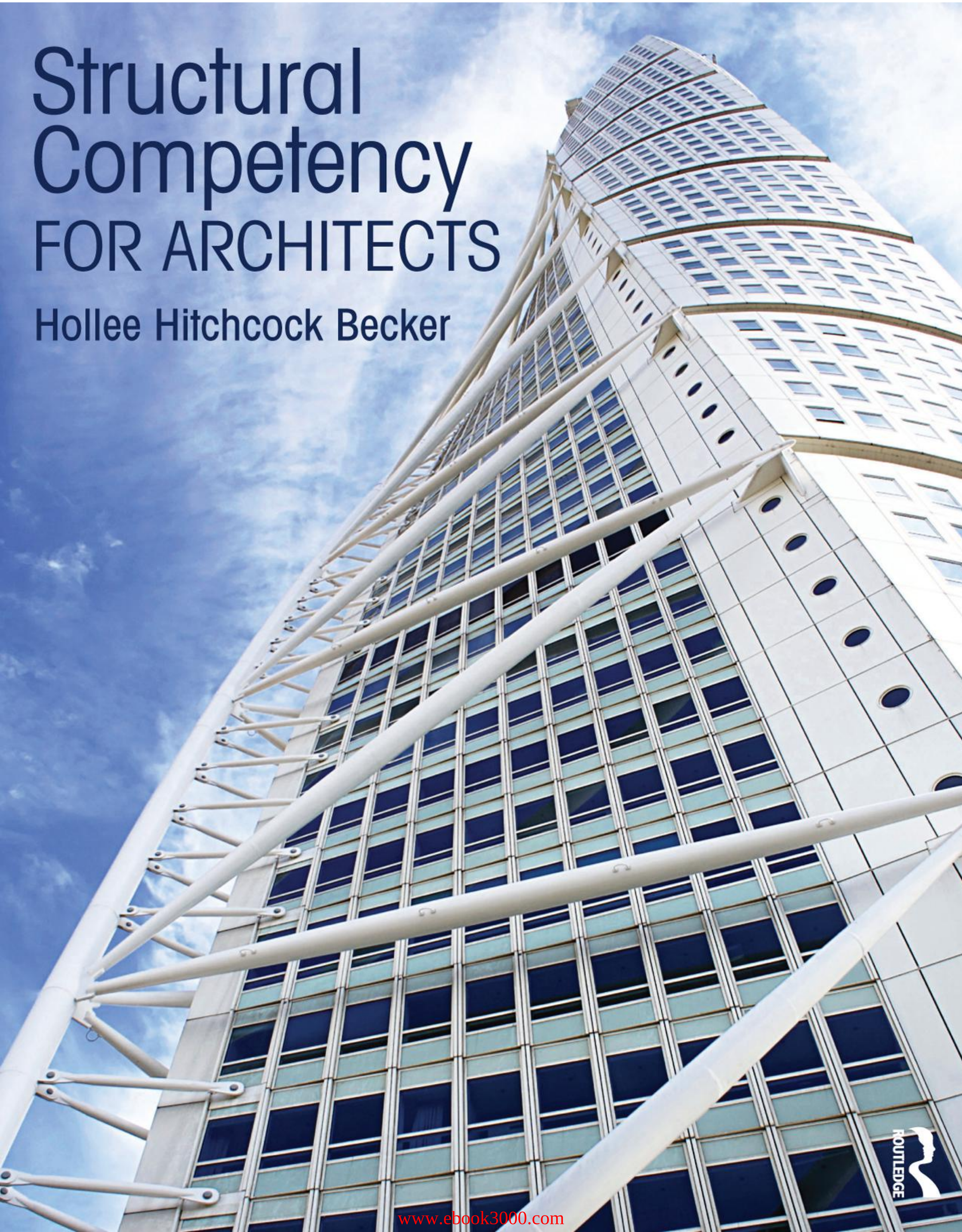
Structural Competency FOR ARCHITECTS

Hollee Hitchcock Becker



ROUTLEDGE

Structural Competency for Architects

Structural Competency for Architects is a comprehensive volume covering topics from structural systems and typologies to statics, strength of materials, and component design. The book includes everything you need to know about structures for the design of components, as well as the logic for design of structural patterns, and selection of structural typologies.

Organized into six key modules, each chapter includes examples, problems, and labs, so that you learn the fundamentals. *Structural Competency for Architects* will also help you pass your registration examinations.

Hollee Hitchcock Becker is an Assistant Professor at the Catholic University of America in Washington, DC where she teaches Structures and Environmental Design. With a BSCE in engineering and a Masters of Architecture, as well as a 30-year career in engineering, business and education, she understands the differences in learning styles between architects and engineers.

Hollee's research includes the structural possibilities of laminated veneer bamboo and replacement structures for at-risk or destroyed housing. She is also developing a Pattern Typology for the integration of structural systems with design intent.

‘Architects learn structural calculations in school, but have little opportunity to practice those skills in daily work. Becker’s book is the clearest text I have seen to help practicing architects remember “How do I do that?” *Structural Competency for Architects* offers step-by-step instructions to address all the common structural design tasks involved in typical buildings.’

Jeremy Fretts, Senior Project Architect, Niles Bolton Associates

‘This is a comprehensive text that addresses every aspect of structural design in one volume […] It is a welcome addition to the literature of structure in architecture.’

Deborah Oakley, Associate Professor, School of Architecture, University of Nevada

Structural Competency for Architects

Hollee Hitchcock Becker

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For Lydia – my sunshine on a rainy day.

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Contents

Introduction

A structure is an assembly of interrelated components that serve a common purpose. Structure may present itself with a hierarchy of components as in the structure of a corporation or rely on the patterns and relationships between similar components as in the structure of molecules. In Architecture, structure is a system of interrelated components that is capable of supporting itself and transferring all loads safely to the ground.

Architects and indeed all designers should understand structures in order to communicate effectively with contractors and consultants or to design component sizes. But the most important reason to understand structures is to express the design intent or concept through the structure. Only by understanding how different structural types and materials behave will the structural system become fully integrated with the design intent.

In this book, the basic concepts of statics and strength of materials are presented first, followed by discussion of structural systems. This order allows the reader to understand how components of various systems behave in terms of the stresses they receive. After discussion of structural types, design methods for components for specific materials of wood, steel and concrete are presented.

If chemical and heat reactions are ignored, there are five basic ways to physically break an object:

1. Tension—pulling
2. Compression—pushing, crushing, squeezing
3. Flexure—bending
4. Shear—chopping, cutting, slicing, punching through
5. Torsion—twisting.

Other types of failure are a refined definition based on these basic five types. Metal fatigue, for example, is caused by the repeated bending in alternating opposite directions.

Try this experiment: Collect five identical pieces of chalk, five identical rubber bands and five identical paper clips. Test each of the three objects for tension, compression, flexure, shear and torsion by trying to break one of the identical objects by pulling, another by crushing, etc. What is noticed about the behavior of chalk compared to rubber?

The forces and reactions in tension, compression, flexure, shear and torsion are determined by statics. Statics is the physical state in which all components are at rest and in equilibrium. How or when or if a component will fail under a particular force or stress depends on the properties of the material from which it is made; the strength of the material.

This book is intended to be a simple explanation of the structural problems architecture students, designers and architects may encounter whether designing in steel, wood, concrete or an alternate material.

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Part I

Statics and Strength of Materials

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one

Finding Reactions

Newton's Three Laws of Motion:

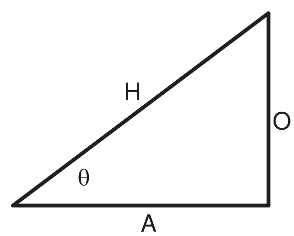
1. A body at rest will remain at rest and a body in motion will continue uniformly in a straight line unless acted upon by a force.
2. F = ma: that is, the rate of change of momentum (mv) is equal to the force producing it and in the direction of that force.
3. Every force acting upon a body at rest has an equal and opposite reaction.

Newton's third law of motion is the basis for static structural analysis. For a structure to remain static, that is, at rest and not in motion, the sum of all forces must equal zero. This means that any force applied to a component must be resisted by that component with an equal and opposite force. In order to do that, the structural component will internalize the force and transfer it to a support or another component of the structural system. The force will be transferred from component to component until it reaches the ground.

1.1 Vectors

It is important to understand basic trigonometric functions in order to work with vectors.

Below is a quick review.



1.1
Basic trigonometric functions

Basic trigonometric functions:

$\sin\theta = O/H$, $\cos\theta = A/H$ and $\tan\theta = O/A$

$O = H\sin\theta$ and $A = H\cos\theta$

1.1.1 Vectors

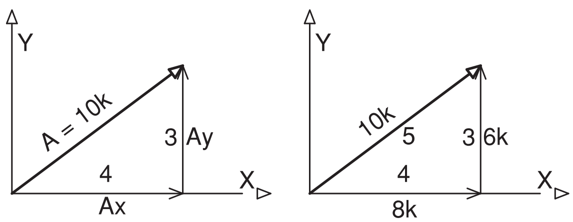
Loads or forces in architecture are described in terms of vectors. There are three necessary components that define a vector:

1. Origin or starting point
2. Direction
3. Magnitude.

The origin is the point of contact. Vector direction is expressed by its x and y relationships. Normal convention for vector direction is that a vector moving to the right is +X, a vector moving to the left is –X, a vector moving up is +Y and a vector moving down is –Y.

A vector direction can be expressed by its x and y relationships or by its angle from an axis. When a vector is expressed in terms of rise and run the ratio of the X and Y components to the full vector magnitude are equal to the ratio of the rise and run of the direction to the hypotenuse they create. This is important to remember, because it allows the vector components or magnitude to be found when only partial information is available.

Example 1-1: Find the X and Y components of the force vector A = 10k with a rise/run of $\frac{3}{4}$.



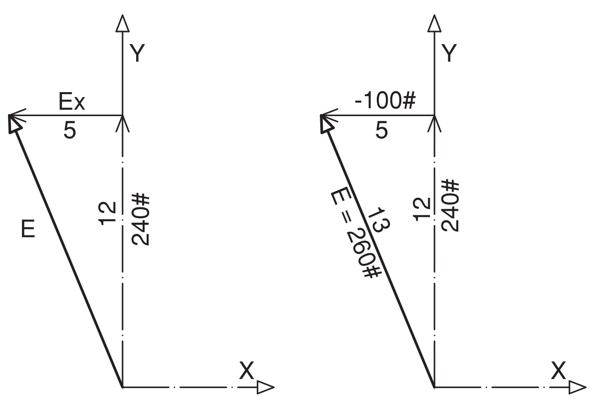
1.2

Vector components defined by rise and run

$$\frac{10k}{5} = \frac{A_y}{3} = \frac{A_x}{4} \ldots A_x = \left(\frac{10k}{5}\right)(4) = 8 \text{ and } A_y = \left(\frac{10k}{5}\right)(3) = 6k$$

Notice that vector components are tip to tail; directed so that they form an alternate route from the origin to the endpoint, indicating component directions.

Example 1-2: Find magnitude of vector E if E_y = 240.



1.3

Vector magnitude defined by rise/run

1. Determine the hypotenuse of the triangle:

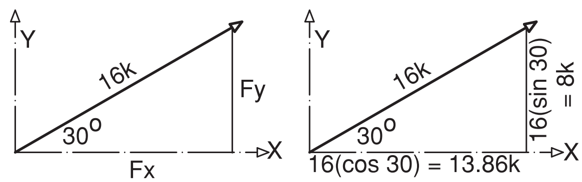
$$H = \sqrt{(5^2 + 12^3)} = 13$$

2. Use ratios to determine the vector component E:

$$\frac{E}{13} = \frac{240\#}{12} = \frac{-E_x}{5} \ldots E = \left(\frac{240\#}{12}\right)(13) = 260$$

Example 1-3: When a vector is expressed in terms of its angle relative to an axis, use trigonometric functions to determine the components.

The 16K force is in a direction 30° above the positive y. Because sin30° and cos30° are known, the ratio of sine or cosine to the whole is equal to the ratio of F_y or F_x to the vector force F = 16k.



1.4

Vector components defined by angle

$$\frac{F_x}{\sin 30} = \frac{16k}{1} \ldots F_x = 16k(\sin 30) = 16k(0.5) = 8k$$

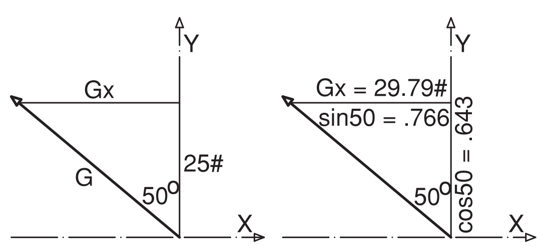
$$\frac{F_y}{\cos 30} = \frac{16k}{1} \ldots F_y = 16k(\cos 30) = 16k(0.866) = 13.86k$$

If the vector direction is expressed in terms of the angle from the Y-axis, the results will be the same.

$$\frac{F_x}{\cos 60} = \frac{16k}{1} \ldots F_x = 16k(0.5) = 8k$$

$$\frac{F_y}{\sin 60} = \frac{16k}{1} \ldots F_y = 16k(0.866) = 13.86k$$

Example 1-4: The ratios of sine and cosine can be used to find a vector force magnitude when only the angle from an axis and one of the components are known.



1.5

Vector magnitude defined by angle

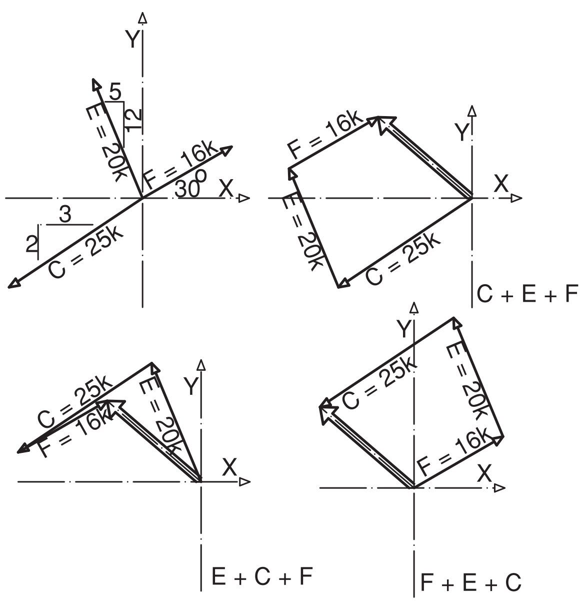
Find magnitude of force G and the horizontal component G_x if the vector G is directed 50° left of the positive Y-axis and the vertical component G_y = 25#.

$$\frac{G}{1} = \frac{25\#}{\cos 50} = \frac{25\#}{0.643} = 38.89\#$$

$$\frac{25\#}{\cos 50} = G_x \sin 50 \ldots G_x = (25\#)\left(\frac{0.766}{0.643}\right) = 29.79\#$$

1.1.2 Adding Vectors

The sum of vectors passing through a common point is called a resultant vector. Vectors traveling through a common point may be added graphically by connecting vectors tip to tail, in any order, starting at the origin, then finding the resultant vector by drawing a line from the origin to the endpoint. The independence of order is demonstrated in Figure 1.6 by adding three vectors in different orders. The resultant vector is always the same. Although an easy way to check an answer, it is only as accurate as the scale of drawing.



1.6
Graphically added vectors

To add vectors mathematically:

1. Break each vector into X and Y components.
2. Sum X direction components; sum Y direction components.
3. Find magnitude of resultant vector by using Pythagorean's theorem.

$$F = \sqrt{(\Sigma f_x)^2 + (\Sigma f_y)^2}$$

4. Find direction of the resultant vector relative to the X-axis by using:

$$\tan^{-1}\left(\frac{f_y}{f_x}\right)$$

Example 1-5: Adding vectors.

Add the three vectors in Figure 1.6:

1. Find vector components:

$$C_x = \left(\frac{25k}{\sqrt{13}}\right)(-3) = -20.80k$$

$$C_y = \left(\frac{25k}{\sqrt{13}}\right)(-2) = -13.87k$$

$$E_x = (20k)\left(\frac{-5}{13}\right) = -7.69k$$

$$E_y = (20k)\left(\frac{12}{13}\right) = -18.46k$$

$$F_x = 16\cos 30 = 13.86k$$

$$F_y = 16\sin 30 = 8.00k$$

2. Sum the X components and sum the Y components.

$$\Sigma f_x = -20.8 - 7.69 + 13.86 = -14.63k$$

$$\Sigma f_y = -13.78 + 18.46 + 8.00 = 12.59k$$

3. Resultant magnitude:

$$R = \sqrt{(14.63^2 + 12.59^2)} = 19.30k$$

4. Resultant direction:

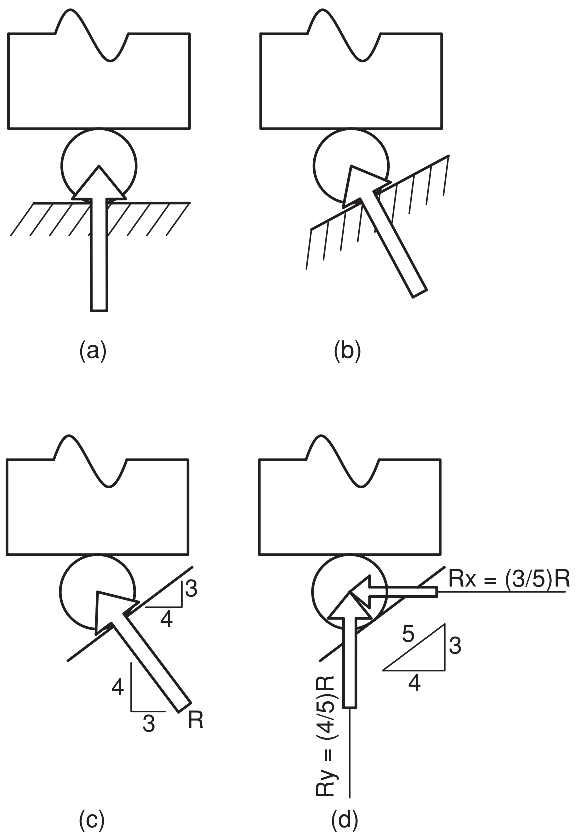
$$\theta = \tan^{-1}\left(\frac{12.59}{-14.63}\right) = -40.71 \text{ or } 40.71°$$

above the negative X-axis.

1.2 Supports

There are three basic types of supports to consider: rollers, pins and fixed connections.

Rollers: The reaction is a force through the roller center perpendicular to the surface on which the roller sits, whether horizontal (a) or at an angle (b).



1.7

Roller support

If the slope of the reaction surface is in terms of a rise (Y) over a run (X), then the slope of the reaction vector, which is perpendicular to the surface, has a rise (X) over a run (Y). Knowing this, the reaction vector components can be calculated.

If the roller support rests on a surface with a rise/run of $\frac{3}{4}$, the slope of the vector R is $\frac{4}{3}$.

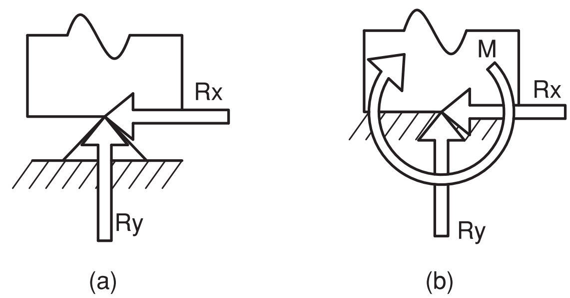
1. Determine the hypotenuse of the triangle:

$$h = \sqrt{3^2 + 4^2} = 5$$

2. Use ratios to determine the vector components A_x and A_y:

$$\frac{R}{5} = \frac{R_y}{4} = \frac{R_x}{3} \ldots R_x = \left(\frac{R}{5}\right)(3) = 0.6R \text{ and } R_y\left(\frac{R}{5}\right)(4) = 0.8R$$

Pinned support: The reaction is a force through the pin in a direction opposite to the resultant of forces applied to the pin. It is important to remember that both pins and rollers are free to rotate. Because of this they do not transfer any rotational force called a moment through the support.



1.8

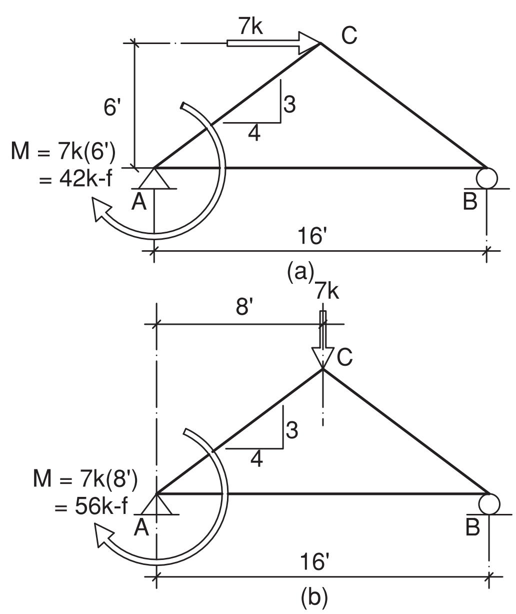
Pinned support (a) and fixed support (b).

Fixed support: A fixed support has a reaction in a direction opposite to the resultant of forces applied. Unlike a pinned support, a fixed support resists rotation and has a moment reaction equal to the moment applied to the support, but in an opposite direction.

1.3 Moments

Moment: $M_A = F(d)$

A moment about some point A is caused by a force, F, acting at a perpendicular distance, d, to the point. The units for a moment are: kip-feet (k-f), kip-inches (k-in), pound-feet (lb-ft) or pound-in (lb-in). Convention for the direction of moment is positive for a clockwise rotation and negative for a counter-clockwise rotation.



1.9

Direction affects moment

The rigid frame in Figure 1.9 has a horizontal 7k force applied at point C. The perpendicular distance between the line of that force and point A is 6′.

$$M_A = f(d) = 7k(6') = 42k\text{-}f$$

The rotation is clockwise, which is considered positive, therefore M_A = 42k-f.

The moment about point B (M_B) is also 42k-f because the perpendicular distance between the line of force and Point B remains 6′. The direction is still clockwise.

By rotating the 7k (b) applied at point C, the moment about point A (M_A) changes because the perpendicular distance between the line of the force and point A changes.

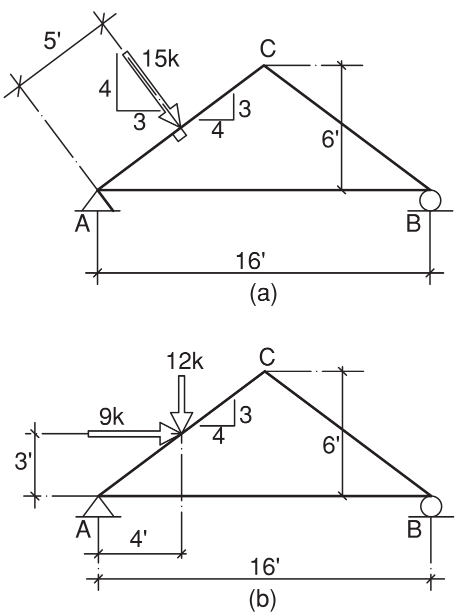
$$M_A = 7k(8') = 56k\text{-}f \text{ clockwise} = 56k\text{-}f$$

$$M_B = 7k(8') = 56k\text{-}f \text{ counter-clockwise} = -56k\text{-}f$$

The direction of a moment can be easily shown by holding a pencil loosely at the point of rotation and pushing in the direction of the applied force. The pencil will rotate in the direction of the moment.

Example 1-6: Summing moments.

The 15k force is applied perpendicular to and at the center of the AC leg. Find the moment about point A and B.



1.10
Summing moments

M_A can be solved easily because the force is perpendicular to the leg AC.

$$\Sigma M_A = 0 = 15k(5') = 75k\text{-}f$$

The 15k must be broken into components to solve for M_B.

$$15k\left(\frac{4}{5}\right) = 12k\downarrow \text{ and } 15k\left(\frac{3}{5}\right) = 9k \rightarrow$$

$$\Sigma M_B = 0 = -12k(12') + 9k(3') = -117k\text{-}f$$

counter-clockwise

1.4 Reactions

Structure transmits loads to the ground through a series of reactions to applied forces. Before any component can be designed to handle the transfer of applied loads, the reactions at the support(s) must be found.

To solve for reactions:

1. Identify the unknowns
2. Break all forces into X and Y components
3. Sum the forces and moments at the supports:

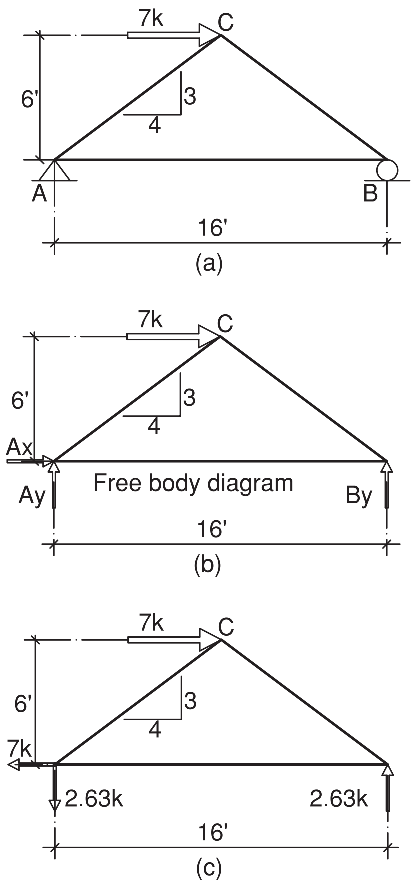
$$\Sigma M = 0,\ \Sigma f_y = 0,\ \Sigma f_x = 0$$

1.4.1 Concentrated Loads

A concentrated load is a load that is applied at a single point. It is handled as a vector with magnitude (force in lb or K), direction and origin (the point at which it is applied).

Example 1-7: Finding reactions.

1. Identify the unknowns: The support at point A is a pin and therefore may have a reaction in the X and Y directions (A_x and A_y). A pin cannot resist rotation and therefore has no moment transfer. The support at point B is a roller and therefore the only reaction is a force perpendicular to the support surface (B_y). The free body diagram (b) shows applied forces and unknown reactions.
2. Break all forces into X and Y components: The applied force is a horizontal force; it does not have a Y component.
3. Sum the forces and moments at the supports: Start by summing the moments about a pin.



1.11

Example 1–7

$\Sigma M = 0 = A_x\,(0') + A_y(0') + 7k(6') - B_y(16')$

$0 = 42k\text{-}f - B_y(16')$

$B_y = 42k\text{-}f/16' = 2.63k$

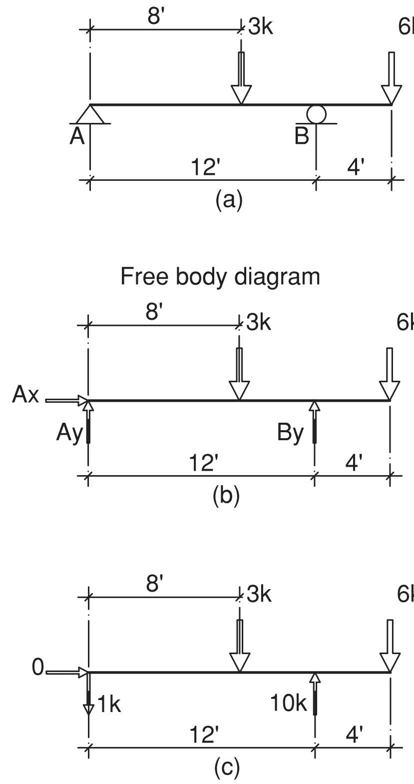
$\Sigma f_y = 0 = A_y + B_y = A_y + 2.63k$

$A_y = -2.63k$. Because the answer is negative and A_y was assumed to be positive, the answer is $A_y = 2.63k \downarrow$

$\Sigma f_x = 0 = 7k + A_x$

$A_x = -7k$. Because the answer is negative and A_x was assumed to be to the right, the answer is $A_x = 7k \leftarrow$

Example 1-8: Find reactions in a 12′ beam with a 4′ overhang.



1.12

Example 1-8

1. The unknowns at pin A are A_x and A_y. The unknown at the roller B is B_y.
2. Since the only applied forces are vertical forces, there is no need to break them into X and Y components.
3. Sum moments about the pin

$\Sigma M_A = 0 = A_x(0') + A_y(0') + 3k(8') - B_y(12') + 6k(16')$

$0 = 24k\text{-}f - B_y(12') + 96k\text{-}f$

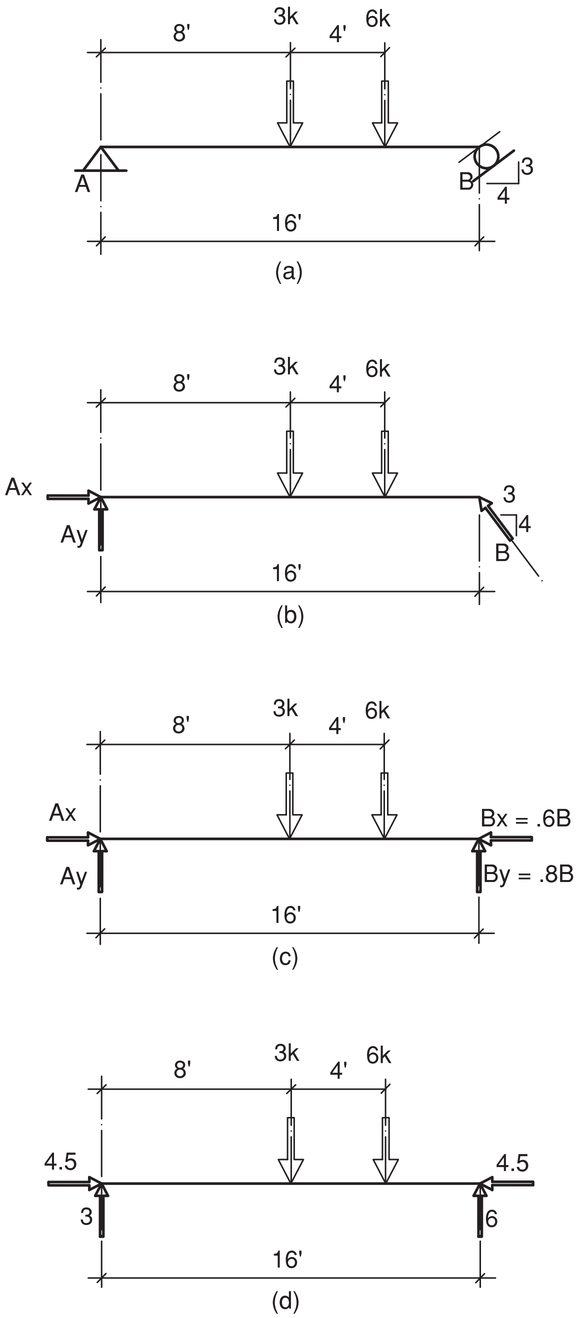
$B_y = 120k\text{-}f/12' = 10k = 10k\uparrow$

$f_y = 0 = A_y + B_y - 3k - 6k = A_y + 10k - 9k$

$A_y = -1k\uparrow$. Because the answer is negative and A_y was assumed to be up, the answer is $A_y = 1k\downarrow$

$\Sigma f_x = 0 = A_x \ldots A_x = 0$

Example 1-9: Find reactions for a 16′ beam with a roller on an angle at B.



1.13

Roller on an angle

1. The unknowns at pin A are A_x and A_y. The unknown at the roller B is a vector B in a direction perpendicular to the surface on which the roller sits. Surface slope = $\frac{3}{4}$ … vector slope = $\frac{4}{3}$. Because we know the direction of the vector in terms of the rise and run, we can break it into its components.

$$\sqrt{3^2+4^2}=5$$

$$\frac{B_y}{B}=\frac{4}{5}\ldots B_y=\frac{4B}{5}=0.8B$$

$$\frac{B_x}{B}=\frac{4}{5}\ldots B_x=\frac{3B}{5}=0.6B$$

2. No forces need breaking into X and Y components.
3. Sum the forces and moments at the supports.

$$\Sigma M_A = 0 = A_x(0') + A_y(0') + 3k(8') + 6k(12') - B_y(16')$$

$$0 = 24k\text{-f} + 72k\text{-f} - B_y(16')$$

$$B_y = 96k\text{-f}/16' = 6k = 6k\uparrow$$

Using the ratios of the vector B:

$$\frac{6}{B}=\frac{4}{5}\ldots B=\frac{6(5)}{4}=7.5k$$

$$B_x = 0.6B = 0.6(7.5) = 4.5k$$

$$\Sigma f_y = 0 = A_y + B_y - 3k - 6k = A_y + 6k - 9k$$

$$A_y = 3k = 3k\uparrow$$

$$\Sigma f_x = 0 = A_x - B_x = A_x - 4.5k$$

$$A_x = 4.5k = 4.5k \rightarrow$$

1.4.2 Distributed Loads

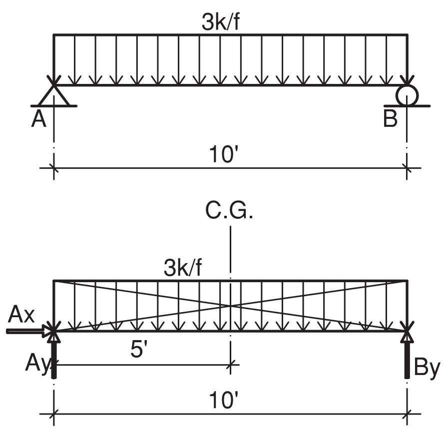
A distributed load is exactly what it sounds like. It is a load distributed over a length and it is expressed in terms of the force per unit of length. Distributed loads may be uniform or non-uniform. Uniform loads are distributed evenly across a portion of a member. As such there are two parameters that define the load condition: the length over which it is distributed and w, the force per unit of length, usually in units of k/f or #/f.

To find the moment about a point caused by a uniform load:

1. Calculate the total load: multiply w by the length of load application.
2. Find the center of gravity for the load. This occurs at the center of the length of load application.
3. Calculate the moment caused by the uniform load by multiplying the total load from step one by the distance

from the point of interest to the center of gravity located in step two.

Example 1-10: Finding reactions with a uniform load.



1.14

Reactions for a uniform load

The uniform load, w = 3k/f, the applied length is 10′

The total load, W = 3k/f(10′) = 30k

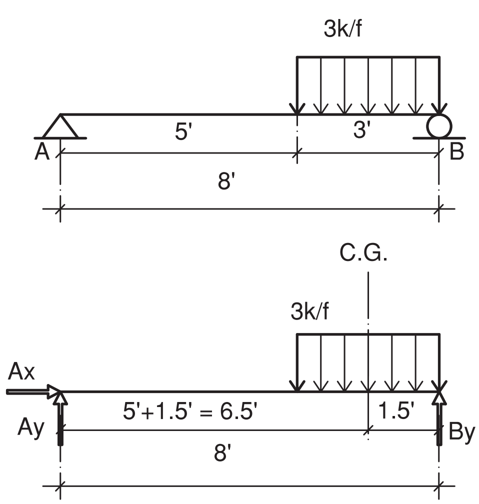
The distance from the center of gravity to point A is 5′ or half of the applied length.

$\Sigma M_A = 0 = 30k(5') - B_y(10') \ldots B_y = 15k$

$\Sigma F_y = 0 = A_y - 30k + 15k \ldots A_y = 15k$

$\Sigma F_x = 0 = A_x$

Example 1-11: Finding reactions with a partial uniform load.



1.15

Reactions for a partial uniform load

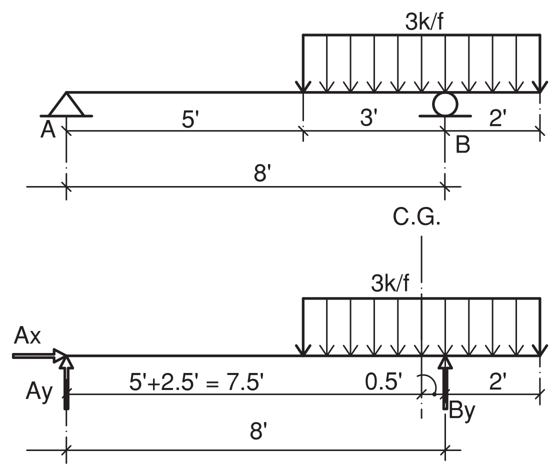
The uniform load is w = 3k/f. The applied length is 3′. The total load, W = (3k/f)(3′) = 9k. The distance from the center of gravity to point A is the applied load plus half of the applied length = 5′ + 3′/2 = 6.5′ from point A.

$\Sigma M_A = 0 = 9k(6.5') - B_y(8') \ldots B_y = 7.31k$

$\Sigma F_y = 0 = A_y - 9k + 7.31k \ldots A_y = 1.69k$

$\Sigma F_x = 0 = A_x$

Example 1-12: Reactions for a partial uniform load and overhang.



1.16

Reactions for a partial uniform load with an overhang

w = 3k/f

W = 3k/f(5′) = 15k

The distance from the center of gravity to point A is the applied load plus half of the applied length = 5′ + 5′/2 = 7.5′ from point A.

$\Sigma M_A = 0 = 15k(7.5') - B_y(8') \ldots B_y = 14.06k$

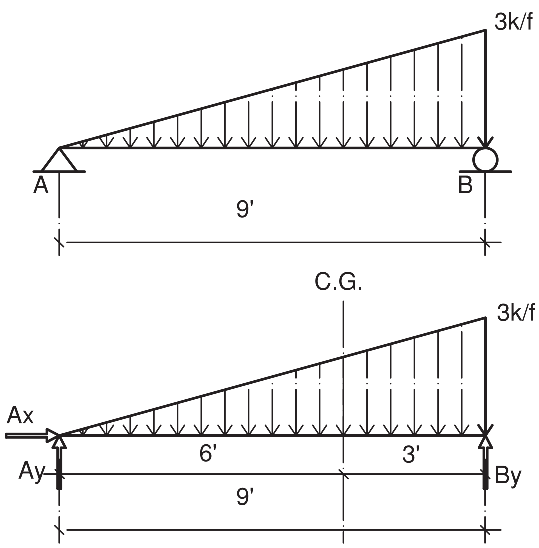
$\Sigma F_y = 0 = A_y - 15k + 14.06 \ldots A_y = 0.94k$

$\Sigma F_x = 0 = A_x$

Not all distributed loads are uniform loads. To find the total load and the center of gravity when the distributed load is non-uniform, the geometry of the shape of the load must be considered.

Example 1-13: Finding reactions with a triangular load.

One example of a non-uniformly distributed load is a triangular load, named for its geometric shape. The area of a triangle is equal to half the base times the height. The center of gravity of a triangle is located one third of the base length from the heaviest end.



1.17

Triangular load

$$W = \frac{(3k / f)(9')}{2} = 13.5k$$

The center of gravity is $\frac{1}{3}$ of the applied length from the heavy end or 3′ from point B. Therefore, the distance from the center of gravity to point A is 9′ – 3′ = 6′.

$\Sigma M_A = 0 = 13.5k(6') - B_y(9') \ldots B_y = 9k$

$\Sigma F_y = 0 = A_y - 13.5k + 9k \ldots A_y = 4.5k$

$\Sigma F_x = 0 = A_x$

Example 1-14: Finding reactions with multiple distributed loads.

Another example of a non-uniformly distributed load is a load that varies linearly from one amount at one end to another amount at the other end as shown in Figure 1.18.

Break the load into one uniform load and one triangular load:

The uniform load is 3k/f. The applied length is 6′.

$W_1 = (3k/f)(6') = 18k$

The center of gravity is 3′ from point A.

The triangular load tapers from 3k/f to 0.

The applied length is 6′. $W_2 = ((3''k/f'')(6'))/2 = 9k$

The center of gravity is 2′ from point A.

$\Sigma M_A = 0 = 18k(3') + 9k(2') - B_y(9') \ldots B_y = 8k$

$\Sigma F_y = 0 = A_y - 18k - 9k + 8k \ldots A_y = 19k$

$\Sigma F_x = 0 = A_x$

Practice Exercises:

1-1 through 1-3: Find the resultant vector magnitude and direction for the forces shown in the diagrams in Figure 1.19.

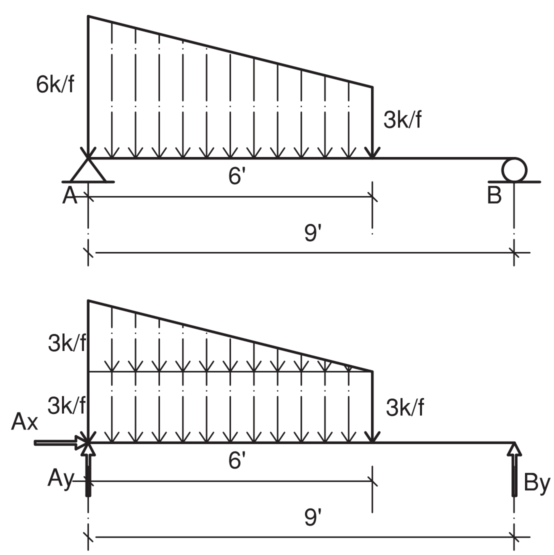
1-4: Find the moment about point A.

1-5: Find the moment about support A.

1-6: Find the moment caused by the force:

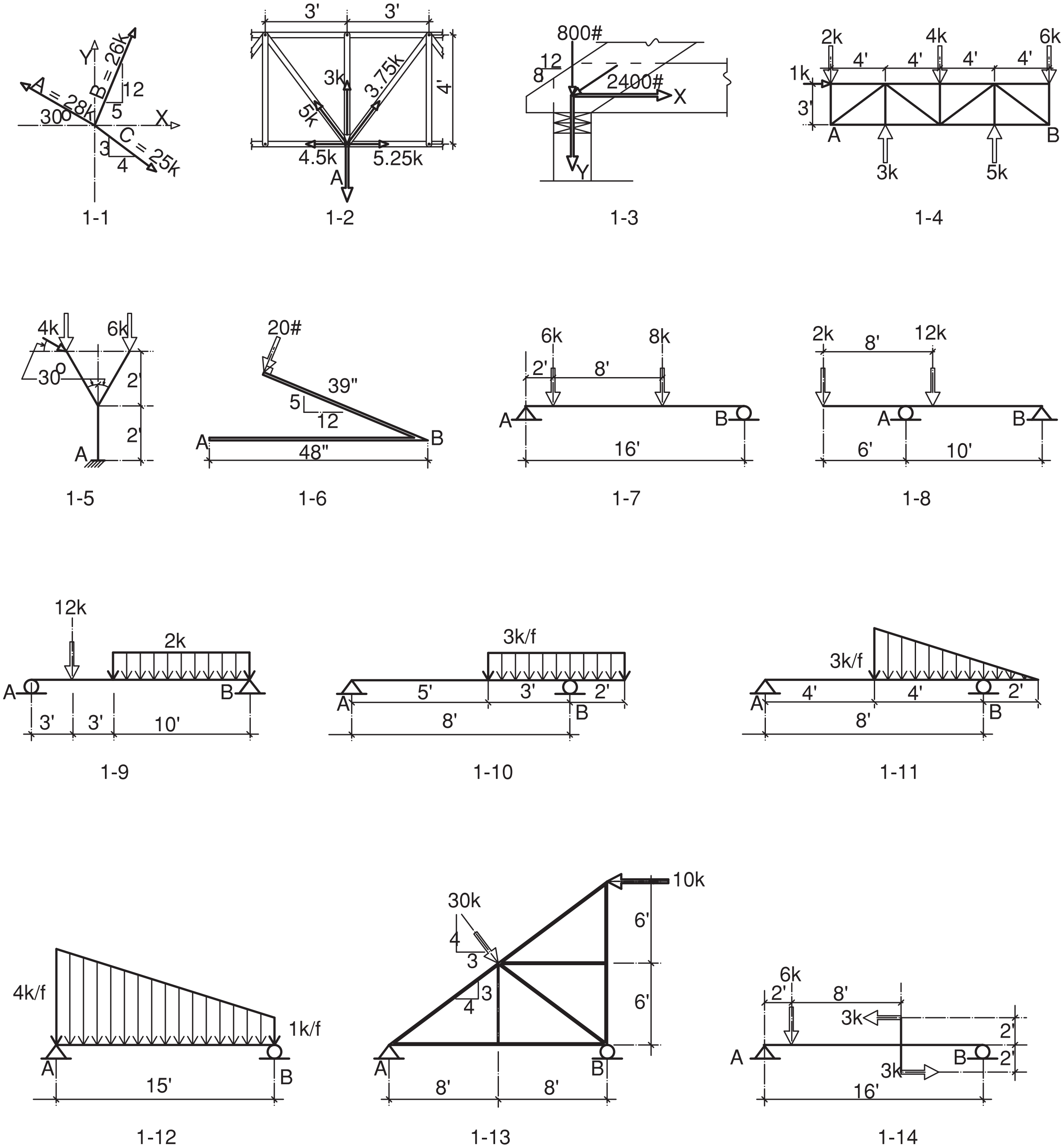
a) about point A
b) about point B.

1-7 through 1-14: Find the reactions for the forces applied.



1.18

Combined distributed loads



1.19

Chapter 1 Practice exercises.

two

Bar Forces in Trusses

A true truss is a stable configuration of bars connected by pinned joints. Because the joints are pinned, no moment is transferred along a bar. Therefore, the direction of any bar force is along its axis. Each bar transfers an axial force in either compression or tension. Bar forces in compression have arrows pointing away from each other ← → and bar forces in tension are indicated by arrows pointing toward each other → ←.

Truss analysis assumes four things:

1. All members are linear.
2. Members are pinned connected at the ends.
3. The weight of the members is neglected.
4. Loads are only applied at the joints.

2.1 Method of Joints

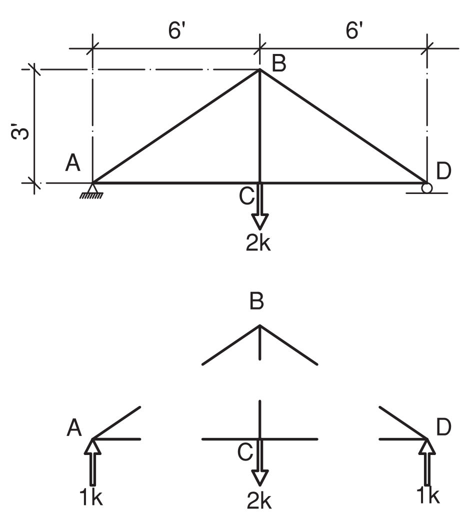
The Method of Joints uses the logic that if a joint is isolated by cutting through the bars, the joint remains in equilibrium due to the bar forces.

To use the Method of Joints:

1. Solve for reactions at the supports.
2. Break the truss into individual joints.
3. Sum the forces in the x and y directions for each joint. $\Sigma f_x = 0$, $\Sigma f_y = 0$. Note the bar forces on the other side of the break as equal in force and opposite in direction.
4. Find resultant bar forces:

$$F = \sqrt{\left(f_x^2 + f_y^2\right)}$$

Example 2-1.



2.1

Break truss into individual joints

1. Solve for reactions:

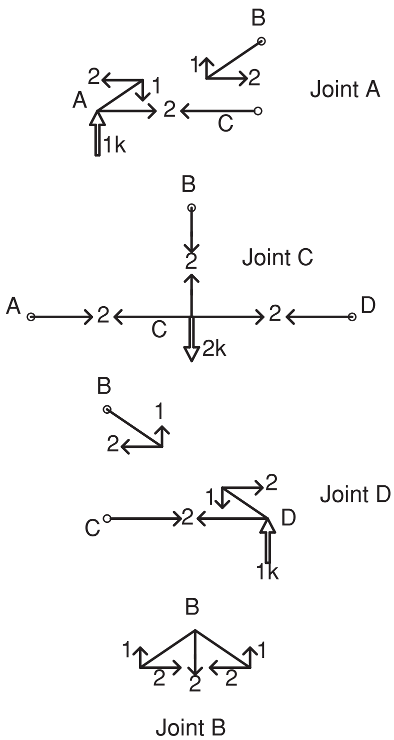
   Unknowns = A_x, A_y, D_y

   $\Sigma M_A = 0 = 2k(6') - D_y(12') \ldots D_y = 1k$

   $\Sigma f_y = 0 = A_y - 2k + 1k \ldots A_y = 1k$

   $\Sigma f_x = 0 = A_x$

2. Break truss into individual joints:



2.2

Sum forces at each joint

3. Sum the forces in the x and y directions for each joint. $\Sigma f_x = 0$, $\Sigma f_y = 0$.

Joint A:

$\Sigma f_y = 0 = 1k - AB_y \ldots AB_y = 1k\downarrow$

$\frac{AB_y}{AB_x} = \frac{3'}{6'} \ldots AB_x = 2k \leftarrow$

$\Sigma f_x = 0 = -2k + AC_x \ldots AC_x = 2k\rightarrow$

Joint C:

$\Sigma f_y = 0 = -2k + BC_y \ldots BC_y = 2k\uparrow$

$\Sigma f_x = 0 = -2k + CD_x \ldots CD_x = 2k\rightarrow$

Joint D:

$\Sigma f_y = 0 = 1 - BD_y \ldots BD_y = 1\downarrow$

$\Sigma f_x = 0 = -2 + BD_x \ldots BD_x = 2\rightarrow$

Joint B: Once all of the bar forces are found, the last joint can be checked to ensure equilibrium.

$\Sigma f_x = 0 = 1k - 2k + 1k \ldots$ okay

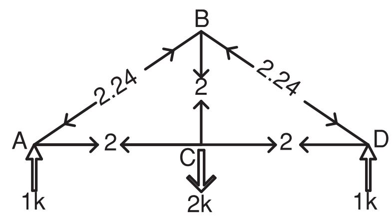
$\Sigma f_x = 0 = -2k + 2k \ldots$ okay

4. Find resultant bar forces:

$F = \sqrt{(f_x^{\,2} + f_y^{\,2})}$

$AB = BD = \sqrt{(22 + 12)} = 2.24$ Compression $\leftarrow\ \rightarrow$

$AC = CD = BC = 2$ Tension $\rightarrow\ \leftarrow$



2.3

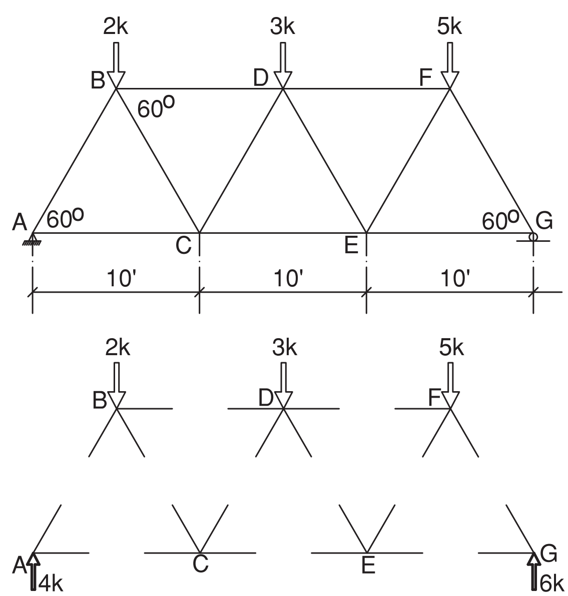
Find total bar forces

Some trusses have diagonals set at an angle θ from the horizontal. For any bar for ratio:

$$\frac{f_y}{f_x} = \frac{F\sin\theta}{F\cos\theta} = \tan\theta$$

$$f_y = f_x \tan\theta \quad \text{and} \quad f_x = \frac{f_y}{\tan\theta}$$

Example 2-2.



2.4

Truss defined by angles

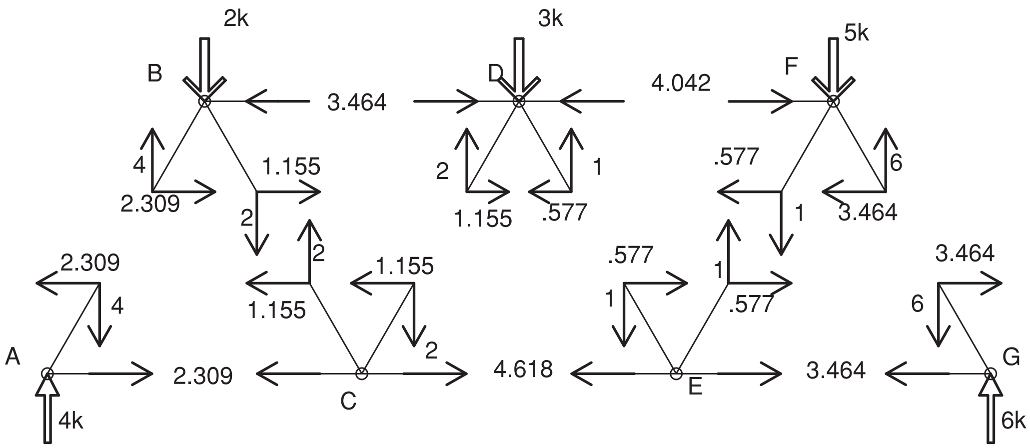
1. Solve for reactions:

   Unknowns are A_x, A_y and G_y

   $\Sigma M_A = 0 = 2k(5') + 3k(15') + 5k(25') - G_y(30') \ldots$
   $G_y = 6k$

   $\Sigma f_y = 0 = 6k - 5k - 3k - 2k + A_y \ldots A_y = 4k$

   $\Sigma f_x = 0 = A_x$



2.5
Sum forces at each joint

2. Break into individual joints:
3. Sum the forces in the x and y directions for each joint.

   $\Sigma f_x = 0, \Sigma f_y = 0.$

Joint A: $\Sigma f_y = 0 = 4k - AB_y \ldots AB_y = 4k\downarrow$

$$AB_x = \frac{AB_y}{\tan 60} = \frac{4}{1.732} = 2.309k \leftarrow$$

$\Sigma f_x = 0 = -2.309 + AC \ldots C = 2.309k\rightarrow$

Joint B: $\Sigma f_y = 0 = 4k - 2 - AC_y \ldots AC_y = 2k\downarrow$

$$AC_x = \frac{AC_y}{\tan 60} = \frac{2}{1.732} = 1.155k \leftarrow$$

$\Sigma f_x = 0 = 2.309 + 1.155 + BD \ldots BD = 3.464\leftarrow$

Joint C: $\Sigma f_y = 0 = 2k - CD_y \ldots CD_y = 2k\downarrow$

$$CD_x = \frac{CD_y}{\tan 60} = \frac{2}{1.732} = 1.155k \leftarrow$$

$\Sigma f_x = 0 = -2.309 - 1.155 - 1.155 + CE \ldots CE = 4.618k\rightarrow$

Joint D: $\Sigma f_y = 0 = 2k - 3 - DE_y \ldots DE_y = 1k\uparrow$

$$DE_x = \frac{DE_y}{\tan 60} = \frac{1}{1.732} = 0.577k \leftarrow$$

$\Sigma f_x = 0 = 3.464 + 1.155 - 0.577 - DF \ldots DF = 4.042k\leftarrow$

Joint E: $\Sigma f_y = 0 = -1k + EF_y \ldots EF_y = 1k\uparrow$

$$EF_x = \frac{EF_y}{\tan 60} = \frac{1}{1.732} = 0.577k \leftarrow$$

$\Sigma f_x = 0 = -4.618 + 0.577 + 0.577 + EG \ldots EG = 3.464k\rightarrow$

Joint F: $\Sigma f_y = 0 = -1k - 5 + FG_y \ldots FG_y = 6k\uparrow$

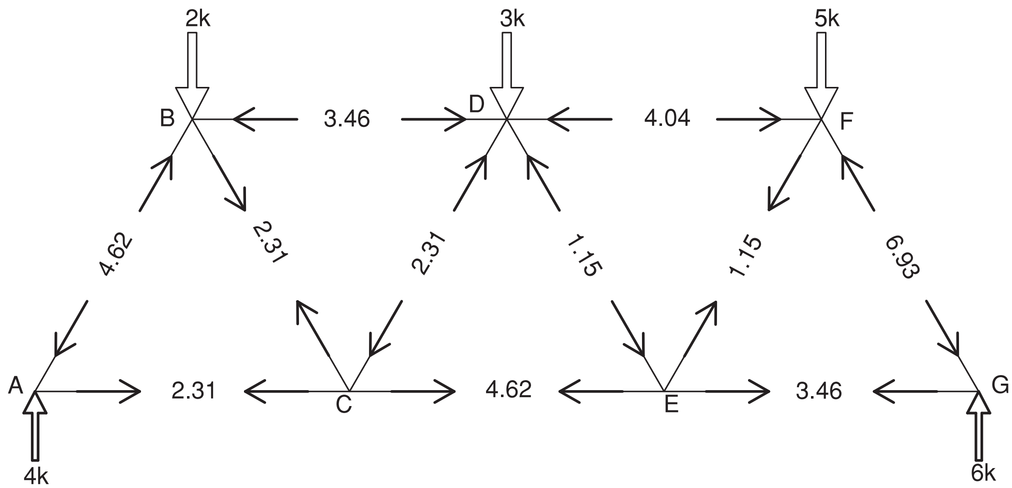
$$FG_x = \frac{FG_y}{\tan 60} = \frac{6}{1.732} = 3.464k \leftarrow$$

Joint G: $\Sigma f_y = 0 = -6k + 6k \ldots$ okay

$\Sigma f_x = 0 = 3.464k - 3.464k \ldots$ okay

4. Find resultant bar forces:

$$F = \sqrt{\left(f_x^{\,2} + f_y^{\,2}\right)}$$



2.6
Find total bar forces

C = compression, T = tension

$$AB = \sqrt{(2.309^2 + 4^2)} = 4.62k\ C$$

AC = 2.31k T

$$BC = \sqrt{(1.155^2 + 2^2)} = 2.31k\ T$$

BD = 3.46 C

$$CD = \sqrt{(1.155^2 + 2^2)} = 2.31k\ C$$

CE = 4.62 T

$$DE = \sqrt{(0.577^2 + 1^2)} = 1.15k\ C$$

DF = 4.04k C

$$EF = \sqrt{(0.5772^2 + 1^2)} = 1.15k\ C$$

EG = 3.46 T

$$FG = \sqrt{(3.464^2 + 6^2)} = 6.93k\ C$$

2.2 Method of Sections

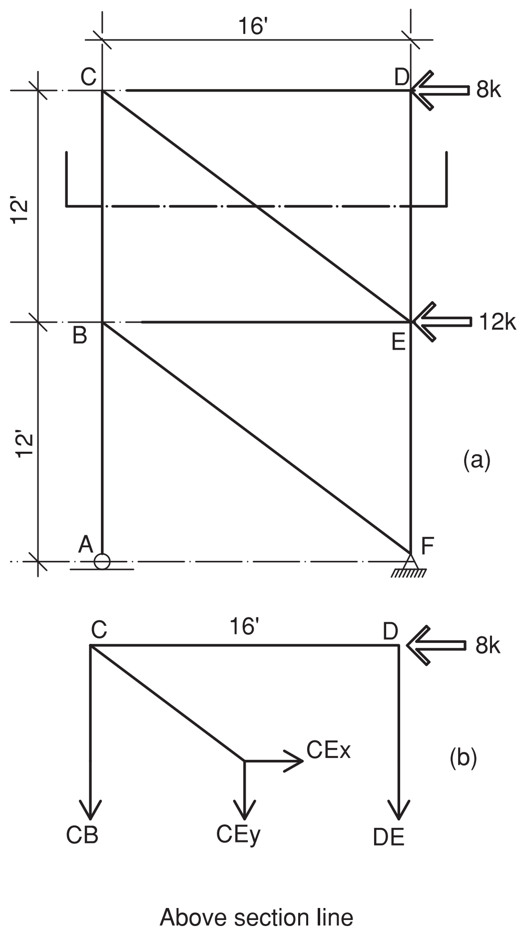
Just as any joint can be isolated and examined to be found in equilibrium, any section of a truss that is isolated will also be in equilibrium, meaning that the sum of forces and moments will equal zero. Isolating a section of a truss is very useful when only a few bar forces need to be found.

To use the Method of Sections:

1. Draw a section line through one or more bars of interest. The section line must cut through the entire truss.
2. Consider only one side of the section line.
   a. If all supports are located on one side of the section cut, then use the other side. This will eliminate the need to solve for reactions first.
   b. If all supports are not located on one side of the section line, then solve for reactions before isolating the section.
3. To each severed bar, assign a bar force variable with components, assuming a direction (AB_x, AB_y, etc.)
4. Solve for the bar forces using $\Sigma M = 0$, $\Sigma f_y = 0$ and $\Sigma f_x = 0$. In order to decide which equation to use, observe the isolated section. Count the number of unknown variables in the x direction. If only one unknown exists, it may be found by using $\Sigma f_x = 0$. The same is true of the y direction. If there is more than one unknown in both directions, use $\Sigma M = 0$, taking the moment about the intersection of two severed bars to find the forces in the third. It is also useful to remember that bar forces are axial and therefore the ratio of f_y/f_x = rise/run.
5. Find resultant bar forces:

$$F = \sqrt{(f_x^2 + f_y^2)}$$

Example 2-3: Find the bar forces in members CB and CE using Method of Sections.



2.7
Method of Sections

1. Draw a section line through CB and CE.
2. Consider only one side of the section line. Since all of the supports are located on the bottom, isolate the top section. This will eliminate the need to solve for reactions first.
3. The variables are CB↓, CE_x→, CE_y↓ and DE↓. All are assumed to be in tension. If the answer is negative, the direction will change and the bar force will be in compression.
4. There is only one X direction variable, CE_x, therefore use $\Sigma f_x = 0$.

$\Sigma f_x = 0 = -8k + CE_x \ldots CE_x = 8k$→

$CE_y = 12'(8k)/16' = 6k$

There are still two variables in the y direction: CB and DE. Therefore, $\Sigma f_y = 0$ cannot be used yet. Use instead, $\Sigma M = 0$.

Since CB is the variable to be found, take a moment about the point where CE and DE intersect; at point E. It does not matter that point E is not part of the section.

$\Sigma M_E = 0 = -8k(12') - CB(16') \ldots CB = -6k$ or 6k in compression

If envisioning a point not in the isolated section is difficult, sum the moments about point C, then sum vertical forces.

$\Sigma M_C = 0 = DE(16') \ldots DE = 0$

$\Sigma f_y = 0 = -6k + 0 - CB \ldots CB = -6$ or 6k in compression.

5. Find resultant bar forces:

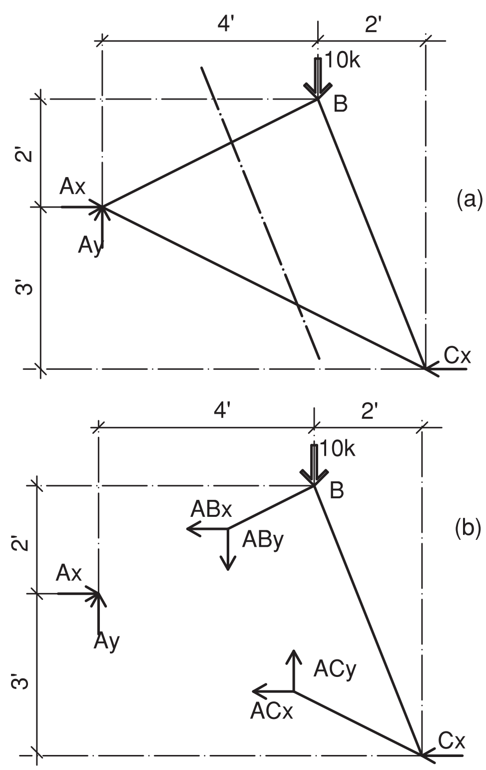
$$F = \sqrt{(f_x^2 + f_y^2)}$$

CB = 6k C

DE = 0

$$CE = \sqrt{(8^2 + 6^2)} = 10k\ T$$

Example 2-4: Find the bar forces in AC using Method of Sections.



2.8
Example 2-4

1. Draw a section line through AC. Note that wherever the section line is drawn, an isolated side will contain a support. Therefore it is necessary to solve for reactions.

$\Sigma M_a = 0 = 10k(4') + C_x(3') \ldots$
$C_x = (-40)/3 = -13.33k = 13.33k \rightarrow$

$\Sigma f_x = 0 = 13.33k + A_x \ldots A_x = -13.33k = 13.33k \leftarrow$

$\Sigma f_y = 0 = -10k + A_y \ldots A_y = 10k \uparrow$

2. Consider only one side of the section line.
3. The variables are AB_x, AB_y, AC_x and AC_y. Since there are four variables and only three available equations, the relationships between the variables must be defined.

$$\frac{AB_y}{AB_x} = \frac{2'}{4'} \ldots 2AB_y = AB_x$$

$$\frac{AC_y}{AC_x} = \frac{3'}{6'} \ldots 2AC_y = AC_x$$

4. There are two variables in each of the x and y directions, therefore use $\Sigma M = 0$. Since AC is the bar of interest, sum the moments about the only point not connected to AC, point B.

$\Sigma M_B = 0 = -13.33k(5') + 5'(AC_x) - 2'(AC_y)$ and since $2AC_y = AC_x$

$\Sigma M_B = 0 = -13.33k(5') + 5'(2AC_y) - 2'(AC_y) \ldots$
$AC_y = 8.33k$

$AC_x = 2(8.33k) = 16.66k$

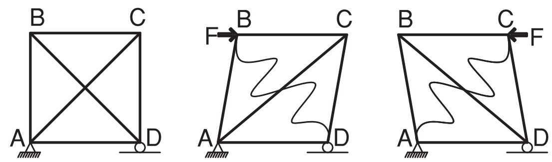
5. Find resultant bar forces:

$$F = \sqrt{(f_x^2 + f_y^2)}$$

$$AC = \sqrt{(16.66^2 + 8.33^2)} = 18.63k\ T$$

2.3 Diagonal Tension Bracing

Diagonal Tension Counters are sets of cables or slender components that stabilize a frame by acting in tension only. Although the tension counters are placed in sets, only one is active given any particular load scenario. For example, if the box is subjected to a force F from the left, the frame wants to lean towards the right. The cable AC is in tension and counteracts the force F. Cable BD cannot resist in compression and becomes inactive. If the box is subjected to a force from the right, Cable BD is active and cable AC is inactive.



2.9
Only one tension brace is active at a time

To analyze diagonal tension counters:
1. Solve for reactions.
2. Cut a section through both tension counters and assume bar fragments are in tension. Isolate one side.
3. Sum forces in the direction parallel to the section line adding only some value T_y if the line is vertical, and T_x if the line is horizontal, for the tension counter variable. Solve for T_y or T_x. A positive answer indicates T_y is up. A negative answer indicates T_y is down.
4. Choose the active tension counter by noting the direction of T_y. Solve for T_x using the ratios T_y/T_x = rise/run.
5. Find the Tension in the active tension counter using

$$T = \sqrt{(T_x^2 + T_y^2)}$$

Example 2-5: Find the tension in the active tension counters.

1. Solve for reactions.

$\Sigma M_A = 0 = 4k(4') + 6k(8') + 8k(12') - H_y(16') \ldots$
$H_y = 10k$

$\Sigma f_y = 0 = A_y - 4 - 6 - 8 + 10 \ldots A_y = 8k$

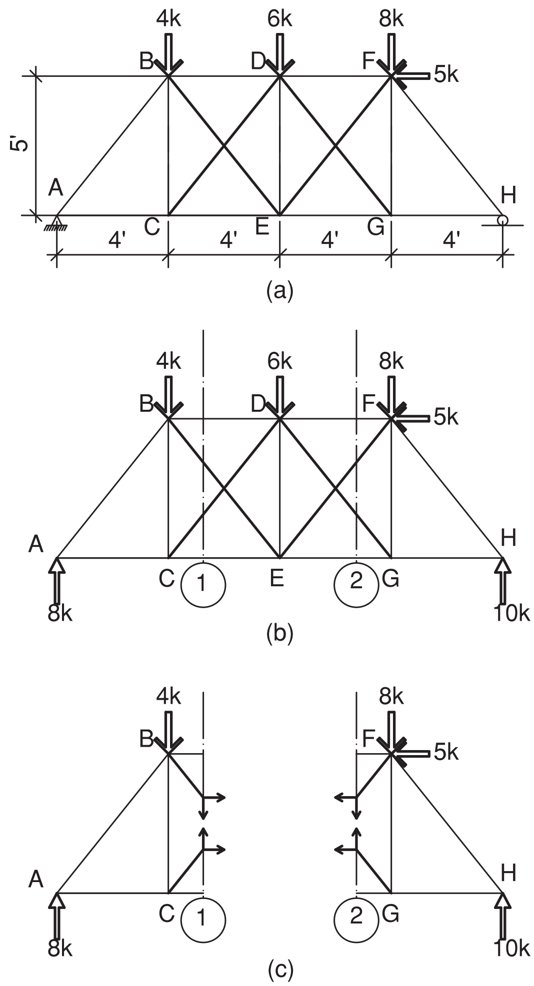
$\Sigma f_x = 0 = A_x$

2. Cut a section through both tension counters and assume bar fragments are in tension. Isolate one side.

Section 1, left side:

3. $\Sigma f_y = 0 = 8k - 4k + T_y \ldots T_y = -4k$ or $4k\downarrow$
4. BE is the active tension counter because the T_y is downward.

$$\frac{4k}{T_x} = \frac{5'}{4'} \ldots T_x = \frac{4k(4')}{5'} = 3.2k$$



2.10

Diagonal tension bracing

5. $T = \sqrt{(T_x{}^2 + T_y{}^2)} = \sqrt{(3.2^2 + 4^2)} = 5.12k$

Section 2, right side:

3. $\Sigma f_y = 0 = 10k - 8k + T_y \ldots T_y = -2k$ or $2k\downarrow$

4. EF is the active tension counter because the T_y is downward.

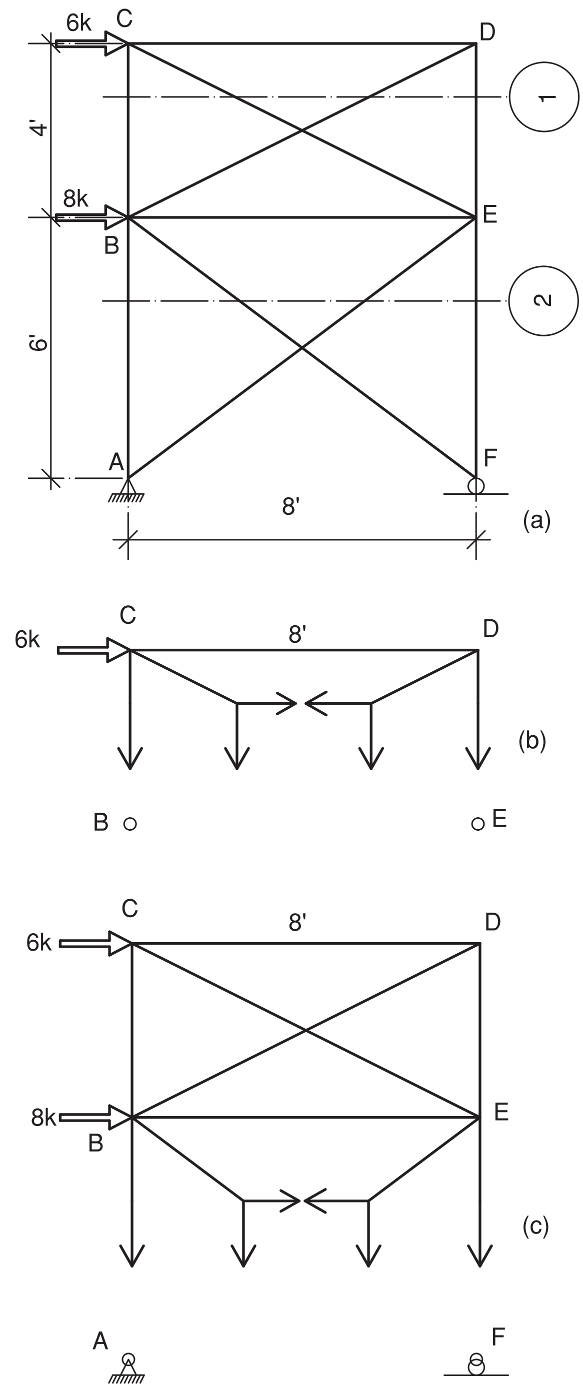
$$\frac{2k}{T_x} = \frac{5'}{4'} \ldots T_x = \frac{2k(4')}{5'} = 1.6k$$

5. $T = \sqrt{(T_x{}^2 + T_y{}^2)} = \sqrt{(1.6^2 + 2^2)} = 2.56k$

Example 2-6.

1. In this example, there is no need to solve for reactions if the area above each section line is isolated.

2. Cut a section through both tension counters and assume bar fragments are in tension. Isolate one side.



2.11

Horizontal section cuts

Section 1, top:

3. $\Sigma f_x = 0 = 6k + T_x \ldots T_x = -6k$ or $6k\leftarrow$

4. DB is the active tension counter because the T_x is toward the left.

$$\frac{6k}{T_x} = \frac{8'}{4'} \ldots T_y = \frac{6k(4')}{8'} = 3k$$

5. $T = \sqrt{(T_x^2 + T_y^2)} = \sqrt{(6^2 + 3^2)} = 6.71k$

Section 2, top:

3. $\Sigma f_x = 0 = 6k + 8k + T_x \ldots T_x = -14k$ or $14k \leftarrow$

4. EA is the active tension counter because the T_x is toward the left.

$$\frac{14k}{T_y} = \frac{8'}{6'} \ldots T_y = \frac{14k(6')}{8'} = 10.5k$$

5. $T = \sqrt{(T_x^2 + T_y^2)} = \sqrt{(14^2 + 10.5^2)} = 17.5k$

Practice Exercises:

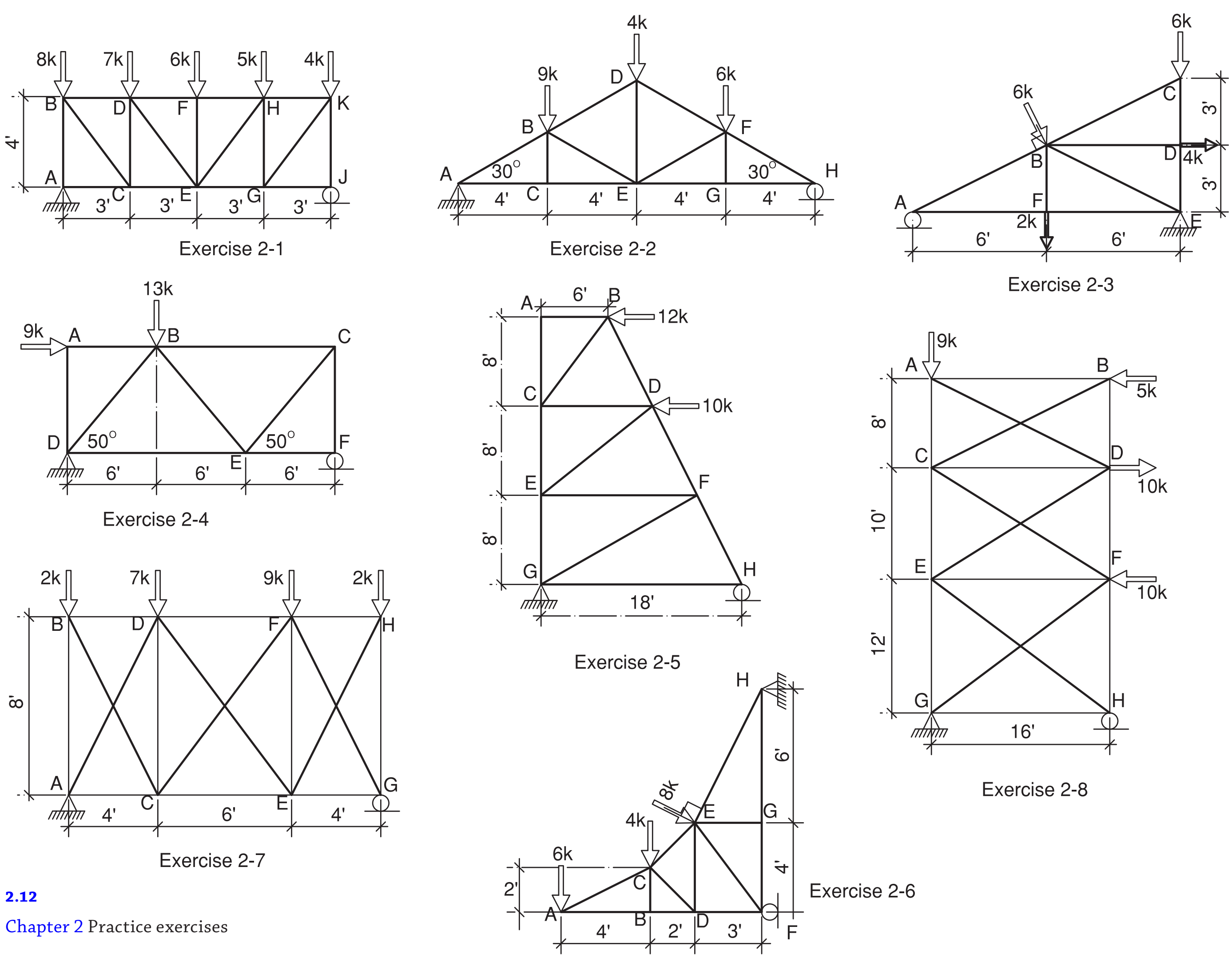
2-1 through 2-3: Solve for the bar forces using Method of Joints.

2-4: Find the axial forces in bars BE and BC using Method of Sections.

2-5: Find the axial forces in bars DE and DF using Method of Sections.

2-6: Find the axial forces in bars CE, CD and CB using Method of Sections.

2-7 through 2-8: Find the tension in the active diagonal tension counters.



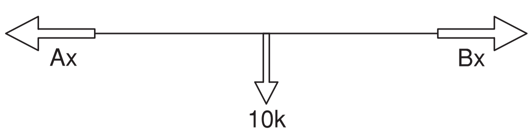
2.12
Chapter 2 Practice exercises

three

Statics in Simple Systems

3.1 Cables

Cables can only transfer load through tension.
All cables must have some sag in order to support a load. This is because the resultant force through a cable is in the direction of its axis and because a cable, in theory, cannot transfer loads through shear. Imagine a cable with no sag. The reactions at the cable supports must be in the same direction as the axis of the cable. If a cable has no sag, the direction and therefore the reactions are only in the horizontal or x direction. When forces are summed in the y direction: $\Sigma f_y = 0 = W$, therefore the load W must be 0.

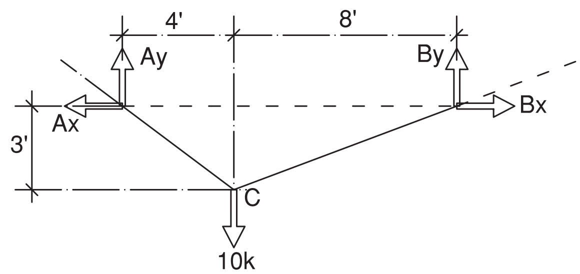


3.1
A cable must have some sag

This is because the ratio of the reactions $A_y/A_x = h/a$. If h = 0, then Ay = 0 and therefore, W = 0.

To solve for the tension in a cable when the sag is known:

Example 3-1: Find the tension in cable segments AC and CB.



3.2
Finding tension in cable segments

1. Find reactions.

$$\Sigma M_a = 0 = 10k(4') - B_y(12') \ldots B_y = 3.33k\uparrow$$

$$\Sigma F_y = 0 = A_y - 10k + 3.33k \ldots A_y = 6.67k\uparrow$$

$$\frac{B_y}{B_x} = \frac{3}{8} \ldots B_x = 3.33k\left(\frac{8}{3}\right) = 8.89k \rightarrow$$

$$\frac{A_y}{A_x} = \frac{3}{4} \ldots A_x = 6.67k\left(\frac{4}{3}\right) = 8.89k \rightarrow$$

2. Sum forces at each point of load.

$AC_x = A_x = 8.89k$

$AC_y = A_y = 6.67k$

$BC_x = B_x = 8.89k$

$BC_y = B_y = 3.33k$

Note that the force in the X-direction remains constant throughout the cable.

3. Find tension in cable legs.

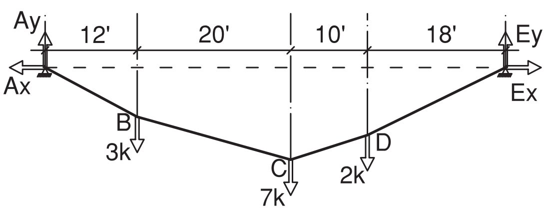
$$AC = \sqrt{(8.89^2 + 6.67^2)} = 11.12\text{k}$$

$$BC = \sqrt{(8.89^2 + 3.33^2)} = 9.50\text{k}$$

Note that the tension is greater in the segment with the steepest slope.

Example 3-2: Find the sag at points B and D and find tension in all segments of cable.

Any portion of the cable may be isolated and resultant forces found.



3.3

Finding sag in cables

$$\Sigma M_A = 0 = 3(12) + 7(32) + 2(42) - E_y(60) \ldots E_y = 5.73\text{k}\uparrow$$

$$\Sigma f_y = 0 = A_y + 5.73 - 12 \ldots A_y = 6.27\text{k}\uparrow$$

A_x and E_x cannot be solved without taking a section cut. Cut cable at point C and isolate right side:

$$\Sigma M_C = 0 = 2(10) - 5.73(28) + E_x(12) \ldots E_x = 11.71\text{k}$$

$$\frac{E_y}{E_x} = \frac{5.73}{11.71} = \frac{h_D}{18'} \ldots h_D = 8.81'$$

Consider entire cable:

$$\Sigma f_x = 0 = 11.71 - A_x \ldots A_x = 11.71\text{k}\leftarrow$$

$$\frac{A_y}{A_x} = \frac{6.27}{11.71} = \frac{h_B}{12'} \ldots h_B = 6.42'$$

$$AB_x = BC_x = CD_x = DE_x = 11.71\text{k}$$

$$\frac{AB_y}{11.71} = \frac{h_B}{12'} \ldots AB_y = 6.27\text{k} \ldots$$

$$AB = \sqrt{(6.27^2 + 11.71^2)} = 13.28\text{k}$$

$$\frac{BC_y}{11.71} = \frac{h_B - 12}{20'} \ldots BC_y = 6.27\text{k} - 3\text{k} \ldots BC_y = 3.27\text{k} \ldots$$

$$BC = \sqrt{(3.27^2 + 11.71^2)} = 12.16\text{k}$$

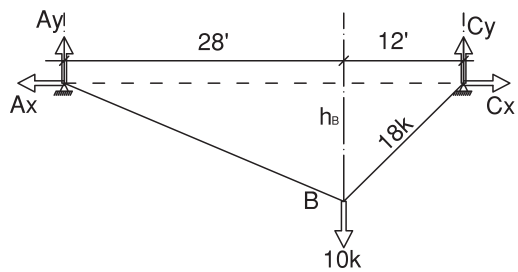
$$\frac{CD}{11.71} = \frac{h_D - 12}{20'} \ldots CD_y = 5.73 - 2 \ldots$$

$$CD = \sqrt{(3.73^2 + 11.71^2)} = 12.29\text{k}$$

$$\frac{DE}{11.71} = \frac{h_D}{12'} \ldots DE_y = 5.73 \ldots$$

$$DE = \sqrt{(5.73^2 + 11.71^2)} = 13.04\text{k}$$

Example 3-3: Find the sag, h, in the cable given the maximum cable tension, T = 18k.



3.4

Finding sag for maximum tension

$$\Sigma M_A = 0 = 28(10) - 40C_y \ldots C_y = 7\text{k}$$

$$\Sigma f_y = 0 = A_y - 10 + 7 \ldots A_y = 3\text{k}$$

The steepest slope will have the greatest tension.

Slope of AB = h/28 and Slope of BC = h/12.

Therefore, BC has the greatest tension and BC = 18k. Comparing the ratios of force to length in the triangle on the right side yields:

$$\frac{h_B}{7} = \frac{12}{BC_x} = \frac{\sqrt{(h_B{}^2 + 12^2)}}{18\text{k}}$$

$$h_B = \frac{\sqrt{(h_B{}^2 + 12^2)}}{18\text{k}} \ldots 18h_B{}^2 = 7h_B{}^2 + 168h_B + 1008 \ldots$$

$$h_B = 5.07'$$

Alternatively, BC_x can be found first and then the used in the equation:

$$\frac{h_B}{7} = \frac{12}{BC_x}$$

$$BC_x^2 + 7^2 = 18^2 \ldots BC_x = 16.58k$$

$$h_B = 7(12)/16.58 = 5.07'$$

3.2 Arches and Pinned Frames

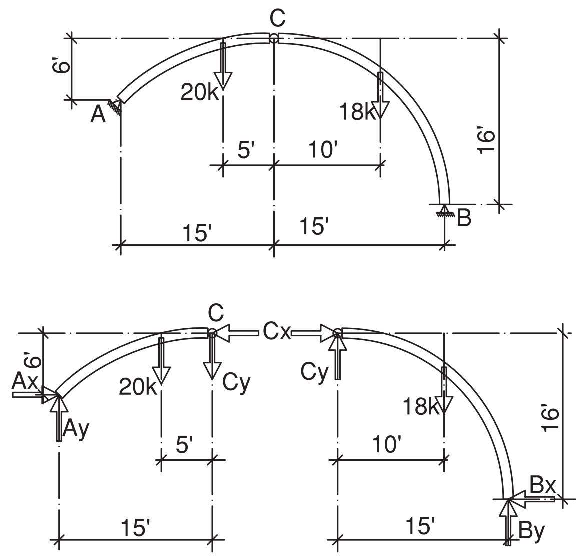
3.2.1 Hinged Arches

Three-hinge arches consist of two arched segments connected by a pin and supported by a pinned connection at each end. Because there are 4 unknowns and only 3 equations, the arch must be separated into segments to solve. Note that the pin forces on the right side are equal and opposite to the pin forces on the left side. The assignment of the direction of pin forces is arbitrary. If the wrong direction is chosen, the answer will appear to be negative, meaning that the direction is opposite of that assumed.

To analyze three-hinge arches:

1. Break into left and right segment. Assign P_x and P_y variables to either side of the pin in opposite directions.
2. Using only the left side, sum the moments about the left support. Find the P_y in terms of the P_x.
3. Using only the right side, sum the moments about the right support. Find the P_y in terms of P_x.
4. Set the P_y in terms of P_x equations from steps 2 and 3 equal to each other. Solve for P_x.
5. Using P_x, solve for P_y.
6. Using only the left side, sum y-direction forces, then x-direction forces to find reactions at the left support.
7. Using only the right side, sum y-direction forces, then x-direction forces to find reactions at the right support.

Example 3-4: Solve for the support reactions and the resultant force in the pin.



3.5

Three-hinged arch

Left side:

$$\Sigma M_A = 0 = 20k(10') - C_x(6') + C_y(15') \ldots$$

$$C_y = (6C_x - 200)/15 = 0.4C_x - 13.33$$

Right side:

$$\Sigma M_B = 0 = C_y(15') + C_x(16') - 18k(5') \ldots$$

$$C_y = (90 - 16C_x)/15 = 6 - 1.07C_x$$

$$0.4C_x - 13.33 = 6 - 1.07Cx \ldots C_x = 19.33/1.47 = 13.15k$$

$$C_y = 6 - 1.07(13.15) = -8.07k$$

Left side:

$$\Sigma f_y = 0 = A_y - 20k + 8.07k \ldots A_y = 11.93k$$

$$\Sigma f_x = 0 = A_x - 13.15k \ldots A_x = 13.15k$$

Right side:

$$\Sigma f_y = 0 = B_y - 18k - 8.07k \ldots B_y = 26.07k$$

$$\Sigma f_x = 0 = -B_x + 13.15k \ldots B_x = 13.15k$$

$$\text{Resultant pin force} = \sqrt{(13.15^2 + 8.07^2)} = 15.43k$$

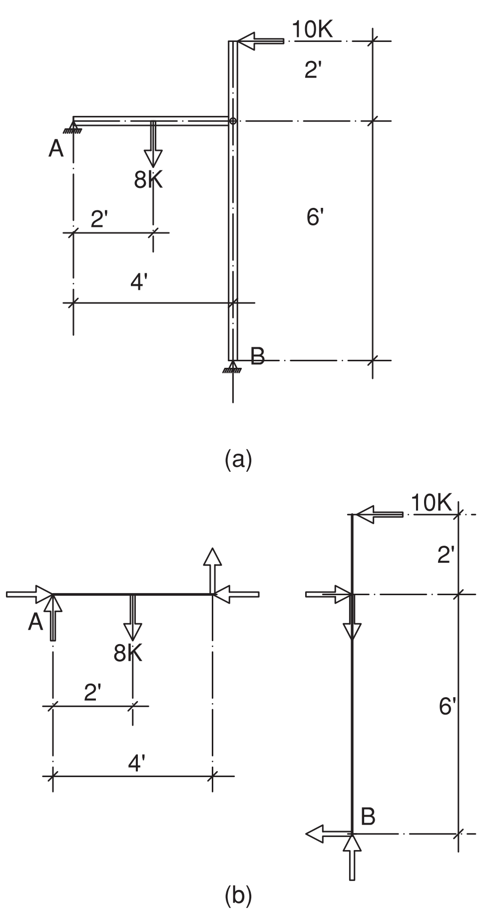
3.2.2 Pinned Frames

Unlike trusses, where bar forces are directed along the bar axis, pinned frames have bar forces that carry shear and therefore the bar force direction is unknown until analyzed.

To analyze pinned frames:

1. Find reactions at supports for entire pinned frame system, if possible.
2. Separate the frame at pins into individual members.
3. Solve for forces in each member remembering that the force at the pin in one member will be equal and opposite to the force at the same pin in the connected member.

Example 3-5: Find the support reactions and the resultant pin force.



3.6

Pinned frame

2 pinned supports = 4 unknowns. Therefore, it is impossible to solve for reactions by looking at the whole system.

Left side:

$$\Sigma M_A = 0 = 8k(2') - C_y(4') \ldots C_y = 4k\uparrow$$

$$\Sigma f_y = 0 = A_y - 8k + 4k \ldots A_y = 4k\uparrow$$

$$\Sigma f_x = 0 = A_x - C_x \ldots A_x = C_x$$

Right side:

$$\Sigma M_B = 0 = -10k(8') + C_x(6') \ldots C_x = 12.67k = 12.67k\rightarrow$$

$$\Sigma f_y = 0 = -4k + B_y \ldots B_y = 4k\uparrow$$

$$\Sigma f_x = 0 = B_x + 12.67k - 10k \ldots B_x = -2.67k = 2.67k\leftarrow$$

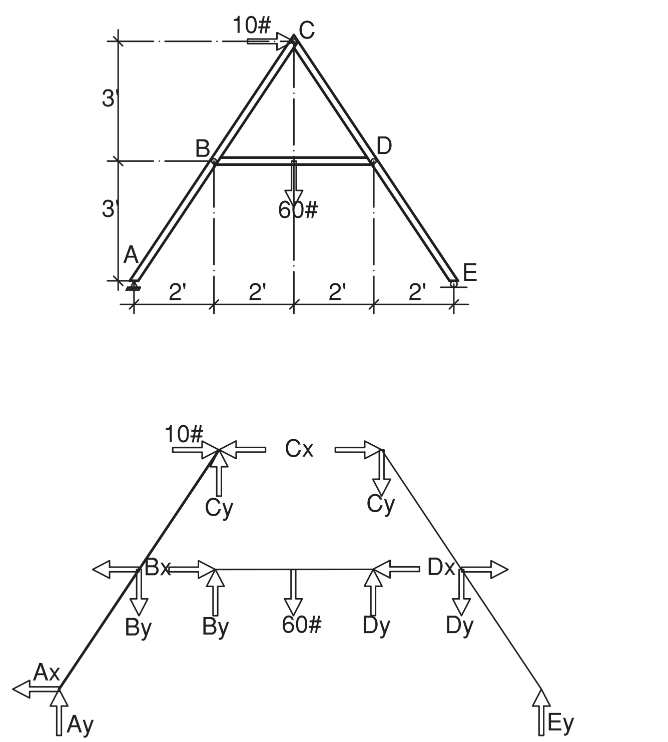
Resultant pin force:

$$C = \sqrt{(12.67^2 + 4^2)} = 13.29k$$

ANSWER:

$$A_x = 2.67k\rightarrow,\ A_y = 4k\uparrow,\ B_x = 2.67k\leftarrow,\ B_y = 4k\uparrow,\ C = 13.29k$$

Example 3-6: Find the forces in the pinned A-Frame shown in Figure 3.7.



3.7

Pinned A-frame

$\Sigma M_A = 0 = 60\#(4') + 10\#(6') - E_y(8') \ldots E_y = 37.5\#\uparrow$

$\Sigma f_y = 0 = A_y - 60\# + 37.5\# \ldots A_y = 22.5\#\uparrow$

$\Sigma f_x = 0 = 10\# - A_x \ldots A_x = 10\#\leftarrow$

Isolate bars.

Bar BD:

$\Sigma M_B = 0 = 60\#(2') - D_y(4') \ldots D_y = 30\#\uparrow$
and on bar CDE $B_y = 30\#\downarrow$

$\Sigma f_y = 0 = B_y - 60\# + 30\# \ldots B_y = 30\#\uparrow$
and on bar ABC $B_y = 30\#\downarrow$

Bar ABC:

$\Sigma M_C = 0 = 10\#(6') + 22.5\#(4') - 30\#(2') + B_x\,(3') \ldots$

$B_x = -30\# = 30\#\rightarrow$ and on bar BD, $B_x = 30\#\leftarrow$

$\Sigma f_y = 0 = 22.5\# - 30\# + C_y \ldots C_y = 7.5\#\uparrow$
and on bar CDE $C_y = 7.5\#\downarrow$

$\Sigma f_x = 0 = 10\# - 10\# + 30\# - C_x \ldots C_x = 30\# \leftarrow$
and on bar CDE $C_x = 30\#\rightarrow$

Bar CDE:

$\Sigma f_x = 0 = 30\# - D_x \ldots D_x = 30\#$

Find pin forces:

$$B = D = \sqrt{(30^2 + 30^2)} = 42.43\text{k}$$

$$C = \sqrt{(7.5^2 + 30^2)} = 30.92\text{k}$$

ANSWER:

$A_x = 10\# \leftarrow$, $A_y = 22.5\#\uparrow$, $E_y = 37.5\#\uparrow$, $B = D = 42.43\#$, $C = 30.92\#$

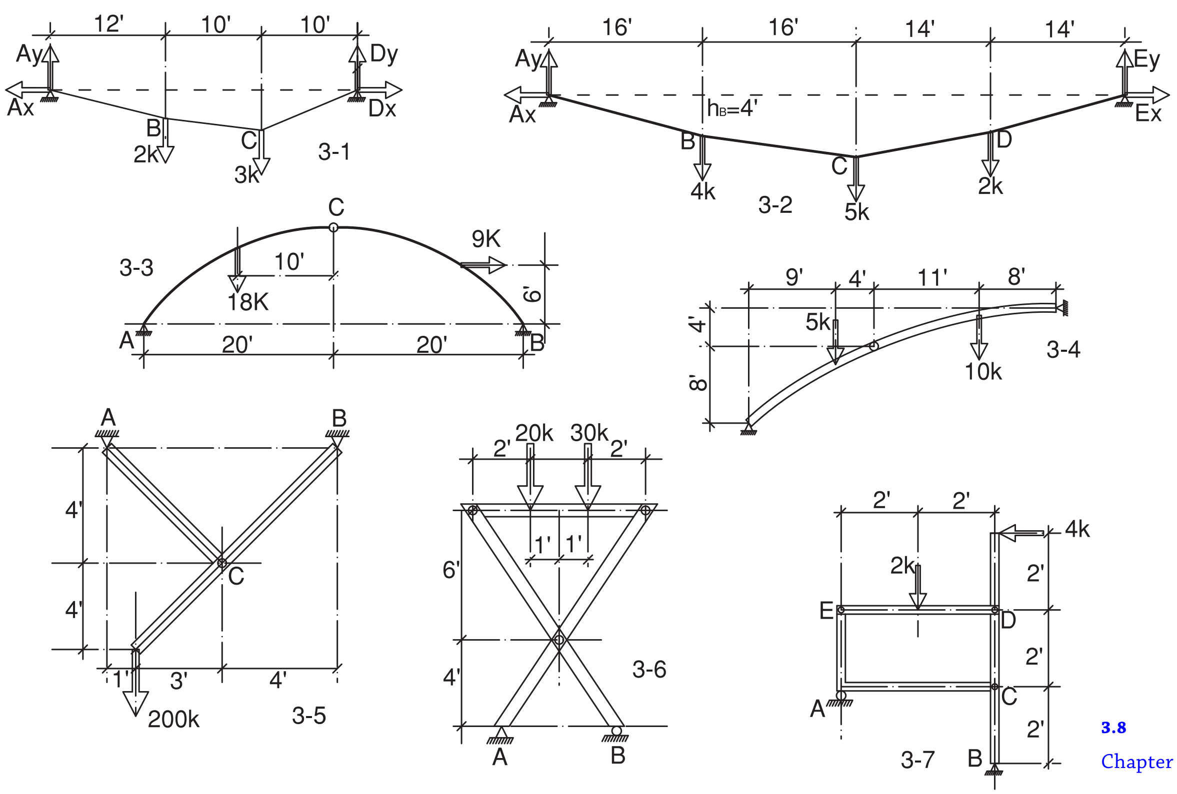
Practice Exercises:

3-1: For the diagram 1-1 in Figure 3.8:

a) Find the sag (h) and the reactions at the support if $h_B = 3'$.
b) Find the sag (h) and the reactions at the supports if the maximum tension in leg CD is 8k.

3-2: Find the tension in each leg of the cable.

3-3 through 3-7: Find the reactions at the supports and the resultant pin forces.



3.8

Chapter 3 Practice exercises

four

Shear and Moment in Beams

There is a mathematical relationship between the load on a beam and the shear and moment forces incurred by that load. This means that given a particular load, the shear and moment can be calculated at any point along the beam.

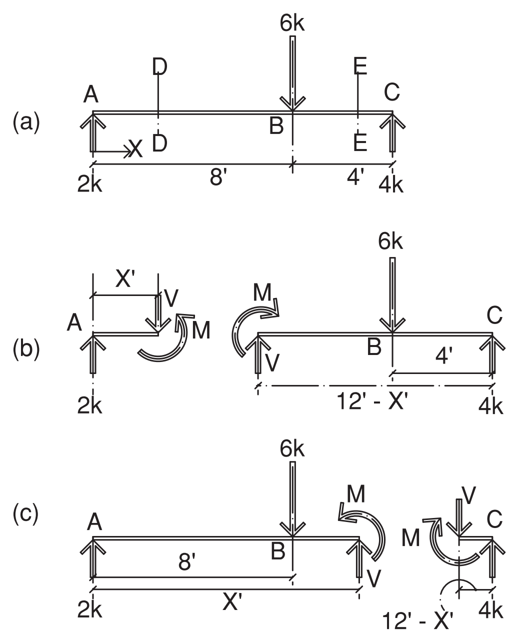
V = shear force (k or lb)

Shear is a chopping action; a force inside the beam that transfers a load, occurring perpendicular to the axis of the beam, to the supports. The shear force, V, at any point along a horizontal beam can be found by summing the forces in the Y direction on either side of that point.

M = moment (k-f, k-in, lb-ft, or lb-in)

Moment is a bending action caused by the shear. The accumulation of shear across a beam determines the amount of moment created in the beam. The moment at any point can be found by summing moments on either side of that point.

The free-body diagram of a simply supported, 12ft beam in Figure 4.1(a) with a concentrated load of 6k located at a distance of 8ft from support A, shows reactions of 2k and 4k at supports A and B, respectively.



4.1

Shear and moment at any point in a beam

The Free Body Diagram is the starting point for finding the shear and moment at any given point along the beam. To determine the shear and moment, take a section at the point of interest. The internal shear (V) and moment (M) may then be calculated by summing forces and moments about any point.

In Figure 4.1(b) the beam is cut at section D–D and the two halves separated, the internal shear force (V) and the internal moment (M) can be calculated for the section of the beam to the left of the point load. Because a vertical load will change the shear, and as a result change the moment in the beam, a different section line, section E–E, must be evaluated for points to the right of the load.

To determine the shear and moment at some point to the left of the point load, break the beam at section line D–D. Section line D–D occurs at some distance X from support A, meaning that the values for shear and moment will be found in terms of the variable X.

Assume a direction for shear (V) and moment (M) on one side of the break. Since the point is static, the forces and moments at the point must be in equilibrium. Therefore, the shear (V) and moment (M) on the other side of the break will be of equal magnitude, but in the opposite direction.

Consider only the left side of section D–D:

$$\Sigma F_y = 0 = 2 - V \ldots V = 2k$$

$$\Sigma M_A = 0 = V(X) - M = 2(X) - M \ldots M = 2X \text{ k-f}$$

Consider only the right side of section D–D:

$$\Sigma F_y = 0 = V - 6k + 4 \ldots V = 2k$$

$$\Sigma M_D = 0 = M + 6(8 - X) - 4(12 - X)$$

$$M = -48 + 6X + 48 - 4X = 2X \text{ k-f}$$

If the moment is taken about point B or point C, the answer will remain the same.

$$\Sigma M_B = 0 = M + 2(8-X) - 4(4)$$

$$M = -16 + 2X + 16 = 2X \text{ k-f}$$

$$\Sigma M_C = 0 = M + 6(8-X) - 4(12-X)$$

$$M = -48 + 6X + 48 - 4X = 2X \text{ k-f}$$

From point A to point B, the shear will remain at 2k and the moment will remain at 2X k-f for any distance X from support A up to the point of load.

Point A: $X = 0$, $V = 2k$, $M = 2(0) = 0$k-f

Point B: $X = 8'$, $V = 2k$, $M = 2(8) = 16$k-f

Past the point of load, section E–E must be considered. Consider only the left side of section E–E as shown in Figure 4.1(c):

$$\Sigma F_y = 0 = 2k - 6k + V \ldots V = 4k$$

$$\Sigma M_A = 0 = 6k(8') - 4k(X) - M \ldots M = 48 - 4X \text{ k-f''}$$

Consider only the right side of section E–E:

$$\Sigma F_y = 0 = -V + 4k \ldots V = 4k$$

$$\Sigma M_B = 0 = M - 4(12 - X) \ldots M = 48 - 4X \text{ k-f}$$

From point B to point C, the shear will remain at 4k and the moment will remain at 48 – 4X k-f for any distance X from point B to point C.

Point B: $X = 8'$, $V = 4k$, $M = 48 - 4(8) = 16$k-f

Point C: $X = 12'$, $V = 4k$, $M = 48 - 4(12) = 0$k-f

4.1 Shear and Moment Diagrams

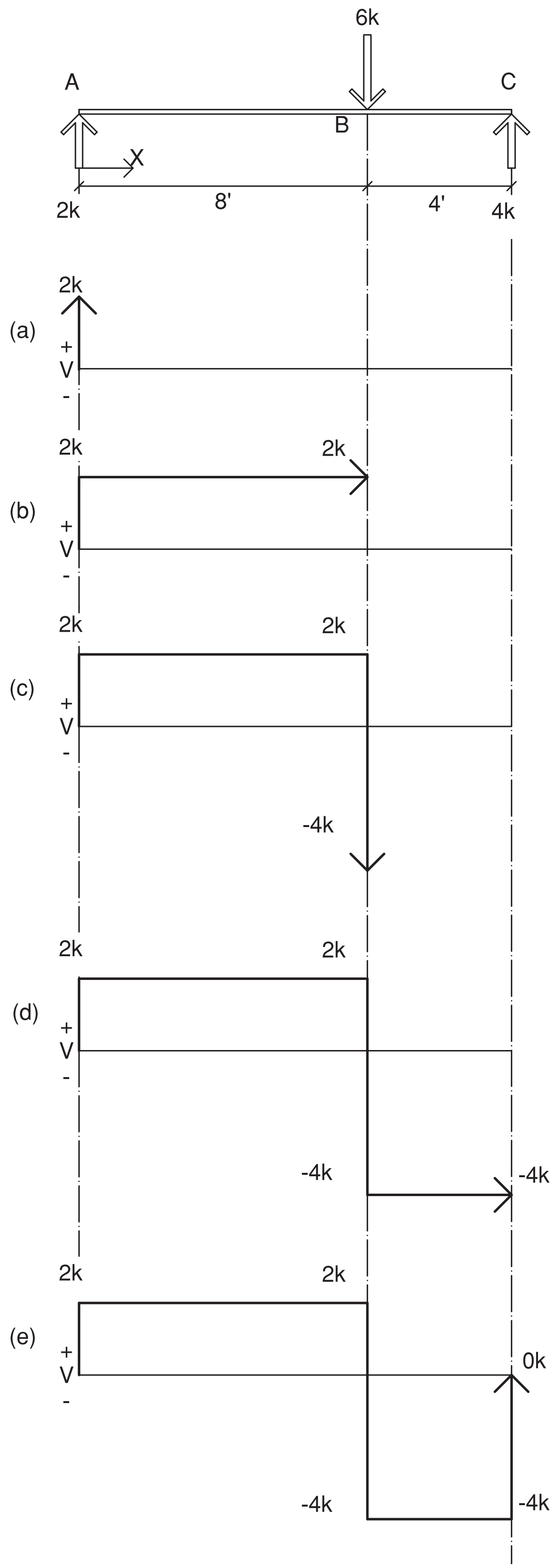
In Beam design, the maximum shear and moment must be considered in order to find the maximum stresses in shear and flexure. By illustrating the shear or moment at any point on the beam in terms of a diagram, it becomes easy to assess the areas of maximum and minimum stress in the beam without drawing a new section at every change in loading.

4.1.1 Diagrams with Concentrated Loads

To draw a shear diagram, begin at X = 0 and move vertically only as a vertical force is encountered. Because reactions are considered, the shear will begin and end at zero. A positive or upward force will cause a positive increase in shear of the same magnitude or amount of force. Likewise, a negative or downward force will cause a decrease in shear of the same amount of force. The change in shear due to a concentrated or point load occurs completely at the point of load and is represented by a vertical line extending from the value of shear on one side to the value of shear on the other side. This vertical line length equals the amount of force encountered at the point.

Example 4-1: A simply supported beam with a concentrated load.

Consider the simple beam discussed at the beginning of the chapter, shown in Figure 4.2:



4.2

Example 4-1: Shear diagram.

(a) Starting at X = 0, which in this example is point A, the reaction of +2K is immediately encountered. The shear changes from 0 to 2k. V = 0 + 2k = 2k. Therefore, draw a line from 0 to 2k at X = 0.

(b) From X = 0 to X = 8′, no vertical forces are encountered. Therefore the shear does not change. It remains at 2K.

(c) At X = 8′, which is point B at the load, there is a downward force of 6k. Therefore, the shear will change by −6k. Since the shear is 2k, it must drop to V = 2k − 6k = −4k.

(d) From point B at X = 8′ to the support point C, at X = 12′, there are no vertical forces encountered. Therefore, the shear remains at −4k.

(e) At point C, the support reaction of 4k upward is encountered. The shear increases to V = −4k + 4k = 0. This is what is expected at the end of the beam.

Just as the shear diagram is influenced by the loads on the beam, the moment diagram is influenced by the shear on the beam. Therefore, once the shear diagram is drawn, it can be used to create the moment diagram. The mathematical relationship between shear and moment is described as: $M = \int V dx$.

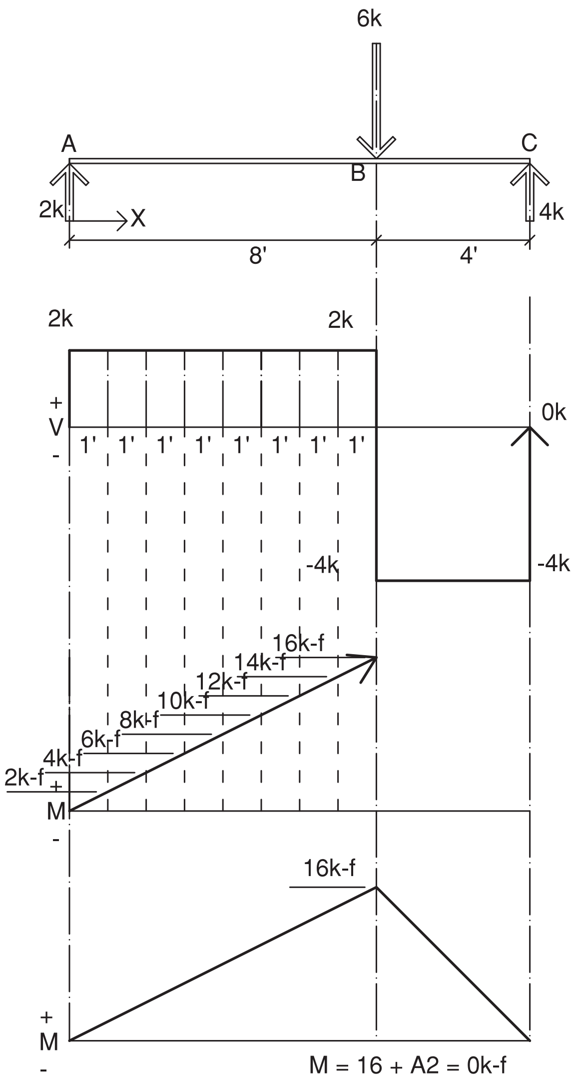
Because the moment is the integral of the shear, it is equal to the area under the shear curve. This means that the moment at any point a distance X from the left is equal to the sum of all shear areas, positive or negative up to that distance X. This yields the same result as cutting a section line D–D and finding that M = 2X k-f. The area under the shear curve at distance X to section D–D is 2k(X′) = M = 2X k-f. Plotting the results for M at points X = 0 through X = 8′ yields Figure 4.3(c). The slope of the moment line equals the shear.

Figure 4.3 illustrates why the total area under the shear curve from X = 0 to 8′ equals the moment at X = 8′, which is M = 16k-f. Therefore, in drawing the moment diagram, it is not necessary to examine every point along the beam, but to calculate the areas as they appear in simple geometric forms.

To create a moment diagram, first calculate the areas below the shear curve. Remember that areas above the zero line will be positive while areas below the zero line will be negative.

$A_1 = 2k(8') = 16k\text{-}f$

$A_2 = -4k(4') = -16k\text{-}f$



4.3

Example 4-1: Moment diagram

To draw the moment diagram, begin at X = 0. The moment will equal zero unless there is a fixed support or an applied moment at that point.

(a) The first shear area, A_1 = 16k-f, extends from X = 0 to X = 8′. Therefore, the moment line will extend from M = 0 at X = 0 to M = 0 + A_1 = 16k-f at X = 8.

(b) The second shear area, A_2 = –16k-f, extends from X = 8′ to X = 12′. Therefore, the moment line will extend from M = 16k-f at X = 8′ to M = 16 + A_2 = 16 – 16 = 0 at X = 12′.

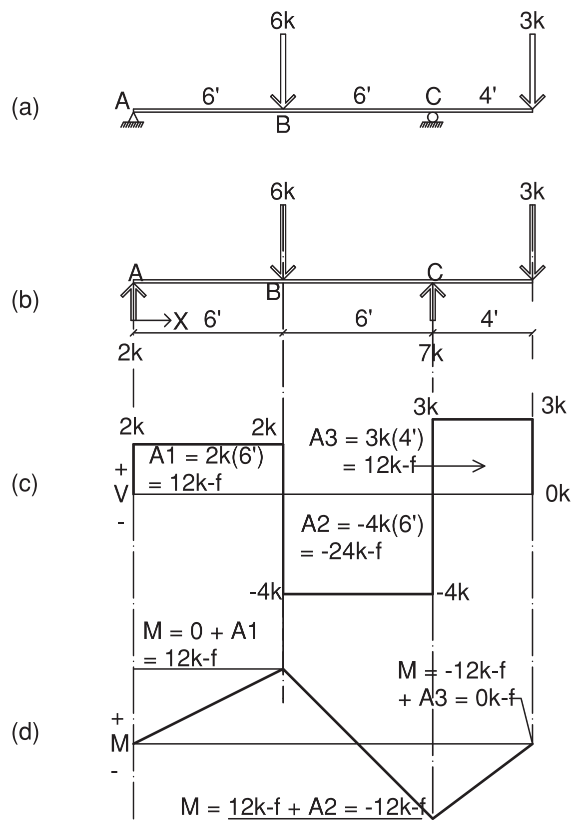
The maximum shear in the beam is 4k. Direction does not matter, simply the magnitude of the shear. The maximum moment in the beam is 16k-f and occurs at X = 8′. The moment in the beam is 0 at X = 0, 12′.

V_{max} = 4k

M_{max} = 16k-f

M = 0 @ X = 0, 12′

Example 4-2: Concentrated loads on a beam with an overhang.



4.4

Example 4-2: Shear diagram

The moment at pinned or roller supports is NOT always zero. Consider the beam in Figure 4.4(a). This beam has a span between supports of 12′ and an overhang of 4′. The free body diagram shown in Figure 4.4(b) is used to find the reactions as follows:

$\Sigma M_A = 0 = 6k(6') - C_y(12') + 3k(16') = 84k\text{-}f - C_y(12') \ldots$

$C_y = 7k$

$\Sigma F_y = 0 = A_y - 6k + 7k - 3k \ldots A_y = 2k$

The shear diagram in Figure 4.4(c) begins with the reaction at the support at X = 0 or point A, but does not return to 0 until it reaches the end of the overhang. The shear values can be summarized as follows:

Shear diagram:

$0 < X < 6'$: $V = 0 + 2k$

$6' < X < 12'$: $V = 2k - 6k = -4k$

$12' < X < 16'$: $V = -4k + 7k = 3k$

Shear areas:

A_1: 2k(6′) = 12k-f

A_2: −4k(6′) = −24k-f

A_3: 3k(4′) = 12k-f

The moment diagram in Figure 4.4(d) begins at zero because support A is a pinned support and there is no applied moment. From point A to point B the moment will increase 2k-f for every foot of beam length because the shear is 2k in this zone. From point B to point C, the shear is −4k and so the moment begins at 12k and decreases 4k-f for every foot of beam in this zone. From point C to the end of the overhang, the moment begins at −12k-f and increases 3k-f per foot of beam until it reaches 0. The moment at key points can be summarized as follows:

Moment diagram:

X = 0: M = 0

X = 6′: $M = 0 + A_1 = 0 + 12$k-f = 12k-f

X = 12′: $M = 0 + A_1 + A_2 = 0 + 12$k-f − 24k-f = −12k-f

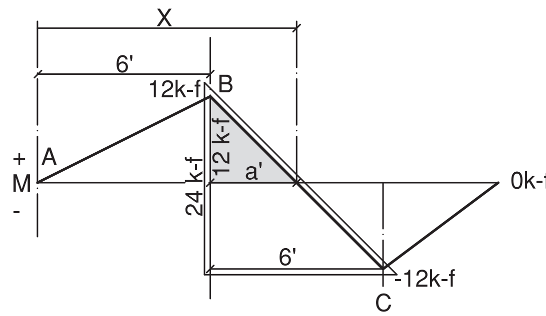
X = 16′: $M = 0 + A_1 + A_2 + A_3$
= 0 + 12k-f − 24k-f + 12k-f = 0

The moment diagram shows that the moment at support C, which occurs at X = 12′, is not 0, but −24k-f. A negative moment at the overhang support is typical.

$V_{max} = 4k$

$M_{max} = 12$k-f

M = 0 @ X = 0, 16′ and some point between 6 and 12′



4.5
Example 4-2: Moment diagram

The distance X where the moment crosses the 0 line may be determined either geometrically as shown in Figure 4.5 or algebraically as discussed in section 4.2.

The moment curve is a straight line at the point where it crosses the zero line. Therefore, equivalent triangles may be used to determine the distance, X′, from the left.

Let "a" equal the distance Point B to the point where M = 0. It is known that during that distance, a, the moment drops from 12k-f to 0k-f. It is also known that from point B to point C the moment drops from 12k-f to −12k-f, a change of 24k-f.

The slope of the small triangle = rise/run = (12k-f)/a

The slope of the large triangle = rise/run = (24k-f)/6′

Since the slope is defined by the same line on the moment diagram, the slopes of the two triangles are equal.

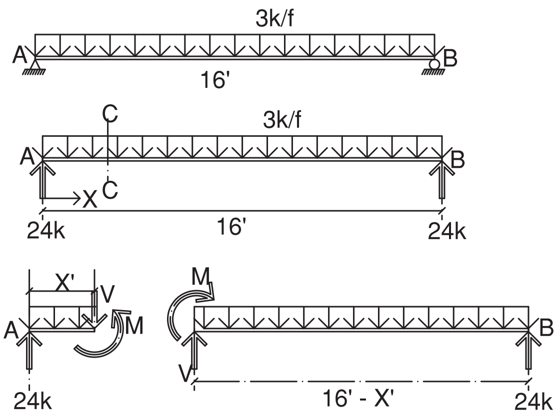
12/a = 24/6 … a = 12/4 = 3′

X = 6′ + a = 6′ + 3′ = 9′

M = 0 @ X = 0, 9′ and 16′

4.1.2 Diagrams with Distributed Loads

A uniform load can be thought of as a series of point loads placed at very small intervals. For illustration purposes, one foot intervals are used. Consider the 16′ long beam in Figure 4.6. It carries a uniformly distributed load of 3k/f. At any distance X (ft) from the left, the load to the left will be 3k/f(X′) = 3Xk. The shear and moment at any given point may be found by drawing a section line and considering one side of the section cut.



4.6

Shear and moment in beams with distributed loads

The reactions are found as follows:

$$\Sigma M_A = 0 = 3\text{ k/f } (16')(8') - B_y\,(16') \ldots B_y = 24\text{k}$$

$$\Sigma F_y = 0 = A_y - (3\text{ k/f})(16') + 24\text{k} \ldots A_y = 24\text{k}$$

Consider only the left side of section C–C:

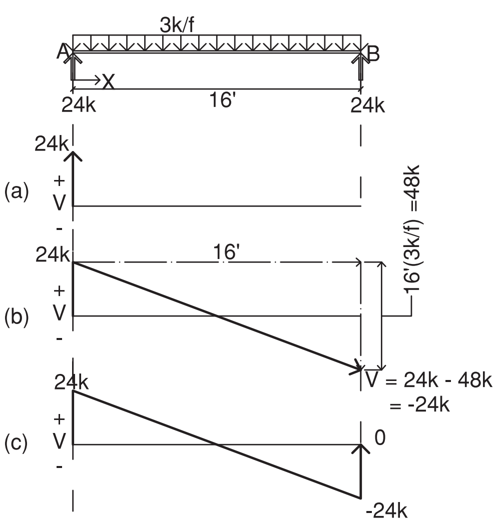
$$\Sigma F_y = 0 = 24 - 3X - V \ldots V = (24 - 3X)\text{k}$$

$$\Sigma M_A\,(\text{k-f}) = 0 = 3X(X/2) + V(X) - M$$

$$M = 3X^2/2 + (24 - 3X)X\text{k-f} = 24X - 3X^2/2\text{k-f}$$

Example 4-3: A uniform load is applied on a simply supported beam.

Draw the shear and moment diagram for a uniformly distributed load, using the Free Body Diagram as shown in Figure 4.7.



4.7

Example 4-3: Shear diagram

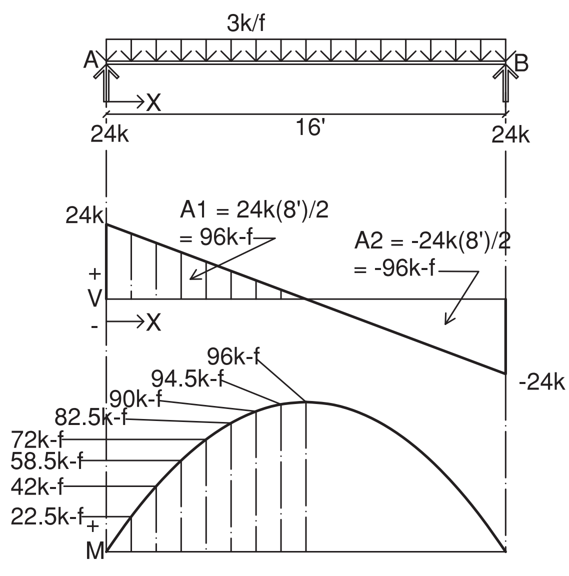
(a) Starting at X = 0, which in this example is point A, the reaction of +24K is immediately encountered. The shear changes from 0 to 24k. V = 0 + 24k = 24k. Therefore, draw a line from 0 to 24k at X = 0.

(b) The uniform load is also immediately encountered. In this example the uniform load is 3k/f. This means that for every foot, there is a downward force of 3k. Plotting the uniform load on the shear diagram, results in a line with a slope of –3k/f. As a result, the shear (V) decreases from V = 24k @ X = 0 to V = 24k – (3k/f)(1′) = 21k @ X = 1 and to V = 24 – (3k/f)(16′) = –24k @ X = 16′. Notice that the total drop in shear due to the uniform load is equal to the area under the load curve.

(c) At point B, the support reaction of 24k upward is encountered. The shear increases to V = –24k + 24k = 0. This is what is expected at the end of the beam.

The distance X at which the shear crosses the zero line can be found using the shear equation:

$$V = 24\text{k} - 3\text{k/f}(X') = 0 \ldots X = 24\text{k}/3\text{k/f} = 8'$$



4.8

Example 4-3: Moment diagram

As discussed in section 4.1.1, the moment is integral of the shear. Therefore, the moment at any point on the simple beam is equal to the accumulated area under the shear curve. In the case of uniformly distributed loads, the shear is not constant, it varies linearly.

Distance X (ft)	*Shear (k)*	*Area under shear line*
X	V = 24k – 3X	$M = Vk(X') + (24k - V)\left(\frac{X'}{2}\right)$
0'	V = 24k – 3k/f(0) = 24k	$M = 24k(0) + (24k - 24k)\left(\frac{0'}{2}\right) = 0.0k\text{-}f$
1'	V = 24k – 3k/f(1') = 21k	$M = 21k(1') + (24k - 21k)\left(\frac{1'}{2}\right) = 22.5k\text{-}f$
2'	V = 24k – 3k/f(2') = 18k	$M = 18k(2') + (24k - 18k)\left(\frac{2'}{2}\right) = 0.0k\text{-}f$
3'	V = 24k – 3k/f(3') = 15k	$M = 15k(3') + (24k - 15k)\left(\frac{3'}{2}\right) = 0.0k\text{-}f$
4'	V = 24k – 3k/f(4') = 12k	$M = 12k(4') + (24k - 12k)\left(\frac{4'}{2}\right) = 0.0k\text{-}f$
5'	V = 24k – 3k/f(5') = 9k	$M = 9k(5') + (24k - 9k)\left(\frac{5'}{2}\right) = 0.0k\text{-}f$
6'	V = 24k – 3k/f(6') = 6k	$M = 6k(6') + (24k - 6k)\left(\frac{6'}{2}\right) = 0.0k\text{-}f$
7'	V = 24k – 3k/f(7') = 3k	$M = 3k(7') + (24k - 3k)\left(\frac{7'}{2}\right) = 0.0k\text{-}f$
8'	V = 24k – 3k/f(8') = 0k	$M = 0k(8') + (24k - 0k)\left(\frac{8'}{2}\right) = 0.0k\text{-}f$

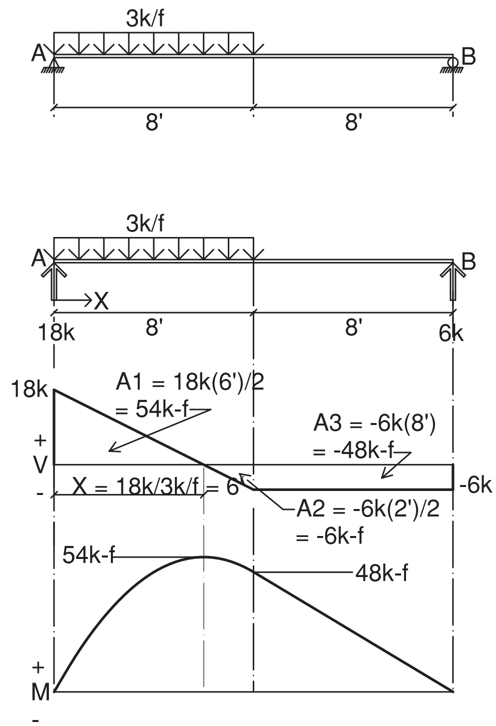
If the moment is plotted at one foot intervals, as shown above, the shape of the moment curve can be seen to be parabolic. The largest increase in moment, and therefore the steepest slope, occurs where the shear is largest.

To draw the moment diagram, calculate the area below the shear curve and consider the shape of the moment curve. In Figure 4.8, the areas under the shear curve are triangles. The area of a triangle is equal to the base times the height divided by two. A_1 = 24k(8′)/2 = 96k-f; A_2 = –24k(8″)/2 = –96k-f. The moment diagram begins at 0k-f because there is no fixed support or applied moment. At X = 8′, the moment increases from 0k-f to M = 0 + A_1 = 96k-f, following a parabolic curve that levels off to a slope of zero at X = 8′. At X = 16′, the moment decreases from 96k-f to M = 96k-f + A_2 = 96k-f – 96k-f = 0.

Example 4-4: A partial uniform load on a simply supported beam.

Often a distributed load does not extend across the entire length of a beam.

Consider the beam in Figure 4.9. The uniform load of 3k/f occurs only between X = 0 and X = 8′. The reactions are found as follows:



4.9

Example 4-4: A partial uniform load on a simply supported beam

$$\Sigma M_A = 0 = (3''k/f'')(8')(4') - B_y(16') \ldots B_y = 6k$$

$$\Sigma F_y = 0 = A_y - (3''k/f'')(8') + 6k \ldots A_y = 18k$$

To draw the shear diagram, begin at X = 0′. The upward concentrated load of 18k that is the reaction at support A is immediately encountered. The shear increase at X = 0 from 0 to V = 0 + 18k = 18k. The uniform load of 3k/f occurs from X = 0 to X = 8′. Therefore, the shear will decrease linearly from V = 18k at X = 0 to V = 18k –(3k/f)(8′) = 18k – 24k = –6k at X = 8′. There are no loads encountered between X = 8′ and X = 16′; therefore the shear remains constant at –6k. At X = 16′, the upward concentrated load of 6k that is the reaction at point B is encountered, increasing the shear at X = 16′ from –6k to V = –6k + 6k = 0k. This is what is expected at the end of the beam. To draw the moment diagram, the areas below the shear curve must be calculated. In order to calculate A_1, the distance X from point A to where the shear crosses the zero line (V = 0) must be calculated. X = $18^k/3^{k/f}$ = 6′.

$$A_1 = \text{base(height)}/2 = 6'(18k)/2 = 54k\text{-}f$$

$$A_2 = \text{base(height)}/2 = (8' - 6')(-6k)/2 = 2'(-6k)/2 = -6k\text{-}f$$

A_3 = base(height) = 8′(−6k) = −48k-f

The moment curve begins at X = 0 with a value of 0. No fixed support or applied moment is encountered. At X = 6′, the moment is M = 0 + A_1 = 54k-f. The increase in the moment from 0 to 54k-f at X = 6′ follows a parabolic curve with the steepest slope at X = 0 and tapering off to a slope of 0 at X = 6′.

From X = 6′ to X = 8′, the moment decreases by the value of A_2 = −6k-f, again following a parabolic curve. This time, however, the slope begins at 0 and becomes steeper until X = 8′ and M = 54k-f + A_2 = 54k-f − 6k-f = 48k-f.

From X = 8′ to X = 16′, the moment decreases linearly by the value of A_3 = −48k-f because the shear is constant in this section. At X = 16′, M = 48k-f + A_3 = 48k-f − 48k-f = 0. This is expected at the end of a simply supported beam.

Example 4-5: Uniform and concentrated loads on a beam.

The 16′ long beam in Figure 4.10 has a partial uniform load and a concentrated load. A combination of concentrated and distributed loads has the same methodology for drawing the shear and moment diagrams as that discussed in the previous examples. The reactions are found as follows:

ΣM_A = 0 = 3 k/f (8′)(4′) + 12k(12′) − By(16′) … B_y = 15k

ΣF_y = 0 = A_y − 3 k/f (8′) − 12k + 15k … A_y = 21k

X = 0: V = 0 + 21k = 21k: point load = vertical line

0 < X < 8′: V drops from V = 21k @ X = 0 to
V = 21 − 3k/f(8′) = −3k @ X = 8′
Uniform load = sloped line

8′ < X < 12′: V remains constant.
No loads = no change in shear.

X = 12′: V = −3k − 12k = −15k: point load = vertical line.

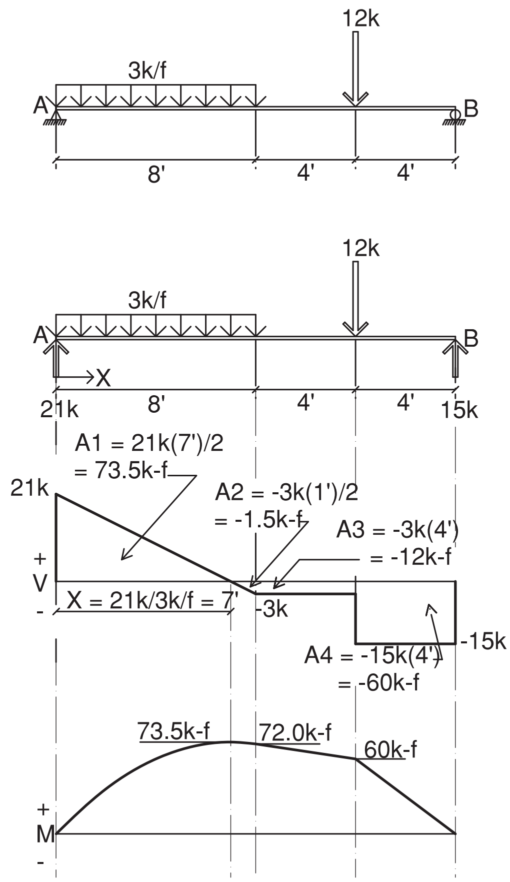
12′ < X < 16′: V remains constant.
No loads = no change in shear.

X = 16′: V = −15k + 15k = 0: point load = vertical line.

Distance to V = 0 is X = 21k/3k/f = 7′

A1 = 7′(21k)/2 = 73.5k-f

A2 = 1′(−3k)/2 = −1.5k-f



4.10

Example 4-5: Uniform and concentrated loads on a beam

A3 = 4′(−3k) = −12k-f

A4 = 4′(−15k) = −60k-f

X = 0: M = 0.

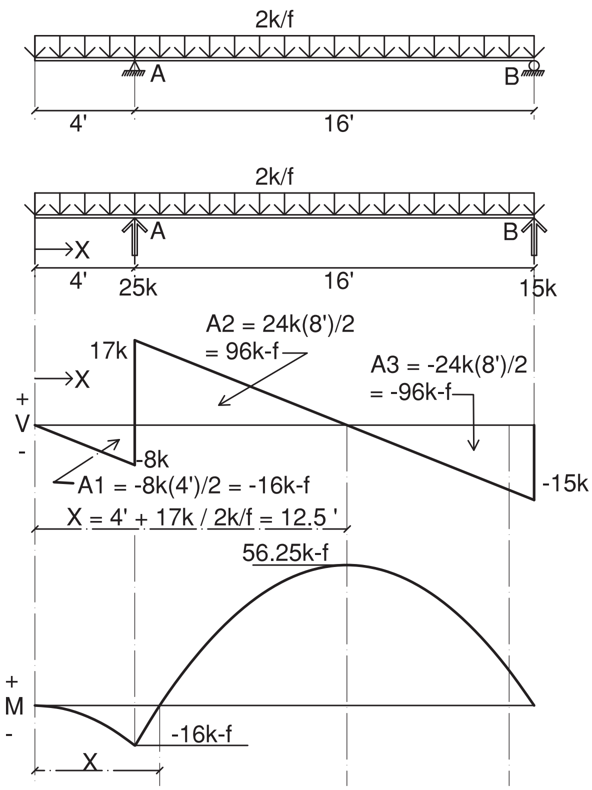
X = 7′: M = 0 + A_1 = 73.5 k-f
Triangular area = parabolic curve.

X = 8′: M = 73.5 k-f + A_2 = 73.5 − 1.5 = 72.0 k-f
Triangular area = parabolic curve.

X = 12′: M = 72 k-f + A_3 = 72.0 − 12 = 60k-f
Rectangular area = sloped line.

X = 16′: M = 60 k-f − A_4 = 60 k-f − 60 k-f = 0
Rectangular area = sloped line.

Example 4-6: A uniform load on a beam with an overhang.



4.11
Example 4-6: A uniform load on a beam with an overhang

Consider the uniformly loaded beam with an overhang (Figure 4.11). The span between supports is 16′ and the overhang is 4′. In this example, the overhang is on the left, meaning the shear diagram will not encounter the reaction at support A until X = 4′. Therefore, the shear will begin at zero and slope downward. The reactions are found as follows:

$\Sigma M_A = 0 = (2\ k/f)(20')(6') - B_y(16') \ldots B_y = 15k$

$\Sigma F_y = 0 = A_y - (2\ k/f)(20') + 15k \ldots A_y = 25k$

X = 0: V = 0 No point load at end of overhang.

0 < X < 4′: V drops from V = 0 @ X = 0 to
V = 0 – 2k/f(4′) = –8k @ X = 4′
Uniform load = sloped line.

X = 4′: V = –8k + 25k = 17k point load = vertical line.

4′ < X < 16′: V drops from V = 17k @ X = 4′ to
V = 17 – 2 k/f(16′) = –15k @ X = 20′
Uniform load = sloped line.

X = 20′: V = –15k + 15k = 0 point load = vertical line.

Distance to V = 0 is X = 4′ + 17k/(2k/f) = 12.5′

$A_1 = 4'(-8k)/2 = -16k\text{-}f$

$A_2 = 8.5'(17k)/2 = 72.25k\text{-}f$

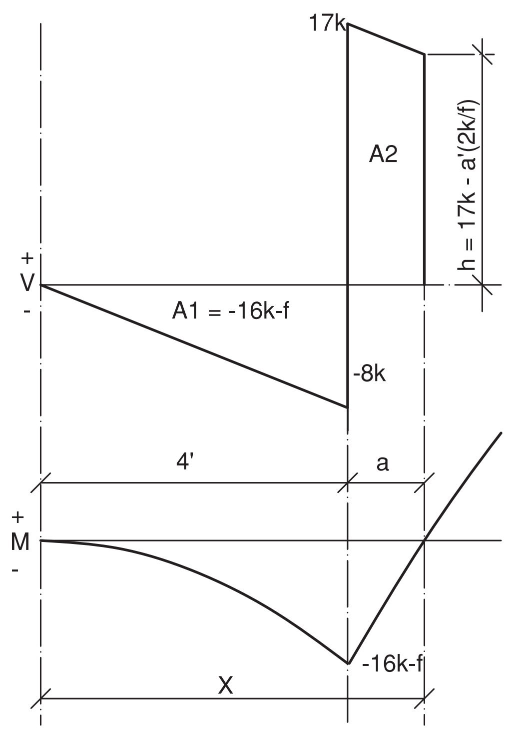
$A_3 = 7.5'(-15k)/2 = -56.25k\text{-}f$

X = 0: M = 0

X = 4′: M = 0 + A_1 = 0 – 16k-f = –16k-f
Triangular area = parabolic curve.

X = 12.5′: M = –16k-f + A_2 = –16 + 72.25 = 56.25k-f
Triangular area = parabolic curve.

X = 20′: M = 56.25 k-f + A_3 = 56.25 – 56.25 = 0
Triangular area = parabolic curve.



4.12
Finding where M = 0

The problem of finding the point on the beam where M = 0 becomes more difficult to solve geometrically when the moment curve contains parabolic curves. The distance cannot be found by comparing similar triangles as in Figure 4.7. However, A1 = –16k-f is the area on the shear diagram required to drop the moment value from 0 to –16k-f. It is logical then, that an area of 16k-f, in the left portion of A_2, will cause the moment value to increase 16k-f. Looking at an enlarged portion of the shear diagram, in Figure 4.14, the variable "a" represents the distance past the support where M = 0.

a = X – 4′ and h = height of the shear curve at X = 4 + a.

$A_2 = a(h) + a(17 - h)/2 = a(h + 17)/2$

$h = 17k - a(2k/f)$

Substituting h into the equation for A_2:

$A_2 = a(17-a) = 16$ or $a^2 - 17a + 16 = 0 = (a - 16)(a - 1)$

a = 16′, X = 4 + 16 = 20′ which is support B.

a = 1′, X = 4 + 1 = 5′ is the point where M = 0.

$V_{max} = 17k$

$M_{max} = 56.25k\text{-}f$

M = 0 @ X = 0, 5′, 20′

4.1.3 Diagrams with Applied Moments

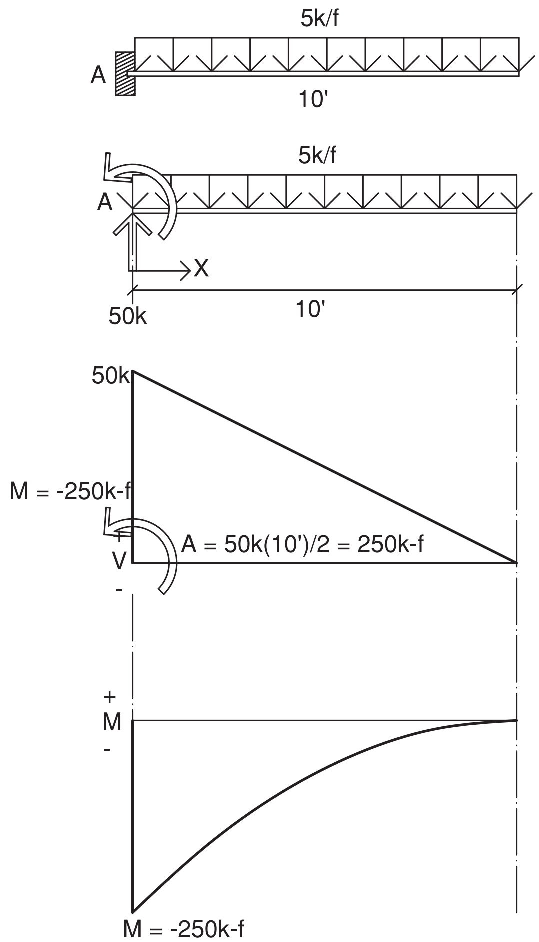
Moments occur at fixed supports. They also occur elsewhere along a beam whenever there is a rotational influence such as a horizontal force offset from the axis of the beam or a couple: two equal but opposite forces acting at a distance apart (See Chapter 1, section 1.1.3). When these moments occur, the influence is immediate and results in a vertical change along the moment line diagram.

Example 4-7: A uniform load on a cantilevered beam.

A beam with a fixed connection, such as the cantilevered beam in Figure 4.13, has a moment that occurs at that connection. That moment is found by summing the moments about point A.

$\Sigma M_A = 0 = -M + 5\text{ k/f }(10')(5')$

M = 250k-f. M was assumed counter-clockwise. The answer is positive and therefore M is 250k-f counter-clockwise. Draw the shear diagram without regard to the moment at the support.



4.13
Example 4-7: A uniform load on a cantilevered beam

X = 0: V = 0 + 50k point load = vertical line.

0 < X < 10′: V drops from V = 50k @ X = 0 to V = 50k – 5k/f(10′) = 0k

@ X = 8′ uniform load = sloped line.

$A_1 = 10'(50k/2) = 250k\text{-}f$

X = 0: M = 0 + M = 0 – 250k-f
Applied moment = straight line.

X = 10′: M = –250k-f + A_1 = –250 + 250 = 0k-f
Triangular area = parabolic curve.

$V_{max} = 50k$

$M_{max} = -250k\text{-}f$

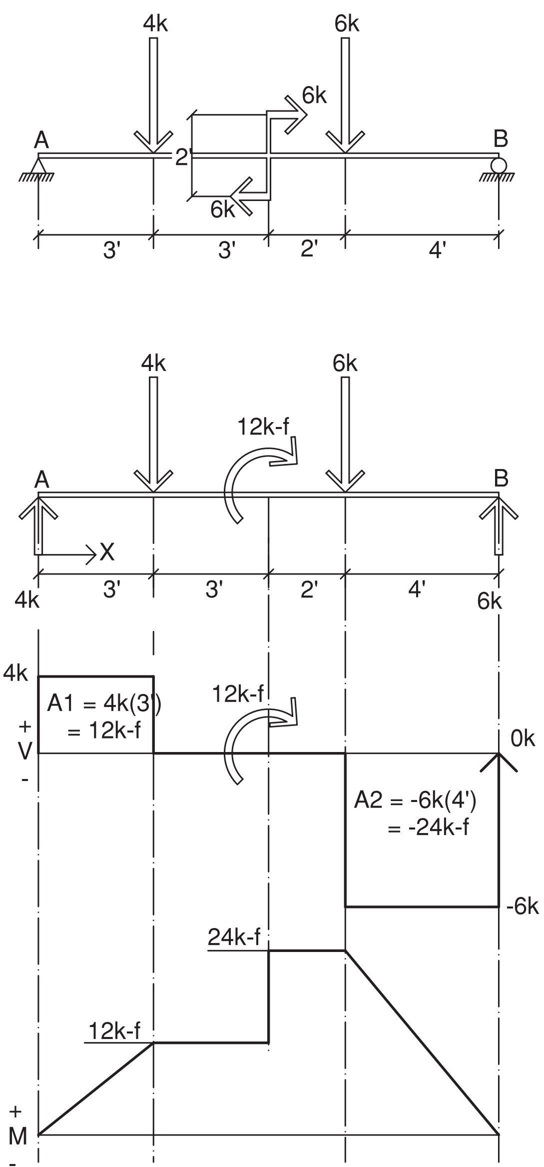
M = 0 @ X = 10′

Example 4-8: A beam with an applied moment due to a couple.

Consider an applied moment at the mid-span of the beam in Figure 4.14. The two horizontal 6k loads at a distance of 2′ apart form a couple with a positive moment of 12k-f. The reactions are found as follows:

$\Sigma M_A = 0 = 4k(3') + 6k(1') + 6k(1') + 6k(8') - B_y\,(12')$ …
$B_y = 6k$

$\Sigma F_y = 0 = Ay - 4k - 6k + 6k$ … $A_y = 4k$



4.14

Example 4-8: A beam with an applied moment due to a couple

$X = 0$: $V = 0 + 4k = 4k$
Point load = vertical line.

$0 < X < 3$: V remains constant
No loads = no change in shear.

$X = 3'$: $V = 4k - 4k = 0$
Point load = vertical line.

$3' < X < 8'$: V remains constant.
No vertical loads = no change in shear.

$X = 8'$: $V = 0 - 6k$
Point load = vertical line.

$8' < X < 12$: V remains constant.
No loads = no change in shear.

$X = 12'$: $V = -6k + 6k = 0$
Point load = vertical line.

$A_1 = 3'(4k) = 12k\text{-}f$

$A_2 = 4'(6k) = 24k\text{-}f$

$X = 0$: $M = 0$
No moment at support.

$X = 4'$: $M = 0 + A_1 = 12$ k-f
Rectangular area = sloped line.

$4' < X < 6'$: $M = 12k\text{-}f + 0 = 12$ k-f
No shear, moment is constant.

$X = 6'$: $M = 12k\text{-}f + 6K(2') = 24k\text{-}f$
Applied moment, vertical line.

$6' < X < 8'$: $M = 24k\text{-}f + 0 = 24k\text{-}f$
No shear, moment is constant.

$X = 12'$: $M = 24k\text{-}f + A_2 = 24 - 24 = 0k\text{-}f$
Rectangular area = sloped line.

$V_{max} = -6k$

$M_{max} = 24k\text{-}f$

$M = 0$ @ $X = 0, 12'$

Summation of process:

Shear diagrams:

1. Begin at X = 0.
2. Add loads as they are encountered. Concentrated loads will cause a vertical change in the shear curve. Uniform loads will cause a linear change in the shear curve with a slope equal to the load.
3. For any portion of the beam that does not encounter a load, uniform or concentrated, the shear remains the same.
4. The shear should return to zero at the end of the beam.

Moment diagrams:

1. Find the areas under the shear curve.
2. Begin at X = 0.
3. Add shear areas as encountered. Areas with constant shear will cause the moment line to change linearly with a slope equal to the shear. Shear areas with a triangular area will cause the moment curve to be parabolic with the steepest slope being at the end with the most shear.
4. Add applied moments when encountered. Applied moments will cause a vertical line on the moment curve.

4.2 Writing Moment Equations

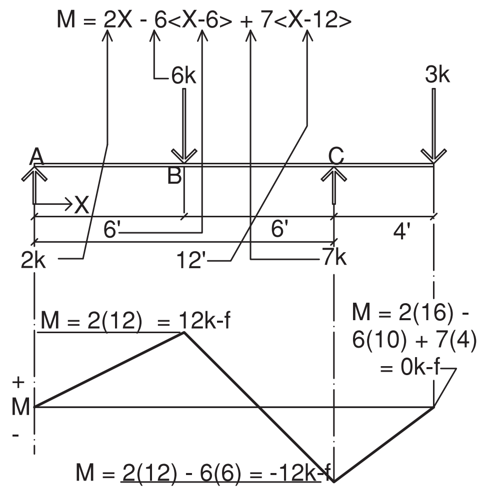
Writing the moment equation is helpful when the loads become complex and it is desired to find where the moment equals zero without geometrically calculating the areas of parabolas and the like. As discussed earlier, the moment is the integral of the shear in a beam. $M = \int V\,dx$. Another reason for writing a moment equation is to find the deflection in a beam. Deflection is the double integral of the moment. $\Delta EI = \iint M\,dx$.

In writing the moment equation, < > brackets indicate contents that are only considered if greater than zero. If the contents inside the < > brackets are less than zero, use zero as the bracketed amount.

When the shear is constant, as from a point load, the moment, $M = \int V\,dx = Vx + C$. When the shear is a uniformly distributed load, *w*, the moment, $M = \int V\,dx + C_1 = \iint wx\,dx + C_1x + C_2$.

Example 4-9: Writing moment equations for concentrated load.

Consider the beam from Figure 4.4 again.



4.15

Writing moment equations for concentrated loads

$M = 2X - 6 < X - 6> + 7<X - 12>$ is the moment equation.

$X = 0$: $M = 0 - 6(0) + 7(0) = 0$k-f

$X = 12'$: $M = 2(12) - 6(6) + 7(0) = -12$k-f

$X = 16'$: $M = 2(16) - 6(10) + 7(4) = 0$

To find where M crosses the zero line, set the moment equation equal to zero.

$M = 0 = 2X - 6<X - 6> + 7<X - 12>$

If $X \geq 12$, $M = 0 = 2X - 6(X - 6) + 7(X - 12) \ldots X = 16'$

If $6' \leq X \leq 12'$, $0 = 2X - 6(X - 6) \ldots X = 9'$

If $X \leq 6'$, $0 = 2X \ldots X = 0$

To find where M is maximum when a beam contains only concentrated loads, look at the points where the shear changes from positive to negative. First, set the derivative of the moment, the shear equation, equal to zero.

$V = 2 - <6 \text{ if } X>6> + < 7 \text{ if } X > 12 > = 0$

If $X \geq 12$, $0 = 2 - 6 + 7 = 3$ (positive)

If $6' \leq X \leq 12'$, $0 = 2 - 6 = -4$k (negative)

If $X \leq 6'$, $0 =$ (positive)

Therefore, the shear crosses the zero line at X = 12′ and X = 6′.

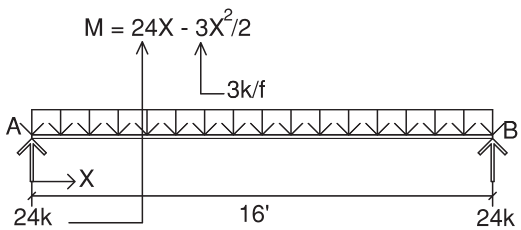
$X = 6'$: $M = 2(6) - 6(0) + 7(0) = 12$k-f

$X = 12'$: $M = 2(12) - 6(6) + 7(0) = -12$k-f

$M_{max} = 12$k-f

Example 4-10: Writing moment equations for full uniform loads.

Consider the beam from Figure 4.7 again. This 16ft beam has a uniform load of 3k/f over the entire length of the beam.



4.16

Writing moment equations for full uniform loads

$M = 24X - 3X^2/2$ where 24 is the reaction of 24k at support A and 3 is the uniform load of 3k/f.

