 for ASD, 1.0 for LRFD)										
	0	5	10	20	30	40	50	60	80	100	120
$H/100$	1	1.1	1.1	1.3	**1.5**/1.4	When ratio exceeds 1.5, simplified method requires a stiffer structure.					
$H/200$	1	1	1.1	1.1	1.2	1.3	**1.4**/1.3	**1.5**/1.4			
$H/300$	1	1	1	1.1	1.1	1.2	1.2	1.3	**1.5**/1.4		
$H/400$	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	**1.4**/1.3	1.5
$H/500$	1	1	1	1	1.1	1.1	1.1	1.2	1.2	1.3	1.4

Note: Where two values are provided, the value in bold is the value associated with $R_m = 0.85$.

The required forces are given by

$$M_r = B_2 M = (1.14)(147 \text{ ft-kips})$$
$$= 168 \text{ ft-kips}$$
$$P_r = B_2 P = (1.14)(267 \text{ kips})$$
$$= 304 \text{ kips}$$

Combined Compression and Flexure[8]

The adequacy of a member to sustain combined compression and flexure is determined by means of the interaction equations given in AISC 360 Sec. H1.1 as follows.

For $P_r/P_c \geq 0.2$,

$$\frac{P_r}{P_c} + \left(\tfrac{8}{9}\right)\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{[AISC H1-1a]}$$

This expression reduces to

$$pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0$$

For $P_r/P_c < 0.2$,

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{[AISC H1-1b]}$$

This expression reduces to

$$\frac{pP_r}{2} + \left(\tfrac{9}{8}\right)\left(b_x M_{rx} + b_y M_{ry}\right) \le 1.0$$

Values of p, b_x, and b_y are tabulated in *AISC Manual* Table 6-1 for W shapes with a yield stress of 50 kips/in^2 and assuming a bending coefficient of $C_b = 1.0$.

Example 4.25

Determine the adequacy of the W12 × 79 column in the frame analyzed by the effective length method in Ex. 4.21. The beams consist of W21 × 62 sections with a length of 25 ft. All members have a yield stress of 50 kips/in^2, and no intermediate bracing is provided to the column about either axis.

Solution

The frame was analyzed for second-order effects in Ex. 4.21 using the effective length method. The required forces in the column are

$$P_r = 414 \text{ kips (LRFD)}, \ 277 \text{ kips (ASD)}$$

$$M_{rx} = 237 \text{ ft-kips (LRFD)}, \ 159 \text{ ft-kips (ASD)}$$

$$M_{ry} = 0 \text{ ft-kips}$$

Calculate the effective column length.

At both joint 1 and joint 2, the stiffness ratio is

$$
\begin{aligned}
G_2 &= \frac{\sum \dfrac{I_c}{L_c}}{\sum \dfrac{I_g}{L_g}} \\[2mm]
&= \frac{(2)\left(\dfrac{662 \text{ in}^4}{12 \text{ ft}}\right)}{\dfrac{1330 \text{ in}^4}{25 \text{ ft}}} \\[2mm]
&= 2.07
\end{aligned}
$$

From the alignment chart for sway frames (see Fig. 4.10), the effective length factor about the x-axis of column 12 is

$$K_x = 1.65$$

The effective length of the column about the x-axis is

$$
\begin{aligned}
(KL)_x &= (1.65)(12 \text{ ft}) \\
&= 19.8 \text{ ft}
\end{aligned}
$$

The effective length of the column about the y-axis is

$$
\begin{aligned}
(KL)_y &= (1.0)(12 \text{ ft}) \\
&= 12 \text{ ft}
\end{aligned}
$$

From *AISC Manual* Table 4-1, a W12 × 79 column has a value of

$$\frac{r_x}{r_y} = 1.75$$

The equivalent effective length about the major axis with respect to the y-axis is

$$
\begin{aligned}
(KL)_{y(\text{equiv})} &= \frac{(KL)_x}{\dfrac{r_x}{r_y}} \\[2mm]
&= \frac{19.8 \text{ ft}}{1.75} \\[2mm]
&= 11.31 \text{ ft} \quad [\text{does not govern}] \\[2mm]
&< K_y L_y
\end{aligned}
$$

The effective length about the minor axis governs.

Apply the interaction equation.

LRFD Method

From *AISC Manual* Table 4-1, a W12 × 79 column with an effective length of $(KL)_y = 12$ ft has a design axial strength of

$$
\begin{aligned}
\phi_c P_n &= 887 \text{ kips} \\[2mm]
\frac{P_r}{\phi_c P_n} &= \frac{414 \text{ kips}}{887 \text{ kips}} \\[2mm]
&= 0.467 \\[2mm]
&> 0.2
\end{aligned}
$$

The AISC 360 Eq. H1-1a applies, and the reduced form of AISC 360 Eq. H1-1a is

$$pP_r + b_x M_{rx} + b_y M_{ry} \le 1.0$$

Substituting in the left hand side of this expression and using the design parameters from *AISC Manual* Table 6-1 gives

$$
\begin{aligned}
&(1.13 \times 10^{-3} \text{ kips}^{-1})(414 \text{ kips}) \\
&\quad + (2.02 \times 10^{-3} \text{ ft-kips}^{-1})(237 \text{ ft-kips}) \\
&\quad + (4.37 \times 10^{3} \text{ ft-kips}^{-1})(0 \text{ ft-kips}) \\
&\quad = 0.95 \\
&\quad < 1.0 \quad [\text{satisfactory}]
\end{aligned}
$$

ASD Method

From *AISC Manual* Table 4-1, a W12 × 79 column with an effective length of $(KL)_y = 12$ ft has a design axial strength of

$$\frac{P_n}{\Omega_c} = 590 \text{ kips}$$

$$\frac{P_r}{\dfrac{P_n}{\Omega_c}} = \frac{277 \text{ kips}}{590 \text{ kips}}$$

$$= 0.469$$

$$> 0.2$$

The AISC 360 Eq. H1-1a applies, and the reduced form of AISC 360 Eq. H1-1a is

$$pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0$$

Substituting in the left hand side of this expression and using the design parameters from *AISC Manual* Table 6-1 gives

$$(1.69 \times 10^{-3} \text{ kips}^{-1})(277 \text{ kips})$$

$$+ (3.04 \times 10^{-3} \text{ ft-kips}^{-1})(159 \text{ ft-kips})$$

$$+ (6.56 \times 10^3 \text{ ft-kips}^{-1})(0 \text{ ft-kips})$$

$$= 0.95$$

$$< 1.0 \quad \text{[satisfactory]}$$

Column Base Plates

The design of column base plates is covered in *AISC Manual* Part 14 and AISC 360 Sec. J8. As shown in Fig. 4.14, the base plate is assumed to cantilever about axes a distance m or n from the edge of the plate. The required base plate thickness is given by the largest of the three values obtained from

$$t_{\text{req}} = m \sqrt{\frac{2P_u}{0.9 F_y BN}} \quad \text{[LRFD]}$$

$$t_{\text{req}} = n \sqrt{\frac{2P_u}{0.9 F_y BN}} \quad \text{[LRFD]}$$

$$t_{\text{req}} = \lambda n' \sqrt{\frac{2P_u}{0.9 F_y BN}} \quad \text{[LRFD]}$$

$$t_{\text{req}} = m \sqrt{\frac{3.33 P_a}{F_y BN}} \quad \text{[ASD]}$$

$$t_{\text{req}} = n \sqrt{\frac{3.33 P_a}{F_y BN}} \quad \text{[ASD]}$$

$$t_{\text{req}} = \lambda n' \sqrt{\frac{3.33 P_a}{F_y BN}} \quad \text{[ASD]}$$

Figure 4.14 *Column Base Plate*
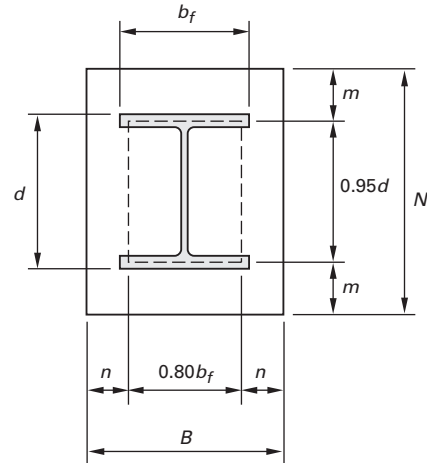
In all the preceding equations,

$$n' = \frac{\sqrt{d b_f}}{4}$$

λ may conservatively be taken as 1.0.

Example 4.26

A 19 in × 19 in grade A36 base plate is proposed for a W12 × 106 column that supports a factored load of 850 kips (LRFD) or 570 kips (ASD). Determine the minimum required base plate thickness.

Solution

From *AISC Manual* Table 1-1, the relevant dimensions are as follows.

$$d = 12.9 \text{ in}$$

$$b_f = 12.2 \text{ in}$$

$$\lambda n' = \frac{\lambda \sqrt{d b_f}}{4}$$

$$= \frac{1.0 \sqrt{(12.9 \text{ in})(12.2 \text{ in})}}{4}$$

$$= 3.14 \text{ in}$$

$$m = \frac{N - 0.95d}{2}$$

$$= \frac{19 \text{ in} - (0.95)(12.9 \text{ in})}{2}$$

$$= 3.37 \text{ in}$$

$$n = \frac{B - 0.8 b_f}{2}$$

$$= \frac{19 \text{ in} - (0.8)(12.2 \text{ in})}{2}$$

$$= 4.62 \text{ in} \quad \text{[governs]}$$

LRFD Method

$$t_{\text{req}} = n\sqrt{\frac{2P_u}{0.9F_yBN}}$$

$$= 4.61 \text{ in} \sqrt{\frac{(2)(850 \text{ kips})}{(0.9)\left(36 \frac{\text{kips}}{\text{in}^2}\right)(19 \text{ in})(19 \text{ in})}}$$

$$= 1.76 \text{ in}$$

ASD Method

$$t_{\text{req}} = n\sqrt{\frac{3.33P_a}{F_yBN}}$$

$$= 4.61 \text{ in} \sqrt{\frac{(3.33)(570 \text{ kips})}{\left(36 \frac{\text{kips}}{\text{in}^2}\right)(19 \text{ in})(19 \text{ in})}}$$

$$= 1.76 \text{ in}$$

6. PLASTIC DESIGN

Nomenclature

A	area of section	in^2
D	degree of indeterminacy of a structure	–
E	modulus of elasticity	kips/in^2
F_y	yield stress	kips/in^2
h	clear distance between flanges less the corner radius at each flange	in
H	horizontal force	kips
l	length of span	ft
L	unbraced length	ft
L_{pd}	limiting laterally unbraced length for plastic analysis	ft or in
m_i	number of independent collapse mechanisms in a structure	–
M_s	bending moment produced by factored loads acting on the cut-back structure	ft-kips
M_y	yield moment	ft-kips
M_1	smaller moment at end of unbraced length of beam	ft-kips
M_2	larger moment at end of unbraced length of beam	ft-kips
M_1/M_2	positive when moments cause reverse curvature and negative for single curvature	–
p	number of possible hinge locations in a structure	–
P	axial force	kips
P_u	required axial strength in compression	kips
P_y	axial yield strength	kips
r	radius of gyration	in
t_w	web thickness	in
V	shear force	kips
w_u	factored uniformly distributed load	kips/ft
W	total load	lbf

Symbols

ϕ_b	resistance factor for flexure	–
ϕ_c	resistance factor for compression	–

Design Considerations

The plastic method of structural analysis is used to determine the maximum loads a structure can support prior to collapse. The plastic method has several advantages over the ASD and LRFD design techniques, because it

- produces a more economical structure
- provides a simple and direct design technique
- accurately models the structure at ultimate loads
- realistically predicts the ultimate strength

The plastic method is applicable to structures constructed with a ductile material possessing ideal elastic-plastic characteristics. As shown in Fig. 4.15, such a material initially exhibits a linear relationship between stress and strain until the yield point is reached.

After this point, the stress remains constant at the yield stress, F_y, while the strain continues to increase indefinitely as plastic yielding of the material occurs. The plastic hinge, formed at the location where yielding occurs, has a plastic moment of resistance, M_p, and rotation continues at the hinge without any increase in the resisting moment. As shown in Fig. 4.16, increasing the applied bending moment on a steel beam eventually causes the extreme fibers to reach the yield stress. The resisting moment developed in the section is the yield moment and is given by

$$M_y = SF_y$$

As the moment on the section continues to increase, the yielding at the extreme fibers progresses towards the equal area axis, until finally the whole of the section has yielded. The resisting moment developed in the section is the plastic moment and is given by

$$M_p = ZF_y$$

Hinge Formation

A plastic hinge is formed in a structure as the bending moment at a specific location reaches the plastic moment of resistance. In Fig. 4.17, a fixed-ended beam supports a uniformly distributed service total load W. The bending moments produced in the beam are shown in Fig. 4.17(a) with moments at the ends of the beam twice that of the center's. As the load W is progressively increased to W', plastic hinges are formed simultaneously at both ends of the beam, and the bending moments in the beam are shown in Fig. 4.17(b). The system is now equivalent to a simply supported beam with an applied load W' and moment M_p at both ends, as shown in Fig. 4.17(c). Progressively increasing the applied load causes the two plastic hinges to rotate

Steel

Figure 4.15 *Elastic-Plastic Material*

Figure 4.17 *Formation of Plastic Hinges*
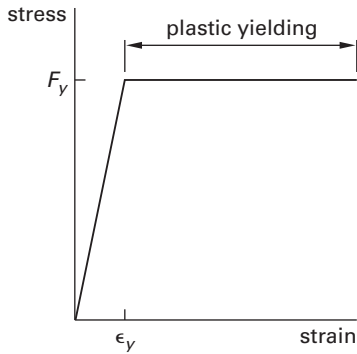
Figure 4.16 *Plastic Moment of Resistance*

(a) initial yield   (b) plastic hinge

while the moments at both ends remain constant. Finally, as the applied load is increased to the value λW, a third plastic hinge forms in the center of the span, giving the distribution of bending moment shown in Fig. 4.17(d). The system is now an unstable mechanism, shown in Fig. 4.17(e), and collapse occurs under λW. Immediately prior to collapse, the system is statically determinate and the ultimate load may be calculated.

Taking moments about the center of the span for the left half of the beam gives

$$2M_p = \left(\frac{\lambda W}{2}\right)\left(\frac{l}{2}\right) - \left(\frac{\lambda W}{2}\right)\left(\frac{l}{4}\right)$$

$$\lambda W = \frac{16M_p}{l}$$

The ratio of the collapse load to the service load, where λ is the load factor, is

$$\frac{\lambda W}{W} = \lambda$$

Design Procedure

In accordance with AISC 360 App. 1.1, plastic design is permitted using LRFD principles only.

Statical Design Method

The statical design procedure is a simple and convenient method for applying inelastic analysis to continuous beams. Figure 4.18 illustrates the procedure that follows for a three-span continuous beam.

step 1: Cut back the continuous beam to three simply supported, statically determinate beams as shown in Fig. 4.18(a), and apply the factored load, λw, to each beam.

step 2: Draw the free bending moment diagram for each beam as shown in Fig. 4.18(b). The maximum moment in each beam is

$$M_s = \frac{\lambda w l^2}{8}$$

step 3: Superimpose the fixing moment line as shown in Fig. 4.18(c). Adjust this line to make the moments at supports 2 and 3 and in spans 12 and 34 equal to M_p.

step 4: The collapse mechanism is shown in Fig. 4.18(d). Collapse occurs simultaneously in the two end spans, with the plastic hinges $0.414l$ from the end supports, and at the interior supports.

This is a partial collapse mechanism, as collapse does not occur in the center span. From the geometry of the figure, $M_p = 0.686M_s$.

To produce a complete collapse mechanism, use a nonuniform beam section as shown in Fig. 4.18(e). The end spans have a plastic moment of resistance of $M_{p2} = 0.766M_s$ that is greater than the plastic moment of resistance

$M_{p1} = 0.5M_s$ of the center span. Figure 4.18(e) shows the superimposed fixing moment line, and the complete collapse mechanism is shown in Fig. 4.18(f).

Example 4.27

The factored loading, including the beam self-weight, acting on a three-span continuous beam of uniform section is shown. Assuming that adequate lateral support is provided to the beam, determine the lightest adequate W12 shape using grade 50 steel.
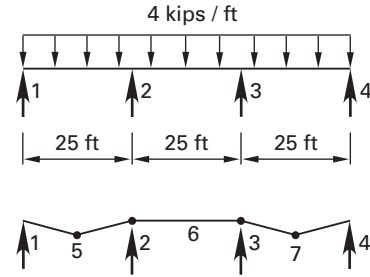
Figure 4.18 *Statical Design Method*
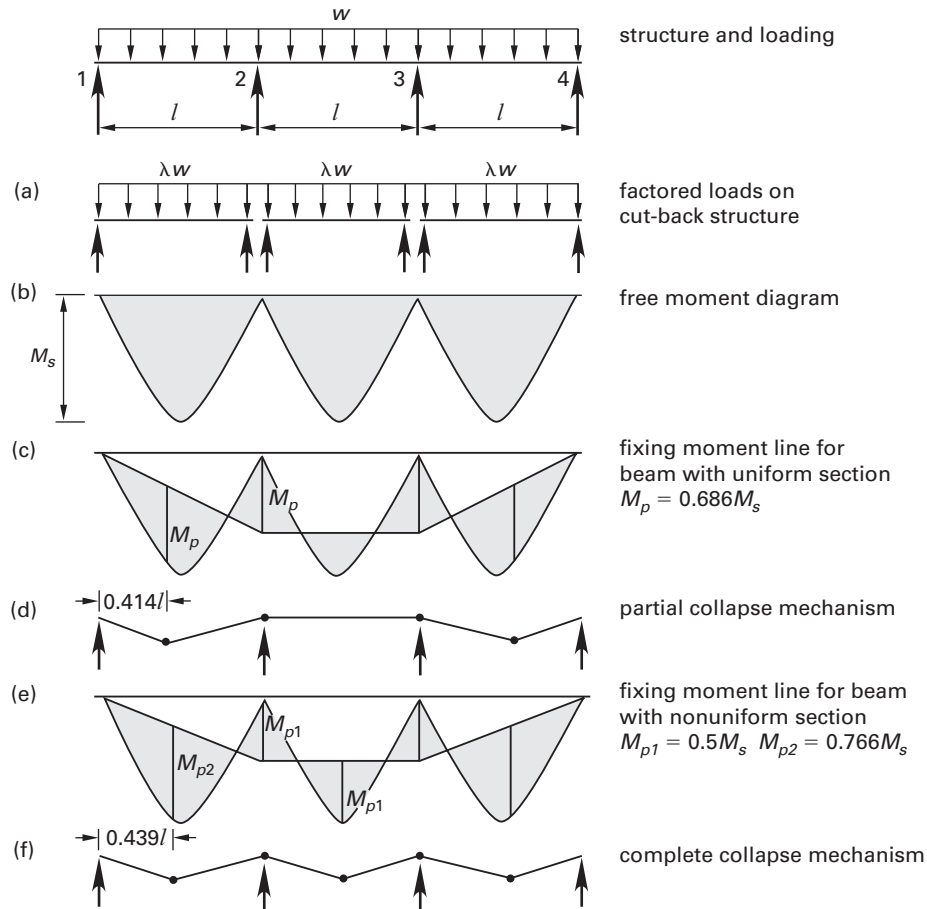
Solution

The free moment in each span is

$$M_s = \frac{w_u l^2}{8} = \frac{\left(4 \ \frac{\text{kips}}{\text{ft}}\right)(25 \ \text{ft})^2}{8}$$

$$= 312 \ \text{ft-kips}$$

Partial collapse occurs as shown in Fig. 4.18(c), with hinges forming in the end spans, and the required plastic moment of resistance is

$$M_p = 0.686 M_s$$

$$= 214 \ \text{ft-kips}$$

The required plastic section modulus is

$$Z = \frac{M_p}{\phi F_y}$$

$$= \frac{(214 \ \text{ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{(0.9)\left(50 \ \frac{\text{kips}}{\text{in}^2}\right)}$$

$$= 57 \ \text{in}^3$$

From *AISC Manual* Table 3-6, a W12 × 40 has a plastic section modulus of

$$Z = 57 \ \text{in}^3 \quad [\text{satisfactory}]$$

Beam Design Requirements

To ensure adequate ductility at plastic hinges, AISC 360 App. 1.2.1 specifies that the yield stress of members subject to plastic hinging must not exceed 65 kips/in^2. In addition, at plastic hinge locations, members must be doubly symmetric and compact with additional slenderness requirements given by AISC 360 Eq. A-1-1 and Eq. A-1-2.

When $P_u/\phi_c P_y \leq 0.125$,

$$\lambda_{pd} = 3.76\sqrt{\frac{E}{F_y}}\left(1 - \frac{2.75 P_u}{\phi_c P_y}\right) \qquad [\textit{AISC A-1-1}]$$

When $P_u/\phi_c P_y > 0.125$,

$$\lambda_{pd} = 1.12\sqrt{\frac{E}{F_y}}\left(2.33 - \frac{P_u}{\phi_c P_y}\right) \geq 1.49\sqrt{\frac{E}{F_y}}$$

$$[\textit{AISC A-1-2}]$$

For member segments that contain plastic hinges, the unbraced length of the segment is restricted to L_{pd}, as given by AISC 360 Eq. A-1-5.

$$L_{pd} = \left(0.12 - 0.076\frac{M_1'}{M_2}\right)\left(\frac{E}{F_y} r_y\right)$$

M_2 is the larger moment at the end of the unbraced length and is taken as positive in all cases. When the magnitude of the bending moment at any location within the unbraced length exceeds M_2,

$$\frac{M_1'}{M_2} = +1 \qquad [\textit{AISC A-1-6a}]$$

Otherwise, when $M_{\text{mid}} \leq (M_1 + M_2)/2$, case (ii) applies,

$$M_1' = M_1 \qquad [\textit{AISC A-1-6b}]$$

When $M_{\text{mid}} > (M_1 + M_2)/2$, case (iii) applies,

$$M_1' = 2M_{\text{mid}} - M_2 < M_2 \qquad [\textit{AISC A-1-6c}]$$

M_1 is the smaller moment at the end of the unbraced length, M_{mid} is the moment at the middle of the unbraced length, and M_1' is the effective moment at the end of the unbraced length opposite from M_2. The moments M_1 and M_{mid} are individually taken as positive when they cause compression in the same flange as the moment M_2, and negative otherwise.

In the case of the last hinge to form, rotation does not occur, and the bracing requirements of AISC 360 Sec. F2.2 are applicable. AISC 360 Sec. F2.2 also applies to segments remote from a plastic hinge.

In accordance with AISC 360 App. 1.3, continuous beams not subject to axial compression may be analyzed using a first-order inelastic procedure, and $P\text{-}\delta$ effects may be ignored.

Example 4.28

The three-span continuous beam of Ex. 4.27 is laterally braced at the midpoint of the central span and, in the end spans, at supports and at the locations of plastic hinges. Determine whether this bracing is adequate.

Solution

From *AISC Manual* Table 1-1 and *AISC Manual* Table 3-6, the relevant properties of a W12 × 40 are

$$r_y = 1.94 \ \text{in}$$

$$L_p = 6.85 \ \text{ft}$$

$$L_r = 21.1 \ \text{ft}$$

$$\phi_b M_p = M_2 = 214 \ \text{ft-kips}$$

The relevant unbraced lengths in the end spans are

$$L_{15} = 0.414 L_{12} = (0.414)(25 \ \text{ft})$$

$$= 10.35 \ \text{ft}$$

$$L_{25} = 0.586 L_{12} = (0.586)(25 \ \text{ft})$$

$$= 14.65 \ \text{ft}$$

For Segment 15

The hinges at 5 and 7 are the last to form, rotation does not occur at these hinges, and the bracing requirements of AISC 360 Sec. F2.2 are applicable to segments 15 and 47. The limiting laterally unbraced length for full plastic bending capacity is

$$L_p = 6.85 \text{ ft}$$
$$< L_{15} = 10.35 \text{ ft}$$

Bracing is inadequate in segments 15 and 47 and an additional brace is required 5 ft from each end support.

For Segment 25

Apply AISC 360 App. 1.2.3(a) to segment 25.

The larger moment at the end of segment 25 is

$$M_2 = M_{52}$$
$$= +M_p$$

The smaller moment at the end of segment 25 is

$$M_1 = M_{25}$$
$$= -M_p$$
$$\frac{M_1 + M_2}{2} = \frac{-M_p + M_p}{2}$$
$$= 0$$
$$> M_{\text{mid}}$$

Since $M_{\text{mid}} < (M_1 + M_2)/2$, case (ii) applies.

$$M_1' = M_1$$
$$= -M_p$$
$$\frac{M_1'}{M_2} = \frac{-M_p}{M_p}$$
$$= -1$$

The required unbraced length is given by AISC 360 Eq. A-1-5 as

$$L_{pd} = \left(0.12 - 0.076\frac{M_1'}{M_2}\right)\frac{E}{F_y}r_y$$
$$= \left(0.12 - (0.076)(-1)\right)\left(\frac{29{,}000\ \frac{\text{kips}}{\text{in}^2}}{50\ \frac{\text{kips}}{\text{in}^2}}\right)\left(\frac{1.94\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right)$$
$$= 18.4 \text{ ft}$$
$$> L_{25} \quad \text{[satisfactory]}$$

The maximum moment in the central span is

$$M_6 = M_s - M_p$$
$$= 312 \text{ ft-kips} - 214 \text{ ft-kips}$$
$$= 98 \text{ ft-kips}$$

The unbraced length in the central span is

$$L_{26} = 12.5 \text{ ft}$$

For Segment 26

Apply AISC 360 App. 1.2.3(a) to segment 26. The larger moment at the end of segment 26 is

$$M_2 = M_{26} = +M_p$$

The smaller moment at the end of segment 26 is

$$M_1 = M_{62}$$
$$= -98 \text{ ft-kips}$$
$$\frac{M_1 + M_2}{2} > M_{\text{mid}}$$

Since $M_{\text{mid}} < (M_1 + M_2)/2$, case (ii) applies.

$$M_1' = M_1$$
$$= -98 \text{ ft-kips}$$
$$\frac{M_1'}{M_2} = \frac{-98 \text{ ft-kips}}{214 \text{ ft-kips}}$$
$$= -0.46$$

The required unbraced length is given by

$$L_{pd} = \left(0.12 - 0.076\frac{M_1'}{M_2}\right)\frac{E}{F_y}r_y$$
$$= \left(0.12 - (0.076)(-0.46)\right)\left(\frac{29{,}000\ \frac{\text{kips}}{\text{in}^2}}{50\ \frac{\text{kips}}{\text{in}^2}}\right)\left(\frac{1.94\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right)$$
$$= 14.5 \text{ ft}$$
$$> L_{26} \quad \text{[satisfactory]}$$

Mechanism Design Method

The locations that plastic hinges may form in a structure include the

- ends of a member
- point of application of a concentrated load
- point of zero shear in a member
- weaker of two members meeting at a joint
- end of each member when three or more members meet at a joint

An independent collapse mechanism corresponds to a condition of unstable equilibrium in a structure. The number of possible independent mechanisms is

$$m_i = p - D$$

p is the number of possible hinge locations in the structure and D is the degree of indeterminacy in the structure.

For the rigid frame shown in Fig. 4.19, the number of possible independent mechanisms is

$$m_i = p - D$$
$$= 5 - 3$$
$$= 2$$

These independent mechanisms are the beam mechanism shown in Fig. 4.19(b) and the sway mechanism shown in Fig. 4.19(c). In addition, these may be combined to form the combined mechanism shown in Fig. 4.19(d).

Figure 4.19 *Mechanism Design Method*

(a) possible hinge locations

(b) beam mechanism

(c) sway mechanism

(d) combined mechanism
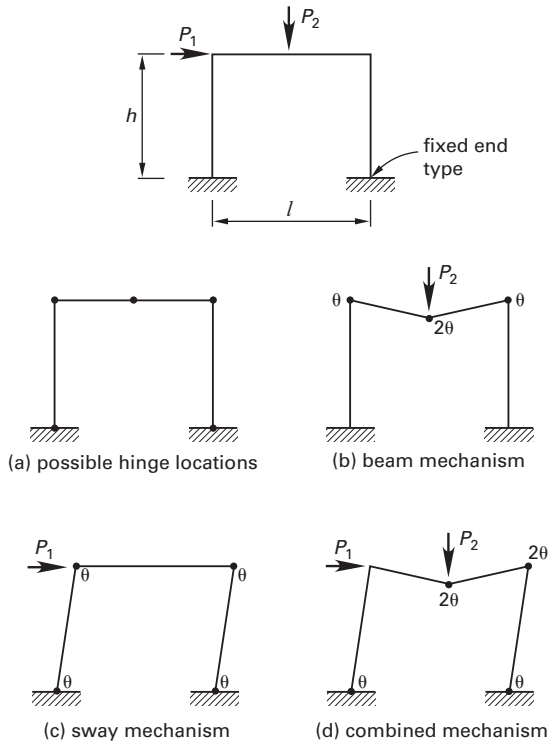
Applying a virtual displacement to each of these mechanisms in turn and equating internal and external work yields three equations, from each of which a value of M_p may be obtained. The largest value of M_p governs.

For the beam mechanism,

$$4M_p\theta = P_2\left(\frac{l}{2}\right)\theta$$
$$M_p = \frac{P_2 l}{8}$$

For the sway mechanism,

$$4M_p\theta = P_1 h\theta$$
$$M_p = \frac{P_1 h}{4}$$

For the combined mechanism,

$$6M_p\theta = P_2\left(\frac{l}{2}\right)\theta + P_1 h\theta$$
$$M_p = \frac{P_1 h + \dfrac{P_2 l}{2}}{6}$$

For the situation where $P_2 = 2P_1$ and $h = l$, the combined mechanism controls and

$$M_p = \frac{P_1 l}{3}$$

Example 4.29

The rigid frame shown is fabricated from members of a uniform section in grade 50 steel. For the factored loading indicated, ignoring the member self-weight and assuming adequate lateral support, determine the lightest adequate W shape.
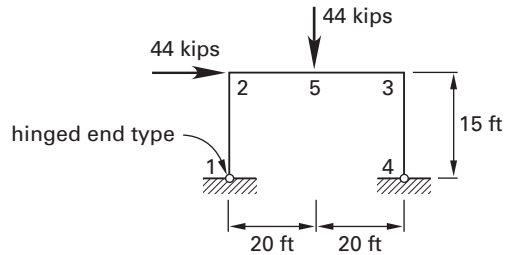
Solution

The three possible collapse mechanisms are shown.

beam mechanism   sway mechanism   combined mechanism
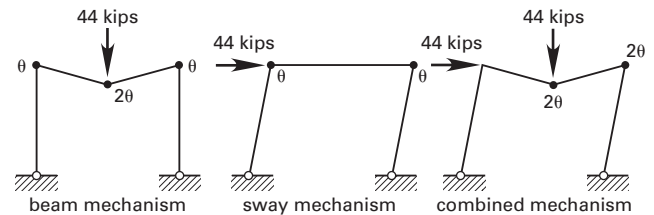
The beam mechanism gives

$$4M_p = (44 \text{ kips})(20 \text{ ft})$$
$$M_p = 220 \text{ ft-kips}$$

The sway mechanism gives

$$2M_p = (44 \text{ kips})(15 \text{ ft})$$
$$M_p = 330 \text{ ft-kips}$$

The combined mechanism gives

$$4M_p = 880 \text{ ft-kips} + 660 \text{ ft-kips}$$

$$M_p = 385 \text{ ft-kips} \quad \text{[governs]}$$

From *AISC Manual* Table 3-6, a W21 × 50 has

$$\phi_b M_p = 413 \text{ ft-kips}$$

$$> 385 \text{ ft-kips} \quad \text{[satisfactory]}$$

A W21 × 48 is noncompact and may not be used.

Static Equilibrium Check

Mechanism methods lead to upper bounds on the collapse load. To confirm that the correct mechanism has been selected, it is necessary to check that the assumed plastic moment is not anywhere exceeded by constructing a moment diagram obtained by static equilibrium methods.

Example 4.30

Draw the bending moment diagram for the assumed collapse mechanism of Ex. 4.29.

Solution
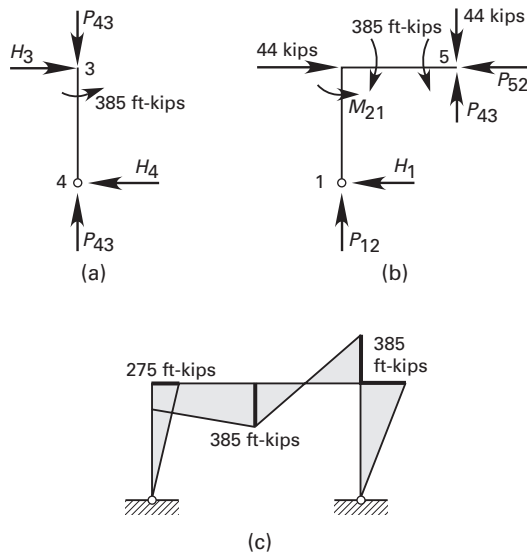
For member 34, as shown at (a) in the illustration,

$$H_4 = \frac{385 \text{ ft-kips}}{15 \text{ ft}} = 25.67 \text{ kips}$$

$$= H_3$$

$$= P_{52}$$

For member 125, as shown at (b) in the figure,

$$H_1 = 44 \text{ kips} - P_{52}$$

$$= 44 \text{ kips} - 25.67 \text{ kips}$$

$$= 18.33 \text{ kips}$$

$$P_{12} = \frac{M_p - H_1 h}{l_{25}}$$

$$= \frac{385 \text{ ft-kips} - (18.33 \text{ kips})(15 \text{ ft})}{20 \text{ ft}}$$

$$= 5.50 \text{ kips}$$

$$M_{21} = H_1 h$$

$$= (18.33 \text{ kips})(15 \text{ ft})$$

$$= 275 \text{ ft-kips}$$

$$P_{43} = 44 \text{ kips} - P_{12}$$

$$= 44 \text{ kips} - 5.50 \text{ kips}$$

$$= 38.50 \text{ kips}$$

The bending moment diagram is shown at (c) in the illustration. Because $M_p = 385$ ft-kips is not exceeded at any point in the frame, the combined mechanism is the correct failure mode.

Column Design Requirements

Flanges and webs of members subjected to combined flexure and compression shall be compact with width-thickness ratios not exceeding the values defined in AISC 360 Table B4.1b. In addition, the webs of W sections shall also comply with AISC 360 Eq. A-1-1 and Eq. A-1-2, which are as follows.

For $P_u/\phi_b P_y \le 0.125$,

$$\frac{h}{t_w} \le 3.76 \sqrt{\frac{E}{F_y}} \left(1 - \frac{2.75 P_u}{\phi_b P_y}\right)$$

For $P_u/\phi_b P_y > 0.125$,

$$\frac{h}{t_w} \le 1.12 \sqrt{\frac{E}{F_y}} \left(2.33 - \frac{P_u}{\phi_b P_y}\right)$$

$$\ge 1.49 \sqrt{\frac{E}{F_y}}$$

The member yield strength is

$$P_y = A F_y$$

The resistance factor for flexure is

$$\phi_b = 0.90$$

The maximum permitted slenderness ratio of a column is specified in AISC 360 App. 1.2.3(b) as

$$\frac{L}{r} = 4.71 \sqrt{\frac{E}{F_y}}$$

$$= 113 \quad [\text{for } F_y = 50 \text{ kips/in}^2]$$

In accordance with AISC 360 App. 1.2.4, the axial load in a column with plastic hinges may not exceed $0.75\phi_c A_g F_y$. As for beams, the maximum unbraced

length is controlled by AISC 360 Eq. A-1-5, and for combined axial force and flexure, the interaction expressions of AISC 360 Eq. H1-1a and Eq. H1-1b govern. In practice, it has been the custom that second-order effects may be neglected for low rise frames with small axial loads.

Example 4.31

Determine whether column 34 of the rigid frame in Ex. 4.29 is satisfactory. The column consists of a grade 50 W21 × 50 section and is laterally braced about its weak axis at 3.75 ft centers and at joint 3. Neglect secondary effects.

Solution

For the pinned connection at joint 4, AISC 360 Comm. App. 7.2 specifies a stiffness ratio of $G_4 = 10$.

At joint 3,

$$G_3 = \frac{\sum \dfrac{I_c}{L_c}}{\sum \dfrac{I_g}{L_g}} = \frac{\dfrac{I}{15}}{\dfrac{I}{40}}$$

$$= 2.7$$

From the alignment chart for sway frames, the effective length factor is

$$K_{34} = 2.2$$

From *AISC Manual* Table 1-1, a W21 × 50 has

$$A_g = 14.7 \text{ in}^2$$
$$r_y = 1.30 \text{ in}$$
$$r_x = 8.18 \text{ in}$$

The slenderness ratio about the x-axis is

$$\frac{K_{34}L_x}{r_x} = \frac{(2.2)(15 \text{ ft})\left(12 \dfrac{\text{in}}{\text{ft}}\right)}{8.18 \text{ in}}$$

$$= 48.4 \quad [\text{governs for axial load}]$$

$$\frac{L_x}{r_x} = \frac{180 \text{ in}}{8.18 \text{ in}}$$

$$= 22.0$$

$$< 113 \quad [\text{satisfies AISC 360 App. 1.2.3(b)}]$$

The slenderness ratio about the y-axis is

$$\frac{K_{34}L_y}{r_y} = \frac{(1.0)(3.75 \text{ ft})\left(12 \dfrac{\text{in}}{\text{ft}}\right)}{1.30 \text{ in}}$$

$$= 34.6$$

$$\frac{L_y}{r_y} = \frac{45 \text{ in}}{1.30 \text{ in}}$$

$$= 34.6$$

$$< 113 \quad [\text{satisfies AISC 360 App. 1.2.3(b)}]$$

In accordance with AISC 360 App. 1.2.4, the maximum axial load in the column is restricted to

$$P_{\max} = 0.75\phi_c A_g F_y$$

$$= (0.75)(0.90)(14.7 \text{ in}^2)\left(50 \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 496 \text{ kips}$$

$$> P_{43} \quad [\text{satisfactory}]$$

From *AISC Manual* Table 3-6, for a W21 × 50,

$$L_p = 4.59 \text{ ft}$$

$$> 3.75 \text{ ft} \quad [\text{full plastic bending capacity available}]$$

$$\phi_b M_{nx} = \phi_b M_p$$

$$= 413 \text{ ft-kips}$$

From *AISC Manual* Table 4-22, for a $K_{34}L_x/r_x$ value of 48.4, the design stress for axial load is

$$\phi_c F_{cr} = 37.9 \text{ kips/in}^2$$

The design axial strength is

$$\phi_c P_n = \phi_c F_{cr} A_g = \left(37.9 \frac{\text{kips}}{\text{in}^2}\right)(14.7 \text{ in}^2)$$

$$= 557 \text{ kips}$$

$$\frac{P_{3\text{-}4}}{\phi_c P_n} = \frac{38.50 \text{ kips}}{557 \text{ kips}}$$

$$= 0.07$$

$$< 0.20 \quad [\text{AISC Eq. H1-1b governs}]$$

Since secondary effects may be neglected, AISC 360 Eq. H1-1b reduces to

$$\frac{P_{34}}{2\phi_c P_n} + \frac{M_p}{\phi_b M_{nx}} \le 1.0$$

$$= \frac{38.50 \text{ kips}}{(2)(557 \text{ kips})} + \frac{385 \text{ ft-kips}}{413 \text{ ft-kips}}$$

$$= 0.97$$

$$< 1.0 \quad [\text{satisfactory}]$$

7. DESIGN OF TENSION MEMBERS

Nomenclature

A_e	effective net area	in²
A_g	gross area	in²
A_n	net area	in²

A_R	area of section required for fatigue loading	in^2
C_f	fatigue constant	–
d	nominal bolt diameter	in
d_h	specified hole diameter	in
f_{max}	maximum tensile stress in member at service load	$kips/in^2$
f_{min}	minimum stress in member at service load (compression negative)	$kips/in^2$
f_{SR}	actual stress range	$kips/in^2$
F_{SR}	allowable stress range for fatigue loading	$kips/in^2$
F_{TH}	threshold stress range	$kips/in^2$
F_u	specified minimum tensile strength	$kips/in^2$
g	transverse center-to-center spacing between fasteners (gage)	in
l	length of connection	in
l	length of weld	in
n_{SR}	number of stress range fluctuations	–
P_n	nominal axial strength	kips
P_u	required axial strength	kips
s	bolt spacing in direction of load	in
t	plate thickness	in
T_{max}	maximum tensile force in member at service load	kips
T_{min}	minimum force in member (compression negative) at service load	kips
U	shear lag factor used in calculating effective net area	–
w	plate width, distance between welds	in
$\bar{x}$	connection eccentricity	in

Plates in Tension

The available strength for plates in tension is either the limit state of yielding or the limit state of rupture, whichever is lower. The *limit state of yielding* occurs when the gross cross-section becomes excessively elongated, which causes the whole structure to be unstable. The *limit state of rupture* occurs when the net cross-section fails at bolted joints, which causes sudden and catastrophic failure.

For yielding of the gross section, AISC 360 Sec. D2 gives the design strength as

$$P_c = \phi_t P_n = 0.9 F_y A_g \quad \text{[LRFD]}$$

The allowable strength is

$$P_c = \frac{P_n}{\Omega_t} = \frac{F_y A_g}{1.67} \quad \text{[ASD]}$$

As shown in Fig. 4.20, the gross area is given by

$$A_g = wt$$

For tensile rupture at the connection, AISC 360 Sec. D2 gives the design strength as

$$P_c = \phi_t P_n = 0.75 F_u A_e \quad \text{[LRFD]}$$

Figure 4.20 *Effective Net Area of Bolted Connection*
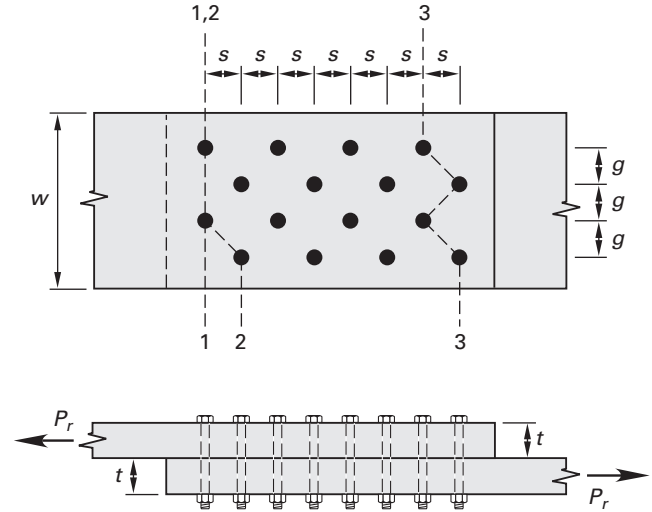
The allowable strength is

$$P_c = \frac{P_n}{\Omega_t} = \frac{F_u A_e}{2.00} \quad \text{[ASD]}$$

Effective Net Area

The shear lag factor, given in AISC 360 Table D3.1, accounts for the effects of eccentricity and shear lag on tension members connected through only part of their cross-sectional elements. The effective net area, A_e, resisting the tensile force is given by AISC 360 Eq. D3-1 as

$$A_e = U A_n$$

A_n is the net area of the member and U is the shear lag factor.

Bolt Hole Diameter

The nominal diameter of a standard hole is detailed in AISC 360 Table J3.3 as $^1/_{16}$ in larger than the bolt diameter. As the hole is formed, some deterioration occurs in the surrounding material, and AISC 360 Sec. B4.3b specifies that the effective hole diameter must be taken as $^1/_{16}$ in larger than the nominal hole diameter. The effective hole diameter when d is the diameter of the fastener is

$$d_h = d + \frac{1}{16} \text{ in} + \frac{1}{16} \text{ in} = d + \frac{1}{8} \text{ in}$$

Plates with Bolted Connections

For a flat plate with bolted connection, the total net cross-section is assumed to transfer the load without shear lag, and AISC 360 Table D3.1, Case 1 indicates that $U = 1.0$. For a flat plate with bolted connection,

$$A_e = U A_n = A_n$$

For the straight perpendicular fracture 1-1 of the bolted plates shown in Fig. 4.20, the effective net area is

$$A_e = t(w - 2d_h)$$

The effective hole diameter is

$$d_h = d + \frac{1}{8} \text{ in}$$

For a staggered fracture, the effective net width is obtained by deducting from the gross plate width the sum of the bolt holes in the failure path and adding, for each gage space traversed by a diagonal portion of the failure path, the quantity $s^2/(4g)$. g is the transverse center-to-center spacing between fasteners (gage) and s is the bolt spacing in direction of load (pitch).

For the staggered fracture 2-2 shown in Fig. 4.20, the effective net area of the plate is

$$A_e = t\left(w - 3d_h + \frac{s^2}{4g}\right)$$

For the staggered fracture 3-3 shown in Fig. 4.20, the effective net area of the plate is

$$A_e = t\left(w - 4d_h + \frac{3s^2}{4g}\right)$$

The length of a bolted splice plate is small and inelastic deformation is limited. AISC 360 Sec. J4.1 limits the effective net area of the connection to a value of

$$A_e \leq 0.85A_g$$

Plates with Welded Connections

For a flat plate with a welded connection, the effective net area is given by AISC 360 Eq. D3-1 as

$$A_e = A_n U = A_g U$$

For a flat plate with a transverse fillet welded connection as shown in Fig. 4.21(a), the total cross-section is assumed to transfer the load without shear lag, and AISC 360 Table D3.1, Case 3 states that $U = 1.0$. For a flat plate with a transverse fillet welded connection,

$$A_e = A_n$$

Figure 4.21 Welded Connections for Plates

(a) transverse weld   (b) longitudinal welds
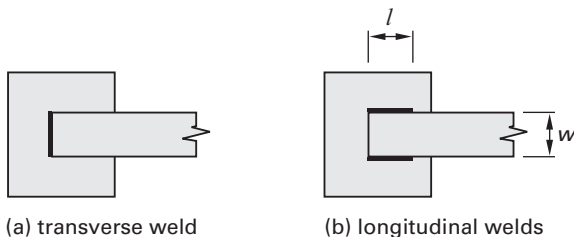
For the longitudinal fillet welded connection shown in Fig. 4.21(b), shear lag occurs at the ends of the plate and the shear lag factor is defined in AISC 360 Table D3.1, Case 4 under the following conditions.

When $l \geq 2w$,

$$U = 1.00$$

When $2w > l > 1.5w$,

$$U = 0.87$$

When $1.5w > l > w$,

$$U = 0.75$$

Example 4.32

As shown, two plates each $^1\!/_2$ in thick $\times$ 9 in wide are connected by three rows of bolts. The distance between rows is 3 in, the distance between bolts in a row is 3 in, and the center row of bolts is staggered. Determine the available axial strength of the plates in direct tension. The relevant properties of the plates are $F_y = 36$ kips/in^2, $F_u = 58$ kips/in^2, and specified hole diameter $d_h = 1.0$ in.
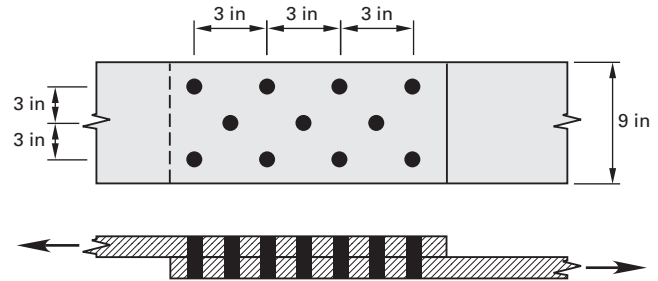
Solution

The gross area of each plate is given by

$$A_g = wt = (9 \text{ in})(0.5 \text{ in})$$
$$= 4.5 \text{ in}^2$$

LRFD Method

The design axial strength for yielding is

$$\phi_t P_n = 0.9 F_y A_g$$
$$= (0.9)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(4.5 \text{ in}^2)$$
$$= 146 \text{ kips}$$

For a straight perpendicular fracture, the effective net area of the plate is given by

$$A_e = t(w - 2d_h) = (0.5)\big(9 \text{ in} - (2)(1.0 \text{ in})\big)$$
$$= 3.5 \text{ in}^2$$
$$< 0.85A_g \quad \text{[satisfactory]}$$

The design axial strength for tensile rupture is

$$\phi_t P_n = 0.75 F_u A_e = (0.75)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(3.5 \ \text{in}^2)$$
$$= 152 \ \text{kips}$$

For a staggered fracture, the effective net area of the plate is given by

$$A_e = t\left(w - 3d_h + \frac{2s^2}{4g}\right)$$
$$= (0.5 \ \text{in})\left(9 \ \text{in} - (3)(1.0 \ \text{in}) + \frac{(2)(1.5 \ \text{in})^2}{(4)(3 \ \text{in})}\right)$$
$$= 3.19 \ \text{in}^2$$

The corresponding design axial strength is

$$\phi_t P_n = 0.75 F_u A_e = (0.75)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(3.19 \ \text{in}^2)$$
$$= 139 \ \text{kips} \quad \text{[governs]}$$

ASD Method

The allowable axial strength for yielding is

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{1.67} = \frac{\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(4.5 \ \text{in}^2)}{1.67}$$
$$= 97 \ \text{kips}$$

For a straight perpendicular fracture, the effective net area of the plate is given by

$$A_e = t(w - 2d_h) = (0.5)\big(9 \ \text{in} - (2)(1.0 \ \text{in})\big)$$
$$= 3.5 \ \text{in}^2$$
$$< 0.85 A_g \quad \text{[satisfactory]}$$

The allowable axial strength for tensile rupture is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{2.00} = \frac{\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(3.5 \ \text{in}^2)}{2.00}$$
$$= 102 \ \text{kips}$$

For a staggered fracture, the effective net area of the plate is given by

$$A_e = t\left(w - 3d_h + \frac{2s^2}{4g}\right)$$
$$= (0.5 \ \text{in})\left(9 \ \text{in} - (3)(1.0 \ \text{in}) + \frac{(2)(1.5 \ \text{in})^2}{(4)(3 \ \text{in})}\right)$$
$$= 3.19 \ \text{in}^2$$

The corresponding allowable axial strength is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{2.00} = \frac{\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(3.19 \ \text{in}^2)}{2.00}$$
$$= 93 \ \text{kips} \quad \text{[governs]}$$

Rolled Sections in Tension

For bolted connections, when rolled structural shapes are connected through only part of their cross-sectional elements, the effective net area is given by AISC 360 Sec. D3 as

$$A_e = A_n U \qquad \text{[\textit{AISC D3-1}]}$$

The value of the shear lag factor is defined in AISC 360 Table D3.1, Case 2 as

$$U = 1 - \frac{\overline{x}}{l}$$

The length of the connection l is defined in AISC 360 Comm. Sec. D3.3 as the distance, parallel to the line of force, between the first and last fasteners in a line. The connection eccentricity $\overline{x}$ is defined as the distance from the connection plane to the centroid of the member resisting the connection force. In lieu of applying this expression for U, AISC 360 Table D3.1 permits the adoption of the following values for the shear lag factor.

$U = 0.90$ for T, W, M, and S shapes
   with $b_f \geq 2d/3$, connected by
   the flange, with not fewer than
   three bolts in line in the direction
   of stress
$U = 0.85$ for T, W, M, and S shapes
   with $b_f < 2d/3$ connected by
   the flange, with not fewer than
   three bolts in line in the direction
   of stress
$U = 0.70$ for T, W, M, and S shapes
   connected by the web, with
   not less than four bolts in line
   in the direction of stress
$U = 0.80$ for single or double angles
   with not less than four bolts in line
   in the direction of stress
$U = 0.60$ for single or double angles with
   two or three bolts in line in the direction
   of stress

For a welded connection, when the axial force is transmitted only by transverse welds, as shown in Fig. 4.22(a), the effective net area is given by AISC 360 Table D3.1 as

$$A_e = \text{area of directly connected elements}$$

For a welded connection, when the axial force is transmitted only by longitudinal welds or in combination

Steel

Figure 4.22 *Welded Connections for Rolled Sections*

(a) transverse weld   (b) longitudinal welds
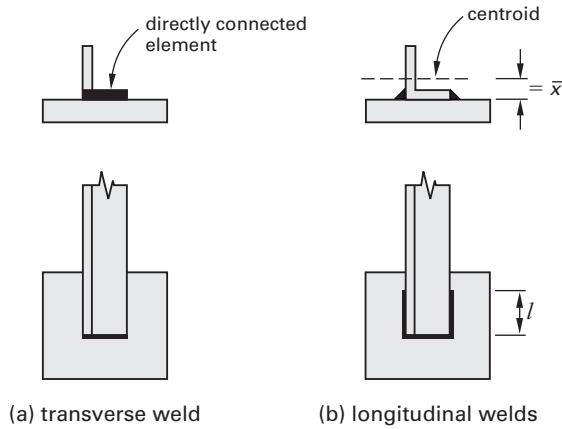
with transverse welds, as shown in Fig. 4.22(b), the effective net area is given by AISC 360 Sec. D3 as

$$A_e = A_g U \qquad \text{[AISC D3-1]}$$

The value of the shear lag factor is defined in AISC 360 Table D3.1, Case 2 as

$$U = 1 - \frac{\overline{x}}{l}$$

The length of the connection l is shown in Fig. 4.22(b) and is defined in AISC 360 Comm. Sec. D3.3 as the length of the weld, parallel to the line of force.

Example 4.33

Assuming that the welds are adequate, determine the design axial strength of the grade A36 W12 × 65 member connected as shown.

(a) welded all round

(b) both flanges welded to gusset plates with transverse welds

(c) both flanges welded to gusset plates with longitudinal welds
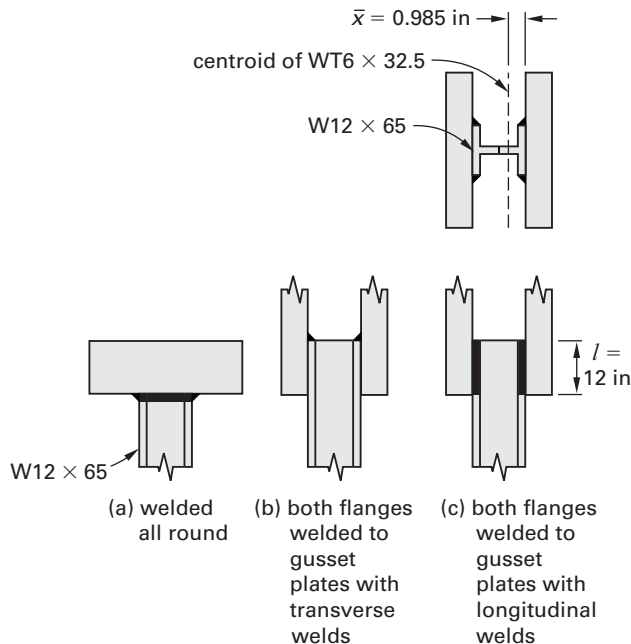
Solution

LRFD Method

The relevant properties of the W12 × 65 are obtained from *AISC Manual* Table 1-1 and are

$$A_g = 19.1 \text{ in}^2$$
$$b_f = 12 \text{ in}$$
$$t_f = 0.605 \text{ in}$$

(a) The W12 × 65 is welded all around to the supporting member.

$$A_e = \text{area of directly connected elements}$$
$$= A_g$$

The available axial strength for rupture is

$$\phi_t P_n = 0.75 F_u A_e$$
$$= (0.75)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(19.1 \text{ in}^2)$$
$$= 830 \text{ kips}$$

The available axial strength for yielding is

$$\phi_t P_n = 0.9 F_y A_g$$
$$= (0.9)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(19.1 \text{ in}^2)$$
$$= 619 \text{ kips} \quad \text{[governs]}$$

(b) Both flanges are welded by transverse welds to gusset plates.

$$A_e = \text{area of directly connected elements}$$
$$= 2 b_f t_f$$
$$= (2)(12 \text{ in})(0.605 \text{ in})$$
$$= 14.5 \text{ in}^2$$

The available axial strength for fracture is

$$\phi_t P_n = 0.75 F_u A_e$$
$$= (0.75)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(14.5 \text{ in}^2)$$
$$= 631 \text{ kips}$$

The available axial strength for yielding is

$$\phi_t P_n = 619 \text{ kips} \quad \text{[governs]}$$

(c) Both flanges are welded by longitudinal welds to gusset plates.

In accordance with AISC 360 Comm. Sec. D3.3, the W section is treated as two WT sections as shown in the figure. The centroidal height of a WT6 × 32.5 cut from

a W12 × 65 is obtained from *AISC Manual* Table 1-8 and is

$$\overline{x} = 0.985 \text{ in}$$

The value of the shear lag factor is defined in AISC 360 Table D3.1, Case 2 as

$$U = 1 - \frac{\overline{x}}{l} = 1 - \frac{0.985 \text{ in}}{12 \text{ in}}$$
$$= 0.92$$

The effective net area is

$$A_e = A_g U = (19.1 \text{ in}^2)(0.92)$$
$$= 17.6 \text{ in}^2$$

The available axial strength for fracture is

$$\phi_t P_n = 0.75 F_u A_e$$
$$= (0.75)\left(58 \frac{\text{kips}}{\text{in}^2}\right)(17.6 \text{ in}^2)$$
$$= 766 \text{ kips}$$

The available axial strength for yielding is

$$\phi_t P_n = 619 \text{ kips} \quad [\text{governs}]$$

ASD Method

The relevant properties of the W12 × 65 are obtained from *AISC Manual* Table 1-1 and are

$$A_g = 19.1 \text{ in}^2$$
$$b_f = 12 \text{ in}$$
$$t_f = 0.605 \text{ in}$$

(a) The W12 × 65 is welded all around to the supporting member.

$$A_e = \text{area of directly connected elements}$$
$$= A_g$$

The available axial strength for rupture is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{2.00} = \frac{\left(58 \frac{\text{kips}}{\text{in}^2}\right)(19.1 \text{ in}^2)}{2.00}$$
$$= 554 \text{ kips}$$

The available axial strength for yielding is

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{1.67} = \frac{\left(36 \frac{\text{kips}}{\text{in}^2}\right)(19.1 \text{ in}^2)}{1.67}$$
$$= 412 \text{ kips} \quad [\text{governs}]$$

(b) Both flanges are welded by transverse welds to gusset plates.

$$A_e = \text{area of directly connected elements}$$
$$= 2b_f t_f$$
$$= (2)(12 \text{ in})(0.605 \text{ in})$$
$$= 14.5 \text{ in}^2$$

The available axial strength for fracture is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{2.00}$$
$$= \frac{\left(58 \frac{\text{kips}}{\text{in}^2}\right)(14.5 \text{ in}^2)}{2.00}$$
$$= 421 \text{ kips}$$

The available axial strength for yielding is

$$\frac{P_n}{\Omega_t} = 412 \text{ kips} \quad [\text{governs}]$$

(c) Both flanges are welded by longitudinal welds to gusset plates.

In accordance with AISC 360 Comm. Sec. D3.3, the W section is treated as two WT sections as shown in the figure. The centroidal height of a WT6 × 32.5 cut from a W12 × 65 is obtained from *AISC Manual* Table 1-8 and is

$$\overline{x} = 0.985 \text{ in}$$

The value of the shear lag factor is defined in AISC 360 Table D3.1, Case 2 as

$$U = 1 - \frac{\overline{x}}{l} = 1 - \frac{0.985 \text{ in}}{12 \text{ in}}$$
$$= 0.92$$

The effective net area is

$$A_e = A_g U = (0.92)(19.1 \text{ in}^2)$$
$$= 17.6 \text{ in}^2$$

The available axial strength for fracture is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{2.00} = \frac{\left(58 \frac{\text{kips}}{\text{in}^2}\right)(17.6 \text{ in}^2)}{2.00}$$
$$= 510 \text{ kips}$$

The available axial strength for yielding is

$$\frac{P_n}{\Omega_t} = 412 \text{ kips} \quad [\text{governs}]$$

Steel

Design for Fatigue

Fatigue failure is caused by fluctuations of tensile stress that cause crack propagation in the parent metal. Fatigue must be considered for tensile stresses, stress reversals, and shear when the number of loading cycles exceeds 20,000 and is based on the stress level at service loads. Fatigue effects are analyzed using service level loads. The maximum permitted stress due to unfactored loads is $0.66F_y$.

Fatigue failure is influenced by the number of applied load cycles, the magnitude of the stress range produced by service live loads, and stress concentrations produced by the fabrication details.

Fatigue will not occur when the

- number of applied load cycles is less than 20,000
- live load stress range is less than the threshold stress range
- fluctuations in stress do not involve tensile stress
- members are HSS sections in buildings subjected to wind loads

The design procedure is given in AISC 360 App. 3 and consists of establishing the applicable loading condition from AISC 360 Table A-3.1.

The applicable values of the fatigue constant, the threshold stress range, and the stress category are obtained from the table.

The stress range is defined as the magnitude of the change in stress due to the application or removal of the unfactored live load. Fatigue must be considered if the stress range in the member exceeds the threshold stress range. The actual stress range, at service level values, is given by

$$f_{SR} = f_{\max} - f_{\min}$$
$$< F_{SR}$$

Eleven stress categories are defined in AISC 360 Table A-3.1. For stress categories A, B, B′, C, D, E, and E′, the design stress range in the member must not exceed the value given by AISC 360 Eq. A-3-1 as

$$F_{SR} = \left(\frac{C_f}{n_{SR}}\right)^{0.333}$$
$$\geq F_{TH}$$

For stress category F, the design stress range in the member must not exceed the value given by AISC 360 Eq. A-3-2.

$$F_{SR} = \left(\frac{C_f}{n_{SR}}\right)^{0.167}$$
$$\geq F_{TH}$$

Example 4.34

A tie member in a steel truss consists of a pair of grade A36 5 in $\times$ 5 in $\times$ $^3/_8$ in angles fillet welded to a gusset plate. The force in the member, due to dead load only, is 90 kips tension. The additional force in the member, due to live load only, varies from a compression of 7 kips to a tension of 50 kips. During the design life of the structure, the live load may be applied 600,000 times. Determine whether fatigue effects are a concern.

Solution

From AISC 360 Table A-3.1, the loading condition of Sec. 4.1 is applicable and the relevant factors are

$$E = \text{stress category}$$

$$F_{SR} = \text{allowable stress range}$$

$$= \left(\frac{C_f}{n_{SR}}\right)^{0.333} = \left(\frac{11 \times 10^8}{6 \times 10^5}\right)^{0.333}$$

$$= 12.21 \text{ kips/in}^2$$

The area of the tie is

$$A_s = 7.22 \text{ in}^2$$

The actual stress range is

$$f_{SR} = \frac{T_{\max} - T_{\min}}{A_s}$$

$$= \frac{50 \text{ kips} - (-7 \text{ kips})}{7.22 \text{ in}^2}$$

$$= 7.9 \text{ kips/in}^2 \quad [\text{exceeds } F_{TH}]$$

$$< F_{SR}$$

This is within the allowable stress range, so fatigue effects need not be considered.

8. DESIGN OF BOLTED CONNECTIONS

Nomenclature

A_b	nominal unthreaded body area of bolt	in^2
C	coefficient for eccentrically loaded bolt and weld groups	–
d	nominal bolt diameter	in
d_m	moment arm between resultant tensile and compressive forces due to an eccentric force	in
D_u	a multiplier that reflects the ratio of the mean installed bolt tension to the specified minimum bolt pretension	–
f_v	computed shear stress	kips/in^2
f_{rv}	required shear stress	kips/in^2
F_{nt}	nominal tensile stress of bolt	kips/in^2
F'_{nt}	nominal tensile stress of a bolt subjected to combined shear and tension	kips/in^2
F_{nv}	nominal shear stress of bolt	kips/in^2
F_u	specified minimum tensile strength	kips/in^2

h_{sc}	modification factor for type of hole	–
k_s	slip-critical combined tension and shear coefficient	–
l_c	clear distance, in the direction of force, between the edge of the hole and the edge of the adjacent hole or edge of the material	in
l_e	edge distance between the bolt center and the edge of the connected part	in
n	number of bolts in a connection	–
n'	number of bolts above the neutral axis (in tension)	–
n_s	number of slip planes	–
n_b	number of bolts carrying strength level tension T_u	–
P_r	load on connection	kips
R_n	nominal strength	kips
s	center-to-center pitch of two consecutive bolts	in
t	thickness of connected part	in
T_a	applied tensile force (ASD)	kips
T_b	minimum pre-tension force	kips
T_u	applied tensile force (LRFD)	kips

Symbols

μ	mean slip coefficient for the applicable surface	–
ϕ	resistance factor	–

Types of Bolts

There are two categories of bolts: common bolts and high-strength bolts. High-strength bolts are additionally grouped by strength levels into two categories: group A bolts (A325, F182, A354 grade BC, and A449) and group B bolts (A490, F2280, and A354 grade BD).

Common bolts of grade A307 with a nominal tensile strength of 45 kips/in^2 are used in snug-tight (bearing-type) connections only.

High-strength bolts in group A with a nominal tensile strength of 90 kips/in^2, or group B with a nominal tensile strength of 113 kips/in^2, are used in bearing-type, pretensioned, and slip-critical connections.

Bolts are installed in the following three types of connections.

1. *Bearing-type* or *snug-tight connections* require the bolts to be tightened sufficiently to bring the plies into firm contact. Levels of installed tension are not specified. Transfer of the load from one connected part to another depends on the bearing of the bolts against the side of the holes. This type may be used when pretensioned or slip-critical connections are not required.

2. *Pretensioned connections* require the bolts to be pre-tensioned to a minimum value of 70% of the bolt's minimum tensile strength and the faying surfaces may be uncoated, coated, or galvanized regardless of the slip coefficient. Transfer of the load from one connected part to another depends on the bearing of the bolts against the side of the holes. Pretensioned connections are required when bearing-type connections are used in

- column splices in buildings over 125 ft in height
- bracing members in buildings over 125 ft in height
- structures carrying cranes of over 5 ton capacity
- supports of machinery causing impact or stress reversal

3. *Slip-critical connections* require the bolts to be pre-tensioned to a minimum value of 70% of the bolt's tensile strength, and the faying surfaces must be prepared to produce a specific value of the slip coefficient. Transfer of the load from one connected part to another depends on the friction induced between the parts. Slip-critical connections are required where

- fatigue load occurs
- bolts are used in oversize holes or slotted holes parallel to the direction of load
- slip at the faying surfaces will affect the performance of the structure
- bolts are used in conjunction with welds

Bearing-Type Bolts in Shear

The minimum permissible distance and the preferred distance between the centers of holes is given by AISC 360 Sec. J3.3 as

$$s_{\min} = 2.67d$$

$$s_{\text{pref}} = 3.0d$$

The nominal shear strength is based on the nominal unthreaded cross-sectional area of the bolt, A_b, and the nominal shear stress, F_{nv}. Nominal shear stress of fasteners and threaded parts is given in AISC 360 Table J3.2, and for high-strength bolts a reduced nominal stress is applicable when threads are not excluded from the shear planes. No reduction is made for A307 bolts. For connections longer than 38 in, the nominal stress is reduced. The bolt's available shear capacity is obtained from AISC 360 Eq. J3-1.

$$\phi R_n = \phi F_{nv} A_b$$
$$= 0.75 F_m A_b \quad \text{[LRFD]}$$
$$\frac{R_n}{\Omega} = \frac{F_{nv} A_b}{\Omega}$$
$$= \frac{F_{nv} A_b}{2.00} \quad \text{[ASD]}$$

Bearing-Type Bolts in Tension and Combined Shear and Tension

The available strength in tension is given by AISC 360 Sec. J3.6 as

$$\phi R_n = \phi F_{nt} A_b$$
$$= 0.75 F_{nt} A_b \quad \text{[LRFD]}$$
$$\frac{R_n}{\Omega} = \frac{F_{nt} A_b}{\Omega}$$
$$= \frac{F_{nt} A_b}{2.00} \quad \text{[ASD]}$$

Values of the nominal tensile stress F_{nt} are given in AISC 360 Table J3.2 for all types of bolts. Values of ϕR_n and R_n/Ω are given in *AISC Manual* Table 7-2.

When a bearing-type bolt is subjected to combined shear and tension, the available strength in shear is unaffected, and the available strength in tension is reduced in accordance with AISC 360 Sec. J3.7.

The value of the reduced nominal tensile capacity is

$$\phi R_n = \phi F'_{nt} A_b$$
$$= 0.75 F'_{nt} A_b \quad \text{[LRFD]}$$
$$\frac{R_n}{\Omega} = \frac{F'_{nt} A_b}{\Omega}$$
$$= \frac{F'_{nt} A_b}{2.00} \quad \text{[ASD]}$$

F'_{nt}, the modified nominal tensile stress, is calculated using AISC 360 Eq. J3-3a for the LRFD method or AISC 360 Eq. J3-3b for the ASD method.

$$F'_{nt} = 1.3 F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_{rv} \quad \text{[LRFD]}$$
$$= 1.3 F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv} \quad \text{[ASD]}$$

The required shear stress, f_{rv}, is determined using appropriate load combinations, and must be equal to or less than the available shear stress. Values of the nominal shear stress F_{nv} are given in AISC 360 Table J3.2 for all types of bolts.

When either $f_{rv} \leq 30\%$ of the available shear stress or $f_t \leq 30\%$ of the available tensile stress, the effects of combined stress do not need to be considered.

Example 4.35

The connection analyzed in Ex. 4.32 consists of 11 grade A307 $^7/_8$ in diameter bolts. Determine the design shear strength of the bolts in the connection.

Solution

From *AISC Manual* Table 7-1, the available strength of the 11 bolts in single shear is

LRFD Method

$$\phi R_n = \phi F_{nv} A_b n = \left(12.2 \ \frac{\text{kips}}{\text{bolt}} \right) (11 \text{ bolts})$$
$$= 134 \text{ kips}$$

ASD Method

$$\frac{R_n}{\Omega} = \frac{F_{nv} A_b n}{\Omega} = \left(8.11 \ \frac{\text{kips}}{\text{bolt}} \right) (11 \text{ bolts})$$
$$= 89 \text{ kips}$$

Slip-Critical Bolts in Shear

Slip-critical bolts are high strength group A or group B bolt pretensioned to the value specified in AISC 360 Table J3.1 of

$$T_b = 0.70 F_u$$

The pretension produces a clamping force between the parts, and transfers the shear load from one connected part to another by friction. At the strength limit state, the connection may slip sufficiently to place the bolts in bearing and AISC 360 Sec. J3.10 requires slip-critical connections to also comply with the requirements of snug-tight connections.

The *frictional resistance* developed in a slip-critical connection depends on the condition of the faying surfaces. The values of the slip coefficient, μ, for two types of surface conditions (class A and class B) are given in AISC 360 Sec. J3.8.

- Class A surface conditions consist of unpainted clean mill scale surfaces or blast-cleaned surfaces with class A coatings. The slip coefficient is

$$\mu = 0.30$$

- Class B surface conditions consist of unpainted blast-cleaned surfaces or blast-cleaned surfaces with class B coatings. The slip coefficient is

$$\mu = 0.50$$

The *nominal slip resistance* is identical for the cases of threads included or excluded from the shear plane, and is given by AISC 360 Eq. J3-4 as

$$R_n = \mu D_u h_f T_b n_s$$

The *bolt tension multiplier* reflects the ratio of the mean installed bolt tension to the specified minimum bolt pretension and is given by

$$D_u = 1.13$$

The *modification factor for fillers*, h_f, is

- 1.00 where bolts are added to distribute loads in the filler

- 1.00 for one filler between connected parts

- 0.85 for two or more fillers between connected parts

The resistance factor and safety factor adopted depend on the type of hole used in the connection. Connections allowing a large amount of slip may cause unacceptable slip and will require a higher safety factor than a connection with little slip. The resistance factors and safety factors that modify the available slip resistance are specified in AISC 360 Sec. J3.8 as follows.

- For standard size and short-slotted holes perpendicular to the direction of the load,

$$\phi = 1.00 \quad \text{[LRFD]}$$
$$\Omega = 1.50 \quad \text{[ASD]}$$

- For oversized and short-slotted holes parallel to the direction of the load,

$$\phi = 0.85 \quad \text{[LRFD]}$$
$$\Omega = 1.76 \quad \text{[ASD]}$$

- For long-slotted holes,

$$\phi = 0.70 \quad \text{[LRFD]}$$
$$\Omega = 2.14 \quad \text{[ASD]}$$

Values for the available slip-critical shear resistance for a class A faying surface for group A and group B bolts are given in *AISC Manual* Table 7-3. These values are multiplied by 1.67 for a class B faying surface.

Slip-Critical Bolts in Tension and Combined Shear and Tension

The available tensile strength of slip-critical bolts is independent of the pretension in the bolt.

The nominal tensile strength is given by AISC 360 Eq. J3-1 as

$$R_n = F_{nt}A_b$$

The available tensile strength is

$$\phi R_n = 0.75 F_{nt}A_b \quad \text{[LRFD]}$$
$$\frac{R_n}{\Omega} = \frac{F_{nt}A_b}{2.00} \quad \text{[ASD]}$$

Values of the nominal tensile stress, F_{nt}, are given in AISC 360 Table J3.2 for all types of bolts. Values of ϕR_n and R_n/Ω are given in *AISC Manual* Table 7-2.

When a slip-critical bolt is subjected to combined shear and tension, the available strength in tension is unaffected. However, in accordance with AISC 360 Sec. J3.9, the available resistance to shear is reduced by being multiplied by the factor

$$k_{sc} = 1 - \frac{T_u}{D_u T_b n_b} \quad \text{[LRFD]} \qquad \text{[\textit{AISC J3-5a}]}$$
$$k_{sc} = 1 - \frac{1.5 T_a}{D_u T_b n_b} \quad \text{[ASD]} \qquad \text{[\textit{AISC J3-5b}]}$$

Example 4.36

The connection analyzed in Ex. 4.32 consists of 11 grade A490 $^7/_8$ in diameter slip-critical bolts. Determine the available resistance to shear of the bolts in the connection. The bolts are in standard holes with a class A faying surface.

Solution

For group B bolts in standard holes and a class A faying surface, *AISC Manual* Table 7-3 gives the available single shear strength of the 11 bolts in shear.

LRFD Method

$$\phi R_n = (16.6 \text{ kips})(11 \text{ bolts})$$
$$= 183 \text{ kips}$$

ASD Method

$$\frac{R_n}{\Omega} = (11.1 \text{ kips})(11 \text{ bolts})$$
$$= 122 \text{ kips}$$

Bolts in Bearing

The bearing strength of connected parts is specified in AISC 360 Sec. J3.10 and is dependent on the diameter, spacing, edge distance of a bolt, the material of the connected parts and the acceptable deformation at the bolt hole. For all equations in AISC 360 Sec. J3.10, use

$$\phi = 0.75$$
$$\Omega = 2.00$$

High-strength bolts in slip-critical connections must also be checked for bearing strength since the connection at the strength limit state may slip sufficiently to place the bolts in bearing.

When deformation at the hole is a design consideration, the nominal bearing strength of connected parts is given by AISC 360 Eq. J3-6a as

$$R_n = 1.2 l_c t F_u \quad \text{[when tear-out strength governs]}$$
$$\leq 2.4 d t F_u \quad \text{[when bearing strength governs]}$$

To ensure that tear out does not occur, the clear distance between adjacent holes or between the edge of a hole and the edge of a connected part is obtained from AISC 360 Eq. J3-6a as

$$l_c \geq 2d$$

The clear distance between adjacent holes is

$$l_c = s - d_n$$

The clear distance between the edge of a hole and the edge of a connected part is

$$l_c = l_e - 0.5d_n$$

l_e is the edge distance between the bolt center and the edge of the connected part.

The nominal hole diameter given in AISC 360 Table J3.3 is

$$d_n = d + \frac{1}{16} \text{ in}$$

The minimum distance between the centers of standard holes to ensure that tear out does not occur is

$$s \geq 3d + \frac{1}{16} \text{ in}$$

When deformation at the hole is not a design consideration, the nominal bearing capacity of each bolt is given by AISC 360 Eq. J3-6b as

$$R_n = 1.5 l_c t F_u \quad \text{[when tear-out strength governs]}$$
$$\leq 3.0 dt F_u \quad \text{[when bearing strength governs]}$$

To ensure that tear out does not occur, the clear distance between adjacent holes or between the edge of a hole and the edge of the connected part is obtained from AISC 360 Eq. J3-6b as

$$l_c \geq 2d$$

The minimum distance between the centers of standard holes to ensure that tear out does not occur is

$$s \geq 3d + \frac{1}{16} \text{ in}$$

For long-slotted holes with the slot perpendicular to the direction of force, the nominal bearing capacity of each bolt is given by AISC 360 Eq. J3-6c as

$$R_n = 1.0 l_c t F_u \quad \text{[when tear-out strength governs]}$$
$$\leq 2.0 dt F_u \quad \text{[when bearing strength governs]}$$

To ensure that tear out does not occur, the clear distance between adjacent holes or between the edge of a hole and the edge of a connected part is obtained from AISC 360 Eq. J3-6b as

$$l_c \geq 2d$$

The minimum distance between the centers of standard holes to ensure that tear out does not occur is

$$s \geq 3d + \frac{1}{16} \text{ in}$$

The available bearing capacity at bolt holes that considers deformation of the connected parts is given in *AISC Manual* Table 7-4 and is based on bolt center-to-center spacing. The available bearing capacity that considers

deformation of the connected parts is given in *AISC Manual* Table 7-5 based on bolt edge distances, measured from the center of bolt to the edge of the connected part. Tables are not provided for the bearing capacity when hole deformation is not considered.

Example 4.37

The connection analyzed in Ex. 4.32 consists of 11 grade A307 $^{7}/_{8}$ in diameter bolts in standard holes. Determine the available bearing strength of the bolts in the A36 plates ($F_u = 58$ kips/in^2) if the edge distance is $L_c = 2.5$ in and $s = 3$ in.

Solution

From *AISC Manual* Table 7-5, the minimum edge distance for full bearing strength is

$$L_c = 2.25 \text{ in}$$
$$< 2.5 \text{ in provided}$$

The edge distance does not govern.

AISC Manual Table 7-4 gives the available strength of the 11 bolts in bearing.

LRFD Method

$$\phi R_n = \left(72.9 \ \frac{\frac{\text{kips}}{\text{in}}}{\text{bolt}} \right) \left(\tfrac{1}{2} \text{ in} \right) (11 \text{ bolts})$$
$$= 401 \text{ kips}$$

ASD Method

$$\frac{R_n}{\Omega} = \left(48.6 \ \frac{\frac{\text{kips}}{\text{in}}}{\text{bolt}} \right) \left(\tfrac{1}{2} \text{ in} \right) (11 \text{ bolts})$$
$$= 267 \text{ kips}$$

Bolt Group Eccentrically Loaded in Plane of Faying Surface

Eccentrically loaded bolt groups of the type shown in Fig. 4.23 may be conservatively designed by means of the elastic unit area method. The moment of inertia of the bolt group about the x-axis is

$$I_x = \sum y^2$$

The moment of inertia of the bolt group about the y-axis is

$$I_y = \sum x^2$$

Figure 4.23 *Eccentrically Loaded Bolt Group*
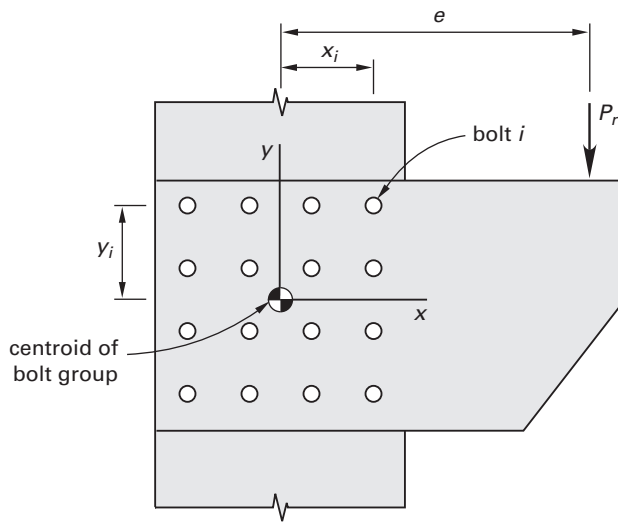
The polar moment of inertia of the bolt group about the centroid is

$$I_o = I_x + I_y$$

The vertical force on bolt i due to the applied load, P_r, is

$$V_p = \frac{P_r}{n}$$

The vertical force on bolt i due to the eccentricity, e, is

$$V_e = \frac{P_r e x_i}{I_o}$$

The horizontal force on bolt i due to the eccentricity, e, is

$$H_e = \frac{P_r e y_i}{I_o}$$

The resultant force on bolt i is

$$R = \sqrt{(V_p + V_e)^2 + H_e^2}$$

The instantaneous center of rotation method of analyzing eccentrically loaded bolt groups affords a more realistic estimate of a bolt group's capacity. *AISC Manual* Table 7-6 through Table 7-13 provide a means of designing common bolt group patterns by this method.

Example 4.38

Determine the diameter of the A325 bearing-type bolts required in the bolted bracket shown. Use the elastic unit area method and compare with the instantaneous center of rotation method.
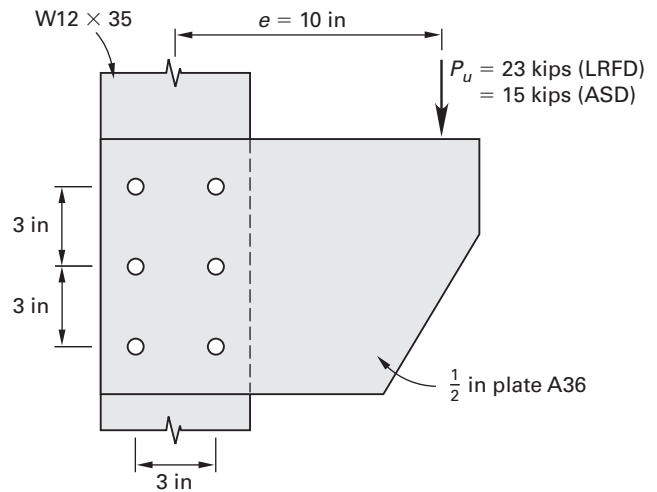
Solution

The geometric properties of the bolt group are obtained by applying the unit area method.

The moment of inertia about the x-axis is

$$I_x = \sum y^2 = (4)(3 \text{ in})^2 = 36 \text{ in}^4/\text{in}^2$$

The moment of inertia about the y-axis is

$$I_y = \sum x^2 = (6)(1.5 \text{ in})^2 = 13.5 \text{ in}^4/\text{in}^2$$

The polar moment of inertia about the centroid is

$$I_o = I_x + I_y = 49.5 \text{ in}^4/\text{in}^2$$

The top right bolt is the most heavily loaded, and the coexistent forces on this bolt are as follows.

LRFD Method

- vertical force due to applied load

$$V_p = \frac{P_r}{n}$$
$$= \frac{23 \text{ kips}}{6}$$
$$= 3.83 \text{ kips}$$

- vertical force due to eccentricity

$$V_e = \frac{P_r e x_i}{I_o}$$
$$= \frac{(23 \text{ kips})(10 \text{ in})(1.5 \text{ in})}{49.5 \ \frac{\text{in}^4}{\text{in}^2}}$$
$$= 6.98 \text{ kips}$$

- horizontal force due to eccentricity

$$H_e = \frac{P_r e y_i}{I_o} = \frac{(23 \text{ kips})(10 \text{ in})(3 \text{ in})}{49.5 \ \frac{\text{in}^4}{\text{in}^2}}$$

$$= 13.94 \text{ kips}$$

- resultant force

$$R = \sqrt{(V_p + V_e)^2 + {H_e}^2}$$
$$= \sqrt{(3.83 \text{ kips} + 6.98 \text{ kips})^2 + (13.94 \text{ kips})^2}$$
$$= 17.6 \text{ kips}$$

Shear controls, and from *AISC Manual* Table 7-1 the design shear strength of a $^3/_4$ in diameter A325N bolt in a standard hole in single shear is

$$\phi R_n = 17.9 \text{ kips}$$
$$> 17.6 \text{ kips} \quad [\text{satisfactory}]$$

From *AISC Manual* Table 7-7, the coefficient C is given as 1.46, and the required design strength of an individual bolt, based on the instantaneous center of rotation method, is

$$\phi R_n = \frac{P_r}{C} = \frac{23 \text{ kips}}{1.46}$$
$$= 15.8 \text{ kips}$$

Shear controls, and from *AISC Manual* Table 7-1 the design shear strength of a $^3/_4$ in diameter A325N bolt in a standard hole in single shear is

$$\phi R_n = 17.9 \text{ kips}$$
$$> 15.8 \text{ kips} \quad [\text{satisfactory}]$$

ASD Method

- vertical force due to applied load

$$V_p = \frac{P_r}{n}$$
$$= \frac{15 \text{ kips}}{6}$$
$$= 2.50 \text{ kips}$$

- vertical force due to eccentricity

$$V_e = \frac{P_r e x_i}{I_o} = \frac{(15 \text{ kips})(10 \text{ in})(1.5 \text{ in})}{49.5 \ \frac{\text{in}^4}{\text{in}^2}}$$

$$= 4.55 \text{ kips}$$

- horizontal force due to eccentricity

$$H_e = \frac{P_r e y_i}{I_o} = \frac{(15 \text{ kips})(10 \text{ in})(3 \text{ in})}{49.5 \ \frac{\text{in}^4}{\text{in}^2}}$$

$$= 9.09 \text{ kips}$$

- resultant force

$$R = \sqrt{(V_p + V_e)^2 + H_e^2}$$
$$= \sqrt{(2.50 \text{ kips} + 4.55 \text{ kips})^2 + (9.09 \text{ kips})^2}$$
$$= 11.50 \text{ kips}$$

Shear controls, and from *AISC Manual* Table 7-1 the allowable shear strength of a $^3/_4$ in diameter A325N bolt in a standard hole in single shear is

$$\frac{R_n}{\Omega} = 11.9 \text{ kips}$$
$$> 11.50 \text{ kips} \quad [\text{satisfactory}]$$

From *AISC Manual* Table 7-7, the coefficient C is given as 1.46, and the required allowable strength of an individual bolt, based on the instantaneous center of rotation method, is

$$\frac{R_n}{\Omega} = \frac{P_r}{C} = \frac{15 \text{ kips}}{1.46}$$
$$= 10.3 \text{ kips}$$

Shear controls, and from *AISC Manual* Table 7-1 the design shear strength of a $^3/_4$ in diameter A325N bolt in a standard hole in single shear is

$$\frac{R_n}{\Omega} = 11.9 \text{ kips}$$
$$> 10.3 \text{ kips} \quad [\text{satisfactory}]$$

Bolt Group Eccentrically Loaded Normal to the Faying Surface (LRFD)

Eccentrically loaded bolt groups of the type shown in Fig. 4.24 may be conservatively designed by assuming that the neutral axis is located at the centroid of the bolt group and that a plastic stress distribution is produced in the bolts. The tensile force in each bolt above the neutral axis due to the eccentricity is given by

$$T_u = \frac{P_u e}{n' d_m}$$

The shear force in each bolt due to the applied load is given by

$$V_p = \frac{P_u}{n}$$

Figure 4.24 *Bolt Group Eccentrically Loaded Normal to Faying Surface (LRFD)*

neutral axis   bolted bracket   plastic stress distribution
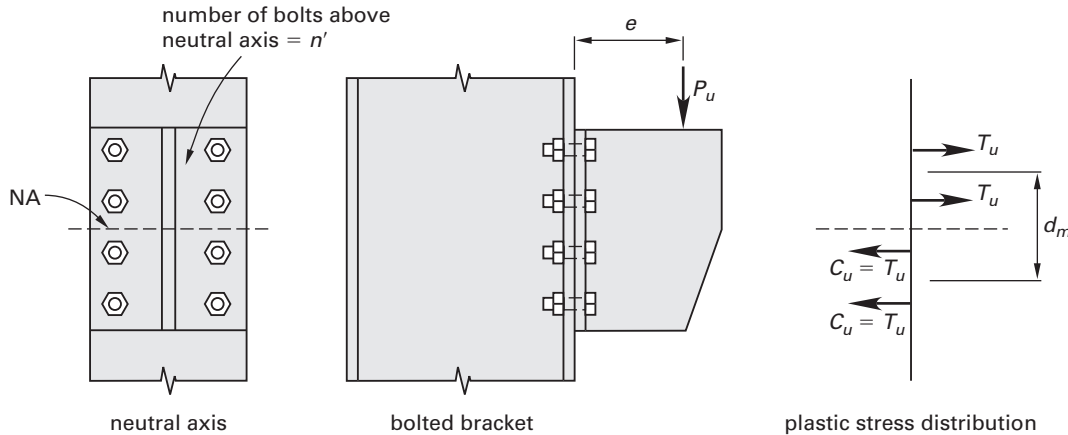
Example 4.39

Determine whether the $\frac{7}{8}$ in diameter A325N bearing-type bolts in the bolted bracket shown are adequate. Prying action may be neglected.
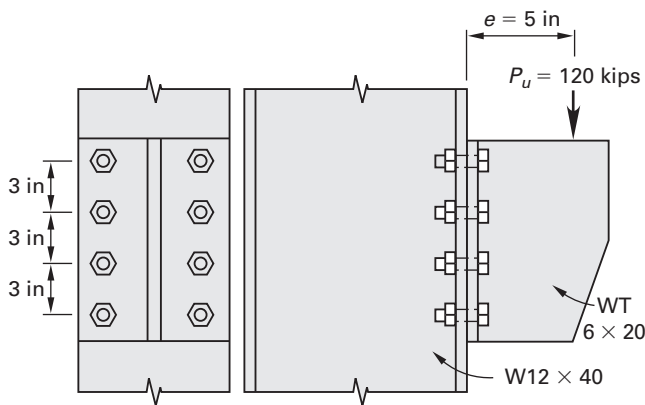
Solution

The tensile force in each bolt above the neutral axis due to the eccentricity is given by

$$T_u = \frac{P_u e}{n' d_m} = \frac{(120 \text{ kips})(5 \text{ in})}{(4)(6 \text{ in})}$$
$$= 25 \text{ kips}$$

The shear force in each bolt due to the applied load is given by

$$V_p = \frac{P_u}{n} = \frac{120 \text{ kips}}{8}$$
$$= 15 \text{ kips}$$

The required shear stress on each bolt is

$$f_{rv} = \frac{V_p}{A_b} = \frac{15 \text{ kips}}{0.601 \text{ in}^2}$$
$$= 25 \text{ kips/in}^2$$

The design shear stress for grade A325 bolts, with threads not excluded from the shear plane, is obtained from AISC 360 Table J3.2 as

$$\phi F_{nv} = (0.75)\left(54 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 40.5 \text{ kips/in}^2$$
$$> f_{rv} \quad \text{[satisfactory]}$$
$$f_{rv} > 0.3 \phi F_{nv}$$

The design tensile stress for grade A325 bolts is obtained from AISC 360 Table J3.2 as

$$\phi F_{nt} = (0.75)\left(90 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 67.5 \text{ kips/in}^2$$

The factored tensile stress in each $\frac{7}{8}$ in diameter bolt is

$$f_t = \frac{T_u}{A_b} = \frac{25 \text{ kips}}{0.601 \text{ in}^2}$$
$$= 41.60 \text{ kips/in}^2$$
$$< \phi F_{nt} \quad \text{[satisfactory]}$$
$$> 0.3 \phi F_{nt}$$

Therefore, it is necessary to investigate the effects of the combined shear and tensile stress. The nominal tensile stress, F'_{nt}, of a bolt subjected to combined shear and tension is given by AISC 360 Eq. J3-3a as

$$F'_{nt} = 1.3 F_{nt} - \frac{f_{rv} F_{nt}}{\phi F_{nv}}$$

$$= (1.3)\left(90 \ \frac{\text{kips}}{\text{in}^2}\right) - \left(\frac{90 \ \frac{\text{kips}}{\text{in}^2}}{40.5 \ \frac{\text{kips}}{\text{in}^2}}\right)\left(25 \ \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 61.44 \text{ kips/in}^2$$

Steel

The design tensile stress, $\phi F'_{nt}$, of a bolt subjected to combined shear and tension is given by AISC 360 Eq. J3-2 as

$$\phi F'_{nt} = (0.75)\left(61.44 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 46.08 \ \text{kips/in}^2$$
$$> f_t \quad \text{[satisfactory]}$$

Bolt Group Eccentrically Loaded Normal to the Faying Surface (ASD)

Eccentrically loaded bolt groups of the type shown in Fig. 4.25 may be conservatively designed by assuming that the neutral axis is located at the centroid of the bolt group. The tensile force in a bolt a distance y_i from the neutral axis is

$$T_i = \frac{P_a e y_i}{I_x}$$

$$I_x = \sum y^2$$

The shear force in each bolt due to the applied load is

$$V_p = \frac{P_a}{n}$$

Example 4.40

Determine whether the $^7/_8$ in diameter A325N bearing-type bolts in the bolted bracket shown are adequate. Prying action may be neglected.
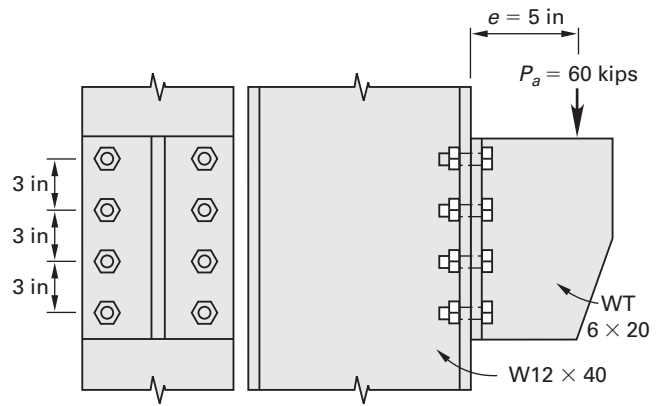
Solution

$$I_x = \sum y^2 = (4)(4.5 \ \text{in})^2 + (4)(1.5 \ \text{in})^2$$
$$= 90 \ \text{in}^4/\text{in}^2$$

The tensile force in each of the top bolts due to the eccentricity is given by

$$T_i = \frac{P_a e y_i}{I_x} = \frac{(60 \ \text{kips})(5 \ \text{in})(4.5 \ \text{in})}{90 \ \dfrac{\text{in}^4}{\text{in}^2}}$$

$$= 15 \ \text{kips}$$

The shear force in each bolt due to the applied load is given by

$$V_p = \frac{P_a}{n} = \frac{60 \ \text{kips}}{8}$$
$$= 7.5 \ \text{kips}$$

Figure 4.25 *Bolt Group Eccentrically Loaded Normal to the Faying Surface (ASD)*

neutral axis   bolted bracket   elastic stress distribution
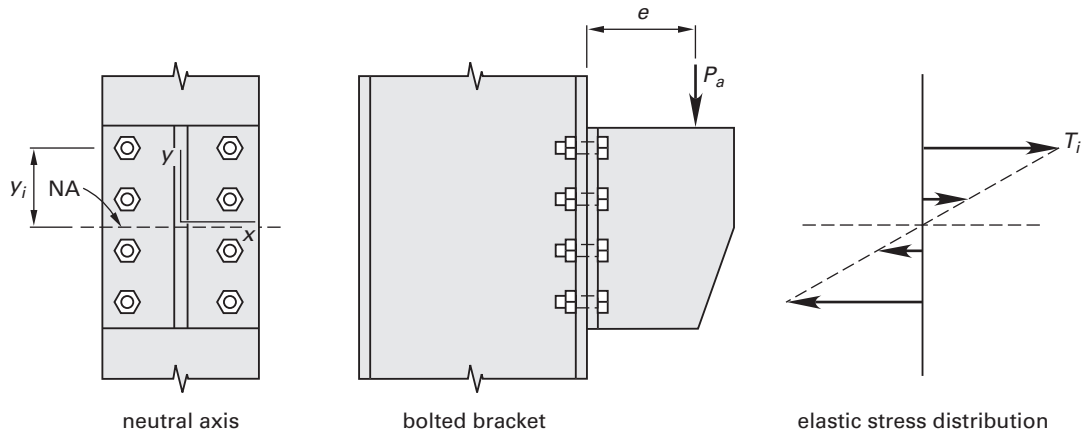
The required shear stress on each bolt is

$$f_{rv} = \frac{V_p}{A_b} = \frac{7.5 \text{ kips}}{0.601 \text{ in}^2}$$

$$= 12.5 \text{ kips/in}^2$$

The allowable shear stress for grade A325 bolts, with threads not excluded from the shear plane, is obtained from AISC 360 Table J3.2 as

$$\frac{F_{nv}}{\Omega} = \frac{54 \dfrac{\text{kips}}{\text{in}^2}}{2.00} = 27 \text{ kips/in}^2$$

$$> f_{rv} \quad [\text{satisfactory}]$$

$$f_{rv} > \frac{0.3 F_{nv}}{\Omega}$$

The allowable tensile stress for grade A325 bolts is obtained from AISC 360 Table J3.2 as

$$\frac{F_{nt}}{\Omega} = \frac{90 \dfrac{\text{kips}}{\text{in}^2}}{2.00} = 45 \text{ kips/in}^2$$

The tensile stress in each of the top $^7/_8$ in diameter bolts is

$$f_t = \frac{T_i}{A_b} = \frac{15 \text{ kips}}{0.601 \text{ in}^2} = 25.0 \text{ kips/in}^2$$

$$< \frac{F_{nt}}{\Omega} \quad [\text{satisfactory}]$$

$$> \frac{0.3 F_{nt}}{\Omega}$$

It is necessary to investigate the effects of the combined shear and tensile stress. The nominal tensile stress, F'_{nt}, of a bolt subjected to combined shear and tension is given by AISC 360 Eq. J3-3b as

$$F'_{nt} = 1.3 F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_{rv}$$

$$= (1.3)\left(90 \ \frac{\text{kips}}{\text{in}^2}\right)$$

$$- \left(\frac{(2.00)\left(90 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{54 \ \dfrac{\text{kips}}{\text{in}^2}}\right)\left(12.5 \ \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 75.33 \text{ kips/in}^2$$

The allowable tensile stress, $\phi F'_{nt}$, of a bolt subjected to combined shear and tension is given by AISC 360 Eq. J3-2 as

$$\frac{F'_{nt}}{\Omega} = \frac{75.33 \ \dfrac{\text{kips}}{\text{in}^2}}{2.00}$$

$$= 37.67 \text{ kips/in}^2$$

$$> f_t \quad [\text{satisfactory}]$$

9. DESIGN OF WELDED CONNECTIONS

Nomenclature

a	coefficient for eccentrically loaded weld group	–
A_{we}	effective area of the weld	in^2
A_{BM}	effective area for base metal	in^2
C	coefficient for eccentrically loaded weld group	–
D	number of sixteenths-of-an-inch in the weld size	–
F_{nBM}	nominal stress for base metal	kips/in^2
F_{EXX}	classification of weld metal	kips/in^2
F_{nw}	nominal strength of weld electrode	kips/in^2
k	coefficient for eccentrically loaded weld group	–
l	characteristic length of weld group used in tabulated values of instantaneous center method	in
$\bar{l}$	total length of weld	in
q	allowable strength	kips/in
q_u	design strength	kips/in
R_n	nominal strength	–
R_{wl}	total nominal strength of longitudinally loaded fillet welds, as determined in accordance with AISC 360 Table J2.5	–
R_{wt}	total nominal strength of transversely loaded fillet welds, as determined in accordance with AISC 360 Table J2.5 without the amplification of the weld shear strength given by AISC 360 Eq. J2-5	–
t_e	effective throat thickness	in
t_e	effective plate thickness	in
w	fillet weld size	in
w	width of plate	in

Symbols

θ	angle of inclination of loading measured from the weld longitudinal axis	degree
ϕ	resistance factor	–

Weld Design Strength

The strength of a welded connection depends on both the strength of the base metal and the strength of the weld metal. Welded connections must be made using a "matching" weld metal of sufficient strength, given the

type of base metal used. These weld strengths are listed in the user note in AISC 360 Sec. J2.6. For typical structural steel applications that use A36 and grade 50, the matching weld metal is E70XX, which has a tensile strength of 70 ksi. Lower strength welded metal can be used in certain situations as specified in AISC 360 Sec. J2.6, however, E70XX is the industry standard and is recommended. Weld nominal stress values, effective areas, resistance factors, and safety factors are tabulated in AISC 360 Table J2.5.

The nominal strength of the base metal is the product of the nominal stress and the effective area of the base metal, and is given by AISC 360 Eq. J2-2 as

$$R_n = F_{nBM} A_{BM}$$

The nominal strength of the weld metal is the product of the nominal stress and the effective area of the weld metal, and is given by AISC 360 Eq. J2-3 as

$$R_n = F_{nw} A_{we}$$

Complete-Penetration Groove Weld

In accordance with AISC 360 Table J2.5, the nominal strength of a complete-penetration groove weld is governed by the base metal and computation of the strength of the weld is not required. The effective thickness of the joint is the thickness of the thinner part joined, shown as t_e in Fig. 4.26.

Figure 4.26 *Complete-Penetration Groove Weld*
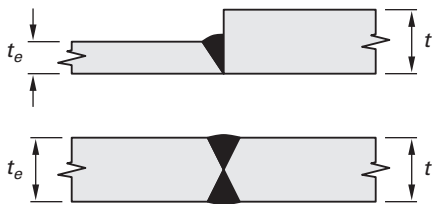
Example 4.41

The two $^1/_2$ in thick by 2 in wide plates shown are connected with a complete-penetration groove weld as indicated. The plates are grade A36 steel and the electrodes are E70XX. Determine the available tensile capacity of the connection.
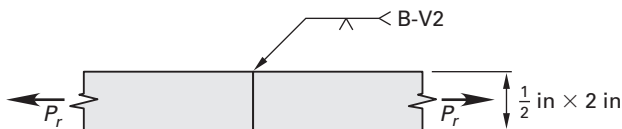
Solution

The effective area of the joint is

$$A_{BM} = wt_e = (2 \text{ in})\left(\frac{1}{2} \text{ in}\right)$$
$$= 1.0 \text{ in}^2$$

LRFD Method

The design axial strength for tensile rupture is

$$\phi R_n = \phi F_u A_{BM}$$
$$= (0.75)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(1.0 \text{ in}^2)$$
$$= 43.5 \text{ kips}$$

The design axial strength for yielding is

$$\phi R_n = \phi F_y A_{BM}$$
$$= (0.90)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(1.0 \text{ in}^2)$$
$$= 32.4 \text{ kips} \quad \text{[governs]}$$

ASD Method

The allowable axial strength for tensile rupture is

$$\frac{R_n}{\Omega} = \frac{F_u A_{BM}}{\Omega}$$
$$= \frac{\left(58 \ \dfrac{\text{kips}}{\text{in}^2}\right)(1.0 \text{ in}^2)}{2.00}$$
$$= 29 \text{ kips}$$

The allowable axial strength for yielding is

$$\frac{R_n}{\Omega} = \frac{F_y A_{BM}}{\Omega}$$
$$= \frac{\left(36 \ \dfrac{\text{kips}}{\text{in}^2}\right)(1.0 \text{ in}^2)}{1.67}$$
$$= 21.6 \text{ kips} \quad \text{[governs]}$$

Partial-Penetration Groove Weld

The nominal strength of a partial-penetration groove weld is governed by the effective throat thickness, given in AISC 360 Table J2.1 and shown as t_e in Fig. 4.27.

Example 4.42

The two $4 \times 1^1/_2$ in plates shown are connected with a double V-groove weld with a $^1/_2$ in penetration into each face, as indicated. The plates are grade A36 steel and the electrodes are E70XX. Determine the available tensile capacity of the connection.
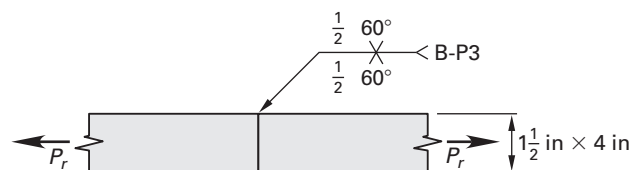
Figure 4.27 *Partial-Penetration Groove Weld*

(a) shield metal arc, gas metal arc,
and flux cored arc

(b) all other welding processes
and positions
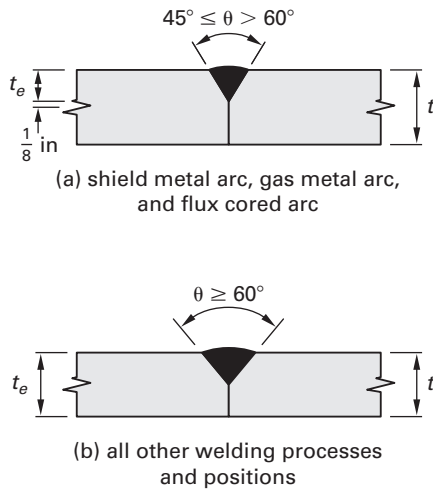
Solution

For an included angle of $60°$, the effective throat thickness equals the depth of the weld, which is $1/2$ in. The total effective area of the weld is

$$A_{we} = wt_e$$
$$= (4 \text{ in})\left(\frac{1}{2} \text{ in}\right)(2 \text{ sides})$$
$$= 4.0 \text{ in}^2$$

The effective area of the base metal is

$$A_{BM} = wt_e$$
$$= (4 \text{ in})\left(1\frac{1}{2} \text{ in}\right)$$
$$= 6.0 \text{ in}^2$$

LRFD Method

The design axial strength for yielding in the base metal is

$$\phi R_n = \phi F_y A_{BM}$$
$$= (0.90)\left(36 \frac{\text{kips}}{\text{in}^2}\right)(6.0 \text{ in}^2)$$
$$= 194.4 \text{ kips}$$

By inspection, tensile rupture of the base metal does not govern. (See Ex. 4.41.) The design tensile capacity of the weld, normal to the effective area, is given by AISC 360 Table J2.5 as

$$\phi R_n = \phi(0.60 F_{\text{EXX}} A_{we})$$
$$= (0.8)\left((0.6)\left(70 \frac{\text{kips}}{\text{in}^2}\right)(4.0 \text{ in}^2)\right)$$
$$= 134.4 \text{ kips} \quad \text{[governs]}$$

ASD Method

The allowable axial strength for yielding in the base metal is

$$\frac{R_n}{\Omega} = \frac{F_y A_{BM}}{\Omega}$$
$$= \frac{\left(36 \frac{\text{kips}}{\text{in}^2}\right)(6.0 \text{ in}^2)}{1.67}$$
$$= 129.3 \text{ kips}$$

By inspection, tensile rupture of the base metal does not govern. (See Ex. 4.41.) The allowable tensile capacity of the weld, normal to the effective area, is given by AISC 360 Table J2.5 as

$$\frac{R_n}{\Omega} = \frac{0.60 F_{\text{EXX}} A_{we}}{\Omega}$$
$$= \frac{(0.60)\left(70 \frac{\text{kips}}{\text{in}^2}\right)(4.0 \text{ in}^2)}{1.88}$$
$$= 89.4 \text{ kips} \quad \text{[governs]}$$

Fillet Weld

The leg length, w, of a fillet weld is used to designate the nominal size of the weld. For fillet welds made by the shielded metal arc process, the effective throat thickness, t_e, is given in AISC 360 Sec. J2.2a and shown in Fig. 4.28. The *effective throat thickness* is the shortest distance between the root and the weld face, and is given by

$$t_e = 0.707 w$$

Figure 4.28 *Fillet Weld*
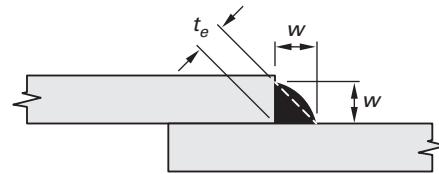
The minimum permitted size of fillet welds is specified in AISC 360 Sec. J2.2b and shown in Table 4.2.

The maximum permitted size of fillet welds along the edges of connected parts is specified in AISC 360 Sec. J2.2b and shown in Table 4.3.

The minimum permissible length of a fillet weld is four times the nominal weld size. When longitudinal fillet welds are used alone in a connection, the length of each fillet weld must not be less than the perpendicular distance between them because of shear lag. See AISC 360 Sec. D3 for shear lag provisions.

Table 4.2 *Minimum Size of Fillet Welds*

thickness of thinner part joined (in)	minimum size of fillet weld (in)
$t \leq {}^1\!/_4$	$w \geq {}^1\!/_8$
${}^1\!/_4 < t \leq {}^1\!/_2$	$w \geq {}^3\!/_{16}$
${}^1\!/_2 < t \leq {}^3\!/_4$	$w \geq {}^1\!/_4$

Table 4.3 *Maximum Size of Fillet Welds*

thickness of part (in)	maximum size of fillet weld (in)
$t < {}^1\!/_4$	$w \leq t$
$t \geq {}^1\!/_4$	$w \leq t - {}^1\!/_{16}$

Available Strength of a $^1/_{16}$ in Fillet Weld

To simplify calculations, it is appropriate to determine the available strength of a $^1/_{16}$ in fillet weld per inch run of E70XX grade electrodes.

LRFD Method

The design strength of a $^1/_{16}$ in fillet weld per inch run of E70XX grade electrodes is

$$
\begin{aligned}
q_u &= \phi F_{nw} A_{we} \\
&= (0.75)(0.60)\left(70 \ \frac{\text{kips}}{\text{in}^2}\right)(0.707)\left(\frac{1}{16} \ \text{in}\right)(1 \ \text{in}) \\
&= 1.39 \ \text{kips/in per } {}^1/_{16} \ \text{in}
\end{aligned}
$$

ASD Method

The allowable strength of a $^1/_{16}$ in fillet weld per inch run of E70XX grade electrodes is

$$
\begin{aligned}
q &= \frac{F_{nw} A_{we}}{\Omega} \\
&= \frac{(0.60)\left(70 \ \frac{\text{kips}}{\text{in}^2}\right)(0.707)\left(\frac{1}{16} \ \text{in}\right)(1 \ \text{in})}{2.00} \\
&= 0.928 \ \text{kips/in per } {}^1/_{16} \ \text{in}
\end{aligned}
$$

Example 4.43

The $^1/_2$ in plate shown is connected to a $^5/_8$ in gusset plate with $^1/_4$ in E70XX fillet welds as indicated. Both plates are A36 material. Determine the available strength of the welds.
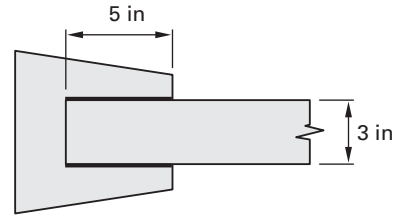
Solution

The total length of weld provided on the $^1/_2$ in plate is

$$
\begin{aligned}
l &= (2)(5 \ \text{in}) \\
&= 10 \ \text{in}
\end{aligned}
$$

From Table 4.2, the minimum permitted size of fillet weld for the $^5/_8$ in thick gusset plate is

$$
\begin{aligned}
w_{\min} &= \frac{1}{4} \ \text{in} \\
w_{\text{provided}} &= \frac{1}{4} \ \text{in} \quad \text{[satisfactory]}
\end{aligned}
$$

From Table 4.3, the maximum permitted size of fillet weld along the edge of the $^1/_2$ in thick plate is

$$
\begin{aligned}
w_{\max} &= \frac{1}{2} \ \text{in} - \frac{1}{16} \ \text{in} \\
&= {}^7/_{16} \ \text{in} \\
&> w_{\text{provided}} \quad \text{[satisfactory]}
\end{aligned}
$$

The distance between the two fillet welds is

$$
s = 3 \ \text{in}
$$

The length of each fillet weld is

$$
\begin{aligned}
l_{\text{each}} &= 5 \ \text{in} \\
&> s \quad \text{[satisfies AISC 360 Sec. J2.2b]}
\end{aligned}
$$

LRFD Method

The design shear capacity of the total length of weld is

$$
\begin{aligned}
\phi R_n &= l D q_u \\
&= (10 \ \text{in})(4 \ \text{sixteenths})\left(1.39 \ \frac{\text{kips}}{\text{in}} \ \text{per} \ \frac{1}{16} \ \text{in}\right) \\
&= 55.6 \ \text{kips}
\end{aligned}
$$

ASD Method

The allowable shear capacity of the total length of weld is

$$
\begin{aligned}
\frac{R_n}{\Omega} &= l D q \\
&= (10 \ \text{in})(4 \ \text{sixteenths})\left(0.928 \ \frac{\text{kips}}{\text{in}} \ \text{per} \ \frac{1}{16} \ \text{in}\right) \\
&= 37.1 \ \text{kips}
\end{aligned}
$$

Fillet Weld Size Governed by Base Metal Thickness

In accordance with AISC 360 Sec. J2.4, the design of a welded connection is governed by the capacity of the weakest shear plane. This is either through the weld or through the base material. Providing a weld size with a strength in excess of the base material strength will not increase the strength of the connection as the base material strength governs. The design shear strength of the weld per linear inch is

$$Q_{uw} = q_u D$$
$$= \left(1.39 \ \frac{\text{kips}}{\text{in}}\right) D$$

The design shear rupture strength per linear inch of grade 50 base material of thickness, t, is derived from AISC 360 Eq. J4-4 as

$$Q_{uBM} = \phi(0.60 F_u A_{nv})$$
$$= (0.75)\left((0.60)\left(65 \ \frac{\text{kips}}{\text{in}^2}\right)(1.0 \ \text{in})\right)t$$
$$= (29.25 \ \text{kips/in})t$$

The largest effective weld size is given by

$$Q_{uw} = Q_{uBM}$$
$$\left(1.39 \ \frac{\text{kips}}{\text{in}}\right)D = \left(29.25 \ \frac{\text{kips}}{\text{in}}\right)t$$
$$D = (21.01 \ \text{sixteenths})t$$

The corresponding values of D for A36 base material and for welds on both sides of the base material are shown in Table 4.4.

Table 4.4 *Effective Weld Size*

base metal	welds on one side of base	welds on both sides of base
A36	$(18.75 \ \text{sixteenths})t$	$(9.38 \ \text{sixteenths})t$
grade 50	$(21.01 \ \text{sixteenths})t$	$(10.51 \ \text{sixteenths})t$

Example 4.44

For the welded connection analyzed in Ex. 4.43, determine the maximum effective weld size applicable.

Solution

For A36 base metal with welds on one side only of the $^5/_8$ in gusset plate, the maximum effective weld size is obtained from Table 4.4 as

$$w = (18.75 \ \text{sixteenths})t$$
$$= \left(\frac{18.75 \ \text{sixteenths}}{16 \ \text{sixteenths}}\right)(0.625 \ \text{in})$$
$$= 0.73 \ \text{in}$$
$$> \ ^1/_4 \ \text{in} \quad [\text{satisfactory}]$$

Strength of Fillet Weld Groups

AISC 360 Sec. J2.4 describes three methods for determining the strength of fillet weld groups, depending on the configuration of the weld elements.

- For a linear weld group with a uniform leg size loaded through the center of gravity, use AISC 360 Sec. J2.4(a).

- For weld elements within a weld group that are analyzed using the instantaneous center of rotation method, use AISC 360 Sec. J2.4(b).

- For fillet weld groups with concentric loading that are made of elements with a uniform leg size and oriented either longitudinally or transversely to the direction of the applied load use AISC 360 Sec. J2.4(c).

The strength of linear weld groups, in which all the elements are in line or are parallel, may be analyzed by the method specified in AISC 360 Sec. J2.4(a). This method accounts for the angle of inclination of the applied loading to the longitudinal axis of the weld. For an angle of inclination θ, the nominal strength in shear is given by AISC 360 Eq. J2-4 as

$$R_n = F_{nw} A_{we}$$

The nominal strength of the weld metal is given by AISC 360 Eq. J2-5 as

$$F_{nw} = 0.60 F_{\text{EXX}}(1.0 + 0.50 \sin^{1.5}\theta)$$

For the usual situation with $\theta = 0°$, $F_{nw} = 0.60 F_{\text{EXX}}$.

The resistance factor is given by

$$\phi = 0.75$$

The safety factor is given by

$$\Omega = 2.00$$

For concentrically loaded weld groups, with elements oriented both longitudinally or transversely to the direction of the applied load, the strength is determined as specified in AISC 360 Sec. J2.4(c). The combined nominal strength of the weld group is given by the greater of

$$R_n = R_{wl} + R_{wt} \qquad [\textit{AISC J2-10a}]$$
$$R_n = 0.85 R_{wl} + 1.5 R_{wt} \qquad [\textit{AISC J2-10b}]$$

Example 4.45

The two grade A36 plates shown are connected by E70XX fillet welds as indicated. Determine the size of weld required to develop the full available axial strength of the $^5/_8$ in plate.

plate 4 in $\times$ $\frac{1}{2}$ in

4 in

fillet weld

P_r

P_r

plate 3 in $\times$ $\frac{5}{8}$ in
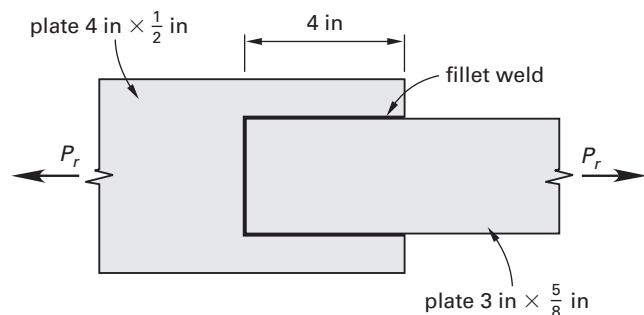
Solution

The total length of the longitudinally loaded weld is

$$l_{wl} = (2)(4 \text{ in})$$
$$= 8 \text{ in}$$

The total length of the transversely loaded weld is

$$l_{wt} = 3 \text{ in}$$

LRFD Method

The design axial strength of the $^5/_8$ in plate is

$$P_u = \phi_t P_n = 0.9 F_y A_g$$
$$= (0.9)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(3 \text{ in})(0.625 \text{ in})$$
$$= 60.75 \text{ kips}$$

The design shear capacity of a $^1/_4$ in fillet weld is

$$Q_w = q_u D$$
$$= \left(1.39 \ \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}\right)(4 \text{ sixteenths})$$
$$= 5.56 \text{ kips/in}$$

Applying AISC 360 Eq. J2-10a, the design strength of the connection is

$$\phi R_n = \phi(R_{wl} + R_{wt}) = l_{wl} Q_w + l_{wt} Q_w$$
$$= (8 \text{ in})\left(5.56 \ \frac{\text{kips}}{\text{in}}\right) + (3 \text{ in})\left(5.56 \ \frac{\text{kips}}{\text{in}}\right)$$
$$= 61.16 \text{ kips}$$

Applying AISC 360 Eq. J2-10b, the design strength of the connection is

$$\phi R_n = \phi(0.85 R_{wl}) + \phi(1.5 R_{wt})$$
$$= 0.85 l_{wl} Q_w + 1.5 l_{wt} Q_w$$
$$= (0.85)(8 \text{ in})\left(5.56 \ \frac{\text{kips}}{\text{in}}\right) + (1.5)(3 \text{ in})\left(5.56 \ \frac{\text{kips}}{\text{in}}\right)$$
$$= 62.83 \text{ kips} \quad \text{[governs]}$$
$$> 61.16 \text{ kips}$$
$$> P_u$$

ASD Method

The allowable axial strength of the $^5/_8$ in plate is

$$P_a = \frac{P_n}{\Omega} = F_y A_g = \frac{\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(3 \text{ in})(0.625 \text{ in})}{1.67}$$
$$= 40.42 \text{ kips}$$

The allowable shear capacity of a $^1/_4$ in fillet weld is

$$Q_w = qD$$
$$= \left(0.928 \ \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}\right)(4 \text{ sixteenths})$$
$$= 3.71 \text{ kips/in}$$

Applying AISC 360 Eq. J2-10a, the allowable strength of the connection is

$$\frac{R_n}{\Omega} = \frac{R_{wl} + R_{wt}}{\Omega} = l_{wl} Q_w + l_{wt} Q_w$$
$$= (8 \text{ in})\left(3.71 \ \frac{\text{kips}}{\text{in}}\right) + (3 \text{ in})\left(3.71 \ \frac{\text{kips}}{\text{in}}\right)$$
$$= 40.81 \text{ kips}$$

Applying AISC 360 Eq. J2-10b, the allowable strength of the connection is

$$\frac{R_n}{\Omega} = \frac{0.85 R_{wl}}{\Omega} + \frac{1.5 R_{wt}}{\Omega}$$
$$= 0.85 l_{wl} Q_w + 1.5 l_{wt} Q_w$$
$$= (0.85)(8 \text{ in})\left(3.71 \ \frac{\text{kips}}{\text{in}}\right) + (1.5)(3 \text{ in})\left(3.71 \ \frac{\text{kips}}{\text{in}}\right)$$
$$= 41.92 \text{ kips} \quad \text{[governs]}$$
$$> 40.81 \text{ kips}$$
$$> P_a$$

Steel

From AISC 360 Table J2.4, the minimum size of fillet weld required for the $1/2$ in plate is

$$w_{\min} = \frac{3}{16} \text{ in}$$

$$< 1/4 \text{ in} \quad \text{[satisfactory]}$$

From AISC 360 Sec. J2.2b, the maximum size of fillet weld permitted at the edge of the $5/8$ in plate is

$$w_{\max} = \frac{5}{8} \text{ in} - \frac{1}{16} \text{ in}$$

$$= 9/16 \text{ in}$$
$$> 1/4 \text{ in} \quad \text{[satisfactory]}$$

A $1/4$ in fillet weld is adequate.

Weld Group Eccentrically Loaded in Plane of Faying Surface

Eccentrically loaded weld groups of the type shown in Fig. 4.29 may be conservatively designed by means of the elastic vector analysis technique assuming unit size of weld. The polar moment of inertia of the weld group about the centroid is

$$I_o = I_x + I_y$$

Figure 4.29 *Eccentrically Loaded Weld Group*
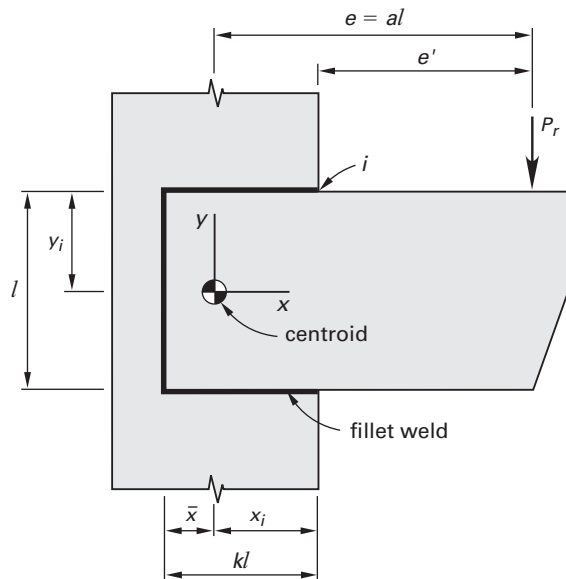
For a total length of weld $\bar{l}$, the vertical force per linear inch of weld due to the applied load, P_r, is

$$V_p = \frac{P_r}{\bar{l}}$$

The vertical force at point i due to the eccentricity, e, is

$$V_e = \frac{P_r e x_i}{I_o}$$

The horizontal force at point i due to the eccentricity, e, is

$$H_e = \frac{P_r e y_i}{I_o}$$

The resultant force at point i is

$$R = \sqrt{(V_p + V_e)^2 + H_e^2}$$

The instantaneous center of rotation method of analyzing eccentrically loaded weld groups affords a more realistic estimate of a weld group's capacity. *AISC Manual* Table 8-4 through Table 8-11a provide a means of designing common weld group patterns by this method.

Example 4.46

Determine the size of E70XX fillet weld required in the welded bracket shown. Use the elastic unit area method and compare with the instantaneous center of rotation method.
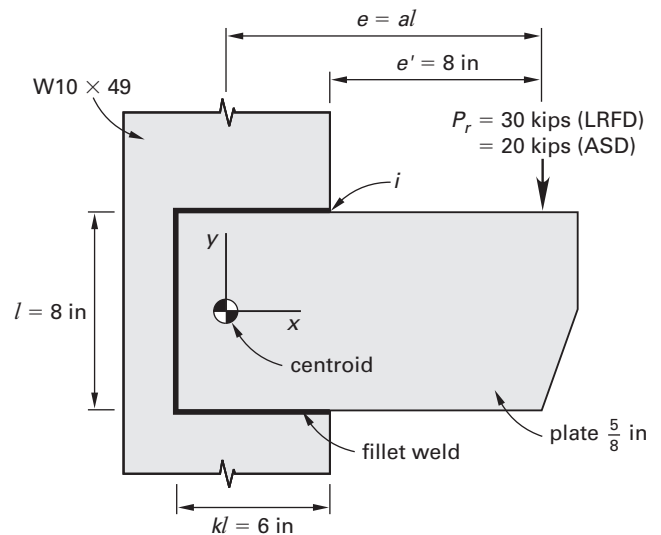
Solution

Assuming unit size of weld, the properties of the weld group are obtained by applying the elastic vector technique. The total length of the weld is

$$\bar{l} = l + 2kl = 8 \text{ in} + (2)(6 \text{ in})$$
$$= 20 \text{ in}$$
$$k = \frac{kl}{l} = \frac{6}{8}$$
$$= 0.75$$

For a value of $k = 0.75$, the centroid location is given by *AISC Manual* Table 8-8 as

$$\overline{x} = xl = (0.225)(8 \text{ in})$$
$$= 1.8 \text{ in}$$

The moment of inertia about the x-axis is

$$I_x = \frac{l^3}{12} + 2(kl)\left(\frac{l}{2}\right)^2$$
$$= \frac{(8 \text{ in})^3}{12} + (2)(6 \text{ in})(4 \text{ in})^2$$
$$= 235 \text{ in}^4/\text{in}$$

The moment of inertia about the y-axis is

$$I_y = \frac{2(kl)^3}{12} + 2(kl)\left(\frac{kl}{2} - \overline{x}\right)^2 + l\overline{x}^2$$
$$= \frac{(2)(6 \text{ in})^3}{12} + (2)(6 \text{ in})(1.2 \text{ in})^2 + (8 \text{ in})(1.8 \text{ in})^2$$
$$= 79 \text{ in}^4/\text{in}$$

The polar moment of inertia is

$$I_o = I_x + I_y$$
$$= 235 \frac{\text{in}^4}{\text{in}} + 79 \frac{\text{in}^4}{\text{in}}$$
$$= 314 \text{ in}^4/\text{in}$$

The eccentricity of the applied load about the centroid of the weld profile is

$$e = e' + kl - \overline{x} = 8 \text{ in} + 6 \text{ in} - 1.8 \text{ in}$$
$$= 12.2 \text{ in}$$
$$a = \frac{e}{l} = \frac{12.2}{8}$$
$$= 1.53$$

The top right corner of the weld profile is the most highly stressed, and the coexistent forces acting at this point in the x-direction and y-direction are as follows.

LRFD Method

- vertical force due to applied load

$$V_p = \frac{P_r}{l} = \frac{30 \text{ kips}}{20 \text{ in}}$$
$$= 1.5 \text{ kips/in}$$

- vertical force due to eccentricity

$$V_e = \frac{P_r e x_i}{I_o}$$
$$= \frac{(30 \text{ kips})(12.2 \text{ in})(4.2 \text{ in})}{314 \frac{\text{in}^4}{\text{in}}}$$
$$= 4.9 \text{ kips/in}$$

- horizontal force due to eccentricity

$$H_e = \frac{P_r e y_i}{I_o}$$
$$= \frac{(30 \text{ kips})(12.2 \text{ in})(4 \text{ in})}{314 \frac{\text{in}^4}{\text{in}}}$$
$$= 4.7 \text{ kips/in}$$

- resultant force

$$R = \sqrt{(V_p + V_e)^2 + H_e^2}$$
$$= \sqrt{\left(1.5 \frac{\text{kips}}{\text{in}} + 4.9 \frac{\text{kips}}{\text{in}}\right)^2 + \left(4.7 \frac{\text{kips}}{\text{in}}\right)^2}$$
$$= 7.9 \text{ kips/in}$$

The required fillet weld size per $^1/_{16}$ in is

$$D = \frac{R}{q_u} = \frac{7.9 \text{ kips}}{1.39 \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}}$$
$$= 5.7 \text{ sixteenths}$$

ASD Method

- vertical force due to applied load

$$V_p = \frac{P_r}{l} = \frac{20 \text{ kips}}{20 \text{ in}}$$
$$= 1.0 \text{ kips/in}$$

- vertical force due to eccentricity

$$V_e = \frac{P_r e x_i}{I_o}$$
$$= \frac{(20 \text{ kips})(12.2 \text{ in})(4.2 \text{ in})}{314 \frac{\text{in}^4}{\text{in}}}$$
$$= 3.3 \text{ kips/in}$$

- horizontal force due to eccentricity

$$H_e = \frac{P_r e y_i}{I_o} = \frac{(20 \text{ kips})(12.2 \text{ in})(4 \text{ in})}{314 \frac{\text{in}^4}{\text{in}}}$$
$$= 3.1 \text{ kips/in}$$

Steel

- resultant force

$$R = \sqrt{(V_p + V_e)^2 + H_e^2}$$

$$= \sqrt{\left(1.0 \ \frac{\text{kips}}{\text{in}} + 3.3 \ \frac{\text{kips}}{\text{in}}\right)^2 + \left(3.1 \ \frac{\text{kips}}{\text{in}}\right)^2}$$

$$= 5.3 \ \text{kips/in}$$

The required fillet weld size per $1/16$ in is

$$D = \frac{R}{q} = \frac{5.3 \ \text{kips}}{0.928 \ \frac{\text{kips}}{\text{in}} \ \text{per} \ \frac{1}{16} \ \text{in}}$$

$$= 5.7 \ \text{sixteenths}$$

Use a weld size of

$$w = \frac{3}{8} \ \text{in}$$

The flange thickness of the W10 × 49 is

$$t_f = 0.560 \ \text{in}$$

From AISC 360 Table J2.4, the minimum size of fillet weld is

$$w_{\min} = \frac{1}{4} \ \text{in}$$

$$< \frac{3}{8} \ \text{in} \quad [\text{satisfactory}]$$

From AISC 360 Sec. J2.2b, the maximum size of fillet weld for the $5/8$ in plate is

$$w_{\max} = \frac{5}{8} \ \text{in} - \frac{1}{16} \ \text{in}$$

$$> \frac{3}{8} \ \text{in} \quad [\text{satisfactory}]$$

From *AISC Manual* Table 8-3, $C_1 = 1.0$. From *AISC Manual* Table 8-8, for values of $a = 1.53$ and $k = 0.75$, the coefficient C is given as 1.59, and the required fillet weld size per $1/16$ in, based on the instantaneous center of rotation method, is as follows.

LRFD Method

$$D = \frac{P_r}{\phi C l C_1} = \frac{30 \ \text{kips}}{(0.75)\left(1.59 \ \frac{\text{kips}}{\text{in}} \ \text{per} \ \frac{1}{16} \ \text{in}\right)(8 \ \text{in})(1.0)}$$

$$= 3.1 \ \text{sixteenths}$$

ASD Method

$$D = \frac{\Omega P_r}{C l C_1} = \frac{(2.00)(20 \ \text{kips})}{\left(1.59 \ \frac{\text{kips}}{\text{in}} \ \text{per} \ \frac{1}{16} \ \text{in}\right)(8 \ \text{in})(1.0)}$$

$$= 3.1 \ \text{sixteenths}$$

Use a weld size of

$$w = \frac{1}{4} \ \text{in}$$

Weld Group Eccentrically Loaded Normal to Faying Surface

Eccentrically loaded weld groups of the type shown in Fig. 4.30 may be conservatively designed by means of the elastic vector analysis technique assuming unit size of weld. For a total length of weld $\bar{l}$, the vertical force per linear inch of weld due to the applied load, P_r, is

$$V_p = \frac{P_r}{\bar{l}} = \frac{P_r}{2l}$$

Figure 4.30 *Weld Group Eccentrically Loaded Normal to Faying Surface*
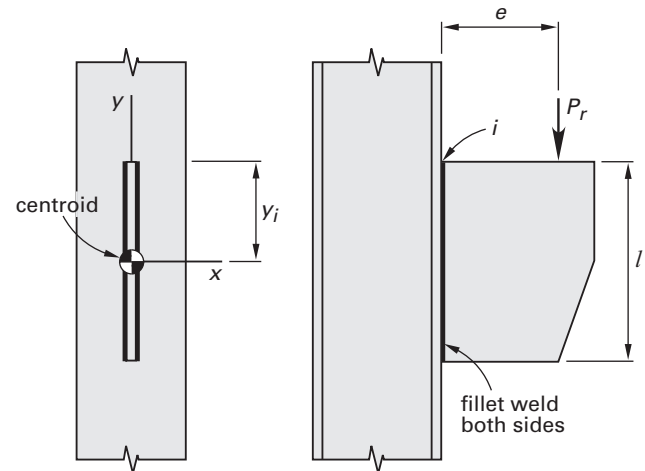
Moment of inertia about the x-axis is

$$I_x = \frac{2l^3}{12} = \frac{l^3}{6}$$

The horizontal force at point i due to the eccentricity, e, is

$$H_e = \frac{P_r e y_i}{I_x} = \frac{3 P_r e}{l^2}$$

The resultant force at point i is

$$R = \sqrt{V_p^2 + H_e^2}$$

The instantaneous center of rotation method of analyzing eccentrically loaded weld groups may also be used to determine a weld group's capacity. *AISC Manual* Table 8-4, with $k = 0$, provides a means of designing weld groups by this method.

Example 4.47

Determine the size of E70XX fillet weld required in the welded gusset plate shown. Use the elastic unit area method and compare with the instantaneous center of rotation method.
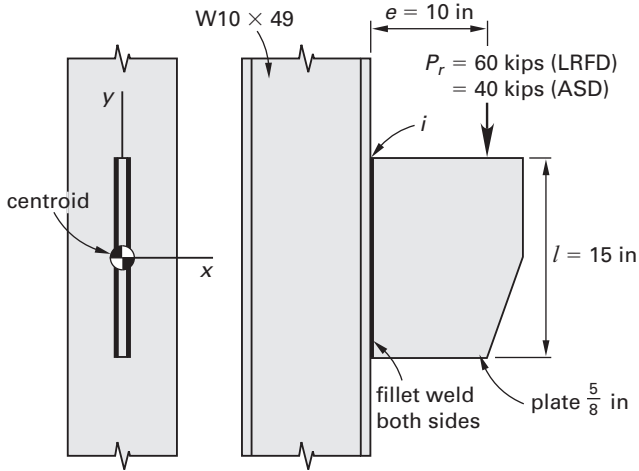
Solution

Assuming unit size of weld, the properties of the weld group are obtained by applying the elastic vector technique. The total length of the weld is

$$\bar{l} = 2l = 30 \text{ in}$$

Moment of inertia about the x-axis is

$$I_x = \frac{2l^3}{12} = \frac{(2)(15 \text{ in})^3}{12} = 563 \text{ in}^3$$

LRFD Method

The vertical force per linear inch of weld due to the applied load, P_r, is

$$V_p = \frac{P_r}{2l} = \frac{60 \text{ kips}}{(2)(15 \text{ in})} = 2.0 \text{ kips/in}$$

The horizontal force at point i due to the eccentricity, e, is

$$H_e = \frac{P_r e y_i}{I_x}$$
$$= \frac{(60 \text{ kips})(10 \text{ in})(7.5 \text{ in})}{563 \text{ in}^3}$$
$$= 8.0 \text{ kips/in}$$

The resultant force at point i is

$$R = \sqrt{V_p^2 + H_e^2} = \sqrt{\left(2.0 \ \frac{\text{kips}}{\text{in}}\right)^2 + \left(8.0 \ \frac{\text{kips}}{\text{in}}\right)^2}$$
$$= 8.2 \text{ kips/in}$$

The required fillet weld size per $^1/_{16}$ in is

$$D = \frac{R}{q_u} = \frac{8.2 \ \dfrac{\text{kips}}{\text{in}}}{1.39 \ \dfrac{\text{kips}}{\text{in}} \text{ per } \dfrac{1}{16} \text{ in}}$$
$$= 5.9 \text{ sixteenths}$$

Use a weld size of

$$w = {}^3/_8 \text{ in}$$

From *AISC Manual* Table 8-3, $C_1 = 1.0$. From *AISC Manual* Table 8-4, for values of $a = 0.67$ and $k = 0$, the coefficient C is given as 1.83, and the required fillet weld size per $^1/_{16}$ in, based on the instantaneous center of rotation method, is

$$D = \frac{P_r}{\phi C l C_1} = \frac{60 \text{ kips}}{(0.75)\left(1.83 \ \dfrac{\text{kips}}{\text{in}} \text{ per } \dfrac{1}{16} \text{ in}\right)(15 \text{ in})(1.0)}$$
$$= 2.9 \text{ sixteenths}$$

Use $w_{\min} = {}^1/_4$ in.

ASD Method

The vertical force per linear inch of weld due to the applied load, P_r, is

$$V_p = \frac{P_r}{2l} = 1.3 \text{ kips/in}$$

The horizontal force at point i due to the eccentricity, e, is

$$H_e = \frac{P_r e y_i}{I_x} = \frac{(40 \text{ kips})(10 \text{ in})(7.5 \text{ in})}{563 \text{ in}^3}$$
$$= 5.3 \text{ kips/in}$$

The resultant force at point i is

$$R = \sqrt{V_p^2 + H_e^2}$$
$$= \sqrt{\left(1.3 \ \frac{\text{kips}}{\text{in}}\right)^2 + \left(5.3 \ \frac{\text{kips}}{\text{in}}\right)^2}$$
$$= 5.5 \text{ kips/in}$$

The required fillet weld size per $^1/_{16}$ in is

$$D = \frac{R}{q} = \frac{5.5 \ \dfrac{\text{kips}}{\text{in}}}{0.928 \ \dfrac{\text{kips}}{\text{in}} \text{ per } \dfrac{1}{16} \text{ in}}$$
$$= 5.9 \text{ sixteenths}$$

Use a weld size of

$$w = {}^3/_8 \text{ in}$$
$$a = \frac{al}{l} = \frac{e}{l} = \frac{10}{15}$$
$$= 0.67$$

From *AISC Manual* Table 8-4, for values of $a = 0.67$ and $k = 0$, the coefficient C is given as 1.83, and the required fillet weld size per $^1/_{16}$ in, based on the instantaneous center of rotation method, is

$$D = \frac{\Omega P_r}{ClC_1}$$
$$= \frac{(2.00)(40 \text{ kips})}{\left(1.83 \dfrac{\text{kips}}{\text{in}} \text{ per } \dfrac{1}{16} \text{ in}\right)(15 \text{ in})(1.0)}$$
$$= 2.9 \text{ sixteenths}$$

Use $w_{\min} = {}^1/_4$ in.

10. PLATE GIRDERS

Nomenclature

a	clear distance between transverse stiffeners	in
a_w	ratio of web area to compression flange area	–
A_f	compression flange area	in^2
A_{pb}	bearing area of stiffener after allowing for corner snip	in^2
A_{sc}	cross-sectional area of a stud shear connector	in^2
A_{st}	area of transverse stiffener	in^2
A_T	area of compression flange plus $^1/_6$ web	in^2
A_w	web area	in^2
b_f	flange width	in
b_{st}	width of transverse stiffener	in
C_b	bending coefficient	–
C_v	shear coefficient for tension field action	–
d	overall depth	in
D_s	factor dependent on the type of transverse stiffener used	–
E_c	modulus of elasticity of concrete	kips/in^2
f'_c	specified compressive strength of concrete	kips/in^2
F_{cr}	critical column axial compression stress	kips/in^2
F_{cr}	critical plate girder compression flange stress	kips/in^2
F_r	compressive residual stress in the flange	kips/in^2
F_u	minimum specified tensile strength of stud shear connector	kips/in^2
F_{uv}	required design shear strength of the stiffener-to-web weld	kips/in
F_{yf}	specified minimum yield stress of flange material	kips/in^2
F_{yst}	specified minimum yield stress of the stiffener material	kips/in^2
F_{yw}	specified minimum yield stress of the web material	kips/in^2
h	clear distance between flanges of a welded plate girder	in

I_{oy}	moment of inertia of flange plus $^1/_6$ web referred to the y-axis	in^4
I_{st}	moment of inertia of transverse stiffener	in^4
I_x	moment of inertia referred to the x-axis	in^4
j	factor used to define moment of inertia of transverse stiffener	–
k_v	web plate buckling coefficient	–
K	effective length factor	–
l	largest unbraced length along either flange at the point of load	in
l	laterally unbraced length of column	in
L_b	unbraced length of compression flange	in or ft
L_p	maximum unbraced length for the limit state of yielding	ft
L_r	maximum unbraced length for the limit state of inelastic lateral-torsional buckling	ft
M_n	nominal flexural capacity	ft-kips
P_u	factored end reaction	kips
q_u	design strength of a $^1/_{16}$ in fillet weld per inch run of E70XX grade electrodes per $^1/_{16}$ in	kips/in per $^1/_{16}$ in
R_g	stud group coefficient	–
R_p	stud position coefficient	–
r_{st}	radius of gyration of bearing stiffener	in
r_t	radius of gyration of compression flange plus $^1/_6$ web referred to the y-axis	in
R_{pg}	plate girder flexural coefficient	–
R_u	nominal bearing strength	kips/in^2
S_x	elastic section modulus referred to the x-axis	in^3
t_f	flange thickness	in
t_{st}	stiffener thickness	in
t_w	web thickness	in
V_n	nominal shear capacity	kips
w	fillet weld size	in
w	unit weight of concrete	lbf/ft^3

Symbols

λ	slenderness parameter	–
λ_p	limiting slenderness parameter for compact element	–
λ_r	limiting slenderness parameter for noncompact element	–
ϕ_b	resistance factor for flexure	–
ϕ_v	resistance factor for shear	–

Girder Proportions

The typical components of a welded, doubly symmetric, nonhybrid plate web girder are shown in Fig. 4.31.

Girder proportions are given in AISC 360 Sec. F5 and Sec. G2 and are summarized as follows.

- Overall girder depth is usually in the range $L/12 < d < L/10$ where L is the span length.

- Assuming the flanges provide all the moment of resistance, flange area is approximately given by

$$A_f \approx \frac{M_u}{\phi_b h F_y} \quad \text{[LRFD]}$$

$$A_f \approx \frac{\Omega M_a}{h F_y} \quad \text{[ASD]}$$

Figure 4.31 *Plate Web Girder*
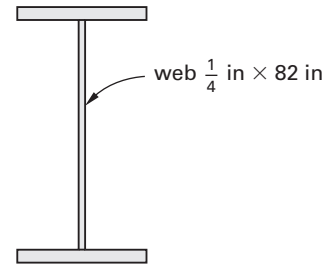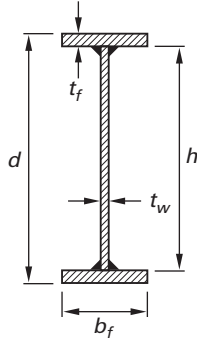
- The flange width is usually in the range $h/5 < b_f < h/3$.

- To ensure a compact compression flange the slenderness parameter is limited by AISC 360 Table B4.1b, Case 10 to

$$\lambda = \frac{b_f}{2t_f} = 0.38\sqrt{\frac{E}{F_y}}$$

- For an unstiffened web, $h/t_w \le 260$ and $ht_w/A_f \le 10$ in accordance with AISC 360 Sec. F13.2.

- In accordance with AISC 360 Sec. G2.2, intermediate stiffeners are not required, provided

$$\frac{h}{t_w} \le 2.46\sqrt{\frac{E}{F_y}}$$

- When intermediate stiffeners are provided and the panel aspect ratio is $a/h > 1.5$, where a is the clear distance between stiffeners, the limiting web depth-to-thickness ratio is

$$\left(\frac{h}{t_w}\right)_{max} = \frac{0.40E}{F_y} \qquad [\textit{AISC F13-4}]$$

- When intermediate stiffeners are provided and $a/h \le 1.5$, the limiting web depth-to-thickness ratio is

$$\frac{h}{t_w} \le 12.0\sqrt{\frac{E}{F_y}} \qquad [\textit{AISC F13-3}]$$

- To utilize tension field action in the web, the panel aspect ratio is limited by AISC 360 Sec. G3.1 to a maximum value given by the lesser of

$$\frac{a}{h} = 3.0$$

$$\frac{a}{h} = \left(\frac{260}{\frac{h}{t_w}}\right)^2$$

Example 4.48

For the welded plate girder of grade A36 steel shown, determine if the web thickness is adequate. The panel aspect ratio is $a/h = 2$ and tension field action is not utilized.

web $\frac{1}{4}$ in $\times$ 82 in

Solution

From the illustration,

$$\frac{h}{t_w} = \frac{82 \text{ in}}{\frac{1}{4}\text{ in}} = 328$$

For an aspect ratio of $a/h = 2$, AISC 360 Eq. F13-4 applies and the maximum permitted web slenderness ratio is

$$\left(\frac{h}{t_w}\right)_{max} = \frac{0.40E}{F_y} = \frac{(0.40)\left(29{,}000 \ \frac{\text{kips}}{\text{in}^2}\right)}{36 \ \frac{\text{kips}}{\text{in}^2}}$$

$$= 322$$

$$< 328$$

The web thickness is not adequate.

Design for Flexure

Doubly symmetric, nonhybrid beams with slender webs and values of $h/t_w > 5.7\sqrt{E/F_y}$, are classified as slender by AISC 360 Table B4.1b, Case 15, and the nominal flexural capacity is given by AISC 360 Sec. F5 as

$$M_n = S_x R_{pg} F_{cr} \qquad [\textit{AISC F5-2}]$$

$$R_{pg} = 1 - \frac{a_w\left(\frac{h}{t_w} - 5.7\sqrt{\frac{E}{F_y}}\right)}{1200 + 300a_w} \qquad [\textit{AISC F5-6}]$$

$$\le 1.0$$

$$a_w \le 10$$

For the limit state of lateral-torsional buckling, the relevant parameters are

$$L_p = 1.1r_t\sqrt{\frac{E}{F_y}} \qquad [\textit{AISC F4-7}]$$

$$L_r = \pi r_t\sqrt{\frac{E}{0.7F_y}} \qquad [\textit{AISC F5-5}]$$

Compression Flange Yielding Governs

For an unbraced length of $L_b \le L_p$, the critical stress is given by

$$F_{cr} = F_y \qquad [\textit{AISC F5-1}]$$

Inelastic Buckling Governs

When $L_p < L_b \leq L_r$, the critical stress is given by

$$F_{cr} = C_b F_y \left(1 - \frac{0.3(L_b - L_p)}{L_r - L_p} \right) \quad [AISC \; F5\text{-}3]$$

$$\leq F_y$$

Elastic Buckling Governs

When $L_b > L_r$, the critical stress is given by

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\dfrac{L_b}{r_t}\right)^2} \leq F_y \quad [AISC \; F5\text{-}4]$$

For the limit state of flange local buckling, the relevant parameters are given by AISC 360 Sec. F5.3 as

$$\lambda = \frac{b_f}{2t_f}$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} \quad [AISC \; Table \; B4.1b, \; Case \; 11]$$

$$= 11 \quad [\text{for } F_y = 36 \text{ kips/in}^2]$$

$$\lambda_r = 0.95\sqrt{\frac{E}{\dfrac{0.7F_y}{k_c}}} \quad \begin{bmatrix} AISC \; Table \; B4.1b, \\ Case \; 11 \end{bmatrix}$$

$$k_c = \frac{4}{\sqrt{\dfrac{h}{t_w}}}$$

$$\geq 0.35$$

$$\leq 0.76$$

Compact Flange

Flange local buckling does not occur in a compact flange with $\lambda \leq \lambda_p$, and the critical compression flange stress is given by AISC 360 Sec. F5.3(a) as

$$F_{cr} = F_y$$

Noncompact Flange

Inelastic local buckling of the flange occurs in a non-compact flange with $\lambda_p < \lambda \leq \lambda_r$, and the critical compression flange stress is given by AISC 360 Sec. F5.3(b) as

$$F_{cr} = F_y \left(1 - \frac{0.3(\lambda - \lambda_p)}{\lambda_r - \lambda_p} \right)$$

Slender Flange

Elastic local buckling of the flange occurs in a slender flange with $\lambda > \lambda_r$, and the critical compression flange stress is given by AISC 360 Sec. F5.3(c) as

$$F_{cr} = \frac{0.9Ek_c}{\lambda^2}$$

The available flexural capacity is given by AISC 360 Sec. F1 as

$$\phi_b M_n = 0.90 M_n \quad [\text{LRFD}]$$

$$\frac{M_n}{\Omega_b} = \frac{M_n}{1.67} \quad [\text{ASD}]$$

Example 4.49

Determine the available flexural capacity for the welded plate girder of grade A36 steel shown. Lateral support to the compression flange is provided at $L_b = 10$ ft centers, and $C_b = 1.0$.

flange 1 in × 18 in
web $\frac{5}{16}$ in × 52 in
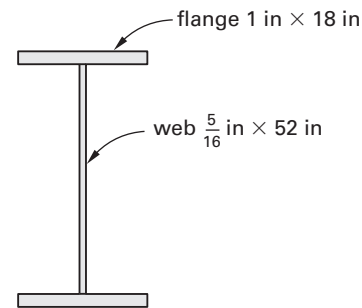
Solution

From the illustration,

$$\frac{h}{t_w} = \frac{52 \text{ in}}{\dfrac{5}{16} \text{ in}} = 166$$

$$< 260 \quad \begin{bmatrix} \text{stiffeners not mandatory} \\ \text{by AISC 360 Sec. F13.2} \end{bmatrix}$$

$$> 5.70\sqrt{\frac{E}{F_y}}$$

The web is classified as slender by AISC 360 Table B4.1b, Case 15, and AISC 360 Sec. F5 applies.

$$ht_w = (52 \text{ in})\left(\frac{5}{16} \text{ in}\right)$$

$$= 16.25 \text{ in}^2$$

$$A_f = b_f t_f = (18 \text{ in})(1.0 \text{ in})$$

$$= 18 \text{ in}^2$$

$$a_w = \frac{ht_w}{A_f} = \frac{16.25 \text{ in}^2}{18 \text{ in}^2}$$

$$= 0.903$$

The moment of inertia of the flange plus $^1\!/_6$ web about the y-axis is

$$I_{oy} = \frac{t_f b_f^3}{12} = \frac{(1.0 \text{ in})(18 \text{ in})^3}{12}$$

$$= 486 \text{ in}^4$$

$$A_T = A_f + \frac{ht_w}{6} = 18 \text{ in}^2 + \frac{16.25 \text{ in}^2}{6}$$

$$= 20.71 \text{ in}^2$$

$$r_t = \sqrt{\frac{I_{oy}}{A_T}} = \sqrt{\frac{486 \text{ in}^4}{20.71 \text{ in}^2}}$$

$$= 4.84 \text{ in}$$

The section modulus referred to the x-axis is

$$S_x = \frac{(2)\left(\begin{array}{c}\left(\frac{1}{12}\right)(18 \text{ in})(54 \text{ in})^3 \\ -\left(\frac{1}{12}\right)\left(18 \text{ in} - \frac{5}{16} \text{ in}\right)(52 \text{ in})^3\end{array}\right)}{54 \text{ in}}$$

$$= 1072 \text{ in}^3$$

For the limit state of lateral-torsional buckling,

$$L_p = 1.1 r_t \sqrt{\frac{E}{F_y}}$$

$$= (1.1)(4.84 \text{ in})\sqrt{\frac{29{,}000 \dfrac{\text{kips}}{\text{in}^2}}{36 \dfrac{\text{kips}}{\text{in}^2}}}$$

$$= 151 \text{ in}$$

$$> 120 \text{ in} \quad [L_b = 120 \text{ in}]$$

Compression flange yielding governs.

$$F_{cr} = F_y$$

For the limit state of flange local buckling,

$$\lambda = \frac{b_f}{2t_f} = \frac{18 \text{ in}}{(2)(1 \text{ in})} \qquad [\textit{AISC Sec. F5.3}]$$

$$= 9$$

$$< \lambda_p = 11$$

Flange local buckling does not occur. Therefore,

$$R_{pg} = 1 - \frac{a_w\left(\dfrac{h}{t_w} - 5.7\sqrt{\dfrac{E}{F_y}}\right)}{1200 + 300 a_w}$$

$$= 1 - \frac{(0.903)\left(166 - 5.7\sqrt{\dfrac{29{,}000 \dfrac{\text{kips}}{\text{in}^2}}{36 \dfrac{\text{kips}}{\text{in}^2}}}\right)}{1200 + (300)(0.903)}$$

$$= 0.997$$

LRFD Method

The design flexural capacity is given by AISC 360 Sec. F5.1 as

$$\phi_b M_n = \phi_b S_x R_{pg} F_{cr}$$

$$= (0.9)(1072 \text{ in}^3)(0.997)\left(36 \frac{\text{kips}}{\text{in}^2}\right)$$

$$= 34{,}629 \text{ in-kips}$$

ASD Method

The allowable flexural capacity is given by AISC 360 Sec. F5.1 as

$$\frac{M_n}{\Omega} = \frac{S_x R_{pg} F_{cr}}{\Omega}$$

$$= \frac{(1072 \text{ in}^3)(0.997)\left(36 \dfrac{\text{kips}}{\text{in}^2}\right)}{1.67}$$

$$= 23{,}040 \text{ in-kips}$$

Providing intermediate transverse stiffeners in a girder increases the elastic buckling strength of the web. When the enhanced elastic critical load is reached, tension field action is induced. Additional load can be carried by the girder through diagonal tension in the web and a corresponding compression in the stiffeners. The top and bottom flanges of the girder act as chords and produce an equivalent Pratt truss as shown in Fig. 4.32.

Figure 4.32 *Tension Field Action*
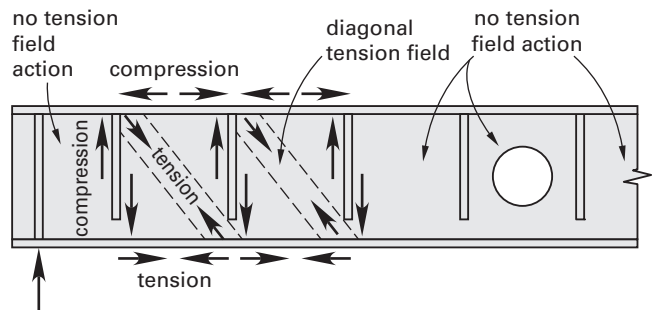
Design using tension field action is not permitted in end panels as a web panel is not available to anchor the tension field. Similarly, design using tension field action is not permitted in a panel with a large hole or in the adjacent panels. Large panel aspect ratios inhibit tension field action and design using tension field action is not permitted when $a/h > 3.0$ or when $a/h > (260 t_w/h)^2$. The design of plate girders for shear may be accomplished by the three following methods.

- *Design using an unstiffened web: AISC Manual* Table 3-16a and Table 3-17a provide values of available shear stress for a range of values of h/t_w and a/h for steel with a yield stress of 36 kips/in^2 and

50 kips/in^2. By entering these tables with a value of 3.0 for a/h the available shear stress in an unstiffened web may be obtained.

- *Design using a stiffened web without utilizing tension field action: AISC Manual* Table 3-16a and Table 3-17a provide values of available shear stress for a range of values of h/t_w and a/h for steel with a yield stress of 36 kips/in^2 and 50 kips/in^2. By entering these tables with the appropriate values of h/t_w and a/h the available shear stress in a stiffened web may be obtained.

- *Design using a stiffened web with tension field action utilized: AISC Manual* Table 3-16b and Table 3-17b provide values of available shear stress for a range of values of h/t_w and a/h for steel with a yield stress of 36 kips/in^2 and 50 kips/in^2. By entering these tables with the appropriate values of h/t_w and a/h the available shear stress may be obtained.

Design for Shear Without Utilizing Tension Field Action

For values of $h/t_w \leq 1.10\sqrt{k_v E/F_y}$, the nominal shear capacity, based on shear yielding of the stiffened or unstiffened web, is given by AISC 360 Sec. G2.1 as

$$V_n = 0.6F_y A_w C_v \qquad \text{[\textit{AISC G2-1}]}$$

$$C_v = 1.0 \qquad \text{[\textit{AISC G2-3}]}$$

For values of $1.10\sqrt{k_v E/F_y} < h/t_w \leq 1.37\sqrt{k_v E/F_y}$, the nominal shear capacity, based on inelastic buckling of the web, is given by AISC 360 Sec. G2.1 as

$$V_n = 0.6F_y A_w \left(\frac{1.10\sqrt{\dfrac{k_v E}{F_y}}}{\dfrac{h}{t_w}} \right)$$

For values of $h/t_w > 1.37\sqrt{k_v E/F_{yw}}$, the nominal shear capacity, based on elastic buckling of the web, is given by AISC 360 Sec. G2.1 as

$$V_n = \frac{A_w(0.91Ek_v)}{\left(\dfrac{h}{t_w}\right)^2}$$

The web plate buckling coefficient is

$$k_v = 5 + \frac{5}{\left(\dfrac{a}{h}\right)^2}$$

$$= 5 \text{ when } a/h > 3$$

$$= 5 \text{ when } a/h > (260t_w/h)^2$$

$$= 5 \text{ for unstiffened webs with } h/t_w < 260$$

AISC Manual Table 3-16a and Table 3-17a provide values of $\phi_v V_n/A_w$ for a range of values of h/t_w and a/h.

Example 4.50

Determine the available shear capacity for the welded plate girder of Ex. 4.48 by using AISC 360 Sec. G2.1. Check the solution by using *AISC Manual* Table 3-16a.

Solution

From Ex. 4.48, for an unstiffened web,

$$A_w = dt_w = (54 \text{ in})(0.313 \text{ in})$$

$$= 16.90 \text{ in}^2$$

$$k_v = 5$$

$$\frac{h}{t_w} = 166$$

$$> 1.37\sqrt{\frac{k_v E}{F_y}}$$

$$V_n = \frac{A_w(0.91Ek_v)}{\left(\dfrac{h}{t_w}\right)^2}$$

$$= \frac{(16.90 \text{ in}^2)(0.91)\left(29{,}000 \, \dfrac{\text{kips}}{\text{in}^2}\right)(5)}{(166)^2}$$

$$= 80.92 \text{ kips}$$

LRFD Method

The design shear capacity is then

$$\phi_v V_n = 0.9 V_n = (0.9)(80.92 \text{ kips})$$

$$= 73 \text{ kips}$$

From *AISC Manual* Table 3-16a, for a value of $a/h > 3.0$ and a value of $h/t_w = 166$,

$$\phi_v V_n = 4.6A_w = \left(4.6 \, \frac{\text{kips}}{\text{in}^2}\right)(16.90 \text{ in}^2)$$

$$= 78 \text{ kips}$$

ASD Method

The allowable shear capacity is then

$$\frac{V_n}{\Omega} = \frac{V_n}{1.67} = \frac{80.92 \text{ kips}}{1.67}$$

$$= 48 \text{ kips}$$

From *AISC Manual* Table 3-16a, for a value of $a/h > 3.0$ and a value of $h/t_w = 166$,

$$\frac{V_n}{\Omega} = 3.1A_w = \left(3.1 \, \frac{\text{kips}}{\text{in}^2}\right)(16.90 \text{ in}^2)$$

$$= 52 \text{ kips}$$

Steel

Design for Shear with Tension Field Action Utilized

When tension field action is used, the nominal shear strength is determined in accordance with AISC 360 Sec. G3.2. *AISC Manual* Table 3-16b and Table 3-17b provide values of $\phi_v V_n/A_w$ and $V_n/(\Omega_b A_w)$ for a range of values of h/t_w and a/h.

Tension field action is not permitted in end panels and when $a/h > 3.0$ or $a/h > (260t_w/h)^2$, in which case, in accordance with AISC 360 Sec. G2.1, the nominal shear strength is given by

$$V_n = 0.6A_w F_y C_v \qquad \text{[AISC G2-1]}$$

Example 4.51

The welded plate web girder of Ex. 4.48 has intermediate stiffeners provided at 100 in centers. Determine the design shear capacity.

Solution

From Ex. 4.48,

$$\frac{h}{t_w} = 166$$

For intermediate stiffeners provided at 100 in centers,

$$\frac{a}{h} = \frac{100 \text{ in}}{52 \text{ in}}$$
$$= 1.92$$

From *AISC Manual* Table 3-16b, for a value of $a/h = 1.92$ and a value of $h/t_w = 166$, the available shear strength is

LRFD Method

$$V_c = \phi_v V_n = 11.5A_w$$
$$= \left(11.5 \ \frac{\text{kips}}{\text{in}^2}\right)(16.90 \text{ in}^2)$$
$$= 194 \text{ kips}$$

ASD Method

$$V_c = \frac{V_n}{\Omega} = 7.7A_w$$
$$= \left(7.7 \ \frac{\text{kips}}{\text{in}^2}\right)(16.90 \text{ in}^2)$$
$$= 130 \text{ kips}$$

Design of Intermediate Stiffeners

Tension Field Action Excluded

The required moment of inertia of a single stiffener about the face in contact with the web plate or of a pair

of stiffener plates about the web centerline is given by AISC 360 Sec. G2.2 as

$$I_{st} = bt_w^3 j \qquad \text{[AISC G2-7]}$$
$$j = \frac{2.5}{\left(\dfrac{a}{h}\right)^2} - 2$$
$$\geq 0.5 \qquad \text{[AISC G2-8]}$$

b is the smaller of the dimensions a and h.

The maximum allowable width-to-thickness ratio of a stiffener plate is given by AISC 360 Table B4.1a, Case 3 as

$$\frac{b_{st}}{t_{st}} = 0.45\sqrt{\frac{E}{F_y}}$$
$$= 12.77 \quad \text{[for } F_y = 36 \text{ kips/in}^2]$$

As specified in AISC 360 Sec. G2.2, the weld used to attach the stiffener to the web must terminate between four times and six times the web thickness from the near toe of the web-to-flange weld.

Tension Field Action Included

The maximum width-to-thickness ratio of a stiffener plate subject to tension field action is given by AISC 360 Eq. G3-3 as

$$\left(\frac{b}{t}\right)_{st} \leq 0.56\sqrt{\frac{E}{F_{yst}}}$$

The minimum required moment of inertia of a single stiffener about the face in contact with the web plate or of a pair of stiffener plates about the web center line is specified by AISC 360 Eq. G3-4 as

$$I_{st} \geq I_{st1} + (I_{st2} - I_{st1})\left(\frac{V_r - V_{c1}}{V_{c2} - V_{c1}}\right)$$

V_{c1} is the smaller of the available shear strengths in the adjacent panels when the nominal shear strength, V_n, is defined by AISC 360 Sec. G2.1. V_{c2} is the smaller of the available shear strengths in the adjacent panels when V_n is defined by AISC 360 Sec. G3.2. V_r is the larger of the required shear strengths in the adjacent panels determined using LRFD or ASD load combinations.

I_{st1} is the moment of inertia as defined in AISC 360 Eq. G2-7.

$$I_{st1} = bt_w^3 j$$

Determine the factor j.

$$j = \frac{2.5}{\left(\dfrac{a}{h}\right)^2} - 2$$
$$\geq 0.5$$

b is the smaller of the dimensions a and h.

I_{st2} is the moment of inertia required for the development of the web shear buckling resistance plus the web tension field resistance.

$$I_{st2} = \frac{h^4 \rho_{st}^{1.3}}{40} \left(\frac{F_{yw}}{E} \right)^{1.5}$$

ρ_{st} is the larger of F_{yw}/F_{yst} and 1.0. F_{yw} is the yield stress of the web material.

Example 4.52

Design the intermediate stiffeners, using a pair of stiffener plates, for the plate web girder of Ex. 4.48, which excludes tension field action. Stiffeners are provided at 100 in centers. The required shear strength is

$$V_r = 120 \text{ kips} \quad \text{[LRFD]}$$
$$V_r = 80 \text{ kips} \quad \text{[ASD]}$$

Solution

For a value of $a/h = 1.92$, the moment of inertia factor is given by AISC 360 Sec. G2.2 as

$$j = \frac{2.5}{\left(\frac{a}{h}\right)^2} - 2 = \frac{2.5}{(1.92)^2} - 2$$
$$= -1.324 \quad \text{[0.5 minimum]}$$
$$b = h = 52 \text{ in}$$

The required moment of inertia of a pair of stiffener plates about the web center line is given by AISC 360 Sec. G2.2 as

$$I_{st} = bt_w^3 j = (52 \text{ in})\left(\frac{5}{16} \text{ in}\right)^3 (0.5)$$
$$= 0.80 \text{ in}^4$$

For a pair of 4 in $\times \, ^3/_8$ in stiffener plates, the width-to-thickness ratio is

$$\frac{b_{st}}{t_{st}} = \frac{4 \text{ in}}{\frac{3}{8} \text{ in}} = 10.7$$
$$< 0.45 \sqrt{\frac{E}{F_y}} \quad \text{[satisfactory]}$$

In accordance with AISC 360 Sec. G2.2, the stiffener may terminate 2 in above the bottom flange.

The moment of inertia provided by the pair of plates is

$$I_{st} = \frac{t_{st}(2b_{st} + t_w)^3}{12} = \frac{\left(\frac{3}{8} \text{ in}\right)\left((2)(4 \text{ in}) + \frac{5}{16} \text{ in}\right)^3}{12}$$
$$= 17.95 \text{ in}^4$$
$$> 0.80 \text{ in}^4 \quad \text{[satisfactory]}$$

Design of Bearing Stiffeners

Bearing stiffeners are required on a plate girder when an applied load exceeds the web's yielding, crippling, or sidesway buckling capacity. The following provisions on the design of bearing stiffeners are stipulated by AISC 360 Sec. J10.8.

- Stiffeners are placed in pairs on opposite sides of the web at the location of the load. (See Fig. 4.33.)

Figure 4.33 Bearing Stiffeners
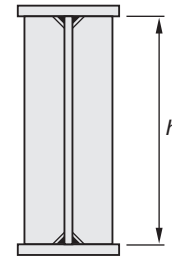
- The column section is composed of the two stiffener plates plus a strip of web having a width of $25t_w$ at interior stiffeners and $12t_w$ at end stiffeners. (See Fig. 4.34.)

Figure 4.34 Stiffener Cross-Section
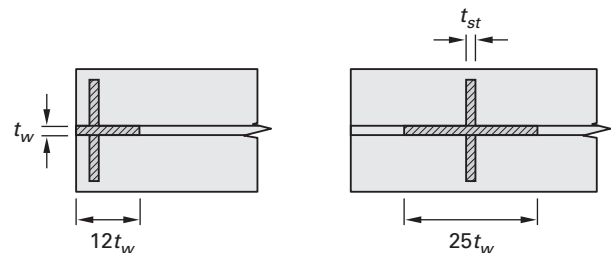
- In accordance with AISC 360 Sec. J10.8, the stiffener is designed as an axially loaded cruciform column.

- The effective length factor of the column is given by AISC 360 Sec. J10.8 as $K = 0.75$ and the slenderness ratio is

$$\frac{KL}{r} = 0.75 \frac{h}{r}$$

When $KL/r \leq 25$, the nominal axial strength is given by AISC 360 Eq. J4-6 as

$$P_n = F_y A_g$$

The available axial strength is

$$\phi P_n = 0.90 P_n \quad \text{[LRFD]}$$
$$\frac{P_n}{\Omega} = \frac{P_n}{1.67} \quad \text{[ASD]}$$

When $KL/r > 25$, the provisions of AISC 360 Chap. E apply.

- Bearing stiffeners must extend the full height of the web and provide close bearing on, or be welded to, the loaded flange.

- Each stiffener plate should extend approximately to the edge of the flanges.

- In accordance with AISC 360 Table B4.1a, Case 3, the limiting width-thickness ratio of each plate is given by

$$\frac{b_{st}}{t_{st}} = 0.45\sqrt{\frac{E}{F_y}}$$

- The nominal bearing strength on the area of the stiffener plate, A_{pb}, in contact with the flange is given by AISC 360 Sec. J7 as

$$R_n = 1.8 F_y A_{pb}$$

- The available bearing strength is

$$\phi R_n = 0.75 R_n \quad \text{[LRFD]}$$

$$\frac{R_n}{\Omega} = \frac{R_n}{2.00} \quad \text{[ASD]}$$

Example 4.53

The welded plate web girder of Ex. 4.48 is provided with bearing stiffeners at each end consisting of a pair of $^1/_2$ in $\times$ 8 in plates of grade A36 steel. Determine the maximum reaction that may be applied to the girder.

Solution

Allowing for a 1 in corner snip to clear the weld, the nominal bearing area of the stiffener plates is

$$A_{pb} = 2t_{st}(b_{st} - 1) = (2)\left(\frac{1}{2}\text{ in}\right)(8\text{ in} - 1\text{ in})$$

$$= 7\text{ in}^2$$

The available bearing strength is given by AISC 360 Sec. J7 as follows.

For LRFD,

$$\phi R_n = \phi(1.8 F_y A_{pb}) = (0.75)(1.8)\left(36\ \frac{\text{kips}}{\text{in}^2}\right)(7\text{ in}^2)$$

$$= 340\text{ kips}$$

For ASD,

$$\frac{R_n}{\Omega} = \frac{1.8 F_y A_{pb}}{\Omega} = \frac{(1.8)\left(36\ \frac{\text{kips}}{\text{in}^2}\right)(7\text{ in}^2)}{2.00}$$

$$= 227\text{ kips}$$

The moment of inertia provided by the pair of plates is

$$I_{st} = \frac{t_{st}(2b_{st} + t_w)^3}{12}$$

$$= \frac{\left(\frac{1}{2}\text{ in}\right)\left((2)(8\text{ in}) + \frac{5}{16}\text{ in}\right)^3}{12}$$

$$= 181\text{ in}^4$$

The effective area of the bearing stiffener is

$$A_{st} = 2t_{st}b_{st} + 12t_w^2$$

$$= (2)\left(\frac{1}{2}\text{ in}\right)(8\text{ in}) + (12)\left(\frac{5}{16}\text{ in}\right)^2$$

$$= 9.18\text{ in}^2$$

The radius of gyration of the bearing stiffener is

$$r_{st} = \sqrt{\frac{I_{st}}{A_{st}}} = \sqrt{\frac{181\text{ in}^4}{9.18\text{ in}^2}} = 4.44\text{ in}$$

The slenderness ratio of the bearing stiffener is

$$\frac{Kl}{r_{st}} = \frac{(0.75)(52\text{ in})}{4.44\text{ in}} = 8.78$$

From AISC 360 Sec. J4.4, the nominal axial strength is

$$P_n = F_y A_g$$

For LRFD, the maximum end reaction is

$$P_u = \phi_c F_y A_{st} = (0.9)\left(36\ \frac{\text{kips}}{\text{in}^2}\right)(9.18\text{ in}^2)$$

$$= 297\text{ kips} \quad \text{[governs]}$$

For ASD, the maximum end reaction is

$$P_a = \frac{F_y A_{st}}{\Omega_c}$$

$$= \frac{\left(36\ \frac{\text{kips}}{\text{in}^2}\right)(9.18\text{ in}^2)}{1.67}$$

$$= 198\text{ kips} \quad \text{[governs]}$$

11. COMPOSITE BEAMS

Nomenclature

a	depth of compression block	in
a	distance between connectors	in
A_c	area of concrete slab within the effective width	in^2
A_s	cross-sectional area of structural steel	in^2
A_{sa}	cross-sectional area of a stud shear connector	in^2
b	effective concrete flange width	in

C_{con}	compressive force in slab at ultimate load	kips
d	depth of steel beam	in
d	diameter of stud shear connector	in
E_c	modulus of elasticity of concrete	kips/in²
$e_{mid\text{-}ht}$	distance from the edge of the stud shank to the steel deck web, measured at mid-height of the deck rib, in the direction of maximum moment for a simply supported beam	–
f'_c	specified compressive strength of the concrete	kips/in²
F_u	minimum specified tensile strength of stud shear connector	kips/in²
F_y	specified minimum yield stress of the structural steel section	kips/in²
h_r	nominal steel deck rib height	in
H_s	length of shear connector, not to exceed $(h_r + 3\text{ in})$ in computations	in
I_{LB}	lower bound moment of inertia	in⁴
L	span length	ft
M_n	nominal flexural strength of member	in-kips or ft-kips
n	number of shear connectors between point of maximum positive moment and point of zero moment	–
N_r	number of studs in one rib at a beam intersection, not to exceed 3 in calculations	–
Q_n	nominal shear strength of single shear connector	kips
R_g	stud group coefficient	–
R_p	stud position coefficient	–
s	beam spacing	ft or in
t_c	actual slab thickness	in
T_{stl}	tensile force in steel at ultimate load	kips
V'	total factored horizontal shear between point of maximum moment and point of zero moment	kips
w	unit weight of concrete	lbf/ft³
w_r	average width of concrete rib	in
y	moment arm between centroids of tensile force and compressive force	in

Y_{con}	distance from top of steel beam to top of concrete	in
Y_1	distance from top of steel beam to plastic neutral axis	in
Y_2	distance from top of steel beam to concrete flange force	in

Symbols

ρ	reduction factor for studs in ribbed steel deck	–
$\sum Q_n$	summation of Q_n between point of maximum moment and point of zero moment on either side	kips

Section Properties

The composite beam shown in Fig. 4.35 consists of a concrete slab supported by a formed metal deck, with the slab acting compositely with a steel beam. In accordance with AISC 360 Sec. I3.1, the effective width of the concrete slab on either side of the beam centerline shall not exceed

- one-eighth of the beam span
- one-half of the beam spacing
- the distance to the edge of the slab

For the composite beam shown in Fig. 4.35, at the ultimate load the depth of the concrete stress block is less than the depth of the slab.[10] For this situation, the plastic neutral axis is located at the top of the steel beam and

$$Y_1 = 0$$

$$Y_2 = Y_{con} - \frac{a}{2}$$

When sufficient shear connectors are provided to ensure full composite action, the depth of the stress block is given by

$$a = \frac{F_y A_s}{0.85 f'_c b}$$

Figure 4.35 *Fully Composite Beam Section Properties*
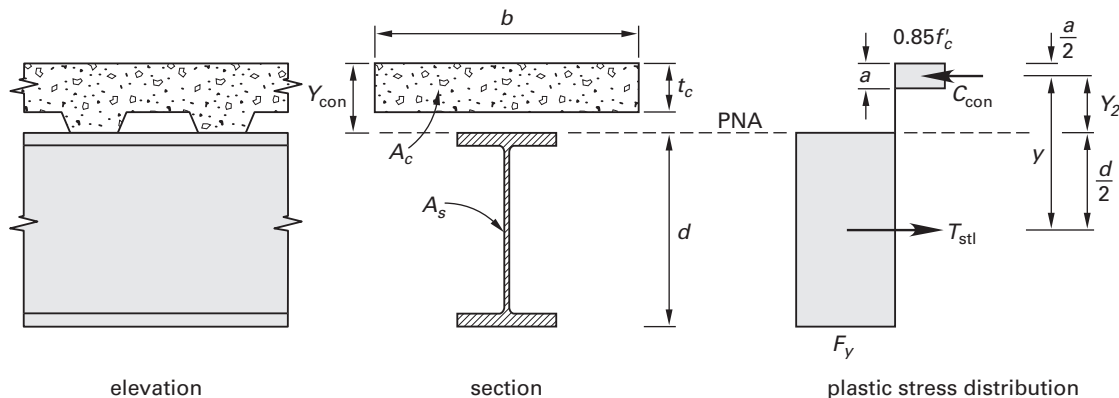
When insufficient shear connectors are provided to ensure full composite action, the depth of the stress block is given by

$$a = \frac{\sum Q_n}{0.85 f'_c b}$$

Using this value of the depth of the stress block to define an equivalent slab depth provides a lower bound on the actual moment of inertia based on elastic principles. *AISC Manual* Table 3-20 provides values of I_{LB} for a range of values of Y_1 and Y_2. This moment of inertia is used to determine the deflection of the composite member.

Example 4.54

A simply supported composite beam consists of a 3 in concrete slab cast on a 3 in formed steel deck over a W21 × 50 grade 50 steel beam. The beams are spaced at 8 ft centers and span 30 ft; the slab consists of 4000 lbf/in^2 normal weight concrete. Determine the lower bound moment of inertia if full composite action is provided.

Solution

The effective width of the concrete slab is the lesser of

$$s = (8 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) = 96 \text{ in}$$

$$\frac{L}{4} = \frac{(30 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{4}$$

$$= 90 \text{ in} \quad [\text{governs}]$$

For full composite action, the depth of the stress block is

$$a = \frac{F_y A_s}{0.85 f'_c b} = \frac{\left(50 \ \frac{\text{kips}}{\text{in}^2}\right)(14.7 \text{ in}^2)}{(0.85)\left(4 \ \frac{\text{kips}}{\text{in}^2}\right)(90 \text{ in})}$$

$$= 2.40 \text{ in} \quad [\text{within the slab}]$$

The distance from the top of the steel beam to the line of action of the concrete slab force is

$$Y_2 = Y_{\text{con}} - \frac{a}{2}$$

$$= 3 \text{ in} + 3 \text{ in} - \frac{2.40 \text{ in}}{2}$$

$$= 4.80 \text{ in}$$

$$Y_1 = 0$$

From *AISC Manual* Table 3-20,

$$I_{LB} = 2686 \text{ in}^4$$

Shear Connection

Shear connectors are provided to transfer the horizontal shear force across the interface. The nominal shear strengths Q_n of different types of shear connectors are given in *AISC Manual* Table 3-21. The required number of connectors may be uniformly distributed between the point of maximum moment and the support on either side, with the total horizontal shear being determined by the lesser value given by AISC 360 Eq. I3-1a and Eq. I3-1b as

$$V' = 0.85 f'_c A_c \qquad [\textit{AISC I3-1a}]$$

$$V' = F_y A_s \qquad [\textit{AISC I3-1b}]$$

To provide complete shear connection and full composite action, the required number of connectors on either side of the point of maximum moment is given by

$$n = \frac{V'}{Q_n}$$

If a smaller number of connectors is provided, only partial composite action can be achieved, and the nominal flexural strength of the composite member is reduced. The number of shear connectors placed between a concentrated load and the nearest support shall be sufficient to develop the moment required at the load point.

The nominal strength of one stud shear connector embedded in a solid slab is given by AISC 360 Eq. I8-1 as

$$Q_n = 0.5 A_{sa} \sqrt{f'_c E_c}$$

$$\leq R_g R_p A_{sa} F_u$$

The minimum tensile strength of a type B shear stud connector made from ASTM A108 material is

$$F_u = 65 \text{ kips/in}^2$$

The stud group coefficient for flat soffit, solid slabs with the stud welded directly to the girder flange is

$$R_g = 1.0$$

The stud position coefficient for flat soffit, solid slabs with the stud welded directly to the girder flange is

$$R_p = 0.75$$

The modulus of elasticity of concrete is given by

$$E_c = w^{1.5} \sqrt{f'_c}$$

The unit weight of normal weight concrete is given by

$$w = 145 \text{ lbf/ft}^3$$

$$Q_n = 0.5 A_{sa} (w f'_c)^{0.75}$$

When the concrete is cast on a formed metal deck, the limitations imposed on the spacing and placement of shear connectors are given in AISC 360 Sec. I3.2c and are summarized in Fig. 4.36.

When the concrete slab is cast on a formed metal deck, the values of R_g and R_p are modified as detailed in AISC 360 Sec. I8.2a.

Deck Ribs Parallel to Steel Beam

When the deck ribs are parallel to the steel beam as shown in Fig. 4.37, the stud group coefficient for any number of studs welded in a row through the steel deck is given by

$$R_g = 1.0 \quad \text{[when } w_r \geq 1.5h_r]$$
$$R_g = 0.85 \quad \text{[when } w_r < 1.5h_r]$$

When the deck ribs are parallel to the steel beam as shown in Fig. 4.37, the stud position coefficient for studs welded through the steel deck is given by

$$R_p = 0.75$$

Deck Ribs Perpendicular to Steel Beam

When the deck ribs are perpendicular to the steel beam as shown in Fig. 4.38, the stud group coefficient for studs welded through the steel deck is given by

$R_g = 1.0$ [for one stud welded in a steel deck rib]

$R_g = 0.85$ [for two studs welded in a steel deck rib]

$R_g = 0.70 \quad \begin{bmatrix} \text{for three or more studs} \\ \text{welded in a steel deck rib} \end{bmatrix}$

When the deck ribs are perpendicular to the steel beam as shown in Fig. 4.39, the stud position coefficient for studs welded through the steel deck is given by

$$R_p = 0.75 \quad \begin{bmatrix} \text{for studs welded in a steel deck rib} \\ \text{with } e_{\text{mid-ht}} \geq 2 \text{ in} \end{bmatrix}$$

$$R_p = 0.60 \quad \begin{bmatrix} \text{for studs welded in a steel deck rib} \\ \text{with } e_{\text{mid-ht}} < 2 \text{ in} \end{bmatrix}$$

The nominal strength of different stud diameters in 3 kips/in^2 and 4 kips/in^2 normal weight and lightweight concrete is given in *AISC Manual* Table 3-21.

Example 4.55

Determine the number of $^3/_4$ in diameter stud shear connectors required in the composite beam of Ex. 4.53 to provide full composite action. The ribs of the formed steel deck are perpendicular to the steel beams with $h_r = 3$ in, $w_r = 3^1/_2$ in, $H_s = 5$ in, and $N_r = 2$. The beam is loaded with a uniformly distributed load.

Solution

The total horizontal shear is given by

$$V' = F_y A_s$$
$$= \left(50 \ \frac{\text{kips}}{\text{in}^2}\right)(14.7 \text{ in}^2)$$
$$= 735 \text{ kips}$$

Two $^3/_4$ in diameter studs are located in each rib in the weak position.

The nominal shear strength of each $^3/_4$ in diameter stud is obtained from *AISC Manual* Table 3-21, as

$$Q_n = 14.6 \text{ kips}$$

Figure 4.36 *Placement of Shear Connectors*
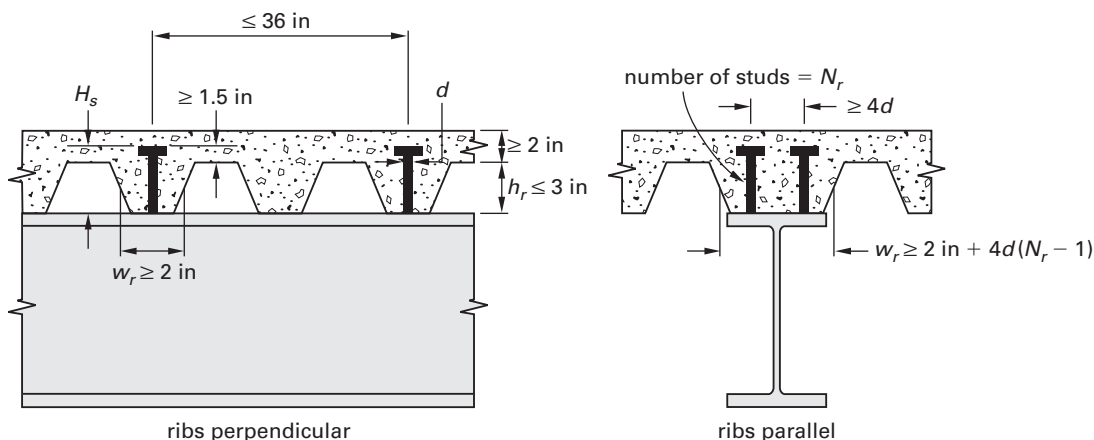
ribs perpendicular   ribs parallel

Figure 4.37 *Deck Ribs Parallel to Steel Beam, R_g and R_p Values*
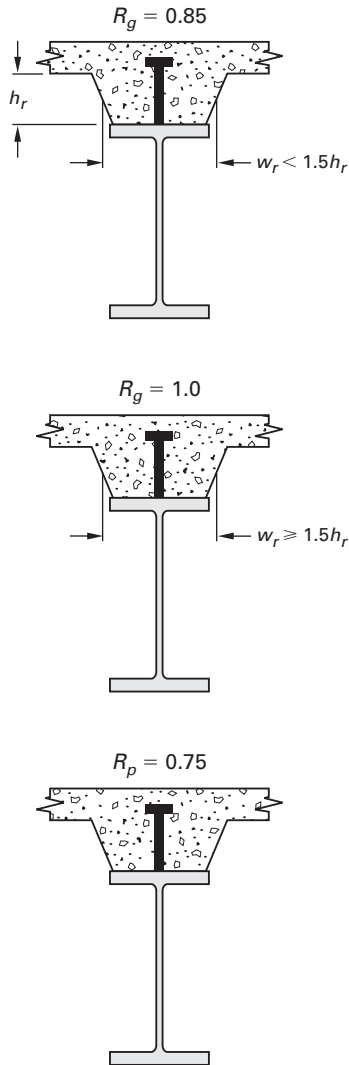
Figure 4.38 *Deck Ribs Perpendicular to Steel Beams, R_g Values*
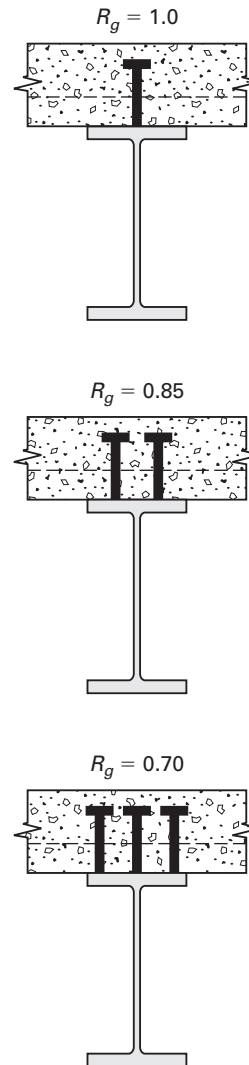
The required number of studs in the beam is

$$2n = \frac{2V'}{Q_n} = \frac{(2)(735 \text{ kips})}{14.6 \text{ kips}}$$
$$= 100 \text{ studs}$$

Figure 4.39 *Deck Ribs Perpendicular to Steel Beams, R_p Values*

Design for Flexure

AISC Manual Table 3-19 provides values of ϕM_n for a range of values of Y_1, Y_2, and $\sum Q_n$. The value of $\sum Q_n$ is given by AISC 360 Sec. I3.2d as the least of

- $0.85 f'_c A_c$
- $F_y A_s$
- nQ_n
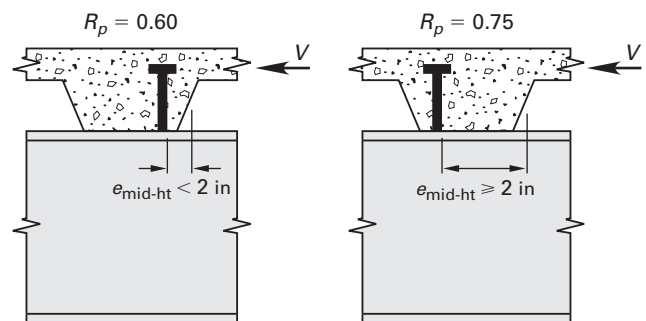
Because of redistribution of stresses at the ultimate load, the composite section is designed to support the total factored loads, due to all dead and live loads, for both shored and unshored construction. In addition, for unshored construction, the steel beam alone must be adequate to support all loads applied before the concrete has attained 75% of its required strength.

Example 4.56

Determine the available flexural strength of the composite beam of Ex. 4.54.

Solution

Because sufficient shear connectors are provided to ensure full composite action,

$$\sum Q_n = F_y A_s = 735 \text{ kips}$$

From Ex. 4.53,

$$Y_2 = 4.80 \text{ in}$$
$$Y_1 = 0$$

From *AISC Manual* Table 3-19, the available strength is

$$\phi M_n = 839 \text{ ft-kips} \quad \text{[LRFD]}$$
$$\frac{M_n}{\Omega} = 558 \text{ ft-kips} \quad \text{[ASD]}$$

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Steel

PRACTICE PROBLEMS

(Answer options for ASD are given in parentheses.)

1. The pair of shear legs shown consists of two nonstandard steel tubes 13 ft long, of 3.5 in outside diameter and 3 in inside diameter, pinned together at the top and inclined to each other at an angle of 45°. The yield stress of the tubes is $F_y = 36$ kips/in². The legs are laterally braced at the top and are pinned at the base. The self-weight of the pipes is neglected. What is most nearly the maximum load the shear legs can lift? (For LRFD, use factored loads.)
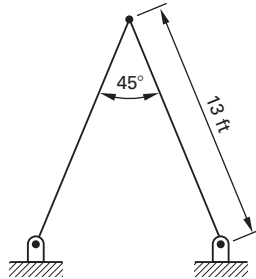
(A) 29 kips (19 kips)

(B) 31 kips (21 kips)

(C) 44 kips (30 kips)

(D) 58 kips (39 kips)

2. Both flanges of a W8 × 24 grade A36 steel section are each connected by six ³⁄₄ in diameter bolts to a steel bracket. A single row of three bolts is provided on each side of the beam web to both flanges, as shown. What is most nearly the capacity of the W section in direct tension?
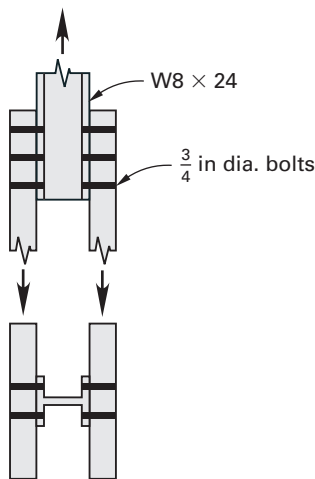
(A) 180 kips (120 kips)

(B) 220 kips (150 kips)

(C) 230 kips (160 kips)

(D) 280 kips (190 kips)

3. A fixed-ended steel beam is shown, with the factored loads indicated. The distributed load shown includes the beam self-weight. Full lateral support is provided to the beam, which is of grade 50 steel. What is the lightest W section beam that can support the factored loads?
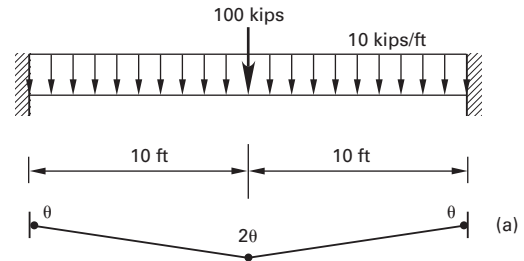
(A) W24 × 55

(B) W24 × 62

(C) W24 × 94

(D) W27 × 94

4. The simply supported composite beam shown consists of a 7¹⁄₂ in normal weight concrete slab cast on a W21 × 57 grade 50 steel beam. The beam forms part of a floor system with the beams spaced at 10 ft centers, and the concrete strength is 3000 lbf/in². The loads are indicated in the illustration; these include the weight of the concrete slab and the self-weight of the steel beam. (LRFD loads are factored.) Seventeen stud shear connectors of ³⁄₄ in diameter are provided between sections 1 and 2. Is the number of connectors adequate, and what is the available flexural strength?
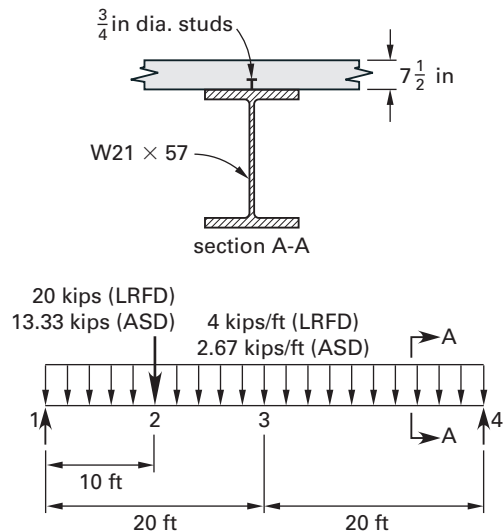
(A) No, $\phi M_n = 750$ ft-kips ($M_n/\Omega = 500$ ft-kips).

(B) No, $\phi M_n = 829$ ft-kips ($M_n/\Omega = 553$ ft-kips).

(C) Yes, $\phi M_n = 750$ ft-kips ($M_n/\Omega = 500$ ft-kips).

(D) Yes, $\phi M_n = 829$ ft-kips ($M_n/\Omega = 553$ ft-kips).

5. Both plates shown are of grade A36 steel, and the plates are connected with E70XX fillet welds. What is most nearly the available block shear capacity of the welded connection?
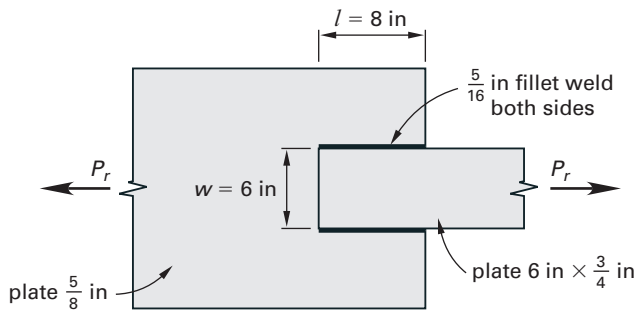
- (A) 330 kips (220 kips)
- (B) 360 kips (240 kips)
- (C) 390 kips (260 kips)
- (D) 420 kips (280 kips)

6. What is most nearly the available tensile capacity of the fillet welds in the welded connection shown in Prob. 5?

- (A) 110 kips (70 kips)
- (B) 180 kips (120 kips)
- (C) 250 kips (160 kips)
- (D) 330 kips (220 kips)

7. What is most nearly the available tensile capacity of the $^3/_4$ in plate in the welded connection shown in Prob. 5?

- (A) 90 kips (60 kips)
- (B) 110 kips (70 kips)
- (C) 150 kips (100 kips)
- (D) 230 kips (160 kips)

SOLUTIONS

1. The area of each pipe is

$$A = \pi(a^2 - b^2)$$
$$= \pi\left((1.75 \text{ in})^2 - (1.5 \text{ in})^2\right)$$
$$= 2.55 \text{ in}^2$$

The radius of gyration of each pipe is

$$r = \frac{\sqrt{a^2 + b^2}}{2} = \frac{\sqrt{(1.75 \text{ in})^2 + (1.5 \text{ in})^2}}{2} = 1.15 \text{ in}$$

The slenderness ratio of each pipe is given by

$$\frac{Kl}{r} = \frac{(1.0)(13 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{1.15 \text{ in}}$$
$$= 135.7$$
$$< 200 \quad \text{[satisfactory]}$$

LRFD Method

From *AISC Manual* Table 4-22, the design axial stress is given by

$$\phi_c F_{cr} = 12.3 \text{ kips/in}^2$$

The design axial strength of each pipe is given by

$$\phi_c P_n = \phi_c F_{cr} A$$
$$= \left(12.3 \frac{\text{kips}}{\text{in}^2}\right)(2.55 \text{ in}^2)$$
$$= 31.37 \text{ kips}$$

The maximum factored load the pair of shear legs can lift is given by

$$P_u = 2\phi_c P_n \cos\phi$$
$$= (2)(31.37 \text{ kips})\cos 22.5°$$
$$= 58 \text{ kips}$$

The answer is (D).

ASD Method

From *AISC Manual* Table 4-22, the allowable axial stress is given by

$$\frac{F_{cr}}{\Omega_c} = 8.2 \text{ kips/in}^2$$

The allowable axial strength of each pipe is given by

$$\frac{P_n}{\Omega_c} = \frac{F_{cr}A}{\Omega_c}$$

$$= \left(8.2 \ \frac{\text{kips}}{\text{in}^2}\right)(2.55 \ \text{in}^2)$$

$$= 20.91 \ \text{kips}$$

The allowable load the pair of shear legs can lift is given by

$$P_a = 2\left(\frac{P_n}{\Omega_c}\right)\cos\phi$$

$$= (2)(20.91 \ \text{kips})\cos 22.5°$$

$$= 38.64 \ \text{kips} \quad (39 \ \text{kips})$$

The answer is (D).

2. The relevant properties of the W8 × 24 are obtained from *AISC Manual* Table 1-1 and are

$$A_g = 7.08 \ \text{in}^2$$

$$b_f = 6.5 \ \text{in}^2$$

$$t_f = 0.40 \ \text{in}$$

$$d = 7.93 \ \text{in}$$

The hole diameter is

$$d_h = d_b + \frac{1}{8} \ \text{in}$$

$$= \frac{3}{4} \ \text{in} + \frac{1}{8} \ \text{in}$$

$$= 0.875 \ \text{in}$$

The ratio is

$$\frac{b_f}{d} = \frac{6.50 \ \text{in}}{7.93 \ \text{in}}$$

$$> 2/3$$

Three bolts are in line in the direction of stress, however, and hence from AISC 360 Sec. D3,

$$U = 0.90$$

The net area is given by

$$A_n = A_g - 4d_h t_f$$

$$= 7.08 \ \text{in}^2 - (4)(0.875 \ \text{in})(0.40 \ \text{in})$$

$$= 5.68 \ \text{in}^2$$

LRFD Method

The design axial strength based on the gross section is

$$\phi_t P_n = 0.9F_y A_g$$

$$= (0.9)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(7.08 \ \text{in}^2)$$

$$= 229 \ \text{kips}$$

The design axial strength based on the net section is

$$\phi_t P_n = 0.75 U F_u A_n$$

$$= (0.75)(0.90)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(5.68 \ \text{in}^2)$$

$$= 222 \ \text{kips} \quad (220 \ \text{kips}) \quad [\text{governs}]$$

The answer is (B).

ASD Method

The allowable axial strength based on the gross section is

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{1.67}$$

$$= \frac{\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(7.08 \ \text{in}^2)}{1.67}$$

$$= 152.62 \ \text{kips}$$

The allowable axial strength based on the net section is

$$\frac{P_n}{\Omega_t} = \frac{U F_u A_n}{2.00}$$

$$= \frac{(0.90)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(5.68 \ \text{in}^2)}{2.00}$$

$$= 148 \ \text{kips} \quad (150 \ \text{kips}) \quad [\text{governs}]$$

The answer is (B).

3. From the collapse mechanism shown in part (a) of the illustration, the required plastic moment of resistance is given by the equation

$$4M_p = (100 \ \text{kips})(10 \ \text{ft}) + \frac{(200 \ \text{kips})(10 \ \text{ft})}{2}$$

$$M_p = \frac{2000 \ \text{ft-kips}}{4}$$

$$= 500 \ \text{ft-kips}$$

From *AISC Manual* Table 3-2, a W24 × 55 has

$$\phi M_p = 503 \text{ ft-kips}$$

$$> 500 \text{ ft-kips} \quad [\text{satisfactory}]$$

The answer is (A).

4. *LRFD Method*

The factored bending moment at section 2 due to distributed load is

$$M_w = \frac{0.75 w L^2}{8} = \frac{(0.75)\left(4 \dfrac{\text{kips}}{\text{ft}}\right)(40 \text{ ft})^2}{8}$$

$$= 600 \text{ ft-kips}$$

Due to point load,

$$M_W = \frac{W a b}{L} = \frac{(20 \text{ kips})(10 \text{ ft})(30 \text{ ft})}{40 \text{ ft}}$$

$$= 150 \text{ ft-kips}$$

The total moment, M_2, is

$$M_w = M_w + M_W$$

$$= 600 \text{ ft-kips} + 150 \text{ ft-kips}$$

$$= 750 \text{ ft-kips}$$

The effective width of the concrete slab is the lesser of

- $s = (10 \text{ ft})\left(12 \dfrac{\text{in}}{\text{ft}}\right)$

  $= 120 \text{ in}$

- $\dfrac{L}{4} = \dfrac{(40 \text{ ft})\left(12 \dfrac{\text{in}}{\text{ft}}\right)}{4}$

  $= 120 \text{ in} \quad [\text{governs}]$

The nominal shear strength of a $^3/_4$ in diameter stud shear connector in 3000 lbf/in^2 concrete is obtained from *AISC Manual* Table 3-21 as

$$Q_n = 21.0 \text{ kips}$$

The total horizontal shear transferred between sections 1 and 2 is given by

$$V_h = \sum Q_n$$

$$= n Q_n$$

$$= (17 \text{ studs})(21.0 \text{ kips})$$

$$= 357 \text{ kips}$$

$$< F_y A_s$$

For this value of ΣQ_n, the plastic neutral axis lies below the top of the W21 × 57 at a distance, obtained from *AISC Manual* Table 3-19, of

$$Y_1 = 1.84 \text{ in}$$

The depth of the concrete stress block is given by

$$a = \frac{\sum Q_n}{0.85 f'_c b}$$

$$= \frac{357 \text{ kips}}{(0.85)\left(3 \dfrac{\text{kips}}{\text{in}^2}\right)(120 \text{ in})}$$

$$= 1.17 \text{ in}$$

The distance between the top of the steel beam and the centroid of the concrete slab force is

$$Y_2 = Y_{\text{con}} - \frac{a}{2}$$

$$= 7.5 \text{ in} - \frac{1.17 \text{ in}}{2}$$

$$= 6.92 \text{ in}$$

For these values of Y_1 and Y_2, the design flexural strength at section 2 is obtained from *AISC Manual* Table 3-19 as

$$\phi M_n = 829 \text{ ft-kips}$$

$$> 750 \text{ ft-kips} \quad [\text{satisfactory}]$$

The shear connectors provided are adequate.

The answer is (D).

ASD Method

The factored bending moment at section 2 due to distributed load is

$$M_w = \frac{0.75 w L^2}{8} = \frac{(0.75)\left(2.67 \dfrac{\text{kips}}{\text{ft}}\right)(40 \text{ ft})^2}{8}$$

$$= 400 \text{ ft-kips}$$

Due to point load,

$$M_W = \frac{W a b}{L} = \frac{(13.33 \text{ kips})(10 \text{ ft})(30 \text{ ft})}{40 \text{ ft}}$$

$$= 100 \text{ ft-kips}$$

The total moment, M_2, is

$$M_w = M_w + M_W$$

$$= 400 \text{ ft-kips} + 100 \text{ ft-kips}$$

$$= 500 \text{ ft-kips}$$

Steel

The effective width of the concrete slab is the lesser of

- $s = (10 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)$

  $= 120 \text{ in}$

- $\dfrac{L}{4} = \dfrac{(40 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{4}$

  $= 120 \text{ in}$ [governs]

The nominal shear strength of a $^3\!/_4$ in diameter stud shear connector in 3000 lbf/in^2 concrete is obtained from *AISC Manual* Table 3-21 as

$$Q_n = 21.0 \text{ kips}$$

The total horizontal shear transferred between sections 1 and 2 is given by

$$\begin{aligned} V_h &= \sum Q_n \\ &= nQ_n \\ &= (17 \text{ studs})(21.0 \text{ kips}) \\ &= 357 \text{ kips} \\ &< F_y A_s \end{aligned}$$

For this value of ΣQ_n, the plastic neutral axis lies below the top of the W21 $\times$ 57 at a distance, obtained from *AISC Manual* Table 3-19, of

$$Y_1 = 1.84 \text{ in}$$

The depth of the concrete stress block is given by

$$\begin{aligned} a &= \frac{\sum Q_n}{0.85 f'_c b} \\ &= \frac{357 \text{ kips}}{(0.85)\left(3 \ \frac{\text{kips}}{\text{in}^2}\right)(120 \text{ in})} \\ &= 1.17 \text{ in} \end{aligned}$$

The distance between the top of the steel beam and the centroid of the concrete slab force is

$$\begin{aligned} Y_2 &= Y_{\text{con}} - \frac{a}{2} \\ &= 7.5 \text{ in} - \frac{1.17 \text{ in}}{2} \\ &= 6.92 \text{ in} \end{aligned}$$

For these values of Y_1 and Y_2, the allowable flexural strength at section 2 is obtained from *AISC Manual* Table 3-19 as

$$\begin{aligned} \frac{M_n}{\Omega} &= 553 \text{ ft-kips} \\ &> 500 \text{ ft-kips} \quad \text{[satisfactory]} \end{aligned}$$

The shear connectors provided are adequate.

The answer is (D).

5. From the illustration, the gross shear area is

$$\begin{aligned} A_{gv} &= (2)(8 \text{ in})(0.625 \text{ in}) \\ &= 10 \text{ in}^2 \\ &= A_{nv} \end{aligned}$$

From the illustration, the net tension area is

$$\begin{aligned} A_{nt} &= (6 \text{ in})(0.625 \text{ in}) \\ &= 3.75 \text{ in}^2 \\ &= A_{gt} \end{aligned}$$

The tensile stress is uniform and the reduction coefficient is

$$U_{bs} = 1.0$$

The rupture strength in tension is given by

$$\begin{aligned} U_{bs} F_u A_{nt} &= (1.0)\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(3.75 \text{ in}^2) \\ &= 218 \text{ kips} \end{aligned}$$

The yield strength in shear is given by

$$\begin{aligned} 0.6 F_y A_{gv} &= (0.6)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(10 \text{ in}^2) \\ &= 216 \text{ kips} \\ &< 0.6 F_u A_{nv} \end{aligned}$$

LRFD Method

Shear yielding governs and the resistance to block shear is given by AISC 360 Eq. J4-5 as

$$\begin{aligned} \phi R_n &= \phi(0.6 F_y A_{gv} + U_{bs} F_u A_{nt}) \\ &= (0.75)(216 \text{ kips} + 218 \text{ kips}) \\ &= 326 \text{ kips} \quad (330 \text{ kips}) \end{aligned}$$

The answer is (A).

Steel

ASD Method

Shear yielding governs and the allowable strength for block shear is given by AISC 360 Eq. J4-5 as

$$\frac{R_n}{\Omega} = \frac{0.6F_y A_{gv} + U_{bs}F_u A_{nt}}{\Omega}$$
$$= \frac{216 \text{ kips} + 218 \text{ kips}}{2.00}$$
$$= 217 \text{ kips} \quad (220 \text{ kips})$$

The answer is (A).

6. The total length of longitudinally loaded weld is

$$l_{wl} = (2)(8 \text{ in})$$
$$= 16 \text{ in}$$

For a linear weld group with $\theta = 0°$,

$$F_w = 0.60 F_{\text{EXX}}$$
$$R_n = F_w A_w$$

LRFD Method

The design shear capacity of a $^5/_{16}$ in fillet weld is

$$Q_w = q_u D$$
$$= \left(1.39 \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}\right)(5 \text{ sixteenths})$$
$$= 6.95 \text{ kips/in}$$

Applying AISC 360 Eq. J2-4, the design strength of the weld is

$$\phi R_n = \phi R_{wl} = l_{wl} Q_w$$
$$= (16 \text{ in})\left(6.95 \frac{\text{kips}}{\text{in}}\right)$$
$$= 111 \text{ kips} \quad (110 \text{ kips})$$

The answer is (A).

ASD Method

The allowable shear capacity of a $^5/_{16}$ in fillet weld is

$$Q_w = q_u D$$
$$= \left(0.928 \frac{\text{kips}}{\text{in}} \text{ per } \frac{1}{16} \text{ in}\right)(5 \text{ sixteenths})$$
$$= 4.64 \text{ kips/in}$$

Applying AISC 360 Eq. J2-4, the allowable strength of the weld is

$$\frac{R_n}{\Omega} = \frac{R_{wl}}{\Omega} = l_{wl} Q_w$$
$$= (16 \text{ in})\left(4.64 \frac{\text{kips}}{\text{in}}\right)$$
$$= 74 \text{ kips} \quad (70 \text{ kips})$$

The answer is (A).

7. The width of the $^3/_4$ in plate is

$$w = 6 \text{ in}$$

The length of the weld is

$$l = 8 \text{ in}$$

Find the ratio of the length to the width.

$$\frac{l}{w} = \frac{8 \text{ in}}{6 \text{ in}}$$
$$= 1.33$$

From AISC 360 Table D3.1, Case 4, the reduction coefficient is

$$U = 0.75$$

The gross area of the $^3/_4$ in plate is given by

$$A_g = wt$$
$$= (6 \text{ in})(0.75 \text{ in})$$
$$= 4.5 \text{ in}^2$$
$$= A_n$$

The effective net area is given by AISC 360 Eq. D3-1 as

$$A_e = A_n U$$
$$= (4.5 \text{ in}^2)(0.75)$$
$$= 3.38 \text{ in}^2$$

LRFD Method

The corresponding design tensile capacity for tensile rupture is given by AISC 360 Sec. D2 as

$$\phi_t P_n = \phi_t F_u A_e$$
$$= (0.75)\left(58 \frac{\text{kips}}{\text{in}^2}\right)(3.38 \text{ in}^2)$$
$$= 147 \text{ kips}$$

Steel

The design tensile capacity for yielding of the gross section is given by AISC 360 Sec. D2 as

$$\phi_t P_n = \phi_t F_y A_g$$
$$= (0.9)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(4.5 \ \text{in}^2)$$
$$= 146 \ \text{kips} \quad (150 \ \text{kips}) \quad \text{[governs]}$$

The answer is (C).

ASD Method

The corresponding allowable tensile capacity for tensile rupture is given by AISC 360 Sec. D2 as

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t}$$
$$= \frac{\left(58 \ \frac{\text{kips}}{\text{in}^2}\right)(3.38 \ \text{in}^2)}{2.00}$$
$$= 98 \ \text{kips}$$

The allowable tensile capacity for yielding of the gross section is given by AISC 360 Sec. D2 as

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{\Omega_t}$$
$$= \frac{\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)(4.5 \ \text{in}^2)}{1.67}$$
$$= 97 \ \text{kips} \quad (100 \ \text{kips}) \quad \text{[governs]}$$

The answer is (C).

5 Timber Design

H	load due to lateral pressure	kips/ft^2
L	live load	kips or kips/ft
L_r	roof live load	kips or kips/ft
Q	load effect due to service load	kips
R	rain load	kips or kips/ft
S	snow load	kips or kips/ft
W	wind load	kips or kips/ft

Symbols

ϕ	resistance factor	–
γ	load factor	–

1. ASD AND LRFD METHODS

In accordance with the *National Design Specification* (NDS) *for Wood Construction*[1] Sec. 1.4, wood structures must be designed using either the provisions for load and resistance factor design (LRFD) or the provisions for allowable stress design (ASD).

The ASD method is the traditional method of designing wood structures. Working loads, also referred to as nominal or service loads, are the actual dead loads plus the live loads applied to a member. The ASD method, based on elastic theory, is used to calculate the stresses produced by the factored working loads. Factored working loads, using ASD load combinations, are applied to the member; the stresses produced in the member must not exceed the specified allowable stress. The allowable stress is determined by applying adjustment factors to the reference design values tabulated in the *NDS Supplement*[2]. The application of load duration factors to the reference design values allow for the effects of transient loads, such as wind and seismic loads.

In the LRFD method, working loads are factored using LRFD load combinations before being applied to the member to determine the required strength. This required strength must not exceed the design strength, which is calculated as the member nominal strength multiplied by a resistance factor, ϕ.

2. LOAD COMBINATIONS

Nomenclature

D	dead load	kips or kips/ft
E	earthquake load	kips or kips/ft

ASD Required Strength

The required strength of a member consists of the most critical combination of factored loads applied to the member. In accordance with NDS Sec. 1.4.4, load combinations must be as specified in the applicable building code. The required strength, $\Sigma \gamma Q$, is defined by nine combinations given in IBC[3] Sec. 1605.3.1. The seismic and wind loads specified in the IBC are at the strength design level in contrast to other loads, which are at the service level. In ASD load combinations, the load factor for seismic loads is 0.7, and the load factor for wind loads is 0.6 to reduce the factored loads to service level values. The combinations, with uncommon load conditions (self-straining loads and fluid pressure) omitted, are as follows.

$$\Sigma \gamma Q = D \qquad [\textit{IBC 16-8}]$$

$$\Sigma \gamma Q = D + H + L \qquad [\textit{IBC 16-9}]$$

$$\Sigma \gamma Q = D + H + L(L_r \text{ or } S \text{ or } R) \qquad [\textit{IBC 16-10}]$$

$$\Sigma \gamma Q = D + H + 0.75L + 0.75(L_r \text{ or } S \text{ or } R) \qquad [\textit{IBC 16-11}]$$

$$\Sigma \gamma Q = D + H + (0.6W \text{ or } 0.7E) \qquad [\textit{IBC 16-12}]$$

$$\Sigma \gamma Q = D + H + 0.75(0.6W) + 0.75L \\ + 0.75(L_r \text{ or } S \text{ or } R) \qquad [\textit{IBC 16-13}]$$

$$\Sigma \gamma Q = D + H + 0.75(0.7E) + 0.75L \\ + 0.75S \qquad [\textit{IBC 16-14}]$$

$$\Sigma \gamma Q = 0.6D + 0.6W + H \qquad [\textit{IBC 16-15}]$$

$$\Sigma \gamma Q = 0.6D + 0.7E + H \qquad [\textit{IBC 16-16}]$$

Timber

LRFD Required Strength

The required ultimate strength of a member consists of the most critical combination of factored loads applied to the member. The required strength, $\Sigma \gamma Q$, is defined by seven combinations given in IBC Sec. 1605.2. The combinations, with uncommon load conditions (self-straining loads and fluid pressure) omitted, are as follows.

$$\Sigma \gamma Q = 1.4D \qquad \qquad \text{[IBC 16-1]}$$

$$\Sigma \gamma Q = 1.2D + 1.6(L + H) \qquad \text{[IBC 16-2]}$$
$$+ \, 0.5(L_r \text{ or } S \text{ or } R)$$

$$\Sigma \gamma Q = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) \qquad \text{[IBC 16-3]}$$
$$+ \, 1.6H + (f_1 L \text{ or } 0.5W)$$

$$\Sigma \gamma Q = 1.2D + 1.0W + f_1 L \qquad \text{[IBC 16-4]}$$
$$+ \, 1.6H + 0.5(L_r \text{ or } S \text{ or } R)$$

$$\Sigma \gamma Q = 1.2D + 1.0E + f_1 L + 1.6H + f_2 S \quad \text{[IBC 16-5]}$$

$$\Sigma \gamma Q = 0.9D + 1.0W + 1.6H \qquad \text{[IBC 16-6]}$$

$$\Sigma \gamma Q = 0.9D + 1.0E + 1.6H \qquad \text{[IBC 16-7]}$$

For IBC Eq. 16-3, 16-4, and 16-5, use $f_1 = 1.0$ for garages, places of public assembly, and areas where $L > 100 \text{ lbf/ft}^2$. Use $f_1 = 0.5$ for all other live loads. For IBC Eq. 16-5, use $f_2 = 0.7$ for roof configurations that do not shed snow, and use 0.2 for other roof configurations.

3. DEFINITIONS AND TERMINOLOGY

A description of the wood products available, and of the terminology used, is as follows.

Decking consists of solid sawn lumber or glued laminated members with 2–4 in nominal thickness, and 4 in or more wide. For 2 in thicknesses, it is usually single tongue and groove, and for 3–4 in thicknesses, it may be double tongue and groove.

Dimension lumber consists of solid sawn lumber members with 2–4 in nominal thickness, and 2 in or more wide.

Dressed size refers to the dimensions of a lumber member after it has been surfaced with a planing machine. It is usually $^1/_2$–$^3/_4$ in less than nominal size.

Grade indicates the classification wood products are given with respect to strength in accordance with specific grading rules.

A *joist* is a lumber member with 2–4 in nominal thickness, and 5 in or wider. A joist is typically loaded on the narrow face and used as framing in floors and roofs.

Lumber is cut to size in the sawmill and surfaced in a planing machine, and it is not further processed.

Mechanically graded lumber is dimension lumber that has been individually evaluated in a testing machine. Load is

applied to the piece of lumber, the deflection is measured, and the modulus of elasticity is calculated. The strength characteristics of the lumber are directly related to the modulus of elasticity and can be determined. A visual check is also made on the lumber to detect visible flaws.

Nominal size is the term used to specify the undressed size of a lumber member. The finished size of a member after dressing is normally $^1/_2$–$^3/_4$ in smaller than the original size. Thus, a 2 in nominal $\times$ 4 in nominal member has actual dimensions of $1^1/_2$ in $\times$ $3^1/_2$ in.

Structural glued laminated timber, or *glulams*, are built up from wood laminations bonded together with adhesives. The grain of all laminations is parallel to the length of the beam, and the laminations are typically $1^1/_2$ in thick.

Timbers are lumber members of nominal 5 in $\times$ 5 in or larger.

Visually stress-graded lumber are lumber members that have been graded visually to detect flaws and defects, and to assess the inherent strength of the member.

Wood structural panels are manufactured from veneers or wood strands bonded together with adhesives. Examples are plywood, oriented strand board, and composite panels.

4. REFERENCE DESIGN VALUES

Nomenclature

b	width of rectangular bending member	in
c	column parameter	–
C_b	bearing area factor	–
C_c	curvature factor for structural glued laminated member	–
C_D	load duration factor	–
C_F	size factor for sawn lumber	–
C_{fu}	flat use factor	–
C_H	shear stress adjustment factor	–
C_i	incising factor	–
C_I	stress interaction factor	–
C_L	beam stability factor	–
C_M	wet service factor	–
C_P	column stability factor	–
C_r	repetitive member factor for dimension lumber	–
C_t	temperature factor	–
C_T	buckling stiffness factor	–
C_V	volume factor for structural glued laminated timber	–
C_{vr}	shear reduction factor	–
d	least depth dimension of rectangular compression member	in
E, E'	reference and adjusted modulus of elasticity	lbf/in^2
$E_{\min}, E'_{\min}$	reference and adjusted modulus of elasticity for beam stability and column stability calculations	lbf/in^2

Timber

F	ratio of F_{bE} to F_b^*	–
F'	ratio of F_{cE} to F_c^*	–
F_b^*	tabulated bending design value multiplied by all applicable adjustment factors except C_{fu}, C_V, and C_L	lbf/in^2
F_b, F_b'	reference and adjusted design value	lbf/in^2
F_{bE}	critical buckling design value for bending member	lbf/in^2
F_c^*	tabulated compressive design value multiplied by all applicable adjustment factors except C_P	lbf/in^2
F_c, F_c'	reference and adjusted design value parallel to grain	lbf/in^2
F_{cE}	critical buckling design value	lbf/in^2
K_F	format conversion factor	–
l_b	length of bearing parallel to the grain of the wood	in
l_e	effective length of compression member	ft or in
l_e	effective span length of bending member	ft or in
l_u	laterally unsupported length of beam	ft or in
L	span length of bending member	ft or in
R	radius of curvature of inside face of lamination	in
R_B	slenderness ratio of bending member	–
t	thickness of lamination	in
x	species parameter for volume factor	–

Symbols

λ	time effect factor	–
ϕ	resistance factor	–

NDS Supplement

The *NDS Supplement* provides reference design values for various wood species and for several types of wood products.

NDS Supplement table	reference design values for wood
Table 4A	visually graded dimension lumber, all species except southern pine
Table 4B	visually graded southern pine dimension lumber
Table 4C	mechanically graded dimension lumber
Table 4D	visually graded timbers
Table 4E	visually graded decking
Table 4F	non-North American visually graded dimension lumber
Table 5A	structural glued laminated softwood timber, stressed primarily in bending
Table 5B	structural glued laminated softwood timber, stressed primarily in axial tension or compression
Table 5C	structural glued laminated hardwood timber, stressed primarily in bending
Table 5D	structural glued laminated hardwood timber, stressed primarily in axial tension or compression

The allowable, or adjusted, design values for a wood member depend on the application of the member and on the service conditions under which it is utilized. The reference design values are applicable to normal dry use conditions of service and normal load duration as specified in NDS Sec. 2.2. To determine the relevant adjusted values for other conditions of service, the reference design values are multiplied by adjustment factors specified in NDS Sec. 2.3. The applicability of each adjustment factor to the reference design values for sawn lumber or glued laminated timber is given in NDS Table 4.3.1 and Table 5.3.1.

5. ADJUSTMENT OF REFERENCE DESIGN VALUES

Adjusted design values for wood members and connections must be appropriate for the conditions under which the wood is used. These values take into account the differences in wood strength properties with different moisture content, load durations, and treatment types. Reference design values are for normal load durations under the moisture service conditions specified in NDS Sec. 2.2. The applicability of adjustment factors to the reference design values for sawn lumber and glued laminated timber is summarized in Table 5.1.

Three additional factors are used solely in the LRFD method. These are the

- format conversion factor, K_F
- resistance factor, ϕ
- time effect factor, λ

Adjusted design values are obtained by multiplying reference design values by the applicable adjustment factors in accordance with the equations given.

Sawn Lumber

Adjusted design values for sawn lumber are as follows.

- bending

$$F_b' = F_b C_F C_r C_i C_D C_M C_t C_{fu} C_L \quad \text{[ASD]}$$

$$F_b' = F_b C_F C_r C_i C_M C_t C_{fu} C_L K_F \lambda \phi_b \quad \text{[LRFD]}$$

- tension

$$F_t' = F_t C_F C_i C_D C_M C_t \quad \text{[ASD]}$$

$$F_t' = F_t C_F C_i C_M C_t K_F \lambda \phi_t \quad \text{[LRFD]}$$

- shear

$$F_v' = F_v C_i C_D C_M C_t \quad \text{[ASD]}$$

$$F_v' = F_v C_i C_M C_t K_F \lambda \phi_v \quad \text{[LRFD]}$$

- compression parallel to the grain

$$F_c' = F_c C_F C_i C_D C_M C_t C_P \quad \text{[ASD]}$$

$$F_c' = F_c C_F C_i C_M C_t C_P K_F \lambda \phi_c \quad \text{[LRFD]}$$

Timber

Table 5.1 *Applicability of Adjustment Factors*

adjustment factor	sawn lumber	glued laminated	F_b	F_t	F_v	$F_{c\perp}$	F_c	E	E_{min}
C_F size	yes	no	√	√	–	–	√	–	–
C_r repetitive member	yes	no	√	–	–	–	–	–	–
C_I incising	yes	no	√	√	√	√	√	√	√
C_V volume*	no	yes	√	–	–	–	–	–	–
C_c curvature	no	yes	√	–	–	–	–	–	–
C_D load duration	yes	yes	√	√	√	–	√	–	–
C_M wet service	yes	yes	√	√	√	√	√	√	√
C_b bearing area	yes	yes	–	–	–	√	–	–	–
C_t temperature	yes	yes	√	√	√	√	√	√	√
C_{fu} flat use	yes	yes	√	–	–	–	–	–	–
C_T buckling	yes	no	–	–	–	–	–	–	√
C_L beam stability*	yes	yes	√	–	–	–	–	–	–
C_P column stability	yes	yes	–	–	–	–	√	–	–
C_I stress interaction	no	yes	√	–	–	–	–	–	–
C_{vr} shear reduction	no	yes	–	–	√	–	–	–	–

*When applied to glued laminated members, only the lesser value of C_L or C_V is applicable. C_D is applicable to the ASD method only.

- compression perpendicular to the grain

$$F'_{c\perp} = F_{c\perp} C_b C_i C_M C_t \quad [\text{ASD}]$$
$$F'_{c\perp} = F_{c\perp} C_b C_i C_M C_t K_F \lambda \phi_c \quad [\text{LRFD}]$$

The adjusted modulus of elasticity for sawn lumber is

$$E' = E C_i C_M C_t \quad [\text{ASD}]$$
$$E' = E C_i C_M C_t \quad [\text{LRFD}]$$

The adjusted modulus of elasticity for beam and column stability calculations for sawn timber is

$$E'_{min} = E_{min} C_i C_M C_t C_T \quad [\text{ASD}]$$
$$E'_{min} = E_{min} C_i C_M C_t C_T K_F \phi_s \quad [\text{LRFD}]$$

Glued Laminated Timber

Adjusted design values for glued laminated timber are as follows.

- bending

$$F'_b = F_b C_c C_I C_D C_M C_t C_{fu} (C_L \text{ or } C_V) \quad [\text{ASD}]$$
$$F'_b = F_b C_c C_I C_M C_t C_{fu} (C_L \text{ or } C_V) K_F \lambda \phi_b \quad [\text{LRFD}]$$

- tension

$$F'_t = F_t C_D C_M C_t \quad [\text{ASD}]$$
$$F'_t = F_t C_M C_t K_F \lambda \phi_t \quad [\text{LRFD}]$$

- shear

$$F'_v = F_v C_{vr} C_D C_M C_t \quad [\text{ASD}]$$
$$F'_v = F_v C_{vr} C_M C_t K_F \lambda \phi_v \quad [\text{LRFD}]$$

- compression parallel to the grain

$$F'_c = F_c C_D C_M C_t C_P \quad [\text{ASD}]$$
$$F'_c = F_c C_M C_t C_P K_F \lambda \phi_c \quad [\text{LRFD}]$$

- compression perpendicular to the grain

$$F'_{c\perp} = F_{c\perp} C_b C_M C_t \quad [\text{ASD}]$$
$$F'_{c\perp} = F_{c\perp} C_b C_M C_t K_F \lambda \phi_c \quad [\text{LRFD}]$$

The adjusted modulus of elasticity for glued laminated timber is

$$E' = E C_M C_t \quad [\text{ASD}]$$
$$E' = E C_M C_t \quad [\text{LRFD}]$$

The adjusted modulus of elasticity for beam and column stability calculations for glued laminated timber is

$$E'_{min} = E_{min} C_M C_t \quad [\text{ASD}]$$
$$E'_{min} = E_{min} C_M C_t K_F \phi_s \quad [\text{LRFD}]$$

6. ADJUSTMENT FACTORS

Adjustment Factors Applicable to Sawn Lumber and Glued Laminated Members

Format Conversion Factor, K_F (LRFD Method)

The format conversion factor converts ASD reference design values to LRFD reference resistances. Values of K_F are specified in NDS Table 2.3.5 and are given in Table 5.2.

Table 5.2 Format Conversion Factor, K_F

property	F_b	F_t	F_v	$F_{c\perp}$	F_c	E	$E_{\min}$
format conversion factor, $K_F{}^*$	2.54	2.70	2.88	1.67	2.40	1.00	1.76

*For connections (all design) values, $K_F = 3.32$.

Resistance Factor, ϕ (LRFD Method)

The resistance factor converts nominal values to design values. Values of ϕ are specified in NDS Table 2.3.6 and are given in Table 5.3.

Table 5.3 Resistance Factor, ϕ

property	F_b	F_t	F_v	$F_{c\perp}$	F_c	E	$E_{\min}$
resistance factor, ϕ^*	0.85	0.80	0.75	0.90	0.90	1.0	0.85

*For connections (all design) values, $\phi = 0.65$.

Time-Effect Factor, λ (LRFD Method)

The time-effect factor provides higher allowable stresses for short term transient loads. Values of λ are specified NDS Table N.3.3 for the load combinations indicated and are given in Table 5.4.

Table 5.4 Time-Effect Factor, λ

load combination	λ
$1.4D$	0.6
$1.2D+1.6(L+H)$ $+0.5(L_r \text{ or } S \text{ or } R)$	0.7 when L is from storage
	0.8 when L is from occupancy
	1.25 when L is from impact
$1.2D+1.6(L_r \text{ or } S \text{ or } R)+1.6H$ $+(0.5L^a \text{ or } 0.5W)$	0.8
$1.2D+1.0W+0.5L^a+1.6H$ $+0.5(L_r \text{ or } S \text{ or } R)$	1.0
$1.2D+1.0E+0.5L^a+0.2S^b+1.6H$	1.0
$0.9D+1.0W+1.6H$	1.0
$0.8D+1.0E+1.6H$	1.0

Note: Where the effect of H resists the primary variable load effect, a load factor of 0.9 must be included with H where H is permanent. H must be set to zero for all other conditions.
[a]Replace $0.5L$ with $1.0L$ for garages, places of public assembly, and areas where $L > 100$ lbf/ft^2.
[b]Replace $0.2S$ with $0.7S$ for roof configurations that do not shed snow.

Load Duration Factor, C_D (ASD Method)

The load duration factor, C_D, is applicable to all reference design values with the exception of compression perpendicular to the grain and modulus of elasticity. Values of the load duration factor are given in Table 5.5.

In a combination of loads, the load duration factor for the shortest duration load applies for that combination.

Table 5.5 Load Duration Factors

design load	C_D
dead load	0.90
occupancy live load	1.00
snow load	1.15
construction load	1.25
wind or earthquake load	1.60
impact load	2.00

Wet Service Factor, C_M

When the moisture content of sawn lumber exceeds 19%, the adjustment factors given in NDS Supp. Table 4A and Table 4B are applicable to visually graded dimension lumber; in NDS Supp. Table 4C to machine-graded dimension lumber; in NDS Supp. Table 4D to visually graded timbers; in NDS Supp. Table 4E to decking; and in NDS Supp. Table 4F to non-North American visually graded dimension lumber. When the moisture content of glued laminated members exceeds 16%, the adjustment factors given in NDS Supp. Table 5A through Table 5D are applicable.

The applicability of the wet service factor to the reference design values for sawn lumber and glued laminated timber is summarized in Table 5.6.

Table 5.6 Wet Service Factor, C_M

classification	F_b	F_t	F_v	$F_{c\perp}$	F_c	E	$E_{\min}$
sawn lumber < 5 $\times$ member	0.85^a	1.00	0.97	0.67	0.8^b	0.90	0.90
sawn lumber ≥ 5 $\times$ member	1.00	1.00	1.00	0.67	0.91	1.00	1.00
decking	0.85^a	–	–	0.67	–	0.90	0.90
glued laminated member	0.80	0.80	0.875	0.53	0.73	0.833	0.833

[a]When $F_bC_P \leq 1150$ lbf/in^2, $C_M = 1.00$.
[b]When $F_bC_P \leq 750$ lbf/in^2, $C_M = 1.00$.

For sawn lumber, C_M is applicable when the moisture content exceeds 19%. For glued laminated members, C_M is applicable when the moisture content exceeds 16%.

Example 5.1

A 2×10 visually graded, select structural, southern pine, sawn lumber member's moisture content exceeds 19%. The governing load combination is the sum of the dead load, the live load, and the wind load. Determine the allowable shear capacity (ASD) and the factored shear capacity (LRFD) of the member.

Solution

The reference design value for shear stress from NDS Supp. Table 4B is

$$F_v = 175 \text{ lbf/in}^2$$

The applicable wet service factor for shear stress from Table 5.6 is

$$C_M = 0.97$$

Timber

The temperature factor and the incising factor are

$$C_t = 1.0$$
$$C_i = 1.0$$

ASD Method

The applicable load duration factor for a load combination including the wind load from Table 5.5 is

$$C_D = 1.60$$

The adjusted allowable design value in shear is

$$\begin{aligned}
F'_v &= F_v C_i C_D C_M C_t \\
&= \left(175 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.60)(0.97)(1.0) \\
&= 271.60 \ \text{lbf/in}^2
\end{aligned}$$

By rearranging NDS Eq. 3.4-2, the allowable shear capacity of the member is

$$\begin{aligned}
V &= \frac{2bdF'_v}{3} = \frac{(2)(1.5 \ \text{in})(9.25 \ \text{in})\left(271.60 \ \frac{\text{lbf}}{\text{in}^2}\right)}{3} \\
&= 2512 \ \text{lbf}
\end{aligned}$$

LRFD Method

From Table 5.4, the applicable time effect factor for IBC Eq. 16-4 is

$$\lambda = 1.0$$

From Table 5.3, the applicable resistance factor for shear is

$$\phi_v = 0.75$$

From Table 5.2, the applicable format conversion factor for shear is

$$K_F = 2.88$$

The adjusted factored design value in shear is

$$\begin{aligned}
F'_v &= F_v C_i C_M C_t K_F \lambda \phi_v \\
&= \left(175 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(0.97)(1.0)(2.88)(1.0)(0.75) \\
&= 366.66 \ \text{lbf/in}^2
\end{aligned}$$

By rearranging NDS Eq. 3.4-2, the factored shear capacity of the member is

$$\begin{aligned}
V &= \frac{2bdF'_v}{3} = \frac{(2)(1.5 \ \text{in})(9.25 \ \text{in})\left(366.66 \ \frac{\text{lbf}}{\text{in}^2}\right)}{3} \\
&= 3392 \ \text{lbf}
\end{aligned}$$

Bearing Area Factor, C_b

For bearings less than 6 in long and not less than 3 in from the end of a member, the reference design values for compression perpendicular to the grain are modified by the adjustment factor C_b. This is specified in NDS Sec. 3.10.4 as

$$C_b = \frac{l_b + 0.375}{l_b} \qquad \text{[\textit{NDS 3.10-2}]}$$

For round bearing areas, the bearing length, l_b, equals the diameter.

Beam Stability Factor, C_L

The beam stability factor is applicable to the reference bending design value for sawn lumber and glued laminated members. For glued laminated members, C_L is not applied simultaneously with the volume factor C_V, and the lesser of these two factors is applicable. The beam stability factor is given by NDS Sec. 3.3.3 as

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}} \qquad \text{[\textit{NDS 3.3-6}]}$$

The variables are defined as

$$F = \frac{F_{bE}}{F_b^*}$$

$F_b^* = $ reference bending design value multiplied

by all applicable adjustment factors

except C_{fu}, C_V, and C_L

$$= F_b C_D C_M C_t C_i C_F C_r C_c \quad \text{[ASD]}$$
$$= F_b C_M C_i C_t C_F C_r C_c K_F \lambda \phi_b \quad \text{[LRFD]}$$

$$\begin{bmatrix}
C_F \text{ applies to visually graded sawn} \\
\text{lumber only, } C_r \text{ applies to dimension} \\
\text{lumber only, } C_c \text{ applies to curved glued} \\
\text{laminated members only, and } C_i \text{ applies to} \\
\text{sawn lumber only.}
\end{bmatrix}$$

$F_{bE} = $ critical buckling design value

$$= \frac{1.20 E'_{\min}}{R_B^2}$$

$E'_{\min}$ = adjusted modulus of elasticity for stability calculations

$$= E_{\min} C_M C_t C_i C_T \quad \text{[ASD]}$$

$$= E_{\min} C_M C_T C_t C_i K_F \lambda \phi_s \quad \text{[LRFD]}$$

$$[C_i \text{ and } C_T \text{ apply to sawn lumber only.}]$$

R_B = slenderness ratio

$$= \sqrt{\frac{l_e d}{b^2}} \le 50 \qquad \text{[NDS 3.3-5]}$$

As specified in NDS Sec. 3.3.3, $C_L = 1$ when the depth of the beam does not exceed its breadth, or when continuous lateral restraint is provided to the compression edge of a beam with the ends restrained against rotation.

The effective span length, l_e, is determined in accordance with NDS Table 3.3.3. The value of l_e depends on the loading configuration and the distance between lateral restraints, l_u. Typical values for l_e are given in Fig. 5.1.

In accordance with NDS Sec. 4.4.1, $C_L = 1.0$ when, based on nominal dimensions, any of the following conditions are met,

- $d/b \le 2$
- $2 < d/b \le 4$ and full depth bracing is provided at the ends of the member
- $4 < d/b \le 5$ and the compression edge is continuously restrained
- $5 < d/b \le 6$ and the compression edge is continuously restrained with full depth bracing provided at a maximum of 8 ft centers
- $6 < d/b \le 7$ and both edges are continuously restrained

Column Stability Factor, C_P

The column stability factor is applicable to the reference compression design values parallel to the grain, and it is specified by NDS Sec. 3.7.1 as

$$C_p = \frac{1.0 + F'}{2c} - \sqrt{\left(\frac{1.0 + F'}{2c}\right)^2 - \frac{F'}{c}}$$

$$\text{[NDS 3.7-1]}$$

The variables are defined as

$$F' = \frac{F_{cE}}{F_c^*}$$

F_c^* = reference compression design value multiplied by all applicable adjustment factors except C_P

$$= F_c C_D C_M C_t C_i C_F \quad \text{[ASD]}$$

$$= F_c C_M C_t C_i C_F K_F \lambda \phi_c \quad \text{[LRFD]}$$

$$\left[\begin{array}{l} C_F \text{ applies to visually graded sawn} \\ \text{lumber only, and } C_i \text{ applies to sawn} \\ \text{lumber only} \end{array}\right]$$

F_c = reference compression design value parallel to grain

F_{cE} = critical buckling design value

$$= \frac{0.822 E'_{\min}}{\left(\frac{l_e}{d}\right)^2}$$

$E'_{\min}$ = adjusted modulus of elasticity for stability calculations

$$= E_{\min} C_M C_T C_i C_t \quad \text{[ASD]}$$

$$= E_{\min} C_M C_T C_t C_i K_F \lambda \phi_s \quad \text{[LRFD]}$$

$$[C_i \text{ and } C_T \text{ apply to sawn lumber only.}]$$

$E_{\min}$ = reference modulus of elasticity

c = column parameter

$$= 0.8 \quad \text{[for sawn lumber]}$$

$$= 0.9 \quad \text{[for glued laminated timber]}$$

$$= 0.85 \quad \text{[for round timber poles and piles]}$$

Temperature Factor, C_t

The temperature factor is applicable to all reference design values for members exposed to sustained temperatures up to 150°F and is specified by NDS Sec. 2.3.3.

Values for the temperature factor are given in Table 5.7.

Example 5.2

A glued laminated beam of combination 24F-V10 western species with 1.5 in thick laminations has a width of $6^3/4$ in and a depth of 30 in. The beam has a moisture content exceeding 16% and is subjected to sustained temperatures between 100°F and 125°F. The governing load combination is dead load plus live load plus wind load. Determine the allowable design value for the modulus of elasticity about the x-x axis.

Table 5.7 Temperature Factor, C_t

reference design values	in-service moisture conditions	temperature factor, C_t		
		$T \le 100°F$	$100°F < T \le 120°F$	$125°F < T \le 150°F$
F_t, E, $E_{\min}$	wet or dry	1.0	0.9	0.9
F_b, F_v, $F_{c\perp}$, F_c	dry	1.0	0.8	0.7
	wet	1.0	0.7	0.5

Figure 5.1 *Typical Values of Effective Length, l_e*

$1.63l_u + 3d \quad [l_u/d \geq 7]$
$2.06l_u \quad [l_u/d < 7]$

$1.63l_u + 3d \quad [l_u/d \geq 7]$
$2.06l_u \quad [l_u/d < 7]$

$1.37l_u + 3d \quad [l_u/d \geq 7]$
$1.8l_u \quad [l_u/d < 7]$

$1.11l_u$

$1.68l_u$

$1.54l_u$

$0.9l_u + 3d \quad [l_u/d \geq 7]$
$1.33l_u \quad [l_u/d < 7]$

$1.44l_u + 3d \quad [l_u/d \geq 7]$
$1.87l_u \quad [l_u/d < 7]$

Timber
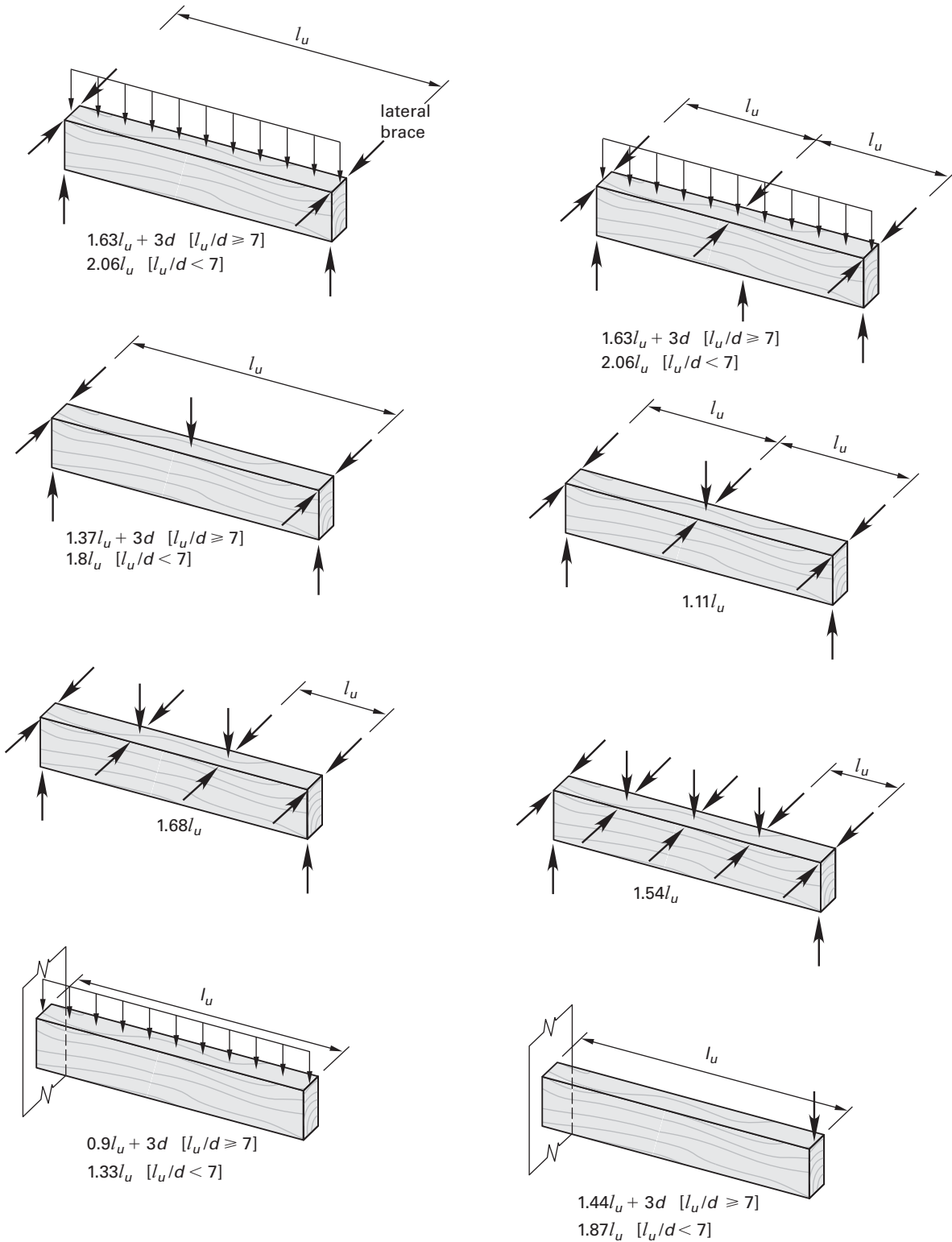
Solution

The reference design value for the modulus of elasticity about the x-x axis is tabulated in NDS Supp. Table 5A as

$$E_x = 1.8 \times 10^6 \text{ lbf/in}^2$$

The applicable adjustment factors for the modulus of elasticity are C_M, for wet service conditions, and C_t, for elevated temperature in wet conditions.

$$C_M = 0.833 \qquad [\textit{NDS Supp. Table 5A}]$$
$$C_t = 0.9 \qquad [\textit{NDS Supp. Table 2.3.3}]$$

The adjusted modulus of elasticity is

$$\begin{aligned}
E'_x &= E_x C_M C_t \\
&= \left(1.8 \times 10^6 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.833)(0.9) \\
&= 1.3 \times 10^6 \text{ lbf/in}^2
\end{aligned}$$

Flat Use Factor, C_{fu}

When wood members are loaded flatwise, flat use adjustment factors are applied to the bending stress. The adjustment factors given in NDS Supp. Table 4A and Table 4B are applicable to visually graded dimension lumber, in NDS Supp. Table 4C to machine-graded dimension lumber, and in NDS Supp. Table 5A through Table 5D to glued laminated members.

The reference design values for visually graded decking given in NDS Supp. Table 4E already incorporate the applicable flat use factor.

Adjustment Factors Applicable to Sawn Lumber

Size Factor, C_F

The size factor is applicable to visually graded dimension lumber, visually graded timbers, and visually graded decking. It is not applied to mechanically graded dimension lumber. The reference design values for bending, tension, and compression are multiplied by the size factor, C_F, to give the appropriate design values. For visually graded dimension lumber, 2 in to 4 in thick, values of the size factor are given in NDS Supp. Table 4A, Table 4B, and Table 4F. For visually graded decking, values of the size factor are given in NDS. For visually graded timbers exceeding 12 in depth and 5 in thickness, the size factor is specified in NDS Sec. 4.3.6 as

$$C_F = \left(\frac{12}{d}\right)^{1/9} \qquad [\textit{NDS 4.3-1}]$$
$$\leq 1.0$$

Repetitive Member Factor, C_r

The repetitive member factor is applicable to visually graded dimension lumber and mechanically graded dimension lumber. It is not applied to visually graded timbers. The design values for visually graded decking in NDS Supp. Table 4E already incorporate the applicable repetitive member factor. The reference design value for bending is multiplied by the repetitive member factor, C_r, when three or more sawn lumber elements, not more than 4 in thick and spaced not more than 24 in apart, are joined by a transverse load distributing element. The value of the repetitive member factor is given in NDS Supp. Table 4A to Table 4C, and Table 4F, and in NDS Sec. 4.3.9, as

$$C_r = 1.15$$

Incising Factor, C_i

Values of the incising factor, C_i, for a prescribed incising pattern are provided in NDS Sec. 4.3.8. These values are applicable to all reference design values for all sawn lumber. The prescribed incising pattern consists of incisions made parallel to the grain at a maximum depth of 0.4 in, a maximum length of $3/8$ in, and at a density of $1100/\text{ft}^2$.

Values of the incising factor are given in Table 5.8.

Table 5.8 Incising Factor, C_i

design value	C_i
E, $E_{\min}$	0.95
F_b, F_t, F_c, F_v	0.80
$F_{c\perp}$	1.00

Buckling Stiffness Factor, C_T

The buckling stiffness factor applies to 2×4 and smaller compression chords in trusses with $3/8$ in or thicker plywood nailed to the narrow face of the chord. Multiply $E_{\min}$ by the buckling stiffness factor to account for the increased chord stiffness. The effective chord length is represented by l_e.

NDS Eq. 4.4-1 defines the buckling stiffness factor as

$$C_T = \frac{1 + K_M l_e}{K_T E}$$

The value of K_M is 2300 for wood seasoned to 19% moisture content or less at the time of plywood attachment. The value of K_M is 1200 for unseasoned or partially seasoned wood at the time of plywood attachment. Values of K_T are

- $1 - 1.645(\text{COV}_E)$
- 0.59 for visually graded lumber
- 0.75 for machine evaluated lumber
- 0.82 for products with $\text{COV}_E \leq 0.11$

COV_E is the coefficient of variation for the modulus of elasticity.

Timber

Example 5.3

Selected Douglas fir-larch, visually graded 3×6 decking is incised with the prescribed incising pattern. Normal service live load, temperature, and dry service conditions are applicable. Determine the adjusted allowable bending stress and the adjusted factored bending stress.

Solution

The reference design value for bending stress tabulated in NDS Supp. Table 4E, which includes the repetitive member and flat use adjustment factors, is

$$F_b = 2000 \text{ lbf/in}^2$$

From Table 5.8, the applicable incising factor for bending is

$$C_i = 0.80$$

From NDS Supp. Table 4E, the applicable size factor for a 3 in thick member is

$$C_F = 1.04$$

From Table 5.7, the applicable temperature factor for normal temperature conditions is

$$C_t = 1.0$$

From NDS Supp. Table 4E, the applicable wet service factor for decking where moisture content does not exceed 19% is

$$C_M = 1.0$$

In accordance with NDS Sec. 3.3.3, the beam stability factor is

$$C_L = 1.0$$

ASD Method

For normal occupancy live load, the load duration factor given by Table 5.5 is

$$C_D = 1.00$$

The adjusted allowable bending stress is

$$
\begin{aligned}
F_b' &= F_b C_F C_r C_i C_D C_M C_t C_{fu} C_L \\
&= \left(2000 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.04)(1.0)(0.8)(1.00) \\
&\quad \times (1.0)(1.0)(1.0)(1.0) \\
&= 1664 \text{ lbf/in}^2
\end{aligned}
$$

LRFD Method

From Table 5.4, the applicable time effect factor for dead load plus occupancy live load is

$$\lambda = 0.8$$

From Table 5.3, the applicable resistance factor for bending is

$$\phi_b = 0.85$$

From Table 5.2, the applicable format conversion factor for bending is

$$K_F = 2.54$$

The adjusted factored bending stress is

$$
\begin{aligned}
F_b' &= F_b C_F C_r C_i C_M C_t C_{fu} C_L K_F \lambda \phi_b \\
&= \left(2000 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.04)(1.0)(0.8)(1.0) \\
&\quad \times (1.0)(1.0)(1.0)(2.54)(0.8)(0.85) \\
&= 2874 \text{ lbf/in}^2
\end{aligned}
$$

Adjustment Factors Applicable to Glued Laminated Members Only

Volume Factor, C_V

The volume factor is applicable to the reference design value for bending and is not applied simultaneously with the beam stability factor, C_L; the lesser of these two factors is applicable. The volume factor is dimensionless and defined in NDS Sec. 5.3.6 as

$$C_V = \left(\frac{1291.5}{b_{\text{in}} d_{\text{in}} L_{\text{ft}}}\right)^{1/x} \qquad [\textit{NDS 5.3-1}]$$

The variables are defined as

$L =$ length of beam between points of zero moment, in feet
$b =$ beam width, in inches
$d =$ beam depth, in inches
$x = 20$ [for southern pine]
$ = 10$ [for all other species]

Curvature Factor, C_c

To account for residual stresses in curved, glued laminated members, the curvature factor is specified in NDS Sec. 5.3.8 as

$$C_c = 1 - 2000\left(\frac{t}{R}\right)^2 \qquad [\textit{NDS 5.3-3}]$$

The variables are defined as

$t =$ thickness of lamination, in inches
$R =$ radius of curvature of inside face of
 lamination in inches
$\dfrac{t}{R} \le \dfrac{1}{100}$ [for hardwoods and southern pine]
$\phantom{\dfrac{t}{R}} \le \dfrac{1}{125}$ [for other softwoods]

The curvature factor does not apply to design values in the straight portion of a member, regardless of curvature elsewhere.

Stress Interaction Factor, C_I

NDS Sec. 5.3.9 provides values for the stress interaction factor for tapered bending members. The stress interaction factor, C_I, is applied to the reference bending design value, F_b. For members tapered on the compression face, the stress interaction factor does not apply simultaneously with the volume factor, and the lesser factor applies. For members tapered on the tension face, the stress interaction factor does not apply simultaneously with the beam stability factor, and the lesser factor applies.

Shear Reduction Factor, C_{vr}

The shear reduction factor, $C_{vr} = 0.72$, is applied to the reference shear design value, F_v, where members are nonprismatic, subject to impact or repetitive cyclic loading, at notches, or at connections.

Example 5.4

A curved, glued laminated beam of stress class 24F-1.7E western species with 1.5 in thick laminations has a radius of curvature of 30 ft, a width of $6\,^3/_4$ in, and a depth of 30 in. The beam has continuous lateral support, a moisture content exceeding 16%, and is subjected to sustained temperatures between 100°F and 125°F. The governing loading combination is dead plus live load. The span is 40 ft. The beam is simply supported, and all loading is uniformly distributed. Determine the adjusted allowable bending stress and the adjusted factored bending stress.

Solution

The reference design value for bending is tabulated in NDS Supp. Table 5A and is

$$F_b = 2400 \text{ lbf/in}^2$$

The applicable adjustment factors for bending stress are as follows.

$C_M =$ wet service factor

$\quad = 0.8$ [Table 5.6]

$C_t =$ temperature factor for wet conditions

$\quad = 0.7$ [Table 5.7]

$C_V =$ volume factor

$\quad = \left(\dfrac{1291.5}{b_{\text{in}}\, d_{\text{in}}\, L_{\text{ft}}} \right)^{1/x}$ [*NDS 5.3-1*]

$\quad = \left(\dfrac{1291.5 \text{ in}^2\text{-ft}}{(6.75 \text{ in})(30 \text{ in})(40 \text{ ft})} \right)^{1/10}$

$\quad = 0.832$

$C_L =$ stability factor

$\quad = 1.0$ [continuous lateral support]

$\quad > $ volume factor

The volume factor governs.

$C_c =$ curvature factor

$\quad = 1 - 2000\left(\dfrac{t}{R}\right)^2$ [*NDS Sec. 5.3.3*]

$\quad = 1 - (2000)\left(\dfrac{1.5 \text{ in}}{(30 \text{ ft})\left(12 \,\dfrac{\text{in}}{\text{ft}}\right)} \right)^2$

$\quad = 0.965$

$$\frac{t}{R} = \frac{1}{240}$$

$$< \frac{1}{125} \quad \text{[satisfactory]}$$

The beam is not tapered and the stress interaction factor is

$$C_I = 1.0$$

The flat use factor is not applicable and

$$C_{fu} = 1.0$$

ASD Method

For normal occupancy live load, the load duration factor given by Table 5.5 is

$$C_D = 1.00$$

The adjusted allowable bending stress is

$$F_b' = F_b C_c C_I C_D C_M C_t C_{fu}(C_L \text{ or } C_V)$$

$$= \left(2400 \,\frac{\text{lbf}}{\text{in}^2}\right)(0.965)(1.0)(1.00)(0.8)(0.7)(1.0)(0.832)$$

$$= 1079 \text{ lbf/in}^2$$

LRFD Method

From Table 5.4, the applicable time effect factor for dead load plus occupancy live load is

$$\lambda = 0.8$$

From Table 5.3, the applicable resistance factor for bending is

$$\phi_b = 0.85$$

From Table 5.2, the applicable format conversion factor for bending is

$$K_F = 2.54$$

Timber

The adjusted factored bending stress is

$$F'_b = F_b C_c C_I C_M C_t C_{fu}(C_L \text{ or } C_V) K_F \lambda \phi_b$$
$$= \left(2400 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.965)(1.0)(0.8)(0.7)$$
$$\times (1.0)(0.832)(2.54)(0.8)(0.85)$$
$$= 1864 \ \text{lbf/in}^2$$

7. DESIGN FOR FLEXURE

General Requirements Applicable to Sawn Lumber and Glued Laminated Members

For all flexural members, in accordance with NDS Sec. 3.2.1, the beam span is taken as the clear span plus one-half the required bearing length at each end.

When the depth of a beam does not exceed its breadth, or when continuous lateral restraint is provided to the compression edge of a beam with the ends restrained against rotation, the beam stability factor, C_L, is 1.0. For other situations, the value of C_L is calculated in accordance with NDS Sec. 3.3.3, and the effective span length, l_e, is determined in accordance with NDS Table 3.3.3. The value of l_e depends on the loading configuration and the distance between lateral restraints, l_u. Typical values for l_e are given in Fig. 5.1.

Requirements Applicable to Sawn Lumber

For sawn lumber, both the stability factor, C_L, and the size factor, C_F, must be applied concurrently.

Example 5.5

A select structural Douglas fir-larch 4×12 beam is simply supported over a span of 20 ft. The governing load combination is a uniformly distributed dead plus live load, and the beam is laterally braced at midspan. Full depth bracing is not provided at the ends of the beam.

The beam is incised with the prescribed pattern specified in NDS Sec. 4.3.8. Determine the adjusted allowable bending stress and the adjusted factored bending stress.

Solution

The reference design values for bending and modulus of elasticity for beam stability calculations are tabulated in NDS Supp. Table 4A and is

$$F_b = 1500 \ \text{lbf/in}^2$$
$$E_{\min} = 0.69 \times 10^6 \ \text{lbf/in}^2$$
$$C_M = 1.0, \ C_t = 1.0, \ C_r = 1.0, \ C_{fu} = 1.0, \ C_T = 1.0$$

The applicable incising factor for *modulus of elasticity* is obtained from NDS Sec. 4.3.8 and is

$$C_i = 0.95$$

The applicable incising factor for flexure is obtained from NDS Sec. 4.3.8 and is

$$C_i = 0.80$$

The applicable size factor for flexure is obtained from NDS Supp. Table 4A and is

$$C_F = 1.1$$

The distance between lateral restraints is

$$l_u = \frac{20 \ \text{ft}}{2}$$
$$= 10 \ \text{ft}$$
$$\frac{l_u}{d} = \frac{(10 \ \text{ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{11.25 \ \text{in}}$$
$$= 10.7$$
$$> 7$$

For a uniformly distributed load and a l_u/d ratio > 7, the effective length is obtained from Fig. 5.1 as

$$l_e = 1.63 l_u + 3d$$
$$= (1.63)(10 \ \text{ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) + (3)(11.25 \ \text{in})$$
$$= 229.4 \ \text{in}$$

The slenderness ratio is given by NDS Sec. 3.3.3 as

$$R_B = \sqrt{\frac{l_e d}{b^2}}$$
$$= \sqrt{\frac{(229.4 \ \text{in})(11.25 \ \text{in})}{(3.5 \ \text{in})^2}}$$
$$= 14.52$$
$$< 50 \quad \begin{bmatrix} \text{satisfies criteria} \\ \text{of NDS Sec. 3.3.3} \end{bmatrix}$$

ASD Method

The adjusted modulus of elasticity for stability calculations with $C_t = C_T = C_M = 1.0$ is

$$E'_{\min} = E_{\min} C_M C_t C_i C_T$$
$$= \left(0.69 \times 10^6 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(0.95)(1.0)$$
$$= 0.656 \times 10^6 \ \text{lbf/in}^2$$

The critical buckling design value is

$$F_{bE} = \frac{1.20E'_{\min}}{R_B^2}$$

$$= \frac{(1.20)\left(0.656 \times 10^6 \, \frac{\text{lbf}}{\text{in}^2}\right)}{(14.52)^2}$$

$$= 3734 \, \text{lbf/in}^2$$

The load duration factor for occupancy live load is

$$C_D = 1.00$$

The reference flexural design value multiplied by all applicable adjustment factors except C_L and C_{fu} is

$$F_b^* = F_b C_D C_M C_t C_F C_i C_r$$

$$= \left(1500 \, \frac{\text{lbf}}{\text{in}^2}\right)(1.00)(1.0)(1.0)(1.1)(0.8)(1.0)$$

$$= 1320 \, \text{lbf/in}^2$$

$$F = \frac{F_{bE}}{F_b^*}$$

$$= \frac{3734 \, \frac{\text{lbf}}{\text{in}^2}}{1320 \, \frac{\text{lbf}}{\text{in}^2}}$$

$$= 2.83$$

The beam stability factor is given by NDS Sec. 3.3.3 as

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}}$$

$$= 0.974$$

The adjusted allowable flexural stress is

$$F_b' = F_b C_M C_T C_L C_F C_{fu} C_i C_r$$

$$= \left(1500 \, \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(0.974)(1.1)(1.0)(0.80)(1.0)$$

$$= 1286 \, \text{lbf/in}^2$$

LRFD Method

From Table 5.4, the time effect factor for dead load plus occupancy live load is

$$\lambda = 0.8$$

The adjusted modulus of elasticity for LRFD stability calculations with $C_t = C_T = 1.0$ is

$$E'_{\min} = E_{\min} C_M C_t C_i C_T K_F \phi_s$$

$$= \left(0.69 \times 10^6 \, \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(0.95)(1.0)(1.76)(0.85)$$

$$= 0.981 \times 10^6 \, \text{lbf/in}^2$$

The critical buckling LRFD design value is

$$F_{bE} = \frac{1.20E'_{\min}}{R_B^2} = \frac{(1.20)\left(0.981 \times 10^6 \, \frac{\text{lbf}}{\text{in}^2}\right)}{(14.52)^2}$$

$$= 5584 \, \text{lbf/in}^2$$

The reference flexural design value multiplied by all applicable LRFD adjustment factors except C_L and C_{fu} is

$$F_b^* = F_b C_M C_t C_F C_i C_r K_F \phi_b \lambda$$

$$= \left(1500 \, \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.1)(0.80)(1.0)$$

$$\times (2.54)(0.85)(0.8)$$

$$= 2280 \, \text{lbf/in}^2$$

$$F = \frac{F_{bE}}{F_b^*} = \frac{5584 \, \frac{\text{lbf}}{\text{in}^2}}{2280 \, \frac{\text{lbf}}{\text{in}^2}}$$

$$= 2.45$$

The LRFD beam stability factor given by NDS Sec. 3.3.3 is

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}}$$

$$= \frac{1.0 + 2.45}{1.9} - \sqrt{\left(\frac{1.0 + 2.45}{1.9}\right)^2 - \frac{2.45}{0.95}}$$

$$= 0.968$$

The adjusted factored bending stress is

$$F_b' = F_b C_M C_t C_L C_F C_{fu} C_i C_r K_F \phi_b \lambda$$

$$= \left(1500 \, \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(0.968)(1.1)(1.0)(0.8)$$

$$\times (1.0)(2.54)(0.85)(0.80)$$

$$= 2207 \, \text{lbf/in}^2$$

Requirements Applicable to Glued Laminated Members Only

For glued laminated members, both the stability factor, C_L, and the volume factor, C_V, must be determined. Only the lesser of these two factors is applicable in determining the allowable design value in bending.

Example 5.6

A glued laminated $6^3/_4 \times 30$ beam of stress class 24F-1.7E western species is simply supported over a span of 40 ft. The governing load combination is a uniformly distributed dead plus live load, and the beam is laterally braced at midspan. Determine the adjusted allowable bending stress and the adjusted factored bending stress.

Solution

$$C_M = 1.0, \ C_t = 1.0,$$
$$C_c = 1.0, \ C_{fu} = 1.0, \ C_I = 1.0,$$

From NDS Supp. Table 5A,

$$F_b = 2400 \ \text{lbf/in}^2$$

$$E_{y(\text{min})} = 0.69 \times 10^6 \ \text{lbf/in}^2$$

From Ex. 5.4, the volume factor is

$$C_V = 0.832$$

The distance between lateral restraints is

$$l_u = \frac{40 \ \text{ft}}{2}$$
$$= 20 \ \text{ft}$$

$$\frac{l_u}{d} = \frac{(20 \ \text{ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{30 \ \text{in}}$$
$$= 8.0$$
$$> 7$$

For a uniformly distributed load and an l_u/d ratio > 7, the effective length is obtained from Fig. 5.1 as

$$l_e = 1.63 l_u + 3d$$
$$= (1.63)(20 \ \text{ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) + (3)(30 \ \text{in})$$
$$= 481 \ \text{in}$$

The slenderness ratio is given by NDS Eq. 3.3-5.

$$R_B = \sqrt{\frac{l_e d}{b^2}}$$
$$= \sqrt{\frac{(481 \ \text{in})(30 \ \text{in})}{(6.75 \ \text{in})^2}}$$
$$= 17.80$$
$$< 50 \quad \begin{bmatrix} \text{satisfies criteria} \\ \text{of NDS Sec. 3.3.3} \end{bmatrix}$$

ASD Method

The adjusted modulus of elasticity for stability calculations with $C_M = C_t = 1.0$ is

$$E'_{\text{min}} = E_{y(\text{min})} C_M C_t$$
$$= \left(0.69 \times 10^6 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)$$
$$= 0.69 \times 10^6 \ \text{lbf/in}^2$$

The critical buckling design value is

$$F_{bE} = \frac{1.20 E'_{\text{min}}}{R_B^2}$$

$$= \frac{(1.20)\left(0.69 \times 10^6 \ \frac{\text{lbf}}{\text{in}^2}\right)}{(17.80)^2}$$

$$= 2613 \ \text{lbf/in}^2$$

The load duration factor for occupancy live load is

$$C_D = 1.00$$

The reference flexural design value multiplied by all applicable adjustment factors except C_L, C_V, and C_{fu} is

$$F_b^* = F_b C_M C_t C_c C_I$$
$$= \left(2400 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)(1.0)(1.0)$$
$$= 2400 \ \frac{\text{lbf}}{\text{in}^2}$$

$$F = \frac{F_{bE}}{F_b^*}$$

$$= \frac{2613 \ \frac{\text{lbf}}{\text{in}^2}}{2400 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 1.09$$

The beam stability factor is given by NDS Sec. 3.3.3 as

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}}$$

$$= \frac{1.0 + 1.09}{1.9} - \sqrt{\left(\frac{1.0 + 1.09}{1.9}\right)^2 - \frac{1.09}{0.95}}$$

$$= 0.85$$
$$> \text{volume factor derived in Ex. 5.4}$$

The volume factor governs. The adjusted allowable flexural stress is

$$F_b' = F_b C_D C_M C_t C_{fu} C_c C_I C_V$$
$$= \left(2400 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.00)(1.0)(1.0)(1.0)(1.0)(1.0)$$
$$\times (0.832)$$
$$= 1997 \ \text{lbf/in}^2$$

LRFD Method

From Table 5.4, the time effect factor for dead load plus occupancy live load is

$$\lambda = 0.8$$

The adjusted modulus of elasticity for LRFD stability calculations with $C_t = C_M = 1.0$ is

$$E'_{\min} = E_{\min} C_M C_t K_F \phi_s$$
$$= \left(0.69 \times 10^6 \; \frac{\text{lbf}}{\text{in}^5}\right)(1.0)(1.0)(1.76)(0.85)$$
$$= 1.032 \times 10^6 \; \text{lbf/in}^2$$

The critical buckling LRFD design value is

$$F_{bE} = \frac{1.20 E'_{\min}}{R_B^2} = \frac{(1.2)\left(1.032 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)}{(17.80)^2}$$
$$= 3909 \; \text{lbf/in}^2$$

The reference flexural design value multiplied by all applicable LRFD adjustment factors except C_V, C_L, and C_{fu} is

$$F_b^* = F_b C_M C_t C_c C_I K_F \phi_b \lambda$$
$$= \left(2400 \; \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)(1.0)(2.54)$$
$$\times (0.85)(0.8)$$
$$= 4145 \; \text{lbf/in}^2$$

$$F = \frac{F_{bE}}{F_b^*} = \frac{3909 \; \frac{\text{lbf}}{\text{in}^2}}{4145 \; \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.943$$

The LRFD beam stability factor given by NDS Sec. 3.3.3 is

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}}$$
$$= \frac{1.0 + 0.943}{1.9} - \sqrt{\left(\frac{1.0 + 0.943}{1.9}\right)^2 - \frac{0.943}{0.95}}$$
$$= 0.792$$
$$< 0.832 \quad \text{[volume factor derived from Ex. 5.4]}$$

The stability factor governs, and the adjusted factored bending stress is

$$F'_b = F_b C_M C_t C_L C_{fu} C_c C_I K_F \phi_b \lambda$$
$$= \left(2400 \; \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(0.792)(1.0)(1.0)(1.0)$$
$$\times (2.54)(0.85)(0.8)$$
$$= 3283 \; \text{lbf/in}^2$$

8. DESIGN FOR SHEAR

Nomenclature

d	depth of unnotched bending member	in
d_e	depth of member, less the distance from the unloaded edge of the member to the nearest edge of the nearest split ring or shear plate connector	in
d_e	depth of member, less the distance from the unloaded edge of the member to the center of the nearest bolt or lag screw	in
d_n	depth of member remaining at a notch	in
e	distance a notch extends past the inner edge of a support	in
f_v	actual shear stress parallel to grain	lbf/in^2
l_n	length of notch	in
V	shear force	lbf
V_r, V'_r	reference and adjusted design shear	lbf
x	distance from beam support face to load	in

General Requirements

The shear stress in a rectangular beam is defined in NDS Sec. 3.4.2 as

$$f_v = \frac{3V}{2bd} \qquad \text{[NDS 3.4-2]}$$

In determining the shear force on the member, uniformly distributed loads applied to the top of the beam within a distance from either support equal to the depth of the beam are ignored, as shown in Fig. 5.2.

Figure 5.2 Shear Determination in a Beam

neglect loads for shear calculations

d

d d

shear caused by distributed loads

x W

d

d d

shear caused by concentrated loads
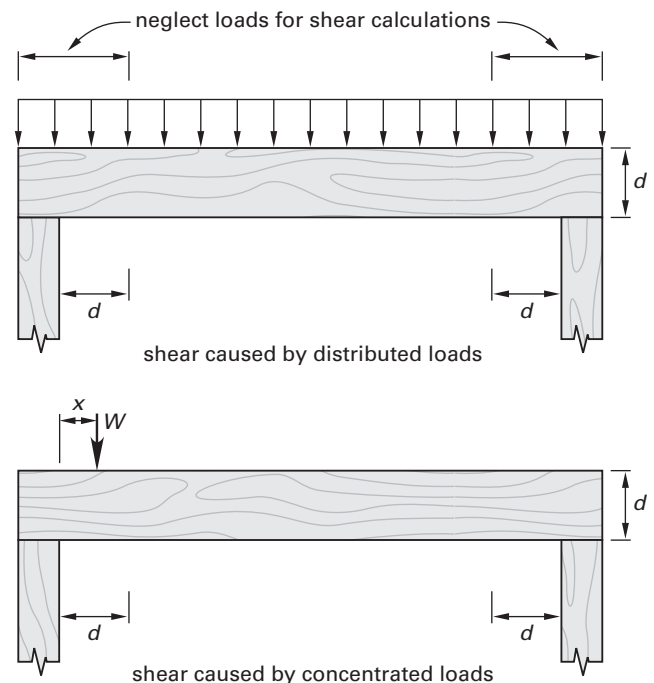
The procedure for a beam with concentrated loads is also illustrated in Fig. 5.2. Concentrated loads within a distance from either support equal to the depth of the beam are multiplied by x/d, where x is the distance from the support to the load, to give an equivalent shear force.

Example 5.7

A glued laminated beam of combination 24F-V10 western species with a width of $6^3/_4$ in and a depth of 30 in is loaded with the service levels shown. The loading consists of a distributed dead load of 2.0 kips/ft, which includes the self-weight of the beam, plus a concentrated occupancy live load of 15 kips. Normal dry use conditions of service are applicable. Determine whether the beam is adequate in shear.
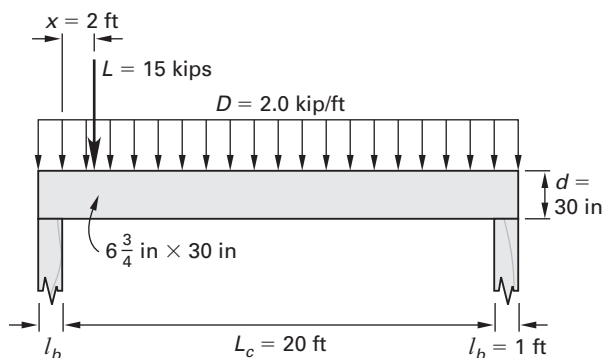
Solution

From NDS Supp. Table 5A, the reference design value for shear stress is

$$F_v = 215 \text{ lbf/in}^2$$

From the illustration, the clear span is

$$L_c = 20 \text{ ft}$$

Also from the illustration, the bearing length is

$$l_b = 1.0 \text{ ft}$$

The design span is defined in NDS Sec. 3.2.1 as

$$L = L_c + 2\left(\frac{l_b}{2}\right)$$
$$= 20 \text{ ft} + (2)\left(\frac{1.0 \text{ ft}}{2}\right)$$
$$= 21 \text{ ft}$$

ASD Method

The adjustment factors for shear stress are

$$C_D = C_M = C_t = C_{vr} = 1.0$$

$$F'_v = F_v = 215 \text{ lbf/in}^2$$

Applying IBC Eq. 16-9, the factored distributed dead load is

$$w = 1.0D = (1.0)\left(2.0 \; \frac{\text{kips}}{\text{ft}}\right)$$
$$= 2 \text{ kips/ft}$$

In calculating shear force due to distributed load, the distributed load within a distance, d, from either support is ignored. In accordance with NDS Sec. 3.4.3.1(a), the shear caused by the distributed load at the location of the concentrated load is given by

$$V_D = 0.5w(L_c - 2d)$$
$$= (0.5)\left(2 \; \frac{\text{kips}}{\text{ft}}\right)\left(20 \text{ ft} - (2)\left(\frac{30 \text{ in}}{12 \; \frac{\text{in}}{\text{ft}}}\right)\right)$$
$$= 15 \text{ kips}$$

Applying IBC Eq. 16-9, the factored concentrated live load is

$$W = 1.0L = (1.0)(15 \text{ kips})$$
$$= 15 \text{ kips}$$

The concentrated load is less than a distance, d, from the face of the left support. In accordance with NDS Sec. 3.4.3.1, this is equivalent to a load of

$$W' = \frac{Wx}{d} = \frac{(15 \text{ kips})(2 \text{ ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{30 \text{ in}}$$
$$= 12 \text{ kips}$$

The shear at the left support caused by the equivalent load is

$$V_C = \frac{W'(L - x - 0.5l_b)}{L}$$
$$= \frac{(12 \text{ kips})\big(21 \text{ ft} - 2 \text{ ft} - (0.5)(1.0 \text{ ft})\big)}{21 \text{ ft}}$$
$$= 10.57 \text{ kips}$$

The total shear at the left support is

$$V = V_D + V_C$$
$$= 15 \text{ kips} + 10.57 \text{ kips}$$
$$= 25.57 \text{ kips}$$

The shear stress parallel to the grain given by NDS Sec. 3.4.2 is

$$F_v = \frac{3V}{2bd} = \frac{(3)(25.57 \text{ kips})\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(2)(6.75 \text{ in})(30 \text{ in})}$$
$$= 189 \text{ lbf/in}^2$$
$$< F_v' \quad [\text{satisfactory}]$$

The beam is adequate.

LRFD Method

The adjustment factors for shear stress are

$$C_D = C_M = C_t = C_{vr} = 1.0$$

From Table 5.2, the format conversion factor is

$$K_F = 2.88$$

From Table 5.4, the time effect factor for occupancy live load is

$$\lambda = 0.8$$

From Table 5.3, the resistance factor for shear is

$$\phi_v = 0.75$$

The adjusted factored shear stress is

$$F_v' = F_v C_M C_t C_{vr} K_F \phi_v \lambda$$
$$= \left(215 \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)(2.88)(0.75)(0.8)$$
$$= 372 \text{ lbf/in}^2$$

Applying IBC Eq. 16-2, the factored distributed dead load is

$$w = 1.2D = (1.2)\left(2.0 \frac{\text{kips}}{\text{ft}}\right) = 2.4 \text{ kips/ft}$$

The shear at the left support caused by the distributed load given by NDS Sec. 3.4.3.1(a) is

$$V_D = 0.5w(L_c - 2d)$$
$$= (0.5)\left(2.4 \frac{\text{kips}}{\text{ft}^2}\right)\left(20 \text{ ft} - (2)\left(\frac{30 \text{ in}}{12 \frac{\text{in}}{\text{ft}}}\right)\right)$$
$$= 18 \text{ kips}$$

Applying IBC Eq. 16-2, the factored concentrated live load is

$$W = 1.6L = (1.6)(15 \text{ kips}) = 24 \text{ kips}$$

The concentrated load is less than a distance, d, from the face of the left support. In accordance with NDS Sec. 3.4.3.1, this is equivalent to a load of

$$W' = \frac{Wx}{d}$$
$$= \frac{(24 \text{ kips})(2 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{30 \text{ in}}$$
$$= 19.2 \text{ kips}$$

The shear at the left support caused by the equivalent live load is

$$V_C = \frac{W'(L - x - 0.5l_b)}{L}$$
$$= \frac{(19.2 \text{ kips})\big(21 \text{ ft} - 2 \text{ ft} - (0.5)(1.0 \text{ ft})\big)}{21 \text{ ft}}$$
$$= 16.91 \text{ kips}$$

The total shear at the left support is

$$V = V_D + V_C$$
$$= 18 \text{ kips} + 16.91 \text{ kips}$$
$$= 34.91 \text{ kips}$$

The shear stress parallel to the grain given by NDS Sec. 3.4.2 is

$$F_v = \frac{3V}{2bd}$$
$$= \frac{(3)(34.91 \text{ kips})\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(2)(6.75 \text{ in})(30 \text{ in})}$$
$$= 259 \text{ lbf/in}^2$$
$$< F_v' \quad [\text{satisfactory}]$$

The beam is adequate.

Timber

Notched Beams

Notches in a beam reduce the shear capacity, and NDS Sec. 3.2.3 imposes restrictions on their size and location, as shown in Fig. 5.3. The adjusted design shear at a notch on the tension side of a beam is given by NDS Sec. 3.4.3.2 as

$$V_r' = \left(\tfrac{2}{3}F_v' b d_n\right)\left(\frac{d_n}{d}\right)^2 \qquad [NDS\ 3.4\text{-}3]$$

When $e \le d_n$, the adjusted design shear at a notch on the compression side of a beam is given by NDS Sec. 3.4.3.2 as

$$V_r' = \left(\tfrac{2}{3}F_v' b\right)\left(d - \left(\frac{d - d_n}{d_n}\right)e\right) \qquad [NDS\ 3.4\text{-}5]$$

When $e > d_n$,

$$V_r' = \tfrac{2}{3}F_v' b d_n$$

Shear at Connections

For a connection less than five times the depth of the member from its end, as shown in Fig. 5.4, the adjusted design shear is given by NDS Sec. 3.4.3.3 as

$$V_r' = \left(\tfrac{2}{3}F_v' b d_e\right)\left(\frac{d_e}{d}\right)^2 \qquad [NDS\ 3.4\text{-}6]$$

Figure 5.3 *Notched Beams*
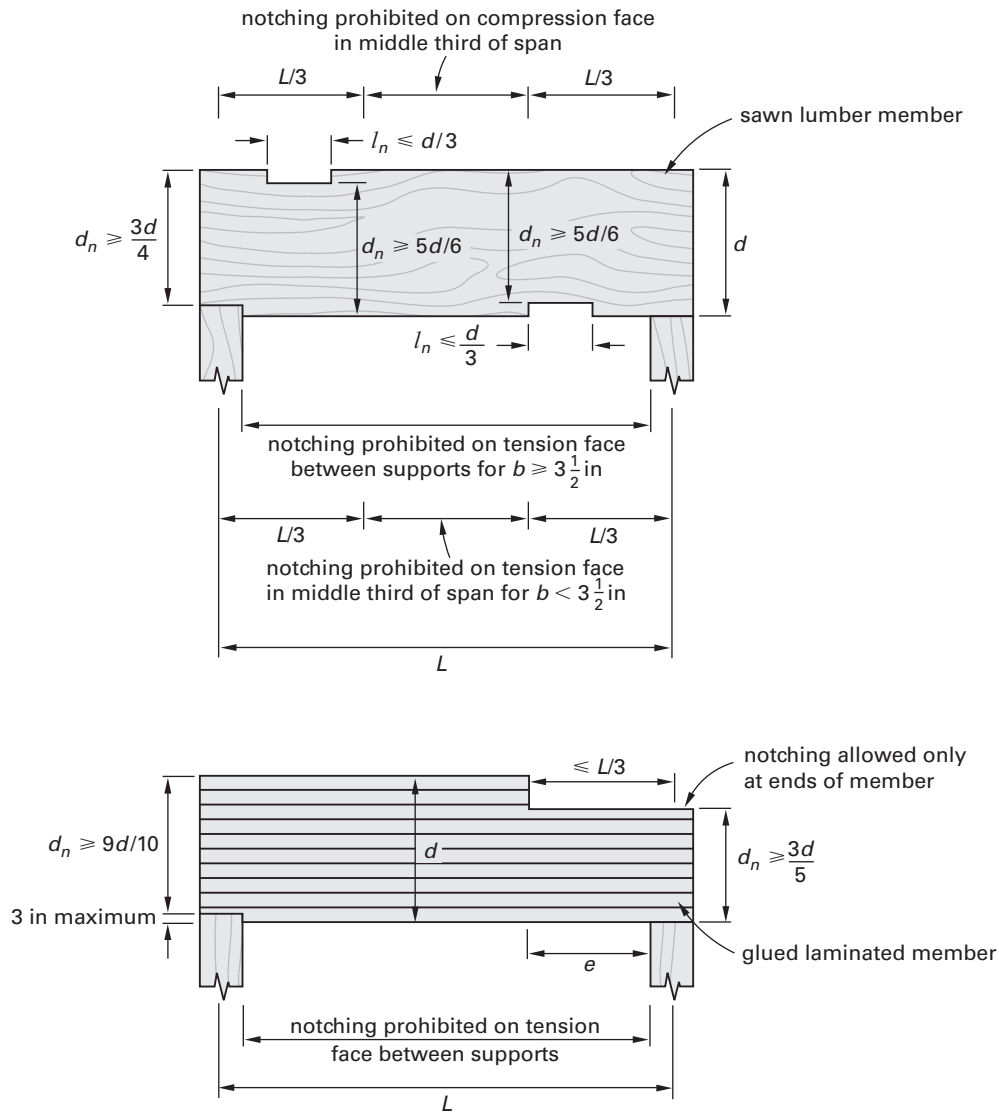
Figure 5.4 *Bolted Connections*
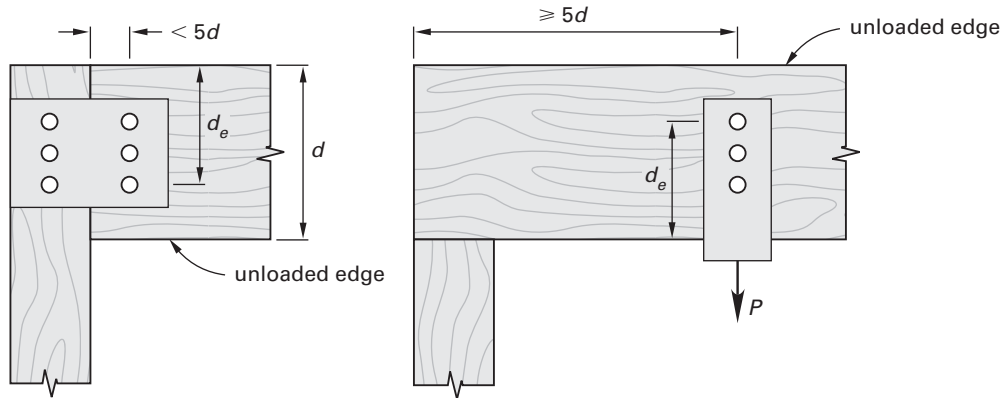
When the connection is at least five times the depth of the member from its end, the adjusted design shear is given by

$$V'_r = \tfrac{2}{3}F'_v b d_e \qquad [\textit{NDS 3.4-7}]$$

To facilitate the selection of glued laminated beam sections, tables are available[4] that provide shear and bending capacities of sections. For lumber joists, tables are available[5] that assist in the selection of a joist size for various span and live load combinations.

Example 5.8

A glued laminated $6^3/_4 \times 30$ beam of stress class 24F-1.7E western species is notched and loaded as shown in the following illustration. The beam has a moisture content exceeding 16% and is subjected to sustained temperatures between 100°F and 125°F. The governing load combination is dead plus occupancy live load. Determine the maximum allowable shear force at each support and at the hanger connection.
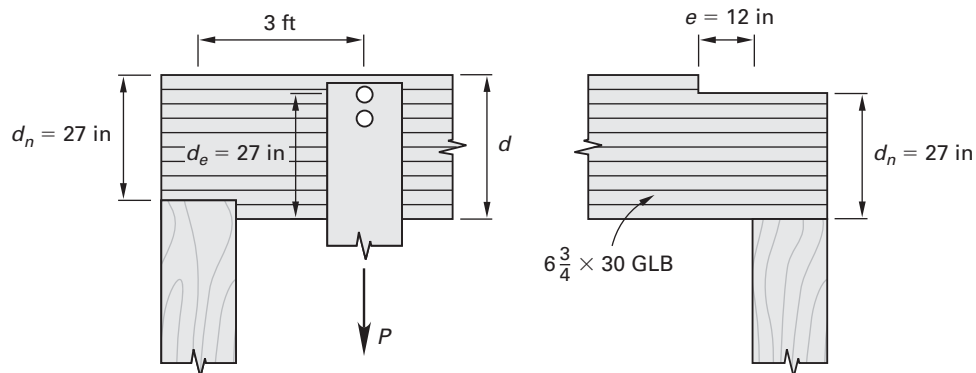
Solution

The reference design value for shear stress, tabulated in NDS Supp. Table 5A, is

$$F_v = 210 \text{ lbf/in}^2$$

The applicable shear reduction factor for the design of members at a notch is

$$C_{vr} = 0.72$$

From Table 5.7, the applicable temperature factor for wet conditions between 100°F and 125°F is

$$C_t = 0.7$$

From Table 5.6, the applicable wet service factor for moisture content exceeding 19% is

$$C_M = 0.875$$

Illustration for Ex. 5.8

ASD Method

For normal occupancy live load, the load duration factor given in Table 5.5 is

$$C_D = 1.00$$

The adjusted allowable shear stress is

$$\begin{aligned}
F_v' &= F_v C_D C_M C_t C_{vr} \\
&= \left(210 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.00)(0.875)(0.7)(0.72) \\
&= 93 \ \text{lbf/in}^2
\end{aligned}$$

At the left support and from NDS Eq. 3.4-3, the allowable shear force is

$$\begin{aligned}
V_r' &= \left(\tfrac{2}{3}F_v' b d_n\right)\left(\frac{d_n}{d}\right)^2 \\
&= \left(\frac{(2)\left(93 \ \frac{\text{lbf}}{\text{in}^2}\right)(6.75 \ \text{in})(27 \ \text{in})}{3}\right)\left(\frac{27 \ \text{in}}{30 \ \text{in}}\right)^2 \\
&= 9153 \ \text{lbf}
\end{aligned}$$

At the right support, $e < d_n$, and from NDS Eq. 3.4-5, the allowable shear force is

$$\begin{aligned}
V_r' &= \tfrac{2}{3}F_v' b\left(d - \left(\frac{d-d_n}{d_n}\right)e\right) \\
&= \left(\frac{(2)\left(93 \ \frac{\text{lbf}}{\text{in}^2}\right)(6.75 \ \text{in})}{3}\right) \\
&\quad \times \left(30 \ \text{in} - \left(\frac{30 \ \text{in} - 27 \ \text{in}}{27 \ \text{in}}\right)(12 \ \text{in})\right) \\
&= 11{,}997 \ \text{lbf}
\end{aligned}$$

The hanger connection is less than $5d$ from the end of the beam, and from NDS Eq. 3.4-6, the allowable shear force is

$$\begin{aligned}
V_r' &= \left(\tfrac{2}{3}F_v' b d_e\right)\left(\frac{d_e}{d}\right)^2 \\
&= \left(\frac{(2)\left(93 \ \frac{\text{lbf}}{\text{in}^2}\right)(6.75 \ \text{in})(27 \ \text{in})}{3}\right)\left(\frac{27 \ \text{in}}{30 \ \text{in}}\right)^2 \\
&= 9153 \ \text{lbf}
\end{aligned}$$

LRFD Method

From Table 5.4, the applicable time effect factor for dead load plus occupancy live load is

$$\lambda = 0.8$$

From Table 5.3, the applicable resistance factor for shear is

$$\phi = 0.75$$

From Table 5.2, the applicable format conversion factor for shear is

$$K_F = 2.88$$

The adjusted factored shear stress is

$$\begin{aligned}
F_v' &= F_v C_M C_t C_{vr} K_F \lambda \phi_v \\
&= \left(210 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.875)(0.7)(0.72) \\
&\quad \times (2.88)(0.8)(0.75) \\
&= 160 \ \text{lbf/in}^2
\end{aligned}$$

At the left support and from NDS Eq. 3.4-3, the allowable shear force is

$$\begin{aligned}
V_r' &= \left(\tfrac{2}{3}F_v' b d_n\right)\left(\frac{d_n}{d}\right)^2 \\
&= \left(\frac{(2)\left(160 \ \frac{\text{lbf}}{\text{in}^2}\right)(6.75 \ \text{in})(27 \ \text{in})}{3}\right)\left(\frac{27 \ \text{in}}{30 \ \text{in}}\right)^2 \\
&= 15{,}747 \ \text{lbf}
\end{aligned}$$

At the right support, $e < d_n$, and from NDS Eq. 3.4-5, the allowable shear force is

$$\begin{aligned}
V_r' &= \tfrac{2}{3}F_v' b\left(d - \left(\frac{d-d_n}{d_n}\right)e\right) \\
&= \left(\frac{(2)\left(160 \ \frac{\text{lbf}}{\text{in}^2}\right)(6.75 \ \text{in})}{3}\right) \\
&\quad \times \left(30 \ \text{in} - \left(\frac{30 \ \text{in} - 27 \ \text{in}}{27 \ \text{in}}\right)(12 \ \text{in})\right) \\
&= 20{,}640 \ \text{lbf}
\end{aligned}$$

The hanger connection is less than $5d$ from the end of the beam, and from NDS Eq. 3.4-6, the allowable shear force is

$$\begin{aligned}
V_r' &= \left(\tfrac{2}{3}F_v' b d_e\right)\left(\frac{d_e}{d}\right)^2 \\
&= \left(\frac{(2)\left(160 \ \frac{\text{lbf}}{\text{in}^2}\right)(6.75 \ \text{in})(27 \ \text{in})}{3}\right)\left(\frac{27 \ \text{in}}{30 \ \text{in}}\right)^2 \\
&= 15{,}747 \ \text{lbf}
\end{aligned}$$

Timber

9. DESIGN FOR COMPRESSION

Nomenclature

A	area of cross section	in^2
C_{m1}	moment magnification factor for biaxial bending and axial compression, $1.0 - f_c/F_{cE1}$	–
C_{m2}	moment magnification factor for biaxial bending and axial compression, $1.0 - f_c/F_{cE2} - (f_{b1}/F_{bE})^2$	–
C_{m3}	moment magnification factor for axial compression and flexure with load applied to narrow face, $1.0 - f_c/F_{cE1}$	–
C_{m4}	moment magnification factor for axial compression and flexure with load applied to wide face, $1.0 - f_c/F_{cE2}$	–
C_{m5}	moment magnification factor for biaxial bending, $1.0 - (f_{b1}/F_{bE})^2$	–
d_1	dimension of wide face	in
d_2	dimension of narrow face	in
f_{b1}	actual edgewise bending stress for load applied to the narrow face	lbf/in^2
f_{b2}	actual flatwise bending stress for load applied to the wide face	lbf/in^2
f_c	actual compression stress parallel to grain	lbf/in^2
F_{bE}	critical buckling design value for bending member, $1.20 E'_{\min}/R_B^2$	lbf/in^2
F'_{b1}	allowable bending design value for load applied to the narrow face, including adjustment for slenderness ratio	lbf/in^2
F'_{b2}	allowable bending design value for load applied to the wide face, including adjustment for slenderness ratio	lbf/in^2
F'_c	allowable compression design value, including adjustment for largest slenderness ratio	lbf/in^2
F'_{cE1}	critical buckling design value in plane of bending for load applied to the narrow face, $0.822 E'_{\min}/(l_{e1}/d_1)^2$	lbf/in^2
F'_{cE2}	critical buckling design value in plane of bending for load applied to the wide face, $0.822 E'_{\min}/(l_{e2}/d_2)^2$	lbf/in^2
K_e	buckling length coefficient for compression members	–
l_1	distance between points of lateral support restraining buckling about the strong axis of compression member	ft or in
l_2	distance between points of lateral support restraining buckling about the weak axis of compression member	ft or in
l_{e1}	effective length between supports restraining buckling in plane of bending from load applied to narrow face of compression member, $K_e l_1$	ft or in
l_{e2}	effective length between supports restraining buckling in plane of bending from load applied to wide face of compression member, $K_e l_2$	ft or in
l_{e1}/d_1	slenderness ratio about the strong axis of compression member	–
l_{e2}/d_2	slenderness ratio about the weak axis of compression member	–
P	total concentrated load or total axial load	lbf or kips

Axial Load Only

The effective length of a column is defined in NDS Sec. 3.7.1.2 as $l_e = K_e l$ where l is the distance between lateral supports. The slenderness ratio is defined in NDS Sec. 3.7.1.3 as $l_e/d \leq 50$. When the distance between lateral supports about the x-x axis and the y-y axis is different, as shown in Fig. 5.5, two values of the slenderness ratio are obtained. These values are l_{e1}/d_1 and l_{e2}/d_2. The larger of these values governs.