$X = 0$: $M = 0 - 0 = 0$

$X = 8'$: $M = 24(8) - 3(8^2)/2 = 96$k-f

$X = 16'$: $M = 24(16) - 3(16^2)/2 = 0$

To find where M = 0, set the moment equation equal to zero.

$M = 0 = 24X - 3X^2/2 = -1.5X^2 + 24X = X(-1.5X + 24) = 0$

$M = 0$ @ $X = 0$ and @ $X = 24/1.5 = 16'$

To find where M is maximum, set the derivative of the moment, the shear equation, equal to zero.

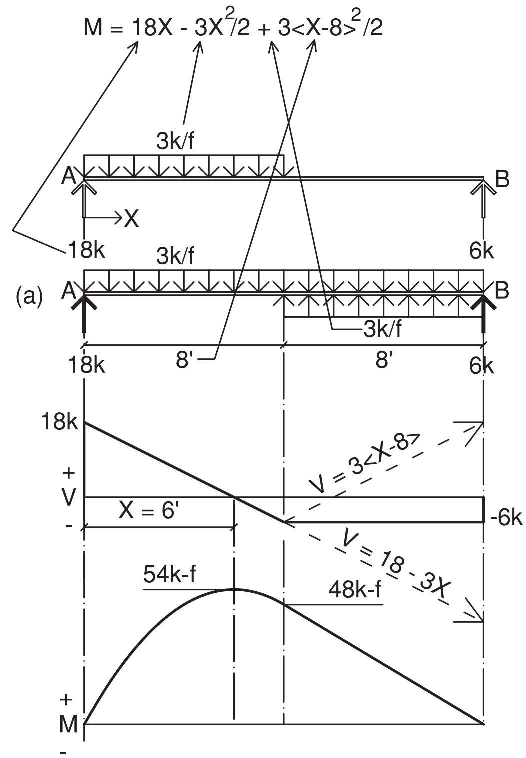
$V = 0 = 24 - 3X$. $X = 24/3 = 8'$

$X = 8'$: $M = 24(8) - 3(8^2)/2 = 96$k-f

$M_{max} = 96$k-f

Example 4-11: Writing moment equations for partial uniform loads.

Consider the beam from Figure 4.9 again. This 16′ beam has a uniform load of 3k/f over one half of its span. Because the uniform load does not continue past X = 8′, its effect must be counteracted. This is the equivalent of taking a uniform load over an entire span and adding an equal but opposite load at $8' < X < 16'$.



4.17

Writing moment equations for partial uniform loads

$M = 18X - 3X^2/2 + 3<X - 8>^2/2$

$X = 0$: $M = 0 - 3(0)/2 + 3(0)/2 = 0$

$X = 6'$: $M = 18(6) - 3(36)/2 + 3(0/2) = 54$k-f

$X = 8'$: $M = 18(8) - 3(64)2 + 3(0)/2 = 48$k-f

$X = 16'$: $M = 18(16) - 3(256)/2 + 3(64)/2 = 0$

To find where M crosses the zero line, set the moment equation equal to zero.

$M = 0 = 18X - 3X^2/2 + 3 <X - 8>^2/2$

If $X \geq 8$, $0 = 18X - 3X^2/2 + 3(X - 8)^2/2 = 18X - 1.5X^2 + 1.5(X^2 - 16X + 64) = 18X - 1.5X^2 + 1.5X^2 - 24X + 96 = -6X + 96 \ldots X = 16'$

If $X \leq 8'$, $0 = 18X - 3X^2/2 = X(18 - 1.5X) \ldots X = 0$ or $12'$

$12'$ is NOT $\leq 8'$ therefore $X = 0$.

To find where M is maximum, set the derivative of the moment, the shear equation, equal to zero.

$M = 18X - 3X^2/2 + 3<X - 8>^2/2$

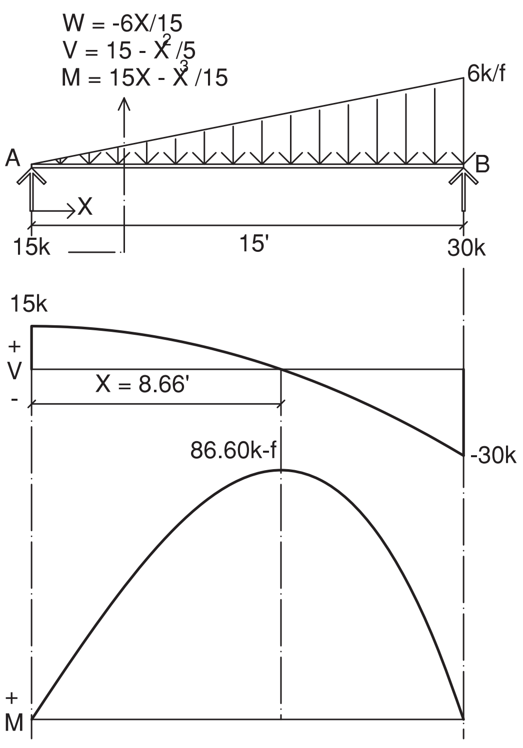
$V = 0 = 18 - 3X + 3<X - 8>$

If $X \geq 8$: $0 = 18 - 3X + 3X - 24 = -6$ but this is impossible since $0 \neq -6$

If $X \leq 8'$: $0 = 18 - 3X \ldots X = 6'$

Example 4-12: Writing moment equations for triangular loads.

Consider the beam in Figure 4.18. It has a triangular load spanning the length of the beam. Often with complex loads such as triangular loads, it is easier to express the load in terms of X and then take the integral of the load to find the shear and the integral of the shear to find the moment.



4.18

Writing moment equations for triangular loads

$W = -6k/f\ (X')/15' = -6X/15k/f$

$V = \int W\,dx = -6X^2/(15(2)) + C = -X^2/5k + C$

At $X = 0$, $V = 15k$ because the reaction at support A, at $X = 0$, is 15k.

Therefore, $C = 15k$

$V = -X^2/5 + 15 = 15 - X^2/5$

$M = \int V\,dx = 15X - X^3/15 + C_1$

At $X = 0$, $M = 0k$ because there is no applied moment at support A and support A is not a fixed support. Therefore, $C_1 = 0k\text{-f}$ and

$M = 15X - X^3/15$

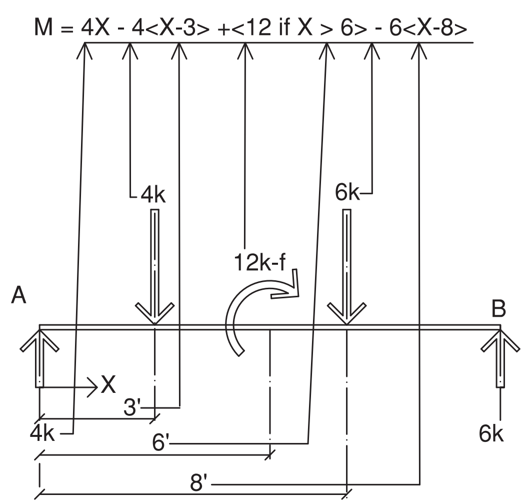
The moment is zero at $M = 0 = 15X - X^3/15$
$= X(15 - X^2/15) \ldots X = 0$ or $15'$
The moment is maximum where $V = 0 = 15 - X^2/5$

$X = \sqrt{75} = 8.66'$

$M_{max} = 15(8.66) - (8.66)^3/15 = 86.60$ k-f

Example 4-13: Writing moment equations for an applied moment.

Consider the beam from Figure 4.14 again. This is the beam with the 12k-f moment at $X = 6'$.



4.19

Writing moment equations for an applied moment

$M = 4X - 4<X - 3> + <12 \text{ if } X > 6> - 6<X - 8>$

To find where $M = 0$, set the moment equation equal to zero.

$M = 0 = 4X - 4<X - 3> + <12 \text{ if } X > 6> - 6<X - 8>$

If $X \geq 8$: $0 = 4X - 4(X - 3) + 12 - 6(X - 8)$
$= 24 + 48 - 6X \ldots X = 12'$

If 6′< X < 8′: 0 = 4X – 4(X – 3) + 12 = 24 impossible

If 3′< X < 6′: 0 = 4X – 4(X – 3) … X = 12 impossible

If X ≤ 3′: 0 = 4X … X = 0

M = 0 @ X = 0, 12′

To find where M is maximum, set the derivative of the moment, the shear equation, equal to zero.

$M = 4X - 4\langle X-3\rangle + \langle 12 \text{ if } X>6\rangle - 6\langle X-8\rangle$

$V = 0 = 4 - \langle 4 \text{ if } X>3\rangle - \langle 6 \text{ if } X>8\rangle$

V = 0 @ 3′ < X < 8′

It is impossible to find the location of M_{max} from this information. But if the Moment equation is re-examined for this zone, it becomes clear.

$M = 4X - 4(X-3) + \langle 12 \text{ if } X>6\rangle = 12 + \langle 12 \text{ if } X>6\rangle$

This means that M = 12k-f @ 3′ < X < 6′ and M_{max} = 24k-f @ 6′ < X < 8′.

Things to remember:

Concentrated load → Uniform Load → Triangular Load

Horizontal Shear Line → Sloped Shear Line → Parabolic Shear Curve

Sloped Moment Line → Parabolic Moment Curve → Third-degree Moment Curve

V = ∫W dx

Shear at any point is equal to the sum of loads on either side of that point.

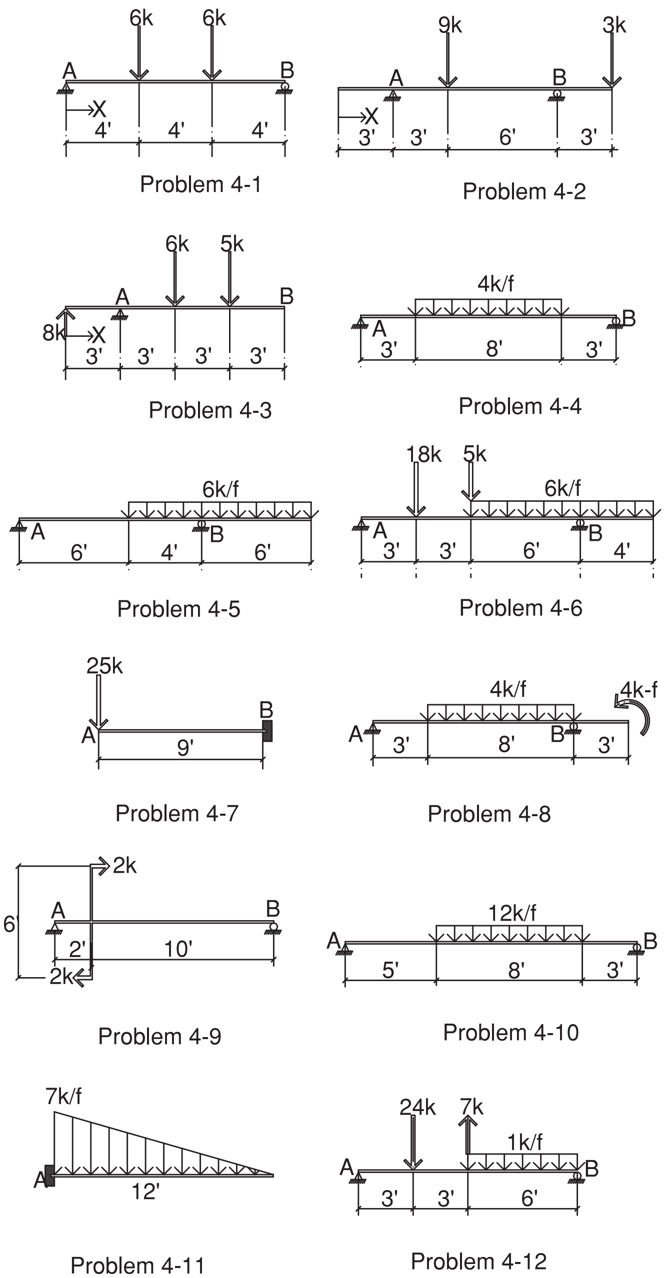
M = ∫V dx

Moment at any point is equal to the sum of shear areas on either side of that point.

Practice Exercises:

4-1 through 4-9: Find the reactions, draw and label the shear and moment diagrams for the beams shown in Figure 4.20 and identify M_{max}, V_{max} and the points where V = 0 and M = 0 for the beams.

4-10 through 4-12: Find the reactions, write the moment equation and find M_{max}, V_{max} and the points where V = 0 and M = 0 for the beams shown in Figure 4.20.



4.20
Chapter 4 Practice exercises

five

Load Tracing

The purpose of structure is to safely transfer all loads to the ground. The path that loads take to reach the ground depends on the structural system design. Load tracing follows the path of applied loads through a structural system, from one component to the next. Most building loads are expressed as uniform loads in pounds per square foot (psf) applied to floors, roofs or walls.

5.1 Finding Floor Loads on Columns

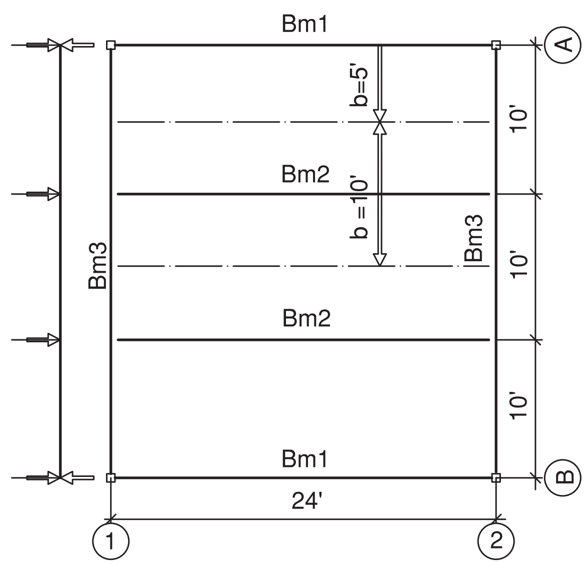
Tributary area is the area of surface with an applied uniform load that is transferred to a building component such as a beam or a column. The load on a beam (w) in #/f is the product of tributary width (b) in feet and the uniform load (U) in psf: W = b(U). Tributary width is defined as the sum of half the distance to the adjacent beam or wall in each direction.

The load on a column (P), when neglecting beam weight, is the tributary area (A) in square feet multiplied by the uniform load (U) in psf yielding a load in #: P = A(U). The tributary area can be found by multiplying the tributary width between columns in the x and y directions.

Example 5-1: Finding column loads.

For a uniform load (U) of 120psf:

Col. A1: A = 24′/2(30′/2) = 180f^2; P = 180f^2(120psf) = 21,600#



5.1

Finding column loads

Bm1: b = 10′/2 = 5′; w_1 = b(U) = 5′(120psf) = 600$^{\#/f}$

Bm2: b = 10′/2 + 10′/2 = 10′; w_2 = b(U) = 10′(120psf) = 1200$^{\#/f}$

The reactions at the ends of beams Bm2 become point loads on beams Bm3.

Bm1: R = 600$^{\#/f}$(24′/2) = 7200# @ x = 0′ and 30′ and bears directly on columns

Bm2: R = 1200$^{\#/f}$(24′/2) = 14400# @x = 10′ and 20′ and bears on Bm3

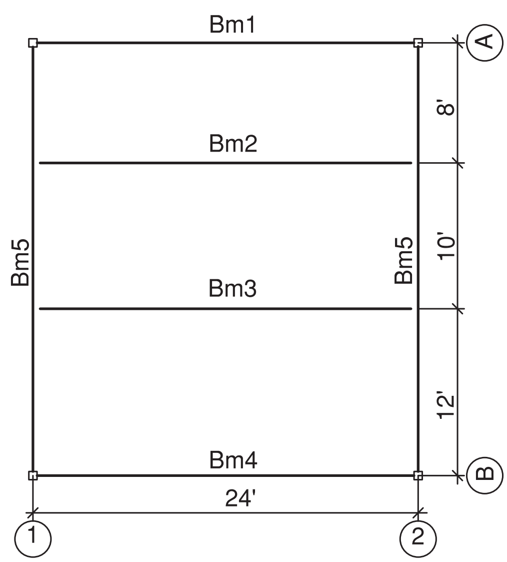
The reaction at either end of Bm3 = (14400#(10′) + 14400#(20′))/30′ = 14400#

The total load on each column equals the reaction at the end of Bm3 plus the reaction at the end of Bm1.

P = 14400# + 7200# = 21600# which is the same value found using the tributary area multiplied by the uniform load.

Example 5-2: Simple bay with unevenly spaced beams and no openings.

This exercise will show that using tributary area to find the load on the columns is not dependent on beam spacing.



5.2

Simple bay with unevenly spaced beams and no openings

Bm1: b = 8′/2 = 4′; w = 4′(120psf) = 480$^{\#/f}$ … R_1 = 480$^{\#/f}$(24′/2) = 5760#

Bm2: b = 10′/2 + 8′/2 = 9′; w = 9′(120psf) = 1080$^{\#/f}$ … R_2 = 1080$^{\#/f}$(24′/2) = 12,960#

Bm3: b = 12′/2 + 10′/2 = 11′; w = 11′(120psf) = 1320$^{\#/f}$ … R_3 = 1320$^{\#/f}$(24′/2) = 15,840#

Bm4: b = 12′/2 = 6′; w = 6′(120psf) = 720$^{\#/f}$ … R_4 = 720$^{\#/f}$(24′/2) = 8640#

Bm5: ΣM_B = 0 = 15,840#(12′) + 12,960#(22′) – Ay(30′) … A_y = 15,840#

Σf_y = 0 = B_y – 15,840 – 12,960 + 15,840 …

B_y = 12,960

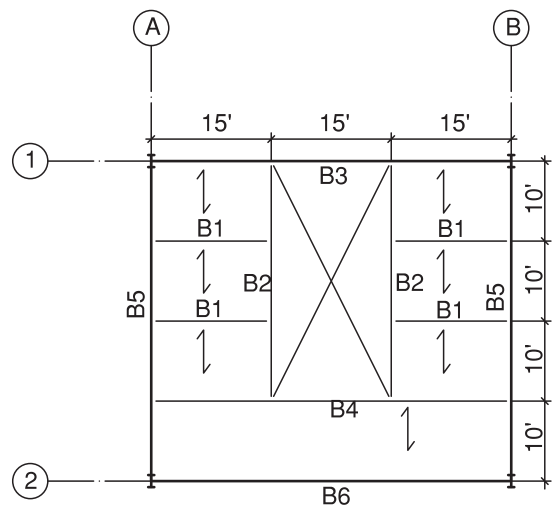
Col A1: The column load equals the sum of the Bm5 reaction at A plus the reaction from Bm1. P = 15,840# + 5760# = 21,600#

Col B1: The column load equals the sum of the Bm5 reaction at B plus the reaction from Bm4. P = 12,960# + 8640# = 21,600#

Note that this answer is the same as found in Example 5-1.

Example 5-3: A simple bay with an opening.

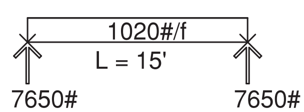
Find the column loads based on 100psf uniform load. Include beam weights: w_{B1} = 20$^{\#/f}$; w_{B2} = 32$^{\#/f}$; w_{B3} = 48$^{\#/f}$; w_{B4} = 64$^{\#/f}$; w_{B5} = 60$^{\#/f}$; w_{B6} = 42$^{\#/f}$.



5.3

A simple bay with an opening

Bm1: b = 10′; w = 10′(100) + 20 = 1020$^{\#/f}$; L = 15′; R1 = 1020$^{\#/f}$(15′/2) = 7650#



5.4

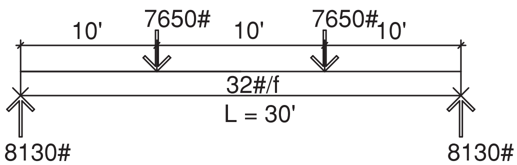
Load diagram for Bm1

Bm2: w = 32#′f; P = R_1 = 7650# @ x = 10′ and 20′

ΣM_A = 0 = 7650#(10′) + 7650#(20′) + 32$^{\#/f}$(30′)(15′) – B_y(30′) … B_y = 8130#

Σf_y = 0 = A_y – 7650# – 7650# – 32(30) + 8130# …

$A_y = 8130\#$



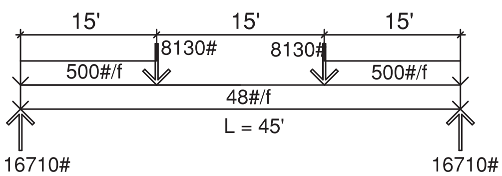
5.5

Load diagram for Bm2

Bm3: $b = 5'$; $w_1 = 5'(100psf) = 500^{\#/f}$; $w_2 = 48^{\#/f}$;
$P = R_2 = 8130\#$ @ $x = 15'$ and $30'$

$\Sigma M_A = 0 = 8130\#(15') + 8130\#(30') + 48^{\#/f}(45')(22.5') + 500^{\#/f}(15')(7.5') + 500^{\#/f}(15')(37.5') - B_y(45') \ldots$
$B_y = 16710\#$

$\Sigma f_y = 0 = A_y - 8130\# - 8130\# - 500^{\#/f}(30') - 48^{\#/f}(45') + 16710\# \ldots A_y = 16710\#$



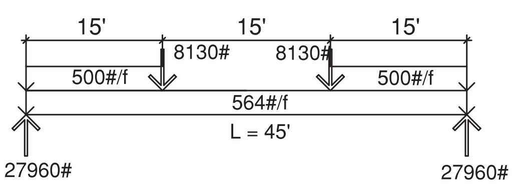
5.6

Load diagram for Bm3

Bm4: $b1 = b = 5'$; $w_1 = 5'(100psf) = 500^{\#/f}$;
$w_2 = 5'(100psf) + 64^{\#/f} = 564^{\#/f}$; $P = R_2 = 8130\#$
/@ 00x = 15′ and 30′

$\Sigma M_A = 0 = 8130\#(15') + 8130\#(30') + 548^{\#/f}(45')(22.5') + 500^{\#/f}(15')(7.5') + 500^{\#/f}(15')(37.5') - B_y(45') \ldots$
$B_y = 27960\#$

$\Sigma fy = 0 = Ay - 8130\# - 8130\# - 500^{\#/f}(30') - 548^{\#/f}(45') + 16710\# \ldots Ay = 27960\#$



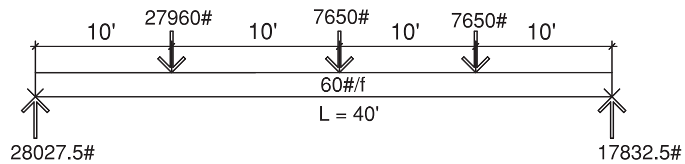
5.7

Load diagram for Bm4

Bm5: $w = 60^{\#/f}$; $P_1 = R_4 = 27{,}960\#$ @ $x = 10'$;
$P_2 = R_1 = 7650\#$ @ $x = 20'$ and $30'$

$\Sigma M_B = 27{,}960\#(10') + 7650\#(20') + 7650\#(30') + 64^{\#/f}(40')(20') - A_y(40') \ldots A_y = 17{,}832.5\#$

$\Sigma f_y = 0 = B_y - 27{,}960 - 7650 - 7650 - 64(40) + 17{,}832.5 \ldots B_y = 28{,}027.5\#$



5.8

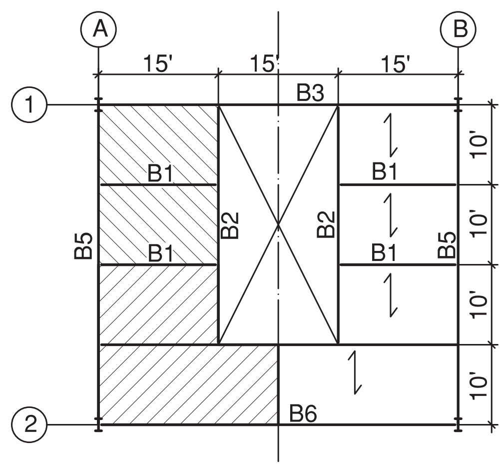
Load diagram for Bm5

Bm6: $w = 100psf(5') + 60^{\#/f} = 560^{\#/f} \ldots$
$A_y = B_y = 560^{\#/f}(45'/2) = 12{,}600\#$

Col A1 and A2: $P = 17{,}832.5 + 16710 = 34542.5\#$

Col B1 and B2: $P = 28{,}027.5 + 12{,}600 = 40{,}627.5\#$

Using tributary area:



5.9

Bay with opening using tributary area

Col A1 and A2: $P = 100psf(15')(20') + 20^{\#/f}(15') + 10^{\#/f}(15') + 32^{\#/f}(20') + 48^{\#/f}(22.5') + 60^{\#/f}(20') = 33{,}370\#$

Col B1 and B2: $P = 100psf[15'(20') + 7.5'(10')] + 10^{\#/f}(15') + 64^{\#/f}(22.5') + 42^{\#/f}(22.5') + 60^{\#/f}(20') = 41{,}235\#$

The difference between the calculated load tracing and using the tributary area is as follows:

On columns A1 and A2:
(33,370# – 34542.5#)/34542.5# = –3.39%

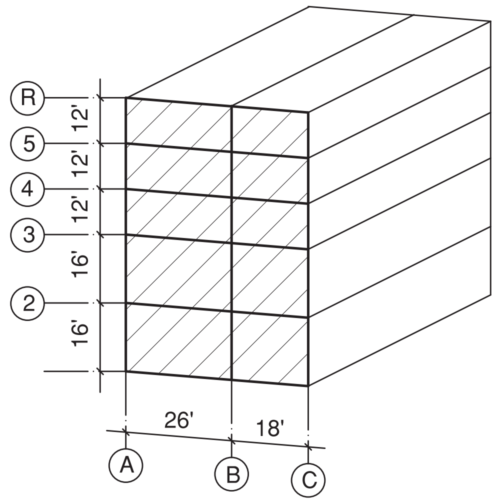
On columns B1 and B2:
(41,235 – 40265.5)/40,617.5 = 2.39%

Note that while the loads on the columns using the tributary area are not accurate when there is an opening in the bay, the margin of error is only 3.39%.

Tributary width and area can be used for lateral loads that act horizontally against a façade. The same methods are applied using the elevation. The loads are transferred to the column lines resisting lateral forces. Lateral loads are usually limited to wind and seismic forces, but may also include hydrostatic pressure from soil or horizontal components of transferred gravity loads. See Chapter 14: Lateral Bracing Systems for lateral design loads and resistance systems.

Example 5-4: A building façade receives a uniform wind pressure of 20psf.

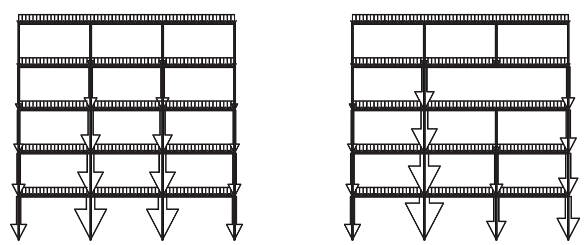
The wind force is resisted by column lines A, B and C. The façade panels transfer loads to the floor plates. Find the wind force applied to each column at each level. The solution is shown in the table below.



5.10
Tributary area for wind pressure

5.2 Accumulation of Column Loads

The load on any segment of a column is equal to the sum of all the loads on that column from levels above that segment. This means that loads accumulate from the top to the bottom of the column, resulting in the heaviest load at the base of C.



5.11
Accumulation of column loads

	Tributary height	*Column A Loads*	*Column B Loads*	*Column C Loads*
Trib. width		$\frac{26}{2} = 13'$	$\frac{26+18}{2} = 22'$	$\frac{18}{2} = 9'$
R	$\frac{12}{2} = 6'$	6'(13')(20psf) = 1560#	6'(22')(20psf) = 2640#	6'(9')(20psf) = 1080#
5	$\frac{12+12}{2} = 12'$	12'(13')(20psf) = 3120#	12'(22')(20psf) = 5280#	12'(9')(20psf) = 2160#
4	$\frac{12+12}{2} = 12'$	12'(13')(20psf) = 3120#	12'(22')(20psf) = 5280#	12'(9')(20psf) = 2160#
3	$\frac{12+16}{2} = 14'$	14'(13')(20psf) = 3640#	14'(22')(20psf) = 6160#	14'(9')(20psf) = 2520#
2	$\frac{16+16}{2} = 16'$	16'(13')(20psf) = 4160#	12'(22')(20psf) = 5280#	16'(9')(20psf) = 2880#

Notice that the interior columns carry more load because the tributary width for interior columns is larger than exterior columns. If there is a discontinuity of a column, as seen on the right, the loads normally carried by that column segment must be transferred by the floor system to neighboring columns. This affects the loads on columns on all levels below the discontinuation. The chart below shows the change in loads when the level 4 to 5 segment of column C is removed.

Column	*A*	*B*	*C*	*D*
5 – R	P	2P	2P2P	PP
4 – 5	2P	4P→6P	4P→0	2P→4P
3 – 4	3P	6P→8P	6P→2P	3P→5P
2 – 3	4P	8P→10P	8P→4P	4P→6P
1 – 2	5P	10P→12P	10P→6p	5P→7P

Practice Exercises:

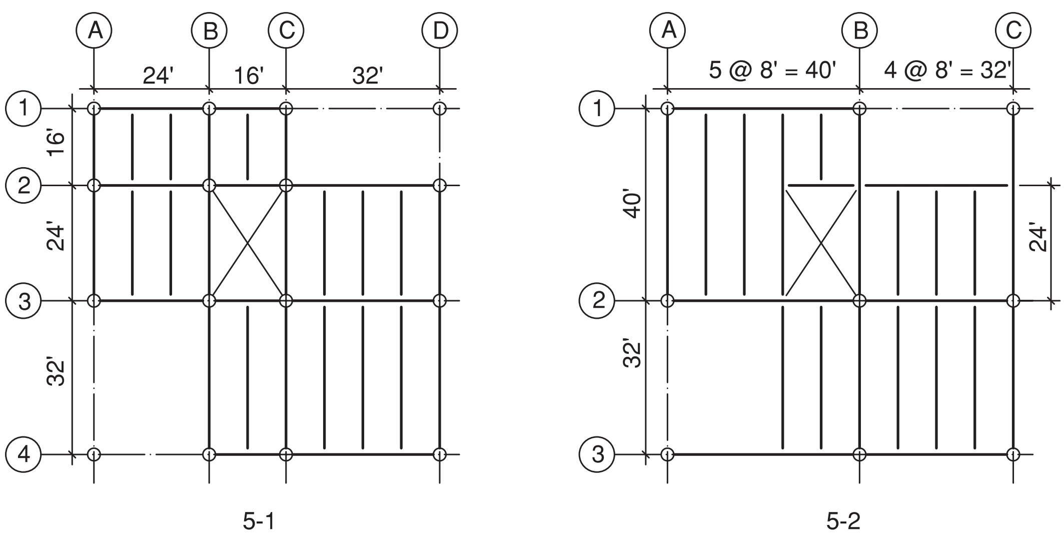
5-1: Find the loads on the columns given a uniform floor load of 80psf using tributary area.

5-2: Find the loads on the columns given a uniform floor load of 80psf

a) using tributary area;

b) by calculating beam reactions.

5-3: A uniform wind load of 30psf is resisted by columns A, B and C in 5-2 at each level. Determine the wind load on each column at each level if levels are 12o.c.



5.12

Chapter 5 Practice exercises

six

Simple Stress and Strain

Chapter 6 discusses strength of materials and the relationship between stress and strain.
Refer to Table A1.1: Materials Properties Table in the Appendix for properties of typical structural materials.

6.1 Force Induced Stress and Strain

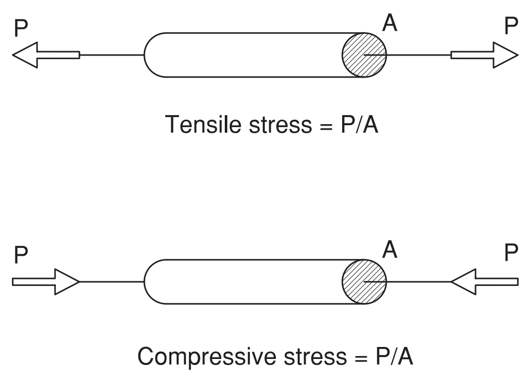
Stress is the expression of a force distributed over the area on which it bears. The basic formula for stress (f) is:

$$\text{stress} = \sigma = f = P/A$$

In this text f will signify actual stress and F will signify allowable stress. For axial forces of tension and compression, f = P/A. The units for stress are psi (pounds per square inch) or ksi (kips per square inch).

6.1.1 Tensile and Compressive Stress

Axial loads of tension and compression act on a stress area that is perpendicular to the line of the force, as seen in Figure 6.1. The stress area is the cross-sectional area for the member under tension or compression.



6.1
Axial stress

Example 6-1: A 3.5″×3.5″ square wood post has an allowable compressive stress F_c of 1000psi.

What is the maximum axial load the post can safely handle?

The allowable compressive stress, F_c = 1000psi, must be greater than the actual compressive stress = f_c.

$A = 3.5''(3.5'') = 12.25\text{in}^2$

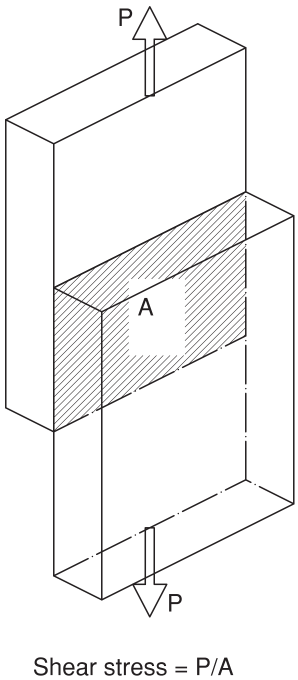
$f = P/A = P/12.25\text{in}^2 \leq 1000\text{psi} \ldots$
$P \leq 1000\text{psi}(12.25\text{in}^2) = 12{,}250\#$

Example 6-2: What size diameter rod is required to support a 200# load if the allowable tensile stress = F_t = 625psi?

$A = 200\#/625\text{psi} = 0.32\text{in}^2 = \pi d^2/4 \ldots$
$d = 0.638''$, round up to $\frac{3}{4}''$ diameter rod.

6.1.2 Shear Stress

Shear stress is caused by a load that is parallel to the stress area $f_v = P/A$.



6.2

Shear stress

Example 6-3: Two 1″×4″ boards are glued with an overlap of 3″ on the wide edge and subjected to an axial tension force of 800#.

If the adhesive is rated with an allowable shear stress of F_v = 40psi, is the overlap adequate?

P = 800# and A = 3″(4″) = 12in^2

f_v = P/A = 800#/12in^2 = 66.67psi > F_v = 40psi … overlap is not adequate.

How much overlap, h, is required?

F_v = 40psi = P/A = 800#/A = 800#/4h … h ≥ 800#/(4″(40psi)) = 5″

Example 6-4: Two steel plates are bolted together with four 1/2″ diameter bolts having an allowable shear stress of 14.4ksi.

What is the maximum axial tensile load, P, that the bolts can resist?

F_v = 14.4ksi, A_v = (4bolts)π(0.5)^2/4 = 0.785in^2

F_v = 14.4ksi ≥ f_v = P/A = P/0.785in^2 … P ≤ 14.4ksi(0.785in^2) = 18.34k

How many $\frac{1}{2}$″ diameter bolts are required to resist a shear force of 25k?

$$F_v = 14.4\text{ksi} > f_v \frac{25\text{k}}{A} \ldots A > \frac{25\text{k}}{14.4\text{ksi}} = 1.736\,\text{in}^2$$

F_v = 14.4ksi ≥ f_v = 25k/A … A ≥ 25k/14.4ksi = 1.736in^2

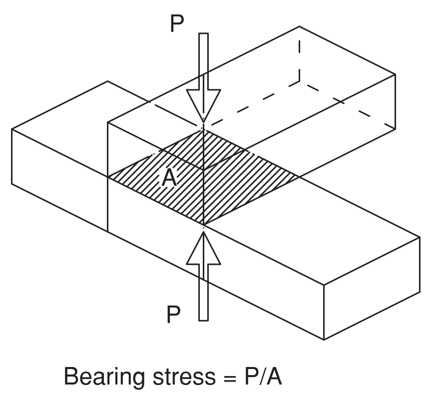
Let N = the # of bolts required

A = 1.736in^2 ≤ Nπ(0.5)^2/4 = 0.196N … N ≥ 1.736/0.196 = 8.857

Answer: Round up to N = 9 bolts.

6.1.3 Bearing Stress

Bearing stress is the stress caused by the transfer of load from one component to another on which it rests. The stress area is perpendicular to the direction of force. $f_{c\perp}$ = P/A where A is the area of bearing.

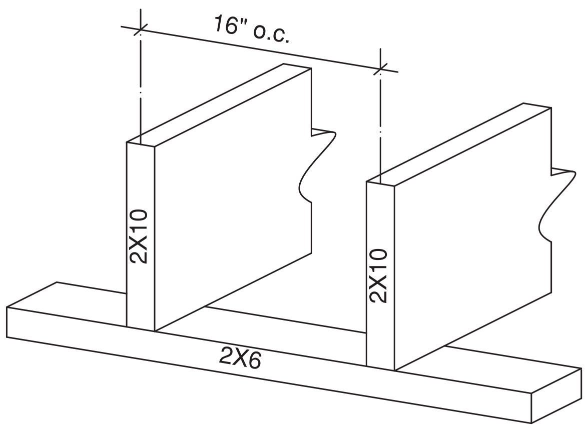


6.3

Bearing stress

Example 6-5: A series of 2×10 joists, spaced at 16″o.c. and 12′ long, with a uniform load of 100psf, bear on a flat 2×6 sill with an allowable bearing stress $F_{c\perp}$ = 975psi.

Is the 2×6 adequate? Actual dimensional lumber sizes: 2×6: 1.5″×5.5″, 2×10: 1.5″×9.25″.



6.4
Joists bearing on header

$F_{c\perp}$ = 975psi ≥ $f_{c\perp}$ = P/A

P = reaction at end of 2 × 10 joist

W = 100psf(16″/12″$^{/f}$) = 133.33$^{\#/f}$

P = WL/2 = 133.33$^{\#/f}$(12′)/2 = 800#

A = (thickness of 2 × 10)(width of 2 × 6) = 1.5″(5.5) = 8.25in^2

$f_{c\perp}$ = P/A = 800#/8.25in^2 = 96.97psi < 975psi

… 2 × 6 is adequate for bearing.

6.1.4 Strain and Modulus of Elasticity

Strain is the ratio of change in length to original length. As a ratio (inches per inch or feet per feet), it has no units.

Strain = ε = dL/L where L = original length and dL or δ = change in length

Modulus of Elasticity is the ratio of stress to strain. The units are the same as those for stress: psi or ksi.

Modulus of Elasticity = E = f/ε

Using the three equations, f = P/A, ε = dL/L and E = f/ε problems of simple stress and strain can be solved.

Example 6-6: What is the change in length of a 2″ square steel bar, 12″ long, subjected to an axial compressive force of 200k if E = 29,000ksi?

From the problem, it is known that L = 12″, A = 2″(2″) = 4in^2, P = 200k and E = 29,000ksi.

ε = dL/L … dL = L(ε)

E = f/ε … ε = f/E … dL = L(ε) = L(f)/E

$$f = \frac{P}{A} \ldots dL = \frac{PL}{EA} = \frac{200k(12")}{29{,}000ksi\ (4in^2)} = 0.0207"$$

Example 6-7: A 12′ long beam has a uniform load of 2k/ft.

It is supported at one end by a 1″ diameter steel rod (E_s = 29000ksi) and at the other end by a 1/2″ diameter titanium rod (E_t = 15000ksi). The steel rod is 2′ long. How long must the titanium rod be for the beam to remain level?

L = 12′, w = 5k/f … P = wL/2 = 2k/f(12′)/2 = 12k

A_s = π(1)^2/4 = 0.785in^2 E_s = 29,000ksi L_s = 2′ = 24″

A_t = π(.5)^2/4 = 0.196in^2 E_t = 15,000ksi L_t = ?

If beam remains level, dL_s = dL_t and since dL = PL/EA

$$L_t = \frac{P_sL_sE_tA_t}{P_tE_sA_s} = \frac{12k(24")(15{,}000ksi)(0.196in^2)}{12k(29{,}000ksi)(0.785in^2)} = 3.10"$$

Is this design adequate given an allowable tensile stress for steel of F_t = 30ksi and for titanium of F_t = 138ksi?

The stress in the steel rod = f_s = P/A = 12k/0.785in^2 = 15.29ksi < 30ksi … okay

The stress in the titanium rod = f_t = P/A = 12k/0.196in^2 = 61.22ksi < 138ksi … okay

6.2 Temperature Induced Stress and Strain

Every material has a coefficient of thermal expansion, α, expressed in terms of strain over change in temperature. Since strain, ε = dL/L, the coefficient of thermal expansion can be expressed as:

α = ε/ΔT = dL/LΔT

The change in length due to thermal expansion is: dL = αL(ΔT)

Because E = stress/strain = fL/dL, the stress from thermal expansion can be defined as:

$f = EdL/L = E\alpha L(\Delta T)/L = E\alpha\Delta T$

Example 6-8: A 1200′ tall high-rise has an exposed steel structure with a coefficient of expansion $\alpha = 6.5 \times 10^{-6}$in/in/°F.

The temperature of the steel is 85°F on the north side of the structure and 165°F on the south side. What is the difference in height between the north and south sides?

$dL = \alpha L(\Delta T) = (6.5 \times 10^{-6})$in/in/°F)(1200′)(12″/f)(165 – 85°F) = 7.49″

Example 6-9: 8′ wide aluminum panels with $\alpha = 12.8 \times 10^{-6}$ are installed on a façade during 50°F weather.

The highest design temperature for the aluminum panels is 200°F. What size expansion joint should be used?

$dL = \alpha L(\Delta T) = (12.8 \times 10^{-6}$in/in/°F)(8′)(12″/f)(200 – 50°F) = 0.184″ … round up to 3/16″ = 0.1875″

Given a value of E = 10,000ksi and F_c = 16ksi for aluminum, what is the maximum change in temperature the panels could handle without expansion joints?

$f = E\alpha(\Delta T)$ … $\Delta T = f/E\alpha$
= 16ksi/[10,000ksi(12.8×10^{-6}in/in/°F) = 125°F

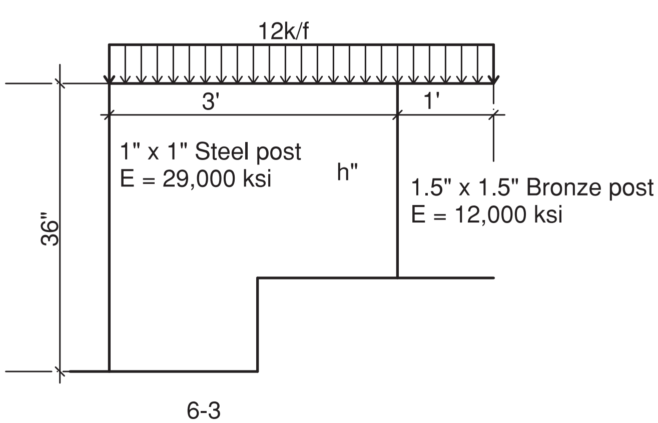
Practice Exercises:

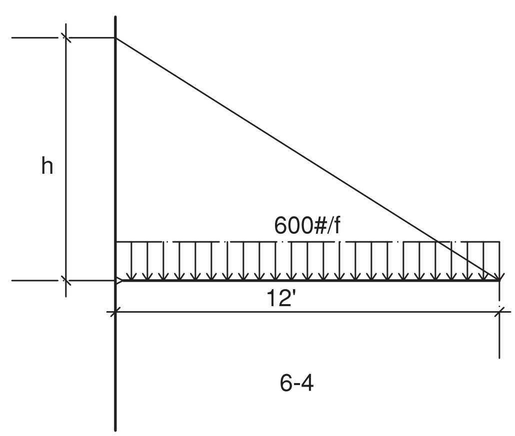
6-1: A diagonal tension brace, 15′ long and having a round cross-section with a diameter of $\frac{3}{4}$″ is subjected to 10k of tension. What is the change in length of the brace if E = 29,000ksi?

6-2: A W14 × 22 with an area, A = 6.49in^2 and a length of 24′ is installed on the roof of a building when the temperature is 80°F. What will be the change in length when the temperature drops to 15°F if the coefficient of thermal expansion for steel is 6.5×10^{-6}in/in/°F?

6-3: What is the required length of the bronze post if the beam must remain level?

6-4: A 12′ canopy supports a load of 600#/f with a hinge at the wall and a cable at the end. The cable is attached to the wall at some distance h above the canopy. Determine the distance h so that the canopy remains level given the cable properties of: E = 29,000ksi, A = 1in^2.





6.5
Chapter 6 Practice exercises

seven

Shear and Flexure in Beams

7.1 Neutral Axis and Moment of Inertia

The shear and bending stresses in a beam are dependent on the shape and size of the cross-section of the beam. In order to determine the shear and bending stresses, the neutral axis of the beam must be located. The neutral axis is located at the center of gravity.

Table 7.1 lists the center of gravity for some common geometric shapes. For a beam with a simple geometric form, finding the neutral axis is as simple as referring to the table.

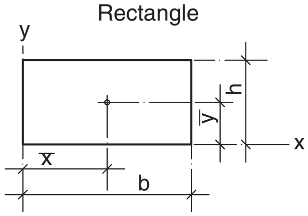
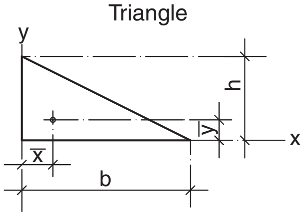
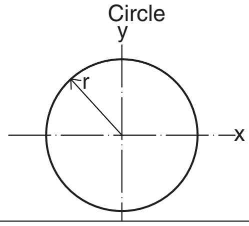
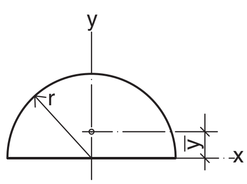
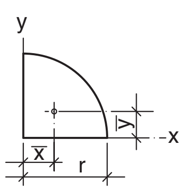
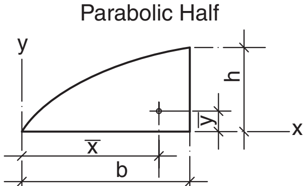
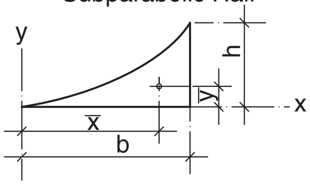
Often, however, a cross-section is not a simple geometric shape. For complex cross-sections, the center of gravity can be found by using the following equations:

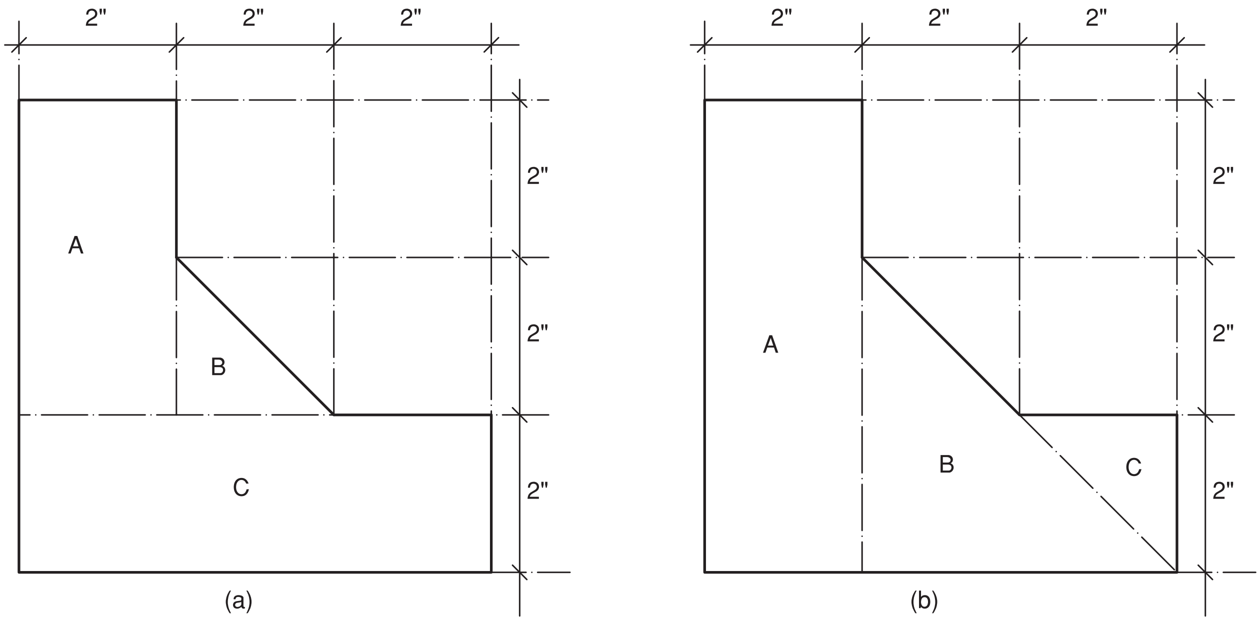
$$X_{ave} = \Sigma A_i X_i / \Sigma A_i \text{ and } Y_{ave} = \Sigma A_i Y_i / \Sigma A_i$$

where the center of gravity is located at a distance X_{ave} from the Y-axis and a distance Y_{ave} from the X-axis and where X_i is the distance X from the Y-axis to the center of gravity of an individual component and where Y_i is the distance Y from the x-Axis to the center of gravity of an individual component and where A_i is the area of an individual component.

Example 7-1: Find the center of gravity for the L-shaped cross-section in Figure 7.1a.

Table 7.1: Properties of simple geometric shapes

Shape	Area	$\overline{x}$	$\overline{y}$	I_x	I_y
Rectangle 	bh	b/2	h/2	$bh^3/12$	$hb^3/12$
Triangle 	bh/2	b/3	h/3	$bh^3/36$	$hb^3/36$
Circle 	πr^2	0	0	$pr^4/4$	$pr^4/4$
Semicircle 	$\pi r^2/2$	0	$4r/3\pi$	$\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$	$\pi^4/8$
Quarter Circle 	$\pi r^2/4$	$4r/3\pi$	$4r/3\pi$	$\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)r^4$	$\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)r^4$
Parabolic Half 	2bh/3	5b/8	2h/5	$8bh^3/175$	$19hb^3/480$
Subparabolic Half 	bh/3	3b/4	3h/10	$37bh^3/2100$	$hb^3/80$



7.1

Finding the center of gravity

Consider the cross-section above that has been broken into three simple geometric shapes, labeled A, B and C in Figure 7.1(a).

The best way to solve for the center of gravity involving multiple geometric shapes is to create a table:

Comp.	A_i	X_i	A_iX_i	Y_i	A_iY_i
A	2(4) = 8	2/2 = 1	8(1) = 8	2 + 4/2 = 4	8(4) = 32
B	2(2)/2 = 2	2 + 2/3 = 2.67	2(2.67) = 5.33	2 + 2/3 = 2.67	2(2.67) = 5.33
C	6(2) = 12	6/2 = 3	12(3) = 36	2/2 = 1	12(1) = 12
Totals	$\Sigma A_i = 22$		$\Sigma A_iX_i = 49.33$		$\Sigma A_iY_i = 49.33$

$$X_{ave} = \Sigma A_iX_i/\Sigma A_i = 49.33/22 = 2.2''$$

The neutral axis Y–Y is located 2.24″ to the right of the origin.

$$Y_{ave} = \Sigma A_iY_i/\Sigma A_i = 49.33/22 = 2.24''$$

The neutral axis X–X is located 2.24″ above the origin

Note: The center of gravity for a given cross-section will remain the same regardless of how the shape is divided into geometric components.

Consider the cross-section has been broken into three simple geometric shapes, labeled A, B and C in Figure 7.1(b).

Comp.	A_i	X_i	A_iX_i	Y_i	A_iY_i
A	2(6) = 12	2/2 = 1	12	6/2 = 3	36
B	4(4)/2 = 8	2 + 4/3 = 3.33	26.67	4/3 = 1.33	10.67
C	2(2)/2 = 2	6 – 2/3 = 5.33	10.66	2 – 2/3 = 1.33	2.66
Totals	$\Sigma A_i = 22$		$\Sigma A_iX_i = 49.33$		$\Sigma A_iY_i = 49.33$

$$X_{ave} = \Sigma A_iX_i/\Sigma A_i = 49.33/22 = 2.24''$$

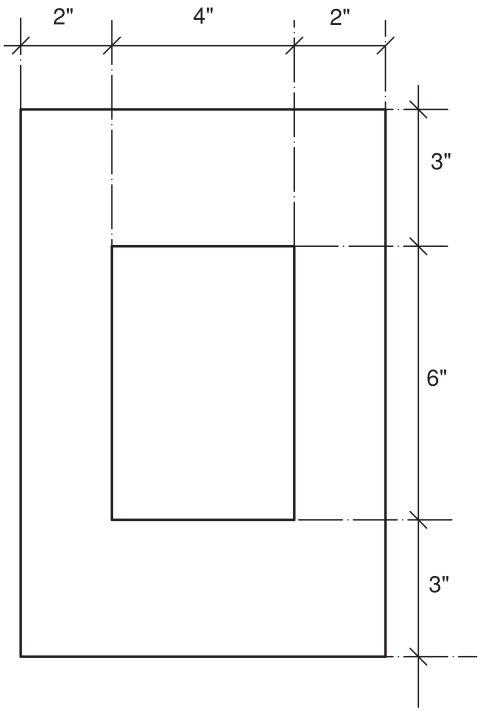
The neutral axis Y–Y is located 2.24in to the right of the origin.

$$Y_{ave} = \Sigma A_iY_i/\Sigma A_i = 49.33/22 = 2.24''$$

The neutral axis X–X is located 2.24″ above the origin.

When a cross-section contains a void, the void is a component with a negative area or an area that may be subtracted from the solid portion of the cross-section.

Example 7-2: Find the center of gravity for the 8 × 12 rectangle with a 4 × 6 void.



7.2

Finding the center of gravity in a shape with a void

Comp.	A_i	X_i	A_iX_i	Y_i	A_iY_i
Solid	8(12) = 96	8/2 = 4	384	12/2 = 6	576
Void	4(6) = −24	2 + 4/2 = 4	-96	4 + /2 = 7	−168
	$\Sigma A_i = 72$		ΣA_iX_i = 288		ΣA_iY_i = 408

$$X_{ave} = \Sigma A_iX_i/\Sigma A_i = 288/72 = 4''$$

$$Y_{ave} = \Sigma A_iY_i/\Sigma A_i = 408/72 = 5.67''$$

7.1.2 Moment of Inertia

Moment of inertia defines the ability of a cross-section to resist bending and deflection.

$$I_x = \int y^2 dA \text{ and } I_y = \int x^2 dA$$

This formula is easy for simple shapes such as a b × h rectangle where

$$A = b(y) \text{ and } dA = bdy$$

$$\int y^2 dA = \int y^2 bdy = by^3/3 \text{ from } y = h/2 \text{ to } -h/2$$

$$\int y^2 bdy = bh^3/24 + bh^3/24 = bh^3/12$$

It's not so easy for more complicated shapes.

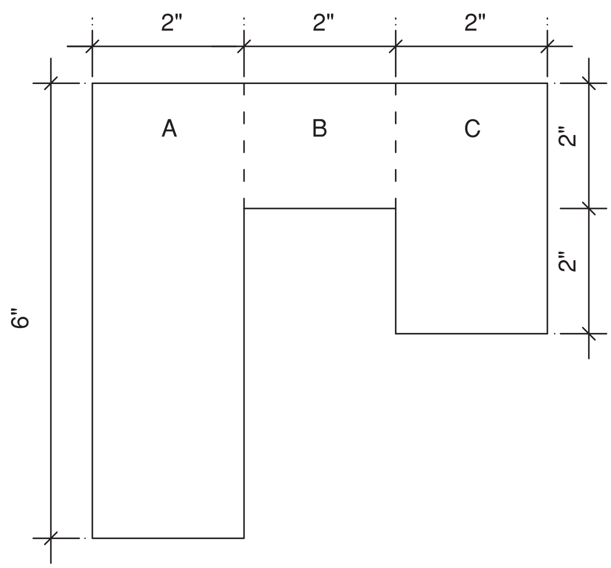
By breaking a complex shape into simple geometric components, and by finding the neutral axis, the formula below can be used to find the moment of inertia:

$$I_x = \Sigma I_{xi} + \Sigma Ady^2 \quad I_y = \Sigma I_{yi} + \Sigma Adx^2$$

where $dy = Y_i - Y_{ave}$ and $dx = X_i - X_{ave}$

Example 7-3: Find I_x for the cross-section in Figure 7.3.

Note that for rectangles, $I_x = bh^3/12$.



7.3

Finding moment of inertia

Comp.	A_i	Y_i	A_iY_i	Ix_i	*dy*	Ady^2
A	2(6) = 12	6/2 = 3	36	$2(6^3)/12$ = 36	3.67 − 3 = .67	5.39
B	2(2) = 4	4 + 2/2 = 5	20	$2(2^3)/12$ = 1.33	3.67 − 5 = −1.33	7.08
C	2(4) = 8	2 + 4/2 = 4	32	$2(4^3)/12$ = 10.67	3.67 − 4 = −.33	0.87
	$\Sigma A_i = 24$		ΣA_iY_i = 88	$\Sigma I_{xi} = 48$		ΣAdy^2 = 13.34

$$Y_{ave} = \Sigma A_iY_i/\Sigma A_i = 88/24 = 3.67''$$

$$I_x = \Sigma I_{xi} + \Sigma Ady^2 = 48 + 13.34 = 61.34\text{in}^4$$

Find I_y for the cross-section in Figure 7.3. Note that for rectangles, $I_y = hb^3/12$.

Comp.	A_i	X_i	A_iX_i	Iy_i	dx	Adx^2
A	2(6) = 12	2/2 = 1	12	$6(2^3)/12 = 4$	2.67 – 1 = 1.67	33.47
B	2(2) = 4	2 + 2/2 = 3	12	$2(2^3)/12 = 1.33$	2.67 – 3 = -0.33	0.44
C	2(4) = 8	4 + 2/2 = 5	40	$4(2^3)/12 = 2.67$	2.67 – 5 = -2.33	43.43
	$\Sigma A_i = 24$		$\Sigma A_iY_i = 64$	$\Sigma I_{yi} = 8$		$\Sigma Ady^2 = 77.34$

$$X_{ave} = \Sigma A_iX_i/\Sigma A_i = 64/24 = 2.67''$$

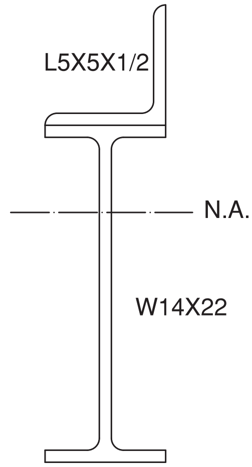
$$I_y = \Sigma I_{yi} + \Sigma Adx^2 = 8 + 77.34 = 85.34\text{in}^4$$

What I_x and I_y reveal about the cross-section in Figure 7.3 is that a load placed vertically, and therefore causing bending about the X–X axis will create more deflection of the beam than a load acting horizontally, and therefore causing bending about the Y–Y axis. This is because $I_x < I_y$ and so the resistance to bending around the X–X axis is less than that around the Y–Y axis.

7.1.3 Moment of Inertia in Rolled Steel Components

The AISC Steel Manual lists section properties for all standard rolled steel components. Among the section properties listed are the moment of inertia values I_x and I_y. When using a standard rolled member, there is no calculation necessary. But, if the cross-section is built up using rolled sections and/or plates, then the equations $I_x = \Sigma I_{xi} + \Sigma Ady^2$ and $I_y = \Sigma I_{yi} + \Sigma Adx^2$ must be used.

Example 7-4: Find the moment of inertia about the X–X axis for a W14×22 with an L5×5×$\frac{1}{2}$ welded to the top flange as shown in Figure 7.4.



7.4
Finding moment of inertia in steel shapes

Section properties:

$$W14\times22: A = 6.49\text{in}^2, d = 13.74'', I_x = 199\text{in}^4$$

$$L5\times5\times\tfrac{1}{2}: A = 4.75\text{in}^2, y = 1.43'', I_x = 11.3\text{in}^4$$

Comp.	A_i	Y_i	A_iY_i	Ix_i	dy	Ady^2
W14×22	6.49	= 13.74/2 = 6.87	44.59	199	10.38 – 6.87 = 3.51	79.96
L5×5×1/2	4.75	= 13.74 + 1.43 = 15.17	72.06	11.3	10.38 – 15.17 = 4.79	109.08
	Σ = 11.24		Σ = 116.65	Σ = 210.3		Σ = 189.04

$$Y_{ave} = 116.65/11.24 = 10.38''$$

$$I_x = 210.3 + 189.04 = 399.34\text{in}^4$$

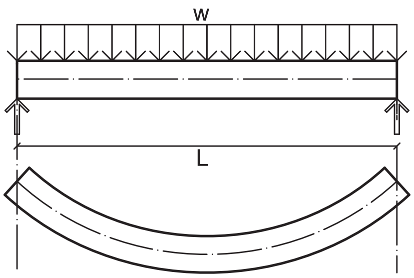
Note: If all components are symmetrical about the bending axis X-X, dy = 0 and the equation for I_x reduces to $I_x = \Sigma I_{xi}$.

7.2 Bending Stress

The basic equation for bending stress is:

$$f_b = Mc/I = M/S$$

The derivation of this equation comes from examination of particles in a beam subjected to bending.

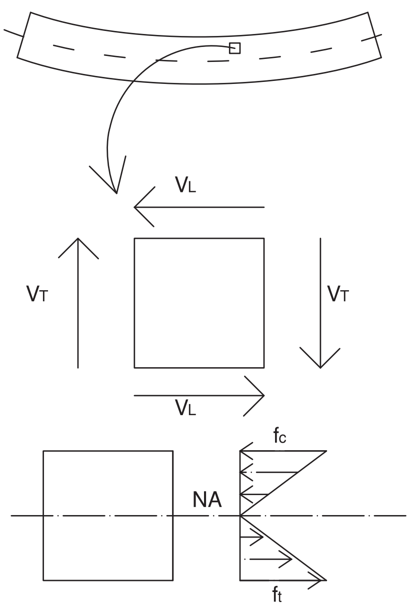


7.5
Bending stress

Consider a beam subjected to bending, as shown in Figure 7.5. The beam wants to deform under the load. The area above the neutral axis is in compression and the area below the neutral axis is in tension.

c = distance from neutral axis to outer most point of cross-section

The greatest stress will occur at the greatest distance from the neutral axis, c. The stress due to bending at any point is f = fc(y/c).



7.6
Internal couples

Every particle in the cross-section is at some distance y from the neutral axis and has some area, dA. The force in tension or compression acting on each particle is F = dA(f). The moment caused by the force acting on any particle at a distance y from the neutral axis is:

$$M_i = Fy = ydA(f)(y/c) = y^2dA(f)/c$$

The bending stress, f_b, on any particle is $f_b = M_ic/y^2dA$ and the total bending stress is the sum of the bending stress on all particles:

$$f_b = \Sigma M_ic/y^2dA = Mc/\Sigma y^2dA$$

Since moment of inertia = Σy^2dA, the value I can be substituted into the equation, giving the bending stress formula:

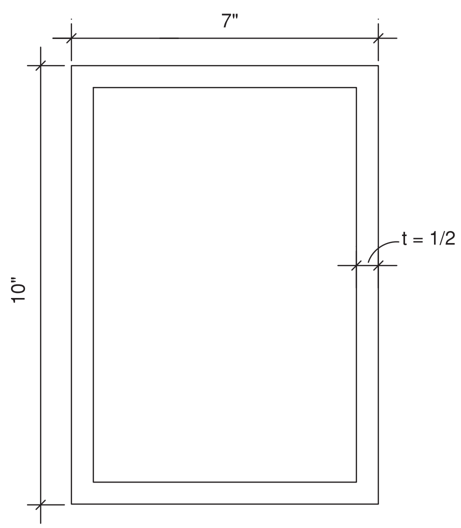
$$f_b = Mc/I$$

Section modulus is defined as I/c, further simplifying the equation to:

$$f_b = M/S$$

Note: Be careful to reconcile the units in the bending stress equations. If the moment found is in units of #-f or k-f, it must be multiplied by a factor of 12 inches per foot to obtain a stress in pounds per square foot (psf) or kips per square foot (ksi), respectively. For example, if M = 48k-f and $S = 16in^3$, $f_b = 48k\text{-}f(12''/f)/16in^3 = 36ksi$.

Example 7-5: Find the maximum bending stress in a simply supported beam carrying a uniform load of 2k/f over a span of 14′ given the cross-section shown in Figure 7.7.



7.7
Example 7-5

$$M_{max} = wL^2/8 = 2k/f(14')^2(12''/f)/8 = 588k\text{-}in$$

$$I_x = 7(10^3)/12 - 6(9^3)/12 = 218.83in^4$$

$c = 10''/2 = 5''$

$f_b = Mc/I = 588\text{k-in}(5'')/218.83\text{in}^4 = 13.44\text{ksi}$

What is the bending stress at a distance of 4′ from a support?

$@x = 4'$, $M = wx^2/2 = 2\text{k/f}(4')^2(12''^{/f})/2 = 192\text{k-in}$

$f_b = Mc/I = 192\text{k-in}(5'')/218.83\text{in}^4 = 4.39\text{ksi}$

For rolled steel, standard size tables usually include the value of the section modulus, S.

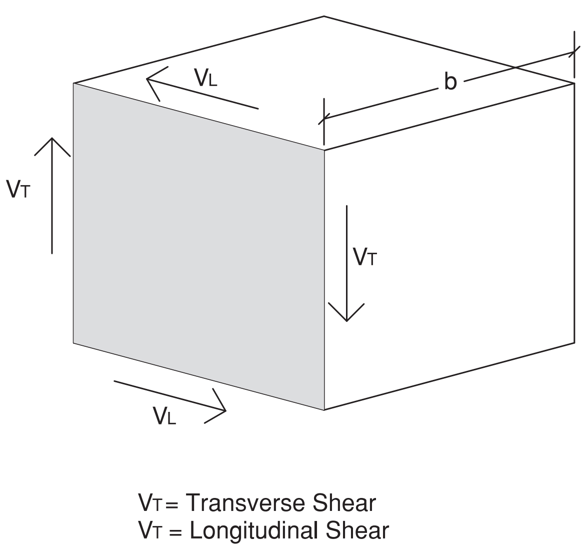
Example 7-6: Find the maximum bending stress for the beam in Example 7-5 if the cross-section is a W16×31 with $S_x = 47.20\text{in}^3$.

$f_b = Mc/I = M/S = 588\text{k-in}/47.20\text{in}^3 = 12.46\text{ksi}$

7.3 Shear Stress

7.3.1 Shear in Beams with Geometric Cross-sections

Unlike shear stress caused by an axial load in which fv = P/A as described in Chapter 6, a beam with bending causes both transverse and longitudinal shear forces within the beam. This occurs because the transverse shear action creates a moment within particles that must be resisted by an equal and opposite moment.



7.8

Shear stress in beams with geometric cross-sections

The equation for shear in beams is:
The shear stress in each particle is:

$f_{vi} = Vt/dy(b)$

For the entire cross-section:

$f_v = V/\Sigma yb$

This can be multiplied by $1 = \Sigma dAy/\Sigma dAy$ to yield:

$f_v = V\Sigma dAy/\Sigma y^2 dAb$

Recognizing $I = \Sigma y^2 dA$, the equation can be reduced to:

$f_v = V\Sigma dAy/Ib$

Let $Q = \Sigma dAy = \Sigma A_i dy$ when considering individual geometrical entities in the cross-section, this will yield the standard shear stress formula:

fv = VQ/Ib where

V = shear from the shear diagram

$Q = \Sigma A_i dy$

A_i = area above or below the shear plane

dy = distance from the neutral axis to the center of gravity of the area A

I = moment of inertia

b = the width of the cross-section at the shear plane.

Example 7-7: Find the shear stress at the neutral axis for a 4″×6″, 12′ beam with a uniform load of $500^{\#/f}$.

$V = wL/2 = 500^{\#/f}(12')/2 = 3000\#$

$I_x = 4(6^3)/12 = 72\text{in}^4$

$b = 4''$

$Y_{ave} = 3''$ = the location of the neutral axis X–X

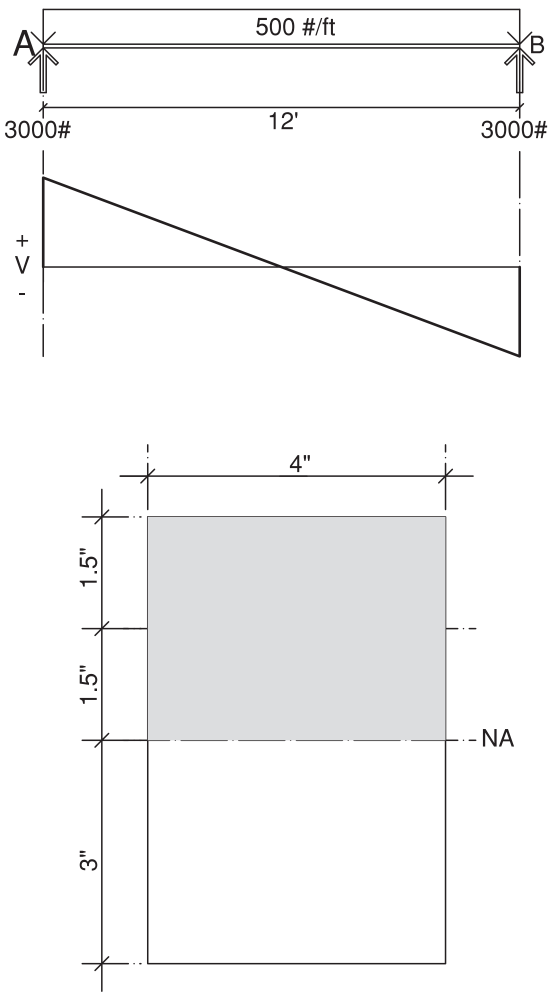
$A_v = 4''(3') = 12\text{in}^2$

$y_A = 3 + 1.5 = 4.5''$ = location of center of gravity of Av

$dy = y_A - Y = 4.5 - 3 = 1.5''$

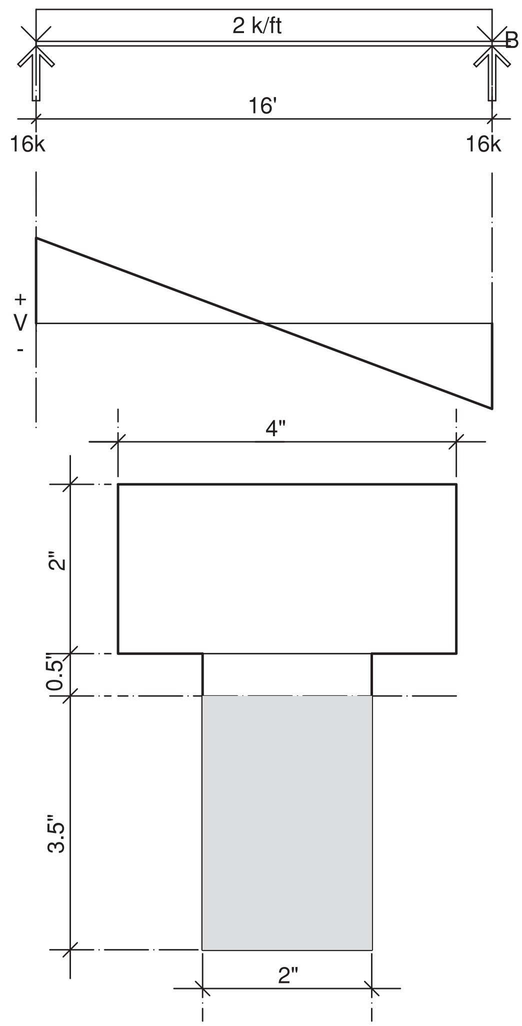
$Q = \Sigma Avdy = 12\text{in}^2(1.5'') = 18\text{in}^3$

$f_v = VQ/Ib = 3000\#(18\text{in}^3)/[72\text{in}^4(4'')] = 187.5\text{psi}$



7.9

Shear stress at neutral axis



7.10

Shear stress in a T-shape

Note that for a rectangular cross-section b × h,

$I = bh^3/12$

$A_i = bh/2$

$dy = 3h/4 - h/2 = h/4$

$Q = (bh/2)(h/4) = bh^2/8$

$f_v = V(bh^2/8)/[(bh^3/12)(b)] = 3V/2bh = 3V/2A$

Example 7-8: The T-shape in Figure 7.10 spans 16′ and carries a uniform load of 2k/f over its entire span.

Find the maximum shear stress at the neutral axis.

$V = wL/2 = 2k/f(16')/2 = 16k$

Comp.	A_i	Y_i	A_iY_i	Ix_i	*dy*	Ady^2
Flange	4(2) = 8	4 + 2/2 = 5	40	$4(2^3)/12$ = 2.67	5 − 3.5 = 1.5	18
Web	2(4) = 8	4/2 = 2	16	$2(4^3)/12$ = 10.67	2 − 3.5 = −1.5	18
	Σ = 16		Σ = 56	Σ = 13.33		Σ = 36

$Y_{ave} = 56/16 = 3.5''$

$I_x = 13.33 + 36 = 49.33 in^4$

$Q = \Sigma Ady = 8(1.5) + 2(.5)(.25) = 12.25 in^3$

$f_v = VQ/Ib = 16k(12.25 in^3)/[49.33 in^4(2'')] = 1.99 psi$

Find the maximum shear stress at the bottom of the flange.

$A_v = 8in^2$ (from the table on page 57: Flange A_i)

$y_A = 5''$ = (from the table on page 57: Flange Y_i)

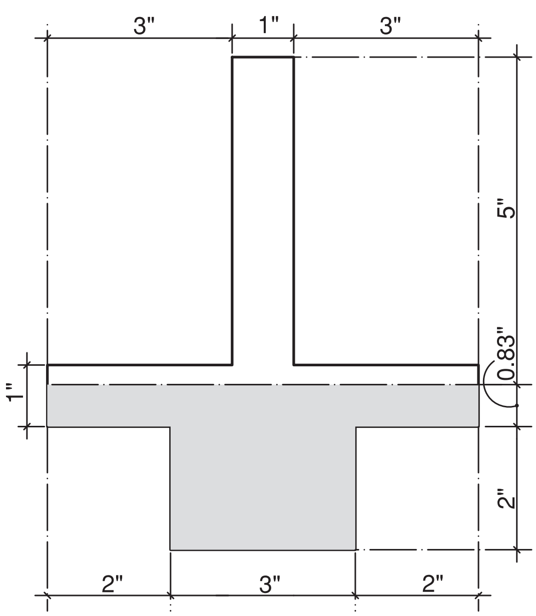
$dy = 5 - 3.5 = 1.5''$ (from the table on page 57: Flange dy)

$Q = \Sigma A_i dy = 8in^2(1.5'') = 12.0in^3$

$f_v = VQ/Ib = 16k(12in^3)/[49.33in^4(2'')] = 1.95psi$

Note: Always check the shear stress at points where the width, b, changes, especially when b decreases in a direction away from the neutral axis.

Example 7-9: Find the shear stress for the cross-section in Figure 7.11 if V = 100k.



7.11

Finding shear stress in composite shapes

Comp.	A_i	Y_i	A_iY_i	Ix_i	dy	Ady^2
Top	5	5.5	27.5	10.42	2.67	35.64
Flange	7	2.5	17.5	0.58	0.33	0.76
Base	6	1	6	2.0	1.83	20.09
	Σ = 18		Σ = 51	Σ = 13.0		Σ = 56.49

$Y_{ave} = 51/18 = 2.83''$

$I_x = 13.0 + 56.49 = 69.49in^4$

$A_v = 5 + .17(7) = 6.19in^2$

$y_A = [5in^2(3 + 5/2) + .17(7)(2.83 + .17/2)/6.19 = 5.00$
= location of center of gravity of A_v from bottom

$dy = y_A - Y_{ave} = 5.00 - 2.83 = 2.17''$

$Q = \Sigma Avdy = 6.19in^2(2.17'') = 13.43in^3$

$f_v = VQ/Ib = 100k(13.43in^3)/[49.33in^4(7'')] = 3.89ksi$

Find the maximum shear stress at the top of the flange.

$A_v = 5in^2$

$dy = 3 + 2.5 - 2.83 = 2.67''$

$Q = Avdy = 5in^2(2.67'') = 13.35in^3$

$f_v = VQ/Ib = 100k(12in^3)/[49.33in^4(2'')] = 24.33ksi$

7.3.2 Shear in Rolled Steel

The AISC (American Institute of Steel Construction) recommends using a value for actual shear stress of $f_v = V/twd$. By using this value, the flange components and fillets connecting flanges to webs are ignored, making the calculation much simpler.

Example 7-10: Find the shear stress in a W8×40, 8′ long carrying a concentrated load at center of 50k if the web thickness, $t_w = 0.36''$ and the depth, d = 8.25″.

$V = wL/2 + P/2 = 40^{\#/f}(1k/1000\#)(8')/2 + 50k/2$
$= 25.16k$

$f_v = V/t_wd = 25.16k/[0.36''(8.25'')] = 8.47ksi$

Practice Exercises:

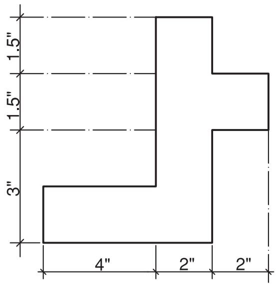
7-1 through 7-6: Find I_x and I_y for the cross-sections shown:

7-7 through 7-10: Find the maximum bending stress in the beams and cross-sections shown.

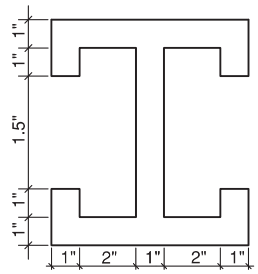
7-11 through 7-14: Find the maximum shear stress in the beams and cross-sections shown.

7.12

Chapter 7 Practice exercises



7-1



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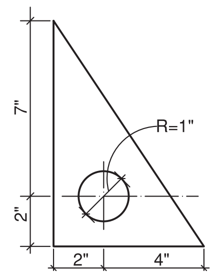

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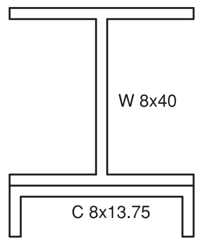

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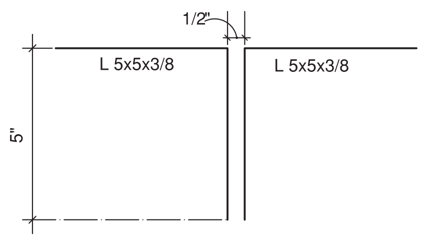

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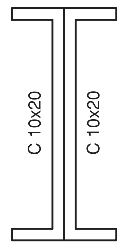

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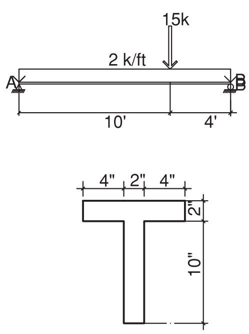

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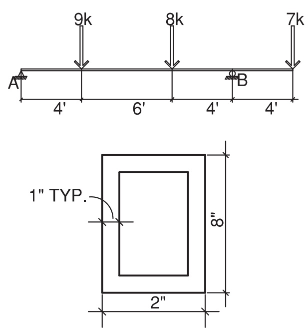

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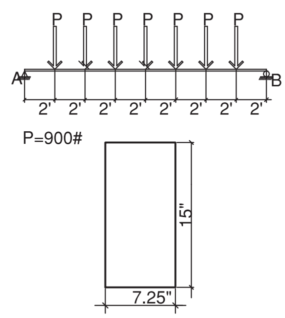

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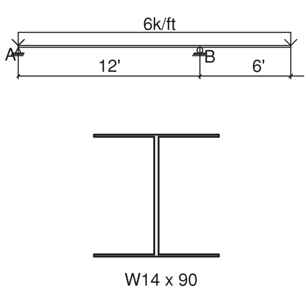

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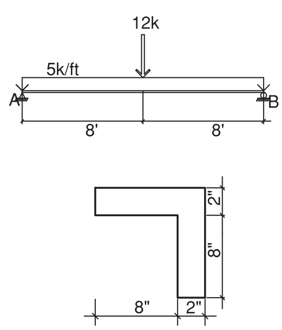

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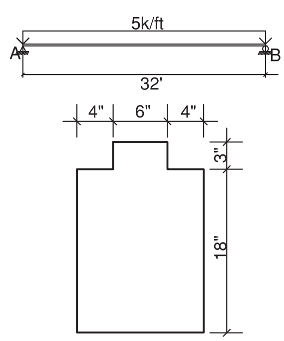

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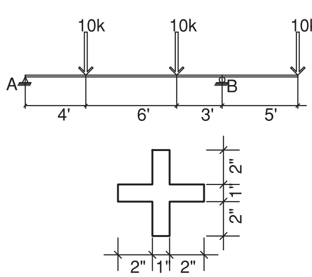

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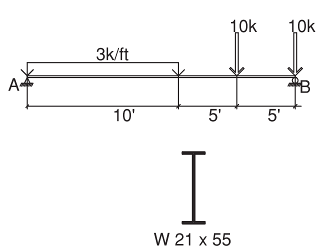


7-14

eight

Deflection in Beams

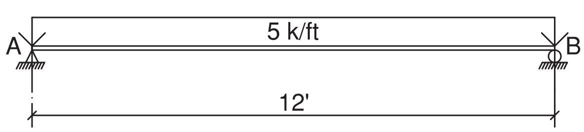
8.1 Deflection Charts

Most architects find the deflection in beams through the use of deflection charts. Deflection charts can be found in such publications as the AISC Steel Manual, or readily in online sources. A sample of shear, moment and deflections for some typical beam loading scenarios can be found in Appendix A1.2.

Note: All deflection charts assume that the length of the beam is in inches. If the length of the beam is not converted to inches, if it is used in units of feet, the deflection equation must be multiplied by $1728in^3/ft^3$ in order to find a deflection in inches.

Example 8-1: Find the maximum deflection in a 12¢ beam with a 5k/f load given E = 29,000ksi and I = $300in^4$.

From A1.2 , load type 1:



8.1

Example 8-1

$\Delta_{max} = 5wL^4/384EI$ @ $x = L/2$

$w = 5k/f$, $L = 12'$, $E = 29000ksi$, $I = 300in^4$

$\Delta_{max} = 5(5k/f)(12')^4(1728in^3/ft^3)/[384(29{,}000ksi)(300in^4)] = 0.27''$

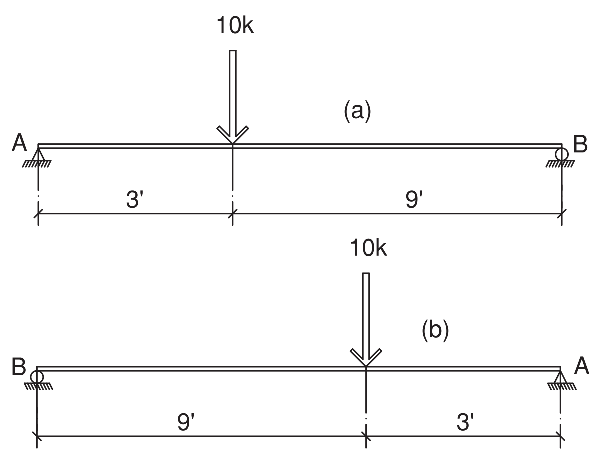
What is the deflection at a point 8′ from the support?

$\Delta x = wx(L^3 - 2Lx^2 + x^3)(1728)/24EI$

$= 5k/f(8')[12'^3 - 2(12')(8'^2) + 8'^3](1728in^3/ft^3)/[24(29000ksi)(300in^4)] = 0.23''$

Example 8-2: Not all loads are symmetrical.

Consider the beam in Figure 8.2(a) with a concentrated load of 10k placed at 3′ from the left support of a 12′ span. What is the maximum deflection if E = 15,000ksi and I = $600in^4$? The placement of the concentrated load is such that the equation in A1.2, load type 3 cannot be used because a < b (3 < 9) and the equation is valid when a > b. If this situation occurs, consider the beam from the other side as shown in Figure 8.2(b).



8.2

Example 8-2

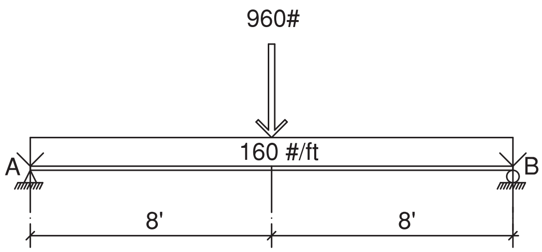
$P = 10k$, $L = 12'$, $a = 9'$, $b = 3'$, $E = 15{,}000ksi$, $I = 600in^4$

$$\Delta_{max} = Pab(a + 2b)\sqrt{(3a(a + 2b))}/27EIL$$

$$= 10(9)(3)(9 + 2(3))\sqrt{[3(9)(9 + 2(3))]}(1728)/[27(15{,}000)(600)] = 0.58''$$

Example 8-3: Combining loads.

Many times a beam will have a combination of load scenarios. For example, a beam may have a uniform load from a floor loading and from its own weight plus a concentrated load from the reaction of a beam it supports. Find the maximum deflection of the beam in Figure 8.3 if E = 1,500,000psi and the cross-section is 8″ wide by 12″ deep.



8.3
Example 8-3

$$I = bh^3/12 = 8(12)^3/12 = 1152in^4$$

Using load scenarios 1 and 3:

$w = 160^{\#/f}$, $L = 16'$, $E = 1{,}500{,}000psi$, $I = 1152in^4$, $P = 960\#$

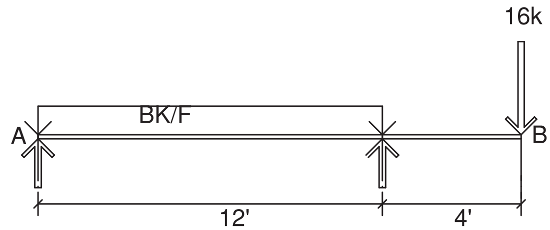
$$\Delta_{max} = 5wL^4/384EI + PL^3/48EI$$

$$= 5(160)(16^4)(1728)/[384(1{,}500{,}000)(1152)] + 960(16^3)(1728)/[48(1{,}500{,}000)(1152)]$$

$$= 0.22''$$

Note: Deflection charts list the absolute value of deflection without regard to the direction. Care must be taken to note in what direction a load will cause a deflection, especially when adding deflections from different loading scenarios.

Example 8-4: Find the deflection at the end of the overhang for the beam in Figure 8.4 if E = 29,000ksi and I = 199in⁴.



8.4
Example 8-4

$W = 3k/f$, $L = 12'$, $x1 = a = 4'$, $P = 16k$, $E = 29{,}000ksi$, $I = 199in^4$

$$\Delta = -(wL^3x1)/24EI + Pa^2(L + a)/3EI$$

Notice that the deflection at the end of the overhang caused by the uniform load will be upwards. Because of this, the equation is entered as a negative value when adding it to the equation for the point load at the end of the overhang, which will be downward.

$$\Delta = -(3(12^3)(4)(1728)/[24(29000)(199)] + 16(4^2)(12 + 4)(1728)/[3(29000)(199)] = -0.259'' + 0.409'' = 0.15''\downarrow$$

8.2 Double Integration Method

$$\Delta = \iint Mdx/EI$$

Deflection is the second integral of the moment equation. The first integral of the moment equation, $\int Mdx$ is the slope of the deflected beam. The Double Integration Method may not seem as easy to use as deflection charts, but it is useful when the location of the maximum deflection is unknown and when there are many combined loading scenarios.

Example 8-5: Find the maximum deflection of a simple beam with a length, L, and a uniform load, w.

1. Begin by writing the moment equation:
   $M = wLx/2 - wx^2/2$
2. Take the first integral of the moment equation. Remember to add the constant to the equation.

$$dEI\Delta = slope = \int Mdx = wLx^2/4 - wx^3/6 + C_1$$

In many cases, it is not know where the slope will equal zero; but with a symmetrical load on a simple beam, it is known that the maximum deflection will be at the center of the beam and that is the point at which the slope will equal zero.

Slope = 0 at x = L/2

$0 = wL(L/2)^2/4 - w(L/2)^3/6 + C_1$

$= wL^3/16 - wL^3/48 + C_1 = 0$

$C_1 = -wL^3/24$

But, for this example, assume that the location of the maximum deflection is unknown.

$dEI\Delta = \int Mdx = wLx^2/4 - wx^3/6 + C_1$

3. Take the second integral of the moment equation:

$EI\Delta = \iint Mdx = wLx^3/12 - wx^4/24 + C_1x + C_2$

$EI\Delta$ = 0 at the supports, @ x = 0, L

$0 = wL(0)/12 - w(0)/24 + C_1(0) + C_2 \ldots C_2 = 0$

$0 = wL(L^3)/12 - w(L^4)/24 + C_1(L) = wL^4/24 + C_1L \ldots$
$C_1 = -wL^3/24$

Inserting the value of C_1 into the ∫Mdx equation and setting the equation equal to zero will reveal where the slope equals zero and therefore points of maximum deflection.

$dEI\Delta = \int Mdx = wLx^2/4 - wx^3/6 - wL^3/24 = 0$

$6Lx^2 - 4x^3 - L^3 = 0 \ldots x = L/2$

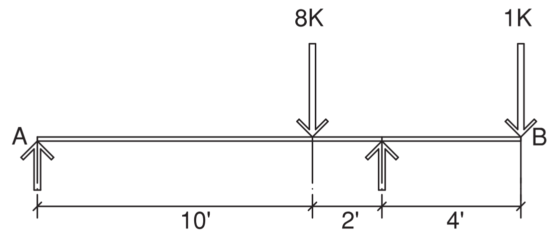
$EI\Delta = \iint Mdx = wLx^3/12 - wx^4/24 - wxL^3/24$ @ $x = L/2$

$EI\Delta = \iint Mdx = wL4/96 - wL4/384 - wL^4/48$
$= wL^4(4 - 1 - 8)/384 = -5wL^4/384$

$\Delta_{max} = -5wL^4/384EI$

Notice that the Double Integration Method gives the direction of the defection. A negative value indicates that the deflection is downward. $\Delta_{max} = 5wL^4/384EI$ is the value given in the deflections charts.

Example 8-6: Find the maximum deflection in the beam shown in Figure 8.5, if E = 29,000ksi and I = 199in⁴.



8.5
Example 8-6

Find reactions:

$\Sigma M_A = 0 = 8k(10') - B_y(12') + 1k(16') \ldots B_y = 8k\uparrow$

$\Sigma f_y = 0 = A_y - 8k + 8k - 1k \ldots A_y = 1k\uparrow$

Write the moment equation:

$M = 1x - 8\langle x - 10\rangle + 8\langle x - 12\rangle$

Find the first and second integral of the moment equation:

$dEI\Delta = x^2/2 - 8\langle x - 10\rangle^2/2 + 8\langle x - 12\rangle^2/2 + C_1$

$EI\Delta = x^3/6 - 8\langle x - 10\rangle^3/6 + 8\langle x - 12\rangle^3/6 + C_1x + C_2$

Solve for C_2 and C_1:

$\Delta = 0$ @ x = 0 and @ x = 12

@x = 0, $EI\Delta = 0 - 0 + 0 + 0 + C_2 \ldots C_2 = 0$

@ x = 12, $EI\Delta = (12)^3/6 - 8(2)^3/6 + 0 + 12C_1 = 0$

$C_1 = [10.666 - 288]/12 = -23.11$

Set $dEI\Delta = 0$ to find point of maximum deflection:

$EI\Delta = x^3/6 - 8\langle x - 10\rangle^3/6 + 8\langle x - 12\rangle^3/6 - 23.11x$

$dEI\Delta = x^2/2 - 8\langle x - 10\rangle^2/2 + 8\langle x - 12\rangle^2/2 - 23.11$
= 0 where deflection changes direction

If Δmax occurs between x = 0 and x = 10′:

$0 = x^2/2 - 23.11 \ldots x = 6.80'$

If Δmax occurs between x = 10′ and x = 12′:

$0 = x^2/2 - 8(x - 10)^2/2 - 23.11 = x^2 - 22.86x + 120.89$
… x = 11.43 – 3.12 = 14.55′ or 8.21′; neither of which falls in the range between x = 10′ and x = 12′ and therefore are not valid.

If Δmax occurs at some point where $x > 12'$: $0 = x^2/2 - 8\langle x - 10\rangle^2/2 + 8\langle x - 12\rangle^2/2 - 23.11$ $X = 16 + 7.06i$ which is an unreal answer meaning that the slope never equals zero in this range, but proceeds to increase.

Find deflection:

@ $x = 6.8'$, $\Delta = [(6.8)^3/6 - 8(0)^3/6 + 8(0)^3/6 - 23.11(6.8)](1728)/[29000(199)] = -0.03'' = 0.03''$↓

@$x = 16$, $\Delta = [(16)^3/6 - 8(6)^3/6 + 8(4)^3/6 - 23.11(16)](1728)/[29000(199)] = 0.03'' = 0.03''$↑

8.3 Moment Area Method

The Moment Area Method is a useful tool to find the deflection in beams with concentrated loads, especially if the moment diagram has already been drawn. Just as the accumulated area under the shear curve equals the moment, the accumulated area under the moment curve equals the slope of the deflected beam. Using theorems developed by Mohr, the deflection at a given point can be found by creating a second moment diagram with a virtual load at the point of interest and then summing the product of the moment from the second diagrams at the center of gravity of areas from the first diagram multiplied by those areas.

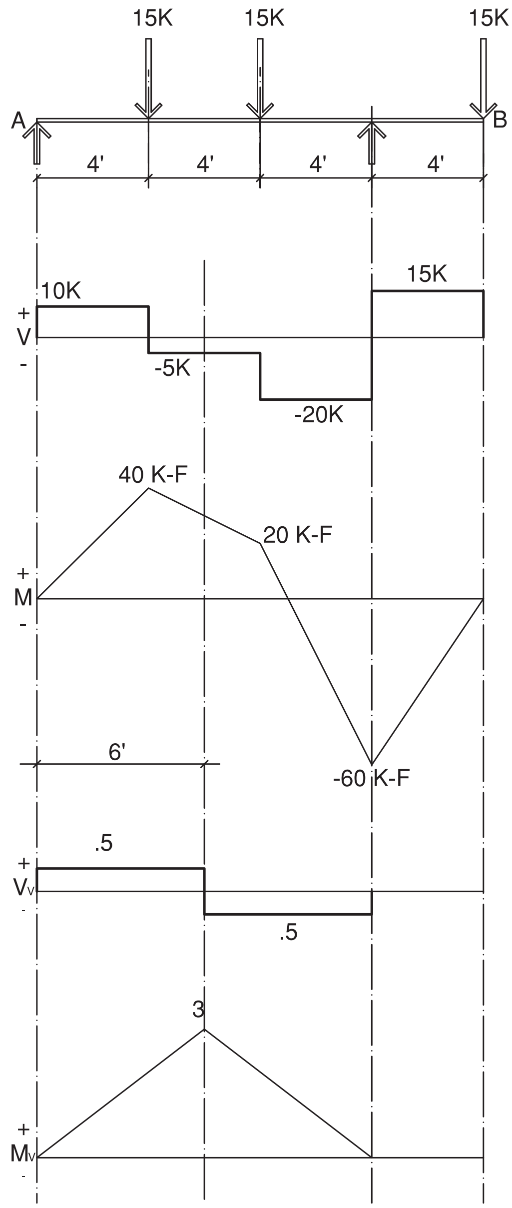
Example 8-7: Find the deflection at the mid-span for the beam shown in Figure 8.6 if E = 29,000 and I = 53.8in⁴.

1. Find reactions:

   $\Sigma M_A = 0 = 15k(4') + 15k(8') - B_y(12') + 15k(16')$ … $B_y = 35k$

   $\Sigma f_y = 0 = A_y - 15k - 15k + 35k - 15k$ … $A_y = 10k$

2. Draw the shear and moment diagrams.
3. Redraw the beam with only a virtual load of 1 at the mid-span.
4. Find Virtual reactions: $A_y = B_y = 0.5$
5. Draw virtual shear and moment diagrams.
6. Divide the real moment diagram vertically where the virtual load is placed and at any point where the virtual moment changes direction. In this case it will be at x = 6′ and x = 12′. Divide the real moment areas into simple geometric shapes and number them as components.



8.6

Finding deflection using Moment Area Method

7. Calculate the area of each geometric shape (A_i) and locate the center of gravity (x).
8. Calculate the virtual moment (M_i) at the centers of gravity (x). In this case: $M = .5x - 1\langle x - 6\rangle + .5\langle x - 12\rangle$. Therefore:

   $x < 6'$, $M_i = 0.5x$

   $6' < x_i < 12'$, $M_i = 6 - 0.5x$

   $12' < x_i$, $M_i = 0$

9. Multiply each value for A_i by the corresponding value for M_i. Remember to consider the positive or negative nature of each value of A_i.

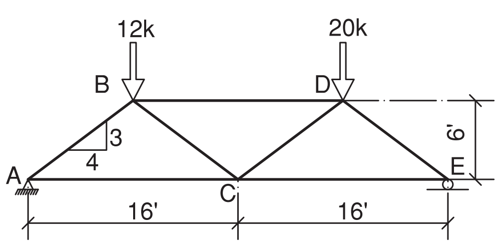
Comp.	A_i	x	M_i	A_iM_i
1	4'(40k-f)/2 = 80	4'(2/3) = 2.67'	0.5(2.67) = 1.33	106.67
2	2'(10k-f)/2 = 10	4 + 2/3 = 4.67'	0.5(4.67) = 2.33	23.33
3	2'(30k-f) = 60	4 + 2/2 = 5.00'	0.5(5.00) = 2.50	150.00
4	2'(10k-f)/2 = 10	6 + 2/3 = 6.67'	6 – .5(6.67) = 2.67	26.67
5	2'(20k-f) = 40	4 + 2 + 2/2 = 7.00'	6 – .5(7) = 2.50	100.00
6	1'(20k-f)/2 = 10	4 + 4 + 1/3 = 8.33'	6 – .5(8.33) = 1.83	18.33
7	3'(–60k-f)/2 = –90	4 + 4 + 1 + 3(2/3) = 11'	6 – .5(11) = 0.50	–45.00
8	4'(–60k-f)/2 = –120	4 + 4 + 4 + 4/3 = 13.33'	0	0
			TOTAL:	380.00

10. $\Delta = \Sigma A_iM_i/EI = 380(1728)/29000(53.8) = 0.42''$

8.4 Method of Virtual Work

The Method of Virtual Work is a useful tool in finding deflections in trusses. As in the Moment Area Method for beams, calculate the forces in the bars of the real truss. Next, apply a virtual load at the point of interest and recalculate the bar forces considering only the virtual load.

Example 8-8: Find the deflection of joint C in the truss shown in Figure 8.7 if all members are 1² diameter steel rods with E = 29,000.



8.7

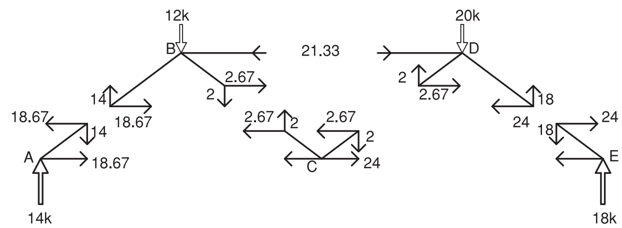
Finding deflection using Method of Virtual Work

1. Find reactions:

$$\Sigma M_A = 0 = 12k(8') + 20k(24') - B_y(32') \ldots B_y = 18k$$

$$\Sigma f_y = 0 = A_y - 12k - 20k + 18k \ldots A_y = 14k$$

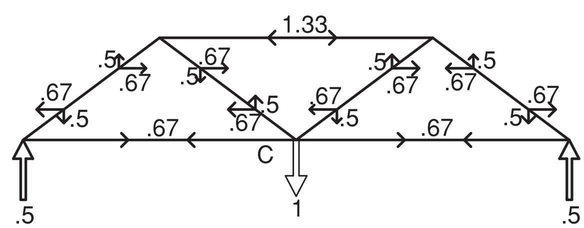
2. Solve for bar forces using Method of Joints:



8.8

Find actual forces in truss bars

3. Redraw truss with a virtual load at point C. Solve for reactions and bar forces in the virtual truss.



8.9

Find virtual forces in truss bars

4. Enter the real and virtual bar forces into the chart assigning compression a negative value and tension a positive value. AE values of the bars. In this case, the bar area $A = \pi(1)^2/4 = 0.785in^2$ and E = 29,000ksi for every bar. Therefore, $AE = 0.785in^2(29{,}000ksi) = 22{,}765k$.

Bar	*Real Force (P_1) in kips*	*Virtual Force (P_2)*	*Length (L)*	*AE*	*P_1P_2L/AE*
AB	–23.34	–.83	10'(12) = 120"	22,765k	0.102
AC	18.67	.67	16'(12) = 192"	22,765k	0.106
BC	3.34	.83	10'(12) = 120"	22,765k	0.015
BD	–21.33	–1.33	16'(12) = 192"	22,765k	0.239
CD	–3.34	.83	10'(12) = 120"	22,765k	–0.015
CE	24.00	.67	16'(12) = 192"	22,765k	0.136
DE	–30.00	–.83	10'(12) = 120"	22,765k	0.131
					$\Delta = \Sigma P_1P_2L/AE = 0.714''$

Since the answer is positive, the deflection is in the direction of the virtual force, which in this case is downward.

$$\Delta = \Sigma P_1P_2L/AE = 0.714''\downarrow$$

Practice Exercises:

8-1: Use deflection charts to find the maximum deflection for the W10 × 45 beam shown if E = 29,000ksi and I = 248.

8-2: Use deflection charts to find the deflection at the end of the overhang for the 7.25″ wide by 15″ deep beam with E = 1,200,000psi

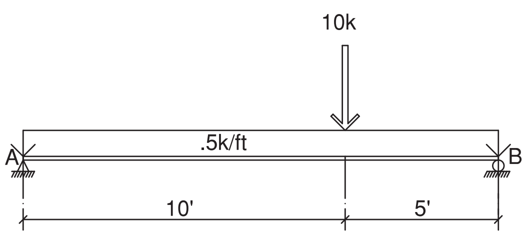
8-3: Use deflection charts to find the deflection at the mid-span between supports for the W14 × 22 beam with E = 29,000ksi and I = 199.

8-4: Use the Double Integration Method to find the deflection at the mid-span between supports for the W8 × 10 beam with E = 29,000ksi and I = 30.8. Check your answer using deflection charts.

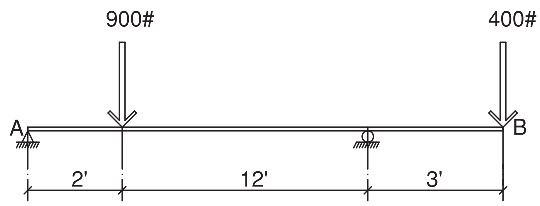
8-5: Use the Double Integration Method to find the deflection at the mid-span between supports for the 5.5″ × 11.25″ beam with E = 1,100,000psi. Check your answer using deflection charts.

8-6: Use the Moment Area Method to find the deflection at X = 4′ for the Titanium beam with E = 15,000ksi and I = 132.4in^4.

8-7: Use the Method of Virtual Work to find the deflection at Joint E for the truss shown. The cross-sectional area of each bar is 4in^2.



8-1



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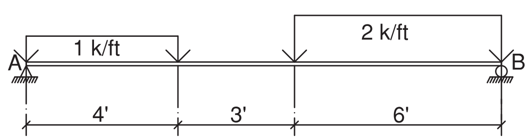

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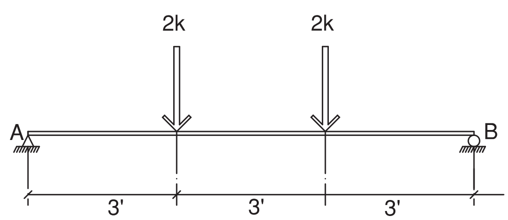
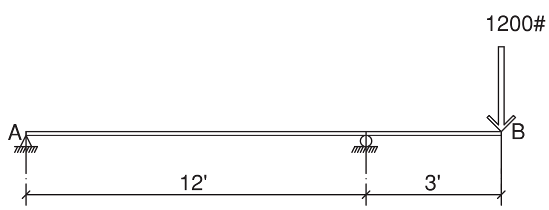


8-4



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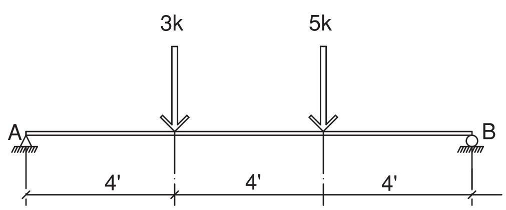

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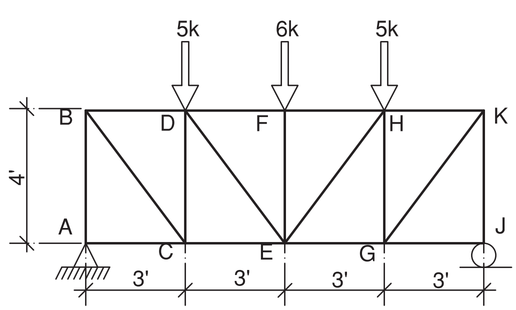


8-8

8.10
Chapter 8 Practice exercises

nine

Design of Beams

This chapter will discuss the fundamental criteria for the design of beams. It does not cover the specifics of design or finding allowable stresses in wood, steel or concrete. For these topics, the reader should refer to Chapters 16 through 30.

9.1 Overview of Design Limitations

9.1.1 Allowable Stresses

Allowable stresses are based on the material of the component and specified by organizations specific to the material. Allowable stresses for steel can readily be found in the AISC steel manual. Allowable stresses for wood can be found in the American Wood Council National Design Specifications. For alternate materials, online sites such as Matweb.com list material properties for various metals and alloys, ceramics, polymers and carbon fibers. Manufacturers are always a good resource as material variations between manufacturers can be great.

Table A1.3 in the Appendix includes a sample guide of material properties for use in solving practice exercises in this text. In actual design situations, refer to the governing codes for the referred allowable stresses.

9.1.2 Allowable Deflections

Allowable deflections are governed by local building codes. Because beams fail due to stress and not deflection, deflection limitations are defined by serviceability. Typical allowable deflections are defined in terms of the beam length L in inches. For example, if a beam is 25′ long, the allowable deflection, $\Delta_{all} = L/240 = 25'(12''^{/f})/240 = 1.25''$.
For $\Delta_{all} = L/360 = 25'(12''^{/f})/360 = 0.83''$.

9.2 Design of Beams for Flexure, Shear and Deflection

The three basic criteria for the design of beams are:

f_b = actual bending stress ≤ F_b = allowable bending stress

f_v = actual shear stress ≤ F_v = allowable shear stress

Δ = actual deflection ≤ Δ_{all} = allowable deflection

Setting the equations for bending stress, shear stress and deflection equal to the allowable stress and deflections will yield the section properties required for the beam design.

$f_b = M/S \le F_b \ldots S \ge M/F_b$

$f_v = VQ/Ib \le F_v$ for geometric shapes … $I \ge VQ/F_v b$

$f_v = 3V/2A \le F_v$ for rectangular sections … $A \ge 3V/2F_v$

$f_v = V/t_w d \le F_v$ for rolled steel shapes … $t_w d \ge V/F_v$

Δ = [some equation]/I $\le \Delta_{all}$ … I $\ge$ [some equation]/Δ_{all}

Example 9-1: Design a 2² X_ joist spanning 14¢ with a load of 160#/f.

E = 1,100,000psi, F_b = 1400psi, F_v = 170psi and Δ_{all} = L/240. Do not consider beam weight.

Bending:

$M_{max} = wL^2/8 = 160^{\#/f}(14)^2(12''^{/f})/8$ = 47,040#-in

$f_b = M/S \le F_b \ldots S \ge M/F_b$ = 47,040#-in/1400psi = 33.6in³

For a rectangle, $S = bh^2/6 = 2h^2/6 \ge 33.6 \ldots$
$h \ge \sqrt{(33.6(6)/2)} = 10.04''$

Shear:

Reactions = Vmax = wL/2 = $160^{\#/f}(14')/2$ = 1120#

$f_v = 3V/2A \le F_v$ for rectangular sections …
$A \ge 3V/2F_v$ = 3(1120#)/[2(170psi)] = 9.88in²

For a rectangle, A = bh = 2h ≥ 9.88in2 …
h ≥ 9.88/2 = 4.94″

Deflection:

$\Delta_{max} = 5wL^4/384EI = 5(160^{\#/f})(14')^4(1728in^3/f^3)/$
[384(1,100,000psi(I))] = 125.73/I

$\Delta_{all} = L/240 = 14'(12''^{/f})/240 = 0.7''$

Δ = [some equation]/ I $\le \Delta_{all}$ …
I ≥ [some equation]/Δ_{all} = 125.73/0.7″ = 179.61

For a rectangle, $I = bh^3/12 = 2h^3/12 \ge 179.61 \ldots$
$h \ge \sqrt[3]{[179.61(12)/2]} = 10.25''$

Deflection governs with the highest value of h required:
h = 10.25″
USE a rectangular section 2″ × 10.25″

Example 9-2: Design a steel W14 section, 40¢ long carrying concentrated loads of 10k every 8¢.

Do not consider beam weight. E = 29,000ksi, F_b = 30ksi, F_v = 20ksi and Δ_{all} = L/240.

Bending:

From the multiple point load Table A1.1, for 4 point loads evenly spaced:

$M_{max} = 3PL/5 = 3(10k)(40')(12''^{/f})/5$ = 2880k-in

$f_b = M/S \le F_b \ldots S \ge M/F_b$ = 2880k-in/30ksi = 96in³

Shear:

V_{max} = 2P = 2(10k) = 20k

$f_v = V/t_w d \le F_v$ for rolled steel shapes …
$t_w d \ge V/F_v$ = 20k/20ksi = 1in²

Deflection:

$\Delta_{max} = .063PL^3/EI = .063(10k)(40')^3(1728in^3/f^3)/$
[29,000ksi(I)] = 2402.52/I

$\Delta_{all} = L/240 = 40'(12''^{/f})/240 = 2.0''$

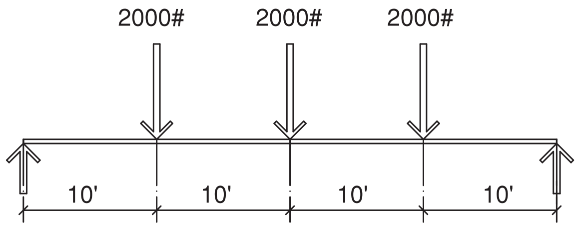
Δ = [some equation]/ I $\le \Delta_{all}$ … I ≥ [some equation]/Δ_{all}
= 2402.52/2″ = 1201.26

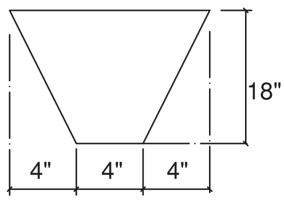
Beam selection: Go to the W14 section properties in Appendix A3.1 to select a size where I ≥ 1201.26in⁴, S ≥ 96in³ and $t_w d$ ≥ 1in². Note that a W14 × 68 would work for bending because S_x = 103 > 96; but it fails for deflection because I_x = 722 < 1201.24. Therefore, A larger size must be used to satisfy the deflection criteria.

USE W14 × 109:

I = 1240 > 1201.26in⁴, S = 173 > 96in³, and
$t_w d$ = 0.525(14.3) = 7.5 > 1in².

Example 9-3: Check whether the cross-section design in Figure 9.1 is adequate for the beam and loading shown if F_b = 1800psi, F_v = 175psi, E = 1,100,000psi and Δ_{all} = L/240.





9.1

Trapezoidal cross-section

Determine cross-section properties:

Comp	*A*	*y*	*Ay*	I_x	*dy*	Ady^2
Left	36	12	432	648	1.5	81
Mid	72	9	648	1944	1.5	162
Right	36	12	432	648	1.5	81
	144		1512	3240		324

N.A. = Y_{ave} = 1512/144 = 10.5″

I_x = 3240 + 324 = 3564in^4

c = 10.5″

$S_x = I_x/c$ = 3564/10.5″ = 339.43in^3

b at N.A. = 4 + 2(4)(10.5/18) = 8.67″

Find Q:

Comp	*Area above neutral axis*	*dy*	*Ady*
Left	7.5”[(12” – 8.67”)/2]/2 = 6.25	7.5(2/3) = 5	31.25
Mid	7.5”(8.67”) = 65	7.5/2 = 3.75	243.75
Right	7.5[(12” – 8.67”)/2]/2 = 6.25	7.5(2/3) = 5	31.25
			Q = 306.25

Bending:

M = PL/2 = 2000#(40’)(12″/f)/2 = 480,000#-in

f_b = M/S = 480,000/339.43 = 1414.14psi < 1800psi … okay for bending

Shear:

V = 3P/2 = 3(2000#)/2 = 3000#

f_v = VQ/Ib = 3000#(306.25in^3)/(3564in^4)(8.67″) = 29.74 < 170psi … okay for shear

Deflection:

Δall = L/240 = 40’(12″/f)/240 = 2″

Δ_{max} = .0495PL^3/EI = .0495(2000#)(40’)^3(1728in^3/f^3)/[1,100,000psi(3564in^4) = 2.79″ > 2″ … NO GOOD for deflection.

At this point, the designer must make a decision about how to modify the cross-section to satisfy the deflection criteria. Enlarging the cross-section proportionally will increase I_x, S_x and Q. Since the new I_x > 3564(2.79/2) = 4971.78 it is an increase by a factor of 2.79/2 = 1.4.

Since I_x involves b and h^3, increasing both dimensions by 1.4$^{0.25}$ = 1.09 should satisfy the criteria. By changing the top width to 13″, the bottom width to 4.5″ and the height to 19.75″; the values change to:

Y = 11.47, I_x = 5175.56, S_x = 451.08, b = 8.65, Q = 395.53

f_b = 1064.12 < 1800psi … okay for bending

f_v = 26.51 < 170psi … okay for shear

Δ_{max} = 1.92″ < 2″ … okay for deflection.

Practice Exercises:

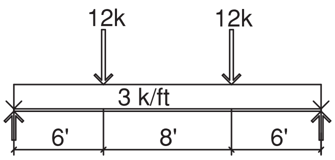
9-1: Design the lightest W12 for the beam shown if E = 29,000ksi, F_b = 30ksi, F_v = 20ksi and Δ_{all} = L/240.

9-2: Design a 4″ wide × h″ deep beam with a rectangular cross-section for the beam shown if E = 1,200,000psi, F_b = 1800psi, F_v = 180psi and Δ_{all} = L/240.

9-3: Find the most economical W14 for the beam shown if E = 29,000ksi, F_b = 21.6ksi, F_v = 14.4ksi and Δ_{all} = L/360.

9-4: Design the most economical (lightest weight) HSS rectangular shape for the beam shown if E = 29,000ksi, F_b = 21.6ksi, F_v = 14.4ksi and Δ_{all} = L/240.

9-5: Find the maximum load, P, the cross-section shown can carry for the beam and loading shown if E = 900,000psi, F_b = 1600psi, F_v = 190psi and Δ_{all} = L/240.



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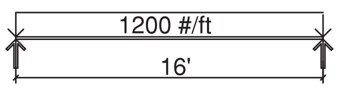

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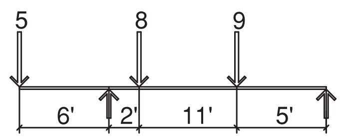

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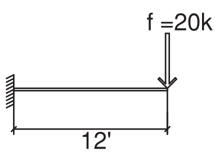

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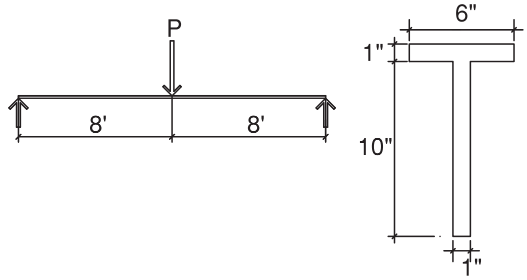


9-5

9.2
Chapter 9 Practice exercises

ten

Design of Columns

Columns are designed to prevent failure in two modes: crushing and buckling.

Crushing occurs when the load distributed on the cross-section is higher than the compressive stress that can be resisted by the column material.

$$f_c = P/A$$

Buckling is compressive failure due to the lateral deflection in a column caused by compression in slender members. The lateral deflection curve will vary depending on the type of support at each end of the column.

10.1 Axial Loads on Columns

Axial loads are theoretically at the center of gravity of a cross-section in the direction of the axis of the column. Theoretically, an axial load should produce no bending stress on a column. But, in reality, either the load is not perfectly placed at the center of gravity or even if it is, the material imperfections of the column will cause an imbalance in stresses. Euler noticed that slender compression members tend to buckle while compact members tend to crush under compression loads.

10.1.1 Critical Buckling Stress

Euler developed an equation for critical buckling stress:

$$f_{crit} = \pi^2 E/(L/r)^2$$

where L and r are both in inches and $r = \sqrt{(I/A)}$ = radius of gyration. The higher the value of (L/r), the more susceptible a column is to buckling. Both directions must be considered. Unless a column is symmetrical along both axes, both L_x/r_x and L_y/r_y must be considered and the higher value used. For steel components, the values for r_x and r_y can be found in the AISC Steel Construction Manual. For geometric shapes, r can be determined by finding I_x and I_y. For example, for a rectangular column b″ × h″: $I_x = bh^3/12$ and $I_y = hb^3/y$. $r_x = \sqrt{I_x/A} = \sqrt{bh^3/12bh} = h/\sqrt{12}$ and $r_y = \sqrt{I_y/A} = \sqrt{hb^3/12bh} = b/\sqrt{12}$.

Example 10-1: Find the critical buckling stress for a W14×90 pinned column with a length of 18′.

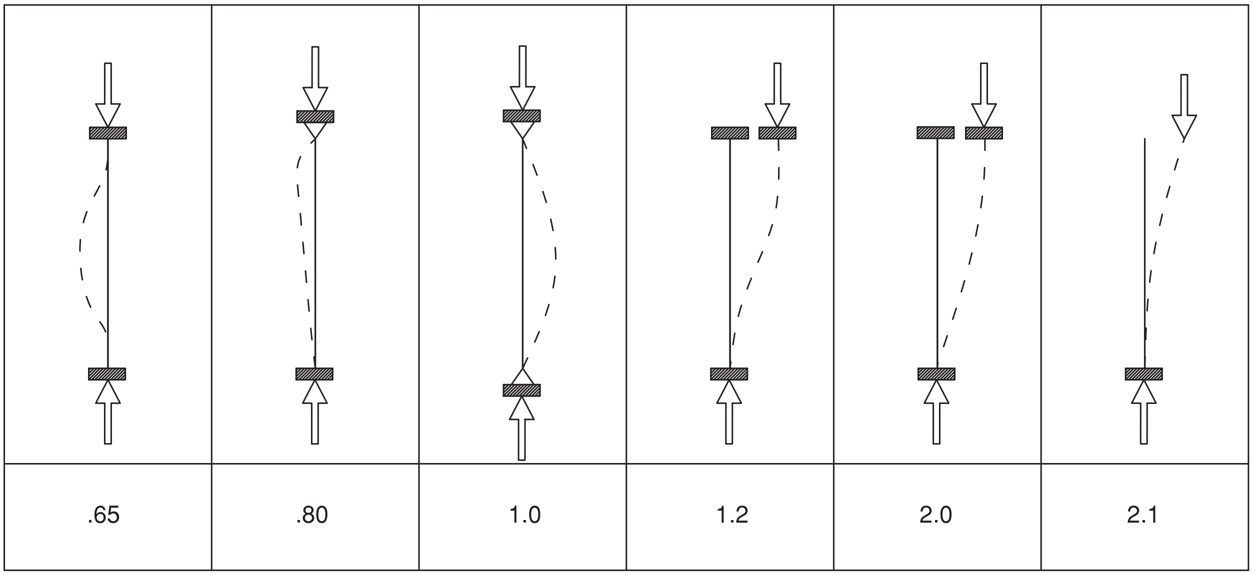
E = 29,000ksi, r_x = 6.14″, r_y = 3.70″, A = 26.5in^2.

$$f_{crit} = \pi^2 E/(L/r)^2 = \pi^2(29{,}000\text{ksi})/((18')(12''^{/f})/3.70'')^2$$
$$= 83.98 \text{ ksi}$$

What is the critical buckling load = P_{crit}? f = P/A …

$$P_{crit} = f_{crit}A = 83.98\text{ksi}(26.5\text{in}^2) = 2{,}225.47\text{k}$$

It is important to note that the critical buckling stress is not the allowable compressive stress, but only one factor in determining the allowable compressive stress. For short columns, crushing will govern the value of allowable compressive stress and for long columns, buckling will govern. Every type of material has its own rules governing the determination of the allowable compressive stress for these rules include the use of an Effective Length Factor, k. The Effective Length Factor, k, is determined by evaluating lateral deflection over the length of the column.



10.1

Effective Length Factor, k, based on Table C.1.8.1, AISC Steel Construction Manual, 8th edition

The effective length of a column = kL, where the value of k is the recommended design value when ideal conditions are approximated in the chart above, NOT the theoretical value.

The slenderness ratio = kL/r

In steel columns the slenderness ratio is limited to $kL/r \leq 200$. To find the allowable compressive stress in steel based on the AISC guidelines and the LRFD Method, see Chapter 22.

Example 10-2: Find the slenderness ratio of a W14×90 column, 20′ long.

a) pinned connections at both ends
b) pinned at one end and fixed at the other
c) fixed connections at both ends
if $r_x = 6.14$ and $r_y = 3.7$:

a) k = 1.0,

$kL/r_x = 1.0(20')(12''/f)/6.14'' = 39.09$

$kL/r_y = 1.0(20')(12''/f)/3.7'' = 75.71$

Use the larger value: 75.71

b) k = 0.8

$kL/r_x = .8(20')(12''/f)/6.14'' = 31.27$

$kL/r_y = .8(20')(12''/f)/3.7'' = 51.89$

Use the larger value: 51.89

c) k = 0.65

$kL/r_x = .65(20')(12''/f)/6.14'' = 25.41$

$kL/r_y = .65(20')(12''/f)/3.7'' = 42.16$

Use the larger value: 42.16

It is easy to see that the larger slenderness value is in the weak direction when the unbraced length is equal in both directions. But, if the unbraced length is different in each direction, be sure to check both.

Example 10-3: Find the slenderness ratio of a W14×90 column, 20′ long and braced at the mid-point in the weak direction.

a) pinned connections at both ends
b) pinned at one end and fixed at the other
c) fixed connections at both ends
if $r_x = 6.14''$ and $r_y = 3.7''$:

a) k = 1.0,
$L_x = 20'$, $L_y = 10'$

$kL/r_x = 1.0(20')(12''/f)/6.14'' = 39.09$

$kL/r_y = 1.0(10')(12''/f)/3.7'' = 32.43$

Use the larger value: 39.09

b) $k = 0.8$

$kL/r_x = .8(20')(12''/') / 6.14'' = 31.27$

$kL/r_y = .8(10')(12''/') / 3.7'' = 25.95$

Use the larger value: 31.27

c) $k = 0.65$

$kL/r_x = .65(20')(12''/') / 6.14'' = 25.41$

$kL/r_y = .65(10')(12''/') / 3.7'' = 21.08$

Use the larger value: 25.41

What is the maximum allowable length of the W14×90 column if pinned connections are used?

$kL/r_y = 1.0(L)(12''/') / 3.07 \leq 200 \ldots L \leq 51.17'$

If the allowable compressive stress, F_{cr}, at $kL/r = 200$ is 3.73ksi, what load can the 51.17′ column carry?

$A = 26.5in^2$

$P = F_{cr}A = 3.73ksi(26.5in^2) = 98.8k$

In wood columns the slenderness is kL/the smallest side $= L_e/d \leq 50$. For the LRFD method to find the allowable compressive stress in wood based on the AWC National Design Specifications, see Chapters 16 through 18.

Example 10-4: Find the maximum allowable unbraced length for a 4×6 column with actual size dimensions 3.5″×5.5″ if the connections are pinned at both ends.

$L_e = kL = 1.0L = L$

$L/3.5'' \leq 50 \ldots L \leq 50(3.5'') = 175'' = 14.58'$

If the allowable compressive stress $F'_c = 400psi$, what load can the column carry?

$P = F'_cA = 400psi(3.5'')(5.5'') = 7700\#$

It may be noted that the slenderness limitations for steel and wood are very similar.

For wood: $L_e/d \leq 50$. Where $L_e = kL \ldots kL/d \leq 50$

For steel: $kL/r \leq 200$. For rectangular cross-sections, $r = d/\sqrt{12} \ldots kL\sqrt{12}/d \leq 200$ or $kL/d \leq 57.74$

In concrete columns, slenderness is much more limited because unlike steel and wood, the tensile strength of concrete is only about 10 % of the compressive strength.

Therefore, concrete columns are categorized and designed as short columns if:

$kL/r < 22$ for pinned connections or
$kL/r < 34 - 12(M_1/M_2)$ for fixed connections where
M_1 = smaller end moment and M_2 = larger end moment.

Again, $r = d/\sqrt{12}$ for a rectangular section, and a comparable look at the slenderness limitations of concrete to those of wood and steel would be: $kL/d \leq 6.35$ for pinned concrete columns. For design of short concrete columns using the LRFD Method and the ACI code, see Chapter 30.

Example 10-5: Is a 20′, 36″ square concrete column with pinned connections a short column?

$kL/r = 1.0(20')(12''/')/(36/\sqrt{12}) = 23.09 > 22.$
No, it is not short.

What unbraced length would make this size column short?

$L < 22(36/\sqrt{12})/(12''/') = 19.05'$

What width would make the 20′ column short?

$kL/r < 22 \ldots r = d/\sqrt{12} > 1(20')(12''/')/22 = 10.909$ and $d > 37.79''$ round up to 38″.

10.2 Column Design

1. Select a trial size and determine the slenderness ratio in each direction. (Alternately, select a trial slenderness ratio.)
2. Determine the allowable compressive stress for the given material and slenderness ratio.
3. Find the actual compressive stress in the column: $f_c = P/A$.
4. Check that the actual stress is less than the allowable stress.

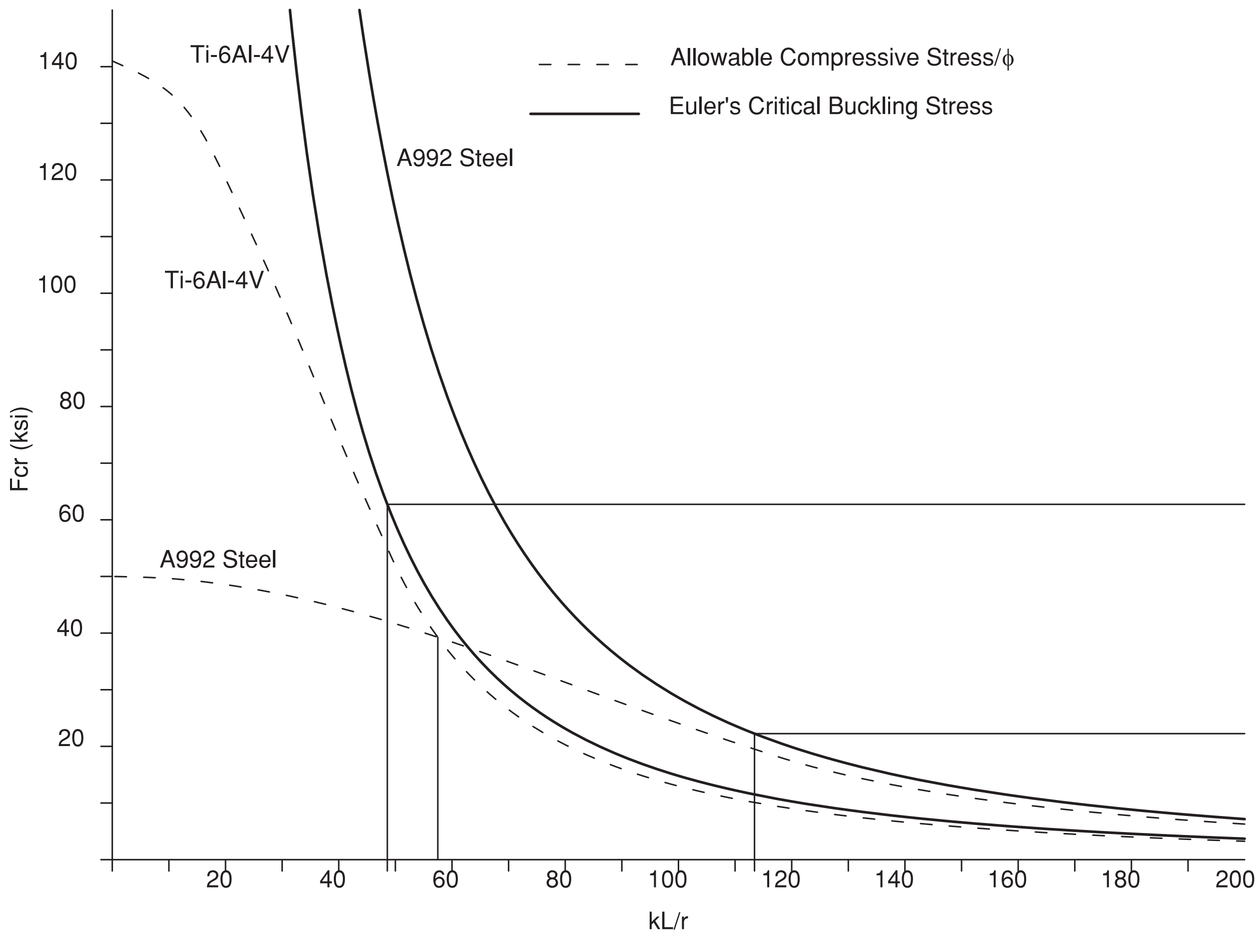
10.2.1 Design of Columns for Metals

For Steel, the AISC determines the allowable compressive stress $= \phi F_{cr}$ by the following equations:

E3-4: $F_e = \pi^2E/(kL/r)^2$. This is Euler's equation for critical buckling stress

E3-2: if $kL/r \leq 4.71\sqrt{(E/F_y)}$, then $F_{cr} = (.658^{Fy/Fe})F_y$

E3-3: if $kL/r > 4.71\sqrt{(E/F_y)}$, then $F_{cr} = 0.877F_e$.



10.2

F_{cr} compared to Euler's formula

These equations can be used with any metal as they will yield a curve such as those shown in Figure 10.2 for steel and titanium alloy. Note that as kL/r approaches 0, F_{cr} approaches F_y. Because F_{cr} is used with the Resistance Factor, ϕ in the LRFD Method and because the loads in the LRFD Method are ultimate loads or factored loads, the value of ϕF_{cr} = allowable compressive stress has an adequate factor of safety built into it.

Example 10-6: Design a titanium Ti-6Al-4V alloy column for a factored compressive load of P_u = 500k with an unbraced length of 12′ and fixed ends if E = 15,000ksi and f_y = 141ksi.

1. Select a trial size: Hollow core 3″×3″×.25″ thick:

   $I_x = I_y = 3^4/12 - 2.5^4/12 = 3.49in^4$

   $A = 3^2 - 2.5^2 = 2.75in^2$

   $r_x = r_y = \sqrt{(3.49/2.75)} = 1.127''$

   k = 0.65

   $kL/r = 0.65(12')(12''^{/f})/1.127 = 83.05$

2. $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(15000/141)} = 48.58$, $F_e = (\pi^2)E/(kL/r)^2$ = 21.464
3. E3-3: if $kL/r > 4.71\sqrt{(E/F_y)}$, then

   $F_{cr} = 0.877F_e = .877(21.464) = 18.82ksi$
4. $P = 18.81ksi(2.75in^2) = 51.73k$

1A. Select a larger trial size: Hollow core 6″×6″×.5″ thick:

   $I_x = I_y = 6^4/12 - 5^4/12 = 55.92in^4$

   $A = 6^2 - 5^2 = 11in^2$

   $r_x = r_y = \sqrt{(55.92/11)} = 2.25''$

   k = 0.65

   $kL/r = 0.65(12')(12''^{/f})/1.127 = 41.6$

2A. $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(15000/141)} = 48.58$,
$F_e = (\pi^2)E/(kL/r)^2 = 85.55$

3A. E3-2: if $kL/r \le 4.71\sqrt{(E/F_y)}$, then $F_{cr} = (.658^{Fy/Fe})F_y = 70.73$

4A. $P = 70.73ksi(11in^2) = 778.03k > 500k$ but not very efficient.

1B. Select a smaller trial size: Hollow core 5″ × 5″ × .5″ thick:

$I_x = I_y = 5^4/12 - 4^4/12 = 30.75in^4$

$A = 5^2 - 4^2 = 9in^2$

$r_x = r_y = \sqrt{(30.75/9)} = 1.85''$

$k = 0.65$

$kL/r = 0.65(12')(12''^{/f})/1.85 = 50.59$

2B. $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(15000/141)} = 48.58$,
$F_e = (\pi^2)E/(kL/r)^2 = 57.833$

3B. E3-3: if $kL/r > 4.71\sqrt{(E/F_y)}$, then $F_{cr} = 0.877F_e = .877(57.83) = 50.72ksi$

4B. $P = 50.72ksi(9in^2) = 456.48k < 500k$ no good.

1C. Select a slightly larger trial size: Hollow core 5″ × 5″ × .625″ thick:

$I_x = I_y = 5^4/12 - 3.75^4/12 = 35.6in^4$

$A = 5^2 - 3.75^2 = 10.94in^2$

$r_x = r_y = \sqrt{(35.6/10.94)} = 1.8''$

$k = 0.65$

$kL/r = 0.65(12')(12''^{/f})/1.8 = 52$

2C. $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(15000/141)} = 48.58$, $F_e = (\pi^2)E/(kL/r)^2 = 54.75$

3C. E3-3: if $kL/r > 4.71\sqrt{(E/F_y)}$, then $F_{cr} = 0.877F_e = .877(54.75) = 48.02ksi$

4C. $P = 48.02ksi(10.94in^2) = 525.29k > 500k$ … okay
USE 5″ × 5″ × 5/8″ HSS in Ti-Al6-4V

Example 10-7: Design an A992 steel column for a factored compressive load of P_u = 500k with an unbraced length of 12′ and fixed ends if E = 29,000ksi and F_y = 50ksi.

1. Select a trial size: Hollow core 6″ × 6″ × .5″ thick:

$I_x = I_y = 6^4/12 - 5^4/12 = 55.92in^4$

$A = 6^2 - 5^2 = 11in^2$

$r_x = r_y = \sqrt{(55.92/11)} = 2.25''$

$k = 0.65$

$kL/r = 0.65(12')(12''^{/f})/1.127 = 41.6$

2. $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/50)} = 113.43$,
$F_e = (\pi^2)E/(kL/r)^2 = 165.39$

3. E3-2: if $kL/r \le 4.71\sqrt{(E/Fy)}$, then $F_{cr} = (.658^{Fy/Fe})F_y = 44.06$

4. $P = 44.06ksi(11in^2) = 484.63 < 500k$ go larger.

1A. Select a trial size: Hollow core 6″ × 6″ × .625″ thick:

$I_x = I_y = 6^4/12 - 4.75^4/12 = 65.58in^4$

$A = 6^2 - 4.75^2 = 13.44in^2$

$r_x = r_y = \sqrt{(65.58/13.44)} = 2.21''$

$k = 0.65$

$kL/r = 0.65(12')(12''^{/f})/2.21 = 42.35$

2A. $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/50)} = 113.43$,
$F_e = (\pi^2)E/(kL/r)^2 = 159.58$

3A. E3-2: if $kL/r \le 4.71\sqrt{(E/F_y)}$, then $F_{cr} = (.658^{Fy/Fe})F_y = 43.85$

4A. $P = 43.85ksi(13.44in^2) = 589.41 > 500k$ … okay
USE 6″ × 6″ × 5/8″ HSS in A992 steel.

Note that the titanium alloy column uses less material with an area of 10.94in² compared to the steel column that requires an area of 13.44in². Given the current cost of titanium alloy at about six times the cost of steel, the steel column is the economical choice at about 21% of the cost of the titanium alloy column. Given the density of steel is 490pcf and the density of Ti-Al6-4v is 276.48pcf, the respective weights of the steel and titanium alloy 12ft columns are 548.8# and 252.06#. The steel column has more than double the weight of the titanium alloy column. It should be noted that for kL/r > 57.5, the allowable compression in steel is higher than that of Ti-Al6-4v. Changing the column length in the previous two examples to 20′ yields:

Example 10-8: Design a titanium Ti-6Al-4V alloy column for a factored compressive load of P_u = 500k with an unbraced length of 20′ and fixed ends if E = 15,000ksi and F_y = 141ksi.

1. Select a trial size: Hollow core 6.25″ × 6.25″ × 1″ thick:

$I_x = I_y = 6.25^4/12 - 4.25^4/12 = 99.97in^4$

$A = 6.25^2 - 4.25^2 = 21in^2$

$r_x = r_y = \sqrt{(99.97/21)} = 2.18''$

$k = 0.65$

$kL/r = 0.65(20')(12''/')/2.18 = 71.56$

2. $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(15000/141)} = 48.58$, $F_e = (\pi^2)E/(kL/r)^2$ $= 28.91$
3. E3-3: if $kL/r > 4.71\sqrt{(E/Fy)}$, then $F_{cr} = 0.877Fe$ $= .877(28.91) = 25.35ksi$
4. $P = 235.35ksi\ (21in^2) = 532.44k > 500k$ … okay
   USE 6.25″ × 6.25″ × 1″ HSS in Ti-Al6-4V

Example 10-9: Design an A992 steel column for a factored compressive load of $P_u = 500k$ with an unbraced length of 20′ and fixed ends if E = 29,000ksi and $F_y = 50ksi$.

1. Select a trial size: Hollow core 6″ × 6″ × .75″ thick:

   $I_x = I_y = 6^4/12 - 4.5^4/12 = 73.82in^4$

   $A = 6^2 - 4.5^2 = 15.75in^2$

   $r_x = r_y = \sqrt{(73.82/15.75)} = 2.17''$

   $k = 0.65$

   $kL/r = 0.65(20')(12''/')/2.17 = 72.05$

2. $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/50)} = 113.43$, $F_e = (\pi^2)E/(kL/r)^2$ $= 55.13ksi$
3. E3-2: if $kL/r \le 4.71\sqrt{(E/Fy)}$, then $F_{cr} = (.658^{Fy/Fe})F_y$ $= 34.21ksi$
4. $P = 34.21ksi(15.75in^2) = 538.76k > 500k$ … okay
   USE 6 × 6″ × ¾″ HSS for Steel.

10.2.2 Design of Wood Columns

The allowable compressive strength in wood columns depends on the species and grade of wood, the moisture, temperature and incising conditions as well as the actual size of the column. A builder's rule of thumb for wood columns is $F_c' = 0.3E/(L/d)^2$. The accuracy of this rule of thumb is shown below:

Example 10-10: Design a 12ft column of structural Select Red Oak with pinned ends, a square cross-section and a factored compressive load of 20,000#.

The LRFD Method and NDS specifications, as shown in Chapter 17, yield an answer of 6 × 6.

Using the builder's rule of thumb, $F_c' = 0.3E/(L/d)^2$, and a trial size of 6 × 6 (5.5″ × 5.5″ actual dimensions) yields the following:

$E = 1{,}300{,}000psi$

$F_c' = 0.3(1{,}300{,}000psi)/(12'(12''/')/5.5'')^2 = 568.94psi$

$F_c'A = P = 568.94psi(5.5)^2 = 17210.44\# < 20{,}000\#$ no good.

Try a larger size: 6 × 8

$F_c' = 0.3(1{,}300{,}000psi)/(12'(12''/')/5.5'')^2 = 568.94psi$

$F_c'A = P = 568.94psi(7.5)(5.5) = 23468.78\# > 20{,}000\#$ … okay

Using the rule of thumb as a quick estimating tool generally yields a larger size, but the exact size required should always be determined, or at least the rule of thumb size should be verified, using the AWC National Design Specifications as outlined in Chapters 16 through 18.

Practice Exercises:

10-1: Determine the critical buckling stress and critical buckling load for a 14ft, W14 × 108 column with pinned ends.

10-2: Given a 4″ × 6″ (actual dimensions) 10ft wood column with E = 1,600,000psi:

a) Determine the critical buckling load.
b) If $F_c' = 1600psi$, what is the load that will cause the column to crush?
c) Will the column buckle or crush first?

10-3: Determine the critical buckling stress of a W21 × 55 column, with E = 29,000ksi and an unbraced length of 20′ in the strong direction and 12′ in the weak direction.

10-4: A 16ft metal column has a hollow circular cross-section with an outside diameter of 18″ and a thickness of 1″. Which metal will hold more load? Metal 1 (E = 10,000ksi and $F_y = 35ksi$) or Metal 2 (E = 12,000ksi and $F_y = 25ksi$)?

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Part II

Structural Design Principles

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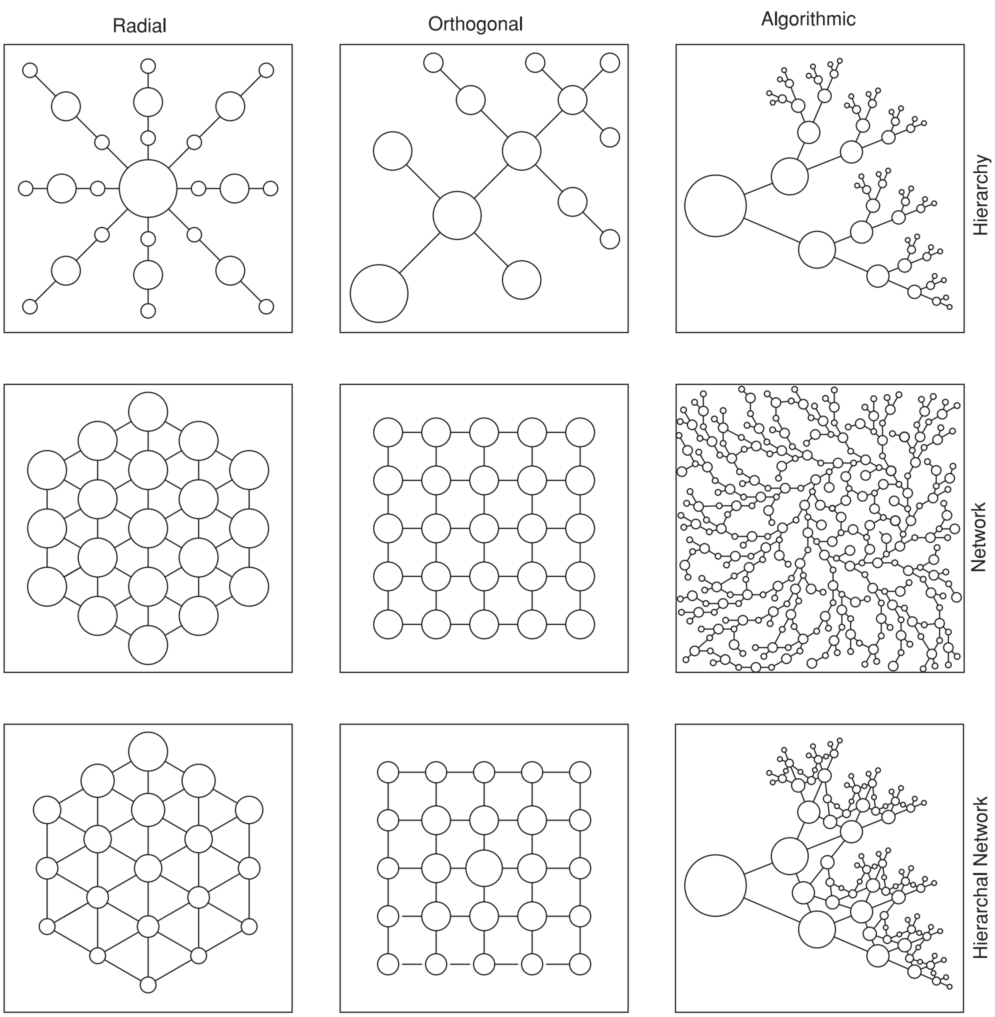
eleven

Structural Patterns

Patterns exist everywhere, whether in an architectural context, a natural context, or an organizational context. Human beings easily recognize, and utilize patterns.

Two general types of patterns are hierarchy and network patterns. These types can further be identified as radial, orthogonal, or algorithmic.



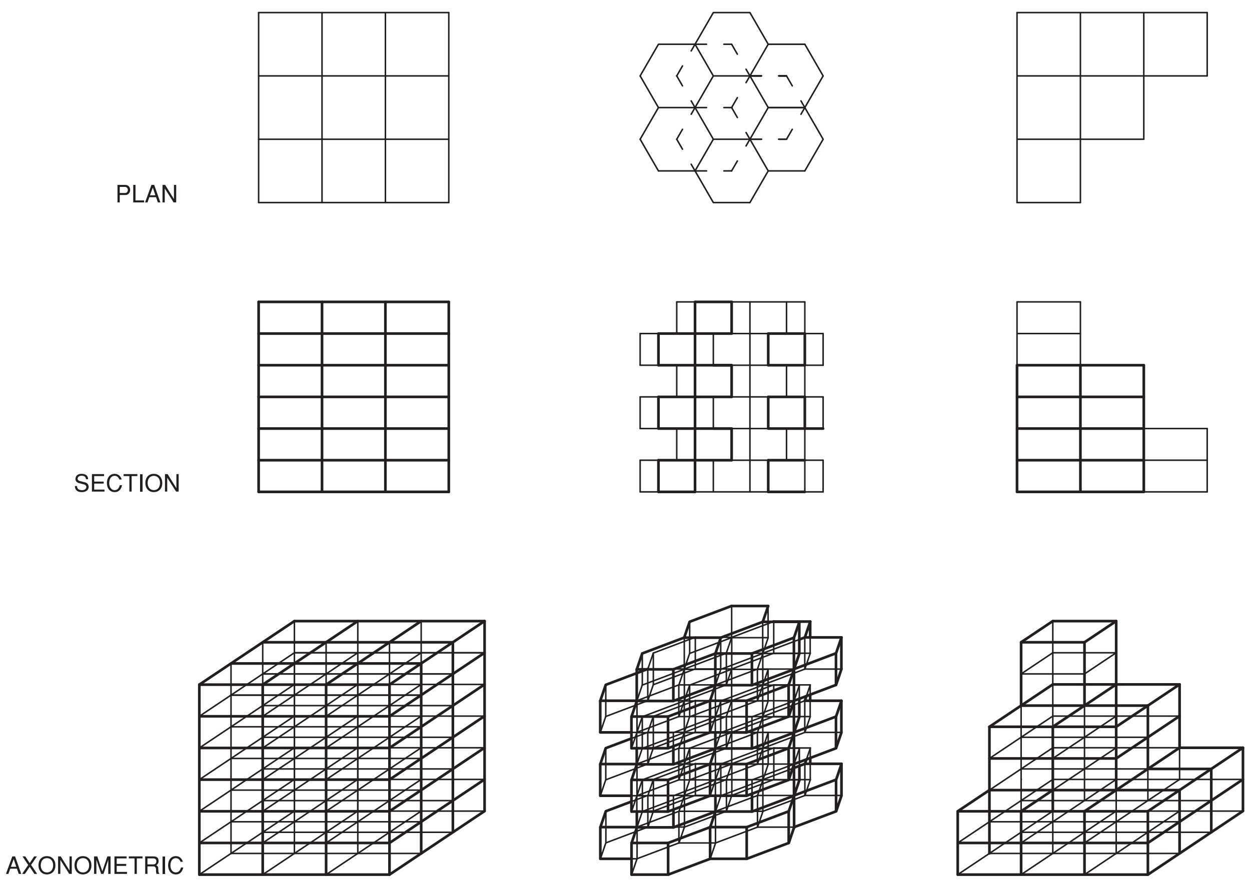
11.1

Pattern types

Hierarchal patterns are systems in which components, members or elements have varying size, status or contributing characteristics that are ordered accordingly. In hierarchal patterns, there is a defined source with subsequent lesser components as a branch from the source. A corporate personnel structure or a tree is an example of a hierarchy.