Values of the buckling length coefficient, K_e, for various end restraint conditions are given in NDS Table G1 and are summarized in Fig. 5.6.

The maximum slenderness ratio of a column determines the critical buckling design value and the adjusted compressive stress, F'_c. The allowable axial column load is $P = A F'_c$.

Example 5.9

The select structural 2×6 Douglas fir-larch top chord of a truss is loaded with the service level loads shown in the illustration. The governing load combination consists of dead plus occupancy live load, and the moisture content exceeds 19%. The chord is laterally braced at midlength about the weak axis, and the self-weight of the chord and bracing members may be neglected. Determine whether the member is adequate.

Solution

The reference design values for compression and modulus of elasticity are tabulated in NDS Supp. Table 4A and are

$$F_c = 1700 \text{ lbf/in}^2$$

$$E_{\min} = 0.69 \times 10^6 \text{ lbf/in}^2$$

$$C_T = 1.0, \ C_t = 1.0, \ C_i = 1.0$$

Figure 5.5 *Axially Loaded Column*

Figure 5.6 *Buckling Length Coefficients*

illus.	end conditions	K_e theoretical	design
(a)	both ends pinned	1	1.00
(b)	both ends built in	0.5	0.65
(c)	one end pinned, one end built in	0.7	0.8
(d)	one end built in, one end free	2	2.10
(e)	one end built in, one end fixed against rotation but free to translate	1	1.20
(f)	one end pinned, one end fixed against rotation but free to translate	2	2.40
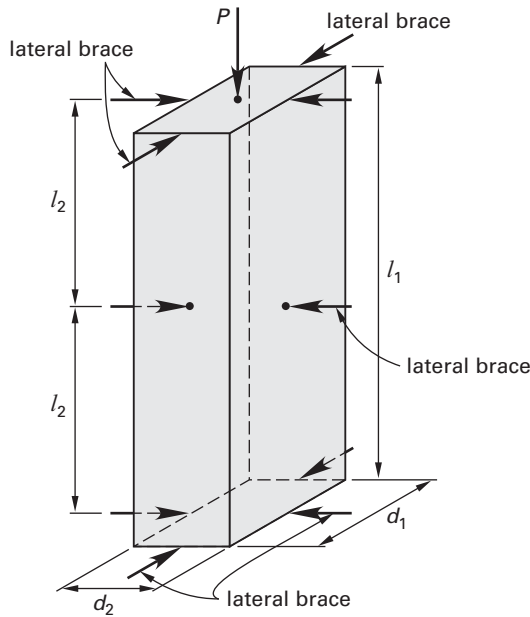
The applicable adjustment factors for compression and modulus of elasticity are as follows.

$$C_M = \text{wet service factor from Table 5.6}$$

$$= 0.8 \quad \text{[compression member]}$$

$$= 0.9 \quad \text{[modulus of elasticity]}$$

$$C_F = \text{size factor from NDS Supp. Table 4A}$$

$$= 1.1 \quad \text{[compression member]}$$

From the illustration, the slenderness ratio about the strong axis is

$$\frac{K_e l_1}{d_1} = \frac{(1.0)(8 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{5.5 \text{ in}}$$

$$= 17.46$$

From the illustration, the slenderness ratio about the weak axis is

$$\frac{K_e l_2}{d_2} = \frac{(1.0)(4 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{1.5 \text{ in}}$$

$$= 32.00 \quad \text{[governs]}$$

ASD Method

The adjusted modulus of elasticity for stability calculations is

$$E'_{\min} = E_{\min} C_M = \left(0.69 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right)(0.9)$$

$$= 0.62 \times 10^6 \text{ lbf/in}^2$$

The load duration factor for dead load plus occupancy live load from Table 5.5 is

$$C_D = 1.00$$

The reference compression design value multiplied by all applicable adjustment factors except C_P is given by

$$F_c^* = F_c C_D C_M C_F$$

$$= \left(1700 \frac{\text{lbf}}{\text{in}^2}\right)(1.00)(0.8)(1.1)$$

$$= 1496 \text{ lbf/in}^2$$

The critical buckling design value is

$$F_{cE2} = \frac{0.822 E'_{\min}}{\left(\frac{l_{e2}}{d_2}\right)^2} = \frac{(0.822)\left(0.62 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right)}{\left(\frac{(4 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{1.5 \text{ in}}\right)^2}$$

$$= 498 \text{ lbf/in}^2$$

The ratio of F_{cE2} to F_c^* is

$$F' = \frac{F_{cE2}}{F_c^*} = \frac{498 \frac{\text{lbf}}{\text{in}^2}}{1496 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.333$$

The column parameter is obtained from NDS Sec. 3.7.1.5 as

$$c = 0.8 \quad \text{[for sawn lumber]}$$

The column stability factor is specified by NDS Sec. 3.7.1 as

$$C_P = \frac{1.0 + F'}{2c} - \sqrt{\left(\frac{1.0 + F'}{2c}\right)^2 - \frac{F'}{c}}$$

$$= \frac{1.0 + 0.333}{(2)(0.8)} - \sqrt{\left(\frac{1.0 + 0.333}{(2)(0.8)}\right)^2 - \frac{0.333}{0.8}}$$

$$= 0.31$$

The allowable compression design value parallel to grain is

$$F_c' = F_c C_M C_F C_P$$

$$= \left(1700 \frac{\text{lbf}}{\text{in}^2}\right)(0.8)(1.1)(0.31)$$

$$= 464 \text{ lbf/in}^2$$

Applying IBC Eq. 16-9, the factored load is

$$P = D + L$$

$$= 1000 \text{ lbf} + 1800 \text{ lbf}$$

$$= 2800 \text{ lbf}$$

The actual compression stress on the chord is given by

$$f_c = \frac{P}{A} = \frac{2800 \text{ lbf}}{(1.5 \text{ in})(5.5 \text{ in})}$$

$$= 339 \text{ lbf/in}^2$$

$$< F_c'$$

The chord is adequate.

LRFD Method

The adjusted factored modulus of elasticity for stability calculations is

$$E'_{\min} = E_{\min} C_M C_t K_F \phi_s$$

$$= \left(0.69 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$\times (0.9)(1.0)(1.76)(0.85)$$

$$= 0.93 \times 10^6 \text{ lbf/in}^2$$

The time effect factor for dead load plus occupancy live load is obtained from Table 5.4 as $\lambda = 0.8$.

The reference compression design value multiplied by all applicable adjustment factors except C_P is given by

$$F_c^* = F_c C_M C_F C_t C_i K_F \lambda \phi_c$$

$$= \left(1700 \frac{\text{lbf}}{\text{in}^2}\right)(0.8)(1.1)(1.0)(1.0)(2.40)(0.8)(0.90)$$

$$= 2585 \text{ lbf/in}^2$$

The critical buckling design value is

$$F_{cE2} = \frac{0.822 E'_{\min}}{\left(\frac{l_{e2}}{d_2}\right)^2} = \frac{(0.822)\left(0.93 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right)}{\left(\frac{(4 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{1.5 \text{ in}}\right)^2}$$

$$= 747 \text{ lbf/in}^2$$

The ratio of F_{cE2} to F_c^* is

$$F' = \frac{F_{cE2}}{F_c^*} = \frac{747 \frac{\text{lbf}}{\text{in}^2}}{2585 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.289$$

The column parameter is obtained from NDS Sec. 3.7.1.5 as

$$c = 0.8 \quad \text{[for sawn lumber]}$$

The column stability factor is specified by NDS Sec. 3.7.1 as

$$C_P = \frac{1.0 + F'}{2c} - \sqrt{\left(\frac{1.0 + F'}{2c}\right)^2 - \frac{F'}{c}}$$

$$= \frac{1.0 + 0.289}{(2)(0.8)} - \sqrt{\left(\frac{1.0 + 0.289}{(2)(0.8)}\right)^2 - \frac{0.289}{0.8}}$$

$$= 0.27$$

Timber

The adjusted factored compression design value parallel to grain is

$$F'_c = F_c C_M C_F C_P C_i C_t K_F \lambda \phi_c$$

$$= \left(1700 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.8)(1.1)(0.27)(1.0)(1.0)(2.40)$$

$$\times (0.8)(0.90)$$

$$= 698 \ \text{lbf/in}^2$$

Applying IBC Eq. 16-2, the factored load is

$$P = 1.2D + 1.6L$$

$$= (1.2)(1000 \ \text{lbf}) + (1.6)(1800 \ \text{lbf})$$

$$= 4080 \ \text{lbf}$$

The actual compression stress on the chord is given by

$$f_c = \frac{P}{A} = \frac{4080 \ \text{lbf}}{(1.5 \ \text{in})(5.5 \ \text{in})}$$

$$= 495 \ \text{lbf/in}^2$$

$$< F'_c$$

The chord is adequate.

Combined Axial Compression and Flexure

Members subjected to combined compression and flexural stresses due to axial and transverse loading must satisfy the interaction equations given in NDS Sec. 3.9.2 as

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1}}{F'_{b1} C_{m1}} + \frac{f_{b2}}{F'_{b2} C_{m2}} \leq 1.00 \qquad [\textit{NDS 3.9-3}]$$

For bending load applied to the narrow face of the member and concentric axial compression load, the interaction equation reduces to

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1}}{F'_{b1} C_{m3}} \leq 1.00$$

For bending load applied to the wide face of the member and concentric axial compression load, the equation reduces to

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b2}}{F'_{b2} C_{m4}} \leq 1.00$$

For bending loads applied to the narrow and wide faces of the member and no concentric axial load, the equation reduces to

$$\frac{f_{b1}}{F'_{b1}} + \frac{f_{b2}}{F'_{b2} C_{m5}} \leq 1.00$$

Example 5.10

The select structural 2×6 Douglas fir-larch top chord of a truss is loaded with the service level loads shown in the illustration. The governing load combination consists of dead plus live load, and the moisture content exceeds 19%. The chord is laterally braced at midlength about the weak axis, and the self-weight of the chord and bracing members may be neglected. Determine whether the member is adequate.
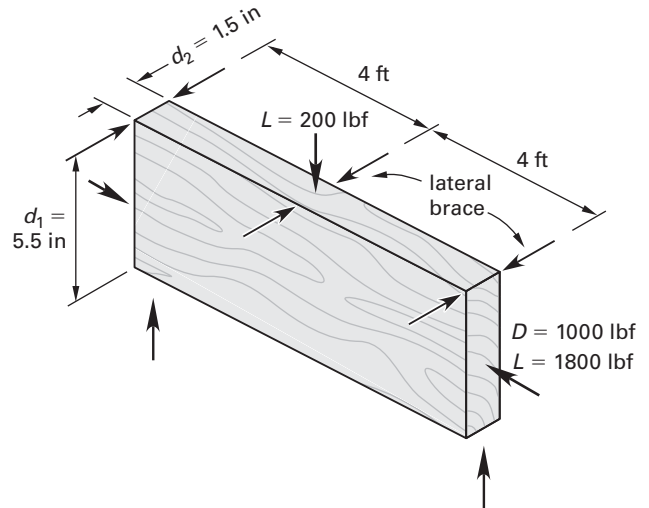
Solution

The reference design values for bending and modulus of elasticity are tabulated in NDS Supp. Table 4A, and they are

$$F_b = 1500 \ \text{lbf/in}^2$$

$$E_{\min} = 0.69 \times 10^6 \ \text{lbf/in}^2$$

$$C_{fu} = 1.0, \ C_t = 1.0, \ C_i = 1.0, \ C_r = 1.0$$

The distance between lateral restraints is

$$l_u = \frac{8 \ \text{ft}}{2}$$

$$= 4 \ \text{ft}$$

From Fig. 5.1, for a concentrated load at midspan and with lateral restraint at midspan, the effective length for flexure is

$$l_e = 1.11 l_u$$

$$= (1.11)(4 \ \text{ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)$$

$$= 53.28 \ \text{in}$$

The slenderness ratio for flexure is given by NDS Sec. 3.3.3 as

$$R_B = \sqrt{\frac{l_e d_1}{d_2^2}} = \sqrt{\frac{(53.28 \text{ in})(5.5 \text{ in})}{(1.5 \text{ in})^2}}$$

$$= 11.41$$

$$< 50 \quad \begin{bmatrix} \text{satisfies criteria of} \\ \text{NDS Sec. 3.3.3} \end{bmatrix}$$

From Table 5.6, the applicable wet service factor for flexure is

$$C_M = 0.85$$

From NDS Supp. Table 4A, the applicable size factor for flexure is

$$C_F = 1.3$$

ASD Method

From Table 5.5, the load duration factor for dead load plus occupancy live load is

$$C_D = 1.00$$

From Ex. 5.9, the adjusted modulus of elasticity for stability calculations is

$$E'_{\min} = E_{\min} C_M = \left(0.69 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.9)$$

$$= 0.62 \times 10^6 \text{ lbf/in}^2$$

The critical buckling design value for flexure is

$$F_{bE} = \frac{1.20 E'_{\min}}{R_B^2}$$

$$= \frac{(1.20)\left(0.62 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)}{(11.41)^2}$$

$$= 5715 \text{ lbf/in}^2$$

The reference flexural design value multiplied by all applicable adjustment factors except C_L is

$$F_b^* = F_b C_M C_F$$

$$= \left(1500 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.85)(1.3)$$

$$= 1657 \text{ lbf/in}^2$$

$$F = \frac{F_{bE}}{F_b^*}$$

$$= \frac{5715 \; \frac{\text{lbf}}{\text{in}^2}}{1657 \; \frac{\text{lbf}}{\text{in}^2}}$$

$$= 3.45$$

The beam stability factor is given by NDS Sec. 3.3.3 as

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}}$$

$$= \frac{1.0 + 3.45}{1.9} - \sqrt{\left(\frac{1.0 + 3.45}{1.9}\right)^2 - \frac{3.45}{0.95}}$$

$$= 0.98$$

The allowable flexural design value for load applied to the narrow face is

$$F'_{b1} = F_b C_M C_L C_F$$

$$= \left(1500 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.85)(0.98)(1.3)$$

$$= 1626 \text{ lbf/in}^2$$

Applying IBC Eq. 16-9, the factored vertical load is

$$W = 1.0L$$

$$= (1.0)(200 \text{ lbf})$$

$$= 200 \text{ lbf}$$

The actual edgewise bending stress is

$$f_{b1} = \frac{WL}{4S}$$

$$= \frac{(200 \text{ lbf})(8 \text{ ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{(4)(7.56 \text{ in}^3)}$$

$$= 635 \text{ lbf/in}^2$$

From Ex. 5.9,

$$\frac{K_e l_1}{d_1} = 17.46$$

$$F'_c = 464 \text{ lbf/in}^2$$

$$f_c = 339 \text{ lbf/in}^2$$

The critical buckling design value, in the plane of bending, for load applied to the narrow face is

$$F_{cE1} = \frac{0.822 E'_{\min}}{\left(\frac{l_{e1}}{d_1}\right)^2}$$

$$= \frac{(0.822)\left(0.62 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)}{\left(\frac{(8 \text{ ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{5.5 \text{ in}}\right)^2}$$

$$= 1672 \text{ lbf/in}^2$$

Timber

The moment magnification factor for axial compression and flexure with load applied to the narrow face is

$$C_{m3} = 1.0 - \frac{f_c}{F_{cE1}}$$

$$= 1.0 - \frac{339 \; \frac{\text{lbf}}{\text{in}^2}}{1672 \; \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.797$$

The interaction equation for bending load applied to the narrow face of the member and concentric axial compression load is given in NDS Sec. 3.9.2 as

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1}}{F'_{b1} C_{m3}} \le 1.0$$

The left side of the expression is

$$\left(\frac{339 \; \frac{\text{lbf}}{\text{in}^2}}{464 \; \frac{\text{lbf}}{\text{in}^2}}\right)^2 + \frac{635 \; \frac{\text{lbf}}{\text{in}^2}}{\left(1626 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.797)} = 0.535 + 0.490$$

$$= 1.025$$

$$\approx 1.0$$

The chord is adequate.

LRFD Method

From Table 5.4, the time effect factor for dead load plus occupancy live load is

$$\lambda = 0.8$$

From Ex. 5.9, the adjusted modulus of elasticity for stability calculations is

$$E'_{\min} = E_{\min} C_M C_t K_F \phi_s$$

$$= \left(0.69 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)$$

$$\times (0.9)(1.0)(1.76)(0.85)$$

$$= 0.93 \times 10^6 \; \text{lbf/in}^2$$

The critical buckling design value for flexure is

$$F_{bE} = \frac{1.20 E'_{\min}}{R_B^2}$$

$$= \frac{(1.20)\left(0.93 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)}{(11.41)^2}$$

$$= 8573 \; \text{lbf/in}^2$$

The reference flexural design value multiplied by all applicable adjustment factors except C_L is

$$F_b^* = F_b C_M C_F C_t C_{fu} C_i C_r K_F \lambda \phi_b$$

$$= \left(1500 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.85)(1.3)(1.0)(1.0)(1.0)(1.0)$$

$$\times (2.54)(0.8)(0.85)$$

$$= 2863 \; \text{lbf/in}^2$$

$$F = \frac{F_{bE}}{F_b^*}$$

$$= \frac{8573 \; \frac{\text{lbf}}{\text{in}^2}}{2863 \; \frac{\text{lbf}}{\text{in}^2}}$$

$$= 3.0$$

The beam stability factor is given by NDS Sec. 3.3.3 as

$$C_L = \frac{1.0 + F}{1.9} - \sqrt{\left(\frac{1.0 + F}{1.9}\right)^2 - \frac{F}{0.95}}$$

$$= \frac{1.0 + 3.0}{1.9} - \sqrt{\left(\frac{1.0 + 3.0}{1.9}\right)^2 - \frac{3.0}{0.95}}$$

$$= 0.98$$

The adjusted factored flexural design value for load applied to the narrow face is

$$F'_{b1} = F_b C_M C_L C_F C_t C_{fu} C_i C_r K_F \lambda \phi_b$$

$$= \left(1500 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.85)(0.98)(1.3)(1.0)(1.0)(1.0)(1.0)$$

$$\times (2.54)(0.8)(0.85)$$

$$= 2806 \; \text{lbf/in}^2$$

Applying IBC Eq. 16-2, the factored vertical load is

$$W = 1.6L = (1.6)(200 \; \text{lbf})$$

$$= 320 \; \text{lbf}$$

The actual edgewise bending stress is

$$f_{b1} = \frac{WL}{4S}$$

$$= \frac{(320 \; \text{lbf})(8 \; \text{ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{(4)(7.56 \; \text{in}^3)}$$

$$= 1016 \; \text{lbf/in}^2$$

From Ex. 5.9,

$$\frac{K_e l_1}{d_1} = 17.46$$

$$F'_c = 698 \; \text{lbf/in}^2$$

$$f_c = 495 \; \text{lbf/in}^2$$

The critical buckling design value, in the plane of bending, for load applied to the narrow face is

$$F_{cE1} = \frac{0.822 E'_{\min}}{\left(\frac{l_{e1}}{d_1}\right)^2}$$

$$= \frac{(0.822)\left(0.93 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)}{\left(\dfrac{(8 \text{ ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{5.5 \text{ in}}\right)^2}$$

$$= 2509 \text{ lbf/in}^2$$

The moment magnification factor for axial compression and flexure with load applied to the narrow face is

$$C_{m3} = 1.0 - \frac{f_c}{F_{cE1}}$$

$$= 1.0 - \frac{495 \; \frac{\text{lbf}}{\text{in}^2}}{2509 \; \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.803$$

The interaction equation for bending load applied to the narrow face of the member and concentric axial compression load is given in NDS Sec. 3.9.2 as

$$\left(\frac{f_c}{F'_c}\right)^2 + \frac{f_{b1}}{F'_{b1}\,C_{m3}} \le 1.0$$

The left side of the expression is

$$\left(\frac{495 \; \frac{\text{lbf}}{\text{in}^2}}{698 \; \frac{\text{lbf}}{\text{in}^2}}\right)^2 + \frac{1016 \; \frac{\text{lbf}}{\text{in}^2}}{\left(2806 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.803)} = 0.503 + 0.451$$

$$= 0.95$$
$$< 1.0$$

The chord is adequate.

10. DESIGN FOR TENSION

Nomenclature

f_t	actual tension stress parallel to grain	lbf/in^2
F_b^*	reference bending design value multiplied by all applicable adjustment factors except C_L	lbf/in^2
F_b^{**}	reference bending design value multiplied by all applicable adjustment factors except C_V	lbf/in^2
F_t, F'_t	reference and adjusted tension design value parallel to grain	lbf/in^2
T	tensile force on member	lbf

Combined Axial Tension and Flexure

Members subjected to combined tension and flexural stresses due to axial and transverse loading must satisfy the two expressions given in NDS Sec. 3.9.1 as

$$\frac{f_t}{F'_t} + \frac{f_b}{F_b^*} \le 1.0 \qquad \text{[NDS 3.9-1]}$$

$$\frac{f_b - f_t}{F_b^{**}} \le 1.0 \qquad \text{[NDS 3.9-2]}$$

Example 5.11

The select structural 2×6 Douglas fir-larch bottom chord of a truss is loaded with the service level loads shown in the illustration. The governing load combination consists of dead plus occupancy live load, and the moisture content exceeds 19%. The chord is laterally braced at midlength about the weak axis, and the self-weight of the chord and bracing members may be neglected. Determine whether the member is adequate.
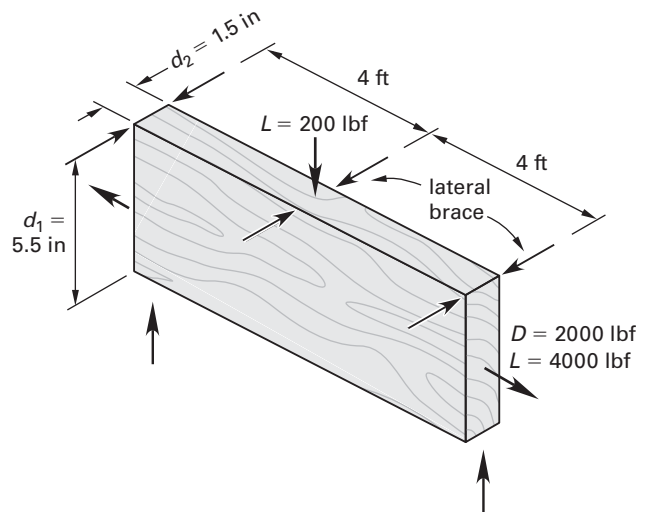
Solution

The reference design value for tension is tabulated in NDS Supp. Table 4A and is

$$F_t = 1000 \text{ lbf/in}^2$$
$$C_t = 1.0, \; C_i = 1.0$$

The applicable adjustment factors for tension are as follows.

$C_M = 1.00$ [wet service factor from Table 5.6]

$C_F = 1.3$ [size factor from NDS Supp. Table 4A]

The applicable adjustment factors for flexure are as follows.

$C_M = 0.85$ [wet service factor from Table 5.6]

$C_F = 1.3$ [size factor from NDS Supp. Table 4A]

ASD Method

From Table 5.5, the load duration factor for dead load plus occupancy live load is

$$C_D = 1.00$$

The adjusted allowable tension design value parallel to grain is

$$F_t' = F_t C_M C_F C_D$$
$$= \left(1000 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.00)(1.3)(1.00)$$
$$= 1300 \ \text{lbf/in}^2$$

Applying IBC Eq. 16-9, the factored tensile load is

$$T = 1.0D + 1.0L$$
$$= (1.0)(2000 \ \text{lbf}) + (1.0)(4000 \ \text{lbf})$$
$$= 6000 \ \text{lbf}$$

The actual tension stress on the chord is given by

$$f_t = \frac{T}{A}$$
$$= \frac{6000 \ \text{lbf}}{8.25 \ \text{in}^2}$$
$$= 727 \ \text{lbf/in}^2$$
$$< F_t' \quad [\text{satisfactory}]$$

From Ex. 5.10, the beam stability factor is

$$C_L = 0.98$$

The actual edgewise bending stress is

$$f_{b1} = 635 \ \text{lbf/in}^2$$

The reference bending design value multiplied by all applicable adjustment factors except C_L is

$$F_b^* = F_b C_M C_F C_D$$
$$= \left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.85)(1.3)(1.0)$$
$$= 1657 \ \text{lbf/in}^2$$

The reference bending design value multiplied by all applicable adjustment factors except C_V is

$$F_b^{**} = F_b C_M C_L C_F C_D$$
$$= \left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.85)(0.98)(1.3)(1.0)$$
$$= 1626 \ \text{lbf/in}^2$$

Substituting in the two expressions given in NDS Sec. 3.9.1 gives

$$\frac{f_t}{F_t'} + \frac{f_{b1}}{F_b^*} = \frac{727 \ \frac{\text{lbf}}{\text{in}^2}}{1300 \ \frac{\text{lbf}}{\text{in}^2}} + \frac{635 \ \frac{\text{lbf}}{\text{in}^2}}{1658 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.942$$
$$< 1.0 \quad [\text{satisfactory}]$$

$$\frac{f_{b1} - f_t}{F_b^{**}} = \frac{635 \ \frac{\text{lbf}}{\text{in}^2} - 727 \ \frac{\text{lbf}}{\text{in}^2}}{1626 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$< 1.0 \quad [\text{satisfactory}]$$

The chord is adequate.

LRFD Method

From Table 5.4, the time effect factor for dead load and occupancy live load is

$$\lambda = 0.8$$

The adjusted factored tension design value parallel to grain is

$$F_t' = F_t C_M C_F C_t C_i K_F \lambda \phi_t$$
$$= \left(1000 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.3)(1.0)(1.0)(2.70)(0.8)(0.80)$$
$$= 2246 \ \text{lbf/in}^2$$

Applying IBC Eq. 16-2, the factored tension load is

$$T = 1.2D + 1.6L$$
$$= (1.2)(2000 \ \text{lbf}) + (1.6)(4000 \ \text{lbf})$$
$$= 8800 \ \text{lbf}$$

The actual tension stress on the chord is given by

$$f_t = \frac{T}{A}$$
$$= \frac{8800 \ \text{lbf}}{(1.5 \ \text{in})(5.5 \ \text{in})}$$
$$= 1067 \ \text{lbf/in}^2$$
$$< F_t' \quad [\text{satisfactory}]$$

From Ex. 5.10, the beam stability factor is

$$C_L = 0.98$$

The actual edgewise bending stress is

$$f_{b1} = 1016 \ \text{lbf/in}^2$$

The reference bending design value multiplied by all applicable adjustment factors except C_L is

$$F_b^* = F_b C_M C_F C_t C_{fu} C_i C_r K_F \lambda \phi_b$$

$$= \left(1500\ \frac{\text{lbf}}{\text{in}^2}\right)(0.85)(1.3)(1.0)(1.0)(1.0)(1.0)$$

$$\times (2.54)(0.8)(0.85)$$

$$= 2863\ \text{lbf/in}^2$$

The reference bending design value multiplied by all applicable adjustment factors except C_V is

$$F_b^{**} = F_b C_M C_L C_F C_t C_{fu} C_i C_r K_F \lambda \phi_b$$

$$= \left(1500\ \frac{\text{lbf}}{\text{in}^2}\right)(0.85)(0.98)(1.3)(1.0)(1.0)(1.0)(1.0)$$

$$\times (2.54)(0.8)(0.85)$$

$$= 2806\ \text{lbf/in}^2$$

Substituting in the two expressions given in NDS Sec. 3.9.1 gives

$$\frac{f_t}{F_t'} + \frac{f_{b1}}{F_b^*} = \frac{1067\ \frac{\text{lbf}}{\text{in}^2}}{2246\ \frac{\text{lbf}}{\text{in}^2}} + \frac{1016\ \frac{\text{lbf}}{\text{in}^2}}{2863\ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.83$$

$$< 1.0 \quad [\text{satisfactory}]$$

$$\frac{f_{b1} - f_t}{F_b^{**}} = \frac{1016\ \frac{\text{lbf}}{\text{in}^2} - 1067\ \frac{\text{lbf}}{\text{in}^2}}{2806\ \frac{\text{lbf}}{\text{in}^2}}$$

$$< 1.0 \quad [\text{satisfactory}]$$

The chord is adequate.

11. DESIGN OF CONNECTIONS

Nomenclature

a	center-to-center spacing between adjacent rows of fasteners	in
a_e	minimum edge distance with load parallel to grain	in
a_p	minimum end distance with load parallel to grain	in
a_q	minimum end distance with load perpendicular to grain	in
A	area of cross section	in²
A_m	gross cross-sectional area of main wood member(s)	in²
A_n	net area of member	in²
A_s	sum of gross cross-sectional areas of side member(s)	in²
C_d	penetration depth factor for connections	–
C_{di}	diaphragm factor for nailed connections	–
C_{eg}	end grain factor for connections	–
C_g	group action factor for connections	–
C_{st}	metal side plate factor for 4 in shear plate connections	–
C_{tn}	toe-nail factor for nailed connections	–
C_Δ	geometry factor for connections	–
d	pennyweight of nail or spike	–
d_e	effective depth of member at a connection	in
D	diameter	in
e_p	minimum edge distance unloaded edge	in
e_q	minimum edge distance loaded edge	in
E	length of tapered tip	in
g	gage of screw	–
l_m	length of bolt in wood main member	in
l_s	total length of bolt in wood side member(s)	in
L	length of nail	in
L	length of screw	in
n	number of fasteners in a row	–
N, N'	reference and adjusted lateral design values at an angle of α to the grain for a single split ring connector unit or shear plate connector unit	lbf
p	depth of fastener penetration into wood member	in
P, P'	reference and adjusted lateral design values parallel to grain for a single split ring connector unit or shear plate connector unit	lbf
Q, Q'	reference and adjusted lateral design values perpendicular to grain for a single split ring connector unit or shear plate connector unit	lbf
s	center-to-center spacing between adjacent fasteners in a row	in
S	unthreaded shank length	in
t_m	thickness of main member	in
t_s	thickness of side member	in
T	minimum thread length	in
W, W'	reference and adjusted withdrawal design values for fastener	lbf/in
Z_α'	allowable design values for lag screw with load applied at an angle α to the wood surface	lbf
Z, Z'	reference and adjusted lateral design values for a single fastener connection	lbf
$Z_\parallel$	reference lateral design value for a single bolt or lag screw connection with all wood members loaded parallel to grain	lbf
$Z_{m\perp}$	reference lateral design value for a single bolt or lag screw wood-to-wood connection with main member loaded perpendicular to grain and side member loaded parallel to grain	lbf
$Z_{s\perp}$	reference lateral design value for a single bolt or lag screw wood-to-wood connection with main member loaded parallel to grain and side member loaded perpendicular to grain	lbf
$Z_\perp$	reference lateral design value for a single bolt or lag screw, wood-to-wood, wood-to-metal, or wood-to-concrete connection with all wood member(s) loaded perpendicular to grain	lbf

Timber

Symbols

α	angle between wood surface and direction of applied load	degree
γ	load/slip modulus for a connection	lbf/in
θ	angle between direction of load and direction of grain (longitudinal axis of member)	degree

Adjustment of Design Values

The reference design values for fasteners are given in NDS Part 10 through Part 13. These design values are applicable to single fastener connections and normal conditions of use as defined in NDS Sec. 2.2. For other conditions of use, these values are multiplied by adjustment factors, specified in NDS Sec. 10.3, to determine the relevant design values. A summary of the adjustment factors follows, and the applicability of each to the nominal design values is shown in Table 5.9.

Load Duration Factor, C_D

With the exception of the impact load duration factor, values of the load duration factor given in Table 5.5 are applicable to connections.

Wet Service Factor, C_M

When the moisture content of the member exceeds 19%, the adjustment factors given in NDS Table 10.3.3 are applicable.

Temperature Factor, C_t

The temperature factor is applicable to all connectors and is specified by NDS Table 10.3.4.

Group Action Factor, C_g

The group action factors for various connection geometries and fastener types are given in NDS Table 10.3.6A

through Table 10.3.6D. This factor is dependent on the ratio of the area of the side members in a connection to the area of the main member, A_s/A_m. A_m and A_s are calculated by using gross areas without deduction for holes. When adjacent rows of fasteners are staggered, as shown in Fig. 5.7, the adjacent rows are considered a single row.

Figure 5.7 *Staggered Fasteners*
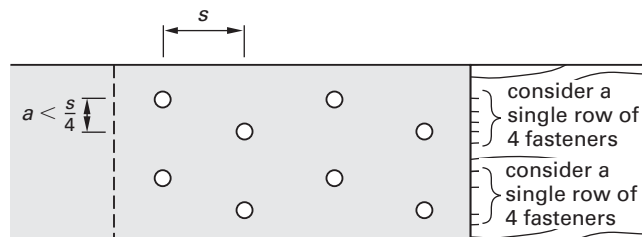
Geometry Factor, C_Δ

The geometry factor applies to bolts, lag screws, split rings, and shear plates. The factor is applied, in accordance with NDS Sec. 11.5.1 and Sec. 12.3.2, when end or edge distances or spacing are less than the specified minimum.

The geometry factor and group action factor are not applied to nails or screws. NDS Comm. Table C11.1.5.7 provides recommended spacing requirements for screws. NDS Comm. Table C11.1.6.6 provides recommended spacing requirements for nails.

Penetration Depth Factor, C_d

The penetration depth factor applies to lag screws, split rings, shear plates, screws, and nails. The factor is applied in accordance with NDS Table 12.2.3 and footnotes to NDS Table 11J through Table 11R when the penetration is less than the minimum specified.

Table 5.9 *Adjustment Factors for Connections*[*]

adjustment factor	bolts	lag screws		split rings and shear plates		screws		nails	
design value	Z	W	Z	P	Q	W	Z	W	Z
C_D load duration factor	√	√	√	√	√	√	√	√	√
C_M wet service factor	√	√	√	√	√	√	√	√	√
C_t temperature factor	√	√	√	√	√	√	√	√	√
C_g group action factor	√	–	√	√	√	–	–	–	–
C_Δ geometry factor	√	–	√	√	√	–	–	–	–
C_d penetration depth factor	–	–	√	√	√	–	√	–	√
C_{eg} end grain factor	–	√	√	–	–	–	√	–	√
C_{st} metal side plate factor	–	–	–	√	–	–	–	–	–
C_{di} diaphragm factor	–	–	–	–	–	–	–	–	√
C_{tn} toe-nail factor	–	–	–	–	–	–	–	√	√

[*]Z=lateral design value; W=withdrawal design value; P=parallel to grain design value; Q=perpendicular to grain design value

End Grain Factor, C_{eg}

The end grain factor applies to lag screws, screws, and nails. The factor is applied in accordance with NDS Sec. 11.5.2 when the fastener is inserted in the end grain of a member.

C_{eg} is 0.75 for lag screws loaded in withdrawal. C_{eg} is 0.67 for laterally loaded dowel-type fasteners.

Metal Side Plate Factor, C_{st}

The metal side plate factor is applicable to split rings and shear plates. The factor is applied in accordance with NDS Sec. 12.2.4 when metal side plates are used instead of wood side members.

The effect of metal side plates on the lateral design values of bolts, lag screws, wood screws, and nails are incorporated into the appropriate tables of reference design values.

Diaphragm Factor, C_{di}

The diaphragm factor applies to nails and spikes. The factor is applied in accordance with NDS Sec. 11.5.3 when the fasteners are used in diaphragm construction and $C_{di} = 1.1$.

Toe-Nail Factor, C_{tn}

The toe-nail factor applies to nails and spikes. The factor is applied in accordance with NDS Sec. 11.5.4 when toe-nailed connections are used and $C_{tn} = 0.83$ for lateral design values.

Three additional factors are applied in the LRFD method. The format conversion factor given by NDS Table 10.3.1 is $K_F = 3.32$. The resistance factor given by NDS Table 10.3.1 is $\phi = 0.65$. The time effect factor, λ, is given in Table 5.4.

Example 5.12

A bolted connection in tension consists of a single row of eight $3/4$ in diameter bolts in two select structural 2×6 Douglas fir-larch members in single shear. The governing load combination consists of dead plus occupancy live load, and the moisture content exceeds 19%. The bolt spacing and end distance are 4 in. Determine the capacity of the connection.

Solution

The $3/4$ in diameter bolt reference design value for single shear is tabulated in NDS Table 11A as

$$Z_{\parallel} = 720 \text{ lbf}$$
$$C_t = 1.0$$
$$A_s = A_m = 8.25 \text{ in}^2$$

The specified minimum end distance for the full bolt design value is specified in NDS Sec. 11.5.1 as

$$a_p = 7D$$
$$= (7)(0.75 \text{ in})$$
$$= 5.25 \text{ in}$$

The applicable adjustment factors for the bolts are as follows.

$C_M = 0.7$ [wet service factor from NDS Table 10.3.3]

$C_g = 0.71$ [group action factor from NDS Table 10.3.6A]

$$C_\Delta = \frac{\text{actual end distance}}{\text{specified minimum end distance}}$$
$$= \frac{4 \text{ in}}{5.25 \text{ in}}$$
$$= 0.76 \quad \text{[geometry factor from NDS Sec. 11.5.1]}$$

ASD Method

From Table 5.5,

$$C_D = 1.00$$

From Ex. 5.11, the allowable tension capacity of the members is

$$T = F'_t A_n$$
$$= F'_t \left(A - \left(D + \frac{1}{16} \right) b \right)$$
$$= \left(1300 \ \frac{\text{lbf}}{\text{in}^2} \right) \left(8.25 \text{ in}^2 - \left(\frac{3}{4} \text{ in} + \frac{1}{16} \text{ in} \right) (1.5 \text{ in}) \right)$$
$$= 9140 \text{ lbf}$$

The allowable capacity for eight bolts is

$$T = n Z_{\parallel} C_M C_g C_\Delta C_D C_t$$
$$= (8)(720 \text{ lbf})(0.7)(0.71)(0.76)(1.00)(1.0)$$
$$= 2176 \text{ lbf} \quad \text{[governs]}$$

LRFD Method

From Table 5.4, for dead load plus occupancy live load, λ is 0.8. From Ex. 5.11, the strength capacity of the members in tension is

$$T = F'_t A_n$$
$$= F'_t \left(A - \left(D + \frac{1}{16} \right) b \right)$$
$$= \left(2246 \ \frac{\text{lbf}}{\text{in}^2} \right) \left(8.25 \text{ in}^2 - \left(\frac{3}{4} \text{ in} + \frac{1}{16} \text{ in} \right) (1.5 \text{ in}) \right)$$
$$= 15{,}791 \text{ lbf}$$

Timber

The strength lateral capacity for eight bolts is

$$T = nZ_{\parallel} C_M C_g C_\Delta C_t K_F \lambda \phi$$
$$= (8)(720 \text{ lbf})(0.7)(0.71)(0.76)(1.0)(3.32)(0.8)(0.65)$$
$$= 3756 \text{ lbf} \quad [\text{governs}]$$

Bolted Connections

Installation Requirements

In accordance with NDS Sec. 11.1.3, bolt holes must be $1/32$ in to $1/16$ in larger than the bolt diameter, and a metal washer or plate is required between the wood and the nut and bolt head. To ensure that the full design values of bolts are attained, spacing and edge and end distances are specified in NDS Sec. 11.5 and are illustrated in Fig. 5.8.

Single Shear Connection

Reference design values for single shear connections are specified in NDS Sec. 11.3 and are tabulated in NDS Table 11A for two sawn lumber members of identical species, in NDS Table 11B for a sawn lumber member with a steel side plate, in NDS Table 11C for a glued

laminated member with sawn lumber side member, in NDS Table 11D for a glued laminated member with a steel side plate, and in NDS Table 11E for connections to concrete.

Example 5.13

A 3×8 select structural Douglas fir-larch ledger attached to a concrete wall with $3/4$ in hook bolts at 4 ft centers is shown. What is the maximum dead load plus occupancy live load that the ledger can support?
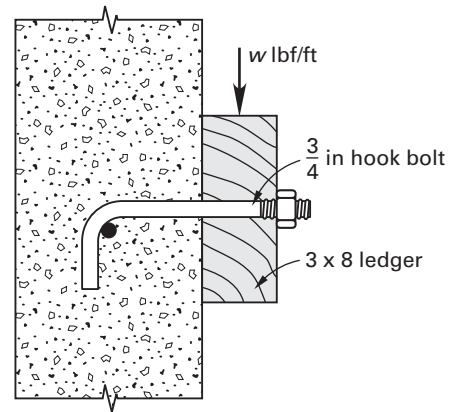
Figure 5.8 *Bolt Spacing Requirements for Full Design Values*
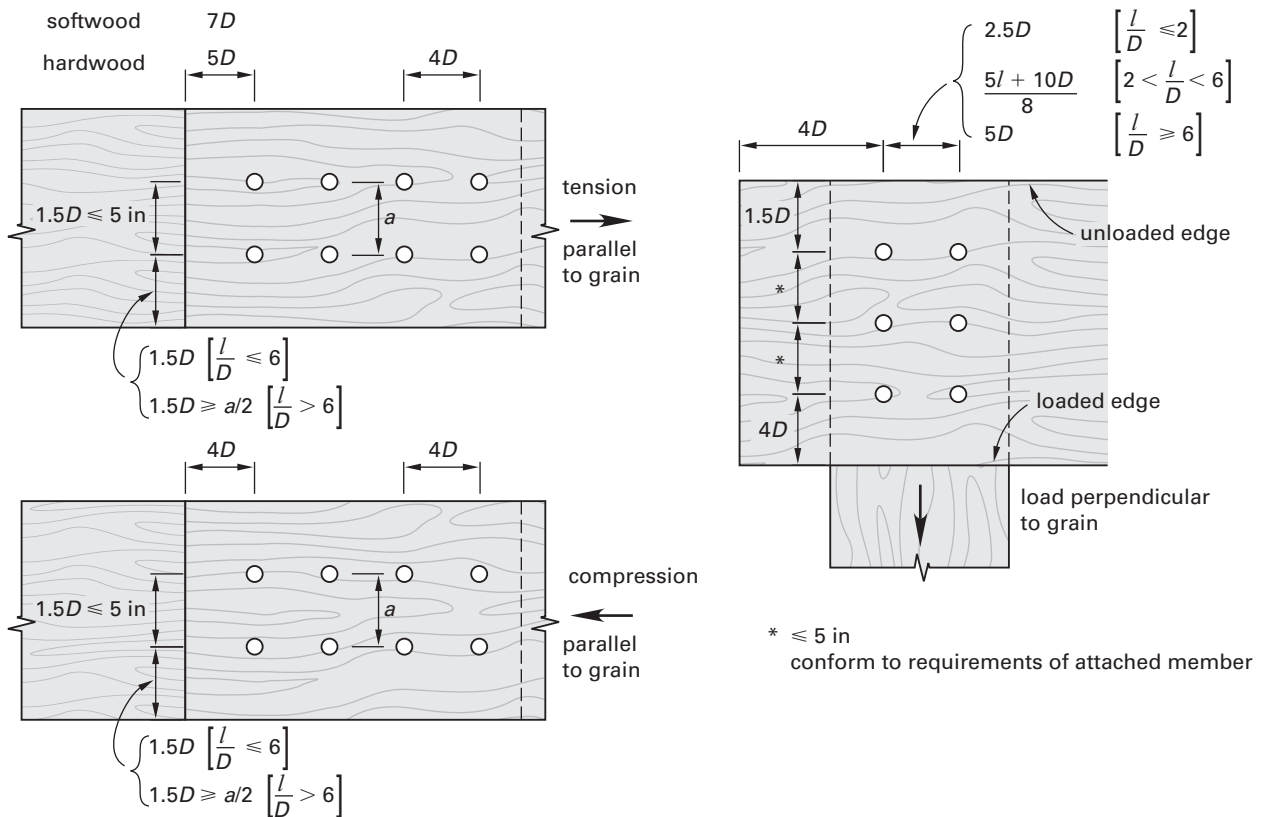
l = lesser of length of bolt in main member or total length of bolt in side member(s)
D = diameter of bolt

Solution

The applicable adjustment factors are

$$C_M = C_g = C_t = 1.0$$

From NDS Table 11E, the reference lateral design value of a $^3/_4$ in diameter bolt in a $2^1/_2$ in thick member loaded perpendicular to the grain and attached to a concrete wall is

$$Z_\perp = 800 \text{ lbf}$$

From NDS Table 11.5.1C, the minimum edge distance for the full bolt design value, for loading perpendicular to grain, is

$$e_{q,\text{full}} = 4D = (4)(0.75 \text{ in}) = 3.0 \text{ in}$$

The actual edge distance is

$$e_q = \frac{7.5 \text{ in}}{2}$$
$$= 3.75 \text{ in} > e_{q,\text{full}} \quad [\text{satisfactory}]$$

The geometry factor given by NDS Sec. 11.5.1 is

$$C_\Delta = 1.0$$

ASD Method

From Table 5.5, for dead load plus occupancy live load, the load duration factor is

$$C_D = 1.00$$

The adjusted allowable lateral design value is

$$Z'_\perp = Z_\perp C_M C_g C_\Delta C_t C_D$$
$$= (800 \text{ lbf})(1.0)(1.0)(1.0)(1.0)(1.00)$$
$$= 800 \text{ lbf}$$

The maximum allowable load that the ledger can support is

$$w = \frac{Z'_\perp}{4} = \frac{800 \text{ lbf}}{4 \text{ ft}}$$
$$= 200 \text{ lbf/ft}$$

LRFD Method

From Table 5.4, for dead load plus occupancy live load, the time effect factor is

$$\lambda = 0.8$$

From NDS Table 10.3.1, the format conversion factor is

$$K_F = 3.32$$

The resistance factor given by NDS Table 10.3.1 is

$$\phi = 0.65$$

The adjusted factored lateral design value is

$$Z'_\perp = Z_\perp C_M C_g C_\Delta C_t K_F \lambda \phi$$
$$= (800 \text{ lbf})(1.0)(1.0)(1.0)(1.0)(3.32)(0.8)(0.65)$$
$$= 1381 \text{ lbf}$$

The maximum strength level load that the ledger can support is

$$w = \frac{Z'_\perp}{4} = \frac{1381 \text{ lbf}}{4 \text{ ft}} = 345 \text{ lbf/ft}$$

Double Shear Connection

Reference design values for double shear connections are tabulated in NDS Table 11F for three sawn lumber members of identical species, in NDS Table 11G for a sawn lumber member with steel side plates, in NDS Table 11H for a glued laminated member with sawn lumber side members, and in NDS Table 11I for a glued laminated member with steel side plates.

Example 5.14

Determine the minimum values for the dimensions A, B, C, and D, shown in the illustration, that will allow the full design values to be applied to the $^3/_4$ in diameter bolts. Determine the maximum tensile force, T, due to wind load that can be resisted by the connection. The $5^1/_8 \times 12$ glued laminated member is of Douglas fir-larch species.

Illustration for Ex. 5.11
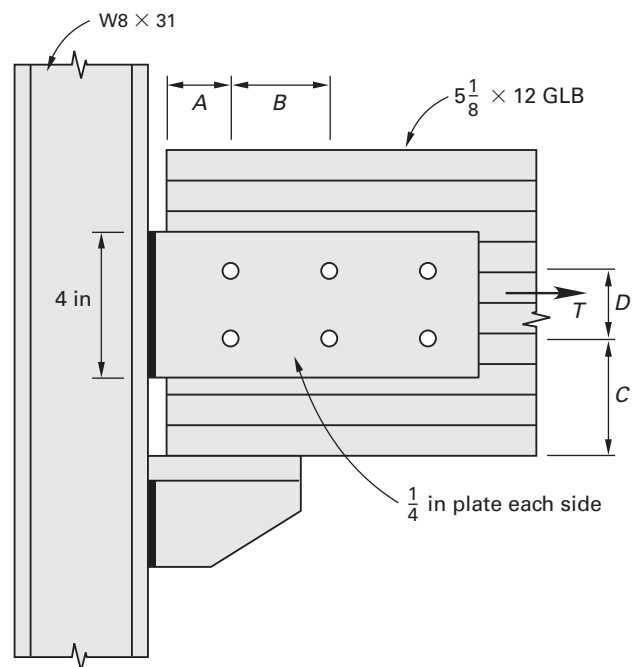
(not to scale)

Solution

The $^3/_4$ in diameter bolt reference design value for double shear is tabulated in NDS Table 11I as

$$Z_\| = 3340 \text{ lbf}$$

$$C_M = 1.0, \ C_t = 1.0$$

$$A_s = 2 \text{ in}^2, \ A_m = 61.5 \text{ in}^2$$

$$\frac{A_m}{A_s} = 30.75$$

The applicable adjustment factors for the bolts are as follows.

$C_g = $ group action factor from NDS Table 10.3.6C
$\quad = 0.99$
$C_\Delta = $ geometry factor from NDS Sec. 11.5.1
$\quad = 1.0 \quad \begin{bmatrix} \text{All dimensions conform} \\ \text{to the specified minimums.} \end{bmatrix}$

The specified minimum end distance, A, for the full bolt design value is specified in NDS Table 11.5.1B as

$$a_p = 7D$$

$$= (7)(0.75 \text{ in})$$

$$= 5.25 \text{ in}$$

The specified minimum spacing, B, for the full bolt design value is specified in NDS Table 11.5.1C as

$$s = 4D$$

$$= (4)(0.75 \text{ in})$$

$$= 3.00 \text{ in}$$

The specified minimum spacing between rows, D, for the full bolt design value is specified in NDS Table 11.5.1D as

$$a = 1.5D$$

$$= (1.5)(0.75 \text{ in})$$

$$= 1.125 \text{ in}$$

The ratio of the length of the bolt in the main member to the bolt diameter is

$$\frac{l_m}{D} = \frac{5.125 \text{ in}}{0.75 \text{ in}}$$

$$= 6.83$$

$$> 6$$

The specified minimum edge distance, C, for the full bolt design value is specified in NDS Table 11.5.1A as the greater of

- $a_e = \dfrac{a}{2}$

$$= \frac{1.125 \text{ in}}{2}$$

$$= 0.563 \text{ in}$$

- $a_e = 1.5D$

$$= (1.5)(0.75 \text{ in})$$

$$= 1.125 \text{ in} \quad \text{[governs]}$$

ASD Method

$C_D = $ load duration factor for wind load
$\qquad$ from Table 5.5
$\quad = 1.60$

The allowable lateral capacity for six bolts is

$$T = nZ_\| C_D C_g C_\Delta C_M C_t$$

$$= (6)(3340 \text{ lbf})(1.60)(0.99)(1.0)(1.0)(1.0)$$

$$= 31{,}743 \text{ lbf}$$

LRFD Method

From Table 5.4, the time effect factor for wind load is

$$\lambda = 1.0$$

From NDS Table 10.3.1, the format conversion factor is

$$K_F = 3.32$$

From NDS Table 10.3.1, the resistance factor is

$$\phi = 0.65$$

The strength lateral capacity for six bolts is

$$T = nZ_\| C_M C_t C_g C_\Delta K_F \lambda \phi$$

$$= (6)\Big(3340 \text{ lbf}\Big)(1.0)(1.0)(0.99)(1.0)(3.32)(1.0)(0.65)$$

$$= 42{,}814 \text{ lbf}$$

Lag Screw Connections

Installation Requirements

In accordance with NDS Sec. 11.1.4, a clearance hole matching the diameter of the shank must be bored for the full length of the unthreaded shank in the member. A lead hole at least equal in length to the threaded portion of the screw must be provided. For wood with a specific gravity greater than 0.6, the lead hole diameter must equal 65–85% of the shank diameter. For wood with a specific gravity of 0.5 or less, the lead hole

diameter must equal 40–70% of the shank diameter. For wood with an intermediate specific gravity, the lead hole diameter must equal 60–75% of the shank diameter.

Lag screws are inserted into the lead hole using a wrench. Lead holes and clearance holes are not required when lag screws with a diameter $3/8$ in or less are loaded primarily in withdrawal in wood that has a specific gravity of 0.5 or less.

Lateral Design Values in Side Grain

Minimum edge distances, end distances, spacing, and geometry factors are identical with those for bolts with a diameter equal to the shank diameter of the lag screw. As specified in NDS Table 11J, for full design values to be applicable, the depth of bolt penetration must not be less than

$$p = 8D$$

The minimum allowable penetration is $4D$. When the penetration is between $4D$ and $8D$, the nominal design value is multiplied by the penetration factor, which is defined in NDS Table 11J as

$$C_d = \frac{p}{8D}$$
$$\leq 1.0$$

Reference design values for single shear connections are specified in NDS Sec. 11.3 and are tabulated in NDS Table 11J for connections with a wood side member and in NDS Table 11K for connections with a steel side plate.

Withdrawal Design Values in Side Grain Without Lateral Load

Minimum edge distance, end distance, and spacing are specified in NDS Table 11.5.1E and are

$$\text{edge distance} = 1.5D$$
$$\text{end distance} = 4D$$
$$\text{spacing} = 4D$$

Reference withdrawal design values in pounds per inch of thread penetration (not including the length of the tapered tip) are tabulated in NDS Table 11.2A.

Combined Lateral and Withdrawal Design Values

When the load applied to a lag screw is at an angle, α, to the wood surface, the lag screw is subjected to combined lateral and withdrawal loading. The design value is determined by the Hankinson formula given by NDS Sec. 11.4.1 as

$$Z'_\alpha = \frac{W'pZ'}{W'p\cos^2\alpha + Z'\sin^2\alpha}$$

Example 5.15

A 3 in long, $3/8$ in diameter lag screw inserted into a Douglas fir-larch joist with a 10-gage steel side plate is subjected to a force inclined at an angle of 30° to the wood surface. Determine the maximum force, due to occupancy live load, that may be applied.

Solution

From NDS Table 11K, the nominal lateral design value for load applied parallel to the grain is

$$Z_{||} = 220 \text{ lbf}$$
$$C_M = 1.0, \ C_t = 1.0, \ C_g = 1.0, \ C_\Delta = 1.0$$

From NDS App. Table L2, the penetration into the main member of the screw shank, plus the threaded length is

$$p = S + T - t_s$$
$$= 1.0 \text{ in} + 2.0 \text{ in} - 0.134 \text{ in}$$
$$= 2.866 \text{ in}$$

From NDS Table 11K, footnote 3, the penetration factor is obtained as

$$C_d = \frac{p}{8D} = \frac{2.866 \text{ in}}{(8)(0.375 \text{ in})}$$
$$= 0.955$$

From NDS Table 11K, the reference lateral design value is $Z_{||} = 220$ lbf.

From NDS Table 11.3.3A, the specific gravity of the Douglas fir-larch joist is

$$G = 0.50$$

From NDS App. Table L2, the penetration into the main member of the threaded length, less the length of the tapered tip, is

$$p = T - E$$
$$= 1.781 \text{ in}$$

ASD Method

For occupancy live load, $C_D = 1.0$.

The adjusted allowable lateral design value is

$$Z'_{||} = Z_{||} C_d C_D C_M C_t C_g C_\Delta$$
$$= (220 \text{ lbf})(0.955)(1.0)(1.0)(1.0)(1.0)(1.0)$$
$$= 210 \text{ lbf}$$

From NDS Table 11.2A, the reference withdrawal design value is

$$W = 305 \text{ lbf/in}$$

The adjusted allowable withdrawal design value is

$$W' = W C_D C_M C_t$$
$$= \left(305 \ \frac{\text{lbf}}{\text{in}}\right)(1.0)(1.0)(1.0)$$
$$= 305 \ \text{lbf/in}$$

The maximum allowable force that may be applied is determined by NDS Sec. 11.4.1 as

$$Z'_\alpha = \frac{(W'p)Z'_\|}{(W'p)\cos^2\alpha + Z'_\| \sin^2\alpha}$$
$$= \frac{\left(305 \ \dfrac{\text{lbf}}{\text{in}}\right)(1.781 \ \text{in})(210 \ \text{lbf})}{\left(305 \ \dfrac{\text{lbf}}{\text{in}}\right)(1.781 \ \text{in})(\cos^2 30°)}$$
$$\qquad + (210 \ \text{lbf})(\sin^2 30°)$$
$$= 248 \ \text{lbf}$$

LRFD Method

From Table 5.4, the time effect factor for occupancy live load is

$$\lambda = 0.8$$

The format conversion factor from NDS Table 10.3.1 is

$$K_F = 3.32$$

The resistance factor given by NDS Table 10.3.1 is

$$\phi = 0.65$$

The adjusted factored lateral design value is

$$Z'_\| = Z_\| C_M C_t C_g C_\Delta C_d K_F \lambda \phi$$
$$= (220 \ \text{lbf})(1.0)(1.0)(1.0)(1.0)(0.955)$$
$$\qquad \times (3.32)(0.8)(0.65)$$
$$= 363 \ \text{lbf}$$

The adjusted factored withdrawal design value is

$$W' = W C_M C_t K_F \lambda \phi$$
$$= (305 \ \text{lbf})(1.0)(1.0)(3.32)(0.8)(0.65)$$
$$= 527 \ \text{lbf}$$

From NDS Sec. 11.4.1, the maximum strength level force that may be applied is

$$Z'_\alpha = \frac{(W'p)Z'_\|}{(W'p)\cos^2\alpha + Z'_\| \sin^2\alpha}$$
$$= \frac{\left(527 \ \dfrac{\text{lbf}}{\text{in}}\right)(1.781 \ \text{in})(363 \ \text{lbf})}{\left(527 \ \dfrac{\text{lbf}}{\text{in}}\right)(1.781 \ \text{in})\cos^2 30° + (363 \ \text{lbf})\sin^2 30°}$$
$$= 429 \ \text{lbf}$$

Split Ring and Shear Plate Connections

Installation Requirements

Edge and end distances, spacing, and geometry factors, C_Δ, for various sizes of split ring and shear plate connectors are specified in NDS Table 12.3.2.2 and Table 12.3.3.1-1 through Table 12.3.3.1-4. When lag screws are used instead of bolts, nominal design values shall, where appropriate, be multiplied by the penetration depth factors specified in NDS Table 12.2.3 for various sizes of connectors and wood species. NDS Table 12.2.4 provides metal side plate factors, C_{st}, for 4 in shear plate connectors, loaded parallel to the grain, when metal side plates are substituted for wood side members. Group action factors, C_g, for 4 in split ring or shear plate connectors with wood side members are tabulated in NDS Table 10.3.6B. Group action factors, C_g, for 4 in shear plate connectors with steel side plates are tabulated in NDS Table 10.3.6D. Dimensions for split ring and shear plate connectors are provided in NDS App. K.

Lateral Design Values

Reference design values for split ring connectors are provided in NDS Table 12.2A and for shear plate connectors in NDS Table 12.2B. When a load acts in the plane of the wood surface at an angle θ to the grain, the allowable design value is given by NDS Sec. 12.2.5 as

$$N' = \frac{P'Q'}{P' \sin^2\theta + Q' \cos^2\theta} \qquad [\textit{NDS 12.2-1}]$$

Example 5.16

The Douglas fir-larch select structural members shown in the following illustration are connected with $2\tfrac{5}{8}$ in shear plate connectors. The governing load combination consists of dead plus occupancy live loads. The connector spacing and end distances are as shown. Determine the capacity of the connection.
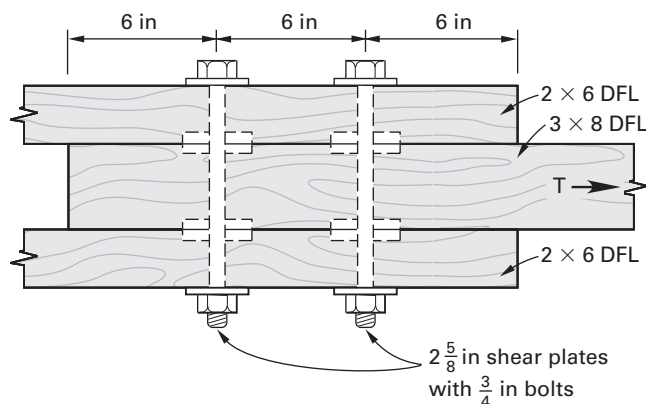
Solution

The reference $2^5/_8$ in shear plate design value for the $2^1/_2$ in thick main member of group B species with a connector on two faces is tabulated in NDS Table 12.2B as

$$P_{\text{main}} = 2860 \text{ lbf}$$

The reference $2^1/_2$ in shear plate design value for a $1^1/_2$ in thick side member of group B species with a connector on one face is tabulated in NDS Table 12.2B as

$$P_{\text{side}} = 2670 \text{ lbf} \quad [\text{governs}]$$

$$C_M = 1.0, \ C_t = 1.0$$

$$A_s = (2)(8.25 \text{ in}^2) = 16.5 \text{ in}^2$$

$$A_m = 18.13 \text{ in}^2$$

$$\frac{A_s}{A_m} = 0.91$$

The specified minimum spacing for the full shear plate design value is given in NDS Table 12.3 as

$$s = 6.75 \text{ in}$$

The applicable adjustment factors for the bolts are as follows.

$$C_g = \text{group action factor from NDS}$$
$$\text{Table 10.3.6B}$$
$$= 0.98$$

$$C_\Delta = \text{geometry factor from NDS}$$
$$\text{Sec. 12.3.2.1 for a spacing of 6 in}$$
$$= 0.5 + \frac{(0.5)(6 \text{ in} - 3.5 \text{ in})}{6.75 \text{ in} - 3.5 \text{ in}}$$
$$= 0.885$$

ASD Method

From Table 5.5, the load duration factor is

$$C_D = 1.0$$

The allowable design value for four shear plates is

$$T = nP_{\text{side}} C_g C_\Delta C_D C_M C_t$$
$$= (4)(2670 \text{ lbf})(0.98)(0.885)(1.00)(1.0)(1.0)$$
$$= 9263 \text{ lbf}$$

LRFD Method

From Table 5.4, the time effect factor for occupancy live load is

$$\lambda = 0.8$$

The format conversion factor from NDS Table 10.3.1 is

$$K_F = 3.32$$

The resistance factor given by NDS Table 10.3.1 is

$$\phi = 0.65$$

The strength level design value for four shear plates is

$$T = nP_{\text{side}} C_g C_\Delta C_M C_t K_F \lambda \phi$$
$$= (4)(2670 \text{ lbf})(0.98)(0.885)(1.0)(1.0)(3.32)$$
$$\times (0.8)(0.65)$$
$$= 15{,}991 \text{ lbf}$$

Wood Screw Connections

Installation

In accordance with NDS Sec. 11.1.5.3, wood screws loaded laterally in wood with a specific gravity in excess of 0.6 must have a clearance hole approximately equal in diameter to the diameter of the shank. The clearance hole must be bored in the member for the full length of the unthreaded shank. The lead hole receiving the threaded portion of the screw must have a diameter approximately equal to the wood screw root diameter. Wood with a specific gravity not exceeding 0.6 must have a clearance hole approximately equal in diameter to $^7/_8$ the diameter of the shank. The clearance hole must be bored in the member for the full length of the unthreaded shank. The lead hole receiving the threaded portion of the screw must have a diameter approximately equal to $^7/_8$ the diameter to the wood screw root diameter.

Lateral Design Values in Side Grain

Recommended edge distances, end distances, and spacing are tabulated in NDS Comm. Table C11.1.5.7 for wood and steel side plates with and without pre-bored holes. Wood screws are not subject to the group action factor, C_g, or the geometry factor.

As specified in NDS Table 11L, for full design values to be applicable, the depth of penetration shall not be less than

$$p = 10D$$

The minimum allowable penetration is $6D$. When the penetration is between $6D$ and $10D$, the reference design value is multiplied by the penetration factor, which is defined in NDS Table 11L as

$$C_d = \frac{p}{10D}$$
$$\leq 1.0$$

Reference design values for single shear connections are specified in NDS Sec. 11.3 and tabulated in NDS Table 11L for connections with a wood side member and in NDS Table 11M for connections with a steel side plate.

Withdrawal Design Values in Side Grain

Withdrawal design values in pounds per inch of thread penetration are tabulated in NDS Table 11.2B. The length of thread is specified in App. L as two-thirds the total screw length or four times the screw diameter, whichever is greater.

Combined Lateral and Withdrawal Loads

When the load applied to a wood screw is at an angle, α, to the wood surface, the wood screw is subjected to combined lateral and withdrawal loading, and the design value is determined by the Hankinson formula given by NDS Sec. 11.4.1 as

$$Z'_\alpha = \frac{W'pZ'}{W'p\cos^2\alpha + Z'\sin^2\alpha}$$

Example 5.17

A 7g steel strap is secured to a select structural Douglas fir-larch collector with ten $14g \times 3$ in wood screws. Edge and end distances and spacing are sufficient to prevent splitting of the wood. Determine the maximum tensile force, T, due to wind load that can be resisted by the connection.

Solution

The reference design value for single shear is tabulated in NDS Table 11M as

$$Z = 202 \text{ lbf}$$
$$C_M = 1.0, \ C_t = 1.0$$

The penetration of the screw shank plus the threaded length is

$$p = L - t_s$$
$$= 3 \text{ in} - 0.179 \text{ in}$$
$$= 2.821 \text{ in}$$

This is greater than $10D$, and from NDS Table 11M, the penetration depth factor is

$$C_d = 1.0$$

ASD Method

From Table 5.5, the load duration factor is

$$C_D = 1.60$$

The allowable lateral design value for 10 screws is

$$T = nZC_DC_dC_MC_t$$
$$= (10)(202 \text{ lbf})(1.60)(1.0)(1.0)(1.0)$$
$$= 3232 \text{ lbf}$$

LRFD Method

From Table 5.4, the time effect factor for wind load is

$$\lambda = 1.0$$

The format conversion factor given in NDS Table 10.3.1 is

$$K_F = 3.32$$

The resistance factor given in NDS Table 10.3.1 is

$$\phi = 0.65$$

The strength level lateral design value for the 10 screws is

$$T = nZC_MC_tC_dK_F\lambda\phi$$
$$= (10)(220 \text{ lbf})(1.0)(1.0)(1.0)(3.32)(1.0)(0.65)$$
$$= 4748 \text{ lbf}$$

Connections with Nails and Spikes

Installation

NDS specifications apply to common nails and spikes, box nails, sinker nails, and threaded hardened-steel nails. The tabulated nominal design values apply to nailed connections with or without pre-bored holes. As specified in NDS Sec. 11.1.6.3, pre-bored holes may be used to prevent the splitting of wood. For wood with a specific gravity greater than 0.6, the hole diameter may not exceed 90% of the diameter of the nail. For wood with a specific gravity less than or equal to 0.6, the hole diameter may not exceed 75% of the diameter of the nail.

As shown in Fig. 5.9 and as specified in NDS Sec. 11.1.6, toe nails are driven into the member at an angle of approximately 30 degrees with the point of penetration approximately one-third the length of the nail from the member end.

$$t_s = \frac{L}{3}$$

Figure 5.9 *Toe-Nailed Connection*
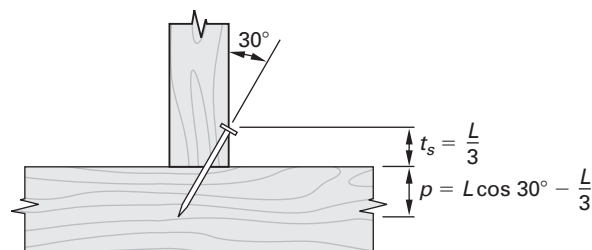
Lateral Design Values in Side Grain

Recommended edge distances, end distances, and spacing are tabulated in NDS Comm. Table C11.1.6.6 for wood and steel side plates with and without pre-bored holes. Nails and spikes are not subject to the group action factor, C_g.

As specified in NDS Table 11N, for full design values to be applicable, the depth of penetration shall not be less than

$$p = 10D$$

The minimum allowable penetration is $6D$. When the penetration is between $6D$ and $10D$, the reference design value is multiplied by the penetration factor, which is defined in NDS Table 11N as

$$C_d = \frac{p}{10D}$$
$$\leq 1.0$$

Reference design values for nails and spikes used in diaphragm construction shall be multiplied by the diaphragm factor $C_{di} = 1.1$.

Reference lateral design values for nails and spikes used in toe-nailed connections shall be multiplied by the toe-nail factor $C_{tn} = 0.83$.

Reference design values for single shear connections for two sawn lumber members of identical species are tabulated in NDS Table 11N.

Reference design values for single shear connections for a sawn lumber member with steel side plates are tabulated in NDS Table 11P.

The reference double shear value for a three-member sawn lumber connection is twice the lesser of the nominal design value for each shear plane. The minimum penetration into the side member shall be six times the connector diameter, or when the side member is at least $^3/_8$ in thick and 12d or smaller nails extend at least three diameters beyond the side member the nails shall be clinched.

Example 5.18

A 4×8 select structural Douglas fir-larch collector is secured to the Douglas fir-larch top plate of a shear wall with a 12-gage steel strap, as shown. Fourteen 16d common nails $2^1/_2$ in long are on each side of the strap. Edge and end distances and spacing are sufficient to prevent splitting of the wood. $C_M = C_t = 1.0$. Determine the maximum tensile force due to wind load that can be resisted by the nails.
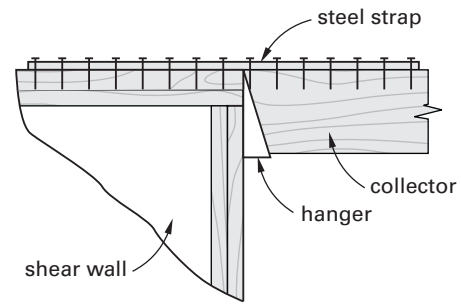
Solution

From NDS Table 11P, the reference single shear design value for a 16d common nail in a Douglas fir-larch member with a 12-gage side plate is

$$Z = 149 \text{ lbf}$$

As specified in NDS Table 11N, for full design values to be applicable, the depth of penetration must not be less than

$$p_{\text{full}} = 10D = (10)(0.162 \text{ in})$$
$$= 1.62 \text{ in}$$

The actual penetration of the nails is

$$p = L - t_s$$
$$= 2.5 \text{ in} - 0.105 \text{ in}$$
$$= 2.395 \text{ in}$$
$$> p_{\text{full}}$$

Since $p > p_{\text{full}}$, the penetration depth factor is given by

$$C_d = 1.0$$

ASD Method

From Table 5.5, the load duration factor for wind load is

$$C_D = 1.60$$

The allowable lateral design value for fourteen 16d nails is

$$T = nZC_DC_MC_dC_t$$
$$= (14)(149 \text{ lbf})(1.60)(1.0)(1.0)(1.0)$$
$$= 3338 \text{ lbf}$$

LRFD Method

From Table 5.4, the time effect factor for wind load is

$$\lambda = 1.0$$

The format conversion factor given in NDS Table 10.3.1 is

$$K_F = 3.32$$

Timber

The resistance factor given in NDS Table 10.3.1 is

$$\phi = 0.65$$

The strength level lateral design value for the 14 nails is

$$
\begin{aligned}
T &= nZC_MC_tC_dK_F\lambda\phi \\
&= (14)(149 \text{ lbf})(1.0)(1.0)(1.0)(3.32)(1.0)(0.65) \\
&= 4502 \text{ lbf}
\end{aligned}
$$

Withdrawal Design Values in Side Grain

Reference withdrawal design values in pounds per inch of penetration are tabulated in NDS Table 11.2C. When toe-nailed connections are used, the reference design values shall be multiplied by the toe-nail factor $C_{tn} = 0.67$.

Combined Lateral and Withdrawal Loads

When the load applied to a nail or spike is at an angle, α, to the wood surface, the nail or spike is subjected to combined lateral and withdrawal loading, and in accordance with the NDS Sec. 11.4.2, the design value is determined by the interaction equation

$$Z'_\alpha = \frac{W'pZ'}{W'p\cos\alpha + Z'\sin\alpha}$$

Example 5.19

Determine the lateral design value for the 3 in long 10d common wire nail in the toe-nailed connection shown. Loading applied to the connection is due to wind load, and all members are Douglas fir-larch.
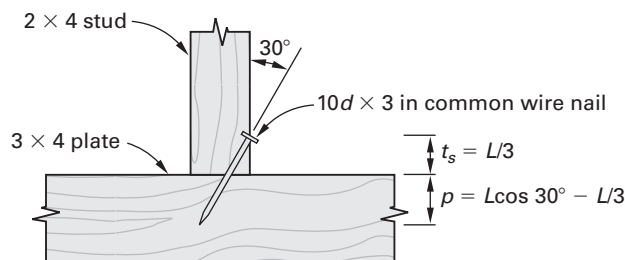
Solution

As specified in NDS Sec. 11.1.5, toe nails are driven at an angle of 30 degrees from the face of the member, with the point of penetration one-third the length of the nail from the member end. In accordance with NDS Comm. Sec. C11.1.5, the side member thickness is taken to be equal to this end distance and

$$
\begin{aligned}
t_s &= \frac{L}{3} = \frac{3 \text{ in}}{3} \\
&= 1 \text{ in}
\end{aligned}
$$

The nominal design value for single shear is tabulated in NDS Table 11N as

$$Z = 118 \text{ lbf}$$

$$C_M = 1.0, \ C_t = 1.0$$

The applicable adjustment factors for the nail are as follows.

- The penetration of the nail into the main member, in accordance with NDS Comm. Sec. C11.1.6, is taken as the vertically projected length of the nail in the member and

$$
\begin{aligned}
p &= L\cos 30° - \frac{L}{3} \\
&= (3 \text{ in})(0.866) - \frac{3 \text{ in}}{3} \\
&= 1.60 \text{ in}
\end{aligned}
$$

This is greater than $10D$, and from NDS Table 11N, the penetration depth factor is

$$C_d = 1.0$$

- $C_{tn} =$ toe-nail factor from NDS Sec. 11.5.4

$$= 0.83$$

ASD Method

From Table 5.5, the load duration factor is

$$C_D = 1.60$$

The allowable lateral design value for the nail is

$$
\begin{aligned}
Z' &= ZC_DC_dC_{tn}C_MC_t \\
&= (118 \text{ lbf})(1.60)(1.0)(0.83)(1.0)(1.0) \\
&= 157 \text{ lbf}
\end{aligned}
$$

LRFD Method

From Table 5.4, the time effect factor for wind load is

$$\lambda = 1.0$$

The format conversion factor given in NDS Table 10.3.1 is

$$K_F = 3.32$$

The resistance factor given in NDS Table 10.3.1 is

$$\phi = 0.65$$

The strength level lateral design value for the nail is

$$
\begin{aligned}
T &= ZC_MC_tC_dC_{tn}K_F\lambda\phi \\
&= (118 \text{ lbf})(1.0)(1.0)(1.0)(0.83)(3.32)(1.0)(0.65) \\
&= 211 \text{ lbf}
\end{aligned}
$$

REFERENCES

1. American Forest & Paper Association. *National Design Specification for Wood Construction ASD/ LRFD with Commentary.* Washington, DC: American Forest & Paper Association, 2012.

2. American Forest & Paper Association. *National Design Specification Supplement: Design Values for Wood Construction.* Washington, DC: American Forest & Paper Association, 2012.

3. International Code Council. *2012 International Building Code.* Country Club Hills, IL: International Code Council, 2012.

4. American Plywood Association. *Glued Laminated Beam Design Tables.* Tacoma, WA: Engineered Wood Systems, 2007.

5. Western Wood Products Association. *Western Lumber Span Tables.* Portland, OR: Western Wood Products Association.

PRACTICE PROBLEMS

1. The select structural 3 × 10 Douglas fir-larch rafter shown in the following illustration is notched over a supporting 3 in wall. Based on the bearing stress in the rafter, what is most nearly the maximum available reaction at the support caused by snow loading? (ASD options are shown first. LRFD options are given in parentheses.)
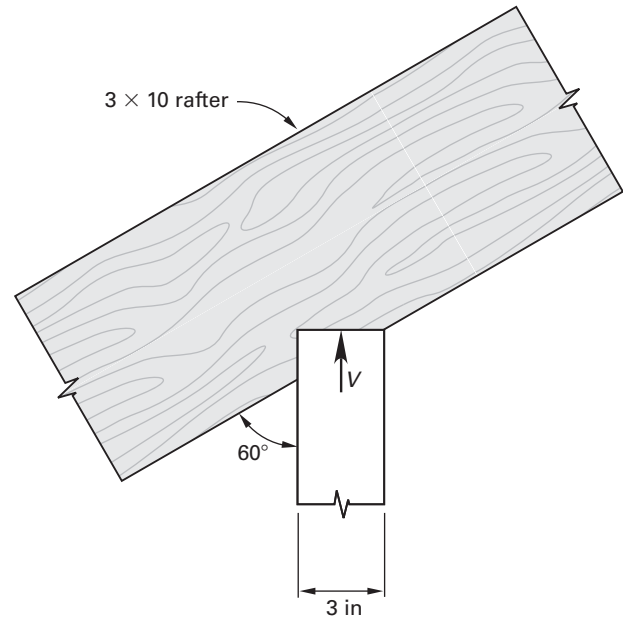
(A) 5600 lbf (6800 lbf)

(B) 6300 lbf (7700 lbf)

(C) 6800 lbf (8300 lbf)

(D) 7500 lbf (9200 lbf)

2. The select structural 4×10 Douglas fir-larch ledger shown in the following illustration supports a dead plus floor live load of 250 lbf/ft (ASD) or 390 lbf/ft (LRFD). Based on the $^3/_4$ in bolt design value in the ledger, what is most nearly the maximum allowable bolt spacing?
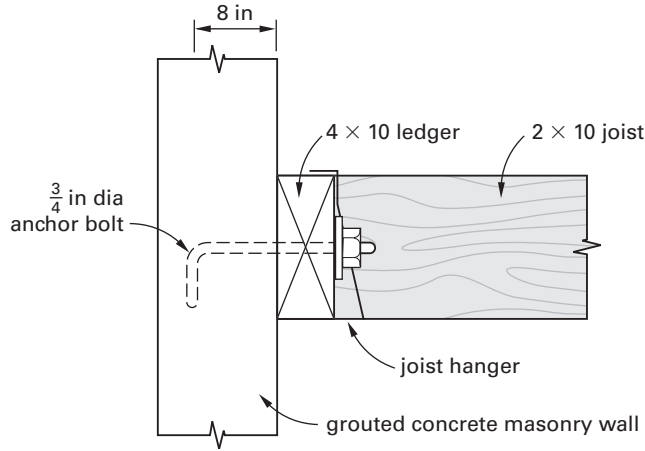
(A) 2.8 ft

(B) 3.2 ft

(C) 3.6 ft

(D) 4.0 ft

3. The floor system in an office building consists of select structural $2 \times$ Douglas fir-larch joists at 16 in centers with $^{19}/_{32}$ plywood sheathing. Each joist supports a dead load of $w_D = 33.33$ lbf/ft plus floor live load of $w_L = 66.67$ lbf/ft over a span of $L = 16$ ft. Acceptable deflection due to live load is $\Delta_{ST} = L/360$ and acceptable deflection due to total load is $\Delta_T = L/240$. The depth of joist necessary to give acceptable stresses and deflection is most nearly

(A) 8 in

(B) 10 in

(C) 12 in

(D) 14 in

4. A select structural 6×6 Douglas fir-larch column is subjected to axial load due to dead plus floor live load. The column is 10 ft high and may be considered pin ended. What is most nearly the maximum load that may be applied? (ASD options are shown first. LRFD options are given in parentheses.)

(A) 18,000 lbf (31,000 lbf)

(B) 19,000 lbf (32,000 lbf)

(C) 21,000 lbf (34,000 lbf)

(D) 22,000 lbf (35,000 lbf)

SOLUTIONS

1. The reference design value for compressive bearing parallel to grain is tabulated in NDS Supp. Table 4A and is

$$F_c = 1700 \text{ lbf/in}^2$$

The applicable adjustment factors for compressive bearing parallel to grain are

$$C_t = 1.0$$
$$C_M = 1.0$$
$$C_F = 1.0$$
$$C_i = 1.0$$

The reference design value for compression perpendicular to grain is tabulated in NDS Supp. Table 4A and is

$$F_{c\perp} = 625 \text{ lbf/in}^2$$

$$C_M = 1.0, \ C_t = 1.0, \ C_i = 1.0$$

The bearing area factor for compression perpendicular to grain is specified in NDS Sec. 3.10.4 as

$$C_b = \frac{l_b + 0.375}{l_b}$$
$$= \frac{3 \text{ in} + 0.375}{3 \text{ in}}$$
$$= 1.125$$

ASD Method

From Table 5.5, the load duration factor for snow is

$$C_D = 1.15$$

The adjusted compressive bearing design value parallel to grain is

$$F_c^* = F_c C_t C_M C_F C_i C_D$$
$$= \left(1700 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)(1.0)(1.15)$$
$$= 1955 \text{ lbf/in}^2$$

The adjusted compression design value perpendicular to grain is

$$F'_{c\perp} = F_{c\perp} C_b C_M C_t C_i$$
$$= \left(625 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.125)(1.0)(1.0)(1.0)$$
$$= 703 \text{ lbf/in}^2$$

The allowable bearing design value at an angle θ to the grain is given by NDS Sec. 3.10.3 as

$$F'_\theta = \frac{F^*_c F'_{c\perp}}{F^*_c \sin^2\theta + F'_{c\perp}\cos^2\theta}$$

$$= \frac{\left(1955\ \dfrac{\text{lbf}}{\text{in}^2}\right)\left(703\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{\left(1955\ \dfrac{\text{lbf}}{\text{in}^2}\right)(\sin^2 60^\circ) + \left(703\ \dfrac{\text{lbf}}{\text{in}^2}\right)(\cos^2 60^\circ)}$$

$$= 837\ \text{lbf/in}^2$$

The allowable reaction at the support is

$$V = F'_\theta b l_b = \left(837\ \frac{\text{lbf}}{\text{in}^2}\right)(2.5\ \text{in})(3\ \text{in})$$

$$= 6278\ \text{lbf} \quad (6300\ \text{lbf})$$

The answer is (B).

LRFD Method

From Table 5.4, the time effect factor for snow load is

$$\lambda = 0.8$$

The format conversion factor given in Table 5.2 is

$$K_F = 1.67 \quad [\text{for } F_{c\perp}]$$

$$K_F = 2.4 \quad [\text{for } F_c]$$

The resistance factor given in Table 5.3 is

$$\phi = 0.90 \quad [\text{for } F_{c\perp}]$$
$$\phi = 0.90 \quad [\text{for } F_c]$$

The adjusted factored compressive bearing design value parallel to grain is

$$F^*_c = F_c C_M C_t C_i C_F K_F \lambda \phi$$

$$= \left(1700\ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)(1.0)$$

$$\times (2.40)(0.8)(0.90)$$

$$= 2938\ \text{lbf/in}^2$$

The adjusted factored compression design value perpendicular to grain is

$$F'_{c\perp} = F_{c\perp} C_b C_t C_i C_F K_F \lambda \phi$$

$$= \left(625\ \frac{\text{lbf}}{\text{in}^2}\right)(1.125)(1.0)(1.0)(1.0)$$

$$\times (1.67)(0.8)(0.90)$$

$$= 845\ \text{lbf/in}^2$$

From NDS Sec. 3.10.3, the strength level bearing design value at an angle θ to the grain is

$$F'_\theta = \frac{F^*_c F'_{c\perp}}{F^*_c \sin^2\theta + F'_{c\perp}\cos^2\theta}$$

$$= \frac{\left(2938\ \dfrac{\text{lbf}}{\text{in}^2}\right)\left(845\ \dfrac{\text{lbf}}{\text{in}^2}\right)}{\left(2938\ \dfrac{\text{lbf}}{\text{in}^2}\right)\sin^2 60^\circ + \left(845\ \dfrac{\text{lbf}}{\text{in}^2}\right)\cos^2 60^\circ}$$

$$= 1028\ \text{lbf/in}^2$$

The available strength level reaction at the support is

$$V = F'_\theta b l_b$$

$$= \left(1028\ \frac{\text{lbf}}{\text{in}^2}\right)(2.5\ \text{in})(3\ \text{in})$$

$$= 7710\ \text{lbf} \quad (7700\ \text{lbf})$$

The answer is (B).

2. The nominal $^3/_4$ in diameter bolt design value for single shear perpendicular to grain into concrete is tabulated in NDS Table 11E as

$$Z_\perp = 900\ \text{lbf}$$
$$C_M = 1.0,\ C_t = 1.0,\ C_g = 1.0$$

From NDS Sec. 11.5.1, the geometry factor for the bolts is

$$C_\Delta = 1.0$$

ASD Method

From Table 5.5, the load duration factor is

$$C_D = 1.00$$

The allowable lateral design value is

$$Z'_\perp = Z_\perp C_\Delta C_M C_g C_t C_D$$

$$= (900\ \text{lbf})(1.0)(1.0)(1.0)(1.0)(1.00)$$

$$= 900\ \text{lbf}$$

The maximum allowable bolt spacing is

$$s = \frac{900\ \text{lbf}}{250\ \dfrac{\text{lbf}}{\text{ft}}}$$

$$= 3.6\ \text{ft}$$

The answer is (C).

LRFD Method

From Table 5.4, the time effect factor for dead load plus floor live load is

$$\lambda = 0.8$$

The format conversion factor given in NDS Table 10.3.1 is

$$K_F = 3.32$$

The resistance factor given in NDS Table 10.3.1 is

$$\phi = 0.65$$

The adjusted factored lateral design value is

$$Z'_\perp = Z_\perp C_M C_g C_\Delta C_t K_F \lambda \phi$$
$$= (900 \text{ lbf})(1.0)(1.0)(1.0)(1.0)(3.32)(0.8)(0.65)$$
$$= 1554 \text{ lbf}$$

The available maximum bolt spacing is

$$s = \frac{1554 \text{ lbf}}{390 \ \dfrac{\text{lbf}}{\text{ft}}}$$
$$= 4.0 \text{ ft}$$

The answer is (D).

3. The basic design values for bending and the modulus of elasticity are tabulated in NDS Supp. Table 4A and are

$$F_b = 1500 \text{ lbf/in}^2$$
$$E = 1.9 \times 10^6 \text{ lbf/in}^2$$
$$C_M = 1.0, \ C_L = 1.0, \ C_t = 1.0 \ C_i = 1.0$$

The applicable adjustment factors for bending stress are as follows.

$C_F =$ size factor from NDS Supp. Table 4A, assuming a 10 in joist
$\quad = 1.1$
$C_r =$ repetitive member factor from NDS Supp. Table 4A
$\quad = 1.15$

ASD Method

Applying IBC Eq. 16-9, the applied ASD load is

$$w = D + L$$
$$= 33.33 \ \frac{\text{lbf}}{\text{ft}} + 66.67 \ \frac{\text{lbf}}{\text{ft}}$$
$$= 100 \text{ lbf/ft}$$

From Table 5.5, the load duration factor is

$$C_D = 1.00$$

The adjusted bending stress is

$$F'_b = F_b C_D C_F C_r$$
$$= \left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.00)(1.1)(1.15)$$
$$= 1898 \text{ lbf/in}^2$$

The applied moment on the joist is

$$M = \frac{wL^2}{8}$$
$$= \frac{\left(100 \ \frac{\text{lbf}}{\text{ft}}\right)(16 \text{ ft})^2 \left(12 \ \frac{\text{in}}{\text{ft}}\right)}{8}$$
$$= 38{,}400 \text{ in-lbf}$$

The required section modulus is

$$S_{xx} = \frac{M}{F'_b} = \frac{38{,}400 \text{ in-lbf}}{1898 \ \dfrac{\text{lbf}}{\text{in}^2}}$$
$$= 20.24 \text{ in}^3$$

Therefore, a 2×10 is adequate for acceptable stresses $(S_{xx} = 21.39 \text{ in}^3)$.

LRFD Method

From Table 5.4, the time effect factor for dead load plus floor load is

$$\lambda = 1.0$$

The format conversion factor for bending given in Table 5.2 is

$$K_F = 2.54$$

The resistance factor given in Table 5.3 is

$$\phi = 0.85$$

Applying IBC Eq. 16-2, the applied LRFD load is

$$
\begin{aligned}
w &= 1.2D + 1.6L \\
&= (1.2)\left(33.33 \; \frac{\text{lbf}}{\text{ft}}\right) + (1.6)\left(66.67 \; \frac{\text{lbf}}{\text{ft}}\right) \\
&= 146.67 \; \text{lbf/ft}
\end{aligned}
$$

The adjusted factored bending stress is

$$
\begin{aligned}
F'_b &= F_b C_F C_r C_M C_t C_L C_i K_F \lambda \phi \\
&= \left(1500 \; \frac{\text{lbf}}{\text{in}^2}\right)(1.1)(1.15)(1.0)(1.0) \\
&\quad \times (1.0)(1.0)(2.54)(1.0)(0.85) \\
&= 4097 \; \text{lbf/in}^2
\end{aligned}
$$

The applied moment on the joint is

$$
\begin{aligned}
M &= \frac{wL^2}{8} \\
&= \frac{\left(146.67 \; \frac{\text{lbf}}{\text{ft}}\right)(16 \; \text{ft})^2 \left(12 \; \frac{\text{in}}{\text{ft}}\right)}{8} \\
&= 56{,}321 \; \text{in-lbf}
\end{aligned}
$$

The required section modulus is

$$
\begin{aligned}
S_{xx} &= \frac{M}{F'_b} = \frac{56{,}321 \; \text{in-lbf}}{4097 \; \frac{\text{lbf}}{\text{in}^2}} \\
&= 13.75 \; \text{in}^3
\end{aligned}
$$

A 2×10 is adequate for acceptable stresses ($S_{xx} = 21.39 \; \text{in}^3$).

Check the deflection. The adjusted modulus of elasticity is

$$
\begin{aligned}
E' &= E \\
&= 1.9 \times 10^6 \; \text{lbf/in}^2
\end{aligned}
$$

The floor live load is

$$
w_L = 66.67 \; \text{lbf/ft}
$$

The floor dead load is

$$
w_D = 33.33 \; \text{lbf/ft}
$$

The required live load deflection is

$$
\begin{aligned}
\Delta_{ST} &= \frac{L}{360} = \frac{(16 \; \text{ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{360} \\
&= 0.53 \; \text{in}
\end{aligned}
$$

The corresponding required moment of inertia is

$$
\begin{aligned}
I_{xx} &= \frac{5 w_L L^4}{384 E \Delta_{ST}} \\
&= \frac{(5)\left(66.67 \; \frac{\text{lbf}}{\text{ft}}\right)(16 \; \text{ft})^4 \left(12 \; \frac{\text{in}}{\text{ft}}\right)^3}{(384)\left(1.9 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.53 \; \text{in})} \\
&= 97.63 \; \text{in}^4
\end{aligned}
$$

Therefore, a 2×10 is acceptable for live load deflection ($I_{xx} = 98.9 \; \text{in}^4$).

The required deflection for total load is given as

$$
\begin{aligned}
\Delta_T &= \frac{L}{240} = \frac{(16 \; \text{ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{240} \\
&= 0.80 \; \text{in}
\end{aligned}
$$

For long-term loads, NDS Sec. 3.5.2 specifies a creep factor, K_{cr}, of 1.5 for seasoned lumber. Therefore, to determine the total deflection, Δ_T, the applicable equivalent total load is

$$
\begin{aligned}
w_T &= w_L + K_{cr} w_D \\
&= 66.67 \; \frac{\text{lbf}}{\text{ft}} + (1.5)\left(33.33 \; \frac{\text{lbf}}{\text{ft}}\right) \\
&= 116.67 \; \text{lbf/ft}
\end{aligned}
$$

The corresponding required moment of inertia is

$$
\begin{aligned}
I_{xx} &= \frac{5 w_T L^4}{384 E \Delta_T} \\
&= \frac{(5)\left(116.67 \; \frac{\text{lbf}}{\text{ft}}\right)(16 \; \text{ft})^4 \left(12 \; \frac{\text{in}}{\text{ft}}\right)^3}{(384)\left(1.9 \times 10^6 \; \frac{\text{lbf}}{\text{in}^2}\right)(0.80 \; \text{in})} \\
&= 113.18 \; \text{in}^4 \quad (110 \; \text{in}^4) \quad [\text{governs}]
\end{aligned}
$$

Therefore, a 2×12 is necessary to control deflection ($I_{xx} = 178 \; \text{in}^4$).

The answer is (C).

4. The reference design values for compression and modulus of elasticity are tabulated in NDS Supp. Table 4D and are

$$
\begin{aligned}
F_c &= 1100 \; \text{lbf/in}^2 \\
E_{\min} &= 0.58 \times 10^6 \; \text{lbf/in}^2 \\
C_M &= 1.0, \; C_t = 1.0, \; C_i = 1.0
\end{aligned}
$$

Timber

The applicable adjustment factor for compression is

$$C_F = \text{size factor from NDS Supp. Table 4D}$$
$$= 1.0$$

The slenderness ratio is

$$\frac{K_e l}{d} = \frac{(1.0)(10 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{5.5 \text{ in}}$$
$$= 21.82$$

ASD Method

The adjusted modulus of elasticity is

$$E'_{\min} = E_{\min} C_M C_t C_i$$
$$= \left(0.58 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)$$
$$= 0.58 \times 10^6 \text{ lbf/in}^2$$

From Table 5.5, the load duration factor for dead load and floor load is

$$C_D = 1.00$$

The reference compression design value multiplied by all applicable adjustment factors except C_P is given by

$$F_c^* = F_c C_F C_D C_f C_M C_i$$
$$= \left(1100 \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.00)(1.0)(1.0)(1.0)$$
$$= 1100 \text{ lbf/in}^2$$

The critical buckling design value is

$$F_{cE} = \frac{0.822 E'_{\min}}{\left(\frac{l_e}{d}\right)^2}$$
$$= \frac{(0.822)\left(0.58 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right)}{\left(\frac{(10 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{5.5 \text{ in}}\right)^2}$$
$$= 1001 \text{ lbf/in}^2$$

The ratio of F_{cE} to F_c^* is

$$F' = \frac{F_{cE}}{F_c^*} = \frac{1001 \frac{\text{lbf}}{\text{in}^2}}{1100 \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.91$$

The column parameter for sawn lumber is obtained from NDS Sec. 3.7.1.5 as

$$c = 0.8$$

The column stability factor specified by NDS Sec. 3.7.1 is

$$C_P = \frac{1.0 + F'}{2c} - \sqrt{\left(\frac{1.0 + F'}{2c}\right)^2 - \frac{F'}{c}}$$
$$= \frac{1.0 + 0.91}{(2)(0.8)} - \sqrt{\left(\frac{1.0 + 0.91}{(2)(0.8)}\right)^2 - \frac{0.91}{0.8}}$$
$$= 0.658$$

The allowable compression design value parallel to grain is

$$F_c' = F_c C_D C_M C_t C_F C_i C_P$$
$$= \left(1100 \frac{\text{lbf}}{\text{in}^2}\right)(1.00)(1.0)(1.0)(1.0)(1.0)(0.658)$$
$$= 724 \text{ lbf/in}^2$$

The allowable load on the column is given by

$$P = F_c' A$$
$$= \left(724 \frac{\text{lbf}}{\text{in}^2}\right)(30.25 \text{ in}^2)$$
$$= 21{,}900 \text{ lbf} \quad (22{,}000 \text{ lbf})$$

The answer is (D).

LRFD Method

The adjusted factored modulus of elasticity is

$$E'_{\min} = E_{\min} C_M C_t C_i K_F \lambda \phi_s$$
$$= \left(0.58 \times 10^6 \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)(1.76)(0.85)$$
$$= 0.87 \times 10^6 \text{ lbf/in}^2$$

From Table 5.4, the time effect factor for dead load and floor load is

$$\lambda = 0.8$$

The reference compression design value multiplied by all applicable adjustment factors except C_P is given by

$$F_c^* = F_c C_F C_t C_M C_i K_F \lambda \phi_c$$
$$= \left(1100 \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)(1.0)$$
$$\quad \times (2.40)(0.8)(0.90)$$
$$= 1901 \text{ lbf/in}^2$$

The critical buckling design value is

$$F_{cE} = \frac{0.822 E'_{\min}}{\left(\frac{l_e}{d}\right)^2}$$

$$= \frac{(0.822)\left(0.87 \times 10^6 \ \frac{\text{lbf}}{\text{in}^2}\right)}{\left(\frac{(10 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{5.5 \text{ in}}\right)^2}$$

$$= 1502 \text{ lbf/in}^2$$

The ratio of F_{cE} to F_c^* is

$$F' = \frac{F_{cE}}{F_c^*} = \frac{1502 \ \frac{\text{lbf}}{\text{in}^2}}{1901 \ \frac{\text{lbf}}{\text{in}^2}}$$

$$= 0.790$$

The column parameter for sawn lumber is obtained from NDS Sec. 3.7.1.5 as

$$c = 0.8$$

The column stability factor specified by NDS Sec. 3.7.1 is

$$C_P = \frac{1.0 + F'}{2c} - \sqrt{\left(\frac{1.0 + F'}{2c}\right)^2 - \frac{F'}{c}}$$

$$= \frac{1.0 + 0.790}{(2)(0.8)} - \sqrt{\left(\frac{1.0 + 0.790}{(2)(0.8)}\right)^2 - \frac{0.790}{0.8}}$$

$$= 0.605$$

The adjusted factored compression design value parallel to grain is

$$F_c' = F_c C_F C_t C_M C_i C_P K_F \lambda \phi_c$$

$$= \left(1100 \ \frac{\text{lbf}}{\text{in}^2}\right)(1.0)(1.0)(1.0)(1.0)(0.605)$$

$$\times (2.40)(0.8)(0.90)$$

$$= 1150 \text{ lbf/in}^2$$

The strength level load on the column is given by

$$P = F_c' A$$

$$= \left(1150 \ \frac{\text{lbf}}{\text{in}^2}\right)(30.25 \text{ in}^2)$$

$$= 34{,}788 \text{ lbf} \quad (35{,}000 \text{ lbf})$$

The answer is (D).

Timber

6 Reinforced Masonry Design

1. CONSTRUCTION DETAILS

Requirements for the placement of reinforcement are given in MSJC[1] Sec. 1.16 and *MSJC Specification*[1] Sec. 3.4. Reinforcing bars must be securely supported to prevent displacement during grout placement and be completely embedded in grout. Grout proportioning and mixing must comply with ASTM C476 and have a slump between 8 in and 11 in. Small cells or cavities require grout with a higher slump than larger cells or cavities. Grout is classified as either fine or coarse depending on the maximum aggregate size used. Grout with a maximum aggregate size less than $3/8$ in is classified as fine. Grout with a maximum aggregate size exceeding $3/8$ in, but smaller than $1/2$ in, is classified as coarse. When suitable, coarse grout is preferable as it shrinks less and requires less cement than fine grout. Fine grout is preferable for small cells to ensure that the grout adequately fills the confined space. The least clear dimension for grouting between wythes, the minimum cell dimensions when grouting hollow units, and the appropriate type of grout to use are given by MSJC Table 1.20.1. Obstructions into the grout space and the diameter of horizontal reinforcing bars must be considered when determining the minimum dimension.

Grout may be placed by either pumping or pouring. In accordance with *MSJC Specification* Sec. 3.5D, grout may be placed in one continuous operation, or lift, not exceeding 12.67 ft in height, provided that

- the masonry has cured for a minimum of four hours to minimize potential displacement of the units

- the grout slump is maintained between 10 in and 11 in to ensure the grout will flow into and completely fill all spaces

- there are no intermediate reinforced bond beams between the top and bottom of the lift

Grout pours not exceeding 1 ft in height may be consolidated by puddling; otherwise, mechanical vibration is required to fill the grout space completely. After consolidation, water loss and settlement of the grout occurs, and reconsolidation is necessary to eliminate any voids formed in the grout. After reconsolidation, additional lifts may be placed, provided that the total height of the pour does not exceed the value given in MSJC Table 1.20.1. After the completion of the pour, additional masonry may be constructed.

2. ASD AND SD METHODS

In accordance with MSJC Sec. 1.1.3, masonry structures may be designed using the provisions for either the allowable stress design of masonry (ASD) method or the strength design of masonry (SD) method.

The ASD method is the traditional method of designing masonry structures, and it is based on elastic theory to calculate the stresses produced in the member. Factored loads using ASD load combinations are applied to the member, and the stresses produced in the member must not exceed the specified allowable stress. MSJC Chap. 2 details the ASD method. IBC[2] Sec. 2107.1 adopts MSJC Chap. 1 and Chap. 2 with the exception of the maximum permitted reinforcement requirements of MSJC Sec. 1.16.2 and the development length requirements of MSJC Sec. 2.1.7.3.

In the SD method, factored loads using SD load combinations are applied to the member to determine the required ultimate strength. This required strength must not exceed the design strength, which is calculated as the member nominal strength multiplied by a resistance factor, ϕ. MSJC Chap. 3 details the SD method. IBC Sec. 2108.1 adopts the SD method of MSJC Chap. 3 with the exception of the development length requirements of MSJC Sec. 3.3.3.3 and the reinforcement splice requirements of MSJC Sec. 3.3.3.4.

Masonry

3. LOAD COMBINATIONS

Nomenclature

D	dead load	lbf or kips
E	earthquake load	lbf or kips
E_m	modulus of elasticity of masonry in compression	lbf/in^2
E_s	modulus of elasticity of steel reinforcement	lbf/in^2
f'_m	specified masonry compressive strength	lbf/in^2
F_b	allowable compressive stress in masonry due to flexure	lbf/in^2
F_s	allowable tensile stress in reinforcement due to flexure	lbf/in^2
H	load due to lateral pressure	lbf or kips
L	live load	lbf or kips
L_r	roof live load	lbf or kips
Q	load effect due to service load	lbf or kips
R	rain load	lbf or kips
S	snow load	lbf or kips
W	wind load	lbf or kips

Symbols

ϕ	resistance factor	–
γ	load factor	–

ASD Required Strength

The required strength of a member consists of the most critical combination of factored loads applied to the member. Factored loads consist of working, or service, loads multiplied by the appropriate ASD load factors. In accordance with MSJC Sec. 1.7.2, load combinations must be as specified in the applicable building code. The required strength, $\Sigma\gamma Q$, is defined by nine load combinations given in IBC Sec. 1605.3.1. The seismic and wind loads specified in the IBC are at the strength design level, in contrast to other loads that are at the service level. In the ASD load combinations, the load factor for seismic loads is 0.7, and the load factor for wind loads is 0.6 to reduce them to service-level values. The combinations, with uncommon load conditions (self-straining loads and fluid pressure) omitted, are as follows.

$$\sum \gamma Q = D \qquad \text{[IBC 16-8]}$$

$$\sum \gamma Q = D + H + L \qquad \text{[IBC 16-9]}$$

$$\sum \gamma Q = D + H + (L_r \text{ or } S \text{ or } R) \qquad \text{[IBC 16-10]}$$

$$\sum \gamma Q = D + H + 0.75L + 0.75(L_r \text{ or } S \text{ or } R) \qquad \text{[IBC 16-11]}$$

$$\sum \gamma Q = D + H + (0.6W \text{ or } 0.7E) \qquad \text{[IBC 16-12]}$$

$$\sum \gamma Q = D + H + 0.75(0.6W) + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$$
$$\text{[IBC 16-13]}$$

$$\sum \gamma Q = D + H + 0.75(0.7E) + 0.75L + 0.75S \qquad \text{[IBC 16-14]}$$

$$\sum \gamma Q = 0.6D + 0.6W + H \qquad \text{[IBC 16-15]}$$

$$\sum \gamma Q = 0.6D + 0.7E + H \qquad \text{[IBC 16-16]}$$

In accordance with MSJC Comm. Sec. 2.1.1, allowable stresses may not be increased by one-third for wind for seismic load combinations.

LRFD Required Strength

The required ultimate strength of a member consists of the most critical combination of factored loads applied to the member. Factored loads consist of working, or service, loads multiplied by the appropriate SD load factors. In accordance with MSJC Sec. 1.7.2, load combinations must be as specified in the applicable building code. The required strength, $\Sigma\gamma Q$, is defined by seven combinations given in IBC Sec. 1605.2. The combinations, with uncommon load conditions (self-straining loads and fluid pressure) omitted, are as follows.

$$\sum \gamma Q = 1.4D \qquad \text{[IBC 16-1]}$$

$$\sum \gamma Q = 1.2D + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R) \qquad \text{[IBC 16-2]}$$

$$\sum \gamma Q = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + 1.6H + (f_1 L \text{ or } 0.5W) \qquad \text{[IBC 16-3]}$$

$$\sum \gamma Q = 1.2D + 1.0W + f_1 L + 1.6H + 0.5(L_r \text{ or } S \text{ or } R) \qquad \text{[IBC 16-4]}$$

$$\sum \gamma Q = 1.2D \pm 1.0E + f_1 L + 1.6H + f_2 S \qquad \text{[IBC 16-5]}$$

$$\sum \gamma Q = 0.9D + 1.0W + 1.6H \qquad \text{[IBC 16-6]}$$

$$\sum \gamma Q = 0.9D + 1.0E + 1.6H \qquad \text{[IBC 16-7]}$$

For IBC Eq. 16-3, Eq. 16-4, and Eq. 16-5, use $f_1 = 1.0$ for garages, places of public assembly, and areas where $L > 100$ lbf/ft^2. Use $f_1 = 0.5$ for all other live loads. For IBC Eq. 16-5, use $f_2 = 0.7$ for roof configurations that do not shed snow, and use 0.2 for other roof configurations.

Where the effect of H resists the primary variable load effect, a load factor of 0.9 must be included with H where H is permanent. H must be set to zero for all other conditions.

Masonry

ASD Allowable Stresses

Allowable stresses are determined by applying safety factors to material nominal stresses. Allowable tensile stress in bar reinforcement is given in MSJC Sec. 2.3.3 as

$$F_s = 32,000 \text{ lbf/in}^2 \quad \text{[for grade 60 reinforcement]}$$

$$F_s = 20,000 \text{ lbf/in}^2 \quad \begin{bmatrix} \text{for grade 40 or grade 50} \\ \text{reinforcement} \end{bmatrix}$$

Allowable compressive stress in masonry due to flexure is given in MSJC Sec. 2.3.4.2.2 as

$$F_b = 0.45 f'_m$$

The modulus of elasticity of steel reinforcement is given by MSJC Sec. 1.8.2.1 as

$$E_s = 29,000,000 \text{ lbf/in}^2$$

The modulus of elasticity of concrete masonry is based on the chord modulus of elasticity. It is between 0.05 and 0.33 of the maximum compressive strength of the masonry prism. Alternatively, from MSJC Sec. 1.8.2.2.1, the modulus of elasticity may be derived from the equation

$$E_m = 900 f'_m$$

SD Design Strength

The design strength of a member consists of the nominal, or theoretical ultimate, strength of the member multiplied by the appropriate strength reduction factor, ϕ. The design strength must equal or exceed the required strength, which consists of combinations of service loads multiplied by the appropriate load factors. The reduction factors for reinforced masonry are defined in MSJC Sec. 3.1.4 as

$$\phi = 0.90 \quad \begin{bmatrix} \text{flexure, axial load, or combinations} \\ \text{of flexure and axial load} \end{bmatrix}$$

$$\phi = 0.80 \quad \text{[shear]}$$

$$\phi = 0.50 \quad \begin{bmatrix} \text{anchor bolts, strength governed by} \\ \text{masonry breakout, crushing, or pryout} \end{bmatrix}$$

$$\phi = 0.90 \quad \begin{bmatrix} \text{anchor bolts, strength governed by} \\ \text{anchor bolt steel} \end{bmatrix}$$

$$\phi = 0.65 \quad \begin{bmatrix} \text{anchor bolts, strength governed by} \\ \text{anchor pullout} \end{bmatrix}$$

$$\phi = 0.60 \quad \text{[bearing on masonry surfaces]}$$

4. MASONRY BEAMS IN FLEXURE

Nomenclature

a	depth of equivalent rectangular stress block	in
$a_{\max}$	maximum allowable value of a	in
A_s	area of tension reinforcement	in^2
b	width of beam	in
c	distance from extreme compression fiber to neutral axis	in
$c_{\max}$	maximum allowable value of c	in
C	compressive force	lbf
d	effective depth, distance from extreme compression fiber to centroid of tension reinforcement	in
d_b	diameter of reinforcement	in
E_m	modulus of elasticity of masonry in compression	lbf/in^2
E_s	modulus of elasticity of steel reinforcement	lbf/in^2
f_b	calculated compressive stress in masonry due to flexure	lbf/in^2
f'_m	specified masonry compressive strength	lbf/in^2
f_r	modulus of rupture of masonry	lbf/in^2
f_s	calculated stress in reinforcement	lbf/in^2
f_y	yield strength of reinforcement	lbf/in^2
F_b	allowable compressive stress in masonry due to flexure	lbf/in^2
F_s	allowable stress in reinforcement	lbf/in^2
h	overall dimension of member	in
j	lever-arm factor, ratio of distance between centroid of flexural compression forces and centroid of tensile forces to effective depth, $1 - k/3$ (ASD)	–
j	lever-arm factor, ratio of distance between centroid of flexural compression forces and centroid of tensile forces to effective depth, $(d - a/2)/d$ (SD method)	–
k	neutral axis depth factor, $\sqrt{2\rho n + (\rho n)^2} - \rho n$	–
K	lesser of masonry cover, clear spacing between adjacent reinforcement, or $5d_b$	in
K_u	design moment factor, $M_u/b_w d^2$	lbf/in^2
l	clear span length of beam	ft
l_c	distance between points of lateral support	ft
l_d	development length of straight reinforcement	in
l_e	effective span	ft
l_e	equivalent development length provided by a standard hook	in
l_s	distance between centers of supports	ft
M	applied moment	ft-kips or in-lbf

Masonry

M_{cr}	nominal cracking moment strength of a member	ft-kips
$M_{\max}$	maximum design flexural strength	ft-kips
M_n	nominal flexural strength of a member	ft-kips
n	modular ratio, E_s/E_m	–
s_c	clear spacing of reinforcement	in
S_n	section modulus of the beam	in^3
T	tensile force	lbf

Symbols

γ	reinforcement size factor	–
ϵ_m	strain in masonry	–
ϵ_{mu}	maximum usable compressive strain of masonry, 0.0035 for clay masonry and 0.0025 for concrete masonry	–
ϵ_s	strain in reinforcement	–
ϵ_y	strain at yield in tension reinforcement	–
ρ	tension reinforcement ratio, $A_s/b_w d$	–
$\rho_{\max}$	maximum tension reinforcement ratio in a rectangular beam with tension reinforcement only	–
$\rho_{\min}$	minimum allowable reinforcement ratio	–
ϕ	strength reduction factor	–

Reinforcement Requirements

The size of reinforcement allowed is specified by MSJC Sec. 1.16.2 and Sec. 3.3.3 and by IBC Sec. 2107.4. To control bond stresses in the bars, and to reduce congestion and aid in grout consolidation, the bar diameter must not exceed the lesser of

- one-eighth of the nominal member thickness

- one-quarter of the least clear dimension of the cell, course, or collar joint

- no. 11 bar (ASD) or no. 9 bar (SD)

The area of reinforcing bars placed in a cell must not exceed 4% of the cell area in accordance with MSJC Sec. 3.3.3.1.

In accordance with MSJC Sec. 1.16.3, the clear distance between parallel bars must not be less than the nominal diameter of the bars, with a minimum distance of 1 in. In columns and pilasters, the clear distance between vertical bars must not be less than 1.5 times the nominal bar diameter, with a minimum distance of $1^{1}/_{2}$ in. The thickness of grout between the reinforcement and the masonry unit must be a minimum of $^{1}/_{4}$ in for fine grout, or $^{1}/_{2}$ in for coarse grout. Not more than two reinforcing bars may be bundled. MSJC Sec. 3.3.4.2.2.1 specifies that not more than two bar sizes may be used in a beam and that variations in bar size cannot exceed one bar size.

Example 6.1

A nominal 8 in beam is shown. Determine the maximum permissible reinforcement bar size.
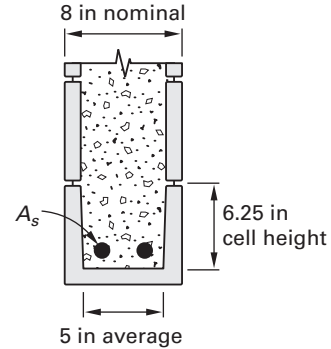
Solution

ASD Method

From MSJC Sec. 1.16.2, the maximum reinforcement bar size permitted is

$$d_b = \text{no. 11 bar}$$

From IBC Sec. 2107.4, the maximum reinforcement bar size permitted is

$$d_b = \text{one-quarter least dimension of cell}$$
$$= \frac{5 \text{ in}}{4}$$
$$= 1.25 \text{ in}$$

Or,

$$d_b = \text{one-eighth nominal member thickness}$$
$$= \frac{8 \text{ in}}{8}$$
$$= 1 \text{ in} \quad [\text{governs}]$$

To conform to the governing bar size of 1 in, two no. 8 bars may be used. This provides a clear spacing between bars of 2 in and the thickness of grout between the reinforcement and the masonry unit of $^{1}/_{2}$ in, which satisfies MSJC Sec. 1.16.3.

SD Method

From MSJC Sec. 3.3.3.1, the maximum reinforcement bar size permitted is

$$d_b = \text{no. 9 bar}$$
$$= \text{one-quarter least dimension of cell}$$
$$= \frac{5 \text{ in}}{4}$$
$$= 1.25 \text{ in}$$

Or,

$$d_b = \text{one-eighth nominal member thickness}$$
$$= \frac{8 \text{ in}}{8}$$
$$= 1 \text{ in} \quad [\text{governs}]$$

From MSJC Sec 3.3.3.1, the maximum area of reinforcing bars permitted is

$$A_s = 4\% \text{ cell area}$$
$$= (0.04)(5 \text{ in average cell width})(6.25 \text{ in cell height})$$
$$= 1.25 \text{ in}^2 \quad [\text{governs}]$$

To conform to the governing reinforcing bar area of 1.25 in^2, two no. 7 bars that give an area of 1.20 in^2 are satisfactory. This provides a clear spacing between bars of 2 in and the thickness of grout between the reinforcement and the masonry unit of $^5/_8$ in, which satisfies MSJC Sec. 1.16.3.

Dimensional Limitations

The maximum permitted unbraced length on the compression side of a masonry beam is given by MSJC Sec. 1.13.1.2 as

$$l_c = 32b$$
$$\leq \frac{120b^2}{d}$$

The minimum permitted bearing length of a masonry beam is given by MSJC Sec. 1.13.1.3 as

$$b_r = 4 \text{ in}$$

In accordance with MSJC Sec. 3.3.4.2.5(b), the nominal depth of a beam must not be less than

$$h = 8 \text{ in}$$

MSJC Sec. 3.3.4.2.4 requires all beams to be solid grouted.

Development Length and Splice Length of Reinforcement

The basic development length of compression and tension reinforcement is given by MSJC Eq. 2-12 and Eq. 3-16 as

$$l_d = \frac{0.13d_b^2 f_y \gamma}{K\sqrt{f'_m}}$$
$$\geq 12 \text{ in}$$

K is the lesser of masonry cover, clear spacing of reinforcement, or nine times the bar diameter, d_b. γ is 1.0 for no. 3 through no. 5 bars; 1.3 for no. 6 through no. 7 bars; and 1.5 for no. 8 through no. 11 bars (ASD) or no. 9 bars (SD).

The equivalent development length of a standard hook in tension is specified in MSJC Sec. 2.1.7.5.1 and Sec. 3.3.3.2 as

$$l_e = 13d_b$$

In accordance with IBC Sec. 2107.2.1, the lap splice length of straight reinforcing bars is

$$l_d = 0.002d_b f_s$$
$$\geq 12 \text{ in}$$
$$\geq 40d_b$$

When the design tensile strength in the reinforcement exceeds 80% of the allowable stress, lap splice length must be increased 50%. When epoxy coated bars are used, lap splice length must be increased 50%. Welded or mechanical splices are required to develop a minimum of $1.25f_y$ in accordance with MSJC Sec. 2.1.7.7 and Sec. 3.3.3.4.

Example 6.2

The nominal 8 in beam shown has a masonry compressive strength of 1500 lbf/in^2 and reinforcement consisting of two grade 60 no. 7 bars. The clear distance between bars is 2 in. Determine the required development length for straight bars and for bars provided with a standard hook.
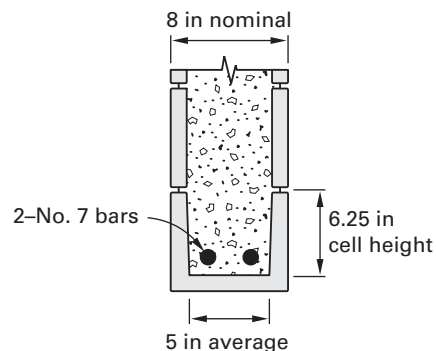
Solution

ASD and SD Method

The development parameter, K, is the lesser of the masonry cover, the clear spacing of reinforcement, or $9d_b$.

For the masonry cover,

$$K = \frac{b - s_c - 2d_b}{2}$$
$$= \frac{7.63 \text{ in} - 2 \text{ in} - (2)(0.875 \text{ in})}{2}$$
$$= 1.94 \text{ in} \quad [\text{governs}]$$

For the clear spacing of reinforcement,

$$K = s_c = 2 \text{ in}$$

For the bar diameter,

$$K = 9d_b$$
$$= (9)(0.875 \text{ in})$$
$$= 7.88 \text{ in}$$

The reinforcement size factor for a no. 7 bar is

$$\gamma = 1.3$$

The required development length for a straight bar is given by MSJC Eq. 2-12 as

$$l_d = \frac{0.13 d_b^2 f_y \gamma}{K \sqrt{f_m'}}$$
$$= \frac{(0.13)(0.875 \text{ in})^2 \left(60{,}000 \ \dfrac{\text{lbf}}{\text{in}^2}\right)(1.3)}{(1.94 \text{ in}) \sqrt{1500 \ \dfrac{\text{lbf}}{\text{in}^2}}}$$
$$= 103 \text{ in}$$

The equivalent development length provided by a standard hook is given by MSJC Sec. 2.1.7.5.1 as

$$l_e = 13d_b$$
$$= (13)(0.875 \text{ in})$$
$$= 11 \text{ in}$$

The required development length of the no. 7 bars, for bars provided with a standard hook, is

$$l_d = 103 \text{ in} - 11 \text{ in}$$
$$= 92 \text{ in}$$

Effective Span Length of Masonry Beams

The effective span length of a simply supported beam is defined in MSJC Sec. 1.13.1.1.1 and is illustrated in Fig. 6.1.

The effective span is defined as the clear span plus the depth of member, as shown in Fig. 6.1(b).

$$l_e = l + h$$

The effective span must not exceed the distance between centers of supports, as shown in Fig. 6.1(a).

$$l_e = l_s = l + b_s$$

Alternatively, as specified in MSJC Sec. 1.13.2.1, the effective span length may be taken as the distance between centers of supports, or 1.15 times the clear span, whichever is the smaller.

For a continuous beam, MSJC Sec. 1.13.1.1.2 defines the effective span as the distance between centers of supports, as shown in Fig. 6.1(c).

$$l_e = l_s = l + b_s$$

As shown in Fig. 6.1(d), the effective span length of beams built integrally with supports is customarily taken as equal to the clear span.

$$l_e = l$$

Beams with Tension Reinforcement Only

ASD Method

For grade 40 or grade 50 reinforcement, the allowable tensile stress and the allowable compressive stress are given by MSJC Sec. 2.3.3 as

$$F_s = 20{,}000 \text{ lbf/in}^2$$

For grade 60 reinforcement, the allowable tensile stress and the allowable compressive stress are given by MSJC Sec. 2.3.3 as

$$F_s = 32{,}000 \text{ lbf/in}^2$$

The allowable compressive stress in masonry due to flexure is given by MSJC Sec. 2.3.4.2.2 as

$$F_b = 0.45 f_m'$$

The elastic design method, illustrated in Fig. 6.2, is used to calculate the stresses in a masonry beam under the action of the applied service loads, and to ensure that these stresses do not exceed allowable values.

The basic assumptions adopted[3] in the elastic, or allowable stress, design method are as follows.

- The strain distribution over the depth of the member is linear, as shown in Fig. 6.2.

- Stresses in the masonry and in the reinforcement are proportional to the induced strain.

- Tensile stress in the masonry is neglected.

Figure 6.1 *Effective Span Length*

(a)

(b)

(c)

(d)
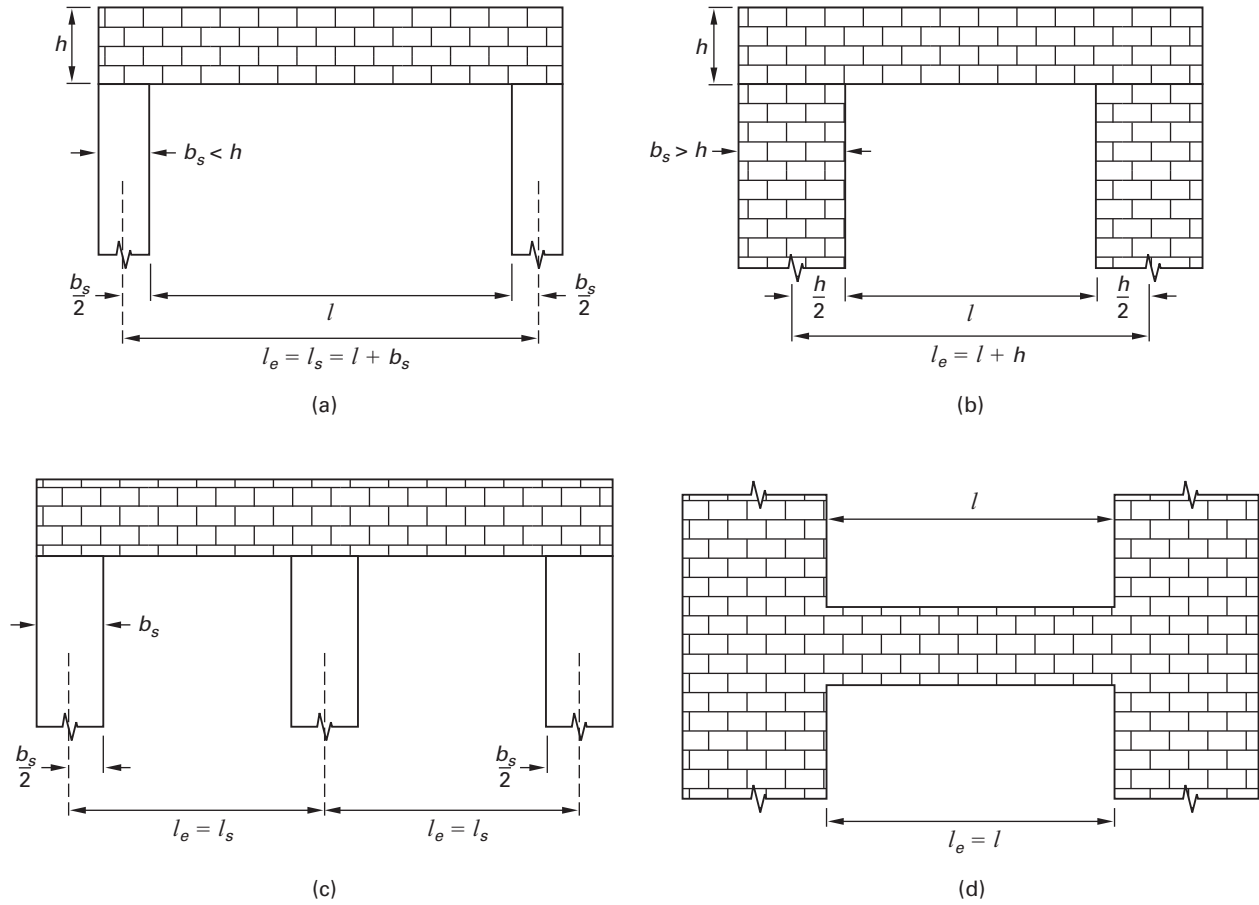
Figure 6.2 *Elastic Design of Reinforced Masonry Beam*

cross section

strain

concrete stress (shaded) and force

The modular ratio is defined as

$$n = \frac{E_s}{E_m}$$

The tension reinforcement ratio is
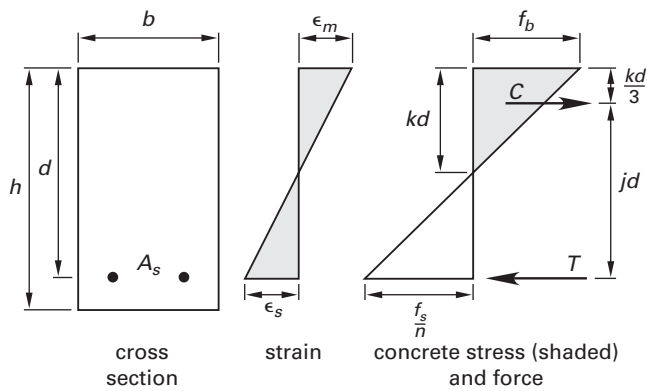
$$\rho = \frac{A_s}{bd}$$

From the strain diagram, with a neutral axis depth of kd, as shown in Fig. 6.2,

$$\frac{\epsilon_s}{\epsilon_m} = \frac{d - kd}{kd}$$

$$= \frac{1 - k}{k}$$

$$\frac{f_s E_m}{f_b E_s} = \frac{1 - k}{k}$$

$$\frac{f_s}{f_b} = \frac{n(1 - k)}{k}$$

A triangular compressive stress distribution is formed in the masonry, above the neutral axis, and the line of action of the compressive force, C, acts at a depth of $kd/3$ with

$$C = \frac{f_b k d b}{2}$$

The tensile force developed in the reinforcement is

$$T = f_s A_s$$

$$= f_s \rho b d$$

Masonry

Equating tensile and compressive forces acting on the section gives

$$f_s \rho b d = \frac{f_b k d b}{2}$$

$$\frac{f_s}{f_b} = \frac{k}{2\rho}$$

$$= \frac{n(1-k)}{k}$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

The lever arm of the internal resisting moment is obtained from Fig. 6.2 as

$$jd = d - \frac{kd}{3}$$

The lever arm factor is

$$j = 1 - \frac{k}{3}$$

The resisting moment of the masonry is

$$M_m = Cjd$$

$$= \frac{f_b j k b d^2}{2}$$

The resisting moment of the reinforcement is

$$M_s = Tjd$$

$$= f_s j \rho b d^2$$

For an allowable masonry stress of F_b and an allowable reinforcement stress of F_s, the allowable moment capacity of the section, M_R, is the lesser of M_m and M_s as follows.

$$M_m = \frac{F_b j k b d^2}{2}$$

$$M_s = F_s j \rho b d^2$$

To facilitate the determination of k, App. B tabulates values of k against ρn.

The stress in the reinforcement due to an applied moment, M, is

$$f_s = \frac{M}{A_s j d}$$

The stress in the masonry due to an applied moment, M, is

$$f_b = \frac{2M}{j k b d^2}$$

ASD Design Procedure

The ASD design procedure consists of the following steps.

step 1: Assume beam dimensions and masonry strength.

step 2: Assume that $j = 0.9$.

step 3: Calculate $A_s = M/(F_s j d)$.

step 4: Select bar size and number required.

step 5: Calculate ρ and ρn.

step 6: Determine k from the $\rho n/k$ table in MSJC App. B.

step 7: Calculate j.

step 8: Calculate M_m.

step 9: If $M_m < M$, increase beam size or f'_m.

step 10: Calculate M_s.

step 11: If $M_s < M$, increase A_s.

step 12: If both M_m and M_s are greater than M, the beam is satisfactory.

ASD Analysis Procedure

The ASD analysis procedure consists of the following steps.

step 1: Calculate ρ and ρn.

step 2: Determine k from the $\rho n/k$ table in MSJC App. B.

step 3: Calculate j.

step 4: Calculate f_b.

step 5: Calculate f_s.

SD Method

The strength design method[7], illustrated in Fig. 6.3 for a beam reinforced in tension, is used to calculate the design strength of a concrete masonry beam under the action of the applied factored loads, to ensure that the design strength is greater than the most critical load combination. In accordance with MSJC Sec. 3.3.2, during the loading to failure of a reinforced masonry beam, the strain distribution over the depth of the beam is linear. MSJC Sec. 3.3.2 provisions also state that an equivalent rectangular stress distribution has a depth of 0.8 times the depth to the neutral axis and a stress of 0.8 times the specified masonry compressive strength; that at failure, the tension reinforcement has yielded; and that the maximum compressive strain in the masonry is 0.0035 for clay masonry and 0.0025 for concrete masonry. The tensile strength of the masonry is neglected.

Figure 6.3 *Strength Design of a Reinforced Masonry Beam*
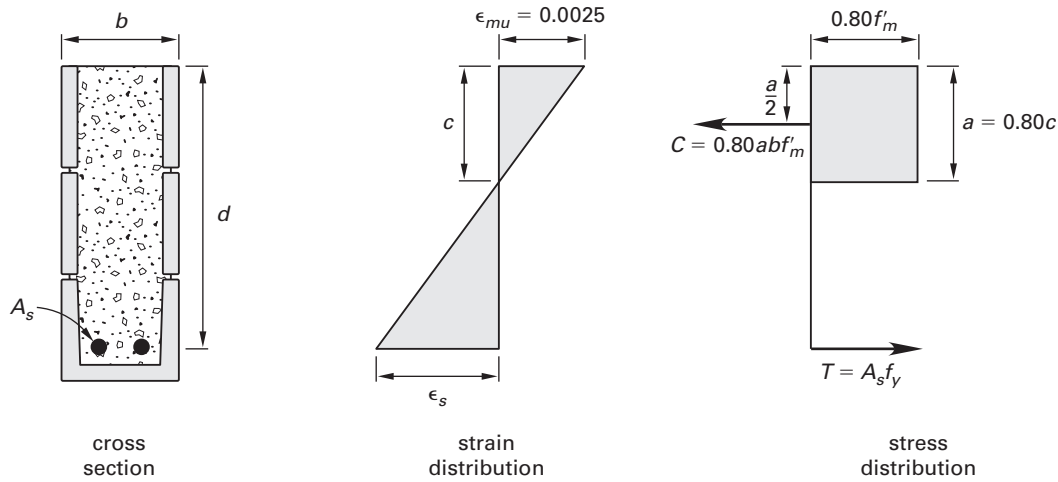
The depth of the equivalent rectangular stress block is obtained by equating the tensile and compressive forces acting on the section.

$$a = \frac{A_s f_y}{0.80 b f'_m}$$
$$= \frac{1.25 \rho d f_y}{f'_m}$$
$$= 0.80c$$

From Fig. 6.3, the nominal flexural strength of the member is derived as

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$
$$= A_s f_y \left(d - \frac{A_s f_y}{(2)(0.80) b f'_m} \right)$$
$$= A_s f_y d \left(\frac{1 - 0.625 A_s f_y}{b d f'_m} \right)$$
$$= \rho f_y b d^2 \left(\frac{1 - 0.625 \rho f_y}{f'_m} \right)$$

In addition, the expression for nominal strength is

$$M_n = \frac{M_u}{\phi}$$
$$= \frac{M_u}{0.9}$$
$$= \rho f_y b d^2 - \frac{0.625 \rho^2 b d^2 f_y^2}{f'_m}$$
$$= \rho^2 - \rho \left(\frac{f'_m}{0.625 f_y} \right) + \frac{f'_m M_u}{0.563 b d^2 f_y^2}$$

The reinforcement ratio required to provide a given factored moment, M_u, is then

$$\rho = 0.80 f'_m \left(\frac{1 - \sqrt{1 - \dfrac{M_u}{0.36 b d^2 f'_m}}}{f_y} \right)$$
$$= 0.80 f'_m \left(\frac{1 - \sqrt{1 - \dfrac{K_u}{0.36 f'_m}}}{f_y} \right)$$

The design moment factor is

$$K_u = \frac{M_u}{b d^2}$$

Minimum Reinforcement Area

To prevent brittle failure of a lightly reinforced beam, MSJC Sec. 3.3.4.2.2.2 requires the nominal flexural strength of a beam to be not less than 1.3 times the nominal cracking moment strength of the beam. This provision is imposed so that there is a minimum area of reinforcement in a beam to ensure compliance. The required nominal moment is

$$M_n \geq 1.3 M_{cr}$$

The nominal cracking moment of the beam is

$$M_{cr} = f_r S_n$$

S_n is the section modulus of the beam, and f_r is the modulus of rupture of the masonry. The modulus of rupture is given in MSJC Table 3.1.8.2.

Masonry

Example 6.3

The nominal 8 in beam shown has a masonry compressive strength of 1500 lbf/in^2. Its reinforcement consists of two grade 60 no. 5 bars with an effective depth of 21 in. The cells are solid grouted, and the masonry is laid in running bond with type S mortar cement. Use the SD method to determine if the minimum reinforcement meets MSJC requirements.

b = 8 in nominal

h = 24 in

A_s = 2–No. 5 bars
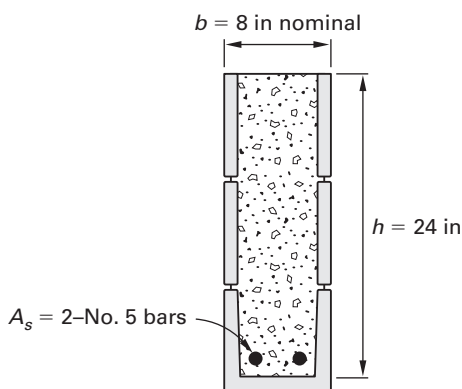
Solution

Use the SD method only.

The section modulus of the beam is

$$S_n = \frac{bh^2}{6}$$
$$= \frac{(7.63 \text{ in})(24 \text{ in})^2}{6}$$
$$= 732 \text{ in}^3$$

The modulus of rupture of the solid grouted masonry, for tension parallel to bed joints and type S mortar, is given by MSJC Table 3.1.8.2 as

$$f_r = 200 \text{ lbf/in}^2$$

The cracking moment of the beam is

$$M_{cr} = f_r S_n$$
$$= \frac{\left(200 \frac{\text{lbf}}{\text{in}^2}\right)(732 \text{ in}^3)}{\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}$$
$$= 12.20 \text{ ft-kips}$$
$$1.3M_{cr} = (1.3)(12.20 \text{ ft-kips})$$
$$= 15.86 \text{ ft-kips}$$

The area of tension reinforcement is given as two no. 5 bars, which provide an area of

$$A_s = 0.62 \text{ in}^2$$

The depth of the equivalent rectangular stress block is obtained by equating tensile and compressive forces acting on the section. The depth is given by

$$a = \frac{A_s f_y}{0.80 b f'_m}$$
$$= \frac{(0.62 \text{ in}^2)\left(60,000 \frac{\text{lbf}}{\text{in}^2}\right)}{(0.80)(7.63 \text{ in})\left(1500 \frac{\text{lbf}}{\text{in}^2}\right)}$$
$$= 4.06 \text{ in}$$

The lever arm of the compressive and tensile forces is given by

$$jd = d - \frac{a}{2}$$
$$= 21 \text{ in} - \frac{4.06 \text{ in}}{2}$$
$$= 18.97 \text{ in}$$

The nominal flexural strength of the beam is given by

$$M_n = A_s jd f_y$$
$$= \frac{(0.62 \text{ in}^2)(18.97 \text{ in})\left(60 \frac{\text{kips}}{\text{in}^2}\right)}{12 \frac{\text{in}}{\text{ft}}}$$
$$= 58.81 \text{ ft-kips}$$
$$> 1.3M_{cr} \quad \text{[satisfies MSJC Sec. 3.3.4.2.2.2]}$$

Maximum Reinforcement Ratio

In order to provide adequate ductile response in a beam, MSJC Sec. 3.3.3.5.1 limits the maximum reinforcement ratio, $\rho_{\max}$, in accordance with a prescribed strain distribution. The area of tensile reinforcement must not exceed the area required to develop a strain of $1.5\epsilon_y$ in the extreme tensile reinforcement, simultaneously with a maximum masonry compressive strain of 0.0025 for concrete masonry or 0.0035 for clay masonry. This is applicable when the structure is designed using a value for the response modification factor, R, greater than 1.5, as defined by ASCE/SEI7[5] Table 12.2-1. Since the yield strain for grade 60 reinforcement is 0.00207, as shown in Fig. 6.4, this requirement effectively limits the depth of the neutral axis for reinforced concrete masonry to a maximum of

$$c_{\max} = d\left(\frac{\epsilon_{mu}}{\epsilon_{mu} + 1.5\epsilon_y}\right)$$
$$= d\left(\frac{0.0025}{0.0025 + (1.5)(0.00207)}\right)$$
$$= 0.446d$$

Masonry

Figure 6.4 *Maximum Reinforcement in Concrete Masonry Beams*
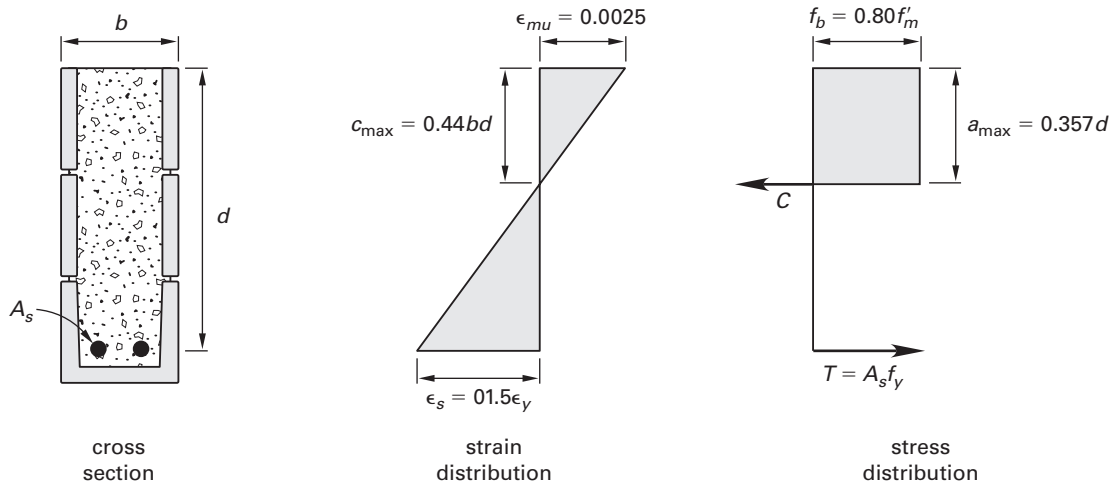
The depth of the equivalent rectangular stress block is limited to a maximum of

$$a_{\max} = 0.8c_{\max}$$
$$= 0.8(0.446d)$$
$$= 0.357d$$

When calculating the maximum reinforcement area, it is assumed that the stress in the reinforcement equals the product of the modulus of elasticity of the steel and the strain in the reinforcement, and that the stress is not greater than f_y. The maximum reinforcement area corresponding to the maximum depth of the equivalent rectangular stress block is given by

$$A_{\max} = \frac{0.80 a_{\max} b f'_m}{f_y}$$
$$= \frac{0.286 b d f'_m}{f_y}$$

The corresponding maximum reinforcement ratio is given by

$$\rho_{\max} = \frac{A_{\max}}{bd}$$
$$= \frac{0.286 f'_m}{f_y}$$

Example 6.4

The nominal 8 in beam shown has a masonry compressive strength of 1500 lbf/in² and reinforcement consisting of two grade 60 no. 5 bars. Use the SD method to determine if the maximum reinforcement meets MSJC requirements.
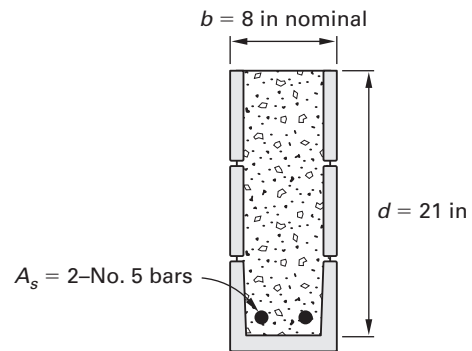
Solution

Use the SD method only.

The maximum allowable reinforcement ratio is

$$\rho_{\max} = \frac{0.286 f'_m}{f_y} = \frac{(0.286)\left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)}{60{,}000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.0072$$

The actual reinforcement ratio provided is

$$\rho = \frac{A_s}{bd}$$
$$= \frac{(2)(0.31 \text{ in}^2)}{(7.63 \text{ in})(21 \text{ in})}$$
$$= 0.0039$$
$$< \rho_{\max} \quad \text{[satisfies MSJC Sec. 3.3.3.5.1]}$$

SD Design Procedure

The SD design procedure consists of the following steps.

step 1: Assume beam dimensions and masonry strength.

step 2: Calculate $K_u = M_u/bd^2$.

step 3: Calculate the reinforcement ratio.

$$\rho = 0.80 f'_m \left(\frac{1 - \sqrt{1 - \dfrac{K_u}{0.36 f'_m}}}{f_y} \right)$$

step 4: Select the bar size and number required.

step 5: Check that the beam complies with the maximum reinforcement requirements of MSJC Sec. 3.3.3.5. Increase beam size or f'_m if necessary.

step 6: Check that $M \geq 1.3 M_{cr}$. Increase beam size or f'_m if necessary.

SD Analysis Procedure

The SD analysis procedure consists of the following steps.

step 1: Calculate stress block depth.

$$a = \frac{A_s f_y}{0.80 b f'_m}$$

step 2: Calculate nominal strength.

$$M_n = A_s f_y \left(\frac{d - a}{2} \right)$$

step 3: Calculate design strength, ϕM_n.

Illustration for Ex. 6.5
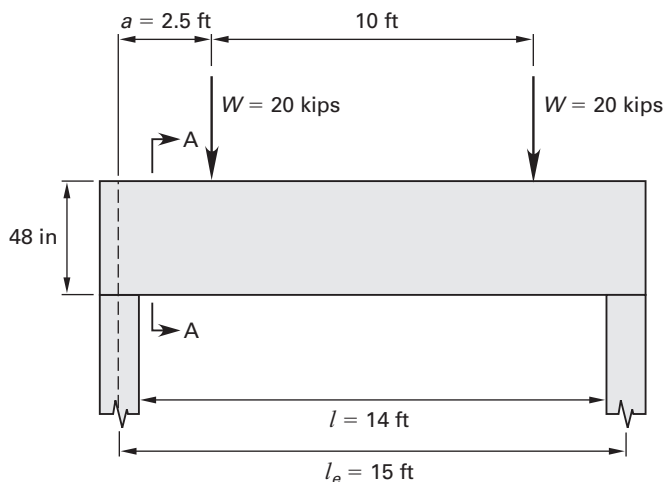
Example 6.5

The 8 in solid grouted, concrete block masonry beam shown is simply supported over an effective span of 15 ft. The masonry has a compressive strength of 1500 lbf/in² and a modulus of elasticity of 1,000,000 lbf/in². Reinforcement consists of four no. 6 grade 60 bars. The effective depth is 45 in, the overall depth is 48 in, and the beam is laterally braced at both ends. The 20 kips concentrated loads are floor live loads. The self-weight of the beam is 69 lbf/ft². Determine whether the beam is adequate.