Network patterns are systems in which all components, members or elements have relatively the same size, status or contributing characteristics. In network patterns, multiple relationships between relatively similar components exist. A honeycomb or a checkerboard is an example of a network pattern. In architecture, geodesic domes and space frames are good examples of network patterns.

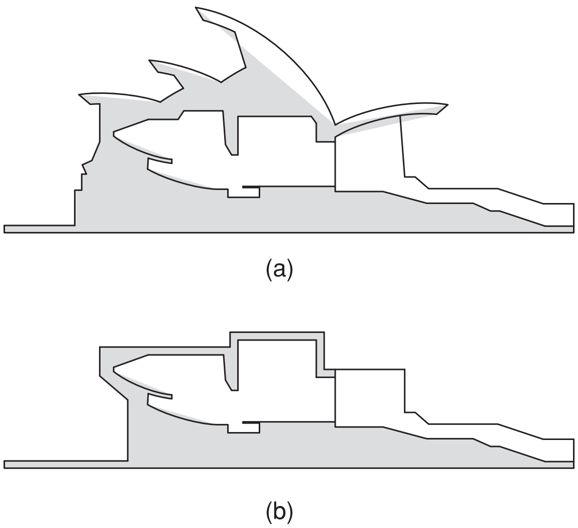
Patterns can also be a combination of hierarchy and networks. Whether hierarchal or networked, patterns allow the designer to identify grid systems that may be used for vertical support systems such as columns, bearing walls or vertical trusses. Orthogonal, radial and algorithmic grids are usually based on a mathematical principle. Grids may also be random or appear random but follow spatial and contextual input. In plan, structural patterns consist of the configuration of supports, horizontal spanning members and lateral force resisting systems. In section or elevation, structural patterns reveal the relationship between the grids from each level and either respond to or mimic the horizontal grids.



11.2

Horizontal and vertical pattern relationship

Structural patterns either correspond with or remain independent of the building shape. In the latter, the structural pattern of the building gives no indication of what takes place spatially or contextually in the building. Likewise, the exterior and interior views are not indicative of the structural patterns in place.



11.3

Structural pattern (a) independent from form and (b) integrated with form

Structural patterns are not usually independent from the spatial and contextual patterns of a project. Because the structure is the skeleton, the physical strength of a building, structural patterns that integrate with spatial, contextual and conceptual patterns help the designer to create a holistic solution.

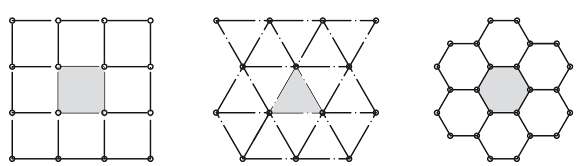
Spatial patterns are usually defined by program and design intent or by environmental comfort factors such as natural lighting, acoustics or thermal convection. Contextual patterns are patterns dictated by topography, site boundaries or context of the site including views, circulation, solar shadows, prevalent winds and the like. Conceptual patterns are the product of creative diagramming of the concept or big idea behind the project. A concept may derive from a social or cultural statement, a natural metaphor for a project or an independent idea conceived by the designer. Once a concept is defined, the tools of defining form such as weaving, sliding, expanding, twisting and the like become tools for the structural patterns as well. Structural patterns that respond to the spatial, conceptual and contextual requirements of a project find the best solution for the parameters given.

11.1 Defining the Structural Grid

The first decision regarding structure is the pattern of support. Pattern of support is determined by several factors, most of which influence the location and distance between columns, walls or other vertical support systems.

The site context defines the perimeter within which a structure is placed. Once the perimeter is defined, consider activity, circulation and materials to determine a preliminary grid pattern. The type of activity dictates the options for width, length and height of spaces to be included. Larger clear spaces require structural systems that can handle large spans. Multilevel spaces often prevent horizontal bracing at the levels between floor and ceiling. The programmatic relationships that exist between types of activity determine the connections and circulation between spaces. Circulation affects the structural grid because columns and other vertical support systems can either define or interfere with a pathway. A colonnade is a perfect example of circulation defined by the structural support system. Conversely, it can be argued that the line of columns in the colonnade is placed for structural support in order to define a line of circulation. Another influence of site context on structural system choices is one of views and privacy. Where views are important, vertical support systems that allow large or multiple perforations are desired. When privacy is important, vertical support systems could be massive bearing walls. The material choices for the structural system will also dictate the allowable spans of beams and size of components.

A grid is a pattern of lines that denotes the placement of columns or bearing walls. An area enclosed by the least number of connected columns is called a bay. In orthogonal systems, bays are rectangular. In diagrid systems, bays are triangular and in honeycomb grids, bays are hexagonal.



11.4

Bays in grid systems

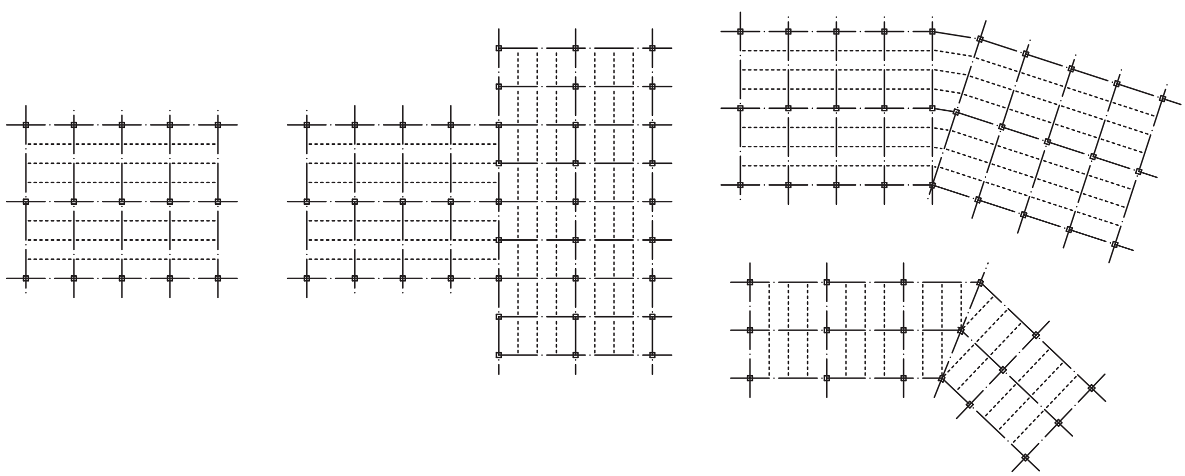
11.1.1 Orthogonal and Radial Grids

Most grids are orthogonal because orthogonal grids are easier to design and construct than other types of grids. Orthogonal grids have many identical members, reducing the number of beams or columns to be designed. The square or rectangular bays in orthogonal grids mean that in construction, connections are at 90°. This means connections are simple to design, fabricate and construct. In the many cases, the bays are of uniform size, but it should be noted that bays may be of varied size. Orthogonal grids may be combined to suit design needs. When combining orthogonal grids, align column lines from each grid to create lateral stability in the system.

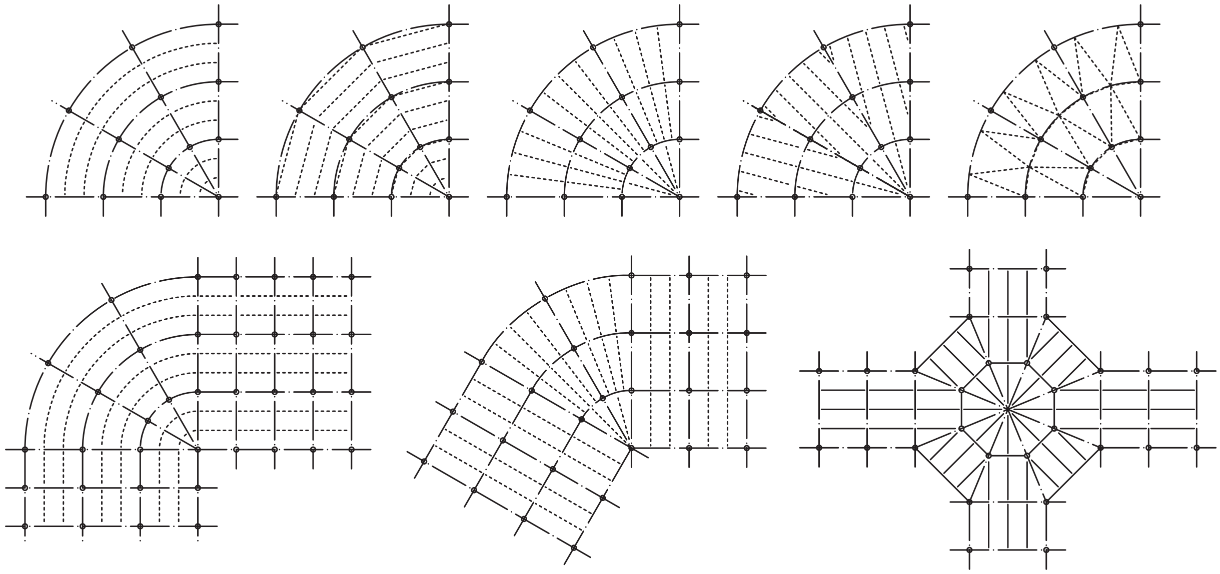
Radial grid lines may be connected by circumferential, radial or diagrid patterns of beams. Radial grids may stand alone, be used as a connector between grids or used as a focal point. Connections become more complicated in a radial grid. Creating regularity in the radial grid, and designing the connections for ease of installation will make a radial grid nearly as practical as an orthogonal grid.

11.1.2 Complex or Irregular Grids

Complex grids may be a combination of orthogonal and radial grids or may involve algorithmic, geometric or natural patterns. For example, a set of grid lines may follow topographic lines, perimeter lines or circulation patterns. In the grid shown in Figure 11.6, longitudinal grid lines follow topographic curves. Transverse grid lines are perpendicular to outer longitudinal lines and evenly space along the center longitudinal line. The grid is strengthened by the triangulated pattern of bracing. Complex grids can be created digitally and components can be manufactured from the digital model meaning that there is an ease of creating double curvature forms. Consider carefully how such forms are to be constructed rather than committing to a structural design based solely on form.

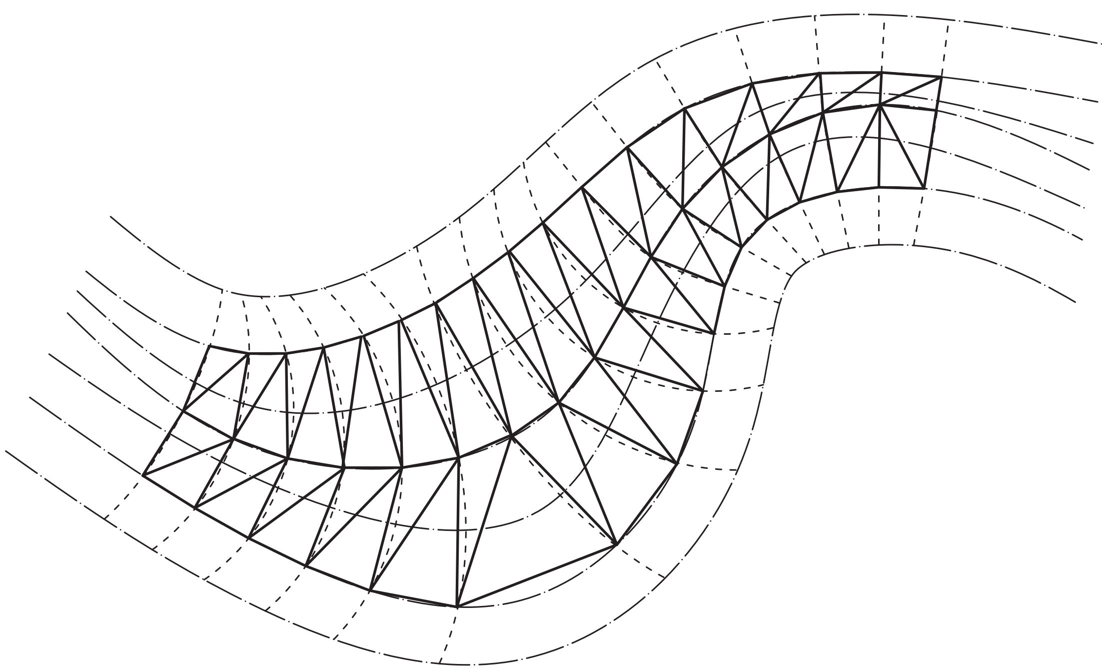


11.5
Orthogonal grids



11.6

Radial grids



11.7

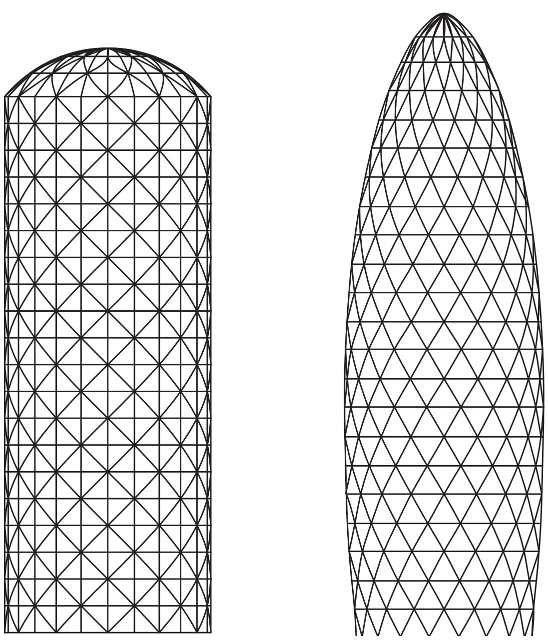
Topography controlled grid

11.1.3 Integration of Structural, Spatial and Contextual Patterns

A structural grid must be developed with regard to the volumes it supports. It is important to define the structural grid early so that spaces may be designed to fit the grid, but it is also equally important to understand that grids may need to be altered to fit the design of spaces or the relationship to site context. While regular grids with equidistant column lines are economical, there is no rule that states grids must be uniform. Do not be afraid to vary distances between column lines or create combined or complex grids in order to achieve the best solution for pattern of support.

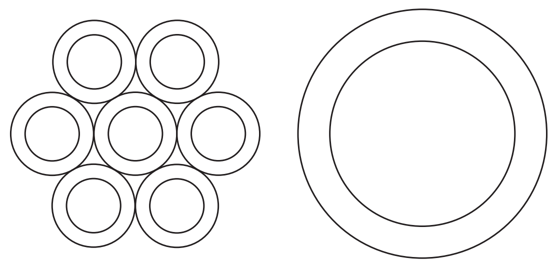
11.2 Natural Design and Structural Form

Nature is endowed with structure. Plants, trees, animals, rock formations and most other nature forms such as cobwebs have a structure and a structural pattern. Natural forms are very efficient and very effective. By observing structural patterns in nature, the designer may find inspiration for solutions to structural design problems.



11.8
Structural comparisons of (a) silica sea sponge (b) 30 St. Mary Axe Building, Foster & Partners

Silica sea sponge is a deceptively strong creature with an exoskeleton made of tiny glass rods that bundle to form struts in an elaborate cylindrical truss wrapped in helical nutrient tubes. The glass rods not only provide a structural skeleton and conduit for nutrients, but also transmit light through fiber optic qualities of the glass rods. Silica sea sponge bears a remarkable resemblance to the Foster & Partners building at 30 St. Mary Axe in London. This building utilizes a diagrid structure with helical circulation, atrium and mechanical system schemes. From a structural perspective, the most interesting thing about silica sea sponge is its strength. It raises the question: Are bundled tubes stronger than individual tubes? If so, is it because bundled tubes have more area or do they have a higher moment of inertia?



11.9
Bundled tubes

Consider the seven bundled tubes in Figure 11.9(a). If all tubes have the same outer radius (r_o) and inner radius (r_i), the area of the seven bundled tubes $A = 7\pi(r_o^2 - r_i^2)$. If a single tube in Figure 11.9(b) has an outer radius equal to $3r_o$, and the same amount of material, and therefore the same area, the thickness of the tube (t) can be found. The single tube would have an outer diameter that is equal to the three diameters of the bundled tubes, or $6r_o$. The inner radius would then be $3r_o - t$. If the total area of the bundled tubes equals the area of the single tube, we get:

$$A = 7\pi(r_o^2 - r_i^2) = \pi((3r_o)^2 - (3r_o - t)^2)$$

$$t = 3r_o - \sqrt{(2r_o^2 + 7r_i^2)}$$

The moment of inertia for the bundled tubes is:

$$I_{xBUNDLED} = 7\pi(r_o^4 - r_i^4)/4 + 4\,\pi(r_o^2 - r_i^2)(1.732r_o)^2$$
$$= (\pi/4)(55r_o^4 - 48r_o^2r_i^2 - 7r_i^4)$$

$$I_{xBUNDLED}/A = (55r_o^2 + 7r_i^2)/28$$

And the moment of inertia for the single tube is:

$$I_{xSINGLE} = \pi((3r_o)^4 - (3r_o - t)^4)/4$$
$$= (\pi/4)(77r_o^4 - 28r_o^2r_i^2 - 49r_i^4)$$

$$I_{xSINGLE}/A = (77r_o^2 + 49r_i^2)/28$$

Clearly, $I_{xSINGLE}/A$ is greater than $I_{xBUNDLED}/A$. Since the radius of gyration $r = \sqrt{(I/A)}$, the single tube will also have a higher value of r, meaning that the slenderness ratio, kL/r will be smaller and therefore the allowable compressive stress will be higher. So, when bundled tubes have the same area as a single tube, the single tube will perform better in compression.

If all tubes have the same thickness (t) where $r_i = r_o - t$, then the area and moment of inertia of the seven bundled tubes remains the same:

$$A_{BUNDLED} = 7\pi(r_o^2 - r_i^2)$$

$$Ix_{BUNDLED} = (\pi/4)(55r_o^4 - 48r_o^2r_i^2 - 7r_i^4)$$

$$Ix_{BUNDLED}/A_{BUNDLED} = (55r_o^2 + 7r_i^2)/28$$

The single tube has an outer radius, $3r_o$, and thickness t. The inner radius is $3r_o - t = 2r_o + r_i$ and the area of the single tube is:

$$A_{SINGLE} = \pi((3r_o)^2 - (3r_o - t)^2) = \pi t(6r_o - t)$$
$$= \pi(r_o - r_i)(5r_o + r_i)$$

And the moment of inertia for the single tube is:

$$I_{xSINGLE} = \pi((3r_o)^4 - (3r_o - t)^4)/4$$
$$= (\pi/4)(65r_o^4 - 32r_o^3r_i - 24r_o^2r_i^2 - 8r_or_i^3 - r_i^4)$$

$$I_{xSINGLE}/A_{SINGLE} = (13r_o^2 + 4r_or_i + r_i^2)/4$$
$$= (91r_o^2 + 28r_or_i + 7r_i^2)/28$$

In this case, the radius of gyration for the single tube is still larger, but the area is smaller and so while the allowable compressive stress will be larger for the single tube, the allowable compressive loads would have to be compared on a case by case basis.

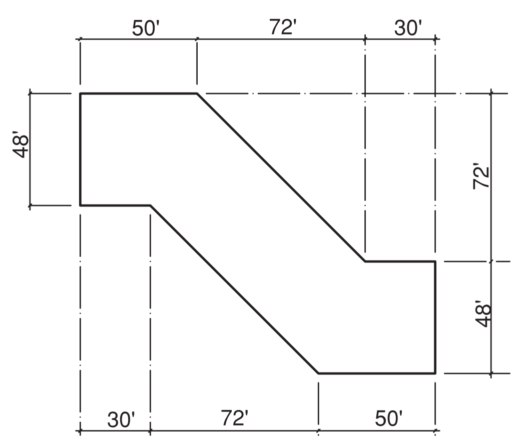
For example, if designing in steel, when $kL/r > 4.71\sqrt{(E/Fy)}$ the allowable compressive stress $= \phi F_{CR} = (.877\pi^2E/(kL)^2)(r^2) = (.877\pi^2E/(kL)^2)(I/A)$ and the allowable compressive load is $\phi F_{CR}(A) = (.877\pi^2E/(kL)^2)(I)$. $I_{xBUNDLED}$ is greater than Ix_{SINGLE} whenever the $r_i \geq 0.4483r_o$. But if $kL/r < 4.71\sqrt{(E/F_y)}$, the allowable compressive stress $= \phi F_{CR} = (.658^{Fy/Fe})(F_y)$ and the equations become complicated and dependent on the kL and Fy values of the problem.

The point is, when observing natural phenomena, it is important to observe, but equally important not to imitate unless fully understanding why a system works. Understanding why something works in nature allows the designer to employ the strategy successfully. The idea of bundled tubes was analyzed for compression, but what about flexure? It is important to analyze a natural system for all of the conditions under which it may be used. It is not the form observed in nature, but how the form behaves that influences structural thinking.

Biomimicry is a term coined by Janine Benyus in her book *Biomimicry: Innovation Inspired by Nature.* Benyus goes beyond observing structural systems found in nature to using natural solutions to inspire innovation in design. For example, shark denticles are a pattern of raised bony scales on the skin of sharks that serve not only as a form of protection but also provide hydrodynamic qualities. It is believed that the denticle pattern allows the shark to move noiselessly through the water. There have been studies of shark denticle patterns to explore textural patterns on the hulls of ships. Imagine using a shark denticle pattern on a metro train or metro station wall to reduce the noise levels in metro stations. That would be an example of biomimic design.

Practice Exercises:

11-1: For the perimeter in Figure 11.10, design a pattern of support for the perimeter shapes below. Maximum beam spacing is 8′ and maximum spacing between columns is 24′.

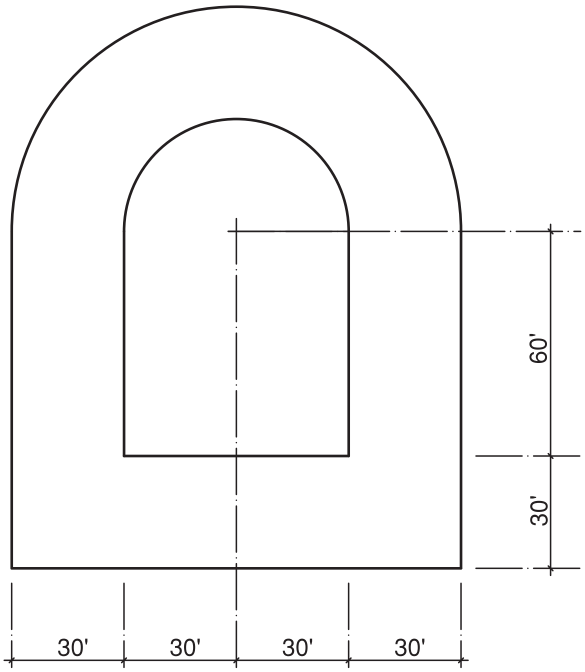


11.10

Practice exercise 11-1

11-2: For the perimeter in Figure 11.11, frame the outer shape with a maximum beam length of 30′ and maximum beam spacing of 10′. Frame the inner shape with a maximum beam length of 60′ and a maximum beam spacing of 10′.

11-3: Create your own shape to enclose 14,000 – 16,000sf within the limits of a 120′ by 150′ site. Include in your enclosed area a 2000 – 4000sf atrium and frame around it. Maximum beam length = 40′ and maximum beam spacing = 10′.



11.11

Practice exercise 11-2

twelve

Design Loads

Design loads are the forces used in the design of structural components. The building code that governs in the location of the project defines what design loads must be used. Most building codes are based on the International Building Code (IBC) although other codes do exist. States typically adopt a building code based on the IBC and may include modifications or more stringent requirements. The State of Florida, for example, developed the 2004 Florida Building Code with higher 3-second gust wind speeds than found in the IBC. Local municipalities usually refer to a state building code. However in some cases, local building codes may be stricter than the state building code. Be certain to use the code that applies to the site location.

Most design loads defined in the IBC are directly based on the *ASCE Minimum Design Loads for Buildings and Other Structures* (ASCE). The building program and site also play a role in determining design loads as they affect importance factors and exposure factors. The first step in determining design loads is to identify the occupancy category in ASCE Table 1-1. Note that Category II is for building types not covered in the other categories.

The occupancy category will determine the importance factor for various load calculations.

Table 12.1 ASCE Table 1.5-1 Occupancy Category, recreated with permission from ASCE

RISK CATEGORY	NATURE OF OCCUPANCY
I	Structures that represent a low hazard to human life in the event of failure
II	All Structures except those listed in Occupancy Categories, I, III and IV
III	Structures that represent a substantial hazard to human life in the event of failure
	Structures not included in Occupancy Category IV, with potential to cause a substantial economic impact and/or mass disruption of day–to–day civilian life in the event of failure
	Structures not included in Occupancy Category IV containing sufficient quantities of toxic or explosive substances that would be dangerous to the public if released
IV	Structures designated as essential facilities
	Structures containing sufficient quantities of toxic or explosive substances that would be dangerous to the public if released
	Structures that represent a substantial hazrd to the community in the even of failure
	Structures required to maintain the functionality of other Risk Category IV structures.

Table 12.2: Importance factors

Occupancy Category	I_S for Snow Loads	I_W for Wind with V = 85-100mph	I_W for Wind with V > 100mph	I_e for Seismic Loads
I	0.8	0.87	0.77	1
II	1	1	1	1
III	1.1	1.15	1.15	1.25
IV	1.2	1.15	1.15	1.5

There are three basic categories of design loads: live loads, dead loads and lateral loads. Live and dead loads are loads that will have a vertical impact on the design and lateral loads are those that will have a horizontal impact on the design.

12.1 Live and Dead Loads

Live and dead loads are gravity loads because they are forces induced by gravity on mass, in other words—weight. The difference between live and dead loads is as follows:

12.1.1 Dead Loads

Dead loads are the weights of all the materials permanently attached to the structure. They are considered dead loads because they do not move or change. Calculating dead loads requires an understanding of the structural system, façade, partition walls and mechanical systems to be used as well as an ability to estimate sizes of components not yet designed. Many handbooks have simple material guides that will give the density of common building materials. For specific materials, manufacturer's typically supply the density of a material or weight of a unit. Be careful to convert all dead loads to the same units whether pounds, pounds per foot, pounds per square foot, or kips per square foot or kilograms per square meter, etc.

Example 12-1: Calculate the dead load on a series of beams spaced at 8′o.c. if the beams carry a 4″ concrete slab made of normal weight concrete at 150pcf and 1″ wood flooring at 4psf.

$$D = 150\text{pcf}(4''/12''^{/f})(8') + 4\text{psf}(8') = 432^{\#/f}$$

12.1.2 Live Loads

Live loads are all gravity loads not permanently attached to the structure. Live loads include people, furnishings, movable equipment, plantings and installations or displays. The IBC is a source code for many local and state codes. The ASCE Table 4-1 and IBC Table 1607.1 list the minimum design live loads by occupancy type. It is important to remember that codes list the minimum allowable live load, but not necessarily the live load that should be considered in special cases. The IBC live loads are listed in units of pounds per square foot (psf) for most cases, although areas such as elevator machine rooms may have a concentrated load listed.

Most buildings have more than one occupancy type to consider and even within one occupancy type, there can be multiple conditions listed in the ASCE Table 4-1. An office building, for example, has an occupancy type: 25 – Office Buildings, that states that lobbies and first floor corridors have a minimum 100psf live load, while corridors above the first floor have a minimum 80psf live load and offices have a 50psf minimum live load. Further, a file room would be considered light storage with a minimum 125psf live load, and stairs and exits have a 100psf minimum live load.

12.1.3 Live Load Reduction

ASCE Chapter 4.8 states that if a member has a tributary area of more than 400 square feet, the original live load (LLo) can be reduced.
Effective LL:

$$LL = LLo(.25 + 15/\sqrt{(K_{LL}A_T)})$$

where K_{LL} is found in IBC Table 1607.9.1, shown here in Table 12.3:

Limits: $LL \geq 0.5LLo$

No LL reduction for Class A occupancy

No LL reduction for LLo > 100psf

Table 12.3: ASCE Table 4-2 Live load element factor, K_{LL}, with permission from ASCE

ELEMENT	K_{LL}
Interior columns	4
Exterior columns without cantilever slabs	4
Edge columns with cantilever slabs	3
Corner Columns with cantilever slabs	2
Edge Beams without cantilever slabs	2
Interior beams	2
All other members not identified including: Edge beams with cantilever slabs, Cantilever beams, One-way Slabs, Two-way Slabs, Members without provisions for continuous shear transfer normal to their span.	1

Example 12-2: If LLo = 95psf and tributary area (A_T) is 600sf, what is reduced live load on an interior column?

From Table 1607.9.1 we find K_{LL} = 4.

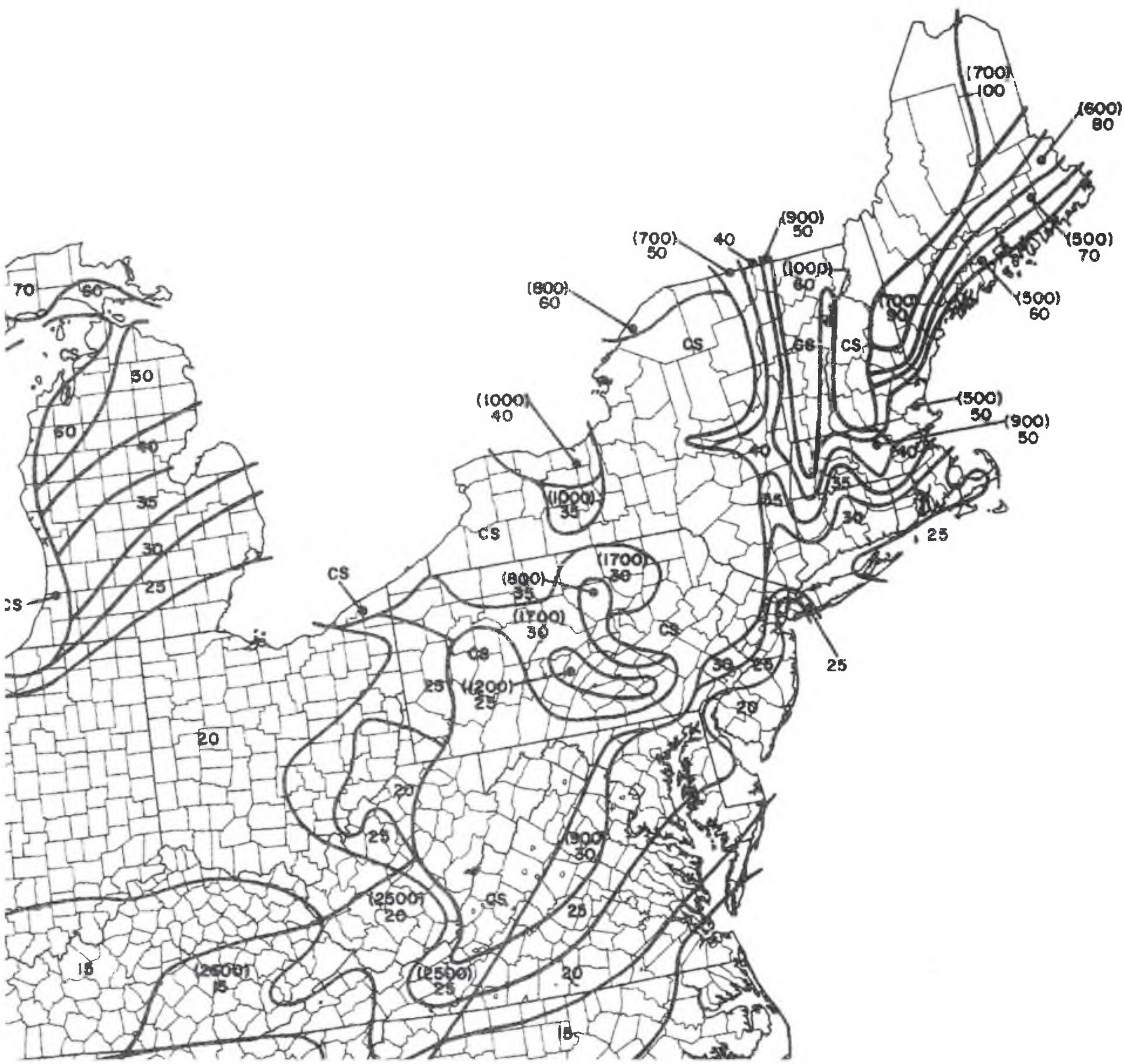
$LL = LLo(.25 + 15/\sqrt{(K_{LL}A_T)}) = 95(.25 + 15/\sqrt{(4)(600)})$ = 52.84psf

0.5LLo = 0.5(95psf) = 47.5psf < 52.84psf … okay

LL = 52.84psf.

12.2 Snow Loads

The procedure to find design snow loads can be found in ASCE Chapter 7. The ASCE Figure 7-1 map, a section of which is shown in Figure 12.1, gives minimum ground snow loads (p_g) in units of psf. The numbers in parentheses are the upper elevation limits in feet. Beyond these elevations, and where CS is shown on the map, specific case studies are required to establish ground snow loads due to potential extreme local variations.



12.1

ASCE 7-1 Snow loads in northeastern U.S. With permission from ASCE

Because snow may drift and create uneven loads, calculations will vary depending on the building design. But for flat roofs, the snow load = $S = 0.7C_eC_tI_sp_g$ where:

p_g = ground snow load from ASCE Figure 7-1

C_e = Exposure Factor from ASCE Table 7-2 where terrain categories are defined as:

B: Urban, suburban and wooded (closely spaced large obstructions)

C: Open spaces with scattered obstructions

D: Flat unobstructed spaces

Table 12.4: ASCE Table 7-2 Exposure factor, with permission from ASCE

TERRAIN CATEGORY	FULLY EXPOSED	EXPOSURE OF ROOF PARTIALLY EXPOSED	SHELTERED
B	0.9	1.0	1.2
C	0.9	1.0	1.1
D	0.8	0.9	1
Above the treeline in windswept mountainous areas	0.7	0.8	N/A
In Alaska, in areas where trees do not exist within a 2-mile radius of the site	0.7	0.8	N/A

C_t = Thermal factor from ASCE Table 7-3

I_s = Importance factor for snow from ASCE Table 7-4

Table 12.5: ASCE Table 7-3 Thermal factor, with permission from ASCE

THERMAL CONDITION	C_t
All structures except as indicated below	1
Structures kept just above freezing and others with cold, ventilated roofs in which the thermal resistance between the ventilated space and the heated space exceeds R-25.	1.1
Unheated structures and structures intentionally kept below freezing	1.2
Continuously heated greenhouses, with a roof having a thermal resistance less than R-2	0.85

Example 12-3: Calculate the design snow load on a flat roof of a hospital in Montpelier, Vermont.

$p_g = 60psf$, $C_e = 0.9$, $C_t = 1.0$, $I_s = 1.2$

$S = 0.7C_eC_tI_sp_g = 0.7(0.9)(1)(1.2)(60) = 45.36psf$

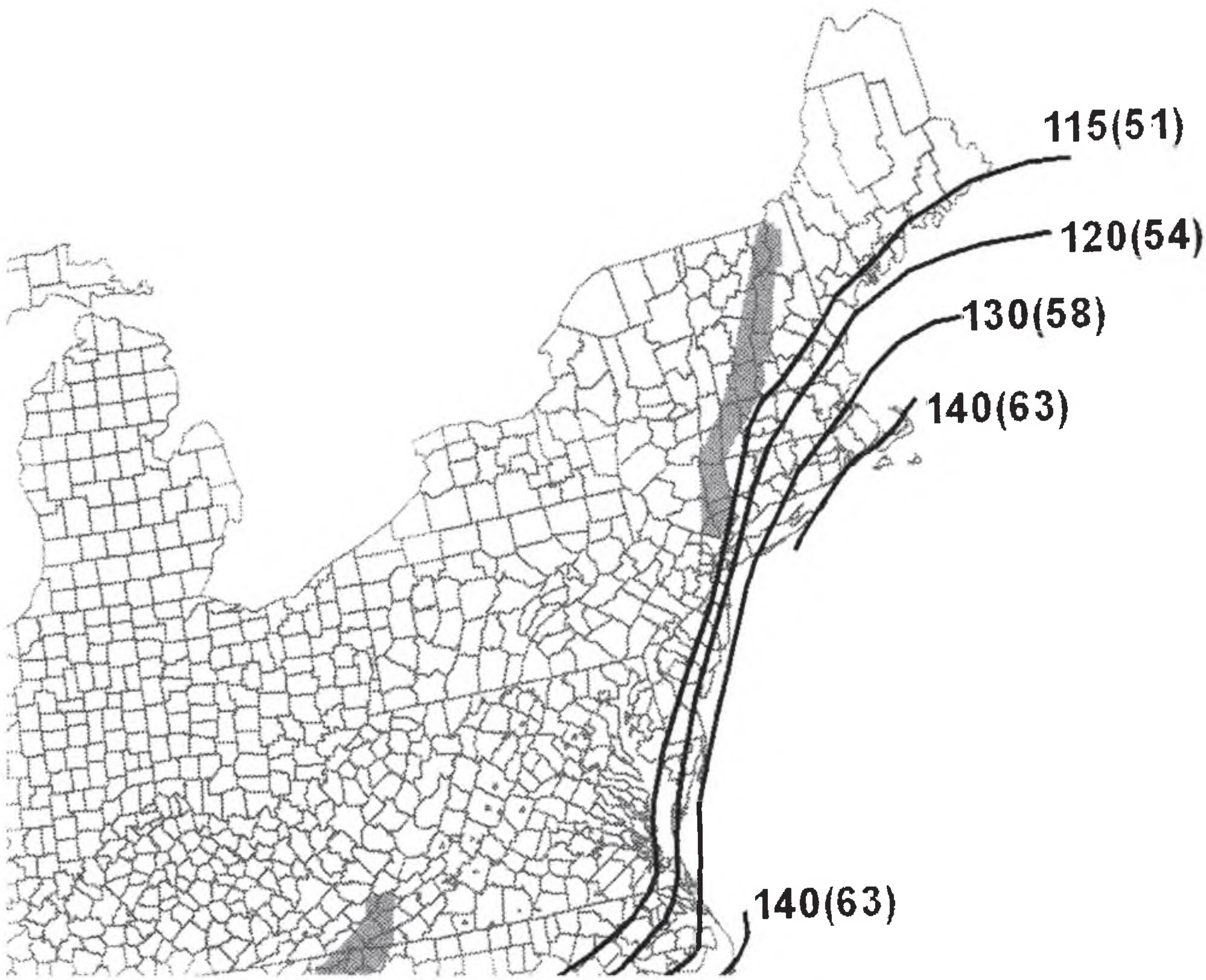
12.3 Lateral Loads

Lateral loads are any loads exerting a lateral or horizontal force on the structure. The most common lateral loads are wind and seismic loads, but there are also other horizontal forces that must be considered. For example, hydrostatic pressure in the soil pushes horizontally against a retaining wall or the weight of a pitched roof pushes outward against the top of a bearing wall supporting it. This section will discuss how to calculate wind and seismic loads.

12.3.1 Wind Loads

Follow the procedures in ASCE Chapter 27 for the calculation of wind loads. There are many scenarios described and care must be taken to use the correct diagrams and tables. The basics of calculating wind design loads are listed below, but are not inclusive of all wind load conditions.

1. Determine the Risk Category based on ASCE Table 1-5.1 shown in Table 12.1.
2. Determine the Design Wind Load Speed, V(mph) at the site location. The maps in ASCE Figures 26.5-1A, B or C provide basic wind speeds for all areas of the U.S. by risk categories. ASCE Figure 26.5-1A is for Risk Category II, a section of which is shown in Figure 12.2. For projects outside of the U.S., refer to local building codes.
3. The directionality factor for the Main Force Resisting System in Buildings is $K_d = 0.85$. For directionality factors for other conditions, see ASCE Table 26.6-1.
4. Determine the Exposure Category in ASCE section 26.7.3. In general terms, the categories are as follows:
   - Category B: Urban and suburban buildings with a mean roof height ≤ 30′
   - Category C: All structures not covered in categories B and D
   - Category D: Unobstructed (open) terrain structures
5. Determine Topography Factor K_{Z1} using ASCE Table 26.8-1. A topographic factor must be included when the building is located on or near a hill, ridge or escarpment. If the site does not meet the conditions described in ASCE Section 26.8.1, the Topgraphic Factor, $K_{Z1} = 1$.



12.2

Excerpt from ASCE Table 26.5-1A: Basic wind speeds for Risk Category II buildings and other structures. With permission from ASCE

6. The Gust-Effect Factor for a rigid building or other structure is G = 0.85. Low-rise buildings (buildings under 100′ in height) are considered rigid. For high-rise buildings, follow ASCE Section 26.9 to determine rigidity and the value for G.
7. This text discusses wind load calculations for enclosed buildings only. See ASCE Sections 26.2 and 26.10 for definitions of enclosure. For enclosed buildings, $GC_{pi} = -0.18$
8. Determine the values of K_h and K_z for each level using ASCE Table 27.3-1 as shown in Figure 12.7. K_h is the coefficient at the mean roof height. K_z is the coefficient at heights where lateral loads can be transferred through structure. For values of height not listed, linear interpolation is allowed.

Table 12.6: ASCE Table 27.3-1 Velocity pressure exposure coefficients K_h and K_z, with permission from ASCE

HEIGHT ABOVE GROUND LEVEL, z (f)	EXPOSURE		
	B	C	D
0-15	0.57	0.85	1.03
20	0.62	0.90	1.08
25	0.66	0.94	1.12
30	0.70	0.98	1.16
40	0.76	1.04	1.22
50	0.81	1.09	1.27
60	0.85	1.13	1.31
70	0.89	1.17	1.34
80	0.93	1.21	1.38
90	0.96	1.24	1.40
100	0.99	1.26	1.43

9. The velocity pressure q_z at any given height is: $q_z = 0.00256K_zK_{zt}K_dV^2 = 0.00256K_z(1)(0.85)V^2$
10. From ASCE Figure 27.4-1, $C_p = 0.8$ for windward walls and $C_p = -0.7$ for side walls. For leeward walls, the value depends on the ratio of L/B where L is the length of the building parallel to the wind direction and B is the width of the building perpendicular to the wind direction. See ASCE Figure 27.4-1 for roof values.

Table 12.7: C_p values for walls

Surface	L/B	Cp	Use with
Windward	All values	0.8	qz
Leeward	0-1	-0.5	qh
	2	-0.3	
	e4	-0.2	
Side Wall	All Values	-0.7	qh

11. Design wind pressure = $p = qGC_p - q_iGC_{pi}$ where:

    $q = q_z$ at each level as found in step 9

    $G = 0.85$ for rigid buildings or value found in step 6

    $C_p = 0.8$ for windward walls of value found in step 10

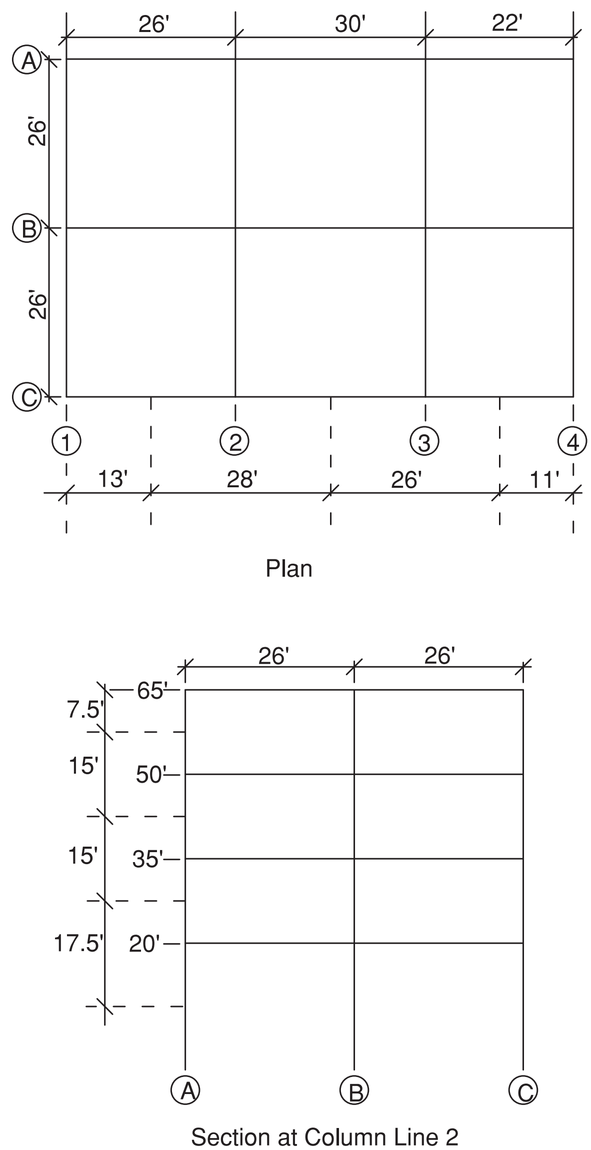
    $q_i = q_h$ for enclosed buildings

    $GC_{pi} = -0.18$ from step 7.

12. Wind load is equal to the design wind pressure multiplied by the tributary area, a. P = pA.

Example 12-4: Find the wind loads for column line 2 if the fully enclosed rigid structure in Pittsburg, PA, shown in Figure 12.3 resists wind with column lines 1, 2, 3 and 4.

Use Exposure Category B. The building is an office building.



12.3
Example 12-5 structure

1. From Figure 12.1, Risk Category II.
2. From Figure 12.6, V = 115mph
3. $K_d = 0.85$
4. Exposure Category B: (given)

5. $K_{Z1} = 1$
6. $G = 0.85$
7. $GC_{pi} = -0.18$
8. Determine the values of K_h and K_z for each level using Table 12.6:

    K_h is at z = 65′

    Interpolate between $K_z = 0.85$ @ 60′ and $K_z = 0.89$ @ 70′
    Use ratios: $(.89 - .85)/(70 - 60) = (K_h - .85)/(65 - 60)$ …
    $K_h = 0.87$

The same method is used to find Kz at each level resisting wind loads.

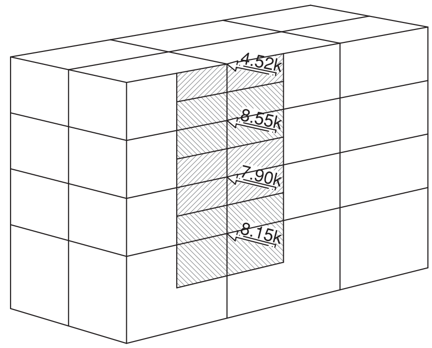
    @ z = 50′: $K_z = 0.81$
    @ z = 35′: $K_z = (.76 - .70)(35 - 30)/(40 - 30) + .70 = 0.73$
    @ z = 20′: $K_z = 0.62$
    At this point it is helpful to create a table as shown at the end of the example.

9. The velocity pressure q_z at any given height is: $q_z = 0.00256K_zK_{zt}K_dV^2 = 0.00256K_z(1)(0.85)(115)^2 = 28.78K_z$
10. From ASCE Figure 27.4-1, $C_p = 0.8$ for windward walls
11. Design wind pressure = $p = qGC_p - q_iGC_{pi} = q_z(.85)(.8) + q_h(.18) = .68q_z + 4.51$
12. Wind load is equal to the design wind pressure multiplied by the tributary area, a. P = pA
    For Column Line 2, the tributary width = (26′ + 30′)/2 = 28′
    @ z = 65′: tributary height = 7.5 … A = 28′(7.5′) = 210sf
    @ z = 50′: tributary height = 15.0 … A = 28′(15.0′) = 420sf
    @ z = 35′: tributary height = 15.0 … A = 28′(15.0′) = 420sf
    @ z = 20′: tributary height = 17.5 … A = 28′(17.5′) = 490sf

SOLUTION:

Table 12.8: Wind load spreadsheet

Height (f)	K_z	$q_z = 28.78K_z$	$p = .68q_z + 4.51$(psf)	$A(f^2)$	P(kips)
65	0.87	25.04	21.54	210	4.52
50	0.81	23.31	20.36	420	8.55
35	0.73	21.01	18.8	420	7.9
20	0.62	17.84	16.64	490	8.15



12.4
Wind loads on Example 12-4 structure

12.3.2 Seismic Loads

Seismic loads are caused by the horizontal shear force induced on buildings by earthquakes. Seismic Design Loads are covered in Chapters 11 through 23 of the ASCE. The basics for determining seismic loads using the Equivalent Lateral Force Procedure are covered below and through Example 12-5.

$V = C_sW$ = seismic base shear

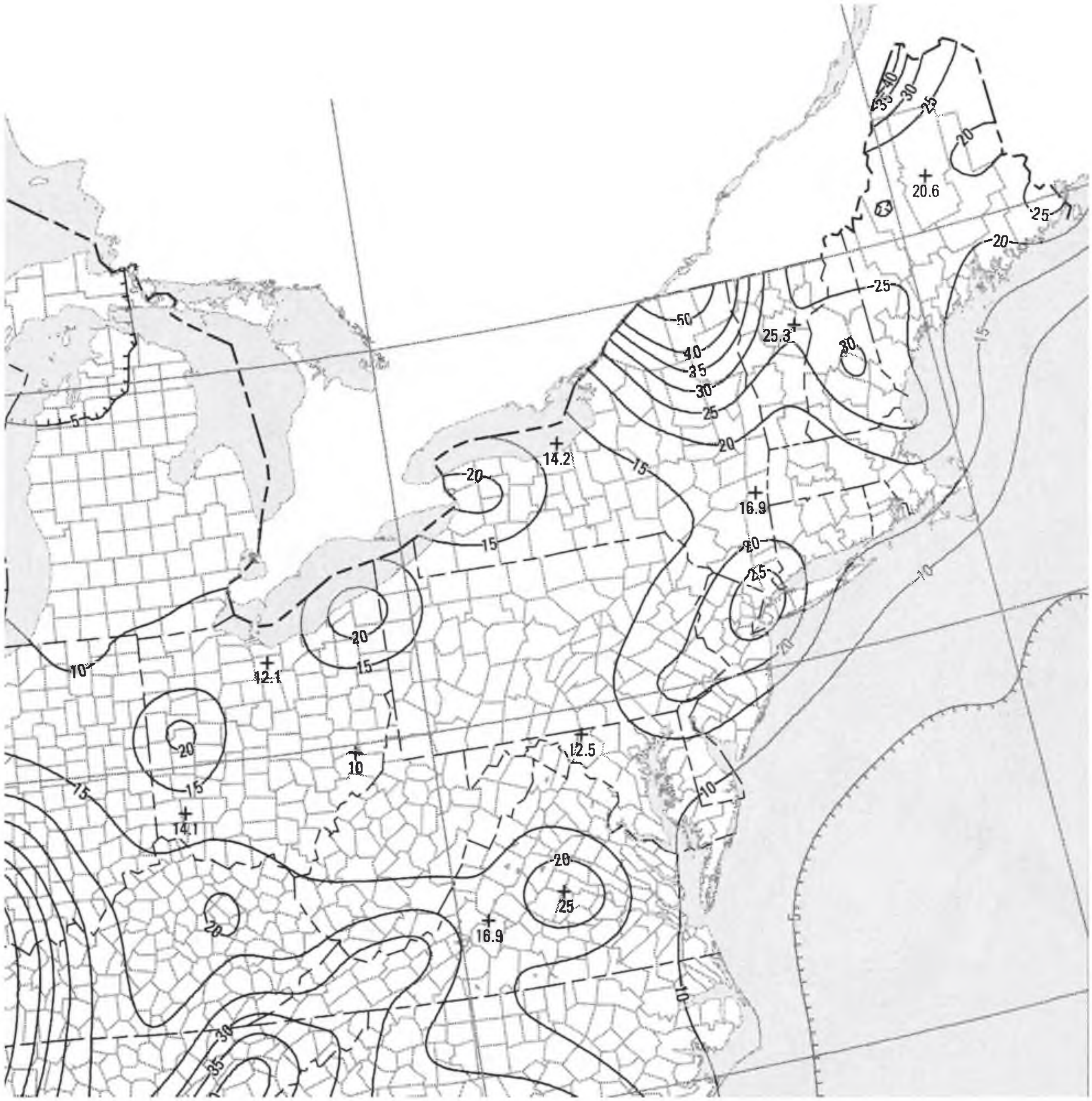
W = effective seismic weight = total dead load f floors and walls plus weights specified in ASCE Section 12.7.2.

$C_s = S_{DS}/[R/I_e]$ = the seismic response coefficient

R = the Response Modification Factor found in ASCE Table 12.2-1, a small portion of which is shown in Table 12.9.

Table 12.9: Sample of ASCE Table 12.2-1, with permission from ASCE

Category and System	Response Modification Coefficient, R
A. Bearing Walls	
Ordinary reinforced concrete shear walls	4
Ordinary reinforced masonry shear walls	2
Light-framed walls sheathed with wood panels rated for shear resistance or steel sheets.	6.5
B. Building Frame Systems	
Steel eccentrically braced frames, non-moment-resisting connections at columns away from links	7
Ordinary steel concentrically braced frames	3.25
Ordinary reinforced concrete shear walls	6
C. Moment-Resisting Frame Systems	
Ordinary Steel Moment frames	3.5
Ordinary Reinforced concrete moment frames	3



12.5

A sample of ASCE 22-1 0.2sec Seismic Response Map for Site Class B, with permission from ASCE

I_e = Importance factor for earthquakes based on Building Risk Category (Table 12.10).

Table 12.10: Importance factor for seismic loads, with permission from ASCE

Risk Category	I_e
I	1
II	1
III	1.25
IV	1.3

$$S_{DS} = (2/3)(F_a)(S_S)$$

S_S = mapped spectral response acceleration for short periods from ASCE Figure 22-1, a portion of which is shown in Figure 12.5.

F_a = site coefficient from ASCE Table 11.4-1, summarized as in Table 12.11:

Table 12.11: Site coefficient, F_a from ASCE Table 11.4-1, with permission from ASCE

Site Class	Ss ≤ 0.25	Ss = 0.5	Ss = 0.75	Ss = 1.0	Ss ≥ 1.25
A	0.8	0.8	0.8	0.8	0.8
B	1	1	1	1	1
C	1.2	1.2	1.1	1	1
D	1.6	1.4	1.2	1.1	1
E	2.5	1.7	1.2	0.9	0.9
F	See ASCE section 11.4.7				

$$S_{D1} = (2/3)(F_v)(S_1)$$

S_1 = mapped spectral response acceleration for a 1-sec period from ASCE Figure 22-2, a portion of which is shown in Figure 12.6.

F_v = site coefficient from ASCE Table 11.4-2, summarized as in Table 12.12:

Table 12.12: Site coefficient F_v from ASCE Table 11.4-2, with permission from ASCE

Site Class	S_1 d 0.1	S_1 = 0.2	S_1 = 0.3	S_1 = 04	S_1 e 0.5
A	0.8	0.8	0.8	0.8	0.8
B	1	1	1	1	1
C	1.7	1.6	1.5	14	1.3
D	2.4	2	1.8	1.6	1.5
E	3.5	3.2	2.8	2.4	2.4
F	See ASCE section 11.4.7				

Example 12-5: Determine the seismic loads for column line 2 of the building in Figure 12.3.

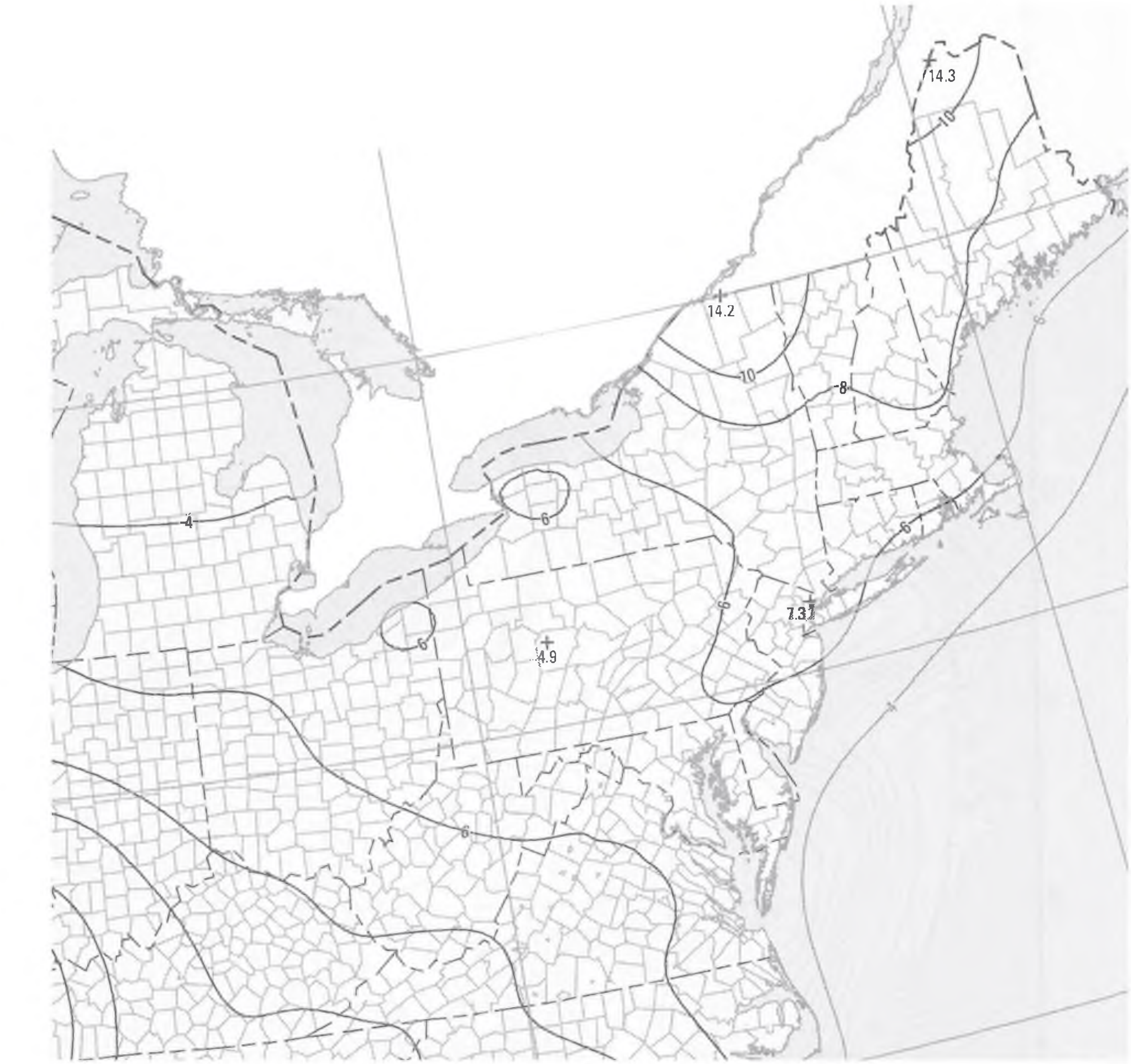
The site has very dense soil. The dead loads are 80psf for floors and 15psf for walls. The structure is a steel braced frame with pinned connections. Assume Risk Category IV.

1. Using ASCE Table 20.3-1 or IBC Table 1613.5.2, determine the Site Class. The site classifications are as follows:
   Site Class A: Hard rock
   Site Class B: Rock
   Site Class C: Very dense soil and soft rock
   Site Class D: Stiff soil
   Site Class E: Soft clay soil
   Site Class F: Soils requiring site response analysis
   If unsure of site conditions, use Site Class D. For this example, very dense soil is Class C.
2. Find the mapped spectral acceleration for short period (0.2sec) (Ss) from Figure 12.5. The values on the map are shown as percentages. For calculations, Ss is used in decimal form. Therefore the map value for Pittsburgh of 15% means that $S_S = 0.15$.
3. Find the mapped spectral acceleration for 1-sec period (S_1) from Figure 12.6. $S_1 = .05$.
4. Find the site coefficient (F_a) from Table 12.10. For Site Class C and $S_S < 0.25$, $F_a = 1.2$.
5. Find the site coefficient (F_v) from Table 12.11. For Site Class C and $S_1 < 0.1$, $F_V = 1.7$.
6. Calculate S_{DS} and S_{D1}:

   $$S_{DS} = (2/3)(F_a)(S_S) = (2/3)(1.2)(0.15) = 0.12$$

   $$S_{D1} = (2/3)(F_v)(S_1) = (2/3)(1.7)(.05) = 0.057$$

7. Find the Response Modification Factor from ASCE Table 12.2-1. For Case B-2, steel braced frames with non-moment-resisting connections, R = 7. Some sample values for R factors are given below. Note that there are limitations for buildings in Seismic Design Categories B, C, D, E and F. See ASCE sections 11.6 and 11.7 to determine the Seismic Design Category before choosing a value for R. In this example, the structure is in category A because $S_{DS} = 0.12 < 0.167$ and the requirements of section 11.6 are met.
8. From Table 12.10, $I_E = 1.5$.



12.6

A sample of ASCE 22-2 1.0sec Seismic Response Map for Site Class B, with permission from ASCE

9. Find the coefficient for the upper limit on calculated period (C_U) in ASCE Table 12.8-1 shown in Table 12.14. $C_U = 1.7$ when $S_{D1} \leq 0.1$.

Table 12.13: Sample Seismic Response Modification Factors from ASCE Table 4-2. With permission from ASCE

S_{D1}	C_U
≥ 0.4	1.4
0.3	1.4
0.2	1.5
0.15	1.6
≤ 0.1	1.7

Structure type	C_t	x
Steel Moment Frame	0.028	0.8
Concrete Moment Frame	0.016	0.9
Steel eccentrically braced frame	0.03	0.75
Steel buckling-restrained braced frar	0.03	0.75
All other systems	0.02	0.75

10. Find C_T and X from ASCE Table 12.8-2. $C_T = .03$ and X = 0.75 for eccentrically braced steel frames.
11. h_n = height in feet to highest point of building = 65′

    $T_a = (C_T)(h_n^{x}) = (.03)(65^{0.75}) = 0.687$ = approximate fundamental period.

12. $T = (C_U)(T_a)$ = the structure period (inverse of frequency of oscillation).

    $T = 1.7(.687) = 1.168$

13. $C_S = S_{DS}(I)/R = 0.12\ (1.5)/7 = 0.0257$
14. $C_{SMIN} = 0.01 < Cs = 0.021$ … okay
15. $C_{SMAX} = S_{DI}/(T(R/I_E)) = .057/[1.168(7)/1.5] = .011$
16. $C_S = .011$
17. K = an exponent related to the structure period
    = 1 if T < 0.5
    = 2 if T > 2.5
    If 0.5 < T > 2.5, you may use 2 or interpolate between 1 and 2… K = 2
18. Make a spreadsheet where:

    W_x = total weight of building at given level
    $= W_{walls} + W_{floor}$

    At each level, the weight of the floor = 80psf(52′)(78′) = 324,480# = 324.48k
    At each level the weight of the walls = 15psf(2(78 + 52)) (tributary height) = 3.9k/f(h_t)
    @ z = 65′: W_{walls} = 3.9k/f(7.5′) = 29.25k … W_x = 353.73k
    @ z = 50′: W_{walls} = 3.9k/f(15.0′) = 58.5k … W_x = 382.98k
    @ z = 35′: W_{walls} = 3.9k/f(15.0′) = 58.5k … W_x = 382.98k
    @ z = 20′: W_{walls} = 3.9k/f(17.5′) = 68.25k … W_x = 392.73k
    @ z = 0′: W_{walls} = 3.9k/f(10.0′) = 39.0k … W_x = 363.48k

    W = total weight of building(dead loads) = 1875.9k

    C_{vx} = vertical distribution factor $= W_x h_x^k/(\Sigma w_i h_i^k)$

    $= W_x h_x^k/(3078202)$

    $V = C_s W = .011(1875.9) = 20.635$

    $F_x = C_{vx}(V)$ = lateral force in kips.

Table 12.14: Seismic load spreadsheet

h	Wx	h^2	$W_x h^2$	Cvx	V	Fx
65	353.73	4225	1494509	0.486	20.635	10.03
50	382.98	2500	957450	0.311	20.635	6.42
35	382.98	1225	469151	0.152	20.635	3.14
20	392.73	400	157092	0.051	20.635	1.05
0	363.48	0	0	0	20.635	0

12.4 Factored Loads

The LRFD (Load Resistance Factor Design) Method uses load factors to create an ultimate or factored load that is the design load. It also uses Resistance Factors (ϕ) which are discussed in chapters related to design with specific materials.

Ultimate or design loads are based on the following types of loads.

U = The design or ultimate load = factored load

W_u = factored uniform load

P_u = factored concentrated load

D = dead load

L = live load

Lr = roof live load

S = snow load

R = rainwater/ice load, (not ponding)

W = wind load

E = earthquake load

All loads are placed into one of these categories and factored using the six equations below. The largest result from the six equations is used as the design load.

1. U = 1.4D
2. U = 1.2D + 1.6L + 0.5 (Lr or S or R)

3. $U = 1.2D + 1.6$ (Lr or S or R) + (L or 0.8W)
4. $U = 1.2D + 1.6W + 0.5L + 0.5$(Lr or S or R)
5. $U = 1.2D \pm E + L + 0.2S$
6. $U = 0.9D \pm$ (1.6W or E)

Example 12-6: Beams weighing $22^{\#/f}$ spaced 8′ on center, support a 50psf dead load, 150^{psf} live load and a $20^{\#/f}$ seismic load.

Find W_u:

$D = 50^{psf}(8') + 22^{\#/f} = 422^{\#/f}$

$L = 150^{psf}(8') = 1200^{\#/f}$

$E = 20^{\#/f}$

1. $W_u = 1.4(422) = 590.8^{\#/f}$
2. $W_u = 1.2(422) + 1.6(1200) + 0.5(0) = 2426.4^{\#/f}$
3. $W_u = 1.2(422) + 1.6(0) + (1200 \text{ OR } 0) = 1706.4^{\#/f}$
4. $W_u = 1.2(422) + 1.6(0) + 0.5(1200) + 0.5(0) = 1106.4^{\#/f}$
5. $W_u = 1.2(422) \pm 20^{\#/f} + 1200 + 0.2(0) = 1726.4^{\#/f}$ & $1686.4^{\#/f}$
6. $W_u = 0.9(422) \pm (1.6(0) \text{ OR } 20) = 399.8^{\#/f}$ & $359.8^{\#/f}$
   $W_u = 2426.4^{\#/f}$ (highest governs)

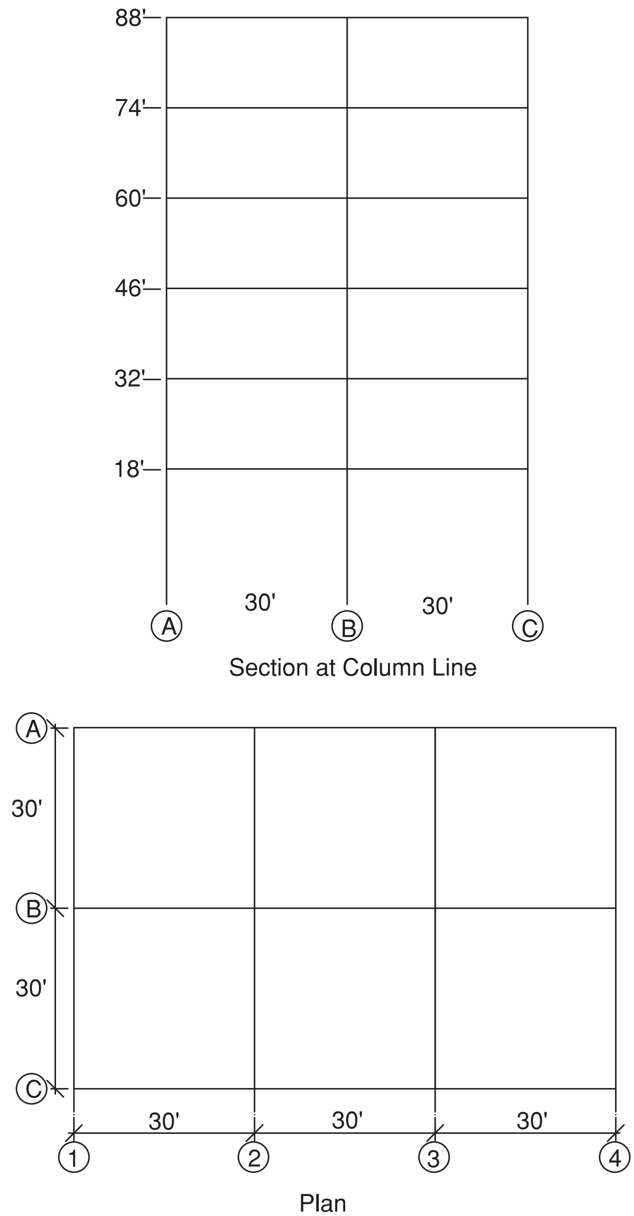
Practice Exercises:

12-1: If LLo = 80psf and tributary area (A_T) is 750^{f2}, what is reduced live load on a corner column with a cantilevered slab?

12-2: Calculate the design snow load on a flat roof of an office building in Denver, Colorado.

12-3: Find the wind loads for Column Line 2 if the fully enclosed structure in Melbourne, Florida, shown in Figure 12.7 resists wind with column lines 1, 2, and 3. Use Exposure Category D and Occupancy Category III.

12-4: Determine the seismic loads for column line 2 of the building in Figure 12.17. The site has very dense soil. The dead loads are 100psf for floors and 50psf for walls. The structure is a reinforced concrete moment frame.



12.7

Chapter 12 Practice exercises

thirteen

Horizontal Framing Systems

Horizontal framing systems are required to carry floor loads and usually to carry roof loads. The main idea of a horizontal framing system is to transfer all floor or roof loads as well as any lateral loads to the vertical support system. To do this, structural bays in orthogonal, radial or other patterns as discussed in Chapter 11 are employed to suit the individual project. The structural materials will define the limitations of the horizontal framing system.

Horizontal spanning systems consist of a deck that supports the floor or roof load and spans between and is supported by beams or joists. The deck not only distributes the loads to the beams, but provides a continuous stiff medium that enables the horizontal spanning systems to act as a horizontal diaphragm, meaning it acts as one rigid body. Decking material ranges can be any material capable of transferring the floor or roof loads to the beams or joists.

The beams and/or joists transfer the loads from the deck to either carrier beams or girders or directly to a vertical support system. Beam spacing is dependent on the allowable span of the deck. While some beams or joists may frame into the walls or columns of the vertical support system, many will frame into carrier beams or girders.

Most horizontal spanning systems employ an orthogonal grid pattern that allows for efficient use of materials and ease of connections. However, this is not required. As discussed in Chapter 11, grids can also be radial, complex or organic in form.

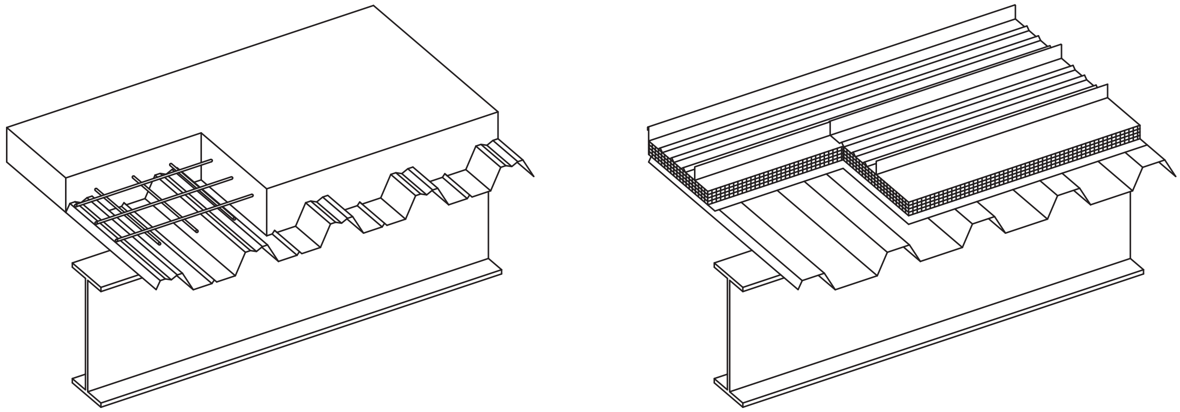
13.1 Typical Steel Framing Systems

Components of a horizontal steel framing system include the decking material, steel beams or joists, and the angles, plates and bolts used for connections. The design of steel components is covered in Chapters 21 through 24. Horizontal framing systems in steel may also take the form of a space frame or space truss as discussed in Chapter 15.

13.1.1 Decking in Steel Framing Systems

Decking can be comprised of almost any material that will support and safely transfer the floor or roof loads to the joists or beams. In steel framing, the decking material is most commonly steel deck, although grating is often used for catwalks and mezzanines in industrial applications. Other choices for decking on steel framing systems include precast concrete slabs and in some cases wood planking.

Steel deck is often covered with a concrete slab. In composite decking, the concrete and metal deck work together to support the loads. The concrete handles the compression forces and the steel deck handles the tension forces. In order for this to happen the deck must be bonded to the concrete through the cross-sectional pattern of the deck and also through the use of steel shear studs welded to the top of the metal deck.



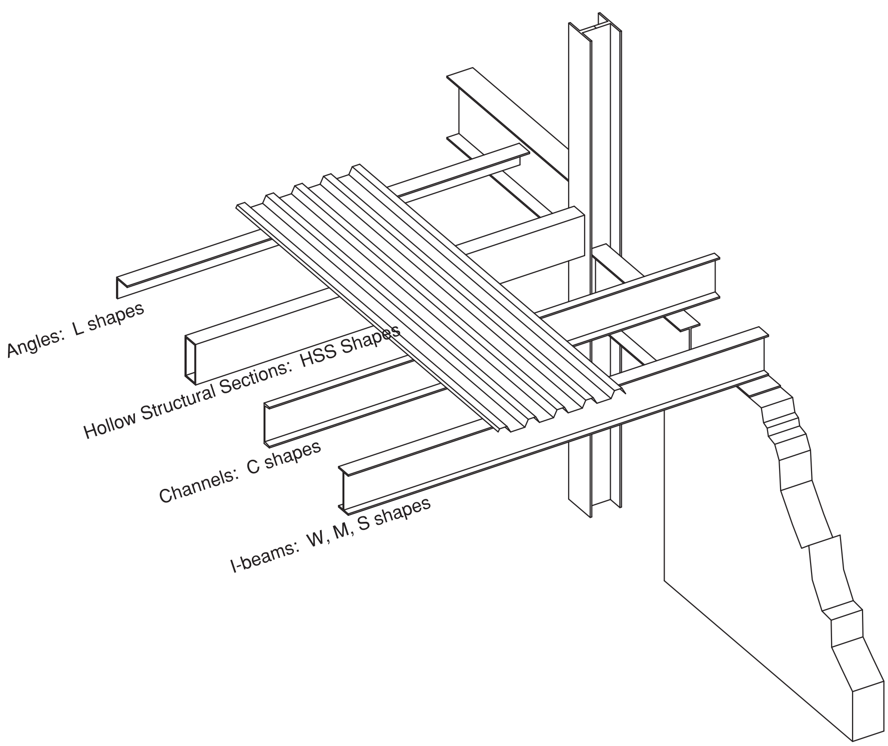
13.1
Steel decks

In non-composite or form decking, the metal deck is simply a form that supports the concrete when it is placed and until it cures. The concrete carries the entire floor or roof load.

Steel deck is selected by reviewing manufacturers' catalogs. Consider the proximity of the manufacturer's facility to the job site; closer is better because less energy will be used for transportation. Consider the recycled content in the steel by observing the total scrap steel, post-consumer recycled content and pre-consumers recycled content of each facility.

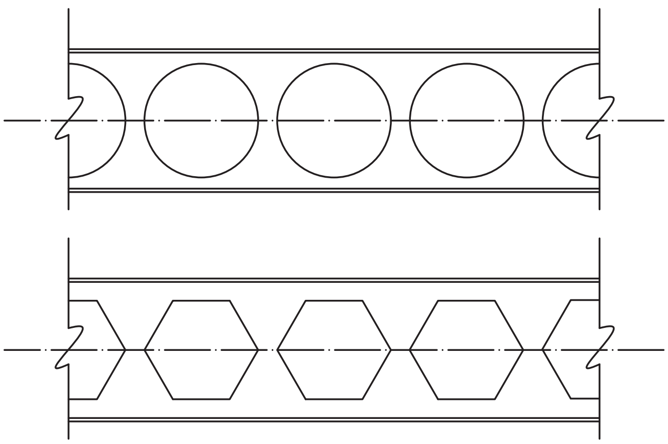
13.1.2 Steel Joists and Beams

The deck is supported by a series of joists, beams or bearing walls. Most steel beams are wide flange beams (W-shapes). W shapes have callout names based on depth and weight. For example, a W14×22 will have an approximate depth of 14″ and a weight of $22^{\#/f}$. M- and S-shapes are other I-beam shapes that may be used. Steel I-beam shapes are most efficient when spanning distances between 20′ and 40′. For lighter loads and shorter spans, channel sections (C-shapes), hollow structural sections (HSS-shapes) or angles (L-shapes) may be used.



13.2
Steel beams

Castellated beams are beams with a perforated web with holes usually in a series of circles or hexagons. They are constructed by combining the top half of one W-shape with the bottom half of another W-shape so that the beam becomes highly efficient. The top half is designed for compression and the bottom for tension. If a castellated beam is used in a scenario involving an overhang, the beam must be checked for compression in the bottom section near the overhang support. Castellated beams are used when it is desired that the openings in the web accommodate ducts, pipes or conduit or when it is necessary to reduce weight.



13.3

Castellated beams

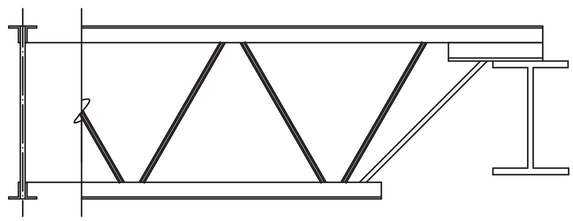
Open-web joists (OWJ) are another spanning option for steel framing systems. OWJ consist of a top and bottom flange usually made of double angles with bar struts placed in a truss configuration forming the web. OWJ are most efficient when used in spans over 30′. The open web not only provides an economical solution, but allows space for ducts, pipes or conduit to pass through the web.

There are three classes of OWJ with depths and spans as shown in Table 13.1. Design may vary by manufacturer, making it important to consider manufacturers close to the site before designing and calling out an OWJ component.

Table 13.1: Classes of open-web joists

Series		depth	span
K	Single bar web, suitable for light loads	12 to 30"	to 60'
LH	Suitable for heavy loads or long spans	18 to 48"	to 96'
DLH	Suitable for heavy loads and long spans	52 to 72"	to 144' (roof), 120' (floor)

OWJs must be braced laterally using horizontal or diagonal bracing to prevent displacement that could cause torsion in the joist. Further, OWJs must be bridged to prevent lateral sway.



13.4

Open-web joists

Rules of thumb for preliminary planning of steel framing systems are as follows where L = span in feet and d = depth in inches:

Steel form deck: d = L/35, L_{max} = 12′

Steel Composite deck and roof deck: d = L/35, L_{max} = 15′

Steel I-beams: d = L/20

Steel carrier beams or girders: d = L/15

13.2 Concrete Framing Systems

Concrete framing systems consist of steel-reinforced concrete components that are either cast in place or precast. The design of concrete components is covered in Chapters 25 through 33. Concrete systems can be designed to any shape for which a form can be fabricated. However, because concrete is heavy, weighing about 150pcf, concrete design should strive for efficiency of material.

13.2.1 Slabs

Concrete slabs can have either a uniform, tapered or ribbed cross-section depending on the span and loads carried. The most common type of slab is one with a uniform depth. A rule of thumb for slab depth is d = L/20, although slabs may be designed to be much thinner by calculating deflection, allowing the slab to be continuous over multiple spans or by employing pre-stressing methods. Thinner slabs can also be achieved by using a high-strength or ultra-high strength concrete.

13.2.2 Concrete Beams and T-beams

Concrete beams are typically rectangular in cross-section, but this is not a requirement. The shape of the cross-section is dependent only on its ability to carry the load over its span and the ability to create formwork to support it while it cures. The depth of a concrete beam is dependent on the span and loads carried as well as the amount of steel reinforcement and strength of the concrete mix. A rule of thumb for the ratio of the width (b) to the effective depth (d) of a concrete beam is 1.5 < d/b < 2.2. If shallower, the beam will begin to behave like a slab and if deeper, the beam will need additional reinforcement along the sides.

13.3 Wood Framing Systems

There are two basic methods for framing in wood: Western Framing and Post and Beam Framing, both of which are discussed in Chapter 15 “Structural Typology.” The difference between the two in horizontal framing systems is as follows. Western Framing Systems use closely spaced wood joists with a plywood or thin plank deck while post and beam systems use timber beams with a thicker plank deck.

13.3.1 Wood Deck

For Western Framing Systems, plywood decking is used as the subfloor and then topped with a finished floor product. The thickness of plywood is typically $\frac{3}{4}$” to 1” depending on the span and the grade of plywood used. OSB, particleboard and other non-veneer construction products may be used if rated for use by the manufacturer.

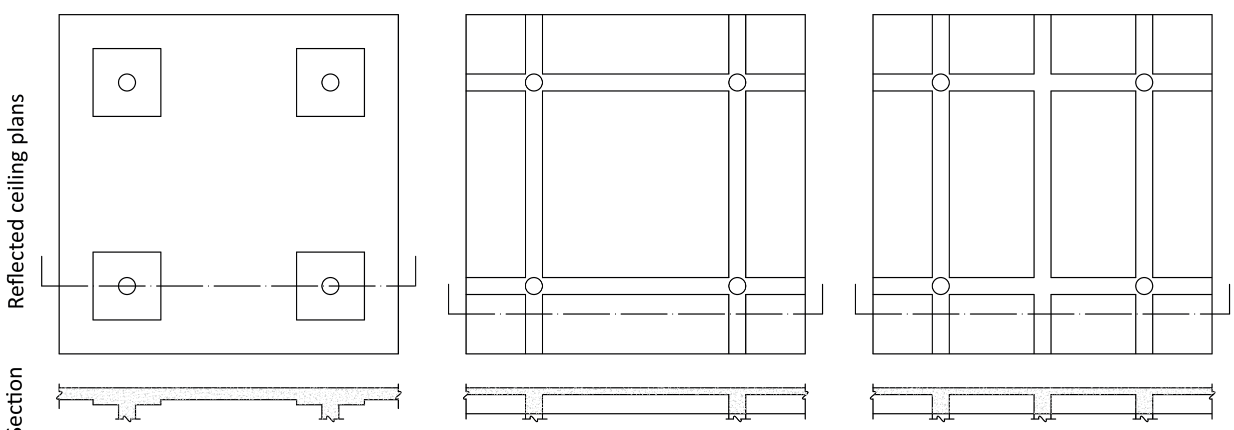


Plate slab -

Typical spans to 20'.

Slab spans in two directions.
Slab thickened around columns to prevent punching shear.

Rule of thumb for slab depth = span/30.

Two-way beamed slab -

Typical spans up to 30' or more if prestressed.

Slab spans in two directions.
Best suited for square or near square bays.

Rule of thumb for slab depth = span /30.

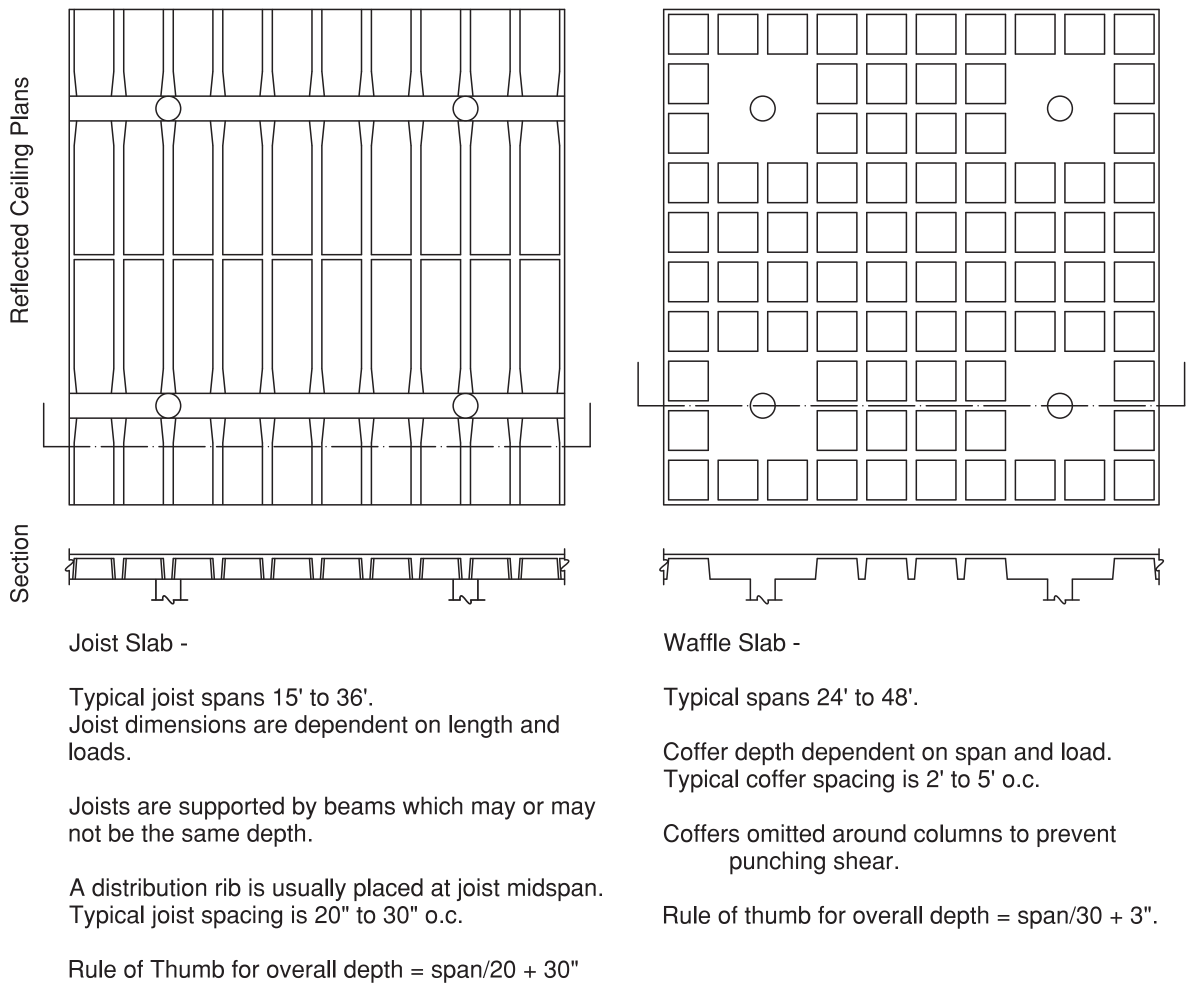
One-way beamed slab -

Typical spans 8' to 12', but may be longer.

Slab spans in short direction.

Rule of thumb for slab depth = span/20.

13.5 Concrete beam and slab systems



13.6
Ribbed concrete systems

Diagonal sheathing may be used in Western Framing Systems. Diagonal sheathing consists of a thin plank board $\frac{3}{4}''$ or greater spanning between joists and used as a subfloor. To ensure the sheathing acts as one unit, it is recommended that the boards have a ship-lap or tongue and groove connection.

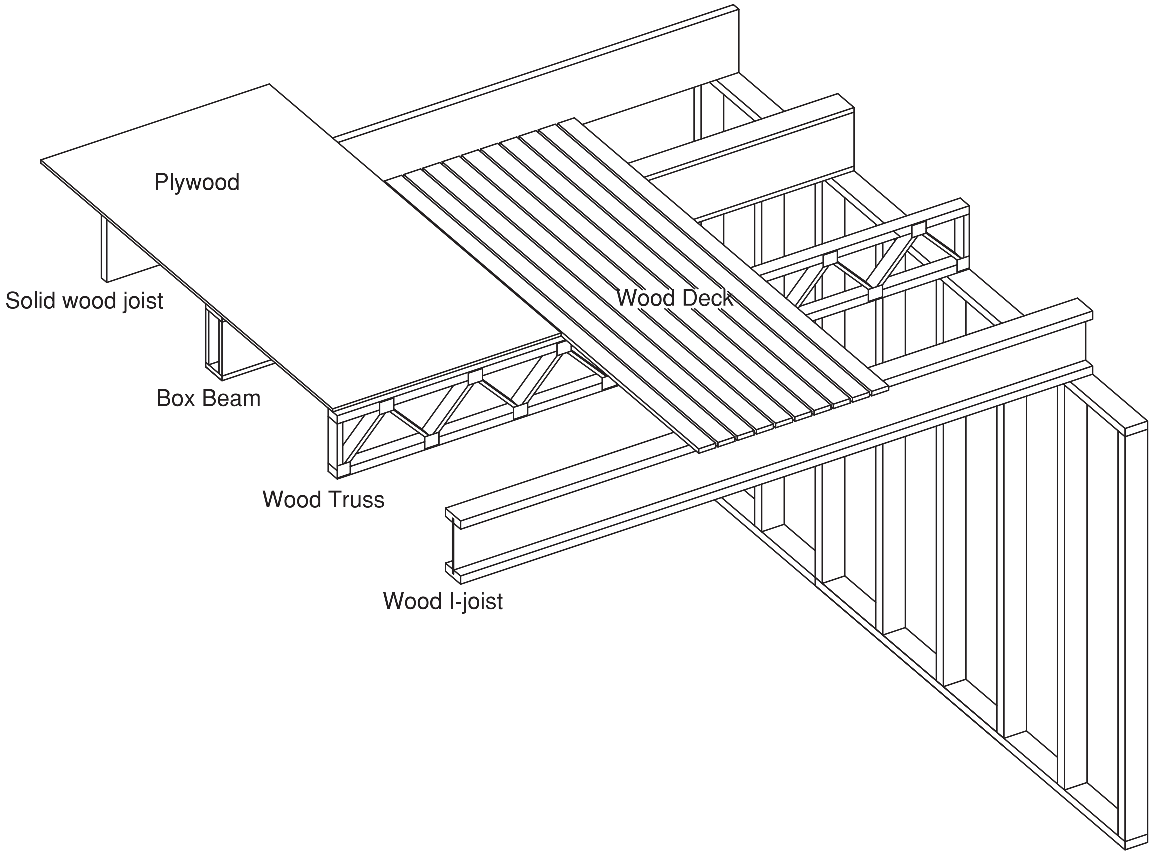
Decking consists of planks placed either diagonally or perpendicular to the span of the beam or joist. Decking planks are typically 1.5″ thick or greater to span 4′ to 8′ with tongue and groove connections between planks.

Outdoor decking consists of 2× dimensional lumber spaced to leave a small gap between the boards for drainage. The span of the decking depends on the ability of a single board to carry the entire weight of a person who might step on a single board. This means that the span between deck joists is much smaller than when the decking is made of plywood or tongue and groove planks of the same thickness.

13.3.2 Wood Joists and Beams

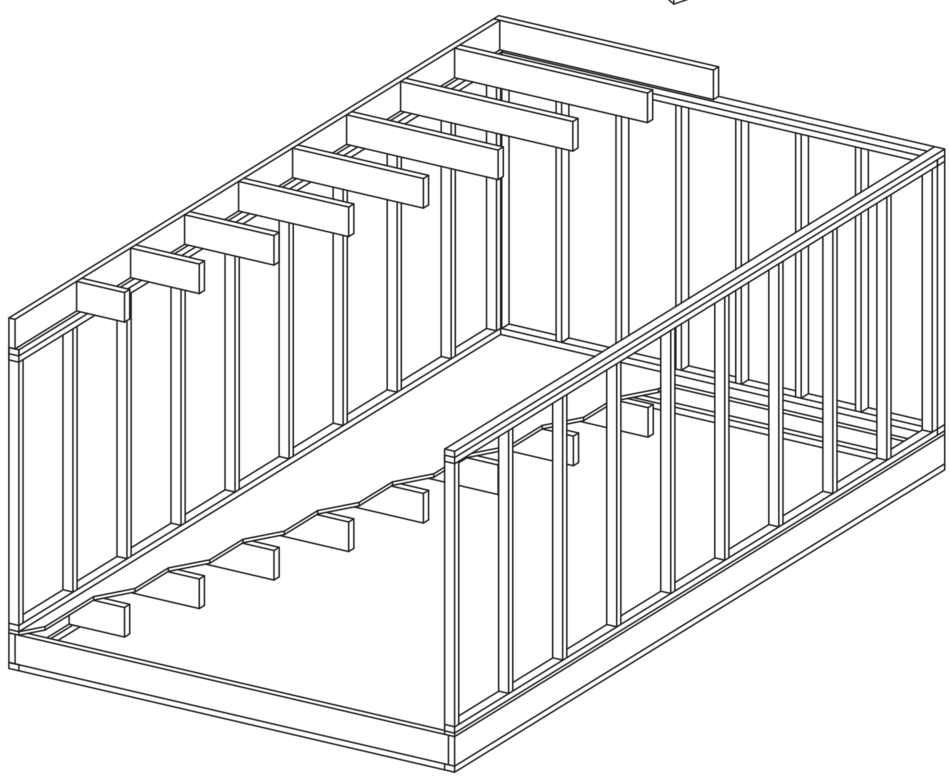
Wood joists and timber beams typically have standard-sized, rectangular cross-sections for economy. But wood is easily shaped and so custom sizes and non-prismatic members are sometimes used to convey a style or design concept. Sizes of wood joist and timber beams are dependent on the loads, spans, species of wood and factors such as water content, termite protection and heat. See Chapters 16 through 18 for design of wood beams.

Wood framing systems may also employ glue-laminated beams. Glue-laminated beams are manufactured by gluing thin layers of wood together to form a particular size and shape. Laminations may be vertical or horizontal, and horizontal laminations may employ cross-laminating, a process of alternating laminations at 90 degree angles to



13.7

Built-up wood beams



13.8

Western framing systems

create a stronger beam. Glue-laminated beams are costlier than sawn lumber, but are capable of longer spans.

13.3.3 Wood Built-up Members, I-beams and Trusses

Wood joists and beams may have cross-sections built up from sawn lumber or fabricated wood materials such as OSB or a combination of both. Typical built-up members include box beams, I-beams and trusses as shown in Figure 13.7.

13.4 Bay Framing

A bay is an area with a perimeter defined by a set of vertical support components, typically four columns. Often there are openings with a bay due to vertical shafts for stairwells, elevators, or MEP services. These openings cause discontinuity in the transfer of loads from deck to beams except in cases where the opening is very small or the deck can handle an overhang or cantilever.

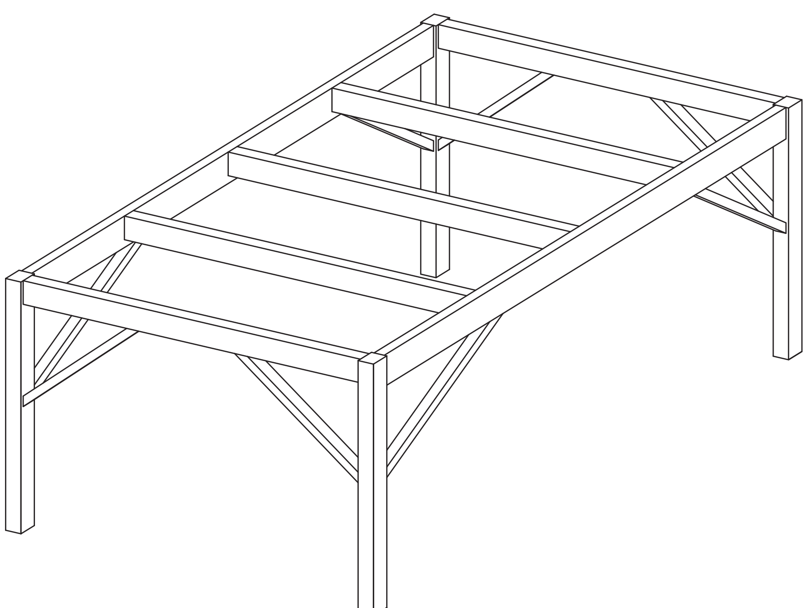
To frame a bay, begin at the perimeter and frame between the columns. Decide in what direction the deck spans and evenly space beams to span perpendicular to the deck span. Consider the typical spacing and spans for the material to be used. See Table 13.2 as a starting point.

Table 13.2: Typical spacing and spans

Horizontal Spanning Member	Rule of thumb for depth	Typical Span Range
Dimensional Lumber		
5/4 Wood decking	-	2' - 4'
Solid wood Joist	L/16	8' - 14'
Box beam	L/18	20' - 60'
I joist	L/18	15' - 60'
Joist truss	L/18	15' - 60'
Timber		
Timber Beam	L/15	8' - 20'
Timber truss	-	20' - 60'
Glue laminated wood		
PSL	L/20	10' - 45'
LVL	L/20	10' - 45'
Glu-lams	L/20	20' - 90'
Steel		
Steel Deck	-	2' - 12'
Light-gaguge steel joists	L/20	8' - 30'
W- shapes	L/20	20' - 40'
Deep W-Shapes	L/15	40' - 80'
Castellated beams	L/24	40' - 80'
OWJ	L/24	20' - 144'
Concrete		
Rectangular beam	L/16	12'-48'
T-beams	L/20	20' - 80'
Doubly reinforced beams	L/20	20' - 50'
Pre-stressed beams	L/20	20' -
Solid one-way slabs	L/28	8' - 18'
Solid two-way slabs	L/45	15 - 40'
Waffle Slabs	L/24	20' - 50'
Joist Slabs	L/24	20' - 40'
Precast Hollow Core Slabs	L/40	12' - 24'

Where openings exist, place a beam along each side of the opening not already framed by a beam and choose the order of the load transfer.

For example, in Figure 13.10 (a) the deck spans E–W and the beams span N–S to create four spans. The opening is $10' \times 20'$ and so the beam on the right is truncated before it



Post and Beam - beams supported by braced columns

13.9

Post and beam systems

penetrates the stairwell. A header beam is placed supported by the center beam and the beam along the column line. The N–S edge of the opening is framed, supported by the header beam and a girder.

In Figure 13.10 (b) the deck spans N–S and the beams span E–W. Two beams must be truncated before before penetrating the stairwell. A header beam is placed spanning N–S and the E–W edge of the opening is framed, supported by the header beam and the girder.

13.5 Framing Process

The process to frame a roof or floor is as follows:

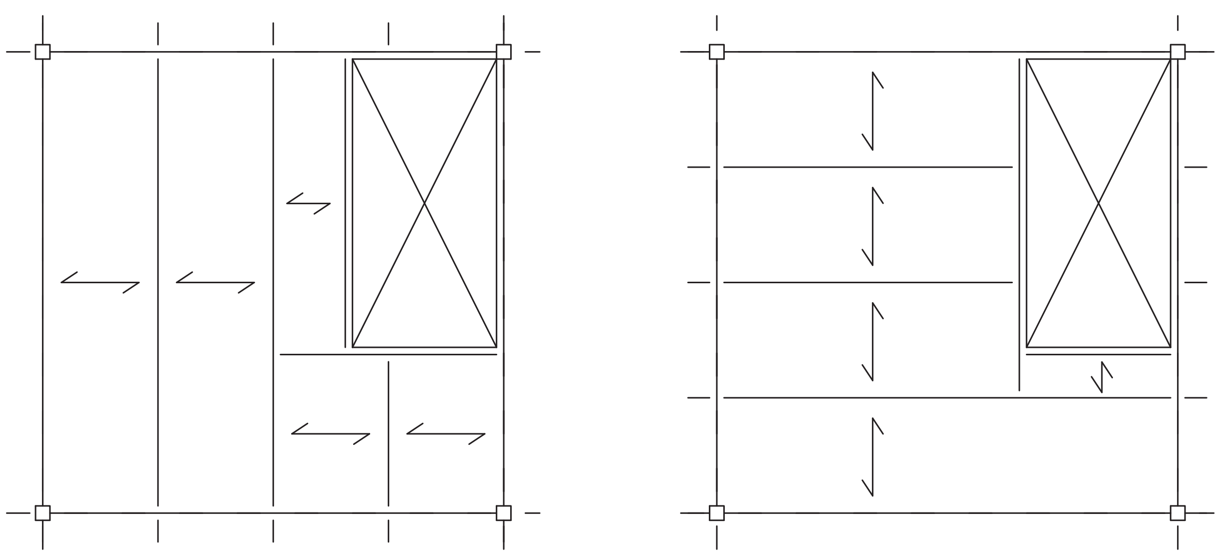
1. Define the perimeter.
2. Locate stairwells, elevator shafts, ventilation shafts and any other large perforation in the floor or roof.
3. Locate multilevel spaces
4. Define circulation patterns and other areas where columns should be avoided.
5. Create the pattern of support using columns or bearing walls. Columns cannot be spaced farther apart than the maximum span of the beams.
6. Frame between vertical supports. Do not place framing members through vertical shafts such as stairwells or elevator shafts.
7. Provide additional beams to support the deck. Beams cannot be spaced farther apart than the maximum span of the deck.
8. Frame around all openings.

Example 13-1: Create a framing plan for the perimeter defined in Figure 13.11 (a) if using structural steel with a maximum beam spacing of 10′ and a maximum column spacing of 40′.

Include an atrium of approximately 5000ft^2 and an 8′ × 20′ stairwell at each end of the building.

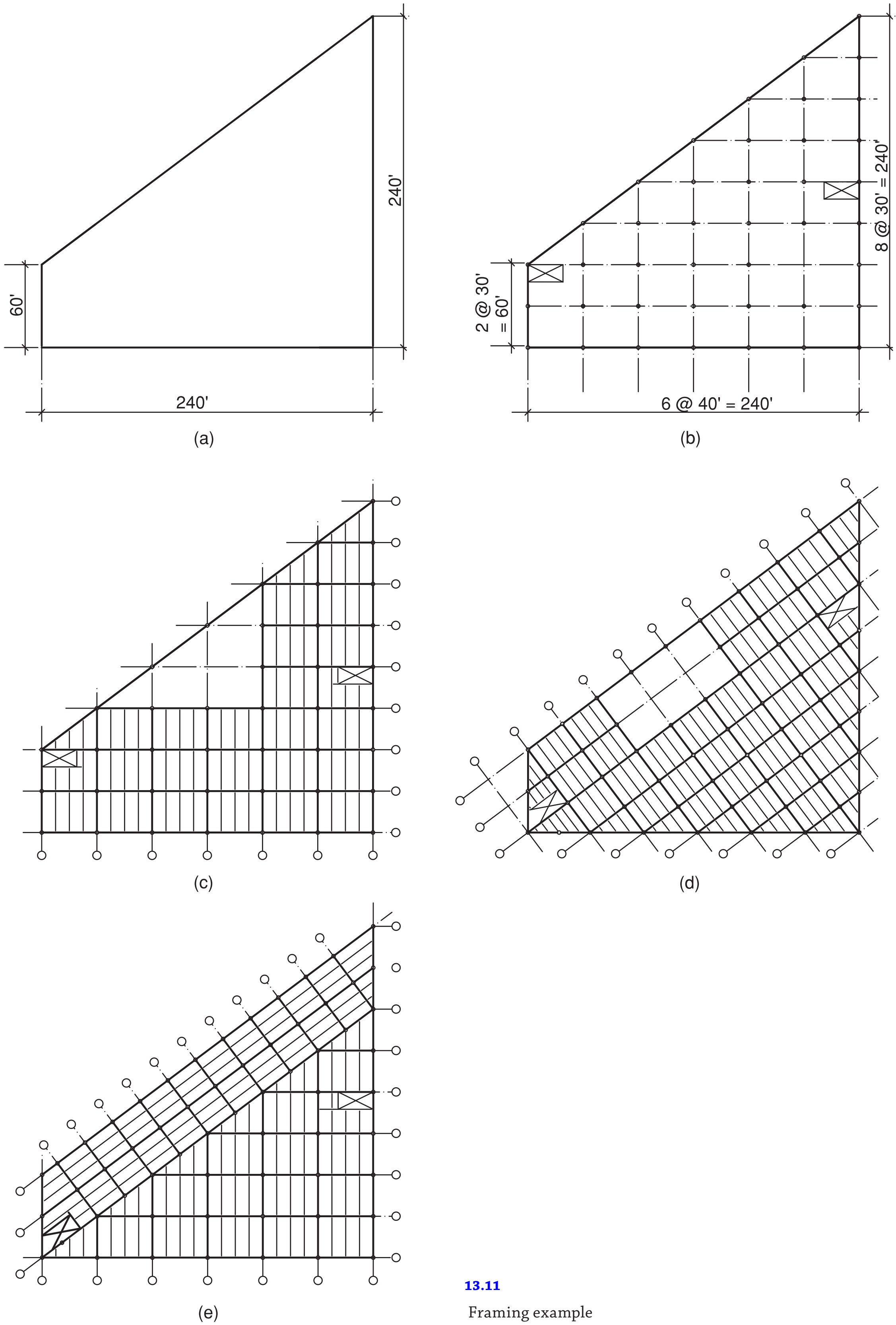
1. Draw column lines in one direction. Do not exceed maximum column spacing. Try to:
    a. place column lines near corners of the building
    b. evenly space column lines for economy or create a pattern of space for a design concept
    c. if stairwells or other vertical shafts fall on a column line, adjust shaft location if possible. If not, adjust column lines or plan to frame around the opening.
2. Create column lines in the perpendicular direction.
3. Frame each bay.
4. Frame around openings.
5. Add beams to support decking. See Figure 13.11 (b).

Possible solutions are shown in Figures 13.11 (c, d, e and f).



13.10

Bay framing (a) E–W deck span (b) N–S deck span



13.11

Framing example

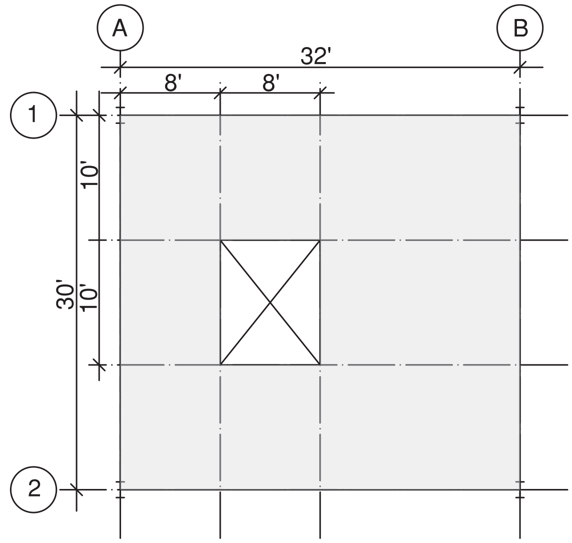
Practice Exercises:

13-1 through 13-3. Frame the bay shown if the maximum deck span is a) 8ft and b) 10ft.

13-4: Frame a structural floor plan that lies within a 96′ by 144′ rectangle.

The plan must include:

1. at least 11,000sf of enclosed space (including the atrium and stairwells listed below);
2. one atrium space between 800 and 1200sf, located anywhere you choose;
3. two 8′ × 20′ stairwells along the perimeter and spaced at opposite ends of the building;
4. maximum slab span = 12′ = maximum beam spacing;
5. maximum beam span = 40′ = maximum column spacing.



13.12

Chapter 13 Practice exercises

fourteen

Lateral Bracing Systems

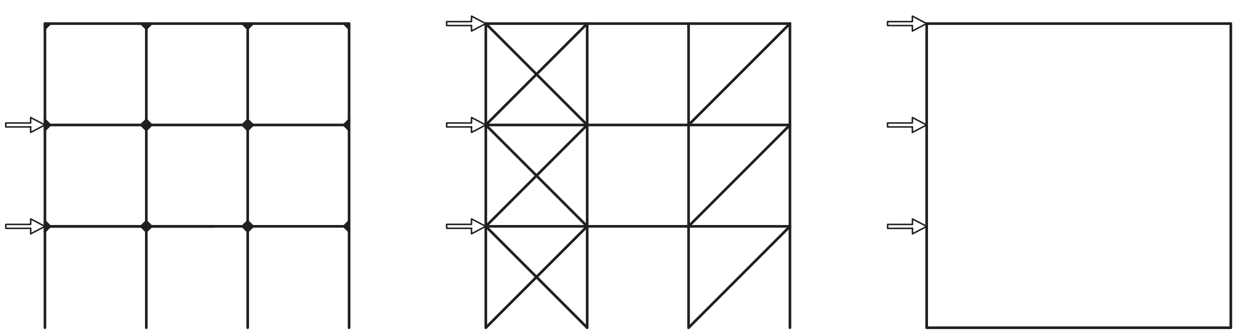
There are three basic methods to resist lateral loads. Trusses, trussed tubes and braced frames with diagonal tension counters, all rely on diagonal bracing to resist lateral loads. Moment frames rely on rigid connections to resist lateral loads. Shear walls, whether made of reinforced concrete, masonry or sheathed stud walls, rely on the stiffness of the wall to resist lateral loads.

When resisting lateral loads, whether wind or seismic, it is important to maintain a balance of resistance throughout the structural system. Otherwise, the building will be subjected to torsion as one portion resists a lateral motion while another is free to deflect.

14.1 Braced Frames

This section explains the Diagonal Truss Method for determining the loads on components due to lateral loads when using diagonal tension bracing. Note that while the diagrams only show the active diagonal tension braces, there are in fact diagonal tension braces in both directions. Because diagonal tension braced systems become indeterminate when using multiple bays and multiple levels, the following assumption must be made.

The Diagonal Truss Method assumes each diagonal on any given level equally resists the sum of horizontal forces above



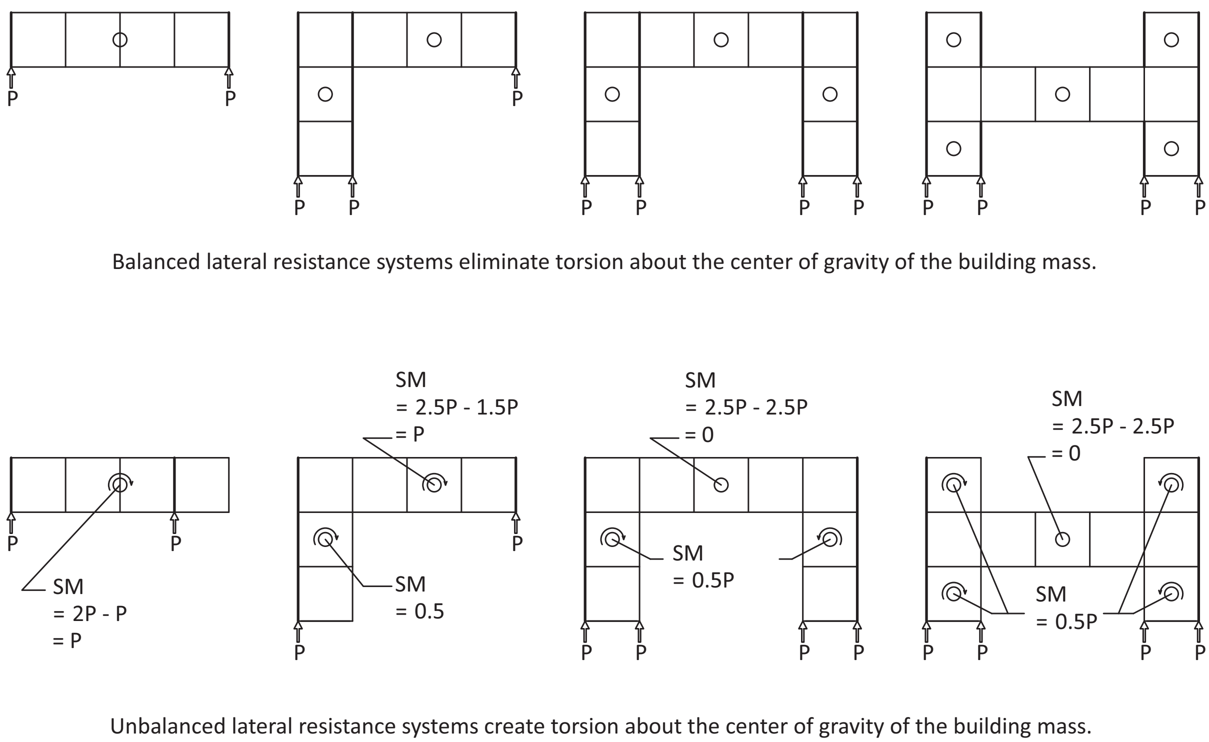
Moment Frame -
transfers loads to the ground through rigid connections

Braced Frame -
transfers loads to the ground through a series of braces

Shear Wall -
transfers loads to the ground through the bulk of the material

14.1

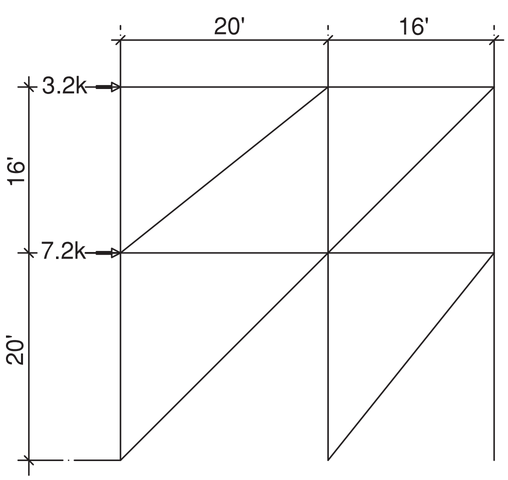
Three basic types of lateral resistance



14.2
Balance in later resistance systems

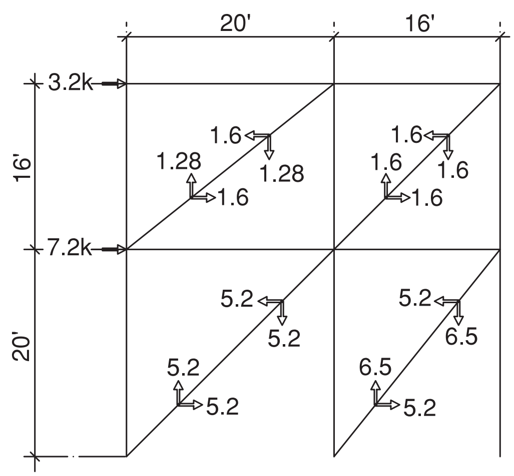
that level. Once this assumption is made, the axial loads in the system can be solved by summing forces in the x and y directions.

Example 14-1: Find the additional loads in each member due to the lateral loads shown.



14.3
Diagonal Truss Method

In the top row the total force to be resisted is 3.2k. Each diagonal will resist 3.2k/2 = 1.6k. In the bottom row, each diagonal will resist (3.2k + 7.2k)/2 = 5.2k.



14.4
Diagonal Truss Method assumes each bar in a row resists lateral loads with equal force

Because the forces are axial in every member, the vertical component in the diagonal can be found using the ratio of rise/run: rise/run = f_y/f_x; or $f_y = f_x$(rise/run).

Top left: $f_y = 1.6(16)/20 = 1.28$

Top right: $f_y = 1.6(16)/16 = 1.6$

Bottom left: $f_y = 5.2(20)/20 = 5.2$

Bottom right: $f_y = 5.2(20)/16 = 6.5$

The forces at the other end of each bar are equal and opposite.

Sum forces at each joint to find the additional loads in the beams and columns.

A: $\Sigma F_x = 0 = 3.2 + x$ .. $x = -3.2$ and $\Sigma F_y = 0 = y \ldots y = 0$

B: $\Sigma F_x = 0 = 3.2 - 1.6 + x \ldots x = -1.6$ and
$\Sigma F_y = 0 = -1.28 + y \ldots y = 1.28$

C: $\Sigma F_x = 0 = 1.6 - 1.6 + x \ldots x = 0$ … okay and
$\Sigma F_y = 0 = -1.6 + y \ldots y = 1.6$

D: $\Sigma F_x = 0 = 7.2 + 1.6 + x \ldots x = -8.8$ and
$\Sigma F_y = 0 = 1.28 + y \ldots y = -1.28$

E: $\Sigma F_x = 0 = -5.2 + 8.8 + 1.6 + x \ldots x = -5.2$ and
$\Sigma F_y = 0 = -5.2 - 1.28 + 1.6 + y \ldots y = 4.88$

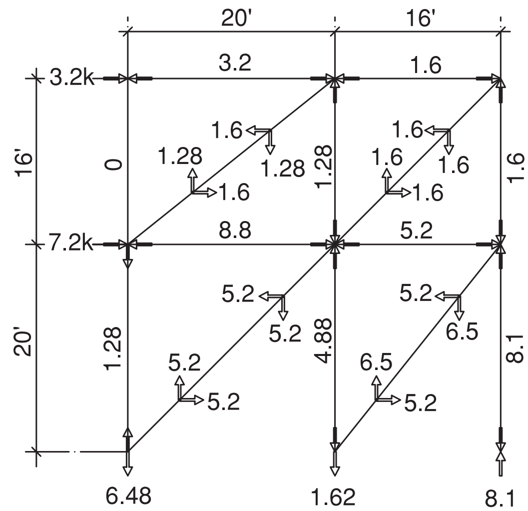
F: $\Sigma F_x = 0 = -5.2 + 5.2 + x \ldots x = 0$ … okay and
$\Sigma F_y = 0 = -6.5 - 1.6 + y \ldots y = 8.1$

G: $\Sigma F_x = 0 = 5.2 + R_x \ldots R_x = -5.2$ and
$\Sigma F_y = 0 = 1.28 + 5.2 + R_y \ldots R_y = -6.48$

H: $\Sigma F_x = 0 = 5.2 + R_x \ldots R_x = -5.2$ and
$\Sigma F_y = 0 = 6.5 - 4.88 + R_y \ldots R_y = -1.62$

I: $\Sigma F_y = 0 = 8.1 + R_y \ldots R_y = -8.1$

It is always a good idea to check the sum of all external horizontal forces.



14.5

Horizontal and vertical bar forces

$\Sigma F_y = 0 = -6.48 - 1.62 + 8.1 = 0$ … okay

$\Sigma F_x = 0 = 3.2k + 7.2k - 5.2k - 5.2k = 0$ … okay

Likewise, the moment about any point should equal zero.

$\Sigma M_G = 3.2(36) + 7.2(20) + 1.62(20) - 8.1(36) = 0$

$\Sigma M_E = 3.2(16) - 6.48(20) - 8.1(16) + 5.2(20) + 5.2(20) = 0$

Calculate the tension in the diagonals:

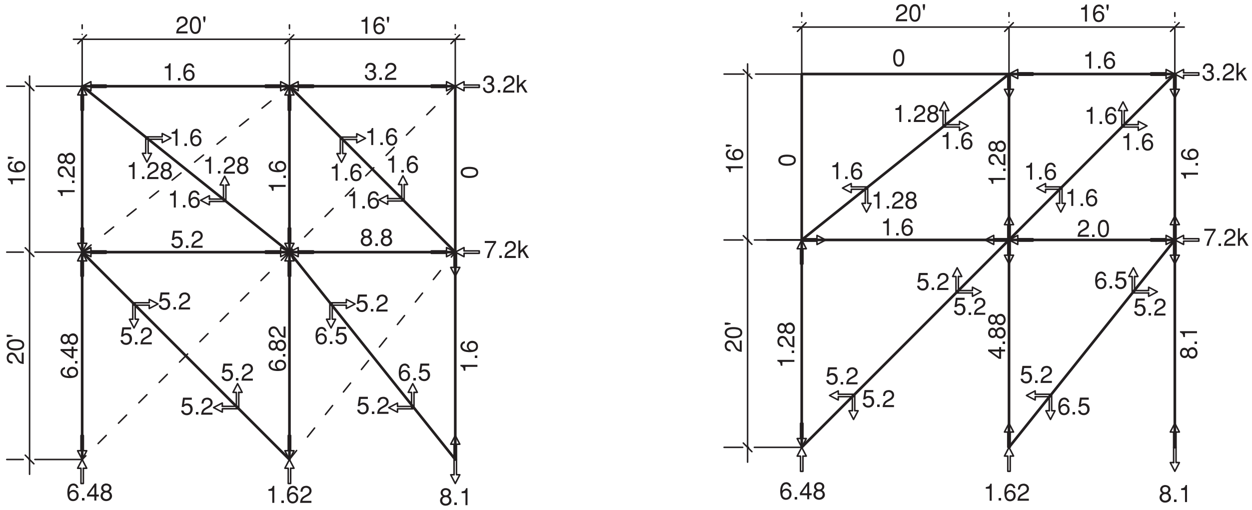
$T_{BD} = \sqrt{(1.6^2 + 1.28^2)} = 2.05$

$T_{CE} = \sqrt{(1.6^2 + 1.6^2)} = 2.26$

$T_{EG} = \sqrt{(5.2^2 + 5.2^2)} = 7.35$

$T_{FH} = \sqrt{(5.2^2 + 6.5^2)} = 8.32$

Note: there are two diagonals in each bay, but only one is active at any given time because the bracing is designed for tension alone. Only one brace is shown for analysis because the force is assumed from one direction. If the lateral loads are reversed, the diagonals shown with dashes in Figure 14.6(a) would be inactive and the following beam and column values can be found using the active tension braces. If only one set of diagonals is preferred, the diagonals will need to be designed for the compression forces created by reversed lateral loads as shown in Figure 14.6(b).



14.6
Reversed lateral loads (a) tensions braces (b) compression braces

The absolute value of the largest loads from both scenarios must be added to the beams and columns. Knowing the tension in the diagonals allows them to be designed.

Assume F_t = 30ksi. Since F_t = P/A, using maximum tension to size all rods the same, A ≥ 8.32k/30ksi = 0.28in^2. And since A = .28 ≤ $\pi d^2/4$, d ≥ 0.60″. Rounding up to the next $\frac{1}{8}$″ yields a $\frac{5}{8}$″ diameter rod. Other structural shapes that could be used include:

L1-1/4 × 1-1/4 × 3/16, A = 0.43

Or C3 × 3.5, A = 1.09

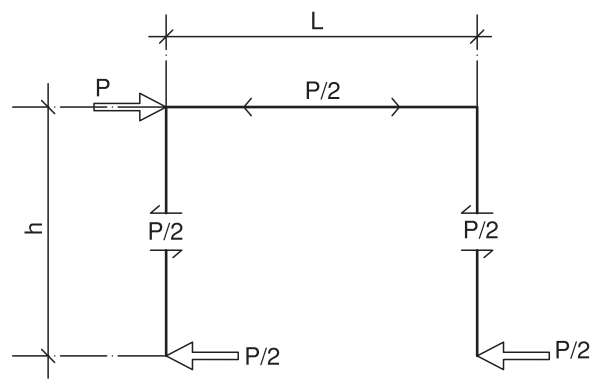
Or HSS2 × 1 × 1/8, A = 0.61

14.2 Moment Frames

Moment frames resist lateral forces by virtue of the rigid connections at each joint. Although the connections are rigid, a moment frame is actually more flexible than a braced frame. This section explains how to use the Portal Method to solve for additional shear and axial forces and additional moment in beams and columns of a moment frame subjected to lateral forces.

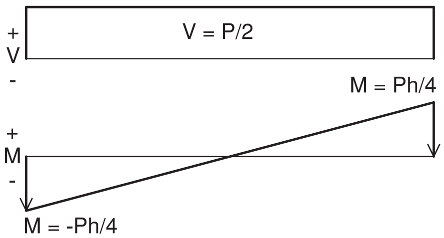
The Portal Method has six basic steps:

1. Find shear in each column
2. Sum x-direction forces
3. Find moment caused by shear in columns
4. Balance moments at each joint
5. Find shear in beams caused by moment
6. Sum y-direction forces



14.7
The basic portal frame

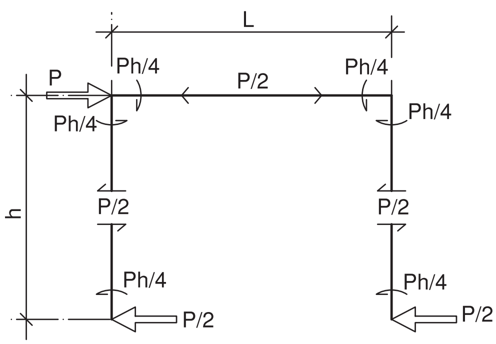
Consider a portal of height h and length L subjected to a lateral force P. It is assumed that each leg is equally capable of resisting the force P and so the reaction at the base of each leg is P/2. The horizontal force is transferred through the vertical leg by shear force. At any given point in the leg, there is a force of P/2 in shear. Summing the horizontal forces at the point of load we find that the top of the portal has an axial force = P/2 in compression.



14.8

Moment in the vertical legs

The moment at either end of the leg will be M = −(P/2)(h/2) = −Ph. And sum of moments at the connection between leg and top must equal zero or else the connection will rotate.



14.9

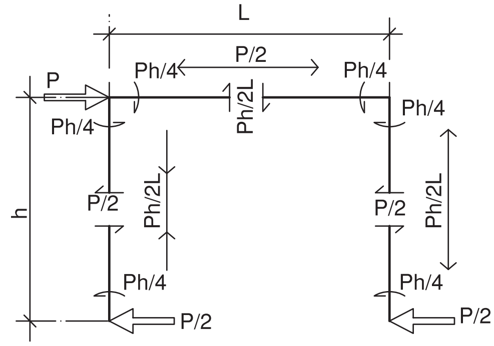
Moment in portal frame members

The moment at either end of the top will create shear equal to the moment divided by half the length of the top or (Ph/4)/(L/2) = Ph/2L.

Multiple portals in a frame.

Each leg on any given level equally resists the sum of all loads above that level.

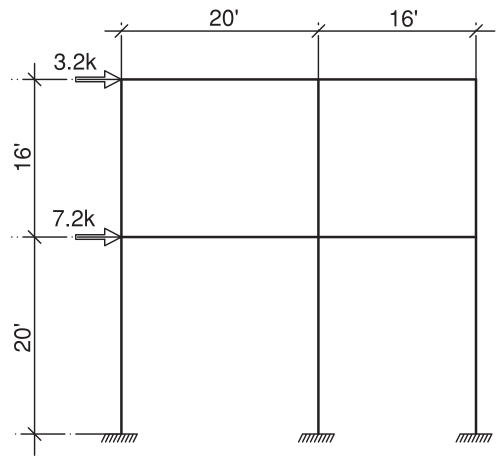
#legs = 2(#bays across)



14.10

Moment, shear and axial forces in a simple portal frame

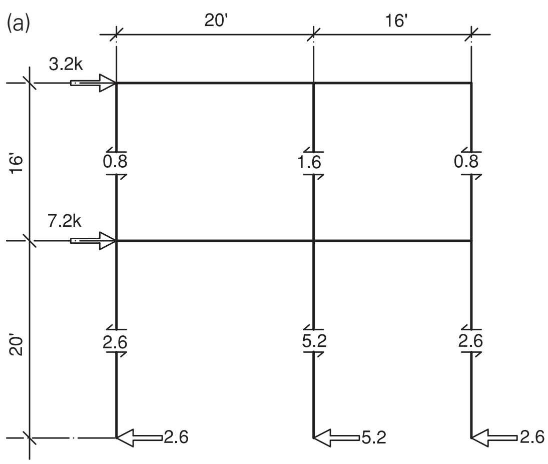
Example 14-2: Use the Portal Method to determine the additional shear, moment and axial forces in the moment frame components shown in Figure 14.11.



14.11

Portal Method example

1. Find shear in each column. Each row has 2 portals and each portal has 2 legs for a total of 4 legs. This means that the exterior columns resist (1leg/4legs total) or $\frac{1}{4}$ the lateral loads above them and the interior column resists (2legs/4legs total) or $\frac{1}{2}$ the lateral loads above them.



14.12

Shear in columns

2 bays = 4 legs:

For the 16′ segment in the columns, the total forces above are P_1 = 3.2k

P_1/#legs = 3.2k/4legs = .8k/leg

Outside columns have 1 leg, interior columns have 2 legs:

$V_A = V_C$ = .8k shear on exterior columns

V_B = .8(2) = 1.6 on interior column

For the 20′ segment in the columns, the total forces above are:

$P_1 + P_2$ = 3.2k + 7.2k = 10.4k

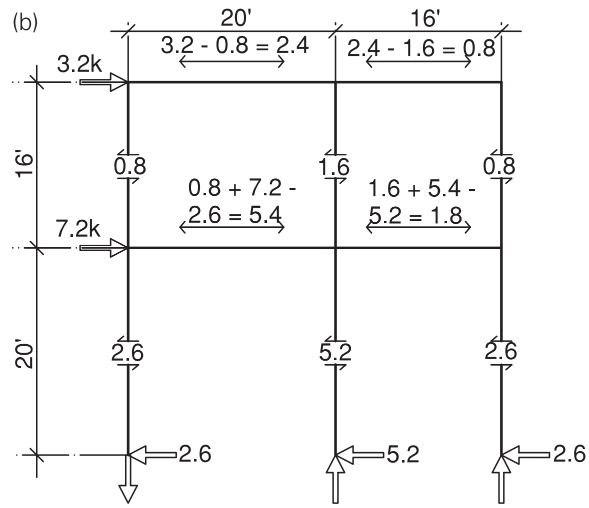
(P1 + P2)/# legs = 10.4/4legs = 2.6k/leg

Outside columns have 1 leg, interior columns have 2 legs:

$V_A = V_C$ = 2.6k shear on exterior columns

V_B = 2.6k(2) = 5.2k on interior column

2. Sum x-direction forces. Start at the top and sum the x-direction forces at each joint.



14.13

Horizontal forces in beams

Joint A2: ΣF_x = 0 = 3.2k − 0.8k + AB2 …
AB2 = 0.8 − 3.2 = −2.4k.

Because the forces in the beam are pointed toward the joints (← →), the beam is in compression. The force exerted on Joint B2 is equal and opposite and therefore positive.

Joint B2: ΣF_x = 0 = 2.4k − 1.6k + BC2 …
BC2 = 1.6 − 2.4 = −0.8k

Joint C2: ΣF_x = 0 = 0.8k − 0.8k. This is correct. The sum should equal zero, although sometimes there will be a small difference at the last joint in a row due to the rounding of values.

Joint A1: ΣF_x = 0 = 0.8k + 7.2k − 2.6k + AB1 …
AB1 = −5.4k = 5.4k←

Joint B1: ΣF_x = 0 = 1.6k + 5.4k − 5.2k + BC1 …
BC1 = −1.8k = 1.8k←

Joint C1: ΣF_x = 0 = 0.8 + 1.8 − 2.6 = 0

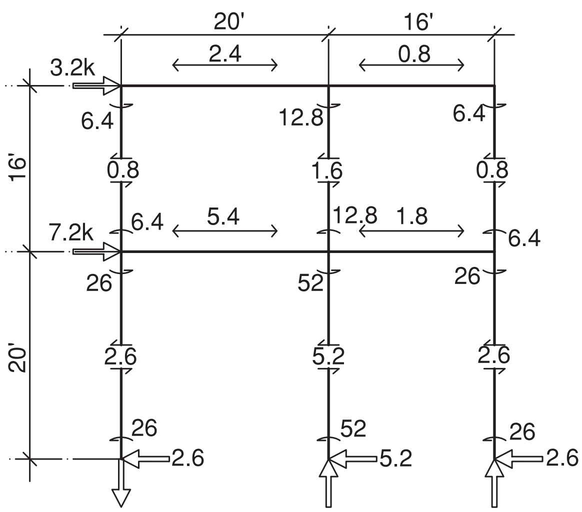
Horizontal reactions at the column bases are equal to the shear in the column:

Joint A0: $\Sigma F_x = 0 = 2.6k + A_x \ldots A_x = -2.6k = 2.6k\leftarrow$

Joint B0: $\Sigma F_x = 0 = 5.2k + B_x \ldots B_x = -5.2k = 5.2k\leftarrow$

Joint C0: $\Sigma F_x = 0 = 2.6k + C_x \ldots C_x = -2.6k = 2.6k\leftarrow$

3. Find moment caused by shear in columns. There is a negative moment at the end of each column segment = M = Vh/2 where V = the shear in the column segment and h = height of the column segment.



14.14

Moment in columns

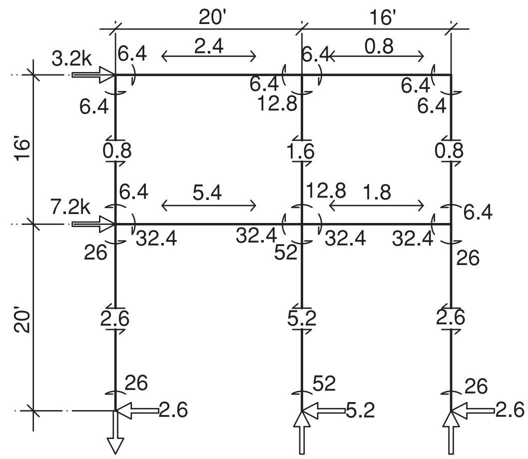
For the 16′ segments on the exterior columns: .8k (16′/2) = 6.4 k-f

For the 16′ segment on the interior column: 1.6(16/2) = 12.8

For the 20′ segments on the exterior columns: 2.6 (20/2) = 26

For the 20′ segment on the interior column: 5.2 (20/2) = 52

4. Balance moments at each joint. ΣM = 0 at each joint. Moments are equal at both ends of a beam segment because shear is constant throughout the beam.



14.15

Moment in beams

Joint A2: $\Sigma M = 0 = -6.4 + M_{AB2} \ldots M_{AB2} = 6.4$k-f

Joint B2: $\Sigma M = 0 = 6.4 - 12.8 + M_{BC2} \ldots M_{BC2} = 6.4$k-f

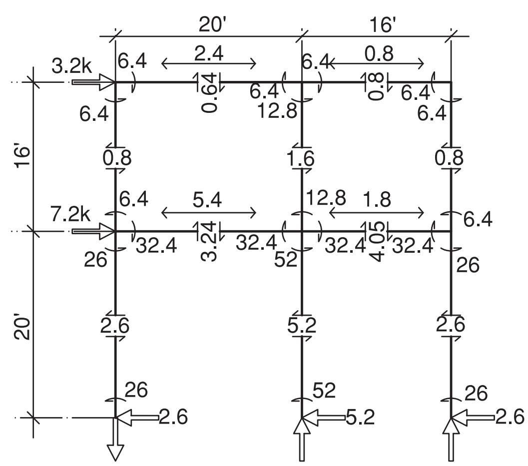
Joint C2: $\Sigma M = 0 = 6.4 - 6.4$. This is correct.

Joint A1: $\Sigma M = 0 = -26 - 6.4 + M_{AB1} \ldots M_{AB1} = 32.4$k-f

Joint B1: $\Sigma M = 0 = -12.8 + 32.4 - 52 + M_{BC2} \ldots$ $M_{BC2} = 32.4$k-f

Joint C1: $\Sigma M = 0 = 32.4 - 6.4 - 26$. This is correct.

5. Find shear in beams caused by moment. V = M/(L/2) where M = moment in the beam segment and L = length of the beam segment.



14.16

Shear in beams

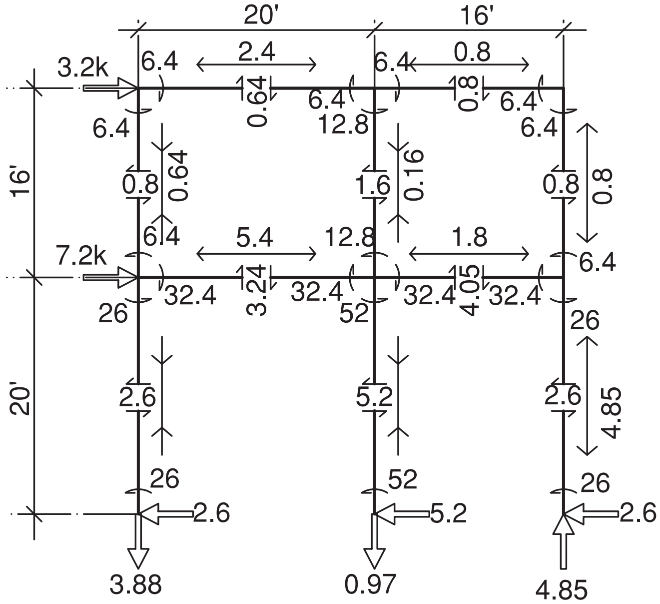
V_{AB2} = 6.4k-f/(20′/2) = 0.64k

V_{BC2} = 6.4k-f/(16′/2) = 0.8k

V_{AB1} = 32.4k-f/(20′/2) = 3.24k

V_{BC1} = 32.4k-f/(16′/2) = 4.05k

6. Sum y-direction forces



14.17

Axial forces in columns

Joint A2: $\Sigma f_y = 0 = 0.64 + F_y \ldots F_y = -0.64k = 0.64k\downarrow$

Joint A1: $\Sigma f_y = 0 = 0.64 + 3.24 + F_y \ldots$
$F_y = -3.88k = 3.88k\downarrow$

Joint A0: $\Sigma f_y = 0 = 3.88 + A_y \ldots A_y = -3.88k = 3.88k\downarrow$

Joint B2: $\Sigma f_y = 0 = -0.64 + 0.8 + F_y \ldots$
$F_y - 0.16k = 0.16k\downarrow$

Joint B1: $\Sigma f_y = 0 = -3.24 + .16 + 4.05 + F_y \ldots$
$F_y = -0.97k = 0.97k\downarrow$

Joint B0: $\Sigma f_y = 0 = 0.97 + B_y \ldots B_y = -0.97k = 097\downarrow$

Joint C2: $\Sigma f_y = 0 = -0.8 + F_y \ldots F_y = .8k = .8k\uparrow$

Joint C1: $\Sigma f_y = 0 = -0.8 - 4.05 + F_y \ldots$
$F_y = 4.85k = 4.85k\uparrow$

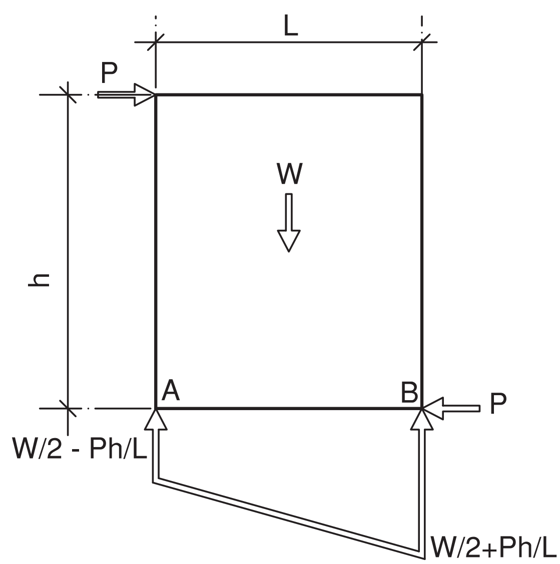
Joint C0: $\Sigma f_y = 0 = -4.85 + C_y \ldots C_y = 4.85k = 4.85k\uparrow$

Sum Y reactions: $\Sigma f_y = 0 = -3.88 - .97 + 4.85 = 0 \ldots$ okay

When you design your beams and columns for shear and flexure and deflection, you must add the values you obtain for shear and moment from this chart. Remember the lateral forces may act in either direction.

14.3 Shear Walls

A shear wall acts as a rigid body capable of transferring lateral loads to the foundation through internal moment. Most shear walls are made of dense material such as masonry or reinforced concrete. But shear walls can also be created by lighter materials such as plywood on Western Framing if adequate tie-downs are provided to resist turnover. If no tie-downs are used, the resisting moment caused by the weight of the wall must be 50% greater than the overturning moment: $M_R \geq 1.5M_O$.



14.18

Shear wall

Consider the shear wall in Figure 14.18. The lateral loads push against the wall and if not counteracted, will overturn the wall about Point B called the toe with an Overturning Moment = M_o = Ph. The weight of the wall (W) helps to counteract the overturning moment by creating a negative moment due to the weight of the wall acting vertically at the center of gravity. If the shear wall is fully connected to columns as in case A, the column reactions due to wall weight W and lateral load P will be A_y = W/2 − Ph/L and B_y = W/2 + Ph/L. But if the wall is not connected to columns, then the weight of the wall is uniformly distributed and the lateral load causes a uniform change from tension to compression along the base of the wall as in Case B. The wall will require tie-downs wherever the net reaction is in tension.

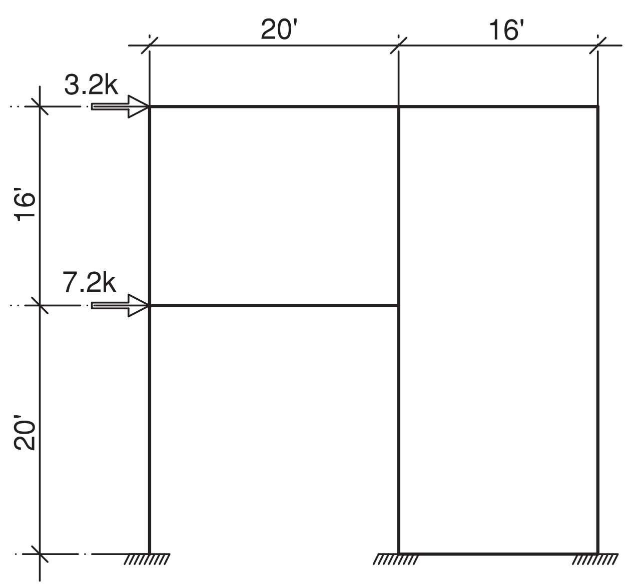
Example 14-3: Determine the reactions on the columns if the normal-weight concrete, 8″ thick shear wall in Figure 14.19 is fully connected to the columns.

Determine a density for the wall based on material: concrete density = 150pcf

W = 16′(36′)(8″)(1′/12″)(150pcf) = 57,600# = 57.6k

ΣM_B = 0 = 10k(36′) + 20k(20′) − 57.6k(8′) + A_y … A_y = −299.2k = 299.2k↓

Σf_y = 0 = −299.2k − 57.6k + B_y … B_y = 356.8k = 356.8k↑



14.19
Shear wall examples

Example 14-4: Determine the required thickness of the normal-weight concrete wall in Figure 14.19 in order to avoid tie-downs if the wall is not connected to columns.

Find weight of wall in terms of some thickness t:

W = 16f(36f)(t)(150pcf)/1000#/k = 86.4t

M_o = 10(36) + 20(20) = 760k-f

Check the moment about the toe to ensure that $M_r/M_o \geq 1.5$

M_r = 86.4(t)(8ft) = 691.2(t)k-f

691.2(t)/760 ≥ 1.5

t ≥ 1.649′ = 19.78″ use a 20″ wall

Check reactions along the base of the wall:

T = (ΣPiHi)(3)/b^2 = 3M_o/b^2 = 3(760k-f)/(16^2) = 8.906k/f

W/b = 86.4k/f(20″/12″/f)/16′ = 9.0k/f

Reaction at point A = W/b − T = 9.0 − 8.906 = 0.094k↑ therefore no tie-downs are required.

Multiple shear walls along a plane of resistance may be used. In such cases, the portion of the load carried by an individual wall proportional to the total load may be assumed equal to the width of that wall divided by the total width of all the walls in that plane.

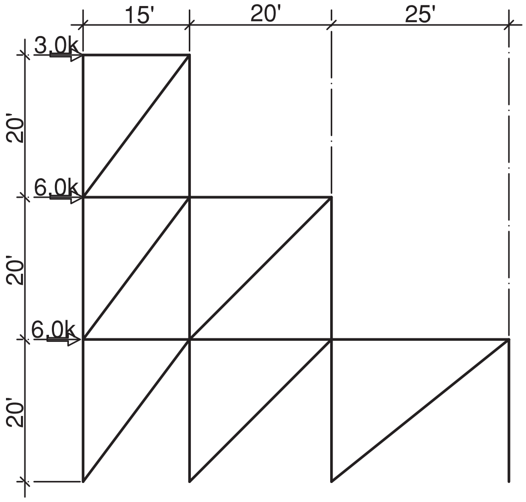
Practice Exercises:

14-1: For the braced frame shown in Figure 14.20, find the additional axial loads in the beams, columns and diagonals caused by the lateral loads. Use the Diagonal Truss Method.

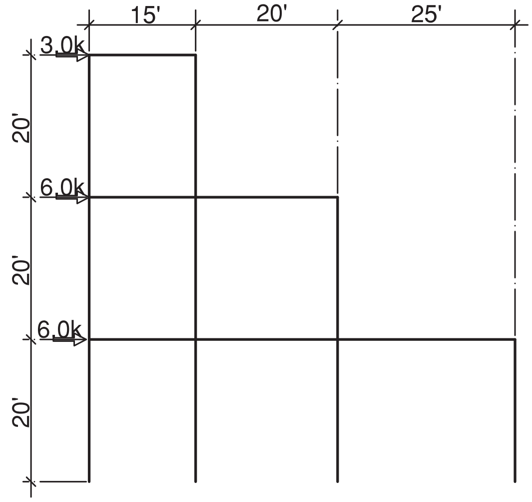
14-2: For the moment frame shown in Figure 14.20, find all additional shear, moment and axial forces in all components caused by the lateral loads. Use the Portal Method.

14-3: Determine the additional axial loads on the columns connected to the shear wall shown in Figure 14.20 if the density of the wall = 90pcf and the wall thickness is 12″.

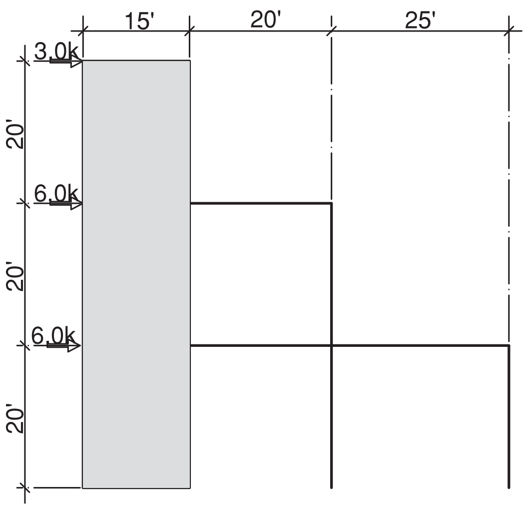
14-4: Determine the required thickness of the unconnected shear wall shown if the wall density is 120pcf.



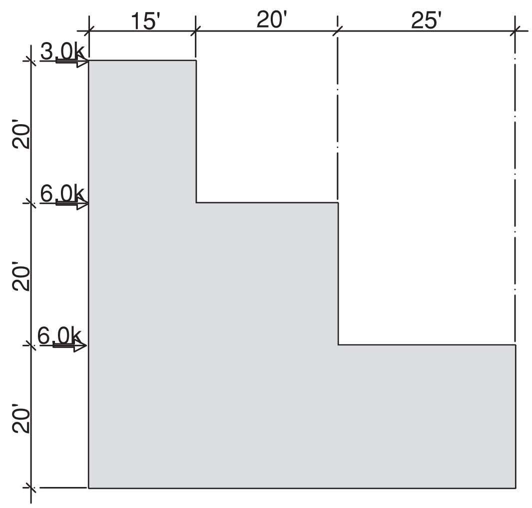
14-1



14-2



14-3



14-4

14.20

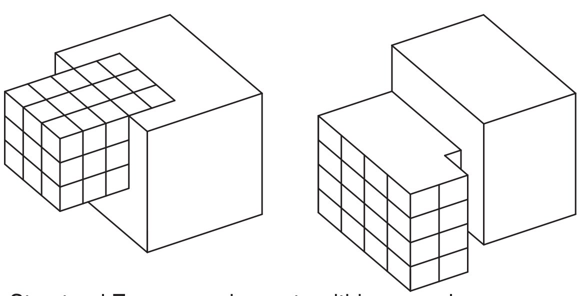
Chapter 14 Practice exercises

fifteen

Structural Typology

Most structures are unique in that they are composed of a set of components that are sized for specific requirements of spatial and contextual conditions. However, many buildings have very similar structural systems that can be grouped as a type. Typology is the study of types. In this chapter structural systems are grouped by type and the basic characteristics of each explained.

A building may have one structural system or it may have multiple different structural systems grouped as structural zones within the building. A building may also have multiple but similar type structural systems grouped or massed within the same building.



Structural Zones may be parts within one volume or may be separate volumes.

15.1

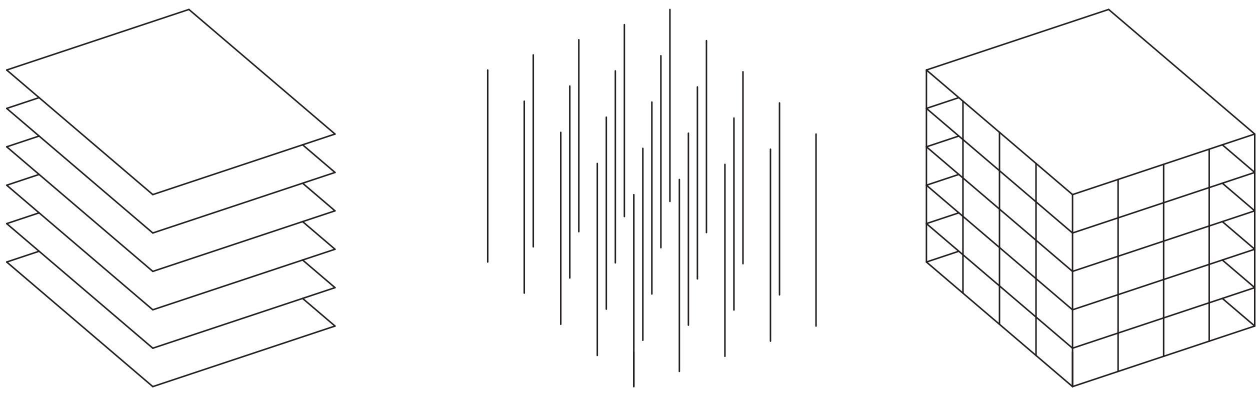
Structural zones

15.1 Beam and Column Systems

Beam and column systems are the most common of structural types. Often called bulk active systems because loads are transferred through the components by virtue of their material qualities, this type of system has distinct subsystems: the horizontal spanning system which is usually a set of floor and roof assemblies; and the vertical support system which is comprised of a pattern of columns, bearing walls or vertical truss or frame assemblies. The components and their connections may be subject to axial and/or shear forces as well as moment during the transfer of loads. These systems tend to be, but are not necessarily orthogonal in vertical section with a combination of vertical support systems and horizontal spanning systems.

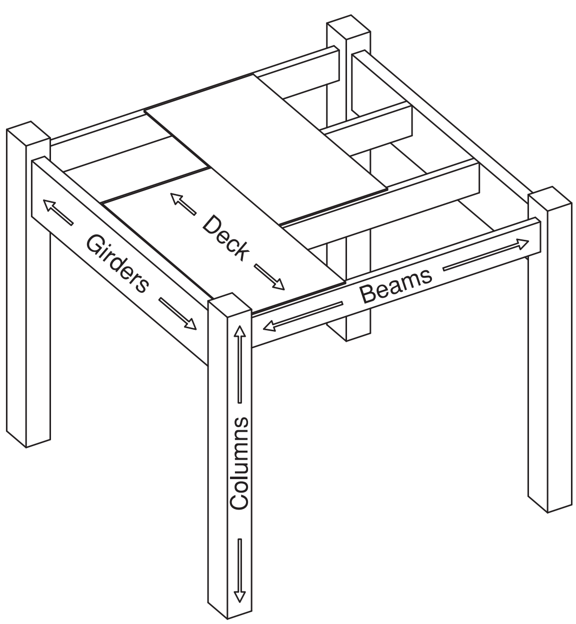
15.1.1 Horizontal Spanning Systems

Most horizontal spanning systems consist of a deck that supports the floor or roof load and spans between and is supported by beams or joists. The deck not only distributes the loads to the beams, but provides a continuous stiff medium that enables the horizontal spanning system to act as a horizontal diaphragm, meaning it acts as one rigid body. Decking material can be $\frac{3}{4}$″ plywood, dimensional lumber, metal deck, grating, concrete slabs or any other material capable of transferring the floor or roof loads to the beams or joists.



15.2
Subsystems of beam and column systems

The beams and/or joists transfer the loads from the deck to either carry beams or girders or directly to a vertical support system. Beam spacing is dependent on the allowable span of the deck. While some beams or joists may frame into the walls or columns of the vertical support system, many will frame into carrier beams or girders.



15.3
Hierarchy in horizontal spanning systems

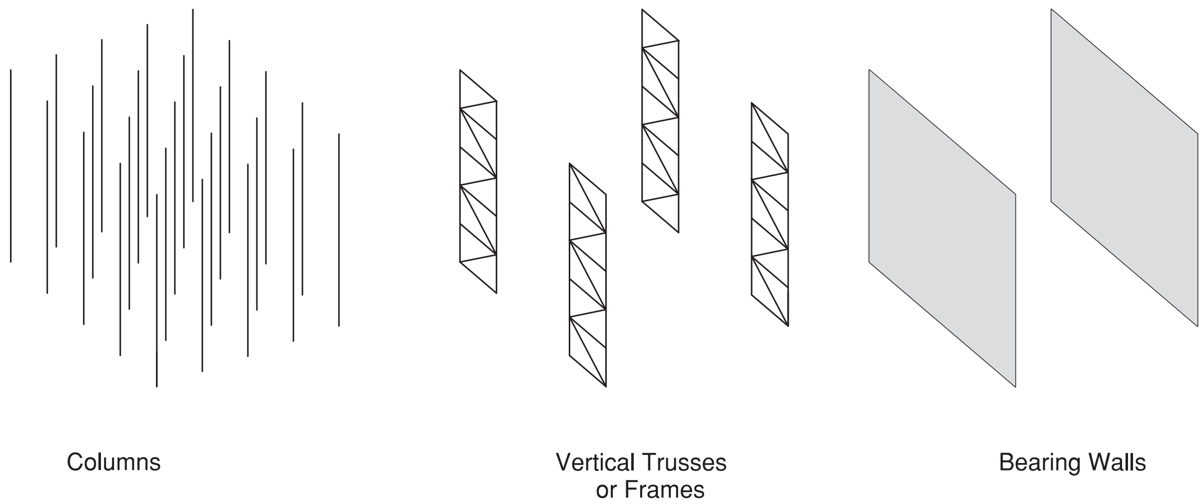
Most horizontal spanning systems employ an orthogonal grid pattern that allows for efficient use of materials and ease of connections. However, this is not required. Beam and column systems can easily follow a non-orthogonal pattern. As discussed in Chapter 11, grids can be radial, complex or organic in form. See Chapter 13 for specific examples of horizontal framing systems using wood, steel or concrete materials.

15.1.2 Vertical Support Systems

In a true beam and column system, the vertical support system consists of columns and/or bearing walls. The walls and columns are configured in a support pattern that follows a grid as described in Chapter 11.

The important thing to remember about beam and column systems is that both gravity and lateral loads are transferred through the connections. Transfer can be made through a bearing when a component rests, or bears, on a support. Where components do not bear on a support, they must have a connection capable of resisting shear.

All systems need lateral stability to prevent failure due to horizontal forces such as wind and seismic shear. Because beam and column systems transfer gravity loads through compression, shear and bearing, they do not necessarily incorporate lateral resistance assemblies as part of the load tracing path. Therefore, it is particularly important in beam



15.4

Vertical support systems

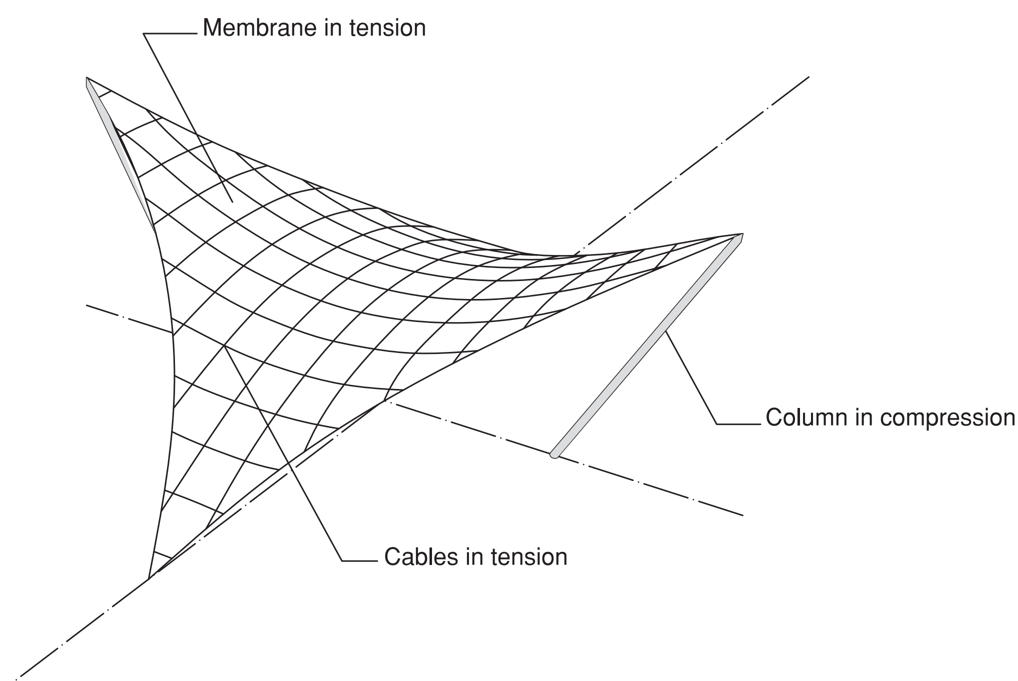
and column systems to remember to plan for lateral stability by including bracing, moment frames or shear walls along strategic column lines.

15.2 Form-Active Systems

Form-active systems are systems in which there is little or no bending stress either because the structure adapts its form as in cables, tents or membrane structures or pneumatic (inflatable) structures or because the form is efficient and has little or no bending stress as in arches, vaults and domes.

15.2.1 Tension Structures

Tension structures, often called membrane or cable structures, are form-active because the external forces and support reactions dictate the shape or form. Because the structural material responds to external forces, the shape becomes efficient for transfer of loads through tension.



15.5

Tension structure components

Tension structures have three basic components: a membrane, a cable system and a compression component. Cables and membrane fabrics do not transfer loads through compression. However, as mentioned above, tension structures do have a compression component. Because cables are hung, the loads are transferred from the cable to the earth via a compression element such as a column, wall, or foundation support.

When a cable hangs, it forms a catenary curve due to its own weight or when subjected to a uniform load. A catenary curve can be described by the formula y = (1/a)cosh(ax). Graphically, a catenary curve is the curve made by the movement of the focal point of a parabola when it is rolled along a linear surface. The difference between a catenary curve and a parabola is small and as a result, many designers choose to design tension structures as if the cables are parabolic in nature.

It is important to remember that cables subjected to concentrated loads will theoretically form line segments between the loads and between loads and supports. Because the cable has some weight and that weight is a uniform load, there will be a contribution to the sage from the weight of the cable. Because cables do not handle shear, the forces are axial along the cable.

A catenary arch with a = 0.2, having a uniform load, w, and span, L, will have the following reactions at any point along the arch at some distance x from the left support:

$$f_x = wL/5.52\leftarrow \text{ and } f_y = wL/2 - wx\downarrow$$

Drawing the reaction vectors at increments of L/20 shows how the resultant forces follow the curve as shown in Figure 15.7. Smaller increments yield even closer results.

Tension structures employ a cable system with a pattern that can be orthogonal, radial, triangulated or any other shape that will safely transfer the load. The cable system pattern should be chosen based on the compression support system pattern. Several cable and support patterns are illustrated in Figure 15.8.

The membrane of a tension structure is the equivalent of a roof deck. It must transfer the roof loads to the cable system. The cable system is the equivalent of a beam or joist system. Three materials that are typically used for membrane include:

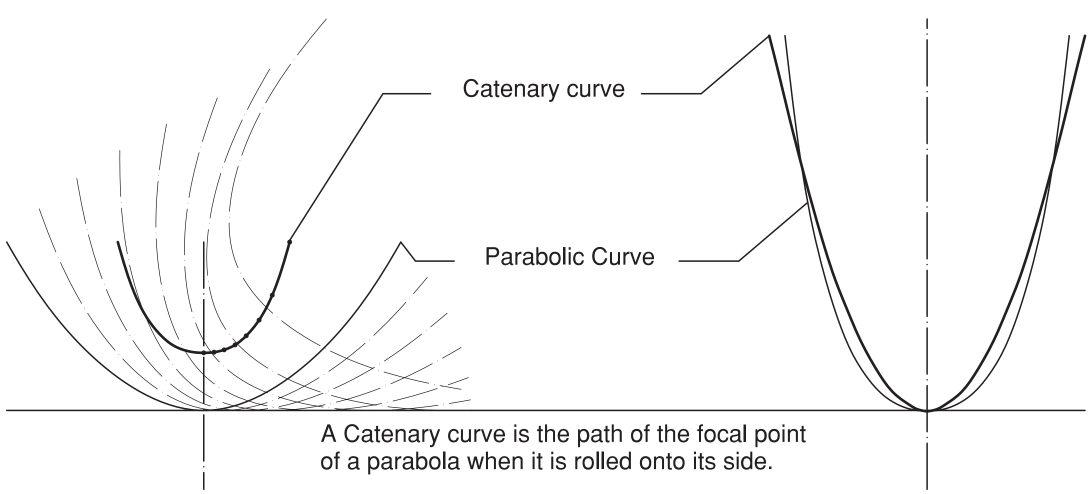
Polytetrafluorethylene (PTFE)—coated glass fiber

Polyvinylchloride (PVC)—coated polyester fabric

Ethylenetetrafluorethylene (ETFE) foil

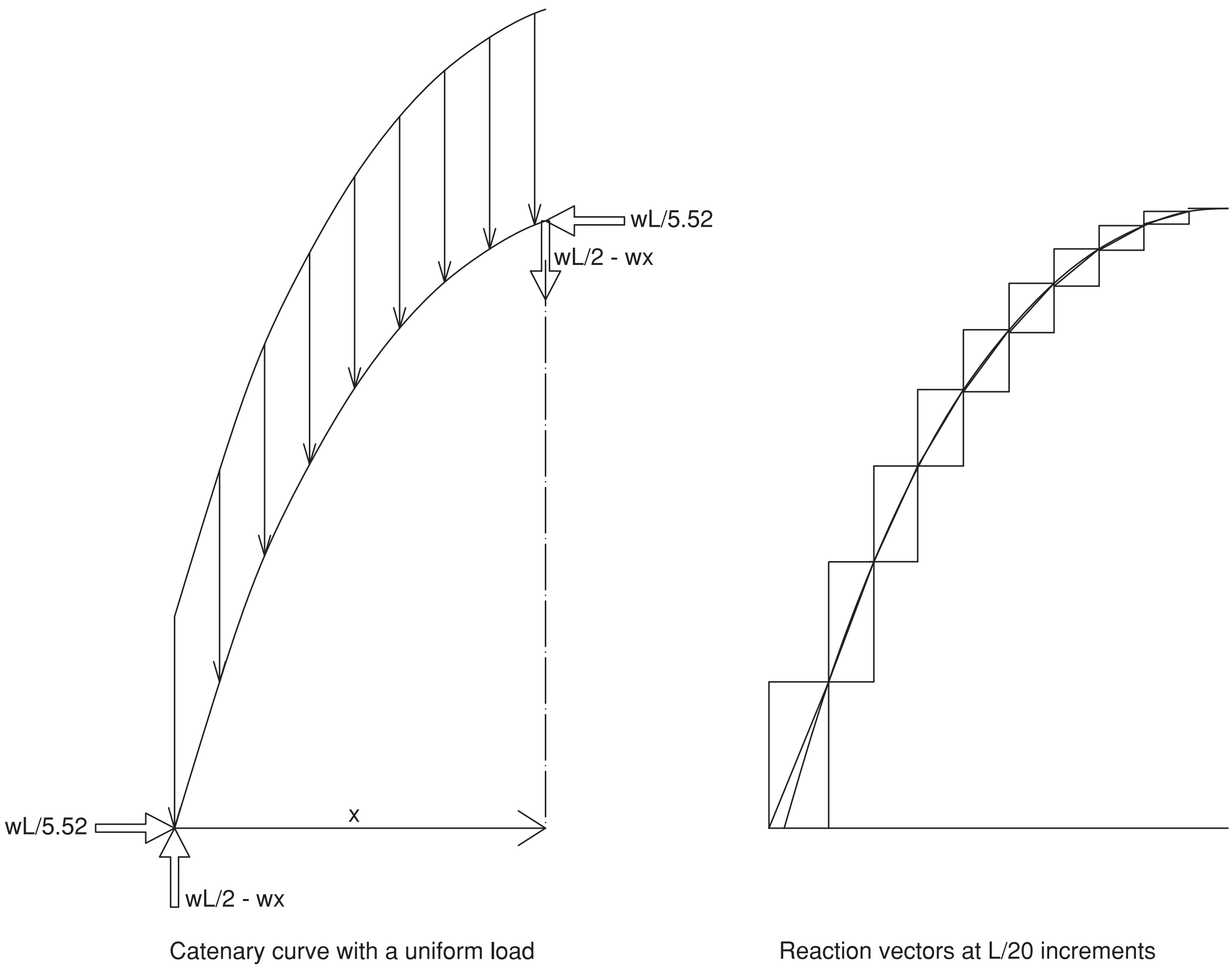
Consider differences in cost, UV protection, life span, fire rating, sound characteristics and thermal qualities in choosing a membrane material. Check with individual manufacturers for allowable spans and loads.

Tension structures are economical, easy to assemble and disassemble, capable of spanning long distances, and most are durable. Also, most allow light transmission while also reflecting heat, making the system energy efficient.



15.6

Catenary curve and comparison to parabola



15.7

Forces in catenary curves

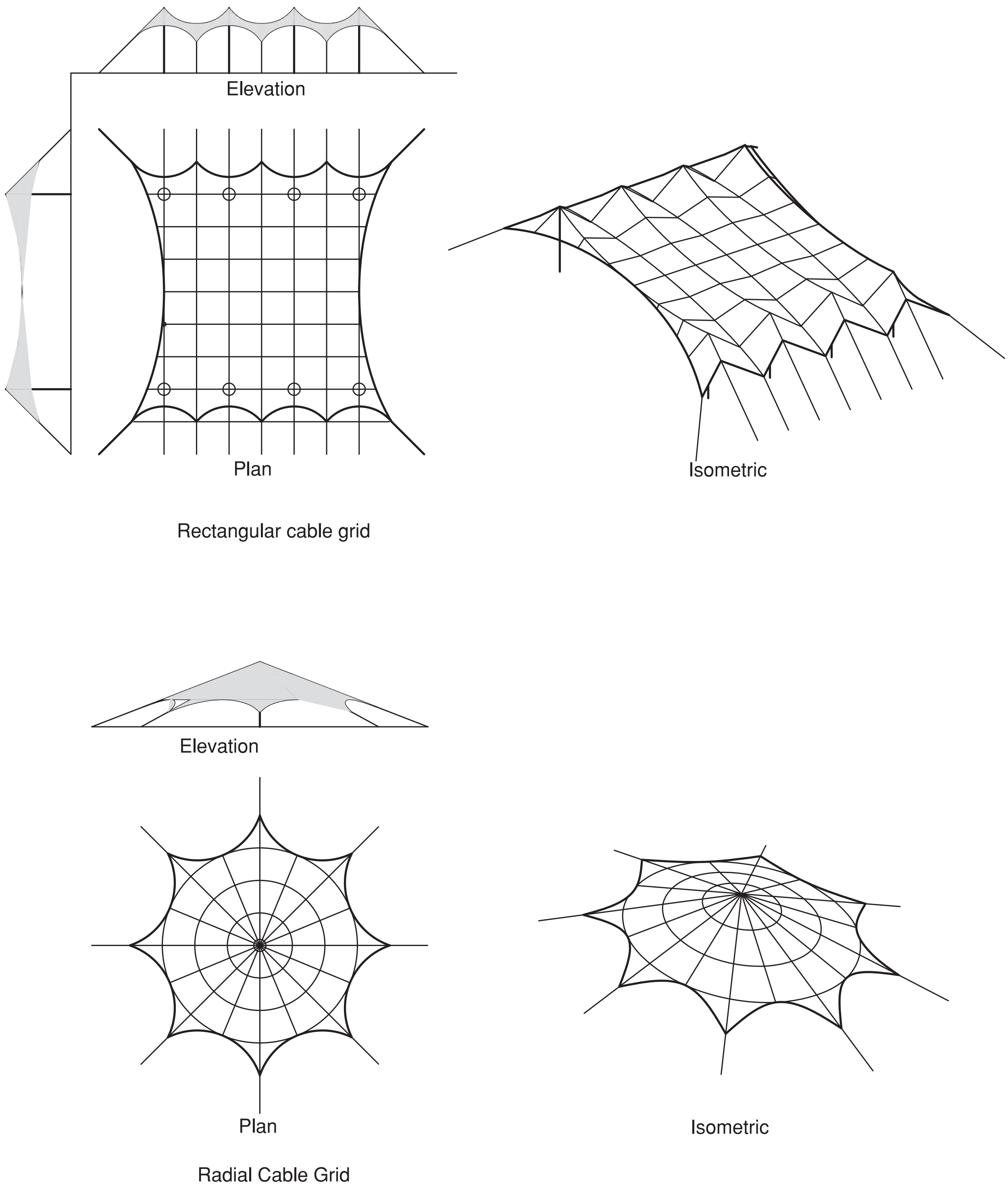
Some interesting examples of tension structures include the O_2 Arena by Populous in Greenwich, England; The Hajj Terminal by SOM in Jeddah, Saudi Arabia; and the Munich Olympic Stadium by Frei Otto.

To design a tension structure:

1. Determine the perimeter of the covered area.
2. Determine where the compression members can be placed (pattern of support).
3. Draw a cable network that will support the membrane and stabilize the compression members.
4. Add connector cables as needed to support the membrane.

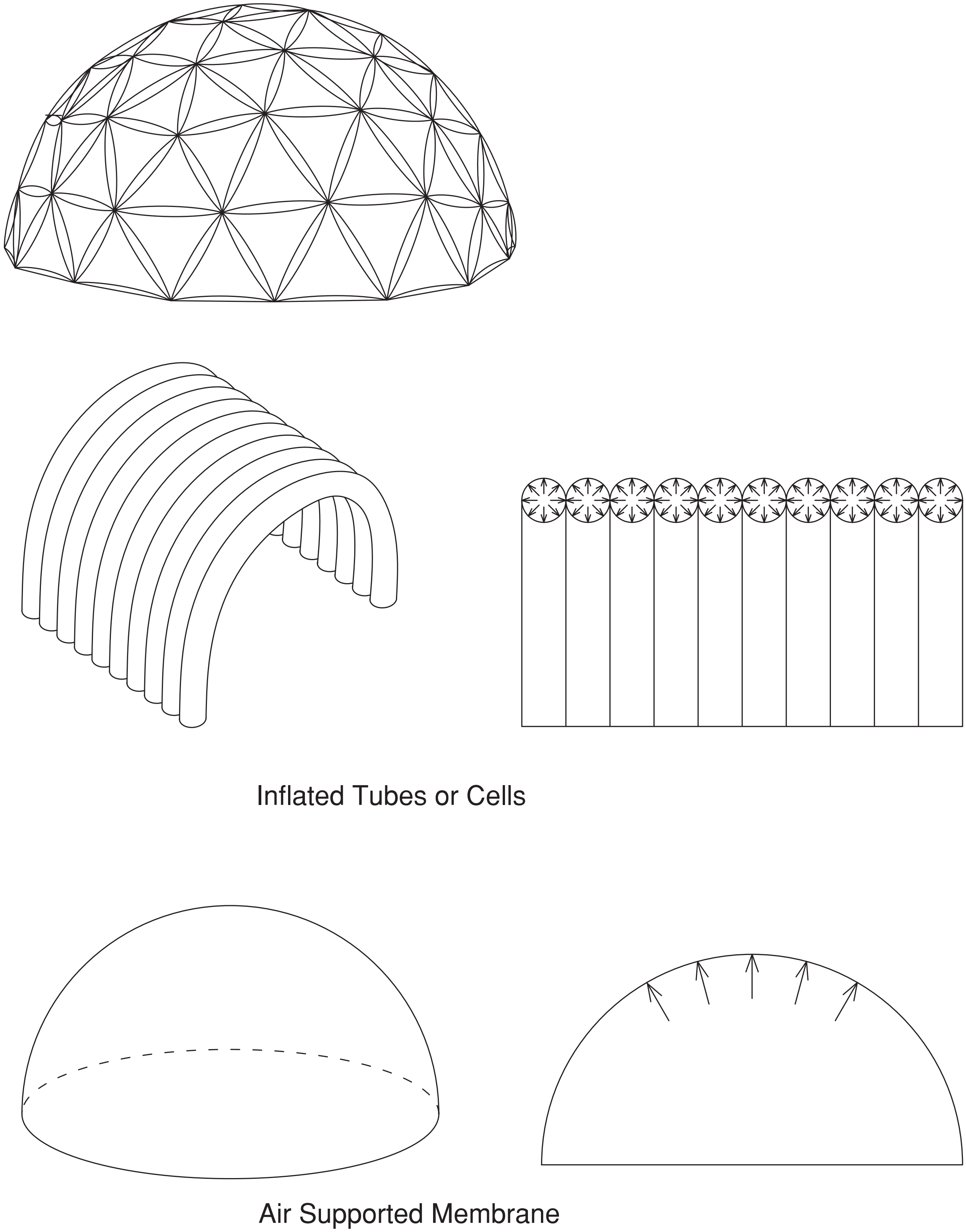
15.2.2 Pneumatic Structures

Pneumatic or inflatable structures have a form that responds to both external forces and internal pressure where load transfer is dependent on tension in the membrane. As such, they are form-active. There are two basic types of pneumatic structures: air-supported and cellular. The pattern of the membrane is the key to its ability to support a load. Cellular or compartmentalized pneumatic structures have the ability to change shape by regulating the pressure in individual cells. Further, cellular pneumatic structures can be sealed to reduce the amount of energy used in keeping the membrane inflated, making them more energy efficient than air-supported systems.



15.8

Cable and support patterns



15.9

Pneumatic structures

Pneumatic structures have some clear advantages such as ease of assembly and disassembly, cost and lightness. But they have a major disadvantage of requiring energy to inflate and in the case of air-supported systems, require constant energy to maintain inflation. Another disadvantage is that unlike tension structures that will remain structurally sound with a puncture in the membrane, a pneumatic cell will deflate once punctured and may cause failure.

15.2.3 Arches, Vaults and Domes

Arches are considered form-active although they do not change shape with load changes. This is because the load transfer, which is dependent on the shape of the arch, has little bending stress. A catenary arch, as discussed in section 15.1, has no bending stress and is the most efficient form for uniform loads applied vertically.

Arches are probably the category with the largest range of attributes. Arches may be formed by bending a single member along its axis or by compiling a series of wedge-shaped compression members along the arch axis line. Many arches are built in segments and assembled with pinned connections on the site. The span can range from a doorway to the width of a stadium. The materials can include anything capable of handling compression. Arches can be used alone to support a load above an opening or can be extruded to form vaults or rotated to form domes.

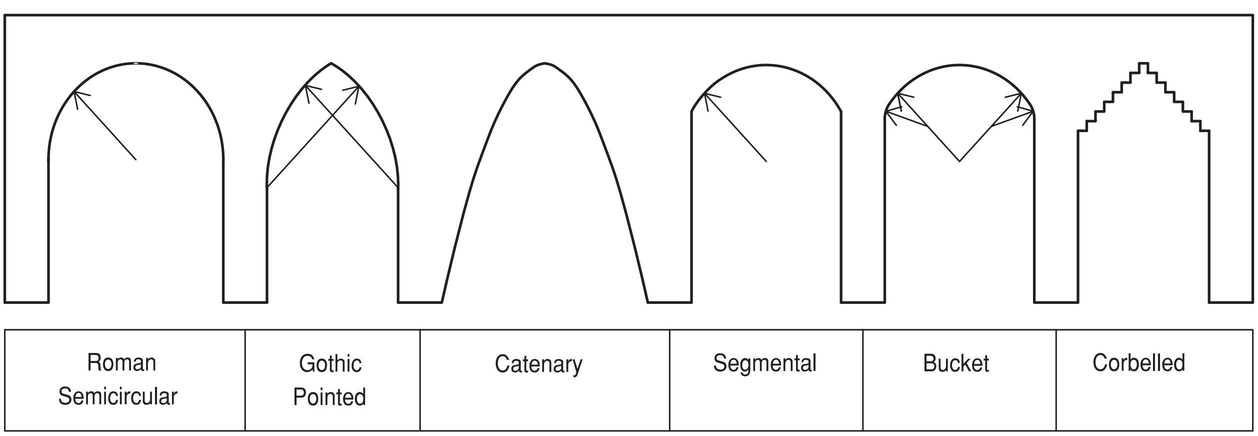
A barrel vault is the linear extrusion of an arch and as such is form-active. And while most vaults maintain the same cross-sectional shape along the length of the vault, this is not a requirement. When two barrel vaults intersect, a groin vault is formed and the loads are transferred along the intersection line called the groin.

A rib vault is a vault created by a series of ribs that have an arch shape. The spaces between the ribs have a deck that transfers loads to the ribs. The ribs are not necessarily parallel or straight members. Curved ribs and intricate patterns are possible.

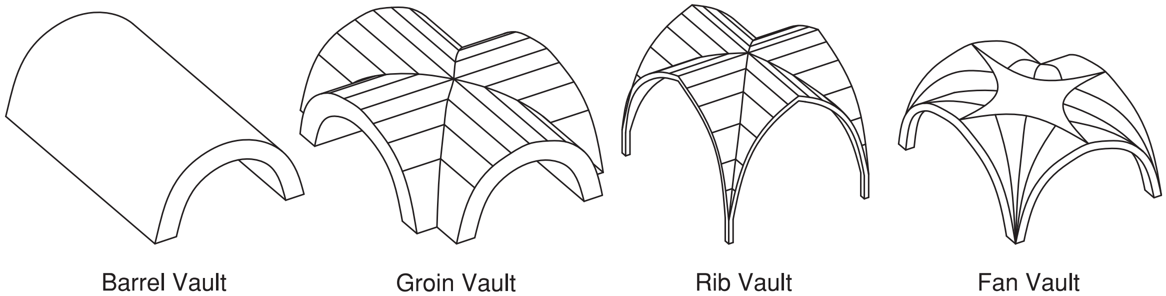
A fan vault uses ribs that fan out from the column supports toward a horizontal plate at the top. The horizontal plate, called a lozenge, is useful for supporting vertical loads above the vault.

Vaults can be tessellated to create a horizontal spanning system. One advantage to tessellating vaults is that the thrust, or outward horizontal force, on the columns is counteracted by the adjacent vault. Another advantage is that with smaller spans, the thickness of surface or ribs is reduced. But it must be remembered that each vault will need vertical support and that tessellations with very small spans may produce awkward, column-filled spaces.

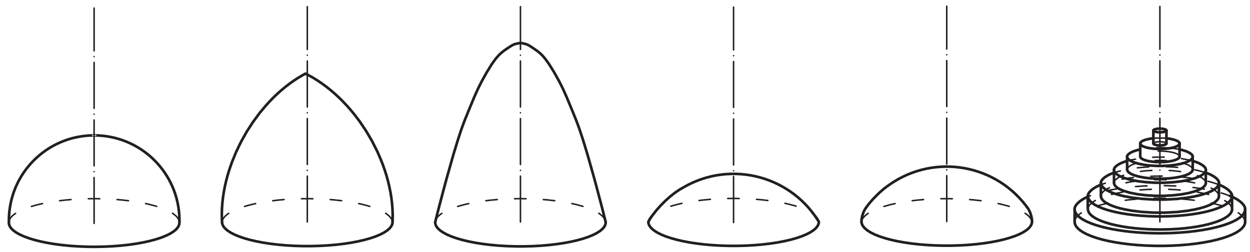
A dome shape is formed by the rotation of an arch shape, but not all domes behave as a series of rotated arches. Radial rib domes are form-active, with each rib transferring loads through compression with little or no flexural stress. Thin shell domes are surface-active (see thin shells, section 15.3) and geodesic domes are vector-active space trusses (see space trusses, section 15.5). Domes are inherently strong structures because of the double-curvature of the shape.



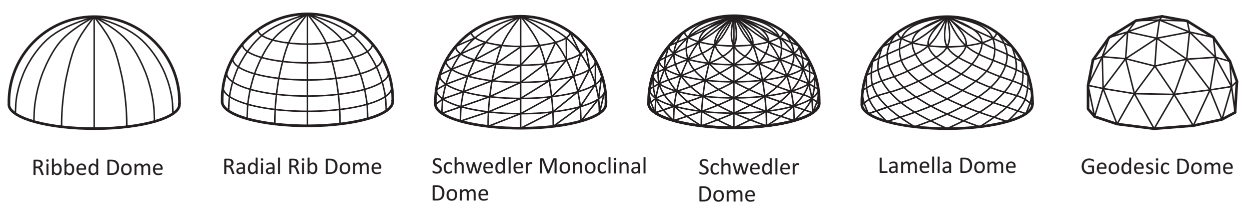
15.10
Arch types



15.11
Vaults



Domes created by rotation of arches.



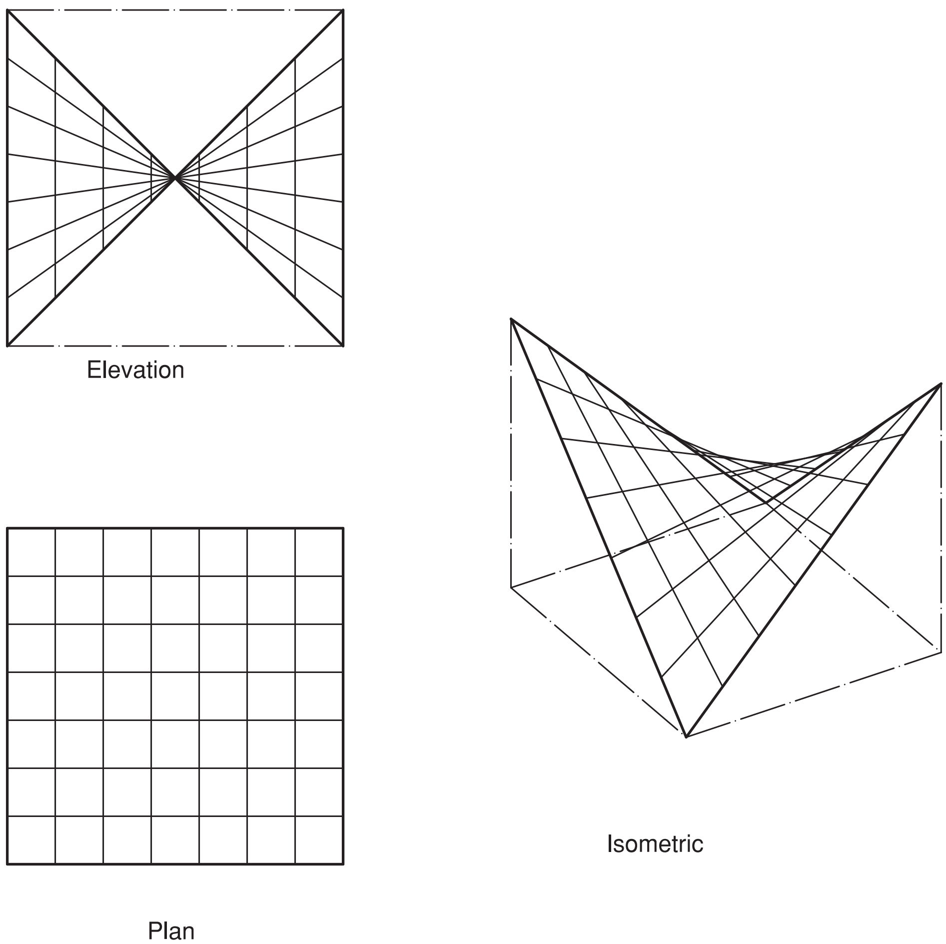
15.12
Domes

15.3 Thin Shells

Thin shells are surface-active, although they seem to be form-active because their shapes are generally efficient like form-active shapes. They have a fixed shape and transfer the load through the surface shape, and not through a particular cross-section, and therefore are surface-active. They differ from arches, in that the load transfer is in two-directions although it can be argued that any cross-section produces some sort of arch. Transfer follows the surface shape, not a particular cross-section. There are many wonderful examples of thin shell structures such as the TWA Terminal at JFK by Eero Saarinen, or the Deitingen Service Station by Heinz Isler, or Loas Manatiales by Feliz Candela, to mention just a few.

15.3.1 Hy-pars

A hy-par is a thin shell that forms the shape of a hyperbolic paraboloid. This means that when a section is taken in one direction, a hyperbola is seen and in a perpendicular section, a parabola is seen. Felix Candela is credited with development of the hy-par and used it successfully in numerous projects. Because the cross-section is a parabola, which is very close to a catenary curve, the shell has very little bending stress. This efficiency accommodates a very thin shell thickness. Because loads are transferred through compression, concrete is a suitable material for the hy-pars.



15.13
Hyperbolic paraboloid

15.4 Folding Plates

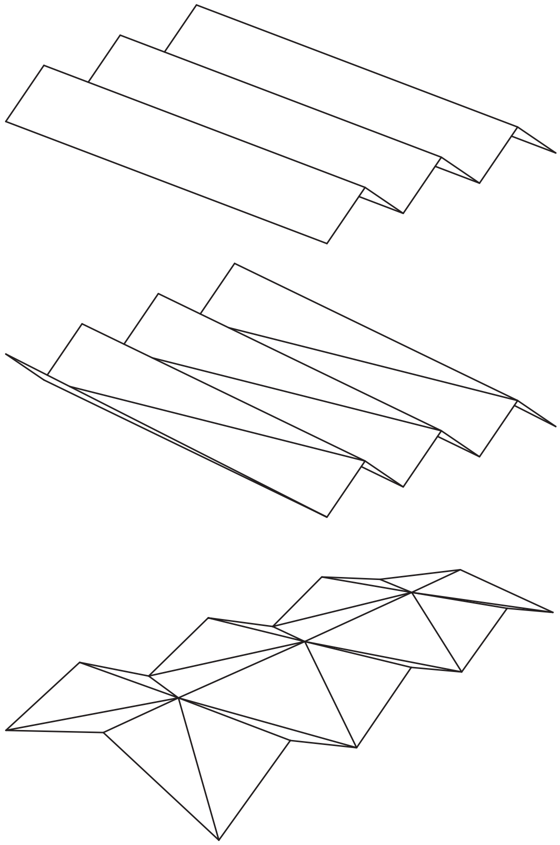
Folding plates transfer loads from one plate to another along the intersection line between the two plates. Individual plates must be capable of transferring loads to the edges through shear and moment and typically can be designed as a horizontal spanning system. It is the connection between plates that is important. Adjacent plates, like adjacent vaults, can counteract each other's horizontal forces and transfer vertical loads along the intersection line.

Plates may be assembled in any workable pattern and tessellation of plate groups is often used. Figure 15.14 describes a few typical plate patterns. Folded plate systems may be used on any scale with plates varying from floor or roof diaphragms to pieces of a façade. By varying the shape of tessellations, either through the use of algorithms or randomly, the tessellated folded plate structure can take on double curvature shapes.

15.5 Trusses and Space Frames

Trusses and space trusses are vector-active systems. A vector-active system, in theory, is one in which loads are transferred through compression or tension and the members are not subject to shear and moment. In reality, many trusses are not true trusses. As mentioned in Chapter 2, bars in a true truss are connected by pinned joints at each end and the forces on the true truss are only applied at the joints. In most construction, however, the top and bottom chords of a truss are usually single members that span multiple bar lengths. Further, most trusses carry a uniform load applied to one or more of the chords.

Trusses may be used as beams or joists and placed in a parallel configuration. But, just as open-web joists need lateral bracing to prevent sway, trusses should be braced laterally as well.

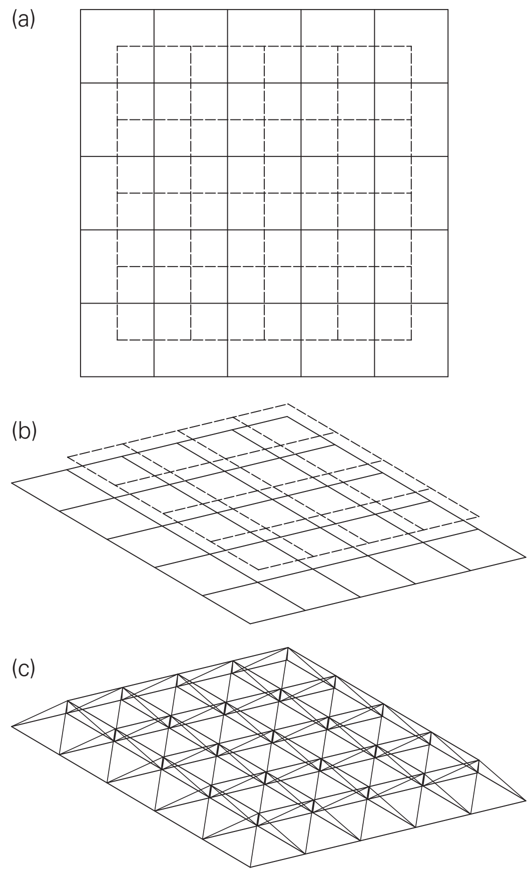


15.14

Folded plate structures

Below is a simple method to create a space truss or space frame.

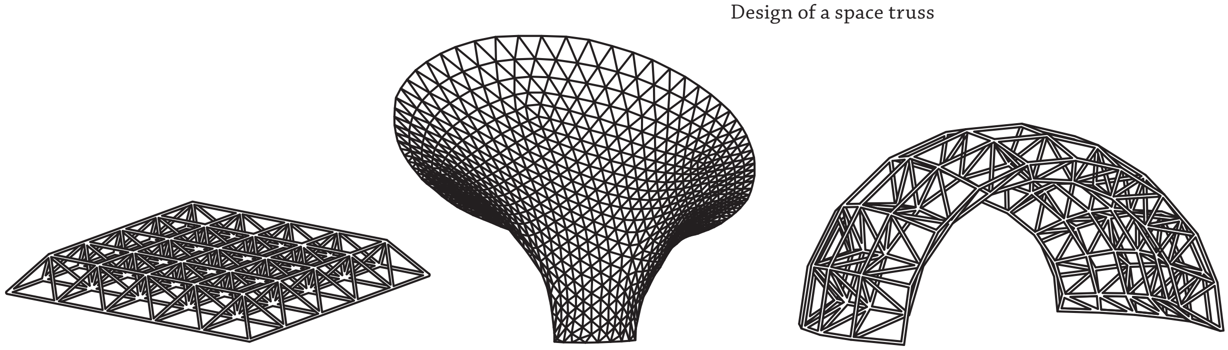
1. Create a pattern—Figure 15.16(a).
2. Locate the center points or the spaces; they become the new vertex points. Connect the vertices on the offset layer to form a new pattern. (green)—Figure 15.16 (b).
3. Offset the layer.
4. Connect the vertices of both layers. (black)—Figure 15.16 (c).



15.16

Design of a space truss

To make truss systems more efficient, the trusses span in two directions, creating a grid of trusses. This is a space truss. Typical space truss configurations are shown in Figure 15.15.



15.15

Space trusses

Space trusses can be stacked or combined in a folded system. Space trusses do not need to be orthogonal in either plan or section as in Figure 15.17 below.

15.6 Moment Frames

Moment frames are systems in which lateral forces are resisted by virtue of the rigidity of the connections. See Chapter 14 for an explanation of the shear, moment and axial forces in moment frames. It is the moment created in the fixed or rigid connections that gives the moment frame its name. The moment frame is often called a rigid frame, but a rigid frame is actually more flexible than a braced frame.

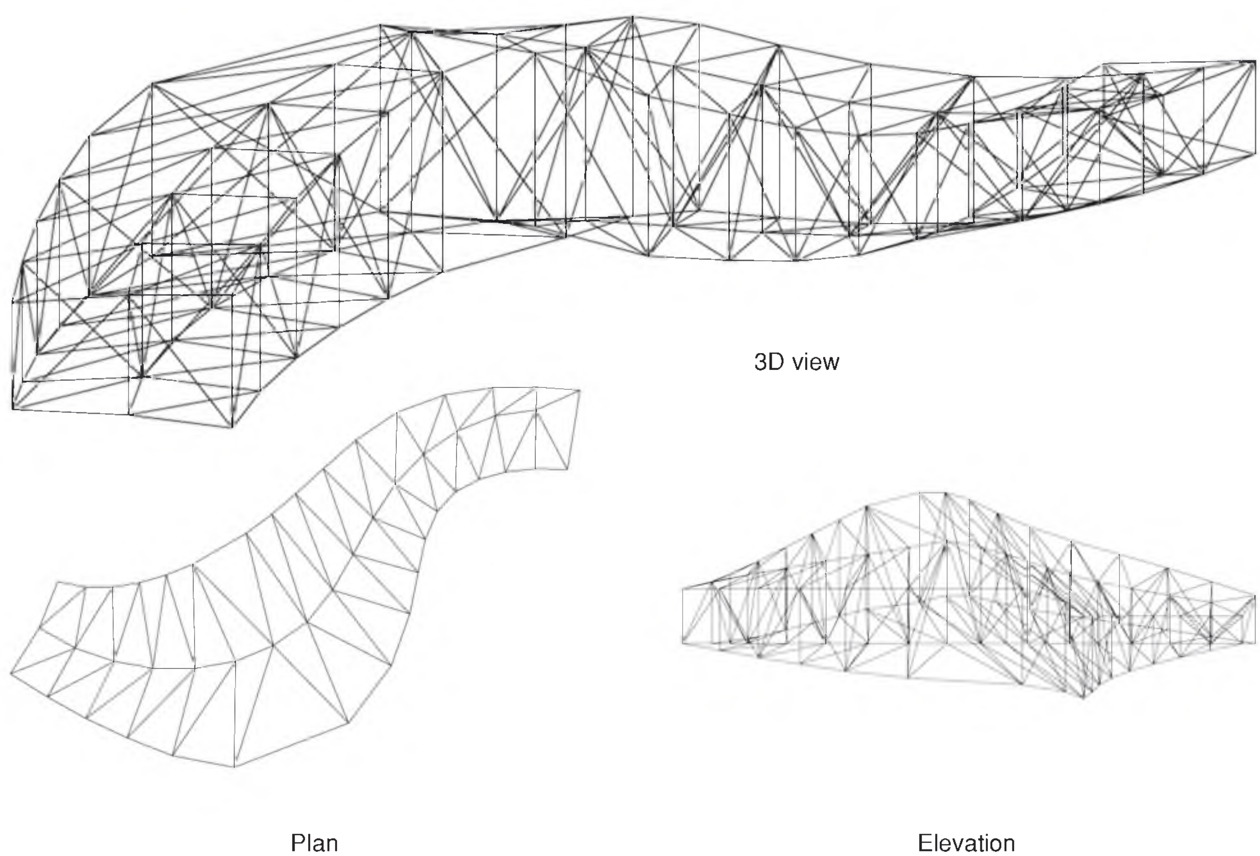
The primary advantage to a moment frame is that no diagonal bracing or shear walls are required. Moment frames allow for full unblocked views, making them ideal for use with curtain wall façades.

15.6.1 Steel Moment Frames

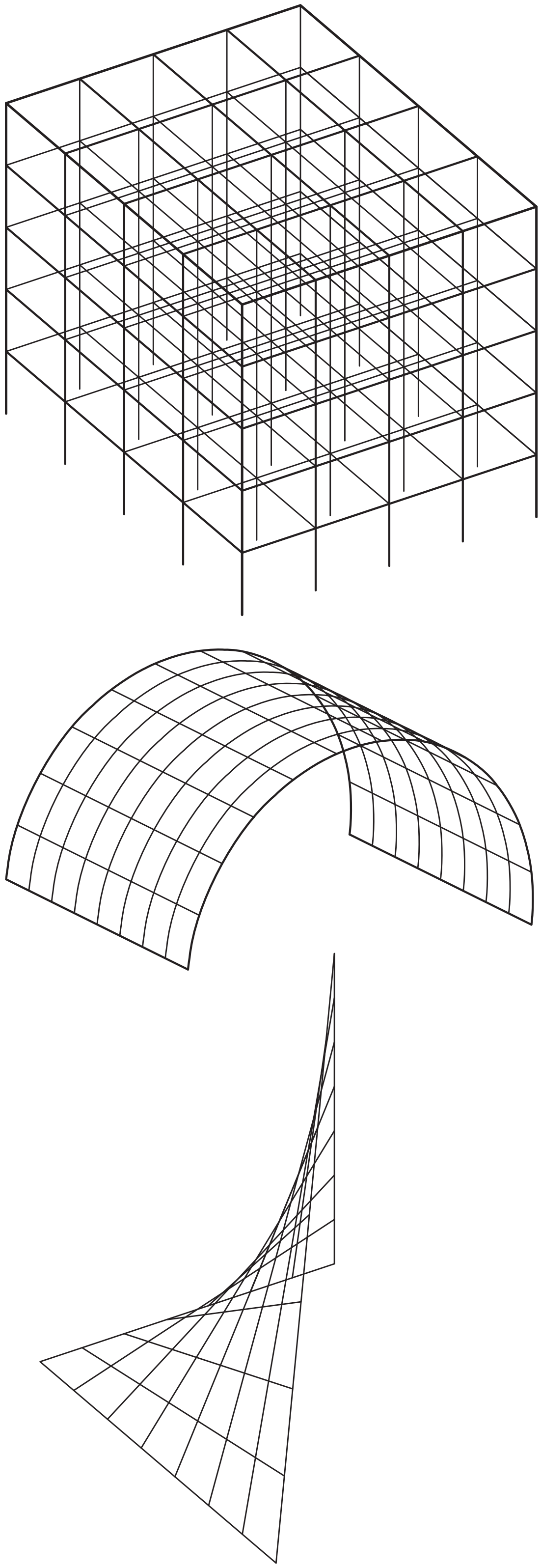
Early moment frame connections used riveted connections. Welding of connections was introduced in the 1950s but popularity faded in the 1980s due to economic concerns about the cost of welding and inspecting welds. Further, many welded fixed connections have failed during earthquakes due to brittle fracture around the weld. After 2001, federal building requirements for blast resistance prompted the design of new moment connection systems.

15.6.2 Non-steel Moment Frames

Moment frames may be designed in concrete or other materials. The logic of load transfer is the same. Small moment frames may be designed using glue-laminated timber or laminated bamboo.



15.17
Design of a non-orthogonal space truss



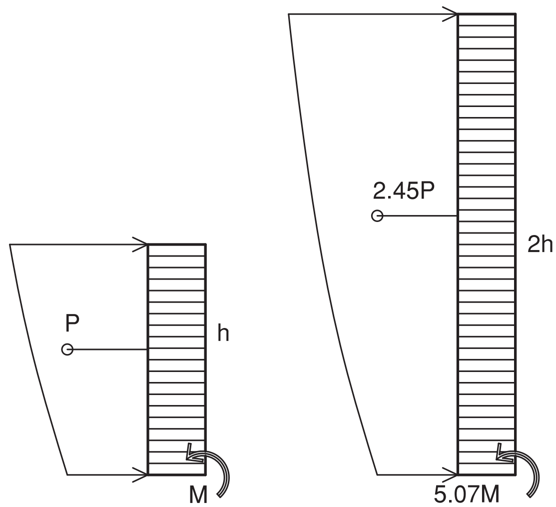
15.18

Moment frames

15.7 High-rise Typology

A high-rise is defined as a building over ten stories in height or at least 100ft in height. A tall building is defined by the fact that its tallness is integral to the design. The height is atypical compared to the vernacular or for the time period in which it is built.

Lateral forces on high-rises are an important concern. Seismic forces increase exponentially with height. Wind forces are constantly present and, as shown in Figure 15.19, a doubling of the height of a building can increase wind loads significantly, causing significant moment and deflection.



15.19

Wind on a high-rise

Lateral loads create another concern for high-rise structures. The P-delta effect is the additional moment created by lateral displacement in high-rise buildings. This may be as high as 10% of moment caused by lateral forces before displacement.

The bending of the structure as a whole unit creates a moment at the base or foundation. To resist overturning, the moment at the base must be resisted.

Lateral loads that are not resisted symmetrically will be subjected to torsion or twisting of the structure.

Temperature differential is another concern. The south side of a structure expands more on a sunny day, compared to the shaded north side, causing flexure in the structure.

$\Delta = \alpha L \Delta T$ and for steel, the coefficient of thermal expansion, $\alpha = 6.5 \times 10^{-6}$. This means that for 100′ tall building with 40°F temperature differential between the sunny and shaded sides, the change in length is Δ = .0000065(100′)(40) = 0.026′ = 0.312″

For a 2000′ tall building with 80°F temperature differential between sunny and shaded sides:
Δ = .0000065(2000′)(80) = 1.04′ = 12.48″

Although these numbers seem small, the effect of thermal expansion is significant enough to warrant expansion joints in tall or long structures.

With a large number of levels, even a few inches of depth in a typical floor design can have a major impact on the structure. A 12″ additional depth per floor in a 100-floor building yields an additional 100ft of height. This means greater wind loads; therefore greater structural member sizes, taller elevators, more HVAC ducts and therefore greater cost per square foot occupied space. Alternately, where there are limitations to building height, a reduction in floor depth allows for more levels and therefore more leasable space. A 1200′ tall building with floor-to-floor height of 12′ yields 100 floors. A one foot reduction in floor depth would yield 1200/11′ = 109 floors allowing nine additional levels of space.

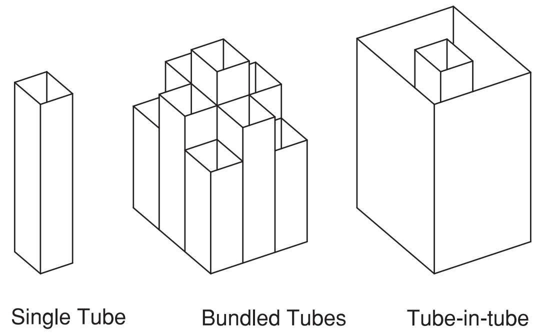
Gravity loads are another concern. Every column must support the accumulated loads on its tributary area from all of the levels above it. An increase in gravity loads per floor therefore has a greater impact on column design in taller buildings. For a given height, gravity loads can be reduced by choosing lighter, more efficient structural materials.

15.7.1 Tubes and Bundled Tubes

A tube structure is a design in which the gravity and lateral forces are primarily handled by the perimeter structure. Flexural stress in the overall structure is reduced because the moment of inertia for the system increases when the members are distanced from the neutral axis.

When bundled, each tube has its own structural integrity. Tied together, they form a unified network capable of resisting large lateral forces. Because loads and moments from each level are accumulated as they transfer to the ground, the number of tubes is often larger at the base than at the top.

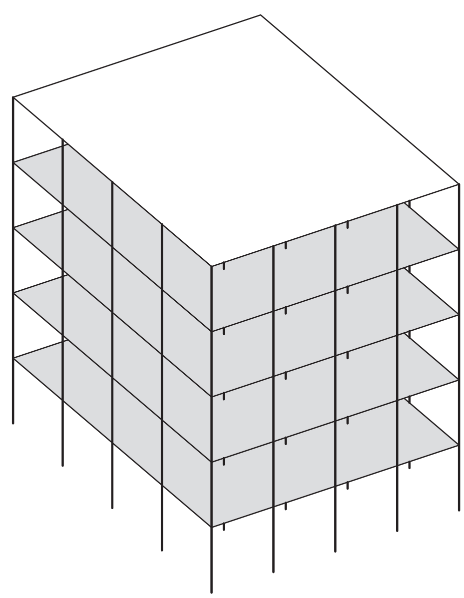
Tube-in-tube structures are exactly as stated. There are the inner and outer tubes that work together to create a structure with a thick "wall" tube.



15.20
Tube structures

15.7.2 Rigid Frame

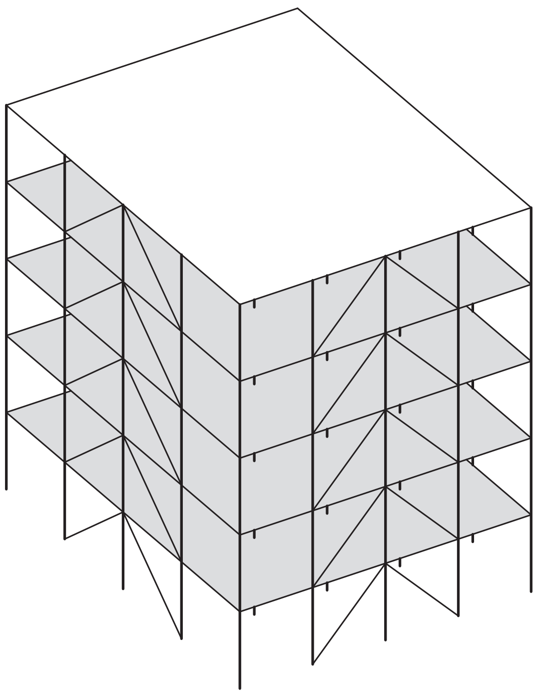
Moment frames can be designed to about 30 stories in height. Above that, member sizes become too large to make the design practical. The primary advantage to a moment frame is the unobstructed view it allows when used with a curtain wall system. Although moment frames are often called rigid frames, they are actually more flexible than braced frames. They are called rigid frames because the system transfers moment from the beams to the columns via rigid connections.



15.21
Rigid frame

15.9.3 Braced Frames

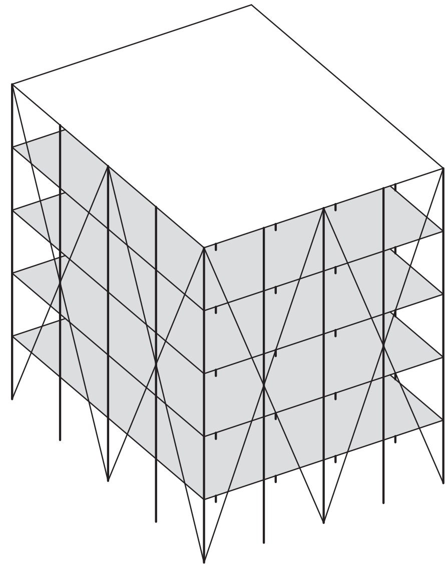
Braced frames work well to a height of about 80 stories. Beyond that, the forces accumulated in the diagonals make the member sizes bulky. Diagonals help reduce the moment in individual members and reduce drift (lateral deflection). Diagonals in braced frames span only one floor level. The disadvantages of braced frames include the possible obstruction of view at window locations and the expense of fabrication and installation of the diagonal connections.



15.22
Braced frames

15.9.4 Trussed Tube

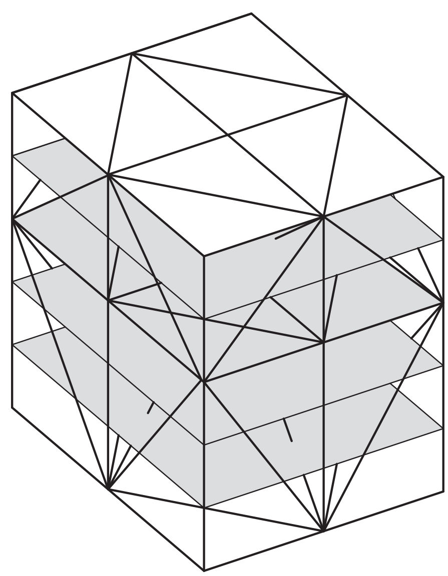
In a trussed tube, sometimes called a braced tube, the tube is braced with diagonals that span multiple stories creating a giant truss system. There may be columns in the core, but they support gravity loads only and not lateral loads. The John Hancock Building in Chicago by Fazul Kahn is a good example of a trussed tube.



15.23
Trussed tube

15.7.5 Space Truss

Space trusses in high-rise systems follow the same logic as discussed in section 15.5 but at a very large scale. Space truss components can vary from single floor height to multiple floor height. The floor loads are transferred to the truss components, which in turn transfer loads vector-actively to the ground.



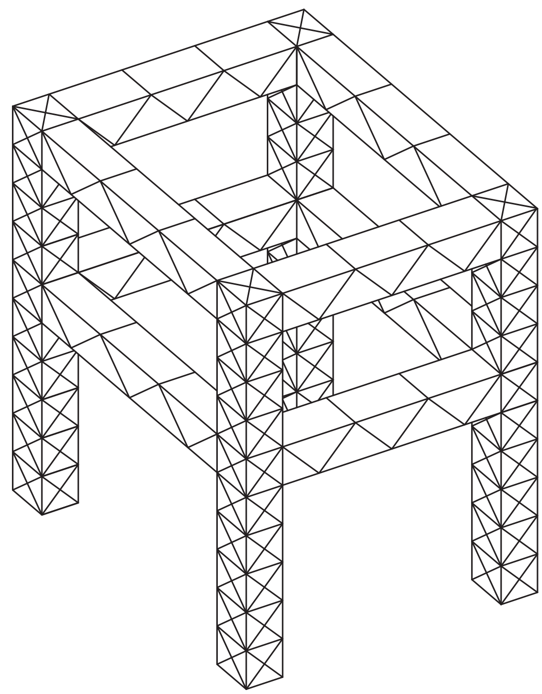
15.24
Space truss

15.7.6 Diagrids

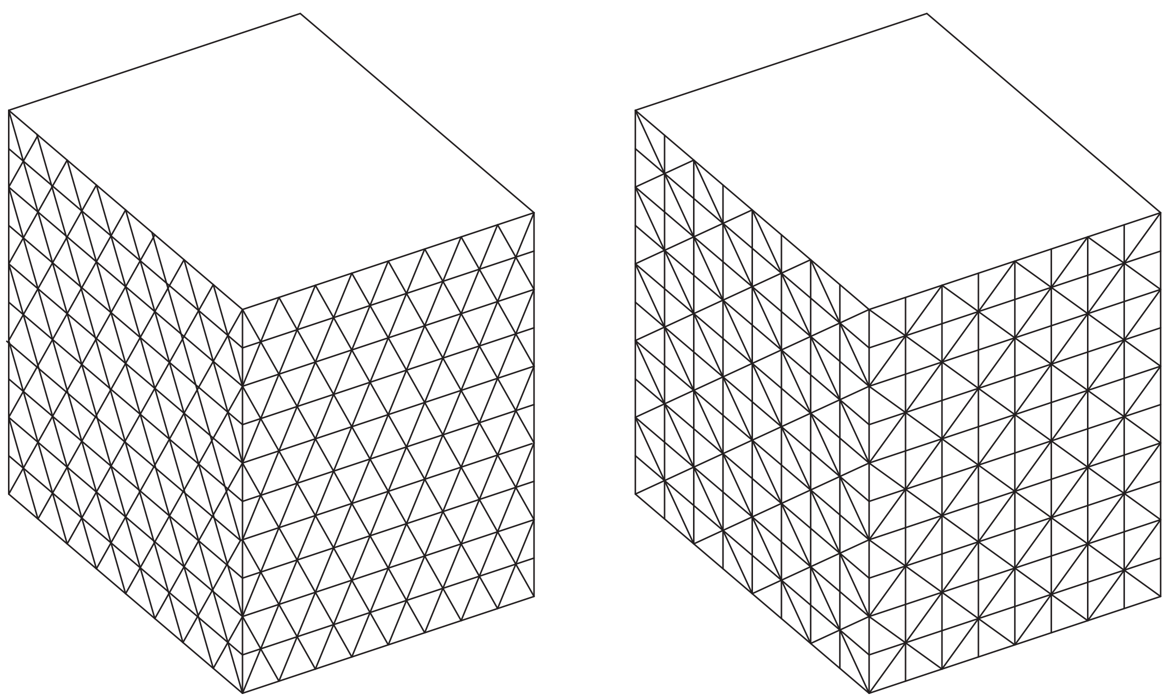
Diagrid systems are truss-like in nature, transferring loads through vector action. Like space frames or space trusses, diagrids are redundant—meaning there are multiple pathways the load can travel. If one pathway becomes fully stressed, another can handle the load transfer. The main difference between a diagrid system and a space frame is that the space frame acts in three dimensions while the diagrid vector action takes place along a surface.

15.7.7 Megaframe

A megaframe is a frame in which components are a subsystem, usually a truss. Trusses or moment frames act as the frame components of the megaframe. This type is a useful method to reduce materials used by allowing lighter weight vector-active systems to replace heavy components.



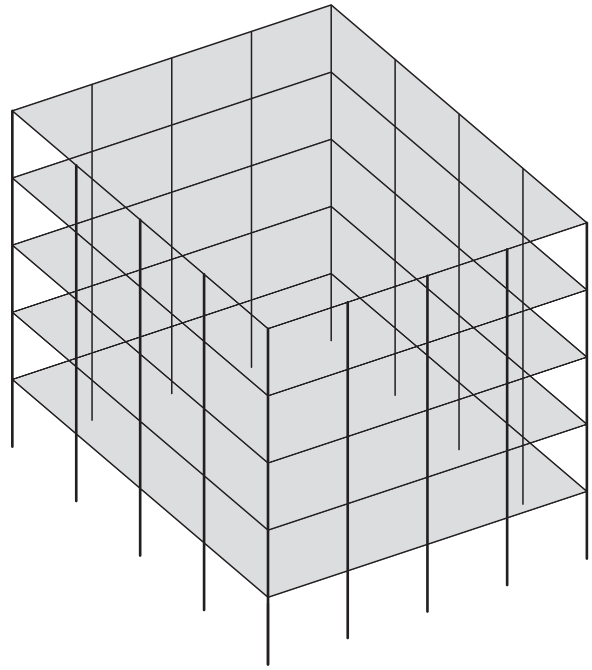
15.26
Megaframe

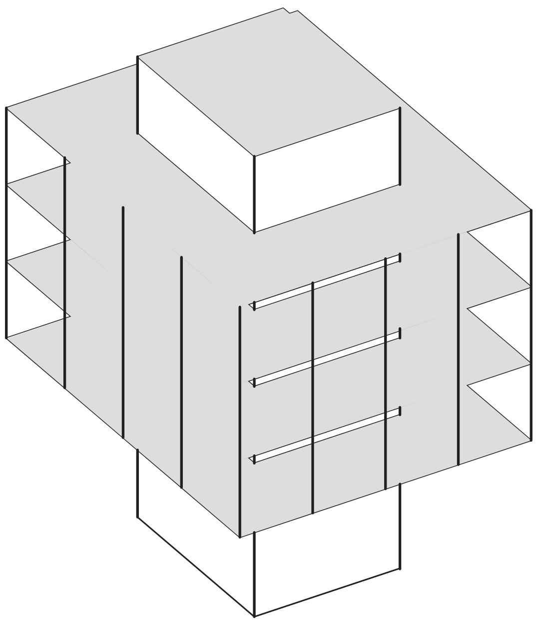


15.25
Diagrid

15.7.8 Core Suspended

Core suspended structures are those in which the structure for the enclosed building space is suspended from a few large compression members. The challenge to this type is that, unlike beam and column design, the entire suspended structure must act as a rigid body resting on support connections. Another challenge is that there are very large shear forces in the connections between the suspended structures and the compression elements.

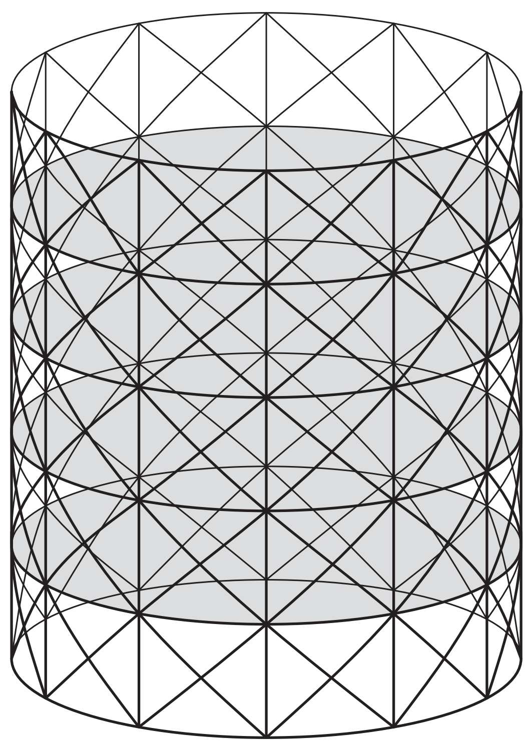




15.27
Core suspended structures

15.8 Exoskeletons

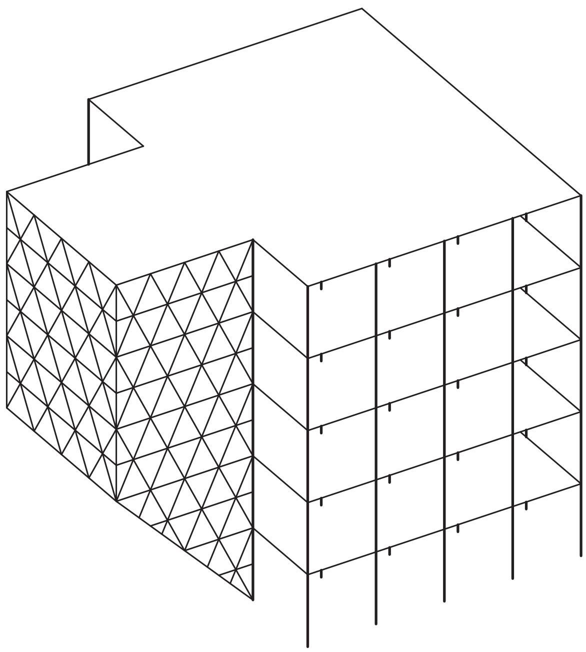
Exoskeletons are systems in which all the loads are transferred along the perimeter of the structure. The skeleton may use any system type. The definition relies on the fact that there are no interior vertical support systems other than core requirements for stairwells, elevators shafts and the like.



15.28
Exoskeletons

15.9 Hybrid Structures

Hybrid structures employ more than one system type in the design. The structure could be as simple as a barrel vault on top of a moment frame, or it could be a complex grouping of structural zones.



15.29
Hybrid structures

Practice Exercises:

15-1: Creatively build a tension structure model with a clear height of 2″ over the cover zone (4″ × 10″). Do not extend beyond the site limits (12″ × 18″), or exceed 6″ in height. Compression members and cables may be glued to a base. Compression members must not span the covered area. Draw the concept idea, and the cable and support pattern used.

15-2: Using $\frac{1}{16}$″ maximum thickness plates only, create a 12″ wide structure that can support itself and a full water bottle over a span of 12". No plate shall have a length greater than 3" measured from any point to any other point on the plate. No adjoining plates may occupy the same plane. As a challenge, include perforations in the design for day lighting from one direction.

15-3: Draw and build a simple space truss to support a full water bottle over a span of 18″. The maximum space truss depth is 2″. Maximum strut size is $\frac{1}{16}$″ × $\frac{1}{16}$″.

15-4: Draw and build a non-orthogonal space truss with varied thickness, varied clear height from base capable of supporting its own weight of a clear span of 1″. Maximum strut size = $\frac{1}{32}$″ × $\frac{1}{32}$″.

Part III

Wood Design

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sixteen

Dimensional Lumber Design

Chapter 16 explains the LRFD Method for analysis and design of dimensional lumber using factors derived by the American Wood Council (AWC). The LRFD (Load Resistance Factor Design) Method uses load factors to create an ultimate or factored load that is the design load. It also uses Resistance Factors (ϕ). Chapter 12 discusses the factor of loads for the LRFD Method.

Categories of Wood Construction Types:

SAWN LUMBER:

Boards: $\frac{3}{4} \times 2$ to $1\frac{1}{2} \times 16$

Dimension Lumber: 2×2 to $4\frac{1}{2} \times 16$

Timber: 5×5 and larger

GLUE-LAMINATED TIMBER: any size

It is important to understand that sawn lumber sizes are nominal sizes not actual sizes. The nominal sizes are rough cut sizes, before planning. The actual dimension of any edge of a standard size piece of sawn lumber will be $\frac{1}{4}''$ to $1''$ less than the nominal edge stated. For example, a 2×4 is actually $1\frac{1}{2}'' \times 3\frac{1}{2}''$. When designing sawn lumber, always use the actual size for values of width and thickness. Because wood is easily ripped and planed, custom sizes can be made at a relatively low cost compared to other materials. In this chapter, only standard sizes will be used. See Table A2.1: Section Properties for Dimensional Lumber for a list of standard dimensional lumber sizes and section properties.

16.1 Adjustment Factors for Dimensional Lumber

Table 16.1: Adjustment factors of sawn lumber, with permission from the American Wood Council

ADJUSTMENT FACTORS FOR SAWN LUMBER

		Temperature Factor	Wet Service Factor	Incising Factor	Size Factor	Beam Stability Factor	Flat Use Factor	Repetition Factor	Column Stability Factor	Bearing Factor	$K_{F}\phi$	Time Effect Factor
$F_b' = F_b$	x	C_t	C_m	C_i	C_F	C_L	C_{fu}	C_r			2.16	λ
$F_t' = F_t$	x	C_t	C_m	C_i	C_F						2.16	λ
$F_c' = F_c$	x	C_t	C_m	C_i	C_F				C_P		2.16	λ
$F_v' = F_v$	x	C_t	C_m	C_i							2.16	λ
$F_{c\perp}' = F_{c\perp}$	x	C_t	C_m	C_i						C_b	1.5	
$E' = E$	x	C_t	C_m	C_i								
$E_{min}' = E_{min}$	x	C_t	C_m	C_i							1.5	

Allowable stresses, F′, in Table 16.1 are found by multiplying the design values listed for a given species of wood from Table 4A and 4B of the National Design Specifications Supplement by the applicable factors. Tables A2.2 and A2.3 contain sample values for use with examples and exercises in this book. Note that there are separate tables, 4A for Western species of wood and 4B for Southern Pine. Southern Pine is unique in that the design values vary by the width of lumber used. Likewise, in some formulas for design, Southern Pine will have a different factor than Western species.

λ is the Time Effect Factor. Values of λ are correlated with the six equations for factored loads as shown in Table 16.2.

The most common case is a dead load and a live load from occupancy, resulting in a value of λ = 0.8.

Table 16.2: λ Time Effect Factor with permission from the American Wood Council

λ Time Effect Factor	
LRFD Load Combination	λ
1.4D	0.6
1.2D + 1.6L + 0.5(Lr or S or R) where L is from storage	0.7
1.2D + 1.6L + 0.5(Lr or S or R) where L is from occupancy	0.8
1.2D + 1.6L + 0.5(Lr or S or R) where L is from impact	1.25
1.2D + 1.6(Lr or S or R) + (L or 0.8W)	0.8
1.2D + 1.6W + 0.5L + 0.5(Lr or S or R)	1
1.2D + E + L + 0.2S	1
0.9D + (1.6W or E)	1

C_t is the temperature factor. It is used with all dimensional lumber, timber and glu-lams.

C_i is the incising factor. It is used with dimensional lumber only. Incising is the injection of treatment into the wood. Examples of incising are termite, fungus or other preservative treatments.

Table 16.5: C_i Incising Factor, with permission from the American Wood Council

C_i Incising Factor	
Design Value	C_i
F_b, F_c, F_t, F_v	0.80
E, E_{min}	0.95
$Fc_{\perp}$	1.00

C_F is the Size Factor. For dimensional lumber, C_F can be found in Table A2.1: Dimensional Lumber Section Properties.

C_L is the Beam Stability Factor. This factor takes some consideration as it is dependent on the unbraced length of the beam, the type of loading, the size and the species of wood. The first thing to do is to establish whether or not C_L = 1.

Table 16.3: C_t Temperature Factor, with permission from the American Wood Council

C_t Temperature Factor				
Reference Design Values	In-Service Moisture Conditions	C_t		
		T ≤ 100°F	100°F < T ≤ 125°F	125°F < T ≤ 150°F
F_t, E, E_{min}	Wet or Dry	1	0.9	0.9
F_b, F_v, F_c, $F_{c\perp}$	Dry	1	0.8	0.7
	Wet or Dry	1	0.7	0.5

C_m is the Wet Service Factor. In dimensional lumber, C_m is used when the moisture content is greater than 19%.

Table 16.4: C_m Wet Service Factor for sawn lumber, with permission from the American Wood Council

C_m, Wet Service Factor	
DIMENSIONAL LUMBER > 19% moisture content	
Design Values	C_m
F_b	0.85
F_b when $F_b(C_F)$ ≤1150 psi	1
F_t	1
F_c	0.8
F_c when $F_c(C_F)$ ≤ 750 psi	1
F_v	0.97
$F_{c\perp}$	0.67
E	0.9
E_{min}	0.9

C_L = 1 if:

a) there is continuous lateral bracing of the compression member;
b) d/b ≤ 2;
c) 2 ≤ d/b ≤ 4 AND edges are secured by blocking or X-bracing;
d) 4 ≤ d/b ≤ 5 AND there is full sheathing AND blocking at ends;
e) 5 ≤ d/b ≤ 6 AND there is full sheathing AND blocking ≤ 8ft o.c.;
f) 6 ≤ d/b ≤ 7 AND there is full sheathing AND blocking at all points of bearing.

If none of these conditions are met, C_L must be calculated using the following steps:

1. Determine the effective length using Table 16.6. For a combination of load types, use the highest value obtained.

Table 16.6: Effective length, with permission from the American Wood Council

EFFECTIVE LENGTH, L_e for bending members			
SIMPLE SPAN BEAM			
Load type:	When $L_u/d < 7$		when $L_u/d > 7$
Uniform load	$2.06L_u$		$1.63L_u + 3d$
Point load at center with no lateral bracing at loa d	$1.8L_u$		$1.37L_u + 3d$
Point load at center with lateral bracing	$1.11L_u$		
2 equal Point loads at L/3 with lateral support at loads	$1.68L_u$		
3 equal Point loads at L/4 with lateral support at loads	$1.54L_u$		
4 equal Point loads at L/5 with lateral support at loads	$1.68L_u$		
5 equal Point loads at L/6 with lateral support at loads	1.73Lu		
6 equal Point loads at L/7 with lateral support at loads	$1.78L_u$		
7 or more equal point loads evenly spaced with l ateral support at loads	$1.84L_u$		
Equal end moments	$1.84L_u$		
CANTILEVER BEAM			
Load type:	when $L_u/d < 7$		when $L_u/d > 7$
Uniform load	$1.33L_u$		$0.90L_u + 3d$
Point load at cantilever end	$1.87L_u$		$1.44L_u + 3d$
SIMPLE SPAN OR CANTILEVER BEAM	when $L_u/d < 7$	when $7 < L_u/d < 14.3$	when $L_u/d > 14.3$
Loading not listed above	$2.06L_u$	$1.63L_u + 3d$	$1.84L_u$

2. $R_b^2 = L_e(d)/b^2$ where L_e was determined in step 1, d = depth of beam and b = thickness of beam.
3. Check that $R_b^2 \le 2500$. If $R_b^2 > 2500$, choose a larger size.
4. Calculate $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
5. $F_{bE} = 1.2(E_{min}')/R_b^2$
6. $F_b^* = F_b$(all factors EXCEPT C_{fu}, C_v and C_L)
7. $F = F_{bE}/F_b^*$
8. $C_L = (1 + F)/1.9) - \sqrt{[((1 + F)/1.9)^2 - (F/0.95)]}$

C_{fu} is the Flat Use Factor. It is used when the lumber is laid flat; meaning that the depth, d is less than the width, b. Examples of flat use are decking and headers or sills. It is only used in finding the allowable bending stress, F_b'. Factors for C_{fu} are listed in Table A2.1: Dimensional Lumber Section Properties.

C_r is the Repetition Factor. It is only used with dimensional lumber and only when the criteria below are met.

C_r = 1.15 when:

1. using dimensional lumber (2–4″ thick);
2. spacing of joists, rafters, studs, etc. is not more than 24″o.c.;
3. there are three or more members in repetition;
4. members are joined by sheathing, subfloor or other load distributing elements adequate to support the applied loads.

C_P is the Column Stability Factor. It is used with all dimensional lumber, timber and glu-lams. If a compression member is laterally supported along its entire length, C_P = 1.0. If not, follow the steps in Table 16.7 to obtain C_P.

1. Determine the Effective Length Factor, k, based on end conditions:

Table 16.7: Effective Length Factor, k

k EFFECTIVE LENGTH FACTOR		
k	end condition	end condition
0.65	fixed	fixed
0.8	pinned	fixed
1	pinned	pinned
1.2	fixed	rotation fixed, translation free
2	pinned	rotation fixed, translation free
2.1	fixed	rotation and translation free

2. Effective length, $L_e = k(L_u)$ where Lu is the unbraced length and k is determined in step 1. For rectangular columns, find L_e in both directions.
3. Check that $L_e/d \le 50$ in each direction. If not, choose a larger size. Use the larger value of L_e/d for step 5.
4. Calculate $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
5. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2$
6. $F_c^* = F_c$(all factors EXCEPT C_P)
7. c = 0.8 for sawn lumber, c = 0.9 for glue-laminated or structural composite lumber.
8. $F = F_{cE}/F_c^*$
9. $C_P = (1 + F)/2c) - \sqrt{[((1 + F)/2c)^2 - (F/c)]}$

C_b is the Bearing Area Factor. It is used for bearing length less than 6″ and not nearer than 3″ to the end of a member.

$C_b = (L_b + 0.375)/L_b$ where L_b = bearing length in inches measured parallel to the grain.

16.2 Design of Dimensional Lumber Components

16.2.1 Flexure

1. Identify the species of wood:
   NOT Southern Pine → step 2
   Southern Pine → step 20
2. Western species dimensional lumber: refer to Table A2.1 Dimensional Lumber Section Properties for sample species
   Identify F_b, F_v, E, E_{min}, G for species and grade
3. Assume trial size = 2 × 12: C_F = 1, A = 16.88in^2, S = 31.64in^3, I = 177.98in^4
4. $F_b' = F_b(C_m)(Ct)(C_L)(C_F)(C_{fu})(C_i)(Cr)(2.16)(\lambda)$
   C_F : Is size 2 × 12?
      Yes: C_F = 1.0
      No: Determine C_F from Table A2.1 Dimensional Lumber Section Properties
   C_m: Is moisture content over 19%?
      No: C_m = 1.0
      Yes: Determine C_m from Table 16.4
   C_t: Is temp. above 100°F?
      No: C_t = 1.0
      Yes: Determine C_t from Table 16.3
   C_{fu}: Is beam laid flat like a plank?
      No: C_{fu} = 1.0
      Yes: Find C_{fu} in Table A2.1 Dimensional Lumber Section Properties
   C_i: Is there preservative or termite treatment or any other incising?
      No: C_i = 1.0
      Yes: C_i = 0.80
   C_r: Are beams repeated at a spacing ≤ 24″o.c.?
      No: C_r = 1.0
      Yes: C_r = 1.15
   λ: Determine λ from Table 16.2:
   Calculate $F_b' = F_b(C_m)(Ct)(C_L)(C_F)(C_{fu})(C_i)(Cr)(2.16)(\lambda)$
      $= (F_b^*)(C_L)$
   C_L: find d/b and determine if C_L = 1. If not, calculate C_L using the steps described earlier in this section
   $F_b' = F_b^*\ (C_L)$
5. Find weight of beam: W_{BM} = (specific gravity)(62.4pcf) [(Ain^2)/(144in^2/ft^2)]
6. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

   $W_u = 1.2(W_{BM} + W_{DL}) + 1.6(W_{LL})$ OR if NO LIVE LOAD: $W_u = 1.4(W_{BM} + W_{DL})$

   $P_u = 1.2P_D + 1.6\ P_L$ OR if NO LIVE LOAD: $P_u = 1.4P_D$
7. Find the maximum moment in the beam. Remember to multiply by 12″/f to obtain an answer in #-in.
8. f_b = M/S where M from step 7, and S from step 3.
9. Is $f_b \le F_b'$?
   Yes → step 10
   No → estimate $S_{req} = M/F_b'$ and go back to step 3 and try larger size.
10. Is $f_b/F_b' \ge 0.90$?
   Yes → step 11
   No → estimate $S_{req} = M/F_b'$ and go back to step 3 and try smaller size.
11. $F_v' = F_v(C_m)(C_t)(C_i)(2.16)\ (\lambda)$
   C_m: Is moisture content over 19%?
      No: C_m = 1.0
      Yes: C_m = .97
   C_t: Is temp. above 100°F?
      No: C_t = 1.0
      Yes: Determine C_t from Table 16.3
   C_i: Is there preservative or termite treatment or any other incising?
      No: C_i = 1.0
      Yes: C_i = 0.8
12. Determine the maximum shear, V, in the beam.
13. f_v = 3V/2A
14. Is $f_v \le F_v'$?
   Yes → step 14
   No → Estimate $A_{req} = 3V/2F_v'$ choose a larger size. If b and d are both greater than or equal to the original precious size, it is not necessary to check bending stress again. If not, Go back to step 3 and check bending stress.
15. Δ_{all} = L(12″/f)/240
16. Unfactored loads: remember to use unfactored loads for deflection. W_{BM} is listed in step 5, and the applied loads are

given. If an applied load is already factored, it may be used as is. Using a factored load will not create a safety issue; it will simply yield a larger required moment of inertia.

17. $E' = E(C_m)(C_t)(C_i)$
    - C_m: Is moisture content over 19%?
        - No: $C_m = 1.0$
        - Yes: $C_m = .90$
    - C_t: Is temp. above 100°F?
        - No: $C_t = 1.0$
        - Yes: Determine C_t from Table 16.3
    - C_i: Is there preservatives, termite treatment or any other incising?
        - No: $C_i = 1.0$
        - Yes: $C_i = 0.95$
18. Find Δ_{act} using deflection charts, by Double Integration Method or by Moment Area Method. Remember to multiply the equations by $1728in^3/ft^3$ in order to obtain an answer in inches when using a length, L in feet. I_x is from step 3.
19. Is $\Delta_{act} \le \Delta_{all}$?
    - Yes → done.
    - No → find $I_{req} = \Delta_{act}(I_x \text{ from step 3})/\Delta_{all}$. Select final size based on I_{req}.
20. Southern Pine dimensional lumber: Assume trial size $= 2 \times 12$: $C_F = 1$, $A = 16.88in^2$
    $S = 31.64in^3$, $I = 178in^4$
21. Refer to Table A2.3 for sample Southern Pine sizes. Identify F_b, F_v, E, E_{min} and G for size and grade.
22. $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda)$
    - C_F:
        - $C_F = 1.0$ for $2 \times 2 - 3 \times 12$ and $4 \times 4 - 4 \times 6$
        - $C_F = (12/d)\ 1/9$ for $d > 12''$
        - $C_F = 1.1$ for $4 \times 8 - 4 \times 12$
    - C_m: Is moisture content over 19%?
        - No: $C_m = 1.0$
        - Yes: Determine C_m from Table 16.4
    - C_t: Is temp. above 100°F?
        - No: $C_t = 1.0$
        - Yes: Determine C_t from Table 16.3
    - C_{fu}: Is beam laid flat like a plank?
        - No: $C_{fu} = 1.0$
        - Yes: Determine C_{fu} from Table A2.1: Dimensional Lumber Sectional Properties
    - C_i: Is there preservatives, termite treatment or any other incising?
        - No: $C_i = 1.0$
        - Yes: $C_i = 0.80$
    - C_r: Are beams repeated at a spacing ≤ 24″o.c.?
        - No. $C_r = 1.0$
        - Yes: $C_r = 1.15$
    - λ: Determine λ from Table 16.2.
    - Calculate $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda)$
        $= (F_b^*)(C_L)$
    - C_L: find d/b and determine if $C_L = 1$. If not, calculate C_L using the steps described earlier in this section.
    - $F_b' = F_b^*(C_L)$
23. Find weight of beam: W_{BM} = (specific gravity)(62.4pcf) $[(Ain^2)/(144in^2/ft^2)]$
24. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

    $W_u = 1.2(W_{BM} + W_{DL}) + 1.6(W_{LL})$ OR if NO LIVE LOAD: $W_u = 1.4(W_{BM} + W_{DL})$

    $P_u = 1.2P_D + 1.6\ P_L$ OR if NO LIVE LOAD: $P_u = 1.4P_D$
25. Find the maximum moment in the beam. Remember to multiply by $12''^{/f}$ to obtain an answer in #-in.
26. $f_b = M/S$ where M from step 7, and S from step 3.
27. Is $f_b \le F_b'$?
    - Yes → step 28
    - No → estimate $S_{req} = M/F_b'$ and go back to step 20 and try larger size.
28. Is $f_b/F_b' \ge 0.90$?
    - Yes → step 29
    - No → estimate $S_{req} = M/F_b'$ and go back to step 20 and try smaller size.
29. $F_v' = F_v(C_m)(C_t)(C_i)(2.16)(\lambda)$
    - C_m: Is moisture content over 19%?
        - No: $C_m = 1.0$
        - Yes: $C_m = .97$
    - C_t: Is temp. above 100°F?
        - No: $C_t = 1.0$
        - Yes: Determine C_t from Table 16.3
    - C_i: Is there preservatives, termite treatment or any other incising?

No: $C_i = 1.0$

Yes: $C_i = 0.8$

30. Determine maximum shear, V, in the beam.
31. $f_v = 3V/2A$
32. Is $f_v \le F_v'$?

Yes → step 33

No → estimate $A_{req} = 3V/2F_v'$ and go back to step 20 and try larger size.

33. $\Delta_{all} = L(12''^{/f})/240$
34. Unfactored loads: remember to use unfactored loads for deflection. W_{BM} is listed in step 5, and the applied loads are listed in problem. If an applied load is already factored, it may be used as is. Using a factored load will not create a safety issue; it will simply yield a larger required moment of inertia.
35. $E' = E(C_m)(C_t)(C_i)$

C_m: Is moisture content over 19%?

No: $C_m = 1.0$

Yes: $C_m = .90$

C_t: Is temp. above 100°F?

No: $C_t = 1.0$

Yes: Determine C_t from Table 16.3

C_i: Is there preservatives, termite treatment or any other incising?

No: $C_i = 1.0$

Yes: $C_i = 0.95$

36. Find Δ_{act} using deflection charts, by Double Integration Method or by Moment Area Method. Remember to multiply the equations by 1728in³/ft³ in order to obtain an answer in inches when using a length, L in feet. I_x is from step 20.
37. Is $\Delta_{act} \le \Delta_{all}$?

Yes → done.

No → find $I_{req} = \Delta_{act}(I_x \text{ from } 20)/\Delta_{all}$. Select final size based on I_{req}.

Example 16-1: Design of a Western species joist: Design a series of construction grade Douglas Fir Larch (north) joists with a moisture content of 16%, spaced at 16″o.c. with X-bracing at 4′o.c. to carry a dead load of 15psf and a live load of 40psf with a span of 12ft.

Average temperature = 105°F. Termite treatment is incised in joists. Max. deflection = L/240.

1. Identify the species of wood: Douglas Fir Larch (north) NOT Southern Pine → step 2.
2. Western species dimensional lumber: refer to Tables A2.2 for sample species.

   $F_b = 950\text{psi}$, $F_v = 180\text{psi}$, $E = 1{,}500{,}000\text{psi}$,

   $E_{min} = 550{,}000\text{psi}$, $G = 0.49$

3. Assume trial size = 2×12: $C_F = 1$, $A = 16.88\text{in}^2$, $S = 31.64\text{in}^3$, $I = 177.98\text{in}^4$
4. $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda)$

   C_F : Is size 2×12? Yes: $C_F = 1.0$

   C_m: Is moisture content over 19%? No: $C_m = 1.0$

   C_t: Is temp. above 100°F? Yes: From Table 16.3, $C_t = 0.8$

   C_{fu}: Is beam laid flat like a plank? No: $C_{fu} = 1.0$

   C_i: Are there preservatives, termite treatment or any other incising? Yes: $C_i = 0.80$

   C_r: Are beams repeated at a spacing ≤ 24″o.c. and are there more than 2 spans? Yes: $C_r = 1.15$

   λ: from Table 16.2, $\lambda = 0.8$

   Calculate $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda)$

   $= (F_b^*)(C_L)$

   $= 950(1)(0.8)(1)(1)(0.8)(1.15)(2.16)(0.8)C_L = 1208.21C_L$

   C_L: $d/b = 11.25/1.5 = 7.5$ … calculate C_L:

   a) $L_u = 48''$ … $L_u/d = 48/11.25 = 4.27 < 7$

      From Table 16.6, $L_e = 2.06L_u = 2.06(48) = 98.88''$

   b) $R_b^2 = L_e(d)/b^2 = 98.88(11.25)/1.5^2 = 494.4$

   c) Check that $R_b^2 = 494.4 \le 2500$. Yes … okay

   d) $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5) = 550{,}000(1)(0.9)(0.95)(1.5) = 705{,}375\text{psi}$

   e) $F_{bE} = 1.2(E_{min}')/R_b^2 = 1.2(705{,}375)/494.4 = 1712.08\text{psi}$

   f) $F_b^* = 1208.21\text{psi}$

   g) $F = F_{bE}/F_b^* = 1712.08/1208.21 = 1.417$

   h) $C_L = (1 + F)/1.9) - \sqrt{[((1 + F)/1.9)^2 - (F/0.95)]} = 0.916$

      $F_b' = F_b^*(C_L) = 1208.21(0.916) = 1106.72\text{psi}$

5. Find weight of beam: W_{BM} = (specific gravity)(62.4pcf) $[(A\text{in}^2)/(144\text{in}^2/\text{ft}^2)] = .49*62.4*16.88/144 = 3.58^{\#/f}$
6. Find factored loads: $W_u = 1.2(DL) + 1.6(LL)$

   $= 1.2[15\text{psf}(16''/12''^{/f}) + 3.58^{\#/f}] + 1.6[40\text{psf}(16''/12''^{/f})]$

   $= 113.63^{\#/f}$

7. $M_u = wL^2/8 = 113.63^{\#/f}(12')^2/8 = 2045.34\#\text{-f}$

   $= 24544.08\#\text{-in}$

8. $f_b = M/S = 24522.08\#\text{-in}/31.64\text{in}^3 = 775.03\text{psi}$
9. Is $f_b \le F_b'$? 775.03psi < 1106.72psi … okay

10. Is $f_b/F_b' \geq 0.90$? 775.03/1106.72 = 0.70
No → estimate $S_{req} = M/F_b'$ = 24544.08/1106.72
= 22.18

3A. Assume 2 × 10: C_F = 1.1, A = 13.88in², S = 21.39in³,
I = 98.93in⁴

4A. $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda)$
C_F = 1.1
C_m: Is moisture content over 19%? No: C_m = 1.0
C_t: Is temp. above 100°F? Yes: From Table 16.3,
C_t = 0.8
C_{fu}: Is beam laid flat like a plank? No: C_{fu} = 1.0
C_i: Are there preservatives, termite treatment or any other incising? Yes: C_i = 0.80
C_r: Are beams repeated at a spacing ≤ 24″o.c. and are there more than 2 spans? Yes: C_r = 1.15
λ: from Table 16.2, λ = 0.8
Calculate $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda)$
$= (F_b^*)(C_L)$
= 950(1.1)(0.8)(1)(1)(0.8)(1.15)(2.16)(0.8)C_L = 1329.03C_L

C_L: d/b = 9.25/1.5 = 6.17 … calculate C_L :
a) L_u = 48″ … L_u/d = 48/11.25 = 4.27 < 7
From Table 16.6, $L_e = 2.06L_u$ = 2.06(48) = 98.88″
b) $R_b^2 = L_e(d)/b^2$ = 98.88(9.25)/1.5² = 406.51
c) Check that R_b^2 = 406.51 ≤ 2500. Yes … okay
d) $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$ = 550,000(1)(0.9)(0.95)(1.5) = 705,375psi
e) $F_{bE} = 1.2(E_{min}')/R_b^2$ = 1.2(705,375)/406.51
= 2082.24psi
f) F_b^* = 1329.03psi
g) $F = F_{bE}/F_b^*$ = 2082.24/1329.03 = 1.567
h) $C_L = (1 + F)/1.9) - \sqrt{[((1 + F)/1.9)^2 - (F/0.95)]}$ = 0.932
$F_b' = F_b^* (C_L)$ = 1329.03(0.932) = 1238.66psi

5A. Find weight of beam: W_{BM} = (specific gravity)(62.4pcf) [(Ain²)/(144in²/ft²)] = .49*62.4*13.88/144 = 2.95#/f

6A. Find factored loads: W_u = 1.2(DL) + 1.6(LL)
= 1.2[15psf(16″/12″/f) + 2.95#/f] + 1.6[40psf(16″/12″/f)]
= 112.87#/f

7A. $M_u = wL^2/8$ = 112.87#/f (12′)²/8 = 2031.72#-f
= 24380.64#-in

8A. f_b = M/S = 24380.64#-in/21.39in³ = 1139.81psi

9A. Is $f_b \leq F_b'$? 1139.81psi < 1238.66psi … okay

10A. Is $f_b/F_b' \geq 0.90$? 1139.81/1238.66 = 0.92 … okay for flexure

11. $F_v' = F_v(C_m)(C_t)(C_i)(2.16)$ (λ)
C_m: Is moisture content over 19%? No: C_m = 1.0
C_t: Is temp. above 100°F? Yes: C_t = 0.8
C_i: Is there preservative or termite treatment or any other incising?
Yes: C_i = 0.8
F_v' = 180(1)(0.8)(0.8)(2.16)(0.8) = 199.07psi

12. V = wL/2 = 112.87#/f(12′)/2 = 677.22#

13. f_v = 3V/2A = 3(677.24#)/[2(10.88in²)] = 93.37psi

14. Is $f_v \leq F_v'$? 93.37 < 199.07 … okay for shear

15. Δ_{all} = L(12″/f)/240 = 12′(12″/f)/240 = 0.6″

16. Unfactored loads: = 2.95#/f + (15psf + 40psf)(16″/(12″/f))
= 76.28#/f

17. $E' = E(C_m)(C_t)(C_i)$
C_m: Is moisture content over 19%? No: C_m = 1.0
C_t: Is temp. above 100°F? Yes: Determine C_t = 0.9
C_i: Is there preservatives, termite treatment or any other incising?
Yes: C_i = 0.95
E′ = 1,500,000psi(1)(.9)(.95) = 1,282,500psi

18. Δact = 5wL⁴/384EI = 5(76.28#/f)(12′⁴)(1728in³/ft³)/[384(1,282,500psi)(98.93in⁴)] = 0.28″

19. Is $\Delta_{act} \leq \Delta_{all}$? Yes. 0.28″ < 0.6″

ANSWER: USE 2 × 10

Example 16-2: Design of a Southern Pine beam: Design a No. 2 Southern Pine beam, 16′ long, with a moisture content of 20%, full lateral bracing and dead loads of 100# applied every 48″. Max. deflection = L/240.

Identify the species of wood: Southern Pine → step 20.

20. Assume trial size = 2 × 12: C_F = 1, A = 16.88in²,
S = 31.64in³, I = 178in⁴

21. F_b = 975, F_v = 175, E = 1,400,000, E_{min} = 580,000,
G = 0.55

22. $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda)$
C_F = 1.0 for 2 × 2 – 3 × 12

C_m: Is moisture content over 19%? Yes: from Table 16.4
C_m = 1 when $F_b(C_F)$ ≤ 1150psi
C_t: Is temp. above 100°F? No: C_t = 1.0
C_{fu}: Is beam laid flat like a plank? No: C_{fu} = 1.0
C_i: Is there preservatives, termite treatment or any other incising? No: C_i = 1.0

C_r: Are beams repeated at a spacing ≤ 24″o.c.?

No: C_r = 1.0

λ: Determine λ from Table 16.2. λ = 0.6 (dead loads only)

Calculate $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda) = (F_b^*)(C_L)$
$= 975(1)(1)(1)(1)(1)(1)(2.16)(.6)C_L = 1263.6C_L$

C_L = 1 (full lateral bracing)

$F_b' = F_b^*$ (C_L) = 1263.6psi

23. Find weight of beam: W_{BM} = (specific gravity)(62.4pcf) [(Ain^2)/(144in^2/ft^2)] = .55(62.4)(16.88)/144 = 4.02$^{\#/f}$
24. Find factored loads:

    Wu = 1.4(4.02) = 5.63$^{\#/f}$

    Pu = 1.4(100) = 140#
25. M = wL^2/8 + PL/2 = 5.63(16^2)/8 + 140(16)/2 = 1300.16#-f = 15,601.92#-in
26. f_b = M/S = 15,601.92/31.64 = 493.11psi
27. Is $f_b \le F_b'$? Yes: 493.11psi < 1263.6psi
28. Is $f_b/F_b' \ge 0.90$? No: 493.11/1263.6 = 0.39 … estimate $S_{req} = M/F_b'$ = 15601.92/1263.6 = 12.35in^3

GO BACK TO STEP 20

20A. Assume trial size = 2×8: C_F = 1, A = 10.88in^2, S = 12.14in^3, I = 47.63in^4

21A. F_b = 1200, F_v = 175, E = 1,600,000, E_{min} = 580,000, G = 0.55

22A. $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda)$

C_F = 1.0 for 2×2 – 3×12

C_m: Is moisture content over 19%? Yes: from Table 16.4

C_m = 0.85

C_t: Is temp. above 100°F?

No: C_t = 1.0

C_{fu}: Is beam laid flat like a plank?

No: C_{fu} = 1.0

C_i: Is there preservatives, termite treatment or any other incising?

No: C_i = 1.0

C_r: Are beams repeated at a spacing ≤ 24″o.c.?

No: C_r = 1.0

λ: Determine λ from Table 16.2. λ = 0.6 (dead loads only)

Calculate $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(C_{fu})(C_i)(C_r)(2.16)(\lambda) = (F_b^*)(C_L)$
$= 1200(0.85)(1)(1)(1)(1)(1)(2.16)(.6)C_L = 1321.92C_L$

C_L = 1(full lateral bracing)

$F_b' = F_b^*$ (C_L) = 1321.92psi

23A. Find weight of beam: W_{BM} = (specific gravity)(62.4pcf) [(Ain^2)/(144in^2/ft^2)] = .55(62.4)(10.88)/144 = 2.59$^{\#/f}$

24A. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

W_u = 1.4(2.59) = 3.63$^{\#/f}$

P_u = 1.4(100) = 140#

25A. M = wL^2/8 + PL/2 = 3.63(16^2)/8 + 140(16)/2 = 1236.16#-f = 14,833.92#-in

26A. f_b = M/S = 14,833.92/12.14 = 1221.90psi

27A. Is $f_b \le F_b'$? Yes 1221.90psi < 1321.92psi

28A. Is $f_b/F_b' \ge 0.90$? Yes. 1221.9/1321.92 = 0.92 … okay for flexure

29. $F_v' = F_v(C_m)(C_t)(C_i)(2.16)$ (λ)

    C_m: Is moisture content over 19%?

    Yes: C_m = .97

    C_t: Is temp. above 100°F?

    No: C_t = 1.0

    C_i: Is there preservatives, termite treatment or any other incising? No: C_i = 1.0

    F_v' = 175(.97)(1)(1)(2.16)(.6) = 220.00psi
30. V = wL/2 + 3P/2 = 3.63(16/2) + 3(140)/2 = 239.04#
31. f_v = 3V/2A = 3(239.04)/2(10.88) = 32.96psi
32. Is $f_v \le F_v'$? Yes 32.96psi < 220.00psi
33. Δ_{all} = L(12″/f)/240 = 16(12)/240 = 0.8″
34. Unfactored loads: w = 2.59$^{\#/f}$. P = 100#
35. $E' = E(C_m)(C_t)(C_i)$

    C_m: Is moisture content over 19%?

    Yes: C_m = .90

    C_t: Is temp. above 100°F?

    No: C_t = 1.0

    C_i: Is there preservatives, termite treatment or any other incising?

    No: C_i = 1.0

    E′ = 1,600,000(0.9) = 1,440,000psi
36. Find Δ_{act} = 5wL^4/384EI + 19PL^3/384EI = [5(2.59)(16^4) + 19(100)(16^3)](1728in^3/f^3)/[384(1,440,000)(47.63)] = 0.57″
37. Is $\Delta_{act} \le \Delta_{all}$? Yes 0.57″ < 0.8″

ANSWER: USE 2×8

16.2.2 Compression

In this section, the term column refers to all members under compression. This section discusses the design of Simple Solid Wood Columns which are columns made of one piece or made of multiple pieces glued together to act as one piece. For a review of Critical Buckling Stress and slenderness ratio, see Chapter 10.

From Table 16.1: Adjustment factors of sawn lumber, the equation for allowable compressive stress is:

$$F_c' = F_c(C_m)(C_t)(C_F)(C_i)(C_P)(2.16)(\lambda)$$

where the factors are described at the beginning of this chapter.

Like the design of wood beams, the design of columns is an iterative process based on an assumed trial size. In the case of wood columns, a good starting point is $A_{trial} = P_u/F_c^*$ where $F_c' = F_c^* C_P$.

Design of wood columns:

1. Look up F_c and E_{min} for the given species and grade of lumber.
2. $F_c' = F_c(C_m)(C_t)(C_F)(C_i)(C_P)2.16\ (\lambda) = F_c^*(C_P)$
   - C_m: Is moisture content over 19%?
     - No: $C_m = 1.0$
     - Yes: $C_m = .8$ unless $F_c(C_F) \leq 750$psi, in which case $C_m = 1$
   - C_t: Is temp. above 100°F?
     - No: $C_t = 1.0$
     - Yes: Determine C_t from Table 16.3
   - C_i: Is there preservatives, termite treatment or any other incising?
     - No: $C_i = 1.0$
     - Yes: $C_i = 0.8$
     - Assume $C_P = 1$ and $C_F = 1$ for now.
3. Calculate $L_e = kL(12''/')$ in each direction. Effective Length Factor, k can be found in Figure 10.1. k = 1.0 for pin–pin, k = 0.8 for pin–fix, k = .65 for fix–fix conditions

   Determine minimum width in each direction based on $L_e/d < 50$.

   $d_{min} = L_{ex}/50$ and $b_{min} = L_{ey}/50$
4. $A_{trial} = P/F_c^*$

   Select a size with $A \geq A_{trial}$, $b \geq b_{min}$, and $d \geq d_{min}$. Note A, b and d.
5. Use larger of L_e/d or L_{ey}/b and L_{ex}/d.
6. $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
   - C_m: Is moisture content over 19%?
     - No: $C_m = 1.0$
     - Yes: $C_m = .9$
   - C_t: Is temp. above 100°F?
     - No: $C_t = 1.0$
     - Yes: Determine C_t from Table 16.3
   - C_i: Is there preservatives, termite treatment or any other incising?
     - No: $C_i = 1.0$
     - Yes: $C_i = 0.95$
7. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2$
8. $F = F_{cE}/F_c^*$
9. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2}$
11. $F_c' = F_c^*(C_P)$ = allowable compressive stress
12. $f_c = P/A$ = actual compressive stress
13. Is $f_c < F_c'$?
    - Yes → step 15
    - No → go back to step 4 and choose larger size.
14. Is $f_c/F_c' \geq 0.90$? If not, go back to step 4 and try smaller size.

Example 16-3: Design a 9′ high column to be made using a nominal 3″ thick No.2 DFL with pinned supports to carry a factored load of 10,000# if termite treatment is incised into the wood.

Use a standard size depth.

1. $F_c = 1350$psi and $E_{min} = 560{,}000$psi.
2. $F_c' = F_c(C_m)(C_t)(C_F)(C_i)(C_P)2.16(\lambda) = F_c^*(C_P)$

   C_m: Is moisture content over 19%? No: $C_m = 1.0$

   C_t: Is temp. above 100°F? No: $C_t = 1.0$

   C_i: Is there preservatives, termite treatment or any other incising? Yes: $C_i = 0.8$

   Assume $C_P = 1$ and $C_F = 1$ for now.

   $F_c' = 1350(1)(1)(0.8)C_F C_P(2.16)(.8) = 1866.24 C_F C_P$
3. $k = 1.0$, $L_e = kL(12''/') = 1(9)(12) = 108''$ in both directions

   Determine min width in each direction based on $L_e/d < 50$.

   $d_{min} = b_{min} = L_{ex}/50 = 108/50 = 2.16''$
4. $A_{trial} = P/F_c^* = 10{,}000/1866.24 = 5.36\text{in}^2$

   Try 3×4: $A = 8.75\text{in}^2$, $b = 2.5''$, $d = 3.5''$
5. $L_e/d = 108/2.5 = 43.2$

6. $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
   C_m: Is moisture content over 19%? No: $C_m = 1.0$
   C_t: Is temp. above 100°F? No: $C_t = 1.0$
   C_i: Is there preservatives, termite treatment or any other incising? Yes: $C_i = 0.95$
   $E_{min}' = 560{,}000(1)(1)(0.95)(1.5) = 798{,}000$psi
7. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2 = .822(798{,}000\text{psi})/(43.2)^2 = 351.49$psi
8. $C_F = 1.15$, $F = F_{cE}/F_c^* = 351.49/1866.24(1.15) = 0.164$
9. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = 0.158$
11. $F_c' = F_c^*(C_P)$ = allowable compressive stress $= 0.158(1866.24)(1.15) = 339.1$
12. $f_c = P/A$ = actual compressive stress $= 10{,}000/8.75 = 1142.86$
13. Is $f_c < F_c'$? No → go back to step 4 and choose larger size.
   $A_{trial} = 8.71(1142.86/344.32) = 29\text{in}^2$

4A. Try 3×12: $A = 28.13\text{in}^2$, $b = 2.5''$, $d = 11.25''$
5A. $L_e/d = 108/2.5 = 43.2$
6A. $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
   C_m: Is moisture content over 19%? No: $C_m = 1.0$
   C_t: Is temp. above 100°F? No: $C_t = 1.0$
   C_i: Is there preservatives, termite treatment or any other incising? Yes: $C_i = 0.95$
   $E_{min}' = 560{,}000(1)(1)(0.95)(1.5) = 798{,}000$psi
7A. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2 = .822(798{,}000\text{psi})/(43.2)^2 = 351.49$psi
8A. $C_F = 1$, $F = F_{cE}/F_c^* = 351.49/1866.24(1) = 0.188$
9A. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10A. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = 0.264$
   11A. $F_c' = F_c^*(C_P)$ = allowable compressive stress $= 0.264\ (1866.24)(1) = 492.69$psi
12A. $f_c = P/A$ = actual compressive stress $= 10{,}000/28.13 = 355.49$
13. Is $f_c < F_c'$? Yes.
14. Is $f_c/F_c' \geq 0.90$? 355.49/492.69 = 0.72 … go back to step 4 and try smaller size.

4B. Try 3×10: $A = 23.13\text{in}^2$, $b = 2.5''$, $d = 9.25''$
5B. $L_e/d = 108/2.5 = 43.2$
6B. $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
   C_m: Is moisture content over 19%? No: $C_m = 1.0$
   C_t: Is temp. above 100°F? No: $C_t = 1.0$
   C_i: Is there preservatives, termite treatment or any other incising? Yes: $C_i = 0.95$
   $E_{min}' = 560{,}000(1)(1)(0.95)(1.5) = 798{,}000$psi
7B. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2 = .822(798{,}000\text{psi})/(43.2)^2 = 351.49$psi
8B. $C_F = 1$, $F = F_{cE}/F_c^* = 351.49/1866.24(1) = 0.188$
9B. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10B. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = 0.264$
11B. $F_c' = F_c^*(C_P)$ = allowable compressive stress $= 0.264(1866.24)(1) = 492.69$psi
12B. $f_c = P/A$ = actual compressive stress $= 10{,}000/23.13 = 432.34$psi
13B. Is $f_c < F_c'$? Yes.
14B. Is $f_c/F_c' \geq 0.90$? 432.34/492.69 = 0.88 … go back to step 4 and try smaller size.

4C. Try 3×8: $A = 18.13\text{in}^2$, $b = 2.5''$, $d = 7.25''$
5C. $L_e/d = 108/2.5 = 43.2$
6C. $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
   C_m: Is moisture content over 19%? No: $C_m = 1.0$
   C_t: Is temp. above 100°F? No: $C_t = 1.0$
   C_i: Is there preservatives, termite treatment or any other incising? Yes: $C_i = 0.95$
   $E_{min}' = 560{,}000(1)(1)(0.95)(1.5) = 798{,}000$psi
7C. $F_{CE} = 0.822(E_{min}')/(L_e/d)^2 = .822(798{,}000\text{psi})/(43.2)^2 = 351.49$psi
8C. $C_F = 1.05$, $F = F_{cE}/F_c^* = 351.49/1866.24(1.05) = 0.179$
9C. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10C. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = 0.172$
11C. $F_c' = F_c^*(C_P)$ = allowable compressive stress $= 0.172(1866.24)(1.05) = 337.04$psi
12C. $f_c = P/A$ = actual compressive stress $= 10{,}000/18.13 = 551.57$psi
13C. Is $f_c < F_c'$? No

ANSWER: USE 3x10

This method may also be used to check the adequacy or spacing of columns of a given size by using the given size in step 4.

Example 16-4: An 8′ high Western frame greenhouse wall has 2×4, No. 2 DFL studs with bracing at 4′ in the weak direction and pinned ends.

The moisture content is 22%, the average temperature is 103°F and the wood is treated with preservatives. If the wall must carry a factored load of 1500#/f, what is the maximum spacing of the studs (in multiples of 3″)?

1. F_c = 1350psi and E_{min} = 560,000psi.
2. $F_c' = F_c(C_m)(C_t)(C_F)(C_i)(C_P)2.16\ (\lambda) = F_c^*(C_P)$
   C_m: Is moisture content over 19%? Yes: C_m = .8
   C_t: Is temp. above 100°F? Yes: From Table 16.3,
   C_t = 0.7
   C_i: Is there preservatives, termite treatment or any other incising? Yes: C_i = 0.8
   Assume C_P = 1and C_F = 1.5
   $F_c' = 1350(.8)(.7)(1.5)(.8)(2.16)(0.8)C_P = 1567.64(C_P)$psi
3. Calculate $L_e = kL(12''^{/f})$ in each direction.
   k = 1
   $d_{min} = L_{ex}/50 = 1(8')(12''^{/f})/50 = 1.92''$
   $b_{min} = L_{ey}/50 = 1(4')(12''^{/f})/50 = 0.96''$
4. 2×4 = given size: A = 5.25in^2, b = 1.5″, d = 3.5″
5. Use larger of L_e/d or L_{ey}/b and L_{ex}/d. 96/3.5 = 27.43 and 48/1.5 = 32 … okay
6. $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
   C_m: Is moisture content over 19%? Yes: C_m = .9
   C_t: Is temp. above 100°F? Yes: Determine C_t = 0.9
   C_i: Is there incising? Yes: C_i = 0.95
   E_{min}' = 560,000(.9)(.9)(.95) = 430,920psi
7. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2 = 0.822(430,920)/32^2$ = 345.91psi
8. $F = F_{cE}/F_c^*$ = 345.91/1045.09 = 0.331
9. c = 0.8 for sawn lumber
10. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = 0.304$
11. $F_c' = F_c^*(C_P)$ = 1567.64(0.331) = 518.89 = allowable compressive stress
12. Let spacing of the studs = s″. P = 1500#/f(s)/(12″/f) = 125s
    f_c = P/A = actual compressive stress = 125s/5.25in^2 = 23.81s psi
13. $f_c \le F_c'$ … 23.81s ≤ 518.89 and s ≤21.79″

ANSWER: space 2×4 studs at 21″o.c.

Example 16-5: Design a built-up column using standard sizes with a nominal 2″ thickness for an unbraced length of 10′ and fixed ends to support a factored load of 5000# using No.1 Southern Pine.

1. Because the values of F_c and E_{min} are higher for 2×4s and 2×3s than for other sizes, it will be most efficient to use 2×3s or 2×4s. F_c = 1850psi, E_{min} = 620,000psi.
2. $F_c' = F_c(C_m)(C_t)(C_F)(C_i)(C_P)2.16\ (\lambda) = Fc^*(C_P)$
   C_m: Is moisture content over 19%? No: C_m = 1.0
   C_t: Is temp. above 100°F? No: C_t = 1.0
   C_i: Is there preservatives, termite treatment or any other incising? No: C_i = 1.0
   Assume C_P = 1 and C_F = 1.5
   $F_c' = 1850(1)(1)(1.5)(1)(2.16)(0.8)C_P = 4795.2$
3. Calculate $L_e = kL(12''^{/f})$ in each direction.
   Effective Length Factor, k can be found in Figure 10.1.
   k = 1.0 for pin–pin, k = 0.8 for pin–fix, k = 0.65 for fix–fix conditions.
   Determine min. width in each direction based on $L_e/d < 50$.
   $d_{min} = b_{min} = L_e/50 = 0.65(10')(12)/50 = 1.56''$
4. $A_{trial} = P/F_c^*$ = 24000/4795.2 = 5
   Try two 2×3: A = 2(3.75) = 7.5in^2, b = 2.5″, d = 2(1.5″) = 3″
5. L_e/d = 0.65(10)(12)/2.5 = 31.2
6. $E_{min}' = E_{min}(C_m)(C_t)(C_i)(1.5)$
   C_m: Is moisture content over 19%? No: C_m = 1.0
   C_t: Is temp. above 100°F? No: C_t = 1.0
   C_i: Is there preservatives, termite treatment or any other incising? No: C_i = 1.0
   E_{min}' = 620,000(1)(1)(1)(1.5) = 930,000psi
7. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2 = .822(930,000)/31.2^2$ = 785.32psi
8. $F = F_{cE}/F_c^*$ = 785.32/4795.2 = 0.164
9. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = 0.158$
11. $F_c' = F_c^*(C_P)$ = allowable compressive stress = 0.158(4795.2) = 757.64psi
12. f_c = P/A = actual compressive stress = 5000#/7.5in^2 = 666.67psi
13. Is $f_c < F_c'$? Yes: 666.67psi < 757.64psi
14. Is $f_c/F_c' \ge 0.90$? 666.67/757.64 = 0.88, however, smaller size will not meet d_{min} requirements.

ANSWER: 2- 2×3s

16.2.3 Bearing

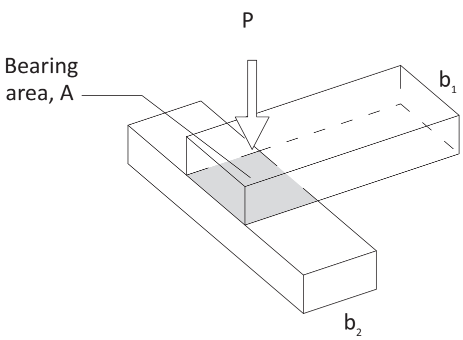
Bearing is compression perpendicular to the grain. It occurs during the transfer of load from one member to another upon which it rests. Bearing must be considered when it occurs within 3″ of the end of a member or has more than 6″ of bearing length at any other point.

For sawn lumber:

$$F_{c\perp}' = F_{c\perp}\,(C_m)(C_t)(C_i)(C_b)(1.5)$$

$$C_b = (L_b + 0.375)/L_b$$

where L_b is the bearing length measured parallel to the grain.



16.1

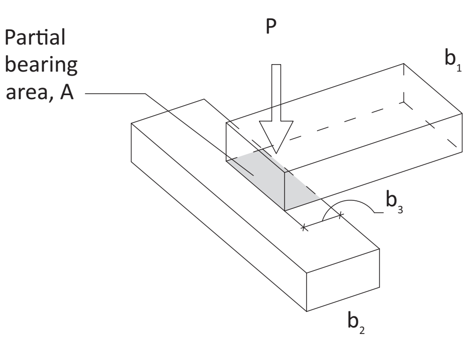
Full bearing

Bearing area = A = $b_1(b_2)$ where

b_1 = width of loaded member

b_2 = width of supporting (bearing) member

L_b = bearing length = b_2 when inspecting loaded component for bearing and b_1 when inspecting the supporting member.



16.2

Partial bearing

Bearing area = A = $b_1(b_3)$ where

b_1 = width of loaded member

b_3 = bearing depth

$b_3 < b_2$ = width of supporting (bearing) member

L_b = bearing length = b_3 when inspecting loaded component for bearing and b_1 when inspecting the supporting member.

Example 16-6: A 2×10 joist carrying a factored load of 300$^{\#/f}$ and having a span of 12′ fully bears on a flat 2×6 No. 2 SP top plate. Is this acceptable?

1. What is P? P = reaction at end of joist = wL/2 = 300$^{\#/f}$(12′)/2 = 1800#
2. What is A? A = (1.5″)(5.5″) = 8.25si
3. What is actual stress? $F_{C\perp}$= P/A = 1800/8.25 = 218.18
4. What is allowable stress? $F_{c\perp}' = F_{c\perp}(C_m)(C_t)(C_i)(C_b)(1.5)$

   $C_m = C_t = C_i = 1.0$

   $C_b = (L_b + 0.375)/L_b = (1.5 + 0.375)/1.5 = 1.25$

   where L_b is the bearing length measured parallel to the grain.

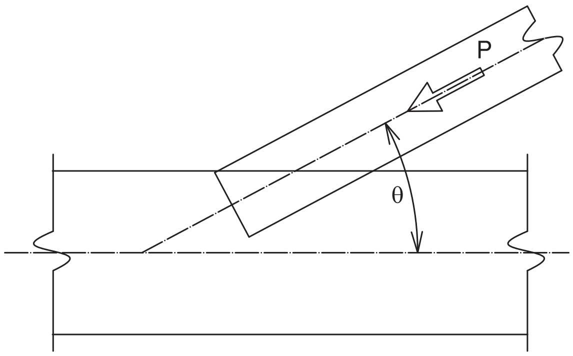
   $F_{c\perp}$ = 565psi

   $F_{c\perp}' = F_{c\perp}(C_m)(C_t)(C_i)(C_b)(1.5) = 565(1.25)(1.5)$ = 1059.375psi
5. 1059.375 allowable bearing stress > 218.18 actual bearing stress … okay

Example 16-7: What is the partial bearing length required for a No. 2 DFL 2×10 bearing on a flat No. 2 DFL 2×8 with a factored load of 3000#?

1. P = 3000#
2. $C_b = (L_b + 0.375)/L_b = (1.5 + 0.375)/1.5 = 1.25$
3. $F_{c\perp}' = F_{c\perp}(C_m)(C_t)(C_i)(C_b)(1.5) = 625(1)(1)(1)(1.25)(1.5)$ = 1171.88psi
4. $f_{c\perp}$ = 3000#/A ≤ 1171.88psi … A ≥ 3000#/1171.88psi = 2.56in^2
5. A = 1.5″(b_3) = 2.56 … b_3 = 2.56/1.5 = 1.71″

Bearing at an angle:



16.3
Bearing at an angle

If bearing is not perpendicular to the direction of the grain; if it occurs at an angle other than 90°, then the stress is a combination of compression and bearing stress where

$$F_\theta' = F_c{}^*F_{c\perp}/[F_c{}^*\sin^2\theta + F_{c\perp}\cos^2\theta]$$

θ = angle between the direction of the load and the direction of the grain in degrees.

Example 16-8: A 2 ×12 No. 2 Southern Pine rafter at a 30° incline carries a vertical load of 800$^{\#/f}$, spans 18′ and bears on a 2×6 flat top plate. Check the bearing stress in the top plate.

1. 2×6 – #2 SP: F_c = 1600psi, $F_{c\perp}$ = 565psi, E_{min} = 580,000psi
2. $F_\theta' = F_c{}^*F_{c\perp}/[F_c{}^*\sin^2\theta + F_{c\perp}\cos^2\theta]$
   $F_c{}^* = F_c(C_m)(C_t)(C_F)(C_i)(2.16)(\lambda) = 1600(1)(1)(1)(1)(2.16)(.8)$
   = 2764.8psi
   C_b = (5.5 + .375)/5.5 = 1.068
   $F_{c\perp} = F_{c\perp}\ (C_m)(C_t)(C_i)(C_b)(1.5) = 565(1.068)(1.5)$
   = 905.284psi
3. $F_\theta' = F_c{}^*(F_{c\perp})/[F_c{}^*\sin^2\theta + F_{c\perp}\cos^2\theta]$
   = 2764.8(905.284) /[2764.8(.5)^2 + 905.284(.866)^2]
   = 1826.79psi
4. $f_{c\perp}$ = P/a = (800$^{\#/f}$)(18/2′)/ [(1.5)(5.5)]
   = 872.7 3 < 1737.66psi … okay

16.2.4 Tension

$$F_t' = F_t(C_t)(C_m)(C_i)(C_F)(2.16)(\lambda)$$

Example 16-9: Design a tension member with a factored tension load of 18,000# using a 2×__ in select structural Douglas Fir Larch, with 18% moisture content and at room temperature.

1. Assume C_F = 1
2. F_t' = 1000(1)(1)(1)(1)(2.16)(.8) = 1728psi
3. f_t = P/A = 18,000#/A … A_{req} = 18,000#/1728psi = 10.42in^2
5. Try 2×8: A = 10.88in^2, C_F = 1.2
6. F_t' = 1728(1.2) = 2073.6psi
7. f_t = P/A = 18,000/10.88 = 1654.41psi
8. Is $f_t \le F_t'$? Yes: 1654.41 < 2073.6
9. Is $f_t/F_t' \le 0.9$? No: 1654.41/2073.6 = 0.80 … Try smaller size.

5A. Try 2×6: A = 8.25in^2, C_F = 1.3
6A. F_t' = 1728(1.3) = 2246.4ps
7A. f_t = P/A = 18,000/8.25 = 2181.82psi
8A. Is $f_t \le F_t'$? Yes: 2181.82 < 2246.4
9A. Is $f_t/F_t' \le 0.9$? Yes: 2181.82/2246.4 = 0.97
ANSWER: USE 2×6

Example 16-10: Find the maximum factored tension that can be carried by a No.2 So. Pine 2×12 in a greenhouse (MC > 20%, temperature < 100°F).

C_F = 1, C_m = 1, C_t = 1. Note that moisture does not affect allowable tensile stress.

F_t' = 550(1)(1)(1)(1)(2.16)(.8) = 950.4psi

f_t = P/[1.5(11.25)] ≤ F_t' … P_{max} = 950.4(5.5)(7.5)
= 39,197.5#

16.2.5 Combined Stresses:

Combined flexure and axial compression:

$$[f_c/F_c']^2 + f_{b1}/\{F_{b1}'[1 - (f_c/F_{CE1})] + f_{b2}/\{F_{b2}'[1 - (fc/F_{CE2}) - (f_{b1}/F_{bE})^2]\} \le 1.0$$

Where

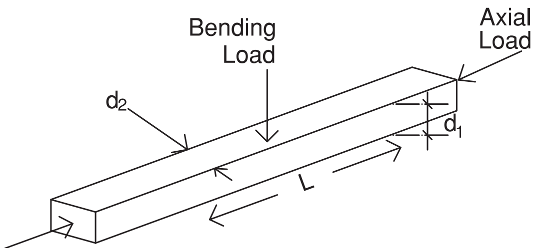
$f_c < F_{CE1} = 0.822\ E_{min}'/(L_{e1}/d_1)^2$ for edge-wise or biaxial bending (d_1 = wide face)

AND

$F_c < F_{CE2} = 0.822\ E_{min}'/(L_{e2}/d_2)^2$ for flatwise or biaxial bending (d_2 = narrow face)

AND

$f_{b1} < F_{bE} = 1.20\ E_{min}'/R_b^2$



16.4

Bending and axial load on dimensional lumber

Example 16-11: A 4.5×5.5 column, built up using three 2×6s of structural Select Red Oak, is 20′ long with fixed ends and has a factored axial load of 5000#, a factored M_x of 800 #-in and M_y of 400 #-in. Is this column adequate?

1. Find values for species and grade:

   $F_b = 1150\text{psi},\ F_c = 1000\text{psi},\ E_{min} = 510{,}000\text{psi}$

2. Find section properties: $A = 3(8.25) = 24.74\text{in}^2$, $S_x = 3(7.56) = 22.68\text{in}^3$,

   $I_y = \Sigma I_{yi} + \Sigma A d_y^2 = 3(1.547) + 2(8.25)(1.5)^2 = 41.766\text{in}^4$

   $c = 1.5 + .75 = 2.25''$

   $S_y = 41.766/2.25 = 18.56\text{in}^3$

3. Find: f_c, f_{b1}, f_{b2}, F_c', F_{CE1}, F_{CE2}, F_{b1}', F_{b2}', F_{bE}

   $f_c = P/A = 5000/24.74 = 202.1\text{psi}$

   $f_{b1} = M_x/S = 800/17.65 = 45.33\text{psi}$

   $f_{b2} = M_y/S = 400/18.56 = 21.55\text{psi}$

4. Find F_{CE1}, F_{CE2}:

   $E_{min}' = 510{,}000(1.5) = 765{,}000\text{psi}$

   $L_u = 20\text{ft}(12'') = 240''$

   $L_e = kL = 0.65(240) = 156$

   $L_e/d_1 = 156/5.5 = 28.36$

   $F_{CE1} = 0.822(765{,}000)/28.36^2 = 781.85$

   $F_{CE1} = 781.85 > 202.1 = f_c$ … okay

   $L_e/d_2 = 156/4.5 = 34.67$

   $F_{CE2} = 0.822(765{,}000)/34.67^2 = 523.15$

   $F_{CE2} = 523.15 > 259.74 = f_c$ … okay

5. Find C_P:

   $F_c' = F_c(C_F)(C_P)(2.16)\lambda = 1000(1.3)(2.16)(.8)C_P$
   $= 2246.4C_P$

   $F_{CE1}/F_C^* = 781.85/2246.4 = 0.348 = F$

   $C_{P1} = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = (1.348/1.6)$
   $- [(1.348/1.6)^2 - (0.348/0.8)]^{1/2} = 0.318$

   $F_{CE2}/F_C^* = 523.15/2246.4 = 0.233$

   $C_{P2} = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2}$
   $= (1.233/1.6) - [(1.233/1.6)^2 - (0.233/0.8)]^{1/2} = 0.221$

   Use lesser value of $C_P = 0.221$

6. Check compression:

   $F_c' = 0.221(2246.4) = 496.50\text{psi} > 202.1 = f_c$ … okay for compression.

7. Find C_L, F_{b1}', F_{b2}':

   $F_b' = F_b(C_L)(C_F)(2.16)\lambda = 1150(1.3)(2.16)(.8)\ C_L$
   $= 2583.36\ C_L$

   $L_e = 1.84L_u$ (equal end moments) $= 1.84(240) = 441.6$

   $R_{b1}^2 = L_e d_1/d_2^2 = 441.6\ (5.5)/4.5^2 = 119.94$

   $F_{bE1} = 1.2(765{,}000)/119.94 = 7653.78$

   $F_{bE1}/F_b^* = 7653.78/2583.36 = 2.963$

   $C_{L1} = (1 + F)/1.9) - \sqrt{[((1 + F)/1.9)^2 - (F/0.95)]}$
   $= (3.963)/1.9) - \sqrt{[(3.963/1.9)^2 - (2.963/0.95)]} = 0.976$

   $F_{b1}' = 0.976(2583.36) = 2521.36 > 45.33$ … okay

   $R_{b2}^2 = Led_2/d_1^2 = 441.6(4.5)/5.5^2 = 65.69$

   $F_{bE2} = 1.2(765{,}000)/65.69 = 13974.73$

   $F_{bE2}/F_b^* = 13974.73/2583.36 = 5.41$

   $C_{L2} = (1 + F)/1.9) - \sqrt{[((1 + F)/1.9)^2 - (F/0.95)]}$
   $= (6.41)/1.9) - \sqrt{[(6.41/1.9)^2 - (5.41/0.95)]} = 0.989$

   $F_{b2}' = 0.989(2583.36) = 2554.94 > 21.55\text{psi} = f_{b2}$ … okay

8. F_{bE} = lesser of F_{bE1} and F_{bE2}:
   F_{bE} = 7653.78psi

Summary of values found		
$f_c = 202.10$psi	$f_{b1} = 45.33$psi	$f_{b2} = 21.55$
$F_c' = 496.50$psi	$F_{CE1} = 781.85$	$F_{CE2} = 523.15$
$F_{b1}' = 2521.36$	$F_{b2}' = 2554.94$psi	$F_{bE} = 7653.78$

9. $[f_c/F_c']^2 + f_{b1}/\{F_{b1}'[1 - (f_c/F_{CE1})]\} + f_{b2}/\{F_{b2}'[1 - (f_c/F_{CE2}) - (f_{b1}/F_{bE})^2]\} \leq 1.0$

   $[202.1/496.5]^2 + 45.33/\{2521.36[1 - (202.1/781.85)]\} + 21.55/\{2554.94[1 - 202.1/523.15 - 45.33/7653.78]\}$
   $= 0.166 + 0.024 + 0.014 = 0.204 < 1.0$ … okay

Combined axial tension and flexure:

$f_t/F_t' + f_b/F_b^* \leq 1.0$ where $F_b^* = F_b$ times all factors but C_L

Example 16-12: Check the adequacy of a 4×16 dimensional lumber beam with L = 16′, one concentrated load at mid-span of 3000# and a tension load of 1500#, structural Select Northern Red Oak with full lateral bracing.

F_b = 1400psi, F_t = 800psi, F_v = 220psi,
E = 1400000psi, E_{min} = 510000psi, G = 0.68

4×16: A = 53.38in^2, S = 135.66in^3, I = 1034in^4

1. Check flexure:

   $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(2.16)(\lambda) = 1400(1)(1)(C_L)(1)(2.16)(0.8) = 2419.2C_L$

   C_L: d/b = 15.25/3.5 = 4.357, but with full lateral bracing, C_L = 1.
   F_b' = 2419.2psi

   weight of beam = 1.2(.68)(62.4)(53.38/144) = 18.88$^{\#/f}$

   $M = wL^2/8 + PL/4 = 18.88(16)^2(12)/8 + 3000(16)(12)/4$
   = 151249.92#-in

   S_x = 135.66

   f_b = M/S = 151249.92/135.66 = 1114.92psi < 2419.2psi … okay for flexure

2. Check tension:

   F_t' = 800(2.16)(0.8) = 1382.4psi

   f_t = P/A = 1500/53.38 = 28.1psi < 1382.4psi … okay for tension

3. Check flexure and tension combined:

   $f_t/F_t + f_b/F_b^*$ = 28.1/1382.4 + 1114.92/ 2419.2
   = 0.48 < 1.0 … okay

4. Check shear:

   $F_v' = F_v(C_m)(C_t)(2.16)(\lambda) = 220(1)(1)(2.16)(0.8) = 380.16$

   V = 18.88(16)/2 + 3000/2 = 1651.05

   f_v = 3V/2A = 3(1651.05)/[2(3.5)(15.25)]
   = 46.40 < 354.24psi … okay for shear

5. Check deflection:

   Δ_{all} = L/240 = 16(12)/240 = 0.8″

   $E' = E(C_m)(C_t)$ = 1,400,000(1)(1) = 1,400,000 psi

   I = 1034in^4

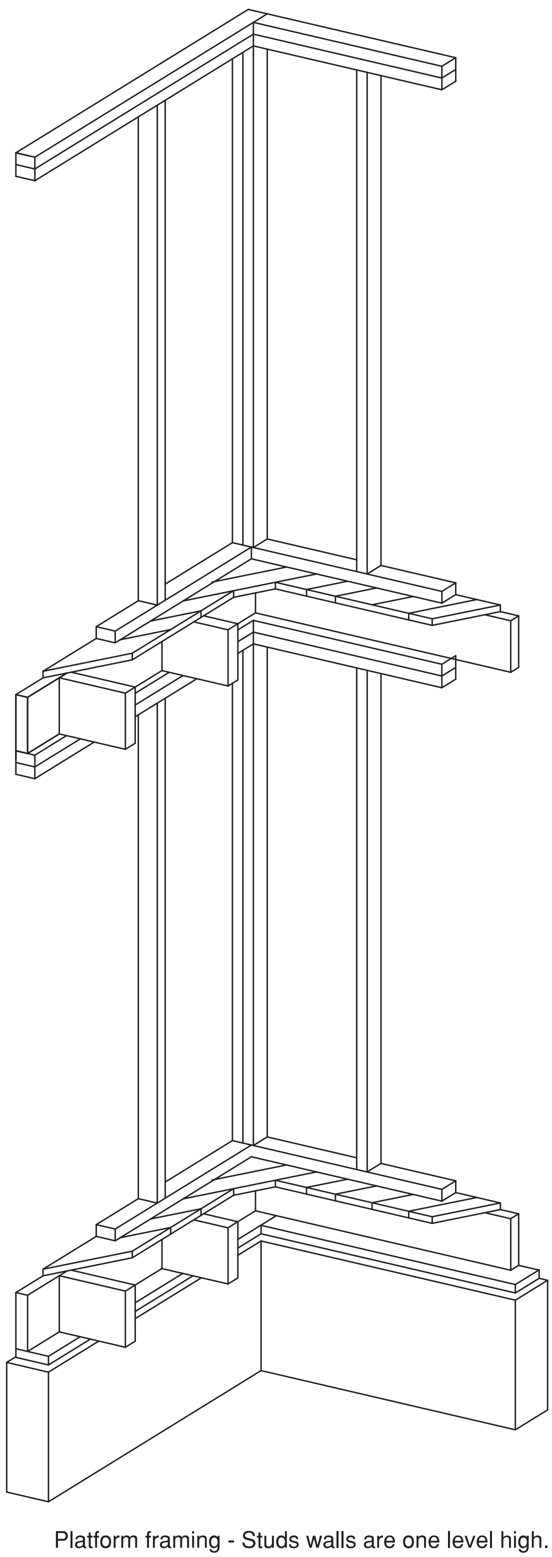
   unfactored load: P = 3000, W = .68(62.4)(53.38)/144
   = 14.99

   $\Delta_{max} = 5wl^4/384EI + PL^3/48EI$ = 5(14.99)(16)^4(1728)/384(1400000)(1034) + 3000(16)^3(1728)/48(1400000)(1034)
   = 0.32 < 0.8 … okay

16.3 Western Framing Considerations

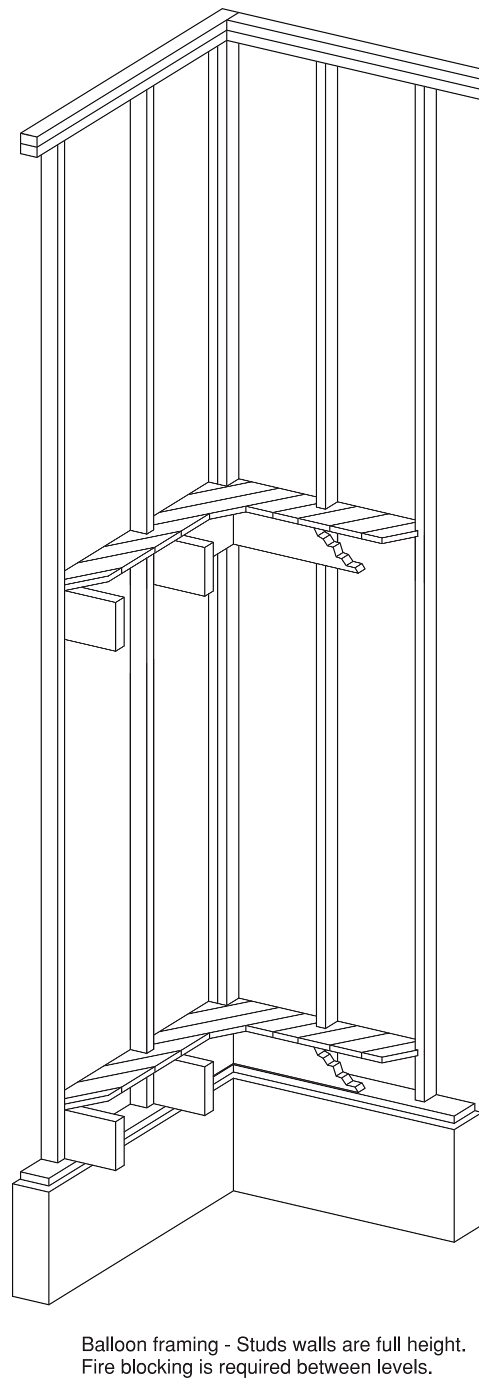
There are two types of Western Framing: platform framing and balloon framing. Platform framing, as shown in Figure 16.5, uses the method of framing a single level at one time, then building a horizontal framing system or platform on which to set the next level walls.

Balloon framing uses studs that have multiple level lengths as shown in Figure 16.6. This method is often employed when there are multilevel height spaces involved in the design. The disadvantage to balloon framing is that longer studs are more expense and more prone to warping and bending.



16.5

Platform framing



16.6

Balloon framing

16.3.1 Choosing a Stud Wall Size

Stud walls are historically made of 2 × 4 @ 16″o.c., although that standard has changed to 2 × 6 @ 24″o.c. for reasons of strength, economy and thermal comfort. Compare the efficiency of a 2 × 4 @ 16″o.c. with 2 × 6 @ 24″o.c.:

If a wall carries w $^{\#/f}$, the axial force P on each stud = (w$^{\#/f}$)(spacing f) and the compressive stress on each stud = f_c = P/A. If the allowable compressive stresses in the studs are equal, as they would be for the same Western species and grade, then the following comparison yields:

2 × 4 @16″ wall:	2 × 6 @ 24″ wall:
P = 16W/12 = 1.333W	P = 24W/12 = 2W
f_c = 1.333W/(1.5)(3.5)	f_c = 2W/(1.5)(5.5)
= 0.254W	= 0.242W

A wall with 2 × 6 @ 24″ can actually carry more load than 2 × 4 @ 16″.

Further, a comparison of insulation yields:

2 × 4 @16″ wall:	2 × 6 @ 24″ wall:
d = 3.5″ … R-13	d = 5.5″ … R-21

A 2 × 6 wall can hold R-21 fiberglass batt insulation between studs compared to R-13 for a 2 × 4 wall. The thermal transfer through the stud is reduced because stud surface is reduced from 1.5″/16 in a 2 × 4 wall to 1.5″/24 in a 2 × 6 wall, a 33% reduction.

A 2 × 6 wall will have more material cost, but less labor cost. Energy savings will counteract the material cost whenever fuel prices are high enough.

16.3.2 Limitations in Western Framing

Most building codes limit Western frame construction to four levels in height. Even at only four levels, studs on ground level often need to be doubled or tripled to carry the gravity loads. Remember that double studs will reduce energy efficiency.

Practice Exercises:

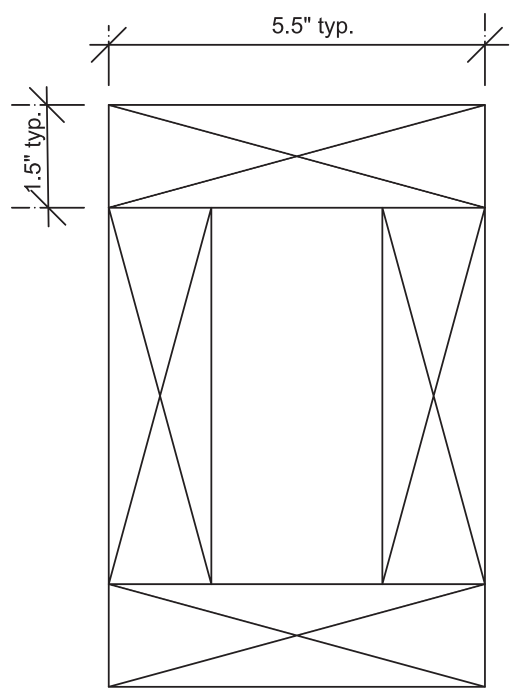
16-1: Design a series of No. 2 DFL floor joists spaced @ 24″o.c., with a moisture content of 20%, termite treatment and a span of 12′. There is a dead load of 15psf and a live load of 40psf.

16-2: Design a series of No. 1 Southern Pine floor joists spaced @ 16″o.c., with a moisture content of 18%, and a span of 15′. There is a dead load of 15psf and a live load of 80psf.

16-3: Determine how many select structural Southern Pine 2 × 12′s must be joined together to support a factored load of 300$^{\#/f}$ over a span of 18′.

16-4: Find the maximum factored compressive load an 8′, No. 1 Southern Pine 2 × 6 can support without bracing.

16-5: Determine the maximum unbraced length of the box column shown in Figure 16.7, subjected to a load of 2000# if the 2 × 6s are No. 2 DFL and there is incising and a moisture content of 21%.



16.7

Practice exercise 16-5

16-6: A 2 × 12 joist carrying a factored load of $600^{\#/f}$ and having a span of 14′ fully bears on a flat 2 × 6 No. 2 Southern Pine top plate. Is this acceptable?

16-7: What is the partial bearing length required for a No. 2 Southern Pine 2 × 10 bearing on a flat No. 2 Southern Pine 2 × 4 with a factored load of 2000#?

16-8: Find the maximum allowable tension in a No. 2 Southern Pine 2 × 4 with 18% moisture content and at room temperature.

16-9: A 3 × 5.5 column, built up using two 2 × 6s of No. 2 Southern Pine, is 16′ long with one end fixed and the other pinned. It has a factored axial load of 3000#, a factored Mx of 750 #-in and My of 150 #-in. Is this column adequate?

16-10: Check the adequacy of a 2 × 8 dimensional lumber beam with L = 12′, L_u = 4′, with two concentrated loads of 2000# at 4′o.c. and a tension load of 500#, using no. 1 DFL.

seventeen

Timber Design

Chapter 17 explains the LRFD Method for analysis and design of timber using factors derived by the American Wood Council (AWC). The LRFD (Load Resistance Factor Design) Method uses load factors to create an ultimate or factored load that is the design load. It also uses Resistance Factors (ϕ). To review finding ultimate loads, see Chapter 16.

Timber is sawn lumber in nominal sizes 5×5 and larger. As with dimensional lumber, timber nominal sizes are 0.5″ to 1″ larger than the actual sizes. Use actual sizes for design purposes.

17.1 Adjustment Factors for Timber

Adjustment factors for timber are the same as those for dimensional lumber although the values vary in some cases. See Table 16.1 for the adjustment factors for sawn lumber.

Values for λ and C_t can be found in Tables 16.2 and 16.3 respectively. Note that values for C_L and C_P are found using the same method as with dimensional lumber.

$C_r + C_i = 1.0$ for timber

$C_m = 1.0$ if moisture content is < 19%. Otherwise, for Western species, $C_m = 0.67$ for $F_{c\perp}$ and 0.91 for F_c (NOT So. Pine)

$C_F = (12/d)^{1/9}$ for $d \geq 12''$ in flexure only,

$C_F = 1$ otherwise

Table 17.1: Flat Use Factor for timber, with permission from the American Wood Council

C_{fu} Flat use factor for Timber		
Grade	F_b	E, E_{min}
Structural Select	0.86	1
No. 1	0.74	.9
No.2	1	1

17.2 Design of Timber Components

17.2.1 Design of Timber Beams:

1. Identify F_b, F_v, E, E_{min} for species and grade. See Table A2.5 for Material Properties of selected Timber Species
2. Assume FACTORED beam weight = $W_{FBM} = L(10)^{\#/f}$; Assume d < 12″
3. $F_b' = F_b(C_t)(C_L)(C_F)(C_{fu})(C_i)(2.16)(\lambda)$
   - C_F: is d < 12″?
     - Yes: $C_F = 1.0$
     - No: $C_F = (12/d)^{1/9}$
   - C_t: is temp. above 100°F?
     - No: $C_t = 1.0$
     - Yes: $C_t \rightarrow$ Table 16.3
   - C_{fu}: is beam laid flat like a plank?
     - No: $C_{fu} = 1.0$
     - Yes: $C_{fu} \rightarrow$ Table 17.1

λ: Find value from Table 16.2

Calculate $F_b' = F_b(C_t)(C_L)(C_F)(C_{fu})(2.16)(\lambda) = (F_b^*)(C_L)$

Assume $C_L = 1$ for now.

4. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

   $W_u = W_{FBM} + 1.2(W_{DL}) + 1.6(W_{LL})$ or if NO live load:

   $W_u = W_{FBM} + 1.4(W_{DL})$

   $P_u = 1.2P_D + 1.6\ P_L$ OR if NO LIVE LOAD:

   $P_u = 1.4P_D$

5. Find the maximum moment, M_u in the beam. Remember to multiply by 12″/f to obtain an answer in #-in.
6. $S_{req} > M_u/F_b'$ (F_b' from step 3)
7. Choose size based on S_{req}: Note A, S_x, I_x. See Table A2.4 Timber Section Properties.
8. C_L: find d/b and determine if $C_L = 1$. If not, calculate C_L using the steps described in Chapter 16.

   $F_b' = F_b^*\ (C_L)$

9. Check C_F for size from step 7.

   C_F : is d< 12″?

   Yes: $C_F = 1.0$

   No: $C_F = (12/d)^{1/9}$

10. Adjust F_b' for new C_F: F_b' = (F_b' from step 3)(C_F from step 9/C_F from step 3)
11. Find actual weight of beam: W_{BM} = (specific gravity) (62.4pcf)(A/144)
12. Find actual factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

    $W_u = 1.2(W_{BM} + W_{DL}) + 1.6(W_{LL})$ OR if NO LIVE LOAD:

    $W_u = 1.4(W_{BM} + W_{DL})$

    $P_u = 1.2P_D + 1.6\ P_L$ OR if NO LIVE LOAD: $P_u = 1.4P_D$

13. Find the maximum moment in the beam. Remember to multiply by 12″/f to obtain an answer in #-in.
14. $f_b = M/S$ M from step 13, S from step 7.
15. Is $f_b \le F_b'$?