Solution

The beam self-weight is

$$w = \left(69 \ \frac{\text{lbf}}{\text{ft}^2} \right) \left(\frac{48 \ \text{in}}{12 \ \frac{\text{in}}{\text{ft}}} \right)$$

$$= 276 \ \text{lbf/ft}$$

At midspan, the bending moment produced by this self-weight is

$$M_s = \frac{wl^2}{8} = \frac{\left(276 \ \dfrac{\text{lbf}}{\text{ft}} \right) (15 \ \text{ft})^2}{(8) \left(1000 \ \dfrac{\text{lbf}}{\text{kip}} \right)}$$

$$= 7.76 \ \text{ft-kips}$$

At midspan, the bending moment produced by the concentrated loads is

$$M_c = Wa = \frac{(20 \ \text{kips})(15 \ \text{ft} - 10 \ \text{ft})}{2}$$

$$= 50 \ \text{ft-kips}$$
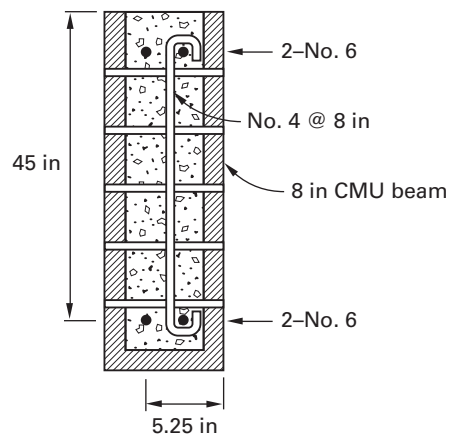
section A-A

ASD Method

At midspan, the total ASD moment is given by IBC Eq. 16-9 as

$$M_y = 1.0M_s + 1.0M_c$$
$$= (1.0)(7.76 \text{ ft-kips}) + (1.0)(50 \text{ ft-kips})$$
$$= 57.76 \text{ ft-kips}$$

The allowable stresses, in accordance with MSJC Sec. 2.3.3 and Sec. 2.3.4.2.2, are

$$F_b = 0.45f'_m$$
$$= (0.45)\left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)$$
$$= 675 \text{ lbf/in}^2$$
$$F_s = 32{,}000 \text{ lbf/in}^2$$

The relevant parameters of the beam are

$$E_m = 1{,}000{,}000 \text{ lbf/in}^2$$
$$E_s = 29{,}000{,}000 \text{ lbf/in}^2$$
$$b = 7.63 \text{ in}$$
$$d = 45 \text{ in}$$
$$l_e = 15 \text{ ft}$$
$$A_s = (2)(0.44 \text{ in}^2) = 0.88 \text{ in}^2$$

$$\frac{l_e}{b} = \frac{(15 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{7.63 \text{ in}}$$
$$= 23.6$$
$$< 32 \quad \begin{bmatrix} \text{satisfies MSJC} \\ \text{Sec. 1.13.1.2} \end{bmatrix}$$
$$n = \frac{E_s}{E_m} = \frac{29{,}000{,}000 \ \dfrac{\text{lbf}}{\text{in}^2}}{1{,}000{,}000 \ \dfrac{\text{lbf}}{\text{in}^2}}$$
$$= 29$$

$$\rho = \frac{A_s}{bd} = \frac{0.88 \text{ in}^2}{(7.63 \text{ in})(45 \text{ in})}$$
$$= 0.00256$$
$$\rho n = (0.00256)(29)$$
$$= 0.0743$$

From App. B, the beam parameters and stresses, in accordance with MSJC Sec. 2.3.2, are

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$
$$= \sqrt{(2)(0.0743) + (0.0743)^2} - 0.0743$$
$$= 0.318$$
$$j = 1 - \frac{k}{3} = 1 - \frac{0.318}{3}$$
$$= 0.894$$
$$f_b = \frac{2M_y}{jkbd^2}$$
$$= \frac{(2)(57.76 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(0.894)(0.318)(7.63 \text{ in})(45 \text{ in})^2}$$
$$= 315 \text{ lbf/in}^2$$
$$< F_b \quad [\text{satisfactory}]$$
$$f_s = \frac{M_y}{jdA_s}$$
$$= \frac{(57.76 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(0.894)(45 \text{ in})(0.88 \text{ in}^2)}$$
$$= 19{,}580 \text{ lbf/in}^2$$
$$< F_s \quad [\text{satisfactory}]$$

The beam is adequate.

SD Method

The total factored moment at midspan is given by IBC Eq. 16-2 as

$$M_u = 1.2M_s + 1.6M_c$$
$$= (1.2)(7.76 \text{ ft-kips}) + (1.6)(50 \text{ ft-kips})$$
$$= 89.31 \text{ ft-kips}$$

The stress block depth is

$$a = \frac{A_s f_y}{0.80bf'_m}$$
$$= \frac{(0.88 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(0.80)(7.63 \text{ in})\left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)}$$
$$= 5.77 \text{ in}$$

Masonry

The nominal strength is

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$$= (0.88 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)\left(\frac{45 \text{ in} - \dfrac{5.77 \text{ in}}{2}}{12 \ \dfrac{\text{in}}{\text{ft}}}\right)$$

$$= 185 \text{ ft-kips}$$

The design strength is

$$\phi M_n = (0.9)(185 \text{ ft-kips})$$

$$= 167 \text{ ft-kips}$$

$$> M_u \quad \text{[satisfactory]}$$

The beam is adequate.

Biaxial Bending

Using the ASD method, the combined stresses produced by biaxial bending must not exceed the allowable values. Using the SD method, the interaction equation is used to determine the adequacy of a member.

Example 6.6

The masonry beam described in Ex. 6.5, in addition to the vertical loads indicated, is subjected to a lateral force of 140 lbf/ft due to wind. Determine whether the beam is adequate.

Solution

At midspan, the bending moment produced by the wind load is

$$M_x = \frac{q l_e^2}{8} = \frac{\left(140 \ \dfrac{\text{lbf}}{\text{ft}}\right)(15 \text{ ft})^2}{(8)\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}$$

$$= 3.94 \text{ ft-kips}$$

The relevant parameters of the beam in the transverse direction are

$$b = 48 \text{ in}$$
$$d = 5.25 \text{ in}$$
$$l_e = 15 \text{ ft}$$
$$A_s = 0.88 \text{ in}^2$$
$$n = 29$$
$$\rho = \frac{A_s}{bd} = \frac{0.88 \text{ in}^2}{(48 \text{ in})(5.25 \text{ in})}$$
$$= 0.00349$$
$$\rho n = (0.00349)(29)$$
$$= 0.101$$

ASD Method

From App. B, the beam parameters and stresses caused by the wind load, in accordance with MSJC Sec. 2.3.2, are

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$= \sqrt{(2)(0.101) + (0.101)^2} - 0.101$$

$$= 0.360$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.360}{3} = 0.88$$

$$f_{bw} = \frac{2M_x}{jkbd^2}$$

$$= \frac{(2)(3.94 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.88)(0.360)(48 \text{ in})(5.25 \text{ in})^2}$$

$$= 225 \text{ lbf/in}^2$$

$$f_{sw} = \frac{M_x}{jdA_s}$$

$$= \frac{(3.94 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.88)(5.25 \text{ in})(0.88 \text{ in}^2)}$$

$$= 11,629 \text{ lbf/in}^2$$

From Ex. 6.5, the allowable stresses are

$$F_b = 675 \text{ lbf/in}^2$$
$$F_s = 32,000 \text{ lbf/in}^2$$

The bending moment produced by the beam self-weight is

$$M_s = 7.76 \text{ ft-kips}$$

The stresses produced in the masonry and in the reinforcement by the beam self-weight are

$$f_{bs} = \frac{2M_s}{jkbd^2}$$

$$= \frac{(2)(7.76 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.894)(0.318)(7.63 \text{ in})(45 \text{ in})^2}$$

$$= 42 \text{ lbf/in}^2$$

$$f_{ss} = \frac{M_s}{jdA_s}$$

$$= \frac{(7.76 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.894)(45 \text{ in})(0.88 \text{ in}^2)}$$

$$= 2630 \text{ lbf/in}^2$$

The bending moment produced by the live load is

$$M_c = 50 \text{ ft-kips}$$

The stresses produced in the masonry and in the reinforcement by the live load are

$$f_{bc} = \frac{2M_c}{jkbd^2}$$

$$= \frac{(2)(50 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(0.894)(0.318)(7.63 \text{ in})(45 \text{ in})^2}$$

$$= 273 \text{ lbf/in}^2$$

$$f_{sc} = \frac{M_c}{jdA_s}$$

$$= \frac{(50 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(0.894)(45 \text{ in})(0.88 \text{ in}^2)}$$

$$= 16{,}948 \text{ lbf/in}^2$$

Applying IBC Eq. 16-12 gives the combined stresses caused by the beam self-weight and the wind load as

$$f_b = 1.0f_{bs} + 0.6f_{bw}$$

$$= (1.0)\left(42 \frac{\text{lbf}}{\text{in}^2}\right) + (0.6)\left(225 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 177 \text{ lbf/in}^2$$

$$< F_b \quad [\text{satisfactory}]$$

$$f_s = 1.0f_{ss} + 0.6f_{sw}$$

$$= (1.0)\left(2630 \frac{\text{lbf}}{\text{in}^2}\right) + (0.6)\left(11{,}629 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 9607 \text{ lbf/in}^2$$

$$< F_s \quad [\text{satisfactory}]$$

Applying IBC Eq. 16-13 gives the combined stresses caused by the beam self-weight, the wind load, and the live load as

$$f_b = 1.0f_{bs} + 0.75(0.6)f_{bw} + 0.75f_{bc}$$

$$= (1.0)\left(42 \frac{\text{lbf}}{\text{in}^2}\right) + (0.75)(0.6)\left(225 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$\quad + (0.75)\left(273 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 348 \text{ lbf/in}^2$$

$$< F_b \quad [\text{satisfactory}]$$

$$f_s = 1.0f_{ss} + 0.75(0.6)f_{sw} + 0.75f_{sc}$$

$$= (1.0)\left(2630 \frac{\text{lbf}}{\text{in}^2}\right) + (0.75)(0.6)\left(11{,}629 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$\quad + (0.75)\left(16{,}948 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 20{,}574 \text{ lbf/in}^2$$

$$< F_s \quad [\text{satisfactory}]$$

The beam is adequate.

SD Method

The service bending moments are from the solution for Ex. 6.6.

The bending moment produced by the beam self-weight is

$$M_s = 7.76 \text{ ft-kips}$$

The bending moment produced by the live load is

$$M_c = 50 \text{ ft-kips}$$

The bending moment produced by the wind load is

$$M_x = 3.94 \text{ ft-kips}$$

IBC Eq. 16-4 governs. Applying this load combination to the vertical loads gives the combined factored moment caused by the beam self-weight and the live load as

$$M_u = 1.2M_s + 0.5M_c$$

$$= (1.2)(7.76 \text{ ft-kips}) + (0.5)(50 \text{ ft-kips})$$

$$= 34.3 \text{ ft-kips}$$

The design strength for vertical loads from Ex. 6.6 is

$$\phi M_x = 167 \text{ ft-kips}$$

Applying IBC Eq. 16-4 to the wind load gives the factored moment as

$$M_w = 1.0M_x$$

$$= (1.0)(3.94 \text{ ft-kips})$$

$$= 3.94 \text{ ft-kips}$$

For lateral loads, the stress block depth is

$$a = \frac{A_s f_y}{0.80bf'_m}$$

$$= \frac{(0.88 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(0.80)(48 \text{ in})\left(1500 \frac{\text{lbf}}{\text{in}^2}\right)}$$

$$= 0.92 \text{ in}$$

Masonry

The nominal strength in the lateral direction is

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$$= \frac{(0.88 \text{ in}^2)\left(60 \ \dfrac{\text{kips}}{\text{in}^2} \right)\left(5.25 \text{ in} - \dfrac{0.92 \text{ in}}{2} \right)}{12 \ \dfrac{\text{in}}{\text{ft}}}$$

$$= 21.1 \text{ ft-kips}$$

The design strength is

$$\phi M_{nx} = (0.9)(21.1 \text{ ft-kips})$$

$$= 19 \text{ ft-kips}$$

Applying the interaction equation gives

$$\frac{M_u}{\phi M_y} + \frac{M_w}{\phi M_{nx}} = \frac{34.3 \text{ ft-kips}}{167 \text{ ft-kips}} + \frac{3.94 \text{ ft-kips}}{19 \text{ ft-kips}}$$

$$= 0.41$$

$$< 1.0 \quad [\text{satisfactory}]$$

The beam is adequate.

5. BEAMS IN SHEAR

Nomenclature

A_n	net area of cross-section	in^2
A_{nv}	net shear area	in^2
A_v	area of shear reinforcement	in^2
d	distance from extreme compression fiber to centroid of tension reinforcement	in
d_v	actual depth of masonry in the direction of shear considered	in
f_v	calculated shear stress in masonry	in
F_s	allowable stress in reinforcement	lbf/in^2
F_v	allowable shear stress	lbf/in^2
F_{vm}	allowable shear stress resisted by the masonry	lbf/in^2
F_{vs}	allowable shear stress resisted by the shear reinforcement	lbf/in^2
M	moment at the section under consideration	ft-kips
M_u	factored moment	ft-kips
P	axial force on a beam	kips
P_u	factored axial force	kips
s	spacing of shear reinforcement	in
V	shear force at the section under consideration	kips
V_n	nominal shear strength	kips
V_{nm}	nominal shear strength provided by the masonry	kips
V_{ns}	nominal shear strength provided by the shear reinforcement	kips
V_u	factored shear force	kips

Shear Reinforcement

When shear reinforcement (see Fig. 6.5) is provided, the reinforcement requirements are specified in MSJC Sec. 3.3.4.2.3 and are as follows.

- Shear reinforcement should consist of a single bar with a standard 180° hook at each end to reduce congestion in the member.

- Shear reinforcement must be hooked round longitudinal reinforcement at each end so as to develop the shear reinforcement.

- The first shear reinforcing bar must be located no more than one-fourth the beam depth from the beam end so as to intersect any diagonal crack formed at the support.

- The spacing of shear reinforcing bars must not exceed one-half the beam depth or 48 in to improve ductility.

As specified in MSJC Sec. 2.3.6.4, the maximum design shear may be calculated at a distance of $d/2$ from the face of the support. This location of the critical section is applicable provided that no concentrated load occurs between the face of the support and a distance $d/2$ from the face, and that the support reaction introduces compression into the end regions of the beam.

Figure 6.5 *Shear Reinforcement*
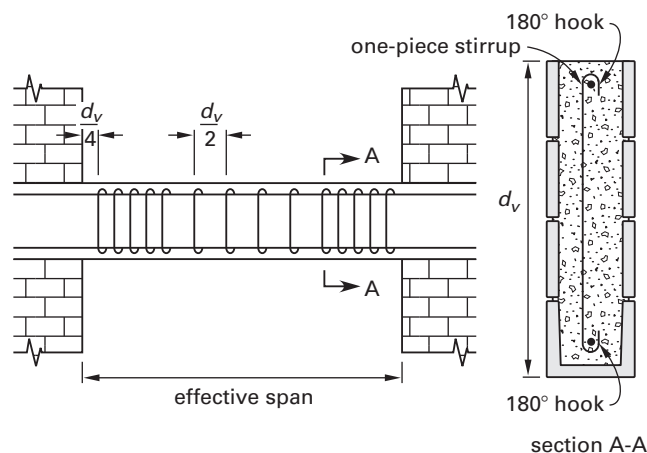
Design for Shear in Beams

ASD Method

In accordance with MSJC Sec. 2.3.6.1.1, the shear stress in a masonry beam is

$$f_v = \frac{V}{bd}$$

The allowable shear stress in a beam without shear reinforcement is given by MSJC Eq. 2-28 as

$$F_{vm} = \tfrac{1}{2}\left(\left(4.0 - 1.75\left(\frac{M}{Vd} \right) \right) \sqrt{f'_m} \right) + 0.25\left(\frac{P}{A_n} \right)$$

When $f_v > F_{vm}$, shear reinforcement is provided to carry the residual shear stress. The area of shear reinforcement required is given by MSJC Eq. 2-30 as

$$F_{vs} = 0.5\left(\frac{A_v F_s d}{A_n s}\right)$$

For a solid grouted masonry beam, the net cross-sectional area of the beam is

$$A_n = bd$$

In accordance with MSJC Eq. 2-25, the shear stress resisted by the masonry and the shear stress resisted by the shear reinforcement are additive to give a combined allowable shear stress of

$$F_v = F_{vm} + F_{vs}$$

The allowable shear stress, when $M/(Vd) \leq 0.25$, is limited by MSJC Eq. 2-26 to

$$F_v \leq 3\sqrt{f'_m}$$

The allowable shear stress, when $M/(Vd) \geq 1.0$, is limited by MSJC Eq. 2-27 to

$$F_v \leq 2\sqrt{f'_m}$$

To simplify the procedure, MSJC Comm. Sec. 2.3.6.1.2 permits $M/(Vd)$ to be 1.0 in MSJC Eq. 2-28 and Sec. 2.3.6.1.2.

SD Method

The nominal shear strength of a beam without shear reinforcement is given by MSJC Eq. 3-23 as

$$V_{nm} = \left(4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right) A_{nv}\sqrt{f'_m} + 0.25 P_u$$

The nominal shear strength provided by shear reinforcement is given by MSJC Eq. 3-24 as

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right) f_y d_v$$

For a solid grouted masonry beam, the net shear area of the beam is

$$A_{nv} = bd_v$$

In accordance with MSJC Eq. 3-20, the nominal shear strength provided by the masonry and the nominal shear strength provided by the shear reinforcement are additive to give a combined nominal shear strength of

$$V_n = V_{nm} + V_{ns}$$

The nominal shear strength, when $M_u/(V_u d_v) \leq 0.25$, is limited by MSJC Eq. 3-21 to

$$V_n \leq 6 A_{nv}\sqrt{f'_m}$$

The nominal shear strength, when $M_u/(V_u d_v) \geq 1.0$, is limited by MSJC Eq. 3-22 to

$$V_n \leq 4 A_{nv}\sqrt{f'_m}$$

To simplify the procedure, MSJC Comm. Sec. 3.3.4.1.2 permits $M_u/(V_u d_v)$ to be 1.0 in MSJC Eq. 3-23 and Sec. 3.3.4.1.2.

Example 6.7

For the masonry beam of Ex. 6.5, determine whether the shear reinforcement provided is adequate.

Solution

ASD Method

The maximum permitted shear stress, assuming $M/Vd = 1$, is given by MSJC Eq. 2-27 as

$$F_v = 2\sqrt{f'_m}$$
$$= 2\sqrt{1500\ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 77.5\ \text{lbf/in}^2$$

The shear force at a distance of $d/2$ from each support is given by

$$V = \frac{w(l - d)}{2} + W$$
$$= \frac{\left(276\ \frac{\text{lbf}}{\text{ft}}\right)\left(14\ \text{ft} - \left(\frac{45\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right)\right)}{(2)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)} + 20\ \text{kips}$$
$$= 21.4\ \text{kips}$$

The shear stress at a distance of $d/2$ from each support is given by MSJC Eq. 2-24 as

$$f_v = \frac{V}{A_{nv}} = \frac{V}{bd}$$
$$= \frac{(21.4\ \text{kips})\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)}{(7.63\ \text{in})(45\ \text{in})}$$
$$= 62.4\ \text{lbf/in}^2$$
$$< 77.5\ \text{lbf/in}^2 \quad \left[\begin{array}{c} \text{satisfies MSJC} \\ \text{Sec. 2.3.6.1.2} \end{array}\right]$$

Masonry

The allowable shear stress in a beam without shear reinforcement is given by MSJC Eq. 2-28. Since $P = 0$ lbf/in^2,

$$F_{vm} = \tfrac{1}{2}\left(\left(4.0 - 1.75\left(\frac{M}{Vd}\right)\right)\sqrt{f'_m}\right) + 0.25\left(\frac{P}{A_n}\right)$$

$$= \left(\tfrac{1}{2}\right)\left(4.0 - (1.75)(1.0)\right)\sqrt{1500\ \frac{\text{lbf}}{\text{in}^2}} + 0\ \frac{\text{lbf}}{\text{in}^2}$$

$$= 43.6\ \text{lbf/in}^2$$

$$< f_v = 62.4\ \text{lbf/in}^2 \quad \text{[shear reinforcement is required]}$$

The reinforcement provided is 0.20 in^2 at 8 in centers. This provides an allowable shear stress, as specified by MSJC Eq. 2-30, of

$$F_{vs} = 0.5\left(\frac{A_v F_s d}{A_n s}\right)$$

$$= \frac{(0.5)(0.20\ \text{in}^2)\left(32{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)(45\ \text{in})}{(7.63\ \text{in})(45\ \text{in})(8\ \text{in})}$$

$$= 52.5\ \text{lbf/in}^2$$

The combined allowable shear stress is given by MSJC Eq. 2-25 as

$$F_v = F_{vm} + F_{vs}$$

$$= 43.6\ \frac{\text{lbf}}{\text{in}^2} + 52.5\ \frac{\text{lbf}}{\text{in}^2}$$

$$= 96\ \text{lbf/in}^2$$

$$> f_v = 62.4\ \text{lbf/in}^2 \quad \text{[satisfactory]}$$

The shear reinforcement provided is adequate.

SD Method

The total factored shear force at a distance of $d/2$ from each support is given by IBC Eq. 16-2 as

$$V_u = 1.2 V_s + 1.6 V_c$$

$$= (1.2)(1.42\ \text{kips}) + (1.6)(20\ \text{kips})$$

$$= 33.7\ \text{kips}$$

If $M_u/(V_u d_v) = 1.0$, the maximum nominal shear capacity permitted is limited by MSJC Eq. 3-22 to

$$V_n \leq 4 A_{nv}\sqrt{f'_m}$$

$$= \frac{(4)(7.63\ \text{in})(48\ \text{in})\sqrt{1500\ \frac{\text{lbf}}{\text{in}^2}}}{1000\ \frac{\text{lbf}}{\text{kip}}}$$

$$= 56.7\ \text{kips}$$

The maximum design shear capacity permitted is

$$\phi V_n = (0.8)(56.7\ \text{kips})$$

$$= 45.4\ \text{kips}$$

$$> V_u = 33.7\ \text{kips} \quad \text{[satisfactory]}$$

The nominal shear capacity of the beam without shear reinforcement is given by MSJC Eq. 3-23. Since $P_u = 0$ lbf,

$$V_{nm} = \left(4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right)A_{nv}\sqrt{f'_m} + 0.25 P_u$$

$$= \frac{\left(4.0 - (1.75)(1.0)\right)(7.63\ \text{in})(48\ \text{in})}{1000\ \frac{\text{lbf}}{\text{kip}}}$$
$$\times \sqrt{1500\ \frac{\text{lbf}}{\text{in}^2}} + 0\ \text{lbf}$$

$$= 31.9\ \text{kips}$$

$$\phi V_{nm} = (0.8)(31.9\ \text{kips})$$

$$= 25.5\ \text{kips}$$

$$< V_u = 33.7\ \text{kips} \quad \text{[shear reinforcement is required]}$$

The reinforcement provided is 0.20 in^2 at 8 in centers. This provides a nominal shear capacity, as specified by MSJC Eq. 3-24, of

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right)f_y d_v$$

$$= \frac{(0.5)\left(\frac{0.20\ \text{in}^2}{8\ \text{in}}\right)\left(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)(48\ \text{in})}{1000\ \frac{\text{lbf}}{\text{kip}}}$$

$$= 36.0\ \text{kips}$$

The combined nominal shear capacity is given by MSJC Eq. 3-20 as

$$V_n = V_{nm} + V_{ns}$$

$$= 31.9\ \text{kips} + 36.0\ \text{kips}$$

$$= 67.9\ \text{kips}$$

The design shear capacity is

$$\phi V_n = (0.8)(67.9)$$

$$= 54.3\ \text{kips}$$

$$> V_u = 33.7\ \text{kips} \quad \text{[satisfactory]}$$

The shear reinforcement provided is adequate.

Masonry

6. DESIGN OF MASONRY COLUMNS

Nomenclature

a	distance between column reinforcement	in
A	area of reinforcement in compression	in^2
A_{max}	maximum area of the tension reinforcement that will satisfy MSJC Sec. 3.2.3.5.1	in^2
A_n	net effective area of column	in^2
A_s	area of reinforcement	in^2
A'_s	area of reinforcement in tension	in^2
A_{st}	area of laterally tied longitudinal reinforcement	in^2
A_t	transformed area of column, $A_n\Big(1 + (2n-1)\rho\Big)$	in^2
A_{ts}	transformed area of reinforcement, $A_s(2n-1)$	in^2
b	width of section	in
c	depth of neutral axis	in
C_m	force in masonry stress block	kips
C_s	force is compression steel	kips
d	effective depth of tension reinforcement	in
d'	depth of compression reinforcement	in
E	modulus of elasticity	lbf/in^2
f_a	calculated compressive stress due to axial load only	lbf/in^2
f_m	calculated stress in the masonry	lbf/in^2
f_s	stress in tension reinforcement	lbf/in^2
f'_s	stress in compression reinforcement	lbf/in^2
F_a	allowable compressive stress due to axial load only	lbf/in^2
h	effective height of column	in
I_n	net effective moment of inertia of column	in^4
I_t	transformed moment of inertia of column	in^4
M	bending moment	ft-kips
M_n	nominal flexural strength of a member	ft-kips
n	modular ratio, E_s/E_m	–
P	axial load	kips
P_a	allowable compressive force due to axial load only	kips
P_m	allowable compressive force on the masonry due to axial load only	kips
P_n	nominal axial strength	kips
P_s	allowable compressive force on the reinforcement due to axial load only	kips
P_u	factored axial load	kips
r	radius of gyration	in
R	response modification factor	–
s	center-to-center spacing of items	in
S_t	transformed section modulus of column	in^3
t	nominal thickness of member	in
T	force in tension steel	kips

Symbols

ϵ_s	strain in tension reinforcement	–
ϵ'_s	strain in compression reinforcement	–
ϵ_y	strain at yield in tension reinforcement	–
ρ	ratio of tension reinforcement	–

Dimensional Limitations

Dimensional requirements for columns, as shown in Fig. 6.6, are specified in MSJC Sec. 1.6, Sec. 1.14.1, and Sec. 2.3.4.3 as follows.

- The minimum nominal column width is 8 in.

- The distance between lateral supports is limited to 99 multiplied by the least radius of gyration, r.

- The maximum depth must not exceed three times the nominal width.

- The height must not be less than four times the thickness.

- The area of longitudinal reinforcement is limited to a maximum of 4% and a minimum of 0.25% of the net column area of cross section.

- At least four longitudinal reinforcing bars must be provided, one in each corner of the column.

- Lateral ties must not have a diameter of less than $^1/_4$ in.

- Lateral ties must be placed at a spacing not exceeding the lesser of 16 longitudinal bar diameters, 48 lateral tie diameters, or the least cross-sectional dimension of the column.

- Lateral ties must be arranged so that every corner and alternate longitudinal bar has support provided by the corner of a lateral tie, and no bar is farther than 6 in clear on each side from a supported bar.

- Lateral ties must be located not more than one-half the lateral tie spacing above the top of footing or slab in any story. Lateral ties must be placed not more than one-half the lateral tie spacing below the horizontal reinforcement in the beam or slab reinforcement above.

- Where beams or brackets frame into a column from four directions, lateral ties may be terminated not more than 3 in below the lowest reinforcement in the shallowest beam or bracket.

Example 6.8

The nominal 16 in square, solid grouted concrete block masonry column shown is reinforced with four no. 4 grade 60 bars. Determine the required size and spacing of lateral ties.
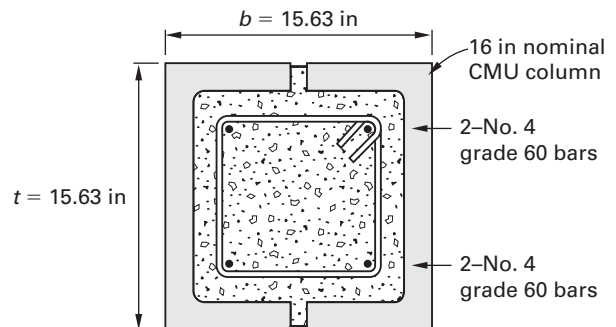
Figure 6.6 *Column Dimensions*

section A-A
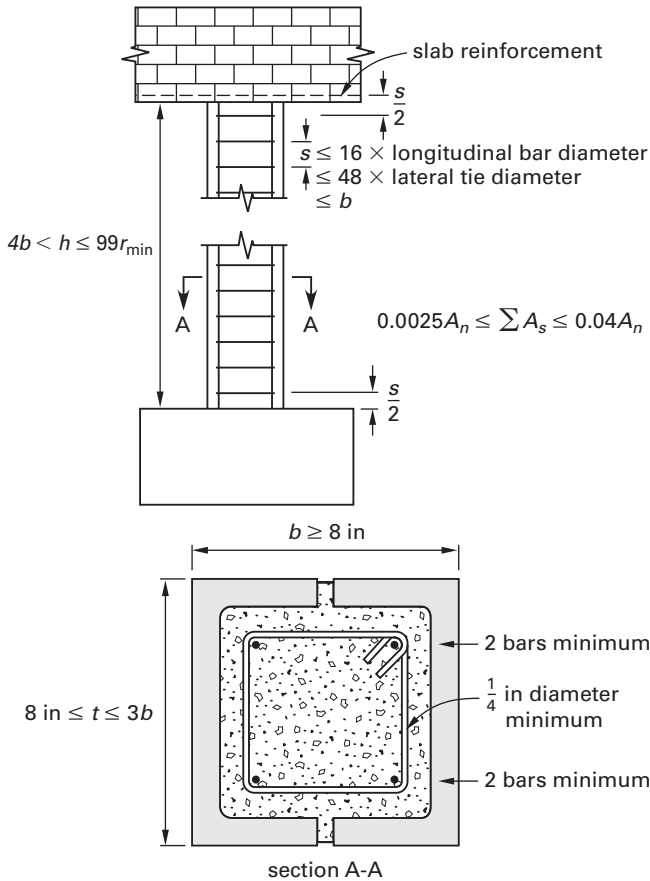
Solution

As specified by MSJC Sec. 1.14.1, lateral ties for the confinement of longitudinal reinforcement must not have a diameter of less than $^{1}/_{4}$ in. Using no. 3 bars for the ties, the spacing must not exceed the lesser of the following.

$$s = 48d_{\text{lateral}} = (48)(0.375 \text{ in})$$

$$= 18 \text{ in}$$

$$s = 16 \text{ in} \quad [\text{least cross-sectional column dimension}]$$

$$s = 16d_{\text{longitudinal}} = (16)(0.5 \text{ in})$$

$$= 8 \text{ in} \quad [\text{governs}]$$

Axial Compression in Columns

ASD Method

The allowable compressive stress in an axially loaded reinforced masonry column is given by

$$F_a = \frac{P_a}{A_n}$$

For columns having an h/r ratio not greater than 99, the allowable axial load is given by MSJC Sec. 2.3.4.2.1 as

$$P_a = (0.25f'_m A_n + 0.65A_{st}F_s)\left(1 - \left(\frac{h}{140r}\right)^2\right)$$

[*MSJC 2-21*]

For columns having an h/r ratio greater than 99, the allowable axial load is given by MSJC Sec. 2.3.4.2.1 as

$$P_a = (0.25f'_m A_n + 0.65A_{st}F_s)\left(\frac{70r}{h}\right)^2 \quad [\textit{MSJC 2-22}]$$

To allow for accidental eccentricities, MSJC Sec. 2.3.4.3 requires that a column be designed for a minimum eccentricity equal to 0.1 times each side dimension. When actual eccentricity exceeds the minimum eccentricity, the actual eccentricity should be used.

For columns with lateral reinforcement, the allowable steel stress is given by MSJC Sec. 2.3.3 as

$$F_s = 20,000 \text{ lbf/in}^2 \quad [\text{grade 40 or grade 50 reinforcement}]$$

$$F_s = 32,000 \text{ lbf/in}^2 \quad [\text{grade 60 reinforcement}]$$

SD Method

The nominal allowable axial compressive strength in an axially loaded reinforced masonry column, as given by MSJC Sec. 3.3.4.1.1, allows for slenderness effects and accidental eccentricity of the applied load. The nominal axial compressive strength must not exceed MSJC Eq. 3-18 or Eq. 3-19, as appropriate. For members having an h/r ratio not greater than 99, MSJC Eq. 3-18 applies and is given by

$$P_n = 0.80\big(0.80f'_m(A_n - A_{st}) + f_y A_{st}\big)\left(1 - \left(\frac{h}{140r}\right)^2\right)$$

For members having an h/r ratio greater than 99, MSJC Eq. 3-19 applies and is given by

$$P_n = 0.80\Big(80f'_m(A_n - A_{st}) + f_y A_{st}\Big)\left(\frac{70r}{h}\right)^2$$

Example 6.9

The nominal 16 in square, solid grouted concrete block masonry column shown has a specified strength of 1500 lbf/in², a modulus of elasticity of 1,000,000 lbf/in², and is reinforced with four no. 4 grade 60 bars. The column has a height of 15 ft and is pinned at each end. Neglecting accidental eccentricity, determine the available column strength.

Masonry
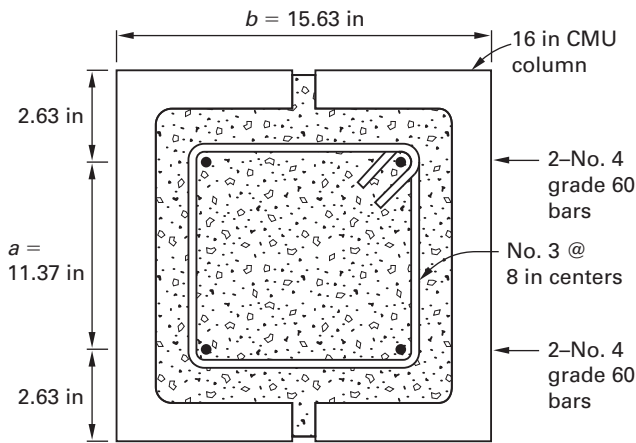
Solution

The relevant properties of the column are

$$b = \text{effective column width}$$
$$= 15.63 \text{ in}$$
$$h = \text{effective column height}$$
$$= 15 \text{ ft}$$
$$A_{st} = \text{reinforcement area}$$
$$= (4)(0.20 \text{ in}^2)$$
$$= 0.80 \text{ in}^2$$
$$A_n = \text{effective column area}$$
$$= b^2$$
$$= (15.63 \text{ in})^2$$
$$= 244 \text{ in}^2$$
$$\rho = \frac{A_{st}}{A_n}$$
$$= \frac{0.80 \text{ in}^2}{244 \text{ in}^2}$$
$$= 0.0033$$
$$< 0.04$$
$$> 0.0025 \quad \begin{bmatrix} \text{satisfies MSJC} \\ \text{Sec. 1.14.1} \end{bmatrix}$$

The radius of gyration of the column is

$$r = \sqrt{\frac{I_n}{A_n}}$$
$$= \sqrt{\frac{\left(\frac{1}{12}\right)(15.63 \text{ in})(15.63 \text{ in})^3}{244 \text{ in}^2}}$$
$$= 4.51 \text{ in}$$

The slenderness ratio of the column is

$$\frac{h}{r} = \frac{(15 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{4.51 \text{ in}}$$
$$= 39.9$$
$$< 99 \quad \begin{bmatrix} \text{MSJC Eq. 2-21} \\ \text{is applicable.} \end{bmatrix}$$

ASD Method

The allowable steel stress is given by MSJC Sec. 2.3.3.3 as

$$F_s = 32,000 \text{ lbf/in}^2 \quad \text{[for grade 60 bars]}$$

The allowable column load is given by

$$P_a = (0.25 f'_m A_n + 0.65 A_{st} F_s)\left(1 - \left(\frac{h}{140r}\right)^2\right)$$
$$= \frac{\left(\begin{array}{c}(0.25)\left(1500 \frac{\text{lbf}}{\text{in}^2}\right)(244 \text{ in}^2) \\ + (0.65)(0.80 \text{ in}^2)\left(32,000 \frac{\text{lbf}}{\text{in}^2}\right)\end{array}\right)}{1000 \frac{\text{lbf}}{\text{kip}}}$$
$$\times \left(1 - \left(\frac{(15 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(140)(4.51 \text{ in})}\right)^2\right)$$
$$= (91.5 \text{ kips} + 16.6 \text{ kips})(0.919)$$
$$= 84.1 \text{ kips} + 15.3 \text{ kips}$$
$$= P_m + P_s \quad \begin{bmatrix} \text{allowable loads on the} \\ \text{masonry and reinforcement} \end{bmatrix}$$
$$= 99.4 \text{ kips}$$

The allowable column strength is 99.4 kips.

SD Method

The slenderness ratio of the column is

$$\frac{h}{r} = 39.9$$
$$< 99 \quad \text{[MSJC Eq. 3-18 is applicable]}$$

From MSJC Eq. 3-18, the nominal axial strength is

$$P_n = 0.80\Big(0.80f'_m(A_n - A_{st}) + f_y A_{st}\Big)\Big(1 - \Big(\frac{h}{140r}\Big)^2\Big)$$

$$= \frac{(0.80)\left(\begin{array}{c}(0.80)\Big(1500\ \frac{\text{lbf}}{\text{in}^2}\Big)(244\ \text{in}^2 - 0.80\ \text{in}^2) \\[2mm] + \Big(60{,}000\ \frac{\text{lbf}}{\text{in}^2}\Big)(0.80\ \text{in}^2)\end{array}\right)}{1000\ \frac{\text{lbf}}{\text{kip}}}$$

$$\times\left(1 - \left(\frac{(15\ \text{ft})\Big(12\ \frac{\text{in}}{\text{ft}}\Big)}{(140)(4.51\ \text{in})}\right)^2\right)$$

$$= (0.80)(291.8\ \text{kips} + 48\ \text{kips})(0.919)$$

$$= 250\ \text{kips}$$

The design column strength is

$$\phi P_n = (0.9)(250\ \text{kips})$$
$$= 225\ \text{kips}$$

Combined Compression and Flexure

ASD Method

The allowable compressive stress in masonry due to combined axial load and flexure is given by MSJC Sec. 2.3.4.2.2 as

$$F_b = 0.45f'_m$$

In addition, the calculated compressive stress due to the axial load cannot exceed the allowable values given in MSJC Sec. 2.2.3.1. For columns having an h/r ratio not greater than 99, this is

$$F_a = (0.25f'_m)\Big(1.0 - \Big(\frac{h}{140r}\Big)^2\Big) \qquad [\textit{MSJC 2-16}]$$

For columns having an h/r ratio greater than 99, the allowable value is

$$F_a = (0.25f'_m)\Big(\frac{70r}{h}\Big)^2 \qquad [\textit{MSJC 2-17}]$$

When the axial load on the column causes a compressive stress larger than the tensile stress produced by the applied bending moment, the section is uncracked and stresses may be calculated by using the transformed section properties.[3] To allow for creep in the masonry,[6] the transformed reinforcement area is taken as $A_s(2n - 1)$,

and the resultant stresses at the extreme fibers of the section, as shown in Fig. 6.7, are given by

$$f_m = f_a \pm f_b = \frac{P}{A_t} \pm \frac{M}{S_t}$$

Stress in the reinforcement is equal to $2n$ times the stress in the adjacent masonry.

When the applied moment produces cracking in the section, the principle of superposition is no longer applicable. To determine the stresses on the section, the strain distribution over the section is estimated, and forces are determined as shown in Fig. 6.8. Internal forces on the section are compared with the applied loads, and the procedure is repeated until external and internal forces balance.

SD Method

As specified in MSJC Sec. 3.1.2, columns must be designed for the maximum design moment accompanying the axial load. As in the case of flexural members, the requirements of MSJC Sec. 3.2.2 are also applied to columns, and are shown in Fig. 6.9. The design assumptions are as follows.

- Stress in the reinforcement in the compression zone is based on a linear strain distribution.

- Stress in the reinforcement below the yield strain is taken as E_s multiplied by strain. Stress above the yield strain is taken as f_y.

- The extreme compressive fiber strain is 0.0025 for concrete masonry and 0.0035 for clay masonry.

- A stress of $0.8f'_m$ and an equivalent rectangular stress distribution are assumed in the masonry with a depth of 0.8 times the depth to the neutral axis.

- The tensile strength of the masonry is neglected.

To analyze a given column section, the neutral axis depth, c, is assumed. Using the notation in Fig. 6.9 and equating compressive and tensile forces acting on the section gives

$$\begin{aligned} P_n &= C_m + C_s - T \\ &= 0.64cbf'_m + A'_s\epsilon'_s E_s - A_s f_y \\ &= 0.64cbf'_m + A'_s f'_s - A_s f_y \quad [f'_s \le f_y] \end{aligned}$$

The neutral axis depth is adjusted until this equation is balanced. The nominal flexural strength is then determined by summing moments about the mid-depth of the section.

$$M_n = C_m\Big(\frac{b}{2} - \frac{a}{2}\Big) + C_s\Big(\frac{b}{2} - d'\Big) + T\Big(d - \frac{b}{2}\Big)$$

Figure 6.7 *Uncracked Section Properties*
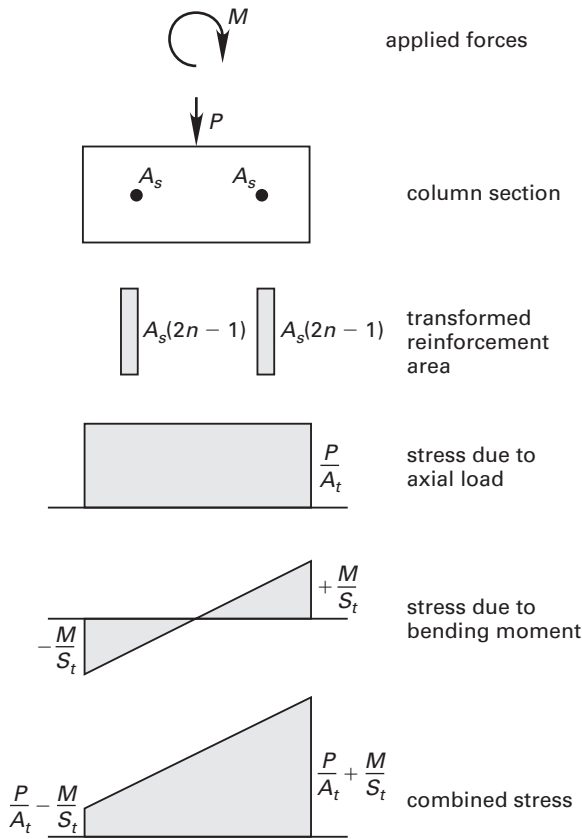
Figure 6.9 *Combined Compression and Flexure*
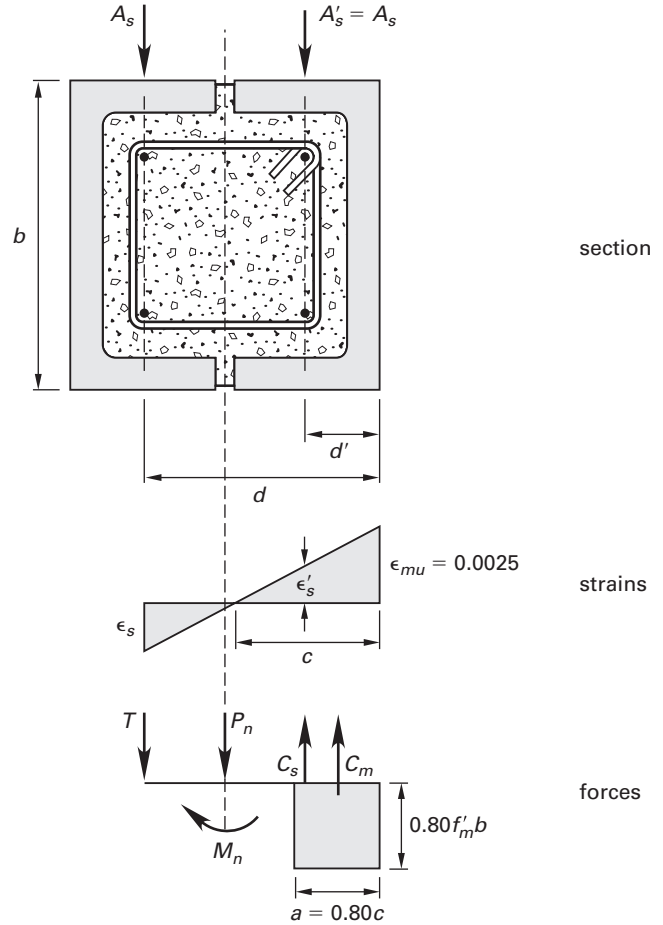
Figure 6.8 *Cracked Section Properties*
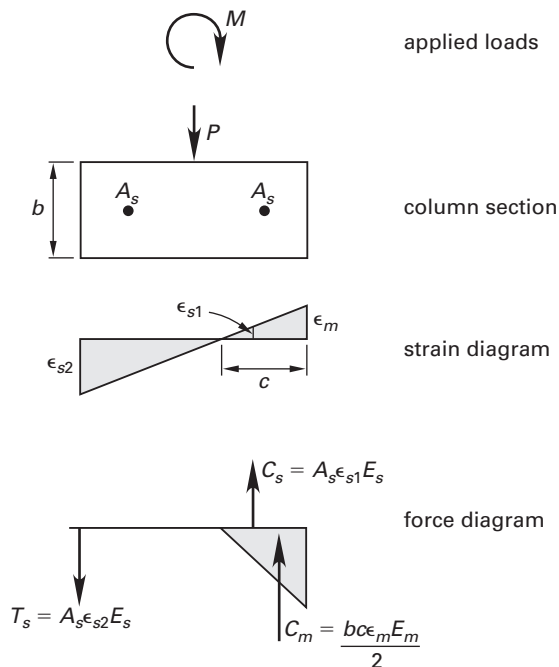
Example 6.10

The masonry column described in Ex. 6.9 is subjected to factored forces of $P = 75$ kips and $M = 10$ ft-kips (ASD), or $P_u = 76$ kips and $M_u = 50$ ft-kips (SD). The axial load includes the column weight. Determine whether the column is adequate.

Solution

From Ex. 6.9 and MSJC Eq. 2-16, the allowable compressive stress in the masonry due to axial load is

$$F_a = \frac{P_m}{A_n} = \frac{(84.1 \text{ kips})\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}{244 \text{ in}^2}$$

$$= 345 \text{ lbf/in}^2$$

As shown in Ex. 6.9, the distance between reinforcement bars is

$$a = 11.37 \text{ in}$$

The modular ratio is

$$n = \frac{E_s}{E_m} = \frac{29{,}000{,}000 \ \frac{\text{lbf}}{\text{in}^2}}{1{,}000{,}000 \ \frac{\text{lbf}}{\text{in}^2}} = 29$$

If the section is not cracked, the transformed area of two no. 4 bars is

$$A_{st} = A_s(2n - 1) = ((2)(0.2 \ \text{in}^2))((2)(29) - 1)$$
$$= 22.8 \ \text{in}^2$$

The transformed area of the column is

$$A_t = A_n\big(1 + (2n - 1)\rho\big)$$
$$= (244 \ \text{in}^2)\big(1 + ((2)(29) - 1)(0.0033)\big)$$
$$= 290 \ \text{in}^2$$

The transformed moment of inertia of the column is

$$I_t = \frac{A_{st}a^2}{2} + \frac{b^4}{12}$$
$$= \frac{(22.8 \ \text{in}^2)(11.37 \ \text{in})^2}{2} + \frac{(15.63)^4}{12}$$
$$= 6447 \ \text{in}^4$$

The stress in the masonry due to the axial load is

$$f_a = \frac{P}{A_t} = \frac{(75 \ \text{kips})\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{290 \ \text{in}^2}$$
$$= 259 \ \text{lbf/in}^2$$
$$< F_a \quad \begin{bmatrix} \text{MSJC Sec. 2.3.4.2.2} \\ \text{is satisfied.} \end{bmatrix}$$

The stresses in the extreme fibers of the column due to the applied moment are

$$f_b = \frac{Mb}{2I_t} = \pm \frac{\substack{(10 \ \text{ft-kips})(15.63 \ \text{in})\left(12 \ \frac{\text{in}}{\text{ft}}\right) \\ \times \left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}}{(2)(6447 \ \text{in}^4)}$$
$$= \pm 145 \ \text{lbf/in}^2$$
$$< f_a \quad \text{[The section is uncracked.]}$$

From Ex. 6.5, the allowable stresses due to the bending moment are

$$F_b = 675 \ \text{lbf/in}^2$$
$$F_s = 32{,}000 \ \text{lbf/in}^2$$

The maximum stress in the masonry due to combined axial and flexural load is

$$f_m = f_a + f_b = 259 \ \frac{\text{lbf}}{\text{in}^2} + 145 \ \frac{\text{lbf}}{\text{in}^2}$$
$$= 404 \ \text{lbf/in}^2$$
$$< F_b \ \text{lbf/in}^2 \quad \text{[satisfactory]}$$

The maximum compressive stress in the reinforcement due to combined axial and flexural load is given by

$$f_s = 2n\left(f_a + \frac{f_b a}{b}\right)$$
$$= (2)(29)\left(259 \ \frac{\text{lbf}}{\text{in}^2} + \frac{\left(145 \ \frac{\text{lbf}}{\text{in}^2}\right)(11.37 \ \text{in})}{15.63 \ \text{in}}\right)$$
$$= 21{,}140 \ \text{lbf/in}^2$$
$$< F_s \quad \text{[satisfactory]}$$

The column is adequate.

SD Method

The required nominal axial strength is given by MSJC Sec. 3.1.4.4 as

$$P_n = \frac{P_u}{\phi} = \frac{76 \ \text{kips}}{0.90}$$
$$= 84.4 \ \text{kips}$$

From the illustration for Ex. 6.9, the depth of the compression reinforcement and the effective depth of the tension reinforcement are, respectively,

$$d' = 2.63 \ \text{in}$$
$$d = 2.63 \ \text{in} + 11.37 \ \text{in}$$
$$= 14.0 \ \text{in}$$

Assuming the depth of the neutral axis is 6.0 in, the depth of the equivalent rectangular stress block is

$$a = 0.80c$$
$$= (0.80)(6.0 \ \text{in})$$
$$= 4.80 \ \text{in}$$

The strain in the tension steel is

$$\epsilon_s = \frac{\epsilon_{mu}(d - c)}{c}$$
$$= \frac{(0.0025)(14.0 \ \text{in} - 6.0 \ \text{in})}{6.0 \ \text{in}}$$
$$= 0.00333$$
$$> \epsilon_y = 0.00207$$

The stress in the tension reinforcement is

$$f_s = f_y = 60 \text{ kips/in}^2$$

The strain in the compression steel is given by

$$\epsilon'_s = \frac{\epsilon_{mu}(c - d')}{c}$$
$$= \frac{(0.0025)(6.0 \text{ in} - 2.63 \text{ in})}{6.0 \text{ in}}$$
$$= 0.00140$$

The stress in the compression steel is

$$f'_s = \epsilon'_s E_s$$
$$= (0.00140)\left(\frac{29{,}000{,}000 \dfrac{\text{lbf}}{\text{in}^2}}{1000 \dfrac{\text{lbf}}{\text{kip}}}\right)$$
$$= 40.6 \text{ kips/in}^2$$

The compressive strength of the masonry block is

$$C_m = 0.64cbf'_m$$
$$= \frac{(0.64)(6.0 \text{ in})(15.63 \text{ in})\left(1500 \dfrac{\text{lbf}}{\text{in}^2}\right)}{1000 \dfrac{\text{lbf}}{\text{kip}}}$$
$$= 90.0 \text{ kips}$$

The compressive strength of the steel is

$$C_s = A'_s \epsilon'_s E_s = A'_s f'_s$$
$$= ((2)(0.2 \text{ in}^2))\left(40.6 \dfrac{\text{kips}}{\text{in}^2}\right)$$
$$= 16.2 \text{ kips}$$

The tensile strength of the steel is

$$T = A_s f_y$$
$$= ((2)(0.2 \text{ in}^2))\left(60 \dfrac{\text{kips}}{\text{in}^2}\right)$$
$$= 24 \text{ kips}$$

The nominal axial load capacity for this strain condition is

$$P_n = C_m + C_s - T$$
$$= 90.0 \text{ kips} + 16.2 \text{ kips} - 24 \text{ kips}$$
$$= 82.2 \text{ kips}$$
$$\approx P_n = 84.44 \text{ kips} \quad \text{[satisfactory]}$$

The nominal flexural strength is given by

$$M_n = C_m\left(\frac{b}{2} - \frac{a}{2}\right) + C_s\left(\frac{b}{2} - d'\right) + T\left(d - \frac{b}{2}\right)$$

$$= \frac{\begin{aligned}&(90.0 \text{ kips})\left(\frac{15.63 \text{ in}}{2} - \frac{4.80 \text{ in}}{2}\right)\\ &\quad + (16.2 \text{ kips})\left(\frac{15.63 \text{ in}}{2} - 2.63 \text{ in}\right)\\ &\quad + (24 \text{ kips})\left(14.0 \text{ in} - \frac{15.63 \text{ in}}{2}\right)\end{aligned}}{12 \dfrac{\text{in}}{\text{ft}}}$$

$$= 60.0 \text{ ft-kips}$$

The design moment strength is given by MSJC Sec. 3.1.4.4 as

$$\phi M_n = (0.9)(60.0 \text{ ft-kips})$$
$$= 54.0 \text{ ft-kips}$$
$$> M_u = 50 \text{ ft-kips}$$

The column is adequate.

Maximum Reinforcement Ratio for Columns

As in the case of flexural members, the requirements of MSJC Sec. 3.3.3.5.1 are also applied to columns. These requirements are shown in Fig. 6.10. When the structure is designed using a value for the response modification factor, R, greater than 1.5, as defined by ASCE/SEI7 Table 12.2-1, the maximum reinforcement ratio is determined using the following design assumptions.

- Strain in the extreme tension reinforcement is 1.5 times the strain associated with the reinforcement yield stress, f_y.

- Flexural strength is calculated by assuming that

$$f_s = \epsilon_s E_s \leq f_y$$

- Axial loads, P, are included in the analysis with $P = D + 0.75L + 0.525Q_E$.

From the strain distribution shown in Fig. 6.10, the neutral axis depth is

$$c = 0.446d$$

The depth of the equivalent rectangular stress block is

$$a = 0.357d$$

The force in the equivalent rectangular stress block is

$$C_m = 0.80abf'_m$$
$$= 0.286bdf'_m$$

Figure 6.10 *Maximum Reinforcement Requirements for Columns*
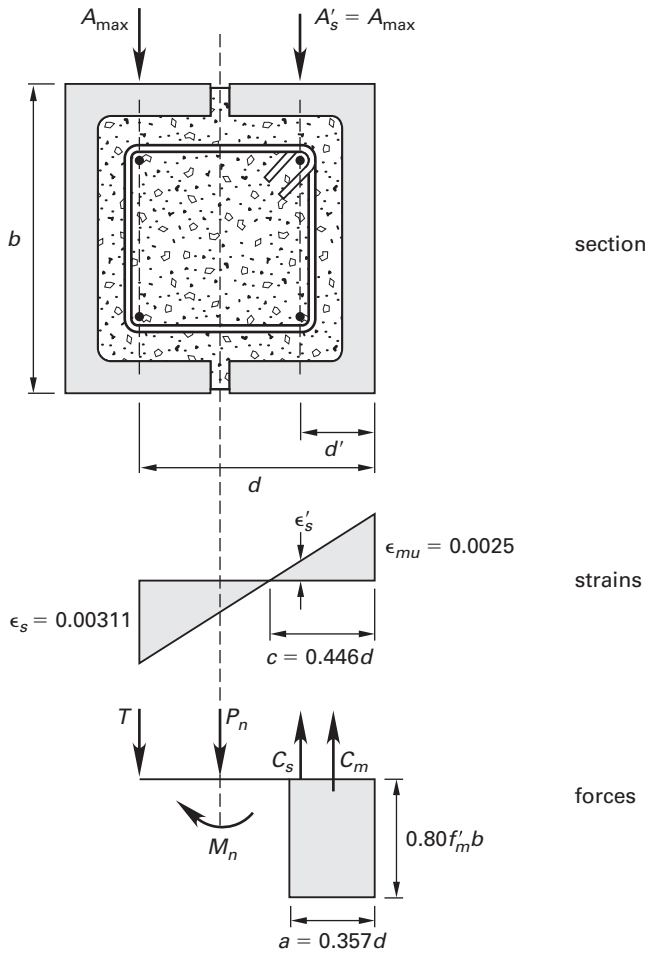
The force in the reinforcing bars on the tension side of the column is given by MSJC Sec. 3.3.3.5.1 as

$$T = A_{\max}f_y$$

The force in the reinforcing bars on the compression side of the column is given by MSJC Sec. 3.3.3.5.1 as

$$\begin{aligned}
C_s &= A'_s\epsilon'_s E_s \\
&= A_{\max}\epsilon'_s E_s \\
&= A_{\max}f'_s \quad [f'_s \le f_y]
\end{aligned}$$

Equating compressive and tensile forces acting on the section gives

$$\begin{aligned}
P &= C_m + C_s - T \\
&= 0.286bdf'_m + A_{\max}f'_s - A_{\max}f_y
\end{aligned}$$

The maximum area of the tension reinforcement that will satisfy MSJC Sec. 3.3.3.5.1 is

$$A_{\max} = \frac{0.286bdf'_m - P}{f_y - f'_s}$$

Example 6.11

The nominal 16 in square, solid grouted concrete block masonry column described in Ex. 6.9 has a specified strength of 1500 lbf/in^2. It is reinforced with four no. 4 grade 60 bars. The column has a height of 15 ft. It is pinned at each end. The axial force is 65 kips. Determine whether the reinforcement area provided satisfies MSJC Sec. 3.3.3.5.1. Use the SD method.

Solution

Use the SD method only.

The relevant dimensions are obtained from Ex. 6.9 as

$$\begin{aligned}
d &= 14.0 \text{ in} \\
c &= 0.446d \\
&= (0.446)(14.0 \text{ in}) \\
&= 6.244 \text{ in} \\
\epsilon'_s &= \frac{\epsilon_{mu}(c - d')}{c} \\
&= (0.0025)\left(\frac{6.244 \text{ in} - 2.63 \text{ in}}{6.244 \text{ in}}\right) \\
&= 0.00145 \\
f'_s &= \epsilon'_s E_s \\
&= (0.00145)\left(29{,}000 \ \frac{\text{kips}}{\text{in}^2}\right) \\
&= 41.9 \text{ kips/in}^2
\end{aligned}$$

The maximum area of tension reinforcement that will satisfy MSJC Sec. 3.3.3.5.1 is

$$\begin{aligned}
A_{\max} &= \frac{0.286bdf'_m - P}{f_y - f'_s} \\
&= \frac{(0.286)(15.63 \text{ in})(14.0 \text{ in})\left(\dfrac{1500 \ \dfrac{\text{lbf}}{\text{in}^2}}{1000 \ \dfrac{\text{lbf}}{\text{kip}}}\right) - 65 \text{ kips}}{60 \ \dfrac{\text{kips}}{\text{in}^2} - 41.9 \ \dfrac{\text{kips}}{\text{in}^2}} \\
&= 1.60 \text{ in}^2 \\
&> 0.40 \text{ in}^2 \text{ provided} \quad [\text{satisfactory}]
\end{aligned}$$

7. DESIGN OF SHEAR WALLS

Nomenclature

A_g	gross cross-sectional area of masonry	in²
A_g	gross cross-sectional area of the wall using specified dimensions	in²
A_{max}	maximum area of reinforcement	in²
A_n	net effective area of shear wall	in²
A_s	area of reinforcement	in²
A_{sh}	area of horizontal reinforcement in shear wall	in²
A_{sv}	area of vertical reinforcement in shear wall	in²
A_v	area of shear reinforcement	in²
b	width of section	in
c	depth of neutral axis	in
C_m	force in masonry stress block	kips
C_s	force in compression steel	kips
d	effective depth of tension reinforcement	in
d_v	actual depth of masonry in direction of shear	in
f'_m	specified masonry compressive strength	lbf/in²
f_s	stress in reinforcement	lbf/in²
f_y	yield strength of reinforcement	lbf/in²
h	height of masonry shear wall	in
M	moment occurring simultaneously with V at the section under consideration	in-lbf
M_n	nominal bending moment strength	ft-kips
M_u	factored bending moment	ft-kips
P	unfactored axial load	kips
P_n	nominal axial strength	kips
P_u	factored axial load	kips
s	spacing of reinforcement	in
T	force in tension steel	kips
V	shear force	lbf
V_m	shear strength provided by masonry	kips
V_n	total nominal shear strength of shear wall	kips
V_s	shear strength provided by shear reinforcement	kips
V_u	factored shear force	kips

Symbols

ρ_{max}	maximum reinforcement ratio	kips

Shear Wall Types

Several types of shear walls are classified in MSJC Sec. 1.6, and the determination of which type to adopt depends on the seismic design category of the structure. Design of shear walls must comply with MSJC Sec. 2.3 or Sec. 3.3. The different types of shear walls are described as follows.

- *Ordinary plain (unreinforced) masonry shear walls* are shear walls designed to resist lateral forces without reinforcement, or where stresses in the reinforcement,

if present, are neglected. This type of wall may be used only in seismic design categories A and B.

- *Detailed plain (unreinforced) masonry shear walls* are shear walls with specific minimum reinforcement and connection requirements that are designed to resist lateral forces. Stresses in the reinforcement are neglected. This type of wall may be used only in seismic design categories A and B. The reinforcement requirements are specified in MSJC Sec. 1.18.3.2.3.1 as horizontal and vertical reinforcement of at least no. 4 bars at a maximum spacing of 120 in. Additional reinforcement is required at wall openings and corners.

- *Ordinary reinforced masonry shear walls* are shear walls having the minimum reinforcement as specified in MSJC Sec. 1.18.3.2.3.1 and that are designed to resist lateral forces while considering the stresses in the reinforcement. This type of wall may be used only in seismic design categories A, B, and C. The maximum permitted height in seismic design category C is 160 ft.

- *Intermediate reinforced masonry shear walls* are shear walls having the minimum reinforcement as specified in MSJC Sec. 1.18.3.2.3.1, with the exception that the spacing of vertical reinforcement is limited to a maximum of 48 in. Walls are designed to resist lateral forces while considering the stresses in the reinforcement. This type of wall may be used only in seismic design categories A, B, and C. There is no limitation on height in seismic design category C.

- *Special reinforced masonry shear walls* are shear walls having the minimum reinforcement as specified in MSJC Sec. 1.18.3.2.6 and that are designed to resist lateral forces while considering the stresses in the reinforcement. This type of wall must be used in seismic design categories D, E, and F. When used in bearing wall or building frame systems, the maximum permitted height in seismic design categories D and E is 160 ft, and in seismic design category F is 100 ft.

Masonry

Special Reinforced Shear Wall Reinforcement Requirements

In accordance with MSJC Sec. 1.18.3.2.6.1.2, special reinforced shear walls designed to resist seismic forces, using the allowable stress method, shall be designed to resist 1.5 times the seismic forces calculated by IBC Chap. 16.

The reinforcement requirements for a shear wall depend on the shear force in the wall and on the seismic design category assigned to the structure. In accordance with MSJC Sec. 1.18.3.2.6, shear reinforcement in special reinforced shear walls shall be anchored around vertical reinforcement with a standard hook. The reinforcement

requirements for a special reinforced wall are given in MSJC Sec. 1.18.3.2.6 and are shown in Fig. 6.11.

The reinforcement requirements for a special reinforced shear wall in stack bond masonry are given in MSJC Sec. 1.18.3.2.6 and are shown in Fig. 6.12.

For special reinforced shear walls designed using the strength design method, MSJC Sec. 1.18.3.2.6.1.1 specifies that the design shear strength of the wall, ϕV_n, must exceed the shear corresponding to 1.25 times the nominal flexural strength, M_n, except that the nominal shear strength, V_n, need not exceed the value given by

$$V_n = 2.5 V_u$$

Figure 6.11 *Reinforcement Details for Special Reinforced Shear Wall Laid in Running Bond*
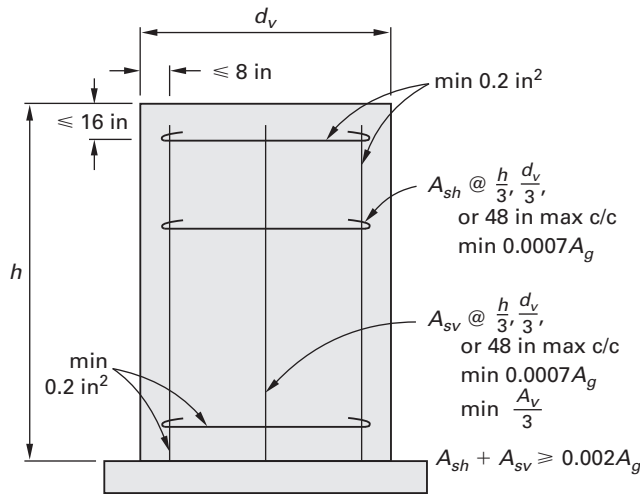
A_v = area of required shear reinforcement

Figure 6.12 *Reinforcement Details for Stack Bond Special Reinforced Shear Wall*
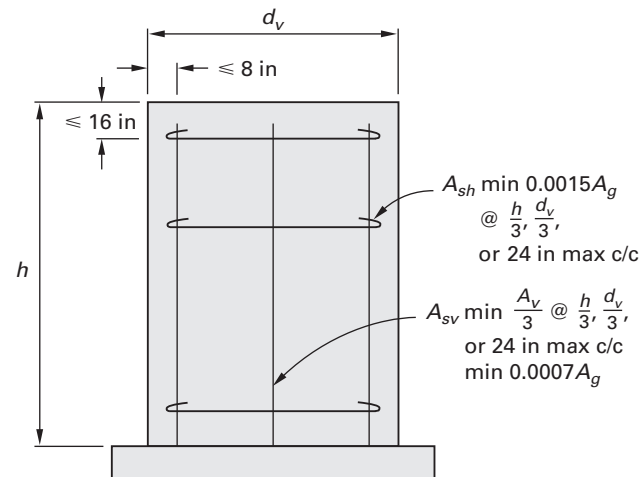
A_v = area of required shear reinforcement

Design for Shear in Shear Walls

ASD Method

In accordance with MSJC Eq. 2-24, the shear stress in a masonry shear wall is determined using

$$f_v = \frac{V}{A_{nv}}$$

The allowable shear stress in a shear wall without shear reinforcement is given by MSJC Eq. 2-28 as

$$F_{vm} = \tfrac{1}{2}\left(\left(4.0 - 1.75\left(\frac{M}{Vd}\right)\right)\sqrt{f'_m}\right) + 0.25\frac{P}{A_n}$$

For a special reinforced masonry shear wall, degradation of the masonry shear strength may occur in plastic hinge regions. The allowable shear stress in a shear wall without shear reinforcement is then given by MSJC Eq. 2-29 as

$$F_{vm} = \tfrac{1}{4}\left(\left(4.0 - 1.75\left(\frac{M}{Vd}\right)\right)\sqrt{f'_m}\right) + 0.25\frac{P}{A_n}$$

When $f_v > F_{vm}$, shear reinforcement is provided to carry the residual shear stress. The area of shear reinforcement required is given by MSJC Eq. 2-30 as

$$F_{vs} = 0.5\left(\frac{A_v F_s d}{A_n s}\right)$$

For a solid grouted masonry shear wall, the net cross-sectional area of the beam is

$$A_n = bd$$

In accordance with MSJC Eq. 2-25, the shear stress resisted by the masonry and the shear stress resisted by the shear reinforcement are additive to give a combined allowable shear stress of

$$F_v = F_{vm} + F_{vs}$$

The allowable shear stress, when $M/(Vd) \leq 0.25$, is limited by MSJC Eq. 2-26 to

$$F_v \leq 3\sqrt{f'_m}$$

The allowable shear stress, when $M/(Vd) \geq 1.0$, is limited by MSJC Eq. 2-27 to

$$F_v \leq 2\sqrt{f'_m}$$

To simplify the procedure, MSJC Comm. Sec. 2.3.6.1.2 permits $M/(Vd)$ to be 1.0 in MSJC Eq. 2-28 and Sec. 2.3.6.1.2.

In accordance with MSJC Sec. 2.3.6.2, shear reinforcement is provided parallel to the direction of the applied shear force with a spacing not exceeding $d/2$ or 48 in.

Reinforcement is provided perpendicular to the shear reinforcement with an area not less than one-third the area of the shear reinforcement and with a spacing not exceeding 8 ft.

SD Method

The nominal shear strength of a shear wall without shear reinforcement is given by MSJC Eq. 3-23 as

$$V_{nm} = \left(4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right) A_{nv}\sqrt{f'_m} + 0.25 P_u$$

The nominal shear strength provided by shear reinforcement is given by MSJC Eq. 3-24 as

$$V_{ns} = 0.5\left(\frac{A_v}{s}\right) f_y d_v$$

For a solid grouted masonry shear wall, the net shear area is

$$A_{nv} = b d_v$$

d_v is the actual depth of the wall in the direction of the applied shear and is equal to the length of shear wall, l_w.

In accordance with MSJC Eq. 3-20, the nominal shear strength provided by the masonry and the nominal shear strength provided by the shear reinforcement are additive to give a combined nominal shear strength of

$$V_n = V_{nm} + V_{ns}$$

The nominal shear strength, when $M_u/(V_u d_v) \le 0.25$, is limited by MSJC Eq. 3-21 to

$$V_n \le 6 A_{nv}\sqrt{f'_m}$$

The nominal shear strength, when $M_u/(V_u d_v) \ge 1.0$, is limited by MSJC Eq. 3-22 to

$$V_n \le 4 A_{nv}\sqrt{f'_m}$$

To simplify the procedure, MSJC Commentary Sec. 3.3.4.1.2 permits $M_u/(V_u d_v)$ to be 1.0 in MSJC Eq. 3-23 and Sec. 3.3.4.1.2.

Design for Flexure in Shear Walls

ASD Method

When flexural reinforcement is concentrated at the ends of a shear wall and axial loads are comparatively light, the shear wall may be designed in the same manner as a beam in bending. When these suppositions are not valid, it is necessary to analyze the wall using basic principles. The compressive resistance of steel reinforcement is

neglected, in accordance with MSJC Sec. 2.3.3.3, unless lateral tie reinforcement is provided.

The maximum flexural reinforcement ratio for special reinforced masonry shear walls with $M/(Vd) \ge 1.0$ and with $P > 0.05 f'_m A_n$ is given by MSJC Eq. 2-23 as

$$\rho_{max} = \frac{n f'_m}{2 f_y \left(n + \dfrac{f_y}{f'_m}\right)}$$

SD Method

When flexural reinforcement is concentrated at the ends of a shear wall and axial loads are comparatively light, the shear wall may be designed in the same manner as a beam in bending. When these conditions are not valid, it is necessary to analyze the wall using basic principles.

To prevent brittle failure of a lightly reinforced shear wall, MSJC Sec. 3.3.4.2.2.2 requires the nominal flexural strength of the shear wall to be greater than 1.3 times the nominal cracking moment strength. The required nominal moment is

$$M \ge 1.3 M_{cr}$$

In order to provide adequate ductile response in a shear wall, MSJC Sec. 3.3.3.5.1 through MSJC Sec. 3.3.3.5.4 limit the maximum reinforcement ratio in accordance with a prescribed strain distribution. The masonry compressive strain is defined as 0.0025 for concrete masonry. The tensile strain, in the extreme tension reinforcement, depends on the seismic design category, the shear wall type, the response modification factor, R, and the value of $M/(Vd)$. The tensile strain is limited to the following values.

- $\epsilon_s = 1.5\epsilon_y$ for $R \ge 1.5$ and $M_u/(V_u d_v) \le 1.0$

- ϵ_s is not limited for $R \le 1.5$ and $M_u/(V_u d_v) \le 1.0$

- $\epsilon_s = 3.0\epsilon_y$ for an intermediate reinforced masonry wall with $M_u/(V_u d_v) \ge 1.0$

- $\epsilon_s = 4.0\epsilon_y$ for a special reinforced masonry wall with $M_u/(V_u d_v) \ge 1.0$

The response modification factor, R, is defined in ASCE/SEI7[5] Table 12.2-1. For masonry shear walls used in bearing wall structural systems, the values of R are given in Table 6.1.

Table 6.1 *Response Modification Factor*

masonry shear wall type	R
special reinforced	5.0
intermediate reinforced	3.5
special reinforced	5.0
ordinary reinforced	2.0
detailed plain	2.0
ordinary plain	1.5

Masonry

The limit on maximum tensile reinforcement ratio for shear walls is waived if special boundary elements are provided to the shear wall in compliance with MSJC Sec. 3.3.6.5. Special boundary elements need not be provided in shear walls meeting the following conditions.

The factored axial load does not exceed the value $P_u \leq 0.10 A_g f'_m$ for geometrically symmetrical wall sections, or $P_u \leq 0.05 A_g f'_m$ for geometrically unsymmetrical wall sections. In addition, one of the following conditions must apply.

$$\frac{M_u}{V_u d_v} \leq 1.0$$

$$V_u \leq 3 A_n \sqrt{f'_m} \quad \left[\text{when } \frac{M_u}{V_u d_v} \leq 3.0\right]$$

Example 6.12

The nominal 8 in solid grouted concrete block masonry shear wall shown in the following illustration has a specified strength of 1500 lbf/in² and a modulus of elasticity of 1,000,000 lbf/in². An in-plane wind load of 32 kips acts at the top of the wall, as shown, and this is the governing shear load. The wall is located in a structure assigned to seismic design category C and is laid in running bond. Determine the reinforcement required in the wall. Axial load may be neglected.
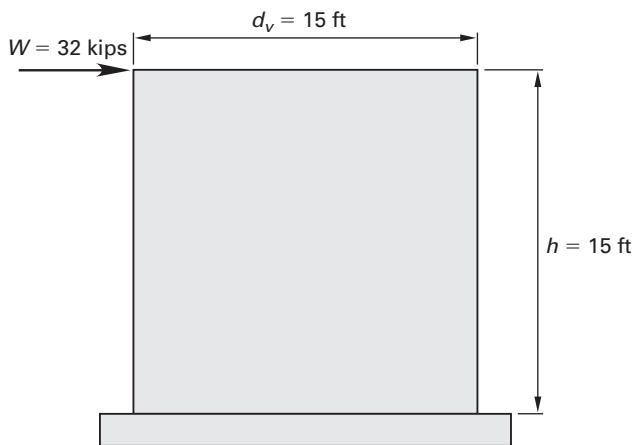
Solution

ASD Method

The wall is located in seismic design category C. An ordinary reinforced masonry wall may be used since $h < 100$ ft.

The allowable stresses, in accordance with MSJC Sec. 2.3.3 and Sec. 2.3.4.2.2, are

$$F_b = 0.45 f'_m = (0.45)\left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 675 \ \text{lbf/in}^2$$

$$F_s = 32{,}000 \ \text{lbf/in}^2$$

Using IBC Eq. 16-15, the factored load is

$$V = 0.6\,W = (0.6)(32 \ \text{kips})$$

$$= 19.2 \ \text{kips}$$

The bending moment, produced by the wind load, at the base of the wall is

$$M = V h = (19.2 \ \text{kips})(15 \ \text{ft})$$

$$= 288 \ \text{ft-kips}$$

Use $M/(Vd) = 1.0$ per MSJC Comm. Sec. 2.3.6.1.2. Assuming that two no. 6 reinforcing bars are located 4 in from each end of the wall, the relevant parameters of the wall are

$$b = 7.63 \ \text{in}$$

$$d = (15 \ \text{ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) - 4 \ \text{in} = 176 \ \text{in}$$

$$A_s = (2)(0.44 \ \text{in}^2) = 0.88 \ \text{in}^2$$

$$n = \frac{E_s}{E_m} = \frac{29{,}000{,}000 \ \dfrac{\text{lbf}}{\text{in}^2}}{1{,}000{,}000 \ \dfrac{\text{lbf}}{\text{in}^2}} = 29$$

$$\rho = \frac{A_s}{bd} = \frac{0.88 \ \text{in}^2}{(7.63 \ \text{in})(176 \ \text{in})}$$

$$= 0.000655$$

$$\rho n = (0.000655)(29)$$

$$= 0.0190$$

From App. B, the wall stresses caused by the wind load, in accordance with MSJC Sec. 2.3, are

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$= \sqrt{(2)(0.0190) + (0.0190)^2} - 0.0190$$

$$= 0.177$$

$$j = 1 - \frac{k}{3}$$

$$= 1 - \frac{0.177}{3}$$

$$= 0.941$$

$$f_b = \frac{2M}{jkbd^2}$$

$$= \frac{(2)(288 \ \text{ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(0.941)(0.177)(7.63 \ \text{in})(176 \ \text{in})^2}$$

$$= 176 \ \text{lbf/in}^2$$

$$< 675 \ \text{lbf/in}^2 \quad \text{[satisfactory]}$$

$$f_s = \frac{M}{jdA_s}$$

$$= \frac{(288 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(0.941)(176 \text{ in})(0.88 \text{ in}^2)}$$

$$= 23{,}713 \ \text{lbf/in}^2$$

$$< 32{,}000 \ \text{lbf/in}^2 \quad [\text{satisfactory}]$$

The flexural reinforcement provided is adequate.

The shear stress in the masonry wall is given by MSJC Eq. 2-24 as

$$f_v = \frac{V}{A_{nv}} = \frac{(19.2 \text{ kips})\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(7.63 \text{ in})(15 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}$$

$$= 14 \ \text{lbf/in}^2$$

The allowable stress is obtained by applying MSJC Sec. 2.3.6.1.

The allowable shear stress in a shear wall without shear reinforcement is given by MSJC Eq. 2-28 as

$$F_{vm} = \tfrac{1}{2}\left(\left(4.0 - 1.75\frac{M}{Vd}\right)\sqrt{f'_m}\right) + 0.25\left(\frac{P}{A_n}\right)$$

$$= \left(\tfrac{1}{2}\right)\left(\left(4.0 - (1.75)(1.0)\right)\sqrt{1500 \ \frac{\text{lbf}}{\text{in}^2}}\right) + 0 \ \frac{\text{lbf}}{\text{in}^2}$$

$$= 43.6 \ \text{lbf/in}^2$$

$$> f_v = 14 \ \text{lbf/in}^2 \quad [\text{satisfactory}]$$

Masonry takes all the shear force, and nominal reinforcement is required as described in MSJC Sec. 1.18.3.2.3.1.

SD Method

The structure is assigned to seismic design category C and an ordinary reinforced masonry wall may be used since $h < 100$ ft.

Using IBC Eq. 16-6, the factored load is

$$V_u = 1.0W$$

$$= (1.0)(32 \text{ kips})$$

$$= 32 \text{ kips}$$

The bending moment produced by the wind load at the base of the wall is

$$M_u = V_u h$$

$$= (32 \text{ kips})(15 \text{ ft})$$

$$= 480 \ \text{kips-ft}$$

Assuming two no. 6 reinforcing bars are located 4 in from each end of the wall, the relevant parameters of the wall are

$$b = 7.63 \text{ in}$$

$$d = (15 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) - 4 \text{ in} = 176 \text{ in}$$

$$A_s = (2)(0.44 \text{ in}^2) = 0.88 \text{ in}^2$$

The stress block depth is

$$a = \frac{A_s f_y}{0.80 b f'_m}$$

$$= \frac{(0.88 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(0.80)(7.63 \text{ in})\left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)}$$

$$= 5.77 \text{ in}$$

The nominal strength is

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$= \frac{(0.88 \text{ in}^2)\left(60 \ \frac{\text{kips}}{\text{in}^2}\right)\left(176 \text{ in} - \frac{5.77 \text{ in}}{2}\right)}{12 \ \frac{\text{in}}{\text{ft}}}$$

$$= 762 \ \text{ft-kips}$$

The design strength is

$$\phi M_n = (0.9)(762 \text{ ft-kips})$$

$$= 686 \ \text{ft-kips}$$

$$> M_u \quad [\text{satisfactory}]$$

The flexural reinforcement provided is adequate.

MSJC Comm. Sec. 3.3.4.1.2 permits the adoption of $M_u/(V_u d_v) = 1.0$. The nominal shear strength of a shear wall without shear reinforcement is given by MSJC Eq. 3-23. Since $P_u = 0$ lbf,

$$V_{nm} = \left(4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right)A_{nv}\sqrt{f'_m} + 0.25 P_u$$

$$= \frac{\left(4.0 - (1.75)(1.0)\right)(7.63 \text{ in})(15 \text{ ft})}{1000 \ \frac{\text{lbf}}{\text{kip}}}$$
$$\times \sqrt{1500 \ \frac{\text{lbf}}{\text{in}^2}}\left(12 \ \frac{\text{in}}{\text{ft}}\right) + 0 \text{ lbf}$$

$$= 120 \text{ kips}$$

$$> V_u = 32 \text{ kips} \quad [\text{satisfactory}]$$

The masonry takes all the shear force, and nominal reinforcement is required as detailed in MSJC Sec. 1.18.3.2.3.1.

Masonry

8. DESIGN OF SLENDER WALLS

Nomenclature

a	depth of equivalent rectangular stress block	in
A_g	gross cross-sectional area of member	in^2
$A_{\max}$	maximum area of reinforcement that will satisfy MSJC Sec. 3.3.3.5.1	in^2
A_s	cross-sectional area of reinforcing steel	in^2
A_{se}	equivalent area of reinforcing steel	in^2
b	width of section	in
c	depth of neutral axis	in
C_m	force in masonry stress block	lbf
d	effective depth	in
D	dead load or related internal moments or forces	–
D_w	wall dead load	lbf/ft
e_u	eccentricity of applied axial load	in
E_s	modulus of elasticity of reinforcement	lbf/in^2
E_m	modulus of elasticity of masonry	lbf/in^2
f_a	compressive stress due to axial load only	lbf/in^2
f_m	stress in masonry	lbf/in^2
f'_m	specified masonry compressive strength	lbf/in^2
f_r	modulus of rupture of masonry	lbf/in^2
f_s	stress in reinforcement	lbf/in^2
f_y	yield strength of reinforcement	lbf/in^2
h	wall height	in
I_{cr}	moment of inertia of cracked transformed section about the neutral axis	in^4
I_g	moment of inertia of gross wall section	in^4
j	lever arm factor, $1 - k/3$	–
k	neutral axis depth factor	–
L	live load or related internal moments or forces	–
M_{cr}	cracking moment	in-lbf
M_n	nominal bending moment strength	in-lbf
M_{ser}	service moment at midheight of wall, including P-delta effects	in-lbf
M_u	factored bending moment at midheight of wall, including P-delta effects	in-lbf
n	modular ratio	–
P	nonfactored axial load	lbf
P_d	nonfactored dead load from tributary floor or roof loads	lbf
P_f	nonfactored load from tributary floor or roof loads, sum of P_d and P_r	lbf
P_r	nonfactored live load from tributary floor or roof loads	lbf
P_u	sum of P_{uw} and P_{uf}	lbf
P_{uf}	factored load from tributary floor or roof loads	lbf
P_{uw}	factored weight of wall tributary to section considered	lbf
P_w	nonfactored weight of wall tributary to section considered	lbf
Q_E	the effect of horizontal seismic forces	–
S_n	section modulus of net wall section	in^3
t	nominal thickness of wall	in
T	tensile force on section	lbf
w	nonfactored lateral load	lbf/ft^2
w_u	factored lateral load	lbf/ft^2
W	wind load or related internal moments or forces	–

Symbols

γ_w	weight of masonry wall	lbf/ft^2
δ_s	deflection at midheight of wall due to service loads and including P-delta effects	in
δ_u	deflection at midheight of wall due to factored loads and including P-delta effects	in
ϵ_{mu}	maximum usable compressive strain of masonry	–
ϵ_s	strain in reinforcement	–
ϵ_{su}	maximum strain in reinforcement	–
ϵ_y	strain at yield in tension reinforcement	–
ρ_e	equivalent tension reinforcement ratio, A_{se}/bd	–
$\rho_{\max}$	maximum reinforcement ratio that will satisfy MSJC Sec. 3.3.3.5.1	–
ϕ	strength reduction factor	–

Design Basis

The design of slender masonry walls with out-of-plane loads is covered in MSJC Sec. 3.3.5. Only the strength design method is permitted. The principle requirements are as follows.[7]

- The P-delta effect caused by the wall deflection is considered in the design analysis. Since the applied moment on the wall is augmented by the deflection and the deflection is produced by the applied moment, the solution requires an iterative procedure.

- In order to minimize residual deflections, the maximum allowable deflection of the wall due to service loads is limited by MSJC Eq. 3-28 to

$$\delta_s = 0.007h$$

- To minimize P-delta effects, limits are imposed by MSJC Sec. 3.3.5.3 on the factored axial stress, $f_a = P_u/A_g$.

    For $h/t > 30$,

    $$f_a \leq 0.05f'_m$$

    For $h/t \leq 30$,

    $$f_a \leq 0.20f'_m$$

- The maximum reinforcement ratio is limited by MSJC Sec. 3.3.3.5.1 to ensure ductility and prevent brittle compression failure.

- The design of the wall must follow strength design procedures.

- The generally accepted minimum wall thickness is given by MSJC Sec. 5.6.2 as

    $t_{\min} = 6$ in for one-story buildings

    $t_{\min} = 8$ in for buildings with more than one story

Masonry

Strength Design Method

Figure 6.13 illustrates the design assumptions of MSJC Sec. 3.3.2 for a concrete masonry wall with a factored axial load of P_u and with the reinforcement area A_s located in the center of the wall. The effective depth of the section is

$$d = \frac{t}{2}$$

Figure 6.13 *Flexural Capacity of a Slender Concrete Masonry Wall*
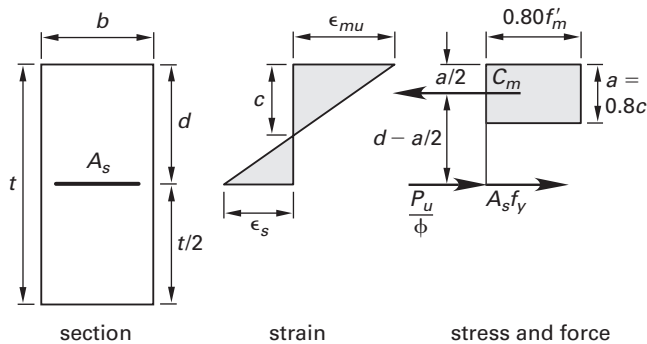
Allowing for the axial load on the wall, the equivalent reinforcement area is

$$A_{se} = \frac{\dfrac{P_u}{\phi} + A_s f_y}{f_y}$$

The stress in the reinforcement is assumed to be equal to the yield strength, f_y, and the equivalent tensile force on the section is

$$T = A_{se} f_y$$

$$= \frac{P_u}{\phi} + A_s f_y$$

The strain in the extreme compression fiber is 0.25% for concrete masonry and 0.35% for clay masonry, and the depth of the neutral axis is c. A rectangular compression stress block is assumed with a magnitude of $0.80 f'_m$ and a depth of

$$a = 0.80c$$

The compression force on the stress block acts at mid-depth of the stress block, and is given by

$$C_m = 0.80 a b f'_m$$

In accordance with MSJC Sec. 3.3.5.4, the depth of the stress block is obtained by equating forces on the section to give

$$a = \frac{\dfrac{P_u}{\phi} + A_s f_y}{0.80 b f'_m}$$

In accordance with MSJC Sec. 3.3.5.4, the nominal moment capacity of the section is obtained by taking moments about the line of action of the compression force to give

$$M_n = \left(\frac{P_u}{\phi} + A_s f_y \right) \left(d - \frac{a}{2} \right)$$

The design moment capacity is given by MSJC Sec. 3.1.4 as

$$\phi M_n = 0.9 \left(\frac{P_u}{\phi} + A_s f_y \right) \left(d - \frac{a}{2} \right)$$

Example 6.13

A nominal 8 in solid grouted, concrete block masonry wall is shown. It has a specified strength of 1500 lbf/in² and is reinforced longitudinally with no. 4 grade 60 bars at 16 in centers placed centrally in the wall. The wall has an effective height of $h = 20$ ft and is simply supported at the top and bottom. The unfactored roof live load is $P_r = 200$ lbf/ft, and the unfactored roof dead load is $P_d = 400$ lbf/ft. The masonry wall has a weight of $\gamma_w = 69$ lbf/ft². Wind load governs and has a value of $W = 30$ lbf/ft². Determine the design flexural strength of the wall.
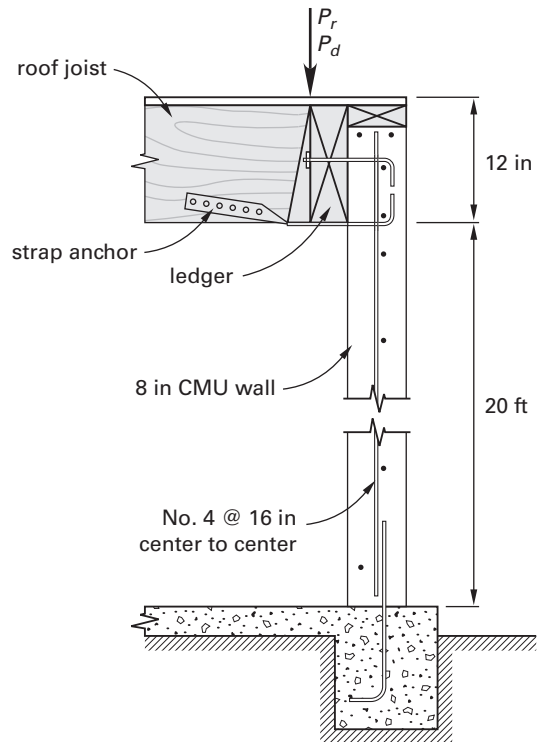
Solution

In determining the moment demand on the wall, the governing load combination is IBC Eq. 16-6, which is

$$Q_u = 0.9D + 1.0W$$

The same load factors will be used to determine the design flexural capacity of the wall.

The critical section of the wall is at midheight of the wall between supports. Use a wall width of $b = 1$ ft. The weight of wall tributary to the critical section is

$$
\begin{aligned}
P_w &= \gamma_w\left(\frac{h}{2} + 1 \text{ ft}\right)b \\
&= \left(69 \ \frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{20 \text{ ft}}{2} + 1 \text{ ft}\right)(1 \text{ ft}) \\
&= 759 \text{ lbf}
\end{aligned}
$$

The dead load from the roof over a 1 ft width is given as

$$
\begin{aligned}
P_d &= 400 \text{ lbf} \\
&= P_f \quad \left[\begin{array}{c} \text{since roof live load is not} \\ \text{included in ASCE/SEI7 Sec. 2.3.2} \\ \text{combination no. 6} \end{array}\right]
\end{aligned}
$$

The total nonfactored gravity load at the critical section is

$$
\begin{aligned}
P = P_w + P_f &= \frac{759 \text{ lbf} + 400 \text{ lbf}}{1000 \ \frac{\text{lbf}}{\text{kip}}} \\
&= 1.16 \text{ kips}
\end{aligned}
$$

The total factored axial load at the critical section of the wall is

$$
\begin{aligned}
P_u &= 0.9(P_w + P_f) \\
&= \frac{(0.9)(759 \text{ lbf} + 400 \text{ lbf})}{1000 \ \frac{\text{lbf}}{\text{kip}}} \\
&= 1.04 \text{ kips} \\
\frac{P_u}{\phi} &= \frac{1.04 \text{ kips}}{0.9} = 1.16 \text{ kips}
\end{aligned}
$$

The reinforcement area over a 1 ft width of the wall is

$$
\begin{aligned}
A_s &= \frac{(1 \text{ ft})(0.20 \text{ in}^2)\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{16 \text{ in}} \\
&= 0.15 \text{ in}^2
\end{aligned}
$$

For strength level loads, the equivalent reinforcement area is given by

$$
\begin{aligned}
A_{se} &= \frac{\dfrac{P_u}{\phi} + A_s f_y}{f_y} \\
&= \frac{1.16 \text{ kips} + (0.15 \text{ in}^2)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)}{60 \ \dfrac{\text{kips}}{\text{in}^2}} \\
&= 0.169 \text{ in}^2
\end{aligned}
$$

The depth of the rectangular stress block is

$$
\begin{aligned}
a &= \frac{A_{se}f_y}{0.80f'_m b} \\
&= \frac{(0.169 \text{ in}^2)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)\left(1000 \ \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.80)\left(1500 \ \dfrac{\text{lbf}}{\text{in}^2}\right)(1 \text{ ft})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)} \\
&= 0.70 \text{ in}
\end{aligned}
$$

The nominal moment strength is

$$
\begin{aligned}
M_n &= A_{se}f_y\left(d - \frac{a}{2}\right) \\
&= \frac{(0.169 \text{ in}^2)\left(60 \ \dfrac{\text{kips}}{\text{in}^2}\right)\left(\dfrac{7.63 \text{ in}}{2} - \dfrac{0.70 \text{ in}}{2}\right)}{12 \ \dfrac{\text{in}}{\text{ft}}} \\
&= 2.93 \text{ ft-kips}
\end{aligned}
$$

The design moment strength is

$$
\begin{aligned}
\phi M_n &= (0.9)(2.93 \text{ ft-kips}) \\
&= 2.64 \text{ ft-kips}
\end{aligned}
$$

Flexural Demand on a Slender Masonry Wall

Slender masonry walls are designed for the factored applied loads, which must include the axial loads and eccentricities, the *P*-delta effects caused by the vertical loads and the lateral deflection of the wall, and the lateral loads. The design method of MSJC Sec. 3.3.5 assumes the wall is simply supported at the top and bottom and is uniformly laterally loaded with the load w_u to produce the lateral deflection δ_u at the midheight of the wall. The critical section then occurs at the midheight of the wall. The factored moment at the critical section is derived from the free-body diagram in Fig. 6.14 to give MSJC Eq. 3-26, which is

$$M_u = \frac{w_u h^2}{8} + \frac{P_{uf}e_u}{2} + P_u\delta_u$$

Figure 6.14 *Analysis of a Slender Concrete Masonry Wall*

note: $H = \dfrac{w_u h}{2} + \dfrac{4 P_{uw} \delta_u}{3h} - \dfrac{P_{uf} e_u}{h}$
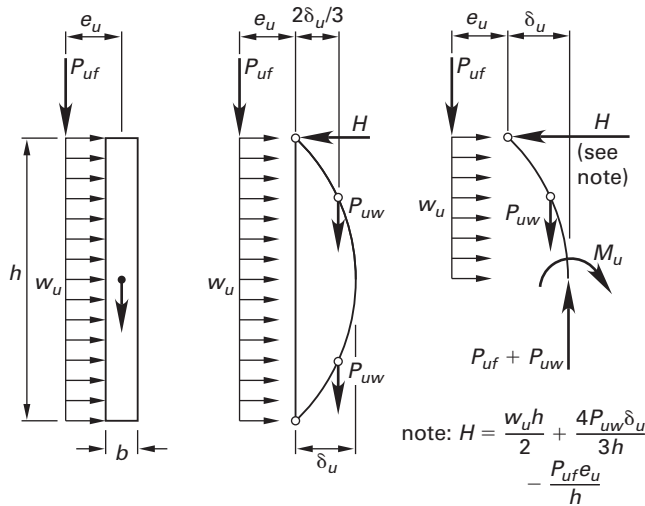
The deflection at midheight of the wall due to the factored loads, and including *P*-delta effects, is given by MSJC Eq. 3-29 and Eq. 3-30. When the moment demand is less than the cracking moment, MSJC Eq. 3-29 governs, which is

$$\delta_u = \frac{5 M_u h^2}{48 E_m I_g}$$

When the moment demand is greater than the cracking moment, MSJC Eq. 3-30 governs, which is

$$\delta_u = \frac{5 M_{cr} h^2}{48 E_m I_g} + \frac{5 h^2 (M_u - M_{cr})}{48 E_m I_{cr}}$$

Calculation of the lateral deflection requires the determination of several additional factors, including the following.

The modulus of elasticity of the reinforcement is given by MSJC Sec. 1.8.2.1 as

$$E_s = 29{,}000 \text{ kips/in}^2$$

The modulus of elasticity of the concrete masonry is given by MSJC Sec. 1.8.2.2.1 as

$$E_m = 900 f'_m$$

The modular ratio is given by MSJC Sec. 1.5 as

$$n = \frac{E_s}{E_m}$$

The modulus of rupture for out-of-plane forces on a fully grouted masonry wall with type N masonry cement mortar is given in MSJC Table 3.1.8.2 as

$$f_r = 145 \text{ lbf/in}^2$$

The moment of inertia of the cross-sectional area of the wall prior to cracking is

$$I_g = \frac{bt^3}{12}$$

The section modulus of the cross-sectional area of the wall prior to cracking is

$$S_n = \frac{bt^2}{6}$$

The depth to the neutral axis is given by MSJC Sec. 3.3.2 as

$$c = \frac{a}{0.80}$$

As shown in Fig. 6.15, the effective reinforcement depth is

$$d = \frac{t}{2}$$

Figure 6.15 *Transformed Section of a Slender Concrete Masonry Wall*
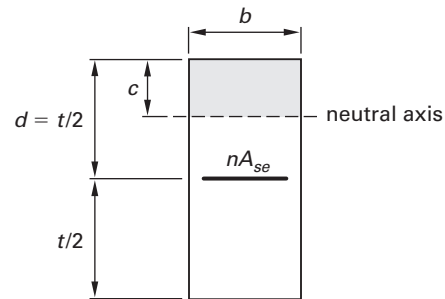
The distance of a reinforcing bar from the neutral axis is $d - c$.

Allowing for the axial load on the wall, the equivalent reinforcement area is

$$A_{se} = \frac{\dfrac{P_u}{\phi} + A_s f_y}{f_y}$$

The transformed reinforcement area is $n A_{se}$, and as shown in Fig. 6.15, the moment of inertia of the cracked transformed section about the neutral axis is

$$I_{cr} = \frac{bc^3}{3} + n A_{se}(d - c)^2$$

Since the moment demand depends on the lateral deflection, and the lateral deflection depends on the moment demand, an iterative process is required until the values for M_u and δ_u converge.

Example 6.14

The nominal 8 in solid grouted, concrete block masonry wall described in Ex. 6.13, has type N masonry cement mortar and a specified strength of 1500 lbf/in². It is reinforced longitudinally with no. 4 grade 60 bars at 16 in centers placed centrally in the wall. The wall has an effective height of $h = 20$ ft and is simply supported at the top and bottom. The unfactored roof live load is $P_r = 200$ lbf/ft. The unfactored roof dead load is $P_d = 400$ lbf/ft. The masonry wall has a weight of $\gamma_w = 69$ lbf/ft². Wind load governs and has a value of $W = 30$ lbf/ft². Determine if the flexural demand on the wall is adequate.
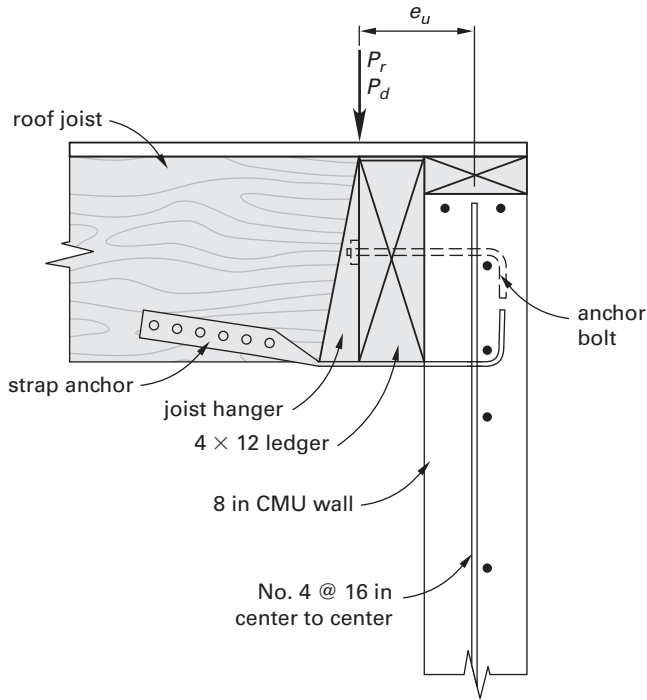
Solution

From Ex. 6.13, the nominal flexural capacity for a 1 ft width of wall is

$$M_n = 2.93 \text{ ft-kips}$$

The design moment strength is

$$\phi M_n = 2.64 \text{ ft-kips}$$

The factored tributary roof dead load over a 1 ft width using IBC Eq. 16-6 is

$$P_{uf} = 0.9 P_d \quad \begin{bmatrix} \text{since roof live load} \\ \text{is not included in} \\ \text{IBC Eq. 16-6} \end{bmatrix}$$

$$= (0.9) \left(\dfrac{400 \ \frac{\text{lbf}}{\text{ft}}}{1000 \ \frac{\text{lbf}}{\text{kip}}} \right) (1 \text{ ft})$$

$$= 0.36 \text{ kip}$$

The total factored axial load at the critical section of the wall is

$$P_u = 1.04 \text{ kips}$$

Using IBC Eq. 16-6, the factored wind load over a 1 ft width is

$$w_u = 1.0 \, W = (1.0) \left(\dfrac{30 \ \frac{\text{lbf}}{\text{in}^2}}{1000 \ \frac{\text{lbf}}{\text{kip}}} \right) (1 \text{ ft}) = 0.03 \text{ kip/ft}$$

The equivalent reinforcement area is

$$A_{se} = 0.169 \text{ in}^2$$

From the illustration, the eccentricity of the tributary roof dead load is

$$e_u = \text{ledger width} + \text{half wall thickness}$$

$$= 3.5 \text{ in} + 4.0 \text{ in}$$

$$= 7.5 \text{ in}$$

The modulus of rupture for out-of-plane forces on a fully grouted masonry wall, with type N masonry cement mortar, is given by MSJC Table 3.1.8.2 as

$$f_r = 145 \text{ lbf/in}^2$$

The section modulus of the net wall section is

$$S_n = \frac{bt^2}{6}$$

$$= \frac{(1 \text{ ft})(7.63 \text{ in})^2 \left(12 \ \frac{\text{in}}{\text{ft}} \right)}{6}$$

$$= 116.4 \text{ in}^3$$

Ignoring the effects of axial load, the nominal cracking moment strength is

$$M_{cr} = S_n f_r$$

$$= \frac{(116.4 \text{ in}^3) \left(145 \ \frac{\text{lbf}}{\text{in}^2} \right)}{\left(12 \ \frac{\text{in}}{\text{ft}} \right) \left(1000 \ \frac{\text{lbf}}{\text{kip}} \right)}$$

$$= 1.41 \text{ ft-kips}$$

The nominal wall moment is

$$M_n = 2.93 \text{ ft-kips}$$

$$\frac{M_n}{M_{cr}} = \frac{2.93 \text{ ft-kips}}{1.41 \text{ ft-kips}}$$

$$= 2.1$$

$$> 1.3 \quad \begin{bmatrix} \text{complies with MSJC} \\ \text{Sec. 3.3.4.2.2.2} \end{bmatrix}$$

Assume a deflection at midheight due to factored loads of

$$\delta_{u1} = 0.44 \text{ in}$$

The applied strength level moment at midheight of the wall is given by MSJC Eq. 3-26 as

$$M_{u1} = \frac{w_u h^2}{8} + \frac{P_{uf} e_u}{2} + P_u \delta_{u1}$$

$$= \frac{\left(0.03 \ \dfrac{\text{kip}}{\text{ft}}\right)(20 \text{ ft})^2}{8} + \frac{(0.36 \text{ kip})(7.5 \text{ in})}{(2)\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}$$

$$+ \frac{(1.04 \text{ kips})(0.44 \text{ in})}{12 \ \dfrac{\text{in}}{\text{ft}}}$$

$$= 1.65 \text{ ft-kips}$$

$$< \phi M_n \quad [\text{satisfactory}]$$

$$> M_{cr} \quad [\text{MSJC Eq. 3-32 applies}]$$

From MSJC Sec. 3.3.2, the strength level moment at midheight is

$$M_{u1} = C_m \left(d - \frac{a}{2}\right)$$

$$= 0.80 a b f'_m \left(d - \frac{a}{2}\right)$$

$$(1.65 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right) = \frac{0.80 a (12 \text{ in})\left(1500 \ \dfrac{\text{lbf}}{\text{in}^2}\right)}{1000 \ \dfrac{\text{lbf}}{\text{kip}}}$$

$$\times \left(\frac{7.63 \text{ in}}{2} - \frac{a}{2}\right)$$

$$a = 0.38 \text{ in}$$

The depth of the neutral axis is

$$c = \frac{a}{0.80}$$

$$= \frac{0.38 \text{ in}}{0.80}$$

$$= 0.475 \text{ in}$$

The deflection corresponding to the factored moment is determined in accordance with MSJC Sec. 3.3.5.3. The moment of inertia of the gross wall section is

$$I_g = \frac{b t^3}{12}$$

$$= \frac{(1 \text{ ft})(7.63 \text{ in})^3 \left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}{12}$$

$$= 444 \text{ in}^4$$

The modulus of elasticity of reinforcement is given by MSJC Sec. 1.8.2.1.1 as

$$E_s = 29{,}000 \text{ kips/in}^2$$

The modulus of elasticity of concrete masonry is given by MSJC Sec. 1.8.2.2 as

$$E_m = 900 f'_m$$

$$= (900) \left(\frac{1500 \ \dfrac{\text{lbf}}{\text{in}^2}}{1000 \ \dfrac{\text{lbf}}{\text{kip}}} \right)$$

$$= 1350 \text{ kips/in}^2$$

The modular ratio is

$$n = \frac{E_s}{E_m} = \frac{29{,}000 \ \dfrac{\text{kips}}{\text{in}^2}}{1350 \ \dfrac{\text{kips}}{\text{in}^2}}$$

$$= 21.5$$

The moment of inertia of the cracked transformed section about the neutral axis is

$$I_{cr} = \frac{b c^3}{3} + n A_{se}(d - c)^2$$

$$= \frac{(1 \text{ ft})(0.475 \text{ in})^3 \left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}{3}$$

$$+ (21.5)(0.169 \text{ in}^2)\left(\frac{7.63 \text{ in}}{2} - 0.475 \text{ in}\right)^2$$

$$= 41.0 \text{ in}^4$$

Since $M_{u1} > M_{cr}$, the midheight deflection corresponding to the factored moment is derived from MSJC Eq. 3-30 as

$$\delta_u = \frac{5 M_{cr} h^2}{48 E_m I_g} + \frac{5(M_{u1} - M_{cr}) h^2}{48 E_m I_{cr}}$$

$$= \frac{(5)(1.41 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)\left((20 \text{ ft})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)\right)^2}{(48)\left(1350 \ \dfrac{\text{kips}}{\text{in}^2}\right)(444 \text{ in}^4)}$$

$$+ \frac{(5)(1.65 \text{ ft-kips} - 1.41 \text{ ft-kips})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)}{(48)\left(1350 \ \dfrac{\text{kips}}{\text{in}^2}\right)(41.0 \text{ in}^4)}$$
$$\times \left((20 \text{ ft})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)\right)^2$$

$$= 0.48 \text{ in}$$

$$\neq \delta_{u1} \quad [\text{unsatisfactory}]$$

Masonry

The first assumed deflection is incorrect and a second iteration is necessary. Assume a revised deflection at midheight of

$$\delta_{u2} = 0.48 \text{ in}$$

Repeating the calculations using this revised value for the deflection gives

$$M_{u2} = 1.654 \text{ ft-kips}$$
$$\delta_u = 0.49 \text{ in}$$
$$\approx \delta_{u2}$$

So,

$$M_u = 1.654 \text{ ft-kips}$$
$$< \phi M_n \quad \text{[satisfactory]}$$

The flexural capacity is adequate.

Maximum Reinforcement Limit for a Slender Masonry Wall

The amount of tensile reinforcement allowed in slender masonry walls is limited so as to provide adequate ductility in the wall by ensuring that the tensile reinforcement yields prior to the masonry compressive zone crushing. To achieve this, MSJC Sec. 3.3.3.5.1 stipulates the following conditions, as shown in Fig. 6.16.

- Strain in the extreme tension reinforcement is 1.5 times the strain associated with the reinforcement yield stress, f_y, and for grade 60 reinforcement is

$$\epsilon_{su} = (1.5)(0.00207)$$
$$= 0.00311$$

- Maximum strain in the extreme masonry compression fiber, for concrete masonry, is

$$\epsilon_{mu} = 0.00250$$

- Unfactored gravity axial loads are included in the analysis using the combination

$$P = D + 0.75L + 0.525Q_E$$

Figure 6.16 *Maximum Reinforcement Requirements for a Slender Concrete Masonry Wall*
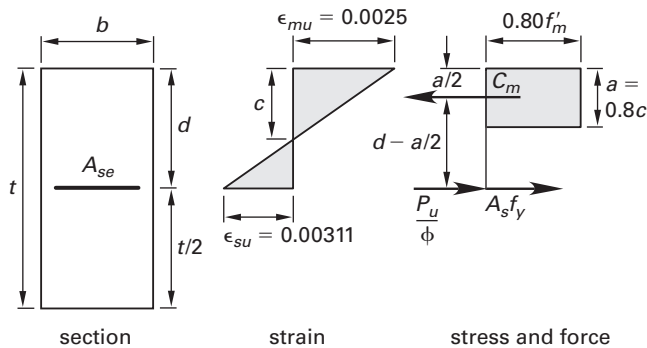
From the strain distribution shown in Fig. 6.16, the neutral axis depth is obtained as

$$c = \frac{0.0025d}{0.00561}$$
$$= 0.446d$$

The depth of the equivalent rectangular stress block is

$$a = 0.80c = (0.80)(0.446d)$$
$$= 0.357d$$

The force in the equivalent rectangular stress block is

$$C_m = 0.80abf'_m = (0.80)(0.357d)bf'_m$$
$$= 0.286bdf'_m$$

The force in the reinforcing bars is given by MSJC Sec. 3.3.3.5.1 as

$$T = A_{\max}f_y$$

Equating compressive and tensile forces acting on the section gives

$$P = C_m - T$$
$$= 0.286bdf'_m - A_{\max}f_y$$

So, the maximum area of the tension reinforcement that will satisfy MSJC Sec. 3.3.3.5.1 is

$$A_{\max} = \frac{0.286bdf'_m - P}{f_y}$$

Example 6.15

The nominal 8 in solid grouted, concrete block masonry wall, with type N masonry cement mortar and described in Ex. 6.13, has a specified strength of 1500 lbf/in^2. It is reinforced longitudinally with no. 4 grade 60 bars at 16 in centers placed centrally in the wall. The wall has an effective height of $h = 20$ ft and is simply supported at the top and bottom. The unfactored roof live load is $P_r = 200$ lbf/ft. The unfactored roof dead load is $P_d = 400$ lbf/ft. The masonry wall has a weight of $\gamma_w = 69$ lbf/ft^2. Wind load governs and has a value of $W = 30$ lbf/ft^2. Determine if the reinforcement area provided satisfies MSJC Sec. 3.3.3.5.1.

Solution

From Ex. 6.13, the reinforcement area over a 1 ft width of the wall is

$$A_s = 0.15 \text{ in}^2$$

The tributary roof dead load on a 1 ft width of wall is

$$P_d = 0.40 \text{ kip}$$

The tributary roof live load on a 1 ft width of wall is

$$P_r = 0.20 \text{ kip}$$

The weight of wall tributary to the critical section is

$$P_w = 0.759 \text{ kip}$$

The specified gravity axial load combination is

$$
\begin{aligned}
P &= D + 0.75L + 0.525Q_E \\
&= P_d + P_w + 0.75P_r + 0 \text{ kips} \\
&= 0.40 \text{ kip} + 0.759 \text{ kip} + (0.75)(0.20 \text{ kip}) + 0 \text{ kips} \\
&= 1.31 \text{ kips}
\end{aligned}
$$

The maximum area of the tension reinforcement that will satisfy MSJC Sec. 3.3.3.5.1 is

$$
\begin{aligned}
A_{\max} &= \frac{0.286bdf'_m - P}{f_y} \\
&= \frac{(0.286)(12 \text{ in})(3.82 \text{ in})\left(1.5\ \dfrac{\text{kips}}{\text{in}^2}\right) - 1.31 \text{ kips}}{60\ \dfrac{\text{kips}}{\text{in}^2}} \\
&= 0.31 \text{ in}^2 \\
&> A_s \text{ provided} \quad [\text{satisfactory}]
\end{aligned}
$$

Lateral Deflection of a Slender Masonry Wall Under Service Loads

The maximum permissible deflection at midheight of the wall due to service level vertical and lateral loads, and including P-delta effects, is given by MSJC Eq. 3-28 as

$$\delta_s = 0.007h$$

When the applied service moment is less than the cracking moment, the service deflection is given by MSJC Eq. 3-29 as

$$\delta_s = \frac{5M_{\text{ser}}h^2}{48E_m I_g}$$

When the applied service moment, M_{ser}, exceeds the cracking moment, M_{cr}, the service deflection is given by MSJC Eq. 3-30 as

$$\delta_s = \frac{5M_{cr}h^2}{48E_m I_g} + \frac{5(M_{\text{ser}} - M_{cr})h^2}{48E_m I_{cr}}$$

The service moment at midheight of wall, including P-delta effects, is

$$M_{\text{ser}} = \frac{wh^2}{8} + \frac{P_f e}{2} + P\delta_s$$

The cracked moment of inertia of the wall section is

$$I_{cr} = \frac{bc^3}{3} + nA_{se}(d - c)^2$$

As shown in Fig. 6.17, the effective reinforcement depth is

$$d = \frac{t}{2}$$
$$c = \text{depth to the neutral axis}$$

The distance of a reinforcing bar from the neutral axis is $d - c$.

Figure 6.17 *Elastic Analysis of a Slender Concrete Masonry Wall*
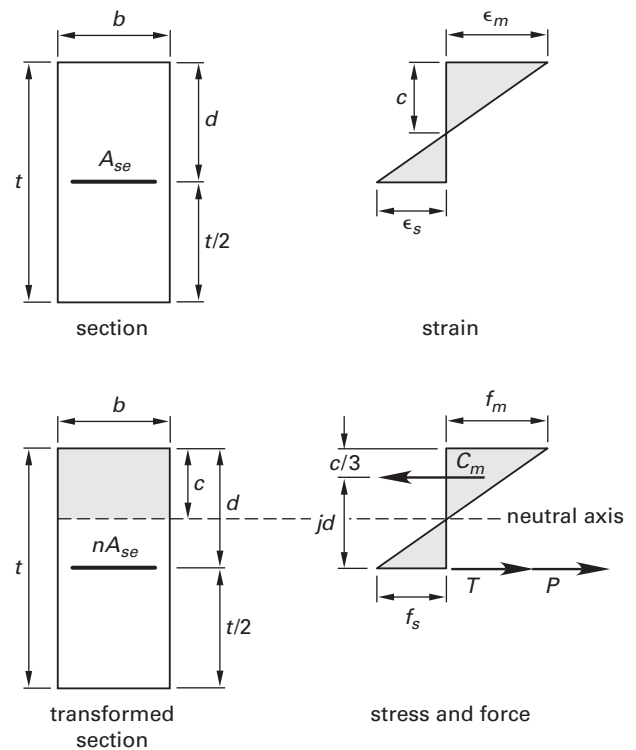
Assuming the stress in the masonry is essentially elastic,

$$
\begin{aligned}
c &= kd \\
k &= \sqrt{2n\rho_e + (n\rho_e)^2} - n\rho_e \\
n &= \frac{E_s}{E_c} \\
\rho_e &= \frac{A_{se}}{bd}
\end{aligned}
$$

A_{se} = equivalent reinforcement area at working load

$$= \frac{P + A_s f_y}{f_y}$$

An iterative process is required until the values for δ_s and M_{ser} converge. However, this process can be time-consuming. Therefore, two alternative procedures are available.

- The deflection, δ_u, under strength level loads has already been determined as part of the design process. If $\delta_u \leq \delta_s$, the deflection under service loads is satisfactory.

- Assume the wall deflection equals the maximum allowable value for δ_s, and calculate the corresponding service load moment, M_{ser}. Using this value of M_{ser}, calculate the corresponding deflection, δ_1. If $\delta_1 \leq \delta_s$, the deflection under service loads is satisfactory.

Example 6.16

The nominal 8 in solid grouted, concrete block masonry wall described in Ex. 6.13 has type N masonry cement mortar and a specified strength of 1500 lbf/in^2. It is reinforced longitudinally with no. 4 grade 60 bars at 16 in centers placed centrally in the wall. The wall has an effective height of $h = 20$ ft and is simply supported at the top and bottom. The unfactored roof live load is $P_r = 200$ lbf/ft. The unfactored roof dead load is $P_d = 400$ lbf/ft. The masonry wall has a weight of $\gamma_w = 69$ lbf/ft^2. Wind load governs and has a factored value of $w_u = 30$ lbf/ft^2. Determine if the midheight deflection of the wall under service level loads is within the permissible limits.

Solution

The maximum permissible deflection at midheight of the wall due to service level vertical and lateral loads, and including P-delta effects, is given by MSJC Eq. 3-28 as

$$\delta_s = 0.007h$$
$$= (0.007)(240 \text{ in})$$
$$= 1.68 \text{ in}$$

From Ex. 6.14, the midheight deflection produced by the factored loads is

$$\delta_u = 0.492 \text{ in}$$
$$< \delta_s$$

So, the deflection under service loads is less than that permissible.

9. DESIGN OF ANCHOR BOLTS

Nomenclature

A_b	effective cross-sectional area of an anchor bolt	in^2
A_o	overlap of projected areas	in^2
A_{pt}	projected area of tensile breakout surface	in^2
A_{pv}	projected area of shear breakout surface	in^2
b_a	applied tensile force on an anchor bolt	lbf
b_{af}	factored tensile force on an anchor bolt	kips
b_v	applied shear force on an anchor bolt	lbf
b_{vf}	factored shear forced on an anchor bolt	kips
B_a	allowable axial load on an anchor bolt	lbf
B_{ab}	allowable axial strength in tension of an anchor bolt when governed by masonry breakout	lbf
B_{an}	nominal axial capacity in tension of an anchor bolt	kips
B_{ap}	allowable axial strength in tension of an anchor bolt when governed by anchor pullout	lbf
B_{as}	allowable axial strength in tension of an anchor bolt when governed by steel yielding	lbf
B_v	allowable shear load on an anchor bolt	lbf
B_{vb}	allowable strength in shear of an anchor bolt when governed by masonry breakout	lbf
B_{vc}	allowable strength in shear of an anchor bolt when governed by masonry crushing	lbf
B_{vn}	nominal capacity in shear of an anchor bolt	kips
B_{vpry}	allowable strength in shear of an anchor bolt when governed by anchor pryout	lbf
B_{vs}	allowable strength in shear of an anchor bolt when governed by steel yielding	lbf
d_b	nominal bolt diameter	in
e_b	projected leg extension of a bent-bar anchor measured from inside edge of anchor at bend to farthest point of anchor in the plane of the hook	in
f_y	yield strength of the anchor bolt	$kips/in^2$
l_b	effective embedment depth of anchor bolt	in
l_{be}	anchor bolt edge distance measured from edge of masonry to center of the cross section of anchor bolt	in
r	radius of projected area	in
s	bolt spacing	in
T	tension force	kips
V	shear force	kips

Symbols

θ	half the angle subtended by the chord at the intersection of overlapping projected areas	degree

Placement Details

Details of headed and bent-bar anchor bolts are given in MSJC Sec. 1.17 and are illustrated in Fig. 6.18.

Figure 6.18 *Anchor Bolts in Concrete Masonry*

headed anchor bolt   bent-bar anchor bolt

Masonry
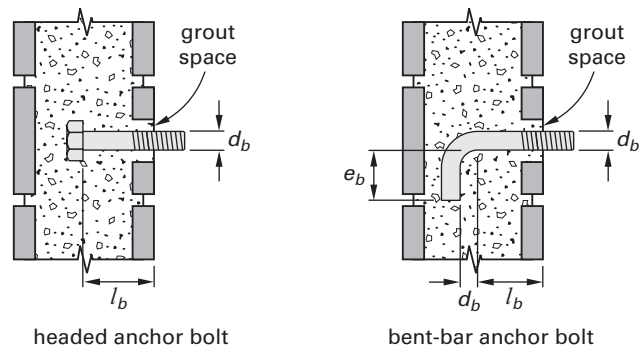
The dimensional limitations specified for anchor bolts in MSJC Sec. 1.17 are as follows.

- Anchor bolts in excess of $1/4$ in diameter must be embedded in grout.

- A minimum of either $1/2$ in coarse grout or $1/4$ in fine grout is required between bolts and the masonry units.

- Anchor bolts of $1/4$ in diameter must be placed in mortar bed joints that are at least $1/2$ in thick.

- The clear distance between parallel anchor bolts must not be less than the diameter of the bolt or less than 1 in.

- The minimum permissible effective embedment length is specified in MSJC Sec. 1.17.6 as $l_b = 4d_b \geq 2$ in.

- For a headed anchor bolt, the effective embedment length is the length of embedment measured from the masonry surface to the bearing surface of the anchor head.

- For a bent-bar anchor bolt, the effective embedment length is the length of embedment measured from the masonry surface to the bearing surface of the bent end, minus one anchor bolt diameter.

Anchor Bolt in Tension

Anchor bolts may fail under tensile forces by[8]

- tensile yielding of the steel anchor
- masonry tensile breakout
- straightening of the hook followed by pullout from the masonry in the case of bent-bar anchors

In accordance with MSJC Comm. Sec. 1.17.2, masonry breakout of an anchor that is solidly grouted in masonry occurs by the pullout of a conically shaped section of masonry. As shown in Fig. 6.19, the failure surface slopes at $45°$. The projected area of the cone on the masonry surface is a circle with a radius equal to the embedment length of the bolt, and with an area of

$$A_{pt} = \pi l_b^2$$

Figure 6.19 *Masonry Breakout of Anchor Bolt in Tension*
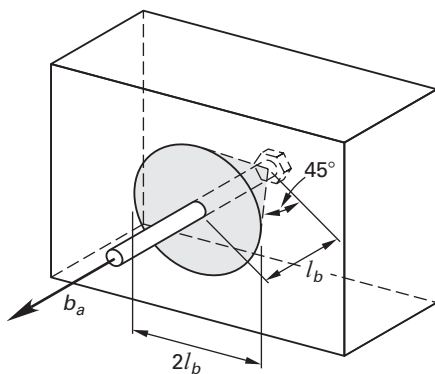
As indicated in MSJC Comm. Sec. 1.17.2, if the projected areas of anchor bolts overlap, an adjustment must be made so as not to overstress the masonry. When bolts are spaced at less than twice the embedment length apart, the projected areas of the bolts overlap, and the anchors must be treated as an anchor group. As specified in MSJC Sec. 1.17.2, the combined projected area of the group is reduced by the overlapping areas. In effect, the projected areas are adjusted so that no area is included more than once. For two anchor bolts, the overlapping area is shown in Fig. 6.20, and is given by

$$A_o = \left(\frac{\pi 2\theta}{180°} - \sin 2\theta\right) r^2$$

Figure 6.20 *Overlap of Projected Areas*
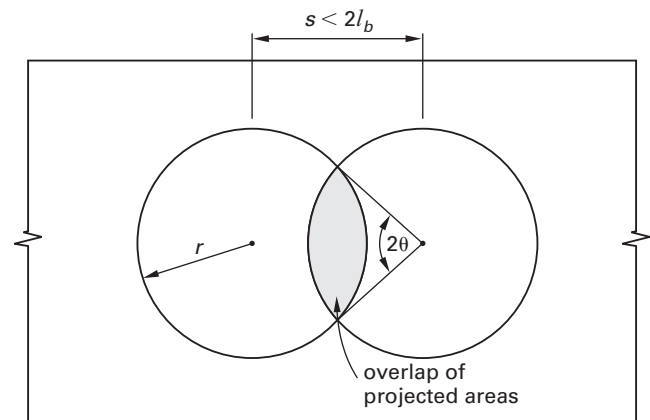
The angle subtended at the center of the projected area by the chord of the intersecting circles is 2θ, where

$$\theta = \arccos \frac{s}{2r}$$
$$r = l_b$$

Similarly, when a bolt is located less than the embedment length from the edge of a member, that portion of the projected area falling outside the masonry member must be deducted from the calculated area. As shown in Fig. 6.21, where the anchor bolt edge distance is l_{be}, the projected area falling outside the masonry wall is

$$A_o = \frac{\left(\frac{\pi 2\theta}{180°} - \sin 2\theta\right) r^2}{2}$$

The angle subtended at the center of the projected area by the chord formed by the member edge is 2θ, where

$$\theta = \arccos \frac{l_{be}}{r}$$

Figure 6.21 *Projected Area Extends Beyond Wall Edge*
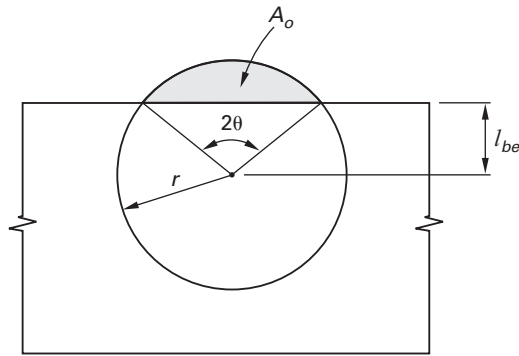
ASD Method

For tensile strength governed by the tensile yielding of a steel anchor, the allowable strength is given by MSJC Eq. 2-2 as

$$B_{as} = 0.6A_b f_y$$

For tensile strength governed by masonry breakout, the allowable strength is given by MSJC Eq. 2-1 as

$$B_{ab} = 1.25A_{pt}\sqrt{f'_m}$$

For bent-bar anchors, when the tensile strength is governed by straightening the hook followed by pullout from the masonry, the allowable strength is given by MSJC Eq. 2-4 as

$$B_{ap} = 0.6f'_m e_b d_b + 120\pi(l_b + e_b + d_b)d_b$$

The allowable strength, B_a, for anchor bolts is the smallest value given by MSJC Eq. 2-1, Eq. 2-2, or Eq. 2-4.

SD Method

For tensile strength governed by the tensile yielding of a steel anchor, the design capacity is given by MSJC Eq. 3-2 as

$$\phi B_{ans} = \phi A_b f_y$$
$$\phi = 0.9$$

For tensile strength governed by masonry breakout, the design capacity is given by MSJC Eq. 3-1 as

$$\phi B_{anb} = \phi(4A_{pt})\sqrt{f'_m}$$
$$\phi = 0.5$$

For bent-bar anchors, when the tensile strength is governed by straightening the hook followed by pullout from the masonry, the design capacity is given by MSJC Eq. 3-4 as

$$\phi B_{anp} = \phi\left(1.5f'_m e_b d_b + 300\pi(l_b + e_b + d_b)d_b\right)$$
$$\phi = 0.65$$

The design capacity, ϕB_{an}, is the smallest value given by MSJC Eq. 2-1, Eq. 2-2, and Eq. 2-4.

Example 6.17

A glulam crosstie between diaphragm chords is supported at one end in a steel beam bucket as shown in the following illustration. The bucket is attached to a solid grouted concrete block masonry wall with a masonry compressive strength of 1500 lbf/in². The four headed anchor bolts are $^3/_4$ in diameter ASTM A 307 type C, with an effective minimum specified yield strength of 36 kips/in² and an effective cross-sectional area of 0.334 in². The effective embedment length of the anchor bolts is 6 in. The ASD governing load combination gives a tension force on the beam bucket of $T = 13$ kips and a shear force of $V = 1.5$ kips. The SD governing load combination gives a tension force of $T_u = 21.7$ kips and a shear force of $V_u = 2.25$ kips. Determine if the bolts are adequate for the tension force on the bucket.
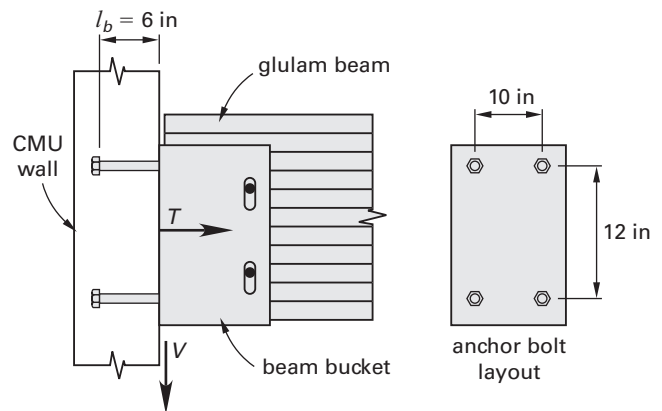
Solution

Masonry breakout

The effective embedment length of an anchor bolt measured from the surface of the masonry to the bearing surface of the bolt head is given as

$$l_b = 6 \text{ in}$$
$$> 4d_b \quad \text{[satisfies MSJC Sec. 1.17.6]}$$
$$> 2 \text{ in} \quad \text{[satisfies MSJC Sec. 1.17.6]}$$

The projected area of one bolt, before considering overlapping areas, is

$$A_{pt} = \pi l_b^2 = \pi(6 \text{ in})^2$$
$$= 113 \text{ in}^2$$

The vertical spacing of the bolts is

$$s = 12 \text{ in}$$
$$= 2l_b$$

The projected areas of the bolts do not overlap in the vertical direction. The horizontal spacing of the bolts is

$$s = 10 \text{ in}$$
$$< 2l_b$$

The projected areas of the bolts overlap in the horizontal direction. The angle subtended at the center of the projected area by the chord of the intersecting circles is

$$2\theta = 2 \arccos \frac{s}{2r}$$
$$= 2 \arccos \frac{10 \text{ in}}{(2)(6 \text{ in})}$$
$$= 67.1°$$

For the top two anchor bolts, the overlapping area is

$$A_o = \left(\frac{\pi 2\theta}{180°} - \sin 2\theta\right) r^2$$
$$= \left(\frac{\pi(67.1°)}{180°} - \sin 67.1°\right)(6 \text{ in})^2$$
$$= 9 \text{ in}^2$$

The reduced projected area for the top two bolts is

$$A'_{pt} = 2A_{pt} - A_o$$
$$= (2)(113 \text{ in}^2) - 9 \text{ in}^2$$
$$= 217 \text{ in}^2$$

For all four bolts the reduced projected area is

$$A''_{pt} = (2)(217 \text{ in}^2)$$
$$= 434 \text{ in}^2$$

ASD Method

For tensile strength governed by masonry breakout, the allowable strength of the four bolts is given by MSJC Eq. 2-1 as

$$B_{ab} = 1.25 A''_{pt} \sqrt{f'_m}$$
$$= \frac{(1.25)(434 \text{ in}^2)\sqrt{1500 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}}$$
$$= 21.0 \text{ kips}$$
$$> T \quad \text{[satisfactory]}$$

For tensile strength governed by the tensile yielding of a steel anchor, the allowable strength of each anchor bolt is given by MSJC Eq. 2-2 as

$$B_{as} = 0.6 A_b f_y$$
$$= (0.6)(0.334 \text{ in}^2)\left(36 \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 7.21 \text{ kips}$$

The allowable strength of the four bolts is

$$4B_{as} = (4)(7.21 \text{ kips})$$
$$= 28.8 \text{ kips} > B_{ab} = 21.0 \text{ kips} \quad \text{[governs]}$$

The bolts are adequate for the tension force on the bucket. The allowable tension force on one bolt in the absence of shear force is

$$B_{as} = \frac{21.0 \text{ kips}}{4} = 5.25 \text{ kips}$$

SD Method

For tensile strength governed by the tensile yielding of a steel anchor, the design capacity is given by MSJC Eq. 3-2 as

$$\phi B_{ans} = \phi A_b f_y = (0.9)(0.334 \text{ in}^2)\left(36 \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 10.8 \text{ kips}$$

The design capacity of the four bolts is

$$4\phi B_{ans} = (4)(10.8 \text{ kips})$$
$$= 43.2 \text{ kips} > T_u \quad \text{[satisfactory]}$$

For tensile strength governed by masonry breakout, the design capacity of the four bolts is given by MSJC Eq. 3-1 as

$$4\phi B_{anb} = \phi(4A_{pt})\sqrt{f'_m}$$
$$= \frac{(0.5)(4)(434 \text{ in}^2)\sqrt{1500 \frac{\text{lbf}}{\text{in}^2}}}{1000 \frac{\text{lbf}}{\text{kip}}}$$
$$= 33.6 \text{ kips} \quad \text{[governs]}$$
$$< 4\phi B_{ans}$$
$$> T_u \quad \text{[satisfactory]}$$

The bolts are adequate for the tension force on the bucket. The design capacity of one bolt in the absence of shear force is

$$\phi B_{an} = \frac{33.6 \text{ kips}}{4}$$
$$= 8.40 \text{ kips}$$

Anchor Bolt in Shear

Anchor bolts may fail under shear forces by

- steel anchor shear yielding
- masonry shear breakout
- masonry shear crushing
- anchor bolt shear pryout

The four modes of failure are shown in Fig. 6.22.

Masonry

Figure 6.22 *Shear Failure Modes for Anchor Bolts*

steel anchor shear yielding   masonry shear breakout

anchor bolt shear pryout   masonry shear crushing
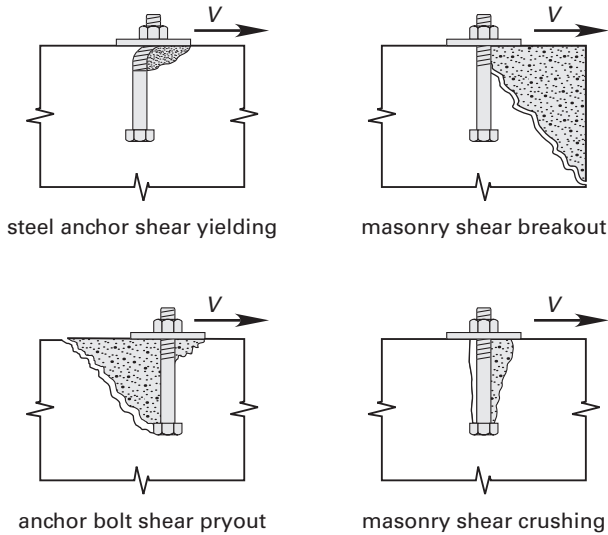
Anchors near an edge in solidly grouted masonry fail in shear by the breakout of one-half of a conically shaped section of masonry. (See Fig. 6.23.) The failure surface slopes at 45°. The bolt's projected area is a semicircle of radius equal to the anchor bolt edge distance with an area given by MSJC Eq. 1-5 of

$$A_{pv} = \frac{\pi l_{be}^2}{2}$$

As indicated in MSJC Sec. 1.17.3, if the projected area of anchor bolts overlap or portions of the projected area fall outside the masonry member, an adjustment must be made to the projected area.

Figure 6.23 *Masonry Breakout in Shear*
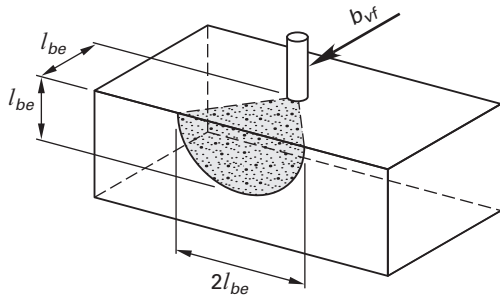
ASD Method

For shear strength governed by masonry breakout, the allowable strength is given by MSJC Eq. 2-6 as

$$B_{vb} = 1.25 A_{pv} \sqrt{f'_m}$$

For shear strength governed by masonry crushing, the allowable strength is given by MSJC Eq. 2-7 as

$$B_{vc} = 350 \sqrt[4]{f'_m A_b}$$

For shear strength governed by masonry pryout, the allowable strength is given by MSJC Eq. 2-8 as

$$B_{vpry} = 2.5 A_{pt} \sqrt{f'_m}$$
$$= 2 B_{ab}$$

For shear strength governed by the shear yielding of a steel anchor, the allowable strength is given by MSJC Eq. 2-9 as

$$B_{vs} = 0.36 A_b f_y$$

The allowable strength is the smallest value given by MSJC Eq. 2-6, Eq. 2-7, Eq. 2-8, and Eq. 2-9.

SD Method

For shear strength governed by masonry breakout, the design capacity is given by MSJC Eq. 3-6 as

$$\phi B_{vnb} = \phi(4 A_{pv} \sqrt{f'_m})$$
$$\phi = 0.5$$

For shear strength governed by masonry crushing, the design capacity is given by MSJC Eq. 3-7 as

$$\phi B_{vnc} = \phi(1050 \sqrt[4]{f'_m A_b})$$
$$\phi = 0.5$$

For shear strength governed by masonry pryout, the design capacity is given by MSJC Eq. 3-8 as

$$\phi B_{vnpry} = \phi(8 A_{pt} \sqrt{f'_m})$$
$$= \phi(2 B_{anb})$$
$$\phi = 0.5$$

The design strength of the anchor bolt in shear yielding is given by MSJC Eq. 3-9 as

$$\phi B_{vns} = \phi(0.6 A_b f_y)$$
$$\phi = 0.9$$

Example 6.18

As indicated in Ex. 6.17, a glulam crosstie between diaphragm chords is supported at one end in a steel beam bucket. The bucket is attached to a solid grouted concrete block masonry wall with a masonry compressive strength of 1500 lbf/in^2. The four headed anchor bolts are $3/4$ in diameter ASTM A 307 type C, with an effective minimum specified yield strength of 36 kips/in^2 and an effective cross-sectional area of 0.334 in^2. The effective embedment length of the anchor bolts is 6 in, and the threaded portion of the bolts is not located within the shear plane. The ASD governing load combination gives a tension force on the beam bucket of $T = 13$ kips and a shear force of $V = 1.5$ kips. The bolts are far from a free edge of the wall and masonry

breakout in shear cannot occur. Determine if the bolts are adequate for the shear force on the bucket.

Solution

From Ex. 6.17, the reduced projected area for the top two bolts is

$$A'_{pt} = 217 \text{ in}^2$$

For all four bolts, the reduced projected area is

$$A''_{pt} = (2)(217 \text{ in}^2)$$
$$= 434 \text{ in}^2$$

ASD Method

Anchor yielding

For shear strength governed by the shear yielding of the steel anchor, the allowable strength of each anchor bolt is given by MSJC Eq. 2-9 as

$$B_{vs} = 0.36 A_b f_y$$
$$= (0.36)(0.334 \text{ in}^2)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 4.33 \text{ kips}$$

The allowable strength of the four bolts is

$$4B_{vs} = (4)(4.33 \text{ kips})$$
$$= 17.3 \text{ kips}$$

Masonry pryout

For shear strength governed by masonry pryout, the allowable strength of all four bolts is given by MSJC Eq. 2-8 as

$$4B_{vpry} = 2.5 A''_{pt}\sqrt{f'_m}$$
$$= \frac{(2.5)(434 \text{ in}^2)\sqrt{1500 \ \frac{\text{lbf}}{\text{in}^2}}}{1000 \ \frac{\text{lbf}}{\text{kip}}}$$
$$= 42.0 \text{ kips}$$

Masonry crushing

For shear strength governed by masonry crushing, the allowable strength of each anchor bolt is given by MSJC Eq. 2-7 as

$$B_{vc} = 350\sqrt[4]{f'_m A_b}$$
$$= \frac{(350)\sqrt[4]{\left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.334 \text{ in}^2)}}{1000 \ \frac{\text{lbf}}{\text{kip}}}$$
$$= 1.66 \text{ kips} \quad \text{[governs]}$$

The allowable strength of the four bolts is

$$4B_{vc} = (4)(1.66 \text{ kips})$$
$$= 6.64 \text{ kips}$$
$$< 4B_{vpry}$$
$$< 4B_{vs}$$
$$> V \quad \text{[satisfactory]}$$

The bolts are adequate for the shear force on the bucket. The allowable shear force on one bolt in the absence of tensile force is

$$B_v = \frac{6.64 \text{ kips}}{4} = 1.66 \text{ kips}$$

SD Method

Anchor yielding

The design strength of each anchor bolt in shear yielding is given by MSJC Eq. 3-9 as

$$\phi B_{vns} = \phi(0.6 A_b f_y)$$
$$= (0.9)(0.6)(0.334 \text{ in}^2)\left(36 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 6.49 \text{ kips}$$

The design strength of the four bolts is

$$4\phi B_{vns} = (4)(6.49 \text{ kips})$$
$$= 26.0 \text{ kips}$$

Masonry pryout

For shear strength governed by masonry pryout, the design capacity for all four bolts is given by MSJC Eq. 3-8 as

$$\phi B_{vnpry} = \phi(2B_{anb})$$
$$= (2)(33.6 \text{ kips})$$
$$= 67.2 \text{ kips}$$

Masonry crushing

For shear strength governed by masonry crushing, the design capacity of each anchor bolt is given by MSJC Eq. 3-7 as

$$\phi B_{vnc} = \phi(1050\sqrt[4]{f'_m A_b})$$
$$= \frac{(0.5)(1050)\sqrt[4]{\left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)(0.334 \text{ in}^2)}}{1000 \ \frac{\text{lbf}}{\text{kips}}}$$
$$= 2.48 \text{ kips} \quad \text{[governs]}$$

Masonry

The design shear strength of the four bolts is

$$4\phi B_{vnc} = (4)(2.48 \text{ kips})$$
$$= 9.92 \text{ kips}$$
$$> V_u \quad \text{[satisfactory]}$$

The bolts are adequate for the shear force on the bucket. The design capacity of one bolt in the absence of tensile force is

$$\phi B_{vnc} = 2.48 \text{ kips}$$

Headed Anchor Bolts in Combined Tension and Shear

ASD Method

Anchor bolts subjected to combined tension and shear must comply with MSJC Eq. 2-10, which is

$$\frac{b_a}{B_a} + \frac{b_v}{B_v} \leq 1$$

In addition, the allowable strength in shear and tension must each exceed the applied loads.

SD Method

The design axial capacity, ϕB_{an}, is the smallest value given by MSJC Eq. 3-1, Eq. 3-2, and Eq. 3-4.

The design shear capacity, ϕB_{vn}, is the smallest value given by MSJC Eq. 3-6 through Eq. 3-9.

For combined tension and shear, anchor bolts must comply with MSJC Eq. 3-10, which is

$$\frac{b_{af}}{\phi B_{an}} + \frac{b_{vf}}{\phi B_{vn}} \leq 1$$

In addition, the design capacity in shear and tension must each exceed the factored applied loads.

Example 6.19

As given in Ex. 6.17, a glulam crosstie between dia-phragm chords is supported at one end in a steel beam bucket. The bucket is attached to a solid grouted concrete block masonry wall with a masonry compressive strength of 1500 lbf/in². The four headed anchor bolts are $^3/_4$ in diameter ASTM A 307 type C, with a minimum specified yield strength of 36 kips/in² and an effective cross-sectional area of 0.334 in². The effective embedment length of the anchor bolts is 6 in, and the threaded portion of the bolts is not located within the shear plane. The ASD governing load combination gives a tension force on the beam bucket of $T = 13$ kips and a shear force of $V = 1.5$ kips. The SD governing load combination gives a tension force of $T_u = 21.7$ kips and a shear force of $V_u = 2.25$ kips. Determine if the bolts are adequate for the combined tension and shear forces.