    Yes → step 16

    No → estimate $S_{req} = M/F_b'$ and go back to step 7 and try larger size.

16. Is $f_b/F_b' \ge 0.90$?

    Yes → step 17

    No → estimate $S_{req} = M/F_b'$ and go back to step 7 and try smaller size.

17. $F_v' = F_v(C_t)(2.16)\ (\lambda)$

    C_t: Is temp. above 100°F?

    No: $C_t = 1.0$

    Yes: C_t → Table 16.3

18. Determine the maximum shear, V, in the beam.
19. $f_v = 3V/2A$
20. is $f_v \le F_v'$?

    Yes → step 21

    No → Estimate $A_{req} = 3V/2F_v'$ choose a larger size. If b and d are both greater than or equal to the original precious size, it is not necessary to check bending stress again. If not, go back to step 7 and check bending stress.

21. Δ_{all} = L(12″/f)/240 (check your local building codes for allowable deflections)
22. Unfactored loads: remember to use unfactored loads for deflection. W_{BM} is listed in step 11, and the applied loads are given. If an applied load is already factored, it may be used as is. Using a factored load will not create a safety issue; it will simply yield a larger required moment of inertia.
23. $E' = E(C_t)$

    C_t: Is temp. above 100°F?

    No: $C_t = 1.0$

    Yes: C_t → Table 16.3

24. Find Δ_{act} using deflection charts, by Double Integration Method or by Moment Area Method. Remember to multiply the equations by 1728in³/ft³ in order to obtain an answer in inches when using a length, L in feet. I_x is from step 3.
25. Is $\Delta_{act} \le \Delta_{all}$?

    Yes → done.

    No → find $I_{req} = \Delta_{act}$ (I_x from step 24)/ Δ_{all}

    Select final size based on I_{req}.

Example 17-1: Design the most efficient 36′ long timber beam of No. 2 Douglas Fir Larch 12×_ to support 3 point loads of LL = 800# and DL = 200 at 9′o.c. with lateral bracing only at point loads.

1. F_b = 875psi, F_v = 170psi, E = 1,300,000psi, E_{min} = 470,000psi, G = 0.5
2. Assume factored beam weight = L(10) = 36(10) = 360#/f; d < 12″
3. $F_b' = F_b(C_t)(C_L)(C_F)(C_{fu})(C_i)(2.16)(0.8) = 875(1)(C_L)(1)(1)(2.16)(0.8) = 1512C_L$

   Assume $C_L = 1$, F_b^* = 1512psi

4. Find factored loads:

$W_u = (W_{fbm}) = 360^{\#/f}$

$P_u = 1.2(200\#) + 1.6(800\#) = 1424\#$

5. $M = wL^2/8 + PL/2 = 360(36)^2/8 + (1424)(36)/2$
   $= 83{,}952$#-f $= 1{,}007{,}424$#-in
6. $S_{req} \geq M/F_b' = 1{,}007{,}424$#-in$/1512 = 666.29in^3$
7. Choose size based on Sreq: Try 12×20 $A = 213.75in^2$, $S = 676.88in^3$, $I_x = 6430.31in^4$.
8. $d/b = 19/11.25 = 1.69 < 2 \ldots C_L = 1$

   $F_b' = F_b^* (C_L) = 1512(1) = 1512psi$
9. Check C_F for size from step 7.
   C_F : is d< 12″? yes: $C_F = (12/19)^{1/9} = 0.95$
10. Adjust F_b' for new C_F: $F_b' = 1512(0.95) = 1436.74psi$
11. Find actual weight of beam: W_{BM} = (specific gravity) (62.4pcf)(A/144) $= .5(62.4)(213.75)/144 = 46.31^{\#/f}$
12. Find actual factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

    $W_u = 1.2(W_{BM}) = 1.2(46.31) = 55.58^{\#/f}$

    $P_u = 1424\#$ (same as in step 4)
13. $M_u = wL^2/8 + PL/2 = 55.58(36)^2/8 + (1424)(36)/2$
    $= 34{,}635.96$#-f $= 415{,}631.52$#-in
14. $f_b = M_u/S = 415{,}613.52/676.88 = 614.04psi$
15. Is $f_b \leq F_b'$? Yes → step 16 614.04 < 1436.74psi
16. Is $f_b/F_b' \geq 0.90$? No: 614.04/1436.74 = 0.43 estimate Sreq $= M_u/F_b' = 415{,}631.52/1436.74 = 282.33in^3$

7A. Try 12×14 $A = 149.06in^2$, $S = 329.18in^3$, $I_x = 2180.82in^4$.

8A. $d/b = 13.25/11.25 = 1.18 < 2 \ldots C_L = 1$
$F_b' = F_b^* (C_L) = 1512(1) = 1512psi$

9A. Check C_F for size from step 7.
C_F : is d < 12″? Yes: $C_F = (12/13.25)^{1/9} = 0.989$

10A. Adjust F_b' for new C_F: $F_b' = 1512(0.989) = 1495.37psi$

11A. Find actual weight of beam: W_{BM} = (specific gravity) (62.4pcf)(A/144) $= .5(62.4)(149.06)/144 = 32.3^{\#/f}$

12A. Find actual factored loads:

$W_u = 1.2(W_{BM}) = 1.2(32.3) = 38.76^{\#/f}$

$P_u = 1424\#$ (same as in step 4)

13A. $M_u = wL^2/8 + PL/2 = 38.76(36)^2/8 + (1424)(36)/2$
$= 31{,}911.12$#-f $= 382{,}933.44$#-in

14A. $f_b = M_u/S = 382{,}933.44/329.18 = 1163.29psi$

15A. Is $f_b \leq F_b'$? Yes → step 16 1163.29 < 1495.37psi

16A. Is $f_b/F_b' \geq 0.90$? No: 1163.29/1495.37 = 0.78
estimate $S_{req} = M_u/F_b' = 382{,}933.44/1495.37 = 256.08in^3$

7B. Try 12×12 $A = 126.54in^2$, $S = 237.3in^3$, $I_x = 1334.84in^4$.

8B. $d/b = 11.25/11.25 = 1 < 2 \ldots C_L = 1$
$F_b' = F_b^* (C_L) = 1512(1) = 1512psi$

9B. Check C_F for size from step 7.
C_F :is d < 12″? No: $C_F = 1$

10B. Adjust F_b' for new C_F: $F_b' = 1512(1) = 1512psi$

11B. Find actual weight of beam: W_{BM} = (specific gravity) (62.4pcf)(A/144) $= .5(62.4)(126.54)/144 = 27.42^{\#/f}$

12B. Find actual factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:
$W_u = 1.2(W_{BM}) = 1.2(27.42) = 32.9^{\#/f}$

$P_u = 1424\#$ (same as in step 4)

13B. $M_u = wL^2/8 + PL/2 = 32.9(36)^2/8 + (1424)(36)/2$
$= 30{,}961.8$#-f $= 371{,}541.6$#-in

14B. $f_b = M_u/S = 371{,}541.6/237.3 = 1565.70psi$

15B. Is $f_b \leq F_b'$? No: 1565.70 > 1512psi … use 12×14 for flexure.

17. $F_v' = F_v(C_t)(2.16)(\lambda)$ C_t: Is temperature above 100°F? No: $C_t = 1.0$
    $F_v' = 170(1)(2.16)(.8) = 293.76psi$
18. $V = wL/2 + 3P/2 = 38.76^{\#/f}(36'/2) + 3(1424\#)/2$
    $= 2833.68\#$
19. $f_v = 3V/2A = 3(2833.68\#)/[2(149.06in^2)] = 114.06psi$
20. Is $f_v \leq F_v'$? Yes: 114.06psi < 293.76psi … okay for shear.
21. $\Delta_{all} = L(12''^{/f})/240 = 36(12)/240 = 1.8''$
22. $w = 32.3^{\#/f}$, $P = 200\# + 800\# = 1000\#$
23. $E' = E(C_t)$
    C_t: is temp. above 100°F? No: $C_t = 1.0$
    $E' = 1{,}300{,}000psi(1) = 1{,}300{,}000psi$
24. $\Delta_{act} = 5wL^4/384EI + .0495PL^3/EI = [5(32.3)(36^4)/384 + .0495(1000)(36^3)](1728)/[1{,}300{,}000(2180.82)] = 1.842''$
25. Is $\Delta_{act} \leq \Delta_{all}$? No → 1.842″ > 1.8″ … $I_{req} = 1.842\,(2180.82)/1.8 = 2231.95$, Use 12×16: $I_x = 3164.06in^4$

ANSWER: USE 12×16

Example 17-2: Design a short, heavily-loaded Douglas Fir Larch (DFL) No.1 timber beam where L = 4′, P_u = 40,000# DL at center, fixed ends, no repetition and no lateral support.

1. F_b = 1350psi, F_v = 170psi, E = 1,600,000psi, E_{min} = 580,000psi, G = 0.5
2. Assume FACTORED beam weight = W_{FBM} = L(10)$^{\#/f}$ = 40$^{\#/f}$; Assume d < 12″
3. $F_b' = F_b(C_t)(C_L)(C_F)(C_{fu})(C_i)(2.16)(\lambda)$

   C_F : is d < 12″? Yes: C_F = 1.0

   C_t: is temp. above 100°F? No: C_t = 1.0

   C_{fu}: is beam laid flat like a plank? No: C_{fu} = 1.0

   λ = 0.6

   Calculate $F_b' = F_b(C_t)(C_L)(C_F)(C_{fu})(2.16)(\lambda) = (F_b{}^*)(C_L)$

   $= 1350(C_F)(C_L)(2.16)(0.6) = 1749.6C_L$

   Assume C_L = 1 for now.
4. W_u = 1.4 W_{FBM} = 1.4(40$^{\#/f}$) = 56$^{\#/f}$, P_u = 1.4P_D = 1.4(40,000#) = 56,000#
5. M_u = wL^2/8 + PL/4 = 56$^{\#/f}$(4′)^2/8 + 56,000(4′/4) = 56,112#-f = 673,344#-in.
6. $S_{req} > M_u/F_b'$ = 673,344#-in/1749.6psi = 384.86
7. Because of short length and heavy load, choose a bulky size with a large area.

   Try 12 × 16: A = 168.75in^2, S = 421.88in^3,

   I_x = 3164.06in^4
8. d/b = 15.25/11.25 = 1.36 < 2 … C_L = 1

   $F_b' = F_b{}^*$ (C_L) = 1749.6(1) = 1749.6psi
9. Check C_F for size from step 7.

   C_F : is d< 12″? No: $C_F = (12/15.25)^{1/9}$ = 0.974
10. Adjust F_b' for new C_F: F_b' = 1749.6(.974) = 1704.11psi
11. W_{BM} = (specific gravity)(62.4pcf)(A/144) = .5(62.4)(168.75/144) = 36.56$^{\#/f}$
12. W_u = 1.4(36.56) = 51.19$^{\#/f}$

    P_u = 1.4(40,000) = 56,000#
13. M_u = wL^2/8 + PL/4 = 51.19$^{\#/f}$(4′)^2/8 + 56,000(4′/4) = 56,102.38#-f = 673,228.56#-in
14. $f_b = M_u/S$ = 673,228.56/421.88 = 1595.78psi
15. Is $f_b \le F_b'$? Yes: 1595.78psi < 1704.11psi
16. Is $f_b/F_b' \ge 0.90$? Yes: 1595.78/1704.11 = 0.94 … okay for flexure
17. $F_v' = F_v(C_t)(2.16)$ (λ)

    C_t: Is temp. above 100°F? No: C_t = 1.0

    F_v' = 170(1)(2.16)(.6) = 220.32psi
18. V = wL/2 + P/2 = 51.19(4/2) + 56,000/2 = 28,103.8#
19. f_v = 3V/2A = 3(28,103.8)/[2(168.75)] = 249.81psi
20. Is $f_v \le F_v'$? No → estimate $A_{req} = 3V/2F_v'$ = 3(28,103.8)/[2(220.320] = 191.34in^2

    Use 14 × 16: A = 198.75in^2, S = 496.08in^3,

    I = 3726.56in^4 because both dimension are equal or larger than previous size, there is no need to recheck for flexure.
21. Δ_{all} = L(12″/f)/240 = 4(12)/240 = 0.20″
22. w = .5(62.4)(198.75/144) = 43.06$^{\#/f}$, P = 40,000#
23. E′ = E(C_t)

    C_t: is temp. above 100°F? No: Ct = 1.0

    E′ = E(1) = 1,600,000psi
24. Δ_{act} = 5wL^4/384EI + PL^3/48EI = 5(43.05)(4^4)(1728)/[384(1,600,000)(3726.56)] + 40,000(43)(1728)/[48(1,600,000)(3726.56)] = .015″
25. Is $\Delta_{act} \le \Delta_{all}$? Yes: 0.015″ < 0.2″

ANSWER: USE 14 × 16

17.2.2 Compression in Timber

Adjustment factors of sawn lumber:

The equation for allowable compressive stress is:

$$F_c' = F_c(C_m)(C_t)(C_F)(C_i)(C_P)(2.16)(\lambda)$$

where the factors are described at the beginning of this chapter. Remember that C_i is not used with timber and therefore C_i = 1. C_P is described in Chapter 16.

$$F_c' = F_c(C_m)(C_t)(C_F)(C_P)(2.16)(\lambda)$$

Like the design of wood beams, the design of columns is an iterative process based on an assumed trial size. In the case of wood columns, a good starting point is $A_{trial} = P_u/F_c{}^*$ where $F_c' = F_c{}^*C_{P.}$

Design of wood columns:

1. Look up F_c and E_{min} for the given species and grade of lumber.
2. $F_c' = F_c(C_m)(C_t)(C_F)(C_P)2.16\ (\lambda) = F_c{}^*(C_P)$

   C_m: Is moisture content over 19%?

   No: C_m = 1.0

   Yes: C_m = 91

   C_t: Is temp. above 100°F?

   No: C_t = 1.0

   Yes: Determine C_t from Table 16.3

   Assume C_P = 1 and C_F = 1 for now.
3. Calculate L_e = kL(12″/f) in each direction. Effective Length Factor, k can be found in Figure 10.1 (k = 1.0 for pin–pin, k = .8 for pin–fix, k = .65 for fix–fix).

Determine minimum width in each direction based on $L_e/d < 50$.

$d_{min} = L_{ex}/50$ and $b_{min} = L_{ey}/50$

4. $A_{trial} = P/F_c{}^*$
   Select a size with $A \geq A_{trial}$, $b \geq b_{min}$, and $d \geq d_{min}$. Note A, b and d.
5. Use larger of L_e/d or L_{ey}/b and L_{ex}/d.
6. $E_{min}' = E_{min}(C_t)(1.5)$
   C_t: Is temp. above 100°F?
   No: $C_t = 1.0$
   Yes: Determine C_t from Table 16.3
7. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2$
8. $F = F_{cE}/F_c{}^*$
9. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2}$
11. $F_c' = F_c{}^*(C_P)$ = allowable compressive stress
12. $f_c = P/A$ = actual compressive stress
13. Is $f_c < F_c'$? Yes → step 15
    No → go back to step 4 and choose larger size.
14. Is $f_c/F_c' \geq 0.90$? If not, go back to step 4 and try smaller size.

Example 17-3: Design a square No.1 Southern Pine column 12′ long to carry a factored axial load of 50,000# with pinned connections and a moisture content of 20%.

1. F_c = 825psi and E_{min} = 550,000psi.
2. $F_c' = F_c(C_m)(C_t)(C_F)(C_P)2.16\ (\lambda) = F_c{}^*(C_P)$
   C_m: Is moisture content over 19%? Yes: $C_m = .91$
   C_t: Is temp. above 100°F? No: $C_t = 1.0$
   Assume $C_P = 1$ and $C_F = 1$ for now.
   $F_c' = 825(.91)(1)(1)(2.16)(.8) = 1297.3(C_P)$psi …
   $F_c{}^* = 1297.3$psi
3. $L_e = kL(12''^{/f}) = 1.0(12')(12''^{/f}) = 144''$
   $d_{min} = b_{min} = 144''/50 = 2.88''$
4. $A_{trial} = P/F_c{}^* = 50{,}000\#/1297.3 = 38.54\text{in}^2$
   Try 8×8: $A = 52.56\text{in}^2$, $d = b = 7.25''$, $S = 63.51\text{in}^3$, $I = 230.23\text{in}^4$
5. Use larger of $L_e/d = 144''/7.25 = 19.86$
6. $E_{min}' = E_{min}(C_t)(1.5)$
   C_t: Is temp. above 100°F? No: $C_t = 1.0$
   $E_{min}' = 550{,}000(1)(1.5) = 825{,}000$psi
7. $F_{CE} = 0.822(E_{min}')/(L_e/d)^2 = 0.822(825{,}000)/19.86^2 = 1719.36$psi
8. $F = F_{CE}/F_c{}^* = 1719.36/1297.3 = 1.325$
9. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = 0.778$
11. $F_c' = F_c{}^*(C_P)$ = allowable compressive stress = 1297.3(0.778) = 1009.3psi
12. $f_c = P/A$ = actual compressive stress = $50{,}000\#/52.56\text{in}^2$ = 951.29psi
13. Is $f_c < F_c'$? Yes: 951.29psi < 1009.3psi
14. Is $f_c/F_c' \geq 0.90$? Yes: 951.29/1009.3 = 0.94

ANSWER: USE 8×8

17.2.3 Bearing in Timber:

Bearing in timber uses the same method and value for C_b as bearing in dimensional lumber. See section 16.2.3. Remember to use timber values for $F_{c\perp}$ and C_m.

17.2.4 Tension in Timber:

Tension in timber uses the same method as tension in dimensional lumber. See section 16.2.4. Remember to use timber values for F_t and C_F. Also remember that $C_m = 1$ and $C_i = 1$ for timber in tension.

17.2.5 Combined Stresses in Timber:

Combined flexure and axial compression:

$$[f_c/F_c']^2 + f_{b1}/\{F_{b1}'[1 - (f_c/F_{CE1})] + f_{b2}/\{F_{b2}'[1 - (f_c/F_{CE2}) - (f_{b1}/F_{bE})^2]\} \leq 1.0$$

Where

$f_c < F_{CE1} = 0.822\ E_{min}'/(L_{e1}/d_1)^2$ for edge-wise or biaxial bending (d_1 = wide face)

AND

$F_c < F_{CE2} = 0.822\ E_{min}'/(L_{e2}/d_2)^2$ for flatwise or biaxial bending (d_2 = narrow face)

AND

$f_{b1} < F_{bE} = 1.20E_{min}'/R_b{}^2$

Example 17-4: A 6×8 column of structural Select Red Oak is 20′ long with fixed ends and has a factored axial load of 20,000#, a factored M_x of 800 #-in and M_y of 400 #-in. Is this column adequate?

1. Find values for species and grade:

   F_b = 1350psi, F_c = 875psi, E_{min} = 440,000psi

2. Find section properties: A = 39.88in^2, S_x = 48.18in^3, S_y = 36.55in^3

3. Find: f_c, f_{b1}, f_{b2}, F_C', F_{CE1}, F_{CE2}, F_{b1}', F_{b2}', F_{bE}

   f_c = P/A = 20000/39.88 = 501.50psi

   f_{b1} = Mx/S = 800/48.18 = 21.89psi

   f_{b2} = My/S = 400/36.55 = 10.94psi

4. Find F_{CE1}, F_{CE2}:

   E_{min}' = 440,000(1.5) = 660,000psi

   L_u = 20ft(12″) = 240″

   L_e = kL = 0.65(240″) = 156″

   L_e/d_1 = 156/7.5 = 20.8

   F_{CE1} = 0.822(660,000)/20.8^2 = 1253.98psi

   F_{CE1} = 1253.98psi > 501.50psi = f_c … okay

   L_e/d_2 = 156/5.5 = 28.36

   F_{CE2} = 0.822(660,000)/28.36^2 = 674.36psi

   F_{CE2} = 674.36psi > 501.50psi = f_c … okay

5. Find C_P:

   $F_c' = F_c(C_F)(C_P)(2.16)\lambda$ = 875(1)(2.16)(.8)C_P = 1512C_P

   F_{CE1}/F_C^* = 1253.98/1512 = 0.829

   C_{P1} = 0.623

   F_{CE2}/F_C^* = 674.36/1512 = 0.446

   C_{P2} = 0.394

   Use lesser value of C_P = 0.394

6. Check compression:

   F_c' = 0.394(1512) = 596.60psi > 501.50 = f_c … okay for compression.

7. Find C_L, F_{b1}', F_{b2}':

   $F_b' = F_b(C_L)(C_F)(2.16)\lambda$ = 1350(1)(1)(2.16)(.8) C_L = 2332.8 C_L

   C_L = 1 because d_1/d_2 = 7.5/5.5 = 1.36 < 2

   F_{b1}' = (1)(2332.8) = 2332.8psi > 21.89psi = f_{b1} … okay

   F_{b2}' = (1)(23328) = 2332.8psi > 10.94psi = f_{b2} … okay

8. F_{bE} = lesser of F_{bE1} and $F_{bE2:}$ F_{bE} = 2332.8psi

Summary of values found

f_c = 501.50psi	f_{b1} = 21.89psi	f_{b2} = 10.94psi
F_c' = 596.60psi	F_{CE1} = 1253.98psi	F_{CE2} = 674.36psi
F_{b1}' = 2332.8psi	F_{b2}' = 2332.8psi	F_{bE} = 2332.8psi

9. $[f_c/F_c']^2 + f_{b1}/\{F_{b1}'[1 - (f_c/F_{CE1})]\} + f_{b2}/\{F_{b2}'[1 - (f_c/F_{CE2}) - (f_{b1}/F_{bE})^2]\} \le 1.0$

   [501.50/596.60]^2 + 21.89/{2332.8[1 − (501.50/1253.98)] + 10.94/{2332.8[1 − 501.50/674.36 − (21.89/2332.8)^2] = 0.707 + 0.016 + 0.018 = 0.741 < 1.0 … okay

Combined axial tension and flexure:

$f_t/F_t' + f_b/F_b^* \le 1.0$ Where F_b^* = F_b times all factors but C_L

Example 17-5: Check the adequacy of a 6×10 timber beam with L = 16′, L_u = 8′, one concentrated load at midspan of 3000#, a tension load of 1500#, structural Select Northern Red Oak.

F_b = 1600psi, F_t = 950psi, F_v = 205psi, E = 1300000psi, E_{min} = 470000psi, G = .68

6×10: A = 50.88, S = 78.43, I = 362.75

1. Check flexure:

   $F_b' = F_b(C_m)(C_t)(C_L)(C_F)(2.16)(\lambda)$ = 1600(1)(1)(C_L)(1)(2.16)(0.8) = 2764.8C_L

   C_L: d/b = 9.25/5.5 = 1.68 < 2 … C_L = 1

   F_b' = 2764.8(1) = 2764.8psi

   weight of beam = 1.2(.68)(62.4)(50.88/144) = 17.99$^{\#/f}$

   M = wL^2/8 + PL/4 = 17.99(16)^2(12)/8 + 3000(16)(12)/4 = 150908.16#-in

   S_x = 78.43

   f_b = M/S = 150908.16/78.43 = 1924.11< 2748.21 … okay for flexure

2. Check tension:

   $F_t' = 950(2.16)(0.8) = 1231.2psi$

   $f_t = P/A = 1500/50.88 = 29.48 < 1231.2$ psi … okay for tension

3. Check flexure and tension combined:

   $f_t/F_t + f_b/F_b^* = 29.48/1231.2 + 1924.11/2764.8 = 0.72 < 1.0$ … okay

4. Check shear:

   $F_v' = F_v(C_m)(C_t)(2.16)(\lambda) = 205(1)(1)(2.16)(0.8) = 354.24$

   $V = 17.99(16)/2 + 3000/2 = 1643.92\#$

   $fv = 3V/2A = 3(1643.92)/[2(5.5)(9.25)] = 48.47 < 354.24$ … okay for shear

5. Check deflection:

   $\Delta_{all} = L/240 = 16(12)/240 = 0.8''$

   $E' = E(C_m)(C_t) = 1{,}300{,}000(1)(1) = 1{,}300{,}000$ psi

   $I = bh^3/12 = 5.5(9.25)^3/12 = 363$

   unfactored load: $P = 3000\#$, $W = .68(62.4)(5.5)(9.25)/144 = 14.99\#/f$

   $\Delta_{max} = 5wl^4/384EI + PL^3/48EI = 5(14.99)(16)^4(1728)/384(1300000)(363) + 3000(16)^3(1728)/48(1300000)(363) = 0.98 > 0.8$ no good.

Practice Exercises:

17-1: Design the most efficient 20′ long timber beam of No. 2 Douglas Fir Larch 8×_ with a uniform dead load, $w_D = 30^{\#/f}$ and a uniform live load $w_L = 640^{\#/f}$ with full lateral bracing.

17-2: Design the most efficient, 16′ long timber beam of No. 1 Douglas Fir Larch 6×_ with a uniform dead load, $w_D = 20^{\#/f}$ and a uniform live load $w_L = 600^{\#/f}$ with lateral bracing at 4′o.c.

17-3: Design a square select structural DFL column 16′ long to carry a factored axial load of 80,000# with fixed connections and a moisture content of 16%.

17-4: Design a 6×_ No. 2 Southern Pine column 12′ long to carry a factored axial load of 90,000# with pinned connections and bracing at 4′ from top in the weak direction.

17-5: A 6×10 column of select structural DFL is 20′ long with fixed ends and has a factored axial load of 50000#, a factored M_x of 400 #-in and M_y of 200 #-in. Is this column adequate?

17-6: Check the adequacy of an 8×16 timber beam with L = 24′, L_u = 8′, two concentrated loads of 3000# each at 8′o.c. and, a tension load of 1500#, structural Select Northern Red Oak.

eighteen

Glue-Laminated Lumber Design

Chapter 18 explains the LRFD method for analysis and design of glue-laminated lumber (glu-lams) using factors derived by the American Wood Council (AWC). The LRFD (Load Resistance Factor Design) Method uses load factors to create an ultimate or factored load that is the design load. It also uses Resistance Factors (ϕ). To review finding ultimate loads, see Chapter 16.

Glu-lams are specified by flexural stress and Modulus of Elasticity when used for beams. For example, a 24F-1.8E specifies F_b = 2400psi and E = 1,800,000psi or 1.8×10^6psi. Designations for glu-lams used primarily in tension or compression consist of a combination symbol followed by species designation and grade. For example, 47/SP/N2M12 refers to a glu-lam with a combination number of 47, Southern Pine species, and a grade designation of N2M12. Softwood glue-lams used in compression can be found in Table 5B of the NDS Supplement. Sample values for problem solving are given in this text in Table A2.8.

18.1 Adjustment Factors for Glu-Lams

Glu-lams can be manufactured to specific sizes and shapes using lamination thicknesses that can vary from .125″ to 1.5″. When using the section properties chart for glu-lams, the nominal sizes are the actual sizes for design purposes. The charts list standard sizes for Western species and for Southern Pine, but again, glu-lams can be custom made to any size.

Note: Only the lesser value of C_v and C_L is applied, NOT BOTH!

Allowable stresses, F′, in Table 18.1 are found by multiplying the design values listed for a given species of wood from Table 6A and 6B of the National Design Specifications Supplement by the applicable factors. Table A2.8 contains sample values for use with examples and exercises in this book.

> Values for λ, and C_t can be found in Tables 16.2 and 16.3, respectively.
>
> Note that values for C_L, C_P and C_b are found using the same method as with dimensional lumber. See Chapter 16.
>
> There is no value of C_r or C_i for glu-lams.

C_m is the Wet Service Factor. In glu-lams, C_m is used when the moisture content is greater than 16%.

Table 18.1: Adjustment factors for glu-lams, with permission from the American Wood Council

ADJUSTMENT FACTORS FOR GLU-LAMS											
		Temperature Factor	Wet Service Factor	Beam Stability Factor	Volume Factor	Flat Use Factor	Curvature Factor	Column Stability Factor	Bearing Factor	$K_F\phi$	Time Effect Factor
$F_b' = F_b$	x	C_t	C_m	C_L	C_V	C_{fu}	C_c			2.16	λ
$F_t' = F_t$	x	C_t	C_m							2.16	λ
$F_c' = F_c$	x	C_t	C_m					C_P		2.16	λ
$F_v' = F_v$	x	C_t	C_m							2.16	λ
$F_{c\perp}' = F_{c\perp}$	x	C_t	C_m						C_b	1.5	
$F_{rt}' = F_{rt}$	x	C_t	C_m							2.16	λ
$E' = E$	x	C_t	C_m								
$E_{min}' = E_{min}$	x	C_t	C_m							1.5	

Table 18.2: C_m Wet Service Factor for glu-lams, with permission from the American Wood Council

C_m Wet Service Factor for Glu-Lams	
C_m = 1.0 if moisture content is < 16%.	
Design Values	C_m
F_b, F_t	0.8
F_c	0.73
F_v	0.875
$F_{c\perp}$	0.53
E, E_{min}	0.833

C_V is the Volume Factor. It is only used with glu-lams.

$C_v = (21/L)^{1/X}(12/d)^{1/X}(5.125/b)^{1/X} \le 1.0$

Where:

L = length in feet of bending member between points of zero moment.

d = depth of bending member in inches

b = width of bending member in inches. When the width is made of multiple pieces, b = width of widest piece in inches and ≤ 10.75″

x = 20 for Southern Pine

x = 10 for all other species.

C_{fu} is the Flat Use Factor. In glu-lams, C_{fu} is used if the laminations are vertical and the depth of the beam, dy < 12″.

$C_{fu} = (12/dy)^{1/9}$

C_c is the curvature factor.

$C_c = 1 - 2000(t/R)^2${/eq

Where:

t = thickness of laminations in inches

R = radius of curvature of the inside face of the member in inches.

$t/R \leq 1/100$ for hardwoods and Southern Pine

$t/R \leq 1/125$ for other softwoods

For tapered and other non-prismatic members, there are also factors C_I, the stress interaction factor and C_{vr}, the shear reduction factor that are not addressed in this book.

18.2 Design of Glu-Lam Components

Component design using glue-laminated wood follows the same basic method of designing any wood component. Find the allowable stress using the design values and adjustment factors given for each condition and then compare the allowable stress to the actual stress. If the actual stress is greater than the allowable stress, the component needs to be resized.

18.2.1 Design of Glu-Lam Beams

Below is a step-by-step method for the design of beams using glue-laminated wood.

1. Using Table 5A of NDS Supplement, or Table A2.8, identify F_b, F_v, E, E_{min} and specific gravity for grade.
2. Assume FACTORED beam weight = $W_{FBM} = L(10)^{\#/f}$
3. $F_b' = F_b(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(2.16)(\lambda)$
   - C_m: Is moisture content over 16%?
     - No: $C_m = 1.0$
     - Yes: $C_m = .80$
   - C_t: Is temp. above 100°F?
     - No: $C_t = 1.0$
     - Yes: Determine C_t from Table 16.3
   - C_{fu}: Are laminations vertical and depth, dy < 12″?
     - No: $C_{fu} = 1.0$
     - Yes: $C_{fu} = (12/dy)^{1/9}$
   - $C_c = 1 - 2000(t/R)^2$
   - λ: Is there a live load or a factored load stated in the problem?
     - No: $\lambda = 0.6$
     - Yes: $\lambda = 0.8$
   - Calculate $F_b' = F_b(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(2.16)(\lambda) = (F_b^*)(C_L)(C_v)$
   - Assume C_L and $C_v = 1$ for now
4. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

   $W_u = W_{FBM} + 1.2(W_{DL}) + 1.6(W_{LL})$ OR if NO LIVE LOAD: $W_u = W_{FBM} + 1.4(W_{DL})$

   $P_u = 1.2P_D + 1.6\ P_L$ OR if NO LIVE LOAD: $P_u = 1.4P_D$
5. Find maximum moment (M). Remember to multiply by $12^{\prime\prime/f}$ to obtain an answer in #-in.
6. $S_{req} > M/F_b'$
7. Choose size based on S_{req} find: A, S_x, and I_x from Table A2.6 for Southern Pine and Table A2.7 for Western species.
8. C_L: find d/b and determine if $C_L = 1$. If not, calculate C_L using the steps described in Chapter 16.
9. $C_v = (21/L)^{1/X}(12/d)^{1/X}(5.125/b)^{1/X} \leq 1.0$
   X = 10 (not Southern Pine) X = 20 (Southern Pine)
10. $F_b' = [F_b^*]$[lesser of (C_L) or C_v] where F_b^* is from step 3.
11. Find actual weight of beam: W_{BM} = (specific gravity)(62.4pcf)$(A/144)^{\#/f}$
12. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

    $W_u = 1.2(W_{BM} + W_{DL}) + 1.6(W_{LL})$ OR if NO LIVE LOAD: $W_u = 1.4(W_{BM} + W_{DL})$

    $P_u = 1.2P_D + 1.6\ P_L$ OR if NO LIVE LOAD: $P_u = 1.4P_D$
13. Find maximum moment (M). Remember to multiply by $12^{\prime\prime/f}$ to obtain an answer in #-in.
14. $f_b = M/S$ M from step 13, S from step 7.
15. Is $f_b \leq F_b'$?
    - Yes → step 9
    - No → estimate $S_{req} = M/F_b'$ and go back to step 3 and try larger size.
16. Is $f_b/F_b' \geq 0.90$?
    - Yes → step 10
    - No → estimate $S_{req} = M/F_b'$ and go back to step 3 and try smaller size.
17. $F_v' = F_v(C_m)(C_t)(2.16)(\lambda)$
    - C_m: Is moisture content over 16%?
      - No: $C_m = 1.0$
      - Yes: $C_m = .875$
    - C_t: Is temp. above 100°F?

No: $C_t = 1.0$

Yes: Determine C_t from Table 16.3

18. Determine V using equations for V_u from Table A1.2.
19. $f_v = 3V/2A$
20. Is $f_v \le F_v'$?

    Yes → step 14

    No → estimate $S_{req} = M/F_b'$ and go back to step 10 and try larger size.
21. $\Delta_{all} = L(12''^{/f})/240$
22. Unfactored loads: use W_{BM} from step 11, loads are given
23. $E' = E(C_m)(C_t)$

    C_m: is moisture content over 16%?

    No: $C_m = 1.0$

    Yes: $C_m = .833$

    Ct: is temp. above 100°F?

    No: $C_t = 1.0$

    Yes: Determine C_t from Table 16.3
24. Find maximum deflection = Δ_{act} Remember to multiply by $1728in^3/ft^3$.
25. Is $\Delta_{act} \le \Delta_{all}$?

    Yes → done.

    No → find $I_{req} = \Delta_{act}$ (I_x from step 3)/ Δ_{all}. Select final size based on I_{req}.

Example 18-1: Design a 12.25″ wide Douglas Fir, 24F-1.8E glu-lam, spanning 80′ with concentrated live loads of 3000# and concentrated dead loads of 4000# spaced 10′o.c.

The beam is curved such that the midpoint of the beam is 8′ above the supports. The laminations are 0.75″ thick. Blocking occurs at points of load and at ends.

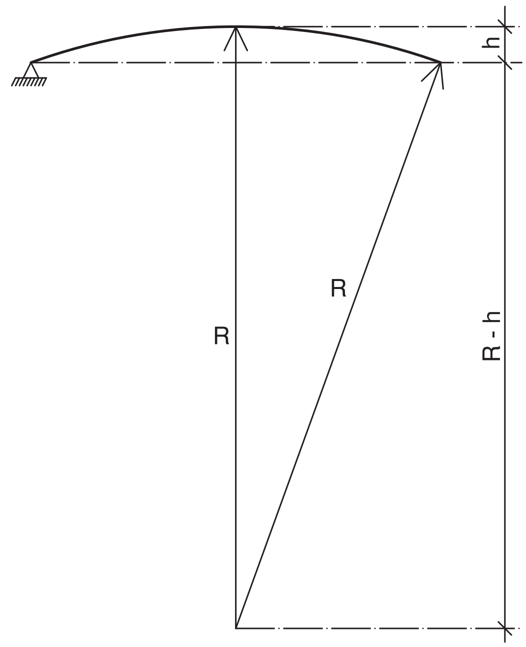
1. $F_b = 2400psi$, $F_v = 265psi$, $E = 1{,}800{,}000psi$, $E_{min} = 950{,}000psi$
2. Assume factored beam weight = $W_{FBM} = 80'(10)^{\#/f} = 800^{\#/f}$
3. $F_b' = F_b(C_m)(C_t)(C_L)(C_v)(C_{fu})(Cc)(2.16)(\lambda)$

    C_m: Is moisture content over 16%? No: $C_m = 1.0$

    C_t: Is temp. above 100°F? No: $C_t = 1.0$

    C_{fu}: Are laminations vertical and depth, dy < 12″? No: $C_{fu} = 1.0$

    $C_c = 1 - 2000(t/R)^2$: find R



18.1

Radius of curvature

Using the Pythagorean Theorem:

$$40^2 + (R - 8)^2 = R^2$$

$$40^2 + R^2 - 16R + 64 = R^2$$

$$R = (1600 + 64)/16 = 104' = 1248''$$

$$t/R = 0.75/1248 = .0006 < 1/125 = .008 \ldots \text{okay}$$

$$C_c = 1 - 2000(.0006)^2 = 0.999$$

λ: Is there a live load or a factored load stated in the problem? Yes: $\lambda = 0.8$

$$F_b' = F_b(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(2.16)(\lambda) = 2400(1)(1)(1)(.999)(2.16)(.8) = (F_b^*)(C_L)(C_V) = 4143.05(C_L)(C_V)$$

4. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

    $$W_u = W_{FBM} = 800^{\#/f}$$

    $$P_u = 1.2(4000\#) + 1.6\,(3000\#) = 9600\#$$
5. $M_{max} = wL^2/8 + PL = 800^{\#/f}(80')^2/8 + 9600\#(80') = 1{,}408{,}000\#\text{-f} = 16{,}896{,}000\#\text{-in}$

6. $S_{req} > M/F_b' = 16{,}896{,}000$#-in/4143.05psi = 4078.15in^3
7. Try 12.25″×45″: A = 551.3in^2, S_x = 4134in^3, and I_x = 93020in^4 from Table A2.7.
8. d/b = 45/12.25 = 3.67 Condition states C_L = 1 if c) 2≤ d/b ≤ 4 AND edges are secured by blocking or X-bracing. Therefore, C_L = 1
9. $C_v = (21/L)^{1/X}(12/d)^{1/X}(5.125/b)^{1/X} = (21/80)^{1/10}(12/45)^{1/10}(5.125/12.25)^{1/10} = 0.703 \le 1.0$
   X = 10 (not Southern Pine)
10. $F_b' = [F_b^*]$[lesser of (C_L) or C_v] = 4143.05(0.703) = 2912.56psi
11. W_{BM} = (specific gravity)(62.4pcf)(A/144)$^{\#/f}$ = (.5)(62.4)(551.3/144) = 119.45$^{\#/f}$
12. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

    $W_u = 1.2(119.45^{\#/f}) = 143.34$

    P_u = 1.2(4000#) + 1.6 (3000#) = 9600#

13. $M_{max} = wL^2/8 + PL = 143.34^{\#/f}(80')^2/8 + 9600\#(80')$ = 882,672#-f = 10,592,064#-in
14. f_b = M/S = 10,592,064/4134 = 2562.18psi
15. Is $f_b \le F_b'$? Yes: 2562.18psi < 2912.56psi
16. Is $f_b/F_b' \ge 0.90$?
    No → 2562.18/2912.56 = 0.88 < 0.9 try smaller size:

7A. Try 12.25″×43.5″: A = 532.9in^2, S_x = 3863in^3, and I_x = 84030in^4

8A. d/b = 43.5/12.25 = 3.55 Condition states C_L = 1 if c) 2 ≤ d/b ≤ 4 AND edges are secured by blocking or X-bracing. Therefore, C_L = 1

9A. $C_v = (21/L)^{1/X}(12/d)^{1/X}(5.125/b)^{1/X} = (21/80)^{1/10}(12/43.5)^{1/10}(5.125/12.25)^{1/10} = 0.705 \le 1.0$

10A. $F_b' = [F_b^*]$ [lesser of (C_L) or C_v] = 4143.05(0.705) = 2920.51psi

11A. W_{BM} = (specific gravity)(62.4pcf)(A/144)$^{\#/f}$ = (.5)(62.4)(532.9/144) = 115.46$^{\#/f}$

12A. Find factored loads using the six equations at the beginning of this chapter. If there are only dead and live loads:

    $W_u = 1.2(115.46^{\#/f}) = 138.55$

    P_u = 1.2(4000#) + 1.6 (3000#) = 9600#

13A. $M_{max} = wL^2/8 + PL = 143.34^{\#/f}(80')^2/8 + 9600\#(80')$ = 878,840#-f = 10,546,080#-in

14A. f_b = M/S = 10,546,080/3863 = 2730.02psi

15A. Is $f_b \le F_b'$? Yes → 2730.02psi < 2920.51psi

16A. Is $f_b/F_b' \ge 0.90$? No → 2730.02/2920.51 = 0.93 > 0.9 … okay

Note: 12.25×42 also works and is more efficient for flexure.

16. $F_v' = F_v(C_m)(C_t)(2.16)(\lambda)$
    C_m: Is moisture content over 16%? No: C_m = 1.0
    C_t: Is temp. above 100°F? No: C_t = 1.0
    F_v' = 265psi(1)(1)(2.16)(.8) = 457.92psi
17. V = wL/2 + 7P/2 = 115.46$^{\#/f}$(80′/2) + 7(9600#)/2 = 38,218.4#
18. f_v = 3V/2A = 3(38,218.4)/[2(532.9)] = 107.58psi
19. Is $f_v \le F_v'$? Yes: 107.58psi < 457.92psi
20. Δ_{all} = L(12″$^{/f}$)/240 = 80′(12″$^{/f}$)/240 = 4″
21. w = 115.46$^{\#/f}$, P = 4000# + 3000# = 7000#
22. $E' = E(C_m)(C_t)$ = 1,800,000psi(1)(1) = 1,800,000psi
    C_m: Is moisture content over 16%? No: C_m = 1.0
    C_t: Is temp. above 100°F? No: C_t = 1.0
23. $\Delta_{act} = 5wL^4/384EI + 79PL^3/768EI = 5(115.46)(80^4)(1728in^3/ft^3)/[384(1{,}800{,}000)(84030)] + 79(7000)(80^3)(1728)/[768(1{,}800{,}000)(84030)] = 0.703 + 4.212$ = 4.915″{/eq
24. Is $\Delta_{act} \le \Delta_{all}$? No → find $I_{req} = \Delta_{act}(I_x$ from step 3)/Δ_{all} = 4.915(84030)/4 = 103251.86in^4

USE 12.25×48: I = 112,900in^4

Example 18-2: Design a 10.5″ wide Southern Pine, 28F-2.1E SP glu-lam, spanning 40′ with a dead load of 1500$^{\#/f}$.

There is no sheathing and blocking is at 8′o.c. The beam is subjected to 110°F average temperature and 19% water content.

1. F_b = 2800psi, F_v = 300psi, E = 2,100,000psi, E_{min} = 1,110,000psi, G = 0.55
2. Assume FACTORED beam weight = W_{FBM} = 40′(10)$^{\#/f}$ = 400$^{\#/f}$
3. $F_b' = F_b(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(2.16)(\lambda)$
   C_m: Is moisture content over 16%? Yes: C_m = .80
   C_t: Is temp. above 100°F? Yes: From Table 16.3, C_t = 0.7
   C_{fu}: Are laminations vertical and depth, dy < 12″? No: C_{fu} = 1.0

$C_c = 1$

λ: Is there a live load or a factored load stated in the problem? No: $\lambda = 0.6$

$F_b' = F_b(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(2.16)(\lambda) = 2800(.8)(.7)(1)(1)(2.16)(.6) = 2032.13\text{psi}\ (F_b^*)(C_L)(C_v) = 2032.13(C_L)(C_v)$

4. Find factored loads using the six equations at the beginning of this chapter.

   $1.4(1500 + 400) = 2660^{\#/f}$

5. $M_{max} = wL^2/8 = 2660^{\#/f}(40')^2/8 = 532{,}000\text{\#-f} = 6{,}384{,}000\text{\#-in}$
6. $S_{req} > M/F_b' = 6{,}384{,}000\text{\#-in}/2032.13\text{psi} = 3141.53\text{in}^3$
7. Try 10.5″ × 42.625″: $A = 447.6\text{in}^2$, $S_x = 3180\text{in}^3$, and $I_x = 67760\text{in}^4$
8. d/b = 42.625/10.5 = 4.06 There is no sheathing therefore C_L must be calculated.

   $L_u = 8' = 96''$

   $L_u/d = 96/42.625 = 2.25 < 7$

   $L_e = 2.06L_u = 2.06(96) = 197.76$

   $R_b^2 = L_e(d)/b^2 = 197.76(42.625)/(10.5^2) = 76.46 < 2500$ … okay

   $E_{min}' = E_{min}(C_m)(C_t)(1.5) = 1{,}110{,}000(.833)(0.9)(1.5) = 1{,}248{,}250.5\text{psi}$

   $F_{bE} = 1.2(E_{min}')/R_b^2 = 1.2(1{,}248{,}250.5)/76.46 = 19{,}590.64\text{psi}$

   $F = F_{bE}/F_b^* = 19{,}590.64/2032.13 = 9.64$

   $C_L = (1 + F)/1.9) - \sqrt{[((1 + F)/1.9)^2 - (F/0.95)]} = 10.64/1.9 - \sqrt{[((10.64)/1.9)^2 - (9.64/0.95)]} = 0.994$

9. $C_v = (21/L)^{1/X}(12/d)^{1/X}(5.125/b)^{1/X} = (21/40)^{1/20}(12/42.625)^{1/20}(5.125/10.5)^{1/20} = 0.877 \le 1.0$

   X = 20 (Southern Pine)

10. $F_b' = [\ F_b^*]$ [lesser of (C_L) or Cv] = 2032.13(0.877) = 1782.18psi
11. $W_{BM} = (\text{specific gravity})(62.4\text{pcf})(A/144)^{\#/f} = (.55)(62.4)(447.6/144) = 106.68^{\#/f}$
12. $W_u = 1.4(1500 + 106.68^{\#/f}) = 2249.35^{\#/f}$
13. $M_{max} = wL^2/8 = 2249.35^{\#/f}(40')^2/8 = 449{,}870\text{\#-f} = 5{,}398{,}440\text{\#-in}$
14. $f_b = M/S = 5{,}398{,}440/3180 = 1697.62\text{psi}$
15. Is $f_b \le F_b'$? Yes: 1697.62psi < 1782.18psi
16. Is $f_b/F_b' \ge 0.90$?

    Yes → 1697.62/1782.18 = 0.95 > 0.9 … okay
17. $F_v' = F_v(C_m)(C_t)(2.16)\ (\lambda)$

    C_m: Is moisture content over 16%? Yes: $C_m = 0.875$

    C_t: Is temp. above 100°F? Yes: $C_t = 0.7$

    $F_v' = 300\text{psi}(0.875)(0.7)(2.16)(.6) = 238.14\text{psi}$
18. $V = wL/2 = 2249.35^{\#/f}(40'/2) = 44{,}987\#$
19. $f_v = 3V/2A = 3(44{,}987)/[2(447.6)] = 150.76\text{psi}$
20. Is $f_v \le F_v'$? Yes: 150.76psi < 238.14psi
21. $\Delta_{all} = L(12''^{/f})/240 = 40'(12''^{/f})/240 = 2''$
22. $w = 1500 + 106.68 = 1606.68^{\#/f}$
23. $E' = E(C_m)(C_t) = 2{,}100{,}000\text{psi}(0.833)(0.9) = 1{,}574{,}370\text{psi}$

    C_m: Is moisture content over 16%? Yes: $C_m = 0.833$

    C_t: Is temp. above 100°F? Yes: $C_t = 0.9$
24. $\Delta_{act} = 5wL^4/384EI = 5(1606.68)(40^4)(1728\text{in}^3/\text{ft}^3)/[384(1{,}574{,}370)(67760)] = 0.868''$
25. Is $\Delta_{act} \le \Delta_{all}$? Yes → 0.868″ < 2″

USE: 10.5 × 42.625

18.2.2 Compression in Glu-Lams

Using adjustment factors for glu-lams, the equation for allowable compressive stress is:

$$F_c' = F_c(C_m)(C_t)(C_P)(2.16)(\lambda)$$

where C_m is described at the beginning of this chapter and C_t and C_P are described in Chapter 16.

Like the design of wood beams, the design of columns is an iterative process based on an assumed trial size. In the case of wood columns, a good starting point is $A_{trial} = P_u/F_c^*$ where $F_c' = F_c^*C_P$.

Design of wood columns:

1. Look up F_c and E_{min} for the given species and grade of lumber.
2. $F_c' = F_c(C_m)(C_t)(C_P)2.16\ (\lambda) = F_c^*(C_P)$

   C_m: Is moisture content over 19%?

   No: $C_m = 1.0$

   Yes: $C_m = .73$

   C_t: Is temp. above 100°F?

   No: $C_t = 1.0$

   Yes: Determine C_t from Table 16.3

   Assume $C_P = 1$ for now.

3. Calculate $L_e = kL(12''/')$ in each direction. Effective Length Factor, k, can be found in Figure 10.1 (k = 1.0 for pin–pin, k = 0.8 for pin–fix, k = 0.65 for fix–fix). Determine min. width in each direction based on $L_e/d < 50$. $d_{min} = L_{ex}/50$ and $b_{min} = L_{ey}/50$
4. $A_{trial} = P/F_c^*$
   Select a size with $A \geq A_{trial}$, $b \geq b_{min}$, and $d \geq d_{min}$.
   Note A, b and d.
5. Use larger of L_e/d or L_{ey}/b and L_{ex}/d.
6. $E_{min}' = E_{min}(C_t)(C_m)(1.5)$
   C_t: Is temp. above 100°F?
   No: $C_t = 1.0$
   Yes: Determine C_t from Table 16.3
   C_m: Is moisture content over 19%?
   No: $C_m = 1.0$
   Yes: $C_m = .833$
7. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2$
8. $F = F_{cE}/F_c^*$
9. c = 0.8 for sawn lumber, c = 0.9 for glu-lams
10. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2}$
11. $F_c' = F_c^*(C_P)$ = allowable compressive stress
12. $f_c = P/A$ = actual compressive stress
13. Is $f_c < F_c'$?
    Yes → step 15
    No → go back to step 4 and choose larger size.
14. Is $f_c/F_c' \geq 0.90$? If not, go back to step 4 and try smaller size.

Example 18-3: Design a 10.5″ wide, 48/SP/N2D12 column, 20′ long, to carry a factored axial load of 50,000# with pinned connections and a moisture content of 15%.

1. F_c = 2200psi and E_{min} = 900,000psi
2. $F_c' = F_c(C_m)(C_t)(C_P)2.16\ (\lambda) = Fc^*(C_P) = 2200(1)(1)(2.16)(.8)(C_P) = 3801.6(C_P)$
   C_m: Is moisture content over 16%? No: $C_m = 1.0$
   C_t: Is temp. above 100°F? No: $C_t = 1.0$
3. $L_e = kL(12''/') = 1(20')(12''/') = 240''$
   $d_{min} = b_{min} = L_e/50 = 240/50 = 4.8''$
4. $A_{trial} = P/F_c^* = 50{,}000\#/3801.6 = 13.15$ Try 10.5″ × 11:
   A = 115.5, b = 10.5, d = 11
5. $L_e/b = 240/10.5 = 22.86$
6. $E_{min}' = E_{min}(C_t)(C_m)(1.5)$ = 900,000psi(1)(1)(1.5) = 1,350,000psi
   C_t: Is temp. above 100°F? No: $C_t = 1.0$
   C_m: Is moisture content over 16%? No: $C_m = 1.0$
7. $F_{cE} = 0.822(E_{min}')/(L_e/d)^2 = 0.822(1{,}350{,}000)/22.86^2$ = 2123.50psi
8. $F = F_{cE}/F_c^* = 2123.50/3801.6 = 0.56$
9. c = 0.9 for glu-lams
10. $C_P = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2}$
    $= (1.56)/1.8 - [((1.56)/1.8)^2 - (.56/.9)]^{1/2} = 0.508$
11. $F_c' = F_c^*(C_P) = 3801.6(0.508)$ = 1931.21psi
12. $f_c = P/A = 50{,}000/115.5$ = 432.9psi
13. Is $f_c < F_c'$? Yes → 432.9 < 1931.21psi
14. Is $f_c/F_c' \geq 0.90$? No: 432.9/1931.2 = .224 but 10.5 × 11 is the smallest available 10.5″ wide size.

ANSWER: USE 10.5″ × 11″

18.2.3 Bearing in Glu-Lams

Bearing in glu-lams uses the same method and value for C_b as bearing dimensional lumber except that there is no C_i factor. See section 16.2.3. Remember to use glu-lam values for $F_{c\perp}$ and C_m.

$F_{C\perp}' = F_{C\perp}(C_m)(Ct)(Cb)(1.5)$

$C_b = (L_b + 0.375)/L_b$

C_t is found in Table 16.3

C_m = 0.53 if the moisture content > 16% (see Table 18.2)

Example 18-4: Check the bearing in a 10.75 × 30 16F-1.3E glu-lam beam that supports a 5.125 × 18 16F-1.3E, beam with a factored load of 20,000# at the bearing point.

$C_m = 1$, $C_t = 1$

$L_b = 5.125''$

$C_b = (L_b + 0.375)/L_b = (5.125 + 0.375)/5.125 = 1.073$

$F_{C\perp}' = F_{C\perp}(C_m)(Ct)(Cb)(1.5)$ = 315psi(1)(1)(1.073)(1.5) = 506.99psi

$f_{c\perp} = P/A = 20{,}000\#/(5.125(10.75))$ = 363.02psi < 506.99psi … okay

18.2.4 Tension in Glu-Lams

Tension in glu-lams uses the same method as tension in dimensional lumber except that there is no C_i or C_F factor. See section 16.2.4. Remember to use glu-lam values for $Fc_\perp$ and C_m.

$F_t' = F_t(C_m)(C_t)(2.16)(\lambda)$ where

λ is found in Table 16.2

C_t is found in Table 16.3

$C_m = 0.8$ if the moisture content > 16% (see Table 18.2)

Example 18-5: Design a 5/DF/L1 beam, 5.125″ wide, with a factored tension load of 150,000# and a moisture content of 18%.

$F_t = 1600$psi

$C_m = 0.8$

$F_t' = F_t(C_m)(C_t)(2.16)(\lambda) = 1600(0.8)(1)(2.16)(0.8) = 2211.84$psi

Required Area = $A = P/F_t' = 150{,}000\#/2211.84$psi $= 67.82\text{in}^2$

USE: 5.125″ × 13.5″: $A = 69.19\text{in}^2$

18.2.5 Combined Stresses in Glu-Lams

The combined stresses formulae below apply to all types of wood: dimensional lumber, timber and glue-laminated lumber. Combined flexure and axial compression:

$[f_c/F_c']^2 + f_{b1}/\{F_{b1}'[1 - (fc/F_{CE1})] + f_{b2}/\{F_{b2}'[1 - (fc/F_{CE2}) - (f_{b1}/F_{bE})^2]\} \le 1.0$

Where

$f_c < F_{CE1} = 0.822E_{min}'/(Le_1/d_1)^2$ for edge-wise or biaxial bending (d_1 = wide face)

AND

$F_c < F_{CE2} = 0.822\,E_{min}'/(L_{e2}/d_2)^2$ for flatwise or biaxial bending (d_2 = narrow face)

AND

$f_{b1} < F_{bE} = 1.20\,E_{min}'/R_b^2$

Example 18-6: A 17/HF/L1D, 12.25″ × 18″ column is 20′ long with fixed ends and has a factored axial load of 500,000#, a factored M_x of 80,000 #-in and M_y of 60,000 #-in.

Is this column adequate?

1. Find values for species and grade:
   $F_{bx} = 1900$psi, $F_{by} = 2000$psi, $F_c = 1750$,
   $E_{min} = 900{,}000$psi
2. Find section properties: $A = 220.5\text{in}^2$, $S_x = 661.5\text{in}^3$, $S_y = 450.2\text{in}^3$
3. Find: f_C, f_{b1}, f_{b2}, F_C', F_{CE1}, F_{CE2}, F_{b1}', F_{b2}', F_{bE}

   $f_C = P/A = 500{,}000/220.5 = 2267.57$psi

   $f_{b1} = M_x/S_x = 80{,}000/661.5 = 12.09$psi

   $f_{b2} = M_y/S_y = 60{,}000/450.2 = 133.28$psi
4. Find F_{CE1}, F_{CE2}:

   $E_{min}' = 900{,}000(1.5) = 1{,}350{,}000$psi

   $L_u = 20'(12'') = 240''$

   $L_e = KL = 0.65(240) = 156$

   $L_e/d_1 = 156/18 = 8.67$

   $F_{CE1} = 0.822(1.350{,}000)/8.67^2 = 14{,}774.11$psi

   $F_{CE1} = 14{,}774.11 > 2267.57 = fc$ … okay

   $Le/d_2 = 156/12.25 = 12.73$

   $F_{CE2} = 0.822(1.350{,}000)/12.73^2 = 6842.72$

   $F_{CE2} = 6842.72 > 2267.57 = f_c$ … okay
5. Find C_P:

   $F_c' = F_c(C_m)(C_t)(C_P)(K_F\phi)\lambda = 1750(2.16)(.8)C_P = 3024C_P$

   $F_{CE1}/F_C^* = 14{,}774.11/3024 = 4.89$

   C = 0.9 for glu-lams

   $C_{P1} = (1 + F)/2c - [((1 + F)/2c)^2 - (F/c)]^{1/2} = (5.89)/1.8 - [(5.89/1.8)^2 - (4.89/.9)]^{1/2} = 0.976$

   $F_{CE2}/F_C^* = 6842.72/3024 = 2.26$

   $C_{P2} = 0.934$

   Use lesser value of $C_P = 0.934$

6. Check compression:

   $F_c' = 0.934(3024) = 2825.31\text{psi} > 2267.57\text{psi} = f_c$ … okay for compression.

7. Find C_L, F_{b1}', F_{b2}':

   $F_{b1}' = F_{b1}(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(K_F\phi)\lambda = 1900(2.16)(.8)C_L = 3283.2C_LC_v$

   $L_e = 1.84L_u$ (equal end moments) $= 1.84(240) = 441.6$

   $R_{b1}{}^2 = L_ed_1/d_2{}^2 = 441.6\ (18)/12.25^2 = 52.97$

   $F_{bE1} = 1.2(1,350,000)/52.97 = 30,583.39\text{psi}$

   $F_{bE1}/F_b{}^* = 30,583.39/3283.2 = 9.32$

   $C_{L1} = 0.988$

   $C_v = (21/20)^{1/10}(12/12.25)^{1/10}(5.125/18)^{1/10} = 0.884$

   $F_{b1}' = .884(3283.2) = 2903.77\text{psi} > 12.09\text{psi} = f_{b1}$ … okay

   $F_{b2}' = F_{b1}(C_m)(C_t)(C_L)(C_v)(C_{fu})(C_c)(K_F\phi)\lambda = 2000(2.16)(.8)C_L = 3456C_LC_v$

   $R_{b2}{}^2 = L_ed_2/d_1{}^2 = 441.6\ (12.25)/18^2 = 16.70$

   $F_{bE2} = 1.2(1,350,000)/16.7 = 97,027.51\text{psi}$

   $F_{bE2}/F_b{}^* = 97,027.51/3456 = 28.08$

   $C_{L2} = .996$

   $C_v = (21/20)^{1/10}(12/18)^{1/10}(5.125/12.25)^{1/10} = 0.884$

   $F_{b2}' = 0.884(3456) = 3056.6 > 133.28 = f_{b2}$ … okay

8. F_{bE} = lesser of F_{bE1} and F_{bE2}: $F_{bE} = 30,583.39\text{psi}$

Summary of values found

f_c = 2267.57psi	f_{b1} = 120.94psi	f_{b2} = 133.28psi
Fc' = 2825.31psi	F_{CE1} = 14,774.11psi	F_{CE2} = 6842.72psi
F_{b1}' = 2903.77psi	F_{b2}' = 3056.6psi	F_{bE} = 30,583.39psi

9. $[f_c/F_c']^2 + f_{b1}/\{F_{b1}'[1 - (f_c/F_{CE1})]\} + f_{b2}/\{F_{b2}'[1 - (f_c/F_{CE2}) - (f_{b1}/F_{bE})^2]\} \le 1.0$

   $[2267.57/2825.31]^2 + 120.94/2903.77[1 - (2267.57/14,774.11)] + 133.28/3056.6[1 - 2267.57/6842.72 - 120.94/30,583.39] = 0.759 < 1.0$ … okay

Combined axial tension and flexure:

$f_t/F_t' + f_b/F_b{}^* \le 1.0$ Where $F_b{}^* = F_b$ times all factors but C_L

Example 18-7: Check the adequacy of a 17/HF/L1D, 12.25×54 glue-laminated beam with L = 80′, L_u = 8′, factored concentrated loads at 8ft o.c. of 3000#, and a tension load of 15000#.

$F_{bx} = 1900\text{psi}$, $F_t = 1400\text{psi}$, $F_{vx} = 215\text{psi}$,
$E = 1,700,000\text{psi}$, $E_{min} = 900,000$, $G = .43$

12.25×54: $A = 661.5\text{in}^2$, $S = 5954\text{in}^3$, $I = 160,700\text{in}^4$

1. Check flexure:

   $F_b' = F_b(C_m)(C_t)(C_L)(C_v)(Cfu)(Cc)(2.16)(\lambda) = 1900(1)(1)(C_L)(C_v)(1)(1)(2.16)(0.8) = 3283.3C_LCv$

   C_L: d/b = 54/12.25 = 4.41 (condition d)

   Full sheathing is not indicated in the problem, therefore C_L must be calculated:

   $L_e = 1.84L_u = 1.84(8')(12''/') = 176.64$

   $R_b{}^2 = Le(d)/b^2 = 176.64(54)/(12.25^2) = 63.56$

   $E_{min}' = 900,000\text{psi}(1.5) = 1,350,000\text{psi}$

   $F_bE = 1.2\ E_{min}'/R_b{}^2 = 1.2(1,350,000)/63.56 = 25,487.73\text{psi}$

   $F_bE/F^* = 25,487.73/3283.3 = 7.76$

   $C_L = (1 + F)/1.9) - \sqrt{[((1 + F)/1.9)^2 - (F/0.95)]} = 8.76/1.9 - \sqrt{[((8.76)/1.9)^2 - (7.76/0.95)]} = 0.993$

   $C_v = [21(12)(5.125)/(80(12.25)(54))]^{1/10} = 0.690$

   Use lesser of C_v and C_L

   $F_b' = 0.69(3283.3) = 2265.48$

   weight of beam $= 1.2(.43)(62.4)(661.5/144) = 147.91^{\#/f}$

   $M = wL^2/8 + 5PL/4 = 147.91^{\#/f}(80')^2(12''^{/f})/8 + 5(3000\#)(80')(12''^{/f})/4 = 5019936\#\text{-in}$

   $S_x = 5954\text{in}^3$

   $f_b = M/S = 5019936\#\text{-in}/5954\text{in}^3 = 843.12\text{psi} < 2265.48\text{psi} = F_b'$ … okay for flexure

2. Check tension:

   $F_t' = Ft(C_m)Ct)(2.16)(\lambda) = 1400(1)(1)(2.16)(0.8) = 2419.2\text{psi}$

   $f_t = P/A = 15,000\#/661.5\text{in}^2 = 22.68\text{psi} < 2419.2$ psi … okay for tension

3. Check flexure and tension combined:

   $f_t/F_t + f_b/F_b{}^* = 22.68/2419.2 + 843.12/\ 2265.48 = 0.38 < 1.0$ … okay

4. Check shear:

   $F_v' = F_v(C_m)\ (C_t)(2.16)(\lambda) = 215(1)(1)(2.16)(0.8) = 371.52psi$

   $V = 147.91(80)/2 + 3000(9)/2 = 19{,}416.4\#$

   $f_v = 3V/2A = 3(19{,}416.4\#)/[2(661.5in^2)] = 44.03psi < 371.52 = F_v'$ … okay for shear

5. Check deflection:

   $\Delta_{all.} = L/240 = 16(80)/240 = 4''$

   $E' = E(C_m)\ (C_t) = 1{,}700{,}000(1)(1) = 1{,}700{,}000$ psi

   $I = 160{,}700in^4$

   unfactored load: P = 3000#(factoring unknown), $w = .43(62.4)(661.5)/144 = 123.26^{\#/f}$

   $\Delta_{max} = 5wl^4/384EI + 31PL^3/240EI = 5(123.26)(80)^4(1728)/384(1700000)(160{,}700) + 31(3000)(80)^3(1728)/240(1700000)(160{,}700) = 1.67'' < 4''$ … okay for deflection.

ANSWER: Beam is adequate.

Practice Exercises:

18-1: Design a 12.25″ wide Douglas Fir, 24F-1.8E glu-lam, spanning 80′ with concentrated live loads of 3000# and concentrated dead loads of 4000# spaced 10′o.c. The beam is curved such that the midpoint of the beam is 8ft above the supports. The laminations are 0.75″ thick. Blocking occurs at points of load and at ends.

18- 2. Design a 10.5″ wide, 48/SP/N2D12 column, 20ft long, to carry a factored axial load of 50,000# with pinned connections and a moisture content of 15%.

nineteen

Wood Connections

Wood to wood connections can be formed by creating interlocking shapes with the wood components, or by using mechanical connectors made of wood or metal.

This text describes the types of connectors available and suggests some rules of thumb for connections. Design of wood connections should follow the National Design Specifications of the American Wood Council, and these procedures are not covered in this text.

19.1 Mechanical Connections

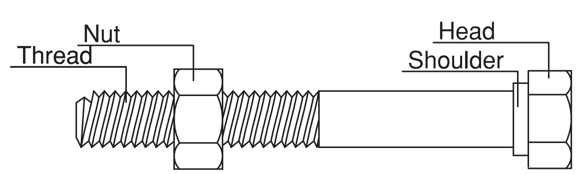
Mechanical connections are those that rely only on the physical nature of a connection without the use of adhesives or pressure and utilize a connector separate from the actual pieces joined.

19.1.1 Bolts

Bolts are dowel-type mechanical connectors that have a threaded end with a nut. The head can have a number of shapes as depicted in Figure 19.1, but the most common is a hexagonal head. Bolts may have a shoulder, an unthreaded portion below the head that has a wider diameter than the rod. Bolts are usually made of steel and the shear and bearing strength can be determined as outlined in Chapter 25. A307 or common steel bolts have strength roughly equivalent to A36 steel. Bolt diameters vary from $\frac{1}{2}''$ to 1.5″ in $\frac{1}{8}''$ increments. Bolt lengths vary in size, but thread lengths on bolts typically are determined by diameter and the overall bolt length as shown in Table 19.1:

Table 19.1: Bolt sizes

Bolt Diameter (in)	Standard Thread Length (in)	
	Bolt length ≤ 6"	Bolt Length > 6"
1/4	3/4	1
5/16	7/8	1-1/8
3/8	1	1-1/4
7/16	1-1/8	1-3/8
1/2	1-1/4	1-1/2
5/8	1-1/2	1-3/4
3/4	1-3/4	2
7/8	2	2-1/4
1	2-1/4	2-1/2



19.1

Parts of a bolt

Bolts must be used with a washer or a steel connector plate. The washer size area is $A \geq T/F_{c\perp}$, where $F_{c\perp}$ is the allowable bearing stress in the wood and T is the tension in the bolt.

Further $F_v' \geq V/nAv$ where:

F_v' = Allowable shears stress in the wood

V = the total shear

N = number of bolts resisting the shear

Av = b(D) where

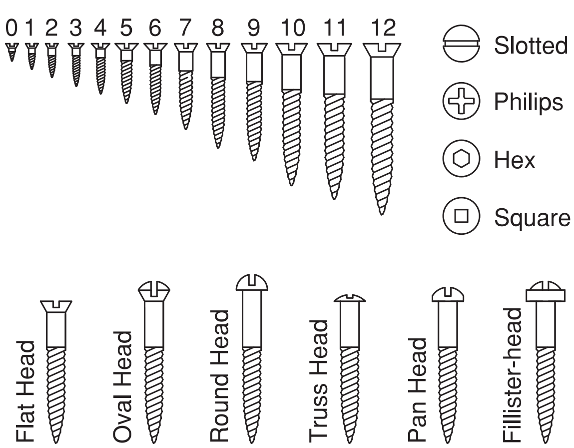
b = thickness of wood

D = bolt diameter.

Where bolts are grouped, see section 24.2 Eccentric Bolted Connections to find the resultant force on each bolt as well as the forces parallel and perpendicular to the grain. In general, bolts can be designed using the same logic of bolt design for steel as long as the correct material properties of the bolt are used.

Lag screws and wood screws:

Wood screws have larger threads than sheet metal or drywall screws and can be steel or brass. Sizes range from 18 to 8 and head types vary as shown in Figure 19.2. The term Phillips refers to an X shape drive for use with a Phillips screwdriver and slotted refers to a simple slot drive for flat bladed screwdrivers. Screw heads may be flat, round or oval. Most flat and oval heads have a conical shape under the head to allow for countersinking. Round heads are usually flat under the head and cannot be countersunk. Wood screw diameters range from $\frac{1}{8}$″ to $\frac{3}{8}$″.



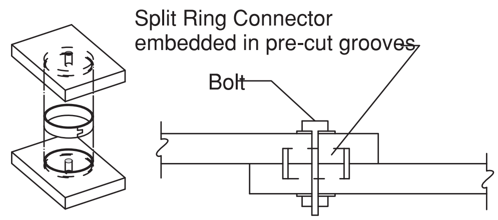
19.2

Wood screws

Lag screws are also called lag bolts. They have a hex or square head like a bolt and a tapered thread like a screw. The diameter ranges from $\frac{1}{4}$″ to $1\frac{1}{4}$″ and the length ranges from 1″ to 12″.

19.1.2 Split Ring Connectors

Split ring connectors are rings that sit in cut grooves formed in two mating surfaces. The purpose of the split ring connector is to handle lateral or shear loads too high for bolts and lag screws. It does so by creating a larger shear area, A_v. Since Shear stress $f_v = P/A_v$, a larger area reduces stress. The diameter of a split ring connector will be either $2\frac{1}{2}$″ or 4″. The connection requires a bolt to hold the mating pieces together.

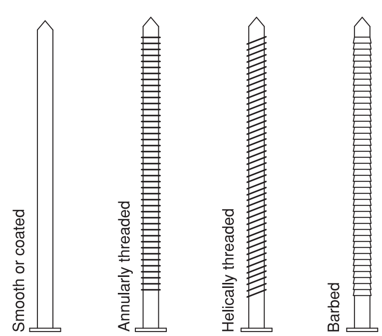


19.3

Split ring connectors

19.1.3 Nails and Spikes

Nails are traditionally the most common connector for dimensional lumber and are made efficient through the use of nail-guns. Nails have a high shear strength and resist withdrawal due to friction between the nail and the wood. However, nailing patterns must be designed for withdrawal forces per American Wood Council National Design Specifications section 11.2. Spikes are longer, larger nail-like fasteners used to connect large wood components.



19.4

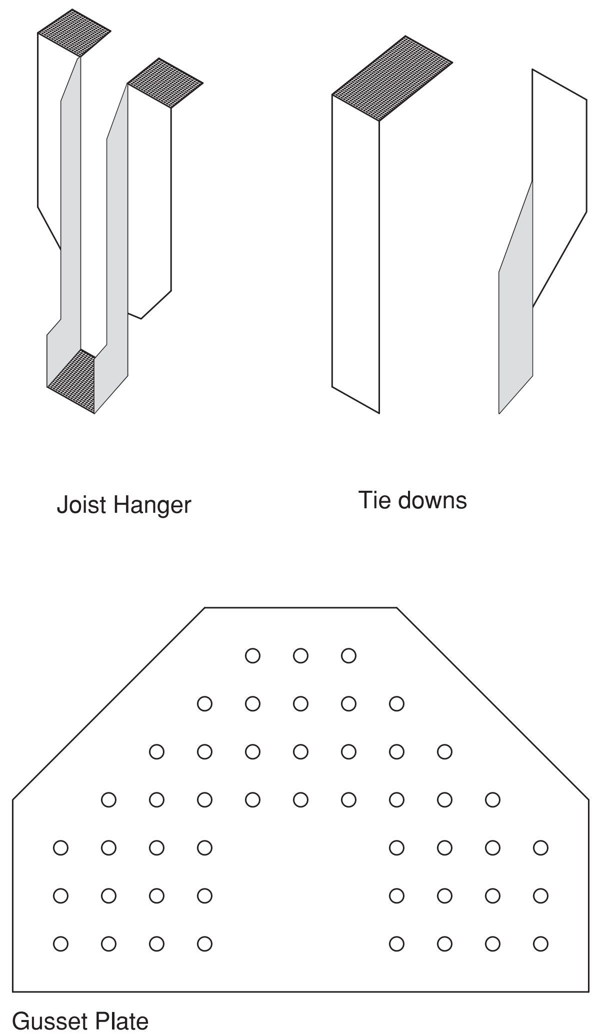
Nails and spikes

Table 19.2: Nail sizes

Nail Sizes			
Size	Length	Gauge	Number per pound (Approx.)
2d	1"	No. 15	845
3d	1"	No. 14	540
4d	1"	No. 12	290
5d	1"	No. 12	250
6d	2"	No. 11	165
7d	2"	No. 11	150
8d	2"	No. 10	100
9d	2"	No. 10	90
10d	3"	No. 9	65
12d	3"	No. 9	60
16d	3"	No. 8	45
20d	4'	No. 6	30
30d	4"	No. 5	20
40d	5"	No. 5	17
50d	5'	No. 3	13
60d	6"	No. 2	10

19.1.4 Metal Connector Plates

Metal plate connectors are available in a large number of standard shapes and sizes, some of which are shown in Figure 19.5. Metal plate connectors add strength as well as ease of construction in many cases, such as providing a seat for joists. Metal plate connectors are strong in tension and are essential connectors for tie-down applications. See individual manufacturer specifications for allowable loads.



19.5
Metal connector plates

19.1.5 Drift Pins

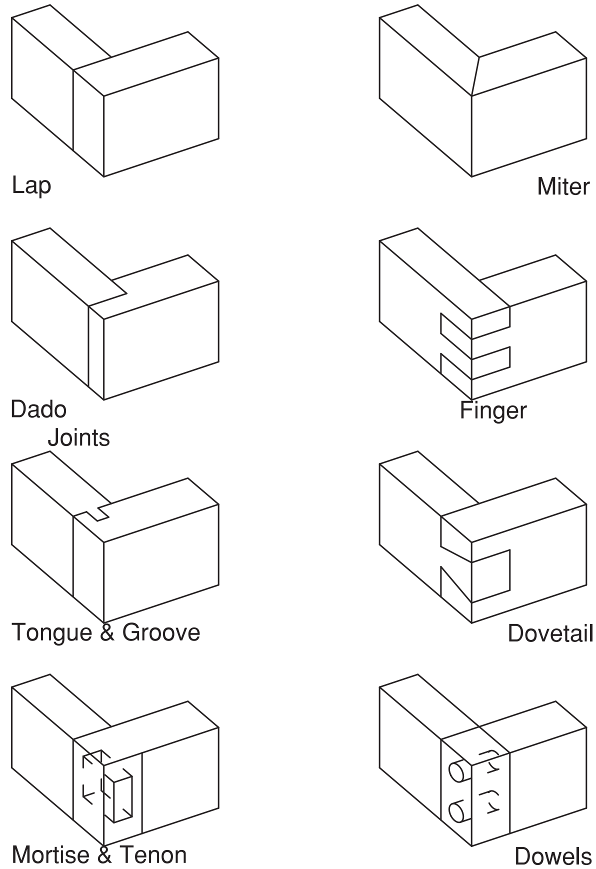
Drift pins are smooth metal rods that are driven into pre-bored holes. They can resist lateral loads, but not withdrawal loads. Drift pins are often used as guides to support components of a connection in place while fastening occurs in other holes.

19.1.6 Wood Dowels

Wood dowels are much larger in diameter than steel dowel-type fasteners such as bolts. This is because steel has much higher allowable stresses than wood. Wood dowel connections are tested for the same four failure modes as steel bolts: gross yielding, tensile rupture, dowel shear and dowel bearing. See American Wood Council National Design Specifications for design procedures for wood dowel components. Humidity must be considered in wood dowel connections. Dry conditions will cause shrinkage in the dowel and the connector members. Shrinkage parallel to the grain is much less than shrinkage perpendicular to the grain meaning that the dowel may shrink and become loose if installed with a high moisture content.

19.2 Wood Joinery

Typical wood joints include lap, dado or rabbet, tongue-and-groove, mitre, mortise-and-tenon, finger and dovetail joints. Examples of these wood joints are illustrated in Figure 19.6. With the exception of dovetail joints, wood joints require a fastener, either mechanical or adhesive, to hold the connection together. Historically, wood structures relied on wood joinery to improve the connection rigidity and minimize the need for dowel-type connections. For example, pioneers in colonial America employed dovetail joints as a way of creating log cabins by notching wedges in logs. The availability of steel connectors and the high cost of labor involved in the making of detailed wood joints has reduced the instances of all wood connections.



19.6
Wood joinery

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Part IV

Steel Design

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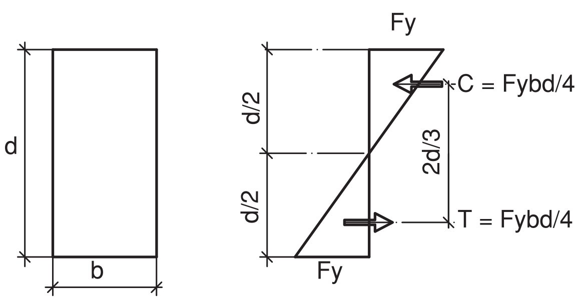
twenty

Steel Beam Design

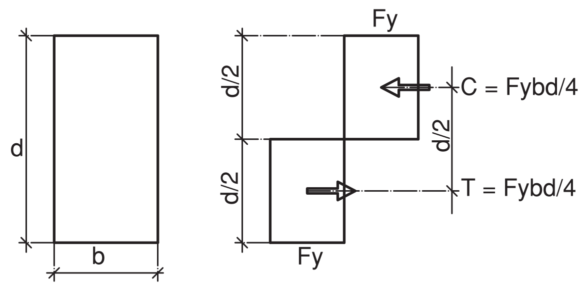
The ASD Method designs beams using only the elastic region. As a result, the allowable moment is limited to the yield moment, $M_y = F_y(I/c) = F_yS$ where S is the elastic modulus, also known as the section modulus. The LRFD Method considers stresses beyond the yield stress because failure doesn't occur until a great deal of yielding occurs. The allowable moment therefore becomes the plastic moment, $M_p = F_yZ$, where Z is the plastic modulus. To illustrate, consider the cross-section of a rectangular beam when yield stress is reached at the extreme fibers, the moment is derived by the compression and tension couple using the internal couple method as shown in Figure 20.1 The yield moment, $M_y = (F_ybd/4)(2d/3) = F_ybd^2/6$.



20.1

Yield moment

The section modulus, S, for a rectangular cross-section $= bd^2/6$. The yield moment, M_y, can then be simplified to $M_y = F_yS$.



20.2

Plastic moment

If the moment increases in the cross-section, the stress increases until all fibers are fully stressed as shown in Figure 20.2. In this scenario, the plastic moment can be defined using the internal couple method as $M_p = (F_ybd/2)(d/2) = F_ybd^2/4 = F_yZ$. Therefore $Z = bd^2/4$. In the case of the rectangular cross-section, $M_p = 1.5M_y$. The ratio of M_p to M_y is called the shape factor and varies with cross-section.

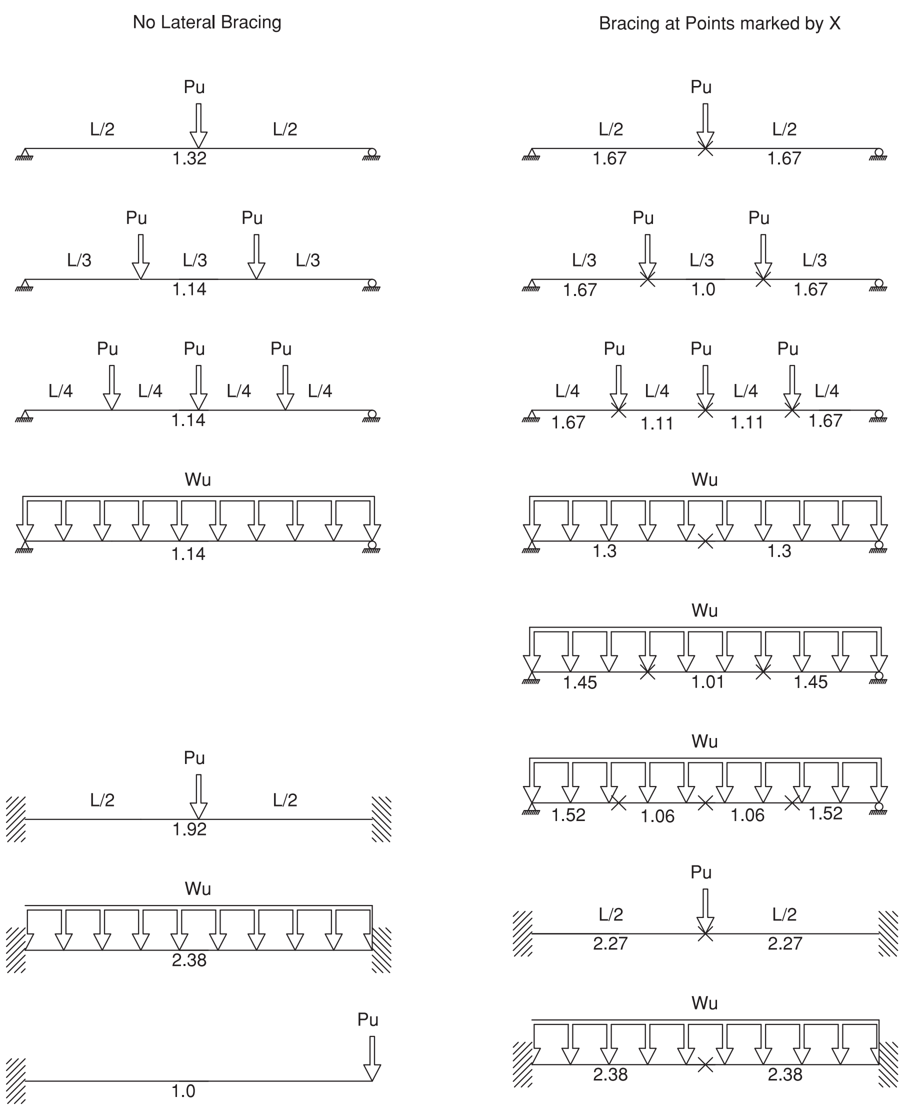
20.1 Designing Beams for Flexure Using LRFD Method

There are three types of behavior to consider in the design of beams, and each type is associated with a zone as follows:

> Zone 1: Plastic behavior (most beams): compact beams that can reach M_p without buckling. Beams are determined to be in Zone 1 if the unbraced length, L_b is less than the value $L_p = 1.76r_y\sqrt{(E/F_y)}$. When $L_b < L_p$, the allowable design moment, $\phi M_n = \phi M_{px} = 0.9F_yZ$.

Zone 2: Inelastic buckling: some but not all of the compression members buckle before reaching the yield stress, F_y. Beams are determined to be in Zone 2 when $L_p < L_b < L_r$ where $L_r = 1.95(rts)E/0.7F_y)(Jc/S_x h_o)^{1/2}(1 + (1 + 6.76(0.7F_y S_x h_o/EJc)^2)^{\wedge}0.5)^{\wedge}0.5)$ = maximum unbraced length in LRFD design for inelastic lateral–torsional buckling. $\phi M_n = C_b[\phi_b M_p - (\phi_b M_p - F_y S_x)(L_b - L_p)] \leq \phi_b M_p$ where C_b is the lateral–torsional buckling modification factor.

AISC Equation F1-1 defines $C_b = 12.5M_{max}(R_m)/[2.5M_{max} + 3M_A + 4M_B + 3M_C] \leq 3.0$ where M_{max} is the largest moment in an unbraced segment of a beam and M_A, M_B and M_C are the moments at the $\frac{1}{4}$ point, $\frac{1}{2}$ point and $\frac{3}{4}$ point, respectively, in the segment. R_m = the factor for the degree of bending. R_m = 1.0 for single curvature bending. The value of C_b must be computed for each unbraced segment in the beam and the lowest value used. A chart for the C_b values of beams with typical loadings can be found in the AISC Steel Manual at the beginning of Table 3-2 and in Figure 20.3 of this text.



20.3

Lateral-torsional buckling modification factor, C_b.

Zone 3: Elastic buckling: buckling occurs before the yield stress, F_y, can be reached anywhere in member. Beams are determined to be in Zone 3 when $L_b > L_r$. $\phi M_n = 0.9 F_{cr}S_x$ where

$F_{cr} = [\ C_b\pi^2E/(L_b/r_{ts})^2]\sqrt{}[1 + 0.078(Jc/S_xh_o)(L_b/r_{ts})^2]$.

Design procedure for steel beams:

1. Determine each ultimate load by factoring load types using the six equations given in Chapter 16. Enter the factored load or ultimate load into the diagram for the beam.
2. Find the ultimate moment, M_u either by drawing the shear and moment diagrams or by using equations in the deflection charts A1.2.
3. Find $Z_{req'd}$ and choose trial size.

   $\phi_bM_{px} = 0.9F_yZ$ where Z = plastic modulus for a cross-section.

   By setting $M_u = 0.9F_yZ$, the required plastic modulus can be found: $Z_{req'd} = M_u/0.9F_y$

   For example, if the ultimate or factored moment is M_u = 350k-f and F_y = 50ksi, $Z_{req'd} = M_u/0.9F_y$ = 350k-f(12″/f)/0.9(50ksi) = 93.33in³

   A W14×61 would work with Z_x = 102in³.
4. Add the moment caused by beam weight (w) to M_u found in step 2.

   $M_{uBM} = wL^2/8$k-f where w_{BM} = 1.2(bm. wt. designation/1000) k/ft

   New $M_u = M_{uSTEP2} + M_{uBM}$
5. What zone to use?

   L_b = unbraced length of the beam.

   $L_p = 1.76r_y\sqrt{}(E/F_y)$

   $L_r = 1.95r_{ts}(E/0.7F_y)(Jc/S_xh_o)^{1/2}\sqrt{}[1 + \sqrt{}(1 + 6.76(.7F_yS_xh_o/Ejc)^2)]$

   The values of L_p and L_r are listed in the AISC Steel Manual Table 3-2 or for W14s in Table 20.1.

   ZONE 1: $L_b \le L_p$

   ZONE 2: $L_p < L_b \le L_r$

   ZONE 3: $L_r < L_b$

Table 20.1: Sample values for LRFD design of W14 beams

SIZE	Zx (in³)	φMpx (k-f)	BF (k)	Lp (ft)	Lr (ft)	φVx (k)
W14X22	33.2	125	7.14	3.67	10.40	94.8
W14X26	40.2	151	7.99	3.81	11.10	106
W14X30	47.3	177	6.99	5.26	14.90	112
W14X34	54.6	205	7.59	5.40	15.60	120
W14X38	61.5	231	8.10	5.47	16.20	131
W14X43	69.6	261	7.24	6.68	20.00	125
W14X48	78.4	294	7.66	6.75	21.10	141
W14X53	87.1	327	7.93	6.78	22.20	155
W14X61	102	383	7.46	8.65	27.50	156
W14X68	115	431	7.81	8.69	29.30	175
W14X74	126	473	8.03	8.76	31.00	191
W14X82	139	521	8.16	8.76	33.10	219
W14X90	157	573	7.22	15.20	42.60	185
W14X99	173	646	7.35	13.50	45.30	206
W14X109	192	720	7.54	13.20	48.40	226
W14X120	212	795	7.64	13.20	52.00	256
W14X132	234	863	14.60	9.40	31.80	332
W14X145	260	975	7.68	14.10	61.70	302
W14X159	287	1080	7.79	14.10	66.70	335
W14X176	320	1200	7.84	14.20	73.20	379
W14X193	355	1330	7.92	14.30	79.70	413
W14X211	390	1460	7.99	14.40	86.40	462
W14X233	436	1640	8.09	14.50	94.90	515
W14X257	487	1830	8.21	14.60	104.00	577
W14X283	542	2030	8.31	14.70	114.00	648
W14X311	603	2260	8.46	14.80	125.00	724
W14X342	672	2520	8.64	15.00	137.00	810
W14X370	736	2760	8.80	15.10	148.00	890
W14X398	801	3000	8.96	15.20	158.00	971
W14X426	869	3260	9.16	15.30	169.00	1050
W14X455	936	3510	9.31	15.50	179.00	1150
W14X500	1050	3940	9.65	15.60	196.00	1290

6. Choose equation: NOTE: If the shape chosen in step 3 is not a compact shape, see Appendix F of the AISC Steel Manual for modification to the following equations. The modifications account for the slenderness of the flanges in certain W shapes.

   ZONE 1: $\phi_bM_n = \phi_bM_p$

   ZONE 2: $\phi_bM_n = C_b[\phi_bM_p - (\phi_bM_p - F_yS_x)(L_b - L_p)] \le \phi_bM_p$

   Look up worst case value for C_b in Figure 20.3 or calculate C_b.

   ZONE 3: $\phi_bM_n = 0.9\ F_{cr}S_x$ and

   $F_{cr} = [\ C_b\pi^2E/(L_b/rts)^2]\sqrt{}[1 + 0.078(Jc/S_xh_o)(L_b/rts)^2]$

   Zone 3 beams have a severely reduced allowable moment. Therefore, it is suggested to choose a heavier beam size to find $L_b < L_r$.
7. Check that ϕ_bM_n is greater than M_u: if ϕ_bM_n (allowable from step 6) > M_u (actual moment from step 4) it is okay for bending stress, if not, try a larger size and go back to step 3.

8. Check shear: New $V_u = V_u$ from step 2 + $w_{BM}L/2$
   note: w_{BM} is found in step 4.
9. Compare V_u to the value of ϕV_{nx}. If $Vu < \phi V_{nx}$ the beam is okay for shear. If not, try a larger size and go back to step 3.

   $\phi V_{nx} = 0.6F_y(t_w d)$

10. Check deflection: Allowable deflection = $L(12''/f)/240 = \Delta_{all}$
11. Calculate or look up deflection equations from charts. Using unfactored loads ($P_D + P_L$ or $W_D + W_L$), calculate the actual deflection. Remember to include the $1728in^3/f^3$ factor when using L in units of feet. Remember to include deflection due to the beam weight: $\Delta_{actual} = 5wL^4(1728)/[384EI]$. For multiple load types on a beam, it may be easier to use the double-integration method to find the maximum deflection.
12. Compare allowable deflection from step 10 to actual deflection from step 11.
    a. If $\Delta_{all} > \Delta_{act}$, okay and you are finished.
    b. If $\Delta_{all} < \Delta_{act}$, find a larger size of same nominal depth with $I_{xnew} = [Ix_{used}][\Delta_{act}]/[\Delta_{all}]$

Example 20-1: Find the most economical size W14 for an A992 steel beam spanning 28′ and spaced 8′o.c. with a dead load W_D = 20psf and a live load W_L = 80psf.

The beam has full lateral bracing.

1. $W_u = 1.2W_D + 1.6W_L = 1.2(20psf)(8') + 1.6(80psf)(8')$
   $= 1216^{\#/f} = 1.216k/f$
2. $M_u = wL^2/8 = 1.216k/f(28')^2(12in/f)/8 = 1430.02k\text{-}in$
3. $Z_{req'd} = M_u/0.9F_y = 1430.02k\text{-}in/[0.9(50ksi)] = 31.78in^3$
   See Table A3.1 for Section Properties of Selected W14 Shapes.
   Try a W14×22: $Z = 33.2in^3$, $I_x = 199in^4$
4. $M_{uBM} = wL^2/8 = 1.2(22/1000)(28)^2(12''/f)/8 = 31.05k\text{-}in$

   New $M_u = M_{uSTEP2} + M_{uBM} = 1430.02k\text{-}in + 31.05k\text{-}in = 1461.07k\text{-}in$.

5. What zone to use?
   $L_b = 0$ because the beam has full lateral bracing. Therefore, $L_b < L_p$ and the beam is in ZONE 1: $L_b \leq L_p$
6. Choose equation:
   ZONE 1: $\phi_b M_n = \phi_b M_p = 0.9F_y Z = 0.9(50ksi)(33.2in^3)$
   $= 1494k\text{-}in$
7. Check that $\phi_b M_n$ is greater than M_u: if $\phi_b M_n$ = 1494 k-in > M_u = 1461.07 k-in, therefore okay
8. Check Shear:
   $V_u = (1.216k/f + 1.2(.022k/f))(28')/2 = 17.39k$
9. $\phi V_{nx} = 0.6Fy(t_w d) = 0.6(50)(0.23)(13.7) = 94.8k$

   $\phi V_{nx} = 94.8k > V_u = 17.39$, therefore the beam is okay for shear.

10. Check deflection: Allowable deflection = $L(12''/f)/240 = \Delta_{all}$
    $= 28'(12''/f)/240 = 1.4''$
11. $w = ((20 + 80psf)(8') + 22^{\#/f})/1000\#/k = 0.822k/f$ Δ_{actual}
    $= 5wL^4(1728)/[384EI] = 5(0.822k/f)(28^4)(1728in^3/f3)/[384(29000ksi)(199in^4)] = 1.97''$
12. Compare allowable deflection from step 10 to actual deflection from step 11.
    $\Delta_{all} = 1.4'' < \Delta_{actual} = 1.97''$
    $I_{xnew} = [I_{xused}][\Delta_{actual}]/[\Delta_{all}] = (199in^4)(1.97'')/1.4''$
    $= 280in^4$
    Choices: W14×30: $I_x = 291in^4$ or W16×26: $I_x = 301in^4$
    W16×26 is most economical, but W14×30 has less depth.

Example 20-2: Find the most economical W14 for an A992 steel beam spanning 24′ with point loads of P_D = 8k and P_L = 16k placed at 8′o.c. with lateral bracing only at the point loads.

1. $P_u = 1.2P_D + 1.6P_L = 1.2(8k) + 1.6(16k) = 35.2k$ @ 8′o.c.
2. $M_u = P_uL/3 = 35.2k(24')(12in/f)/3 = 3379.20k\text{-}in$
3. $Z_{req'd} = M_u/0.9F_y = 3379.2k\text{-}in/[0.9(50ksi)] = 75.09in^3$
   Try a W14×48: $Z = 78.4in^3$, $S_x = 70.2in^3$, $I_x = 484in^4$, $r_y = 1.91''$, $t_w = 0.38''$, $d = 13.8''$
4. $M_{uBM} = wL^2/8 = 1.2(48/1000)(24)^2(12''/f)/8 = 49.77k\text{-}in$
   New $M_u = M_{uSTEP2} + M_{uBM} = 3379.20k\text{-}in + 49.77k\text{-}in$
   $= 3428.97k\text{-}in$

5. What zone to use?
   $L_b = 8'$ as stated in the problem. $L_p = 1.76r_y\sqrt{(E/F_y)}$
   $= 1.76(1.91'')\sqrt{(29000/50)} = 80.96'' = 6.75'$. The values of L_p and L_r can also be found in the AISC Steel Manual Table 3-2: $L_p = 6.75'$, $L_r = 21.1'$
   ZONE 2: $L_p \leq L_b \leq L_r$
6. Choose equation:
   ZONE 2: $\phi_b M_n = C_b[\phi_b M_p - (\phi_b M_p - F_y S_x)(L_b - L_p)] \leq \phi_b M_p$
   From Figure 20.3: In the outer unbraced segments, $C_b = 1.67$ and in the middle unbraced segment $C_b = 1.0$.

Use C_b = 1.0 because the maximum moment occurs at the center. If the location of the maximum moment is not calculated, use the lesser value for a more conservative answer.

$\phi_b M_p = 0.9F_yZ = 0.9(50ksi)(78.4in^3) = 3810.24k\text{-}in$

$\phi_b M_n = C_b[\phi_b M_p - (\phi_b M_p - F_yS_x)(L_b - L_p)] = 1.0[3810.24k\text{-}in - (3810.24 - 50ksi(70.2in^3))(8.0 - 6.75)] = 3434.94k\text{-}in \leq 3810.24\ \phi_b M_p \ldots \phi_b M_n = 3434.94k\text{-}in$

7. Check that $\phi_b M_n$ is greater than M_u: $\phi_b M_n$ = 3434.94k-in > M_u = 3428.97k-in, therefore okay
8. Check Shear: V_u = 35.2k + 1.2(.048k/f)(24′)/2 = 35.89k
9. $\phi V_{nx} = 0.6F_y(t_wd)$ = 0.6(50)(0.38)(13.8) = 157.32k
ϕV_{nx} = 157.32k > V_u = 35.89, therefore the beam is okay for shear.
10. Check deflection: Allowable deflection = L(12″/f)/240 = Δ_{all} = 24′(12″/f)/240 = 1.2″
11. P = 8k +16k = 24k, w = .048k/f Δ_{actual} = 23PL³(1728)/[648EI] + 5wL⁴(1728)/[384EI] = 23(24k)(24′3)(1728)/[648(29000)(484)] + 5(0.048k/f)(24⁴)(1728in³/f3)/[384(29000ksi)(484in⁴)] = 1.45″ + 0.03″ = 1.48″
12. Compare allowable deflection from step 10 to actual deflection from step 11.

Δ_{all} = 1.2″ < Δ_{actual} = 1.48″

$I_{xnew} = [I_{xused}][\Delta_{actual}]/[\Delta_{all}]$ = (484in⁴)(1.48″)/1.2″ = 596.93in⁴

USE: W14×61: I_x = 640in⁴

Example 20-3: Find the most economical W14 for an A992 steel beam spanning 24′ with point loads of P_D = 8k and P_L = 16k placed at 8′o.c. with no lateral bracing.

1. $P_u = 1.2P_D + 1.6P_L$ = 1.2(8k) + 1.6(16k) = 35.2k @ 8′o.c.
2. $M_u = P_uL/3$ = 35.2k(24′)(12in/f)/3 = 3379.20k-in
3. $Z_{req'd} = M_u/0.9F_y$ = 3379.2k-in/[0.9(50ksi)] = 75.09in³
Try a W14×48: Z = 78.4in³, S_x = 70.2in³, I = 484in⁴, r_y = 1.91″, t_w = 0.38″, d = 13.8″
4. $M_{uBM} = wL^2/8$ = 1.2(48/1000)(24)²(12″/f)/8 = 49.77k-in

New $M_u = M_{uSTEP2} + M_{uBM}$ = 3379.20k-in + 49.77k-in = 3428.97k-in

5. What zone to use?
L_b = 24′ as stated in the problem.
L_p = 6.75′, L_r = 21.1′
ZONE 3: $L_r \leq L_b$
6. Choose equation:
ZONE 3: $\phi_b M_n = 0.9F_{cr}S_x$ and $F_{cr} = [C_b\pi^2E/(L_b/rts)^2]\sqrt{[1 + 0.078(Jc/S_xho)(L_b/rts)^2]}$
From Figure 20.3 : In the outer unbraced segments, C_b = 1.67 and in the middle unbraced segment C_b = 1.0.
From Table 1-1 of the AISC Steel Manual, the following values are obtained:
rts = 2.20, J = 1.45, S_x = 70.2, h_o = 13.2 and c = 1.0 because W shapes are doubly-symmetrical.
$(L_b/rts)^2$ = [(24′)(12″/f)/2.20]² = 17137.19
Jc/S_xh_o = 1.45/[70.2(13.2)] = 0.00156
F_{cr} = [1.0(3.14159)²(29000)/(17137.19)]√[1 + 0.078(.00156)(17137.19)] = 29.34ksi
$\phi_b M_n = 0.9F_{cr}S_x$ = 0.9(29.34ksi)(70.2in³) = 1853.46 k-in
7. Check $\phi_b M_n$ against M_u: $\phi_b M_n$ = 1853.46k-in < M_u = 3428.97k-in, therefore, the beam is inadequate. Therefore, go back to step 3 and try a larger size.

3A. Try a W14×61: Z = 102in³, S_x = 92.1in³, I = 640in⁴, r_y = 2.45″, t_w = 0.375″, d = 13.9″

4A. $M_{uBM} = wL^2/8$ = 1.2(61/1000)(24)²(12″/f)/8 = 63.24k-in

New $M_u = M_{uSTEP2} + M_{uBM}$ = 3379.20k-in + 63.24k-in = 3442.44k-in

5A. What zone to use?
L_b = 24′ as stated in the problem.
From Table 20.1: L_p = 8.65′, L_r = 27.5′
ZONE 2: $L_p \leq L_b \leq L_r$

6A. Choose equation:
ZONE 2: $\phi_b Mn = C_b[\phi_b M_p - (\phi_b M_p - F_yS_x)(L_b - L_p)] \leq \phi_b M_p$
C_b = 1.0
$\phi_b M_p = 0.9F_yZ$ = 0.9(50ksi)(102in³) = 4590k-in
$\phi_b M_n = C_b[\phi_b M_p - (\phi_b M_p - F_yS_x)(L_b - L_p)]$ = 1.0[4590k-in − (4590 −50ksi(92.1in³))(24.0 − 8.65)] = 4820.25k-in > 4590 = $\phi_b M_p \ldots \phi_b M_n$ = 4590k-in

7A. Check $\phi_b M_n$ against M_u: $\phi_b M_n$ = 4590k-in > M_u = 3442.44 k-in, therefore, the beam is okay for flexure

8. Check Shear: V_u = 35.2k + 1.2(.061k/f)(24′)/2 = 35.89k
9. $\phi V_{nx} = 0.6F_y(t_wd)$ = 0.6(50)(0.375)(13.9) = 156.38k
ϕV_{nx} = 156.38k > V_u = 35.89, therefore the beam is okay for shear.

10. Check deflection: Allowable deflection = L(12″/f)/240 = Δ_{all} = 24′(12″/f)/240 = 1.2″
11. P = 8k +16k = 24k, w = .061k/f Δ_{actual} = 23PL^3(1728)/[648EI] + 5$_w$L^4(1728)/[384EI] = 23(24k)(24^3)(1728)/[648(29000)(640)] + 5(0.061k/f)(24^4)(1728in^3/f3)/[384(29000ksi)(640in^4)] = 1.10″ + 0.02″ = 1.12″
12. Compare allowable deflection from step 10 to actual deflection from step 11.
    Δ_{all} = 1.2″ > Δ_{actual} = 1.12″ therefore the beam is adequate for deflection.
    USE: W14 × 61.

Practice Exercises:

20-1: Find the most economical W14 for an A992 steel beam spanning 40′ with a dead load of 50psf and a live load of 80psf if the beams are spaced at 12′o.c. and full lateral bracing is provided.

20-2: Find the most economical W14 for an A992 steel beam spanning 35′ with concentrated dead loads of 1k and concentrated live loads of 2k spaced at 5′o.c. if lateral bracing is only provided at the point loads.

20-3: Find the most economical W14 for an A992 steel beam spanning 30′ with concentrated dead loads of 10k and concentrated live loads of 20k at midspan if

a) no lateral bracing is provided;
b) lateral bracing is provided at midspan.

twenty one

Design of Steel Compression Members

There are three types of failure that can occur in steel under axial compression.

1. Flexural buckling: Flexural buckling occurs when the bending stress is too high. The compression member fails as a whole.
2. Local buckling: Local buckling is a condition in which part of the cross-section buckles before the entire section, causing eccentricities.
3. Flexural torsional buckling: Flexural torsional buckling is a condition in which localized buckling causes bending in multiple directions, causing torsion about the axis of the compression member. Flexural torsional buckling is not covered in this book

See section 10.1.1, critical buckling stress, for information on allowable stresses in other metals, effective length factors and general design guidelines.

21.1 Axial Loads on Steel Columns

Column Design Process for LRFD Method:

1. Calculate the factored load, P_u.
2. Assume a value for the slenderness ratio, kL/r. Any number between 1 and 200 may be used, however, the farther from the final ratio of the designed column, the more iterations will be required to find the best choice of column size. Guidelines for selecting an assumed kL/r ratio are as follows:
   for columns 10 to 15′ use kl/r = 50;
   for longer columns use kl/r = 70;
   for short columns or heavy loads use kl/r = 30.
3. Find $\phi_c F_{cr}$ from Table 4-22 of the AISC Steel Manual or by using AISC equations E3-2, E3-3 and E3-4:
   E3-4: $F_e = \pi^2E/(kL/r)^2$. This is Euler's equation for critical buckling stress
   E3-2: if $kL/r \leq 4.71\sqrt{(E/F_y)}$, then $F_{cr} = (.658^{Fy/Fe})F_y$
   E3-3: if $kL/r > 4.71\sqrt{(E/F_y)}$, then $F_{cr} = 0.877F_e$.
4. Calculate a trial area: $A_{trial} = P_u/\phi_c F_{cr}$
5. Choose a trial size based on A_{trial} from step 4.
6. Calculate actual kL_x/r_x and kL_y/r_y. Use larger value.
7. Find allowable compressive stress, $\phi_c F_{cr}$, from Table 4-22 of the AISC Steel Manual or use AISC equations E3-2, E3-3 and E3-4:
8. Calculate the actual compressive stress = f_c = P/A.
9. If $\phi_c F_{cr} > f_c$, the size is adequate. If $\phi_c F_{cr} < f_c$, go back to step 5 and try larger size.
10. Check the efficiency of the column: If $f_c/\phi_c F_{cr} \geq 0.90$, the size is efficient. If $fc/\phi_c F_{cr} < 0.90$, go back to step 5 and try a smaller size.

Example 21-1: Design the most economical W21, pinned at top, fixed at base, L = 30′, P_D = 600k, P_L = 100k using A992 steel.

1. P_u = 1.2(600) + 1.6(100) = 880k
2. Assume kL/r = 70 (long and heavy)
3. Find $\phi_c Fc_r$

   E3-4: $F_e = \pi^2E/(kL/r)^2 = \pi^2(29{,}000)/(70)^2 = 58.412$ksi

   $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29{,}000/50)} = 113.43$

E3-2: $kL/r \leq 4.71\sqrt{(E/F_y)}$, therefore $F_{cr} = (.658^{Fy/Fe})F_y$
$= (.658^{50/58.412})(50) = 34.94$ksi

$\phi_c F_{cr} = 0.9(34.94) = 31.45$ksi

4. $A_{trial} = P_u/\phi_c F_{cr} = 880k/31.45ksi = 22.98in^2$
5. Try W21 × 101: A = 29.8in², $r_y = 2.89''$
6. kL/r = 0.8(30)(12)/2.89 = 99.65
7. Find actual $\phi_c F_{cr}$:

E3-4: $F_e = \pi^2 E/(kL/r)^2 = \pi^2(29{,}000)/(99.65)^2 = 28.82$ksi

$4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29{,}000/50)} = 113.43$

E3-2: $kL/r \leq 4.71\sqrt{(E/F_y)}$, therefore $F_{cr} = (.658^{Fy/Fe})F_y$
$= (.658^{50/28.82})(50) = 24.19$ksi

$\phi_c Fc_r = 0.9(24.19) = 21.77$ksi

8. Calculate the actual compressive stress = f_c = P/A = 880k/29.8in² = 29.53ksi
9. $\phi_c F_{cr} = 21.77 < f_c = 29.53$ therefore the column is not adequate. Go back to step 5 and try larger size. At this point, one may estimate the area of the next trial size by multiplying the area tried by the ratio of the actual stress to the allowable stress. $A_{trial} = 29.53(29.8)/21.77 = 40.42in^2$

5A. Try W21 × 147: A = 43.2, $r_y = 2.95$
6A. kL/r = 0.8(30)(12)/2.95 = 97.627
7A. Find actual $\phi_c F_{cr}$:

E3-4: $F_e = \pi^2 E/(kL/r)^2 = \pi^2(29{,}000)/(97.63)^2 = 30.03$ksi

$4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29{,}000/50)} = 113.43$

E3-2: $kL/r \leq 4.71\sqrt{(E/F_y)}$, therefore $F_{cr} = (.658^{Fy/Fe})F_y$
$= (.658^{50/30.03})(50) = 24.91$ksi

$\phi_c F_{cr} = 0.9(24.91) = 22.42$ksi

8A. Calculate the actual compressive stress = f_c = P/A = 880k/43.2in² = 20.37ksi
9A. $\phi_c F_{cr} = 22.42 > f_c = 20.37$ therefore the column is adequate.
10. $f_c/\phi_c F_{cr} = 20.37/22.42 = .909 \geq 0.90$, the size is efficient.

Compact sections
The design method above assumes that trial shapes have compact sections and therefore $\phi_c = 0.90$. Compact section simply means that the section is sufficient to withstand buckling until it reaches its yield stress, after which it is in plastic deformation. For a section to be compact, the width–thickness ratios must be $\leq \lambda_p$ in the AISC Steel Manual Table B4.1. The standard shapes listed in the AISC Steel Manual Table 1-1 are compact unless noted with a superscript c such as W16 × 31^c, in which case the shape is slender for F_y = 50ksi steel.

Non-compact sections
For sizes with non-compact sections, the yield stress can be reached in some parts of the cross-section before buckling is reached for the entire section. Width–thickness ratios are greater than λ_p but must be less than λ_r from Table B4.1 of the AISC Steel Manual. $\phi_c = 0.85$ for non-compact members.
Slender compression elements
Design of slender columns is complex and generally strength is very low.

Most W, S shapes etc. are compact or non-compact. Slender shapes are noted with superscripts following the shape name. In slender shapes, the width–thickness ratio is greater than λ_r from Table B4.1 and as with non-compact shapes, $\phi_c = 0.85$ for non-compact members. Slender members can buckle locally. This is common in HSS members where local buckling may govern failure and so requires a reduction factor Q. Where:

$$Q = A_e/A$$

$$A_e = A - 2(t)(h - b_{eh}) - 2(t)(b - b_{eb})$$

$$b_{eb} = 1.92t[\sqrt{E/f_y}][1 - (0.38/(b/t))\sqrt{E/f_y}]$$

$$b_{eh} = 1.92t[\sqrt{E/f_y}][1 - (0.38/(h/t))\sqrt{E/f_y}]$$

21.1.1 Process to Find Reduced $\phi_c F_{cr}$ in Slender HSS Rectangular Sections

Find $\phi_c F_{cr}$ of an HSS H × B × T with a slenderness factor K, yield stress F_y(ksi) and a length L(f).

1. Find the section properties: A r_x r_y t b/t h/t
2. $b_{eb} = 1.92t[\sqrt{E/F_y}][1 - (0.38/(b/t))\sqrt{E/F_y}]$
3. b = B − 3t
4. $b_{eh} = 1.92t\ [\sqrt{E/f_y}][1 - (0.38/(h/t))\sqrt{E/f_y}]$
5. h = H − 3t
6. $A_e = A - 2(t)(h - b_{eh}) - 2(t)(b - b_{eb})$
7. $Q = A_e/A$
8. Find: kL/r

9. $F_e = \pi^2E/(kL/r)^2$
10. Find F_{CR}:
    if $4.71\sqrt{29{,}000/(Q(F_y))} > kL/r$, use $F_{cr} = Q[0.658^{Qfy/Fe}]F_y$
    if $4.71\sqrt{29{,}000/(Q(F_y))} < kL/r$, use $F_{cr} = 0.877F_e$
11. $\phi_cF_{cr} = 0.85F_{CR}$

Example 21-2: Find ϕ_cP_n of 24′, HSS 14×10×$\frac{1}{4}$ fixed at one end, pinned at other and F_y = 50ksi.

1. $A = 10.8$, $r_x = 5.35$, $r_y = 4.14$, $t = 0.233$, $b/t = 39.9$, $h/t = 57.1$
2. $b_{eb} = 1.92t[\sqrt{E/F_y}][1 - (0.38/(b/t))\sqrt{E/F_y}] = 1.92(.233)[\sqrt{29{,}000/50}][1 - (0.38/(39.9))\sqrt{29000/50}] = 8.3$
3. $b = B - 3t = 10 - 3(.233) = 9.3''$
4. $b_{eh} = 1.92t[\sqrt{E/F_y}][1 - (0.38/(h/t))\sqrt{E/F_y}] = 1.92(.233)[\sqrt{29{,}000/50}][1 - (0.38/(57.1))\sqrt{(29{,}000/50)}] = 9.05$
5. $h = H - 3t = 14 - 3(.233) = 13.3$
6. $A_e = A - 2(t)(h - b_{eh}) - 2(t)(b - b_{eb})$

    $b - b_{eb} = 9.3 - 8.3 = 1.0$

    $h - b_{eh} = 13.3 - 9.05 = 4.25$

    $A_e = A - 2(t)(h - b_{eh}) - 2(t)(b - b_{eb}) = 10.8 - 2(.233)(4.25) - 2(.233)(1.0) = 8.35$

7. $Q = A_e/A = 8.35/10.8 = 0.773$
8. Find: $kL/r = 0.8(24)(12)/4.14 = 55.65$
9. $F_e = \pi^2E/(kL/r)^2 = \pi^2E/(kL/r)^2 = 92.42$
10. Find F_{cr}:

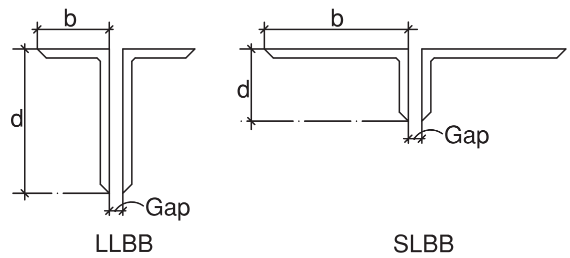
    $4.71\sqrt{(E/QF_y)} = 4.71\sqrt{29{,}000/(.773(50))} = 129.02$

    $4.71\sqrt{(29{,}000/QF_y)} > kL/r$ therefore $F_{cr} = Q[0.658^{Qfy/Fe}]F_y = 0.773[.658^{.418}]50 = 32.45$ksi

11. $\phi_cF_{cr} = 0.85F_{cr} = 0.85(32.45) = 27.58$ksi
12. $\phi_cP_n = \phi_cF_{cr}A_g = 27.58(10.8) = 297.89$

21.1.2 Process to Find Reduced F_{cr} in Slender Double Angles with a Back-to-Back Separation of either 0″, $\frac{3}{8}$″ or $\frac{3}{4}$″

1. Look up A, r_x, r_y and Q_s in Table 1-15 Double Angle Section Properties of the AISC Steel Manual. Be sure to use the correct value of Q_s dependent on whether the long sides or short sides of the angles are back-to-back.



21.1
Double angles

2. Find kL/r.
3. Find F_{cr}:
    $F_e = \pi^2E/(kL/r)^2$
    If $kL/r \le 4.71\sqrt{(E/QF_y)}$ then $F_{cr} = Q[0.658^{QFy/Fe}]F_y$
    If $kL/r > 4.71\sqrt{(E/QF_y)}$ then $F_{cr} = 0.877F_e$

Example 21-3: Find ϕ_cP_n for 2L6×6×$\frac{1}{2}$ LLBB (long legs back-to-back) with a back-to-back separation of $\frac{3}{8}$″, fixed ends, L = 20′, and F_y = 36ksi.

1. Sections properties: $A = 11.5$, $r_x = 1.86$, $r_y = 2.63$, $Q_s = 1.0$
2. Find: $kL/r = 0.65(20)(12)/1.86 = 83.87$
3. $F_e = \pi^2E/(kL/r)^2 = (\pi)^2(29{,}000)/(83.87)^2 = 40.69$
4. $4.71\sqrt{E/QF_y} = 4.71\sqrt{(29{,}000/1.0(36))} = 133.68$
    $59.32 \le 133.68$ therefore
    $F_{cr} = Q[0.658^{QFy/Fe}]Fy = 1.0[.658^{36/40.69}][36] = 24.86$
5. $P_u = \phi_cP_n = \phi_cF_{cr}A_g = 0.85F_{cr}A = 0.85(24.86)(11.5) = 243.01$k

21.2 Combined Axial Compression and Flexure

Steel components subjected to compression and flexure act as both a column and a beam. The component may be a beam with an axial load due to lateral forces or a column that has fixed supports, such as columns in a moment frame. The best strategy in component design for these cases is to design for the primary function and then check for the combination of loads.

For doubly and singly symmetric shapes, the AISC Specification Section H1.1 governs. For unsymmetrical shapes, AISC Specification Section H.2 governs.

AISC H1.1:

1. If $P_u/\phi P_n \geq 0.2$ $P_u/\phi P_n + (8/9)[(M_{ux}/\phi M_{nx}) + (M_{uy}/\phi M_{ny})] \leq 1.0$
2. If $P_u/\phi P_n < 0.2$ $P_u/2\phi P_n + [(M_{ux}/\phi M_{nx}) + (M_{uy}/\phi M_{ny})] \leq 1.0$

Example 21-4: Design a W14 column with fixed supports at each end to carry an axial factored load of P_u = 1200k and a moment in the strong direction of M_x = 200k-f and a moment in the weak direction of M_y = 180k-f. L_x = 24′, L_y = 12′.

Use the steel column design guide in section 21.1, except use $A_{trial} = P_u/[0.6(\phi_c F_{cr})]$ in step 4 and do not test for efficiency.

1. P_u = 1200k
2. Assume kl/r = 70
3. Find $\phi_c F_{cr}$ from Table 4-22 of the AISC Steel Manual or by using AISC equations E3-2, E3-3 and E3-4:
   E3-4: $F_e = \pi^2E/(kL/r)^2 = \pi^2(29000ksi)/(70)^2 = 58.41ksi$
   $4.71\sqrt{(E/Fy)} = 4.71\sqrt{(29000/50)} = 113.43$
   $F_{cr} = (.658^{Fy/Fe})F_y = (.658^{50/58.41})(50) = 34.94ksi$
4. Calculate a trial area: $A_{trial} = P_u/[0.6(\phi_c F_{cr})] = 1200/[0.6(34.94)] = 57.24in^2$
5. Try W14 × 193: A = 56.8in², r_x = 6.5″, r_y = 4.05″, Z_x = 355in³, Z_y = 180in³, S_x = 310in³, S_y = 119in³
6. kL/r_x = .65(24′)(12″/′)/6.5″ = 28.8 and kL/r_y = .65(12′)(12″/′)/4.05″ = 23.11
7. Find $\phi_c F_{cr}$:

   $F_e = \pi^2E/(kL/r)^2 = \pi^2(29000ksi)/(28.8)^2 = 345.07ksi$

   $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/50)} = 113.43$

   $F_{cr} = (.658^{Fy/Fe})F_y = (.658^{50/345.07})(50) = 47.06ksi$

8. Calculate the actual compressive stress = f_c = P/A = 1200/56.8 = 21.13ksi
9. If $\phi_c F_{cr}$ = 47.06ksi > f_c = 21.13ksi, therefore the size is adequate.
10. Moment in strong direction = M_x = 200k-f.
11. $\phi_b M_{px} = 0.9F_yZ_x = 0.9(50ksi)(355in^3)/12^{in/f} = 1331.25k\text{-}f$
12. $L_p = 1.76r_y\sqrt{(E/F_y)} = 1.76(4.05)\sqrt{(29000/50)} = 171.66''$ = 14.31′ < L_x = 24′
13. S_xho/Jc = 34.8(1)/[310(14)] = 124.71
14. $L_r = 1.95(rts)E/0.7F_y)(Jc/S_xh_o)^{1/2}(1 + (1 + 6.76(0.7F_yS_xh_o/EJc)^2)$^0.5)^0.5) = 1.95(4.59)E/0.7(50))(1/124.71)$^{1/2}$(1 + (1 + 6.76(0.7(50)(124.71)/29000)^2)^0.5)^0.5) = 956.35″ = 79.7′
15. L_p = 14.31′ < L_x = 24′ < L_r = 79.7′, therefore the column is in Zone 2.
16. $\phi_b M_n = C_b[\phi_b M_p - (\phi_b M_p - F_yS_x)(L_b - L_p)] \leq \phi_b M_p$, where C_b = 1.0 (Moment constant throughout column length)

    $\phi_b M_{nx}$ = 1.0[1331.25 – (1331.25 – (50ksi(310in³)/12in/f)) (24′ – 14.31′)] = 947.69k-f > 200k-f … okay

17. Moment in weak direction = M_y = 180-f.
18. $\phi_b M_{py} = 0.9F_yZ_y = 0.9(50ksi)(180in^3)/12^{in/f} = 675k\text{-}f$
19. $L_p = 1.76r_y\sqrt{(E/F_y)} = 1.76(4.05)\sqrt{(29000/50)} = 171.66''$ = 14.31′ > L_y = 12′, therefore the column is in Zone 2.
20. $\phi_b M_n = \phi_b M_{py}$ = 675k-f > 180k-f … okay
21. AISC EQTN H-1.1:

    $P_u/\phi P_n$ = 1200/(47.06(56.8)) = 0.449 > 0.2 … use

    $P_u/\phi P_n + (8/9)[(M_{ux}/\phi M_{nx}) + (M_{uy}/\phi M_{ny})] \leq 1.0$

    0.499 + (8/9)[(200/947.69) + (180/675)] = 0.953 <1.0 … okay

ANSWER: W14 × 193

21.3 Built-up Columns

Built-up columns are columns created by assembling and connecting shapes into a desired design. If the components are welded together to form one cross-section without gaps, then the moment of inertia, and the radius of gyration can be calculated and the column may be designed as previously discussed with one exception. The AISC Steel manual calls for modification to the slenderness ratio of built-up columns where the members are connected by bolts and welds. AISC equation E6-1 must be used to calculate slenderness ratio for built-up columns with bolted connections.

E6-1: The modified slenderness ratio = $(kL/r_m) = \sqrt{[(kL/r)_o^2 + (a/r_i)^2]}$ where

$(kL/r)_o$ = column slenderness of built-up column acting as a whole;

a = distance between connectors;

r_i = minimum radius of gyration for individual component.

AISC equation E6-2 must be used to calculate the slenderness ratio for built-up columns with welded connections.

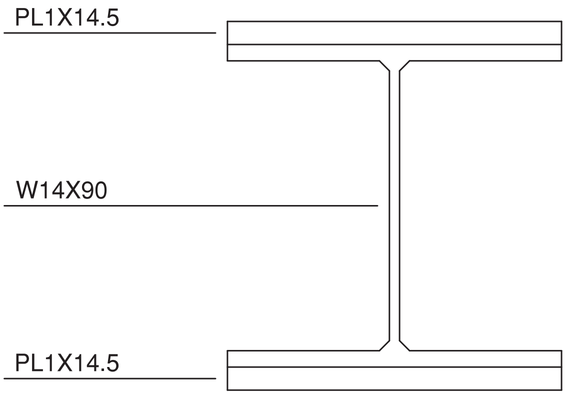
E6-2: The modified slenderness ratio = $(kL/r_m) = \sqrt{[(kL/r)_o^2 + 0.82(\alpha^2/(1 + \alpha^2))(a/r_{ib})^2]}$ where

r_{ib} = radius of gyration of individual component in direction parallel to the weak axis of the built-up column;

h = distance between center of gravity of bracing members;

A = $h/2r_{ib}$

Example 21-5: Find the maximum compressive load, ϕP_n for a W14×90 with a 1″×14.5″ plate bolted to each flange with a bolt spacing of 12″ along the length of the 20′ column.



21.2

Built-up column example

1. Find section properties of built-up member:
   W14×90: A = 26.6in^2, d = 14.0″, r_x = 6.14″, r_y = 3.70″
   PL1×14.5: A = 14.5, $I_x = 14.5(1^3)/12 = 1.21\text{in}^4$
   $r_i = \sqrt{[I/A]} = \sqrt{[1.21/14.5]} = 0.289''$
   a = spacing of bolts = 12″
   Built-up properties:

Comp.	*Ai*	*Ixi*	*dy*	*Ady2*
W14×90	26.5	999	0	0
1×14.5	14.5	1.21	7.5	815.625
1×14.5	14.5	1.21	7.5	815.625
	ΣA_i = 55.5	ΣI_{xi} = 1001.42		ΣAdy^2 = 1631.25

$I_x = \Sigma I_{xi} + \Sigma Ady^2 = 1001.42 + 1631.25 = 2632.67\text{in}^4$

$r = \sqrt{[I/A]} = \sqrt{[2632.67/55.5]} = 6.89''$

2. Find modified slenderness ratio: $kL/r_m = \sqrt{[(kL/r)_o^2 + (a/r_i)^2]}$ $= \sqrt{[(1(20(12\text{in/f})/6.89'')^2 + (12/.289)^2]} = 54.20$

4. Find $\phi_c F_{cr}$:

$F_e = \pi^2 E/(kL/r)^2 = \pi^2(29000\text{ksi})/(54.2)^2 = 97.43\text{ksi}$

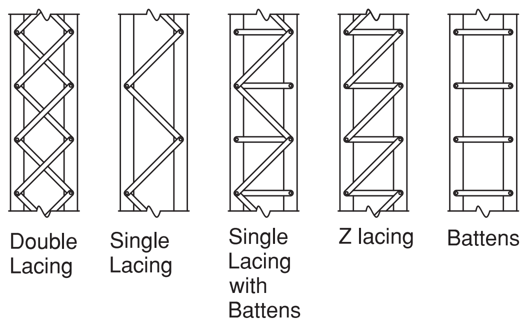
$4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/50)} = 113.43$

$F_{cr} = (.658^{Fy/Fe})F_y = (.658^{50/97.43})(50) = 40.34\text{ksi}$

$\phi_c F_{cr} = .9(40.34) = 36.31\text{ksi}$

$\phi_c P_n = \phi_c F_{cr}\, A = 36.31(55.5) = 2015.21\text{k}$

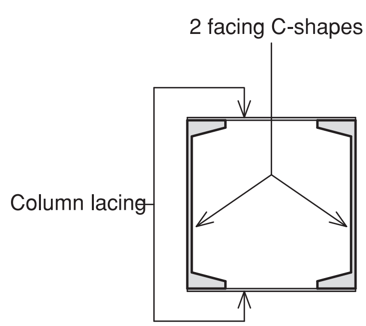
This section discusses the design of a column consisting of separate, clustered components connected only at distinct intervals. Equations E6.1 and E6.2 do not apply because the members are not in contact with each other. The connections are made by using diagonal bracing, called lacing in single or double layers as shown in Figure 21.3 or by covering the open edge with a perforated plate.



21.3

Built-up column lacing

The main thing to remember when designing built-up columns is that there are two cases to be examined. First consider the column as a whole and second consider the individual components as columns with an unbraced length equal to the horizontal connection spacing. A typical built-up column is one made of two facing channels as shown in Figure 21.4.



21.4

Two channel built-up column

Example 21-6: Design two 15″ channels used to form a 15″ square with F_y = 50ksi, P_u = 800k, and L = 18′.

1. Assume kL/r = 50 which means $\phi_c F_c = 37.5$ksi
2. $A_{trial} = 800/37.5 = 21.33in^2$ total area. $A = 21.33/2 = 10.67in^2$ for each channel
3. Try C15×40: $A = 11.8in^2$, $I_x = 348in^4$, $I_y = 9.17in^4$, x = .778″, d = 15″
4. Consider column as a whole and find I, r and kL/r values.

$A = 11.8(2) = 23.6in^2$

$I_x = 348(2) = 696in^4$ and
$I_y = 9.17(2) + 2(11.8)(15/2 - .778)^2 = 1084.71in^4$

$r_x = \sqrt{(I_x/A)} = \sqrt{(696/23.6)} = 5.43''$

$r_y = \sqrt{(I_y/A)} = \sqrt{(1084.71/23.6)} = 6.78''$

$kL/r = 1.0(18)(12)/5.43 = 39.78$

5. Find $\phi_c F_{cr}$:

$F_e = \pi^2 E/(kL/r)^2 = \pi^2(29000ksi)/(39.78)^2 = 180.87ksi$

$4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/50)} = 113.43$

$F_{cr} = (.658^{Fy/Fe})Fy = (.658^{50/180.87})(50) = 44.17ksi$

$\phi_c F_{cr} = .9(44.17) = 39.76$

6. $f_c = P/A = 800k/23.6in^2 = 33.9ksi < \phi_c F_{cr} = 39.76$ therefore okay
7. efficiency = $f_c/\phi_c F_{cr} = 33.9/39.76 = .85 < 0.9$ … try smaller size

3A. Try C15×33.9: $A = 10in^2$, $I_x = 315in^4$, $I_y = 8.07in^4$, x = .788″, d = 15″

4A. Consider column as a whole and find I, r and kL/r values.

$A = 10(2) = 20in^2$

$I_x = 315(2) = 630in^4$ and
$I_y = 8.07(2) + 2(10)(15/2 - .788)^2 = 917.16in^4$

$r_x = \sqrt{(I_x/A)} = \sqrt{(630/20)} = 5.61''$

$r_y = \sqrt{(I_y/A)} = \sqrt{(917.16/20)} = 6.77''$

$kL/r = 1.0(18)(12)/5.61 = 38.5$

5A. Find $\phi_c F_{cr}$:

$F_e = \pi^2 E/(kL/r)^2 = \pi^2(29000ksi)/(38.5)^2 = 193.1ksi$

$4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/50)} = 113.43$

$F_{cr} = (.658^{Fy/Fe})Fy = (.658^{50/193.1})(50) = 44.86ksi$

$\phi_c F_{cr} = .9(44.86) = 40.38$

6A. $f_c = P/A = 800k/20in^2 = 40ksi < \phi_c F_{cr} = 40.38$ therefore okay

7A. efficiency = $f_c/\phi_c F_{cr} = 40/40.38 = .99 > 0.9$ … okay

8. Consider individual C15×33.9 as columns to find allowable unbraced length and find kL/r.
C15×33.9: A = 10, $r_x = 5.62$, $r_y = 0.901$
The two channels must be interconnected because each individual channel carrying half the load would yield: kL/r = 12(20)/.901 = 266.37 > 200 which is the allowable slenderness limit for compression.
Assume single lacing at 45° with bolt holes 1.5″ from inside edge of channels.

$L = 2(\text{built-up column width} - 2(b_f - 1.5''))$
$= 2(18 - 2(3.40 - 1.5)) = 28.4''$

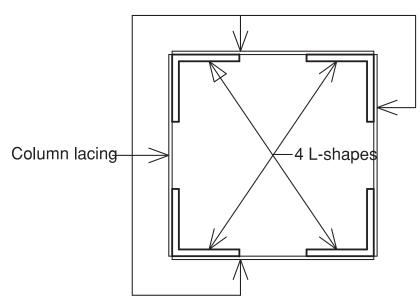
$kL/r_y = 1.0(28.4''/0.901'') = 31.52$

9. Find $\phi_c F_{cr}$:
$F_e = \pi^2 E/(kL/r)^2 = \pi^2(29000ksi)/(31.52)^2 = 288.09ksi$
$4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/50)} = 113.43$
$F_{cr} = (.658^{Fy/Fe})Fy = (.658^{50/288.09})(50) = 46.5ksi$
$A = 10in^2$ and P = 800k/2 = 400k on each channel section.
$f_c = P/A = 400/10 = 40ksi \leq \phi_c F_{cr} = 46.5ksi$ … okay

Built-up columns made of angles are a special case. Please note the different equations used to calculate the slenderness ratio in step 3 of the next example.

Example 21-7: Check the adequacy of the column in Figure 21.2 for a factored axial load, P = 800k.

The total unbraced height is 30′ in both directions. The individual L4×4×1/2 L-shapes form an 18″ square column and are braced at 4′o.c. F_y = 36ksi



21.5
Four angle built-up column

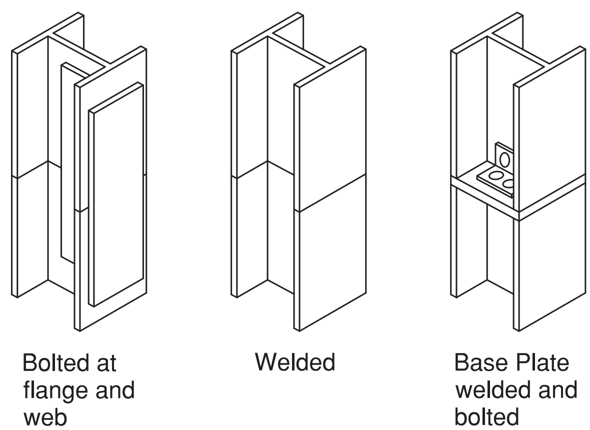
1. Consider the column as a whole and determine the Area, Moment of Inertia and Radius of Gyration in each direction:
   $L4 \times 4 \times \frac{1}{2}$: $A = 3.75$, $I_x = I_y = 5.52$, $r_x = r_y = 1.21$,
   $y = x = 1.18$
   $A = 4(3.75) = 15.0$
   $I_x = I_y = 4(5.52) + 15(9 - 1.18)^2 = 939.37in^4$.
   See Chapter 7 for method to find I_x and I_y.
   $r = \sqrt{(I/A)} = \sqrt{(939.37/15)} = 7.91''$
2. Find $\phi_c F_{cr}$ for column as a whole
   $kL/r = 30(12)/7.914 = 45.49$
   $F_e = \pi^2 E/(kL/r)^2 = \pi^2(29000ksi)/(45.49)^2 = 138.31ksi$
   $4.71\sqrt{(E/F_y)} = 4.71\sqrt{(29000/36)} = 133.68 > kL/r$
   therefore use Equation E3-2: $F_{cr} = (.658^{Fy/Fe})F_y$
   $= (.658^{36/138.31})(36) = 32.28ksi$
   $\phi_c F_{cr} = .9(32.28) = 29.05ksi$
   $\phi_c P_n = \phi_c F_{cr}(A) = 29.05(15) = 435.75k$
3. Consider individual angles in compression. As shown in Figure 21.2, the column has single lacing with bolts at 1.5″ from inside edge (2.5 from outside edge).

   $L = 2(18 - 2(2.5)) = 26''$

   Because single angles often have large eccentricities when loaded, the AISC has two equations for a modified slenderness ratio when the angles are members of a box truss or space truss. The built-up column shown is essentially a vertical box truss and so AISC E5-3 and E5-4 govern.
   If $L/r_x \le 75$, use E5-3: $kL/r = 60 + 0.8L/r_x$.
   If $L/_{rx} > 75$, use E5-2: $kL/r = 45 + 1.25L/r_{x.} \le 200$.
   $L/r_x = 26''/1.21'' = 21.49 < 75$, therefore use E5-3.
   $kL/r = 60 + 0.8L/r_x = 60 + 0.8(21.49) = 77.19$
   Find $\phi_c F_{cr}$:
   $F_e = \pi^2 E/(kL/r)^2 = \pi^2(29000ksi)/(77.19)^2 = 48.04ksi$
   $4.71\sqrt{(E/Fy)} = 4.71\sqrt{(29000/36)} = 133.68 > kL/r$
   therefore use Equation E3-2: $F_{cr} = (.658^{Fy/Fe})F_y$
   $= (.658^{36/48.04})(36) = 26.31ksi$
   $\phi_c F_{cr} = .9(26.31) = 23.68$
   $\phi_c P_n = 23.68(3.75)(4) = 355.2k$
4. $\phi_c P_n$ is the lesser of the values obtained for the column as a whole and for individual angles. In this case, the column as a whole can safely carry 435.75k while the individual angles can only carry 355.2k. This means that the column will fail by the buckling of an individual angle at $\phi_c P_n = 355.2k$.

21.4 Column Splices

The length of a column may be longer than the length that can be manufactured or transported to the site. In such a case, the column segments must be spliced together so that the transfer of loads between components can safely occur. Column splices may also occur because it may be more economical to use smaller columns at the top and increase in size as loads are accumulated. In either scenario, it is recommended to place column splices at 4′ above finished floor to allow room for beam connections. See AISC Steel Manual Table 14-3 for typical column splice details. There are nine splicing scenarios covered by the AISC Steel Manual Table 14-3, but the three most common are shown in Figure 21.6.



21.6
Column splices

General guidelines for column splices include:
1. Use a welded splice plate for splicing columns of same depth.
2. Use a bearing plate when splicing columns of different depths.
3. Plates may be applied at the flanges, web or both.
4. When there is moment in the column, the plates may have to carry up to 75% of the design load.

Practice Exercises:

21-1: Design most economical W14, L = 30′, P_D = 200k, P_L = 400k using A992 steel for the following end conditions:

a) both ends fixed;
b) one end fixed and one end pinned;
c) both ends pinned.

21-2: Find $\phi_c P_n$ of 18′ HSS $10 \times 18 \times \frac{1}{2}$ fixed at both ends and F_y = 46ksi.
21-3: Find the maximum compressive load, ϕP_n for a W14 × 120 with a 1/2″ × 12″ plate bolted to each flange with a bolt spacing of 16″ along the length of the 24′ column. F_y = 50ksi.

21-4: Repeat exercise 21-3 if the bolts at 16″o.c. are replaced with 2″welds at 18″o.c.

21-5: Design two 12″ channels used to form a 12″ square with F_y = 50ksi, P_u = 400k, and L = 16′. Assume single lacing at 45° with bolts at 1″ from inside edge.

21-6: Design a 16 × 16″ column made of 4 angles to support a factored load, P_u = 500k if L = 14′. Assume double lacing at 45° with bolts 1.5″ from inside edges.

twenty two

Steel Tension Design

In designing steel components subjected to tension, the tensile stress, $f_t = P/A$ is a good starting point to determine how much cross-section area is required to prevent the component from pulling apart. Most components in tension, however, are connected to another member with a bolted connection. In these cases, not only must gross yielding in tension be considered, but also tensile rupture, block shear and the shear and bearing strength of the bolts. Bolt design is covered in Chapter 24: Steel Connections.

Bolt hole sizes:

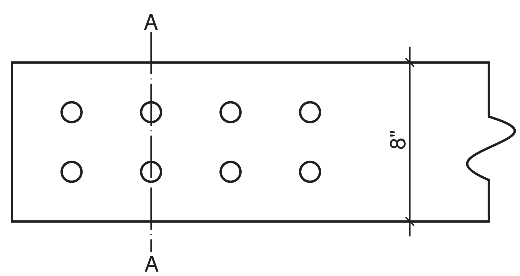
Standard bolt holes are punched or drilled $\frac{1}{16}''$ larger than the bolt diameter. But punching holes may damage steel beyond the hole perimeter. Therefore, for design with punched holes, $\frac{1}{8}''$ is added to the bolt size to determine the design size of the bolt hole. Drilled holes only require a $\frac{1}{16}''$ addition, but for consistency, in this text $\frac{1}{8}''$ is added to the bolt diameter in all cases to find bolt hole size.

Before discussing the analysis and design methodologies for tensile rupture and block shear, it is necessary to understand how to find the net area of a cross-section. Net area (A_n) is the cross-sectional gross area of a component minus the area of the bolt holes. In calculating A_n, every possible path of fracture must be examined.

A_g = gross area

A_{bh} = area of bolt holes where A_{bh} = (number of bolt holes)(bolt hole diameter)(thickness)

A_n = net area = $A_g - A_{bh}$

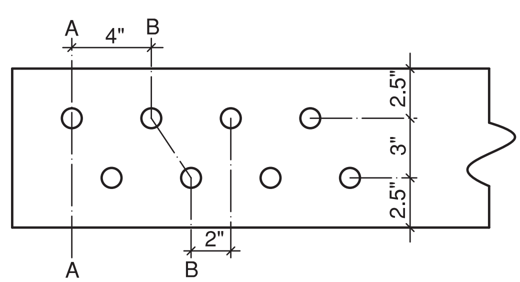


22.1

1″ × 8″ plate with holes aligned

In Figure 22.1, the gross area of the 1″ × 8″ plate = A_g = 1″(8″) = 8in². The bolt size is $\frac{3}{4}''$. The bolt hole size is $\frac{3}{4}'' + \frac{1}{8}'' = \frac{7}{8}''$ diameter.

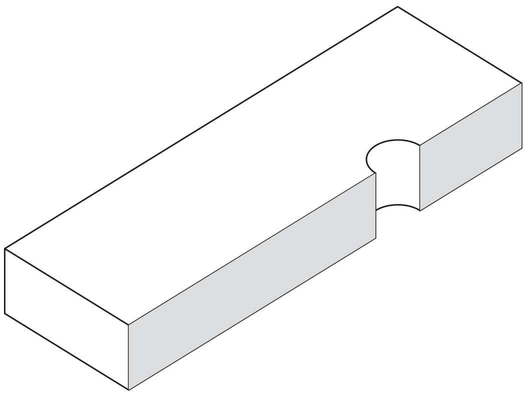
$A_n = A_g - A_{bh} = 8\text{in}^2 - (2\text{bolts})(\frac{7}{8}''\text{ diameter})(1''\text{ thickness}) = 6.25\text{in}^2$



22.2

1" × 8" plate with staggered holes

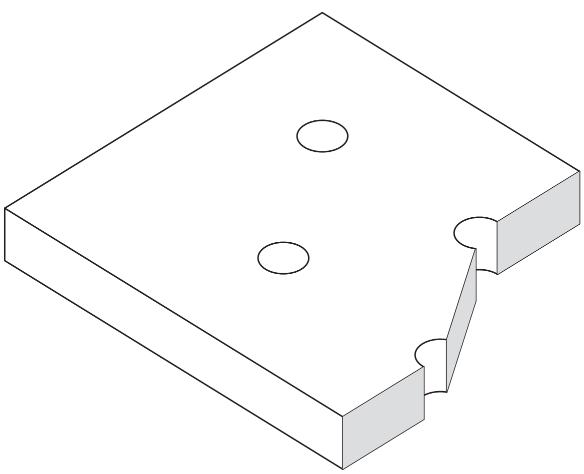
In Figure 22.2, the plate has staggered holes and there are two possible paths through holes to be considered in determining A_n. Path A goes through one bolt hole, creating a section of Path A as shown in Figure 22.3.



22.3
Path A

Path A: $A_n = A_g - A_{bh} = 8in^2 - (1bolt)(\frac{7}{8}''\text{ diameter})(1''\text{ thickness}) = 7.125in^2$

Path B goes through two bolts that are separated by a diagonal as shown in the section in Figure 22.4.



22.4
Path B

Because shear and tensile stresses occur together in diagonals between staggered holes, the actual length of the diagonal cannot be used to determine A_n. Instead, use the $S^2/4G$ rule where:

G = gauge = the distance between the rows of bolt holes (G = 3″ in this example);

S = spacing = the distance between diagonal holes measured parallel to the line of the rows. (S = 2.00 in this example);

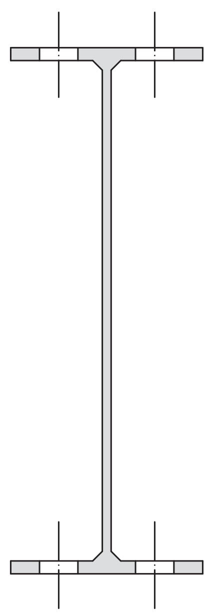
d_{bh} = diameter of bolt hole;

$A_n = A_g - (d_{bh})(\#\text{holes in path})(t) + \Sigma(S^2/4G)(t) = 8in^2 - (\frac{7}{8}'')(2)(1'') + (2^2/4(3))(1) = 8 - 1.75 + 0.33 = 6.58''$

Once A_n has been determined for all paths, use the lesser value in determining tensile rupture.

To find A_n in standard shapes, look up the value for the cross-sectional gross area, listed in the AISC Steel manual Table 1-1 under the heading Area. Next note the flange thickness, t_f and/or the web thickness, t_w depending on where the holes are located in the cross-section.

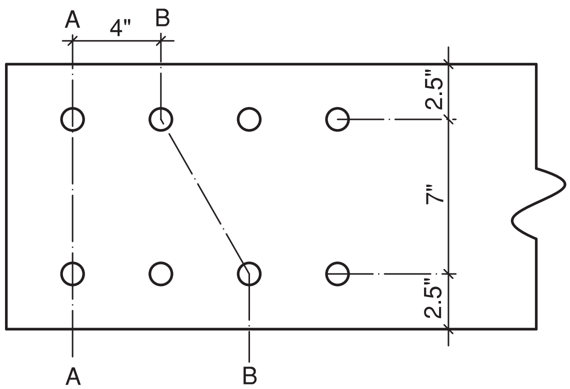
Example 22-1: Find the net area of a W14×22 with 2 – 1.125″ diameter bolt holes through each flange.



22.5
W14 × 22 with four bolt holes

From Table A3.1: $A = 6.49in^2$, $t_f = 0.335''$

$A_n = A_g - (\#\text{ holes})(\text{bolt hole dia.})(t_f)$
$= 6.49 - 4(1.125'')(0.335'') = 4.98in^2$



22.6

Channel back with four bolts (a) Path A (b) Path B

Example 22-2: Find the net area of a C12×30 with two rows of two $\frac{3}{4}''$ bolts in the web with 7″ gauge, 4″ pitch.

Note that when bolt holes are aligned, the smallest value of A_n will be the straight path through the member as shown in Figure 23.6(a). This is because Path B as shown in Figure 23.6(b) will go through the same number of holes but will add the value of $S^2/4g$ for the diagonal, making the net area, A_n, that much larger. A_n for Path A and Path B are shown below to demonstrate this point; however, in analysis and design with aligned holes, Path B need not be considered.

From AISC Steel Manual Table 1-1: t_w = 0.510″, A = 8.81″
Path A: $A_{nA} = A_g - (d_{hole})$(#holes)(t) = 8.81 – (.875)(2)(.51)
= 7.92in^2
Path B: $A_{nB} = A_g - (d_{bh})$(#holes in path)(t) + ($S^2/4G$)(#diagonals)(t) = 8.81 – (0.875)(2)(.51) + (16/28)(1)(.51) = 8.21in^2
Use Path A: A_n = 7.92in^2

22.1 Gross Yielding in Tension

Using the LRFD method where:

ϕ = Resistance Factor,

F_y = the yield stress of the steel and

A_g = gross area of the cross-section,

ϕ_t = 0.9 = tensile Resistance Factor

$P_n = A_gF_y$ = nominal load

$P_u = \phi_tA_gF_y$ = ultimate load or design strength
= gross yielding.

Example 22-3: Determine ultimate tensile load of an A992 steel 1″×4″ plate.

ϕ_t = 0.9, A_g = 4in^2, F_y = 50ksi

P_u = (0.9)(4in^2)(50ksi) = 180kips

Example 22-4: Determine the ultimate tensile load of an A992 steel W14×90.

ϕ_t = 0.9, A_g = 26.5in^2, F_y = 50ksi

P_u = (0.9)(26.5in^2)(50ksi) = 1192.5kips

22.2 Tensile Rupture Strength

In cases where members in tension have bolted connections, tensile rupture must be considered. Using the LRFD method where ϕ = Resistance Factor,
F_u = the ultimate stress of the steel
= 65ksi for A992 steel
= 60ksi for steel pipe
= 58ksi for A36 steel
A_e = effective area of the cross-section
ϕ = 0.75 = tensile rupture Resistance Factor
$P_n = A_eF_u$ = nominal load
$P_u = \phi A_eF_u$ = ultimate load or design strength
= tensile rupture strength.
$A_e = A_nU$ where:
U is the shear lag factor from Figure 22.7 or Table D-3.1 of the AISC Steel Manual;
U = larger of either Table D-3.1 or the equation U = 1 – x/L;
L = distance between the first and last bolts in the line;
x = distance from line of bolts to the N.A. of the portion of the member supported by the bolts.
If a W10×45 has four bolt lines, one on each side of each flange, then the value of x is the value of y-bar taken from the WT5×22.5 section properties, which = .907. But if the W10×45 has bolt lines on only one flange, then the value of x is the y-bar value from a W10×45 which is 5″. The AISC allows either U = 1 – x/L or Case 7 for W, M, S and HP shapes.

For example, if the W10×45 has four lines on bolts with four bolts per line at 3″o.c.,
L = 3(3″o.c.) = 9″ and X = 0.907. This means that U = 1 − x/L = 1 − .907/9 = 0.90.
Using Case 7, AISC Table D-3.1 states that for a flange connected with three or more fasteners per line in direction of loading, if

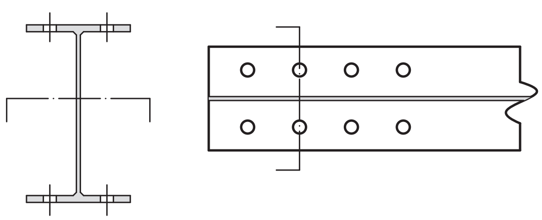
$b_f \geq 2/3d$, U = 0.90 and if

$b_f < 2/3d$, U = 0.85

$b_f = 8.02$, 2d/3 = 2(10.1)/3 = 6.73 therefore U = 0.90.

But if the number of bolts per line is reduced to two:
L = 3″ and U = 1 − .907/3 = 0.70.

Example 22-5: Find the ultimate allowable load P_u for the A992 steel W14×43 with 2 lines of holes in each flange for $\frac{7}{8}$″ bolts as shown in Figure 22.7.



22.7
W14×43 with four lines of holes

From Table A3.1: W14×43: A = 12.60in², t_f = 0.53″, b_f = 8.00″, d = 13.7″
Gross yielding:

F_y = 50ksi for A992 steel
A_g = A = 12.6in²
$P_u = \phi_t A_g F_y$ = 0.9(12.6in2)(50ksi) = 567k

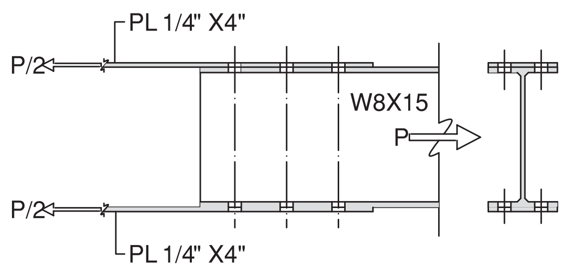
Tensile rupture:

Bolt hole diameter = d_{bh} = bolt diameter + 1/8″ = 7/8″ + 1/8″ = 1″
$A_n = A_g - A_{bh} = A_g$ − (# bolt holes)(bolt hole diameter)(thickness) = 12.6in² − 4(1″)(0.53″) = 10.48in²
To determine the value of U for a W shape with 3 or more bolts per row, see case 7 of Table 22.1. U = 0.9 if $b_f \geq 2d/3$. U = 0.85 if $b_f < 2d/3$.
b_f = 8.0″ and 2d/3 = 2(13.7)/3 = 9.13″. Therefore, $b_f < 2d/3$ and U = 0.85.
$A_e = UA_n$ = 0.85(10.48in²) = 8.91in²
F_u = 65ksi = ultimate stress for A992 steel.
$P_u = \phi P_n = .75F_uA_e$ = 0.75 (65ksi)(8.91in²) = 434.36k

Tensile design strength is the lesser value of the gross yielding strength and the tensile rupture strength.
434.6k < 567k, therefore: P_u = 434.36k.

Tensile connections may have more than one shear plane. Multiple shear planes in a connection may occur because more than two structural members are connected or because symmetry is desired or because several plates are required to carry the tensile load. When dealing with multiple components, remember that tensile forces must be balanced.

Example 22-6: Tension connection with multiple shear planes.



22.8
Tension connection with multiple shear planes

Table 22.1: U-factors.

SHEAR LAG FACTOR, U				
CASE	DESCRIPTION			U
1	Plates and built-up members connected by bolts			1.00
7	W, M, S or HP Shapes or Tees cut from these shapes	3 or more bolts in flange in direction of load	bf ≥ 2d/3	0.90
			bf < 2d/3	0.85
		4 or more bolts in web in direction of load		0.70
8	Single Angles	4 or more bolts in direction of loading		0.80
		2 or 3 bolts in direction of loading		0.80

Find the maximum design load, P_u, for the connection in Figure 22.8.

1. Check W8 × 15 for gross yielding:
   $A = 4.44in^2$, $t_f = 0.315''$, $b_f = 4.01''$, $d = 8.11''$
   $P_u = \phi P_n = 0.9P_n = 0.9F_yA_g = .9(50ksi)(4.44in^2)$
   $= 199.8k$ for gross yielding in the W8 × 15.
2. Check W8 × 15 for tensile rupture:
   $A = 4.44in^2$, $t_f = 0.315''$, $b_f = 4.01''$, $d = 8.11''$
   $P_u = \phi P_n = 0.75Pn = 0.75F_uA_e = 0.75(65)A_n(U)$
   $A_n = A_g - (\#lines)(d_{BH})(t_f) = 4.44 - 4(7/8)(.315)$
   $= 3.338in^2$
   Check U, Case 7: $(2/3)d = (2/3)(8.11) = 5.41 > b_f = 4.01$
   therefore, $U = 0.85$
   $A_e = A_nU = 3.338in^2(0.85) = 2.84in^2$
   $P_u = .75(65)(2.84) = 138.45k$ for tensile rupture in the W8 × 15.
3. Check $\frac{1}{4} \times 4''$ plates for gross yielding: Note that because there are two equal size plates carrying the total load P_u, each plate will carry half the load or $P_u/2$.
   $P_u/2 = \phi P_n = 0.9P_n = 0.9F_yA_g = .9(36)(4)(.25) = 32.4k$
   $P_u = 32.4k(2) = 64.8k$ for gross yield in plates
4. Check plates for tensile rupture:
   $P_u/2 = \phi P_n = 0.75P_n = 0.75F_uA_e = 0.75(58)A_n(U)$
   $A_n = 4(.25) - 2(7/8)(.25) = 0.5625in^2$
   $U = 1.0$ for plates … $A_e = A_nU = 0.5625in^2$
   $P_u/2 = .75(58)(0.5625) = 24.47k$ $P_u = 48.94k$ for tensile rupture in plates.
5. The smallest value governs, therefore, the design strength of the connection is 48.94k. Beyond that, the connection will fail by tensile rupture in the plates.
   If the design strength is not adequate, the size of the plates could be increased so that plate rupture becomes equal to beam rupture.

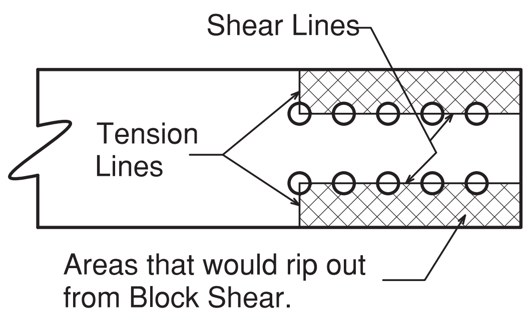
Find area of plates so that $P_u = 138.45k$ in tensile rupture:

1. Assume plate width b and plate thickness t
2. Find desired tensile rupture strength per plate: $P_u/2 = \phi P_n = 0.75P_n = 0.75F_uA_e = 0.75(58)A_n = 138.45/2 = 69.23$
3. Find A_n in terms of b and t:
   $A_n = A_g - (\#lines)(d_{bh})(t_f) = b(t) - 2(7/8)(t) = t(b - 1.75)$
4. Set A_n equal to desired $A_n = (P_u/2)/.75F_u$
   $t(b - 1.75) = 69.23/[.75(58)] = 1.591in^2$

   if $b = 4$, $t = 1.591/(2.25) = 0.707''$
   Option 1: increase thickness to $\frac{3}{4}''$ PL $4'' \times \frac{3}{4}''$
   OR if $t = .25''$ $b = 1.591/.25 + 1.75 = 8.114$
   Option 2: increase width to 8.25″ PL $8.25'' \times \frac{1}{4}''$

22.3 Block Shear

Block shear is a type of rupture—a tearing out of a section of steel at the corner of a member. This type of tearing occurs through the bolt holes and involves both shear and tension in the process; shear parallel to the line of force and tension perpendicular to the line of force.



22.9

Block shear

$\phi = 0.75$ for block shear

$P_u = \phi R_n = .75R_n$

$R_n = .6F_UA_{nv} + U_{BS}F_UA_{nt} < .6F_YA_{gv} + U_{BS}F_UA_{nt}$

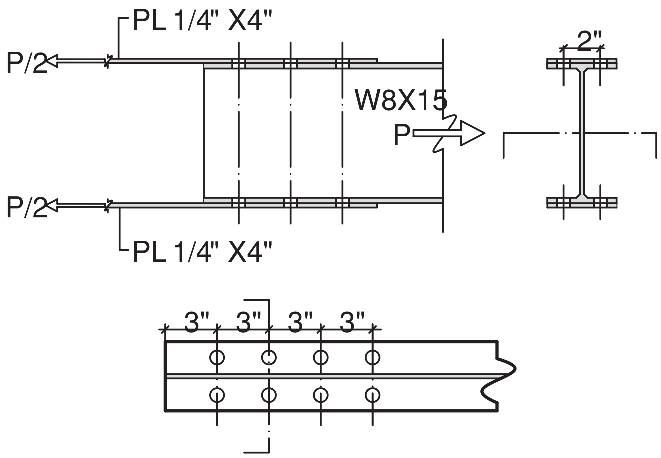
A_{gv} = gross area subjected to shear = (#lines)(distance from center of farthest bolt to end)

A_{nv} = net area subjected to shear = A_{gv} − (#lines)(#holes per line − 0.5)(d_{bh})

A_{nt} = net area subjected to tension = (#lines)(distance from centerline of bolt to edge − $d_{bh}/2$)

To check for block shear, members should first be checked for gross yielding and tensile rupture.

Example 22-7: Find the block shear in the connection from Example 22-6: If there are three bolts spaced at 3″o.c. per line and 3″ from the end, with lines 2″ apart.



22.10

Block shear example 22-7

From Example 22-6, the following values were obtained for gross yielding and tensile rupture:

W8 × 15 in gross yielding:	P_u = 199.8k
W8 × 15 in tensile rupture:	P_u = 138.45k
Plates in gross yielding:	P_u = 68.4k
Plates in tensile rupture:	P_u = 48.94k

1. A_{gv} = #lines(length of shear line)(t_f) = 4(3″ + 3″ + 3″)(.315″) = 11.34in^2
2. A_{nv} = A_{gv} – (#lines)(#holes per line – 0.5) (d_{BH})(t_f) = 11.34in^2 – (4)(2.5)(3/4″ + 1/8″)(.315″) = 8.584in^2
3. The length of the tension line is (bf – distance between lines)/2 = (4.01 – 2)/2 = 1.005″.
   A_{nt} = (#line)[length of tension line – (0.5) (hole dia.)](t_f) = 4[1.005″ – 0.5(7/8″)](.315″) = 0.72in^2
4. Use lesser of both equations:
   $P_u = \phi R_n$ = 0.75 (0.6$F_y A_{gv}$ + $F_u A_{nt}$) = .75[.6(50ksi)(11.34in^2) + 65ksi(0.72in^2) = 290.25k
   OR
   $P_u = \phi R_n$ = 0.75 (0.6$F_u A_{nv}$ + $F_u A_{nt}$) = .75[.6(65ksi)(8.58in^2) + 65ksi(0.72in^2) = 286.07k
5. Block shear in the W8 × 15: P_u = 286.07k

Check the block shear in the PL $\frac{1}{4} \times 4''$:

1. A_{gv} = 2(9″)(.25″) = 4.5in^2
2. A_{nv} = 4.5in^2 – (2)(2.5)(3/4″ + 1/8″)(.25) = 3.41in^2
3. The length of the tension line = (4 – 2)/2 = 1″
   A_{nt} = 2[1 – 0.5(7/8)](.25) = 0.28in^2
4. $P_u/2 = \phi R_n$ = 0.75 (0.6$F_y A_{gv}$ + $F_u A_{nt}$) = .75[.6(36ksi)(4.5in^2) + 58ksi(0.28in^2)] = 85.08k … P_u = 2(85.08k) = 170.16k
   <listpara >OR
   $P_u/2 = \phi R_n$ = 0.75 (0.6$F_u A_{nv}$ + $F_u A_{nt}$) = .75[.6(58ksi)(3.41in^2) + 58ksi(0.28in^2)] = 101.18k … Pu = 2(101.18k) = 202.36k
5. Block shear in the plates: P_u = 170.16k.

Failure will occur in plate rupture at 48.94k.

22.4 Design of Tension Members

To design tension members, the goal is to find the gross area required for gross yielding and tensile rupture. Block shear is checked in the designed member and the size is adjusted if needed. Since $P_u \le 0.9F_y A_g$ for gross section yielding, dividing both sides of the equation yields $A_g \ge P_u/0.9F_y$.
For example: Design a W section for a tensile load P_u = 240kips using A992 steel.

$A_g \ge$ 240k/[(0.9)(50ksi)] = 5.33in^2

A W10 × 19 would work with A_g = 5.62in^2

For tension members with bolts, tensile rupture must also be considered.

Since $P_u = \phi P_n = 0.75F_u Ae$ for tensile rupture, $A_e \ge P_u/0.75F_u$. And since $A_e = UA_n$, $A_n \ge P_u/0.75F_u U$. But it is the gross area, not the net area that must be determined. $A_g = A_n + A_{bh.}$ At this point, the area of bolt holes (A_{bh}) must be estimated. Because the number of lines of bolts and the bolt size are typically decided, A_{bh} can be expressed in terms of t_f.

A_{bh} = (#lines)(d_{bh})(t_f)

$A_g = A_n$ + (#lines)(d_{bh})(t_f)

$A_g \ge P_u/0.75F_u U + A_{bh}$

When designing tension members, the slenderness ratio, L/r ≤ 300.

The procedure for design of tensile connections:

1. Determine the factored load, P_u.
2. $A_g > P_u/(.9(F_y))$

3. Choose assumed value for U: 1.0 for plates, 0.85 for W shapes.
4. Assume a value for t_f. $A_g \geq A_n + A_{bh} = P_u/(0.75F_uU) +$ (#lines)(bolt hole dia.)(t_f)
5. $r \geq L(12in/f)/300$
6. Select a trial size and note: A_g, t_f and r_y. Check that $A_{gREQD} = P_u/(0.75F_uU) +$ (#lines)(bolt hole dia.)(t_f) $> A_{g\ ACTUAL}$
7. A_{gv} = (#lines)(shear line length)(t_f)

   $A_{nv} = A_{gv}$ – (#lines)(#bolt holes)(d_{bh})(t_f)

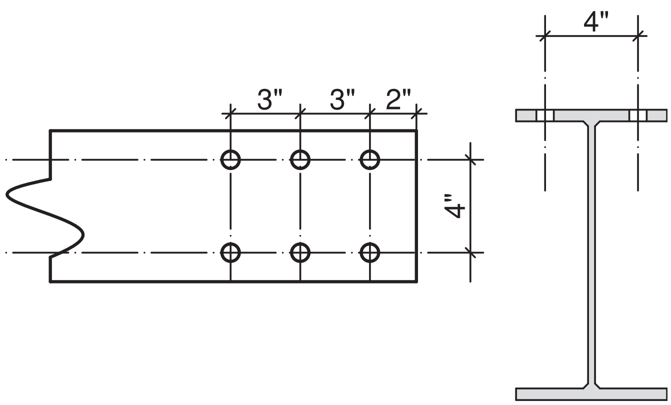
   A_{nt} = (#line)[length of tension line – (0.5)(d_{bh})](t_f)

8. Check both Shear Block Equations:

   $P_u \leq .75[.6F_UA_{nv} + U_{BS}F_UA_{nt}]$

   $P_u \leq .75[.6F_YA_{gv} + U_{BS}F_UA_{nt}]$

Example 22-8: Design a 20′ long W12 section for a tensile load P_u = 240kips using A992 steel assuming two rows of three $\frac{3}{4}$″ bolts @ 3″o.c. on one flange, 2″ from end and lines at 4″o.c.



22.11
Block shear in a W12 with holes in one flange

1. P_u = 240kips
2. Gross yielding: $A_g > P_u/(.9(F_y))$ = 240k/(.9(50ksi)) = 5.33in^2
   The size must be at least a W12×19 (A = 5.57in^2)
3. Choose assumed value for U = 0.85
4. Assume t_f = .4, $A_g \geq A_n + A_{bh} = P_u/(0.75F_uU)$ + (#lines)(bolt hole dia.)(t_f)

   = 240k/0.75(65ksi)(0.85) + 2(3/4″ + 1/8″)(0.4″)
   = 5.792 + 1.75t_f = 6.49in^2

5. r ≥ 20′(12in/f)/300 = 0.8″
6. Try W12×26: A_g = 7.65, t_f = 0.38, r_y = 1.51, b_f = 6.49, d = 12.2
7. r_y = 1.51″ > 0.8″ … okay
8. Check the U value and adjust the equation for A_n if necessary:

   2d/3 = (2/3)(12.2) = 8.13 > 6.49 = b_f … U = 0.85 … okay

9. A_g = 7.65 > 5.792 + 1.75(0.38) = 6.46 … okay
10. Block shear:

    A_{gv} = (#lines)(shear line length)(t_f) = 2(3 + 3 + 2)(0.38) = 6.08
    $A_{nv} = A_{gv}$ – (#lines)(#bolt holes)(d_{bh})(t_f) = 6.08 – 2(2.5)(0.875)(0.38) = 4.42in^2
    b_f = 6.49, tension line = (6.49 – 4)/2 = 1.245″
    A_{nt} = 2(1.245 – .5(.875)) = 1.615in^2
    P_u = 0.75[0.6(65ksi)(4.42) + 1.0(65ksi)(1.615)] = 208.02 k < 240k
    The W12×26 is not adequate. Go back to step 6 and try a larger size.

6A. Try W12×35: A_g = 10.3, t_f = 0.52, r_y = 1.54, d = 12.5, b_f = 6.56
7A. r_y = 1.54″ > 0.8″ … okay
8A. Check the U value and adjust the equation for A_n if necessary:

   2d/3 = (2/3)(12.5) = 8.33 > 6.56 = b_f … U = 0.85 … okay

9A. A_g = 10.3in^2 > 5.792 + 1.75(0.52) = 8.06in^2 … okay
10A. Block shear:

   A_{gv} = (#lines)(shear line length)(t_f) = 2(3 + 3 + 2)(0.52) = 8.32in^2

   $A_{nv} = A_{gv}$ – (#lines)(#bolt holes – .5)(d_{bh})(t_f) = 8.32 – 2(2.5)(0.875)(0.52) = 6.05in^2

   b_f = 6.56, tension line = (6.56 – 4)/2 = 1.28″

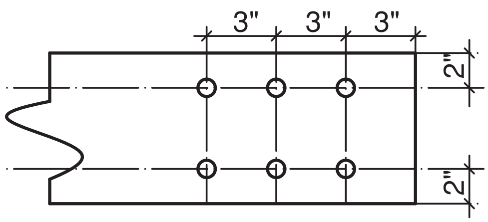
   A_{nt} = 2(1.28 – .5(.875)) = 1.69″

$P_u = 0.75[0.6(50ksi)(8.32) + 1.0(65ksi)(1.69)]$
$= 269.59k > 240k$

$P_u = 0.75[0.6(65ksi)(6.05) + 1.0(65ksi)(1.69)]$
$= 259.35k > 240k$ … okay

ANSWER: Use W12 × 35

It is often possible to reduce the weight of a tension member whose size is governed by block shear by increasing the distance from the holes to the end or the edges. For example, if the problem is changed so that the distance from the holes to the end is 3″ instead of 2″ and the distance between lines of bolts is changed so that the length of the tension line is 1.5″ instead of 1.28″, the answer would change.



22.12
Changed bolt hole spacing

Example 22-9: Design a 20′ long W12 section for a tensile load P_u = 240kips using A992 steel assuming two rows of three $\frac{3}{4}$″ bolts @ 3″o.c. on one flange, 3″ from end and 1.5″ from edges.

1. P_u = 240kips
2. Gross yielding: $A_g > P_u/(.9(F_y)) = 240k/(.9(50ksi)) = 5.33in^2$
   Must be at least a W12 × 19 ($A = 5.57in^2$)
3. Choose assumed value for U = 0.85
4. Assume $t_f = .4$, $A_g \geq A_n + A_{bh} = P_u/(0.75F_uU)$ + (#lines)(bolt hole dia.)(t_f) = 240k/0.75(65ksi)(0.85) + 2(3/4″ + 1/8″)(0.4″) = 5.792 + 1.75t_f = 6.49in²
5. r ≥ 20′(12in/f)/300 = 0.8″
6. Try W12 × 26: $A_g = 7.65$, $t_f = 0.38$, $r_y = 1.51$, $b_f = 6.49$, d = 12.2
7. r_y = 1.51″ > 0.8″ … okay
8. Check the U value and adjust the equation for A_n if necessary:
   2d/3 = (2/3)(12.2) = 8.13 > 6.49 = b_f … U = 0.85 … okay
9. A_g = 7.65 > 5.792 + 1.75(0.38) = 6.46 … okay
10. Block shear:
    A_{gv} = (#lines)(shear line length)(t_f) = 2(3 + 3 + 3)(0.44) = 7.92
    $A_{nv} = A_{gv}$ − (#lines)(#bolt holes)(d_{bh})(t_f) = 7.92 − 2(2.5)(0.875)(0.44) = 6.0
    b_f = 6.52, tension line = 1.5″
    $A_{nt} = 2(1.5 - .5(.875)) = 2.13in^2$
    $P_u = 0.75[0.6(65ksi)(6.0) + 1.0(65ksi)(2.13)] = 279.34k > 240k$
    $P_u = 0.75[0.6(50)(7.92) + 65(2.13)] = 282.04k > 240k$
    USE: W12 × 26

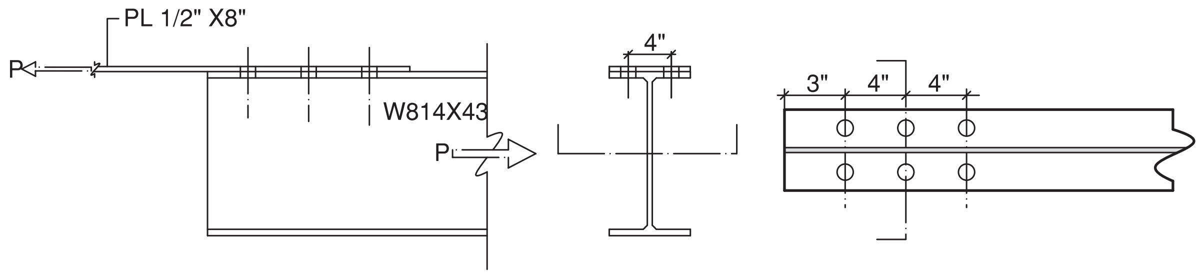
Practice Exercises:

22-1 and 22-2: Find the design strength of the connections shown.

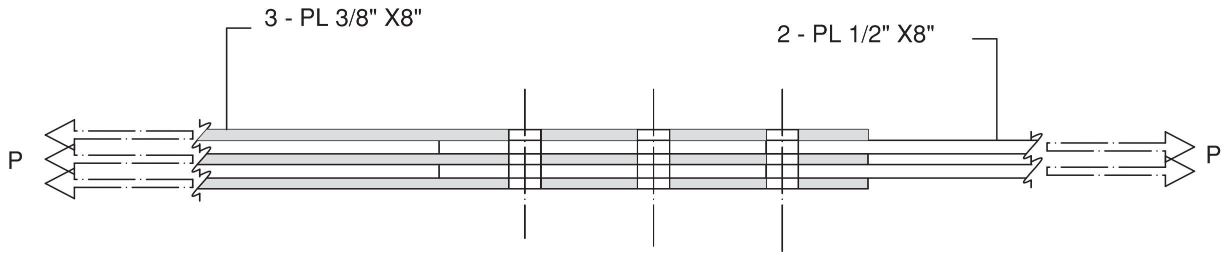
22-3: Find the narrowest 6″ plate thickness, t, for the connection shown if P_u = 500k.

22-4: Find the most economical W14 for a connection with a tensile load of 1200# if there are four lines of bolts (2 in each flange). Each line has four bolts with 1″ diameter bolt holes spaced at 3″o.c. and 3″ from the end. The lines of bolts are b_f/2″ apart.

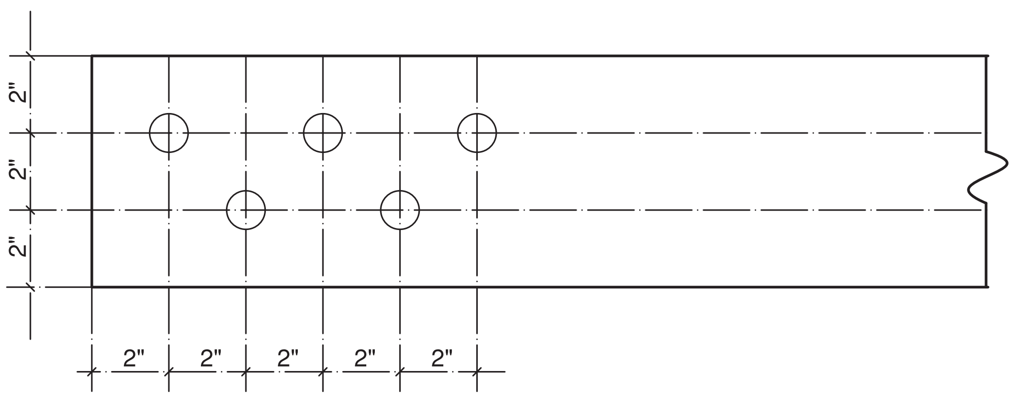
22-5: Repeat problem 22-4 using $\frac{7}{8}$″ bolt holes spaced 4″o.c. and 3″ from the end.



22-1



22-2



22-3

22.13

Chapter 22 Practice exercises

twenty three

Steel Baseplates

A baseplate is a steel plate that is welded or bolted to the bottom of a steel column. The purpose of a baseplate is to distribute the load over an area larger than the column cross-sectional area so that the concrete footing below can support the load. If a heavily loaded column was placed directly on a concrete footing, the compressive stress carried by the steel would be too great for the concrete and the column would punch through the footing.

There are two scenarios to consider in designing a baseplate: full and partial coverage of the concrete. If the baseplate covers the entire area of the concrete, the nominal load (P_p) is $P_p = 0.85f'_cA_1$ where:

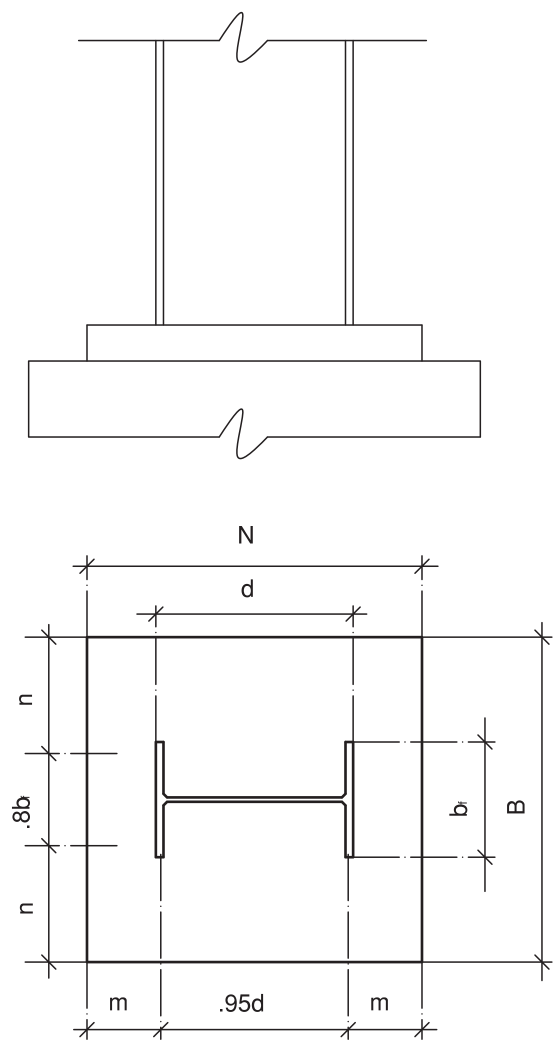
> f'_c = compressive strength of the concrete and
>
> A_1 = gross area of the base plate = BN.
>
> The design load is $P_u = \phi_c P_p$ where $\phi_c = 0.60$ for base plates.

If the area of the concrete is greater than the area of the baseplate, the nominal load, $P_p = (0.85f'_cA_1)\sqrt{(A_2/A_1)}$ where:

> A_2 = the area of the concrete
>
> A_1 = the area of the baseplate = BN and
>
> the value of $\sqrt{(A_2/A_1)} \leq 2$.
>
> f'_c = compressive strength of the concrete

By rearranging the equation for P_p:

> $A_1 = P_u/[\phi_c(0.85f'_c)\sqrt{(A_2/A_1)}]$ where
>
> $\phi_c = 0.60$ for base plates



23.1
Steel baseplate

To design a base plate:

1. Determine P_u
2. Determine footing area in square inches
3. The base plate $A_1 = P_u/[\phi_c(0.85f'_c)(2)]$
4. The base plate must be at least as large as the column dimensions: d by b_f. Check that $A_1 > d(b_f)$, $B > b_f$ and $N > d$.
5. Round B and N up to whole numbers
6. $A_1 = BN$
7. Check the bearing strength of the concrete:
   $P_u < \phi_c P_p = 0.6(0.85f'_c A_1)\sqrt{(A_2/A_1)}$
8. $m = [N - 0.95d]/2$
   $n = [B - 0.80b_f]/2$
   $n' = [\sqrt{db_f}]/4$ = limitations in determining thickness requirement in order to account for columns with light loads.
   l = largest of m,n,n′
9. $t_{req} = l\sqrt{[2P_u/.9F_y BN]}$

Example 23-1: Design a baseplate for a W14×90 column carrying an axial load of P_u = 900k and bearing on a 6′ by 6′ concrete footing with f'_c = 3ksi. d = 14, b_f = 14.5.

1. $P_u = 900k$
2. $A_2 = 6'(12in/f)(6')(12in/f) = 5184in^2$
3. $A_1 = P_u/[\phi_c(0.85f'c)(2)] = 900/[.6(.85)(3)(2)] = 294.12in^2$
4. $db_f = 14(14.5) = 203$. Check that $A_1 > db_f$
5. 294.12 > 203 … okay
6. Round B and N up to whole numbers: $\sqrt{294} = 17.146$
   use 17×18:
   $A_1 = BN = 306in^2$
   Note: $\sqrt{(A_2/A_1)} = 2$
7. Check the bearing strength of the concrete:
   $P_u < \phi_c P_p = 0.6(0.85(3)(306))(2) = 936.36 > 900k$
   … okay
8. Find base plate thickness:

   $m = [N - 0.95d]/2 = [17 - .95(14)]/2 = 1.85$

   $n = [B - 0.80b_f]/2 = 18 - .8(14.4)]/2 = 3.24$

   $n' = [\sqrt{db_f}]/4 = [\sqrt{14(14.5)}]/4 = 3.56 \quad l = 3.56$

9. $t_{req} = l\sqrt{[2P_u/.9F_y BN]} = 3.56\sqrt{[2(900)/.9(36)(17)(18)]} = 1.52''$

Base plate: PL $18\times17\times1\frac{5}{8}''$

Example 23-2: Design the thickness for a base plate of a given size where:

P_u = 900k, column W14×90: d = 14, b_f = 14.5
Footing: f'_c = 3ksi, 25″×25″ pedestal with baseplate covering pedestal

1. $P_u = 900k$
2. $A_2 = 25(25) = 625in^2$
3. $P_p = 0.85F'_c A_1 = .85(3)(625) = 1593.75$ … go right to step 7
7. Check the bearing strength of the concrete:
   $P_u < \phi_c P_p = 0.6(1593.75) = 956.25 > 900k$ … okay
8. Find base plate thickness
   $M = [N - 0.95d]/2 = [25 - .95(14)]/2 = 5.85$

   $N = [B - 0.80b_f]/2 = [25 - .8(14.5)]/2 = 6.7$

   $n' = [\sqrt{db_f}]/4 = [\sqrt{14(14.5)}]/4 = 3.56 \quad l = 6.7$

9. $t_{req} = l\sqrt{[2P_u/.9F_y BN]} = 6.7\sqrt{[2(900)/.9(36)(25)(25)]} = 2''$

Base Plate: PL 25×25×2″

Practice Exercises:

23-1: Design a baseplate for a W24×192 column carrying an axial load of P_u = 2400k and bearing on a 8′×8′ concrete footing with f'_c = 4ksi, d = 25.5, and b_f = 13.0.

23-2: Design the thickness of a 30″×30″ base plate fully covering a pedestal of f'_c = 3ksi concrete and supporting a W14×120 columns with an axial load of P_u = 1200k, d = 14.5, b_f = 14.7.

twenty four

Steel Connections

24.1 Bolted Connections

Bolted connections are the most common type of connection used in steel. The strength of the connections depends not only on the strength of the connected components as discussed in Chapter 22, but also on the shear and bearing strength of the bolts.

In bearing connections, some slippage is assumed and the bolt is checked for bearing and shear. In bearing conditions, the bolts are snug-tight or pre-tensioned without inspection. A snug-tight bolt is tightened by hand and then wrench tightened 1.5 turns. The threads are seated together but do not have to be in continuous contact with each other.

In slip-critical connections, the bolt strength is based on surface conditions, pre-tensioning of the bolt and hole size. In skip-critical connections, bolts are tightened up to 70% of the tensile design strength, threads are in constant contact and contact surfaces as well as the tension must be inspected.

The three most common types of bolts are the A307, A325 and A490. The A307 is a common bolt and has a strength roughly equivalent to A36 steel. The A325 and A490 are high strength bolts and therefore are the most commonly used.

Bolt sizes range from 0.5″ to 1.5″ in diameter in $\frac{1}{8}$″ increments. A designation of X or N is given to high strength bolts. X indicates threads are excluded from the shear planes and N indicates threads are NOT excluded from the shear planes.

24.1.1 Bearing Connection Analysis

Four conditions must be considered for bearing connections:

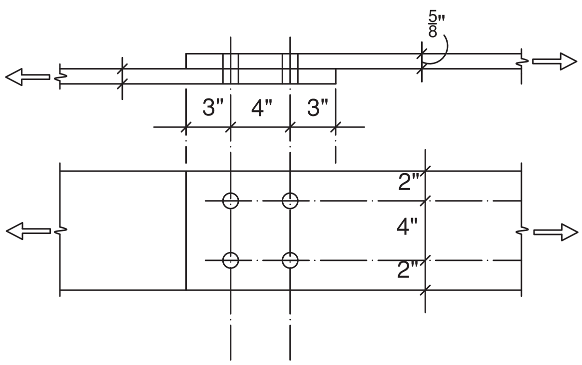
1. Gross yielding of plates: $\phi P_n = 0.9F_yA_g$ as discussed in Chapter 22.
2. Tensile rupture of plates: $\phi P_n = 0.75F_uA_e$ as discussed in Chapter 22.
3. Bearing in bolts: $\phi R_n = 0.75\ R_n$ where R_n is the smaller value of
   $R_n = 1.2\ L_c(t)(F_u)(\#\text{ of bolts})$
   OR
   $R_n = 2.4(d)(t)(\ F_u)(\#\text{ of bolts})$
   L_c = the smallest clear distance between the edges of holes and edges of adjacent holes or material edges in direction of force.
   t = plate thickness
   d = diameter of bolt
4. Shear in bolts: $\phi R_n = \phi F_nA_b = 0.75F_nA_b$ where
   F_n is found in Table 24.1
   A_b = total area of bolts = (TOTAL # bolts)(π)(bolt diameter)^2/4

The design tension is the smallest value from the four cases.

Example 24-1: Determine the design strength ϕ_cP_n for bearing connection in Figure 24.1 if plates are A36 steel and bolts are A-325 with $\frac{3}{4}$″ diameter bolts with threads excluded from the shear plane.

Table 24.1: Nominal shear in bearing connections: F_n (ksi) from AISC Steel Construction Manual Table J3.2.

Bolt Type	Nominal Shear in Bearing Connections: F_n (ksi)
A307	24
A325-N	48
A325-X	60
A490-N	60
A490-X	75



24.1

Finding design strength of a bearing connection

1. Gross yielding in plates:
   $\phi P_n = 0.9F_yA_g = 0.9(36ksi)(0.625)(8) = 162k$
2. Tensile rupture: $\phi P_n = 0.75F_uA_e$
   $A_e = UA_n$ and U = 1 for plates
   $A_e = A_n = (.625'')(8'') - (.625)(3/4 + 1/8)(2) = 4.02in^2$
   Where .625″ is plate thickness and 8″ is plate width
   $\frac{3}{4}'' + \frac{1}{8}'' = 0.785''$ = bolt hole diameter
   2 = # bolt holes in the cross-section
   $\phi P_n = 0.75F_uA_e = 0.75(58\ ksi)(4.02in^2) = 174.87k$
3. Bearing in bolts: $\phi R_n = 0.75R_n$ where R_n is the smaller value of both R_n equations below:
   L_c = minimum clear distance in direction of force
   = smaller of
   3 – .872/2″ = 2.56″ or 4″ – 0.875″ = 3.125″
   $L_c = 2.56''$
   t = 0.625″ = thickness of plate,
   d = 0.75″ = diameter of bolt
   Total number of bolts = 4
   $R_n = 1.2L_c(t)(F_u)$(# of bolts) = 1.2(2.56)(0.625)(58)(4)
   = 445.44k
   OR
   $R_n = 2.4(d)(t)(F_u)$(# of bolts) = 2.4(.75)(.625)(58)(4)
   = 261k
   $\phi R_n = 0.75R_n = 0.75(261k) = 195.75$
4. Shear in bolts: $\phi R_n = \phi F_nA_b = 0.75F_nA_b$
   From Table 24.1 For A325-X bolt, $f_{nv} = 60ksi$
   A_b = total area of bolts = (TOTAL # bolts)(π)(bolt diameter)^2/4 = 4(3.14159)(0.75)^2/4 = 1.767in^2
   $\phi R_n = \phi F_nA_b = 0.75(60)(1.767) = 79.52k$
   The design tension = smallest of four cases
   = 79.52k

24.1.2 Bearing Connection Design

In design, the number of bolt holes required needs to be determined for a given load. This is done after the plates or other connection components are designed.

1. Determine bearing strength in one bolt
2. Determine shear strength in one bolt
3. Determine number of bolts needed

Example 24-2: If the plates in Figure 24.1 are designed for a tensile design load, P_u = 160k, how many rows of A325-N bolts are required?

1. Determine bearing strength in one bolt:
   $R_n = 1.2L_c(t)(F_u)$ (1 bolt)
   L_c = minimum clear distance in direction of force
   = smaller of
   3 – .872/2″ = 2.56″ or 4″ – 0.875″ = 3.125″
   $L_c = 2.56''$
   t = 0.625″ = thickness of plate
   d = 0.75″ = diameter of bolt
   $R_n = 1.2L_c(t)(F_u)$(1 bolt) = 1.2(2.56)(.625)(58)(1)
   = 111.36k/bolt
   OR
   $R_n = 2.4(d)(t)(F_u)$(1 bolt) = 2.4(.75)(.625)(58)(1)
   = 65.25k/bolt
   $\phi R_n = 0.75\ R_n = 0.75(65.25) = 48.94k/bolt$
2. Determine shear strength in one bolt: $\phi R_n = \phi F_nA_b$
   $= 0.75F_nA_b$

From Table 24.1 For A325-X bolt,

f_{nv} = 60ksi

A_b = area of ONE bolt = (π)(bolt diameter)^2/4

= (3.14159)(0.75)^2/4 = 0.442in^2

$\phi R_n = \phi F_n A_b$ = 0.75(60)(0.442) = 19.88k/bolt

3. Determine number of bolts needed:

P_u = 160k (given)

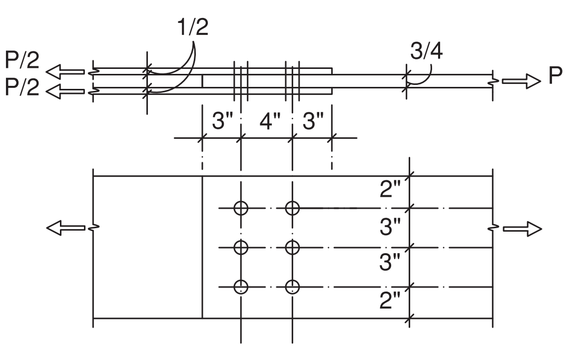
ϕR_n = 19.88k/bolt (#bolts) = 160k … #bolts

= 160/19.88 = 8.05 … round up to 10 bolts.

24.1.3 Analysis for More than One Shear Plane

Analysis for connections with more than one shear plane follows the same procedure as in section 22.1.1. However, it must be remembered that if the number of plates carrying a force in one direction is different than the number of plates carrying a force in the opposite direction, then the thickness to be used is equal to the smaller of the sums of plate thicknesses on each side.

Example 24-3: Determine the design strength $\phi_c P_n$ for bearing connection in Figure 24.3 if plates are A992 steel and bolts are A-490 with $\frac{7}{8}''$ diameter bolts with threads NOT excluded from the shear plane.



24.2

Multiple shear planes

1. Thickness to the left = 0.5(2) = 1 and thickness of plates to the right = 0.75(1) = 0.75 … t = 0.75
2. Gross yielding in plates: $\phi P_n = 0.9F_yA_g$ = 0.9(50ksi)(0.75)(10) = 337.5k
3. Tensile rupture: $\phi P_n = 0.75F_uA_e$

   $A_e = UA_n$ and U = 1 for plates.

   $A_e = A_n$ = (.75″)(10″) – (.75)(7/8 + 1/8)(3) = 5.25in^2

   $\phi P_n = 0.75F_uA_e$ = 0.75(65ksi)(5.25in^2) = 255.94k
4. Bearing in bolts: $\phi R_n = 0.75R_n$ where R_n is the smaller value of both R_n equations below:

   L_c = minimum clear distance in direction of force

   = smaller of

   3 – 1/2″ = 2.5″ or 4″ – 1″ = 3.0″

   L_c = 2.5″

   t = 0.75″ = thickness of plate,

   d = 0.875″ = diameter of bolt

   Total number of bolts = 4

   $R_n = 1.2L_c(t)(F_u)$(# of bolts) = 1.2(2.5)(0.75)(65)(6)

   = 877.5k

   OR

   R_n = 2.4(d)(t)(F_u)(# of bolts) = 2.4(.875)(.75)(65)(6)

   = 614.25k

   $\phi R_n = 0.75R_n$ = 0.75(614.25k) = 460.69k
5. Shear in bolts: $\phi R_n = \phi F_nA_b = 0.75F_nA_b$

   From Table 24.1 For A490-N bolt, f_{nv} = 60ksi

   A_b = total area of bolts = (TOTAL # bolts)(π)(bolt diameter)^2/4 = 6(3.14159)(0.875)^2/4 = 3.608in^2

   $\phi R_n = \phi F_nA_b$ = 0.75(60)(3.608) = 162.34k

The design tension = smallest of four cases = 162.34k. The connection will fail through shear in the bolts. If the design strength is not adequate, the design can be altered to improve the strength. If the bolt size is increased to 1″ diameter, the bolt holes increased to 1.125″ diameter. Steps 3, 4 and 5 would need to be re-evaluated.

3. Tensile rupture: $\phi P_n = 0.75F_uA_e$

   $A_e = UA_n$ and U = 1 for plates.

   $A_e = A_n$ = (.75″)(10″) – (.75)(1.125)(3) = 4.97in^2

   $\phi P_n = 0.75F_uA_e$ = 0.75(65 ksi)(4.97in^2) = 223.59k
4. Bearing in bolts: ϕR_n = 0.75 R_n where R_n is the smaller value of both R_n equations below:

   L_c = minimum clear distance in direction of force

   = smaller of

   3 – 1.125/2″ = 2.44″ or 4″ – 1.125″ = 2.875″

   L_c = 2.44″

   t = 0.75″ = thickness of plate

   d = 1″ = diameter of bolt

   Total number of bolts = 4

   R_n = 1.2 L_c (t)(F_u)(# of bolts) = 1.2(2.44)(0.75)(65)(6)

   = 856.44k

OR

$R_n = 2.4(d)(t)(F_u)(\# \text{ of bolts}) = 2.4(1)(.75)(65)(6) = 702k$

$\phi R_n = 0.75 R_n = 0.75(702k) = 526.5k$.

5. Shear in bolts: $\phi R_n = \phi F_n A_b = 0.75 F_n A_b$

   From Table 24.1 For A490-N bolt, $f_{nv} = 60ksi$

   A_b = total area of bolts = (TOTAL # bolts)(π)(bolt diameter)^2/4 = 6(3.14159)(1)^2/4 = 4.712in^2

   $\phi R_n = \phi F_n A_b = 0.75(60)(4.712) = 212.06k$

The design tension = smallest of four cases = 212.06k
The connection still fails by shear in the bolts, but the increase in size by 14% increased the design strength of the connection by 30.6%. The next smallest value for design strength is in tensile rupture where the design strength was 255.94k using $\frac{7}{8}$″ bolts. The number of rows of bolts could be changed to increase the design strength. To find how many rows of bolts are needed to make connection fail by tensile rupture, there is no need to reconsider bearing in bolts as it will only increase with addition of bolts. Therefore, only the shear in bolts needs to be considered.

5. Shear in bolts: $\phi R_n = \phi F_n A_b = 0.75 F_n A_b$

   From Table 24.1 for A490-N bolt, $f_{nv} = 60ksi$

   A_b = total area of ONE bolt = (1)(π)(bolt diameter)^2/4 = 1(3.14159)(0.875)^2/4 = 0.601in^2

   $\phi R_n = \phi F_n A_b = 0.75(60)(0.601) = 27.06k$

bolts required = 255.94k/27.06k/bolt = 9.46 bolts … use 10 bolts.

24.1.4 Slip-critical Connections

ϕ = 1.0 for connections preventing slip at serviceability limit state

ϕ = 0.85 for connections preventing slip at the required strength level

Design of slip-critical connection:

1. P_u = factored loads
2. Nominal Strength of one bolt:

   $R_n = \mu D_u h_{sc} T_b N_s$

   #bolts required = $P_u/\phi R_n$

Table 24.2: Minimum bolt pretension, T_b(k) from Table J3.1 of the AISC Steel Construction Manual, reprinted with permission

Bolt Diameter (in)	A-325 Bolts	A-490 Bolts
0.5"	12	15
0.625"	19	24
0.75"	28	35
0.875"	39	49
1"	51	64
1.125"	56	80
1.25"	71	102
1.375"	85	121
1.5"	103	148

3. Bearing in bolts. NOTE that the constants from bearing connections equations have changed from 1.2 and 2.4 to 1.5 and 3.0, respectively.

   $R_n = 1.5 L_c t F_u$ (#bolts)

   or

   $R_n = 3dtF_u$ (# bolts)

   Check that $\phi R_n = 0.75 R_n > P_u$

4. Shear in bolts

   $R_n = F_n A_b$ (#bolts)

   Check that $\phi R_n = 0.75 R_n > P_u$

Example 24-4: Determine the number of 1″ A325 slip-critical bolts in standard size holes needed for the serviceability limit; state whether the faying surface is Class A.

The edge distance is 1.75″ and the c.c. spacing of bolts is 3(in). F_y = 50ksi, F_u = 65ksi, P_L = 30k, P_D = 50k.

1. $P_u = 1.2(50) + 1.6(30) = 108k$
2. Nominal strength for slip-critical design for serviceability state:

   $R_n = \mu D_u h_{sc} T_b N_s$

   μ = 0.35 for Class A faying surfaces (unpainted, mill scale or with Class A coatings)

   D_u = 1.13

   h_{sc} = a hole factor = 1.00 for standard size holes

   T_b = minimum fastener tension from Table 24.2 = 51 kips

N_s = number of slip planes = 1

$R_n = 0.35(1.13)(1.0)(51)(1) = 20.17$ k/bolt

$\phi R_n = 1.0(20.17) = 20.17$k

#bolts required = 108/20.17 = 5.35

USE: 6 bolts.

3. Check bearing strength in all bolts:

   $L_c = 3 - (1 + 1/8) = 1.875''$

   Or

   $L_c = 1.75 - (1 + 1/8)/2 = 1.187''$

   $R_n = 1.5\ L_c\ t\ F_u$ (#bolts) $= 1.5(1.187)(5/8)(65)(6)$
   $= 433.98$

   Or

   $R_n = 3dt\ F_u$ (# bolts) $= 3(1)(5/8)(65)(6) = 731.25$

   Check that $\phi R_n = 0.75R_n = 0.75(433.98) = 325.485 > P_u$
   $= 108$ … okay

4. Check shear strength in all bolts:

   $R_n = F_nA_b$(#bolts) $= 60[(3.14159)(1)^2/4](6) = 282.743$k

   Check that $\phi R_n = 0.75R_n = .75(282.743) = 212.057 > P_u$
   $= 108$k

Example 24-5: Find number of bolts needed for slip-critical connection for the following conditions for the connections in Figure 24.2.

A325-N, $\frac{3}{4}$ inch bolts, Class A surfaces, $P_D = 60$k, $P_L = 90$k, standard hole sizes, $L_c = 3$, design state, A36 steel

1. $P_u = 1.2(60) + 1.6(90) = 216$k
2. Nominal strength of one bolt:

   $R_n = \mu D_u h_{sc} T_b N_s$

   $\mu = 0.35$, $D_u = 1.13$, $h_{sc} = 1.0$, $T_b = 28$, $N_s = 2$, $\phi = 0.85$

   $R_n = 0.35(1.13)(1.0)(28)(2) = 22.148$k/bolt

   #bolts required $= P_u/\phi R_n = 216\text{k}/(0.85(22.148\text{k/bolt}))$
   $= 11.47$

   USE: 12 bolts minimum (4 rows of 3)

3. Bearing in bolts

   $R_n = 1.5L_ctF_u$ (#bolts) $= 1.5(3.0)(.75)(58)(12) = 2349$k

   or

   $= 3dtF_u$(#bolts) $= 3(3/4)(.75)(58)(12) = 1174.5$k

   Check that $\phi R_n = 0.75R_n = 0.75(1174.5) = 880.88 > P_u$
   $= 216$ … okay

4. Shear in bolts

   $R_n = F_nA_b$ (#bolts) $= 60[(3.14159)(3/4)^2/4][12] = 318.09$k

   Check that $\phi R_n = 0.75R_n = 0.75(318.09\text{k}) = 238.56 > P_u$
   $= 216$k … okay

   USE: 12 bolts: 4 rows of 3.

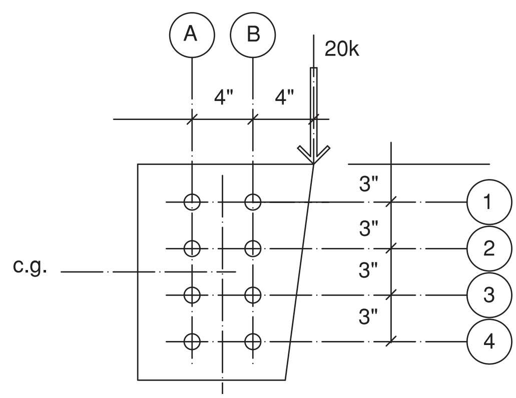
24.2 Eccentric Bolted Connections

This text uses the Elastic Method for analyzing eccentric bolted connections.

$$M = Pe$$

Where P is the load applied at an offset (eccentricity) of e from the center of gravity of the bolt group. The horizontal and vertical components of shear forces on each bolt are calculated by summing the moments about the center of gravity of the bolt grouping.

Example 24-6: For the $\frac{1}{2}''$ plate shown in Figure 25.3, check the adequacy of using 1″, A325-X bolts.



24.3

Eccentric bolt group

1. Determine e: because the center of gravity is half the distance between the two lines of bolts, e = 4″ + 4″/2 = 6″
2. Determine the horizontal (h) and vertical (v) distances from each bolt to the center of gravity:

BOLT	*h*	*h2*	*v*	*v2*
A1	2	4	4.5	20.25
A2	2	4	1.5	2.25
A3	2	4	1.5	2.25
A4	2	4	4.5	20.25
B1	2	4	4.5	20.25
B2	2	4	1.5	2.25
B3	2	4	1.5	2.25
B4	2	4	4.5	20.25
TOTAL	–	32.0	–	90.0

$\Sigma d^2 = \Sigma h^2 + \Sigma v^2 = 32 + 90 = 122$

3. Determine the resultant force on each bolt:

   H = horizontal force due to moment on each bolt

   $= Mv/\Sigma d^2$

   $M = 20k(6'') = 120k\text{-in}$

   For bolts in rows 1 and 4: $H_1 = 120(4.5)/122 = 4.427k$

   For bolts in rows 2 and 3: $H_2 = 120(1.5)/122 = 1.475k$

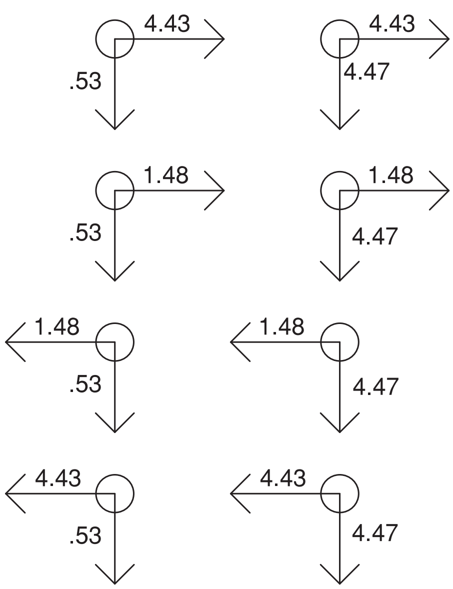
   $V = Mh/\Sigma d^2 = 120(2)/122 = 1.967k$ for all bolts

   And vertical force due to load

   $= P/\#bolts = 20/8 = 2.5k \downarrow$

   R = resultant force on bolt $= \sqrt{[H^2 + (V + P/\#bolts)^2]}$

BOLT	*H*	*V*	*P/#bolts*	*V + P/#bolts*	*R*
A1	4.43	1.97	−2.5	−0.53	4.46
A2	1.48	1.97	−2.5	−0.53	1.57
A3	−1.48	1.97	−2.5	−0.53	1.57
A4	−4.48	1.97	−2.5	−0.53	4.46
B1	4.43	−1.97	−2.5	−4.47	6.29
B2	1.48	−1.97	−2.5	−4.47	4.70
B3	−1.48	−1.97	−2.5	−4.47	4.70
B4	−4.43	−1.97	−2.5	−4.47	6.29



24.4

Forces on eccentric bolt group

The upper right hand (B1) and lower right hand (B4) bolts have the greatest force. $R_{max} = 6.289k$

4. Check that R_{max} < the bearing for one bolt: Determine bearing strength in one bolt:

   L_c = minimum clear distance in direction of force = smaller of

   $3'' - \frac{1}{2}'' = 2.5''$

   or

   $3'' - \frac{1}{2}(2) = 2'' \ldots L_c = 2''$

   $t = \frac{1}{2}''$ = thickness of plate, $d = 1''$ = diameter of bolt

   $R_n = 1.2L_c(t)(F_u)(1 \text{ bolt}) = 69.6$

   OR

   $R_n = 2.4(d)(t)(F_u)(1 \text{ bolt}) = 69.6$

   $\phi R_n = 0.75 R_n = 0.75(69.6) = 52.2k$ = design load per bolt

   Highest load on a bolt = 6.29k < 52.2k … okay for bearing

5. Check that R_{max} < shear strength in one bolt: Determine shear strength in one bolt:

   From Table 24.1, for A325-X: $f_{nv} = 60$

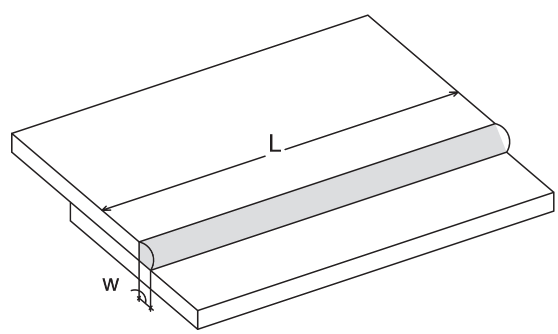
   A_b = total area of 1 bolt $= (\pi)(1)^2/4 = 0.785$

   $\phi R_n = \phi F_n A_b = 0.75(60)(0.785) = 35.343k$

   Highest load on a bolt = 6.29k < 35.343k … okay

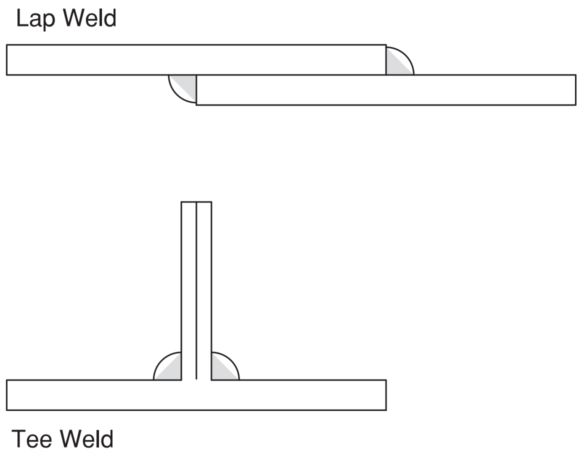
24.3 Welded Connections

While plug and slot welds can be used to transmit shear in overlapping components, they are not generally used in structural steel design. This text will outline the process for designing fillet welds as they are the most commonly used. Fillet welds can be used to join components that overlap or meet at an angle.



24.5

Fillet weld



24.6

Weld locations

AISC LRFD Specifications are as follows:

1. Length of weld (L) must be greater than 4 times nominal leg size (w) of weld.
2. Max. size of fillet weld = material thickness for material $\leq \frac{1}{4}''$ thick
   = material thickness $- \frac{1}{16}''$ for material $> \frac{1}{4}''$ thick
3. Min. size of fillet weld: Material thickness of thicker part joined:

$\frac{1}{8}''$	$\frac{1}{4}''$
$\frac{3}{16}''$	$\frac{1}{4}''$ to $\frac{1}{2}''$
$\frac{1}{4}''$	$\frac{1}{2}''$ to $\frac{3}{4}''$
$\frac{5}{16}''$	over $\frac{3}{4}''$

4. For longitudinal fillet welds connecting plates or bars, length may not be less than the perpendicular distance between them.
5. For lap joints, the minimum overlap permitted = 5 times thickness of thinner part joined and not less than 1″.
6. If length L of an end loaded fillet weld is greater than 100 times its leg size (w), effective length of weld = $\beta L = L[1.2 - 0.002(L/w) \leq L$. If the length L is greater than 300w, then $\beta L = 0.6L$.

Design of Longitudinal Fillet Welds: (welds parallel to direction of force)

$$\text{Design strength} = \phi R_n \beta L = \phi F_w A_w \beta L$$

Example 24-7: Determine design strength of a 20″ long $\frac{1}{4}''$ fillet weld using E70 electrodes with a minimum tensile strength 70ksi. Load is applied parallel to weld length.

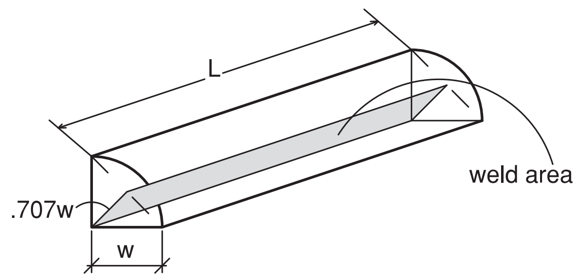
$L = 20''$. $b = \frac{1}{4}''$

$L/b = 20/.25 = 80 < 100$; therefore $\beta = 1$

Nominal strength of weld $= F_w = 0.6(70\text{ksi}) = 42\text{ksi}$

Area of weld $= A_w = L[\,0.707b] = 20\text{in}\,[0.707][0.25'']$

$= 3.535$



24.7

Area of weld

$$\phi R_n \beta L = 0.75[42\text{ksi}][1][3.535] = 111.35$$

Design of transverse fillet welds (welds at an angle θ to direction of force)

For a transverse weld, $F_w = (0.6\,F_n)(1.0 + 0.5\sin^{1.5}\theta)$

When the weld is perpendicular to direction of force this equation becomes

$$F_w = (0.6\,F_n)(1.5)$$

but if the transverse weld is at a 45 degree angle from the direction of force, then the F_w would equal

$$F_w = (0.6\,F_n)(1 + 0.5(0.707^{1.5})) = (0.6F_n)(1.2973)$$

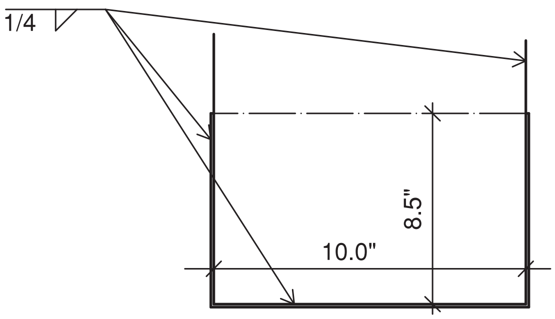
When combining longitudinal and transverse welds use the LARGER of the two equations below:

$$R_n = R_{WL} + R_{WT}$$

or

$$R_n = 0.85R_{WL} + 1.5R_{WT}$$

Example 24-8: Find LRFD design strength for the $\frac{1}{4}''$ E70 weld shown.



24.8

Weld design example

1. $F_w = 0.6(70) = 42\text{ksi}$
2. $A_w = 0.707(0.25)L$
3. Longitudinal welds: L = 2 welds @ 8.5″ each = 17″
   $A_w = 0.707(0.25)(17) = 3.005\text{in}^2$
   $R_{WL} = 42\text{ksi}(3.005\text{in}^2) = 126.20\text{k}$
4. Transverse weld: L = 10″
   $A_w = 0.707(0.25)(10) = 1.768\text{in}^2$
   $R_{WT} = 42\text{ksi}(1.768\text{in}^2) = 74.26\text{k}$
5. USE: LARGER OF
   $R_n = R_{WL} + R_{WT} = 126.20 + 74.26 = 200.46\text{k}$

   or

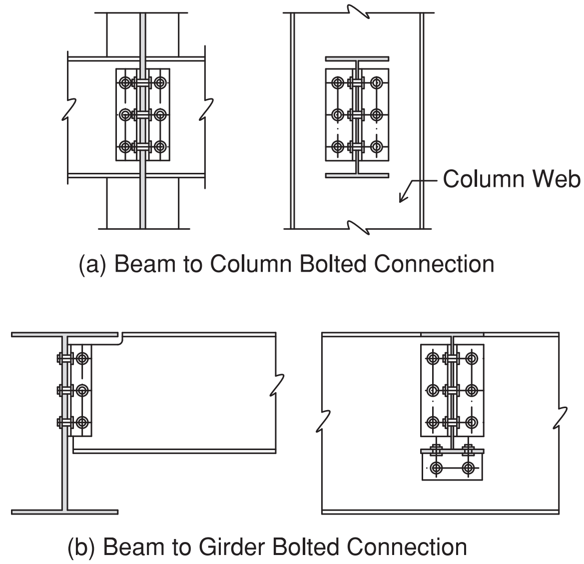
   $R_n = 0.85\, R_{WL} + 1.5R_{WT} = 0.85(126.2) + (111.384)$
   $= 218.65\text{k}$
6. $\phi R_n = 0.75(218.65\text{k}) = 163.99\text{k}$ = LRFD design strength

24.4 Standard Bolted Connections

Simple shear connections can be designed using the tables in Part 10 of the AISC Steel Manual.

Example 24-9: Select the proper double angle for the connection in Figure 24.9 (a) if $P_D = 50\text{k}$ and $P_L = 70\text{k}$, $F_y = 36$ and $F_u = 58$ for angles and $F_y = 50$ and $F_u = 65\text{ksi}$ for the beam and column.

Use $\frac{3}{4}$″ A325-N bolts.



24.9

Simple bolted connection

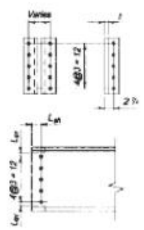
1. $R_u = 1.2(50) + 1.6(70) = 172\text{k}$
2. Look through Table 10-1 of the AISC Steel Manual to find the least number of rows allowed for a W30 beam. Five rows is the least number of rows. The list of W sizes available is found in the fourth box from the top on the left side, just under the number of rows. Note: this table is for $\frac{3}{4}$″ bolts.
3. Look at A325-N bolts. Maximum load carried is 159k < 172k needed, therefore go to the chart for six rows of bolts.
4. A 5/16″ angle will work in shear because the LRFD value listed is 187k which is greater than the 172k needed.
5. Calculate length of the angle for holes spaced at 3″o.c. and 1.25: from each end as shown in Figure 24.10:
   $L = 5(3) + 2(1.25) = 17.5''$
6. Size the angle legs:
   on beam web: 2.5″ gage + 1″ minimum distance = 3.5″
   On col. flange: .3125 + 1.375 + 1.25 + 1 = 3.9375
   Use 4″

Use: LL4 × 3.5 × 5/16 × 17.5″ long with six rows of bolts

Beam $F_y = 50$ ksi, $F_u = 65$ ksi
Angle $F_y = 36$ ksi, $F_u = 58$ ksi

Table 10–1 (continued)
All-Bolted Double-Angle Connections
3/4-in. Bolts

5 Rows — W30, 27, 24, 21, 18



Bolt and Angle Available Strength, kips

ASTM Desig.	Thread Cond.	Hole Type	1/4 ASD	1/4 LRFD	5/16 ASD	5/16 LRFD	3/8 ASD	3/8 LRFD	1/2 ASD	1/2 LRFD
A325/F1852	N	—	83.3	125	104	156	106	159	106	159
	X	—	83.3	125	104	156	125	187	133	199
	SC Class A	STD	73.8	111	73.8	111	73.8	111	73.8	111
		OVS	53.3	80.0	53.3	80.0	53.3	80.0	53.3	80.0
		SSLT	62.8	94.1	62.8	94.1	62.8	94.1	62.8	94.1
	SC Class B	STD	83.3	125	104	156	105	158	105	158
		OVS	76.2	114	76.2	114	76.2	114	76.2	114
		SSLT	82.0	123	89.6	134	89.6	134	89.6	134
A490	N	—	83.3	125	104	156	125	187	133	199
	X	—	83.3	125	104	156	125	187	166	249
	SC Class A	STD	83.3	125	92.3	138	92.3	138	92.3	138
		OVS	66.7	100	66.7	100	66.7	100	66.7	100
		SSLT	78.4	118	78.4	118	78.4	118	78.4	118
	SC Class B	STD	83.3	125	104	156	125	187	132	198
		OVS	82.4	124	95.2	143	95.2	143	95.2	143
		SSLT	82.0	123	102	154	112	168	112	168

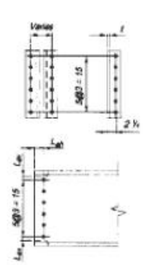
Beam Web Available Strength per Inch Thickness, kips/in.

24.10
Simple bolted connection with five rows of bolts from AISC Steel Construction Manual, Table 10-1, reprinted with permission

Beam $F_y = 50$ ksi, $F_u = 65$ ksi
Angle $F_y = 36$ ksi, $F_u = 58$ ksi

Table 10–1 (continued)
All-Bolted Double-Angle Connections
3/4-in. Bolts

6 Rows — W40, 36, 33, 30, 27, 24, 21



Bolt and Angle Available Strength, kips

ASTM Desig.	Thread Cond.	Hole Type	1/4 ASD	1/4 LRFD	5/16 ASD	5/16 LRFD	3/8 ASD	3/8 LRFD	1/2 ASD	1/2 LRFD
A325/F1852	N	—	99.5	149	124	187	127	191	127	191
	X	—	99.5	149	124	187	149	224	159	239
	SC Class A	STD	88.6	133	88.6	133	88.6	133	88.6	133
		OVS	64	96	64	96	64	96	64	96
		SSLT	75.3	113	75.3	113	75.3	113	75.3	113
	SC Class B	STD	99.5	149	124	187	127	190	127	190
		OVS	91.4	137	91.4	137	91.4	137	91.4	137
		SSLT	98.2	147	108	161	108	161	108	161
A490	N	—	99.5	149	124	187	149	224	159	239
	X	—	99.5	149	124	187	149	224	199	298
	SC Class A	STD	99.5	149	111	166	111	166	111	166
		OVS	80	120	80	120	80	120	80	120
		SSLT	94.1	141	94.1	141	94.1	141	94.1	141
	SC Class B	STD	99.5	149	124	187	149	224	158	237
		OVS	98.6	148	114	171	114	171	114	171
		SSLT	98.2	147	123	184	134	202	134	202

Beam Web Available Strength per Inch Thickness, kips/in.

24.11
Simple bolted connection with six rows of bolts from the AISC Steel Construction Manual, Table 10-1, reprinted with permission.

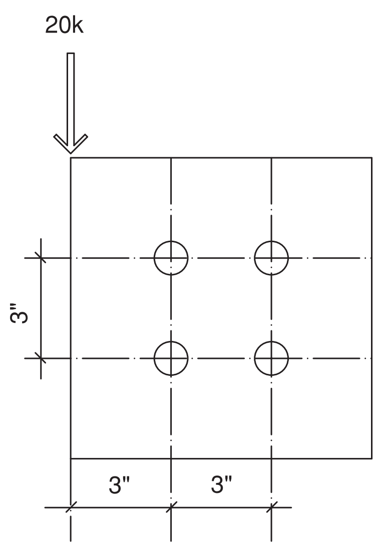
Practice Exercises:

24-1: Find the number of A325 bolts required for a bearing connection with a load P_u = 300k connecting 2 – A36(F_u = 58ksi) plates, each $\frac{3}{4}$″ thick with $\frac{7}{8}$″ bolts spaced at 3″ on center and 3″ from each edge. The plates are 9″ wide and there are two bolts per row. Bolt threads are excluded from shear plane.

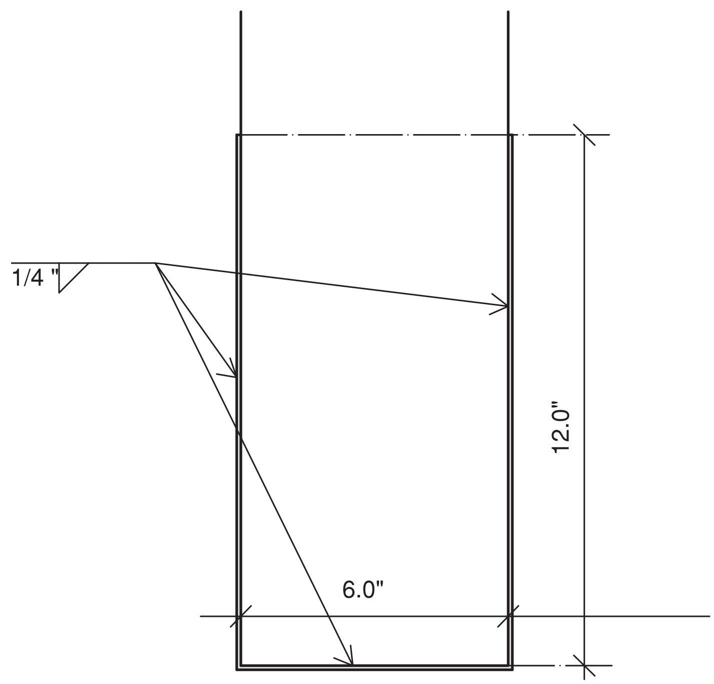
24-2: Repeat exercise 24-1 for a slip-critical connection, assuming standard bolt hole size and class A coatings.

24-3: Find design load for eccentric connection shown, using $\frac{3}{4}$″ bolts, $\frac{3}{4}$″ thick, A36 plate.

24-4: Find design strength for E70xx weld shown.



24-3



24-4

24.12
Chapter 24 Practice exercises

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Part V

Concrete Design

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Concrete Beam Design

Basic Concrete Information:

Advantages of concrete as a structural material include fire resistance, vibration resistance, flexibility of shape, ease of maintenance, and availability of the mixture components. Concrete is a suitable material for almost every type of structural component including slabs, beams, columns, bearing or shear walls and foundations. The disadvantages of concrete as a structural material include the need for formwork, the need for time to allow the concrete to cure before subjection to load and the fact that the strength depends on the mix of ingredients.

Concrete density may vary depending on the weight of the aggregate. Concrete densities are categorized as:

normal weight concrete: 140 – 150pcf

lightweight concrete: 90 – 112pcf

heavyweight concrete: > 200pcf.

The design strength of concrete is specified in terms of its compressive strength at 28 days after placement and designated as f'_c.

low strength: $f'_c = 3000$psi

moderate strength: $f'_c = 3000 – 6000$psi

high strength: $f'_c > 6000$psi

Concrete gains strength over time:

7 days – 70% f'_c

14 days – 85–90% f'_c

28 days – 100% f'_c

5 years – approximately 150% f'_c, but the values are dependent on curing conditions of temperature and humidity, wet or dry surface, and additives to the concrete.

Modulus of Elasticity:

$E_c = w_c^{1.5} 33\sqrt{f'_c}$ where:

w_c = the density of the concrete in pcf

f'_c = 28 day compressive strength in psi

NOTE: for normal weight concrete, $E = 57000\ f'_c$.

25.1 The Internal Couple

25.1.1 Modulus of Rupture and Cracking Moment

When a beam is stressed in bending due to a downward load, the portion of the cross-section above the neutral axis is in compression and the area of the cross-section below the neutral axis is in tension. Concrete handles compression well, but its tensile strength is only 10–15% of the compressive strength. Once concrete is stressed in tension beyond the modulus of rupture, it will crack.

modulus of rupture = $f_r = 7.5\lambda\sqrt{f'_c}$ where:

λ is a factor based on concrete type.

λ = 1.0 for normal weight concrete

λ = 0.85 for sand-lightweight concrete

λ = 0.75 for all-lightweight concrete.

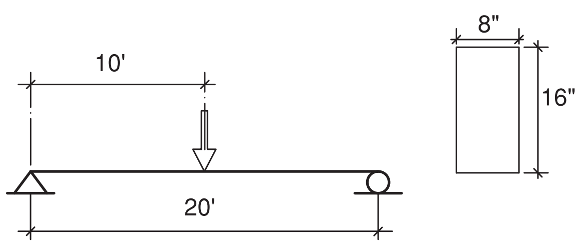
Given that bending stress f_b = MI/c, setting the bending stress equal to the modulus of rupture allows the moment at the point of rupture to be found.

$f_r = f_b = M_{cr}(I)/c$

Cracking moment = $M_{cr} = f_r (I)/c = 7.5\lambda(\sqrt{f'_c})(I)/c$

Once the moment on a beam reaches the cracking moment (M_{cr}), it will require tension reinforcement.

Example 25-1: At what factored load, P, applied to the beam and cross-section in Figure 25.1 will require tension reinforcement if the density of the concrete is 150pcf and f'_c = 4,000psi?



25.1

Example 25-1

W_u = 150pcf(8″)(16″)/(144in²/f²) = 133.33#/f

$M_u = PL/4 + wL^2/8 = P(20')/4 + 133.33^{\#/f}(20')^2/8$
= 5P + 6666.5#-f = 60P + 79998 #-in

$I/c = (bh^3/12)/(h/2) = bh^2/6 = 8(16)^2/6 = 341.33in^3$

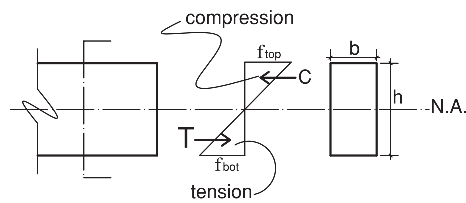
$f_r = 7.5\lambda\sqrt{f'_c}(I)/c = 7.5(1)\sqrt{4,000}$ = 474.34psi

$M_{cr} = f_r(I)/c = 474.34psi(341.33in^3)$ = 161907.04#-in

M_{cr} = 161907.04 = 60P + 79998 = M_u … P = 1365.15#

25.1.2 The Internal Couple

Concrete does not behave elastically over its entire cross-section when stressed beyond the modulus of rupture (f_r) in tension. As a result, the external method of measuring bending stress, f_b = MI/c becomes invalid and an internal look at the stress must be used. A look at the rectangular cross-section in Figure 25.2 with width b and depth h results in a neutral axis located at h/2.



25.2

The internal couple

The stress in the cross-section varies linearly from the f_{top} to f_{bottom}. The area above the neutral axis is in compression and the area below the neutral axis is in tension. The applied moment on the beam (M_u) equals the internal couple. Remember a couple is a moment caused by two equal and opposite forces acting at a distance Z apart. In this case, the equal and opposite forces are compression, C, and tension, T.

M_u = CZ = TZ = internal couple

For a rectangular cross-section, the center of gravity for the compression triangle is $\frac{1}{3}$(h/2) = h/6 from the top and the center of gravity for the tension triangle is h/6 from the bottom. Therefore, the distance between the two forces,

Z = h − 2(h/6) = 2h/3

$M_u = M_{cr}$ = TZ = CZ

$T = C = M_u/Z = M_u/(2h/3) = 3\,M_u/2h$

The stress at the top equals the stress at the bottom = f_{top} = f_{bot} and

the average stress = f = $f_{top}/2 = f_{bot}/2$ = C/A = T/A

where A is the area upon which the force is distributed. For a rectangular section, A = bh/2.

$f_{bot} = 2T/A = 4T/bh = 4[3\,M_u/2h]/bh = 6\,M_u/bh^2$
$= M_u/[bh^2/6]$

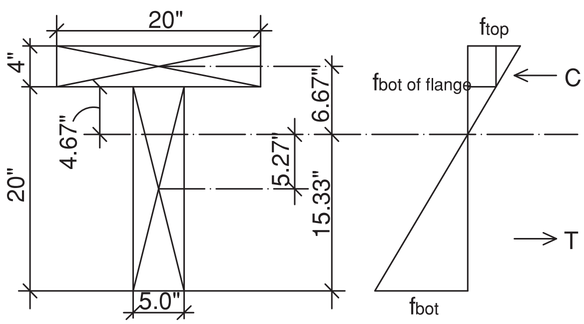
This is the same as the answer found by the external method $f_b = M_u/S$ where $S = bh^2/6$ for a rectangular section.

Not all cross-sections are rectangular. The generic equation for the cracking moment is:

$$M_{cr} = ZAf_r/2$$

Example 25-2: Find the cracking moment for the T-shaped cross-section shown in Figure 25.3.

Because the area above the neutral axis, the compression area, is not uniform in width, the force and moment arm for the web and flange will need to be calculated separately.



25.3

Cracking Moment in T-shape

Find the neutral axis:

Component	*A(in2)*	*Y(in)*	*Ay(in3)*
Flange	4(20) = 80	20 + 4/3 = 22	1760
Web	5(20) = 100	20/2 = 10	1000
	ΣA = 180		ΣAy = 2760

N.A. at Y = 2760/180 = 15.33in

f_{bot} is in tension – note f_{bot} is greater than f_{top}

$f_{bot} = f_r = 7.5(1)\sqrt{4000} = 474.34$psi

$f_{top} = 474.34$psi$(8.67/15.33) = 268.1$psi

Stress at bottom of flange: $f_{bot\ of\ flange}$ = 474.34(4.67/15.33) = 144.36psi

1. T = average tensile stress × area = $(f_{bot}/2)(5(15.33))$ = (474.34/2)(5(15.33))psi

   T = 18183.1# acting at 10.22″ below the neutral axis.

2. C_1 = (144.36/2)(5)(4.67) = 1685.4 acting at (2/3)(4.67) = 3.11″ above the neutral axis.
3. C_2 = (144.36)(20)(4) = 11548.8 acting at 4.67 + 4/2 = 6.67″ above the neutral axis.
4. C_3 = ((268.1 – 144.36)/2)(20)(4) = 4949.6 acting at 4.67 + (2/3)(4) = 7.33″ above the neutral axis.

Z_1 = 10.22 + 3.11 = 13.33″

Z_2 = 10.22 + 6.67 = 16.89″

Z_3 = 10.22 + 7.33 = 17.55″

$M_{cr} = Z_1C_1 + Z_2C_2 + Z_3C_3$

= 13.33(1685.4) + 16.89(11548.8) + 17.77(4949.6)
= 305,480#-in

= 25,456.67#-ft.

25.2 Reinforced Concrete Beams

Concrete beams are reinforced with steel rebar to carry the tension load. This allows the concrete in the beam to carry a load in compression equal to the load that the steel carries in tension. Rebar is designated by bar number. In bar numbers 3 through 8, the number corresponds to the diameter of the bar (d_b) in eighths of an inch. For bar diameters of bar numbers 9 through 16, refer to Table A4.1. This table lists bar diameters and bar areas for bar sizes 3 through 11.

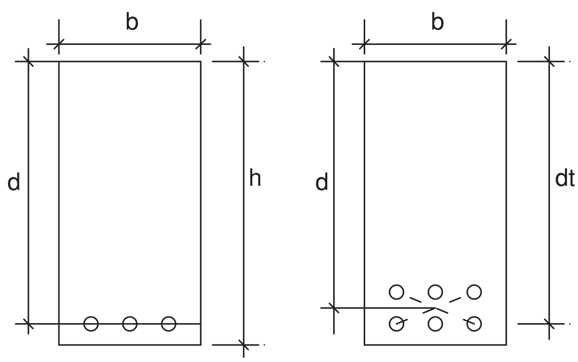
25.2.1 Assumptions for Reinforced Concrete Design:

1. The strain remains linear throughout the cross-section of the beam.
2. If $f_c \le f'_c/2$, stress and strain are proportional … $\varepsilon_c/\varepsilon_s = f_c/f_s$.
3. Ignore concrete in tension; assume the reinforcing steel handles all tension in the beam.
4. If $f_s < f_y$, $f_s = E_s\varepsilon_s$.
5. $\varepsilon_c \le 0.003$in/in.
6. There is no slip between the concrete and the steel.

Nomenclature for concrete beam cross-sections:

d = effective depth = the distance from the top of the beam to the center of gravity of the reinforcing steel.

d_t = the depth from the top of the beam to the center line of the bottom-most row of steel.



25.4

Typical concrete beam cross-section

Notice that $d = d_t$ when there is only one row of steel.

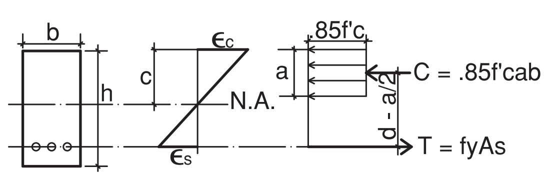
The standard minimum cover for beams, meaning the clear distance from the outermost steel to the edge of the beam, is 1.5″. See American Concrete Institute (ACI) standards for beams on grade or beams exposed to exterior or adverse conditions. Most beams have a stirrup that reinforces against shear. This can be assumed to be a #3 rebar with a diameter of $\frac{3}{8}$″.

d = h – cover – stirrup diameter – half the diameter of rebar ($d_b/2$).

In most cases,

$d = h - 1.5'' - 0.375'' - d_b/2$

25.2.2 The Equivalent Stress Block



25.5

Equivalent stress block

When stresses are low in a reinforced concrete beam, the line of stress for compression remains linear, ranging from 0 at the neutral axis to f_c at the top. But the addition of reinforcement allows the beam to carry higher stresses. As a result, the area in compression, the stress block, takes on a curved shape. Because the area and center of gravity of the stress block is difficult to calculate when curved, an equivalent stress block is used with a unit stress of $0.85f'_c$ and a depth,

$a = \beta_1 c$ where:

$\beta_1 = 0.85$ for 2500psi $\leq f'_c \leq$ 4000psi

$\beta_1 = 0.85 - 0.05(f'_c - 4000)/1000 \geq 0.65$ for $f'_c >$ 4000psi

c = distance from the top of the beam to the Neutral Axis.

Since stress = force/area, force = stress(area). If b = width of cross-section:

$C = 0.85f'_c ab$ acting at a distance a/2 from the top of the beam

$T = f_y A_s$ acting at a distance d from the top of the beam, where

f_y = yield stress of reinforcement

A_s = cross-sectional area of reinforcement.

The internal couple or internal moment

$M_n = TZ = f_y A_s(d - a/2)$

Summing forces horizontally yields C = T or

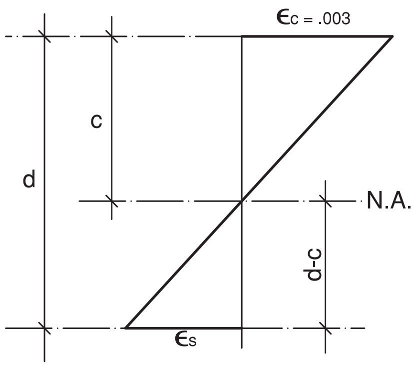
$(0.85f'_c)ab = f_y A_s$ … $a = f_y A_s/(0.85f'_c b)$

M_n = practical nominal moment = $f_y A_s(d - (.5f_y A_s/(0.85f'_c b))$

Strain in concrete and steel:

When the maximum steel strain is less than 0.002 (ε_y = .002 for grade 60 steel), the beam will fail in compression. Failure due to compression of concrete is sudden and without warning. Therefore, we want $\varepsilon_t \geq 0.002$ when concrete reaches ε_c = .003.

When the maximum steel strain is greater than 0.004 ($\varepsilon_y < \varepsilon_t$ for all grades of steel rebar), the beam will fail ductility. Failure due to tension in steel is gradual and with warning. This is the desired failure mode. Therefore, ε_t must be greater than or equal to 0.004. Because the strain is linear, and because the maximum allowable strain in concrete is 0.003, using equivalent triangles gives:



$.003/c = \epsilon_s/(d-c)$

$\epsilon_s = .003(d-c)/c$

25.6

Strain in a concrete beam

$$\varepsilon_t/(d - c) = .003/c \ldots \varepsilon_t = .003(d - c)/c$$

M_u = design moment = ϕM_n where ϕ is the LRFD Resistance Factor per ACI Code, section 9.3. ϕ is a strength-reduction factor that takes into account workmanship, dimensional variations on site and material variations.

$\phi = 0.90$ for tension-controlled sections

$\phi = 0.90$ in beams, if the steel strain $\varepsilon_t \geq 0.005$

$\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$ if $.004 \leq \varepsilon_t < 0.005$

$\phi = 0.75$ in compression-controlled sections that are spirally reinforced

$\phi = 0.65$ for other reinforcement in compression-controlled sections

$\phi = 0.75$ for shear and torsion

$\phi = 0.65$ for bearing on concrete

Minimum reinforcement steel:

$$A_{s\ min} = bd(3\sqrt{f'_c})/ f_y \geq 200bd/f_y$$

To check the adequacy of a beam:

1. Determine b, d, A_s, f'_c, f_y, and M_u
2. $a = f_yA_s/(0.85f'_cb)$
3. $M_n = f_yA_s(d - a/2)$
4. Check $A_s \geq A_{s\ min} = bd(3\sqrt{f'_c})/f_y \geq 200bd/f_y$
5. Find $\varepsilon_t = .003(d - c)/c$ using $c = a/\beta_1$
6. Check $\varepsilon_t \geq 0.004$
7. Determine ϕ where $\phi = 0.90$ if $\varepsilon_t \geq 0.005$ and $\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$ if $0.004 \leq \varepsilon_t < 0.005$.
8. If $M_u \leq \phi M_n$, beam is adequate.

Example 25-3: Determine the adequacy of a 24′ beam with a cross-section 16″ wide by 24″ deep, reinforcement of four #8 rebars, an effective depth, d = 21″, carrying a 1k/f uniform live load and a .5k/f uniform dead load exclusive of beam weight. f'_c = 4ksi, f_y = 60ksi.

1. b = 16″, d = 21″, A_s = 3.142, f'_c = 4,000psi, f_y = 60,000psi
   $W_u = 1.2(500^{\#/f} + 150pcf(16/12)(24/12)) + 1.6(1{,}000^{\#/f})$
   $= 2680^{\#/f}$

   $M_u = wL^2/8 = 2680^{\#/f}(24')^2/8 = 192{,}960\#\text{-f}$
   $= 2{,}315{,}520\#\text{-in}$
2. $a = f_yA_s/(0.85f'_cb) = 60{,}000psi(3.142in^2)/[.85(4{,}000psi)(16'')] = 3.465''$
3. $M_n = f_yA_s(d - a/2) = 60{,}000(3.142)(21 - 3.465/2)$
   $= 3{,}632{,}309.1\#\text{-in}$
4. Check $A_s \geq A_{s\ min} = bd(3\sqrt{f'_c})/f_y \geq 200bd/f_y$.

   $A_{s\ min} = 16(21)(3)(\sqrt{4{,}000})/60{,}000$
   $= 1.063 \geq 200(16)(21)/60{,}000 = 1.12$

   $A_{s\ min} = 1.12 < A_s = 3.142in^2 \ldots$ okay
5. $c = a/\beta_1 = 3.465/.85 = 4.076''$, $\varepsilon_t = .003(d - c)/c$
   $= .003(21 - 4.076)/4.076 = 0.012$
6. $\varepsilon_t = 0.012 \geq 0.004 \ldots$ okay
7. $\varepsilon_t = 0012 > 0.005 \ldots \phi = 0.90$
8. $\phi M_n = 0.9(3{,}632{,}309.1) = 3{,}269{,}078\#\text{-in} > M_u$
   $= 2{,}315{,}529\ \#\text{-in} \ldots$ beam is adequate.

In designing beam reinforcement for a beam of a given size, the goal is to find the required area of steel, A_s. Setting the applied moment equal to ϕM_n:

$$M_u = \phi M_n = \phi[f_yA_s(d - a/2)]$$

where $a = f_yA_s/(.85f'_cb)$

$$M_u = \phi[f_yA_s(d - f_yA_s/(1.7f'_cb))]$$

$$[M_u/\phi f_y][1.7f'_cb/f_y] = A_s[d(1.7f'_cb/f_y) - A_s]$$

$$A_s^2 - A_s[1.7f'_cbd/f_y] + [M_u/\phi f_y][1.7f'_cb/f_y] = 0$$

$$A_s = 1.7f'_cbd/2f_y - (1/2)\sqrt{[1 - 4M_u(1.7f'_cb/\phi f_y^2]}$$

$$A_s = 0.85f'_cbd/f_y [1 - \sqrt{[1 - 2M_u/\phi(.85f'_cbd^2)]}$$

To design reinforcement in a rectangular beam of a given size:

1. Determine f'_c, f_y, b and h.

2. Calculate M_u including beam weight.
3. Estimate d = h – 3
4. Assume ϕ = 0.9
5. $A_s = [.85f'_c bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f'_c bd^2]}$
6. Check that $A_s \geq A_{s\ min} = bd(3\sqrt{f'_c})/f_y \geq 200bd/f_y$. If not, use $A_{s\ min}$.
7. Select bars from A4.2. Note actual A_s. Calculate the actual value of d. Check the required width for the number and size of bars chosen from A4.2.
8. $a = f_y A_s/(.85f'_c b)$ $c = a/\beta_1$
9. Check $\varepsilon_t = .003(d - c)/c > 0.004$.
10. Check ϕ = 0.9 assumption. If $\varepsilon_t < 0.005$, recalculate ϕ and that check $\phi M_n \geq M_u$.

    $\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$ if $0.004 \leq \varepsilon_t < 0.005$.

11. Using actual ϕ, d and A_s, check that $M_u \leq \phi[f_y A_s(d - f_y A_s/(1.7f'_c b))]$, if not, go back to step 5. Using new value for ϕ.

Example 25-4: Design reinforcement for a 12″ by 20″ concrete beam with a simple span of 20′, a dead load of 600#/f and a live load of 1200#/f using f'_c = 3,000psi and f_y = 40,000psi.

1. f'_c = 3,000psi, f_y = 40,000psi, b = 12″, h = 20″
2. W_u = 1.2(600 + 150(12/12)(20/12)) + 1.6(1200#/f) = 2940#/f
   $M_u = 2940(20')^2/8$ = 147,000#-f = 1,764,000#-in
3. Estimate d = h – 3 = 20 – 3 = 17″
4. Assume ϕ = 0.9
5. $R = .85f'_c bd$ = .85(3000)(12)(17) = 520,200
   $A_s = [R/f_y][1 - \sqrt{[1 - 2M_u/\phi Rd]}$ = [520,200/40,000][1 – $\sqrt{[1 - 2(1{,}764{,}000)/(.9(520{,}200)(17)]}$ = 3.30in²
6. $A_{s\ min} = bd(3\sqrt{f'_c})/f_y$ = 12(17)(3)($\sqrt{3000}$)/40,000 = 0.838 ≥ $200bd/f_y$ = 200(12)(17)/40,000 = 1.02

   $A_{s\ min}$ = 1.02in² < 3.30in² = A_s … okay
7. Use three #10: A_s = 3.8 > 3.30in², b_{req} = 9.75″ < 12″ … okay

   d_{actual} = 20 – 1.5 – .375 – 1.27/2 = 17.79″
8. $a = f_y A_s/(.85f'_c b)$ = 40,000(3.8)/[.85(3000)(12)] = 4.97″
   $c = a/\beta_1$ = 4.97″/0.85 = 5.84″
9. Check $\varepsilon_t = .003(d - c)/c$ = .003(17.49 – .84)/5.84 = .006 > 0.004 … okay
10. ε_t = .006 > 0.005 … ϕ = 0.9
11. M_u = 1,764,000# – in ≤ $\phi[f_y A_s(d - a/2))]$ = .9[40,000(3.8)(17.49 – 4.97/2)] = 2,052,684#-in … okay

ANSWER: 12″×20″ beam with three #10

To design a beam with a given width, the goal is to find the most efficient depth, d and the area of reinforcing steel, A_s. Because these two variables are related, an assumption must be made. Assume a = 0.2d. The depth of the equivalent stress block is assumed to be about 20% of the effective depth of the beam. This puts the bottom of the equivalent stress block well above the neutral axis and yields a value for d where the ratio of b/d will most likely fit into the recommend range of 1.5 ≤ b/d ≤ 2.2.

$$A_s f_y = T = C = 0.85f'_c ab \ \ldots \ A_s = 0.85f'_c ab/f_y$$

$M_u = \phi M_n = \phi f_y A_s(d - a/2)$ … $(d - a/2) = M_u/[\phi f_y A_s]$ and $d = M_u/[\phi f_y A_s] + a/2$. Substituting $A_s = 0.85f'_c ab/f_y$ yields:

$$d = M_u/[\phi 0.85f'_c ab] + a/2$$

Inserting the assumption that a = 0.2d yields:

$$d = M_u/[\phi 0.85f'_c(.2d)b] + (.2d)/2$$

$$0.9d = M_u/[\phi 0.85f'_c(.2d)b]$$

$$d = \sqrt{\{M_u/[.153\phi f'_c b]\}}$$

To design a rectangular beam of a given width, but unknown depth:

1. Determine f'_c, f_y, and b.
2. Calculate M_u excluding beam weight.
3. Assume ϕ = 0.9
4. $d = \sqrt{M_u/[.153\phi f'_c b]}$
5. Check proportions of d/b. 1.5 < d/b < 2.2 if not, change b and recalculate d in step 4.
6. Estimate h = d + 2.5 and round up to next whole inch.
7. Determine factored beam weight.
8. Calculate Actual M_u
9. $A_s = [R/f_y][1 - \sqrt{[1 - 2M_u/\phi Rd]}$ where $R = .85f'_c bd$.
10. Check that $A_s \geq A_{s\ min} = bd(3\sqrt{f'_c})/f_y \geq 200bd/f_y$. If not, use $A_{s\ min}$.
11. Select bars from Table A4.2. Note actual A_s. Calculate the actual value of d. Check the required width for the number and size of bars chosen from A4.2.

12. $a = f_yA_s/(.85f'_cb)$ and $c = a/\beta_1$
13. Calculate d_{actual} = h – cover – stirrup diameter – $d_b/2$
12. Check $\varepsilon_t = .003(d - c)/c > 0.004$.
14. Check $\phi = 0.9$ assumption. If $\varepsilon_t < 0.005$, recalculate ϕ and check $\phi M_n \geq M_u$.

$$\phi = 0.65 + (\varepsilon_t - 0.002)(250/3) \text{ if } 0.004 \leq \varepsilon_t < 0.005$$

15. Using actual ϕ, d and A_s, check that $M_u \leq \phi[f_yA_s(d - f_yA_s/(1.7f'_cb))]$, if not, go back to step 3 using new value for ϕ.

Example 25-5: Design a simple beam, 10″ wide, with a span of 3′t to carry a live load of 1k/f. f'_c = 4,000psi and f_y = 60,000psi.

1. f'_c = 4,000psi, f_y = 60,000psi, b = 10″
2. $M_u = 1.6(1000^{\#/f})(30')^2/8$ = 180,000#-f = 2,160,000#-in
3. Assume $\phi = 0.9$
4. $d = \sqrt{M_u/[.153\phi f'_cb]} = \sqrt{[2{,}160{,}000/(.153(0.9)(4000)(10))]}$ = 19.80″
5. d/b = 19.80″/10″ = 1.98″ and 1.5 < 1.98 < 2.2 … b is okay
6. Estimate h = d + 2.5 = 19.8″ + 2.5″ = 22.3″. Round up to 23″.
7. W_{bm} = 150pcf(10″/12)(23″/12) = $239.58^{\#/f}$

   $W_u = 1.2(239.58^{\#/f}) + 1.6(1000^{\#/f}) = 1887.5^{\#/f}$

8. $M_u = wL^2/8 = 1887.5^{\#/f}(30')^2/8$ = 212,343.75#-f = 2,548,125#-in
9. R = .85f'_cbd = .85(4000)(10)(19.8) = 673,200

   $A_s = [R/f_y][1 - \sqrt{[1 - 2M_u/\phi Rd]} = [673{,}200/60{,}000][1 - \sqrt{(2(2{,}548{,}125)/(0.9(673{,}200)(19.8))}] = 2.71in^2$

10. Check that $A_s \geq A_{s\ min} = bd(3\sqrt{f'_c})/f_y$ = 10(19.8)(3√4000)/60,000 = 0.626 ≥ 200bd/f_y = 200(10)(19.8)/60,000 = 0.66

    $A_{s\ min}$ = 0.66. $A_s > A_{s\ min}$ … okay

11. Use three #9: A_s = 2.998, b_{req} = 9.25 < 10 = b.
12. $a = f_yA_s/(.85f'_cb)$ = 60000(2.998)/[.85(4000)(10)] = 5.29″
    $c = a/\beta_1$ = 5.29/0.85 = 6.22″
13. Calculate d_{actual} = h – cover – stirrup diameter – $d_b/2$
    = 23 – 1.5 – 0.375 – 1.128/2 = 20.56″
14. Check $\varepsilon_t = .003(d - c)/c$ = .003(20.56 – 6.22)/6.22
    = .0069 > 0.004 … okay
15. $\phi = 0.9$ assumption is correct because .0069 > 0.005
16. $\phi[f_yA_s(d - a/2]$ = .9[60000(2.998)(20.56′ – 6.22/2)]
    = 2,825,0154#-in allowable moment > 2,548,125#-in actual moment … beam is okay

USE: 10″×23″ beam with three #9.

Practice Exercises:

25-1: Determine whether a concrete 14″ by 30″ beam with a simple span of 20′ will need reinforcing to carry a $500^{\#/f}$ dead load and a $900^{\#/f}$ live load if f'_c = 3,000psi.

25-2: An unreinforced concrete beam has a rectangular cross-section 12″ wide by 20″ deep. If it is made using concrete with f'_c = 4000psi, at what length will it fail under its own weight?

25-3: Design for flexure: a 14″ wide, 24″ deep concrete beam with a simple span of 24′ and a uniform live load of $960^{\#/f}$.

25-4: Design for flexure: a 16″ wide by 30″ deep concrete beam with a simple span of 32′ to carry two point loads, each 3000# dead load, evenly spaced.

25-5: Design for flexure: a 12″ wide beam with a simple span of 26′ to carry P_D = 1000# and P_L = 2000# at the center of the span.

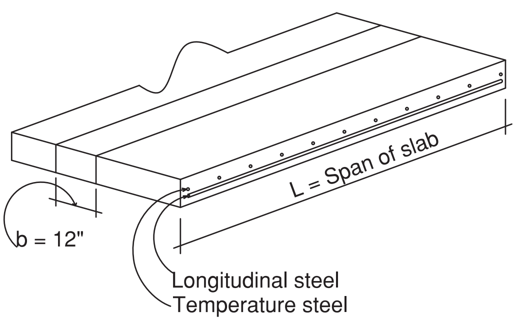
25-6: Design the lightest beam (ignore weight of reinforcement steel) with a maximum width of 16″ to carry $1500^{\#/f}$ uniform dead load and a $2000^{\#/f}$ uniform live load over a span of 16′.

twenty six

Concrete Slab Design

26.1 One-way Slabs

When analyzing or designing a slab, think of a one-foot section that is treated like a beam where h = the depth of the slab and b = 12″.



26.1

One-way slab

Minimum slab thickness:
ACI Table 9.5a defines the minimum allowable thickness of a slab for which deflections are not checked. For a simply supported one-way slab, using normal weight concrete and f_y = 60,000 psi steel, the minimum slab thickness:

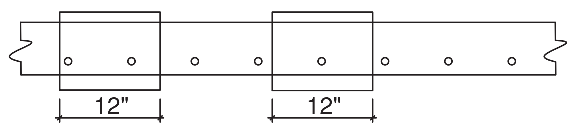
h = L/20 where L = slab span in inches.

If another steel strength is used, the found values for h are multiplied by (0.4 + 60,000/f_y).

If lightweight concrete is used, where 90 ≤ w_c ≤ 120pcf, the values are multiplied by (1.65 − .005w_c) but never by more than 1.09.

w_c = weight of concrete in pcf.

Steel area per foot:



26.2

Steel area per foot

As can be seen in Figure 26.2, the choice of where to cut a 12″ section would determine whether one or two bars are in the section. When designing or analyzing a slab, do not consider the steel in an exact 12″ section, but rather the average steel in any 12″ section.

A_s = area of steel per foot of slab and when steel is selected it is designated by the size of the bar and the spacing. For example, knowing the bar size and spacing, the area of steel can be found:

#5 @ 7″o.c.

A_s = (area of one bar)(12/spacing) = 0.307in^2(12$^{\prime\prime/f}$)/7″ = 0.53in^2/f

Knowing an area of steel required, A_s, and a desired bar size area, the spacing can be determined:

A_s = 0.42, #4 bars (area = .196)

S = (area of one bar)(12/ A_s) = 0.196in^2(12$^{\prime\prime/f}$)/0.42in^2/f = 5.6

Minimum steel in slabs:

$A_{s\ min}$ = 0.002bh for f_y = 40,000 or 50,000psi

$A_{s\ min}$ = 0.0018bh for 60,000psi

Note that the full cross-section, bh is used in these equations, not the effective area, bd, as used to find minimum steel area in beams.

Example 26-1: Slab analysis.

Find service live load in psf for a 10″ deep, one-way slab with a 16′ span, $\frac{3}{4}$″ cover, with f'_c = 4000psi and f_y = 60,000psi and longitudinal steel = #5 @ 6″o.c.

1. A_s = (.307in²)(12″/f)/6″ = 0.61in²/f
   d = 10 – .75 – .625/2 = 8.94″, b = 12″
2. $a = f_yA_s/(0.85f'_cb)$ = 60,000psi(0.61in²/f)/[.85(4000psi)(12″/f)] = 0.897″
3. $M_n = f_yA_s(d - a/2)$ = 60,000psi(0.61in²/f)(8.94 – 0.897/2) = 310788.9#-in
4. Check $A_s \geq A_{s\ min}$ = .0018bh = .0018(12″/f)(10″) = 0.216in²/f < 0.61in²/f … okay
5. $c = a/\beta_1$ = 0.897/.85 = 1.06
   ε_t = .003(d – c)/c = .003(10 – 1.06)/1.06 = 0.0253
6. 0.0253 ≥ 0.004
7. ϕ = 0.90 because 0.253 > 0.005
8. $M_u = \phi M_n$ = 0.9(310788.9#-in) = 279710#-in = 23309.17#-f

   M_u 23309.17#-f = w(16′)²/8 … w_u = 23309.17(8)/16² = 728#/f

   One foot section of slab weight = w_{bm} = 150pcf(10″/12″/f)(1′) = 125#/f

   w_u = 1.2(125#/f) + 1.6(LL) = 728#/f … LL = 361.35#/f per foot of slab = 361.35psf

Example 26-2: Determine adequate steel reinforcement for an 8″ deep slab with a 1″ cover having a span of 12′ and a live load of 250psf if f'_c = 4000psi and f_y = 40,000psi.

1. W = 1.6(250) + 1.2(150)(8/12)(12/12) = 520psf
2. M_u = 520k/ft(12)²/8 = 9360 #-ft = 112,320 #-in
3. Assume #5 reinforcement … d = 8 – 1 – .625/2 = 6.69
4. Assume ϕ = 0.9
5. $A_s = (0.85f'_cbd/f_y)\ [1 - \sqrt{[1 - 2M_u/\phi(.85f'_cbd^2)]}$
   = [.85(4000)(12)(6.69)/40000][1 – √[1 – 2(112,320)/(.9(.85)(4000)(12)(6.69²)] = 0.483in²/f
6. $A_{s\ min}$ = .002bh = .002(12)(8) = .192 < 0.483 … okay
7. #5 longitudinal steel spacing: s = 0.307(12/0.483) = 7.63″ round down to 7.5″
8. #5 temperature steel: s = .307(12/.192) = 19.19 > 18″ max … use 18″.

One-way slab design without calculating deflection:

1. Calculate h_{min} based on ACI Table 9.5a; round up to next 1/4″ for h< 6″ or up to next $\frac{1}{2}$″ for h>6″.
2. Find M_u.
3. Assume d = h – 1.12 (#6 bars & 3/4″ cover)
4. Assume ϕ = 0.9
5. $A_s = 0.85f'_cbd/f_y\ [1 - \sqrt{[1 - 2\ M_u/\phi(.85f'_cbd^2)]}$
6. $a = f_yA_s/(.85f'_cb)$, $c = a/\beta_1$, ε_t = .003(d – c)/c
   If $\varepsilon_t \geq 0.005$, ϕ = 0.9; if 0.005 > $\varepsilon_t \geq 0.004$, $\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$; if $\varepsilon_t \leq 0.004$, make the slab thinner.
7. $A_{s\ min}$ = .002bh for f_y = 40 or 50

   $A_{s\ min}$ = .0018bh for f_y = 60
8. Longitudinal steel spacing: s = (bar area)(12/ A_s) , round down to next .5″
9. Temperature steel: s = (bar area)(12/ $A_{s\ min}$), round down to next .5″
10. Check maximum spacing of 5h or 18″.

Example 26-3: Design a slab to span 12′ and carry a live load of 225psf.

f'_c = 4ksi, f_y = 40ksi use # 6 rebars

1. h_{min} = L/20(.4 + 40/100) = 12(12)(.8)/20 = 5.76″ round up to 6″
2. w_u = 1.2(150)(6/12)(12/12) + 1.6(225) = 450#/f

   M_u = 450(12)²/8 = 8100#-f = 97,200 #-in
3. d = 6 – 1.12 = 4.88″
4. Assume ϕ = 0.9
5. $A_s = (0.85f'_cbd/f_y)[1 - \sqrt{[1 - 2\ M_u/\phi(.85f'_cbd^2)]}$ = [.85(4000)(12)(4.88)/40000][1 – √[1 – 2(97,200)/(.9(.85)(4000)(12)(4.88)²)] = 0.588
6. $a = f_yA_s/(.85f'_cb)$ = 40,000(.588)/[.85(4000)(12)] = 0.576″
   c = .576/.85 = 0.677

   ε_t = .003(d – c)/c = .003(4.88 – .677)/.677 = .0186 > .005 … ϕ = 0.9

7. $A_{s\ min}$ = .002bh = .002(12)(6) = 0.144
8. Longitudinal steel spacing: s = (.442)(12/.588) = 9.02″ round down to 9″
9. Temperature steel: s = (.442)(12/.144) = 36.83
10. Check maximum spacing of 5h or 18″.
    5(6) = 30″ > 18″ … max. spacing = 18″

Answer:

Temperature steel: #6 @ 18″

Longitudinal steel: #6 @ 9″

Slab thickness = 6″

Design a slab for minimum h, where deflection will be checked:

1. Determine f'_c, f_y
2. Assume h = 6″ for weight, calculate w_u, M_u
3. Assume ϕ = 0.9
4. $d = \sqrt{[M_u/.153\phi f'_c b]} = \sqrt{[88{,}560/(.153(.9)(4000)(12))]}$
5. Estimate h = d + 1.12 and round up to next 1/4″ if h < 6, next 1/2″ if h > 6″.
6. Calculate actual w_u and M_u using slab thickness
7. $A_s = [.85f'_c bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f'_c bd^2]}]$
8. $a = f_y A_s/(.85f'_c b)$, $c = a/\beta_1$, $\varepsilon_t = .003(d - c)/c$
   If $\varepsilon_t \geq 0.005$, ϕ = 0.9; if $0.005 > \varepsilon_t \geq 0.004$, $\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$; if $\varepsilon_t \leq 0.004$, make the slab thinner.
   If ϕ ≠ 0.9, recalculate A_s with the new value of ϕ.
9. $A_{s\ min}$ = .0018bh for f_y = 40,000 or 50,000psi,
   $A_{s\ min}$ = .002bh for f_y = 60,000psi
10. Longitudinal steel spacing: s = (bar area)(12/A_s)
11. Temperature steel: s = (bar area)(12/$A_{s\ min}$)
12. Check maximum spacing of 5h or 18″.

Example 26-4.

Design a one-way slab for L = 12′, f'_c = 4ksi, f_y = 60ksi, live load = 200psf:

1. f'_c = 4000psi, f_y = 60,000psi, b = 12″
2. assume h = 6″ for weight

   $w_u = 1.6(200) + 1.2(150)(6/12) = 410^{\#/f}$

   $M_u = 410(12)^2/8 = 7380\#\text{-f} = 88{,}560\#\text{-in}$

3. Assume ϕ = 0.9
4. $d = \sqrt{[M_u/(.153\phi f'_c b)]} = \sqrt{[88{,}560/(.153(.9)(4000)(12))]}$ = 3.66″
5. Estimate h = d + 1.12 = 3.66 + 1.12 4.78″ round up to 5″
6. $w_u = 1.6(200) + 1.2(150)(5/12) = 395^{\#/f}$
   $M_u = 395(12)^2(12)/8 = 85{,}320\#\text{-in}$
7. $A_s = [.85f'_c bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f'_c bd^2]}] = [.85(4000)(12)(3.66)/60{,}000][1 - \sqrt{[1 - 2(85{,}320)/(.9(.85)(4000)(12)(3.66)^2]}] = 0.478$
8. $a = f_y A_s/(.85f'_c b) = 60{,}000(.478)/(.85(4000)(12)) = 0.703$, $c = 0.703/.85 = 0.827$, $\varepsilon_t = .003(3.66 - 0.827)/0.827 = 0.010 > .005$ … ϕ = 0.9
9. $A_{s\ min}$ = .0018bh = .0018(12)(5) = 0.108
10. Longitudinal steel spacing: (#5) s = (.307)(12/.478) = 7.7″ round down to 7.5″
11. Temperature steel using #5rebar: s = (.307)(12/.108) = 34.11″
12. Check maximum spacing of 5h or 18″.
    s = 5(5) = 25″ > 18″ … max. spacing = 18″

Answer:

Temperature steel: #5 @ 18″

Longitudinal steel: #5 @ 7.5″

Slab thickness = 5″

26.2 Continuous Slabs

Limitations:

There must be two or more spans not varying by more than 20% in length

Slab must have uniformly distributed loads

Ratio of LL to DL ≤ 3

Members must be prismatic (orthorhombic).

Minimum thickness: ACI Table 9.5a dictates minimum slab thickness for continuous slabs not checked for deflection as follows:

Both ends continuous: h ≥ L/28

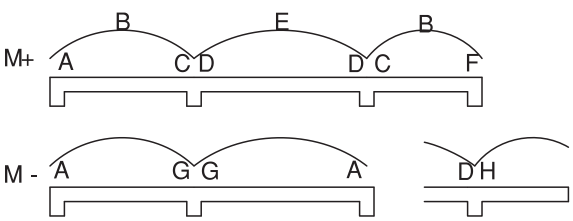
One end Continuous: h ≥ L/24

Cantilever: h ≥ L/10

Design of continuous one-way slab:

1. Determine minimum slab thickness

2. Determine slab weight and total factored load w_u = 1.2DL + 1.6LL
3. Calculate the value of wL^2
4. Calculate M_u for each location:
5. Calculate $A_s = [.85f'_c bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f'_c bd^2]}$ for each location.
6. Calculate a, c, ε_t each case. Note: start with the smallest value of M_u and work up. Once the strain is above 0.005 for a particular case, all other cases with a higher value of M_u will have higher strain and ε_t will not need to be calculated.
   If $\varepsilon_t \geq 0.005$, $\phi = 0.9$; if $0.005 > \varepsilon_t \geq 0.004$, $\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$; if $\varepsilon_t \leq 0.004$, make the slab thinner.
   If $\phi \neq 0.9$, recalculate A_s with new ϕ.
7. $A_{s\ min}$ = .002bh
8. Longitudinal steel spacing: s = (bar area)(12/A_s)
9. Temperature steel: s = (bar area)(12/$A_{s\ min}$)
10. Check maximum spacing of 5h or 18″.
11. Check $\phi Vn = \phi 2\sqrt{(f'_c bd)} \geq Vu = 1.15wL/2$. If not, increase h and go back to step 1.



26.3
Moment conditions in continuous slabs

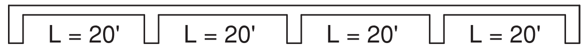
It is helpful in designing continuous slabs to create a table (Table 26.1).

Table 26.1: Continuous slab design template

Location	M_u eqtn	M_u	A_s	a	c	ε_t	$A_{s\ min}$	$s_{Long.}$	$s_{Temp.}$
A	$wL^2/24$								
B,D	$wL^2/11$								
C	$wL^2/10$								
E,F	$wL^2/16$								
G	$wL^2/9$								
H	$wL^2/2$								

Example 26-5: Design a continuous one-way slab for the plan shown in Figure 26.4.

LL = 80psf, f'_c = 4ksi, f_y = 60ksi. Use #3 bars



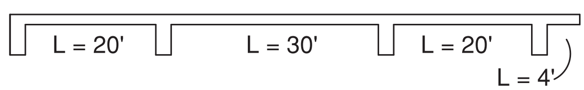
26.4
Example 26-5

1. Determine minimum slab thickness: 10′(12)/24 = 5″, d = 5 – 1.12 = 3.88″
2. w_u = 1.2(150pcf)(5″/12)(12″/12) + 1.6(80psf)(1′) = $203^{\#/f}$
3. $wL^2(12^{\prime\prime/f}) = 203(10^2)(12)$ = 243,600#-in
4. Calculate M_u for each location: see Table 26.1.
5. Calculate $A_s = [.85f'_c bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f'_c bd^2]}$ $= 2.638[1 - \sqrt{(1 - M_u/276398.8)}]$
6. Calculate a, c, ε_t for each case.
7. $A_{s\ min}$ = .0018bh = .0018(12)(5) = .108
8. Longitudinal steel spacing: s = (.307)(12/A_s)
9. Temperature steel : s = (.11)(12/.108) = 12.22 round down to 12″
10. Check maximum spacing of 5h or 18″.

    5h = 5(6) = 30″ > 18″ … max. spacing = 18″

11. Check $\phi V_n = \phi 2\sqrt{(f'_c bd)} > V_u = 1.15wL/2$. If not, increase h and go back to step 1. $\phi V_n = (.75)2\sqrt{4000}(12)(3.88)$ = 4,420# > 1.15(203)(10)/2 = 1,167# … okay

Example 26-6: Design a continuous one-way slab with 20′ spans as shown in Figure 26.5. Live load = 80psf, f'_c = 4ksi, f_y = 60ksi, use #5 steel.



26.5
Example 26-6

Location	*Mu eqtn*	*Mu*	*As*	*a*	*c*	*εt*	*As min*	*sLong.*	*sTemp.*
A	$wL^2/24$	10150	~~0.049~~ .108	.072	.085	.135	.108	12	12
B, D	$wL^2/11$	22145	0.108	.159	.187	.059	.108	12	12
C	$wL^2/10$	24360	0.119	.175	.206	.054	.108	11	12
E, F	$wL^2/16$	15225	~~0.074~~ .108	.108	.129	.088	.108	12	12
G	$wL^2/9$	n/a							
H	$wL^2/2$	n/a							

1. Determine minimum slab thickness: 20′(12)/24 = 10″, d = 10 – 1.12 = 8.88″
2. w_u = 1.2(150pcf)(10″/12)(12″/12) + 1.6(80psf)(1′) = 278$^{\#/f}$
3. wL^2(12$^{\prime\prime/f}$) = 278(20^2)(12) = 1,334,400#-in
4. Calculate M_u for each location:
5. Calculate $A_s = [.85f'_c bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f'_c bd^2]}$ = 6.038[1 – $\sqrt{(1 - M_u/1{,}447{,}776.8)}$]
6. Calculate a, c, ε_t each case.
7. $A_{s\ min}$ = .0018bh = .0018(12)(10) = .216
8. Longitudinal steel spacing: s = (.307)(12/A_s)
9. Temperature steel: s = (.307)(12/.216) = 17.06 round down to 17″
10. Check maximum spacing of 5h or 18″.
    5h = 5(10) = 50″ > 18″ … max. spacing = 18″
11. Check $\phi V_n = \phi 2\sqrt{(f'_c bd)}$ > Vu = 1.15wL/2. If not, increase h and go back to step 1. ϕV_n = (.75)2√4000(12)(8.88) = 10,109.2# > 1.15(278)(20)/2 = 3,197# … okay

Location	*Mu eqtn*	*Mu*	*As*	*a*	*c*	*εt*	*As min*	*sLong.*	*sTemp.*
A	$wL^2/24$	55600	~~0.117~~ .216	.172	.203	.129	.216	17	17
B, D	$wL^2/11$	121309	0.258	.380	.447	.057	.216	14	17
C	$wL^2/10$	133440	0.285	.419	.493	.051	.216	12.5	17
E, F	$wL^2/16$	83400	~~0.176~~ .216	.260	.305	.084	.216	17	17
G	$wL^2/9$	n/a							
H	$wL^2/2$	n/a							

Practice Exercises:

26-1: Find the allowable service live load in psf for an 8″ deep, one-way slab with a 12ft span, 3/4″ cover, with f'_c = 4000psi and f_y = 60,000psi and longitudinal steel = #5 @ 9″o.c.

26-2: Design a slab to span 14ft and carry a live load = 120psf where deflection is not checked. f'_c = 3,000psi, f_y = 40,000psi, use # 5 rebars.

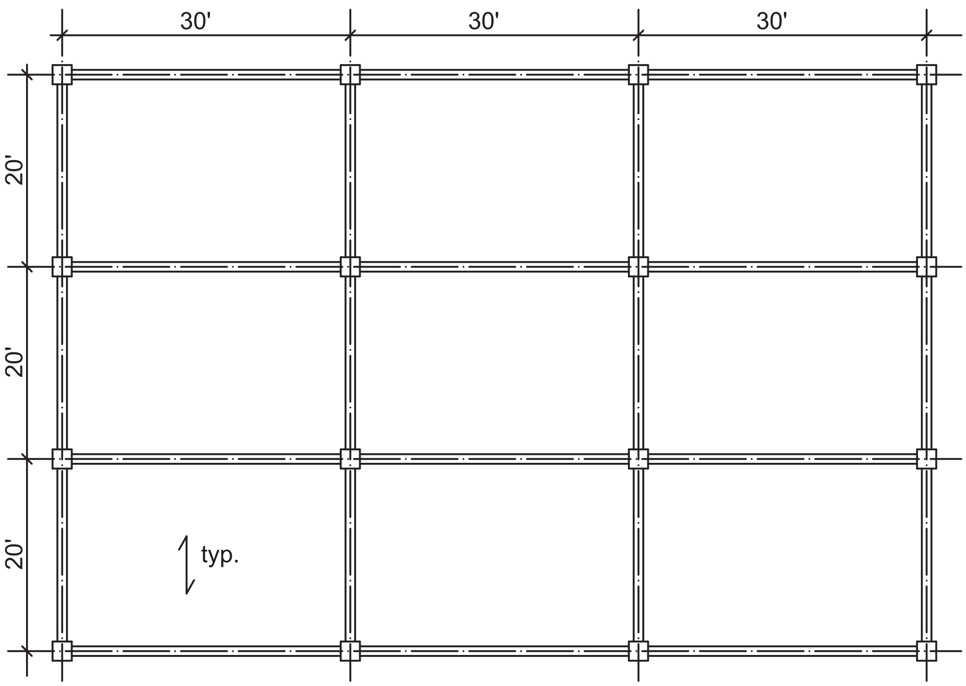
26-3: Design a slab to span 15ft and carry a live load = 90psf where deflection is not checked. f'_c = 4,000psi, f_y = 60,000psi{, use # 5 rebars.

26-4: Design a slab with minimum thickness to span 14ft and carry a LL = 120psf where deflection will be checked. f'_c = 3,000psi, f_y = 40,000psi, use # 5 rebars.

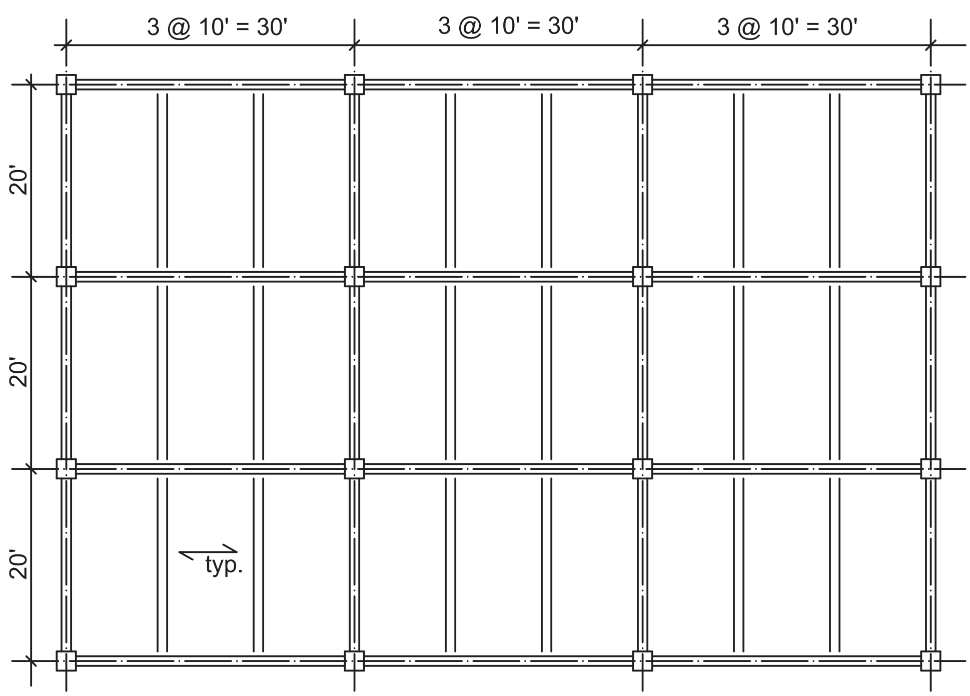
26-5: Design a slab with minimum thickness to span 15ft and carry a LL = 90psf where deflection will be checked. f'_c = 4,000psi, f_y = 60,000psi, use # 5 rebars.

26-6: Design a continuous slab for the plan shown in Figure 26.6 if the floor carries a LL of 90psf and a dead load of 15psf. f'_c = 4,000psi, f_y = 60,000psi use # 5 rebars.

26-7: Design a continuous slab for the plan shown in Figure 26.6 if the floor carries a LL of 90psf and a dead load of 15psf. f'_c = 4,000psi, f_y = 60,000psi use # 5 rebars.



26-6



26-7

26.6

Exercise 26-6

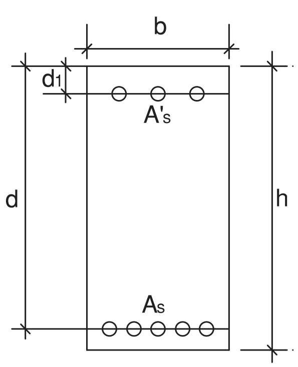
twenty seven

Doubly Reinforced Beams and T-beams

27.1 Doubly Reinforced Beams

If the practical nominal moment in a concrete beam needs to be increased and the beam size cannot be increased, ACI code, section 10.3.5.1 allows for additional steel to be added in tension provided it is also added in compression. Two conditions can exist when top steel is added to a beam.

> Condition 1: Both the tensile and compressive steels yield before the concrete strain reaches 0.003.
>
> Condition 2: The tensile steel yields but the compressive steel does not yield before the concrete strain reaches 0.003.



27.1
Doubly reinforced concrete beams

27.1.1 Condition 1

In order to ensure both the top and bottom steel yield before the concrete, ε_t must not go below .005.

> A_s = area of tensile and compressive steel at bottom (depth = d)
>
> A_s' = area of compressive steel at top (depth of d′)

For condition 1, when $\varepsilon_c = .003$, $\varepsilon_s = .005$
$N_{T1} = A_{s1}f_y$ where A_{s1} is the portion of the bottom steel that allows the concrete to reach its full compressive strain of 0.003.
The internal couple produced is the same as in singly reinforced beams:

$$\phi M_{n1} = N_{T1}Z_1 = (A_{s1}f_y)(d - a/2)$$

$$N_{T1} = N_{C1} \ldots A_{s1}f_y = \beta f'_c ab \ldots a = A_{s1}f_y/\beta f'_c b$$

$$\beta = .85 \text{ for } f'_c \leq 4\text{ksi}.$$

In analysis, the top steel area A_s', is known and it is assumed that the steel yields. Therefore, $f_s' = f_y$.

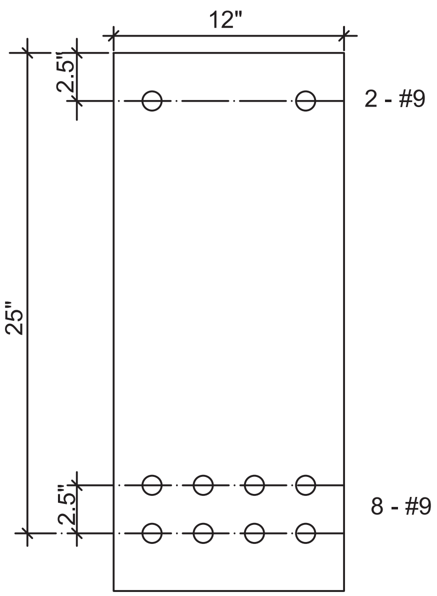
$$N_{C2} = N_{T2} = A_s'f_y = A_{S2}f_y \ldots . A_s' = A_{S2}$$

$$A_s = A_{S1} + A_{S2}. \text{ Therefore, } A_{S1} = A_s - A_s'$$

$$A = A_{S1}f_y/.85f'_c b \text{ and } c = a/.85$$

Once c is calculated, the strain can be determined and the beam can be verified to be in condition 1 or 2. If $\varepsilon_s' \geq \varepsilon_y$ then the beam is in Condition 1; all the steel yields and the assumption is correct. $\varepsilon_y = f_y/E$ for f_y = 60ksi, ε_y = 60/29,000 = 0.00207.

Example 27-1: Find ϕM_n for the beam shown in Figure 27.2. f'_c = 4ksi, f_y = 60ksi.



27.2
Example 27-1

$A_s = 8in^2$ $A_s' = A_{s2} = 2.0in^2$

$A_{s1} = A_s - A_{s2} = 8 - 2 = 6in^2$

Assume condition 1: $f_s = f_s' = f_y$

$a = A_{s1}f_y/\beta f'_c b = 6(60)/[.85(4)(12)] = 8.82''$ $c = 8.82/.85 = 10.38''$

$d = 25 - 2.5/2 = 23.75''$, $d_t = 25$

Using similar triangles:

$\varepsilon_s' = .003(10.38 - 2.5)/10.38 = .00228 > .00207$

$\varepsilon_t = .003(25 - 10.38)/10.38 = .00423 > .00207$ … Condition 1

$\phi = 0.65 + (.00423 - .002)(250/3) = 0.836$

$M_{n1} = A_{s1}f_y(d - a/2) = 6(60)(23.75 - 8.82/2) = 6962.4k\text{-}in = 580.2k\text{-}f$

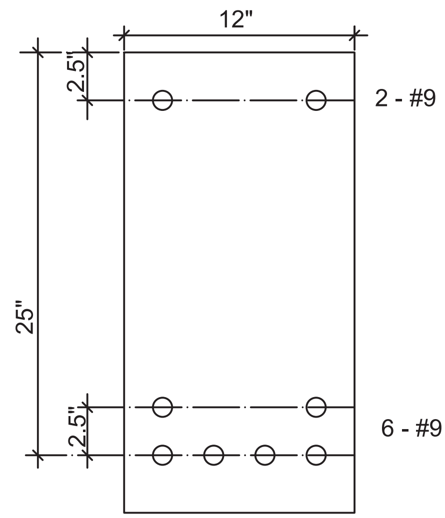
$M_{n2} = A_s'f_y(d - d') = 2(60)(23.75 - 2.5) = 2550k\text{-}in = 212.5k\text{-}f$

$\phi M_n = \phi(M_{n1} + M_{n2}) = .836(580.2 + 212.5) = 662.7k\text{-}f$

27.1.2 Condition 2

Condition 2 occurs when $\varepsilon_s' < \varepsilon y$ and $\varepsilon s > \varepsilon y$.

Example 27-2: Find ϕM_n for the beam shown in Figure 27.3. f'_c = 5ksi, f_y = 60ksi.



27.3
Condition 2 example of doubly reinforced beam

Assume all steel yields.

$A_{s2} = A_s' = 2.0in^2$

$A_s = 6.0in^2$, $A_{s1} = 6 - 2 = 4in^2$

$a = 4(60{,}000)/[.85(5000)(12)] = 4.71''$

$c = a/\beta$

Remember that, for $f'_c > 4000psi$, $\beta 1 = 0.85 - 0.05(f'_c - 4000)/1000 \geq 0.65$

$= .85 - .05(5000 - 4000)/1000 = 0.8$

$c = 4.71/.8 = 5.89''$

$\varepsilon_s' = .003(c - d')/c = .003(5.89 - 2.5)/5.89 = .00173 < .00207$

The compression steel has not yielded ($\varepsilon_s' < \varepsilon_y$)

$\varepsilon_t = .003(d_t - c)/c = .003(21 - 5.89)/5.89 = .0077 > .00207$

The tensile steel had yielded ($\varepsilon_t > \varepsilon_y$)

Condition 2 exists:

$$N_T = N_{C1} + N_{C2} \ldots A_s f_y = .85f'_c ba + f_s' A_s'$$

a and f_s' have changed because the assumption that the beam was in Condition 1 was wrong.
$a = \beta c$ and $f_s' = \varepsilon' c E_s = [.003(c - d')/c]E_s$. The only unknown is c. Solve for c by substituting these equations into the original and forming a quadratic equation

$$A_s f_y = (.85f'_c)(b)(a) + f_s' A_s'$$

$$A_s f_y = .85f'_c b\beta c + [.003(c - d')/c]E_s A_s' \text{ where:}$$

$A_s = 6$, $f_y = 60$ksi, $f'_c = 5$ksi, $b = 12''$, $\beta = 0.8$, $d' = 2.5''$, $E_s = 29{,}000$ksi, $A_s' = 2.0$

Note: Be careful to use consistent units: If using $E_s = 29{,}000$ksi, use f'_c and f_y in ksi.

$$A_s f_y = 6(60) = (.85)(5)(12)(.8)(c) + [.003(c - 2.5)/c](29{,}000)(2) = .85f'_c b\beta c + [.003(c - d')/c]E_s A_s'$$

$$360 = 40.8c + 174 - 435/c$$

$$0 = 40.8c^2 - 186c - 435 = c^2 - 4.559c - 10.662$$

Use quadratic equation formula:

$$c = 4.559/2 \pm .5\sqrt{(4.559^2 + 4(10.662))} = 2.28 \pm 3.98 = 6.26''$$

Check that assumptions are correct.

$$f_s' = [.003(c - d')/c]E_s$$

$$= [.003(6.26 - 2.5)/6.26][29{,}000]$$

$$= 52.26 < f_y = 60\text{ksi} \ldots \text{assumption is correct}$$

Knowing $c = 6.26''$, check $\varepsilon_t \geq 0.004$

$$\varepsilon_t = .003(d_t - c)/c = .003(21 - 6.26)/6.26 = 0.0071 > 0.004 \ldots \text{okay}$$

$$\phi = 0.9 \text{ because } .0071 > .005$$

Solve for ϕM_n:

$$d = 21'' - 2.5''/2 = 19.75''$$

$$M_{n1} = N_{C1}Z_1 = N_{C1}(d - a/2)$$

$$= .85f'_c ab(d - a/2)$$

$$= (.85)(5\text{ksi})(.8(6.26)(12)(19.75 - .8(6.26)/2)$$

$$= 4494.77\text{k-in} = 367.06\text{k-f}$$

$$M_{n2} = N_{C2}Z_2 = N_{C2}(d - d') = A_s' f_s'(d - d')$$

$$\{eq = 2(52.26)(19.75 - 2.5) = 1802.97\text{k-in} = 150.25\text{k-f}$$

$$\phi M_n = 0.9(367.06 + 150.25) = 465.58\text{k-f}$$

27.1.3 Doubly Reinforced Beam Design

To design a doubly reinforced beam, begin by designing a singly reinforced beam. If the moment requirements cannot be met without enlarging the beam, design a doubly reinforced beam. Design each of the two internal couples (M_{n1} and M_{n2}) separately so that the total satisfies the required moment.

1. Determine M_u if unknown.
2. Assume $d = h - 3$
3. Assume $\phi = 0.9$
4. $A_s = (.85f'_c bd/f_y)[1 - \sqrt{(1 - 2M_u/\phi.85f'_c bd^2)}]$
5. $a = f_y A_s/(0.85f'_c b)$, $c = a/\beta_1$, $\varepsilon_t = .003(d - c)/c$. If $\varepsilon_t < 0.004$, then beam needs double reinforcement.
6. At $\varepsilon_t = 0.005$, $c = 3d/8$, $a = .375d\beta_1$
7. $\phi M_{n1} = .9(.85f'_c)ab(d - a/2)$
8. $A_{S1} = .85f'_c ab/f_y$
9. $\phi M_{n2} = M_u - \phi M_{n1}$
10. $N_{C2} = \phi M_{n2}/\phi(d - d')$
11. $\varepsilon_s' = .003(c - d')/c < \varepsilon_y = .00207 \ldots f_s' = \varepsilon_s' f_y$
12. $A_s' = N_{C2}/f_s'$
13. $A_{s2} = N_{C2}/f_y$
14. $A_s = A_{s1} + A_{s2}$
15. Check actual d > assumed d
16. Check ε_s', ε_t and ϕ using selected steel:

$$A = (A_s - A_s')f_y/.85f'_c b$$

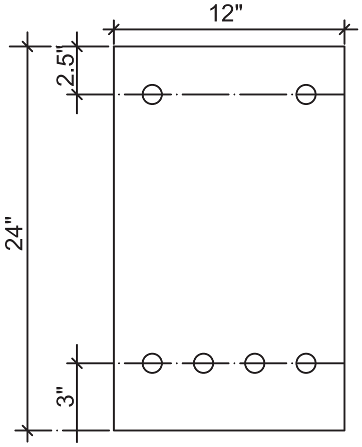
$$\varepsilon_s' = .003(c - d')/c$$

$$f_s' = \varepsilon_s' E_s$$

$$\varepsilon_t = .003(dt - c)/c \text{ if } \varepsilon_t > .005 \ldots \phi = .9$$

17. Check $\phi M_n > M_u$:

Example 27-3: Design the steel for a beam with: $M_u = 350$k-f, $b = 12$, $h = 24$, $f'_c = 3$ksi, $f_y = 60$ksi, $d' = 2.5''$.



27.4
Example 27-3

1. Determine M_u if unknown. M_u = 350k-f = 4200k-in
2. Assume d = h − 3 … d = 24 − 3 = 21″
3. Assume $\phi = 0.9$
4. $A_s = [.85f'_c bd/f_y][1 - \sqrt{[1 - 2M_u/\phi .85f'_c bd^2}] = [.85(3)(12)(21)/60][1 - \sqrt{[1 - 2(4200)/(.9(.85)(3)(12)(21)^2}] = 4.762\text{in}^2$
5. $a = f_y A_s/(0.85f'_c b) = 60(4.762)/(.85(3)(12)) = 9.34″$

   $c = a/\beta_1 = 9.34/.85 = 10.99″$

   $\varepsilon_t = .003(d - c)/c = .003(21 - 10.99)/10.99 = 0.0027 < 0.004$ … beam needs to be enlarged or needs double reinforcement.

6. Let $\varepsilon_t = .003(d - c)/c = 0.005$ … c = 3d/8 … $a = .375d\beta_1 = .375(21)(.85) = 6.69″$
7. $\phi M_{n1} = .9[(.85f'_c)ab](d - a/2) = .9(.85)(3)(6.69)(12)(21 - 6.69/2) = 3252.8$k-in = 271.07k-f
8. $A_{s1} = .85f'_c ab/f_y = .85(3)(6.69)(12)/60 = 3.41\text{in}^2$
9. $\phi M_{n2} = M_u - \phi M_{n1} = 350 - 271.07 = 78.93$k-f
10. $N_{C2} = \phi M_{n2}/\phi\ (d - d') = 78.93$k-f(12in/f)/.9(21 − 2.5) = 56.89k
11. a = 6.69″ (from step 6) … $c = a/\beta_1 = 6.69/.85 = 7.87″$ $\varepsilon_s' = .003(7.87 - 2.5')/7.87 = .00205 < \varepsilon y = .00207$ … $f_s' = .00205(29000) = 59.36$ksi
12. $A_s' = N_{C2}/f_s' = 56.89\text{k}/59.36\text{ksi} = 0.96\text{in}^2$ use two #7 for $A_s' = 1.2$
13. $A_{s2} = N_{C2}/f_y = 56.89/60 = .95$
14. $A_s = A_{s1} + A_{s2} = 3.41 + .95 = 4.36$ use 3 – #11 for $A_s = 4.68$
15. Check actual d > 21(assumed d)

    d = 24 − 1.5 − .375 − 1.41/2 = 21.42 > 21 … okay

    $A_s' = 1.2$, $A_s = 4.68$
16. Check ε_s', ε_t and ϕ using selected steel:

    $A = (A_s - A_s')f_y/.85f'_c b = (4.68 - 1.2)(60)/.85(3)(12)$…

    $c = 6.63/.85 = 7.8″$

    $\varepsilon_s' = .003(7.8 - 2.5)/7.8 = .00204 < .00207$ … $f_s' < f_y$

    $f_s' = .00204(29000) = 59.16$

    $\varepsilon_t = .003(21.42 - 7.8)/7.8 = .00524 > .005$ … $\phi = .9$
17. Check $\phi M_n > M_u$:

    $M_{n1} = A_{s1}f_y(d - a/2) = (3.48)(60)(21.42 - 6.63/2) = 3780.32$k-in = 315k-f

    $M_{n2} = A_s'f_s'(d - d') = 1.2(59.16)(21.42 - 2.5) = 1343.17$k-in = 111.93k-f

    $\phi M_n = .9(315 + 111.93) = 384.24 > M_u = 350$k-f … okay

Answer: Use three #11 on the bottom, two #7 on the top.

27.2 T-beams

The term T-beam describes a concrete beam that utilizes the floor slab as a compression flange. The T-beam can be a simple beam with uniform load or a carrier beam or girder. The effective flange width, b, is the width of slab that is allowed to be a part of the T-beam. ACI Code section 8-12 limits effective flange length as follows:

1. $b \le L/4$ (span length/4)
2. $b \le b_w + 16h_f$ (web thickness + slab thickness)
3. $b \le s$ = center-to-center spacing of beams.

If flange is only on one side then effective flange width is limited to:

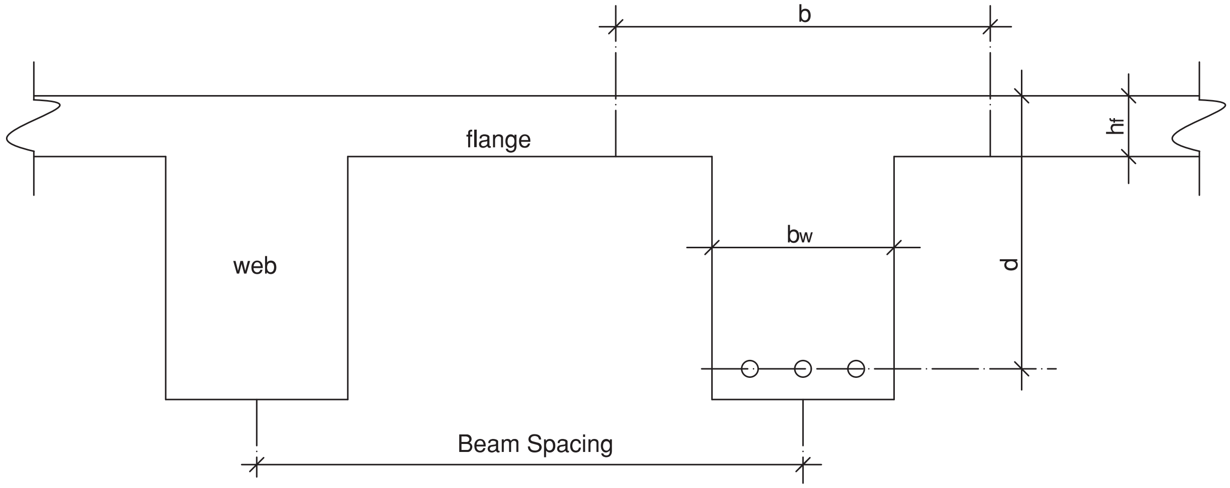
1. $b \le L/12$
2. $b \le 6hf$
3. $b \le 1/2$ clear distance to next beam.

If the T-shape is isolated and not part of a slab:

$$b_w/2 \le b \le 4\,b_w$$

Minimum steel reinforcement for T-beams is:

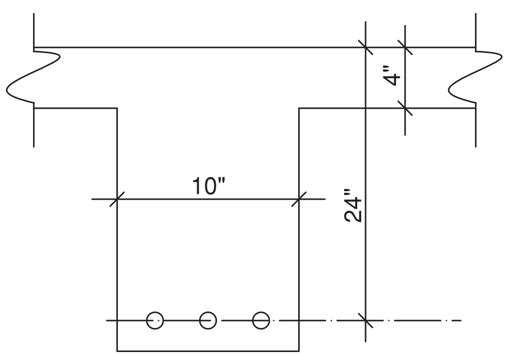
$$A_{s\ min} = 3(\sqrt{f'_c})b_w d/f_y \ge 200b_w d/f_y$$



27.5
T-beam

27.2.1 The Practical Moment Strength ϕM_n

Example 27-4: Find the practical moment strength, ϕM_n for the T-beam shown in Figure 27.6 if the span is 24ft and the center-to-center beam spacing is 5ft. f'_c = 4ksi and f_y = 60 ksi.



27.6
Example 27-4

1. Find effective flange length:

   L/4 = 24(12)/4 = 72″

   bw + 16 hf = 10 + 64 = 74″

   beam spacing = 60″

   use b = 60″

2. Check A_s min = .0033bwd = .0033(10)(24) = 0.79 < 2.37 (3 #8) … okay
3. Assume the steel yields. Find N_T:

   $N_T = A_s f_y$ = 2.37(60,000psi) = 142,200#

4. Find whether the flange can handle the compressive force:
   $N_{Cf} = (.85f'_c)(b)(h_f)$ = .85(4000)(60)(4) = 816,000 > 142,200 … the compression is handled by the flange and the analysis is the same as for a rectangular beam with a width b = 60″.
5. Find a = $A_s f_y / .85f'_c b$ = 2.37(60,000)/.85(4000)(60) = 0.697″
   Note: the ratio of $a/h_f = N_T/N_{Cf}$ … $a = N_T h_f / N_{Cf}$ = 142,200(4)/816,000 = .697
6. Find tensile strain ε_t = .003(d − c)/c

   c = a/.85 = .697/.85 = 0.82

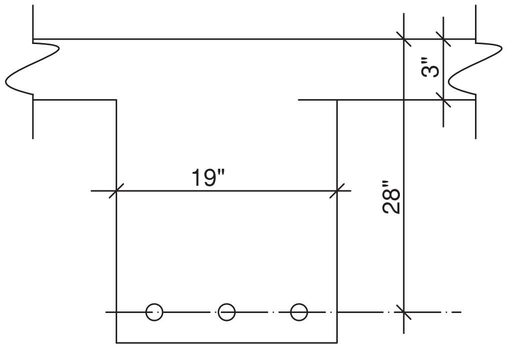
   ε_t = .003(24 − .82)/.82 = 0.0848

   .0848 > .005 therefore tension controls yielding and ϕ = 0.9.

7. $\phi M_n = \phi A_s f_y (d - a/2)$ = 0.9(2.37)(60ksi) (24 − .697/2) = 3026.92k-in = 252.243 k-f

Compression in the web occurs when there is not adequate area in the flange, as in the next example.

Example 27-5: Find the ϕM_n for the T-beam shown in Figure 27.7. f'_c = 3ksi and f_y = 60ksi. The span is 16ft and the center-to-center beam spacing is 6ft.



27.7
Example 27-5

1. Find b:

   $L/4 = 16(12)/4 = 48''$

   $b_w + 16\ h_f = 19 + 48 = 67''$

   beam spacing = 72″

   USE: b = 48″
2. $A_{s\ min} = .0033b_wd = .0033(19)(28) = 1.76 < 7.8$ (5 #11) … okay
3. Assume the steel yields. Find N_T:

   $N_T = A_sf_y = 7.8(60{,}000psi) = 468{,}000\#$
4. Find if the flange can handle the compressive force.

   $N_{Cf} = (.85f'_c)(b)(h_f) = .85(3000)(48)(3) = 367{,}200$

   367,200 < 468,000 therefore the web must help handle the compression.
5. Find the compression carried by the web.

   $N_{CW} = 468{,}000 - 367{,}200 = 100{,}800\#$
6. Find the distance the compression block extends below the flange $(a - h_f)$.

   $(a - h_f) = 100{,}800/.85(3000)(19) = 2.08$

   $a = 2.08 + h_f = 5.08$ and $c = 5.08/.85 = 5.976$
7. $M_{nf} = N_{cf}Z_f = 367.2k\ (28 - 3/2) = 9730.8$ k-in
8. $M_{nw} = N_{cw}Z_w = 100.8k(28 - 3 - 2.08/2) = 2415.17$k-in
9. $M_n = 9730.8 + 2415.17 = 12145.97$k-in = 1012.16k-f
10. $\varepsilon_t = .003(28 - c)/c = .003(28 - 5.976)/5.976 = .0111 > .005$ therefore $\phi = 0.9$
11. $\phi M_n = 0.9(1012.16) = 910.944$ k-f

27.2.3 To Design a T-beam

1. Determine M_u
2. Assume $d = h - 3$, and $\phi = 0.9$
3. Find effective width b: $b \leq L/4$, beam spacing and $b_w + 16h_f$.
4. $M_{nf} = .85f'_cbh_f(d - h_f/2)$
5. If $\phi M_{nf} \geq M_u$, go to step 6. If $\phi M_{nf} < M_u$, go to step 12.
6. $A_s = [.85f'_cbd/f_y][1 - \sqrt{}[1 - 2M_u/\phi.85f'_cbd^2]$
7. Check that $A_s \geq A_{s\ min} = b_wd(3\sqrt{f'_c})/f_y \geq 200b_wd/f_y$. If not, use $A_{s\ min}$.
8. Select bars based on A_s values from Table A4.2. Note actual A_s. Calculate the actual value of d. Check the required width for the number and size of bars chosen from Table 26.1.
9. $a = f_yA_s/(.85f'_cb)c = a/\beta_1$
10. Check $\varepsilon_t = .003(d - c)/c > 0.004$.
11. Check $\phi = 0.9$ assumption. If $\varepsilon_t < 0.005$, recalculate ϕ and check that $\phi M_n \geq M_u$.

    $\phi = 0.65 + (\varepsilon_t - 0.002)(250/3)$ if $0.004 \leq \varepsilon_t < 0.005$.