Solution

The applied tension force on one bolt is

$$b_a = \frac{T}{4}$$
$$= \frac{13 \text{ kips}}{4}$$
$$= 3.25 \text{ kips}$$

The applied shear force on one bolt is

$$b_v = \frac{V}{4}$$
$$= \frac{1.5 \text{ kips}}{4}$$
$$= 0.38 \text{ kip}$$

For combined tension and shear, MSJC Eq. 2-10 must be satisfied.

$$\frac{b_a}{B_a} + \frac{b_v}{B_v} \leq 1$$
$$\frac{3.25 \text{ kips}}{5.25 \text{ kips}} + \frac{0.38 \text{ kip}}{1.66 \text{ kips}} = 0.85$$
$$< 1 \quad \text{[satisfactory]}$$

SD Method

The factored tension force on one bolt is

$$b_{af} = \frac{T_u}{4}$$
$$= \frac{21.7 \text{ kips}}{4}$$
$$= 5.43 \text{ kips}$$

The factored shear force on one bolt is

$$b_{vf} = \frac{V_u}{4}$$
$$= \frac{2.25 \text{ kips}}{4}$$
$$= 0.56 \text{ kips}$$

For combined tension and shear, anchor bolts must comply with MSJC Eq. 3-10, which is

$$\frac{b_{af}}{\phi B_{an}} + \frac{b_{vf}}{\phi B_{vn}} \leq 1$$
$$\frac{5.43 \text{ kips}}{8.41 \text{ kips}} + \frac{0.56 \text{ kip}}{2.48 \text{ kips}} = 0.87$$
$$< 1 \quad \text{[satisfactory]}$$

10. DESIGN OF PRESTRESSED MASONRY

Nomenclature

A_n	net cross-sectional area of masonry	in^2
A_{ps}	area of prestressing steel	in^2
A_s	area of nonprestressed reinforcement	in^2
b	width of section	in
C_m	force in masonry stress block	lbf
d	distance from extreme compression fiber to centroid of prestressing tendon or reinforcement	in
D	dead load or related internal moments or forces	lbf
e	eccentricity of axial load	in
E_m	modulus of elasticity of masonry	lbf/in^2
E_{mi}	modulus of elasticity of masonry at transfer	lbf/in^2
E_{ps}	modulus of elasticity of prestressing steel	lbf/in^2
f_a	calculated compressive stress in masonry due to axial load	lbf/in^2
f_{ai}	calculated compressive stress in masonry at transfer due to axial load	lbf/in^2
f_b	calculated compressive stress in masonry due to flexure	lbf/in^2
f_{bi}	calculated compressive stress in masonry at transfer due to flexure	lbf/in^2
f'_m	specified compressive strength of masonry	lbf/in^2
f'_{mi}	specified compressive strength of masonry at time of prestress transfer	lbf/in^2
f_{pj}	stress in prestressing tendon due to jacking force	lbf/in^2
f_{ps}	stress in prestressing tendon at nominal strength	lbf/in^2
f_{psi}	initial stress in prestressing tendon	lbf/in^2
f_{pu}	specified tensile strength of prestressing tendons	lbf/in^2
f_{py}	specified yield strength of prestressing tendons	lbf/in^2
f_{se}	effective stress in prestressing tendon after allowance for all prestress losses	lbf/in^2
f_y	specified yield stress of nonprestressed reinforcement	lbf/in^2
F_a	allowable compressive stress in masonry due to axial load	lbf/in^2
F_{ai}	allowable compressive stress in masonry at transfer due to axial load	lbf/in^2
F_b	allowable compressive stress in masonry due to flexure	lbf/in^2
F_{bi}	allowable compressive stress in masonry at transfer due to flexure	lbf/in^2
F_t	allowable tensile stress in masonry due to flexure	lbf/in^2
h	effective height of wall	ft
I_n	moment of inertia of net cross-sectional area	in^4
l_p	clear span of the prestressed member in the direction of the prestressing tendon	ft
M	maximum moment at the section under consideration	ft-kips
M_n	nominal flexural strength	ft-kips
M_u	factored moment at the section under consideration	ft-kips
P	nonfactored axial load	lbf
P_d	nonfactored dead load from tributary floor or roof loads	lbf
P_e	Euler critical load	lbf
P_f	nonfactored load from tributary floor or roof loads, sum of P_d and P_r	lbf
P_i	axial load at transfer	lbf
P_{ps}	prestressing tendon force at time and location relevant for design	kips
P_{psi}	prestressing tendon force at transfer	kips
P_{psj}	prestressing force at the jack	kips
P_r	nonfactored live load from tributary floor or roof loads	lbf
P_u	factored axial load	lbf
P_w	nonfactored weight of wall tributary to section considered	lbf
Q	load combination	–
r	radius of gyration	in
S_n	section modulus of net cross-sectional area	in^3
t	nominal thickness of member	in
t_f	face-shell thickness	in
w	wind load	lbf/ft^2
W	wind load or related internal moments or forces	lbf

Symbols

γ_w	weight of wall	lbf/ft^2
ϕ	strength reduction factor	–

General Considerations

A masonry wall is prestressed by tensioning a prestressing tendon that is located in the mid-plane of the wall. This produces compression in the member, which increases its flexural tensile capacity and improves its resistance to lateral loading.[9] Prestressed masonry members must be designed for three distinct design stages.

- the *transfer design stage*, when a prestressing force is applied to the wall and immediate prestress losses occur due to elastic deformation of the masonry and seating of the anchorage

- the *serviceability design stage*, when all time-dependent prestress losses have occurred as a result of creep and shrinkage of the masonry and relaxation of the tendons

- the *strength design stage*, when the design capacity of a wall is checked to ensure that it is not less than the demand produced by the factored loads

By comparison with traditional reinforced masonry construction, prestressed masonry has the advantage of not requiring grouting of the cores in which the prestressing tendons are placed. This results in

- a reduced wall weight, requiring smaller footings and reduced seismic forces

- the elimination of grout, requiring less material and labor

- a reduction in construction times

Construction Details

Prestressing tendons must be of high tensile steel as required by *MSJC Specification* Sec. 2.4B. Tendons may consist of wire, rods with threaded ends, or strand. Rods may be connected during construction using couplers. Strand is provided in the full required length. A typical wall using rods with threaded ends is shown in Fig. 6.24.

Masonry

Figure 6.24 *Prestressed Masonry Details*
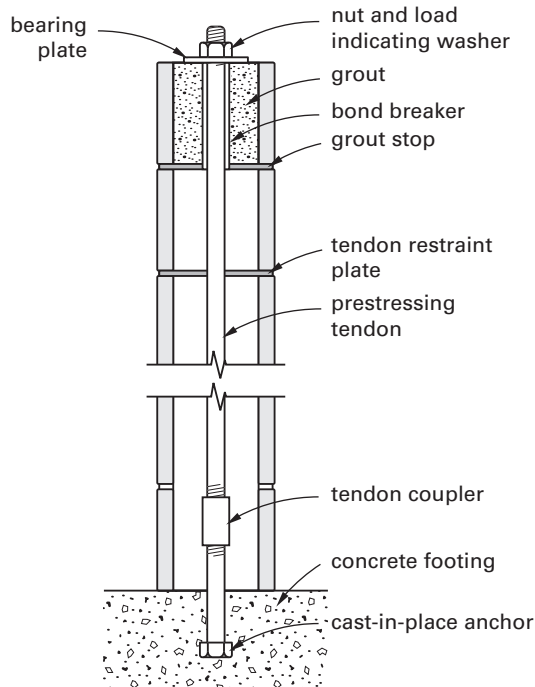
Walls are usually post-tensioned and ungrouted, as shown in Fig. 6.24. Walls subject to a moist and corrosive environment require corrosion protection for the tendon. Corrosion protection may be provided by coating the tendon with a corrosion-inhibiting material and enclosing the tendon in a continuous plastic sheath. Alternatively, tendons may be galvanized threaded high-tensile steel rods.

The prestressing force is transferred to the masonry by means of anchorages at the foot of the wall and at the top of the wall. As specified in MSJC Sec. 4.8.1, tendons may be anchored by

- mechanical devices bearing directly on masonry or placed inside an end block of concrete or fully grouted masonry

- bond in reinforced concrete end blocks or members

An example of an anchor bolt embedded in the concrete foundation is shown in Fig. 6.24. The prestressing rod is attached to the anchor bolt by a threaded coupler. An inspection port is provided at the location of the coupler to enable the rod to be connected to the coupler after the block wall has been constructed to partial or full height. In accordance with MSJC Sec. 4.8.2, couplers must develop 95% of the specified tensile strength of the prestressing rods. When a moment connection is not required at the base of the wall, anchor bolts may be located in the first block course of masonry.

The tendons are normally laterally restrained so as to deflect laterally with the wall. The advantage of laterally restraining the tendons is that the prestressing

force does not contribute to the elastic instability of the wall, as lateral displacement of the wall is counterbalanced by an equal and opposite restraint from the prestressing tendons. So, the wall cannot buckle under its own prestressing force. In addition, the ultimate strength of the wall is increased by ensuring that the wall and the tendon deform together. MSJC Comm. Sec. 4.4.2 stipulates that three restraints along the length of a tendon are sufficient to provide adequate restraint. As shown in Fig. 6.24, restraint is provided by a steel plate with a central hole. The sides of the plate are embedded in the joint between courses. Alternatively, a cell can be filled with grout, and a bond breaker is applied to the tendon to allow it to move freely within the cell.

A mechanical anchorage is used at the top of the wall shown in Fig. 6.24. The topmost cell is filled with grout and a steel bearing plate is bedded on top. The size of the bearing plate is determined from the requirement of MSJC Sec. 4.8.4.2, which stipulates that bearing stresses must not exceed 50% of the masonry compressive strength at the time of transfer. Reinforcement is required in the grouted cell, as specified in MSJC Sec. 4.8.3, to resist the bursting and tensile forces produced when the tendon is stressed. The tendon may be stressed by means of a hydraulic jack or by tightening a nut with a standard wrench. In the former method, the load is measured by a load cell or a calibrated gauge on the jack. In the latter method, the force produced by tightening a nut may be measured with a direct tension indicator washer (a washer with dimples formed on its top face). A hardened steel washer is placed on top of the indicator washer and as the nut is tightened, the dimples are compressed. The required force in the tendon has been produced when the gap between the two washers reaches a specified amount as measured by a feeler gauge. *MSJC Specification* Sec. 3.6B requires that the elongation of the tendons be measured and compared with the elongation anticipated for the applied prestressing force. If the discrepancy between the two methods exceeds 7% for post-tensioned walls, the cause must be determined and corrected.

Masonry blocks are usually laid in running bond in face-shell mortar bedding, using type S mortar to provide early strength gain. The small percentage of prestressing steel in the wall results in a highly ductile system with the neutral axis entirely within the face shell. Properties of concrete masonry units with face-shell mortar bedding are given in the *Masonry Designer's Guide*[10] and in Table 6.2.

Table 6.2 *Properties of CMU Walls With Face-Shell Mortar Bedding and Unit Density of 110 lbf/ft^3*

nominal wall thickness	6 in	8 in	10 in	12 in
face-shell thickness	1.00	1.25	1.38	1.5
area, A_n (in^2/ft)	24	30	33	36
moment of inertia, I_n (in^4/ft)	130	309	567	929
section modulus, S_n (in^3/ft)	46.3	81	118	160
radius of gyration, r (in/ft)	2.33	3.21	4.14	5.08
weight of wall, γ_w (lbf/ft^2)	28.5	37	45	51

Transfer Design Stage

The transfer design stage occurs immediately after a prestressing force is applied to the wall. The prestressing force is usually applied to the wall three days after construction of the wall when the masonry strength is f'_{mi}. Since the masonry will continue to gain strength, MSJC Sec. 4.4.1.2 permits a 20% overstress in the customary allowable axial and flexural stresses, and in the customary allowable axial force. The allowable compressive stress at transfer in a member having an h/r ratio not greater than 99 and that is subjected to axial load and flexure is given by MSJC Eq. 2-16 as

$$F_{ai} = 1.2(0.25f'_{mi})\left(1 - \left(\frac{h}{140r}\right)^2\right)$$

For members having an h/r ratio greater than 99, the allowable compressive stress at transfer is given by MSJC Eq. 2-17 as

$$F_{ai} = 1.2(0.25f'_{mi})\left(\frac{70r}{h}\right)^2$$

The allowable compressive stress at transfer in a member due to flexure is given by MSJC Eq. 2-18 as

$$F_{bi} = \frac{1.2f'_{mi}}{3}$$

The axial load at transfer is due entirely to the self-weight of the wall and the prestressing force at transfer, since additional floor or roof loads are not present and the axial force in the masonry is

$$P_i = P_w + P_{psi}$$

The axial stress in the masonry at transfer is

$$f_{ai} = \frac{P_i}{A_n}$$

In the event that the full lateral load occurs at transfer, the bending compressive stress is

$$f_{bi} = \frac{M}{S_n}$$

The resulting compressive stress due to combined axial load and flexure must satisfy the interaction expression of MSJC Eq. 2-14, which is

$$\frac{f_{ai}}{F_{ai}} + \frac{f_{bi}}{F_{bi}} \le 1.0$$

In applying this expression, the governing load combination is IBC Eq. 16-12, which is

$$Q = D + 0.6W$$

The allowable tensile stress, F_t, due to flexure is given by MSJC Table 2.2.3.2 and, in accordance with MSJC Sec. 2.2.3.2, is

$$f_{bi} - f_{ai} \le F_t$$

In applying this expression, the governing load combination is IBC Eq. 16-15, which is

$$Q = 0.6D + 0.6W$$

To ensure elastic stability at transfer, the axial force is limited by MSJC Eq. 2-15 and MSJC Sec. 4.4.1.2 to

$$P_i \le \frac{1.2P_{ei}}{4}$$

The Euler critical load, P_{ei}, is given by MSJC Sec. 4.4.2.2 and MSJC Eq. 2-19 as

$$P_{ei} = \frac{\pi^2 E_{mi}I_n\left(1 - \dfrac{0.577e}{r}\right)^3}{h^2}$$

Using laterally restrained tendons, the wall cannot buckle under its own prestressing force and, in accordance with MSJC Sec. 4.4.2.2, for stability calculations, P_{psi} is not considered in the determination of P_i. At transfer, additional floor or roof loads are not present, and the axial force in the masonry is

$$P_i = P_w$$

The stress in a prestressing tendon due to the jacking force is limited by MSJC Sec. 4.3.1 to

$$f_{pj} \le 0.94f_{py}$$
$$\le 0.80f_{pu}$$

Immediately after transfer, when initial loss of prestress has occurred, the stress in a post-tensioned tendon at anchorages and couplers is limited by MSJC Sec. 4.3.3 to

$$f_{psi} \le 0.78f_{py}$$
$$\le 0.70f_{pu}$$

The initial loss of prestress in a tendon at transfer due to elastic deformation of the masonry and seating of the anchorage is given by MSJC Comm. Sec. 4.3.4 as 5% to 10%.

Example 6.20

A nominal 8 in ungrouted concrete block masonry wall with face-shell mortar bedding of type S portland cement/lime mortar has a specified strength of $f'_m = 2500$ lbf/in^2 at 28 days, and a specified strength of $f'_{mi} = 2000$ lbf/in^2 at transfer. The wall is post-tensioned with $^7/_{16}$ in diameter steel rods at 16 in centers placed centrally in the wall, and the rods are laterally restrained. The wall has an effective height of $h = 20$ ft and is simply supported at the top and bottom. The roof

Masonry

live load is 200 lbf/ft and the roof dead load is 400 lbf/ft applied to the wall without eccentricity. The masonry wall has a weight of $\gamma_w = 37$ lbf/ft^2. The properties of the steel rod are $A_{ps} = 0.142$ in^2, $f_{py} = 100$ kips/in^2, and $f_{pu} = 122$ kips/in^2. Wind load governs and has a value of $w = 25$ lbf/ft^2. Assume the loss of prestress at transfer is 5% and the final loss of prestress is 30%. Determine if the wall under transfer level loads is within the permissible limits. Ignore P-delta effects.

Solution

The calculations are based on a 1 ft length of wall. The relevant wall section properties are obtained from Table 6.2 as

$$A_n = 30 \text{ in}^2$$
$$I_n = 309 \text{ in}^4$$
$$S_n = 81 \text{ in}^3$$
$$r = 3.21 \text{ in}$$

The weight of the wall above midheight is

$$P_w = \frac{\gamma_w h}{2} = \frac{\left(37 \, \frac{\text{lbf}}{\text{ft}^2}\right)(20 \text{ ft})(1 \text{ ft})}{2}$$
$$= 370 \text{ lbf}$$

Prestress details

The equivalent area of prestressing tendon per foot of wall is

$$A_{ps} = \frac{(0.142 \text{ in}^2)(1 \text{ ft})(12 \text{ in})}{16 \text{ in}} = 0.107 \text{ in}^2$$

The stress in a prestressing tendon due to the jacking force is limited by MSJC Sec. 4.3.1 to

$$f_{pj} \leq 0.80 f_{pu} = (0.80)\left(122 \, \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 97.6 \text{ kips/in}^2$$
$$\leq 0.94 f_{py}$$
$$= (0.94)\left(100 \, \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 94.0 \text{ kips/in}^2 \quad [\text{governs}]$$

After 5% initial losses, the stress in the prestressing tendon is

$$f_{psi} = 0.95 f_{pj}$$
$$= (0.95)\left(94.0 \, \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 89.3 \text{ kips/in}^2$$

The stress in a post-tensioned tendon after transfer is limited by MSJC Sec. 4.3.3 to

$$f_{psi} \leq 0.70 f_{pu}$$
$$= (0.70)\left(122 \, \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 85.4 \text{ kips/in}^2$$
$$\leq 0.78 f_{py}$$
$$= (0.78)\left(100 \, \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 78.0 \text{ kips/in}^2 \quad [\text{governs}]$$
$$< 89.3 \text{ kips/in}^2$$

Therefore, the initial jacking force per foot of wall is restricted to

$$P_{psj} = 1.05 f_{psi} A_{ps}$$
$$= (1.05)\left(78.0 \, \frac{\text{kips}}{\text{in}^2}\right)(0.107 \text{ in}^2)$$
$$= 8.76 \text{ kips}$$

After 5% initial losses, the prestressing tendon force per foot of wall is

$$P_{psi} = 0.95 P_{psj}$$
$$= (0.95)(8.76 \text{ kips})$$
$$= 8.32 \text{ kips}$$

Compressive stress check

The slenderness ratio of the wall is

$$\frac{h}{r} = \frac{(20 \text{ ft})\left(12 \, \frac{\text{in}}{\text{ft}}\right)}{3.21 \text{ in}} = 74.8$$

After allowing for the 20% increase in allowable stress at transfer, the allowable compressive stress in a member having an h/r ratio not greater than 99 and subjected to axial load is given by MSJC Eq. 2-16 as

$$F_{ai} = 1.2\left(0.25 f'_{mi}\right)\left(1 - \left(\frac{h}{140r}\right)^2\right)$$
$$= (1.2)(0.25)\left(2000 \, \frac{\text{lbf}}{\text{in}^2}\right)\left(1 - \left(\frac{74.8}{140}\right)^2\right)$$
$$= 429 \text{ lbf/in}^2$$

After allowing for the 20% increase in stress at transfer, the allowable compressive stress in a member due to flexure is given by MSJC Eq. 2-18 as

$$F_{bi} = \frac{1.2 f'_{mi}}{3} = \frac{(1.2)\left(2000 \, \frac{\text{lbf}}{\text{in}^2}\right)}{3}$$
$$= 800 \text{ lbf/in}^2$$

Masonry

The compressive stress due to combined axial load and flexure must be checked using IBC Eq. 16-12, which is

$$Q = D + 0.6W$$

The total axial load at midheight of the wall at transfer is

$$\begin{aligned} P_i &= P_w + P_{psi} \\ &= 370 \text{ lbf} + (8.32 \text{ kips})\left(1000 \; \frac{\text{lbf}}{\text{kip}}\right) \\ &= 8690 \text{ lbf} \end{aligned}$$

The compressive stress produced in the wall by the wall self-weight and the effective prestressing force after initial losses at transfer is

$$\begin{aligned} f_{ai} &= \frac{P}{A_n} = \frac{8690 \text{ lbf}}{30 \text{ in}^2} \\ &= 290 \text{ lbf/in}^2 \end{aligned}$$

Assuming the full lateral load occurs at transfer, the bending moment acting on the wall is

$$\begin{aligned} M &= \frac{0.6wh^2}{8} = \frac{(0.6)\left(25 \; \frac{\text{lbf}}{\text{ft}}\right)(20 \text{ ft})^2}{8} \\ &= 750 \text{ ft-lbf} \end{aligned}$$

The flexural stress produced in the wall by applied lateral load is

$$\begin{aligned} f_{bi} &= \frac{M}{S_n} \\ &= \frac{(750 \text{ ft-lbf})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{81 \text{ in}^3} \\ &= 111 \text{ lbf/in}^2 \end{aligned}$$

The combined stresses must satisfy the interaction equation MSJC Eq. 2-14, which is

$$\begin{aligned} \frac{f_{ai}}{F_{ai}} + \frac{f_{bi}}{F_{bi}} &\leq 1.0 \\ \frac{290 \; \frac{\text{lbf}}{\text{in}^2}}{429 \; \frac{\text{lbf}}{\text{in}^2}} + \frac{111 \; \frac{\text{lbf}}{\text{in}^2}}{800 \; \frac{\text{lbf}}{\text{in}^2}} &= 0.82 \\ &\leq 1.0 \quad \text{[satisfactory]} \end{aligned}$$

Tensile stress check

Per MSJC Table 2.2.3.2, the allowable tensile stress for type S portland cement/lime mortar due to flexure is

$$F_t = 33 \text{ lbf/in}^2$$

The tensile stress due to combined axial load and flexure must be checked using IBC Eq. 16-15, which is

$$\begin{aligned} Q &= 0.6D + 0.6W \\ P_i &= 0.6P_w + P_{psi} \\ &= (0.6)(370 \text{ lbf}) \\ &\quad + (8.32 \text{ kips})\left(1000 \; \frac{\text{lbf}}{\text{kip}}\right) \\ &= 8542 \text{ lbf} \end{aligned}$$

The compressive stress produced in the wall by the wall self-weight and the effective prestressing force after initial losses at transfer is

$$\begin{aligned} f_{ai} &= \frac{P}{A_n} \\ &= \frac{8542 \text{ lbf}}{30 \text{ in}^2} \\ &= 285 \text{ lbf/in}^2 \end{aligned}$$

The combined stresses must satisfy MSJC Sec. 2.2.3.2, which requires

$$\begin{aligned} f_{bi} - f_{ai} &\leq F_t \\ 111 \; \frac{\text{lbf}}{\text{in}^2} - 285 \; \frac{\text{lbf}}{\text{in}^2} &= -174 \; \frac{\text{lbf}}{\text{in}^2} \\ &< 33 \text{ lbf/in}^2 \quad \text{[satisfactory]} \end{aligned}$$

Stability check

The modulus of elasticity of the concrete masonry at transfer is given by MSJC Sec. 1.8.2.2.1 as

$$\begin{aligned} E_{mi} &= 900f'_{mi} = (900)\left(\frac{2000 \; \frac{\text{lbf}}{\text{in}^2}}{1000 \; \frac{\text{lbf}}{\text{kip}}}\right) \\ &= 1800 \text{ kips/in}^2 \end{aligned}$$

For laterally restrained tendons, the Euler critical load is given by MSJC Eq. 2-19 as

$$\begin{aligned} P_{ei} &= \frac{\pi^2 E_{mi} I_n \left(1 - \frac{0.577e}{r}\right)^3}{h^2} \\ &= \frac{\pi^2 \left(1800 \; \frac{\text{kips}}{\text{in}^2}\right)(309 \text{ in}^4)\left(1 - \frac{(0.577)(0 \text{ in})}{3.21 \text{ in}}\right)^3}{\left((20 \text{ ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right)\right)^2} \\ &= 95 \text{ kips} \end{aligned}$$

To ensure elastic stability, the axial load on a wall, not including prestressing force, is limited by MSJC Eq. 2-15 and MSJC Sec. 4.4.1.2 to a maximum value of

$$P_i = \frac{1.2P_{ei}}{4}$$
$$= \frac{(1.2)(95 \text{ kips})}{4}$$
$$= 28.5 \text{ kips}$$
$$> P_w \quad \text{[satisfactory]}$$

The wall is adequate at transfer.

Serviceability Design Stage

At the serviceability design stage, all time-dependent prestress losses have occurred as a result of creep and shrinkage of the masonry and relaxation of the tendons. The total loss of prestress after long-term service for concrete masonry is given by MSJC Comm. Sec. 4.3.4 as 30% to 35%.

The design procedure for the serviceability design stage is similar to that used for the transfer design stage.

Example 6.21

The nominal 8 in ungrouted concrete block masonry wall described in Ex. 6.20 has face-shell mortar bedding of type S portland cement/lime mortar, a specified strength of $f'_m = 2500 \text{ lbf/in}^2$ at 28 days, and a specified strength of $f'_{mi} = 2000 \text{ lbf/in}^2$ at transfer. The wall is post-tensioned with $^7/_{16}$ in diameter steel rods at 16 in centers placed centrally in the wall, and the rods are laterally restrained. The wall has an effective height of $h = 20$ ft and is simply supported at the top and bottom. The roof live load is 200 lbf/ft, and the roof dead load is 400 lbf/ft applied to the wall without eccentricity. The wall has a weight of $\gamma_w = 37 \text{ lbf/ft}^2$. The properties of the steel rod are $A_{ps} = 0.142 \text{ in}^2$, $f_{py} = 100 \text{ kips/in}^2$, $f_{pu} = 122 \text{ kips/in}^2$. Wind load governs and has a value of $w = 25 \text{ lbf/ft}^2$. Assume the loss of prestress at transfer is 5% and the final loss of prestress is 30%. Determine if the wall under service level loads is within the permissible limits. Ignore P-delta effects.

Solution

The following calculations are based on a 1 ft length of wall. The relevant details from Ex. 6.20 are

$$A_n = 30 \text{ in}^2$$
$$I_n = 309 \text{ in}^4$$
$$S_n = 81 \text{ in}^3$$
$$r = 3.21 \text{ in}$$

The weight of the wall above midheight is

$$P_w = 370 \text{ lbf}$$

The roof dead load is

$$P_d = 400 \text{ lbf}$$

Roof live load is not included in IBC Eq. 16-12 and Eq. 16-15, so

$$P_f = P_d + P_r$$
$$= 400 \text{ lbf} + 0 \text{ lbf}$$
$$= 400 \text{ lbf}$$

Prestress details

The initial jacking force per foot of wall is

$$P_{psj} = 8.76 \text{ kips}$$

After 30% total losses, the prestressing tendon force per foot of wall is

$$P_{ps} = 0.70P_{psj} = (0.70)(8.76 \text{ kips})$$
$$= 6.13 \text{ kips}$$

Compressive stress check

The slenderness ratio of the wall is

$$\frac{h}{r} = 74.8$$

The allowable compressive stress in a member having an h/r ratio not greater than 99 and subjected to combined axial load and flexure is given by MSJC Eq. 2-16 as

$$F_a = \left(0.25f'_m\right)\left(1 - \left(\frac{h}{140r}\right)^2\right)$$
$$= (0.25)\left(2500 \ \frac{\text{lbf}}{\text{in}^2}\right)\left(1 - \left(\frac{74.8}{140}\right)^2\right)$$
$$= 447 \text{ lbf/in}^2$$

The allowable compressive stress in a member due to flexure is given by MSJC Eq. 2-18 as

$$F_b = \frac{f'_m}{3}$$
$$= \frac{2500 \ \dfrac{\text{lbf}}{\text{in}^2}}{3}$$
$$= 833 \text{ lbf/in}^2$$

The compressive stress due to combined axial load and flexure must be checked using IBC Eq. 16-12, which is

$$Q = D + 0.6W$$

The total axial load at midheight of the wall under service loads is

$$P = P_f + P_w + P_{ps}$$
$$= 400 \text{ lbf} + 370 \text{ lbf} + (6.13 \text{ kips})\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)$$
$$= 6900 \text{ lbf}$$

The compressive stress produced in the wall by the wall self-weight, roof dead load, and effective prestressing force after all losses is

$$f_a = \frac{P}{A_n}$$
$$= \frac{6900 \text{ lbf}}{30 \text{ in}^2}$$
$$= 230 \text{ lbf/in}^2$$

The bending moment produced on the wall by the lateral wind load is

$$M = \frac{0.6wh^2}{8}$$
$$= \frac{(0.6)\left(25 \ \frac{\text{lbf}}{\text{ft}}\right)(20 \text{ ft})^2}{8}$$
$$= 750 \text{ ft-lbf}$$

The flexural stress produced in the wall by applied lateral load is

$$f_b = \frac{M}{S_n}$$
$$= \frac{(750 \text{ ft-lbf})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{81 \text{ in}^3}$$
$$= 111 \text{ lbf/in}^2$$

The combined stresses must satisfy the interaction equation MSJC Eq. 2-14, which is

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0$$
$$\frac{230 \ \frac{\text{lbf}}{\text{in}^2}}{447 \ \frac{\text{lbf}}{\text{in}^2}} + \frac{111 \ \frac{\text{lbf}}{\text{in}^2}}{833 \ \frac{\text{lbf}}{\text{in}^2}} = 0.65$$
$$\leq 1.0 \quad \text{[satisfactory]}$$

Tensile stress check

The allowable tensile stress for type S portland cement/lime mortar due to flexure is given by MSJC Table 2.2.3.2 as

$$F_t = 33 \text{ lbf/in}^2$$

The tensile stress due to combined axial load and flexure must be checked using IBC Eq. 16-15, which is

$$Q = 0.6D + 0.6W$$
$$P = 0.6(P_f + P_w) + P_{ps}$$
$$= (0.6)(400 \text{ lbf} + 370 \text{ lbf}) + (6.13 \text{ kips})\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)$$
$$= 6592 \text{ lbf}$$

The compressive stress produced in the wall by the wall self-weight and the effective prestressing force after total losses at transfer is

$$f_a = \frac{P}{A_n}$$
$$= \frac{6592 \text{ lbf}}{30 \text{ in}^2}$$
$$= 220 \text{ lbf/in}^2$$

The combined stresses must satisfy MSJC Sec. 2.2.3.2, which is

$$f_b - f_a \leq F_t$$
$$111 \ \frac{\text{lbf}}{\text{in}^2} - 220 \ \frac{\text{lbf}}{\text{in}^2} = -109 \text{ lbf/in}^2$$
$$< 33 \text{ lbf/in}^2 \quad \text{[satisfactory]}$$

Stability check

The modulus of elasticity of the concrete masonry at the service load stage is given by MSJC Sec. 1.8.2.2.1 as

$$E_m = 900f'_m$$
$$= (900)\left(\frac{2500 \ \frac{\text{lbf}}{\text{in}^2}}{1000 \ \frac{\text{lbf}}{\text{kip}}}\right)$$
$$= 2250 \text{ kips/in}^2$$

For laterally restrained tendons, the Euler critical load is given by MSJC Eq. 2-19 as

$$P_e = \frac{\pi^2 E_m I_n \left(1 - \frac{0.577e}{r}\right)^3}{h^2}$$
$$= \frac{\pi^2 \left(2250 \ \frac{\text{kips}}{\text{in}^2}\right)(309 \text{ in}^4)\left(1 - \frac{(0.577)(0 \text{ in})}{3.21 \text{ in}}\right)^3}{\left((20 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\right)^2}$$
$$= 119 \text{ kips}$$

To ensure elastic stability, the axial load on a wall, not including prestressing force, is limited by MSJC Eq. 2-15 to a maximum value of

$$P = \frac{P_e}{4}$$
$$= \frac{119 \text{ kips}}{4}$$
$$= 30 \text{ kips}$$
$$> P_f + P_w \quad \text{[satisfactory]}$$

The wall is adequate under service level loads.

Strength Design Stage

The strength design method, illustrated in Fig. 6.25 for a prestressed concrete masonry wall, is used to calculate the design strength of the wall under the action of the applied factored loads so as to ensure that the design strength is not less than the most critical load combination. Figure 6.25 shows the face shell of the concrete masonry unit and the centrally placed prestressing tendon. The forces and stresses developed at ultimate load are indicated.

Figure 6.25 *Strength Design of Prestressed Masonry*

section   stress and force
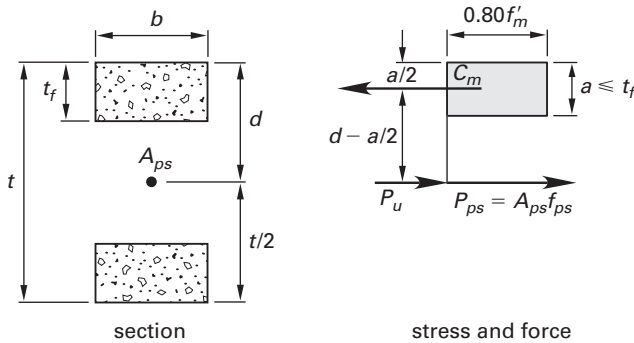
For a centrally located, laterally restrained or unrestrained, unbonded prestressing tendon, the stress in the tendon at nominal load has been determined empirically, and it is given by MSJC Eq. 4-3 as

$$f_{ps} = f_{se} + 0.03 \left(\frac{E_{ps} d}{l_p} \right) \left(1 - 1.56 \left(\frac{A_{ps} f_{ps} + P}{f'_m b d} \right) \right)$$
$$\geq f_{se}$$
$$\leq f_{py}$$

The force in the prestressing tendon at nominal load is

$$P_{ps} = f_{ps} A_{ps}$$

A rectangular compression stress block is assumed in the face shell on the compression face of the wall with a magnitude of $0.80f'_m$ and a depth of a. The compression

force on the stress block acts at mid-depth of the stress block and is given by

$$C_m = 0.80 a b f'_m$$

Stress in the face shell on the tension face of the wall is ignored. Allowing for the factored axial load, P_u, on the wall, the depth of the stress block is obtained by equating forces on the section to give

$$a = \frac{f_{ps} A_{ps} + P_u}{0.80 f'_m b}$$

When the wall also contains concentrically placed, bonded, nonprestressed reinforcement in grouted cores, the depth of the stress block is given by MSJC Eq. 4-1 as

$$a = \frac{f_{ps} A_{ps} + f_y A_s + P_u}{0.80 f'_m b}$$

The nominal flexural capacity of the section is obtained by taking moments about the line of action of the compression force to give MSJC Eq. 4-2 which is

$$M_n = \left(f_{ps} A_{ps} + f_y A_s + P_u \right) \left(d - \frac{a}{2} \right)$$

The design flexural capacity is given by MSJC Sec. 4.4.3.3 as

$$\phi M_n = 0.8 M_n$$

To ensure a ductile failure, MSJC Sec. 4.4.3.6 specifies a maximum depth for the stress block of

$$a = 0.38d$$

In addition, the depth of the stress block may not exceed the thickness of the face shell.

Example 6.22

The nominal 8 in ungrouted concrete block masonry wall described in Ex. 6.20 has face-shell mortar bedding of type S portland cement/lime mortar, a specified strength of $f'_m = 2500 \text{ lbf/in}^2$ at 28 days, and a specified strength of $f'_{mi} = 2000 \text{ lbf/in}^2$ at transfer. The wall is post-tensioned with $^7/_{16}$ in diameter steel rods at 16 in centers placed centrally in the wall, and the rods are laterally restrained. The wall has an effective height of $h = 20 \text{ ft}$ and is simply supported at the top and bottom. The roof live load is 200 lbf/ft, and the roof dead load is 400 lbf/ft applied to the wall without eccentricity. The masonry wall has a weight of $\gamma_w = 37 \text{ lbf/in}^2$. The properties of the steel rod are $E_{ps} = 29{,}000 \text{ kips/in}^2$, $A_{ps} = 0.142 \text{ in}^2$, $f_{py} = 100 \text{ kips/in}^2$, $f_{pu} = 122 \text{ kips/in}^2$. Wind load governs and has a value of $w = 25 \text{ lbf/ft}^2$. Assume the loss of prestress at transferis 5% and the final loss of prestress is 30%. Determine if the wall design flexural capacity is adequate. Ignore P-delta effects.

Solution

The following calculations are based on a 1 ft length of wall and on information obtained from Ex. 6.20 and Ex. 6.21.

The weight of the wall above midheight plus the roof dead load is

$$P = 770 \text{ lbf}$$

In determining the moment demand on the wall, the governing load combination is IBC Eq. 16-6, which is

$$Q_u = 0.9D + 1.0W$$

The factored axial load on the wall is

$$
\begin{aligned}
P_u &= 0.9P \\
&= (0.9)(770 \text{ lbf}) \\
&= 693 \text{ lbf}
\end{aligned}
$$

The bending moment produced on the wall by the factored lateral wind load is

$$
\begin{aligned}
M_u &= \frac{wh^2}{8} \\
&= \frac{\left(25 \dfrac{\text{lbf}}{\text{ft}}\right)(20 \text{ ft})^2}{8} \\
&= 1250 \text{ ft-lbf}
\end{aligned}
$$

The equivalent area of prestressing tendon per foot of wall is

$$A_{ps} = 0.107 \text{ in}^2$$

After 30% total losses, the prestressing tendon force per foot of wall is

$$P_{ps} = 6.13 \text{ kips}$$

After 30% total losses, the prestressing tendon stress per foot of wall is

$$
\begin{aligned}
f_{se} &= \frac{P_{ps}}{A_{ps}} \\
&= \frac{6.13 \text{ kips}}{0.107 \text{ in}^2} \\
&= 57.3 \text{ kips/in}^2
\end{aligned}
$$

The stress in the tendon at nominal load is given by MSJC Eq. 4-3 as

$$
f_{ps} = f_{se} + 0.03\left(\frac{E_{ps}d}{l_p}\right)\left(1 - 1.56\left(\frac{A_{ps}f_{ps} + P}{f'_m bd}\right)\right)
$$

$$
\begin{aligned}
&= \left(57.3 \frac{\text{kips}}{\text{in}^2}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right) \\
&\quad + (0.03)\left(\frac{\left(29,000,000 \dfrac{\text{lbf}}{\text{in}^2}\right)\left(\dfrac{7.63 \text{ in}}{2}\right)}{(20 \text{ ft})\left(12 \dfrac{\text{in}}{\text{ft}}\right)}\right) \\
&\quad \times \left(1 - (1.56)\left(\frac{(0.107 \text{ in}^2)f_{ps} + 770 \text{ lbf}}{\left(2500 \dfrac{\text{lbf}}{\text{in}^2}\right)(12 \text{ in})} \right.\right. \\
&\qquad\qquad\qquad\qquad \left.\left. \times \left(\dfrac{7.63 \text{ in}}{2}\right)\right)\right) \\
&= 63,783 \text{ lbf/in}^2 \\
&> f_{se} \\
&< f_{py}
\end{aligned}
$$

The depth of the stress block is given by MSJC Eq. 4-1 as

$$
\begin{aligned}
a &= \frac{f_{ps}A_{ps} + f_y A_s + P_u}{0.80f'_m b} \\
&= \frac{\left(63,783 \dfrac{\text{lbf}}{\text{in}^2}\right)(0.107 \text{ in}^2) + \left(0 \dfrac{\text{lbf}}{\text{in}^2}\right)(0 \text{ in}^2) + 693 \text{ lbf}}{(0.80)\left(2500 \dfrac{\text{lbf}}{\text{in}^2}\right)(12 \text{ in})}
\end{aligned}
$$

$$
\begin{aligned}
&= 0.31 \text{ in} \quad \text{[satisfactory]} \\
&< t_f/2 \\
&< 0.38d \quad \text{[satisfies MSJC Sec. 4.4.3.7.2]}
\end{aligned}
$$

The nominal moment is given by MSJC Eq. 4-2 as

$$
M_n = \left(f_{ps}A_{ps} + f_y A_s + P_u\right)\left(d - \frac{a}{2}\right)
$$

$$
\begin{aligned}
&= \frac{\left(\begin{array}{l}\left(63,783 \dfrac{\text{lbf}}{\text{in}^2}\right)(0.107 \text{ in}^2) \\ \quad + \left(0 \dfrac{\text{lbf}}{\text{in}^2}\right)(0 \text{ in}^2) + 693 \text{ lbf}\end{array}\right)}{12 \dfrac{\text{in}}{\text{ft}}} \\
&\qquad \times \left(\dfrac{7.63 \text{ in}}{2} - \dfrac{0.31 \text{ in}}{2}\right) \\
&= 2291 \text{ ft-lbf}
\end{aligned}
$$

The design flexural strength for a prestressed member is given by MSJC Sec. 4.4.3.3 as

$$\phi M_n = 0.8 M_n$$
$$= (0.8)(2291 \text{ ft-lbf})$$
$$= 1833 \text{ ft-lbf}$$
$$> M_u \quad [\text{satisfactory}]$$

The design moment strength is adequate.

11. QUALITY ASSURANCE, TESTING, AND INSPECTION

A quality assurance program is used to ensure that the constructed masonry is in compliance with the construction documents. The quality assurance plan specifies the required level of quality for the work, and testing and inspection determines the acceptability of the final construction. For masonry construction, these issues are addressed in IBC Sec. 110, Sec. 1704.5, Sec. 1705.4, and Sec. 2105, as well as MSJC Sec. 1.12.

A quality assurance plan, in conformance with IBC requirements, is developed by the engineer of record and incorporated into the contract documents. An approved testing agency is appointed to sample and test the materials used on the project and to determine compliance with the contract documents. The test results are reported to the engineer of record and to the contractor. An approved inspection agency is appointed to perform the inspection and evaluation of the construction as required by the contract documents. The inspection reports are provided to the engineer of record and to the contractor. Both the testing agency and the inspection agency are required to draw any observed deficiencies to the attention of the engineer of record, the building official, and the contractor.

Types of Inspections

Inspections may be of two types, periodic or continuous. *Periodic inspection* is the part-time or intermittent observation of work requiring periodic inspection by an inspector who is present at selected stages of construction and at the completion of the work. *Continuous inspection* is the full-time observation of work requiring continuous inspection by an inspector who is continuously present in the area where the work is being performed. In addition, inspections may be classified as standard and special. Standard inspections are the basic inspection requirements specified in IBC Sec. 110 and are applicable to all projects. *Special inspections* are required by IBC Sec. 1704.2 for the installation of critical components and connections. These inspections are performed by special inspectors, with special expertise in the specific work involved, to ensure compliance with the contract documents.

Level of Inspection

The level of inspection required by the IBC depends on the design process used for the project and on the occupancy category assigned to the completed structure. The project may be designed using either an empirical approach or by an engineered design procedure. Engineered masonry comprises projects designed by allowable stress design, strength design, and prestressed design as specified in Chap. 1 through Chap. 4 in the MSJC. Empirically designed masonry is specified in Chap. 5 of the MSJC. Also included in this category is veneer and glass unit masonry, as specified in MSJC Chap. 6 and Chap. 7. Empirical design is, of necessity, more conservative than engineered design.

Risk Category

The *risk category* is a designation used to determine the structural requirements of a building based on the nature of its occupancy. IBC Table 1604.5 lists risk categories, and an abbreviated listing is given in Table 6.3.

- Category I structures are facilities that represent a low hazard to human life in the event of failure. These include agricultural facilities, temporary facilities, and minor storage facilities.

- Category II structures are all facilities except those listed in risk categories I, III, and IV.

- Category III structures are facilities that represent a substantial hazard to human life in the event of failure because of their high occupant load. These include facilities where more than 300 people congregate in one area, school or daycare facilities with a capacity exceeding 250, colleges and universities with a capacity exceeding 500, healthcare facilities with a capacity of 50 or more resident patients but not having surgery or emergency treatment facilities, correctional centers, power stations, and water treatment facilities. Also included are facilities not included in risk category IV containing sufficient quantities of toxic or explosive substances to be dangerous to the public if released.

- Category IV structures are facilities housing essential equipment that is required for post-disaster recovery. These include healthcare facilities with a capacity of 50 or more resident patients requiring surgery or emergency treatment; fire, rescue, ambulance and police stations; emergency vehicle garages; emergency shelters; emergency response centers; air-traffic control centers; structures having critical national defense functions; water storage facilities and pump structures required to maintain water pressure for fire suppression; and buildings housing equipment and utilities required as emergency backup facilities for risk category IV structures. Also included are facilities containing highly toxic materials where the quantity of the material exceeds the maximum allowable quantities of IBC Table 307.1(2).

Table 6.3 *Risk Category of Buildings*

occupancy category	nature of occupancy
I	low hazard structures
II	standard occupancy structures
III	assembly structures
IV	essential structures or structures housing hazardous materials

Table 6.4 *Masonry Quality Assurance Requirements*

risk category	empirically designed masonry	glass unit masonry or masonry veneer	engineered design of masonry
I, II, and III	level A quality assurance	level A quality assurance	level B quality assurance
IV	not permitted	level B quality assurance	level C quality assurance

In accordance with IBC Sec. 1705.4, special inspections are not required for empirically designed masonry, glass unit masonry, or masonry veneer designed by Chap. 5, Chap. 6, or Chap. 7 of the MSJC, when they are part of structures classified as risk category I, II, or III. Also exempted are masonry foundation walls, chimneys, and fireplaces conforming to the prescriptive design requirements of the MSJC. The basic inspection requirements for these projects are specified in IBC Sec. 110 and are applicable on these and on all masonry projects.

As specified by IBC Sec. 110, a preliminary inspection of the proposed project site may be required by the building official prior to issuing a permit. The basic inspections applicable to masonry construction are given by IBC Sec. 110.3. These include the following.

- Footing and foundation inspections must be made after excavations for footings are complete and any required reinforcing steel is in place.

- For concrete foundations, all required forms must be in place prior to inspection, and materials for the foundation shall be on the job, except where concrete is ready mixed.

- Concrete slab and under-floor inspections must be made after in-slab or under-floor reinforcing steel and building service equipment items are in place, but before any concrete is placed.

- In addition to the inspections already specified, the building official is authorized to require other inspections of any construction work to ascertain compliance with the code.

- The final inspection must be made after completion of all work required by the building permit.

For empirically designed masonry, on-site testing is generally not required. Certificates for the materials used in the work must be provided to show compliance with the contract documents.

In accordance with IBC Sec. 1705.4 and MSJC Sec. 1.19, there are three levels of quality assurance required for masonry. *Level A quality assurance* requires verification of compliance with approved submittals. *Level B quality assurance* mainly requires periodic inspections. *Level C quality assurance* is similar to level B quality assurance with more items requiring continuous inspection. Table 6.4 indicates the applicability of the different levels of quality assurance for types of masonry.

Level A quality assurance requirements are detailed in MSJC Table 1.19.1, level B quality assurance requirements are detailed in MSJC Table 1.19.2, and level C quality assurance requirements are detailed in MSJC Table 1.19.3.

Structural Observation

Structural observation is required by IBC Sec. 1704.5 for some structures assigned to seismic design category D, E, or F, or in an area where the basic wind speed exceeds 110 mph. Structural observation consists of the visual observation of the structural system, usually by the engineer of record, to ensure general conformance to the contract documents. Structural observation, when required, is additional to normal inspection procedures. IBC Sec. 1704.5 requires structural observation for a structure assigned to seismic design category D, E, or F when

- the structure is classified as risk category III or IV

- the height of the structure is greater than 75 ft

- the structure is assigned to seismic design category E, is classified as risk category I or II, and is greater than two stories above grade plane

- such observation is stipulated by the registered design professional responsible for the structural design

- such observation is specifically required by the building official

IBC Sec. 1704.5.2 requires structural observation for structures located where the basic wind speed exceeds 110 mph when

- the structure is classified as risk category III or IV

- the height of the structure is greater than 75 ft

- such observation is stipulated by the registered design professional responsible for the structural design

- such observation is specifically required by the building official

Masonry

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Masonry

PRACTICE PROBLEMS

(Answer options for SD are given in parentheses.)

1. The nominal 8 in solid grouted concrete block masonry bearing wall shown in the following illustration has a specified strength of 1500 lbf/in^2 and a modulus of elasticity of 1,000,000 lbf/in^2. The wall supports an axial load, including its own weight, as shown in the illustration. The wall, has a height of 15 ft, and may be considered pinned at the top and bottom. Ignore accidental eccentricity. The wall is not part of the lateral-force resisting system. Determine if the wall is adequate. What is most nearly the minimum required vertical reinforcement in the wall?

P_u = 40 kips/ft (SD)
P = 20 kips/ft (ASD)

8 in CMU wall

h = 15 ft
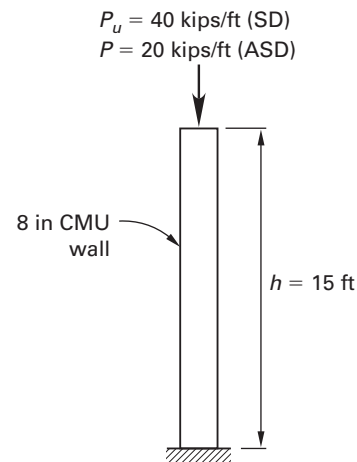
(A) no. 4 at 48 in on center

(B) no. 4 at 72 in on center

(C) no. 4 at 96 in on center

(D) no. 4 at 120 in on center

2. The nominal 8 in solid grouted concrete block masonry retaining wall shown in the following illustration has a specified strength of 1500 lbf/in² and a modulus of elasticity of 1,000,000 lbf/in². The reinforcement consists of no. 4 grade 60 bars at 16 in centers. The wall retains a soil with an equivalent fluid pressure of 30 lbf/ft²/ft, and the self-weight of the wall may be neglected. Is the wall adequate?
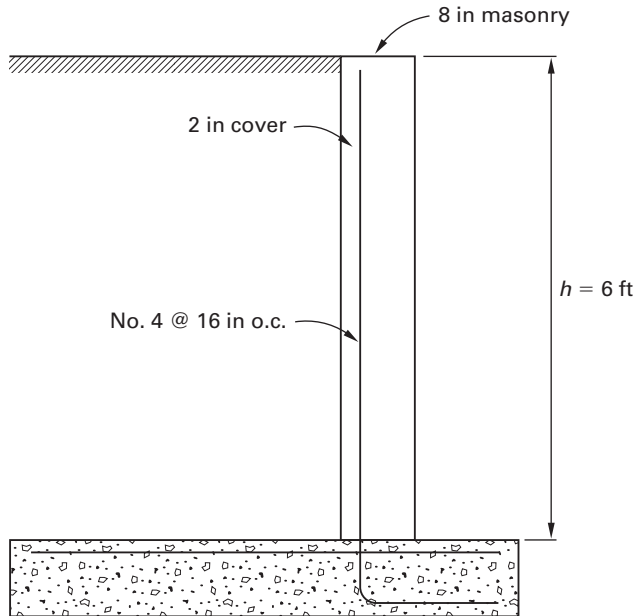
(A) No, $f_b = 272$ lbf/in², $f_s = 25,000$ lbf/in².
    (No, $\phi M_n = 4100$ ft-lbf.)

(B) No, $f_b = 300$ lbf/in², $f_s = 18,000$ lbf/in².
    (No, $\phi M_n = 3500$ ft-lbf.)

(C) Yes, $f_b = 300$ lbf/in², $f_s = 24,000$ lbf/in².
    (Yes, $\phi M_n = 4000$ ft-lbf.)

(D) Yes, $f_b = 272$ lbf/in², $f_s = 17,880$ lbf/in².
    (Yes, $\phi M_n = 3400$ ft-lbf.)

3. The 8 in solid grouted concrete block masonry beam shown in the following illustration may be considered simply supported over an effective span of 15 ft. The masonry has a compressive strength of 1500 lbf/in² and a modulus of elasticity of 1,000,000 lbf/in², and the reinforcement consists of two no. 7 grade 60 bars. The effective depth is 36 in, the overall depth is 40 in, and the beam is laterally braced at both ends. Is the beam adequate in flexure to support a uniformly distributed load, including its own weight, of 2000 lbf/ft?
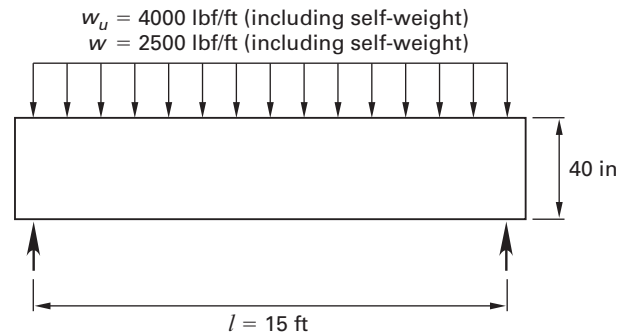
(A) No, $f_b = 250$ lbf/in², $f_s = 20,150$ lbf/in².
    (No, $\phi M_n = 200$ ft-kips.)

(B) No, $f_b = 425$ lbf/in², $f_s = 16,870$ lbf/in².
    (No, $\phi M_n = 150$ ft-kips.)

(C) Yes, $f_b = 500$ lbf/in², $f_s = 22,470$ lbf/in².
    (Yes, $\phi M_n = 170$ ft-kips.)

(D) Yes, $f_b = 250$ lbf/in², $f_s = 15,975$ lbf/in².
    (Yes, $\phi M_n = 130$ ft-kips.)

4. For the masonry beam of Prob. 3, the shear reinforcement required at each support is most nearly

(A) 0.05 in²/ft (0.113 in²/ft)

(B) 0.06 in²/ft (0.128 in²/ft)

(C) 0.07 in²/ft (0.156 in²/ft)

(D) 0.08 in²/ft (0.167 in²/ft)

SOLUTIONS

1. For axial loading, considering a 1 ft length of wall, the relevant parameters of the wall are

$$b = 12 \text{ in}$$

$$d_n = \text{nominal depth of wall}$$
$$= 7.63 \text{ in}$$

$$h = \text{effective column height}$$
$$= 15 \text{ ft}$$

$$A_s = \text{reinforcement area}$$
$$= 0.23 \text{ in}^2$$

$$A_n = \text{effective column area}$$
$$= bd_n$$
$$= (12 \text{ in})(7.63 \text{ in})$$
$$= 91.5 \text{ in}^2$$

$$\rho = \frac{A_s}{A_n}$$
$$= \frac{0.23 \text{ in}^2}{91.5 \text{ in}^2}$$
$$= 0.00251$$
$$< 0.04$$
$$> 0.0025 \quad \begin{bmatrix} \text{satisfies MSJC} \\ \text{Sec. 1.14.1} \end{bmatrix}$$

$$P = 20 \text{ kips}$$

The radius of gyration of the wall is

$$r = \sqrt{\frac{I_n}{A_n}}$$
$$= 0.289 d_n$$
$$= (0.289)(7.63 \text{ in})$$
$$= 2.21 \text{ in}$$

The slenderness ratio of the wall is

$$\frac{h}{r} = \frac{(15 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{2.21 \text{ in}}$$
$$= 81.4$$
$$< 99$$

ASD Method

Ignoring vertical reinforcement in conformity with MSJC Sec. 2.3.3.3, the allowable wall load is given by MSJC Eq. 2-16 as

$$P_a = (0.25 f'_m A_n)\left(1.0 - \left(\frac{h}{140r}\right)^2\right)$$

$$= (0.25)\left(\frac{1500 \ \frac{\text{lbf}}{\text{in}^2}}{1000 \ \frac{\text{lbf}}{\text{kip}}}\right)(91.5 \text{ in}^2)\left(1.0 - \left(\frac{81.4}{140}\right)^2\right)$$

$$= 22.7 \text{ kips}$$
$$> P \quad [\text{satisfactory}]$$

The wall is adequate for axial loading, and in accordance with MSJC Sec. 1.18.4.3.1, the necessary minimum vertical reinforcement is no. 4 bars at 120 in centers. The required reinforcement details are shown in the following illustration.
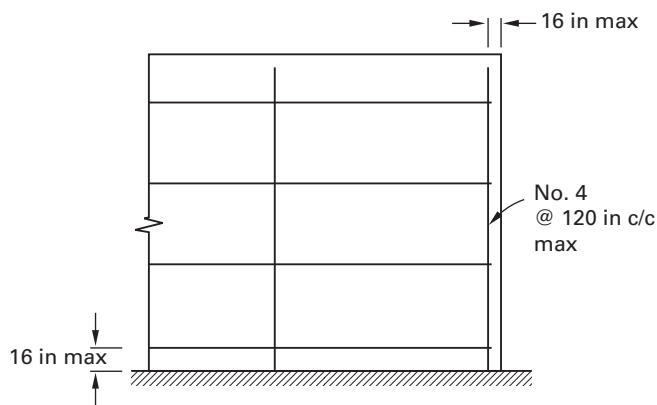
The answer is (D).

SD Method

The slenderness ratio of the wall is

$$\frac{h}{r} = 81.4$$
$$< 99 \quad [\text{MSJC Eq. 3-11 is applicable}]$$

Masonry

The nominal axial strength, as given in MSJC Eq. 3-11, is

$$P_n = 0.80\left(0.80A_nf'_m\left(1-\left(\frac{h}{140r}\right)^2\right)\right)$$

$$= (0.80)(0.80)(91.5 \text{ in}^2)\left(\frac{1500 \ \frac{\text{lbf}}{\text{in}^2}}{1000 \ \frac{\text{lbf}}{\text{kip}}}\right)$$

$$\times \left(1-\left(\frac{81.4}{140}\right)^2\right)$$

$$= 58.1 \text{ kips}$$

The design wall strength is

$$\phi P_n = (0.9)(58.1 \text{ kips})$$

$$= 52.3 \text{ kips}$$

$$> P \quad [\text{satisfactory}]$$

The wall is adequate for axial loading and, in accordance with MSJC Sec. 1.18.4.3.1, the necessary minimum vertical reinforcement requires no. 4 bars at 120 in centers. The required reinforcement details are shown.
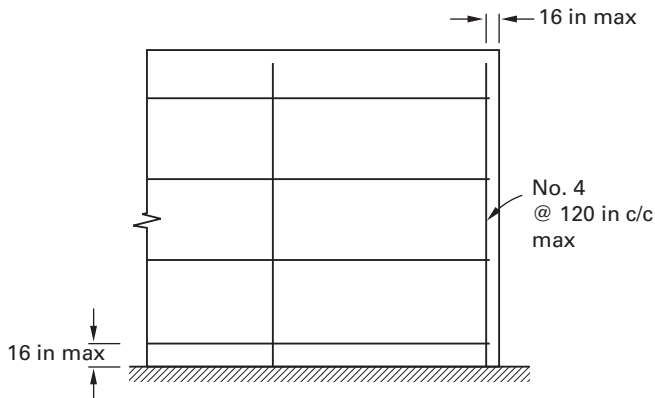
The answer is (D).

2. The allowable stresses, in accordance with MSJC Sec. 2.3.2, Sec. 2.3.3, and Sec. 2.3.4.2.2 are

$$F_b = 0.45f'_m$$

$$= (0.45)\left(1500 \ \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 675 \text{ lbf/in}^2$$

$$F_s = 24{,}000 \text{ lbf/in}^2$$

At the base of the wall, the bending moment produced in a 1 ft length of wall by the backfill is

$$M = \frac{qh^3}{6}$$

$$= \frac{\left(30 \ \frac{\text{lbf}}{\text{ft}^2}\right)(6 \text{ ft})^3}{6}$$

$$= 1080 \text{ ft-lbf}$$

With vertical reinforcement consisting of no. 4 bars at 16 in centers located with 2 in cover to the earth face, the relevant parameters of the wall are

$$b_w = 12 \text{ in}$$

$$d = 7.63 \text{ in} - 2 \text{ in} - 0.25 \text{ in}$$

$$= 5.38 \text{ in}$$

$$A_s = 0.15 \text{ in}^2$$

$$n = 29$$

$$\rho = \frac{A_s}{b_w d}$$

$$= \frac{0.15 \text{ in}^2}{(12 \text{ in})(5.38 \text{ in})}$$

$$= 0.00232$$

$$\rho n = (0.00232)(29)$$

$$= 0.0674$$

ASD Method

From App. B, the stresses, in accordance with MSJC Sec. 2.3.2, are

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$= \sqrt{(2)(0.0674) + (0.0674)^2} - 0.0674$$

$$= 0.306$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.306}{3}$$

$$= 0.898$$

$$f_b = \frac{2M}{jkb_w d^2}$$

$$= \frac{(2)(1080 \text{ ft-lbf})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{(0.898)(0.306)(12 \text{ in})(5.38 \text{ in})^2}$$

$$= 272 \text{ lbf/in}^2$$

$$< F_b \quad [\text{satisfactory}]$$

$$f_s = \frac{M}{jdA_s}$$

$$= \frac{(1080 \text{ ft-lbf})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(0.898)(5.38 \text{ in})(0.15 \text{ in}^2)}$$

$$= 17{,}880 \text{ lbf/in}^2$$

$$< F_s \quad [\text{satisfactory}]$$

The wall is adequate.

The answer is (D).

SD Method

The factored moment at the base of the wall is given by IBC Eq. 16-2 as

$$M_u = 1.6M$$

$$= (1.6)(1080 \text{ ft-lbf})$$

$$= 1728 \text{ ft-lbf}$$

The stress block depth is

$$a = \frac{A_s f_y}{0.80 b_w f'_m}$$

$$= \frac{(0.15 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)\left(1000 \frac{\text{lbf}}{\text{kips}}\right)}{(0.80)(12 \text{ in})\left(1500 \frac{\text{lbf}}{\text{in}^2}\right)}$$

$$= 0.63 \text{ in}$$

The nominal strength is

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$= \frac{(0.15 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)\left(5.38 \text{ in} - \frac{0.63 \text{ in}}{2}\right)}{12 \frac{\text{in}}{\text{ft}}} \times \left(1000 \frac{\text{lbf}}{\text{kips}}\right)$$

$$= 3799 \text{ ft-lbf}$$

The design strength is

$$\phi M_n = (0.9)(3799 \text{ ft-lbf})$$

$$= 3419 \text{ ft-kips} \quad (3400 \text{ ft-lbf})$$

$$> M_u \quad [\text{satisfactory}]$$

The wall is adequate.

The answer is (D).

3. The allowable stresses, in accordance with MSJC Sec. 2.3.2 and Sec. 2.3.4.2.2, are

$$F_b = 0.45 f'_m$$

$$= (0.45)\left(1500 \frac{\text{lbf}}{\text{in}^2}\right)$$

$$= 675 \text{ lbf/in}^2$$

$$F_s = 32{,}000 \text{ lbf/in}^2$$

The relevant parameters of the beam are

$$E_m = 1{,}000{,}000 \frac{\text{lbf}}{\text{in}^2}$$

$$E_s = 29{,}000{,}000 \frac{\text{lbf}}{\text{in}^2}$$

$$b = 7.63 \text{ in}$$

$$d = 36 \text{ in}$$

$$l = 15 \text{ ft}$$

$$A_s = 1.20 \text{ in}^2$$

$$\frac{l}{b} = \frac{(15 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{7.63 \text{ in}}$$

$$= 23.6$$

$$< 32 \quad \left[\begin{array}{c} \text{satisfies MSJC} \\ \text{Sec. 1.13.2} \end{array}\right]$$

$$n = \frac{E_s}{E_m}$$

$$= \frac{29{,}000{,}000 \frac{\text{lbf}}{\text{in}^2}}{1{,}000{,}000 \frac{\text{lbf}}{\text{in}^2}}$$

$$= 29$$

$$\rho = \frac{A_s}{bd}$$

$$= \frac{1.20 \text{ in}^2}{(7.63 \text{ in})(36 \text{ in})}$$

$$= 0.00437$$

$$\rho n = (0.00437)(29)$$

$$= 0.127$$

ASD Method

At midspan, the bending moment produced by the distributed load is

$$M = \frac{wl^2}{8} = \frac{\left(2500 \frac{\text{lbf}}{\text{ft}}\right)(15 \text{ ft})^2}{(8)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}$$

$$= 70.3 \text{ ft-kips}$$

Masonry

From App. B, the beam stresses, in accordance with MSJC Sec. 2.3, are

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$
$$= \sqrt{(2)(0.127) + (0.127)^2} - 0.127$$
$$= 0.393$$
$$j = 1 - \frac{k}{3} = 1 - \frac{0.393}{3}$$
$$= 0.869$$
$$f_b = \frac{2M}{jkbd^2}$$
$$= \frac{(2)(70.3 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(0.869)(0.393)(7.63 \text{ in})(36 \text{ in})^2}$$
$$= 500 \text{ lbf/in}^2$$
$$< F_b \quad [\text{satisfactory}]$$
$$f_s = \frac{M}{jdA_s}$$
$$= \frac{(70.3 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(0.869)(36 \text{ in})(1.20 \text{ in}^2)}$$
$$= 22{,}472 \text{ lbf/in}^2 \quad \left(22{,}470 \frac{\text{lbf}}{\text{in}^2}\right)$$
$$< F_s \quad [\text{satisfactory}]$$

The beam is adequate in flexure.

The answer is (C).

SD Method

The factored moment produced by the distributed load at midspan is

$$M_u = \frac{w_u l^2}{8} = \frac{\left(4000 \frac{\text{lbf}}{\text{ft}}\right)(15 \text{ ft})^2}{(8)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}$$
$$= 112.50 \text{ ft-kips}$$

The stress block depth is

$$a = \frac{A_s f_y}{0.80 b f'_m}$$
$$= \frac{(1.20 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(0.80)(7.63 \text{ in})\left(1500 \frac{\text{lbf}}{\text{in}^2}\right)}$$
$$= 7.86 \text{ in}$$

The nominal strength is

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$
$$= \frac{(1.20 \text{ in}^2)\left(60 \frac{\text{kips}}{\text{in}^2}\right)\left(36 \text{ in} - \frac{7.86 \text{ in}}{2}\right)}{12 \frac{\text{in}}{\text{ft}}}$$
$$= 192 \text{ ft-kips}$$

The design strength is

$$\phi M_n = (0.9)(192 \text{ ft-kips})$$
$$= 173 \text{ ft-kips} \quad (170 \text{ ft-kips})$$
$$> M_u \quad [\text{satisfactory}]$$

The beam is adequte in flexure.

The answer is (C).

4. *ASD Method*

The maximum permitted shear stress, assuming $M/(Vd) = 1$, is given by MSJC Eq. 2-27 as

$$F_v = 2\sqrt{f'_m} = 2\sqrt{1500 \frac{\text{lbf}}{\text{in}^2}}$$
$$= 77.5 \text{ lbf/in}^2$$

The shear force at a distance of $d/2$ from each support is given by

$$V = \frac{w(l - d)}{2}$$
$$= \frac{\left(2500 \frac{\text{lbf}}{\text{ft}}\right)\left(15 \text{ ft} - \left(\frac{36 \text{ in}}{12 \frac{\text{in}}{\text{ft}}}\right)\right)}{(2)\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}$$
$$= 15.0 \text{ kips}$$

The shear stress at a distance of $d/2$ from each support is given by MSJC Eq. 2-24 as

$$f_v = \frac{V}{A_{nv}}$$
$$= \frac{(15.0 \text{ kips})\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{(7.63 \text{ in})(36 \text{ in})}$$
$$= 54.6 \text{ lbf/in}^2$$
$$< 77.5 \text{ lbf/in}^2 \quad \begin{bmatrix} \text{satisfies MSJC} \\ \text{Sec. 2.3.6.1.2} \end{bmatrix}$$

Masonry

The allowable shear stress in a beam without shear reinforcement is given by MSJC Eq. 2-28. Since $P = 0$ lbf/in^2,

$$F_{vm} = \tfrac{1}{2}\left(\left(4.0 - 1.75\left(\frac{M}{Vd}\right)\right)\sqrt{f'_m}\right) + 0.25\left(\frac{P}{A_n}\right)$$

$$= \left(\tfrac{1}{2}\right)\left(\left(4.0 - (1.75)(1.0)\right)\sqrt{1500\ \frac{\text{lbf}}{\text{in}^2}}\right) + 0\ \frac{\text{lbf}}{\text{in}^2}$$

$$= 43.6\ \text{lbf/in}^2$$

$$< f_v = 54.6\ \text{lbf/in}^2 \quad \begin{bmatrix} \text{shear reinforcement} \\ \text{is required} \end{bmatrix}$$

The shear stress required from shear reinforcement is given by MSJC Eq. 2-25 as

$$F_{vs} = f_v - F_{vm}$$

$$= 54.6\ \frac{\text{lbf}}{\text{in}^2} - 43.6\ \frac{\text{lbf}}{\text{in}^2}$$

$$= 11.0\ \text{lbf/in}^2$$

The area of shear reinforcement required per foot is given by MSJC Eq. 2-30 as

$$A_v = \frac{2F_{vs}A_n s}{F_s d}$$

$$= \frac{(2)\left(11.0\ \frac{\text{lbf}}{\text{in}^2}\right)(7.63\ \text{in})(36\ \text{in})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{\left(32{,}000\ \frac{\text{lbf}}{\text{in}^2}\right)(36\ \text{in})}$$

$$= 0.063\ \text{in}^2/\text{ft} \quad (0.06\ \text{in}^2/\text{ft})$$

The answer is (B)

SD Method

The factored shear force at a distance, $d/2$, from each support is

$$V_u = \frac{w_u(l - d)}{2}$$

$$= \frac{\left(4000\ \frac{\text{lbf}}{\text{ft}}\right)\left(15\ \text{ft} - \left(\frac{36\ \text{in}}{12\ \frac{\text{in}}{\text{ft}}}\right)\right)}{(2)\left(1000\ \frac{\text{lbf}}{\text{kip}}\right)}$$

$$= 24\ \text{kips}$$

The maximum nominal shear capacity permitted, assuming $M_u/(V_u d_v) = 1.0$, is limited by MSJC Eq. 3-22 to

$$V_n \le 4A_{nv}\sqrt{f'_m} = \frac{(4)(7.63\ \text{in})(36\ \text{in})\sqrt{1500\ \frac{\text{lbf}}{\text{in}^2}}}{1000\ \frac{\text{lbf}}{\text{kip}}}$$

$$= 42.5\ \text{kips}$$

The maximum design shear capacity permitted is

$$\phi V_n = (0.8)(42.5\ \text{kips})$$

$$= 34.0\ \text{kips}$$

$$> V_u = 24\ \text{kips} \quad [\text{satisfactory}]$$

The nominal shear capacity of the beam without shear reinforcement is given by MSJC Eq. 3-23. Since $P_u = 0$ lbf,

$$V_{nm} = \left(4.0 - 1.75\left(\frac{M_u}{V_u d_v}\right)\right)A_{nv}\sqrt{f'_m} + 0.25P_u$$

$$= \frac{\left(4.0 - (1.75)(1.0)\right)(7.63\ \text{in})(36\ \text{in})}{\times\sqrt{1500\ \frac{\text{lbf}}{\text{in}^2}} + 0\ \text{lbf}}{1000\ \frac{\text{lbf}}{\text{kip}}}$$

$$= 23.9\ \text{kips}$$

$$\phi V_{nm} = (0.8)(23.9\ \text{kips})$$

$$= 19.1\ \text{kips}$$

$$< V_u = 24\ \text{kips} \quad \begin{bmatrix} \text{shear reinforcement} \\ \text{is required} \end{bmatrix}$$

The design shear capacity required from shear reinforcement is given by MSJC Eq. 2-25 as

$$\phi V_{ns} = V_u - \phi V_{nm}$$

$$= 24\ \text{kips} - 19.1\ \text{kips}$$

$$= 4.9\ \text{kips}$$

The area of shear reinforcement required per foot is given by MSJC Eq. 2-30 as

$$A_v = \frac{2\phi V_{ns} s}{\phi F_y d}$$

$$= \frac{(2)(4.9\ \text{kips})\left(12\ \frac{\text{in}}{\text{ft}}\right)}{(0.8)\left(32\ \frac{\text{kips}}{\text{in}^2}\right)(36\ \text{in})}$$

$$= 0.128\ \text{in}^2/\text{ft}$$

The answer is (B).

Masonry

7 Lateral Forces

PART 1: LATERAL FORCE-RESISTING SYSTEMS

Nomenclature

b_s	length of shear wall	ft
B	width of diaphragm	ft
C	compression chord force in diaphragm or shear wall	kips
C_o	shear capacity adjustment factor	–
d_{ij}	distribution factor at end i of member ij	–
E	modulus of elasticity	$kips/in^2$
h	height of shear wall	ft
I	moment of inertia	in^4
L	length of diaphragm, shear wall or wall segment, or member	ft
M_D	bending moment in a diaphragm	ft-kips
s_{ij}	stiffness factor at end i of member ij	–
t	uniform uplift force in perforated shear wall	lbf/ft
T	tensile chord force in diaphragm or shear wall, or hold-down force on shear wall	kips
v	induced unit shear	lbf/ft
V	shear force	lbf
w	distributed load	lbf/ft

Symbols

$\sum L_i$	sum of perforated shear wall segment lengths	ft
ϕ_D	resistance factor	–

1. INTRODUCTION

The IBC[1] adopts by reference the American Society of Civil Engineers' *Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI7[2]) for many of its code requirements. However, ASCE/SEI7 Chap. 14 and ASCE/SEI7 App. 11A are not adopted, and ASCE/SEI7 Sec. 17.5.4.2, which covers seismically isolated structures, is modified. The ASCE/SEI7 provisions are derived from the NEHRP[3] provisions.

2. BASIC COMPONENTS

The basic function of a *lateral force-resisting system* is to transfer the lateral forces acting on the structure to the foundation. Vertical and horizontal resisting components are used to provide a continuous and competent load path from the top of the structure to the foundation.

For the single-story, light-framed steel structure shown in Fig. 7.1, the steel roof deck forms the horizontal resisting component or diaphragm.[6] The lateral wind

Figure 7.1 *Lateral Force-Resisting Components*

(not to scale)
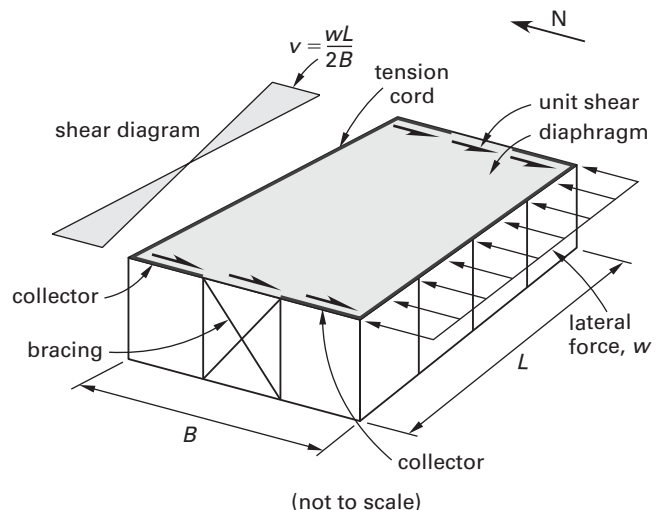
or seismic force acting on the south wall of the structure delivers a line load of w (in units of lbf/ft) to the roof diaphragm. The diaphragm acts as a deep beam with end reactions of

$$V = \frac{wL}{2}$$

These reactions produce a maximum unit shear at the ends of the diaphragm of

$$v = \frac{V}{B} = \frac{wL}{2B}$$

The edges of the diaphragm normal to the direction of the lateral force are known as *chords* and act in a manner similar to the flanges of a steel beam to resist the bending moment produced by the lateral force. As shown in Fig. 7.1, one chord is in tension and the other in compression. The force in a chord is given by the equation

$$T = C = \frac{M_D}{B} = \frac{wL^2}{8B}$$

As shown in Fig. 7.1, the diaphragm must be designed with a capacity to resist the maximum shear at the ends, but may be designed for a smaller shear nearer midspan. Diaphragms are typically constructed from concrete,[6] composite and non-composite formed steel deck,[7] or wood structural panels.[8] Alternatively, in place of a diaphragm, horizontal bracing may be used to resist the lateral load.

The diaphragm shear force is transferred by the collector elements, or *drag struts*, to the vertical resisting components. In Fig. 7.1, these are the bracing elements in the end walls. The bracing elements transfer the lateral force to the structure's foundation.

Structural Irregularities

The structure illustrated in Fig. 7.1 is defined as a *regular building*. A regular building has the following characteristics.

- a continuous load path that is provided to transfer the applied lateral forces to the foundation

- a plan shape that is symmetrical and vertical lateral force-resisting elements that have similar strengths so as to minimize torsion

- vertical lateral force-resisting elements that are located so as to provide the maximum torsional capacity

- uniformly distributed mass, stiffness, and strength to minimize stress concentrations.

- no geometric irregularities, discontinuities, or reentrant corners that will produce stress concentrations

Buildings that have structural irregularities are known as *non-regular buildings*. *Structural irregularities* produce stress concentrations and increased torsional effects that may cause collapse of the building. In order to discourage the use of irregular features in a design and encourage regularity and redundancy, ASCE/SEI7 Chap. 12 imposes penalties on non-regular structures. These penalties include the following.

- an increase of 30% in design forces for buildings that lack redundancy (redundancy factor of 1.3)

- an increase of up to three times the normal design force in critical members to ensure an elastic response in the member (overstrength factor of 3.0)

- the prohibition of irregularities such as extreme soft stories, weak stories, or extreme torsional irregularities in buildings assigned to seismic design categories E and F

- the required use of dynamic analysis for buildings assigned to seismic design categories D, E, and F that have irregularities such as soft stories, mass irregularities, vertical geometric irregularities, or torsional irregularities, with the exception of occupancy category I and II buildings no more than two stories high and light-framed construction no more than three stories high

3. STRUCTURAL SYSTEMS

The *lateral force-resisting systems* listed in ASCE/SEI7 Table 12.2-1 consist of the following eight types.

- bearing wall systems

- moment-resisting frame systems

- building frame systems with shear walls

- building frame systems with braced frames

- dual systems

- shear wall-frame interactive system with ordinary reinforced concrete moment frames and ordinary reinforced concrete shear walls

- cantilever column systems

- steel systems not specifically detailed for seismic resistance

Bearing Wall Systems

A *bearing wall system*, as shown in Fig. 7.2, has shear walls that are designed to support most of the gravity load of the building and to resist all lateral forces. The shear walls receive the shear force from the floor and roof diaphragms and transfer the lateral force to the building foundation. The system lacks redundancy, since a failure of the lateral capacity of the walls will also produce collapse of the gravity load carrying capacity. Therefore, the system has a comparatively low value of the response modification coefficient, R, and a correspondingly high lateral design force. The system has considerable stiffness and low inelastic deformation, which results in negligible damage to architectural features and nonstructural elements in the event of an earthquake. Shear walls must extend from the roof to

the foundation without offsets or extensive openings, and this restricts architectural design freedom. In seismic design categories D, E, and F, concrete and masonry shear walls must be specially reinforced and detailed, and they are restricted in height. Wood framed residential and small commercial and industrial structures are also constructed using this system, with shear walls sheathed with wood structural panels.

Figure 7.2 Bearing Wall System
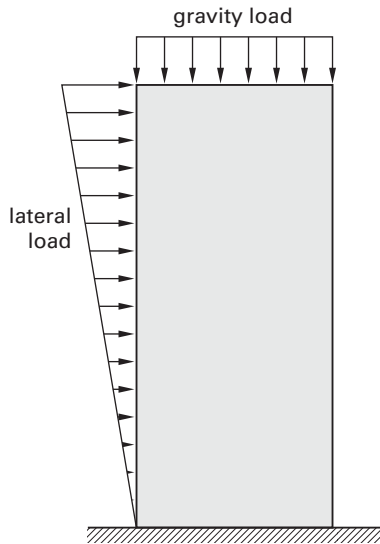
Moment-Resisting Frame Systems

A *moment-resisting frame system*, as shown in Fig. 7.3, provides support for both gravity and lateral loads by flexural action induced by rigid connections at the beam/column joints. In seismic design categories D, E, and F, joints must be capable of developing an interstory drift angle of 0.04 rad at a flexural strength of 80%

Figure 7.3 Moment-Resisting Frame System
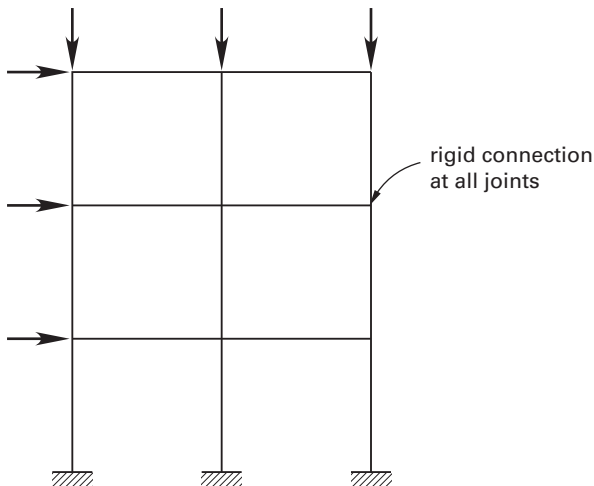
of the plastic moment capacity of the beam. For special steel moment-resisting frames, details of prequalified joints that meet this requirement have been published.[9,10] Special reinforced concrete moment-resisting frames are detailed to ensure that large inelastic displacements can occur without impairing the structural integrity of the frame.[11,12]

In moment-resisting frame systems, no diagonal bracing or shear walls are necessary, and the open bays allow for great freedom with architectural designs. No restrictions are placed on the height of special moment-resisting frames. The system has considerable flexibility and high inelastic deformation capacity, which may result in significant damage to architectural features and nonstructural elements in the event of an earthquake. Therefore, the design of moment-resisting frames is often governed by code requirements to limit interstory drift rather than to provide minimum strength. The system is highly redundant and has a high value of the response modification coefficient, R, and a correspondingly low lateral design force.

A uniform distribution of stiffness, strength, and mass is required over the height of the frame in order to prevent P-delta instability. In addition, a strong-column/weak-beam design approach is required to ensure that column flexural strength exceeds the beam flexural strength at each joint. Inelastic drift must be uniformly distributed over the height of the frame, and inelastic deformations are concentrated at the ends of the beams. The resulting collapse mechanism is shown in Fig. 7.4.

Figure 7.4 Strong-Column/Weak-Beam Frame Collapse Mechanism
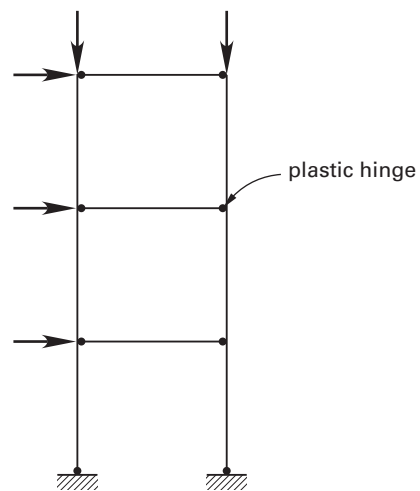
Where the stiffness of the columns in a story is reduced, a *soft story irregularity* is produced. Large inelastic drift occurs in the story, and plastic hinges are formed at the top and bottom of the columns in the soft story. P-delta effects cause a *story mechanism* to form, and the collapse mechanism is shown in Fig. 7.5.

Figure 7.5 Story Mechanism
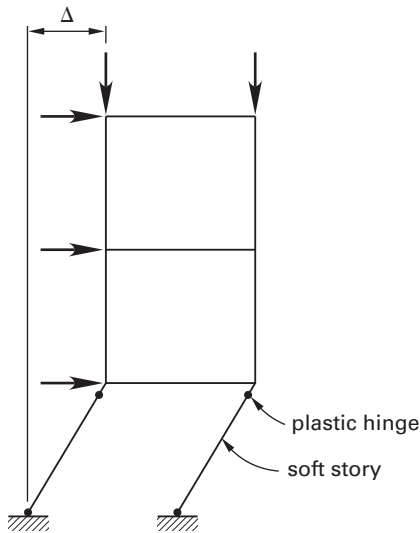
Several methods are available for determining an approximate estimate of the forces in a rigid frame subjected to lateral loads.[13] The simplest of these is the *portal method*. The portal method assumes the following.

- A point of inflection occurs at the mid-height of each column.

- A point of inflection occurs at the midpoint of each girder.

- The shear and moment in each interior column is twice that of an exterior column.

- The axial force in an interior column is zero.

An accurate determination of the member forces in a rigid frame, with axial effects neglected, may be obtained using the *moment distribution procedure*. In the case of lateral loads applied to a *symmetrical single bay frame*, as shown in Fig. 7.6, the skew symmetry may be used to simplify the process.[14] Allowing for skew symmetry, the modified stiffness of each column is EI/L, and the modified stiffness of each beam is $6EI/L$. The carryover factor in the columns is -1, and there is no carryover between girder ends. The initial fixed-end moments are obtained by imposing unit virtual sway displacement on each story in turn.

Figure 7.6 Sway Distribution for Single Bay Frame
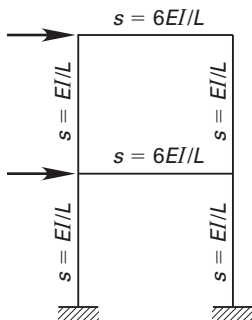
Multi-bay frames of the type shown in Fig. 7.7, where the relative EI/L values are shown ringed, satisfy the principle of multiples. Using the *principle of multiples*, the original frame is replaced by two substitute frames, with the ratio of applied load to member stiffness the same in each frame. Joint rotations and sway displacements are the same in the substitute frames and the original frame, and member forces in the substitute frames can be added to give the corresponding member force in the original frame.

Figure 7.7 Principle of Multiples
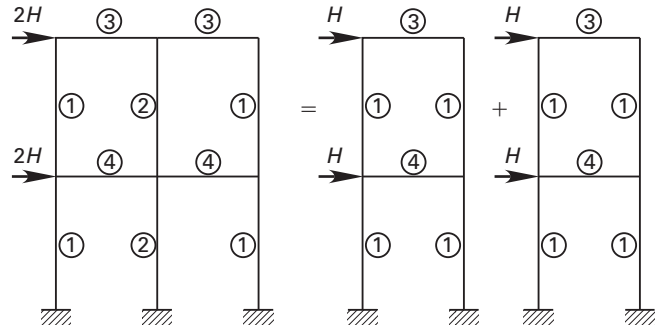
Example 7.1

Using the portal method, determine the bending moments and support reactions produced in the two-story rigid frame shown.
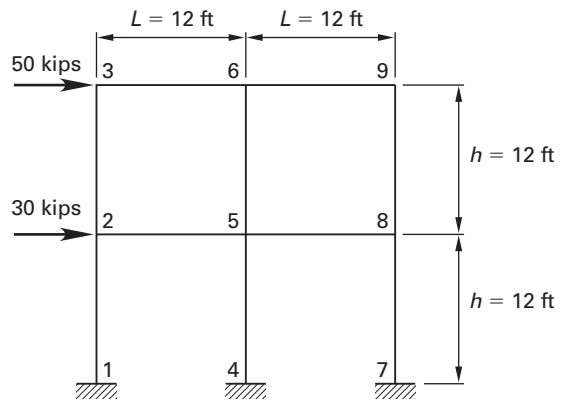
Solution

Equating the reactions at the column bases with the applied horizontal forces gives

$$H_1 + H_4 + H_7 = H_2 + H_3$$
$$= 30 \text{ kips} + 50 \text{ kips}$$
$$= 80 \text{ kips}$$

Since the shear in an interior column is twice that of an exterior column,

$$H_1 + H_4 + H_7 = 0.5H_4 + H_4 + 0.5H_4$$
$$= 2H_4$$
$$= 80 \text{ kips}$$
$$H_4 = 40 \text{ kips}$$
$$H_1 = H_7 = 0.5H_4$$
$$= (0.5)(40 \text{ kips})$$
$$= 20 \text{ kips}$$

A point of inflection occurs at the midpoint of each column, and the moment in each interior column is twice that of an exterior column. So,

$$M_{12} = M_{21} = M_{78} = M_{87}$$
$$= \frac{H_1 h}{2}$$
$$= \frac{(20 \text{ kips})(12 \text{ ft})}{2}$$
$$= 120 \text{ ft-kips}$$
$$M_{45} = M_{54} = 2M_{12}$$
$$= (2)(120 \text{ ft-kips})$$
$$= 240 \text{ ft-kips}$$

Since the axial force in the central column is zero, taking moments about support 1 gives

$$\sum M = V_7(2L) + M_{12} + M_{45} + M_{78}$$
$$- H_2 h - H_3(2h) = 0$$
$$V_7 = \frac{H_2 h + H_3(2h) - M_{12} - M_{45} - M_{78}}{2L}$$
$$= \frac{\begin{array}{c}(30 \text{ kips})(12 \text{ ft}) + (50 \text{ kips})(2)(12 \text{ ft}) \\ - 120 \text{ ft-kips} - 240 \text{ ft-kips} \\ \hline - 120 \text{ ft-kips}\end{array}}{(2)(12 \text{ ft})}$$
$$= 45 \text{ kips} \quad [45 \text{ kips up}]$$
$$V_1 = -V_7 = -45 \text{ kips} \quad [45 \text{ kips down}]$$

Imposing unit virtual sway displacement on the top story gives

$$M_{23} + M_{56} + M_{89} + M_{32} + M_{65} + M_{98}$$
$$= \text{moment of } H_3 \text{ about node 2}$$
$$= H_3 h$$
$$= (50 \text{ kips})(12 \text{ ft})$$
$$= 600 \text{ ft-kips}$$

Each of the two interior moments, M_{56} and M_{65}, is twice as large as each of the four exterior moments, so

$$M_{23} + M_{56} + M_{89} + M_{32} + M_{65} + M_{98}$$
$$= 600 \text{ ft-kips}$$
$$= M_{23} + 2M_{23} + M_{23} + M_{23}$$
$$+ 2M_{23} + M_{23}$$
$$= 8M_{23}$$
$$M_{23} = \frac{600 \text{ kips}}{8}$$
$$= 75 \text{ kips}$$
$$= M_{89} = M_{32} = M_{98}$$
$$2M_{23} = (2)(75 \text{ kips})$$
$$= 150 \text{ kips}$$
$$= M_{56} = M_{65}$$

Equating moments at node 2 gives

$$M_{25} = M_{21} + M_{23}$$
$$= 120 \text{ ft-kips} + 75 \text{ ft-kips}$$
$$= 195 \text{ ft-kips}$$
$$= M_{52} = M_{58} = M_{85}$$

Equating moments at node 3 gives

$$M_{36} = M_{32}$$
$$= 75 \text{ ft-kips}$$
$$= M_{63} = M_{69} = M_{96}$$

The member forces (kips) are shown in the following illustration with moments (ft-kips) drawn on the tension side of the members.
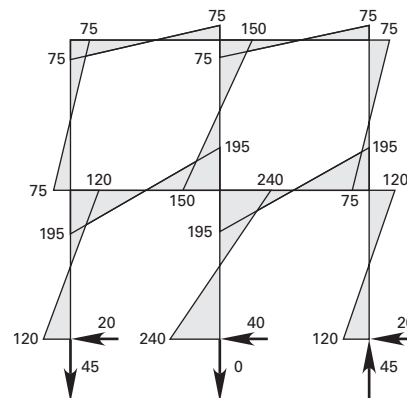
Example 7.2

Using the principle of multiples, determine the bending moments and support reactions produced in the two-story rigid frame analyzed in Ex. 7.1. The relative EI/L values for columns 45 and 56 are twice the values for each of the other members.

Solution

The frame satisfies the principle of multiples, and each of the two substitute frames is identical with the single-bay frame shown.
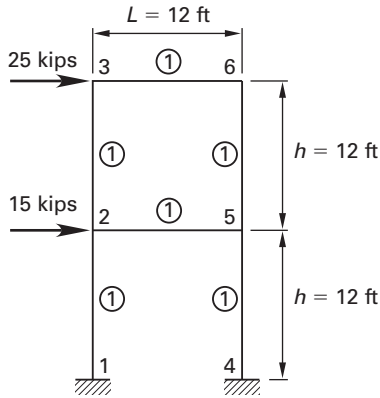
The skew symmetry in the substitute frame allows automatically for sidesway. For all members,

$$\frac{EI}{L} = k$$

Allowing for skew symmetry, the modified stiffness of the members is

$$s_{21} = s_{23} = s_{32} = k$$

$$s_{25} = s_{36} = 6k$$

The distribution factors at joint 2 are

$$d_{25} = \frac{s_{25}}{s_{25} + s_{21} + s_{23}}$$

$$= \frac{6k}{8k}$$

$$= \frac{3}{4}$$

$$d_{21} = d_{23} = \frac{1}{8}$$

The distribution factors at joint 3 are

$$d_{36} = \frac{s_{36}}{s_{36} + s_{32}}$$

$$= \frac{6k}{7k}$$

$$= \frac{6}{7}$$

$$d_{32} = \frac{1}{7}$$

Allowing for skew symmetry, the carry-over factor in the columns is -1, and there is no carry-over between girder ends. The initial fixed-end moments are obtained by imposing unit virtual sway displacement on each story.

For the upper story,

$$M_{F23} = M_{F32} = \frac{-V_3 h_{23}}{4}$$

$$= \frac{-(25 \text{ kips})(12 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{4}$$

$$= -900 \text{ kips-in}$$

For the lower story,

$$M_{F12} = M_{F21} = \frac{-(V_3 + V_2)h_{12}}{4}$$

$$= \frac{-(40 \text{ kips})(12 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)}{4}$$

$$= -1440 \text{ kips-in}$$

The distribution of moments is given in the following table, with distribution occurring in the left half of the substitute frame only.

member	12	21	25	23	32	36
distribution factor	0	1/8	3/4	1/8	1/7	6/7
fixed-end moments	−1440	−1440			−900	−900
distribution		293	1754	293	129	771
carry-over	−293			−129	−293	
distribution		16	97	16	42	251
carry-over	−16			−42	−16	
distribution		5	32	5	2	14
carry-over	−5			−2	−5	
distribution			2		1	4
final moments (in-kips)	−1754	−1126	1885	−759	−1040	1040
final moments (ft-kips)	−146	−94	157	−63	−87	87

The final bending moments (in ft-kips) produced in the members are shown in the following illustration with the moments drawn on the tension sides of the members.
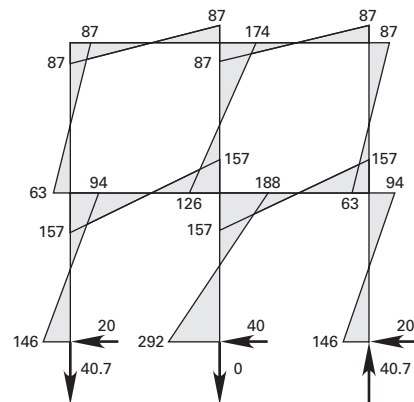
Building Frame Systems with Shear Walls

A *building frame system with shear walls*, as shown in Fig. 7.8, consists of two independent systems: one to provide support for gravity loads, the other to resist lateral loads. The shear walls resist all lateral loads, and a column and beam frame supports most gravity loads. Since the gravity frame does not contribute to lateral resistance, it does not require special ductile detailing, and simple shear connections are used to connect the beams and columns. However, the gravity frame is required to satisfy deformation compatibility requirements. Since the gravity and lateral-force systems are independent, the building frame system is more redundant than the bearing wall system. It has a higher value of the response modification coefficient, R, and a correspondingly lower lateral design force. The system has considerable stiffness and low inelastic deformation resulting in negligible damage to architectural features and nonstructural elements in the event of an earthquake. Shear walls must extend from the roof to the foundation without offsets or extensive openings, and this restricts architectural design freedom.

Figure 7.8 *Building Frame System with Shear Walls*
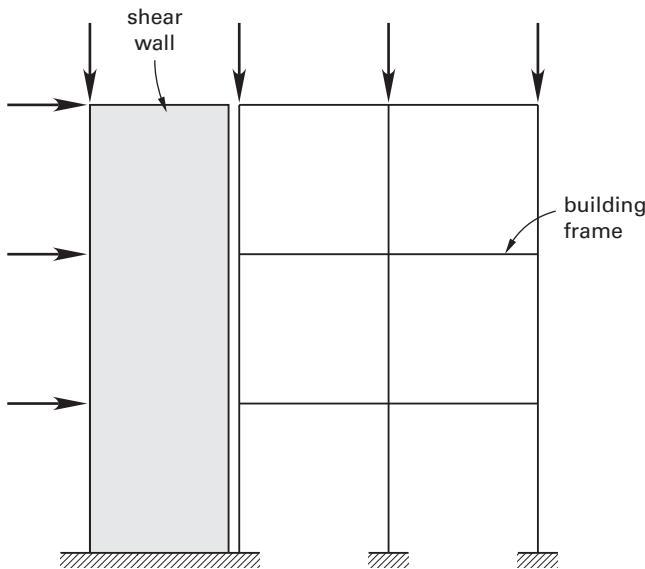
In seismic design categories D, E, and F, concrete and masonry shear walls must be specially reinforced and detailed, and they are restricted in height. Special steel plate shear walls are also used in a building frame system, and they are restricted in height in seismic design categories D, E, and F. Steel plate shear walls consist of a steel frame stiffened by thin steel plates. Lateral loads are resisted by buckling of the plate mobilizing diagonal tension-field action. Wood structural panel shear walls in wood framed small commercial and industrial structures may also be used in a building frame system and are restricted to a height of 65 ft in seismic design categories D, E, and F.

IBC Sec. 2306.3 requires wood structural panel shear walls be designed and constructed in accordance with the American Forest & Paper Association's *Special Design Provisions for Wind and Seismic* (SDPWS[15]). The construction details of a typical *plywood sheathed shear wall* are shown in Fig. 7.9. The shear capacity of a shear wall depends on the thickness and grade of the plywood sheathing, width of framing members, support of the panel edges, and nail spacing and penetration. Nominal shear capacities are given in SDPWS Table 4.3A for seismic and wind loading of walls with plywood sheathing on one side, all panel edges blocked, and 2 in nominal framing of Douglas fir-larch or southern pine.

Figure 7.9 *Shear Wall Details*
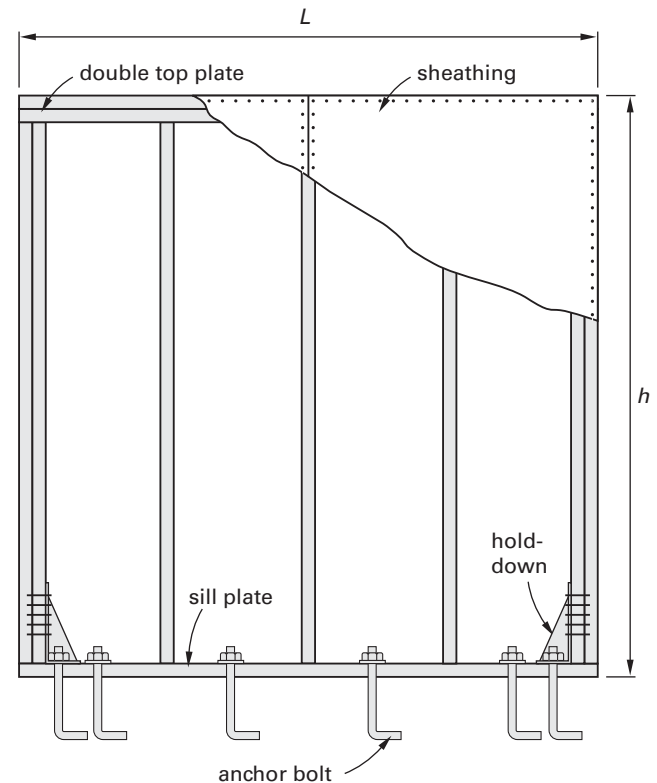
The nominal unit shear capacities tabulated in SDPWS Table 4.3A specifically relate to shear walls with plywood panels attached to one side of the wall using 6d, 8d, or 10d common or galvanized box nails. The table provides *nominal unit shear capacity* values for seismic design in column A and values for wind design are provided in column B. The corresponding ASD *allowable unit shear capacity* values are obtained by dividing the tabulated nominal unit shear capacity values by the ASD reduction factor of 2.0. The LRFD *factored unit resistance* values are obtained by multiplying the tabulated nominal unit shear capacity values by the resistance factor, ϕ, of 0.80.

SDPWS Table 4.3B, Table 4.3C, and Table 4.3D provide nominal unit shear capacities for shear walls sheathed with wood structural panels applied over

Lateral Forces

gypsum wallboard; gypsum and Portland cement plaster; and lumber, respectively.

IBC Table 2306.3(1) tabulates allowable unit shear capacity values for shear walls sheathed with plywood panels using *staple fasteners*. IBC Table 2306.3(2) and Table 2306.3(3) provide allowable unit shear capacities for shear walls sheathed with fiberboard and gypsum board, respectively, using staple fasteners.

To control the stiffness of wood structural panel shear walls, SDPWS Table 4.3.4 limits the maximum permitted aspect ratio. For blocked panels subjected to wind forces, the maximum aspect ratio is 3.5:1. For blocked panels subjected to seismic forces, the maximum aspect ratio is 2:1, unless the nominal unit shear capacity is multiplied by $2b_s/h$. To resist the uplift due to overturning moments on the shear wall, a *hold-down* must be provided at each end of each wall. Transfer of the lateral force to the foundation is achieved with anchor bolts at a maximum spacing of 6 ft. A minimum of two bolts is required.

Since the shear wall is considered non-rigid, gravity loads along the top of the wall do not provide a restoring moment to the wall. Therefore, the tension force, T, in the hold-down, which is also the force in the end posts, or chords, is given by SDPWS Eq. 4.3-7 as

$$T = C = vh$$

C is the compression force; v is the induced unit shear; and h is the shear wall height.

To accommodate the bolts or screws in the hold-down, a double end post is usually provided. Similarly, to provide continuity for the top plate and to provide overlapping at intersections, a double top plate is normally used. IBC Sec. 2308.6 requires sill plates to be anchored to the foundation with not less than $1/2$ in diameter steel bolts or approved anchors spaced not more than 6 ft apart. Bolts must be embedded at least 7 in into concrete or masonry, and there must be a minimum of two bolts or anchor straps per wall, with one bolt or anchor strap located not more than 12 in, or less than 4 in, from the end of each wall. As shown in Fig. 7.10, to minimize the potential for cross grain bending in the sill plate, SDPWS Sec. 4.3.6.4.3 requires a steel plate washer under each nut not less than 0.229 in × 3 in × 3 in in size. The plate washer must extend to within $1/2$ in of the edge of the sill plate on the sheathed side when the required nominal unit shear capacity exceeds 400 lbf/ft for wind or seismic. Standard cut washers may be used when anchor bolts are designed to resist shear only and the following requirements are met.

- The shear wall is designed as an individual full-height wall segment with required uplift anchorage at shear wall ends sized to resist overturning, neglecting the dead load stabilizing moment.

- The shear wall aspect ratio does not exceed 2:1.

- The nominal unit shear capacity of the shear wall does not exceed 980 lbf/ft for seismic, or 1370 lbf/ft for wind.

Figure 7.10 *Anchor Bolt Detail*
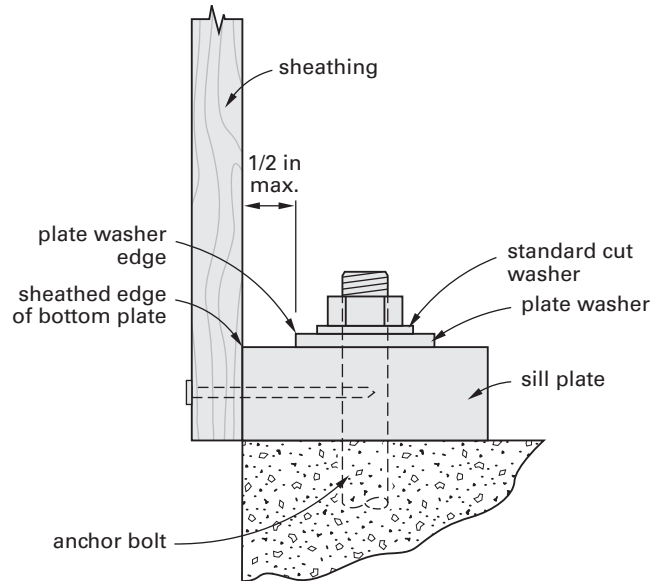
Adapted with permission from *Special Design Provisions for Wind and Seismic with Commentary*, copyright © 2008, by the American Wood Council.

In accordance with SDPWS Sec. 4.3.7, the maximum stud spacing is 24 in. At intermediate framing members, the maximum permitted nail spacing is 6 in, except that 12 in is permitted when stud spacing is less than 24 in, or panel thickness is $7/16$ in or more. Nails along intermediate framing members must be the same as nails specified for panel edge nailing. The width of the nailed face of framing members and blocking is required to be 2 in nominal or greater at adjoining panel edges, except that a 3 in nominal or greater width at adjoining panel edges and staggered nailing at all panel edges are required where any of the following conditions exist.

- Nail spacing of 2 in or less at adjoining panel edges is specified.

- 10d common nails having penetration into framing members and blocking of more than $1\frac{1}{2}$ in are specified at 3 in on center or less at adjoining panel edges.

- The nominal unit shear capacity on either side of the shear wall exceeds 700 lbf/ft in seismic design categories D, E, or F.

In accordance with SDPWS Sec. 4.3.3, for walls sheathed with plywood of equal shear capacity on opposite sides of the wall, the shear capacity of the wall may be taken as twice the value permitted for one side. For shear walls sheathed with dissimilar materials on opposite sides, the combined shear capacity may be taken as the maximum value given by twice the smaller capacity or equal to the larger capacity. Summing capacities of dissimilar materials applied to shear walls in the same wall line is not permitted.

Example 7.3

The shear wall shown in Fig. 7.9 has dimensions of $L = 8$ ft and $h = 8$ ft, and the sheathing is $^{15}/_{32}$ in Structural I. The dead load on the wall from the roof diaphragm is $w = 80$ lbf/ft, and the strength level wind load transmitted by the diaphragm is $V = 11.6$ kips. Determine the nailing requirements for the shear wall, the force in the hold-downs, and the number of $^5/_8$ in diameter anchor bolts required in the 4 in × 3 in Douglas fir-larch sill plate. Neglect the self-weight of the shear wall.

Solution

The strength level unit shear acting on the shear wall is

$$v = \frac{V}{L} = \frac{(11.6 \text{ kips})\left(1000 \frac{\text{lbf}}{\text{kip}}\right)}{8 \text{ ft}} = 1450 \text{ lbf/ft}$$

The required spacing of 10d common nails with $1^1/_2$ in penetration is obtained from SDPWS Table 4.3A. The required nail spacing is

all panel edges	3 in
intermediate framing members	12 in
capacity provided	$(0.8)\left(1860 \frac{\text{lbf}}{\text{ft}}\right) = 1488$ lbf/ft
capacity required	1450 lbf/ft

From SDPWS Sec. 4.3.7, the framing at adjoining panel edges must be nominal 3 in, and staggered nailing at all panel edges is required.

The service level value of the unit shear is given by IBC Eq. 16-15 as

$$v' = 0.6v$$
$$= (0.6)\left(1450 \frac{\text{lbf}}{\text{ft}}\right)$$
$$= 870 \text{ lbf/ft}$$
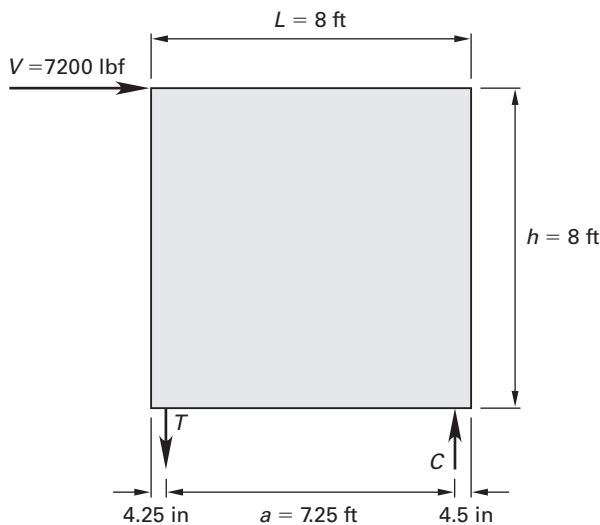
SDPWS Eq. 4.3-7 may be used to obtain the hold-down force. This ignores the vertical load on the wall and the distance of the hold-down anchor rod from the end of the wall. SDPWS Eq. 4.3-7 gives the hold-down force as

$$T = v'h = \left(870 \frac{\text{lbf}}{\text{ft}}\right)(8 \text{ ft}) = 6960 \text{ lbf}$$

The allowable parallel-to-grain load, $Z_\parallel$, on a $^5/_8$ in diameter bolt in the $2^1/_2$ in thick Douglas fir-larch sill plate is obtained from NDS[16] Table 11E as

$$Z_\parallel = 1180 \text{ lbf}$$

The load duration factor, C_D, for wind load is given by NDS Table 2.3.2 as

$$C_D = 1.6$$

Therefore, the number, N, of $^5/_8$ in diameter bolts required is

$$N = \frac{V}{C_D Z_\parallel} = \frac{6960 \text{ lbf}}{(1.6)(1180 \text{ lbf})}$$
$$= 3.69 \quad [\text{use 4 bolts}]$$

Shear Walls with Openings

A shear wall with openings has been designed traditionally by considering each full height segment of the wall as an individual shear wall. As shown in Fig. 7.11, this results in a wall with a single opening being designed as two separate shear walls requiring a total of four hold-downs, one at either end of the two shear walls. Sheathing above and below the opening is not considered to contribute to the overall shear capacity of the wall, and the shear capacity of the wall is calculated as the sum of the capacities of the individual segments.

Figure 7.11 *Segmented Shear Wall*
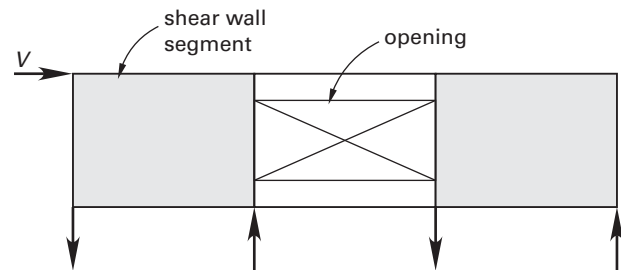
An alternative design method is the *perforated shear wall method.*[17] The shear capacity of a perforated shear wall is calculated as a percentage of the capacity of the wall without openings, and the method is specified in SDPWS Sec. 4.3.3.5. As shown in Fig. 7.12, the advantage of the method is that only two hold-downs are necessary, one at either end of the wall. Sheathed areas above and below openings are not designed for force transfer and are considered to provide only local restraint at their ends. The shear capacity of a perforated wall depends on the maximum opening height and on the percentage of full height sheathing.

Figure 7.12 *Perforated Shear Wall*
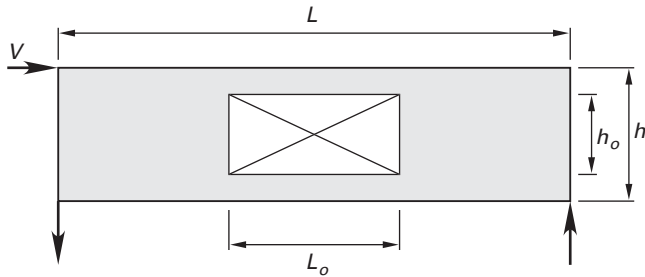
SDPWS Sec. 4.3.5.3 requires the following for perforated walls.

- A segment without openings must be located at each end of the perforated shear wall.
- The aspect ratio limitations of SDPWS Sec. 4.3.4.1 apply.
- The required nominal unit shear capacity for a single sided wall is limited to a maximum of 1740 lbf/ft for seismic or 2435 lbf/ft for wind.
- Where out-of-plane offsets occur, portions of the wall on each side of the offset must be considered separate perforated shear walls.
- Collectors for shear transfer must be provided through the full length of the wall.
- A perforated shear wall must have uniform top-of-wall and bottom-of-wall elevations.
- The height must not exceed 20 ft.

The design shear capacity, V, of a perforated shear wall is

$$V = v C_o \sum L_i$$

The variables are defined as

v = allowable unit shear capacity in a segmented shear wall (lbf/ft)
C_o = shear capacity adjustment factor given in SDPWS Table 4.3.3.5
$\sum L_i$ = sum of perforated shear wall segment lengths (ft)

Values of C_o are given in Table 7.1.

The force in a hold-down, which is also the force in the end posts, is given by SDPWS Eq. 4.3-8 as

$$T = C$$
$$= \frac{Vh}{C_o \sum L_i}$$

The unit shear force in a perforated shear wall is given by SDPWS Eq. 4.3-9 as

$$v_{max} = \frac{V}{C_o \sum L_i}$$

Table 7.1 *Capacity Adjustment Factors*

wall height, h	maximum opening height				
	$h/3$	$h/2$	$2h/3$	$5h/6$	h
8'-0" wall	2'-8"	4'-0"	5'-4"	6'-8"	8'-0"
10'-0" wall	3'-4"	5'-0"	6'-8"	8'-4"	10'-0"
full-height sheathing (%)	shear capacity adjustment factor				
10	1.00	0.69	0.53	0.43	0.36
20	1.00	0.71	0.56	0.45	0.38
40	1.00	0.77	0.63	0.53	0.45
60	1.00	0.83	0.71	0.63	0.56
80	1.00	0.91	0.83	0.77	0.71
100	1.00	1.00	1.00	1.00	1.00

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Anchor bolts, in addition to resisting the horizontal shear force $v_{max}\Sigma L_i$, must also resist a uniformly distributed uplift force. This force is given by SDPWS Sec. 4.3.6.4.2.1 as

$$t = v_{max}$$

Example 7.4

The perforated shear wall shown in Fig. 7.12 has an overall length, L, of 24 ft and a height, h, of 8 ft. The sheathing is $^{15}/_{32}$ in Structural I. The centrally placed opening has dimensions of $L_o = 8$ ft and $h_o = 4$ ft. The dead load on the wall from the roof diaphragm is $w = 80$ lbf/ft, and the strength level wind load transmitted by the diaphragm is $V = 11.6$ kips. Determine the nailing requirements for the shear wall, the force in the hold-downs, and the number of $^5/_8$ in diameter anchor bolts required in the 4 in × 3 in Douglas fir-larch sill plate. Neglect the self-weight of the shear wall.

Solution

The aspect ratio of each segment is

$$a = \frac{L - L_o}{2h} = \frac{24 \text{ ft} - 8 \text{ ft}}{(2)(8 \text{ ft})}$$
$$= 1.0 \quad [\text{complies with SDPWS Table 4.3.4}]$$

The percentage of full height sheathing is

$$\frac{\sum L_i}{L} = \frac{8 \text{ ft} + 8 \text{ ft}}{24} \times 100\%$$
$$= 67\%$$

The maximum opening height ratio is

$$\frac{h_o}{h} = \frac{4 \text{ ft}}{8 \text{ ft}}$$
$$= 0.5$$

From SDPWS Table 4.3.3.5, the shear capacity adjustment factor is

$$C_o = 0.86$$

The strength level unit shear acting on the shear wall is

$$
\begin{aligned}
v &= \frac{V}{L - L_o} \\
&= \frac{(11.6 \text{ kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{24 \text{ ft} - 8 \text{ ft}} \\
&= 725 \text{ lbf/ft}
\end{aligned}
$$

Allowing for the shear capacity adjustment factor, the equivalent unit shear for a perforated shear wall is

$$
\begin{aligned}
v' &= \frac{v}{C_o} \\
&= \frac{725 \dfrac{\text{lbf}}{\text{ft}}}{0.86} \\
&= 843 \text{ lbf/ft}
\end{aligned}
$$

The required spacing of 10d common nails with $1^{1}/_{2}$ in penetration is obtained from SDPWS Table 4.3A as

all panel edges	4 in
intermediate framing members	12 in
capacity provided	$(0.8)(1430 \text{ lbf/ft}) = 1144 \text{ lbf/ft}$
capacity required	843 lbf/ft [satisfactory]

The allowable parallel-to-grain load on a $^{5}/_{8}$ in diameter bolt in the $2^{1}/_{2}$ in thick Douglas fir-larch sill plate is obtained from NDS Table 11E as

$$Z_{\parallel} = 1180 \text{ lbf}$$

The load duration factor for wind load is given by NDS Table 2.3.2 as

$$C_D = 1.6$$

Converting to service level values using IBC Eq. 16-15, the number of $^{5}/_{8}$ in diameter bolts required is

$$
\begin{aligned}
N &= \frac{0.6V}{C_D Z_{\parallel}} \\
&= \frac{(0.6)(11.6 \text{ kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{(1.6)(1180 \text{ lbf})} \\
&= 3.7 \quad \text{[use 4 bolts]}
\end{aligned}
$$

Two bolts in each segment must be provided. The strength level shear force on each bolt is

$$
\begin{aligned}
P_v &= \frac{V}{4} = \frac{(11.6 \text{ kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{4 \text{ bolts}} \\
&= 2900 \text{ lbf/bolt}
\end{aligned}
$$

The available shear force on a $^{5}/_{8}$ in diameter A307 bolt is given by *AISC Manual*[18] Table 7-1 as

$$
\begin{aligned}
\phi r_n &= 6230 \text{ lbf} \\
&> P_v \quad \text{[satisfactory]}
\end{aligned}
$$

The uniformly distributed design uplift anchorage force on the sill plate of each full height, perforated shear wall segment is given by SDPWS Sec. 4.3.6.4.2.1 as

$$
\begin{aligned}
t &= v_{\max} \\
&= \frac{V}{C_o \sum L_i} \\
&= \frac{(11.6 \text{ kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{(0.86)(8 \text{ ft} + 8 \text{ ft})} \\
&= 843 \text{ lbf/ft}
\end{aligned}
$$

The net uplift on the shear wall is given by IBC Eq. 16-6 as

$$
\begin{aligned}
u &= 0.9w + t \\
&= (0.9)\left(-80 \dfrac{\text{lbf}}{\text{ft}}\right) + 843 \dfrac{\text{lbf}}{\text{ft}} \\
&= 771 \text{ lbf/ft}
\end{aligned}
$$

The strength level tensile force in each bolt is

$$
\begin{aligned}
P_t &= \frac{u \sum L_i}{4 \text{ bolts}} = \frac{\left(771 \dfrac{\text{lbf}}{\text{ft}}\right)(8 \text{ ft} + 8 \text{ ft})}{4 \text{ bolts}} \\
&= 3084 \text{ lbf/bolt}
\end{aligned}
$$

The available tensile force on a $^{5}/_{8}$ in diameter A307 bolt is given by *AISC Manual* Table 7-2 as

$$
\begin{aligned}
\phi r_n &= 10{,}400 \text{ lbf} \\
&> P_t \quad \text{[satisfactory]}
\end{aligned}
$$

The anchor bolts provided are adequate for both shear and uplift.

Lateral Forces

The force in a hold-down is given by SDPWS Eq. 4.3-8 as

$$
\begin{aligned}
T &= C \\
&= \frac{Vh}{C_o \sum L_i} \\
&= v_{\max} h \\
&= \left(843 \ \frac{\text{lbf}}{\text{ft}}\right)(8 \ \text{ft}) \\
&= 6744 \ \text{lbf} \quad \text{[at strength level]} \\
&= 4046 \ \text{lbf} \quad \text{[at service level]}
\end{aligned}
$$

Building Frame System with Braced Frames

A *building frame system with braced frames*, as shown in Fig. 7.13, consists of a braced frame that resists lateral loads by truss action and an independent column and beam frame that supports most gravity loads. Since the gravity frame does not contribute to lateral resistance, it does not require special ductile detailing, and simple shear connections are used to connect the beams and columns. However, the gravity frame is required to satisfy deformation compatibility requirements. A braced frame is more ductile than a shear wall and provides more architectural freedom than a shear wall.

Figure 7.13 *Building Frame System with Braced Frame*
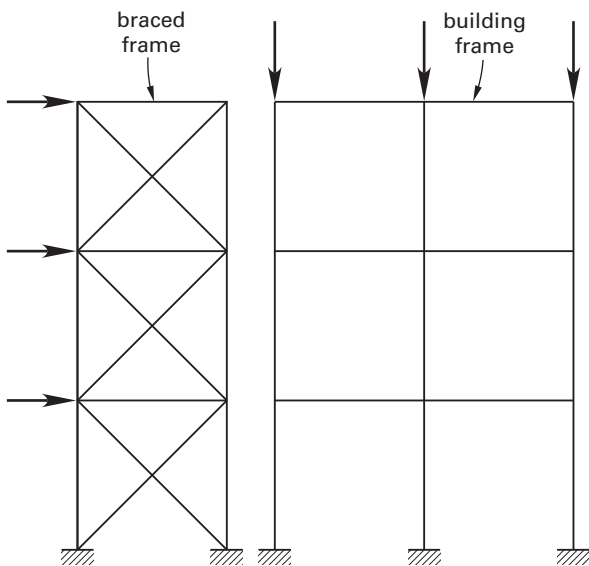
There are four general types of braced frames.

- special steel concentrically braced frames
- ordinary steel concentrically braced frames
- buckling-restrained braced frames
- steel eccentrically braced frames

In *special steel concentrically braced frames*, the center-line of all the frame members, beams, columns, and braces are coincident. Therefore, applied lateral loads

are resisted by axial forces in the members and flexure is eliminated. Many different configurations are possible, and several are shown in Fig. 7.14. To ensure ductile behavior in the brace, limits are placed on the slenderness and compactness of the brace. Also, to ensure that the full strength of the brace can be developed, brace connections must be designed to resist the yield strength of the brace.

Figure 7.14 *Special Steel Concentrically Braced Frames*
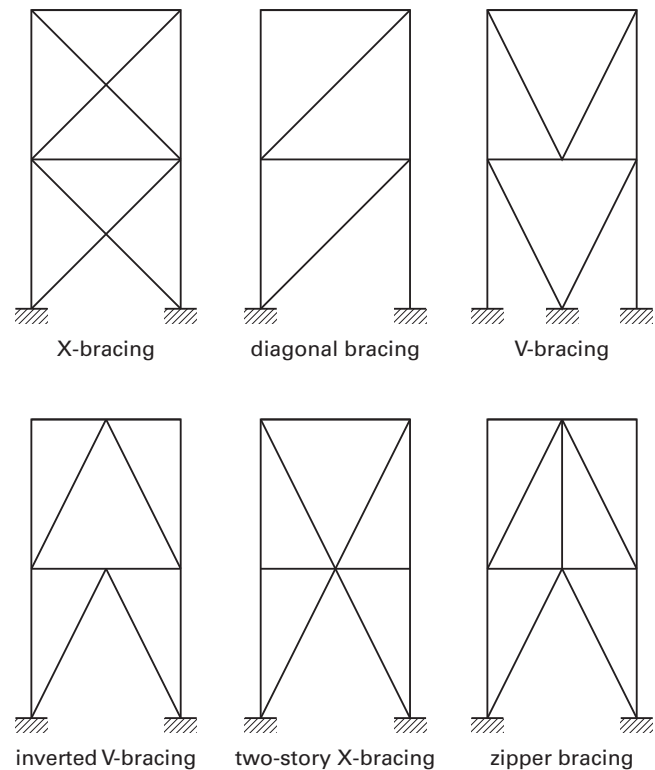
In *V-bracing* and *inverted V-bracing systems*, the intersecting beam must be designed to carry all gravity loads without support from the braces. In addition, the beam must be designed for the unbalanced force produced in the event of buckling of the compression brace and yielding of the tensile brace. In *X-bracing* and *zipper bracing systems*, this unbalanced force on the beam is eliminated.

A building frame system with special steel concentrically braced frames is assigned the same value of the response modification coefficient, R, as a building frame system with special reinforced concrete shear walls and is subject to the same height restrictions in seismic design categories D, E, and F. A building frame system with special steel concentrically braced frames is generally less expensive than a special moment-resisting frame system because it avoids the high cost of the special rigid joints in moment-resisting frames.

Ordinary steel concentrically braced frames are similar to special steel concentrically braced frames in configuration and may utilize all of the systems indicated in

Fig. 7.14. However, the value of the response modification coefficient, R, is much lower and, consequently, structures are designed for a higher seismic force. Therefore, the system remains essentially elastic under a seismic event and does not require the special detailing necessary for the special steel concentrically braced frame system. In addition to the bracing configurations shown in Fig. 7.14, *K-bracing* is also permitted for ordinary steel concentrically braced frames. K-bracing is not allowed in special steel concentrically braced frames because of the unbalanced force produced in the column in the event of buckling of the compression brace. The K-bracing system is shown in Fig. 7.15.

Figure 7.15 *K-Bracing System*

K-bracing
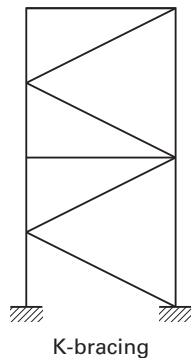
Ordinary steel concentrically braced frames are subject to a height restriction of 35 ft in seismic design categories D and E. The system is not permitted in seismic design category F, except for single-story buildings less than or equal to 60 ft high.

Buckling-restrained braced frames are a special type of concentrically braced frame. In this type of frame, special braces are used that consist of a steel core surrounded by a casing of steel and mortar to prevent the core from buckling. Therefore, compression yielding of the core can occur, as well as tensile yielding. This ensures significantly better ductility than a special steel concentrically braced frame.

The system has a higher value of the response modification coefficient, R, than a special steel concentrically braced frame and a correspondingly lower lateral design force. The buckling-restrained braced frame is subject to the same height restrictions as a special steel concentrically braced frame in seismic design categories D, E, and F.

In an *eccentrically braced frame*, as shown in Fig. 7.16, one end of the brace is connected to the beam so as to form a short link between the brace and the column, or between two opposing braces. The link acts as a structural fuse by providing the inelastic behavior necessary to absorb the input seismic energy while the other framing elements remain elastic. The eccentrically braced frame provides lateral stiffness equivalent to that of a concentrically braced frame, and ductility equivalent to that of a special moment-resisting frame.

Figure 7.16 *Steel Eccentrically Braced Frames*
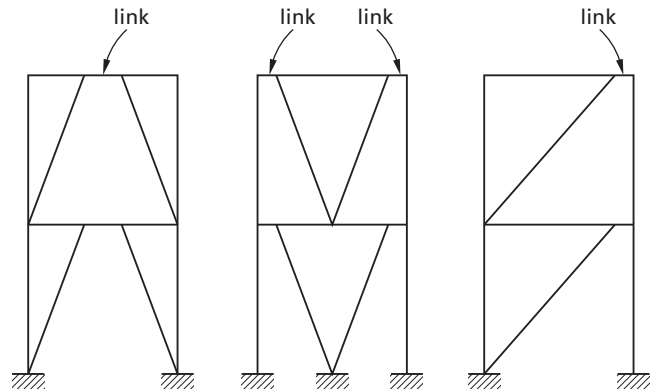
The larger spaces between braces give greater freedom in architectural design than there is with concentrically braced frames. The increased stiffness of the system, compared with special moment-resisting frames, limits damage to architectural features and nonstructural elements in the event of an earthquake. The eccentrically braced frame is subject to the same height restrictions as a special steel concentrically braced frame in seismic design categories D, E, and F.

The system has a higher value of the response modification coefficient, R, than a special steel concentrically braced frame, and a correspondingly lower lateral design force. However, the additional design and detailing requirements for the link beams is generally more expensive than a special steel concentrically braced frame configuration because of the high cost of the link beam.

Dual System with Moment-Resisting Frames

As shown in Fig. 7.17, a *dual system* has a secondary lateral support system coupled with the primary non-gravity-load-bearing lateral support system. Shear walls or braced frames provide the primary lateral support system with a special or intermediate moment-resisting frame providing primary support for gravity loads, and act as a backup for the lateral support system. The moment-resisting frame must provide resistance to at least 25% of the seismic forces.

The special moment-resisting frame system has a high value of the response modification coefficient, R, and no restrictions are placed on height. The ordinary moment-resisting frame system has a lower value of the response modification coefficient, R, and restrictions are placed on the height of systems in seismic design categories D, E, and F.

Shear Wall-Frame Interactive System

A shear wall-frame interactive system with ordinary reinforced concrete moment frames and ordinary reinforced concrete shear walls is a dual system and it is used for structures assigned to seismic design categories A and B. The shear walls and frames are designed to resist lateral forces in proportion to their rigidities

Lateral Forces

Figure 7.17 *Dual System with Moment-Resisting Frames*

(a)

(b)
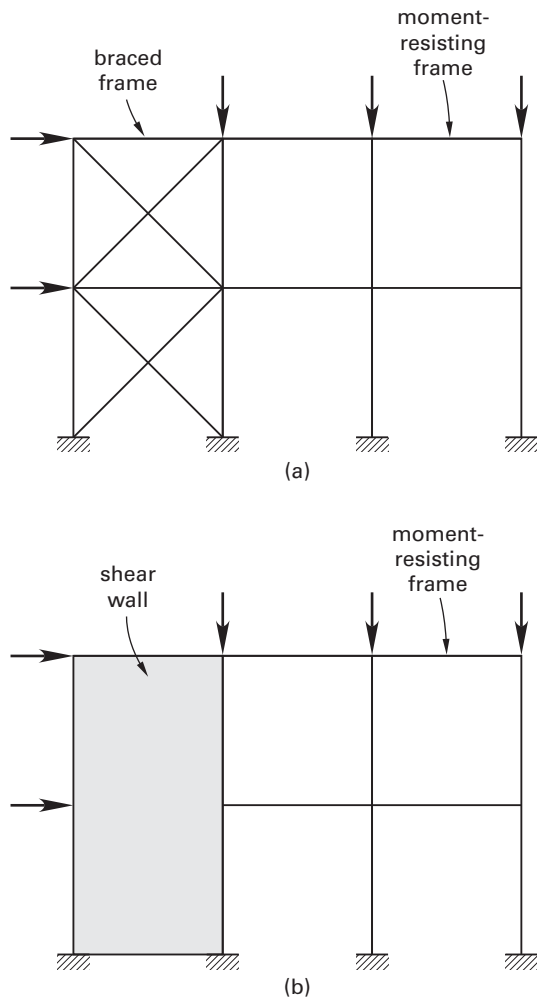
considering the interaction between shear walls and frames on all levels. In accordance with ASCE/SEI7 Sec. 12.2.5.8, the shear strength of the shear wall must be at least 75% of the design story shear at each story. The frames of the shear wall-frame interactive system must be capable of resisting at least 25% of the design story shear at each story.

Structures utilizing this design system are located in zones of low seismicity, so it is not necessary to use the special seismic detailing required for a dual system with moment-resisting frames. For structures assigned to seismic design category A, ACI Sec. 21.1.1.7 specifies that both ordinary reinforced concrete moment frames and ordinary reinforced concrete shear walls must comply with ACI Chap. 1 through Chap. 18, but compliance with Chap. 21 is not required. Ordinary reinforced concrete shear walls assigned to seismic design category B do not need to comply with Chap. 21 requirements. For ordinary reinforced concrete moment frames assigned to seismic design category B, compliance requirements are given in ACI Sec. 21.2. These requirements are that beams must have at least two longitudinal bars continuous along both the top and bottom faces, and that

columns with a clear height less than or equal to five times the column width must be designed for shear in accordance with ACI Sec. 21.3.3.2.

Cantilever Column Systems

As shown in Fig. 7.18, a *cantilevered column system* consists of a structure supported on columns cantilevering from their base. This system lacks redundancy, as the inelastic behavior necessary to absorb the input seismic energy is concentrated only at the base of the columns, and this produces a sidesway collapse mechanism. Additionally, the excessive flexibility of the system leads to excessive drift and consequent P-delta instability. The system has a low value for the response modification coefficient, R, and is restricted in height in all seismic design categories.

Figure 7.18 *Cantilever Column System*

cantilever column
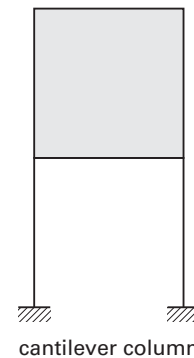
Steel Systems Not Specifically Detailed for Seismic Resistance

AISC 360[19] provides criteria for the design of structural steel buildings. It is specifically intended for low-seismic applications where design is based on a seismic response modification coefficient, R, less than 3. AISC 341[20] provides criteria for the design of structural steel buildings and is specifically intended for high-seismic applications where design is based on a seismic response modification coefficient greater than 3. In accordance with IBC Sec. 2205.2.2, steel building structures assigned to seismic design categories D, E, or F, must be designed and detailed as specified by AISC 341. In accordance with IBC Sec. 2205.2.1, AISC 341 may also be used to design and detail steel building structures assigned to seismic design categories B or C. When AISC 341 is used, the seismic loads are computed using the response modification coefficient given in ASCE/SEI7 Table 12.2-1.

In accordance with IBC Sec. 2205.2.1, steel building structures assigned to seismic design categories B or C, with the exception of cantilever column systems, may also be designed and detailed as specified by AISC 360. When AISC 360 is used, the seismic loads are computed using a response modification coefficient of 3, which

ensures a nominally elastic response to the applied loads. This may often result in a structure that is less expensive to build. For seismic design category A, special detailing is not required, and steel building structures may be designed and detailed as specified by AISC 360.

When wind effects exceed seismic effects, the building elements must still be detailed in accordance with AISC 341 provisions. These provisions provide the design requirements for structural steel seismic force-resisting systems to sustain the large inelastic deformations necessary to dissipate the seismic-induced demand. The AISC's *Seismic Design Manual*[21] provides guidance on the application of the provisions to the design of structural steel seismic force-resisting systems.

4. DIAPHRAGMS

Nomenclature

A_x	amplification factor	–
B	width of diaphragm	ft
F	force	kips
J	$\Sigma r_i^2 R_i$	ft-kips
r_i	distance of wall i from the center of rigidity	ft
R	rigidity	kips/ft
R_i	rigidity of wall i	kips/ft
T	torsion	ft-kips
V	shear	kips

Symbols

δ_{avg}	average of displacements at extreme points of the structure at level x computed assuming $A_x = 1$	in
δ_{max}	maximum displacement at level x computed assuming $A_x = 1$	in
Δ_{avg}	average story drift	in
Δ_D	deflection of diaphragm	in

In accordance with ASCE/SEI7 Sec. 12.3.1, diaphragms are classified as flexible, semirigid, or rigid.

Flexible Diaphragms

A *flexible diaphragm*, as defined in ASCE/SEI7 Sec. 12.13.1.3, is a diaphragm whose lateral deformation under a lateral load is more than twice the average story drift of the adjoining vertical elements of the lateral force-resisting system. This is illustrated in Fig. 7.19. To qualify as a flexible diaphragm, the diaphragm must satisfy the following requirement.

$$\Delta_D > 2\Delta_{avg}$$

When subjected to a transverse force, a flexible diaphragm undergoes lateral displacement without rotation, and a lateral force is distributed to the vertical seismic-load-resisting elements based on tributary areas.

Figure 7.19 Flexible Diaphragm
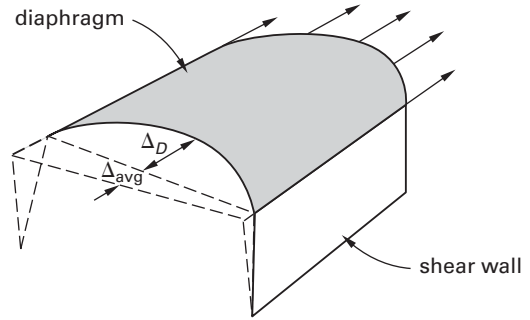
In accordance with ASCE/SEI7 Sec. 12.3.1.1, the following types of diaphragms may be considered flexible.

- untopped steel decking or wood structural panels supported by vertical elements of steel or composite braced frames, or by concrete, masonry, steel, or composite shear walls

- untopped steel decking or wood structural panels in one- and two-family residential buildings of light-frame construction

In addition, in structures of light-frame construction, diaphragms of untopped steel decking or wood structural panels are considered flexible, provided all of the following conditions are met.

- Toppings of concrete or similar materials are not placed over wood structural panel diaphragms, except for nonstructural toppings no greater than 1.5 in thick.

- Each line of the lateral force-resisting system complies with the allowable story drift of ASCE/SEI7 Table 12.12-1.

Semirigid Diaphragm

Diaphragms not satisfying the requirements of ASCE/SEI7 Sec. 12.3.1.1, Sec. 12.3.1.2, or Sec. 12.3.1.3 are considered semirigid. Analysis of a structure with semirigid diaphragms must include consideration of the actual stiffness of the diaphragm.

Rigid Diaphragm

A *rigid diaphragm*, as defined in IBC Sec. 202, is a diaphragm that, for the purpose of distributing story shear and torsional moment, has a lateral deformation that is less than or equal to twice the average story drift. Diaphragms of concrete slabs or of concrete filled metal decks with span-to-depth ratios of 3 or less in structures that have no horizontal irregularities are considered by ASCE/SEI7 Sec. 12.3.1.2 to be rigid.

How a rigid diaphragm distributes lateral force to the vertical seismic-load-resisting elements depends on two things: first, the relative rigidity of these elements, and second, the torsional displacements produced by the rigid-body rotation of the diaphragm. To calculate the

Lateral Forces

torsional displacements, the center of rigidity and the center of mass for the structure must both be known. The *center of rigidity* is the point about which a structure rotates when subjected to a torsional moment. The *center of mass* is the point through which the lateral force, V, acts. For the structure shown in Fig. 7.20(a), the torsional moment acting on the diaphragm is defined as $T = Ve$, where e is the eccentricity of the center of mass with respect to the center of rigidity.

Figure 7.20 *Rigid Diaphragm*

(a) layout   (b) in-plane   (c) torsional
                          forces   forces
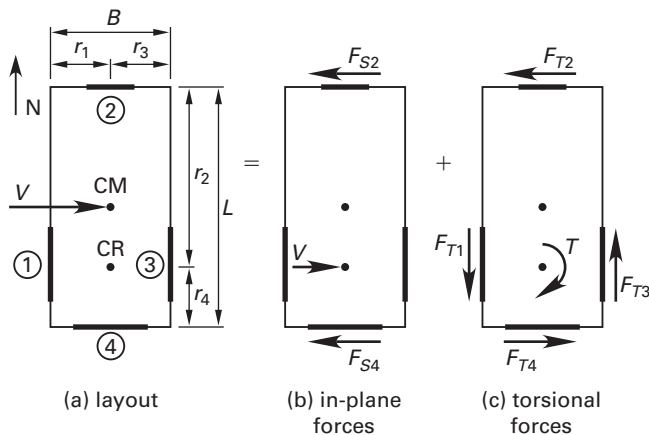
The displacement of the building consists of an east-west translation and a clockwise rotation about the center of rigidity. As shown in Fig. 7.20(b), the translation produces in-plane forces in vertical elements 2 and 4 proportional to their relative translational stiffness. The in-plane shear force, F_{Si}, in wall i is

$$F_{Si} = \frac{VR}{\sum R_i}$$

No forces are produced in vertical elements 1 and 3 by this translation, and $\Sigma R_i = R_2 + R_4$. The clockwise rotation produces forces in all four walls, proportional to their torsional stiffnesses, as shown in Fig. 7.20(c). The torsional shear force in wall i is

$$F_{Ti} = \frac{Tr_iR_i}{J}$$

The total force in wall i is

$$F_i = F_{Si} + F_{Ti}$$

In a perfectly symmetric building, the centers of mass and rigidity coincide, and torsion is not produced. However, the centers of mass and rigidity may not be calculated accurately because of uncertainties in determining the mass and stiffness distribution in the building. In addition, torsional components of the ground motion may also cause torsion to develop. Therefore, accidental eccentricity may in fact exist even in a nominally symmetric structure. Torsion resulting from this accidental eccentricity is referred to as *accidental torsion*. To

account for accidental torsion, ASCE/SEI7 Sec. 12.8.4.2 specifies that the center of mass is assumed displaced each way from its actual location by a distance equal to 5% of the building dimension perpendicular to the direction of the applied force.

When a building that is assigned to a seismic design category C through F has a torsional irregularity as defined in ASCE/SEI7 Table 12.3-1 (horizontal structural irregularity type 1a or 1b), the accidental torsion is amplified as specified in ASCE/SEI7 Sec. 12.8.4.3. The *amplification factor* is given by ASCE/SEI7 Eq. 12.8-14 as

$$A_x = \left(\frac{\delta_{\max}}{1.2\delta_{\mathrm{avg}}}\right)^2$$

In accordance with ASCE/SEI7 Sec. 12.3.3.1, structures assigned to seismic design category E or F with torsional irregularity type 1b are not permitted.

Example 7.5

The single story building shown in Fig. 7.20 has a rigid roof diaphragm that is acted on by an east-west force of 40 kips. Determine the force produced on shear wall 2 if the center of gravity is located at the center of the building. The building dimensions and the relative wall rigidities are as follows.

$$L = 80 \text{ ft}$$
$$B = 40 \text{ ft}$$
$$R_4 = 3R$$
$$R_1 = R_2 = R_3 = R$$

Neglect accidental eccentricity.

Solution

From the problem statement, the distance from wall 4 to the center of mass is

$$x = \frac{80 \text{ ft}}{2} = 40 \text{ ft}$$

Due to symmetry, the center of rigidity is located midway between wall 1 and wall 3.

$$r_1 = r_3$$
$$= 20 \text{ ft}$$

The distance from wall 4 to the center of rigidity is

$$r_4 = \frac{R_2 L}{R_2 + R_4}$$
$$= \frac{(1R)(80 \text{ ft})}{1R + 3R}$$
$$= 20 \text{ ft}$$

The distance of the center of rigidity from wall 2 is

$$r_2 = L - r_4 = 80 \text{ ft} - 20 \text{ ft}$$
$$= 60 \text{ ft}$$

The sum of the values of $r^2 R$ for the four walls is

$$J = \sum r_i^2 R_i = r_1^2 R_1 + r_2^2 R_2 + r_3^2 R_3 + r_4^2 R_4$$
$$= (20 \text{ ft})^2 (1R) + (60 \text{ ft})^2 (1R) + (20 \text{ ft})^2 (1R)$$
$$\quad + (20 \text{ ft})^2 (3R)$$
$$= (5600 \text{ ft}^2) R$$

For a seismic load in the east-west direction, the eccentricity is

$$e = x - r_4 = 40 \text{ ft} - 20 \text{ ft}$$
$$= 20 \text{ ft}$$

The torsional moment acting about the center of rigidity is

$$T = Ve = (40 \text{ kips})(20 \text{ ft})$$
$$= 800 \text{ ft-kips}$$

The torsional shear force in wall 2 is

$$F_{T2} = \frac{Tr_2 R_2}{J} = \frac{(800 \text{ ft-kips})(60 \text{ ft})(1R)}{(5600 \text{ ft}^2) R}$$
$$= 8.57 \text{ kips}$$

Illustration for Ex. 7.6
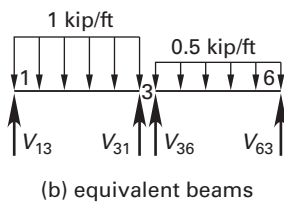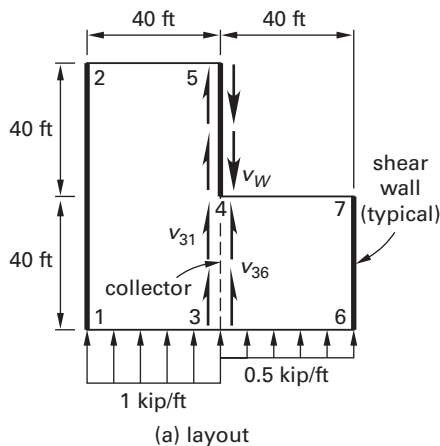
(a) layout

(b) equivalent beams

(c) unit shear

(d) net shear

(e) drag force

The in-plane shear force in wall 2 is

$$F_{S2} = \frac{VR_2}{R_2 + R_4}$$
$$= \frac{(40 \text{ kips})(1R)}{1R + 3R}$$
$$= 10 \text{ kips}$$

The total force in wall 2 is

$$F_2 = F_{S2} + F_{T2}$$
$$= 10 \text{ kips} + 8.57 \text{ kips}$$
$$= 18.57 \text{ kips}$$

Collectors

A *collector*, also known as a *drag strut*, is defined in IBC Sec. 202 as a horizontal diaphragm element, parallel and in line with the applied force, that collects and transfers diaphragm shear forces to the vertical elements of the lateral force-resisting system. Collectors are required where shear walls or braced frames terminate along the boundary of a diaphragm.

Example 7.6

A single-story building has a flexible roof diaphragm and masonry shear walls, and has a north-south seismic force acting on the building, as shown. Determine the force in collector 34.
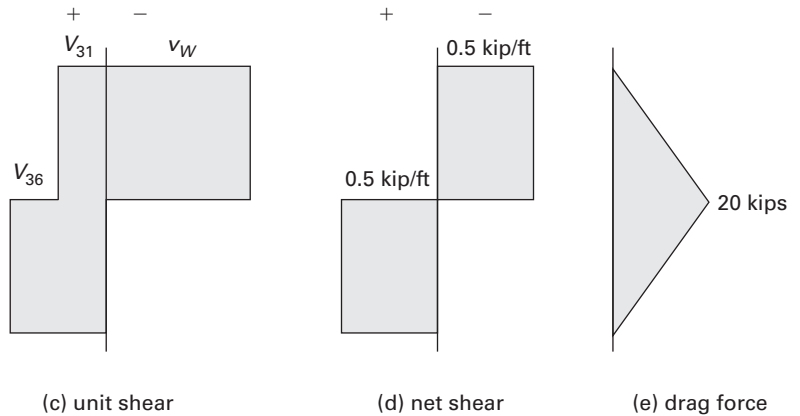
Solution

As shown in the illustration part (a), shear wall 45 terminates at the diaphragm boundary, and drag strut 34 is required to anchor the diaphragm to the shear wall. The diaphragm is flexible, and shear wall 45 effectively subdivides the diaphragm into two simply supported segments 12543 and 3476. These may be treated as two independent simply supported beams, 13 and 36, as shown in the illustration part (b). The equivalent beam reactions, which represent the shear force at the boundaries of each diaphragm, are

$$V_{31} = \frac{\left(1\ \dfrac{\text{kip}}{\text{ft}}\right)(40\ \text{ft})}{2}$$
$$= 20\ \text{kips}$$

$$V_{36} = \frac{\left(0.5\ \dfrac{\text{kip}}{\text{ft}}\right)(40\ \text{ft})}{2}$$
$$= 10\ \text{kips}$$

The unit shears at the boundaries of each diaphragm are

$$v_{31} = \frac{V_{31}}{L_{35}}$$
$$= \frac{20\ \text{kips}}{80\ \text{ft}}$$
$$= 0.25\ \text{kip/ft}$$

$$v_{36} = \frac{V_{36}}{L_{34}}$$
$$= \frac{10\ \text{kips}}{40\ \text{ft}}$$
$$= 0.25\ \text{kip/ft}$$

The unit shears acting on the diaphragms are shown in the illustration part (c).

The shear wall 45 resists a shear force of

$$V_W = V_{31} + V_{36}$$
$$= 20\ \text{kips} + 10\ \text{kips}$$
$$= 30\ \text{kips}$$

The unit shear in the shear wall is

$$v_W = \frac{V_W}{L_{45}}$$
$$= \frac{30\ \text{kips}}{40\ \text{ft}}$$
$$= 0.75\ \text{kip/ft}$$

The unit shear acting on the shear wall is shown in the illustration part (c).