When $\phi M_{nf} < M_u$:

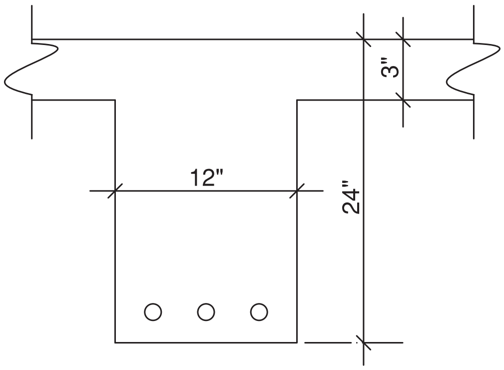
12. $Z_f = d - h_f/2$ where Z_f is the distance from the center of gravity of the steel to the center of gravity of the flange.
13. $A_{sf} = M_{nf}/f_yZ_f$
14. $d_w = d - h_f$
15. $M_{nw} = (M_u - \phi M_{nf})/\phi$
16. a_w = depth of stress block in the web.

    $a_w = d_w \pm \sqrt{}[(d_w{}^2 - 2M_{nw}/(.85f'_cb_w)]$
17. $A_{sw} = .85f'_ca_wb_w/\ f_y$
18. $A_s = A_{sf} + A_{sw}$
19. Calculate actual value of d:
20. $A_{s\ min} = b_wd(3\sqrt{f'_c})/f_y \geq 200b_wd/f_y$
21. $a = a_w + h_f$, $c = a/\beta 1$, $\varepsilon_t = .003(d - c)/c$

    If $\varepsilon_t > .005$, $\phi = 0.9$,

    $0.004 < \varepsilon_t < .005$, $\phi = 0.65 + (\varepsilon_t - .002)(250/3)$ and check $\phi\ M_n > M_u$.

Example 27-6: Design reinforcement for the T-beam shown in Figure 27.8. f'_c = 4ksi, f_y = 60ksi, b_w = 12″, h = 24″, h_f = 3″, Beam spacing = 8ft, L = 20′, Live Load = 125psf, Dead Load = 200psf (includes concrete wt).



27.8

T-beam design

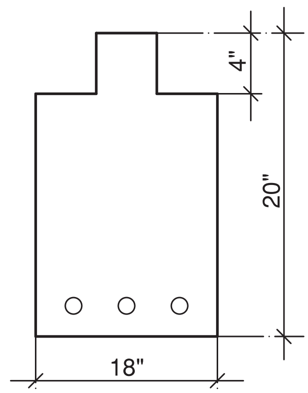
1. W_u = [1.2(200) + 1.6(125)]8ft = $3520^{\#/f}$ = 3.52k/f
   M_u = $(3.52k/f)(20)^2/8$ = 176k-f
2. d = 24 – 3 = 21″, Assume ϕ = 0.9
3. b ≤ L/4 = 20(12)/4 = 60″
   b ≤ b_w + $16h_f$ = 12 + 16(3) = 60″
   b ≤ beam spacing = 96″
   use b = 60″
4. M_{nf} = $.85f'_c bh_f(d - h_f/2)$ = .85(4ksi)(60″)(3″)(21 – 3/2)
   = 11934k-in = 994.5k-f
5. 0.9(994.5) = 895.05k-f > M_u = 176k-f … Rectangular T-beam
6. $A_s = [.85f'_c bd/f_y][1 - \sqrt{[1 - 2M_u/\phi .85f'_c bd^2]}]$ = [.85(4)(60)(21)/60][1 $\sqrt{(1 - 2(176k\text{-}f)(12^{\prime\prime/f})/(.9(.85)(4)(60)(21)^2)}$]
   = $1.89in^2$
7. Check that $A_s \geq A_{s\ min} = b_w d(3\sqrt{f'_c})/f_y \geq 200b_w d/f_y$ = 12(21)(3)($\sqrt{4000}$)/60000 = 0.797 ≥ 200(12)(21)/60000 = 0.84 …
   $A_{s\ min}$ = 0.84 < 1.89 … okay
8. Use two #9 A_s = 2.0. d = 24 – 1.5 – .375 – 1.128/2
   = 21.561 > 21 … okay
9. $a = f_y A_s/(.85f'_c b)$ = 60(2)/(.85(4)(60) = 0.588 c = a/β_1
   = 0.588/.85 = 0.692
10. Check ε_t = .003(d – c)/c = .003(21.561 – .692)/.692
    = 0.09 > 0.004.
11. 0.09 > 0.005 … ϕ = 0.9
    ANSWER: Use two #9

27.2.3 Irregular Shapes

Irregular shapes can be designed using the same logic as with T-beams. Divide the beam into sections based on width and work from the top down.

Example 27-7: Design reinforcement for the inverted T-beam shown.

f'_c = 4ksi, f_y = 60ksi, b_w = 18″, h = 20″, h_f = 4″, b = 6″,
L = 20′, M_u = 176k-f



27.9

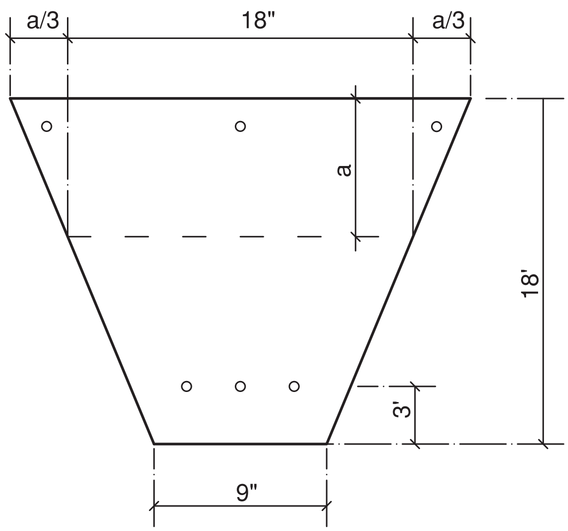
Inverted T-beam

1. Determine M_u = 176k-f
2. Assume d = 20 – 3 = 17″, ϕ = 0.9
3. Find effective width b: b ≤ L/4, beam spacing and b_w + $16h_f$. In this case, b is given and is smaller than all conditions: b = 6″
4. M_{nf} = $.85f'_c bh_f(d - h_f/2)$ = .85(4)(6)(4)(17 – 4/2) = 1224k-in
   = 102k-f
5. f ϕM_{nf} = 0.9(102) = 91.8k-f < M_u = 176k-f … go to step 12
12. Z_f = d – $h_f/2$ = 17 – 4/2 = 15″
13. A_{sf} = $M_{nf}/f_y Z_f$ = 102k-f(12in/f)/(60ksi)(15″) = $1.36in^2$
14. d_w = d – h_f = 17 – 4 = 13″
15. M_{nw} = $(M_u - \phi M_{nf})/\phi$ = (176 – 91.8)/0.9 = 93.56k-f
    = 1122.67k-in
16. a_w = depth of stress block in the web.
    $a_w = d_w \pm \sqrt{[d_w^2 - 2M_{nw}/(.85f'_c b_w)]}$
    = 13 + $\sqrt{[169 - 2(1122.67)/(.85(4)(18))]}$ = $1.497in^2$
17. A_{sw} = $.85f'_c a_w b_w/f_y$ = .85(4)(1.497)(18)/60 = $1.527in^2$
18. A_s = A_{sf} + A_{sw} = 1.36 + 1.527 = 2.887, use three #9
19. Calculate actual value of d: d = 20″ – 1.5″ – .375″ – 1.128″/2 = 17.56″ > 17″ … okay

20. $A_{s\ min} = b_w d(3\sqrt{f'_c})/f_y = 18(17.56)(3)(\sqrt{4000})/60{,}000$
$= 1.0 \geq 200 b_w d/f_y = 200(18)(17.56)/60000 = 1.05 \ldots A_{s\ min}$
$= 1.05 < 3.0$ … okay
21. $a = a_w + h_f = 5.497''$, $c = a/\beta 1 = 5.497/.85 = 6.467''$,
$\varepsilon_t = .003(d - c)/c = .003(17.56 - 6.467)/6.467 = 0.00514$
$> 0.005 \ldots \phi = 0.9$.
ANSWER: Use three #9

If an irregular shape has a cross-section not easily adaptable to the previous method, make an assumption about the ratio of a/d. Then check whether the assumption is adequate. If not, go through another iteration with a higher ratio.

Example 27-8: Design reinforcement for the wedge-shaped beam shown.



27.10
Wedge-shaped beam

$M_u = 150\text{k-f} = 1800\text{k-in}, d = 15'', \phi = 0.9$

Assume $a = 0.2d$, let A_c = area in compression. Note: for larger values of M_u, a larger assumption for the ratio of a/d will be needed.

$A_c = a(18 - a/3) = 0.2d(18 - .2d/3)$
$= .2(15)(18 - .2(15/3)) = 51\text{in}^2$

$A_s f_y = .85 f'_c A_c \ldots A_s = .85(4)(51)/60 = 2.89\text{in}^2$

Use three #9, $A_s = 3.0$

$A_c = 3.0(60)/(.85(4)) = 52.94 = a(18 - a/3)$

$a = 54/2 \pm .5\sqrt{[54^2 - 4(3)(52.94)]} = 3.122''$

$c = 3.122''/0.85 = 3.672''$

$\varepsilon_t = 0.003(15 - 3.672)/3.762 = 0.009 > 0.005 \ldots$
$\phi = 0.9$

Check $\phi M_n = \phi f_y A_s (d - a/2) = 0.9(60)(3.0)(15 - 3.122/2)$
$= 2177.118\text{k-in} > 1800\text{k-in} = M_u$ … Okay.
ANSWER: Use three #9.

Practice Exercises:

27-1: Find ϕM_n for the beam shown in Figure 27.11. $f'_c = 4$ksi, $f_y = 60$ksi.

27-2: Find ϕM_n for the beam shown in Figure 27.11. $f'_c = 5$ksi, $f_y = 60$ksi.

27-3: Design the steel for a beam with: $M_u = 450$k-f, $b = 14''$, $h = 26''$, $f'_c = 3$ksi, $f_y = 60$ksi, $d' = 2.5''$.

27-4: Design the steel for a beam with: $M_u = 600$k-f, $b = 16''$, $h = 30''$, $f'_c = 4$ksi, $f_y = 60$ksi, $d' = 2.5''$.

27-5: Find the ϕM_n for the T-beam show in Figure 27.11. $f'_c = 3$ksi and $f_y = 60$ksi. The span is 20′ and the center-to-center beam spacing is 5′.

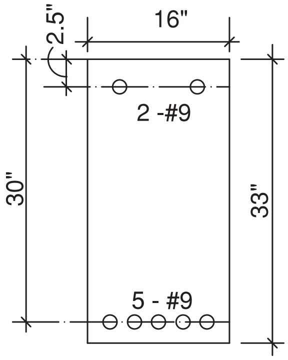
27-6: Find the ϕM_n for the T-beam show in Figure 27.11. $f'_c = 4$ksi and $f_y = 60$ksi. The span is 24′ and the center-to-center beam spacing is 8′.

27-7: Design reinforcement for a T-beam with $f'_c = 4$ksi, $f_y = 60$ksi, $b_w = 14''$, $h = 27''$, $h_f = 4''$, beam spacing = 7′, beam span = 18′ and $M_u = 250$k-f.

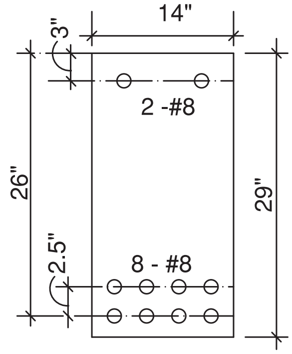
27-8: Design reinforcement for a T-beam with $f'_c = 4$ksi, $f_y = 60$ksi, $b_w = 16''$, $h = 27''$, $h_f = 3''$, beam spacing = 5′, beam span = 22′ and $M_u = 300$k-f.

27-9: Design reinforcement for the inverted T-beam shown in Figure 27.11. $f'_c = 4$ksi, $f_y = 60$ksi, beam span = 20′ and $M_u = 200$k-f.

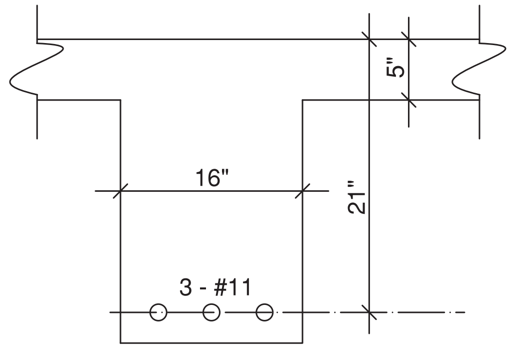
27-10: Design reinforcement for the box beam shown in Figure 27.11. $f'_c = 4$ksi, $f_y = 60$ksi, beam span = 20′ and $M_u = 300$k-f.



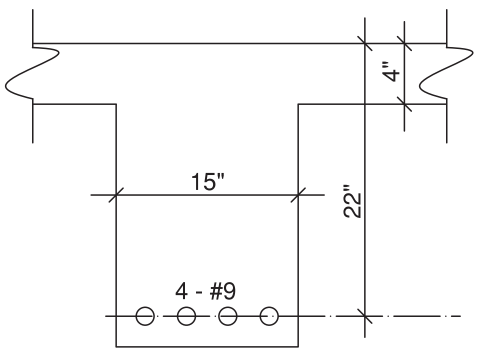
27-1



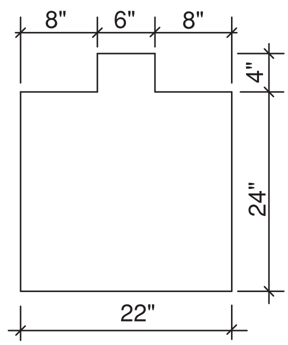
27-2



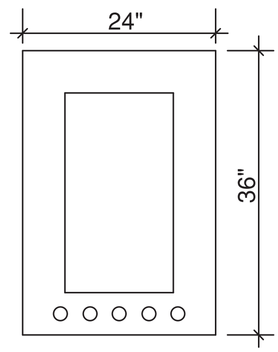
27-5



27-6



27-9



27-10

27.11

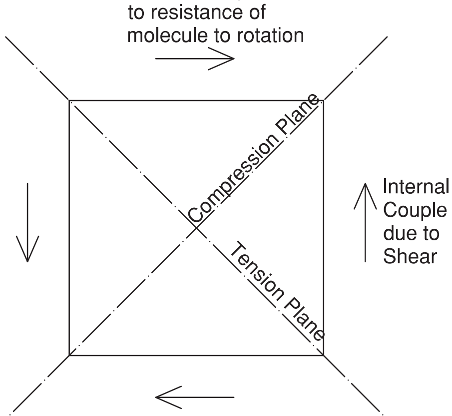
Chapter 27 Practice exercises

twenty eight

Shear and Deflection in Concrete Beams

28.1 Shear in Concrete Beams

When a beam is subjected to a vertical load, each unit element transfers the shear through a shear couple (the vertical forces shown in Figure 28.1) that must be counteracted by a counteracting couple (the horizontal forces shown) in order for the unit to remain stable. These forces cause planes of compression and planes of tension. When shear forces cause diagonal tension greater than the tensile strength of the concrete, shear cracks appear. Because shear is usually greatest at the support, shear cracks most often occur at the bottom of the beam near the edge of a support and work diagonally upwards and toward the center of the beam.



28.1

Tension and compression planes caused by shear

V_c = amount of shear force unreinforced concrete can resist

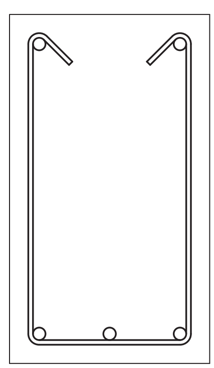
$V_c = 2\lambda(\sqrt{f'_c})b_w d$

λ = weight modification factor (1.0 for normal weight concrete)

b_w = web thickness = b for rectangular sections

Minimum shear reinforcement is required except when the following conditions exist:

1. in slabs and footings;
2. in concrete joist construction defined by ACI Code, Section 8.13;
3. in beams with a total depth less than:
   10.5″
   2.5 times the flange thickness
   one-half the width of the web.



28.2

Shear stirrups

A_v = minimum shear reinforcement = total cross sectional area of stirrup steel = $2A_s$ where:

$A_v = 0.75(\sqrt{f'_c})b_w s/f_{yt} \geq 50b_w s/f_{yt}$

A_s = cross-sectional area of stirrup steel.

s = center to center spacing of stirrups in direction parallel to longitudinal reinforcement.

f_{yt} = yield stress of stirrup steel

Note for $f'_c \leq 4444$psi $A_v = 50b_w s/f_{yt}$

28.1.1 Shear Reinforcement:

Minimum shear steel required:

If $V_u \geq \phi V_c/2$, $A_v = 0.75(\sqrt{f'_c})b_w s/f_{yt} \geq 50b_w s/f_{yt}$

When $V_u > \phi$ web reinforcement must be designed for $\phi V_s = V_u - \phi V_c$ and

$A_v = V_s s/f_{yt}d$

V_s = nominal shear provided by the shear reinforcement

Maximum spacing of stirrups

Check spacing for minimum steel requirement:

$s_{max} = A_v f_{yt}/50b_w$

Check ACI 11.5.4 maximum spacing requirement:

if $V_s \leq 4(\sqrt{f'_c})b_w d$ … $s_{max} \leq d/2 \leq 24''$
if $V_s \geq 4(\sqrt{f'_c})b_w d$ … $s_{max} \leq d/4 \leq 12''$

Design procedure for shear:

1. Calculate V_u = factored shear at distance d from support.
2. Calculate $\phi V_c = \phi 2\sqrt{f'_c}b_w d$
3. Is $V_u \geq \phi V_c/2$? If no – you're done. No shear reinforcement required.
   If yes – go to step 4.
4. Is $\phi V_c/2 \leq V_u \leq \phi V_c$? If yes, go to step 8
   If no, go to step 5
5. If $V_u > \phi V_c$, calculate $V_s = V_u/\phi - V_c$ or $(V_u - \phi V_c)/\phi$
6. Check that $V_s \leq 8\sqrt{f'_c}b_w d$ (otherwise against code)
7. Assume a stirrup size and solve for $s \leq A_v f_{yt} d/V_s$
8. Check spacing for minimum steel requirement:

   $s_{max} = A_v f_{yt}/50b_w$

9. Check ACI 11.5.4 maximum spacing requirement:

   if $V_s \leq 4\sqrt{f'_c}b_w d$ … $s_{max} \leq d/2 \leq 24''$

   if $V_s \geq 4\sqrt{f'_c}b_w d$ … $s_{max} \leq d/4 \leq 12''$

10. Check minimum spacing $s_{min} = 4''$
11. Locate where ϕV_c and $\phi V_c/2$ are located on shear diagram in terms of x.
12. Indicate what shear reinforcement is required and where.

Example 28-1: Design shear reinforcement for a 28′ beam with b = 14″, d = 27″, f'_c = 4ksi, f_y = 60ksi, and a factored uniform load of 6k/f.

1. $V_u = 28(6)/2 - 6(27/12) = 70.5$
2. $\phi V_c = \phi 2(\sqrt{f'_c})b_w d = .75(2)(\sqrt{4000})(14)(27)/1000 = 35.86$
3. Is $V_u \geq \phi V_c/2$? 70.5 > 35.86/2 = 17.93 … go to step 4.
4. Is $\phi V_c/2 \leq V_u \leq \phi V_c$? No, go to step 5.
5. If $V_u > \phi V_c$, calculate $V_s = (V_u - \phi V_c)/\phi = (70.5 - 35.86)/.75 = 46.19$
6. Check that $V_s \leq 8(\sqrt{f'_c})b_w d$: $46.19 < 8(\sqrt{4000})(14)(27)/1000 = 191.25$ … okay
7. Assume #3 stirrup, $s \leq A_v f_{yt} d/V_s = .22(60)(27)/46.19 = 7.72''$
8. Check spacing for minimum steel requirement:

   $s_{max} = A_v f_{yt}/50b_w = .22(60000)/(50(14)) = 18.86 > 7.72$ … okay

9. Check ACI 11.5.4 maximum spacing requirement:

   $46.19 < 4(\sqrt{4000})(14)(27)/1000 = 95.63$

   if $V_s \leq 4(\sqrt{f'_c})b_w d$ … $s_{max} \leq d/2 \leq 24'' = 27/2 = 13.5''$ > 7.72 … okay

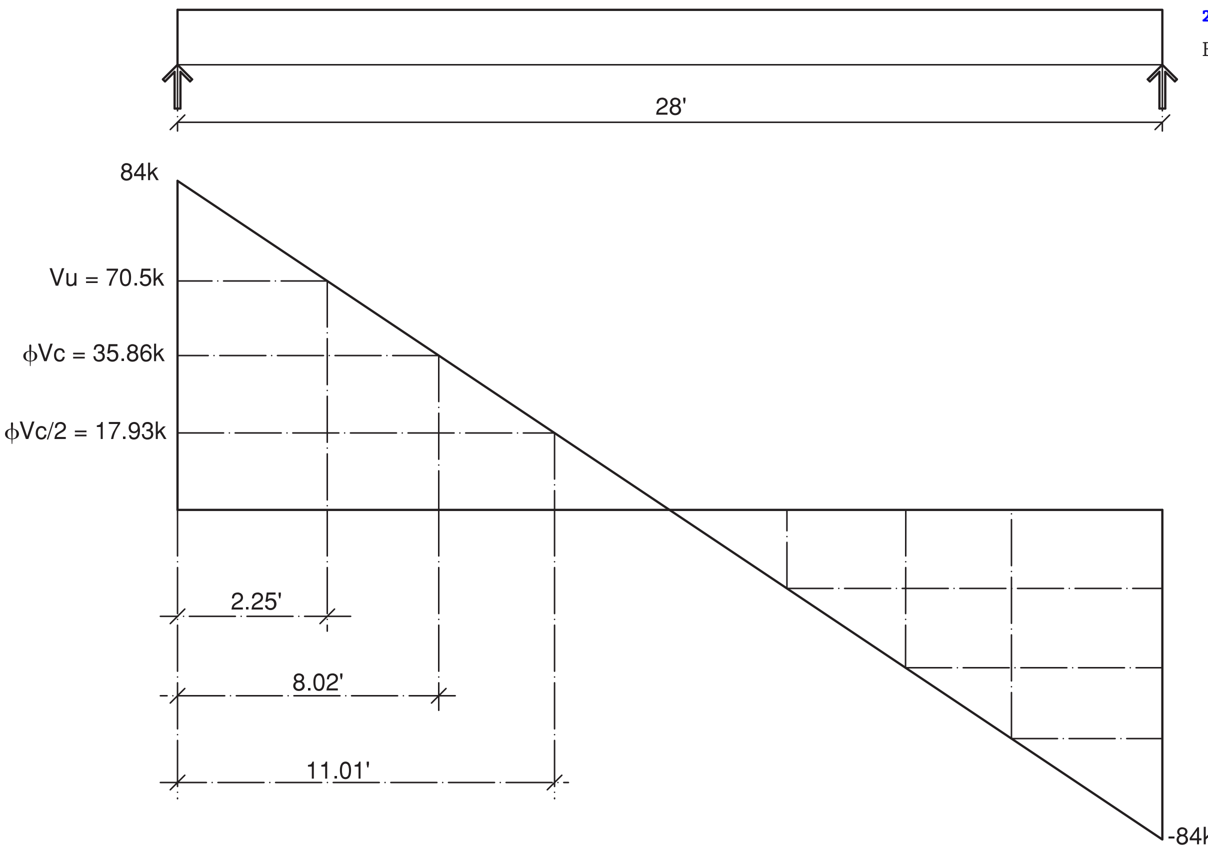
10. Check Minimum spacing $s_{min} = 4'' < 7.72$ … okay
11. Locate where ϕV_c and $\phi V_c/2$ are located on shear diagram in terms of x.

    ϕV_c is @ x where $28(6)/2 - 6x = 35.86$ … x = 8.02′

    $\phi V_c/2$ is @ x where $28(6)/2 - 6x = 17.93$ … x = 11.01′

12. Indicate what shear reinforcement is required and where.
    Use #3 stirrups @ 7″ 0 < x < 8.02 and 19.98 < x < 28
    Use #3 stirrups @ 13.5″

Example 28-2: Design shear reinforcement for a 40ft beam with b = 16″, d = 32″, f'_c = 4ksi, f_y = 60ksi, a factored uniform load of 1k/f and a concentrated load every 5′ of 24k.



28.3
Beam shear example

Draw and label shear diagram.

1. $V_u = 104 - 1(32/12) = 101.33$
2. $\phi V_c = \phi 2\sqrt{f'_c} b_w d = .75(2)(\sqrt{4000})(16)(32)/1000 = 48.57$
3. Is $V_u \geq \phi V_c/2$? $101.33 > 48.57/2 = 24.29$ … go to step 4.
4. Is $\phi V_c/2 \leq V_u \leq \phi V_c$? No, go to step 5.
5. If $V_u > \phi V_c$, calculate $Vs = (V_u - \phi V_c)/\phi = (101.33 - 48.57)/.75 = 70.34$
6. Check that $V_s \leq 8\sqrt{f'_c} b_w d$: $70.34 < 8(\sqrt{4000})(16)(32)/1000 = 259.05$ … okay
7. Assume #3 stirrup, $s \leq A_v f_{yt} d/V_s = .22(60)(32)/70.34 = 6.01''$
8. Check spacing for minimum steel requirement:

   $s_{max} = A_v f_{yt}/50b_w = .22(60000)/(50(16)) = 16.5 > 7.72$ … okay

9. Check ACI 11.5.4 maximum spacing requirement:

   $70.34 < 4 (\sqrt{4000})(16)(32)/1000 = 129.53$

   if $V_s \leq 4\sqrt{f'_c} b_w d$ … $s_{max} \leq d/2 \leq 24'' = 32/2 = 16'' > 7.72$ … okay

10. Check minimum spacing $s_{min} = 4'' < 7.72$ … okay
11. Locate where ϕV_c and $\phi V_c/2$ are located on shear diagram in terms of x.

    ϕV_c is @ $x = 10'$

    $\phi V_c/2$ is @ $x = 15'$

12. Indicate what shear reinforcement is required and where.
    Use #3 stirrups @ 7″ $0 < x < 10$ and $30 < x < 40$
    Use #3 stirrups @ 16″ $10 < x < 15$ and $25 < x < 30$

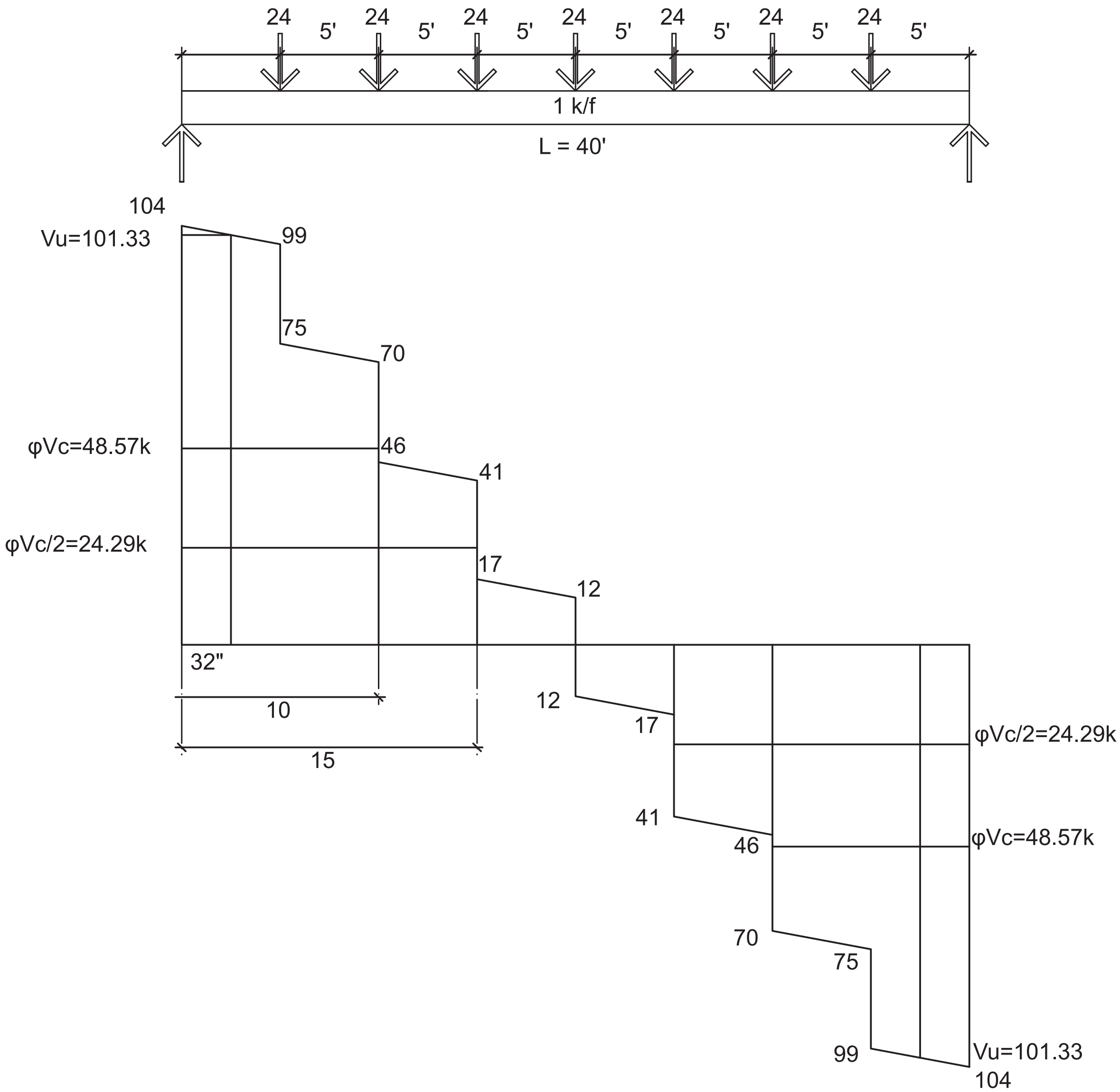
28.2 Deflection in Concrete Beams

ACI equation 9-8 states that the moment of inertia to be used in calculating deflection in concrete is:

$I_e = \{(M_{cr}/M_a)^3 I_g + [1 - (M_{cr} / M_a)^2] I_{cr} \} \leq I_g$, where:

M_a = maximum moment where deflection is being calculated

M_{cr} = cracking moment for the given cross-section



28.4
Shear diagram

Ig = gross area moment of inertia = $bh^3/12$ for rectangular beams

I_{cr} = moment of inertia of the cracked concrete section

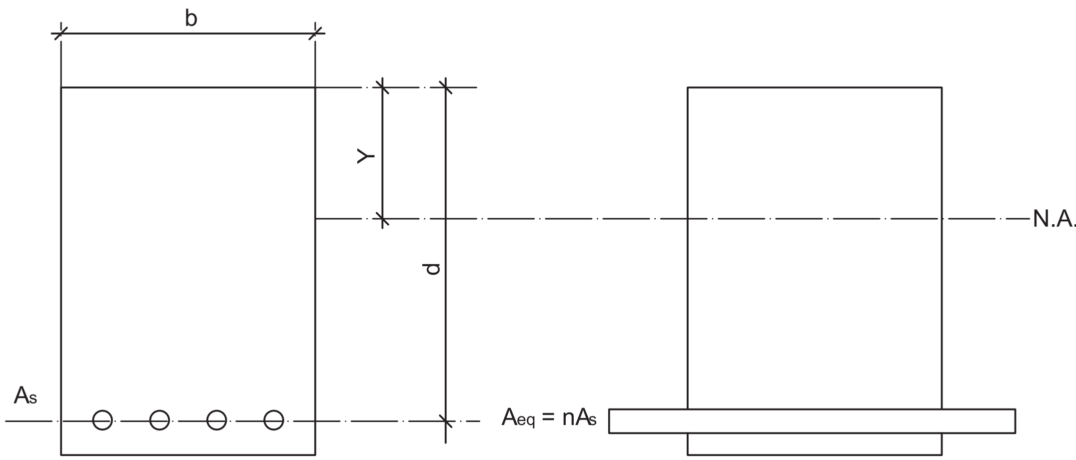
$I_{cr} = bY^3/3 + nA_s(d - Y)^2$ where:

Y = the distance from the top of the beam to the neutral axis = $A_s(\sqrt{[1 + 2bd/nA_s]} - 1)/b$

$n = E_s/E_c$ = modular ratio

The modular ratio, n, is derived by setting the strain in the steel equal to the strain in the concrete in tension. It is used to find the equivalent concrete area, A_{eq}, that may replace the steel area A_s.

Using $A_{eq} = nA_s$ allows for the neutral axis, Y, to be located and then I to be determined using the general equation of $I_x = \Sigma I_{xi} + \Sigma A d_y^2$.



28.5

Equivalent area

$$Y = \Sigma Ay/\Sigma A = [bY(Y/2) + nA_s d]/[bY + nA_s]$$

This equation can be reformulated into a quadratic equation:

$$bY^2/2 + nA_s Y - nA_s d = 0$$

$$Y = (nA_s/b)(\sqrt{(1 + 2bd/nA_s)} - 1)$$

$$I_{cr} = \Sigma I_{xi} + \Sigma A d_y{}^2$$
$$= bY^3/12 + b(nA_s/b)^3/12 + bY(Y/2)^2 + nA_s(d - Y)^2$$

I_{cr} for doubly reinforced beam:
The neutral axis can be located by using the equation:

$$bY^2/2 + nA_s{}'Y - nA_s{}'d' - nA_s d + nA_s Y = 0$$

or

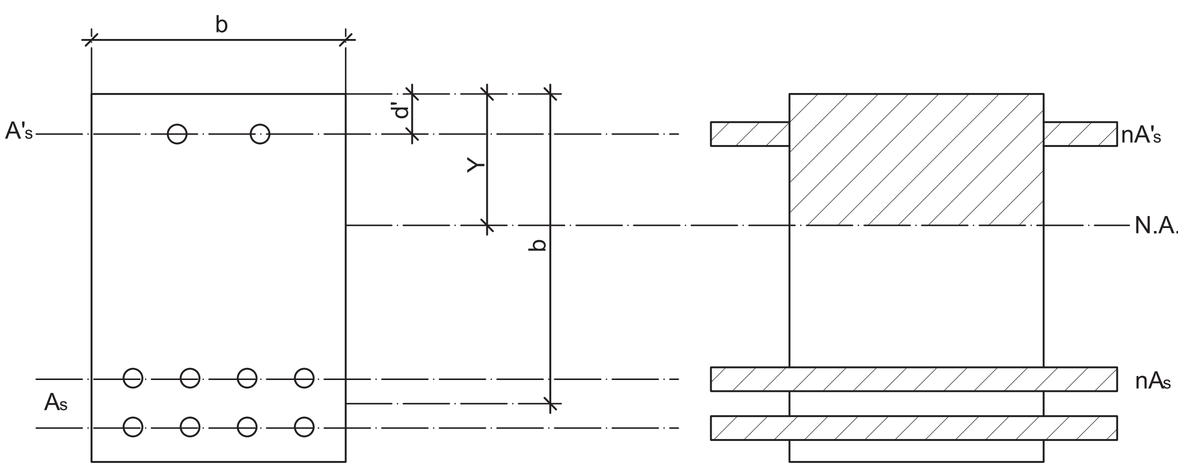
$$(b/2)Y^2 + n(As' + As)Y - n(A_s{}'d' + A_s d) = 0$$

Using the quadratic equation formula:

$$Y = -n(As' + As)/b \pm (1/b)\sqrt{[n^2(As' + As)^2 + (4bn/2)(A_s{}'d' + A_s d)]}$$

$$I_{cr} = bY^3/3 + nA_s(d - Y)^2 + nA_s{}'(Y - d')^2$$

$$I_g = bh^3/12$$



28.6

Moment of inertia for doubly reinforced beam

The moment at the point of rupture = M_{cr} = cracking moment and $f_r = M_{cr}/S$

$$M_{cr} = f_r S = f_r I_g / y_t$$

y_t = distance from the neutral axis of uncracked cross-section neglecting steel to extreme outside fiber.

$$y_t = h/2 \text{ for rectangular beams}$$

Example 28-3: Find the deflection in a 16×32 beam with five #9 rebars @ d = 29″, spanning 30′ and carrying a live load of 2000#/f if f'_c = 4ksi and f_y = 60ksi.

1. $n = E_s/E_c = 29000/[57\sqrt{4000}] = 8.04$
2. $y = nA_s[\sqrt{(1 + 2bd/nA_s)} - 1]/b$
   $= 8.04(5)[\sqrt{(1 + 2(16)(29)/8.04(5))} - 1]/16 = 9.82''$
3. $I_{cr} = by^3/3 + nA_s(d - y)^2$
   $= 16(9.82)^3/3 + 8.04(5)(29 - 9.82)^2 = 19839$
4. $I_g = 16(32)^3/12 = 43691$

   $f_r = (7.5\sqrt{4000})/1000 = 0.474\text{ksi}$

   $M_{cr} = f_r I_g/y_t = .474\text{ksi } (43690.67) /(32/2) = 1294.3 \text{ k-in}$

   $w = (150)(16/12)(32/12) + (2000) = 2533.33^{\#/f}$

   $M_a = 2.53(30)^2(12)/8 = 3415.5 \text{ k-in}$

   $M_{cr}/M_a = 1294.3/3415.5 = 0.379$

9. $I_e = \{[M_{cr}/M_a]^3 I_g + [1 - (M_{cr}/M_a)^3] I_{cr}\}$
   $= \{[.379]^3(43691) + [1 - (.379)^3](19839) = 21138\text{in}^4$
10. $\Delta_{max} = 5wL^4(1728)/384EI = 5(2.53k/f)(30)^4(1728\text{in}^3/f^3)/[384(57\sqrt{4000})(21138)] = 0.61''$

Allowable deflections: ACI Code Table 9.5b sets the criteria for allowable deflections in concrete beams as follows:

> L/180: Immediate deflection due to live load on flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections.
>
> L/240: The sum of the long term deflection due to sustained loads plus immediate deflection due to any additional live loads on roofs or floors supporting or attached to nonstructural elements not likely to be damaged by large deflections.
>
> L/360: Immediate deflection due to live load on floors not supporting or attached to nonstructural elements likely to be damaged by large deflections.
>
> L/480: The sum of the long term deflection due to sustained loads plus immediate deflection due to any additional live loads on roofs or floors supporting or attached to nonstructural elements likely to be damaged by large deflections.

Checking the beam from example 28-3:

$$L/240 = 30'(12)/240 = 1.5''$$

$$L/360 = 30'(12)/360 = 1''$$

$$L/480 = 30'(12)/480 = 0.75''$$

This beam would work in any scenario.

Example 28-4: Find the immediate deflection in a concrete girder that spans 40ft carrying a concentrated load of 60k @ x = 10′, 20′ and 30′.

The beam is 16″×36″ with eight #10 in two rows on the bottom and four #10 at 2.5″ from top. f'_c = 4ksi and f_y = 60ks. A_s' = 5.08, A_s = 10.16, d = 32.36″, allowable deflection = Δ_{all} = L/240.

1. $n = E_s/E_c = 29000/[57\sqrt{4000}] = 8.04$
2. $Y = -n(As' + As)/b \pm (1/b)\sqrt{[n^2(As' + As)^2 + (4bn/2)(A_s'd' + A_s d)]} = -8.04(5.08 + 10.16)/16 \pm (1/16)\sqrt{[8.04^2(5.08 + 10.16)^2 + (4(16)(8.04/2)(5.08(2.5) + 10.16(32.36))]}$
   $= -7.658 + 20.05 = 12.39''$
3. $I_{cr} = by^3/3 + nA_s(d - y)^2 + nA's(y - d')^2 = 16(12.39)^3/3 + 8.04(10.16)(32.36 - 12.39)^2 + 8.04(5.08)(12.39 - 2.5)^2$
   $= 46715.65\text{in}^4$
4. $I_g = 16(36)^3/12 = 62208\text{in}^4$
5. $f_r = (7.5\sqrt{4000})/1000 = 0.474\text{ksi}$
6. $M_{cr} = f_r I_g/y_t = .474\text{ksi}(62208)/(36/2) = 1638.14\text{k-in}$
7. $w_{bm} = .15(16/12)(36/12) = 0.6\text{k/f}$
8. $M_a = wL^2/8 + PL/2 = 0.6(40)^2/8 + 60(40/2) = 1320\text{k-f}$
   $= 15840\text{k-in}$
9. $M_{cr}/M_a = 1638.14/15840 = .103$
10. $I_e = \{[M_{cr} / M_a]^3 Ig + [1 - (M_{cr} / M_a)^3] I_{cr}\}$
    $= \{[.103]^3(62208) + [1 - (.103)^3](46716) = 46733$
11. $\Delta_{max} = 5wl^4/384EI + 19PL^3/384EI$
    $= 5(.6)(40)^4(1728)/[384(57\sqrt{4000})(46733)]$
    $+ 19(60)(40)^3(1728)/[384(57\sqrt{4000})(46733)] = 2.154''$
12. $\Delta_{max} = 2.154 > \Delta_{all} = L/240 = 40(12)/240 = 2''$ … not okay

28.2.1 Long-Term Deflection

$\Delta_{LT} = \Delta i \xi / (1 + 50\rho')$ is applied only to sustained loads.

Δi = immediate deflection

ξ = time-dependent factor for sustained loads:

= 2.0 for 5 years or more
= 1.4 for 1 year
= 1.2 for 6 months
= 1.0 for 3 months

ρ' = non-prestressed compression reinf. (A′s/bd)

Example 28-5: A 12″ by 18″ beam has L = 20f, w = 2.67k/f exclusive of beam weight, d = 15.56″, A_s = 4.0, A_s' = 0, f'_c = 3ksi, f_y = 60ksi.

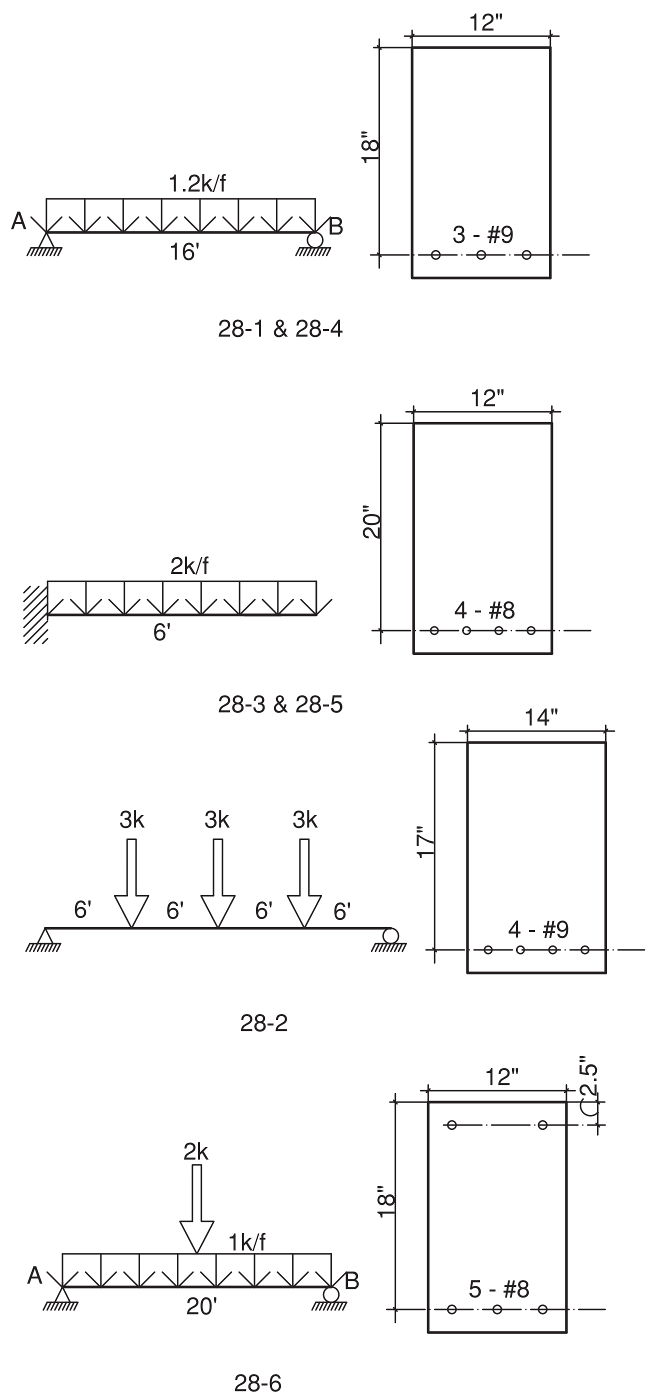
Find the immediate and 5 year deflection if only the beam weight and 1k/f are sustained loads.

1. $n = E_s/E_c = 29000/[57\sqrt{3000}] = 9.29$
2. $y = nA_s[\sqrt{(1 + 2bd/nA_s)} - 1]/b = 10.29''$
3. $I_{cr} = by^3/3 + nA_s(d - y)^2$
   $= 12(10.29)^3/3 + 9.29(4)(15.56 - 10.29)^2 = 5394\text{in}^4$
4. $I_g = 12(18)^3/12 = 5832\text{in}^4$
5. $f_r = (7.5\sqrt{3000})/1000 = 0.411\text{ksi}$
6. $M_{cr} = f_r I_g / y_t = .411\text{ksi}\ (5832)/(18/2) = 266.2$ k-in
7. $w = .15(12/12)(18/12) + 2.67 = 2.895$k/f
8. $M_a = 2.895(20^2)(12''^{/f})/8 = 1737$k-in $M_{cr}/M_a = 266.2/1737$
   $= .153$
9. $I_e = \{[M_{cr}/M_a]^3 Ig + [1 - (M_{cr}/M_a)^3] I_{cr}\}$
   $= \{.153^3(5832) + [1 - .153^3]5394\} = 5396\text{in}^4$
10. $\Delta_i = 5wL^4/384EI$
    $= 5(2.895)(20)^4(1728)/[384(57\sqrt{3000})(5396) = 0.619''$
11. $\xi = 2.0$ for 5 years or more.
12. $w_{sustained} = .15(12/12)(18/12) + 1 = 1.225$k/f
13. $\Delta_i = 0.619(1.225/2.895) = 0.262''$
14. $\rho' = 0$
15. $\Delta_{LT} = \Delta i \xi / (1 + 50\rho') = .262(2)/(1 + 0) = 0.524''$
16. Total deflection $= \Delta = 0.619 + .525 = 1.14''$

Practice Exercises:

28-1 through 28-3: Design shear reinforcement for the concrete beams shown in Figure 28.7.

28-4 through 28-6: Find the immediate and long-term deflections of the concrete beams shown in Figure 28.7.



28.7
Chapter 28 Practice exercises

twenty nine

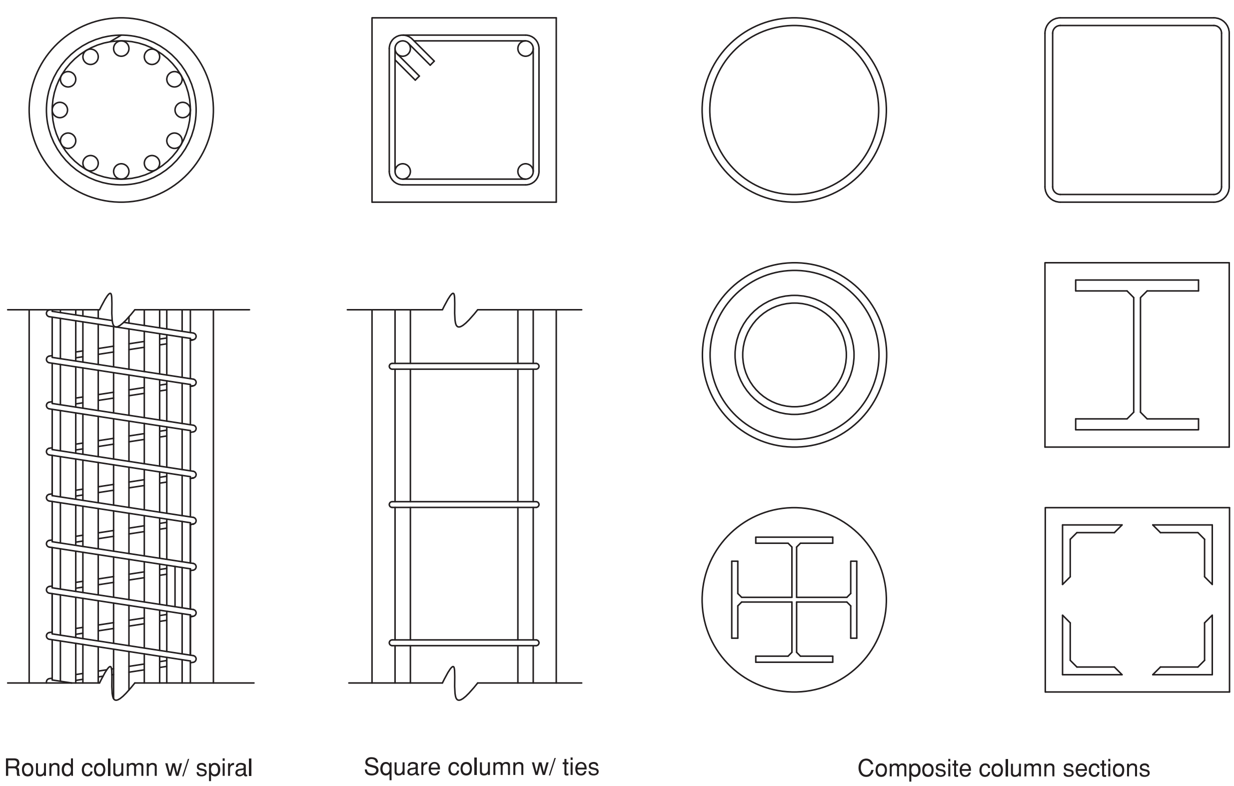
Concrete Columns

Concrete columns are either reinforced concrete columns or composite columns. Composite columns are columns made of steel sections that are either encased in concrete or filled with concrete as in Figure 29.1.

There are two types of reinforcement in concrete columns: spiral and ties. Spirals are used in columns with a circular cross-section. Ties are used in concrete columns or other shapes. This text discusses the design of columns with rectangular and round cross-sections. Note the following terms:

A_g = gross area of the column

A_{ch} = core area of the column where the core is defined by the area enclosed by and including the transverse steel.



29.1

Concrete column types

For square columns with a width h, $A_{ch} = (h - 2(cover))^2$. For round columns with a diameter h, $A_{ch} = \pi(h - 2(cover))^2/4$.

Axial loads with small eccentricities are those with a small ratio of eccentricity to column width.

Ties column: $e/h \le 0.1$

Spiral columns $e/h \le 0.05$

h = column dimension perpendicular to bending axis

29.1 Design of Short Axially Loaded Columns

A concrete column is considered to be short if its slenderness ratio meets the following requirements:

$kL_u/r < 22$ for pinned connections
$kL_u/r < 34 - 12(M_1/M_2)$ for fixed connections where M_1 = smaller end moment and M_2 = larger end moment

Is a 36″ square column pinned at both ends and with unbraced length of 20ft short?

$r = [36(36)^3/12(36)^2]^{1/2} = 10.392$

$kL_u/r = 1.0(20)(12)/10.392 = 23.095 > 22$ …

No, the column is not short.

What is the required width for a 20′ square column pinned at both ends to be short?

r = [h4/12(h)2]1/2 = 0.289h

$kL_u/r = 1.0(20)(12)/0.289h < 22$ …

$h > 240/.289(22) = 37.75''$

In generic terms, for square pinned columns to be short, $h > 1.887L_u$ and for fixed columns with equal moments at each end, $h > 1.51L_u$.

29.1.1 Design Loads for Short Concrete Columns

P_o = nominal axial load strength at e = 0

$P_o = .85f'_c(A_g - A_{st}) + f_y(A_{st})$

A_{st} = area of longitudinal steel

A_g = gross area of column.

Design axial load strength = ϕP_n
For spiral columns: $\phi = 0.75$

$\phi P_n = \phi(.85P_o) = .75(.85)[.85f'_c(A_g - A_{st}) + f_y(A_{st})]$

For tied columns: $\phi = 0.65$

$\phi P_n = \phi(.8P_o) = .65(.8)[.85f'_c(A_g - A_{st}) + f_y(A_{st})]$

Example 29-1: Find allowable axial load on an 18″×18″ tied column with a maximum unbraced length of 12′, f'_c = 4ksi, f_y′60ksi with 12 #8 longitudinal bars.

From Table A4.1, 12 #8s have an area of steel = A_{st} = 9.48in²

$A_g = 18^2 = 324\text{in}^2$

$\phi P_n = .8(.65)[.85f'_c(A_g - A_{st}) + f_y(A_{st})]$
= .8(.65)[.85(4)(324 – 9.48) + 60(9.48)] = 851.8k

29.1.2 Code Requirements for Column Details

Longitudinal Reinforcement:

$0.01 < \rho_g = A_{st} / A_g < 0.08$

Minimum 4 longitudinal bars for rectangular or circular ties

Minimum 6 longitudinal bars for spirals.

Minimum recommended size #5

Clear distance between longitudinal bars > 1.5 bar diameter (d_b) and > 1.5″
Cover > 1.5″
Ties:

minimum #3 for #10 and smaller longitudinal steel bars

minimum #4 for #11 and greater longitudinal steel bars

#5 maximum bar size.

Center-to-center distance between ties < $16d_b$ < 48 tie-bar diameters, or least column dimension where d_b = diameter of longitudinal steel bar.

Spirals:

minimum 3/8″ diameter, maximum 5/8″ diameter

1″ < clear spacing between spirals < 3″

$\rho_s = 4A_{sp}/d_{ch}s$ = volume of spiral steel in one turn/ volume of column core in height s. $d_{ch} = h - 2(\text{cover})$

$\rho_{s\ min} = 0.45((A_g/A_{ch}) - 1)(f'_c/f_y t)$ where

A_{ch} = cross-sectional area of core (out-to-out of spiral) $= \pi d_{ch}^2/4$

29.1.3 Analysis of Short Columns

The method to analyze the strength in short columns is as follows:

1. Check that $0.01 < \rho_g = A_{st}/A_g < 0.08$. If not, the column is not adequate.
2. Check that the number of longitudinal bars will fit in the core space of the column with clear spacing limits (Table A4.2) and that there is a minimum of 4 bars when using ties and 6 bars when using spirals.
3. Check that $P_u < \phi P_n$.

   $\phi P_n = .75(.85)[.85f'_c(A_g - A_{st}) + f_y(A_{st})]$ for columns with spiral reinforcement

   $\phi P_n = .65(.8)[.85f'_c(A_g - A_{st}) + f_y(A_{st})]$ for columns with ties

4. Check tie size, spacing and arrangement or check spiral size, ρs and clear distance.
5. Check clear spacing between longitudinal bars on one face < 6″. If not, additional ties are required.

Example 29-2: Check the adequacy of a short 28″×28″ tied column with a 1.5″ cover, f'_c = 4ksi, f_y = 60ksi, 16 #11 and P_u = 2000k. The column ties are #4 bars at 22″o.c.

1. Check that $0.01 < \rho_g = A_{st}/A_g < 0.08$.
   For 16 #11 from Table A4.1, $A_{st} = 25.0\text{in}^2$

   $A_g = 282 = 784\text{in}^2$

   $\rho_g = A_{st}/A_g = 25/785 = .032$

   $0.01 < \rho_g = .032 < 0.08$, therefore the column is adequate for ρ_g.

2. Core width h – 2(cover) = 28 – 2(1.5) = 25″.
   From Table A4.3, 16 #11 will fit in a core space of 25″×25″ and there are greater than 4 bars, therefore column is adequate for steel placement.
3. Check that $P_u < \phi P_n$. P_u = 2000k

   $\phi P_n = .65(.8)[.85f'_c(A_g - A_{st}) + f_y(A_{st})]$
   $= .65(.8)\ [.85\ (4)(784 - 25) + 60(25)] = 2121.9\text{k}$

   $P_u = 2000 < 2121.9 = \phi P_n$, therefore column is adequate for load.

4. Check tie size:
   Okay for minimum #4 for #11 and greater longitudinal steel bars
   Tie spacing criteria:

   $16d_b = 16(1.41) = 22.56″$

   $48d_{tie} = 48(.5) = 24″$

   least column dimension = 28″

   22″ < 22.56″ … okay for tie spacing

5. Check clear spacing between longitudinal bars on one face = (28 – 3 – 2(.5) – 5(1.41))/4 = 4.24″ < 6″, therefore column is adequate for longitudinal bar spacing.

29.1.4 Design of Short Columns

The method for design of short, axially loaded columns is as follows:

1. Decide material strengths and ρ_g .
   f'_c = 4ksi, f_y = 60ksi, ρ_g = .03 are recommended values.
2. Determine factored axial load, P_u.
3. Determine A_g:
   For rectangular columns:

   $A_g = P_u/\{.8(.65)[.85f'_c(1 - \rho_g) + f_y\rho_g]\}$

   For spiral columns:

   $A_g = P_u/\{.85(.75)[.85f'_c(1 - \rho_g) + f_y\rho_g]\}$

4. Determine column size and actual A_g.
5. Determine load on concrete:

   $\phi P_c = .65(.8)A_g[.85f'_c(1 - \rho_g)]$ (tied)

   $\phi P_c = .75(.85)A_g[.85f'_c(1 - \rho_g)]$ (spiral)

6. Determine load on steel $\phi P_s = P_u - \phi P_c$
7. Determine A_{st}:

$A_{st} = \phi P_s/.8(.65)f_y$ (tied)

$A_{st} = \phi P_s/.85(.75)f_y$ (spiral)

8. Select longitudinal bar size and number and check against Table A4.3 (maximum allowed in one layer)

9A (tied columns). Select tie size and determine spacing: $s < 48d_{tie}$ or $16d_b$ or least dimension

9B (spiral columns). Select spiral size and determine spacing:

$A_{ch} = \pi d_{ch}^2/4$

$\rho_{s\ min} = .45((A_g/A_{ch}) - 1)(f'_c/f_{yt})$ where f_{yt} is the yield strength of the spiral.

10A (tied columns). Check clear spacing between longitudinal bars on one face:
If clear distance between longitudinal bars > 6″, additional ties are required.

10B (spiral columns) Clear spacing between spirals,

$s_{max} = 4A_{sp}/d_{ch}\rho_{s\ min}$ where A_{sp} = area of spiral

and

1″ < clear s < 3″ or $1 + d_{sp} < s < 3 + d_{sp}$″

Example 29-3: Design a short square column to carry a dead load of 1000k and a live load of 500k.

1. Use f'_c = 4ksi, f_y = 60ksi, ρ_g = .03
2. P_u = 1.2(1000) + 1.6(500) = 2000k
3. $A_g = P_u/\{.65(.8)[.85f'_c(1 - \rho_g) + f_y\rho_g]\}$
   = 2000/{.65(.8)[.85(4)(.97) + 60(.03)]} = 754.44in²
4. √754.44 = 27.47″ round up to next whole inch. Use 28×28″ column A_g = 784in²
5. $\phi P_c = .65(.8)A_g[.85f'_c(1 - \rho_g)]$ = .65(.8)(784)(.85)(4)(.97) = 1344.53k
6. Determine load on steel $\phi P_s = P_u - \phi P_c$ = 2000 − 1344.53 = 655.47k
7. $A_{st} = \phi P_s/.8(.65)\ f_y$ = 655.47/.8(.65)(60) = 21in²
8. From Table A4.1, the area of 16 #11 = 25.0 > 21in² and this is a multiple of 4 (required for even distribution in a square column). From Table A4.3, for a core width d_{ch} = 28 − 1.5(2) = 25″, 16 #11 will fit.
9. From A4.3, choose recommended tie size: Use #5 tie

   s < 48(.625) = 30 or 16(1.41) = 22.56 or 28″ …
   s = 22.5″

10. Check clear spacing between longitudinal bars on one face = (h − 2(cover) − $2d_{tie}$ − (#bars/4 + 1)d_b)/(#bars/4) = (28 − 3 − 2(.625) − 5(1.41))/4 = 4.175″ < 6″ therefore no additional ties are required.

Example 29-4: Design a round column for 1000k DL and 500k LL.

1. Use f'_c = 4ksi, f_y = 60ksi, ρ_g = .03
2. P_u = 1.2(1000) + 1.6(500) = 2000k
3. $A_g = P_u/\{.75(.85)[.85f'_c(1 - \rho_g) + f_y\rho_g]\}$
   = 2000/{.75(.85)[.85(4)(.97) + 60(.03)]} = 615.39 = $\pi h^2/4$
4. h = √[615.39(4)/π] = 27.99. Round up to next whole number and use 28″ dia. column $A_g = \pi 28^2/4$ = 615.75in²
5. $\phi P_c = .75(.85)A_g[.85f'_c(1 - \rho_g)]$
   = .75(.85)(615.75)(.85)(4)(.97) = 1294.6k
6. $\phi P_s = P_u - \phi P_c$ = 2000 − 1294.6 = 705.4k
7. $A_{st} = \phi P_s/.85(.75)\ f_y$ = 705.4/.85(.75)(60) = 18.44in²
8. From Table A4.1, choose 12 #11 = 19.7in² From Table A4.3, for a core diameter of 28 − 1.5(2) = 25″, 12 #11 will fit.
9. Using a $\frac{5}{8}$″ diameter spiral, $A_{ch} = \pi d_{ch}2/4$ = 452.4in²

   $\rho_{s\ min} = .45((A_g/A_{ch}) - 1)(f'_c/f_{yt})$
   = .45((615.75/452.4) − 1)(4/60) = 0.0108

   $s_{max} = 4A_{sp}/d_{ch}\rho s$ = 4(.31)/25(.0108) = 4.59″

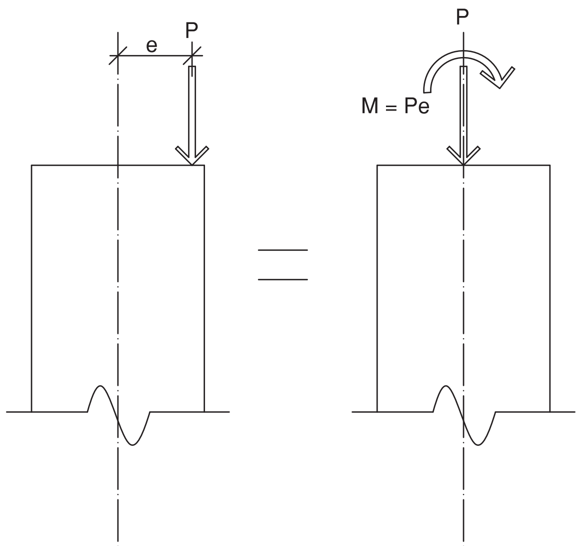
   $1 + d_{sp} < s < 3 + d_{sp}$″ … s < 3 + 0.625 = 3.625″

Use s = 3.5″

29.2 Columns with Large Eccentric loads

When the eccentricity of a load is larger than e = .1h in rectangular columns and e = .05h in round columns, ϕP_n must be reduced.

For axial loads with small eccentricity, all the steel is in compression.

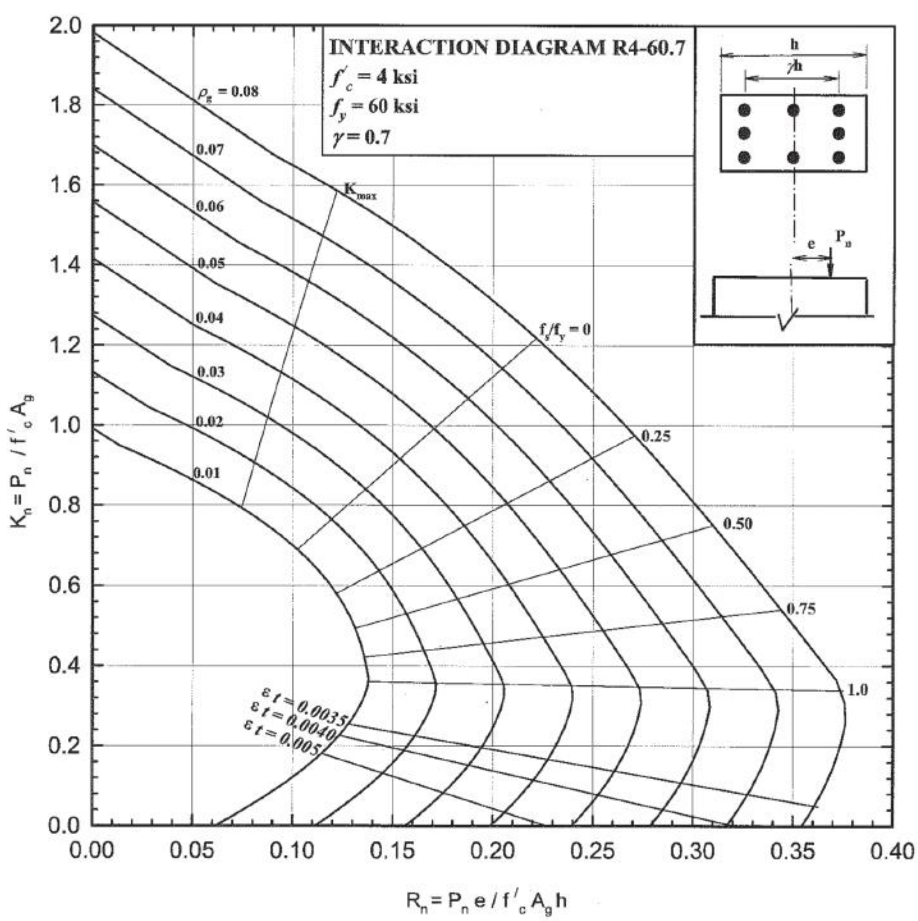


29.2

Column with eccentric load

But as axial loads in columns with large eccentricity increase, the steel on the side away from the load decreases in compression and eventually goes into tension. Because of the large eccentricity, strain values change and corresponding strength reduction factors, ϕ change. For ease of analysis, the ACI Design Handbook SP17(11) Volume 1 provides a series of interaction diagrams for the analysis and design of columns with large eccentricities. Sample interaction diagrams are supplied in Appendix A4.4.

Each diagram is created based on a column type and longitudinal bar configuration as shown in the top-right corner and the material values f'_c and f_y. Each diagram is also based on a value of γ equal to the ratio of the center-to-center distance between bars to the column width, h, in the direction of bending.



29.3

Typical interaction diagram. Reproduced with permission from the American Concrete Institute

The horizontal axis of the interaction diagram measures the value of $R_n = P_n e/f'_c A_g h$ where $P_n e = M_n$. The vertical axis measures the value of $K_n = P_n/f'_c A_g$. ρ_g and is indicated by the curved lines and the strain, ε is indicated by the diagonal lines that radiate through the ρ_g curves.

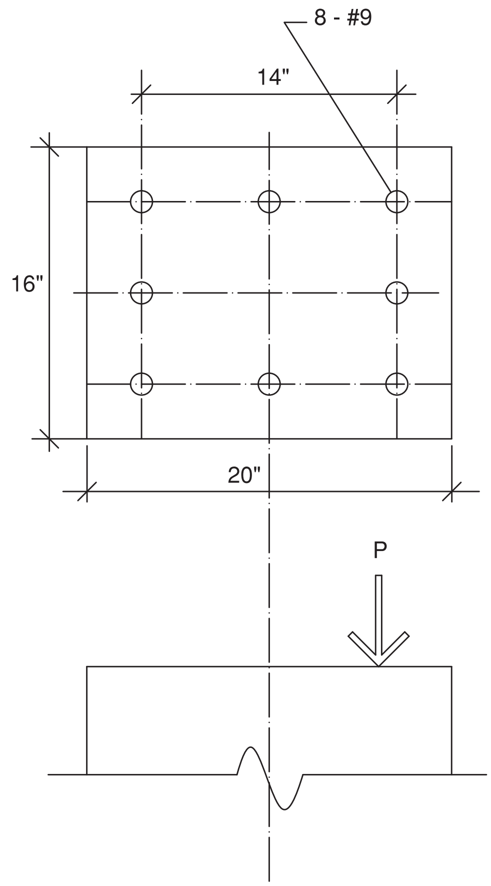
29.2.1 Analysis of columns with large eccentricity

The method to analyze a column with large eccentricity is as follows:

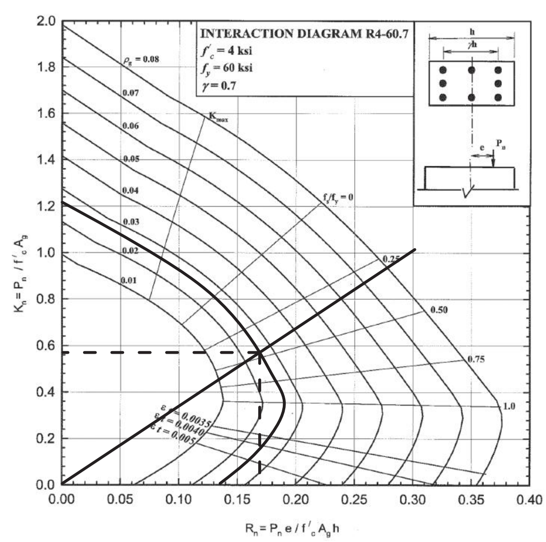
1. Choose the correct interaction diagram based on f'_c, f_y, γ, column shape and bar configuration from section A4.4.
2. Calculate $\rho_g = A_{st}/A_g$
3. Locate ρ_g on diagram chosen in step 1.
4. Calculate slope of the h/e. Draw line originating at bottom left (0,0) and following the slope = h/e.
5. Find the intersection of the ρ_g curve and the h/e line from steps 3 and 4. Draw a horizontal line through the intersection to locate K_n, and a vertical line through the intersection to locate R_n.
6. Determine ϕ by checking strain.
   a) If the point of intersection is above 1.0 line for f_s/f_y, then the column steel is in compression and $\phi = 0.65$ for tied columns and 0.75 for spiral columns.
   b) If the point of intersection falls below the $\varepsilon_t = 0.0050$ line, then the column is in tension and $\phi = 0.9$.
   c) If the point of intersection falls between the lines from cases a and b, then the column is in transition. $\phi = 0.65 + (\varepsilon_t - .002)(250/3)$ for tied columns and $\phi = 0.75 + (\varepsilon_t - .002)(250/3)$ for spiral columns.
7. $\phi P_n = \phi K_n f'_c A_g$ and $\phi M_n = \phi R_n f'_c A_g h = \phi P_n e$

Example 29-5: Find the practical nominal moment for the column shown in Figure 29.4. Eight #9 bars, f'_c = 4ksi and f_y = 60ksi, e = 6″.

1. c.c. bars = 14″, h = 20″ … γ = 14/20 = 0.7 … use Diagram A4.4.3
2. $\rho_g = A_{st}/A_g$ = 8/16(20) = 0.025
3. Locate ρ_g = 0.025 on Diagram A4.4.1. Drawn as heavy curve on Figure 29.5.
4. slope = h/e = 20/6 = 3.33. Drawn from origin as heavy line on Figure 29.5.



29.4
Example 29-5



29.5
Finding K_n and R_n. Interaction diagram reproduced with permission from the American Concrete Institute

5. Find the intersection of the ρ_g curve and the h/e line from steps 3 and 4. Draw a horizontal line through the intersection to locate K_n = .56, and a vertical line through the intersection to locate R_n = .17.
6. Determine ϕ by checking strain. The point of intersection is above 1.0 line for fs/fy, therefore the column steel is in compression and ϕ = 0.65.
7. $\phi P_n = \phi K_n f'_c A_g$ = .65(.56)(4)(20)(16) = 466k

   $\phi M_n = \phi R_n f'_c A_g h$ = .65(.17)(4)(320)(20)/12in/f = 236k-f

   Or $\phi M_n = \phi P_n e$ = 466(6)/12 = 233k-f

The difference between the two values of ϕM_n is due to the accuracy of estimating R_n and K_n from the interaction diagram. A more accurate reading of the chart gives R_n = .167 and K_n = .555 yielding…

Kn = .555 … $\phi P_n = \phi K_n f'_c A_g$ = .65(.558)(4)(20)(16) = 464

Rn = .167 … $\phi M_n = \phi R_n f'_c A_g h$ = .65(.167)(4)(320)(20)/12in/f = 232k-f

Or $\phi M_n = \phi P_n e$ = 464(6)/12 = 233k-f

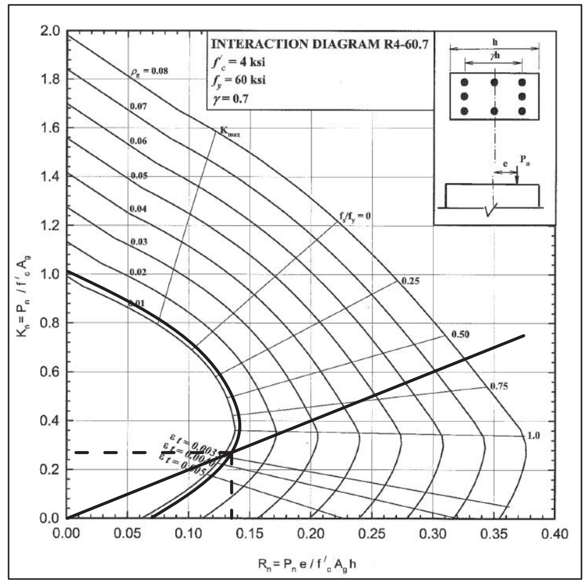
Example 29-6: Find the practical nominal moment for the column shown in Figure 29.4. if eight #6 bars, f'_c = 4ksi and f_y = 60ksi, e = 10″.

Assume there is a large applied moment or eccentricity such that slope = h/e = 1.0

1. c.c. bars = 14″, h = 20″ … γ = 14/20 = 0.7 … use Diagram A4.4.3
2. $\rho_g = A_{st} / A_g$ = 3.52/16(20) = 0.011
3. Locate ρ_g = 0.011 on Diagram A4.4.3. Drawn as heavy curve on Figure 27.6.
4. slope = h/e = 20/10 = 2. Drawn from origin as heavy line on Figure 27.6.
5. Find the intersection of the ρ_g curve and the h/e line from steps 3 and 4. Draw a horizontal line through the intersection to locate K_n = .26, and a vertical line through the intersection to locate R_n = .13
6. Determine ϕ by checking strain. The strain, ε = .0035, therefore ϕ = 0.65 + (.0035 − .002)(250/3) = 0.775
7. $\phi P_n = \phi K_n f'_c A_g$ = .775(.26)(4)(20)(16) = 257.92k

   $\phi M_n = \phi R_n f'_c A_g h$ = .775(.13)(4)(320)(20)/12in/f = 214.93k-f

   Or $\phi M_n = \phi P_n e$ = 257.92(10)/12 = 214.63k-f



29.6

Example 29-6. Interaction Diagram reproduced with permission from the American Concrete Institute.

Example 29-7: Find the practical nominal moment for a 24″ diameter column with 1.5″ cover, #3 spiral, 14 #9 evenly spaced bars, e = 8″, f'_c = 4ksi, f_y = 60ksi, e = 8″.

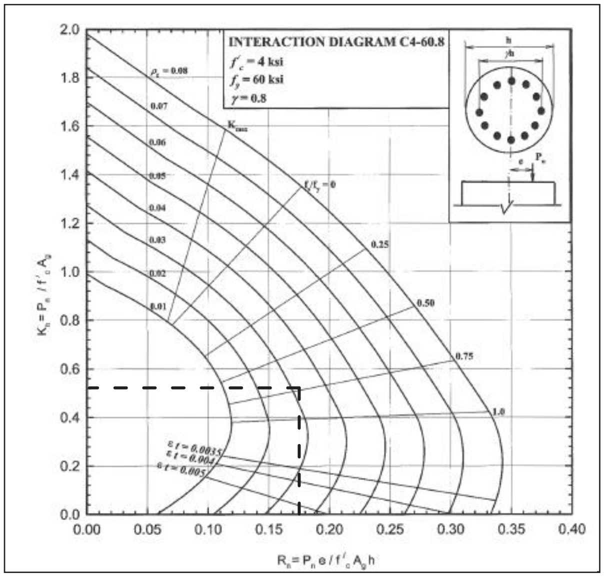
1. Center-to-center distance longitudinal bars = 24 − 2(1.5) − 2(.375) − 1.128 = 19.122

   γ = 19.122/24 = .797 … use Fig. A4.4.6
2. $A_g = \pi(24)^2/4$ = 452.39

   $\rho_g = A_{st} / A_g$ = 14/452.39 = 0.03
3. Locate ρ_g on diagram chosen in step 1.
4. Slope = h/e = 24/8 = 3. Draw line originating at bottom left (0,0) and following the slope = 3.
5. K_n = .49, R_n = .195
6. The point of intersection is just above 1.0 line for f_s/f_y, therefore the column steel is in compression and ϕ = 0.75 for spiral columns.
7. $\phi P_n = \phi K_n f'_c A_g$ = .75(.51)(4)(144π) = 692k

   $\phi M_n = \phi R_n f'_c A_g h$ = .75(.17)(4)(144π)(24)/12in/f = 461k-f

   Or $\phi M_n = \phi P_n e$ = 692(8)/12 = 461k-f



29.7

Example 29-7. Interaction diagram reproduced with permission from the American Concrete Institute

29.2.2 Design of Columns with Large Eccentricity

The method to design a column with large eccentricity is as follows:

1. Determine the factored load, P_u.
2. Estimate the column size based on $\rho_g = 0.01$ and ignoring the eccentricity:

   $A_g = P_u/[.85(.75)(.85f'_c(.99) + .01f_y)]$ for spiral columns

   $A_g = P_u/[.8(.65)(.85f'_c(.99) + .01f_y)]$ for tied columns

3. Choose a trial size and calculate A_g.
4. Assume a bar size and tie or spiral size.
5. Choose the correct interaction diagram based on f'_c, f_y, γ, column shape and bar configuration from section A.4.4.
6. Required $K_n = P_u/[\phi f'_c A_g]$ and required $R_n = Mu/[\phi f_c' A_g h]$. Locate the point of intersection on the diagram.
7. Determine ρ_g and ϕ at the point of intersection.
8. $A_s = \rho_g A_g$
9. Select bars.

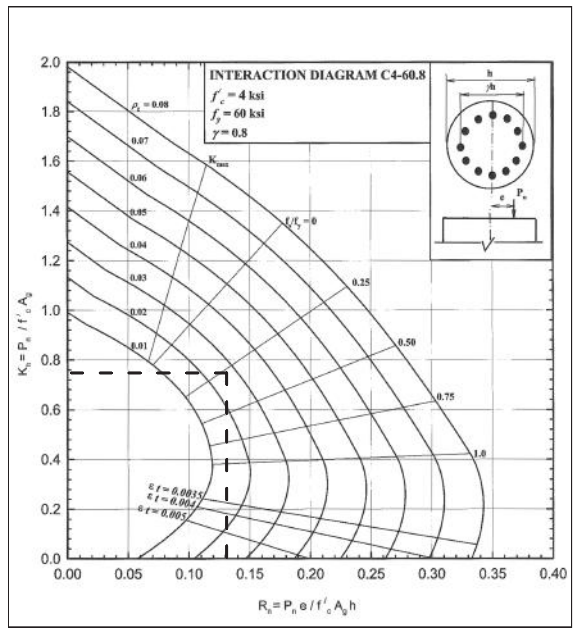
10A. For tied columns, design ties.

10B. For spiral columns, design spirals.

Example 29-8: Design a circular column with spiral reinforcement, P_u = 1100kips, e = 4″, f'_c = 4ksi and f_y = 60ksi.

1. $P_u = 1100k$, $M_u = P_u e = 1100k(4'') = 4400k\text{-in}$
2. $A_g = P_u/[.85(.75)(.85f'_c(.99) + .01f_y)]$
   $= 1100/[.85(.75)(.85(4)(.99) + .01(60))] = 435.07in^2$
3. $h = \sqrt{(435.07(4)/\pi)} = 23.54''$ use $h = 24''$, $A_g = \pi(24^2)/4$
   $= 452.39in^2$
4. Assume #9 size and a 3/8″ spiral size.
5. c.c. long. Bars = 24 – 3 – 2(.375) – 1.128 = 19.122″

   $\gamma = 19.122/24 = .79675$, use Table A4.4.6



29.8

Example 29-8. Interaction diagram reproduced with permission from the American Concrete Institute

6. Required $K_n = P_u/[\phi f'_c A_g] = 1100/.75(4)(452.39) = .811$
   Required $R_n = M_u/[\phi f_c' A_g h] = 4400/[.75(4)(452.39)(24)]$
   $= 0.135$

   Locate the point of intersection on the diagram.
7. $\rho g = .032$ and $\phi = 0.75$ (above $f_s/f_y = 1.0$ line)
8. $A_s = \rho_g A_g = .032(452.59) = 14.48in^2$
9. From Table A4.2, 15 #9 would work, and from Table A4.3, the maximum number of #9s that can be placed in a single row within a core diameter of (24 – 3) 21″ is 15; therefore okay

10B. For spiral columns, design spirals.

$d_{ch} = 24 - 3 = 21''$ & $A_{ch} = \pi(21^2)/4 = 346.36in^2$

ρ_s required $= .45(A_g/A_{ch} - 1)(f_c'/f_y)$
$= .45(452/346.36 - 1)(4/60) = .0091$

$s = 4A_{sp}/d_{ch}\rho_s = 4(.11)/21(.0091) = 2.3''$

USE: spiral spacing @ 2.25″
Clear spacing = 2.25 – .375 = 1.875 > 1″ and < 3″
… okay

ANSWER: 24″ diameter column with 14 #9 and 3/8″ spiral at 2.25″o.c.

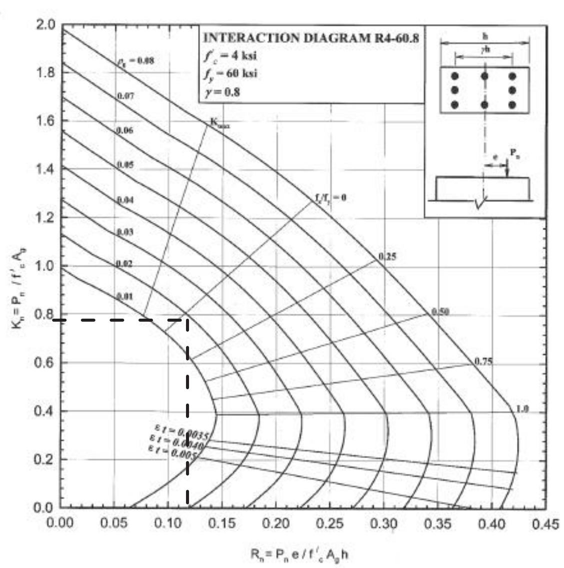
Example 29-9: Design a square column with ties P_u = 1100kips, e = 4″, f'_c = 4ksi and f_y = 60ksi.

1. $P_u = 1100k$, $M_u = P_u e = 1100k(4'') = 4400k\text{-}in$
2. Estimate the column size based on $\rho_g = 0.01$ and ignoring the eccentricity:

   $A_g = P_u/[.8(.65)(.85f'_c(.99) + .01f_y)] = 1100/[.8(.65)(.85(4)(.99) + .01(60))] = 533.38in^2$

3. $h = \sqrt{533.38} = 23.1$ use 24″ $A_g = 24^2 = 576in^2$.
4. Assume #9 size and #4 tie.
5. c.c long. Bars = 24 – 3 – 2(.375) – 1.128 = 19.122″

   $\gamma = 19.122/24 = .79675$, use Diagram A4.4.2



29.9

Example 29-9. Interaction diagram reproduced with permission from the American Concrete Institute

6. Required $K_n = P_u/[\phi f_c' A_g] = 1100/.65(4)(576) = .735$
   Required $R_n = M_u/\phi f_c' A_g h = 4400/[.65(4)(576)(24)] = 0.122$
   Locate the point of intersection on the diagram.
7. g = .018 and $\phi = 0.65$
8. $A_s = \rho_g A_g = .018(576) = 10.37in^2$
9. Because the column is square, the number of bars must be a multiple of 4. Therefore, use 12 #9, $A_s = 12.0$. Checking with Table A4.3 shows 16 #9 are allowed in a 24″ square column.

10A. For tied columns, design ties.

Design ties:

$d_{ch} = 24 - 3 = 21''$ and $A_{ch} = 441$

s = smallest of 16(1.128) = 18.05″ or 48(.5) = 24″ or 16″ Use #4 ties @16″

Check clear spacing of longitudinal bars:

$(h - 2(\text{cover}) - 2d_{tie} - (\#bars/4 + 1)d_b)/(\#bars/4)$
$= (24 - 3 - 2(.5) - 4(1.128))/3 = 5.16'' < 6''$, therefore no additional ties are required.

Practice Exercises:

29-1: Find allowable axial load on a 12 × 12″ tied column with a maximum unbraced length of 14′, f'_c = 4ksi, f_y = 60ksi with eight #8 longitudinal bars.

29-2: Check the adequacy of a short 22″ × 22″ tied column with a 1.5″ cover, f'_c = 4ksi, f_y = 60ksi, 16 #9 and P_u = 1200k. The column ties are #4 bars at 22″o.c.

29-3: Design a short square column to carry a dead load of 500k and a live load of 800k.

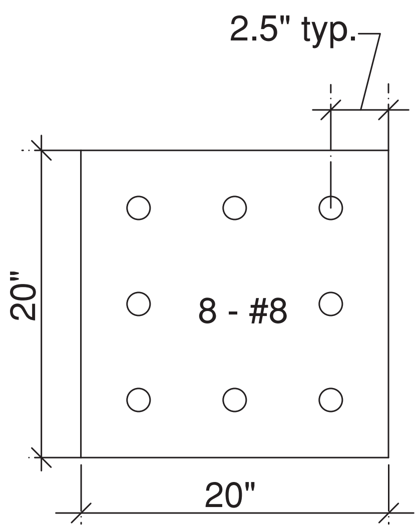
29-4: Design a short round column with spiral reinforcement to carry a dead load of 500k and a live load of 800k.

29-5: Find the practical nominal moment for the column shown below. Eight #8 bars, f'_c = 4ksi and f_y = 60ksi, e = 3″.

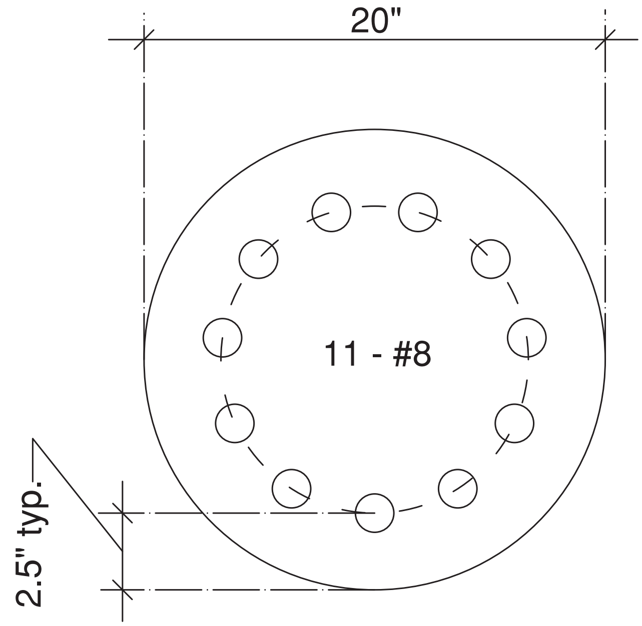
29-6: Find the practical nominal moment for the column shown in Figure 29.10. Eleven #8 bars, f'_c = 4ksi and f_y = 60ksi, e = 6″.

29-7: Design a round column with spiral reinforcement. P_u = 800kips, e = 6″, f'_c = 4ksi and f_y = 60ksi.

29-8: Design a square column with ties P_u = 1500kips, e = 10″, f'_c = 4ksi and f_y = 60ksi.



29-5



29-6

29.10

Chapter 29 Practice exercises

thirty

Development Length

Reinforced concrete design is based on the assumption that there is an adequate bond between the concrete and the reinforcement bars to prevent slippage and ensure the two elements act together as one system. Development length is the length of reinforcement bar required to ensure that the bond between the concrete and steel is perfect and without slippage.

Factors affecting development length include:

Ψ_t = reinforcement location factor as dictated by ACI 12.2.4:

Ψ_t = 1.3 if there is more than 12″ of concrete below the bar

Ψ_t = 1.0 all other cases

Ψ_e = coating factor

Ψ_e = 1.0 uncoated & galvanized

Ψ_e = 1.5 epoxy coated with cover < 3db or clear spacing 6db between bars

Ψ_e = 1.2 all other conditions

Ψs = size factor

Ψ_s = .8 for #6 and smaller

Ψ_s = 1.0 for #7 and larger

c_b = spacing factor is the smaller of:

c_b = the distance from the center of the bar to the nearest edge

or

c_b = (center to center distance of bar spacing)/2

K_{tr} = transverse reinforcement index = $40A_{tr}/sn$ where

A_{tr} = the total cross-sectional area of the transverse reinforcement

s = the spacing of the transverse bars in the area where the development length is being calculated.

n = the number of transverse bars in the area where the development length is being calculated.

The ACI code allows K_{tr} = 0. This is conservative and simplifies the equations and therefore, K_{tr} = 0 will be used in this text.

d_b = diameter of the bar

λ = modification factor based on concrete weight:

λ = 1.0 for normal weight concrete

λ = 0.85 for san lightweight concrete

λ = 0.75 for all other lightweight concrete

$K_{ER} = A_{s\ required}/A_{s\ used}$: This reduction factor is not included in the ACI equation for development length, but may be used as a factor with the development length found from ACI Equation 12-1.

30.1 Development Length in Tension Bars

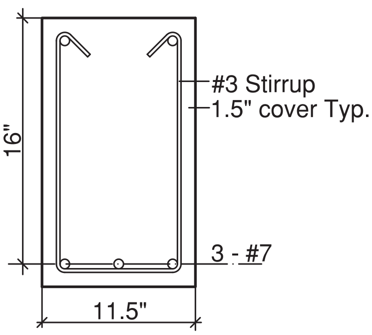
$L_d = K_{ER}K_D[\rho_t\rho_e\rho_s/(c_b + K_{tr})/d_b](d_b)$ where

$K_D = (3/40)(f_y/\lambda\sqrt{f'c})$ and

$(c_b + K_{tr})/d_b < 2.5$

The method for calculating L_d in tension bars is as follows:

1. Calculate $K_D = (3/40)(f_y/\lambda\sqrt{f'_c})$
2. Determine Ψ_t, Ψ_e, Ψ_s, c_b
3. Assume $K_{tr} = 0$, find Calculate c_b/d_b
4. Calculate $K_{ER} = A_{s\ required}/A_{s\ used}$
5. $L_d = K_{ER}K_D[\rho_t\rho_e\rho_s/(c_b/d_b)](d_b)$



30.1
Example 30-1

Example 30-1: Find L_d for the uncoated #7s in Figure 30.1, for f'_c = 4ksi, f_y = 60ksi and required A_s = 1.77in².

1. Calculate $K_D = (3/40)(f_y/\lambda\sqrt{f'_c}) = (3/40)(60000/(1(\sqrt{4000}))$ $= 71.15$
2. Determine Ψ_t, Ψ_e, Ψ_s, c_b.

   $\Psi_t = 1.0$ (less than 12″ below reinforcing steel)

   $\Psi_e = 1.0$ (uncoated & galvanized)

   $\Psi_s = 1.0$ (#7 and larger)

   c_b:

   center to edge = .875/2 + .375 + 1.5 = 2.31

   $\frac{1}{2}$ center to center = .5(11.5 – 2(1.5 + .375 + .875/2))/2 = 1.72

   $c_b = 1.72$
3. Assume $K_{tr} = 0$, calculate $c_b/d_b = 1.72/0.875 = 1.96$ and 2.5 … use 1.96

   If c_b/d_b was found to be greater than 2.5, then 2.5 would be used.
4. Calculate $K_{ER} = A_{s\ required}/A_{s\ used} = 1.77/1.8 = 0.983$
5. $L_d = K_{ER}K_D[\rho_t\rho_e\rho_s/(c_b/d_b)](d_b) = 0.983(71.15)[1/1.96](.875) = 31.23''$

30.2 Development Length in Tension Bars with Hooks

If development length cannot be reached, use a mechanical fastener such as a hook. Cover for hooks varies with the degree of bending and the size of the hook. Some standard ACI hooks are shown in Figure 30.2. D = $6d_b$ for #3 to #8 bars and D = $8d_b$ for #9 to #11 bar for hooks in primary reinforcement. For ties and stirrups with hooks, D ≥ $4d_b$ for #3 to #5 bars and $6d_b$ for #6 to #8 bars.
Use the ACI Code, Section 12.5 formula for bars in tension with a standard hook:

$L_{dh} = .02\rho_e f_y d_b/\lambda\sqrt{f'_c}$ where

$\Psi_e = 1.2$ for epoxy coating, 1.0 otherwise

$\lambda = 0.75$ for lightweight concrete, 1.0 otherwise

$L_d = L_{dh}(C_{cover})(C_{encl})$
C_{cover} = Cover factor = 0.7

If using # 11 and smaller bars with a side cover ≥ 2.5″

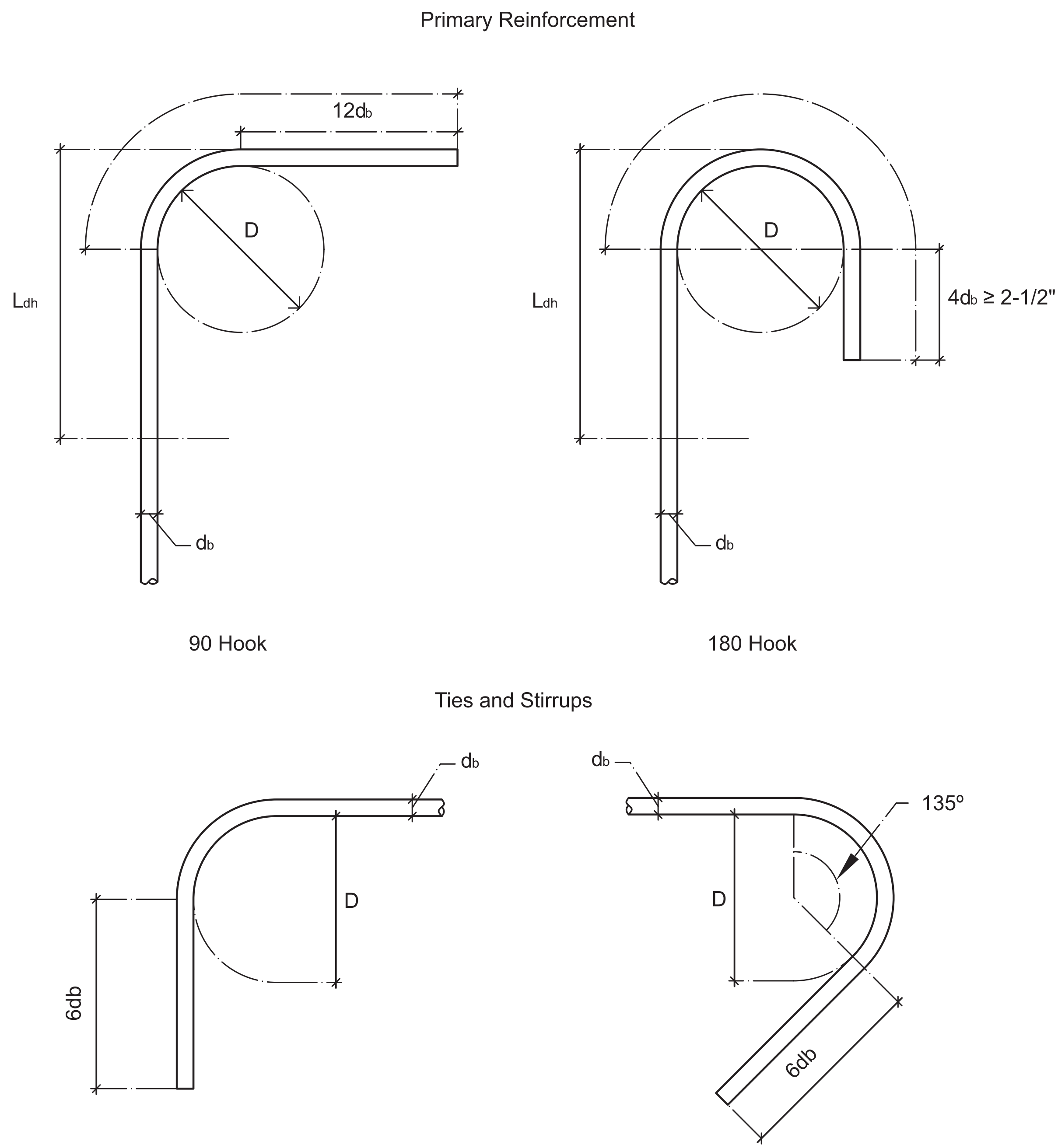
For 90° hooks: extension cover ≥ 2.0″

C_{encl} = Enclosure factor = 0.8

90° hooks with # 11 and smaller bars within perpendicular ties or stirrups spaced ≤ 3_{db} along L_{dh} or within parallel ties or stirrups spaced ≤ 3_{db} along bend and tail extension.

180° hooks with # 11 and smaller bars within perpendicular ties or stirrups spaced ≤ 3db along Ldh.

Example 30-2: Find L_d for a 90° hook of uncoated #7 in Figure 30.1, for f'_c = 4ksi, f_y = 60ksi, side cover = end cover = 1.5″ and required A_s = 1.77in².



30.2

Standard hook shapes

$\rho_e = 1$

$L_{dh} = .02\rho_e f_y d_b/\lambda\sqrt{f'_c} = .02(1)(60{,}000)(.875)/\sqrt{4000}$
$= 16.6''$

$C_{cover} = 1$, $C_{encl} = 0.8$ (#7 bars)

$L_d = L_{dh}(C_{cover})(C_{encl}) = 16.6(1)(0.8) = 13.28''$

30.3 Tension Splices

Class A: lap length = $1.0L_d$ if the area of reinforcement is twice that for the length of splice and not more than 50% of total reinforcement is spliced within the required lap length.

Class B: lap length = 1.3Ld (everything not class A)
Minimum Lap Length = 12″

Splices in tension tie members must be full welds or full mechanical splice, staggered at least 30″, ACI recommends all members have staggered splices.

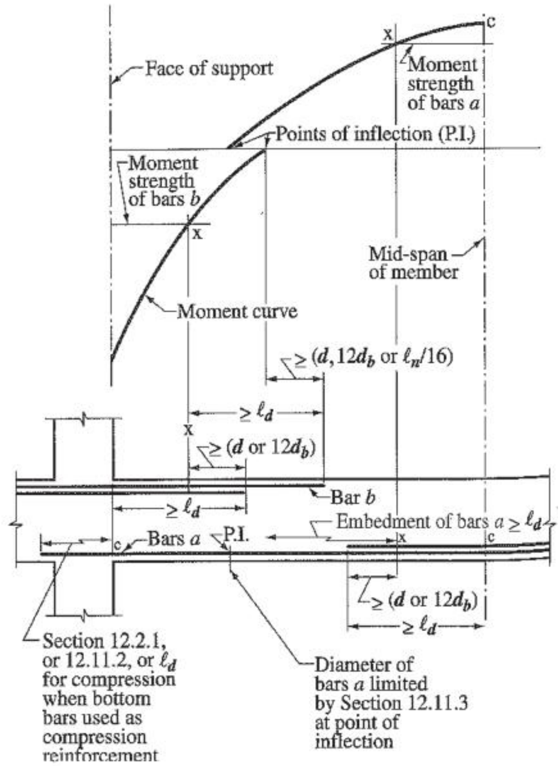
30.4 Development Length in Compression Bars

$$8'' > L_{dc} = K_{ER}(.02 d_b f_y / \lambda\sqrt{f'_c}) \geq .0003 f_y d_b$$

L_{dc} may be reduced by a factor of .75 if enclosed by a spiral not less than $\frac{1}{4}$″ in diameter, not more than 4″ pitch or if enclosed by #4 ties spaced not more than 4″

30.5 Bar Cut-offs

Beam reinforcement is based on on the design moment, M_u. The maximum moment in a beam usually occurs around the midspan. Near the supports, the moment is reduced and fewer reinforcing bars may be used. The stopping point for reinforcing bars can be determined by examining the moment diagram. Development length and tension splice lengths need to be considered in this process. Further, ACI code has bar cut-off requirements as detailed in Figure 30.3.



30.3
Bar cut-off and splice requirements. Reproduced with permission from the American Concrete Institute

Example 30-3: A simple beam 16″ by 30″ with a span of 30′ carries a dead load of 1k/f and a live load of 1k/f.

The reinforcement is six #7 evenly spaced inside a #3 stirrup. f'_c = 4ksi, f_y = 60ksi. Find at what point bars can be cut off.

1. $w_u = 1.6(1) + 1.2[1 + .15(16/12)(30/12)] = 3.4$k/f
2. $M_u = 3.4(30)^2/8 = 382.5$k-f = 4590k-in
3. For six #7, $A_s = 3.60\text{in}^2$
4. $A_{s\ min} = .0033(16)(27.69) = 1.46\text{in}^2$
5. Find the moment, ϕM_n for four #7:$A_s = 2.40\text{in}^2 > A_{s\ min}$) $= 1.46\text{in}^2$ … okay

   $d = 30 - 1.5 - .375 - .875/2 = 27.69''$

   $a = f_y A_s/(0.85 f'_c b) = 60(2.4)/(.85(4)(16) = 2.65''$

   $M_n = f_y A_s(d - a/2) = 60(2.4)(27.69 - 2.65/2)$
   $= 3796.56$k-in $= 316.38$k-f

   $\phi M_n = .9(316.38) = 284.74$k-f $= wx^2/2$

   $x = \sqrt{[284.74(2)/3.4]} = 12.94'$ where x is the distance from the support.

6. Check the development length to see if two bars can be cut at x = 12.9′:

   $K_D = (3/40)(f_y/\lambda\sqrt{f'_c}) = (3/40)(60000/(1(\sqrt{4000})) = 71.15$

   $d_b = .875''$

   $\Psi_t = 1.0$, $\Psi_e = 1.0$, $\Psi_s = 1.0$

   c_b = lesser of:
   center to edge = .875/2 + .375 + 1.5 = 2.31
   Or $\frac{1}{2}$ center to center = .5(16 − 2(1.5 + .375 + .875/2))/5 = 1.1375″

   $c_b/d_b = 1.14/.875 = 1.30\ 2.5$ … okay

   $L_d = 71.2(.875)/1.30 = 47.92''$

   This means the two bars cannot be cut until at least 4ft from centerline; or to x = 15 − 4 = 11′.

7. The cut-off must also be closer to the support than the theoretical cut-off point by the larger of d or $12d_b$: d = 27.69″ = 2.3′ and $12d_b$ = 12(.875)/12 = .875′. 2.3′ governs … x = 12.9 – 2.3 = 10.6′.
8. Terminate two bars at the lesser of answers from step 6 and step 7:
   Terminate two bars at 10.6′.
9. Find the moment, ϕM_n for two #7: $A_s = 1.20in^2 < A_{s\ min} = 1.46in^2$ … cannot cut down to two bars.

30.6 Development Length for Positive Moment at Simple Supports

An additional rule for development length in simply supported members must be considered.

$L_d \leq L_a + M_n/V_u$ at the point of inflection and

$L_d \leq L_a + 1.3M_n/V_u$ at the support

Where L_a = the greater of effective depth d or $12d_b$ and V_u = total applied shear at the section.

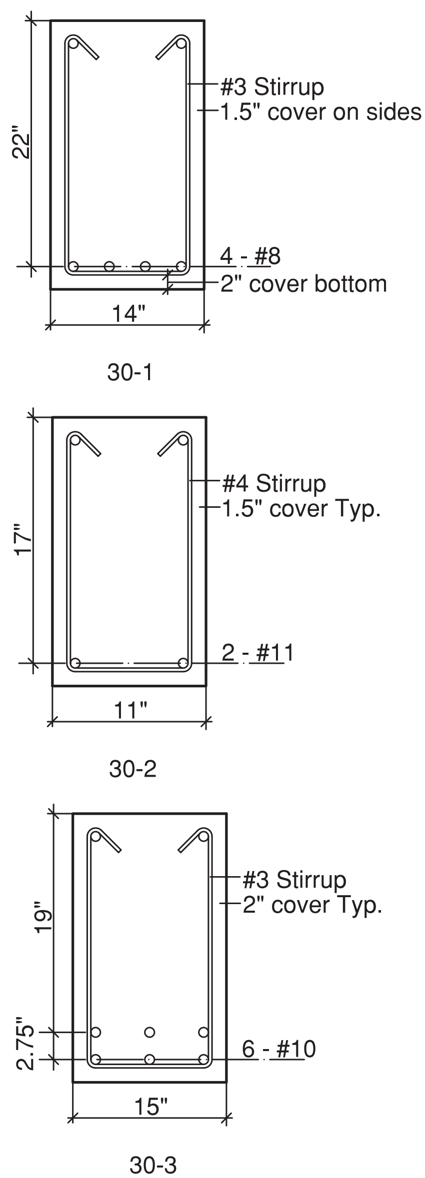
Example 30-4: Consider the beam in Example 30-3. Check the development requirements for positive moment at the supports if bars extend 4″ past centerline of support.

1. $V_u = W_uL/2$ = 3.4k/f(30′)/2 = 51k
2. M_n = 316.38k-f (remember 2 bars were cut off before reaching the support)
3. L_a = d = 27.69″ or $12d_b$ = 12(.875) = 10.5″ use greater length: L_a = 27.69″
4. $L_d \leq L_a + 1.3M_n/V_u$ = 27.69 + 1.3(316.38(12in/f)/51) = 126.4″
5. From example 30-3, L_d for the four #7 was determined to be L_d = 47.92″ < 126.4″ … okay

Practice Exercises:

30-1 through 30-3: Find the development length for the reinforcement bars shown in Figure 30.4.

30-4: A simple beam 15″ by 28″ with a span of 24′ three point loads, P_u = 50k at x = 6′, 12′ and 18′. The reinforcement is six #8 evenly spaced inside a #3 stirrup. f'_c = 4ksi, f_y = 60ksi. Find at what point bars can be cut off.



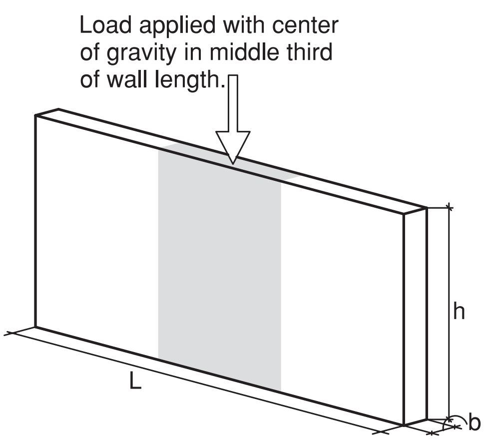
30.4
Chapter 30 Practice exercises

thirty one

Concrete Walls

This chapter explains the design methods for retaining walls, bearing walls and shear walls. While all three types may have significant vertical or gravity loads, shear walls are designed to handle lateral loads parallel to the face of the wall and retaining walls are designed to handle lateral loads normal to the face of the wall.

31.1 Bearing Walls



31.1

Bearing Walls

Bearing walls carry applied vertical loads. When the vertical loads are applied in the middle third of the wall cross-section, the axial load strength, ϕP_n, is expressed as:

$\phi P_n = 0.55\phi f'_c A_g[1 - (kh/32b)^2]$ where:

$\phi = 0.65$

b = wall thickness (in)

b_{min} = 1/25 unsupported height or length > 4″

b_{min} = 7.5″ for exterior basement or foundation walls

h = Vertical distance between supports

A_g = gross area in section (in^2)

k = Effective Length Factor

= 0.8 if restrained against rotation at one or both ends

= 1.0 if unrestrained against rotation at both ends

= 2.0 for wall not braced against translation

L_e = The effective length of the wall is the smaller of:

= the center-to-center distance between loads or

= width of bearing plus four times wall width.

= 12″ for uniform loads

A_{shmin} = minimum horizontal reinf.

= .0025(12b) = .03b in^2/f – for #6 and larger

= .002(12b) = .024b in^2/f – for # 5 or smaller bars & $f_y \geq 60$ksi

A_{svmin} = minimum vertical reinf.

= .0015(12b) = .018b in^2/f – for #6 and larger

= .0012(12b) = .0144b in^2/f – for #5 or smaller bars & $f_y \geq 60$ksi

If b > 10″, bearing walls other than basement walls must have reinforcement in each direction on each face.
The bearing design strength = $\phi(.85f'_cA_1)$ where A_1 is the bearing area.

Max. spacing of bars, s = 3b < 18″

Method for design of bearing walls:

1. Find minimum wall thickness, b_{min}. b = b_{min} rounded up to next whole inch.
2. Check that bearing strength of concrete = $\phi(.85f'_c)A_1 \leq P_u$. If not, increase b.
3. Find effective length of wall, L_e.
4. Check that axial load strength, $\phi P_n = 0.55\phi f'_cA_g[1 - (kh/32b)^2] \geq P_u$.
5. Select steel based on: A_{svmin} and A_{shmin}
6. Check that maximum spacing of bars, s = 3b ≤ 18″.
7. Check if more than one layer of reinforcement is necessary.

Example 31-1: Design a reinforced concrete bearing wall to support 12″ wide beams spaced at 10′o.c.

The beams bear on the full thickness of the wall. The bottom of the wall is a fixed connection, the top is a pinned connection (braced against lateral movement but not against rotation). The wall is 20′ high and the load from each beam, P_u = 30k. f'_c = 4ksi, f_y = 60ksi.

1. b_{min} = h/25 = (1/25)(20)(12) = 9.6″ … b = 10″
2. Bearing strength of concrete = $\phi(.85f'_c)A_1$ = 0.65(.85)(4)(8)(12) = 212.16k
   212.6k > 30k = P_u … okay
3. Effective length of wall is lesser of:
   distance between loads = 10′ = 120″ or
   width of bearing + 4b = 12 + 4(10) = 52″ Use 52″
   L_e = 52″
4. $\phi P_n = 0.55\phi f'_cA_g[1 - (kh/32b)^2]$ = .55(.65)(4)(10)(52)[1 – (.8(20)(12)/32(10))^2] = 475.9k > 30k … okay
5. Reinforcing steel: (assume #5 or smaller)
   Vertical steel: A_s = .0144)(10) = .144 use #4 @16″
   Horizontal Steel: A_s = .024(10) = .24 use #4 @ 10″
6. Check max. spacing of bars, s = 3(10) ≤ 18 … s = 18″ … okay
7. One layer of reinforcement may be used because the wall thickness, h ≤ 10″.

Example 31-2: Design a 24′ long reinforced concrete bearing wall to support a slab bearing on the full thickness of the wall.

The bottom and top of the wall are fixed connections.
The wall is 16′ high and the load from the slab, W_u = 6k/f, f'_c = 4ksi, f_y = 60ksi.

1. b_{min} = (1/25)(16)(12) = 7.68″ … b = 8″.
2. $\phi(.85f'_c)$A1 = 0.65(.85)(4)(8)(12″) = 212.16k > 6k/f(1′) = 6k = P_u … okay
3. L_e = 12″
4. Check that axial load strength, $\phi P_n = 0.55\ \phi f'_cA_g[1 - (kh/32b)^2]$ = .55(.65)(4)(8)(12)[1 – (.8(16)(12)/32(8))^2] = 87.86k > 6k/f(1′) = 6k … okay
5. Select steel based on:
   Vertical steel: A_s = .0144(8) = .12 use #4 @18″
   Horizontal Steel: A_s = .024(8) = .20 use #4 @ 12″
6. Check that maximum spacing of bars, s = 3b = 3(8″) = 24″ or s ≤ 18″ … s = 18″.
7. One layer of reinforcement may be used because the wall thickness, b ≤ 10″.

31.2 Shear Walls

Shear walls are capable of resisting lateral loads as well as supporting vertical loads. As such, they require additional reinforcement and a different design method.

h = overall height of wall

L = overall length of wall

b = thickness of wall

A = bL = area of wall cross-section

G = shear Modulus of Elasticity = E/2.4 for concrete

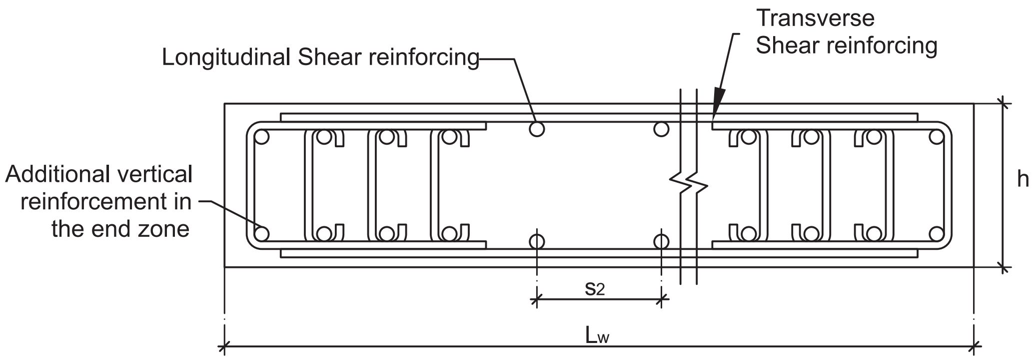
$I = bL^3/12$

$k_i = 3EI/h^3 + GA/1.2h = (Eh/12)(L/h)^3 + (Eh/2.88)(L/h)$

$F_i = F_{total}(k_i/\Sigma k_i)$ where the stiffness, k, is determined by the size and material of each wall. If the lateral resistance system uses shear walls with equal values for k, $F_i = F_{total}$/number of walls.

d = 0.8(L)

ϕV_n = maximum allowable shear strength = $10\phi(\sqrt{f'_c})$



31.2

Shear walls

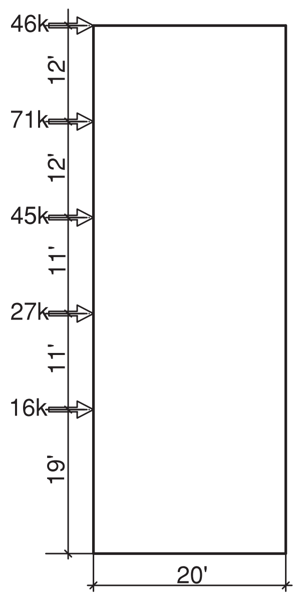
$bd > V_u$

$\phi V_c = 2\phi(\sqrt{f'_c})bd$ If $V_u > \phi V_c$, shear reinforcement is required.

Method for design of shear walls:

1. Determine factored lateral loads.
2. Determine the portion of the lateral load carried by each shear wall, $F_i = F_{total}(k_i/k_{total})$.
3. Assume the wall thickness, b = 8″. Assume placement of reinforcement on both faces.
4. Check the maximum allowable shear strength of wall where d = 0.8(L). $\phi V_n = 10\phi(\sqrt{f'_c})bd > V_u$ (shear at base of wall = sum of lateral loads)
5. $\phi V_c = 2\phi(\sqrt{f'_c})bd$ If $V_u > \phi V_c$, shear reinforcement is required.
6. Select trial size and determine A_v (for example, #4 on both faces = $.2in^2 \times 2 = .4in^2 = A_v$)
7. Determine spacing horizontal shear reinforcement, $s_h = \phi f_y d A_v/[V_u - \phi V_c]$
8. Maximum spacing is smallest of: L/5, 3b or 18″
9. Determine $\rho_L = .0025 + .5(2.5 - h/L)(A_v/s_h b - .0025) \geq 0.0025$
10. Determine spacing of vertical shear reinforcement, $s_v = A_v \rho_L/b$
11. Maximum spacing is smallest of: L/3, 3b or 18″
12. Calculate M_u = moment at base of wall due to factored lateral loads.
13. Assume $\phi = 0.9$ for flexure
14. $A_s = [.85f_c'bd/f_y][1 - \sqrt{[1 - 2\,M_u/\phi.85f_c'bd^2]}$
15. Check that $A_s \geq A_{s\,min} = 3bd\sqrt{f_c'}/f_y \geq 200bd/f_y$. If not, use $A_{s\,min}$.
16. Select bars from A4.2. These bars are to be placed at each end of the wall.

Example 31-3: Design reinforcement for the shear wall shown in Figure 31.3. Use f'_c = 4ksi, f_y = 60ksi.



31.3

Shear wall example

1. Determine factored lateral loads. F = 46k + 71k + 45k + 27k + 16k = 205k
2. Determine the portion of the lateral load carried by each shear wall, $F_i = F_{total}(k_i/k_{total})$ = 205k
3. Assume the wall thickness, b = 8″. Assume placement of reinforcement on both faces.
4. d = 0.8(L) = .8(20′)(12″/f) = 192″,

   $\phi V_n = 10\phi(\sqrt{f'_c})bd = 10(.75)(\sqrt{4000})(8)(192)/1000\#/k$ = 728.6k

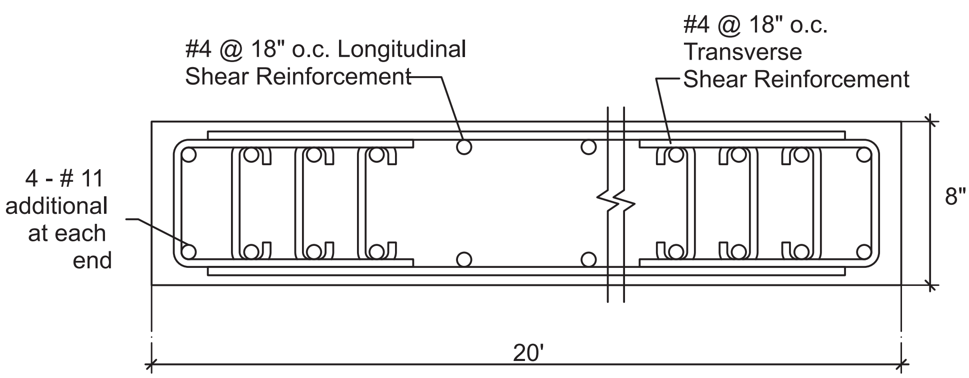
   ϕV_n = 728.6k > V_u = 205k … okay

5. $\phi V_c = 2\phi(\sqrt{f'_c})bd = 2(.75)(\sqrt{4000})(8)(192)/1000 = 145.7k$
   $V_u = 205k > 145.7k = \phi V_c$ … shear reinforcement is required.
6. Use #4 on both faces $= .2in^2 \times 2 = .4in^2 = A_v$.
7. Determine spacing horizontal shear reinforcement, $s_h = \phi f_y dA_v/[V_u - \phi V_c] = .75(60)(192)(.4)/(205 - 145.7) = 58.28''$
8. $s_{max} = 20'(12'/f)/5 = 48''$ or $= 3b = 3(8) = 24''$ or $18''$ … $s_h = 18''$, use #4@18″
9. Determine $\rho_L = .0025 + .5(2.5 - h/L)(A_v/s_h b - .0025) = .0025 + .5(2.5 - 65'/20')(.4/(18(8)) - .0025) = .0024 < 0.0025$ … $\rho_L = .0025$
10. Determine spacing of vertical Shear reinforcement, $s_v = A_v/\rho_L b = .4/.0025(8) = 20''$
11. $s_{max} = 20'(12'/f)/5 = 48''$ or $= 3b = 3(8) = 24''$ or $18''$ … $s_v = 18''$, use #4@18
12. $M_u = 46(65) + 71(53) + 45(41) + 27(30) + 16(19) = 9712k\text{-}f = 116544k\text{-}in$
13. Assume $\phi = 0.9$ for flexure
14. $A_s = [.85f_c'bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f_c'bd^2]} = [.85(4)(8)(192)/60][1 - \sqrt{[1 - 2(116544)/[.9(.85)(4)(8)(192)^2]}] = 12.08in^2$
15. Check that $A_s \geq A_{s\ min} = 3bd\sqrt{f_c'}/f_y = 3(8)(192)(\sqrt{4000}/60000) = 4.86 \geq 200bd/f_y = 200(8)(192)/60000 = 5.12$ … therefore $A_{s\ min} = 5.12 < 12.13$ … okay
16. From Table A4.1 choose 8 #11 with $A_s = 12.5 > 12.13in^2$. Place 4 #11 VEF (vertical each face) at each end.

31.3 Retaining Walls

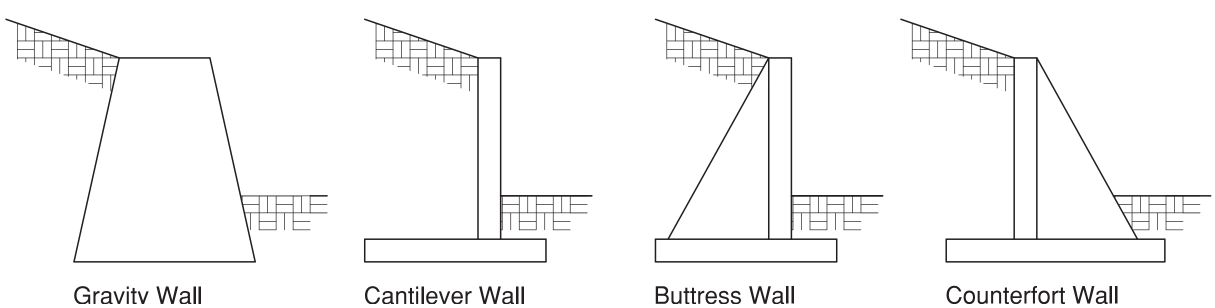
Retaining walls are used to resist the lateral forces of hydrostatic soil pressure. A retaining wall may also carry a vertical load as in the case of basement walls, but in this section only the design for the lateral forces is considered. Retaining walls can fail by sliding, sinking, overturning or buckling.

Basic types of retaining walls are shown in Figure 31.5. A gravity wall is one in which the bulk of the material is the deterrent to lateral forces. Its weight creates enough friction to prevent sliding and enough moment about the toe to prevent overturning. Its base is often wide to be able to distribute the loads to the ground without sinking.



31.4

Example 31-3

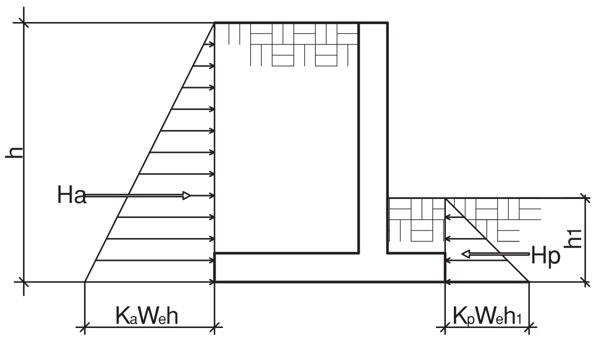


31.5

Retaining walls

A cantilever wall is a retaining wall in which a moment connection between the stem and the footing allow the stem to be thinner than in a gravity wall. The heel utilizes the load of the soil sitting on it to counteract overturning.

Forces on a retaining wall with level backfill:



31.6

Forces on a retaining wall with level backfill

$H_a = K_a w_e h^2/2$ = the horizontal equivalent force of the soil

W_e = density of soil

h = height of wall

K_a = coefficient of active earth pressure

ϕ = internal friction angle of soil

$K_a = (1 - \sin\phi)/(1 + \sin\phi)$

$K_a W_e$ = equivalent fluid pressure in pcf

$H_p = K_p w_e h_1^2/2$ = horizontal resisting force of the soil in front of the retaining wall.

W_e = density of soil

h_1 = height of earth on resisting side

$K_p = 1/K_a$ = coefficient of passive earth pressure

31.3.1 Checking a Wall for the Four Modes of Failure

The first three modes of failure are Overturning, Sliding and Sinking. The fourth mode of failure is failure of the concrete itself. This is covered in section 31.3.2.

31.3.1.1 Overturning

The factored overturning moment = $M_o = 1.6H_a(h/3)$
$= 1.6K_a W_e h^3/6$

The factored resisting moment = M_r
$= 1.6H_p(h_1/3) + 1.2W_{wall}(d_1) + 1.2W_{soil}(d_2)$ where:

d_1 = distance from toe to center of gravity of the wall

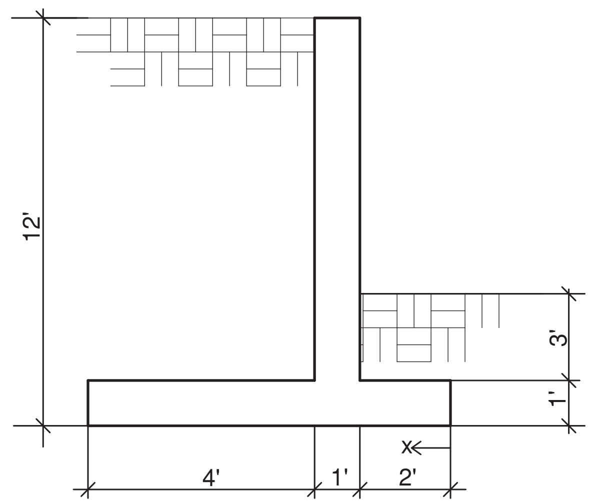
d_2 = distance from toe to center of gravity of soil resting on heel

Factor of safety:

$M_r/M_o \geq 1.5$

Example 31-4: Check the adequacy of the retaining wall in Figure 31.7 against overturning.

Soil density = 80pcf, concrete density = 150pcf, ϕ = 23 and equivalent fluid pressure, $K_a W_e$ = 35pcf.



31.7

Example 31-4

$M_o = 1.6H_a(h/3) = 1.6K_a W_e h^3/6 = 1.6(35)(12)^3/6$
$= 16128\#\text{-f}$

$K_p = 80\text{pcf}/35\text{pcf} = 2.286$

$M_{r1} = 1.6(2.286)(80\text{pcf})(4)^3/6 = 3121.15\#\text{-f}$

$M_{r2} = 1.2(150)(1)(11)(2.5') = 6187.5\#\text{-f}$

$M_{r3} = 1.2(150)(7)(1)(3.5) = 4410\#\text{-f}$

$M_{r4} = 1.2(80)(4)(11)(5') = 21120\#\text{-f}$

$M_{r5} = 1.2(80)(2)(3)(1) = 576\text{\#-f}$

$M_r = 3121.15 + 6187.5 + 4410 + 21120 + 576$
$= 35415\text{\#-f}$

Factor of safety:

$M_r/M_o = 35415/16128 = 2.20 > 1.5$ … retaining wall will not overturn.

31.3.1.2 Sliding

Sliding occurs when the horizontal forces against the wall are not counteracted by sufficient resisting forces. Resisting forces are created by the friction between the soil and the concrete.

Typical coefficient of friction $f = 0.5$ between soil and concrete

Resisting force $F = f\Sigma W$

Sliding force $S = H_a - H_p$

Factor of safety: $F/S \geq 1.5$

Example 31-5: Check the retaining wall in Figure 31.7 against sliding. Soil density = 80pcf, concrete density = 150pcf, equivalent fluid pressure = 35pcf.

$K_p = 80\text{pcf}/35\text{pcf} = 2.286$

$W_1 = 1.2(150\text{pcf})(1')(11') = 1980\text{\#/f}$

$W_2 = 1.2(150)\text{pcf}(7')(1') = 1260\text{\#/f}$

$W_3 = 1.2(80\text{pcf})(4')(11') = 4224\text{\#/f}$

$W_4 = 1.2(80\text{pcf})(2')(3') = 576\text{\#/f}$

$\Sigma W = (1980 + 1260 + 4224 + 576)(1' \text{ thickness of wall})$
$= 8040\text{\#}$

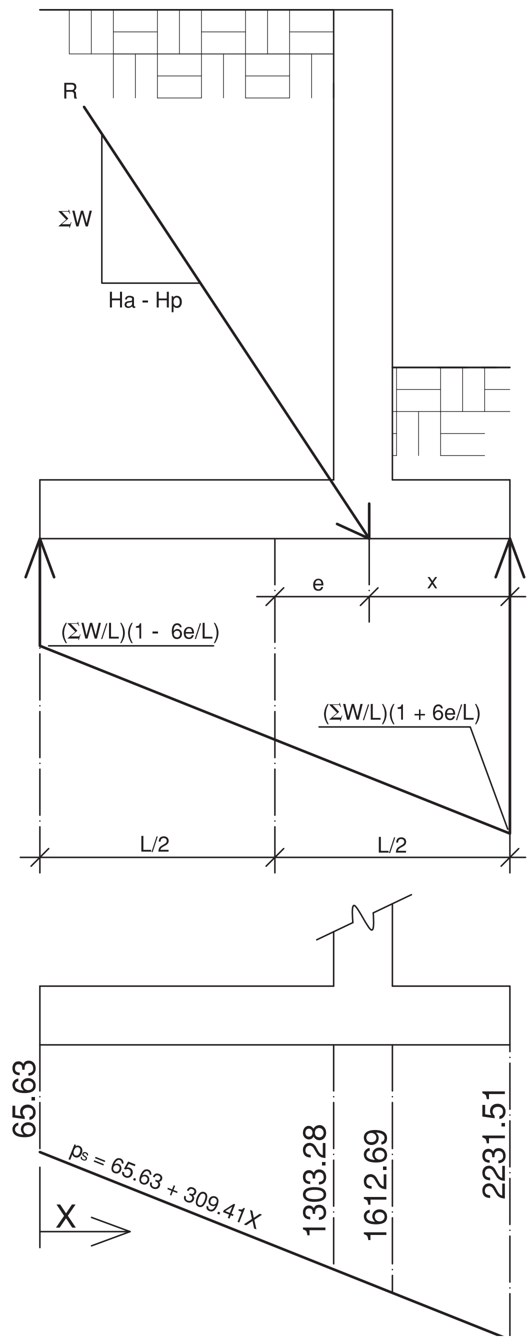
$F = 0.5(8040\text{\#}) = 4020\text{\#}$

$S = H_a - H_p = 1.6K_a w_e h^2/2 - 1.6K_p w_e h_1^2/2$
$= 1.6(35)(12)^2/2 - 1.6(2.286)(80)(4)^2/2 = 2276.35\text{\#}$

Factor of safety $= F/S = 4020/2276.35 = 1.77 > 1.5$
… okay

31.3.1.3 Sinking

Sinking occurs if the downward pressure caused by weight of the soil and the wall distributed along the footing of the wall are greater than the upward allowable soil bearing pressure.



31.8
Example 31-6

p_a = The allowable soil bearing pressure in psf.
$p_s = P/A \pm Mc/I$ = soil pressure where

$P = \Sigma W$

A = area of the footing acting on soil = (L′)(1′ swath) = Lft²

e = distance from the centerline of footing to the point of the resultant vector

$M = Pe = (\Sigma W)(e)$

c = distance from centerline to edge = L/2

$I = L^3/12$

$p_s = \Sigma W/L \pm 6e\Sigma W/L^2 = (\Sigma W/L)(1 \pm 6e/L)$

There is a resultant vector that acts at the base of the footing at a distance X from the toe. The horizontal component of that vector is $H_a - H_p$ and the vertical component is ΣW.

$X = (M_r - M_o)/\Sigma W$ = the distance from the toe to the point where the resultant vector acts at the base of the footing.

Example 31-6: Determine whether the retaining wall in Figure 31.7 is adequate against sinking if the allowable soil bearing pressure, p_a = 2500psf.

From example 31-4, M_r = 35415#-f and M_o = 16128#-f.
From example 31-5, ΣW = 8040#

$X = (M_r - M_o)/\Sigma W = (35415 - 16128)/8040 = 2.40'$

Centerline of the footing = 7′/2 = 3.5′

e = 3.5 − 2.4 = 1.1′

Soil pressure = $p_s = P/A \pm Mc/I$

$p_s = (\Sigma W/L)(1 \pm 6e/L) = (8040/7)(1+ 6(1.1/7) = 2231.51$ @ the toe and

$p_s = (8040/7)(1 - 6(1.1/7) = 65.63$ @ the heel

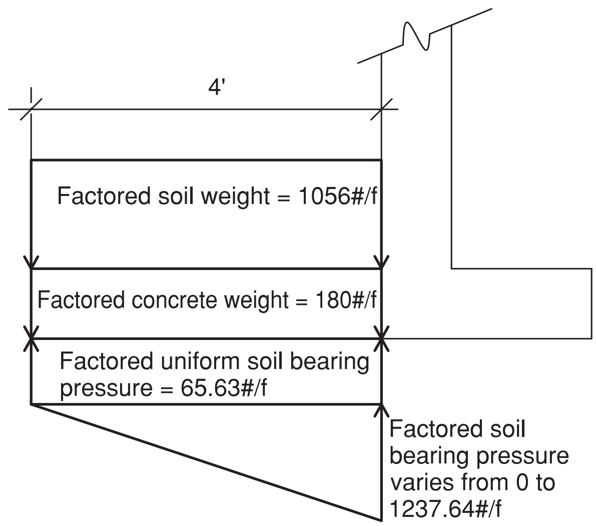
$p_s = 65.63 + 309.41x$

$p_s = 2231.51\text{psf} < p_a = 2500\text{psf}$ … retaining wall is adequate against sinking.

31.3.2 Retaining wall design

Reinforcement for each component of a cantilever retaining wall—the heel, toe and stem—can be designed as a cantilevered beam.

Example 31-7: Design the reinforcement for the retaining wall in Figure 31.7 using f'_c = 3000psi and f_y = 60,000psi.



31.9
Shear and flexure in heel

Shear in heel:

Factored concrete weight: = 1.2(150pcf)(1′)(1′) = 180#/f↓

Factored soil weight: = 1.2(80pcf)(1′)(11′) = 1056#/f↓

$p_s = 65.63 + 309.41x$

Soil bearing pressure changes from 65.63psf @ x = 0′ to 1303.28psf @ x = 4′, a difference of 309.41(4) = 1237.64psf.

$W_1 = -1056 - 180 + 65.63 = -1170.37\#/f$

$W_2 = 309.41x$

$W_x = -1170.37 + 309.41x$

$V_x = -1170.37x + 154.71x^2$

Vmax is at Wx = 0. X = 1170.37/309.41 = 3.78′

$V_u = -1170.37(3.78) + 154.71(3.78)^2 = -2213.44k$

Assume #8 bars

d = 12″ − 2″ cover(steel at top of footing) − .5 = 9.5″

$\phi V_c = \phi(2\sqrt{f'_c})bd = .75(2)(\sqrt{3000})(12)(9.5) = 9366\# > V_u$ … okay

$\phi V_c/2 = 9366/2 = 4683\# > 2213.44\# = V_u$ … Stirrups are not required.

Flexure in heel: Take the moment about x = 4′.

$W_u = 4'(180+1056) = 4944\#\downarrow$ and c.g. is at x = 2′

$P_1 = (1')(65.63psf)(4') = 2400\#\uparrow$ and c.g. is at x = 2′

$P_2 = (1')(1237.64)(4')/2 = 2475.28\#\uparrow$ and c.g. is at x = 2(4′)/3 = 2.67′

$M_u = 4944\#(2') - 2400\#(2') - 2475.28\#(4' - 2.67')$ = 1787.63#-f = 21451.52k-in

$A_s = [.85f_c'bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f_c'bd^2]} = [.85(3000)(12)(9.5)/(60,000)][1 - \sqrt{[1 - 2(21451.52)/(.9(.85)(3000)(12)(9.5)^2)]} = 0.042in^2$

$A_{s\ min}$ = greater of:

$3bd\sqrt{f_c'}/f_y = 3(12)(9.5)\sqrt{3000}/60,000 = 0.312$

Or $200bd/f_y = 0.38$

$A_s < A_{s\ min} = 0.38$, therefore $A_s = 0.38in^2$

Use # 6 steel: A = 0.442in²

Spacing ≥ .442(12)/.38 = 13.96″

Main Steel: #6@ 14″

Shrinkage steel: .0018(12″)(12″) = 0.26

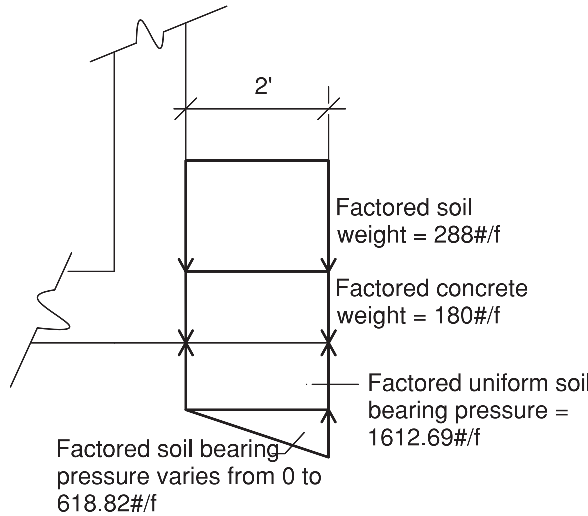
18″ ≥ Spacing ≥ .442(12)/.26 = 20.4″

Shrinkage Steel: #6 @ 18″

Shear in toe:

Factored concrete weight: = 1.2(150pcf)(1′)(1′) = 180#/f↓

Factored soil weight: = 1.2(80pcf)(1′)(3′) = 288#/f↓



31.10

Shear and flexure in toe

Soil bearing pressure changes from 1612.69psf @ x = 0′ to 2231.51psf @ x = 2′, a difference of 2231.51 − 1612.69 = 618.82psf

$W_1 = -180 - 288 + 1612.69 = 1144.69\#/f\uparrow$

$W_2 = 309.41x\#/f\uparrow$

$W_x = 1144.69 + 309.41x$

$V_x = 1144.69x + 154.71x^2$

V_{max} is at x = 2′

$V_u = 1144.69(2) + 154.71(2)^2 = 2908.22k$

Assume #8 bars

d = 12″ − 2″ cover(steel at top of footing) − .5 = 9.5″

$\phi V_c = \phi(2\sqrt{f'_c})bd = .75(2)(\sqrt{3000})(12)(9.5) = 9366\# > V_u$ … okay

$\phi V_c/2 = 9366/2 = 4683\# > 2908.22\# = V_u$ … Stirrups are not required.

Flexure in toe: Take the moment about x = 0.

$W_u = 2'(180 + 288) = 936\#\downarrow$ and c.g. is at x = 1′

$P_1 = (1')(1612.69psf)(2') = 3225.38\#\uparrow$ and c.g. is at x = 1′

$P_2 = (1')(618.82)(2')/2 = 618.82\#\uparrow$ and c.g. is at $x = 2(2')/3 = 1.33'$

$M_u = 936\#(1') - 3225.38\#(1') - 618.82\#(1.33')$
$= -3114.47\#\text{-f} = 37373.68\text{k-in}$

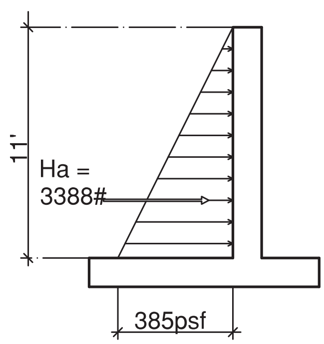
$A_s = [.85f_c'bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f_c'bd^2]} = [.85(3000)(12)(9.5)/(60{,}000)][1 - \sqrt{[1 - 2(37373.68)/(.9(.85)(3000)(12)(9.5)^2)]} = .073\text{in}^2$

$A_{s\ min} = 0.38$ as calculated above

$A_s < A_{s\ min} = 0.38$, therefore $A_s = 0.38$. And the steel is the same as in the heel:

Main Steel: #6@ 14″

Shrinkage steel: #6 @ 18″



31.11
Shear and flexure in stem

Shear in stem:
The horizontal force $k_{aWe}h$ varies from 0 @ y = 11′ to 35(11) = 385psf @ y = 0′

$H_a = 1.6(385\text{psf})(1')(11'/2) = 3388\#$

H_p is not considered because there may be a time when this soil is removed.

$V_u = H_a = 3388\#$

d = 12 – 2″ cover(steel at outside of footing) – .5(#8 bars) = 9.5″

$\phi V_c = \phi(2\sqrt{f'_c})bd = .75(2)(\sqrt{3000})(12)(9.5) = 9366\# > V_u$ … okay

$\phi V_c/2 = 9366/2 = 4683\# > 3388\#$ … no stirrups required.

Flexure in stem: Take moment at y = 0.

$M_u = [(3388\#)(11'/3)] = 12422.67\#\text{-f} = 149{,}072\#\text{-in}$

$A_s = [.85f_c'bd/f_y][1 - \sqrt{[1 - 2M_u/\phi.85f_c'bd^2]} = [.85(3000)(12)(9.5)/(60{,}000)][1 - \sqrt{[1 - 2(149072)/(.9(.85)(3000)(12)(9.5)^2)]} = .30\text{in}^2$

$A_{s\ min} = 0.38$

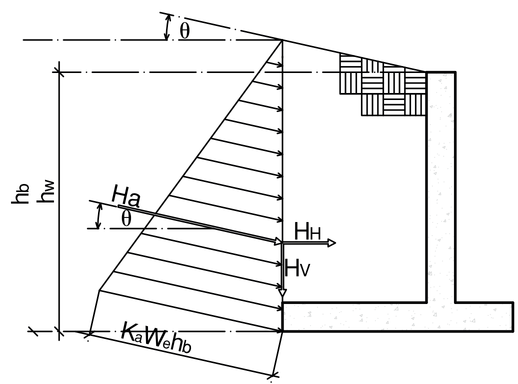
$A_s < A_{s\ min} = 0.38$, therefore $A_s = 0.38$. and the steel in the stem is the same as in the heel and toe:

Main steel: #6@ 14″

Shrinkage steel: #6 @ 18″

31.3.3 Sloped backfill

A sloped backfill affects the forces on the wall. Not only does the weight of soil on the heel increase, but the hydrostatic pressure is not horizontal, meaning that there is both a horizontal and vertical component to it.



31.12
Retaining wall with sloped backfill

h_b = height of backfill at end of heel taken from bottom of heel

θ= slope of backfill

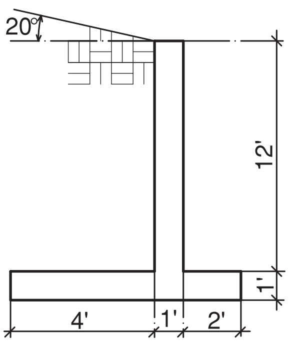
ϕ = internal friction angle of soil

$$K_a = \cos\theta\left(\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right)$$

$H_s = K_a W_e hb^2/2$

$H_H = H_s\cos\theta$

$H_V = H_s\sin\theta$



31.13
Example 31-8

Example 31-8: Check the adequacy of the retaining wall in Figure 31.13 with a backfill angle of 20° and no soil above the toe.

Use $\phi = 30$, $W_e = 80$pcf, soil bearing capacity of 2500psf.

θ = slope of backfill = 20°

$h_b = 12' + 4'\tan 20 = 13.46'$

$\sqrt{(\cos^2 20 - \cos^2 30)} = \sqrt{(.883 - .75)} = .3647$

$K_a = .9397(.9397 - .3647)/(.9397 + .3647) = 0.4142$

$H_s = K_a W_e h_b b^2/2 = .4142(80)(13.46)^2/2 = 3001.9\#$

$H_H = H_s \cos\theta = 3001.9(\cos 20) = 2820.86\#$

$Hv = H_s \sin\theta = 3001.9(\sin 20) = 1026.71\#$

Overturning:

$M_o = 1.6H_H(h_b/3) = 1.6(2820.86\#)(13.46')/3$
$= 20250.0\#\text{-f}$

$M_{r1} = 1.6Hv(7') = 1.5(1026.71\#)(7') = 11499.15\#\text{-f}$

$M_{r2} = 1.2(150\text{pcf})(1')(1')(11')(2.5') = 4950\#\text{-f}$

$M_{r3} = 1.2(150\text{pcf})(7')(1')(1')(3.5') = 4410\#\text{-f}$

$M_{r4} = 1.2(80\text{pcf})(1')(4')(11')(5') = 21120\#\text{-f}$

$M_{r5} = 1.2(80\text{pcf})(1')(13.46' - 12')(4'/2)(2' + 1' + (2/3)(4')) = 1588.48\#\text{-f}$

$M_r = 11499.15 + 4950 + 4410 + 21120 + 1588.48$
$= 43567.63\#\text{-f}$

Factor of safety: $M_r/M_o = 43567.63/20250 = 2.15 > 1.5$ … okay

Sliding:

$w_1 = 1.6H_v = 1.6(1026.71\#) = 1642.74\#$

$w_2 = 1.2(150)(1)(11) = 1980\#$

$w_3 = 1.2(150)(7)(1) = 1260\#$

$w_4 = 1.2(80)(4)(11) = 4224\#$

$w_5 = 1.2(80)(13.46 - 12)(4/2) = 280.8\#$

$\Sigma w = 9387.54\#$

$F = .5(9387.54) = 4693.77\# > H_H = 1.6(2820.86)$
$= 4513.38\#$

Factor of safety = 4693.77/4513.38 = 1.04 < 1.5 … no good

Add soil to top of toe at some height, h_1, from bottom of footing.

Note that overturning need not be rechecked because the soil at the front will add more resisting moment about the toe, further increasing the factor of safety.

Let $h_1 = 4'$

$W_6 = 1.2(80\text{pcf})(2')(3') = 576\#$

$\Sigma w = 9387.54 + 576 = 9963.54\#$

$K_p = 1/K_a = 1/.4142 = 2.414$

$H_p = 1.6K_p W_e h_1^2/2 = 1.6(2.414)(80)(4)^2/2 = 2471.94\#$

$F = .5(9963.54) = 4981.77 > H_H - Hp$
$= 4513.38 - 2471.94 = 2041.44$

Factor of safety = 4981.77/2041.44 = 2.44 > 1.5 … okay

Soil bearing pressure:

New $M_r = 43567.63 + 2471.94(4'/3) + 576(1')$
$= 47439.55\#\text{-f}$

$X = (Mr - Mo)/\Sigma w = (47439.55 - 20250)/9963.54$
$= 2.73'$

$e = 3.5 - 2.73 = 0.77'$

Soil pressure $= p = (\Sigma w/L)(1 \pm 6e/L)$
$= (9963.54/7)(1 \pm 6(.77)/7) = 1423.36 \pm 939.42$

$p_{max} = 2362.78 < 2500$psf … okay

Practice Exercises:

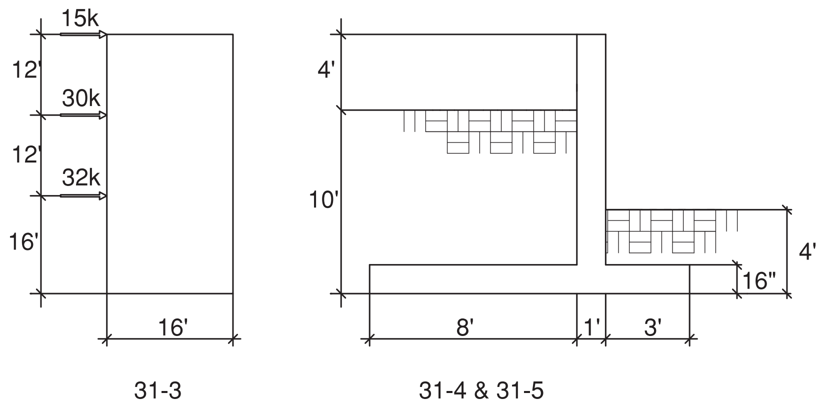
31-1: Design a reinforced concrete bearing wall to support 16″ steel beams spaced at 8′o.c. The beams bear on the full thickness of the wall with b_f = 14.5″. The bottom and top of the wall are fixed connections. The wall is 18′ high and the load from each beam, P_u = 40k. f'_c = 4ksi, f_y = 60ksi.

31-2: Design a 20′ long reinforced concrete bearing wall to support a slab bearing on the full thickness of the wall. The bottom and top of the wall are fixed connections. The wall is 22′ high and the load from the slab, W_u = 2k/f. f'_c = 4ksi, f_y = 60ksi.

31-3: Design reinforcement for the shear wall shown in Figure 31.14. Use f'_c = 4ksi, f_y = 60ksi.

31-4: Check the adequacy of the retaining wall in Figure 31.14 against overturning, sliding and sinking. Soil density = 80pcf, concrete density = 150pcf, ϕ = 23 and equivalent fluid pressure, K_aW_e = 35pcf.

31-5: Design reinforcement for the retaining wall shown in Figure 31.14. Soil density = 80pcf, concrete density = 150pcf, ϕ = 23 and equivalent fluid pressure, K_aW_e = 35pcf.



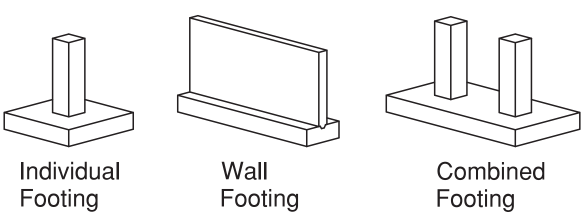
31.14
Chapter 31 Practice exercises

thirty two

Footings

The purpose of a footing is to distribute the loads of the structure sufficiently so that the soil can support the loads. Concrete has a compressive strength that ranges from 3000psi to over 6000psi depending on the mix. Soils have a soil bearing capacity that can range from 1000psf (6.94psi) for loose sandy soils to over 4000psf (27.78psi) for rock or hardpan. As a result, a structure without a footing to distribute the loads over a larger area would likely sink into the soil.