The net shear along the diaphragm boundary at 54 is

$$v_{54} = v_{31} - v_W$$
$$= 0.25\ \frac{\text{kip}}{\text{ft}} - 0.75\ \frac{\text{kip}}{\text{ft}}$$
$$= -0.50\ \text{kip/ft}$$

The net shear along the diaphragm interface at 34 is

$$v_{34} = v_{31} + v_{36}$$
$$= 0.25\ \frac{\text{kip}}{\text{ft}} + 0.25\ \frac{\text{kip}}{\text{ft}}$$
$$= 0.50\ \text{kip/ft}$$

The net shears are plotted as shown in the illustration part (d).

The maximum drag force occurs at the end of the collector, at the connection to the shear wall. It is given by

$$F = v_{34} L_{34}$$
$$= \left(0.50\ \frac{\text{kip}}{\text{ft}}\right)(40\ \text{ft})$$
$$= 20\ \text{kips}$$

The drag force diagram is shown in the illustration part (e).

Subdiaphragms[22]

Nomenclature

a_p	component amplification factor from ASCE/SEI7 Table 13.5-1 is 2.5 for an unbraced parapet	–
F_P	force on diaphragm, force on wall	lbf
h	height of roof above the base	ft
I_e	occupancy importance factor	–
I_p	component importance factor given in ASCE/SEI7 Sec. 13.1.3	–
k_a	amplification factor for diaphragm flexibility	–
L_f	span of a flexible diaphragm	ft
R_p	component response modification factor from ASCE/SEI7 Table 13.5-1 is 2.5 for an unbraced parapet	–
S_{DS}	design response acceleration at a period of 0.2 second	–
W_P	weight of the wall tributary to the anchor	lbf
z	height of point of attachment of parapet above the base $= h$	ft

In seismic design categories C through F, ASCE/SEI7 Sec. 12.11.2.2.1 requires that continuous ties be provided across the complete depth of the diaphragm. This is to transfer the diaphragm anchorage forces across the depth of the diaphragm and to prevent the walls and diaphragm from separating. To reduce the number of full depth ties required, *subdiaphragms* and added chords are used to span between the full depth ties. The maximum permitted length-to-width ratio of the subdiaphragm is 2.5 to 1.

A typical arrangement of subdiaphragms and crossties is shown in Fig. 7.21. IBC Sec. 1604.8.2 requires all structural walls to be anchored to the diaphragm to prevent separation of the walls from the diaphragm. Wall anchors are provided at the ends of the subdiaphragm ties, and must have a capacity to resist the minimum horizontal force. For walls of structures assigned to seismic design category A, this force is given in ASCE/SEI7 Sec. 1.4.5 as

$$F_p = 0.2 W_p$$
$$\geq 5 \text{ lbf/ft}^2$$

The subdiaphragm ties transfer the anchor force to the subdiaphragm chords which, in turn, transfer the anchor force to the continuous crossties. In accordance with ASCE/SEI7 Sec. 12.11.2.2.3 and Sec. 12.11.2.2.4, the continuous ties must be in addition to plywood sheathing or metal deck that is considered ineffective in providing the ties.

Figure 7.21 *Subdiaphragms and Crossties*

subdiaphragm
subdiaphragm ties (typical)
crossties
subdiaphragm chord
plan view
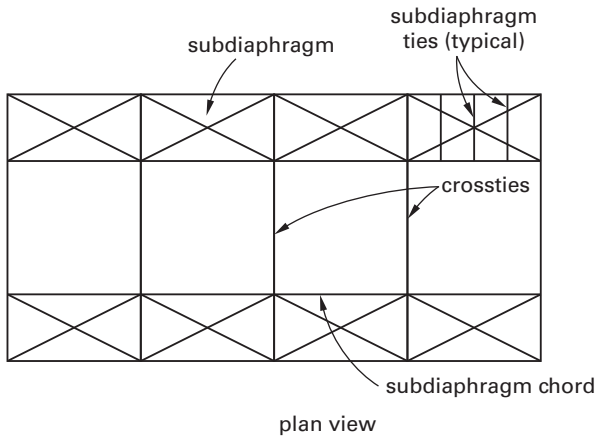
Lateral Design Force on Walls and Parapets

The out-of-plane seismic force on a wall is specified in ASCE/SEI7 Sec. 12.11.1 as

$$F_p = 0.4 S_{DS} I_e W_p$$
$$\geq 0.1 W_p$$

ASCE/SEI7 Sec. 13.3.1 requires parapets in seismic design category B through F to be designed as architectural components using

$$F_p = \frac{0.4 a_p S_{DS} W_p}{\dfrac{R_p}{I_p}} \left(1 + 2\frac{z}{h}\right) \quad [\textit{ASCE/SEI7 13.3-1}]$$
$$\leq 1.6 S_{DS} I_p W_p \quad [\textit{ASCE/SEI7 13.3-2}]$$
$$\geq 0.3 S_{DS} I_p W_p \quad [\textit{ASCE/SEI7 13.3-3}]$$

The poor seismic performance and the lack of redundancy of parapets can create a safety hazard to the public. Therefore, parapets are designed for a higher design load than walls, and a high value of 2.5 is assigned to the component amplification factor, a_p. The lateral force is considered uniformly distributed over the height of the parapet.

Anchorage of Structural Walls to Flexible Diaphragms

During past earthquakes, a major cause of failure has been the separation of flexible diaphragms from concrete and masonry supporting walls. This separation is due to the diaphragm flexibility amplifying out-of-plane accelerations. To prevent separation from occurring, supporting walls must be securely anchored to the subdiaphragm ties. Where the wall anchor spacing exceeds 4 ft, in accordance with ASCE/SEI7 Sec. 12.11.2.1, the wall must be designed to span between anchors. In accordance with ASCE/SEI7 Sec. 12.11.2.2.2, steel elements in the anchorage system are required to resist 1.4 times the calculated anchorage force.

For buildings assigned to seismic design categories B through F, ASCE/SEI7 Sec. 12.11.2.1 requires anchors to be designed for the force as

$$F_p = 0.4 S_{DS} k_a I_e W_p \quad [\textit{ASCE/SEI7 12.11-1}]$$
$$\geq 0.2 k_a I_e W_p$$
$$k_a = 1.0 + \frac{L_f}{100} \quad [\textit{ASCE/SEI7 12.11-2}]$$

k_a is the amplification factor for diaphragm flexibility. L_f is the span, in feet, of a flexible diaphragm measured between vertical elements. These vertical elements provide lateral support to the diaphragm in the direction considered. For rigid diaphragms, $L_f = 0$.

Anchorage of Structural Walls to Rigid Diaphragms

For buildings with rigid diaphragms assigned to seismic design categories B through F, ASCE/SEI7 Sec. 12.11.2.1 requires anchors that are not at roof level to be designed for the force as

$$F_p = 0.4 S_{DS} I_e W_p \left(\frac{1 + \dfrac{2z}{h}}{3}\right)$$
$$\geq 0.2 I_e W_p$$

Anchorage force for rigid diaphragms at roof level is determined from ASCE/SEI7 Eq. 12.11-1 with $k_a = 1.0$, which gives

$$F_p = 0.4 S_{DS} I_e W_p$$
$$\geq 0.2 I_e W_p$$

Example 7.7

For the north-south direction, design a suitable subdiaphragm arrangement for the roof diaphragm shown in the following illustration. The blocked plywood diaphragm is flexible, and the concrete walls have a weight, w, of 75 lbf/ft^2. The building is assigned to seismic design category D with an importance factor of $I_e = 1.0$ and a design response acceleration, S_{DS}, of 1.0g.

plan view

section A-A
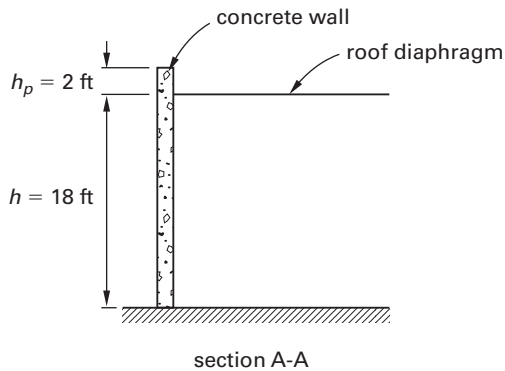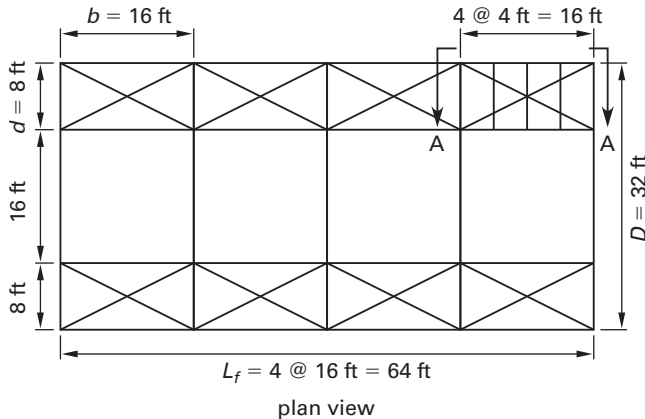
Solution

For the layout shown in the illustration, the aspect ratio of the whole diaphragm is

$$\frac{B}{D} = \frac{64 \text{ ft}}{32 \text{ ft}}$$
$$= 2$$
$$< 4 \quad [\text{conforms to SDPWS Sec. 4.2.4}]$$

The aspect ratio of the subdiaphragm is

$$\frac{b}{d} = \frac{16 \text{ ft}}{8 \text{ ft}}$$
$$= 2$$
$$< 2.5 \quad [\text{conforms to ASCE/SEI7 Sec. 12.11.2.2.1}]$$

Provide wall anchors at each subdiaphragm tie. Spacing, s, is 4 ft on center. The equivalent area of wall tributary to each anchor is obtained by taking moments about the base of the wall.

$$A = \frac{s(h + h_p)^2}{2h}$$
$$= \frac{(4 \text{ ft})(20 \text{ ft})^2}{(2)(18 \text{ ft})}$$
$$= 44.44 \text{ ft}^2$$

The weight of wall tributary to each anchor is

$$W_p = wA$$
$$= \left(75 \ \frac{\text{lbf}}{\text{ft}^2}\right)(44.44 \text{ ft}^2)$$
$$= 3333 \text{ lbf}$$

The span of the flexible diaphragm is

$$L_f = 64 \text{ ft}$$

The amplification factor for diaphragm flexibility is

$$k_a = 1.0 + \frac{L_f}{100}$$
$$= 1.0 + \frac{64 \text{ ft}}{100 \text{ ft}}$$
$$= 1.64$$

For seismic design category D, the seismic lateral force on an anchor is given by ASCE/SEI7 Eq. 12.11-1 as

$$F_p = 0.4 S_{DS} k_a I_e W_p$$
$$= (0.4)(1.0)(1.64)(1.0)(3333 \text{ lbf})$$
$$= 2186 \text{ lbf} \quad [\text{governs}]$$

The minimum permissible force on an anchor is

$$F_p = 0.2 k_a I_e W_p$$
$$= (0.2)(1.64)(1.0)(3333 \text{ lbf})$$
$$= 1093 \text{ lbf}$$
$$< 2186 \text{ lbf}$$

The required seismic design force for the anchors is

$$F_p = 2186 \text{ lbf}$$

Therefore, the pull-out force, p, along the wall is

$$p = \frac{F_p}{s} = \frac{2186 \text{ lbf}}{4 \text{ ft}}$$
$$= 547 \text{ lbf/ft}$$

The unit shear stress at the subdiaphragm boundary is

$$v = \frac{pb}{2d}$$

$$= \frac{\left(547 \ \dfrac{\text{lbf}}{\text{ft}}\right)(16 \text{ ft})}{(2)(8 \text{ ft})}$$

$$= 547 \text{ lbf/ft}$$

The subdiaphragm chord force is

$$P_c = \frac{pb^2}{8d}$$

$$= \frac{\left(547 \ \dfrac{\text{lbf}}{\text{ft}}\right)(16 \text{ ft})^2}{(8)(8 \text{ ft})}$$

$$= 2188 \text{ lbf}$$

The force in the crossties is

$$P_t = pb$$

$$= \left(547 \ \dfrac{\text{lbf}}{\text{ft}}\right)(16 \text{ ft})$$

$$= 8752 \text{ lbf}$$

PART 2: SEISMIC DESIGN

Nomenclature

A_x	area of diaphragm immediately above the story	ft^2
C_d	deflection amplification factor from ASCE/SEI7 Table 12.2-1	–
C_s	seismic response coefficient specified in ASCE/SEI7 Sec. 12.8.1	–
C_u	coefficient for upper limit on calculated period from ASCE/SEI7 Table 12.8-1	–
D	dead load applied to a structural element	lbf or kips
E	calculated seismic load on an element of a structure resulting from both horizontal and vertical earthquake induced forces as given by ASCE/SEI7 Eq. 12.4-1 and Eq. 12.4-2	lbf or kips
f_i	design seismic lateral force at level i	lbf or kips
F_a	short-period amplification factor	–
F_p	force on diaphragm	lbf or kips
F_v	long-period amplification factor	–
F_x	design seismic lateral force at level x as specified in ASCE/SEI7 Sec. 12.8.3	lbf or kips
g	gravitational acceleration, 32.2	ft/sec^2
h_i	height above the base to level i	ft
h_n	height of the roof above the base, not including the height of penthouses or parapets	ft
h_{sx}	story height below level x	ft
h_x	height above the base to level x	ft

I_e	seismic importance factor	–
k	distribution exponent given in ASCE/SEI7 Sec. 12.8.3	–
L	superimposed floor live load	lbf or kips
L_r	superimposed roof live load	lbf or kips
M_P	primary moment	kip-ft
M_S	secondary moment	kip-ft
N	number of stories	–
P_x	total unfactored vertical design load at and above level x	lbf or kips
Q_E	effect of horizontal seismic forces	lbf or kips
R	response modification coefficient for a specific structural system from ASCE/SEI7 Table 12.2-1	–
s_i	stiffness of story i	lbf/in or kips/in
S	snow load applied to a structural element	lbf or kips
S_1	maximum considered response acceleration for a period of 1.0 sec	–
S_a	design spectral response acceleration	–
S_{DS}	design spectral response acceleration at a period of 0.2 sec	–
S_{D1}	design spectral response acceleration at a period of 1.0 sec	–
S_{MS}	modified spectral response acceleration at a period of 0.2 sec	–
S_{M1}	modified spectral response acceleration at a period of 1.0 sec	–
S_S	maximum considered response acceleration for a period of 0.2 sec	–
T	fundamental period of vibration, defined in ASCE/SEI7 Sec. 12.8.2	sec
T_a	approximate fundamental period of vibration determined using ASCE/SEI7 Sec. 12.8.2.1	sec
T_L	long-period transition period	sec
T_0	defined in ASCE/SEI7 Sec. 11.4.5 as $0.2S_{D1}/S_{DS}$	–
T_S	defined in ASCE/SEI7 Sec. 11.4.5 as S_{D1}/S_{DS}	–
V	total seismic base shear	lbf or kips
V_x	total shear force at level x	lbf or kips
V_S	design base shear	–
V_Y	base shear at formation of the collapse mechanism	–
w_i	seismic dead load located at level i	lbf or kips
w_p	seismic dead load tributary to diaphragm	lbf or kips
w_x	seismic dead load located at level x	lbf or kips
W	wind load applied to a structural element	lbf or kips
W	effective seismic weight defined in ASCE/SEI7 Sec. 12.7.2	lbf or kips
$\sum F_i$	total shear force at level i	lbf or kips
$\sum w_i$	total seismic dead load at level i and above	lbf or kips

Symbols

β	ratio of shear demand to shear capacity for the story between levels x and x-1 as defined in ASCE/SEI7 Sec. 12.8.7	–
δ_x	amplified horizontal deflection at level x, defined in ASCE/SEI7 Sec. 12.8.6	in
δ_{xe}	horizontal deflection at level x, determined by an elastic analysis, as defined in ASCE/SEI7 Sec. 12.8.6	in
Δ	design story drift, occurring simultaneously with the story shear V_x, defined in ASCE/SEI7 Sec. 12.8.6, and calculated using the amplification factor C_d	in
Δ_a	allowable story drift, defined in ASCE/SEI7 Table 12.12-1	in
Ω_0	overstrength factor tabulated in ASCE/SEI7 Table 12.2-1	–
θ	stability coefficient defined in ASCE/SEI7 Sec. 12.8.7	–

5. EQUIVALENT LATERAL FORCE PROCEDURE

Determination of the seismic response of a structure depends on several factors, including ground motion parameters, site classification, site coefficient, adjusted response acceleration, design spectral response acceleration, importance factor, seismic design category, classification of the structural system, response modification coefficient, deflection amplification factor, overstrength factor, effective seismic weight, fundamental period of vibration, and seismic response coefficient. A summary of these factors follows.

Ground Motion Parameters

Ground motion parameters defined in ASCE/SEI7 Sec. 11.4.1 are values of the maximum considered ground acceleration that may be experienced at a specific location. As defined in ASCE/SEI7 Sec. 11.2, these are the most severe earthquake effects considered by the code. The parameters are risk-adjusted to provide a uniform risk with a 1% probability of collapse in 50 years. Two values of the ground acceleration are required, and these are designated S_S and S_1. S_S represents the 5% damped, maximum considered earthquake spectral response acceleration for a period of 0.2 sec for structures founded on rock (site classification B) and is applicable to short-period structures. S_1 represents the 5% damped, maximum considered earthquake spectral response acceleration for a period of 1 sec for structures founded on rock and is applicable to structures with longer periods. Values of the ground accelerations S_S and S_1 are mapped in ASCE/SEI7 Fig. 22-1 through Fig. 22-17. The parameters are given as a percentage of the acceleration due to gravity.

Site Classification Characteristics

Site classification is defined in ASCE/SEI7 Sec. 11.4.2 and ASCE/SEI7 Table 20.3-1. Six different soil types are specified and range from site class A, which consists of hard rock, through site class F, which consists of peat, highly plastic clay, or collapsible soil. The soil profile may be determined on site from the average shear wave velocity in the top 100 ft of material. Alternatively, for site classification types C, D, or E, the classification may be made by measuring the standard penetration resistance or undrained shear strength of the material. An abbreviated listing of the site classifications is provided in Table 7.2.

Table 7.2 *Site Classification Definitions*

site classification	soil profile name	shear wave velocity (ft/sec)
A	hard rock	> 5000
B	rock	2500 to 5000
C	soft rock	1200 to 2500
D	stiff soil	600 to 1200
E	soft soil	< 600
F	–	–

Soil classification type A has the effect of reducing the ground response by 20%. Soil classification type E is defined as soft soil and has the effect of increasing the long period ground response by up to 350%. When soil parameters are unknown, in accordance with ASCE/SEI7 Sec. 11.4.2, soil classification type D may be assumed unless the building official determines that soil classification types E or F are likely to be present at the site.

Site Coefficients

Site coefficients are amplification factors applied to the maximum considered ground acceleration and are a function of the site classification. F_a is the short-period or acceleration-based amplification factor and is tabulated in ASCE/SEI7 Table 11.4-1. F_v is the long-period or velocity-based amplification factor and is tabulated in ASCE/SEI7 Table 11.4-2. ASCE/SEI7 Table 11.4-1 and Table 11.4-2 are combined and reproduced in Table 7.3. Linear interpolation may be used to obtain intermediate values.

Adjusted Response Accelerations

The maximum considered ground accelerations must be adjusted by the site coefficients to allow for the site classification effects. ASCE/SEI7 Sec. 11.4.3 defines the modified spectral response accelerations at short periods and at a period of 1 sec as

$$S_{MS} = F_a S_S \qquad [\textit{ASCE/SEI7 11.4-1}]$$

$$S_{M1} = F_v S_1 \qquad [\textit{ASCE/SEI7 11.4-2}]$$

Table 7.3 *Site Coefficients F_a corresponding to S_S, and F_v corresponding to S_1*

site classification	response acceleration, S_S					response acceleration, S_1				
	≤ 0.25	0.50	0.75	1.00	≥ 1.25	≤ 0.1	0.2	0.3	0.4	≥ 0.5
A	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0	1.7	1.6	1.5	1.4	1.3
D	1.6	1.4	1.2	1.1	1.0	2.4	2.0	1.8	1.6	1.5
E	2.5	1.7	1.2	0.9	0.9	3.5	3.2	2.8	2.4	2.4
F	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)	(a)

Note: (a) Site-specific geotechnical investigation and dynamic site response analysis required except for structures with $T \leq 0.5$ sec.

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Design Spectral Response Acceleration Parameters

The relevant design parameters are defined in ASCE/SEI7 Sec. 11.4.4 and are given by

$S_{DS} = 5\%$ damped design spectral response

acceleration for a period of 0.2 sec

$$= \frac{2S_{MS}}{3} \qquad [ASCE/SEI7\ 11.4\text{-}3]$$

$S_{D1} = 5\%$ damped design spectral response

acceleration for a period of 1 sec

$$= \frac{2S_{M1}}{3} \qquad [ASCE/SEI7\ 11.4\text{-}4]$$

Example 7.8

The two-story, reinforced concrete, moment-resisting frame shown in the following illustration is located on a site with a soil profile of stiff soil having a shear wave velocity of 600 ft/sec. The 5% damped, maximum considered earthquake spectral response accelerations are obtained from the ASCE/SEI7 standard. They are $S_S = 1.5g$ and $S_1 = 0.7g$. Determine the 5% damped design spectral response accelerations S_{DS} and S_{D1}.
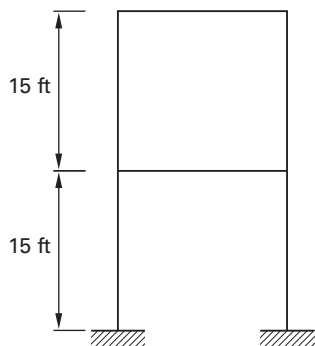
Solution

From ASCE/SEI7 Table 20.3-1 or from Table 7.2, the applicable site classification for stiff soil with a shear wave velocity of 600 ft/sec is site classification D. The site coefficients for this site classification and for the given values of the 5% damped, maximum considered earthquake spectral response accelerations are obtained from ASCE/SEI7 Table 11.4-1 and Table 11.4-2 or from Table 7.3 as

$$F_a = 1.0$$
$$F_v = 1.5$$

The adjusted spectral response accelerations are given by ASCE/SEI7 Sec. 11.4.3 as

$$S_{MS} = F_a S_S$$
$$= (1.0)(1.5g)$$
$$= 1.5g$$
$$S_{M1} = F_v S_1$$
$$= (1.5)(0.7g)$$
$$= 1.05g$$

The 5% damped design spectral response accelerations are given by ASCE/SEI7 Sec. 11.4.4 as

$$S_{DS} = \frac{2S_{MS}}{3}$$
$$= \frac{(2)(1.5g)}{3}$$
$$= 1.0g$$
$$S_{D1} = \frac{2S_{M1}}{3}$$
$$= \frac{(2)(1.05g)}{3}$$
$$= 0.7g$$

Lateral Forces

Risk Category and Importance Factors

In accordance with ASCE/SEI7 Table 1.5-1, each structure is assigned to a risk category, depending on the nature of its *occupancy*, with the corresponding *importance factor* indicated in ASCE/SEI7 Table 1.5-2. Table 7.4 lists the risk categories, seismic use groups, and seismic importance factors.

Table 7.4 *Occupancies and Importance Factors*

risk category	occupancy type	importance factor, I_e
I	low hazard structures	1.00
II	standard occupancy structures	1.00
III	assembly structures	1.25
IV	essential or hazardous structures	1.50

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Category IV structures are those housing essential facilities that are required for post-earthquake recovery. Also included in category IV are structures containing substantial quantities of highly toxic substances that would endanger the safety of the public if released. Essential facilities are defined in IBC Table 1604.5 as hospitals, fire and police stations, emergency response centers, and buildings housing utilities and equipment required for these facilities. In order to ensure that category IV facilities remain functional after an upper level earthquake, an importance factor, I_e, of 1.5 is assigned to these facilities. This has the effect of increasing the design seismic forces by 50%, and raises the seismic level at which inelastic behavior occurs and the level at which the operation of essential facilities is compromised.

Category III structures are facilities that, if they failed, would become a substantial public hazard because of their high occupant load. These facilities are buildings where more than 300 people congregate in one area, schools with a capacity exceeding 250, colleges with a capacity exceeding 500, health care facilities with a capacity of 50 or more that do not have emergency treatment facilities, jails, and power stations. Also included are facilities containing explosive or toxic substances in a quantity exceeding the exempt amounts in IBC Table 307.1(1). These structures are allocated a seismic importance factor, I_e, of 1.25.

Category II structures comprise standard occupancy structures and are allocated an importance factor, I_e, of 1.00. Standard occupancy structures consist of residential, commercial, and office buildings.

Category I structures comprise low-hazard structures and are allocated an importance factor, I_e, of 1.00.

Low-hazard structures consist of agricultural facilities, temporary facilities and minor storage facilities.

Determination of Seismic Design Category

Structures are assigned to a *seismic design category* A through F based on their risk category and the design spectral response coefficients S_{DS} and S_{D1}. The seismic design category is defined in ASCE/SEI7 Sec. 11.6 and ASCE/SEI7 Table 11.6-1 and Table 11.6-2, and establishes the design and detailing requirements necessary in a structure. The seismic design category is determined twice, first as a function of S_{DS} using ASCE/SEI7 Table 11.6-1, and then as a function of S_{D1} using ASCE/SEI7 Table 11.6-2. The most severe seismic design category governs. ASCE/SEI7 Table 11.6-1 and ASCE/SEI7 Table 11.6-2 are combined and reproduced in Table 7.5.

Table 7.5 *Seismic Design Categories*

		risk category		
S_{DS}	S_{D1}	I or II	III	IV
$S_{DS} < 0.167g$	$S_{D1} < 0.067g$	A	A	A
$0.167g \leq S_{DS} < 0.33g$	$0.067g \leq S_{D1} < 0.133g$	B	B	C
$0.33g \leq S_{DS} < 0.50g$	$0.133g \leq S_{D1} < 0.20g$	C	C	D
$0.50g \leq S_{DS}$	$0.20g \leq S_{D1}$	D	D	D

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Six seismic design categories are defined, categories A through F, and these establish the design and detailing requirements necessary in a structure. Seismic design category A is applicable to structures in locations where anticipated ground movements are minimal. ASCE/SEI7 Sec. 11.7 specifies requirements to ensure the integrity of the structure in the event of a minor earthquake. Seismic design category B is applicable to structures in risk categories I, II, and III in regions of moderate seismicity. Seismic design category C is applicable to category IV structures in regions of moderate seismicity as well as structures in risk categories I, II, and III in regions of somewhat more severe seismicity. The use of some structural systems is restricted in this design category. Plain concrete and masonry structures are not permitted. Seismic design category D includes structures in risk categories I, II, III, and IV in regions of high seismicity, but not located close to a major active fault, as well as risk category IV structures in regions with less severe seismicity. In this design category some types of structural systems must be designed by dynamic analysis methods. Seismic design category E includes structures in risk categories I, II, and III located close to a major active fault. Seismic design category F includes risk category IV structures located close to a major active fault. In this design category restrictions are imposed on the use of structural systems and analysis methods.

Lateral Forces

Example 7.9

The two-story, reinforced concrete, moment-resisting frame analyzed in Ex. 7.8 is used as a residential building. Determine the applicable risk category, importance factor, and seismic design category.

Solution

The 5% damped design spectral response accelerations are obtained from Ex. 7.8 as

$$S_{DS} = 1.0g$$
$$S_{D1} = 0.7g$$

A residential building is classified as a standard occupancy structure. The applicable risk category is obtained from ASCE/SEI7 Table 1.5-1 or Table 7.4. The risk category is II. For risk category II, the seismic importance factor is obtained from ASCE/SEI7 Table 1.5-2 or Table 7.4 as

$$I_e = 1.00$$

The design spectral response acceleration at short periods is

$$S_{DS} = 1.0g$$
$$> 0.50g$$

For a risk category of II, the seismic design category for this acceleration is obtained from ASCE/SEI7 Table 11.6-1 or Table 7.5. The seismic design category is D.

The design spectral response acceleration at a period of 1 sec is

$$S_{D1} = 0.7g$$
$$> 0.20g$$

For a risk category of II, the seismic design category for this acceleration is obtained from ASCE/SEI7 Table 11.6-2 or Table 7.5. The category is D. Therefore, the seismic design category for this building is D.

Classification of the Structural System

ASCE/SEI7 Sec. 12.2.1 and ASCE/SEI7 Table 12.2-1 detail eight major categories of building types characterized by the method used to resist the lateral force. These categories consist of bearing walls, building frames, moment-resisting frames, dual systems with a special moment-resisting frame, dual systems with a reinforced concrete intermediate moment frame or a steel ordinary moment frame, shear wall-frame interactive, inverted pendulum structures, and steel systems not detailed for seismic resistance.

A *bearing wall system* consists of shear walls that provide support for the gravity loads and resist all lateral loads.

A *building frame system* consists of shear walls or braced frames that resist all lateral loads, and a separate framework that provides support for gravity loads. *Moment-resisting frames* provide support for both lateral and gravity loads by flexural action. In a *dual system*, nonbearing walls or braced frames supply the primary resistance to lateral loads, with a moment frame providing primary support for gravity loads plus additional resistance to lateral loads. A *shear wall-frame interactive system* is a dual system using ordinary reinforced concrete moment frames and ordinary reinforced concrete shear walls. A *cantilevered column structure* consists of a building supported on column elements to produce an inverted pendulum structure. A *steel system* not detailed for seismic resistance is designed using AISC 360.

Response Modification Coefficient

The structure *response modification coefficient*, *R*, is a measure of the ability of a specific structural system to resist lateral loads without collapse. ASCE/SEI7 Table 12.2-1 lists the different structural framing systems, with the height limitations, response modification coefficients, and deflection amplification factors for each. An abbreviated listing of structural systems, response modification coefficients, overstrength factors, and deflection amplification factors is provided in Table 7.6.

Deflection Amplification Factor

The *deflection amplification factor* is tabulated in ASCE/SEI7 Table 12.2-1 and in Table 7.6 and is given by

$$C_d = \frac{\delta_x}{\delta_{xe}}$$

After allowing for the risk importance factor, ASCE/SEI7 Eq. 12.8-15 gives the value of the actual displacement as

$$\delta_x = \frac{C_d \delta_{xe}}{I_e}$$

Overstrength Factor

The *overstrength factor* is a measure of the actual strength of a structure compared to the design seismic force. Values of the overstrength factor for various building systems are tabulated in ASCE/SEI7 Table 12.2-1 and shown in Table 7.6. The overstrength factor is given by

$$\Omega_0 = \frac{V_Y}{V_S}$$

The system overstrength is produced by the following factors: conservative design methods, system redundancy, material overstrength, oversized members, application of load factors, and drift limitations controlling design.

Lateral Forces

Table 7.6 *Seismic Parameters and Building Height*

| | | | | system and height limitations | | | | |
| | | | | seismic design category | | | | |
structural system	R	Ω_0	C_d	B	C	D	E	F
bearing wall								
light-framed walls sheathed with wood shear panels	6.5	3.0	4.0	NL	NL	65	65	65
special reinforced concrete shear walls	5.0	2.5	5.0	NL	NL	160	160	10
special reinforced masonry shear walls	5.0	2.5	3.5	NL	NL	160	160	100
building frame								
steel eccentrically braced frame	8.0	2.0	4.0	NL	NL	160	160	100
special steel concentrically braced frames	6.0	2.0	5.0	NL	NL	160	160	100
ordinary steel concentrically braced frames	3.25	2.0	3.25	NL	NL	35	35	NP
light frame walls sheathed with wood shear panels	7.0	2.5	4.5	NL	NL	65	65	65
buckling-restrained braced frame	8.0	2.5	5.0	NL	NL	160	160	100
special steel plate shear walls	7.0	2.0	6.0	NL	NL	160	160	100
special reinforced concrete shear walls	6.0	2.5	5.0	NL	NL	160	160	100
special reinforced masonry shear walls	5.5	2.5	4.0	NL	NL	160	160	100
moment-resisting frame								
steel or concrete special moment frames	8.0	3.0	5.5	NL	NL	NL	NL	NL
special steel truss moment frames	7.0	3.0	5.5	NL	NL	160	100	NP
intermediate steel moment frames	4.5	3.0	4.0	NL	NL	35	NP	NP
ordinary steel moment frames	3.5	3.0	3.0	NL	NL	NP	NP	NP
intermediate moment frames of reinforced concrete	5.0	3.0	4.5	NL	NL	NP	NP	NP
ordinary moment frames of reinforced concrete	3.0	3.0	2.5	NL	NP	NP	NP	NP
dual system with special moment-resisting frames								
steel eccentrically braced frames	8.0	2.5	4.0	NL	NL	NL	NL	NL
special steel concentrically braced frames	7.0	2.5	5.5	NL	NL	NL	NL	NL
buckling-restrained braced frames	8.0	2.5	5.0	NL	NL	NL	NL	NL
special steel plate shear walls	8.0	2.5	6.5	NL	NL	NL	NL	NL
special reinforced concrete shear walls	7.0	2.5	5.5	NL	NL	NL	NL	NL
special reinforced masonry shear walls	5.5	3.0	5.0	NL	NL	NL	NL	NL
dual system with intermediate moment frames								
special steel concentrically braced frames	6.0	2.5	5.0	NL	NL	35	NP	NP
steel and concrete composite special concentrically braced frames	5.5	2.5	4.5	NL	NL	160	100	NP
special reinforced concrete shear walls	6.5	2.5	5.0	NL	NL	160	100	100
intermediate reinforced masonry shear walls	3.5	3.0	3.0	NL	NL	NP	NP	NP
shear wall-frame interactive system with ordinary reinforced concrete moment frames and ordinary reinforced shear walls	4.5	2.5	4	NL	NP	NP	NP	NP
cantilevered column								
special steel cantilever column systems	2.5	1.25	2.5	35	35	35	35	35
ordinary steel cantilever column systems	1.25	1.25	1.25	35	35	NP	NP	NP
special reinforced concrete moment frames	2.5	1.25	2.5	35	35	35	35	35
steel systems not specifically detailed for seismic resistance, excluding cantilever column systems	3.0	3.0	3.0	NL	NL	NP	NP	NP

Note: NL = not limited and NP = not permitted.

Adapted with permission from *Minimum Design Loads for Buildings and Other Structures*, Table 12.2-1, copyright © 2010, by the American Society of Civil Engineers.

Effective Seismic Weight

The *effective seismic weight, W,* as specified in ASCE/SEI7 Sec. 12.7.2, is the total dead load of the structure and the part of the service load that may be expected to be attached to the building. The effective seismic weight consists of the following.

- 25% of the floor live load for storage and warehouse occupancies

- a minimum allowance of 10 lbf/ft^2 for moveable partitions

- flat roof snow loads exceeding 30 lbf/ft^2, which may be reduced by 80%

- the total weight of permanent equipment and fittings

Roof and floor live loads, except as noted above, are not included in the value of W.

Fundamental Period of Vibration

ASCE/SEI7 Sec. 12.8.2 and Sec. 12.8.2.1 provide three methods for determining the *fundamental period* of a structure. These three methods are the two approximate methods given in ASCE/SEI7 Sec. 12.8.2.1 and the properly substantiated analysis method, such as the Rayleigh method, given in ASCE/SEI7 Sec. 12.8.2.

From ASCE/SEI7 Eq. 12.8-7, the approximate fundamental period is given by

$T_a = 0.028 h_n^{0.8}$ for steel moment-resisting frames

$T_a = 0.016 h_n^{0.9}$ for reinforced concrete moment-resisting frames

$T_a = 0.030 h_n^{0.75}$ for eccentrically braced steel frames

$T_a = 0.020 h_n^{0.75}$ for all other structural systems

Alternatively, for moment-resisting frames not exceeding 12 stories in height and with a story height not less than 10 ft, the approximate fundamental period may be determined by ASCE/SEI7 Eq. 12.8-8 as

$$T_a = 0.1N$$

Example 7.10

A two-story, reinforced concrete, moment-resisting frame is shown in the following illustration. Calculate the natural period of vibration T_a.
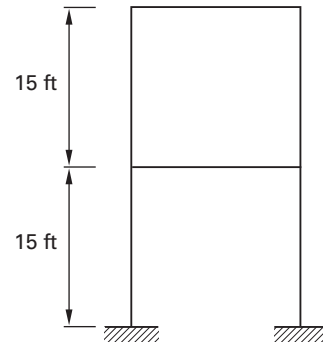
Solution

The number of stories is

$$N = 2$$
$$< 12$$

Then, for a moment-resisting frame, ASCE/SEI7 Eq. 12.8-8 specifies a value for a building period of

$$T_a = 0.1N$$
$$= \left(0.1 \ \frac{\sec}{\text{story}}\right)(2 \text{ stories})$$
$$= 0.20 \text{ sec}$$

Example 7.11

For the two-story, reinforced concrete, moment-resisting frame analyzed in Ex. 7.10, calculate the fundamental period of vibration T_a by using ASCE/SEI7 Eq. 12.8-7.

Solution

For a reinforced concrete frame, the fundamental period is given by ASCE/SEI7 Eq. 12.8-7 as

$$T_a = (0.016)(30 \text{ ft})^{0.9}$$
$$= 0.342 \text{ sec}$$

Rayleigh Procedure

ASCE/SEI7 Sec. 12.8.2 permits the fundamental period to be determined by a "properly substantiated analysis." The *Rayleigh procedure* is an acceptable method, and the fundamental period is given by

$$T = 2\pi \sqrt{\frac{\sum w_i \delta_i^2}{g \sum f_i \delta_i}}$$
$$= 0.32 \sqrt{\frac{\sum w_i \delta_i^2}{\sum f_i \delta_i}}$$

Lateral Forces

The terms in this expression are illustrated in Fig. 7.22, where δ_i represents the elastic displacements due to a lateral force distribution f_i increasing approximately uniformly with height.

Figure 7.22 *Application of the Rayleigh Procedure*

lateral   story   lateral
force   weights   displacements
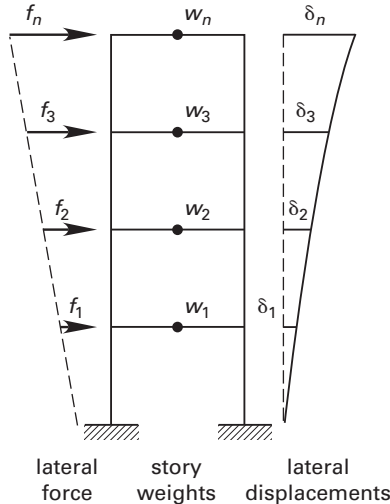
To allow for a possible underestimation of the stiffness of the structure, ASCE/SEI7 Sec. 12.8.2 specifies that the value of the natural period determined by this method may not exceed the value of

$$T = C_u T_a$$

Values of C_u are given in ASCE/SEI7 Table 12.8-1 and are shown in Table 7.7.

Table 7.7 *Coefficient for Upper Limit on the Calculated Period*

S_{D1}	≥ 0.40	0.30	0.20	0.15	≤ 0.10
C_u	1.4	1.4	1.5	1.6	1.7

Example 7.12

Using ASCE/SEI7 Sec. 12.8.2, determine the fundamental period of vibration of the two-story frame of Ex. 7.10, which is located in an area with a value for S_{D1} exceeding 0.4. The force system shown in the following illustration may be used; the effective seismic weight at each level and the total stiffness of each story are indicated.
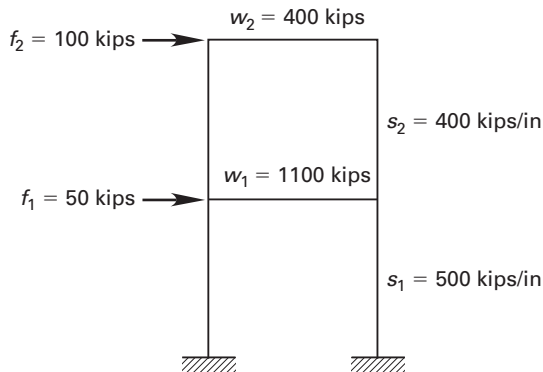
Solution

For the force system indicated, the displacements at each level are given by

$$\delta_1 = \frac{f_2 + f_1}{s_1} = \frac{100 \text{ kips} + 50 \text{ kips}}{500 \ \dfrac{\text{kips}}{\text{in}}}$$

$$= 0.30 \text{ in}$$

$$\delta_2 = \frac{f_2}{s_2} + \delta_1 = \frac{100 \text{ kips}}{400 \ \dfrac{\text{kips}}{\text{in}}} + 0.30 \text{ in}$$

$$= 0.55 \text{ in}$$

The natural period is given by Rayleigh's procedure as

$$T = 0.32 \sqrt{\frac{\sum w_i \delta_i^2}{\sum f_i \delta_i}}$$

The relevant values are given in the following table.

Rayleigh's procedure

level	w_i (kips)	f_i (kips)	δ_i (in)	$w_i \delta_i^2$ (kips-in^2)	$f_i \delta_i$ (in-kips)
2	400	100	0.55	121	55
1	1100	50	0.30	99	15
total	1500	–	–	220	70

$$T = \left(0.32 \ \frac{\text{sec}}{\sqrt{\text{in}}}\right) \sqrt{\frac{220 \text{ kips-in}^2}{70 \text{ in-kips}}}$$

$$= 0.567 \text{ sec}$$

In an area with a value for $S_{D1} > 0.4$, the value of the coefficient for the upper limit on the calculated period is obtained from ASCE/SEI7 Table 12.8-1 or Table 7.7 as

$$C_u = 1.4$$

The fundamental period, in accordance with ASCE/SEI7 Sec. 12.8.2, is limited to

$$T = 1.4 T_a$$

$$= (1.4)(0.2 \text{ sec})$$

$$= 0.28 \text{ sec}$$

$$< 0.567 \text{ sec}$$

Use the maximum value of

$$T = 0.28 \text{ sec}$$

Alternatively, the value obtained for T_a in Ex. 7.11 may be used to give

$$T = (1.4)(0.342 \text{ sec})$$

$$= 0.479 \text{ sec}$$

General Procedure Response Spectrum

The *general procedure response spectrum* is defined in ASCE/SEI7 Sec. 11.4.5 and shown in ASCE/SEI7 Fig. 11.4-1. The response spectrum is reproduced in Fig. 7.23.

Figure 7.23 *Construction of ASCE/SEI7 Response Spectra*
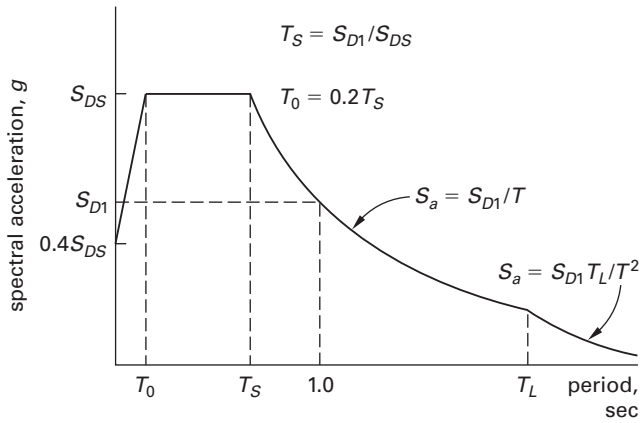
The response spectrum is constructed as shown using the following functions.

$$T_S = \frac{S_{D1}}{S_{DS}}$$

$$T_0 = \frac{0.2S_{D1}}{S_{DS}}$$

For periods less than or equal to T_0, the design spectral response acceleration is given by ASCE/SEI7 Eq. 11.4-5 as

$$S_a = \frac{0.6S_{DS}T}{T_0} + 0.4S_{DS}$$

At $T = 0$,

$$S_a = 0.4S_{DS}$$

For periods greater than or equal to T_0 and less than or equal to T_S, the design response acceleration is given by

$$S_a = S_{DS}$$

For periods greater than T_S and less than or equal to T_L, the design response acceleration is given by ASCE/SEI7 Eq. 11.4-6 as

$$S_a = \frac{S_{D1}}{T}$$

For periods greater than T_L, the design spectral response acceleration is given by ASCE/SEI7 Eq. 11.4-7 as

$$S_a = \frac{S_{D1}T_L}{T^2}$$

Values of T_L range from 4 sec to 6 sec, and are mapped in ASCE/SEI7 Fig. 22-12 through Fig. 22-16.

Seismic Response Coefficient

The *seismic response coefficient*, C_s, given in ASCE/SEI7 Sec. 12.8.1.1 represents the code design spectrum and is given by ASCE/SEI7 Eq. 12.8-3 for values of T not greater than T_L as

$$C_s = \frac{S_{D1}I_e}{RT}$$

For values of T greater than T_L, the seismic response coefficient is given by ASCE/SEI7 Eq. 12.8-4 as

$$C_s = \frac{S_{D1}T_LI_e}{RT^2}$$

The maximum value of the seismic response coefficient is given by ASCE/SEI7 Eq. 12.8-2 as

$$C_s = \frac{S_{DS}I_e}{R}$$

This latter expression controls for shorter periods up to approximately 1 sec. For longer periods, the expression provides conservative values.

In accordance with ASCE/SEI7 Eq. 12.8-5, the value of the seismic response coefficient must not be taken less than

$$C_s = 0.044S_{DS}I_e \geq 0.01$$

For those structures for which the 1 sec spectral response value is $S_1 \geq 0.6g$, the minimum value of the seismic response coefficient is given by ASCE/SEI7 Eq. 12.8-6 as

$$C_s = \frac{0.5S_1I_e}{R}$$

Example 7.13

The two-story, reinforced concrete, special moment-resisting frame of Ex. 7.12 is used for a residential building. Calculate the seismic response coefficient by using the alternative value for the fundamental period determined in Ex. 7.12.

Solution

From previous examples, the relevant parameters are

$$S_{DS} = 1.0g$$
$$S_{D1} = 0.7g$$
$$I_e = 1.00$$
$$T = 0.479 \text{ sec}$$
$$T_S = 0.70 \text{ sec}$$

The value of the response modification coefficient for a special moment-resisting frame is obtained from Table 7.6 as

$$R = 8.0$$

The seismic response coefficient is given in ASCE/SEI7 Sec. 12.8.1.1 as

$$C_s = \frac{S_{D1}I_e}{RT}$$
$$= \frac{(0.7)(1.00)}{(8.0)(0.479 \text{ sec})}$$
$$= 0.183$$

The maximum value of the seismic response coefficient is

$$C_s = \frac{S_{DS}I_e}{R}$$
$$= \frac{(1.0)(1.00)}{8.0}$$
$$= 0.125 \quad [\text{governs}]$$

This follows since $T < T_S$. The minimum values of C_s do not govern.

Seismic Base Shear

ASCE/SEI7 Eq. 12.8-1 specifies the *seismic base shear* as

$$V = C_s W$$

Example 7.14

Calculate the seismic base shear for the two-story, reinforced concrete, moment-resisting frame of Ex. 7.13.

Solution

The value of the effective seismic weight was derived in Table 7.6 as

$$W = 1500 \text{ kips}$$

The value of the seismic response coefficient was derived in Ex. 7.13 as

$$C_s = 0.125$$

The base shear is given by ASCE/SEI7 Eq. 12.8-1 as

$$V = C_s W$$
$$= (0.125)(1500 \text{ kips})$$
$$= 188 \text{ kips}$$

Building Configuration Requirements

The static lateral force procedure is applicable to structures that satisfy prescribed conditions of regularity, occupancy, location, and height. A regular structure has mass, stiffness, and strength uniformly distributed over the height of the structure and is without irregular features that will produce stress concentrations. Vertical irregularities are defined in ASCE/SEI7 Table 12.3-2 and horizontal irregularities in ASCE/SEI7 Table 12.3-1. As defined in ASCE/SEI7 Table 12.6-1, the equivalent lateral force method may be used in the design of a structure when a structure is assigned to seismic design category B or C. Additionally, the equivalent lateral force method may be used when a structure is assigned to seismic design category D, E, or F and conforms to one of the following conditions.

- light-frame construction
- a risk category of I or II and does not exceed two stories in height
- height not exceeding 160 ft and is a regular building
- height not exceeding 160 ft and has neither horizontal irregularities 1a (torsional) or 1b (extreme torsional), nor vertical irregularities 1a (soft story), 1b (extreme soft story), 2 (mass), or 3 (geometric)
- height exceeding 160 ft, a fundamental period $T < 3.5T_S$, and has no structural irregularities

All other structures not described above that are assigned to seismic design categories D, E, and F need a modal analysis.

Redundancy Factor

To improve the seismic performance of structures in seismic design categories D, E, and F, redundancy is incorporated in the structures by providing multiple load-resisting paths. The *redundancy factor*, ρ, is a factor that penalizes structures with relatively few lateral load-resisting elements and is specified in ASCE/SEI7 Sec. 12.3.4. The redundancy factor is further defined in this text in Sec. 7.11.

6. VERTICAL DISTRIBUTION OF SEISMIC FORCES

The distribution of base shear over the height of a building is obtained from ASCE/SEI7 Sec. 12.8.3, and the design lateral force at level x is given by

$$F_x = \frac{V w_x h_x^k}{\sum w_i h_i^k}$$

The terms in this expression are illustrated in Fig. 7.24, where h_i represents the height above the base to any level i, h_x represents the height above the base to a specific level x, and $\Sigma w_i h_i^k$ represents the summation over the whole structure, of the product of w_i and h_i^k. To allow for higher mode effects in long period buildings, when T has a value of 2.5 sec or more, the distribution exponent k is given by

$$k = 2$$

When T has a value not exceeding 0.5 sec, the distribution exponent is

$$k = 1$$

For intermediate values of T, a linear variation of k may be assumed.

Figure 7.24 *Vertical Force Distribution*
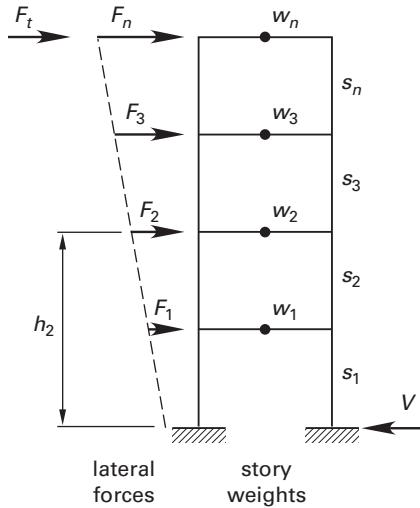
Example 7.15

Determine the vertical force distribution for the two-story reinforced concrete, moment-resisting frame of Ex. 7.13.

Solution

The fundamental period was derived in Ex. 7.12 as

$$T = 0.479 \text{ sec}$$
$$< 0.5 \text{ sec}$$

The value of the distribution exponent factor is obtained from ASCE/SEI7 Sec. 12.8.3 as

$$k = 1.0$$

In accordance with ASCE/SEI7 Sec. 12.8.3, the expression for F_x reduces to

$$F_x = \frac{V w_x h_x}{\sum w_i h_i}$$

The effective seismic weights located at levels 1 and 2 are obtained from Ex. 7.12, and the relevant values are given in the following table.

Vertical Force Distribution

level	w_x (kips)	h_x (ft)	$w_x h_x$ (ft-kips)	F_x (kips)
2	400	30	12,000	79
1	1100	15	16,500	109
total	1500	–	28,500	188

From Ex. 7.14, the base shear is given by

$$V = 188 \text{ kips}$$

The design lateral force at level x is

$$
\begin{aligned}
F_x &= \frac{V w_x h_x}{\sum w_i h_i} \\
&= \frac{(188 \text{ kips}) w_x h_x}{28,500 \text{ ft-kips}} \\
&= 0.00660 w_x h_x
\end{aligned}
$$

The values of F_x are given in the previous table.

7. DIAPHRAGM LOADS

The load acting on a horizontal diaphragm is given by ASCE/SEI7 Sec. 12.10.1.1 as

$$
\begin{aligned}
F_{px} &= \frac{w_{px} \sum F_i}{\sum w_i} \qquad [ASCE/SEI7 \ 12.10\text{-}1] \\
&\geq 0.2 S_{DS} I w_{px} \\
&\leq 0.4 S_{DS} I w_{px}
\end{aligned}
$$

The terms in the expression are illustrated in Fig. 7.25, where ΣF_i represents the total shear force at level i, Σw_i represents the total seismic weight at level i and above, and w_{px} represents the seismic weight tributary to the diaphragm at level x, not including walls parallel to the direction of the seismic load.

Figure 7.25 *Diaphragm Loads*
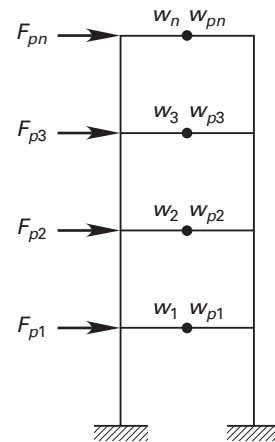
For a single-story structure, the expression reduces to

$$F_{px} = \frac{V w_{px}}{W} = C_s w_{px}$$

For a multistory structure, at the second-floor level,

$$\frac{\sum F_i}{\sum w_i} = \frac{V}{W} = C_s$$

Example 7.16

Determine the diaphragm loads for the two-story reinforced concrete, moment-resisting frame of Ex. 7.15. The effective seismic weight tributary to the diaphragm at roof level is 300 kips and at the second-floor level is 600 kips.

Solution

From Ex. 7.8, the design response coefficient is

$$S_{DS} = 1.0g$$

The diaphragm load is given by ASCE/SEI7 Eq. 12.10-1 as

$$F_{px} = \frac{w_{px} \sum F_i}{\sum w_i}$$

level	w_i (kips)	$\sum w_i$ (kips)	F_i (kips)	$\sum F_i$ (kips)	$\dfrac{\sum F_i}{\sum w_i}$
2	400	400	79	79	0.198
1	1100	1500	109	188	0.125

level	max	min	w_{px} (kips)	F_{px} (kips)
2	0.40	0.20	300	60
1	0.40	0.20	600	120

The maximum value for the diaphragm load is given by ASCE/SEI7 Sec. 12.10.1.1 as

$$F_{p(\max)} = 0.4 S_{DS} I_e w_{px}$$
$$= (0.4)(1.0)(1.00) w_{px}$$
$$= 0.40 w_{px}$$

The minimum value for the diaphragm load is given by ASCE/SEI7 Sec. 12.10.1.1 as

$$F_{p(\min)} = 0.2 S_{DS} I_e w_{px}$$
$$= (0.2)(1.0)(1.00) w_{px}$$
$$= 0.20 w_{px} \quad \text{[governs at both levels]}$$

The relevant values are given in the previous table.

8. STORY DRIFT

Story drift is defined in ASCE/SEI7 Sec. 12.8.6 as the lateral displacement of one level of a multistory structure relative to the level below. The maximum allowable story drift, Δ_a, is given in ASCE/SEI7 Table 12.12-1 and is shown in Table 7.8.

To allow for inelastic deformations, drift is determined by using the deflection amplification factor C_d defined in

Table 7.8 *Maximum Allowable Story Drift,* Δ_a

	risk category		
building type	I or II	III	IV
one-story buildings with fittings designed to accomodate drift	no limit	no limit	no limit
buildings other than masonry buildings of four stories or less with fittings designed to accommodate drift	$0.025 h_{sx}$	$0.020 h_{sx}$	$0.015 h_{sx}$
masonry cantilever shear wall buildings	$0.010 h_{sx}$	$0.010 h_{sx}$	$0.010 h_{sx}$
other masonry shear wall buildings	$0.007 h_{sx}$	$0.007 h_{sx}$	$0.007 h_{sx}$
all other buildings	$0.020 h_{sx}$	$0.015 h_{sx}$	$0.010 h_{sx}$

Table 7.6. The amplified deflection at level x is given by ASCE/SEI7 Eq. 12.8-15 as

$$\delta_x = \frac{C_d \delta_{xe}}{I_e}$$

The term δ_{xe} represents the horizontal deflection at level x, determined by an elastic analysis using the code-prescribed design level forces. In accordance with ASCE/SEI7 Sec. 12.8.7, P-delta effects need not be included in the calculation of drift when the stability coefficient θ does not exceed 0.10.

Using the nomenclature from Fig. 7.24, the elastic lateral displacement in the bottom story is

$$\delta_{1e} = \frac{F_n + F_3 + F_2 + F_1}{s_1}$$

The elastic lateral displacement in the second story is

$$\delta_{2e} = \frac{F_n + F_3 + F_2}{s_2} + \delta_{1e}$$

The elastic lateral displacement in the third story is

$$\delta_{3e} = \frac{F_n + F_3}{s_3} + \delta_{2e}$$

The elastic lateral displacement in the top story is

$$\delta_{ne} = \frac{F_n}{s_n} + \delta_{3e}$$

The drift in the bottom story is

$$\Delta_1 = \frac{C_d \delta_{1e}}{I_e}$$

The drift in the second story is

$$\Delta_2 = \frac{C_d (\delta_{2e} - \delta_{1e})}{I_e}$$

Lateral Forces

The drift in the third story is

$$\Delta_3 = \frac{C_d(\delta_{3e} - \delta_{2e})}{I_e}$$

The drift in the top story is

$$\Delta_n = \frac{C_d(\delta_{ne} - \delta_{3e})}{I_e}$$

For the calculation of drift, in accordance with ASCE/SEI7 Sec. 12.8.6.2, the full value of T, the fundamental period determined by using the Rayleigh procedure, may be used to determine the seismic base shear.

Example 7.17

Determine the drift in the bottom story of the two-story, reinforced concrete, moment-resisting frame of Ex. 7.15, which is used as a residential building. The relevant details are shown. Fittings are designed to accommodate drift.
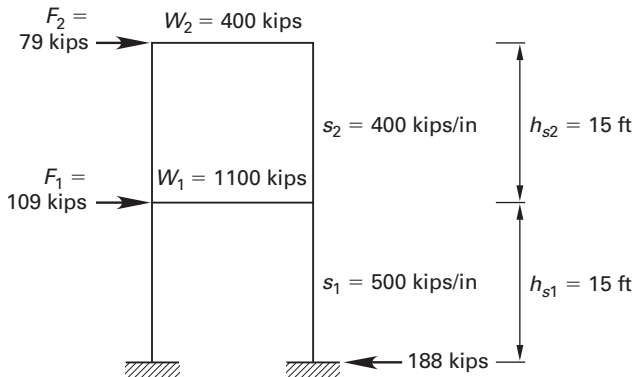
Solution

From Ex. 7.12, the fundamental period obtained by using the Rayleigh procedure is

$$T = 0.567 \text{ sec}$$

The seismic response coefficient is given in ASCE/SEI7 Sec. 12.8.1.1 as

$$C_s = \frac{S_{D1}I_e}{RT} = \frac{(0.7)(1.00)}{(8.0)(0.567 \text{ sec})} = 0.154$$

The maximum value of the seismic design coefficient is

$$C_s = \frac{S_{DS}I_e}{R} = \frac{(1.0)(1.00)}{8.0} = 0.125 \quad \text{[governs]}$$

The seismic base shear is given by ASCE/SEI7 Sec. 12.8.1 and is identical with the value calculated in Ex. 7.14 as

$$V = 188 \text{ kips}$$

In addition, the lateral forces are identical with the values calculated in Ex. 7.15.

For a moment-resisting frame, the amplification factor is obtained from ASCE/SEI7 Table 12.2-1 or Table 7.6 as

$$C_d = 5.5$$

From the lateral forces determined in Ex. 7.15, the drift in the bottom story, allowing for $I_e = 1.0$, is

$$\begin{aligned}
\Delta_1 &= C_d \delta_{xe} \\
&= \frac{C_d(F_2 + F_1)}{s_1} \\
&= \frac{(5.5)(79 \text{ kips} + 109 \text{ kips})}{500 \ \dfrac{\text{kips}}{\text{in}}} \\
&= 2.07 \text{ in}
\end{aligned}$$

In accordance with ASCE/SEI7 Table 12.12-1, the maximum allowable drift for a two-story structure in risk category II is

$$\begin{aligned}
\Delta_a &= 0.025 h_{s1} = (0.025)(15 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right) \\
&= 4.50 \text{ in} \\
&> 2.07 \text{ in}
\end{aligned}$$

The drift is acceptable.

9. *P*-DELTA EFFECTS

The *P-delta effects* are calculated by using the design level seismic forces and elastic displacements determined in accordance with ASCE/SEI7 Sec. 12.8.1. *P*-delta effects in a given story are a result of the secondary moments caused by the eccentricity of the gravity loads above that story. The *secondary moment in a story* is defined as the product of the total dead load, floor live load, and snow load above the story multiplied by the elastic drift of that story. The *primary moment in a story* is defined as the seismic shear in the story multiplied by the height of the story.

The ratio of the secondary moment to primary moment is termed the *stability coefficient* and is given by ASCE/SEI7 Sec. 12.8.7 as

$$\begin{aligned}
\theta &= \frac{P_x \Delta}{V_x h_{sx} C_d} \\
&= \frac{P_x(\delta_{xe} - \delta_{(x-1)e})}{V_x h_{sx}}
\end{aligned}$$

The stability coefficient shall not exceed the value

$$\begin{aligned}
\theta_{\text{max}} &= \frac{0.5}{\beta C_d} \\
&\leq 0.25
\end{aligned}$$

The term β is the ratio of the shear demand to the shear capacity in a story and may conservatively be considered equal to 1.0. If the stability coefficient in any story exceeds 0.1, the effects of the secondary moments must

be included in the analysis of the whole structure. The revised story drift, allowing for *P*-delta effects, is obtained as the product of the calculated drift and the factor $1/(1 - \theta)$.

As shown in Fig. 7.26, with the designated lateral forces and story drift and with the combined dead load plus floor live load indicated by W_1 and the combined dead load plus roof snow load indicated by W_2, the primary moment in the second story of the frame is

$$M_{P2} = F_2 h_{s2}$$

Figure 7.26 *P-delta Effects*
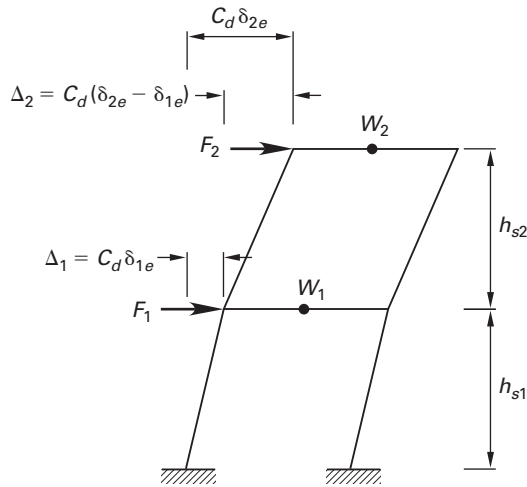
The secondary moment in the second story is

$$M_{S2} = \frac{P_2 \Delta_2}{C_d}$$
$$= W_2(\delta_{2e} - \delta_{1e})$$

The stability coefficient in the second story is

$$\theta_2 = \frac{M_{S2}}{M_{P2}}$$

The primary moment in the first story of the frame is

$$M_{P1} = (F_1 + F_2)h_{s1}$$

The secondary moment in the first story is

$$M_{S1} = \frac{P_1 \Delta_1}{C_d}$$
$$= (W_2 + W_1)\delta_{1e}$$

The stability coefficient in the first story is

$$\theta_1 = \frac{M_{S1}}{M_{P1}}$$

Example 7.18

Determine the stability coefficient for the bottom story of the two-story reinforced concrete, moment-resisting frame of Ex. 7.17.

Solution

The drift in the bottom story is derived in Ex. 7.17 as

$$\Delta_1 = 2.07 \text{ in}$$

The primary moment in the bottom story is

$$M_{P1} = (F_2 + F_1)h_{s1}$$
$$= (79 \text{ kips} + 109 \text{ kips})(15 \text{ ft})\left(12 \text{ } \frac{\text{in}}{\text{ft}}\right)$$
$$= 33{,}840 \text{ in-kips}$$

The secondary moment is

$$M_{S1} = \frac{(W_2 + W_1)\Delta_1}{C_d}$$
$$= \frac{(1500 \text{ kips})(2.07 \text{ in})}{5.5}$$
$$= 565 \text{ in-kips}$$

The stability coefficient is

$$\theta_1 = \frac{M_{S1}}{M_{P1}}$$
$$= \frac{565 \text{ in-kips}}{33{,}840 \text{ in-kips}}$$
$$= 0.017$$
$$< 0.1 \quad \text{[Secondary moments need not be considered.]}$$

10. SIMPLIFIED LATERAL FORCE PROCEDURE

For some low-rise structures, ASCE/SEI7 Sec. 12.14 permits an alternative, conservative design method. The simplified method is applicable to a structure in which the following 12 limitations are met.

1. The structure does not exceed three stories in height.

2. The structure is assigned to risk category I or II.

3. The structure is located at a site with a soil profile of site class A through D.

4. The structure's lateral force-resisting system is either a bearing wall system or a building frame system.

5. The structure must have at least two lines of lateral resistance in each of two major axis directions. (See Fig. 7.27.)

6. At least one line of resistance shall be provided on each side of the center of mass (CM) in each direction. (See Fig. 7.27.)

Figure 7.27 *Lines of Lateral Resistance*

Figure 7.28 *Flexible Diaphragm Overhang*
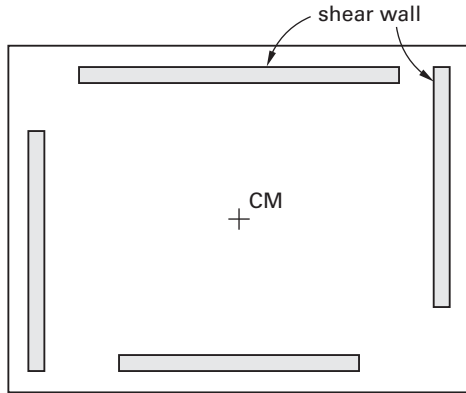
7. A structure with flexible diaphragms, overhangs beyond the outside line of shear walls, or braced frames must satisfy ASCE/SEI7 Eq. 12.14-1. (See Fig. 7.28.) The distance, a, perpendicular to the forces being considered from the extreme edge of the diaphragm to the line of vertical resistance closest to that edge shall not exceed one-fifth the depth, d, of the diaphragm parallel to the forces being considered at the line of vertical resistance closest to the edge.

$$a \leq \frac{d}{5} \qquad [ASCE/SEI7\ 12.14\text{-}1]$$

8. For a structure with a diaphragm that is nonflexible, the distance between the center of rigidity and the center of mass parallel to each major axis must not exceed 15% of the greatest width of the diaphragm parallel to that axis. In addition, as shown in Fig. 7.29, the building layout shall satisfy both of the following equations.

$$\sum_{i=1}^{m} k_{1i} d_i^2 + \sum_{j=1}^{n} k_{2j} d_j^2 \geq 2.5 \left(0.05 + \frac{e_1}{b_1} \right) b_1^2 \sum_{i=1}^{m} k_{1i}$$

$$[ASCE/SEI7\ 12.14\text{-}2A]$$

$$\sum_{i=1}^{m} k_{1i} d_i^2 + \sum_{j=1}^{n} k_{2j} d_j^2 \geq 2.5 \left(0.05 + \frac{e_2}{b_2} \right) b_2^2 \sum_{i=1}^{m} k_{1i}$$

$$[ASCE/SEI7\ 12.14\text{-}2B]$$

The stiffness of wall i or braced frame i parallel to major axis 1 is

$$k = k_{1i}$$

The stiffness of wall j or braced frame j parallel to major axis 2 is

$$k = k_{2j}$$

The distance from the wall i or braced frame i to the center of rigidity, perpendicular to major axis 1 is

$$d = d_{1i}$$

Figure 7.29 *Torsion Check for Nonflexible Diaphragms*
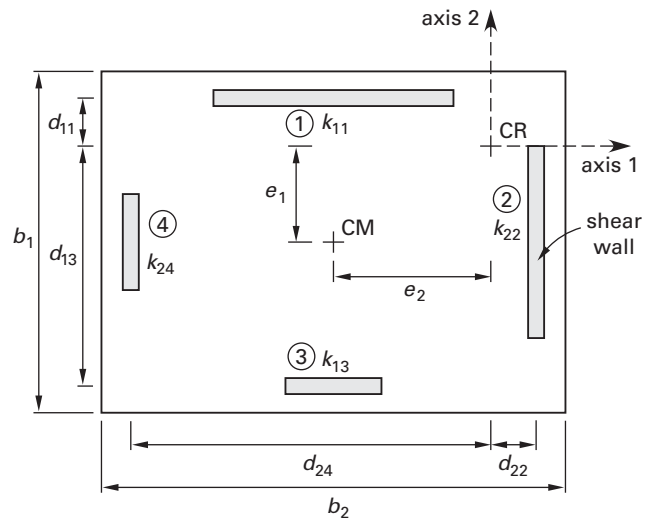
The distance from the wall j or braced frame j to the center of rigidity, perpendicular to major axis 2 is

$$d = d_{2j}$$

The distance perpendicular to major axis 1 between the center of rigidity and the center of mass is

$$e = e_1$$

The width of the diaphragm perpendicular to major axis 1 is

$$b = b_1$$

The distance perpendicular to major axis 2 between the center of rigidity and the center of mass is

$$e = e_2$$

The width of the diaphragm perpendicular to major axis 2 is

$$b = b_2$$

m is the number of walls and braced frames resisting lateral force in direction 1.

Lateral Forces

n is the number of walls and braced frames resisting lateral force in direction 2.

As shown in Fig. 7.30, these two equations need not be checked provided that the structure fulfills all of the following three conditions.

- *Condition 1:* The arrangement of walls or braced frames is symmetric about each major axis.

- *Condition 2:* The distance between the two most separated lines of walls or braced frames is at least 90% of the dimension of the structure perpendicular to that axis direction.

- *Condition 3:* The stiffness along each of the lines is at least 33% of the total stiffness in that axis direction.

Figure 7.30 *Torsion Check Unnecessary*
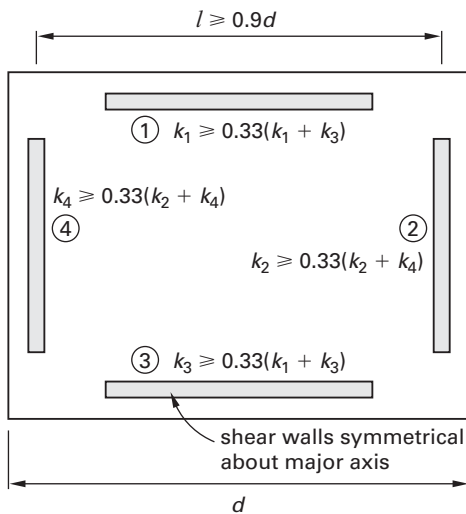
9. Lines of resistance of the structure's lateral force-resisting system must be oriented at angles of no more than 15° from alignment with the major orthogonal horizontal axes of the building.

10. The simplified design procedure must be used for each major orthogonal horizontal axis direction of the structure.

11. System irregularities caused by in plane or out of plane offsets of lateral force-resisting elements are not permitted, except in two-story structures of light frame construction provided that the upper wall is designed for a factor of safety of 2.5 against overturning.

12. The lateral load resistance of any story must not be less than 80% of the story above it.

When using the simplified design procedure, ASCE/SEI7 Sec. 12.14.3.1.1 states that the redundancy factor may be taken as

$$\rho = 1.0$$

In accordance with ASCE/SEI7 Sec. 12.14.3.2.1, the over-strength factor is

$$\Omega_0 = 2.5$$

When the simplified design procedure is used, ASCE/SEI7 Sec. 12.14.8.5 specifies that structural drift need not be calculated. If a drift value is required for design of cladding or to determine building separation, it may be assumed to be 1% of building height.

Example 7.19

The two-story structure shown is an office building located on a site with a soil classification type D. The structure consists of a bearing wall system with the reinforced concrete shear walls continuous through both stories, and the roof and floor diaphragms are rigid. Determine whether the simplified lateral force procedure is applicable.

section A-A
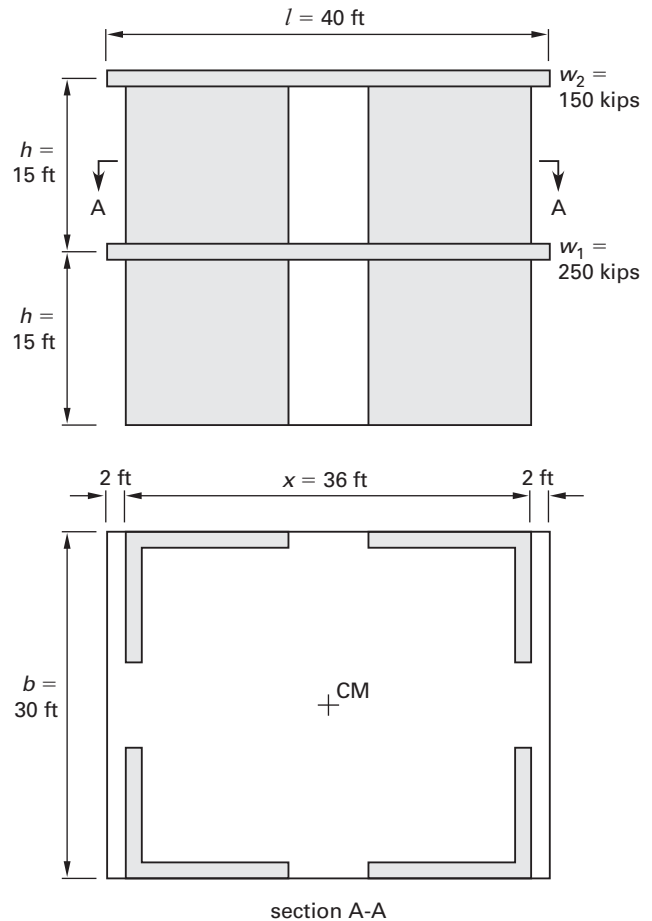
Solution

The following hold true for the office building and are satisfactory.

- The risk category for an office building is II.

- It is situated on a site with a soil classification type D.

- It is two stories in height, which is less than three stories.

- The seismic force-resisting system is a bearing wall system.

- It has two lines of lateral resistance in each of the two major axis directions.

- One line of resistance is provided on each side of the center of mass in each direction.

- The lines of resistance of the lateral force-resisting system are parallel to the major orthogonal horizontal axes of the building.

- There are no irregularities caused by in plane or out of plane offsets of lateral force-resisting elements.

- The lateral load resistance is identical in both stories.

Since the building has rigid diaphragms, it is necessary to check ASCE/SEI7 Eq. 12.14-2A and Eq. 12.14-2B, or determine whether the limitations are such that doing so is not necessary. The relevant limitations are as follows.

The arrangement of the shear walls is symmetric about each major axis, and this is satisfactory.

The length of the building is

$$l = 40 \text{ ft}$$

The separation between the east and west shear walls is

$$x = 36 \text{ ft}$$

The ratio of separation of walls to length of building is

$$\frac{x}{l} = \frac{36 \text{ ft}}{40 \text{ ft}}$$
$$= 0.9 \quad [\text{satisfactory}]$$

The stiffness along each line of shear walls is 50% of the total stiffness in that axis direction. Therefore, it is not necessary to check ASCE/SEI7 Eq. 12.14-2A and Eq. 12.14-2B. The simplified lateral force procedure is applicable.

Simplified Determination of Seismic Base Shear

The simplified seismic base shear is given by ASCE/SEI7 Eq. 12.14-11 as

$$V = \left(\frac{F S_{DS}}{R} \right) W$$

The design spectral response acceleration at short periods is given by ASCE/SEI7 Sec. 12.14.8.1 as

$$S_{DS} = \frac{2 F_a S_S}{3}$$

The 5% damped, maximum considered earthquake spectral response acceleration, for a period of 0.2 sec for structures founded on rock, is given by ASCE/SEI7 Sec. 11.4.1 with the limitation

$$S_S \leq 1.5g$$

The short-period site coefficient is obtained from ASCE/SEI7 Table 11.4-1, or may be taken as

$$F_a = 1.0 \quad [\text{rock sites}]$$
$$F_a = 1.4 \quad [\text{soil sites}]$$

ASCE/SEI7 Sec. 12.14.8.1 defines a rock site as having the height of the soil between the rock surface and the bottom of the building's foundations no greater than 10 ft.

The modification factor for building type is

$$F = 1.0 \quad [\text{one-story buildings}]$$
$$F = 1.1 \quad [\text{two-story buildings}]$$
$$F = 1.2 \quad [\text{three-story buildings}]$$

The effective seismic weight, W, as specified in ASCE/SEI7 Sec. 12.14.8.1, includes the total dead load of the structure plus the following loads.

- 25% of the floor live load for storage and warehouse occupancies

- a minimum allowance of 10 lbf/ft^2 for moveable partitions

- snow loads exceeding 30 lbf/ft^2, which may be reduced by 80% when approved by the building official

- the total weight of permanent equipment and fittings

The response modification factor, R, and the limitations on the use of the various lateral force-resisting systems are given in ASCE/SEI7 Table 12.14-1 and are summarized in Table 7.9.

Example 7.20

Determine the seismic base shear using the simplified procedure for the two-story special reinforced concrete bearing wall structure of Ex. 7.19. The 5% damped, maximum considered earthquake spectral response acceleration, for a period of 0.2 sec, is $S_S = 1.2g$. The effective seismic weight of the building is 400 kips.

Solution

The 5% damped, maximum considered earthquake spectral response acceleration for a period of 0.2 sec is given as

$$S_S = 1.2g$$

Lateral Forces

Table 7.9 Design Factors for Simplified Lateral Force Procedure

structural system	response modification coefficient, R	limitations seismic design category		
		B	C	D, E
bearing wall				
special reinforced concrete shear walls	5	P	P	P
ordinary reinforced concrete shear walls	4	P	P	NP
special reinforced masonry shear walls	5	P	P	P
intermediate reinforced masonry shear walls	3.5	P	P	NP
ordinary reinforced masonry shear walls	2	P	NP	NP
light-framed walls with wood structural panels	6.5	P	P	P
building frame				
eccentrically braced frame, moment-resisting connections at column away from link	8	P	P	P
eccentrically braced frame, non-moment-resisting connections at column away from link	7	P	P	P
special steel concentrically braced frames	6	P	P	P
ordinary steel concentrically braced frames	3.25	P	P	P
special reinforced concrete shear walls	6	P	P	P
ordinary reinforced concrete shear walls	5	P	P	P
composite steel concentrically braced frames	5	P	P	P
special reinforced masonry shear walls	5.5	P	P	P
intermediate reinforced masonry shear walls	4	P	P	NP
light-framed walls with wood structural panels	7	P	P	P

P = permitted, NP = not permitted

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The short-period site coefficient for a soil site is given by ASCE/SEI7 Sec. 12.14.8.1 as

$$F_a = 1.4$$

The design spectral response acceleration at short periods is given by ASCE/SEI7 Sec. 12.14.8.1 as

$$S_{DS} = \frac{2F_a S_S}{3} = \frac{(2)(1.4)(1.2g)}{3}$$
$$= 1.12g$$

For a bearing wall structure with special reinforced concrete shear walls, the response modification factor is obtained from ASCE/SEI7 Table 12.14-1, or from Table 7.9, as

$$R = 5$$

The modification factor for a two-story building is

$$F = 1.1$$

The effective seismic weight of the building is given as

$$W = 400 \text{ kips}$$

Therefore, the simplified base shear is given by ASCE/SEI7 Eq. 12.14-11 as

$$V = \left(\frac{FS_{DS}}{R}\right)W$$
$$= \left(\frac{(1.1)(1.12g)}{5}\right)(400 \text{ kips})$$
$$= 98.6 \text{ kips}$$

Simplified Vertical Distribution of Base Shear

When the simplified procedure is used to determine the seismic base shear, the forces at each level may be determined from ASCE/SEI7 Sec. 12.14.8.2 as

$$F_x = \frac{w_x V}{W} \qquad [\textit{ASCE/SEI7 12.14-12}]$$

Example 7.21

Determine the vertical force distribution by using the simplified procedure for the two-story, reinforced concrete, bearing wall structure of Ex. 7.20.

Solution

From Ex. 7.20, the following values are obtained.

$$V = 98.6 \text{ kips}$$
$$W = 400 \text{ kips}$$

The forces at each level are determined from ASCE/SEI7 Eq. 12.14-12 as

$$F_x = \frac{w_x V}{W}$$
$$= \frac{w_x(98.6 \text{ kips})}{400 \text{ kips}}$$
$$= 0.25w_x$$

The values of F_x are given in the following table.

level	W_x (kips)	F_x (kips)
2	150	37.0
1	250	61.6
total	400	98.6

Simplified Determination of Drift

In accordance with ASCE/SEI7 Sec. 12.14.8.5, when the simplified procedure is used to determine the seismic base shear, the design story drift in any story must be taken as

$$\Delta_x = 0.01 h_{sx}$$

Example 7.22

Using the simplified procedure, determine the drift in the bottom story of the two-story, reinforced concrete, bearing wall structure of Ex. 7.20.

Solution

The design story drift in the bottom story is given by

$$\Delta_1 = 0.01 h_{s1}$$
$$= (0.01)(15 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)$$
$$= 1.80 \text{ in}$$

11. SEISMIC LOAD ON AN ELEMENT OF A STRUCTURE

The *seismic load, E*, is a function of both horizontal and vertical earthquake-induced forces and is given by ASCE/SEI7 Sec. 12.4.2 as

$$E = \rho Q_E + 0.2 S_{DS} D$$

The term Q_E is the lateral force produced by the calculated base shear V. The term $0.2 S_{DS} D$ is the vertical force due to the effects of vertical acceleration. The redundancy factor, ρ, is a factor that penalizes structures with relatively few lateral load-resisting elements and is defined by ASCE/SEI7 Sec. 12.3.4.

The seismic performance of a structure is enhanced by providing multiple lateral load-resisting elements. Therefore, the yield of one element will not result in an unstable condition that may lead to collapse of the structure. For this situation, the value of the redundancy factor is $\rho = 1.0$. When the yield of an element will result in an unstable condition, the value of the redundancy factor is $\rho = 1.3$.

In accordance with ASCE/SEI7 Sec. 12.3.4.1, the value of the redundancy factor is equal to 1.0 for the following.

- structures assigned to seismic design categories B and C

- drift calculations and *P*-delta effects

- design of nonstructural components

- design of nonbuilding structures that are not similar to buildings

- design of collector elements, splices, and their connections for which load combinations with overstrength factors are used

- design of members and connections for which load combinations with overstrength factors are required

- diaphragm loads that are determined using ASCE/SEI7 Eq. 12.10-1

- structures with damping systems designed in accordance with ASCE/SEI7 Chap. 18

For structures assigned to seismic design categories D, E, and F, $\rho = 1.3$ unless the requirements of ASCE/SEI7 Sec. 12.3.4.2a or Sec. 12.3.4.2b are met.

In accordance with ASCE/SEI7 Sec. 12.3.4.2a, the value of the redundancy factor may be taken equal to 1.0, provided that each story resisting more than 35% of the base shear complies with the following.

- For a braced frame, the removal of an individual brace, or connection thereto, does not result in more than a 33% reduction in story strength, nor create an extreme torsional irregularity (horizontal structural irregularity type 1b).

- For a moment frame, loss of moment resistance at the beam to column connections at both ends of a single beam does not result in more than a 33% reduction in story strength, nor create an extreme torsional irregularity (horizontal structural irregularity type 1b).

- For a shear wall or a wall-pier system with a height to length ratio greater than 1.0, removal of a wall or pier, or collector connections thereto, does not result in more than a 33% reduction in story strength, nor create an extreme torsional irregularity (horizontal structural irregularity type 1b).

- For a cantilever column, loss of moment resistance at the base connections of any single cantilever column does not result in more than a 33% reduction in story strength, nor create an extreme torsional irregularity (horizontal structural irregularity type 1b).

- There are no requirements for all other structural systems.

In accordance with ASCE/SEI7 Sec. 12.3.4.2b, the value of the redundancy factor may be taken equal to 1.0 provided that the building is regular in plan at all levels with not less than two bays of lateral load-resisting perimeter framing on each side of the building in each orthogonal direction at each story resisting more than 35% of the base shear. The number of bays for a shear wall is calculated as the length of the shear wall divided by the story height. For light-framed construction, the number of bays for a shear wall is calculated as twice the length of the shear wall divided by the story height.

Lateral Forces

Example 7.23

Determine the redundancy factor for the moment-resisting framed structure shown. The stiffness of all frames is identical. The roof diaphragm is flexible.
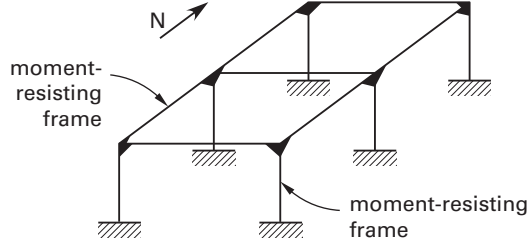
Solution

The building is regular in plan, and this complies with ASCE/SEI7 Sec. 12.3.4.2b.

In the north-south direction, two bays of moment-resisting perimeter frames are provided on each side of the building. The frames on each perimeter resist 50% of the base shear. This complies with ASCE/SEI7 Sec. 12.3.4.2b.

In the east-west direction, only one bay of moment-resisting perimeter frames is provided on each side of the building. This does not comply with ASCE/SEI7 Sec. 12.3.4.2b.

In the north-south direction, removing one frame results in a reduction of shear strength of

$$\frac{1}{4} = 0.25$$
$$< 0.33$$

This complies with ASCE/SEI7 Sec. 12.3.4.2a.

In the east-west direction, removing one frame results in a reduction of shear strength of

$$\frac{1}{3} = 0.33$$

This complies with ASCE/SEI7 Sec. 12.3.4.2a.

The redundancy factor is given by ASCE/SEI7 Sec. 12.3.4 as

$$\rho = 1.0$$

Example 7.24

Determine the redundancy factor for the structure shown. The stiffness of all braced frames in the north-south direction is identical, and the stiffness of all moment-resisting frames in the east-west direction is identical. The roof diaphragm is flexible.
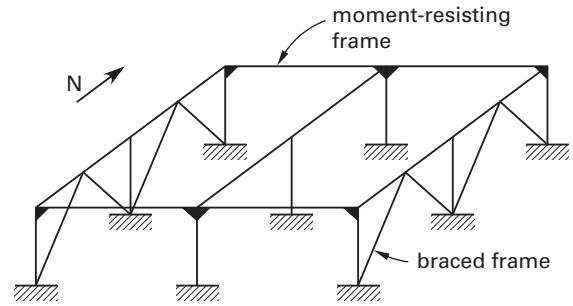
Solution

The building is regular in plan and this complies with ASCE/SEI7 Sec. 12.3.4.2b.

In the north-south direction, two bays of chevron bracing are provided on each side of the building. The frames on each perimeter resist 50% of the base shear. This complies with ASCE/SEI7 Sec. 12.3.4.2b.

Similarly, in the east-west direction, two bays of moment-resisting frames are provided on each side of the building. The frames on each perimeter resist 50% of the base shear. This complies with ASCE/SEI7 Sec. 12.3.4.2b.

The redundancy factor is given by ASCE/SEI7 Sec. 12.3.4 as

$$\rho = 1.0$$

PART 3: WIND DESIGN

Nomenclature

a	width of pressure coefficient zone	ft
A	effective wind area	ft^2
A_g	gross area of wall in which A_o is identified	ft^2
A_{gi}	sum of gross surface areas of building envelope (walls and roof), excluding A_g	ft^2
A_o	total area of openings in a wall receiving positive external pressure	ft^2
A_{og}	total area of openings in building envelope	ft^2
A_{oi}	sum of areas of all openings in building envelope (walls and roof), excluding A_o	ft^2
B	horizontal dimension of building measured normal to wind direction	ft
C_{net}	net pressure coefficient based on $K_d((GC_p) - (GC_{pi}))$ per IBC Table 1609.6.2	–
C_p	external pressure coefficient	–
C_{pi}	internal pressure coefficient	–
G	gust effect factor	–
(GC_p)	product of gust effect factor and external pressure coefficient	–

(GC_{pf})	product of gust effect factor and equivalent external pressure coefficient for determining wind loads in MWFRS of low-rise buildings	–
(GC_{pi})	product of internal pressure coefficient and gust effect factor	–
h	mean roof or eave height	ft
I	importance factor	–
K_1, K_2, K_3	multipliers from ASCE/SEI7 Fig. 26.8-1 used to obtain K_{zt}	–
K_d	wind directionality factor given in ASCE/SEI7 Table 26.6-1	–
K_h	velocity pressure exposure coefficient evaluated at height $z = h$	–
K_z	velocity pressure exposure coefficient evaluated at height z	–
K_{zt}	topographic factor as defined in ASCE/SEI7 Sec. 26.8.1	–
L	horizontal dimension of a building measured parallel to wind direction	ft
MWFRS	main wind force-resisting system	–
p	design pressure for determining wind loads	lbf/ft^2
P_{net}	design wind pressure for determining wind loads on buildings, building components, or cladding	lbf/ft^2
q	velocity pressure	lbf/ft^2
q_h	velocity pressure evaluated at height $z = h$	lbf/ft^2
q_s	wind stagnation pressure	lbf/ft^2
q_z	velocity pressure evaluated at height z above ground	lbf/ft^2
V	basic wind speed	mi/hr
z	height above ground level	ft
z_g	nominal height of atmospheric boundary layer	ft

Symbols

| θ | angle of plane of roof from horizontal | degree |

12. WIND LOADS

Design Procedures

Two basic wind design procedures, the directional procedure and the envelope procedure, are defined in ASCE/SEI7 Sec. 26.2. The *directional procedure* determines the wind loads on buildings for specific wind directions. It uses external pressure coefficients based on wind tunnel testing of prototypical building models for the corresponding direction of wind. The *envelope procedure* determines the wind load cases on buildings. It uses pseudo-external pressure coefficients derived from wind tunnel testing of prototypical building models successively rotated through 360 degrees. These pseudo-pressure cases produce key structural actions (uplift, horizontal shear, bending moments, etc.) that envelop the actions' maximum values among all possible wind directions.

Building Characteristics

The following terms are defined in ASCE/SEI7 Sec. 26.2 and are used to designate building types and components.

- The *main wind-force resisting system* is an assemblage of structural elements assigned to provide support and stability for the overall structure. The system generally receives wind loading from more than one surface.

- *Components* and *cladding* are elements of the building envelope that do not qualify as part of the main wind-force resisting system. *Building cladding* receives wind loading directly. Examples of cladding include wall and roof sheathing, windows, and doors. *Building components* receive wind loading from the cladding and transfer the load to the main wind force-resisting system. Components include purlins, studs, girts, fasteners, and roof trusses. Some elements, such as roof trusses and sheathing, may also form part of the main wind force-resisting system and must be designed for both conditions.

- *Low-rise buildings* are enclosed or partially enclosed buildings that comply with the following conditions: the mean roof height, h, is less than or equal to 60 ft; and the mean roof height also does not exceed the least horizontal dimension.

- *Simple diaphragm buildings* are buildings in which both windward and leeward wind loads are transmitted by roof and vertically spanning wall assemblies, through continuous floor and roof diaphragms, to the main wind-force resisting system.

- *Rigid buildings* have a fundamental frequency greater than or equal to 1 Hz. Most buildings with a height-to-minimum-width ratio of less than 4 may be considered rigid.

- *Regular-shaped buildings* have no unusual geometrical irregularity in spatial form.

Wind Load Determination

In accordance with IBC Sec. 1609.1.1, wind loads on buildings are determined by the provisions of ASCE/SEI7 or by the alternate method of IBC Sec. 1609.6. Several different procedures for determining wind loads on the main wind-force resisting system of buildings are specified as follows.

- The analytical directional design method of ASCE/SEI7 Sec. 27.4 is applicable to enclosed, partially enclosed, and open buildings of all heights and roof geometry.

- The simplified method of ASCE/SEI7 Sec. 27.5 is based on the analytical method of ASCE/SEI7 Chap. 27, Part 1 and is applicable to enclosed, simple diaphragm buildings of any roof geometry with a height not exceeding 160 ft.

- The envelope design method of ASCE/SEI7 Sec. 28.4 is applicable to enclosed, partially enclosed, and

Lateral Forces

open low-rise buildings having a flat, gable, or hip roof with a height not exceeding 60 ft.

- The simplified method of ASCE/SEI7 Sec. 28.6 is based on the envelope procedure of ASCE/SEI7 Chap. 28, Part 1 and is applicable to enclosed, simple diaphragm low-rise buildings having a flat, gable, or hip roof with a height not exceeding 60 ft.

- The wind tunnel procedure of ASCE/SEI7 Chap. 31 may be used for any structure.

- The simplified alternate all-heights method of IBC Sec. 1609.6 is based on the analytical method of ASCE/SEI7 Sec. 27.4. The building must be a simple diaphragm building with dimensions such that either $h \leq 75$ ft and $h/L \leq 4$, or the building is a rigid structure.

Several different procedures for determining wind loads on components and cladding of buildings are specified as follows.

- The analytical envelope design method of ASCE/SEI7 Sec. 30.4 is applicable to enclosed and partially enclosed low-rise buildings.

- The simplified envelope design method of ASCE/SEI7 Sec. 30.5 is applicable to enclosed low-rise buildings.

- The analytical directional design method of ASCE/SEI7 Sec. 30.6 is applicable to enclosed and partially enclosed buildings with $h > 60$ ft.

- The simplified directional design method of ASCE/SEI7 Sec. 30.7 is applicable to enclosed buildings with $h \leq 160$ ft.

- The analytical directional design method of ASCE/SEI7 Sec. 30.8 is applicable to open buildings of all heights.

- The alternate method of IBC Sec. 1609.6 is applicable to simple diaphragm buildings with $h \leq 75$ ft with a height-to-least-width ratio of 4 or less.

Wind loads on parapets and roof overhangs are covered in ASCE/SEI7 Chap. 29.

The following parameters must be known in order to determine a structure's wind load.

- exposure category of the site
- basic wind speed at the location of the structure
- velocity pressure exposure coefficient
- topographic effects
- directionality factor
- wind importance factor
- wind velocity pressure

Surface Roughness Category

Ground surface roughness is defined in ASCE/SEI7 Sec. 26.7.2. (See Table 7.10.)

Table 7.10 *Surface Roughness Categories*

surface roughness category	applicable ground surface roughness
B	urban and suburban areas, wooded areas, or other terrain with numerous closely spaced obstructions the size of single-family dwellings or larger
C	open terrain containing scattered obstructions that have heights generally less than 30 ft; includes flat open country and grasslands
D	flat, unobstructed areas and water surfaces; includes smooth mud flats, salt flats, and unbroken ice

Site Exposure Category

A site's exposure category accounts for the effect of terrain roughness on wind speed and is described in ASCE/SEI7 Sec. C26.7. Table 7.11 lists the three exposure categories defined in ASCE/SEI7 Sec. 26.7.3.

Table 7.11 *Site Exposure Categories*

site exposure category	applicable site exposure
B	applies (1) for building height $h \leq 30$ ft and where surface roughness B prevails in the upwind direction for more than 1500 ft, or (2) for building height $h > 30$ ft, and where surface roughness B prevails in the upwind direction for more than the greater of 2600 ft or $20h$
C	applies in all cases where exposures B or D do not apply
D	applies (1) where surface roughness D prevails in the upwind direction for more than the greater of 5000 ft or $20h$, or (2) where surface roughness immediately upwind of the site is B or C for a distance not exceeding the greater of 600 ft or $20h$ is followed by surface roughness D for more than the greater of 5000 ft or $20h$

Basic Wind Speed

Basic wind speed, V, is used to determine the design wind loads on buildings and other structures, and it is determined from ASCE/SEI7 Fig. 26.5-1A, Fig. 26.5-1B, and Fig. 26.5-1C. Wind is assumed to come from any horizontal direction. Basic wind speed values are based on the 3 sec gust wind speed (in miles per hour). They are adjusted to a reference height of 33 ft for exposure category C. Drag effects slow down wind flow close to the ground, and wind speed increases with height above ground level until the gradient height is reached and the speed becomes constant. The gradient heights, z_g, for different exposure conditions are given in ASCE/SEI7 Table 26.9-1.

The wind speed is given at the strength level that gives a load factor of 1.0 for wind loads in the strength design load combinations. ASCE/SEI7 uses three wind speed maps for buildings with different risk categories. This ensures that high-risk facilities are designed for higher loads so as to reduce possible structural damage. Details of the different risk categories and corresponding wind speed maps are given in Table 7.12.

Table 7.12 *Risk Category and Wind Speed Maps*

risk category	nature of occupancy	ASCE/SEI7 wind speed maps
I	low hazard structures	26.5-1C
II	standard occupancy structures	26.5-1A
III	assembly structures	26.5-1B
IV	essential or hazardous structures	26.5-1B

Velocity Pressure Exposure Coefficients

The velocity pressure exposure coefficient, K_z, reflects the change in wind speed with height and exposure category. K_z values for different exposure conditions are given in ASCE/SEI7 Table 27.3-1 and Table 28.3-1. These are tabulated for a limited number of heights in Table 7.13 for main wind force-resisting systems for the purpose of determining overall wind loads on the building.

Table 7.13 *Velocity Pressure Exposure Coefficients for Main Wind Force-Resisting Systems*

	height above ground level (ft)					
exposure	0–15	20	25	30	40	50
B[a]	0.57	0.62	0.66	0.70	0.76	0.81
B[b]	0.70	0.70	0.70	0.70	0.76	0.81
C	0.85	0.90	0.94	0.98	1.04	1.09
D	1.03	1.08	1.12	1.16	1.22	1.27

[a]directional procedure
[b]envelope procedure

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Minimum Design Wind Loads

The minimum design wind loads for an enclosed or partially enclosed building are given in ASCE/SEI7 Sec. 27.1.5 and are shown in ASCE/SEI7 Fig. C27.4-1. The loading consists of an external pressure of 16 lbf/ft^2 on wall areas and 8 lbf/ft^2 on roof areas projected onto a vertical plane normal to the wind direction. Details are shown in Fig. 7.31. The minimum loads are to be applied as a separate load case in addition to the normal load cases specified.

Figure 7.31 *Minimum Design Wind Loads*
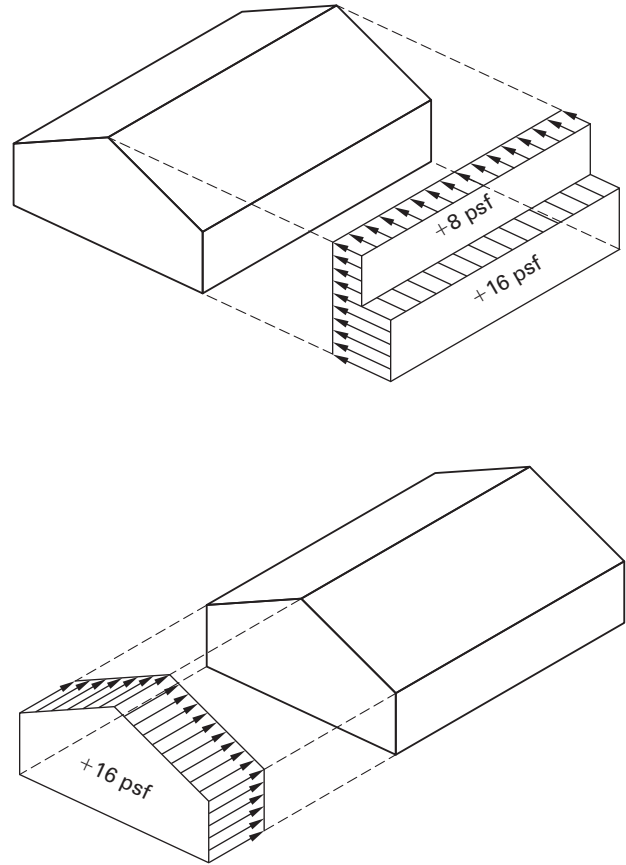
Topographic Effects

Increased wind speed effects are produced at isolated hills, ridges, and escarpments constituting abrupt changes in the general topography. To account for this, the velocity pressure exposure coefficient is multiplied by the *topographic factor*, K_{zt}. The topographic factor is a function of the following criteria.

- slope of the hill
- distance of the building from the crest
- height of the building above the local ground surface

These three criteria are represented by the topographic multipliers K_1, K_2, and K_3, and are tabulated in ASCE/SEI7 Fig. 26.8-1. The topographic factor is given by ASCE/SEI7 Eq. 26.8-1 as

$$K_{zt} = (1 + K_1 K_2 K_3)^2$$

When the topographic effect need not be considered, it is given by

$$K_{zt} = 1.0$$

Directionality Factor

The *wind directionality factor*, K_d, is determined from ASCE/SEI7 Table 26.6-1. For buildings, it is given as 0.85. The directionality factor accounts for the reduced probability of the following.

- extreme winds occurring in any specific direction

- the peak pressure coefficient occurring for a specific wind direction

Wind Velocity Pressure

The *wind velocity pressure*, q_z (in units of lbf/ft^2), is evaluated at a height, z, using ASCE/SEI7 Eq. 28.3-1. (This formula is not dimensionally consistent.)

$$q_{z,\text{lbf/ft}^2} = 0.00256 K_z K_{zt} K_d V^2_{\text{mi/hr}}$$

The constant 0.00256 reflects the mass density of air at a temperature of 59°F and a pressure of 29.92 in of mercury. This value should be used unless sufficient data is available to justify a different value. The velocity pressure varies as the value of the velocity pressure exposure coefficient varies with the height above ground level.

Example 7.25

The steel framed factory shown is located in a suburban area 25 miles inland from the coast of Southern California. The factory is not located in a special wind region. Determine the wind velocity pressure at roof height for the main wind force-resisting system.

(not to scale)
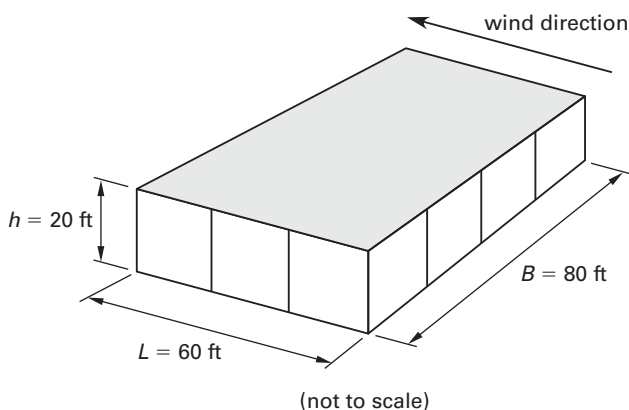
Solution

For a factory building, the risk category is II. From Table 7.12, wind speed map ASCE/SEI7 Fig. 26.5-1A applies.

From ASCE/SEI7 Fig. 26.5-1A, the basic wind speed is

$$V = 110 \text{ mi/hr}$$

The height-to-minimum-width ratio is

$$\frac{h}{L} = \frac{20 \text{ ft}}{60 \text{ ft}} = 0.33$$

$$< 1 \quad \text{[low-rise building]}$$

$$< 4 \quad \text{[rigid building]}$$

The mean roof height is

$$h = 20 \text{ ft}$$

$$< 60 \text{ ft} \quad \text{[low-rise building]}$$

So, the building qualifies as a low-rise, rigid building. For a building designed using ASCE/SEI7 Fig. 28.4-1 values from ASCE/SEI7 Table 28.3-1 are applicable for the velocity pressure exposure coefficients.

For a suburban area, the exposure is category B and the relevant parameters are obtained as follows.

$$
\begin{aligned}
K_z &= \text{velocity pressure exposure coefficient from ASCE/} \\
&\quad \text{SEI7 Table 28.3-1 for a height of 20 ft for the main} \\
&\quad \text{wind force-resisting system and exposure} \\
&\quad \text{category B} \\
&= 0.70 \\
K_{zt} &= \text{topographic factor from ASCE/SEI7 Fig. 26.8-1} \\
&= 1.0 \\
K_d &= \text{wind directionality factor from ASCE/SEI7} \\
&\quad \text{Table 26.6-1} \\
&= 0.85
\end{aligned}
$$

The wind velocity pressure, q_h, at a roof height of 20 ft above the ground, is given by ASCE/SEI7 Eq. 28.3-1.

$$q_{h,\text{lbf/ft}^2} = 0.00256 K_z K_{zt} K_d V^2_{\text{mi/hr}}$$

$$= (0.00256)(0.70)(1.0)(0.85)\left(110 \, \frac{\text{mi}}{\text{hr}}\right)^2$$

$$= 18.43 \text{ lbf/ft}^2$$

13. DESIGN WIND PRESSURE

In order to determine the design wind pressures on a structure, it is necessary to convert the wind velocity pressure to *design wind pressure*. To do so, the following items must be determined.

- rigidity of the structure

- gust effect factor

- enclosure classification

Rigidity of the Structure

A *flexible structure* is defined in ASCE/SEI7 Sec. 26.2 as a slender structure with a fundamental frequency less than 1 Hz. Therefore, a *rigid structure* is a structure with a fundamental frequency greater than or equal to 1 Hz. A flexible structure exhibits a significant dynamic resonant response to wind gusts. According to ASCE/SEI7

Sec. C26.2, most structures with a height-to-minimum-width ratio less than 4 qualify as rigid. Where necessary, the fundamental frequency may be determined using the procedures given in ASCE/SEI7 Sec. 26.9.3.

Gust Effect Factor

For a rigid structure, the *gust effect factor* is taken as 0.85. Alternatively, the gust effect factor may be calculated using the procedure given in ASCE/SEI7 Sec. 26.9.5. The gust effect factor accounts for along-wind loading effects (that is, loading effects in the direction of the wind) caused by dynamic amplification in flexible structures and for wind turbulence-structure interaction.

Enclosure Classification

A structure's *enclosure classification* is determined in accordance with ASCE/SEI7 Sec. 26.2, and is based on the number of openings in its building envelope. ASCE/SEI7 Sec. 26.2 defines *openings* as apertures or holes in the building envelope that allow air to flow through the building envelope. The *building envelope* is defined as cladding, roofing, exterior walls, glazing, door assemblies, window assemblies, skylight assemblies, and other components enclosing the structure or building. Per ASCE/SEI7 Sec. 26.2, there are three types of building enclosures: enclosed, partially enclosed, and open.

An *enclosed building* is defined as one that does not comply with the requirements for open or partially enclosed buildings.

A *partially enclosed building* is defined as satisfying both of the following conditions.

- *condition 1:* The total area of openings in a wall that receives positive external pressure must exceed the sum of the areas of openings in the balance of the building envelope (walls and roof) by more than 10%.

- *condition 2:* The total area of openings in a wall that receives positive external pressure exceeds 4 ft^2 or 1% of the area of that wall, whichever is smaller, and the percentage of openings in the balance of the building envelope does not exceed 20%.

These conditions are expressed by the following equations.

- $A_o > 1.10 A_{oi}$

- $A_o > 4$ ft^2 or $> 0.01 A_g$, whichever is smaller, and $A_{oi}/A_{gi} \leq 0.20$

An *open building* is defined as a building where each wall is at least 80% open. This condition is expressed for each wall by

$$A_o \geq 0.8 A_g$$

ASCE/SEI7 Sec. 26.10.3.1 states that if a building is in a wind-borne debris region, then all glazing in the lower 60 ft of the building must be impact resistant or protected with an impact-resistant covering. The same requirement applies to glazing that is less than 30 ft above any aggregate surface roof located within 1500 ft of the building. Glazing in category I buildings may be unprotected.

A *wind-borne debris region* is defined in ASCE/SEI7 Sec. 26.10.3.1 as being

- within 1 mi of the coastal mean high water line and where the basic wind speed is at least 130 mph

- any region where the basic wind speed is at least 140 mph

A structure's *internal pressure* (i.e., the pressure that is produced in a structure by wind) depends on the size and location of openings in the external walls of the structure. Figure 7.32 shows an opening in the windward wall of a structure, which produces an internal pressure, and an opening in the leeward wall of a structure, which produces an internal suction.

Figure 7.32 *Building Openings*
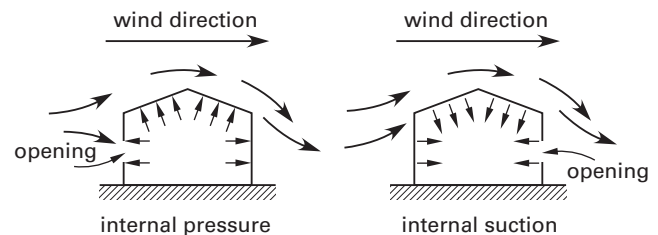
14. LOW-RISE REGULAR BUILDING, MAIN WIND FORCE-RESISTING SYSTEM

Nomenclature

(GC_{pf})	product of the equivalent external pressure coefficient and gust effect factor as given in ASCE/SEI7 Fig. 28.4-1	–
(GC_{pi})	product of the internal pressure coefficient and gust effect factor as given in ASCE/SEI7 Table 26.11-1	–
q_h	wind velocity pressure at mean roof height h for the applicable exposure category	lbf/ft^2

Envelope Design Method for MWFRS

The analytical envelope procedure (Part 1) defined in ASCE/SEI7 Sec. 28.4 is applicable to a low-rise structure that meets both of the following requirements.

- The structure must be regular-shaped as defined in ASCE/SEI7 Sec. 26.2, without any irregularities such as projections or indentations.

- Either the structure does not have response characteristics that make it subject to across-wind loading, vortex shedding, or instability due to galloping or flutter; or it does not have a site location for which channeling effects or buffeting in the wake of upwind obstructions warrant special consideration.

For low-rise buildings, a simplification is introduced into the analytical procedure by combining the gust effect factor with the pressure coefficient and treating the combination as a single factor. The design wind pressure, p, on the main system is given by ASCE/SEI7 Eq. 28.4-1 as

$$p = q_h\big((GC_{pf}) - (GC_{pi})\big)$$

Internal Pressure Coefficients

For the envelope procedure of ASCE/SEI7 Chap. 28, Part 1, the gust effect factor is combined with the internal pressure coefficient and denoted by (GC_{pi}). Values of (GC_{pi}) are tabulated in ASCE/SEI7 Table 26.11-1 for the three different building enclosure classifications and are given in Table 7.14.

Table 7.14 *Values of Internal Pressure Coefficients*

enclosure classification	(GC_{pi})
open buildings	0.00
partially enclosed buildings	±0.55
enclosed buildings	±0.18

The pressure acting on internal surfaces is obtained from the second term of ASCE/SEI7 Eq. 28.4-1 as

$$p_i = \pm q_h(GC_{pi})$$

Pressures act normal to wall and roof surfaces. They are positive when acting toward the surface and negative when acting away from the surface. The conditions that produce internal suction and internal pressure are shown in Fig. 7.32. Both cases must be considered for any building and added algebraically to external pressures to determine the most critical loading condition.

Design Wind Load Cases

In the envelope procedure, a building is designed for all wind directions by considering each corner of the building as the windward corner. At each corner, wind acting in both the transverse and longitudinal directions must be considered, giving eight basic load cases as illustrated in ASCE/SEI7 Fig. 28.4-1. Two of these load cases are shown in Fig. 7.33. Load case A covers wind in the transverse direction, and load case B covers wind in the longitudinal direction.

For each of the eight basic load cases, both positive and negative internal pressure must be considered, resulting in a total of 16 combinations. When the building is symmetrical about one axis, only two corners need to be investigated. If the building is doubly symmetrical, only one corner needs to be investigated. When torsion must be considered, each of these load cases is also modified as indicated in ASCE/SEI7 Fig. 28.4-1, Note 5. Torsion need not be considered if the building is one story with a height less than 30 ft, or if the building is no more than two stories and either

Figure 7.33 *Load Case A and Load Case B*
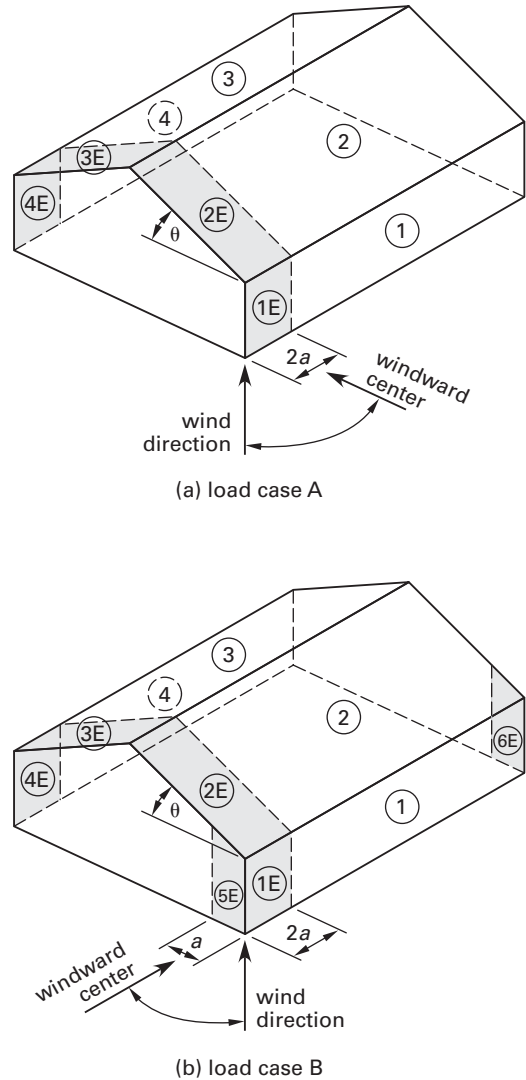
(a) load case A

(b) load case B

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designed with flexible diaphragms or framed with light frame construction.

Local turbulence at building corners and at the roof eaves produces an increase in pressure in these areas. To allow for the differences in pressure, the method subdivides the building surface into distinct zones. As shown in Fig. 7.33, eight zones are designated for transverse load, and twelve zones are designated for longitudinal wind. External pressure coefficients are tabulated for each zone.

External Pressure Coefficients

For the envelope procedure described in ASCE/SEI7 Chap. 28, Part 1, the gust effect factor is combined with the external pressure coefficients and denoted by

(GC_{pf}). Values of (GC_{pf}) are tabulated in ASCE/SEI7 Fig. 28.4-1 and given in Table 7.15 and Table 7.16. Values are given for two separate loading conditions—case A for wind acting transversely and case B for wind acting longitudinally to the building.

The pressure acting on external surfaces is obtained from the first term of ASCE/SEI7 Eq. 28.4-1 as

$$p_e = q_h(GC_{pf})$$

On each wall and roof surface external pressure coefficients are given for two zones: an end zone and an interior zone. The end zone width is given by ASCE/SEI7 Fig. 28.4-1, Note 9 as $2a$, where a is the lesser of the following.

$$a = (0.1)(\text{least horizontal dimension})$$

$$a = 0.4h$$

However, a cannot be less than either of the following.

$$a = (0.04)(\text{least horizontal dimension})$$

$$a = 3 \text{ ft}$$

Pressures act normal to wall and roof surfaces and are positive when acting toward the surface. They are negative when acting away from the surface.

Envelope Procedure

The following information is needed to determine wind loads using the envelope procedure.

- risk category I, II, III, or IV from Table 7.12 [ASCE/SEI7 Table 1.5-1]
- basic wind speed, V, for the applicable risk category from ASCE/SEI7 Fig. 26.5-1A, Fig. 26.5-1B, and Fig. 26.5-1C

- exposure category B, C, or D from Table 7.11 [ASCE/SEI7 Sec. 26.7]
- velocity pressure exposure coefficients, K_z, for the applicable exposure category from Table 7.13 [ASCE/SEI7 Table 28.3-1]
- topographic factor, K_{zt}, from ASCE/SEI7 Fig. 26.8-1
- directionality factor, K_d, from ASCE/SEI7 Table 26.6-1
- enclosure classification from ASCE/SEI7 Sec. 26.10
- internal pressure coefficient, (GC_{pi}), from Table 7.14 [ASCE/SEI7 Table 26.11-1]
- wind velocity pressure, q_h, from ASCE/SEI7 Eq. 28.3-1
- external pressure coefficient, (GC_{pf}), from Table 7.15 and Table 7.16 [ASCE/SEI7 Fig. 28.4-1]
- internal wind pressure, $p_i = q_h(GC_{pi})$, from ASCE/SEI7 Eq. 28.4-1
- external wind pressure, $p_e = q_h(GC_{pf})$, from ASCE/SEI7 Eq. 28.4-1
- combined internal and external wind pressures, p, from ASCE/SEI7 Eq. 28.4-1
- minimum applicable design wind loads from Fig. 7.31 [ASCE/SEI7 Sec. 28.4.4]

The wind pressures are applied to each building corner in turn, as shown in ASCE/SEI7 Fig. 28.4-1. Torsional effects are evaluated if necessary.

Example 7.26

The steel framed factory analyzed in Ex. 7.25 may be considered a closed structure. Determine the design wind pressure for the main wind force-resisting system at the ends of the 80 ft side, for transverse wind load.

Table 7.15 *External Pressure Coefficients for Load Case A*

roof angle, θ (degrees)	load case A building surface							
	1	2	3	4	1E	2E	3E	4E
0–5	0.40	−0.69	−0.37	−0.29	0.61	−1.07	−0.53	−0.43
20	0.53	−0.69	−0.48	−0.43	0.80	−1.07	−0.69	−0.64
30–45	0.56	0.21	−0.43	−0.37	0.69	0.27	−0.53	−0.48
90	0.56	0.56	−0.37	−0.37	0.69	0.69	−0.48	−0.48

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Table 7.16 *External Pressure Coefficients for Load Case B*

roof angle, θ (degrees)	load case B building surface											
	1	2	3	4	5	6	1E	2E	3E	4E	5E	6E
0–90	−0.45	−0.69	−0.37	−0.45	0.40	−0.29	−0.48	−1.07	−0.53	−0.48	0.61	−0.43

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Lateral Forces

Solution

From Ex. 7.25, the velocity pressure at mean roof height is obtained as

$$q_h = 18.43 \text{ lbf/ft}^2$$

The height-to-minimum-width ratio is

$$\frac{h}{L} = \frac{20 \text{ ft}}{60 \text{ ft}} = 0.33$$
$$< 1$$

The mean roof height is

$$h = 20 \text{ ft}$$
$$< 60 \text{ ft}$$

So, the building qualifies as a low-rise, regular building as defined by ASCE/SEI7 Sec. 26.2, and the low-rise building analytical method of ASCE/SEI7 Sec. 28.4 is applicable. Values of (GC_{pf}) may be obtained from Table 7.15.

For a one-story building with a height less than 30 ft, ASCE/SEI7 Fig. 28.4-1 specifies that torsional load cases may be neglected. To determine the design wind pressure at the ends of the building, the pressures on surfaces 1E, 2E, 3E, and 4E must be determined for load case A. For an enclosed building, the product of the internal pressure coefficient and gust effect factor is given by Table 7.14 as

$$(GC_{pi}) = \pm 0.18$$

For surface 1E, the product of the equivalent external pressure coefficient and gust effect factor is given by Table 7.15 as

$$(GC_{pf}) = 0.61$$

The design wind pressure is given by ASCE/SEI7 Eq. 28.4-1 as

$$p = q_h\big((GC_{pf}) - (GC_{pi})\big)$$
$$= \left(18.43 \; \frac{\text{lbf}}{\text{ft}^2}\right)(0.61 - (\pm 0.18))$$
$$= 14.56 \text{ lbf/ft}^2 \quad \begin{bmatrix} \text{for negative internal} \\ \text{pressure (suction)} \end{bmatrix}$$
$$= 7.93 \text{ lbf/ft}^2 \quad \text{[for positive internal pressure]}$$

For surface 2E, the product of the equivalent external pressure coefficient and gust effect factor is given by ASCE/SEI7 Fig 28.4-1 as

$$(GC_{pf}) = -1.07$$

The design wind pressure is given by ASCE/SEI7 Eq. 28.4-1 as

$$p = q_h\big((GC_{pf}) - (GC_{pi})\big)$$
$$= \left(18.43 \; \frac{\text{lbf}}{\text{ft}^2}\right)(-1.07 - (\pm 0.18))$$
$$= -16.40 \text{ lbf/ft}^2 \quad \begin{bmatrix} \text{for negative internal} \\ \text{pressure (suction)} \end{bmatrix}$$
$$= -23.04 \text{ lbf/ft}^2 \quad \text{[for positive internal pressure]}$$

For surface 3E, the product of the equivalent external pressure coefficient and gust effect factor is given by ASCE/SEI7 Fig. 28.4-1 as

$$(GC_{pf}) = -0.53$$

The design wind pressure is given by ASCE/SEI7 Eq. 28.4-1 as

$$p = q_h\big((GC_{pf}) - (GC_{pi})\big)$$
$$= \left(18.43 \; \frac{\text{lbf}}{\text{ft}^2}\right)(-0.53 - (\pm 0.18))$$
$$= -6.45 \text{ lbf/ft}^2 \quad \begin{bmatrix} \text{for negative internal} \\ \text{pressure (suction)} \end{bmatrix}$$
$$= -13.09 \text{ lbf/ft}^2 \quad \text{[for positive internal pressure]}$$

For surface 4E, the product of the equivalent external pressure coefficient and gust effect factor is given by ASCE/SEI7 Fig. 28.4-1 as

$$(GC_{pf}) = -0.43$$

The design wind pressure is given by ASCE/SEI7 Eq. 28.4-1 as

$$p = q_h\big((GC_{pf}) - (GC_{pi})\big)$$
$$= \left(18.43 \; \frac{\text{lbf}}{\text{ft}^2}\right)(-0.43 - (\pm 0.18))$$
$$= -4.61 \text{ lbf/ft}^2 \quad \begin{bmatrix} \text{for negative internal} \\ \text{pressure (suction)} \end{bmatrix}$$
$$= -11.24 \text{ lbf/ft}^2 \quad \text{[for positive internal pressure]}$$

The wind pressure diagrams for both internal suction and internal pressure are shown.

internal pressure

internal suction

Lateral Forces
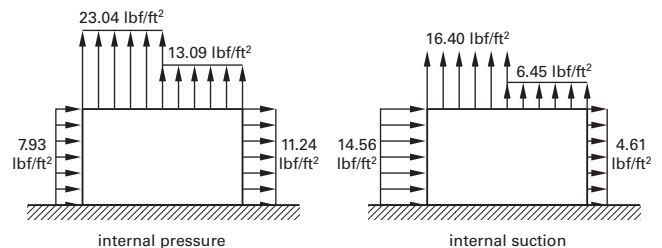
15. LOW-RISE REGULAR BUILDING, COMPONENTS AND CLADDING

Nomenclature

b_e	effective tributary width, $\geq l/3$	ft
(GC_p)	product of the equivalent external pressure coefficient and gust effect factor, as given in ASCE/SEI7 Fig. 30.4-1 through Fig. 30.4-7	–
(GC_{pi})	product of the internal pressure coefficient and gust effect	–
l	element span length	ft
q_h	wind velocity pressure at mean roof height, h	lbf/ft^2

Envelope Design Procedure for Components and Cladding

The envelope design procedure of ASCE/SEI7 Sec. 30.4 is applicable to enclosed and partially enclosed buildings which are low-rise buildings as defined in ASCE/SEI7 Sec. 26.2; buildings with $h \leq 60$ ft; or buildings with flat roofs, gable roofs, multispan gable roofs, hip roofs, monoslope roofs, stepped roofs, or sawtooth roofs.

This procedure is simplified by combining the gust effect factor with the pressure coefficient and treating the combination as a single factor. Then the design wind pressure, p, on the main system is given by ASCE/SEI7 Eq. 30.4-1 as

$$p = q_h\big((GC_p) - (GC_{pi})\big)$$

Velocity Pressure Exposure Coefficients

The velocity pressure exposure coefficient, K_z, reflects the change in wind speed with height and exposure category and is tabulated for different exposure categories given in ASCE/SEI7 Table 30.3-1. Table 7.17 lists the velocity pressure exposure coefficients for a limited number of heights.

Table 7.17 *Velocity Pressure Exposure Coefficients for Components and Cladding Systems*

exposure category	height above ground level (ft)					
	0–15	20	25	30	40	50
B	0.70	0.70	0.70	0.70	0.76	0.81
C	0.85	0.90	0.94	0.98	1.04	1.09
D	1.03	1.08	1.12	1.16	1.22	1.27

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Minimum Design Wind Loads

The minimum design wind loads for an enclosed or partially enclosed building are given in ASCE/SEI7 Sec. 30.2.2. The loading consists of a net pressure of 16 lbf/ft^2 applied in either direction normal to the surface.

Internal Pressure Coefficients

For the envelope procedure of ASCE/SEI7 Sec. 30.4, the gust effect factor is combined with the internal pressure coefficient and denoted by (GC_{pi}). Refer to Table 7.14 for (GC_{pi}) values for enclosed and partially enclosed buildings.

The pressure acting on internal surfaces is obtained from the second term of ASCE/SEI7 Eq. 30.4-1 as

$$p_i = \pm q_h(GC_{pi})$$

Pressures act normal to wall and roof surfaces and are positive when acting towards the surface and negative when acting away from the surface. The conditions that produce internal suction and internal pressure are shown in Fig. 7.32. Both cases must be considered for any building and added algebraically to external pressures to determine the most critical loading condition.

Effective Wind Area

Because of local turbulence, which may occur over small areas and at ridges and corners of buildings, components and cladding are designed for higher wind pressures than the main wind force-resisting system. An effective wind area is used to determine the external pressure coefficient. This is defined in ASCE/SEI7 Sec. 26.2 as the span length multiplied by an effective tributary width and is given by

$$A = b_e l$$
$$b_e \geq l/3$$

l is the element span length and b_e is the effective tributary width.

For cladding fasteners, the effective wind area must not be greater than the area that is tributary to an individual fastener.

In accordance with ASCE/SEI7 Sec. 30.2.3, component and cladding elements with tributary areas greater than 700 ft^2 may be designed using provisions for the main wind force-resisting system.

External Pressure Coefficients

For the envelope procedure of ASCE/SEI7 Sec. 30.4, the gust effect factor is combined with the external pressure coefficients and denoted by (GC_p). The pressure acting on external surfaces is obtained from the first term of ASCE/SEI7 Eq. 30.4-1 as

$$p_e = \pm q_h(GC_p)$$

Values of (GC_p) for walls are tabulated in ASCE/SEI7 Fig. 30.4-1 and for roofs in ASCE/SEI7 Fig. 30.4-2 through Fig. 30.4-7. The values of (GC_p) are a function of the effective area attributed to the element considered. Because turbulence at wall corners and at roof eaves produces a large increase in pressure, the building surface is divided into distinct zones as shown in Fig. 7.34.

Lateral Forces

Figure 7.34 *Components and Cladding External Pressure Zones*
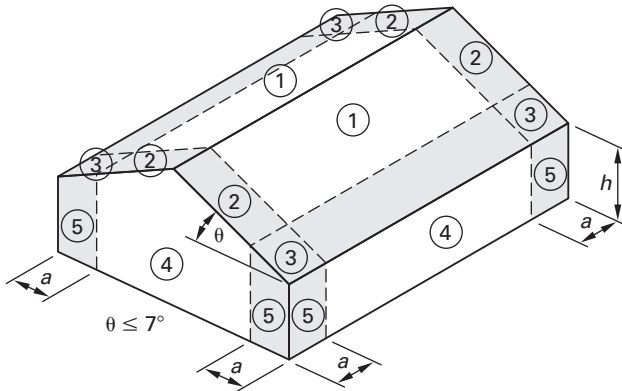
Adapted with permission from *Minimum Design Loads for Buildings and Other Structures*, copyright © 2010, by the American Society of Civil Engineers.

Walls are divided into two zones, an end zone and an interior zone. Roofs are divided into three zones, an end zone, an interior zone, and a corner zone, with a different wind pressure coefficient assigned to each. ASCE/SEI7 Fig. 30.4-1, Note 6 gives the end-of-wall zone width, labeled 5, and the eave zone width, labeled 2, as a, where a is the lesser of the following.

$$a = (0.1)(\text{least horizontal dimension})$$

$$a = 0.4h$$

However, a cannot be less than either of the following.

$$a = (0.04)(\text{least horizontal dimension})$$

$$a = 3 \text{ ft}$$

Pressures act normal to wall and roof surfaces and are positive when acting toward the surface, and negative when acting away from the surface. In accordance with ASCE/SEI7 Fig. 30.4-1, Note 5, the values of (GC_p) must be reduced by 10% for the walls of buildings with a roof slope of 10° or less.

Envelope Procedure for Components and Cladding

The following information is needed to determine wind loads for components and cladding using the envelope procedure.

- risk category I, II, III, or IV from Table 7.12 [ASCE/SEI7 Table 1.5-1]
- basic wind speed, V, for the applicable risk category from ASCE/SEI7 Fig. 26.5-1A, Fig. 26.5-1B, or Fig. 26.5-1C
- exposure category B, C, or D from Table 7.11 [ASCE/SEI7 Sec. 26.7]
- velocity pressure exposure coefficients, K_z, for the applicable exposure category from Table 7.17 [ASCE/SEI7 Table 30.3-1]
- topographic factor, K_{zt}, from ASCE/SEI7 Fig. 26.8-1

- directionality factor, K_d, from ASCE/SEI7 Table 26.6-1
- enclosure classification from ASCE/SEI7 Sec. 26.10
- internal pressure coefficient, (GC_{pi}), from Table 7.14 [ASCE/SEI7 Table 26.11-1]
- wind velocity pressure, q_h, from ASCE/SEI7 Eq. 30.3-1
- external pressure coefficient, (GC_p), from ASCE/SEI7 Fig. 30.4-1 through Fig. 30.4-7
- internal wind pressure, $p_i = q_h(GC_{pi})$ from ASCE/SEI7 Eq. 30.4-1
- external wind pressure, $p_e = q_h(GC_p)$ from ASCE/SEI7 Eq. 30.4-1
- combined internal and external wind pressures, p, from ASCE/SEI7 Eq. 30.4-1
- minimum design wind loads from ASCE/SEI7 Sec. 30.2.2

Example 7.27

The roof framing of the factory analyzed in Ex. 7.25 consists of open web joists spaced at 5 ft centers that span 20 ft parallel to the long side of the building. For wind flowing normal to the 80 ft long side of the building, determine the design wind pressure acting on a roof joist in interior zone 1, and the width of eave zone 2. The building is enclosed.

Solution

From Ex. 7.25, the velocity pressure at mean roof height using case 1 values for K_z is

$$q_h = 18.43 \text{ lbf/ft}^2$$

The product of the internal pressure coefficient and the gust effect factor is given by Table 7.14 as

$$(GC_{pi}) = \pm 0.18$$

The mean roof height is

$$h = 20 \text{ ft}$$

$$< 60 \text{ ft}$$

Therefore, the low-rise building method of ASCE/SEI7 Sec. 30.4 is applicable.

The width of eave zone 2 is given by ASCE/SEI7 Fig. 30.4-2A, Note 7 as the lesser of the following.

$$a = 0.1L$$
$$= (0.1)(60 \text{ ft})$$
$$= 6 \text{ ft} \quad [\text{governs}]$$

Or,

$$a = 0.4h$$
$$= (0.4)(20 \text{ ft})$$
$$= 8.0 \text{ ft}$$

Lateral Forces

However, the width cannot be less than either of the following.

$$a = 0.04L$$
$$= (0.04)(60 \text{ ft})$$
$$= 2.4 \text{ ft}$$
$$a = 3 \text{ ft}$$

Therefore,

$$a = 6 \text{ ft}$$

The effective tributary width of a roof joist is defined in ASCE/SEI7 Sec. 26.2 as the larger of the following.

$$b_e = \text{joist spacing}$$
$$= 5 \text{ ft}$$

Or,

$$b_e = \frac{l}{3}$$
$$= \frac{20 \text{ ft}}{3}$$
$$= 6.67 \text{ ft} \quad \text{[governs]}$$

The effective wind area attributed to the roof joist is

$$A = b_e l$$
$$= (6.67 \text{ ft})(20 \text{ ft})$$
$$= 133 \text{ ft}^2$$

The negative external pressure coefficient for roof interior zone 1, for an effective wind area of 133 ft^2, is obtained from ASCE/SEI7 Fig. 30.4-2A as

$$(GC_p) = -0.9$$

The negative design wind pressure on a roof joist for interior zone 1 is obtained from ASCE/SEI7 Eq. 30.4-1 as

$$p = q_h\big((GC_p) - (GC_{pi})\big)$$
$$= \left(18.43 \ \frac{\text{lbf}}{\text{ft}^2}\right)(-0.9 - 0.18)$$
$$= -19.90 \text{ lbf/ft}^2$$

The upward load on the roof joist over interior zone 1 is

$$w = ps$$
$$= \left(-19.90 \ \frac{\text{lbf}}{\text{ft}^2}\right)(5 \text{ ft})$$
$$= -99.50 \text{ lbf/ft}$$

The positive external pressure coefficient for roof interior zone 1, for an effective wind area of 133 ft^2, is obtained from ASCE/SEI7 Fig. 30.4-2A as

$$(GC_p) = 0.2$$

The positive design wind pressure on a roof joist for interior zone 1 is obtained from ASCE/SEI7 Eq. 30.4-1 as

$$p = q_h\big((GC_p) - (GC_{pi})\big)$$
$$= \left(18.43 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.2 - (-0.18))$$
$$= 7.00 \text{ lbf/ft}^2$$

The downward load on the roof joist over interior zone 1 is

$$w = ps$$
$$= \left(7.00 \ \frac{\text{lbf}}{\text{ft}^2}\right)(5 \text{ ft})$$
$$= 35.00 \text{ lbf/ft}$$

The wind loading acting on the roof joist is shown.

upward load   downward load
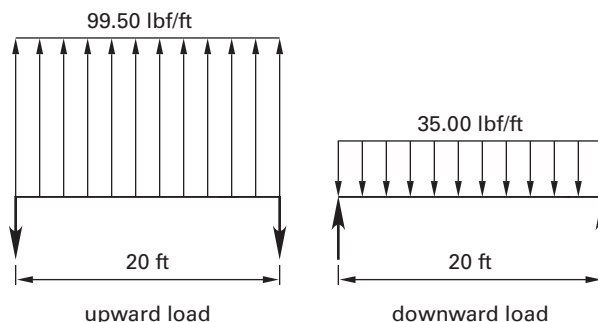
16. IBC ALTERNATE ALL-HEIGHTS PROCEDURE

The *IBC alternate all-heights wind design provisions* (known also as the *IBC alternate procedure*) are simplifications of the directional design method of ASCE/SEI7 Sec. 27.4. The IBC Sec. 1609.6 alternate procedure[4,5] may be used to determine the wind effects on regularly shaped structures that meet all of the following conditions.

- The structure is less than or equal to 75 ft in height with a height-to-least-width ratio of 4 or less, or the structure has a fundamental frequency greater than or equal to 1 Hz.

- The structure is not sensitive to dynamic effects.

- The structure is not located on a site for which channeling effects or buffeting in the wake of upwind obstructions warrant special consideration.

- The structure meets the requirements of a simple diaphragm building as defined in ASCE/SEI7 Sec. 26.2, in which both windward and leeward wind loads are transmitted through floor and roof diaphragms to the same vertical main wind force-resisting system (i.e., there are no structural separations).

Lateral Forces

In order to determine wind loads by the IBC alternate procedure, the following must be derived.

- velocity pressure exposure coefficient
- topographic factor
- wind stagnation pressure
- net pressure coefficient
- design wind pressure

Velocity Pressure Exposure Coefficient

Wind speed increases with height and as the exposure changes from category B (terrain with closely spaced obstructions) to category D (flat, open terrain). Based on the exposure category, a *velocity pressure exposure coefficient*, K_z, is determined from ASCE/SEI7 Table 27.3-1. For the windward wall, K_z is based on the actual height above ground level of each floor of the building. For leeward walls, side walls, and roofs, $K_z = K_h$. K_z is evaluated at mean roof height only and is a constant value over the height of the building.

Topographic Factor

Increased wind speed effects are produced at isolated hills, ridges, and escarpments constituting abrupt changes in the general topography. To account for this, the velocity pressure exposure coefficient is multiplied by the *topographic factor*, K_{zt}. The topographic factor is a function of the following three criteria.

- slope of the hill
- distance of the building from the crest
- height of the building above the local ground surface

These three criteria are represented by the multipliers K_1, K_2, and K_3 that are tabulated in ASCE/SEI7 Fig. 26.8-1. The topographic factor is given by ASCE/SEI7 Eq. 26.8-1 as

$$K_{zt} = (1 + K_1 K_2 K_3)^2$$

When no topographic effect is to be considered, the topographic factor is

$$K_{zt} = 1.0$$

Wind Stagnation Pressure

The basic wind speed may be converted to a *stagnation pressure*, q_s, at a standard height of 33 ft by the following formula. (This formula is not dimensionally consistent.)

$$q_{s,\text{lbf/ft}^2} = 0.00256 V^2$$

Values of q_s are provided in Table 7.18.

Table 7.18 *Wind Stagnation Pressure*

basic wind speed, V (mph)	pressure, q_s (lbf/ft^2)
85	18.5
90	20.7
100	25.6
105	28.2
110	31.0
120	36.9
125	40.0
130	43.3
140	50.2
150	57.6
160	65.5

Net-Pressure Coefficient

The inherent simplicity of the IBC alternate procedure lies in the derivation of the net pressure coefficient given by IBC Sec. 1609.6.2 as

$$C_{\text{net}} = K_d \big(G C_p - (G C_{pi}) \big)$$

This equation adds internal and external pressures as is appropriate for a simple diaphragm building where the internal pressures cancel out. In addition, the wind directionality factor and gust effect factor are taken as constants with the following.

$$
\begin{aligned}
K_d &= \text{wind directionality factor from ASCE/SEI7} \\
&\quad \text{Table 26.6-1} \\
&= 0.85 \quad \text{[appropriate for all buildings]} \\
G &= \text{gust effect factor from ASCE/SEI7 Sec. 26.9.1} \\
&= 0.85 \quad \text{[appropriate for a rigid structure]} \\
C_p &= \text{external pressure coefficient from ASCE/SEI7} \\
&\quad \text{Fig. 27.4-1} \\
(G C_{pi}) &= \text{product of internal pressure coefficient and gust} \\
&\quad \text{effect factor from ASCE/SEI7 Fig. 26.11-1}
\end{aligned}
$$

Therefore, the net pressure coefficient expression reduces to

$$
\begin{aligned}
C_{\text{net}} &= K_d \big(G C_p - (G C_{pi}) \big) \\
&= 0.85 \big(0.85 C_p - (G C_{pi}) \big)
\end{aligned}
$$

Values of C_{net} are provided in IBC Table 1609.6.2. Values are given for enclosed and partially enclosed buildings and for roofs of varying slopes. Values are given for windward wall and roof, leeward wall and roof, and side wall.

Design Wind Pressure

The design wind pressure is calculated from IBC Eq. 16-35 as

$$P_{\text{net}} = q_s K_z C_{\text{net}} K_{zt}$$

Variables are defined as follows.

K_z = velocity pressure exposure coefficient evaluated at height z and listed in ASCE/SEI7 Table 27.3-1

K_{zt} = topographic factor as defined in ASCE/SEI7 Sec. 26.8.2

In ASCE/SEI7 Eq. 26.8-1, the topographic factor is defined as

$$K_{zt} = (1 + K_1 K_2 K_3)^2$$

The topographic multipliers K_1, K_2, and K_3 are tabulated in ASCE/SEI7 Fig. 26.8-1. When no topographic effect is to be considered, the topographic factor is

$$K_{zt} = 1.0$$

Values of P_{net} are applicable for the design of both main frames and components and cladding.

In accordance with IBC Sec. 1609.6.3, design wind forces for the main system may not be less than 16 lbf/ft^2 multiplied by the area of the building projected on a plane normal to the wind direction. Design net wind pressure for components and cladding cannot be less than 16 lbf/ft^2 acting in either direction normal to the surface.

Wind Pressure on Main System: Alternate Design Method

In determining wind pressure in the main wind force-resisting system, the IBC alternate procedure method does not provide net pressure coefficients to account for turbulence at wall corners or roof ridge and eaves. Wind pressure is constant over the surface for leeward walls and side walls. For windward walls, wind pressure varies with height since K_z varies with height. For the roof, separate coefficients are given for the windward and leeward portions.

IBC Alternate Procedure for Main Wind Force-Resisting System

The following information is needed to determine wind loads using the IBC alternate procedure.

- risk category I, II, III, or IV from Table 7.12 [ASCE/SEI7 Table 1.5-1]

- basic wind speed, V, for the applicable risk category from ASCE/SEI7 Fig. 26.5-1A, Fig. 26.5-1B, or Fig. 26.5-1C

- exposure category B, C, or D from Table 7.11 [ASCE/SEI7 Sec. 26.7]

- velocity pressure exposure coefficients, K_z, for the applicable exposure category from Table 7.13 [ASCE/SEI7 Table 27.3-1]

- topographic factor, K_{zt}, from ASCE/SEI7 Fig. 26.8-1

- net-pressure coefficients at walls and roofs, C_{net}, from IBC Table 1609.6.2

- design wind pressure, P_{net}, from IBC Eq. 16-35

- minimum design wind pressure from IBC Sec. 1609.6.3

The following example demonstrates how the IBC alternate design method may be used to solve for wind pressure on main systems.

Example 7.28

The steel framed factory analyzed in Ex. 7.25 meets the requirements of a simple diaphragm building and may be considered a closed structure. The building is not sensitive to dynamic effects, and it is not located on a site at which channeling or buffeting occurs. For wind flowing normal to the 80 ft side of the building, determine the design wind pressure for the main wind force-resisting system at the windward wall, the leeward wall, and the roof.

Solution

The height-to-minimum-width ratio is

$$\frac{h}{L} = \frac{20 \text{ ft}}{60 \text{ ft}}$$
$$= 0.33$$
$$< 4$$

So, the structure is rigid as defined by ASCE/SEI7 Sec. 26.2.

The mean roof height is

$$h = 20 \text{ ft}$$
$$< 75 \text{ ft}$$

Therefore, the alternate method of IBC Sec. 1609.6 is applicable and values of C_{net} may be obtained from IBC Table 1609.6.2.

The value of the wind stagnation pressure for a wind speed, V, of 110 mi/hr is obtained from Table 7.18 as

$$q_s = 31.0 \text{ lbf/ft}^2$$

For the main wind force-resisting system of a building designed using the alternate method, per IBC Sec. 1609.6, values in ASCE/SEI7 Table 27.3-1 may be used for the velocity pressure exposure coefficients.

For a suburban area, the exposure is category B and the relevant parameters are as follows.

K_h = velocity pressure exposure coefficient at roof height from ASCE/SEI7 Table 27.3-1 for a height of 20 ft for the main wind force-resisting system and exposure category B

= 0.62

Lateral Forces

K_z = velocity pressure exposure coefficient at a
height of 15 ft for the main wind force-resisting
system and exposure category B
= 0.57
K_{zt} = topographic factor
= 1.0 from ASCE/SEI7 Fig. 26.8-1

For a one-story building with a height not exceeding
30 ft, ASCE/SEI7 Fig. 28.4-1 specifies that torsional
load cases may be neglected.

For the windward wall

IBC Table 1609.6.2 gives the values of the net pressure
coefficient as

C_{net} = 0.43 for positive internal pressure
= 0.73 for negative internal pressure (suction)

The design wind pressure at roof height, given by IBC
Eq. 16-35, is

$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.62)(0.73)(1.0)$$
$$= 14.03 \ \text{lbf/ft}^2 \quad [\text{negative internal pressure (suction)}]$$
$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.62)(0.43)(1.0)$$
$$= 8.27 \ \text{lbf/ft}^2 \quad [\text{positive internal pressure}]$$

The design wind pressure at a height of 15 ft is

$$P_{net} = q_s K_z C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.57)(0.73)(1.0)$$
$$= 12.90 \ \text{lbf/ft}^2 \quad [\text{negative internal pressure (suction)}]$$
$$P_{net} = q_s K_z C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.57)(0.43)(1.0)$$
$$= 7.60 \ \text{lbf/ft}^2 \quad [\text{positive internal pressure}]$$

For the leeward wall

IBC Table 1609.6.2 gives the values of the net pressure
coefficient as

C_{net} = −0.51 [positive internal pressure]
= −0.21 [negative internal pressure (suction)]

The design wind pressure, given by IBC Eq. 16-35, is

$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.62)(-0.21)(1.0)$$
$$= -4.04 \ \text{lbf/ft}^2 \quad [\text{negative internal pressure (suction)}]$$
$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.62)(-0.51)(1.0)$$
$$= -9.80 \ \text{lbf/ft}^2 \quad [\text{positive internal pressure}]$$

For the windward roof

IBC Table 1609.6.2 gives the values of the net pressure
coefficient as

C_{net} = −1.09 [positive internal pressure]
= −0.79 [negative internal pressure (suction)]

The design wind pressure, given by IBC Eq. 16-35, is

$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.62)(-0.79)(1.0)$$
$$= -15.18 \ \text{lbf/ft}^2 \ [\text{negative internal pressure (suction)}]$$
$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.62)(-1.09)(1.0)$$
$$= -20.95 \ \text{lbf/ft}^2 \quad [\text{positive internal pressure}]$$

For the leeward roof

IBC Table 1609.6.2 gives the values of the net pressure
coefficient as

C_{net} = −0.66 [positive internal pressure]
= −0.35 [negative internal pressure (suction)]

The design wind pressure, given by IBC Eq. 16-35, is

$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.62)(-0.35)(1.0)$$
$$= -6.73 \ \text{lbf/ft}^2 \quad [\text{negative internal pressure (suction)}]$$
$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.62)(-0.66)(1.0)$$
$$= -12.69 \ \text{lbf/ft}^2 \quad [\text{positive internal pressure}]$$

The wind pressure diagrams for both cases, internal suction and internal pressure, are shown.

internal pressure

internal suction
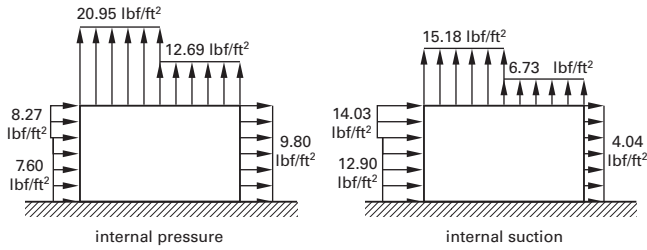
Wind Pressure on Components and Cladding: Alternate Design Method

In determining wind pressure for components and cladding, the alternate design method provides net pressure coefficients to account for turbulence at roof corners, ridges, and eaves. Reference is made to the zones illustrated in ASCE/SEI7 Fig. 30.4-2, Fig. 30.4-5, and Fig. 30.8-1. Velocity pressure exposure coefficients, K_z, for the applicable exposure category are obtained from ASCE/SEI7 Table 30.3-1.

The design procedure for components and cladding is similar to that used for the main wind force-resisting system.

The following example demonstrates how the alternate design method may be used to find the wind pressure on components and cladding.

Example 7.29

The roof framing of the building analyzed in Ex. 7.25 consists of open web joists spaced at 5 ft centers that span 20 ft parallel to the long side of the building. For wind flowing normal to the 80 ft long side of the building, determine the design wind pressure acting on a roof joist in interior zone 1. The building is enclosed.

Solution

The effective tributary width of a roof joist is defined in ASCE/SEI7 Sec. 26.2 as the larger of the following.

$$b_e = \text{joist spacing}$$
$$= 5 \text{ ft}$$

Or,

$$b_e = \frac{l}{3} = \frac{20 \text{ ft}}{3}$$
$$= 6.67 \text{ ft} \quad [\text{governs}]$$

The effective wind area attributed to the roof joist is

$$A = b_e l$$
$$= (6.67 \text{ ft})(20 \text{ ft})$$
$$= 133 \text{ ft}^2$$

The value of the wind stagnation pressure for a wind speed, V, of 110 mi/hr is obtained from Table 7.18 as

$$q_s = 31.0 \text{ lbf/ft}^2$$

Case 1 values are applicable for the velocity pressure exposure coefficients for the components and cladding of a building designed using the alternate method of IBC Sec. 1609.6. For a suburban area, the exposure is category B and the relevant parameters are obtained as

K_h = velocity pressure exposure coefficient at roof height from ASCE/SEI7 Table 30.3-1 for a height of 20 ft for components and cladding for exposure category B

$\quad = 0.70$

K_{zt} = topographic factor from ASCE/SEI7 Fig. 28.6-2

$\quad = 1.0$

From Ex. 7.27, the tributary wind area attributed to the roof truss is $A = 133 \text{ ft}^2$.

The negative net pressure coefficient for interior zone 1 with a tributary area of 133 ft^2 and a slope of $< 27°$ is obtained from IBC Table 1609.6.2 as

$$C_{\text{net}} = -0.92$$

The net negative design wind pressure on a roof joist for interior zone 1 is obtained from IBC Eq. 16-35, which gives

$$P_{\text{net}} = q_s K_h C_{\text{net}} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.70)(-0.92)(1.0)$$
$$= -19.96 \text{ lbf/ft}^2$$

The upward load on the roof joist in interior zone 1 is

$$w = P_{\text{net}} s$$
$$= \left(-19.96 \ \frac{\text{lbf}}{\text{ft}^2}\right)(5 \text{ ft})$$
$$= -99.80 \text{ lbf/ft}$$

The positive net pressure coefficient for interior zone 1 is obtained from IBC Table 1609.6.2 for a tributary area of 133 ft^2 and a slope of less than 27° as

$$C_{\text{net}} = 0.41$$

The net positive design wind pressure on a roof joist for interior zone 1 is obtained from IBC Eq. 16-35, which gives

$$P_{\text{net}} = q_s K_h C_{\text{net}} K_{zt}$$
$$= \left(31.0 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.70)(0.41)(1.0)$$
$$= 8.90 \text{ lbf/ft}^2$$

Lateral Forces

The downward load on the roof joist in interior zone 1 is

$$w = P_{net} s$$
$$= \left(8.90 \ \frac{lbf}{ft^2}\right)(5 \ ft)$$
$$= 44.50 \ lbf/ft$$

The wind loading acting on the roof joist is shown.

−99.80 lbf/ft

44.50 lbf/ft

20 ft

20 ft

upward load

downward load
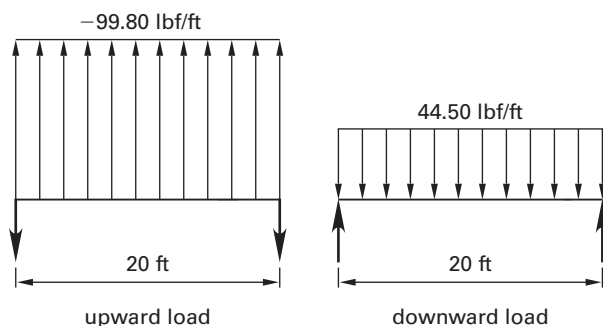
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Lateral Forces

PRACTICE PROBLEMS

Lateral Force-Resisting Systems

1. A dual system with steel moment frames and special steel concentrically braced frames is subjected to a seismic force of 280 kips. The relative stiffnesses of the braced frame and the moment frame are 100:40. What is most nearly the design lateral force for each frame?

    (A) 200 kips (braced frame); 80 kips (moment frame)

    (B) 210 kips (braced frame); 90 kips (moment frame)

    (C) 220 kips (braced frame); 100 kips (moment frame)

    (D) 230 kips (braced frame); 110 kips (moment frame)

2. A dual system with steel moment frames and special steel concentrically braced frames is subjected to a seismic force of 280 kips. The relative stiffnesses of the braced frame and the moment frame are 100:15. What is most nearly the design lateral force for each frame?

    (A) 230 kips (braced frame); 60 kips (moment frame)

    (B) 240 kips (braced frame); 70 kips (moment frame)

    (C) 250 kips (braced frame); 80 kips (moment frame)

    (D) 260 kips (braced frame); 90 kips (moment frame)

3. The single-story building shown has a rigid roof diaphragm that is acted on by an east-west force of 40 kips. The building's center of gravity is located at its center.

(a) layout   (b) in-plane forces   (c) torsional forces
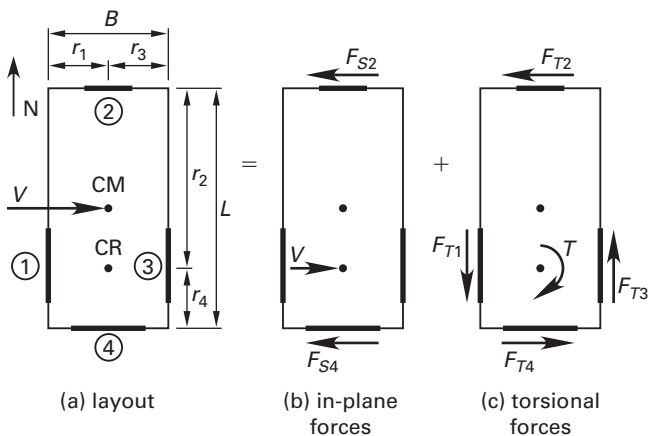
The following parameters apply.

- $L = 80$ ft
- $B = 40$ ft
- $R_4 = 3R$
- $R_1 = R_2 = R_3 = 1R$
- $e_y = 20$ ft
- allow for accidental eccentricity
- $r_1 = r_3 = 20$ ft
- $r_4 = 20$ ft
- $r_2 = 60$ ft
- $J = (5600 \text{ ft}^2)R$
- neglect the amplification factor

The force produced in wall 2 is most nearly

    (A) 17 kips

    (B) 18 kips

    (C) 19 kips

    (D) 20 kips

Seismic Design

4. A one-story industrial building is assigned to seismic design category D and is located in an area with a 0.2 sec acceleration coefficient of $S_{DS}=1.0$ and a 1.0 sec acceleration coefficient of $S_{D1}=0.7$. Details are shown in the illustration. The weight of the wood roof is 15 lbf/ft^2, and the weight of the masonry walls is 75 lbf/ft^2. The roof sheathing is $^{15}/_{32}$ in Structural I grade plywood and is blocked. For north-south seismic loads, what is most nearly the service level unit shear along the diaphragm boundaries?

plan

detail 1   section 1-1
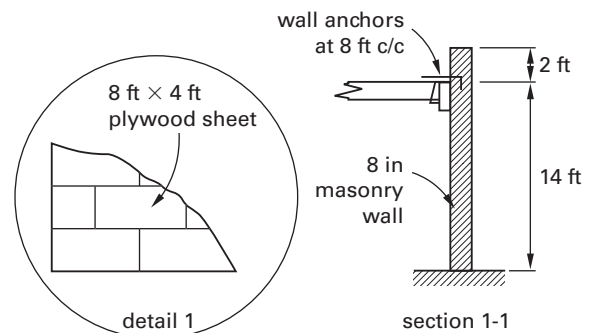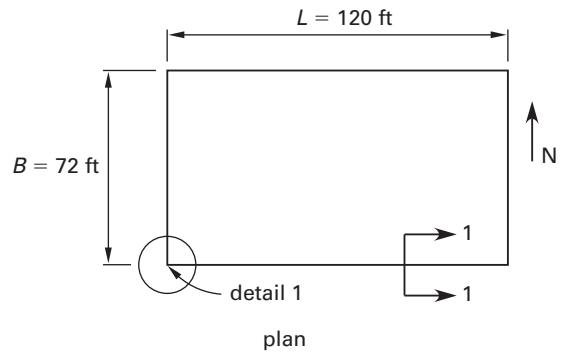
    (A) 285 lbf/ft

    (B) 320 lbf/ft

    (C) 380 lbf/ft

    (D) 410 lbf/ft

5. For the building of Prob. 4, wall anchors are provided to the masonry wall at 8 ft centers. What is most nearly the service level design force in each anchor?

    (A) 4600 lbf

    (B) 4700 lbf

    (C) 4800 lbf

    (D) 4900 lbf

Wind Design

6. The office building shown is located in a suburban area, which is subjected to a wind speed of $V = 100$ mi/hr. Wind flows normal to the 60 ft long side of the building. The building's risk category is II. The building is enclosed, has flexible roof and floor diaphragms, and it is not sensitive to dynamic effects, nor is it located on a site at which channeling or buffeting occur. What are most nearly the approximate wind loads (in lbf/ft^2) at the center of the windward wall for the main wind force-resisting system under positive internal force only? Use the IBC alternate method, if applicable.

wind direction

15 ft

15 ft

$B = 60$ ft

$L = 45$ ft

(not to scale)
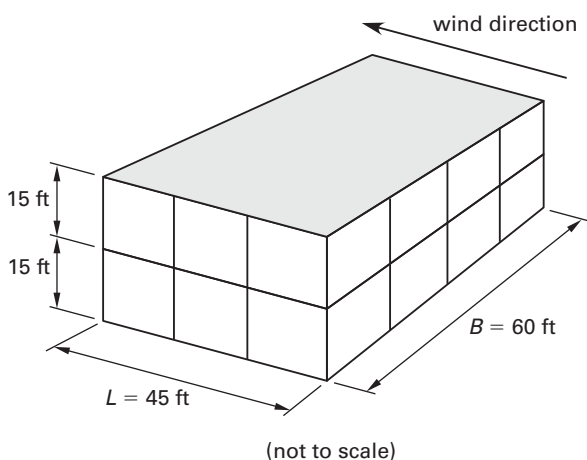
    (A) 6.5 lbf/ft^2 (roof height); 6.0 lbf/ft^2 (15 ft height)

    (B) 7.1 lbf/ft^2 (roof height); 6.6 lbf/ft^2 (15 ft height)

    (C) 7.4 lbf/ft^2 (roof height); 6.9 lbf/ft^2 (15 ft height)

    (D) 7.7 lbf/ft^2 (roof height); 6.3 lbf/ft^2 (15 ft height)

7. The roof framing of the building in Prob. 5 consists of joists spaced at 5 ft centers that span 15 ft parallel to the long side of the building. Wind flows normal to the 60 ft long side of the building. What is most nearly the design wind load that acts on a roof beam in interior zone 1? Use the IBC alternate method.

    (A) 37 lbf/ft (down); 76 lbf/ft (up)

    (B) 39 lbf/ft (down); 80 lbf/ft (up)

    (C) 41 lbf/ft (down); 84 lbf/ft (up)

    (D) 43 lbf/ft (down); 88 lbf/ft (up)

SOLUTIONS

1. The frames will resist applied force in proportion to their relative stiffnesses. Find the design force for the braced frame.

$$F = \frac{(100)(280 \text{ kips})}{100 + 40}$$
$$= 200 \text{ kips}$$

Find the design force for the moment frame.

$$F = 280 \text{ kips} - 200 \text{ kips}$$
$$= 80 \text{ kips}$$

ASCE/SEI7 Table 12.2-1 states that a moment frame must resist a minimum of 25% of the applied lateral force. Therefore, the minimum force for the moment frame is

$$F = (0.25)(280 \text{ kips})$$
$$= 70 \text{ kips} < 80 \text{ kips} \quad \text{[satisfactory]}$$

The answer is (A).

2. In accordance with ASCE/SEI7 Table 12.2-1, the moment frame in a dual system must be designed for 25% of the applied lateral force. Therefore, the moment frame must be designed for the force

$$F = (0.25)(280 \text{ kips})$$
$$= 70 \text{ kips}$$

The braced frame will resist the applied force proportional to its stiffness. Therefore, the braced frame must be designed for the force

$$F = \frac{(100)(280 \text{ kips})}{100 + 15}$$
$$= 243 \text{ kips} \quad (240 \text{ kips})$$

The answer is (B).

3. Accidental eccentricity, in accordance with ASCE/SEI7 Sec. 12.8.4.2, is

$$e_a = \pm 0.05L$$
$$= \pm (0.05)(80 \text{ ft})$$
$$= \pm 4 \text{ ft}$$

An accidental displacement of the center of mass to the north gives a maximum eccentricity of

$$e = e_y + e_a$$
$$= 20 \text{ ft} + 4.0 \text{ ft}$$
$$= 24 \text{ ft}$$

Lateral Forces

The maximum eccentricity governs for the force in wall 2 since the torsional force and the in-plane force act in the same direction and are additive. The maximum torsional moment acting about the center of rigidity is

$$T = Ve$$
$$= (40 \text{ kips})(24 \text{ ft})$$
$$= 960 \text{ ft-kips}$$

The torsional shear force in wall 2 is

$$F_{T2} = \frac{Tr_2 R_2}{J}$$
$$= \frac{(960 \text{ ft-kips})(60 \text{ ft})(1R)}{(5600 \text{ ft}^2)R}$$
$$= 10.29 \text{ kips}$$

The in-plane shear force in wall 2 is

$$F_{S2} = \frac{VR_2}{R_2 + R_4}$$
$$= \frac{(40 \text{ kips})1R}{1R + 3R}$$
$$= 10 \text{ kips}$$

The total force in wall 2 is

$$F_2 = F_{S2} + F_{T2}$$
$$= 10 \text{ kips} + 10.29 \text{ kips}$$
$$= 20.29 \text{ kips} \quad (20 \text{ kips})$$

The answer is (D).

4. The relevant dead load tributary to the roof diaphragm in the north-south direction is due to the north and south wall and the roof dead load and is obtained as

$$w_r = \left(15 \ \frac{\text{lbf}}{\text{ft}^2}\right)(72 \text{ ft})$$
$$= 1080 \text{ lbf/ft}$$

$$w_{\text{N+S walls}} = \frac{(2 \text{ walls})\left(75 \ \frac{\text{lbf}}{\text{ft}^2}\right)(16 \text{ ft})^2}{(2)(14 \text{ ft})}$$
$$= 1371 \text{ lbf/ft}$$

The total dead load tributary to the roof diaphragm is

$$w_{px} = \frac{\left(1080 \ \frac{\text{lbf}}{\text{ft}} + 1371 \ \frac{\text{lbf}}{\text{ft}}\right)(120 \text{ ft})}{1000 \ \frac{\text{lbf}}{\text{kip}}}$$
$$= 294 \text{ kips}$$

For a standard occupancy structure, the importance factor is

$$I_e = 1.0$$

For a bearing wall structure with reinforced masonry shear walls, the value of the response modification factor is obtained from Table 7.6 as

$$R = 5.0$$

For this type of structure, the maximum value of the seismic response coefficient controls, and the value of the seismic response coefficient is given by ASCE/SEI7 Sec. 12.8.1.1 as

$$C_s = \frac{S_{DS}I_e}{R}$$
$$= \frac{(1.0)(1.0)}{5}$$
$$= 0.2$$

In accordance with ASCE/SEI7 Sec. 12.10.1.1, the force acting on the roof diaphragm of a one-story building may be taken as

$$F_{px} = C_s w_{px}$$
$$= 0.2 w_{px}$$

The minimum allowable force acting on the roof diaphragm is given by ASCE/SEI7 Sec. 12.10.1.1 as

$$F_{px(\text{min})} = 0.2 S_{DS} I_e w_{px}$$
$$= (0.2)(1.0)(1.0) w_{px}$$
$$= 0.2 w_{px}$$

The force on the diaphragm is given by

$$F_{px} = (0.2)(294 \text{ kips})$$
$$= 59 \text{ kips}$$

This value of F_{px} is at the strength level, and the equivalent service level value for design of the diaphragm using allowable stress design is given by ASCE/SEI7 Sec. 2.4.1 as

$$F'_{px} = 0.7 F_{px}$$
$$= (0.7)(59 \text{ kips})$$
$$= 41 \text{ kips}$$

Lateral Forces

The service level design unit shear along the diaphragm boundary is

$$q = \frac{F'_{px}}{2B}$$

$$= \frac{(41 \text{ kips})\left(1000 \dfrac{\text{lbf}}{\text{kip}}\right)}{(2)(72 \text{ ft})}$$

$$= 285 \text{ lbf/ft}$$

The required nail spacing is obtained from SDPWS Table 4.2A with a case 1 plywood layout applicable, all edges blocked, and 2 in framing. Using both $^{15}/_{32}$ in Structural I grade plywood and 10d nails with $1^1/_2$ in penetration, a nail spacing of 6 in at the diaphragm boundaries, and 6 in at all other panel edges gives an allowable unit shear in accordance with SDPWS Sec. 4.2.3 of

$$q_a = \frac{640 \dfrac{\text{lbf}}{\text{ft}}}{2} = 320 \text{ lbf/ft}$$

$$> 285 \text{ lbf/ft} \quad \text{[satisfactory]}$$

The answer is (A).

5. The relevant weight of the element tributary to the wall anchors is obtained from Prob. 4 as

$$W_w = \frac{1371 \dfrac{\text{lbf}}{\text{ft}}}{2}$$

$$= 685.5 \text{ lbf/ft}$$

The weight of wall tributary to each anchor is

$$W_p = W_w s$$

$$= \left(685.5 \dfrac{\text{lbf}}{\text{ft}}\right)(8 \text{ ft})$$

$$= 5484 \text{ lbf}$$

From the illustration in Prob. 4, the span of the flexible diaphragm is $L_f = 120$ ft

The amplification factor for diaphragm flexibility is

$$k_a = 1.0 + \frac{L_f}{100}$$

$$= 1.0 + \frac{120 \text{ ft}}{100 \text{ ft}}$$

$$= 2.2$$

For seismic design category D, the seismic lateral force on an anchor is given by ASCE/SEI7 Eq. 12.11-1 as

$$F_p = 0.4 S_{DS} k_a I_e W_p$$

$$= (0.4)(1.0)(2.2)(1.0)(5484 \text{ lbf})$$

$$= 4826 \text{ lbf} \quad \text{[governs]}$$

The minimum permissible force on one anchor is

$$F_p = 0.2 k_a I_e W_p$$

$$= (0.2)(2.2)(1.0)(5484 \text{ lbf})$$

$$= 2413 \text{ lbf}$$

$$< 4826 \text{ lbf}$$

The required seismic design force for the anchors is

$$F_p = 4826 \text{ lbf} \quad (4800 \text{ lbf})$$

The answer is (C).

6. Determine whether the IBC alternate method, as defined in IBC Sec. 1609.6, is applicable. The height to minimum width ratio is

$$\frac{h}{L} = \frac{30 \text{ ft}}{45 \text{ ft}}$$

$$= 0.67$$

$$< 4 \quad \left[\begin{array}{c}\text{The structure is rigid as defined} \\ \text{by ASCE/SEI7 Sec. 26.2.}\end{array}\right]$$

The mean roof height is

$$h = 30 \text{ ft}$$

$$< 75 \text{ ft} \quad \left[\begin{array}{c}\text{IBC Sec. 1609.6 alternate} \\ \text{method is applicable.}\end{array}\right]$$

Since the IBC alternate method is applicable, values of C_{net} may be obtained from IBC Table 1609.6.2.

The value of the wind stagnation pressure for a wind speed V of 100 mi/hr is obtained from Table 7.18 as

$$q_s = 25.6 \text{ lbf/ft}^2$$

For the main wind force-resisting system of a building using the IBC alternate method, values for the velocity pressure exposure coefficients may be obtain from ASCE/SEI7 Table 27.3-1. The exposure category for a suburban area is B.

K_h = velocity pressure exposure coefficient at a roof height of 30 ft
   = 0.70
K_z = velocity pressure exposure coefficient at a height of 15 ft
   = 0.57

Lateral Forces

From ASCE/SEI7 Fig. 26.8-1, the topographic factor is

$$K_{zt} = 1.0$$

For a two story building with flexible diaphragms, ASCE/SEI7 Fig. 28.4.1 specifies that torsional load cases may be neglected.

For the windward wall, IBC Table 1609.6.2 gives the values of the net-pressure coefficient as

$$C_{net} = 0.43 \text{ for positive internal pressure}$$
$$= 0.73 \text{ for negative internal pressure (suction)}$$

IBC Eq. 16-35 gives the design wind pressure at roof height as

$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(25.6 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.70)(0.73)(1.0)$$
$$= 13.08 \ \frac{\text{lbf}}{\text{ft}^2} \quad \left[\begin{array}{c} \text{for negative internal} \\ \text{pressure (suction)} \end{array}\right]$$
$$P_{net} = \left(25.6 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.70)(0.43)(1.0)$$
$$= 7.71 \ \text{lbf/ft}^2 \quad (7.7 \ \text{lbf/ft}^2) \quad \left[\begin{array}{c} \text{for positive internal} \\ \text{pressure} \end{array}\right]$$

The design wind pressure at a height of 15 ft is

$$P_{net} = q_s K_z C_{net}(I K_{zt})$$
$$= \left(25.6 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.57)(0.73)(1.0)$$
$$= 10.65 \ \frac{\text{lbf}}{\text{ft}^2} \quad \left[\begin{array}{c} \text{for negative internal} \\ \text{pressure (suction)} \end{array}\right]$$
$$P_{net} = \left(25.6 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.57)(0.43)(1.0)$$
$$= 6.28 \ \text{lbf/ft}^2 \quad (6.3 \ \text{lbf/ft}^2) \quad \left[\begin{array}{c} \text{for positive internal} \\ \text{pressure} \end{array}\right]$$

The answer is (D).

7. The effective tributary width of a roof joist is defined in ASCE/SEI7 Sec. 6.2 as the larger of

$$b_e = \text{joist spacing}$$
$$= 5 \text{ ft}$$

Or,

$$b_e = \frac{I}{3}$$
$$= \frac{15 \text{ ft}}{3}$$
$$= 5 \text{ ft} \quad [\text{governs}]$$

The effective wind area attributed to the roof joist is then

$$A = b_e I$$
$$= (5 \text{ ft})(15 \text{ ft})$$
$$= 75 \text{ ft}^2$$

The value of the wind stagnation pressure for a wind speed V of 100 mi/hr is obtained from Table 7.18 as

$$q_s = 25.6 \ \text{lbf/ft}^2$$

Using the IBC alternate method, for a building's components and cladding, the velocity pressure exposure coefficient may be obtained from ASCE/SEI7 Table 27.3-1. The exposure category for a suburban area is B. Therefore,

$$K_h = \text{velocity pressure exposure coefficient at a roof}$$
$$\text{height of 30 ft}$$
$$= 0.70$$

From ASCE/SEI7 Fig. 26.8-1, the topographic factor is

$$K_{zt} = 1.0$$

IBC Table 1609.6.2 gives the negative net-pressure coefficient for interior zone 1 for a tributary area of 75 ft^2 and a slope of $< 27°$ as

$$C_{net} = -0.94$$

IBC Eq. 16-35 gives the net negative design wind pressure on a roof joist for interior zone 1 as

$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(25.6 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.70)(-0.94)(1.0)$$
$$= -16.85 \ \text{lbf/ft}^2$$

The upward load on the roof joist in interior zone 1 is

$$w = P_{net} s$$
$$= \left(-16.85 \ \frac{\text{lbf}}{\text{ft}^2}\right)(5 \text{ ft})$$
$$= -84.25 \ \text{lbf/ft} \quad (84 \ \text{lbf/ft})$$

IBC Table 1609.6.2 gives the positive net-pressure coefficient for interior zone 1 for a tributary area of 75 ft^2 and a slope of $< 27°$ as

$$C_{net} = 0.46$$

IBC Eq. 16-35 gives the net positive design wind pressure on a roof joist for interior zone 1 as

$$P_{net} = q_s K_h C_{net} K_{zt}$$
$$= \left(25.6 \ \frac{\text{lbf}}{\text{ft}^2}\right)(0.70)(0.46)(1.0)$$
$$= 8.25 \ \text{lbf/ft}^2$$

Lateral Forces

The downward load on the roof joist in interior zone 1 is

$$w = P_{net}\, s$$
$$= \left(8.25\ \frac{\text{lbf}}{\text{ft}^2}\right)(5\ \text{ft})$$
$$= 41.25\ \text{lbf/ft} \quad (41\ \text{lbf/ft})$$

The answer is (C).

8 Bridge Design

1. DESIGN LOADS

Nomenclature

a	tabulated force coefficient for distributed loads	–
A	area of beam	in^2
b	width of beam	in
B_s	width of equivalent strip	in
d	depth of beam	in
d_e	depth to the resultant of the tensile force	in
d_v	distance between the resultants of the tensile and compressive forces due to flexure	in
D	dead load	kips
DC	dead load of components and attachments	kips
DW	dead load of wearing surface and utilities	kips
e_g	distance between the centers of gravity of the beam and the deck slab	in
E	width of equivalent strip for a slab bridge	in
EQ	earthquake load	lbf or kips
g	distribution factor for moment or shear	–
I	dynamic factor	–
I	moment of inertia of beam	in^4
IM	dynamic load allowance	–
k_s	stiffness of equivalent strip	in^4
K_g	longitudinal stiffness parameter	in^4
L	span length	ft
L_1	modified span length	ft
LL	vehicular live load	kips
m	multiple presence factor	–
M	bending moment	ft-kips
M_D	dead load moment	ft-kips
M_L	live load moment	ft-kips
M_s	service load moment	ft-kips
M_u	factored design moment	ft-kips
n	ratio of the modulus of elasticity of the beam and the deck slab	–
N_b	number of beams	–
N_L	number of traffic lanes	–
P	wheel load	kips
Q	factored force effect	–
R_n	nominal resistance capacity	ft-kips
S	spacing of beams	ft
t_s	deck slab thickness	in
V	shear force	kips
w	distributed load	kips/ft
w	roadway width between curbs	ft
w_D	dead load	kips/ft
w_L	lane width	ft
W	concentrated load	kips
W	edge-to-edge width of the bridge	ft
W_1	modified edge-to-edge width of the bridge	ft
W_L	live load	kips
W_n	nominal load	kips
W_s	service load	kips
W_u	factored design load	kips
x	distance from the center line of a stringer to the face of the stringer	in

Symbols

γ	load factor	–
η	load modifier	–
ϕ	strength reduction factor	–

Design Lanes

A bridge deck is divided into design lanes as defined in AASHTO[1] Sec. 3.6. For deck widths between 20 ft and 24 ft, two design lanes are specified, each equal to one-half the deck width. For all other deck widths, design lanes are defined as being 12 ft wide, with fractional parts of a lane discounted, and the number of design lanes is given by

$$N_L = \text{INT}\left(\frac{w}{12}\right)$$

INT is the integer part of the ratio.

Design lanes are positioned on the deck to produce the maximum effect. The determination of the number of design lanes is illustrated in Fig. 8.1.

Bridges

Figure 8.1 *Design Traffic Lanes*

$N_L = 1 : w_L = 12$ ft

$w < 20$ ft

$N_L = 2 : w_L = \dfrac{w}{2}$

20 ft $\leq w \leq 24$ ft

$N_L = \text{INT} \left(\dfrac{w}{12} \right) : w_L = 12$ ft

$w > 24$ ft
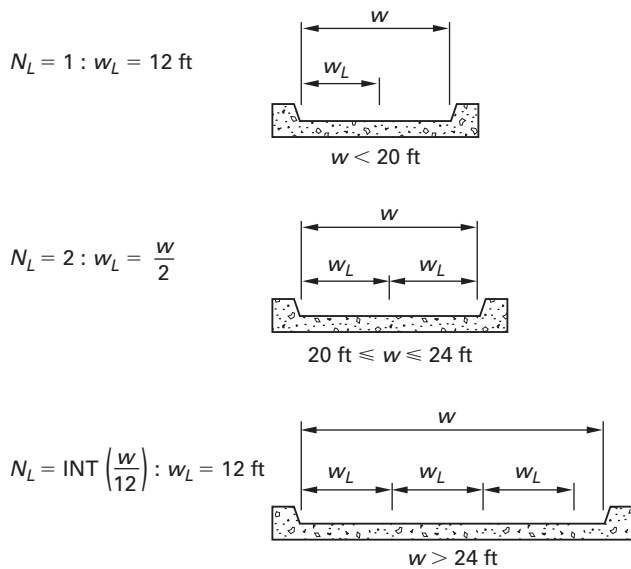
Example 8.1

For the bridge deck shown in the following illustration, determine the number of design lanes.

$w = 36$ ft

$w_L = 12$ ft $w_L = 12$ ft $w_L = 12$ ft

9 in
30 in

12 in

1.5 ft   11 ft   11 ft   11 ft   1.5 ft

39 ft
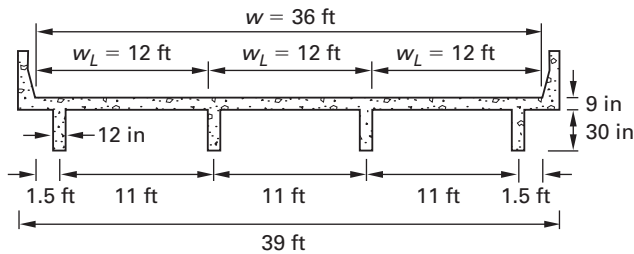
Solution

From AASHTO Sec. 3.6, the number of design lanes is given by

$$N_L = \frac{w}{12} = \frac{36 \text{ ft}}{12 \ \dfrac{\text{ft}}{\text{lane}}}$$

$$= 3 \text{ lanes}$$

Live Loads

The vehicular live loading for bridges is designated HL-93 and is specified in AASHTO Sec. 3.6.1.2. The loading consists of the more critical of the following two loading types.

- a design lane load combined with a design truck
- a design lane load combined with a design tandem

As shown in Fig. 8.2, the design lane load consists of a load of 0.64 kip/ft uniformly distributed in the longitudinal direction. In considering the design lane load in the design of continuous spans, as many spans shall be

loaded with the 0.64 kip/ft uniform load as is necessary to produce the maximum effect. The design lane load is placed longitudinally only on those portions of the spans of a bridge to give the most critical effect. Transversely, the design lane load is uniformly distributed over a 10 ft width. The 10 ft loaded width is placed in the design lane to give the most critical effect without encroaching on the adjacent lane. A dynamic load allowance is not applied to the design lane load.

Figure 8.2 *Design Lane Load*

0.64 kip/ft
(length to produce maximum effect)
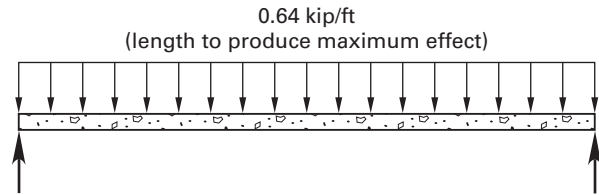
As shown in Fig. 8.3, the design truck load consists of three axles—the lead axle of 8 kips and the two following axles of 32 kips. The spacing between the two 32 kip axles is varied between 14 ft and 30 ft to produce the most critical effect. The transverse spacing of the wheels is 6 ft. Transversely, the design truck is positioned in a lane, as specified in AASHTO Sec. 3.6.1.3.1, so that the center of any wheel load is not closer than

- 1 ft from the face of a curb for the design of a deck overhang
- 2 ft from the edge of the design lane for all other components

A dynamic load allowance is applied to the design truck load.

Figure 8.3 *Design Truck Load*

14 ft   14 ft to 30 ft

8 kips   32 kips   32 kips

axle locations

6 ft   2 ft*

*1 ft for deck overhang

wheel locations
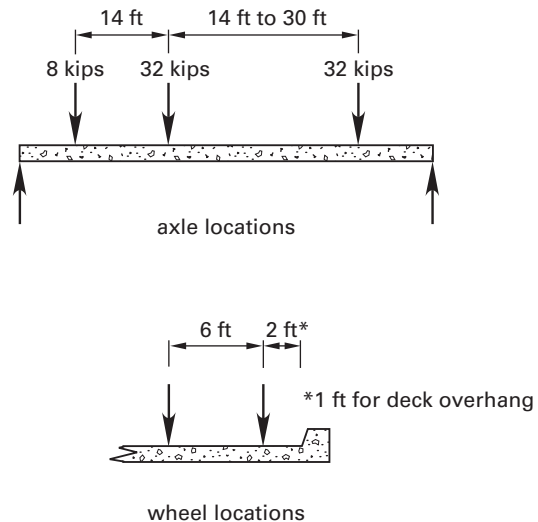
As shown in Fig. 8.4, the design tandem load consists of a pair of 25 kip axles spaced 4 ft apart. The transverse spacing of the wheels is 6 ft. Transversely, the design tandem is positioned in a lane in the same manner as the design truck. A dynamic load allowance is applied to the design tandem load.

Figure 8.4 *Design Tandem Load*

axle locations   wheel locations

*1 ft for deck overhang
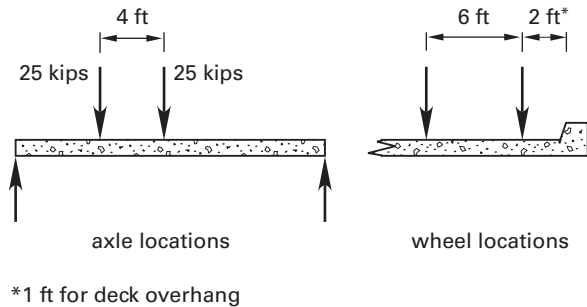
In accordance with AASHTO Sec. 3.6.1.1.2, the number of loaded lanes is selected to produce the most critical effect. As specified in AASHTO Sec. 3.6.1.2.1, each lane under consideration shall be occupied by either the design truck or tandem combined with the lane load. Figure 8.5 shows the location of the design truck and the design lane load to produce the maximum positive moment at midspan of the end span of a three span continuous deck. The design lane load is placed on both end spans so as to produce the maximum effect. The design truck is placed with its central axle at the midspan of the end span.

Figure 8.5 *Design Truck Positioned for Maximum Positive Moment at Point 5*

loading positions for maximum M_5

influence line for M_5
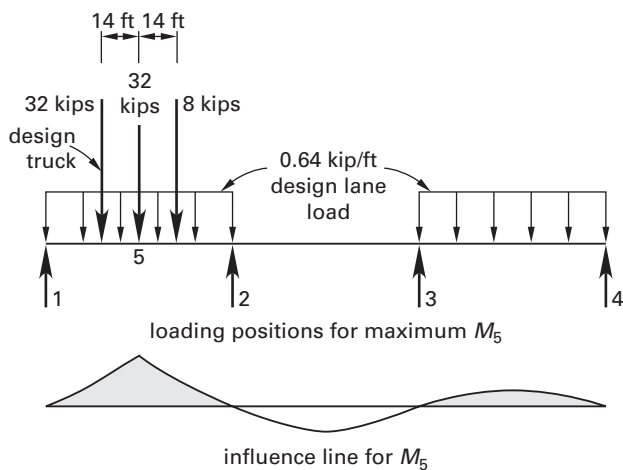
Similarly, Fig. 8.6 shows the design tandem positioned to produce the maximum positive moment at midspan of the end span of a three span continuous deck.

Figure 8.6 *Design Tandem Positioned for Maximum Positive Moment at Point 5*

loading positions for maximum M_5

influence line for M_5
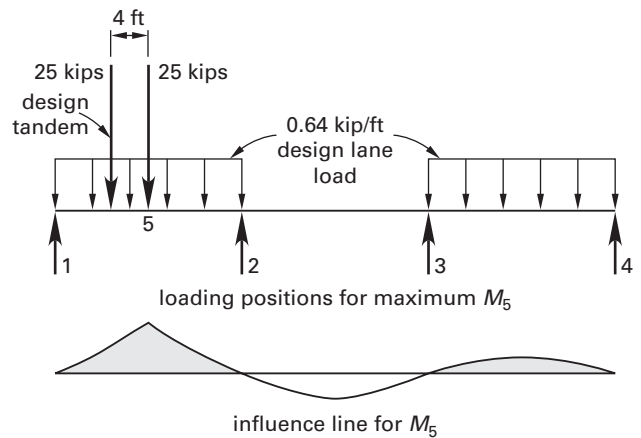
When several lanes are loaded, the force effect determined is multiplied by a multiple presence factor to account for the probability of simultaneous lane occupation by the full HL-93 design live load. AASHTO Table 3.6.1.1.2-1 gives the multiple presence factor, m, as

- 1.2 for one loaded lane

- 1.0 for two loaded lanes

- 0.85 for three loaded lanes

- 0.65 for more than three loaded lanes

For the determination of maximum negative moments in a continuous deck, AASHTO Sec. 3.6.1.3.1 specifies that two design trucks may be located in each lane with a minimum distance of 50 ft between the lead axle of one truck and the rear axle of the other truck. The distance between the 32 kip axles of each truck is 14 ft. The two design trucks are placed in adjacent spans to produce maximum force effects. Axles that do not contribute to the negative moment are neglected. The truck loading is combined with the design lane load using patch loading to produce the maximum effect. The total combined moment is multiplied by a reduction factor of 0.9 to give the design moment. The same procedure is used to determine the reaction at interior piers.

Similarly, as shown in Fig. 8.7, two design tandems may be applied, spaced 26–40 ft apart, and combined with the design lane load. This represents the loading caused by "low-boy" type vehicles weighing in excess of 110 kips. The total combined moment obtained is the required design moment without multiplying by a reduction factor.

For continuous spans, influence lines may be used to determine the maximum effect, and these are available[2,3] for standard cases. For nonstandard situations, several methods[4,5] may be used to determine the required influence lines.

Bridges

Figure 8.7 *Design Lane Load and Two Design Tandems Positioned for Maximum Moment at Support 3*

loading positions for maximum M_3

influence line for M_3

Example 8.2

The four-span bridge shown in the following illustration has the superstructure analyzed in Ex. 8.1. Determine the maximum moment at support 2 produced by loading one design lane with the design lane load combined with the design truck. Neglect the multiple presence factor and dynamic load allowance.
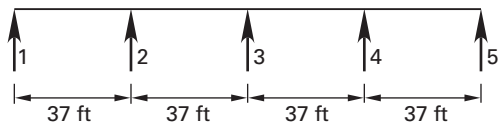
Solution

The locations of the design lane load to produce the maximum moment at support 2 are obtained[2] as shown in the following illustration. Span 34 is not loaded.

loading

influence line
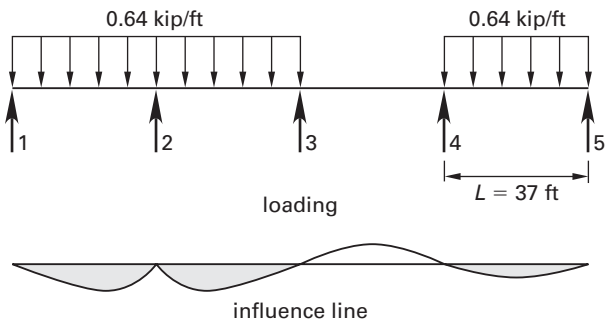
The bending moment at support 2, produced by one design lane load, is given by[2]

$$M_2 = awL^2$$

$$= (0.1205)\left(0.64 \ \frac{\text{kip}}{\text{ft}}\right)(37 \ \text{ft})^2$$

$$= 106 \ \text{ft-kips}$$

The axle locations of the two design trucks to produce the maximum moment at support 2 are obtained[2] as

shown in the following illustration. The 8 kip lead axle on each truck is neglected.

The bending moment at support 2, produced by the two design trucks, is given by[2]

$$M_2 = \sum \gamma WL$$

$$= \left(\begin{array}{l} 0.083 + 0.090 \\ \quad + 0.0064 + 0.0051 \end{array}\right)(32 \ \text{kips})(37 \ \text{ft})$$

$$= 218 \ \text{ft-kips}$$

The combined moment produced by the design lane load and the two standard trucks is

$$M_2 = 106 \ \text{ft-kips} + 218 \ \text{ft-kips}$$

$$= 324 \ \text{ft-kips}$$

AASHTO Sec. 3.6.1.3.1 gives the design moment as

$$M_2 = (0.9)(324 \ \text{ft-kips})$$

$$= 292 \ \text{ft-kips}$$

Dynamic Load Allowance

In accordance with AASHTO Sec. 3.6.2, an allowance for dynamic effects is applied to the static axle loads of the design truck and the design tandem. AASHTO Table 3.6.2.1-1 gives the dynamic load allowance, *IM*, as

- 75% for deck joints for all limit states

- 15% for all other components for fatigue and fracture limit states

- 33% for all other components for all other limit states

The dynamic factor to be applied to the static load is

$$I = 1 + \frac{IM}{100}$$

The dynamic factor is not applied to

- the design lane load
- pedestrian loads
- centrifugal forces and braking forces
- retaining walls not subject to vertical loads from the superstructure
- foundation components that are entirely below ground level
- wood structures

The dynamic load allowance for culverts and other buried structures is given by AASHTO Eq. 3.6.2.2-1 as

$$IM = 33(1.0 - 0.125D_E)$$
$$\geq 0\%$$

D_E, the minimum depth of earth cover over the structure, is in ft.

Example 8.3

For the four-span bridge of Ex. 8.1, determine the maximum moment at support 2 produced by loading one design lane with the design lane load combined with the design truck. Include the effect of the dynamic load allowance.

Solution

The dynamic load allowance for the support moment is given by AASHTO Table 3.6.2.1-1 as

$$IM = 33\%$$

This is applied to the static axle loads of the design trucks to give the dynamic factor

$$I = 1 + \frac{IM}{100}$$
$$= 1 + \frac{33\%}{100}$$
$$= 1.33$$

The static moment at support 2 caused by the two design trucks is given by Ex. 8.2 as

$$M_2 = 218 \text{ ft-kips}$$

The moment at support 2 caused by the two design trucks, including the dynamic load allowance, is

$$M_2 = (1.33)(218 \text{ ft-kips})$$
$$= 290 \text{ ft-kips}$$

The combined moment produced by the design lane load and the two standard trucks, including the dynamic load allowance, is

$$M_2 = 106 \text{ ft-kips} + 290 \text{ ft-kips}$$
$$= 396 \text{ ft-kips}$$

The final design moment is given by AASHTO Sec. 3.6.1.3.1 as

$$M_2 = (0.9)(396 \text{ ft-kips})$$
$$= 356 \text{ ft-kips}$$

Lateral Distribution of Loads

In accordance with AASHTO Sec. 4.6.2.2.1 in the calculation of bending moments for T-beam bridges, permanent loads of and on the deck may be distributed uniformly to all beams.

The distribution of live load depends on the torsional stiffness of the bridge deck system and, if necessary, may be determined by several[6,7,8,9] analytical methods. In accordance with AASHTO Sec. 4.6.2.2, however, the distribution of live load may be calculated by empirical expressions, depending on the superstructure type and the stringer spacing.

The types of superstructure for which the distribution factor method may be used are illustrated in AASHTO Table 4.6.2.2.1-1. The method may be applied provided that the following conditions are met.

- A single lane of live loading is analyzed.
- Multiple lanes of live loading producing approximately the same force effect per lane are analyzed.
- The deck width is constant.
- The number of beams is not less than four (with some exceptions).
- Beams are parallel and have approximately the same stiffness.
- The roadway part of the overhang does not exceed 3 ft (with some exceptions).
- The curvature of the superstructure is less than the limit specified in AASHTO Sec. 4.4.

Additional requirements are specified for each specific superstructure illustrated in AASHTO Table 4.6.2.2.1-1, and these are listed in AASHTO Table 4.6.2.2.2b-1.

A monolithic T-beam superstructure is listed as case (e) in AASHTO Table 4.6.2.2.1-1. For this type of bridge, the limitation on the beam spacing is

$$3.5 \text{ ft} \leq S \leq 16.0 \text{ ft}$$

Bridges

The limitation on the deck slab thickness is

$$4.5 \text{ in} \le t_s \le 12.0 \text{ in}$$

The limitation on the superstructure span is

$$20 \text{ ft} \le L \le 240 \text{ ft}$$

The limitation on the number of beams is

$$N_b \ge 4$$

The limitation on the longitudinal stiffness parameter is

$$10,000 \text{ in}^4 \le K_g \le 7,000,000 \text{ in}^4$$

The longitudinal stiffness parameter is defined by AASHTO Eq. 4.6.2.2.1-1 as

$$K_g = n(I + Ae_g^2)$$

The ratio of the modulus of elasticity of the beam and the deck slab is defined by AASHTO Eq. 4.6.2.2.1-2 as

$$n = \frac{E_B}{E_D}$$

The moment of inertia of the beam is

$$I = \frac{bd^3}{12}$$

The area of the beam is

$$A = bd$$

The distance between the centers of gravity of the beam and the deck slab is

$$e_g = \frac{t_s + d}{2}$$

When these conditions are complied with, AASHTO Table 4.6.2.2.2b-1 gives the distribution factor for moment for one design lane loaded as

$$g_1 = 0.06 + \left(\frac{S}{14}\right)^{0.4} \left(\frac{S}{L}\right)^{0.3} \left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$

When two or more design lanes are loaded, the distribution factor for moment is

$$g_m = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2} \left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$

In accordance with AASHTO Sec. 3.6.1.1.2, the multiple presence factors specified in AASHTO Table 3.6.1.1.2-1 are not applicable as these factors are already incorporated in the distribution factors. The dynamic load allowance must be applied to that portion of the bending moment produced by design trucks and design tandems.

These distribution factors are not applicable for the determination of bending moments in exterior beams. For exterior beams, with one lane loaded, and for interior beams in decks with less than four beams, the lever-rule method specified in AASHTO Sec. C4.6.2.2.1 may be used. For these analyses, both the multiple presence factor and the dynamic load allowance must be applied. Irrespective of the calculated moment, an exterior beam shall have a carrying capacity not less than that of an interior beam.

Example 8.4

For the four-span, concrete T-beam bridge of Ex. 8.2, determine the maximum live load moment for design of an interior beam at support 2. The ratio of the modulus of elasticity of the beam and the deck slab is $n = 1.0$.

Solution

From Ex. 8.3, the maximum live load moment produced at support 2 by loading one design lane with two design trucks, plus dynamic load allowance, and the design lane load is

$$M_2 = 356 \text{ ft-kips}$$

The ratio of the modulus of elasticity of the beam and the deck slab is given as

$$n = \frac{E_B}{E_D}$$
$$= 1.0$$

The moment of inertia of the beam is

$$I = \frac{bd^3}{12}$$
$$= \frac{(12 \text{ in})(30 \text{ in})^3}{12}$$
$$= 27,000 \text{ in}^4$$

The area of the beam is

$$A = bd$$
$$= (12 \text{ in})(30 \text{ in})$$
$$= 360 \text{ in}^2$$

The distance between the centers of gravity of the beam and the deck slab is

$$e_g = \frac{t_s + d}{2}$$
$$= \frac{9 \text{ in} + 30 \text{ in}}{2}$$
$$= 19.5 \text{ in}$$

Bridges

The longitudinal stiffness parameter of the deck is defined by AASHTO Eq. 4.6.2.2.1-1 as

$$K_g = n(I + Ae_g^2)$$
$$= (1.0)\big(27{,}000 \text{ in}^4 + (360 \text{ in}^2)(19.5 \text{ in})^2\big)$$
$$= 163{,}890 \text{ in}^4 \quad \begin{bmatrix} \text{complies with} \\ \text{AASHTO Table 4.6.2.2.2b-1} \end{bmatrix}$$
$$> 10{,}000 \text{ in}^4$$
$$< 7{,}000{,}000 \text{ in}^4$$

The beam spacing is

$S = 11$ ft   [complies with AASHTO Table 4.6.2.2.2b-1]
> 3.5 ft
< 16.0 ft

The deck slab thickness is

$t_s = 9$ in   [complies with AASHTO Table 4.6.2.2.2b-1]
> 4.5 in
< 12.0 in

The superstructure span is

$L = 37$ ft   [complies with AASHTO Table 4.6.2.2.2b-1]
> 20 ft
< 240 ft

The number of beams in the deck is

$N_b = 4$ [complies with AASHTO Table 4.6.2.2.2b-1]

Thus, the configuration of the deck completely conforms to the requirements of AASHTO Table 4.6.2.2.2b-1.

With one lane loaded, AASHTO Table 4.6.2.2.2b-1 gives the distribution factor for moment as

$$g_1 = 0.06 + \left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$
$$= 0.06 + \left(\frac{11 \text{ ft}}{14}\right)^{0.4}\left(\frac{11 \text{ ft}}{37 \text{ ft}}\right)^{0.3}$$
$$\times \left(\frac{163{,}890 \text{ in}^4}{(12.0)(37 \text{ ft})(9 \text{ in})^3}\right)^{0.1}$$
$$= 0.650$$

With two lanes loaded, as shown in the illustration, AASHTO Table 4.6.2.2.2b-1 gives the distribution factor for moment as

$$g_m = 0.075 + \left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$$
$$= 0.075 + \left(\frac{11 \text{ ft}}{9.5}\right)^{0.6}\left(\frac{11 \text{ ft}}{37 \text{ ft}}\right)^{0.2}$$
$$\times \left(\frac{163{,}890 \text{ in}^4}{(12.0)(37 \text{ ft})(9 \text{ in})^3}\right)^{0.1}$$
$$= 0.875 \quad \text{[governs]}$$

The live load moment for the design of an interior beam at support 2 is

$$M_L = g_m M_2$$
$$= (0.875)(356 \text{ ft-kips})$$
$$= 312 \text{ ft-kips}$$

Shear Determination

The distribution factor method is also used to calculate design shear in interior beams. The distribution factors and the range of applicability are listed in AASHTO Table 4.6.2.2.3a-1. For a monolithic T-beam superstructure, the limitations on beam spacing, slab thickness, span length, and number of beams are identical with those for determining the distribution factor for moment. There is no requirement specified for the longitudinal stiffness parameter.

When these conditions are complied with, AASHTO Table 4.6.2.2.3a-1 gives the distribution factor for shear for one design lane loaded as

$$g_1 = 0.36 + \frac{S}{25}$$

Bridges

When two or more design lanes are loaded, the distribution factor for shear is

$$g_m = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0}$$

In accordance with AASHTO Sec. 3.6.1.1.2, the multiple presence factors that are specified in AASHTO Table 3.6.1.1.2-1 are not applicable as these factors are already incorporated in the distribution factors. The dynamic load allowance must be applied to that portion of the shear produced by design trucks and design tandems.

These distribution factors are not applicable for the determination of shear in exterior beams. For exterior beams, with one lane loaded, and for interior beams in decks with less than four beams, the lever rule method specified in AASHTO Sec. C4.6.2.2.1 may be used. For these analyses, both the multiple presence factor and the dynamic load allowance must be applied. Irrespective of the calculated shear, an exterior beam shall not have less resistance than an interior beam.

The application of the lever rule is illustrated in Fig. 8.8 for the determination of the distribution factor for shear in the exterior girder of a T-beam superstructure. For one lane loaded, the center of one wheel of an axle of the design truck or the design tandem is located 2 ft from the edge of the design lane as specified in AASHTO Sec. 3.6.1.3.1. A notional hinge is introduced into the deck slab at the position of beam 2 and moments are taken about this hinge. The reaction at beam 1, in terms of one wheel load, is

$$V_1 = \frac{P(4.5 \text{ ft} + 10.5 \text{ ft})}{11 \text{ ft}}$$
$$= 1.364P$$

The distribution factor for shear for one lane loaded with one axle is

$$g_1 = \frac{V_1}{2P}$$
$$= \frac{1.364P}{2P}$$
$$= 0.682$$

Applying the multiple presence factor for one lane loaded gives a distribution factor of

$$g = 1.2g_1$$
$$= (1.2)(0.682)$$
$$= 0.818$$

Figure 8.8 *Lever Rule for Shear in an Exterior Girder*
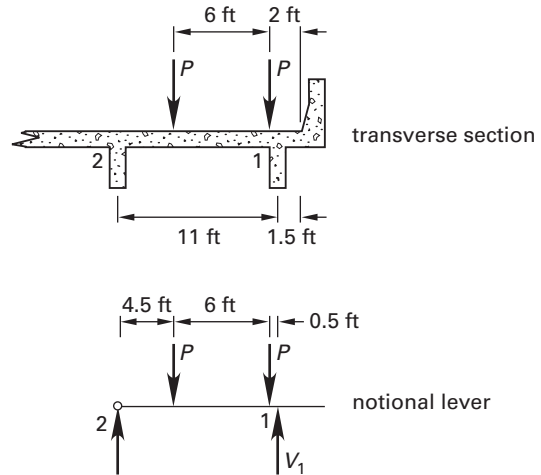
Example 8.5

For the four-span concrete T-beam bridge of Ex. 8.2, determine the live load shear, V_{23}, for design of an interior beam. Use the design truck load combined with the design lane load.

Solution

The influence line for V_{23} is shown in the following illustration.
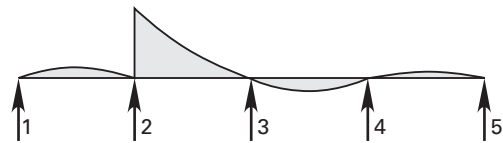
To produce the maximum value of V_{23}, the truck is positioned as shown in the following illustration.
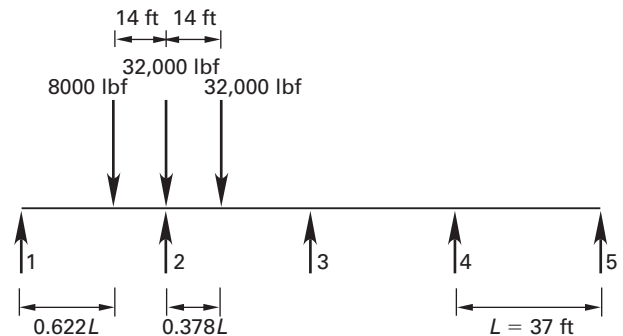
The shear at end 2 of span 23 produced by one standard truck is given by[2]

$$V_{23} = \sum \gamma W$$
$$= \frac{(0.128)(8000 \text{ lbf})}{1000 \frac{\text{lbf}}{\text{kip}}} + \frac{(1.635)(32{,}000 \text{ lbf})}{1000 \frac{\text{lbf}}{\text{kip}}}$$
$$= 53.34 \text{ kips}$$

As determined in Ex. 8.4, the superstructure dimensions are within the allowable range of applicability.

AASHTO Table 4.6.2.2.3a-1 gives the distribution factor for shear for one design lane loaded as

$$g_1 = 0.36 + \frac{S}{25}$$
$$= 0.36 + \frac{11 \text{ ft}}{25 \text{ ft}}$$
$$= 0.800$$

When two or more design lanes are loaded, the distribution factor for shear is

$$g_m = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0}$$
$$= 0.2 + \frac{11 \text{ ft}}{12 \text{ ft}} - \left(\frac{11 \text{ ft}}{35 \text{ ft}}\right)^{2.0}$$
$$= 1.018 \quad [\text{governs}]$$

The dynamic load allowance for the shear is given by AASHTO Table 3.6.2.1-1 as

$$IM = 33\%$$

This is applied to the static axle load of the design truck to give the dynamic factor

$$I = 1 + \frac{IM}{100}$$
$$= 1 + \frac{33\%}{100}$$
$$= 1.33$$

The shear at support 2 caused by the design truck, including the dynamic load allowance, is

$$V_{23} = (1.33)(53.34 \text{ kips})$$
$$= 70.94 \text{ kips}$$

The design lane load is positioned in spans 12, 23, and 45 in order to produce the maximum shear at support 2.

The shear produced by one design lane load is given by[2]

$$V_{23} = awL$$
$$= (0.6027)\left(0.64 \ \frac{\text{kip}}{\text{ft}}\right)(37 \text{ ft})$$
$$= 14.27 \text{ kips}$$

The combined shear produced by one lane of the design lane load and the standard truck, including the dynamic load allowance, is

$$V_{23} = 70.94 \text{ kips} + 14.27 \text{ kips}$$
$$= 85.21 \text{ kips}$$

The live load shear, produced by two loaded lanes for the design of an interior beam at support 2, is

$$V_L = g_m V_{23}$$
$$= (1.018)(85.21 \text{ kips})$$
$$= 86.74 \text{ kips}$$

Design of Concrete Deck Slabs

The bending moments, caused by wheel loads, in concrete deck slabs supported by longitudinal stringers and transverse girders may be obtained by the methods proposed by Westergaard[10] and Pucher.[11] AASHTO Sec. 4.6.2.1 provides an equivalent strip method for the design of concrete deck slabs.

This method consists of dividing the deck into strips perpendicular to the supporting stringers and transverse girders. The principal features of the method are as follows.

- The extreme positive moment in any deck panel shall be applied to all positive moment regions.

- The extreme negative moment over any supporting component shall be applied to all negative moment regions.

- Where the deck slab spans primarily in the transverse direction, only the axles of the design truck or the design tandem shall be applied to the deck slab.

- Where the deck slab spans primarily in the longitudinal direction, and the span does not exceed 15 ft, only the axles of the design truck or the design tandem shall be applied to the deck slab.

- Where the deck slab spans primarily in the longitudinal direction, and the span exceeds 15 ft, the design truck combined with the design lane load, or the design tandem combined with the design lane load, shall be applied to the deck slab and the provisions of AASHTO Sec. 4.6.2.3 shall apply.

- Where the deck slab spans primarily in the longitudinal direction, the width of the equivalent strip supporting an axle load shall not be taken greater than 40 in for open grids.

- Where the deck slab spans primarily in the transverse direction, the equivalent strip is not subject to width limits.

- Where the spacing of supporting components in the secondary direction exceeds 1.5 times the spacing in the primary direction, all of the wheel loads may be considered to be applied to the primary strip. Distribution reinforcement that complies with AASHTO Sec. 9.7.3.2 may be applied in the secondary direction.

- Where the spacing of supporting components in the secondary direction is less than 1.5 times the spacing in the primary direction, the deck shall be modeled as a system of intersecting strips.

- Wheel loads are distributed to the intersecting strips in proportion to their stiffnesses.

- The stiffness of a strip is specified as $k_s = EI_s/S^3$.

- Strips are treated as simply supported or continuous beams as appropriate with a span length equal to the center-to-center distance between the supporting components.

- Wheel loads may be modeled as concentrated loads or as patch loads whose length along the span is equal to the length of the tire contact area plus the depth of the deck slab.

- Both the multiple presence factor and the dynamic load allowance must be applied to the bending moments calculated.

- In lieu of determining the width of the equivalent strip, the moments may be obtained directly from AASHTO Table A4-1 and these values include an allowance for both the multiple presence factor and the dynamic load allowance.

AASHTO Table 4.6.2.1.3-1 defines the width of an equivalent strip. For cast-in-place deck slabs, the width, in inches, of both longitudinal and transverse strips for calculating positive moment is

$$B_s = 26 \text{ in} + 6.6S_{\text{ft}}$$

The width of both longitudinal and transverse strips for calculating negative moment is

$$B_s = 48 \text{ in} + 3.0S_{\text{ft}}$$

Example 8.6

For the four-span concrete T-beam bridge of Ex. 8.2, determine the maximum negative live load moment in the slab and the width of the equivalent strip. The layout of longitudinal and transverse girders is shown in the following illustration.
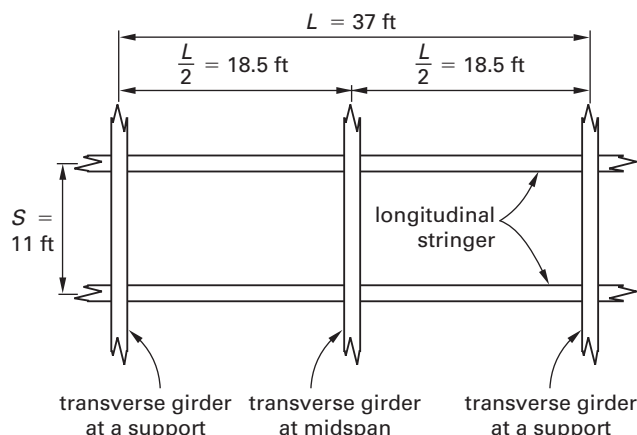
Solution

The aspect ratio of the slab is

$$AR = \frac{\dfrac{L}{2}}{S} = \frac{18.5 \text{ ft}}{11 \text{ ft}}$$
$$= 1.68$$
$$> 1.5$$

Therefore, all of the wheel loads may be considered to be applied to the primary strip in the transverse direction. Distribution reinforcement that complies with AASHTO Sec. 9.7.3.2 may be applied in the secondary direction. Since the deck slab spans primarily in the transverse direction, only the axles of the design vehicle shall be applied to the deck slab.

The width, in inches, of the transverse strip, which is used for calculating negative moment, is given by AASHTO Table 4.6.2.1.3-1 as

$$B_s = 48 \text{ in} + 3.0S_{\text{ft}}$$
$$= \frac{48 \text{ in} + (3.0)(11 \text{ ft})}{12 \dfrac{\text{in}}{\text{ft}}}$$
$$= 6.75 \text{ ft}$$

The required moment may be determined from AASHTO Table A4-1. The span length of the transverse strip is

$$S = 11 \text{ ft}$$

The distance from the center line of a longitudinal stringer to the face of the stringer is

$$x = 6 \text{ in}$$

Therefore, from AASHTO Table A4-1, the maximum negative bending moment is

$$M_s = 7.38 \text{ ft-kips/ft}$$

Design of Slab-Type Bridges

The bending moments and shears in concrete slab-type decks, caused by axle loads, may be obtained by an equivalent strip method that is defined in AASHTO Sec. 4.6.2.3. The equivalent width of a longitudinal strip with two lines of wheels in one lane is given by AASHTO Eq. 4.6.2.3-1 as

$$E = 10.0 + 5.0(L_1 W_1)^{0.5}$$

The modified span length, L_1, is equal to the lesser of the actual span length or 60 ft. The modified edge-to-edge width of the bridge, W_1, is equal to the lesser of the actual width, W, or 30 ft.

The equivalent width of a longitudinal strip with more than one lane loaded is given by AASHTO Eq. 4.6.2.3-2 as

$$E = 84.0 + 1.44(L_1 W_1)^{0.5}$$
$$\leq 12.0\,W/N_L$$

The modified span length, L_1, is equal to the lesser of the actual span length or 60 ft. The modified edge-to-edge width of the bridge, W_1, is equal to the lesser of the actual width, W, or 60 ft. The number of design lanes, N_L, is determined as specified in AASHTO Sec. 3.6.1.1.1.

An allowance for the multiple presence factor is included in the equivalent strip width. The dynamic load allowance must be applied to the bending moments calculated.

Example 8.7

A prestressed concrete slab bridge has a simply supported span of $L = 37$ ft. The overall width of the bridge is $W = 39$ ft, and the distance between curbs is $w = 36$ ft. Determine the width of the equivalent strip.

Solution

From AASHTO Sec. 3.6, the number of design lanes is

$$N_L = \frac{w}{12}$$
$$= \frac{36 \text{ ft}}{12 \, \dfrac{\text{ft}}{\text{lane}}}$$
$$= 3 \text{ lanes}$$

For one design lane loaded, the modified span length is equal to the lesser of the actual span length or 60 ft, and

$$L_1 = L$$
$$= 37 \text{ ft}$$

The modified edge-to-edge width of the bridge is equal to the lesser of the actual width or 30 ft, and

$$W_1 = 30 \text{ ft}$$

The equivalent width of a longitudinal strip is given by AASHTO Eq. 4.6.2.3-1 as

$$E = 10.0 + 5.0(L_1 W_1)^{0.5}$$
$$= 10.0 \text{ in} + (5.0)\big((37 \text{ ft})(30 \text{ ft})\big)^{0.5}$$
$$= 176.6 \text{ in}$$

For more than one design lane loaded, the modified span length is equal to the lesser of the actual span length or 60 ft, and

$$L_1 = L$$
$$= 37 \text{ ft}$$

The modified edge-to-edge width of the bridge is equal to the lesser of the actual width or 60 ft, and

$$W_1 = W$$
$$= 39 \text{ ft}$$

The equivalent width of a longitudinal strip is given by AASHTO Eq. 4.6.2.3-2 as

$$E = 84.0 + 1.44(L_1 W_1)^{0.5}$$
$$= 84.0 \text{ in} + (1.44)\big((37 \text{ ft})(39 \text{ ft})\big)^{0.5}$$
$$= 138.7 \text{ in}$$
$$\frac{12.0\,W}{N_L} = \frac{(12.0)(39 \text{ ft})}{3 \text{ lanes}}$$
$$= 156 \text{ in}$$
$$> E \quad [E = 138.7 \text{ in}]$$

The equivalent width for more than one design lane loaded governs, and

$$E = 138.7 \text{ in}$$

Combinations of Loads

The load and resistance factor design method presented in AASHTO Sec. 1.3.2, defines four limit states: the service limit state, the fatigue and fracture limit state, the strength limit state, and the extreme event limit state.

The *service limit state* governs the design of the structure under regular service conditions to ensure satisfactory stresses, deformations, and crack widths. Four service limit states are defined, with service I limit state comprising the load combination relating to the normal operational use of the bridge with a 55 mph wind, and all loads taken at their nominal values.

The *fatigue limit state* governs the design of the structure loaded with a single design truck for a given number of stress range cycles. The fracture limit state is

defined as a set of material toughness requirements given in the AASHTO Materials Specifications.

The *strength limit state* ensures the structure's strength and structural integrity under the various load combinations imposed on the bridge during its design life. Five strength limit states are defined, with strength I limit state comprising the load combination relating to the normal vehicular use of the bridge without wind.

The *extreme event limit state* ensures the survival of the structure during a major earthquake or flood, or when subject to collision from a vessel, vehicle, or ice flow. Two extreme limit states are defined, with extreme event I limit state comprising the load combination that includes earthquake.

The factored load is influenced by the ductility of the components, the redundancy of the structure, and the operational importance of the bridge based on social or defense requirements. It is preferable for components to exhibit ductile behavior, as this provides warning of impending failure by large inelastic deformations. Brittle components are undesirable because failure occurs suddenly, with little or no warning, when the elastic limit is exceeded. For the strength limit state, the load modifier for ductility is given by AASHTO Sec. 1.3.3 as

$$\eta_D = 1.05 \quad \text{[nonductile components]}$$
$$= 1.00 \quad \text{[conventional designs and details]}$$
$$\geq 0.95 \quad \begin{bmatrix} \text{components with} \\ \text{ductility-enhancing features} \end{bmatrix}$$

For all other limit states, the load modifier for ductility is given by AASHTO Sec. 1.3.3 as

$$\eta_D = 1.00$$

The component redundancy classification is based on the contribution of the component to the bridge safety. Major components, whose failure will cause collapse of the structure, are designated as failure-critical, and the associated structural system is designated nonredundant. Alternatively, components whose failure will not cause collapse of the structure are designated as nonfailure-critical, and the associated structural system is designated redundant. For the strength limit state, the load modifier for redundancy is given by AASHTO Sec. 1.3.4 as

$$\eta_R = 1.05 \quad \text{[nonredundant components]}$$
$$= 1.00 \quad \text{[conventional levels of redundancy]}$$
$$\geq 0.95 \quad \text{[exceptional levels of redundancy]}$$

For all other limit states, the load modifier for redundancy is given by AASHTO Sec. 1.3.4 as

$$\eta_R = 1.00$$

A bridge may be declared to be of operational importance based on survival or security reasons. For the strength limit state, the load modifier for operational importance is given by AASHTO Sec. 1.3.5 as

$$\eta_I = 1.05 \quad \text{[for important bridges]}$$
$$= 1.00 \quad \text{[for typical bridges]}$$
$$\geq 0.95 \quad \text{[for relatively less important bridges]}$$

For all other limit states, the load modifier for importance is given by AASHTO Sec. 1.3.5 as

$$\eta_I = 1.00$$

For loads where a maximum value is appropriate, the combined load modifier relating to ductility, redundancy, and operational importance is given by AASHTO Eq. 1.3.2.1-2 as

$$\eta_i = \eta_D \eta_R \eta_I$$
$$\geq 0.95$$

For loads where a minimum value is appropriate, the combined load modifier is given by AASHTO Eq. 1.3.2.1-3 as

$$\eta_i = \frac{1}{\eta_D \eta_R \eta_I}$$
$$\leq 1.0$$

The load factors applicable to permanent loads are listed in AASHTO Table 3.4.1-2 and are summarized in Table 8.1.

Table 8.1 *Load Factors for Permanent Loads*

type of load	load factor, γ_p	
	max	min
components and attachments, DC	1.25	0.90
wearing surfaces and utilities, DW	1.5	0.65

The actual value of permanent loads may be less than or more than the nominal value, and both possibilities must be considered by using the maximum and minimum values given for the load factor.

Load combinations and load factors are listed in AASHTO Table 3.4.1-1, and those applicable to gravity and earthquake loads are summarized in Table 8.2.

Table 8.2 *Load Factors and Load Combinations*

load combination limit state	DC and DW	LL and IM	EQ
strength I	γ_p	1.75	–
extreme event I	γ_p	γ_{EQ}	1.00
service I	1.00	1.00	–
fatigue I	–	1.50	–
fatigue II	–	0.75	–

The value of the load factor γ_p for the dead load of components and wearing surfaces is obtained from Table 8.1. The load factor γ_{EQ} in extreme event limit state I has traditionally been taken as 0.0. However, partial live load should be considered, and a reasonable value for the load factor is

$$\gamma_{EQ} = 0.50$$

The total factored force effect is given by AASHTO Eq. 3.4.1-1 as

$$Q = \sum \eta_i \gamma_i Q_i$$
$$\leq \phi R_n$$

Both positive and negative extremes must be considered for each load combination. For permanent loads, the load factor that produces the more critical effect is selected from Table 8.1. In strength I limit state, when the permanent loads produce a positive effect and the live loads produce a negative effect, the appropriate total factored force effect is

$$Q = 0.9DC + 0.65DW + 1.75(LL + IM)$$

In strength I limit state, when both the permanent loads and the live loads produce a negative effect, the appropriate total factored force effect is

$$Q = 1.25DC + 1.50DW + 1.75(LL + IM)$$

Example 8.8

For the four-span concrete T-beam bridge of Ex. 8.1, determine the strength I factored moment for design of an interior beam at support 2. Each concrete parapet has a weight of 0.5 kip/ft, and the parapets are constructed after the deck slab has cured. Assume a unit weight of concrete of 0.15 kip/ft[3].

Solution

The dead load acting on an interior beam consists of the beam self-weight, plus the applicable portion of the deck slab, plus the applicable portion of the two parapets. The dead load of a beam is

$$w_B = \left(0.15 \ \frac{\text{kip}}{\text{ft}^3}\right)(2.5 \ \text{ft})(1 \ \text{ft})$$
$$= 0.375 \ \text{kip/ft}$$

The dead load of the applicable portion of the deck slab, in accordance with AASHTO Sec. 4.6.2.2.1, is

$$w_S = \frac{\left(0.15 \ \frac{\text{kip}}{\text{ft}^3}\right)(39 \ \text{ft})(0.75 \ \text{ft})}{4 \ \text{beams}}$$
$$= 1.097 \ \text{kips/ft}$$

In accordance with AASHTO Sec. 4.6.2.2.1, the weights of the two concrete parapets are distributed equally to the four beams. Then, the applicable weight distributed to an interior beam is

$$w_P = \frac{\left(0.5 \ \frac{\text{kip}}{\text{ft}}\right)(2 \ \text{parapets})}{4 \ \text{beams}}$$
$$= 0.25 \ \text{kip/ft}$$

The total dead load supported by an interior beam is

$$w_D = w_B + w_S + w_P$$
$$= 0.375 \ \frac{\text{kip}}{\text{ft}} + 1.097 \ \frac{\text{kips}}{\text{ft}} + 0.25 \ \frac{\text{kip}}{\text{ft}}$$
$$= 1.722 \ \text{kips/ft}$$

The bending moment produced in an interior beam at support 2 by the uniformly distributed dead load is given by[2]

$$M_D = aw_D L^2$$
$$= (0.1071)\left(1.722 \ \frac{\text{kips}}{\text{ft}}\right)(37 \ \text{ft})^2$$
$$= 252 \ \text{ft-kips}$$

The live load bending moment plus impact at support 2 is obtained from Ex. 8.4 as

$$M_L = 312 \ \text{ft-kips}$$

The factored design moment for a strength I limit state is given by AASHTO Eq. 3.4.1-1 and by AASHTO Table 3.4.1-1 as

$$M_u = \eta_i(\gamma_p M_D + \gamma_{LL+IM} M_L)$$
$$= 1.0(1.25 M_D + 1.75 M_L)$$
$$= (1.0)\big((1.25)(252 \ \text{ft-kips}) + (1.75)(312 \ \text{ft-kips})\big)$$
$$= 861 \ \text{ft-kips}$$

Critical Section for Shear

AASHTO Sec. 5.8.3.2 specifies that when the support reaction produces a compressive stress in a reinforced concrete beam, the critical section for shear is located at a distance from the support equal to the depth, d_v. The depth, d_v, is defined in AASHTO Sec. 5.8.2.9 as the distance between the resultants of the tensile and compressive forces due to flexure. d_e is the depth to the resultant of the tensile force.

$$d_v \geq 0.9 d_e$$
$$\geq 0.72 h$$

Example 8.9

For the four-span concrete T-beam bridge of Ex. 8.1, determine the factored shear force, V_{23}, for design of an interior beam at support 2. The depth $d_v = 31.4$ in.

Solution

The live load shear force, including impact, on an interior beam at support 2 is obtained from Ex. 8.5 as

$$V_L = 86.74 \text{ kips}$$

The dead load supported by an interior beam is obtained from Ex. 8.8 as

$$w_D = 1.722 \text{ kips/ft}$$

The dead load shear at the support of an interior beam is given by[2]

$$V_s = aw_D L$$
$$= (0.536)\left(1.722 \ \frac{\text{kips}}{\text{ft}}\right)(37 \text{ ft})$$
$$= 34.15 \text{ kips}$$

In accordance with AASHTO Sec. 5.8.3.2, the design shear for a distributed load may be determined at a distance d_v from the support and is given by

$$V_D = V_s - w_D d_v$$
$$= 34.15 \text{ kips} - \frac{\left(1.722 \ \frac{\text{kips}}{\text{ft}}\right)(31.4 \text{ in})}{12 \ \frac{\text{in}}{\text{ft}}}$$
$$= 29.64 \text{ kips}$$

The factored design shear for strength I limit state is given by AASHTO Eq. 3.4.1-1 and AASHTO Table 3.4.1-1 as

$$V_{23} = \eta_i(\gamma_p V_D + \gamma_{LL+IM} V_L)$$
$$= 1.0(1.25 V_D + 1.75 V_L)$$
$$= (1.0)\big((1.25)(29.64 \text{ kips}) + (1.75)(86.74 \text{ kips})\big)$$
$$= 189 \text{ kips}$$

Service Limit State

The service limit state governs stresses, deformations, and crack widths under regular service conditions. The service I limit state comprises the load combination relating to the normal operational use of a bridge with a 55 mph wind and all loads taken at their nominal values.

Example 8.10

For the four-span concrete T-beam bridge of Ex. 8.1, determine the service I design moment for an interior beam at support 2. Each concrete parapet has a weight of 0.5 kip/ft, and the parapets are constructed after the deck slab has cured. Wind effects may be neglected.

Solution

From Ex. 8.8, the bending moment at support 2 produced by the uniformly distributed dead load is

$$M_D = 252 \text{ ft-kips}$$

The live load bending moment plus impact at support 2 is obtained from Ex. 8.8 as

$$M_L = 312 \text{ ft-kips}$$

The service I design moment is given by AASHTO Sec. 3.4.1 as

$$M_s = \eta_i(\gamma_p M_D + \gamma_{LL+IM} M_L)$$
$$= 1.0(1.0 M_D + 1.0 M_L)$$
$$= 252 \text{ ft-kips} + 312 \text{ ft-kips}$$
$$= 564 \text{ ft-kips}$$

2. REINFORCED CONCRETE DESIGN

Design for Flexure

Nomenclature

a	depth of equivalent rectangular stress block	in
A_{max}	maximum area of tension reinforcement	in^2
A_s	area of tension reinforcement	in^2
A_{sk}	area of skin reinforcement per unit height in one side face	in^2/ft
b	width of compression face of member	in
b_w	web width	in
c	distance from extreme compression fiber to neutral axis	in
$\overline{c}$	distance from extreme tension fiber to centroid of tension reinforcement	in
d	distance from extreme compression fiber to centroid of tension reinforcement	in
d_b	diameter of bar	in
d_c	thickness of concrete cover measured from extreme tension fiber to center of nearest bar	in
f'_c	compressive strength of concrete	kips/in^2
f_f	allowable stress range	kips/in^2
f_{max}	maximum stress in reinforcement	kips/in^2
f_{min}	minimum stress in reinforcement	kips/in^2
f_r	modulus of rupture of concrete	kips/in^2
f_{ss}	calculated stress in tension reinforcement at service loads	kips/in^2
f_y	yield strength of reinforcement	kips/in^2
h	overall dimension of member	in
h_f	flange depth	in
h_{min}	recommended minimum depth of superstructure	ft
I_g	moment of inertia of gross concrete section	in^4
K_u	design moment factor	lbf/in^2
l_a	lever arm for elastic design	in
M_{cr}	cracking moment	ft-kips

M_{dnc}	bending moment due to noncomposite dead load acting on the precast section	ft-kips
M_D	dead load moment	ft-kips
$M_{\max}$	maximum moment	ft-kips
$M_{\min}$	minimum design flexural strength	ft-kips
M_{mr}	maximum moment range	ft-kips
M_n	nominal flexural strength of a member	ft-kips
M_u	factored moment on the member	ft-kips
n	number of tensile reinforcing bars	–
s	spacing of reinforcement	in
S_c	section modulus of the composite section referred to the bottom fiber	in^3
S_{nc}	section modulus of the noncomposite section referred to the bottom fiber	in^3

Symbols

β_1	compression zone factor	–
β_s	ratio of flexural strain at the extreme tension face to the strain of the centroid of the reinforcement layer nearest to the tension face	–
γ	influence line coefficient, load factor	–
γ_1	flexural cracking variability factor	–
γ_2	prestress variability factor	–
γ_3	ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement	–
γ_e	exposure factor	–
ρ	ratio of tension reinforcement	–
$\rho_{\max}$	maximum allowable tension reinforcement ratio	–
ϕ	strength reduction factor	–
ω	tension reinforcement index	–
Δf	live load stress range	kips/in^2
$(\Delta F)_{\text{TH}}$	constant-amplitude fatigue threshold	kips/in^2

Strength Design Method

The procedure specified in AASHTO Sec. 5.7 is similar to the procedure adopted in the ACI[12] building code. In addition, stresses at service load shall be limited to ensure satisfactory performance under service load conditions, and the requirements for deflection, cracking moment, flexural cracking, skin reinforcement, and fatigue must be satisfied.

Load Factor Design

When the depth of the equivalent stress block is not greater than the flange depth of a reinforced concrete T-beam, the section may be designed as a rectangular beam. The resistance factor for a tension-controlled reinforced concrete section is given by AASHTO Sec. 5.5.4.2.1 as

$$\phi = 0.90$$

Example 8.11

For the four-span concrete T-beam bridge of Ex. 8.1, determine the tensile reinforcement required in an interior beam in the end span 12. The concrete strength is 4 kips/in^2, and the reinforcement consists of no. 9 grade 60 bars. Assume that the strength I factored moment is $M_u = 1216$ ft-kips.

Solution

The effective compression flange width is given by AASHTO Sec. 4.6.2.6.1 as the tributary width, which is

$$b = S$$
$$= (11 \text{ ft})\left(12 \ \frac{\text{in}}{\text{ft}}\right)$$
$$= 132 \text{ in}$$

The factored design moment is given as

$$M_u = 1216 \text{ ft-kips}$$

Assuming that the stress block lies within the flange and the effective depth, d, is 34.6 in, the required tension reinforcement is determined from the principles of AASHTO Sec. 5.7. The design moment factor is

$$K_u = \frac{M_u}{bd^2}$$
$$= \frac{(1216 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(132 \text{ in})(34.6 \text{ in})^2}$$
$$= 92.3 \text{ lbf/in}^2$$

$$\frac{K_u}{f_c'} = \frac{92.3 \ \frac{\text{lbf}}{\text{in}^2}}{4000 \ \frac{\text{lbf}}{\text{in}^2}}$$
$$= 0.0231$$

From App. A, the corresponding tension reinforcement index is

$$\omega = 0.026$$
$$< 0.319\beta_1$$
$$= (0.319)(0.85)$$
$$= 0.271$$

Hence, the section is tension controlled, and $\phi = 0.9$.

The required reinforcement ratio is

$$\rho = \frac{\omega f_c'}{f_y}$$
$$= \frac{(0.026)\left(4 \ \frac{\text{kips}}{\text{in}^2}\right)}{60 \ \frac{\text{kips}}{\text{in}^2}}$$
$$= 0.00173$$

The reinforcement area required is

$$A_s = \rho b d$$
$$= (0.00173)(132 \text{ in})(34.6 \text{ in})$$
$$= 7.90 \text{ in}^2$$

Bridges

Using eight no. 9 bars as shown in the following illustration, the reinforcement area provided is

$$A_s = 8 \text{ in}^2$$
$$> 7.90 \text{ in}^2 \quad \text{[satisfactory]}$$
$$\phi M_n = \frac{M_u(8 \text{ in}^2)}{7.90 \text{ in}^2} = \frac{(1216 \text{ ft-kips})(8 \text{ in}^2)}{7.90 \text{ in}^2}$$
$$= 1231 \text{ ft-kips}$$
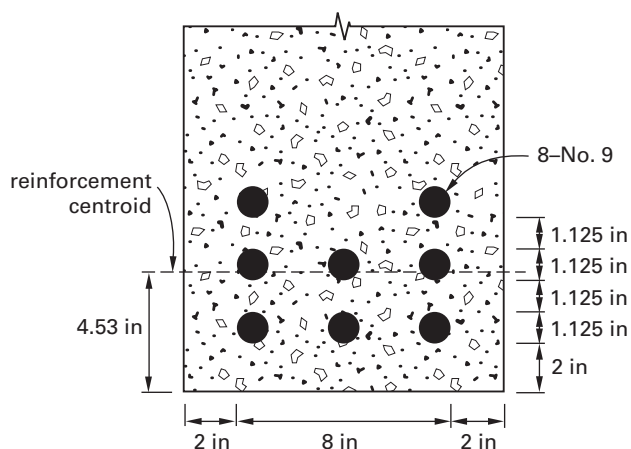
The height of the centroid of the tensile reinforcement is

$$\overline{c} = \frac{(3)(2.563 \text{ in} + 4.813 \text{ in}) + (2)(7.063 \text{ in})}{8}$$
$$= 4.53 \text{ in}$$

The effective depth is

$$d = h - \overline{c}$$
$$= 39 \text{ in} - 4.53 \text{ in}$$
$$= 34.47 \text{ in}$$
$$\approx 34.6 \text{ in} \quad \text{[assumed value of } d \text{ satisfactory]}$$

The stress block depth is

$$a = \frac{A_s f_y}{0.85 b f_c'}$$
$$= \frac{(8 \text{ in}^2)\left(60{,}000 \dfrac{\text{lbf}}{\text{in}^2}\right)}{(0.85)(132 \text{ in})\left(4000 \dfrac{\text{lbf}}{\text{in}^2}\right)}$$
$$= 1.07 \text{ in}$$
$$< h_f \quad \left[\begin{array}{c} \text{The stress block is contained} \\ \text{within the flange.} \end{array}\right]$$

Deflection Requirements

Deflections due to service live load plus impact are limited by AASHTO Sec. 2.5.2.6.2 to

$$\delta_{\max} = \frac{L}{800}$$

To achieve these limits, AASHTO Table 2.5.2.6.3-1 provides expressions for the determination of minimum superstructure depths. These are summarized in Table 8.3.

Table 8.3 Recommended Minimum Depths

superstructure type	minimum depth (ft)	
	simple spans	continuous spans
slabs spanning in direction of traffic	$(1.2)(L+10)/30$	$(L+10)/30 \geq 0.54$
T-beams	$0.070L$	$0.065L$
box girders	$0.060L$	$0.055L$

Actual deflections may be calculated in accordance with AASHTO Sec. 5.7.3.6.2, with the modulus of elasticity of normal weight concrete given by AASHTO Eq. C5.4.2.4-1 as

$$E_c = 1820\sqrt{f_c'}$$

In determining deflections, the effective moment of inertia may be taken as the moment of inertia of the gross concrete section.

Example 8.12

Determine whether the deflection under live load of the four-span concrete T-beam bridge of Ex. 8.1 is satisfactory.

Solution

The recommended minimum depth of the T-beam superstructure, in accordance with AASHTO Table 2.5.2.6.3-1, is

$$h_{\min} = 0.065L$$
$$= (0.065)(37 \text{ ft})$$
$$= 2.4 \text{ ft}$$

The depth provided is

$$h = 3.25 \text{ ft}$$
$$> 2.4 \text{ ft} \quad \text{[satisfactory]}$$

Cracking Moment Requirements

The *cracking moment* is the moment that when applied to a reinforced concrete member, will produce cracking

in the tension face of the member. In determining the cracking moment, AASHTO Sec. 5.7.3.6.2 allows the use of the gross section properties neglecting reinforcement. In the case of T-beam construction, it is appropriate to include the full width of the flange, tributary to the web, in determining the gross moment of inertia, I_g. The modulus of rupture of normal weight concrete is given by AASHTO Sec. 5.4.2.6 as

$$f_r = 0.24\sqrt{f_c'}$$

For a noncomposite reinforced concrete member, $\gamma_2 = 0$, $S_c/S_{nc} = 1$, and M_{dnc} is not applicable. When the neutral axis of the section is a distance $\overline{y}$ from the tension face, the cracking moment is given by the reduced version of AASHTO Eq. 5.7.3.3.2-1 as

$$M_{cr} = \frac{\gamma_1 \gamma_3 f_r I_g}{\overline{y}}$$

The applicable factors for the cracking moment are as follows.

$\gamma_1 =$ flexural cracking variability factor

    $= 1.2$ [precast segmental structures]

    $= 1.6$ [other concrete structures]

$\gamma_3 =$ ratio of specified minimum yield strength to

      ultimate tensile strength of the reinforcement

    $= 0.67$ [A615, grade 60 reinforcement]

    $= 0.75$ [A706, grade 60 reinforcement]

To prevent sudden tensile failure of a flexural member, AASHTO Sec. 5.7.3.3.2 requires the member to have a moment capacity at least equal to the lesser of

$$\phi M_n = M_{cr}$$
$$\phi M_n = 1.33 M_u$$

Example 8.13

Determine whether the interior beam in the end span 12 of the four-span concrete T-beam bridge of Ex. 8.11 complies with AASHTO Sec. 5.7.3.3.2. The bridge has A615, grade 60 reinforcement.

Solution

The gross moment of inertia of an interior beam is obtained as shown in the following table.

part	A (in^2)	y (in)	I (in^4)	Ay (in^3)	Ay^2 (in^4)
beams	360	15.0	27,000	5400	81,000
flange	1188	34.5	8019	40,986	1,414,017
total	1548	–	35,019	46,386	1,495,017

The height of the neutral axis of the section is

$$\overline{y} = \frac{\sum Ay}{\sum A} = \frac{46{,}386 \text{ in}^3}{1548 \text{ in}^2}$$
$$= 30 \text{ in}$$
$$I_g = \sum I + \sum Ay^2 + \overline{y}^2 \sum A - 2\overline{y}\sum Ay$$
$$= 35{,}019 \text{ in}^4 + 1{,}495{,}017 \text{ in}^4$$
$$\quad + (1548 \text{ in}^2)(30 \text{ in})^2 - (60 \text{ in})(46{,}386 \text{ in}^3)$$
$$= 140{,}074 \text{ in}^4$$

The modulus of rupture of the concrete is given by AASHTO Sec. 5.4.2.6 as

$$f_r = 0.24\sqrt{f_c'} = 0.24\sqrt{4 \ \frac{\text{kips}}{\text{in}^2}}$$
$$= 0.48 \text{ kip/in}^2$$

For a nonsegmental concrete structure,

$$\gamma_1 = 1.6$$

For a nonprestressed concrete structure with A615, grade 60 reinforcement,

$$\gamma_3 = 0.67$$

Therefore,

$$\gamma_1 \gamma_3 = 1.072$$

The cracking moment of an interior beam is given by the reduced version of AASHTO Eq. 5.7.3.3.2-1 as

$$M_{cr} = \frac{\gamma_1 \gamma_3 f_r I_g}{\overline{y}}$$
$$= \frac{(1.072)\left(0.48 \ \frac{\text{kip}}{\text{in}^2}\right)(140{,}074 \text{ in}^4)}{(30 \text{ in})\left(12 \ \frac{\text{in}}{\text{ft}}\right)}$$
$$= 200 \text{ ft-kips}$$

From Ex. 8.11, the factored applied moment is

$$M_u = 1216 \text{ ft-kips}$$
$$1.33 M_u = (1.33)(1216 \text{ ft-kips})$$
$$= 1617 \text{ ft-kips}$$
$$> M_{cr} \quad [M_{cr} \text{ governs}]$$

From Ex. 8.11, the design strength of an interior beam is

$$\phi M_n = 1231 \text{ ft-kips}$$
$$> M_{cr} \quad [\text{satisfactory}]$$

The beam complies with AASHTO Sec. 5.7.3.3.2.

Bridges

Control of Flexural Cracking

To control flexural cracking of the concrete, the size and arrangement of tension reinforcement must be adjusted.

Two exposure conditions are defined in AASHTO Sec. 5.7.3.4. Class 1 exposure condition applies when cracks can be tolerated because of reduced concern for appearance or corrosion. Class 2 exposure condition applies when there is greater concern for appearance or corrosion.

The anticipated crack width depends on the following factors.

- the spacing, s, of reinforcement in the layer closest to the tension face

- the tensile stress, f_{ss}, in reinforcement at the service limit state

- the thickness of concrete cover, d_c, measured from the extreme tension fiber to center of reinforcement in the layer closest to the tension face

The exposure factor is defined as

$$\gamma_e = 1.00 \quad \text{[class 1 exposure conditions]}$$

$$\gamma_e = 0.75 \quad \text{[class 2 exposure conditions]}$$

The ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer nearest to the tension face is defined as

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

The spacing of reinforcement in the layer closest to the tension face is given by AASHTO Eq. 5.7.3.4-1 as

$$s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$$

Example 8.14

For an interior beam in the end span 12 of the four-span concrete T-beam bridge of Ex. 8.1, determine the allowable spacing of tension reinforcement. Assume that the service I moment is $M_s = 639$ ft-kips.

Solution

The concrete cover measured to the center of the reinforcing bar closest to the tension face of the member is obtained from Ex. 8.11 as

$$d_c = 2.56 \text{ in}$$

The lever-arm for elastic design may conservatively be taken as

$$l_a = d - \frac{h_f}{2} = 34.47 \text{ in} - 4.5 \text{ in}$$
$$= 29.97 \text{ in}$$

The maximum service dead plus live load moment in an interior beam in the end span 12 is given as

$$M_s = 639 \text{ ft-kips}$$

The stress in the reinforcement is given by

$$f_{ss} = \frac{M_s}{l_a A_s}$$
$$= \frac{(639 \text{ ft-kips})\left(12 \; \frac{\text{in}}{\text{ft}}\right)}{(29.97 \text{ in})(8 \text{ in}^2)}$$
$$= 31.98 \text{ kips/in}^2$$

The exposure factor for class 1 exposure conditions is given by AASHTO Sec. 5.7.3.4 as

$$\gamma_e = 1.00$$

The ratio of flexural strain at the extreme tension face to the strain at the centroid of the reinforcement layer nearest to the tension face is

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$
$$= 1 + \frac{2.56 \text{ in}}{(0.7)(39 \text{ in} - 2.56 \text{ in})}$$
$$= 1.10$$

The spacing of reinforcement in the layer closest to the tension face is given by AASHTO Eq. 5.7.3.4-1 as

$$s \leq \frac{700\gamma_e}{\beta_s f_{ss}} - 2d_c$$
$$= \frac{(700)(1.00)}{(1.10)\left(31.98 \; \frac{\text{kips}}{\text{in}^2}\right)} - (2)(2.56 \text{ in})$$
$$= 14.8 \text{ in}$$

The spacing provided is

$$s = 2.3 \text{ in} \quad \text{[satisfactory]}$$

Longitudinal Skin Reinforcement

Longitudinal skin reinforcement is required in the side faces of members exceeding 3 ft in effective depth to control cracking. In accordance with AASHTO Sec. 5.7.3.4, skin reinforcement shall be provided over a distance of $d/2$ nearest the flexural tension reinforcement, and the area in each face, per foot of height, shall be not less than

$$A_{s(\text{min})} = (0.012)(d - 30) \quad [\text{in}^2/\text{ft}]$$
$$\leq A_s/4$$

The spacing of this reinforcement shall not exceed

$$s = \frac{d}{6}$$
$$\leq 12 \text{ in}$$

Example 8.15

Determine the skin reinforcement required, in an interior beam, in the end span 12 of the four-span concrete T-beam bridge of Ex. 8.1.

Solution

The effective depth of the beam is

$$d = 34.47 \text{ in}$$
$$< 36 \text{ in}$$

Hence, in accordance with AASHTO Sec. 5.7.3.4, skin reinforcement is not required.

Fatigue Limits

Fatigue stress limits are defined in AASHTO Sec. 5.5.3 and depend on the stress in the reinforcement and the range of stress resulting from the fatigue I load combination. The constant-amplitude fatigue threshold, $(\Delta F)_{\text{TH}}$, for straight reinforcement, is defined by AASHTO Sec. 5.5.3.2-1 as

$$(\Delta F)_{\text{TH}} = 24 - 0.33 f_{\min} \qquad [AASHTO\ 5.5.3.2\text{-}1]$$

$f_{\min}$ is the minimum live-load stress (in kips/in²) resulting from the fatigue I load combination combined with the more severe stress from either the permanent loads, or the permanent loads, shrinkage, and creep-induced external loads. The minimum stress is positive if tension, negative if compression.

The factored live load stress range due to the passage of the fatigue I vehicle is

$$\gamma(\Delta f) \leq (\Delta F)_{\text{TH}}$$

Stress levels are determined at the fatigue I limit state load. In accordance with AASHTO Table 3.4.1-1, this consists of 150% of the design vehicle live load including dynamic load allowance. As specified in AASHTO Sec. 3.6.1.4, the design vehicle consists of a single design truck with a constant spacing of 30 ft between the 32 kip axles. In accordance with AASHTO Sec. 3.6.1.3.1, axles that do not contribute to the maximum force under consideration are neglected.

Example 8.16

Determine whether the fatigue stress limits, in an interior beam, at the midspan of span 12 of the four-span concrete T-beam bridge of Ex. 8.1 are satisfactory. Assume a value of $g = 0.65$ for the load distribution to the beam. Ignore the effects of creep and shrinkage.

Solution

The maximum moment at the midspan of an interior beam in span 12 caused by the design truck, plus the dynamic load allowance, is derived[2] as

$$M_{\max} = IWL\gamma g$$
$$= (1.15)(32 \text{ kips})(37 \text{ ft})(0.1998)(0.65)$$
$$= 177 \text{ ft-kips}$$

150% of the maximum moment is

$$M_{1.5,\max} = (1.5)(177 \text{ ft-kips})$$
$$= 266 \text{ ft-kips}$$

The influence line for the bending moment at midspan of beam 12 is shown in the following illustration.
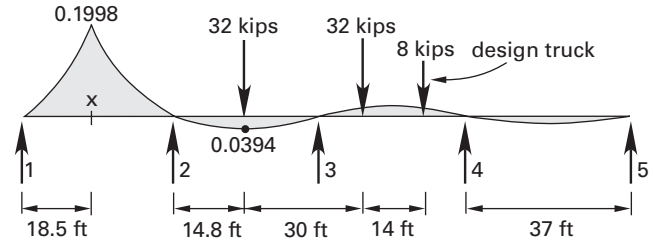
The location of the design truck to produce the minimum live load moment at point x is shown in the illustration. In accordance with AASHTO Sec. 3.6.1.3.1, the leading axle and the first 32 kips axle are ignored. Hence, the minimum moment caused by the design truck plus the dynamic load allowance is derived[2] as

$$M_{\min} = IWL\gamma g$$
$$= (1.15)(32 \text{ kips})(37 \text{ ft})(-0.0394)(0.65)$$
$$= -35 \text{ ft-kips}$$

150% of the minimum moment is

$$M_{1.5,\min} = (1.5)(-35 \text{ ft-kips})$$
$$= -53 \text{ ft-kips}$$

The maximum moment range is

$$M_{mr} = M_{1.5,\max} - M_{1.5,\min}$$
$$= 266 \text{ ft-kips} - (-53 \text{ ft-kips})$$
$$= 319 \text{ ft-kips}$$

The lever arm for elastic design is obtained from Ex. 8.14 as

$$l_a = 29.97 \text{ in}$$

The actual factored stress range is

$$\gamma(\Delta f) = \frac{M_{mr}}{l_a A_s} = \frac{(319 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(29.97 \text{ in})(8 \text{ in}^2)}$$
$$= 15.97 \text{ kips/in}^2$$

From Ex. 8.8, the dead load acting on beam 12 is $w = 1.722$ kips/ft

The permanent dead load moment at section x is derived[2] as

$$M_D = wL^2\gamma$$
$$= \left(1.722 \frac{\text{kips}}{\text{ft}}\right)(37 \text{ ft})^2(0.0714)$$
$$= 168 \text{ ft-kips}$$

The minimum live load moment resulting from the fatigue load combined with the permanent dead load moment is

$$M = M_D + M_{1.5,\text{min}}$$
$$= 168 \text{ ft-kips} - 53 \text{ ft-kips}$$
$$= 115 \text{ ft-kips}$$

The corresponding stress is

$$f_{\min} = \frac{M}{l_a A_s}$$
$$= \frac{(115 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(29.97 \text{ in})(8 \text{ in}^2)}$$
$$= 5.76 \text{ kips/in}^2$$

The constant-amplitude fatigue threshold is given by AASHTO Eq. 5.5.3.2-1 as

$$(\Delta F)_{\text{TH}} = 24 - 0.33 f_{\min}$$
$$= 24 - (0.33)\left(5.76 \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 22.10 \text{ kips/in}^2$$
$$> \gamma(\Delta f)$$

The fatigue stress limits are satisfactory.

Design for Shear

Nomenclature

A_s	area of tension reinforcement	in^2
A_v	area of shear reinforcement perpendicular to flexural tension reinforcement	in^2
b_v	web width	in
d_e	effective depth from extreme compression fiber to the centroid of the tensile force in the tensile reinforcement	in
d_v	effective shear depth	in
f_c'	specified compressive strength of concrete	kips/in^2
f_y	specified yield strength of reinforcing bars	kips/in^2
M_n	nominal flexural resistance	ft-kips
M_u	factored moment at the section	ft-kips
s	spacing of transverse reinforcement	in
v_u	average factored shear stress	kips/in^2
V_c	nominal shear strength provided by concrete	kips
V_s	nominal shear strength provided by shear reinforcement	kips
V_u	factored shear force at section	kips

Symbols

β	factor relating effect of longitudinal strain on the shear capacity of concrete	–
θ	angle of inclination of diagonal compressive stress	degrees
ϕ	resistance factor	–

Design Methods

Three design methods are described in the *AASHTO LRFD Bridge Design Specifications.* For members in which the strain distribution is nonlinear, AASHTO Sec. 5.6.3 specifies the use of a strut-and-tie model. This method is applicable to pile caps and deep footings, and to members with abrupt changes in cross section. The traditional sectional model is applicable where engineering beam theory is valid, as is the case for typical bridge girders and slabs. The sectional model is specified in AASHTO Sec. 5.8.3. For nonprestressed concrete sections, not subjected to axial tension and with the minimum area of transverse reinforcement specified in AASHTO Eq. 5.8.2.5-1, a simplified procedure is permissible as specified in AASHTO Sec. 5.8.3.4.1.

Simplified Design Method

In the simplified method, the value for the longitudinal strain factor is taken as $\beta = 2.0$, and the value of the angle of inclination of diagonal compressive stress is taken as $\theta = 45°$. This produces the simplified expressions for nominal concrete shear capacity, V_c, and nominal stirrup shear capacity, V_s, used in the procedure. This is similar to the approach adopted in the ACI[12] building code.

The nominal shear capacity of the concrete section is given by AASHTO Sec. 5.8.3.3 as

$$V_c = 0.0632 b_v d_v \sqrt{f_c'} \quad [\textit{AASHTO 5.8.3.3-3}]$$

The effective shear depth, d_v, is taken as the distance between the resultants of the tensile and compressive forces due to flexure.

The effective shear depth is given by AASHTO Eq. C5.8.2.9-1 as

$$d_v = \frac{M_n}{A_s f_y}$$

The effective shear depth need not be taken to be less than the greater of $0.9 d_e$ or $0.72 h$. The effective web width, b_v, is taken as the minimum web width between the resultants of the tensile and compressive forces due to flexure.

The nominal shear capacity of vertical stirrups is given by AASHTO Sec. 5.8.3.3 as

$$V_s = \frac{A_v f_y d_v}{s} \quad [\textit{AASHTO 5.8.3.3-4}]$$

The shear stress on the concrete is calculated by AASHTO Eq. 5.8.2.9-1 as

$$v_u = \frac{V_u}{\phi b_v d_v}$$

For a value of v_u less than $0.125f'_c$, AASHTO Sec. 5.8.2.7 limits the spacing of transverse reinforcement to the lesser of $0.8d_v$, or 24 in. When the value of v_u is not less than $0.125f'_c$, the spacing is reduced to the lesser of $0.4d_v$, or 12 in.

A minimum area of transverse reinforcement is required to control diagonal cracking and is specified by AASHTO Eq. 5.8.2.5-1 as

$$A_v = \frac{0.0316\sqrt{f'_c}b_v s}{f_y}$$

The combined nominal shear resistance of the concrete section and the shear reinforcement is given by AASHTO Sec. 5.8.3.3 as the lesser of

$$V_n = V_c + V_s$$
$$V_n = 0.25f'_c b_v d_v$$

The combined shear capacity of the concrete section and the shear reinforcement is

$$\phi V_n = \phi V_c + \phi V_s$$
$$\geq V_u$$

The resistance factor for shear and torsion is given by AASHTO Sec. 5.5.4.2.1 as

$$\phi = 0.90 \quad \text{[normal weight concrete]}$$
$$\phi = 0.80 \quad \text{[lightweight concrete]}$$

Example 8.17

For the four-span concrete T-beam bridge of Ex. 8.1, determine the shear reinforcement required in an interior beam at end 2 of span 23. The concrete strength is 4 kips/in^2, and the shear reinforcement consists of no. 4 grade 60 bars. The depth $d_v = 31.4$ in.

Solution

From Ex. 8.9, the factored shear at a distance d_v from the support is
$$V_{23} = 189 \text{ kips}$$

The shear strength provided by the concrete is given by

$$\phi V_c = 0.0632\phi b_v d_v \sqrt{f'_c}$$
$$= (0.0632)(0.90)(12 \text{ in})(31.4 \text{ in})\sqrt{4 \frac{\text{kips}}{\text{in}^2}}$$
$$= 42.87 \text{ kips}$$
$$< V_{23}$$

The factored shear force exceeds the shear strength of the concrete, and the shear strength required from shear reinforcement is given by

$$\phi V_s = V_{23} - \phi V_c$$
$$= 189 \text{ kips} - 42.87 \text{ kips}$$
$$= 146.13 \text{ kips}$$

The shear stress is given by AASHTO Eq. 5.8.2.9-1 as

$$v_u = \frac{V_{23}}{\phi b_v d_v} = \frac{189 \text{ kips}}{(0.9)(12 \text{ in})(31.4 \text{ in})}$$
$$= 0.56 \text{ kips/in}^2$$
$$> 0.125f'_c$$

Therefore, stirrups are required at a maximum spacing of 12 in. The area of shear reinforcement required is given by AASHTO Eq. 5.8.3.3-4 as

$$\frac{A_v}{s} = \frac{\phi V_s}{\phi d_v f_y}$$
$$= \frac{(146.13 \text{ kips})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(0.90)(31.4 \text{ in})\left(60 \frac{\text{kips}}{\text{in}^2}\right)}$$
$$= 1.03 \text{ in}^2/\text{ft}$$

Shear reinforcement consisting of two arms of no. 4 bars at 4 in spacing provides a reinforcement area of

$$\frac{A_v}{s} = 1.2 \frac{\text{in}^2}{\text{ft}}$$
$$> 1.03 \text{ in}^2/\text{ft} \quad \text{[satisfactory]}$$

3. PRESTRESSED CONCRETE DESIGN

Design for Flexure

Nomenclature

a	depth of equivalent rectangular stress block	in
A	area of concrete section	in^2
A_c	area of composite section	in^2
A_{ps}	area of prestressing steel	in^2
A_s	area of nonprestressed tension reinforcement	in^2
b	width of compression face of member	in
c	distance from the extreme compression fiber to the neutral axis	in
d_e	effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement	in
d_p	distance from extreme compression fiber to centroid of prestressing tendons	in
d_s	distance from the extreme compression fiber to the centroid of nonprestressed reinforcement	in
d_t	distance from extreme compression fiber to centroid of extreme tensile reinforcement	in
e	eccentricity of prestressing force	in

f_{be}	bottom fiber stress at service load after allowance for all prestress losses	kips/in^2
f_{bi}	bottom fiber stress immediately after prestress transfer and before time-dependent prestress losses	kips/in^2
f'_c	specified compressive strength of concrete	kips/in^2
f'_{ci}	compressive strength of concrete at time of prestress transfer	kips/in^2
f_{cpe}	bottom fiber stress due only to effective prestressing force after allowance for all prestress losses	kips/in^2
f_{pbt}	allowable stress in prestressing steel immediately prior to prestress transfer	kips/in^2
f_{pe}	effective stress in prestressing steel after allowance for all prestress losses	kips/in^2
f_{pj}	stress in the prestressing steel at jacking	kips/in^2
f_{ps}	stress in prestressing steel at ultimate load	kips/in^2
f_{pt}	stress in the prestressing steel immediately after transfer	kips/in^2
f_{pu}	specified tensile strength of prestressing steel	kips/in^2
f_{py}	specified yield strength of prestressing steel	kips/in^2
f_r	modulus of rupture of concrete	kips/in^2
f_s	stress in the tension reinforcement at nominal flexural resistance	kips/in^2
f_{te}	top fiber stress at service loads after allowance for all prestress losses	kips/in^2
f_{ti}	top fiber stress immediately after prestress transfer and before time-dependent prestress losses	kips/in^2
f_y	specified yield strength of reinforcing bars	kips/in^2
h	overall depth of section	in
h_f	compression flange thickness	in
k	prestressing steel factor	–
L	span length	ft
M_{cr}	cracking moment	in-kips
M_D	bending moment due to superimposed dead load	in-kips
M_{DC}	bending moment due to superimposed dead load on composite section	in-kips
M_{dnc}	bending moment due to noncomposite dead load acting on the precast section, $M_g + M_S$	in-kips
M_g	bending moment due to self-weight of girder	in-kips
M_L	bending moment due to superimposed live load	in-kips
M_n	nominal flexural strength	in-kips
M_r	factored flexural resistance	in-kips
M_S	bending moment due to weight of deck slab	in-kips
M_u	factored moment	in-kips
P_e	force in prestressing steel at service loads after allowance for all losses	kips
P_i	force in prestressing steel immediately after prestress transfer	kips
S_b	section modulus of the concrete section referred to the bottom fiber	in^3
S_c	section modulus of the composite section referred to the bottom fiber	in^3
S_{ci}	section modulus of the composite section referred to the interface of girder and slab	in^3
S_{nc}	section modulus of the noncomposite section referred to the bottom fiber	in^3

S_t	section modulus of the concrete section referred to the top fiber	in^3
w	distributed load	kips/ft
y_b	height of centroid of the concrete section	in
y_s	height of centroid of the prestressing steel	in

Symbols

β_1	compression zone factor	–
γ_1	flexural cracking variability factor	–
γ_2	prestress variability factor	–
γ_3	ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement	–
ϵ_{cu}	failure strain of concrete in compression	–
ϵ_t	net tensile strain in extreme tension steel at nominal resistance	–
ϕ	strength reduction factor	–
ϕ_w	reduction factor for slender members	–

Conditions at Transfer

The allowable stresses in the concrete at transfer, in other than segmentally constructed bridges, are specified in AASHTO Sec. 5.9.4.1 and are

$$f_{ti} \geq -0.0948\sqrt{f'_{ci}}$$
$$\geq -0.2 \text{ kips/in}^2 \quad \text{[without bonded reinforcement]}$$
$$f_{ti} \geq -0.24\sqrt{f'_{ci}} \quad \text{[with bonded reinforcement]}$$
$$f_{bi} \leq 0.60f'_{ci} \quad \text{[pretensioned members]}$$
$$f_{bi} \leq 0.60f'_{ci} \quad \text{[post-tensioned members]}$$

In accordance with AASHTO Sec. 5.9.3, the maximum allowable stress in pretensioned tendons immediately prior to transfer is

$$f_{pbt} = 0.75f_{pu} \quad \text{[low-relaxation strand]}$$
$$f_{pbt} = 0.70f_{pu} \quad \text{[stress-relieved strand]}$$

The maximum allowable stress in post-tensioned tendons immediately after transfer is

$$f_{pt} = 0.70f_{pu} \quad \text{[at the anchorage]}$$
$$f_{pt} = 0.74f_{pu} \quad \begin{bmatrix} \text{elsewhere, low-} \\ \text{relaxation strand} \end{bmatrix}$$

The maximum allowable stress at jacking is

$$f_{pj} = 0.90f_{py}$$

Example 8.18

The post-tensioned girder, with grouted tendons, shown in the following illustration is simply supported over a span of 100 ft and has the following properties.

A	S_t	S_b	y_b	f'_{ci}
800 in^2	$14,700 \text{ in}^3$	$15,600 \text{ in}^3$	37.8 in	4500 lbf/in^2

The concrete strength at transfer is $f'_{ci} = 4.5$ kips/in^2. The prestressing force immediately after transfer is 1000 kips, and the centroid of the tendons is 7 in above the bottom of the beam. Determine the actual and allowable stresses in the girder at midspan immediately after transfer if only prestressed reinforcement is provided.
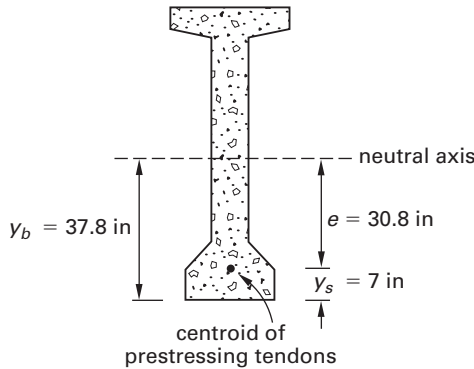
Solution

At midspan, the minimum allowable tensile stress in the top fiber without bonded reinforcement is given by AASHTO Sec. 5.9.4.1 as

$$f_{ti} = -0.0948\sqrt{f'_{ci}}$$
$$= -0.0948\sqrt{4.5 \ \frac{\text{kips}}{\text{in}^2}}$$
$$= -0.201 \ \text{kip/in}^2$$

Use the minimum allowable value of

$$f_{ti} = -0.200 \ \text{kip/in}^2$$

At midspan, the maximum allowable compressive stress in the bottom fiber is given by AASHTO Sec. 5.9.4.1 as

$$f_{bi} = 0.60'_{ci}$$
$$= (0.60)\left(4.5 \ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 2.70 \ \text{kips/in}^2$$

At midspan, the self-weight moment is

$$M_g = \frac{wL^2}{8}$$
$$= \frac{\left(150 \ \frac{\text{lbf}}{\text{ft}^3}\right)\left(\dfrac{800 \ \text{in}^2}{\left(12 \ \frac{\text{in}}{\text{ft}}\right)^2}\right)(100 \ \text{ft})^2\left(12 \ \frac{\text{in}}{\text{ft}}\right)}{(8)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}$$
$$= 12{,}500 \ \text{in-kips}$$

At midspan, the eccentricity of the prestressing force is

$$e = y_b - y_s = 37.8 \ \text{in} - 7 \ \text{in}$$
$$= 30.8 \ \text{in}$$

At midspan, the actual stress in the top fiber is given by

$$f_{ti} = \frac{P_i}{A} - \frac{P_i e}{S_t} + \frac{M_g}{S_t}$$
$$= \frac{1000 \ \text{kips}}{800 \ \text{in}^2} - \frac{(1000 \ \text{kips})(30.8 \ \text{in})}{14{,}700 \ \text{in}^3}$$
$$\quad + \frac{12{,}500 \ \text{in-kips}}{14{,}700 \ \text{in}^3}$$
$$= +0.005 \ \text{kip/in}^2$$
$$> -0.20 \ \text{kip/in}^2 \quad [\text{satisfactory}]$$

At midspan, the actual compressive stress in the bottom fiber is given by

$$f_{bi} = \frac{P_i}{A} + \frac{P_i e}{S_b} - \frac{M_g}{S_b}$$
$$= \frac{1000 \ \text{kips}}{800 \ \text{in}^2} + \frac{(1000 \ \text{kips})(30.8 \ \text{in})}{15{,}600 \ \text{in}^3}$$
$$\quad - \frac{12{,}500 \ \text{in-kips}}{15{,}600 \ \text{in}^3}$$
$$= 2.42 \ \text{kips/in}^2$$
$$< 2.70 \ \text{kips/in}^2 \quad [\text{satisfactory}]$$

Service Load Conditions

The allowable stresses in the concrete under service loads, in other than segmentally constructed bridges, after all prestressing losses have occurred are specified in AASHTO Sec. 5.9.4.2 as

$f_{te} \leq 0.45 f'_c \quad [\text{for permanent load}]$

$f_{te} \leq 0.60 \phi_w f'_c \quad [\text{for permanent + transient loads}]$

$f_{be} \geq -0.19\sqrt{f'_c} \quad \left[\begin{array}{l}\text{with bonded prestressing tendons}\\ \text{or reinforcement and mild exposure}\end{array}\right]$

$f_{be} \geq -0.0948\sqrt{f'_c} \quad \left[\begin{array}{l}\text{with bonded prestressing}\\ \text{tendons or reinforcement}\\ \text{and severe exposure}\end{array}\right]$

$f_{be} \geq 0 \quad [\text{with unbonded prestressing tendons}]$

In accordance with AASHTO Sec. 5.9.3, the maximum allowable stress in the tendons after all losses is

$$f_{pe} = 0.80 f_{py}$$

Example 8.19

The post-tensioned girder of Ex. 8.18 forms part of a composite deck, as shown in the following illustration, with girders located at 8 ft centers. The resulting composite section properties, allowing for the different moduli

of elasticity of the girder and the slab, are tabulated as follows.

A_c	S_{ci}	S_c	f'_c (girder)
1250 in^2	45,400 in^3	21,200 in^3	6000 lbf/in^2

The concrete strength of the girder at 28 days is $f'_c = 6$ kips/in^2.

The prestressing force after all losses is 800 kips, and the losses occur before the deck slab is cast. The bending moment, M_{DC}, due to dead load imposed on the composite section is 3000 in-kips. The bending moment, M_L, due to live load plus impact is 16,250 in-kips. Determine the actual and allowable stresses in the girder at midspan if the girder is subject to mild exposure. Bonded reinforcement is provided at the bottom of the girder. Assume $\phi_w = 1.0$.

precast girder
at 8 ft centers
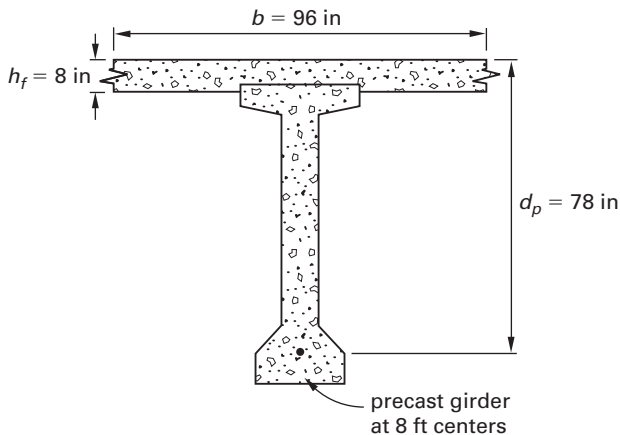
Solution

At midspan, the minimum allowable tensile stress in the bottom fiber with bonded reinforcement is given by AASHTO Sec. 5.9.4.2 as

$$f_{be} = -0.19\sqrt{f'_c}$$
$$= -0.19\sqrt{6\ \frac{\text{kips}}{\text{in}^2}}$$
$$= -0.465\ \text{kip/in}^2$$

At midspan, the maximum allowable compressive stress in the top fiber is given by AASHTO Sec. 5.9.4.2 as

$$f_{te} = 0.45 f'_c \quad \text{[for permanent loads]}$$
$$= (0.45)\left(6\ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 2.7\ \text{kips/in}^2$$
$$f_{te} = 0.60\phi_w f'_c \quad \text{[for permanent and transient loads]}$$
$$= (0.60)(1.0)\left(6\ \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 3.6\ \text{kips/in}^2$$

From Ex. 8.18, the midspan moment due to the self-weight of the girder is

$$M_g = 12{,}500\ \text{in-kips}$$

The resulting stresses in the girder are

$$f_{Gt} = \frac{M_g}{S_t} = \frac{12{,}500\ \text{in-kips}}{14{,}700\ \text{in}^3}$$
$$= 0.850\ \text{kip/in}^2$$
$$f_{Gb} = -\frac{M_g}{S_b} = -\frac{12{,}500\ \text{in-kips}}{15{,}600\ \text{in}^3}$$
$$= -0.801\ \text{kip/in}^2$$

At midspan, the moment due to the weight of the deck slab is

$$M_S = \frac{wL^2}{8}$$

$$= \frac{\left(150\ \dfrac{\text{lbf}}{\text{ft}^3}\right)\left(\dfrac{8\ \text{in}}{12\ \dfrac{\text{in}}{\text{ft}}}\right)(8\ \text{ft})(100\ \text{ft})^2\left(12\ \dfrac{\text{in}}{\text{ft}}\right)}{(8)\left(1000\ \dfrac{\text{lbf}}{\text{kip}}\right)}$$

$$= 12{,}000\ \text{in-kips}$$

The resulting stresses in the girder are

$$f_{St} = \frac{M_S}{S_t} = \frac{12{,}000\ \text{in-kips}}{14{,}700\ \text{in}^3}$$
$$= 0.816\ \text{kip/in}^2$$
$$f_{Sb} = -\frac{M_S}{S_b} = -\frac{12{,}000\ \text{in-kips}}{15{,}600\ \text{in}^3}$$
$$= -0.769\ \text{kip/in}^2$$

The resulting stresses in the girder due to the dead load imposed on the composite section are

$$f_{Dt} = \frac{M_{DC}}{S_{ci}} = \frac{3000\ \text{in-kips}}{45{,}400\ \text{in}^3}$$
$$= 0.066\ \text{kip/in}^2$$
$$f_{Db} = -\frac{M_{DC}}{S_c} = -\frac{3000\ \text{in-kips}}{21{,}200\ \text{in}^3}$$
$$= -0.142\ \text{kip/in}^2$$

The resulting stresses in the girder due to the live load imposed on the composite section are

$$f_{Lt} = \frac{M_L}{S_{ci}} = \frac{16{,}250\ \text{in-kips}}{45{,}400\ \text{in}^3}$$
$$= 0.358\ \text{kip/in}^2$$
$$f_{Lb} = -\frac{M_L}{S_c} = -\frac{16{,}250\ \text{in-kips}}{21{,}200\ \text{in}^3}$$
$$= -0.767\ \text{kip/in}^2$$

The stresses in the girder due to the effective prestressing force after all losses are

$$f_{Pt} = P_e \left(\frac{1}{A} - \frac{e}{S_t} \right)$$

$$= (800 \text{ kips}) \left(\frac{1}{800 \text{ in}^2} - \frac{30.8 \text{ in}}{14{,}700 \text{ in}^3} \right)$$

$$= -0.676 \text{ kip/in}^2$$

$$f_{Pb} = P_e \left(\frac{1}{A} + \frac{e}{S_b} \right)$$

$$= (800 \text{ kips}) \left(\frac{1}{800 \text{ in}^2} + \frac{30.8 \text{ in}}{15{,}600 \text{ in}^3} \right)$$

$$= 2.579 \text{ kips/in}^2$$

The final bottom fiber stress in the girder due to all loads is

$$f_{be} = f_{Gb} + f_{Sb} + f_{Db} + f_{Lb} + f_{Pb}$$

$$= -0.801 \, \frac{\text{kip}}{\text{in}^2} + -0.769 \, \frac{\text{kip}}{\text{in}^2} + -0.142 \, \frac{\text{kip}}{\text{in}^2}$$

$$+ -0.767 \, \frac{\text{kip}}{\text{in}^2} + 2.579 \, \frac{\text{kips}}{\text{in}^2}$$

$$= 0.1 \text{ kip/in}^2$$

$$> -0.465 \text{ kip/in}^2 \quad \text{[satisfactory]}$$

The final top fiber stress in the girder due to all permanent and transient loads is

$$f_{te} = f_{Gt} + f_{St} + f_{Dt} + f_{Lt} + f_{Pt}$$

$$= 0.850 \, \frac{\text{kip}}{\text{in}^2} + 0.816 \, \frac{\text{kip}}{\text{in}^2} + 0.066 \, \frac{\text{kip}}{\text{in}^2}$$

$$+ 0.358 \, \frac{\text{kip}}{\text{in}^2} + -0.676 \, \frac{\text{kip}}{\text{in}^2}$$

$$= 1.414 \text{ kips/in}^2$$

$$< 3.600 \text{ kips/in}^2 \quad \text{[satisfactory]}$$

The final top fiber stress in the girder due to sustained loads is

$$f_t = f_{Gt} + f_{St} + f_{Dt} + f_{Pt}$$

$$= 0.850 \, \frac{\text{kip}}{\text{in}^2} + 0.816 \, \frac{\text{kip}}{\text{in}^2} + 0.066 \, \frac{\text{kip}}{\text{in}^2}$$

$$+ -0.676 \, \frac{\text{kip}}{\text{in}^2}$$

$$= 1.056 \text{ kips/in}^2$$

$$< 2.700 \text{ kips/in}^2 \quad \text{[satisfactory]}$$

Ultimate Load Conditions

Provided that the effective prestress in the tendons after losses, f_{pe}, is not less than half the tensile strength of the tendons, f_{pu}, the stress in bonded tendons at ultimate load is given by AASHTO Eq. 5.7.3.1.1-1 as

$$f_{ps} = f_{pu} \left(1 - \frac{kc}{d_p} \right)$$

This expression is based on the assumption that all of the prestressing steel is concentrated at a distance d_p from the extreme compression fiber. If this assumption is not justified, a method based on strain compatibility must be used.

The prestressing steel factor, k, is given by AASHTO Table C5.7.3.1.1-1 as

- 0.48 for type 2 high-strength bars with $f_{py}/f_{pu} = 0.80$
- 0.38 for stress-relieved strands and type 1 high-strength bars with $f_{py}/f_{pu} = 0.85$
- 0.28 for low-relaxation wire and strands with $f_{py}/f_{pu} = 0.90$

The compression zone factor, β_1, given in AASHTO Sec. 5.7.2.2 is

- 0.85 for $f_c' \leq 4 \text{ kips/in}^2$
- $0.85 - (f_c' - 4)/20$ for $4 \text{ kips/in}^2 < f_c' \leq 8 \text{ kips/in}^2$
- 0.65 for $f_c' > 8 \text{ kips/in}^2$

For a rectangular section, with nonprestressed tension reinforcement, the distance from the extreme compression fiber to the neutral axis is given by AASHTO Eq. 5.7.3.1.1-4 as

$$c = \frac{A_{ps}f_{pu} + A_s f_s}{0.85 f_c' \beta_1 b + \dfrac{k A_{ps} f_{pu}}{d_p}}$$

The previous expression is also applicable to a flanged section with the neutral axis within the flange.

As specified in AASHTO Sec. 5.7.2.1, f_y may replace f_s when, using f_y in the calculation, the resulting ratio c/d_s does not exceed 0.6. If c/d_s exceeds 0.6, strain compatibility shall be used to determine the stress in the mild steel tension reinforcement.

The depth of the equivalent rectangular stress block is given by AASHTO Sec. 5.7.2.2 as

$$a = \beta_1 c$$

The nominal flexural strength of a rectangular section, without nonprestressed compression reinforcement, is given by AASHTO Eq. 5.7.3.2.2-1 as

$$M_n = A_{ps}f_{ps} \left(d_p - \frac{a}{2} \right) + A_s f_s \left(d_s - \frac{a}{2} \right)$$

Bridges

The factored flexural resistance is given by AASHTO Eq. 5.7.3.2.1-1 as

$$M_r = \phi M_n$$

The resistance factor for a tension-controlled prestressed concrete section is given by AASHTO Sec. 5.5.4.2.1 as

$$\phi = 1.0$$

Example 8.20

The area of the low-relaxation strand in the post-tensioned girder of Ex. 8.19 is 5.36 in^2, and the strand has a specified tensile strength of 270 kips/in^2. The 28 day compressive strength of the deck slab is 3 kips/in^2. Determine the maximum factored moment at midspan and the design flexural capacity of the composite section.

Solution

From Ex. 8.19, the total dead load moment on the composite section is

$$M_D = M_g + M_S + M_{DC}$$
$$= 12{,}500 \text{ in-kips} + 12{,}000 \text{ in-kips}$$
$$+ 3000 \text{ in-kips}$$
$$= 27{,}500 \text{ in-kips}$$

The live load moment plus impact is

$$M_L = 16{,}250 \text{ in-kips}$$

The strength I limit state moment is given by AASHTO Eq. 3.4.1-1 as

$$M_u = \gamma_p M_D + \gamma_{LL+IM} M_L$$
$$= (1.25)(27{,}500 \text{ in-kips})$$
$$+ (1.75)(16{,}250 \text{ in-kips})$$
$$= 62{,}813 \text{ in-kips}$$

The effective prestress in the tendons after all losses is obtained from Ex. 8.19 as

$$f_{pe} = \frac{P_e}{A_{ps}}$$
$$= \frac{800 \text{ kips}}{5.36 \text{ in}^2}$$
$$= 149 \text{ kips/in}^2$$
$$> 0.5 f_{pu}$$

Therefore, AASHTO Sec. 5.7.3.1 is applicable.

The compression zone factor for 3 kips/in^2 concrete is

$$\beta_1 = 0.85$$

The prestressing steel factor is given by AASHTO Table 5.7.3.1.1-1 as

$$k = 0.28 \quad \text{[for low-relaxation strand]}$$

Assuming that the neutral axis lies within the flange, for a section without nonprestressed tension reinforcement, the distance from the extreme compression fiber to the neutral axis is given by AASHTO Eq. 5.7.3.1.1-4 as

$$c = \frac{A_{ps} f_{pu}}{0.85 f'_c \beta_1 b + \frac{k A_{ps} f_{pu}}{d_p}}$$

$$= \frac{(5.36 \text{ in}^2)\left(270 \dfrac{\text{kips}}{\text{in}^2}\right)}{(0.85)\left(3 \dfrac{\text{kips}}{\text{in}^2}\right)(0.85)(96 \text{ in})}$$
$$+ \frac{(0.28)(5.36 \text{ in}^2)\left(270 \dfrac{\text{kips}}{\text{in}^2}\right)}{78 \text{ in}}$$

$$= 6.79 \text{ in}$$
$$< 8 \text{ in}$$

Therefore, the neutral axis does lie within the flange.

The depth of the equivalent rectangular stress block is given by AASHTO Sec. 5.7.2.2 as

$$a = \beta_1 c$$
$$= (0.85)(6.79 \text{ in})$$
$$= 5.8 \text{ in}$$

The stress in bonded tendons at ultimate load is given by AASHTO Eq. 5.7.3.1.1-1 as

$$f_{ps} = f_{pu}\left(1 - \frac{kc}{d_p}\right)$$
$$= \left(270 \dfrac{\text{kips}}{\text{in}^2}\right)\left(1 - \frac{(0.28)(6.79 \text{ in})}{78 \text{ in}}\right)$$
$$= 263.42 \text{ kips/in}^2$$

The nominal flexural strength of the section is given by AASHTO Eq. 5.7.3.2.2-1 as

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right)$$
$$= (5.36 \text{ in}^2)\left(263.42 \dfrac{\text{kips}}{\text{in}^2}\right)\left(78 \text{ in} - \frac{5.8 \text{ in}}{2}\right)$$
$$= 106{,}036 \text{ in-kips}$$

The strain in the prestressing tendons at the nominal flexural strength is

$$\epsilon_t = \epsilon_{cu}\left(\frac{d_p - c}{c}\right) = (0.003)\left(\frac{78\text{ in} - 6.79\text{ in}}{6.79\text{ in}}\right)$$

$$= 0.031$$

$$> 0.005$$

Therefore, from AASHTO Sec. 5.7.2.1, the section is tension controlled and the resistance factor is given by AASHTO Sec. 5.5.4.2.1 as

$$\phi = 1.0$$

The factored flexural resistance is

$$M_r = \phi M_n = (1.0)(106{,}036\text{ in-kips})$$

$$= 106{,}036\text{ in-kips}$$

$$> M_u \quad \text{[satisfactory]}$$

Cracking Moment

The *cracking moment* is the external moment that, when applied to the member after all losses have occurred, will cause cracking in the bottom fiber. This cracking occurs when the stress in the bottom fiber exceeds the modulus of rupture, which is defined in AASHTO Sec. 5.4.2.6 for normal weight concrete as

$$f_r = 0.24\sqrt{f'_c}$$

For a composite section, the cracking moment is defined in AASHTO Sec. 5.7.3.3.2 as

$$M_{cr} = \gamma_3\left(S_c(\gamma_2 f_{cpe} + \gamma_1 f_r) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right)$$

$$\geq S_c f_r$$

The applicable factors for the cracking moment are as follows.

γ_1 = flexural cracking variability factor

    = 1.2   [precast segmental structures]

    = 1.6   [other concrete structures]

γ_2 = prestress variability factor

    = 1.1   [bonded tendons]

    = 1.0   [unbonded tendons]

γ_3 = ratio of specified minimum yield strength to

      ultimate tensile strength of the reinforcement

    = 0.67   [A615, grade 60 reinforcement]

    = 0.75   [A706, grade 60 reinforcement]

    = 1.00   [prestressed concrete structures]

For noncomposite beams, S_{nc} is substituted for S_c in the previous expression. To prevent sudden tensile failure, AASHTO Sec. 5.7.3.3.2 requires that

$$\phi M_n \geq M_{cr}$$

Example 8.21

Determine the cracking moment of the composite section of Ex. 8.19.

Solution

From Ex. 8.19, the bottom fiber stress due only to the effective prestressing force after allowance for all prestress losses is

$$f_{cpe} = 2.579\text{ kips/in}^2$$

In addition, the bending moment due to the noncomposite dead load acting on the precast section is given by

$$M_{dnc} = M_g + M_S = 12{,}500\text{ in-kips} + 12{,}000\text{ in-kips}$$

$$= 24{,}500\text{ in-kips}$$

The modulus of rupture is given by AASHTO Sec. 5.4.2.6 as

$$f_r = 0.24\sqrt{f'_c} = 0.24\sqrt{6\ \frac{\text{kips}}{\text{in}^2}}$$

$$= 0.588\text{ kip/in}^2$$

From Ex. 8.18, the section modulus of the noncomposite section referred to the bottom fiber is

$$S_{nc} = S_b = 15{,}600\text{ in}^3$$

The cracking moment is given by AASHTO Sec. 5.7.3.3.2 as

$$M_{cr} = \gamma_3\left(S_c(\gamma_2 f_{cpe} + \gamma_1 f_r) - M_{dnc}\left(\frac{S_c}{S_{nc}} - 1\right)\right)$$

$$= (1.0)\left(\begin{array}{l}(21{,}200\text{ in}^3)\left(\begin{array}{l}(1.1)\left(2.579\ \dfrac{\text{kips}}{\text{in}^2}\right)\\[6pt] + (1.6)\left(0.588\ \dfrac{\text{kip}}{\text{in}^2}\right)\end{array}\right)\\[14pt] - (24{,}500\text{ in-kips})\left(\dfrac{21{,}200\text{ in}^3}{15{,}600\text{ in}^3} - 1\right)\end{array}\right)$$

$$= 71{,}292\text{ in-kips}$$

$$< \phi M_n \quad \left[\begin{array}{l}\text{satisfies AASHTO}\\ \text{Sec. 5.7.3.3.2}\end{array}\right]$$

Design for Shear

Nomenclature

A_v	area of shear reinforcement	in^2
b_v	web width	in
d_e	effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement	in
d_p	distance from the extreme compression fiber to the centroid of the prestressing tendons	in

d_v	effective shear depth	in
f'_c	specified compressive strength of concrete	kips/in^2
f_{cpe}	compressive stress in the concrete, due to the final prestressing force only, at the bottom fiber of the section	lbf/in^2
f_d	tensile stress at bottom fiber of precast member due to unfactored dead load acting on the precast member	lbf/in^2
f_{pc}	compressive stress in the concrete, due to the final prestressing force and applied loads resisted by precast member, at the centroid of the composite section	lbf/in^2
f_r	modulus of rupture	kips/in^2
f_y	specified yield strength of reinforcing bars	kips/in^2
g	drape of the prestressing cable	in
h	depth of section	in
M_{cre}	moment causing flexural cracking at section due to externally applied loads	in-lbf or in-kips
M_d	moment due to unfactored dead load	in-kips
M_{dnc}	total unfactored dead load moment acting on the precast member	in-kips
M_L	bending moment due to live load	in-kips
M_{max}	maximum factored moment at section due to externally applied loads	in-lbf or in-kips
M_u	factored moment at the section due to total factored loads	in-kips
s	longitudinal spacing of shear reinforcement	in
S	section modulus at the centroid of the composite section	in^3
S_c	section modulus at the bottom of the composite member	in^3
S_{ci}	section modulus at the interface of the composite member	in^3
S_{nc}	section modulus at the bottom of the precast member	in^3
V_c	nominal shear strength provided by concrete	kips
V_{ci}	nominal shear strength provided by concrete when diagonal cracking results from combined shear and moment	kips
V_{cw}	nominal shear strength provided by concrete when diagonal cracking results from excessive principal tensile stress in the web	kips
V_d	shear force at section due to unfactored dead load	kips
V_i	factored shear force at section due to externally applied loads occurring simultaneously with M_{max}	kips
V_L	unfactored shear due to live load	kips
V_p	vertical component of effective prestress force at section	kips
V_s	nominal shear strength provided by shear reinforcement	kips
V_u	factored shear force at section	kips
y_s	height of cable above beam soffit	in
x	horizontal distance	ft
z	half the beam length	ft

Symbols

γ_c	specific weight of concrete	kips/ft^3
θ	angle of inclination of diagonal compressive stresses	degree
ϕ	strength reduction factor	–

Ultimate Load Design for Shear

The simplified procedure of AASHTO Sec. 5.8.3.4.3 is permissible for prestressed concrete sections that are not subjected to significant axial tension and with the minimum amount of transverse reinforcement specified in AASHTO Eq. 5.8.2.5-1.

The nominal shear capacity of the concrete, V_c, is provided by the lesser value of V_{ci} or V_{cw} given by AASHTO Sec. 5.8.3.4.3. For flexural-shear cracking, the nominal shear capacity is given by AASHTO Eq. 5.8.3.4.3-1.[13]

$$V_{ci} = 0.02 b_v d_v \sqrt{f'_c} + V_d + \frac{V_i M_{cre}}{M_{max}}$$
$$\geq 0.06 b_v d_v \sqrt{f'_c}$$

The moment causing flexural cracking at the section due to externally applied loads is defined in AASHTO Eq. 5.8.3.4.3-2 as

$$M_{cre} = S_c \left(f_r + f_{cpe} - \frac{M_{dnc}}{S_{nc}} \right)$$

For web-shear cracking, the nominal shear capacity is given by AASHTO Eq. 5.8.3.4.3-3.[13]

$$V_{cw} = b_v d_v \left(0.06 \sqrt{f'_c} + 0.3 f_{pc} \right) + V_p$$

In a composite member, f_{pc} is the resultant compressive stress at the centroid of the composite section, or at the junction of the web and flange when the centroid lies within the flange, due to both the final prestress and moments resisted by the precast member acting alone.

The effective shear depth, d_v, to the centroid of the pre-stressing steel need not be taken as less than the greater of

$$d_v = 0.72h$$
$$d_v = 0.9 d_e$$

The nominal shear capacity of vertical shear reinforcement is given by AASHTO Eq. 5.8.3.3-4 as

$$V_s = A_v f_y \left(\frac{d_v}{s} \right) \quad [\text{for } V_{ci} < V_{cw}]$$
$$= A_v f_y \left(\frac{d_v \cot \theta}{s} \right) \quad [\text{for } V_{ci} > V_{cw}]$$
$$\cot \theta = 1.0 + \frac{3 f_{pc}}{\sqrt{f'_c}}$$
$$\leq 1.8$$

For a composite section constructed in two stages, some of the dead load is resisted by the precast section and the remainder by the composite section. In AASHTO Eq. 5.8.3.4.3-1, V_d is the total shear force due to unfactored dead loads acting on the precast section, plus the unfactored superimposed dead load acting on the composite section. V_i is the factored shear force at a section caused by the externally applied loads occurring simultaneously with the maximum factored moment at the section, $M_{\max}$. In accordance with AASHTO Sec. C5.8.3.4.3, where V_u and M_u represent the factored shear and moment at the section, these values may be taken as

$$V_i = V_u - V_d$$

$$M_{\max} = M_u - M_d$$

The effective shear depth, d_v, is taken as the distance between the resultants of the tensile and compressive forces due to flexure. The effective shear depth is given by AASHTO Eq. C5.8.2.9-1 as

$$d_v = \frac{M_n}{A_s f_y + A_{ps} f_{ps}}$$

The effective shear depth need not be taken to be less than the greater of $0.9d_e$ or $0.72h$. The effective web width, b_v, is taken as the minimum web width between the resultants of the tensile and compressive forces due to flexure.

In AASHTO Eq. 5.8.3.4.3-2, the stress in the bottom fiber due to unfactored dead load acting on the precast section only is given by

$$f_d = \frac{M_{dnc}}{S_{nc}}$$

To account for the effects of differential shrinkage and thermal gradients, AASHTO Eq. 5.8.3.4.3-2 uses a reduced value of the modulus of rupture, and this is given by AASHTO Sec. 5.4.2.6 as

$$f_r = 0.20\sqrt{f'_c}$$

The combined nominal shear resistance is given by AASHTO Sec. 5.8.3.3 as the lesser of

$$V_n = V_c + V_s + V_p$$

$$V_n = 0.25f'_c b_v d_v + V_p$$

The combined shear capacity is

$$\phi V_n \geq V_u$$

The resistance factor for normal weight concrete is given by AASHTO Sec. 5.5.4.2.1 as

$$\phi = 0.90$$

The shear stress on the concrete is calculated by AASHTO Eq. 5.8.2.9-1 as

$$v_u = \frac{V_u - \phi V_p}{\phi b_v d_v}$$

For a value of v_u less than $0.125f'_c$, AASHTO Sec. 5.8.2.7 limits the spacing of transverse reinforcement to the lesser of $0.8d_v$ or 24 in. When the value of v_u is not less than $0.125f'_c$, the spacing is reduced to the lesser of $0.4d_v$ or 12 in.

When the support reaction produces a compressive stress in the member, AASHTO Sec. 5.8.3.2 specifies that the critical section for shear may be taken at a distance from the support equal to the effective shear depth, d_v.

Example 8.22

The tendon centroid of the post-tensioned girder of Ex. 8.18 is parabolic in shape, as shown in the following illustration. At section A-A, the unfactored shear and moment due to live load plus impact are $V_L = 58$ kips and $M_L = 2340$ in-kips. The moment of inertia of the precast section is $I = 589{,}680$ in^4. Determine the required spacing of no. 3 grade 60 stirrups.

Solution

The equation of the parabolic cable profile is

$$y = \frac{gx^2}{z^2}$$

At section A-A, the rise of the cable is given by

$$y = \frac{(30 \text{ in})(600 \text{ in} - 61.2 \text{ in})^2}{(600 \text{ in})^2}$$

$$= 24.2 \text{ in}$$

At section A-A, the effective depth of the prestressing cable referred to the composite section is

$$d_p = h - y - y_s$$

$$= 85 \text{ in} - 24.2 \text{ in} - 7 \text{ in}$$

$$= 53.8 \text{ in}$$

$$= d_e \quad \left[\begin{array}{c}\text{effective depth from the extreme compression} \\ \text{fiber to the centroid of the tensile force}\end{array}\right]$$

From Ex. 8.20, the depth of the stress block at midspan of the composite section is

$$a = 5.8 \text{ in}$$

The value of a may be conservatively taken as the stress block depth at section A-A and the effective shear depth is

$$d_v = d_e - \frac{a}{2}$$

$$= 53.8 \text{ in} - \frac{5.8 \text{ in}}{2}$$

$$= 50.9 \text{ in}$$

Illustration for Ex. 8.22
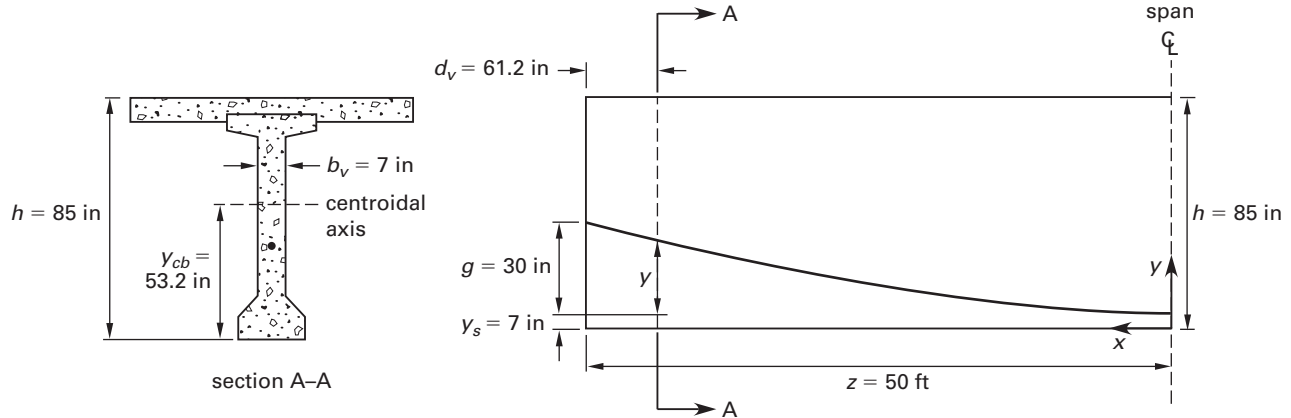
section A–A

The effective shear depth need not be taken to be less than the greater of

$$0.9d_e = (0.9)(53.8 \text{ in})$$
$$= 48.4 \text{ in}$$
$$0.72h = (0.72)(85 \text{ in})$$
$$= 61.2 \text{ in} \quad \text{[governs]}$$

Therefore, as specified by AASHTO Sec. 5.8.3.2, the critical section for shear is located a distance of 61.2 in from the support.