Types of footings include individual footings, wall footing, combined footings, caissons and piles as shown in Figure 33.1. Piles are a footing scenario that does not rely on a larger area. Instead, the piles are driven to bedrock or to a depth where the friction between the sides of the pile and the soil will resist the load.

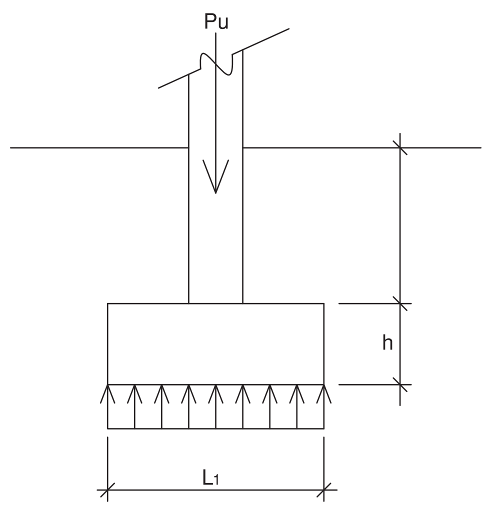


32.1

Footing types

32.1 Wall Footings

Just as in the design of a concrete wall or a slab, the design of a wall footing considers a one-foot thickness of wall, therefore, b = 12″.



32.2

Wall footing

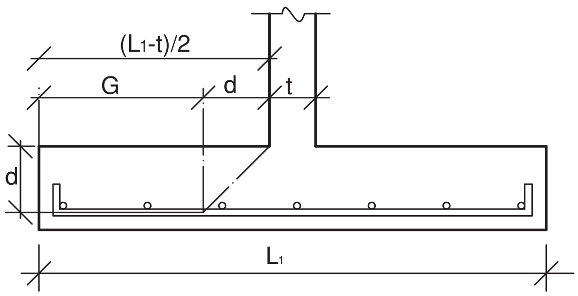
32.1.1 Wall Footing Design Method:

1. Compute factored loads = $P_u(^{k/f})$ and unfactored loads = $P(^{k/f})$
2. Assume a footing thickness: h (in)
3. Calculate the weight of the footing per foot of width: $w_{ftg} = 0.15kcf(h''/12''^{/f})$
4. Calculate the weight of the soil on the footing $= w_s = \gamma_s(h_{ftg} - h)$
5. Calculate net allowable soil pressure $= p_{net} = p_s - w_{ftg} - w_s$
6. Calculate maximum allowable soil pressure $= p_{max} = (P_u/P)(p_{net})$
7. Calculate required footing width $= t_{ftg} = P_u/p_{max}$ and round up to the next inch.

8. If $P_u/b < p_{max}$, recalculate factored soil pressure = P_u/b
9. Find effective depth d = h″ – 3″ cover – .5′ (assuming #8 bar)
10. Check whether shear reinforcement is required:

    Shear reinforcement is not required in footings if $\phi V_c > V_u$

    $V_u = p_{max}(G)$ where $G = (L_1 - t)/2 - d$



32.3

Shear in a wall footing

$\phi V_c = .75(2)\sqrt{(f'_c\ psi)}\ bd/1000^{\#/k}$

11. The moment is maximum at 1/4 of the wall thickness into the wall, therefore the moment arm = L = $(L_1 - t)/2 + t/4 = L_1/2 - t/4$

    $M_u = p_{max}L^2/2$
12. Find area s steel required:

    $A_s = 0.85f'_c bd/f_y[1 \pm \sqrt{[1 - 2M_u/\phi(.85f'_c bd^2)]}$in²/ft of wall

    $A_{s\ min} = bd(3\sqrt{f'_c})/f_y \geq 200bd/f_y$ for beams and

    $A_{s\ min} = .0018bh$ for slabs

    Use the larger of the three values:
13. Check development length of the transverse bars: (see Chapter 30 for explanation of development length)
14. Longitudinal steel: As min = .0018bh

Example 32-1: Design a wall footing for an 8″ concrete wall (t = 8″) DL = 8k/f, LL = 16k/f, f'_c = 3ksi, f_y = 60ksi, soil density = γ_s = 80pcf, allowable soil pressure = 4000psf.

The bottom of the footing must be 3.5′ below grade.

1. Compute factored loads: (1.2(8) + 1.6(16))(1′) = 35.2k = P_u

   Unfactored loads = P = (8 + 16)(1′) = 24k
2. Assume footing thickness: h = 18″
3. w_{ftg} = 0.15kcf(1.5′) = 0.225ksf
4. w_s = 80pcf(3.5′ – 1.5′) = 160psf = 0.16ksf
5. Net allowable soil pressure = $p_{net} = p_s - w_{ftg} - w_s$ = 4.0ksf – .225ksf – .16ksf = 3.615 ksf
6. Maximum allowable soil pressure = $p_{max} = (P_u/P)(p_{net})$ = (35.2/24)(3.615ksf) = 5.302ksf
7. Required footing width = $L_1 = P_u/p_{max}$ = 35.2/5.302 = 6.639′, round up to 6′–8″ = 6.67′ = 80″
8. Recalculate factored soil pressure: = $p_u = P_u/L_1$ = 35.2^k/f^/6.67′ = 5.277ksf < 5.302ksf … okay
9. Find effective depth assuming #8 bars:

   d = 18″ – 3″ cover – .5″ = 14.5″
10. Shear reinforcement is not required in footings if $\phi V_c > V_u$. Since the footing width is 6.67′ and wall width is 8″/12″^/f^ = 0.67′, the length of the footing on either side = (6.67 – .67)/2 = 3′.

    d = 14.5/12 = 1.208′

    V_u = (3.0 – 1.208′)(1′)(5.302ksf) = 9.50k

    $\phi V_c = .75(2)\sqrt{(f'_c)}\ d(12'')/1000^{\#/k}$ = 14.3k

    Since ϕV_c = 14.3 > 9.501 = V_u … No shear reinforcement necessary.
11. M_{max} is at $\frac{1}{4}$ of the wall thickness into the wall. Wall thickness = 8″ … M_{max} is 2″ into the wall.

    Moment arm = 2″/12″^/f^ + 3′ = 3.16′

    M_u = 5.302ksf(3.16ft)²/2 = 26.47k-f = 26.47k-f(12″^/f^)(1000#/k) = 317,640#-in
12. $A_s = 0.85f'_c bd/fy[1 \pm \sqrt{[1 - 2M_u/\phi(.85f'cbd^2)]}$ in²/ft of wall = [(.85(3000)(12)(14.5))/60000][1 – √[1 – 2(317,640#-in)/.9(.85(3000)(12)(14.5)#-in)(14.5″)] = .417in²/ft of wall

    $A_{s\ min} = bd(3\sqrt{f'_c})/f_y \geq 200bd/f_y$ for beams $3\sqrt{f'_c}$ = 164.32 < 200 … use 200, $A_{s\ min}$ = 200(12″)(14.5″)/60000 = 0.58 for beams and

    $A_{s\ min}$ = .0018(12″)(18″) = 0.389 for slabs

    Use larger of the three values: A_s = 0.58in²/f

    #6: A = 0.442, spacing = 12″(0.442/0.58) = 9.14″ round down to 9.0″

    Use #6 @ 9″o.c. A_s = 0.589in²
13. Check development length of the transverse bars: (see Chapter 30 for explanation of development length)

    $K_d = 3f_y/40\sqrt{f'_c} = 3(60000)/[40(\sqrt{3000})] = 82.16$

    ρ_t = 1.0 (no top reinforcement)

    ρ_e = 1.0 (no epoxy coating on bars)

$\rho_s = 0.8$ (#6 or smaller bars)

$\lambda = 1.0$ (normal weight concrete)

$\rho_t \rho_e = 1.0 < 1.7$ … okay

c_b = smaller of cover or half spacing = 3.38″

$K_{tr} = 0$

$(c_b + K_{tr})/d_b = 3.38/0.75 = 4.51'' > 2.5''$ … Use 2.5″

$L_d = (K_d/\lambda)(\rho t \rho e \rho s)(d_b)/[(c_b + K_{tr})/d_b] = 82.8(.8)(.75)/2.5 = 19.728$

You may use K_{er} factor $= A_{s\ req'd}/A_{s\ used} = .574/.589 = .975$

$L_d = 19.728(.975) = 19.23''$

L_d provided = critical length for moment – 3″ cover = 3.08′(12″/f) – 3″ = 33.96″ > 19.06″ … okay

14. Longitudinal steel: $A_{s\ min} = .0018bh = .0018(6.67')(12''/f)(18'') = 2.59in^2$
    USE: six #6 bars spaced equally

32.1.2 Rules of Thumb for Non-reinforced Wall Footings with Light Loads:

8″ wall minimum

Footing depth ≥ wall width

Footing width = 2 × wall width

32.2 Individual Footings

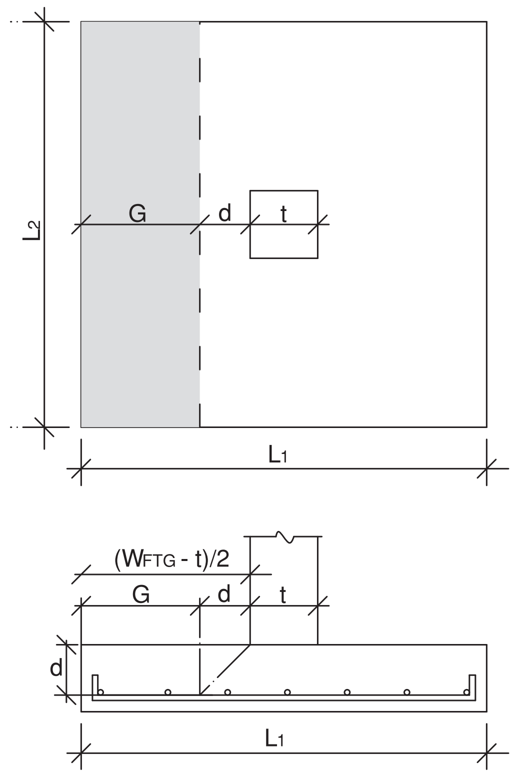
Individual footings may be any shape, but are typically square or rectangular. They may support any type of column or vertical truss system and may or may not have a concrete pedestal.

Design of square footings with width = L_1 = length = L_2

1. Assume footing thickness h = 24″
2. Find net allowable soil pressure = p_{net} = allowable soil pressure – concrete weight – soil weight
3. Find required footing area = A_{REQ} = unfactored loads/net allowable soil pressure. Round up and compute actual area, A.
4. Calculate factored soil pressure $p_u = P_u/A$
5. Calculate d: d = h – 3″ cover – 2rows(d_b/2)
6. Check one-way shear: if $\phi V_c > V_u$, no shear reinforcement necessary.
   One-way shear in a footing – beam shear

   $V_u = p_u L_2 G$

   $\phi V_c = .75(2\sqrt{f'_c})bd$



32.4
One-way shear in a footing

7. Check two-way shear in a footing – punching shear

   $B = t + 2(d/2) = t + d$

   $V_u = p_u(L_1^2 - B^2)$

   V_c = smallest of:

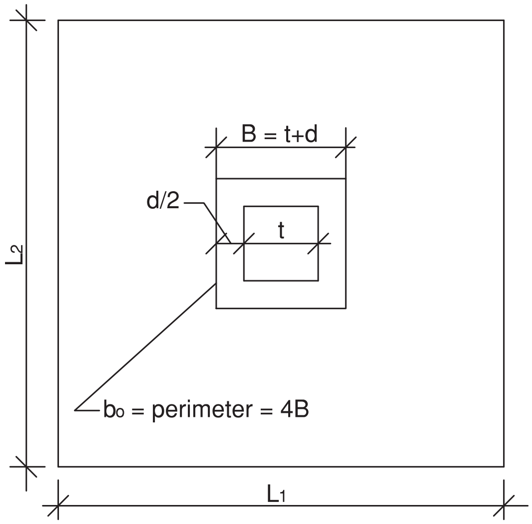
   $V_c = (2 + 4/\beta_c)\sqrt{f'_c} b_o d$ Where $\beta_c = L_1/L_2 = 1$ for square footings and $b_o = 4B$

   Or

   $V_c = (\alpha_s d/b_o + 2)\sqrt{f'_c} b_o d$ where $\alpha_s = 40$ for interior columns, 30 for edge columns and 20 for corner columns.

Or

$V_c = 4\lambda\sqrt{f'_c}b_o d$

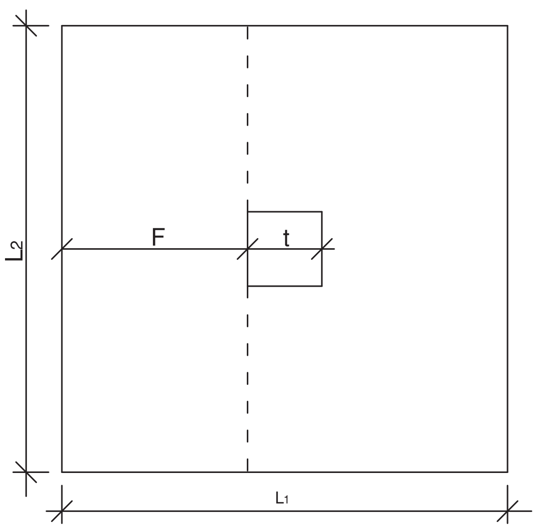


32.5

Two-way shear in a footing

8. Calculate moment: $M_u = p_u L_2(F^2/2)$ where:

   $F = (L_1 - t)/2$



32.6

Moment in a footing

9. Find Area s steel required: $(b = L_2)$

   $A_s = 0.85f'_c bd/f_y[1 \pm \sqrt{[1 - 2M_u/\phi(.85f'_c bd^2)]}$ in^2

   $A_{s\ min} = bd(3\sqrt{f'_c})/f_y \geq 200bd/f_y$ for beams and

   $A_{s\ min} = .0018bh$ for slabs

   Use larger of the three values:
10. Check development length of steel
11. Check that the concrete bearing strength at base of column in the column and the footing are $> P_u$

    $A_1 = t$ = area of column

    $A_2 = L_1^2$ = area of footing

    Col. bearing strength $= \phi(.85f'_c A_1)$

    Ftg. Bearing Strength $= \phi(.85f'_c A_1)\sqrt{(A_2/A_1)} < \phi(.85f'_c A_1)$ (2)
12. Calculate dowel Area $= A_s d = .005A_1$ and check development length:

    $L_{dc} = (.02f_y/\lambda\sqrt{f'_c})(d_b)$(Required A_sd/providedA_sd) $\geq .0003f_y d_b$

Example 32-2: Design an individual column where: DL = 400k, LL = 150k, allowable soil pressure = 5ksf, f'_{ccol} = 4ksi, f'_{cftg} = 3ksi, soil density = 100pcf, 24″×24″ column, bottom of footing is 4′ below grade, supporting interior column.

1. Assume footing thickness h = 24″
2. Find net allowable soil pressure $= p_{net}$

   = 5ksf – .15(24/12) – .1(24/12) = 4.5ksf
3. Required footing area $= A_{REQ}$ = (400 + 150)/4.5 = 122.22f^2

   Round up: Footing size = 11.25′ by 11.25′ = 126.56f^2
4. Find factored soil pressure $p_u = P_u/A$

   = (1.2(400) + 1.6(150))/126.56 = 720/126.56 = 5.689ksf
5. Calculate $d = h - 3'' - d_b$ = 24 – 3 – 1 = 20″ (assuming #8bars)
6. Shear: if $\phi V_c > V_u$, no shear reinforcement necessary.

   One-way shear – beam shear

   G = (135 – 24)/2 – 20 = 35.5″

   $V_u = p_u L_2 G$ = 5.689ksf(11.25′)(35.5/12) = 189.34k

   $V_c = 2\sqrt{f'_c}bd = 2\sqrt{3000\text{psi}}(135\text{in})(20\text{in})/1000\#/\text{k}$ = 295.77k

   189.34k < .75(295.77k) = 221.83k … okay

   Two-way shear – punching shear

   B = 24 + 20 = 44″

   $V_u = p_u(W^2 - B^2) = 5.689(11.25^2 - 3.67^2)$ = 643.53k

V_c = smallest of:

$V_c = (2 + 4/\beta_c)\sqrt{f'_c} b_o d = 6\sqrt{3000}(4)(44)(20)/1000 = 1376.67k$

Or

$V_c = (\alpha_s d/b_o + 2)\sqrt{f'_c} b_o d = (40(20)/4(44) + 2)\sqrt{3000}(4)(44)(20)/1000 = 1261.95k$

Or

$V_c = 4\lambda\sqrt{f'_c} b_o d = 4(1)\sqrt{3000}(4)(44)(20)/1000 = 771.2k$

$V_u = 643.53 > \phi V_c = .75(771.2) = 578.4k$ NO GOOD!

The footing depth must be increased, go back to step 1

1A. Increase footing depth to 27″

2A. $p_{net} = 5ksf - .15(27/12) - .1(21/12) = 4.49ksf$

3A. $A_{REQ} = (400 + 150)/4.49 = 122.49f^2$. Use 11.25′ × 11.25′ $= 126.56f^2$

4A. $p_u = P_u/A = (1.2(400) + 1.6(150))/126.56 = 720/126.56 = 5.689ksf$

5A. d = 27 − 3 − 1 = 23″ (assuming #8bars)

6A. Shear: if $\phi V_c > V_u$, no shear reinforcement necessary. One-way shear – beam shear

$G = (135 - 24)/2 - 23 = 32.5$

$V_u = p_u L_2 G = 5.689ksf(11.25')(32.5/12) = 173.34k$

$V_c = 2\sqrt{f'_c} bd = 2\sqrt{3000psi}(135in)(23in)/1000\#/k = 340.14$

173.34 < .75(340.14) = 255.1 … the footing is adequate for one-way shear.

7A. Two-way shear – punching shear

B = 24 + 23 = 47″

$V_u = p_u(L_1^2 - B^2) = 5.689(11.25^2 - 3.92^2) = 632.59k$

V_c = smallest of:

$V_c = (2 + 4/\beta_c)\sqrt{f'_c} b_o d = 6\sqrt{3000}(4)(47)(23)/1000 = 1421.01$

Or

$V_c = (\alpha_s d/b_o + 2)\sqrt{f'c} b_o d = (40(23)/4(47) + 2)\sqrt{3000}(4)(47)(23)/1000 = 1632.65$

Or

$V_c = 4\lambda\sqrt{f'c} b_o d = 4(1)\sqrt{3000}(4)(47)(23)/1000 = 947.34$

$V_u = 632.59 < \phi V_c = .75(947.34) = 710.51$ … the footing is adequate for punching shear.

7. Calculate moment: $M_u = p_u L_2(F^2/2)$ where:

$F = (L_1 - t)/2 = (11.25 - 2)/2 = 4.625'$

$M_u = p_u L_2(F^2/2) = 5.689(11.25)(4.625^2/2) = 684.51k\text{-}f = 8214.16k\text{-}in$

8. Find Area s steel required: ($b = L_2$)

$A_s = 0.85f'_c bd/f_y[1 \pm \sqrt{[1 - 2M_u/\phi(.85f'_c bd^2)]}]\ in^2$
$= [.85(3000)(11.25')(12''/f)(23'')/60000]\ [1 \pm \sqrt{[1 - 2(8214.16k\text{-}in)(1000\#/k)/.9(.85(3000psi)(11.25')(12''/f)(23''d)^2)]}] = 6.788in^2$

$A_{s\ min} = bd(3\sqrt{f'_c})/f_y \geq 200bd/f_y = 200(11.25)(12)(23)/60000 = 10.35in^2$

$A_{s\ min} = .0018bh = .0018(11.25')(12''/f)(27'') = 6.561in^2$

USE: larger of the three values:

$A_s = 10.35in^2$: USE: 24 #6 evenly spaced

9. Check development length of steel

$K_d = 3f_y/40\sqrt{f'_c} = 3(60000)/[40(\sqrt{3000})] = 82.16$

$\rho_t = 1.0$ (no top reinforcement)

$\rho_e = 1.0$ (no epoxy coating on bars)

$\rho_s = 1$ (#8 bars)

$\lambda = 1.0$ (normal weight concrete)

$\rho_t\rho_e = 1.0 < 1.7$ … okay

c_b = smaller of cover (= 3″) or half spacing (= (11.25′(12″/f) − 6″ − .75″)/24bars − 0.75″ = 4.59″ spacing) … $c_b = 3''$}

$K_{tr} = 0$

$(c_b + K_{tr})/d_b = 3.0/.75 = 4'' > 2.5''$ … Use 2.5″

$L_d = (K_d/\lambda)(\rho_t\rho_e\rho_s)(d_b)/[(c_b + K_{tr})/d_b]\ [A_{s\ req'd}/A_{s\ used}] = [82.8(1)(1)/2.5][10.25/10.3] = 32.96''$

L_d provided = critical length for moment – 3″ cover = 4.635(12) – 3 = 52.5″ > 32.96″ … okay

10. Check that concrete bearing strength at base of column for column and for footing are $> P_u$.

$A_1 = t^2 = 24^2 = 576in^2$

$A_2 = L_1^2 = 135^2 = 18225in^2$

*Use f'_c for column

** Use f'_c for footing

Col. bearing strength = $\phi(.85f'_cA_1) = .65(.85)(4ksi^*)(576) = 1272.96 > 740K = P_u$ … okay

Footing bearing strength = $\phi(.85f'_cA_1)\sqrt{(A_2/A_1)} < \phi(.85f'_cA_1)(2)$

$\sqrt{(A2/A1} = 5.625 > 2$ … use 2

$\phi(.85f'_cA1)(2) = .65(.85)(3ksi^{**})(576)(2) = 1909.44 > 740k = P_u$ … okay

11. Calculate dowel area = $A_{sd} = .005A_1 = .005(576) = 2.88in^2$ … use four # 8, $A_s = 3.16in^2$
    Check development length:

    $L_{dc} = (.02f_y/\lambda\sqrt{f'_c})(d_b)(\text{Required } A_{sd})/\text{provided } A_{sd}) \geq .0003f_yd_b$

    $L_{dc} = (.02(60000)/(1)\sqrt{3000})(0.75)(2.88)/3.16) = 14.976'' \geq .0003f_yd_b = 13.5''$

    USE: four #8 × 13.5″ long.

32.3 Combined Footings

There are times when due to heavy loads on adjacent footings or due to the close proximity of a footing to the site line, it becomes necessary to create a combined footing. A combined footing acts like a beam with two concentrated loads that is supported by a uniform load.

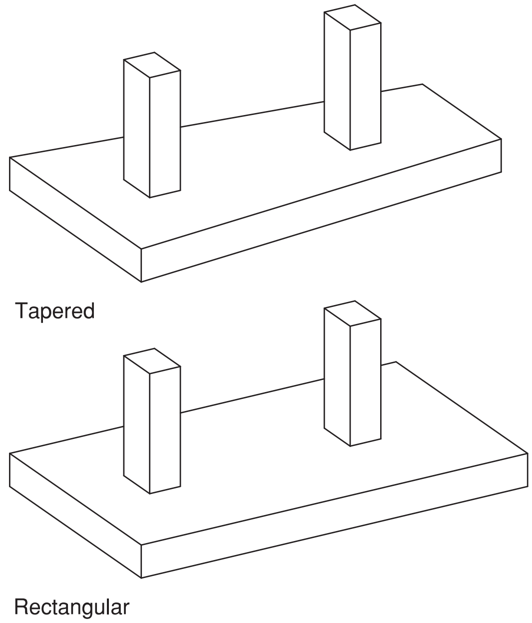
Example 32-3: Design a rectangular combined footing for the two columns shown in Figure 32.8.

The allowable soil pressure = 4ksf, f'_c = 3ksi and f_y = 60ksi. Soil density = 80pcf. Column A is 16″ × 16″ and carries a dead load of 50k and live load of 200k. Column B is 20″ × 20″ and carries a dead load of 80k and a live load of 300k.

1. R = resultant of the column loads. NOTE: Do not factor loads

    50 + 200 + 80 + 300 = 630k = R

    X = the location of the resultant, R



32.7
Combined footing

630X = (50 + 250)(0) + (80 + 300)(16′) = 6080k-f

X = 6080/630 = 9.65′

2. Find the length of footing:
    maximum distance to left = 9.65 + 2′ = 11.65′
    maximum footing length = 11.65′(2) = 23.3′, round down to 23.25′ = 279″

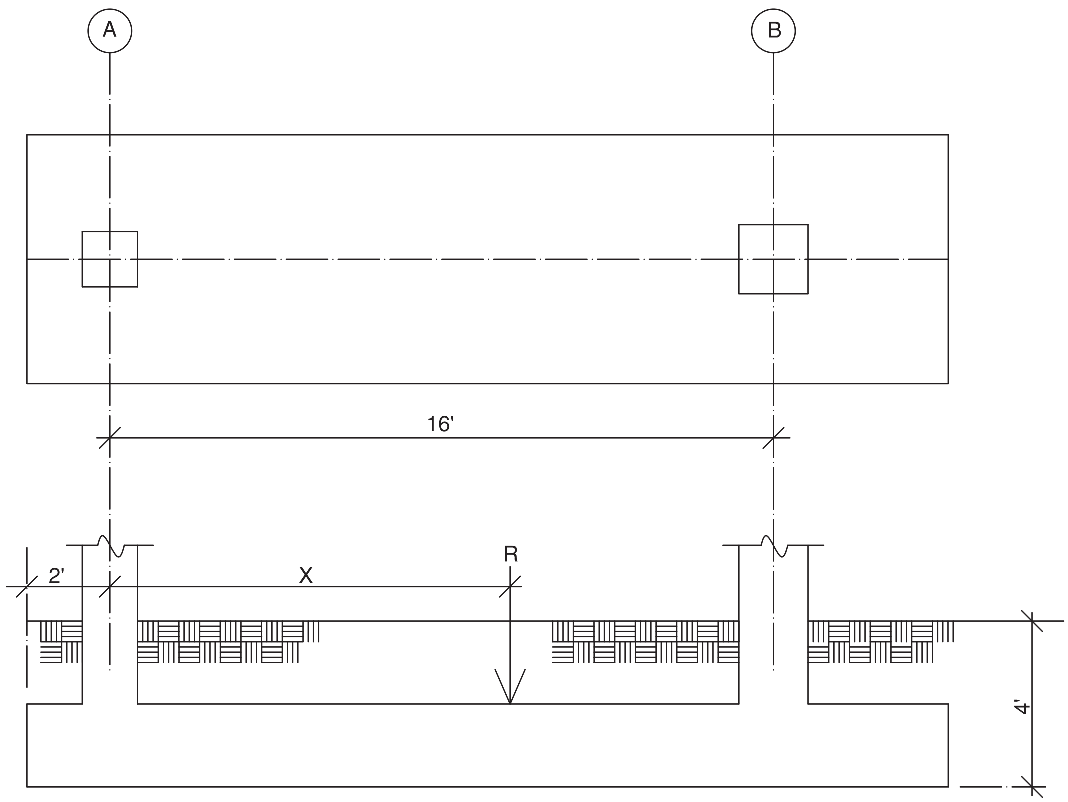
    L = 23.25ft.

3. Find footing width:
    assume h = 24in
    net soil pressure: p_{net} = 4ksf – 2(.15) – (.08ksf)(2′) = 3.54ksf

    $A = R/p_{net} = 630/3.54 = 177.97sf$

    b = A/L = 177.97/23.25 = 7.65′

    round up to W = 7.75′ = 93″
4. Draw shear diagram and find M_u in longitudinal direction:
    USE: FACTORED LOADS:
    Column A: P_L = 1.2(50) + 1.6(200) = 380k
    Column B: P_R = 1.2(80) + 1.6(300) = 576k
    Soil weight and beam weight can be ignored because their effect is offset by an equivalent uniform soil pressure.



32.8

Example 32-3

The uniform reaction in response to the column loadings

= (380 + 576)/23.25f = 41.12k/f

M_u = 2′(82.24k)/2 – 297.75k(7.24′)/2 = –995.62k-f

= 11,947.44k-in

5. Find required depth of footing: Assuming that the depth of the equivalent stress block, a = 0.2d, (see chapter 26):

 d = √{M_u/[.153ϕf′cb]} = √{11947.44/[.153(.9)(3ksi)(93″)]}

 = 17.63″

6. Find depth for one-way and punching shear: Consider Column A to have L_1 = 2′ + 7.24′ = 9.24′ (zero point on moment diagram), and Column B to have

 L_1 = 23.25′ – 9.24′ = 14.01′

 One-way shear – beam shear

 Column A: V_u = 297.75k – (9″ + d″)(41.12k/f)/(12″/f)

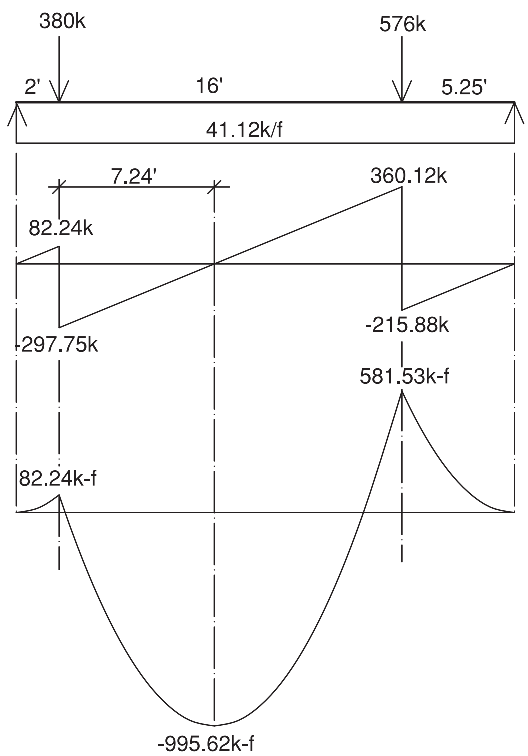
 = 297.75 – 30.84 – 3.427d = 266.91 – 3.427d

ϕV_c = 2√f′cbd = .75(2)√3000psi(93)d/1000#/k = 7.64d

d = 266.91/(7.64 + 3.427) = 24.1″

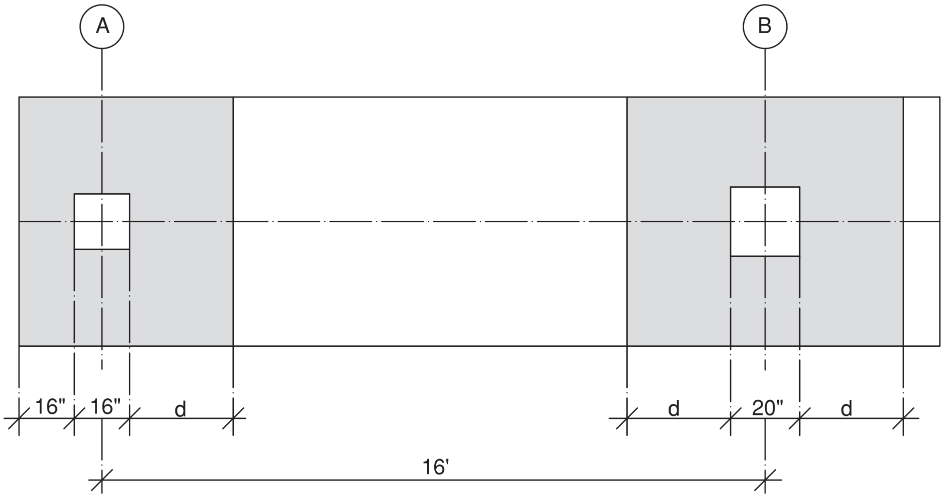
 Column B: V_u = 360.12 – (10 + d)(41.12)/12

 = 325.73 – 3.427d



32.9

Reaction and moment in combined column



32.10

One-way shear in combined column

$\phi V_c = 2\sqrt{f'c}bd = .75(2)\sqrt{3000psi}(93)d/1000\#/k = 7.64d$

$d = 325.73/(7.64 + 3.427) = 29.43''$

USE: $d = 30''$, $h = 33''$

Check two-way shear – punching shear

Column A: (edge column)

$A_1 = 9.24'(7.75') = 71.61f^2$

$A_2 = (16'' + 30'')^2 /(144in^2/f^2) = 14.69f^2$

$\sqrt{(71.61/14.69)} = 2.21 > 2.0 \ldots \beta_c = 2$

$V_u = p_u(A_1 - A_2) = ((41.12k/f)/7.75')(71.61 - 14.69)$
$= 302.01k$

$b_o = 4(16 + 30) = 184''$

V_c = smallest of:

$V_c = (2 + 4/\beta_c)\sqrt{f'c}b_0d = (2 + 4/2)(\sqrt{3000})(184)(30)/1000$
$= 1209.37k$

Or

$V_c = (\alpha_s d/b_0 + 2)\sqrt{f'c}b_0d = (30(30)/184 + 2)(\sqrt{3000})$
$(184)(30)/1000 = 2083.54k$

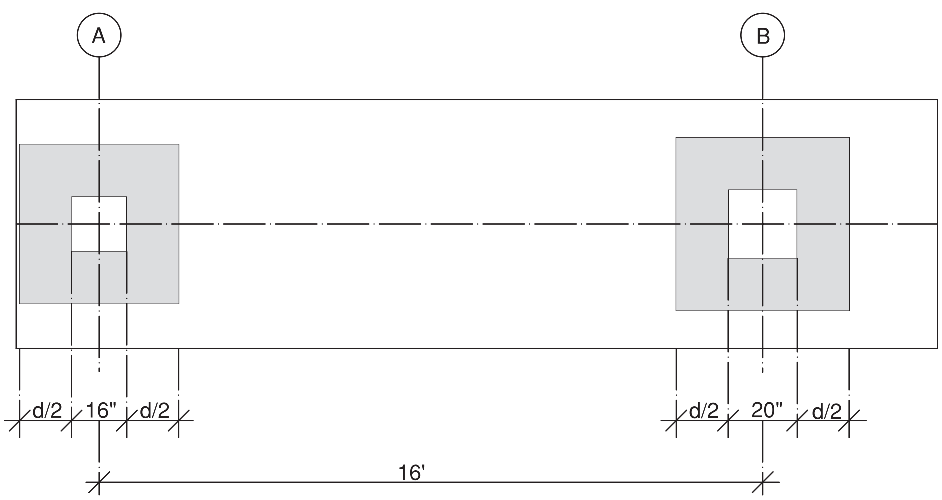
Or

$V_c = 4\lambda\sqrt{f'c}b_0d = (4)(\sqrt{3000})(184)(30)/1000 = 1209.37k$

$\phi V_c - .75(1209.37) = 907.03k > 302.01k \ldots$ okay

Column B: (interior column)

$A_1 = 14.01'(7.75') = 108.58f^2$



32.11

Punching shear in combined column

$A_2 = (20'' + 30'')^2 / (144in^2/f^2) = 17.36f^2$

$\sqrt{(108.58/17.36)} = 2.5 > 2.0 \ldots \beta_c = 2$

$V_u = p_u(A_1 - A_2) = ((41.12k/f)/7.75')(108.58 - 17.36)$
$= 484.00k$

$b_o = 4(20 + 30) = 200''$

V_c = smallest of:

$V_c = (2 + 4/\beta_c)\sqrt{f'c}b_0d = (2 + 4/2)(\sqrt{3000})(200)(30)/1000$
$= 1314.53k$

Or

$V_c = (\alpha_s d/b_o + 2)\sqrt{f'c}b_0d = (40(30)/200 + 2)(\sqrt{3000})$
$(200)(30)/1000 = 2629.07k$

Or

$V_c = 4\lambda\sqrt{f'c}b_0d = (4)(\sqrt{3000})(200)(30)/1000 = 1314.53k$

$\phi V_c - .75(1314.53) = 985.9 > 484k \ldots$ okay

7. Compute flexural steel for positive moment:
Column A: $M_u = 82.24k\text{-}f = 82.24(12000)\#\text{-}in$
$= 986{,}880\#\text{-}in$

$A_s = 0.85f'_c bd/fy[1 \pm \sqrt{[1 - 2M_u/\phi(.85f'cbd^2)]}$
$= [0.85(3000)(93)(30)/60000][1 \pm \sqrt{[1 - 2(986{,}800)/}$
$((.9)(.85(3000)(93)(30)^2))] = 0.61in^2$

USE: two #5
Column B: $M_u = 581.53k\text{-}f = 581.53(12000)\ \#\text{-}in$
$= 6{,}978{,}360\#\text{-}in$

$A_s = 0.85f'_c bd/f_y[1 \pm \sqrt{[1 - 2M_u/\phi(.85f'_c bd^2)]}$
$= [0.85(3000)(93)(30)/60000][1 \pm \sqrt{[1 - 2(6{,}978{,}360)/}$
$((.9)(.85(3000)(93)(30)^2))] = 4.39in^2$

USE: six #8

8. Compute flexural steel for negative moment

$M_u = 995.62k\text{-}f = 995.62(12000)\#\text{-}in = 11{,}947{,}440\#\text{-}in$

$A_s = 0.85f'_c bd/f_y[1 \pm \sqrt{[1 - 2M_u/\phi(.85f'cbd^2)]}$
$= [0.85(3000)(93)(30)/60000][1 \pm \sqrt{[1 - 2(11{,}947{,}440)/}$
$((.9)(.85(3000)(93)(30)^2))] = 7.62in^2$

USE: ten #8
Compute transverse steel:

$M_u = [(37.5''/12''/f)2/2](1')[41.12k/f/7.775'] = 25.91k\text{-}f$
$= 310{,}877\#\text{-}in$

$A_s = 0.85f'_c bd/f_y[1 \pm \sqrt{[1 - 2M_u/\phi(.85f'_c bd^2)]}$
$= [0.85(3000)(12)(30)/60000][1 \pm \sqrt{[1 - 2(310{,}877)/((.9)}$
$(.85(3000)(12)(30)^2))] = 0.193in^2/f$

$A_{s\ min} = .0033(12''/f)(30'') = 1.18in^2/f$

$A_{s\ min} = .0018(12''/f)(33'') = 0.71in^2/f$

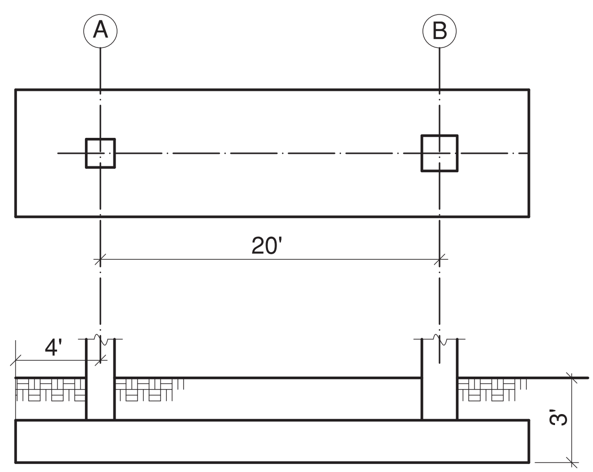
USE: #8 @ 8"o.c.

Practice Exercises:

32-1: Design a wall footing for an 8" concrete wall (t = 8") DL = 6k/f, LL = 12k/f, f'_c = 3ksi, f_y = 60ksi, soil density = γ_s = 80pcf, allowable soil pressure = 4000psf. The bottom of the footing must be 4.0' below grade.

32-2: Design an individual column where: DL = 200k, LL = 500k, allowable soil pressure = 3ksf, $f'_{c\ col}$ = 4ksi, $f'_{c\ ftg}$ = 3ksi, soil density = 80pcf, 28"×28" column, bottom of footing is 3' below grade, supporting interior column.

32-3: Design a rectangular combined footing for the two columns shown in Figure 32.12. The allowable soil pressure = 3.5ksf, f'_c = 3ksi and f_y = 60ksi. Soil density = 90pcf. Column A is 18"×18" and carries a dead load of 100k and live load of 300k. Column B is 20"×20" and carries a dead load of 100k and a live load of 500k.



32.12
Chapter 32 Practice exercises

thirty three

Precast and Precast and Prestressed Concrete

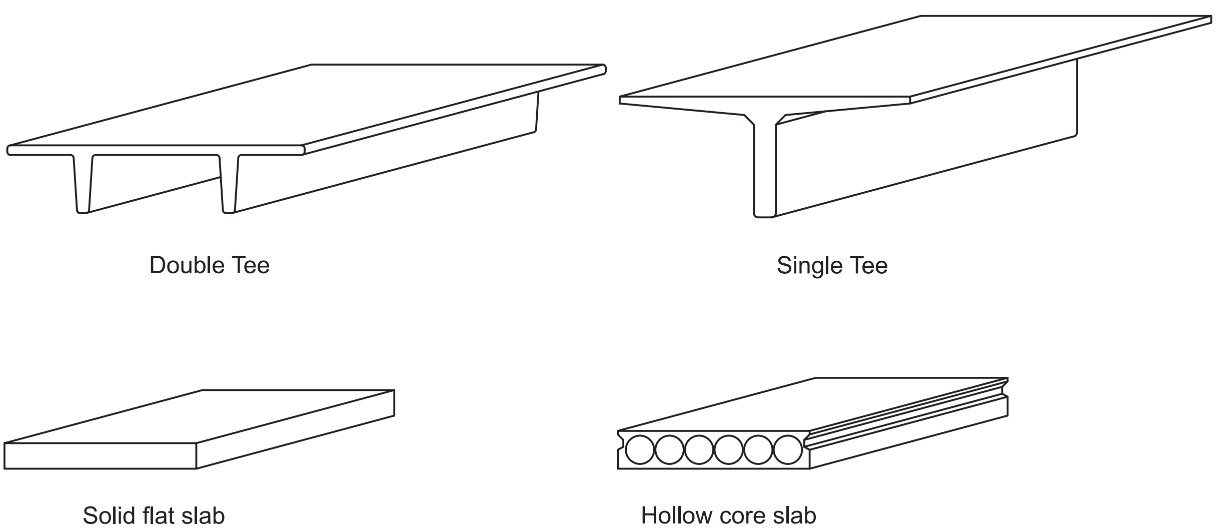
33.1 Precast Concrete

Precast concrete is concrete that is cast and cured prior to installation as a solid component of a structural system or as a non-structural component. In this text, only precast components that support and transfer applied loads are discussed.

Precast concrete is commonly used as floor slabs, shear walls, bearing walls, lintels, staircases and columns in structural systems. Many other precast components can be used in building design to form railings, decorative façade elements and the like.

Limitations in size are typically the limitations provided by transportation such as the length of a truck bed. There are also design limitations based on the amount of manipulation during transport. For example, a unit designed as a vertical wall panel must be able to support its own weight if laid flat for transport.

Precast concrete has the advantage of reducing on-site construction time. Because the unit is cast independently, there is no delay on the construction site to allow for curing; units can support weight immediately after installation. Precast concrete also has the advantage of being mixed,



33.1

Typical precast structural components

placed and finished in a controlled environment. As such, any defective units can be discarded before reaching the building site. High strength concrete and the use of reactive powder concrete can result in lighter weight and longer spans. Precasting allows for the economical creation and reuse of intricate formwork for detailed units, allowing architectural details to remain economical.

The strength of precast units is determined, as with site-cast components, by the strength of the materials (f'_c and f_y), the cross-sectional properties, and the placement of reinforcement. But because precast units are cast and cured independently of each other, there is no continuity of material. Therefore, connections and joints must be carefully designed to consider gravity and lateral loads, expansion and contraction, and product tolerances. Bolted or welded connections can be made between adjacent precast elements or between precast and site-cast or metal components by embedding a steel plate or angle into the precast unit. Figure 33.2 shows some typical precast connections. Design of these connections follows the same methods as discussed earlier in this text.

33.1.1 Precast Concrete Floor Slabs

Precast concrete floor slabs are generally hollow core slabs (HC slabs). The advantages of a hollow core slab include the fact that HC slabs are lightweight relative to cast-in-place slabs to the same depth. Most manufacturers supply widths of 2′, 4′ and 8′, although some offer 10′ and 12′ widths. Slabs are available in depths from 6 to 16″. As a rule of thumb, floor slabs can span 30 × depth and roof slabs can span 40 × depth. They maximize ceiling height when the voids are used for conduit runs. Further, when prestressed, the slabs can be delivered to site with a camber that will negate deflections.

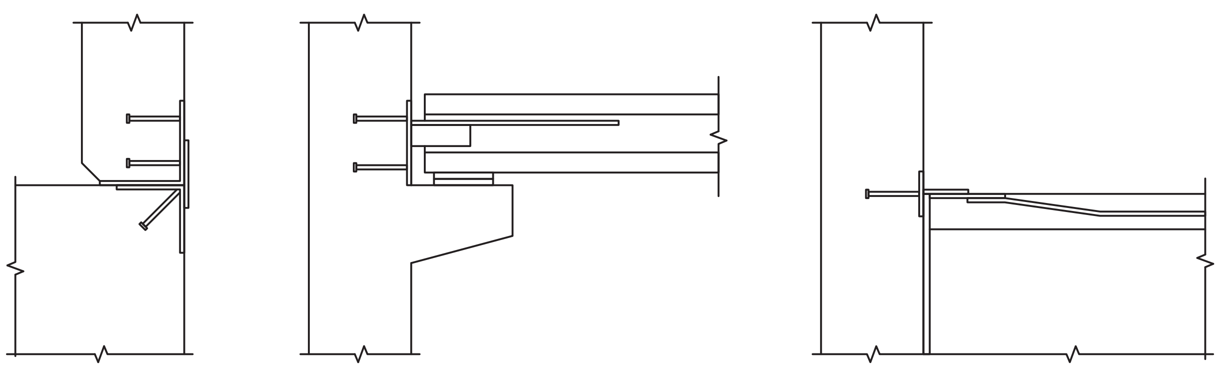
Standard HC slabs are manufactured in depths from 4″ to 15″, although deeper slabs have been created for specialized purposes or by specific manufacturers. Load tables for HC slabs are provided by the manufacturer using an equivalent uniform load: $w_{equivalent} = (1.4/1.7)DL + LL$ when uniform design loads are present and $w_{equivalent} = 8M_u/L^2$ when other loads are present.

As with all precast components, connection details are important to prevent disaster. HC slabs may have an embedded plate that welds to a bearing plate or the voids may be filled at the end to create a sufficient development length in which to insert a rebar from the bearing wall or beam. HC slabs may also be embedded into a masonry wall.

Follow ACI codes and refer to the Precast Concrete Institute's (PCI) manual for the design of hollow core slabs.

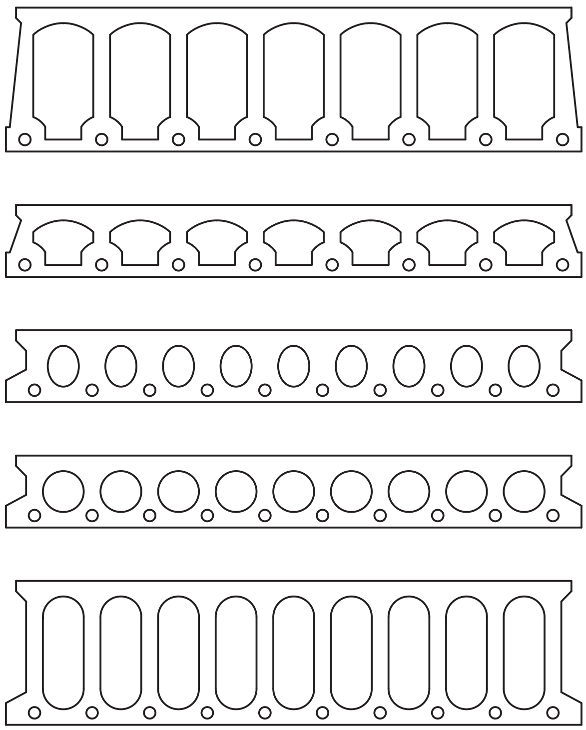
33.1.2 Precast Concrete Beams and Tees

Precast beams are usually prestressed and have a cross-section that is either rectangular, an inverted Tee or an L shape. Although they typically vary in width from 12 to 24″



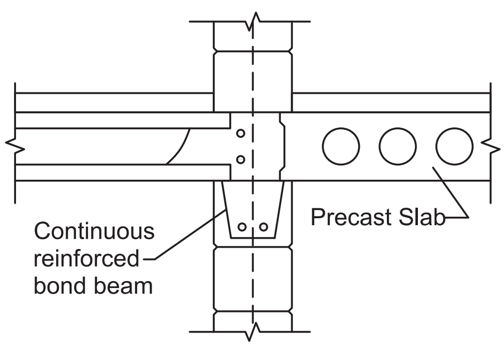
33.2

Typical precast connections



33.3

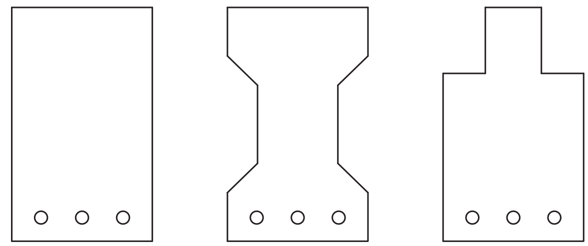
Typical precast hollow core slab cross-sections



33.4

Precast hollow core slab embedded in masonry wall, reproduced with permission from the American Concrete Institute

and in depth from 16 to 40″, they can be made in any size specified. The inverted Tee and L shape are often used to support slabs that will have a concrete cover. As a rule of thumb, spans are between 10 and 20 times the depth.



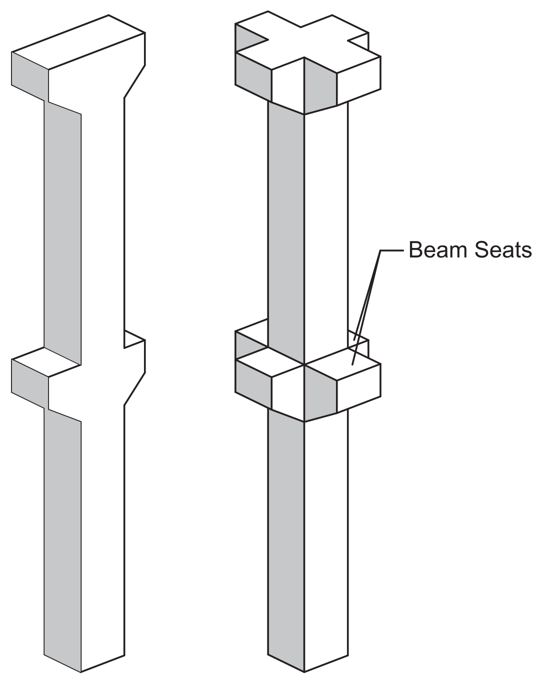
33.5

Precast beams

Double Tees are used for floor and roof systems. They are usually supplied in widths of 8, 10, 12 or 15′ and depths from 24 to 34″. Double Tees used in floors can span to about 35′ and in roofs to about 40′ depending on the loads.

33.1.3 Precast Columns

Precast columns are typically used to support beams in precast systems in low to mid-rise buildings. The columns are designed to be stacked and often have side ledges to support beams. They range from 12 × 12″ to 24 × 48″ in size and can be almost any shape. It should be noted, however, that the columns are cast in a horizontal position, so one side will be flat and troweled to match the surfaces of the other three sides.



33.6

Precast columns

33.1.4 Other Precast Components

Shear and load-bearing walls can be designed using precast components. They range in size from 15′ to 30′ widths, 10′ to 30′ heights, and 8″ to 16″ thicknesses. Because of the large size, they are often cast at the site in a horizontal position and then tilted into place.

Concrete piles are designed to be stacked and connected together to form longer piles when needed. They are typically 18 by 18″ but can be made smaller or larger and in any shape. Large piles often have a hollow core to eliminate unnecessary weight.

Precast stair units are commonly supplied in a large variety of standard or custom sizes. Cast either upside down or on end, the underside of the stair is smooth. The top riser is installed flush with the top of the finished floor.

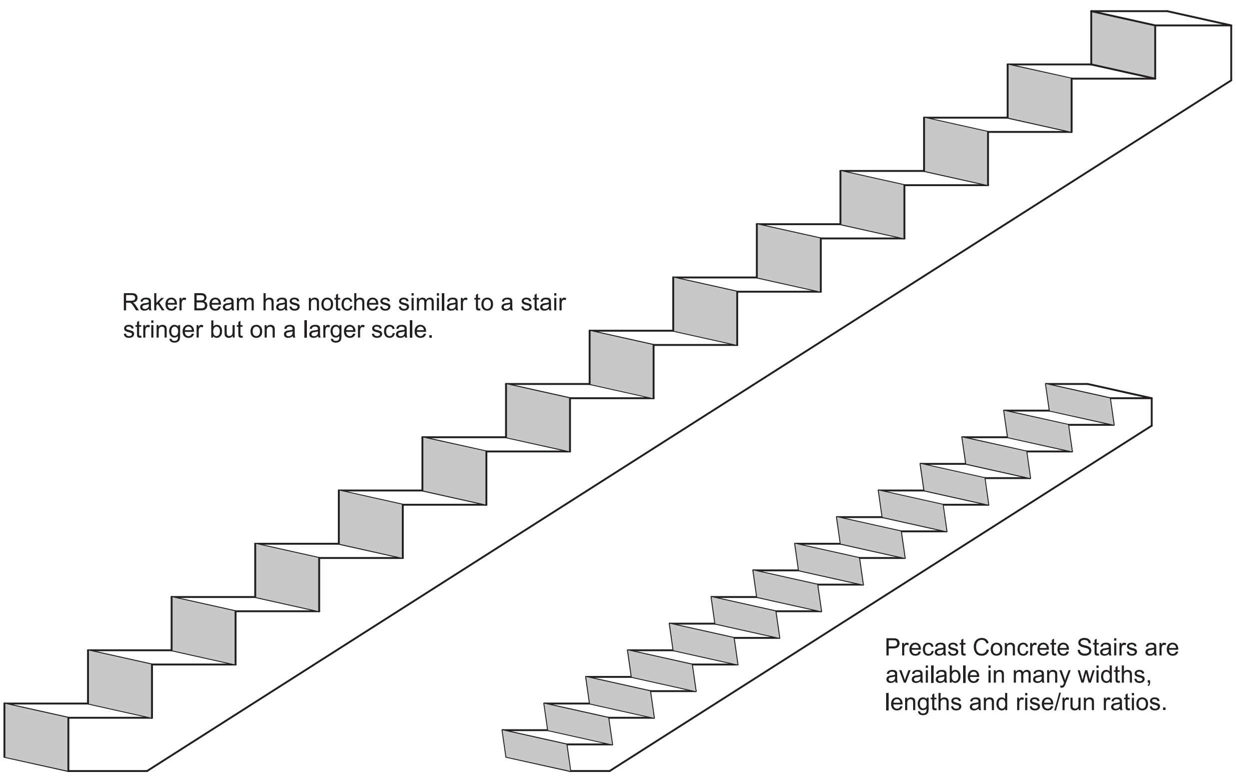
Stadiums often utilize precast raker beams or stadium risers which are similar to a stair stringer in that they support the horizontal floor decks under the seating in a stepped fashion as shown in Figure 33.7.

33.2 Prestressed Concrete

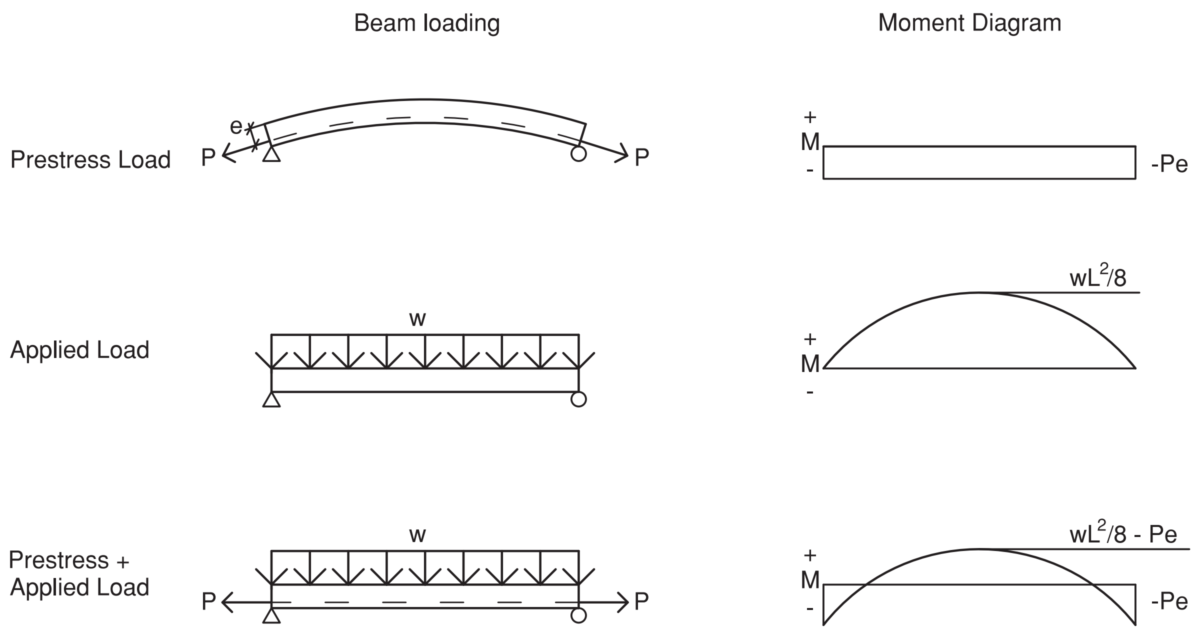
Prestressing creates compressive stress in concrete to counteract the tension that will develop due to applied loads. Prestressing can be achieved either by post-tensioning or by pre-tensioning.

In post-tensioning, cables called tendons are stressed in tension after concrete is placed. Concrete members are cast with hollow tubes, through which tendons are pulled. After the concrete strengthens to a certain point, the tendons are tensioned.

In pre-tensioning, tendons are stressed in tension before the concrete is placed. After the concrete cures, the tension is released in the cable, but because the cable has bonded to the concrete, it transfers that release into the concrete as compression. Components are usually cast off-site and then shipped to the site. The cable is typically a 7-wire, uncoated cable with an ultimate stress, f_{pu} ranging from 250 to 270ksi.



33.7
Precast stairs and rakers



33.8
Prestressing in a concrete beam

One important factor to consider when designing a prestressed beam is the mode of failure. As discussed in Chapter 25, concrete beams must be designed to fail in tension as failure in compression is sudden and without warning. If too much prestressing is used, the concrete beam risks failure in compression. Another factor to consider is creep. Creep is long-term deflection and as such reduces the effect of prestressing.

Design of a prestressed concrete beam must consider both the bending stress due to applied loads and the axial stress due to the prestressing.

1. Applied loads create a positive moment that is counteracted with an internal couple creating compression at the top and tension at the bottom. The compression force due to applied loads $= C_{app} = T_{app} = M_u/[\phi(d - a/2)]$
2. Prestressing force, P, creates compression over the entire cross-section. The compressive stress resulting from prestressing $= f_{pre} = P/bh$ acts over the entire cross-section. Therefore, the maximum stress in the equivalent stress block that can be used to counteract M_u is $f_{app} = .85f'_c - f_{pre}$. This means that the depth of the equivalent stress block (a) will vary with different values of prestressing force P.
3. The goal of prestressing is to eliminate the tensile strain at the bottom of the beam that is produced by the applied loads. When tendons have a sag, (e) in inches, there is a negative moment produced by the eccentricity of the sag. $M_{pre} = Pe$. When $M_{pre} = M_u$, the resultant moment on the beam is $M_u - M_{pre} = 0$.

   $P = M_u^{k\text{-}in}/[e(\# \text{ of tendons})]$
4. It is necessary to check the depth, a of the equivalent stress block to ensure that it is within the top half of the depth of the beam. Since $C_{app} = abf_{app} = M_u/[\phi(d - a/2)]$. Solving for a yields $a = d - \sqrt{[d^2 - 2M_u/\phi bf_{app}]}$

Example 33-1: Given a 40′ long 16″×24″ beam with f'_c = 4000psi, a DL of .1k/f and a LL of .8k/f.

1. Determine prestressing force required if two parabolic tendons with a sag of 9″ are used.

   Beam weight = w = .15pcf(16/12)(24/12) = 0.4 k/f

   Factored load = 1.2(0.4 + .1) + 1.6(.8) = 1.88k/f

   Moment = M_u = 1.88k/f$(40)^2$/8 = 376k-f = 4,512,000#-in

   Prestressing force = P = M_u/sag = 4512000/[(9″)(2tendons)] = 250,666.7#
2. Check equivalent stress block depth, a:

   $f_{app} = .85f'_c - P/bh$ = .85(4000) − 250,666.7/(16(24) = 2747.22psi

$a = d - \sqrt{[d^2 - 2M_u/\phi bf_{app}]}$
$= 21 - \sqrt{[21^2 - 2(4512000)/(.9)(16)(2747.22)]} = 6.41''$

$c = a/.85 = 7.54'' < h/2 = 24/2 = 12''$ … okay

Example 33-2: Given a 40′ long, 9″ deep slab with a DL of .01ksf and a LL of .08ksf.

1. Determine the prestressing force required for parabolic tendons with a sag of 3″ every 6″. d = 7.875″

   Slab weight = w = .15pcf(9/12) = 0.1125k/f

   Factored load for 12″ swath = [1.2(0.1125 + .01) + 1.6(.08)] 12/12 = 0.275 k/f

   Moment = $M_u = .275\text{k/f}(40)^2/8 = 55\text{k-f} = 660{,}000\#\text{-in}$

   Prestressing force = $P = M_u/\text{sag} = 660000\text{k-in}/(3(2)) = 110{,}000\#$

2. Check equivalent stress block depth, a:

   $f_{app} = .85f'_c - P/bh = .85(4000) - 110000/(12(9))$
   $= 2381.48\text{psi}$

   $a = d - \sqrt{[d^2 - 2M_u/\phi bf_{app}]} = 7.875 - \sqrt{[7.875^2 - 2(660000)/(.9)(12)(2381.48)]} = 4.6''$

   $c = 4.6''/.85 = 5.42'' > 9/2 = 4.5''$ … okay

Practice Exercises:

33-1: Design a 40′ long 14″×30″ beam with $f'_c = 4000$psi, a DL of .1k/f and a LL of .8k/f using two tendons with a sag of 12″.

33-2: Design a 48′ long, 12″ deep slab with a LL of .1ksf using tendons every 6″.

Part VI

Masonry and Alternate Materials

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thirty four

Masonry Design

Masonry is a structural system made of units of clay, concrete or stone connected by mortar. The units may vary in size and shape and in pattern of arrangement. Masonry is used structurally as vertical compression members. Deep wall reinforced masonry wall beams can handle flexure, but are not discussed in this text.

34.1 Masonry Load Bearing Walls

Based on the MSJC 'Building Code Requirements for Masonry Structures' developed by the Masonry Standards Joint Committee, ACI503/ASCE 5/TMS402 and the IBC, the height/thickness ratio = length/thickness ratio = 20 for solid, unreinforced, load bearing walls. For example, a 24′ high masonry load bearing wall would have a minimum thickness, $t = 24'(12''/f)/20 = 14.4''$.

Mortar types M, S, N are used in load bearing walls. Although mortar provides binding between units, the allowable tensile stresses are very low and vary between 40 and 70 psi.

f'_m = masonry prism test compressive strength. Masonry distributes loads applied on a single unit to successive supporting units as shown in Figure 34.1. The prism test evaluates compressive strength based on the distribution of load in a given bond pattern of a wall.

Allowable bending stress:

$F_b = f'_m/3$ for unreinforced masonry

$F_b = 0.45f'_m$ for reinforced masonry

Allowable shear stress:

$F_v = 1.5\sqrt{f'_m} \leq 120$psi for unreinforced masonry

$F_v = 2.0\sqrt{f'_m}$ for reinforced masonry where $M/Vd \geq 1.0$

$F_v = 3.0\sqrt{f'_m}$ for reinforced masonry where $M/Vd \leq 0.25$ and may be interpolated for values of $.25 < M/Vd < 1.0$.

Flexure in masonry walls is usually created by lateral loads or a vertical load with an eccentricity. The maximum stress in the equivalent stress block for reinforced masonry is $0.8f'_m$.

$M_u = \phi A_s f_y(d - a/2)$ and $T = A_s f_y = C = .08f'_m ab$ …
$a = A_s f_y/.8f'_m b$

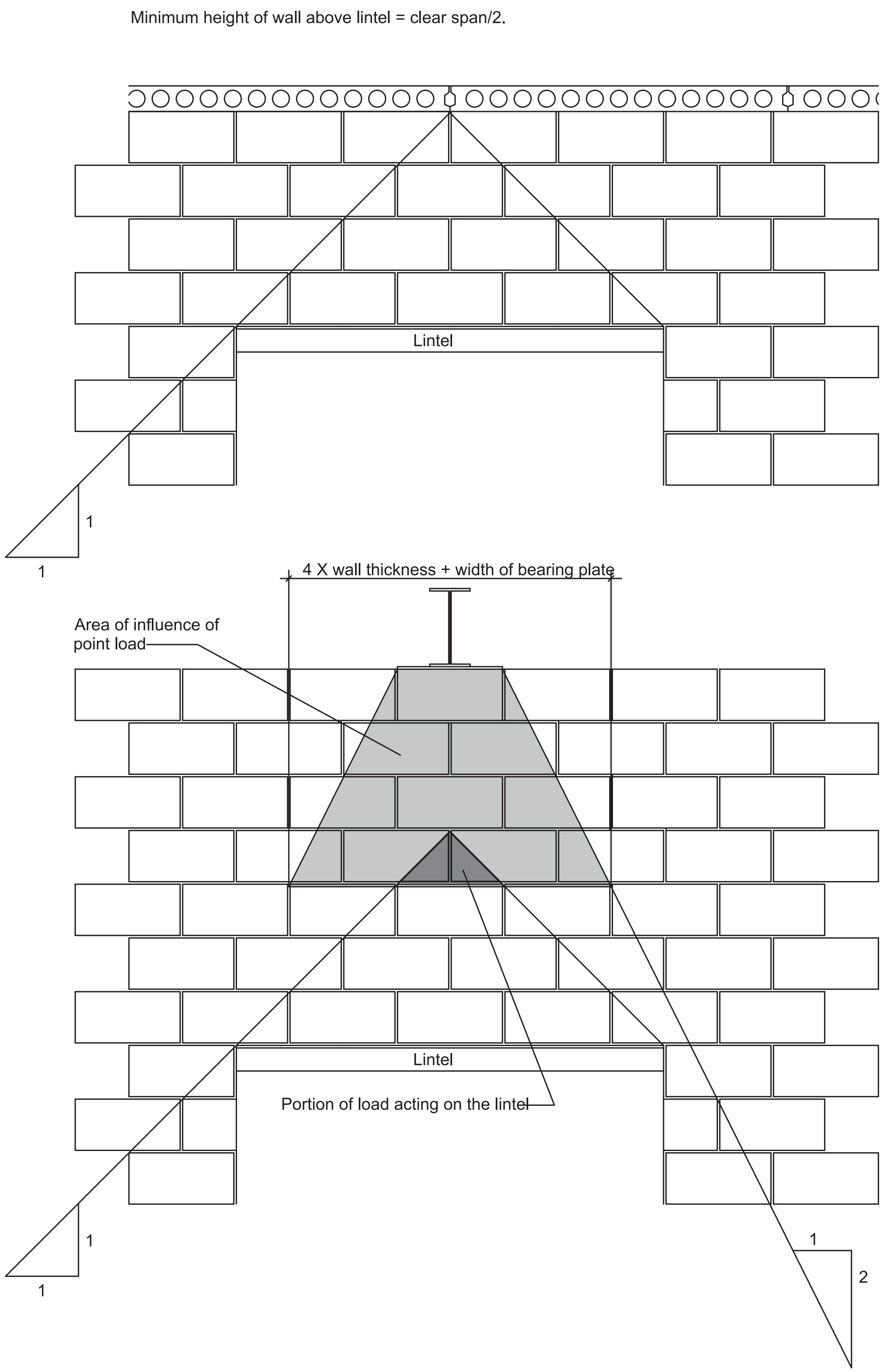
$M_u = \phi A_s f_y(d - A_s f_y/1.6f'_m b)$

$A_s = (.8f'_m b/f_y)(d \pm \sqrt{[d^2 - (2f_y M_u/.8f'_m b\phi f_y)]}$

The combined axial and flexural stresses must be checked to satisfy $f_b/F_b + f_a/F_a \leq 1$.

Example 34-1: Design reinforcement for an 8″ thick masonry wall filled and reinforced with vertical 60ksi rebar at 16″o.c.

There is a uniform load of 1800#/f with an eccentricity of 2″. The wall is 16′ high and the filled weight = 145pcf.
$F'_m = 2000$psi



34.1
Load distribution in a masonry wall

1. Determine M_u for one foot swath of wall:
   M_u = Pe = 1800#/f(12″/f)(2″) = 43200#-in.
2. $A_s = (.8f'_m b/f_y)(d \pm \sqrt{[d^2 - (2f_y M_u/.8f'_m b\phi f_y)]}$ = (.8(2000)(12)/(60000))(4 ± √[4² − (2(60000)(43200)/.8(2000)(12)(.9)(60000))] = .22in²/f = #5 @ 16″o.c.
3. f_b = M/S = 43200#-in/[(12″)(8²)/6] = 337.5psi
4. $F_b = 0.45f'_m$ = .45(2000) = 900psi
5. f_a = P/A = 145pcf(16′)/144in²/f² + 1800#/f/[12″/f(8″)] = 34.86psi
6. $F_a = .8f'_m$ = .8(2000) = 1600psi
7. $f_a/F_a + f_b/F_b$ = 34.86/1600 + 337.5/900 = 0.397 <1.0 … okay
   USE: #5 @ 16″o.c.

Example 34-2: Design reinforcement for a 12″ thick masonry wall filled and reinforced with vertical 60ksi rebar at 16″o.c.

There is a uniform load of 2400#/f centered on the wall and a lateral force of 20psf on the surface of the wall. The wall is 16′ high and the filled weight = 145pcf. F'_m = 3000psi

1. Determine M_u for one foot swath of wall:
   M_u = 20psf(12″/f)(16′)²/8 = 7680#-f = 92160#-in.
2. $A_s = (.8f'_m b/f_y)(d \pm \sqrt{[d^2 - (2f_y M_u/.8f'_m b\phi f_y)]}$ = (.8(2000)(12)/(60000))(4 ± √[4² − (2(60000)(9216000)/.8(3000)(12)(.9)(60000))] = .49in²/f = #8 @ 16″o.c.
3. f_b = M/S = 92160#-in/[(12″)(8²)/6] = 720psi
4. $F_b = 0.45f'_m$ = .45(3000) = 1350psi
5. f_a = P/A = 145pcf(16′)/144in²/f² + 2400#/f/[12″/f(8″)] = 41.11psi
6. $F_a = .8f'_m$ = .8(3000) = 2400psi
7. $f_a/F_a + f_b/F_b$ = 41.11/2400 + 720/1350 = 0.55 <1.0 … okay
   USE: #8 @ 16″o.c.

Practice Exercise:

34-1: Design reinforcement for a 16″ thick masonry wall filled and reinforced with vertical 60ksi rebar. There is a uniform load of 4800#/f centered on the wall and a lateral force of 20psf on the surface of the wall. The wall is 30′ high and the filled weight = 145pcf. F'_m = 3000psi.

thirty five

Alternate Structural Materials

35.1 Concrete, Steel and Wood

Concrete, steel and wood are the basic structural materials covered in most architectural programs. See Table 35.1 for a summary of the advantages and disadvantages of steel, concrete and wood as structural materials.

35.1.1 Fly Ash

Fly ash is a waste product of burning coal. In an effort to find a use for fly ash, it was added to concrete mixes with surprising results. The benefits of fly ash in concrete include higher strength, workability, durability and decreased bleeding, segregation, efflorescence, and permeability. As shown in Figure 35.1, the compressive strength of concrete is lower in the first 28 days and higher after 28 days than plain concrete. The increase in compressive strength continues to grow over time. Check sites such as www.flyash.com for more information on the use of fly ash in concrete. LEED 2.2 offers credits for the use of fly ash in concrete dependent on the amount used.

35.1.2 Reactive Powder Concrete

Reactive powder concrete , also known as ultra-high performance concrete uses fine quartz sand as the largest aggregate mixed with up to 10% steel fibers by volume. It has compressive strengths as high as 120,000 psi and tensile strengths up to 7000 psi. It also has less deflection due to a higher Modulus of Elasticity and less creep (long-term deflection). The drawbacks to reactive powder concrete are that it requires applied pressure while setting, is very expensive and to date there are no codes governing its use.

35.1.3 Air-scrubbing concrete

Concrete that uses titanium dioxide to reduce nitrous oxide pollution. It has the aesthetic benefit of a very white concrete. The disadvantage is the cost which is about 30% higher than plain concrete,

35.2 Alternate Metals

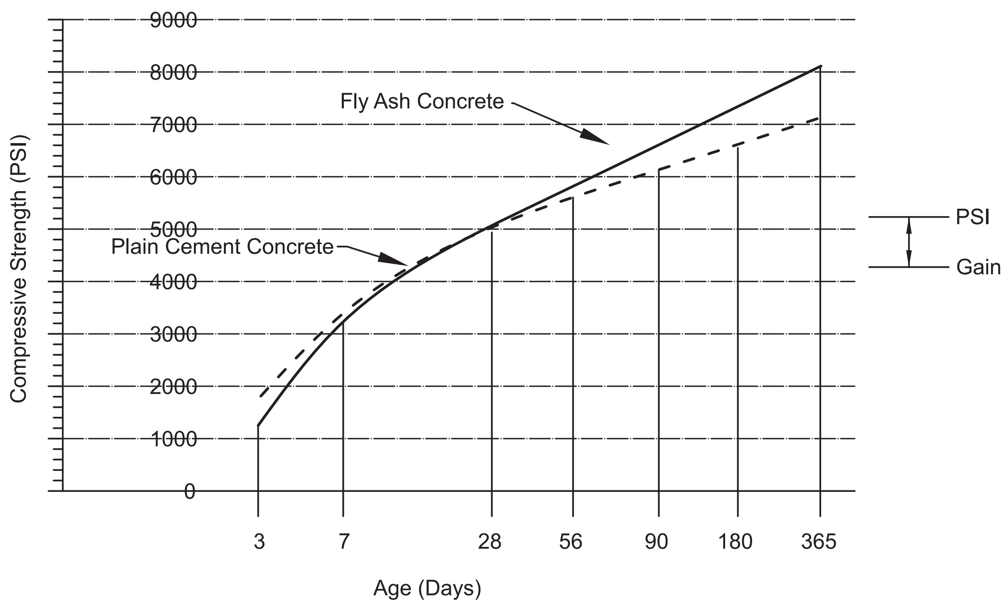
35.2.1 Titanium

Titanium is a high strength metal usually used in alloy form such as Ti-6Al-4V with compressive yield strengths ranging from 125 to 155ksi, shear strength = 79,800psi and a tensile yield strength of 128,000psi.

The advantages of titanium include corrosion resistance. Titanium has a self-healing oxide film that forms spontaneously when exposed to air. This oxide film is stable and protects titanium from corrosion. Titanium

Table 35.1: Advantages and disadvantages of steel, concrete and wood as structural materials

Material	Steel	Concrete	Wood
Advantages	High strength to weight ratio	Strength increases with time	Renewable resource
	High recycle content	May be formed to any shape	Economical
	Easily assembled and disassembled		Easily customized shapes
	Uniformity		
	Elasticity		
Disadvantages	Must be fire-protected	Requires formwork	Flammability
	Corrosion (rust)	Not easily recycled	Must be protected from Fungus and pests
	Fatigue	Strength is dependent on mix of components	
		Strength is dependent on site conditions	



35.1

Compressive strength of fly ash concrete and plain concrete over time.

has the highest strength to weight ratio of all metals. It is lightweight at 283pcf; it has only 58% of the weight of steel. Another advantage to titanium is its low coefficient of thermal expansion. At 4.78μin/in°F, the coefficient of thermal expansion of titanium alloy is only half that of steel, one-third of aluminum and equivalent to glass or concrete. Its thermal conductivity of 10 Btu/hr.-°F/ft. is very low at one-tenth of aluminum. Titanium is an innoxious metal meaning it does not interact with humans. It is non-magnetic and has a high melting point.

The disadvantage of titanium is that it has a low Modulus of Elasticity of about E = 16,000ksi depending on the alloy used, compared to E = 29,000ksi for steel. In identical cross-sections and loads, deflection in titanium will be 29/16 or 1.81 times the deflection in steel. This indicates that titanium is best used in vector-active systems where loads are transferred by compression and tension and flexure is avoided or minimized.

35.2.2 Aluminum

Aluminum is a lightweight metal suitable for light loads. Most often used as metal studs or façade panels, aluminum has the advantage of being lightweight and easily extruded. Although aluminum doesn't rust in the sense that it does not contain iron, it does oxidize, especially when in contact with saltwater. Other disadvantages include a high thermal conductivity. Aluminum transfers heat at roughly three times the rate of steel. Aluminum has a low melting point at 1220°F, compared to steel at 2460°F or titanium at 3000°F. It also has a rate of electrical conductivity six times that of steel, and nearly double the coefficient of linear expansion. But the most significant disadvantage of aluminum from a structural point of view is its low Modulus of Elasticity at E = 10,000ksi. This means that an aluminum beam would have nearly three times the defection of a steel beam with a comparable cross-section and load scenario. Another significant disadvantage is aluminum's high embodied energy. Although aluminum is an element, it does not occur naturally in nature. Like steel, it must be manufactured; but the production of aluminum involves roughly four times the embodied energy of the production of steel. Aluminum is expensive, generally about three times the cost of steel. It does not weld easily and it has high galvanic action. Galvanic action is a corrosion that occurs when two different metals are in contact. And although the galvanic action between aluminum and steel is not as high as between steel and brass, for example, it is significant enough to require the use of gaskets to prevent galvanic action.

35.3 Plant-based Materials

35.3.1 Laminated Bamboo

Raw bamboo has been used as a structural material for centuries. Traditionally, the bamboo pole is used intact and tethered to adjacent poles to create a structure. Mechanical connections are difficult because of the hollow cylindrical shape of the bamboo pole and because of the variability of pole diameters and wall thicknesses.

Bamboo is a rapidly renewable material with a 3–5 year regrowth rate compared to a 20–25 year renew rate for timber. Bamboo yields measured in lb/acre are four times that of wood (Lugt 2006). But perhaps the most significant advantage bamboo has over timber is found in its structural properties. All allowable stresses except for compression parallel to the grain are greater for raw bamboo than those of most wood species. This information indicates that raw bamboo poles are a good material for beams, but not necessarily for columns or other compression members such as top struts in a horizontal truss.

If bamboo is laminated to form structural components, the material properties become significantly better than those of laminated wood. Laminated bamboo (LBL) is ten times stronger in tension and six times stronger in compression and flexure than laminated timber (LVL). And yet, laminated bamboo is only recently becoming a material of interest to designers. Other advantages of LBL are that it has 15% less embodied energy in processing than wood and is 20% more stable than wood in moisture and temperature changes.

35.3.2 Paper

Before Shigeru Ban became famous for his paper tube houses, Martin Pawley built a house of recycled materials with a paper tube structure at Rensselaer Polytechnic Institute in 1976. A 4″ inside diameter paper tube has an ultimate bending stress of 1727psi at 10% moisture content, but paper tubes must be protected from moisture with a full coverage of a waterproof coating or strength diminishes rapidly.

35.4 Plastics

Plastics are man-made materials, usually a petroleum derivative. Biodegradable plastics are obviously not for structural use. There are many types of plastics with varied strengths and mechanical properties. In choosing a plastic, consider not only the compressive and flexural strength, but also ductility and fatigue. While plastics have the advantage of low maintenance, they also have many disadvantages such as high embodied energy and ultraviolet deterioration. Plastics are not a renewable resource and many are not recyclable. If choosing a recyclable plastic, note that mechanical properties vary depending on the recycled content. If using recycled plastic lumber, look for single polymer made from high density polyethylene (HDPE).

35.5 Carbon Fiber

Carbon fiber reinforced polymer (CFRP) is a composite that is extremely lightweight. At 95pcf, it is less than one-fifth the density of steel. Carbon fiber is a tensile material with an ultimate tensile strength of 602ksi and a modulus of elasticity of 33,500ksi. Carbon fiber reinforced polymers utilize the high tensile strength of carbon fiber and combine it with a material high in compression. For example, a CFRP such as HexPly M49 has a tensile yield strength of 107ksi, compressive yield strength of 88.5ksi and a flexural yield strength of 134ksi.

Carbon fibers may be used as tension reinforcement in concrete to increase the flexural strength of a beam, but the amount of fibers used, the orientation and the quality of the bond influences the results.

35.6 Glass

Most glass products including glass block are self-supporting but not considered structural elements. Glass façade panels can transfer wind loads to structural support systems. Structural glass floor systems utilize glass as a decking material with a span up to 48′.

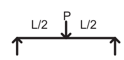
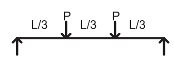
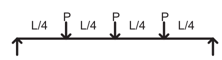
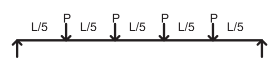
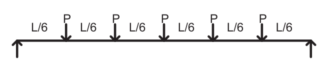
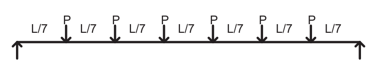
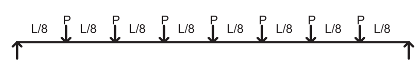
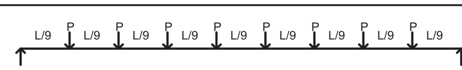
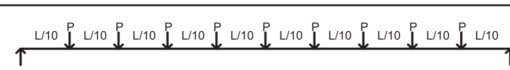
Conclusions

Things to remember:

1. Structural design and analysis are based on Newton's Third Law of Motion. Remember that all forces and moments must be in balance in a static system.
2. Design the structural system from the relationships between spatial, contextual and conceptual patterns.
3. Choose the materials for the system. Every material has inherent strengths. Identify the materials that have the strengths necessary for the chosen structural system.
4. Build redundancy into the structural system.
5. Follow all building codes. Remember this does not mean designing for minimum loads. Anticipate load conditions one hundred years from now and design for the worst case scenario.
6. Make sustainable choices in materials and methods.
7. Be true to your design intent. Do not allow your design to be compromised by a lack of structural understanding and creativity.

Appendices

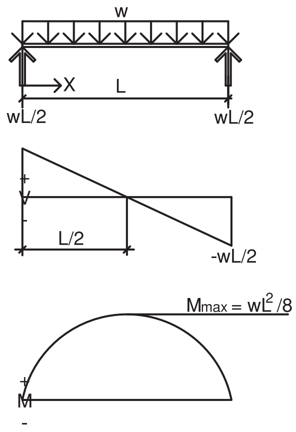
A1: General Information

M_{MAX}	Δ_{MAX}-actual	Δ_{MAX}-EQ. UNIFORM LOAD	Diagram
PL/4	$PL^3/48EI$ $= .0208PL^3/E1$	$5(2P/L)L^4/384EI$ $= .02604PL^3/E1$	
PL/3	$23PL^3/648EI$ $= .0355PL^3/EI$	$5(3P/L)L^4/384EI$ $= .03906PL^3/EI$	
PL/2	$19PL^3/384EI$ $= .0495PL^3/EI$	$5(4P/L)L^4/384EI$ $=.0521PL^3/EI$	
3PL/5	$63PL^3/1000EI$ $=.063PL^3/EI$	$5(5P/L)L^4/384EI$ $= .0651PL^3/EI$	
3PL/4	$11PL^3/144EI$ $= .0764PL^3/EI$	$5(6P/L)L^4/384EI$ $= .0781PL^3/EI$	
6PL/7	$123PL^3/1372EI$ $= .0897PL^3/EI$	$5(7P/L)L^4/384EI$ $= .0911PL^3/EI$	
PL	$79PL^3/768EI$ $= .1029PL^3/EI$	$5(8P/L)L^4/384EI$ $= .1042PL^3/EI$	
10PL/9	$1015PL^3/8748EI$ $= .1160PL^3/EI$	$5(9P/L)L^4/384EI$ $= .1172PL^3/EI$	
5PL/4	$31PL^3/240EI$ $= .1292PL^3/EI$	$5(10P/L)L^4/384EI$ $= .1302PL^3/EI$	

A1.1

Multiple point load chart

1. Simple Beam - uniformly distributed load



$V_{max} = wL/2$

$V_x = w(L/2 - x)$

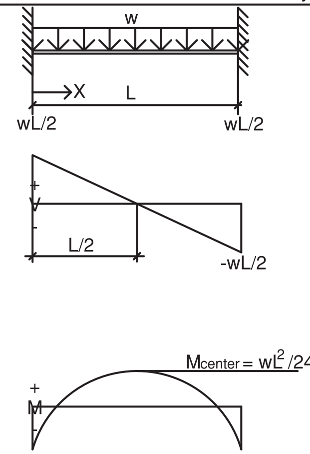
$M_{max} = wL^2/8$

$M_x = wx(L - x)/2$

$\Delta_{max} = \dfrac{5wL^4}{384EI}$

$\Delta_x = \dfrac{wx(L^3 - 2Lx^2 + x^3)}{24EI}$

5. Beam Fixed at Both Ends - uniformly distributed load



$V_{max} = wL/2$

$V_x = w(L/2 - x)$

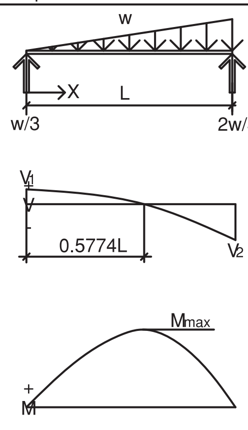
M_{max} (at ends) $= wL^2/12$

$M_x = w(6Lx - L^2 - 6x^2)/12$

$\Delta_{max} = \dfrac{wL^4}{384EI}$

$\Delta_x = \dfrac{wx^2(L - x)^2}{24EI}$

2. Simple Beam - load increasing uniformly to one end



$V_{max} = 2w/3$

$V_x = w/3 - wx^2/L^2$

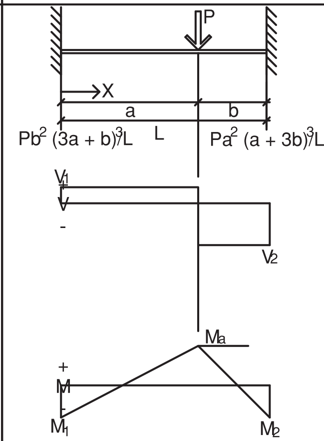
M_{max} (at x = .5574L) = .128wL

$M_x = \dfrac{wx(L^2 - x^2)}{3L^2}$

Δ_{max} (at x = 0.519L) $= \dfrac{0.0130wL^3}{EI}$

$\Delta_x = \dfrac{wx(7L^4 - 10L^2x^2 + 3x^4)}{180EIL^2}$

6. Beam Fixed at Both Ends - concentrated load at any point



V_1 (= V_{max} when a < b) $= Pb^2(3a + b)/L^3$

V_2 (= V_{max} when a > b) $= Pa^2(a + 3b)/L^3$

M_a (at point of load) $= 2Pa^2b^2/L^3$

M_1 (= M_{max} when a < b) $= Pab^2/L^2$

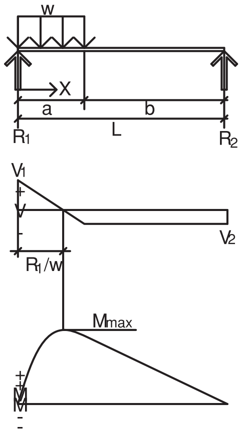
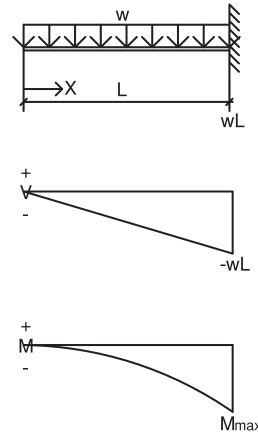
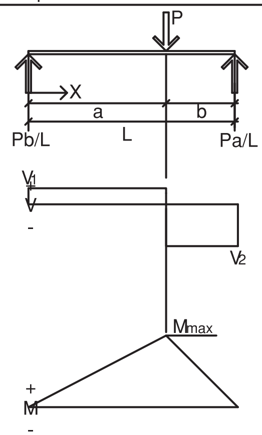
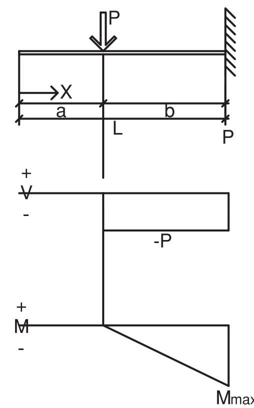
M_2 (= M_{max} when a > b) $= Pa^2b/L^2$

M_x (when x < a) $= V_1x - Pab^2/L^2$

Δ_{max} (at $x = \dfrac{2aL}{3a + b}$, when a > b) $= \dfrac{2Pa^3b^2}{3EI(3a + b)^2}$

Δ_a (at point of load) $= \dfrac{Pa^3b^3}{3EIL^3}$

Δ_x (when x < a) $= \dfrac{Pb^2x^2}{6EIL^3}(3aL - 3ax - bx)$

3. Simple Beam - uniform load partially distributed at one end	7. Cantilevered Beam - uniformly distributed load
 $R_1 = V_{max} = (2L-a)wa/2L$ $R_2 = V_2 = wa^2/2L$ V_x (when $x < a$) $= R_1 - wx$ M_{max} (at $x = R_1/w$) $= R_1^2/2w$ M_x (when $x < a$) $= R_1 x - wx^2/2$ M_x (when $x > a$) $= R_2 (L - x)$ Δ_x (when $x > a$) $= \frac{wx(a^2(2L-a)^2 - 2ax^2(2L-a) + Lx^3)}{24EIL}$ Δ_x (when $x < a$) $= \frac{wa^2(L-x)(4xL - 2x^2 - a^2)}{24EIL}$	 $V_{max} = wL$ $V_x = wx$ M_{max} (at fixed end) $= wL^2/2$ $M_x = wx^2/2$ Δ_{max} (at free end) $= \frac{wL^4}{8EI}$ $\Delta_x = \frac{w(x^4 - 4L^3 x + 3L^4)}{24EI}$
4. Simple Beam - concentrated load at any point	**8. Cantilevered Beam - concentrated load at any point**
 V_1 ($= V_{max}$ when $a < b$) $= Pb/L$ V_2 ($= V_{max}$ when $a > b$) $= Pa/L$ M_{max} (at point of load) $= Pab/L$ M_x (when $x < a$) $= Pbx/L$ Δ_{max} (at $x = \sqrt{a(a + 2b)/3}$, when $a > b$) $= \frac{Pab(a + 2b)\sqrt{3a(a + 2b)}}{27EIL}$ Δ_a (at point of load) $= \frac{Pa^2b^2}{3EIL}$ Δ_x (when $x < a$) $= \frac{Pbx(L^2 - b^2 - x^2)}{6EIL}$	 $V = P$ M_{max} (at fixed end) $= Pb$ M_x (when $x > a$) $= P(x - a)$ Δ_{max} (at free end) $= \frac{Pb^2}{6EI}(3L - b)$ Δ_a (at point of load) $= \frac{Pb^3}{3EI}$ Δ_x (when $x < a$) $= \frac{Pb^2}{6EI}(3L - 3x - b)$ Δ_x (when $x > a$) $= \frac{P(L - x)^2}{6EI}(3b - L + x)$

A1.2

Deflection charts

Table A1.3: Materials properties of selected metal materials

MATERIAL PROPERTIES FOR SELECTED MATERIALS						
MATERIAL	F_b	F_v	F_t	F_C	E	Density
A36 Steel	21.6 ksi	14.4 ksi	21.6 ksi	21.6 ksi	29,000 ksi	0.49kcf
A992 Steel	30 ksi	20 ksi	30 ksi	30 ksi	29,000 ksi	0.49 kcf
Aluminum	40 ksi	30 ksi	40 ksi	40 ksi	10,000 ksi	0.168 kcf
Titanium Alloy	138 ksi	80 ksi	138 ksi	141 ksi	16,500 ksi	0.276 kcf
Douglas Fir Larch	1,200 psi	85 psi	700 psi	1,000 psi	1,700,000 psi	35pcf
Southern Pine	1,500 psi	90 psi	900 psi	1,000 psi	1,600,000 psi	35pcf
Glu-lams	2,400 psi	165 psi	1,100 psi	1,650 psi	1,800,000 psi	45pcf

A2: Wood Design

Table A2.1: Section properties for dimensional lumber

DIMENSIONAL LUMBER SECTION PROPERTIES												
DIMENSIONAL LUMBER									Western Species			So. Pine
NOMINAL SIZE	b	d	A	Sx	Ix	Sy	Iy	Cfu	C_F for F_b	C_F for F_t	C_F for F_c	C_F for Fb
2X3	1.5	2.50	3.75	1.56	1.95	0.94	0.70	1.00	1.50	1.50	1.15	1.0
2X4	1.5	3.50	5.25	3.06	5.36	1.31	0.98	1.10	1.50	1.50	1.15	1.0
2X6	1.5	5.50	8.25	7.56	20.80	2.06	1.55	1.15	1.30	1.30	1.20	1.0
2X8	1.5	7.25	10.88	13.14	47.63	2.72	2.04	1.15	1.20	1.20	1.05	1.0
2X10	1.5	9.25	13.88	21.39	98.93	3.47	2.60	1.20	1.10	1.10	1.00	1.0
2X12	1.5	11.25	16.88	31.64	177.98	4.22	3.16	1.20	1.00	1.00	1.00	1.0
2X14	1.5	13.25	19.88	43.89	290.78	4.97	3.73	1.20	0.90	0.90	0.90	1.0
3X4	2.5	3.50	8.75	5.10	8.93	3.65	4.56	1.10	1.50	1.50	1.15	1.0
3X6	2.5	5.50	13.75	12.60	34.66	5.73	7.16	1.15	1.30	1.30	1.20	1.0
3X8	2.5	7.25	18.13	21.90	79.39	7.55	9.44	1.15	1.20	1.20	1.05	1.0
3X10	2.5	9.25	23.13	35.65	164.89	9.64	12.04	1.20	1.10	1.10	1.00	1.0
3X12	2.5	11.25	28.13	52.73	296.63	11.72	14.65	1.20	1.00	1.00	1.00	1.0
3X14	2.5	13.25	33.13	73.15	484.63	13.80	17.25	1.20	0.90	0.90	0.90	1.0
3X16	2.5	15.25	38.13	96.90	738.87	15.89	19.86	1.20	0.90	0.90	0.90	1.0
4X4	3.5	3.50	12.25	7.15	12.51	7.15	12.51	1.00	1.50	1.50	1.15	1.0
4X6	3.5	5.50	19.25	17.65	48.53	11.23	19.65	1.05	1.30	1.30	1.10	1.0
4X8	3.5	7.25	25.38	30.66	111.15	14.80	25.90	1.05	1.30	1.20	1.05	1.1
4X10	3.5	9.25	32.38	49.91	230.84	18.89	33.05	1.10	1.20	1.10	1.00	1.1
4X12	3.5	11.25	39.38	73.83	415.28	22.97	40.20	1.10	1.10	1.00	1.00	0.9
4X14	3.5	13.25	46.38	102.41	678.48	27.05	47.34	1.10	1.00	0.90	0.90	0.9
4X16	3.5	15.25	53.38	135.66	1034.42	31.14	54.49	1.10	1.00	0.90	0.90	0.9

Table A2.2: Material properties for selected dimensional lumber species, courtesy American Wood Council, Leesburg, VA

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)							Specific gravity
		Bending	Tension parallel to grain	Shear parallel to grain	Compression perpendicular to grain	Compression parallel to grain	Modulus of Elasticity		
		F_b	F_t	F_v	$F_{c\perp}$	F_c	E	E_{min}	G
Douglas Fir–Larch									
Select Structural		1,500	1,000	180	625	1,700	1,900,000	690,000	
No. 1 & Btr		1,200	800	180	625	1,550	1,800,000	660,000	
No. 1	2" & wider	1,000	675	180	625	1,500	1,700,000	620,000	
No. 2		900	575	180	625	1,350	1,600,000	580,000	
No. 3		525	325	180	625	775	1,400,000	510,000	0.50
Stud	2" & wider	700	450	180	625	850	1,400,000	510,000	
Construction		1,000	650	180	625	1,650	1,500,000	550,000	
Standard	2" – 4" wide	575	375	180	625	1,400	1,400,000	510,000	
Utility		275	175	180	625	900	1,300,000	470,000	
Hem–Fir									
Select Structural		1,400	925	150	405	1,500	1,600,000	580,000	
No. 1 & Btr		1,100	725	150	405	1,350	1,500,000	550,000	
No. 1	2" & wider	975	625	150	405	1,350	1,500,000	550,000	
No. 2		850	525	150	405	1,300	1,300,000	470,000	
No. 3		500	300	150	405	725	1,200,000	440,000	0.43
Stud	2" & wider	675	400	150	405	800	1,200,000	440,000	
Construction		975	600	150	405	1,550	1,300,000	470,000	
Standard	2" – 4" wide	550	325	150	405	1,300	1,200,000	440,000	
Utility		250	150	150	405	850	1,100,000	400,000	
Northern Red Oak									
Select Structural		1,400	800	220	885	1,150	1,400,000	510,000	
No. 1	2" & wider	1,000	575	220	885	925	1,400,000	510,000	
No. 2		975	575	220	885	725	1,300,000	470,000	
No. 3		550	325	220	885	425	1,200,000	440,000	0.68
Stud	2" & wider	750	450	220	885	450	1,200,000	440,000	
Construction		1,100	650	220	885	975	1,200,000	440,000	
Standard	2" – 4" wide	625	350	220	885	750	1,100,000	400,000	
Utility		300	175	220	885	500	1,000,000	370,000	
Red Maple									
Select Structural		1,300	750	210	615	1,100	1,700,000	620,000	
No. 1	2" & wider	925	550	210	615	900	1,600,000	580,000	
No. 2		900	525	210	615	700	1,500,000	550,000	
No. 3		525	300	210	615	400	1,300,000	470,000	0.58
Stud	2" & wider	700	425	210	615	450	1,300,000	470,000	
Construction		1,050	600	210	615	925	1,400,000	510,000	
Standard	2" – 4" wide	575	325	210	615	725	1,300,000	470,000	
Utility		275	150	210	615	475	1,200,000	440,000	
Redwood									
Clear Structural		1,750	1,000	160	650	1,850	1,400,000	510,000	0.44
Select Structural		1,350	800	160	650	1,500	1,400,000	510,000	0.44
Select Structural, open		1,100	625	160	425	1,100	1,100,000	400,000	0.37
No. 1		975	575	160	650	1,200	1,300,000	470,000	0.44
No. 1, open grain	2" & wider	775	450	160	425	900	1,100,000	400,000	0.37
No. 2		925	525	160	650	950	1,200,000	440,000	0.44
No. 2, open grain		725	425	160	425	700	1,000,000	370,000	0.37
No. 3		525	300	160	650	550	1,100,000	400,000	0.44
No. 3, open grain		425	250	160	425	400	900,000	330,000	0.37
Stud	2" & wider	575	325	160	425	450	900,000	330,000	0.44
Construction		825	475	160	425	925	900,000	330,000	0.44
Standard	2" – 4" wide	450	275	160	425	725	900,000	330,000	0.44
Utility		225	125	160	425	475	800,000	290,000	0.44

Table A2.3: Material properties for selected dimensional Southern Pine lumber, Courtesy American Wood Council, Leesburg, VA

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)							
		Bending	Tension parallel to grain	Shear parallel to grain	Compression perpendicular to grain	Compression parallel to grain	Modulus of Elasticity		Specific gravity
		F_b	F_t	F_v	F_c	F_c	E	E_{min}	G
Southern Pine									
Select Structural	2" - 4" wide	2,850	1,600	175	565	2,100	1,800,000	660,000	0.55
No. 1		1,850	1,050	175	565	1,850	1,700,000	620,000	
No. 2		1,500	825	175	565	1,650	1,600,000	580,000	
Construction	4" wide	1,100	625	175	565	1,800	1,500,000	550,000	0.55
Standard		625	350	175	565	1,500	1,300,000	470,000	
Utility		300	175	175	565	975	1,300,000	470,000	
Select Structural	5" - 6" wide	2,550	1,400	175	565	2,000	1,800,000	660,000	0.55
No. 1		1,650	900	175	565	1,750	1,700,000	620,000	
No. 2		1,250	725	175	565	1,600	1,600,000	580,000	
Select Structural	8" wide	2,300	1,300	175	565	1,900	1,800,000	660,000	0.55
No. 1		1,500	825	175	565	1,650	1,700,000	580,000	
No. 2		1,200	650	175	565	1,550	1,600,000	580,000	
Select Structural	10" wide	2,050	1,100	175	565	1,850	1,800,000	660,000	0.55
No. 1		1,300	725	175	565	1,600	1,700,000	620,000	
No. 2		1,050	575	175	565	1,500	1,600,000	580,000	
Select Structural	12" wide	1,900	1,050	175	565	1,800	1,800,000	660,000	0.55
No. 1		1,250	675	175	565	1,600	1,700,000	620,000	
No. 2		975	550	175	565	1,450	1,600,000	580,000	

Table A2.4: Timber section properties

SECTION PROPERTIES FOR TIMBER								
NOMINAL SIZE	b	d	A	Sx	Ix	Sy	Iy	C_F for Fb
5X5	4.50	4.50	20.25	15.19	34.17	15.19	34.17	1.00
6X6	5.50	5.50	30.25	27.73	76.26	27.73	76.26	1.00
6X8	5.50	7.25	39.88	48.18	174.66	36.55	100.52	1.00
6X10	5.50	9.25	50.88	78.43	362.75	46.64	128.25	1.00
6X12	5.50	11.25	61.88	116.02	652.59	56.72	155.98	1.00
6X14	5.50	13.25	72.88	160.93	1066.18	66.80	183.71	0.99
6X16	5.50	15.00	82.50	206.25	1546.88	75.63	207.97	0.98
6X18	5.50	17.00	93.50	264.92	2251.79	85.71	235.70	0.96
6X20	5.50	19.00	104.50	330.92	3143.71	95.79	263.43	0.95
6X22	5.50	21.00	115.50	404.25	4244.63	105.88	291.16	0.94
6X24	5.50	23.00	126.50	484.92	5576.54	115.96	318.89	0.93
8X8	7.25	7.25	52.56	63.51	230.23	63.51	230.23	1.00
8X10	7.25	9.25	67.06	103.39	478.17	81.03	293.75	1.00
8X12	7.25	11.25	81.56	152.93	860.23	98.55	357.26	1.00
8X14	7.25	13.25	96.06	212.14	1405.41	116.08	420.77	0.99
8X16	7.25	15.00	108.75	271.88	2039.06	131.41	476.35	0.98
8X18	7.25	17.00	123.25	349.21	2968.27	148.93	539.86	0.96
8X20	7.25	19.00	137.75	436.21	4143.98	166.45	603.37	0.95
8X22	7.25	21.00	152.25	532.88	5595.19	183.97	666.89	0.94
8X24	7.25	23.00	166.75	639.21	7350.90	201.49	730.40	0.93
10X10	9.25	9.25	85.56	131.91	610.08	131.91	610.08	1.00
10X12	9.25	11.25	104.06	195.12	1097.53	160.43	741.99	1.00
10X14	9.25	13.25	122.56	270.66	1793.11	188.95	873.90	0.99
10X16	9.25	15.00	138.75	346.88	2601.56	213.91	989.32	0.98
10X18	9.25	17.00	157.25	445.54	3787.10	242.43	1121.23	0.96
10X20	9.25	19.00	175.75	556.54	5287.15	270.95	1253.13	0.95
10X22	9.25	21.00	194.25	679.88	7138.69	299.47	1385.04	0.94
10X24	9.25	23.00	212.75	815.54	9378.73	327.99	1516.95	0.93
12X12	11.25	11.25	126.56	237.30	1334.84	237.30	1334.84	1.00
12X14	11.25	13.25	149.06	329.18	2180.82	279.49	1572.14	0.99
12X16	11.25	15.00	168.75	421.88	3164.06	316.41	1779.79	0.98
12X18	11.25	17.00	191.25	541.88	4605.94	358.59	2017.09	0.96
12X20	11.25	19.00	213.75	676.88	6430.31	400.78	2254.39	0.95
12X22	11.25	21.00	236.25	826.88	8682.19	442.97	2491.70	0.94
12X24	11.25	23.00	258.75	991.88	11406.56	485.16	2729.00	0.93
14X14	13.25	13.25	175.56	387.70	2568.52	387.70	2568.52	0.99
14X16	13.25	15.00	198.75	496.88	3726.56	438.91	2907.75	0.98
14X18	13.25	17.00	225.25	638.21	5424.77	497.43	3295.45	0.96
14X20	13.25	19.00	251.75	797.21	7573.48	555.95	3683.15	0.95
14X22	13.25	21.00	278.25	973.88	10225.69	614.47	4070.86	0.94
14X24	13.25	23.00	304.75	1168.21	13434.40	672.99	4458.56	0.93
16X16	15.00	15.00	225.00	562.50	4218.75	562.50	4218.75	0.98
16X18	15.00	17.00	255.00	722.50	6141.25	637.50	4781.25	0.96
16X20	15.00	19.00	285.00	902.50	8573.75	712.50	5343.75	0.95
16X22	15.00	21.00	315.00	1102.50	11576.25	787.50	5906.25	0.94
16X24	15.00	23.00	345.00	1322.50	15208.75	862.50	6468.75	0.93
18X18	17.00	17.00	289.00	818.83	6960.08	818.83	6960.08	0.96
18X20	17.00	19.00	323.00	1022.83	9716.92	915.17	7778.92	0.95
18X22	17.00	21.00	357.00	1249.50	13119.75	1011.50	8597.75	0.94
18X24	17.00	23.00	391.00	1498.83	17236.58	1107.83	9416.58	0.93
20X20	19.00	19.00	361.00	1143.17	10860.08	1143.17	10860.08	0.95
20X22	19.00	21.00	399.00	1396.50	14663.25	1263.50	12003.25	0.94
20X24	19.00	23.00	437.00	1675.17	19264.42	1383.83	13146.42	0.93

Table A2.5: Material properties for selected timber species, courtesy American Wood Council, Leesburg, VA

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)							Specific gravity
		Bending	Tension parallel to grain	Shear parallel to grain	Compression perpendicular to grain	Compression parallel to grain	Modulus of Elasticity		
		F_b	F_t	F_v	$F_{c\perp}$	F_c	E	E_{min}	G
Douglas Fir-Larch									
Dense Select Structural	Beams and Stringers	1,900	1,100	170	730	1,300	1,700,000	620,000	0.50
Select Structural	Beams and Stringers	1,600	950	170	625	1,100	1,600,000	580,000	0.50
Dense No. 1	Beams and Stringers	1,550	775	170	730	1,100	1,700,000	620,000	0.50
No. 1	Beams and Stringers	1,350	675	170	625	925	1,600,000	580,000	0.50
No. 2 Dense	Beams and Stringers	1,000	500	170	730	700	1,400,000	510,000	0.50
No. 2	Beams and Stringers	875	425	170	625	600	1,300,000	470,000	0.50
Dense Select Structural	Posts and Timbers	1,750	1,150	170	730	1,350	1,700,000	620,000	0.50
Select Structural	Posts and Timbers	1,500	1,000	170	625	1,150	1,600,000	580,000	0.50
Dense No. 1	Posts and Timbers	1,400	950	170	730	1,200	1,700,000	620,000	0.50
No. 1	Posts and Timbers	1,200	825	170	625	1,000	1,600,000	580,000	0.50
No. 2 Dense	Posts and Timbers	850	550	170	730	825	1,400,000	510,000	0.50
No. 2	Posts and Timbers	750	475	170	625	700	1,300,000	470,000	0.50
Hem-Fir									
Select Structural	Beams and Stringers	1,300	750	140	405	925	1,300,000	470,000	0.43
No. 1	Beams and Stringers	1,050	525	140	405	750	1,300,000	470,000	0.43
No. 2	Beams and Stringers	675	350	140	405	500	1,100,000	400,000	0.43
Select Structural	Posts and Timbers	1,200	800	140	405	975	1,300,000	470,000	0.43
No. 1	Posts and Timbers	975	650	140	405	850	1,300,000	470,000	0.43
No. 2	Posts and Timbers	575	375	140	405	575	1,100,000	400,000	0.43
Northern Red Oak									
Select Structural	Beams and Stringers	1,600	950	205	885	950	1,300,000	470,000	0.68
No. 1	Beams and Stringers	1,350	675	205	885	800	1,300,000	470,000	0.68
No. 2	Beams and Stringers	875	425	205	885	500	1,000,000	370,000	0.68
Select Structural	Posts and Timbers	1,500	1,000	205	885	1,000	1,300,000	470,000	0.68
No. 1	Posts and Timbers	1,200	800	205	885	875	1,300,000	470,000	0.68
No. 2	Posts and Timbers	700	475	205	885	400	1,000,000	370,000	0.68
Red Maple									
Select Structural	Beams and Stringers	1,500	875	195	615	900	1,500,000	550,000	0.58
No. 1	Beams and Stringers	1,250	625	195	615	750	1,500,000	550,000	0.58
No. 2	Beams and Stringers	800	400	195	615	475	1,200,000	440,000	0.58
Select Structural	Posts and Timbers	1,400	925	195	615	950	1,500,000	550,000	0.58
No. 1	Posts and Timbers	1,150	750	195	615	825	1,500,000	550,000	0.58
No. 2	Posts and Timbers	650	425	195	615	375	1,500,000	440,000	0.58
Redwood									
Clear Structural	5"x5" and Larger	1,850	1,250	145	650	1,650	1,300,000	470,000	0.44
Select Structural	5"x5" and Larger	1,400	950	145	650	1,200	1,300,000	470,000	0.44
No. 1	5"x5" and Larger	1,200	800	145	650	1,050	1,300,000	470,000	0.44
No. 1, open grain	5"x5" and Larger	950	650	145	420	800	1,000,000	370,000	0.37
No. 2	5"x5" and Larger	1,000	525	145	650	900	1,100,000	400,000	0.44
No. 2, open grain	5"x5" and Larger	750	400	145	420	650	900,000	330,000	0.37
Southern Pine									
Select Structural	5"x5" and Larger	1,500	1,000	165	375	950	1,500,000	550,000	0.55
No. 1	5"x5" and Larger	1,350	900	165	375	825	1,500,000	550,000	0.55
No. 2	5"x5" and Larger	850	550	165	375	525	1,200,000	440,000	0.55

Table A2.6: Section properties for Southern Pine glu-lams

Depth	Area	X-X Axis			Y-Y Axis	
d (in.)	A (in.^{2})	I_x (in.^{4})	S_x (in.^{3})	r_x (in.)	I_y (in.^{4})	S_y (in.^{3})
3-1/2 in. Width					(r_y = 1.010 in.)	
5 1/2	19.25	48.53	17.65	1.588	19.65	11.23
6 7/8	24.06	94.78	27.57	1.985	24.56	14.04
8 1/4	28.88	163.8	39.70	2.382	29.48	16.84
9 5/8	33.69	260.1	54.04	2.778	34.39	19.65
11	38.50	388.2	70.58	3.175	39.30	22.46
12 3/8	43.31	552.7	89.33	3.572	44.21	25.27
13 3/4	48.13	758.2	110.3	3.969	49.13	28.07
15 1/8	52.94	1009	133.4	4.366	54.04	30.88
16 1/2	57.75	1310	158.8	4.763	58.95	33.69
17 7/8	62.56	1666	186.4	5.160	63.87	36.49
19 1/4	67.38	2081	216.2	5.557	68.78	39.30
20 5/8	72.19	2559	248.1	5.954	73.69	42.11
22	77.00	3106	282.3	6.351	78.60	44.92
23 3/8	81.81	3725	318.7	6.748	83.52	47.72
5-1/2 in. Width					(r_y = 1.588 in.)	
6 7/8	37.81	148.9	43.33	1.985	95.32	34.66
8 1/4	45.38	257.4	62.39	2.382	114.4	41.59
9 5/8	52.94	408.7	84.92	2.778	133.4	48.53
11	60.50	610.0	110.9	3.175	152.5	55.46
12 3/8	68.06	868.6	140.4	3.572	171.6	62.32
13 3/4	75.63	1191	173.3	3.969	190.6	69.32
15 1/8	83.19	1586	209.7	4.366	209.7	76.26
16 1/2	90.75	2059	249.6	4.763	228.8	83.19
17 7/8	98.31	2618	292.9	5.160	247.8	90.12
19 1/4	105.9	3269	339.7	5.557	266.9	97.05
20 5/8	113.4	4021	389.9	5.954	286.0	103.0
22	121.0	4880	443.7	6.351	305.0	110.9
23 3/8	128.6	5854	500.9	6.748	324.1	117.8
24 3/4	136.1	6949	561.5	7.145	343.1	124.8
26 1/8	143.7	8172	625.6	7.542	362.2	131.7
27 1/2	151.3	9532	693.2	7.939	381.3	138.6
28 7/8	158.8	11030	764.3	8.335	400.3	145.6
30 1/4	166.4	12690	838.8	8.732	419.4	152.5
31 5/8	173.9	14500	916.8	9.129	438.5	159.4
33	181.5	16470	998.3	9.526	457.5	166.4
34 3/8	189.1	18620	1083	9.923	476.6	173.3
35 3/4	196.6	20940	1172	10.32	495.7	180.2
8-1/2 in. Width					(r_y = 2.454 in.)	
9 5/8	81.81	631.6	131.2	2.778	492.6	115.9
11	93.50	942.8	171.4	3.175	562.9	132.5
12 3/8	105.2	1342	216.9	3.572	633.3	149.0
13 3/4	116.9	1841	267.8	3.969	703.7	165.6
15 1/8	128.6	2451	324.1	4.366	774.1	182.1
16 1/2	140.3	3182	385.7	4.763	844.4	198.7
17 7/8	151.9	4046	452.6	5.160	914.8	215.2
19 1/4	163.6	5053	525.0	5.557	985.2	231.8
20 5/8	175.3	6215	602.6	5.954	1056	248.4
22	187.0	7542	685.7	6.351	1126	264.9
23 3/8	198.7	9047	774.1	6.748	1196	281.5
24 3/4	210.4	10740	867.8	7.145	1267	298.0
26 1/8	222.1	12630	966.9	7.542	1337	314.6
27 1/2	233.8	14730	1071	7.939	1407	331.1
28 7/8	245.4	17050	1181	8.335	1478	347.7
30 1/4	257.1	19610	1296	8.732	1548	364.3
31 5/8	268.8	22400	1417	9.129	1618	380.8
33	280.5	25460	1543	9.526	1689	397.4
34 3/8	292.2	28770	1674	9.923	1759	413.9

Depth	Area	X-X Axis			Y-Y Axis	
d (in.)	A (in.^{2})	I_x (in.^{4})	S_x (in.^{3})	r_x (in.)	I_y (in.^{4})	S_y (in.^{3})
8-1/2 in. Width cont'd.					(r_y = 2.454 in.)	
35 3/4	303.9	32360	1811	10.32	1830	430.5
37 1/8	315.6	36240	1953	10.72	1900	447.0
38 1/2	327.3	40420	2100	11.11	1970	463.6
39 7/8	338.9	44910	2253	11.51	2041	480.2
41 1/4	350.6	49720	2411	11.91	2111	496.7
42 5/8	362.3	54860	2574	12.30	2181	513.3
44	374.0	60340	2743	12.70	2252	529.8
45 3/8	384.7	66170	2917	13.10	2322	546.4
46 3/4	397.4	72370	3096	13.50	2393	562.9
48 1/8	409.1	78950	3281	13.89	2463	579.5
49 1/2	420.8	85910	3471	14.29	2533	596.1
50 7/8	432.4	93270	3667	14.69	2604	612.6
52 1/4	444.1	101000	3868	15.08	2674	629.2
53 5/8	455.8	109200	4074	15.48	2744	645.7
55	467.5	117800	4285	15.88	2815	662.3
56 3/8	479.2	126900	4502	16.27	2885	678.8
57 3/4	490.9	136400	4725	16.67	2955	695.4
59 1/8	502.6	146400	4952	17.07	3026	712.0
60 1/2	514.3	156900	5185	17.46	3096	728.5
10-1/2 in. Width					(r_y = 3.031 in.)	
11	115.5	1165	211.8	3.175	1061	202.1
12 3/8	129.9	1658	268.0	3.572	1194	227.4
13 3/4	144.4	2275	330.9	3.969	1326	252.7
15 1/8	158.8	3028	400.3	4.366	1459	277.9
16 1/2	173.3	3931	476.4	4.763	1592	303.2
17 7/8	187.7	4997	559.2	5.160	1724	328.5
19 1/4	202.1	6242	648.5	5.557	1857	353.7
20 5/8	216.6	7677	744.4	5.954	1990	379.0
22	231.0	9317	847.0	6.351	2122	404.3
23 3/8	245.4	11180	956.2	6.748	2255	429.5
24 3/4	259.9	13270	1072.0	7.145	2388	454.8
26 1/8	274.3	15600	1194.0	7.542	2520	480.0
27 1/2	288.8	18200	1323.0	7.939	2653	505.3
28 7/8	303.2	21070	1459.0	8.335	2786	530.6
30 1/4	317.6	24220	1601.0	8.732	2918	555.8
31 5/8	332.1	27680	1750.0	9.129	3051	581.1
33	346.5	31440	1906.0	9.526	3183	606.4
34 3/8	360.9	35540	2068.0	9.923	3316	631.6
35 3/4	375.4	39980	2237.0	10.32	3449	656.9
37 1/8	389.8	44770	2412.0	10.72	3581	682.2
38 1/2	404.3	49930	2594.0	11.11	3714	707.4
39 7/8	418.7	55480	2783.0	11.51	3847	732.7
41 1/4	433.1	61420	2978.0	11.91	3979	758.0
42 5/8	447.6	67760	3180.0	12.30	4112	783.2
44	462.0	74540	3388.0	12.70	4245	808.5
45 3/8	476.4	81740	3603.0	13.10	4377	833.8
46 3/4	490.9	89400	3825.0	13.50	4510	859.0
48 1/8	505.3	97530	4053.0	13.89	4643	884.3
49 1/2	519.8	106100	4288.0	14.29	4775	909.6
50 7/8	534.2	115200	4529.0	14.69	4908	934.8
52 1/4	548.6	124800	4778.0	15.08	5040	960.1
53 5/8	563.1	134900	5032.0	15.48	5173	985.4
55	577.5	145600	5294.0	15.88	5306	1011.0
56 3/8	591.9	156800	5562.0	16.27	5438	1036.0
57 3/4	606.4	168500	5836.0	16.67	5571	1061.0
59 1/8	620.8	180900	6118.0	17.07	5704	1086.0
60 1/2	635.3	193800	6405.0	17.46	5836	1112.0

Table A2.7: Section properties for Western species glu-lams

Depth	Area	X-X Axis			Y-Y Axis	
d (in.)	A (in.^{2})	I_x (in.^{4})	S_x (in.^{3})	r_x (in.)	I_y (in.^{4})	S_y (in.^{3})
		3-1/2 in. Width			(r_y = 1.010 in.)	
6	21.00	63.0	21.00	1.732	21.44	12.25
7 1/2	26.25	123.0	32.81	2.165	26.80	15.31
9	31.50	212.6	47.25	2.598	32.16	18.38
10 1/2	36.75	337.6	64.31	3.031	37.52	21.44
12	42.00	504.0	84.00	3.464	42.88	24.50
13 1/2	47.25	717.6	106.3	3.897	48.23	27.56
15	52.50	984.4	131.3	4.330	53.59	30.63
16 1/2	57.75	1310	158.8	4.763	58.95	33.69
18	63.00	1701	189.0	5.196	64.31	36.75
19 1/2	68.25	2163	221.8	5.629	69.67	39.81
21	73.50	2701	257.3	6.062	75.03	42.88
22 1/2	78.75	3322	295.3	6.495	80.39	45.94
24	84.00	4032	336.0	6.928	85.75	49.00
		5-1/2 in. Width			(r_y = 1.588 in.)	
6	33.00	99.0	33.00	1.732	83.19	30.25
7 1/2	41.25	193.4	51.56	2.165	104.0	37.81
9	49.50	334.1	74.25	2.598	124.8	45.38
10 1/2	57.75	530.6	101.1	3.031	145.6	52.94
12	66.00	792.0	132.0	3.464	166.4	60.50
13 1/2	74.25	1128	167.1	3.897	187.2	68.06
15	82.50	1547	206.3	4.330	208.0	75.63
16 1/2	90.75	2059	249.6	4.763	228.8	83.19
18	99.00	2673	297.0	5.196	249.6	90.75
19 1/2	107.3	3398	348.6	5.629	270.4	98.31
21	115.5	4245	404.3	6.062	291.2	105.9
22 1/2	123.8	5221	464.1	6.495	312.0	113.4
24	132.0	6336	528.0	6.928	332.8	121.0
25 1/2	140.3	7600	596.1	7.361	353.5	128.6
27	148.5	9021	668.3	7.794	374.3	136.1
28 1/2	156.8	10610	744.6	8.227	395.1	143.7
30	165.0	12380	825.0	8.660	415.9	151.3
31 1/2	173.3	14330	909.6	9.093	436.7	158.8
33	181.5	16470	998.3	9.526	457.5	166.4
34 1/2	189.8	18820	1091	9.959	478.3	173.9
36	198.0	21380	1188	10.39	499.1	181.5
		8-3/4 in. Width			(r_y = 2.526 in.)	
9	78.75	531.6	118.1	2.598	502.4	114.8
10 1/2	91.88	844.1	160.8	3.031	586.2	134.0
12	105.0	1260	210.0	3.464	669.9	153.1
13 1/2	118.1	1794	265.8	3.897	753.7	172.3
15	131.3	2461	328.1	4.330	837.4	191.4
16 1/2	144.4	3276	397.0	4.763	921.1	210.5
18	157.5	4253	472.5	5.196	1005	229.7
19 1/2	170.6	5407	554.5	5.629	1089	248.8
21	183.8	6753	643.1	6.062	1172	268.0
22 1/2	196.9	8306	738.3	6.495	1256	287.1
24	210.0	10080	840.0	6.928	1340	306.3
25 1/2	223.1	12090	948.3	7.361	1424	325.4
27	236.3	14350	1063	7.794	1507	344.5
28 1/2	249.4	16880	1185	8.227	1591	363.7
30	262.5	19690	1313	8.660	1675	382.8
31 1/2	275.6	22790	1447	9.093	1759	402.0
33	288.8	26200	1588	9.526	1842	421.1
34 1/2	301.9	29940	1736	9.959	1926	440.2
36	315.0	34020	1890	10.39	2010	459.4
37 1/2	328.1	38450	2051	10.83	2094	478.5
39	341.3	43250	2218	11.26	2177	497.7
40 1/2	354.4	48440	2392	11.69	2261	516.8
42	367.5	54020	2573	12.12	2345	535.9
43 1/2	380.6	60020	2760	12.56	2428	555.1
45	393.8	66450	2953	12.99	2512	574.2
46 1/2	406.9	73310	3153	13.42	2596	593.4
48	420.0	80640	3360	13.86	2680	612.5
49 1/2	433.1	88440	3573	14.29	2763	631.6
51	446.3	96720	3793	14.72	2847	650.8
52 1/2	459.4	105500	4020	15.16	2931	669.9
54	472.5	114800	4253	15.59	3015	689.1
55 1/2	485.6	124700	4492	16.02	3098	708.2
57	498.8	135000	4738	16.45	3182	727.3
58 1/2	511.9	146000	4991	16.89	3266	746.5
60	525.0	157500	5250	17.32	3350	765.6
		10-3/4 in. Width			(r_y = 3.103 in.)	
12	129.0	1548	258.0	3.464	1242.00	231.1
13 1/2	145.1	2204	326.5	3.897	1398.00	260.0
15	161.3	3023	403.1	4.330	1553.00	288.9
16 1/2	177.4	4024	487.8	4.763	1708.00	317.8
18	193.5	5225	580.5	5.196	1863.00	346.7
19 1/2	209.6	6642	681.3	5.629	2019.00	375.6
21	225.8	8296	790.1	6.062	2174.00	404.5
22 1/2	241.9	10200	907.0	6.495	2329.00	433.4
24	258.0	12380	1032	6.928	2485.00	462.3
25 1/2	274.1	14850	1165	7.361	2640.00	491.1
27	290.3	17630	1306	7.794	2795.00	520.0
28 1/2	306.4	20740	1455	8.227	2950.00	548.9
30	322.5	24190	1613	8.660	3106.00	577.8
31 1/2	338.6	28000	1778	9.093	3261.00	606.7
33	354.8	32190	1951	9.526	3416.00	635.6
34 1/2	370.9	36790	2133	9.959	2572.00	664.5
36	387.0	41800	2322	10.39	3727.00	693.4
37 1/2	403.1	47240	2520	10.83	3882.00	722.3
39	419.3	53140	2725	11.26	4037.00	751.2
40 1/2	435.4	59510	2939	11.69	4193.00	780.0
42	451.5	66370	3161	12.12	4348.00	808.9
43 1/2	467.6	73740	3390	12.56	4503.00	837.8
45	483.8	81630	3628	12.99	4659.00	866.7
46 1/2	499.9	90070	3874	13.42	4814.00	895.6
48	516.0	99070	4128	13.86	4969.00	924.5
49 1/2	532.1	108700	4390	14.29	5124.00	953.4
51	548.3	118800	4660	14.72	5280.00	982.3
52 1/2	564.4	129600	4938	15.16	5435.00	1011
54	580.5	141100	5225	15.59	5590.00	1040
55 1/2	596.6	153100	5519	16.02	5746.00	1069
57	612.8	165900	5821	16.45	5901.00	1098
58 1/2	628.9	179300	6132	16.89	6056.00	1127
60	645.0	193500	6450	17.32	6211.00	1156
		12-1/4 in. Width			(r_y = 3.536 in.)	
13 1/2	165.40	2512	372.1	3.897	2068	337.6
15	183.80	3445	459.4	4.330	2298	375.2
16 1/2	202.10	4586	555.8	4.763	2528	412.7
18	220.50	5954	661.5	5.196	2757	450.2
19 1/2	238.90	7569	776.3	5.629	2987	487.7
21	257.30	9454	900.4	6.062	3217	525.2
22 1/2	275.60	11630	1034	6.495	3447	562.7
24	294.00	14110	1176	6.928	3677	600.3
25 1/2	312.40	16930	1328	7.361	3906	637.8
27	330.80	20090	1488	7.794	4136	675.3
28 1/2	349.10	23630	1658	8.227	4366	712.8
30	367.50	27560	1838	8.660	4596	750.3
31 1/2	385.90	31910	2026	9.093	4825	787.8
33	404.30	36690	2223	9.526	5055	825.3
34 1/2	422.60	41920	2430	9.959	5285	862.9
36	441.00	47630	2646	10.39	5515	900.4
37 1/2	459.40	53830	2871	10.83	5745	937.9
39	477.80	60550	3105	11.26	5974	975.4
40 1/2	469.10	67810	3349	11.69	6204	1013
42	514.50	75630	3602	12.12	6434	1050
43 1/2	532.90	84030	3863	12.56	6664	1088
45	551.30	93020	4134	12.99	6893	1125
46 1/2	569.60	102600	4415	13.42	7123	1163
48	588.00	112900	4704	13.86	7353	1201
49 1/2	606.40	123800	5003	14.29	7583	1238
51	624.80	135400	5310	14.72	7813	1276
52 1/2	643.10	147700	5627	15.16	8042	1313
54	661.50	160700	5954	15.59	8272	1351
55 1/2	679.90	174500	6289	16.02	8502	1388
57	698.30	189100	6633	16.45	8732	1426
58 1/2	716.60	204400	6987	16.89	8962	1463
60	735.00	220500	7350	17.32	9191	1501

Table A2.8: Material properties for glu-lam design

Species and commercial grade	Size classification	Design values in pounds per square inch (psi)							
		Bending	Tension parallel to grain	Shear parallel to grain	Compression perpendicular to grain	Compression parallel to grain	Modulus of Elasticity		Specific gravity
		F_b	F_t	F_v	F_c	F_c	E	E_{min}	G
Southern Pine									
Select Structural	2" - 4" wide	2,850	1,600	175	565	2,100	1,800,000	660,000	0.55
No. 1		1,850	1,050	175	565	1,850	1,700,000	620,000	
No. 2		1,500	825	175	565	1,650	1,600,000	580,000	
Construction	4" wide	1,100	625	175	565	1,800	1,500,000	550,000	0.55
Standard		625	350	175	565	1,500	1,300,000	470,000	
Utility		300	175	175	565	975	1,300,000	470,000	
Select Structural	5" - 6" wide	2,550	1,400	175	565	2,000	1,800,000	660,000	0.55
No. 1		1,650	900	175	565	1,750	1,700,000	620,000	
No. 2		1,250	725	175	565	1,600	1,600,000	580,000	
Select Structural	8" wide	2,300	1,300	175	565	1,900	1,800,000	660,000	0.55
No. 1		1,500	825	175	565	1,650	1,700,000	580,000	
No. 2		1,200	650	175	565	1,550	1,600,000	580,000	
Select Structural	10" wide	2,050	1,100	175	565	1,850	1,800,000	660,000	0.55
No. 1		1,300	725	175	565	1,600	1,700,000	620,000	
No. 2		1,050	575	175	565	1,500	1,600,000	580,000	
Select Structural	12" wide	1,900	1,050	175	565	1,800	1,800,000	660,000	0.55
No. 1		1,250	675	175	565	1,600	1,700,000	620,000	
No. 2		975	550	175	565	1,450	1,600,000	580,000	

A-3 Steel Design

Table A3.1: W14 Section properties

ROLLED STEEL SECTION PROPERTIES														
SIZE	A area (in^2)	d depth (in)	t_w web thickness (in)	b_f flange width (in)	t_f flange thickness (in)	Ix (in^4)	Sx (in^3)	rx (in)	Zx (in^3)	Iy (in^4)	Sy (in^3)	ry (in)	Zy (in^3)	r_{ts}
W14X730	215.00	22.40	3.070	17.90	4.910	14,300	1,280.0	8.17	1,660.00	4,720	527.0	4.69	816	5.68
W14X655	196.00	21.60	2.830	17.70	4.520	12,400	1,150.0	7.98	1,480.00	4,170	472.0	4.62	730	5.57
W14X605	178.00	20.90	2.600	17.40	4.160	10,800	1,040.0	7.80	1,320.00	3,680	423.0	4.55	652	5.46
W14X550	162.00	20.20	2.380	17.20	3.820	9,430	931.0	7.63	1,180.00	3,250	378.0	4.49	583	5.36
W14X500	147.00	19.60	2.190	17.00	3.500	8,210	838.0	7.48	1,050.00	2,880	339.0	4.43	522	5.26
W14X455	134.00	19.00	2.020	16.80	3.210	7,190	756.0	7.33	936.00	2,560	304.0	4.38	468	5.17
W14X426	125.00	18.70	1.880	16.70	3.040	6,600	706.0	7.26	869.00	2,360	283.0	4.34	434	5.11
W14X398	117.00	18.30	1.770	16.60	2.850	6,000	656.0	7.16	801.00	2,170	262.0	4.31	402	5.06
W14X370	109.00	17.90	1.660	16.50	2.660	5,440	607.0	7.07	736.00	1,990	241.0	4.27	370	5.00
W14X342	101.00	17.50	1.540	16.40	2.470	4,900	558.0	6.98	672.00	1,810	221.0	4.24	338	4.94
W14X311	91.40	17.10	1.410	16.20	2.260	4,330	506.0	6.88	603.00	1,610	199.0	4.20	304	4.87
W14X283	83.30	16.70	1.290	16.20	2.070	3,840	459.0	6.79	542.00	1,440	179.0	4.17	274	4.81
W14X257	75.60	16.40	1.180	16.00	1.890	3,400	415.0	6.71	487.00	1,290	161.0	4.13	246	4.75
W14X233	68.50	16.00	1.070	15.90	1.720	3,010	375.0	6.63	436.00	1,150	145.0	4.10	221	4.69
W14X211	62.00	15.70	0.980	15.80	1.560	2,660	338.0	6.55	390.00	1,030	130.0	4.07	198	4.64
W14X193	56.80	15.50	0.890	15.70	1.440	2,400	310.0	6.50	355.00	931	119.0	4.05	180	4.59
W14X176	51.80	15.20	0.830	15.70	1.310	2,140	281.0	6.43	320.00	838	107.0	4.02	163	4.55
W14X159	46.70	15.00	0.745	15.60	1.190	1,900	254.0	6.38	287.00	748	96.2	4.00	146	4.51
W14X145	42.70	14.80	0.680	15.50	1.090	1,710	232.0	6.33	260.00	677	87.3	3.98	133	4.47
W14X132	38.30	14.70	0.645	14.70	1.030	1,530	209.0	6.28	234.00	548	74.5	3.76	113	4.23
W14X120	35.30	14.50	0.590	14.70	0.940	1,380	190.0	6.24	212.00	495	67.5	3.74	102	4.20
W14X109	32.00	14.30	0.525	14.60	0.860	1,240	173.0	6.22	192.00	447	61.2	3.73	93	4.17
W14X99	29.10	14.20	0.485	14.60	0.780	1,110	157.0	6.17	173.00	402	55.2	3.71	84	4.14
W14X90	26.50	14.00	0.440	14.50	0.710	999	143.0	6.14	157.00	362	49.9	3.70	76	4.11
W14X82	24.00	14.30	0.510	10.10	0.855	881	123.0	6.05	139.00	148	29.3	2.48	45	2.85
W14X74	21.80	14.20	0.450	10.10	0.785	795	112.0	6.04	126.00	134	26.6	2.48	41	2.82
W14X68	20.00	14.00	0.415	10.00	0.720	722	103.0	6.01	115.00	121	24.2	2.46	37	2.80
W14X61	17.90	13.90	0.375	10.00	0.645	640	92.1	5.98	102.00	107	21.5	2.45	33	2.78
W14X53	15.60	13.90	0.370	8.10	0.660	541	77.8	5.89	87.10	58	14.3	1.92	22	2.22
W14X48	14.10	13.80	0.340	8.00	0.595	484	70.2	5.85	78.40	51	12.8	1.91	20	2.20
W14X43	12.60	13.70	0.305	8.00	0.530	428	62.6	5.82	69.60	45	11.3	1.89	17	2.18
W14X38	11.20	14.10	0.310	6.80	0.515	385	54.6	5.87	61.50	27	7.9	1.55	12	1.82
W14X34	10.00	14.00	0.285	6.80	0.455	340	48.6	5.83	54.60	23	6.9	1.53	11	1.80
W14X30	8.90	13.80	0.270	6.70	0.385	291	42.0	5.73	47.30	20	5.8	1.49	9	1.77
W14X26	7.70	13.90	0.255	5.03	0.420	245	35.3	5.65	40.20	9	3.6	3.55	6	1.31
W14X22	6.50	13.80	0.230	5.00	0.335	199	29.0	5.54	33.20	7	2.8	1.04	4	1.27

A-4 Concrete Design

Table A4.1: Rebar diameters and areas

Rebar Size	3	4	5	6	7	8	9	10	11
Diameter (in)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410
Number of Bars	Area of Bars (in^2)								
1	0.110	0.196	0.307	0.442	0.601	0.785	0.999	1.267	1.561
2	0.221	0.393	0.614	0.884	1.203	1.571	1.999	2.534	3.123
3	0.331	0.589	0.920	1.325	1.804	2.356	2.998	3.800	4.684
4	0.442	0.785	1.227	1.767	2.405	3.142	3.997	5.067	6.246
5	0.552	0.982	1.534	2.209	3.007	3.927	4.997	6.334	7.807
6	0.663	1.178	1.841	2.651	3.608	4.712	5.996	7.601	9.369
7	0.773	1.374	2.148	3.093	4.209	5.498	6.995	8.867	10.930
8	0.884	1.571	2.454	3.534	4.811	6.283	7.995	10.134	12.492
9	0.994	1.767	2.761	3.976	5.412	7.069	8.994	11.401	14.053
10	1.104	1.963	3.068	4.418	6.013	7.854	9.993	12.668	15.614

Table A4.2: Maximum number of bars in one row

Rebar Size	3	4	5	6	7	8	9	10	11
Diameter (in)	0.375	0.500	0.625	0.750	0.875	1.000	1.128	1.270	1.410
Number of Bars per layer	Minimum Beam Width (in.)								
2	5.5	6.0	6.0	6.5	6.5	7.0	7.0	7.5	8.0
3	7.0	7.5	8.0	8.0	8.5	9.0	9.5	10.0	10.0
4	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5
5	10.0	10.5	11.0	11.5	12.5	13.0	13.5	14.5	15.0
6	11.0	12.0	12.5	13.5	14.0	15.0	15.5	16.5	17.5
7	12.5	13.5	14.5	15.0	16.0	17.0	18.0	19.0	20.0
8	14.0	15.0	16.0	17.0	18.0	19.0	20.0	21.0	22.0

Table A4.3: Minimum and maximum number of longitudinal bars for concrete columns

Minimum - Maximum number of longitudinal bars per single row

Spiral Reinforcement Bar Number							Diameter or Side width	Tie Reinforcement Bar Number						
5	6	7	8	9	10	11		5	6	7	8	9	10	11
4 - 12	4 - 11	4 - 10	4 - 10	4 - 9	4 - 7	4 - 5	12	8 - 12	8 - 12	8 - 12	8 - 8	4 - 8	4 - 8	4 - 4
5 - 13	5 - 11	5 - 12	5 - 11	4 - 10	4 - 8	4 - 6	13	8 - 16	8 - 12	8 - 12	8 - 12	8 - 8	8 - 8	8 - 8
6 - 15	5 - 12	5 - 13	5 - 12	5 - 11	5 - 9	5 - 7	14	8 - 16	8 - 16	8 - 12	8 - 12	8 - 12	8 - 8	8 - 8
6 - 16	6 - 13	6 - 14	5 - 13	5 - 12	5 - 11	5 - 9	15	8 - 20	8 - 16	8 - 16	8 - 16	8 - 12	8 - 12	8 - 8
7 - 18	6 - 15	6 - 16	6 - 15	6 - 14	6 - 12	5 - 10	16	12 - 20	8 - 20	8 - 16	8 - 16	8 - 12	8 - 12	8 - 12
8 - 19	7 - 16	6 - 17	6 - 16	6 - 15	6 - 14	6 - 11	17	12 - 20	8 - 20	8 - 20	8 - 16	8 - 16	8 - 12	8 - 12
9 - 21	7 - 17	7 - 18	7 - 17	7 - 16	6 - 15	6 - 13	18	12 - 24	8 - 24	8 - 20	8 - 20	8 - 16	8 - 16	8 - 12
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25 - 40	18 - 33	13 - 35	13 - 33	12 - 32	12 - 30	12 - 28	31	32 - 48	24 - 44	16 - 44	16 - 40	16 - 36	16 - 32	16 - 28
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34 - 47	24 - 40	17 - 42	15 - 40	15 - 38	14 - 35	14 - 34	36	44 - 56	28 - 56	24 - 52	20 - 48	20 - 44	20 - 36	20 - 32

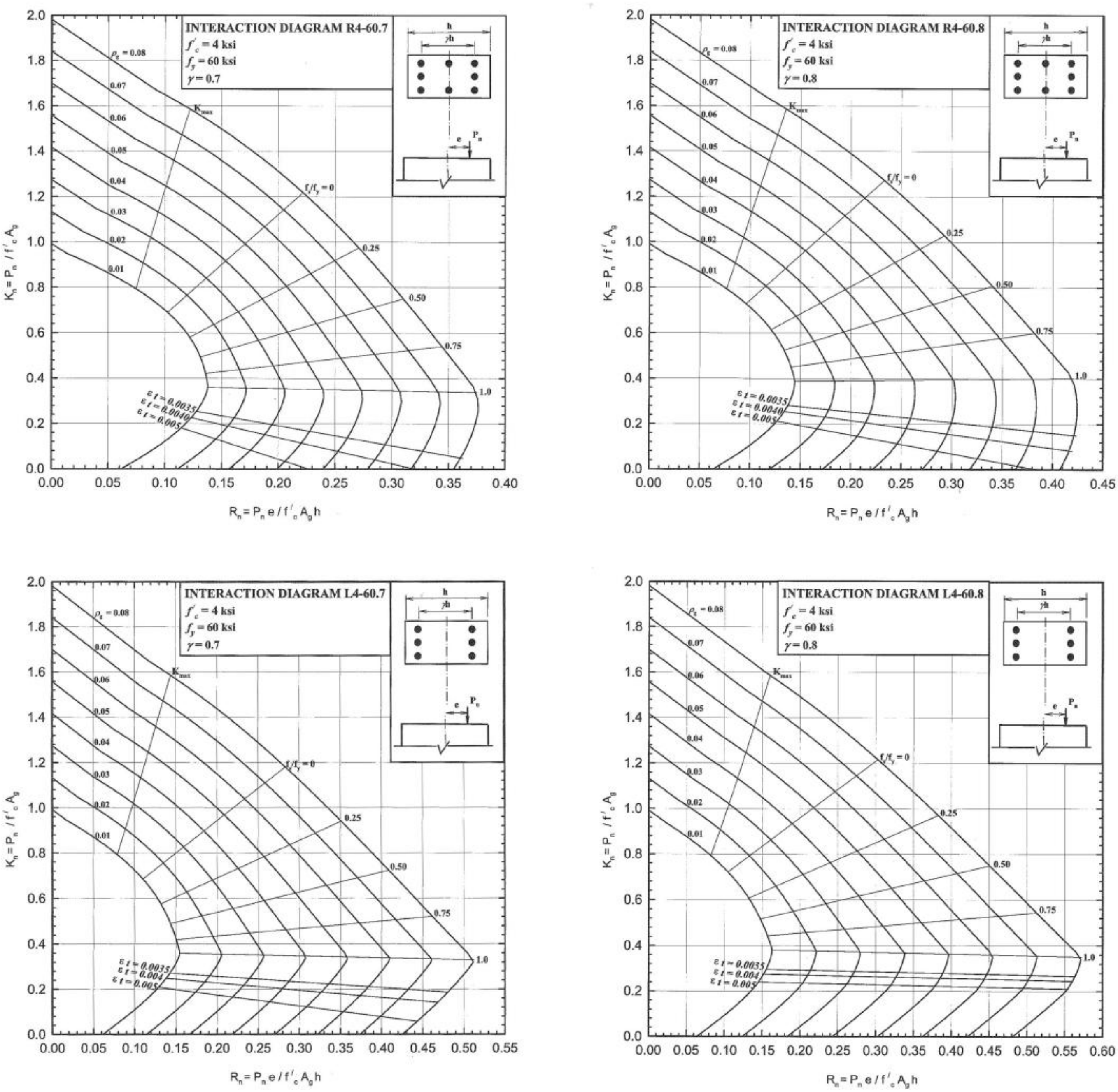
Minimum number of bars is based on 6" maximum clear distance between longitudinal bars and $\rho_s \geq 0.01$

Maximum number of bars is based on 1.5" clear or $1.5d_b$ and $\rho_s \leq 0.08$

Table A4.4: Areas of reinforcing bars per foot

Spacing (in.)	Bar Size								
	3	4	5	6	7	8	9	10	11
3	0.44	0.79	1.23	1.77	2.41	3.14	4.00	*	*
3.5	0.38	0.67	1.05	1.51	2.06	2.69	3.43	4.34	*
4	0.33	0.59	0.92	1.33	1.80	2.36	3.00	3.80	4.68
4.5	0.29	0.52	0.82	1.18	1.60	2.09	2.66	3.38	4.16
5	0.27	0.47	0.74	1.06	1.44	1.88	2.40	3.04	3.75
5.5	0.24	0.43	0.67	0.96	1.31	1.71	2.18	2.76	3.41
6	0.22	0.39	0.61	0.88	1.20	1.57	2.00	2.53	3.12
6.5	0.20	0.36	0.57	0.82	1.11	1.45	1.84	2.34	2.88
7	0.19	0.34	0.53	0.76	1.03	1.35	1.71	2.17	2.68
7.5	0.18	0.31	0.49	0.71	0.96	1.26	1.60	2.03	2.50
8	0.17	0.29	0.46	0.66	0.90	1.18	1.50	1.90	2.34
8.5	0.16	0.28	0.43	0.62	0.85	1.11	1.41	1.79	2.20
9	0.15	0.26	0.41	0.59	0.80	1.05	1.33	1.69	2.08
9.5	0.14	0.25	0.39	0.56	0.76	0.99	1.26	1.60	1.97
10	0.13	0.24	0.37	0.53	0.72	0.94	1.20	1.52	1.87
11	0.12	0.21	0.33	0.48	0.66	0.86	1.09	1.38	1.70
12	0.11	0.20	0.31	0.44	0.60	0.79	1.00	1.27	1.56
13	0.10	0.18	0.28	0.41	0.56	0.72	0.92	1.17	1.44
14	0.09	0.17	0.26	0.38	0.52	0.67	0.86	1.09	1.34
15	0.09	0.16	0.25	0.35	0.48	0.63	0.80	1.01	1.25
16	0.08	0.15	0.23	0.33	0.45	0.59	0.75	0.95	1.17
17	0.08	0.14	0.22	0.31	0.42	0.55	0.71	0.89	1.10
18	0.07	0.13	0.20	0.29	0.40	0.52	0.67	0.84	1.04

* Minimum clear distance = $1.5d_b$ therefore the bar size cannot be placed at the spacing indicated.



A4.5

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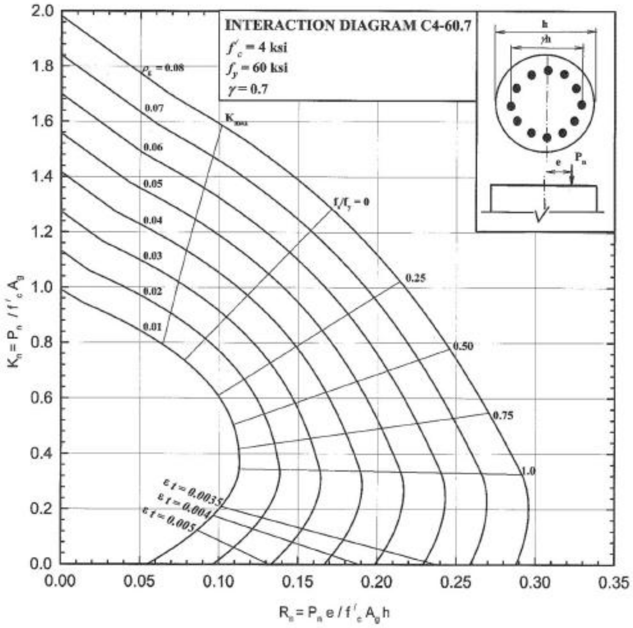

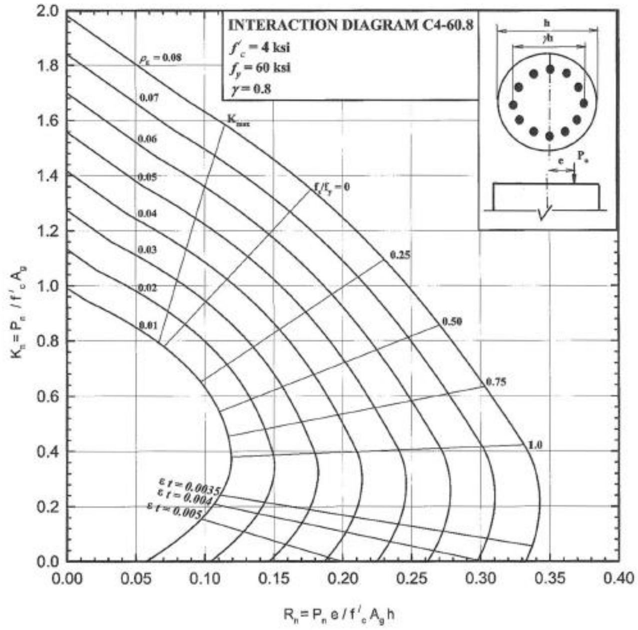

A4.5

Interaction diagrams from ACI SP17(11) Vol. 1: The Reinforced Concrete Design Manual, reproduced with permission from the American Concrete Institute, *continued*

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Figures and Tables

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