At section A-A, the cable eccentricity referred to the precast section is given by

$$e = y_b - y_s - y$$
$$= 37.8 \text{ in} - 7 \text{ in} - 24.2 \text{ in}$$
$$= 6.6 \text{ in}$$

At section A-A, the slope of the cable is given by

$$\frac{dy}{dx} = \frac{2gx}{z^2}$$
$$= \frac{(2)(30 \text{ in})(538.8 \text{ in})}{(600 \text{ in})^2}$$
$$= 0.0898$$

The vertical component of the final effective prestressing force at section A-A is

$$V_P = P_e\left(\frac{dy}{dx}\right)$$
$$= (800 \text{ kips})(0.0898)$$
$$= 72 \text{ kips}$$

The centroid of the composite section is at a height of

$$y_{cb} = 53.2 \text{ in}$$

The section modulus of the precast section at the centroid of the composite section is

$$S = \frac{I}{y_{cb} - y_b} = \frac{589{,}680 \text{ in}^4}{53.2 \text{ in} - 37.8 \text{ in}}$$
$$= 38{,}290 \text{ in}^3$$

At section A-A, the stress in the concrete, at the centroid of the composite section, due to the final prestressing force only is

$$f_p = P_e\left(\frac{1}{A} - \frac{e}{S}\right)$$
$$= (800 \text{ kips})\left(\frac{1}{800 \text{ in}^2} - \frac{6.6 \text{ in}}{38{,}290 \text{ in}^3}\right)$$
$$= 0.862 \text{ kip/in}^2$$

At section A-A, the bending moment due to the girder self-weight is

$$M = M_g\left(1 - \left(\frac{x}{z}\right)^2\right)$$
$$= (12{,}500 \text{ in-kips})\left(1 - \left(\frac{538.8 \text{ in}}{600 \text{ in}}\right)^2\right)$$
$$= 2420 \text{ in-kips}$$

At section A-A, the stress in the concrete at the centroid of the composite section due to the girder self-weight is

$$f_G = \frac{M}{S} = \frac{2420 \text{ in-kips}}{38{,}290 \text{ in}^3}$$
$$= 0.063 \text{ kip/in}^2$$

At section A-A, the bending moment due to the weight of the deck slab is

$$M = M_s\left(1 - \left(\frac{x}{z}\right)^2\right)$$
$$= (12{,}000 \text{ in-kips})\left(1 - \left(\frac{538.8 \text{ in}}{600 \text{ in}}\right)^2\right)$$
$$= 2323 \text{ in-kips}$$

At section A-A, the stress in the concrete at the centroid of the composite section due to the weight of the deck slab is

$$f_s = \frac{M}{S} = \frac{2323 \text{ in-kips}}{38{,}290 \text{ in}^3}$$
$$= 0.061 \text{ kip/in}^2$$

At section A-A, the compressive stress at the centroid of the composite section due to final prestress and the bending moments resisted by the precast member acting alone is

$$f_{pc} = f_p + f_G + f_s$$
$$= 0.862 \frac{\text{kip}}{\text{in}^2} + 0.063 \frac{\text{kip}}{\text{in}^2} + 0.061 \frac{\text{kip}}{\text{in}^2}$$
$$= 0.986 \text{ kip/in}^2$$

The nominal web-shear capacity is given by AASHTO Eq. 5.8.3.4.3-3 as

$$V_{cw} = b_v d_v (0.06\sqrt{f'_c} + 0.3f_{pc}) + V_p$$
$$= (7 \text{ in})(61.2 \text{ in})\left(\begin{array}{l} (0.06)\left(6 \, \frac{\text{kips}}{\text{in}^2} \right)^{0.5} \\ \\ + (0.3)\left(0.986 \, \frac{\text{kip}}{\text{in}^2} \right) \end{array} \right)$$
$$+ 72 \text{ kips}$$
$$= 262 \text{ kips}$$

The compressive stress in the bottom fiber of the precast member at section A-A due to the final prestressing force is

$$f_{cpe} = P_e\left(\frac{1}{A} + \frac{e}{S_{nc}} \right)$$
$$= (800 \text{ kips})\left(\frac{1}{800 \text{ in}^2} + \frac{6.6 \text{ in}}{15{,}600 \text{ in}^3} \right)$$
$$= 1.338 \text{ kips/in}^2$$

At section A-A, the stress in the concrete at the bottom fiber due to the dead load imposed on the precast section is, from Ex. 8.19,

$$f_d = f_{Gb+Sb}\left(1 - \left(\frac{x}{z} \right)^2 \right)$$
$$= \left(1.570 \, \frac{\text{kips}}{\text{in}^2} \right)\left(1 - \left(\frac{538.8 \text{ in}}{600 \text{ in}} \right)^2 \right)$$
$$= 0.304 \text{ kip/in}^2 \quad [\text{tension}]$$

The moment causing flexural cracking at the section due to externally applied loads is given by AASHTO Eq. 5.8.3.4.3-2 as

$$M_{cre} = S_c(f_r + f_{cpe} - f_d)$$
$$= (21{,}200 \text{ in}^3)\left(\begin{array}{c} 0.20\sqrt{6 \, \frac{\text{kips}}{\text{in}^2}} + 1.338 \, \frac{\text{kips}}{\text{in}^2} \\ \\ -0.304 \, \frac{\text{kip}}{\text{in}^2} \end{array} \right)$$
$$= 32{,}307 \text{ in-kips}$$
$$> 2340 \text{ in-kips} \quad \left[\begin{array}{c} \text{the given unfactored applied} \\ \text{moment at section A-A} \end{array} \right]$$

Hence, flexural-shear cracking does not occur at section A-A; the web-shear capacity controls with the nominal shear strength given by

$$V_c = V_{cw}$$
$$= 262 \text{ kips}$$

The design shear capacity is

$$\phi V_c = (0.9)(262 \text{ kips})$$
$$= 236 \text{ kips}$$

From Ex. 8.18, the cross-sectional area of the girder is $A_G = 800 \text{ in}^2$. At section A-A, the shear force due to the girder self-weight is

$$V_G = w_G x = \gamma_c A_G x$$
$$= \frac{\left(0.150 \, \frac{\text{kip}}{\text{ft}^3} \right)\left(\dfrac{800 \text{ in}^2}{\left(12 \, \frac{\text{in}}{\text{ft}} \right)^2} \right)(538.8 \text{ in})}{12 \, \frac{\text{in}}{\text{ft}}}$$
$$= 37.4 \text{ kips}$$

The cross-sectional area of the slab is $A_F = (8 \text{ in})(96 \text{ in}) = 768 \text{ in}^2$. At section A-A, the shear force due to the self-weight of the slab is

$$V_S = w_S x = \gamma_c A_F x$$
$$= \frac{\left(0.150 \, \frac{\text{kip}}{\text{ft}^3} \right)\left(\dfrac{768 \text{ in}^2}{\left(12 \, \frac{\text{in}}{\text{ft}} \right)^2} \right)(538.8 \text{ in})}{12 \, \frac{\text{in}}{\text{ft}}}$$
$$= 36.0 \text{ kips}$$

Bridges

At section A-A, the shear force due to the dead load imposed on the composite section is

$$V_{DC} = w_{DC}x = \frac{\left(0.20 \ \frac{\text{kip}}{\text{ft}}\right)(538.8 \ \text{in})}{12 \ \frac{\text{in}}{\text{ft}}}$$

$$= 9.0 \ \text{kips}$$

At section A-A, the total dead load shear force on the composite section is

$$V_D = V_G + V_S + V_{DC}$$
$$= 37.4 \ \text{kips} + 36.0 \ \text{kips} + 9.0 \ \text{kips}$$
$$= 82.4 \ \text{kips}$$

The live load shear plus impact is given as

$$V_L = 58 \ \text{kips}$$

The strength I limit state shear force is given by AASHTO Eq. 3.4.1-1 as

$$V_u = \gamma_p V_D + \gamma_{LL+IM} V_L$$
$$= (1.25)(82.4 \ \text{kips}) + (1.75)(58 \ \text{kips})$$
$$= 205 \ \text{kips}$$
$$< \phi V_c$$

Hence, a minimum area of shear reinforcement is required and is given by AASHTO Eq. 5.8.2.5-1 as

$$\frac{A_v}{s} = 0.0316\left(\frac{\sqrt{f'_c}b_v}{f_y}\right)$$

$$= (0.0316)\left(\frac{\left(\sqrt{6 \ \frac{\text{kips}}{\text{in}^2}}\right)(7 \ \text{in})}{60 \ \frac{\text{kips}}{\text{in}^2}}\right)$$

$$= 0.0090 \ \text{in}^2/\text{in}$$

The shear stress on the concrete is calculated by AASHTO Eq. 5.8.2.9-1 as

$$v_u = \frac{V_u - \phi V_p}{\phi b_v d_v}$$

$$= \frac{203 \ \text{kips} - (0.9)(72 \ \text{kips})}{(0.9)(7 \ \text{in})(61.2 \ \text{in})}$$

$$= 0.358 \ \text{kip/in}^2$$

$$< 0.125f'_c \quad (0.75 \ \text{kip/in}^2)$$

Therefore, AASHTO Sec. 5.8.2.7 limits the spacing of transverse reinforcement to the lesser of

$$s = 0.8d_v$$
$$= (0.8)(61.2 \ \text{in})$$
$$= 49 \ \text{in}$$
$$s = 24 \ \text{in} \quad [\text{governs}]$$

Providing no. 3 stirrups at the maximum permitted spacing of 24 in gives a value of

$$\frac{A_v}{s} = \frac{0.22 \ \text{in}^2}{24 \ \text{in}}$$
$$= 0.0092 \ \text{in}^2/\text{in} \quad [\text{satisfactory}]$$
$$> 0.0090 \ \text{in}^2/\text{in}$$

Prestress Losses

Nomenclature

A_g	gross area of section	in^2
A_{ps}	area of prestressing steel	in^2
E_{ci}	modulus of elasticity of concrete at time of initial prestress	kips/in^2
E_p	modulus of elasticity of prestressing steel	kips/in^2
f_{cgp}	compressive stress at centroid of prestressing steel due to prestress and self-weight of girder at transfer	kips/in^2
f_{pj}	stress in the prestressing steel at jacking	kips/in^2
g	drape of prestressing steel	in
I_g	moment of inertia of precast girder	in^4
K	wobble friction coefficient per foot of prestressing tendon	–
l_{px}	distance from free end of cable to section under consideration	ft
M_g	bending moment due to self-weight of precast member	in-kips
n_i	modular ratio at transfer	–
N	number of identical prestressing tendons	–
P_{ES}	loss of prestress force due to elastic shortening	kips
P_i	force in prestressing steel immediately after transfer	kips
P_o	force in prestressing steel at anchorage	kips
R	radius of curvature of tendon profile	ft

Symbols

α	angular change of tendon profile from jacking end to any point x	radians
Δf_{pES}	loss of prestress due to elastic shortening	kips/in^2
Δf_{pF}	prestress loss due to friction	kips/in^2
Δf_{pLT}	long-term prestress loss due to creep and shrinkage of concrete and relaxation of steel	kips/in^2
μ	curvature friction coefficient	–

Friction Losses

AASHTO Sec. 5.9.5.2.2 determines friction losses from the equation

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(Kx+\mu\alpha)}\right)$$

Values of the wobble and curvature friction coefficients are given in AASHTO Table 5.9.5.2.2b-1 and for pre-stressing strand are

$$K = 0.0002$$
$$\mu = 0.15 \text{ to } 0.25$$

Example 8.23

The beam of Ex. 8.22 is post-tensioned with low-relaxation strands with a total area of 5.36 in², a yield strength of 243 kips/in², and a tensile strength of 270 kips/in². The strands are located in 4 cables. The centroid of the prestressing steel is parabolic in shape and is stressed simultaneously from both ends with a jacking force, P_o, of 1036 kips. The value of the wobble friction coefficient is 0.0002/ft, and the curvature friction coefficient is 0.25. Determine the force in the prestressing steel at midspan of the member before elastic losses.

Solution

The nominal radius of the profile of the prestressing steel is

$$R = \frac{z^2}{2g} = \frac{(50 \text{ ft})^2}{(2)(2.5 \text{ ft})}$$
$$= 500 \text{ ft}$$

The length along the curve from the jacking end to midspan is

$$l_{px} = z + \frac{g^2}{3z} = 50 \text{ ft} + \frac{(2.5 \text{ ft})^2}{(3)(50 \text{ ft})}$$
$$= 50.04 \text{ ft}$$

The angular change of the cable profile over this length is

$$\alpha = \frac{l_{px}}{R} = \frac{50.04 \text{ ft}}{500 \text{ ft}}$$
$$= 0.100 \text{ radians}$$
$$(Kl_{px} + \mu\alpha) = (0.0002)(50.04 \text{ ft})$$
$$+ (0.25)(0.100 \text{ radians})$$
$$= 0.035$$

The cable force at midspan is given by

$$P_x = P_o e^{-(K_x + \mu\alpha)}$$
$$= (1036 \text{ kips})\left(e^{-0.035}\right)$$
$$= 1000 \text{ kips}$$

Elastic Shortening

Losses occur due to the elastic shortening of the concrete. The concrete stress at the level of the centroid of the prestressing steel after elastic shortening is

$$f_{cgp} = P_i\left(\frac{1}{A_g} + \frac{e^2}{I_g}\right) - \frac{eM_g}{I_g}$$

AASHTO Sec. C5.4.2.4 specifies that the modulus of elasticity of normal weight concrete at transfer is

$$E_{ci} = 1820\sqrt{f'_{ci}}$$

The modulus of elasticity of prestressing strand is given as

$$E_p = 28{,}500 \text{ kips/in}^2$$

The modular ratio at transfer is

$$n_i = \frac{E_p}{E_{ci}}$$

For a pretensioned member, the loss of prestress due to elastic shortening is given by AASHTO Sec. 5.9.5.2.3a as

$$\Delta f_{pES} = n_i f_{cgp}$$

For a post-tensioned member, the loss of prestress is given by AASHTO Sec. 5.9.5.2.3b.

$$\Delta f_{pES} = \frac{N-1}{2N} n_i f_{cgp}$$

Example 8.24

The post-tensioned beam of Ex. 8.23 has a concrete strength at transfer of 4.5 kips/in². The initial force at midspan, after friction losses and before allowance for elastic shortening, is 1000 kips. The moment of inertia of the girder is 589,680 in⁴. Determine the loss of prestress due to elastic shortening.

Solution

From AASHTO Eq. C5.4.2.4-1, the modulus of elasticity of the concrete at transfer is

$$E_{ci} = 1820\sqrt{f'_{ci}}$$
$$= 1820\sqrt{4.5 \ \frac{\text{kips}}{\text{in}^2}}$$
$$= 3861 \text{ kips/in}^2$$

The modular ratio at transfer is

$$n_i = \frac{E_s}{E_{ci}} = \frac{28{,}500 \ \frac{\text{kips}}{\text{in}^2}}{3861 \ \frac{\text{kips}}{\text{in}^2}}$$

$$= 7.38$$

Assuming a 3% loss due to elastic shortening, the initial prestressing force at midspan is

$$P_i = 970 \text{ kips}$$

The compressive stress at the centroid of the prestressing steel immediately after transfer is

$$f_{cgp} = P_i \left(\frac{1}{A_g} + \frac{e^2}{I_g} \right) - \frac{eM_g}{I_g}$$

$$= (970 \text{ kips}) \left(\frac{1}{800 \text{ in}^2} + \frac{(30.8 \text{ in})^2}{589{,}680 \text{ in}^4} \right)$$

$$- \frac{(30.8 \text{ in})(12{,}500 \text{ in-kips})}{589{,}680 \text{ in}^4}$$

$$= 2.120 \text{ kips/in}^2$$

For a post-tensioned member, the loss of prestress is given by AASHTO Sec. 5.9.5.2.3b as

$$\Delta f_{pES} = \frac{N-1}{2N} n_i f_{cgp}$$

$$= \frac{(3)(7.38) \left(2.120 \ \frac{\text{kips}}{\text{in}^2} \right)}{(2)(4)}$$

$$= 5.87 \text{ kips/in}^2$$

The loss of prestressing force is

$$P_{ES} = A_{ps} \Delta f_{pES} = (5.36 \text{ in}^2) \left(5.87 \ \frac{\text{kips}}{\text{in}^2} \right)$$

$$= 31.45 \text{ kips}$$

$$\approx 30 \text{ kips} \quad \begin{bmatrix} \text{assumed value is} \\ \text{sufficiently accurate} \end{bmatrix}$$

Estimated Time-Dependent Losses

An estimate of time-dependent losses for pretensioned members of usual design, with normal loading, using normal weight concrete, and exposed to average exposure conditions may be obtained from AASHTO Sec. 5.9.5.3.

The *long-term prestress loss*, Δf_{pLT}, due to creep of concrete, shrinkage of concrete, and relaxation of steel may be estimated using AASHTO Eq. 5.9.5.3-1.

$$\Delta f_{pLT} = \frac{10.0 \gamma_h \gamma_{st} f_{pi} A_{ps}}{A_g} + 12.0 \gamma_h \gamma_{st} + \Delta f_{pR}$$

Where,

γ_h = correction factor for relative humidity of the ambient air
$= 1.7 - 0.01H$

γ_{st} = correction factor for specified concrete strength at time of transfer
$= \dfrac{5}{1 + f'_{ci}}$

f_{pi} = prestressing steel stress immediately prior to transfer (kips/in^2)

H = the average annual ambient relative humidity (%)

Δf_{pR} = an estimate of relaxation loss taken as 2.4 kips/in^2 for low relaxation strand; 10.0 kips/in^2 for stress relieved strand; and in accordance with manufacturers' recommendations for other types of strand (kips/in^2)

The first term in AASHTO Eq. 5.9.5.3-1 corresponds to creep losses, the second term to shrinkage losses, and the third to relaxation losses. The elastic losses at transfer must be added to the time-dependent losses to determine the total losses.

Example 8.25

A precast pretensioned girder with composite deck has the following properties.

$$A_g = 800 \text{ in}^2$$

$$A_{ps} = 5.36 \text{ in}^2$$

$$f'_{ci} = 4 \text{ kips/in}^2$$

$$f_{pi} = 190 \text{ kips/in}^2$$

The girder is pretensioned with low-relaxation strand and the average annual ambient relative humidity is 70%. Determine the time-dependent prestress losses.

Solution

The correction factor for relative humidity of the ambient air is

$$\gamma_h = 1.7 - 0.01H$$

$$= 1.7 - (0.01)(70\%)$$

$$= 1.0$$

The correction factor for specified concrete strength at the time of transfer is

$$\gamma_{st} = \frac{5}{1 + f'_{ci}} = \frac{5 \ \frac{\text{kips}}{\text{in}^2}}{1 \ \frac{\text{kip}}{\text{in}^2} + 4 \ \frac{\text{kips}}{\text{in}^2}}$$

$$= 1.0$$

The long-term prestress loss is given by AASHTO Eq. 5.9.5.3-1 as

$$\Delta f_{pLT} = 10.0\gamma_h\gamma_{st}f_{pi}\frac{A_{ps}}{A_g} + 12.0\gamma_h\gamma_{st} + \Delta f_{pR}$$

$$= (10.0)(1.0)(1.0)\left(190 \ \frac{\text{kips}}{\text{in}^2}\right)\left(\frac{5.36 \ \text{in}^2}{800 \ \text{in}^2}\right)$$

$$\quad + \left(12.0 \ \frac{\text{kips}}{\text{in}^2}\right)(1.0)(1.0) + 2.4 \ \frac{\text{kips}}{\text{in}^2}$$

$$= 27.1 \ \text{kips/in}^2$$

4. STRUCTURAL STEEL DESIGN

Design for Flexure

Nomenclature

A_{rb}	area of bottom layer of longitudinal reinforcement within the effective concrete deck width	in^2
A_{rt}	area of top layer of longitudinal reinforcement within the effective concrete deck width	in^2
A_s	cross-sectional area of structural steel	in^2
b_c	width of the compression flange of the steel beam	in
b_s	effective concrete flange width	in or ft
b_t	width of the tension flange of the steel beam	in
C	compressive force in slab at ultimate load	kips
d	depth of steel beam	in
d_c	distance from the plastic neutral axis to the midthickness of the compression flange used to compute the plastic moment	in
d_{rb}	distance from the plastic neutral axis to the centerline of the bottom layer of longitudinal concrete deck reinforcement used to compute the plastic moment	in
d_{rt}	distance from the plastic neutral axis to the centerline of the top layer of longitudinal concrete deck reinforcement used to compute the plastic moment	in
d_t	distance from the plastic neutral axis to the midthickness of the tension flange used to compute the plastic moment	in
d_w	distance from the plastic neutral axis to the midthickness of the web used to compute the plastic moment	in
D	depth of the web of the steel beam	in
D_p	distance from the top of slab to the plastic neutral axis	in
D_t	total depth of composite section	in
f'_c	specified compressive strength of the concrete	kips/in^2

F_y	specified minimum yield strength of the structural steel section	kips/in^2
F_{yc}	specified yield strength of the compression flange of the steel beam	kips/in^2
F_{yrb}	specified yield strength of the bottom layer of longitudinal deck reinforcement	kips/in^2
F_{yrt}	specified yield strength of the top layer of longitudinal deck reinforcement	kips/in^2
F_{yt}	specified yield strength of the tension flange of the steel beam	kips/in^2
F_{yw}	specified yield strength of the web of the steel beam	kips/in^2
L	span length	ft
M_n	nominal flexural resistance of the composite beam	in-kips or ft-kips
M_p	full plastic moment of the member	in-kips or ft-kips
M_u	moment due to factored loads	in-kips or ft-kips
P_c	plastic force in the compression flange of the steel beam	kips
P_{rb}	plastic force in the bottom layer of longitudinal deck reinforcement	kips
P_{rt}	plastic force in the top layer of longitudinal deck reinforcement	kips
P_s	plastic force in the full depth of the concrete deck	kips
P_t	plastic force in the tension flange of the steel beam	kips
P_w	plastic force in the web of the steel beam	kips
S	beam spacing	ft or in
t_s	slab thickness	in
y	moment arm between centroids of tensile force and compressive force	in

Symbols

γ	compression zone factor	–
ϕ_f	resistance factor for flexure	–

Strength Design Method

The strength design of a composite member is detailed in AASHTO Sec. 6.10.7 and AASHTO App. D6. As shown in Fig. 8.9, for positive bending with the plastic neutral axis (PNA) within the concrete slab, AASHTO Table D6.1-1 case V provides expressions for the depth of the plastic neutral axis and for the plastic moment of resistance. The depth of the plastic neutral axis is

$$D_p = t_s\left(\frac{P_{rb} + P_c + P_w + P_t - P_{rt}}{P_s}\right)$$

The plastic moment of resistance is

$$M_p = \frac{D_p^2 P_s}{2t_s} + P_{rb}d_{rb} + P_c d_c + P_w d_w + P_t d_t + P_{rt}d_{rt}$$

Figure 8.9 *Determination of M_p*
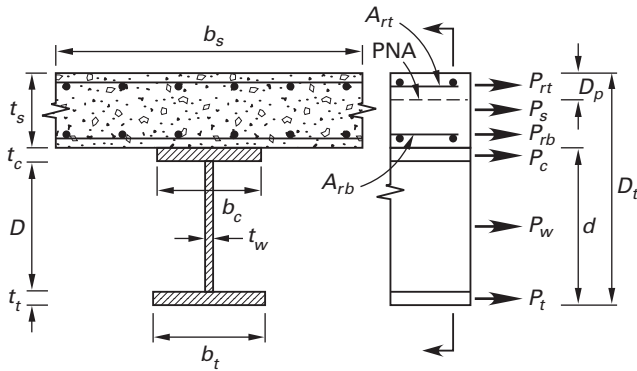
The plastic force in the bottom layer of longitudinal deck reinforcement is given by AASHTO Table D6.1-2 as

$$P_{rb} = F_{yrb}A_{rb}$$

The plastic force in the top layer of longitudinal deck reinforcement is given by AASHTO Table D6.1-2 as

$$P_{rt} = F_{yrt}A_{rt}$$

The plastic force in the top flange of the steel beam is given by AASHTO Table D6.1-2 as

$$P_c = F_{yc}b_ct_c$$

The plastic force in the bottom flange of the steel beam is given by AASHTO Table D6.1-2 as

$$P_t = F_{yt}b_tt_t$$

The plastic force in the web of the steel beam is given by AASHTO Table D6.1-2 as

$$P_w = F_{yw}Dt_w$$

The plastic compressive force in the full depth of the concrete deck is given by AASHTO Table D6.1-2 as

$$P_s = 0.85f'_cb_st_s$$

In the derivation of the expressions, concrete in tension is neglected. The plastic force in the portion of the concrete slab that is in compression is based on a magnitude of the compressive stress equal to $0.85f'_c$.

The forces in the longitudinal reinforcement in the slab may be conservatively neglected. The plastic moment of resistance of the composite section may then be determined as shown in Fig. 8.10.

For positive bending where the top flange of the steel beam is encased in concrete or anchored to the deck slab by shear connectors, the flange is considered continuously braced. Lateral bending stresses are then considered equal to zero. For this condition, at the strength

limit state, the moment due to factored loads shall satisfy the expression

$$M_u \le \phi_f M_n$$

The resistance factor for flexure is given by AASHTO Sec. 6.5.4.2 as

$$\phi_f = 1.0$$

For $D_p \le 0.1D_t$ the nominal flexural resistance of the section is given by AASHTO Eq. 6.10.7.1.2-1 as

$$M_n = M_p$$

For $D_p > 0.1D_t$ the nominal flexural resistance of the section is given by AASHTO Eq. 6.10.7.1.2-2 as

$$M_n = M_p\left(1.07 - \frac{0.7D_p}{D_t}\right)$$

For the composite beam shown in Fig. 8.10, the effective width of the concrete slab is given by AASHTO Sec. 4.6.2.6.1 as the tributary width, which is

$$b_s = S$$

Figure 8.10 *Fully Composite Beam Ultimate Strength*
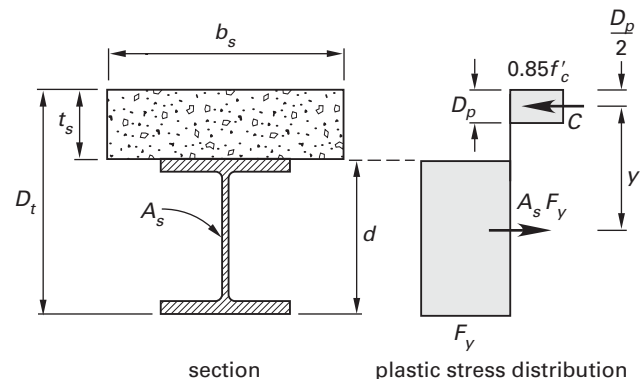
Figure 8.10 shows conditions at the strength limit state when the depth of the compression zone at the ultimate load is less than the depth of the slab. In accordance with AASHTO Sec. D6.1, the depth of the stress block is given by

$$D_p = \frac{F_yA_s}{0.85f'_cb_s}$$

The distance between the centroids of the compressive force in the slab and the tensile force in the girder is

$$y = \frac{d}{2} + t_s - \frac{D_p}{2}$$

The plastic moment capacity in bending is given by

$$M_p = F_y A_s y$$

Example 8.26

The simply supported composite beam shown in the following illustration consists of an 8 in concrete slab cast on W36 × 194 grade A50 steel beams with adequate shear connection. The beams are spaced at 8 ft centers and span 100 ft; the slab consists of 4.5 kips/in^2 normal weight concrete. The bending moment, M_{DC}, due to dead load imposed on the composite section is 250 ft-kips. The bending moment, M_L, due to live load plus impact is 1354 ft-kips. Determine the maximum factored applied moment and the maximum flexural strength of the composite section in bending.

section
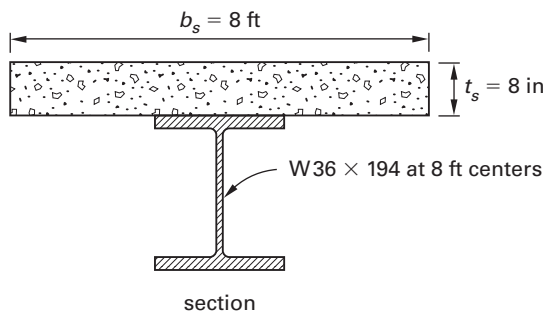
Solution

The W36 × 194 girder has the properties tabulated as follows.

A_s	d	t_w	t_f	D	I
57 in^2	36.5 in	0.765 in	1.26 in	33.98 in	12,100 in^4

The effective width of the concrete slab is given by AASHTO Sec. 4.6.2.6.1 as the tributary width, which is

$$\begin{aligned} b_s &= S \\ &= (8 \text{ ft})\left(12 \; \frac{\text{in}}{\text{ft}}\right) \\ &= 96 \text{ in} \end{aligned}$$

Assuming that the compression zone is contained within the slab, the depth of the stress block is given by AASHTO Sec. D6.1 as

$$\begin{aligned} D_p &= \frac{F_y A_s}{0.85 f'_c b_s} \\ &= \frac{\left(50 \; \frac{\text{kips}}{\text{in}^2}\right)(57 \text{ in}^2)}{(0.85)\left(4.5 \; \frac{\text{kips}}{\text{in}^2}\right)(96 \text{ in})} \\ &= 7.76 \text{ in} \end{aligned}$$

Hence, the compression zone is located within the slab. The distance between the centroids of the compressive force in the slab and the tensile force in the girder is

$$\begin{aligned} y &= \frac{d}{2} + t_s - \frac{D_p}{2} \\ &= \frac{36.5 \text{ in}}{2} + 8 \text{ in} - \frac{7.76 \text{ in}}{2} \\ &= 22.37 \text{ in} \end{aligned}$$

The plastic moment capacity in bending is given by

$$\begin{aligned} M_p &= F_y A_s y \\ &= \frac{\left(50 \; \frac{\text{kips}}{\text{in}^2}\right)(57 \text{ in}^2)(22.37 \text{ in})}{12 \; \frac{\text{in}}{\text{ft}}} \\ &= 5313 \text{ ft-kips} \\ \frac{D_p}{D_t} &= \frac{7.76 \text{ in}}{44.5 \text{ in}} = 0.174 \\ &> 0.1 \end{aligned}$$

Therefore, the nominal flexural resistance of the section is given by AASHTO Eq. 6.10.7.1.2-2 as

$$\begin{aligned} M_n &= \left(1.07 - \frac{0.7 D_p}{D_t}\right) M_p \\ &= \left(1.07 - (0.7)\left(\frac{7.76 \text{ in}}{44.5 \text{ in}}\right)\right)(5313 \text{ ft-kips}) \\ &= 5038 \text{ ft-kips} \end{aligned}$$

At midspan, the self-weight moment is

$$\begin{aligned} M_g &= \frac{wL^2}{8} \\ &= \frac{\left(0.194 \; \frac{\text{kip}}{\text{ft}}\right)(100 \text{ ft})^2}{8} \\ &= 243 \text{ ft-kips} \end{aligned}$$

The moment due to the weight of the deck slab is

$$\begin{aligned} M_s = \frac{wL^2}{8} &= \frac{\left(150 \; \frac{\text{lbf}}{\text{ft}^3}\right)\left(\dfrac{8 \text{ in}}{12 \; \frac{\text{in}}{\text{ft}}}\right)(8 \text{ ft})(100 \text{ ft})^2}{(8)\left(1000 \; \frac{\text{lbf}}{\text{kip}}\right)} \\ &= 1000 \text{ ft-kips} \end{aligned}$$

Bridges

The total dead load moment on the composite section is

$$M_D = M_g + M_S + M_{DC}$$
$$= 243 \text{ ft-kips} + 1000 \text{ ft-kips} + 250 \text{ ft-kips}$$
$$= 1493 \text{ ft-kips}$$

The live load moment plus impact is

$$M_L = 1354 \text{ ft-kips}$$

The factored applied moment is given by AASHTO Eq. 3.4.1-1 as

$$M_u = \gamma_p M_D + \gamma_{LL+IM} M_L$$
$$= (1.25)(1493 \text{ ft-kips}) + (1.75)(1354 \text{ ft-kips})$$
$$= 4236 \text{ ft-kips}$$

Design for Shear

Nomenclature

C	web buckling coefficient	–
D	depth of the web of the steel beam	in
E	modulus of elasticity of the steel beam	kips/in^2
t_w	web thickness	in
V_n	nominal shear strength	kips
V_p	shear yielding strength of the web	kips
V_r	factored shear resistance	kips
V_u	factored applied shear force	kips

Symbols

γ	compression zone factor	–
ϕ_v	resistance factor for shear	–

Strength Design Method

AASHTO Sec. 6.10.9.2 defines the nominal shear strength of a girder with unstiffened web as

$$V_n = C V_p \qquad [\textit{AASHTO 6.10.9.2-1}]$$

The plastic shear force of the web is given by

$$V_p = 0.58 F_{yw} D t_w \qquad [\textit{AASHTO 6.10.9.2-2}]$$

For values of $D/t_w \le 1.12\sqrt{5E/F_{yw}}$, the web buckling coefficient is defined by AASHTO Eq. 6.10.9.3.2-4 as

$$C = 1.0$$

The factored shear resistance is given by AASHTO Eq. 6.12.1.2.3a-1 as

$$V_r = \phi_v V_n$$

The resistance factor for shear is given by AASHTO Sec. 6.5.4.2 as

$$\phi_v = 1.0$$

Example 8.27

The simply supported composite beam of Ex. 8.26 is subjected to a support reaction, V_L, of 58 kips due to live load plus impact and a support reaction, V_{DC}, of 10 kips due to dead load imposed on the composite section. Determine whether the section is adequate.

Solution

The web buckling coefficient for the composite section is

$$C = 1.0$$

The nominal shear strength of the composite section is given by AASHTO Sec. 6.10.9.2 as

$$V_n = C V_p = 0.58 F_{yw} D t_w$$
$$= (0.58)\left(50 \ \frac{\text{kips}}{\text{in}^2}\right)(33.97 \text{ in})(0.765 \text{ in})$$
$$= 754 \text{ kips}$$

The factored shear resistance is

$$V_r = \phi_v V_n$$
$$= (1.0)(754 \text{ kips})$$
$$= 754 \text{ kips}$$

The support reaction due to the self-weight of the girder is

$$V_g = \frac{w_g L}{2} = \frac{\left(0.194 \ \dfrac{\text{kip}}{\text{ft}}\right)(100 \text{ ft})}{2}$$
$$= 9.7 \text{ kips}$$

The support reaction due to the weight of the slab is

$$V_S = \frac{w_S L}{2}$$
$$= \left(0.150 \ \frac{\text{kip}}{\text{ft}^3}\right)\left(\frac{64 \text{ in}}{12 \ \dfrac{\text{in}}{\text{ft}}}\right)(50 \text{ ft})$$
$$= 40 \text{ kips}$$

The total dead load support reaction is

$$V_D = V_g + V_S + V_{DC}$$
$$= 9.7 \text{ kips} + 40 \text{ kips} + 10 \text{ kips}$$
$$= 59.7 \text{ kips}$$

The live load reaction plus impact is

$$V_L = 58 \text{ kips}$$

The factored applied reaction is given by AASHTO Eq. 3.4.1-1 as

$$V_u = \gamma_p V_D + \gamma_{LL+IM} V_L$$
$$= (1.25)(59.7 \text{ kips}) + (1.75)(58 \text{ kips})$$
$$= 176 \text{ kips}$$
$$< V_r \quad [\text{The section is adequate.}]$$

Shear Connection

Nomenclature

A_{ct}	transformed area of concrete slab	in^2
A_s	area of steel beam	in^2
A_{sc}	cross-sectional area of stud shear connector	in^2
b_s	effective width of the concrete deck	in
b_t	transformed width of concrete slab	in
d	diameter of stud shear connector	in
D	depth of the web of the steel beam	in
E	modulus of elasticity of the steel beam	kips/in^2
E_c	modulus of elasticity of concrete	kips/in^2
f'_c	specified 28 day compressive strength of concrete	kips/in^2
F_u	specified tensile strength of the steel beam	kips/in^2
F_y	specified yield strength of the steel beam	kips/in^2
H	stud height	in
I	impact factor	–
I	moment of inertia of transformed composite section	in^4
L	span length	ft
n	modular ratio	–
n	number of shear connectors between point of maximum positive moment and point of zero moment	–
n	number of shear connectors in a cross section	–
N	number of cycles	–
p	connector spacing	in
P	total shear force at interface at ultimate limit state	kips
Q	moment of transformed compressive concrete area about neutral axis	in^3
S_u	ultimate strength of shear connector	lbf
V_f	range of shear force due to live load plus impact	kips
V_r	factored shear resistance	kips
V_{sr}	range of horizontal shear at interface	kips/in
w	unit weight of concrete	lbf/ft^3
y'	distance from slab center to neutral axis of transformed composite	in
Z_r	allowable range of shear for a welded stud	lbf

Symbols

α	stress cycle factor	–
γ	load factor	–
ϕ	reduction factor	–
ϕ_v	resistance factor for shear	–

General

Shear connectors are designed for fatigue and are checked for ultimate strength. Fatigue stresses are caused by the range of shear produced on the connector by live load plus impact and are calculated by using elastic design principles. The ultimate strength of the connectors must be adequate to develop the lesser of the strength of the steel girder or the ultimate strength of the concrete slab. The pitch of the connectors is determined from the fatigue limit state. The total number of connectors required is determined from the strength limit state.

Fatigue Strength

The elastic design properties of the composite section are determined by using the transformed width of the concrete slab. As shown in Fig. 8.11, the transformed width is given by

$$b_t = \frac{b_s}{n}$$

The value of the modular ratio for short-term loads is given by AASHTO Sec. C6.10.1.1.1b as

$$n = 10 \quad [\text{for } 2.4 \le f'_c < 2.9]$$
$$n = 9 \quad [\text{for } 2.9 \le f'_c < 3.6]$$
$$n = 8 \quad [\text{for } 3.6 \le f'_c < 4.6]$$
$$n = 7 \quad [\text{for } 4.6 \le f'_c < 6.0]$$
$$n = 6 \quad [\text{for } 6.0 \le f'_c]$$

The transformed area of the concrete slab is

$$A_{ct} = b_t t_s$$

The statical moment of the transformed concrete area about the neutral axis of the composite section is

$$Q = y' A_{ct}$$

For a straight girder, the range of horizontal shear at the interface is given by AASHTO Eq. 6.10.10.1.2-3 as

$$V_{sr} = \frac{V_f Q}{I}$$

The range of shear force, V_f, due to live load plus impact is the difference between the maximum and minimum applied shear under the fatigue load combination. For a ratio of height to diameter, H/d, of not less than 4, the allowable range of shear for a welded stud is given by AASHTO Sec. 6.10.10.2.

For a projected 75 year, single lane average daily truck traffic greater than or equal to 960 trucks per day, the fatigue I load combination is used. The fatigue shear resistance for infinite life is taken as

$$Z_r = 5.5d^2 \quad [\textit{AASHTO Sec. 6.10.10.2-1}]$$

Figure 8.11 *Composite Section Properties*

actual section

transformed section
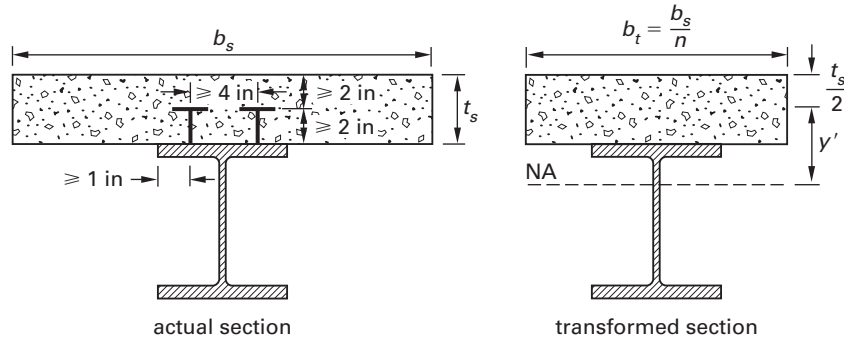
For other loading conditions, the fatigue II load combination is used and the fatigue shear resistance for finite life is taken as

$$Z_r = \alpha d^2 \quad \text{[AASHTO Sec. 6.10.10.2-2]}$$

$$\alpha = 34.5 - 4.28 \log N \quad \text{[AASHTO Sec. 6.10.10.2-3]}$$

The required connector pitch is given by AASHTO Eq. 6.10.10.1.2-1 as

$$p = \frac{nZ_r}{V_{sr}}$$
$$\leq 24 \text{ in}$$
$$\geq 6d$$

Example 8.28

For the composite beam of Ex. 8.27, determine the required spacing of $^3/_4$ in diameter stud shear connectors at the support. The fatigue I limit state applies, and the maximum shear force at the support due to the fatigue I vehicle is $V = 36$ kips, including impact.

Solution

The compressive strength of the concrete slab is 4.5 kips/in^2, and the corresponding modular ratio is

$$n = 8$$

The transformed area of the concrete slab is

$$A_{ct} = \frac{b_s t_s}{n} = \frac{(96 \text{ in})(8 \text{ in})}{8}$$
$$= 96 \text{ in}^2$$

The moment of inertia of the transformed section is derived as shown in the following table.

part	A (in^2)	y (in)	I (in^4)	Ay (in^3)	Ay^2 (in^4)
girder	57	18.25	12,100	1040	18,985
slab	96	40.49	512	3887	157,385
total	153	–	12,612	4927	176,370

The height of the neutral axis of the transformed section is

$$\overline{y} = \frac{\sum Ay}{\sum A} = \frac{4927 \text{ in}^3}{153 \text{ in}^2}$$
$$= 32.2 \text{ in}$$

The moment of inertia of the transformed section is

$$I = \sum I + \sum Ay^2 + \overline{y}^2 \sum A - 2\overline{y}\sum Ay$$
$$= 12,612 \text{ in}^4 + 176,370 \text{ in}^4 + (32.2 \text{ in})^2(153 \text{ in}^2)$$
$$\quad - (64.4 \text{ in})(4927 \text{ in}^3)$$
$$= 30,320 \text{ in}^4$$

The statical moment of the transformed slab about the neutral axis is

$$Q = y'A_{ct} = \left(d + \frac{t_s}{2} - \overline{y}\right)A_{ct}$$
$$= \left(36.5 \text{ in} + \frac{8 \text{ in}}{2} - 32.2 \text{ in}\right)(96 \text{ in}^2)$$
$$= 797 \text{ in}^3$$
$$\frac{Q}{I} = \frac{797 \text{ in}^3}{30,320 \text{ in}^4}$$
$$= 0.0263 \text{ in}^{-1}$$

The transformed section is shown in the following illustration.
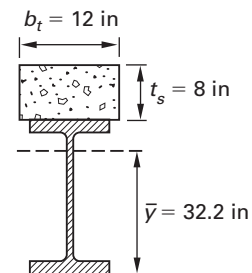
transformed composite section

The fatigue shear resistance for infinite life for a $^3/_4$ in diameter welded stud subjected to fatigue I loading is given by AASHTO Eq. 6.10.10.2-1 as

$$Z_r = 5.5d^2 = \left(5.5 \ \frac{\text{kips}}{\text{in}^2}\right)(0.75 \ \text{in})^2$$
$$= 3.09 \ \text{kips}$$

The maximum factored shear force at the support due to the fatigue I vehicle plus impact is

$$V_{\text{max}} = \gamma V = (1.5)(36 \ \text{kips})$$
$$= 54 \ \text{kips}$$

The minimum shear force at the support is

$$V_{\text{min}} = 0 \ \text{kips}$$

The range of shear force at the support for the fatigue I limit state is

$$V_f = V_{\text{max}} - V_{\text{min}}$$
$$= 54 \ \text{kips} - 0 \ \text{kips}$$
$$= 54 \ \text{kips}$$

The range of horizontal shear at the support at the interface is given by AASHTO Eq. 6.10.10.1.2-3 as

$$V_{sr} = \frac{V_f Q}{I} = (54 \ \text{kips})(0.0263 \ \text{in}^{-1})$$
$$= 1.42 \ \text{kips/in}$$

With three studs per row, the required stud spacing is

$$p = \frac{nZ_r}{V_{sr}} = \frac{(3)(3.09 \ \text{kips})}{1.42 \ \frac{\text{kips}}{\text{in}}}$$
$$= 6.5 \ \text{in}$$

Ultimate Strength

To provide adequate connection at the interface at ultimate load, the number of connectors required on each side of the point of maximum moment is given by AASHTO Sec. 6.10.10.4.1 as

$$n = \frac{P}{\phi_{sc}Q_n}$$

For a straight rolled section girder, the total shear force at the interface at the ultimate limit state is given in AASHTO Sec. 6.10.10.4.2 as the lesser of

$$P = A_s F_y \quad [\text{girder governs}]$$
$$P = 0.85f'_c b_s t_s \quad [\text{deck slab governs}]$$

The resistance factor for a shear connector is given by AASHTO Sec. 6.5.4.2 as

$$\phi_{sc} = 0.85$$

The nominal strength of a welded stud shear connector is given by AASHTO Eq. 6.10.10.4.3-1 as

$$Q_n = 0.5A_{sc}\sqrt{f'_c E_c}$$
$$\leq A_{sc}F_u$$

The modulus of elasticity of normal weight concrete is given by AASHTO Eq. C5.4.2.4-1 as

$$E_c = 1820\sqrt{f'_c}$$

Example 8.29

For the composite beam of Ex. 8.27, determine the required number of $^3/_4$ in diameter stud shear connectors to provide adequate connection at the ultimate load. The shear connectors have a tensile strength of $F_u = 60 \ \text{kips/in}^2$.

Solution

The compressive strength of the concrete slab is 4.5 kips/in^2, and the corresponding modulus of elasticity is given by AASHTO Eq. C5.4.2.4-1 as

$$E_c = 1820\sqrt{f'_c}$$
$$= 1820\sqrt{4.5 \ \frac{\text{kips}}{\text{in}^2}}$$
$$= 3860 \ \text{kips/in}^2$$

The ultimate strength of a $^3/_4$ in diameter welded stud shear connector is given by AASHTO Eq. 6.10.10.4.3-1 as

$$Q_n = 0.5A_{sc}\sqrt{f'_c E_c}$$
$$= (0.5)(0.44 \ \text{in}^2)\sqrt{\left(4.5 \ \frac{\text{kips}}{\text{in}^2}\right)\left(3860 \ \frac{\text{kips}}{\text{in}^2}\right)}$$
$$= 29 \ \text{kips}$$

Maximum strength is

$$Q_{n(\text{max})} = F_u A_{sc}$$
$$= (60)(0.44)$$
$$= 26.40 \ \text{kips} \quad [\text{governs}]$$

The nominal shear force at the interface is governed by the girder, and the total number of connectors required on the girder is

$$2n = \frac{2A_s F_y}{\phi_{sc}Q_n} = \frac{(2)(57 \ \text{in}^2)\left(50 \ \frac{\text{kips}}{\text{in}^2}\right)}{(0.85)(26.40 \ \text{kips})}$$
$$= 254$$

5. WOOD STRUCTURES

Basic Design Values and Adjustment Factors

Nomenclature

a	species parameter for volume factor	–
A	ratio of F_{bE} to F_b	–
b	breadth of rectangular bending member	in
C_d	deck factor	–
C_F	size factor for sawn lumber	–
C_{fu}	flat use factor	–
C_i	incising factor	–
C_{KF}	format conversion factor	–
C_L	beam stability factor	–
C_M	wet service factor	–
C_V	volume factor for structural glued laminated timber	–
C_λ	time effect factor	–
d	depth of member	in
E, E_o	adjusted and reference modulus of elasticity	kips/in^2
F_b	reference bending design value multiplied by all applicable adjustment factors	kips/in^2
F_{bE}	critical buckling design value for bending members	kips/in^2
F_{bo}	reference bending design value	kips/in^2
F_v	adjusted design value of wood in shear	kips/in^2
F_{vo}	reference design value of wood in shear	kips/in^2
K_{bE}	Euler buckling coefficient for beams	–
L	span length of bending member	ft or in
L_e	effective bending member length	ft or in
L_u	laterally unsupported bending member length	ft or in
M_n	nominal flexural resistance	in-kips
M_r	factored flexural resistance	in-kips
R_B	slenderness ratio of bending member	–
S	section modulus	in^3
V_n	nominal shear resistance	kips
V_r	factored shear resistance	kips

Symbols

ϕ	resistance factor	–

Reference Design Values

The reference design values for sawn lumber are given in AASHTO Table 8.4.1.1.4-1, Table 8.4.1.1.4-2, and Table 8.4.1.1.4-3. The reference design values for glued laminated timber are given in AASHTO Table 8.4.1.2.3-1 and Table 8.4.1.2.3-2. These tabulated design values are applicable to normal conditions of use as defined in AASHTO Sec. C8.4.1. For other conditions of use, the tabulated values are multiplied by adjustment factors, specified in AASHTO Sec. 8.4.4, to determine the corresponding adjusted design values. In accordance with AASHTO Sec. 8.4.4.1, the adjusted design value in bending is

$$F_b = F_{bo}C_{KF}C_M(C_F \text{ or } C_V)C_{fu}C_iC_dC_\lambda$$

[*AASHTO 8.4.4.1-1*]

Adjustment Factors

The *time effect factor*, C_λ, given in AASHTO Sec. 8.4.4.9 is applicable to all reference design values with the exception of the modulus of elasticity. Values of the time effect factor are given in Table 8.4 [AASHTO Table 8.4.4.9-1].

Table 8.4 Time Effect Factor, C_λ

limit state	C_λ
strength I	0.8
strength II	1.0
strength III	1.0
strength IV (permanent)	0.6
extreme event I	1.0

The *wet service factor*, C_M, given in AASHTO Table 8.4.4.3-1, is applicable to sawn lumber when the moisture content exceeds 19%. Values of the wet service factor are given in Table 8.5. For a moisture content $\leq 19\%$, $C_M = 1.0$.

Table 8.5 Wet Service Factor, C_M, for Sawn Lumber

design function	$F_{bo}C_F > 1.15$	F_{vo}
members not exceeding 4 in thickness	0.85	0.97
members exceeding 4 in thickness	1.00	1.00

When the moisture content of a glued laminated member exceeds 16%, the adjustment factor given in AASHTO Table 8.4.4.3-2 is applicable. Values of the wet service factor are given in Table 8.6. For a moisture content $\leq 16\%$, $C_M = 1.0$.

Table 8.6 Wet Service Factor, C_M, for Glued Laminated Members

design function	F_{bo}	F_{vo}
wet service factor	0.80	0.875

The *beam stability factor* is applicable to the tabulated bending reference design value for sawn lumber and glued laminated members. For glued laminated members, C_L is not applied simultaneously with the volume factor, C_V, and the lesser of these two values is applicable.

The beam stability factor is given by AASHTO Sec. 8.6.2 as

$$C_L = \frac{1.0 + A}{1.9} - \sqrt{\frac{(1.0 + A)^2}{3.61} - \frac{A}{0.95}}$$

[*AASHTO 8.6.2-2*]

The variables are defined as

$$A = \frac{F_{bE}}{F_b} \qquad \text{[AASHTO 8.6.2-3]}$$

$F_b = $ reference bending design value multi-
plied by all applicable adjustment
factors

$$= F_{bo}C_{KF}C_MC_FC_{fu}C_iC_dC_\lambda \quad \begin{bmatrix} C_F \text{ applies only} \\ \text{to visually graded} \\ \text{sawn lumber} \end{bmatrix}$$

$F_{bE} = $ critical buckling design value

$$= \frac{K_{bE}E}{R_B^2} \qquad \text{[AASHTO 8.6.2-4]}$$

$K_{bE} = $ Euler buckling coefficient

$\qquad = 1.10 \quad$ [for glued laminated timber]

$\qquad = 0.76 \quad$ [for visually graded lumber]

$E = $ adjusted modulus of elasticity

$$= E_oC_MC_i \qquad \text{[AASHTO 8.4.4.1-6]}$$

$R_B = $ slenderness ratio

$$= \sqrt{\frac{L_ed}{b^2}} \qquad \text{[AASHTO 8.6.2-5]}$$

$$\leq 50$$

The term L_e is the effective length of a bending member and is defined in AASHTO Sec. 8.6.2 and tabulated in Table 8.7.

Table 8.7 Effective Length, L_e

member dimensions	L_e
$\frac{L_u}{d} < 7$	$2.06L_u$
$7 \leq \frac{L_u}{d} \leq 14.3$	$1.63L_u + 3d$
$\frac{L_u}{d} > 14.3$	$1.84L_u$

In accordance with AASHTO Sec. 8.6.2, $C_L = 1.0$ when

- $\frac{d}{b} \leq 1.0$
- the compression edge is continuously restrained

The *size factor*, C_F, is applicable to sawn lumber and to glued laminated members with load applied parallel to the wide face of the laminations. For sawn lumber 2–4 in thick, values of the size factor are given in AASHTO Table 8.4.4.4-1. Reference design values for southern pine and mixed southern pine dimension lumber are size-adjusted and no further adjustment for size is required. For members exceeding 12 in depth, the size factor is

$$C_F = \left(\frac{12}{d}\right)^{1/9} \qquad \text{[AASHTO 8.4.4.4-2]}$$

The *volume factor*, C_V, is applicable to the reference design value for bending of glued laminated members

and is not applied simultaneously with the beam stability factor, C_L; the lesser of these two factors is applicable.

The volume factor is defined in AASHTO Sec. 8.4.4.5 as

$$C_V = \left(\frac{1291.5}{bdL}\right)^a \qquad \text{[AASHTO 8.4.4.5-1]}$$

$$\leq 1.0$$

The variables are defined as

$L = $ length of beam between points
of zero moment, ft

$b = $ beam width, in

$d = $ beam depth, in

$a = 0.05 \quad$ [for southern pine]

$\quad = 0.10 \quad$ [for all other species]

The *flat-use factor*, C_{fu}, is applicable to dimension lumber with the load applied to the wide face. Values of the flat-use factor are given in AASHTO Table 8.4.4.6-1. Flat-use factors for glued laminated members, with the load applied parallel to the wide faces of the laminations, are given in AASHTO Table 8.4.4.6-2. The flat-use factor is not applied to dimension lumber graded as decking, as the design values already incorporate the appropriate factor.

Values of the *incising factor*, C_i, for a prescribed incising pattern are given in AASHTO Table 8.4.4.7-1. The prescribed pattern consists of incisions parallel to the grain a maximum depth of 0.4 in, a maximum length of 3/8 in, and a density of incisions of up to $1100/\text{ft}^2$.

The *deck factor*, C_d, is applied to mechanically laminated decks. Values of the deck factor for stressed wood, spike-laminated, and nail-laminated decks are given in AASHTO Table 8.4.4.8-1.

The format conversion factor, C_{KF}, is used to ensure that load and resistance factor design will result in the same size members as allowable stress design. AASHTO Sec. 8.4.4.2 gives the format conversion factor for all loading conditions except compression perpendicular to the grain, as

$$C_{KF} = \frac{2.5}{\phi}$$

Resistance factors are tabulated in AASHTO Sec. 8.5.2.2 and these include

$$\phi = 0.85 \text{ for flexure}$$

$$\phi = 0.75 \text{ for shear}$$

The factored flexural resistance is given by AASHTO Eq. 8.6.1-1 as

$$M_r = \phi M_n$$

Bridges

The nominal flexural resistance is given by AASHTO Eq. 8.6.2-1 as

$$M_n = F_b S C_L$$

For shear, the corresponding values are

$$V_r = \phi V_n \qquad \text{[AASHTO 8.7-1]}$$

$$V_n = \frac{F_v b d}{1.5} \qquad \text{[AASHTO 8.7-2]}$$

Example 8.30

The bridge superstructure shown in the following illustration is simply supported over a span of 35 ft. The moisture content exceeds 16%. Determine the nominal resistance values in bending and shear for the strength I limit state for the glued laminated girders, with combination 24F-V4 Douglas fir.

3 in asphalt wearing surface

6 in glued laminated panel

$8\frac{3}{4}$ in $\times$ 36 in glued laminated girder at 5.5 ft spacing
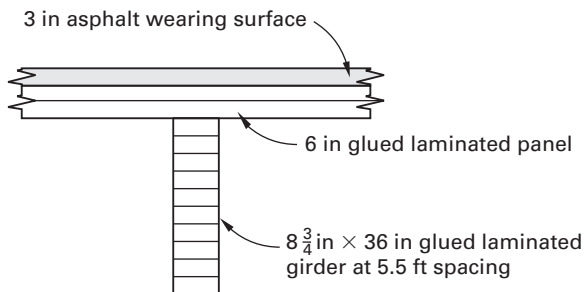
Solution

The reference design bending stress for a Douglas fir 24F-V4 glued laminated stringer is obtained from AASHTO Table 8.4.1.2.3-1 and is

$$F_{bo} = 2.4 \text{ kips/in}^2$$

The applicable adjustment factors for bending stress are as follows.

C_λ = time effect factor for the strength I limit state from Table 8.4.4.9-1

$\quad = 0.8$

C_M = wet service factor from AASHTO Table 8.4.4.3-2

$\quad = 0.80$

C_L = beam stability factor from AASHTO Sec. 8.6.2

$\quad = 1.0 \quad \begin{bmatrix} \text{compression face of the girder} \\ \text{fully supported laterally} \end{bmatrix}$

C_V = volume factor given by AASHTO Eq. 8.4.4.5-1

$\quad = \left(\dfrac{1291.5}{bdL}\right)^a$

$\quad = \left(\dfrac{1291.5 \text{ in}^2\text{-ft}}{(8.75 \text{ in})(36.0 \text{ in})(35 \text{ ft})}\right)^{0.10}$

$\quad = 0.81 \quad \text{[governs]}$

$\quad < C_L$

In accordance with AASHTO Eq. 8.4.4.1-1, the adjusted design bending stress is

$$\begin{aligned} F_b &= F_{bo} C_\lambda C_M C_V C_{KF} \\ &= \left(2.4 \ \frac{\text{kips}}{\text{in}^2}\right)(0.8)(0.80)(0.81)\left(\frac{2.5}{0.85}\right) \\ &= 3.66 \text{ kips/in}^2 \end{aligned}$$

The nominal flexural resistance is given by AASHTO Eq. 8.6.2-1 as

$$\begin{aligned} M_n &= F_b S C_L = (3.66 \text{ kips})(1890 \text{ in}^3)(1.0) \\ &= 6917 \text{ in-kips} \end{aligned}$$

The reference design shear stress for a Douglas fir 24F-V4 glued laminated stringer is obtained from AASHTO Table 8.4.1.2.3-1 and is

$$F_{vo} = 0.265 \text{ kip/in}^2$$

The applicable adjustment factors for shear stress are as follows.

C_λ = time effect factor for the strength I limit state from Table 8.4.4.9-1

$\quad = 0.8$

C_M = wet service factor from AASHTO Table 8.4.4.3-2

$\quad = 0.875$

In accordance with AASHTO Eq. 8.4.4.1-2, the adjusted design shear stress is

$$\begin{aligned} F_v &= F_{vo} C_\lambda C_M C_{KF} \\ &= \left(0.265 \ \frac{\text{kip}}{\text{in}^2}\right)(0.8)(0.875)\left(\frac{2.5}{0.75}\right) \\ &= 0.62 \text{ kip/in}^2 \end{aligned}$$

Design Requirements for Flexure

AASHTO Sec. 8.6.2 specifies that when the depth of a beam does not exceed its breadth or when continuous lateral restraint is provided to the compression edge of a beam with the ends restrained against rotation, the beam stability factor $C_L = 1.0$. For other situations, the value of C_L is calculated in accordance with AASHTO Sec. 8.6.2, and the effective span length, L_e, is determined in accordance with AASHTO Sec. 8.6.2. For visually graded sawn lumber, both the stability factor, C_L, and the size factor, C_F, must be considered concurrently. For glued laminated members, both the stability factor, C_L, and the volume factor, C_V, must be determined. Only the lesser of these two factors is applicable in determining the allowable design value in bending. In accordance with AASHTO Sec. 3.6.2.3, impact need not be considered in timber structures.

Design Requirements for Shear

In accordance with AASHTO Sec. 8.7, shear shall be investigated at a distance from the support equal to the depth of the beam. The governing *shear force, V, for vehicle loads* is determined by placing the live load to produce the maximum shear at a distance from the support given by the lesser of

- $3d$

- $L/4$

In accordance with AASHTO Sec. 3.6.2.3, impact need not be considered in timber structures.

6. SEISMIC DESIGN

Nomenclature

A_S	peak seismic ground acceleration coefficient modified by zero period site factor from AASHTO 3.10.4.2	–
C_{sm}	seismic response coefficient specified in AASHTO Sec. 3.10.4.2	–
D	dead load applied to a structural element	lbf or kips
F_a	site factor for short-period range of acceleration response spectrum from AASHTO Sec. 3.10.3.2	–
F_{pga}	site factor at zero-period on acceleration response spectrum from AASHTO Sec. 3.10.3.2	–
F_v	site factor for long-period range of acceleration response spectrum from AASHTO Sec. 3.10.3.2	–
g	gravitational acceleration, 32.2	ft/sec^2
IC	importance category	–
K	total lateral stiffness of bridge	lbf/in or kips/in
L	length of bridge deck	ft
M_P	primary moment	ft-kips
N	minimum support length for girders	in
$p_e(x)$	intensity of the equivalent static seismic loading used to calculate the period in AASHTO Sec. C4.7.4.3.2b	lbf/in or kips/in
p_o	assumed uniform loading used to calculate the period in AASHTO Sec. C.4.7.4.3.2b	lbf/ft or kips/ft
PGA	peak seismic ground acceleration coefficient on rock (site class B) from AASHTO Sec. 3.10.2.1	–
R	response modification factor from AASHTO Sec. 3.10.7.1	–
S_1	horizontal response spectral acceleration coefficient at 1.0 sec period on rock (site class B) from AASHTO Sec. 3.10.2.1	–
S_{D1}	horizontal response spectral acceleration coefficient at 1.0 sec modified by long-period site factor from AASHTO Sec. 3.10.4.2	–
S_{DS}	horizontal response spectral acceleration coefficient at 0.2 sec period modified by short-period site factor from AASHTO Sec. 3.10.4.2	–

S_S	horizontal response spectral acceleration coefficient at 0.2 sec period on rock (site class B) from AASHTO Sec. 3.10.2.1	–
SPZ	seismic performance zone	–
SC	site class from AASHTO Sec. 3.10.3.1	–
T_0	reference period used to define shape of acceleration response spectrum from AASHTO Sec. 3.10.4.2	–
T_m	fundamental period of vibration, defined in AASHTO Sec. C4.7.4.3.2b	sec
T_S	corner period at which acceleration response spectrum changes from being independent of period to being inversely proportional to period from AASHTO Sec. 3.10.4.2	sec
$v_e(x)$	static displacement profile resulting from applied load p_e used in AASHTO Sec. 4.7.4.3.2c	in
$v_s(x)$	static displacement profile resulting from applied load p_o used in AASHTO Sec. C4.7.4.3.2b	in
$w(x)$	dead weight of bridge superstructure and tributary substructure per unit length	lbf/ft or kips/ft
W	total weight of bridge superstructure and tributary substructure	lbf or kips

Symbols

α	coefficient used to calculate period of the bridge in AASHTO Sec. C4.7.4.3.2b	ft^2
β	coefficient used to calculate period of the bridge in AASHTO Sec. C4.7.4.3.2b	ft-kips
γ	coefficient used to calculate period of the bridge in AASHTO Sec. C4.7.4.3.2b	ft^2-kips

Analysis Procedures

To determine the seismic response of the structure, several factors must be considered. These factors include the ground motion parameters, site class, fundamental period, and response modification factors. Selection of the design procedure depends on the type of bridge, the importance category, and the seismic zone. Four analysis procedures[14,15] are presented in AASHTO Sec. 4.7.4.3.1 and are shown in Table 8.8.

Table 8.8 Analysis Procedures

procedure	method
UL	uniform load elastic
SM	single-mode elastic
MM	multimode elastic
TH	time history

The uniform load and single-mode procedures both assume that the seismic response of a bridge can be represented by a single mode of vibration and are suitable for hand computation. The multimode and time history procedures account for higher modes of vibration and require analysis by computer.

Bridges

Acceleration Coefficients

The acceleration coefficients PGA, S_S, and S_1 are defined in AASHTO Sec. 3.10.4.2, and they are shown in AASHTO Fig. 3.10.2.1-1 to Fig. 3.10.2.1-21. These are an estimate of the site-dependent design ground acceleration expressed as a percentage of the gravity constant, g. The acceleration coefficients correspond to ground acceleration values with a recurrence interval of 1000 yr, which gives a 7% probability of being exceeded in a 75 yr period. This is termed the design earthquake.

Example 8.31

The two-span bridge shown in the following illustration is located at 33.70° north and −117.50° west on a nonessential route. The central circular column is fixed at the top and bottom. The soil profile at the site consists of a stiff soil with a shear wave velocity of 700 ft/sec. The relevant criteria are (a) column moment of inertia $I_c = 60$ ft^4, (b) column modulus of elasticity $E_c = 450{,}000$ kips/ft^2, (c) column height $h_c = 30$ ft, (d) weight of the superstructure and tributary substructure $w = 10$ kips/ft, (e) superstructure moment of inertia $I_s = 4000$ ft^4, and (f) superstructure modulus of elasticity $E_s = 450{,}000$ kips/ft^2. Determine the applicable acceleration coefficients.
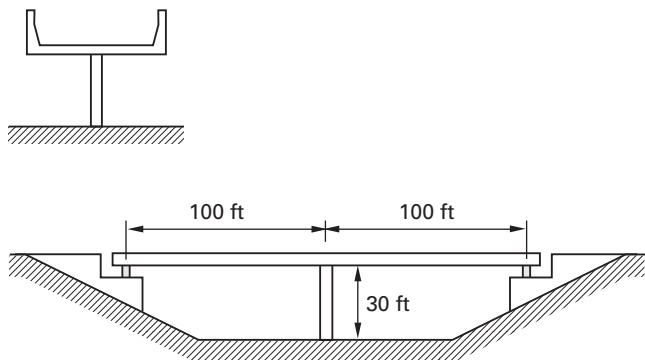
Solution

From AASHTO Fig. 3.10.2.1-4 to Fig. 3.10.2.1-6, the applicable acceleration coefficients are

$$PGA = 0.61$$
$$S_S = 1.45$$
$$S_1 = 0.52$$

Importance Category

The importance category is defined in AASHTO Sec. 3.10.5, and three categories are specified: critical bridges, essential bridges, and other bridges. The importance category of a bridge is determined on the basis of social and security requirements. An importance category of *critical* is assigned to bridges that must remain

functional immediately after a 2500 year return period earthquake. An importance category of *essential* is assigned to bridges that must remain functional immediately after the design earthquake. An importance category of *other* is assigned to nonessential bridges.

Example 8.32

Determine the importance category for the bridge of Ex. 8.31.

Solution

From AASHTO Sec. 3.10.5, for a bridge on a nonessential route, the importance category is

$$IC = other$$

Site Class

Six soil profile types are identified in AASHTO Table 3.10.3.1-1. Table 8.9 gives a summary of the soil profile types.

Table 8.9 *Site Classes*

site class	soil profile name	shear wave velocity
A	hard rock	> 5000
B	rock	2500–5000
C	soft rock	1200–2500
D	stiff soil	600–1200
E	soft soil	< 600
F	(*)	(*)

Note: (*) consists of peat or high plasticity clay requiring a site-specific geotechnical investigation.

Example 8.33

Determine the site class for the bridge of Ex. 8.31.

Solution

From AASHTO Sec. 3.10.3.1 and Table 8.9, the relevant site class for a stiff soil is D.

Site Factors

Site factors are amplification factors applied to the ground accelerations and are a function of the site class. Site factor F_{pga} corresponds to PGA, F_a corresponds to S_S, and F_v corresponds to S_1. Site Class B is the reference site category and has a site factor of 1.0. The site factors generally increase as the soil profile becomes softer (in going from site class A to E). The factors also decrease as the ground motion level increases due to the nonlinear behavior of the soil. Site factors F_{pga}, F_a, and F_v are specified in AASHTO Table 3.10.3.2-1, Table 3.10.3.2-2, and Table 3.10.3.2-3 and are summarized in Table 8.10. Linear interpolation may be used to obtain intermediate values.

Table 8.10 Site Factors (F_{pga} corresponding to PGA; F_a corresponding to S_S; F_v corresponding to S_1)

site class	ground acceleration, PGA					ground acceleration, S_S					ground acceleration, S_1				
	≤ 0.1	0.2	0.3	0.4	≥ 0.5	≤ 0.25	0.50	0.75	1.00	≥ 1.25	≤ 0.1	0.2	0.3	0.4	≥ 0.5
A	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8
B	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
C	1.2	1.2	1.1	1.0	1.0	1.2	1.2	1.1	1.0	1.0	1.7	1.6	1.5	1.4	1.3
D	1.6	1.4	1.2	1.1	1.0	1.6	1.4	1.2	1.1	1.0	2.4	2.0	1.8	1.6	1.5
E	2.5	1.7	1.2	0.9	(*)	2.5	1.7	1.2	0.9	(*)	3.5	3.2	2.8	2.4	(*)
F	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)	(*)

Note: (*) Site-specific geotechnical investigation and dynamic site response analysis is required.

Example 8.34

Determine the site factors for the bridge of Ex. 8.31.

Solution

From Ex. 8.31, the ground motion parameters are PGA = 0.61g, S_S = 1.45g, and S_1 = 0.52g. From Ex. 8.33, the site class at the location of the bridge is SC = D. From Table 8.10 the site factors are

$$F_{pga} = 1.0$$
$$F_a = 1.0$$
$$F_v = 1.5$$

Adjusted Response Parameters

As specified in AASHTO Sec. 3.10.4.2, the ground motion parameters are modified by the site factors to allow for the site class effects. Therefore, the adjusted response parameters are

$$A_S = F_{pga}(\text{PGA})$$
$$S_{DS} = F_a S_S$$
$$S_{D1} = F_v S_1$$

Example 8.35

Determine the adjusted response parameters for the bridge of Ex. 8.31.

Solution

From Ex. 8.31, the ground motion parameters are PGA = 0.61g, S_S = 1.45g,   and S_1 = 0.52g.   From Ex. 8.34, the site factors are F_{pga} = 1.0, F_a = 1.0, and F_v = 1.5. From AASHTO Sec. 3.10.4.2, the adjusted response parameters are

$$A_s = F_{pga}(\text{PGA}) = (1.0)(0.61g)$$
$$= 0.61g$$
$$S_{DS} = F_a S_S = (1.0)(1.45g)$$
$$= 1.45g$$
$$S_{D1} = F_v S_1 = (1.5)(0.52g)$$
$$= 0.78g$$

Design Response Spectrum

The adopted design response spectrum is given by AASHTO Fig. 3.10.4.1-1 and is shown in Fig. 8.12. The spectrum for a specific location is a graph of the elastic seismic response coefficient, C_{sm}, over a range of periods of vibration, T_m.

Elastic Seismic Response Coefficient

As shown in Fig. 8.12, the design response spectrum is composed of three segments demarcated by the periods of vibration:

$$T_m = 0$$
$$T_m = T_0 = 0.2T_S$$
$$T_m = T_S = \frac{S_{D1}}{S_{DS}}$$

The values of the elastic seismic response coefficient are determined using equations given in AASHTO Sec. 3.10.4.2, which are summarized in Table 8.11.

Figure 8.12 Design Response Spectrum
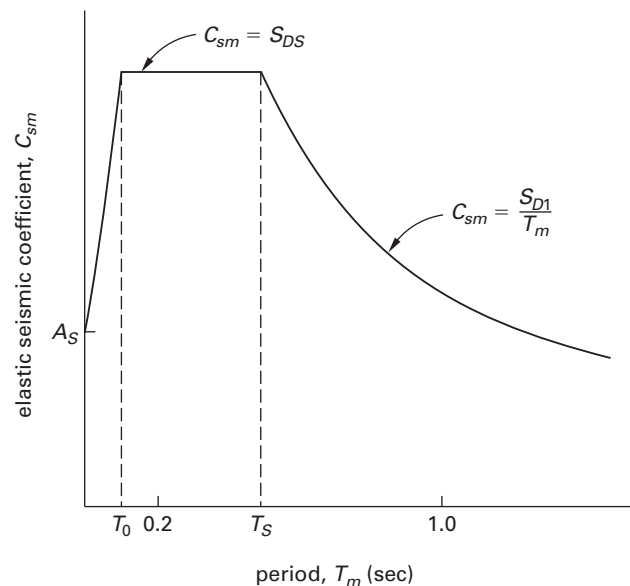
Table 8.11 *Elastic Seismic Response Coefficient Equations*

period, T_m	elastic seismic response coefficient, C_{sm}
$T_m \leq T_0$	$A_S + (S_{DS} - AS)(T_m/T_0)$
$T_0 < T_m \leq T_S$	S_{DS}
$T_S < T_m$	S_{D1}/T_m

Example 8.36

Determine the reference periods used to define the shape of the response spectrum for the bridge of Ex. 8.31.

Solution

From Ex. 8.35, the adjusted response parameters are $S_{DS} = 1.45g$ and $S_{D1} = 0.78g$. From AASHTO Sec. 3.10.4.2, the reference periods are

$$T_S = \frac{S_{D1}}{S_{DS}} = \frac{0.78g}{1.45g}$$
$$= 0.538 \text{ sec}$$
$$T_0 = 0.2T_S = (0.2)(0.538 \text{ sec})$$
$$= 0.108 \text{ sec}$$

Seismic Performance Zone

The seismic performance zone (SPZ) is a function of the acceleration coefficient S_{D1} and is defined in AASHTO Sec. 3.10.6. The four categories are shown in Table 8.12; these determine the necessary requirements for selection of the design procedure, minimum support lengths, and substructure design details.

Table 8.12 *Seismic Performance Zones*

acceleration coefficient, S_{D1}	seismic zone
$S_{D1} \leq 0.15$	1
$0.15 < S_{D1} \leq 0.30$	2
$0.30 < S_{D1} \leq 0.50$	3
$0.50 < S_{D1}$	4

Example 8.37

Determine the seismic zone for the bridge of Ex. 8.31.

Solution

From AASHTO Sec. 3.10.6, for a value of the acceleration coefficient exceeding 0.50, the relevant seismic performance zone is

$$\text{SPZ} = 4$$

Selection of Analysis Procedure

In accordance with AASHTO Sec. 4.7.4.3, the analysis procedure selected depends on the seismic zone, importance category, and bridge regularity. This information is summarized in Table 8.13. A *regular bridge* is defined as having fewer than seven spans with no abrupt

changes in weight, stiffness, or geometry. An *irregular bridge* does not satisfy the definition of a regular bridge, and in this type of structure, the higher modes of vibration significantly affect the seismic response. A detailed seismic analysis is not required for single-span bridges. Minimum support lengths are required, however, to accommodate the maximum inelastic displacement, in accordance with AASHTO Sec. 4.7.4.4.

A seismic analysis is not required for bridges in seismic zone 1.

Table 8.13 *Selection of Analysis Procedure for Multispan Bridges*

seismic zone	other bridges		essential bridges		critical bridges	
	regular	irregular	regular	irregular	regular	irregular
2	SM/UL	SM	SM/UL	MM	MM	MM
3	SM/UL	MM	MM	MM	MM	TH
4	SM/UL	MM	MM	MM	TH	TH

Note: UL = uniform load elastic method; SM = single-mode elastic method; MM = multimode elastic method; TH = time history method.

Example 8.38

Determine the required analysis procedure for the bridge of Ex. 8.31.

Solution

From Ex. 8.37, the seismic performance zone is 4. From AASHTO Sec. 4.7.4.3, for a regular bridge in seismic zone 4 with an importance category of "other," the required analysis procedure is UL or SM.

The Uniform Load Elastic Method

The uniform load elastic method is defined in AASHTO Sec. 4.7.4.3.2c as being suitable for regular bridges that respond principally in their fundamental mode. The method may be used for both transverse and longitudinal earthquake motions. The seven stages in the procedure are as follows.

1. Calculate the maximum lateral displacement, $v_{s(\max)}$, due to a uniform unit load, p_o, as shown in Fig. 8.13 for a transverse load. The uniform load is resisted by the lateral stiffness of the superstructure and by the stiffness of the central column. The abutments are assumed to be rigid and to provide a pinned end restraint at each end of the superstructure. The maximum displacement and the corresponding reaction, V_o, in the column may be determined by the virtual work method.[16]

2. The bridge transverse stiffness is given by

$$K = \frac{p_o L}{v_{s(\max)}} \qquad \text{[AASHTO C4.7.4.3.2c-1]}$$

Figure 8.13 *Transverse Displacement Due to Unit Transverse Load*

Example 8.39

Using the uniform load method, determine the elastic seismic design moment in the column, in the transverse direction, for the bridge of Ex. 8.31.

Solution

The stiffness of the column, fixed at the top and bottom, is given by

$$K_c = \frac{12E_c I_c}{h_c^3}$$

$$= \frac{(12)\left(450{,}000 \ \dfrac{\text{kips}}{\text{ft}^2}\right)(60 \ \text{ft}^4)}{(30 \ \text{ft})^3}$$

$$= 12{,}000 \ \text{kips/ft}$$

3. The total weight of the bridge superstructure and tributary substructure is

$$W = \int w(x)\,dx \quad \text{[AASHTO C4.7.4.3.2c-2]}$$

4. The fundamental period of the bridge is given by

$$T_m = 2\pi\sqrt{\frac{W}{gK}} \quad \text{[AASHTO C4.7.4.3.2c-3]}$$

5. The governing elastic seismic response coefficient, C_{sm}, is determined from AASHTO Sec. 3.10.4.2.

6. The uniform equivalent static seismic load is

$$p_e = \frac{C_{sm} W}{L} \quad \text{[AASHTO C4.7.4.3.2c-4]}$$

7. Apply p_e to the bridge as shown in Fig. 8.14 and determine the member forces due to the seismic load.

The transverse reaction in the column due to a uniform unit transverse load, p_o, on the superstructure is

$$V_o = v_{s(\max)} K_c$$

$$= v_{s(\max)}(12{,}000 \ \text{kips/ft})$$

The maximum transverse displacement of the superstructure alone due to a uniform unit transverse load, p_o, is

$$\delta_p = \frac{5p_o L^4}{384 E_s I_s}$$

$$= \frac{(5)\left(1.0 \ \dfrac{\text{kip}}{\text{ft}}\right)(200 \ \text{ft})^4}{(384)\left(450{,}000 \ \dfrac{\text{kips}}{\text{ft}^2}\right)(4000 \ \text{ft}^4)}$$

$$= 0.0116 \ \text{ft}$$

The maximum transverse displacement of the superstructure due to the column reaction, V_o, is

$$\delta_v = -\frac{V_o L^3}{48 E_s I_s}$$

$$= -\frac{v_{s(\max)}\left(12{,}000 \ \dfrac{\text{kips}}{\text{ft}}\right)L^3}{48 E_s I_s}$$

$$= -\frac{v_{s(\max)}\left(12{,}000 \ \dfrac{\text{kips}}{\text{ft}}\right)(200 \ \text{ft})^3}{(48)\left(450{,}000 \ \dfrac{\text{kips}}{\text{ft}^2}\right)(4000 \ \text{ft}^4)}$$

$$= -v_{s(\max)}(1.1111)$$

Figure 8.14 *Equivalent Static Seismic Load Applied to Bridge*

Bridges

The maximum transverse displacement of the superstructure due to p_o and V_o combined is

$$
\begin{aligned}
v_{s(\max)} &= \delta_p + \delta_v \\
&= 0.0116 \text{ ft} - v_{s(\max)}(1.1111) \\
&= 0.00548 \text{ ft}
\end{aligned}
$$

The bridge transverse stiffness is given by AASHTO Eq. C4.7.4.3.2c-1 as

$$
\begin{aligned}
K &= \frac{p_o L}{v_{s(\max)}} = \frac{\left(1.0 \ \dfrac{\text{kip}}{\text{ft}}\right)(200 \text{ ft})}{(0.00548 \text{ ft})\left(12 \ \dfrac{\text{in}}{\text{ft}}\right)} \\
&= 3041 \text{ kips/in} \quad (3040 \text{ kips/in})
\end{aligned}
$$

The total weight of the bridge superstructure and of the tributary substructure is given by AASHTO Eq. C4.7.4.3.2c-2 as

$$
\begin{aligned}
W &= \int w(x)\,dx = wL \\
&= \left(10 \ \frac{\text{kips}}{\text{ft}}\right)(200 \text{ ft}) \\
&= 2000 \text{ kips}
\end{aligned}
$$

The fundamental period of the bridge is given by AASHTO Eq. C4.7.4.3.2c-3 as

$$
\begin{aligned}
T_m &= 2\pi \sqrt{\frac{W}{gK}} = 0.32\sqrt{\frac{W}{K}} \\
&= 0.32 \ \frac{\text{sec}}{\text{in}^{-1}} \sqrt{\frac{2000 \text{ kips}}{3040 \ \dfrac{\text{kips}}{\text{in}}}} \\
&= 0.26 \text{ sec} > T_0 \\
&\quad\quad\quad\quad < T_S
\end{aligned}
$$

The elastic seismic response coefficient is given by AASHTO Eq. 3.10.4.2-4 as

$$
\begin{aligned}
C_{sm} &= S_{DS} \\
&= 1.45
\end{aligned}
$$

The uniform equivalent static seismic load is given by AASHTO Eq. C4.7.4.3.2c-4 as

$$
\begin{aligned}
p_e &= \frac{C_{sm} W}{L} = \frac{(1.45)(2000 \text{ kips})}{200 \text{ ft}} \\
&= 14.5 \text{ kips/ft}
\end{aligned}
$$

The maximum transverse displacement due to the equivalent seismic load is

$$
\begin{aligned}
v_{e(\max)} &= \frac{p_e v_{s(\max)}}{p_o} = \frac{\left(14.5 \ \dfrac{\text{kips}}{\text{ft}}\right)(0.00548 \text{ ft})}{1.0 \ \dfrac{\text{kip}}{\text{ft}}} \\
&= 0.079 \text{ ft}
\end{aligned}
$$

The elastic transverse shear in the column is

$$
\begin{aligned}
V &= v_{e(\max)} K_c \\
&= (0.079)\left(12{,}000 \ \frac{\text{kips}}{\text{ft}}\right) \\
&= 948 \text{ kips}
\end{aligned}
$$

The elastic transverse moment in the column is

$$
\begin{aligned}
M &= \frac{V h_c}{2} = \frac{(948 \text{ kips})(30 \text{ ft})}{2} \\
&= 14{,}220 \text{ ft-kips}
\end{aligned}
$$

The Single-Mode Elastic Method

The fundamental period and the equivalent static force are obtained by using the technique detailed in AASHTO Sec. 4.7.4.3.2b. The method may be used for both transverse and longitudinal earthquake motions. The six steps in the procedure are as follows.

step 1: Calculate the static displacements, $v_s(x)$, due to a uniform unit load, p_o, as shown in Fig. 8.15 for a longitudinal load. The uniform load is resisted by the lateral stiffness of the central column, with the abutments assumed to provide no restraint.

Figure 8.15 *Longitudinal Displacement Due to Unit Longitudinal Load*
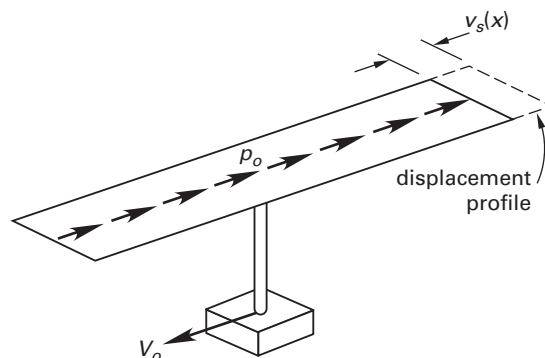
step 2: Calculate the factors α, β, and γ, which are given by

$$\alpha = \int v_s(x)\,dx \qquad \text{[AASHTO C4.7.4.3.2b-1]}$$

$$\beta = \int w(x)v_s(x)\,dx \qquad \text{[AASHTO C4.7.4.3.2b-2]}$$

$$\gamma = \int w(x)v_s^2(x)\,dx \qquad \text{[AASHTO C4.7.4.3.2b-3]}$$

The limits of the integrals extend over the whole length of the bridge.

step 3: The fundamental period is given by

$$T_m = 2\pi\sqrt{\frac{\gamma}{p_o g \alpha}} \qquad \text{[AASHTO C4.7.4.3.2b-4]}$$

step 4: The governing elastic seismic response coefficient, C_{sm}, is used to determine the elastic force in a member and is given by AASHTO Sec. 3.10.4.2, summarized in Table 8.11.

step 5: The equivalent static seismic load is

$$p_e(x) = \frac{\beta C_{sm} w(x) v_s(x)}{\gamma} \qquad \text{[AASHTO C4.7.4.3.2b-5]}$$

step 6: Apply $p_e(x)$ to the bridge as shown in Fig. 8.16 and determine the member forces due to the seismic load.

Figure 8.16 *Equivalent Static Seismic Load Applied to Bridge*
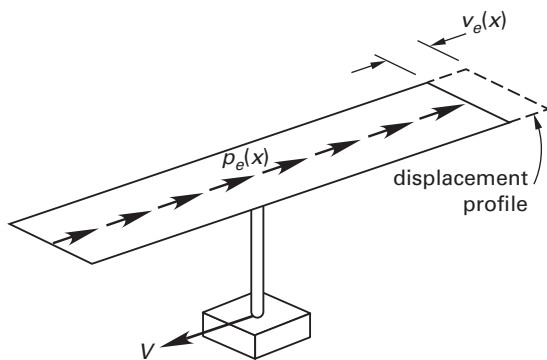
Example 8.40

Using the single-mode elastic method, determine the elastic seismic design moment in the column, in the longitudinal direction, for the bridge of Ex. 8.31.

Solution

The stiffness of the column, fixed at the top and bottom, is obtained from Ex. 8.39 as

$$K_c = 12{,}000 \text{ kips/ft}$$

Applying a uniform load of $p_o = 1.0$ kip/ft along the longitudinal axis of the bridge produces a longitudinal displacement of the superstructure of

$$v_s(x) = \frac{p_o L}{K_c} = \frac{\left(1.0 \ \dfrac{\text{kip}}{\text{ft}}\right)(200 \text{ ft})}{12{,}000 \ \dfrac{\text{kips}}{\text{ft}}}$$

$$= 0.0167 \text{ ft}$$

The factor α is given by AASHTO Eq. C4.7.4.3.2b-1 as

$$\alpha = \int v_s(x)\,dx$$

$$= (0.0167 \text{ ft})(200 \text{ ft})$$

$$= 3.333 \text{ ft}^2$$

The factor β is given by AASHTO Eq. C4.7.4.3.2b-2 as

$$\beta = \int w(x)v_s(x)\,dx$$

$$= \left(10 \ \frac{\text{kips}}{\text{ft}}\right)(3.333 \text{ ft}^2)$$

$$= 33.333 \text{ ft-kips}$$

The factor γ is given by AASHTO Eq. C4.7.4.3.2b-3 as

$$\gamma = \int w(x)v_s^2(x)\,dx$$

$$= \left(10 \ \frac{\text{kips}}{\text{ft}}\right)(0.0167 \text{ ft})^2(200 \text{ ft})$$

$$= 0.557 \text{ ft}^2\text{-kip}$$

The fundamental period is given by AASHTO Eq. C4.7.4.3.2b-4 as

$$T_m = 2\pi\sqrt{\frac{\gamma}{p_o g \alpha}}$$

$$= 2\pi\sqrt{\frac{0.557 \text{ ft}^2\text{-kip}}{\left(1.0 \ \dfrac{\text{kip}}{\text{ft}}\right)\left(32.2 \ \dfrac{\text{ft}}{\text{sec}^2}\right)(3.333 \text{ ft}^2)}}$$

$$= 0.45 \text{ sec} > T_0$$

$$< T_s$$

The elastic seismic response coefficient is given by AASHTO Eq. 3.10.4.2-4 as

$$C_{sm} = S_{DS}$$

$$= 1.45$$

Bridges

The equivalent static seismic load is given by AASHTO Eq. C4.7.4.3.2b-5.

$$
\begin{aligned}
p_e(x) &= \frac{\beta C_{sm} w(x) v_s(x)}{\gamma} \\[6pt]
&= \frac{(33.333 \text{ ft-kips})(1.45)\left(10\ \dfrac{\text{kips}}{\text{ft}}\right)(0.0167 \text{ ft})}{0.557 \text{ ft}^2\text{-kip}} \\[6pt]
&= 14.49 \text{ kips/ft}
\end{aligned}
$$

The longitudinal displacement due to the equivalent seismic load is

$$
\begin{aligned}
v_e(x) &= \frac{p_e(x) v_s(x)}{p_0} \\[6pt]
&= \frac{\left(14.49\ \dfrac{\text{kips}}{\text{ft}}\right)(0.0167 \text{ ft})}{1.0\ \dfrac{\text{kip}}{\text{ft}}} \\[6pt]
&= 0.242 \text{ ft}
\end{aligned}
$$

The elastic shear in the column in the longitudinal direction is

$$
\begin{aligned}
V &= v_e(x) K_c \\[6pt]
&= (0.242 \text{ ft})\left(12{,}000\ \frac{\text{kips}}{\text{ft}}\right) \\[6pt]
&= 2904 \text{ kips}
\end{aligned}
$$

The elastic moment in the column in the longitudinal direction is

$$
\begin{aligned}
M &= \frac{V h_c}{2} = \frac{(2904 \text{ kips})(30 \text{ ft})}{2} \\[6pt]
&= 43{,}560 \text{ ft-kips}
\end{aligned}
$$

Response Modification Factor

The seismic design force for a member is determined by dividing the elastic force by the response modification factor, R. AASHTO Table 3.10.7.1-1 and Table 3.10.7.1-2 list the different structural systems and response modification factors. An abbreviated listing of response modification factors is provided in Table 8.14 and Table 8.15.

Table 8.14 *Response Modification Factors for Substructures*

	importance category		
substructure	critical	essential	other
wall-type pier: strong axis	1.5	1.5	2.0
single column	1.5	2.0	3.0
multiple column bents	1.5	3.5	5.0

Table 8.15 *Response Modification Factors for Connections*

connection	all importance categories
superstructure to abutment	0.8
superstructure to column or pier	1.0
column or pier to foundation	1.0

Example 8.41

Determine the seismic design moment in the column, in the longitudinal and transverse direction, for the bridge of Ex. 8.31.

Solution

From Ex. 8.32, the importance category is "other." The response modification factor for a single column is given in Table 8.14 as

$$
R = 3
$$

The reduced design moment in the column in the longitudinal direction is

$$
\begin{aligned}
M_R &= \frac{M}{R} = \frac{43{,}560 \text{ ft-kips}}{3} \\[6pt]
&= 14{,}520 \text{ ft-kips}
\end{aligned}
$$

The reduced transverse design moment in the column in the transverse direction is

$$
\begin{aligned}
M_R &= \frac{M}{R} = \frac{14{,}220 \text{ ft-kips}}{3} \\[6pt]
&= 4740 \text{ ft-kips}
\end{aligned}
$$

Combination of Orthogonal Seismic Forces

AASHTO Sec. 3.10.8 requires the combination of orthogonal seismic forces to account for the directional uncertainty of earthquake motions and the simultaneous occurrence of earthquake forces in two perpendicular horizontal directions. Two load combinations are specified as follows.

- *load case 1:* 100% of the forces due to a seismic event in the longitudinal direction plus 30% of the forces due to a seismic event in the transverse direction

- *load case 2:* 100% of the forces due to a seismic event in the transverse direction plus 30% of the forces due to a seismic event in the longitudinal direction

Example 8.42

Determine the resultant seismic design moment in the column, due to the longitudinal and transverse forces, for the bridge of Ex. 8.31.

Solution

From Ex. 8.41, load case 1 governs, and the longitudinal moment is

$$M_x = (1.0)(14{,}520 \text{ ft-kips})$$
$$= 14{,}520 \text{ ft-kips}$$

The corresponding transverse moment is

$$M_y = (0.3)(4740 \text{ ft-kips})$$
$$= 1422 \text{ ft-kips}$$

For a circular column, the maximum resultant moment is given by

$$M_R = \sqrt{M_x^2 + M_y^2}$$
$$= \sqrt{(14{,}520 \text{ ft-kips})^2 + (1422 \text{ ft-kips})^2}$$
$$= 14{,}589 \text{ ft-kips}$$

Minimum Seat-Width Requirements

In accordance with AASHTO Sec. 4.7.4.4, minimum support lengths are required at the expansion ends of all girders as shown in Fig. 8.17. For seismic zone 1, with $A_s \geq 0.05$, the minimum support length in inches is given by

$$N = (8 + 0.02L + 0.08H)(1 + 0.000125S^2)$$

[*AASHTO 4.7.4.4-1*]

For seismic zone 1, with $A_s < 0.05$, the minimum support length is

$$N = (6 + 0.015L + 0.06H)(1 + 0.000125S^2)$$

For seismic zones 2, 3, and 4, the minimum support length is

$$N = (12 + 0.03L + 0.12H)(1 + 0.000125S^2)$$

Figure 8.17 *Minimum Seat-Width Requirements*
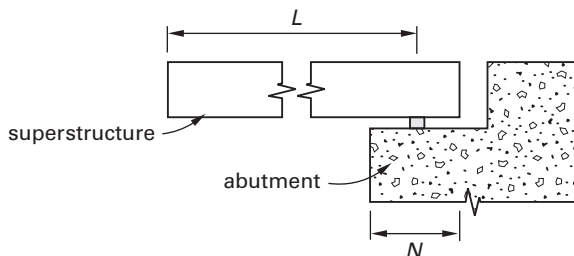
The terms in these expressions are defined as follows.

$L =$ length in feet of the bridge deck to the adjacent expansion joint or the end of the bridge deck
$H =$ average height in feet of the columns
$\quad = 0$ for a single-span bridge
$S =$ angle of skew of the support in degrees measured from a line normal to the span

Example 8.43

Determine the minimum support length for the bridge of Ex. 8.31.

Solution

From Ex. 8.37, the seismic zone is 4.

From AASHTO Sec. 4.7.4.4, the minimum support length is given by

$$N = (12 + 0.03L + 0.12H)(1 + 0.000125S^2)$$
$$= \left(12 \text{ in} + \left(0.03 \tfrac{\text{in}}{\text{ft}}\right)(200 \text{ ft}) + \left(0.12 \tfrac{\text{in}}{\text{ft}}\right)(30 \text{ ft})\right)$$
$$\quad \times \left(1 + (0.000125)(0.0)^2\right)$$
$$= 21.6 \text{ in}$$

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PRACTICE PROBLEMS

1. The reinforced concrete T-beam bridge shown in the following illustration is simply supported over a span of 40 ft. The deck has an overall width of 39 ft with five supporting beams, and has three 12 ft design lanes. The ratio of the modulus of elasticity of the beam and the deck slab is $n = 1.0$. The superimposed dead load on an interior beam due to surfacing is 0.25 kip/ft, and due to parapets is 0.2 kip/ft. For an interior beam, what is most nearly the bending moment produced by the permanent loads?
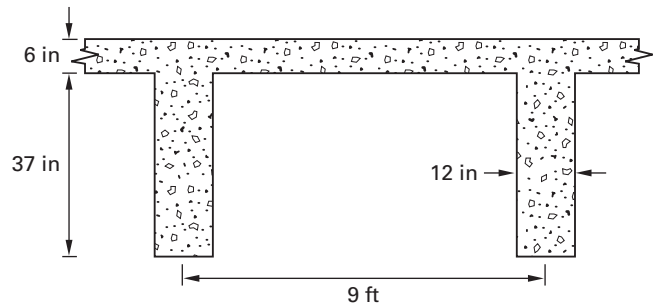
(A) 200 ft-kips

(B) 250 ft-kips

(C) 300 ft-kips

(D) 325 ft-kips

2. For the reinforced concrete T-beam bridge of Prob. 1, the maximum bending moment produced in an interior beam by the design lane load in combination with the design tandem is most nearly

(A) 470 ft-kips

(B) 600 ft-kips

(C) 630 ft-kips

(D) 730 ft-kips

For Prob. 3 through Prob. 5, assume $M_C = 250$ ft-kips, $M_W = 50$ ft-kips, and $M_L = 627$ ft-kips.

3. For the reinforced concrete T-beam bridge of Prob. 1, the strength I limit state factored moment for design of an interior beam is most nearly

(A) 1420 ft-kips

(B) 1485 ft-kips

(C) 1500 ft-kips

(D) 1660 ft-kips

4. For an interior beam of the reinforced concrete T-beam bridge of Prob. 1, the concrete strength is 4000 lbf/in^2, and the reinforcement consists of nine no. 9 grade 60 bars. The stress block depth is most nearly

    (A) 1.25 in

    (B) 1.35 in

    (C) 1.45 in

    (D) 1.55 in

5. For an interior beam of the reinforced concrete T-beam bridge of Prob. 1, are the fatigue stress limits satisfactory? Ignore the effects of the 8 kip axle and assume a value of $g = 0.65$ for the load distribution factor to the interior beam.

    (A) No, $(\Delta F)_{\text{TH}} = 20.25$ kips/in^2.

    (B) No, $(\Delta F)_{\text{TH}} = 21.40$ kips/in^2.

    (C) Yes, $(\Delta F)_{\text{TH}} = 20.25$ kips/in^2.

    (D) Yes, $(\Delta F)_{\text{TH}} = 21.40$ kips/in^2.

SOLUTIONS

1. The dead load acting on one beam due to the weight of the parapets, the weight of the deck slab, and the self-weight of the beam is

$$
\begin{aligned}
w_C = &\ 0.20 \ \frac{\text{kip}}{\text{ft}} \\
&+ \left(0.15 \ \frac{\text{kip}}{\text{ft}^3}\right)\left(\frac{(0.5 \ \text{ft})(39 \ \text{ft})}{5 \ \text{beams}} + (1.0 \ \text{ft})(3.08 \ \text{ft})\right) \\
= &\ 1.25 \ \text{kips/ft}
\end{aligned}
$$

The bending moment produced in an interior beam at the center of the span by the parapets, deck slab, and beam self-weight is

$$
\begin{aligned}
M_C &= \frac{w_C L^2}{8} \\
&= \frac{\left(1.25 \ \dfrac{\text{kips}}{\text{ft}}\right)(40 \ \text{ft})^2}{8} \\
&= 250 \ \text{ft-kips}
\end{aligned}
$$

The bending moment produced in an interior beam at the center of the span by the surfacing is

$$
\begin{aligned}
M_W &= \frac{w_W L^2}{8} \\
&= \frac{\left(0.25 \ \dfrac{\text{kip}}{\text{ft}}\right)(40 \ \text{ft})^2}{8} \\
&= 50 \ \text{ft-kips}
\end{aligned}
$$

The total dead load bending moment produced in an interior beam at the center of the span is

$$
\begin{aligned}
M_D &= M_C + M_W \\
&= 250 \ \text{ft-kips} + 50 \ \text{ft-kips} \\
&= 300 \ \text{ft-kips}
\end{aligned}
$$

The answer is (C).

2. The bending moment produced at the center of the span by the design lane load is

$$M_{LL} = \frac{w_{LL}L^2}{8}$$
$$= \frac{\left(0.64 \ \frac{\text{kip}}{\text{ft}}\right)(40 \ \text{ft})^2}{8}$$
$$= 128 \ \text{ft-kips}$$

As shown in the following illustration, the maximum moment due to the design tandem is produced under the lead axle of the design tandem when it is located 1 ft beyond the center of the span, and is given by

$$M_{DT} = \frac{(25 \ \text{kips})(19 \ \text{ft})(21 \ \text{ft} + 17 \ \text{ft})}{40 \ \text{ft}}$$
$$= 451 \ \text{ft-kips}$$

location of design tandem

bending moment, ft-kips
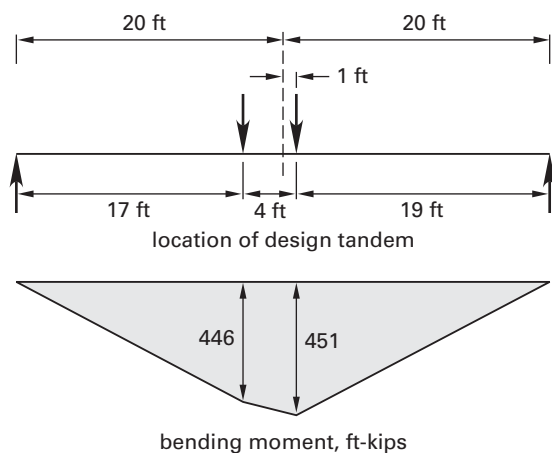
The dynamic load allowance for the span moment is given by AASHTO Table 3.6.2.1-1 as

$$IM = 33\%$$

It is applied to the static axle loads of the design tandem and the dynamic factor to be applied is

$$I = 1 + \frac{IM}{100}$$
$$= 1 + \frac{33\%}{100}$$
$$= 1.33$$

The moment caused by the design tandem, including the dynamic load allowance, is

$$M = (1.33)(451 \ \text{ft-kips})$$
$$= 600 \ \text{ft-kips}$$

The combined moment produced by the design lane load and the design tandem, including the dynamic load allowance, is

$$M_{L+T} = 128 \ \text{ft-kips} + 600 \ \text{ft-kips}$$
$$= 728 \ \text{ft-kips}$$

The ratio of the modulus of elasticity of the beam and the deck slab is given as

$$n = \frac{E_B}{E_D}$$
$$= 1.0$$

The moment of inertia of the beam is

$$I = \frac{bd^3}{12}$$
$$= \frac{(12 \ \text{in})(37 \ \text{in})^3}{12}$$
$$= 50{,}653 \ \text{in}^4$$

The area of the beam is

$$A = bd$$
$$= (12 \ \text{in})(37 \ \text{in})$$
$$= 444 \ \text{in}^2$$

The distance between the centers of gravity of the beam and the deck slab is

$$e_g = \frac{t_s + d}{2}$$
$$= \frac{6 \ \text{in} + 37 \ \text{in}}{2}$$
$$= 21.5 \ \text{in}$$

The longitudinal stiffness parameter of the deck is defined by AASHTO Eq. 4.6.2.2.1-1 as

$$K_g = n(I + Ae_g^2)$$
$$= (1.0)\left(50{,}653 \text{ in}^4 + (444 \text{ in}^2)(21.5 \text{ in})^2\right)$$
$$= 255{,}892 \text{ in}^4 \quad \begin{bmatrix} \text{complies with AASHTO} \\ \text{Table 4.6.2.2.2b-1} \end{bmatrix}$$
$$> 10{,}000 \text{ in}^4$$
$$< 7{,}000{,}000 \text{ in}^4$$

The beam spacing is

$$S = 9 \text{ ft} \quad \text{[complies with AASHTO Table 4.6.2.2.2b-1]}$$
$$> 3.5 \text{ ft}$$
$$< 16.0 \text{ ft}$$

The deck slab thickness is

$$t_s = 6 \text{ in} \quad \text{[complies with AASHTO Table 4.6.2.2.2b-1]}$$
$$> 4.5 \text{ in}$$
$$< 12.0 \text{ in}$$

The superstructure span is

$$L = 40 \text{ ft} \quad \text{[complies with AASHTO Table 4.6.2.2.2b-1]}$$
$$> 20 \text{ ft}$$
$$< 240 \text{ ft}$$

The number of beams in the deck is

$$N_b = 5 \quad \text{[complies with AASHTO Table 4.6.2.2.2b-1]}$$

Therefore, the configuration of the deck is in full conformity with the requirements of AASHTO Table 4.6.2.2.2b-1.

With one lane loaded, AASHTO Table 4.6.2.2.2b-1 gives the distribution factor for moment as

$$g_1 = 0.06 + \left(\frac{S}{14}\right)^{0.4} \left(\frac{S}{L}\right)^{0.3} \left(\frac{K_g}{12.0 L t_s^3}\right)^{0.1}$$
$$= 0.06 + \left(\frac{9 \text{ ft}}{14}\right)^{0.4} \left(\frac{9 \text{ ft}}{40 \text{ ft}}\right)^{0.3}$$
$$\times \left(\frac{255{,}892 \text{ in}^4}{(12.0)(40 \text{ ft})(6 \text{ in})^3}\right)^{0.1}$$
$$= 0.646$$

With two lanes loaded, as shown in the illustration, AASHTO Table 4.6.2.2.2b-1 gives the distribution factor for moment as

$$g_m = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \left(\frac{S}{L}\right)^{0.2} \left(\frac{K_g}{12.0 L t_s^3}\right)^{0.1}$$
$$= 0.075 + \left(\frac{9 \text{ ft}}{9.5}\right)^{0.6} \left(\frac{9 \text{ ft}}{40 \text{ ft}}\right)^{0.2}$$
$$\times \left(\frac{255{,}892 \text{ in}^4}{(12.0)(40 \text{ ft})(6 \text{ in})^3}\right)^{0.1}$$
$$= 0.861 \quad \text{[governs]}$$

The live load moment for the design of an interior beam is

$$M_L = g_m M_{L+T}$$
$$= (0.861)(728 \text{ ft-kips})$$
$$= 627 \text{ ft-kips} \quad (630 \text{ ft-kips})$$

The answer is (C).

3. The relevant service level moments are

$M_C =$ moment produced by the parapets, deck slab, and beam self-weight
$\quad = 250 \text{ ft-kips}$
$M_W =$ moment produced by the wearing surface
$\quad = 50 \text{ ft-kips}$
$M_L =$ moment produced by the design lane load and the design tandem, including the dynamic load allowance
$\quad = 627 \text{ ft-kips}$

The factored design moment for the strength I limit state is given by AASHTO Eq. 3.4.1-1 and AASHTO Table 3.4.1-1 as

$$M_u = \eta_i(\gamma_p M_C + \gamma_p M_W + \gamma_L M_L)$$
$$= (1.0)(1.25 M_C + 1.5 M_W + 1.75 M_L)$$
$$= (1.25)(250 \text{ ft-kips}) + (1.5)(50 \text{ ft-kips})$$
$$+ (1.75)(627 \text{ ft-kips})$$
$$= 1485 \text{ ft-kips}$$

The answer is (B).

4. The effective compression flange width is given by AASHTO Sec. 4.6.2.6.1 as the minimum of

$$b = S$$
$$= (9 \text{ ft})\left(12 \frac{\text{in}}{\text{ft}}\right)$$
$$= 108 \text{ in}$$

The height of the centroid of the tensile reinforcement is

$$\overline{c} = 2 \text{ in} + (2.5)(1.125 \text{ in})$$
$$= 4.81 \text{ in}$$

The effective depth is

$$d = h - \overline{c}$$
$$= 43 \text{ in} - 4.81 \text{ in}$$
$$= 38.19 \text{ in}$$

Assuming that the stress block lies within the flange, the required tension reinforcement is determined from the principles of AASHTO Sec. 5.7. The design moment factor is

$$K_u = \frac{M_u}{b_w d^2}$$
$$= \frac{(1485 \text{ ft-kips})\left(12 \ \frac{\text{in}}{\text{ft}}\right)\left(1000 \ \frac{\text{lbf}}{\text{kip}}\right)}{(108 \text{ in})(38.19 \text{ in})^2}$$
$$= 113 \text{ lbf/in}^2$$
$$\frac{K_u}{f'_c} = \frac{113 \ \dfrac{\text{lbf}}{\text{in}^2}}{4000 \ \dfrac{\text{lbf}}{\text{in}^2}}$$
$$= 0.0283$$

From App. A, the corresponding tension reinforcement index is

$$\omega = 0.032$$
$$< 0.319\beta_1$$
$$= (0.319)(0.85)$$
$$= 0.271$$

Therefore, the section is tension controlled, and $\phi = 0.90$.

The required reinforcement ratio is

$$\rho = \frac{\omega f'_c}{f_y}$$
$$= \frac{(0.032)\left(4000 \ \dfrac{\text{lbf}}{\text{in}^2}\right)}{60,000 \ \dfrac{\text{lbf}}{\text{in}^2}}$$
$$= 0.00213$$

The reinforcement area required is

$$A_s = \rho bd$$
$$= (0.00213)(108 \text{ in})(38.19 \text{ in})$$
$$= 8.79 \text{ in}^2$$

For nine no. 9 bars as shown in the following illustration, the reinforcement area provided is

$$A_s = 9 \text{ in}^2$$
$$> 8.79 \text{ in}^2 \quad [\text{satisfactory}]$$
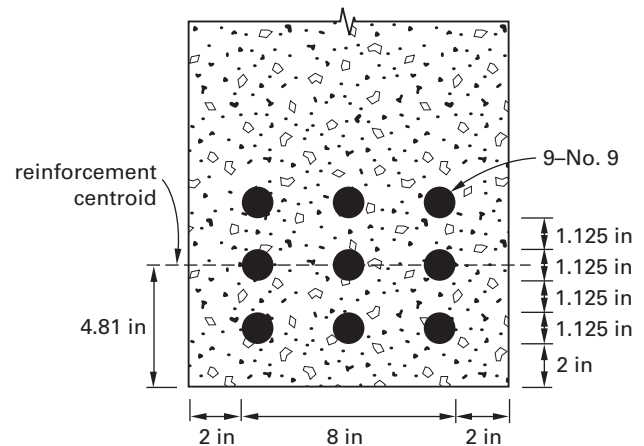
The stress block depth is

$$a = \frac{A_s f_y}{0.85 b f'_c}$$
$$= \frac{(9 \text{ in}^2)\left(60,000 \ \dfrac{\text{lbf}}{\text{in}^2}\right)}{(0.85)(108 \text{ in})\left(4000 \ \dfrac{\text{lbf}}{\text{in}^2}\right)}$$
$$= 1.47 \text{ in} \quad (1.45 \text{ in})$$
$$< h_f \quad \begin{bmatrix} \text{The stress block is contained} \\ \text{within the flange.} \end{bmatrix}$$

The answer is (C).

5. The lever-arm for elastic design is conservatively obtained from Prob. 4 as

$$l_a = d - \frac{h_f}{2}$$
$$= 38.19 \text{ in} - \frac{6 \text{ in}}{2}$$
$$= 35.19 \text{ in}$$

Fatigue limits are determined using 150% of the stress produced by the design truck plus dynamic load allowance. The distance between the 32 kip axles of the design truck is fixed at 30 ft. Ignoring the effect of the

8 kip lead axle, the maximum moment is developed at the location of the 32 kip axle when this is positioned at the center of the span. The bending moment produced in an interior beam is

$$M_{DT} = \frac{gWL}{4}$$
$$= \frac{(0.65)(32 \text{ kips})(40 \text{ ft})}{4}$$
$$= 208 \text{ ft-kips}$$

The dynamic load allowance for the span moment is given by AASHTO Table 3.6.2.1-1 as

$$IM = 15\%$$

This is applied to the static axle loads of the design truck and the dynamic factor to be applied is

$$I = 1 + \frac{IM}{100}$$
$$= 1 + \frac{15\%}{100}$$
$$= 1.15$$

150% of the moment caused by the design truck, including the dynamic load allowance, is

$$M_f = (1.5)(1.15)M_{DT}$$
$$= (1.5)(1.15)(208 \text{ ft-kips})$$
$$= 359 \text{ ft-kips}$$

This is the maximum moment range producing fatigue.

The corresponding maximum factored stress range is

$$\gamma(\Delta f) = \frac{M_f}{l_a A_s}$$
$$= \frac{(359 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(35.19 \text{ in})(9 \text{ in}^2)}$$
$$= 13.60 \text{ kips/in}^2$$

The permanent dead load moment produced in an interior beam at the center of the span is derived in Prob. 1 as

$$M_D = 300 \text{ ft-kips}$$

This is the minimum moment at the center of the span.

The corresponding minimum stress is

$$f_{\min} = \frac{M_D}{l_a A_s}$$
$$= \frac{(300 \text{ ft-kips})\left(12 \frac{\text{in}}{\text{ft}}\right)}{(35.19 \text{ in})(9 \text{ in}^2)}$$
$$= 11.37 \text{ kips/in}^2$$

The constant-amplitude fatigue threshold is given by AASHTO Eq. 5.5.3.2-1 as

$$(\Delta F)_{\text{TH}} = 24 - 0.33 f_{\min}$$
$$= 24 - (0.33)\left(11.37 \frac{\text{kips}}{\text{in}^2}\right)$$
$$= 20.25 \text{ kips/in}^2$$
$$> \gamma(\Delta f) \quad \text{[satisfactory]}$$

The answer is (C).

Appendices
Table of Contents

APPENDIX A
Values of $M_u/f'_c bd^2$ for a Tension-Controlled Section

ω	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0	0.0000	0.0009	0.0018	0.0027	0.0036	0.0045	0.0054	0.0063	0.0072	0.0081
0.01	0.0089	0.0098	0.0107	0.0116	0.0125	0.0134	0.0143	0.0151	0.0160	0.0169
0.02	0.0178	0.0187	0.0195	0.0204	0.0213	0.0222	0.0230	0.0239	0.0248	0.0257
0.03	0.0265	0.0274	0.0283	0.0291	0.0300	0.0309	0.0317	0.0326	0.0334	0.0343
0.04	0.0352	0.0360	0.0369	0.0377	0.0386	0.0394	0.0403	0.0411	0.0420	0.0428
0.05	0.0437	0.0445	0.0454	0.0462	0.0471	0.0479	0.0487	0.0496	0.0504	0.0513
0.06	0.0521	0.0529	0.0538	0.0546	0.0554	0.0563	0.0571	0.0579	0.0588	0.0596
0.07	0.0604	0.0612	0.0621	0.0629	0.0637	0.0645	0.0653	0.0662	0.0670	0.0678
0.08	0.0686	0.0694	0.0702	0.0711	0.0719	0.0727	0.0735	0.0743	0.0751	0.0759
0.09	0.0767	0.0775	0.0783	0.0791	0.0799	0.0807	0.0815	0.0823	0.0831	0.0839
0.10	0.0847	0.0855	0.0863	0.0871	0.0879	0.0887	0.0895	0.0902	0.0910	0.0918
0.11	0.0926	0.0934	0.0942	0.0949	0.0957	0.0965	0.0973	0.0981	0.0988	0.0996
0.12	0.1004	0.1011	0.1019	0.1027	0.1035	0.1042	0.1050	0.1058	0.1065	0.1073
0.13	0.1081	0.1088	0.1096	0.1103	0.1111	0.1119	0.1126	0.1134	0.1141	0.1149
0.14	0.1156	0.1164	0.1171	0.1179	0.1186	0.1194	0.1201	0.1209	0.1216	0.1223
0.15	0.1231	0.1238	0.1246	0.1253	0.1260	0.1268	0.1275	0.1283	0.1290	0.1297
0.16	0.1304	0.1312	0.1319	0.1326	0.1334	0.1341	0.1348	0.1355	0.1363	0.1370
0.17	0.1377	0.1384	0.1391	0.1399	0.1406	0.1413	0.1420	0.1427	0.1434	0.1441
0.18	0.1448	0.1456	0.1463	0.1470	0.1477	0.1484	0.1491	0.1498	0.1505	0.1512
0.19	0.1519	0.1526	0.1533	0.1540	0.1547	0.1554	0.1561	0.1568	0.1574	0.1581
0.20	0.1588	0.1595	0.1602	0.1609	0.1616	0.1623	0.1629	0.1636	0.1643	0.1650
0.21	0.1657	0.1663	0.1670	0.1677	0.1684	0.1690	0.1697	0.1704	0.1710	0.1717
0.22	0.1724	0.1730	0.1737	0.1744	0.1750	0.1757	0.1764	0.1770	0.1777	0.1783
0.23	0.1790	0.1797	0.1803	0.1810	0.1816	0.1823	0.1829	0.1836	0.1842	0.1849
0.24	0.1855	0.1862	0.1868	0.1874	0.1881	0.1887	0.1894	0.1900	0.1906	0.1913
0.25	0.1919	0.1925	0.1932	0.1938	0.1944	0.1951	0.1957	0.1963	0.1970	0.1976
0.26	0.1982	0.1988	0.1995	0.2001	0.2007	0.2013	0.2019	0.2026	0.2032	0.2038
0.27	0.2044	0.2050	0.2056	0.2062	0.2069	0.2075	0.2081	0.2087	0.2093	0.2099
0.28	0.2105	0.2111	0.2117	0.2123	0.2129	0.2135	0.2141	0.2147	0.2153	0.2159
0.29	0.2165	0.2171	0.2177	0.2183	0.2188	0.2194	0.2200	0.2206	0.2212	0.2218
0.30	0.2224	0.2229	0.2235	0.2241	0.2247	0.2253	0.2258	0.2264	0.2270	0.2276
0.31	0.2281	0.2287	0.2293	0.2298	0.2304	0.2310	0.2315	0.2321	0.2327	0.2332
0.32	0.2338	0.2344	0.2349	0.2355	0.2360	0.2366	0.2371	0.2377	0.2382	0.2388
0.33	0.2393	0.2399	0.2404	0.2410	0.2415	0.2421	0.2426	0.2432	0.2437	0.2443
0.34	0.2448	0.2453	0.2459	0.2464	0.2470	0.2475	0.2480	0.2486	0.2491	0.2496
0.35	0.2501	0.2507	0.2512	0.2517	0.2523	0.2528	0.2533	0.2538	0.2543	0.2549
0.36	0.2554	0.2559	0.2564	0.2569	0.2575	0.2580	0.2585	0.2590	0.2595	0.2600
0.37	0.2605	0.2610	0.2615	0.2620	0.2625	0.2631	0.2636	0.2641	0.2646	0.2651
0.38	0.2656	0.2661	0.2665	0.2670	0.2675	0.2680	0.2685	0.2690	0.2695	0.2700
0.39	0.2705	0.2710	0.2715	0.2719	0.2724	0.2729	0.2734	0.2739	0.2743	0.2748

APPENDIX B
Values of the Neutral Axis Depth Factor, k

ρn	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0	0.0000	0.0437	0.0613	0.0745	0.0855	0.0951	0.1037	0.1115	0.1187	0.1255
0.01	0.1318	0.1377	0.1434	0.1488	0.1539	0.1589	0.1636	0.1682	0.1726	0.1769
0.02	0.1810	0.1850	0.1889	0.1927	0.1964	0.2000	0.2035	0.2069	0.2103	0.2136
0.03	0.2168	0.2199	0.2230	0.2260	0.2290	0.2319	0.2347	0.2375	0.2403	0.2430
0.04	0.2457	0.2483	0.2509	0.2534	0.2559	0.2584	0.2608	0.2632	0.2655	0.2679
0.05	0.2702	0.2724	0.2747	0.2769	0.2790	0.2812	0.2833	0.2854	0.2875	0.2895
0.06	0.2916	0.2936	0.2956	0.2975	0.2995	0.3014	0.3033	0.3051	0.3070	0.3088
0.07	0.3107	0.3125	0.3142	0.3160	0.3178	0.3195	0.3212	0.3229	0.3246	0.3263
0.08	0.3279	0.3296	0.3312	0.3328	0.3344	0.3360	0.3376	0.3391	0.3407	0.3422
0.09	0.3437	0.3452	0.3467	0.3482	0.3497	0.3511	0.3526	0.3540	0.3554	0.3569
0.10	0.3583	0.3597	0.3610	0.3624	0.3638	0.3651	0.3665	0.3678	0.3691	0.3705
0.11	0.3718	0.3731	0.3744	0.3756	0.3769	0.3782	0.3794	0.3807	0.3819	0.3832
0.12	0.3844	0.3856	0.3868	0.3880	0.3892	0.3904	0.3916	0.3927	0.3939	0.3951
0.13	0.3962	0.3974	0.3985	0.3996	0.4007	0.4019	0.4030	0.4041	0.4052	0.4063
0.14	0.4074	0.4084	0.4095	0.4106	0.4116	0.4127	0.4137	0.4148	0.4158	0.4169
0.15	0.4179	0.4189	0.4199	0.4209	0.4219	0.4229	0.4239	0.4249	0.4259	0.4269
0.16	0.4279	0.4288	0.4298	0.4308	0.4317	0.4327	0.4336	0.4346	0.4355	0.4364
0.17	0.4374	0.4383	0.4392	0.4401	0.4410	0.4419	0.4429	0.4437	0.4446	0.4455
0.18	0.4464	0.4473	0.4482	0.4491	0.4499	0.4508	0.4516	0.4525	0.4534	0.4542
0.19	0.4551	0.4559	0.4567	0.4576	0.4584	0.4592	0.4601	0.4609	0.4617	0.4625
0.20	0.4633	0.4641	0.4649	0.4657	0.4665	0.4673	0.4681	0.4689	0.4697	0.4705
0.21	0.4712	0.4720	0.4728	0.4736	0.4743	0.4751	0.4758	0.4766	0.4774	0.4781
0.22	0.4789	0.4796	0.4803	0.4811	0.4818	0.4825	0.4833	0.4840	0.4847	0.4855
0.23	0.4862	0.4869	0.4876	0.4883	0.4890	0.4897	0.4904	0.4911	0.4918	0.4925
0.24	0.4932	0.4939	0.4946	0.4953	0.4960	0.4966	0.4973	0.4980	0.4987	0.4993
0.25	0.5000	0.5007	0.5013	0.5020	0.5026	0.5033	0.5040	0.5046	0.5053	0.5059
0.26	0.5066	0.5072	0.5078	0.5085	0.5091	0.5097	0.5104	0.5110	0.5116	0.5123
0.27	0.5129	0.5135	0.5141	0.5147	0.5154	0.5160	0.5166	0.5172	0.5178	0.5184
0.28	0.5190	0.5196	0.5202	0.5208	0.5214	0.5220	0.5226	0.5232	0.5238	0.5243
0.29	0.5249	0.5255	0.5261	0.5267	0.5272	0.5278	0.5284	0.5290	0.5295	0.5301
0.30	0.5307	0.5312	0.5318	0.5323	0.5329	0.5335	0.5340	0.5346	0.5351	0.5357
0.31	0.5362	0.5368	0.5373	0.5379	0.5384	0.5389	0.5395	0.5400	0.5406	0.5411
0.32	0.5416	0.5422	0.5427	0.5432	0.5437	0.5443	0.5448	0.5453	0.5458	0.5464
0.33	0.5469	0.5474	0.5479	0.5484	0.5489	0.5494	0.5499	0.5505	0.5510	0.5515
0.34	0.5520	0.5525	0.5530	0.5535	0.5540	0.5545	0.5550	0.5554	0.5559	0.5564
0.35	0.5569	0.5574	0.5579	0.5584	0.5589	0.5593	0.5598	0.5603	0.5608	0.5613
0.36	0.5617	0.5622	0.5627	0.5632	0.5636	0.5641	0.5646	0.5650	0.5655	0.5660
0.37	0.5664	0.5669	0.5674	0.5678	0.5683	0.5687	0.5692	0.5696	0.5701	0.5705
0.38	0.5710	0.5714	0.5719	0.5723	0.5728	0.5732	0.5737	0.5741	0.5746	0.5750
0.39	0.5755	0.5759	0.5763	0.5768	0.5772	0.5776	0.5781	0.5785	0.5789	0.5794

APPENDIX C
Interaction Diagram: Tied Circular Column
$(f'_c = 4 \text{ kips/in}^2, \ f_y = 60 \text{ kips/in}^2, \ \gamma = 0.60)$
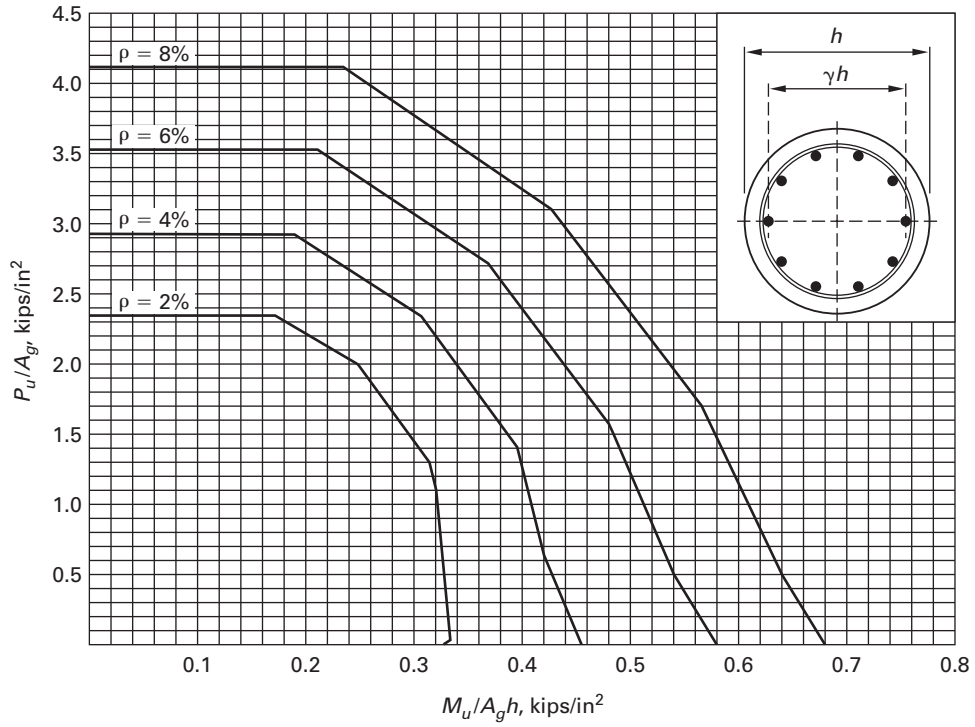
APPENDIX D
Interaction Diagram: Tied Circular Column
$(f'_c = 4 \text{ kips/in}^2,\ f_y = 60 \text{ kips/in}^2,\ \gamma = 0.75)$
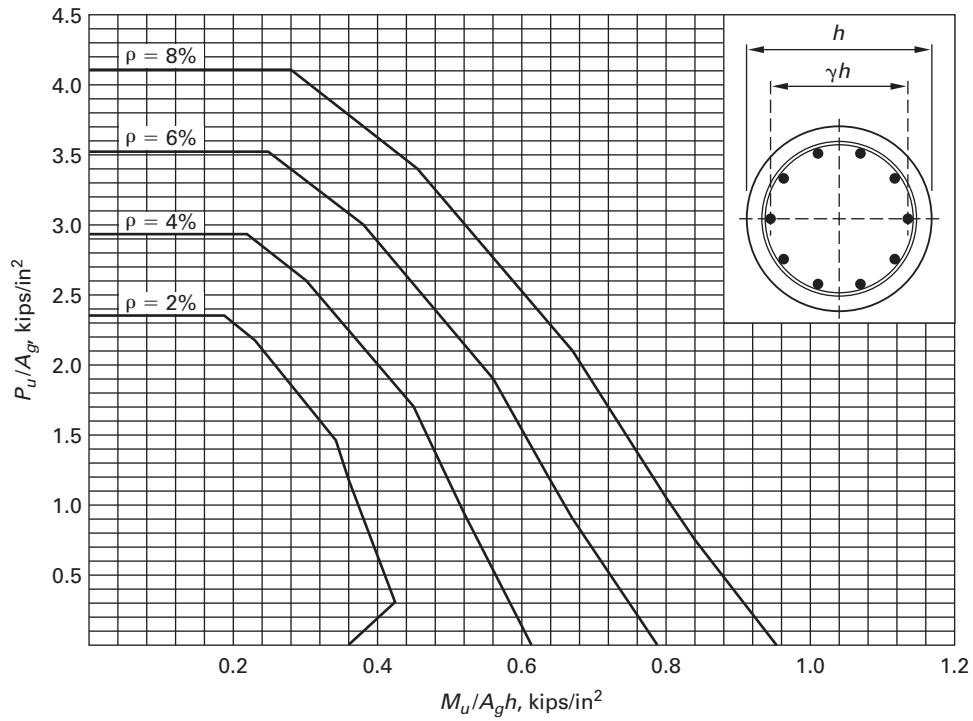
APPENDIX E
Interaction Diagram: Tied Circular Column
$(f'_c = 4 \text{ kips/in}^2, \ f_y = 60 \text{ kips/in}^2, \ \gamma = 0.90)$
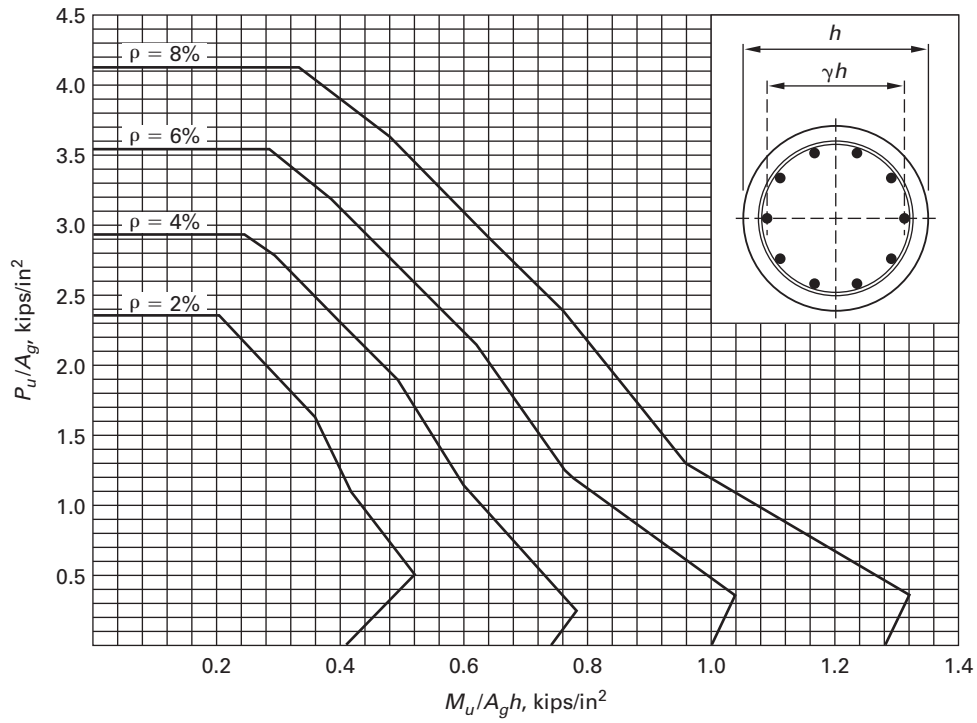
APPENDIX F
Interaction Diagram: Tied Square Column
$(f'_c = 4 \text{ kips/in}^2, \ f_y = 60 \text{ kips/in}^2, \ \gamma = 0.60)$
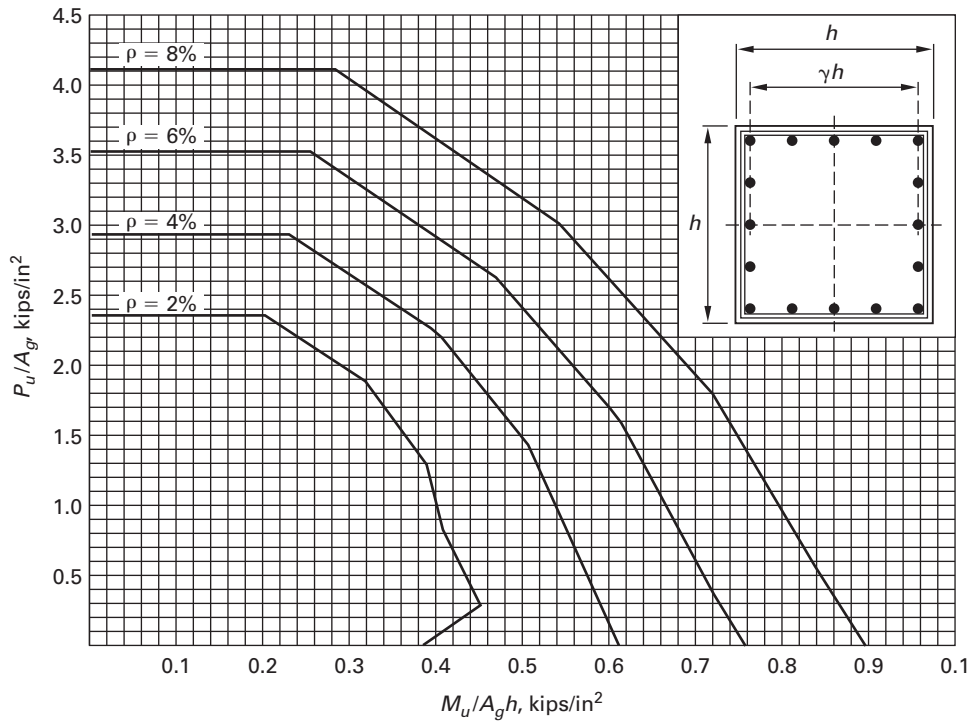
APPENDIX G
Interaction Diagram: Tied Square Column
($f'_c = 4$ kips/in^2, $f_y = 60$ kips/in^2, $\gamma = 0.75$)

APPENDIX H
Interaction Diagram: Tied Square Column
($f'_c = 4$ kips/in^2, $f_y = 60$ kips/in^2, $\gamma = 0.90$)

Index

INDEX - Z

Index of Codes

CODES - A

ACI
  App. A, 1-21, 1-23
  App. D, 1-52
  Eq. 9-1, 1-1, 1-2
  Eq. 9-2, 1-1, 1-3, 1-6, 1-7
  Eq. 9-3, 1-1, 1-3
  Eq. 9-4, 1-1, 1-3
  Eq. 9-5, 1-1, 1-2
  Eq. 9-6, 1-1, 1-3
  Eq. 9-7, 1-1, 1-2
  Eq. 9-8, 1-15
  Eq. 9-9, 1-15
  Eq. 9-10, 1-15, 3-6
  Eq. 9-11, 1-16
  Eq. 10-1, 1-33
  Eq. 10-2, 1-33, 1-34
  Eq. 10-4, 1-12, 1-13
  Eq. 10-5, 1-31
  Eq. 10-6, 1-33
  Eq. 10-7, 1-33, 1-35
  Eq. 10-11, 1-36
  Eq. 10-12, 1-36
  Eq. 10-13, 1-35
  Eq. 10-15, 1-35
  Eq. 10-16, 1-35, 1-36
  Eq. 10-18, 1-36
  Eq. 10-19, 1-36, 1-37
  Eq. 10-21, 1-36, 1-37
  Eq. 11-1, 1-19, 1-20, 3-20
  Eq. 11-2, 1-19, 1-20, 3-16, 3-20
  Eq. 11-3, 1-19, 1-20, 1-50, 2-4, 2-8, 2-16,
    2-21, 2-31
  Eq. 11-5, 1-19
  Eq. 11-9, 3-15, 3-16
  Eq. 11-10, 3-16, 3-17, 3-18
  Eq. 11-11, 3-17, 3-18
  Eq. 11-12, 3-17, 3-18
  Eq. 11-13, 1-18, 1-21, 1-38, 3-15, 3-20
  Eq. 11-14, 3-15, 3-20
  Eq. 11-15, 1-19, 1-20, 1-21, 3-14,
    3-16, 3-20
  Eq. 11-16, 1-20
  Eq. 11-17, 1-20, 1-21
  Eq. 11-21, 1-27, 1-28
  Eq. 11-22, 1-28, 1-29, 3-19
  Eq. 11-23, 1-28, 1-29, 3-19
  Eq. 11-24, 1-28, 1-29, 3-19
  Eq. 11-25, 1-25, 3-27
  Eq. 11-31, 1-48, 1-49, 2-6
  Eq. 11-32, 1-48, 1-49, 2-6
  Eq. 11-33, 1-48, 1-49, 1-50, 2-6, 2-8,
    2-15, 2-30
  Eq. 11-35, 2-6
  Eq. 12-1, 1-38, 1-39, 2-5
  Eq. 12-5, 1-42
  Eq. 13-4, 1-47
  Eq. 17-1, 3-27
  Eq. 18-1, 3-9, 3-10, 3-37
  Eq. 18-2, 3-10, 3-21, 3-22
  Eq. 18-3, 3-11
  Eq. 18-4, 3-11
  Eq. A-2, 1-22
  Eq. A-3, 1-22, 1-25
  Eq. A-7, 1-23
  Eq. A-8, 1-23, 1-25
  Eq. D-2, 1-53, 1-55
  Eq. D-3, 1-53
  Eq. D-4, 1-54, 1-56
  Eq. D-5, 1-53
  Eq. D-6, 1-56
  Eq. D-7, 1-54
  Eq. D-8, 1-54
  Eq. D-13, 1-54, 1-56
  Eq. D-14, 1-54
  Eq. D-16, 1-54
  Eq. D-28, 1-56
  Eq. D-29, 1-56, 1-58
  Eq. D-30, 1-57
  Eq. D-31, 1-57
  Eq. D-32, 1-56
  Eq. D-33, 1-57
  Eq. D-34, 1-57
  Eq. D-36, 1-57
  Eq. D-39, 1-57
  Eq. D-40, 1-57
  Eq. D-41, 1-57, 1-58
  Eq. D-42, 1-58, 1-59
  Fig. R9.3.2, 1-5
  Fig. R10.10.1.1, 1-32
  Sec. 2.1, 3-9
  Sec. 3.3.2, 2-6
  Sec. 7.6.1, 2-6
  Sec. 7.6.5, 2-6
  Sec. 7.7, 1-55
  Sec. 7.7.1, 2-6
  Sec. 7.10, 1-30
  Sec. 7.10.4.2, 1-31
  Sec. 7.10.4.3, 1-31
  Sec. 7.10.5, 1-30
  Sec. 7.10.5.1, 1-33
  Sec. 7.10.5.2, 1-33
  Sec. 7.12, 1-6, 1-39
  Sec. 7.12.2, 2-4, 2-5, 2-9, 2-12, 2-17, 2-31
  Sec. 7.12.2.1, 2-6
  Sec. 7.12.2.2, 2-6
  Sec. 7.13, 1-39
  Sec. 8.3.3, 1-60, 1-62, 2-27
  Sec. 8.5, 1-15, 3-23, 3-24, 3-26
  Sec. 8.5.1, 1-35
  Sec. 8.6.1, 3-1, 3-2
  Sec. 8.12, 3-25
  Sec. 9.2, 2-1, 2-2, 2-25
  Sec. 9.2.1, 1-1, 2-27
  Sec. 9.3, 1-3, 2-30
  Sec. 9.3.2.1, 1-5, 3-8
  Sec. 9.3.2.2, 1-5, 3-8
  Sec. 9.3.2.6, 1-22, 1-24
  Sec. 9.5.2.3, 1-14, 3-7
  Sec. 9.5.2.5, 1-14
  Sec. 9.5.4.2, 3-6
  Sec. 10.2, 1-4, 1-21, 1-42, 2-4
  Sec. 10.2.3, 1-4
  Sec. 10.2.7.3, 3-10, 3-37
  Sec. 10.3.3, 1-5, 3-8
  Sec. 10.3.4, 1-4, 1-7, 3-8, 3-9, 3-10,
    3-12, 3-37
  Sec. 10.3.5, 1-5, 1-7
  Sec. 10.3.6.1, 1-33
  Sec. 10.3.6.2, 1-33
  Sec. 10.5, 1-6, 1-7
  Sec. 10.5.1, 2-21, 2-22
  Sec. 10.5.3, 2-17, 2-21, 2-22
  Sec. 10.5.4, 2-4, 2-6
  Sec. 10.6, 1-12, 1-13
  Sec. 10.6.4, 3-6
  Sec. 10.6.7, 1-13, 3-6
  Sec. 10.7.1, 1-21
  Sec. 10.9, 1-30
  Sec. 10.9.3, 1-30
  Sec. 10.10.1, 1-33, 1-36, 1-37
  Sec. 10.10.1.2, 1-32, 1-33
  Sec. 10.10.5.1, 1-32
  Sec. 10.10.5.2, 1-32
  Sec. 10.10.6, 1-34, 1-35
  Sec. 10.10.6.3, 1-32
  Sec. 10.10.6.4, 1-35
  Sec. 10.10.7, 1-36
  Sec. 10.10.7.4, 1-36
  Sec. 10.14, 2-10, 2-11
  Sec. 10.14.1, 2-9, 2-10
  Sec. 11.1.1, 3-15
  Sec. 11.1.2, 3-15
  Sec. 11.1.3, 3-14
  Sec. 11.1.3.1, 2-3
  Sec. 11.2, 2-3
  Sec. 11.2.1.1, 1-18, 1-48
  Sec. 11.2.2, 1-18
  Sec. 11.3.2, 3-15
  Sec. 11.3.3.1, 3-16
  Sec. 11.3.3.2, 3-16, 3-17
  Sec. 11.4.1.2, 1-20
  Sec. 11.4.3, 3-15
  Sec. 11.4.5, 3-15
  Sec. 11.4.5.1, 1-19, 1-21
  Sec. 11.4.5.3, 1-19, 1-20
  Sec. 11.4.6, 3-15
  Sec. 11.4.6.1, 1-19
  Sec. 11.4.6.3, 1-18, 1-19
  Sec. 11.4.7.2, 1-18, 3-14
  Sec. 11.4.7.4, 1-20
  Sec. 11.4.7.5, 1-20
  Sec. 11.4.7.9, 1-18, 1-20, 3-14
  Sec. 11.5.1, 1-27, 1-28, 3-19
  Sec. 11.5.1.1, 3-18
  Sec. 11.5.2.2, 1-28, 3-19
  Sec. 11.5.2.5, 3-19
  Sec. 11.5.3.6, 1-27, 3-19
  Sec. 11.5.3.7, 1-28, 3-19
  Sec. 11.5.5.2, 1-28, 3-19
  Sec. 11.5.5.3, 1-28
  Sec. 11.5.6.1, 1-28, 1-29, 3-19
  Sec. 11.5.6.2, 1-28, 3-19
  Sec. 11.6.4, 1-26, 3-27
  Sec. 11.6.4.3, 1-25, 3-27
  Sec. 11.7.1, 1-21
  Sec. 11.7.2, 1-21
  Sec. 11.7.3, 1-21
  Sec. 11.7.4, 1-21
  Sec. 11.8, 1-25
  Sec. 11.8.3.1, 1-26
  Sec. 11.8.3.2, 1-25, 1-26
  Sec. 11.8.3.2.1, 1-25
  Sec. 11.8.3.4, 1-25, 1-26
  Sec. 11.8.3.5, 1-26
  Sec. 11.8.4, 1-26, 1-27
  Sec. 11.8.5, 1-26
  Sec. 11.11.1.2, 1-48, 2-6
  Sec. 11.11.2.1, 1-48, 1-49, 2-6, 2-14
  Sec. 11.11.7.1, 2-6
  Sec. 12.1.2, 1-38
  Sec. 12.2.2, 1-38
  Sec. 12.2.3, 1-38
  Sec. 12.2.4, 1-38, 1-39
  Sec. 12.2.5, 1-38, 1-39
  Sec. 12.3.1, 2-11
  Sec. 12.3.2, 1-40, 1-45, 2-10
  Sec. 12.3.3, 1-40, 1-45
  Sec. 12.3.3(a), 1-40
  Sec. 12.3.3(b), 1-40
  Sec. 12.4, 1-40
  Sec. 12.4.1, 1-38
  Sec. 12.4.2, 1-38
  Sec. 12.5.1, 1-41
  Sec. 12.5.2, 1-25, 1-40, 1-41
  Sec. 12.5.3, 1-41
  Sec. 12.10.3, 1-41, 1-42
  Sec. 12.10.4, 1-41, 1-42
  Sec. 12.10.5, 1-41
  Sec. 12.11, 1-39
  Sec. 12.11.1, 1-42, 1-43
  Sec. 12.11.3, 1-42, 1-43
  Sec. 12.12, 1-43
  Sec. 12.14, 1-43
  Sec. 12.15, 1-39, 1-43
  Sec. 12.16.1, 1-44, 1-45
  Sec. 12.16.2, 1-44
  Sec. 12.17.2, 1-44
  Sec. 12.17.2.4, 1-45
  Sec. 13.2, 1-47
  Sec. 13.2.4, 3-18
  Sec. 13.4, 1-48
  Sec. 13.5.3.2, 2-6
  Sec. 13.6, 1-61
  Sec. 13.6.1, 1-46
  Sec. 13.6.2, 1-47, 1-48
  Sec. 13.6.2.5, 1-48
  Sec. 13.6.3, 1-47, 1-48
  Sec. 13.6.4, 1-47, 1-48
  Sec. 13.6.5, 1-47
  Sec. 13.6.6, 1-47, 1-48
  Sec. 13.6.7, 1-47
  Sec. 14.3, 2-23
  Sec. 14.3.3, 2-31
  Sec. 15.4.2, 2-4
  Sec. 15.4.4.2, 2-11, 2-13
  Sec. 15.5.2, 2-3, 2-6
  Sec. 15.8, 2-9
  Sec. 15.8.2.1, 2-10
  Sec. 17.2.4, 3-28

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